Positive Data Languages

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Friedrich-Alexander-Universität
Faculty of Engineering

What is 'Positivity'?

$\mathbb{D}$: Admissible User IDs for a Server ($\rightsquigarrow$ *Infinite Set*)

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What happens if we restrict this to just equalities?

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Definition (*Positive Data Language* **)**

A *positive data language* is closed under arbitrary renamings $\rho\colon \mathbb{D} \to \mathbb{D}$ of data values.

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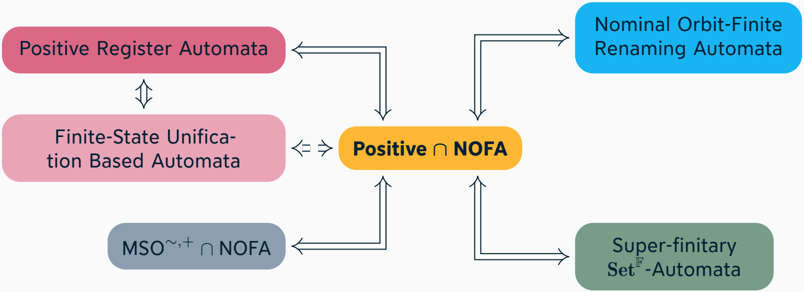

Nominal Sets (Talking about Infinite Data with Finite Means)

T.CS

$\rightsquigarrow$ Fix a (countably infinite) set $\mathbb{D}$ of 'names'. ⟵ Data Values

Definition (***Nominal Sets*** **)** **Gabbay, Pitts '99**

A *nominal set* is a set whose elements depend on a *finite* number of these names.
$\to \mathrm{supp}(x)$

$\rightsquigarrow$ We can change the names of an element using permutations $\pi \colon \mathbb{D} \xrightarrow{\simeq} \mathbb{D}$ which act upon these elements.

```
<book id="bk007">
   <author lstname="Doe"
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- Proper 'finiteness' is now replaced by finiteness up to such permutations. $\rightsquigarrow$ *Orbit-Finiteness*
- Nominal Sets and permutation-preserving maps form a category $\mathbf{Nom}$.

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- Proper ‘finiteness’ is now replaced

What happens if we give up injectivity of these permutations like before?
$\rightarrow$ $\mathbf{RnNom}$ (Renaming Nominal Sets, Gabbay, Hofmann ’08)

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Definition (*NOF A*) **Bojańczyk, Klin, Lasota '14**

A *nondeterministic orbit-finite* *automaton* $A = (Q, \delta, I, F)$ consists of:

- an orbit-finite nominal set $Q \in \mathbf{Nom}$ specifying *states*;
- an equivariant *transition* relation $\delta \subseteq Q \times \mathbb{D} \times Q$;
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Acceptance of words $w \in \mathbb{D}^\star$ is defined classically over runs.

$$L_0 = \{\ d_1 \cdots d_n \in \mathbb{D}^\star \mid d_i \neq d_n \text{ for all } i < n\ \}$$

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L_0 and L_1 are both NOFA-recognizable.

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A *nondeterministic orbit-finite renaming automaton* $A = (Q, \delta, I, F)$ consists of:

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$$\cancel{L_0 = \{\, d_1 \cdots d_n \in \mathbb{D}^\star \mid d_i \neq d_n \text{ for all } i < n \,\}}$$

Only L_1 is NOFRA-recognizable.

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Theorem (*First Equivalence*)

A language is positive and NOFA-recognizable iff it is recognized by a NOFRA.

A Slight Problem

The state set of NOFRAs is not truly finite, but just *orbit-finite*.

Store the names of states now explicitly in a finite amount of registers.

$\rightsquigarrow$ **Idea (**Kaminski, Francez '94, Bojańczyk, Klin, Lasota '14**):**
Change transitions to Boolean formulae of equations of register values and input symbols.

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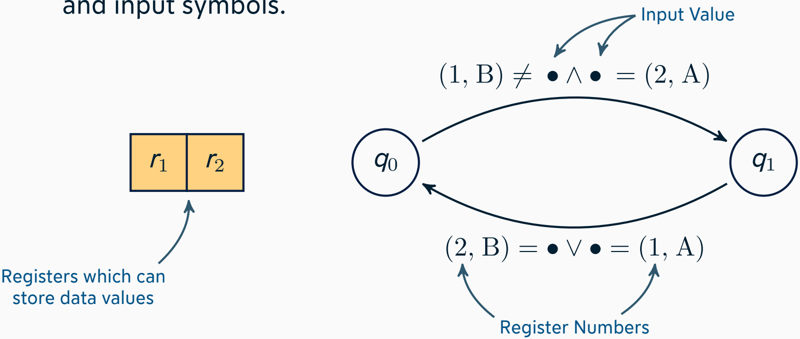

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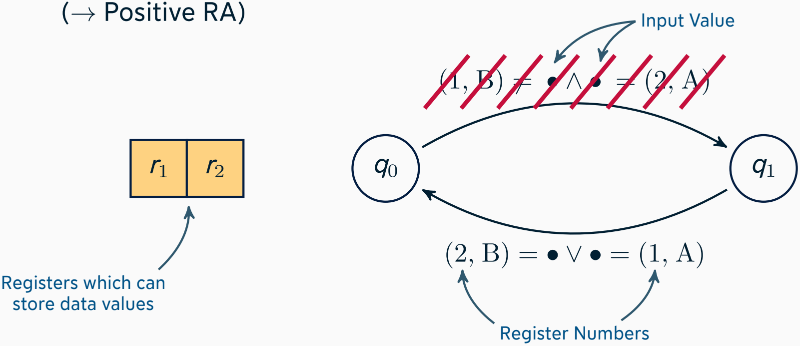
$\rightsquigarrow$ **Our Idea:**
Let the transition formulae be positive, i.e. without negation.
($\to$ Positive RA)



Theorem (***Second Equivalence*** **)**

A language is positive and NOFA-recognizable iff it is accepted by a positive RA.

Theorem (***Third Equivalence*** **)**

Positive register automata are equivalent to *finite-state unification based automata*.

Introduced by
Tal '99 and Kaminski, Tan '06

$\mathsf{MSO}^{\sim}$ Neven, Schwentick, Vianu '04

$$\phi, \psi \quad := \quad x < y \mid x \sim y \mid X(x) \mid \neg\phi \mid \phi \vee \psi \mid \phi \wedge \psi \mid \exists x.\, \phi \mid \exists X.\, \phi \mid \forall x.\, \phi \mid \forall X.\, \phi$$

- Formulae are interpreted over a fixed data word $w = d_1 \cdots d_n \in \mathbb{D}^\star$.

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describes the last position

$$\varphi_0 = \forall y.\ \mathrm{last}(y) \Rightarrow (\forall x.\ x < y \Rightarrow \neg(x \sim y))$$

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Definition ($MSO^{\sim,+}$)

We restrict $\mathsf{MSO}^{\sim}$ formulae to those whose NNF contains no subformula of the form $\neg(x \sim y)$.

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Definition (*$MSO^{\sim,+}$* **)**

We restrict MSO$^\sim$ formulae to those whose NNF contains no subformula of the form $\neg(x \sim y)$.

Theorem (*Fourth Equivalence* **)**

A NOFA-recognizable language is positive iff it is definable within MSO$^{\sim,+}$.

There are equivalences of categories for both $\mathbf{Nom}$ and $\mathbf{RnNom}$:

Pullback-Preserving Presheaves

$$\mathbf{Nom} \simeq \mathsf{Sh}(\mathbf{Set}^{\mathbb{I}}) \qquad \text{and} \qquad \mathbf{RnNom} \simeq \mathsf{Sh}(\mathbf{Set}^{\mathbb{F}})$$

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Categorical Automata

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Pullback-Preserving Presheaves

$\hookrightarrow$ We developed a notion of NA on those presheaf categories as well.

Definition (***Nondeterministic $\mathscr{C}$-Automata*** **)**

A *nondeterministic $\mathscr{C}$-Automaton* $A = (Q, \Sigma, \delta, I, F)$ consists of:

- objects $Q \in \mathscr{C}$ (*states*) and $\Sigma \in \mathscr{C}$ (*input alphabet*);
- a subobject $m_\delta : \delta \rightarrowtail Q \times \Sigma \times Q$ specifying *transitions*; and
- subobjects $m_I : I \rightarrowtail Q$ and $m_F : F \rightarrowtail Q$ for *initial* and *final* states.

The accepted language is then a family of subobjects of Σ^n for each $n \in \mathbb{N}$ defined over generalized runs.

Example (*Instances of Categorical Automata*)

Classical NFA, NOFA, and NOFRA with $\Sigma = \mathbb{D}$ are all instances of categorical automata for $\mathscr{C} = \mathbf{Set}, \mathbf{Nom}, \mathbf{RnNom}.$

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- While presheaf automata accept presheaf languages, we can also look at the accepted *word languages* (subsets of $\mathbb{D}^\star$).

Categorical Automata (Presheaf Automata)

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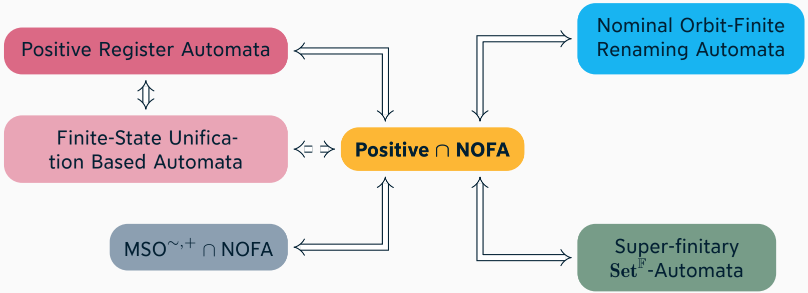
Theorem (*Fifth Equivalence* **)**

A word language is NOFA-recognizable iff it is accepted by a finitely presentable $\mathbf{Set}^{\mathbb{I}}$-automaton.

A word language is positive and NOFA-recognizable iff it is accepted by a finitely presentable $\mathbf{Set}^{\mathbb{F}}$-automaton.

Conclusion

- We looked at a restricted subclass of data languages, which has a rich theory and many equivalent perspectives: ($\rightsquigarrow$ Regular Languages)



- There are still open problems left:
  - Identifying a Suitable Fragment of MSO$^{\sim,+}$
  - Decidability Results
  - Expressive Power of MSO$^{\sim,+}$
  - Equivalences of Different Automata Models

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Register Automata (Make Infinite Things Finite Again)

T.CS

Definition (*Register Automata*) Bojańczyk, Klin, Lasota '14

A *register automaton* $A = (C, m, \delta, I, F)$ consists of:

- a *finite* set C of control states;
- a number $m \in \mathbb{N}$ of registers;
- a transition relation $\delta \subseteq C \times \mathbb{B}(\Phi) \times C$, where

Boolean formulae with Φ as atoms.

$$\Phi = (\{1, \ldots, m\} \times \{\mathrm{BEF}, \mathrm{AFT}\} \cup \{\bullet\})^2 \text{; and}$$

Equations: Compare register values with one another or the input value ($\bullet$).

- sets $I \subseteq C$ and $F \subseteq C$ of *initial* and *final* states.

Configurations: (c, r) with $c \in C$ and $r \in (\mathbb{D} \cup \{\perp\})^m$ (partial assigments to registers)
A move $(c, r) \xrightarrow{a} (c', r')$ is defined iff it is *consistent* with some transition $c \xrightarrow{\varphi} c'$.
Acceptance is defined over runs of moves.

T.CS

Definition (*Positive Register Automata*)

A *positive register automaton* $A = (C, m, \delta, I, F)$ consists of:

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- a transition relation $\delta \subseteq C \times \mathbb{B}^+(\Phi) \times C$, where

Positive Boolean formulae (i.e. *no* negations) with Φ as atoms.

$$\Phi = (\{ 1, \dots, m \} \times \{ \mathrm{BEF}, \mathrm{AFT} \} \cup \{ \bullet \})^2 \text{ ; and}$$

Equations: Compare register values with one another or the input value ($\bullet$).

- sets $I \subseteq C$ and $F \subseteq C$ of *initial* and *final* states.

Configurations: (c, r) with $c \in C$ and $r \in (\mathbb{D} \cup \{ \perp \})^m$ (partial assigments to registers)
A move $(c, r) \xrightarrow{a} (c', r')$ is defined iff it is *consistent* with some transition $c \xrightarrow{\varphi} c'$.
Acceptance is defined over runs of moves.