Practice Exercises

The following exercises are proposed to practice C programming. They are optional. They do not need to be submitted will not be graded.

1. Write a function that calculate the factorial of a positive integer. Display on the screen the Pascal triangle, which contains values of

$$\left(\begin{array}{c} n \\ k \end{array}\right) = \frac{n!}{(n-k)!k!}$$

   Values of n increase along the lines and values of k increase along the columns. You may want to explore what %d, %4d, or %6d do inside "printf".

2. The Taylor series of the sine function and the cosine function are given by

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

   and

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = \sum_{n=0}^{+\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

   respectively. Write functions "sinTaylor(N, x)" and "cosTaylor(N, x)" that calculate the first N terms in the series. Include the mathematical library "math.h" at the beginning of your code so that the standard functions "sin" and "cos" are available. Verify that the errors

$$\left|\sin(x) - \mathrm{sinTaylor}(N, x)\right|$$

   and

$$\left|\cos(x) - \mathrm{cosTaylor}(N, x)\right|$$

   are bounded by the (N+1)-th term (the first dropped term) in the corresponding Taylor series. Remember to use "-lm" at the end of your compilation command.

3. The golden-ratio algorithm can find the local maximum of a function on an interval $[a, b]$. The golden ratio is the number

$$g = \frac{-1 + \sqrt{5}}{2}$$

   which satisfies

$$\frac{1-g}{g} = g$$

   The algorithm has the steps

   (a) Define

$$l = b - g(b-a) \quad \text{and} \quad r = a + g(b-a)$$

   (b) Substitute

$$\begin{array}{lll} b \leftarrow r & \text{if} & f(l) > f(r) \\ a \leftarrow l & \text{if} & f(l) \leq f(r) \end{array}$$

   (c) If $b - a < 10^{-6}$, then $(a+b)/2$ is accepted as the required solution; otherwise, go back to Step (a).

   Implement the algorithm for $f(x) = \cos(x)$, $a = -100$, and $b = 10$.