Three points define a triangle. T =(V_1,V_2,V_3)
Counterclockwise order means the triangle faces up or out. ijk is 123, 231, or 312. $e_i \equiv V_k - V_j$.

A point in the triangle is a convex weighting of its vertices. $V = \sum_{i=1}^{3} b_i V_i$ $b_1, b_2, b_3 \geq 0$ $b_1 + b_2 + b_3 = 1$

The scalar weight of each vertex point is its barycentric coordinate.
Barycentric coordinates map all triangles to each other.

In 2D, $b_i = \begin{vmatrix} V - V_j \\ V_k - V_j \end{vmatrix} / \begin{vmatrix} V_i - V_j \\ V_k - V_j \end{vmatrix} = 2A_i / 2A = (V - V_j) \cdot e_i^L / 2A$ $e_i^L = (y_j - y_k, x_k - y_j)$

∇b_i, is perpendicular to e_i . $\frac{\nabla b_i}{\|\nabla b_i\|^2}$ is the ith height vector.

The symbol $\frac{\partial F}{\partial b_i}$ is defined as $\lim_{\delta \to 0} \frac{F(b_i + \delta)}{\delta}$ where every thing else is held constant.

This can't be done here, $b_1 + b_2 + b_3 = 1$, so the symbol needs definition. $\frac{\partial F}{\partial b_i} \equiv \frac{\nabla F \cdot \nabla b_i}{\nabla b_i \cdot \nabla b_i}$.

$\frac{\partial b_j}{\partial b_i} = \frac{e_j \cdot e_i}{e_i \cdot e_i}$ $\sum_{i=1}^{3} b_i = 1$ and $\frac{\partial b_i}{\partial b_i} = 1$ so $\frac{\partial b_j}{\partial b_i} + \frac{\partial b_k}{\partial b_i} = -1$.

The 6 numbers, $\left\{ \frac{\partial b_j}{\partial b_i} \right\}$, define the shape of a triangle with 2 degrees of freedom.

$\frac{\partial b_j}{\partial b_i} = -1/2, j \neq i$ describes an equilateral triangle.

A uniform partition can be used as the domain for a simple piece-wise defined curve.
A simple piece-wise defined surface can use all equilateral triangles for its domain.
A chord length partition improves the shape of curves.

Applying $\frac{\partial b_j}{\partial b_i} = \frac{e_j \cdot e_i}{e_i \cdot e_i}$ to the range triangle improves the shape of surfaces.