

LIGO Parameter Estimation with MCMC algorithms

Parameter Estimation Problem

- We are given a detector output $d(t) = s(t) + n(t)$ with $s(t)$ a GW signal and $n(t)$ the detector noise. We want to determine the parameters of the binary that produced $s(t)$.
- We have at our disposal a set of models $m(t, \theta)$, with θ the parameters of the binary (e.g. masses, distance, orientation, time/phase, spins,...)
- We want the probability distribution of the parameters θ given the detector output $d(t)$.

Parameter Estimation Problem

- We follow van der Sluys et al. 2008 (arXiv:0805.1689):

$$p(\theta|d) \propto p(\theta)L(d|\theta)$$

**Probability of
 θ given d**

Prior

**Likelihood
of d given θ**

- Likelihood

$$L(d|\theta) \propto \exp \left(-2 \int_0^\infty \frac{|\tilde{d}(f) - \tilde{m}(f, \theta)|^2}{S_n(f)} df \right)$$

with $S(f)$ the LIGO noise power spectrum density

MCMC Algorithm

- Choose prior for the binary parameters θ
- Initialize binary parameters $\theta = \theta_0$, compute $L(\theta_0|d)$
- MCMC loop:
 - Choose $\theta_1 = \theta_0 + d\theta$, with $d\theta$ drawn from a normal distribution of width σ . Compute $L(\theta_1|d)$
 - Pick a random number r . If $p(\theta_1) L(\theta_1|d) > r p(\theta_0) L(\theta_0|d)$, accept the MC step and set $\theta = \theta_1$. Otherwise, reject the step.
 - If the step was successful, multiply σ by 8. Otherwise, divide σ by 2.
- Store all successful θ . They provide us with a Monte-Carlo sampling of the probability distribution $p(\theta)$.

Practical session

- Using the provided code:
 - Produce a waveform with $M=60M_{\odot}$, $\eta=0.24$, $d=10^{22}$ [equivalent to GW150914]
 - Compute the signal-to-noise ratio of this signal
 - Add fake LIGO noise to the signal (using the `GetNoise()` function) to get a simulated detector output
 - As a test, compute the likelihood of the true parameters, and of a few erroneous parameters
- Implement a MCMC algorithm computing the distribution of the total mass M of the binary, assuming that all other parameters are fixed (i.e. a 1D search!).