LIGO Parameter Estimation with MCMC algorithms

Parameter Estimation Problem

- We are given a detector output d(t) = s(t) + n(t) with s(t) a GW signal and n(t) the detector noise. We want to determine the parameters of the binary that produced s(t).

- We have at our disposal a set of models $m(t,\Theta)$, with Θ the parameters of the binary (e.g. masses, distance, orientation, time/phase, spins,…)

- We want the probability distribution of the parameters Θ given the detector output d(t).

Parameter Estimation Problem

- We follow van der Sluys et al. 2008 (arXiv:0805.1689):

$$p(\theta|d) \propto p(\theta) L(d|\theta)$$

Probability of Θ given d	Prior	Likelihood of d given Θ

- Likelihood

$$L(d|\theta) \propto \exp\left(-2\int_0^\infty \frac{|\tilde{d}(f) - \tilde{m}(f,\theta)|^2}{S_n(f)} df\right)$$

with S(f) the LIGO noise power spectrum density

MCMC Algorithm

- Choose prior for the binary parameters Θ

- Initialize binary parameters $\Theta = \Theta_0$, compute $L(\Theta_0|d)$

- MCMC loop:

  - Choose $\Theta_1 = \Theta_0 + d\,\Theta$, with $d\,\Theta$ drawn from a normal distribution of width $\boldsymbol{\sigma}$. Compute $L(\Theta_1|d)$

  - Pick a random number r in [0,1]. If $p(\Theta_1)\, L(\Theta_1|d) > r\, p(\Theta_0)\, L(\Theta_0|d)$, accept the MC step and set $\Theta = \Theta_1$. Otherwise, reject the step.

  - If the step was successful, multiply $\boldsymbol{\sigma}$ by 8. Otherwise, divide $\boldsymbol{\sigma}$ by 2.

- Store all successful Θ. They provide us with a Monte-Carlo sampling of the probability distribution $p(\Theta)$.

Practical session

- Using the provided code:

  - Produce a waveform with $M=60M_\odot$, $\eta=0.24$, $d=10^{22}$ [equivalent to GW150914]

  - Compute the signal-to-noise ratio of this signal

  - Add fake LIGO noise to the signal (using the GetNoise() function) to get a simulated detector output

  - As a test, compute the likelihood of the true parameters, and of a few erroneous parameters

- Implement a MCMC algorithm computing the distribution of the total mass M of the binary, assuming that all other parameters are fixed (i.e. a 1D search!).

Practical session

- Once you have a working MCMC algorithm:
  - Start using 2000 frequency bins instead of 200 bins (the likelihood function will be better approximated, but the code will be slower)
  - Implement parallel tempering (Sec. 2.6 of arXiv:0805.1689), to decrease the chance of getting stuck at local extrema
- Analyzing the results:
  - Discard the beginning of your MCMC chain (~50%-90%). The remaining points Θ_i should sample the posterior distribution
  - In 1D, we can explicitly compute the likelihood function. We can thus verify that our code converges to the correct answer:

$$p(\theta|d) \propto p(\theta)L(d|\theta)$$