Monitoring-geodetic sets in graphs

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The aim of this research internship is to initiate the study of monitoring-geodetic sets in graphs. This notion is motivated by network monitoring applications and is based on other previous research works. Since this is a new concept, we expect to obtain interesting results on this topic.

Context. A well-studied covering problem in graph theory is the geodetic set problem, introduced in [3] (see [2] and references therein for more recent results). A set S of vertices of a graph G is a *geodetic set* if every vertex of G lies on some shortest path betwen two vertices of S.

In the area of *network monitoring* [1], one wishes to monitor the vertices and/or the edges of a network (viewed as a graph) in order to detect and prevent failures.

The goal of this proposal is to strengthen the definition of a geodetic set to enable not only to cover the network, but also to monitor it. A *monitoring-geodetic set* of a graph G is a geodetic set S of G with the additional property that for every vertex v of G, there are two vertices x, y of S such that v lies on all shortest paths between x and y.

Here, the vertices of S are seen as monitoring probes that can measure the distance among each other at any point of time. If the vertex v is removed from the network (for example if it ceases to function), the monitoring probes x and y will detect the failure since the distance between them will increase.

A stronger version, where the edges of the network are monitored as well, could also be investigated, based on the similar notion of *edge-geodetic sets* [4].

Research proposal. The goal of this master thesis is to initiate the study of monitoring-geodetic sets.

The work on this project will be organized as follows. The first part will consist of a bibliographic review of geodetic sets, network monitoring and related topics. Then, the research work will follow the two following paths (depending on progress, one path may be considered more thoroughly than the other):

- Structural issues: extremal values, optimal solutions for specific graph classes, relations to other graph parameters...

- Algorithmic issues: complexity of the associated decision problem, algorithm design (classical, approximation or parameterized) for specific graph classes...

References

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