Contents

Part A – An Active Filter

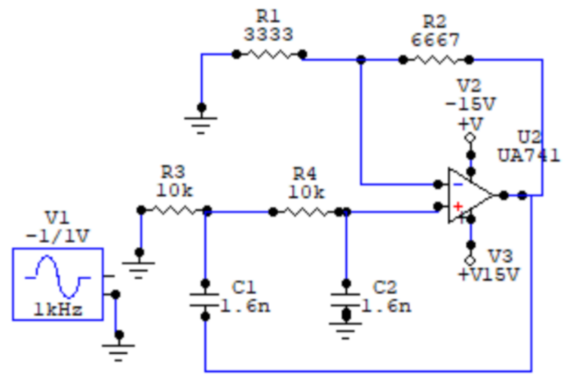
1. 2nd Order Butterworth Filter

The denominator of a 2nd order Butterworth Filter has terms in the form of $s^2 + 2\xi s + 1$, where ξ is the damping factor, and $2\xi = 1.414$. From the transfer function given in the project description, the gain A_M of the circuit is:

$$A_M = 3 - 2\xi = 3 - 1.414 = 1.586\ \frac{V}{V}$$

The value of C can be found using the following relationship:

$\omega_c = \frac{1}{RC}$, where ω_c is the cutoff frequency of 10 kHz.



Circuit 1.1: 2nd Order Low-pass Filter Circuit

Thus,

$$C = \frac{1}{R\omega_c} = \frac{1}{10000 \times 10000 \times 2\pi} = 1.6\ nF$$

To set the values of R_1 and R_2, the following equations were used:

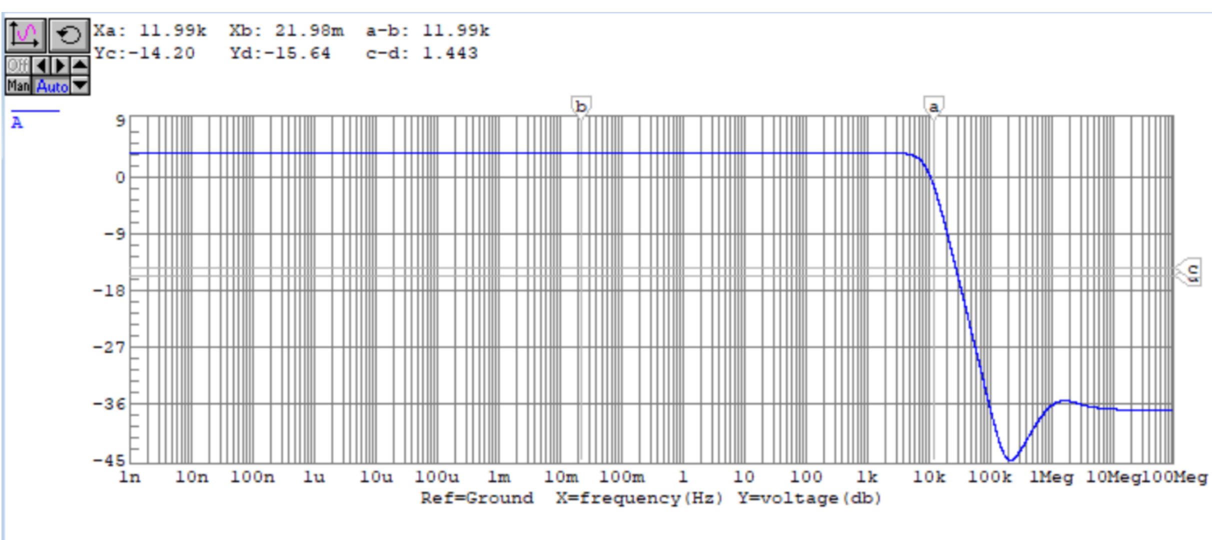
$$R_1 + R_2 = 10\ k\Omega \qquad \textbf{e.q. (1)}$$

$$A_M = 1 + \frac{R_2}{R_1} \qquad \textbf{e.q. (2)}$$

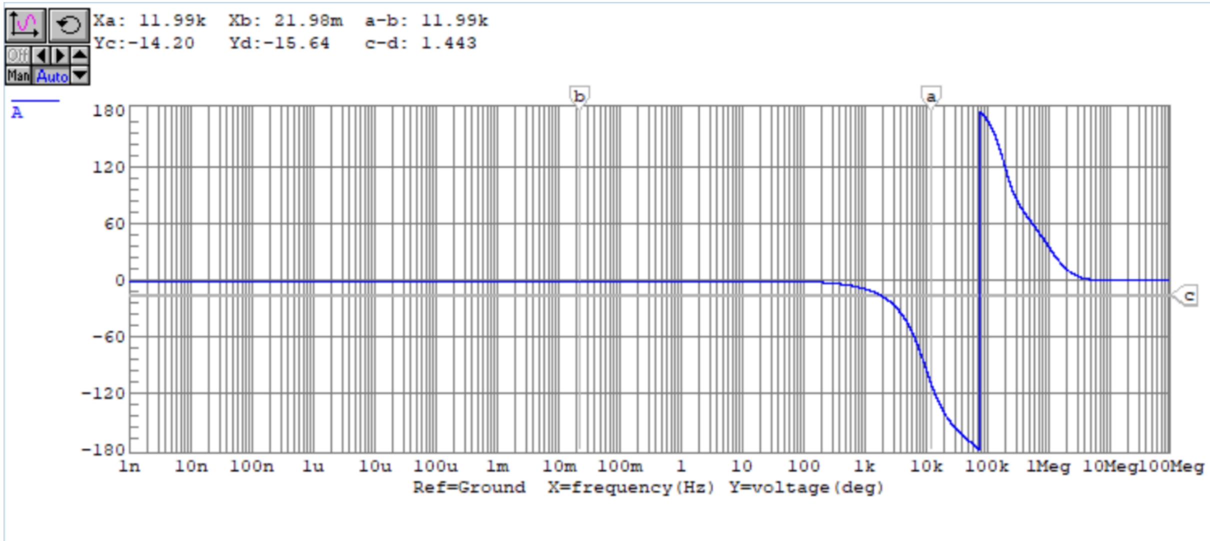
Solving **e.q. (1)** and **e.q. (2)**, we get:

$$R_1 = 6.305\ k\Omega \qquad R_2 = 3.695\ k\Omega$$

The circuit was then wired up, as shown in **Circuit 1.1**, and the following Bode plots for magnitude (**Graph 1.1**) and phase (**Graph 1.2**) were obtained:



Graph 1.1: Bode Magnitude Plot of **Circuit 1.1**



Graph 1.2: Bode Phase Plot of **Circuit 1.1**

From **Graph 1.1**, $\omega_p = 10\ kHz$

The transfer function of our circuit has the form:

$$H(s) = \frac{\frac{A_M}{(RC)^2}}{s^2 + \frac{3 - A_M}{RC} + \frac{1}{(RC)^2}} \qquad \textbf{e.q. 1}$$

2. Value of A_M at which the Circuit Oscillates

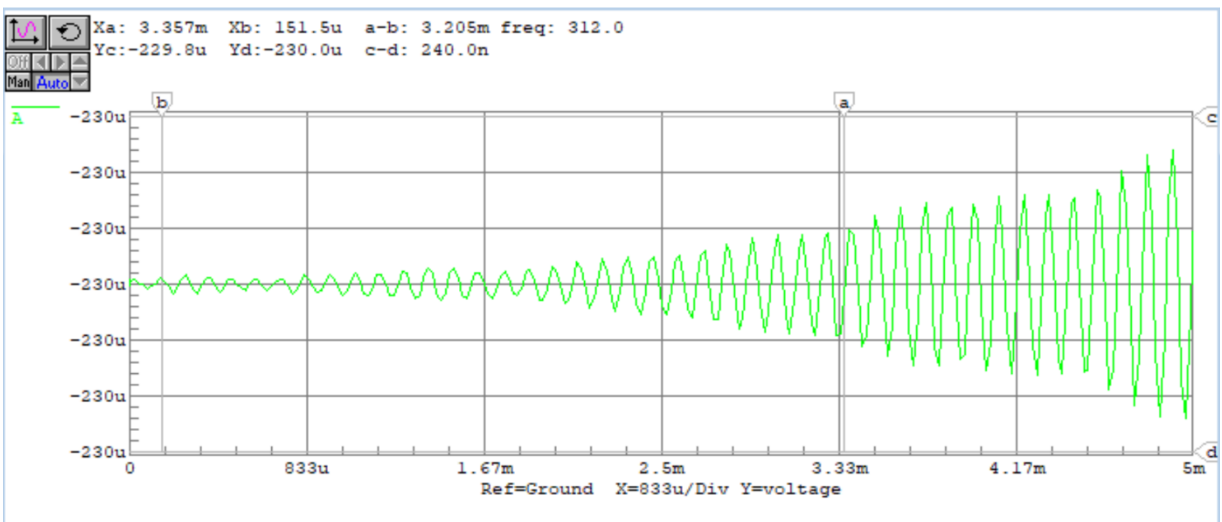
A pure oscillation results from having two complex conjugate poles in our transfer function, located on the $j\omega$-axis to be marginally stable (i.e. critically damped). Recall that the Laplace Transform of a sine wave is in the form $\frac{\omega^2}{s^2+\omega^2}$. In order to make our ransfer function in that form, the s-term must be equal to 0:

$$\frac{3 - A_M}{RC} = 0$$

$$A_M = 3$$

Thus, we expect that our circuit will strat to oscillate at $A_M = 3$, which when used gtin **e.q. (1)** and **e.q. (2)** would yield:

$$R_1 = 3.333\ k\Omega \qquad\qquad R_2 = 6.667\ k\Omega$$



Graph 1.3: Circuit Output when it starts to oscillate

Nevertheless, after varying R_1 and R_2, the values at which the circuit starts to oscillate are slightly different:

$$R_1 = 3.305\ k\Omega \qquad\qquad R_2 = 6.695\ k\Omega$$

Which gives $A_M = 1 + \frac{R_2}{R_1} = 3.03$

The output of the circuit using these values is shown in **Graph 1.3**. The frequnecy at which it oscillates is about 8.25 kHz.

By plugging the values of R, C, and A_M into **e.q. 1** above, we can find the transfer function of the circuit, its poles and plot the root locus. The root locus of the transfer function at $A_M = 1.586\ \frac{V}{V}$ was plotted in **Figure 1.1**. As we can see, the transfer function has two complex conjugate poles, located on the left-hand plane. At this value of A_M, the circuit is oscillating, but decaying. As we increase A_M, the poles move towards the $j\omega$-axis. **Figure 1.2** shows the root locus of the transfer function, when we have a pure, undamped oscillation. This occurs when $A_M = 3\ \frac{V}{V}$, as then our transfer function will have no 's' term in its denominator. **Figure 1.3** shows the location of the poles when A_M exceeds $3\ \frac{V}{V}$, on the right-hand plane, which results in an unstable circuit.

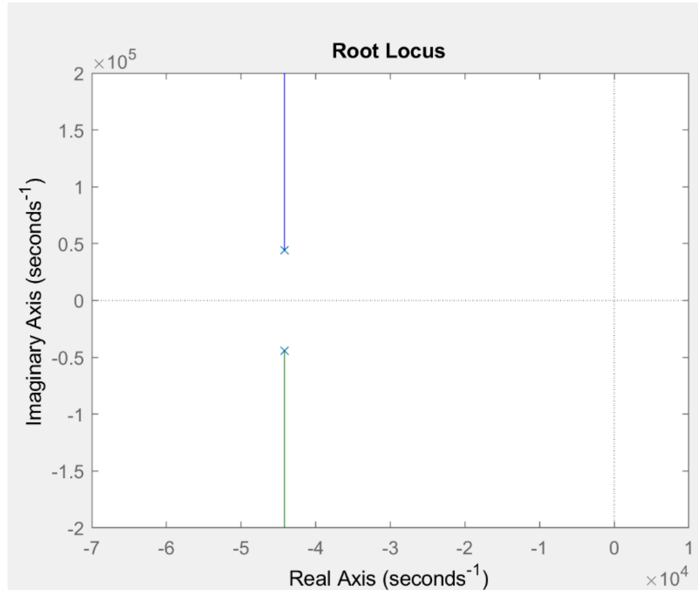


Figure 1.1: Root Locus of the Transfer Function at $A_M = 1.586\frac{V}{V}$

The root locus plots of each value of A_M including the values of the poles are shown on the next page, in **Figure 1.4-6**.

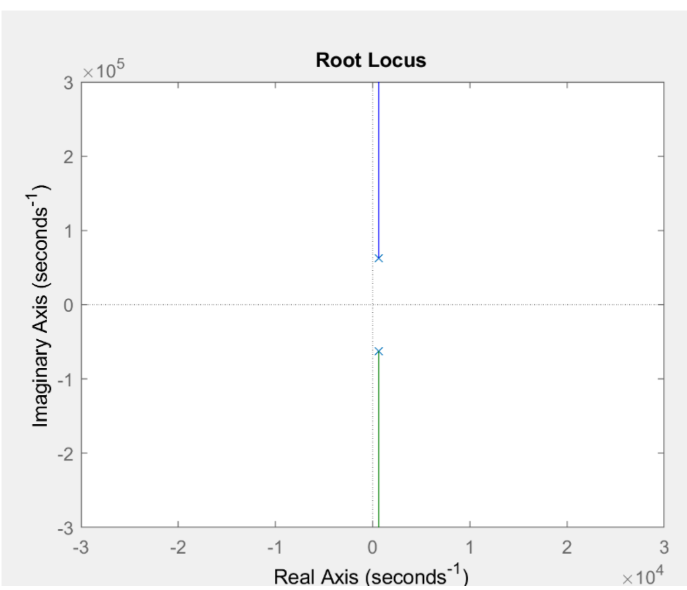


Figure 1.3: Root Locus of the Transfer Function at $A_M = 3.02\frac{V}{V}$

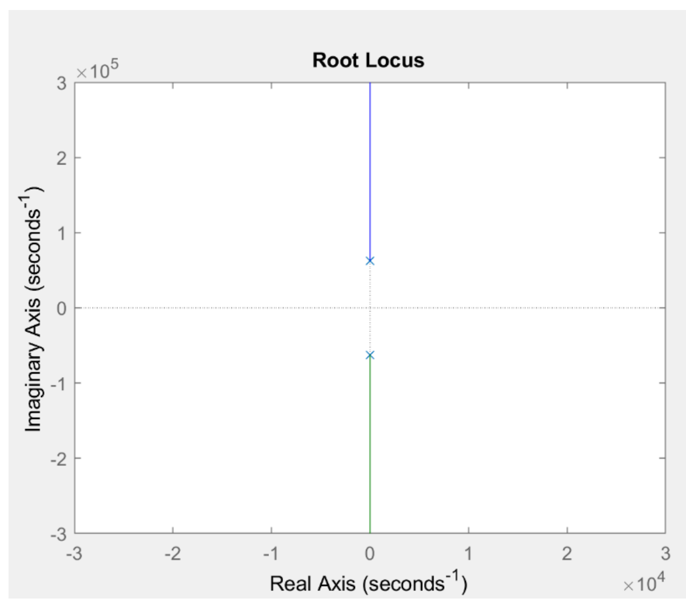


Figure 1.2: Root Locus of the Transfer Function at $A_M = 3\frac{V}{V}$

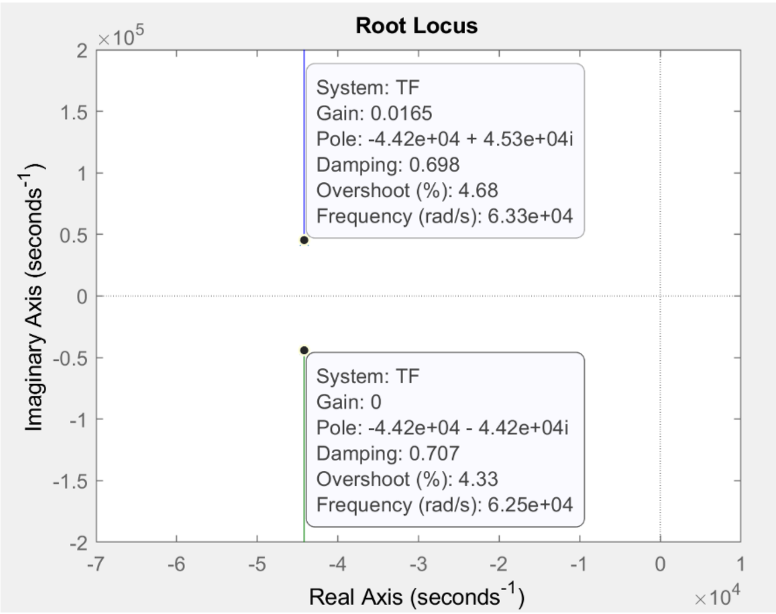


Figure 1.4: Root Locus of the Transfer Function at $A_M = 1.586\frac{V}{V}$

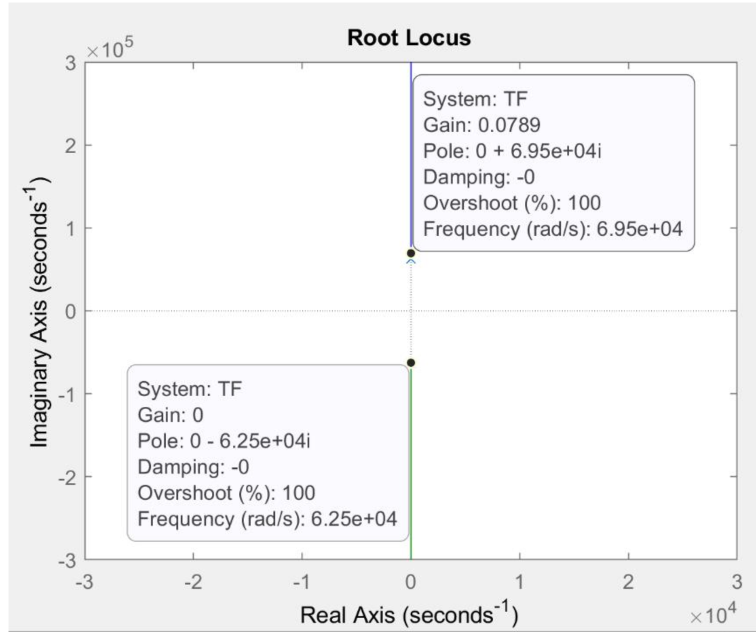


Figure 1.5: Root Locus of the Transfer Function at $A_M = 3\frac{V}{V}$

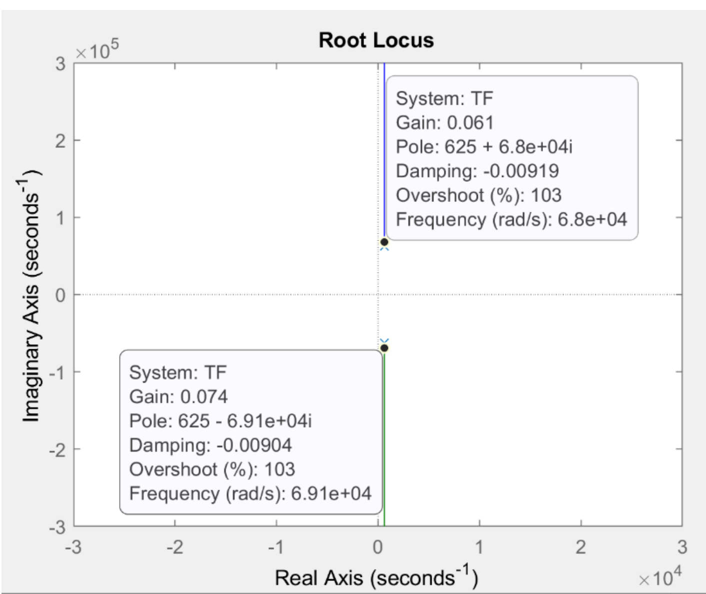
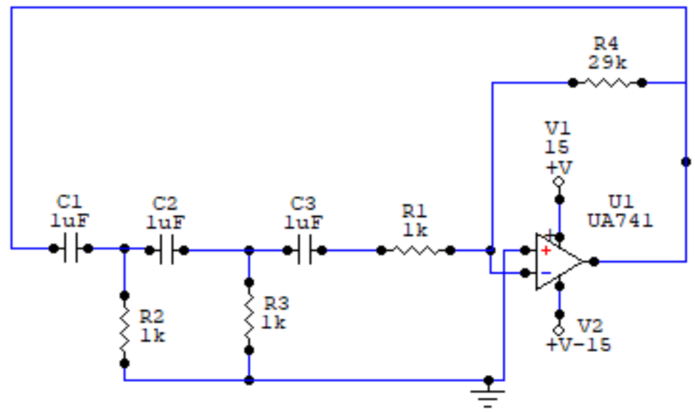


Figure 1.6: Root Locus of the Transfer Function at $A_M = 3.02\frac{V}{V}$

Part B – A Phase Shift Oscillator

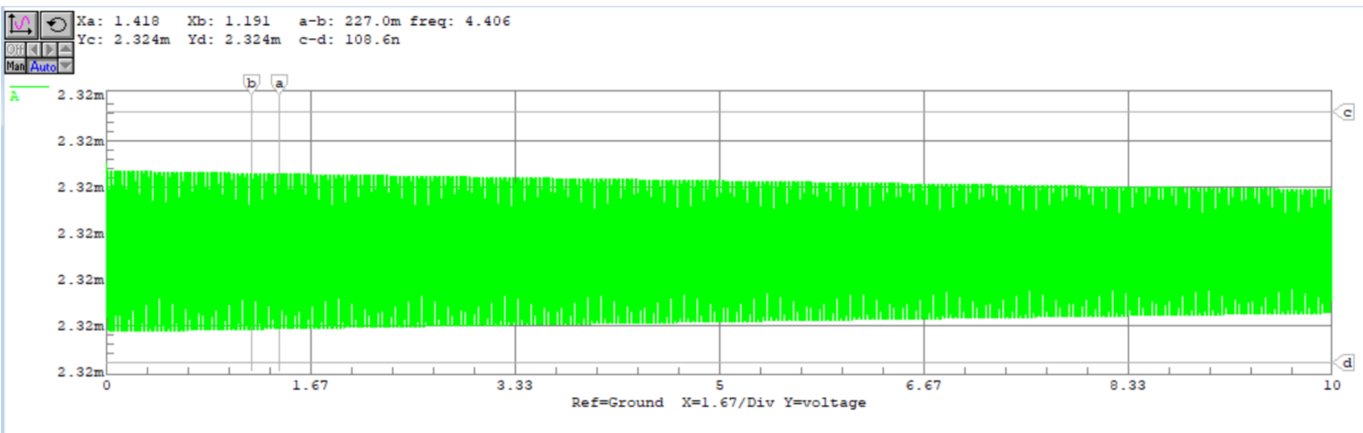
A linear oscillator is a circuit that generates a stable sinusoidal waveform of uniform amplitude at a specific frequency[1]. A phase shift oscillator is a linear oscillator that "shifts" the phase of its amplifier by 180° at the oscillation frequency, giving positive feedback.

Shown in **Circuit 2.1**, a Phase Shift Oscillator consists of a n inverting op amp, with a phase-shift feedback network consisting of capacitors and resistors that "shifts" its output. The oscillator produces a phase shift that is proportional to frequency.



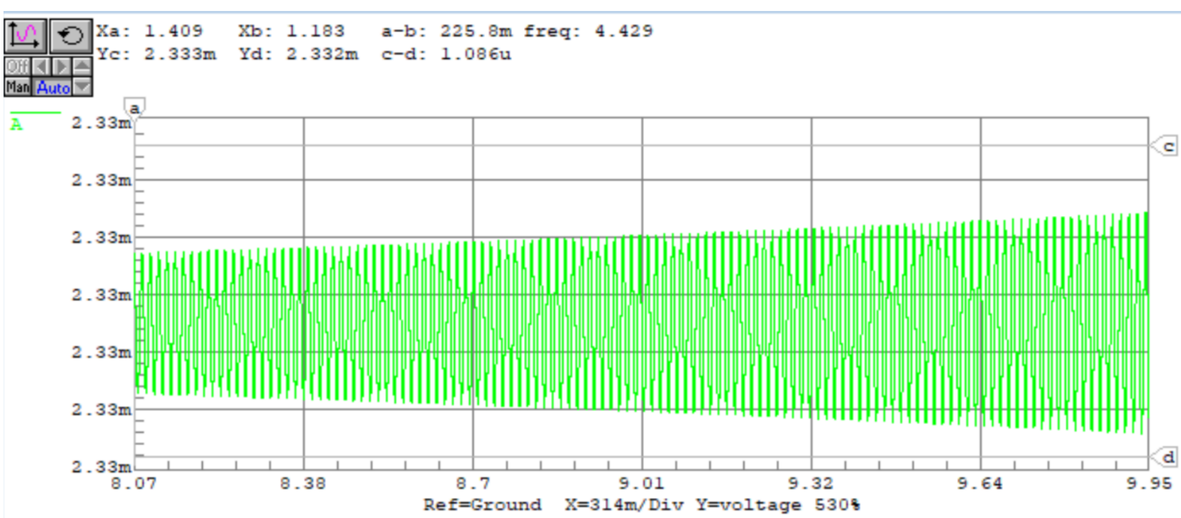
Circuit 2.1: Phase Shift Oscillator

Setting the 29R resistor to 29 $k\Omega$ gives the oscillation in **Graph 2.1**.



Graph 2.1: Circuit Oscillation at 29R =29 $k\Omega$

Setting the 29R resistor to 29.1 $k\Omega$ gives the oscillation in **Graph 2.2.**



Graph 2.1: Circuit Oscillation at 29R =29 $k\Omega$

[1] Nicolas A.F. Jaeger: Oscillator – P 22.1

At 29 $k\Omega$, the circuit oscillates, but slowly decays. On the other hand, at 29.1 $k\Omega$, the circuit oscillates and eventually starts to stabilize.

Table 2.1: Table showing the calculated values of the oscillation frequency with varying R and C values:

	Measured Oscillating Frequency	Calculated Oscillating Frequency	% Error
$Original\ R\ and\ C$	$64.29\ Hz$	$64.97\ Hz$	1.06%
$Decreased\ by\ a\ factor\ of\ 2$	$250\ Hz$	$259.9\ Hz$	3.96%
$Increased\ by\ a\ factor\ of\ 2$	$16.1 Hz$	$16.24\ Hz$	0.87%

The formula:

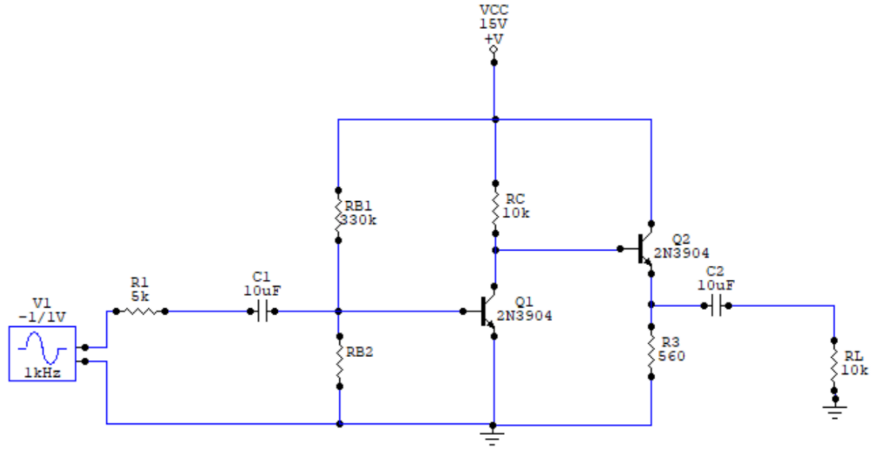
$$f = \frac{1}{2\pi\sqrt{6}RC}$$

was used to calculate the "predicted" oscillation frequency of the circuit at the different RC values.

As shown in **Table 2.1**, the error between the measured values, and the values calculated using the equation above is very negligible. Thus, we can say that there are almost no discrepancies between the measured and calculated values. which makes it a reliable formula to use to predict the oscillating frequency.

Part C – A Feedback Circuit

Since the feedback network samples the output voltage and converts it into a current (as mentioned in the handout), we are looking at y-parameters, which is used with a shunt-shunt topology. Therefore, this circuit has a shunt-shunt topology.

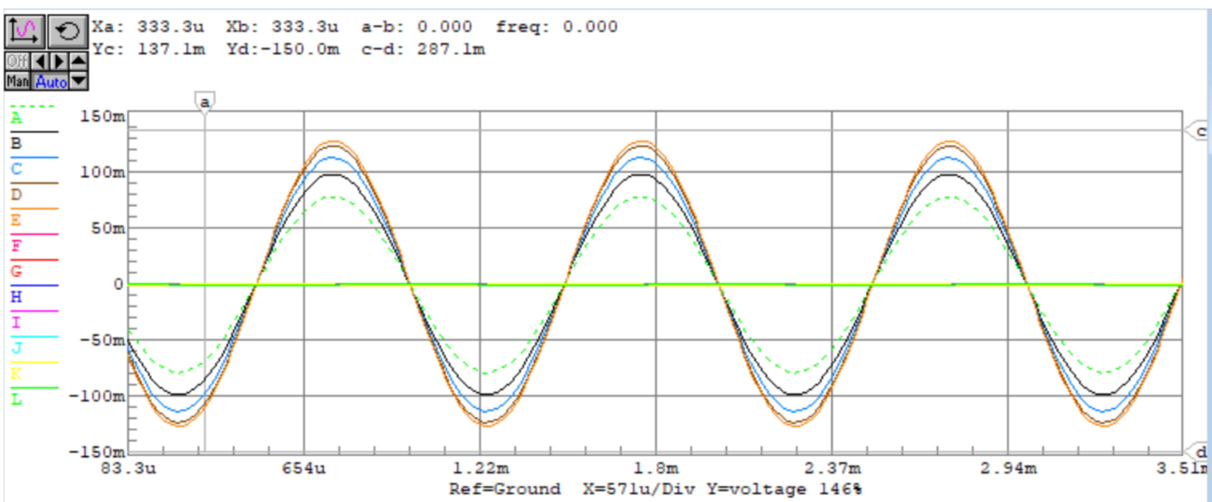


Circuit 3.1: The Open-Loop Circuit

Biasing the Amplifier

The circuit was wired up as shown in **Circuit 3.1**, with $R_f = \infty$. A parameter sweep was then conducted on the variable resistor R_{B2}, for value ranging from 15 $k\Omega$ to 25 $k\Omega$, and the maximum open-loop gain at 1 kHz was at $R_{B2} = 20\ k\Omega$, as shown in **Graph 3.1**.

1. D.C. Bias Values



Graph 3.1: Graph showing Open-Loop Gain for $R_{B2}$from 15 $k\Omega$ to 25 $k\Omega$. The Maximum Amplitude corresponds to 20 $k\Omega$.

<u>**Table 3.1:** Table showing the D.C. bias values for the circuit at $R_{B2} = 20.2\ k\Omega$</u>

Q1	D.C. Operating Point	Q2	D.C. Operating Point
V_{B1}	$0.654\ V$	V_{B2}	$1.9\ V$
I_{B1}	$10.77\ \mu A$	I_{B2}	$15.39\ \mu A$
V_{C1}	$1.9\ V$	V_{C2}	$15\ V$
I_{C1}	$1.295\ mA$	I_{C2}	$2.19\ mA$
V_{E1}	$0\ V$	V_{E2}	$1.235\ V$

I_{E1}	$1.305\ mA$	I_{E2}	$2.205\ mA$

Using the bias value obtained for R_{B2}, the D.C. Operating Points of the two transistors, Q1 and Q2, were obtained and summarized in **Table 3.1** above. The parameters h_{fe}, g_m and r_π were then calculated and summarized in **Table 3.2** below.

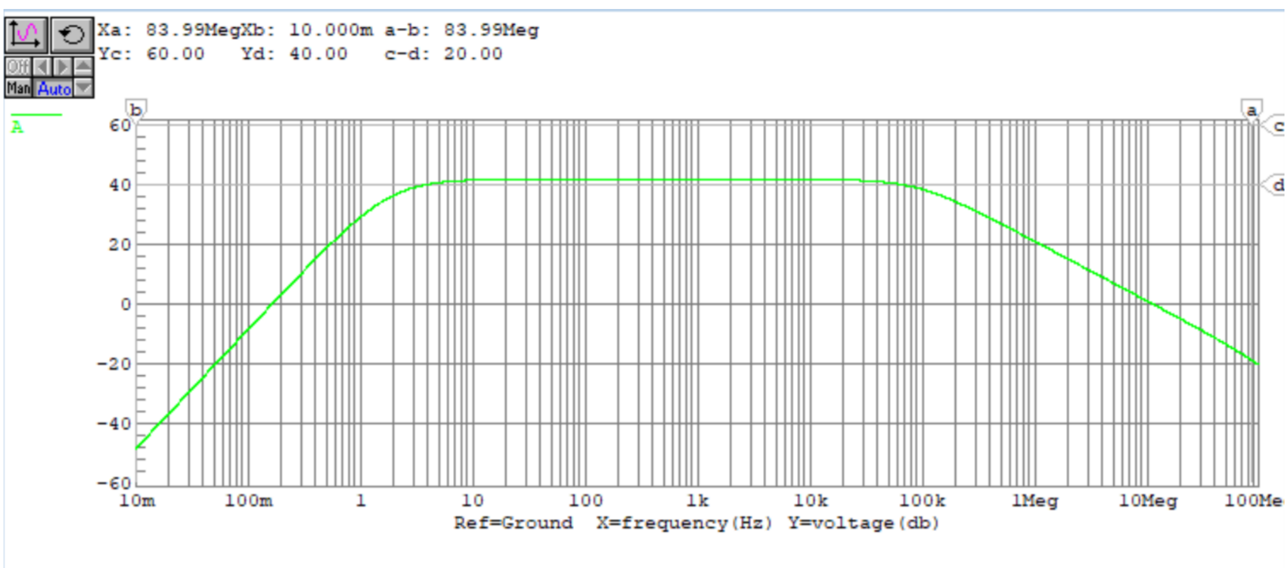
$$h_{fe} = \frac{I_C}{I_B} \qquad g_m = \frac{I_C}{V_T} \qquad r_\pi = \frac{h_{fe}}{g_m}$$

Table 3.2: Table showing the measured values of parameters for the 2N3904 transistor

Parameters for Q1	Measured Value	Parameters for Q1	Measured Value	Unit
h_{fe}	120.24	h_{fe}	142.3	$-$
g_m	0.052	g_m	0.088	S
r_π	2.321	r_π	1.624	$k\Omega$

2. Open-Loop Frequency Response

Using bias value of R_{B2} obtained in part 1 above, the Bode Plot of the circuit was plotted as shown in **Graph 3.2**.



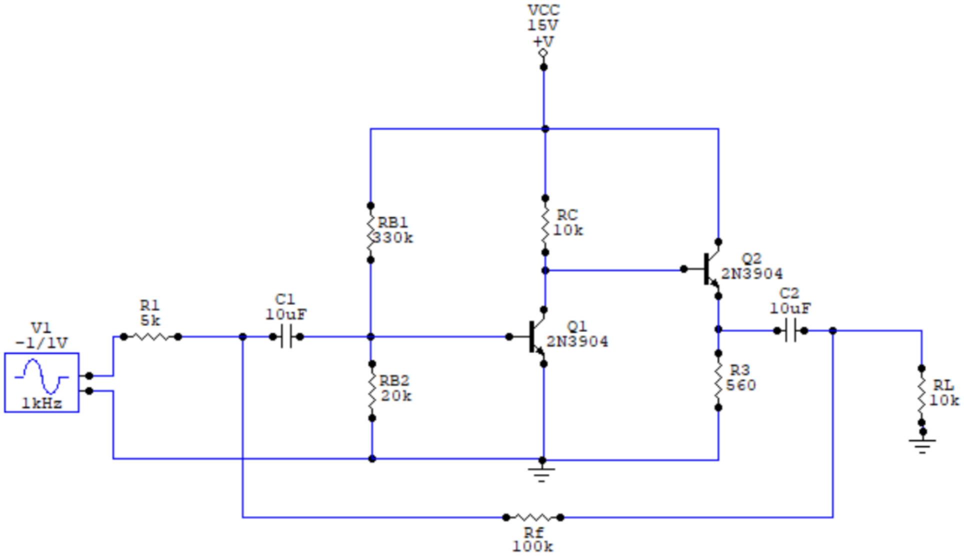
Graph 3.2: Graph showing Open-Loop Frequency Response of the Circuit.

ω_{L3dB} and ω_{H3dB} were measured from **Graph 3.2** to be:

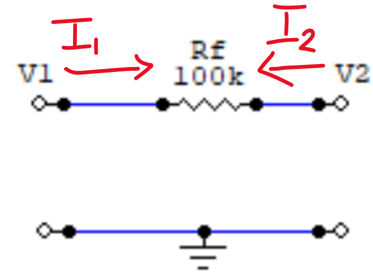
$$\omega_{L3dB} = 2.656\ Hz \qquad \omega_{H3dB} = 96.57\ kHz$$

The open-loop gain is at $41.986\ dB$, equivalent to $-125.6\frac{V}{V}$ (negative for an inverting amplifier).

Now, with $R_f = 100\ k\Omega$, the new Feedback Circuit is wired up as shown in **Circuit 3.2**.



Circuit 3.2: The Feedback Circuit



Circuit 3.3: The Feedback Network

To "predict" the closed-system response, we will use the admittance (y) parameters. The feedback network in **Circuit 3.3** will be used to find the admittance parameters:

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$$

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

Setting $V_2 = 0$:

$I_1 = y_{11}V_1$ $\qquad y_{11} = \frac{I_1}{V_1} = \frac{1}{R_f} = 1 \times 10^{-5}\ S$

$I_2 = y_{21}V_1$ $\qquad y_{21} = \frac{I_2}{V_1} = -\frac{1}{R_f} = -1 \times 10^{-5}\ S$ (We don't really care about y_{21} since it represents feed-forward, which we don't care about)

Setting $V_1 = 0$:

$I_1 = y_{12}V_2$ $\qquad y_{12} = \frac{I_1}{V_2} = -\frac{1}{R_f} = \beta = -1 \times 10^{-5}\ S$

$I_2 = y_{22}V_2$ $\qquad y_{22} = \frac{I_2}{V_2} = \frac{1}{R_f} = 1 \times 10^{-5}\ S$

For a shunt-shunt topology, we have a current-controlled voltage, which means that our open-loop gain is in V/A rather than V/V. We have our open-loop gain in V/V, which is $-125.6\frac{V}{V}$.

To convert it to V/A,

$$\frac{V_o}{i_i} = \frac{V_o}{\frac{V_i}{R_s}} = R_s \times \frac{V_o}{V_i} = 5\ k\Omega \times -125.6\frac{V}{V} = -628k\frac{V}{A} \quad (k\Omega)$$

Now, we can calculate the "predicted" closed-loop gain using the following relationship:

$$A_f = \frac{A}{1 + A\beta} = \frac{-628k\Omega}{1 + (-628k\Omega)(-1 \times 10^{-5}\mu S)} = -86.264\ k\Omega$$

$$R_{if} = \frac{R_i}{1 + A\beta} = \frac{2.575k}{1 + A\beta} = 353.7\Omega$$

$$R_{of} = \frac{R_o}{1 + A\beta} = \frac{62.7}{1 + A\beta} = 8.61\Omega$$

Table 3.3: Table showing the measured vs calculated values of closed-Frequency Response, R_i & R_o:

	Measured Value	Calculated Values	Unit	% Error
A_f	-84.619	-86.264	$k\Omega$	1.94%
ω_{L3dB}	0.515	0.352	Hz	31.65%
ω_{H3dB}	730.5	703	kHz	3.76%
R_{if}	272.3	353.7	Ω	29.89%
R_{of}	8.575	8.61	Ω	0.41%

The values of R_i and R_o were measured to be as follows:

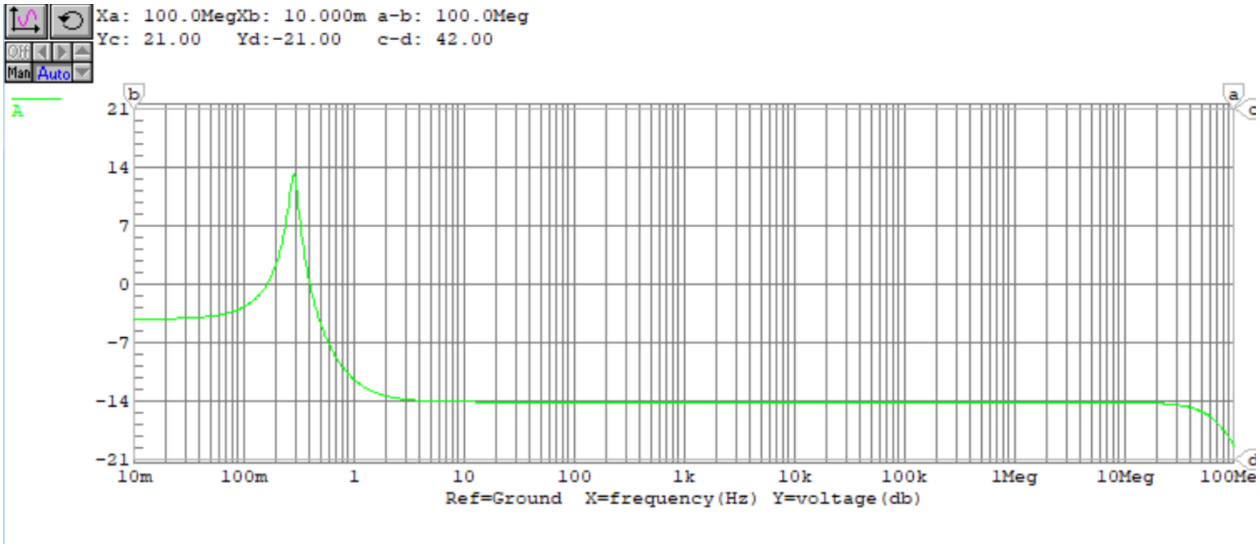
$R_i = \frac{240.1\ \mu V}{93.23\ nA} = 2.575\ k\Omega$ $\qquad$ $R_o = \frac{706.2\ \mu V}{11.27\ \ \mu A} = 62.7\ \Omega$

ω_{L3dB} and ω_{H3dB} were calculated as follows:

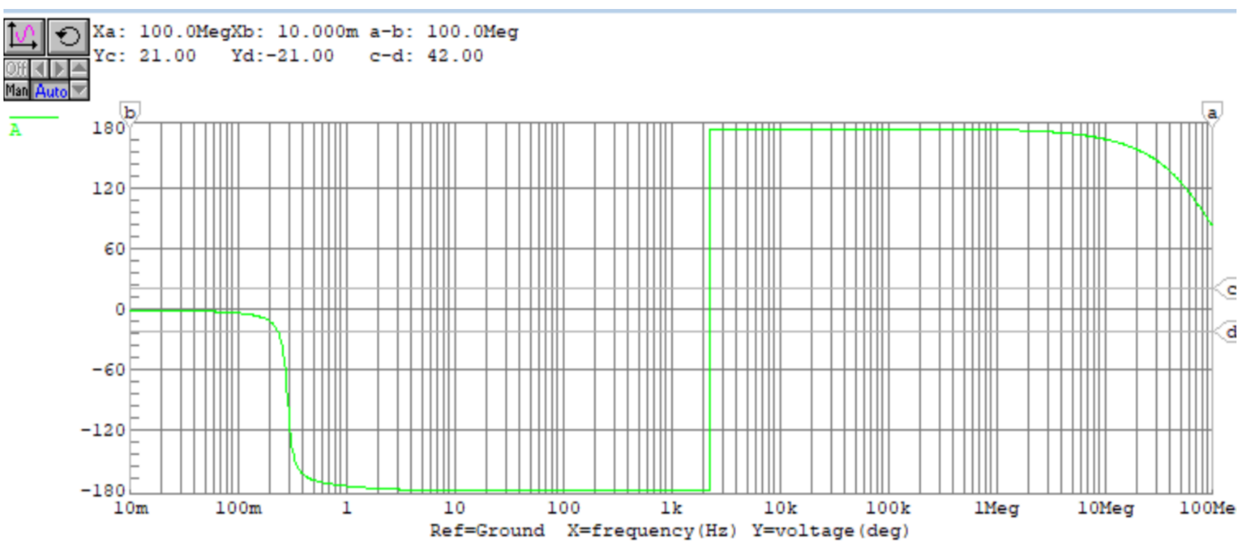
$\omega'_{L3dB} = \frac{\omega_{L3dB}}{1+A\beta} = 0.352\ Hz$ $\qquad$ $\omega'_{h3dB} = \omega_{H3dB}(1 + A\beta) = 703\ kHz$

3. Closed-Loop Frequency Response Over a Range of Values

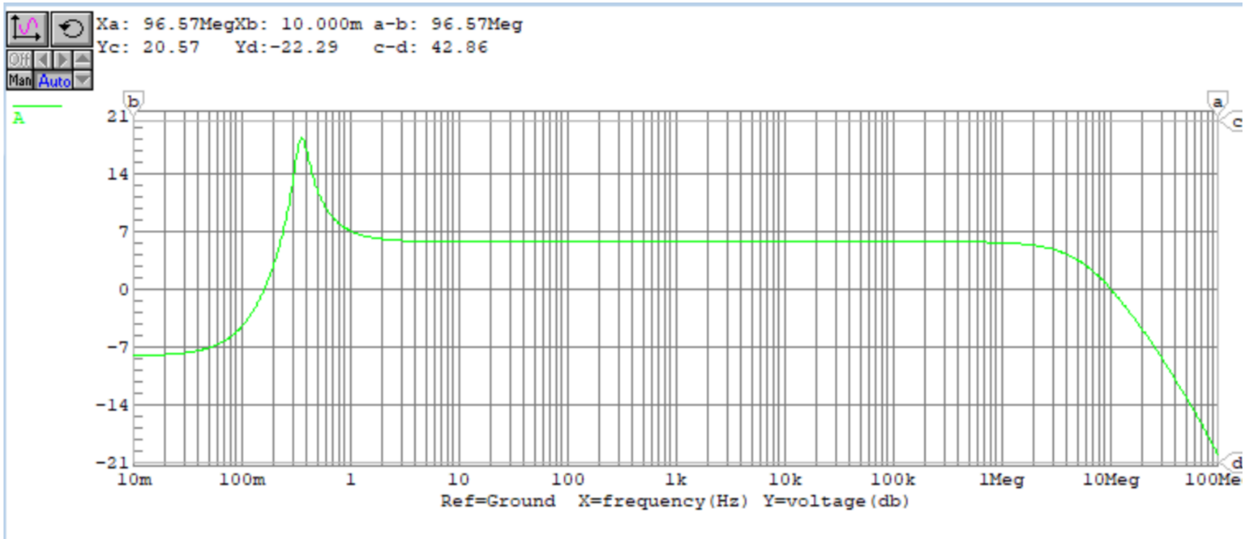
The graphs **Graph 3.3-3.12** show the amplitude and phase responses of the different R_f values.



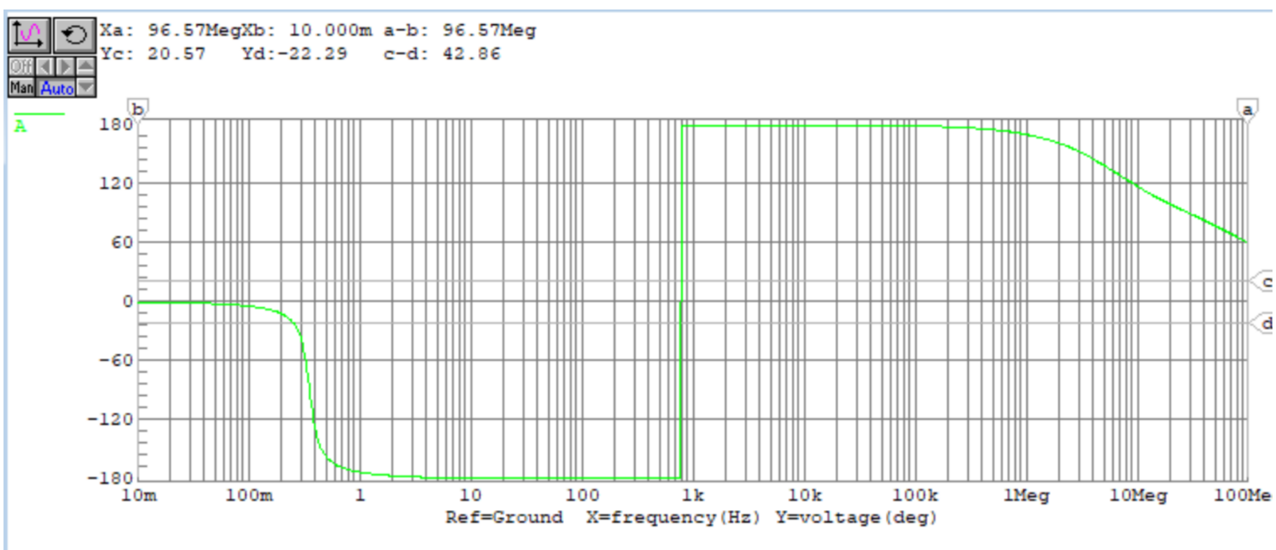
Graph 3.3: Graph showing Closed-Loop Frequency Response for $R_f = 1\ k\Omega$



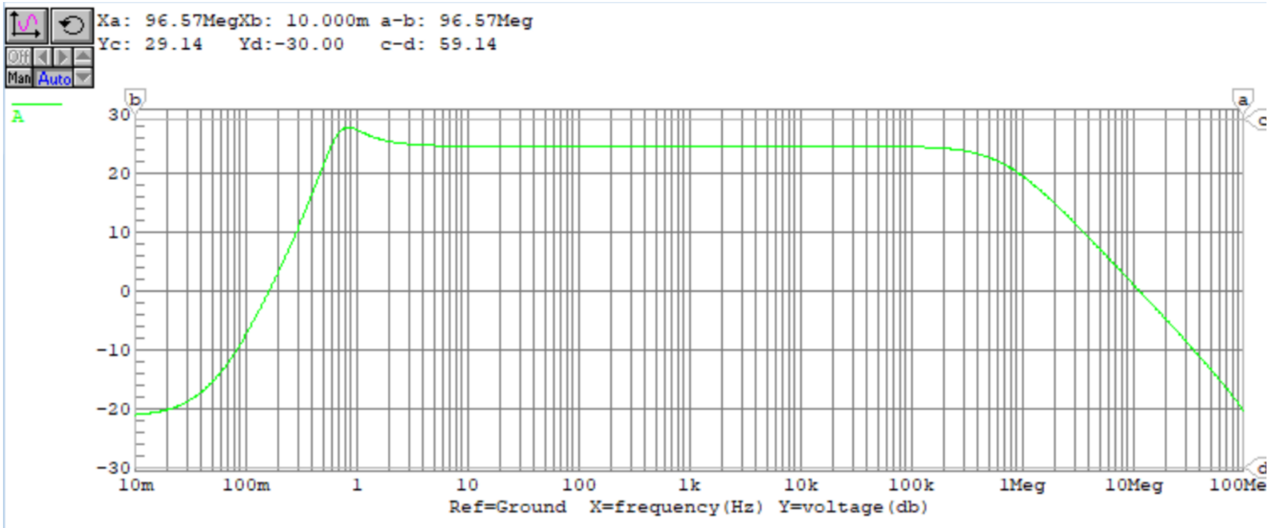
Graph 3.4: Graph showing Closed-Loop Phase Response for $R_f = 1\ k\Omega$



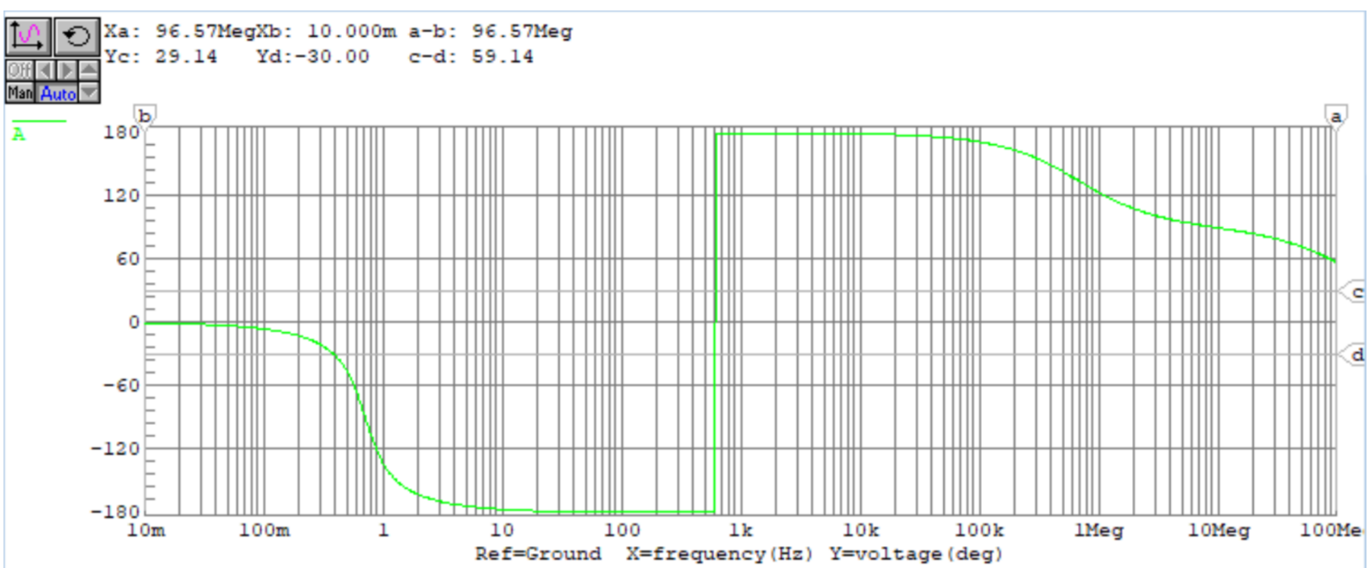
Graph 3.5: Graph showing Closed-Loop Frequency Response for $R_f = 10\ k\Omega$



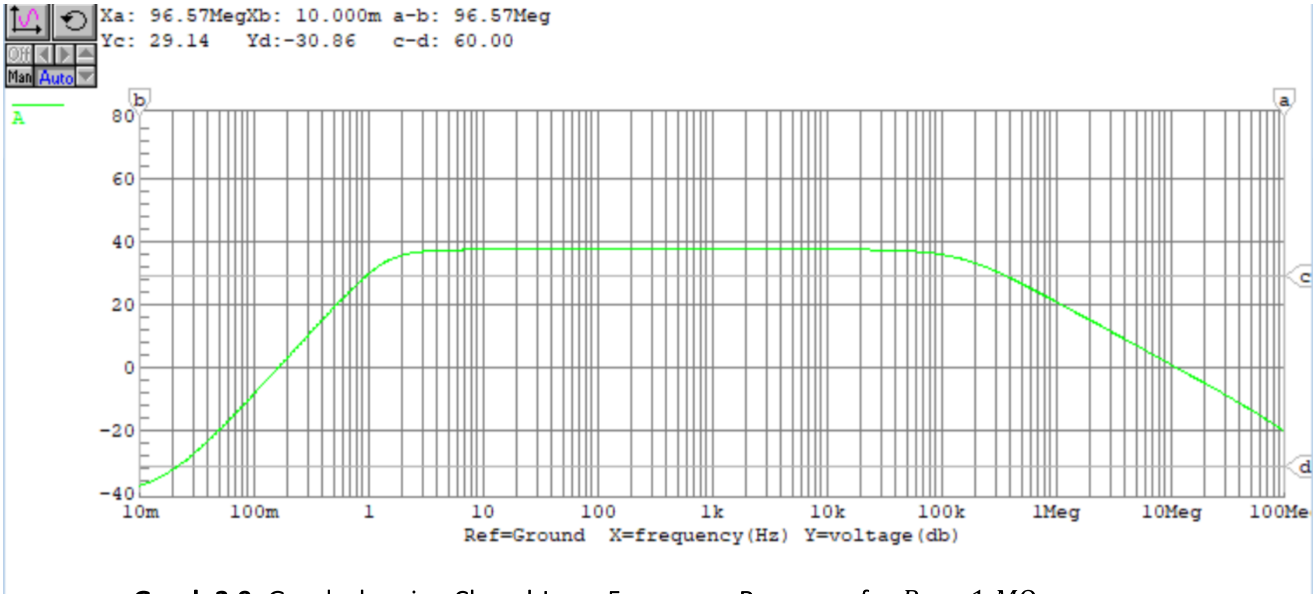
Graph 3.6: Graph showing Closed-Loop Phase Response for $R_f = 10\ k\Omega$



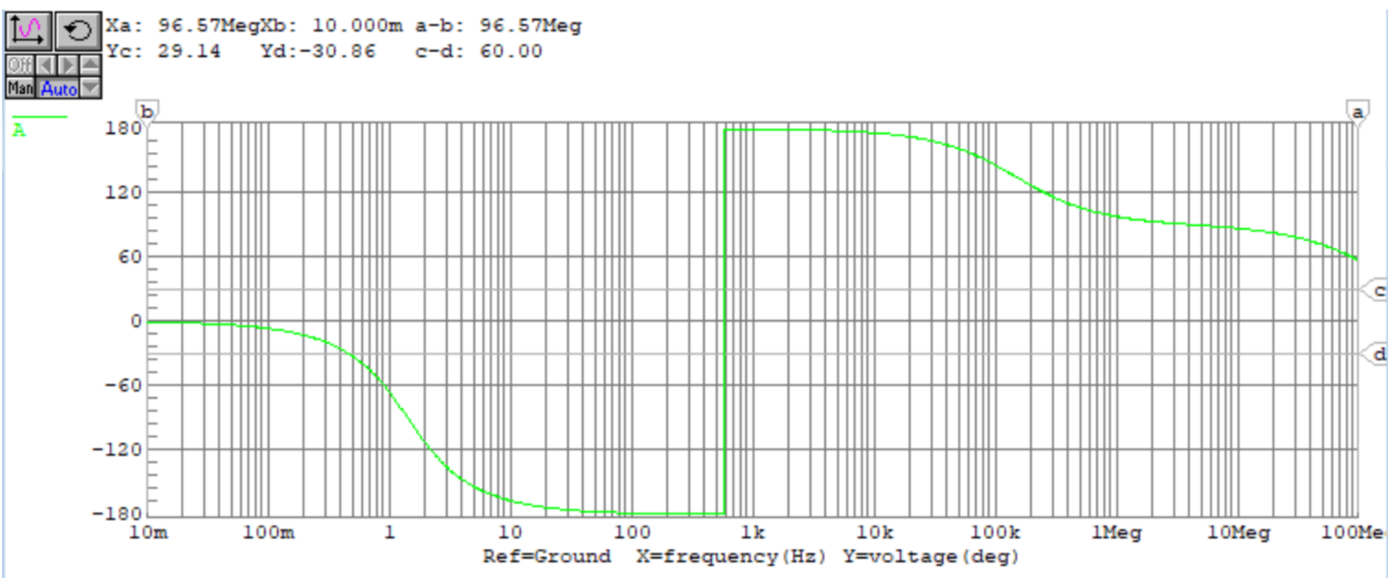
Graph 3.7: Graph showing Closed-Loop Frequency Response for $R_f = 100\ k\Omega$



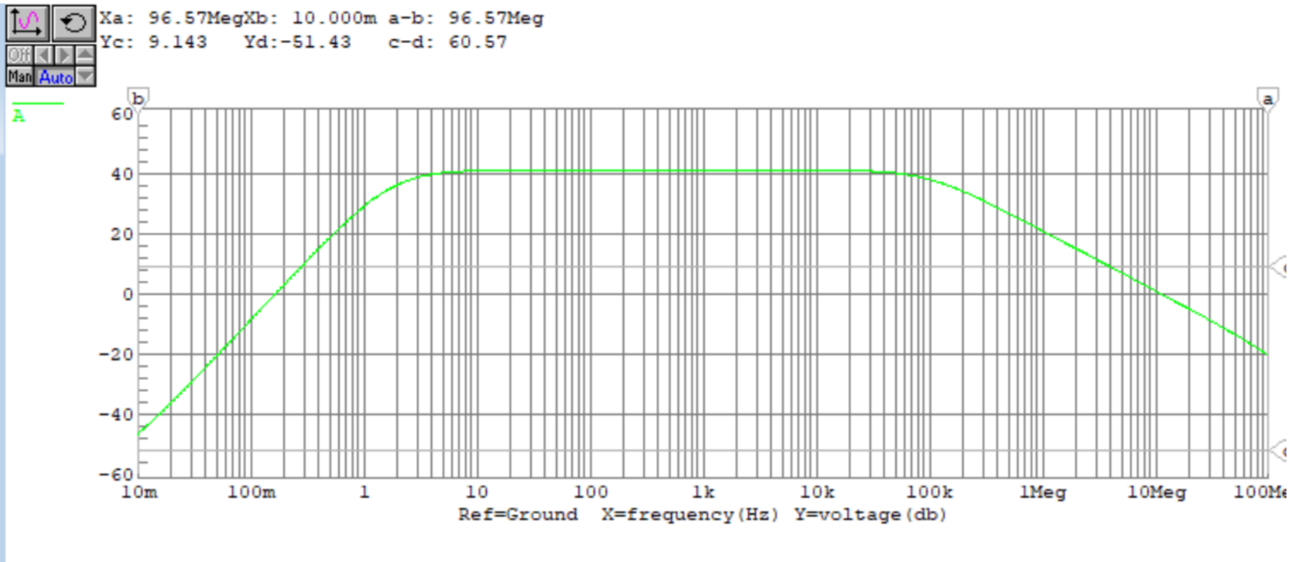
Graph 3.8: Graph showing Closed-Loop Phase Response for $R_f = 100\ k\Omega$



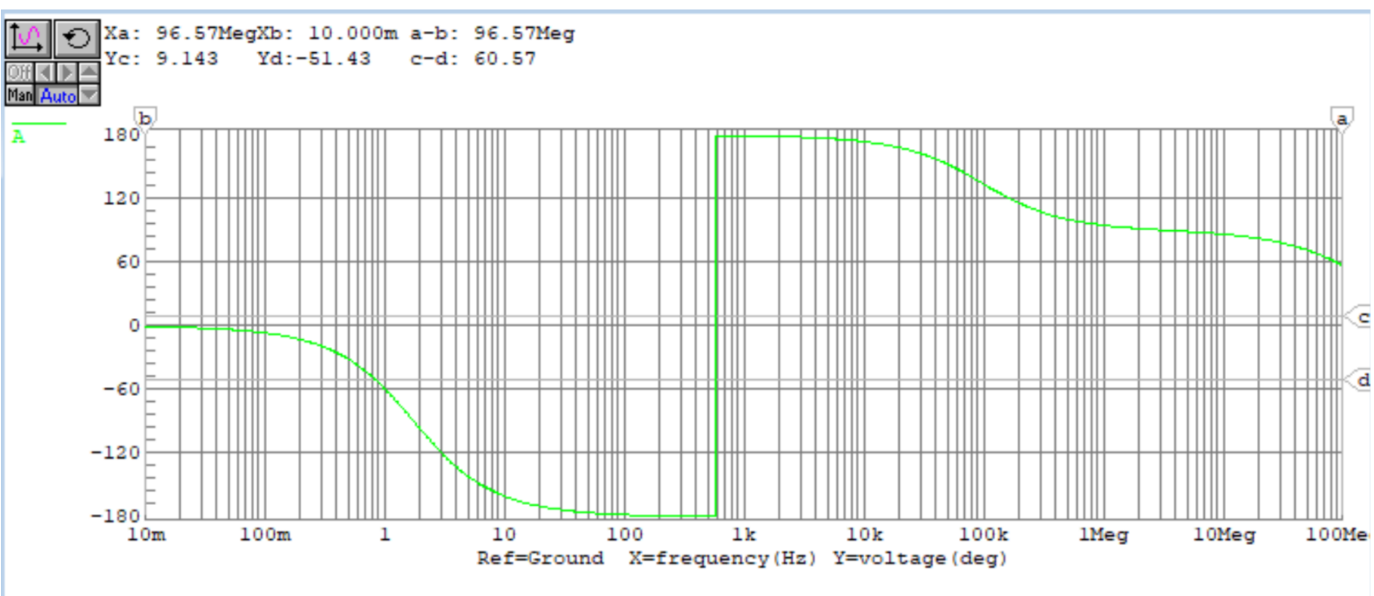
Graph 3.9: Graph showing Closed-Loop Frequency Response for $R_f = 1\ M\Omega$



Graph 3.10: Graph showing Closed-Loop Phase Response for $R_f = 1\ M\Omega$



Graph 3.11: Graph showing Closed-Loop Frequency Response for $R_f = 10\ M\Omega$



Graph 3.12: Graph showing Closed-Loop Phase Response for $R_f = 10\ M\Omega$

Table 3.4: Table showing the measured vs calculated values of the gain and feedback factor for different values of $\boldsymbol{R_f}$:

R_f	Measured β	Calculated β	% Error	Measured A_f (dB)	Measured A_f (V/V)	Measured A_f (V/A)
$1\ k\Omega$	$-0.9 \times 10^{-3}\ S$	$-1\ \times 10^{-3}\ S$	11.11%	$13.4\ dB$	-0.214	-1069
$10\ k\Omega$	$-1.01\ \times 10^{-4}\ S$	$-1\ \times 10^{-4}\ S$	0.99%	$5.77\ dB$	-1.943	$-9.716k$
$100\ k\Omega$	$-0.85\ \times 10^{-5}\ S$	$-1\ \times 10^{-5}\ S$	17.65%	$25.96\ dB$	-19.86	$-99.304\ k$
$1\ M\Omega$	$-1\ \times 10^{-6}\ S$	$-1\ \times 10^{-6}\ S$	0%	$37.94\ dB$	-78.89	$-394.45\ k$
$10\ M\Omega$	$-1.03 \times 10^{-7}\ S$	$-1\ \times 10^{-7}\ S$	2.91%	$41.44\ dB$	-118.0	$-590\ k$

The β values were calculated using the relationship:

$$\beta = -\frac{1}{R_f}$$

The measured β values were found by measuring A_f, and A from part 2 above using the relationship:

$$A_f = \frac{A}{1+A\beta} \qquad \beta = \frac{1}{A_f} - \frac{1}{A}$$

From **Table 3.4**, we can see that the error or uncertainty in the β calculation decreases as the value of R_f increases.

4. Input and Output Resistance of the Feedback Amplifier

Table 3.5: Table showing the measured vs calculated values of β_i & β_o for different values of R_f:

R_f	Measured R_{if}	β_i	Measured R_{of}	β_o	β_{avg}	Feedback	% Error
$10\ k\Omega$	$25.60\ \Omega$	-1.6×10^{-4}	$1.115\ \Omega$	-0.9×10^{-4}	-1.25×10^{-4}	79.5	24.61%
$100\ k\Omega$	$242.2\ \Omega$	-1.5×10^{-5}	$8.46\ \Omega$	-1.02×10^{-5}	-1.26×10^{-5}	8.9128	22.43%
$1\ M\Omega$	$1.311\ \mathrm{k}\Omega$	-1.5×10^{-6}	$37.6\ \Omega$	-1.06×10^{-6}	-1.28×10^{-6}	1.80384	10.80%

The values of β_i and β_o in **Table 3.5** where calculated as follows:

$$R_{if} = \frac{R_i}{1+A\beta} \qquad R_{of} = \frac{R_o}{1+A\beta}$$

Feedback was calculated using $1 + A\beta$, with A from part 2 and calculated β from part 3.

Using R_i, R_o, and A from part 2 above, and β from part 3 above. The feedback amounts were compared to the feedback amounts calculated from β's in part 3 above. The percentage error in the values is not negligible when using β_i and β_o. Thus, approximating the feedback amount using input and output impedances, R_{if} and R_{of}, is less accurate than using the feedback gain, A_f.

5. Desensitivity Factor

The desensitivity factor of the amplifier can be calculated using the following formula:

$$Desensitivity\ Factor = 1 + A\beta$$

Table 3.6: Table showing the measured vs calculated values of the desensitivity factor for different values of R_C for $R_f = \infty$:

R_C	Gain (dB)	Gain (V/V)	Gain (V/A)	Using Measured β	Using Calculated β
$9.9\ k\Omega$	$42.03\ dB$	-126.33	$-63.16\ k$	1	1
$10\ k\Omega$	$42.03\ dB$	-127.06	$-63.53\ k$	1	1
$10.1\ \mathrm{k\Omega}$	$42.03\ dB$	-128.23	$-64.12\ k$	1	1

Since $R_f = \infty$, $\beta = 0$, so desensitivity factor is 1.

Table 3.7: Table showing the measured vs calculated values of the desensitivity factor for different values of R_C for $R_f = 100\ k\Omega$:

R_C	Gain (dB)	Gain (V/V)	Gain (V/A)	Using Measured β	Using Calculated β	% Error
$9.9\ k\Omega$	$24.57\ dB$	-16.92	$-84.62\ k$	7.42	7.28	1.92%
$10\ k\Omega$	$24.69\ dB$	-17.16	$-85.80\ k$	7.32	7.28	0.55%
$10.1\ \mathrm{k\Omega}$	$24.87\ dB$	-17.52	$-87.59\ k$	7.17	7.28	1.51%

Using $A_f = \frac{A}{1+A\beta}$, I solved for $1 + A\beta$. Then using the measured values of A and β from parts 2 and 3 above, I calculated $1 + A\beta$ to be 7.28.

From **Table 3.7**, we can see that the calculated desensitivity factor is 7.28, and the measured desensitivity factor is about 7.30. There is a very small percentage error between the calculated and the measured values of the desensitivity factor, which could be neglected. This shows that approximating the amount of feedback using the measured values of A and β is quite accurate.

In **Graphs 3.5-7**, we notice that for smaller values of R the gain is somewhat larger than at mid band. This seems to be due to the damping at smaller resistor value, which causes the overshoot in frequency at the 3 dB frequency which is higher than at midband.