Contents

Part I

A. Design and Simulation of Circuit

Determining the R values:

From the question, we know the gain is 0.125 = 1/8 = (1/2)^3. At mid-band, our circuit looks like **Figure 1.1**. Using the voltage divider method, we can determine the values of the resistors that produce a gain of 0.125. At mid-band, the circuit looks like **Figure 1.1**:

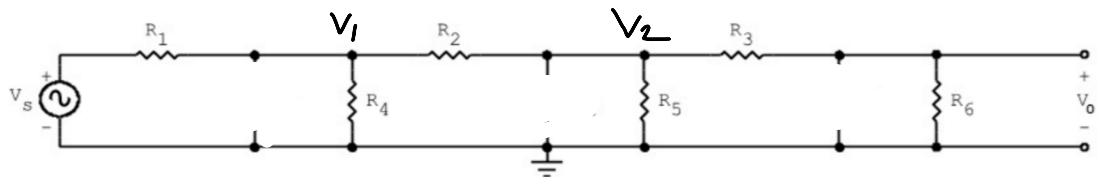


Figure 1.1.

Knowing that $\frac{Vo}{Vs} = (\frac{1}{2})^3 = \frac{Vo}{2V2} \times \frac{V2}{2V1} \times \frac{V1}{2Vs}$

We can break down this equation into the following:

$Vo = \frac{1}{2} \times V2$, so $R3 + R6 = R5$ must be true.

Similarly, $V2 = \frac{1}{2} \times V1$, so $(R3 + R6\ ||R5) + R2 = R4$ and $((R3 + R6\ ||R5) + R2)||R4) = R1$

After solving these equations, the following values of resistors were determined:

$R1 = R2 = R3 = R6 = 500\Omega$ and $R4 = R5 = 1000\Omega$.

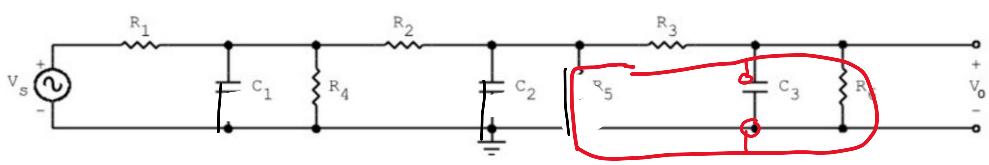
Determining the C values:

To determine the values of the capacitors, open-circuit and short-circuit methods were used. Knowing that $C1 > C2 > C3$, we can match determine that the largest pole (5 x 10^7) will correspond to the smallest capacitor C3, and the smallest pole (5 x 10^5) will correspond tot the largest capacitor C1 (**Figure 1.2**).

$$T(s) = \frac{V_o(s)}{V_s(s)} = 0.125 \times \frac{5\times10^5/\text{sec}}{s + 5\times10^5/\text{sec}} \times \frac{5\times10^6/\text{sec}}{s + 5\times10^6/\text{sec}} \times \frac{5\times10^7/\text{sec}}{s + 5\times10^7/\text{sec}}$$

Figure 1.2: Transfer function from Part I

To calculate τ_{C3}, we will use the short-circuit method, applied in **Circuit 1.1**:

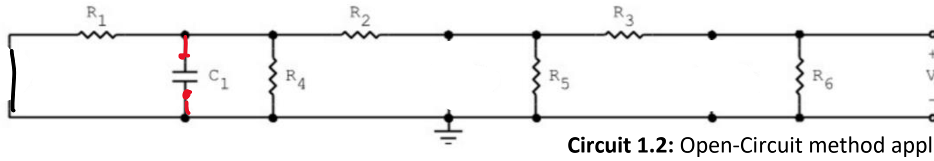


Circuit 1.1: Short-Circuit method applied to find C3

$$\omega_{Hp3} = 5 \times 10^7 = \frac{1}{\tau_{C3}} = \frac{1}{R3||R6\ \times C3} = \frac{1}{250 \times C3}$$

$$C3 = \frac{1}{250 \times 5 \times 10^7} = 80\ pF$$

To calculate τ_{C1}, we will use the open-circuit method, applied in **Circuit 1.2**

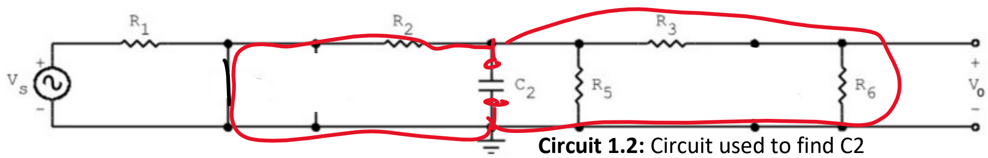


Circuit 1.2: Open-Circuit method applied to find C1

$$\omega_{Hp1} = 5 \times 10^5 = \frac{1}{\tau_{C1}} = \frac{1}{(((R6 + R3)||R5 + R2)||R4 + R1) \times C1} = \frac{1}{250 \times C1}$$

$$C1 = \frac{1}{250 \times 5 \times 10^5} = 8\, nF$$

When calculating τ_{C2}, C3 is seen by C2 as an open circuit, due to its small capacitance value compared to C2, and C1 is seen as a short circuit by C2, due to its large capacitance value compare to C2 (**Circuit 1.3**):



Circuit 1.2: Circuit used to find C2

$$\omega_{Hp2} = 5 \times 10^6 = \frac{1}{\tau_{C2}} = \frac{1}{((R6 + R3)||R5||R2) \times C2} = \frac{1}{250 \times C2}$$

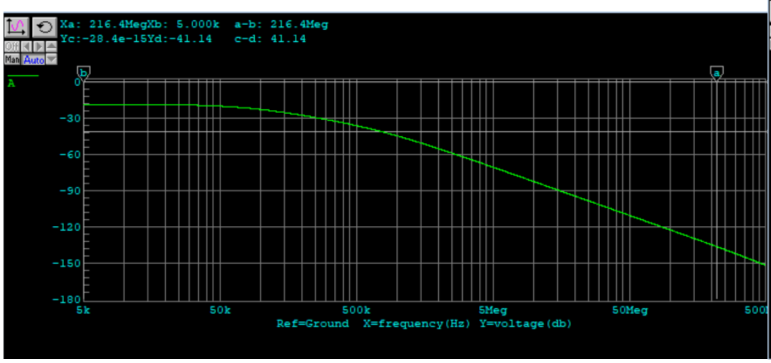
$$C2 = \frac{1}{250 \times 5 \times 10^6} = 0.8\, nF$$

Finally,

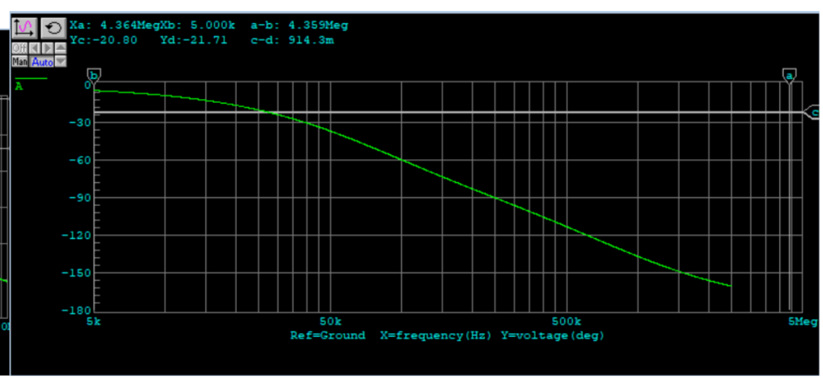
$$\boxed{\boldsymbol{R1 = R2 = R3 = R6 = 500\Omega\,, \qquad R4 = R5 = 1000\Omega.}}$$

$$\boxed{\boldsymbol{C1 = 8nF, \qquad C2 = 0.8\, nF, \qquad C3 = 80\, pF}}$$

When the circuit is simulated using CircuitMaker, the following Bode Plots of the Magnitude (**Graph 1.1**) and the phase (**Graph 1.2**) are generated:



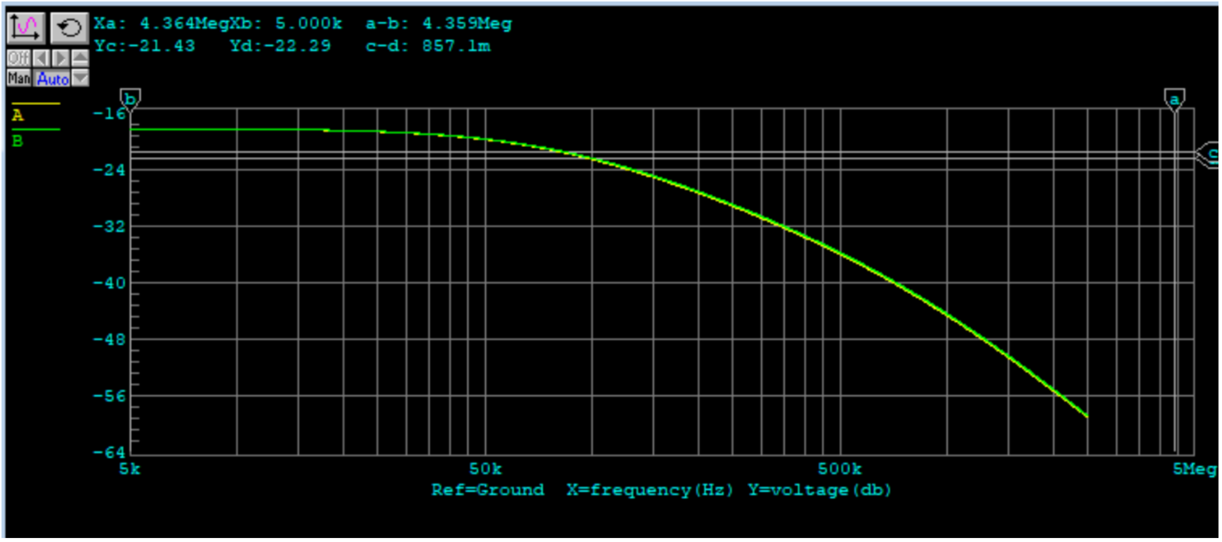
Graph 1.1: Bode Plot of the Magnitude (dB) vs. frequency (Hz)



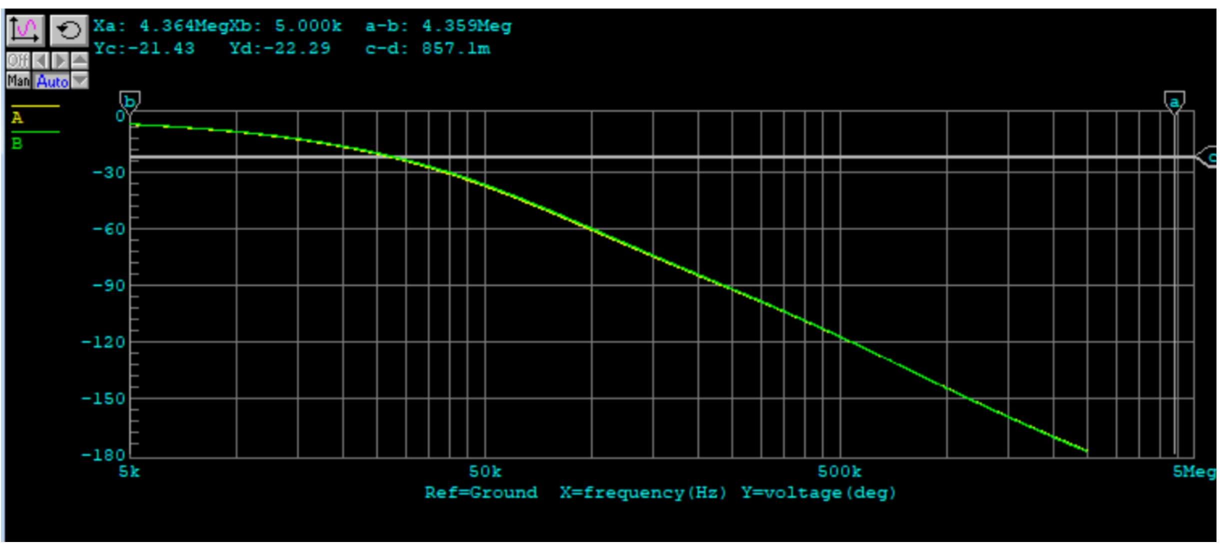
Graph 1.2: Bode Plot of the Phase (Degrees) vs. frequency (Hz)

B. Comparing Bode Plots of the Circuit Design to the Transfer Function

To validate the circuit designed in part A, I modelled it using CircuitMaker, generated a bode plot of its magnitude (**Graph 1.3**) and phase (**Graph 1.4**), and compared them to those of the transfer function given in **Figure 1.1**.



Graph 1.3: Bode Plot of the Magnitude (dB) vs. frequency (Hz)
The green curve shows the Transfer Function, and the Yellow curve shows the Circuit designed in part A.



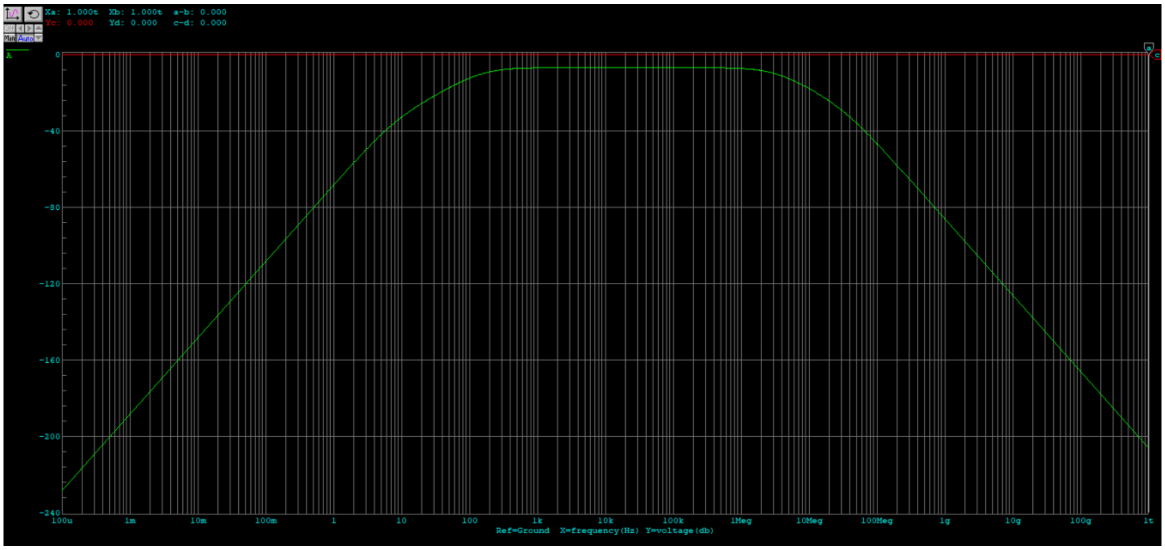
Graph 1.3: Bode Plot of the Phase (Degrees) vs. frequency (Hz)
The green curve shows the Transfer Function, and the Yellow curve shows the Circuit designed in part A.

The graphs above show that our approximation matches the real circuit quite well.

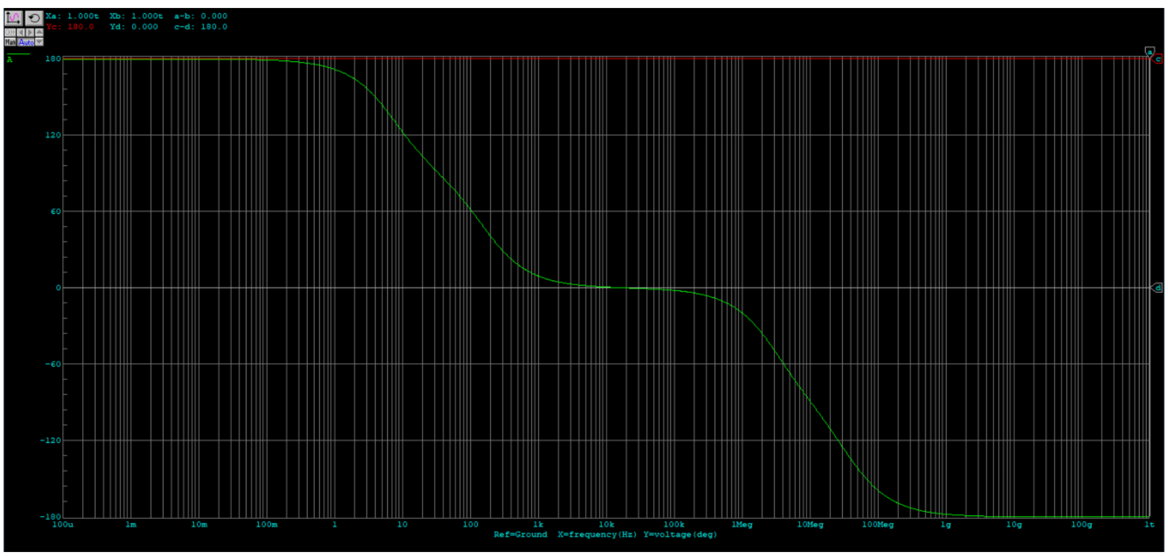
Part II

A. Locating the poles of the Transfer Function

The circuit was simulated using Circuit maker, and the following bode plots of the magnitude (**Graph 2.1**) and phase (**Graph 2.2**) were generated:



Graph 2.1: Bode Plot of the Magnitude (dB) vs. frequency (Hz)

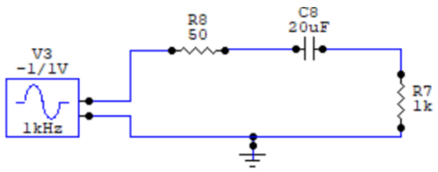


Graph 2.2: Bode Plot of the Phase (Degrees) vs. frequency (Hz)

The poles of the transfer function of the circuit were then graphically located plotting the short-circuit and open-circuit equivalent on Circuit Maker for each capacitor.

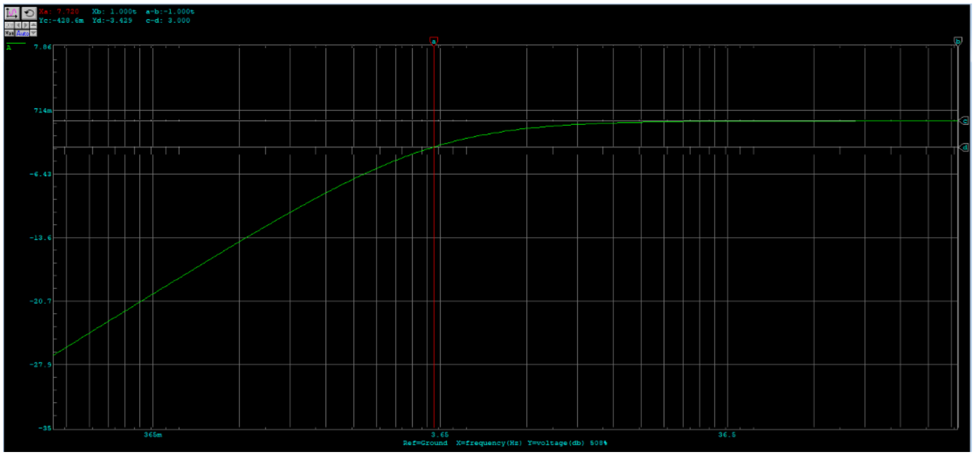
Sample graphical location of the pole corresponding to C1:

The open-circuit method was used to locate the pole for the capacitor C1, producing the following **Circuit 2.1**:



Circuit 2.1: Open-circuit equivalent circuit for C1

The low-pass filter capacitors C2 and C4 were open-circuited due to their relatively very small capacitance value compared to C1. The same logic was applied to the high-pass filter capacitor C3, since its capacitance is 4 orders of magnitude smaller than that of C1.



Graph 2.3: Locating the Pole Corresponding to C1

To locate the pole, the horizontal cursors are placed so that the top one at the midband and the bottom one 3 dB below. The vertical cursor 'a' was then used to locate the point of intersection between the bottom horziontal cursor and the graph, giving $\omega_{Lp1} = 7.72\ Hz$.

The same method was applied to locate the remaining poles which are summarized in **Table 2.1**.

Table 2.1: Table showing the Calculated and Actual values (located graphically) in Hz of the poles of the circuit and their % error:

ω	ω_{Lp1}	ω_{Lp2}	ω_{Hp1}	ω_{Hp2}
Actual Value	7.72	155.2	32.03 M	3.04 M
Calculated Value	7.58	155.6	36.59 M	3.11 M
% Error	1.81%	0.26%	14.24%	2.30%

Percentage Error was calculated using the following formula:

$$\%\ Error = \left|\frac{Actual\ Value - Calculated\ Value}{Actual\ Value}\right| \times 100$$

B. Exploring the effect of increasing C3 on Low Frequency Poles

The values of the high and low 3 dB poles with C3 = 500 nF were calculated using the open and short circuit methods to be as follows (Refer to **Appendix II** for the calculation of individual poles of the circuit):

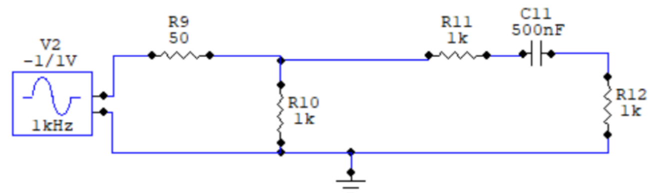
$$\omega_{L3dB} = \sqrt{{\omega_{Lp1}}^2 + {\omega_{Lp2}}^2} = \sqrt{47.62^2 + 977.72^2} = 978.88\frac{rad}{sec} = 155.79\ Hz$$

$$\tau_{H3dB} = \sqrt{{\tau_{Hp1}}^2 + {\tau_{Hp2}}^2} = \sqrt{(4.65 \times 10^{-9})^2 + (5.12 \times 5.12^{-8})^2} = 5.14 \times 10^{-8}\ sec$$

$$\omega_{H3dB} = \frac{1}{\tau_{H3dB}} = \frac{1}{5.14 \times 10^{-8}} = 19.46 \times 10^{6} \frac{rad}{sec} = 3.10 \times 10^{6} Hz$$

Table 2.2 shows the values of ω_{L3dB} for each of the C3 values (500 nF, 1 µF, 2 µF, 5 µF, and 10 µF) calculated using the same method above, compared the actual values located on graphs generated using CircuitMaker, and the percentage error between the two.

To calculate ω_{H3dB}, open-circuit test was applied to find the equivalent circuits for C1 (**Circuit 2.1**) and C3 (**Circuit 2.2**).



Circuit 2.2: Open-circuit equivalent circuit for C3

The new value of ω_{Lp2} for each capacitor value was calculated using the following formula:

$$\omega_{Lp2} = \frac{1}{\tau_{C3}} = \frac{1}{((R1||R2) + R3 + R4) \times C3}$$

Then, this value was used along with the ω_{Lp2} corresponding to C1 to calculate ω_{H3dB} using the following formula:

$$\tau_{H3dB} = \sqrt{\tau_{Hp1}{}^{2} + \tau_{Hp2}{}^{2}}$$

$$\omega_{H3dB} = \frac{1}{\tau_{H3dB}}$$

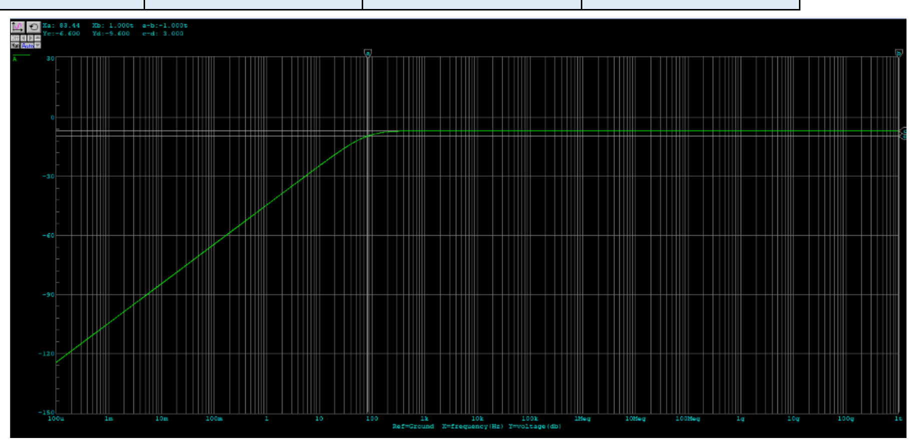
Note: The values obtained graphically are in the unit of Hz, so the proper conversions in the calculations were made to ensure the correct units are achieved.

Table 2.2: Table showing the calculated and actual values of ω_{L3dB} for each C3 value in Hz and their %error:

C3 Value	500nF	1 µF	2 µF	5 µF	10 µF
Actual Value	155.20	79.23	43.70	22.30	15.52
Calculated Value	155.79	78.10	39.59	17.30	10.85
% Error	0.38%	1.43%	9.41%	22.42%	30.09%

As shown in the table, the percentage error in the approximated (calculated) value increases as the value of the capacitor increases. The value of ω_{L3dB} is inversely proportional to the value of the capacitor. **Graph 2.4** shows the new pole of C3 at value 1 µF. The poles for the remaining values of C3 were located similarly.

Note: Only one example is provided to save space.

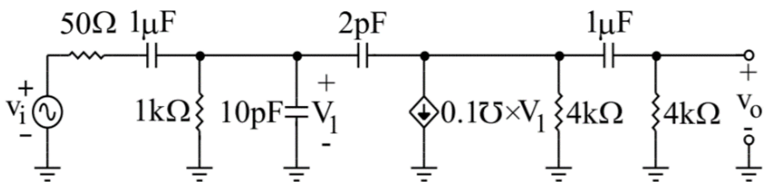


Graph 2.4: Graph showing the open-circuit equivalent of C3 at value 1 µF

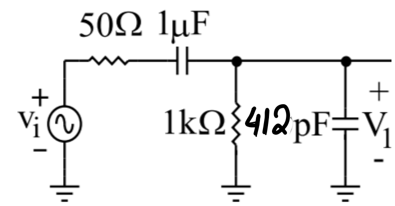
Part III

A. Using Miller's Theorem

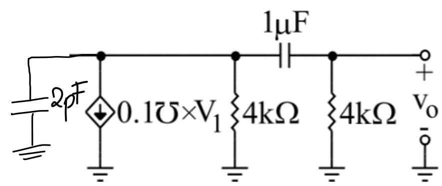
In this section, we will use Miller's Theorem, along with open-circuit and short-circu transfer function of the **Circuit 3.1**.



Circuit 3.1: Transconductance Amplifier



Circuit 3.2: Miller's Equivalent Circuit for the Input stage



Circuit 3.3: Miller's Equivalent Circuit for the Output stage

Miller's gain can be applied to the circuit to break it up into two stages; the input stage (**Circuit 3.2**) and the output stage (**Circuit 3.3**).

Input Stage

For the Input stage, the miller equivalent capacitor is calculated to have a value of approximately 402 pF, which we can then combine in parallel with the 10pF capacitor to give the resultant 412 pF capacitor in the **Circuit 3.2**.

Then the poles of each of the capacitors in the Input stage can be calculated as follows:

For the 1 µF capacitor, the 412 pF capacitor is seen as an open-circuit, due to its relatively very high impedance. Therefore, the equivalent resistance becomes 1050 Ω.

$$\omega_{Lp1} = \frac{1}{\tau_{1\mu F}} = \frac{1}{(50 + 1k)\Omega \times 1\mu F} = 952.38\, rad/sec$$

Similarly, for the 412 pF, the 1 µF is seen as a short circuit due to its relatively low impedance. Therefore, the equivalent impedance is (50||1000) Ω.

$$\omega_{Hp1} = \frac{1}{\tau_{412pF}} = \frac{1}{(50||1k)\Omega \times 412pF} = 51.00 \times 10^{6}\, rad/sec$$

Output Stage

For the output stage, the miller equivalent capacitor can be estimated to be 2 pF, since the gain is 200, so the term $\frac{1}{k}$ in $Z_2 = \frac{1}{1-\frac{1}{k}}$ becomes negligible compared to 1, therefore $Z_2 \approx Z$. As in the input stage, the poles of each capacitor can be calculated as follows:

For the 1 µF capacitor, the 2 pF capacitor is seen as an open-circuit, due to its relatively very high impedance. Therefore, the equivalent resistance becomes (4k||4k) Ω.

$$\omega_{Lp2} = \frac{1}{\tau_{1\mu F}} = \frac{1}{8k\Omega \times 1\mu F} = 125\, rad/sec$$

Similarly, for the 2 pF, the 1 µF is seen as a short circuit due to its relatively low impedance. Therefore, the equivalent impedance is (4k||4k) Ω.

$$\omega_{Hp2} = \frac{1}{\tau_{2pF}} = \frac{1}{2k\Omega \times 2pF} = 250 \times 10^{6}\, rad/sec$$

Mid-band Gain

The mid-band gain has been calculated as follows:

$$\frac{V_o}{V_i} = \frac{V_o}{V_1} \times \frac{V_1}{V_i}$$

From the questions, we know that k = -200, therefore

$$V_o = -200 \times V_1$$

$$\frac{V_o}{V_i} = -200 \times \frac{V_1}{V_i}$$

Then, we simply need to apply voltage divider logic to find the ratio $\frac{V_1}{V_i}$, so

$$V_1 = \frac{V_i \times 1000}{1050}$$

Therefore,

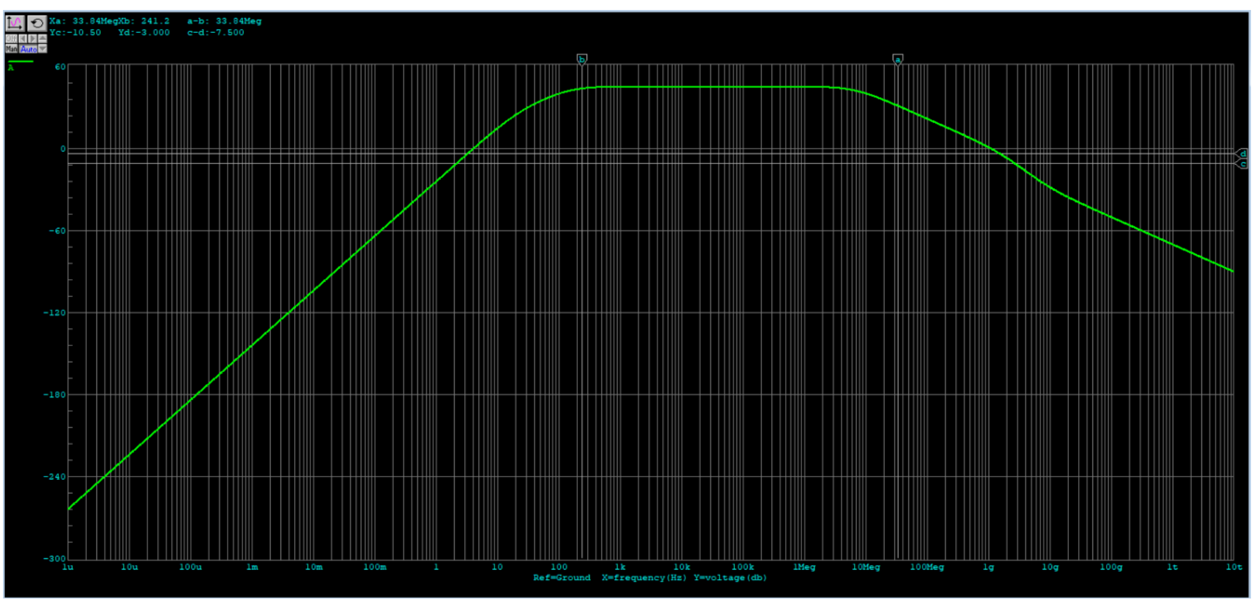
$$\boxed{\frac{V_o}{V_i} = -200 \times \frac{1000}{1050} = -190.48 \frac{V}{V}}$$

Zeros

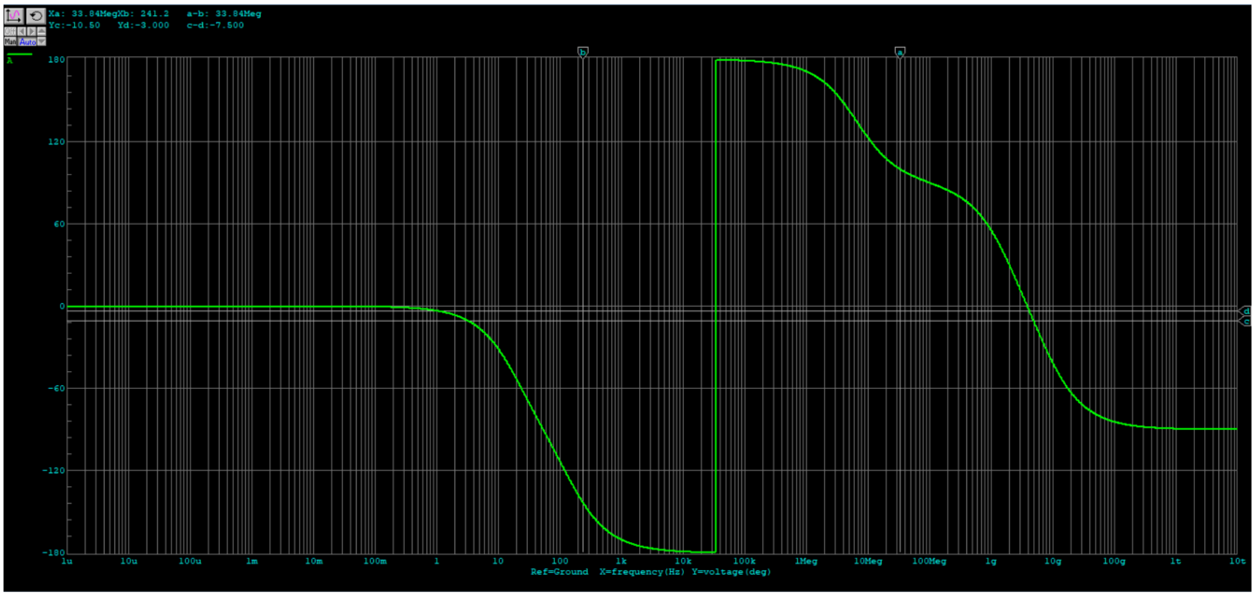
Since there are two high-frequency poles, we know that there is a double Zero at 0. When applying Miller's theorem, the other pole disappears. This is especially since Miller's theorem can only be used to estimate poles, but not zeros.

B. Percentage Error Calculations

An AC simulation was run on the circuit, and a bode plot both the magnitude (**Graph 3.1**) and the phase (**Graph 3.2**) were generated.



Graph 3.1: Bode Plot of the Magnitude (dB) vs. frequency (Hz)



Graph 3.2: Bode Plot of the Phase (Degrees) vs. frequency (Hz)

Table 3.1 shows the values of ω for each pole calculated in part A, compared to the actual values located on graphs generated using CircuitMaker, and the percentage error between the two.

<u>**Table 3.1:** Table showing the calculated values (from part A) and actual values of ω in Hz and their %error:</u>

$\boldsymbol{\omega}$	$\boldsymbol{\omega_{Lp1}}$	$\boldsymbol{\omega_{Lp2}}$	$\boldsymbol{\omega_{Hp1}}$	$\boldsymbol{\omega_{Hp2}}$	$\boldsymbol{\omega_{L3dB}}$	$\boldsymbol{\omega_{H3dB}}$
Actual Value	143.3	20.54	8.05 M	39.24 M	154.00	6.73 M
Calculated Value	151.58	19.90	8.12M	39.79 M	152.88	7.94 M
% Error	5.78%	3.12%	0.87%	1.40%	0.73%	17.98%

Now, we can calculate ω_{L3dB} and ω_{H3dB} as follows:

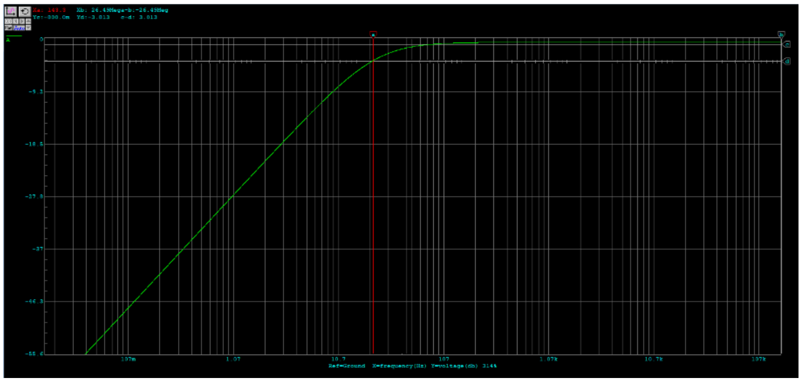
$$\omega_{L3dB} = \sqrt{\omega_{Lp1}{}^2 + \omega_{Lp2}{}^2} = \sqrt{151.58^2+19.9^2} = 152.88\,Hz = 960.58\,rad/sec$$

$$\tau_{H3dB} = \sqrt{\tau_{Hp1}{}^2 + \tau_{Hp2}{}^2} = \sqrt{(\frac{1}{8.12} \times 10^{-6})^2 + (\frac{1}{39.79} \times 10^{-6})^2} = 1.26 \times 10^{-7}\ sec$$

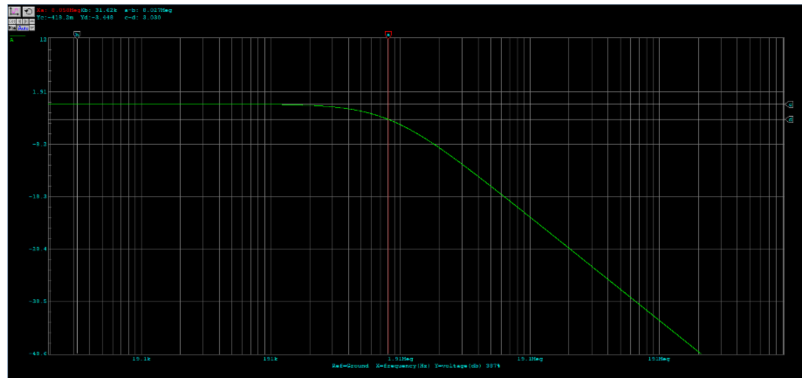
$$\omega_{H3dB} = \frac{1}{\tau_{H3dB}} = \frac{1}{1.26 \times 10^{-7}} = 7.94 \times 10^6 Hz = 49.87 \times 10^6\ rad/sec$$

The poles of the transfer function were obtained graphically using the method as in part II, section A.

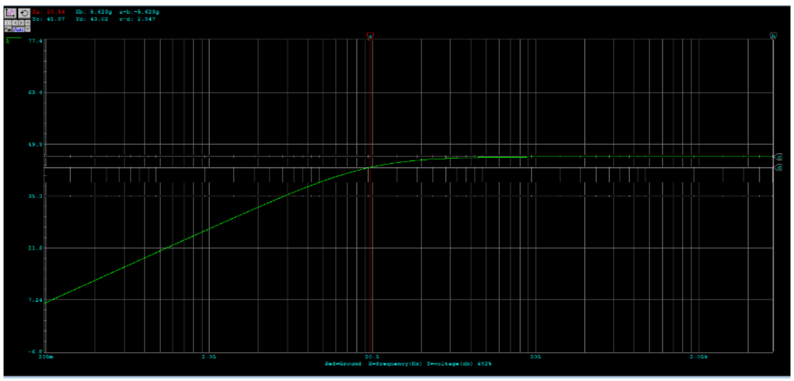
The following are graphs of the Poles of this transfer function:



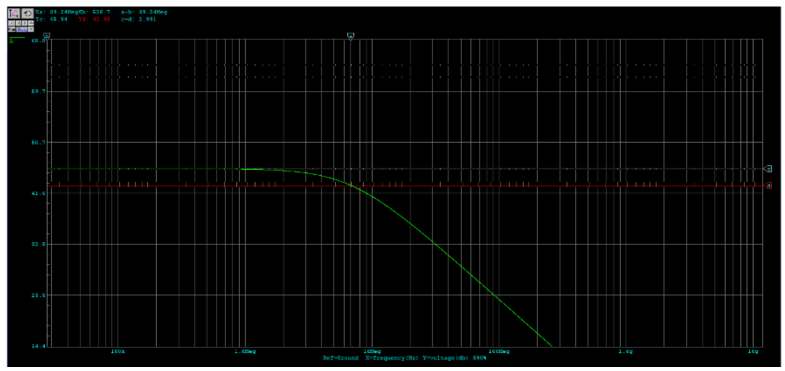
Graph 3.3: Locating the Pole Corresponding to 1 μF capacitor in the Input Stage



Graph 3.4: Locating the Pole Corresponding to 412 pF capacitor in the Input Stage



Graph 3.5: Locating the Pole Corresponding to 1 μF capacitor in the Output Stage



Graph 3.5: Locating the Pole Corresponding to 2pF capacitor in the Output Stage

Conclusion

In conclusion, by using Miller’s theorem, open-circuit and short-circuit tests, we are only approximating the behaviour of the circuit, which is why error arises between the estimated values and the actual values obtained form the graph. However, it is important to note that due to the low resolution of the software used (i.e. CircuitMaker), the error could be greater than it should be if a higher resolution software was used.

Overall, Miller’s theorem and open and short circuit methods are very useful in approximating circuits, producing small and acceptable percentage errors.