exercise 2.5
(a)

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x f(x) dx = \frac{1}{b-a}\int_a^b x * 1 dx = \frac{1}{2} * \frac{b^2 - a^2}{b-a} = \frac{a+b}{2}$$

$$\begin{aligned}
\tilde{x} &= \frac{1}{L}\sum_{l=1}^{L} x_l \\
bias(\tilde{x}) &= \mathbb{E}[\tilde{x}] - \theta \\
&= \mathbb{E}[\frac{1}{L}\sum_{l=1}^{L} x_l] - \theta \\
&= \frac{1}{L}\sum_{l=1}^{L} \mathbb{E}[x_l] - \theta \\
&= \frac{1}{L}\sum_{l=1}^{L} (\frac{0+\theta}{2}) - \theta \\
&= \frac{\theta}{2} - \theta
\end{aligned}$$

This shows that $\frac{\theta}{2} - \theta \neq 0$.

(b)

Unbiased estimator:

$$\tilde{x} = 2 * \frac{1}{L}\sum_{l=1}^{L} x_l$$

(c)

$$\begin{aligned}
SE(\tilde{x}) &= SE(\frac{2}{L}\sum_{l=1}^{L} x_l) \\
&= \frac{2}{L} SE(\sum_{l=1}^{L} x_l) \\
&= \frac{2}{L}\sqrt{\sum_{l=1}^{L} SE(x_l)^2} \\
&= \frac{2}{L}\sqrt{L\sigma^2} \\
&= \frac{2\sigma}{\sqrt{L}}
\end{aligned}$$

$$\lim_{L\to\infty} \frac{2\sigma}{\sqrt{L}} = 0$$

(d)