Convolutional neural networks

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Motivation

People are good at recognizing objects in images.

Computers are bad at it! Why?

Scene conditions

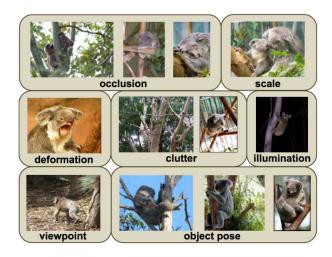


Figure 1: Difficult scene conditions: background clutter, occlusion...

Similarity and variability



Figure 2: Must identify inter-class similarity, while accommodating intra-class variability.

Dimension

- Huge number of classes
- Images can have millions of pixels

For example, CIFAR-10: tiny images of size 32x32x3. One *fully-connected* neuron in a first hidden layer of a regular NN would have 3072 weights!

Object can be anywhere within image

HANDWRITING SAMPLE FORM

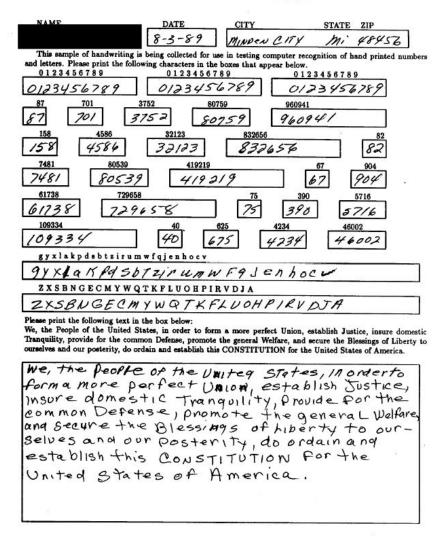


Figure 3: MNIST sample. Find the "3" in the form?

Convolutional neural networks

Key idea

- Neuron is connected to a small part of image at a time (locally connected)
- By having multiple locally connected neurons covering the entire image, we effectively "scan" the image

Locally connected neurons: illustration

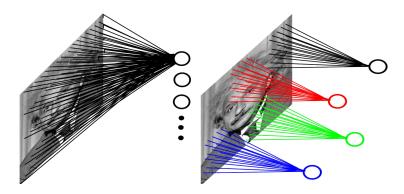


Figure 4: Example: 200x200 image. Fully connected network with 400,000 hidden units, 16 billion parameters. Locally connected network with 400,000 hidden units in 10x10 fields, 40 million parameters.

Spatial arrangement: conventional networks

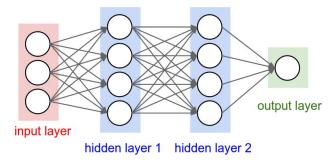


Figure 5: Conventional neural network: neurons don't have spatial arrangement.

Spatial arrangement: convolutional networks

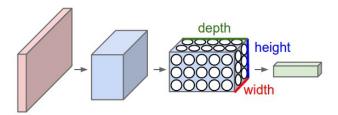


Figure 6: CNN: input and output of each layer is a *tensor*, a multidimensional array with width, height, and depth. Preserves spatial relationships.

Layers in CNN

Different layer types

- Convolutional Layer
- Pooling Layer
- Fully-Connected Layer

Each layer accepts an input 3D volume, transforms it to an output 3D volume.

Convolutional layer

- · Layer has a set of learnable "filters"
- Each filter has small width and height, but full depth
- During forward pass, filter "slides" across width and height of input, and computes dot product
- Effectively performs "convolution"

Convolution example

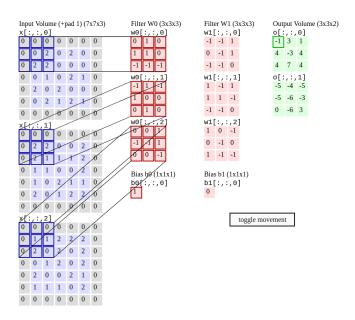


Figure 7: Animated demo at https://cs231n.github.io/assets/conv-demo/index.html

Feature localization via "convolution"

- Given large image X with dimensions $N_1\times N_2$, small filter W with dimensions $K_1\times K_2$

At each offset (j_1, j_2) we compute

$$Z[j_1,j_2] = \sum_{k_1=0}^{K_1-1} \sum_{k_2=0}^{K_2-1} W[k_1,k_2] X[j_1+k_1,j_2+k_2]$$

which is large if "matching" feature is present.

Feature localization via convolution (illustration)



Figure 8: Finding features with convolution.

Local connectivity

- Each filter is fully connected along depth axis, but only locally connected along width and height.
- Example: For CIFAR-10 (32x32x3), a 5x5 filter will have weights to a (5x5x3) region in input volume.
- Parameter dimensions: 75 weights and 1 bias. (much smaller!)

Size of output volume

Size of output volume is determined by

- Input volume size W
- depth
- ullet filter field size F
- ullet stride S
- zero padding P

Size of output volume: depth

Output depth is a hyperparameter: corresponds to number of filters that should "look" at the same region of input at a time.

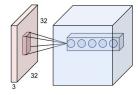


Figure 9: Example: this output depth is 5.

Size of output volume: stride

How many pixels do we slide the filter each time? This is called the stride.

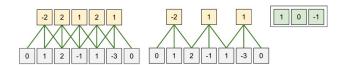


Figure 10: In this example there is one spatial dimension (x-axis), one neuron with F = 3, W = 5, and P = 1. Left: S=1. Right: S=2.

Size of output volume: zero-padding

Use zero padding on border-

- Without padding, size would shrink in each layer.
- Without padding, neurons "touch" the edges less often than the middle

To have output width and height the same as input, use $P = \frac{F-1}{2}$.

Summary of convolutional layer

- Accepts input volume $W_1 \times H_1 \times D_1$ Four hyperparameters: number of filters K, filter size F, stride S, amount of zero padding P
- · Produces volume of size

$$W_2 = \frac{W_1 - F + 2P}{S} + 1, H_2 = \frac{H_1 - F + 2P}{S} + 1$$

$$D_2 = K$$

- With parameter sharing: $F\cdot F\cdot D_1$ weights per filter, for $F\cdot F\cdot D_1\cdot K$ weights and K biases
- Common setting: F = 3, S = 1, P = 1.

Parameter sharing

Basic insight:

- · A particular filter with a set of weights represents a feature to look for
- If it is useful to look for a feature at position x, y, it is probably useful to look for the same feature
- "Depth slice" = all the shifted versions of a filter. All neurons within a depth slice can share the same weights.

Greatly reduces number of parameters.

Example: AlexNet filters

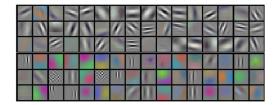


Figure 11: Each of the 96 filters shown here is of size 11x11x3, and each one is shared by the 55x55 neurons in one depth slice.

ReLU activation

· Convolutional typically followed by ReLU activation function

Pooling layer

- Reduces spatial size of image (reduce computation, prevent overfitting)
- Typical example: 2x2 filter size, stride of 2, downsamples by a factor of 2 along width and height
- · Works independently on each depth slice
- Typically uses MAX operation

Pooling: illustration

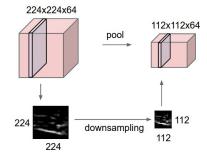


Figure 12: Input volume of size 224x224x64 is pooled with filter size 2, stride 2 into output volume of size 112x112x64 (with same depth).

Pooling: illustration of max operation

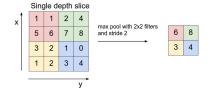


Figure 13: Each max is taken over a 2x2 square.

Summary of pooling layer

- Accepts input volume $W_1 \times H_1 \times D_1$ - Two hyperparameters: filter size F , stride S

· Produces volume of size

$$W_2 = \frac{W_1 - F}{S} + 1, H_2 = \frac{H_1 - F}{S} + 1, D_2 = D_1$$

· No parameters

Fully connected layer

Reshape into matrix

· Output with matrix multiplication

$$Z[i,k] = \sum_{j} W[j,k] U[i,j] + b[k], \quad k = 0, \ldots, N_O \label{eq:Z}$$

Typical architecture

Input

Some number of convolutional + ReLU layers

• Occasional pooling layers

Some number of fully connected + ReLU layers

Fully connected output layer

Example

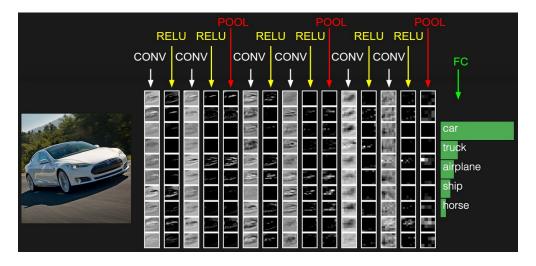


Figure 14: Example of a convolutional network architecture. Live demo link.

Reference

Source of most images here, and excellent set of notes on convolutional neural networks: https://cs231n.github.io/convolutional-networks/