Introduction to Machine Learning
Problem Set: Bias Variance Tradeoff and Model Order Selection

Summer 2021

1. (Based on a question by Prof. Sundeep Rangan) **Model class.**

   For each of the following pairs of true functions $t(x)$ and assumed model classes $f(x, w)$ determine whether the true function can be expressed by the assumed model class, i.e. is there a parameter vector w_t such that $t(x) = f(x, w_t)$?

   If yes, also give the parameter vector w_t.

   (a) $t(x) = 1 + 2x$ and $f(x, w) = w_0 + w_1 x + w_2 x^2$
   Solution: Yes, if we use this w_t: $w_0 = 1, w_1 = 2, w_2 = 0$, then $f(x, w_t) = t(x)$.

   (b) $t(x) = 1 + 2x + 5x^3$ and $f(x, w) = w_1 x + w_2 x^2 + w_3 x^3$
   Solution: No, $f(x, w) = w_1 x + w_2 x^2 + w_3 x^3$ does not have a intercept term and $t(x)$ has a intercept term. There is no w_t for which $f(x, w_t) = t(x)$.

   (c) $t(x) = x/(2 + 3x)$, $f(x, w) = (w_0 + w_1 x)/(w_3 + w_4 x)$
   Solution: Yes, if we use this w_t: $w_0 = 0, w_1 = 1, w_3 = 2, w_4 = 3$ (or any scaled version of these where all the weights are multiplied by the same constant), then $f(x, w_t) = t(x)$.

   (d) $t(x) = (x_1 - x_2)^2$ and $f(x, w) = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_1^2 + + w_4 x_2^2$
   Solution: No, $t(x) = (x_1 - x_2)^2 = x_1^2 - 2x_1 x_2 + x_2^2$ cannot be expressed by the model class. The model class doesn't have any $x_1 x_2$ term. There is no w_t for which $f(x, w_t) = t(x)$.

   (e) $t(x) = (1 - x)^3$ and $f(x, w) = w_0 + w_1 x + w_2 x^2 + w_3 x^3$
   Solution: Yes, for $t(x) = (1 - x)^3 = 1 - 3x + 3x^2 - x^3$, if we use this w_t: $w_0 = 1, w_1 = -3, w_2 = 3, w_4 = -1$, then $f(x, w_t) = t(x)$.

2. **Model order selection on neural data**.

   Please refer to the homework notebook posted on the class site.

   Solution: Refer to the solution notebook.