

Big O Notation.

$$f(n) = O(g(n)) \Rightarrow f(x) \leq c \times g(x)$$

Ex) Let $f(x) = 2x^2 + 3$, $g(x) = x^2$. Then, $f(x) = O(g(x))$?

Sol) Let $f(x) \leq c \times g(x)$, this inequality is as follows $2x^2 + 3 \leq cx^2$.

$$\text{if } c = 1 \rightarrow 2x^2 + 3 \leq x^2 \quad (\text{x})$$

$$\text{if } c = 2 \rightarrow 2x^2 + 3 \leq 2x^2 \quad (\text{x})$$

$$\text{if } c = 3 \rightarrow 2x^2 + 3 \leq 3x^2 \quad (\text{o})$$

In this case, the following inequality forms: $3 \leq x^2$

$$\text{if } x = 1 \rightarrow 3 \leq 1 \quad (\text{x})$$

$$\text{if } x = 2 \rightarrow 3 \leq 4 \quad (\text{o})$$

Therefore, $f(x) = O(g(x))$ is true when $c = 3$ and $x \geq 2$.

Big Ω Notation.

$$f(n) = \Omega(g(n)) \Rightarrow f(n) > c \times g(n) \text{ for all } n > n_0$$

Ex) Let $f(x) = 2x^2 + 3$, $g(x) = x^2$. Then, $f(x) = \Omega(g(x))$?

Sol) Let $f(x) \leq c \times g(x)$.

$$\text{if } c = 1 \rightarrow 2x^2 + 3 > x^2 \quad (\text{o})$$

Therefore, $f(x) = \Omega(g(x))$ is true.

Big Θ Notation.

$$f(n) = \Theta(g(n)) \text{ when, } f(x) = O(g(x)) \text{ and } f(x) = \Omega(g(x)).$$

Ex) Let $f(x) = 2x^2 + 3$, $g(x) = x^2$. Then, $f(x) = \Theta(g(x))$?

Sol) There are $f(x) = O(g(x))$ and $f(x) = \Omega(g(x))$.

Therefore, $f(n) = \Theta(g(n))$ is true.