Supplementary material - linear discriminant analysis

DV

Contents

In the results section of the paper, Roy-Bargmann stepdown analyses are used to dive deeper into the nature of the differences identified by the MANOVAs. As some authors (eg. Field, Miles, & Zoe, 2012; Salkind, 2007; Tabachnick & Fidell, 2012) recommend conducting a linear discriminant analysis as a follow-up to MANOVA, we are reporting these results here.

Hard exclusion criteria

As in the paper itself, the following analyses are going to be conducted on a subset of the collected data which contains 203 cases. We will conduct the analyses specified in the `analysis-plan.md` file and follow them up with linear discriminant analyses.

Interpolated activity effect

Again, we will first conduct a MANOVA with the total number of correct answers and total number of intrusive distractors chosen as dependent variables and the type of interpolated activity as the independent variable.

Note

A decision was made not to check the univariate and multivariate outliers at this point. Regarding the univariate outliers - the boxplots point to only one case which could be an outlier. The scatterplots show no point that's obviously different from the rest. As for the multivariate outliers, Tabachnick and Fidell (2012) warn that the Mahalanobis distance can produce false negatives or false positives. Furthermore, deleting a set of outliers and rerunning the analysis can reveal yet another set of outliers — without a clear-cut and absolute criterion, exclusions are somewhat arbitrary. Finally, cases were excluded based on criteria that are more or less substantively meaningful in the context of the conducted study. Given the above, no statistical criteria is used for exclusion at this point.

MANOVA

Here's the output of `R`'s `manova` function:

```
##
## Type II MANOVA Tests:
##
## Sum of squares and products for error:
##                 totalCorrect totalIntrusors
## totalCorrect             993           -417
## totalIntrusors          -417            435
##
## ----------------------------------------
##
## Term: as.factor(activityFactor)
##
## Sum of squares and products for the hypothesis:
##                 totalCorrect totalIntrusors
## totalCorrect           125.9          -70.5
## totalIntrusors         -70.5           41.0
##
## Multivariate Tests: as.factor(activityFactor)
##                   Df test stat approx F num Df den Df  Pr(>F)
## Pillai             2     0.126     3.99      4    238 0.00376 **
## Wilks              2     0.875     4.07      4    236 0.00327 **
## Hotelling-Lawley   2     0.142     4.16      4    234 0.00285 **
## Roy                2     0.137     8.13      2    119 0.00049 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

As can be seen from the resulting output, Pillai's V indicates that the three groups differ significantly along the linear combination of the two DVs. The other three reported statistics point to the same conclusion. Therefore, we'll proceed with conducting a linear discriminant analysis.

Linear discriminant analysis

We are using the `candisc` function from the eponymous package (Friendly & Fox, 2017) to conduct the LDA.

```
##
## Canonical Discriminant Analysis for as.factor(activityFactor):
##
##    CanRsq Eigenvalue Difference Percent Cumulative
## 1 0.12018     0.1366      0.131   96.13       96.1
## 2 0.00547     0.0055      0.131    3.87      100.0
##
## Test of H0: The canonical correlations in the
## current row and all that follow are zero
```

```
##
##   LR test stat approx F numDF denDF Pr(> F)
## 1        0.875     4.07     4   236  0.0033 **
## 2        0.995     0.65     1   119  0.4202
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

## Class means

##             Can1    Can2
## content    0.501 -0.0104
## general   -0.309 -0.0845
## rereading -0.217  0.0954

## Raw coefficients

##                  Can1  Can2
## totalCorrect    0.255 0.368
## totalIntrusors -0.186 0.651

## Standardized coefficients

##                  Can1 Can2
## totalCorrect    0.736 1.06
## totalIntrusors -0.355 1.24
```

From the above output we can see that the first variate explains most of the variance. Furthermore, Wilks' lambda values inform us that the groups are separated only on the first variate, so that's the only one we'll interpret. Also, we can see that the variation in the grouping variable is almost exclusively explained by the first variate.

Looking at the structure scores, we can see that both the total number of correct answers and the total number of intrusive distractors chosen share a lot of variance with the first variate. The first variate is almost completely defined by the total number of correct answers, but the contribution of the number of chosen intrusors is also considerable. This could be due to the relatively high correlation between those two variables.

To assess the ability of the LDA model to discriminate group membership based on the number of correct answers to the questions and the number of chosen intrusive distractors, we'll re-train the model and evaluate its error rate using the leave-one-out cross-validation technique:

```
## Confusion Matrix and Statistics
##
##            Reference
## Prediction  content general rereading
##   content        27      16        17
##   general        11      19        13
##   rereading       4       5        10
##
## Overall Statistics
##
```

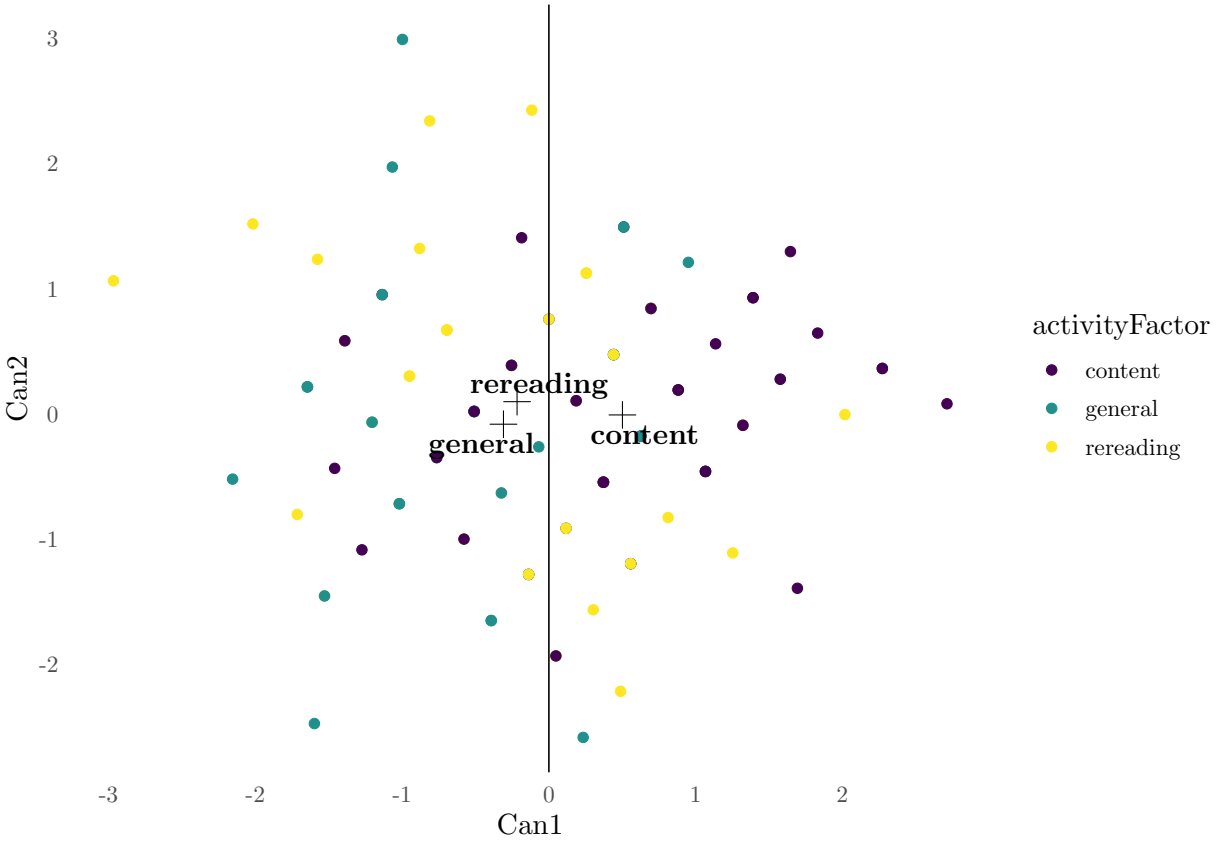

Figure 1: Plot showing the cases' location on the two variates. Group means on the variates are marked by crosses. The vertical line marks the 0 on the first variate.

```
##                Accuracy : 0.459
##                  95% CI : (0.368, 0.552)
##     No Information Rate : 0.344
##     P-Value [Acc > NIR] : 0.00572
##
##                   Kappa : 0.185
##  Mcnemar's Test P-Value : 0.00577
##
## Statistics by Class:
##
##                      Class: content Class: general Class: rereading
## Sensitivity                   0.643          0.475            0.250
## Specificity                   0.588          0.707            0.890
## Pos Pred Value                0.450          0.442            0.526
## Neg Pred Value                0.758          0.734            0.709
## Precision                     0.450          0.442            0.526
## Recall                        0.643          0.475            0.250
## F1                            0.529          0.458            0.339
## Prevalence                    0.344          0.328            0.328
## Detection Rate                0.221          0.156            0.082
## Detection Prevalence          0.492          0.352            0.156
```

```
## Balanced Accuracy             0.615          0.591            0.570
```

As can be seen from the table, the total LOOCV accuracy is 0.459, which is significantly above the no information rate (which is taken to be the largest class percentage in the data). According to the Landis & Koch (1977; as reported in Salkind, 2007) guidelines, this represents only a slight agreement between the predicted and actual classes. Next, we'll drill into the individual predictors to see which are useful for discriminating between different groups.

Evaluating individual predictors

Tabachnick and Fidell (2012) describe the process of sequential discriminant analysis, where predictors are entered one-by-one, and the improvement in classification accuracy is monitored. Therefore, we'll fit an LDA model containing only the number of correct answers as a predictor. Then, we will compare this model's LOOCV accuracy to that of the full model (reported at the end of the previous section). Here are the results for the total-correct-only model:

```
## Confusion Matrix and Statistics
##
##            Reference
## Prediction  content general rereading
##   content        29      15        16
##   general        11      22        18
##   rereading       2       3         6
##
## Overall Statistics
##
##                Accuracy : 0.467
##                  95% CI : (0.376, 0.56)
##     No Information Rate : 0.344
##     P-Value [Acc > NIR] : 0.00334
##
##                   Kappa : 0.198
##  Mcnemar's Test P-Value : 5.87e-05
##
## Statistics by Class:
##
##                      Class: content Class: general Class: rereading
## Sensitivity                   0.690          0.550           0.1500
## Specificity                   0.613          0.646           0.9390
## Pos Pred Value                0.483          0.431           0.5455
## Neg Pred Value                0.790          0.746           0.6937
## Precision                     0.483          0.431           0.5455
## Recall                        0.690          0.550           0.1500
## F1                            0.569          0.484           0.2353
## Prevalence                    0.344          0.328           0.3279
```

```
## Detection Rate               0.238          0.180          0.0492
## Detection Prevalence         0.492          0.418          0.0902
## Balanced Accuracy            0.651          0.598          0.5445
```

As can be seen from the second confusion matrix, the accuracy of this model is actually somewhat higher than in the full model, as is Cohen's κ. Importantly, we notice that adding the total number of intrusors to the model doesn't significantly increase the accuracy of the model (the 95% confidence intervals for the accuracies of the two models completely overlap).

Multivariate contrasts

We've planned to contrast the two test groups with the rereading group, and the two test groups with each other. That's what we'll do here.

```
##           test vs rereading content vs general
## content                    1                  1
## general                    1                 -1
## rereading                 -2                  0
```

```
##                                  totalCorrect totalIntrusors
## (Intercept)                            11.379          4.194
## activityFactortest vs rereading         0.252         -0.216
## activityFactorcontent vs general        1.155         -0.597
```

Now that we've set up the model, let's run the contrasts. The first contrast is between the two test groups (content and general knowledge) and the rereading group.

```
##
## Sum of squares and products for the hypothesis:
##                totalCorrect totalIntrusors
## totalCorrect           15.3          -13.1
## totalIntrusors        -13.1           11.3
##
## Sum of squares and products for error:
##                totalCorrect totalIntrusors
## totalCorrect            993           -417
## totalIntrusors         -417            435
##
## Multivariate Tests:
##                   Df test stat approx F num Df den Df Pr(>F)
## Pillai             1     0.026     1.57      2    118   0.21
## Wilks              1     0.974     1.57      2    118   0.21
## Hotelling-Lawley   1     0.027     1.57      2    118   0.21
## Roy                1     0.027     1.57      2    118   0.21
```

As can be seen from the test statistics, no significant difference is found between the two test groups and the rereading group. Next, we'll look at the contrast between the content test group and the general knowledge test group.

```
##
## Sum of squares and products for the hypothesis:
##                totalCorrect totalIntrusors
## totalCorrect          109.4          -56.5
## totalIntrusors        -56.5           29.2
##
## Sum of squares and products for error:
##                totalCorrect totalIntrusors
## totalCorrect            993           -417
## totalIntrusors         -417            435
##
## Multivariate Tests:
##                   Df test stat approx F num Df den Df Pr(>F)
## Pillai             1     0.102     6.73      2    118 0.0017 **
## Wilks              1     0.898     6.73      2    118 0.0017 **
## Hotelling-Lawley   1     0.114     6.73      2    118 0.0017 **
## Roy                1     0.114     6.73      2    118 0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

This contrast is statistically significant, indicating that the two groups differ on the linear combination of the number of correct answers and number of intrusive distractors chosen. We'll calculate the same effect size indices as for the omnibus model.

The multivariate η^2 is 0.102. The effect size index ξ^2 is 0.051. Finally, we will calculate Tatsuoka's (1970; according to Huberty & Olejnik, 2006) extension of the ω^2 to the multivariate case. In this case, $\omega^2_{mult} = 0.087$. The adjusted value of the ξ^2 statistic is $\xi^2_{adj} = 0.035$

Contrast LDA

Again, to further investigate the nature of the difference between the content and general knowledge test group, we'll conduct a linear discriminant analysis to try and find the variate that best discriminates these two groups. Here's the MANOVA model:

```
##
## Type II MANOVA Tests: Pillai test statistic
##                Df test stat approx F num Df den Df Pr(>F)
## activityFactor  1     0.148     6.85      2     79 0.0018 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

And the LDA:

```
##
## Canonical Discriminant Analysis for activityFactor:
##
##   CanRsq Eigenvalue Difference Percent Cumulative
```

```
## 1 0.1479     0.1735                100        100
##
## Class means:
##
## [1]  0.4015 -0.4216
##
##  std coefficients:
##   totalCorrect totalIntrusors
##         0.7210        -0.3722
```

```
##                   Can1
## totalCorrect    0.964
## totalIntrusors -0.851
```

Again, we see that both predictors are highly correlated with the discriminant function, albeit with different signs. Let's look at the LOOCV prediction accuracy:

```
## Confusion Matrix and Statistics
##
##           Reference
## Prediction content general
##    content      28      16
##    general      14      24
##
##                Accuracy : 0.634
##                  95% CI : (0.52, 0.738)
##     No Information Rate : 0.512
##     P-Value [Acc > NIR] : 0.0175
##
##                   Kappa : 0.267
##  Mcnemar's Test P-Value : 0.8551
##
##             Sensitivity : 0.667
##             Specificity : 0.600
##          Pos Pred Value : 0.636
##          Neg Pred Value : 0.632
##              Prevalence : 0.512
##          Detection Rate : 0.341
##    Detection Prevalence : 0.537
##       Balanced Accuracy : 0.633
##
##        'Positive' Class : content
##
```

As can be seen from the above output, with both predictors, the prediction accuracy is significantly above the no information rate. Here is the model with the total number of intrusive distractors dropped:

```
## Confusion Matrix and Statistics
##
##           Reference
## Prediction content general
##    content      29      15
##    general      13      25
##
##                Accuracy : 0.659
##                  95% CI : (0.546, 0.76)
##     No Information Rate : 0.512
##     P-Value [Acc > NIR] : 0.00523
##
##                   Kappa : 0.316
##  Mcnemar's Test P-Value : 0.85011
##
##             Sensitivity : 0.690
##             Specificity : 0.625
##          Pos Pred Value : 0.659
##          Neg Pred Value : 0.658
##              Prevalence : 0.512
##          Detection Rate : 0.354
##    Detection Prevalence : 0.537
##       Balanced Accuracy : 0.658
##
##        'Positive' Class : content
##
```

As can be seen, the prediction accuracy doesn't drop significantly when we omit the total number of intrusors.

The interaction between feedback and interpolated activity type

Again, we'll first fit the MANOVA model:

```
##
## Type III MANOVA Tests: Pillai test statistic
##                             Df test stat approx F num Df den Df Pr(>F)
## (Intercept)                  1     0.940     1229      2    158 <2e-16 ***
## activityFactor               1     0.071        6      2    158  0.003 **
## giveFeedback                 1     0.003        0      2    158  0.800
## activityFactor:giveFeedback  1     0.001        0      2    158  0.941
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

As was already established in the paper, we find only an effect of the type of interpolated activity. Let's fit the LDA model for this effect:

```
##
## Canonical Discriminant Analysis for activityFactor:
##
##   CanRsq Eigenvalue Difference Percent Cumulative
## 1  0.071     0.0765                100        100
##
## Test of H0: The canonical correlations in the
## current row and all that follow are zero
##
##   LR test stat approx F numDF denDF Pr(> F)
## 1        0.929              2

## Class means

## [1]  0.365 -0.379

## Raw coefficients

##                   Can1
## totalCorrect    0.222
## totalIntrusors -0.216

## Standardized coefficients

##                   Can1
## totalCorrect    0.693
## totalIntrusors -0.388
```

Since there are only two groups in this analysis, the LDA results aren't particularly more informative than the MANOVA output. It is interesting to notice, however, that the standardized structure coefficient for the total number of correct answers is quite larger than the coefficient for the total number of intrusors. Let's take a look at the LOOCV prediction accuracy fort this model:

```
## Confusion Matrix and Statistics
##
##           Reference
## Prediction content general
##    content      57      33
##    general      26      47
##
##                Accuracy : 0.638
##                  95% CI : (0.559, 0.712)
##     No Information Rate : 0.509
##     P-Value [Acc > NIR] : 0.000614
##
##                   Kappa : 0.275
##  Mcnemar's Test P-Value : 0.434724
##
```

```
##             Sensitivity : 0.687
##             Specificity : 0.588
##          Pos Pred Value : 0.633
##          Neg Pred Value : 0.644
##               Precision : 0.633
##                  Recall : 0.687
##                      F1 : 0.659
##              Prevalence : 0.509
##          Detection Rate : 0.350
##    Detection Prevalence : 0.552
##       Balanced Accuracy : 0.637
##
##        'Positive' Class : content
##
```

Again, we find that the prediction accuracy is somewhat above the no information rate. Let's try to tease out which of the two predictors is more important for predicting group membership. To do that, we'll fit a model with only the total number of correct answers as the predictor:

```
## Confusion Matrix and Statistics
##
##           Reference
## Prediction content general
##    content      60      33
##    general      23      47
##
##                Accuracy : 0.656
##                  95% CI : (0.578, 0.729)
##     No Information Rate : 0.509
##     P-Value [Acc > NIR] : 0.000103
##
##                   Kappa : 0.311
##  Mcnemar's Test P-Value : 0.229102
##
##             Sensitivity : 0.723
##             Specificity : 0.588
##          Pos Pred Value : 0.645
##          Neg Pred Value : 0.671
##               Precision : 0.645
##                  Recall : 0.723
##                      F1 : 0.682
##              Prevalence : 0.509
##          Detection Rate : 0.368
##    Detection Prevalence : 0.571
##       Balanced Accuracy : 0.655
```

```
##
##        'Positive' Class : content
##
```

As was the case in the previous section, we find that the prediction accuracy after excluding the total number of chosen intrusors is virtually unchanged. Given all the results above, we may presume that, in our study, different types of interpolated activities caused differences in the total number of correct answers, but we do not find evidence of an effect on the total number of intrusive distractors chosen.

Notes

LDA cross-validation done using `MASS` (Venables & Ripley, 2002). `viridis` (Garnier, 2018) used for color scale. Labelling in plot done with the help of `ggrepel` (Slowikowski, 2018).

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