Relative Direction Change A Topology-based Metric for Layout Stability in Treemaps

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Abstract: This paper presents a topology-based metric for layout stability in treemaps—the Relative Direction Change

(RDC). The presented metric considers the adjacency and arrangement of single shapes in a treemap, and allows for a rotation-invariant description of layout changes between two snapshots of a dataset depicted with treemaps. A user study was conducted that shows the applicability of the Relative Direction Change in comparison and addition to established layout metrics, such as Average Distance Change (ADC) and Average Aspect Ratio (AAR), with respect to human perception of treemaps. This work contributes to the establishment

of a more precise model for the replicable and reliable comparison of treemap layout algorithms.

1 Introduction

Treemaps represent hierarchical data by means of space-constrained, recursively nested sets of convex polygons that express hierarchy nodes. Their sizes are proportional to per-node weights (Johnson and Shneiderman, 1991). Data associated with nodes, the attributes, can be mapped by the visual variables (Bertin, 1983; Carpendale, 2003) of treemaps such as polygon size, color, texture, and shading. Variants of treemaps are applied in a large number of applications and systems to interactively display, explore and analyze multivariate, hierarchical data of, e.g., file systems (Shneiderman, 1992), software systems (Wettel and Lanza, 2008), business data (Vliegen et al., 2006), or stock markets (Wattenberg, 1999). Treemap implementations can be mainly distinguished according to the underlying layout algorithm they apply (Schulz, 2011). Various approaches have been developed over the last decades, e.g., layout algorithms that optimize the aspect ratio of the visual representations, preserve a specific order of the data items and depict this order in the visual counterparts or offer non-rectangular shapes such as polygons. In addition to those properties, the stability of the layout represents a key quality of a treemap implementation. A layout is called stable if small changes of the data only cause small changes of the arrangement and positions of the visual item representations. If treemaps should be used in a consistent and continuous way in applications (e.g., as a visual data interface), layout stability becomes a key

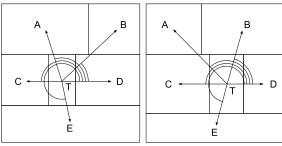


Figure 1: Relative Direction Change presents a topology-based metric for layout stability that takes into account the adjacencies of treemap items.

requirement because an instable layout would distort the users mental map. For detailed definitions of the term *layout stability* we refer to the literature (Table 1). Measuring the stability of a treemap layout algorithm is done either by an image-based comparison of the resulting depictions, or by computing layout metrics that focus on specific properties of the individual treemap items such as position changes.

In this paper, we propose a layout metric that focuses on the topology of treemaps and their items adjacency — the *Relative Direction Change (RDC)* (Figure 1). It extends the idea of the *Average Angular Displacement (AAD)* (Wood and Dykes, 2008) by adding an invariance against rotations. We present results from a user study showing the usefulness of the proposed metric in comparison to well known metrics such as the *Average Distance Change (ADC)*.

Table 1: Definitions of *layout stability*, publishing year, and used metric(s) for the evaluation of the layout algorithm. Underlined metrics were first introduced in the respective publication.

Quote	Evaluation Metric(s)	Ref.
While these algorithms do improve visibility of small items in a single layout, they introduce instability over time in the display of dynamically changing data, fail to preserve order of the underlying data, and create layouts that are difficult to visually search.	Average Distance Change, Readability	(Bederson et al., 2002)
Stability with regard to changing leaf values, stability with regard to changing tree structure, and preservation of ordering information. [] ensure that small changes in the underlying data will lead to small changes in the corresponding layout.	Mathematical Proof	(Wattenberg, 2005)
The stability of the layout is mainly reflected by Average Distance Change, for which the spiral layout is better than the strip layout in most cases, except for the case where the aspect ratio is large.	Average Distance Change, Readability, Continuity	(Tu and Shen, 2007)
We demonstrate that stability is not fully captured by the commonly used "distance change" metric. To address this shortcoming we introduce a new "location drift" metric that better encapsulates stability.	Average Distance Change, Location Drift, Readability, Continuity	(Tak and Cockburn, 2013)
In other words, we refer to such a layout algorithm's "toler- ance" against changes in varying input hierarchy-data with respect to the arrangement and layout of resulting visual representations as layout stability.	Image-based Comparison	(Hahn et al., 2014)

2 Related Work

Treemap Layout Algorithms are published for more than two decades (Schulz, 2011). The initial Slice and Dice treemap (1991) used a linear subdivision of a rectangle in alternating, horizontal and vertical, directions based on the tree depth of an hierarchy item (Johnson and Shneiderman, 1991). This approach, especially if used for sub-hierarchies with a large number of items, results in shapes with high aspect ratios and, therefore, poor readability. Bruls et al. put a high focus on readability with Squarified treemaps (2000), using a treemap algorithm that creates square-like shapes and, hence, it allows for Average Aspect Ratios near one, but as a trade-off shows poor layout stability (Bruls et al., 1999). The trade-off between nicely-shaped regions and layout stability was first mentioned by Bederson et al., introducing the Strip treemap (2002) and a first evaluation that takes into account the change of positions for varying hierarchical data sets (Bederson et al., 2002). Tu and Shen tried to overcome the challenge of layout instability by using a spiral-shaped space-filling curve, Spiral treemap (2007) (Tu and Shen, 2007), that also allows for preserving a specific order of data in the depiction. Tak and Cockburn (Tak and Cockburn, 2013) also use a space-filling curve to compute the initial item positions; their Hilbert & Moore treemaps (2013) create low mean aspect ratio and high stability. They also introduced a new layout metric, the location drift, which overcomes some

of the disadvantages of the distance change metric. Nevertheless, the evaluation of this algorithm against other common ones did not consider hierarchical data sets. In addition to the common rectangular treemap approaches, Balzer and Deussen present generalized Voronoi- (or Power-)diagrams to create *Voronoi treemaps* (2005), using random initial positions for items (Balzer and Deussen, 2005). The algorithm was extended by Hahn et al. to allow for stable distributions, resulting in treemaps that create items with low Average Aspect Ratios and a high visual stability (Hahn et al., 2014). Although they show an image-based comparison, an actual metric-based evaluation of the stability is missing.

The Perception of Treemaps with respect to layout stability is highly connected to the research in mental maps. Misue et al. define the mental map for graphs with a model consisting of three different aspects: orthogonal ordering, proximity relations, and topology (Misue et al., 1995). Their definition of topology focuses on the connections between graph nodes is not directly applicable to implicit hierarchical visualization techniques like treemaps. Nevertheless, the orthogonal ordering and proximity relations propose a direction on how to evaluate the changes in a layout with respect to a user's mental map. Wood and Dykes seize the idea of topology preservation within the abstract depiction of geo-related data by a treemap algorithm (Wood and Dykes, 2008). The ability to preserve the topology of the depicted items

in their geo-space and treemap-space was evaluated by using the Average Angular Displacement metric (Ghoniem et al., 2015). Another common metric for evaluating treemap layout stability is the Average Distance Change introduced by Bederson et al. (Bederson et al., 2002), which only takes into account the change in the Euclidean distance of the absolute position and aspect ratio of depicted items. Several evaluations were performed showing that their respective layout algorithm performs best with respect to layout stability. However, either they introduced algorithm specific metrics or used artificial or non-hierarchical data sets (Bederson et al., 2002; Tu and Shen, 2007; Tak and Cockburn, 2013). Kong et al. evaluate as a prerequisite for a good area estimation in treemaps, the rule of nicely-shaped regions and item orientations (Kong et al., 2010) In a controlled experiment they found, that users can hardly estimate high aspect ratios especially with different orientations, but did not focus on the stability of different treemap algorithms.

3 Relative Direction Change

The mapping stage of the visualization pipeline generates the visual representations of the data items. For each data item a shape (rectangle or convex polygon) is created and positioned with respect to the hierarchy position in the data set. Typically, an attribute of the data item is mapped to the ground area of the visual artifact that represents the proportion with respect to its siblings. In a treemap this step is handled by the layout algorithm. By this, each visual representation of a node has some properties defining the topography of the whole treemap. Those properties are the position of the artifact inside the representation of the root node, the width and height, and consequently the aspect ratio. We refer to metrics that use those properties as intra-node metrics because measuring the change of them would only include each node itself. Since the human perception of maps — and the cognition of mental maps is not only based on the recognition of single item shapes (Misue et al., 1995; Kaas, 1997), but also on the arrangement of sub-structures, the Relative Direction Change is introduced. This inter-node metric also takes into account the adjacency of nodes from each sub-tree. The concept of orthogonal ordering as described by Misue et al. (Misue et al., 1995) and the Average Angular Displacement (Wood and Dykes, 2008) serve as a basis for this metric. Each visual element that occurs in both treemaps has a position p in the first one (p' for the second) defined

by its center (or centroid for polygonal shapes) with cartesian coordinates x_p and y_p (x'_p and y'_p). Here, directions are expressed as angles, computed by the arctangens (atan2(Δy , Δx)) of the differences between the centers (Equation 1). Computing the direction of each items' center towards the other items' center results in a direction matrix M (Equation 2). This results in two direction matrices (M and M'), one for each hierarchical depiction. The absolute difference between two of these matrices shows the absolute change in the treemap. The change between two corresponding elements of these matrices is written as $\Delta_{i,j}$ for brevity (Equation 3. Further the change between two angles is normalized into the range of $(-\pi, \pi]$ (Equation 4). Computing the average change of a row, results in the average change of one item with respect to all other items (Equation 5). The average of all rows finally results in the Relative Direction Change of the two depictions (Equation 6). To make the Relative Direction Change rotation-invariant, the average of all values in a row is subtracted from each single value of the row (Equation 7). With this approach a metric is shown that allows for the computation of similarity between two treemaps based on the actual topography (Figure 2). To ensure comprehensive computation of the RDC metric for non-rectangular shapes the center of a treemap item is defined by the shapes centroid.

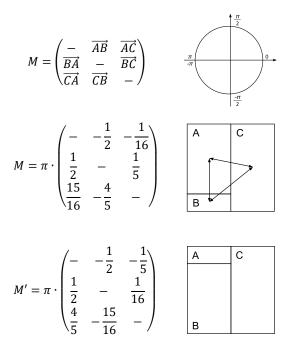


Figure 2: Exemplary computation of the *Relative Direction Change* (RDC). The resulting AAD is 0.318 and the rotation-invariant RDC is 0.159.

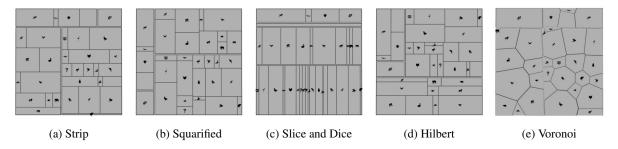


Figure 3: Depictions of the first snapshot of the dataset that was used for the controlled study by each layout algorithm.

$$d_{p,p'} = \operatorname{atan2}(y_{p'} - y_p, x_{p'} - x_p)$$
 (1)

$$M_{i,j} = \begin{pmatrix} d_{1,1} & d_{1,2} & \cdots & d_{1,n} \\ d_{2,1} & d_{2,2} & \cdots & d_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n,1} & d_{n,2} & \cdots & d_{n,n} \end{pmatrix}$$
(2)

$$\Delta_{i,j} = M_{i,j} - M'_{i,j} \tag{3}$$

$$\|\alpha\| = \begin{cases} \alpha - 2\pi, & \text{if } \alpha > \pi \\ \alpha + 2\pi, & \text{if } \alpha \leq -\pi \\ \alpha, & \text{otherwise} \end{cases}$$
 (4)

$$AVG_i = \frac{1}{n-1} \sum_{\substack{j=1 \ i \neq j}}^{n} (\|\Delta_{i,j}\|)$$
 (5)

$$RDC = \frac{1}{n} \sum_{i=1}^{n} |AVG_i|$$
 (6)

$$RDC_{RI} = \frac{1}{n^2 - n} \sum_{i=1}^{n} \sum_{\substack{j=1 \ i \neq j}}^{n} |(\|\Delta_{i,j}\| - AVG_i)| (7)$$

4 Evaluation

The main goal is to investigate mathematical relations between layout metrics and completion times for item-recovering tasks in different treemap layout algorithms and a hierarchical dataset that changes over time. Based on the independent variables Algorithm and Year, the task completion time was measured for both, gaze fixations and mouse interaction. The completion time, measured in two ways (time until click at certain element, time until first fixation of an element), serves as a result for the prediction of different models. The relation to different layout metrics and their interaction was tested by using (multiple) linear models. Since Relative Direction Change is designed to also take into account the topology of treemaps we expect an improvement of the predictions when Relative Direction Change is used in addition to the other layout metrics.

4.1 Data Set

The dataset that was used to create the treemap layouts needed to fulfill a set of requirements. First, it should be a small-sized non-artifical dataset, that could be understood easily, to achieve small tasks completion times. Second, main operations for hierarchical datasets that change over time should be included such as changing attributes associated to the hierarchy data items, as well as adding or removing such items from one snapshot to another. For the attribute changes any values are acceptable, since they are mapped to the input weights for the treemap algorithms. Last, the dataset should also remain a stable basis, meaning the amount of changes should not be too large. This requirement is motivated by the main goal of creating layout stable treemap algorithms to achieve high spatial coherence in the resulting images while small changes in the underlying data occur. A suitable dataset was found in the annual (each year) population measure of the Munich Zoo¹. The population size of animals from different species were extracted for seven consecutive years from 2008 to 2014 from a public business report. The hierarchical structure of the data was given by the taxonomy of the living animals. The results were aggregated and summed up to the second hierarchy level (order), resulting in 31 to 34 order elements belonging to four different classes.

4.2 Participants

We conducted an empirical study with 24 volunteer participants (3 female) recruited from the local university campus. The age of the participants ranged from 19 to 37 (mean = 24.2, SD = 4.1). 20 participants stated, they were familiar with the concept of a treemap, while four were not. Nine participants agreed or strongly agreed in being an expert in computer graphics and visualization (rating a 4 or 5 respectively on a 1 to 5 Likert scale).

¹ http://www.hellabrunn.de/en/



Figure 4: An example sequence of images shown between the first picture and the second picture of a block in the user study. Between each treemap image a cross was displayed that had to be fixated for at least 500 ms.

4.3 Apparatus

The study was conducted with eye tracking technology to support a more precise measurement of the dis- or recovering task using an eye tracking system named EyeFollower from Interactive Minds². It allows for accurate (< 0.4 °deviation) gaze-tracking within natural head movements at a desktop environment. The participants sat in front of a 24 inch display with a FullHD resolution (1920 \times 1080 p). Additionally, an observation display was placed behind a wall, hidden from the view of the participant. This allowed the observer to check for losses of head-tracking during the experiment. The proprietary software NYAN -Architect Edition², was used for calibration, displaying fixation crosses, presentation of stimuli and the recording of data. The gaze samples were recorded with 120 Hz. After the study was completely conducted, the raw gaze data and mouse event data was exported and analyzed separately.

4.4 Procedure

The display was set up on top of the eye tracker. Participants were instructed to take a comfortable seat in a stable chair in front of the eye tracker setup. If, during the calibration process, they had taken a seat outside of the tracking range of the eye-tracker, they were instructed to adjust their position accordingly. Prior to participating in the study, they sign a consent form and complete a questionnaire soliciting demographic data. The tasks (dis- or recovering a certain item) were received in written form. A mid-sized item that appeared in each year of the dataset was randomly picked for the discovering tasks (the item size was not leveled within the study). Within the study a block of depictions for each layout algorithm was shown to the participants in its actual order (2008, 2009, 2010, etc.). Due to this, participants had discover the item in the first depiction of a block, but should only recover it in the following ones. The task sheet specified to solve the task as fast as possible, while avoiding errors. Each participant started the experiment with an initial training session, with datasets that were modified manually to avoid possible learning effect in the following study. First, the eye-tracker was calibrated, then three example blocks of this modified data were shown. The goal of the training session was to bring participants up to speed and make sure the task was well understood. Finally, the study continued with five blocks (one for each algorithm) that took approximately 10 minutes for each participant.

4.5 Design

The experiment was a 5×7 within-subjects design. There were two independent variables:

- Algorithm (Squarified, Slice and Dice, Strip, Hilbert, Voronoi)
- Year (2008, 2009, 2010, 2011, 2012, 2013, 2014)

The conditions of algorithms, resulted in five different treemap depictions (Figure 3) with seven pictures, one for each year (3). To measure the learning effect for a single algorithm, participants would always see the seven pictures of one algorithm, separated by short breaks only. Every time the participant clicked on the target of one picture, the treemap was hidden, and a fixation cross was displayed instead that had to be fixated for at least 500 ms before continuing with the next picture (Figure 4). After completing a block a short break (at least 10 seconds) was taken before the participants were allowed to continue with the next algorithm. The order of the algorithm conditions was fully randomized for each participant to prevent order and learning effects between the different algorithms. Aside from training, the amount of observations was 24 participants \times 5 algorithms \times 7 years. This made a total of 840 observations, 35 per subject. Taken into account, that only 720 trials (24 \times 5 \times 6 years) are used as basis for the measurement of the recovering task.

²http://www.interactive-minds.com

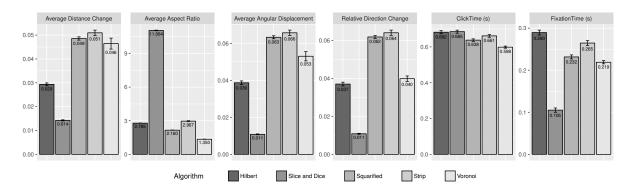


Figure 5: Layout stability metrics and measurements from the user study per layout algorithm.

5 Results

During the experiment the dependent variables *click-Time* and *fixationTime* were measured within the recovering task. The results for these metrics for the different treemap layout algorithms are presented first. These are followed by the results of the layout metrics and the results correlation of user perception from the experiment with the layout metrics.

5.1 ClickTime & FixationTime

The experiment resulted in complete 719 observations (only 1 had to be removed due to imcompleteness). A Shapiro-Wilk test showed that the distributions of both, the clickTime and fixationTime within the different algorithm groups are not appearing to come from a normal distribution (*clickTime*: all *p-Values* < .001; fixationTime: all p-Values < .001). Additionally, a Levene test shows high significance for heteroscedasticity between the groups for both variables (click-Time: p < .002; fixationTime: p < .01). A statistically significant difference (clickTime: H = 14.201, p < .01; fixationTime: H = 180.76, p < .001) was found using a Kruskal-Wallis test. In a post-hoc pairwise comparison of the measurements for clickTime only three groups showed significant differences using a Wilcoxon test (comparison between Voronoi -Hilbert; Voronoi - Slice and Dice and Voronoi -Strip). However, the comparison of *fixationTime* measurements showed significant differences between all groups, but Strip - Squarified (p = 0.36). Table 2 shows a complete overview of the pairwise comparison between the groups. In addition to measuring the completion times representing the users perception we computed the layout metrics for the Average Distance Change (ADC), the Relative Direction Change (RDC) and the Average Aspect Ratio (AAR) for each pair of tested years of the dataset (see Figure 5 for the complete data). The measurements from the

Table 2: p values for post hoc comparison of groups.

Pair	clickTime	fixationTime
Hilbert - Slice and Dice	.860	< .001 **
Hilbert - Squarified	.138	< .001 **
Hilbert - Strip	.557	.006 **
Hilbert - Voronoi	.002 **	< .001 **
Slice and Dice - Squarified	.102	< .001 **
Slice and Dice - Strip	.469	< .001 **
Slice and Dice - Voronoi	.001 **	< .001 **
Squarified - Strip	.361	.356
Squarified - Voronoi	.114	.034 **
Strip - Voronoi	.010 **	< .001 **

trials were used to create a simple linear regression model for the users' perception based on the layout metrics. Finally, different multiple linear regression models were calculated to predict the dependent variables *clickTime* and *fixationTime* based on the three different layout metrics. For both dependent variables a multiple linear regression model was calculated that included either the *Average Distance Change* together with the *Average Aspect Ratio* or *Relative Direction Change* together with the *Average Aspect Ratio*. Additionally, a multiple linear regression model was calculated that included all three layout metrics.

5.2 Models for *ClickTime*

ADC + **AAR**: A multiple linear regression was calculated to predict *clickTime* based on *ADC* and *AAR*. A significant regression equation was found ($F_{2,716} = 30.68, p < .001$), with a R^2 of .07893. Predicted *clickTime* is equal to $0.54788 + 1.45747 \times ADC + 0.01225 \times AAR$.

AAD + **AAR**: A second multiple linear regression was calculated to predict *clickTime* based on *AAD* and *AAR*. A significant regression equation was found ($F_{2,716} = 36.34, p < .001$), with a R^2 of .09215. Predicted *clickTime* is equal to $0.533360 + 1.363092 \times AAD + 0.013902 \times AAR$.

RDC + **AAR**: Another multiple linear regression was calculated to predict *clickTime* based on *RDC* and *AAR*. A significant regression equation was found ($F_{2,716} = 47.06, p < .001$), with a R^2 of .1162. Predicted *clickTime* is equal to $0.505296 + 1.995893 \times RDC + 0.015369 \times AAR$.

ADC + **RDC** + **AAR:** A last multiple linear regression was calculated to predict *clickTime* based on *ADC*, *RDC* and *AAR*. A significant regression equation was found ($F_{3,715} = 33.38, p < .001$), with a R^2 of .1229. Predicted *clickTime* is equal to $0.498811 - 1.118791 \times ADC + 3.072374 \times RDC + 0.016096 \times AAR$.

5.3 Models for *FixationTime*

ADC + **AAR**: A multiple linear regression was calculated to predict *fixationTime* based on *ADC* and *AAR*. A significant regression equation was found ($F_{2,716} = 51.36, p < .001$), with a R^2 of .1255. Predicted *fixationTime* is equal to $0.232953 + 0.813848 \times ADC - 0.010284 \times AAR$.

AAD + **AAR**: A second multiple linear regression was calculated to predict *fixationTime* based on *AAD* and *AAR*. A significant regression equation was found ($F_{2,716} = 59.23, p < .001$), with a R^2 of .142. Predicted *fixationTime* is equal to $0.219946 + 0.838116 \times AAD - 0.009035 \times AAR$.

RDC + **AAR**: Another multiple linear regression was calculated to predict *fixationTime* based on *RDC* and *AAR*. A significant regression equation was found ($F_{2,716} = 73.49, p < .001$), with a R^2 of .1703. Predicted *fixationTime* is equal to $0.196907 + 1.327742 \times RDC - 0.007766 \times AAR$.

ADC + **RDC** + **AAR:** A last multiple linear regression was calculated to predict *fixationTime* based on *ADC*, *RDC* and *AAR*. A significant regression equation was found ($F_{3,715} = 57.42, p < .001$), with a R^2 of .1941. Predicted *fixationTime* is equal to $0.187921 - 1.550314 \times ADC + 2.819427 \times RDC - 0.006758 \times AAR$.

Within all regressions all layout metrics were found significant predictors for both, *clickTime* as well as *fixationTime* (see Figure 6 for an overview).

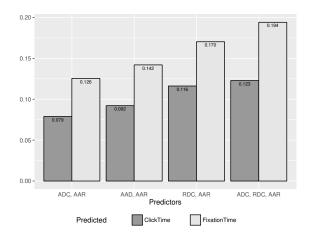


Figure 6: R^2 values for all evaluated models.

6 Conclusion

The presented Relative Direction Change metric is a layout metric that similar to the Average Angular Displacement metric considers the adjacency and arrangement of shapes in a treemap, but also allows for a rotation-invariant consideration of layout changes. Unlike most previously published metrics it focuses not just on the changes of each individual visual artifact but also on the treemap topology.

6.1 Discussion

The evaluation and analysis show comparable results (described in Section 5.1) for the layout metrics Average Aspect Ratio and Average Distance Change within the implemented layout algorithms compared to previously published ones, such as in (Bederson et al., 2002; Tak and Cockburn, 2013). Therefore we assume the correctness of the implementation of the used layouting algorithms. In addition, as the results from Section 5.2 and 5.3 show, Relative Direction Change seems to be a promising candidate as a layout stability metric for treemaps. Since neither Average Distance Change nor Relative Direction Change alone allow for a good reflection of the users recovering tasks — especially for the click time — the use of other layout metrics such as the Average Aspect Ratio is an increasing factor in explaining the variances of the users recovering tasks time. The used regression models are simple, but show three important things:

- the prediction of variance (R² value) for the recovering tasks times increase while using RDC instead of AAD,
- the interaction of *RDC* together with *AAR* increases the *R*² value for both, the *clickTime* and *fixationTime*, compared to the use of *ADC* and *AAR* and
- the interaction of *ADC*, *RDC* and *AAR* additionally increases the *R*² value for both, the *clickTime* and *fixationTime*, compared to the two predictor models.

Nevertheless, explaining a complex process, e.g., the used recovering task, with such a simple model seems to be insufficient and shows the need for a more complex model.

6.2 Future work

The presented experiment gives first hints in finding a model for the prescription of a human perception of treemap layout stability based on layout metrics. However, more measurement with real life datasets from different domains needs to be done to expand the model database and find correlations between suggested measurements. Also, a deeper look in finding a more complex model needs to be done to increase the \mathbb{R}^2 value. Finally, it is possible to implement more algorithms (even non-treemap layouts) and run trials with their resulting depictions.

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