

Modelltheorie II

Homework Sheet 8

Deadline: 26.06.2023, 14 Uhr

Unless explicitly mentioned, we work inside a sufficiently large saturated model \mathbb{U} of an ω -stable complete first-order theory T with infinite models in a fixed language \mathcal{L} .

Exercise 1 (6 Points). Consider a subset C of \mathbb{U} and a C -indiscernible sequence $(a_n)_{n \in \mathbb{N}}$ which is also C -independent.

- a) Given $B \supset C$ with $B \downarrow_C \{a_n\}_{n \in \mathbb{N}}$, show that $a_1, \dots, a_n \equiv_B a_{m_1}, \dots, a_{m_n}$ for all pairwise distinct m_1, \dots, m_n .
- b) Does the above hold in the random graph, setting $A \downarrow_C B$ if and only if $A \cap B \subset C$?

Exercise 2 (7 Points). Consider a definable group G acting definably on a definable set X .

- a) Given an arbitrary (non-empty) subset S of X , show that there are s_1, \dots, s_n in S such that for all $g \in G$

$$g \star s = s \text{ for all } s \text{ in } S \iff g \star s_i = s_i \text{ for } i = 1, \dots, n.$$

- b) Compute the rank $\text{RM}(\text{GL}_n(\mathbb{C}))$, where $\text{Mat}_{n \times n}(\mathbb{C})$ is the collection of square $n \times n$ -matrices.
- c) Let now S be the collection of strictly upper triangular matrices in $\text{Mat}_{n \times n}(\mathbb{C})$ (so each element of S is nilpotent!). Find an explicit set of elements s_i as in (a) for the action of $\text{GL}_n(\mathbb{C})$ on $\text{Mat}_{n \times n}(\mathbb{C})$ by conjugation.

Exercise 3 (7 Points).

- a) Consider a fixed algebraic closure K^{alg} of a field K of characteristic different from 2. Show that the number of intermediate fields $K \subset L \subset K^{alg}$ of degree $[L : K] \leq 2$ equals the index $(K^* : (K^*)^2)$ of the squares.

Consider now an ultraproduct $K = \prod_{\mathcal{U}} \mathbb{F}_p$ with respect to a non-principal ultrafilter \mathcal{U} on the set P of prime numbers, where we view each finite field \mathbb{F}_p as an \mathcal{L}_{ring} -structure.

- b) Determine the characteristic of K and show that the underlying additive group $(K, +)$ is connected.
- c) Is the underlying multiplicative group (K^*, \cdot) connected?

DIE ÜBUNGSBLÄTTER KÖNNEN ZU ZWEIT EINGEREICHT WERDEN. ABGABE DER ÜBUNGSBLÄTTER IM FACH 3.07 IM KELLER DES MATHEMATISCHEN INSTITUTS.