

# Geomod Conference 2023 – Schedule

*Please click on a speakers name to get to the abstract of the talk*

Time	Monday 13.11.		Tuesday 14.11.		Wednesday 15.11.		Thursday 16.11.		Friday 17.11.	
	09:15-09:30	Registration and Welcome								
09:30 - 10:30	Nick Ramsey		Tom Scanlon		Benjamin Castle		Anand Pillay		Raf Cluckers	
10:40-11:15	Coffee Break		Conference Picture		Coffee Break		Coffee Break		Coffee Break	
			Coffee Break							
11:20-12:20	Nadja Hempel Valentin		Adele Padgett		Zoé Chatzidakis		Isabel Müller		Rosario Mennuni	
12:20-13:20	Omar Leon Sánchez		Itay Kaplan		Daniel Palacín		Christian d’Elbée		Konstantinos Kartas	
13:20-15:30	Lunch Break		Lunch Break		Lunch Break		Lunch Break		Lunch Break	
									15:00-16:00	Martin Ziegler
15:30-16:30	Sylvain Rideau-Kikuchi		15:30-15:55	Shezad Mohamed	FREE AFTERNOON		Rémi Jaoui		DEPARTURE	
			16:00-16:25	Neer Bhardwaj						
16:30-17:20	Coffee Break		Coffee Break				Coffee Break			
17:20-17:45	Simone Ramello		Sebastian Eterovic				Giuseppina Terzo			
17:50-18:15	Paul Wang		Haydar Göral				Aris Papadopoulos			
	18:30-20:00	Reception & Poster Session			19:30-22:00	Conference Dinner				

## List of Abstracts

### **Benjamin Castle - Zilber's Restricted Trichotomy for O-minimal Structures in Higher Dimensions**

Peterzil has conjectured that the Zilber trichotomy holds for strongly minimal structures interpreted in o-minimal fields (generalizing Zilber's restricted trichotomy for algebraically closed fields in characteristic zero). Hasson, Onshuus, and Peterzil established this conjecture when the universe of the interpreted structure has o-minimal dimension 1, and the dimension 2 case was shown for expansions of groups by Eleftheriou, Hasson, and Peterzil. In this talk, I will discuss a proof of the conjecture in all dimensions 3 and greater.

### **Zoé Chatzidakis – Some remarks about difference-differential fields**

In an earlier paper, I showed that while the theory ACFA of existentially closed difference fields does not have prime models, kappa-prime models over algebraically closed subfields with fixed field pseudo-finite and kappa-saturated, exist and are unique in characteristic 0.

The object of this talk is to discuss a little the proof, isolate some ingredients, and how one can (hope to?) generalise it to  $\text{DCF}_{\text{mA}}$  (one automorphism, several derivations, all commuting).

Joint work with R. Bustamante and S. Montenegro.

### **Raf Cluckers – Motivic integration and Mellin transforms**

TBA

### **Christian d'Elbée - Free amalgamation of Lazard Lie Algebras**

This talk is a continuation of Isabel Müller's talk on the model theory of nilpotent groups and Lie algebras. The class of finitely generated Lazard Lie algebras (LLA) over a given field  $K$  is a free amalgamation class in the sense of Baudisch. This was originally proven by Baudisch (with traces of earlier results of Maier) but several major gaps were found in his proof. In this talk, I will present a (hopefully) correct construction of the free amalgam. Given three LLAs  $A, B, C$  where  $A, B$  extend  $C$ , the construction of the free amalgam of  $A$  and  $B$  over  $C$  follows the following 3 steps:

Step I : We construct the amalgam in the case where there exists singletons  $a, b$  such that  $A = \langle Ca \rangle$  and  $B = \langle Cb \rangle$  and  $C$  is an ideal of  $A$  and  $B$ .  $A$  and  $B$  are called basic extensions of  $C$  and the amalgam here is explicitly constructed via free Lie algebras. Checking that the construction yields the right object uses heavy Lie algebra computation as well as an induction on the number of iterations in Hall's rewriting algorithm for Lie monomials.

Step II : We construct the free amalgam of  $A$  and  $B$  over  $C$  in the case where  $A$  is a basic extension of  $C$  and  $B$  is arbitrary. This step is particularly tricky and needs a notion of rank which allows an inductive argument.

Step III : We conclude the existence of the general amalgam via a standard inductive argument. If time allows it, we will mention how this construction yields an extra result which is crucial in the proof that the Fraïssé of LLA over a finite field is  $\text{NSOP}_4$ .

### **Nadja Hempel Valentin - Pushing Properties for NIP Groups and Fields up the n-dependent hierarchy**

(joint with Chernikov) 1-dependent theories, better known as NIP theories, are the first class of the strict hierarchy of  $n$ -dependent theories. We proved the existence of strictly  $n$ -dependent groups for all natural numbers  $n$ . On the other hand, there are no known examples of strictly  $n$ -dependent fields and we conjecture that there aren't any.

We were interested which properties of groups and fields for NIP theories remain true in or can be generalized to the  $n$ -dependent context. A crucial fact about (type-)definable groups in NIP theories is the absoluteness of their connected components. Our first aim in this talk is to give examples of  $n$ -dependent groups and discuss an adapted version of absoluteness of the connected component. Secondly, we will review the known properties of NIP fields and see how they can be generalized.

### **Rémi Jaoui - On the Galois group of the equation of one-forms of a differential field extension**

To any differential field extension  $K/k$  of characteristic zero, one can associate the differential module of one-forms on  $K/k$ . In the setting of differential algebra, the construction of this differential module goes back (at least) to Ax's proof of the differential version of Schanuel conjecture and can be traced back even further to the work of (Elie) Cartan in the setting of differential geometry.

Using differential Galois theory, one can then produce a linear algebraic group attached to the extension  $K/k$  from this differential module. The goal of my talk will be to describe this invariant and its relation with several model-theoretic properties of types in the theory of differentially closed fields of characteristic zero.

### **Itay Kaplan - A result on the chromatic number of stable graphs**

Joint work with Halevi and Shelah, I want to present the following theorem: suppose that  $G$  is a graph whose edge relation is stable. Then if there is some  $G' \equiv G$  of size  $\mu^+$  whose chromatic number of  $G$  is  $\mu^+$ , then this is because  $G$  contains a copy of the complete graph on  $n$  vertices for every natural number  $n$ . I will put this result in a bigger context, related to questions on the chromatic number of stable graphs in general.

### **Konstantinos Kartas - On $C_i$ fields of mixed characteristic**

Given  $i \in \mathbb{N}$ , a field  $k$  is called  $C_i$  if every homogeneous polynomial over  $k$  of degree  $d$  in more than  $d^i$  variables has a non-trivial zero. Emil Artin had famously conjectured that  $\mathbb{Q}_p$  is  $C_2$ . While this was refuted by Terjanian, an appropriate asymptotic version for  $p \rightarrow \infty$  was proved by Ax-Kochen. In a somewhat orthogonal direction, we fix  $p$  but instead let the ramification go to infinity. I will present some recent work in progress towards showing that  $\mathbb{Q}_p(p^{1/p^\infty})$  is  $C_2$ .

### **Omar Leon Sánchez - Some remarks on differentially large fields and CODFs**

Abstract: We make some observations around differentially large fields (in characteristic zero). In particular, we note that they can be characterised as those differential fields that are existentially closed in the "differential algebraic" Laurent series ring. We also note that a field admits a differentially large structure iff it is of infinite transcendence degree (over  $\mathbb{Q}$ ). We then turn our attention to the theory CODF (closed ordered differential fields). We observe that a real closed differential field has a prime model extension (in CODF) iff it is already a CODF. This extends a result of Singer showing that CODF has no prime model. We then discuss the question of when a real closed differential field has a CODF extension inside a differential closure.

This is a combination of joint work with Marcus Tressl and Anand Pillay.

### **Rosario Mennuni - O-minimality, domination, and preorders**

In work in progress of P. Andújar Guerrero, P. Eleftheriou, and myself, we address the following question: is it true that, in o-minimal theories, every global invariant type is dominated by finitely many orthogonal 1-types?

I will discuss the background of the problem, its solution in certain special cases, and the connection with recent work of P. Andújar Guerrero, M. Thomas and E. Walsberg on cofinal curves in definable preorders.

### **Isabel Müller – On the Model Theory of Generic Nilpotent Groups and Lie Algebras**

The class of  $c$ -nilpotent Lie algebras with a predicate for a Lazard series admits free amalgamation. In this talk, we will exhibit some model theoretic properties of its limit, the generic  $c$ -nilpotent Lie algebra, which, using the Lazard correspondence, also transfer to  $c$ -nilpotent groups of prime exponent  $p > c$ . In particular, we will argue that the 2-nilpotent limit, which has previously been studied by Baudisch, is 2-dependent and NSOP1, whereas the  $c$ -nilpotent limit over  $F_p$  for any  $c$  larger than 2, is strictly NSOP4 and  $c$ -dependent.

### **Silvain Rideau-Kikuchi - An imaginary AKE principle**

(Joint with M. Vicaria)

In recent work with M. Hils, we showed that if the value group is particularly nice ( $\mathbb{Z}$  or  $\mathbb{Q}$ ) then the imaginaries in an (enriched) equicharacteristic zero henselian field can be understood in terms of the imaginaries of the value group and the imaginaries of some vector spaces of the residue field. On the other hand, M. Vicaria showed that if the residue field is particularly nice (algebraically closed) and the value group has bounded regular rank then the imaginaries can be understood in terms of the codes of modules and the imaginaries in the value group. In this talk I will explain how these two results can be merged together.

My goal is first to present various obstructions to elimination of imaginaries in a valued fields and then explain why in an (enriched) equicharacteristic zero henselian field (with some restrictions on the value group), these are the only obstructions. The main ingredients of the proof are a density result for quantifier free types in a well chosen language, an RV-domination result and a classification of definable sets internal to the residue field.

### **Adele Padgett - Some equations involving the Gamma function**

The Gamma function extends factorials to complex numbers and thus appears in many different mathematical contexts. Though Gamma is a transcendental holomorphic function, it satisfies several important functional equations. A step toward understanding its model theory would be to study any other algebraic properties it may possess. In this talk, I will present recent work with S. Eterović in which we prove certain systems of equations involving addition, multiplication, and the Gamma function must have infinitely many solutions in the complex numbers. Similar results have been obtained for periodic functions such as  $\exp$ , the modular  $j$  function, and others. We combine techniques established for these functions with new ideas in order to study Gamma, a non-periodic function. An immediate corollary is that the Gamma function has infinitely many periodic points of every period.

### **Daniel Palacín - Algebraic structures without the CBP**

The Canonical Base Property (CBP) is a model-theoretic property whose formulation was motivated by the interpretation of Pillay of a result of Campana and Fujiki in compact complex spaces, and similar results of Pillay and Ziegler on differentially closed fields of characteristic zero. While these, and many other, natural examples have the CBP, it is not the case that every finite Morley rank theory does it. However, none of these known examples without the CBP is an algebraic object. The purpose of this talk is to present a large variety of new algebraic structures whose theories have finite Morley rank but do not satisfy the CBP. This is joint work with Michael Loesch.

### **Anand Pillay - Invariant measures on automorphism groups of prime models**

We consider the existence and uniqueness of  $\text{Aut}(M/A)$  invariant measures on the Boolean algebra of "definable subsets" of  $\text{Aut}(M/A)$  when  $M$  is atomic over  $A$  and strongly omega-homogeneous over  $A$ . We have to explain what "definable subset of  $\text{Aut}(M/A)$ " means.

Uniqueness of Haar measure on the profinite group  $\text{Gal}(\mathbb{Q}^{\text{alg}}/\mathbb{Q})$  will be a special case.

We relate the issue to several topics such as differential Galois theory, definable Galois cohomology, and the theory of internality...

### **Nick Ramsey - Higher amalgamation in PAC fields**

Pseudo algebraically closed (PAC) fields, introduced by Ax in his characterization of pseudo-finite fields, are an important class of examples in both model theory and field arithmetic. A recurrent theme in the study of PAC fields is that their analysis often reduces to an analysis of the absolute Galois group. One of the most significant results along these lines is a theorem of Chatzidakis, which relates the independence theorem in a PAC field to the independence theorem in the inverse system of the absolute Galois group, viewed as a first-order structure. We explain how this connection can be generalized to relate independent  $n$ -amalgamation in a PAC field to independent  $n$ -amalgamation in the absolute Galois group. We will describe some corollaries for omega-free PAC fields and, more generally, for Frobenius fields.

### **Tom Scanlon – TBA**

### **Martin Ziegler - Pairs of algebraically closed fields and the Hilbert scheme.**

This is joint work with Amador Martin-Pizarro.

We show that pairs of algebraically closed fields are Noetherian with respect to tame formulas. This amounts to constructing a smallest tame formula in every type, which is done using the theory of Hilbert schemes.

Noetherian theories were defined by Hoffmann and Kowalski in "PAC structures as invariants of finite group actions" (2023), tame formulas by the authors in "Equational theories of fields" (2020).

## **Contributed Talks**

### **Neer Bhardwaj - Approximate Pila-Wilkie type counting for complex analytic sets**

We develop a variation of the Pila-Wilkie counting theorem, where we count rational points that approximate bounded complex analytic sets. A unique aspect of our result is that it does not depend on the analytic set in question. We apply this approximate counting to obtain an effective Pila-Wilkie statement for analytic sets cut out by computable functions. This is joint work with Gal Binyamini.

### **Sebastian Eterovic - Solutions to equations involving the modular $j$ function and its derivatives**

I will present a result showing which kinds of equations in one variable involving the modular  $j$  function and its first two derivatives have a Zariski dense set of solutions. This work extends the now well established research on Existential Closedness problems for various important analytic functions. This is joint work with Vahagn Aslanyan and Vincenzo Mantova.

### **Haydar Göral - Lehmer's conjecture via model theory**

In this talk, we first introduce the height function and the Mahler measure on the field of algebraic numbers. We state and give a survey on Lehmer's conjecture for the Mahler measure, which is still an open problem. Then, we consider the field of algebraic numbers with elements of small Mahler measures in terms of model theory, and we link this theory with Lehmer's conjecture and Bounded Lehmer's conjecture. Our proof applies van der Waerden's theorem from additive combinatorics.

### **Shezad Mohamed -Very slim differential fields**

What are some examples of simple differential fields? Hoffmann and Leon Sanchez show that pseudo-differentially closed fields are simple with nonforking independence given by linear disjointness. In this talk I will outline a method to find others, showing that for a given class of model complete differential fields, whether their theory is simple depends entirely on whether the pure field is simple. This class of fields is the class of very L-slim fields: a slight modification of a notion of Junker and Koenigsmann. I will also briefly show that a similar phenomenon occurs for stability, and for a wider class of operators than just derivations.

### **Aris Papadopoulos - Zarankiewicz's Problem in Presburger Arithmetic**

Put yourself in the following scenario: You're given a napkin on which somebody has written down a symmetric  $n \times n$  matrix consisting of 0's and 1's. They tell you that no  $2 \times 2$  matrix consisting only of 1's appears on said napkin, and ask you to make an educated guess on the maximum number of 1's. After some thought, you guess that the answer should be at most some absolute constant times  $n^{\frac{3}{2}}$ .

Now, they tell you that this matrix is actually the adjacency matrix of a finite subgraph of an infinite bipartite graph which is definable in Presburger arithmetic and does not contain any infinite complete subgraphs. In my talk, I will explain how, jointly with P. Eleftheriou, we can help you improve your answer.

### **Simone Ramello - The Kaplansky theory of non-inversive valued difference fields**

The classical strategy for proving AK/E-type principles for enriched valued fields is developing an understanding of maximal immediate extensions. I will outline the main problems that arise when dealing with non-inversive valued difference fields, and how the uniqueness theorem fails in this setting.

### **Giuseppina Terzo - Generic derivations on Algebraically Bounded Structures**

Let  $K$  be an algebraically bounded structure. If  $K$  is model complete, then the theory of  $K$  endowed with a derivation has a model completion. Similar results hold for several derivations, both commuting and non-commuting.

(Joint work with Antongiulio Fornasiero)

### **Paul Wang - On groups and fields interpretable in various $NTP_2$ fields**

The aim of this work is to give tools to answer the following questions: in a model-theoretically tame field, is it possible to compare definable, and even interpretable, groups to algebraic groups? Similarly, can one classify the interpretable fields?

One crucial point is to be able to construct definable group morphisms from generic data. It appears that one stabilizer theorem of Montenegro, Onshuus and Simon, which is a variant of Hrushovski's stabilizer theorem, is very useful to achieve this.