

RANK CRANK CONGRUENCES MAPLE MANUAL

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> **currentdir()** ;

"C:\cygwin64\home\fgarv\math\research\rank-crank-congruences-mod-p"

(1)

CSH-SCRIPTS

make-rank-crank-modp-data t lastn smod sres

This script

- * creates a FORTAN input data file

- * runs a FORTRAN program that produces three files:

 - o ranksave

 - o cranksave

 - o ptnsave

The rows of ranksave: $N(r, t, \text{smod} \cdot j + \text{sres}) \bmod t$, $r=0 \dots (t-1)/2$
where $j=0 \dots \text{floor}((\text{lastn}-\text{sres})/\text{smod})$

The rows of cranksave: $M(r, t, \text{smod} \cdot j + \text{sres}) \bmod t$, $r=0 \dots (t-1)/2$
where $j=0 \dots \text{floor}((\text{lastn}-\text{sres})/\text{smod})$

The rows of ptnsave: $p(\text{smod} \cdot j + \text{sres}) \bmod t$,
where $j=0 \dots \text{floor}((\text{lastn}-\text{sres})/\text{smod})$

- * Using sed spaces are changed to tabs to make three new files of the form

 - o ranksave-t-lastn-smod-sres

 - o cranksave-t-lastn-smod-sres

 - o ptnsave-t-lastn-smod-sres

EXAMPLE

make-rank-crank-modp-data 13 100000 169 162

STEP 1: Create init file inputdata2

STEP 2: Run fortran program makegfdat

t = 13

n = 100000

smod = 169

sres = 162

init done

computing p(10000) mod 13

computing p(20000) mod 13

computing p(30000) mod 13

computing p(40000) mod 13

computing p(50000) mod 13

computing p(60000) mod 13

computing p(70000) mod 13

computing p(80000) mod 13

computing p(90000) mod 13

computing p(100000) mod 13

0 590

100	590
200	590
300	590
400	590
500	590

STEP 3: Convert fortran output to maple input

SEE new files:

```
-rw-r--r-- 1 fgarr fgarr 9202 Sep 10 18:27 ranksave-13-100000-169-162
-rw-r--r-- 1 fgarr fgarr 9225 Sep 10 18:27 cranksave-13-100000-169-162
-rw-r--r-- 1 fgarr fgarr 1326 Sep 10 18:27 ptnsave-13-100000-169-162
```

`collect-rank-crank-modp-data`

Make maple lists RPL and CPL and save them to the file RPCPLs.txt

EXAMPLE

```
collect-rank-crank-modp-data
-rw-r--r-- 1 fgarr fgarr 182 Sep 10 18:31 RPCPLs.txt
12 12 182 RPCPLs.txt
See contents (y/n)?
y
RPL:=[
[13,10000,13,6],
[13,100000,169,162],
[17,80000,289,277],
[19,100000,361,346],
[]]:
CPL:=[
[13,10000,13,6],
[13,100000,169,162],
[17,80000,289,277],
[19,100000,361,346],
[]]:
```

MAPLE PROCS

The procs are in the file `mprog`.

`EQROWS(mat)`

Determine which rows of the matrix **mat** have identical entries.

`analyzeCRANKRANKMATS(t, lastn, smod, sres)`

This proc imports the matrices **ranksave-t-lastn-smod-sres** and **cranksave-t-lastn-smod-sres**

It returns a list **[CP,RP,Y]**

CP = the rows of **cranksave-t-lastn-smod-sres** with identical rows

RP = the rows of **ranksave-t-lastn-smod-sres** with identical rows

Y = the values **n** for which $p(\text{smod}*(n-1)+\text{sres}) = 0 \bmod t$

It also prints out whether **CP** subset of **RP**, **RP** subset of **CP** and **CP=RP**.

EXAMPLE

```

> read mprog:
> RPL;
[[ [13, 10000, 13, 6], [13, 100000, 169, 162], [17, 80000, 289, 277], [19, 100000, 361, 346],
  [ ] ]
> analyzeCRANKRANKMATS (17, 80000, 17^2, modp(1/24, 17^2)) ;
"CP = ", [6, 9, 15, 58, 63, 64, 69, 72, 75, 78, 95, 100, 103, 118, 149, 153, 160, 169, 202, 215,
  224, 227, 229, 253, 272 ], "no. ", 25
"RP = ", [15, 58, 63, 69, 72, 78, 95, 100, 118, 153, 169, 202, 215, 227, 229, 253, 272 ], "no. ",
  17
"Y = ", [6, 9, 15, 58, 63, 64, 69, 72, 75, 78, 95, 100, 101, 103, 118, 149, 153, 160, 169, 202,
  215, 224, 227, 229, 253, 272 ], "no. ", 26
      "CP subset of RP ", false
      "RP subset of CP ", true
      "RP = CP ", false
[[ [6, 9, 15, 58, 63, 64, 69, 72, 75, 78, 95, 100, 103, 118, 149, 153, 160, 169, 202, 215, 224,
  227, 229, 253, 272 ], [15, 58, 63, 69, 72, 78, 95, 100, 118, 153, 169, 202, 215, 227, 229,
  253, 272 ], [6, 9, 15, 58, 63, 64, 69, 72, 75, 78, 95, 100, 101, 103, 118, 149, 153, 160, 169,
  202, 215, 224, 227, 229, 253, 272 ] ]

```

(3.2)

makerankmomGFmodp(k, t, lastn, smod, sres)

Returns the rank moment generating function $\sum N[k](smod*n + sres)*q^n \mod t$

EXAMPLE

```

> with(qseries) :
> N4136:=makerankmomGFmodp(4, 13, 10000, 13, 6) :
> series(N4136, q, 10) ;
      5 + 7 q + 5 q^2 + q^3 + 3 q^4 + 5 q^5 + 10 q^7 + 12 q^8 + q^9 + O(q^11)

```

(4.1)

```

> with(modforms) :
> modp(series(N4136-etaq(q, 1, 1000)^11*(12 + 6*E4), q, 768), 13) ;
      O(q^768)

```

(4.2)

This confirms an identity for the 4th rank moment mod 13.

makecrankmomGFmodp(k, t, lastn, smod, sres)

Returns the crank moment generating function $\sum M[k](smod*n + sres)*q^n \mod t$

EXAMPLE

```

> with(qseries) :
> M4136:=makecrankmomGFmodp(4, 13, 10000, 13, 6) :
> series(M4136, q, 10) ;
      11 + 3 q + 2 q^2 + 12 q^3 + 7 q^5 + q^8 + 3 q^9 + O(q^11)
> with(modforms) : FL:=map(f->series(f*etaq(q, 1, 1000)^11, q,
  1000), [1, E4]) :
> symFL:=map(f->f*_E^11, [1, _E4]) :

```

(5.1)

```
> findlincombomodp(M4136, FL, symFL, 13, q, 0);
```

$$-E^{11}E^4 - E^{11} \quad (5.2)$$

```
> modp(series(M4136+etaq(q,1,1000)^11*(1 + E4), q, 768), 13);
```

$$O(q^{768}) \quad (5.3)$$

This confirms an identity for the 4th crank moment mod 13.

makerankGFmodp(k, t, lastn, smod, sres)

Returns the rank generating function $\text{Sum } N(k, t, \text{smod} \cdot n + \text{sres}) \cdot q^n \text{ mod } t$

EXAMPLE

```
> with(qseries): with(rank):
```

```
> R0:=makerankGFmodp(0,13,10000,13,6):
```

```
> series(R0,q,10);
```

$$1 + 5q + 11q^3 + 3q^4 + 3q^5 + 5q^6 + 2q^8 + 5q^9 + O(q^{10}) \quad (6.1)$$

```
> floor((3000-6)/13);
```

$$230 \quad (6.2)$$

```
> modp(series(R0-add(N(0,13,13*n+6)*q^n,n=0..230),q,231),13);
```

$$O(q^{231}) \quad (6.3)$$

```
> with(modforms): with(misc):
```

```
> PHI11:=series(Phiq(11,q,3001),q,3001):
```

```
> PPHI11:=series(2*P*PHI11,q,2001):
```

```
> E11:=series(etaq(q,1,800)^11,q,800):
```

```
> symMB2:=[_E11*_E4, _E11*_E6, _E10*_E11, _E6^2*_E11, _DELTA12*_E11, _PHI11*_E11];
```

$$\text{symMB2} := [_E11_E4, _E11_E6, _E10_E11, _E6^2_E11, _DELTA12_E11, _PHI11_E11] \quad (6.4)$$

```
> MB2:=map(f->series(subs({_E11=E11, _PHI11=PHI11, _E4=E4, _E6=E6, _E10=E10, _DELTA12=DELTA12},f),q,250),symMB2):
```

```
> symidR0:=modp(findlincombomodp(R0,MB2,symMB2,13,q,0),13);
```

$$\text{symidR0} := 10_E11_E6^2 + 12_E10_E11 + 11_E11_E4 + 7_E11_E6 + 4_E11_PHI11 + 4_E11_DELTA12 \quad (6.5)$$

```
> idR0:=series(subs({_E11=E11, _PHI11=PHI11, _E4=E4, _E6=E6, _E10=E10, _DELTA12=DELTA12},symidR0),q,800):
```

```
> qdegree(R0);
```

$$768 \quad (6.6)$$

```
> modp(series(R0-idR0,q,769),13);
```

$$O(q^{769}) \quad (6.7)$$

This confirms an identity for the GF of $N(0,13,13 \cdot n + 6) \text{ mod } 13$.

makecrankGFmodp(k, t, lastn, smod, sres)

Returns the rank generating function $\text{Sum } M(k, t, \text{smod} \cdot n + \text{sres}) \cdot q^n \text{ mod } t$

EXAMPLE

```
> with(qseries): with(crank):
```

```
> C0:=makecrankGFmodp(0,13,10000,13,6):
```

```
> series(C0,q,10);
```

$$(7.1)$$

$$1 + 12q + 8q^2 + 11q^3 + q^4 + 12q^5 + 12q^6 + 3q^7 + 12q^8 + 4q^9 + O(q^{10}) \quad (7.1)$$

```
> floor((2000-6)/13);
```

$$153 \quad (7.2)$$

```
> modp(series(C0-add(M(0,13,13*n+6)*q^n,n=0..153),q,154),13);
```

$$O(q^{154}) \quad (7.3)$$

```
> with(modforms): with(misc):
```

```
> PHI11:=series(Phiq(11,q,3001),q,3001):
```

```
> PPHI11:=series(2*P*PHI11,q,2001):
```

```
> E11:=series(etaq(q,1,800)^11,q,800):
```

```
> symMB2:=[_E11*_E4, _E11*_E6, _E11*_E8, _E10*_E11, _E6^2*_E11,
  _DELTA12*_E11, _PHI11*_E11];
```

$$\text{symMB2} := [_E11_E4, _E11_E6, _E11_E8, _E10_E11, _E6^2_E11, _DELTA12_E11, \quad (7.4)$$

$$_PHI11_E11]$$

```
> MB2:=map(f->series(subs({_E11=E11, _PHI11=PHI11, _E4=E4, _E6=E6,
  _E8=E8, _E10=E10, _DELTA12=DELTA12},f),q,250),symMB2):
```

```
> symidC0:=modp(findlincombomodp(C0,MB2,symMB2,13,q,0),13);
```

$$\text{symidC0} := 6_E11_E6^2 + 9_E10_E11 + 9_E11_E4 + 5_E11_E6 + 11_E11_E8 \quad (7.5)$$

$$+ 4_E11_PHI11$$

```
> idC0:=series(subs({_E11=E11, _PHI11=PHI11, _E4=E4, _E6=E6, _E8=E8,
  _E10=E10, _DELTA12=DELTA12},symidC0),q,800):
```

```
> qdegree(C0);
```

$$768 \quad (7.6)$$

```
> modp(series(C0-idC0,q,769),13);
```

$$O(q^{769}) \quad (7.7)$$

[This confirms an identity for the GF of $N(0,13,13*n+6) \bmod 13$.

`makeptnGFmodp(t, lastn, smod, sres)`

Returns the partition generating function $\text{Sum } p(\text{smod}*n + \text{sres}) * q^n \bmod t$

EXAMPLE

```
> with(qseries): read mprog:
```

```
> P0:=makeptnGFmodp(13,10000,13,6):
```

```
> series(P0,q,10);
```

$$11 + 9q + 3q^2 + 6q^3 + 12q^4 + 6q^5 + q^7 + 7q^8 + 11q^9 + O(q^{11}) \quad (8.1)$$

```
> qdegree(P), floor((5000-6)/13);
```

$$5000, 384 \quad (8.2)$$

```
> modp(series(P0-sift(P,q,13,6,5000),q,385),13);
```

$$O(q^{385}) \quad (8.3)$$

```
> E11:=series(etaq(q,1,800)^11,q,800):
```

```
> symidP0:=modp(findlincombomodp(P0,[E11],[_E11],13,q,0),13);
```

$$\text{symidP0} := 11_E11 \quad (8.4)$$

```
> qdegree (P0) ;
```

768

(8.5)

```
> modp (series (P0-11*E11,q,769) ,13) ;
```

$O(q^{769})$

(8.6)

[This confirms an identity for the GF of $p(13*n+6) \bmod 13$.