RANK CRANK CONGRUENCES MAPLE MANUAL

DATE: Fri Sep 10 20:53:04 EDT 2021

```
> currentdir();
```

"C:\cygwin64\home\fgarv\math\research\rank-crank-congruences-mod-p" (1)

CSH-SCRIPTS

make-rank-crank-modp-data t lastn smod sres

This script
* creates a FORTAN input data file
* runs a FORTRAN program that produces three files:
  o ranksave
  o cranksave
  o ptnsave
  The rows of ranksave: N(r, t, smod*j + sres) mod t, r=0 .. (t-1)/2
  where j=0 ... floor((lastn-sres)/smod)
  The rows of cranksave: M(r, t, smod*j + sres) mod t, r=0 .. (t-1)/2
  where j=0 ... floor((lastn-sres)/smod)
  The rows of ptnsave: p(smod*j + sres) mod t,
  where j=0 ... floor((lastn-sres)/smod)
* Using sed spaces are changed to tabs to make three new files of the form
  o ranksave-t-lastn-smod-sres
  o cranksave-t-lastn-smod-sres
  o ptnsave-t-lastn-smod-sres

EXAMPLE

```
make-rank-crank-modp-data 13 100000 169 162

STEP 1:  Create init file inputdata2
STEP 2:  Run fortran program makegfdat

 t =          13
 n =      100000
 smod =         169
 sres =         162
 init done
 computing p(      10000 ) mod          13
 computing p(      20000 ) mod          13
 computing p(      30000 ) mod          13
 computing p(      40000 ) mod          13
 computing p(      50000 ) mod          13
 computing p(      60000 ) mod          13
 computing p(      70000 ) mod          13
 computing p(      80000 ) mod          13
 computing p(      90000 ) mod          13
 computing p(     100000 ) mod          13
           0         590
```

```
100      590
200      590
300      590
400      590
500      590
```

STEP 3: Convert fortran output to maple input
SEE new files:
-rw-r--r-- 1 fgarv fgarv 9202 Sep 10 18:27 ranksave-13-100000-169-162
-rw-r--r-- 1 fgarv fgarv 9225 Sep 10 18:27 cranksave-13-100000-169-162
-rw-r--r-- 1 fgarv fgarv 1326 Sep 10 18:27 ptnsave-13-100000-169-162

`collect-rank-crank-modp-data`
Make maple lists RPL and CPL and save them to the file RPCPLs.txt

EXAMPLE

```
collect-rank-crank-modp-data
-rw-r--r-- 1 fgarv fgarv 182 Sep 10 18:31 RPCPLs.txt
 12  12 182 RPCPLs.txt
See contents (y/n)?
y
RPL:=[
[13,10000,13,6],
[13,100000,169,162],
[17,80000,289,277],
[19,100000,361,346],
[]]:
CPL:=[
[13,10000,13,6],
[13,100000,169,162],
[17,80000,289,277],
[19,100000,361,346],
[]]:
```

MAPLE PROCS

The procs are in the file `mprog`.

`EQROWS`(mat)
Determine which rows of the matrix **mat** have identical entries.

`analyzeCRANKRANKMATS`(t, lastn, smod, sres)
This proc imports the matrices **ranksave-t-lastn-smod-sres** and **cranksave-t-lastn-smod-sres**
It returns a list **[CP,RP,Y]**
CP = the rows of **cranksave-t-lastn-smod-sres** with identical rows
RP = the rows of **ranksave-t-lastn-smod-sres** with identical rows
Y = the values **n** for which **p(smod*(n-1)+sres) = 0 mod t**
It also prints out whether **CP** subset of **RP, RP** subset of **CP** and **CP=RP.**

EXAMPLE

```
> read mprog:
> RPL;
```

[[13, 10000, 13, 6], [13, 100000, 169, 162], [17, 80000, 289, 277], [19, 100000, 361, 346], [ ]] **(3.1)**

```
> analyzeCRANKRANKMATS(17,80000,17^2,modp(1/24,17^2));
```

"CP = ", [6, 9, 15, 58, 63, 64, 69, 72, 75, 78, 95, 100, 103, 118, 149, 153, 160, 169, 202, 215, 224, 227, 229, 253, 272], "no. ", 25

"RP = ", [15, 58, 63, 69, 72, 78, 95, 100, 118, 153, 169, 202, 215, 227, 229, 253, 272], "no. ", 17

"Y = ", [6, 9, 15, 58, 63, 64, 69, 72, 75, 78, 95, 100, 101, 103, 118, 149, 153, 160, 169, 202, 215, 224, 227, 229, 253, 272], "no. ", 26

"CP subset of RP ", *false*

"RP subset of CP ", *true*

"RP = CP ", *false*

[[6, 9, 15, 58, 63, 64, 69, 72, 75, 78, 95, 100, 103, 118, 149, 153, 160, 169, 202, 215, 224, 227, 229, 253, 272], [15, 58, 63, 69, 72, 78, 95, 100, 118, 153, 169, 202, 215, 227, 229, 253, 272], [6, 9, 15, 58, 63, 64, 69, 72, 75, 78, 95, 100, 101, 103, 118, 149, 153, 160, 169, 202, 215, 224, 227, 229, 253, 272]] **(3.2)**

makerankmomGFmodp(k, t, lastn, smod, sres)
Returns the rank moment generating function **Sum N[k](smod*n + sres)*q^n mod t**

EXAMPLE

```
> with(qseries):
> N4136:=makerankmomGFmodp(4,13,10000,13,6):
> series(N4136,q,10);
```

$$5+7q+5q^2+q^3+3q^4+5q^5+10q^7+12q^8+q^9+O(q^{11}) \quad \textbf{(4.1)}$$

```
> with(modforms):
> modp(series(N4136-etaq(q,1,1000)^11*(12 + 6*E4),q,768),13);
```

$$O(q^{768}) \quad \textbf{(4.2)}$$

This confirms an identity for the 4th rank moment mod 13.

makecrankmomGFmodp(k, t, lastn, smod, sres)
Returns the cank moment generating function **Sum M[k](smod*n + sres)*q^n mod t**

EXAMPLE

```
> with(qseries):
> M4136:=makecrankmomGFmodp(4,13,10000,13,6):
> series(M4136,q,10);
```

$$11+3q+2q^2+12q^3+7q^5+q^8+3q^9+O(q^{11}) \quad \textbf{(5.1)}$$

```
> with(modforms):    FL:=map(f->series(f*etaq(q,1,1000)^11,q,
1000),[1,E4]):
> symFL:=map(f->f*_E^11,[1,_E4]):
```

```
> findlincombomodp(M4136,FL,symFL,13,q,0);
```

$$-\_E^{11}\ \_E4 - \_E^{11} \tag{5.2}$$

```
> modp(series(M4136+etaq(q,1,1000)^11*(1 + E4),q,768),13);
```

$$\mathrm{O}\left(q^{768}\right) \tag{5.3}$$

This confirms an identity for the 4th crank moment mod 13.

makerankGFmodp(k, t, lastn, smod, sres)
Returns the rank generating function **Sum N(k,t,smod*n + sres)*q^n mod t**

EXAMPLE

```
> with(qseries): with(rank):
> R0:=makerankGFmodp(0,13,10000,13,6):
> series(R0,q,10);
```

$$1 + 5 q + 11 q^3 + 3 q^4 + 3 q^5 + 5 q^6 + 2 q^8 + 5 q^9 + \mathrm{O}\left(q^{10}\right) \tag{6.1}$$

```
> floor((3000-6)/13);
```

$$230 \tag{6.2}$$

```
> modp(series(R0-add(N(0,13,13*n+6)*q^n,n=0..230),q,231),13);
```

$$\mathrm{O}\left(q^{231}\right) \tag{6.3}$$

```
> with(modforms): with(misc):
> PHI11:=series(Phiq(11,q,3001),q,3001):
> PPHI11:=series(2*P*PHI11,q,2001):
> E11:=series(etaq(q,1,800)^11,q,800):
> symMB2:=[_E11*_E4, _E11*_E6, _E10*_E11, _E6^2*_E11, _DELTA12*
  _E11, _PHI11*_E11];
```

$$symMB2 := \left[\_E11\ \_E4, \_E11\ \_E6, \_E10\ \_E11, \_E6^2\ \_E11, \_DELTA12\ \_E11, \_PHI11\ \_E11\right] \tag{6.4}$$

```
> MB2:=map(f->series(subs({_E11=E11,_PHI11=PHI11,_E4=E4,_E6=E6,
  _E10=E10,_DELTA12=DELTA12},f),q,250),symMB2):
> symidR0:=modp(findlincombomodp(R0,MB2,symMB2,13,q,0),13);
```

$$symidR0 := 10\ \_E11\ \_E6^2 + 12\ \_E10\ \_E11 + 11\ \_E11\ \_E4 + 7\ \_E11\ \_E6 + 4\ \_E11\ \_PHI11 + 4\ \_E11\ \_DELTA12 \tag{6.5}$$

```
> idR0:=series(subs({_E11=E11,_PHI11=PHI11,_E4=E4,_E6=E6,_E10=
  E10,_DELTA12=DELTA12},symidR0),q,800):
> qdegree(R0);
```

$$768 \tag{6.6}$$

```
> modp(series(R0-idR0,q,769),13);
```

$$\mathrm{O}\left(q^{769}\right) \tag{6.7}$$

This confirms an identity for the GF of N(0,13,13*n+6) mod 13.

makecrankGFmodp(k, t, lastn, smod, sres)
Returns the rank generating function **Sum M(k,t,smod*n + sres)*q^n mod t**

EXAMPLE

```
> with(qseries): with(crank):
> C0:=makecrankGFmodp(0,13,10000,13,6):
> series(C0,q,10);
```

(7.1)

$$1+12q+8q^2+11q^3+q^4+12q^5+12q^6+3q^7+12q^8+4q^9+O(q^{10}) \quad (7.1)$$

```
> floor((2000-6)/13);
```

$$153 \quad (7.2)$$

```
> modp(series(C0-add(M(0,13,13*n+6)*q^n,n=0..153),q,154),13);
```

$$O(q^{154}) \quad (7.3)$$

```
> with(modforms): with(misc):
> PHI11:=series(Phiq(11,q,3001),q,3001):
> PPHI11:=series(2*P*PHI11,q,2001):
> E11:=series(etaq(q,1,800)^11,q,800):
> symMB2:=[_E11*_E4, _E11*_E6, _E11*_E8,_E10*_E11, _E6^2*_E11,
  _DELTA12*_E11, _PHI11*_E11];
```

$$symMB2 := [\_E11\ \_E4, \_E11\ \_E6, \_E11\ \_E8, \_E10\ \_E11, \_E6^2\ \_E11, \_DELTA12\ \_E11, \_PHI11\ \_E11] \quad (7.4)$$

```
> MB2:=map(f->series(subs({_E11=E11,_PHI11=PHI11,_E4=E4,_E6=E6,
  _E8=E8,_E10=E10,_DELTA12=DELTA12},f),q,250),symMB2):
> symidC0:=modp(findlincombomodp(C0,MB2,symMB2,13,q,0),13);
```

$$symidC0 := 6\ \_E11\ \_E6^2 + 9\ \_E10\ \_E11 + 9\ \_E11\ \_E4 + 5\ \_E11\ \_E6 + 11\ \_E11\ \_E8 + 4\ \_E11\ \_PHI11 \quad (7.5)$$

```
> idC0:=series(subs({_E11=E11,_PHI11=PHI11,_E4=E4,_E6=E6,_E8=E8,
  _E10=E10,_DELTA12=DELTA12},symidC0),q,800):
> qdegree(C0);
```

$$768 \quad (7.6)$$

```
> modp(series(C0-idC0,q,769),13);
```

$$O(q^{769}) \quad (7.7)$$

This confirms an identity for the GF of N(0,13,13*n+6) mod 13.

makeptnGFmodp(t, lastn, smod, sres)

Returns the partition generating function **Sum p(smod*n + sres)*q^n mod t**

EXAMPLE

```
> with(qseries): read mprog:
> P0:=makeptnGFmodp(13,10000,13,6):
> series(P0,q,10);
```

$$11+9q+3q^2+6q^3+12q^4+6q^5+q^7+7q^8+11q^9+O(q^{11}) \quad (8.1)$$

```
> qdegree(P), floor( (5000-6)/13);
```

$$5000, 384 \quad (8.2)$$

```
> modp(series(P0-sift(P,q,13,6,5000),q,385),13);
```

$$O(q^{385}) \quad (8.3)$$

```
> E11:=series(etaq(q,1,800)^11,q,800):
```

```
> symidP0:=modp(findlincombomodp(P0,[E11],[_E11],13,q,0),13);
```

$$symidP0 := 11\ \_E11 \quad (8.4)$$

```
> qdegree(P0);
```

$$768 \tag{8.5}$$

```
> modp(series(P0-11*E11,q,769),13);
```

$$\mathrm{O}\left(q^{769}\right) \tag{8.6}$$

This confirms an identity for the GF of p(13*n+6) mod 13.