1 Constants

Symbol	Name	value	unit
	Earth radius	6378.14	km
μ	Gravitational parameter	3.986×10^{14}	m^3/s^2

2 Newton's laws of motion

2.1 Newton's first law of motion

A body continues in its state of rest, or of uniform motion in a straight line, unless compelled to change that state by forces impressed upon it.

$$\boxed{\vec{p} = m\vec{V}}$$

$$\begin{aligned}
\vec{p} &= \text{linear momentum vector (kg} \cdot \text{m/s)} \\
m &= \text{mass (kg)} \\
\vec{V} &= \text{velocity vector (m/s)}
\end{aligned}$$

$$\boxed{\vec{H} = I\vec{\Omega}}$$

$$\begin{aligned}
\vec{H} &= \text{angular momentum vector (kg} \cdot \text{m}^2\text{/s)} \\
I &= \text{moment of inertia (kg} \cdot \text{m}^2\text{)} \\
\vec{\Omega} &= \text{angular velocity vector (rad/s)}
\end{aligned}$$

$$\boxed{\vec{H} = \vec{R} \times m\vec{V}}$$

$$\begin{aligned}
\vec{H} &= \text{angular momentum vector (kg} \cdot \text{m}^2\text{/s)} \\
\vec{R} &= \text{position (m)} \\
m &= \text{mass (kg)} \\
\vec{V} &= \text{velocity vector (m/s)}
\end{aligned}$$

2.2 Newton's second law of motion

The time rate of change of an object's momenutm equals the applied force.

$$\boxed{\vec{F} = m\vec{a}}$$

$$\begin{aligned}
\vec{F} &= \text{force vector (kgm/s}^2 = \text{N)} \\
m &= \text{mass (kg)} \\
\vec{a} &= \text{acceleration (m/s}^2\text{)}
\end{aligned}$$

2.3 Newton's third law of motion

When body A exerts a force on body B, body B will exert an equal, but opposite, force on body A

3 Newton's laws of universal gravitation

$$\boxed{F_g = \frac{Gm_1m_2}{R^2}}$$

$$\begin{aligned}
F_g &= \text{force due to gravity (N)} \\
G &= \text{universal gravitational constant} \approx 6.674 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2 \\
m_1, m_2 &= \text{masses of two bodies (kg)} \\
R &= \text{distance between the two bodies (m)}
\end{aligned}$$

$$\boxed{a_g = \frac{\mu_{Earth}}{R^2}}$$

$$\begin{aligned} a_g &= \text{acceleration due to gravity (m/s}^2\text{)} \\ \mu_{Earth} &\equiv G\, m_{Earth} \approx 3.986 \times 10^{14}\ \text{m}^3/\text{s}^2 \\ R &= \text{distance between the two bodies (m)} \end{aligned}$$

4 Laws of conservation

4.1 Conservation of momentum

In the absence of outside forces, linear and angular momentum are conserved.

4.2 Energy

$$\boxed{E = KE + PE}$$

$$\begin{aligned} E &= \text{total mech. energy (kg m}^2/\text{s}^2\text{)} \\ KE &= \text{kinetic energy (kg m}^2/\text{s}^2\text{)} \\ PE &= \text{potential energy (kg m}^2/\text{s}^2\text{)} \end{aligned}$$

$$\boxed{PE = m\, a_g h}$$

$$\begin{aligned} m &= \text{mass (kg)} \\ a_g &= \text{acceleration due to gravity (m/s}^2\text{)} \\ h &= \text{height above ref. point (m)} \end{aligned}$$

$$\boxed{PE = -\frac{m\mu}{R}}$$

$$\begin{aligned} m &= \text{spacecraft's mass (kg)} \\ \mu &= \text{gravitational parameter (km}^3/\text{s}^2\text{)} \\ R &= \text{distance from Earth's center (km)} \end{aligned}$$

$$\boxed{KE = \frac{1}{2} m V^2}$$

$$\begin{aligned} KE &= \text{kinetic energy (kg m}^2/\text{s}^2\text{)} \\ m &= \text{mass (kg)} \\ V &= \text{velocity (km/s)} \end{aligned}$$

$$\boxed{E = \frac{1}{2} m V^2 - \frac{m\mu}{R}}$$

$$\begin{aligned} E &= \text{total mech. energy (kg m}^2/\text{s}^2\text{)} \\ m &= \text{mass (kg)} \\ V &= \text{velocity (km/s)} \\ \mu &= \text{gravitational parameter (km}^3/\text{s}^2\text{)} \\ R &= \text{position (km)} \end{aligned}$$

5 The restricted two-body problem

5.1 Coordinate systems

A coordinate system (figure 2) is:

- **an origin**
- **a fundamental plane**, containing two axes, and the perpendicular to it
- **a principal direction** within the plane
- **the third axis** using the right-hand rule

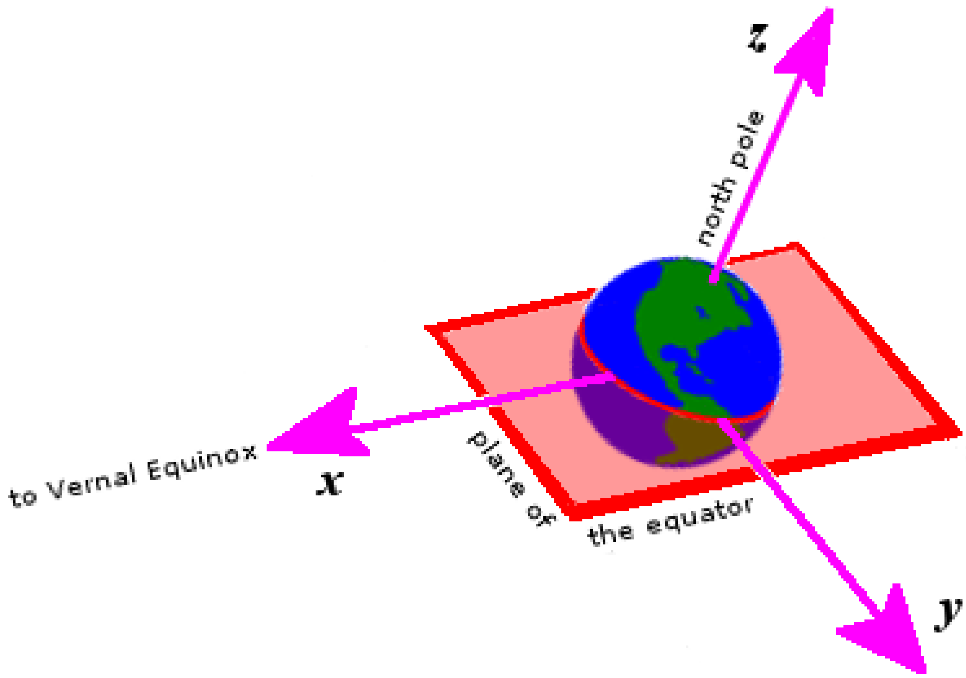


FIGURE 1 – Geocentric equatorial coordinates. The origin is the centre of the Earth. The fundamental plane is the plane of the Earth's equator. The primary direction (the x axis) is the vernal equinox. A right-handed convention specifies a y axis 90° to the east in the fundamental plane; the z axis is the north polar axis. The reference frame does not rotate with the Earth, rather, the Earth rotates around the z axis.

5.2 Equation of motion

$$\boxed{\ddot{\vec{R}} + \frac{\mu}{R^2}\frac{\vec{R}}{R} = 0}$$

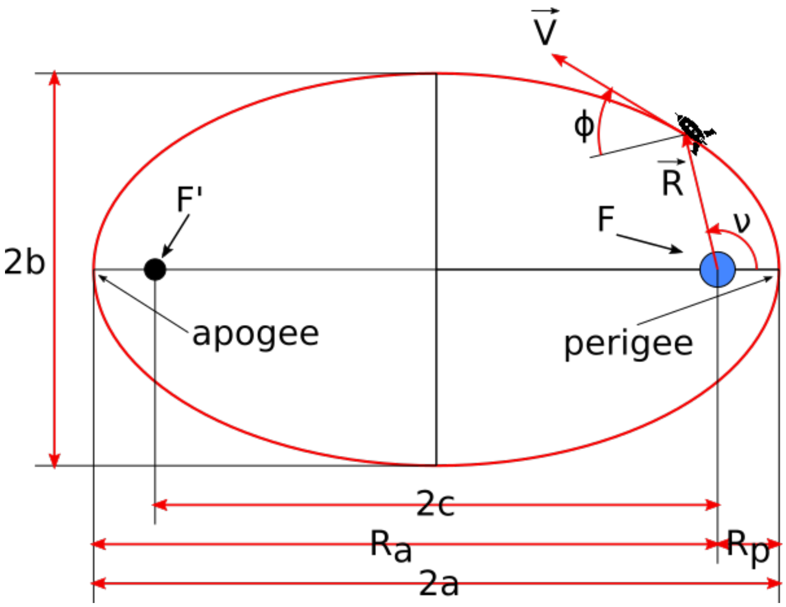
$\ddot{\vec{R}} =$ spacecraft's acceleration (km/s^2)

$\mu =$ gravitational parameter (km^3/s^2)

$\vec{R} =$ spacecraft's position vector (km)

$R =$ magnitude of the spacecraft's position vector (km)

5.3 Orbital geometry



$\vec{R}$ = spacecraft's position vector
$\vec{V}$ = spacecraft's velocity vector
$FandF'$ = primary and vacant foci
R_p = radius of perigee
R_a = radius of apogee
$2a$ = major axis
$2b$ = minor axis
$2c$ = distance between the foci
a = semimajor axis
b = semiminor axis
ν = true anomaly
ϕ = flight-path angle

FIGURE 2 – Geometry of an elliptical orbit

$$\boxed{e = \frac{2c}{2a} = \frac{R_a - R_p}{R_a + R_p}}$$

$e = eccentricity$

$$\boxed{R = \frac{a(1-e^2)}{1 + ecos\nu}}$$

R = magnitude of the spacecraft's position vector (km)
a = semi-major axis (km)
e = eccentricity (unitless)
ν = true anomaly (deg or rad)

Conic section	a = semimajor axis	c = one half the distance between foci	e = eccentricity
circle	a > 0	c = 0	e = 0
ellipse	a > 0	0 < c < a	0 < e < 1
parabola	a = ∞	c = ∞	e = 1
hyperbola	a < 0	$\|a\| < \|c\| > 0$	e > 1

6 Constants of orbital motion

6.1 Specific mechanical energy

$$\boxed{\varepsilon \equiv \frac{E}{m} = \frac{V^2}{2} - \frac{\mu}{R}}$$

$$\boxed{V = \sqrt{2(\frac{\mu}{R} + \varepsilon)}}$$

ε = spacecraft's specific mechanical energy (km^2/s^2)
V = spacecraft's velocity (km/ sec)
μ = Gravitational parameter $km^3/sec^2 \approx 3.986 \times 10^5 km^3/s^2$ for Earth
R = spacecraft's distance from Earth's center (km)

$$\boxed{\varepsilon = -\frac{\mu}{2a}}$$

ε = spacecraft's specific mechanical energy (km^2/s^2)
μ = Gravitational parameter $km^3/sec^2 \approx 3.986 \times 10^5 km^3/s^2$ for Earth
a = semimajor axis (km)

$$\boxed{P = 2\pi\sqrt{\frac{a^3}{\mu}}}$$

$P =$ period (seconds)
$\pi = 3.14159...$ (unitless)
$a =$ semimajor axis (km)
$\mu =$ Gravitational parameter $\text{km}^3/\sec^2 \approx 3.986 \times 10^5 \text{km}^3/\text{s}^2$ for Earth

6.2 Specific angular momentum