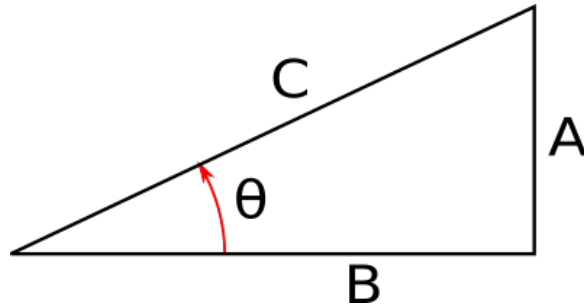


Astronautics Cheat Sheet

April 6, 2021

1 Math review

1.1 Trigonometry



SOH-CAH-TOA

- $\sin \theta = \text{Opposite} / \text{Hypotenuse}$
- $\cos \theta = \text{Adjacent} / \text{Hypotenuse}$
- $\tan \theta = \text{Opposite} / \text{Adjacent}$

Spherical Trigonometry

TODO

1.2 Vector math

Vector components

$$\vec{A} = A_I \hat{I} + A_J \hat{J} + A_K \hat{K}$$

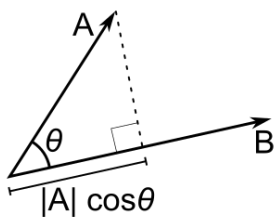
Magnitude of vector

$$\|\vec{A}\| = A = \sqrt{A_I^2 + A_J^2 + A_K^2}$$

Vector addition

$$\vec{A} + \vec{B} = (A_I + B_I)\hat{I} + (A_J + B_J)\hat{J} + (A_K + B_K)\hat{K}$$

Scalar or dot product



$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\vec{A} \cdot \vec{B} = (A_I B_I) + (A_J B_J) + (A_K B_K)$$

FIGURE 1 – Dot product

Vector or cross product

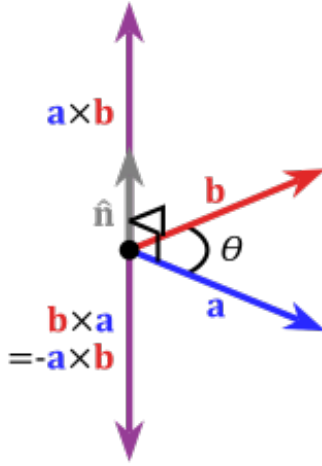


FIGURE 2 – Cross product

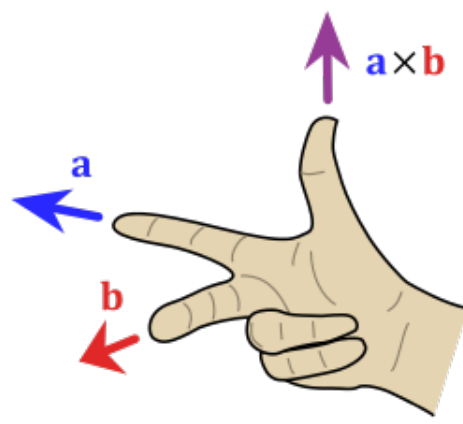


FIGURE 3 – Right hand rule

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https://commons.wikimedia.org/wiki/File:Right_hand_rule_cross_product.svg

$$\vec{A} \times \vec{B} = [(A_J)(B_K) - (B_J)(A_K)]\hat{I} - [(A_I)(B_K) - (B_I)(A_K)]\hat{J} + [(A_I)(B_J) - (B_I)(A_J)]\hat{K}$$

$$\|\vec{A} \times \vec{B}\| = AB \sin \theta$$

2 Constants

Symbol	Name	value	unit
	Earth radius	6378.14	km
μ	Gravitational parameter	3.986×10^{14}	m^3/s^2

3 Newton's laws of motion

3.1 Newton's first law of motion

A body continues in its state of rest, or of uniform motion in a straight line, unless compelled to change that state by forces impressed upon it.

$$\vec{p} = m\vec{V}$$

\vec{p} = linear momentum vector ($\text{kg} \cdot \text{m/s}$)

m = mass (kg)

\vec{V} = velocity vector (m/s)

$$\vec{H} = I\vec{\Omega}$$

\vec{H} = angular momentum vector ($\text{kg} \cdot \text{m}^2/\text{s}$)

I = moment of inertia ($\text{kg} \cdot \text{m}^2$)

$\vec{\Omega}$ = angular velocity vector (rad/s)

$$\vec{H} = \vec{R} \times m\vec{V}$$

\vec{H} = angular momentum vector ($\text{kg} \cdot \text{m}^2/\text{s}$)

\vec{R} = position (m)

m = mass (kg)

\vec{V} = velocity vector (m/s)

3.2 Newton's second law of motion

The time rate of change of an object's momentum equals the applied force.

$$\vec{F} = m\vec{a}$$

\vec{F} = force vector ($\text{kgm}/\text{s}^2 = \text{N}$)

m = mass (kg)

\vec{a} = acceleration (m/s^2)

3.3 Newton's third law of motion

When body A exerts a force on body B, body B will exert an equal, but opposite, force on body A

4 Newton's laws of universal gravitation

$$F_g = \frac{Gm_1m_2}{R^2}$$

F_g = force due to gravity (N)

G = universal gravitational constant $\approx 6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$

m_1, m_2 = masses of two bodies (kg)

R = distance between the two bodies (m)

$$a_g = \frac{\mu_{Earth}}{R^2}$$

a_g = acceleration due to gravity (m/s^2)

$\mu_{Earth} \equiv G m_{Earth} \approx 3.986 \times 10^{14} \text{ m}^3/\text{s}^2$

R = distance between the two bodies (m)

5 Laws of conservation

5.1 Conservation of momentum

In the absence of outside forces, linear and angular momentum are conserved.

5.2 Energy

E = total mechanical energy ($\text{kg m}^2/\text{s}^2$)

KE = kinetic energy ($\text{kg m}^2/\text{s}^2$)

PE = potential energy ($\text{kg m}^2/\text{s}^2$)

$$E = KE + PE$$

$$PE = m a_g h$$

$$PE = -\frac{m\mu}{R}$$

m = mass (kg)

a_g = acceleration due to gravity (m/s^2)

h = height above ref. point (m)

m = spacecraft's mass (kg)

μ = gravitational parameter (km^3/s^2)

R = distance from Earth's center (km)

$$KE = \frac{1}{2}mV^2$$

$$E = \frac{1}{2}mV^2 - \frac{m\mu}{R}$$

KE = kinetic energy ($\text{kg m}^2/\text{s}^2$)

m = mass (kg)

V = velocity (km/s)

E = total mech. energy ($\text{kg m}^2/\text{s}^2$)

m = mass (kg)

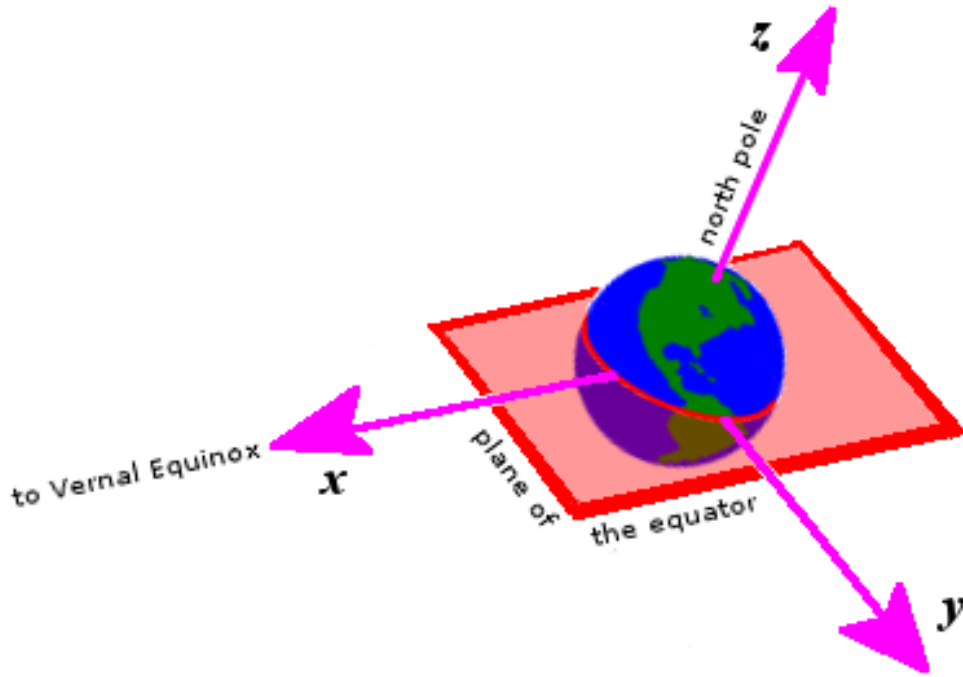
V = velocity (km/s)

μ = gravitational parameter (km^3/s^2)

R = position (km)

6 The restricted two-body problem

6.1 Coordinate systems



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FIGURE 4 – Geocentric equatorial coordinates. The origin is the centre of the Earth. The fundamental plane is the plane of the Earth's equator. The primary direction (the x axis) is the vernal equinox. A right-handed convention specifies a y axis 90° to the east in the fundamental plane; the z axis is the north polar axis. The reference frame does not rotate with the Earth, rather, the Earth rotates around the z axis.

A coordinate system (figure 7) is:

- an origin
- a fundamental plane, containing two axes, and the perpendicular to it
- a principal direction within the plane
- the third axis using the right-hand rule

6.2 Equation of motion

$$\ddot{\vec{R}} + \frac{\mu}{R^2} \frac{\vec{R}}{R} = 0$$

$\ddot{\vec{R}}$ = spacecraft's acceleration (km/s²)

μ = gravitational parameter (km³/s²)

\vec{R} = spacecraft's position vector (km)

R = magnitude of the spacecraft's position vector (km)

6.3 Orbital geometry

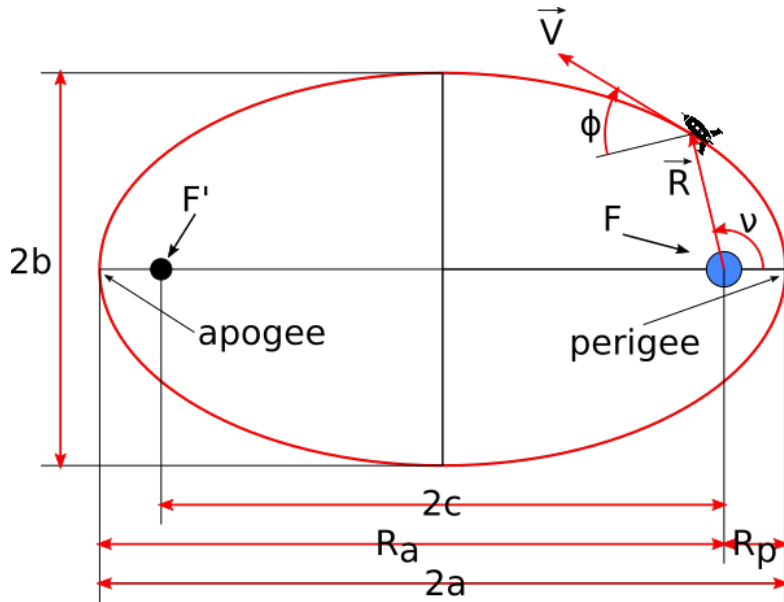


FIGURE 5 – Geometry of an elliptical orbit

$$e = \frac{2c}{2a} = \frac{R_a - R_p}{R_a + R_p}$$

e = eccentricity

\vec{R} = spacecraft's position vector

\vec{V} = spacecraft's velocity vector

F and F' = primary and vacant foci

R_p = radius of perigee

R_a = radius of apogee

$2a$ = major axis

$2b$ = minor axis

$2c$ = distance between the foci

a = semimajor axis

b = semiminor axis

ν = true anomaly

ϕ = flight-path angle

$$R = \frac{a(1 - e^2)}{1 + e \cos \nu}$$

R = magnitude of the spacecraft's position vector (km)

a = semi-major axis (km)

e = eccentricity (unitless)

ν = true anomaly (deg or rad)

Conic section	a = semimajor axis	c = one half the distance between foci	e = eccentricity
circle	a > 0	c = 0	e = 0
ellipse	a > 0	0 < c < a	0 < e < 1
parabola	a = ∞	c = ∞	e = 1
hyperbola	a < 0	a < c > 0	e > 1

7 Constants of orbital motion

7.1 Specific mechanical energy

$$\varepsilon \equiv \frac{E}{m} = \frac{V^2}{2} - \frac{\mu}{R}$$

$$V = \sqrt{2\left(\frac{\mu}{R} + \varepsilon\right)}$$

ε = spacecraft's specific mechanical energy (km²/s²)

V = spacecraft's velocity (km/sec)

μ = Gravitational parameter km³/sec² $\approx 3.986 \times 10^5$ km³/s² for Earth

R = spacecraft's distance from Earth's center (km)

$$\varepsilon = -\frac{\mu}{2a}$$

ε = spacecraft's specific mechanical energy (km²/s²)

μ = Gravitational parameter km³/sec² $\approx 3.986 \times 10^5$ km³/s² for Earth

a = semimajor axis (km)

$$P = 2\pi\sqrt{\frac{a^3}{\mu}}$$

P = period (seconds)

π = 3.14159... (unitless)

a = semimajor axis (km)

μ = Gravitational parameter km³/sec² $\approx 3.986 \times 10^5$ km³/s² for Earth

7.2 Specific angular momentum

$$\vec{h} \equiv \frac{\vec{H}}{m} = \vec{R} \times \vec{V}$$

\vec{h} = spacecraft's specific angular momentum (km²/s)

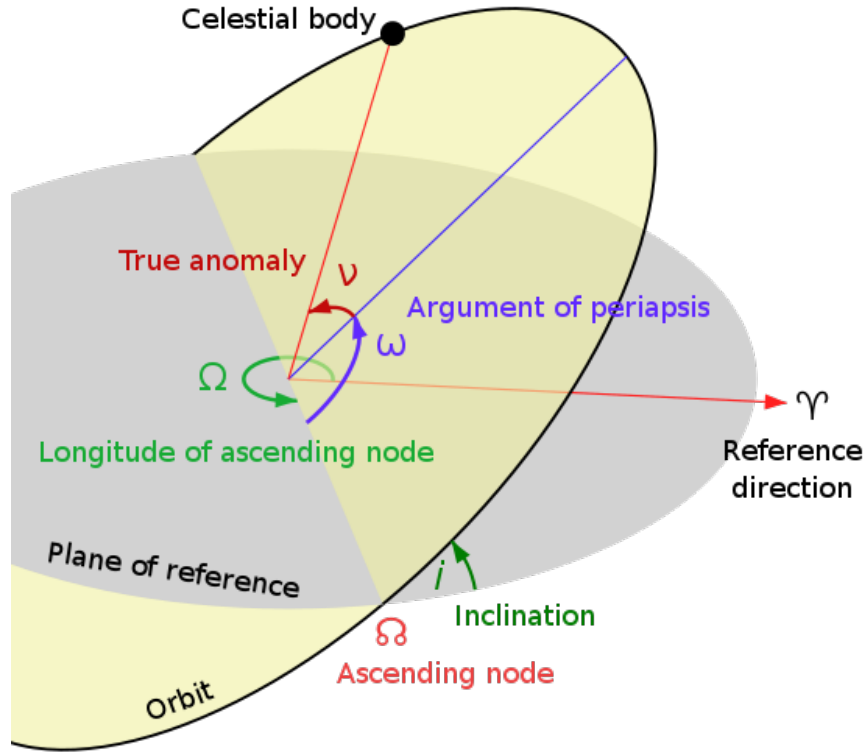
\vec{R} = spacecraft's position vector (km)

\vec{V} = spacecraft's velocity vector (km/s)

8 Describing orbits

8.1 Orbital elements

- Size: semimajor axis, a
- Shape: eccentricity, e
- Tilt: inclination, i
- Angle from vernal equinox to ascending node: right ascension (or longitude) of ascending node, Ω
- Angle from AN to Pe: argument of perigee, ω
- Angle from Pe to spacecraft: true anomaly, ν



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FIGURE 6 – Orbital elements

8.2 Computing orbital elements

Knowing \vec{R} and \vec{V} from ground tracking, we can compute orbital elements:

$$\varepsilon = \frac{V^2}{2} - \frac{\mu}{R}$$

$$a = -\frac{\mu}{2\varepsilon}$$

ε = spacecraft's specific mechanical energy (km^2/s^2)

V = spacecraft's velocity (km/sec)

μ = Gravitational parameter $\text{km}^3/\text{sec}^2 \approx 3.986 \times 10^5 \text{km}^3/\text{s}^2$ for Earth

R = spacecraft's distance from Earth's center (km)

$$\vec{e} = \frac{1}{\mu} \left[\left(V^2 - \frac{\mu}{R} \right) \vec{R} - (\vec{R} \cdot \vec{V}) \vec{V} \right]$$

\vec{e} = eccentricity vector (unitless, points at Pe)

μ = Gravitational parameter $\text{km}^3/\text{sec}^2 \approx 3.986 \times 10^5 \text{km}^3/\text{s}^2$ for Earth

V = magnitude of \vec{V} (km/s)

R = magnitude of \vec{R} (km)

\vec{R} = position vector (km)

\vec{V} = velocity vector (km/s)

$$i = \cos^{-1} \left(\frac{\hat{K} \cdot \vec{h}}{Kh} \right)$$

$$0 \leq i \leq 180^\circ$$

i = inclination (deg or rad)

\hat{K} = unit vector through the North Pole

\vec{h} = specific angular momentum vector (km^2/s)

K = magnitude of $\hat{K} = 1$

h = magnitude of \vec{h} (km^2/s)

$$\vec{n} = \hat{K} \times \vec{h}$$

\vec{n} = ascending node vector (km²/s, points at the ascending node)
 \hat{K} = unit vector through the North Pole
 \vec{h} = specific angular momentum vector (km²/s)

$$\Omega = \cos^{-1} \left(\frac{\hat{I} \cdot \vec{n}}{In} \right)$$

if $n_j \geq 0$ then $0 \leq \Omega \leq 180^\circ$
 if $n_j < 0$ then $180^\circ < \Omega < 360^\circ$

Ω = right ascension of the ascending node (deg or rad)
 \hat{I} = unit vector in the principal direction
 \vec{n} = ascending node vector (km²/s, points at the ascending node)
 I = magnitude of $\hat{I} = 1$
 n = magnitude of \vec{n} (km²/s)

$$\omega = \cos^{-1} \left(\frac{\vec{n} \cdot \vec{e}}{ne} \right)$$

if $e_K \geq 0$ then $0 \leq \omega \leq 180^\circ$
 if $e_K < 0$ then $180^\circ < \omega < 360^\circ$

ω = argument of perigee (deg or rad)
 \vec{n} = ascending node vector (km²/s, points at the ascending node)
 \vec{e} = eccentricity vector (unitless, points at perigee)
 n = magnitude of \vec{n} (km²/s)
 e = magnitude of \vec{e} (unitless)

$$\nu = \cos^{-1} \left(\frac{\vec{e} \cdot \vec{R}}{eR} \right)$$

if $\vec{R} \cdot \vec{V} \geq 0$ then $0 \leq \nu \leq 180^\circ$
 if $\vec{R} \cdot \vec{V} < 0$ then $180^\circ < \nu < 360^\circ$

ν = true anomaly (deg or rad)
 \vec{e} = eccentricity vector (unitless, points at perigee)
 \vec{R} = position vector (km)
 e = magnitude of \vec{e} (unitless)
 R = magnitude of \vec{R} (km)

8.3 Ground tracks

Nodal displacement: displacement of orbit to the west between each revolution

$$\Delta N = 360^\circ - \text{longitude between successive ascending nodes}$$

$$\text{Period (hours)} = \frac{\Delta N}{15^\circ/hr} \text{ for direct orbits only } (0 < i < 90^\circ)$$

$$a = \sqrt[3]{\mu(P/2\pi)}$$

a = semimajor axis (km)
 μ = Gravitational parameter km³/sec² $\approx 3.986 \times 10^5$ km³/s² for Earth
 P = period (s)
 π = 3.14159...(unitless)

inclination = highest latitude

- For a direct orbit ($0 < i < 90^\circ$), inclination = highest north or south latitude.
- For a retrograde orbit ($90^\circ < i < 180^\circ$), inclination = 180 - max latitude

9 Maneuvers

9.1 Hohmann Transfers

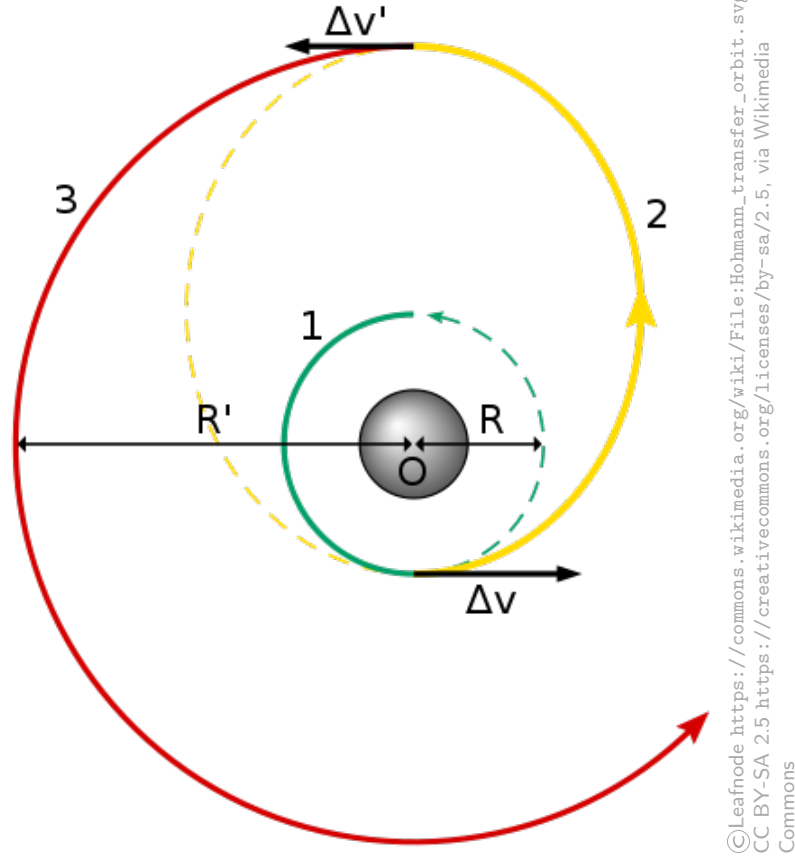


FIGURE 7 – Transfer orbit

Specific mechanical energy of each orbit:

- $\varepsilon_{\text{orbit 1}} = -\frac{\mu}{2a_{\text{orbit 1}}}$
- $\varepsilon_{\text{orbit 2}} = -\frac{\mu}{2a_{\text{orbit 2}}}$
- $\varepsilon_{\text{transfer}} = -\frac{\mu}{2a_{\text{transfer}}}$ with $2a_{\text{transfer}} = R + R'$

Velocity at each maneuver point using specific mechanical energy equation from section 8.2:

- $V_{\text{orbit 1}} = \sqrt{2 \left(\frac{\mu}{R} + \varepsilon_{\text{orbit 1}} \right)}$
- $V_{\text{orbit 2}} = \sqrt{2 \left(\frac{\mu}{R'} + \varepsilon_{\text{orbit 2}} \right)}$
- $V_{\text{transfer at orbit 1}} = \sqrt{2 \left(\frac{\mu}{R} + \varepsilon_{\text{transfer}} \right)}$
- $V_{\text{transfer at orbit 2}} = \sqrt{2 \left(\frac{\mu}{R'} + \varepsilon_{\text{transfer}} \right)}$
- $\Delta V_1 = |V_{\text{transfer at orbit 1}} - V_{\text{orbit 1}}|$
- $\Delta V_2 = |V_{\text{orbit 2}} - V_{\text{transfer at orbit 2}}|$
- $\Delta V_{\text{total}} = \Delta V_1 + \Delta V_2$

Time of flight = half period

$$TOF = \frac{P}{2} = \pi \sqrt{\frac{a_{transfer}^3}{\mu}}$$

TOF = spacecraft's time of flight

P = orbital period (s)

a = semimajor axis of transfer orbit (km)

μ = Gravitational parameter $\text{km}^3/\text{sec}^2 \approx 3.986 \times 10^5 \text{km}^3/\text{s}^2$ for Earth

9.2 Plane changes

9.2.1 Simple plane changes

$$\Delta V_{simple} = 2V_{initial} \sin \frac{\theta}{2}$$

ΔV_{simple} = velocity change for a simple plane change (km/s)

$V_{initial} = V_{final}$ = velocities in the initial and final orbits (km/s)

θ = plane-change angle (deg or rad)

Change of inclination: maneuver at ascending/descending node.

Change of right ascension of ascending node, Ω : maneuver at North or South pole for polar orbit

9.2.2 Combined plane change

Hohmann transfer combined with plane change at apogee (more efficient):

$$\Delta V_{combined} = \sqrt{|\vec{V}_{initial}|^2 + |\vec{V}_{final}|^2 - 2|\vec{V}_{initial}||\vec{V}_{final}|\cos\theta}$$

$\Delta V_{combined}$ = velocity change for a combined plane change (km/s)

$|\vec{V}_{initial}|$ = magnitude of the velocity in the initial orbit (km/s)

$|\vec{V}_{final}|$ = magnitude of the velocity in the final orbit (km/s)

θ = plane-change angle (deg or rad)