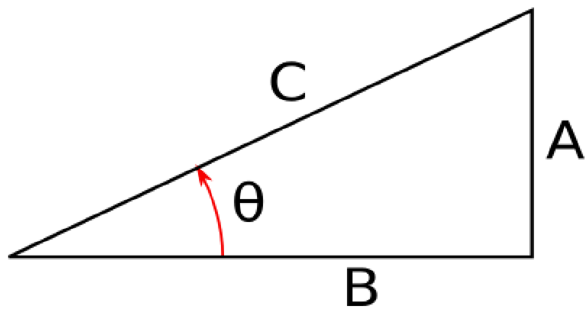
Astronautics Cheat Sheet

March 24, 2021

1 Math review

1.1 Trigonometry



SOH-CAH-TOA

- $\sin\,\theta$ = Opposite / Hypotenuse
- $\cos\,\theta$ = Adjacent / Hypotenuse
- $\tan\,\theta$ = Opposite / Adjacent

Spherical Trigonometry

TODO

1.2 Vector math

Vector components

$\vec{A} = A_I\hat{I} + A_J\hat{J} + A_K\hat{K}$

Magnitude of vector

$\|\vec{A}\| = A = \sqrt{A_I^2 + A_J^2 + A_K^2}$

Vector addition

$\vec{A} + \vec{B} = (A_I + B_I)\hat{I} + (A_J + B_J)\hat{J} + (A_K + B_K)\hat{K}$

Scalar or dot product

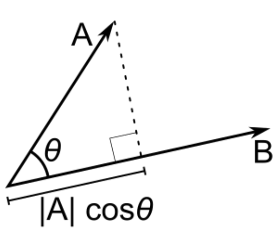


Figure 1 – Dot product

$$\boxed{\vec{A}\cdot\vec{B} = AB\cos\theta}$$

$$\boxed{\theta = \cos^{-}1\frac{\vec{A}\cdot\vec{B}}{AB}}$$

$$\boxed{\vec{A}\cdot\vec{B} = (A_I B_I) + (A_J B_J) + (A_K B_K)}$$

Vector or cross product

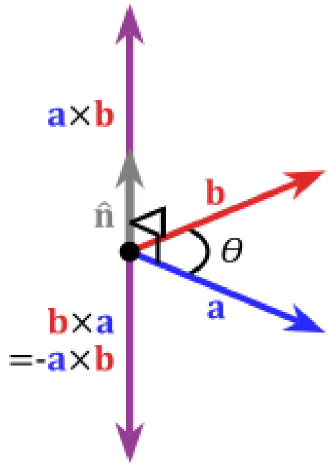


FIGURE 2 – Cross product

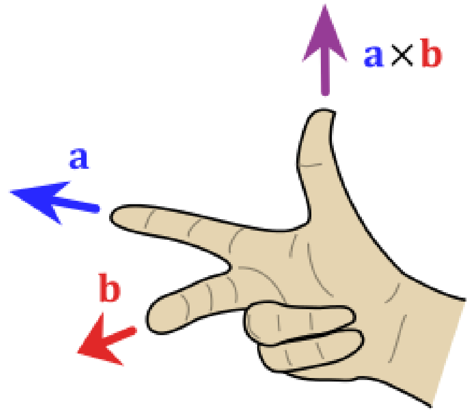


FIGURE 3 – Right hand rule

$$\boxed{\vec{A}\times\vec{B} = [(A_J)(B_K)-(B_J)(A_K)]\hat{I} - [(A_I)(B_K)-(B_I)(A_K)]\hat{J} + [(A_I)(B_J)-(B_I)(A_J)]}$$

$$\boxed{\|\vec{A}\times\vec{B}\| = AB\sin\theta}$$

2 Constants

Symbol	Name	value	unit
	Earth radius	6378.14	km
μ	Gravitational parameter	3.986×10^{14}	$\mathrm{m^3/s^2}$

3 Newton's laws of motion

3.1 Newton's first law of motion

A body continues in its state of rest, or of uniform motion in a straight line, unless compelled to change that state by forces impressed upon it.

$$\boxed{\vec{p} = m\vec{V}}$$

$$\begin{aligned}\vec{p} &= \text{linear momentum vector (kg}\cdot\text{m/s)}\\ m &= \text{mass (kg)}\\ \vec{V} &= \text{velocity vector (m/s)}\end{aligned}$$

$$\boxed{\vec{H} = I\vec{\Omega}}$$

$$\begin{aligned}\vec{H} &= \text{angular momentum vector (kg}\cdot\text{m}^2\text{/s)}\\ I &= \text{moment of inertia (kg}\cdot\text{m}^2)\\ \vec{\Omega} &= \text{angular velocity vector (rad/s)}\end{aligned}$$

$$\boxed{\vec{H} = \vec{R}\times m\vec{V}}$$

$$\vec{H} = \text{angular momentum vector } (\text{kg} \cdot \text{m}^2/\text{s})$$
$$\vec{R} = \text{position (m)}$$
$$m = \text{mass (kg)}$$
$$\vec{V} = \text{velocity vector (m/s)}$$

3.2 Newton's second law of motion

The time rate of change of an object's momentum equals the applied force.

$$\boxed{\vec{F} = m\vec{a}}$$

$$\vec{F} = \text{force vector } (\text{kgm/s}^2 = \text{N})$$
$$m = \text{mass (kg)}$$
$$\vec{a} = \text{acceleration } (\text{m/s}^2)$$

3.3 Newton's third law of motion

When body A exerts a force on body B, body B will exert an equal, but opposite, force on body A

4 Newton's laws of universal gravitation

$$\boxed{F_g = \frac{Gm_1m_2}{R^2}}$$

$$F_g = \text{force due to gravity (N)}$$
$$G = \text{universal gravitational constant} \approx 6.674 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2$$
$$m_1, m_2 = \text{masses of two bodies (kg)}$$
$$R = \text{distance between the two bodies (m)}$$

$$\boxed{a_g = \frac{\mu_{Earth}}{R^2}}$$

$$a_g = \text{acceleration due to gravity } (\text{m/s}^2)$$
$$\mu_{Earth} \equiv G\, m_{Earth} \approx 3.986 \times 10^{14}\,\text{m}^3/\text{s}^2$$
$$R = \text{distance between the two bodies (m)}$$

5 Laws of conservation

5.1 Conservation of momentum

In the absence of outside forces, linear and angular momentum are conserved.

5.2 Energy

$$E = \text{total mechanical energy } (\text{kg}\,\text{m}^2/\text{s}^2)$$
$$KE = \text{kinetic energy } (\text{kg}\,\text{m}^2/\text{s}^2)$$
$$PE = \text{potential energy } (\text{kg}\,\text{m}^2/\text{s}^2)$$

$$\boxed{E = KE + PE}$$

$$\boxed{PE = m\,a_g h}$$

m = mass (kg)
a_g = acceleration due to gravity (m/s^2)
h = height above ref. point (m)

$$\boxed{PE = -\frac{m\mu}{R}}$$

m = spacecraft's mass (kg)
μ = gravitational parameter (km^3/s^2)
R = distance from Earth's center (km)

$$\boxed{KE = \frac{1}{2}mV^2}$$

KE = kinetic energy (kg m^2/s^2)
m = mass (kg)
V = velocity (km/s)

$$\boxed{E = \frac{1}{2}mV^2 - \frac{m\mu}{R}}$$

E = total mech. energy (kg m^2/s^2)
m = mass (kg)
V = velocity (km/s)
μ = gravitational parameter (km^3/s^2)
R = position (km)

6 The restricted two-body problem

6.1 Coordinate systems

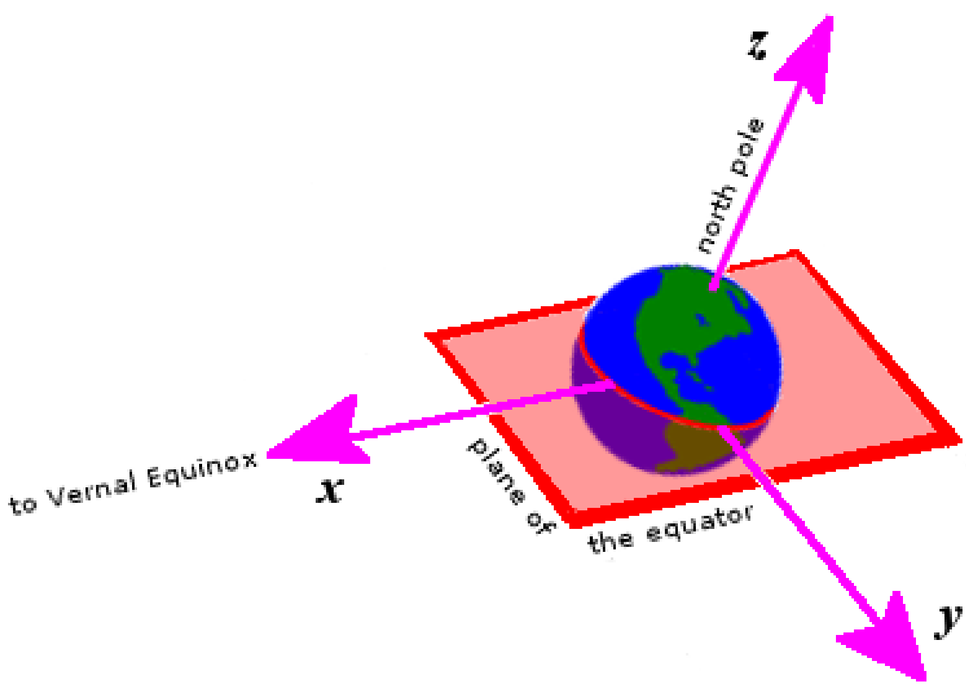


FIGURE 4 – Geocentric equatorial coordinates. The origin is the centre of the Earth. The fundamental plane is the plane of the Earth's equator. The primary direction (the x axis) is the vernal equinox. A right-handed convention specifies a y axis 90° to the east in the fundamental plane; the z axis is the north polar axis. The reference frame does not rotate with the Earth, rather, the Earth rotates around the z axis.

A coordinate system (figure 5) is:

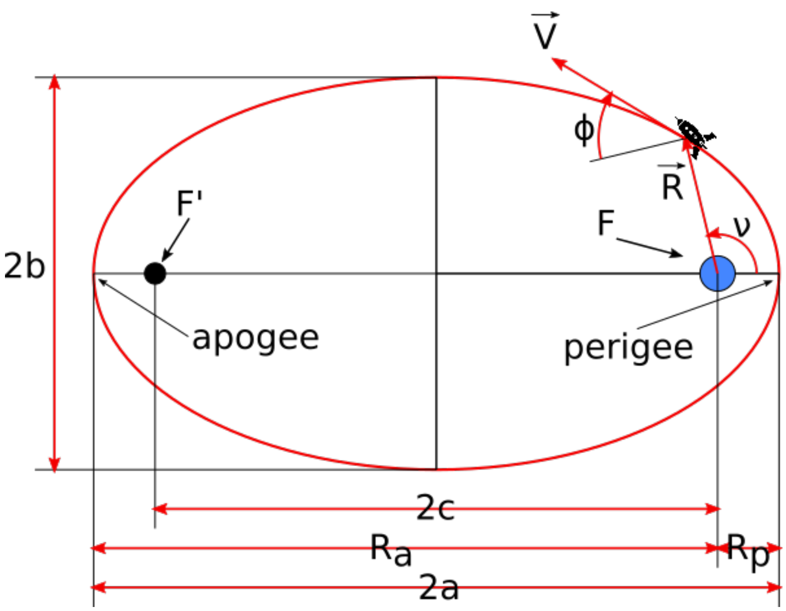
- **an origin**
- **a fundamental plane**, containing two axes, and the perpendicular to it
- **a principal direction** within the plane
- **the third axis** using the right-hand rule

6.2 Equation of motion

$$\boxed{\ddot{\vec{R}} + \frac{\mu}{R^2}\frac{\vec{R}}{R} = 0}$$

$\ddot{\vec{R}} =$ spacecraft's acceleration (km/s^2)
$\mu =$ gravitational parameter (km^3/s^2)
$\vec{R} =$ spacecraft's position vector (km)
$R =$ magnitude of the spacecraft's position vector (km)

6.3 Orbital geometry



$\vec{R} =$ spacecraft's position vector
$\vec{V} =$ spacecraft's velocity vector
$FandF' =$ primary and vacant foci
$R_p =$ radius of perigee
$R_a =$ radius of apogee
$2a =$ major axis
$2b =$ minor axis
$2c =$ distance between the foci
$a =$ semimajor axis
$b =$ semiminor axis
$\nu =$ true anomaly
$\phi =$ flight-path angle

FIGURE 5 – Geometry of an elliptical orbit

$$\boxed{e = \frac{2c}{2a} = \frac{R_a - R_p}{R_a + R_p}}$$

$e = eccentricity$

$$\boxed{R = \frac{a(1-e^2)}{1+ecos\nu}}$$

$R =$ magnitude of the spacecraft's position vector (km)
$a =$ semi-major axis (km)
$e =$ eccentricity (unitless)
$\nu =$ true anomaly (deg or rad)

Conic section	a = semimajor axis	c = one half the distance between foci	e = eccentricity
circle	a > 0	c = 0	e = 0
ellipse	a > 0	0 < c < a	0 < e < 1
parabola	a = ∞	c = ∞	e = 1
hyperbola	a < 0	$\|a\| < \|c\| > 0$	e > 1

7 Constants of orbital motion

7.1 Specific mechanical energy

$$\boxed{\varepsilon \equiv \frac{E}{m} = \frac{V^2}{2} - \frac{\mu}{R}}$$

$$\boxed{V = \sqrt{2(\frac{\mu}{R} + \varepsilon)}}$$

$\varepsilon =$ spacecraft's specific mechanical energy ($\mathrm{km}^2/\mathrm{s}^2$)
$V =$ spacecraft's velocity (km/ sec)
$\mu =$ Gravitational parameter $\mathrm{km}^3/\sec^2 \approx 3.986 \times 10^5 \mathrm{km}^3/\mathrm{s}^2$ for Earth
$R =$ spacecraft's distance from Earth's center (km)

$$\boxed{\varepsilon = -\frac{\mu}{2a}}$$

$\varepsilon =$ spacecraft's specific mechanical energy ($\mathrm{km}^2/\mathrm{s}^2$)
$\mu =$ Gravitational parameter $\mathrm{km}^3/\sec^2 \approx 3.986 \times 10^5 \mathrm{km}^3/\mathrm{s}^2$ for Earth
$a =$ semimajor axis (km)

$$\boxed{P = 2\pi\sqrt{\frac{a^3}{\mu}}}$$

$P =$ period (seconds)
$\pi =$ 3.14159... (unitless)
$a =$ semimajor axis (km)
$\mu =$ Gravitational parameter $\mathrm{km}^3/\sec^2 \approx 3.986 \times 10^5 \mathrm{km}^3/\mathrm{s}^2$ for Earth

7.2 Specific angular momentum

$$\boxed{\vec{h} \equiv \frac{\vec{H}}{m} = \vec{R} \times \vec{V}}$$

$\vec{h} =$ spacecraft's specific angular momentum (km^2/s)
$\vec{R} =$ spacecraft's position vector (km)
$\vec{V} =$ spacecraft's velocity vector (km/s)

8 Describing orbits

8.1 Orbital elements

- Size: semimajor axis, a
- Shape: eccentricity, e
- Tilt: inclination, i
- Angle from vernal equinox to ascending node: right ascension of ascending node, Ω
- Angle from AN to Pe: argument of perigee, ω
- Angle from Pe to spacecraft: true anomaly, ν

8.2 Computing orbital elements

Knowing $\vec{R}$ and $\vec{V}$ from ground tracking, we can compute orbital elements:

$$\boxed{\varepsilon = \frac{V^2}{2} - \frac{\mu}{R}}$$

$$\boxed{a = -\frac{\mu}{2\varepsilon}}$$

$\varepsilon =$ spacecraft's specific mechanical energy (km^2/s^2)
$V =$ spacecraft's velocity (km/ sec)
$\mu =$ Gravitational parameter km^3/ sec^2 $\approx 3.986 \times 10^5$km^3/s^2 for Earth
$R =$ spacecraft's distance from Earth's center (km)

$$\boxed{\vec{e} = \frac{1}{\mu}\left[\left(V^2 - \frac{\mu}{R}\right)\vec{R} - (\vec{R}\cdot\vec{V})\vec{V}\right]}$$

$\vec{e} =$ eccentricity vetor (unitless, points at Pe)
$\mu =$ Gravitational parameter km^3/ sec^2 $\approx 3.986 \times 10^5$km^3/s^2 for Earth
$V =$ magnitude of $\vec{V}$ (km/s)
$R =$ magnitude of $\vec{R}$ (km)
$\vec{R} =$ position vector (km)
$\vec{V} =$ velocity vector (km/s)

$$\boxed{i = \cos^{-1}\left(\frac{\hat{K}\cdot\vec{h}}{Kh}\right)}$$

$i =$ inclination (deg or rad)
$\hat{K} =$ unit vector through the North Pole
$\vec{h} =$ specific angular momentum vector (km^2/s)
$K =$ magnitude of $\hat{K} = 1$
$h =$ magnitude of $\vec{h}$ (km^2/s)

$$\boxed{\vec{n} = \hat{K} \times \vec{h}}$$

$\vec{n} =$ ascending node vector (km^2/s, points at the ascending node)
$\hat{K} =$ unit vector through the North Pole
$\vec{h} =$ specific angular momentum vector (km^2/s)

$$\boxed{\Omega = \cos^{-1}\left(\frac{\hat{I}\cdot\vec{n}}{In}\right)}$$

$\Omega =$ right ascension of the ascending node (deg or rad)
$\hat{I} =$ unit vector in the principal direction
$\vec{n} =$ ascending node vector (km^2/s, points at the ascending node)
$I =$ magnitude of $\hat{I} = 1$
$n =$ magnitude of $\vec{n}$ (km^2/s)

$$\boxed{\omega = \cos^{-1}\left(\frac{\vec{n}\cdot\vec{e}}{ne}\right)}$$

$\omega =$ argument of perigee (deg or rad)
$\vec{n} =$ ascending node vector (km^2/s, points at the ascending node)
$\vec{e} =$ eccentricity vector (unitless, points at perigee)
$n =$ magnitude of $\vec{n}$ (km^2/s)
$e =$ magnitude of $\vec{e}$ (unitless)

$$\boxed{\nu = \cos^{-1}\left(\frac{\vec{e} \cdot \vec{R}}{eR}\right)}$$

$\nu =$ true anomaly (deg or rad)

$\vec{e} =$ eccentricity vector (unitless, points at perigee)

$\vec{R} =$ position vector (km

$e =$ magnitude of $\vec{e}$ (unitless)

$R =$ magnitude of $\vec{R}$ (km)