Astronautics Cheat Sheet

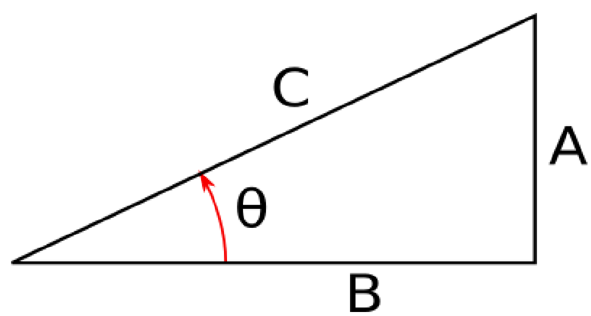
April 20, 2021

1 Constants

Symbol	Name	value	unit
	Earth radius	6378.14	km
G	Universal gravitational constant	6.67×10^{-11}	$\mathrm{N} \cdot \mathrm{m}^2/\mathrm{km}^2$
μ	Gravitational parameter of Earth	3.986×10^5	$\mathrm{km}^3/\mathrm{s}^2$
μ_{sun}	Gravitational parameter of the Sun	1.327×10^{11}	$\mathrm{km}^3/\mathrm{s}^2$
m_{sun}	Sun's mass	1.989×10^{30}	kg

2 Math review

2.1 Trigonometry



SOH-CAH-TOA

- sin θ = Opposite / Hypotenuse
- cos θ = Adjacent / Hypotenuse
- tan θ = Opposite / Adjacent

Inverse functions

- $\sin^{-1}(\sin\theta) = \theta \ or \ \pi - \theta$
- $\cos^{-1}(\cos\theta) = \theta \ or \ 2\pi - \theta$
- $\tan^{-1}(\tan\theta) = \theta \ or \ \theta + n\pi$

Spherical Trigonometry

TODO

2.2 Vector math

Vector components

$\vec{A} = A_I\hat{I} + A_J\hat{J} + A_K\hat{K}$

Magnitude of vector

$\|\vec{A}\| = A = \sqrt{A_I^2 + A_J^2 + A_K^2}$

Vector addition

$\vec{A} + \vec{B} = (A_I + B_I)\hat{I} + (A_J + B_J)\hat{J} + (A_K + B_K)\hat{K}$

Scalar or dot product

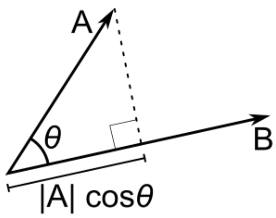


FIGURE 1 – Dot product

$$\boxed{\vec{A} \cdot \vec{B} = AB\cos\theta}$$

$$\boxed{\theta = \cos^{-}1\frac{\vec{A} \cdot \vec{B}}{AB}}$$

$$\boxed{\vec{A} \cdot \vec{B} = (A_I B_I) + (A_J B_J) + (A_K B_K)}$$

Vector or cross product

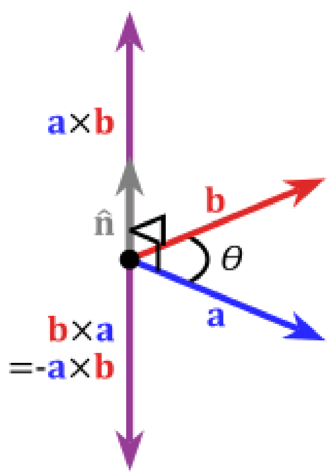


FIGURE 2 – Cross product

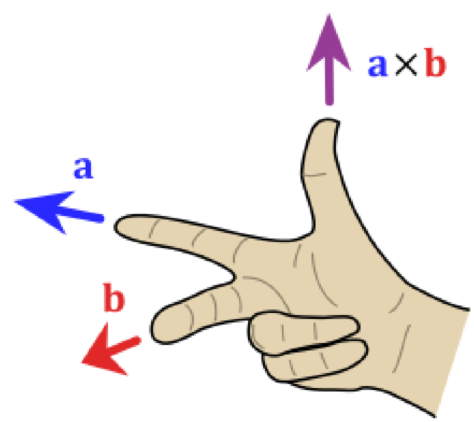


FIGURE 3 – Right hand rule

$$\boxed{\vec{A} \times \vec{B} = [(A_J)(B_K) - (B_J)(A_K)]\hat{I} - [(A_I)(B_K) - (B_I)(A_K)]\hat{J} + [(A_I)(B_J) - (B_I)(A_J)]}$$

$$\boxed{\|\vec{A} \times \vec{B}\| = AB\sin\theta}$$

3 Newton's laws of motion

3.1 Newton's first law of motion

A body continues in its state of rest, or of uniform motion in a straight line, unless compelled to change that state by forces impressed upon it.

$$\boxed{\vec{p} = m\vec{V}}$$

$\vec{p}$ = linear momentum vector (kg · m/s)
m = mass (kg)
$\vec{V}$ = velocity vector (m/s)

$$\boxed{\vec{H} = I\vec{\Omega}}$$

$$\begin{aligned}
\vec{H} &= \text{angular momentum vector (kg} \cdot \text{m}^2\text{/s)} \\
I &= \text{moment of inertia (kg} \cdot \text{m}^2\text{)} \\
\vec{\Omega} &= \text{angular velocity vector (rad/s)}
\end{aligned}$$

$$\boxed{\vec{H} = \vec{R} \times m\vec{V}}$$

$$\begin{aligned}
\vec{H} &= \text{angular momentum vector (kg} \cdot \text{m}^2\text{/s)} \\
\vec{R} &= \text{position (m)} \\
m &= \text{mass (kg)} \\
\vec{V} &= \text{velocity vector (m/s)}
\end{aligned}$$

3.2 Newton's second law of motion

The time rate of change of an object's momentum equals the applied force.

$$\boxed{\vec{F} = m\vec{a}}$$

$$\begin{aligned}
\vec{F} &= \text{force vector (kgm/s}^2 = \text{N)} \\
m &= \text{mass (kg)} \\
\vec{a} &= \text{acceleration (m/s}^2\text{)}
\end{aligned}$$

3.3 Newton's third law of motion

When body A exerts a force on body B, body B will exert an equal, but opposite, force on body A

4 Newton's laws of universal gravitation

$$\boxed{F_g = \frac{Gm_1m_2}{R^2}}$$

$$\begin{aligned}
F_g &= \text{force due to gravity (N)} \\
G &= \text{universal gravitational constant} \approx 6.674 \times 10^{-11}\,\text{N} \cdot \text{m}^2\text{/kg}^2 \\
m_1, m_2 &= \text{masses of two bodies (kg)} \\
R &= \text{distance between the two bodies (m)}
\end{aligned}$$

$$\boxed{a_g = \frac{\mu_{Earth}}{R^2}}$$

$$\begin{aligned}
a_g &= \text{acceleration due to gravity (m/s}^2\text{)} \\
\mu_{Earth} &\equiv G\,m_{Earth} \approx 3.986 \times 10^{14}\,\text{m}^3\text{/s}^2 \\
R &= \text{distance between the two bodies (m)}
\end{aligned}$$

5 Laws of conservation

5.1 Conservation of momentum

In the absence of outside forces, linear and angular momentum are conserved.

5.2 Energy

$$\boxed{E = KE + PE}$$

E = total mechanical energy ($\mathrm{kg\,m^2/s^2}$)
KE = kinetic energy ($\mathrm{kg\,m^2/s^2}$)
PE = potential energy ($\mathrm{kg\,m^2/s^2}$)

$$\boxed{PE = m\,a_g h}$$

m = mass (kg)
a_g = acceleration due to gravity ($\mathrm{m/s^2}$)
h = height above ref. point (m)

$$\boxed{PE = -\frac{m\mu}{R}}$$

m = spacecraft's mass (kg)
μ = gravitational parameter ($\mathrm{km^3/s^2}$)
R = distance from Earth's center (km)

$$\boxed{KE = \frac{1}{2}mV^2}$$

KE = kinetic energy ($\mathrm{kg\,m^2/s^2}$)
m = mass (kg)
V = velocity (km/s)

$$\boxed{E = \frac{1}{2}mV^2 - \frac{m\mu}{R}}$$

E = total mech. energy ($\mathrm{kg\,m^2/s^2}$)
m = mass (kg)
V = velocity (km/s)
μ = gravitational parameter ($\mathrm{km^3/s^2}$)
R = position (km)

6 The restricted two-body problem

6.1 Coordinate systems

A coordinate system (figure 7) is:

- **an origin**
- **a fundamental plane**, containing two axes, and the perpendicular to it
- **a principal direction** within the plane
- **the third axis** using the right-hand rule

6.2 Equation of motion

$$\boxed{\ddot{\vec{R}} + \frac{\mu}{R^2}\frac{\vec{R}}{R} = 0}$$

$\ddot{\vec{R}}$ = spacecraft's acceleration ($\mathrm{km/s^2}$)
μ = gravitational parameter ($\mathrm{km^3/s^2}$)
$\vec{R}$ = spacecraft's position vector (km)
R = magnitude of the spacecraft's position vector (km)

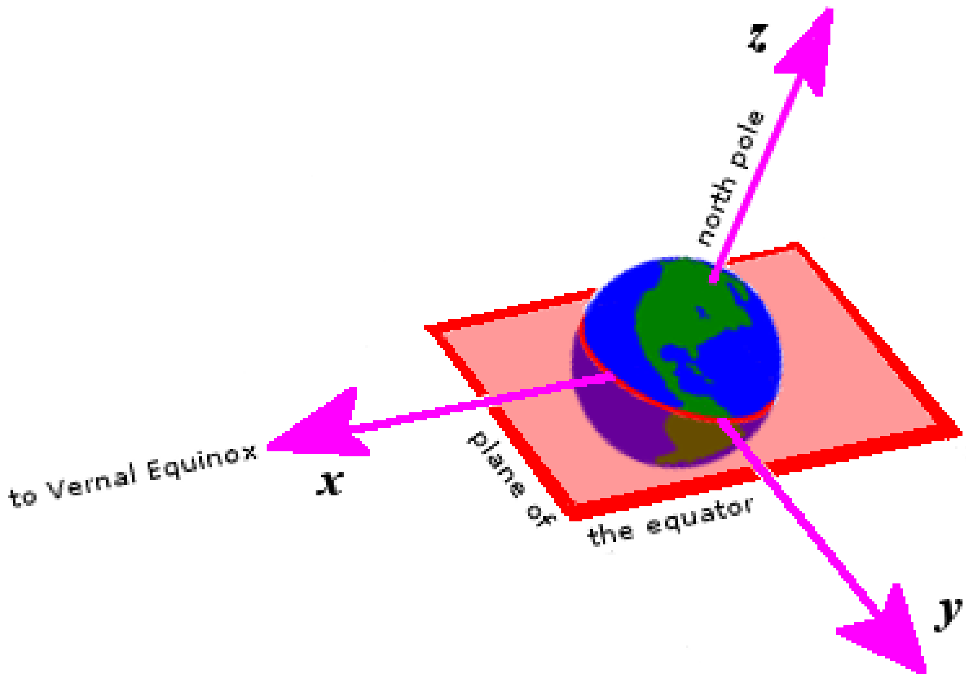


FIGURE 4 – Geocentric equatorial coordinates. The origin is the centre of the Earth. The fundamental plane is the plane of the Earth's equator. The primary direction (the x axis) is the vernal equinox. A right-handed convention specifies a y axis 90° to the east in the fundamental plane; the z axis is the north polar axis. The reference frame does not rotate with the Earth, rather, the Earth rotates around the z axis.

6.3 Orbital geometry

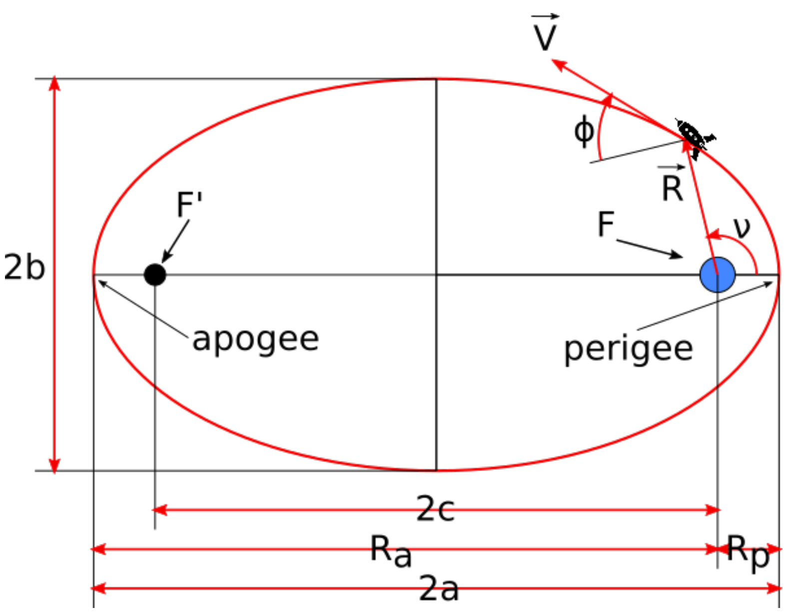


FIGURE 5 – Geometry of an elliptical orbit

$$\begin{aligned}
\vec{R} &= \text{spacecraft's position vector} \\
\vec{V} &= \text{spacecraft's velocity vector} \\
F and F' &= \text{primary and vacant foci} \\
R_p &= \text{radius of perigee} \\
R_a &= \text{radius of apogee} \\
2a &= \text{major axis} \\
2b &= \text{minor axis} \\
2c &= \text{distance between the foci} \\
a &= \text{semimajor axis} \\
b &= \text{semiminor axis} \\
\nu &= \text{true anomaly} \\
\phi &= \text{flight-path angle}
\end{aligned}$$

$$\boxed{e = \frac{2c}{2a} = \frac{R_a - R_p}{R_a + R_p}}$$

$e = eccentricity$

$$\boxed{R = \frac{a(1-e^2)}{1 + ecos\nu}}$$

$$\begin{aligned}
R &= \text{magnitude of the spacecraft's position vector (km)} \\
a &= \text{semi-major axis (km)} \\
e &= \text{eccentricity (unitless)} \\
\nu &= \text{true anomaly (deg or rad)}
\end{aligned}$$

Conic section	a = semimajor axis	c = one half the distance between foci	e = eccentricity
circle	a > 0	c = 0	e = 0
ellipse	a > 0	0 < c < a	0 < e < 1
parabola	a = ∞	c = ∞	e = 1
hyperbola	a < 0	$\lvert a \rvert < \lvert c \rvert > 0$	e > 1

7 Constants of orbital motion

7.1 Specific mechanical energy

$$\boxed{\varepsilon \equiv \frac{E}{m} = \frac{V^2}{2} - \frac{\mu}{R}}$$

$$\boxed{V = \sqrt{2(\frac{\mu}{R} + \varepsilon)}}$$

$\varepsilon =$ spacecraft's specific mechanical energy (km^2/s^2)
$V =$ spacecraft's velocity (km/ sec)
$\mu =$ Gravitational parameter km^3/ sec^2 $\approx 3.986 \times 10^5$km^3/s^2 for Earth
$R =$ spacecraft's distance from Earth's center (km)

$$\boxed{\varepsilon = -\frac{\mu}{2a}}$$

$\varepsilon =$ spacecraft's specific mechanical energy (km^2/s^2)
$\mu =$ Gravitational parameter km^3/ sec^2 $\approx 3.986 \times 10^5$km^3/s^2 for Earth
$a =$ semimajor axis (km)

$$\boxed{P = 2\pi\sqrt{\frac{a^3}{\mu}}}$$

$P =$ period (seconds)
$\pi =$ 3.14159... (unitless)
$a =$ semimajor axis (km)
$\mu =$ Gravitational parameter km^3/ sec^2 $\approx 3.986 \times 10^5$km^3/s^2 for Earth

7.2 Specific angular momentum

$$\boxed{\vec{h} \equiv \frac{\vec{H}}{m} = \vec{R} \times \vec{V}}$$

$\vec{h} =$ spacecraft's specific angular momentum (km^2/s)
$\vec{R} =$ spacecraft's position vector (km)
$\vec{V} =$ spacecraft's velocity vector (km/s)

8 Describing orbits

8.1 Orbital elements

- Size: semimajor axis, a
- Shape: eccentricity, e
- Tilt: inclination, i
- Angle from vernal equinox to ascending node: right ascension (or longitude) of ascending node, Ω
- Angle from AN to Pe: argument of perigee, ω
- Angle from Pe to spacecraft: true anomaly, ν

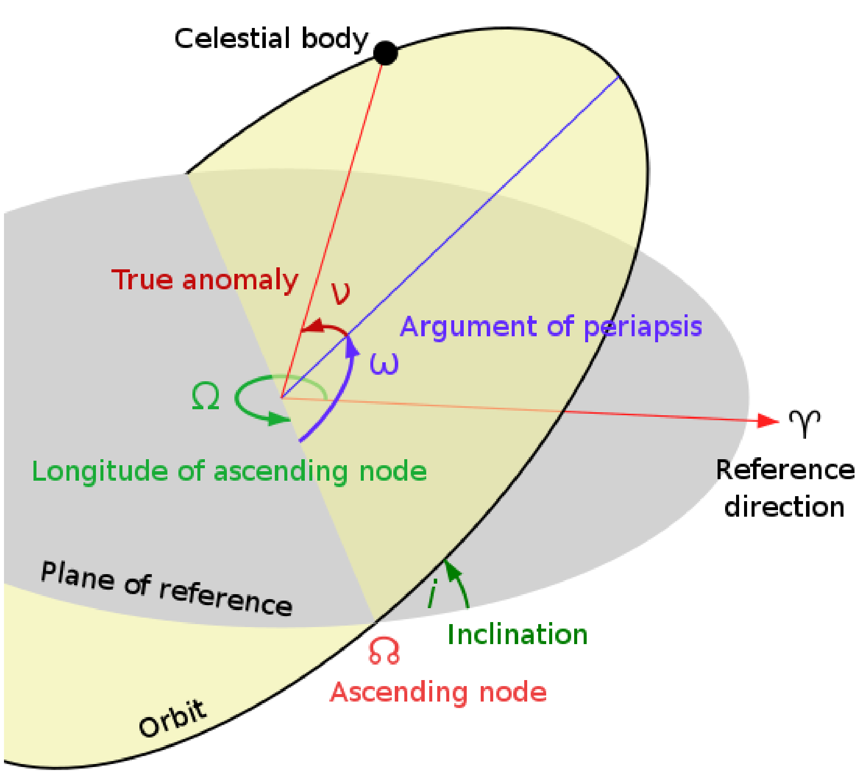


FIGURE 6 – Orbital elements

8.2 Computing orbital elements

Knowing $\vec{R}$ and $\vec{V}$ from ground tracking, we can compute orbital elements:

$$\boxed{\varepsilon = \frac{V^2}{2} - \frac{\mu}{R}}$$

$$\boxed{a = -\frac{\mu}{2\varepsilon}}$$

$\varepsilon =$ spacecraft's specific mechanical energy $(\mathrm{km}^2/\mathrm{s}^2)$

$V =$ spacecraft's velocity $(\mathrm{km}/\sec)$

$\mu =$ Gravitational parameter $\mathrm{km}^3/\sec^2 \approx 3.986 \times 10^5 \mathrm{km}^3/\mathrm{s}^2$ for Earth

$R =$ spacecraft's distance from Earth's center (km)

$$\boxed{\vec{e} = \frac{1}{\mu}\left[\left(V^2 - \frac{\mu}{R}\right)\vec{R} - (\vec{R}\cdot\vec{V})\vec{V}\right]}$$

$\vec{e} =$ eccentricity vetor (unitless, points at Pe)

$\mu =$ Gravitational parameter $\mathrm{km}^3/\sec^2 \approx 3.986 \times 10^5 \mathrm{km}^3/\mathrm{s}^2$ for Earth

$V =$ magnitude of $\vec{V}$ (km/s)

$R =$ magnitude of $\vec{R}$ (km)

$\vec{R} =$ position vector (km)

$\vec{V} =$ velocity vector (km/s)

$$\boxed{i = \cos^{-1}\left(\frac{\hat{K}\cdot\vec{h}}{Kh}\right)}$$

$$0 \leq i \leq 180°$$

$i =$ inclination (deg or rad)

$\hat{K} =$ unit vector through the North Pole

$\vec{h} =$ specific angular momentum vector $(\mathrm{km}^2/\mathrm{s})$

$K =$ magnitude of $\hat{K} = 1$

$h =$ magnitude of $\vec{h}$ $(\mathrm{km}^2/\mathrm{s})$

$$\boxed{\vec{n} = \hat{K} \times \vec{h}}$$

$\vec{n}$ = ascending node vector (km^2/s, points at the ascending node)
$\hat{K}$ = unit vector through the North Pole
$\vec{h}$ = specific angular momentum vector (km^2/s)

$$\boxed{\Omega = \cos^{-1}\left(\frac{\hat{I} \cdot \vec{n}}{In}\right)}$$

if $n_j \geq 0$ then $0 \leq \Omega \leq 180°$

if $n_j < 0$ then $180° < \Omega < 360°$

Ω = right ascension of the ascending node (deg or rad)
$\hat{I}$ = unit vector in the principal direction
$\vec{n}$ = ascending node vector (km^2/s, points at the ascending node)
I = magnitude of $\hat{I} = 1$
n = magnitude of $\vec{n}$ (km^2/s)

$$\boxed{\omega = \cos^{-1}\left(\frac{\vec{n} \cdot \vec{e}}{ne}\right)}$$

if $e_K \geq 0$ then $0 \leq \omega \leq 180°$

if $e_K < 0$ then $180° < \omega < 360°$

ω = argument of perigee (deg or rad)
$\vec{n}$ = ascending node vector (km^2/s, points at the ascending node)
$\vec{e}$ = eccentricity vector (unitless, points at perigee)
n = magnitude of $\vec{n}$ (km^2/s)
e = magnitude of $\vec{e}$ (unitless)

$$\boxed{\nu = \cos^{-1}\left(\frac{\vec{e} \cdot \vec{R}}{eR}\right)}$$

if $\vec{R} \cdot \vec{V} \geq 0$ then $0 \leq \nu \leq 180°$

if $\vec{R} \cdot \vec{V} < 0$ then $180° < \nu < 360°$

ν = true anomaly (deg or rad)
$\vec{e}$ = eccentricity vector (unitless, points at perigee)
$\vec{R}$ = position vector (km
e = magnitude of $\vec{e}$ (unitless)
R = magnitude of $\vec{R}$ (km)

8.3 Ground tracks

Nodal displacement: displacement of orbit to the west between each revolution

$$\boxed{\Delta N = 360° - \text{longitude between successive ascending nodes}}$$

$$\boxed{\text{Period (hours)} = \frac{\Delta N}{15°/hr}} \text{ for direct orbits only} (0 < i < 90°)$$

$$\boxed{a = \sqrt[3]{\mu(P/2\pi)}}$$

a = semimajor axis (km)
μ = Gravitational parameter km^3/sec$^2 \approx 3.986 \times 10^5$km^3/s^2 for Earth
P = period (s)
π = 3.14159...(unitless)

inclination = highest latitude

- For a direct orbit ($0 < i < 90°$), inclination = highest north or south latitude.
- For a retrograde orbit ($90° < i < 180°$), inclination = 180 - max latitude

9 Maneuvers

9.1 Hohmann Transfers

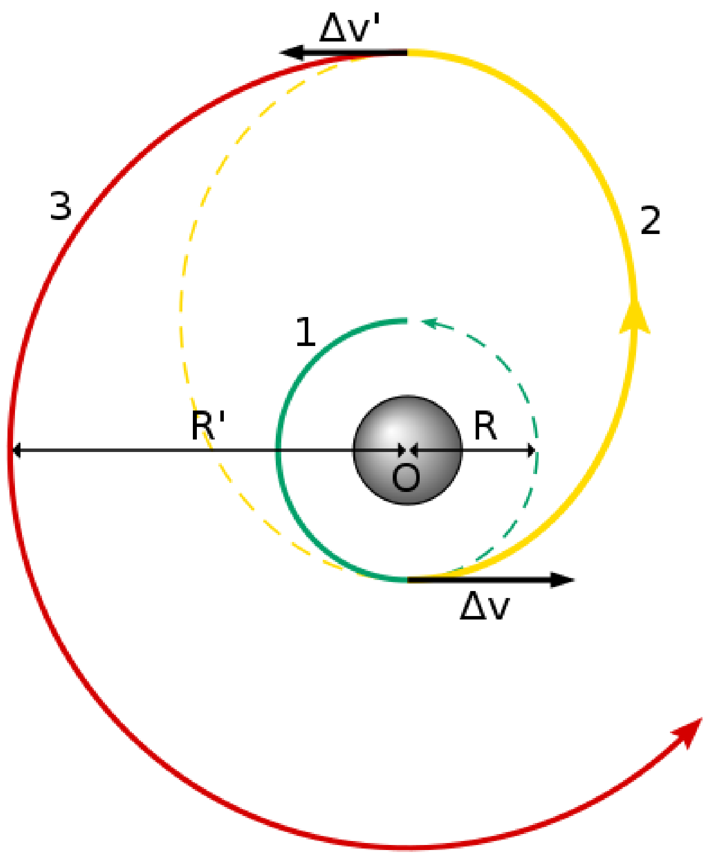


FIGURE 7 – Transfer orbit

Specific mechanical energy of each orbit:

- $\varepsilon_{\text{orbit 1}} = -\dfrac{\mu}{2a_{\text{orbit 1}}}$
- $\varepsilon_{\text{orbit 2}} = -\dfrac{\mu}{2a_{\text{orbit 2}}}$
- $\varepsilon_{transfer} = -\dfrac{\mu}{2a_{transfer}}$ with $2a_{transfer} = R + R'$

Velocity at each maneuver point using specific mechanical energy equation from section 8.2:

- $V_{\text{orbit 1}} = \sqrt{2\left(\dfrac{\mu}{R} + \varepsilon_{\text{orbit 1}}\right)}$
- $V_{\text{orbit 2}} = \sqrt{2\left(\dfrac{\mu}{R'} + \varepsilon_{\text{orbit 2}}\right)}$
- $V_{\text{transfer at orbit 1}} = \sqrt{2\left(\dfrac{\mu}{R} + \varepsilon_{transfer}\right)}$
- $V_{\text{transfer at orbit 2}} = \sqrt{2\left(\dfrac{\mu}{R'} + \varepsilon_{transfer}\right)}$

- $\Delta V_1 = |V_{\text{transfer at orbit 1}} - V_{\text{orbit 1}}|$
- $\Delta V_2 = |V_{\text{orbit 2}} - V_{\text{transfer at orbit 2}}|$
- $\Delta V_{total} = \Delta V_1 + \Delta V_2$

Time of flight = half period

$$\boxed{TOF = \frac{P}{2} = \pi\sqrt{\frac{a_{transfer}^3}{\mu}}}$$

TOF = spacecraft's time of flight
P = orbital period (s)
a = semimajor axis of transfer orbit (km)
μ = Gravitational parameter $\mathrm{km}^3/\sec^2 \approx 3.986 \times 10^5 \mathrm{km}^3/\mathrm{s}^2$ for Earth

9.2 Plane changes

9.2.1 Simple plane changes

$$\boxed{\Delta V_{simple} = 2V_{initial}\sin\frac{\theta}{2}}$$

ΔV_{simple} = velocity change for a simple plane change (km/s)
$V_{initial} = V_{final}$ = velocities in the initial and final orbits (km/s)
θ = plane-change angle (deg or rad)

Change of inclination: maneuver at ascending/descending node.
Change of right ascension of ascending node, Ω: maneuver at North or South pole for polar orbit

9.2.2 Combined plane change

Hohmann transfer combined with plane change at apogee (more efficient):

$$\boxed{\Delta V_{combined} = \sqrt{|\vec{V}_{initial}|^2 + |\vec{V}_{final}|^2} - 2|\vec{V}_{initial}||\vec{V}_{final}|\cos\theta}$$

$\Delta V_{combined}$ = velocity change for a combined plane change (km/s)
$|\vec{V}_{initial}|$ = magnitude of the velocity in the initial orbit (km/s)
$|\vec{V}_{final}|$ = magnitude of the velocity in the final orbit (km/s)
θ = plane-change angle (deg or rad)

9.3 Rendezvous

9.3.1 Coplanar

$$\boxed{\omega = \sqrt{\frac{\mu}{a^3}}}$$

ω = spacecraft's angular velocity (rad/s)
μ = Gravitational parameter $\mathrm{km}^3/\sec^2 \approx 3.986 \times 10^5 \mathrm{km}^3/\mathrm{s}^2$ for Earth
a = semimajor axis (km)

$$\boxed{\alpha_{lead} = \omega_{target} TOF}$$

α_{lead} = amount by which the interceptor must lead the target (rad)
ω_{target} = target's angular velocity (rad/s)
TOF = time of flight (s)

$$\boxed{\phi_{final} = \pi - \alpha_{lead}}$$

ϕ_{final} = phase angle between the interceptor and target as the transfer begins (rad)
α_{lead} = angle by which the interceptor must lead the target (rad)

$$\boxed{\text{wait time} = \frac{\phi_{final} - \phi_{initial}}{\omega_{target} - \omega_{interceptor}}}$$

$$\begin{aligned} \text{wait time} &= \text{time until the interceptor initiates the rendezvous (s)} \\ \phi_{initial}, \phi_{final} &= \text{initial and final phase angle (rad)} \\ \omega_{target}, \omega_{interceptor} &= \text{target and interceptor angular velocities (rad/s)} \end{aligned}$$

9.3.2 Co-orbital

$$\boxed{a_{phasing} = \sqrt[3]{\mu \left(\frac{\phi_{travel}}{2\pi\omega_{target}} \right)^2}}$$

$$\begin{aligned} a_{phasing} &= \text{semimajor axis of the phasing orbit (km)} \\ \mu &= \text{Gravitational parameter } \mathrm{km}^3/\sec^2 \approx 3.986 \times 10^5 \mathrm{km}^3/\mathrm{s}^2 \text{ for Earth} \\ \phi_{travel} &= \text{angular distance between target and rendezvous location (rad)} \\ \omega_{target} &= \text{target's angular velocity (rad/s)} \end{aligned}$$

10 Interplanetary travel using the patched-conic approximation

10.1 Sphere of influence

$$\boxed{R_{SOI} = a_{planet} \left(\frac{m_{planet}}{m_{sun}} \right)^{\frac{2}{5}}}$$

$$\begin{aligned} R_{SOI} &= \text{radius of a planet's SOI (km)} \\ a_{planet} &= \text{semimajor axis of the planet's orbit around the Sun (km)} \\ m_{planet} &= \text{planet's mass (kg)} \\ m_{sun} &= \text{Sun's mass} = 1.989 \times 10^{30} \text{ kg} \end{aligned}$$

10.2 Elliptical Hohmann transfer between planets

Assuming spacecraft on same orbit around the sun as the Earth and an heliocentric-ecliptic coordinate system:

$$\boxed{\varepsilon = \frac{V^2}{2} - \frac{\mu}{R}}$$

$$\begin{aligned} \varepsilon &= \text{spacecraft's specific mechanical energy } (\mathrm{km}^2/\mathrm{s}^2) \\ V &= \text{spacecraft's velocity } (\mathrm{km}/\sec) \\ \mu &= \text{gravitational parameter of the central body } (\mathrm{km}^3/\mathrm{s}^2) \\ &\approx 1.327 \times 10^{11} \mathrm{km}^3/\mathrm{s}^2 \text{ for our sun} \\ R &= \text{magnitude of the spacecraft's position vector (km)} \end{aligned}$$

$$\boxed{\varepsilon = -\frac{\mu}{2a}}$$

$$a = \text{orbit's semimajor axis (km)}$$

$$\boxed{V_{Earth} = \sqrt{2 \left(\frac{\mu_{sun}}{R_{\text{to Earth}}} + \varepsilon_{Earth} \right)}}$$

$$
\begin{aligned}
V_{Earth} &= \text{Earth's orbital velocity with respect to the Sun (km/s)} \\
\mu_{sun} &= \text{Sun's gravitational parameter} \approx 1.327 \times 10^{11}\ \text{km}^3/\text{s}^2 \\
R_{\text{to Earth}} &= \text{distance from the Sun to Earth} \\
&= \text{1 astronomical unit (AU)} \\
&= 1.496 \times 10^{8}\ \text{km} \\
\varepsilon_{Earth} &= \text{specific mechanical energy of Earth's orbit } (\text{km}^2/\text{s}^2)
\end{aligned}
$$

10.3 From Earth to the target

Elliptical Hohmann transfer
Reference Frame: Heliocentric-ecliptic
μ_{sun} = Sun's gravitational parameter $\approx 1.327 \times 10^{11}$ km^3/s^2

Energy

$$\varepsilon_{Earth} = -\frac{\mu_{Sun}}{2a_{Earth}}$$

$$\varepsilon_{transfer} = -\frac{\mu_{Sun}}{2a_{transfer}}$$

$$a_{transfer} = \frac{R_{to\ Earth} + R_{to\ target}}{2}$$

$$\varepsilon_{target} = -\frac{\mu_{Sun}}{2a_{target}}$$

Velocities

$$V_{Earth} = \sqrt{2\left(\frac{\mu_{Sun}}{R_{to\ Earth}} + \varepsilon_{Earth}\right)}$$

$$V_{target} = \sqrt{2\left(\frac{\mu_{Sun}}{R_{to\ target}} + \varepsilon_{target}\right)}$$

$$V_{transfer\ at\ Earth} = \sqrt{2\left(\frac{\mu_{Sun}}{R_{to\ Earth}} + \varepsilon_{transfer}\right)}$$

$$V_{transfer\ at\ target} = \sqrt{2\left(\frac{\mu_{Sun}}{R_{to\ target}} + \varepsilon_{transfer}\right)}$$

$$V_{\infty\ Earth} = |V_{transfer\ at\ Earth} - V_{Earth}|$$

$$V_{\infty\ target} = |V_{target} - V_{transfer\ at\ target}|$$

10.4 Departure from Earth

Hyperbolic trajectory
Reference Frame: Geocentric-equatorial
$\mu_{Earth} = \approx 3.986 \times 10^{5}$ km^3/s^2

Energy

$$\varepsilon_{\infty\ Earth} = \frac{V^2_{\infty\ Earth}}{2}$$

Velocities

$$V_{\infty\ Earth} = \text{from above}$$

$$V_{hyperbolic\ at\ Earth} = \sqrt{2\left(\frac{\mu_{Earth}}{R_{park\ at\ Earth}} + \varepsilon_{\infty\ Earth}\right)}$$

$$V_{park\ at\ Earth} = \sqrt{\frac{\mu_{Earth}}{R_{park\ at\ Earth}}}$$

$$\Delta V_{boost} = |V_{hyperbolic\ at\ Earth} - V_{park\ at\ Earth}|$$

10.5 Arrival at target planet

Hyperbolic trajectory
Reference Frame: Planet-centered equatorial
$\mu_{planet} = Gm_{planet}$ km^3/s^2

Energy

$$\varepsilon_{\infty\ target} = \frac{V^2_{\infty\ target}}{2}$$

Velocities

$$V_{\infty\ target} = \text{from above}$$

$$V_{hyperbolic\ at\ target} = \sqrt{2\left(\frac{\mu_{target}}{R_{park\ at\ target}} + \varepsilon_{\infty\ target}\right)}$$

$$V_{park\ at\ target} = \sqrt{\frac{\mu_{target}}{R_{park\ at\ target}}}$$

$$\Delta V_{retro} = |V_{park\ at\ target} - V_{hyperbolic\ at\ target}|$$

11 Predicting orbit - Kepler's Equation

$$n = \sqrt{\frac{\mu}{a^3}}$$

n = spacecraft's mean motion (rad/s)
μ = Gravitational parameter km^3/ sec^2 $\approx 3.986 \times 10^5$km^3/s^2 for Earth
a = semimajor axis (km)

$$M = nT$$

M = mean anomaly (rad)
n = spacecraft's mean motion (rad/s)
T = the time since last perigee passage (s)

$$\boxed{M_{future} - M_{initial} = n(t_{future} - t_{initial}) - 2k\pi}$$

M_{future} = mean anomaly when the spacecraft is in the future position (rad)
$M_{initial}$ = mean anomaly when the spacecraft is in the initial position (rad)
$t_{future} - t_{initial}$ = time of flight (TOF) between to points in the orbit
t_{future} = time when the spacecraft is in the final position
$t_{initial}$ = time when the spacecraft is in the initial position
k = the number of times the spacecraft passes perigee during the TOF

Kepler's equation

must use radian!

$$\boxed{M = E - e\sin E}$$

M = mean anomaly (rad)
E = eccentric anomaly (rad)
e = eccentricity (unitless)

$$\boxed{\cos E = \frac{e + \cos\nu}{1 + e\cos\nu}}$$

$E =$ eccentric anomaly (rad)
$e =$ eccentricity (unitless)
$\nu =$ true anomaly (rad)

$$\boxed{\cos\nu = \frac{\cos E - e}{1 - e\cos E}}$$

$E =$ eccentric anomaly (rad)
$e =$ eccentricity (unitless)
$\nu =$ true anomaly (rad)