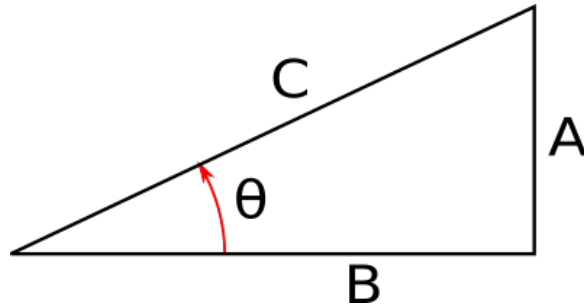


# Astronautics Cheat Sheet

March 29, 2021

## 1 Math review

### 1.1 Trigonometry



#### SOH-CAH-TOA

- $\sin \theta = \text{Opposite} / \text{Hypotenuse}$
- $\cos \theta = \text{Adjacent} / \text{Hypotenuse}$
- $\tan \theta = \text{Opposite} / \text{Adjacent}$

#### Spherical Trigonometry

TODO

### 1.2 Vector math

#### Vector components

$$\vec{A} = A_I \hat{I} + A_J \hat{J} + A_K \hat{K}$$

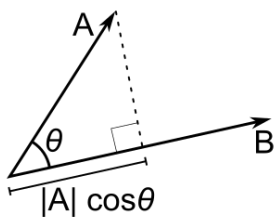
#### Magnitude of vector

$$\|\vec{A}\| = A = \sqrt{A_I^2 + A_J^2 + A_K^2}$$

#### Vector addition

$$\vec{A} + \vec{B} = (A_I + B_I)\hat{I} + (A_J + B_J)\hat{J} + (A_K + B_K)\hat{K}$$

#### Scalar or dot product



$$\vec{A} \cdot \vec{B} = AB \cos \theta$$

$$\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$\vec{A} \cdot \vec{B} = (A_I B_I) + (A_J B_J) + (A_K B_K)$$

FIGURE 1 – Dot product

## Vector or cross product

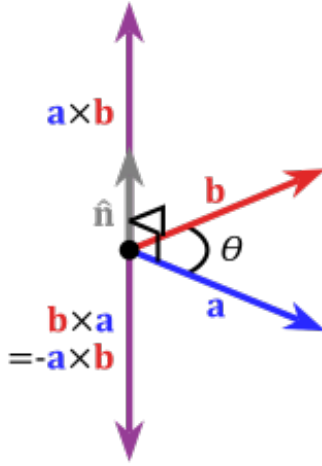


FIGURE 2 – Cross product

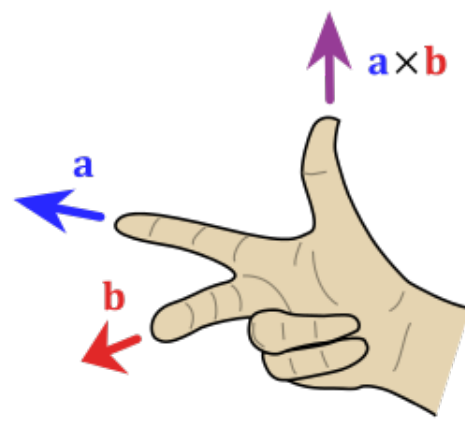


FIGURE 3 – Right hand rule

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[https://commons.wikimedia.org/wiki/File:Right\\_hand\\_rule\\_cross\\_product.svg](https://commons.wikimedia.org/wiki/File:Right_hand_rule_cross_product.svg)

$$\vec{A} \times \vec{B} = [(A_J)(B_K) - (B_J)(A_K)]\hat{I} - [(A_I)(B_K) - (B_I)(A_K)]\hat{J} + [(A_I)(B_J) - (B_I)(A_J)]\hat{K}$$

$$\|\vec{A} \times \vec{B}\| = AB \sin \theta$$

## 2 Constants

Symbol	Name	value	unit
	Earth radius	6378.14	km
$\mu$	Gravitational parameter	$3.986 \times 10^{14}$	$\text{m}^3/\text{s}^2$

## 3 Newton's laws of motion

### 3.1 Newton's first law of motion

A body continues in its state of rest, or of uniform motion in a straight line, unless compelled to change that state by forces impressed upon it.

$$\vec{p} = m\vec{V}$$

$\vec{p}$  = linear momentum vector ( $\text{kg} \cdot \text{m/s}$ )

$m$  = mass (kg)

$\vec{V}$  = velocity vector (m/s)

$$\vec{H} = I\vec{\Omega}$$

$\vec{H}$  = angular momentum vector ( $\text{kg} \cdot \text{m}^2/\text{s}$ )

$I$  = moment of inertia ( $\text{kg} \cdot \text{m}^2$ )

$\vec{\Omega}$  = angular velocity vector (rad/s)

$$\vec{H} = \vec{R} \times m\vec{V}$$

$\vec{H}$  = angular momentum vector ( $\text{kg} \cdot \text{m}^2/\text{s}$ )

$\vec{R}$  = position (m)

$m$  = mass (kg)

$\vec{V}$  = velocity vector (m/s)

### 3.2 Newton's second law of motion

The time rate of change of an object's momentum equals the applied force.

$$\vec{F} = m\vec{a}$$

$\vec{F}$  = force vector ( $\text{kgm}/\text{s}^2 = \text{N}$ )

$m$  = mass (kg)

$\vec{a}$  = acceleration ( $\text{m}/\text{s}^2$ )

### 3.3 Newton's third law of motion

When body A exerts a force on body B, body B will exert an equal, but opposite, force on body A

## 4 Newton's laws of universal gravitation

$$F_g = \frac{Gm_1m_2}{R^2}$$

$F_g$  = force due to gravity (N)

$G$  = universal gravitational constant  $\approx 6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$

$m_1, m_2$  = masses of two bodies (kg)

$R$  = distance between the two bodies (m)

$$a_g = \frac{\mu_{Earth}}{R^2}$$

$a_g$  = acceleration due to gravity ( $\text{m}/\text{s}^2$ )

$\mu_{Earth} \equiv G m_{Earth} \approx 3.986 \times 10^{14} \text{ m}^3/\text{s}^2$

$R$  = distance between the two bodies (m)

## 5 Laws of conservation

### 5.1 Conservation of momentum

In the absence of outside forces, linear and angular momentum are conserved.

### 5.2 Energy

$E$  = total mechanical energy ( $\text{kg m}^2/\text{s}^2$ )

$KE$  = kinetic energy ( $\text{kg m}^2/\text{s}^2$ )

$PE$  = potential energy ( $\text{kg m}^2/\text{s}^2$ )

$$E = KE + PE$$

$$PE = m a_g h$$

$$PE = -\frac{m\mu}{R}$$

$m$  = mass (kg)

$a_g$  = acceleration due to gravity ( $\text{m/s}^2$ )

$h$  = height above ref. point (m)

$m$  = spacecraft's mass (kg)

$\mu$  = gravitational parameter ( $\text{km}^3/\text{s}^2$ )

$R$  = distance from Earth's center (km)

$$KE = \frac{1}{2}mV^2$$

$$E = \frac{1}{2}mV^2 - \frac{m\mu}{R}$$

$KE$  = kinetic energy ( $\text{kg m}^2/\text{s}^2$ )

$m$  = mass (kg)

$V$  = velocity (km/s)

$E$  = total mech. energy ( $\text{kg m}^2/\text{s}^2$ )

$m$  = mass (kg)

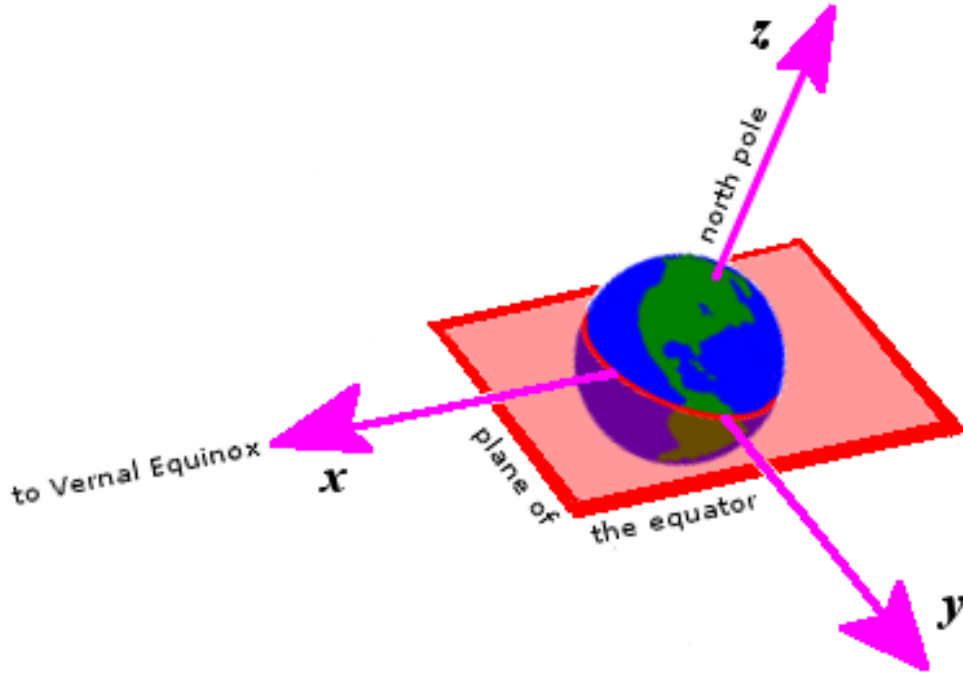
$V$  = velocity (km/s)

$\mu$  = gravitational parameter ( $\text{km}^3/\text{s}^2$ )

$R$  = position (km)

## 6 The restricted two-body problem

### 6.1 Coordinate systems



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FIGURE 4 – Geocentric equatorial coordinates. The origin is the centre of the Earth. The fundamental plane is the plane of the Earth's equator. The primary direction (the x axis) is the vernal equinox. A right-handed convention specifies a y axis 90° to the east in the fundamental plane; the z axis is the north polar axis. The reference frame does not rotate with the Earth, rather, the Earth rotates around the z axis.

A coordinate system (figure 6) is:

- an origin
- a fundamental plane, containing two axes, and the perpendicular to it
- a principal direction within the plane
- the third axis using the right-hand rule

## 6.2 Equation of motion

$$\ddot{\vec{R}} + \frac{\mu}{R^2} \frac{\vec{R}}{R} = 0$$

$\ddot{\vec{R}}$  = spacecraft's acceleration (km/s<sup>2</sup>)

$\mu$  = gravitational parameter (km<sup>3</sup>/s<sup>2</sup>)

$\vec{R}$  = spacecraft's position vector (km)

$R$  = magnitude of the spacecraft's position vector (km)

## 6.3 Orbital geometry

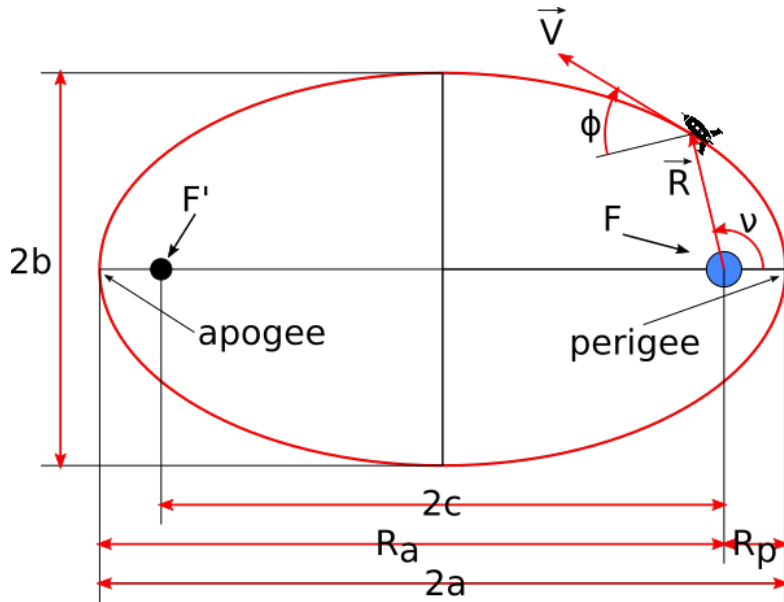


FIGURE 5 – Geometry of an elliptical orbit

$$e = \frac{2c}{2a} = \frac{R_a - R_p}{R_a + R_p}$$

$e$  = eccentricity

$\vec{R}$  = spacecraft's position vector

$\vec{V}$  = spacecraft's velocity vector

$F$  and  $F'$  = primary and vacant foci

$R_p$  = radius of perigee

$R_a$  = radius of apogee

$2a$  = major axis

$2b$  = minor axis

$2c$  = distance between the foci

$a$  = semimajor axis

$b$  = semiminor axis

$\nu$  = true anomaly

$\phi$  = flight-path angle

$$R = \frac{a(1 - e^2)}{1 + e \cos \nu}$$

$R$  = magnitude of the spacecraft's position vector (km)

$a$  = semi-major axis (km)

$e$  = eccentricity (unitless)

$\nu$  = true anomaly (deg or rad)

Conic section	a = semimajor axis	c = one half the distance between foci	e = eccentricity
circle	a > 0	c = 0	e = 0
ellipse	a > 0	0 < c < a	0 < e < 1
parabola	a = ∞	c = ∞	e = 1
hyperbola	a < 0	a  <  c  > 0	e > 1

## 7 Constants of orbital motion

### 7.1 Specific mechanical energy

$$\varepsilon \equiv \frac{E}{m} = \frac{V^2}{2} - \frac{\mu}{R}$$

$$V = \sqrt{2\left(\frac{\mu}{R} + \varepsilon\right)}$$

$\varepsilon$  = spacecraft's specific mechanical energy (km<sup>2</sup>/s<sup>2</sup>)

$V$  = spacecraft's velocity (km/sec)

$\mu$  = Gravitational parameter km<sup>3</sup>/sec<sup>2</sup>  $\approx 3.986 \times 10^5$  km<sup>3</sup>/s<sup>2</sup> for Earth

$R$  = spacecraft's distance from Earth's center (km)

$$\varepsilon = -\frac{\mu}{2a}$$

$\varepsilon$  = spacecraft's specific mechanical energy (km<sup>2</sup>/s<sup>2</sup>)

$\mu$  = Gravitational parameter km<sup>3</sup>/sec<sup>2</sup>  $\approx 3.986 \times 10^5$  km<sup>3</sup>/s<sup>2</sup> for Earth

$a$  = semimajor axis (km)

$$P = 2\pi\sqrt{\frac{a^3}{\mu}}$$

$P$  = period (seconds)

$\pi$  = 3.14159... (unitless)

$a$  = semimajor axis (km)

$\mu$  = Gravitational parameter km<sup>3</sup>/sec<sup>2</sup>  $\approx 3.986 \times 10^5$  km<sup>3</sup>/s<sup>2</sup> for Earth

### 7.2 Specific angular momentum

$$\vec{h} \equiv \frac{\vec{H}}{m} = \vec{R} \times \vec{V}$$

$\vec{h}$  = spacecraft's specific angular momentum (km<sup>2</sup>/s)

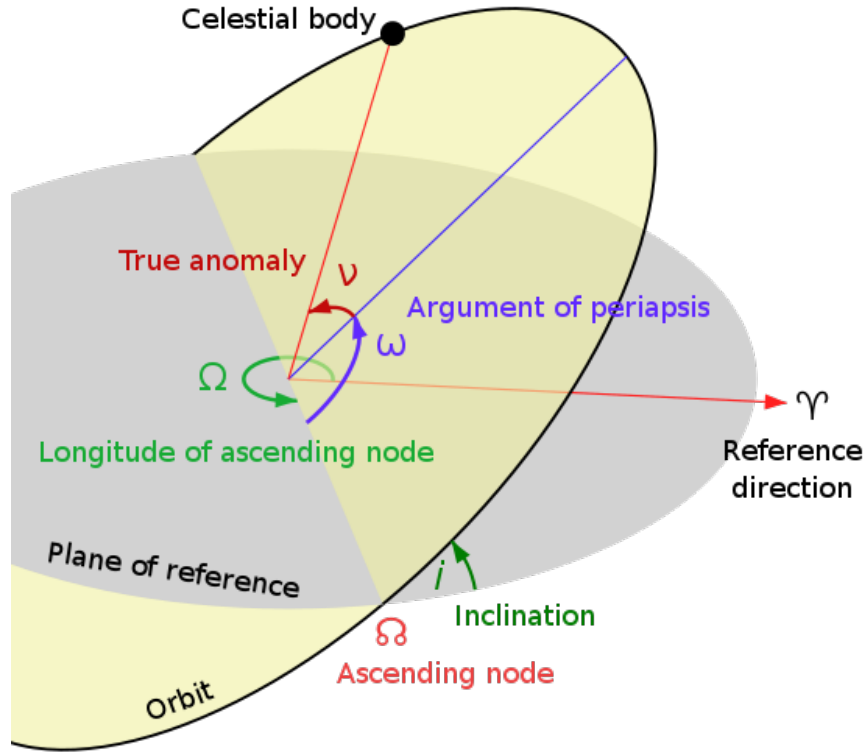
$\vec{R}$  = spacecraft's position vector (km)

$\vec{V}$  = spacecraft's velocity vector (km/s)

## 8 Describing orbits

### 8.1 Orbital elements

- Size: semimajor axis,  $a$
- Shape: eccentricity,  $e$
- Tilt: inclination,  $i$
- Angle from vernal equinox to ascending node: right ascension (or longitude) of ascending node,  $\Omega$
- Angle from AN to Pe: argument of perigee,  $\omega$
- Angle from Pe to spacecraft: true anomaly,  $\nu$



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FIGURE 6 – Orbital elements

## 8.2 Computing orbital elements

Knowing  $\vec{R}$  and  $\vec{V}$  from ground tracking, we can compute orbital elements:

$$\varepsilon = \frac{V^2}{2} - \frac{\mu}{R}$$

$$a = -\frac{\mu}{2\varepsilon}$$

$\varepsilon$  = spacecraft's specific mechanical energy ( $\text{km}^2/\text{s}^2$ )

$V$  = spacecraft's velocity ( $\text{km}/\text{sec}$ )

$\mu$  = Gravitational parameter  $\text{km}^3/\text{sec}^2 \approx 3.986 \times 10^5 \text{km}^3/\text{s}^2$  for Earth

$R$  = spacecraft's distance from Earth's center ( $\text{km}$ )

$$\vec{e} = \frac{1}{\mu} \left[ \left( V^2 - \frac{\mu}{R} \right) \vec{R} - (\vec{R} \cdot \vec{V}) \vec{V} \right]$$

$\vec{e}$  = eccentricity vector (unitless, points at Pe)

$\mu$  = Gravitational parameter  $\text{km}^3/\text{sec}^2 \approx 3.986 \times 10^5 \text{km}^3/\text{s}^2$  for Earth

$V$  = magnitude of  $\vec{V}$  ( $\text{km}/\text{s}$ )

$R$  = magnitude of  $\vec{R}$  ( $\text{km}$ )

$\vec{R}$  = position vector ( $\text{km}$ )

$\vec{V}$  = velocity vector ( $\text{km}/\text{s}$ )

$$i = \cos^{-1} \left( \frac{\hat{K} \cdot \vec{h}}{Kh} \right)$$

$$0 \leq i \leq 180^\circ$$

$i$  = inclination (deg or rad)

$\hat{K}$  = unit vector through the North Pole

$\vec{h}$  = specific angular momentum vector ( $\text{km}^2/\text{s}$ )

$K$  = magnitude of  $\hat{K} = 1$

$h$  = magnitude of  $\vec{h}$  ( $\text{km}^2/\text{s}$ )

$$\vec{n} = \hat{K} \times \vec{h}$$

$\vec{n}$  = ascending node vector (km<sup>2</sup>/s, points at the ascending node)

$\hat{K}$  = unit vector through the North Pole

$\vec{h}$  = specific angular momentum vector (km<sup>2</sup>/s)

$$\Omega = \cos^{-1} \left( \frac{\hat{I} \cdot \vec{n}}{In} \right)$$

if  $n_j \geq 0$  then  $0 \leq \Omega \leq 180^\circ$

if  $n_j < 0$  then  $180^\circ < \Omega < 360^\circ$

$\Omega$  = right ascension of the ascending node (deg or rad)

$\hat{I}$  = unit vector in the principal direction

$\vec{n}$  = ascending node vector (km<sup>2</sup>/s, points at the ascending node)

$I$  = magnitude of  $\hat{I} = 1$

$n$  = magnitude of  $\vec{n}$  (km<sup>2</sup>/s)

$$\omega = \cos^{-1} \left( \frac{\vec{n} \cdot \vec{e}}{ne} \right)$$

if  $e_K \geq 0$  then  $0 \leq \omega \leq 180^\circ$

if  $e_K < 0$  then  $180^\circ < \omega < 360^\circ$

$\omega$  = argument of perigee (deg or rad)

$\vec{n}$  = ascending node vector (km<sup>2</sup>/s, points at the ascending node)

$\vec{e}$  = eccentricity vector (unitless, points at perigee)

$n$  = magnitude of  $\vec{n}$  (km<sup>2</sup>/s)

$e$  = magnitude of  $\vec{e}$  (unitless)

$$\nu = \cos^{-1} \left( \frac{\vec{e} \cdot \vec{R}}{eR} \right)$$

if  $\vec{R} \cdot \vec{V} \geq 0$  then  $0 \leq \nu \leq 180^\circ$

if  $\vec{R} \cdot \vec{V} < 0$  then  $180^\circ < \nu < 360^\circ$

$\nu$  = true anomaly (deg or rad)

$\vec{e}$  = eccentricity vector (unitless, points at perigee)

$\vec{R}$  = position vector (km)

$e$  = magnitude of  $\vec{e}$  (unitless)

$R$  = magnitude of  $\vec{R}$  (km)