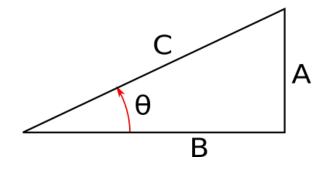
Astronautics Cheat Sheet

April 5, 2021

1 Math review

1.1 Trigonometry



SOH-CAH-TOA

- $\sin \theta = \text{Opposite} / \text{Hypotenuse}$
- $\cos \theta = \text{Adjacent} / \text{Hypotenuse}$
- $\tan \theta = \text{Opposite} / \text{Adjacent}$

Spherical Trigonometry

TODO

1.2 Vector math

Vector components

$$\vec{A} = A_I \hat{I} + A_J \hat{J} + A_K \hat{K}$$

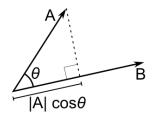
Magnitude of vector

$$\|\vec{A}\| = A = \sqrt{A_I^2 + A_J^2 + A_K^2}$$

Vector addition

$$\vec{A} + \vec{B} = (A_I + B_I)\hat{I} + (A_J + B_J)\hat{J} + (A_K + B_K)\hat{K}$$

Scalar or dot product



$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\vec{A} \cdot \vec{B} = (A_I B_I) + (A_J B_J) + (A_K B_K)$$

$$\theta = \cos^- 1 \frac{\vec{A} \cdot \vec{B}}{AB}$$

Figure 1 - Dot product

Vector or cross product

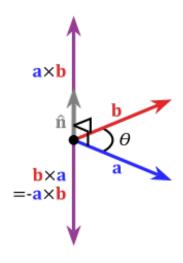


Figure 2 - Cross product

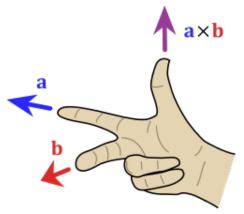


FIGURE 3 – Right hand rule

$$\vec{A} \times \vec{B} = [(A_J)(B_K) - (B_J)(A_K)]\hat{I} - [(A_I)(B_K) - (B_I)(A_K)]\hat{J} + [(A_I)(B_J) - (B_I)(A_J)]$$

$$\|\vec{A} \times \vec{B}\| = AB\sin\theta$$

2 Constants

Symbol	Name	Name value	
	Earth radius	6378.14	km
μ	Gravitational parameter	3.986×10^{14}	$\mathrm{m}^3/\mathrm{s}^2$

Newton's laws of motion

Newton's first law of motion

A body continues in its state of rest, or of uniform motion in a straight line, unless compelled to change that state by forces impressed upon it.

$$\vec{p} = m\vec{V}$$

 $\vec{p} = \text{linear momentum vector } (\text{kg} \cdot \text{m/s})$

m = mass (kg)

 \vec{V} = velocity vector (m/s)

$$ec{H}=Iec{\Omega}$$

 \vec{H} = angular momentum vector (kg·m²/s)

 $I = \text{moment of inertia } (\text{kg} \cdot \text{m}^2)$

 $\vec{\Omega}$ = angular velocity vector (rad/s)

$$\vec{H} = \vec{R} \times m\vec{V}$$

 $\vec{H} = \text{angular momentum vector } (\text{kg} \cdot \text{m}^2/\text{s})$

 $\vec{R} = \text{position (m)}$

m = mass (kg)

 \vec{V} = velocity vector (m/s)

3.2 Newton's second law of motion

The time rate of change of an object's momentum equals the applied force.

$$\vec{F} = m\vec{a}$$

 $\vec{F} = \text{force vector } (\text{kgm/s}^2 = \text{N})$

m = mass (kg)

 $\vec{a} = \text{acceleration (m/s}^2)$

3.3 Newton's third law of motion

When body A exerts a force on body B, body B will exert an equal, but opposite, force on body A

4 Newton's laws of universal gravitation

$$F_q = \frac{Gm_1m_2}{R^2}$$
 | $F_q = \text{force due to gravity (N)}$

 $G = \text{universal gravitational constant} \approx 6.674 \times 10^{-11} \,\mathrm{N} \cdot \mathrm{m}^2/\mathrm{kg}^2$

 $m_1, m_2 = \text{masses of two bodies (kg)}$

R = distance between the two bodies (m)

$$a_g = \frac{\mu_{Earth}}{R^2}$$
 $a_g = \text{acceleration due to gravity (m/s}^2)$ $\mu_{Earth} \equiv G \, m_{Earth} \approx 3.986 \times 10^{14} \, \text{m}^3/\text{s}^2$ $R = \text{distance between the two bodies (m)}$

5 Laws of conservation

5.1 Conservation of momentum

In the absence of outside forces, linear and angular momentum are conserved.

5.2 Energy

$$E = \text{total mechanical energy } (\text{kg m}^2/\text{s}^2)$$

$$KE = \text{kinetic energy } (\text{kg m}^2/\text{s}^2)$$

$$PE = \text{potential energy } (\text{kg m}^2/\text{s}^2)$$

$$E = KE + PE$$

$$PE = m \, a_g h$$

$$PE = -\frac{m\mu}{R}$$

m = mass (kg)

 $a_q = \text{acceleration due to gravity (m/s}^2)$ m = spacecraft's mass (kg)

h = height above ref. point (m) $\mu = \text{gravitational parameter (km}^3/\text{s}^2)$

R = distance from Earth's center (km)

$$KE = \frac{1}{2}mV^2$$

$$E = \frac{1}{2}mV^2 - \frac{m\mu}{R}$$

$$KE = \text{kinetic energy } (\text{kg m}^2/\text{s}^2)$$

$$m = \text{mass } (\text{kg})$$

$$V = \text{velocity } (\text{km/s})$$

$$E = \text{total mech. energy } (\text{kg m}^2/\text{s}^2)$$

$$m = \text{mass } (\text{kg})$$

$$V = \text{velocity } (\text{km/s})$$

 $\mu = \text{gravitational parameter } (\text{km}^3/\text{s}^2)$

R = position (km)

6 The restricted two-body problem

6.1 Coordinate systems

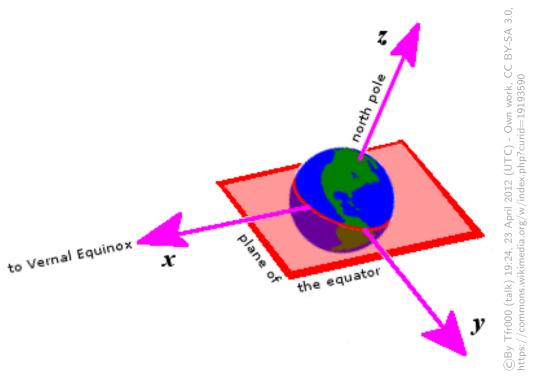


Figure 4 – Geocentric equatorial coordinates. The origin is the centre of the Earth. The fundamental plane is the plane of the Earth's equator. The primary direction (the x axis) is the vernal equinox. A right-handed convention specifies a y axis 90° to the east in the fundamental plane; the z axis is the north polar axis. The reference frame does not rotate with the Earth, rather, the Earth rotates around the z axis.

A coordinate system (figure 7) is:

- an origin
- a fundamental plane, containing two axes, and the perpendicular to it
- a principal direction within the plane
- the third axis using the right-hand rule

6.2 Equation of motion

$$\ddot{\vec{R}} + \frac{\mu}{R^2} \frac{\vec{R}}{R} = 0$$

 $\ddot{\vec{R}} = \text{spacecraft's acceleration } (\text{km/s}^2)$

 $\mu = \text{gravitational parameter } (\text{km}^3/\text{s}^2)$

 $\vec{R} = \text{spacecraft's position vector (km)}$

R = magnitude of the spacecraft's position vector (km)

6.3 Orbital geometry

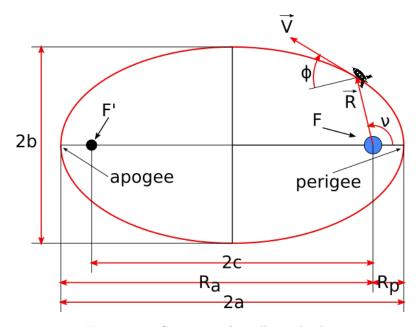


Figure 5 – Geometry of an elliptical orbit

$$e = \frac{2c}{2a} = \frac{R_a - R_p}{R_a + R_p}$$

$$e = eccentricity$$

 $\vec{R} = \text{spacecraft's position vector}$

 $\vec{V} = \text{spacecraft's velocity vector}$

FandF' =primary and vacant foci

 $R_p = \text{radius of perigee}$

 $R_a = \text{radius of apogee}$

2a = major axis

2b = minor axis

2c = distance between the foci

a = semimajor axis

b = semiminor axis

 $\nu = \text{true anomaly}$

$$\phi = \text{flight-path angle}$$

$$R = \frac{a(1 - e^2)}{1 + e\cos\nu}$$

R = magnitude of the spacecraft's position vector (km)

a = semi-major axis (km)

e = eccentricity (unitless)

 $\nu = \text{true anomaly (deg or rad)}$

Conic section	a = semimajor axis	c = one half the distance between foci	e = eccentricity
circle	a > 0	c = 0	e = 0
ellipse	a > 0	0 < c < a	0 < e < 1
parabola	$a = \infty$	$c = \infty$	e = 1
hyperbola	a < 0	a < c > 0	e > 1

7 Constants of orbital motion

7.1 Specific mechanical energy

$$\varepsilon \equiv \frac{E}{m} = \frac{V^2}{2} - \frac{\mu}{R}$$

 $V = \sqrt{2(\frac{\mu}{R} + \varepsilon)}$

 $\varepsilon = \text{spacecraft's specific mechanical energy } (\text{km}^2/\text{s}^2)$

V = spacecraft's velocity (km/sec)

 $\mu = \text{Gravitational parameter km}^3/\sec^2 \approx 3.986 \times 10^5 \text{km}^3/\text{s}^2 \text{ for Earth}$

R = spacecraft's distance from Earth's center (km)

$$\varepsilon = -\frac{\mu}{2a}$$

 $\varepsilon = \text{spacecraft's specific mechanical energy } (\text{km}^2/\text{s}^2)$

 $\mu = \text{Gravitational parameter km}^3/\sec^2 \approx 3.986 \times 10^5 \text{km}^3/\text{s}^2 \text{ for Earth}$

a = semimajor axis (km)

$$P = 2\pi \sqrt{\frac{a^3}{\mu}}$$

P = period (seconds)

 $\pi = 3.14159...$ (unitless)

a = semimajor axis (km)

 $\mu={\rm Gravitational~parameter~km^3/sec^2}\approx 3.986\times 10^5{\rm km^3/s^2}$ for Earth

7.2 Specific angular momentum

$$\boxed{\vec{h} \equiv \frac{\vec{H}}{m} = \vec{R} \times \vec{V}}$$

 $\vec{h} = \text{spacecraft's specific angular momentum } (\text{km}^2/\text{s})$

 $\vec{R} = \text{spacecraft's position vector (km)}$

 $\vec{V} = \text{spacecraft's velocity vector (km/s)}$

8 Describing orbits

8.1 Orbital elements

• Size: semimajor axis, a

• Shape: eccentricity, e

• Tilt: inclination, i

• Angle from vernal equinox to ascending node: right ascension (or longitude) of ascending node, Ω

• Angle from AN to Pe: argument of perigee, ω

• Angle from Pe to spacecraft: true anomaly, ν

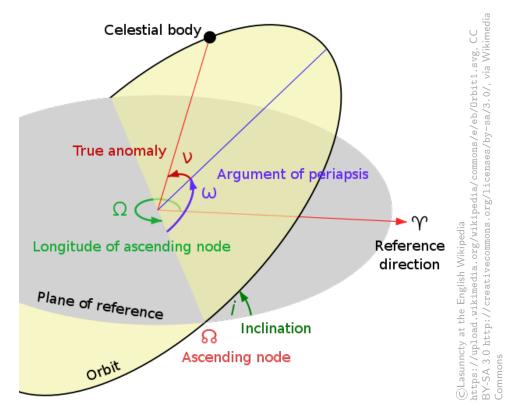


Figure 6 – Orbital elements

8.2 Computing orbital elements

Knowing \vec{R} and \vec{V} from ground tracking, we can compute orbital elements:

$$\varepsilon = \frac{V^2}{2} - \frac{\mu}{R}$$

$$\varepsilon = \text{spacecraft's specific mechanical energy (km²/s²)}$$

$$V = \text{spacecraft's velocity (km/sec)}$$

$$\mu = \text{Gravitational parameter km³/sec²} \approx 3.986 \times 10^5 \text{km³/s² for Earth}$$

$$R = \text{spacecraft's distance from Earth's center (km)}$$

$$\vec{e} = \frac{1}{\mu} \left[\left(V^2 - \frac{\mu}{R} \right) \vec{R} - (\vec{R} \cdot \vec{V}) \vec{V} \right]$$

$$\vec{e} = \text{eccentricity vetor (unitless, points at Pe)}$$

$$\mu = \text{Gravitational parameter km}^3 / \sec^2 \approx 3.986 \times 10^5 \text{km}^3 / \text{s}^2 \text{ for Earth}$$

$$V = \text{magnitude of } \vec{V} \text{ (km/s)}$$

$$R = \text{magnitude of } \vec{R} \text{ (km)}$$

$$\vec{R} = \text{position vector (km)}$$

$$\vec{V} = \text{velocity vector (km/s)}$$

$$i = \cos^{-1}\left(\frac{\hat{K} \cdot \vec{h}}{Kh}\right)$$

$$i = \text{inclination (deg or rad)}$$

$$\hat{K} = \text{unit vector through the North Pole}$$

$$0 \le i \le 180^{\circ}$$

$$\vec{h} = \text{specific angular momentum vector (km²/s)}$$

$$K = \text{magnitude of } \hat{K} = 1$$

$$h = \text{magnitude of } \hat{h} \text{ (km²/s)}$$

$$\vec{n} = \hat{K} \times \vec{h}$$

$$\vec{n} = \text{ascending node vector (km}^2/\text{s, points at the ascending node)}$$

$$\hat{K} = \text{unit vector through the North Pole}$$

$$\vec{h} = \text{specific angular momentum vector (km}^2/\text{s)}$$

$$\Omega = \cos^{-1}\left(\frac{\vec{I} \cdot \vec{n}}{In}\right)$$

$$\Omega = \text{right ascension of the ascending node (deg or rad)}$$

$$\hat{I} = \text{unit vector in the principal direction}$$

$$\vec{n} = \text{ascending node vector (km}^2/\text{s, points at the ascending node)}$$

$$\vec{I} = \text{magnitude of } \hat{I} = 1$$

$$\text{if } n_j < 0 \text{ then } 180^\circ < \Omega < 360^\circ$$

$$\vec{n} = \text{magnitude of } \hat{I} = 1$$

$$\vec{n} = \text{magnitude of } \vec{n} \text{ (km}^2/\text{s)}$$

$$\omega = \cos^{-1}\left(\frac{\vec{n} \cdot \vec{e}}{ne}\right)$$

$$\omega = \text{argument of perigee (deg or rad)}$$

$$\vec{n} = \text{ascending node vector (km}^2/\text{s, points at the ascending node)}$$

$$\vec{e} = \text{eccentricity vector (unitless, points at perigee)}$$

$$\vec{n} = \text{magnitude of } \vec{n} \text{ (km}^2/\text{s)}$$

$$\vec{e} = \text{magnitude of } \vec{e} \text{ (unitless)}$$

$$\nu = \cos^{-1}\left(\frac{\vec{e}\cdot\vec{R}}{eR}\right)$$

$$\nu = \text{true anomaly (deg or rad)}$$

$$\vec{e} = \text{eccentricity vector (unitless, points at perigee)}$$

$$\vec{R} = \text{position vector (km}$$

$$\text{if } \vec{R}\cdot\vec{V} \geq 0 \text{ then } 0 \leq \nu \leq 180^{\circ}$$

$$\vec{e} = \text{magnitude of } \vec{e} \text{ (unitless)}$$

$$\vec{R} = \text{magnitude of } \vec{R} \text{ (km)}$$

8.3 Ground tracks

Nodal displacement: displacement of orbit to the west between each revolution

$$\Delta N = 360^{\circ} - \text{longitude}$$
 between successive ascending nodes

Period (hours) =
$$\frac{\Delta N}{15^{\circ}/hr}$$
 for direct orbits only(0 < i < 90°)

$$a = \text{semimajor axis (km)}$$

$$\mu = \text{Gravitational parameter km}^3/\sec^2 \approx 3.986 \times 10^5 \text{km}^3/\text{s}^2 \text{ for Earth}$$

$$P = \text{period (s)}$$

$$\pi = 3.14159...(\text{unitless})$$

inclination = highest latitude

- For a direct orbit $(0 < i < 90^{\circ})$, inclination = highest north or south latitude.
- For a retrograde orbit (90° < i < 180°), inclination = 180 max latitude

9 Maneuvers

9.1 Hohmann Transfers

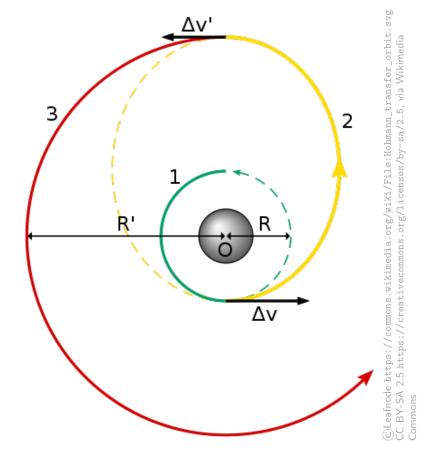


FIGURE 7 – Transfer orbit

Specific mechanical energy of each orbit:

•
$$\varepsilon_{\text{orbit }1} = -\frac{\mu}{2a_{\text{orbit }1}}$$

•
$$\varepsilon_{\text{orbit }2} = -\frac{\mu}{2a_{\text{orbit }2}}$$

•
$$\varepsilon_{transfer} = -\frac{\mu}{2a_{transfer}}$$
 with $2a_{transfer} = R + R'$

Velocity at each maneuver point using specific mechanical energy equation from section 8.2:

•
$$V_{\text{orbit }1} = \sqrt{2\left(\frac{\mu}{R} + \varepsilon_{\text{orbit }1}\right)}$$

•
$$V_{\text{orbit 2}} = \sqrt{2\left(\frac{\mu}{R'} + \varepsilon_{\text{orbit 2}}\right)}$$

•
$$V_{\text{transfer at orbit 1}} = \sqrt{2\left(\frac{\mu}{R} + \varepsilon_{transfer}\right)}$$

•
$$V_{\text{transfer at orbit 2}} = \sqrt{2\left(\frac{\mu}{R'} + \varepsilon_{transfer}\right)}$$

•
$$\Delta V_1 = |V_{\text{transfer at orbit 1}} - V_{\text{orbit 1}}|$$

•
$$\Delta V_2 = |V_{\text{orbit 2}} - V_{\text{transfer at orbit 2}}|$$

•
$$\Delta V_{total} = \Delta V_1 + \Delta V_2$$

Time of flight = half period

$$TOF = \frac{P}{2} = \pi \sqrt{\frac{a_{transfer}^3}{\mu}}$$

TOF =spacecraft's time of flight

P = orbital period (s)

a = semimajor axis of transfer orbit (km)

 $\mu = {\rm Gravitational~parameter~km^3/sec^2} \approx 3.986 \times 10^5 {\rm km^3/s^2}$ for Earth