1 Newton's laws of motion

1.1 Newton's first law of motion

A body continues in its state of rest, or of uniform motion in a straight line, unless compelled to change that state by forces impressed upon it.

$$\boxed{\vec{p} = m\vec{V}}$$

$$\begin{aligned}\vec{p} &= \text{linear momentum vector } (\text{kg}\cdot\text{m/s})\\ m &= \text{mass (kg)}\\ \vec{V} &= \text{velocity vector (m/s)}\end{aligned}$$

$$\boxed{\vec{H} = I\vec{\Omega}}$$

$$\begin{aligned}\vec{H} &= \text{angular momentum vector } (\text{kg}\cdot\text{m}^2/\text{s})\\ I &= \text{moment of inertia } (\text{kg}\cdot\text{m}^2)\\ \vec{\Omega} &= \text{angular velocity vector (rad/s)}\end{aligned}$$

$$\boxed{\vec{H} = \vec{R}\times m\vec{V}}$$

$$\begin{aligned}\vec{H} &= \text{angular momentum vector } (\text{kg}\cdot\text{m}^2/\text{s})\\ \vec{R} &= \text{position (m)}\\ m &= \text{mass (kg)}\\ \vec{V} &= \text{velocity vector (m/s)}\end{aligned}$$

1.2 Newton's second law of motion

The time rate of change of an object's momenutm equals the applied force.

$$\boxed{\vec{F} = m\vec{a}}$$

$$\begin{aligned}\vec{F} &= \text{force vector } (\text{kgm/s}^2 = \text{N})\\ m &= \text{mass (kg)}\\ \vec{a} &= \text{acceleration } (\text{m/s}^2)\end{aligned}$$

1.3 Newton's third law of motion

When body A exerts a force on body B, body B will exert an equal, but opposite, force on body A

2 Newton's laws of universal gravitation

$$\boxed{F_g = \frac{Gm_1m_2}{R^2}}$$

$$\begin{aligned}F_g &= \text{force due to gravity (N)}\\ G &= \text{universal gravitational constant} \approx 6.674\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\\ m_1, m_2 &= \text{masses of two bodies (kg)}\\ R &= \text{distance between the two bodies (m)}\end{aligned}$$

$$\boxed{a_g = \frac{\mu_{Earth}}{R^2}}$$

$$\begin{aligned}a_g &= \text{acceleration due to gravity } (\text{m/s}^2)\\ \mu_{Earth} &\equiv G\,m_{Earth} \approx 3.986\times10^{14}\,\text{m}^3/\text{s}^2\\ R &= \text{distance between the two bodies (m)}\end{aligned}$$

3 Laws of conservation

3.1 Conservation of momentum

In the absence of outside forces, linear and angular momentum are conserved.

3.2 Energy

$$\boxed{E = KE + PE}$$

$$\begin{aligned} E &= \text{total mech. energy } (\mathrm{kg\,m^2/s^2}) \\ KE &= \text{kinetic energy } (\mathrm{kg\,m^2/s^2}) \\ PE &= \text{potential energy } (\mathrm{kg\,m^2/s^2}) \end{aligned}$$

$$\boxed{PE = m\,a_g h}$$

$$\begin{aligned} m &= \text{mass (kg)} \\ a_g &= \text{acceleration due to gravity } (\mathrm{m/s^2}) \\ h &= \text{height above ref. point (m)} \end{aligned}$$

$$\boxed{PE = -\frac{m\mu}{R}}$$

$$\begin{aligned} m &= \text{spacecraft's mass (kg)} \\ \mu &= \text{gravitational parameter } (\mathrm{km^3/s^2}) \\ R &= \text{distance from Earth's center (km)} \end{aligned}$$

$$\boxed{KE = \frac{1}{2}mV^2}$$

$$\begin{aligned} KE &= \text{kinetic energy } (\mathrm{kg\,m^2/s^2}) \\ m &= \text{mass (kg)} \\ V &= \text{velocity (km/s)} \end{aligned}$$

$$\boxed{E = \frac{1}{2}mV^2 - \frac{m\mu}{R}}$$

$$\begin{aligned} E &= \text{total mech. energy } (\mathrm{kg\,m^2/s^2}) \\ m &= \text{mass (kg)} \\ V &= \text{velocity (km/s)} \\ \mu &= \text{gravitational parameter } (\mathrm{km^3/s^2}) \\ R &= \text{position (km)} \end{aligned}$$

4 The restricted two-body problem

4.1 Coordinate systems

A coordinate system (figure 2) is:

- **an origin**
- **a fundamental plane**, containing two axes, and the perpendicular to it
- **a principal direction** within the plane
- **the third axis** using the right-hand rule

4.2 Equation of motion

$$\boxed{\ddot{\vec{R}} + \frac{\mu}{R^2}\frac{\vec{R}}{R} = 0}$$

$$\begin{aligned} \ddot{\vec{R}} &= \text{spacecraft's acceleration } (\mathrm{km/s^2}) \\ \mu &= \text{gravitational parameter } (\mathrm{km^3/s^2}) \\ \vec{R} &= \text{spacecraft's position vector (km)} \\ R &= \text{magnitude of the spacecraft's position vector (km)} \end{aligned}$$

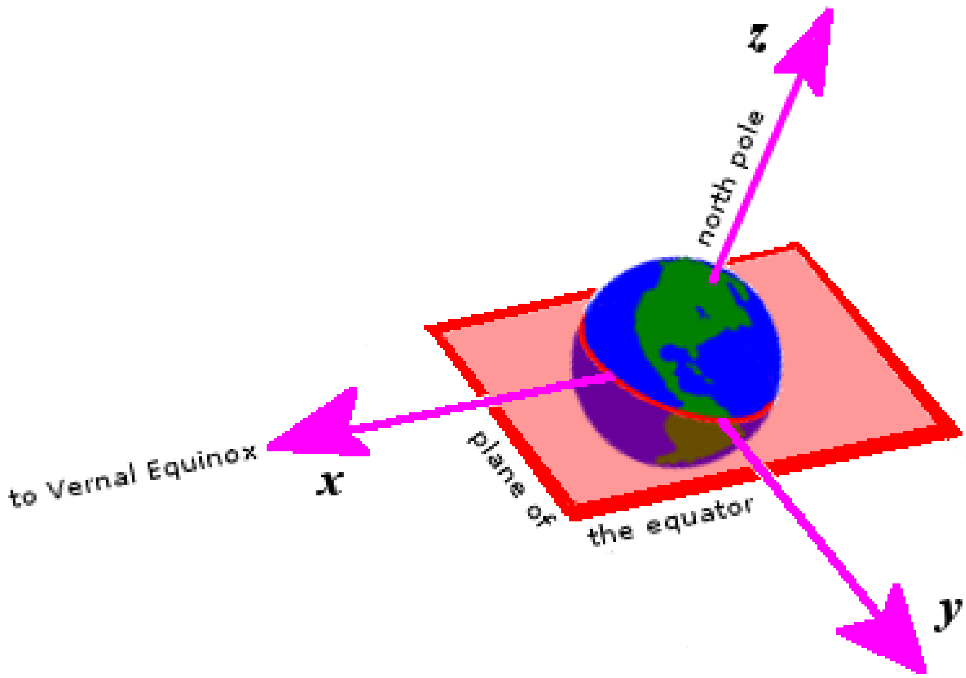

FIGURE 1 – Geocentric equatorial coordinates. The origin is the centre of the Earth. The fundamental plane is the plane of the Earth's equator. The primary direction (the x axis) is the vernal equinox. A right-handed convention specifies a y axis 90° to the east in the fundamental plane; the z axis is the north polar axis. The reference frame does not rotate with the Earth, rather, the Earth rotates around the z axis.

4.3 Orbital geometry

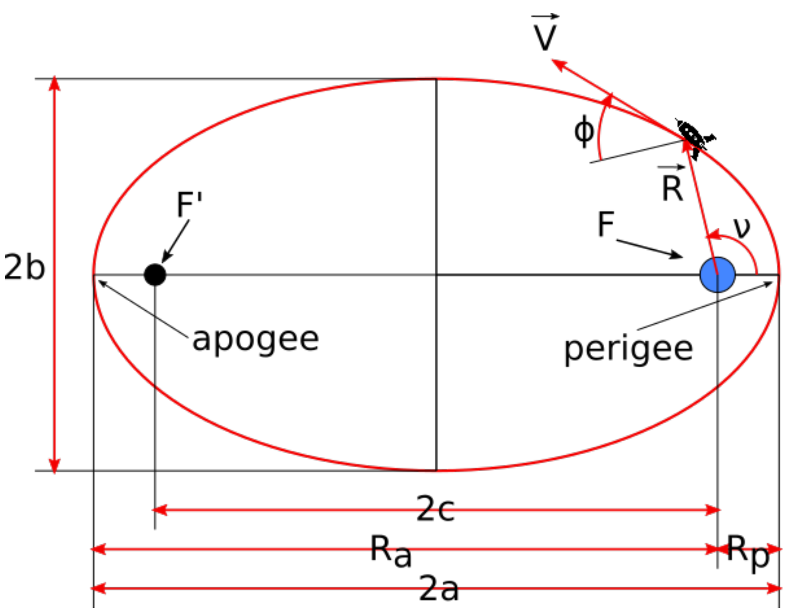

FIGURE 2 – Geometry of an elliptical orbit

$\vec{R}$ = spacecraft's position vector
$\vec{V}$ = spacecraft's velocity vector
$FandF'$ = primary and vacant foci
R_p = radius of perigee
R_a = radius of apogee
$2a$ = major axis
$2b$ = minor axis
$2c$ = distance between the foci
a = semimajor axis
b = semiminor axis
ν = true anomaly
ϕ = flight-path angle