Leverage and Disagreement

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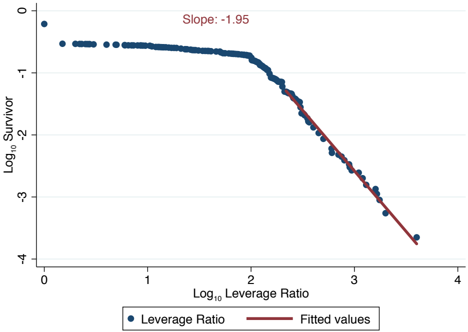
- In this paper, I develop a model of :
  - **Endogenous Leverage**
  - Interest Rates on Collateralized Bonds

  among competitive investors with **heterogenous beliefs**.
- Geanakoplos (1997) and subsequent :
  - Only **one** leverage ratio (simplifying assumption on the structure of beliefs / or on the number of agents).
  - Counterfactual. **Many** leverage ratios, even for same asset: homebuyers, entrepreneurs, hedge funds, investment banks...
- Relaxing the hypotheses leading to one leverage ratio, the model yields two key predictions.

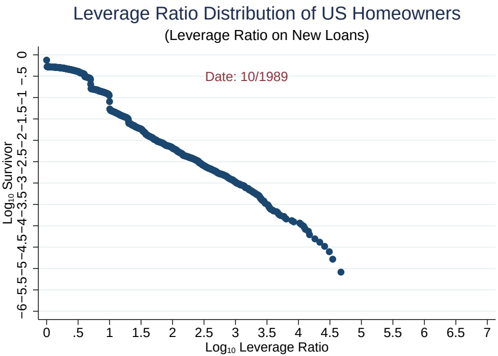
1) When disagreement goes to 0, the **upper tail** of the distribution of leverage ratios goes to a **Pareto** with **endogenous tail coefficient** $\boxed{2}$, for any smooth and bounded away from zero density of beliefs.

- Cross section of Hedge Funds (TASS Lipper, 2006)



- **Pareto in the upper tail** ($l \in [150, 3000]$) Histogram
- Point estimate for **tail coefficient** : $\boxed{\alpha = \mathbf{1.95}}$ (std: 0.2).

- Cross-section of homowners' initial leverage ratios (Dataquick, for example October 1989).



- Pareto of leverage ratios found also for:
  - Entrepreneurs in the SCF.
  - Firms in Compustat.
- $\Rightarrow$ Pareto for borrowers' expected / realized returns, however small belief heterogeneity:
  - Pareto Returns to entrepreneurship.
  - Pareto Returns to speculation in general.

2) Distribution of interest rates adjusts so that borrowers and lenders are matched assortatively : **interest rates are assignment / hedonic prices**, disconnected from expected and true default probability:

- New determinant for pricing fixed income securities. ($\Rightarrow$ Credit Spread Puzzle? / CDS-Bond Basis)
- Investing in high yield not necessarily risk shifting.
- High customization / fragmentation of the market = Endogenous OTC structure. $\Rightarrow$ OTC versus exchanges debate.

Model Ingredients :

- Heterogenous priors asset pricing model with endogenous leverage. Geanakoplos (1997), Simsek (2013).
- Disagreement on mean rather than on default probabilities.

Key Results :

- **Pareto** distributions for leverage ratios / expected and realized returns. Also gives information on:
  - Representativeness of marginal buyer/ Elements of the belief distribution. ($\Rightarrow$ monitoring systemic risk?)
  - Underlying financial structure.
- Credit spreads as **hedonic interest rates**.

Other Theoretical / Methodological contributions:

- Pyramiding Lending Arrangements.
- Endogenous Short-sales:
  - Endogenous rebate rates, without transactions costs / risk aversion.
  - Endogenous short interest.

Literature

- **Heterogeneous Priors.** Miller (1977), Harrison, Kreps (1978), Ofek, Richardson (2003), Hong, Scheinkman, Xiong (2006), Hong, Stein (2007), Hong, Sraer (2012).
- **Heterogeneous Priors & Collateral Constraints.** Geanakoplos (1997, 2003), Geanakoplos, Zame (2002), Geanakoplos (2010), Fostel, Geanakoplos (2012), Simsek (2013).
- **Competitive Assignment Models.** Roy (1950), Rosen (1974), Sattinger (1975), Rosen (1981), Teulings (1995), Gabaix, Landier (2008).
- **Pareto distributions.** Champernowne (1953), Simon (1955), Kesten (1973), Gabaix (1999), Luttmer (2007).
- **Credit Spread Puzzle.** Chen, Colling-Dufresne, Goldstein (2009), Buraschi, Trojani, Vedolin (2011), Huang and Huang (2012), Albagli, Hellwig, Tsyvinski (2012), McQuade (2013).
- **Entrepreneurship.** Moscowitz, Vissing-Jorgensen (2002), Hurst, Lusardi (2004).

Set-up

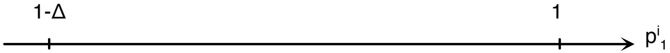
- Two Periods: 0 and 1.
- Continuum of agents. Measure 1.
- Wealth 1.
- Consume in period 1.

Assets

- Storage's Return $R = 1$. $\rightarrow$ **Cash**.
- **Real Asset.** Finite Supply normalized to 1. Exogenous p_1. Endogenous Price: p.
- **Borrowing Contracts** collateralized by the Real Asset.
  - No-recourse.
  - Normalization: 1 unit of Real Asset in Collateral.
  - ϕ: **Face Value** - promised payment in period 1.
  - Notation for contract: (ϕ).
  - Competitive Markets (Anonymous). Price: $q(\phi)$. "Loan amount". Implicit interest rate: $r(\phi) = \phi/q(\phi)$.
  - Payoff: $\min\{\phi, p_1\}$.

Beliefs

- Agents agree to disagree on p_1.
- Agent i: point expectations $p_1^i \in [1-\Delta, 1]$.



  - Key difference with Geanakoplos (1997), where agents agree on value upon default.
  - Generalization:
    - Agents agree on a probability distribution around mean.
    - Risk neutral.
- Density $f(.)$, c.d.f $F(.)$ on $[1-\Delta, 1]$.
- Exogenously given.
- No learning.

Agents' Problem

Given $(p, q(.))$, agent i chooses $(n^i_A, n^i_B(.), n^i_C)$ to max. expected wealth (W) in period 1 under:

- Budget Constraint (BC).
- Collateral Constraint (CC).

$$\max_{(n^i_A, n^i_B(.), n^i_C)} n^i_A p^i_1 + \int_\phi n^i_B(\phi) \min\{\phi, p^i_1\} d\phi + n^i_C \tag{W}$$

$$\text{s.t.} \quad n^i_A p + \int_\phi n^i_B(\phi) q(\phi) d\phi + n^i_C \leq 1 \tag{BC}$$

$$\text{s.t.} \quad \int_\phi \max\{-n^i_B(\phi), 0\} d\phi \leq n^i_A \tag{CC}$$

$$\text{s.t.} \quad n^i_A \geq 0, \quad n^i_C \geq 0$$

Equilibrium

Definition (Competitive Equilibrium for Economy $\mathcal{E}^{\mathcal{B}}$)

A competitive equilibrium is a price system $(p, q(.))$, and portfolios $(n^i_A, n^i_B(.), n^i_C)$ for all i such that:

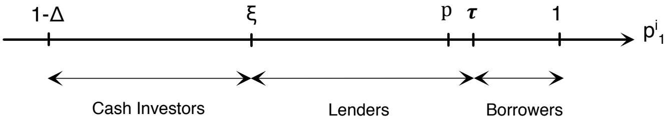
- Given $(p, q(.))$, agent i chooses $(n^i_A, n^i_B(\phi), n^i_C)$ maximizing (W) under (BC) and (CC),
- Markets clear:

$$\int_i n^i_A di = 1,$$

$$\text{and} \quad \forall \phi, \quad \int_i n^i_B(\phi) di = 0.$$

Agents' Types

Agents split into three types depending on optimism:



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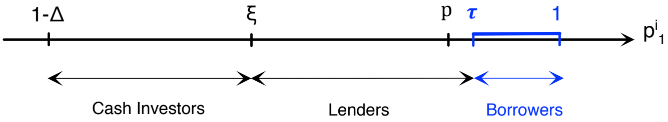

- $p_1^i \in [\tau, 1] \rightarrow$ Borrowers ("Homeowners", "Hedge Funds", "Entrepreneurs").

$$n_A^i > 0 \qquad \exists \phi, \quad n_B^i(\phi) < 0.$$

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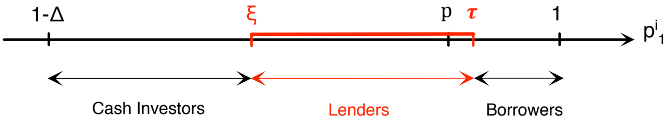

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- $p_1^i \in [\xi, \tau] \rightarrow$ Lenders ("Banks", "Money-Market Fund").

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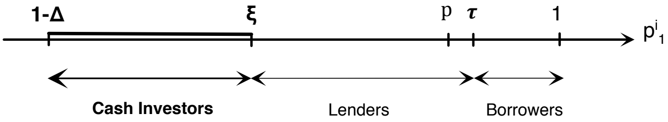

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$$\exists \phi, \quad n_B^i(\phi) > 0.$$

- $p_1^i \in [1-\Delta, \xi] \rightarrow$ Cash Investors.

$$n_C^i = 1.$$

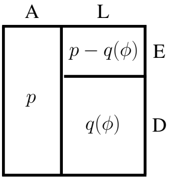
Borrowers' Problem

Lemma

A borrower p_1^i *chooses* (ϕ) *s.t.:* $\phi = \mathbf{arg}\max_\phi \dfrac{\mathbf{p_1^i} - \phi}{\mathbf{p} - \mathbf{q}(\phi)}$.

- Coll. Const. binds: 1 Real asset $\Rightarrow$ 1 Borrowing Contract.
- Number: $1/(p - q(\phi))$ of Real assets / Borrowing Contracts.

- Leverage ratio of (ϕ): $l(\phi) = p/(p - q(\phi))$.



$$\frac{1}{p - q(\phi)}(p_1^i - \phi) = \frac{p_1^i}{p} l(\phi) - \frac{\phi}{q(\phi)}(l(\phi) - 1)$$
$$= \frac{p_1^i}{p} + \left(\frac{p_1^i}{p} - r(\phi)\right)(l(\phi) - 1).$$

- Promise $\phi \nearrow \;\Rightarrow q(\phi) \nearrow \;\Rightarrow \boxed{\mathbf{q}'(\phi) > \mathbf{0}} \Rightarrow \boxed{\mathbf{l}'(\phi) > \mathbf{0}} \Rightarrow$ Leverage rises with face value ϕ.
- Trade-off between higher ϕ but higher $r(\phi) \Rightarrow \boxed{\mathbf{r}'(\phi) > \mathbf{0}}$.

Lenders

Lemma

A lender with beliefs p_1^i chooses contract (p_1^i).

- For lenders: Face value of the loan = Beliefs about the Real Asset.
  - Why not a higher ϕ ? Default for sure.

$$\text{Return:} \quad \frac{\min\{p_1^i, \phi\}}{q(\phi)} = \frac{p_1^i}{q(\phi)} \quad \searrow \phi.$$

  - Why not a lower ϕ ?

$$\text{Return:} \quad \frac{\min\{p_1^i, \phi\}}{q(\phi)} = \frac{\phi}{q(\phi)} = r(\phi) \quad \nearrow \phi.$$

- Leverage rises with ϕ, and $\boxed{\phi = p_1^i}$ of lenders $\Rightarrow$ Leverage rises with beliefs of lenders.
- Lenders think they trade **perfectly safe contracts**.

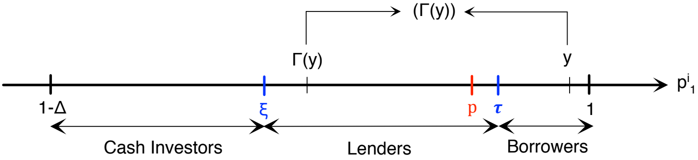
Positive Sorting

- Supermodularity of Expected Wealth of a Borrower with respect to his Beliefs p_1^i and the face value ϕ:

$$\frac{p_1^i - \phi}{p - q(\phi)} = \frac{p_1^i}{p}(1 + l(\phi)) - \frac{\phi}{q(\phi)} l(\phi)$$
$$\Rightarrow \quad \frac{\partial^2}{\partial \phi \partial p_1^i}(.) = \frac{1}{p} l'(\phi) > 0.$$

- **Complementarity** between leverage (ϕ) and expected return on each asset (p_1^i).
- $\phi = p_1^i$ of lenders $\Rightarrow$ **Positive Sorting** of borrowers and lenders . Empirically: Over-The-Counter (OTC) Markets.
- $\Gamma(.)$: Belief of borrower $\rightarrow$ Belief of lender. Sorting: $\Gamma'(.) > 0$.

2 first-order ODE for $\Gamma(.)$ and $q(.)$



- $p_1^i = y$ chooses ϕ s.t. lender choosing same ϕ is $\Gamma(y)$:

$$\Gamma(y) = \arg\max_{\phi} \frac{y-\phi}{p-q(\phi)} \quad \Rightarrow q'(\phi)\frac{y-\phi}{p-q(\phi)} = 1$$

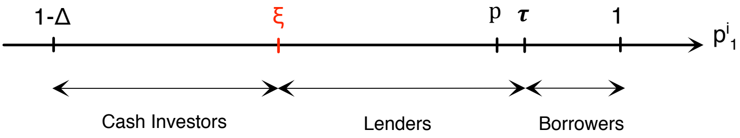
$$\Rightarrow \quad \boxed{(y-\Gamma(y))\, q'(\Gamma(y)) = p - q(\Gamma(y)).}$$

- Market clearing for contract (x):

$$\int_i n_B^i(x)di = 0 \quad \Rightarrow \quad \frac{f(\Gamma(y))d\Gamma(y)}{q(\Gamma(y))} = \frac{f(y)dy}{p-q(\Gamma(y))}$$

$$\Rightarrow \quad \boxed{(p-q(\Gamma(y)))\, f(\Gamma(y))\, \Gamma'(y) = q(\Gamma(y)) f(y)}.$$

- Unknowns: $q(.)$ $(\equiv r(.))$, $\Gamma(.)$, ξ, p, τ.
- 2 First-Order ODEs $\Rightarrow$ Need 5 algebraic equations.



- Indifference Cash / Lending: $r(\xi) = 1$.

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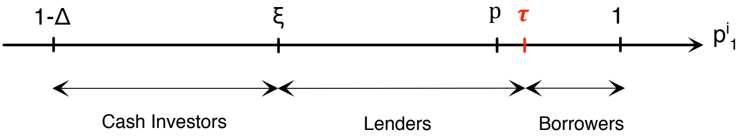

- Indifference Cash / Lending: $r(\xi) = 1.$
- Indifference Lending / Investing: $r(\tau) = \dfrac{\tau - \xi}{p - \xi}.$

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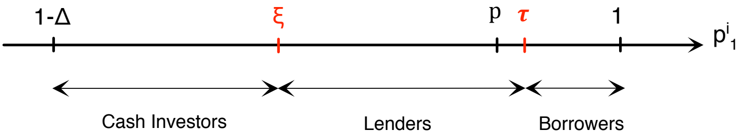

- Indifference Cash / Lending: $r(\xi) = 1.$
- Indifference Lending / Investing: $r(\tau) = \dfrac{\tau - \xi}{p - \xi}.$
- Most pessimistic lenders & borrowers: $\Gamma(\tau) = \xi.$

- Unknowns: $q(.)$ ($\equiv r(.)$), $\Gamma(.)$, ξ, p, τ.
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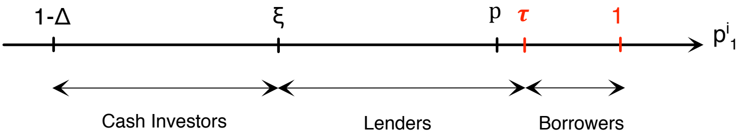

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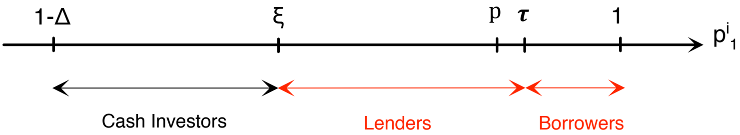
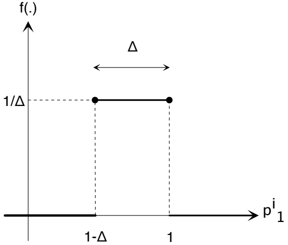
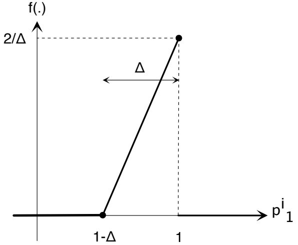


- Indifference Cash / Lending: $r(\xi) = 1.$
- Indifference Lending / Investing: $r(\tau) = \dfrac{\tau - \xi}{p - \xi}.$
- Most pessimistic lenders & borrowers: $\Gamma(\tau) = \xi.$
- Most optimistic lenders & borrowers: $\Gamma(1) = \tau.$
- Market clearing for the real asset: $1 - F(\xi) = p.$

Illustrating examples : *f* uniform, *f* increasing





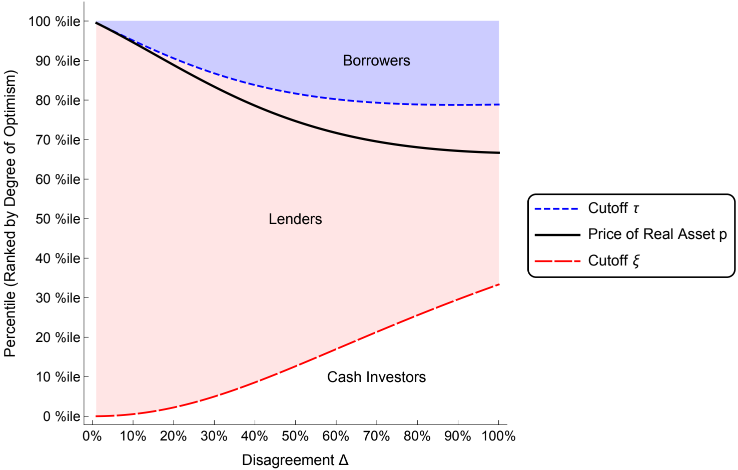
- Uniform : 2 first-order ODE $\rightarrow$ second-order ODE:

$$\boxed{\Gamma''(\Gamma - x) + \Gamma' + \Gamma'^2 = 0} \quad \Rightarrow \quad \Gamma(\mathbf{x}) = -\mathbf{x} - a + b\sqrt{\mathbf{x} + c}.$$

- Closed form: p, ξ, τ, $r(.)$, $q(.)$, $L(.)$, a, b, c. Example:

$$\mathbf{p} = \frac{1 + \Delta + 2\Delta^2 + 2\Delta^3 - \sqrt{(-1+\Delta)^2(1+2\Delta^2)}}{2\Delta + \Delta^2 + 4\Delta^3 + 2\Delta^4} = \mathbf{1} - \mathbf{O}(\ ^{\mathbf{2}}).$$

Cutoffs as a function of Δ (f uniform)



True across bounded away from zero density function:

$$p = 1 - O(\Delta^2), \quad \tau = 1 - O(\Delta^2), \quad \text{and} \quad \xi = 1 - O(\Delta).$$

Limiting Pareto Tail of Endogenous Tail Coefficient 2

- In uniform case, truncated Pareto with coeff 2:

$$\frac{p}{p - Q(\mathbf{y})} = \frac{p}{\sqrt{2\xi}} \sqrt{\frac{p-\xi}{\tau-\xi}} \frac{1}{\sqrt{\frac{(p+\xi)\tau - \xi(p-\xi)}{2\xi} - \mathbf{y}}}.$$

Proposition (Limiting Pareto Distribution for Leverage Ratios of Optimists for smooth $f(.)$)

Let $f(.)$ differentiable, f' continuous, $f(.)$ bounded away from 0. $G_\Delta(.)$ distribution function for the leverage of borrowers for $f_\Delta(.)$:

$$\exists A_\Delta, \quad \left\| l^2(1 - G_\Delta(l)) - A_\Delta \right\|_\infty^{[L_\Delta(1)/2, L_\Delta(1)]} \xrightarrow{\Delta \to 0} 0,$$

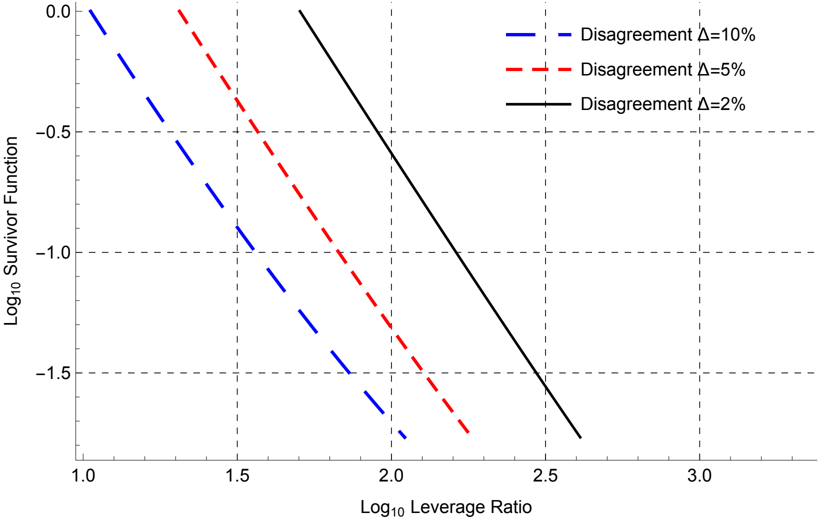
- Heuristically:

$$1 - G_\Delta(l) \sim \frac{A_\Delta}{l^2}.$$

- Upper tail behavior: not dependent on $f(.)$.

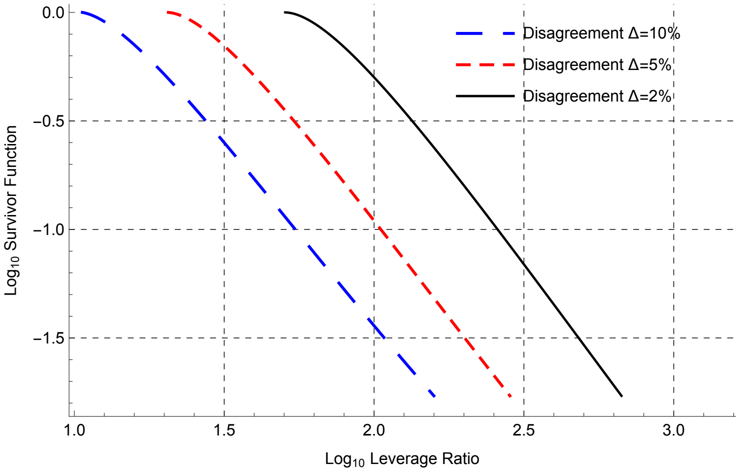
Pareto Distributions for Leverage Ratios, Uniform Distribution

Coefficient: 2.



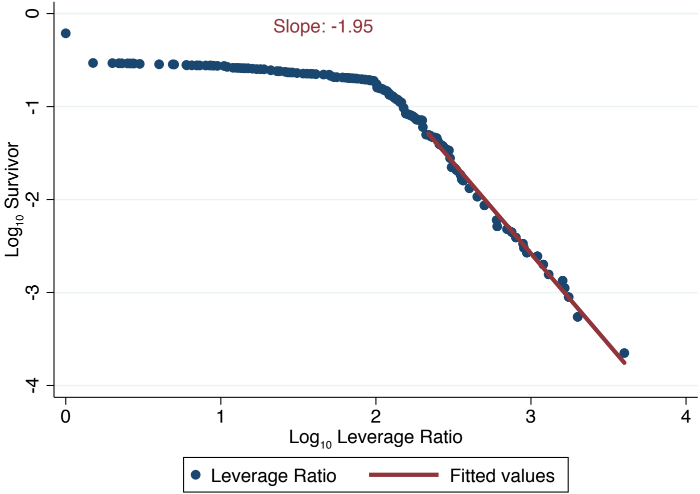
Pareto Distributions for Leverage Ratios, Increasing Distribution

Still Coefficient: 2.



Empirical Counterpart

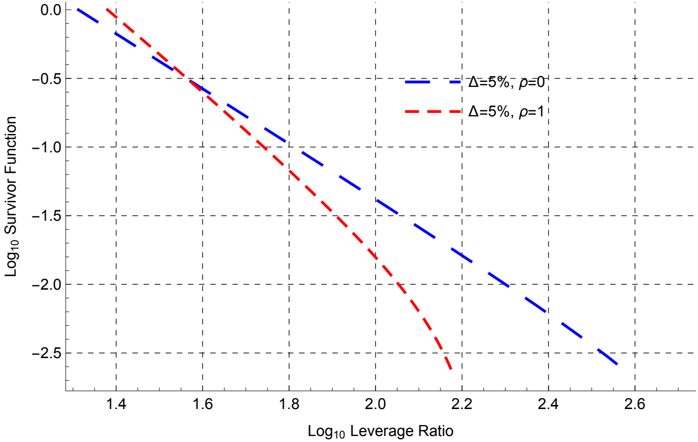
TASS Hedge Fund Database, August 2006.



Calibration: disagreement $\approx 1.8\%$.

Non Bounded away from 0.

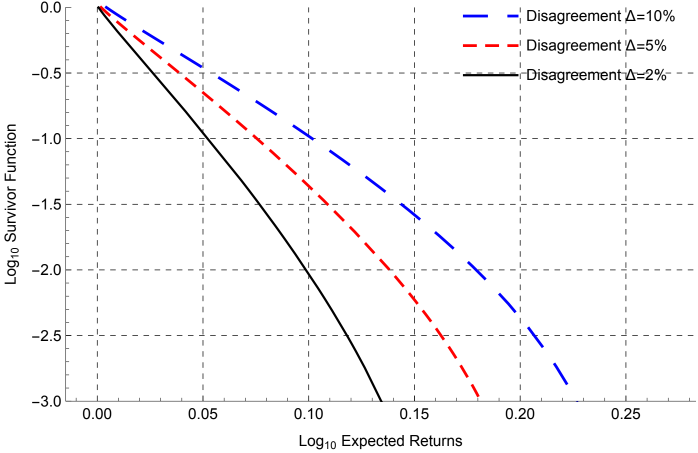
- If $f(x) \sim (1-x)^{\rho} \Rightarrow$ Pareto with coefficient $\mathbf{2} + \rho$.
- Scale Independence Remains.



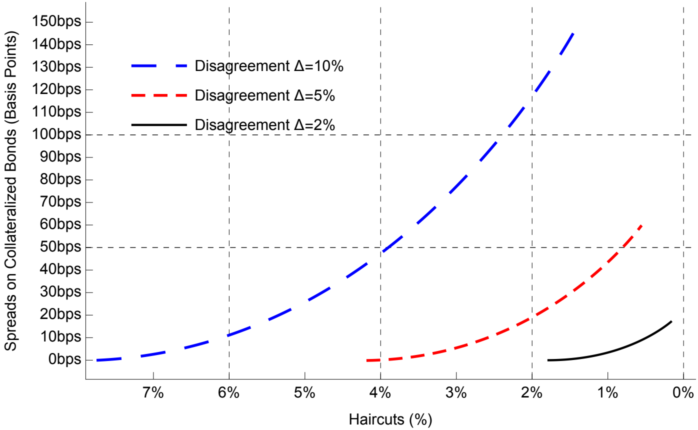
Returns to Entrepreneurship ?

- Expected Returns are Pareto from envelope condition:

$$R'(y) = \frac{1}{p - Q(y)} = \frac{\text{Leverage}(y)}{p}.$$



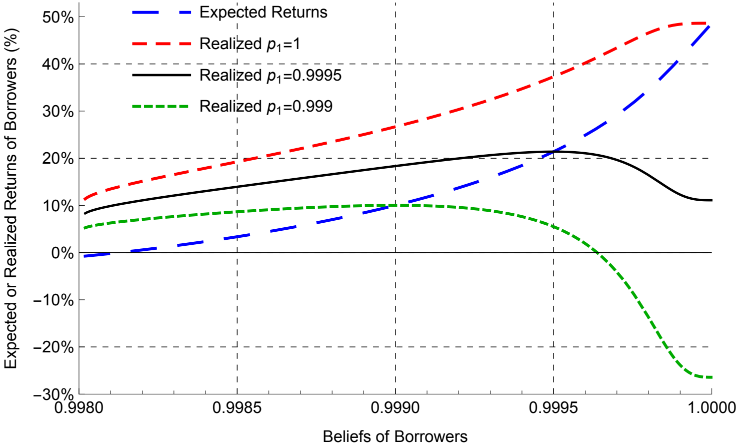
Hedonic Interest Rates



- **Hedonic Interest rates** $r(.)$ on safe bonds for lenders. Can be substantial. Example with $f(x) = 2(1-x)/\Delta$.
- $Corr(r(.), l(.)) > 0$ from disagreement. **But:** no risk shifting $\Rightarrow$ Different regulatory implications.

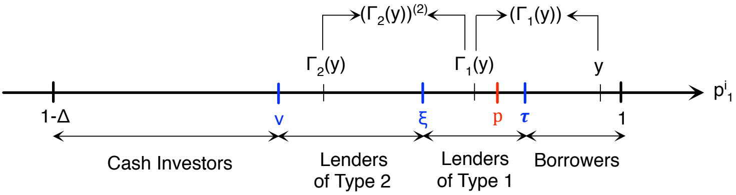
Hedonic Interest Rates

- **Non monotonic relationship between leverage and realized returns** of borrowers, because of spreads.



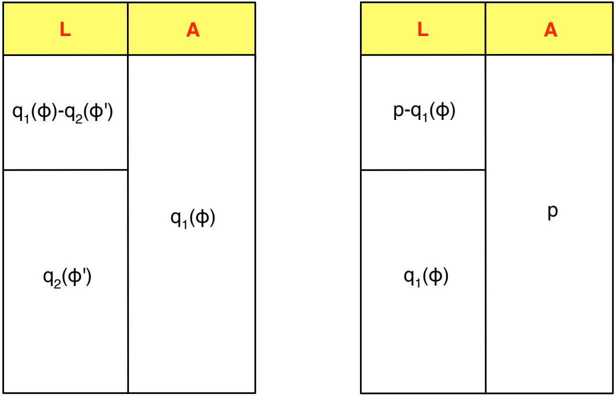
Pyramiding Lending Arrangements

- Allow Borrowing Contracts to be used as collateral.



- Hedonic interest rates ⇒ Lenders want to **leverage into them !**
- Example for houses, loans to SMEs: securitization. Or rehypothecation of collateral, repos of mortgage-backed securities, etc.
- **Price p increases even more.**

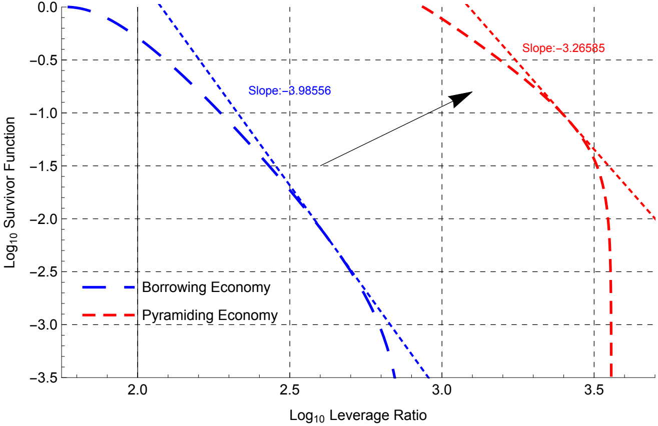
Balance Sheets



- Akin to tranching. The lender of type 2 is repaid until ϕ', then lender of type 1 is repaid on $\phi - \phi'$, then the borrower gets $p_1 - \phi$.

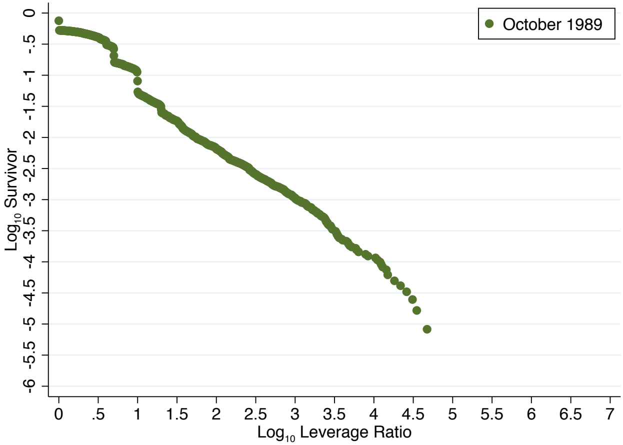
Pyramiding Lending Arrangements

- Pareto Coefficients decrease (leverage distributions are multiplied) $\Rightarrow$ Leverage Ratio distribution shifted to the right.
- Price expresses the opinion of superoptimists.



Empirics

- Leverage Ratios on New Loans. Source: Dataquick.
- $\approx$ 100,000 - 500,000 new loans per month.



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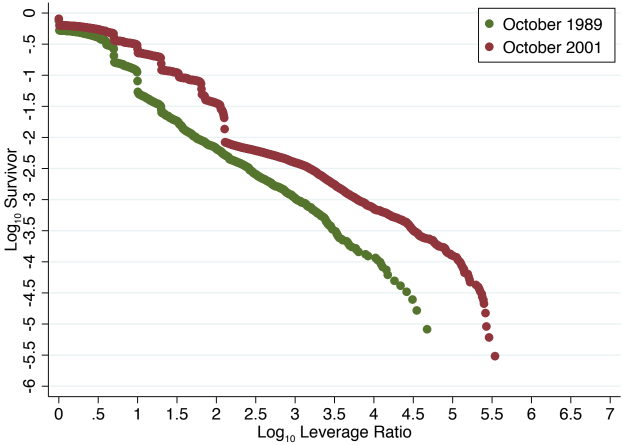

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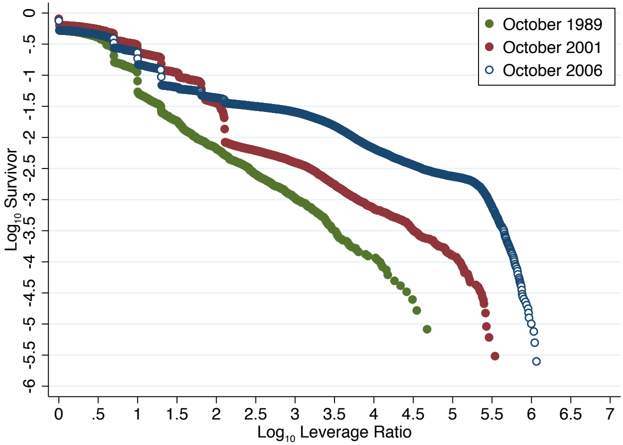

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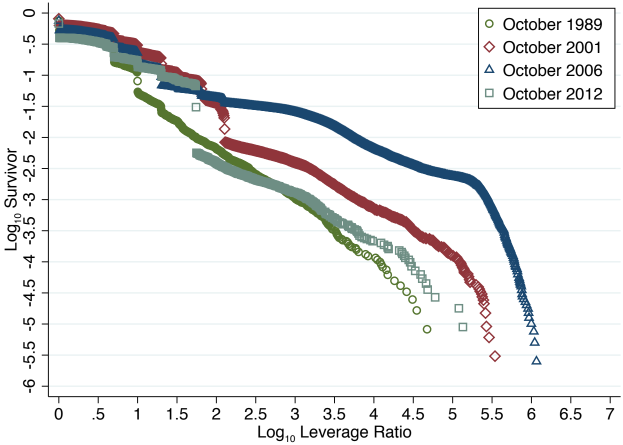
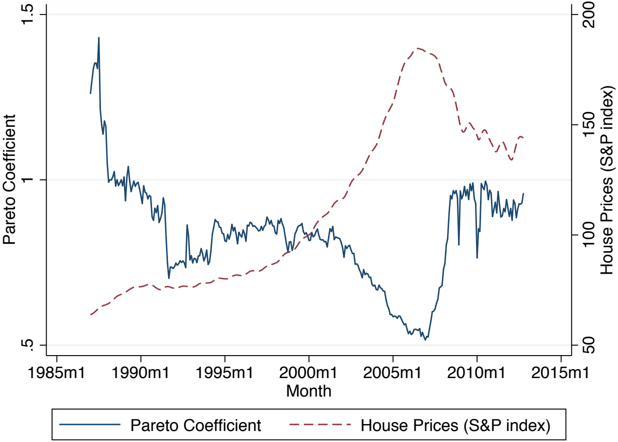
- Leverage Ratios on New Loans. Source: Dataquick.
- ≈ 100,000 - 500,000 new loans per month.

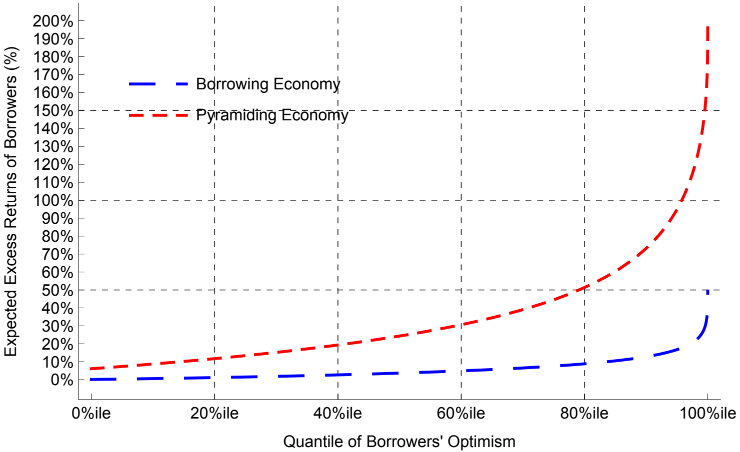


Pyramiding Lending Arrangements



- Video: the leverage ratio distribution from 1987 to 2012.
- Link: `http://www.econ.ucla.edu/fgeerolf/research/geerolf-leverage-video.avi`

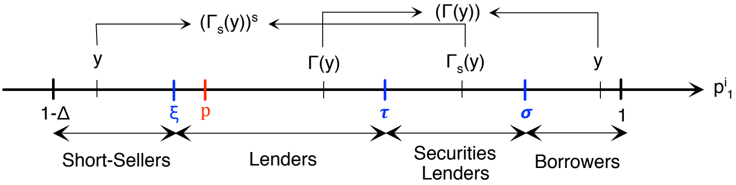
- The model allows to recover the corresponding increase in borrowers' expected returns.



- In a model with a little bit of risk aversion: more risk taking?

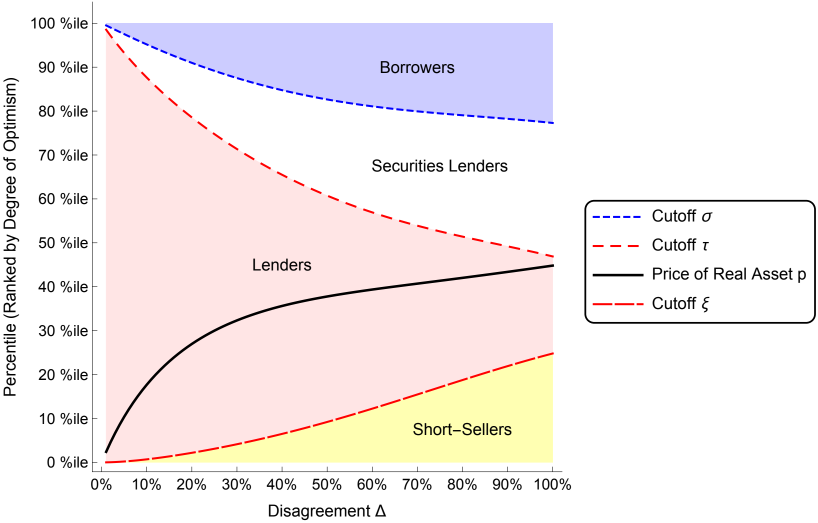
Short-Sales

- Unlike existing disagreement models, the model allows the treatment of short-sales.



- Price = pessimists' valuations ⇒ Systematic undervaluation - similar to noise trader risks in De Long et al. (1990), but risk neutrality. Equity premium, discount of closed-end funds, etc.
- **Endogenous rebate rates** - apparent short-selling costs not evidence of constraints: about 100 bps, larger with more disagreement.
- **Endogenous Short-interest** (a few percent).

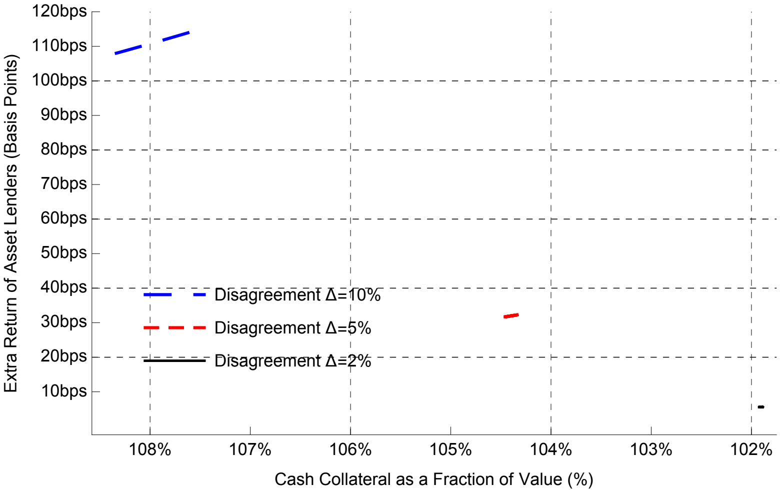
Short-Sales



Borrowers
Securities Lenders
Lenders
Short–Sellers
Percentile (Ranked by Degree of Optimism)
100 %ile
90 %ile
80 %ile
70 %ile
60 %ile
50 %ile
40 %ile
30 %ile
20 %ile
10 %ile
0 %ile
0%
10%
20%
30%
40%
50%
60%
70%
80%
90%
100%
Disagreement Δ
Cutoff σ
Cutoff τ
Price of Real Asset p
Cutoff ξ

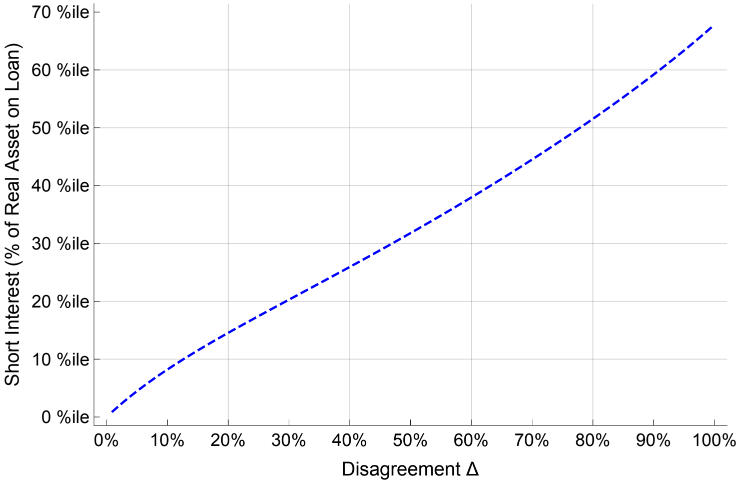
Endogenous Rebate Rates and Cash Collateral

- No short-selling costs or costs of default.

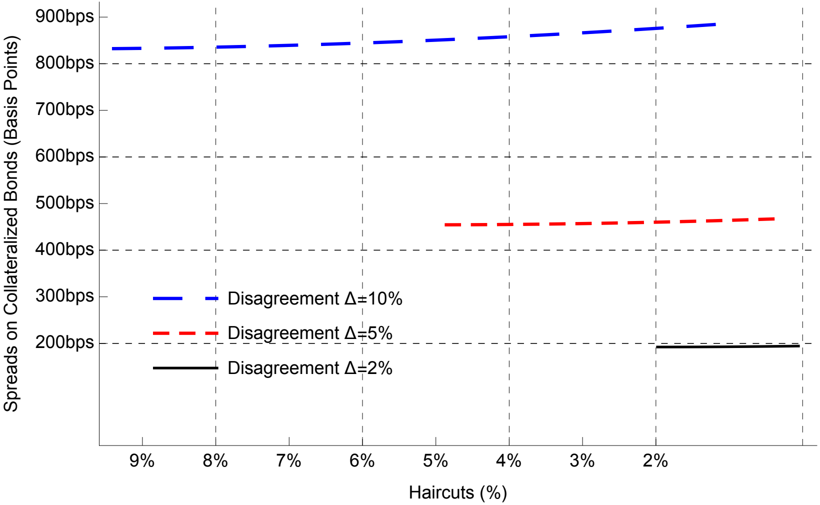


Endogenous Short Interest

- Only a few percent of stocks are on loan in equilibrium, even though all are potentially available.



Larger Spreads on Bonds, even the safest (AAA)



Model with Borrowing Contracts Only

- Setup
- Equilibrium Definition
- Equilibrium Solution
- Equilibrium Properties

Extension 1: "Pyramiding" Lending Arrangements

Extension 2: Short-Sales

Conclusion

Conclusion

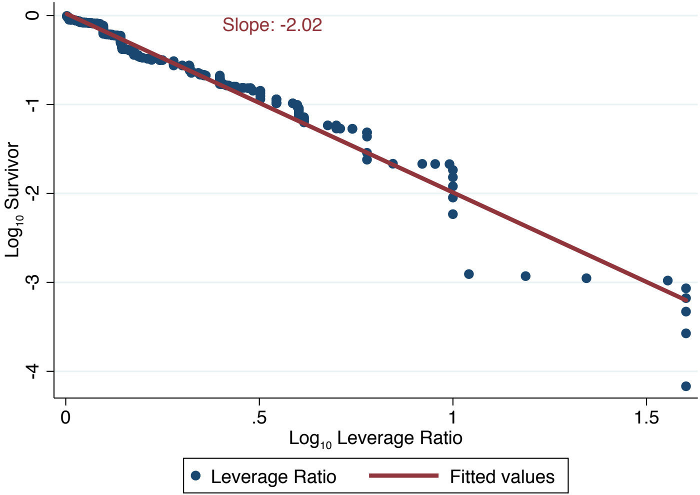
- Homeowners / Entrepreneurs' / Hedge Funds data lend support to a very stylized model.
- New (static) source of Pareto distributions in returns independent from Gibrat's law/ random growth.
- New intuitions on key financial prices / quantities:
  - Returns on Bonds.
  - Short-selling "costs".
  - Short interest

Potential for future work:

- Empirical work on short interest, rebate rates, distributions of leverage ratios to recover disagreement.
- Financial regulation:
  - Costs of moving OTC onto exchanges.
  - Monitoring financial system through ultimate borrowers' leverage ratio distribution ?

Thank you

Leverage Ratios of Entrepreneurs



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