Lecture 3 - Consumption - Intertemporal Optimization

UCLA - Econ 102 - Fall 2018

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Consumption and saving are perhaps the most important and controversial issues in macroeconomics. In the Solow [1956] growth model, saving was a constant fraction s of GDP, by assumption. We now build on Economics 101, in order to derive saving behavior from microeconomic principles. In other words, we work to make saving "endogenous" (that is, explained by the model), while it was previously taken as exogenous (that is, assumed in the model).

Although this discussion may appear somewhat abstract at first, these calculations are the basis of some of the most important controversies in macroeconomics, which we shall come to in the next lectures.

1 The Two-Period Consumption Problem

1.1 Assumptions

There are two periods, $t = 0$ (think of this as "today") and $t = 1$ (think of this as "tomorrow"). The consumer values consumption c_0 in period 0 and c_1 in period 1 according to the following utility function:

$$U(c_0, c_1) = u(c_0) + \beta u(c_1).$$

where $u(.)$ is an increasing and concave function, and $\beta \leq 1$. β captures that people typically have a preference for the present. (they are **present-biased**)

Assume that agents earn (labor) income y_0 in period 0, and (labor) income y_1 in period 1. They also are born with some financial wealth f_0 now, and have financial wealth f_1 in period 1, which they consume entirely because this is the last period. (there is no point keeping more money for after period 1, because there is no future at that point) The amount that agents save in this economy is thus $f_1 - f_0$, and the amount of their accumulated savings is the savings they already had plus what they decided to accumulate, so that $f_0 + (f_1 - f_0) = f_1$.

Therefore, consumption in period 0 is given by:

$$c_0 = y_0 - (f_1 - f_0)$$

The second period consumption ($t = 1$) is given by income plus the return to (accumulated !) savings:

$$c_1 = y_1 + (1 + R)f_1.$$

1.2 Solution

Intertemporal budget constraint. Rewriting f_1 from this second equation: $f_1 = (c_1 - y_1)/(1+R)$, and plugging into the first,

$$c_0 = y_0 - \left(\frac{c_1 - y_1}{1+R} - f_0\right).$$

Rearranging, total wealth is then the sum of financial wealth f_0 and of the present discounted value of human wealth:

$$c_0 + \frac{c_1}{1+R} = \overbrace{f_0 + \underbrace{y_0 + \frac{y_1}{1+R}}_{\text{human wealth}}}^{\text{total wealth}}.$$

The intertemporal budget constraint says that the present discounted value of consumption is equal to total wealth.

Optimization. The problem of the consumer is then simply that of maximizing utility under his budget constraint:

$$\max_{c_0, c_1} \quad u(c_0) + \beta u(c_1)$$
$$\text{s.t.} \quad c_0 + \frac{c_1}{1+R} = f_0 + y_0 + \frac{y_1}{1+R}.$$

You may solve this optimization in four different ways:

1. Apply the well known ratio of marginal utilities formula from Econ 101 and say that:

$$\frac{\partial U/\partial c_1}{\partial U/\partial c_0} = \frac{1}{1+R} \quad \Rightarrow \quad \frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+R}$$

2. Apply the following intuitive economic argument. The marginal utility from consuming in period 1 is $\beta u'(c_1)$. The marginal utility from consuming in period 0 is $u'(c_0)$. By putting one unit of consumption in the bank, one forgoes 1 unit of consumption in period 0 to get $1+R$ units of consumption in period 1. The two have to be equal:

$$u'(c_0) = (1+R)\beta u'(c_1) \quad \Rightarrow \quad \frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+R}.$$

3. Replace c_0 from the intertemporal budget constraint above and optimize with respect to c_1:

$$\max_{c_1} \quad u\left[\left(f_0 + y_0 + \frac{y_1}{1+R}\right) - \frac{c_1}{1+R}\right] + \beta u(c_1)$$

This leads to the following First-Order Condition:

$$-\frac{1}{1+R}u'(c_0) + \beta u'(c_1) = 0 \quad \Rightarrow \quad \frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+R}.$$

4. Alternatively, you may substitute c_1 out and optimize with respect to c_0

$$\max_{c_0} \quad u(c_0) + \beta u \left[(1+R) \left(f_0 + y_0 + \frac{y_1}{1+R} \right) - (1+R) c_0 \right]$$

This again leads to the same First-Order Condition:

$$u'(c_0) - \beta(1+R)u'(c_1) = 0 \quad \Rightarrow \quad \frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+R}.$$

1.3 Some examples

Log utility, no discounting ($\beta = 1$). Log utility implies that $u(c)$ is given by the natural logarithm. Marginal utility is then just:

$$u'(c) = \frac{1}{c},$$

Therefore, the above optimality condition (derived 4 times) can be written as:

$$c_0 = \frac{c_1}{1+R}$$

.

Substituting out in the intertemporal budget constraint allows to calculate consumption at time 0 c_0 as well as consumption at time 1 c_1:

$$c_0 = \frac{1}{2} \left(f_0 + y_0 + \frac{y_1}{1+R} \right),$$
$$c_1 = \frac{1+R}{2} \left(f_0 + y_0 + \frac{y_1}{1+R} \right).$$

According to this expression, the **Marginal Propensity to Consume (MPC)** out of current wealth f_0 is given by $1/2$. When f_0 rises to $f_0 + \Delta f_0$, the corresponding change in consumption is:

$$\Delta c_0 = \frac{1}{2} \Delta f_0.$$

If we were to study a model with more periods, say T periods, we would find that people Marginal Propensity to Consume is approximately equal to $1/T$, at least according to this model. Whether such is actually the case, and people are that rational, is a subject of fierce debate among macroeconomists, and one that we will take up in the next lectures.

Log utility, with discounting ($\beta < 1$). Marginal utility is then $u'(c) = 1/c$, so that the optimality condition gives:

$$\frac{c_1}{1+R} = \beta c_0.$$

Substituting out in the intertemporal budget constraint, you can check that:

$$c_0 = \frac{1}{1+\beta} \left(f_0 + y_0 + \frac{y_1}{1+R} \right)$$
$$c_1 = \frac{\beta(1+R)}{1+\beta} \left(f_0 + y_0 + \frac{y_1}{1+R} \right).$$

Because people are more impatient in this case, they consume more, and their Marginal Propensity to Consume (MPC) is **higher** with $\beta < 1$:

$$\Delta c_0 = \frac{1}{1+\beta}\Delta f_0.$$

Note that the solution with no discounting corresponds to that with discounting when $\beta = 1$, which was expected.

1.4 Generalization

Assume that an individual receives wage w in period 0, and that this wage is expected to grow at rate g in the next T years. What is the present value of his human wealth, assuming that the interest rate is given by R? The answer is that his human wealth H is given as follows:

$$H = w + w\frac{1+g}{1+R} + w\left(\frac{1+g}{1+R}\right)^2 + ... + w\left(\frac{1+g}{1+R}\right)^{T-1}$$

$$H = w\frac{1-\left(\frac{1+g}{1+R}\right)^T}{1-\frac{1+g}{1+R}}$$

References

Robert M. Solow. A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, 70(1):65–94, 1956. ISSN 0033-5533. doi: 10.2307/1884513. URL http://www.jstor.org/stable/1884513.