Problem Set 3

UCLA - Econ 102 - Fall 2018

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3 Problem Set 3

3.1 Two-period Intertemporal Optimization

Consider the model of lecture 3 again. Instead of logarithmic preferences, assume that preferences are given by:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

1. Under what condition on σ is this an increasing and concave utility function?
2. Show using 4 different methods that:
$$\frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+r}$$
3. Using the equation in question 1, what is the ratio c_1/c_0?
4. Replace in the intertemporal budget constraint to find an implicit equation for c_1. Do the same for c_0.
5. Assume that $\sigma = 1/2$, and $f_0 = 0$, $y_0 = \$90,000$, $y_1 = 0$, $\beta = 1$. What are c_0 and c_1 if $r = 1\%$? What about if $r = 2\%$? How much does c_0 change then? How much in percentage terms?
6. Same questions if $\sigma = 1$.
7. Same questions if $\sigma = 2$.
8. Compare the changes in c_0 following an increase in the real interest rates in questions 5, 6, 7. Comment.

3.2 Another Overlapping Generations model

Consider the model of lecture 4 again, with one small twist: agents care only about old age consumption. In other words, their utility functions are given by:

$$U = u(c^o_{t+1}).$$

Our goal is to derive the law of motion for the capital stock K_t, that is, a function relating K_{t+1} to K_t and the parameters of the model.

1. Why can the utility function be left unspecified for computing the level of saving?
2. Derive the law of motion for capital.
3. What is the corresponding value of the saving rate in the Solow model?
4. Provide a condition on α such that the capital stock is below the Golden Rule level.
5. Is that condition likely to be satisfied?