Cobb and Douglas (1928) production function

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The Cobb and Douglas (1928) production function gives output Y as a power function of capital K and labor N :

$$Y = F(K,N) = K^{\alpha} N^{1-\alpha}$$

Take a firm which has this production function, hires labor at price w and rents capital at rate r. Then, this firm maximizes:

$$\max_{K,N} \quad F(K,N) - wN - rK$$

The firm's optimal choice of the capital stock K implies that the marginal productivity of capital is equal to the rental rate of capital r:

$$r = \frac{\partial F(K,N)}{\partial K} = \alpha K^{\alpha-1} N^{1-\alpha}$$

Thus, income from capital is:

$$rK = \alpha K^{\alpha} N^{1-\alpha} \quad \Rightarrow \quad \frac{rK}{Y} = \alpha$$

Similarly, the firms's optimal choice of employment N implies that the marginal productivity of labor is equal to the wage w:

$$w = \frac{\partial F(K,N)}{\partial N} = (1-\alpha) K^{\alpha} N^{-\alpha}$$

Thus, income from labor is:

$$wN = (1-\alpha) K^{\alpha} N^{1-\alpha} \quad \Rightarrow \quad \frac{wN}{Y} = 1-\alpha$$

Note that in equilibrium, firms make zero profits:

$$\frac{rK}{Y} + \frac{wN}{Y} = \alpha + (1-\alpha) = 1 \quad \Rightarrow \quad F(K,N) = wN + rK$$

Therefore, the **production approach to GDP** gives the same as the **income approach to GDP**.

α is the **share of capital in value added**, and is typically equal to $\alpha \approx 1/3$. In contrast, the share of labor in value added is $1-\alpha \approx 2/3$.

References

Cobb, Charles W. and Paul H. Douglas, "A Theory of Production," *The American Economic Review*, 1928, *18* (1), 139–165.