Calculus Review

UCLA - Econ 102 - Fall 2018

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1 Taylor Approximations

Multiplication. If x and y are small, then:

$$\boxed{(1+x)(1+y) \approx 1+x+y}.$$

Proof. We have:

$$(1+x)(1+y) = 1+x+y+xy.$$

When x and y are both small, then xy is negligible (it is a second order term), which gives the result:

$$(1+x)(1+y) \approx 1+x+y.$$

Ratio. If x and y are small, then:

$$\boxed{\frac{1+x}{1+y} \approx 1+x-y}.$$

Proof. We have:

$$\frac{1}{1+y} = 1-y+y^2-y^3+...$$

When x and y are both small, all terms of the product are negligible except for first-order terms:

$$\frac{1+x}{1+y} \approx 1+x-y.$$

Power. If x is small, then:

$$\boxed{(1+x)^n \approx 1+nx}.$$

Proof. For $n=1$, we know that $(1+x)^1 = 1+x$ (obviously). Assume that the approximation is true for n, or that $(1+x)^n \approx 1+nx$, let's prove that it is true for $n+1$:

$$(1+x)^{n+1} = (1+x)^n \approx (1+nx)(1+x) \approx 1+(n+1)x+nx^2 \approx 1+(n+1)x,$$

which proves the proposition for $n+1$. Thus, the Taylor approximation is true for any $n \in \mathbb{N}$.

2 Growth Rates

Multiplication. If g_X and g_Y are small, then:

$$\boxed{g_{XY} = g_X + g_Y}.$$

Proof. The growth rate g_X of X is given by $g_X = X_{t+1}/X_t - 1$. Thus, the growth rate of XY is:

$$\begin{aligned}
g_{XY} &= \frac{X_{t+1}Y_{t+1}}{X_tY_t} - 1 \\
&= \frac{X_{t+1}}{X_t}\frac{Y_{t+1}}{Y_t} - 1 \\
&= (1+g_X)(1+g_Y) - 1 \\
&\approx 1 + g_X + g_Y - 1 \\
g_{XY} &\approx g_X + g_Y,
\end{aligned}$$

where we have used the above Taylor approximation with $(1+g_X)(1+g_Y) \approx 1+g_X+g_Y$.

Ratio. If g_X and g_Y are small, then:

$$\boxed{g_{X/Y} = g_X - g_Y}.$$

Proof. The growth rate g_X of X is given by $g_X = X_{t+1}/X_t - 1$. Thus, the growth rate of XY is:

$$\begin{aligned} g_{X/Y} &= \frac{X_{t+1}/Y_{t+1}}{X_t/Y_t} - 1 \\ &= \frac{X_{t+1}}{X_t}\frac{Y_t}{Y_{t+1}} - 1 \\ &= \frac{1+g_X}{1+g_Y} - 1 \\ &\approx 1 + g_X - g_Y - 1 \\ g_{X/Y} &\approx g_X - g_Y, \end{aligned}$$

where we have used the above Taylor approximation with $(1+g_X)/(1+g_Y) \approx 1+g_X-g_Y$.