The Paradox of Thrift

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As Blanchard (2017) puts it, we are told about the virtues of thrift as we grow up (for example, this boy). Keynes (1936) was revolutionary in many ways, and often times challenged not only academic but sometimes even popular wisdoms. In particular, he argued that when people tried to save more, the result might be a decline in output and unchanged saving, or even declining saving in the aggregate. This is now called the "paradox of thrift." We are able to understand how the paradox of thrift might arise, using two different models that we learned in the lectures (Lectures 2 and 4) and in the recitation sections (Chapter 3, Problem 8).

- The **consumption multiplier model**. (Lecture 2 and 4) This model leads to ***unchanged*** saving as people try to save more. This is the model that is used by Blanchard (2017) in his Focus Box on the paradox of thrift, page 63. Section 1 reviews this model.
- The **consumption and investment multiplier model**. (Problem 3, Chapter 3) This model leads to ***declining*** saving as people try to save more. Section 2 reviews this model.

1 The Paradox of Thrift with unchanged saving

In the consumption multiplier model, we have a paradox of thrift in that efforts by consumers to save more are self-defeating, and saving is constant. We start from the basic goods market model:

$$C = c_0 + c_1 Y_D$$
$$Y_D = Y - T$$

where c_1 is the marginal propensity to consume out of disposable income Y_D, disposable income is income minus taxes, and investment $I = \bar{I}$ and government expenditures G are taken as given, as well as taxes T.

There are two ways that one prove the "paradox of thrift" result: first, as in the class, using that saving = investment. The proof has only one line, but the paradox of thrift feels perhaps a bit "magical" (section 1.1). Therefore, I also provide a longer but more intuitive proof in section 1.2.

1.1 Shorter proof

The first way that one may see the paradox of thrift is by writing that investment $\bar{I}$, which is fixed in this model, has to be equal to total saving, which are themselves equal to private saving S plus public saving $T-G$. Therefore, one can write saving as a function of only exogenous variables:

$$\bar{I} = S + (T-G) \quad \Rightarrow \quad S = \bar{I} - (T-G)\,.$$

This proves the result ! In particular, if there is a change in consumption of $\Delta c_0 < 0$, then private saving does not depend on that so does not move. The next section presents a much longer proof, but one that also explains more clearly why saving does not write despite a change in c_0 given by $\Delta c_0 < 0$.

1.2 Longer but more intuitive proof

We write that output equals demand, to get an expression for output:

$$\begin{aligned} Y &= Z = C + I + G \\ Y &= c_0 + c_1(Y-T) + \bar{I} + G \end{aligned}$$

Therefore:

$$Y = \frac{1}{1-c_1}\left(c_0 - c_1 T + \bar{I} + G\right)$$

This is the usual multiplier: for a given change in Δc_0, the change in output is given by the direct effect on output, but also by all the successive new rounds, which add up to $\frac{\Delta c_0}{1-c_1}$ in total. Thus, such a change in Δc_0 leads to a change in output of:

$$\boxed{\Delta Y = \frac{\Delta c_0}{1-c_1}.}$$

Private saving is given by disposable income $Y-T$ minus consumption (what is earned, not paid in taxes, nor consumed, is saved), and therefore:

$$S = Y - T - C$$

Replacing consumption C in this equation, using disposable income, allows us to write:

$$\begin{aligned} S &= Y - T - c_0 - c_1\,(Y-T) \\ S &= -c_0 + (1-c_1)\,(Y-T)\,. \end{aligned}$$

What happens when people attempt to save more, say by lowering c_0? A change in consumption of $\Delta c_0 < 0$ clearly leads to:

- **a direct effect** on private saving that is given by $-\Delta c_0 > 0$ (private saving rises).
- **an indirect effect** going through the change in output whose magnitude was calculated above given by $\Delta\left[(1-c_1)\,(Y-T)\right]$.

In other words, we may write:

$$\boxed{\Delta S = \underbrace{\Delta(-c_0)}_{\text{direct effect}} + \underbrace{\Delta\left[(1-c_1)\,(Y-T)\right]}_{\text{indirect effect}}}$$

How large is the indirect effect? Some algebra allows to conclude that it is exactly the opposite of the direct effect:

$$
\begin{aligned}
\Delta\left[(1-c_1)(Y-T)\right] &= (1-c_1)\Delta Y \\
&= (1-c_1)\frac{\Delta c_0}{1-c_1} \\
\Delta\left[(1-c_1)(Y-T)\right] &= \Delta c_0.
\end{aligned}
$$

Therefore, the total effect on saving is:

$$
\begin{aligned}
\Delta S &= \Delta(-c_0) + \Delta\left[(1-c_1)(Y-T)\right] \\
&= -\Delta c_0 + \Delta c_0
\end{aligned}
$$

Thus finally:

$$\boxed{\Delta S = 0}.$$

2 The Paradox of Thrift with declining saving

In the consumption and investment multiplier model, we get an even stronger paradox of thrift in that efforts by consumers to save more lead to ***declining saving***. The consumption and investment multiplier model is presented in Chapter 3, Problem 8, where investment is allowed to depend on output. We thus start from the following goods market model:

$$
\begin{aligned}
C &= c_0 + c_1 Y_D \\
Y_D &= Y - T \\
I &= b_0 + b_1 Y
\end{aligned}
$$

where c_1 is the marginal propensity to consume out of disposable income Y_D, disposable income is income minus taxes, government expenditures G is taken as given, as well as taxes T.

Again, there are two ways that one can prove that savings decline when c_0 declines: first, as in the class, using that saving = investment, as in section 2.1. I provide a more intuitive proof in section 2.2.

2.1 Shorter proof

Again, let us write the investment = total saving identity:

$$I = S + (T - G) \quad \Rightarrow \quad S = I - (T - G).$$

We know that a fall in consumption of $\Delta c_0 < 0$ leads to a decline in output $\Delta Y < 0$, and therefore through the equation giving investment as a function of output, to a decline in investment since $\Delta I = b_1 \Delta Y$:

$$I = b_0 + b_1 Y \quad \Rightarrow \quad \Delta I = b_1 \Delta Y.$$

Because T and G are assumed to be fixed (so that public saving is fixed), the change in private saving is equal to the change in investment, and is therefore negative. Therefore, a fall in consumption, leads to a fall in private saving ! Again, that proof is probably not very intuitive. We now turn to the longer proof.

2.2 Longer but more intuitive proof

Again, we write that output equals demand, which allows to get an expression for output:

$$\begin{aligned} Y &= Z = C + I + G \\ Y &= c_0 + c_1(Y - T) + b_0 + b_1 Y + G \end{aligned}$$

Therefore:

$$Y = \frac{1}{1 - c_1 - b_1}\left(c_0 + b_0 - c_1 T + G\right)$$

This is the usual multiplier, compounding the consumption and investment effects (it is assumed here that $c_1 + b_1 < 1$). Therefore, a given change in $\Delta c_0 < 0$ leads to decline in output of:

$$\boxed{\Delta Y = \frac{\Delta c_0}{1 - c_1 - b_1}.}$$

Again, one can show that $S = -c_0 + (1 - c_1)(Y - T)$ – see section 1.2 for a proof – so that we have the following decomposition:

$$\boxed{\Delta S = \underbrace{\Delta(-c_0)}_{\text{direct effect}} + \underbrace{\Delta\left[(1 - c_1)(Y - T)\right]}_{\text{indirect effect}}}$$

However, this time, the two effects do not exactly cancel out as the indirect effect is:

$$\begin{aligned} \Delta\left[(1 - c_1)(Y - T)\right] &= (1 - c_1)\Delta Y \\ &= (1 - c_1)\frac{\Delta c_0}{1 - c_1 - b_1} \\ \Delta\left[(1 - c_1)(Y - T)\right] &= \frac{1 - c_1}{1 - c_1 - b_1}\Delta c_0. \end{aligned}$$

Therefore, the total effect on saving is:

$$\begin{aligned} \Delta S &= \Delta(-c_0) + \Delta\left[(1 - c_1)(Y - T)\right] \\ &= -\Delta c_0 + \frac{1 - c_1}{1 - c_1 - b_1}\Delta c_0 \\ \Delta S &= \frac{b_1}{1 - c_1 - b_1}\Delta c_0 \end{aligned}$$

Thus finally:

$$\boxed{\Delta S < 0}.$$

References

Blanchard, Olivier J., *Macroeconomics*, Pearson Education, 2017.

Keynes, John Maynard, *The General Theory of Employment, Interest, and Money* 1936.