Problem Set 4

UCLA - Econ 102 - Fall 2018

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3.1 The Solow Growth Model with Exogenous Growth

Consider the Solow growth model of Lecture 2, with however two small changes. Assume that the production function is given by:

$$F(K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha},$$

where productivity A_t grows exogenously at rate g and $A_0 = 1$:

$$A_t = (1+g)^t.$$

Moreover, assume that the labor force also grows at a rate n and $L_0 = 1$, so that at any time t:

$$L_t = (1+n)^t.$$

1. Write the law of motion for capital K_t.
2. Define k_t as:
$$k_t \equiv \frac{K_t}{A_t^{1/(1-\alpha)} L_t},$$
and write a law of motion for k_t. Assume that n, and g are small in order to simplify this law of motion. *Hint*: if n and g are small then: $(1+g)^{1/(1-\alpha)}(1+n) \approx 1 + \frac{1}{1-\alpha} g + n$.
3. Show that k_t converges to a steady-state k^*. Compute k^*.
4. When k_t has reached a steady-state, the economy is said to be on a **balanced growth path**. On this balanced growth path, what is the rate of growth of Y_t, C_t, K_t, K_t/Y_t, K_t/L_t, w_t, $w_t L_t$ and $w_t L_t / Y_t$? Denoting by R_t the marginal product of capital, what is the rate of growth of R_t, $R_t K_t$, and $R_t K_t / Y_t$?
5. Compute y^* and c^* corresponding to steady-state k^* with:
$$y_t \equiv \frac{Y_t}{A_t^{1/(1-\alpha)} L_t} \quad \text{and} \quad c_t \equiv \frac{C_t}{A_t^{1/(1-\alpha)} L_t}.$$
6. What is the saving rate which maximizes c^* ? (Golden Rule level of capital accumulation)
7. What is then the value of the marginal product of capital R^* ?

3.2 The Neoclassical Labor Market Model

Consider the neoclassical labor market model of lecture 6. Assume that preferences and the production function are as in lecture 6:

$$U(c,l) = c - B\frac{l^{1+\epsilon}}{1+\epsilon}, \qquad f(l) = A l^{1-\alpha}.$$

Denote the wage by w, and the price of consumption by p.

1. Derive the Labor Demand curve.

2. Assume that $\alpha = 1/3$ and $A = 2$. Using your favorite spreadsheet software, plot this demand curve in a $(l, w/p)$ plane - that is, putting l on the x-axis and w/p on the y-axis.
3. Take logs of both sides. What does the demand curve look like in a $(\log(l), \log(w/p))$ plane? What is the slope of the demand curve equal to? If α is higher, is the demand curve steeper or flatter? What shifts the demand curve to the left or to the right?
4. Derive the Labor Supply curve.
5. Assume that $\epsilon = 5$ and $B = 2$. Using your favorite spreadsheet software, plot this supply curve in a $(l, w/p)$ plane - that is, putting l on the x-axis and w/p on the y-axis. Add the supply curve to the demand curve of question 2.
6. Take logs of both sides. What does the supply curve look like in a $(\log(l), \log(w/p))$ plane? What is the slope of the supply curve equal to? If ϵ is higher, is the supply curve steeper or flatter? What shifts the supply curve to the left, or to the right?
7. Assume that productivity A decreases by 5%, to $A = 1.9$. What is the effect on the quantity of employment, and on the real wage? If α is higher, is that effect larger or smaller? What is the economic intuition?
8. Assume that leisure becomes relatively more attractive relative to working (think of Facebook, Netflix, etc.), so that B increases by 10% (the disutility of work increases). What is the effect on the quantity of employment, and on the real wage? If ϵ is higher, is that effect larger or smaller? What is the economic intuition for this?

3.3 The "Keynesian" Labor Market Model

Consider the neoclassical labor market model of the previous problem.

1. Assume that productivity A decreases by 5%, but that real wages w/p are rigid. Compute the change in the quantity of employment following a fall in productivity.
2. Compare the effect with question 7 in the previous problem. Explain.
3. Assume that leisure becomes relatively more attractive relative to working, so that B increases by 10%. Compute the change in the quantity of employment following a increase in leisure attractiveness.
4. Compare the effect with question 8 in the previous problem. Explain.

3.4 The Bathtub model

Consider the bathtub model of lecture 6. Assume a monthly job separation rate equal to $s = 1\%$, and a monthly job finding rate equal to $f = 20\%$. Assume that the labor force is given by $L = 159$ million.

1. Derive the steady-state unemployment rate. How many people are unemployed in the steady-state? How many people lose their jobs every month? How many people find a job every month?
2. Assume that the economy starts with an unemployment rate equal to $u_0 = 8\%$. Using your favorite spreadsheet software, show the evolution of the unemployment rate over time. How long before the unemployment rate reaches 5%?
3. If $s = 2\%$ instead, which job finding rate f gives the same steady-state unemployment rate?
4. Assuming the separation rate and the job finding rate are given from question 3, answer question 2 again.
5. Explain why an economy with more churning (that is, faster reallocation) - think of the US versus Europe - has a faster recovery in terms of unemployment after a recession. *Note:* A recession could be

coming from a temporary increase in the job separation rate, or a temporary decrease in the job finding rate, which then goes back to its original value.