Intermediate Macroeconomics

UCLA - Econ 102 - Fall 2018

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Contents

Preface

This online book contains most of the class material for *Intermediate Macro (Econ 102)* I teach at UCLA. The Moodle platform should be used for the discussion board as well as some additional readings.

Main Information

Lectures: Mondays and Wednesdays, 3:30-4:45pm, Dodd Hall, Room 147.

Office hours: Tuesdays, 4-6pm. (Bunche 8389)

Moodle Website: https://moodle2.sscnet.ucla.edu/course/view/18F-ECON102-1

Graduate Student Instructors (GSIs): Graduate Student Instructors are all graduate students in the UCLA Economics Department. They will teach sections and hold 2 hours of office hours in the Alper Room every week:

- Sections 1E-1I. Paula Beltran. OH: F 11-12; 2-3. pabeltran90@gmail.com
- Sections 1H-1M. Alvaro Boitier. OH: M 2:30-3:30; T 2-3. alvaro.boitier@gmail.com
- Sections 1N-1K. Conor Foley. OH: T 2-4. conor.teaches.econ@gmail.com
- Sections 1D-1J. Kun Hu. OH: R; 9-11. rickhukun@ucla.edu
- Sections 1G-1O. Ivan Lavrov. OH: W 1-3. ilavrov113@gmail.com
- Sections 1B-1C. Anthony Papac. OH: M 10-11; R 12:30-1:30. anthonypapac@g.ucla.edu
- Sections 1A-1F. Mengbo Zhang. OH: W 10-12. zmbruc@gmail.com

Course description. This course is meant to provide an intermediate-level treatment of macroeconomic topics, including the study of economic growth, business cycle fluctuations, unemployment, inflation, as well as open-economy macroeconomic issues such as trade imbalances and exchange rate policy. Although the title of the class is "Macroeconomic Theory", students will learn both the theory as well as some of the empirical evidence behind the theory, and its practical implications. Special emphasis will be placed on the application of economic tools to contemporary economic problems and policies. Competing schools of thought will be presented, with a particular emphasis on Neoclassical and Keynesian theories, and they will be discussed in the light of macroeconomic data. Class meetings will be highly interactive, with many opportunities for you to both ask and answer questions.

Course Objectives. My objective is that, by the end of the course, you will be able to read, and critically assess writings from *The Economist*, *The Wall Street Journal*, or *The New York Times*. Macroeconomics is everywhere in the news, and I want to walk you through the tools you need to understand it better. Economics is ultimately an empirical subject, so as much as possible I will try to convey not just the theory of how the economy works, but also the evidence supporting, or contradicting the theory. We will not always reach definitive conclusions on most of the issues we will examine, but you should have a more informed opinion on each of them and why these questions are hard and debated scientifically.

Prerequisites. A strict prerequisite for the class is that you have taken Econ 101. If you do not meet this prerequisite, you are advised to take this course during another term. You should also be familiar with

some elementary mathematics. For example, you need to know what a logarithm is, and how to calculate a geometric sum:

$$1 + c_1 + c_1^2 + ... = \frac{1}{1 - c_1} \quad \text{if} \quad 0 < c_1 < 1,$$

because that is really useful to understand how a Keynesian multiplier works, for example. If you do not know that already, that is fine too, but you should at least be willing to learn. If you want a treatment of Econ 102 which is less heavy on algebra, you are best advised to take this class in another term.

Textbook (optional): Olivier Blanchard's *Macroeconomics*, 7th Edition (previous editions should be fine, too).

Class Rules

Questions? If you have any question about the material covered during the course, you should consider the following options in order:

1. First, you should never refrain from asking questions during class.
2. Second, you may ask questions during recitation sections. The smaller group should allow you to ask questions more freely. Our teaching assistants are all passionate graduate students, writing a phD thesis on macroeconomics or other related subjects, so they will be very happy to help you.
3. Third, TAs will hold their office hours. The times for their office hours is reminded here:
   - Paula Beltran. OH: F 11-12; 2-3. pabeltran90@gmail.com
   - Alvaro Boitier. OH: M 2:30-3:30; T 2-3. alvaro.boitier@gmail.com
   - Conor Foley. OH: T 2-4. conor.teaches.econ@gmail.com
   - Kun Hu. OH: R; 9-11. rickhukun@ucla.edu
   - Ivan Lavrov. OH: W 1-3. ilavrov113@gmail.com
   - Anthony Papac. OH: M 10-11; R 12:30-1:30. anthonypapac@g.ucla.edu
   - Mengbo Zhang. OH: W 10-12. zmbruc@gmail.com
4. Fourth, you should feel free to ask questions on the discussion board (not by email). We will never respond to questions about contents by email (in particular those starting with "is X, Y, Z, test material"), because doing so would be unfair to the rest of the class. In contrast, we commit to respond to all questions on the Moodle Website within 24 hours (either me or the TAs will). Beware ! You should start studying for the midterm exam earlier than November 4 – we will stop answering questions at **6pm the day before each exam** (either the midterm on November 5, or the final on December 14).
5. Finally, I will hold regular office hours on Tuesdays, 4-6pm, in my office 8389. Please send me an email prior if you plan to arrive after 5pm.

Class notes. Class notes will be posted *after* each class, so as to encourage you to take notes. Notes might not always be comprehensive, and everything I will say during class is potentially examinable, even if it does not appear in the notes. Thus, to do well it's best if you attend all lectures.

(Optional) Would-be Data scientists. A lot of what we do in the class involves a fair amount of data. I use the *R statistical software* in order to prepare my lecture notes and input the data from official sources, to provide you with the most up-to-date statistics. I will try to provide the required code to replicate all the analysis available in my lecture notes, as much as possible. For example, lecture 1 has the R code added to the lecture notes available here. An introduction to R statistical software is available here. I think that data science, statistics and economics are very complementary skills (so does the Massachusetts Institute of Technology). However, understanding code is not required at all to succeed in that class. You will not be penalized in any way if you skip this.

Exams

Grades. Your final grade has two components: one midterm exam, and a comprehensive final exam. Your final grade will be given by whichever of these two options gives you the best grade: (**Midterm (40%)** + **Final Exam (60%)**) or (**Final Exam (100%)**) at the following dates:

1. November 5, 3:30pm to 4:45pm: Midterm Exam.
2. December 14, 11:30am to 2:30pm: Final Exam.

No make-up exams. In any case, there will be no make-up exams. If a midterm exam is missed due to a documented serious illness or emergency, the final exam will be worth 100 % of your grade. Note that attending the midterm is like an "option value": you are necessarily better off attending the midterm, no matter what your grade is. Please make sure right away that you can be there on November 5 !

Regrade Policy. Students who wish to have their midterm or their final examinations regraded should submit a request in written form to their assigned Graduate Student Instructor, clearly explaining why they think they deserve a regrade. If a student requests a regrade, the whole exam will be regraded. Therefore, the grade can increase or decrease.

Exam content. Everything that I say during the class, that is covered during recitation sections, is potentially exam material. Exams will be a combination of multiple choice and short essay questions. Therefore, it is absolutely necessary that you attend all lectures! I encourage you to take notes during the class.

Exam practicalities. During exams, sufficient space will be provided on the sheets to answer. No notes, no books, no smartphones, no calculators, will be allowed during the exam. You must bring your UCLA ID in order to take the exam. Without a UCLA ID, you will not be allowed to take the exam. You will not need to bring scantrons, as we will be using Scantrons from the Office of Instructional Development (OID).

Other. For more details about policies regarding grading, exams and other matters please refer to the following link: https://www.econ.ucla.edu/undergraduate/. I will adhere to the guidelines specified in this webpage. If you wish to request an accommodation due to a suspected or documented disability, please contact the Center for Accessible Education as soon as possible at A255 Murphy Hall, (310) 825-1501, (310) 206-6083 (telephone device for the deaf). Website: http://www.cae.ucla.edu/

Topics (tentative)

Here is a tentative list of topics that we will cover. A rough correspondance to chapters in Olivier Blanchard's *Macroeconomics* textbook is provided here.

Disclaimer: Teaching Philosophy

To the extent possible, I will strive to emphasize **facts** over **theories**. This is a major difference with the way that I taught this class in the past. Many of the issues that we will look at are politically charged, and various theories have been developed which usually speak to either ideological views. Theory usually does not allow to conclude definitively. This is unfortunate, because macroeconomic questions are debated on both sides of the political spectrum:

- Do advanced economies have too high levels of public debt?
- Should fiscal stimulus be used to fight recessions?
- What is the cause of unemployment? (how much is voluntary or involuntary?)
- etc.

Fortunately, these questions are increasingly studied on the empirical front. Whenever possible, we shall try to "let the data speak", and put the different theories that we will study to the test. Empirical research is

still ongoing, and I will do my best to teach you the most up-to-date findings. In doing so, I will try to be as objective as possible, and try to avoid any conservative or liberal bias. According to this article (link), the latter is more of a risk than the former. I will always try to give you both sides of the debate, and arguments supporting each side. You are welcomed (and even encouraged !) to disagree with what I say during class !

Tentative Timetable

Classes

Oct 1. Lecture 1 - Introduction to Macroeconomics.

Oct 3. Lecture 2 - The Solow Growth Model.

Oct 8. Lecture 3 - Consumption - Intertemporal optimization.

Oct 10. Lecture 4 - The Overlapping Generations Model.

Oct 15. Lecture 5 - Technological Growth.

Oct 17. Lecture 6 - The Labor Market and Unemployment.

Oct 22. Lecture 7 - The Consumption Function and the Multiplier.

Oct 24. Lecture 8 - The Paradox of Thrift.

Oct 29. *Review.*

Oct 31. Lecture 9 - Redistributive Policies.

Nov 5. *Midterm.*

Nov 7. Lecture 10 - Public debt, Say's law.

Nov 12. *No Class (Veteran's day).*

Nov 14. *Pause (Finishing Lecture 10).*

Nov 19. Lecture 11 - The Open Economy and the Multiplier.

Nov 21. *No Class (Thanksgiving).*

Nov 26. Lecture 12 - Twin Deficits.

Nov 28. Lecture 13 - Empirics of Fiscal Policy.

Dec 3. Lecture 14 - Monetary Policy.

Dec 5. Lecture 15 - Summing Up: A Macroeconomic History of the U.S.

Exams

November 5, 3:30pm to 4:45pm: Midterm Exam. (40% or 0%)

December 14, 11:30am to 2:30pm: Final Exam. (60% or 100%)

Chapter 1

Introduction to Macroeconomics

GDP is the value of all final goods and services produced in a country within a given period. There are two sides to GDP, the demand side and the supply side:

- On the **demand side**, the **product approach to GDP** recognizes that total aggregate demand is made of four components:
  - Consumption spending by households (C).
  - Investment spending by households and corporations (I).
  - Government purchases (G).
  - Net exports (NX).
- On the **supply side**, the production of output involves the use of factor of production, often limited to capital and labor. These factors of production receive payment for their use, whose sum equals GDP.[1] The **income approach to GDP** consists in dividing up these payments into the different factors of production. Again, this often simply means a division of total value added into capital income, and labor income.

1.1 GDP: The Product Approach

GDP is equal to the total aggregate demand for goods:

$$Y = C + I + G + X - M.$$

We often define net exports as:[2]

$$NX = X - M,$$

so that GDP is simply:

$$Y = C + I + G + NX.$$

We look at each component of aggregate demand in turn:

- Consumption (C)
- Investment (I)
- Government Purchases (G)
- Net Exports (NX)

[1] Note that we are assuming here that there are no "rents" in the economy, that is that nothing can be obtained without either labor or capital. This is not exactly true, as for example oil is clearly more valuable than the costs of extracting it for the soil. Land is another example of something that is preexisting, but nevertheless earns a payment. It is a good first-order approximation however, as most production in fact requires either labor of capital.

[2] In some textbooks (as well as in earlier versions of these lecture notes), imports are denoted by IM instead of M.

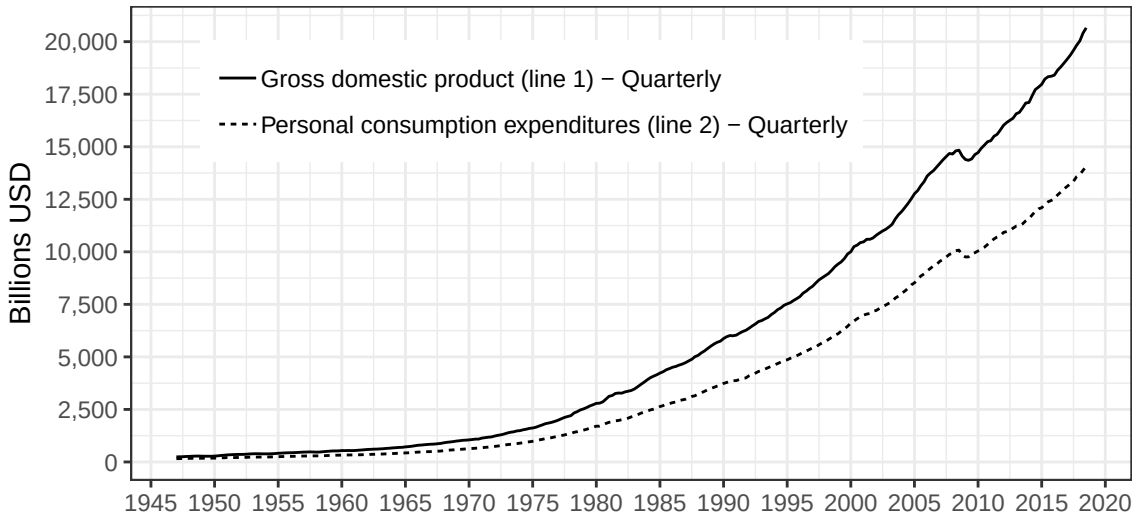

Figure 1.1: US GDP from NIPA (BEA)

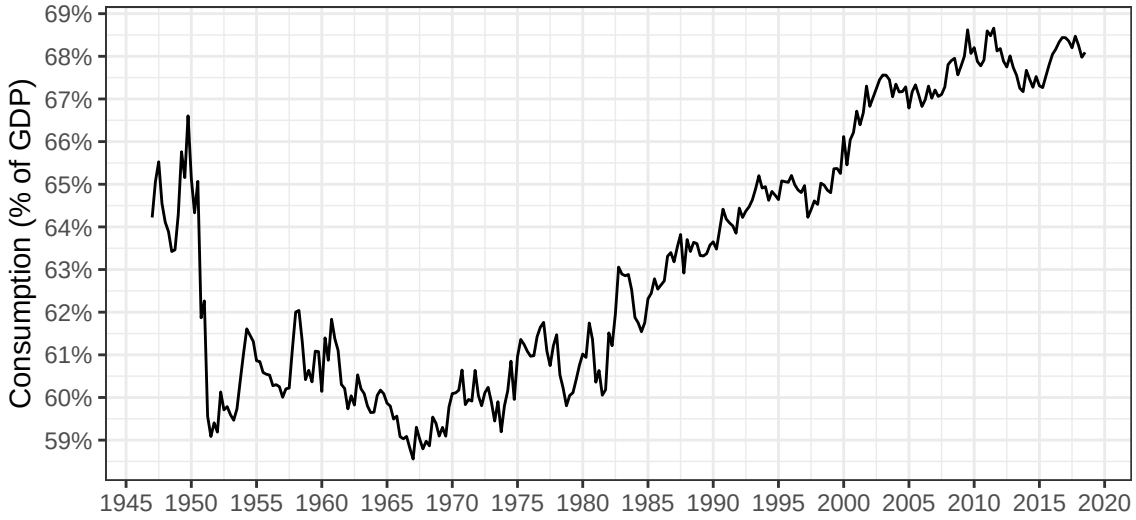

Figure 1.2: Consumption as a share of GDP from NIPA (BEA)

1.1.1 Personal Consumption Expenditures - Consumption (C)

Figure 1.1 plots GDP from the BEA, as well as PCE, in millions of dollars. US GDP being in the vicinity of USD 20 trillion dollars (or USD 20,000 billions, or USD 20,000,000 millions), this looks about right. On this figure, data for GDP is taken from the Bureau of Economic Analysis's National Income and Product Accounts (NIPA) here and data for Personal Consumption Expenditures is taken from there.

To get a better of sense of how big consumption is as a fraction to GDP (although you may eyeball it on this picture), we might plot consumption as a function of GDP, which is what I do below. You can see that Personal Consumption Expenditures are approximately **60 to 70 % of GDP**. You can also see that it's been rising since the end of the sixties. We will discuss that.

Personal Consumption Expenditures are divided up into:

- Durable Goods (more than 3 years of durability): e.g. cars.
- Non-durable Goods (less than 3 years of durability).
- Services.

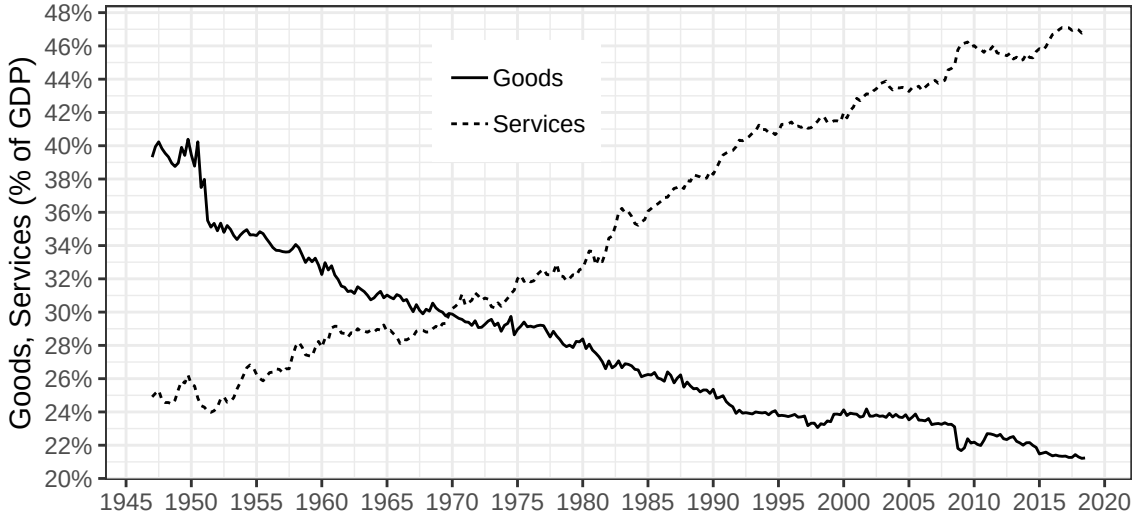

Figure 1.3: Goods and Services Consumption as a share of GDP from NIPA (BEA)

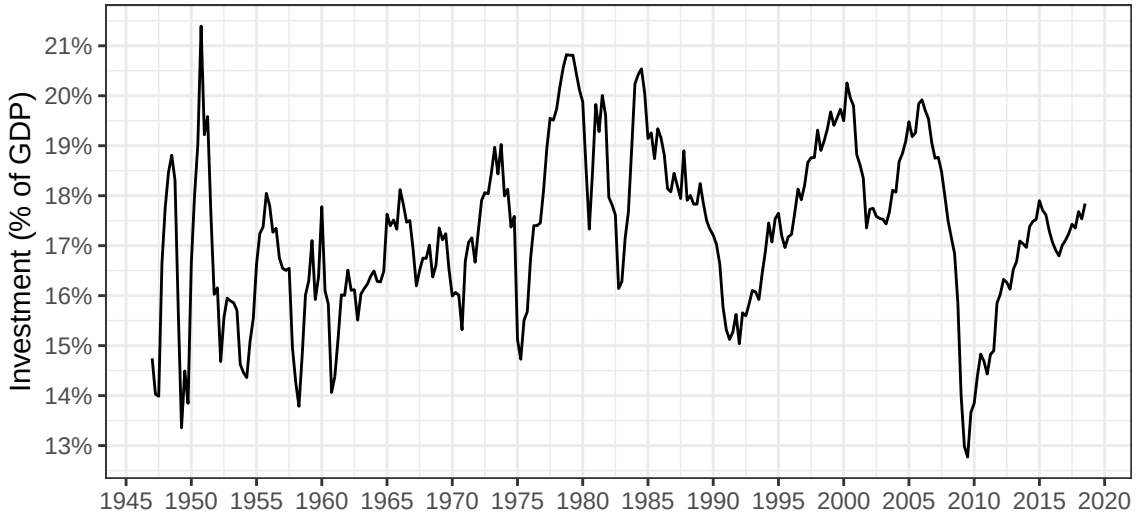

Figure 1.4: Investment as a share of GDP from NIPA (BEA)

Services have become more important than Goods in total consumption since the 1970s.

1.1.2 Gross private domestic investment - Investment (I)

Investment has two components:

- non residential investment is the purchase of new capital goods by firms: structures, new plants.
- residential investment is the purchase of new houses.

Gross private domestic investment is approximately **15 to 20 % of GDP**, as you can see on Figure 1.4. It is also very volatile over the cycle.

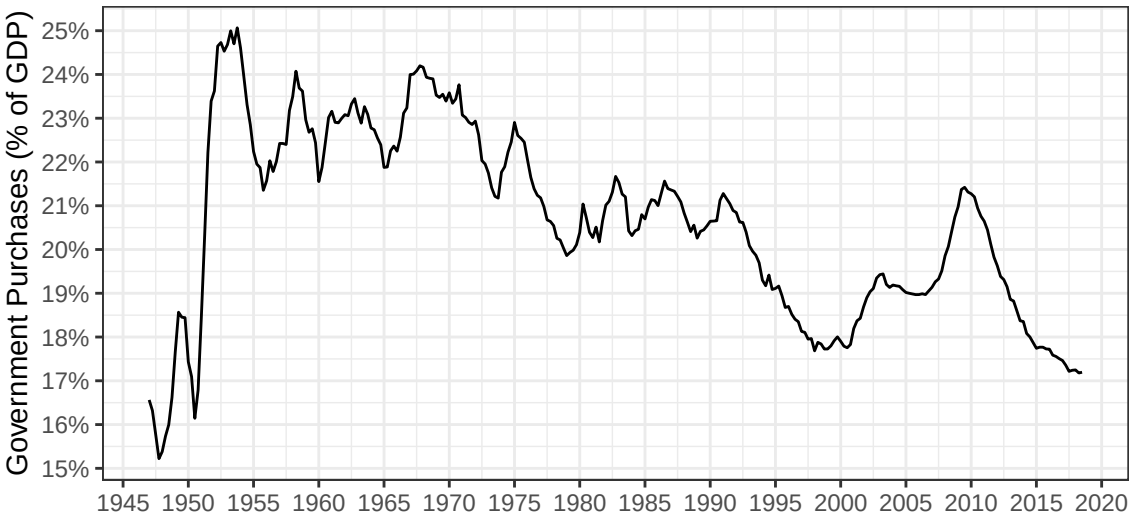

Figure 1.5: Government Purchases as a share of GDP from NIPA (BEA)

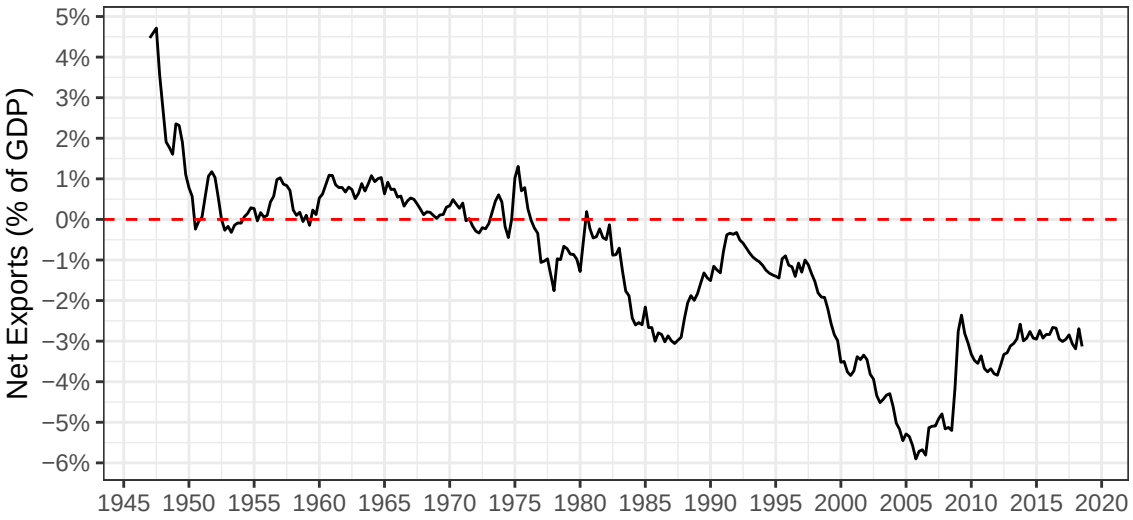

Figure 1.6: Net Exports as a share of GDP from NIPA (BEA)

1.1.3 Government Purchases (G)

Government purchases are composed of purchases of goods by the government plus the compensation of government employees. Overall, they comprise about approximately 20% of GDP, as can be seen on Figure 1.5. Note however that they do not include transfers from the government of interest payments on government debt.

1.1.4 Net Exports (NX)

Net exports of goods and services are approximately **-2 to -6 % of GDP**, at least in the modern period (and in the United States), as you can see on Figure 1.6.

1.2 GDP: The Income Approach

1.2.1 Cobb Douglas Production function

In order to organize our thinking, let's write out a Cobb-Douglas production function, defined as:

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha},$$

where α is a number between 0 and 1. It is useful to then think of a firm which would choose the amount of labor it uses L_t as well as the amount of capital it uses K_t in order to maximize its profits:

$$\max_{K_t, L_t} \quad A_t K_t^{\alpha} L_t^{1-\alpha} - R_t K_t - w_t L_t,$$

In this expression, R_t is the **rental rate** of capital, also called the **gross return** to capital. This represents how much it costs to rent one unit of capital. This cost actually has two components:

1. It includes a conventional interest rate r_t, which is the cost of borrowing money to invest in capital. You should think of this as the real interest rate which is charged by a bank to borrow money.
2. It also includes a component to account for depreciation of the capital stock, often denoted by δ. If capital is bought, then the resale price for each unit of capital is lower by δ, which is a cost to the investor. If capital is rented, then capital needs to be given back to the owner in its original state.

We have that the rental rate or gross return is equal to the net return plus the depreciation rate:

$$\boxed{R_t = r_t + \delta}.$$

From Econ 11, it should be clear that a way to solve this problem is to set the derivative of the profit function equal to 0 with respect to K_t and L_t:

- Differentiating with respect to K_t implies:

$$\alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} - R_t = 0 \quad \Rightarrow \quad \boxed{R_t = \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha}}.$$

- Differentiating with respect to L_t implies:

$$(1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha} - w_t = 0 \quad \Rightarrow \quad \boxed{w_t = (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha}}$$

Note that an alternative, and more direct way to get at that same result, would be to use Econ 11 directly, and write that the marginal products have to be equal to prices:

- The **rental rate of capital** R_t is the marginal product of capital. The marginal product of capital is how much more output is obtained when the capital stock is increased by one unit, which is just the derivative of output with respect to capital $\partial Y_t / \partial K_t$:

$$\begin{aligned} R_t &= \frac{\partial Y_t}{\partial K_t} \\ &= \frac{\partial \left(A_t K_t^{\alpha} L_t^{1-\alpha} \right)}{\partial K_t} \\ R_t &= \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} \end{aligned}$$

- The **wage** w_t is the marginal product of labor. The marginal product of labor is how much more output is obtained when the quantity of labor is increased by one unit, which is just the derivative of output with respect to labor $\partial Y_t / \partial L_t$:

$$\begin{aligned} w_t &= \frac{\partial Y_t}{\partial L_t} \\ &= \frac{\partial \left(A_t K_t^{\alpha} L_t^{1-\alpha} \right)}{\partial L_t} \\ w_t &= (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha} \end{aligned}$$

The total wage bill wL_t is a fraction $1-\alpha$ of output Y_t:

$$\begin{aligned} w_t L_t &= (1-\alpha) A_t K_t^{\alpha} L_t^{-\alpha} \cdot L_t \\ &= (1-\alpha) A_t K_t^{\alpha} L_t^{1-\alpha} \\ w_t L_t &= (1-\alpha) Y_t \end{aligned}$$

The total capital income $R_t K_t$ is a fraction α of output Y_t:

$$\begin{aligned} R_t K_t &= \alpha A_t K_t^{\alpha-1} L_t^{1-\alpha} \cdot K_t \\ &= \alpha A_t K_t^{\alpha} L_t^{1-\alpha} \\ R_t K_t &= \alpha Y_t. \end{aligned}$$

This implies that the share of capital income in output (or equivalently, value added) is:

$$\frac{R_t K_t}{Y_t} = \alpha,$$

while the share of labor income in output (or equivalently, value added) is:

$$\frac{w_t L_t}{Y_t} = 1 - \alpha.$$

Note that capital income plus labor income equals total output:

$$R_t K_t + w_t L_t = Y_t.$$

Another way to say the same thing is that the share of capital income in output and that of labor income in output add up to one:

$$\boxed{\frac{R_t K_t}{Y_t} + \frac{w_t L_t}{Y_t} = 1}.$$

1.2.2 The Income Side in the Data

In practice, how much goes to the compensation of employees (labor income), and how much goes to the returns to capital (capital income)? The answer is that it goes approximately for 1/3 to capital and for 2/3 to labor. In turn, this implies that we will, in numerical applications of our theories, often assume that:

$$\alpha = \frac{1}{3}$$

The calculations for these are less straightforward than for computing the share of consumption, investment, as we did above. The reason is that in practice, the division between labor and capital is not as clear cut in the national accounts as one might hope: for example, someone who owns her/his own business reports most of her/his income in the form of capital income, even when a large part of it is actually labor income, so that compensation of employees is (vastly) understated. Figure 1.7 shows which results are obtained using this understated measure. It needs to be adjusted upwards by about 10% of GDP, for the reasons mentioned above.

For our purposes, we only need to remember that the share of compensation of employees is approximately 2/3 of value added. Therefore, we will very often work with a Cobb-Douglas production function such that:

$$Y_t = A_t K_t^{1/3} L_t^{2/3}.$$

Lecture 2 will walk you through the Solow growth model, where we shall make heavy use of that Cobb-Douglas production function.

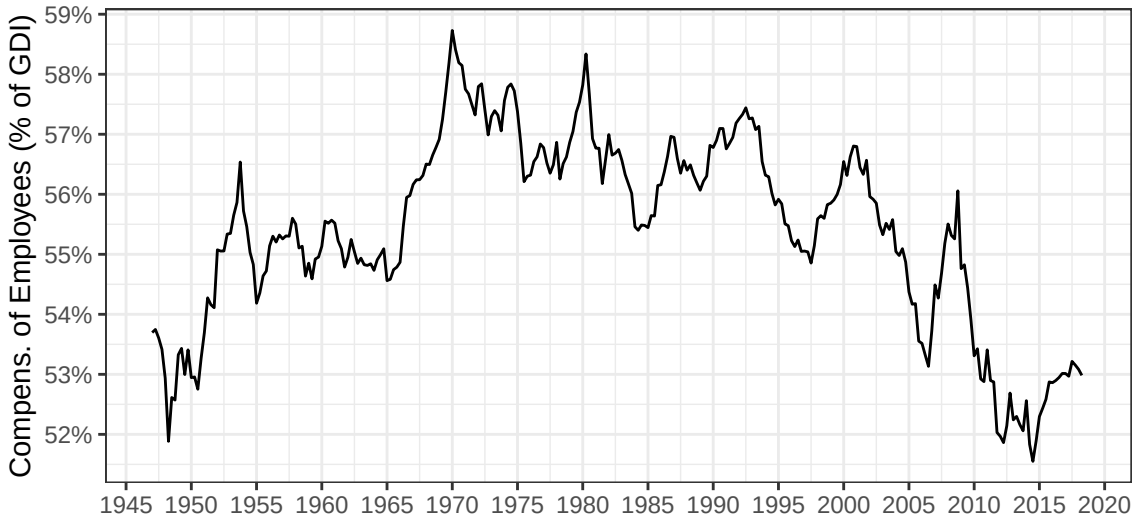

Figure 1.7: Compensation of Employees as a share of GDP from NIPA (BEA)

Readings - To go further

The Economics of Well Being, *Harvard Business Review*.

G.D.P. R.I.P., *The New York Times*, August 9, 2009.

(Gated) Keeping up with the Karumes, *The Economist*, October 29, 2015.

(More Difficult Read) Abraham, Katharine G. "Distinguished Lecture on Economics in Government-What We Don't Know Could Hurt Us: Some Reflections on the Measurement of Economic Activity." Journal of Economic Perspectives 19, no. 3 (September 2005): 3–18.

Chapter 2

Solow Growth model

Introduction

Robert Solow, 1987 Nobel Memorial Prize in Economic Sciences, starts from a general production function, giving at any point in time output Y_t as a function of inputs, capital K_t and labor L_t:

$$Y_t = F(K_t, L_t).$$

Chapter 3

Two-period optimization Problem

3.1 Introduction

Consumption and saving are perhaps the most important and controversial issues in macroeconomics. In the Solow growth model, saving was a constant fraction s of GDP, by assumption. We now build on *Economics 11* (the one where you learn consumer optimization with Lagrangians and all that), in order to derive saving behavior from microeconomic principles. In other words, we work to make saving "endogenous" (that is, explained by the model), while it was previously taken as exogenous (that is, assumed in the model).

Although this discussion may appear somewhat abstract at first, these calculations are the basis of some of the most important controversies in macroeconomics, which we shall come to in the next lectures.

3.2 The Two-Period Consumption Problem

3.2.1 Assumptions

There are two periods, $t = 0$ (think of this as "today") and $t = 1$ (think of this as "tomorrow"). The consumer values consumption c_0 in period 0 and c_1 in period 1 according to the following utility function:

$$U(c_0, c_1) = u(c_0) + \beta u(c_1).$$

where $u(.)$ is an increasing and concave function, and $\beta \leq 1$. β captures that people typically have a preference for the present. (they are **present-biased**)

Assume that agents earn (labor) income y_0 in period 0, and (labor) income y_1 in period 1. They also are born with some financial wealth f_0 now, and have financial wealth f_1 in period 1, which they consume entirely because this is the last period. (there is no point keeping more money for after period 1, because there is no future at that point) The amount that agents save in this economy is thus $f_1 - f_0$, and the amount of their accumulated savings is the savings they already had plus what they decided to accumulate, so that $f_0 + (f_1 - f_0) = f_1$.

Therefore, consumption in period 0 is given by:

$$c_0 = y_0 - (f_1 - f_0)$$

The second period consumption ($t = 1$) is given by income plus the return to (accumulated !) savings:

$$c_1 = y_1 + (1+r)f_1.$$

3.2.2 Solution

Intertemporal budget constraint. Rewriting f_1 from this second equation: $f_1 = (c_1 - y_1)/(1+r)$, and plugging into the first,

$$c_0 = y_0 - \left(\frac{c_1 - y_1}{1+r} - f_0\right).$$

Rearranging, total wealth is then the sum of financial wealth f_0 and of the present discounted value of human wealth:

$$c_0 + \frac{c_1}{1+r} = \overbrace{f_0 + \underbrace{y_0 + \frac{y_1}{1+r}}_{\text{human wealth}}}^{\text{total wealth}}.$$

The intertemporal budget constraint says that the present discounted value of consumption is equal to total wealth.

Optimization. The problem of the consumer is then simply that of maximizing utility under his budget constraint:

$$\begin{aligned} \max_{c_0,c_1} \quad & u(c_0) + \beta u(c_1) \\ \text{s.t.} \quad & c_0 + \frac{c_1}{1+r} = f_0 + y_0 + \frac{y_1}{1+r}. \end{aligned}$$

You may solve this optimization in four different ways:

1. Apply the well known ratio of marginal utilities formula from Econ 11. Let us rewrite this optimization problem as follows:

$$\begin{aligned} \max_{c_0,c_1} \quad & u(c_0) + \beta u(c_1) \\ \text{s.t.} \quad & p_0 c_0 + p_1 c_1 = B. \end{aligned}$$

   where we have defined the price of consumption in period 0 by:

$$p_0 \equiv 1,$$

   the price of consumption in period 1 by:

$$p_1 \equiv \frac{1}{1+r},$$

   and finally the budget B by the present discounted value of lifetime resources:

$$B \equiv f_0 + y_0 + \frac{y_1}{1+r}.$$

   Note that the relative price of consumption in period 1 relative to period 0 is given by $1/(1+r)$: when the interest rate becomes higher, consuming in period 1 becomes relatively cheaper, or consuming in period 0 becomes more expensive (it's really expensive to consume now rather than later if the bank is offering me a really high interest rate). Thus, applying the formula from Econ 11 allows to say that the marginal rate of substitution between consumption in period 1 c_1 and consumption in period 0 c_0 - the ratio of marginal utilities - is equal to the ratio of prices:

$$\frac{\partial U/\partial c_1}{\partial U/\partial c_0} = \frac{p_1}{p_0} = \frac{1}{1+r} \quad \Rightarrow \quad \frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+r}.$$

2. Apply the following intuitive economic argument. The marginal utility from consuming in period 1 is $\beta u'(c_1)$. The marginal utility from consuming in period 0 is $u'(c_0)$. By putting one unit of consumption in the bank, one forgoes 1 unit of consumption in period 0 to get $1+r$ units of consumption in period 1. The two have to be equal, if one is optimizing. If consuming more in period 0 gives a higher marginal utility, or $u'(c_0) > (1+r)\beta u'(c_1)$, then one should consume more and save less. On the contrary, should $u'(c_0) < (1+r)\beta u'(c_1)$, one should consume less and save more. Therefore, in equilibrium, these two options can only be equal:

$$u'(c_0) = (1+r)\beta u'(c_1) \quad \Rightarrow \quad \frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+r}.$$

3. Replace c_0 from the intertemporal budget constraint above and optimize with respect to c_1:

$$\max_{c_1} \quad u\left[\left(f_0 + y_0 + \frac{y_1}{1+r}\right) - \frac{c_1}{1+r}\right] + \beta u(c_1).$$

Taking the derivative of this expression with respect to c_1 leads to:

$$-\frac{1}{1+r}u'\left[\left(f_0 + y_0 + \frac{y_1}{1+r}\right) - \frac{c_1}{1+r}\right] + \beta u'(c_1) = 0$$
$$\Rightarrow \quad -\frac{1}{1+r}u'(c_0) + \beta u'(c_1) = 0 \quad \Rightarrow \quad \frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+r}.$$

where the first substitution uses the intertemporal budget constraint which implies:

$$\left(f_0 + y_0 + \frac{y_1}{1+r}\right) - \frac{c_1}{1+r} = c_0$$

4. Alternatively, you may substitute c_1 out and optimize with respect to c_0:

$$\max_{c_0} \quad u(c_0) + \beta u\left[(1+r)\left(f_0 + y_0 + \frac{y_1}{1+r}\right) - (1+r)c_0\right].$$

Taking the derivative of this expression with respect to c_0 leads to:

$$u'(c_0) - \beta(1+r)u'\left[(1+r)\left(f_0 + y_0 + \frac{y_1}{1+r}\right) - (1+r)c_0\right] = 0$$
$$\Rightarrow \quad u'(c_0) - \beta(1+r)u'(c_1) = 0 \quad \Rightarrow \quad \frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+r}.$$

where the first substitution uses the intertemporal budget constraint which implies (pre-multiplying both sides by $1+r$):

$$(1+r)\left(f_0 + y_0 + \frac{y_1}{1+r}\right) - (1+r)c_0 = c_1$$

3.2.3 Some examples

Log utility, no discounting ($\beta = 1$). Log utility implies that $u(c)$ is given by the natural logarithm. Marginal utility is then just:

$$u'(c) = \frac{1}{c},$$

Since $\beta = 1$, the above optimality condition (derived 4 times) can be written as:

$$\frac{u'(c_1)}{u'(c_0)} = \frac{1}{1+r} \quad \Rightarrow \quad \frac{1/c_1}{1/c_0} = \frac{1}{1+r}$$
$$\Rightarrow \quad \frac{c_0}{c_1} = \frac{1}{1+r} \quad \Rightarrow \quad c_0 = \frac{c_1}{1+r}$$

Substituting out $c_1/(1+r) = c_0$ in the intertemporal budget constraint allows to calculate consumption at time 0 c_0:

$$\begin{aligned} c_0 + \frac{c_1}{1+r} &= f_0 + y_0 + \frac{y_1}{1+r} \\ \Rightarrow \quad c_0 + c_0 &= f_0 + y_0 + \frac{y_1}{1+r} \\ \Rightarrow \quad c_0 &= \frac{1}{2}\left(f_0 + y_0 + \frac{y_1}{1+r}\right) \end{aligned}$$

Finally, we may calculate c_1:

$$c_1 = (1+r)c_0 = \frac{1+r}{2}\left(f_0 + y_0 + \frac{y_1}{1+r}\right).$$

According to this expression, the **Marginal Propensity to Consume (MPC)** out of current wealth f_0 is given by $1/2$. When f_0 rises to $f_0 + \Delta f_0$, the corresponding change in consumption is:

$$\Delta c_0 = \frac{1}{2}\Delta f_0.$$

If we were to study a model with more periods, say T periods, we would find that people Marginal Propensity to Consume is approximately equal to $1/T$, at least according to this model. Whether such is actually the case, and people are that rational, is a subject of fierce debate among macroeconomists, and one that we will take up in the next lectures.

Log utility, with discounting ($\beta < 1$). Marginal utility is then $u'(c) = 1/c$, so that the optimality condition gives:

$$\begin{aligned} \frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+r} \quad &\Rightarrow \quad \frac{\beta/c_1}{1/c_0} = \frac{1}{1+r} \\ \Rightarrow \quad \frac{\beta c_0}{c_1} = \frac{1}{1+r} \quad &\Rightarrow \quad \beta c_0 = \frac{c_1}{1+r} \end{aligned}$$

Substituting out $c_1/(1+r) = \beta c_0$ in the intertemporal budget constraint allows to calculate consumption at time 0 c_0:

$$\begin{aligned} c_0 + \frac{c_1}{1+r} &= f_0 + y_0 + \frac{y_1}{1+r} \\ \Rightarrow \quad c_0 + \beta c_0 &= f_0 + y_0 + \frac{y_1}{1+r} \\ \Rightarrow \quad c_0 &= \frac{1}{1+\beta}\left(f_0 + y_0 + \frac{y_1}{1+r}\right) \end{aligned}$$

Finally, we may calculate c_1:

$$c_1 = \beta(1+r)c_0 = \frac{\beta(1+r)}{1+\beta}\left(f_0 + y_0 + \frac{y_1}{1+r}\right).$$

Because people are more impatient in this case, they consume more, and their Marginal Propensity to Consume (MPC) is **higher** with $\beta < 1$:

$$\Delta c_0 = \frac{1}{1+\beta}\Delta f_0.$$

Note that the solution with no discounting corresponds to that with discounting when $\beta = 1$, which was expected.

3.2.4 Generalization

Assume that an individual receives wage w in period 0, and that this wage is expected to grow at rate g in the next T years. What is the present value of his human wealth, assuming that the interest rate is given by R? The answer is that his human wealth H is given as follows:

$$H = w + w\frac{1+g}{1+r} + w\left(\frac{1+g}{1+r}\right)^2 + ... + w\left(\frac{1+g}{1+r}\right)^{T-1}$$

$$H = w\frac{1-\left(\frac{1+g}{1+r}\right)^T}{1-\frac{1+g}{1+r}}$$

Chapter 4

Applications

Some *significant* applications are demonstrated in this chapter.

4.1 Example one

4.2 Example two

Chapter 5

Final Words

We have finished a nice book.