# Problem Set 3

#### UCLA - Econ 102 - Fall 2018

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## 3 Problem Set 3

### 3.1 Two-period Intertemporal Optimization

Consider the model of lecture 3 again. Instead of logarithmic preferences, assume that preferences are given by:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},$$

- 1. Under what condition on  $\sigma$  is this an increasing and concave utility function?
- 2. Show using 4 different methods that:

$$\frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+r}$$

- 3. Using the equation in question 1, what is the ratio  $c_1/c_0$ ?
- 4. Replace in the intertemporal budget constraint to find an implicit equation for  $c_1$ . Do the same for  $c_0$ .
- 5. Assume that  $\sigma = 1/2$ , and  $f_0 = 0$ ,  $y_0 = \$90,000$ ,  $y_1 = 0$ ,  $\beta = 1$ . What are  $c_0$  and  $c_1$  if r = 1%? What about if r = 2%? How much does  $c_0$  change then? How much in percentage terms?
- 6. Same questions if  $\sigma = 1$ .
- 7. Same questions if  $\sigma = 2$ .
- 8. Compare the changes in  $c_0$  following an increase in the real interest rates in questions 5, 6, 7. Comment.

#### 3.2 Another Overlapping Generations model

Consider the model of lecture 4 again, with one small twist: agents care only about old age consumption. In other words, their utility functions are given by:

$$U = u(c_{t+1}^o).$$

Our goal is to derive the law of motion for the capital stock  $K_t$ , that is, a function relating  $K_{t+1}$  to  $K_t$  and the parameters of the model.

- 1. Why can the utility function be left unspecified for computing the level of saving?
- 2. Derive the law of motion for capital.
- 3. What is the corresponding value of the saving rate in the Solow model?
- 4. Provide a condition on  $\alpha$  such that the capital stock is below the Golden Rule level.
- 5. Is that condition likely to be satisfied?