

Problem Set 7 - Solutions

UCLA - Econ 102 - Fall 2018

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1 Optimal Taxation: The Supply Side View

1. The firm's problem is to maximize:

$$\max_l pf(l) - wl$$

Therefore, the real wage is given by:

$$\boxed{\frac{w}{p} = A}.$$

2. Writing that $c = c_0 + (1 - \tau) \cdot (w/p) \cdot l$ and plugging the value of c into the worker's optimization problem:

$$\max_l (1 - \tau) \frac{w}{p} l + c_0 - B \frac{l^{1+\epsilon}}{1 + \epsilon}$$

The first-order condition implies:

$$(1 - \tau) \frac{w}{p} = Bl^\epsilon \quad \Rightarrow \quad \boxed{l = \frac{(1 - \tau)^{1/\epsilon}}{B^{1/\epsilon}} \left(\frac{w}{p} \right)^{1/\epsilon}}$$

3. The number of hours worked is given by replacing out the real wage w/p from the labor demand equation to the labor supply equation, so that:

$$\boxed{l = (1 - \tau)^{1/\epsilon} \frac{A^{1/\epsilon}}{B^{1/\epsilon}}}.$$

Real pre-tax income is when simply given by the real wage times the number of hours. Since the real wage is simply A , we get:

$$y = \frac{w}{p} l = A \cdot (1 - \tau)^{1/\epsilon} \frac{A^{1/\epsilon}}{B^{1/\epsilon}}.$$

Therefore:

$$\boxed{y = (1 - \tau)^{1/\epsilon} \frac{A^{1/\epsilon+1}}{B^{1/\epsilon}}}.$$

4. A numerical application shown on the Google Spreadsheet implies that:

$$\underline{l} = (1 - \underline{\tau})^{1/\epsilon} \frac{\underline{A}^{1/\epsilon}}{\underline{B}^{1/\epsilon}}$$

Therefore:

$$\underline{l} = \left(1 - \frac{1}{4}\right)^{1/2} \left(\frac{1000000}{28188}\right)^{1/2} \left(\frac{491569855488}{3000000}\right)^{1/2}$$
$$\underline{l} = 2088.$$

Note that under the assumption of 281 business days in a year, this implies **8 hours a day** of work. According to the Google Spreadsheet, income is then given by:

$$\underline{y} = \underline{A} \underline{l}$$
$$= \frac{1000000}{28188} \cdot 2088$$
$$\underline{y} \approx 74074.07$$

5. We note that this group has a lower disutility of working B and also a higher productivity per hour A . Therefore they work more (both because they are more productive and because they have a lower disutility of working):

$$\bar{l} = \left(1 - \frac{1}{2}\right)^{1/2} \left(\frac{1000000}{6264}\right)^{1/2} \left(\frac{218475491328}{1000000}\right)^{1/2}$$

$$\bar{l} = 4176.$$

Note that under the assumption of 281 business days in a year, this implies 16 hours a day of work on average (more likely on weekends, or during Thanksgiving!). Income is then given by:

$$\bar{y} = \bar{A}\bar{l}$$

$$= \frac{1000000}{6264} \cdot 4176$$

$$\bar{y} \approx 666666.66$$

6. Total output is 20 trillion, 10 trillion coming from the bottom 90% and 10 trillion coming from the top 10%. In million dollars, output of the bottom 90% is:

$$\underline{Y} = \lambda N \underline{y}$$

$$= 0.9 \cdot 150 \cdot 74074.07$$

$$= 10,000,000 \text{ million}$$

$$\underline{Y} = 10 \text{ trillion}$$

In million dollars, output of the top 10% is:

$$\bar{Y} = (1 - \lambda) N \bar{y}$$

$$= 0.1 \cdot 150 \cdot 666666.66$$

$$= 10,000,000 \text{ million}$$

$$\bar{Y} = 10 \text{ trillion}$$

7. According to the Google Spreadsheet, an individual earning \underline{y} pays taxes given by:

$$\underline{t} = -5000 + 0.25 * (74074.07 - 25000) \approx 7,268$$

which means that this individual pays \$7268 on average per year in taxes. The total taxes paid by this group are:

$$\underline{T} = \lambda N \underline{t} \approx 981 \text{ billion.}$$

According to the Google Spreadsheet, an individual earning \bar{y} pays taxes given by:

$$\bar{t} = -5000 + 0.25 * (200000 - 25000) + 0.5 * (666666 - 200000) \approx 272,083$$

which means that this individual pays \$272,083 on average in taxes per year. The total taxes paid are:

$$\bar{T} = (1 - \lambda) N \bar{t} \approx 4,081 \text{ billion.}$$

The Google Spreadsheet shows that lowering the bottom marginal tax rate leads to an increase in hours worked by the bottom 90%, which increases GDP by about **328 billion**. The cost of the policy is a fall in tax receipts of **266 billion**. This implies that the supply-side “multiplier” is:

$$-\frac{\Delta Y}{\Delta T} = 1.23.$$

8. The Google Spreadsheet shows that lowering the top marginal tax rate leads to an increase in hours worked by the top 10%, which increases GDP by about **488 billion**. The cost of the policy is a fall in tax receipts of **130 billion**. This implies that the supply-side “multiplier” is:

$$-\frac{\Delta Y}{\Delta T} = 3.74.$$