

# Problem Set 3

UCLA - Econ 102 - Fall 2018

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## 1 Two-period Intertemporal Optimization

Consider the model of lecture 3 again. Instead of logarithmic preferences, assume that preferences are given by:

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

1. Under what condition on  $\sigma$  is this an increasing and concave utility function?
2. Show using 4 different ways that:

$$\frac{\beta u'(c_1)}{u'(c_0)} = \frac{1}{1+R}$$

3. Using the equation in question 1, what is the ratio  $c_1/c_0$ ?
4. Replace in the intertemporal budget constraint to find an implicit equation for  $c_1$ . Do the same for  $c_0$ .
5. Assume that  $\sigma = 1/2$ , and  $f_0 = 0$ ,  $y_0 = \$90000$ ,  $y_1 = 0$ ,  $\beta = 1$ . What are  $c_0$  and  $c_1$  if  $R = 1\%$ ? What about if  $R = 2\%$ ? How much does  $c_0$  change then? How much in percentage terms?
6. Same questions if  $\sigma = 1$ .
7. Same questions if  $\sigma = 2$ .
8. Compare the changes in  $c_0$  following an increase in the real interest rates in questions 5, 6, 7. Comment.

## 2 Another Overlapping Generations model

Consider the model of lecture 4 again, with one small twist: agents care only about old age consumption. In other words, their utility functions is given by:

$$U = u(c_{t+1}^o).$$

Our goal is to derive the law of motion for the capital stock  $K_t$ , that is, a function relating  $K_{t+1}$  to  $K_t$  and the parameters of the model.

1. Why can the utility function be left unspecified for computing the level of saving?
2. Derive the law of motion for capital.
3. What is the corresponding value of the saving rate in the Solow model?
4. Provide a condition on  $\alpha$  such that the capital stock is below the Golden Rule level.
5. Is that condition likely to be satisfied?