Monopoly Pricing

François Geerolf
UCLA

In the class, we alluded to **monopoly pricing** and **markups** as a way to justify a price over the marginal cost (the wage). This note reminds you of the basic microeconomics around monopoly pricing and markups.

Consider a monopolist facing a downward sloping demand curve $q(p)$ which represents consumers' demand as a function of the price. Assuming that $q(.)$ is first differentiable, downward sloping demand implies $q'(p) < 0$. This monopolist chooses a price to maximize his profit (as opposed to taking the price as given):

$$\max_p \quad \underbrace{pq(p)}_{\text{Total Revenue}} - \underbrace{c\left(q(p)\right)}_{\text{Total Cost}}$$

The first order condition for this problem gives the familiar condition that Marginal Revenue equals the marginal cost:

$$\begin{aligned} pq'(p) + q(p) - c'(q(p))q'(p) &= 0 \\ \Rightarrow \qquad \underbrace{p + \frac{q(p)}{q'(p)}}_{\text{Marginal Revenue}} &= \underbrace{c'\left(q(p)\right)}_{\text{Marginal Cost}} \end{aligned}$$

Denote by $\eta(p)$ the elasticity of demand at price p, we have:

$$\eta(p) = -\frac{d\ln q(p)}{d\ln p} = -\frac{pq'(p)}{q(p)}$$

Note that the elasticity of demand is such that when the price increases by 1% starting from p, the demand decreases by $\eta(p)$%. Thus, if the elasticity is high, then demand is very responsive to the price: the good is said to be **elastic**. (for example, because the product has close substitutes) If the elasticity is low, then demand is not responsive to the price, and the good is said to be **inelastic**. If **demand is isoelastic**, then the elasticity of demand does not depend on the price:

$$q(p) = kp^{-\eta} \quad \Rightarrow \quad \eta(p) = -\frac{p\left(-k\eta p^{-\eta-1}\right)}{kp^{-\eta}} = \eta$$

Then, the above marginal revenue equals marginal cost identity can be rewritten in the following way:

$$\begin{aligned} p - c'(q(p)) &= -\frac{q(p)}{q'(p)p}p \\ &= \frac{1}{\eta}p \end{aligned}$$

In this case, the price is equal to the marginal cost times a markup:

$$\boxed{p = c'(q(p))\frac{\eta}{\eta - 1}}$$

In the class, we have denoted the markup by $1 + m$. Thus, with isoelastic demand, m is given as a function of η as follows:

$$1 + m = \frac{\eta}{\eta - 1} \quad \Rightarrow \quad m = \frac{1}{\eta - 1}.$$

Thus, the markup is decreasing with the elasticity of demand. When demand becomes very elastic ($\eta \to \infty$), then the markup approaches 0 ($m \to 0$). Note also that in the class, the marginal cost was also constant and equal to the wage, given that the production function was $Y = N$. This allows us to conclude that:

$$\boxed{P = (1 + m)W}.$$