Lecture 10 - Public Debt, Say's Law

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Introduction

Until now, we have talked about government spending and taxes as if the government could take on as many debt as it wants. Our view of the economy was kind of magic, in that the government could lower taxes and raise government spending as it pleases. For example, in the "accelerator" version of the goods market model we saw in lecture 7, we proceeded to experimenting with taxes and government spending, wondering what would happen if government spending was increased, and taxes were reduced, on GDP. For example, an increase in government spending ΔG leads to an increase in output: $\Delta Y = \frac{\Delta G}{1-c_1-b_1} > 0$ in one version of the model, and a tax cut $\Delta T < 0$ leads to an increase in output as well: $\Delta Y = \frac{c_1 \Delta T}{1-c_1-b_1} > 0$. But then, why doesn't the government actually do more of that spending? A first reason might be that there is a very important issue which we did not take into consideration: it is that of the government deficit, and the impact of government debt on future generations. Indeed, when government spending increases $\Delta G > 0$, this leads to a government deficit of equal magnitude: $\Delta(T - G) = -\Delta G < 0$. Similarly, a tax cut $\Delta T < 0$ leads to increased deficits given by $\Delta(T - G) = \Delta T < 0$. One might worry that this debt will someday have to be repaid. In this case, higher GDP today might only be thought of as leading to lower GDP in the future. Therefore, the dynamics of government debt is very important for Keynesian models. We study the intertemporal budget constraint of the government, and provide conditions under which this government debt is stable.

Today, we make two related points: 1. It is not true that public debt will need to be repaid. In the overlapping generations model, public debt is never repaid, as there are always new generations coming along. 2. A condition for the sustainability of public debt is that $r < g$. Interestingly, we shall see that this condition on the sustainability of public debt happens to be the same condition which ensures that the neoclassical Solow growth model of lecture 2 and the neoclassical overlapping-generations model of lecture 4 are above the Golden Rule level of capital accumulation.

The Keynesian and neoclassical models are different, in that they imply different impacts of higher public debt on investment spending:
- In the **Keynesian model**, investment is not crowded out by public debt (in the simplest model, investment is in fact fixed).
- In the **Neoclassical model**, investment is crowded out by public debt, but this may be a good thing if the economy has too much capital to begin with. This is called the "Treasury View". I am skeptical of this Treasury View, but this is what mainstream economists have come to adopt. Anyway, even if one accepts the

Treasury View, when the capital stock is below the Golden Rule level, both the Neoclassical and Keynesian models converge on their policy prescriptions: deficit spending should be used.

Finally, we discuss Say's law: supply creates its own demand, and we try to reconcile the Keynesian and the Neoclassical models with respect to their elasticities between capital and labor.

1 Sustainability of Public Debt

Law of motion for government debt. Let's denote everything in terms of goods, to avoid talking about the complicated issues surrounding inflation. Let us denote by G_t the government spending at period t, and by T_t the taxes in period t. Let us also denote by $(G_t - T_t)$ the government deficit in period t, which is the excess of government expenditures over taxes levied by the government.

If the interest rate that the government pays is given by r_t, then the law of motion of government debt is given by:

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t$$

Thefore, the law of motion for government debt is given by the sum of the government deficit (also often called primary deficit) and interest payments on the debt. The total government deficit, which is equal to the change in government debt ΔB_t, is equal to the sum of interest payments and the primary deficit $G_t - T_t$.

$$\text{Deficit}_t = \Delta B_t = B_t - B_{t-1} = \underbrace{r_t B_{t-1}}_{\text{Interest Payments}} + \underbrace{G_t - T_t}_{\text{Primary Deficit}}$$

From the above equation, the evolution of the debt to GDP ratio B_t/Y_t:

$$\frac{B_t}{Y_t} = (1 + r_t)\frac{Y_{t-1}}{Y_t}\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Let us denote the debt to GDP ratio by b_t:

$$b_t \equiv \frac{B_t}{Y_t}.$$

Therefore:

$$b_t = (1 + r_t)\frac{Y_{t-1}}{Y_t}b_{t-1} + \frac{G_t - T_t}{Y_t}.$$

Assuming that GDP grows at rate g_Y, we have that:

$$\frac{Y_t}{Y_{t-1}} = 1 + g_Y$$

Therefore:

$$b_t = \frac{1 + r_t}{1 + g_Y}b_{t-1} + \frac{G_t - T_t}{Y_t}.$$

Imagine that all future primary surpluses were equal to zero after $t = t_0$, that is:

$$\text{for all } t \geq t_0, \quad G_t = T_t$$

and that real interest rates are constant:

$$r_t = r$$

We then have that:

$$\boxed{\text{for all } t \geq t_0, \quad b_t = \frac{1 + r}{1 + g_Y}b_{t-1}}$$

Then the debt to GDP ratio would be given by:

$$\text{for all } t \geq t_0, \quad b_t = \left(\frac{1+r}{1+g_Y}\right)^{t-t_0} b_{t_0}$$

There are three possible cases:
1. If $r < g_Y$, the debt to GDP ratio goes to 0. (Indeed, when $a < 1$, $a^t \to 0$ when $t \to +\infty$.) Therefore, the debt to GDP ratio goes to zero mechanically.
2. If $r = g_Y$, the debt to GDP ratio stays constant.
3. If $r > g_Y$, the debt to GDP ratio goes to infinity. Indeed, when $a > 1$, $a^t \to +\infty$ when $t \to +\infty$. Then, the dynamics of government debt are explosive.

Up until now, it has been the case that $r < g_Y$. (nominal GDP growth, inclusive of inflation, is larger than 3%, while the interest rate on government debt is close to 2%. The ratio of government debt to GDP does not appear to be on an unsustainable path so far. However, some economists are worried about the US government debt, and believe it should be reduced as soon as possible.

2 The Government Budget Constraint with an Infinite Number of Periods

To be sure, the government has actually never fully repaid its debt but instead has been rolling it over for a long time. The government debt is a Ponzi scheme, but if the interest rate on debt is lower than the rate of growth, then it is definitely a sustainable Ponzi Scheme.

The question which needs to be asked is: is it a good one?

The answer is that even in the neoclassical growth model, the Diamond model with endogenous saving, it can be a good one provided that the interest arte is lower than the rate of growtgh $R < g$, a situation known as dynamic inefficiency.

Thus, even under neoclassical principles, the Golden Rule level of capital accumulation is lower than than it currently is provided that the return on capital is lower than the rate of growth. This is a question we take up next.

Remmeber that in pproblem Set 4, the Golden Rule level of capital accumulation was such that actually the net interest rate was equal to g: Therefore, in the steady state, using the above expression for k^* (question 3) we get:

A law of motion for k_{t+1} is thus (we use equal signs now, even though it is really an approximation):

$$k_{t+1} = \frac{s}{1+g/(1-\alpha)+n} k_t^\alpha + \frac{1-\delta}{1+g/(1-\alpha)+n} k_t.$$

3. The steady-state is such that:

$$\left(1 + \frac{1}{1-\alpha} g + n\right) k^* = s(k^*)^\alpha + (1-\delta)k^*.$$

Therefore:

$$\left(\delta + \frac{1}{1-\alpha} g + n\right) k^* = s(k^*)^\alpha.$$

Finally, this gives k^*:

$$k^* = \left(\frac{s}{\delta + g/(1-\alpha)+n}\right)^{\frac{1}{1-\alpha}}.$$

The marginal product of capital r_t is:

$$\begin{aligned} r_t &= \alpha A_t \left(\frac{K_t}{L_t}\right)^{\alpha-1} \\ &= \alpha \left(\frac{K_t}{A_t^{1/(1-\alpha)} L_t}\right)^{\alpha-1} \\ r_t &= \alpha k_t^{\alpha-1}. \end{aligned}$$

$$\begin{aligned} r^* &= \alpha \left(\frac{\alpha}{\delta + g/(1-\alpha) + n}\right)^{\frac{1}{1-\alpha}} \\ &= \delta + g/(1-\alpha) + n \\ r^* &= \delta + g_Y. \end{aligned}$$

Finally, the net interest rate $r^* - \delta$ needs to be equal to the rate of growth of output g_Y, to be at the Golden Rule level of capital accumulation. We shall encounter this condition again in lecture 10 when we look at the sustainability of public debt.

2.1 Model

Let us look at a simplified version of the overlapping generations model we looked at in Lecture 3. Let's assume that people only care about old age consumption, and that they work only when young, receiving wage w_t. It does not really matter what the form of their utility function is with respect to old age consumption (in any case, remember that utility is an ordinal concept, used to rationalize people's behavior, not a cardinal concept):

$$U = u(c_{t+1}^o).$$

The intertemporal budget constraint is given by:

$$c_t^y + \frac{c_{t+1}^o}{1+r_t} = w_t.$$

In this very simple environment, and because consumption in young age will always optimally be set to zero ($c_t^y = 0$), as this consumption does not appear in the utility function, this means that:

$$c_{t+1}^o = (1+r_t) w_t.$$

Similarly to the previous time, we assume that the labor force is fixed to unity:Diamond (1965) had population growth in his original model. We do the simplest version of his model. ($L_t = \bar{L} = 1$) There is a Cobb-Douglas, constant returns to scale, production function:

$$Y_t = K_t^\alpha L_t^{1-\alpha}.$$

Together with the previous assumption of constant labor $L_t = 1$, this means that:

$$Y_t = K_t^\alpha.$$

Markets are competitive, so that the wage is just the marginal product of labor:

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha) K_t^\alpha L_t^{-\alpha} = (1-\alpha) K_t^\alpha.$$

Similarly as previously, we also get through firms' optimization on the amount of capital that:

$$r_t + \delta = \frac{\partial Y_t}{\partial K_t} = \alpha K_t^{\alpha-1}.$$

Finally, we assume again that capital depreciates at rate $\delta = 1 = 100\%$. (that is, capital fully depreciates each period - this is reasonable if you take one unit of time to represent one generation, or about 30 years - remember that the depreciation rate for one year was approximately equal to 10%.)

2.2 Without a government

Which equilibrium arises in the absence of a government? In the absence of a government, we get even simpler expressions than the previous time. The law of motion for capital is given as follows:

$$\Delta K_{t+1} = w_t - \delta K_t.$$

Since w_t is a fraction $1-\alpha$ of output, this law of motion corresponds to the Solow growth model with $s = 1-\alpha$. The law of motion for capital is:

$$K_{t+1} = (1-\alpha)K_t^{\alpha} + (1-\delta)K_t.$$

This is a difference equation for sequence K_t which converges to a steady state value for the capital stock K^* such that:

$$\delta K^* = (1-\alpha)(K^*)^{\alpha} K^* = \left(\frac{1-\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}.$$

The steady state value for the interest rate is then:

$$\begin{aligned}
r^* + \delta &= \alpha (K^*)^{\alpha-1} \\
&= \alpha \left[\left(\frac{1-\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}\right]^{\alpha-1} \\
r^* + \delta &= \frac{\delta\alpha}{1-\alpha}
\end{aligned}$$

Therefore:

$$r^* = \frac{2\alpha-1}{1-\alpha}\delta$$

The steady state value for output is then:

$$\begin{aligned}
Y^* &= (K^*)^{\alpha} \\
Y^* &= \left(\frac{1-\alpha}{\delta}\right)^{\frac{\alpha}{1-\alpha}}.
\end{aligned}$$

The value for the wage would be:

$$\begin{aligned}
w^* &= (1-\alpha)\,(K^*)^{\alpha} \\
&= (1-\alpha)\left(\frac{1-\alpha}{\delta}\right)^{\frac{\alpha}{1-\alpha}} \\
w^* &= \frac{(1-\alpha)^{\frac{1}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}}
\end{aligned}$$

Steady-state consumption of the old is thus given by:

$$\begin{aligned}
(c^o)^* &= (1+r^*)w^* \\
(c^o)^* &= \left(1 + \frac{2\alpha-1}{1-\alpha}\delta\right)(1-\alpha)^{\frac{1}{1-\alpha}}
\end{aligned}$$

Example. With $\alpha = 1/3$ and $\delta = 1$:

$$K^* = \left(\frac{2}{3}\right)^{3/2}, \qquad r^* = -\frac{1}{2} = -50\%, \qquad Y^* = \sqrt{\frac{2}{3}}$$

$$w^* = \left(\frac{2}{3}\right)^{3/2} \qquad (c^o)^* = \alpha\,(1-\alpha)^{\frac{\alpha}{1-\alpha}}$$

2.3 With a government taking on debt

Imagine we start at a steady state value for capital calculated above. The government can make every generation better off by taking on debt, for example by giving a transfer to the first generation of old people (like was done after the second world war), and rolling over this debt every period. Indeed, let us look at the level of capital such that $r^* = 0$ (which is called the golden rule interest rate). This level of capital is given by:

$$r^* + \delta = \alpha (K_g^*)^{\alpha - 1}.$$

Therefore:

$$\frac{2\alpha - 1}{1 - \alpha} + 1 = \alpha (K_g^*)^{\alpha-1} \quad \Rightarrow \quad \frac{\alpha}{1-\alpha} = \alpha (K_g^*)^{\alpha-1}$$
$$\Rightarrow \quad K_g^* = (1-\alpha)^{\frac{1}{1-\alpha}}.$$

Example. With $\alpha = 1/3$, with this level of the capital stock, we know that the wage would be given by:

$$w^* = \frac{2}{3} Y^* = \frac{2}{3} (K^*)^{1/3} = \frac{2}{3\sqrt{3}}.$$

The steady state value for output would then be:

$$Y^* = (K^*)^{1/3} = \frac{1}{\sqrt{3}}.$$

Then the steady-state consumption of the old is given by:

$$(c^o)^* = w^* = \frac{2}{3\sqrt{3}}.$$

Note that this is greater than the level of consumption achieved by the old without a government since $2 > \sqrt{2}$. But what is amazing is that the level of capital in this case is actually lower than the level of capital in the previous section. The government can force the economy into this level of capital accumulation by taking on debt. The level of debt B^* that corresponds to that level of capital accumulation is given by:

$$B^* + K^* = w^* \quad \Rightarrow \quad B^* = w^* - K^* = \frac{2}{3\sqrt{3}} - \frac{1}{3\sqrt{3}} = \frac{1}{3\sqrt{3}}.$$

The government can reach that level of debt by giving a transfer to the first generation of old, like the war veterans, who will then consume:

$$c_0^o = \frac{\sqrt{2}}{3\sqrt{3}} + \frac{1}{3\sqrt{3}} = \frac{1+\sqrt{2}}{3\sqrt{3}}.$$

All future generations will then consume more because of the above formula. With a lot of capital, there is such a thing as a free lunch!

3 Readings - To go further

There is no significant budget deficit, Olivier Blanchard, Jeffrey Sachs, *New York Times*, March 6, 1981.

A Note On The Ricardian Equivalence Argument Against Stimulus (Slightly Wonkish), New York Times Blog Post, December 26, 2011.

Paul Krugman, Multipliers and Reality, New York Times Blog Post, June 3, 2015.

In Japan, the Government Gets Paid to Borrow Money, *Wall Street Journal*, March 1, 2016.

(Gated) Say's law: supply creates its own demand. *The Economist*, August 10, 2017.

(Gated) Why is macroeconomics so hard to teach? *The Economist*, August 9, 2018.