Problem Set 8 - Solutions

UCLA - Econ 102 - Fall 2018

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1 Optimal Taxation: The Supply Side View

1. The firm's problem is to maximize:
$$\max_l \quad pf(l) - wl$$
Therefore, the real wage is given by:
$$\boxed{\frac{w}{p} = A}.$$

2. Writing that $c = c_0 + (1-\tau) \cdot (w/p) \cdot l$ and plugging the value of c into the worker's optimization problem:
$$\max_l \quad (1-\tau)\frac{w}{p}l + c_0 - B\frac{l^{1+\epsilon}}{1+\epsilon}$$
The first-order condition implies:
$$(1-\tau)\frac{w}{p} = Bl^{\epsilon} \quad \Rightarrow \quad \boxed{l = \frac{(1-\tau)^{1/\epsilon}}{B^{1/\epsilon}}\left(\frac{w}{p}\right)^{1/\epsilon}}$$

3. The number of hours worked is given by replacing out the real wage w/p from the labor demand equation to the labor supply equation, so that:
$$\boxed{l = (1-\tau)^{1/\epsilon}\frac{A^{1/\epsilon}}{B^{1/\epsilon}}}.$$
Real pre-tax income is when simply given by the real wage times the number of hours. Since the real wage is simply A, we get:
$$y = \frac{w}{p}l = A \cdot (1-\tau)^{1/\epsilon}\frac{A^{1/\epsilon}}{B^{1/\epsilon}}.$$
Therefore:
$$\boxed{y = (1-\tau)^{1/\epsilon}\frac{A^{1/\epsilon+1}}{B^{1/\epsilon}}}.$$

4. A numerical application shown on the Google Spreadsheet implies that:
$$\underline{l} = (1-\underline{\tau})^{1/\epsilon}\frac{\underline{A}^{1/\epsilon}}{\underline{B}^{1/\epsilon}}$$
Therefore:
$$\underline{l} = \left(1 - \frac{1}{4}\right)^{1/2}\left(\frac{1000000}{28188}\right)^{1/2}\left(\frac{491569855488}{3000000}\right)^{1/2}$$
$$\underline{l} = 2088.$$
Note that under the assumption of 281 business days in a year, this implies **8 hours a day** of work. According to the Google Spreadsheet, income is then given by:
$$\begin{aligned}\underline{y} &= \underline{Al} \\ &= \frac{1000000}{28188} \cdot 2088 \\ \underline{y} &\approx 74074.07\end{aligned}$$

5. We note that this group has a lower disutility of working B and also a higher productivity per hour A. Therefore they work more (both because they are more productive and because they have a lower disutility of working):

$$\begin{aligned}\bar{l} &= \left(1-\frac{1}{2}\right)^{1/2}\left(\frac{1000000}{6264}\right)^{1/2}\left(\frac{218475491328}{1000000}\right)^{1/2}\\ \bar{l} &= 4176.\end{aligned}$$

Note that under the assumption of 281 business days in a year, this implies 16 hours a day of work on average (more likely on weekends, or during Thanksgiving!). Income is then given by:

$$\begin{aligned}\bar{y} &= \bar{A}\bar{l}\\ &= \frac{1000000}{6264}\cdot 4176\\ \bar{y} &\approx 666666.66\end{aligned}$$

6. Total output is 20 trillion, 10 trillion coming from the bottom 90% and 10 trillion coming from the top 10%. In million dollars, output of the bottom 90% is:

$$\begin{aligned}\underline{Y} &= \lambda N \underline{y}\\ &= 0.9\cdot 150\cdot 74074.07\\ &= 10,000,000 \text{ million}\\ \underline{Y} &= 10 \text{ trillion}\end{aligned}$$

In million dollars, output of the top 10% is:

$$\begin{aligned}\bar{Y} &= (1-\lambda)N\bar{y}\\ &= 0.1\cdot 150\cdot 666666.66\\ &= 10,000,000 \text{ million}\\ \bar{Y} &= 10 \text{ trillion}\end{aligned}$$

7. According to the Google Spreadsheet, an individual earning $\underline{y}$ pays taxes given by:

$$\underline{t} = -5000 + 0.25 * (74074.07 - 25000) \approx 7,268$$

which means that this individual pays \$7268 on average per year in taxes. The total taxes paid by this group are:

$$\underline{T} = \lambda N \underline{t} \approx 981 \text{ billion}.$$

According to the Google Spreadsheet, an individual earning $\bar{y}$ pays taxes given by:

$$\bar{t} = -5000 + 0.25 * (200000 - 25000) + 0.5 * (666666 - 200000) \approx 272,083$$

which means that this individual pays \$272,083 on average in taxes per year. The total taxes paid are:

$$\bar{T} = (1-\lambda)N\bar{t} \approx 4,081 \text{ billion}.$$

The Google Spreadsheet shows that lowering the bottom marginal tax rate leads to an increase in hours worked by the bottom 90%, which increases GDP by about **328 billion**. The cost of the policy is a fall in tax receipts of **266 billion**. This implies that the supply-side "multiplier" is:

$$-\frac{\Delta Y}{\Delta T} = 1.23.$$

8. The Google Spreadsheet shows that lowering the top marginal tax rate leads to an increase in hours worked by the top 10%, which increases GDP by about **488 billion**. The cost of the policy is a fall in tax receipts of **130 billion**. This implies that the supply-side "multiplier" is:

$$-\frac{\Delta Y}{\Delta T} = 3.74.$$