Course 2 - Gaussian Versus Pareto Distributions

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Contents

Introduction

During this course, we shall try to understand a technical passage in Rosen [1983]'s *The American Scholar* piece:

> Of particular interest here is an observation, first studied systematically by the great Italian economist Vilfredo Pareto in the late nineteenth century, that the distribution of income contains an unusually large proportion of top earners: that is, among the rich rather than the poor. A visual image will perhaps clarify what is meant by "unusual" in this connection. Imagine a graph plotting IQ scores on the horizontal and the frequency of scores on the vertical. The result is a familiar bell-shaped curve. The peak of the bell occurs at a score arbitrarily scaled at 100 and the curve falls symmetrically on either side of 100. Now picture a similar graph, except with earnings on the horizontal. The resulting curve is unbalanced and nonsymmetrical - a bell that is definitely out of whack. To the left of the modal (peak) value it appears much like the IQ frequency curve. However, to the right of the mode it does not fall as fast as it does to the left. It looks as if someone had stood at the right end of the curve, placed it over his back like a rope, and dragged and stretched it out a very long distance. The upper or right-hand tail of the distribution of income is much thicker than the lower, left-hand tail. The extra weight on the right lends a certain skewness to the distribution of income. What this comes down to is that the distribution of earnings is far from proportionate to the distribution of ability. Amazingly, Pareto's observations have been qualitatively duplicated in virtually every era of every society for which data on income distributions can be found.

In this passage, Sherwin Rosen draws a sharp distribution between Gaussian distributions on the one hand (characterized by the well known bell-shaped curve) and Pareto distributions on the other hand:

1. "Imagine a graph plotting IQ scores on the horizontal and the frequency of scores on the vertical. The result is a familiar bell-shaped curve." We first remind ourselves of some properties concerning bell-shaped distributions.
2. "The upper or right-hand tail of the distribution of income is much thicker than the lower, left-hand tail. The extra weight on the right lends a certain skewness to the distribution of income. What this comes down to is that the distribution of earnings is far from proportionate to the distribution of ability." We

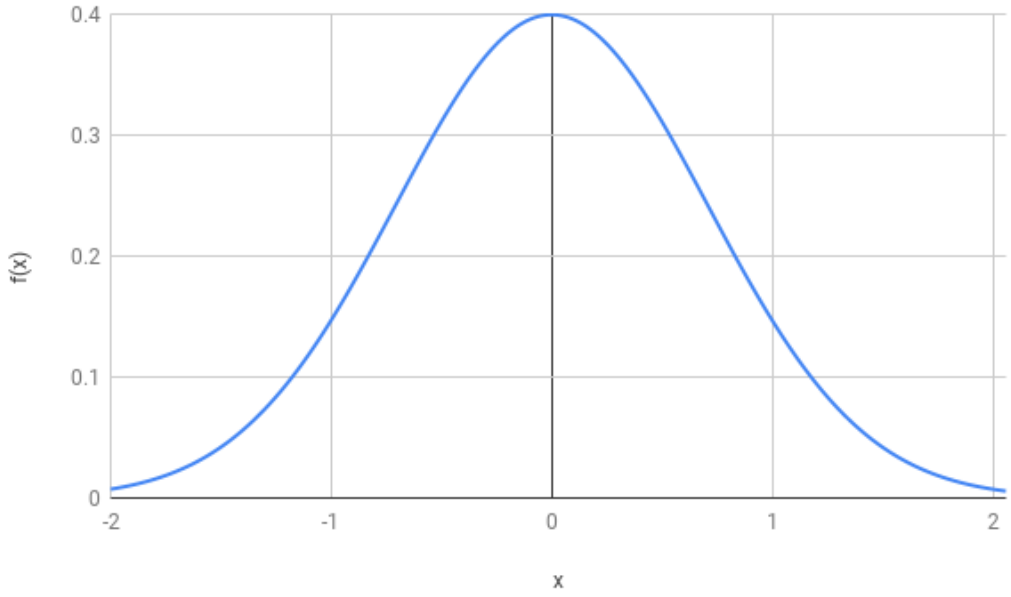


Figure 1: Bell Shaped Curve

will then investigate some properties of these distributions, define Pareto distributions in particular and show that they are ubiquitous in social sciences.

1 Bell-Shaped Distributions

1.1 Natural Sciences

```
height <- read.csv("https://raw.githubusercontent.com/hadley/r4ds/master/data/heights.csv")
```

1.2 Some maths

The Bell shape curve is defined by a density function given by:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{(x-\mu)^2}{\sigma^2}\right).$$

1.3 Google Sheets

Here is a link to the Google Sheets that we created in order to look at the Gaussian distribution. In particular, we were able to plot the density function of a Normal Distribution with $\mu = 0$ and $\sigma = 1$, using the formula above.

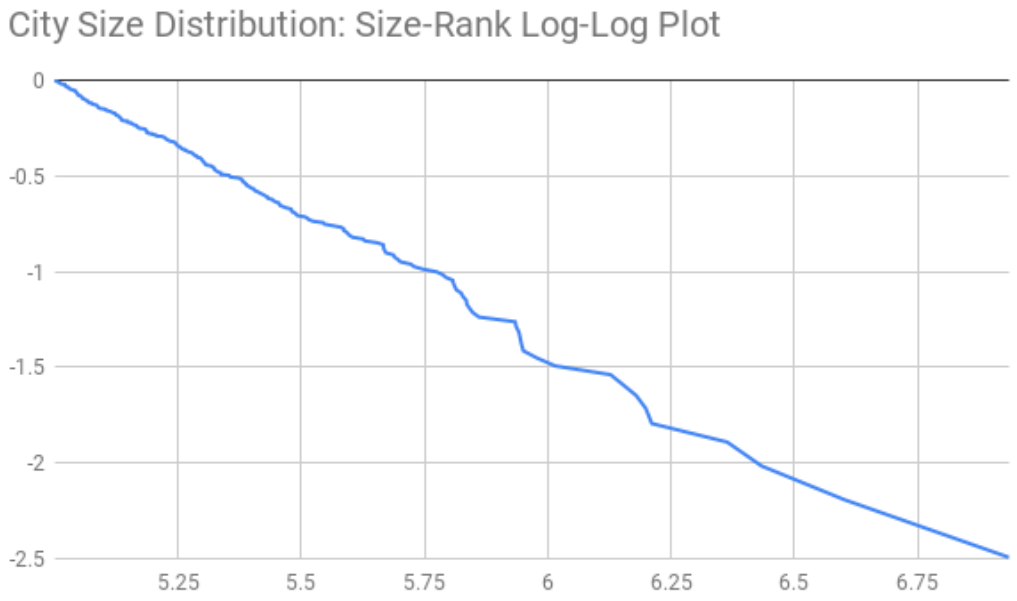


Figure 2: City Size Distribution

2 Pareto Distributions

2.1 Maths of Pareto

A key feature of the Pareto distribution is that the density distribution does not go as fast to 0 as x becomes large. If x is population, his means that there are relatively many cities with a large size. Similarly, there are relatively many incomes that are much larger than the mean. The Pareto Distribution is in fact defined by:

$$f(x) = a\frac{x_m^a}{x^{a+1}}.$$

For the cumulative distribution function, this implies:

$$1 - F(x) = \left(\frac{x_m}{x}\right)^a.$$

2.2 Data on cities using Google Sheets

We have used this Google Spreadsheet in order to plot the city size distribution of cities.

We note that the result is something that is close to a linear relationship, suggestive of Pareto like behavior.

2.3 Data on cities using R Statistical software

```
pklist <- c("tidyverse", "rvest")
source("https://fgeerolf.github.io/code/load-packages.R")
options(tibble.print_max = 100)
```

The data comes from the following Wikipedia entry: List of United States cities by population.

```
data <- "https://en.wikipedia.org/wiki/List_of_United_States_cities_by_population" %>%
  read_html %>%
  html_table(header = TRUE, fill = TRUE)
```

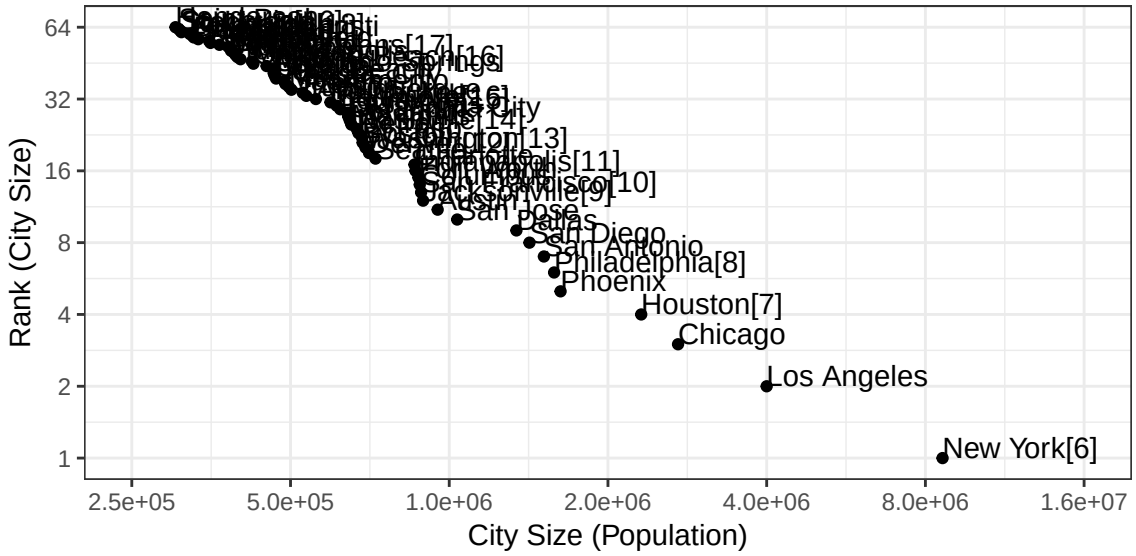
Biggest cities:

```
data[[5]][,c(1:4)] %>%
  as.tibble %>%
  select(rank = "2017rank", "City", state = "State[5]", pop = "2017estimate") %>%
  head(28)
```

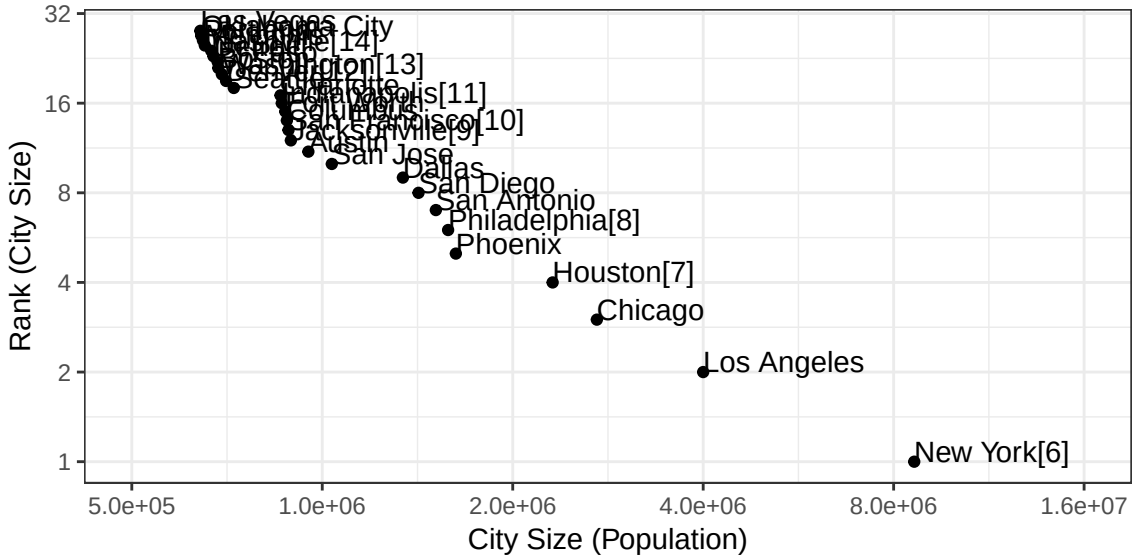
```
## # A tibble: 28 x 4
##     rank City              state                pop
##    <int> <chr>             <chr>                <chr>
##  1     1 New York[6]       New York             8,622,698
##  2     2 Los Angeles       California           3,999,759
##  3     3 Chicago           Illinois             2,716,450
##  4     4 Houston[7]        Texas                2,312,717
##  5     5 Phoenix           Arizona              1,626,078
##  6     6 Philadelphia[8]   Pennsylvania         1,580,863
##  7     7 San Antonio       Texas                1,511,946
##  8     8 San Diego         California           1,419,516
##  9     9 Dallas            Texas                1,341,075
## 10    10 San Jose          California           1,035,317
## 11    11 Austin            Texas                950,715
## 12    12 Jacksonville[9]   Florida              892,062
## 13    13 San Francisco[10] California           884,363
## 14    14 Columbus          Ohio                 879,170
## 15    15 Fort Worth        Texas                874,168
## 16    16 Indianapolis[11]  Indiana              863,002
## 17    17 Charlotte         North Carolina       859,035
## 18    18 Seattle           Washington           724,745
## 19    19 Denver[12]        Colorado             704,621
## 20    20 Washington[13]    District of Columbia 693,972
## 21    21 Boston            Massachusetts        685,094
## 22    22 El Paso           Texas                683,577
## 23    23 Detroit           Michigan             673,104
## 24    24 Nashville[14]     Tennessee            667,560
## 25    25 Memphis           Tennessee            652,236
## 26    26 Portland          Oregon               647,805
## 27    27 Oklahoma City     Oklahoma             643,648
## 28    28 Las Vegas         Nevada               641,676
```

```
data[[5]][,c(1:4)] %>%
  as.tibble %>%
  select(rank = "2017rank", pop = "2017estimate", "City") %>%
  mutate(pop = pop %>% gsub(",", "", .) %>% as.numeric) %>%
  arrange(-pop) %>%
  mutate(rank = 1:n()) %>%
  ggplot(aes(x = pop, y = rank)) + geom_point() + theme_bw() +
  geom_text(aes(label = City), hjust = 0, vjust = 0) +
  scale_y_log10(breaks = 2^(seq(0, 10, 1)),
                limits = c(1, 64)) +
  scale_x_log10(breaks = 250000*2^seq(0, 10, 1),
```

```
                limits = c(250000, 16000000)) +
  ylab("Rank (City Size)") + xlab("City Size (Population)")
```




```
data[[5]][,c(1:4)] %>%
  as.tibble %>%
  select(rank = "2017rank", pop = "2017estimate", "City") %>%
  mutate(pop = pop %>% gsub(",", "", .) %>% as.numeric) %>%
  arrange(-pop) %>%
  mutate(rank = 1:n()) %>%
  filter(pop >= 500000) %>%
  ggplot(aes(x = pop, y = rank)) + geom_point() + theme_bw() +
  geom_text(aes(label = City), hjust = 0, vjust = 0) +
  scale_y_log10(breaks = 2^(seq(0, 10, 1)),
                limits = c(1, 28)) +
  scale_x_log10(breaks = 500000*2^seq(0, 10, 1),
                limits = c(500000, 16000000)) +
  ylab("Rank (City Size)") + xlab("City Size (Population)")
```



References

Sherwin Rosen. The Economics of Superstars. *The American Scholar*, 52(4):449–460, 1983. ISSN 0003-0937. URL http://www.jstor.org/stable/41210977.