

Problem Set 2

UCLA - Econ 102 - Fall 2018

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1 Solow Growth Model if $\alpha = 1/2$ and $A = 1/2$

Suppose that the production function is such that $A = 1/2$ (often, we assume that $A = 1$) and $\alpha = 1/2$:

$$Y = \frac{1}{2}\sqrt{K}\sqrt{L}$$

1. Derive the steady-state levels of output per worker and capital per worker in terms of the saving rate, s , and the depreciation rate, δ .
2. Derive the equation for steady-state output per worker and steady-state consumption per worker in terms of s and δ .
3. Suppose that $\delta = 0.05$. With your favorite spreadsheet software, compute steady-state output per worker and steady-state consumption per worker for $s = 0$; $s = 0.1$; $s = 0.2$; $s = 1$. Explain the intuition behind your results.
4. Use your favorite spreadsheet software to graph the steady-state level of output per worker and the steady-state level of consumption per worker as a function of the saving rate (i.e., measure the saving rate on the horizontal axis of your graph and the corresponding values of output per worker and consumption per worker on the vertical axis).
5. Does the graph show that there is a value of s that maximizes output per worker? Does the graph show that there is a value of s that maximizes consumption per worker? If so, what is this value?

2 Solow Growth Model if $\alpha = 1/3$

Suppose that the economy's production function is given by

$$Y = K^\alpha L^{1-\alpha}$$

and assume that $\alpha = 1/3$.

1. Is this production function characterized by constant returns to scale? Explain.
2. Are there decreasing returns to capital?
3. Are there decreasing returns to labor?
4. Transform the production function into a relation between output per worker and capital per worker.
5. For a given saving rate, s , and depreciation rate, δ , give an expression for capital per worker in the steady state.
6. Give an expression for output per worker in the steady state.
7. Solve for the steady-state level of output per worker when $s = 0.32$ and $\delta = 0.08$.
8. Suppose that the depreciation rate remains constant at $\delta = 0.08$, while the saving rate is reduced by half, to $s = 0.16$. What is the new steady-state output per worker?

3 An increase in the depreciation rate in the Solow growth model

Continuing with the logic from the previous problem, suppose that the economy's production function is given by

$$Y = K^\alpha L^{1-\alpha}$$

with $\alpha = 1/3$ and that both the saving rate, s , and the depreciation rate, δ are equal to 0.10.

1. What is the steady-state level of capital per worker?
2. What is the steady-state level of output per worker? Suppose that the economy is in steady state and that, in period t , the depreciation rate increases permanently from 0.10 to 0.20.
3. What will be the new steady-state levels of capital per worker and output per worker?
4. Compute the path of capital per worker and output per worker over the first three periods after the change in the depreciation rate.

4 Deficits and the capital stock

Suppose that the production function is given by:

$$Y = \sqrt{K}\sqrt{L}$$

1. Show that the steady-state capital stock per worker and output per worker are given by:

$$\frac{K^*}{L} = \left(\frac{s}{\delta}\right)^2 \quad \text{and} \quad \frac{Y^*}{L} = \frac{s}{\delta}.$$

2. Suppose that the saving rate, s , is initially 15 % per year, and the depreciation rate, δ , is 7.5 %. What is the steady-state capital stock per worker? What is steady-state output per worker?
3. Suppose that there is a government deficit of 5% of GDP and that the government eliminates this deficit. Assume that private saving is unchanged so that total saving increases to 20%. What is the new steady-state capital stock per worker? What is the new steady-state output per worker? How does this compare to your answer to part 2?

5 U.S. saving and government deficits.

This question continues the logic of the previous question, to explore the implications of the U.S. government budget deficit for the long-run capital stock.

1. The World Bank reports gross domestic saving rate by country and year. The Web site is <http://data.worldbank.org/indicator/NY.GDS.TOTL.ZS>. Find the most recent number for the United States. What is the total saving rate in the United States as a percentage of GDP? Using the depreciation rate and the logic from the previous problem, what would be the steady-state capital stock per worker? What would be steady-state output per worker?
2. Go to the most recent Economic Report of the President (ERP) and find the most recent federal deficit as a percentage of GDP. In the 2015 ERP, this is found in Table B-20. Using the reasoning from the previous problem, suppose that the federal budget deficit was eliminated and there was no change in private saving. What would be the effect on the long-run capital stock per worker? What would be the effect on long-run output per worker?
3. Return to the World Bank table of gross domestic saving rates. How does the saving rate in China compare to the saving rate in the United States?