Problem Set 1

UCLA - Econ 102 - Fall 2018

François Geerolf

1 Geometric Sums

1. Calculate the following geometric sum, when $x \neq 1$, as a function of x^{n+1} in particular:

$$\sum_{i=0}^{n} x^i = 1 + x + x^2 + ... + x^n$$

2. State a condition on x such that the infinite geometric sum (when $n \to \infty$) has a finite value.
3. Assuming that this geometric sum is finite, calculate:

$$\sum_{i=0}^{\infty} x^i = 1 + x + x^2 + ... + x^n + ...$$

4. More generally, for m a positive integer, calculate:

$$\sum_{i=m}^{n} x^i = x^m + x^{m+1} + ... + x^n$$

5. What is the present discounted value of an infinite stream of incomes, which grows at rate $g = 2$ %, starts at $y_0 = 90000$, if the interest rate is $i = 3$ %?

2 Taylor Approximations

1. If x is small, show that:
$$(1 + x)^n \approx 1 + nx.$$
2. If x and y are small, then show that:
$$(1 + x)(1 + y) \approx 1 + x + y.$$
3. If x and y are small, then show that:
$$\frac{1 + x}{1 + y} \approx 1 + x - y.$$
4. Your savings account offers a nominal interest rate of 1%. Meanwhile, annual inflation is 1.5%. What is an exact value for the real interest rate, defined as the rate of increase in your purchasing power if you leave your money in the bank? What is an approximate value for this real interest rate, using one of the previous formulas?

3 Growth Rates

1. If y_t grows at a constant rate g during a given period, then show that the growth G of y_t after T periods is:
$$G = \frac{y_T}{y_0} - 1 = (1+g)^T - 1.$$

2. Conversely, if the growth rate of y_t after T periods is G, then show that the average growth rate of y_t per period is:
$$g = \frac{y_{t+1}}{y_t} - 1 = (1+G)^{1/T} - 1.$$

3. If your annual return on your savings rate is 1%, what is your daily return (assuming a year is 365 days)? How much do you then give up each day by leaving $100K on a zero interest checking account?