

Lecture 4 - Recommended Exercises Solutions

☆☆☆ Problem 2, Chapter 5

Consider first the goods market model with constant investment that we saw in Chapter 3. Consumption is given by:

$$C = c_0 + c_1(Y - T)$$

and I , G , and T are given.

- a. **Solve for equilibrium output. What is the value of the multiplier for a change in autonomous spending?**

As we saw in Chapter 3, we can solve for equilibrium output as follows:

$$\begin{aligned} Y &= c_0 + c_1Y - c_1T + \bar{I} + G \\ (1 - c_1)Y &= c_0 - c_1T + \bar{I} + G \\ Y_{equil} &= \underbrace{\left(\frac{1}{1 - c_1}\right)}_{\text{multiplier}} \times \underbrace{(c_0 - c_1T + \bar{I} + G)}_{\text{Autonomous Spending}} \end{aligned}$$

Now let investment depend on both sales and the interest rate:

$$I = b_0 + b_1Y - b_2i$$

- b. **Solve for equilibrium output using the methods learned in Chapter 3. At a given interest rate, why is the effect of a change in autonomous spending bigger than what it was in part (a)? Why? (Assume $c_1 + b_1 < 1$.)**

With the new assumption for the behavior of investment, our equation for equilibrium output becomes

$$\begin{aligned} Y &= c_0 + c_1Y - c_1T + b_0 + b_1Y - b_2i + G \\ (1 - c_1 - b_1)Y &= c_0 - c_1T + b_0 - b_2i + G \\ Y_{equil} &= \underbrace{\left(\frac{1}{1 - c_1 - b_1}\right)}_{\text{multiplier}} \times (c_0 - c_1T + b_0 - b_2i + G) \end{aligned}$$

The new multiplier now features a magnifier effect through both consumption (c_1) and through investment (b_1). When we include b_1 , total output is more responsive to shocks in the components of autonomous spending. In a way, there are two multiplier effects.

- c. **Suppose the central bank chooses an interest rate of \bar{i} . Solve for equilibrium output at that interest rate.**

Given our result in part (b), this is just turning i from some (unexplained) variable to a fixed value (\bar{i}) pinned down by the central bank's policy.

$$Y_{equil}(\bar{i}) = \underbrace{\left(\frac{1}{1 - c_1 - b_1} \right)}_{\text{multiplier}} \times (c_0 - c_1T + b_0 - b_2\bar{i} + G)$$

- d. **Draw the equilibrium of this economy using an IS/LM diagram.**

☆☆☆ Problem 3, Chapter 5

The response of the economy to fiscal policy.

- a. **Using an IS/LM diagram, show the effect on output of a decrease in government spending. Can you tell what happens to investment? Why?**

Now consider the following IS/LM model:

$$C = c_0 + c_1(Y - T)$$

$$I = b_0 + b_1Y - b_2i$$

$$Z = C + I + G$$

$$i = \bar{i}$$

- b. **Solve for equilibrium output when the interest rate is \bar{i} . Assume $c_1 + b_1 < 1$. (Hint: You may want to rework through Problem 2 if you are having trouble with this step).**

In addition to the equations listed above, we also need to use the market-clearing condition (or the equilibrium condition) which says that aggregate spending equals aggregate output: $Z = Y$.

The result for this question is the same as that shown in Q5-2(c) above.

- c. **Solve for the equilibrium level of investment.**

Equilibrium investment is just taking our equation for investment and plugging in the equilibrium interest rate (\bar{i}) and the equilibrium level of output (Y_{equil}).

$$\begin{aligned} I_{equil} &= b_0 + b_1Y_{equil} - b_2\bar{i} \\ &= b_0 + b_1 \left(\frac{1}{1 - c_1 - b_1} \right) (c_0 - c_1T + b_0 - b_2\bar{i} + G) - b_2\bar{i} \\ &= \left(\frac{1 - c_1}{1 - c_1 - b_1} \right) (b_0 - b_2\bar{i}) + \left(\frac{b_1}{1 - c_1 - b_1} \right) (c_0 - c_1T + G) \end{aligned}$$

- d. **Let's go behind the scene in the money market. Use the equilibrium in the money market $M/P = d_1Y - d_2i$ to solve for the equilibrium level of the**

real money supply when $i = \bar{i}$. How does the real money supply vary with government spending?

Just as we saw above, the equilibrium level of real money balances (M/P) is just plugging in \bar{i} and Y_{equil} to the equation for (M/P). Looking at the equation for (M/P), we can see that the answer will be that an increase in government spending will increase real money balances. This is because we already know that when G increases, Y_{equil} will increase and thus raise (M/P) while there is no change in the interest rate because the central bank will not allow the interest rate to change. Put another way, in order for the central bank to hit its target for \bar{i} when output increases, they will have to increase the real money supply. From the perspective of a central bank, this is a function that says: “what should I do with the money supply when GDP changes in order to maintain the interest rate target?”

$$\begin{aligned}(M/P)_{equil} &= d_1 Y_{equil} - d_2 \bar{i} \\ &= d_1 \left(\frac{1}{1 - c_1 - b_1} \right) (c_0 - c_1 T + b_0 - b_2 \bar{i} + G) - d_2 \bar{i}\end{aligned}$$

☆☆ Problem 11, Chapter 5

Consumption, investment, and the recession of 2001

This question asks you to examine the movements of investment and consumption before, during and after the recession of 2001. It also asks you to consider the events of September 11, 2001.

Go to the website of the Bureau of Economic Analysis (www.bea.gov). Find the NIPA tables, in particular the quarterly versions of Table 1.1.1, which shows the percentage change in real GDP and its components, and Table 1.1.2, which shows the contribution of the components of GDP to the overall percentage change in GDP. Table 1.1.2 weighs the percentage change of the components by their size. Investment is more variable than consumption, but consumption is much bigger than investment, so smaller percentage changes in consumption can have the same impact on GDP as much larger percentage changes in investment. Note that the quarterly percentage changes are annualized (i.e. expressed as annual rates). Retrieve the quarterly data on real GDP, consumption, gross private domestic investment, and nonresidential fixed investment for years 1999 to 2002 from Tables 1.1.1 and 1.1.2.

- a. Identify the quarters of negative growth in 2000 and 2001.
Overall GDP growth was negative in 2001Q1 and 2001Q3.
- b. Track consumption and investment around 2000 and 2001. From Table 1.1.1 which variable has the bigger percentage change around this time? Compare nonresidential fixed investment with overall investment. Which variable had the bigger percentage change?

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Table 1.1.1. Percent Change From Preceding Period in Real Gross Domestic Product

[Percent] Seasonally adjusted at annual rates

Last Revised on: March 28, 2018 - Next Release Date April 27, 2018

Line		1999	1999	1999	1999	2000	2000	2000	2000	2001	2001	2001	2001
		Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
1	Gross domestic product	3.2	3.3	5.1	7.1	1.2	7.8	0.5	2.3	-1.1	2.1	-1.3	1.1
2	Personal consumption expenditures	3.9	6.1	4.6	6.0	6.2	3.9	3.9	3.6	1.7	1.0	1.5	6.3
3	Goods	5.0	9.7	4.8	7.0	6.1	2.1	4.2	3.5	0.9	1.1	2.9	15.2
4	Durable goods	0.9	20.5	9.9	4.9	24.3	-7.2	7.0	2.8	6.7	-0.3	4.7	37.7
5	Nondurable goods	7.6	3.8	1.9	8.4	-3.5	8.2	2.7	3.9	-2.4	1.9	1.9	3.4
6	Services	3.2	4.0	4.5	5.4	6.3	4.9	3.8	3.6	2.1	1.0	0.7	1.6
7	Gross private domestic investment	11.9	0.4	10.4	12.3	-3.6	26.1	-4.9	0.2	-17.2	-1.4	-6.9	-18.2
8	Fixed investment	7.1	9.8	9.5	1.9	11.6	9.3	1.6	0.9	-2.0	-5.2	-3.2	-8.9
9	Nonresidential	9.1	10.9	11.7	1.5	15.0	13.3	4.3	1.1	-3.2	-8.8	-5.1	-11.1
10	Structures	-1.1	-4.1	-0.2	4.2	8.7	17.8	14.8	1.4	-10.8	2.2	3.2	-33.1
11	Equipment	12.3	15.0	14.2	-2.6	19.3	15.6	1.4	-0.8	-3.0	-15.6	-10.0	-1.6
12	Intellectual property products	11.3	16.0	16.6	7.4	12.1	5.8	2.1	4.2	2.9	-4.5	-3.3	-6.0
13	Residential	1.5	6.4	3.0	3.0	2.0	-2.0	-6.1	0.4	1.7	6.3	2.5	-2.5
14	Change in private inventories	---	---	---	---	---	---	---	---	---	---	---	---
15	Net exports of goods and services	---	---	---	---	---	---	---	---	---	---	---	---
16	Exports	-9.4	4.1	12.3	10.5	6.2	12.2	10.2	-3.4	-6.1	-12.6	-18.1	-11.9
17	Goods	-6.4	5.8	10.8	13.5	8.1	12.6	15.5	-5.4	-5.8	-15.0	-20.3	-8.6
18	Services	-17.1	0.0	16.0	3.1	1.7	11.1	-2.9	2.0	-6.8	-5.7	-12.1	-20.2
19	Imports	6.0	12.0	15.8	10.2	17.0	12.9	14.6	-0.2	-6.5	-11.3	-7.8	-5.4
20	Goods	13.0	14.7	16.5	10.1	16.8	12.0	14.4	1.5	-7.1	-14.6	-7.4	-3.2
21	Services	-24.5	-1.3	11.7	10.8	18.7	18.5	15.2	-9.3	-3.2	8.8	-9.9	-16.2
22	Government consumption expenditures and gross investment	1.5	2.3	4.8	6.1	-3.2	5.0	-0.8	1.2	6.1	8.1	-0.3	6.0
23	Federal	-3.7	2.0	7.2	8.1	-14.0	14.8	-6.5	-2.1	9.0	8.1	3.4	3.2
24	National defense	-7.5	4.2	11.2	8.0	-21.8	18.6	-7.6	0.1	5.5	6.9	4.5	2.8
25	Nondefense	3.0	-1.6	0.8	8.3	0.6	8.8	-4.8	-5.5	14.9	10.0	1.8	3.8
26	State and local	4.4	2.4	3.5	5.0	3.1	0.1	2.3	3.0	4.6	8.1	-2.2	7.4
Addendum:													
27	Gross domestic product, current dollars	5.3	4.7	6.7	9.1	4.3	10.2	3.1	4.5	1.4	5.1	0.0	2.3

Investment had a bigger percentage change, and unlike consumption, growth in investment was negative for every quarter in 2000 and 2001, except 2000:II. Overall investment was generally more variable than nonresidential fixed investment in 2000 and 2001. Moreover, nonresidential fixed investment had positive growth during 2000, but negative growth in 2001.

- c. From Table 1.1.2, get the contribution to GDP growth of consumption and investment for 1999 to 2001. Calculate the average of the quarterly contributions for each variable for each year. Now calculate the change in the contribution of each variable for 2000 and 2001 (i.e., subtract the average contribution of consumption in 1999 from the average contribution of consumption in 2000, subtract the average contribution of consumption in 2000 from the average contribution of consumption in 2001, and do the same for investment in both years.) Which variable had the largest decrease in its contribution to growth? What do you think was the proximate cause of the recession of 2001? (Was it a fall in investment demand or a fall in consumption demand?)

Investment had a substantially larger decline in its contribution to growth in 2000 and 2001. The proximate cause of the recession of 2001 was a fall in investment demand. (due to a fallout from the internet “bubble” in the years 1998-2001, which fueled a large rise in investment spending)

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Table 1.1.2. Contributions to Percent Change in Real Gross Domestic Product

Seasonally adjusted at annual rates

Last Revised on: March 28, 2018 - Next Release Date April 27, 2018

Line		1999	1999	1999	1999	2000	2000	2000	2000	2001	2001	2001	2001
		Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
	Percent change at annual rate:												
1	Gross domestic product	3.2	3.3	5.1	7.1	1.2	7.8	0.5	2.3	-1.1	2.1	-1.3	1.1
	Percentage points at annual rates:												
2	Personal consumption expenditures	2.49	3.88	3.02	3.93	4.01	2.62	2.55	2.35	1.09	0.67	0.96	4.06
3	Goods	1.16	2.21	1.14	1.66	1.43	0.54	0.98	0.82	0.21	0.25	0.67	3.38
4	Durable goods	0.08	1.66	0.86	0.44	1.96	-0.66	0.59	0.24	0.57	-0.03	0.39	2.88
5	Nondurable goods	1.08	0.55	0.28	1.22	-0.53	1.20	0.39	0.58	-0.36	0.28	0.28	0.50
6	Services	1.34	1.67	1.88	2.27	2.58	2.08	1.57	1.53	0.88	0.42	0.29	0.69
7	Gross private domestic investment	2.16	0.05	1.94	2.33	-0.71	4.72	-1.01	0.06	-3.60	-0.24	-1.29	-3.54
8	Fixed investment	1.25	1.73	1.72	0.36	2.10	1.75	0.30	0.15	-0.43	-1.02	-0.60	-1.68
9	Nonresidential	1.17	1.44	1.57	0.22	2.00	1.85	0.60	0.13	-0.51	-1.31	-0.72	-1.55
10	Structures	-0.03	-0.12	0.00	0.13	0.25	0.50	0.43	0.04	-0.36	0.07	0.10	-1.23
11	Equipment	0.81	1.01	0.99	-0.19	1.31	1.12	0.09	-0.08	-0.26	-1.20	-0.69	-0.08
12	Intellectual property products	0.39	0.55	0.59	0.28	0.45	0.23	0.08	0.17	0.12	-0.18	-0.13	-0.24
13	Residential	0.07	0.30	0.15	0.15	0.10	-0.10	-0.29	0.02	0.08	0.29	0.12	-0.13
14	Change in private inventories	0.91	-1.68	0.22	1.97	-2.81	2.97	-1.31	-0.08	-3.17	0.77	-0.69	-1.86
15	Net exports of goods and services	-1.69	-1.00	-0.69	-0.23	-1.54	-0.47	-0.92	-0.34	0.31	0.28	-0.88	-0.48
16	Exports	-0.99	0.42	1.21	1.06	0.64	1.25	1.05	-0.37	-0.65	-1.37	-1.92	-1.17
17	Goods	-0.50	0.40	0.76	0.96	0.59	0.94	1.13	-0.43	-0.45	-1.20	-1.58	-0.60
18	Services	-0.50	0.01	0.45	0.10	0.05	0.31	-0.09	0.06	-0.20	-0.16	-0.35	-0.58
19	Imports	-0.69	-1.42	-1.90	-1.29	-2.18	-1.72	-1.97	0.02	0.96	1.64	1.05	0.69
20	Goods	-1.23	-1.42	-1.66	-1.08	-1.82	-1.36	-1.66	-0.19	0.88	1.82	0.83	0.33
21	Services	0.53	0.00	-0.24	-0.22	-0.36	-0.36	-0.31	0.21	0.07	-0.18	0.22	0.36
22	Government consumption expenditures and gross investment	0.27	0.41	0.86	1.09	-0.59	0.90	-0.15	0.23	1.07	1.43	-0.05	1.07
23	Federal	-0.23	0.13	0.45	0.52	-0.95	0.88	-0.42	-0.13	0.53	0.49	0.21	0.20
24	National defense	-0.31	0.17	0.43	0.33	-0.96	0.67	-0.30	0.00	0.20	0.26	0.17	0.11
25	Nondefense	0.07	-0.04	0.02	0.19	0.01	0.20	-0.11	-0.13	0.33	0.23	0.04	0.09
26	State and local	0.50	0.27	0.40	0.57	0.36	0.02	0.27	0.35	0.54	0.95	-0.26	0.88

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Table 1.1.2. Contributions to Percent Change in Real Gross Domestic Product

Seasonally adjusted at annual rates

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Line		2002	2002	2002	2002
		Q1	Q2	Q3	Q4
	Percent change at annual rate:				
1	Gross domestic product	3.7	2.2	2.0	0.3
	Percentage points at annual rates:				
2	Personal consumption expenditures	0.82	1.37	1.88	1.44
3	Goods	-0.24	0.38	1.19	0.26
4	Durable goods	-0.46	0.35	1.05	-0.50
5	Nondurable goods	0.22	0.03	0.14	0.76
6	Services	1.06	0.99	0.69	1.18
7	Gross private domestic investment	2.50	0.73	0.00	-0.14
8	Fixed investment	-0.38	-0.17	-0.25	-0.27
9	Nonresidential	-1.02	-0.66	-0.31	-0.64
10	Structures	-0.62	-0.60	-0.49	-0.14
11	Equipment	-0.39	-0.27	0.03	-0.42
12	Intellectual property products	-0.01	0.21	0.15	-0.07
13	Residential	0.64	0.49	0.06	0.37
14	Change in private inventories	2.89	0.90	0.25	0.13
15	Net exports of goods and services	-0.69	-0.60	-0.51	-1.60
16	Exports	0.72	0.98	0.26	-0.50
17	Goods	-0.05	0.89	0.17	-0.61
18	Services	0.77	0.09	0.09	0.11
19	Imports	-1.42	-1.58	-0.77	-1.10
20	Goods	-0.78	-1.79	-0.79	-0.62
21	Services	-0.63	0.21	0.02	-0.48
22	Government consumption expenditures and gross investment	1.11	0.73	0.59	0.55
23	Federal	0.64	0.62	0.42	0.50
24	National defense	0.35	0.38	0.28	0.35
25	Nondefense	0.28	0.24	0.14	0.15
26	State and local	0.47	0.11	0.17	0.05

	1999	2000	2001	2002
GDP (% change, 4Q avg)	4.7	3.0	0.2	2.1
annual change		-1.7	-2.8	1.9
Consumption	3.33	2.88	1.70	1.38
annual change		-0.45	-1.19	-0.32
Investment	1.62	0.77	-2.17	0.77
annual change		0.86	-2.93	2.94
Net Exports	-0.90	-0.82	-0.19	-0.85
annual change		0.09	0.63	-0.66
Government Spending	0.66	0.10	0.88	0.75
annual change		-0.56	0.78	-0.14

- d. Now look at what happened to consumption and investment after the events of September 11th in the third and fourth quarters of 2001 and in the first two quarters of 2002. Does the drop in investment at the end of 2001 make

sense to you? How long did this drop in investment last? What happened to consumption about this time? How do you explain, in particular, the change in consumption in the fourth quarter of 2001? Did the events of September 11, 2001 cause the recession of 2001? Use the discussion in the chapter and your own intuition as guides in answers these questions.

Investment fell in the last two quarters of 2001, but began growing again in the first quarter of 2001. Consumption growth was slow for the first three quarters of 2001, but grew rapidly in the fourth quarter. As mentioned in the text, the Fed reduced the federal funds rate several times during the fourth quarter of 2001. Moreover, automobile manufacturers offered large discounts. These actions may have helped to generate strong consumer spending. In any event, it is clear that the events of September 11 did not cause the recession of 2001. The recession had started well before these events.

☆ Problem 5, Chapter 5

Consider the following numerical example of the IS/LM model:

$$C = 200 + 0.25Y_D$$

$$I = 150 + 0.25Y - 1000i$$

$$G = 250$$

$$T = 200$$

$$\bar{i} = .05$$

- a. **Derive the IS relation. (Hint: You want an equation with Y on the left side and everything else on the right.)**

As we saw above, equilibrium output (when i is an unknown variable) will be:

$$Y_{equil} = \left(\frac{1}{1 - c_1 - b_1} \right) (c_0 - c_1T + b_0 - b_2i + G)$$

and plugging in the numerical values from the equation above, we have:

$$\begin{aligned} Y(i) &= \left(\frac{1}{1 - 0.25 - 0.25} \right) (200 - 0.25(200) + 150 - 1000i + 250) \\ &= 1100 - 2000i \end{aligned}$$

And then plugging in the value of the interest rate set by the central bank, $i = \bar{i} = 0.05$, gives us:

$$Y_{equil} = 1100 - 2000(0.05) = 1000$$

- b. **The central bank sets an interest rate of 5%. How is that decision represented in the equations?**

The central bank's choice of the interest rate is shown by $\bar{i} = 0.05$. In the IS relation, this only enters through the fact that we now pin down a value of the interest rate. This shows that that effect of monetary policy on output flows through the response to the lower interest rate.

- c. **What is the level of the real money supply when the interest rate is 5%? Use the expression:**

$$(M/P) = 2Y - 8000i$$

Plugging in the value of Y we found earlier, together with the interest rate, we have:

$$(M/P)_{equil} = 2Y_{equil} - 8000\bar{i} = 2(1000) - 8000(0.05) = 1600$$

- d. **Solve for the equilibrium value of C and I , and verify the value you obtained for Y by adding C , I , and G .**

As we showed earlier, the equilibrium level of output is 1000. So, plugging in equilibrium output, together with the other exogenous parameters (c_0 , c_1 , b_0 , b_1 , b_2 , \bar{i} , and T), we have:

$$C_{equil} = 400 = 200 + 0.25(1000 - 200) = c_0 + c_1(Y_{equil} - T)$$

$$I_{equil} = 350 = 150 + 0.25(1000) - 1000(0.05) = b_0 + b_1Y_{equil} + b_2\bar{i}$$

In addition, we can double check our solution by confirming that the GDP accounting identity is satisfied at our equations.

$$Y_{equil} = C_{equil} + I_{equil} + G$$

$$1000 = 400 + 350 + 250$$

- e. **Now suppose that the central bank cuts the interest rate to 3%. How does this change the LM curve? Solve for Y , I , and C , and describe in words the effects of an expansionary monetary policy. What is the new equilibrium value of M/P supply?**

We've defined the LM curve as the equation $i = \bar{i}$. In our graph of the IS/LM model, where i is on the vertical axis and Y is on the horizontal axis, the LM curve is just a horizontal line at $i = \bar{i}$. Thus, when we lower the interest rate target, this is shown as a downward shift of the LM curve.

The shift in the LM curve represents the effect of the central bank expanding the money supply. At first, when the central bank increases the money supply, this lowers the interest rate and stimulates investment. This initial increase in output is then fed through the multiplier mechanism, further raising output through both consumption and investment. Note that this higher spending also increase liquidity demand. However, the central bank keeps moving the money supply until we reach a new equilibrium where $i = \bar{i}$ and $(M/P)^{supply} = (M/P)^{demand} = d_1Y(\bar{i}) - d_2\bar{i}$.

Using the IS equation we found earlier, together with the behavior assumptions, we have:

$$Y_{equil} = 1040 = 1100 - 2000(0.03)$$

$$I_{equil} = 380 = 150 + 0.25(1040) - 1000(0.03)$$

$$C_{equil} = 410 = 200 + 0.25(1040 - 200)$$

$$(M/P)_{equil} = 1840 = 2(1040) - 8000(0.03)$$

- f. **Return to the initial situation in which the interest rate set by the central bank is 5%. Now suppose that government spending increases to $G = 400$. Summarize the effects of an expansionary fiscal policy on Y , I , and C . What is the effect of the expansionary fiscal policy on the real money supply?**

Returning to the IS relationship, changing government spending will cause an outward (rightward) shift of the IS curve. This shows up in our equations as a larger constant in the IS curve. Specifically, since the multiplier is $\left(\frac{1}{1-0.25-0.25}\right) = 2$, the change in output will be $2\Delta G = 2(400 - 250) = 300$. This gives us a new IS curve of:

$$Y(i) = 1400 - 2000i$$

and then plugging in the interest rate target \bar{i} we have the new values of the endogenous variables (Y , I , C , and M/P):

$$Y_{equil} = 1300 = 1400 - 2000(0.05)$$

$$I_{equil} = 425 = 150 + 0.25(1300) - 1000(0.05)$$

$$C_{equil} = 475 = 200 + 0.25(1300 - 200)$$

$$(M/P)_{equil} = 2200 = 2(1300) - 8000(0.05)$$