

# Problem Set 4

UCLA - Econ 102 - Fall 2018

*François Geerolf*

# Problem Set 4 {#pset4}

## 3.1 The Solow Growth Model with Exogenous Growth

Consider the Solow growth model of Lecture 2, with however two small changes. Assume that the production function is given by:

$$F(K_t, L_t) = A_t K_t^\alpha L_t^{1-\alpha},$$

where productivity  $A_t$  grows exogenously at rate  $g$  and  $A_0 = 1$ :

$$A_t = (1 + g)^t.$$

Moreover, assume that the labor force also grows at a rate  $n$  and  $L_0 = 1$ , so that at any time  $t$ :

$$L_t = (1 + n)^t.$$

1. Write the law of motion for capital  $K_t$ .

2. Define  $k_t$  as:

$$k_t \equiv \frac{K_t}{A_t^{1/(1-\alpha)} L_t},$$

and write a law of motion for  $k_t$ . Assume that  $n$ , and  $g$  are small in order to simplify this law of motion.

*Hint:* if  $n$  and  $g$  are small then:  $(1 + g)^{1/(1-\alpha)}(1 + n) \approx 1 + \frac{1}{1-\alpha}g + n$ .

3. Show that  $k_t$  converges to a steady-state  $k^*$ . Compute  $k^*$ .

4. When  $k_t$  has reached a steady-state, the economy is said to be on a **balanced growth path**. On this balanced growth path, what is the rate of growth of  $Y_t$ ,  $C_t$ ,  $K_t$ ,  $K_t/Y_t$ ,  $K_t/L_t$ ,  $w_t$ ,  $w_t L_t$  and  $w_t L_t/Y_t$ ? Denoting by  $R_t$  the marginal product of capital, what is the rate of growth of  $R_t$ ,  $R_t K_t$ , and  $R_t K_t/Y_t$ ?

5. Compute  $y^*$  and  $c^*$  corresponding to steady-state  $k^*$  with:

$$y_t \equiv \frac{Y_t}{A_t^{1/(1-\alpha)} L_t} \quad \text{and} \quad c_t \equiv \frac{C_t}{A_t^{1/(1-\alpha)} L_t}.$$

6. What is the saving rate which maximizes  $c^*$ ? (Golden Rule level of capital accumulation)

7. What is then the value of the marginal product of capital  $R^*$ ?

## 3.2 The Neoclassical Labor Market Model

Consider the neoclassical labor market model of lecture 6. Assume that preferences and the production function are as in lecture 6:

$$U(c, l) = c - B \frac{l^{1+\epsilon}}{1+\epsilon}, \quad f(l) = A l^{1-\alpha}.$$

Denote the wage by  $w$ , and the price of consumption by  $p$ .

1. Derive the Labor Demand curve.

2. Assume that  $\alpha = 1/3$  and  $A = 2$ . Using your favorite spreadsheet software, plot this demand curve in a  $(l, w/p)$  plane - that is, putting  $l$  on the x-axis and  $w/p$  on the y-axis.
3. Take logs of both sides. What does the demand curve look like in a  $(\log(l), \log(w/p))$  plane? What is the slope of the demand curve equal to? If  $\alpha$  is higher, is the demand curve steeper or flatter? What shifts the demand curve to the left or to the right?
4. Derive the Labor Supply curve.
5. Assume that  $\epsilon = 5$  and  $B = 2$ . Using your favorite spreadsheet software, plot this supply curve in a  $(l, w/p)$  plane - that is, putting  $l$  on the x-axis and  $w/p$  on the y-axis. Add the supply curve to the demand curve of question 2.
6. Take logs of both sides. What does the supply curve look like in a  $(\log(l), \log(w/p))$  plane? What is the slope of the supply curve equal to? If  $\epsilon$  is higher, is the supply curve steeper or flatter? What shifts the supply curve to the left, or to the right?
7. Assume that productivity  $A$  decreases by 5%, to  $A = 1.9$ . What is the effect on the quantity of employment, and on the real wage? If  $\alpha$  is higher, is that effect larger or smaller? What is the economic intuition?
8. Assume that leisure becomes relatively more attractive relative to working (think of Facebook, Netflix, etc.), so that  $B$  increases by 10% (the disutility of work increases). What is the effect on the quantity of employment, and on the real wage? If  $\epsilon$  is higher, is that effect larger or smaller? What is the economic intuition for this?

### 3.3 The “Keynesian” Labor Market Model

Consider the neoclassical labor market model of the previous problem.

1. Assume that productivity  $A$  decreases by 5%, but that real wages  $w/p$  are rigid. Compute the change in the quantity of employment following a fall in productivity.
2. Compare the effect with question 7 in the previous problem. Explain.
3. Assume that leisure becomes relatively more attractive relative to working, so that  $B$  increases by 10%. Compute the change in the quantity of employment following a increase in leisure attractiveness.
4. Compare the effect with question 8 in the previous problem. Explain.

### 3.4 The Bathtub model

Consider the bathtub model of lecture 6. Assume a monthly job separation rate equal to  $s = 1\%$ , and a monthly job finding rate equal to  $f = 20\%$ . Assume that the labor force is given by  $L = 159$  million.

1. Derive the steady-state unemployment rate. How many people are unemployed in the steady-state? How many people lose their jobs every month? How many people find a job every month?
2. Assume that the economy starts with an unemployment rate equal to  $u_0 = 8\%$ . Using your favorite spreadsheet software, show the evolution of the unemployment rate over time. How long before the unemployment rate reaches 5%?
3. If  $s = 2\%$  instead, which job finding rate  $f$  gives the same steady-state unemployment rate?
4. Assuming the separation rate and the job finding rate are given from question 3, answer question 2 again.
5. Explain why an economy with more churning (that is, faster reallocation) - think of the US versus Europe - has a faster recovery in terms of unemployment after a recession. *Note:* A recession could be

coming from a temporary increase in the job separation rate, or a temporary decrease in the job finding rate, which then goes back to its original value.