

# Lecture 4 - The Overlapping Generations Model

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## 4 The Overlapping Generations Model

In the Solow growth model of lecture ??, we assumed that saving was a constant fraction  $s$  of GDP. Lecture ??, in turn, has shown how to use basic microeconomic principles, and in particular optimization and market clearing, in order to derive saving behavior endogenously. In this lecture, we present a model that combines the two previous lectures, in order to “endogenize” the saving rate in the Solow model. In this model, the reason why people save is that they want to provide for their old age, when they are retired.<sup>1</sup>

More specifically, we develop a simplified version of Peter Diamond’s so-called **overlapping-generations model**.<sup>2</sup> In this model, people save for life-cycle reasons, in order to provide for retirement in their old age. This model is based on Maurice Allais’ as well as on Paul Samuelson’s. This model is used not just to give microfoundations to Robert Solow’s growth model and endogenize the saving rate  $s$ , but also to think about social security, public debt. This task will be taken up in the course of lecture ??.

### 4.1 Assumptions

#### 4.1.1 Time

We assume that people in this economy live only for 2 periods: People are called “young” in the first period of their life, and “old” in the second. Thus, you should really think that the length of a period is a generation (approximately 30 years). However, instead of referring to these two periods as 0 and 1, I shall refer to them as  $t$  and  $t + 1$ .

#### 4.1.2 Demographics

People from generation  $t$  are young in period  $t$ , and old in period  $t + 1$ . We denote their consumption when young by  $c_t^y$  and their consumption when old by  $c_{t+1}^o$ . In terms of lecture ??, you should really think of  $c_t^y$  as  $c_0$ , and of  $c_{t+1}^o$  as  $c_1$ .

People work when young, and then receive a wage given by  $w_t$ . They retire when old, and then do not work. Their lifetime utility is logarithmic with  $\beta = 1$ :

$$U = \log(c_t^y) + \log(c_{t+1}^o).$$

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<sup>1</sup>This is an important reason for saving; although it is probably not the primary one, as we discuss in section 4.3.

<sup>2</sup>This model was published in an article titled “National Debt in a Neoclassical Growth Model” in *The American Economic Review* in 1965: <https://www.jstor.org/stable/1809231>. Needless to say, you are not responsible for reading the original paper in the exam!

Their intertemporal budget constraint is given by:

$$c_t^y + \frac{c_{t+1}^o}{1+r} = w_t.$$

There are always two generations living in period  $t$ : the previous period's young, born in period  $t-1$ , now old, consuming the return from their savings; and this period's young, newly born (in period  $t$ ).

### 4.1.3 Production

For simplicity, we shall assume a Cobb-Douglas, constant returns to scale, production function:

$$Y_t = K_t^\alpha L_t^{1-\alpha}.$$

We assume that the labor force is constant and fixed to unity (this is to avoid carrying  $L$  around everywhere - from lecture ??, you should now know that everything can be expressed per capita, because of constant returns to scale), and therefore:

$$L_t = L = 1.$$

Again for simplicity, we shall assume that capital depreciates at rate  $\delta = 1 = 100\%$ . (that is, capital fully depreciates each period - this is not that unreasonable if you take one unit of time to represent one generation, or about 30 years - remember that the depreciation rate for one year was approximately equal to 2% to 30% depending on the type of capital involved.)

## 4.2 Solution

### 4.2.1 Saving

Utility is logarithmic, so that the consumption of the young  $c_t^y$  and consumption of the old  $c_{t+1}^o$  are given as a function of the wage as follows (this is just an application of lecture ??):

$$c_t^y = \frac{w_t}{2} \quad c_{t+1}^o = (1+r)\frac{w_t}{2}.$$

Indeed, if you want to think of this model as the two periods model of lecture ??, think that everything is as if:

$$f_0 = 0, \quad y_0 = w_t, \quad y_1 = 0.$$

### 4.2.2 Capital accumulation

Saving (and savings) is equal to investment, and therefore we have that:

$$S_t = I_t = w_t - c_t^y = \frac{w_t}{2}.$$

The major difference with the Solow model is that saving is here endogenous, and coming from agents' optimizing choices. In the Solow model in contrast, saving was taken as exogenous and equal to a fraction  $s$ .

The wage paid by employers, given that  $L = 1$ , is:

$$w_t = (1-\alpha)K_t^\alpha L^{-\alpha} = (1-\alpha)K_t^\alpha = (1-\alpha)Y_t.$$

Finally:

$$\Delta K_{t+1} = \frac{w_t}{2} - \delta K_t = \frac{1-\alpha}{2} Y_t - \delta K_t.$$

This is the capital accumulation equation (or law of motion for capital) of the Solow model, with  $s = (1-\alpha)/2$ . The new element here of course is to get saving endogenously, from agents' optimal decisions. Note that the value for the saving rate has an economic interpretation: wages are only a fraction  $1-\alpha$  of output, from lecture ???. On the other hand, savers / consumers want to smooth consumption and therefore want to save a half of that. This is why a fraction  $(1-\alpha)/2$  of output is saved.

### 4.2.3 Numerical Application

Note that if  $\alpha = 1/3$ , then the saving rate is equal to  $s = 1/3$ , which happens to be (by coincidence) the Golden Rule level of saving. This does not mean that the Golden Rule level is always satisfied. This only happens by chance in this very stylized model. In particular, saving is not just because of retirement, but also because of precautionary behavior, leaving bequests or simply liking being wealthy. We will come back to these issues in future lectures, but we can look at some data on who owns wealth and how it is divided first, before we move to that.

## 4.3 Why do people save?

In Peter Diamond's overlapping generations model, saving behavior only has one source: planning for retirement. Reality is a bit more nuanced. This section provides data which is suggestive that much of the wealth does not in fact come from young workers saving to provide for their old age. Thus, the overlapping generations model, in which most saving is lifecycle saving, does not capture an important part of the motive to save. We propose other factors at the end of this note.

### 4.3.1 Some data

Figure 1 from Emmanuel Saez and Gabriel Zucman, two economists at the University of California, Berkeley, working on the world distribution of wealth, shows the composition of aggregate US household wealth from 1913 to 2013.<sup>3</sup> The US tax code includes provisions which strongly encourage retirement saving in the form of retirement accounts. However, houses are also clearly a potential source of revenue for older people – because of the flow of rents that owner-occupied housing provides, but also because there is always an option to liquidate one's house when old.

Figure 2 shows the saving rate by wealth class, which echoes the evidence on saving rate by income shown previously in lecture ??.

Figure 3 shows the top 10% wealth share. As you can see, nearly 75% of household wealth is held by the top 10% wealth owners. This is more concentrated than labor income (the top 10% in the United States gets about 50% of pre-tax income, and much less after-tax), and therefore does not appear to be solely accounted for by saving for retirement.

Figure 4 shows the top 1% wealth share, and the top 1-10% wealth share. As you can see, the top 1% now owns nearly 40% of the wealth in the United States, while it only accounts for about 20-25% of pre-tax income. Again, it does not seem like saving for retirement is the whole story.

<sup>3</sup>These pictures are taken from work by Emmanuel Saez and Gabriel Zucman, "Wealth Inequality in the United States since 1913: Evidence from Capitalized Income Tax Data," published in 2016 in *The Quarterly Journal of Economics*. The paper is available here: <https://doi.org/10.1093/qje/qjw004>.



Figure 1: AGGREGATE US HOUSEHOLD WEALTH, 1913–2013. SOURCE: SAEZ, ZUCMAN (2016).

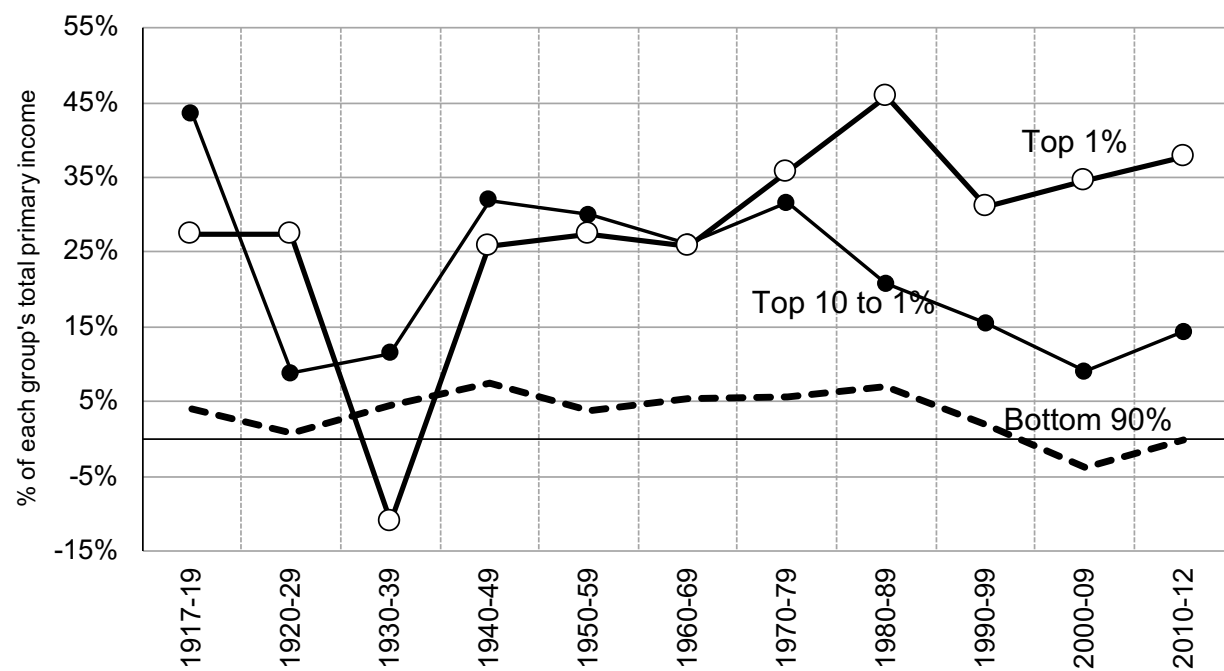


Figure 2: SAVING RATE BY WEALTH CLASS (DECENNIAL AVERAGE). SOURCE: SAEZ, ZUCMAN (2016).



Figure 3: TOP 10% WEALTH SHARE. SOURCE: SAEZ, ZUCMAN (2016).



Figure 4: TOP 1-10% AND TOP 1% WEALTH SHARE. SOURCE: SAEZ, ZUCMAN (2016).

### 4.3.2 Saving of the rich

Understanding the sources of the capital stock amounts to a first order to understand the saving of people with very high net worth. What leads high income and high net worth people to save so much? A number of explanations have been proposed:

1. **Leaving bequests.** One reason why people might want to save over and above what they need to provide for retirement, is to leave bequests. However, it has been shown that even high income workers without children save a lot, more than warranted by their retirement needs.
2. **Prestige.** Wealth brings prestige. Adam Smith has a great passage in *The Theory of Moral Sentiments*, published in 1759:

To what purpose is all the toil and bustle of the world?... It is our vanity which urges us on... It is not wealth that men desire, but the consideration and good opinion that wait upon riches.

3. **Concern for relative wealth.** Related to this explanation is a concern not for the absolute level of wealth per se, but for a relative standing compared to others in society. This is for example echoed in an academic article, published in 1992:<sup>4</sup>

But think for a moment about an already very rich agent such as Donald Trump. Why does he continue to work long days, endure substantial amounts of stress, and take enormous risks? Surely it cannot be that he is savoring the prospect of going to the grocery store with a looser budget constraint next year. He seems to have more money than he could spend in several lifetimes. Even if we are wrong about Trump's net worth, there clearly seem to be wealthy individuals that continue to work very hard and take large risks to increase their net worth. It is hard to reconcile such behavior with the underlying decision making in traditional growth models. We propose that people like Trump continue to care about increasing their net worth because their utility depends not only on the absolute level of their wealth but also on their wealth relative to that of other very rich people.

4. **Religious beliefs and work ethic.** Max Weber has famously proposed the protestant work ethic in *The Protestant Ethic and the Spirit of Capitalism* as one explanation for the emergence of capitalism, and the importance of hard work and saving. John Maynard Keynes, in *The Economic Consequences of the Peace* published in 1919, was thinking very much in these terms:

Europe was so organised socially and economically as to secure the maximum accumulation of capital. While there was some continuous improvement in the daily conditions of life of the mass of the population, society was so framed as to throw a great part of the increased income into the control of the class least likely to consume it. The new rich of the nineteenth century were not brought up to large expenditures, and preferred the power which investment gave them to the pleasures of immediate consumption. In fact, it was precisely the inequality of the distribution of wealth which made possible those vast accumulations of fixed wealth and of capital improvements which distinguished that age from all others. Herein lay, in fact, the main justification of the capitalist system. If the rich had spent their new wealth on their own enjoyments, the world would long ago have found such a régime intolerable. But like bees they saved and accumulated, not less to the advantage of the whole community because they themselves held narrower ends in prospect.

The immense accumulations of fixed capital which, to the great benefit of mankind, were built up during the half century before the war, could never have come about in a society where wealth was divided equitably. The railways of the world, which that age built as a monument to posterity, were, not less than the pyramids of Egypt, the work of labour which was not free to consume in immediate enjoyment the full equivalent of its efforts.

<sup>4</sup>Harold L. Cole, George J. Mailath, and Andrew Postlewaite, "Social Norms, Savings Behavior, and Growth," *Journal of Political Economy* 100, no. 6 (December 1, 1992): 1092–1125, <https://doi.org/10.1086/261855>.

Thus this remarkable system depended for its growth on a double bluff or deception. On the one hand the labouring classes accepted from ignorance or powerlessness, or were compelled, persuaded, or cajoled by custom, convention, authority, and the well-established order of society into accepting, a situation in which they could call their own very little of the cake that they and nature and the capitalists were co-operating to produce. And on the other hand the capitalist classes were allowed to call the best part of the cake theirs and were theoretically free to consume it, on the tacit underlying condition that they consumed very little of it in practice. The duty of 'saving' became nine-tenths of virtue and the growth of the cake the object of true religion. There grew round the non-consumption of the cake all those instincts of puritanism which in other ages has withdrawn itself from the world and has neglected the arts of production as well as those of enjoyment. And so the cake increased; but to what end was not clearly contemplated. Individuals would be exhorted not so much to abstain as to defer, and to cultivate the pleasures of security and anticipation. Saving was for old age or for your children; but this was only in theory – the virtue of the cake was that it was never to be consumed, neither by you nor by your children after you.

5. **A final hypothesis.** An even more mundane explanation (which does not make it wrong!) has been proposed by Lee Iacocca, former CEO from Chrysler. According to him, the rich simply do not know what to do with their money:

Once you reach a certain level in a material way, what more can you do? You can't eat more than three meals a day; you'll kill yourself. You can't wear two suits one over the other. You might now have three cars in your garage-but six! Oh, you can indulge yourself, but only to a point.

Most economists are however general skeptical of this type of explanations. What they find puzzling is that high net worth individuals keep working even when they have achieved a sufficient amount of wealth.<sup>5</sup>

All this discussion may seem like armchair theorizing. At the same time, these are probably the most important questions facing macroeconomics. They actually determine the stance that should be taken on optimal capital accumulation, the optimal level of public debt, etc. We shall come back to these issues repeatedly in the following lectures.

## Readings - To go further

Emmanuel Saez, Gabriel Zucman, "Exploding wealth inequality in the United States", *Vox EU*, October 29, 2014.

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<sup>5</sup>I personally am less sure, as culture and norms certainly play a bigger role than many economists imagine - for example, as explained above, the protestant work ethic, might be one answer.