Week 2 - Recommended Problems
Solutions

UCLA
Intermediate Macroeconomics, Econ 102

Chapter 11, Problem 6

Suppose that the production function is given by:

$$Y = \frac{1}{2}\sqrt{K}\sqrt{L}$$

a. **Derive the steady-state levels of output per worker and capital per worker in terms of the saving rate, s, and the depreciation rate, δ.**

We write the capital accumulation equation:

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + I_t \\ &= (1-\delta)K_t + sY_t \\ K_{t+1} &= (1-\delta)K_t + \frac{s}{2}\sqrt{K_t}\sqrt{L} \end{aligned}$$

Dividing both sides by population L:

$$\frac{K_{t+1}}{N} = (1-\delta)\frac{K_t}{N} + \frac{s}{2}\sqrt{\frac{K_t}{N}}$$

In steady state, $\frac{K_{t+1}}{N} = \frac{K_t}{N} = \frac{K^*}{N}$, so that we have:

$$\delta\frac{K^*}{N} = \frac{s}{2}\sqrt{\frac{K^*}{N}} \quad \Rightarrow \quad \boxed{\frac{K^*}{N} = \left(\frac{s}{2\delta}\right)^2}$$

Finally:

$$\frac{Y^*}{N} = \frac{1}{2}\sqrt{\frac{K^*}{N}}$$

Thus:

$$\boxed{\frac{Y^*}{N} = \frac{s}{4\delta}}$$

b. **Derive the equation for steady-state output per worker and steady-state consumption per worker in terms of s and δ.**

We already know that:

$$\frac{Y^*}{N} = \frac{s}{4\delta}$$

Therefore:

$$\frac{C^*}{N} = (1-s)\frac{Y^*}{N} \quad \Rightarrow \quad \boxed{\frac{C^*}{N} = \frac{s(1-s)}{4\delta}}$$

c. **Suppose that $\delta = 0.05$. With your favorite spreadsheet software, compute steady-state output per worker and steady-state consumption per worker for $s = 0; s = 0.1; s = 0.2; s = 1$. Explain the intuition behind your results.**

I used Google Sheets, and the result is available here (just click).
The intuition for why steady-state output per worker is a monotone function of the saving rate is that more investment always leads to a higher capital stock, which leads to higher output per worker. However, the effect of saving on steady-state consumption is ambiguous. It should be intuitive that if saving is equal to 0%, or 100%, consumption per worker is zero: in the first case, because there is no capital and therefore no production; in the second case, because everything is saved and there is nothing left to consumption. Thus, there is a limit to how much capital should be accumulated, at least for consumption purposes.

d. **Use your favorite spreadsheet software to graph the steady-state level of output per worker and the steady-state level of consumption per worker as a function of the saving rate (i.e., measure the saving rate on the horizontal axis of your graph and the corresponding values of output per worker and consumption per worker on the vertical axis).**

Again, I used Google Sheets, and the result is available here (just click).

s	Y/N	C/N
0	0	0
0.1	0.5	0.45
0.2	1	0.8
1	5	0

e. **Does the graph show that there is a value of s that maximizes output per worker? Does the graph show that there is a value of $s$that maximizes consumption per worker? If so, what is this value?**

It should be clear from the Google Sheets that $s = 1$ maximizes output per worker. $s = 0.5$ maximizes consumption per worker.

Chapter 11, Problem 7

The Cobb-Douglas production function and the steady state. Suppose that the economy's production function is given by

$$Y = K^{\alpha} N^{1-\alpha}$$

and assume that $\alpha = 1/3$.

a. **Is this production function characterized by constant returns to scale? Explain.**

$$Y(xK, xN) = (xK)^{1/3}(xN)^{2/3} = xK^{1/3}N^{2/3} = xY(K, N)$$

So this production function is characterized by constant returns to scale.

b. **Are there decreasing returns to capital?**

$$Y(xK, N) = (xK)^{1/3}N^{2/3} = x^{1/3}Y(K, N) < xY(K, N)$$

So there are decreasing returns to capital.

c. **Are there decreasing returns to labor?**

$$Y(K, xN) = K^{1/3}(xN)^{2/3} = x^{2/3}Y(K, N) < xY(K, N)$$

So there are decreasing returns to labor.

d. **Transform the production function into a relation between output per worker and capital per worker.**

$$\frac{Y}{N} = \frac{K^\alpha N^{1-\alpha}}{N} = K^\alpha N^{-\alpha} = \left(\frac{K}{N}\right)^\alpha$$

e. **For a given saving rate, s, and depreciation rate, δ, give an expression for capital per worker in the steady state.**

Again, we write the evolution of the capital stock as:

$$K_{t+1} = \quad (1-\delta)K_t + I_t = (1-\delta)K_t + sY_t = (1-\delta)K_t + sK_t^{1/3}N^{2/3}$$

Dividing both sides by N :

$$\frac{K_{t+1}}{N} = (1-\delta)\frac{K_t}{N} + s\left(\frac{K_t}{N}\right)^{1/3}$$

In steady state, $\frac{K_{t+1}}{N} = \frac{K_t}{N} = \frac{K^*}{N}$, so we have

$$\delta\frac{K^*}{N} = s\left(\frac{K^*}{N}\right)^{1/3}$$

Therefore:

$$\boxed{\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^{3/2}}.$$

f. **Give an expression for output per worker in the steady state.**

Using that: $Y^* = K^{*1/3}N^{2/3}$, we have:

$$\frac{Y^*}{N} = \left(\frac{K^*}{N}\right)^{1/3} \quad \Rightarrow \quad \boxed{\frac{Y^*}{N} = \sqrt{\frac{s}{\delta}}}.$$

g. **Solve for the steady-state level of output per worker when $s = 0.32$ and $\delta = 0.08$.**

$$\frac{Y^*}{N} = \sqrt{\frac{s}{\delta}} = \sqrt{\frac{0.32}{0.08}} = 2$$

h. **Suppose that the depreciation rate remains constant at $\delta = 0.08$, while the saving rate is reduced by half, to $s = 0.16$. What is the new steady-state output per worker?**

$$\frac{Y^*}{N} = \sqrt{\frac{s}{\delta}} = \sqrt{\frac{0.16}{0.08}} = \sqrt{2}$$

Chapter 11, Problem 8

Continuing with the logic from Problem 7, suppose that the economy's production function is given by $Y = K^{\alpha}N^{1-\alpha}$ with $\alpha = 1/3$ and that both the saving rate, s, and the depreciation rate, δ are equal to 0.10.

a. **What is the steady-state level of capital per worker?**

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^{3/2} = \left(\frac{0.10}{0.10}\right)^{3/2} = 1$$

b. **What is the steady-state level of output per worker?**

$$\frac{Y^*}{N} = \sqrt{\frac{s}{\delta}} = \sqrt{\frac{0.10}{0.10}} = 1$$

Suppose that the economy is in steady state and that, in period t, the depreciation rate increases permanently from 0.10 to 0.20.

c. **What will be the new steady-state levels of capital per worker and output per worker?**

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^{3/2} = \left(\frac{0.10}{0.20}\right)^{3/2} \approx 0.35$$

$$\frac{Y^*}{N} = \sqrt{\frac{s}{\delta}} = \sqrt{\frac{0.10}{0.20}} \approx 0.71$$

d. **Compute the path of capital per worker and output per worker over the first three periods after the change in the depreciation rate.**

From Problem 7, we know the evolution of capital per worker is

$$\frac{K_{t+1}}{N} = (1-\delta)\frac{K_t}{N} + s\left(\frac{K_t}{N}\right)^{1/3}$$

So starting from $\frac{K_0}{N} = 1$, with $s = 0.10$, $\delta = 0.20$, we have

$$\frac{K_1}{N} = (1-\delta)\frac{K_0}{N} + s\left(\frac{K_0}{N}\right)^{1/3} = 0.8 \times 1 + 0.1 \times 1^{1/3} = 0.9$$

$$\frac{K_2}{N} = (1-\delta)\frac{K_1}{N} + s\left(\frac{K_1}{N}\right)^{1/3} = 0.8 \times 0.9 + 0.1 \times (0.9)^{1/3} \approx 0.82$$

$$\frac{K_3}{N} = (1-\delta)\frac{K_2}{N} + s\left(\frac{K_2}{N}\right)^{1/3} = 0.8 \times 0.82 + 0.1 \times (0.82)^{1/3} \approx 0.75$$

For more iterations, you may use Google Sheets: the result is available here (just click). You should not that it indeed converges to the above values.

From there, we may calculate the path of output per worker:

$$\frac{Y_1}{N} = \left(\frac{K_1}{N}\right)^{1/3} = (0.9)^{1/3} \approx 0.97$$

$$\frac{Y_2}{N} = \left(\frac{K_2}{N}\right)^{1/3} = (0.82)^{1/3} \approx 0.93$$

$$\frac{Y_3}{N} = \left(\frac{K_3}{N}\right)^{1/3} = (0.75)^{1/3} \approx 0.91.$$

For more iterations, you may use Google Sheets: the result is available here (just click).

Chapter 11, Problem 9

Deficits and the capital stock. Suppose that the production function is given by:

$$Y = \sqrt{K}\sqrt{N}$$

a. **Show that the steady-state capital stock per worker is given by:**

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^2$$

and that output per worker is given by:

$$\frac{Y^*}{N} = \frac{s}{\delta}$$

By the evolution of capital stock

$$K_{t+1} = \quad (1-\delta)K_t + I_t = (1-\delta)K_t + sY_t = (1-\delta)K_t + s\sqrt{K_t}\sqrt{N}$$

Dividing both sides by N:

$$\frac{K_{t+1}}{N} = (1-\delta)\frac{K_t}{N} + s\sqrt{\frac{K_t}{N}}$$

In steady state, $\frac{K_{t+1}}{N} = \frac{K_t}{N} = \frac{K^*}{N}$, so we have:

$$\delta \frac{K^*}{N} = s\sqrt{\frac{K^*}{N}}$$

Therefore:

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^2$$

Using the production function $Y^* = \sqrt{K^*}\sqrt{N}$:

$$\frac{Y^*}{N} = \sqrt{\frac{K^*}{N}} = \frac{s}{\delta}.$$

b. **Suppose that the saving rate, s, is initially 15% per year, and the depreciation rate, δ, is 7.5%. What is the steady-state capital stock per worker? What is steady-state output per worker?**

The steady-state capital stock per worker is given by:

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^2 = \left(\frac{15\%}{7.5\%}\right)^2 = 4$$

The steady-state output per worker is given by:

$$\frac{Y^*}{N} = \frac{s}{\delta} = \frac{15\%}{7.5\%} = 2$$

c. **Suppose that there is a government deficit of 5% of GDP and that the government eliminates this deficit. Assume that private saving is unchanged so that total saving increases to 20%. What is the new steady-state capital stock per worker? What is the new steady-state output per worker? How does this compare to your answer to part b?**

The new steady-state capital stock per worker is:

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^2 = \left(\frac{20\%}{7.5\%}\right)^2 \approx 7.11$$

The new steady-state output per worker:

$$\frac{Y^*}{N} = \frac{s}{\delta} = \frac{20\%}{7.5\%} \approx 2.67$$

Therefore, both the capital per worker and the output per worker increase.

Chapter 11, Problem 10

U.S. saving and government deficits. This question continues the logic of Problem 9 to explore the implications of the U.S. government budget deficit for the long-run capital stock. The question assumes that the United States will have a budget deficit over the life of this edition of the text.

a. **TheWorld Bank reports gross domestic saving rate by country and year. The Web site is `http://data.worldbank.org/indicator/NY.GDS.TOTL.ZS`. Find the most recent number for the United States. What is the total saving rate in the United States as a percentage of GDP? Using the depreciation rate and the logic from Problem 9, what would be the steady-state capital stock per worker? What would be steady-state output per worker?**

For 2016, the national saving rate was approximately 16.9%. Steady-state capital stock per worker is

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^2 = \left(\frac{16.9\%}{7.5\%}\right)^2 \approx 5.08$$

The steady-state output per worker is

$$\frac{Y^*}{N} = \frac{s}{\delta} = \frac{16.9\%}{7.5\%} \approx 2.25$$

b. **Go to the most recent Economic Report of the President (ERP) and find the most recent federal deficit as a percentage of GDP. In the 2015 ERP, this is found in Table B-20. Using the reasoning from Problem 9, suppose that the federal budget deficit was eliminated and there was no change in private saving. What would be the effect on the long-run capital stock per worker? What would be the effect on long-run output per worker?**

For fiscal year 2017, the federal fiscal deficit was 3.5% percent of GDP. Assuming that the federal budget deficit was eliminated and there was no change in private saving, the saving rate would change from 16.9% to $16.9\% + 3.5\% = 20.4\%$.

The new steady-state capital sock per worker is:

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^2 = \left(\frac{20.4\%}{7.5\%}\right)^2 \approx 7.40$$

which increases by 45.67%.
The new steady-state output per worker is:

$$\frac{Y}{N} = \frac{s}{\delta} = \frac{20.4\%}{7.5\%} = 2.72$$

which increases by 20.89%.

c. **Return to the World Bank table of gross domestic saving rates. How does the saving rate in China compare to the saving rate in the United States?**

The saving rate in China was 46.54% in the year of 2016, which is much higher than the saving rate in the United States.