# TA Notes - Week 1
# Math Review

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## Contents



## 1 Derivative Rules (Single Variable Functions)

Here we go through a number of important rules for derivatives, and give examples of how to apply these rules. We focus on defining the rules for functions of one variable. When we write derivatives with respect to one variable, we will write this as $\frac{df(x)}{dx}$ or $\frac{df}{dx}$. You may also have seen this as $\frac{df(x)}{dx} = f'(x)$.

Recall that we can interpret derivatives as the *rate of change* of the function $f(x)$ at a given value of $x$, or the *slope of the line tangent to* $f(x)$. It's also important to remember that the derivative is also itself a function of $x$ (although it may be a boring function, like a constant or just 0).

The rules for taking derivatives of a single variable function are directly applicable to working with a function of multiple variables by using the partial derivative. Mechanically, when you take $f(x, y)$ and you take the partial derivative with respect to $x$ you treat $y$ as if it were constant. For example, think of the following formula describing points on a circle:

$$f(x, y) = x^2 + y^2 = 1$$

When moving on the circle, if we change $x$ we have to change $y$. However, for the purposes of taking a partial derivative we can ignore that linkage between $x$ and $y$. This gives us:

$$\frac{\partial}{\partial x} f(x, y) = f_1(x, y) = 2x \frac{\partial}{\partial y} f(x, y) \qquad = f_2(x, y) = 2y$$

(i) Constant functions

*For errors or corrections, please email me at conor.teaches.econ@gmail.com.

- A function that does not depend on $x$, i.e. $f(x) = c$ where $c$ is some number.
- Rule: $\frac{df(x)}{dx} = \frac{dc}{dx} = 0$
- Example: $f(x) = 5 \rightarrow \frac{df(x)}{dx} = 0$.

(ii) Power Rule

- $f(x) = x^c$ where $c$ is some constant
- Rule: $\frac{df(x)}{dx} = \frac{d[x^c]}{dx} = cx^{c-1}$
- Example 1: $f(x) = x^9 \rightarrow \frac{df(x)}{dx} = 9x^8$
- Example 2: $f(x) = x^{\frac{1}{2}} \rightarrow \frac{df(x)}{dx} = \frac{1}{2}x^{\frac{-1}{2}}$
- Example 3: $f(x) = x^{-2} \rightarrow \frac{df(x)}{dx} = -2x^{-3}$

(iii) Constant times a function

- $f(x) = cg(x)$ where $g(x)$ is any function of $x$.
- Rule: $\frac{df(x)}{dx} = \frac{d[cg(x)]}{dx} = c\frac{dg(x)}{dx}$
- Example 1: $f(x) = 5x \rightarrow \frac{df(x)}{dx} = 5$.
- Example 2: $f(x) = 7x^2 = 7(x^2) \rightarrow \frac{df(x)}{dx} = 7(2x^1) = 14x$

(iv) Linear functions of $x$

- $f(x) = ax + b$ where $a$ and $b$ are constants
- Rule: $\frac{df(x)}{dx} = a$
- Intuition: Combination of Power rule and rule for constants
- Example: $f(x) = 7x + 2 \rightarrow \frac{df(x)}{dx} = 7$

(v) Addition of two functions

- $f(x) = g_1(x) + g_2(x)$ where $g_1(x)$ and $g_2(x)$ are each their own functions of $x$
- Rule: $\frac{df(x)}{dx} = \frac{d[g_1(x)+g_2(x)]}{dx} = \frac{dg_1(x)}{dx} + \frac{dg_2(x)}{dx}$
- Example: $f(x) = 10x + 3x^2 \rightarrow \frac{df(x)}{dx} = 10 + 6x$

(vi) Product Rule

- $f(x) = g_1(x)g_2(x)$
- Rule: $\frac{df(x)}{dx} = \frac{d[g_1(x)g_2(x)]}{dx} = g_1(x)\frac{dg_2(x)}{dx} + g_2(x)\frac{dg_1(x)}{dx}$
- Example 1:

$$f(x) = 5x^2ln(x) = (5x^2)(ln(x)) \qquad \text{Note: } 5x^2 = g_1(x) \text{ and } ln(x) = g_2(x)$$
$$\frac{df(x)}{dx} = (10x)ln(x) + (5x^2)\frac{1}{x} = (10x)ln(x) + 5x = 5x(2ln(x) + 1)$$

- Example 2:

$$f(x) = (5x^2+4)3x^3 \quad \text{Note: } 5x^2+4 = g_1(x) \text{ and } 3x^3 = g_2(x)$$
$$\frac{df(x)}{dx} = (10x)3x^3 + (5x^2+4)9x^2 = 30x^4 + 45x^4 + 36x^2 = 75x^4 + 36x^2$$

(vii) Quotient Rule

- $f(x) = \frac{g_1(x)}{g_2(x)}$
- Rule:

$$\frac{df(x)}{dx} = \frac{d\left[\frac{g_1(x)}{g_2(x)}\right]}{dx} = \frac{g_1(x)\frac{dg_2(x)}{dx} - g_2(x)\frac{dg_1(x)}{dx}}{[g_2(x)]^2}$$

- Trick to remember this: Quotient rule is **"Low dHigh minus High dLow over Low squared"** where Low is the function on the bottom of the division (the denominator, or $g_2(x)$ above) and High is the function on the top of the division (the numerator, or $g_1(x)$ above).
- Example 1: $f(x) = \frac{x^2+2}{x} \rightarrow \frac{df(x)}{dx} = \frac{(x)(2x)-(x^2+2)(1)}{x^2}$
- Example 2: $f(x) = \frac{(3x+2)(x^2+2)}{x^3-3} \rightarrow \frac{df(x)}{dx} = \frac{(x^3-3)[(3x+2)(2x)+(x^2+2)(3)]-(3x+2)(x^2+2)(3x^2)}{(x^3-3)^2}$

(viii) Chain Rule

- $f(x) = g_1(g_2(x))$
- Rule: $\frac{df(x)}{dx} = \frac{d[g_1(g_2(x))]}{dx} = \frac{dg_1(g_2)}{dg_2}\frac{dg_2(x)}{dx}$
- Rule (using prime notation): $g_1'(g_2(x)) \times g_2'(x)$
- Example 1

$$f(x) = (3x^2+2)^3 \quad \text{Note: } g_1(x) = x^3 \text{ and } g_2(x) = 3x^2+2$$
$$\frac{df(x)}{dx} = 3(3x^2+2)^2(6x)$$

- Example 2

$$f(x) = ln(x^3+2x^2+2) \quad \text{Notes: } g_1(x) = ln(x) \text{ and } g_2(x) = x^3+2x^2+2$$
$$\frac{df(x)}{dx} = \frac{1}{x^3+2x^2+2}(3x^2+4x)$$

An important result in calculus is that the derivative of $e^x$ is just $e^x$ itself, where $e$ refers to Euler's number. Using this result together with the chain rule, we can prove the following results about derivatives for exponentiation and logarithms:

- $\frac{d}{dx}b^x = b^x ln(b)$ (e.g. $\frac{d}{dx}5^x = 5^x \ln(5)$ )
- $\frac{d}{dx}log_b(x) = \frac{1}{xln(b)}$ (e.g. $\frac{d}{dx}\log_{10}(35) = \frac{1}{35\ln(10)}$)

In the case of the natural logarithm ln (or $log_e$) the second equation simplifies to just:

- $\frac{d}{dx}\ln(x) = \frac{1}{x}$