Problem Set 8 - Solutions

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8.1 Optimal Taxation: The Supply Side View

1. The firm's problem is to maximize:
$$\max_l \quad p \cdot f(l) - w \cdot l.$$

Given that $f(l) = A \cdot l$, the firm equivalently maximizes:

$$\max_l \quad p \cdot A \cdot l - w \cdot l.$$

One way to get the result is, as usual, to take the first-order condition to this problem:

$$\frac{w}{p} = A.$$

However, I should warn you here that we should actually not be taking a first-order condition. A mathematician would be right in saying that this is wrong: the objective function is linear, and so the above function does not have a maximum ! At the same time, doing so does lead to the "right solution". The real wage equals labor productivity in the end. What is going on here? One intuitive way to see this is to say that for any α, however small, then with $f(l) = Al^{1-\alpha}$ we would get $w/p = A \cdot l^{-\alpha}$ as a first order condition. Taking then the limit of α to 0 then leads to the above result since then $l^{-\alpha} \approx 1$. However, unfortunately, this is not a valid mathematical proof.[1] So how do we think of what happens if the technology is completely linear? How do we think of an equilibrium in that case? The proper economic reasoning is as follows:

- if we had $p \cdot A > w$, then the marginal gain of hiring an additional worker would be higher than the marginal cost of doing so, whatever l. Therefore, firms would like to hire an infinity of workers, and the solution would be $l = +\infty$. This is not a competitive equilibrium, because this level of demand can never be equal to supply.

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[1] There is no theorem in mathematics which says that the limit of the maxima of many functions converges to the maximum of the limit of these functions. I said "unfortunately", but this is fortunate actually, as we just found a counterexample in this exercise: the limit of the functions does not, in fact, have a maximum!

- if we had $p \cdot A < w$, then the marginal gain of hiring an additional worker would be lower than the marginal cost of doing so, whatever l. Therefore, the optimal thing for the firm to maximize its profit would be to hire no worker $l = 0$. This is not a competitive equilibrum either, because this level of demand is not equal to supply either.

  As a consequence:

  $$p \cdot A = w \quad \Rightarrow \quad \boxed{\frac{w}{p} = A}.$$

  The labor demand curve is said to be infinitely elastic.

  Note. You should note that with $w/p = A$, the firms' profit is 0 regardless of what l it chooses. Thus, the firm produces, potentially at a scale consistent with market clearing, but in terms of profit it is indifferent between producing or not.

2. Writing that $c = c_0 + (1 - \tau) \cdot (w/p) \cdot l$ and plugging the value of c into the worker's optimization problem:

  $$\max_{l} \quad c_0 + (1 - \tau)\frac{w}{p}l - B\frac{l^{1+\epsilon}}{1 + \epsilon}$$

  The first-order condition (which we can take here as long as $\epsilon > 0$) implies:

  $$(1 - \tau) \cdot \frac{w}{p} = B \cdot l^\epsilon \quad \Rightarrow \quad \boxed{l = \frac{(1 - \tau)^{1/\epsilon}}{B^{1/\epsilon}}\left(\frac{w}{p}\right)^{1/\epsilon}}$$

3. The number of hours worked is given by replacing out the real wage w/p from the labor demand equation to the labor supply equation, so that:

  $$\boxed{l = (1 - \tau)^{1/\epsilon}\frac{A^{1/\epsilon}}{B^{1/\epsilon}}}.$$

  Real pre-tax income is when simply given by the real wage times the number of hours. Since the real wage is simply A, we get:

  $$y = \frac{w}{p} \cdot l = A \cdot (1 - \tau)^{1/\epsilon}\frac{A^{1/\epsilon}}{B^{1/\epsilon}}.$$

  Finally:

  $$\boxed{y = (1 - \tau)^{1/\epsilon}\frac{A^{1/\epsilon+1}}{B^{1/\epsilon}}}.$$

4. A numerical application shown on the Google Spreadsheet implies that:

  $$\underline{l} = (1 - \underline{\tau})^{1/\epsilon}\frac{\underline{A}^{1/\epsilon}}{\underline{B}^{1/\epsilon}}$$

  Therefore:

  $$\underline{l} = \left(1 - \frac{1}{4}\right)^{1/2}\left(\frac{1000000}{28188}\right)^{1/2}\left(\frac{491569855488}{3000000}\right)^{1/2}$$

  $$\underline{l} = 2088.$$

  Note that under the assumption of 281 business days in a year, this implies **8 hours a day** of work.

  Note: Of course, this did not happen by random chance. This explains the weird-looking numbers for A and B in the examples. I reversed engineered A and B so that people in the model would happen to

work a realistic number of hours.[2] According to the Google Spreadsheet, income is then given by:

$$\underline{y} = \underline{Al}$$
$$= \frac{1000000}{28188} \cdot 2088$$
$$\underline{y} \approx 74074.07$$

Again, I reversed engineered the examples so that you would get the income of low-income people in lecture **??**.

5. We note that this group has a lower disutility of working B (which could again, provy for many things: education, religion, work ethic, etc.) - and also a higher productivity per hour A. Therefore they work more - both because they are more productive and because they have a lower disutility of working:

$$\bar{l} = \left(1 - \frac{1}{2}\right)^{1/2} \left(\frac{1000000}{6264}\right)^{1/2} \left(\frac{218475491328}{1000000}\right)^{1/2}$$
$$\bar{l} = 4176.$$

Note that under the assumption of 281 business days in a year, this implies 16 hours a day of work on average (more likely on weekends, or during Thanksgiving!). Income is then given by:

$$\bar{y} = \bar{A}\bar{l}$$
$$= \frac{1000000}{6264} \cdot 4176$$
$$\bar{y} \approx 666666.66$$

6. Total output is 20 trillion, 10 trillion coming from the bottom 90% and 10 trillion coming from the top 10%. In million dollars, output of the bottom 90% is:

$$\underline{Y} = \lambda N \underline{y}$$
$$= 0.9 \cdot 150 \cdot 74074.07$$
$$= 10,000,000 \text{ million}$$
$$\underline{Y} = 10 \text{ trillion}$$

In million dollars, output of the top 10% is:

$$\bar{Y} = (1 - \lambda) N \bar{y}$$
$$= 0.1 \cdot 150 \cdot 666666.66$$
$$= 10,000,000 \text{ million}$$
$$\bar{Y} = 10 \text{ trillion}$$

7. According to the Google Spreadsheet, an individual earning $\underline{y}$ pays taxes given by:

$$\underline{t} = -5000 + 0.25 * (74074.07 - 25000) \approx 7,268$$

which means that this individual pays \$7268 on average per year in taxes. The total taxes paid by this group are:

$$\underline{T} = \lambda N \underline{t} \approx 981 \text{ billion.}$$

According to the Google Spreadsheet, an individual earning $\bar{y}$ pays taxes given by:

$$\bar{t} = -5000 + 0.25 * (200000 - 25000) + 0.5 * (666666 - 200000) \approx 272,083$$

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[2]This is also how economists work in practice. Economists rarely observe preferences directly (outside of a lab where they would run experiments), but only the choices that people make. (which comes from a variety of sources: surveys, administrative data, etc.) Therefore, they rationalize people's choices picking values for the parameters in their models, so that their models are able to match people's behavior.

which means that this individual pays $272,083 on average in taxes per year. The total taxes paid are:

$$\bar{T} = (1 - \lambda)N\bar{t} \approx 4,081 \text{ billion.}$$

The Google Spreadsheet shows that lowering the bottom marginal tax rate leads to an increase in hours worked by the bottom 90%, which increases GDP by about **328 billion**. The cost of the policy is a fall in tax receipts of **266 billion**. Note that the tax cut, if workers did not adjust their working hours, would be larger than that. It would be equal to **331 billion**. But because the tax cut leads people to work more, it in turns leads to higher tax receipts - there is higher GDP. Thus, there are two concepts of multipliers: there is the ex-ante multiplier, calculated with respect to the change in tax receipts if hours did not change, and there is the ex-post multiplier, calculated with respect to the actual change in tax receipts. The ex-post is:

$$-\frac{\Delta Y}{\Delta T} = 1.23.$$

The ex-ante is:

$$-\frac{\Delta Y}{\Delta T} = 0.99.$$

8. The Google Spreadsheet shows that lowering the top marginal tax rate leads to an increase in hours worked by the top 10%, which increases GDP by about **488 billion**. The cost of the policy is a fall in tax receipts of **130 billion**. Again, note that the tax cut, if workers did not adjust their working hours, would be larger than that. It would be equal to **350 billion**. The ex-post supply-side multiplier is:

$$-\frac{\Delta Y}{\Delta T} = 3.74.$$

The ex-ante supply-side multiplier is:

$$-\frac{\Delta Y}{\Delta T} = 1.39.$$

In both cases, in the neoclassical view, it is better to cut taxes on high income people. (the opposite as from an aggregate demand perspective)

8.2 Paul Krugman VS Robert Barro on the Bush tax cuts

1. A transcript of the interview is available on Mark Thoma's *Economists's View* blog. Robert Barro's view on taxes is in line with the previous exercise. He believes that what matters is incentives, and he has a purely "supply side" view of taxes:

And the basic way you do that is you cut the taxes where they start out having the highest rates. Now, a lot of that at the moment is among the rich people. It used to be it was more at the poor people. Though we've actually done a lot on that with respect to the earned income tax credits, so it is not so true now. But the way you really enhance the economy and make it work better is by cutting tax rates where they start out being the highest, because that is where the government is initially distorting the situation the most.

This is what we have found in the previous exercise where output was completely determined by the supply side. Cuts on the high income were more efficient than cuts on the low income in the calculations above. Robert Barro says that the reason why tax cuts work so well on the high income, is because taxes on the rich are initially higher so "that is where the government is initially distorting the situation the most." How can we understand the logic of his argument? In fact, there are some mathematics behind Robert Barro's comment (he was a physics major at Caltech). Recall that we found that income (which adjusts based on labor supply) is given as a function of taxes as:

$$y = (1 - \tau)^{1/\epsilon} \frac{A^{1/\epsilon+1}}{B^{1/\epsilon}}$$

Taking logs of this expression, as we often do when we are dealing with power functions (as in the problem set on the labor market):

$$\log(y) = \frac{1}{\epsilon} \log(1 - \tau) + \left(\frac{1}{\epsilon} + 1\right) \log A - \frac{1}{\epsilon} \log B.$$

The derivative of $\log(1 - \tau)$ with respect to τ is:

$$\frac{d \log(1 - \tau)}{d\tau} = -\frac{1}{1 - \tau}$$

Therefore, a change in τ leads to a change in y given by:

$$\boxed{\frac{d \log(y)}{d\tau} = -\frac{1}{\epsilon(1 - \tau)}}.$$

Therefore, the percentage change in income for a person is an increasing function of τ, or what is marginal tax rate initially is. So Robert Barro is validated in his statement that the government should cut taxes where marginal tax rates are initially the highest. For example, for $\epsilon = 2$ as above, and $\tau = 0.5$, we get a percentage change in income of 1% for a change in marginal tax rate of 1%. For $\tau = 0.25$, we get a percentage change in income of

$$1/(2 \cdot 0.75) = 0.66$$

for a change in marginal tax rate of 1%.

On the other hand, Robert Barro does not like Keynesian stimulus at all, which he views as just "heaping money" to people. To him, tax cuts must be used in order to increase the incentives to work (or rather, lower the desincentives to work):

> And I should distinguish a lot between the 2003 and 2001 tax cut plans. They are really quite different. The big thing about the 2003 plan is that it didn't just heap money to people. It didn't just particularly give money to people at increased incentives to do things. It did that particularly by accelerating the marginal income tax rate cuts.

2. On the contrary, Paul Krugman, when asked by Charlie Rose "If, in fact, Senator Kerry is elected, and if, in fact, he is able to go forward with his program to roll back tax cuts for those making more than $200,000 a year, what impact do you think it will have on the economy and reducing the deficit?" responds:

> I don't think it would have a negative effect. Because I think that we have a problem of demand, not supply. And we have a problem of spending, not incentives. And since, in fact, Kerry is proposing to use the bulk of any additional revenue for new government programs, doesn't do much for the deficit, actually. But it also doesn't do anything to depress spending. And at the point about the scale of it, the scale of what Kerry is proposing is, in fact, almost identical to the Clinton '93 tax increase, which did not exactly sink the U.S. economy. So, you know, you can't make the case that this is a major thing one way or the other.

Paul Krugman does not think that higher taxes on the rich are very detrimental to the economy. For him, the economy suffers more from a lack of demand, of spending, than from the types of incentive effects in the previous exercise.

3. Perhaps paradoxically, Paul Krugman worries a lot more about rising public debt than Robert Barro. In particular, he worries that the reason why tax cuts on the rich are made is to later "starve the beast":

> Robert, you have written about and you are supporting starve-the- beast as a doctrine, cut revenues and then we can use that to squeeze down government spending. You know, I'm not in favor of that. I think that the government does important stuff. And I don't think there is a lot of − ... basically, we have this $400 billion deficit. It's not going to go away simply through economic growth, it has to be some combination of either spending cuts or increased revenue. And I don't like these tax cuts, which ... will eventually force a cut in programs that I think are very important to people's lives.

But he does not want to see expenditures phased out:

> I want to maintain the social insurance institutions. I want to maintain Social Security, Medicare and Medicaid. And I want some from further expansion of health insurance. So I say look, you know, we don't have enough revenue as is. I don't want more tax cuts that will further undermine the revenue base that makes it possible to have these programs that sand off some of the rough edges of capitalism. (. . .) I just have to say, what kind of conservative, or better what kind of Republican is Bush? And the answer, of course, is he is a banana Republican. Just look at the fiscal irresponsibility.

On the contrary, Robert Barro is not that worried about public debt. He likes that tax cuts help provide incentives to work, as in the previous exercise. He says:

> You can't go that crazy about the deficit. It's a little over 3 percent of the GDP now.

As we will see during the following lectures, Robert Barro believes in "Ricardian" equivalence: he thinks that rational optimizers anticipate future tax increases to come, and that they will offset by an increase in private saving the tax increases done by the government.[3]

4. Robert Barro suggests at the end of the interview some reforms on the Social Security side "in terms of introducing some kind of private accounts, not changing the pensions for the existing retired people or people close to it, but down the road, people who are younger." This is how he justifies his position:

> Well, that is something that has to be argued out. But you know, you can imagine going up to somebody in early 40, but that's something you would have to go through. But the idea would be to have a transition where a lot more in the next generation is going to be in terms of personal accounts, rather than the kind of plan that we've had since the 1930s.

Paul Krugman, on the other hand, has the following view:

> Well, I think it is a terrible idea, but even aside from that, it is very expensive. Because the real problem in any of these things, is what about somebody – well, my age, your age, we have been paying into the system all our lives. We don't have a private account. Where is the money going to come from to pay for our retirement? You and I don't need it, but a lot of people do. And then you start to say well, we're going to finance that. Anyway you cut it, you need several trillion dollars of additional money injected into the system to make any of these things work, to cover the transition. I don't think it is a good idea to do it in any case, but to cover this transition for all the people who've been paying into what has been a pay-as-you-go program, where each generation supports the previous generation in its retirement.

Finally, Robert Barro compares the pay-as-you-go system the U.S. currently has to implicit public debt:

> What you have now is a situation where there is a big public debt out there in the form of future Social Security payments. So it's not an official part of the public debt, but it's just like it. And, of course, the transition is where you increase the literal debt in order to fund it, you have to borrow. But you're replacing implicit debt, which is the pension payments you are owing, with explicit debt.

As we have seen in problem set 7, this discussion on the pay-as-you-go makes perfect sense in terms of the overlapping-generations model. Robert Barro want to see life-cycle contribute to capital accumulation, and phase-out progressively the current system of pay-as-you-go. Of course, as Paul Krugman points out, promises have been made to the current generation of retirees, which means that public debt would need to be taken on by the government in order to pay these pension benefits (if the new young generation now invests their saving in private accounts, then they cannot at the same time contribute to the pay-as-you-go

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[3]The fact that high income earners do not increase their consumption much following tax cuts is not necessarily due to the fact that they expect future taxes to come. Another candidate explanation is simply that they have a lower marginal propensity to consume - which, in turn, might come from the various reasons outlined in lecture **??**. They may have utility for wealth, in the form of prestige, or just the joy of owning. Or, in the case of the extremely rich, they simply would not know what to do with this money.

system). Therefore, privatizing social security would merely make existing government liabilities more visible, by transforming them from implicit liabilities into explicit liabilities.