The Keynesian Cross and the Keynesian Multiplier

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An important idea of Keynesian economics is the multiplier effect of government spending (or tax cuts) on aggregate demand, coming from the concept of the Marginal Propensity to Consume (Keynes (1936)). Even though, again, *The General Theory of Employment, Interest, and Money* is not an easy read, you might want to take a look at Chapter 10 in Book III (page 77 in this edition), called "The Marginal Propensity to Consume and the Multiplier".

The **Keynesian cross**, or the (YY)-(ZZ) model, represented below in Figure 1, is a pedagogical device introduced in 1948 by Paul Samuelson in his famous *Economics* textbook, to provide a geometric interpretation for Keynes' ideas (Samuelson (1948)). It comes very directly from the goods market model we have developed in Lecture 2. The value for demand is:

$$\begin{aligned} Z &= C + \bar{I} + G \\ &= c_0 + c_1 (Y - T) + \bar{I} + G \\ Z &= \left(c_0 + \bar{I} + G - c_1 T\right) + c_1 Y \end{aligned}$$

The demand equation is the expression for the (ZZ) curve:

$$Z = \underbrace{\left(c_0 + \bar{I} + G - c_1 T\right)}_{\text{Autonomous Spending } z_0} + \underbrace{c_1}_{\text{MPC}} Y$$

The value of autonomous spending is equal to z_0, and is the value of demand when income is equal to $Y = 0$. The steepness of this curve determines the value for the multiplier. The closer to one the slope is, the higher the value of the Keynesian multiplier. Indeed, solving for $Z = Y$:

$$Y = \underbrace{\frac{1}{1 - c_1}}_{\text{Multiplier}} \times \underbrace{\left(c_0 + \bar{I} + G - c_1 T\right)}_{\text{Autonomous Spending } z_0}$$

Therefore, the value of the multiplier is given by:

$$\boxed{\text{Multiplier} = \frac{1}{1 - c_1}}.$$

Consider a change in autonomous spending $\Delta z_0 = z_0' - z_0$ coming from a change in government spending $\Delta z_0 = \Delta G$, or from a change in net taxes $\Delta z_0 = -c_1 \Delta T$. We have:

$$\Delta Y = \frac{\Delta z_0}{1 - c_1}$$

Why is the change in output higher than the change in autonomous spending? There are actually 4 ways to see this, 2 are algebraic, and 2 are geometric:

1. **Algebra.** The first way to see this is just, as above to equate output to demand $Y = Z$, which allows to get at the result.

2. **Infinite sum of a geometric series.** Indeed, this is coming from the fact that 1 dollar of additional autonomous spending brings in a second round c_1 dollar increase in consumption, a third round c_1^2 increase in consumption, which add up to:

$$\sum_{i=0}^{n} c_1^i = 1 + c_1 + c_1^2 + ... + c_1^n = \frac{1 - c_1^{n+1}}{1 - c_1}$$

   Therefore, because $c_1^{n+1} \to 0$ if $n \to +\infty$ and $0 < c_1 < 1$, we have:

$$\text{Multiplier} = \sum_{i=0}^{+\infty} c_1^i = \lim_{n \to +\infty} \sum_{i=0}^{n} c_1^i = \lim_{n \to +\infty} \frac{1 - c_1^{n+1}}{1 - c_1} = \frac{1}{1 - c_1}.$$

3. **Graphical interpretation 1.** The left panel of Figure 1 gives a graphical interpretation to this infinite geometric sum. This graphical interpretation makes it clear that:

$$Y' - Y = \sum_{i=0}^{+\infty} c_1^i = \lim_{n \to +\infty} \sum_{i=0}^{n} c_1^i = \lim_{n \to +\infty} \frac{1 - c_1^{n+1}}{1 - c_1} = \frac{1}{1 - c_1}.$$

4. **Graphical interpretation 2**. The right panel of Figure 1 gives a graphical interpretation which does not use a geometric sum. If m is the unknown value of the multiplier, then the geometry makes clear that m has to satisfy $m = 1 + mc_1$ which also gives the value for the multiplier:

$$m = 1 + mc_1 \quad \Rightarrow \quad m = \frac{1}{1 - c_1}.$$

1 Variations on the Keynesian Cross

There are multiple variations on the theme of the Keynesian cross. Section 1.1 shows that automatic stabilizers lead to a flatter (ZZ) curve, whose slope if then $c_1(1-t_1)$. For $t_1 > 0$, this leads to a lower multiplier. Section 1.2 shows in contrast that the accelerator effect of demand on investment leads to a steeper (ZZ) curve, and therefore a larger multiplier. Finally, section 1.3 considers a case with procyclical government spending. Again, this leads to a steeper (ZZ) curve.

1.1 Automatic Stabilizers

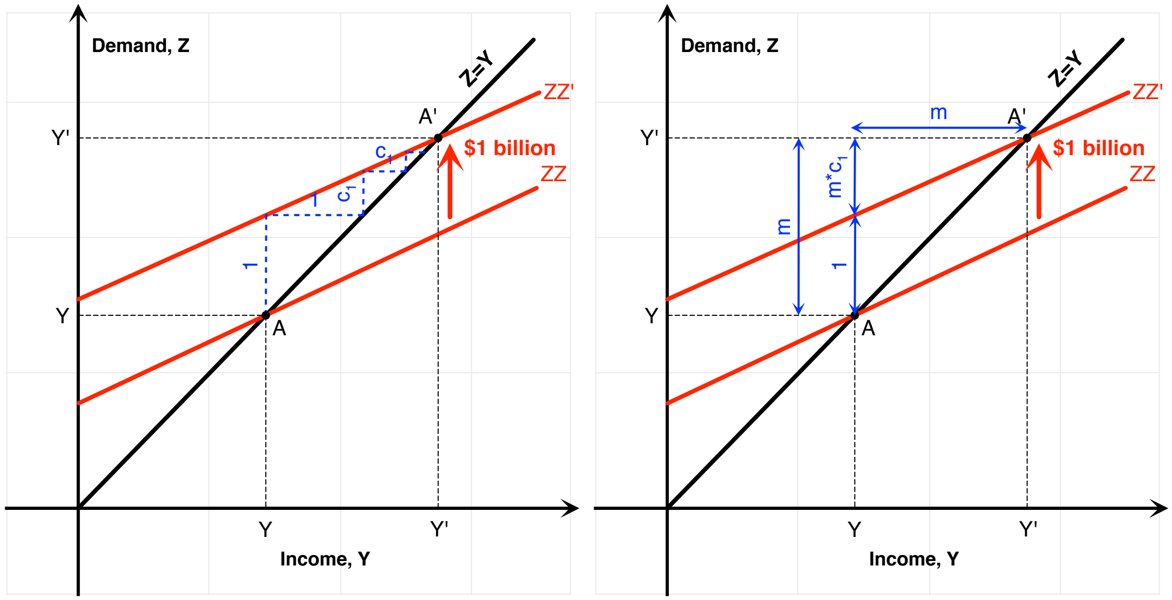
Assume now that:

$$T = t_0 + t_1 Y$$

Therefore:

$$\begin{aligned} Z &= C + \bar{I} + G \\ &= c_0 + c_1 \left(Y - t_0 - t_1 Y\right) + \bar{I} + G \\ Z &= \left(c_0 - c_1 t_0 + \bar{I} + G\right) + \left(\left(1 - t_1\right) c_1\right) Y \end{aligned}$$

Figure 1: THE KEYNESIAN CROSS



The slope of the (ZZ) curve is $(1 - t_1)\, c_1$.

$$\boxed{Y = \underbrace{\frac{1}{1 - c_1 (1 - t_1)}}_{\text{Multiplier}} \times \underbrace{\left(c_0 - c_1 t_0 + \bar{I} + G\right)}_{\text{Autonomous Spending}}}$$

1.2 "Accelerator" effect of demand on investment

Assume that:

$$I = b_0 + b_1 Y$$

Therefore:

$$\begin{aligned} Z &= C + I + G \\ &= c_0 + c_1 (Y - T) + b_0 + b_1 Y + G \\ Z &= (c_0 - c_1 T + b_0 + G) + (c_1 + b_1) Y \end{aligned}$$

The slope if $c_1 + b_1$. Income is equal to demand and therefore, if $c_1 + b_1 < 1$:

$$\boxed{Y = \underbrace{\frac{1}{1 - (c_1 + b_1)}}_{\text{Multiplier}} \times \underbrace{\left(c_0 - c_1 T + b_0 + G\right)}_{\text{Autonomous Spending}}}$$

If instead $c_1 + b_1 \geq 1$, then the multiplier is infinite.

1.3 Procyclical Government spending

Assume that the government systematically spends more when GDP is higher (it builds new roads, hires new teachers, etc.), and conversely when GDP is lower (it then stops construction projects, fires teachers, etc.) – this was the exercise for the midterm:

$$G = g_0 + g_1 Y$$

Therefore:

$$\begin{aligned} Y = Z &= C + \bar{I} + G \\ Y &= c_0 + c_1 (Y - T) + g_0 + g_1 Y + G \\ Y &= \left(c_0 - c_1 T + g_0 + \bar{I}\right) + (c_1 + g_1) Y \end{aligned}$$

Income is equal to demand and therefore, if $c_1 + g_1 < 1$:

$$\boxed{Y = \underbrace{\frac{1}{1 - (c_1 + g_1)}}_{\text{Multiplier}} \underbrace{\left(c_0 - c_1 T + \bar{I} + g_0\right)}_{\text{Autonomous Spending}}}$$

If instead $c_1 + g_1 \geq 1$, then the multiplier is infinite.

References

Keynes, John Maynard, *The General Theory of Employment, Interest, and Money* 1936.

Samuelson, Paul A., *Economics*, New York Toronto London: McGraw-Hill Book Company, 1948.