Lecture 10 - Public Debt, Say's Law

UCLA - Econ 102 - Fall 2018

François Geerolf

Contents

Introduction

Until now, we have been talking about government spending and taxes as if the government can take on as much debt as it wants. We have made our life perhaps a bit easy, by simply assuming that the government can always lower taxes and raise government spending as much as it wants to. In this model, more spending by the government leads to higher GDP. For example, in one of the vintages of the goods market model of lecture 7 - the one where both consumption and investment depend on output:
- An increase in government spending ΔG leads to an increase in output: $\Delta Y = \Delta G/(1 - c_1 - b_1) > 0$.
- A tax cut $\Delta T < 0$ lead to an increase in output: $\Delta Y = -c_1 \Delta T/(1 - c_1 - b_1) > 0$.

But then, why doesn't the government just do more of these tax cuts, or more of that public spending, or both? We alluded to a first reason when we talked about the consequences of having $1 - c_1 < b_1$, or the propensity to save be less than the propensity to invest. We argued then that we would never be in a Keynesian situation of deficient aggregate demand, so that multiplier effects would stop when facing constraints on supply. Similarly, the government would eventually face constraints on what the economy can supply: if fiscal policy was too accommodative, and to the limit if G was set at an astromically high value, then clearly supply constraints would start to bite - one example was given historically in the 1940s when the U.S. engaged in World War II. However, these levels of spending are clearly out of the question, and this is perhaps not what constraints the government from doing a little bit more spending, or a little bit more tax reductions.

An issue which is very often cited is not that one, however, but that of the government deficit, and the impact of government debt on future generations. Indeed, when government spending increases $\Delta G > 0$, this leads to a government deficit of equal magnitude: $\Delta(T - G) = -\Delta G < 0$. Similarly, a tax cut $\Delta T < 0$ leads to increased deficits given by $\Delta(T - G) = \Delta T < 0$. One might worry that this debt will someday have to be repaid, and that the current generation is simply putting a burden on future generations. In this case, higher GDP today might only be thought of as leading to lower GDP in the future, when aggregate demand will be diminished.

During this lecture, we make three related points concerning government deficits and government debt:

1. We show first, without using any economic model, that simple accounting suggests that public debt is on a sustainable path whenever the real interest rate on public debt is lower than the rate of growth of GDP $(r < g)$, a situation called "dynamic inefficiency" for reasons that will become clear later. (from problem set 4 you may already remember that the Golden Rule level of capital accumulation corresponded to $r = g$). I shall argue that real interest rates appear to be below the rate of growth of GDP, at least for now, so there does not seem to be cause for alarm - at least, until interest rates don't rise more.

2. Second, I illustrate using an economic model that it is not true that public debt necessarily will need to be repaid eventually, so that government debt is not necessarily a burden on future generations - an argument which is often made in the public debate. In the overlapping generations model of lecture 4, and provided that capital accumulation is above the Golden Rule level $(r < g)$, so that there is **dynamic inefficiency**, public debt is never repaid, as there are always new generations coming along, who buy government debt when they are young and sell it to the next generation when old. This is sustainable if $r < g$, for reasons laid out in part one.

3. Third and last, we shall discuss the effects of larger government deficits on the economy, and constrast the Keynesian and Neoclassical views on this issue. In particular, Keynesian and Neoclassical economists have very different predictions for the impact of higher public deficits on investment spending. You may already have understood that by contrasting lectures 2 and 4 with lectures 7, 8 and 9. We discuss this and related issues surrounding the so-called Treasury View and Say's law in the last section of this lecture.

1 Sustainability of Public Debt

1.1 Law of motion for Public Debt

In this lecture, we denote everything in terms of goods, to avoid thinking about the complicated issues surrounding inflation. Let us denote by G_t the government spending at period t, and by T_t the taxes in period t. Let us also denote by $(G_t - T_t)$ the government (primary) deficit in period t, which is the excess of government expenditures over taxes levied by the government (thus, when $G_t - T_t > 0$, there is a deficit in the budget, so that the government must borrow). If the interest rate that the government pays is given by r_t, then the law of motion of government debt is given by:

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t$$

Thefore, the law of motion for government debt is given by the sum of the **primary deficit** and **interest payments** on the debt.

The **total** government deficit, which is equal to the change in government debt ΔB_t, is equal to the sum of interest payments and the primary deficit $G_t - T_t$.

$$\text{Deficit}_t = \Delta B_t = B_t - B_{t-1} = \underbrace{r_t B_{t-1}}_{\text{Interest Payments}} + \underbrace{G_t - T_t}_{\text{Primary Deficit}}$$

From the above equation, the evolution of the debt to GDP ratio B_t/Y_t:

$$\frac{B_t}{Y_t} = (1 + r_t)\frac{Y_{t-1}}{Y_t}\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Let us denote the debt to GDP ratio by b_t:

$$b_t \equiv \frac{B_t}{Y_t}.$$

Therefore:

$$b_t = (1 + r_t)\frac{Y_{t-1}}{Y_t}b_{t-1} + \frac{G_t - T_t}{Y_t}.$$

Assuming that GDP grows at rate g_Y, we have that:

$$\frac{Y_t}{Y_{t-1}} = 1 + g_Y$$

Therefore:

$$\boxed{b_t = \frac{1 + r_t}{1 + g_Y}b_{t-1} + \frac{G_t - T_t}{Y_t}.}$$

1.2 Condition for Sustainability

A thought experiment is useful to think about the sustainability of public debt in this environment. Imagine that all future primary surpluses were equal to zero after $t = t_0$, that is:

$$\text{for all } t \geq t_0, \quad G_t = T_t$$

and that real interest rates are constant after $t \geq t_0$:

$$r_t = r.$$

We then have that:

$$\text{for all } t \geq t_0, \quad b_t = \frac{1 + r}{1 + g_Y}b_{t-1}$$

Then the debt to GDP ratio would be given by:

$$\text{for all } t \geq t_0, \quad b_t = \left(\frac{1 + r}{1 + g_Y}\right)^{t - t_0} b_{t_0}$$

There are three possible cases:
1. If $r < g_Y$, the debt to GDP ratio goes to 0. (Indeed, when $a < 1$, $a^t \to 0$ when $t \to +\infty$.) Therefore, the debt to GDP ratio goes to zero mechanically. This situation is said to be **dynamically inefficient**. (for reasons that will be clear later)
2. If $r = g_Y$, the debt to GDP ratio stays constant.
3. If $r > g_Y$, the debt to GDP ratio goes to infinity. Indeed, when $a > 1$, $a^t \to +\infty$ when $t \to +\infty$. Then, the dynamics of government debt are explosive. This situation is said to be **dynamically efficient**.

1.3 Is public debt sustainable in the U.S.?

Which of these three cases is relevant for the U.S. economy? Is public debt sustainable in the U.S.? How do the real interest rate r and the growth rate of GDP g_Y compare? Up until now, I would argue that it's fair to say that $r < g_Y$.

The real interest rate r can be measured in two ways:
1. Either using the nominal interest rate, and substracting an average expected (or realized) inflation rate in order to get to a real interest rate. The nominal interest rate has recently averaged around 2 to 3%, while inflation has been from 1 to 2% on average. This implies a real interest rate which is around 1%, perhaps 2%.
2. Or, one can measure the real interest rate is to look at the rate on the so-called Treasury Inflation Protected Securities (also called TIPS). Measuring the real interest rate in this way leads to an interest rate around 1%.

On the other hand, real GDP growth seems to be hovering about $g_Y \approx 2.5\%$. Therefore, the ratio of government debt to GDP does not appear to be on an unsustainable path so far.

Another way to see this is that the ratio of interest payments to GDP is not particularly high historically. This implies that if the primary deficit was reduced to zero, the debt to GDP ratio would not be on an explosive trajectory.
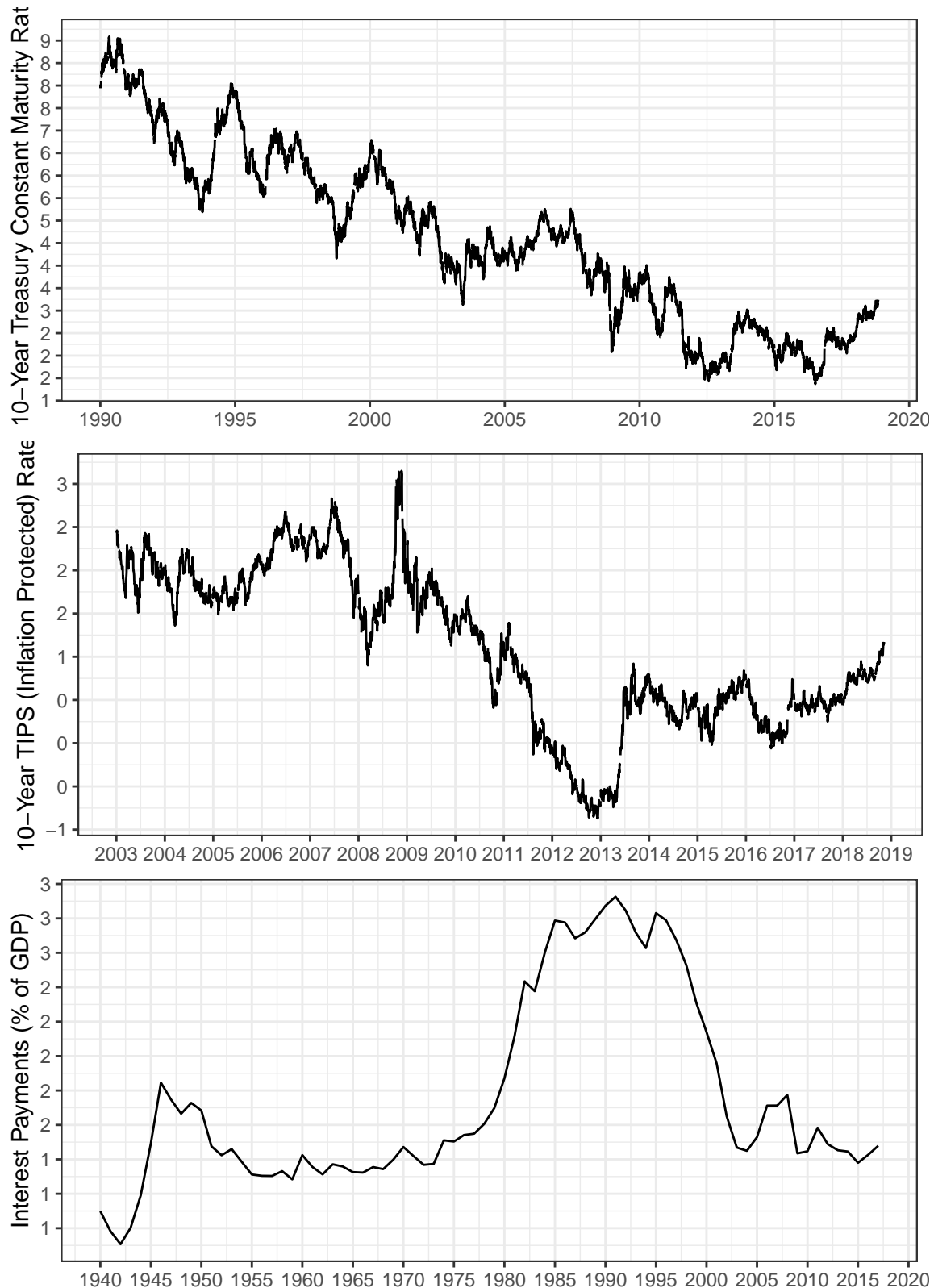
2 Public Debt in the Overlapping Generations Model

2.1 Overlapping Generations Model

Let us look at a simplified version of the overlapping generations model we looked at in Lecture 3, the model that we studied in Problem Set 3 called "Another Overlapping Generations Model". For this model, we shall assume that people only care about old age consumption, and that they work only when young, receiving wage w_t. It does not really matter what the form of their utility function is with respect to old age consumption, because they will save everything anyway:

$$U = u(c^o_{t+1}).$$

Denoting by r_t the (net) real interest rate, their intertemporal budget constraint is then given by:

$$c^y_t + \frac{c^o_{t+1}}{1 + r_t} = w_t.$$

In this very simple environment, and because consumption in young age will always optimally be set to zero ($c^y_t = 0$), this implies:

$$c^o_{t+1} = (1 + r_t)w_t.$$

Similarly to the previous time, we assume that the labor force is fixed to unity:Diamond (1965) had population growth in his original model. We do the simplest version of his model. ($L_t = \bar{L} = 1$) There is a Cobb-Douglas, constant returns to scale, production function:

$$Y_t = K^\alpha_t L^{1-\alpha}_t.$$

Together with the previous assumption of constant labor $L_t = 1$, this means that:

$$Y_t = K^\alpha_t.$$

Markets are competitive, so that the wage is just the marginal product of labor:

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1 - \alpha)K^\alpha_t L^{-\alpha}_t = (1 - \alpha)K^\alpha_t.$$

Similarly as previously, we also get through firms' optimization on the amount of capital that:

$$r_t + \delta = \frac{\partial Y_t}{\partial K_t} = \alpha K^{\alpha-1}_t.$$

Finally, we assume again that capital depreciates at rate $\delta = 1 = 100\%$. (that is, capital fully depreciates each period - this is reasonable if you take one unit of time to represent one generation, or about 30 years - remember that the depreciation rate for one year was approximately equal to 10%.)

2.2 Without Government Debt

Let us first remind ourselves what happens in the absence of government debt in this model. In the absence of a government, we get even simpler expressions than the previous time. The law of motion for capital is given as follows:

$$\Delta K_{t+1} = w_t - \delta K_t.$$

Since w_t is a fraction $1 - \alpha$ of output, this law of motion corresponds to the Solow growth model with $s = 1 - \alpha$. The law of motion for capital is:

$$K_{t+1} = (1 - \alpha)K^\alpha_t + (1 - \delta)K_t.$$

This is a difference equation for sequence K_t which converges to a steady state value for the capital stock K^* such that:

$$\delta K^* = (1-\alpha)(K^*)^\alpha$$

$$\Rightarrow \quad K^* = \frac{(1-\alpha)^{\frac{1}{1-\alpha}}}{\delta^{\frac{1}{1-\alpha}}}.$$

The steady state value for the interest rate r^* is then such that:

$$r^* + \delta = \alpha(K^*)^{\alpha-1}$$

$$= \alpha\left[\left(\frac{1-\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}\right]^{\alpha-1}$$

$$r^* + \delta = \frac{\delta\alpha}{1-\alpha}$$

Therefore, the steady-state value of the interest rate r^*:

$$r^* = \frac{2\alpha-1}{1-\alpha}\delta$$

which, note, is negative for $\alpha < 1/2$. The steady state value for output Y^* is then:

$$Y^* = (K^*)^\alpha$$

$$Y^* = \frac{(1-\alpha)^{\frac{\alpha}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}}.$$

The value for the wage w^* is:

$$w^* = (1-\alpha)(K^*)^\alpha$$

$$= (1-\alpha)\left(\frac{1-\alpha}{\delta}\right)^{\frac{\alpha}{1-\alpha}}$$

$$w^* = \frac{(1-\alpha)^{\frac{1}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}}$$

Steady-state consumption of the old $(c^o)^*$ is thus given by:

$$(c^o)^* = (1+r^*)w^*$$

$$(c^o)^* = \left(1 + \frac{2\alpha-1}{1-\alpha}\delta\right)(1-\alpha)^{\frac{1}{1-\alpha}}$$

Note that if $\delta = 1$ (a reasonable assumption since one period here is one generation), then:

$$(c^o)^* = \left(1 + \frac{2\alpha-1}{1-\alpha}\delta\right)(1-\alpha)^{\frac{1}{1-\alpha}}$$

$$= \frac{\alpha}{1-\alpha}(1-\alpha)^{\frac{1}{1-\alpha}}$$

$$(c^o)^* = \alpha(1-\alpha)^{\frac{\alpha}{1-\alpha}}$$

Numerical Application. With $\alpha = 1/3$ and $\delta = 1$:

$$K^* = \frac{(1-\alpha)^{\frac{1}{1-\alpha}}}{\delta^{\frac{1}{1-\alpha}}} = \left(\frac{2}{3}\right)^{3/2}$$

$$r^* = \frac{2\alpha - 1}{1 - \alpha}\delta = \frac{-1/3}{2/3} = -\frac{1}{2} = -50\%$$

$$Y^* = \frac{(1-\alpha)^{\frac{\alpha}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}} = \left(\frac{2}{3}\right)^{1/2} = \sqrt{\frac{2}{3}}$$

$$w^* = \frac{(1-\alpha)^{\frac{1}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}} = \left(\frac{2}{3}\right)^{3/2}$$

$$(c^o)^* = (1+r^*)w^* = \frac{1}{2}\left(\frac{2}{3}\right)^{3/2}$$

2.3   With Government Debt

As we saw in lecture 2, and then again in lecture 4, because $r^* < 0$ we have that the quantity of capital is higher than the Golden Rule level of the capital stock, which is such that $r_g^* = 0$. Another way to see this is that there is too much saving. As we saw in lectures 2 and 4, one way to solve this problem would be to force everyone to save less; however ths might be thought of as a little bit intruisive. Another way to solve this problem is to use public debt (decrease public saving, to reduce total saving) in order to solve this problem of excess saving and excess investment.

In order to better understand which level of public debt is warranted, we look at the level of capital such that $r_g^* = 0$ - which again, is the golden rule interest rate, since the rate of growth of output is $g_Y = 0$. Thus, the corresponding Golden Rule level of the capital stock K_g^* is such that:

$$r_g^* + \delta = \alpha(K_g^*)^{\alpha-1} \quad \Rightarrow_{r_g^*=0} \quad \delta = \alpha(K_g^*)^{\alpha-1}$$

Therefore:

$$K_g^* = \frac{\alpha^{\frac{1}{1-\alpha}}}{\delta^{\frac{1}{1-\alpha}}}.$$

The Golden rule steady-state value for output Y_g^* would be then:

$$Y_g^* = \left(K_g^*\right)^\alpha$$

$$Y_g^* = \frac{\alpha^{\frac{\alpha}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}}.$$

The value for the steady-state wage w_g^* is then:

$$w_g^* = (1-\alpha)\left(K^*\right)^\alpha$$

$$w_g^* = (1-\alpha)\frac{\alpha^{\frac{\alpha}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}}$$

Steady-state consumption of the old $(c^o)_g^*$ is thus given by:

$$(c^o)_g^* = (1+r^*)w_g^*$$

$$(c^o)_g^* = (1-\alpha)\frac{\alpha^{\frac{\alpha}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}}$$

If $\delta = 1$, then:

$$(c^o)_g^* = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}$$

The question is how to we achieve this quantity of capital K_g^*? The answer is that some public debt needs to be taken on by the government. Again, saving is equal to the wage w_g^*, and to the purchase of total assets, which includes both public debt whose quantity is given by B_g^*, and the capital stock whose quantity is K_g^*. Therefore, we may compute the level of the public debt which allows to reach this Golden-Rule level of capital accumulation:

$$B_g^* + K_g^* = w_g^* \quad \Rightarrow \quad B_g^* = w_g^* - K_g^*.$$

Substituting:

$$B_g^* = w_g^* - K_g^*$$

$$= (1 - \alpha)\frac{\alpha^{\frac{\alpha}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}} - \frac{\alpha^{\frac{1}{1-\alpha}}}{\delta^{\frac{1}{1-\alpha}}}$$

$$B_g^* = \frac{\alpha^{\frac{\alpha}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}}\left(1 - \alpha - \frac{\alpha}{\delta}\right).$$

Note that with $\delta = 1$ which is a reasonable assumption if a generation is about 30 years, and $\alpha < 1/2$ which is generally assumed to be true, then this level of public debt is strictly positive.

Numerical Application. With $\alpha = 1/3$ and $\delta = 1$:

$$K_g^* = \frac{\alpha^{\frac{1}{1-\alpha}}}{\delta^{\frac{1}{1-\alpha}}} = \left(\frac{1}{3}\right)^{3/2} = \frac{1}{3\sqrt{3}}$$

$$r_g^* = 0$$

$$Y_g^* = \frac{\alpha^{\frac{\alpha}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}} = \left(\frac{1}{3}\right)^{1/2} = \frac{1}{\sqrt{3}}$$

$$w_g^* = (1 - \alpha)\frac{\alpha^{\frac{\alpha}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}} = \frac{2}{3}\sqrt{\frac{1}{3}} = \frac{2}{3\sqrt{3}}$$

$$(c^o)_g^* = (1 + r_g^*)w_g^* = w_g^* = \frac{2}{3\sqrt{3}}$$

Note that the steady-state consumption of the old $(c^o)_g^*$ is greater than the level of consumption achieved by the old without government debt since $2 > \sqrt{2}$. But what is amazing is that the level of capital in this case is actually lower than the level of capital in the previous section. The government can force the economy into this level of capital accumulation by taking on debt. The level of debt B_g^* that corresponds to that level of capital accumulation is given by:

$$B_g^* = w^* - K^*$$

$$= \frac{2}{3\sqrt{3}} - \frac{1}{3\sqrt{3}}$$

$$B_g^* = \frac{1}{3\sqrt{3}}.$$

The government can reach that level of debt by giving a transfer to the first generation of old, like the war veterans, who will then consume:

$$c_0^o = \frac{\sqrt{2}}{3\sqrt{3}} + \frac{1}{3\sqrt{3}} = \frac{1 + \sqrt{2}}{3\sqrt{3}}.$$

All future generations thus consume more. With a lot of capital, there is such a thing as a free lunch! Public debt is a Ponzi scheme, but a beneficial one. Public debt allows to increase consumption for everyone, and it can be rolled over every period. This is true even in the neoclassical model, provided that there is dynamic inefficiency ($r^* < g_Y$) to begin with.

3 The Treasury View, and Say's Law

One of the most controversial and also most important questions in macroeconomics revolved around the so-called Treasury View, and Say's law. The question is whether government spending reduces investment,

and whether supply automatically creates "its own demand". These are the most difficult, controversial, but also the most important questions for macroeconomics.

3.1   Treasury View: The Effects of Deficit Spending on Investment

The Treasury View asserts that more saving, either by the government or by households, leads to more investment. Conversely, more government spending, or less government spending, necessarily leads to *crowd out* (reduce) an equal amount of investment spending. This view was advanced in the 1930s (during the Great Depression) by the staff of the British Chancellor of the Exchequer. In his 1929 budget speech, Winston Churchill explained:

> The orthodox Treasury view... is that when the Government borrows in the money market it becomes a new competitor with industry and engrosses to itself resources which would otherwise have been employed by private enterprise, and in the process raises the rent of money to all who have need of it.

The Keynesian and the neoclassical model lead to very different assessments on the Treasury view: Keynesian economics, as we saw in , does not predict such an effect.

- In the **Keynesian model** of lectures 7, 8, and 9, investment is not crowded out by public debt - in the simplest model of the goods market, investment is in fact fixed. In the accelerator model, $I = b_0 + b_1 Y$ so that investment depends only on sales, not on saving.

- In the **neoclassical model** of lectures 2 and 4 in contrast, investment is determined by total saving. In particular, public saving is a component of total saving, so investment is crowded out by public deficits. Note however that this does not mean that in the neoclassical model, government deficits are always bad. As we just saw, public deficits may be a good thing if the economy has too much capital to begin with.

This was the main bone of contention during the financial crisis. While Chicago economists were articulating the Treasury view in various different flavors, Keynesian economists were rejecting this notion very strongly (most notably Paul Krugman). We shall come back in lecture 13 to where the empirical evidence lies on this issue.

3.2   Say's law

Say's law, named after Jean-Baptiste Say (1767 - 1832), says that "supply creates its own demand". This "law", which is more of an assumption, says that there never can be any problem of aggregate demand - what Say calls a "general glut". Say's reasoning is straightforward: people work either because they want to consume now, in which case they create consumption demand, or because they want to consume later, in which case they create investment demand. Therefore, "supply creates its own demand". You can see at least intuitively, that Say's law is very related to the above "Treasury View".

4   Readings - To go further

There is no significant budget deficit, Olivier Blanchard, Jeffrey Sachs, *New York Times*, March 6, 1981.

A Note On The Ricardian Equivalence Argument Against Stimulus (Slightly Wonkish), New York Times Blog Post, December 26, 2011.

Paul Krugman, Multipliers and Reality, New York Times Blog Post, June 3, 2015.

In Japan, the Government Gets Paid to Borrow Money, *Wall Street Journal*, March 1, 2016.

(Gated) Say's law: supply creates its own demand. *The Economist*, August 10, 2017.

(Gated) Why is macroeconomics so hard to teach? *The Economist*, August 9, 2018.