Lecture 4 - Overlapping Generations Model

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Contents

In the Solow [1956] growth model, we assumed that saving was a constant fraction of GDP. Lecture 3 has shown how to use microeconomics, and optimization, in order to derive saving behavior endogenously (that is, to explain it).

This section presents a very simple version of the Diamond [1965] **overlapping-generations model**. This model is used not just to give microfoundations to the Solow [1956] model, but also to think about social security, public debt, which we shall take up in the next lectures.

1 Assumptions

1.1 Time

We assume that people in this economy live only for 2 periods. People are called "young" in the first period of their life, and "old" in the second. Thus, you should really think that the length of a period is a generation (approximately 30 years). However, instead of referring to these two periods as 0 and 1, I shall refer to them as t and $t+1$.

1.2 Demographics

People from generation t are young in period t, and old in period $t+1$. We denote their consumption when young by c_t^y and their consumption when old by c_{t+1}^o. In terms of Lecture 3, you should really think of c_t^y as c_0, and of c_{t+1}^o as c_1.

People work when young, and then receive a wage given by w_t. They retire when old, and then do not work. Their lifetime utility is logarithmic with $\beta = 1$:

$$U = \log(c_t^y) + \log(c_{t+1}^o).$$

Their intertemporal budget constraint is given by:

$$c_t^y + \frac{c_{t+1}^o}{1+R} = w_t.$$

There are always two generations living in period t: the previous period's young, born in period $t-1$, now old, consuming the return from their savings; and this period's young, newly born (in period t).

1.3 Production

For simplicity, we shall assume a Cobb-Douglas, constant returns to scale, production function:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}.$$

We assume that the labor force is constant and fixed to unity (this is to avoid carrying L around everywhere - from lecture 2, you should now know that everything can be expressed per capita, because of constant returns to scale), and therefore:

$$L_t = L = 1.$$

Again for simplicity, we shall assume that capital depreciates at rate $\delta = 1 = 100\%$. (that is, capital fully depreciates each period - this is not that unreasonable if you take one unit of time to represent one generation, or about 30 years - remember that the depreciation rate for one year was approximately equal to 2% to 30% depending on the type of capital involved.)

2 Solution

2.1 Calculating Saving from the Consumption Problem

Utility is logarithmic, so that the consumption of the young c_t^y and consumption of the old c_{t+1}^o are given as a function of the wage as follows (this is just an application of Lecture 3):

$$c_t^y = \frac{w_t}{2} \qquad c_{t+1}^o = (1+R)\frac{w_t}{2}.$$

Indeed, if you want to think of this model as the two periods model of Lecture 3, think that everything is as if:

$$f_0 = 0, \qquad y_0 = w_t, \qquad y_1 = 0.$$

2.2 Capital accumulation

Saving (and savings) is equal to investment, and therefore we have that:

$$S_t = I_t = w_t - c_t^y = \frac{w_t}{2}.$$

The major difference with the Solow model is that saving is here endogenous, and coming from agents' optimizing choices. In the Solow model in contrast, saving was taken as exogenous and equal to a fraction s.

The wage paid by employers, given that $L = 1$, is:

$$w_t = (1-\alpha)K_t^{\alpha} L^{-\alpha} = (1-\alpha)K_t^{\alpha} = (1-\alpha)Y_t.$$

Finally:

$$\Delta K_{t+1} = \frac{w_t}{2} - \delta K_t = \frac{1-\alpha}{2} Y_t - \delta K_t.$$

This is the capital accumulation equation (or law of motion for capital) of the Solow model, with $s = (1-\alpha)/2$. The new element here of course is to get saving endogenously, from agents' optimal decisions. Note that the value for the saving rate has an economic interpretation: wages are only a fraction $1-\alpha$ of output, from lecture 1. On the other hand, savers / consumers want to smooth consumption and therefore want to save a half of that. This is why a fraction $(1-\alpha)/2$ of output is saved.

2.3 Numerical Application

Note that if $\alpha = 1/3$, then the saving rate is equal to $s = 1/3$, which happens to be (by coincdence) the Golden Rule level of saving. This does not mean that the Golden Rule level is always satisfied. This only happens by chance in this very stylized model. In particular, saving is not just because of retirement, but also because of precautionary behavior, leaving bequests or simply liking being wealthy. We will come back to these issues in future lectures.

References

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Robert M. Solow. A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, 70(1):65–94, 1956. ISSN 0033-5533. doi: 10.2307/1884513. URL http://www.jstor.org/stable/1884513.