

# Reassessing Dynamic Efficiency\*

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## Abstract

In a seminal paper, Abel et al. (1989) argue that the United States and six other major advanced economies are dynamically efficient. Updating data on mixed income and land rents, I find in contrast that the criterion for dynamic efficiency is not verified for any advanced economy, and that Japan and South Korea have unambiguously over-accumulated capital. This world “savings glut” can potentially explain otherwise hard-to-understand macroeconomic stylized facts - low interest rates, cash holding by firms, financial bubbles. It is also the macroeconomic counterpart of the microeconomic observation that average firms’ return on investment is lower than their measured cost of capital. Subject to some caveats, an increase of public debt, or a generalization of pay-as-you-go systems could therefore be Pareto-improving.

**Keywords:** Dynamic efficiency, savings glut, marginal returns to capital

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Dynamic efficiency is an important macroeconomic issue. The presumption that increasing investment is always good for the economy relies on the fact that more investment leads to more output in the long run, and that more output helps achieve higher consumption. The first fact is mechanical, but the second is not necessarily true, because more capital also means more investment to maintain the same capital/output ratio. Consequently, there are limits as to how much capital should ideally be accumulated. The amount of capital such that the flow of consumption is maximized is called the Golden Rule level of capital, which is reached when the interest rate  $r$  is equal to the rate of growth of the economy  $g$ .<sup>1</sup> If the capital stock installed is higher than the Golden Rule or  $r < g$ , then every agent could be made better off by consuming the capital so that the Golden-Rule level is restored.<sup>2</sup> A competitive equilibrium with optimizing agents, market clearing and price taking can lead to such a situation of over-accumulation, even without assuming any type of inefficiency. All that is needed is that the economy is expected to run forever.

Dynamic efficiency is ultimately an empirical question. But the real world provides the empiricist with multiple interest rates  $r$  to choose from. Should he use the safe interest rates on government bonds, in which case he would conclude that the economy is strongly inefficient? Or should he use the return on equities, which almost always exceed the rate of growth?<sup>3</sup> Abel et al. (1989) extend the Diamond (1965) overlapping generations capital accumulation model to provide an operational test of dynamic efficiency valid across a general class of stochastic production functions. The cost of generality is that equivalence results are lost; the empiricist is only left with sufficient conditions. If capital income always exceeds investment, then the economy is in a dynamically efficient state. If investment to the contrary always exceeds capital income, then the economy is dynamically inefficient. Empirically, Abel et al. (1989) find that this sufficient criterion for dynamic efficiency is satisfied by a wide margin for the United States (1929 – 1985) and 6 other advanced economies (1960 - 1985).

This paper overturns the result: the sufficient conditions for dynamic efficiency are nowhere satisfied, and Japan and South Korea are even unambiguously inefficient. I use a new dataset coming from recent released of a harmonized system of National Accounts compiled by the OECD, which has a more extensive coverage of mixed income.<sup>4</sup> Mixed

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<sup>1</sup>On the Golden-Rule level of capital, see Ramsey (1928), Phelps (1961), Phelps (1965), Cass (1965), Diamond (1965).

<sup>2</sup>According to a steady-state welfare criterion, capital underaccumulation (with  $r > g$ ) is also suboptimal. However the transitional dynamics to this new steady state are costly for the generations who therefore need to consume less and accumulate capital.

<sup>3</sup>However, calculating returns on equity from stock market returns is a contaminated test of dynamic efficiency because dynamic inefficiency creates the potential for rational bubbles, which increase the rate of return to capital through capital gains. In particular, Fama and French (2002) note that the income return on book equity is much lower than the average stock return. More on this below.

<sup>4</sup>National Income and Product Accounts (NIPA) used by Abel et al. (1989) do not account for all income of unincorporated enterprises, which include labor income.

income consists in part of labor income, so that understating mixed income leads to overstate capital income, and be too sanguine about dynamic efficiency. I also use country-specific data for land rents, while Abel et al. (1989) only had only the numbers for the United States at their disposal. Land is not an accumulated factor of production, so its marginal product should not be treated as investment income. This second adjustment is for many countries quite critical: by means of an example, Abel et al. (1989) estimate land rents in Japan to be roughly equal to 5% of GDP - for lack of better data, the share of land in GDP was assumed to be constant across countries. Both OECD and Goldsmith (1985)'s country-level data estimates suggest in contrast they are in the 17% of GDP range, certainly because land is much scarcer in Japan than in the United States. All in all, using this new dataset, I find that sufficient conditions for dynamic efficiency are verified for none of the advanced economies and to the contrary, that Japan and South Korea verify the criterion for dynamic inefficiency.<sup>5</sup> For Australia and Canada, dynamic inefficiency is confirmed if Tobin's average  $Q$  (which proxy for rents in the corporate sector) is just a bit higher than 1.

Dynamic inefficiency could help explain a number of macroeconomic and financial stylized facts. A microeconomic counterpart to macroeconomic dynamic inefficiency is the claim in Fama and French (2002) and Campbell (2003), that firms seem to sink resources, since they get lower returns on dividends than is implied by their own measured cost of capital. For example, Fama and French (2002) write: "Most important, the average stock return for 1951 to 2000 is much greater than the average income return on book equity. Taken at face value, this says that investment during the period is on average unprofitable: its expected return is less than the cost of capital. In contrast, the lower estimates of the expected stock return from the dividend and earnings growth models are less than the income return on investment, so the message is that investment is on average profitable." Similarly, Campbell (2003) writes: "if one uses average returns as an estimate of the true cost of capital, one is forced to the implausible conclusion that corporations destroyed stockholder value by retaining and reinvesting earnings rather than paying them out." It is perhaps not so surprising that microeconomic data finds that resources are sunk in investment is investment is on average unprofitable. This finding of dynamic inefficiency can potentially explain financial instability, as with dynamic inefficiency, asset prices are no longer pinned down, as is well known since Samuelson (1958) and Tirole (1985). Linked to the possibility of rational bubbles, equity prices can command an equity premium over finitely-lived assets, if an inception of rational bubbles has increased their rate of return by higher-than- $r$  returns from capital gains. Finally, among other examples (see Section 4), dynamic inefficiency can perhaps help understand the Japanese lost decade, during which investment was substantially

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<sup>5</sup>While one could argue that South Korea still is in a capital accumulation phase, it is certainly not the case for Japan.

scaled down but consumption did not go down so much<sup>6</sup>, as well as the current crisis that advanced economies are currently witnessing. Normatively, Samuelson (1958) and Diamond (1965) showed that the government could in this case make every agent better off by borrowing, or running other types of Ponzi schemes, such as extending the breadth of pay-as-you-go systems.

The idea that the economy has accumulated enough capital may seem counterintuitive, to say the least, to many. With an ageing population, shouldn't the old generation care more about the less numerous young generation and help reduce public debt, rather than the other way around? But the idea behind dynamic inefficiency (and the OLG model) is that capital might actually do a poor job at transferring resources ; it might well be more efficient to have the young work for the old in each period (a pay-as-you-go system), rather than having every individual save for retirement through their own retirement account, because capital is not so efficient at producing fruits needed at retirement. More importantly, this analysis is not only normative but also positive. For if the economy is in a dynamically inefficient state then rational bubbles can appear<sup>7</sup>, and transfer resources from young to old agents though in a more unpredictable way.<sup>8</sup> Still on the positive front, dynamically inefficient economies have many of the properties of Keynesian economies as discussed by Geanakoplos (2008): in particular, they are isomorphic to Arrow-Debreu economies where markets do not clear at infinity. Infinite-horizon models of dynamic inefficiency therefore leave room for “animal spirits” to determine asset prices and rates of interest, although many other models can deliver the same kinds of results.

The rest of the paper proceeds as follows. In section 1, I Abel et al. (1989)'s sufficient conditions for dynamic efficiency in the case of a very stylized economy, to remind the reader about the intuition for their results, and highlight the particular role played by mixed income and land rents. In Section 2.1, I reassess dynamic efficiency for the United States, using the same primary dataset as Abel et al. (1989). I then review dynamic efficiency in other advanced economies - 15 additional countries - in Section 2.2. These two sections lead to the conclusion that Japan and South Korea are dynamically inefficient, while dynamic inefficiency cannot be rejected for any advanced economy. Section 3 tries to go further at the cost of more assumptions; but I let the reader make her mind about dynamic inefficiency of the other advanced economies. Finally, I review

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<sup>6</sup>Note that in an overlapping-generations model, GDP is no good measure for welfare, as more investment can lead to lower consumption always, and yet higher GDP.

<sup>7</sup>This is because bubbles can in that case grow at a rate higher than the rate of interest, even with a constant fraction of optimists in the population: their wealth grows at rate  $g$ , while the interest rate is  $r < g$ . Dynamic inefficiency thus provides a strong rationale for buying overvalued assets and chasing capital gains rather than dividends.

<sup>8</sup>For example, when the young buy overvalued houses from the old, it is effectively a transfer of wealth from the young to the old. Note that the bailing out of the financial sector also amounts to such a transfer that the private sector had been achieving on its own until the housing bubble collapsed.

in section 4 some stylized facts consistent with dynamic inefficiency, and I discuss some potential caveats to this study.

**Related literature.** To the best of my knowledge, no paper has reassessed dynamic efficiency since Abel et al. (1989) so far. The breadth of the literature which this paper speaks to is potentially very large, given the importance of dynamic efficiency for the economics of intergenerational transfers. From a methodological standpoint, there has been a renewed interest in national accounts data recently, in particular since the release of harmonized national accounts by the OECD. This has led to reassessing many common wisdoms. Gollin (2002) shows that labor income shares are not so variable across countries than economists once thought, and this paper tries to share the care he gives to the treatment of capital income. Caselli and Feyrer (2007) revisit Lucas’ puzzle, showing that marginal product of capital is equalized across countries, once one accounts in particular for the effect of land and other non-reproducible resources. Piketty and Zucman (2014) use new balance sheet data to investigate the long run evolutions of wealth-income ratios over the courses of the nineteenth and twentieth centuries, and what they draw from this analysis is that “capital is back” A big advantage from using flows instead of stocks to study capital overaccumulation is that I do not have to make assumptions about the relative price of capital and consumption goods. From a substantive standpoint, this paper provides some support for Bernanke (2005)’s view that there indeed is a “savings glut” at the world level. It relates to a broad literature on the lack of safe assets, for example Caballero and Krishnamurthy (2006), Caballero et al. (2008) or Krishnamurthy and Vissing-Jorgensen (2012). The claim of this paper that the world does not so much lack safe assets, but lacks assets per say.<sup>9</sup>

## 1 Theory

In this section, I take an example of a very stylized production economy, which allows me to state, give an intuition and prove Abel et al. (1989)’s results, and at the same time discuss the case of fixed income and land rents, both of which are important in my empirical work. For simplicity, I shall review the case of a deterministic neoclassical production function, with full depreciation, and fixed labor, which is both far less general than Diamond (1965) and Abel et al. (1989) but allows to get quickly and perhaps more intuitively at the key results. I refer the reader to those two papers for the more general proofs. On the other hand, I incorporate land in the production function to show that

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<sup>9</sup>However, the two theories have a lot in common. With dynamic inefficiency, there is a shortage of assets and so rational bubbles can form naturally as discussed above. These bubbles increase the supply of assets but as they are subject to a coordination problem, this supply is not “safe”. Agents therefore would ideally need assets that they perceive as safe, even though they are valued above their fundamental values. In Krishnamurthy and Vissing-Jorgensen (2012), US Treasuries play this role.

its marginal product should be removed from capital income for the purpose of assessing dynamic efficiency.

Time is discrete  $t = 0, 1, 2, \dots$ . Firms operate by means of a neoclassical production function to produce a single consumption good in quantity  $Y_t$  with  $Y_t = A_t K_t^\alpha N_t^\beta L_t^\gamma$ , denoting  $A_t$  total factor productivity, using capital coming from previous investment (with  $Y_t = I_t + C_t$ ), with  $K_t = I_{t-1}$  (full depreciation), labor in fixed supply  $N_t = \bar{N}$ , and  $L_t$  units of land with  $L_t = \bar{L}$  with potentially  $\alpha + \beta + \gamma < 1$  if there are pure rents in this economy. Crucially, note that the amount of land is fixed because land is assumed to be a non-reproducible factor of production. Therefore its marginal product does not come from investment. The representative firm in this economy solves:

$$\max_{K_t, N_t, L_t} A_t K_t^\alpha N_t^\beta L_t^\gamma - r_t^K K_t - w_t N_t - r_t^L L_t.$$

If there are strictly decreasing returns to scale, then profits of the representative firms are non-zero  $\Pi_{pure} > 0$ . From an income approach,

$$\begin{aligned} A_t K_t^\alpha N_t^\beta L_t^\gamma &= Cap.Inc + w_t N_t \\ A_t K_t^\alpha N_t^\beta L_t^\gamma &= \textcolor{red}{r}_t^K K_t + r_t^L L_t + \Pi_{pure} + w_t N_t. \end{aligned}$$

In such an economy, one can state a special case of Abel et al. (1989)'s results, and write the sufficient conditions for dynamic inefficiency. They involve comparing the flow of capital income and investment from some date  $t \geq t_0$ .

## 1.1 Sufficient Conditions for Dynamic Inefficiency

**Proposition 1.** *[Abel et al. (1989)] A sufficient condition for dynamic efficiency is that  $\exists \epsilon > 0, \quad \forall t > t_0, \quad \textcolor{red}{r}_t^K K_t \geq (1 + \epsilon)I_t$ . A sufficient condition for dynamic inefficiency is:  $\exists \epsilon > 0, \quad \forall t > t_0, \quad \textcolor{red}{r}_t^K K_t \leq (1 - \epsilon)I_t$ .*

The intuition is pretty straightforward: the economy is dynamically inefficient if it is always investing more than it is getting from capital income, or sinking resources each period (at the steady state growth path). To the contrary, it is efficient if it is always investing less than it is getting out. But note that crucially, neither pure profits nor land rents enter in such a calculation. The reason is that both of them do not come from previously accumulated investment, and therefore they should not be taken into account when assessing the economy's efficiency. To understand rigorously why pure profits and land rents should be excluded, let me go over the proof for the Abel et al. (1989) result in my stylized economy. For brevity, let us look at the sufficient conditions for dynamic inefficiency. Assume that at the competitive equilibrium of this economy,

the allocation for this economy is such that:<sup>10</sup>

$$\exists \epsilon > 0, \quad \forall t > t_0, \quad r_t^K K_t \leq (1 - \epsilon)I_t.$$

Because I assumed full depreciation  $K_t = I_{t-1}$ :

$$\exists \epsilon > 0, \quad \forall t > t_0, \quad r_t^K I_{t-1} \leq (1 - \epsilon)I_t.$$

Let us look at a different allocation, and show that the old allocation is Pareto-dominated by this other one with higher consumption in all periods. Let us increase the consumption by an amount  $dC_{t_0} > 0$ , financed by  $dK_{t_0+1} = dI_{t_0} = -dC_{t_0}$ . To maintain the same consumption in the period after  $dC_{t_0+1} = 0$ , there must be a reduction in investment in  $t_0 + 1$ , which is at least equal to the fall in output that was due to lower investment:

$$dI_{t_0+1} = \frac{\partial Y_{t_0+1}}{\partial K_{t_0+1}} dI_{t_0} = r_t^K dI_{t_0} \quad \Rightarrow \quad \frac{dI_{t_0+1}}{I_{t_0+1}} = \frac{r_{t_0+1}^K I_{t_0}}{I_{t_0+1}} \frac{dI_{t_0}}{I_{t_0}}.$$

In fact, for all  $t \geq t_0 + 1$ , investment must decrease to maintain  $dC_t = 0$ :

$$\forall t > t_0, \quad \frac{dI_t}{I_t} = \left( \prod_{\tau=t_0+1}^t \frac{r_{\tau}^K I_{\tau-1}}{I_{\tau}} \right) \frac{dI_{t_0}}{I_{t_0}}.$$

In the context of dynamic inefficiency, such a Pareto improving change is feasible, since  $\frac{r_t^K I_{t-1}}{I_t} \leq 1 - \epsilon$ . If the condition was not satisfied, then the change would lead to unbounded changes in investment rates to maintain the same level of consumption, which is not possible since the initial investment was finite. In other words, output would be always decreasing and therefore consumption would go to 0 at one date when there would be not capital left to produce.

## 1.2 $r_t^K K_t$ in the Data

Proposition 1 is regrettably not directly operational, because  $r_t^K K_t$  is not so easy to measure in the data. The first issue is that labor income and capital income are not always easy to distinguish, as unincorporated enterprises (for example, sole partnerships) do not report capital and labor income separately. The income of doctors, lawyers, entrepreneurs nevertheless comes both from the labor they supply, as well as from the machines and office space they use. Such income is reported in the national accounts as “mixed income”. To take the notations used earlier in the stylized economy, output

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<sup>10</sup>Note that underlying this competitive equilibrium, there must be a demographic structure with finite lives for agents; otherwise asset demand (savings) would never be such that interest rates are too low. For conciseness again, I do not write the agents’ problem leading to a choice of consumption and savings. Here the allocation is taken as given.

is given by:

$$A_t K_t^\alpha N_t^\beta L_t^\gamma = \left( A_t K_t^\alpha N_t^\beta L_t^\gamma \right)_{mix} + (Cap.Inc)_{nonmix} + (w_t N_t)_{nonmix}.$$

$r_t^K K_t$  is therefore not readily available but for example in the National Income and Product Accounts data (BEA data),  $r_t^K K_t$  can be inferred from the following equation:

$$r_t^K K_t = f \left( \left( A_t K_t^\alpha N_t^\beta L_t^\gamma \right)_{mix} \right) + (Cap.Inc - r_t^L L_t - \Pi_{pure})_{nonmix}.$$

The quantities which are not given by the national accounts are denoted in blue, and assumptions must be made about them. First, an assumption must be made concerning the flow of income going to capital. A conservative, but standard assumption is to assume that one third of mixed income is capital income  $f(x) = 1/3x$ . We will come back to this assumption later. Second, rents coming from non-reproducible capital are not directly available in the national accounts. Firms' profits however also come from the land they have previously bought when they own their office space (or factories), and rental income of individuals or real estate firms comes both from previous construction (residential investment), as well as from land rents. For example, Abel et al. (1989) estimate the value of land to represent about 25% of assets, which according to their calculations correspond to a dividend flow of about 5% of GDP. Third and finally, pure profits are assumed to equal 0, that is  $\Pi_{pure} = 0$ ; and we will come back to this assumption in section 3.

## 2 Reassessing Dynamic Efficiency

This section looks at this criterion again for the US economy first in section 2.1, and at other economies next, in section 2.2; because the data sources are not the same for the two: the US has its own national accounting system, the NIPA, which Abel et al. (1989) use. Using OECD data for the United States as I do for other economics in section 2.2 will lead to the same conclusions.

### 2.1 In the United States

In this section, I investigate dynamic efficiency in the United States assuming that Tobin's  $Q$  is equal to 1. Since there are monopoly rents and decreasing returns to scale in the real world, this leads me to be too sanguine about dynamic efficiency (see Section 3.1 for different assumptions about Tobin's  $Q$ ); yet the conclusion here will be that even with  $q = 1$ , dynamic inefficiency cannot be rejected in the United States. In order to pinpoint how my assessment differs from the seminal paper by Abel et al. (1989), I



follow them in using the National Income of Product Accounts, even though these are not the harmonized national accounts from OECD I use later.

I use the NIPA data maintained by the Bureau of Economic Analysis (BEA) to compare Gross Private Domestic Investment on the one hand, and Gross Capital Income on the other hand. While investment is available as a series, Gross Capital Income has to be calculated. Again, I follow their methodology in adding profit (including taxes on profit), rental income, interest income, capital income of proprietors, and private Capital Consumption Allowances (which are the difference of total and government Capital Consumption Allowances). On Figure 2 I compare data obtained from their Tables with data I calculate from today's series of the NIPA. I do not systematically over or understate capital income in any way, and the fit is good.

As these results are often remembered, capital income is about 25% of GDP, while investment is 15% of GDP. Therefore, dynamic efficiency seems to be satisfied by a wide margin for the United States. Yet there are 2 ways in which these calculations are being too sanguine about dynamic efficiency of the US economy. The first is that entrepreneurial income is not properly accounted for, as unincorporated enterprises are not taken into account in the NIPA, unlike in OECD mixed income. Second, because land rents are a bit higher than 5% of GDP as revealed by land values. I make both adjustments in turn:

- **Entrepreneurial income.** Quoting Mead et al. (2004), "Some aggregates exist in one system but not in the other. For example, NIPA corporate profits and personal income do not have precise counterparts in the SNA, and the SNA concept of "mixed income" - that is, the residual business income of unincorporated corporations that is attributable to labor and to capital has not been implemented in the NIPAs, pending a review of the sectoring of unincorporated businesses." NIPA only accounts for the income of proprietors, while OECD notion of mixed income includes also that of unincorporated enterprises. In other words, NIPA misses part of mixed income. How should I attribute mixed income between capital and labor income? This is extensively discussed in Gollin (2002), who goes as far as attributing the entirety of mixed income to labor. For robustness, I do not take such an extreme stand here: I only impute 2/3 of mixed income to labor. One should however bear in mind that this estimate is very conservative in many ways. First, this imputation is traditionally used since the work of Christensen (1971). At the time, proprietors' income was mostly that of farmers working in agriculture, where wages were relatively low; today, unincorporated enterprises use more skilled labor. In particular, doctors, lawyers operate in such structures. Furthermore, in most countries there exists a tax incentive to create an incorporated enterprise if the business is capital intensive; this is because unincorporated

enterprise (e.g. LLC in the US) tax capital as personal income.<sup>11</sup> And finally, the share of capital in aggregate output is closer to  $3/4$  than  $2/3$ . Figure 5 should really be seen as a higher bound on capital income excluding mixed income coming from labor.

- **Land rents, non-reproducible assets.** Land is a fixed non-reproducible factor, its return should therefore not be included to assess whether investment is on average productive or not.<sup>12</sup> Abel et al. (1989) use data from Rhee (1991).<sup>13</sup> They settle for 5% of GDP for the US, because it is consistent with an estimate of the aggregate value of land at  $2/3$  of GDP and a return of 8%.<sup>14</sup> Data on land rents is not directly available, as it is tied to residential structures, and both often trade as a bundle. I use two sources for calculating land rents in the United States: Goldsmith (1985) estimates a replacement cost of structure and attributes the remainder to land., which leads him to impute 25% of total assets to land (this is consistent with Davis and Heathcote (2007)).<sup>15</sup> Imputing land rents in this way leads to the results in Figure 6. Note that land is not the only non-reproducible asset: natural resources are another one. For the sake of conservativeness, I will however not make an attempt at subtracting them from capital income.

## 2.2 In other countries

As in section 2.1, I hereby assume that Tobin's  $Q$  is equal to 1, which goes against the conclusions of this paper. I compare capital income and investment for 15 additional countries (those for which OECD releases Gross Investment and Capital Income series): Australia, Belgium, Canada, Denmark, France, Germany, Hungary, Italy, Japan, Norway, Russia, South Korea, Sweden, Switzerland, the United Kingdom.<sup>16</sup>

**Reproducing.** Abel et al. (1989) investigate dynamic efficiency in England, France, Germany, Italy, Canada, Japan. Their results are in Table 3 to the paper, which are plotted on Figure 7 as the difference between gross profit and gross investment. But

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<sup>11</sup>More generally, there is an incentive to treat labor income as capital income for tax reasons. Capital income is thus always more likely to be overestimated than the contrary.

<sup>12</sup>In agricultural societies, one could argue that land needed to be somewhat maintained in order to remain productive. Today, land rents overwhelmingly consist of urban land, which has value because of economic geography considerations. For example, in the Alonso-Muth-Mills monocentric city model, higher land rents in the core are the exact counterpart of lower transportation cost.

<sup>13</sup>Rhee (1991) is using Denison (1962), itself updating Goldsmith (1959).

<sup>14</sup>Notwithstanding the very low assumed aggregate value of land in total assets, calculating land rents using returns is precisely what assessing dynamic efficiency is about, which is a bit circular. Instead, I will impute factor returns using their proportion in total assets.

<sup>15</sup>See Appendix C for a discussion on land data.

<sup>16</sup>I drop Mexico from the sample, because it has relative income to the US (in purchasing-power parity) significantly lower than the rest of my papers: about 30% on average. Dynamic efficiency is a steady-state concept, and such an emerging economy has not reached its steady-state of capital accumulation.

they do not account for mixed income and land, which together account for about 9% of GDP. Note that with the Abel et al. (1989) data, Japan and Germany could already not be confirmed as being dynamically efficient, since investment is not lower than capital income over the whole period.

**Updating.** Land rents and mixed income do vary a lot across countries. OECD provides estimates of both for many countries. In contrast, Abel et al. (1989) used 5% of GDP for every country for lack of better data. When not available, I complement land price data with Goldsmith (1985) estimates from Comparative National Balance Sheets. Since Goldsmith (1985) does not provide data on land since 1978, I assume that land shares were constant ever since, and take the lowest of the 1973 or 1978 to be more conservative. These details are discussed more precisely in Appendix C. Moreover, I perform a number of robustness checks in Section 3.3.

**Results.** I present in the main text the most inefficient economies according to OECD data: Japan and South Korea. I will not put too much of an emphasis on South Korea as it has developed only recently. The criterion for dynamic efficiency applies only at the (stochastic) steady-state, after the period of capital accumulation. Since I do not always have much data on mixed income, I present both capital income correcting for the value of mixed income and capital income containing mixed income. More precisely, capital income containing mixed income is an over-estimation of capital income coming from investment, but an under-estimation of capital income containing mixed income. This is because I calculate the value of land rents through the proportion of land in non-financial assets; so that I overestimate land rents when using capital income containing labor income from mixed income. More precisely, I use the following inequality:

$$\underbrace{\text{CapIncNoLandNoMixed}}_{\text{Capital Income}} = \left( \text{CapInc} - \frac{2}{3} \text{mixed} \right) \cdot \left( 1 - \frac{\text{land}}{\text{assets}} \right)$$

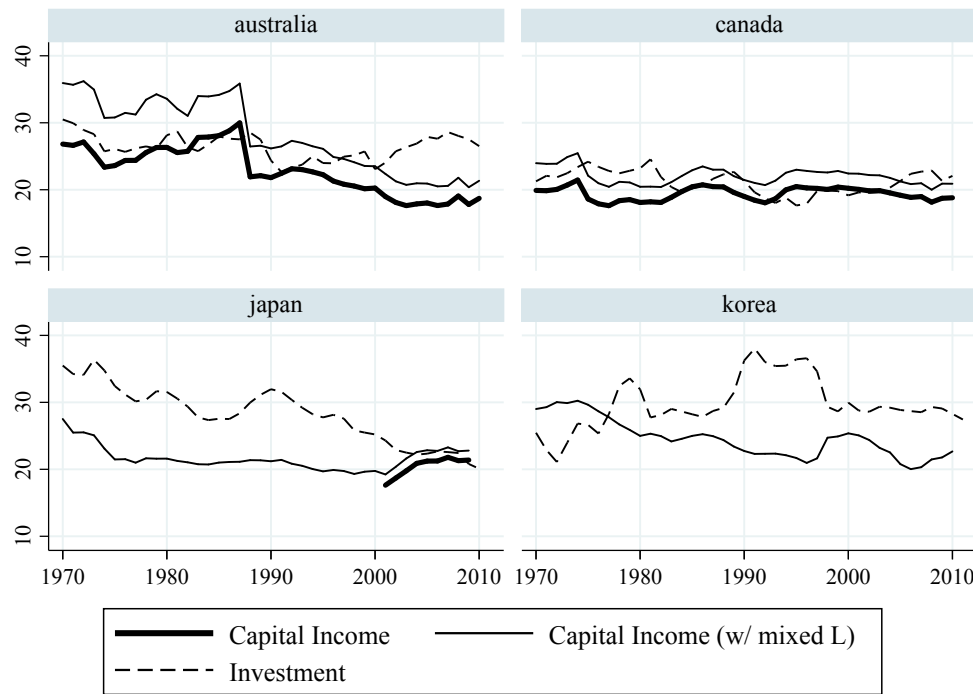
$$\leq \underbrace{\text{CapInc} \cdot \left( 1 - \frac{\text{land}}{\text{assets}} \right)}_{\text{Capital Income with mixed L}}$$

“Capital income with mixed Labor” is therefore a slight abuse of language in the graphs, only a fraction  $1 - \text{land}/\text{assets}$  of mixed labor is actually included.

**Description of the results.** As can be seen on Figure 1, OECD does not provide data on mixed income for Japan before 2001. But the continuous thin line is always a higher bound on the thick line (capital income). Data from the OECD therefore suggests that Japan is inefficient. This contrasts starkly with Abel et al. (1989)’s results. The reason is that Japan has much higher land rents than the United States, because land

is far more scarce. Therefore, by assuming that the United States have 5% of GDP in land revenues, Abel et al. (1989) strongly underestimate Japanese land rents. Data for mixed income in South Korea unfortunately is not available, but capital income including mixed income has been lower than investment since 1980, suggesting strong inefficiency. However, as discussed earlier, South Korea might already be in a stage of capital accumulation. Finally, Australia and Canada have low capital income compared to investment (excluding a few years in the eighties for Australia, and the nineties for Canada). This is surprising as Australia and Canada are not usual suspects for capital over-accumulation. Finally, Figures 12, 13 and 14 show that dynamic inefficiency cannot be rejected for any country (France, Germany, Hungary, Italy, Russia, Norway, Sweden, Switzerland, Belgium, the United Kingdom).

Figure 1: ASSESSING DYNAMIC EFFICIENCY IN AUSTRALIA, CANADA, JAPAN, SOUTH KOREA



**Notes:** The dotted line represents Gross Capital Formation as a % of GDP. The thick continuous line is Capital Income as a % of GDP (including Capital Consumption Allowances), excluding land rents and mixed income coming from labor. Those two are to be compared to assess dynamic efficiency. Because data on mixed income is not always available, I also plot as a thin continuous line Capital Income excluding land rents but including labor income in mixed income (thus overstating economic Capital Income). “Korea” refers to South Korea. Data comes from the OECD, Goldsmith (1985), and author’s calculations.

### 3 Further Calculations

From the upper left hand graph of Figure 1, one could boldly conclude that every advanced economy having an open financial account with Japan is dynamically inefficient. The reasoning is the following. In the absence of financial frictions, any dynamically efficient country could borrow from Japan and invest at a higher rate than Japan.<sup>17</sup> Therefore, a simple arbitrage argument would have all countries be inefficient if one of them is. But why is it then that investment is not always higher than capital income in other advanced economies as well? This section is about discussing other factors influencing the calculation of capital income, without necessarily being able to take a quantitative stance on these factors. In subsection 3.1, I argue that Tobin's average  $Q$  is significantly higher than 1, which has led me to be too sanguine about dynamic efficiency. In subsection 3.2, I review other reasons which might have led me to be too sanguine about dynamic efficiency. In subsection 3.3 I do the opposite and examine the robustness of my calculations to other assumptions, notably about government investment.

#### 3.1 Taking into account Tobin's $Q$

##### 3.1.1 Extension of the Abel et al. (1989) model to decreasing returns

This model differs with Abel et al. (1989) in the production function  $Y_t = F(I_{t-n}^{t-1}, L_t, \theta_t)$ , which has decreasing returns. ( $I_{t-n}^{t-1} = (I_{t-1}, \dots, I_{t-n})$ ) Defining profit:  $\pi_t = Y_t - \frac{\partial F}{\partial L_t} L_t$ , and pure profit:

$$\pi_t^p = Y_t - \frac{\partial F}{\partial L_t} L_t - \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} I_{t-i}.$$

**Proposition 2. (Decreasing returns)** *With decreasing returns to scale, a sufficient condition for dynamic inefficiency is:  $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^p - I_t \leq -\epsilon V_t$*

*Moreover,  $\exists \epsilon > 0, \forall t, \pi - I_t \geq \epsilon V_t$  is not sufficient for dynamic efficiency. A sufficient condition for dynamic efficiency is:  $\exists \epsilon > 0, \forall t, \pi_t - \pi_t^p - I_t \geq \epsilon V_t$ .*

*Proof.* See Appendix A.1. □

##### 3.1.2 Extension of the Abel et al. (1989) model to monopolistic competition

The setup is essentially the same as in the previous section, except that the consumption good is now a constant-elasticity of substitution aggregator of different varieties, and each firm produces one variety monopolistically. More precisely, both young and old consumption goods now are a CES of different varieties  $\omega \in [0, 1]$ , such that:

$$c_t^y = \left[ \int_0^1 c_t^y(\omega)^{\frac{\theta-1}{\theta}} d\omega \right]^{\frac{\theta}{\theta-1}}, \quad c_t^o = \left[ \int_0^1 c_t^o(\omega)^{\frac{\theta-1}{\theta}} d\omega \right]^{\frac{\theta}{\theta-1}}, \quad \theta > 1.$$

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<sup>17</sup>In practice, Japan has a current account surplus but it still is inefficient, perhaps less so today.

Dropping generation subscripts (everything is symmetric for old and young), the demand function for a generic good and the welfare-based price index are:

$$c_t(\omega) = \left( \frac{p_t(\omega)}{p_t} \right)^{-\theta} c_t.$$

$$p_t = \left[ \int_0^1 p_t(\omega)^{1-\theta} d\omega \right]^{\frac{1}{1-\theta}}$$

The model also differs from that of the previous section in that the environment is no longer competitive, but that of monopolistic competition. That is, every variety is produced by a monopolist with a constant-returns-to-scale production function<sup>18</sup>, defining as previously  $I_{t-n}^{t-1}(\omega) = (I_{t-1}(\omega), \dots, I_{t-n}(\omega))$ :

$$y_t(\omega) = F(I_{t-n}^{t-1}(\omega), L_t(\omega), \theta_t).$$

Note that  $\theta_t$  is a productivity shock affecting all corporations equally. Denote the value function of minimizing the labor cost for an individual monopolistic firm  $\omega$ :

$$\Lambda_t(y_t(\omega)) = \min_{L_t(\omega)} \{w_t L_t(\omega) \text{ s.t. } y_t(\omega) = F(I_{t-n}^{t-1}(\omega), L_t(\omega), \theta_t)\}.$$

Again defining aggregate profit as:

$$\pi_t = \int_0^1 \left( \frac{\theta}{\theta-1} \Lambda'_t(y_t(\omega)) y_t(\omega) - \Lambda_t(y_t(\omega)) \right) d\omega,$$

and monopoly profit as:

$$\pi_t^m = \int_0^1 \left( \frac{\theta}{\theta-1} \Lambda'_t(y_t(\omega)) y_t(\omega) - \Lambda_t(y_t(\omega)) \right) d\omega - \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} I_{t-i},$$

allows to state the following proposition:

**Proposition 3. (Monopoly power.)** *With monopolistic competition, a sufficient condition for dynamic inefficiency is  $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^m - I_t \leq -\epsilon V_t$ . Moreover,  $\exists \epsilon > 0, \forall t, \pi - I_t \geq \epsilon V_t$  is not sufficient for dynamic efficiency. A sufficient condition for dynamic efficiency is:  $\exists \epsilon > 0, \forall t, \pi_t - \pi_t^m - I_t \geq \epsilon V_t$ .*

*Proof.* See Appendix A.2. □

### 3.1.3 Discussion of Tobin's $Q$ for the United States

Based on Figure 10 from Hall (2001) (updated by Philippon (2009)), one can notice that Tobin's  $Q$  departs significantly from 1. Unfortunately, Tobin's  $Q$  captures ad-

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<sup>18</sup>Here again, I could generalize to decreasing returns to scale, adding a pure rent component in addition to the monopoly rents profits, but this would only complicate the exposition.

justment costs as well as potential monopoly rents and decreasing returns to scale.<sup>19</sup> Given Hall (2001)'s methodology, the starting value for Tobin's  $q$  is assumed to be 1. Moreover, given Hall (2001) adjustment cost model, which by construction minimizes the distance between Tobin's  $q$  and 1 (subject to constraints), average  $q$  is likely to be underestimated.

Note that in any case, Tobin's  $Q$  cannot be lower than 1 for my purposes - that is, irreversibility of investment which potentially drives Tobin's  $q$  below 1 is of no interest. Decreasing returns to scale or monopoly power cannot be negative.

The discussion about Tobin's  $Q$ , and why it can so persistently depart from 1, would lead us too far for the purpose of this paper. What is important is that while Hall (2001) work in the zero-rent framework, he repeatedly cites monopoly power as a potential microfoundation for adjustment costs. Tobin's  $Q$  might as well capture investment in intangibles that is not taken into account in official investment data - marketing costs are expensed for example, but they bring revenues in the future. In that case, it would be wrong to interpret the entirety of Tobin's  $Q$  as consequential to the presence of rents. However, if investment was abnormally high in the 1990s and early 2000s as some have suggested to explain the "dotcom bubble", then most capital income would have materialized later, which would have dramatically increased the probability of dynamic inefficiency, at least in these years. Bond et al. (2000) question the importance of the intangible channel and instead point to irrational valuations from the part of investment. I believe there is more to the latter story than to the former, therefore I will never consider Tobin's  $Q$  on the order of 3.<sup>20</sup>

### 3.1.4 Robustness to Tobin's $Q$

Because of the difficulties outlined above, I do not want to take an affirmative stand on the value of Tobin's  $Q$ . Given that Japan already provides us with the presumption that advanced economies may well be inefficient, I perform a sensitivity analysis using different parameters of Tobin's  $Q$ .

The results for the United States are on Figure 15, for Japan, Anstralia and Canada on Figure 16. On Figure 11, I plot the fraction of years in which investment exceeds capital income. Graphically, dynamic inefficiency can be rejected for Tobin's  $Q$  equal to 1 in countries for which the line begins from the  $x$  axis. However, note that I here only use the thick line from previous graphs, which means I am missing many years for which those countries were actually inefficient, as suggested by the extrapolated thin line. Dynamic inefficiency cannot be rejected for Tobin's  $Q$  equal to 1 in countries for which the line begins from the  $y$  axis.

<sup>19</sup>Although adjustment costs can be microfounded by patent rights. (?)

<sup>20</sup>Note however that the presence of rational bubbles is possible only with dynamic inefficiency. One could make the following reasoning: either rational bubbles exist and dynamic inefficiency is guaranteed, or they do not and Tobin's  $Q$  should be taken at face value.

### 3.2 Other causes for overstatement of capital income

Capital income is likely to be overestimated for multiple other reasons not mentioned above because they are hard to quantify. However, some evidence suggests that some of them might be of first order significance. To get an idea of the orders of magnitudes involved, overstating capital income by 3% of GDP for the US leads to satisfy the sufficient condition for dynamic inefficiency assuming Tobin's  $Q$  is equal to 1.

**Incidence of corporate taxation.** In the calculations above, taxes on profit are assumed to fall entirely on capital. Hence the revenue they raise is treated as capital income. If instead taxes on profit are borne by workers or consumers, as at least then taxes on profit are not capital income. Given the order of magnitude involved ( $\simeq 2.3\%$  of GDP), tax incidence is not a detail. In fact, if taxes on profit do not fall at all on capital income, then the sufficient condition for dynamic inefficiency is verified in the United States with Tobin's  $Q$  equal to 1.

**Household production in financial services.** Individual investors search for stocks, and put some effort in portfolio management. There is a cost to managing one's wealth, that is not recorded in the national accounts when it is not done professionally. Some individuals indeed spend a lot of time monitoring their financial intermediaries and finding more performing ones, doing their investment in stocks, etc. Other rent their real-estate assets, and provide the labor services of choosing tenants and collecting rents. All this implies that pure capital income tends to be over-estimated. As Piketty and Saez (2013) put it, these efforts should be viewed as informal financial services that are directly supplied and consumed by households; they estimate these financial services to be of the order of **2%-3% of GDP** at the very most.

**Favorable tax treatment of capital income.** As pointed out previously, it is very hard to measure the share of labor in mixed income. This is all the more true that capital income is less heavily taxed than labor income in most tax systems, so that entrepreneurs have a strong incentive to make their labor revenues appear as capital revenues. Hence, revenues from LBOs are usually treated as capital gains, or as revenue accruing to investment, while it usually employs a very qualified workforce to pick these investments and "beat the market".<sup>21</sup> In other words, management fees often understate the return to labor - part of financiers' wages are earned through capital gains. The public finance literature has only begun to investigate this issue (for example Piketty (2011) and Piketty and Saez (2013)).

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<sup>21</sup>Very often, changing the corporate finance structure of the company and loading it with debt helps benefit from the tax deductibility of debt interest.



**Public debt and rational bubbles.** Needless to say, advanced economies have very high levels of debt. To the extent that Ricardian equivalence does not hold in an overlapping generations model, private savings do not perfectly offset these public dissavings. Hence, dynamic inefficiency would likely even be more severe absent our extraordinary levels of public debt. Rational bubbles can similarly crowd out private savings and raise consumption. To the extent that they are a feature of the real world, rational bubbles lessen the severity of dynamic inefficiency.

**Other rents.** Only land rents have been taken out of capital income. But there are other physical rents, of which the World Bank maintains a data series (extensively used by Caselli and Feyrer (2007)). However, it might well be that countries have not reached a steady-state of their resource-extraction path (for example, US oil), so that excluding these rents would actually lead to an underestimation of capital income. For the sake of robustness, I do not attempt at such a calculation here, which would only strengthen my conclusions.

### 3.3 Robustness

**Government investment.** In Appendix G, I take out government investment from Investment series, when available. I also take out Public Capital Consumption Allowances from Capital Income. Note however that such a calculation leads to be too sanguine about dynamic efficiency, as government investment mostly yields private benefits in the form of private capital income. Figures 20, 21, 22 and 23 give the results.

**Data on land.** One might worry that the asset approach leads to overstate the importance of land in value added, especially if land is subject to overvaluation due to animal spirits. This potential limitation applies only to 6 countries for which I use recent OECD data. I perform a number of robustness checks in this direction. In particular I assume that land shares stayed constant after 2000 - when the run-up in house prices began in most advanced economies.

## 4 Discussion

### 4.1 Consistent stylized facts

**Feldstein and Horioka (1980) puzzle.** A puzzle in international finance is that investment and savings are highly correlated in the cross section of countries. This puzzle has been named the Feldstein-Horioka puzzle: investment opportunities have no reason to be present where savings rate are also higher. Current accounts should make up for the differences, but they are relatively small compared to the differences in savings and

investment rates across countries. All this theoretical analysis however assumes that capital is relatively scarce. With dynamic inefficiency, asset supply is in any case too low relative to asset demand, and so even an arbitrarily small amount of home bias translates into so high level of savings-investment correlation.

**Crises.** The severity of dynamic inefficiency seems to be strongly correlated with key macroeconomic events, like banking crises. For example, as the second and third quadrants of Figure 13 show, Norway and Sweden had a dynamic efficiency problem before their banking crises in 1991 – 1993. Japan was the most severely inefficient of all countries in the 80s, and the “lost decade” corresponds to a period of declining investment and increasing consumption.<sup>22</sup>

## 4.2 Potential caveats

An important assumption for the normative implications of dynamic inefficiency is that there be no externalities, or that those are negligible. In contrast, in a capital accumulation model with externalities (e.g. Saint-Paul (1992)), capital income, or private returns to capital accumulation, can be lower than social returns. Increasing public debt in the case of dynamic inefficiency could well lower consumption. A point worthy of note however is that capital externalities are usually a feature of human capital rather than physical capital.

Yet even if one thinks that there are such externalities, the positive consequences of dynamic inefficiency remain, among which the most important one is the possibility of rational bubbles. (even if such are not necessarily Pareto-optimal) With dynamic inefficiency, there is then a strong force that pushes agents to buy overvalued assets, and this *de facto* implements intergenerational transfers from the old to the young, whether they are optimal (neoclassical production function) or not. The question whether policymakers should encourage those transfers is however, no less central.

Finally, it is now well understood that dynamic efficiency does not preclude the existence of rational bubbles. In Farhi and Tirole (2012), dynamic inefficiency is no longer a necessary condition for the existence of bubbles in the presence of financing frictions.

## Conclusion

“Search for yield”, “scarcity of assets”, “abundant liquidity”, all these phrases point to the fact that there might well be too many savings chasing too few assets in the world. Because of high levels of public debt, older people are usually accused of not caring

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<sup>22</sup>Note that going from an above-Golden rule steady state growth path to a Golden-Rule one mechanically decreases output and leads to a recession, even though consumption increases.

enough about their children. In contrast, this paper suggests that they still save too much, relative to the relatively few investment opportunities present in our economies.

Series from the Bureau of Economic Analysis start in 1929, just before the Great Depression; before that, national accounting was very rudimentary. But the difference between capital income and investment was similar in 1929 than it is today. Was capital also overaccumulated at the eve of the Great Depression, after the long period of capital accumulation in the nineteenth century documented in particular by Piketty (2011)?

In any case, dynamic inefficiency invites us to revisit many policy questions. In a world of too much capital accumulation, capital taxation, which is often thought of as a deterrent to capital accumulation, is perhaps not a bad idea after all.<sup>23</sup> At the same time, future capital taxes decrease the value of assets today, and so increase the problem of asset scarcity. Dynamic inefficiency also makes rational bubbles possible (Tirole (1985), Santos and Woodford (1997)): when assets are scarce, any real, financial or even monetary asset can become a locus for bubbles. Other social contracts such as pay-as-you-go systems to replace funded systems can also be Pareto-improving.

Solving the problem of dynamic efficiency is certainly not as straightforward as Diamond (1965) suggested. In practice, the market may have a hard time coordinating on so high implied levels of debt. This is all the more true that this debt is not an infinitely lived asset, and has to be refinanced from time to time. Moreover, it always is subject to the risk of default, even if Hellwig and Lorenzoni (2009) show that the costs of defaulting (being further excluded from the financial markets) far exceeds the benefits with dynamic inefficiency. Moreover, I suspect that markets can very well coordinate on other rational bubbles that are equally fragile; and that public debt does not have a special status in this respect. I leave this very important issue for future research.

Similarly, it is tempting to back out an average return on capital from observed wealth income ratios as measured for example by Piketty and Zucman (2014), and capital income: it would be Capital Income as a percentage of GDP divided by Wealth Income ratios as a % of GDP. This approach is however misleading, as not all future capital income is capitalized in today's capital values, like revenues from future ideas and companies. For example, assume that assets are trees decaying at a rate  $\delta$ , giving a first dividend equal to  $R$ , and that new ideas come up each period to replace those decaying investments, as in Tirole (1985). Then the method outlined above would lead one to overstate return on capital by an amount  $\delta$ , since the capitalized value of assets would be  $R/(r + \delta)$  and capital income would be  $R$  in each period.

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<sup>23</sup>However note that Piketty and Saez (2013) recommend in their Appendix to use public debt for dealing with dynamic inefficiency, to restore the Golden Rule level of capital accumulation; and use capital taxes for redistribution motives. There is a dichotomy with 2 instruments and 2 objectives.

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## A Proofs

### A.1 Proof of Proposition 1

I will go over the sufficient condition for dynamic inefficiency in detail, since the condition is less restrictive as in Abel et al. (1989). The proof for the sufficient condition for dynamic efficiency is very similar.

*Proof.* Assume that:  $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^p - I_t \leq -\epsilon V_t$ . Then, by definition:

$$\exists \epsilon > 0, \forall t \in \mathbb{N}, \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} I_{t-i} - I_t \leq -\epsilon V_t \leq -\epsilon I_t$$

Therefore:

$$\exists \epsilon > 0, \forall t \in \mathbb{N}, \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1 - \epsilon.$$

Let us consider now an increase in consumption financed by a decrease in investment (so that it is resource-feasible), and the size of this increase be  $\delta > 0$ ,  $dc_0 = \delta$  small  $\Rightarrow dC_0 = \delta L_0, dI_0 = -\delta L_0$ . To make this change Pareto-improving, one has to make up for the decrease in output in the following periods by reducing investment as well. From the production function, production in period 1 decreases by  $dY_1 = \frac{\partial F}{\partial I_0} dI_0$ , and so for unchanged consumption  $dC_1 = 0$ ,  $dI_1 = \frac{\partial F}{\partial I_0} dI_0$ . More generally, today's output will be reduced by the  $n$  previous reductions in investment of all vintages. The general formula is:

$$\begin{aligned} dI_t &= \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} dI_{t-i} \\ \Rightarrow \frac{dI_t}{I_t} &= \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \frac{dI_{t-i}}{I_{t-i}}. \end{aligned}$$

Of course, there is a limit to how much one can reduce investment in each period, since there is a non-negativity constraint on investment. Since  $\forall t \in \mathbb{N}, \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1 - \epsilon$ , such a change is feasible for  $\delta > 0$  sufficiently low.  $\square$

Once can similarly extend Abel et al. (1989) to sufficient conditions for efficiency, redefining profit in an adequate manner.

### A.2 Proof of Proposition 2

Again, I will only go over the proof for dynamic inefficiency.

*Proof.* Let us first calculate each firm's profit. Facing demand for its product as:

$$y_t(\omega) = \left( \frac{p_t(\omega)}{p_t} \right)^{-\theta} y_t,$$

a monopolistic firm chooses  $p_t(\omega)$  so as to maximize its profit, that is:

$$\max_{p_t(\omega)} \left\{ p_t(\omega) \left( \frac{p_t(\omega)}{p_t} \right)^{-\theta} y_t - \Lambda \left( \left( \frac{p_t(\omega)}{p_t} \right)^{-\theta} y_t \right) - \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} I_{t-i}(\omega) \right\}.$$

The firm charges a markup over marginal labor cost:

$$p_t(\omega) = \frac{\theta}{\theta - 1} \Lambda'_t(Y_t(\omega)).$$

Monopoly profits for a single firm are therefore:

$$\pi_t^m(\omega) = \frac{\theta}{\theta-1} \Lambda'_t(y_t(\omega)) y_t(\omega) - \Lambda_t(y_t(\omega)) - \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} I_{t-i}(\omega).$$

Total profits for a monopolistic firm, including returns to capital are:

$$\pi_t(\omega) = \frac{\theta}{\theta-1} \Lambda'_t(y_t(\omega)) y_t(\omega) - \Lambda_t(y_t(\omega)).$$

Using that the returns on capital for each firm are equalized because it is supplied competitively, I can sum over  $\omega$  to find:

$$\pi_t^m = \int_0^1 \pi_t^m(\omega) d\omega = \int_0^1 \left( \frac{\theta}{\theta-1} \Lambda'_t(y_t(\omega)) y_t(\omega) - \Lambda_t(y_t(\omega)) \right) d\omega - \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} I_{t-i}.$$

Now as in the previous proof, assume that:  $\exists \epsilon > 0, \forall t \in \mathbb{N}, \pi_t - \pi_t^m - I_t \leq -\epsilon V_t$ . Therefore:

$$\exists \epsilon > 0, \forall t \in \mathbb{N}, \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1 - \epsilon.$$

Let us consider now an increase in consumption financed by a decrease in investment in each monopolistic firm (so that it is resource-feasible), and the size of this increase be  $\delta > 0$ ,  $dc_0 = \delta$  (per-capita consumption) small  $\Rightarrow dC_0 = \delta L_0$ , so that the aggregate decrease in investment must be  $dI_0 = -\delta L_0$ . Let us split this decrease in investment equally among firms so that  $dI_0 = \int_0^1 dI_0(\omega) d\omega$ . To make this change Pareto-improving, one has to make up for the decrease in output in each firm in the following periods by reducing investment as well. From the production function, production in period 1 decreases by  $dY_1(\omega) = \frac{\partial F}{\partial I_0} dI_0(\omega)$  in each firm, and so for unchanged consumption  $dC_1 = 0$ ,  $dI_1(\omega) = \frac{\partial F}{\partial I_0} dI_0(\omega)$  in order to maintain the same level of production in each firm. More generally, today's output will be reduced by the  $n$  previous reductions in investment of all vintages. The general formula is:

$$\forall \omega \in [0, 1], dI_t(\omega) = \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} dI_{t-i}(\omega).$$

Summing, and because marginal returns to capital are equalized across firms:

$$\begin{aligned} dI_t &= \int_0^1 dI_t(\omega) d\omega = \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} \int_0^1 dI_{t-i}(\omega) d\omega = \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} dI_{t-i} \\ &\Rightarrow \frac{dI_t}{I_t} = \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \frac{dI_{t-i}}{I_{t-i}}. \end{aligned}$$

Of course, there is a limit to how much one can reduce investment in each period, since there is a non-negativity constraint on investment. Since  $\forall t \in \mathbb{N}, \sum_{i=1}^n \frac{\partial F}{\partial I_{t-i}} \frac{I_{t-i}}{I_t} \leq 1 - \epsilon$ , such a change is feasible for  $\delta > 0$  sufficiently low.  $\square$

## B Reproducing Abel et al. (1989) step by step

### B.1 United States

Abel et al. (1989) use data from the National Income and Products Accounts (NIPA). Raw data from Table 1 in this paper is reproduced in Figure 2. Figure 2 updates this data with contemporaneous data given by the Bureau of Economic Analysis. The numbers are very similar; this confirms that I am using a very similar methodology for calculating capital income. In Figure 3, I use their estimate of land rents (5% of GDP), which I subtract from capital income. Finally, I use more recent data on land rents in Figure 4.

Figure 2: REPRODUCING AND UPDATING ABEL ET AL. (1989)

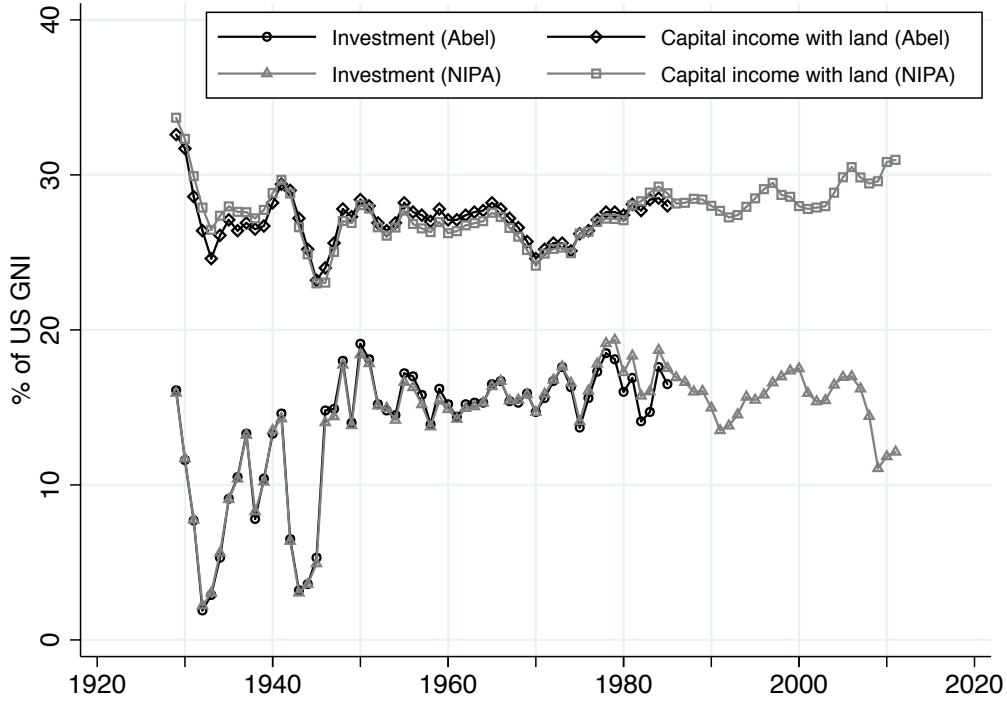




Figure 3: REMOVING LAND RENTS AT 5% OF GDP

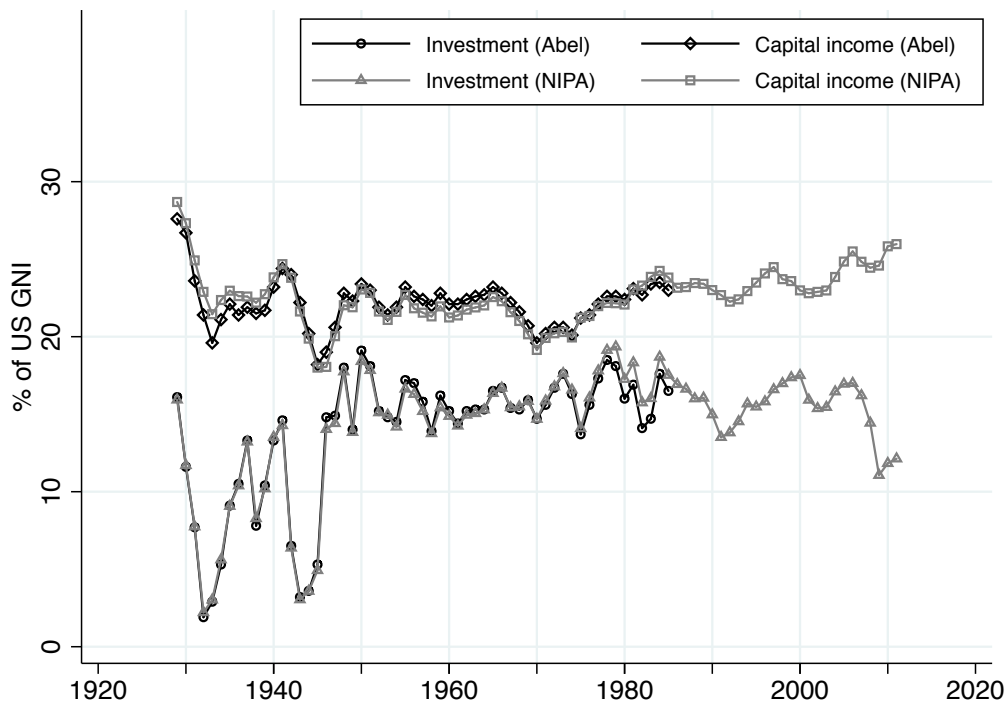


Figure 4: USING OECD DATA FOR LAND RENTS

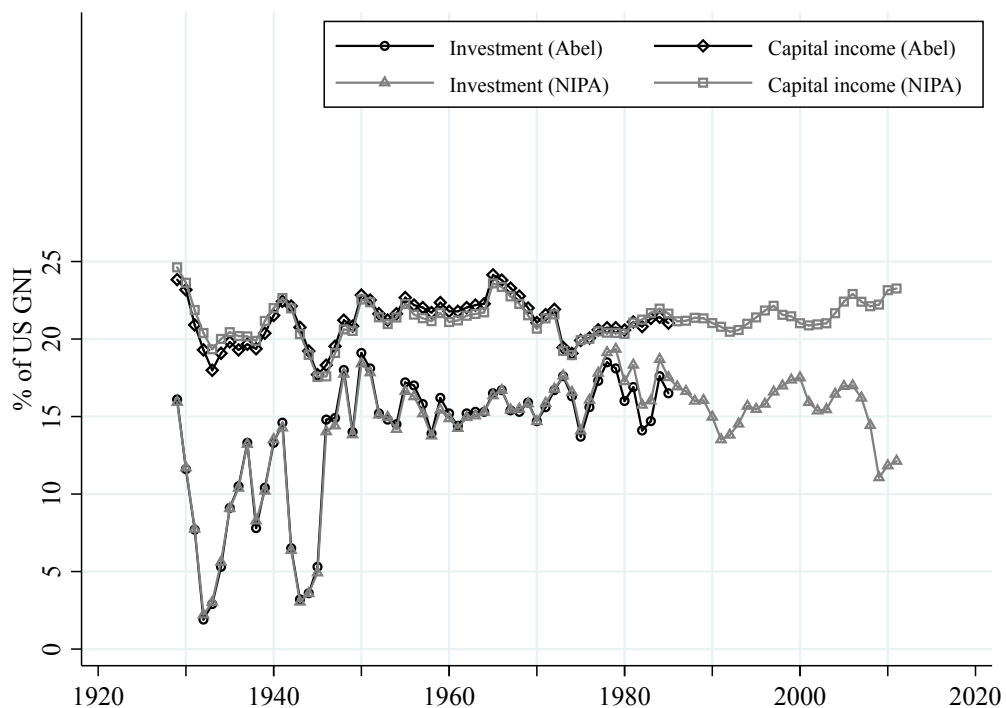


Figure 5: USING OECD DATA FOR MIXED INCOME

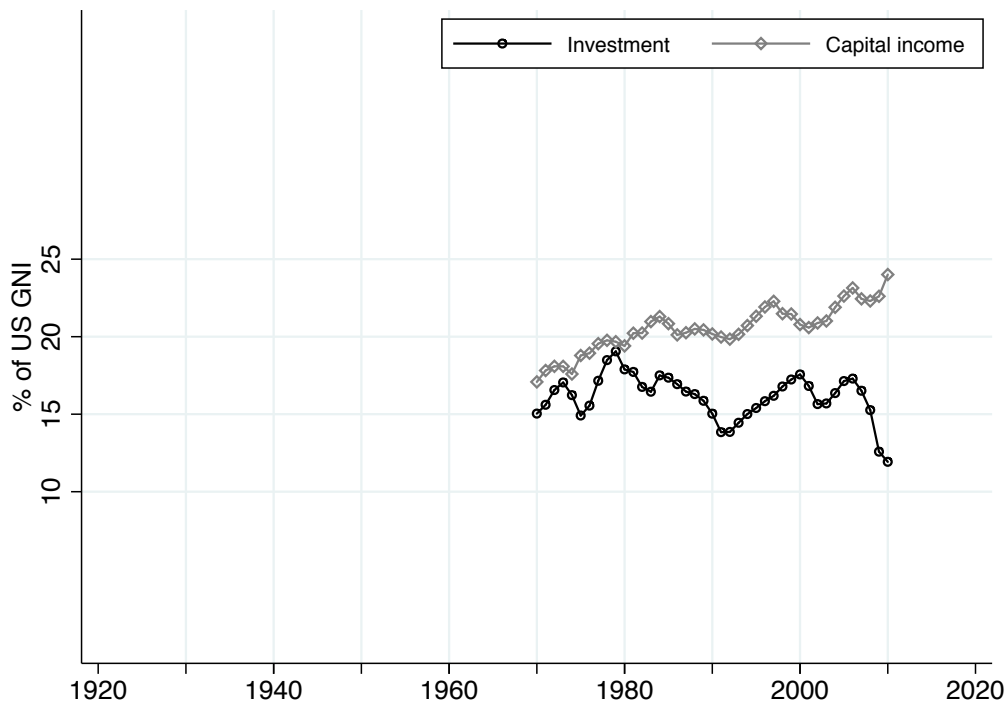


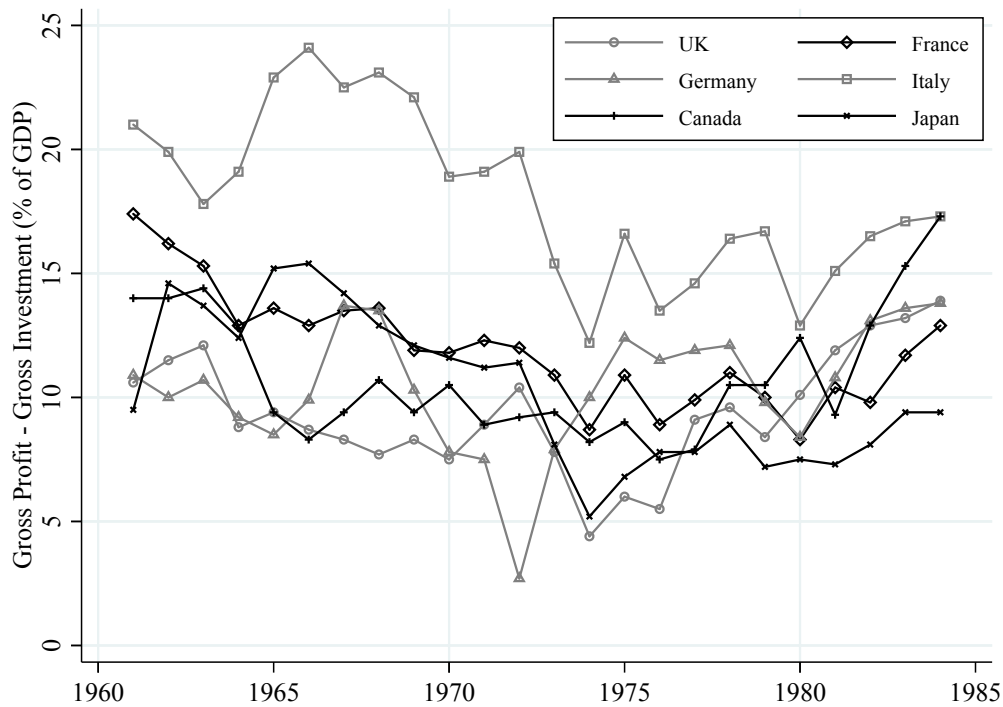
Figure 6: PREFERRED SPECIFICATION, ASSUMING TOBIN'S  $Q = 1$



## B.2 UK, France, Germany, Canada, Italy, Japan

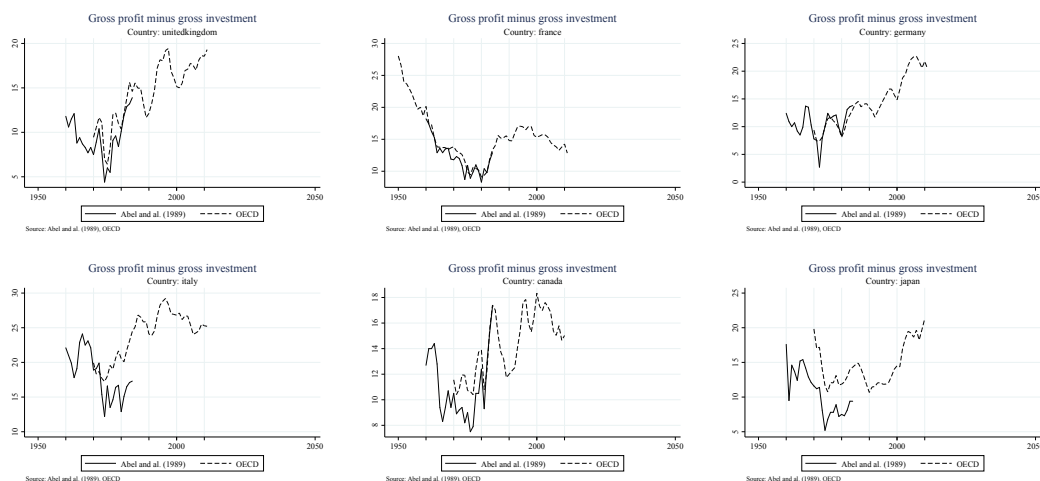
They then use gross investment and gross capital income coming from OECD database. Results from Table 3 are given in Figure 7. Note that even with their data, and taking an estimate of 5% of GDP for land rents and 4% of GDP for labor mixed income, dynamic efficiency could not be confirmed in Japan, Germany, the United Kingdom, and Canada.

Figure 7: PLOTTING RESULTS FROM TABLE 3



As shown on Figure 8, my estimates sometimes coincide, sometimes are higher for capital income than theirs. If anything, overstating capital income goes against the conclusions of this paper, and should only reinforce them.

Figure 8: REPRODUCING FOR 6 OECD ECONOMIES



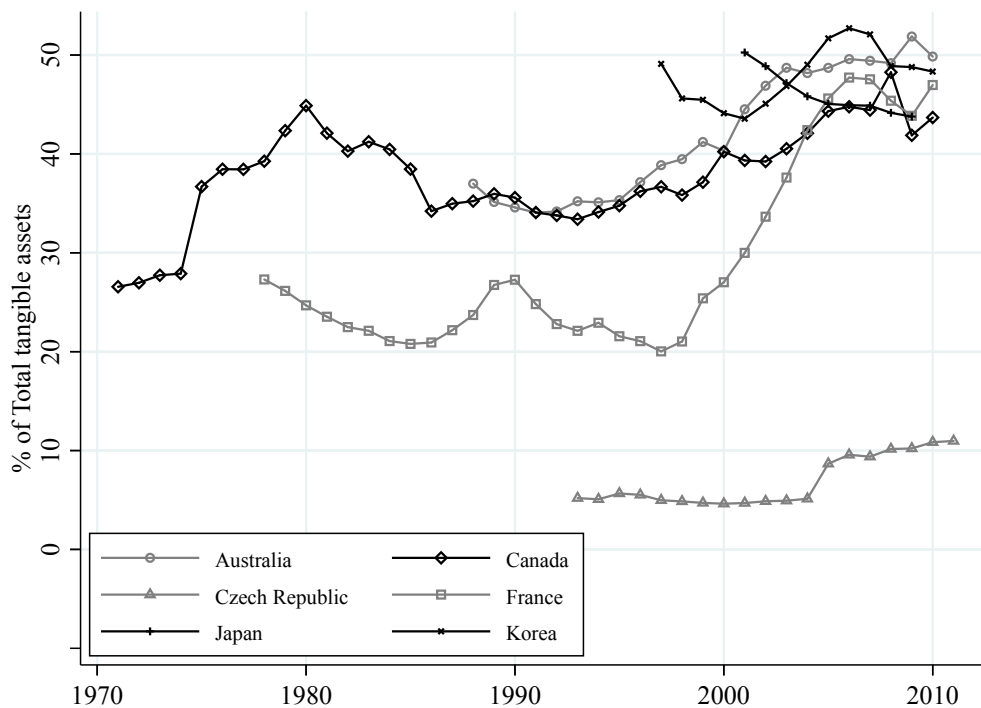
## C Data on land

Data on land comes mainly from two sources:

- OECD for 6 countries (Australia, Canada, Czech Republic, France, Japan, Korea). Data for OECD is plotted on Figure 9.
- Goldsmith (1985) *Comparative National Balance Sheets*, for 20 countries (with two dates 1973 and 1978).

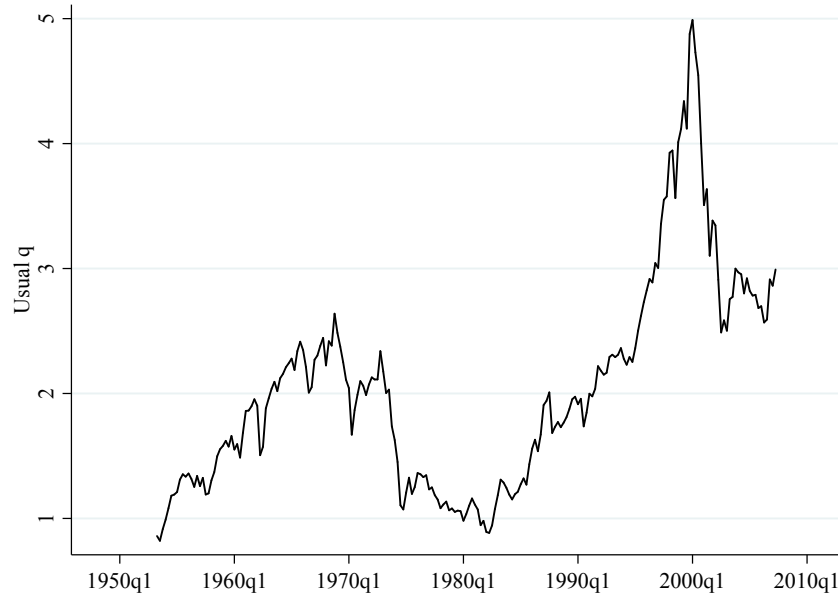
For other countries, I take the lowest value in 1973 or 1978 from Goldsmith for all years. This is very conservative, especially for the last decade. Furthermore, Goldsmith underestimates relative to OECD (although it displays a similar evolution).

Figure 9: LAND AS A % OF TOTAL ASSETS



## D Data on Tobin's $Q$

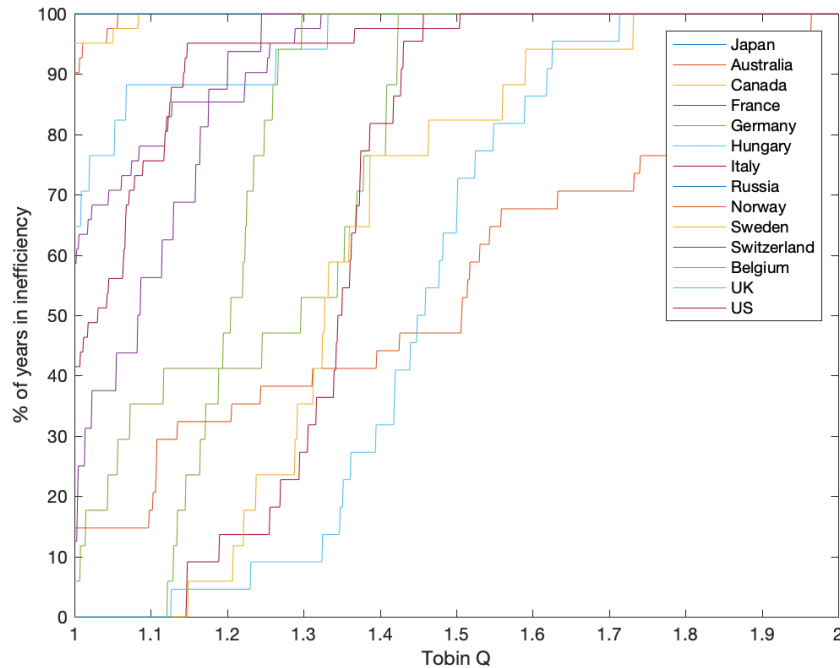
Figure 10: TOBIN'S  $Q$ , USING EQUITY VALUES.



## E Inefficiency as a function of Tobin's $Q$

Figure 11 plots the fraction of years for which investment exceeds capital income in the data, as a function of assumed Tobin's  $Q$ .

Figure 11: FRACTION OF YEARS (IN %) IN WHICH INVESTMENT  $>$  CAPITAL INCOME

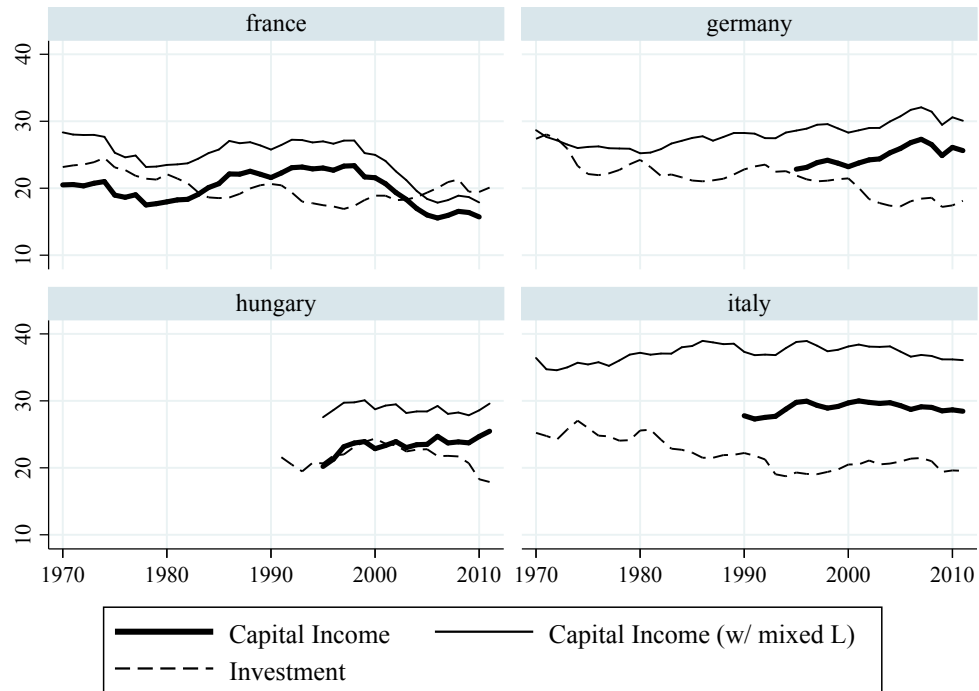


## F More countries

### F.1 Tobin's $Q$ equal to 1

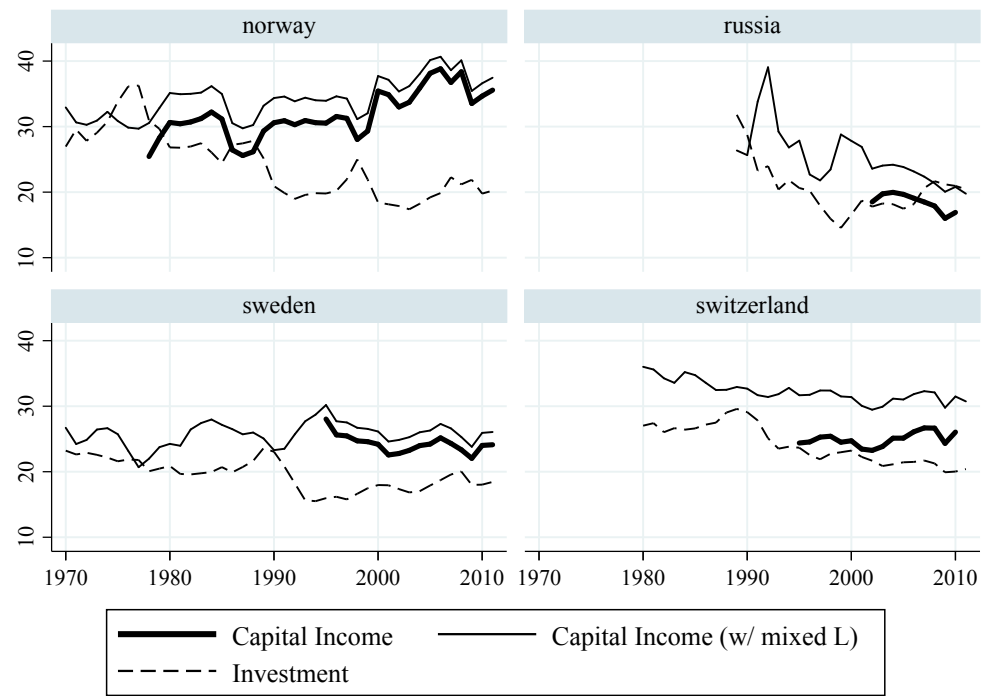
Capital income and investment in countries for which there is data are plotted in Figures 12, 13 and 14 (reminder: data for 4 inefficient economies is plotted in Figure 1).

Figure 12: ASSESSING DYNAMIC EFFICIENCY IN FRANCE, GERMANY, HUNGARY AND ITALY



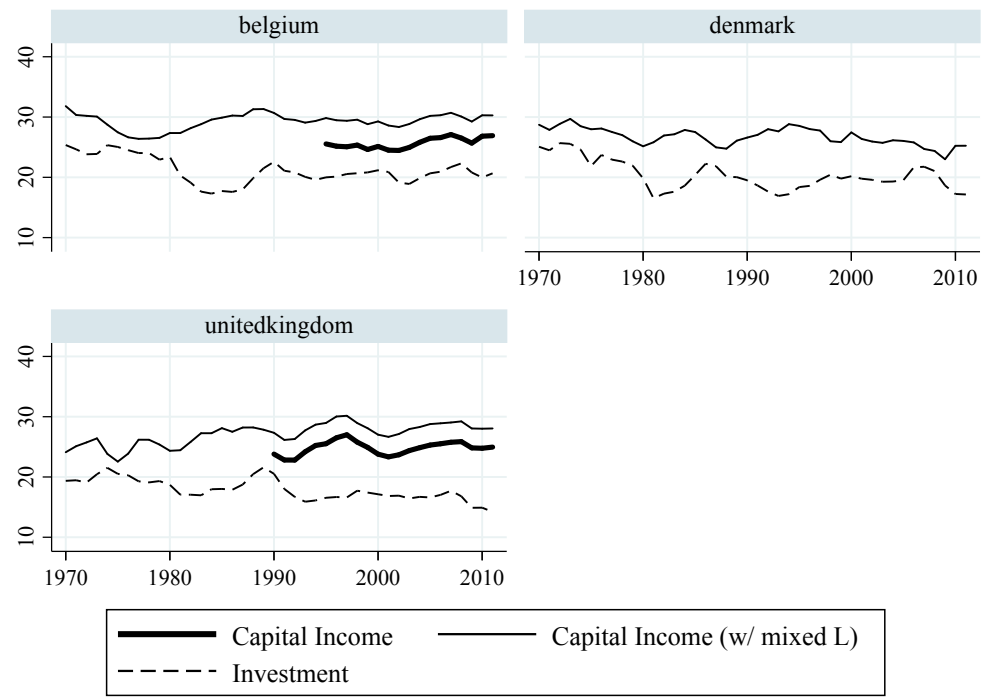
**Notes:** See Figure 1.

Figure 13: ASSESSING DYNAMIC EFFICIENCY IN RUSSIA, NORWAY, SWEDEN, SWITZERLAND



Notes: See Figure 1.

Figure 14: ASSESSING DYNAMIC EFFICIENCY IN BELGIUM, DENMARK, THE UNITED KINGDOM



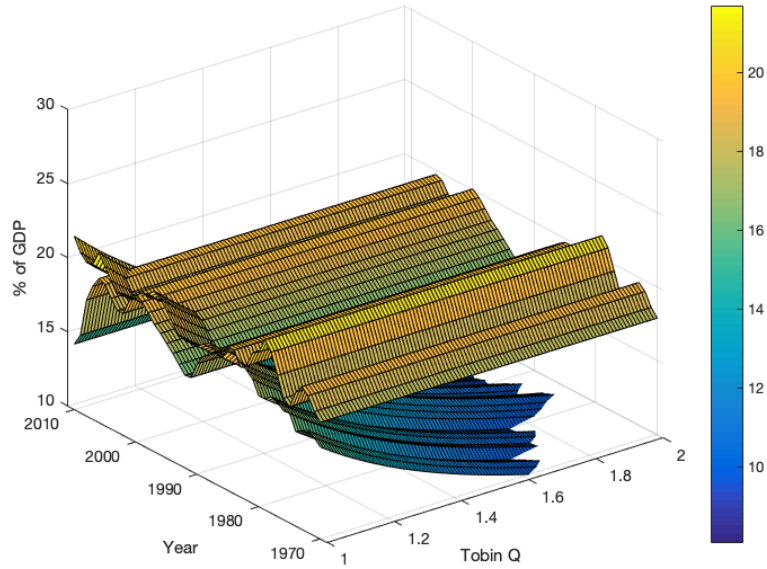
Notes: See Figure 1.



## F.2 Varying the Tobin's $Q$

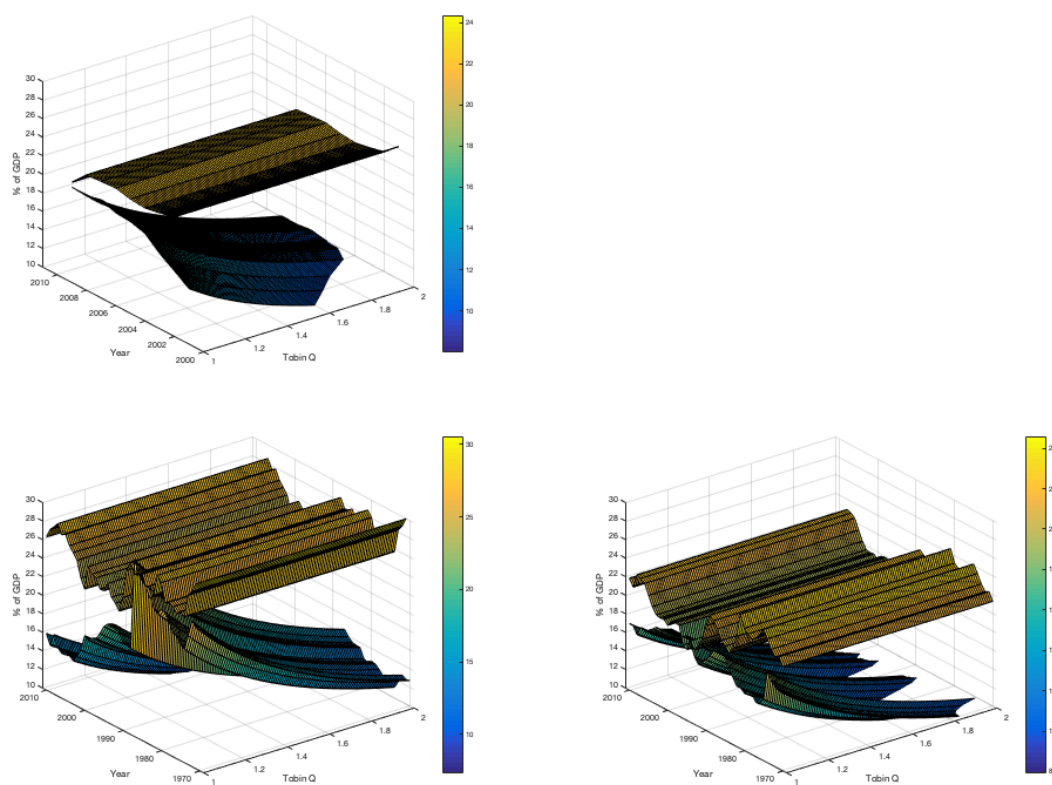
Tobin's  $Q$  is allowed to vary in Figures 17, 18 and 19. Inefficient economies are plotted on Figure 16.

Figure 15: DYNAMIC EFFICIENCY AS A FUNCTION OF TOBIN'S  $Q$  IN THE US



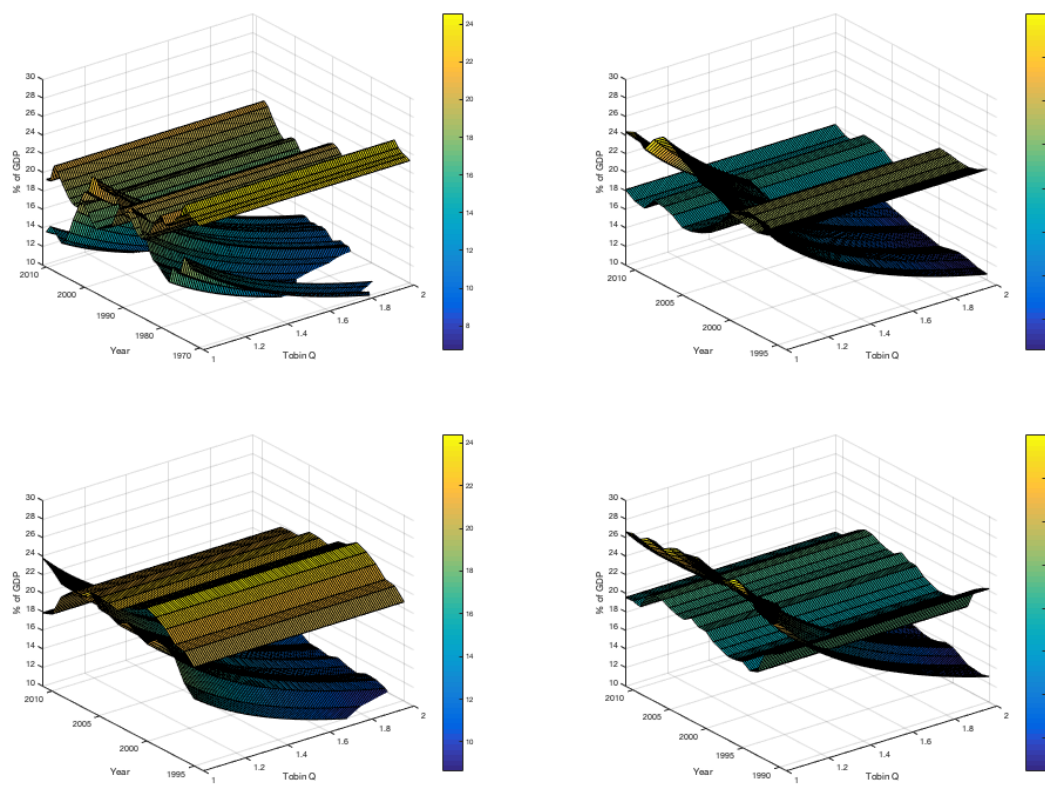
**Notes:** This 3D graph represents capital income and investment (as a % of GDP), capital income coming from investment being an hyperbolic function of Tobin's  $Q$ . If average Tobin's  $Q$  is superior to 1.5, then capital income is lower than investment for all years, and therefore the US economy is dynamically inefficient. The data is from OECD Goldsmith (1985), and author's calculations.

Figure 16: DYNAMIC EFFICIENCY AS A FUNCTION OF TOBIN Q IN JAPAN, AUSTRALIA AND CANADA



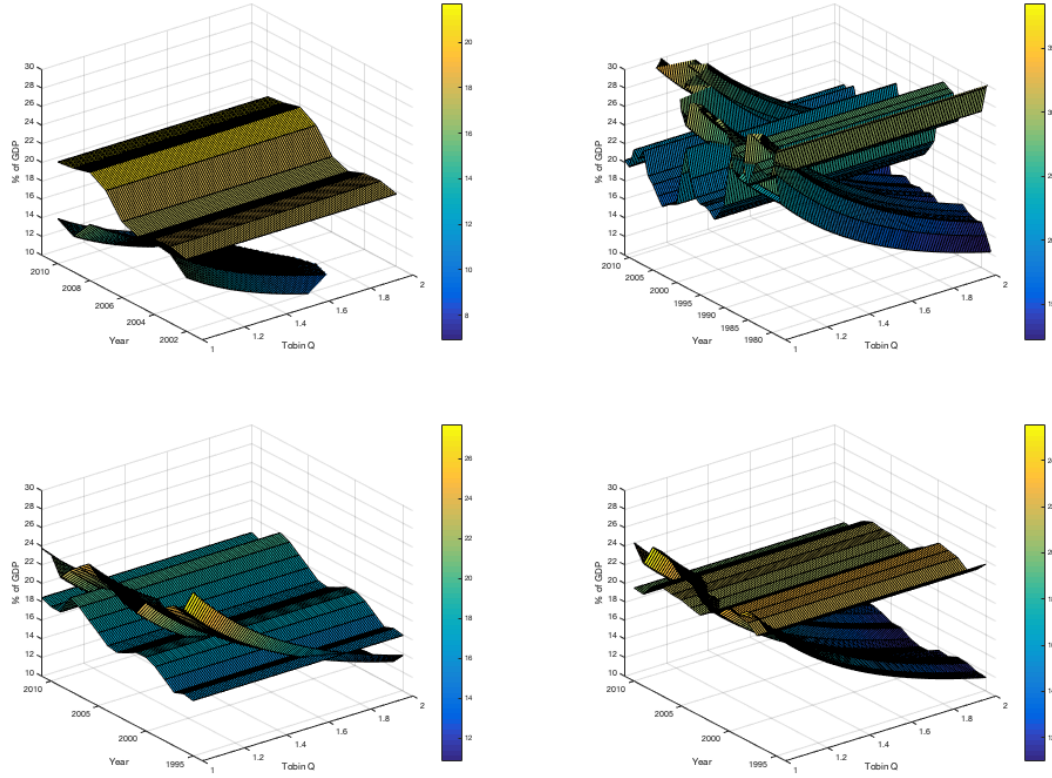
Notes: See Figure 15.

Figure 17: ASSESSING DYNAMIC EFFICIENCY IN FRANCE, GERMANY, HUNGARY AND ITALY



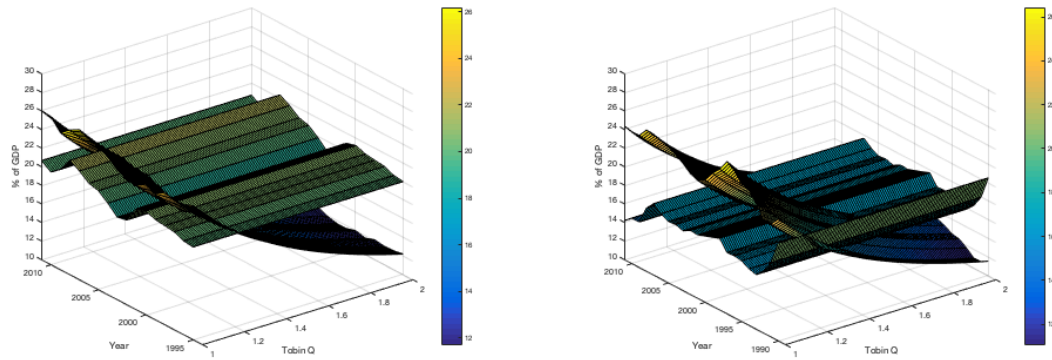
Notes: See Figure 15.

Figure 18: ASSESSING DYNAMIC EFFICIENCY IN RUSSIA, NORWAY, SWEDEN, SWITZERLAND



Notes: See Figure 15.

Figure 19: ASSESSING DYNAMIC EFFICIENCY IN BELGIUM, AND THE UNITED KINGDOM

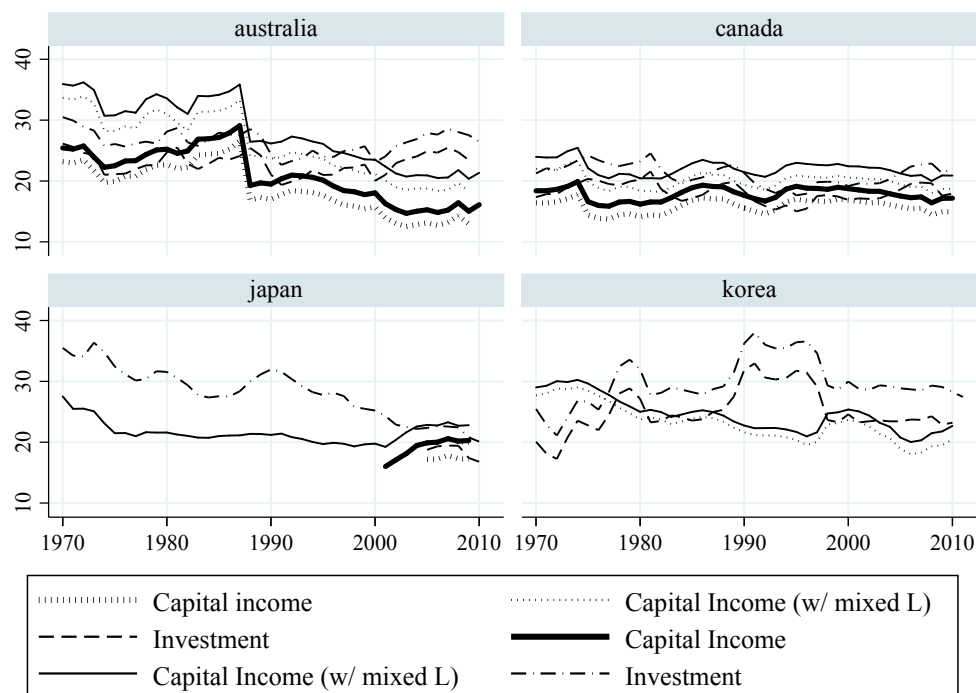


Notes: See Figure 15.

## G Robustness: Government investment

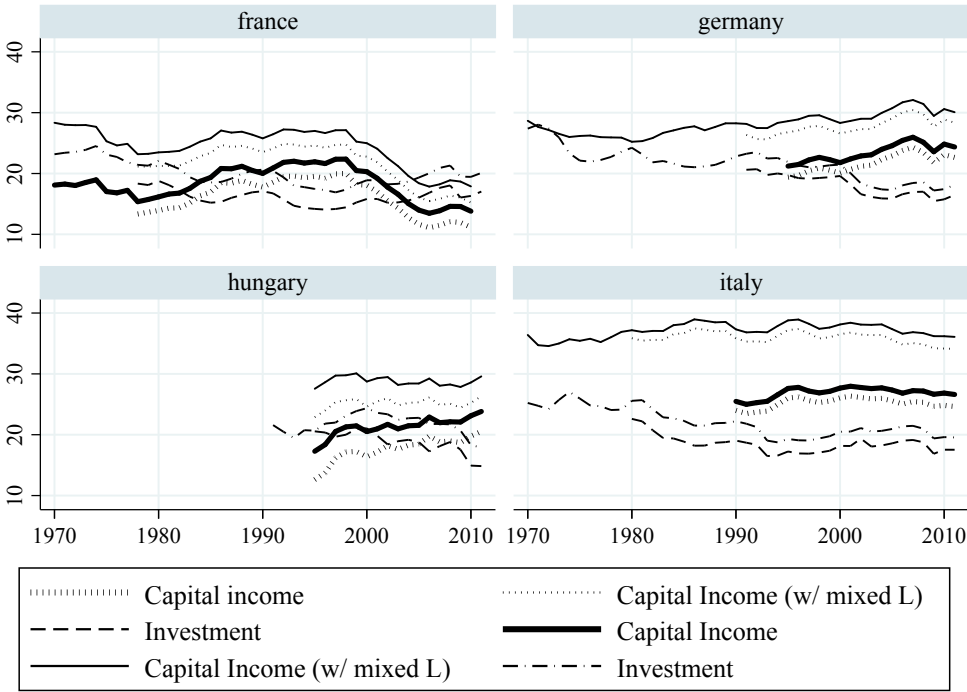
As Abel et al. (1989), I have used private investment when using the Bureau of Economic Analysis (NIPA) data, and investment (including private and public) when using OECD data. This is because government investment series and capital consumption expenditures for government are not always available for all countries. However, I show here that government investment isn't driving dynamic inefficiency. In other words, it is not the case that government investment is so inefficient that it explains why capital income is low compared to investment. Moreover, there are many reasons to believe that private capital income includes some returns to government investment, as firms need roads to operate, for example. Results are shown in Figures 20, 21, 22 and 23.

Figure 20: ASSESSING DYNAMIC EFFICIENCY IN JAPAN, SOUTH KOREA, AUSTRALIA, CANADA



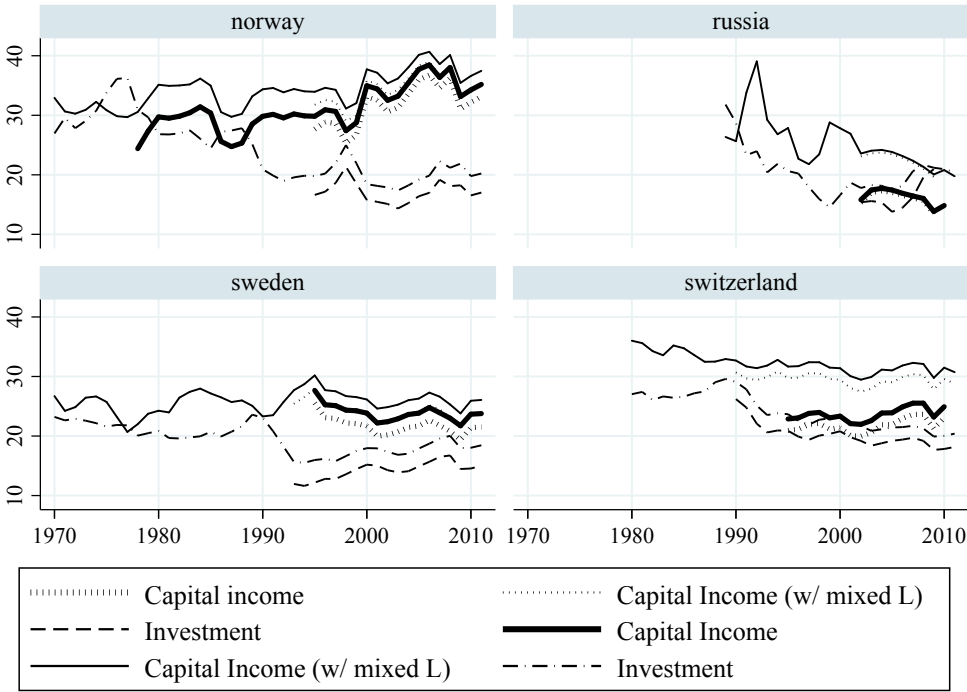
**Notes:** The dotted line represents Gross Capital Formation as a % of GDP. The dash-dotted line is total investment, excluding public investment. The thick continuous line is Capital Income as a % of GDP, excluding land rents and mixed income coming from labor. The thin continuous line is Capital Income excluding land rents but including labor income in mixed income. "Korea" refers South Korea. The thick dotted line is Capital income as a % of GDP excluding Government Consumption Allowances, and the thin line as before includes labor income in mixed income. Data comes from OECD, Goldsmith (1985), and author's calculations.

Figure 21: ASSESSING DYNAMIC EFFICIENCY IN FRANCE, GERMANY, HUNGARY AND ITALY



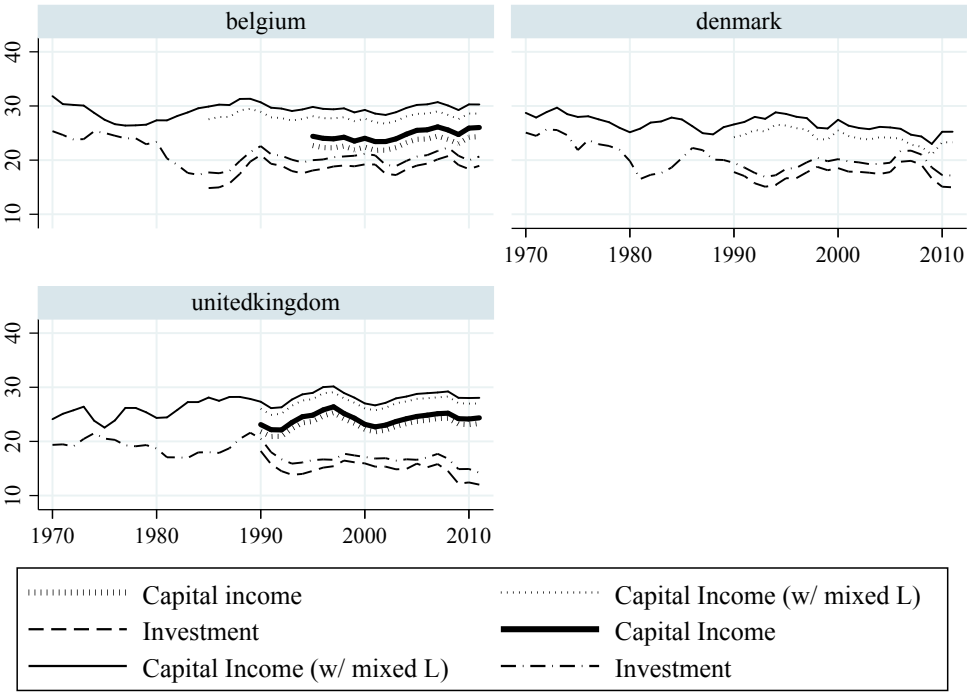
Notes: See Figure 20.

Figure 22: ASSESSING DYNAMIC EFFICIENCY IN RUSSIA, NORWAY, SWEDEN, SWITZERLAND



Notes: See Figure 20.

Figure 23: ASSESSING DYNAMIC EFFICIENCY IN BELGIUM, DENMARK, THE UNITED KINGDOM



Notes: See Figure 20.