Lecture 10: Saving, Capital Accumulation and Output

Intermediate Macroeconomics, Econ 102

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Are Americans saving too little?

- Since 1970, the **saving rate** – the **ratio of saving to GDP** – in the **US** has averaged only **17%**, compared to **22%** in **Germany** and **30%** in **Japan** (note that $CA = S - I$, as we shall see later).
- One reason that is often cited for arguing that the U.S. saves too little is that it leads to too little capital accumulation, which lowers the level of output. Such an effect was not present in the Keynesian model we saw so far: saving was only viewed as a **drag on demand**.
- Indeed, saving feeds investment (in a closed economy), investment feeds the capital stock, and if:

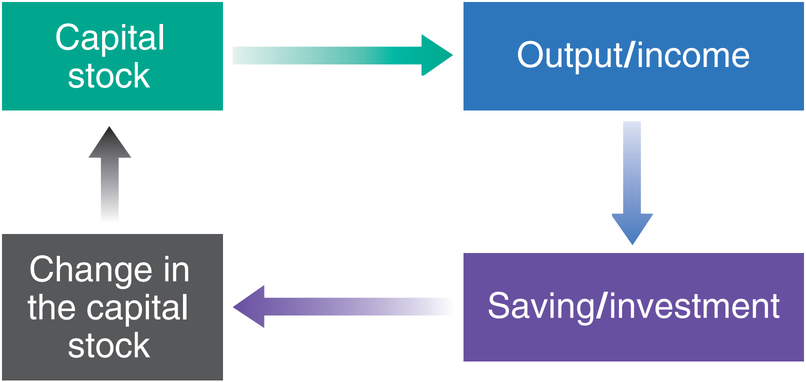
$$Y_t = f(K_t, N_t)$$

  then more capital leads to more output. So if this production function holds, then deferring consumption must also be a good thing, in that it allows to produce more output in the future.
- This is the logic in the Solow (1956) growth model, which we shall go through today.

The idea of the Solow (1956) Model

- Output in the long run depends on two relations between output and capital:
  - The amount of capital determines the amount of output being produced.
  - The amount of output being produced determines the amount of saving, which in turn determines the amount of capital being accumulated over time.
- Therefore, saving, expecially at the beginning of economic development, starts a very beneficial feedback loop. Even if the saving **rate** is constant, then more output leads to more saving.

Capital, Output, and Saving/Investment



The Effects of Capital on Output

- In the previous lecture, we saw that:

$$\frac{Y}{N} = F\left(\frac{K}{N}, 1\right) = f\left(\frac{K}{N}\right)$$

  with $f(x) \equiv F(x, 1)$.

- Assume that N is constant, including:
  - size of the population.
  - participation rate.
  - unemployment rate. (this is why it is a model about the **long run**)
- Assume also that there is no technological progress, so f does not change over time:

$$\frac{Y_t}{N} = f\left(\frac{K_t}{N}\right)$$

- In this model, capital accumulation is good, in that higher capital per worker leads to higher output per worker.

Output and Investment

- Assume:
  - The economy is closed:

$$I = S + (T - G)$$

  - Public saving $(T - G)$ is 0:

$$I = S$$

  - Private saving is proportional to income:

$$S = sY$$

- Combining these two and introducing a time index:

$$\boxed{I_t = sY_t}$$

- Investment is proportional to output.
- The higher (lower) output is, the higher (lower) is saving and so the higher (lower) is investment.

Investment and Capital Accumulation

- The evolution of the capital stock is:

$$K_{t+1} = (1-\delta)\, K_t + I_t$$

- Replace investment by the above expression and divide both sides by N:

$$\frac{K_{t+1}}{N} = (1-\delta)\,\frac{K_t}{N} + s\frac{Y_t}{N}$$
$$\Rightarrow \quad \boxed{\frac{K_{t+1}}{N} - \frac{K_t}{N} = s\frac{Y_t}{N} - \delta\frac{K_t}{N}}$$

- The change in the capital stock per worker is equal to saving per worker minus depreciation.

Dynamics of Capital and Output

- Combining the above equations:

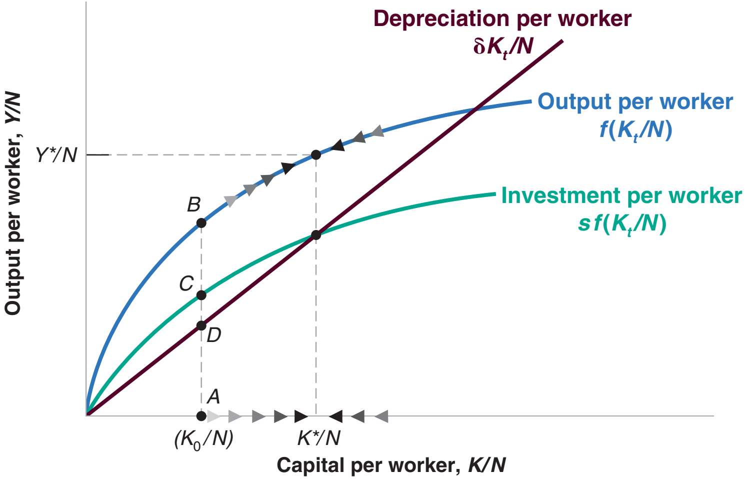
$$\underbrace{\frac{K_{t+1}}{N} - \frac{K_t}{N}}_{\text{Change in capital}} = \underbrace{sf\left(\frac{K_t}{N}\right)}_{\text{Investment}} - \underbrace{\delta\frac{K_t}{N}}_{\text{Depreciation}}$$

- There are 3 cases:
  1. If investment per worker is less than depreciation per worker, the change in capital per worker is positive. Capital per worker increases.
  2. If investment per worker is less than depreciation per worker, the change in capital per worker is negative. Capital per worker decreases.
  3. If investment per worker is equal to depreciation per worker, then the change in capital per worker is negative. Capital per worker stays constant.

- Investment minus depreciation is sometimes called net investment. To avoid any ambiguity, investment is often called **gross investment**.

Capital and Output Dynamics

- When capital and output are low, investment exceeds depreciation and capital increases. When capital and output are high, investment is less than depreciation and capital decreases.



Dynamics of Capital and Output

- The state in which output per worker and capital per worker are no longer changing is called the **steady state** of the economy. The steady-state value of capital per worker is such that the amount of saving per worker is sufficient to cover depreciation of the capital stock per worker:

$$sf\left(\frac{K^*}{N}\right) = \delta \frac{K^*}{N}.$$

- The steady-state value of output per worker Y^*/N, as a function of K^*/N is given by:

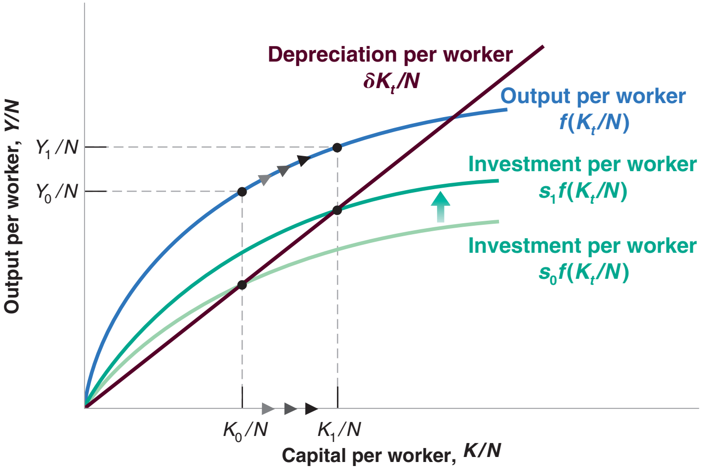
$$\frac{Y^*}{N} = f\left(\frac{K^*}{N}\right).$$

Three lessons from Solow (1956)

1. The saving rate has no effect on the long-run growth rate of output per worker, which is equal to zero:
    - This is coming from convergence.
    - Sustaining a constant growth rate would require capital to grow even faster.
2. The saving rate determines the level of output per worker in the long run:
    - Graphically
    - Algebraically.
3. An increase in the saving rate will lead to higher growth of output per worker for some time, but not forever.

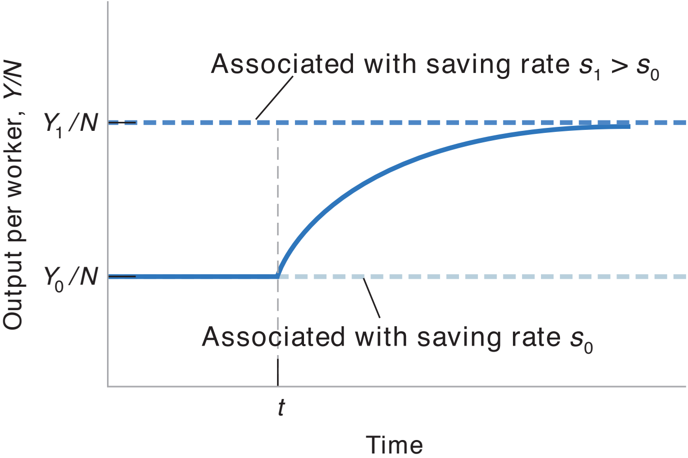
The Effects of Different Saving Rates

- A country with a higher saving rate achieves a higher steady-state level of output per worker.



The Effects of an Increase in the Saving Rate on Output per Worker in an Economy Without Technological Progress

- An increase in the saving rate leads to a period of higher growth until output reaches its new higher steady-state level.



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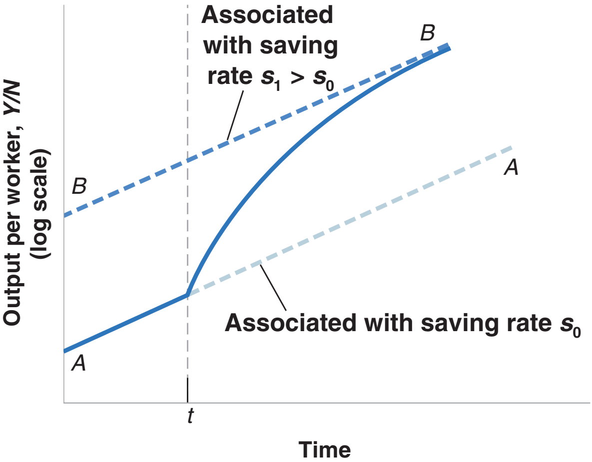
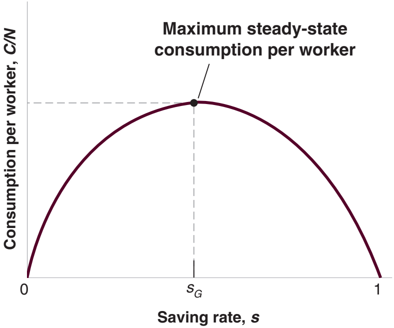


The Saving Rate and Consumption

- What matters to people is not how many is produced, but how much they consume.
- Governments may affect the saving rate by:
  - changing public saving (budget surplus).
  - using capital taxes to affect private saving (untaxed retirement accounts, etc.).
  - redistributing between high and low income earners.
- **Golden-rule level of capital**: The level of capital associated with the saving rate that yields the highest level of consumption in steady state.

The Effects of the Saving Rate on Steady-State Consumption per Worker

- An increase in the saving rate leads to an increase, then to a decrease in steady-state consumption per worker.



What is the Golden Rule Level of Capital

- For a saving rate between zero and the golden-rule level, a higher saving rate leads to higher capital per worker, higher output per worker and higher consumption per worker.
- For a saving rate greater than the golden-rule level, a higher saving rate still leads to higher capital per worker and output per worker, but lower consumption per worker.
- An increase in the saving rate leads to lower consumption for some time but higher consumption later.

FOCUS: Social Security, Saving, and Capital Accumulation in the United States

- Social Security, introduced in 1935, has led to a lower U.S. saving rate and thus lower capital accumulation and lower output per person in the long run.
- Social Security is a **pay-as-you-can system** that taxes workers and redistributes the tax contributions as benefits to current retirees, resulting in lower private saving as workers anticipate receiving benefits when they retire.
- An alternative is a **fully-funded** system that pays back the principal plus interest to the workers when they retire, resulting in lower private saving but higher public saving as the System invests their contributions in financial assets.

Putting some numbers

- Assume the production function f:

$$Y = \sqrt{K}\sqrt{N}$$

so that the previous equation becomes:

$$\frac{K_{t+1}}{N} - \frac{K_t}{N} = s\sqrt{\frac{K_t}{N}} - \delta\frac{K_t}{N}$$

which describes the evolution of capital over time.

- The previous equation implies that capital per worker in the steady state (K^*/N) becomes:

$$\frac{K^*}{N} = \left(\frac{s}{\delta}\right)^2$$

- Combining the previous two equations gives the steady state output per worker:

$$\frac{Y^*}{N} = \sqrt{\frac{K^*}{N}} = \sqrt{\left(\frac{s}{\delta}\right)^2} = \frac{s}{\delta}$$

- In the long run, output per worker doubles when the saving rate doubles.

- What is the **Golden Rule Level of capital**, and the corresponding Golden Rule level of saving?
- This Golden Rule Level of capital maximizes the level of steady state consumption given by:

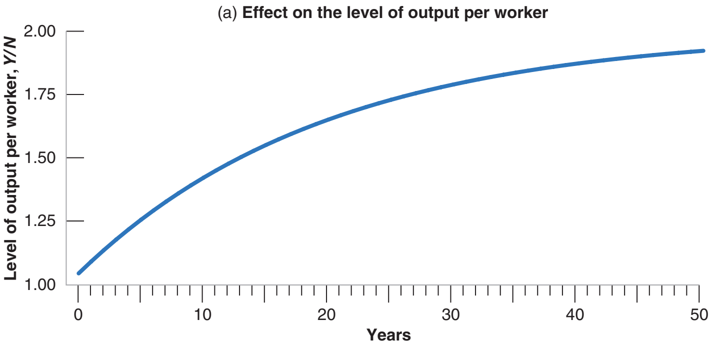
$$\frac{C^*}{N} = (1-s)\frac{Y^*}{N} = (1-s)\frac{s}{\delta}$$

- $s(1-s)$ is maximized for $\boxed{s = 50\%}$.
- The Golden Rule Level of capital is thus:

$$\frac{K^*}{N} = \frac{1}{4\delta^2}$$

The Dynamic Effects of an Increase in the Saving Rate from 10% to 20% on the Level and the Growth Rate of Output per Worker

- It takes a long time for output to adjust to its new higher level after an increase in the saving rate. Put another way, an increase in the saving rate leads to a long period of higher growth.



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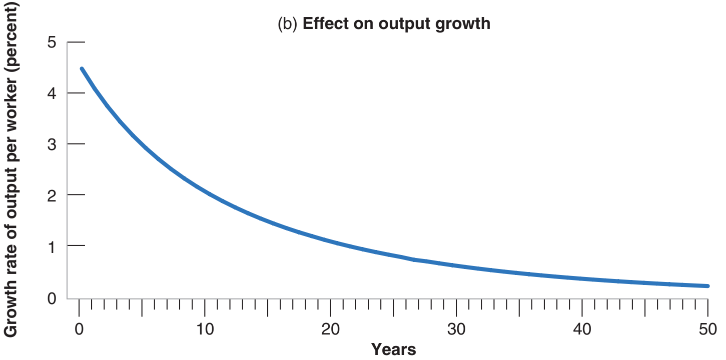

- In the steady state, consumption per worker is:

$$\frac{C}{N} = \frac{Y}{N} - \delta\frac{K}{N}$$

- Therefore, the steady-state consumption per worker is:

$$\frac{C}{N} = \frac{s}{\delta} - \delta\left(\frac{s}{\delta}\right)^2 = \frac{s(1-s)}{\delta}$$

- Table 11-1 gives the steady-state values of capital per worker, output per worker and consumption per worker for different saving rates (given $\delta = 10\%$)

The Saving Rate and the Steady-State Levels of Capital, Output, and Consumption per Worker

Table 11-1 The Saving Rate and the Steady-State Levels of Capital, Output, and Consumption per Worker

Saving Rate s	Capital per Worker K/N	Output per Worker Y/N	Consumption per Worker C/N
0.0	0.0	0.0	0.0
0.1	1.0	1.0	0.9
0.2	4.0	2.0	1.6
0.3	9.0	3.0	2.1
0.4	16.0	4.0	2.4
0.5	25.0	5.0	2.5
0.6	36.0	6.0	2.4
...	...	...	...
1.0	100.0	10.0	0.0

- **Human capital** (H): The set of skills of the workers in the economy built through education and on-the-job training.
- The production function with human capital:

$$\frac{Y}{N} = f\left(\frac{K}{N}, \frac{H}{N}\right)$$

- As for physical capital (K) accumulation, countries that save more or spend more on education can achieve higher steady-state levels of output per worker.

- **Models of endogenous growth**: Steady-state growth in outpour per worker depends on variables such as the saving rate and the rate of spending on education, even without technological progress.
- However, the current consensus is that given the rate of technological progress, higher rates of saving or spending on education do not lead to a permanently higher growth rate.

APPENDIX: The Cobb-Douglas Production Function and the Steady State

- The Cobb-Douglas production function:

$$Y = K^{\alpha} N^{1-\alpha}$$

which gives a good description of the relation between output, physical capital, and labor in the United States from 1899 to 1922.

- In steady state, saving per worker must be equal to depreciation per worker, implying that:

$$s\left(\frac{K^*}{N}\right)^{\alpha} = \delta \frac{K^*}{N}$$

where K^* is the steady-state level of capital.

APPENDIX: The Cobb-Douglas Production Function and the Steady State

- The preceding expression can be rewritten as:

$$s = \delta \left(\frac{K^*}{N} \right)^{1-\alpha}$$

- The steady-state level of capital per worker becomes:

$$\frac{K^*}{N} = \left(\frac{s}{\delta} \right)^{\frac{\alpha}{1-\alpha}}$$

- If $\alpha = 1/2$, then:

$$\frac{K^*}{N} = \frac{s}{\delta}$$

  which implies that a doubling of the saving rate leads to a doubling in steady-state output per worker.

Readings

☞ Chapter 11, *Macroeconomics*, 7th Edition, Olivier Blanchard.
Tu'ppence, Mary Poppins, 1964. Link
Solow, Robert M. "A Contribution to the Theory of Economic Growth." The Quarterly Journal of Economics 70, no. 1 (1956): 65–94. Link
Capital/ Labor Substitution in Practice, *Wall Street Journal*, October 6, 2013. Link
Garon, Sheldon. Beyond Our Means: Why America Spends While the World Saves. Princeton University Press, 2012. Link
The paradox of the paradox of thrift, The Economist, November 18, 2009. Link

Bibliography I

Solow, Robert M., "A Contribution to the Theory of Economic Growth," *The Quarterly Journal of Economics*, 1956, *70* (1), 65–94.