Lecture 9 - Recommended Problems Solutions

François Geerolf
UCLA

☆☆☆ Chapter 10, Problem 3

Consider the production function:

$$Y = \sqrt{K}\sqrt{N}$$

a. **Compute output when $K = 49$ and $N = 81$.**

$$Y = \sqrt{49}\sqrt{81} = 7 \cdot 9 = 63$$

b. **If both capital and labor double, what happens to output?**

Output doubles, because of constant returns to scale. Indeed, one can verify that:

$$\sqrt{2K}\sqrt{2N} = \sqrt{2}\sqrt{2}\left(\sqrt{K}\sqrt{N}\right) = 2Y$$

c. **Is this production function characterized by constant returns to scale? Explain.**

Yes, as the example above shows, if we double inputs (capital and labor), then output also doubles. Similarly, if we triple inputs, the output triples, if we quadruple inputs, it quadruples, etc. More formally, for any number x we have that

$$\sqrt{xK}\sqrt{xN} = \sqrt{x}\sqrt{x}\left(\sqrt{K}\sqrt{N}\right) = xY$$

d. **Write this production function as a relation between output per worker and capital per worker.**

We divide output by total labor (number of workers) and substitute the production function:

$$\frac{Y}{N} = \frac{\sqrt{K}\sqrt{N}}{N} = \sqrt{\frac{K}{N}}$$

e. **Let $K/N = 4$. What is Y/N? Now double K/N to 8. Does Y/N double as a result?**

Using the formula from above, if we have $K/N = 4$

$$\frac{Y}{N} = \sqrt{4} = 2$$

Now if we have $K/N = 8$,

$$\frac{Y}{N} = \sqrt{8} \approx 2.83 < 4$$

So output per worker less than doubles.

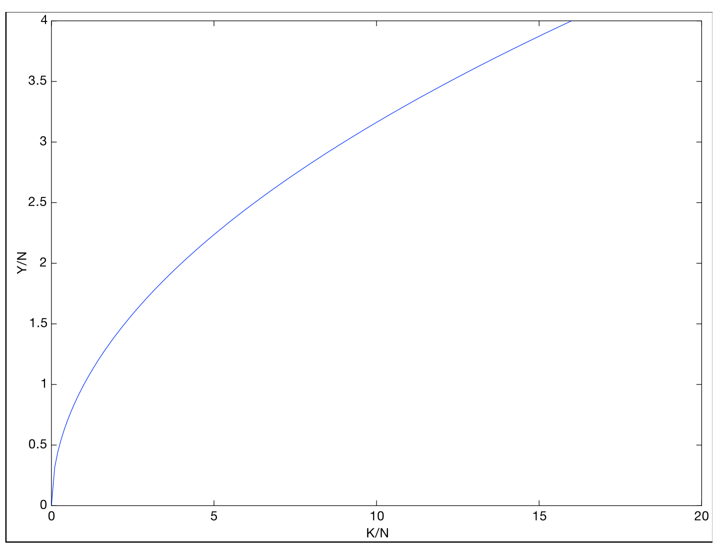
f. **Does the relation between output per worker and capital per worker exhibit constant returns to scale?**

No. If we double capital per worker, output per worker less than doubles. In other words, there are decreasing returns to capital per worker in this relation.

g. **Is your answer to (f) the same as your answer to (c)? Why or why not?**

No. In (f), dividing output by number of workers is equivalent to keeping labor constant in (c). Therefore, an increase in capital has less than proportional effect on output because of decreasing returns to capital: one needs to both double labor and capital in order to double output. If one doubles capital only, keeping labor constant, then capital will run into dimishing marginal returns.
For example, if a car manufacturer uses machines and technicians to repair his machines, adding new machines will be less and less efficient as there are less and less technicians available to repair them.

h. **Plot the relation between output per worker and capital per worker. Does it have the same general shape as the relation in Figure 10-4? Explain.**



This concave shape is characteristic for any production function that exhibits decreasing returns to capital. In this case, we get the graph of a square root function because of the functional form for the production function.

☆☆☆ Chapter 10, Problem 4

The growth rates of capital and output. Consider the production function given in Problem 3 above. Assume that N is constant and equal to 1. Note that if $z = x^a$, then $g_z \approx a g_x$, where g_z and g_x are the growth rates of z and x.

a. **Given the growth approximation here, derive the relation between the growth rate of output and the growth rate of capital.**

Substitute $N = 1$ into the production function. Using the above formula:

$$Y = \sqrt{K} \;\Rightarrow\; g_Y = 0.5 g_K$$

b. **Suppose we want to achieve output growth equal to 2% per year. What is the required rate of growth of capital?**

Substituting for $g_Y = 2\%$, we get:

$$0.02 = 0.5 g_K \;\Rightarrow\; g_K = \frac{0.02}{0.5} = 0.04 = 4\%$$

We can see that faster growth rate of capital stock is needed to achieve a given growth rate of output. This is because of decreasing returns to capital: in order to double output, we need to more than double capital.

c. **In (b), what happens to the ratio of capital to output over time?**

Capital to output ratio grows, since the numerator grows faster than the denominator. This is again a manifestation of decreasing returns to capital. We need a disproportionately larger increase of capital to achieve a given increase of output, so the amount of capital per each unit of output grows over time.

d. **Is it possible to sustain output growth of 2% forever in this economy? Why or why not?**

No, it is not possible. As explained above, if one wants to achieve 2% output growth, the growth rate of capital would have to be 4%. Denoting by I_t time t investment and by $\delta > 0$ the depreciation rate, we can write a law of motion of capital stock:

$$K_{t+1} = K_t - \delta K_t + I_t$$

If K_t grows at 4%, we have that $K_{t+1} = 1.04 K_t$. Substituting this into the equation above, we thus have

$$I_t = (0.04 + \delta) K_t$$

This in turn implies that investment has to grow at the same rate as capital. Formally,

$$1 + g_I = \frac{I_{t+1}}{I_t} = \frac{(0.04 + \delta) K_{t+1}}{(0.04 + \delta) K_t} = 1 + g_K$$

Since Y_t grows at 2%, we reach an impossibility, since investment = saving and saving is necessarily lower than output:

$$Y_t = I_t + C_t \quad \Rightarrow \quad I_t \le Y_t$$

This is an impossibility as any positive number which grows at 4% no matter how small, will eventually be greater than a positive number growing at 2%:

$$\frac{I_t}{Y_t} = \frac{I_0}{Y_0} \frac{(1+0.04)^t}{(1+0.02)^t} \rightarrow_{t \to +\infty} +\infty$$

☆☆ Chapter 10, Problem 2

Assume that the average consumer in Mexico and the average consumer in the United States buy the quantities and pay the prices indicated in the following table:

	Food		**Transportation Services**	
	Price	**Quantity**	**Price**	**Quantity**
Mexico	**5 pesos**	**400**	**20 pesos**	**200**
United States	**\$1**	**1,000**	**\$2**	**2000**

a. **Compute U.S. consumption per capita in dollars.**

U.S. consumption per capita is the total expenditure of the average U.S. consumer on food and transportation services:

$$\$1 \cdot 1000 + \$2 \cdot 2000 = \$5000$$

b. **Compute Mexican consumption per capita in pesos.**

Mexican consumption per capita in pesos is:

$$5\ pesos \cdot 400 + 20\ pesos \cdot 200 = 6000\ pesos$$

c. **Suppose that 1 dollar is worth 10 pesos. Compute Mexico's consumption per capita in dollars.**

Note that this exchange rate, as in the Russian example given in the course, corresponds to the nominal exchange rate which makes the price of the traded good equal (assuming that transportation services' main input are trains, planes, cars, etc. which are traded indeed). This would be thus a likely reference point for the nominal exchange rate, as the price of traded goods is generally not that different across countries.

To answer the question, we need to determine how many dollars are needed to buy the Mexican consumption bundle valued at Mexican prices. The Mexican consumption bundle valued at Mexican prices is what we computed in (b). Therefore, we should divide the result in (b) by the exchange rate:

$$6000\ pesos/10 = \$600$$

d. **Using the purchasing power parity method and U.S. prices, compute Mexican consumption per capita in dollars.**

Now we have have to determine how many dollars are needed to buy the Mexican consumption bundle valued at U.S. prices. So we have to take Mexican quantities and multiply by U.S. prices:

$$\$1 \cdot 400 + \$2 \cdot 200 = \$800$$

e. **Under each method, how much lower is the standard of living in Mexico than in the United States? Does the choice of method make a difference?**

Using the exchange rate method, consumption in Mexico is \$600/\$5000 = 12% of U.S. consumption per capita. Using the PPP method, consumption in Mexico is \$800/\$5000 = 16% of U.S. consumption per capita. PPP method tends to show higher standard of living because it takes into account lower price of food in Mexico compared to the U.S, and the fact that food is a relatively large component of consumption in Mexico, more so than in the United States.

☆☆ Chapter 10, Problem 6

The Bureau of Labor Statistics has a user-friendly Web site of GDP per capita here. Find GDP per capita in Japan and in the United States in 1960, 1990, and the most recent year.

a. **Compute the average annual growth rates of GDP per person for the United States and Japan for two time periods: 1960 to 1990 and 1990 to the most recent year available. Did the level of real output per person in Japan tend to converge to the level of real output per person in the United States in both these periods? Explain.**

	Y/N_{1960}	Y/N_{1990}	Y/N_{2011}	Avg. growth 1960-90	Avg. growth 1990-2011
U.S.	\$17,747	\$36,378	\$48,282	**2.42%**	1.36%
Japan	\$6,109	\$29,679	\$34,294	**5.41%**	0.69%

In the period of 1960-1990, real GDP per capita grew faster in Japan than in the U.S., hence it was catching up (converging) with the U.S. During 1990-2011, real GDP per capita in Japan grew slower than in the U.S., so Japan was actually lagging behind the U.S. After the period of catching up with the US, in part coming from substantial capital accumulation in Japan (as the Solow growth model would suggest), driving growth, it seems that Japan is now at the frontier, both in terms of technology as well as in terms of capital accumulation, so that it's much harder to sustain growth. Note also that Japan is an ageing population, so that GDP per worker comparisons would probably be more meaningful.

Comment: Note that the comparison is based on the PPP method. Note also that the GDP series are converted to 2011 price levels, so that they are comparable across time. The average annual growth rate 1960-90 is equal to $\left(\frac{Y/N_{1990}}{Y/N_{1960}}\right)^{1/30} - 1$, and the average annual growth rate 1990-2011 is equal to $\left(\frac{Y/N_{2011}}{Y/N_{1990}}\right)^{1/21} - 1$. Note that it is preferable not to use approximations here as the changes in GDP per capita were substantial, and the approximations only work well for small changes.

b. **Suppose that in every year since 1990, Japan and the United States had each continued to have their average annual growth rates for the period 1960 to 1990. How would real GDP per person compare in Japan and the United States today?**

If the growth rate in Japan had stayed at 5.41%, then real GDP per capita in Japan in 2011 would have been equal to:

$$\$29,679 \cdot (1 + 0.0541)^{21} = \$89,736$$

Whereas in US it would have been:

$$\$36,378 \cdot (1 + 0.0242)^{21} = \$60,106$$

Japan would have surpassed the United States in output per person. Japanese real GDP per capita would have been equal to 149% of U.S. real GDP per capita by 2011.

c. **What actually happened to growth in real GDP per capita in Japan and the United States from 1990 to 2011?**

Growth slowed down considerably in both countries, but slowdown was stronger in Japan. Given Japan's external surplus, it would seem that Japan's problem come more from the demand side than on the supply side, and that it is suffering from a shortage of aggregate demand. We shall be discussing open economy issues later in the class.

☆ Chapter 10, Problem 8

Growth successes and failures. Go hereto the Web site containing the Penn World Table and collect data on real GDP per capita (chained series) for 1970 for all available countries (a direct link to the excel file is here, and to a (not so) user-friendly interface here). Do the same for a recent year of data, say one year before the most recent year available in the Penn World Table. (If you choose the most recent year available, the Penn World Table may not have the data for some countries relevant to this question.)

a. **Rank the countries according to GDP per person in 1970. List the countries with the 10 highest levels of GDP per person in 1970. Are there any surprises?**

I used the most recent version PWT 9.0. Real GDP is in 2011 dollars. Real GDP per capita is obtained by dividing real GDP (column rgdpe) by total population (column pop). The richest countries tend to be oil exporters or small countries hosting international businesses. U.S. is the only exception in this list.

Country	Real GDP per capita, 1970
United Arab Emirates	$244,668
Brunei Darussalam	$90,760
Qatar	$86,457
Saudi Arabia	$39,417
Kuwait	$31,788
Switzerland	$27,249
Cayman Islands	$26,463
Luxembourg	$26,044
United States	**$23,608**
Bermuda	$20,255

b. **Carry out the analysis in part (a) for the most recent year for which you collected data. Has the composition of the 10 richest countries changed since 1970?**

Country	Real GDP per capita, 2014
Qatar	$144,340
China, Macao SAR	$126,980
Luxembourg	$95,176
Singapore	$72,583
Brunei Darussalam	$68,499
United Arab Emirates	$64,398
Norway	$64,274
Kuwait	$63,886
Switzerland	$58,469
Bermuda	$57,531

c. **Use all the countries for which there are data in both 1970 and the latest year. Which five countries have the highest proportional increase in real GDP per capita?**

Country	Real GDP per capita growth, 1970-2014
Equatorial Guinea	10.06%
Botswana	7.08%
China, Macao SAR	7.04%
Republic of Korea	6.61%
Singapore	5.91%

d. **Use all the countries for which there are data in both 1970 and the latest year. Which five countries have the lowest proportional increase in real GDP per capita?**

Country	Real GDP per capita growth, 1970-2014
United Arab Emirates	-2.99%
Djibouti	-2.01%
Central African Republic	-1.90%
D.R. of the Congo	-1.66%
Liberia	-1.40%

e. **Do a brief Internet search on either the country from part (c) with the greatest increase in GDP per capita or the country from part (d) with the smallest increase. Can you ascertain any reasons for the economic success, or lack of it, for this country?**

The Republic of Korea (South Korea) underwent massive market reforms over this time period, which were incredibly successful. Meanwhile, Equatorial Guinea and Botswana discovered and developed large resource deposits (oil and diamonds).

The United Arab Emirates (UEA) had incredibly fast population growth over the time period, so that even though output was growing, output per capita was falling. Other countries like D.R. of the Congo suffered from civil wars during this time period.