Lecture 7 - Consumption Function, Multiplier

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Contents

Introduction

This lecture opens a set of lectures on Keynesian economics. Keynesian economics is very different from the neoclassical models that we have seen in the 6 first lectures of this class. These models, although they correspond to research that was done after Keynes [1936], are more similar to the models which were popular at the beginning of the twentieth century, and that Keynes [1936] was so firmly against. Keynes [1936] would have disagreed with most of what we have been discussing so far. This lecture opens a set of 6 lectures which gives you an introduction to Keynesian economics. Expect a very different approach:

> The difficulty lies, not in the new ideas, but in escaping from the old ones, which ramify, for those brought up as most of us have been, into every corner of our minds.

In the model that we will study today, consumption does not result from intertemporal optimization, as in Lecture 3 and Lecture 4: Keynes [1936] developed the idea of the "consumption function", according to which consumption was just a constant fraction of current income (instead of present discounted income, as in the two-period intertemporal model, for example). Moreover, unlike in the Solow growth model of Lecture 2, investment will not be determined by saving, but will either be exogenously fixed, or a positive function of GDP. Finally, unlike in the neoclassical labor market model of Lecture 6, there will be involuntary unemployment. As a consequence, any increase in demand will be potentially met by an increase in supply, because some labor is idle and ready to be used in production. It's totally natural for you if this paragraph is still mysterious to you. We will be able to take stock once we are done with this part of the class.

During this lecture, I first go through a presentation of the simple Keynesian cross, and then present a couple of variations on the simple Keynesian cross: automatic stabilizers, and "accelerator" effects of aggregate demand on investment.

1 The Simple Keynesian Cross

The Keynesian cross, or the (YY)-(ZZ) model, represented below in the Figure below, is a pedagogical device introduced in 1948 by Paul Samuelson in his famous *Economics* textbook, to provide a geometric intuitive interpretation of Keynes' ideas (Samuelson [1948]).

1.1 Assumptions

The basic Keynesian model has several assumptions which are at odds with the neoclassical theory we have studies so far. First, it assumes that there exists a "marginal propensity to consume" out of income, which determines consumption. Second, at least in its most simple form, it postulates that investment is simply fixed.

Marginal Propensity to Consume. Keynes [1936] assumes that consumption is simply a function of *current* income, or gdp Y, with:

$$C = c_0 + c_1 Y, \quad 0 < c_1 < 1.$$

Fixed investment. The value of investment is denoted by $\bar{I}$, to emphasize that it is fixed. A key assumption here is that it does not depend on saving. This is very much in contrast of the Solow [1956] growth model, where investment is given by how much saving is available to invest. In the background, the assumption in the Solow [1956] growth model is that the rate of interest may fall sufficiently to induce the "right" level of investment by firms, which allows to use of all the saving capacity in the economy.

1.2 Algebraic derivation of the multiplier

With these assumptions, and assuming a closed economy, the value of demand is:

$$\begin{aligned} Z &= C + \bar{I} + G \\ &= c_0 + c_1 (Y - T) + \bar{I} + G \\ Z &= \left(c_0 + \bar{I} + G - c_1 T\right) + c_1 Y \end{aligned}$$

The demand equation is the expression for the (ZZ) curve is therefore:

$$Z = \underbrace{\left(c_0 + \bar{I} + G - c_1 T\right)}_{\text{Autonomous Spending } z_0} + \underbrace{c_1}_{\text{MPC}} Y$$

The value of autonomous spending is equal to z_0, and is the value of demand when income is equal to $Y = 0$. The steepness of this curve determines the value for the multiplier. The closer to one the slope is, the higher the value of the Keynesian multiplier. Indeed, solving for $Z = Y$:

$$Y = \underbrace{\frac{1}{1 - c_1}}_{\text{Multiplier}} \times \underbrace{\left(c_0 + \bar{I} + G - c_1 T\right)}_{\text{Autonomous Spending } z_0}.$$

Therefore, the value of the multiplier is given by:

$$\boxed{\text{Multiplier} = \frac{1}{1 - c_1}}.$$

Consider a change in autonomous spending $\Delta z_0 = z_0' - z_0$ coming from a change in government spending $\Delta z_0 = \Delta G$, or from a change in net taxes $\Delta z_0 = -c_1 \Delta T$. We have:

$$\Delta Y = \frac{\Delta z_0}{1 - c_1}.$$

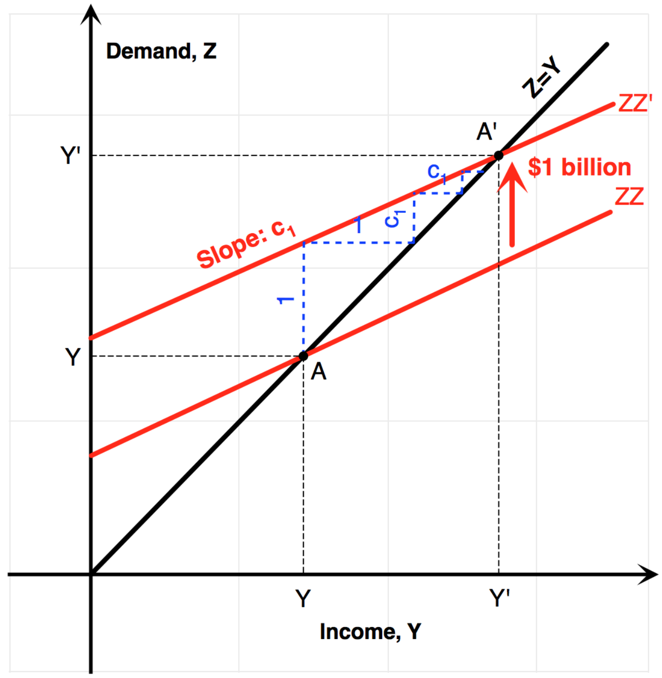

Figure 1: Simple Keynesian Cross: Graphical Interpretation 1

1.3 Four interpretations

Why is the change in output higher than the change in autonomous spending? There are actually 4 ways to see this, 2 are algebraic, and 2 are geometric:

1. **Algebra.** The first way to see this is just, as above to equate output to demand $Y = Z$, which allows to get at the result.

2. **Infinite sum of a geometric series.** 1\$ of additional autonomous spending brings in a second round c_1\$ increase in consumption, a third round c_1^2\$ increase in consumption, which add up to:

$$\sum_{i=0}^{n} c_1^i = 1 + c_1 + c_1^2 + ... + c_1^n = \frac{1 - c_1^{n+1}}{1 - c_1}.$$

   Therefore, because $c_1^{n+1} \to 0$ if $n \to +\infty$ and $0 < c_1 < 1$, we have:

$$\text{Multiplier} = \sum_{i=0}^{+\infty} c_1^i = \lim_{n\to+\infty} \sum_{i=0}^{n} c_1^i = \lim_{n\to+\infty} \frac{1 - c_1^{n+1}}{1 - c_1} = \frac{1}{1 - c_1}.$$

3. **Graphical interpretation 1.** The Figure below gives a graphical interpretation to this infinite geometric sum. This graphical interpretation makes it clear that:

$$Y' - Y = \sum_{i=0}^{+\infty} c_1^i = \lim_{n\to+\infty} \sum_{i=0}^{n} c_1^i = \lim_{n\to+\infty} \frac{1 - c_1^{n+1}}{1 - c_1} = \frac{1}{1 - c_1}.$$

4. **Graphical interpretation 2.** The Figure below gives a graphical interpretation which does not use a geometric sum. If m is the unknown value of the multiplier, then the geometry makes clear that m has to satisfy $m = 1 + mc_1$ which also gives the value for the multiplier:

$$m = 1 + mc_1 \quad \Rightarrow \quad m = \frac{1}{1 - c_1}.$$

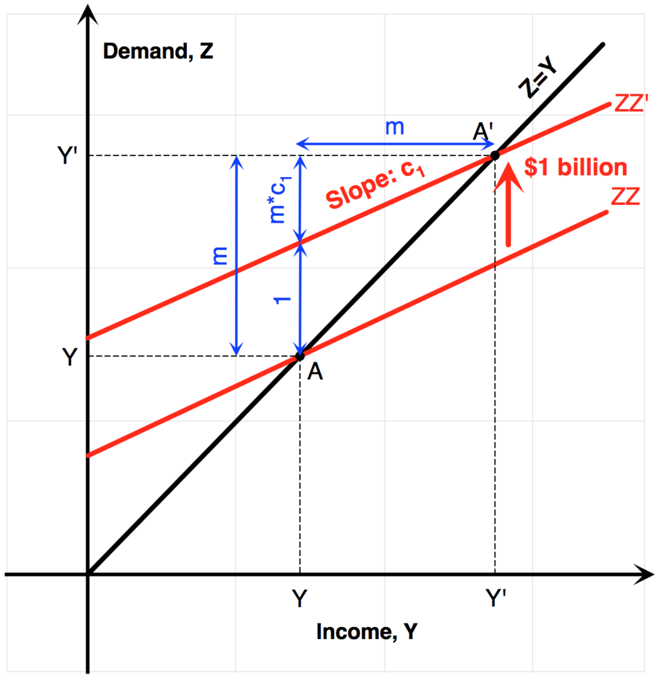

Figure 2: Simple Keynesian Cross: Graphical Interpretation 2

2 Variations on the Keynesian Cross

There are multiple variations on the theme of the Keynesian cross: one with automatic stabilizers, one with an accelerator effect of demand on investment.

2.1 Automatic Stabilizers

Assume now that:

$$T = t_0 + t_1 Y$$

Therefore:

$$\begin{aligned} Z &= C + \bar{I} + G \\ &= c_0 + c_1 \left(Y - t_0 - t_1 Y\right) + \bar{I} + G \\ Z &= \left(c_0 - c_1 t_0 + \bar{I} + G\right) + \left(\left(1 - t_1\right) c_1\right) Y \end{aligned}$$

The slope of the (ZZ) curve is $(1 - t_1)\, c_1$.

$$\boxed{Y = \underbrace{\frac{1}{1 - c_1 \left(1 - t_1\right)}}_{\text{Multiplier}} \times \underbrace{\left(c_0 - c_1 t_0 + \bar{I} + G\right)}_{\text{Autonomous Spending}}}$$

In the Figure below, the (ZZ) curve is less steep, and thus the multiplier $Y' - Y$ is smaller.

2.2 "Accelerator" effect of demand on investment

Assume that:

$$I = b_0 + b_1 Y.$$

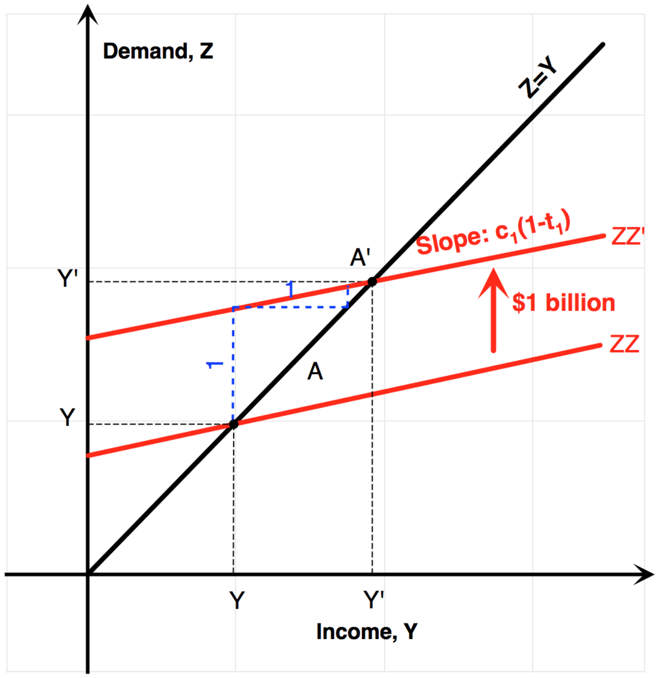

Figure 3: AUTOMATIC STABILIZERS

Therefore:

$$\begin{aligned} Z &= C + I + G \\ &= c_0 + c_1 (Y - T) + b_0 + b_1 Y + G \\ Z &= (c_0 - c_1 T + b_0 + G) + (c_1 + b_1) Y. \end{aligned}$$

The slope is $c_1 + b_1$. Income is equal to demand and therefore, if $c_1 + b_1 < 1$:

$$\boxed{Y = \underbrace{\frac{1}{1 - (c_1 + b_1)}}_{\text{Multiplier}} \times \underbrace{(c_0 - c_1 T + b_0 + G)}_{\text{Autonomous Spending}}}$$

In the Figure below, the (ZZ) curve is steeper, and thus the multiplier $Y' - Y$ is larger. Note that if instead $c_1 + b_1 \geq 1$, then the multiplier is infinite.

3 Required Readings

Robert Barro. Keynes Is Still Dead. *Wall Street Journal*. October 29, 1992.

What would Keynes have done? Mankiw, N. Gregory. *New York Times*, 30 November 2008, 2008

What Is Keynesian Economics?, *Finance & Development*, September 2014, Sarwat Jahan, Ahmed Saber Mahmud, and Chris Papageorgiou.

4 Readings - To go further

Keynes, John Maynard. The Economic Consequences of the Peace, 1919.

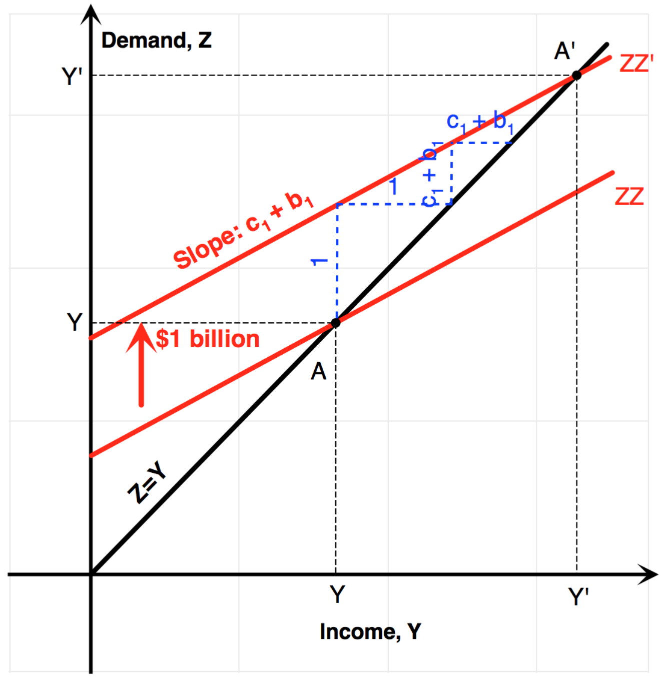

Figure 4: Accelerator Effect of Investment

Keynes, John Maynard. The General Theory of Employment, Interest, and Money, 1936.

References

John Maynard Keynes. *The General Theory of Employment, Interest, and Money.* 1936. ISBN 81-269-0591-3.

Paul A. Samuelson. *Economics.* McGraw-Hill Book Company, New York Toronto London, 1948.

Robert M. Solow. A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*, 70(1):65–94, 1956. ISSN 0033-5533. doi: 10.2307/1884513. URL http://www.jstor.org/stable/1884513.