Problem Set 6 - Solutions

UCLA - Econ 102 - Fall 2018

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Contents

6 Problem Set 6 - Solution **1**

Another Numerical Example 1

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Another Numerical Example

1. For a household, the threshold to be in the top 1% is around **$421,926** according to the Economic Policy Institute. Let me Google that for you: http://bfy.tw/KhF4.

2. Using the given notations (and those of lecture 9), total income is the sum of the top 1% income and that of the bottom 99%:
$$Y = \lambda N \underline{y} + (1-\lambda) N \bar{y}.$$
Since $\bar{y} = \gamma \underline{y}$ we have:
$$Y = \lambda N \underline{y} + (1-\lambda) N \gamma \underline{y}.$$
The total income for the bottom 99% $\underline{Y}$ is given by:
$$\underline{Y} = \lambda N \underline{y}.$$
Therefore, the share of total income captured by the bottom 99% is:
$$\frac{\underline{Y}}{Y} = \frac{\lambda N \underline{y}}{\lambda N \underline{y} + (1-\lambda) N \gamma \underline{y}}$$
$$\frac{\underline{Y}}{Y} = \frac{\lambda}{\lambda + (1-\lambda)\gamma}$$
Denoting by ν the share of income going to the low income:
$$\nu \equiv \frac{\underline{Y}}{Y}$$
$$\nu = \frac{\lambda}{\lambda + (1-\lambda)\gamma}$$
Solving for γ:
$$\frac{\lambda}{\lambda + (1-\lambda)\gamma} = \nu \quad \Rightarrow \quad \lambda = \lambda \cdot \nu + (1-\lambda)\gamma \cdot \nu$$
$$\Rightarrow \quad \lambda \cdot (1-\nu) = \gamma \cdot (1-\lambda) \cdot \nu \quad \Rightarrow \quad \boxed{\gamma = \frac{\lambda}{1-\lambda} \frac{1-\nu}{\nu}}$$
A numerical application is $\nu = 0.8$ and $\lambda = 0.99$ so that:
$$\gamma = \frac{0.99}{1-0.99} \frac{1-0.8}{0.8}$$
$$= \frac{99}{4}$$
$$\gamma = 24.75$$

This implies that on average, high income earners in the top 1% are approximately **25 times richer** (exactly 24.75 times richer) than low income earners in the bottom 99% (note that you can use the above formula to recover the $\gamma = 9$ from the class, using $\lambda = 0.9$ and $\nu = 0.5$ since $\gamma = 0.9/0.1 \cdot 0.5/0.5 = 9$).

3. Total consumption by the low income earners $\underline{C}$ is such that:

$$\begin{aligned}
\underline{C} &= \lambda N \underline{c} \\
&= \lambda N \left(\underline{c}_0 + \underline{c}_1(\underline{y} - \underline{t})\right) \\
&= \lambda N \underline{c}_0 + \lambda N (1-t_1)\underline{c}_1 \underline{y} - \lambda N \underline{c}_1 \underline{t}_0 \\
\underline{C} &= [\lambda N \underline{c}_0 - \lambda N \underline{c}_1 \underline{t}_0] + \frac{\lambda \underline{c}_1}{\lambda + (1-\lambda)\gamma}(1-t_1)Y
\end{aligned}$$

Symmetrically, consumption by the high income earners $\bar{C}$ is such that:

$$\begin{aligned}
\bar{C} &= (1-\lambda) N \bar{c} \\
&= (1-\lambda) N \left(\bar{c}_0 + \bar{c}_1(\bar{y} - \bar{t})\right) \\
&= (1-\lambda) N \bar{c}_0 + (1-\lambda) N (1-t_1)\bar{c}_1 \bar{y} - (1-\lambda) N \bar{c}_1 \bar{t}_0 \\
\bar{C} &= \left[(1-\lambda) N \bar{c}_0 - (1-\lambda) N \bar{c}_1 \bar{t}_0\right] + \frac{(1-\lambda)\gamma \bar{c}_1}{\lambda + (1-\lambda)\gamma}(1-t_1)Y
\end{aligned}$$

Therefore, aggregate consumption $C = \underline{C} + \bar{C}$ is given by:

$$\begin{aligned}
C &= \underline{C} + \bar{C} \\
&= [\lambda N \underline{c}_0 - \lambda N \underline{c}_1 \underline{t}_0] + \frac{\lambda \underline{c}_1}{\lambda + (1-\lambda)\gamma}(1-t_1)Y + \left[(1-\lambda) N \bar{c}_0 - (1-\lambda) N \bar{c}_1 \bar{t}_0\right] + \frac{(1-\lambda)\gamma \bar{c}_1}{\lambda + (1-\lambda)\gamma}(1-t_1)Y \\
&= (\lambda N \underline{c}_0 + (1-\lambda) N \bar{c}_0) - \left(\lambda N \underline{c}_1 \underline{t}_0 + (1-\lambda) N \bar{c}_1 \bar{t}_0\right) + \frac{\lambda \underline{c}_1 + (1-\lambda)\gamma \bar{c}_1}{\lambda + (1-\lambda)\gamma}(1-t_1)Y \\
&= [\lambda N \underline{c}_0 + (1-\lambda) N \bar{c}_0] - \left[\underline{c}_1(\lambda N \underline{t}_0) + \bar{c}_1((1-\lambda) N \bar{t}_0)\right] + \frac{\lambda \underline{c}_1 + (1-\lambda)\gamma \bar{c}_1}{\lambda + (1-\lambda)\gamma}(1-t_1)Y \\
C &= C_0 - \left(\underline{c}_1 \underline{T}_0 + \bar{c}_1 \bar{T}_0\right) + c_1(1-t_1)Y.
\end{aligned}$$

where we have used the suggested notations:

$$\begin{aligned}
C_0 &\equiv \lambda N \underline{c}_0 + (1-\lambda) N \bar{c}_0 \\
\underline{T}_0 &\equiv \lambda N \underline{t}_0 \\
\bar{T}_0 &\equiv (1-\lambda) N \bar{t}_0 \\
c_1 &\equiv \frac{\lambda \underline{c}_1 + (1-\lambda)\gamma \bar{c}_1}{\lambda + (1-\lambda)\gamma}.
\end{aligned}$$

Therefore, aggregate consumption is given by:

$$\boxed{C = C_0 - \left(\underline{c}_1 \underline{T}_0 + \bar{c}_1 \bar{T}_0\right) + c_1(1-t_1)Y}.$$

4. c_1 is the average marginal propensity to consume, where the marginal propensity to consume of each group $\underline{c}_1$ and $\bar{c}_1$ is weighted by their share of income in the population $\underline{Y}/Y$ and $\bar{Y}/Y$:

$$\begin{aligned}
c_1 &= \frac{\lambda \underline{c}_1 + (1-\lambda)\gamma \bar{c}_1}{\lambda + (1-\lambda)\gamma} \\
&= \frac{\lambda}{\lambda + (1-\lambda)\gamma}\underline{c}_1 + \frac{(1-\lambda)\gamma}{\lambda + (1-\lambda)\gamma}\bar{c}_1 \\
c_1 &= \frac{\underline{Y}}{Y}\underline{c}_1 + \frac{\bar{Y}}{Y}\bar{c}_1
\end{aligned}$$

This has a straightforward economic interpretation: for each additional dollar of output, a fraction $\underline{Y}/Y$ goes to low income earners who consume a fraction $\underline{c}_1$, and a fraction $\bar{Y}/Y$ goes to high income earners who consume a fraction $\bar{c}_1$. The average propensity to consume is the sum of these two fractions. We can then simply compute the average marginal propensity to consume when $\underline{c}_1 = 1$ and $\bar{c}_1 = 1/4$:

$$\begin{aligned} c_1 &= \frac{\underline{Y}}{Y}\underline{c}_1 + \frac{\bar{Y}}{Y}\bar{c}_1 \\ &= \frac{4}{5} \cdot 1 + \frac{1}{5} \cdot \frac{1}{4} \\ c_1 &= \frac{17}{20} \end{aligned}$$

Therefore the average propensity to consume is:

$$\boxed{c_1 = 0.85}.$$

5. Using the expression for aggregate consumption C in question 3., and that $I = b_0 + b_1 Y$, and plugging it into total aggregate demand Z yields:

$$\begin{aligned} Z &= C + I + G \\ &= C_0 - \left(\underline{c}_1 \underline{T}_0 + \bar{c}_1 \bar{T}_0\right) + c_1(1-t_1)Y + b_0 + b_1 Y + G \\ Z &= \left[C_0 - \left(\underline{c}_1 \underline{T}_0 + \bar{c}_1 \bar{T}_0\right) + b_0 + G\right] + \left(c_1(1-t_1) + b_1\right) Y \end{aligned}$$

Equating aggregate demand to aggregate income $Z = Y$ gives the value for output (see the lecture notes for details):

$$\boxed{Y = \frac{1}{1 - (1-t_1)\,c_1 - b_1}\left[C_0 - \underline{c}_1 \underline{T}_0 - \bar{c}_1 \bar{T}_0 + b_0 + G\right]}$$

6. As in lecture 9, a 100 billion dollars tax cut on the top 1% $\Delta\bar{T}_0 = -100$ leads to an increase in GDP given by:

$$\begin{aligned} \Delta Y &= \frac{-\bar{c}_1 \Delta \bar{T}_0}{1 - c_1(1-t_1) - b_1} \\ &= \frac{-1/4 * (-100 \text{ billion})}{1 - 0.85 \cdot 0.75 - 1/6} \\ \Delta Y &\approx 127.6 \text{ billion.} \end{aligned}$$

Thus, according to these numbers, we get a **127.6 billion dollars** increase in GDP. The impact on the government surplus is given by:

$$\begin{aligned} \Delta\left(T - G\right) &= \Delta T \\ &= \Delta\bar{T}_0 + \Delta\underline{T}_0 + t_1 \Delta Y \\ &= \Delta\bar{T}_0 + t_1 \Delta Y \\ &\approx -100 + \frac{1}{4} \cdot 127.6 \\ \Delta\left(T - G\right) &\approx -68.1 \text{ billion} \end{aligned}$$

Thus, we get a **68.1 billion dollars** increase in the government deficit.

7. A 100 billion dollars tax cut on the bottom 99% $\Delta\underline{T}_0 = -100$ leads to an increase in GDP given by:

$$\begin{aligned} \Delta Y &= \frac{-\underline{c}_1 \Delta \underline{T}_0}{1 - c_1(1-t_1) - b_1} \\ &= \frac{-1 * (-100 \text{ billion})}{1 - 0.85 \cdot 0.75 - 1/6} \\ \Delta Y &\approx 510.6 \text{ billion.} \end{aligned}$$

Thus, according to these numbers, we get a **510.6 billion dollars** increase in GDP. The impact on the government surplus is given by:

$$
\begin{aligned}
\Delta(T-G) &= \Delta T \\
&= \Delta\bar{T}_0 + \Delta\underline{T}_0 + t_1\Delta Y \\
&= \Delta\underline{T}_0 + t_1\Delta Y \\
&\approx -100 + \frac{1}{4}\cdot 510.6 \\
\Delta(T-G) &\approx 27.6 \text{ billion}
\end{aligned}
$$

Thus, despite the 100 billion dollars tax cut, we get a **27.6 billion dollars** increase in the government surplus, or a reduction in the government deficit. In this situation, tax cuts **more than pay for themselves**. This seems like a much better policy than the tax reduction on the rich. However, we will see in the next lectures that things are not so straightforward.

8. Finally, a 100 billion dollars tax cut on the bottom 99% $\Delta\underline{T}_0 = -100$ financed by a 100 billion tax increase on the top 1% $\Delta\bar{T}_0 = 100$ leads to an increase in GDP given by:

$$
\begin{aligned}
\Delta Y &= \frac{(\underline{c}_1 - \bar{c}_1)\Delta\bar{T}_0}{1 - c_1(1-t_1) - b_1} \\
&= \frac{(1-1/4) * (100 \text{ billion})}{1 - 0.85\cdot 0.75 - 1/6} \\
\Delta Y &\approx 383.0 \text{ billion}
\end{aligned}
$$

Thus, according to these numbers, we get a **383.0 billion dollars** increase in GDP. The impact on the government surplus is given by:

$$
\begin{aligned}
\Delta(T-G) &= \Delta T \\
&= \Delta\bar{T}_0 + \Delta\underline{T}_0 + t_1\Delta Y \\
&= t_1\Delta Y \\
&\approx \frac{1}{4}\cdot 383.0 \\
\Delta(T-G) &\approx 95.7 \text{ billion}
\end{aligned}
$$

We get a **95.7 billion dollars** increase in the government surplus, or a reduction in the government deficit.

9. If there are no automatic stabilizers ($t_1 = 0$), then the multiplier apparently becomes infinite since $1 - c_1 - b_1 = 1 - 0.85 - 1/6 < 0$. However, this is impossible, as there are only finite resources in the economy. Therefore, this implies that output is determined by supply constraints (amount of labor, capital, and technology), as in the Solow growth model, and that the Keynesian type of analysis no longer applies.
In fact, as I said during the lecture, this result is intuitive. If $1 - c_1 < b_1$, then this implies that the marginal propensity to invest b_1 is higher than the marginal propensity to save $1 - c_1$. As a result, we never get to a Keynesian situation of "too much saving".