Lecture 10 - Public Debt, Say's Law

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Introduction

Until now, we have talked about government spending and taxes as if the government could take on as many debt as it wants. Our view of the economy was kind of magic, in that the government could lower taxes and raise government spending as it pleases. For example, in the "accelerator" version of the goods market model we saw in lecture 7, we proceeded to experimenting with taxes and government spending, wondering what would happen if government spending was increased, and taxes were reduced, on GDP. For example, an increase in government spending ΔG leads to an increase in output: $\Delta Y = \frac{\Delta G}{1-c_1-b_1} > 0$ in one version of the model, and a tax cut $\Delta T < 0$ leads to an increase in output as well: $\Delta Y = \frac{c_1 \Delta T}{1-c_1-b_1} > 0$. But then, why doesn't the government actually do more of that spending? A first reason might be that there is a very important issue which we did not take into consideration: it is that of the government deficit, and the impact of government debt on future generations. Indeed, when government spending increases $\Delta G > 0$, this leads to a government deficit of equal magnitude: $\Delta(T-G) = -\Delta G < 0$. Similarly, a tax cut $\Delta T < 0$ leads to increased deficits given by $\Delta(T-G) = \Delta T < 0$. One might worry that this debt will someday have to be repaid. In this case, higher GDP today might only be thought of as leading to lower GDP in the future. Therefore, the dynamics of government debt is very important for Keynesian models. We study the intertemporal budget constraint of the government, and provide conditions under which this government debt is stable.

During this lecture, we take up three related questions:

1. We show first, without using any economic model, that simple accounting suggests that public debt is on a sustainable path whenever the real interest rate on public debt is lower than the rate of growth of GDP: $r < g$, a situation called "dynamic inefficiency" for reasons that will become clear after. Since real interest rates appear to be below the rate of growth of GDP, at least for now, there does not seem to be cause for alarm. (at least, until interest rates don't rise more)

2. Second, we illustrate using an economic model that it is not true that public debt necessarily will need to be repaid eventually (an argument which is often made in the public debate). In the overlapping generations model of lecture 4, and provided that capital accumulation is above the Golden Rule level ($r < g$), so that there is **dynamic inefficiency**, public debt is never repaid, as there are always new generations coming along, who buy government debt when they are young and sell it to the next generation when old.
3. Finally, we shall discuss the effects of larger government deficits on the economy, and constrast the Keynesian and Neoclassical views on this issue. In particular, each school of thought has contrasting predictions for the impact of higher public deficits on investment spending. You may already have understood that by contrasting lectures 2 and 4 with lectures 7, 8 and 9, that those two models have very different logics. We discuss this and related issues surrounding the so-called Treasury View and Say's law in the last section of this lecture.

1 Sustainability of Public Debt

1.1 Law of motion for Public Debt

In this lecture, we denote everything in terms of goods, to avoid thinking about the complicated issues surrounding inflation. Let us denote by G_t the government spending at period t, and by T_t the taxes in period t. Let us also denote by $(G_t - T_t)$ the government (primary) deficit in period t, which is the excess of government expenditures over taxes levied by the government (thus, when $G_t - T_t > 0$, there is a deficit in the budget, so that the government must borrow). If the interest rate that the government pays is given by r_t, then the law of motion of government debt is given by:

$$B_t = (1 + r_t)B_{t-1} + G_t - T_t$$

Thefore, the law of motion for government debt is given by the sum of the **primary deficit** and **interest payments** on the debt.

The **total** government deficit, which is equal to the change in government debt ΔB_t, is equal to the sum of interest payments and the primary deficit $G_t - T_t$.

$$\text{Deficit}_t = \Delta B_t = B_t - B_{t-1} = \underbrace{r_t B_{t-1}}_{\text{Interest Payments}} + \underbrace{G_t - T_t}_{\text{Primary Deficit}}$$

From the above equation, the evolution of the debt to GDP ratio B_t/Y_t:

$$\frac{B_t}{Y_t} = (1 + r_t)\frac{Y_{t-1}}{Y_t}\frac{B_{t-1}}{Y_{t-1}} + \frac{G_t - T_t}{Y_t}$$

Let us denote the debt to GDP ratio by b_t:

$$b_t \equiv \frac{B_t}{Y_t}.$$

Therefore:

$$b_t = (1 + r_t)\frac{Y_{t-1}}{Y_t}b_{t-1} + \frac{G_t - T_t}{Y_t}.$$

Assuming that GDP grows at rate g_Y, we have that:

$$\frac{Y_t}{Y_{t-1}} = 1 + g_Y$$

Therefore:

$$\boxed{b_t = \frac{1 + r_t}{1 + g_Y}b_{t-1} + \frac{G_t - T_t}{Y_t}}.$$

1.2 Condition for Sustainability

A thought experiment is useful to think about the sustainability of public debt in this environment. Imagine that all future primary surpluses were equal to zero after $t = t_0$, that is:

$$\text{for all } t \geq t_0, \quad G_t = T_t$$

and that real interest rates are constant after $t \geq t_0$:

$$r_t = r.$$

We then have that:

$$\text{for all } t \geq t_0, \quad b_t = \frac{1+r}{1+g_Y} b_{t-1}$$

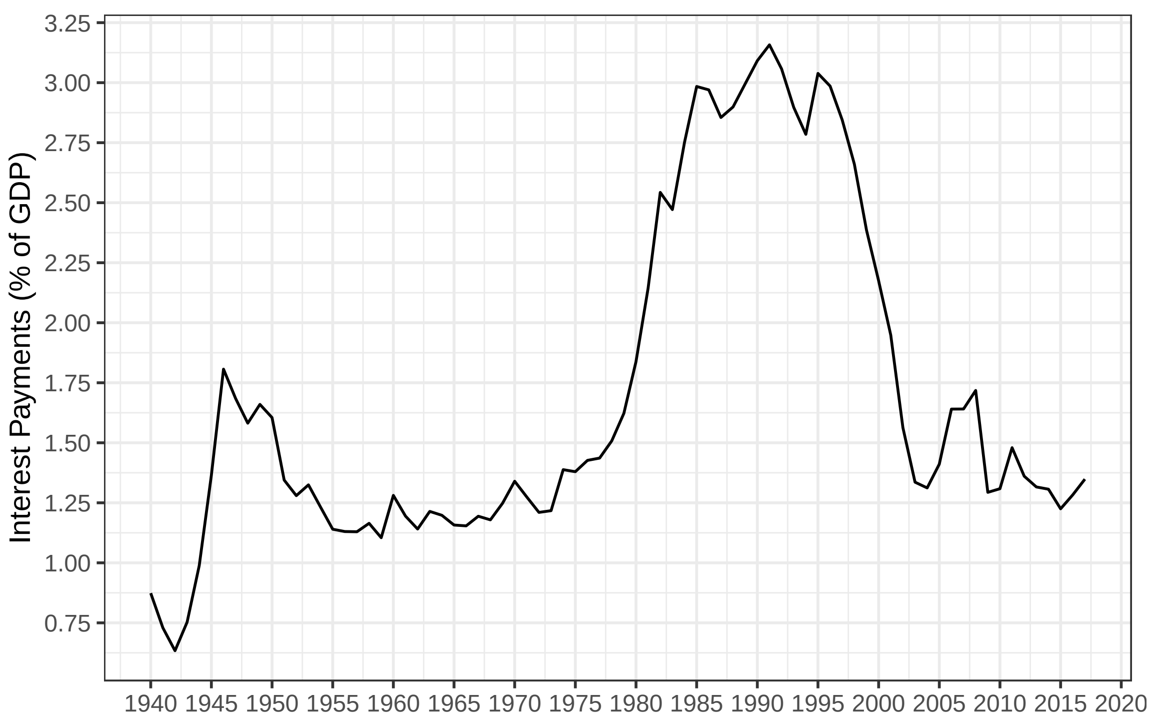
Then the debt to GDP ratio would be given by:

$$\text{for all } t \geq t_0, \quad b_t = \left(\frac{1+r}{1+g_Y}\right)^{t-t_0} b_{t_0}$$

There are three possible cases:
1. If $r < g_Y$, the debt to GDP ratio goes to 0. (Indeed, when $a < 1$, $a^t \to 0$ when $t \to +\infty$.) Therefore, the debt to GDP ratio goes to zero mechanically. This situation is said to be **dynamically inefficient**. (for reasons that will be clear later) 2. If $r = g_Y$, the debt to GDP ratio stays constant.
3. If $r > g_Y$, the debt to GDP ratio goes to infinity. Indeed, when $a > 1$, $a^t \to +\infty$ when $t \to +\infty$. Then, the dynamics of government debt are explosive. This situation is said to be **dynamically efficient**.

1.3 Is public debt sustainable in the U.S.?

Up until now, it has been the case that $r < g_Y$. Indeed, as the Figures below show, nominal GDP growth, inclusive of inflation, is larger than 3%, while the interest rate on government debt is close to 2%. Therefore, the ratio of government debt to GDP does not appear to be on an unsustainable path so far. Similarly, the ratio of interest payments to GDP is not particularly high historically. This implies that if the primary deficit was reduced to zero, the debt to GDP ratio would not be on an explosive trajectory.



2 Public Debt in the Overlapping Generations Model

2.1 Overlapping Generations Model

Let us look at a simplified version of the overlapping generations model we looked at in Lecture 3, the model that we studied in Problem Set 3 called "Another Overlapping Generations Model". For this model, we shall assume that people only care about old age consumption, and that they work only when young, receiving wage w_t. It does not really matter what the form of their utility function is with respect to old age consumption, because they will save everything anyway:

$$U = u(c_{t+1}^o).$$

Denoting by r_t the (net) real interest rate, their intertemporal budget constraint is then given by:

$$c_t^y + \frac{c_{t+1}^o}{1+r_t} = w_t.$$

In this very simple environment, and because consumption in young age will always optimally be set to zero ($c_t^y = 0$), this implies:

$$c_{t+1}^o = (1+r_t)w_t.$$

Similarly to the previous time, we assume that the labor force is fixed to unity:Diamond (1965) had population growth in his original model. We do the simplest version of his model. ($L_t = \bar{L} = 1$) There is a Cobb-Douglas, constant returns to scale, production function:

$$Y_t = K_t^\alpha L_t^{1-\alpha}.$$

Together with the previous assumption of constant labor $L_t = 1$, this means that:

$$Y_t = K_t^\alpha.$$

Markets are competitive, so that the wage is just the marginal product of labor:

$$w_t = \frac{\partial Y_t}{\partial L_t} = (1-\alpha)K_t^\alpha L_t^{-\alpha} = (1-\alpha)K_t^\alpha.$$

Similarly as previously, we also get through firms' optimization on the amount of capital that:

$$r_t + \delta = \frac{\partial Y_t}{\partial K_t} = \alpha K_t^{\alpha-1}.$$

Finally, we assume again that capital depreciates at rate $\delta = 1 = 100\%$. (that is, capital fully depreciates each period - this is reasonable if you take one unit of time to represent one generation, or about 30 years - remember that the depreciation rate for one year was approximately equal to 10%.)

2.2 Without Government Debt

Let us first remind ourselves what happens in the absence of government debt in this model. In the absence of a government, we get even simpler expressions than the previous time. The law of motion for capital is given as follows:

$$\Delta K_{t+1} = w_t - \delta K_t.$$

Since w_t is a fraction $1-\alpha$ of output, this law of motion corresponds to the Solow growth model with $s = 1-\alpha$. The law of motion for capital is:

$$K_{t+1} = (1-\alpha)K_t^\alpha + (1-\delta)K_t.$$

This is a difference equation for sequence K_t which converges to a steady state value for the capital stock K^* such that:

$$\delta K^* = (1-\alpha)(K^*)^\alpha$$
$$\Rightarrow \quad K^* = \left(\frac{1-\alpha}{\delta}\right)^{\frac{1}{1-\alpha}}.$$

The steady state value for the interest rate is then:

$$\begin{aligned}
r^* + \delta &= \alpha (K^*)^{\alpha-1} \\
&= \alpha \left[\left(\frac{1-\alpha}{\delta} \right)^{\frac{1}{1-\alpha}} \right]^{\alpha-1} \\
r^* + \delta &= \frac{\delta\alpha}{1-\alpha}
\end{aligned}$$

Therefore:

$$r^* = \frac{2\alpha - 1}{1-\alpha}\delta$$

The steady state value for output is then:

$$\begin{aligned}
Y^* &= (K^*)^\alpha \\
Y^* &= \left(\frac{1-\alpha}{\delta} \right)^{\frac{\alpha}{1-\alpha}}.
\end{aligned}$$

The value for the wage would be:

$$\begin{aligned}
w^* &= (1-\alpha)\,(K^*)^\alpha \\
&= (1-\alpha) \left(\frac{1-\alpha}{\delta} \right)^{\frac{\alpha}{1-\alpha}} \\
w^* &= \frac{(1-\alpha)^{\frac{1}{1-\alpha}}}{\delta^{\frac{\alpha}{1-\alpha}}}
\end{aligned}$$

Steady-state consumption of the old is thus given by:

$$\begin{aligned}
(c^o)^* &= (1+r^*)w^* \\
(c^o)^* &= \left(1 + \frac{2\alpha-1}{1-\alpha}\delta \right) (1-\alpha)^{\frac{1}{1-\alpha}}
\end{aligned}$$

Example. With $\alpha = 1/3$ and $\delta = 1$:

$$\begin{aligned}
K^* &= \left(\frac{2}{3} \right)^{3/2}, \qquad r^* = -\frac{1}{2} = -50\%, \qquad Y^* = \sqrt{\frac{2}{3}} \\
w^* &= \left(\frac{2}{3} \right)^{3/2} \qquad (c^o)^* = \alpha\,(1-\alpha)^{\frac{\alpha}{1-\alpha}}
\end{aligned}$$

2.3 With Government Debt

Imagine we start at a steady state value for capital calculated above. The government can make every generation better off by taking on debt, for example by giving a transfer to the first generation of old people (like was done after the second world war), and rolling over this debt every period. Indeed, let us look at the level of capital such that $r^* = 0$ (which is called the golden rule interest rate). This level of capital is given by:

$$r^* + \delta = \alpha (K_g^*)^{\alpha-1}.$$

Therefore:

$$\begin{aligned}
\frac{2\alpha-1}{1-\alpha} + 1 = \alpha (K_g^*)^{\alpha-1} \quad &\Rightarrow \quad \frac{\alpha}{1-\alpha} = \alpha (K_g^*)^{\alpha-1} \\
\Rightarrow \quad K_g^* = (1-\alpha)^{\frac{1}{1-\alpha}}.&
\end{aligned}$$

Example. With $\alpha = 1/3$, with this level of the capital stock, we know that the wage would be given by:

$$w^* = \frac{2}{3}Y^* = \frac{2}{3}(K^*)^{1/3} = \frac{2}{3\sqrt{3}}.$$

The steady state value for output would then be:

$$Y^* = (K^*)^{1/3} = \frac{1}{\sqrt{3}}.$$

Then the steady-state consumption of the old is given by:

$$(c^o)^* = w^* = \frac{2}{3\sqrt{3}}.$$

Note that this is greater than the level of consumption achieved by the old without a government since $2 > \sqrt{2}$. But what is amazing is that the level of capital in this case is actually lower than the level of capital in the previous section. The government can force the economy into this level of capital accumulation by taking on debt. The level of debt B^* that corresponds to that level of capital accumulation is given by:

$$B^* + K^* = w^* \quad \Rightarrow \quad B^* = w^* - K^* = \frac{2}{3\sqrt{3}} - \frac{1}{3\sqrt{3}} = \frac{1}{3\sqrt{3}}.$$

The government can reach that level of debt by giving a transfer to the first generation of old, like the war veterans, who will then consume:

$$c_0^o = \frac{\sqrt{2}}{3\sqrt{3}} + \frac{1}{3\sqrt{3}} = \frac{1+\sqrt{2}}{3\sqrt{3}}.$$

All future generations will then consume more because of the above formula. With a lot of capital, there is such a thing as a free lunch! Public debt is a Ponzi scheme, but a beneficial one. Public debt allows to increase consumption for everyone, and it can be rolled over every period.

3 The Treasury View, and Say's Law

Even though the neoclassical model also predicts that government deficits have positive effects when the economy is in a dynamically inefficient state, they disagree on the effects that deficit spending has on investment: to simplify, there are two views: the neoclassical view (also called Treasury View) and the Keynesian view. We then discuss the relation to Say's law.

3.1 Treasury View: The Effects of Deficit Spending on Investment

- In the **Keynesian model**, investment is not crowded out by public debt (in the simplest model, investment is in fact fixed).

- In the **neoclassical model** of lectures 2 and 4, investment is crowded out by public deficits. Indeed, in this model but this may be a good thing if the economy has too much capital to begin with. This is called the "Treasury View", which is criticized by Keynesian economists (and I think they have a point there). Anyway, even if one accepts the Treasury View, when the capital stock is below the Golden Rule level, both the Neoclassical and Keynesian models converge on their policy prescriptions: deficit spending should be used.

This was the main bone of contention during the financial crisis. While Chicago economits were articulating the Treasury view in various different flavors, more Keynesian economists were rejecting this notion very strongly. We shall see in lecture 13 where the empirical evidence lies on this issue.

3.2 Say's law

Finally, we discuss Say's law: supply creates its own demand, and we try to reconcile the Keynesian and the Neoclassical models with respect to their elasticities between capital and labor.

4 Readings - To go further

There is no significant budget deficit, Olivier Blanchard, Jeffrey Sachs, *New York Times*, March 6, 1981.

A Note On The Ricardian Equivalence Argument Against Stimulus (Slightly Wonkish), New York Times Blog Post, December 26, 2011.

Paul Krugman, Multipliers and Reality, New York Times Blog Post, June 3, 2015.

In Japan, the Government Gets Paid to Borrow Money, *Wall Street Journal*, March 1, 2016.

(Gated) Say's law: supply creates its own demand. *The Economist*, August 10, 2017.

(Gated) Why is macroeconomics so hard to teach? *The Economist*, August 9, 2018.