

# Cobb and Douglas (1928) production function

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The Cobb and Douglas (1928) production function gives output  $Y$  as a power function of capital  $K$  and labor  $N$  :

$$Y = F(K, N) = K^\alpha N^{1-\alpha}$$

Take a firm which has this production function, hires labor at price  $w$  and rents capital at rate  $r$ . Then, this firm maximizes:

$$\max_{K, N} F(K, N) - wN - rK$$

The firm's optimal choice of the capital stock  $K$  implies that the marginal productivity of capital is equal to the rental rate of capital  $r$ :

$$r = \frac{\partial F(K, N)}{\partial K} = \alpha K^{\alpha-1} N^{1-\alpha}$$

Thus, income from capital is:

$$rK = \alpha K^\alpha N^{1-\alpha} \Rightarrow \frac{rK}{Y} = \alpha$$

Similarly, the firm's optimal choice of employment  $N$  implies that the marginal productivity of labor is equal to the wage  $w$ :

$$w = \frac{\partial F(K, N)}{\partial N} = (1 - \alpha) K^\alpha N^{-\alpha}$$

Thus, income from labor is:

$$wN = (1 - \alpha) K^\alpha N^{1-\alpha} \Rightarrow \frac{wN}{Y} = 1 - \alpha$$

Note that in equilibrium, firms make zero profits:

$$\frac{rK}{Y} + \frac{wN}{Y} = \alpha + (1 - \alpha) = 1 \Rightarrow F(K, N) = wN + rK$$

Therefore, the production approach to GDP gives the same as the income approach to GDP.

$\alpha$  is the **share of capital in value added**, and is typically equal to  $\alpha \approx 1/3$ . In contrast, the share of labor in value added is  $1 - \alpha \approx 2/3$ .

## References

Cobb, Charles W. and Paul H. Douglas, "A Theory of Production," *The American Economic Review*, 1928, 18 (1), 139–165. (document)