

# Lecture 9 - Redistributive Policies

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## Introduction

Keynesian economics provides a mechanism through which more redistribution might actually increase output overall, at the same time as it reduces inequality. The idea that the economy suffers from a shortage of aggregate demand coming from increases in inequality has been put forward recently by mainstream academics such as Raghuram Rajan, former chief economist of the IMF, and now governor at the Bank of England (Rajan [2010]), as well as by Robert Reich, US Secretary of Labor from 1993 to 1997 (Reich [2011]).

The idea that the MPC is influenced by the distribution of income and wealth comes back to Keynes [1936]:

Since the end of the nineteenth century significant progress towards the removal of very great disparities of wealth and income has been achieved through the instrument of direct taxation— income tax and surtax and death duties—especially in Great Britain. Many people would wish to see this process carried much further, but they are deterred by two considerations; partly by the fear of making skilful evasions too much worth while and also of diminishing unduly the motive towards risk-taking, but mainly, I think, by the belief that the growth of capital depends upon the strength of the motive towards individual saving and that for a large proportion of this growth we are dependent on the savings of the rich out of their superfluity. Our argument does not affect the first of these considerations. But it may considerably modify our attitude towards the second. For we have seen that, up to the point where full employment prevails, the growth of capital depends not at all on a low propensity to consume but is, on the contrary, held back by it; and only in conditions of full employment is a low propensity to consume conducive to the growth of capital. Moreover, experience suggests that in existing conditions saving by institutions and through sinking funds is more than adequate, and that measures for the redistribution of incomes in a way likely to raise the propensity to consume may prove positively favourable to the growth of capital.

This passage from Keynes [1936] is intuitive: as long as saving propensities are no longer an impediment to capital accumulation, redistributing income or wealth from low to high **Marginal Propensity to Consume**

(MPC) should lead to higher output. According to Keynes [1936], this is in fact one reason for restricting the increase in inequality:

The State will have to exercise a guiding influence on the propensity to consume partly through its scheme of taxation. (...) Whilst, therefore, the enlargement of the functions of government, involved in the task of adjusting to one another the propensity to consume and the inducement to invest, would seem to a nineteenth-century publicist or to a contemporary American financier to be a terrific encroachment on individualism, I defend it, on the contrary, both as the only practicable means of avoiding the destruction of existing economic forms in their entirety and as the condition of the successful functioning of individual initiative.

During this lecture, we derive this result using the Keynesian model that was developed in lecture 7 and lecture 8. One appeal of writing the equations is that we are not able to prove these assertions qualitatively, but we are also able to understand how important they are quantitatively. As we go along, we therefore attempt to put some actual numbers on all these arguments, to get a sense of the orders of magnitude. We shall investigate two types of policies:

- Income redistribution, from high to low income earners.
- Deficit-financed decreases in taxes, on high income earners or low income earners, financed by public debt.

## 1 Assumptions

Some minor modifications to the goods market model underlying lecture 7 and lecture 8 are in order, in order to think about stimulus policies in the presence of inequality. Instead of assuming one type of consumer, with the average income  $Y$  and a given MPC  $c_1$ , we shall assume two types of workers. In total, there are  $N$  workers:

- There is a fraction  $\lambda$  of low income earners, who earn income  $y$ , pay net taxes  $t$ , and the MPC of the low income earners is  $c_1$ :

$$c = c_0 + c_1(y - t).$$

- There is a fraction  $1 - \lambda$  of high income earners, they get a higher income  $\bar{y}$  which is a multiple  $\gamma$  of low income earners' income, given by:

$$\bar{y} = \gamma y,$$

where  $\gamma$  indexes inequality. They each pay net taxes  $\bar{t}$ , and the MPC of the high income earners is  $\bar{c}_1$ :

$$\bar{c} = \bar{c}_0 + \bar{c}_1(\bar{y} - \bar{t})$$

Moreover, we assume that high income earners have a lower MPC than low income earners, so that:

$$\bar{c}_1 < c_1$$

We assume that investment depends on output:

$$I = b_0 + b_1 Y$$

We also assume that taxes depend on output, both for low income earners:

$$t = t_0 + t_1 y$$

as well as for high income earners:

$$\bar{t} = \bar{t}_0 + t_1 \bar{y}.$$

## 2 Solving the model

### 2.1 Income

Since there is a fraction  $\lambda$  of low income earners, and the total population is  $N$ , the total income  $\underline{Y}$  captured by low income earners  $\underline{y}$  is:

$$\underline{Y} = \lambda N \underline{y}$$

Symmetrically, the total income  $\bar{Y}$  captured by high income earners is:

$$\bar{Y} = (1 - \lambda) N \bar{y}.$$

Total income is given by the sum of  $\underline{Y}$ , and  $\bar{Y}$ , which allows to express low income earners as a function of total income:

$$\begin{aligned} Y &= \underline{Y} + \bar{Y} \\ &= \lambda N \underline{y} + (1 - \lambda) N \bar{y} \\ &= \lambda N \underline{y} + (1 - \lambda) N \gamma \underline{y} \\ Y &= (\lambda + (1 - \lambda) \gamma) N \underline{y} \end{aligned}$$

This implies that low income earners'  $\underline{y}$  is given as a function of output per person  $Y/N$  and the parameters of the model by:

$$\underline{y} = \frac{1}{\lambda + (1 - \lambda) \gamma} \frac{Y}{N}$$

As a consequence, high income' individual  $\bar{y}$  is given as a function of output per person  $Y/N$  and the parameters of the model by:

$$\bar{y} = \gamma \underline{y} = \frac{\gamma}{\lambda + (1 - \lambda) \gamma} \frac{Y}{N}.$$

The share of income captured by low income earners is:

$$\begin{aligned} \frac{\underline{Y}}{Y} &= \frac{\lambda N \underline{y}}{(\lambda + (1 - \lambda) \gamma) N \underline{y}} \\ \frac{\underline{Y}}{Y} &= \frac{\lambda}{\lambda + (1 - \lambda) \gamma}. \end{aligned}$$

The share of income captured by high income earners is:

$$\begin{aligned} \frac{\bar{Y}}{Y} &= \frac{(1 - \lambda) N \bar{y}}{(\lambda + (1 - \lambda) \gamma) N \underline{y}} \\ &= \frac{(1 - \lambda) \gamma N \underline{y}}{(\lambda + (1 - \lambda) \gamma) N \underline{y}} \\ \frac{\bar{Y}}{Y} &= \frac{(1 - \lambda) \gamma}{\lambda + (1 - \lambda) \gamma}. \end{aligned}$$

**Numerical Application:** Approximate the number of workers in the US to about 150 million:

$$N = 150,000,000$$

and that GDP is 20 trillion. Therefore, GDP per worker on average is:

$$\frac{Y}{N} = \$133,333.33$$

Let us divide the population in two groups, the top 10% income share, and the bottom 90% income share, so that:  $\lambda = 0.9$ . Since the top 10% get approximately 50% of the income in the U.S. (the exact data is available here), this implies, using the formula above  $\underline{Y}/Y = \lambda/(\lambda + (1 - \lambda)\gamma)$ :

$$\frac{0.9}{0.9 + 0.1 \cdot \gamma} = 0.5 \Rightarrow \gamma = 9.$$

This is actually very intuitive: if 90% of the population have the same income as 10% of the population (half of total income), then on average they are 9 times poorer. The average income for someone in the top 10% is then:

$$\bar{y} = \gamma \underline{y} = \frac{\gamma}{\lambda + (1 - \lambda)\gamma} \frac{Y}{N} = \$666,666.66$$

They are:

$$(1 - \lambda)N = 15,000,000.$$

While the average income for someone in the bottom 90% is:

$$\underline{y} = \frac{1}{\lambda + (1 - \lambda)\gamma} \frac{Y}{N} = \$74,074.07.$$

They are:

$$\lambda N = 135,000,000.$$

## 2.2 Taxes

Aggregate taxes  $T$  are the sum of taxes paid by the low income earners  $\underline{T}$  and the high income earners  $\bar{T}$ :

$$\begin{aligned} T &= \underline{T} + \bar{T} \\ &= \lambda N \underline{t} + (1 - \lambda) N \bar{t} \\ &= \lambda N (t_0 + t_1 \underline{y}) + (1 - \lambda) N (\bar{t}_0 + t_1 \bar{y}) \\ &= (\lambda N \underline{t}_0 + (1 - \lambda) N \bar{t}_0) + t_1 (\lambda N \underline{y} + (1 - \lambda) N \bar{y}) \\ T &= (\underline{T}_0 + \bar{T}_0) + t_1 Y \end{aligned}$$

The aggregate baseline level of taxes  $T_0$  is:

$$T_0 \equiv \underline{T}_0 + \bar{T}_0 = \lambda N \underline{t}_0 + (1 - \lambda) N \bar{t}_0,$$

where baseline level of taxes for low and high income earners is given by:

$$\underline{T}_0 \equiv \lambda N \underline{t}_0, \quad \bar{T}_0 \equiv (1 - \lambda) N \bar{t}_0.$$

To conclude, total aggregate taxes are:

$$\boxed{T = (\underline{T}_0 + \bar{T}_0) + t_1 Y.}$$

## 2.3 Consumption

The challenging part, which differs from the models seen in lecture 7 and lecture 8, is to calculate aggregate consumption, which is composed both of the consumption by low income earners, and that by high income earners. Total consumption by the low income earners  $\underline{C}$  is such that:

$$\begin{aligned} \underline{C} &= \lambda N \underline{c} \\ &= \lambda N (\underline{c}_0 + \underline{c}_1 (\underline{y} - \underline{t})) \\ &= \lambda N \underline{c}_0 + \lambda N (1 - t_1) \underline{c}_1 \underline{y} - \lambda N \underline{c}_1 \underline{t}_0 \\ \underline{C} &= [\lambda N \underline{c}_0 - \lambda N \underline{c}_1 \underline{t}_0] + \frac{\lambda \underline{c}_1}{\lambda + (1 - \lambda)\gamma} (1 - t_1) Y \end{aligned}$$

Symmetrically, consumption by the high income earners  $\bar{C}$  is such that:

$$\begin{aligned}\bar{C} &= (1 - \lambda)N\bar{c} \\ &= (1 - \lambda)N(\bar{c}_0 + \bar{c}_1(\bar{y} - \bar{t})) \\ &= (1 - \lambda)N\bar{c}_0 + (1 - \lambda)N(1 - t_1)\bar{c}_1\bar{y} - (1 - \lambda)N\bar{c}_1\bar{t}_0 \\ \bar{C} &= [(1 - \lambda)N\bar{c}_0 - (1 - \lambda)N\bar{c}_1\bar{t}_0] + \frac{(1 - \lambda)\gamma\bar{c}_1}{\lambda + (1 - \lambda)\gamma}(1 - t_1)Y\end{aligned}$$

Therefore, aggregate consumption  $C = \underline{C} + \bar{C}$  is given by:

$$\begin{aligned}C &= \underline{C} + \bar{C} \\ &= [\lambda N\underline{c}_0 - \lambda N\underline{c}_1\underline{t}_0] + \frac{\lambda\underline{c}_1}{\lambda + (1 - \lambda)\gamma}(1 - t_1)Y + [(1 - \lambda)N\bar{c}_0 - (1 - \lambda)N\bar{c}_1\bar{t}_0] + \frac{(1 - \lambda)\gamma\bar{c}_1}{\lambda + (1 - \lambda)\gamma}(1 - t_1)Y \\ &= (\lambda N\underline{c}_0 + (1 - \lambda)N\bar{c}_0) - (\lambda N\underline{c}_1\underline{t}_0 + (1 - \lambda)N\bar{c}_1\bar{t}_0) + \frac{\lambda\underline{c}_1 + (1 - \lambda)\gamma\bar{c}_1}{\lambda + (1 - \lambda)\gamma}(1 - t_1)Y \\ C &= C_0 - (\underline{c}_1\underline{T}_0 + \bar{c}_1\bar{T}_0) + c_1(1 - t_1)Y.\end{aligned}$$

where we have defined the average MPC  $c_1$  by:

$$c_1 \equiv \frac{\lambda\underline{c}_1 + (1 - \lambda)\gamma\bar{c}_1}{\lambda + (1 - \lambda)\gamma}.$$

the aggregate baseline level of consumption  $C_0$  as:

$$C_0 \equiv \underline{C}_0 + \bar{C}_0 = \lambda N\underline{c}_0 + (1 - \lambda)N\bar{c}_0,$$

where the baseline level of consumption for low and high income earners is given by:

$$\underline{C}_0 \equiv \lambda N\underline{c}_0, \quad \bar{C}_0 \equiv (1 - \lambda)N\bar{c}_0.$$

To conclude, aggregate consumption is:

$$C = C_0 - (\underline{c}_1\underline{T}_0 + \bar{c}_1\bar{T}_0) + c_1(1 - t_1)Y.$$

### 3 Aggregate Demand

We want to compute aggregate demand:

$$Z = C + I + G.$$

We know that  $I$  has a very straightforward expression:

$$I = b_0 + b_1Y.$$

Using the expression for aggregate consumption  $C$  as well and plugging in into total demand yields:

$$\begin{aligned}Z &= C + I + G \\ &= C_0 - (\underline{c}_1\underline{T}_0 + \bar{c}_1\bar{T}_0) + c_1(1 - t_1)Y + b_0 + b_1Y + G \\ Z &= [C_0 - (\underline{c}_1\underline{T}_0 + \bar{c}_1\bar{T}_0) + b_0 + G] + (c_1(1 - t_1) + b_1)Y\end{aligned}$$

Equating output to demand  $Z = Y$  gives the value for output:

$$Y = \frac{1}{1 - (1 - t_1)c_1 - b_1} [C_0 - \underline{c}_1\underline{T}_0 - \bar{c}_1\bar{T}_0 + b_0 + G]$$

## 4 Different Fiscal Policies

### 4.1 Income redistribution from high income to low income earners

Assume that transfers to the low income earners are increased (or taxes decreased), so that  $\Delta \underline{T}_0 < 0$ , with an offsetting increase in taxes on high income earners such that  $\Delta T_0 = \Delta \underline{T}_0 + \Delta \bar{T}_0 = 0$ . We then have that  $\Delta \bar{T}_0 = -\Delta \underline{T}_0 > 0$ . This leads to a change in autonomous spending:

$$\Delta z_0 = -c_1 \Delta \underline{T}_0 - \bar{c}_1 \Delta \bar{T}_0 \Rightarrow \Delta z_0 = (c_1 - \bar{c}_1) \Delta \bar{T}_0 > 0.$$

That impulse leads to an increase in output given by:

$$\Delta Y = \frac{c_1 - \bar{c}_1}{1 - (1 - t_1) c_1 - b_1} \Delta \bar{T}_0 > 0.$$

Using the value for aggregate taxes:

$$\begin{aligned} T &= (\underline{T}_0 + \bar{T}_0) + t_1 Y \\ \Rightarrow \Delta T &= \underbrace{\Delta \underline{T}_0 + \Delta \bar{T}_0}_{\Delta T_0=0} + t_1 \Delta Y. \end{aligned}$$

Finally:

$$\Delta T = \frac{t_1 (c_1 - \bar{c}_1)}{1 - (1 - t_1) c_1 - b_1} \Delta \bar{T}_0.$$

Thus, public saving increase, there is a reduction in the deficit, in public debt, and therefore:

$$\Delta (T - G) = \frac{t_1 (c_1 - \bar{c}_1)}{1 - (1 - t_1) c_1 - b_1} \Delta \bar{T}_0$$

**Numerical Application:** On top of the above numerical values, we assume that the marginal tax rate is 25% so that  $t_1 = 1/4$ . Therefore:

$$c_1 = 1, \quad \bar{c}_1 = 1/3, \quad \gamma = 9, \quad \lambda = 0.9, \quad b_1 = 1/6, \quad t_1 = 1/4.$$

As shown above, this implies an average MPC given by  $c_1 = 2/3$ . Thus, a tax cut to low income earners financed by tax increases to high income earners leads to an increase in output given by the following multiplier:

$$\begin{aligned} \frac{c_1 - \bar{c}_1}{1 - (1 - t_1) c_1 - b_1} &= \frac{1 - 1/3}{1 - (1 - 1/4) * 2/3 - 1/6} \\ &= \frac{2/3}{1 - 1/2 - 1/6} \\ \frac{c_1 - \bar{c}_1}{1 - (1 - t_1) c_1 - b_1} &= 2 \end{aligned}$$

This implies that if \$1 billion is transferred from high to low income earners, GDP rises by \$2 billion. Importantly, this does not necessarily imply it should be done: first, high income earners are clearly worse off. Second, this calculation based on high income earners' lower MPC does not take into account that they may be discouraged to create jobs and become entrepreneurs if they are taxed too much.

Because output increases, we get an improvement in the budget surplus as well, given by:

$$\Delta (T - G) = \frac{t_1 (c_1 - \bar{c}_1)}{1 - (1 - t_1) c_1 - b_1} \Delta \bar{T}_0 = \frac{1}{2}$$

or, 500 million dollar improvement.

Therefore, **\$1 billion dollar transfer from high to low income earners** leads to an improvement in the public **surplus of \$0.5 billion dollars**.

## 4.2 Debt-financed tax cuts for high income earners

Assume tax cuts for high income earners  $\Delta \bar{T}_0 < 0$ , then output increases:

$$\Delta Y = -\frac{\bar{c}_1}{1 - (1 - t_1) c_1 - b_1} \Delta \bar{T}_0 > 0$$

The impact on aggregate taxes is however ambiguous:

$$\begin{aligned} \Delta T &= \Delta \bar{T}_0 + \underbrace{\Delta \underline{T}_0}_{=0} + t_1 \Delta Y \\ \Delta T &= \left( 1 - \frac{t_1 \bar{c}_1}{1 - (1 - t_1) c_1 - b_1} \right) \Delta \bar{T}_0 \end{aligned}$$

Therefore, the impact on public saving is similarly ambiguous:

$$\Delta (T - G) = \left( 1 - \frac{t_1 \bar{c}_1}{1 - (1 - t_1) c_1 - b_1} \right) \Delta \bar{T}_0$$

There is an increase in output and, depending on parameters, there can be a government surplus or a government deficit.

**Numerical Application:** We assume:

$$\underline{c}_1 = 1, \quad \bar{c}_1 = 1/3, \quad \gamma = 9, \quad \lambda = 0.9, \quad b_1 = 1/6, \quad t_1 = 1/4.$$

This implies  $c_1 = 2/3$ . Thus, we may calculate the tax multiplier for tax cuts to high income earners, which is given by:

$$\begin{aligned} \frac{\bar{c}_1}{1 - (1 - t_1) c_1 - b_1} &= \frac{1/3}{1 - (1 - 1/4) * 2/3 - 1/6} \\ &= \frac{1/3}{1 - 1/2 - 1/6} \\ \frac{\bar{c}_1}{1 - (1 - t_1) c_1 - b_1} &= 1 \end{aligned}$$

Therefore, the impact on public saving is given by:

$$\begin{aligned} \Delta (T - G) &= \left( 1 - \frac{t_1 \bar{c}_1}{1 - (1 - t_1) c_1 - b_1} \right) \Delta \bar{T}_0 \\ &= \left( 1 - t_1 * \frac{\bar{c}_1}{1 - (1 - t_1) c_1 - b_1} \right) \Delta \bar{T}_0 \\ &= \left( 1 - \frac{1}{4} * 1 \right) * (-1) \\ \Delta (T - G) &= -\frac{3}{4} \end{aligned}$$

Therefore, **\$1 billion dollar tax cut on high income earners** leads to an increase in the public **deficit** of **\$0.75 billion dollars**.

## 4.3 Debt-financed tax cuts for the low income earners

Assume tax cuts for low income earners  $\Delta \underline{T}_0 < 0$ , then output increases:

$$\Delta Y = -\frac{\underline{c}_1}{1 - (1 - t_1) c_1 - b_1} \Delta \underline{T}_0 > 0.$$

The impact on aggregate taxes is however ambiguous:

$$\Delta T = \Delta \underline{T}_0 + \underbrace{\Delta \bar{T}_0}_{=0} + t_1 \Delta Y$$

$$\Delta T = \left( 1 - \frac{t_1 \underline{c}_1}{1 - (1 - t_1) c_1 - b_1} \right) \Delta \underline{T}_0$$

The impact on public saving is similarly ambiguous:

$$\Delta (T - G) = \left( 1 - \frac{t_1 \underline{c}_1}{1 - (1 - t_1) c_1 - b_1} \right) \Delta \underline{T}_0$$

There is an increase in output and, depending on parameters, there can be a government surplus or a government deficit.

**Numerical Application:** We assume:

$$\underline{c}_1 = 1, \quad \bar{c}_1 = 1/3, \quad \gamma = 9, \quad \lambda = 0.9, \quad b_1 = 1/6, \quad t_1 = 1/4.$$

This implies  $c_1 = 2/3$ . Thus, we may calculate the tax multiplier for tax cuts to low income earners, which is given by:

$$\begin{aligned} \frac{\underline{c}_1}{1 - (1 - t_1) c_1 - b_1} &= \frac{1}{1 - (1 - 1/4) * 2/3 - 1/6} \\ &= \frac{1}{1 - 1/2 - 1/6} \\ \frac{\underline{c}_1}{1 - (1 - t_1) c_1 - b_1} &= 3 \end{aligned}$$

Therefore, the impact on public saving is given by:

$$\begin{aligned} \Delta (T - G) &= \left( 1 - \frac{t_1 \underline{c}_1}{1 - (1 - t_1) c_1 - b_1} \right) \Delta \underline{T}_0 \\ &= \left( 1 - t_1 * \frac{\underline{c}_1}{1 - (1 - t_1) c_1 - b_1} \right) \Delta \underline{T}_0 \\ &= \left( 1 - \frac{1}{4} * 3 \right) * (-1) \\ \Delta (T - G) &= -\frac{1}{4} \end{aligned}$$

Therefore, **\$1 billion dollar tax cut on low income earners** leads to an increase in the public **deficit** of **\$0.25 billion dollars**.

## References

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