Problem Set 4

UCLA - Econ 102 - Fall 2018

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1 The Solow Growth Model with Exogenous Growth

Consider the Solow growth model of Lecture 2, with however two small changes. Assume that the production function is given by:

$$F(K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha},$$

where productivity A_t grows exogenously at rate g and $A_0 = 1$:

$$A_t = (1+g)^t.$$

Moreover, assume that the labor force also grows at a rate n and $L_0 = 1$, so that at any time t:

$$L_t = (1+n)^t.$$

1. Write the law of motion for capital K_t.
2. Define k_t as:
$$k_t \equiv \frac{K_t}{A_t^{1/(1-\alpha)} L_t},$$
and write a law of motion for k_t. Assume that n, and g are small in order to simplify this law of motion. *Hint*: if n and g are small then: $(1+g)^{1/(1-\alpha)}(1+n) \approx 1 + \frac{1}{1-\alpha} g + n$.
3. Show that k_t converges to a steady-state k^*. Compute k^*.
4. When k_t has reached a steady-state, the economy is said to be on a **balanced growth path**. On this balanced growth path, what is the rate of growth of Y_t, C_t, K_t, K_t/Y_t, K_t/L_t, w_t, $w_t L_t$, r_t, $r_t K_t$, $w_t L_t / Y_t$, and $r_t K_t / Y_t$?
5. Compute y^* and c^* corresponding to steady-state k^* with:
$$y_t \equiv \frac{Y_t}{A_t^{1/(1-\alpha)} L_t} \quad \text{and} \quad c_t \equiv \frac{C_t}{A_t^{1/(1-\alpha)} L_t}.$$
6. What is the saving rate which maximizes c^*? (Golden Rule)
7. What is the value of r^* that corresponds to the Golden Rule level of capital accumulation?

2 The Neoclassical Labor Market Model

Consider the neoclassical labor market model of lecture 6. Assume that preferences and the production function are as in lecture 6:

$$U(c, l) = c - B\frac{l^{1+\epsilon}}{1+\epsilon}, \qquad f(l) = A l^{1-\alpha}.$$

Denote the wage by w, and the price of consumption by p.

1. Derive the Labor Demand curve.
2. Assume that $\alpha = 1/3$ and $A = 2$. Using your favorite spreadsheet software, plot this demand curve in a $(l, w/p)$ plane - that is, putting l on the x-axis and w/p on the y-axis.

3. Take logs of both sides. What does the demand curve look like in a $(\log(l), \log(w/p))$ plane? What is the slope of the demand curve equal to? If α is higher, is the demand curve steeper or flatter? What shifts the demand curve to the left or to the right?
4. Derive the Labor Supply curve.
5. Assume that $\epsilon = 5$ and $B = 2$. Using your favorite spreadsheet software, plot this supply curve in a $(l, w/p)$ plane - that is, putting l on the x-axis and w/p on the y-axis. Add the supply curve to the demand curve of question 2.
6. Take logs of both sides. What does the supply curve look like in a $(\log(l), \log(w/p))$ plane? What is the slope of the supply curve equal to? If ϵ is higher, is the supply curve steeper or flatter? What shifts the supply curve to the left, or to the right?
7. Assume that productivity A decreases by 5%, to $A = 1.9$. What is the effect on the quantity of employment, and on the real wage? If α is higher, is that effect larger or smaller? What is the economic intuition?
8. Assume that leisure becomes relatively more attractive relative to working (think of Facebook, Netflix, etc.), so that B increases by 10% (the disutility of work increases). What is the effect on the quantity of employment, and on the real wage? If ϵ is higher, is that effect larger or smaller? What is the economic intuition for this?

3 The Keynesian Labor Market Model

Consider the neoclassical labor market model of the previous problem.

1. Assume that productivity A decreases by 5%, but that real wages w/p are rigid. Compute the change in the quantity of employment following a fall in productivity.
2. Compare the effect with question 7 in the previous problem. Explain.
3. Assume that leisure becomes relatively more attractive relative to working, so that B increases by 10%. Compute the change in the quantity of employment following a increase in leisure attractiveness.
4. Compare the effect with question 8 in the previous problem. Explain.

4 The Bathtub model

Consider the bathtub model of lecture 6. Assume a monthly job separation rate equal to $s = 1\%$, and a monthly job finding rate equal to $f = 20\%$. Assume that the labor force is given by $L = 159$ million.

1. Derive the steady-state unemployment rate. How many people are unemployed in the steady-state? How many people lose their jobs every month? How many people find a job every month?
2. Assume that the economy starts with an unemployment rate equal to $u_0 = 8\%$. Using your favorite spreadsheet software, show the evolution of the unemployment rate over time. How long before the unemployment rate reaches 5%?
3. If $s = 2\%$ instead, which job finding rate f gives the same steady-state unemployment rate?
4. Assuming the separation rate and the job finding rate are given from question 4, answer question 2 again.
5. Explain why an economy with more churning (that is, faster reallocation) - think of the US versus Europe - has a faster recovery in terms of unemployment after a recession. *Note:* A recession could be coming from a temporary increase in the job separation rate, or a temporary decrease in the job finding rate, which then goes back to its original value.