

Foundations and Frontiers of Statistical Science  
From Probability to Learning, Complex Systems, and Philosophy

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# Preface



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