Systems Science: From Zero to Research Frontiers

A 70-Chapter Technical Roadmap (Hardcore Theory & Methods)

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Part I

System Language and Axiomatic Modeling

Chapter 1

Minimal Definitions: System, Boundary, and Environment

1.1 Starting with the Ship of Theseus

The Ship of Theseus is famous because it never really goes away. It does not feel like a distant "philosophy-only" riddle. It shows up in labs, in code, and in everyday scientific talk.

Replace one plank: is it still the same ship? You can argue either way, and each answer can sound reasonable: maybe it is the same ship because it still floats and still sails; maybe it is not, because it is no longer *the original*; maybe the "true" ship existed only at one specific moment in spacetime; maybe the ship must carry a particular microbiome, otherwise it is not *that* ship.

This chapter does not try to pick a winner. Instead, we use the Ship of Theseus as a gentle way to surface a bigger issue: when we say "the same," what rule are we using?

1.2 We are really arguing about an equal sign

I increasingly think many disagreements are not about the object itself, but about an invisible equal sign living in our heads.

When we ask "is it the same ship after one plank is replaced?" we are implicitly asking: *what does "=" mean here?*

Is our equal sign something like *functional equivalence* (it behaves the same way), or *material equivalence* (it is made of the same parts), or *historical continuity* (it is the same through time), or *instantaneous identity* (it must match exactly at a time-slice)?

A useful move is to stop treating this as a hidden preference. Instead, we make it explicit: different choices of "=" carve the world into different equivalence classes, and then we end up arguing as if we were talking about the same thing.

So the modest goal of this opening chapter is simple: **make the equal sign visible.** We do not rush to decide which one is "correct." We first admit that different equal signs produce different objects, different notions of essence vs. noise, and different scientific languages.

1.3 From "the same ship" to "essence vs. noise"

Here is a place where the Ship of Theseus quietly becomes a data-analysis problem.

In practice, scientists constantly draw a line between *what matters* (signal, essence, invariants, causal structure) and *what does not* (noise, artifacts, irrelevant fluctuations). But that line is not automatically given by nature. It is a choice—often a reasonable choice, but still a choice.

A concrete feeling (to be expanded later). Consider a recorded time series, say an EEG trace. A common habit is: "the smooth component is the signal, spikes are noise." But we can also ask: why *must* spikes be noise? Why couldn't spikes contain invariants, or structure, or even causal clues? Once you notice this, the essence/noise split starts to look like a cousin of the Ship of Theseus problem.

Draft note for later chapters: we will revisit this with actual tools (filtering, feature choices, generative models), and watch how changing the equivalence choice changes what we call "the system."

1.4 Why boundaries are often fuzzy (outliers as a continuum)

There is another reason these debates do not end quickly: the boundary between "noise" and "essence" is often not crisp.

Think about outliers. If a point is extremely far from the mean, we might confidently call it noise and remove it. But if it is slightly less far, is it still noise? If yes, what about slightly less far again? This is not a joke—it is a genuine continuity problem.

So even if we agree on the *spirit* of a criterion, we may still struggle to write a clean rule that draws a sharp boundary. And when the boundary is fuzzy, our equal sign becomes fuzzy too.

1.5 The awkwardness of natural language

We can talk about "noise" and "essence" very fluently in natural language, but natural language is not a specification language.

We often say things like: "remove artifacts," "keep the meaningful part," "ignore irrelevant variation," "focus on the invariant." These phrases communicate intent, but they do not uniquely determine an equivalence relation.

This is why disputes can persist even among smart, honest people: they may share a vibe, but not share the same formal boundary.

1.6 A learner's truce with the infinite tower

Yes, abstraction can be abstracted again, and again, producing an infinite meta-tower. We acknowledge the tower, but we do not climb it in this book.

The reason is not that the higher levels are meaningless. The reason is that this text is a learning record, and my understanding of systems science is still shallow. If I insist on treating my own current worldview as the judge of all previous work, I will end up rejecting useful tools before I have even learned them.

So we make a humble truce: for each theory we study, we temporarily accept the theory-builder's defaults (their objects, their boundaries, their observables, and their working notion of "the same"). This is not a final philosophical commitment. It is a practical way to stand on the shoulders of giants.

1.7 What we will do in this book

This chapter is not the place to enforce one universal notion of sameness. Doing that too early would exclude too much.

Instead we adopt a pragmatic learning strategy:

- **Meta-level:** we keep the general viewpoint that many scientific disputes can be reframed as disputes about an implicit equal sign (an equivalence relation induced by what we treat as distinguishable).
- **Working-level:** when we learn a specific theory or technique, we temporarily accept the theory-builder's default worldview (their chosen objects, boundaries, observables, and notion of "same"). This is not the final word; it is how we learn from existing knowledge.
- **Later:** when the moment is right, we will deliberately *switch the equivalence choice* and compare what changes. This is where we can see, concretely, what a "worldview" buys us and what it hides.

A small motto. We do not deny the ambiguity. We do not pretend it is solved. We keep it on the table, because it is there—like a mountain that does not disappear just because we refuse to look at it.

1.8 Minimal working definitions (level 1)

We now end this chapter with three minimal definitions. They are not meant to be final. They are meant to be *usable*: stable enough to host technical tools later, while honest about the fact that they depend on a chosen viewpoint.

System. A **system** is the collection of variables/objects whose behavior we take responsibility to model or explain, together with the internal update law we assign to them (dynamics, transition rule, mechanism). Equivalently: the system is what we choose to treat as "inside" under a working notion of sameness.

Boundary. A **boundary** is the cut that separates "inside" from "outside" in our description. Operationally, it is the rule that decides which influences are treated as internal mechanisms and which are external actions. A boundary typically comes with **ports** (interfaces): the quantities allowed to cross the cut (inputs, outputs, exchanged resources).

Environment. An **environment** is everything outside the boundary that can still affect the system (or be affected by it) through the ports. In practice, we often decompose the environment into *inputs* (what we intentionally apply), *disturbances* (unwanted but present influences), and *constraints* (limitations imposed on what can happen).

One-line summary. *The system is what we model; the boundary is where we stop modeling; the environment is what we do not model but still acknowledge as acting through the interface.*

A final reminder. These definitions live at level 1 of the abstraction tower. We keep them as working conventions so that later chapters can be precise, while remembering that changing the equivalence choice can change what counts as "the system," "the boundary," and "the environment."

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