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Technical Note

A Postprocessor Based on the Kinematics Model for General Five-Axis Machine Tools

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Abstract

This paper presents a novel concept to describe the three types of five-axis machine tools by a generalized kinematic structure. A generic postprocessor capable of converting the cutter location (CL) data to machine control data was developed based on the generalized kinematics model of five-axis machine tools. The machine tool's form-shaping function matrix is derived according to the homogeneous coordinate transformation matrix and the kinematic parameters characterizing the configuration of general five-axis machine tools. The analytical equations for NC data are determined by equating the CL data matrix and the form-shaping function matrix. A trial-cut experiment on a typical five-axis machine tool and the verification on the coordinate measurement machine demonstrates the effectiveness of the proposed scheme. The algorithm proposed here can facilitate determination of the postprocessors for various five-axis machine tools more systematically.

Keywords: *Five-axis, Postprocessor, Cutter Location Data, Form-shaping Function, Coordinate Transformation Matrix*

Introduction

Owing to the increasing requirement of enhancing a product's esthetics and performance, free-form surfaces, which can be defined as surfaces with variable curvature by higher-degree polynomials, are conventionally used in automotive, naval, and aeronautic industries, as well as in turbine blades, impellers, and plastic injection dies/molds. With the advent of commercial CAD/CAM systems and CNC machines, free-form surfaces can be machined with three- or five-axis machine tools through the tool path generation by software packages. To reduce preliminary work and increase cutting efficiency as well as workpiece accuracy, employing five-axis machining is increasingly prominent as compared to conventional three-axis machining, owing to the application of adding two rotational degrees of freedom such that spindle orientation with respect to workpiece may be varied while machining. However, industries frequently encounter difficulties in interfacing the CAM system with the machine control system. The interface is a software program known as the postprocessor, which converts the cutter location (CL) data created by CAM systems into the machine control data or "G" codes. *Figure 1* illustrates that the postprocessor acts as the bridge between the CAM system and the machine tool.

The CL data not only include the physical locations of the cutter, composed of cutter tip position and the tool orientation, but also the information regarding a machine tool's operation, such as tool change, feed rate, and coolant on/off. Because individual controls manufacturers do not adhere to an international standard, such as ISO 6983, the machine operation information processed by the postprocessor may be different according to various machine tool controllers; that is, different manufacturers may use different codes to define the same

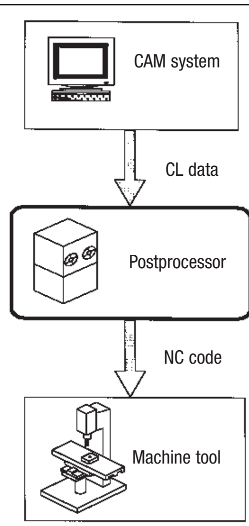

Figure 1
Relationship of CAM System, Postprocessor, and Machine Tool

function. Consequently, the more variety of NC units and machine configurations used, the greater the number of postprocessors. The postprocessor plays a crucial role in a modern machine shop. Owing to the lack of standardization of code format for various machine tool builders, the postprocessor presented here concerns only the transformation from the cutter's physical locations to the desired machine tool's motions, including linear motions and rotary motions, which is the main part of NC programs.

Methods to develop postprocessors for NC machine tools can generally be divided into two categories: three-axis and multi-axis. From the perspective of the three-axis postprocessor, the processing technology is straightforward and no coordinate transformation technique is required because the orientation of the cutter remains unchanged. Further details regarding the three-axis postprocessor can be found in Bedi and Vickers[1] and Lin and Chu.[2] From the perspective of the multi-axis (four- and five-axis) postprocessor, because of the varying cutter's orientation with respect to the workpiece, the coordinate transformation matrix must be introduced to obtain the desired equations for NC code; refer to Suh and Lee,[3] especially for five-axis machining. On the other hand, with the two additional rotary degrees of freedom in the tool motion, various combinations may be synthesized to yield five-axis machine tool configurations. Consequently, the postprocessor for a five-axis machine tool is inevitably developed individually.[4] For instance, Takeuchi and Watanabe[4] presented the postprocessor method on two five-axis configurations. Warkentin, Bedi, and Ismail[5] developed the technique of machining spherical surfaces and derived the five-axis NC code for one configuration. Moreover, Sakamoto and Inasaki[6] classified the configurations of five-axis machine tools into three typical types. Recently, Lee and She[7] presented the method for deriving the complete analytical NC code expressions for the above three kinds of five-axis machine tool configurations.

Worth mentioning is binary cutter location (BCL), which is another approach used as an input to the NC controller with postprocessor function.[8] Rüegg[9] proposed a generalized kinematics model for three to five-axis machine tools and implemented in a CNC controller. However, due to the high cost investment on the controller and the BCL data format not being widely accepted by users and machine tool builders,[10] this concept is not popularly used for universal five-axis machine tools.

The purpose of this paper is to extend the previous research proposed by Lee and She[7] and develop the generalized postprocessor for effectively dealing with the universal five-axis machine tool's configuration. By adding four rotational degrees of freedom where two of them are applied to the fixture table and the other two are applied to the spindle, the generalized kinematics model of general five-axis machines can be established and the corresponding form-shaping function matrix can then be determined through the homogeneous coordinate transformation matrix. In addition, the desired analytical equations for NC data are obtained by equating the known CL data matrix and the form-shaping function matrix. Furthermore, the validity of this methodology is confirmed by implementing a trial cut on the five-axis machining center and verifying on the coordinate measurement machine (CMM) according to the proposed algorithms.

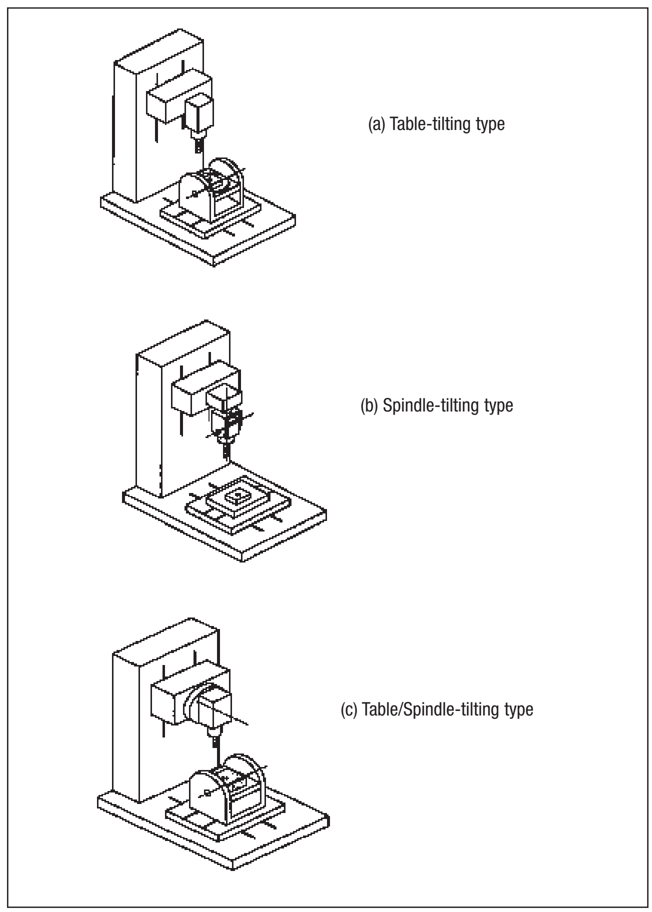

Figure 2
Three Typical Types of Five-Axis Machine Tool Configurations

Classification of Five-Axis Machine Tools

Each machine tool is characterized by combining two basic types of axis movements: linear motion and rotary motion. The five-axis machine tool typically consists of three perpendicular axes plus two more degrees of freedom in rotation along two of its three axes. Different machines may have different rotational axes either on the spindle or part fixture table. As *Figure 2* depicts, the five-axis machine tool can generally be classified into three basic types:[6] table-tilting, spindle-tilting, and table/spindle-tilting. For the table-tilting and spindle-tilting type, two rotational degrees of freedom exist on the table and the spindle, respectively. In addition, there is only one degree of freedom on the table and the spindle for the table/spindle-tilting type.

In this paper, the generalized kinematic structure of a machine tool consisting of three translational movements of X, Y, and Z axes and two rotational movements on part fixture table and two rotational movements on the spindle is proposed to develop the generalized five-axis machine tool's postprocessor.

Link and Joint Modeling with Elementary Matrices

Any machine tool may be thought of as a set of links connected in a chain by joints. Each joint usually exhibits one degree of freedom. Most machine tools have joints that are like hinges, called revolute joints, or have sliding joints called prismatic joints.

The revolute joint shown in *Figure 3* connects two links numbered i and j. Coordinate systems $O_iX_iY_iZ_i$ and $O_jX_jY_jZ_j$ are attached to link i and link j, respectively, where $L_{i,j}$ is the distance from origin O_i to O_j. The relative position and orientation of the jth system with respect to ith system can be obtained from the ith system by a translation $L_{i,j}(<\vec{\mathbf{u}}_{i,j},\vec{\mathbf{u}}_{xi}>,<\vec{\mathbf{u}}_{i,j},\vec{\mathbf{u}}_{yi}>,<\vec{\mathbf{u}}_{i,j},\vec{\mathbf{u}}_{zi}>)$ to O_j and is then rotated ϕ_N angle around the N coordinate axis to its jth system. Mathematically, the general form of the transformation that relates the coordinate systems attached to neighboring links can be expressed as follows:

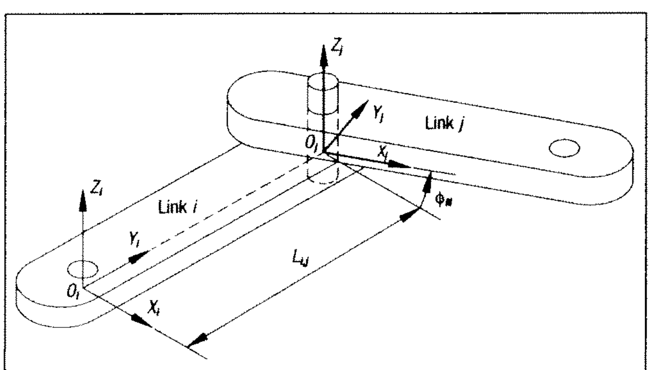

Figure 3
Coordinate Systems for a Revolute Joint

$$
{}^{i}\mathbf{A}_j = \mathbf{Trans}\left(L_{i,j}\langle\vec{\mathbf{u}}_{i,j},\vec{\mathbf{u}}_{xi}\rangle, L_{i,j}\langle\vec{\mathbf{u}}_{i,j},\vec{\mathbf{u}}_{yi}\rangle, L_{i,j}\langle\vec{\mathbf{u}}_{i,j},\vec{\mathbf{u}}_{zi}\rangle\right)\mathbf{Rot}\,(N,\phi_N) \tag{1}
$$

where ${}^{i}\mathbf{A}_j$ represents the relative transformation matrix of system $O_jX_jY_jZ_j$ with respect to system $O_iX_iY_iZ_i$. $\vec{\mathbf{u}}_{i,j}$ is the unit vector determined from O_i to O_j. $\vec{\mathbf{u}}_{xi},\vec{\mathbf{u}}_{yi},\vec{\mathbf{u}}_{zi}$ are the unit vectors of system $O_iX_iY_iZ_i$; $<.\,,.>$ denotes the inner product and symbol $N \equiv x,\ y,\ z$. **Trans** and **Rot** are translation and rotation matrices[11] and can be expressed in 4×4 homogeneous matrices as follows:

$$
\mathbf{Trans}(a,b,c)=\begin{bmatrix}1&0&0&a\\0&1&0&b\\0&0&1&c\\0&0&0&1\end{bmatrix} \tag{2}
$$

$$
\mathbf{Rot}(x,\phi_x)=\begin{bmatrix}1&0&0&0\\0&C\phi_x&-S\phi_x&0\\0&S\phi_x&C\phi_x&0\\0&0&0&1\end{bmatrix} \tag{3}
$$

$$
\mathbf{Rot}(y,\phi_y)=\begin{bmatrix}C\phi_y&0&S\phi_y&0\\0&1&0&0\\-S\phi_y&0&C\phi_y&0\\0&0&0&1\end{bmatrix} \tag{4}
$$

$$
\mathbf{Rot}(z,\phi_z)=\begin{bmatrix}C\phi_z&-S\phi_z&0&0\\S\phi_z&C\phi_z&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} \tag{5}
$$

where "C" and "S" refer to cosine and sine functions, respectively.

Note that Eq. (1) is valid not only for revolute joints but also for prismatic joints, provided that $\phi_N = 0$.

For the special case, if $\vec{\mathbf{u}}_{i,j}$ is parallel or antiparallel to one of the unit vectors $\vec{\mathbf{u}}_{xi},\vec{\mathbf{u}}_{yi},\vec{\mathbf{u}}_{zi}$, the elementary matrices are introduced to simplify the expression of transformation matrix for later use. The elementary matrices are defined as follows:

$$ {}^{i}\mathbf{T}_j^1(L_{i,j},\phi_N) = \mathbf{Trans}(L_{i,j},0,0)\ \mathbf{Rot}(N,\phi_N) \quad \text{if } \vec{\mathbf{u}}_{xi} // \vec{\mathbf{u}}_{i,j} \tag{6} $$

$$ {}^{i}\mathbf{T}_j^2(L_{i,j},\phi_N) = \mathbf{Trans}(0,L_{i,j},0)\ \mathbf{Rot}(N,\phi_N) \quad \text{if } \vec{\mathbf{u}}_{yi} // \vec{\mathbf{u}}_{i,j} \tag{7} $$

$$ {}^{i}\mathbf{T}_j^3(L_{i,j},\phi_N) = \mathbf{Trans}(0,0,L_{i,j})\ \mathbf{Rot}(N,\phi_N) \quad \text{if } \vec{\mathbf{u}}_{zi} // \vec{\mathbf{u}}_{i,j} \tag{8} $$

$$ {}^{i}\mathbf{T}_j^4(L_x,L_y,L_z,\phi_N) = \mathbf{Trans}(L_x,L_y,L_z)\ \mathbf{Rot}(N,\phi_N) \quad \text{otherwise} \tag{9} $$

where $L_{i,j}$ in Eqs. (6) to (8) is a signed quantity and the sign of $L_{i,j}$ depends on $< \vec{\mathbf{u}}_{i,j}, \vec{\mathbf{u}}_{xi} >$, $< \vec{\mathbf{u}}_{i,j}, \vec{\mathbf{u}}_{yi} >$, and $< \vec{\mathbf{u}}_{i,j}, \vec{\mathbf{u}}_{zi} >$. L_x, L_y, and L_z are the components of $L_{i,j}$ and can be determined by the following:

$$ L_x = |L_{i,j}| \langle \vec{\mathbf{u}}_{i,j}, \vec{\mathbf{u}}_{xi} \rangle \tag{10} $$

$$ L_y = |L_{i,j}| \langle \vec{\mathbf{u}}_{i,j}, \vec{\mathbf{u}}_{yi} \rangle \tag{11} $$

$$ L_z = |L_{i,j}| \langle \vec{\mathbf{u}}_{i,j}, \vec{\mathbf{u}}_{zi} \rangle \tag{12} $$

Forward Kinematic Modeling of Generalized Five-Axis Machine Tool Structure

As *Figure 4* indicates, two rotational movements occur on the workpiece, where $\vec{\mathbf{I}}_{wp}$ and $\vec{\mathbf{I}}_{ws}$ are the primary and secondary rotation axes on the workpiece fixture table. In addition, the other two rotational movements are on the spindle with the primary and secondary rotation axes $\vec{\mathbf{I}}_{sp}$ and $\vec{\mathbf{I}}_{ss}$. To describe the relative position and orientation of the cutting tool with respect to the workpiece, appropriate coordinate systems shown in *Figure 5a* should be established first, and *Figure 5b* shows the relationship of consecutive structural elements and coordinate systems. The coordinate system for the workpiece is $O_wX_wY_wZ_w$. Coordinate systems $O_{wp}X_{wp}Y_{wp}Z_{wp}$ and $O_{ws}X_{ws}Y_{ws}Z_{ws}$ are attached to the primary and secondary rotations

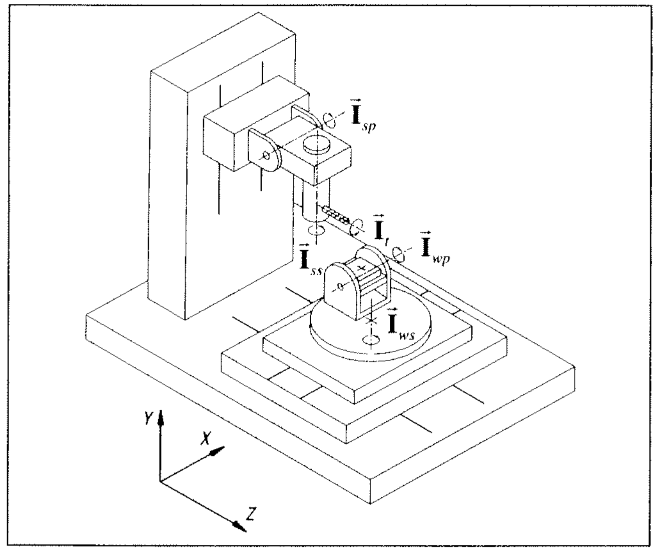

Figure 4
Generalized Kinematic Structure of Five-Axis Machine Tool

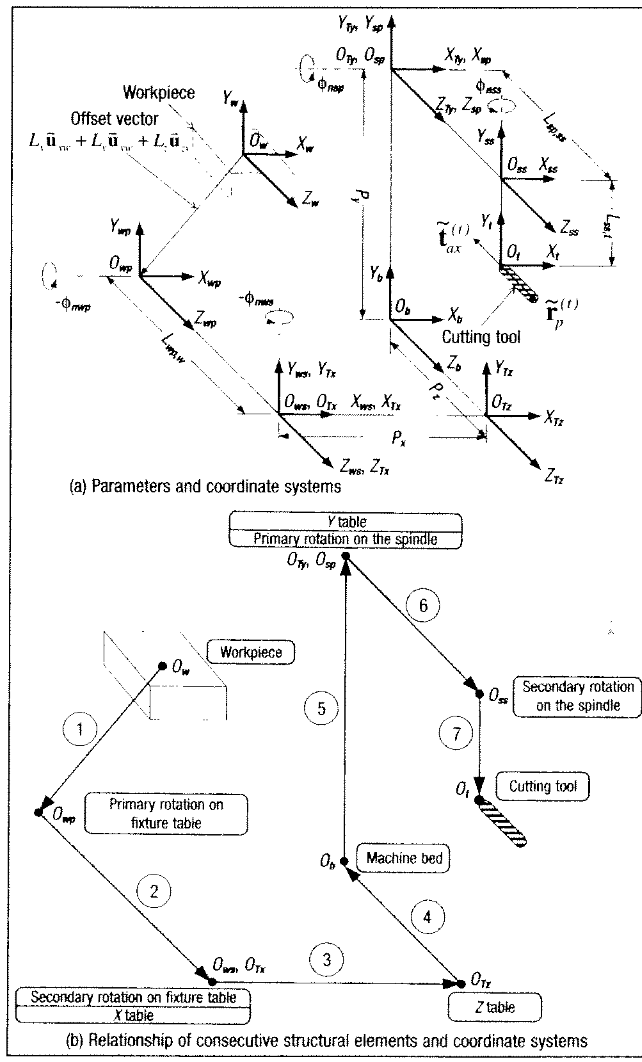

Figure 5
Coordinate Systems for Generalized Five-Axis Machine Tool

on the workpiece fixture table, respectively, because for any two axes in 3D space a line exists that is mutually perpendicular to both axes. The common normal line of axes $\vec{\mathbf{l}}_{wp}$ and $\vec{\mathbf{l}}_{ws}$ intersects with axes $\vec{\mathbf{l}}_{wp}$ and $\vec{\mathbf{l}}_{ws}$ at two points. For simplicity, and without any loss in generality, it is convenient to set the origins O_{wp} and O_{ws} at those two points. Similarly, coordinate systems $O_{sp}X_{sp}Y_{sp}Z_{sp}$ and $O_{ss}X_{ss}Y_{ss}Z_{ss}$ are attached to the primary and secondary rotations on the spindle. The origins O_{sp} and O_{ss} are located at the intersection of axes $\vec{\mathbf{l}}_{sp}$ and $\vec{\mathbf{l}}_{ss}$ and their common normal line, respectively. The coordinate system for the machine bed is $O_bX_bY_bZ_b$, and systems $O_{Tx}X_{Tx}Y_{Tx}Z_{Tx}$, $O_{Ty}X_{Ty}Y_{Ty}Z_{Ty}$, and $O_{Tz}X_{Tz}Y_{Tz}Z_{Tz}$, which describe only the relative pure translation along the X, Y, and Z direction, are attached to the X, Y, and Z table. The coordinate system for the cutting tool is $O_tX_tY_tZ_t$, and origin O_t is located at the intersection of cutting tool axis $\vec{\mathbf{l}}_t$ and axis $\vec{\mathbf{l}}_{ss}$. The detailed flow chart of the structural elements and coordinate systems is shown in *Figure 6*.

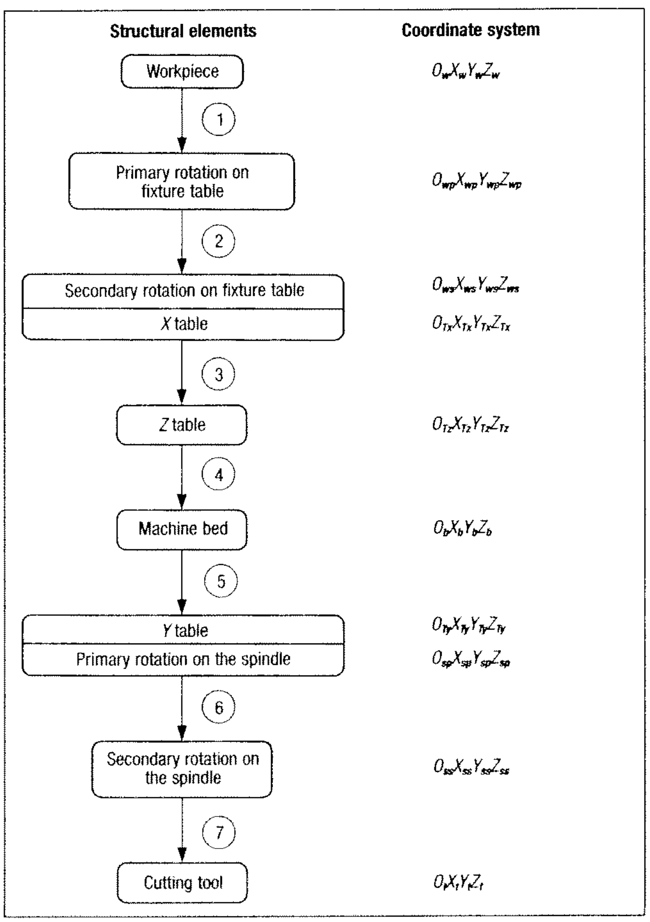

Figure 6
Flow Chart of Structural Elements and Corresponding Coordinate Systems for the Machine Tool

According to the definition of coordinate systems, the following transformation matrices of the coordinate systems can be obtained:

$$^{w}\mathbf{A}_{wp} = {}^{w}\mathbf{T}^{4}_{wp}\left(L_x, L_y, L_z, -\phi_{nwp}\right) \tag{13}$$

$$^{wp}\mathbf{A}_{ws} = {}^{wp}\mathbf{T}^{mws}_{ws}\left(L_{wp,ws}, -\phi_{nws}\right) \tag{14}$$

$$^{Tx}\mathbf{A}_{Tz} = {}^{Tx}\mathbf{T}^{1}_{Tz}\left(P_x, 0\right) \tag{15}$$

$$^{Tz}\mathbf{A}_{b} = {}^{Tz}\mathbf{T}^{3}_{b}\left(P_z, 0\right) \tag{16}$$

$$^{b}\mathbf{A}_{Ty} = {}^{b}\mathbf{T}^{2}_{Ty}\left(P_y, 0\right) \tag{17}$$

$$^{Ty}\mathbf{A}_{sp} = {}^{Ty}\mathbf{T}^{4}_{sp}\left(0, 0, 0, \phi_{nsp}\right) \tag{18}$$

$$^{sp}\mathbf{A}_{ss} = {}^{sp}\mathbf{T}^{mss}_{ss}\left(L_{sp,ss}, \phi_{nss}\right) \tag{19}$$

$$^{ss}\mathbf{A}_{t} = {}^{ss}\mathbf{T}^{mt}_{t}\left(L_{ss,t}, 0\right) \tag{20}$$

where the offset vector $L_x\vec{\mathbf{u}}_{xw} + L_y\vec{\mathbf{u}}_{yw} + L_z\vec{\mathbf{u}}_{zw}$ is determined from origin O_w to O_{wp}. Superscripts mws, mss, and mt = 1, 2, 3, and subscripts nwp, nws, nsp, and $nss \equiv x, y, z$. P_x, P_y, and P_z denote the relative translation distances of the X, Y, Z tables, respectively. In addition, ϕ_{nwp}, ϕ_{nws}, ϕ_{nsp}, and ϕ_{nss} represent the rotation angles for the fixture table and the spindle, respectively. Positive rotation is in the direction to advance a right-hand screw in the $+X$, $+Y$, or $+Z$ axis direction. Moreover, when the fixture table rotates ϕ_{nwp} and ϕ_{nws} angles around the corresponding axis, the relative rotation angles for system $O_{wp}X_{wp}Y_{wp}Z_{wp}$ with respect to system $O_wX_wY_wZ_w$, and system $O_{ws}X_{ws}Y_{ws}Z_{ws}$ with respect to system $O_{wp}X_{wp}Y_{wp}Z_{wp}$, are $-\phi_{nwp}$ and $-\phi_{nws}$, respectively. The above fact accounts for the angle effect arising in Eqs. (13) and (14).

Furthermore, because the secondary rotation on the part fixture is attached to the X table, the relative transformation matrix of system $O_{Tx}X_{Tx}Y_{Tx}Z_{Tx}$ with respect to system $O_{ws}X_{ws}Y_{ws}Z_{ws}$ is an identity transformation, that is,

$$^{ws}\mathbf{A}_{Tx} = \tilde{\mathbf{I}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{21}$$

Now, the relative position and orientation of coordinate system $O_tX_tY_tZ_t$ attached to the cutting tool with respect to the coordinate system $O_wX_wY_wZ_w$ attached to workpiece $^w\mathbf{A}_t$ can be obtained by transforming $^w\mathbf{A}_{wp}$ to $^{ss}\mathbf{A}_t$ in series (see *Figure 6*), and is expressed in a matrices product as follows:

$$^{w}\mathbf{A}_t = {}^{w}\mathbf{A}_{wp}\,{}^{wp}\mathbf{A}_{ws}\,{}^{ws}\mathbf{A}_{Tx}\,{}^{Tx}\mathbf{A}_{Tz}\,{}^{Tz}\mathbf{A}_{b}\,{}^{b}\mathbf{A}_{Ty}\,{}^{Ty}\mathbf{A}_{sp}\,{}^{sp}\mathbf{A}_{ss}\,{}^{ss}\mathbf{A}_{t} \tag{22}$$

Moreover, because

$$\begin{aligned} &{}^{ws}\mathbf{A}_{Tx}\,{}^{Tx}\mathbf{A}_{Tz}\,{}^{Tz}\mathbf{A}_{b}\,{}^{b}\mathbf{A}_{Ty}\,{}^{Ty}\mathbf{A}_{sp} \\ &= \tilde{\mathbf{I}}\,{}^{Tx}\mathbf{T}^1_{Tz}(P_x,0)\,{}^{Tz}\mathbf{T}^3_{b}(P_z,0)\,{}^{b}\mathbf{T}^2_{Ty}(P_y,0)\,{}^{Ty}\mathbf{T}^4_{sp}(0,0,0,\phi_{nsp}) \\ &= {}^{ws}\mathbf{T}^4_{sp}(P_x,P_y,P_z,\phi_{nsp}) \end{aligned} \tag{23}$$

The above equation suggests that the relative transformation matrix of system $O_{sp}X_{sp}Y_{sp}Z_{sp}$ with respect to system $O_{ws}X_{ws}Y_{ws}Z_{ws}$ can be obtained by a translation (P_x, P_y, P_z), which represents the relative X, Y, Z table movements, respectively, to O_{sp} and is then rotated ϕ_{nsp} angle about O_{sp}.

Therefore, Eq. (22) can be represented as follows:

$$\begin{aligned} &{}^{w}\mathbf{A}_t = {}^{w}\mathbf{T}^4_{wp}(L_x, L_y, L_z, -\phi_{nwp}) \\ &{}^{wp}\mathbf{T}^{mws}_{ws}(L_{wp,ws}, -\phi_{nws})\,{}^{ws}\mathbf{T}^4_{sp}(P_x,P_y,P_z,\phi_{nsp}) \\ &{}^{sp}\mathbf{T}^{mss}_{ss}(L_{sp,ss}, \phi_{nss})\,{}^{ss}\mathbf{T}^{mt}_{t}(L_{ss,t},0) \end{aligned} \tag{24}$$

In addition, the tool axis vector, $\tilde{\mathbf{t}}^{(t)}_{ax}$, and the position vector of the cutter tip center, $\tilde{\mathbf{r}}^{(t)}_{p}$, with respect to system $O_tX_tY_tZ_t$ can be transformed to system $O_wX_wY_wZ_w$, and obtain $\tilde{\mathbf{t}}^{(w)}_{ax}$, $\tilde{\mathbf{r}}^{(w)}_{p}$ using Eq. (24). The transformation is as follows:

$$\begin{aligned} &\begin{bmatrix} \tilde{\mathbf{t}}^{(w)}_{ax} & \tilde{\mathbf{r}}^{(w)}_{p} \\ 0 & 1 \end{bmatrix} \\ &= {}^{w}\mathbf{A}_t \begin{bmatrix} \tilde{\mathbf{t}}^{(t)}_{ax} & \tilde{\mathbf{r}}^{(t)}_{p} \\ 0 & 1 \end{bmatrix} \\ &= {}^{w}\mathbf{T}^4_{wp}(L_x, L_y, L_z, -\phi_{nwp})\,{}^{wp}\mathbf{T}^{mws}_{ws}(L_{wp,ws}, -\phi_{nws}) \\ &{}^{ws}\mathbf{T}^4_{sp}(P_x,P_y,P_z,\phi_{nsp})\,{}^{sp}\mathbf{T}^{mss}_{ss}(L_{sp,ss}, \phi_{nss}) \\ &{}^{ss}\mathbf{T}^{mt}_{t}(L_{ss,t},0) \begin{bmatrix} \tilde{\mathbf{t}}^{(t)}_{ax} & \tilde{\mathbf{r}}^{(t)}_{p} \\ 0 & 1 \end{bmatrix} \end{aligned} \tag{25}$$

Equation (25) describes the form-shaping function matrix[12] of generalized five-axis machine tools consisting of three linear axis motions and four rotational axis motions of which only two of them are active; and this matrix will be used to determine the desired joint parameters, that is, ϕ_{nwp}, ϕ_{nws}, ϕ_{nsp}, ϕ_{nss}, P_x, P_y, and P_z.

Inverse Kinematics

A large number of NC-oriented CAD/CAM systems can generate the tool path (CL data) for five-axis machining of free-form surfaces. The CL data consists of the cutter location point $\tilde{\mathbf{Q}}$ and the cutter's axis orientation $\tilde{\mathbf{K}}$ with respect to the workpiece coordinate system and can be expressed in the matrix form as follows:

$$\begin{bmatrix} \tilde{\mathbf{K}} & \tilde{\mathbf{Q}} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} K_x & Q_x \\ K_y & Q_y \\ K_z & Q_z \\ 0 & 1 \end{bmatrix} \tag{26}$$

where K_x, K_y, and K_z are direction cosines of the vector $\tilde{\mathbf{K}}$ and Q_x, Q_y, and Q_z are the components of position point vector $\tilde{\mathbf{Q}}$. Once the CL data matrix is known, determining five reference inputs (three linear motions plus two rotary motions) for the five-axis machine used is referred to as the inverse kinematics transformation. As derived in the preceding section, the form-shaping function matrix that characterizes the trajectory of motion of tool points relative to the workpiece is constructed through the coordinate transformation matrices. Hence, by equating the CL data matrix and form-shaping function matrix, the joint parameters can be solved such that the appropriate NC data employed for five-axis machining are also determined. From this perspective, the following is obtained:

$$\begin{bmatrix} \tilde{\mathbf{K}} & \tilde{\mathbf{Q}} \\ 0 & 1 \end{bmatrix} = {}^{w}\mathbf{T}_{wp}^{4}\left(L_x, L_y, L_z, -\phi_{nwp}\right) {}^{wp}\mathbf{T}_{ws}^{mws}\left(L_{wp,ws}, -\phi_{nws}\right)$$
$$ {}^{ws}\mathbf{T}_{sp}^{4}\left(P_x, P_y, P_z, \phi_{nsp}\right) {}^{sp}\mathbf{T}_{ss}^{mss}\left(L_{sp,ss}, \phi_{nss}\right) \quad (27)$$
$$ {}^{ss}\mathbf{T}_{t}^{mt}\left(L_{ss,t}, 0\right) \begin{bmatrix} \tilde{\mathbf{t}}_{ax}^{(t)} & \tilde{\mathbf{r}}_{p}^{(t)} \\ 0 & 1 \end{bmatrix}$$

The inverse kinematics of Eq. (27) is solved by first finding the joint angles ϕ_{nwp}, ϕ_{nws}, ϕ_{nsp}, and ϕ_{nss} and then three linear joint parameters P_x, P_y, and P_z.

Determination of Rotational Motions

Taking the corresponding elements of the first column of Eq. (27) leads to the following equation:

$$\begin{bmatrix} \tilde{\mathbf{K}} \\ 0 \end{bmatrix} = {}^{w}\mathbf{T}_{wp}^{4}\left(L_x, L_y, L_z, -\phi_{nwp}\right) {}^{wp}\mathbf{T}_{ws}^{mws}\left(L_{wp,ws}, -\phi_{nws}\right)$$
$$ {}^{ws}\mathbf{T}_{sp}^{4}\left(P_x, P_y, P_z, \phi_{nsp}\right) {}^{sp}\mathbf{T}_{ss}^{mss}\left(L_{sp,ss}, \phi_{nss}\right) \quad (28)$$
$$ {}^{ss}\mathbf{T}_{t}^{mt}\left(L_{ss,t}, 0\right) \begin{bmatrix} \tilde{\mathbf{t}}_{ax}^{(t)} \\ 0 \end{bmatrix}$$

Two of the joint angles (ϕ_{nwp}, ϕ_{nws}, ϕ_{nsp}, and ϕ_{nss}) on the right-hand side of Eq. (28) are unknown, and the matrix Eq. (28) yields to three scalar equations of which two of them are independent, owing to the fact that K_x, K_y, and K_z are direction cosines of the vector $\tilde{\mathbf{K}}$ and $K_x^2 + K_y^2 + K_z^2 = 1$ is always true. Note that the linear joint parameters (P_x, P_y, and P_z) in Eq. (28) do not affect the calculation of joint angles. This will be demonstrated later in the illustrative example. Once the joint angles are obtained, the input of rotary motions for the five-axis machine tool can then be described by an axis command consisting of the axis address word (*A*, *B*, or *C*) followed by the magnitude of the corresponding joint angle.

Determination of Linear Motions

As before, using Eq. (27) yields:

$$\begin{bmatrix} \tilde{\mathbf{Q}} \\ 1 \end{bmatrix} = {}^{w}\mathbf{T}_{wp}^{4}\left(L_x, L_y, L_z, -\phi_{nwp}\right) {}^{wp}\mathbf{T}_{ws}^{mws}\left(L_{wp,ws}, -\phi_{nws}\right)$$
$$ {}^{ws}\mathbf{T}_{sp}^{4}\left(P_x, P_y, P_z, \phi_{nsp}\right) {}^{sp}\mathbf{T}_{ss}^{mss}\left(L_{sp,ss}, \phi_{nss}\right) \quad (29)$$
$$ {}^{ss}\mathbf{T}_{t}^{mt}\left(L_{ss,t}, 0\right) \begin{bmatrix} \tilde{\mathbf{r}}_{p}^{(t)} \\ 1 \end{bmatrix}$$

After substituting joint angle values into Eq. (29) and comparing the corresponding elements of the matrix on both sides of Eq. (29), three simultaneous equations involving three unknowns (P_x, P_y, and P_z) are obtained. Therefore, the linear joint parameters can be solved according to the simultaneous equations derived. It is worth mentioning that P_x, P_y, and P_z are not the actual NC code expressions (denoted as *X*, *Y*, and *Z*). The axis command of linear movements of the machine tool should be referred to the program coordinate system. Most machine controllers (for example, FANUC) use the series of G codes from G54 through G59 to define the program coordinate system, which is often coincident with the workpiece coordinate system. To simplify the programming effort, the program coordinate system is assumed here to coincide with the workpiece coordinate system. Consequently, the desired *X*, *Y*, and *Z* values of NC data in programming can then be determined using Eq. (29) under the following condition:

$$\phi_{nwp} = \phi_{nws} = \phi_{nsp} = \phi_{nss} = 0$$
$$\text{and } \begin{bmatrix} Q_x & Q_y & Q_z & 1 \end{bmatrix}^T = \begin{bmatrix} X & Y & Z & 1 \end{bmatrix}^T$$

This leads to

$$\begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = {}^{w}\mathbf{T}_{wp}^{4}\left(L_x, L_y, L_z, 0\right) {}^{wp}\mathbf{T}_{ws}^{mws}\left(L_{wp,ws}, 0\right)$$
$$ {}^{ws}\mathbf{T}_{sp}^{4}\left(P_x, P_y, P_z, 0\right) {}^{sp}\mathbf{T}_{ss}^{mss}\left(L_{sp,ss}, 0\right) \quad (30)$$
$$ {}^{ss}\mathbf{T}_{t}^{mt}\left(L_{ss,t}, 0\right) \begin{bmatrix} \tilde{\mathbf{r}}_{p}^{(t)} \\ 1 \end{bmatrix}$$

The illustrative example outlines the complete derivation procedures for the proposed algorithm.

Implementation

Illustrative example

A table/spindle-tilting type of five-axis machine tool is used to verify the feasibility of the presented methodology. For the kinematic structure in *Figure 7*, the relevant parameters can be specified as follows:

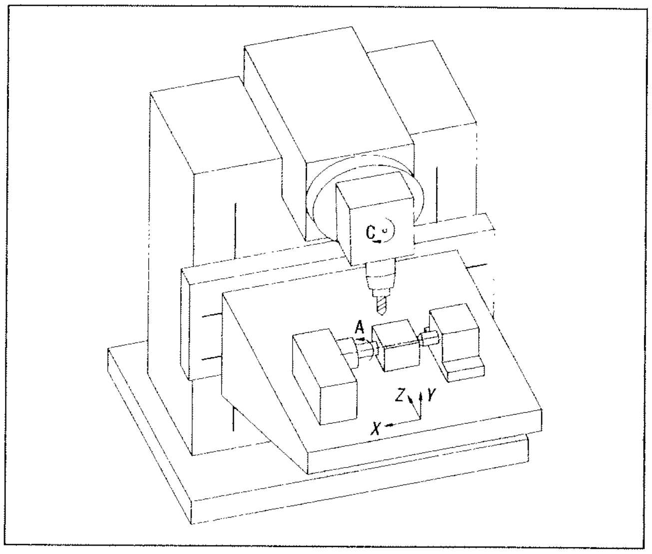

Figure 7
Configuration of a Table/Spindle-Tilting Type of Five-Axis Machining Center

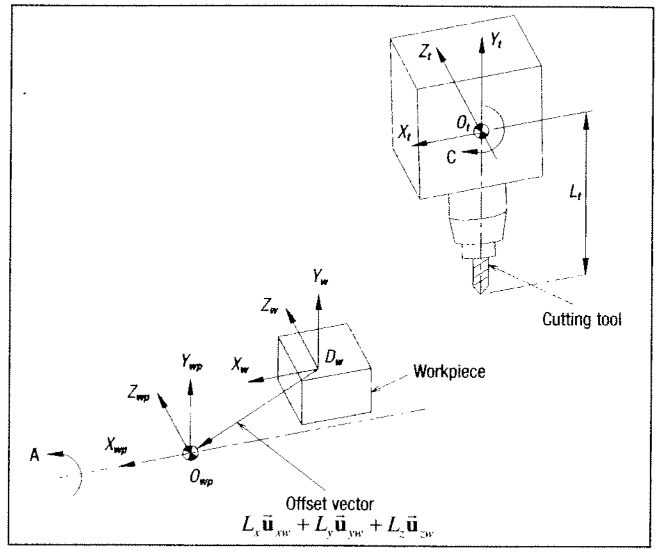

Figure 8
Relevant Coordinate Systems for a Table/Spindle-Tilting Type of Five-Axis Machining Center

$$\phi_{nws}=\phi_{nss}=L_{wp,ws}=L_{sp,ss}=L_{ss,t}=0 \tag{31}$$

$$nwp \equiv x \tag{32}$$

$$nsp \equiv z \tag{33}$$

$$\tilde{\mathbf{r}}_p^{(t)}=\begin{bmatrix}0 & -L_t & 0\end{bmatrix}^T \tag{34}$$

$$\tilde{\mathbf{t}}_{ax}^{(t)}=\begin{bmatrix}0 & 1 & 0\end{bmatrix}^T \tag{35}$$

where L_t is the distance determined from origin O_t to the cutter tip center, as shown in *Figure 8*.

Furthermore, under the conditions in Eq. (31), it follows that

$$^{wp}\mathbf{T}_{ws}^{mws}(0,0)={}^{sp}\mathbf{T}_{ss}^{mss}(0,0)={}^{ss}\mathbf{T}_t^{mt}(0,0)=\tilde{\mathbf{I}} \tag{36}$$

where $\tilde{\mathbf{I}}$ is the identity matrix defined in Eq. (21).

Thus, Eqs. (28), (29), and (30) can be reduced to:

$$\begin{bmatrix}K_x\\K_y\\K_z\\0\end{bmatrix}={}^{w}\mathbf{T}_{wp}^{4}\left(L_x,L_y,L_z,-\phi_x\right){}^{ws}\mathbf{T}_{sp}^{4}\left(P_x,P_y,P_z,\phi_z\right)\begin{bmatrix}0\\1\\0\\0\end{bmatrix} \tag{37}$$

$$\begin{bmatrix}Q_x\\Q_y\\Q_z\\1\end{bmatrix}={}^{w}\mathbf{T}_{wp}^{4}\left(L_x,L_y,L_z,-\phi_x\right){}^{ws}\mathbf{T}_{sp}^{4}\left(P_x,P_y,P_z,\phi_z\right)\begin{bmatrix}0\\-L_t\\0\\1\end{bmatrix} \tag{38}$$

$$\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}={}^{w}\mathbf{T}_{wp}^{4}\left(L_x,L_y,L_z,0\right){}^{ws}\mathbf{T}_{sp}^{4}\left(P_x,P_y,P_z,0\right)\begin{bmatrix}0\\-L_t\\0\\1\end{bmatrix} \tag{39}$$

Multiplying Eqs. (37) to (39) out becomes:

$$\begin{bmatrix}K_x\\K_y\\K_z\\0\end{bmatrix}=\begin{bmatrix}-\mathrm{S}\phi_z\\ \mathrm{C}\phi_x\mathrm{C}\phi_z\\ -\mathrm{S}\phi_x\mathrm{C}\phi_z\\ 0\end{bmatrix} \tag{40}$$

$$\begin{bmatrix}Q_x\\Q_y\\Q_z\\1\end{bmatrix}=\begin{bmatrix}P_x+L_t\mathrm{S}\phi_z+L_x\\ P_y\mathrm{C}\phi_x+P_z\mathrm{S}\phi_z-L_t\mathrm{C}\phi_x\mathrm{C}\phi_z+L_y\\ -P_y\mathrm{S}\phi_x+P_z\mathrm{C}\phi_x+L_t\mathrm{S}\phi_x\mathrm{C}\phi_z+L_z\\ 1\end{bmatrix} \tag{41}$$

$$\begin{bmatrix}X\\Y\\Z\\1\end{bmatrix}=\begin{bmatrix}P_x+L_x\\ P_y+L_y-L_t\\ P_z+L_z\\ 1\end{bmatrix} \tag{42}$$

Therefore, the desired equations for NC data (X, Y, Z, A, and C) of this five-axis machine tool's configuration can be determined by solving the above equations, and are given by:

$$C=\phi_z=arcsin\left(-K_x\right) \quad \text{for } -90° \le \phi_z \le 90° \tag{43}$$

$$A = \phi_x = \arctan 2(-K_z, K_y) \quad \text{for } -180° \le \phi_x \le 180° \tag{44}$$

$$X = P_x + L_x = Q_x - L_t S\phi_z \tag{45}$$

$$Y = P_y + L_y - L_t = (Q_y - L_y)C\phi_x - (Q_z - L_z)S\phi_x + L_t(C\phi_z - 1) + L_y \tag{46}$$

$$Z = P_z + L_z = (Q_y - L_y)S\phi_x + (Q_z - L_z)C\phi_x + L_z \tag{47}$$

where arctan $2(y, x)$ is the function that returns angles in the range of $-180° \le \theta \le 180°$ by examining the sign of both y and x.[11]

Trial-Cut Experiment

The effectiveness of the derived postprocessor was demonstrated in an experiment at Cheng Kung University to machine a Bezier surface with an area 60×60 mm^2 and a maximum depth of 10 mm on a table/spindle-tilting machining center. The tool path for the zigzag cutting pattern (*Figure 9*) is generated based on the differential geometry[13] and then converted to the NC codes according to Eqs. (43) to (47). The workpiece material is aluminum alloy, and the following machining conditions are used: (1) the cutting tool is a ball-end mill with 10 mm diameter; (2) spindle speed = 1000 rpm; (3) feed rate = 500 mm/min.; (4) step-over for the tool path = 0.5 mm; (5) offset vector $L_x = 0$, $L_y = -10$ mm, and $L_z = -20$ mm; and (6) effective tool length $L_t = 410.306$ mm. *Tables 1* and *2* show the examples of CL data points and generated NC data. *Figure 10* shows the actual cutting, indicating that the proposed scheme can be applied to practical five-axis machining.

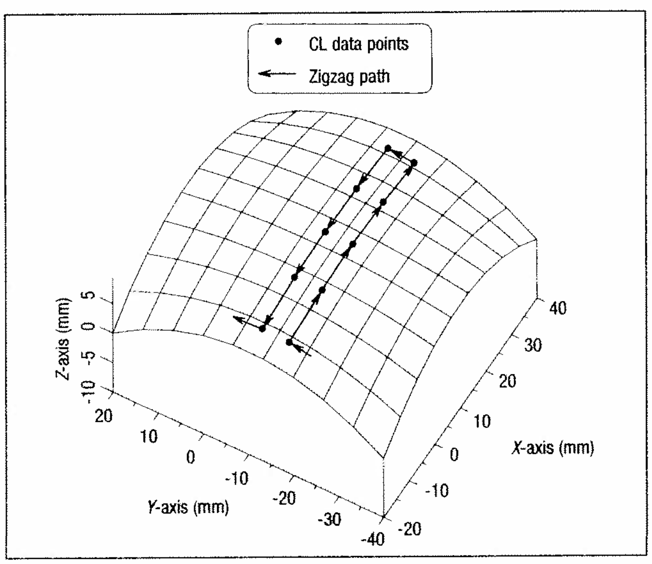

Figure 9
Illustration of Zigzag Tool Path and CL Data Points

Table 1
Examples of CL Data Points

Q_x	Q_y	Q_z	K_x	K_y	K_z
-14.000	-16.000	7.956	-0.1110	-0.0724	0.9912
-2.000	-16.000	8.964	-0.0559	-0.0369	0.9978
10.000	-16.000	9.300	0.0000	-0.0250	0.9997
22.000	-16.000	8.964	0.0559	-0.0369	0.9978
34.000	-16.000	7.956	0.1110	-0.0724	0.9912
34.000	-10.000	8.175	0.0995	0.0000	0.9950
22.000	-10.000	9.075	0.0499	0.0000	0.9988
10.000	-10.000	9.375	0.0000	0.0000	1.0000
-2.000	-10.000	9.075	-0.0499	0.0000	0.9988
-14.000	-10.000	8.175	-0.0995	0.0000	0.9950

Table 2
Examples of Generated NC Data

X-59.544	Y15.783	Z-16.052	A-94.178	C6.373
X-24.936	Y18.524	Z-15.074	A-92.118	C3.205
X10.000	Y19.441	Z-14.734	A-91.433	C0.000
X44.936	Y18.524	Z-15.074	A-92.118	C-3.205
X79.544	Y15.783	Z-16.052	A-94.178	C-6.373
X74.825	Y16.139	Z-20.000	A-90.000	C-5.710
X42.474	Y18.564	Z-20.000	A-90.000	C-2.860
X10.000	Y19.375	Z-20.000	A-90.000	C0.000
X-22.474	Y18.564	Z-20.000	A-90.000	C2.860
X-54.825	Y16.139	Z-20.000	A-90.000	C5.710

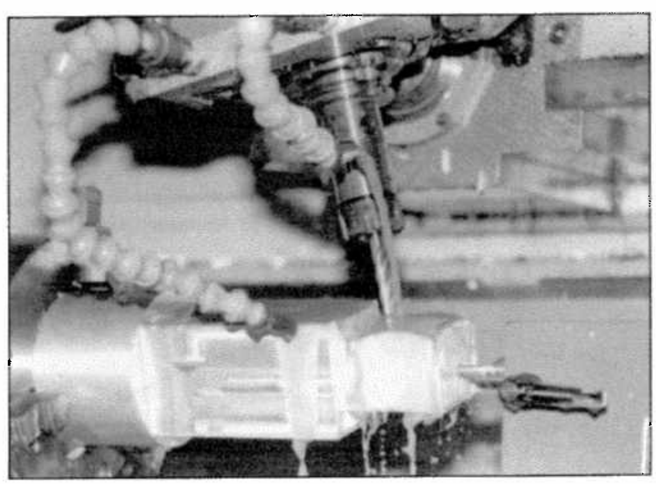

Figure 10
Trial Cut Experiment on Five-Axis Machine Tool

CMM Verification

The finished part is measured on a Mitutoyo (model BHN710) CMM consisting of a bridge-type main body and personal computer. A Renishaw PH-9 touch-trigger probe with 2 mm diameter is used to measure 16 points on the part, and the measured data can be saved in a file. Referring to *Figure 11*, the theoretical sample point T is $[Q_x \quad Q_y \quad Q_z \quad 1]^T$ and the corresponding measured point M is $[M_x \quad M_y \quad M_z \quad 1]^T$. The outer surface normal vector $\tilde{\mathbf{K}}$ is $[K_x \quad K_y \quad K_z \quad 0]^T$. Therefore, the dimensional error vector $\tilde{\mathbf{E}}$, determined from point T to point M, can be expressed as follows:

$$\tilde{\mathbf{E}} = \begin{bmatrix} M_x - Q_x & M_y - Q_y & M_z - Q_z & 0 \end{bmatrix}^T \quad (48)$$

In general, vector $\tilde{\mathbf{E}}$ can be projected onto the surface normal vector, and its magnitude can be obtained by the following:

$$\langle \tilde{\mathbf{E}}, \tilde{\mathbf{K}} \rangle = (M_x - Q_x)K_x + (M_y - Q_y)K_y + (M_z - Q_z)K_z \quad (49)$$

Note that the positive value of Eq. (49) means the measured point M is outside the design surface, whereas point M is inside the design surface if the value of Eq. (49) is negative. *Figure 12* shows the projected dimensional error of the measured sample points with the design surface. This figure reveals that the maximum deviation of the finished surface, compared to the design surface, is 0.02 mm. As the result demonstrated, the proposed postprocessor method is highly reliable.

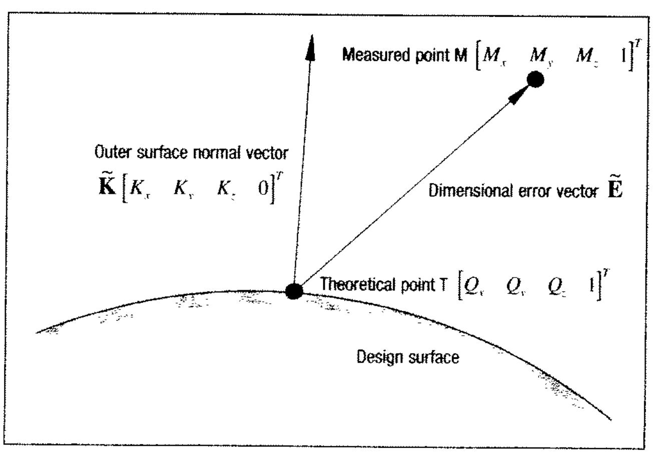

Figure 11
Illustration of Dimensional Error Vector $\tilde{\mathbf{E}}$

Conclusion

Based on the generalized kinematics model, the generalized postprocessor to explicitly determine the equations to generate NC data for general five-axis machine tools is presented. The homogeneous coordinate transformation matrix has been employed to derive the machine tool's form-shaping function matrix. In addition, the analytical equations for NC data have been obtained by equating the form-shaping function matrix and the CL data matrix through inverse kinematics. Implementation with a trial cut on a five-axis machining center and verification on a CMM confirmed that the proposed postprocessor method is reliable. The methodology proposed in this paper is a general model that can be utilized in the derivation of a postprocessor for various types of five-axis machine tools more efficiently and systematically.

The general postprocessor algorithm can be applied not only by machine tool builders but also by CAM vendors and machine tool users. There are more and more companies acquiring various types of five-axis machine tools within one company. The introduction of a generalized postprocessor is crucial for implementing an effective DNC system.

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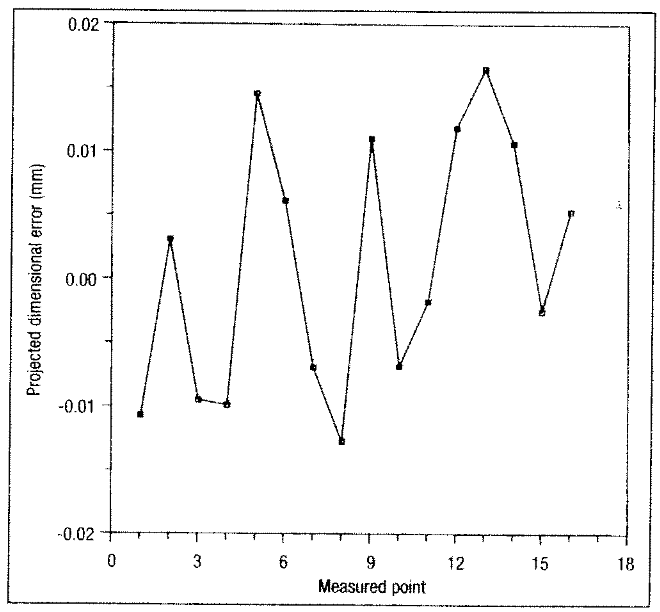

Figure 12
Projected Dimensional Error of Machined Surface from CMM Measurement

Nomenclature

${}^{i}\mathbf{A}_j$	Relative transformation matrix of system $O_jX_jY_jZ_j$ with respect to system $O_iX_iY_iZ_i$
A, B, C	Machine rotary axes
$\tilde{\mathbf{E}}$	Dimensional error vector
$\vec{\mathbf{l}}_i$	Rotation axis for system *i*
K_x, K_y, K_z	Cutting tool's axis orientation
$L_{i,j}$	Translation distance from system *i* to system *j*
L_x, L_y, L_z	Components of offset vector
L_t	Effective tool length
M_x, M_y, M_z	Coordinate of measured point
$O_iX_iY_iZ_i$	Coordinate system *i*
P_x, P_y, P_z	Relative translation distance of the *X*, *Y*, and *Z* tables
Q_x, Q_y, Q_z	Coordinate of cutter tip center
Rot	Rotation matrix
Trans	Translation matrix
${}^{i}\mathbf{T}_j^{m}$	Elementary transformation matrix
$\vec{\mathbf{u}}_i$	Unit vector for system *i*
X, Y, Z	Machine linear axes
ϕ_A, ϕ_B, ϕ_C	Rotation angle about *X*, *Y*, and *Z* axes

Superscripts and Subscripts

b	Machine bed coordinate system
mws, mss, mt	Index for characterizing translation effect of elementary tranformation matrix
nwp, nws, nsp, nss	Index for characterizing rotation effect of elementary transformation matrix
sp	Coordinate system for primary rotation on the spindle
ss	Coordinate system for secondary rotation on the spindle
Tx	Machine *X* table coordinate system
Ty	Machine *Y* table coordinate system
Tz	Machine *Z* table coordinate system
t	Cutting tool coordinate system
w	Workpiece coordinate system
wp	Coordinate system for primary rotation on workpiece fixture table
ws	Coordinate system for secondary rotation on workpiece fixture table

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