STOCHASTIC FEM

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4th Assignment

Bayesian Inference

To perform Bayesian updating to the prior of E which is a normal $\sim N(10^5, 2\times10^4)$, we also have to know the "observation model" pdf or $p(\mathcal{D}|\theta)$. However, we only have $p(\mathcal{D}|\theta)$ for the measurement of u. For u:

$$p_u(\mathcal{D}|\theta) \sim N(0.1, 0.01),$$

a normal with 0.1 m mean and standard deviation of 0.01 m.

Based on the previous assignments, we run the deterministic FEM for $P = 10$ kN and get that for

* $E = 1.32\times10^5$, we get $u = 0.100286 \leftrightarrow (\mu)$
* $E = 1.47\times10^5$, $u = 0.0900523 \leftrightarrow (\mu-\sigma)$
* $E = 1.20\times10^5$, $u = 0.110314 \leftrightarrow (\mu+\sigma)$

Therefore, we can assume that $p(\mathcal{D}|\theta)$ for E follows a normal distribution with mean $\mu = 1.32\times10^5$ and standard deviation $\sigma = 0.135\times10^5$.

Then, we perform Bayesian updating using the ARS algorithm, with:

prior: $p(\theta) \sim N(10^5, 2\times10^4)$

observation model: $p(\mathcal{D}|\theta) \sim N(1.32\times10^5, 0.135\times10^5)$

We used the candidate :

$$q(x) \sim N(1.2\times10^5, 0.2\times10^5)$$

with $M = 13\times10^{-5}$:

$$M\,q(x) \geq p(\mathcal{D}|\theta)\,p(\theta), \quad \forall\, x.$$

The implementation, as well as the posterior pdf, is shown in the Mathematica notebook.
Also, a comparison between the prior and the updated (posterior) pdf is shown at the end.