1st Assignment: Stochastic FEM

F. I. Giasemis

B: Spectral representation method

Parameters

```
ωu = 3; (* Cutoff frequency. *)
M = 200; (* Number of terms in the expansion. *)
R = 5000; (* Number of realizations. *)
```

Terms in the expansion

```
A[0] = 0;
ω[0] = 0;
Δω = ωu / M;
For[n = 0, n < M - 1, n = n + 1;
 ω[n] = n Δω;
 If[ω[n] < 1,
   G[ω_] := 0;
   A[n] = Sqrt[G[ω[n]] Δω];
  ]
  If[1 ≤ ω[n] ≤ 2,
   G[ω_] := ω - 1;
   A[n] = Sqrt[G[ω[n]] Δω];
  ]
  If[2 < ω[n] ≤ 3,
   G[ω_] := 3 - ω;
   A[n] = Sqrt[G[ω[n]] Δω];
  ]
]
```

Random variables Φ

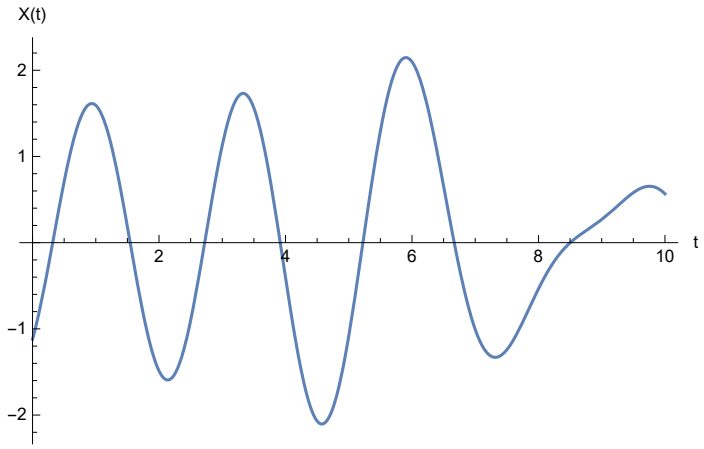
```
For[i = 0, i < R, i = i + 1;
 Φ[i] = RandomVariate[UniformDistribution[{0, 2 Pi}], M - 1]
]
```

Realization

```
Realization[i_, t_] := Sqrt[2] Sum[A[n] Cos[ω[n] t + Φ[i][[n]]], {n, 1, M - 1}];
```

Example plot of a realization of X(t)

```
Plot[Realization[4578, t], {t, 0, 10}, AxesLabel → {"t", "X(t)"}]
```



Ensemble averages and variances

```
EnsembleAverage[t_] := Mean[Table[Realization[i, t], {i, 1, R}]]
EnsembleVariance[t_] := Variance[Table[Realization[i, t], {i, 1, R}]]
```

Example calculation of ensemble average and variance

```
EnsembleAverage[5]
EnsembleVariance[5]
```

```
0.00342159
```

```
1.00844
```

Temporal average and variance from a single realization

```
TempAverage[i_] := NIntegrate[Realization[i, t], {t, 0, 10}] / 10
TempVariance[i_] := NIntegrate[Realization[i, t]^2, {t, 0, 10}] / 10 -
  (NIntegrate[Realization[i, t], {t, 0, 10}] / 10)^2
```

Example calculation of temporal average and variance

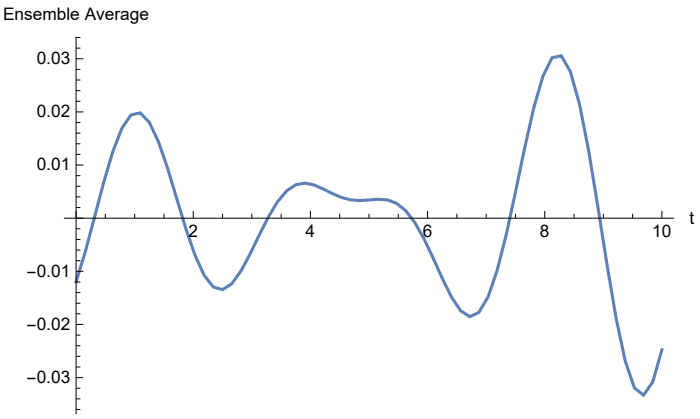
```
TempAverage[2000]
TempVariance[2000]
```

```
-0.000967937
```

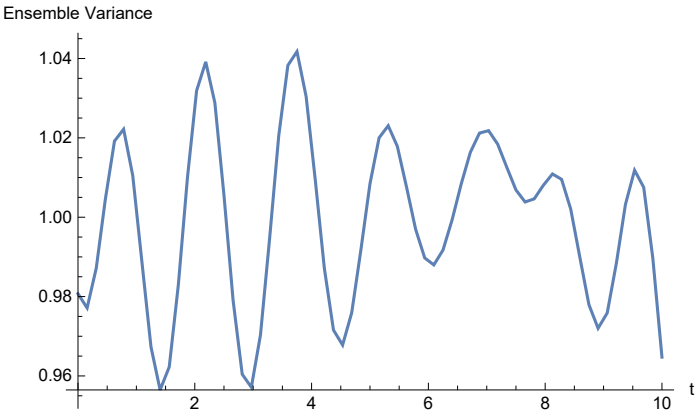
```
0.968735
```

Plot of ensemble average and variance

```
Plot[EnsembleAverage[t], {t, 0, 10}, PlotPoints → 2, AxesLabel → {"t", "Ensemble Average"}]
```

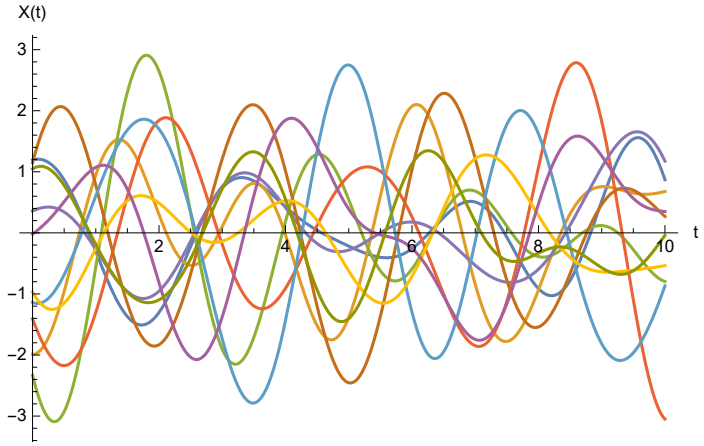



```
Plot[EnsembleVariance[t], {t, 0, 10},
 PlotPoints → 2, AxesLabel → {"t", "Ensemble Variance"}]
```



Plot of 10 realizations

```
list = {};
For[i = 0, i < 10, i = i + 1;
 AppendTo[list, Realization[i, t]]
]
Plot[list, {t, 0, 10}, AxesLabel → {"t", "X(t)"}]
```



Plot of 10 realizations

```
list = {};
For[i = 4100, i < 4110, i = i + 1;
 AppendTo[list, Realization[i, t]]
]
Plot[list, {t, 0, 10}, AxesLabel → {"t", "X(t)"}]
```