1st Assignment: Stochastic FEM

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A: KL expansion

Parameters from the problem

```
b = 2;
a = 2.5;
M = 40; (* Number of terms in the KL expansion. *)
R = 5000; (* Number of realizations. *)
```

Eigenvalues and eigenfunctions for f(x)

```
For[n = 0, n < M / 2, n = n + 1;
 sol = NSolve[{1 / b - x Tan[x a] == 0, (n - 1) Pi / a ≤ x ≤ (n - 1 / 2) Pi / a}, x];
 wodd[n] = Part[x /. sol, 1];
 λodd[n] = 2 b / (1 + wodd[n]^2 b^2);
 codd[n] = 1 / Sqrt[a + Sin[2 wodd[n] a] / (2 wodd[n])];
 φodd[n][x_] := codd[n] Cos[wodd [n] x];

 sol = NSolve[{1 / b Tan[x a] + x == 0, (n - 1 / 2) Pi / a ≤ x ≤ (n) Pi / a}, x];
 weven[n] = Part[x /. sol, 1];
 λeven[n] = 2 b / (1 + weven[n]^2 b^2);
 ceven[n] = 1 / Sqrt[a - Sin[2 weven[n] a] / (2 weven[n])];
 φeven[n][x_] := ceven[n] Sin[weven[n] x]
]
```

Random variables $\xi(\theta)$

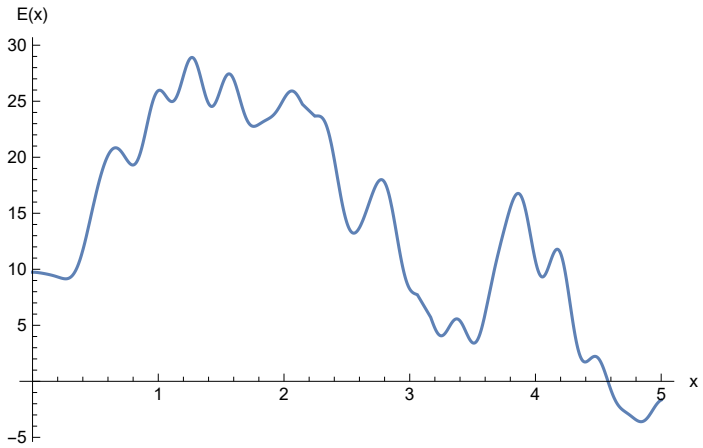
```
For[i = 0, i < R, i = i + 1;
 ξ[i] = RandomVariate[NormalDistribution[], M]
]
```

Realization of f(x) and E(x)

```
RealizationF[i_, x_] := Sum[Sqrt[λodd[n]] φodd[n][x - 2.5] ξ[i][[n]], {n, 1, M / 2}] +
   Sum[Sqrt[λeven[n]] φeven[n][x - 2.5] ξ[i][[M / 2 + n]], {n, 1, M / 2}];
Realization[i_, x_] := 10 (1 + RealizationF[i, x]);
```

Example plot of a realization of E(x)

```
Plot[Realization[567, x], {x, 0, 5}, AxesLabel → {"x", "E(x)"}]
```



Ensemble averages and variances

```
EnsembleAverage[x_] := Mean[Table[Realization[i, x], {i, 1, R}]]
EnsembleVariance[x_] := Variance[Table[Realization[i, x], {i, 1, R}]]
```

Example calculation of ensemble average and variance

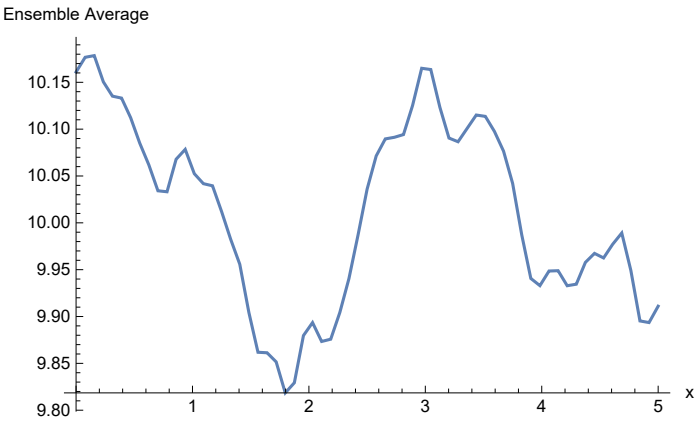
```
EnsembleAverage[2]
EnsembleVariance[2]
```

```
9.89485
```

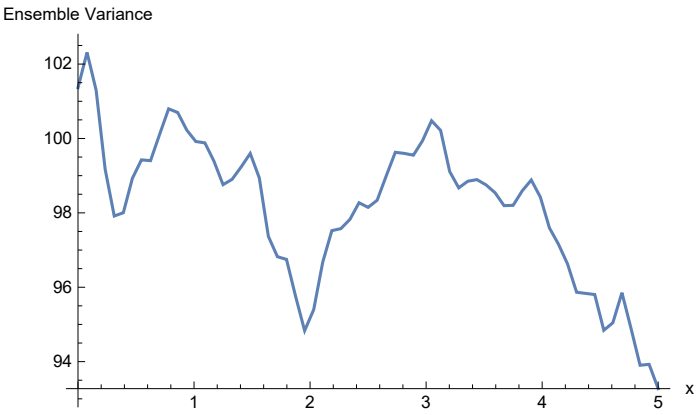
```
94.9847
```

Plot of ensemble average and variance

```
Plot[EnsembleAverage[x], {x, 0, 5}, PlotPoints → 2, AxesLabel → {"x", "Ensemble Average"}]
```

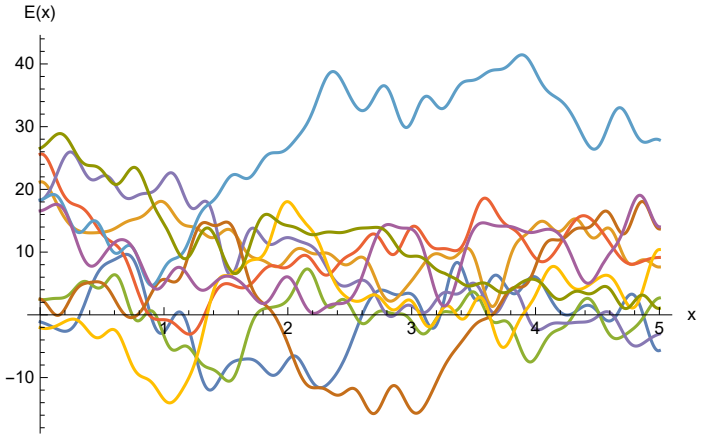



```
Plot[EnsembleVariance[x], {x, 0, 5}, PlotPoints → 2, AxesLabel → {"x", "Ensemble Variance"}]
```



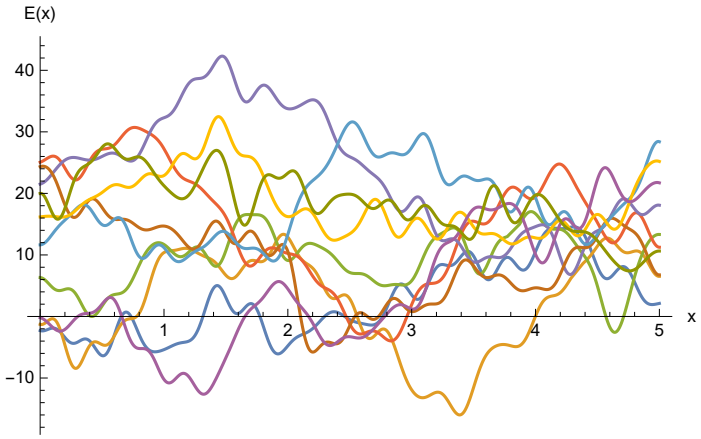
Plot of 10 realizations

```
list = {};
For[i = 0, i < 10, i = i + 1;
 AppendTo[list, Realization[i, x]]
]
Plot[list, {x, 0, 5}, AxesLabel → {"x", "E(x)"}]
```



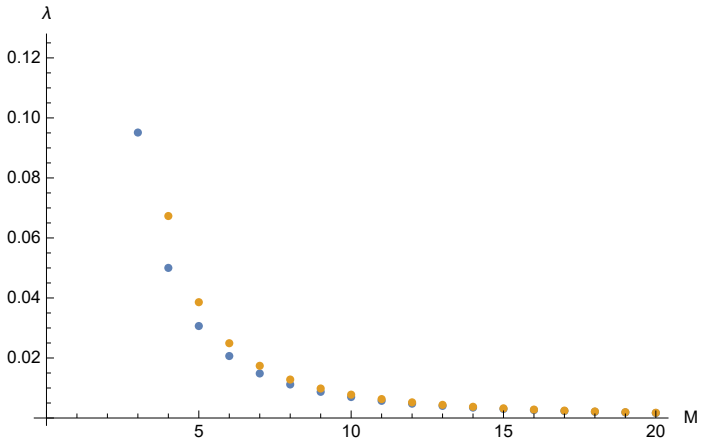
Plot of 10 realizations

```
list = {};
For[i = 4300, i < 4310, i = i + 1;
 AppendTo[list, Realization[i, x]]
]
Plot[list, {x, 0, 5}, AxesLabel → {"x", "E(x)"}]
```



Number of terms in the KL expansion

```
ListPlot[{Table[λeven[n], {n, 1, M}], Table[λodd[n], {n, 1, M}]}, AxesLabel → {"M", "λ"}]
(* Justifying the number of terms in the KL expansion. *)
```




```
λeven[20] / λeven[1] (* Keep terms only with λ > λ_threshold etc. Here,
λ_even_threshold = 0.0016. *)
λodd[20] / λodd[1]
```

```
0.00159695
```

```
0.000680913
```