4th Assignment: Stochastic FEM

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FEM

The stiffness matrix for one element

```
aelem = 0.05;
belem = 0.05;
ν = 0.3;
t = 0.2;
r = aelem / belem;
ρ = (1 - ν) / 2;
μ = 3 (1 + ν) / 2;
λ = 3 (1 - 3 ν) / 2;

kinput = t / (12 (1 - ν^2)) {{4 / r + 4 ρ r, 0, 0, 0, 0, 0, 0, 0}, {μ, 4 r + 4 ρ / r, 0, 0, 0, 0, 0, 0},
    {-4 / r + 2 ρ r, λ, 4 / r + 4 ρ r, 0, 0, 0, 0, 0}, {-λ, 2 r - 4 ρ / r, -μ, 4 r + 4 ρ / r, 0, 0, 0, 0},
    {-2 / r - 2 ρ r, -μ, 2 / r - 4 ρ r, λ, 4 / r + 4 ρ r, 0, 0, 0},
    {-μ, -2 r - 2 ρ / r, -λ, -4 r + 2 ρ / r, μ, 4 r + 4 ρ / r, 0, 0},
    {2 / r - 4 ρ r, -λ, -2 / r - 2 ρ r, μ, -4 / r + 2 ρ r, λ, 4 / r + 4 ρ r, 0},
    {λ, -4 r + 2 ρ / r, μ, -2 r - 2 ρ / r, -λ, 2 r - 4 ρ / r, -μ, 4 r + 4 ρ / r}};
k = kinput + Transpose[kinput] - DiagonalMatrix[
    {kinput[[1, 1]], kinput[[1, 1]], kinput[[1, 1]], kinput[[1, 1]],
     kinput[[1, 1]], kinput[[1, 1]], kinput[[1, 1]], kinput[[1, 1]]}];

klocal[x_] := (
  EY = f[x];
  Return[EY k])
```

Assembling the global stiffness matrix

```
dim = (40 + 1) (10 + 1) 2;
global[i_, j_, m_, n_] := (
  x = Mod[i, 41] 2 aelem - aelem;
  kglobal = ConstantArray[0, {dim, dim}];
kglobal[[2 i - 1, 2 i - 1]] += klocal[x][[1, 1]];
kglobal[[2 i - 1, 2 i]] += klocal[x][[1, 2]];
kglobal[[2 i - 1, 2 j - 1]] += klocal[x][[1, 3]];
kglobal[[2 i - 1, 2 j]] += klocal[x][[1, 4]];
kglobal[[2 i - 1, 2 m - 1]] += klocal[x][[1, 5]];
kglobal[[2 i - 1, 2 m]] += klocal[x][[1, 6]];
kglobal[[2 i - 1, 2 n - 1]] += klocal[x][[1, 7]];
kglobal[[2 i - 1, 2 n]] += klocal[x][[1, 8]];
```

```
kglobal[[2 i, 2 i - 1]] += klocal[x][[2, 1]];
kglobal[[2 i, 2 i]] += klocal[x][[2, 2]];
kglobal[[2 i, 2 j - 1]] += klocal[x][[2, 3]];
kglobal[[2 i, 2 j]] += klocal[x][[2, 4]];
kglobal[[2 i, 2 m - 1]] += klocal[x][[2, 5]];
kglobal[[2 i, 2 m]] += klocal[x][[2, 6]];
kglobal[[2 i, 2 n - 1]] += klocal[x][[2, 7]];
kglobal[[2 i, 2 n]] += klocal[x][[2, 8]];
kglobal[[2 j - 1, 2 i - 1]] += klocal[x][[3, 1]];
kglobal[[2 j - 1, 2 i]] += klocal[x][[3, 2]];
kglobal[[2 j - 1, 2 j - 1]] += klocal[x][[3, 3]];
kglobal[[2 j - 1, 2 j]] += klocal[x][[3, 4]];
kglobal[[2 j - 1, 2 m - 1]] += klocal[x][[3, 5]];
kglobal[[2 j - 1, 2 m]] += klocal[x][[3, 6]];
kglobal[[2 j - 1, 2 n - 1]] += klocal[x][[3, 7]];
kglobal[[2 j - 1, 2 n]] += klocal[x][[3, 8]];
kglobal[[2 j, 2 i - 1]] += klocal[x][[4, 1]];
kglobal[[2 j, 2 i]] += klocal[x][[4, 2]];
kglobal[[2 j, 2 j - 1]] += klocal[x][[4, 3]];
kglobal[[2 j, 2 j]] += klocal[x][[4, 4]];
kglobal[[2 j, 2 m - 1]] += klocal[x][[4, 5]];
kglobal[[2 j, 2 m]] += klocal[x][[4, 6]];
kglobal[[2 j, 2 n - 1]] += klocal[x][[4, 7]];
kglobal[[2 j, 2 n]] += klocal[x][[4, 8]];
kglobal[[2 m - 1, 2 i - 1]] += klocal[x][[5, 1]];
kglobal[[2 m - 1, 2 i]] += klocal[x][[5, 2]];
kglobal[[2 m - 1, 2 j - 1]] += klocal[x][[5, 3]];
kglobal[[2 m - 1, 2 j]] += klocal[x][[5, 4]];
kglobal[[2 m - 1, 2 m - 1]] += klocal[x][[5, 5]];
kglobal[[2 m - 1, 2 m]] += klocal[x][[5, 6]];
kglobal[[2 m - 1, 2 n - 1]] += klocal[x][[5, 7]];
kglobal[[2 m - 1, 2 n]] += klocal[x][[5, 8]];
kglobal[[2 m, 2 i - 1]] += klocal[x][[6, 1]];
kglobal[[2 m, 2 i]] += klocal[x][[6, 2]];
kglobal[[2 m, 2 j - 1]] += klocal[x][[6, 3]];
kglobal[[2 m, 2 j]] += klocal[x][[6, 4]];
kglobal[[2 m, 2 m - 1]] += klocal[x][[6, 5]];
kglobal[[2 m, 2 m]] += klocal[x][[6, 6]];
kglobal[[2 m, 2 n - 1]] += klocal[x][[6, 7]];
kglobal[[2 m, 2 n]] += klocal[x][[6, 8]];
kglobal[[2 n - 1, 2 i - 1]] += klocal[x][[7, 1]];
kglobal[[2 n - 1, 2 i]] += klocal[x][[7, 2]];
kglobal[[2 n - 1, 2 j - 1]] += klocal[x][[7, 3]];
kglobal[[2 n - 1, 2 j]] += klocal[x][[7, 4]];
kglobal[[2 n - 1, 2 m - 1]] += klocal[x][[7, 5]];
kglobal[[2 n - 1, 2 m]] += klocal[x][[7, 6]];
kglobal[[2 n - 1, 2 n - 1]] += klocal[x][[7, 7]];
kglobal[[2 n - 1, 2 n]] += klocal[x][[7, 8]];
kglobal[[2 n, 2 i - 1]] += klocal[x][[8, 1]];
kglobal[[2 n, 2 i]] += klocal[x][[8, 2]];
kglobal[[2 n, 2 j - 1]] += klocal[x][[8, 3]];
```

```
kglobal[[2 n, 2 j]] += klocal[x][[8, 4]];
kglobal[[2 n, 2 m - 1]] += klocal[x][[8, 5]];
kglobal[[2 n, 2 m]] += klocal[x][[8, 6]];
kglobal[[2 n, 2 n - 1]] += klocal[x][[8, 7]];
kglobal[[2 n, 2 n]] += klocal[x][[8, 8]];
Return[kglobal])
```

The global stiffness matrix

```
Kglobal := (
  Kglobal = ConstantArray[0, {dim, dim}];
  For[index = 0, index ≤ 408, index += 1,
   i = 1 + index;
   j = 2 + index;
   m = 43 + index;
   n = 42 + index;
   If[Divisible[i, 41] == False, Kglobal += global[i, j, m, n]]
  ];
  Return[Kglobal])
```

Boundary conditions

```
fix[x_] := (
  For[i = 1, i ≤ dim, i += 1,
   KGLOBAL[[x, i]] = 0;
   KGLOBAL[[i, x]] = 0;];
  KGLOBAL[[x, x]] = 10^10;)

fixall := (
  For[x = 1, x ≤ 821, x += 82,
   fix[x]];
  For[x = 2, x ≤ 822, x += 82,
   fix[x]];)
```

Solve the system

```
aelem = 0.05;
belem = 0.05;
ν = 0.3;
t = 0.2;
r = aelem / belem;
ρ = (1 - ν) / 2;
μ = 3 (1 + ν) / 2;
λ = 3 (1 - 3 ν) / 2;
dim = (40 + 1) (10 + 1) 2;

global[i_, j_, m_, n_] := (
   x = Mod[i, 41] 2 aelem - aelem;
   kglobal = ConstantArray[0, {dim, dim}];
```

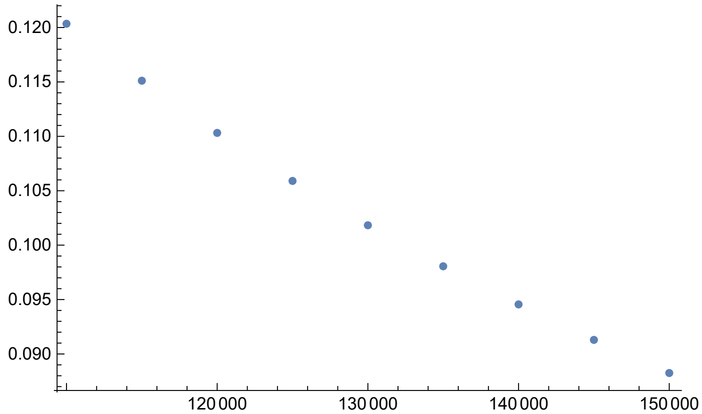
```
kglobal[[2 i - 1, 2 i - 1]] += klocal[x][[1, 1]];
kglobal[[2 i - 1, 2 i]] += klocal[x][[1, 2]];
kglobal[[2 i - 1, 2 j - 1]] += klocal[x][[1, 3]];
kglobal[[2 i - 1, 2 j]] += klocal[x][[1, 4]];
kglobal[[2 i - 1, 2 m - 1]] += klocal[x][[1, 5]];
kglobal[[2 i - 1, 2 m]] += klocal[x][[1, 6]];
kglobal[[2 i - 1, 2 n - 1]] += klocal[x][[1, 7]];
kglobal[[2 i - 1, 2 n]] += klocal[x][[1, 8]];
kglobal[[2 i, 2 i - 1]] += klocal[x][[2, 1]];
kglobal[[2 i, 2 i]] += klocal[x][[2, 2]];
kglobal[[2 i, 2 j - 1]] += klocal[x][[2, 3]];
kglobal[[2 i, 2 j]] += klocal[x][[2, 4]];
kglobal[[2 i, 2 m - 1]] += klocal[x][[2, 5]];
kglobal[[2 i, 2 m]] += klocal[x][[2, 6]];
kglobal[[2 i, 2 n - 1]] += klocal[x][[2, 7]];
kglobal[[2 i, 2 n]] += klocal[x][[2, 8]];
kglobal[[2 j - 1, 2 i - 1]] += klocal[x][[3, 1]];
kglobal[[2 j - 1, 2 i]] += klocal[x][[3, 2]];
kglobal[[2 j - 1, 2 j - 1]] += klocal[x][[3, 3]];
kglobal[[2 j - 1, 2 j]] += klocal[x][[3, 4]];
kglobal[[2 j - 1, 2 m - 1]] += klocal[x][[3, 5]];
kglobal[[2 j - 1, 2 m]] += klocal[x][[3, 6]];
kglobal[[2 j - 1, 2 n - 1]] += klocal[x][[3, 7]];
kglobal[[2 j - 1, 2 n]] += klocal[x][[3, 8]];
kglobal[[2 j, 2 i - 1]] += klocal[x][[4, 1]];
kglobal[[2 j, 2 i]] += klocal[x][[4, 2]];
kglobal[[2 j, 2 j - 1]] += klocal[x][[4, 3]];
kglobal[[2 j, 2 j]] += klocal[x][[4, 4]];
kglobal[[2 j, 2 m - 1]] += klocal[x][[4, 5]];
kglobal[[2 j, 2 m]] += klocal[x][[4, 6]];
kglobal[[2 j, 2 n - 1]] += klocal[x][[4, 7]];
kglobal[[2 j, 2 n]] += klocal[x][[4, 8]];
kglobal[[2 m - 1, 2 i - 1]] += klocal[x][[5, 1]];
kglobal[[2 m - 1, 2 i]] += klocal[x][[5, 2]];
kglobal[[2 m - 1, 2 j - 1]] += klocal[x][[5, 3]];
kglobal[[2 m - 1, 2 j]] += klocal[x][[5, 4]];
kglobal[[2 m - 1, 2 m - 1]] += klocal[x][[5, 5]];
kglobal[[2 m - 1, 2 m]] += klocal[x][[5, 6]];
kglobal[[2 m - 1, 2 n - 1]] += klocal[x][[5, 7]];
kglobal[[2 m - 1, 2 n]] += klocal[x][[5, 8]];
kglobal[[2 m, 2 i - 1]] += klocal[x][[6, 1]];
kglobal[[2 m, 2 i]] += klocal[x][[6, 2]];
kglobal[[2 m, 2 j - 1]] += klocal[x][[6, 3]];
kglobal[[2 m, 2 j]] += klocal[x][[6, 4]];
kglobal[[2 m, 2 m - 1]] += klocal[x][[6, 5]];
kglobal[[2 m, 2 m]] += klocal[x][[6, 6]];
kglobal[[2 m, 2 n - 1]] += klocal[x][[6, 7]];
kglobal[[2 m, 2 n]] += klocal[x][[6, 8]];
kglobal[[2 n - 1, 2 i - 1]] += klocal[x][[7, 1]];
kglobal[[2 n - 1, 2 i]] += klocal[x][[7, 2]];
kglobal[[2 n - 1, 2 j - 1]] += klocal[x][[7, 3]];
```

```
kglobal[[2 n - 1, 2 j]] += klocal[x][[7, 4]];
kglobal[[2 n - 1, 2 m - 1]] += klocal[x][[7, 5]];
kglobal[[2 n - 1, 2 m]] += klocal[x][[7, 6]];
kglobal[[2 n - 1, 2 n - 1]] += klocal[x][[7, 7]];
kglobal[[2 n - 1, 2 n]] += klocal[x][[7, 8]];
kglobal[[2 n, 2 i - 1]] += klocal[x][[8, 1]];
kglobal[[2 n, 2 i]] += klocal[x][[8, 2]];
kglobal[[2 n, 2 j - 1]] += klocal[x][[8, 3]];
kglobal[[2 n, 2 j]] += klocal[x][[8, 4]];
kglobal[[2 n, 2 m - 1]] += klocal[x][[8, 5]];
kglobal[[2 n, 2 m]] += klocal[x][[8, 6]];
kglobal[[2 n, 2 n - 1]] += klocal[x][[8, 7]];
kglobal[[2 n, 2 n]] += klocal[x][[8, 8]];
Return[kglobal]);


kinput = t / (12 (1 - ν^2)) {{4 / r + 4 ρ r, 0, 0, 0, 0, 0, 0, 0}, {μ, 4 r + 4 ρ / r, 0, 0, 0, 0, 0, 0},
    {-4 / r + 2 ρ r, λ, 4 / r + 4 ρ r, 0, 0, 0, 0, 0}, {-λ, 2 r - 4 ρ / r, -μ, 4 r + 4 ρ / r, 0, 0, 0, 0},
    {-2 / r - 2 ρ r, -μ, 2 / r - 4 ρ r, λ, 4 / r + 4 ρ r, 0, 0, 0},
    {-μ, -2 r - 2 ρ / r, -λ, -4 r + 2 ρ / r, μ, 4 r + 4 ρ / r, 0, 0},
    {2 / r - 4 ρ r, -λ, -2 / r - 2 ρ r, μ, -4 / r + 2 ρ r, λ, 4 / r + 4 ρ r, 0},
    {λ, -4 r + 2 ρ / r, μ, -2 r - 2 ρ / r, -λ, 2 r - 4 ρ / r, -μ, 4 r + 4 ρ / r}};
k = kinput + Transpose[kinput] - DiagonalMatrix[
    {kinput[[1, 1]], kinput[[1, 1]], kinput[[1, 1]], kinput[[1, 1]],
     kinput[[1, 1]], kinput[[1, 1]], kinput[[1, 1]], kinput[[1, 1]]}];

F = ConstantArray[0, dim];
F[[dim]] = -10;



U[y_] := (

  EY = y;

  klocal[x_] := EY k;

  Kglobal = ConstantArray[0, {dim, dim}];
  For[index = 0, index ≤ 408, index += 1,
   i = 1 + index;
   j = 2 + index;
   m = 43 + index;
   n = 42 + index;
   If[Divisible[i, 41] == False, Kglobal += global[i, j, m, n]]
  ];

  KGLOBAL = Kglobal;

  fixall;
```

```
  sol = Inverse[KGLOBAL].F;

  Return[sol];)
```

Histogram of response u

```
ListPlot[Table[{x, -U[x][[82]]}, {x, 110000, 150000, 5000}]]
```

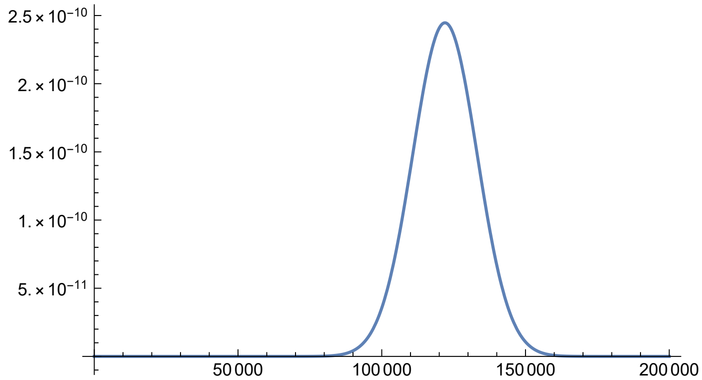



```
-U[1.32 × 10^5][[82]]
0.100286

-U[1.47 × 10^5][[82]]
0.0900523

-U[1.20 × 10^5][[82]]
0.110314

1.47 - 1.32
1.32 - 1.20
0.15

0.12

(0.15 + 0.12) / 2
0.135
```

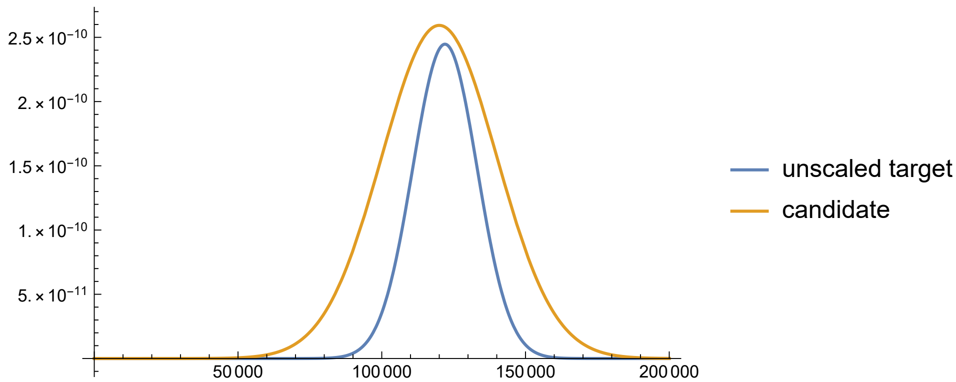
Acceptance-rejection sampling

```
p[x_] := PDF[NormalDistribution[10^5, 2 × 10^4], x];
pd[x_] := PDF[NormalDistribution[1.32 × 10^5, 0.135 × 10^5], x];
likelih[x_] := pd[x] p[x];
```

```
Plot[likelih[x], {x, 0, 2 × 10^5}, PlotRange → Full]
```



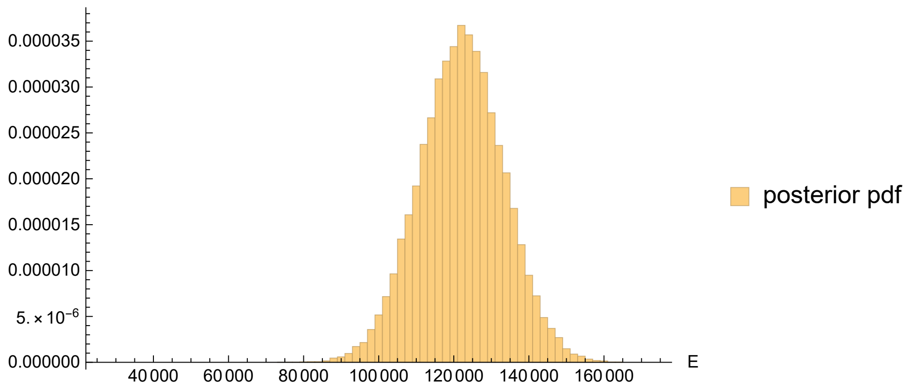

```
g[x_] := PDF[NormalDistribution[1.2 × 10^5, 0.2 × 10^5], x];
M =  1.3 × 10^(-5);
Plot[{likelih[x], M g[x]}, {x, 0, 2 × 10^5},
 PlotRange → Full, PlotLegends → {"unscaled target", "candidate"}]
```




```
posterior = {};
For[i = 1, i < 100000, i++,
 x = RandomVariate[NormalDistribution[1.2 × 10^5, 0.2 × 10^5]];
 w = pd[x] p[x] / (M g[x]);
 u = RandomVariate[UniformDistribution[{0, 1}]];
 If[u < w, AppendTo[posterior, x]]
]
```

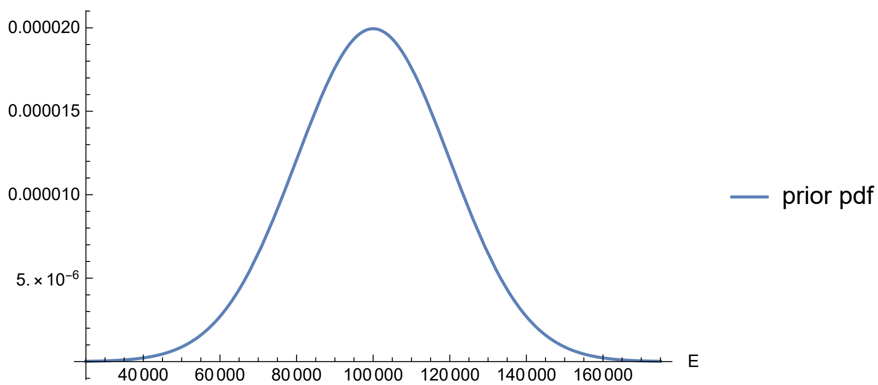
The posterior pdf

```
hist = Histogram[posterior, {0.25 × 10^5, 1.75 × 10^5, 2000},
  "PDF", AxesLabel → {"E"}, ChartLegends → {"posterior pdf"}]
```



The prior pdf

```
plot =
 Plot[p[x], {x, 0.25 × 10^5, 1.75 × 10^5}, AxesLabel → {"E"}, PlotLegends → {"prior pdf"}]
```



Comparison

```
Show[hist, plot]
```