1st Assignment: Stochastic FEM

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A: The KL expansion

Parameters from the problem

```
b = 2;
a = 2.5;
M = 40; (* Number of terms in the KL expansion. *)
R = 5000; (* Number of realizations. *)
```

Eigenvalues and eigenfunctions for f(x)

```
For[n = 0, n < M, n = n + 1;
 If[OddQ[n],
  sol = NSolve[{1 / b - x Tan[x a] == 0, (n - 1) Pi / a ≤ x ≤ (n - 1 / 2) Pi / a}, x];
  w[n] = Part[x /. sol, 1];
  λ[n] = 2 b / (1 + w[n]^2 b^2);
  c[n] = 1 / Sqrt[a + Sin[2 w[n] a] / (2 w[n])];
  φ[n][x_] := c[n] Cos[w [n] x],
  sol = NSolve[{1 / b Tan[x a] + x == 0, (n - 1 / 2) Pi / a ≤ x ≤ (n) Pi / a}, x];
  w[n] = Part[x /. sol, 1];
  λ[n] = 2 b / (1 + w[n]^2 b^2);
  c[n] = 1 / Sqrt[a - Sin[2 w[n] a] / (2 w[n])];
  φ[n][x_] := c[n] Sin[w[n] x]
 ]
]
```

Random variables $\xi(\theta)$

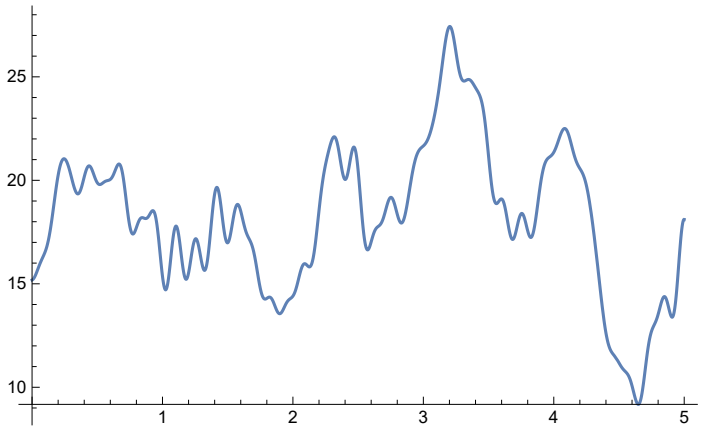
```
For[i = 0, i < R, i = i + 1;
 ξ[i] = RandomVariate[NormalDistribution[], M]
]
```

Realization of f(x) and E(x)

```
RealizationF[i_, x_] := Sum[Sqrt[λ[n]] φ[n][x - 2.5] ξ[i][[n]], {n, 1, M}];
Realization[i_, x_] := 10 (1 + RealizationF[i, x]);
```

Example plot of a realization of E(x)

```
Plot[Realization[567, x], {x, 0, 5}]
```



Ensemble averages and variances

```
EnsembleAverage[x_] := Mean[Table[Realization[i, x], {i, 1, R}]]
EnsembleVariance[x_] := Variance[Table[Realization[i, x], {i, 1, R}]]
```

Variance error

```
VarianceError[x_] := 10^2 - 10^2 Sum[λ[n] φ[n][x - 2.5]^2, {n, 1, M}]
```

Example calculation of ensemble average, variance and variance error

```
EnsembleAverage[2]
EnsembleVariance[2]
VarianceError[2]
```

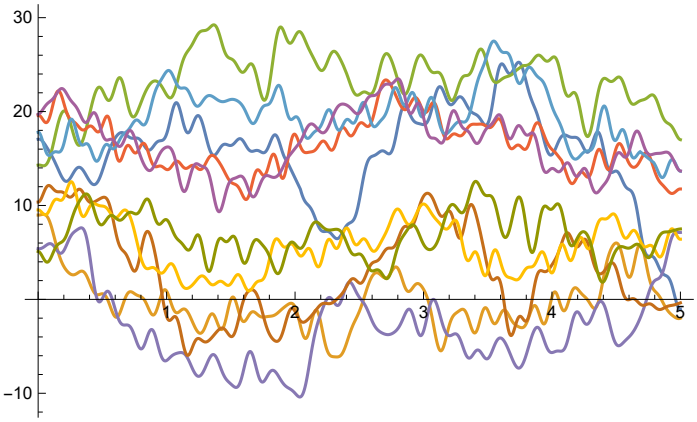
```
9.97326

75.0572

23.155
```

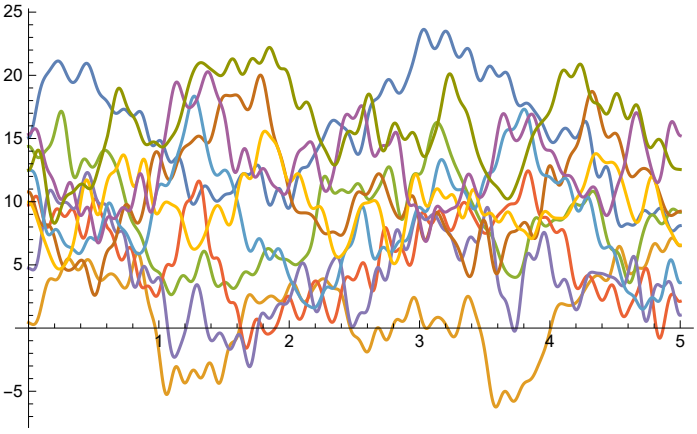
Plot of 10 realizations

```
list = {};
For[i = 0, i < 10, i = i + 1;
 AppendTo[list, Realization[i, x]]
]
Plot[list, {x, 0, 5}]
```



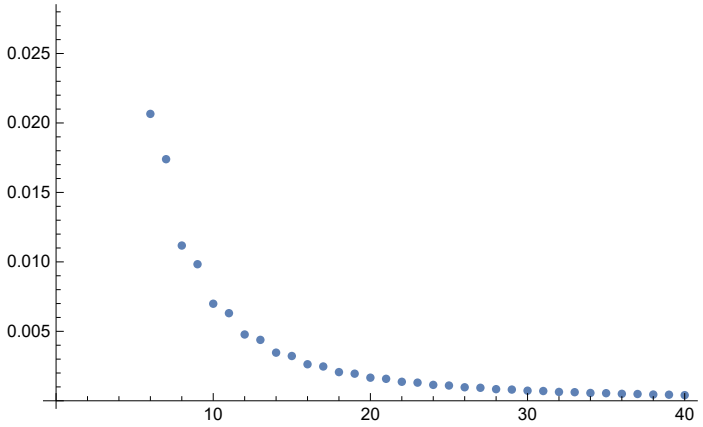
Plot of 10 realizations

```
list = {};
For[i = 4300, i < 4310, i = i + 1;
 AppendTo[list, Realization[i, x]]
]
Plot[list, {x, 0, 5}]
```



Number of terms in the KL expansion

```
ListPlot[Table[λ[n], {n, 1, M}]]
```




```
λ[M] / λ[1]
```

```
0.000157685
```