# CO32: Fourier Optics

F. I. Giasemis
Keble College, University of Oxford

October 2016

## 1 Abstract

The aim of this practical is to investigate the *Discrete Fourier Transform* (DFT). We applied the formulae related to the Discrete Fourier Transform and in this way we calculated various diffraction patterns using MatLab, comparing each result with the previous.

## 2 Introduction

The discrete Fourier transform (DFT) converts a finite sequence of equally-spaced samples of a function into an equivalent-length sequence of equally-spaced samples of the discrete-time Fourier transform (DTFT), which is a complex-valued function of frequency. The interval at which the DTFT is sampled is the reciprocal of the duration of the input sequence. An inverse DFT is a Fourier series, using the DTFT samples as coefficients of complex sinusoids at the corresponding DTFT frequencies. It has the same sample-values as the original input sequence. The DFT is the most important discrete transform, used to perform Fourier analysis in many practical applications.[1][2]

## 3 MatLab code

The MatLab scripts we used for our calculations are the following.

```
%CO32
%Fourier Optics
%Discrete Fourier Transform (DFT)
%Fotios Ioannis Giasemis
%Part 1

clear all;

disp('1 or 2?');
A=input('ANS=');
clc
N=250;
If=zeros(1,500);
Rf=zeros(1,500);

for i=245:255;
    Rf(1,i)=Rf(1,i)+1;
end

S1=0; S2=0;


for i=(-N):(N-1)
```

```
        s1=0; s2=0;
        for j=(-N):(N-1)
        s1=s1+Rf(1,j+N+1)*cos(-pi*j*i/N)-If(1,j+N+1)*sin(-pi*j*i/N);
        s2=s2+Rf(1,j+N+1)*sin(-pi*j*i/N)-If(1,j+N+1)*cos(-pi*j*i/N);
        end
        RF(1,i+N+1)=1/(2*N)*s1;
        IF(1,i+N+1)=1/(2*N)*s2;

end

x=1:500;amp=sqrt(RF.^2+IF.^2);

if A==2
    figure
    subplot(1,2,1)
    plot(x,Rf)
    subplot(1,2,2)
    plot(x,amp)
elseif A==1
    plot(x,amp)
end
```

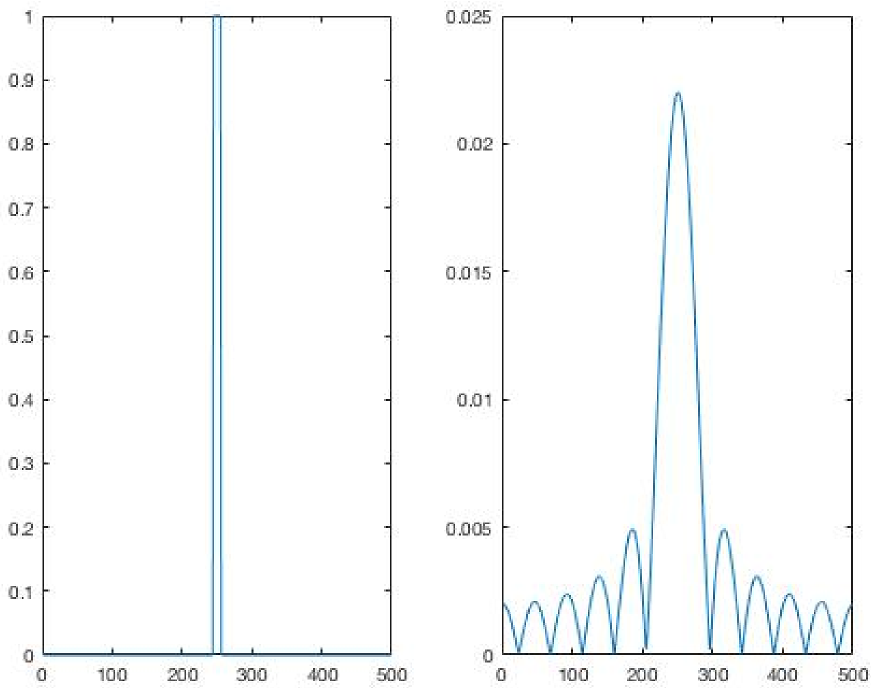


Figure 1: The real part and the amplitude of the transform

```
%CO32
%Fourier Optics
%Discrete Fourier Transform (DFT)
%Fotios Ioannis Giasemis
%Part 2
```

```
clear all;

N=250;
If=zeros(1,500);
Rf=zeros(1,500);

for i=(245-10):(255-10);
    Rf(1,i)=Rf(1,i)+1;
end

S1=0; S2=0;


for i=(-N):(N-1)
    s1=0; s2=0;
    for j=(-N):(N-1)
    s1=s1+Rf(1,j+N+1)*cos(-pi*j*i/N)-If(1,j+N+1)*sin(-pi*j*i/N);
    s2=s2+Rf(1,j+N+1)*sin(-pi*j*i/N)-If(1,j+N+1)*cos(-pi*j*i/N);
    end
    RF(1,i+N+1)=1/(2*N)*s1;
    IF(1,i+N+1)=1/(2*N)*s2;

end

x=1:500;amp=sqrt(RF.^2+IF.^2);
figure
subplot(1,2,1)
plot(x,Rf)
subplot(1,2,2)
plot(x,amp)
```

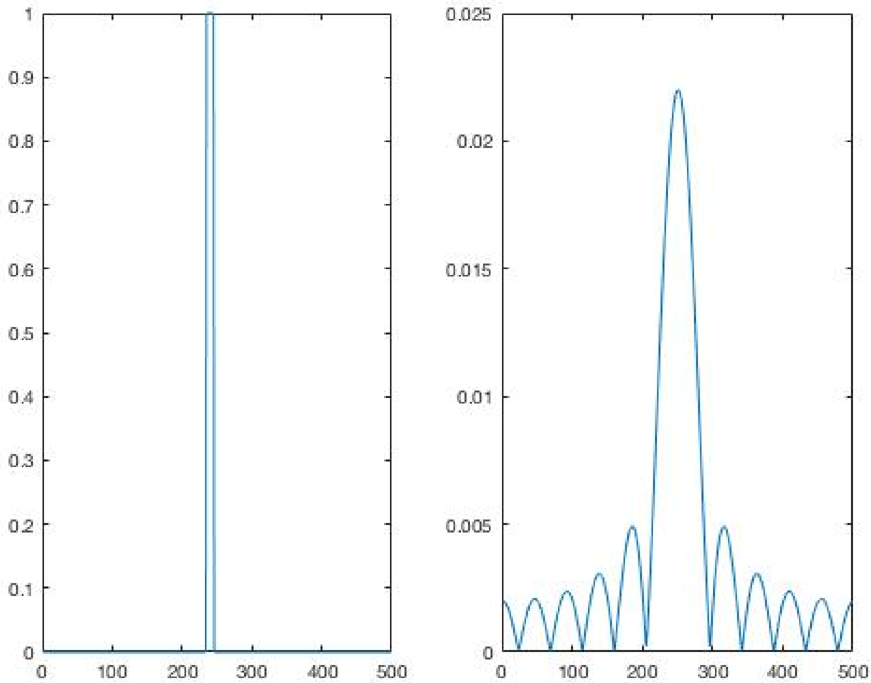


Figure 2: The real part and the amplitude of the transform

```
%CO32
%Fourier Optics
%Discrete Fourier Transform (DFT)
%Fotios Ioannis Giasemis
%Part 3

clear all;

N=250;
If=zeros(1,500);
Rf=zeros(1,500);

for i=240:260;
    Rf(1,i)=Rf(1,i)+1;
end

S1=0; S2=0;


for i=(-N):(N-1)
    s1=0; s2=0;
    for j=(-N):(N-1)
    s1=s1+Rf(1,j+N+1)*cos(-pi*j*i/N)-If(1,j+N+1)*sin(-pi*j*i/N);
    s2=s2+Rf(1,j+N+1)*sin(-pi*j*i/N)-If(1,j+N+1)*cos(-pi*j*i/N);
    end
    RF(1,i+N+1)=1/(2*N)*s1;
    IF(1,i+N+1)=1/(2*N)*s2;

```

```
end
x=1:500;amp=sqrt(RF.^2+IF.^2);

plot(x,amp)
```

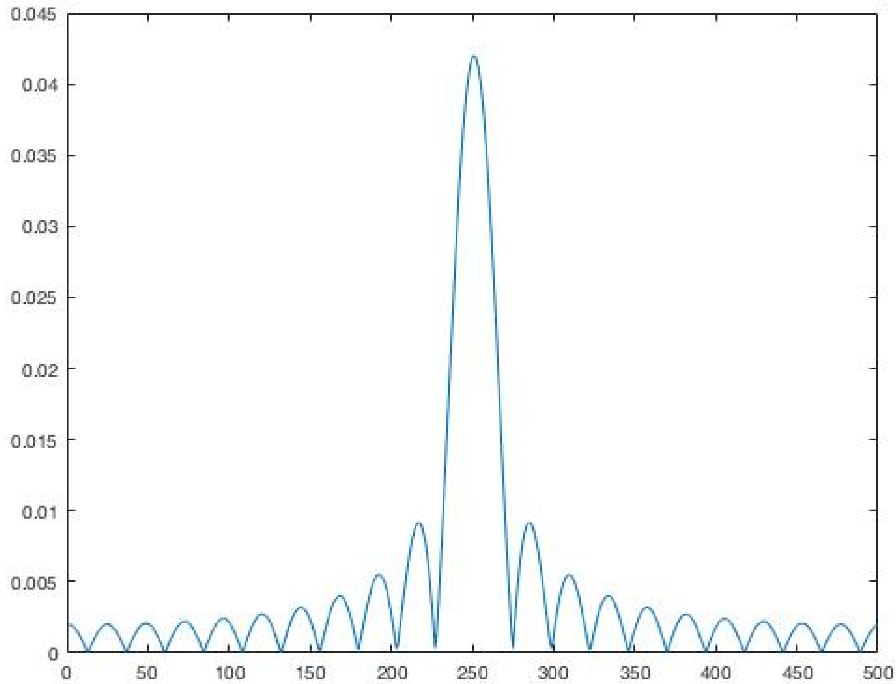


Figure 3: The amplitude of the transform

```
%CO32
%Fourier Optics
%Discrete Fourier Transform (DFT)
%Fotios Ioannis Giasemis
%Part 4

clear all;

N=250;
If=zeros(1,500);
Rf=zeros(1,500);

for i=240:260;
    Rf(1,i)=Rf(1,i)+1/2;
end

S1=0; S2=0;


for i=(-N):(N-1)
    s1=0; s2=0;
    for j=(-N):(N-1)
```

```
      s1=s1+Rf(1,j+N+1)*cos(-pi*j*i/N)-If(1,j+N+1)*sin(-pi*j*i/N);
      s2=s2+Rf(1,j+N+1)*sin(-pi*j*i/N)-If(1,j+N+1)*cos(-pi*j*i/N);
      end
      RF(1,i+N+1)=1/(2*N)*s1;
      IF(1,i+N+1)=1/(2*N)*s2;

end

x=1:500;amp=sqrt(RF.^2+IF.^2);
plot(x,amp)
```

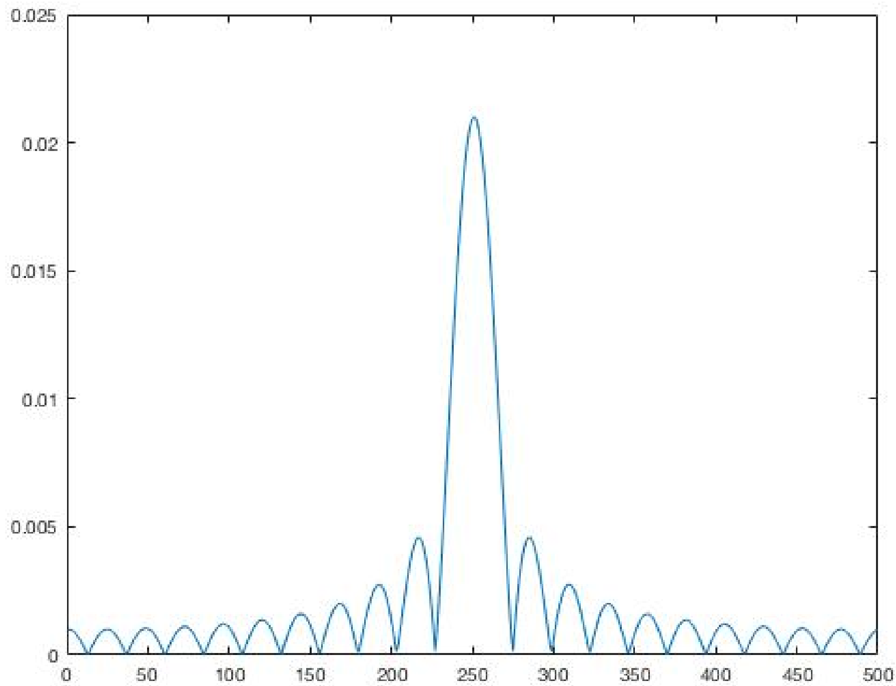


Figure 4: The amplitude of the transform

```
%CO32
%Fourier Optics
%Discrete Fourier Transform (DFT)
%Fotios Ioannis Giasemis
%Part 5

clear all;

N=250;
If=zeros(1,500);
Rf=zeros(1,500);

for i=240:260;
      Rf(1,i)=Rf(1,i)+1;
end

```

```
s1=0;  s2=0;

C=conv(Rf,Rf);  C(1,1000)=0;

N=500;
for k=(-N):(N-1)
        s1=0;

        for j=(-N):(N-1)
                s1=s1+C(1,j+N+1)*exp(-pi*1i*j*k/N);
        end

        FC(1,k+N+1)=1/(2*N)*s1;

end

x=1:1000;
figure
plot(x,FC);
```

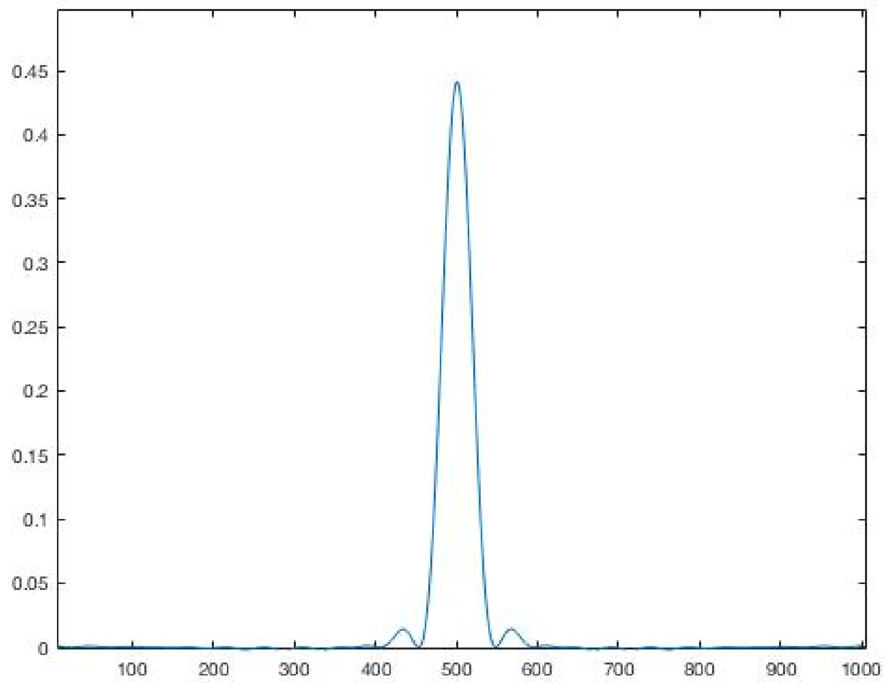


Figure 5: The amplitude of the transform of the convolution

If you do:

```
amp=amp^2;
plot(x,amp)
```

you can easily verify the Convolution theorem.

```
%CO32
%Fourier Optics
```

```
%Discrete Fourier Transform (DFT)
%Fotios Ioannis Giasemis
%Part 6

clear all;

clc
N=25; M=25;
If=zeros(2*N,2*M);
Rf=zeros(2*N,2*M);

Rf(25,25)=Rf(25,25)+1; Rf(25,26)=Rf(25,26)+1; Rf(25,24)=Rf(25,24)+1;
Rf(24,25)=Rf(24,25)+1; Rf(26,25)=Rf(26,25)+1;

for i=(-N):(N-1)
    for j=(-M):(M-1)
        s1=0; s2=0;
        for k=(-N):(N-1)
            for l=(-M):(M-1)
            s1=s1+Rf(k+N+1,l+N+1)*cos(-pi*k*i/N-pi*l*j/M)-If(k+N+1,l+N+1)*sin(-pi*k*i/N-pi*l*j/M);
            s2=s2+Rf(k+N+1,l+N+1)*sin(-pi*k*i/N-pi*l*j/M)+If(k+N+1,l+N+1)*cos(-pi*k*i/N-pi*l*j/M);
            RF(j+N+1,i+N+1)=1/(4*N*M)*s1;
            IF(j+N+1,i+N+1)=1/(4*N*M)*s2;
            end

        end
    end

end

amp=sqrt(RF.^2+IF.^2);

 figure
    subplot(1,2,1)
    surf(Rf)
    subplot(1,2,2)
    surf(amp)
```

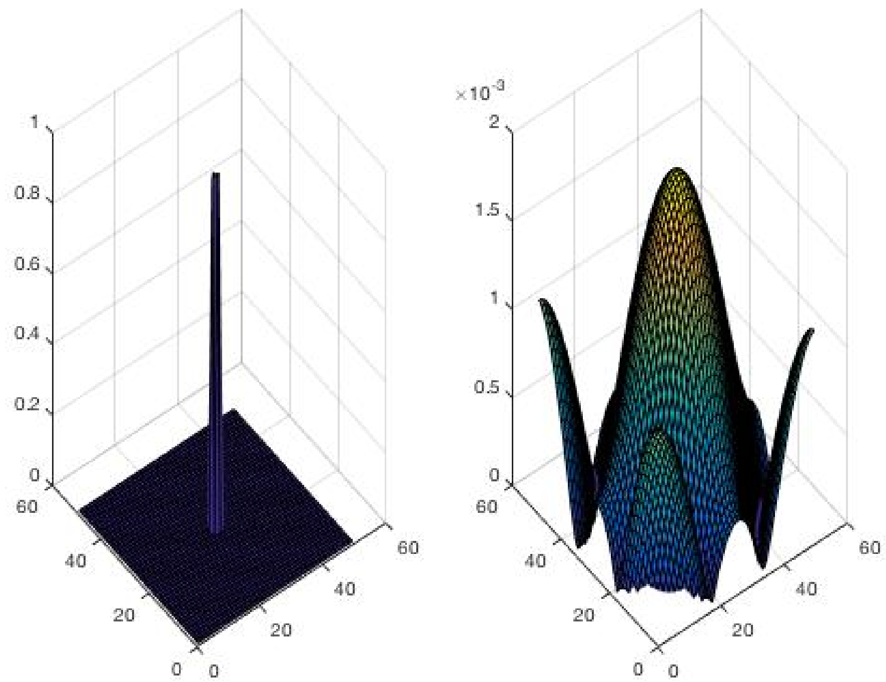


Figure 6: The real part and the amplitude of the transform

```
%CO32
%Fourier Optics
%Discrete Fourier Transform (DFT)
%Fotios Ioannis Giasemis
%Part 7

clear all;

clc
N=25; M=25;
If=zeros(2*N,2*M);
Rf=zeros(2*N,2*M);

for m=24:26
    for n=15:35
        Rf(m,n)=Rf(m,n)+1;
    end
end

for i=(-N):(N-1)
    for j=(-M):(M-1)
        s1=0; s2=0;
        for k=(-N):(N-1)
            for l=(-M):(M-1)
            s1=s1+Rf(k+N+1,l+N+1)*cos(-pi*k*i/N-pi*l*j/M)-If(k+N+1,l+N+1)*
                sin(-pi*k*i/N-pi*l*j/M);
            s2=s2+Rf(k+N+1,l+N+1)*sin(-pi*k*i/N-pi*l*j/M)+If(k+N+1,l+N+1)*
                cos(-pi*k*i/N-pi*l*j/M);
```

```
                    RF( j+N+1,i+N+1)=1/(4*N*M)*s1;
                    IF( j+N+1,i+N+1)=1/(4*N*M)*s2;
                    end

                end
            end

        end

amp=sqrt(RF.^2+IF.^2);

 figure
    subplot(1,2,1)
    surf(Rf)
    subplot(1,2,2)
    surf(amp)
```

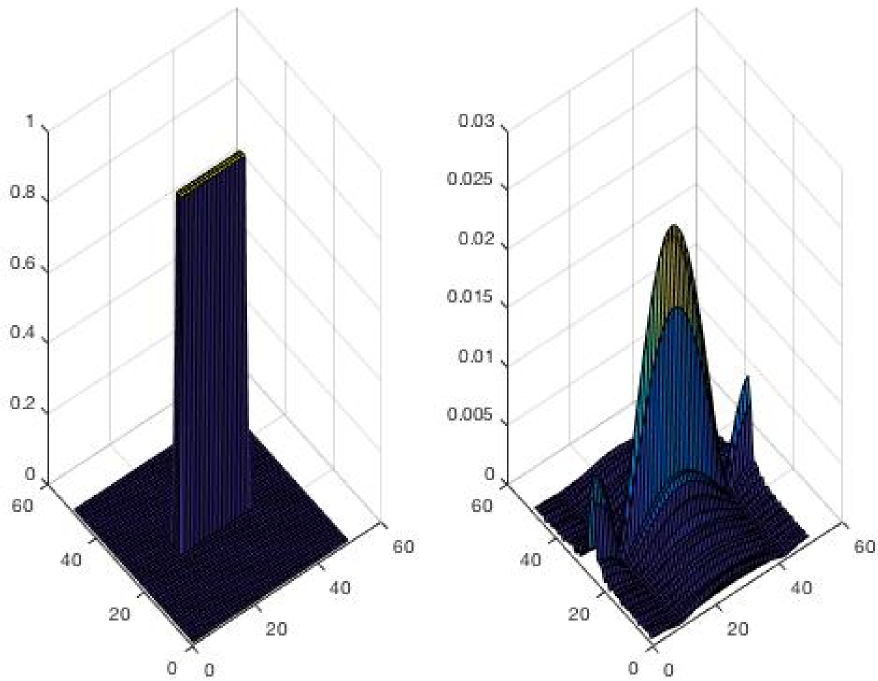


Figure 7: The real part and the amplitude of the transform

# 4 References


[1] Strang, Gilbert (MayJune 1994). "Wavelets". American Scientist. 82 (3): 253. Retrieved 8 October 2013.
[2] https://en.wikipedia.org/wiki/Discrete_Fourier_transform