CO24: Chaos

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1 Abstract

The aim of this practical is to get an insight into how *chaotic systems* behave to slight changes in the initial conditions. First, we solved the Lorenz system using the classical Runge-Kutta method (RK4). Then, we tampered with the initial values and observed how the behaviour changes when you change the constants of the system.

2 Introduction

2.1 Chaos

Small differences in initial conditions (such as those due to rounding errors in numerical computation) yield widely diverging outcomes for such dynamical systems, rendering long-term prediction impossible in general.[1] This behavior is known as deterministic chaos, or simply chaos. The theory was summarized by Edward Lorenz[2] as:

> Chaos: When the present determines the future, but the approximate present does not approximately determine the future.

2.2 Lorenz Equations

The Lorenz system is a system of ordinary differential equations first studied by Edward Lorenz. It is notable for having chaotic solutions for certain parameter values and initial conditions. In particular, the Lorenz attractor is a set of chaotic solutions of the Lorenz system.[3]

The Lorenz equations describe fluid circulation. Therefore, this model can be applied to atmospheric phenomena and meteorology. However, a slight change in the initial values of $\{y_1, y_2, y_3\}$ (temperature, pressure, wind speed etc.) completely changes the outcome, and that is how we can tell that the set of differential equations is chaotic. This can be a demonstration of how difficult weather prediction is.

3 Classical Runge-Kutta Methods

The classical Runge-Kutta method (RK4) is an iterative method for the approximate solutions of ordinary differential equations (ODEs). This method was developed by the German mathematicians C. Runge and M. W. Kutta, together with some other similar ones. These methods are collectively known as Runge-Kutta methods.

The RK methods are based on the Euler method, numerical procedure for solving ODEs, and also on Simpson's rule.

Numerical methods for solving O.D.E.s are categorised according to their global error. The RK4 is classified as a second order method, having a global error of $O(h^2)$.[5]

The main advantages of Runge-Kutta methods are that they are easy to implement, they are very stable, and they are "self-starting". The primary disadvantages of Runge-Kutta methods are that they require significantly more computer time than multi-step methods of comparable accuracy, and they do not easily yield good global estimates of the truncation error.[4]

4 Appendix A: MatLab code

The MatLab code we used for this practical.

```
%CO24
%Chaos
%Runge-Kutta Methods
%Fotios Ioannis Giasemis

clear;

a=10; b=8/3; dt=0.05; t=0;

n=input('Number of steps: n=');
disp('Coordinates {y} of the starting point:')
y1=input('y1=');
y2=input('y2=');
y3=input('y3=');
r=input('Value of r: r=');

f1=@(y1,y2) a*(y2-y1);
f2=@(y1,y2,y3) r*y1-y2-y1*y3;
f3=@(y1,y2,y3) y1*y2-b*y3;

Y1(1,1)=y1;
Y2(1,1)=y2;
Y3(1,1)=y3;

for i=2:n

    f1_0=f1(y1,y2);
    f2_0=f2(y1,y2,y3);
    f3_0=f3(y1,y2,y3);

    f1_1=f1(y1+f1_0*dt/2,y2+f2_0*dt/2);
    f2_1=f2(y1+f1_0*dt/2,y2+f2_0*dt/2,y3+f3_0*dt/2);
    f3_1=f3(y1+f1_0*dt/2,y2+f2_0*dt/2,y3+f3_0*dt/2);

    f1_2=f1(y1+f1_1*dt/2,y2+f2_1*dt/2);
    f2_2=f2(y1+f1_1*dt/2,y2+f2_1*dt/2,y3+f3_1*dt/2);
    f3_2=f3(y1+f1_1*dt/2,y2+f2_1*dt/2,y3+f3_1*dt/2);

```

```
    f1_3=f1(y1+f1_2*dt,y2+f2_2*dt);
    f2_3=f2(y1+f1_2*dt,y2+f2_2*dt,y3+f3_2*dt);
    f3_3=f3(y1+f1_2*dt,y2+f2_2*dt,y3+f3_2*dt);

    y1=y1+dt*(f1_0+2*f1_1+2*f1_2+f1_3)/6;
    y2=y2+dt*(f2_0+2*f2_1+2*f2_2+f2_3)/6;
    y3=y3+dt*(f3_0+2*f3_1+2*f3_2+f3_3)/6;

    Y1(1,i)=y1;
    Y2(1,i)=y2;
    Y3(1,i)=y3;

    t=t+dt;
end

T=linspace(0,t,n);
figure
subplot(1,2,1)
plot(T,Y1)

subplot(1,2,2)
plot(Y2,Y3)

clc; clear;
```

5 Appendix B: The Lorenz Attractor

Using the code in Appendix A (n=1000, r=40):

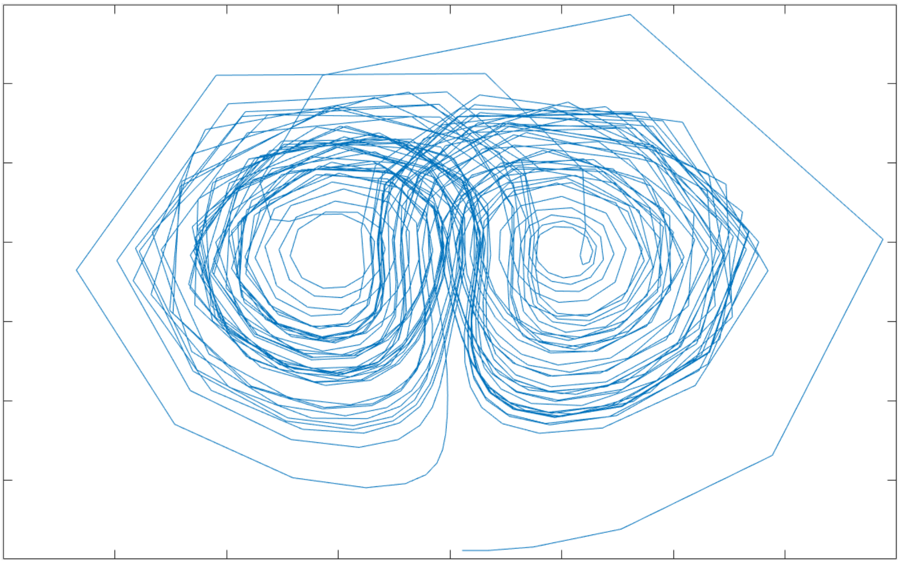

Figure 1: $y_1 = y_2 = y_3 = 1$

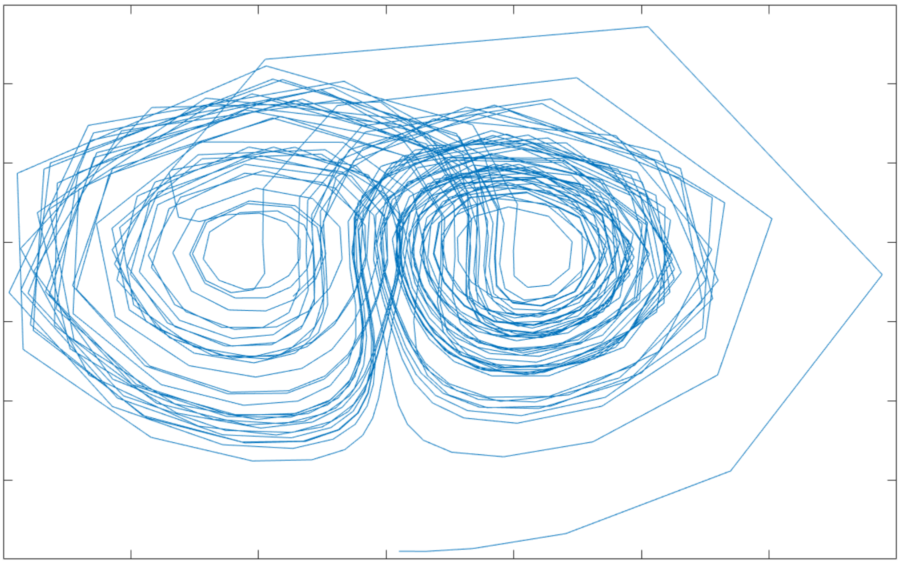

Figure 2: $y_1 = y_2 = y_3 = 1.1$

6 References

[1] Kellert, Stephen H. (1993). In the Wake of Chaos: Unpredictable Order in Dynamical Systems. University of Chicago Press. p. 32. ISBN 0-226-42976-8.
[2] Danforth, Christopher M. (April 2013). "Chaos in an Atmosphere Hanging on a Wall". Mathematics of Planet Earth 2013. Retrieved 4 April 2013.
[3] `https://en.wikipedia.org/wiki/Lorenz_system`
[4] `http://farside.ph.utexas.edu/teaching/329/lectures/node40.html`
[5] A. OHare, Numerical Methods for Physicists, Oxford Physics, 2005.