Multiphase flows
Lecture 2

Equation of motion of a single particle, forces on individual particles

In this lecture we will:

- Analyze the equation of motion of an individual particle
- Identify and discuss forces acting on individual particles
- Discuss specific features related to bubbles

But first:

Let us (re)visit the concepts of Eulerian and Lagrangian frames of reference – fundamental to understand continuum mechanics (including fluid mechanics)

Lagrangian framework: evolution of properties associated with the $i-th$ particle (X, V, R *(radius)* – Lagrangian coordinates)

$$\frac{\mathrm{d}\boldsymbol{X}_{(i)}}{dt} = \boldsymbol{V}_{(i)} \qquad \frac{\mathrm{d}\boldsymbol{V}_{(i)}}{dt} = \boldsymbol{A}_{(i)} \qquad \frac{\mathrm{d}R_{(i)}}{dt} = \Theta_{(i)}$$

$\boldsymbol{A}_{\boldsymbol{i}}$ – acceleration experienced by the $i-th$ particle

$\Theta_{(i)}$ - rate of change of radius due to interphase mass transfer (e.g. change of phase)

Lagrangian vs. Eulerian reference frames

Eulerian reference frame

$$\vec{u}(\vec{x}, t)$$

Eulerian velocity field at location $\vec{x}$ and time t

Lagrangian reference frame

$$\vec{X}\left(\overrightarrow{X_0}, t\right)$$

Position of element $\overrightarrow{X_0}$ at time t

There is something fundamental we have to learn and adopt here

Communication between Lagrangian and Eulerian frameworks

Essence of everything we do in continuum mechanics

The property associated with a fluid particle at a point is the same as the (field) property at a point.

The *Lagrangian (or material)* property associated with a fluid particle at a point is the same as the *Eulerian (or spatial)* property evaluated at the point.

Essential: Lagrangian displacement field

$$\boxed{\vec{x} = \vec{X}(\vec{X}_o, t)}$$

X_0: fluid element

$\boldsymbol{x}$: "action" that displaces the fluid element X_0

How it works: take the density

Lagrangian Eulerian

$$R(\vec{X}_o, t) = \rho(\underbrace{\vec{X}[\vec{X}_o, t]}_{\vec{x}}, t)$$

Density of the fluid element (fluid particle) X_0

…or the velocity

Eulerian velocity field

$$u(\underbrace{\vec{X}(\overrightarrow{X_0}, t)}_{\vec{x}}, t) = \frac{\partial \vec{X}}{\partial t}(\overrightarrow{X_0}, t)$$

Now we can study:

Equation of motion of a single particle

- Assumptions
- Some hints on the derivation procedure
- Analysis of the forces
- Differences between different types of particles

Equation of motion of a single particle (Maxey and Riley, 1983)

Index p – particle
Index f - fluid

Action of the fluid on the particle surface

$$m_P \frac{d\boldsymbol{u}_P}{dt} = \sum \boldsymbol{F}_i \iff m_P \frac{d\boldsymbol{u}_P}{dt} = -\int_{S_P} \sigma_{fij} n_{pj} \mathrm{d}S + m_P \boldsymbol{g}$$

$$m_P \frac{d\boldsymbol{u}_P}{dt} = \sum \boldsymbol{F}_i \iff m_P \frac{du_{P,i}}{dt} = \oint_S \underbrace{\left[-p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)\right]} n_j dS + m_P g_i$$

Stress tensor of the fluid phase
(to be evaluated on the surface of the sphere)

Fundamental assumptions in the process of derivation:

- Incompressible flow
- Translational motion
- Unbounded domain - no influence from walls
- Small sphere – smaller than the smallest length scale of the flow
- $Re_P \ll 1$ – creeping flow (defined with the particle Reynolds number)

$$Re_P = \frac{\rho_f \left(u_f - u_P\right) d_\mathrm{p}}{\mu_f}$$

Some indication on a possible way to do the derivation - Decomposition of the stress tensor

$$-\int_{S_p} \sigma_{fij} n_{1j} dS = \int_{S_p} \overset{o}{\sigma}_{fij}\, n_{2j} dS + \int_{S_p} \delta\sigma_{fij} n_{2j} dS$$

Term 1 Term 2

Term 1- Stress tensor on a surface of a particle (if the particle were not present)

Term 2 – Perturbation (due to the presence of a particle)

Result – identification of different forces

- Drag force
- Lift force
- History force
- Added mass force
- Pressure gradient force
- Force due to Brownian motion
- …

Important to have in mind:

- Analytical solution available only for the Stokes regime (creeping flow)
- Consideration of heat and mass transfer requires solution of two additional PDE (e.g. droplet diameter and droplet temperature)
- What happens for higher particle Re numbers?