Generalization of the forces (virtual mass and lift, similar for other forces)

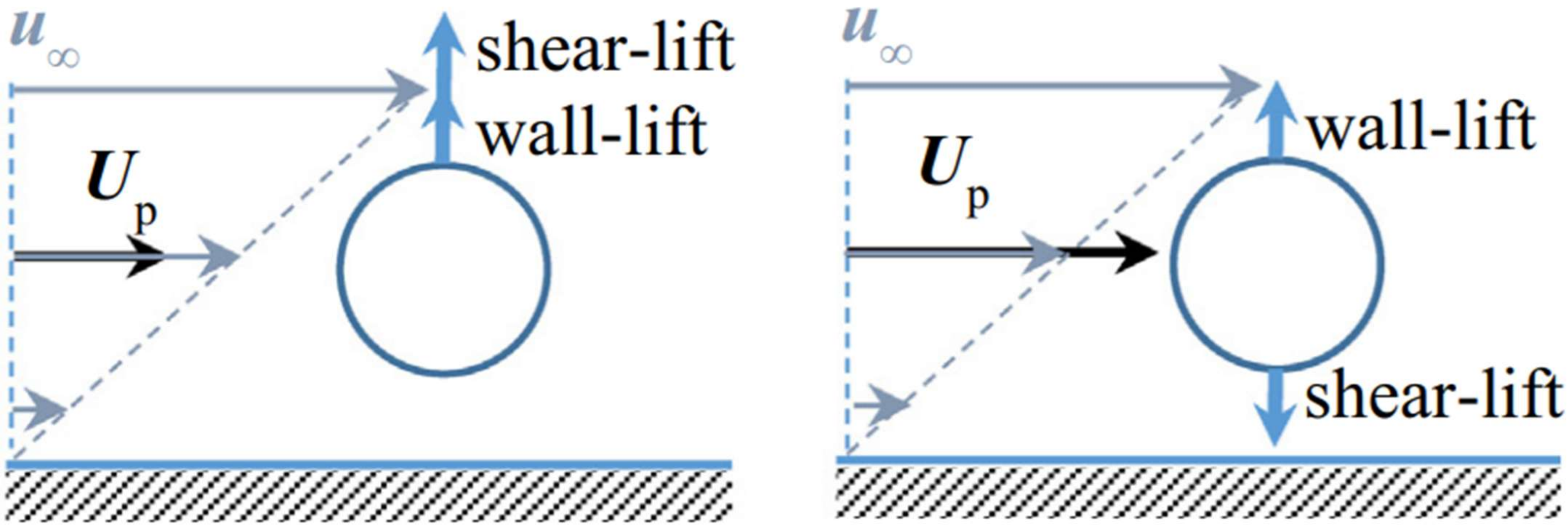
$$\boldsymbol{F}_{\mathbf{vm}} = C_{vm}\rho_l\alpha_g\alpha_l\left(\frac{D\mathbf{u_l}}{Dt} - \frac{D\mathbf{u_g}}{Dt}\right)$$

$$\boldsymbol{F}_{\mathbf{lift}} = C_L\rho_l\alpha_g\alpha_l\left(\mathbf{u_g} - \mathbf{u_l}\right) \times \underbrace{\left(\nabla \times \mathbf{u_l}\right)}_{\omega_l}$$

l, *g*: liquid, gas

Vorticity of the liquid phase

Other things are important as well – influence of the wall



- Corrections of the lift force due to the presence of the wall
- Change of direction (what is different on the sketch above?)

Specific features related to bubbles

Important to have in mind

Absence of a rigid interface between a bubble and fluid

⬇

Consequence: Reduction of the drag force (compared to the corresponding solid particle)

Contamination of the surface of a bubble

⬇

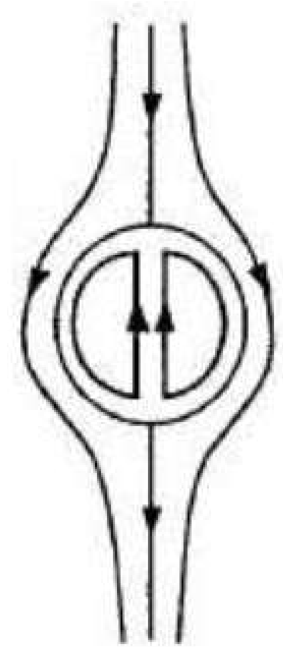
Consequence: Rigid interface again created?

Internal circulation as a fundamental phenomenon (function of viscosity ratio)

Tendency to internal circulation given by:

$$\frac{3\kappa + 2}{3\kappa + 3}$$

1 = particle behaves as a rigid body
2/3 = "full" internal circulation



$$\kappa = \frac{viscosity\ of\ particle\ fluid}{viscosity\ of\ carrier\ fluid}$$

Drag force – what happens for bubbles and drops?

Hadamard-Rybczynski drag law – stresses on the surface induce internal motion - drag coefficient decreased

$$C_{D,HR} = \frac{24}{Re_p}\left(\frac{2/3 + \mu_d/\mu_c}{1 + \mu_d/\mu_c}\right)$$

d- dispersed phase
c – continuous (carrier) phase

Droplet in air : the Stokes law is recovered $(24/Rep)$

Bubble in liquid: drag is reduced by $1/3$

Bubble deformation and oscillation

Weber number: **(inertia/surface tension)**

$$We_b = \frac{\rho_f \, u_{rel}^2 \, d_{eq}}{\sigma}$$

d_{eq}: Equivalent hydraulic diameter

u_{rel}: relative velocity between the phases

σ - surface tension

Can bubbles maintain spherical shape?

$$E_O = \frac{We_b}{Fr_b} = g \frac{\left|\rho_f - \rho_b\right| \, d_{eq}{}^2}{\sigma}$$

Eötvös number **(buoyancy/surface tension)**

Bubble-shape diagram

Capillary number

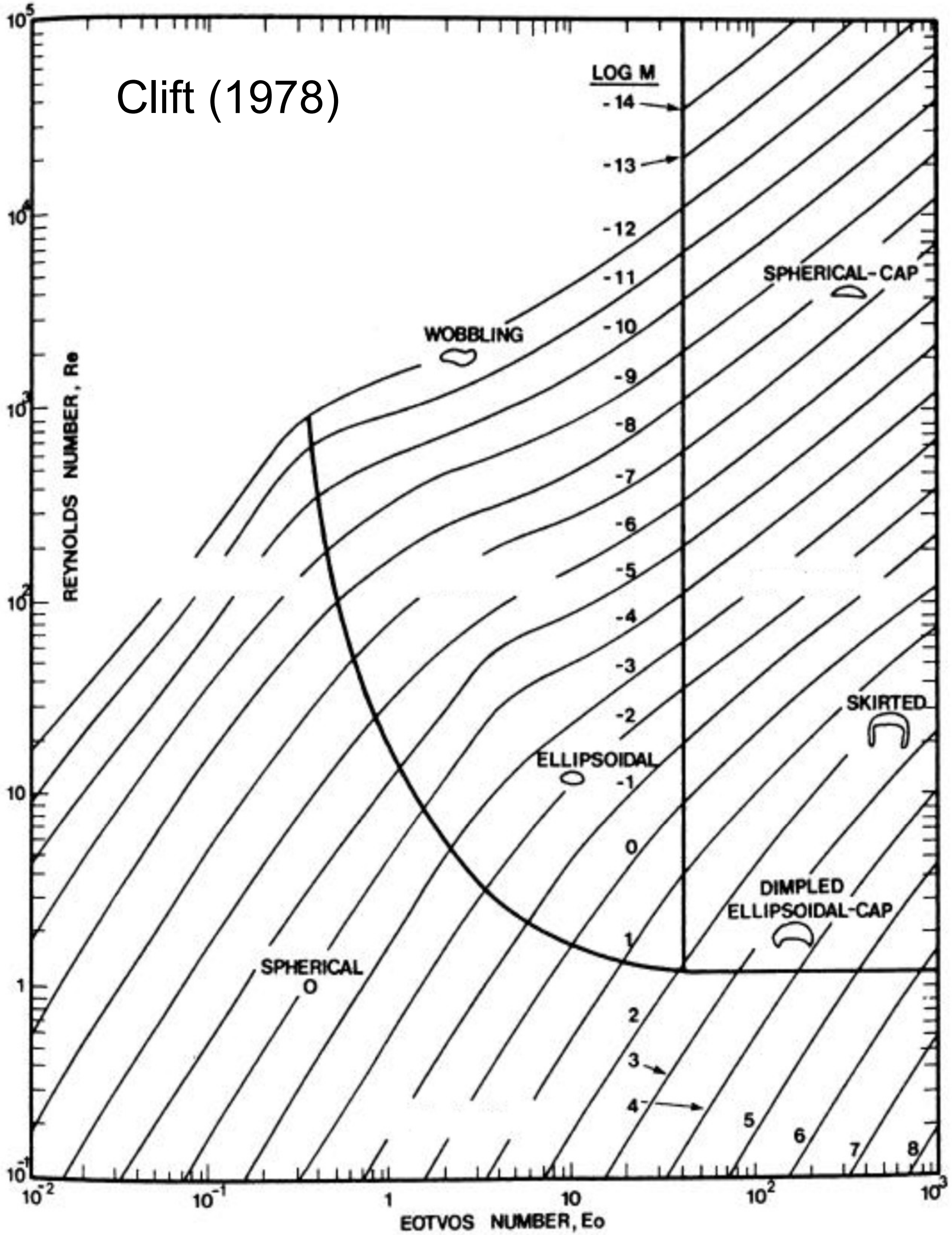
(viscous/surface tension)

$$Ca = \frac{\mu_f U}{\sigma} = \frac{We}{\mathrm{Re}}$$

Morton number

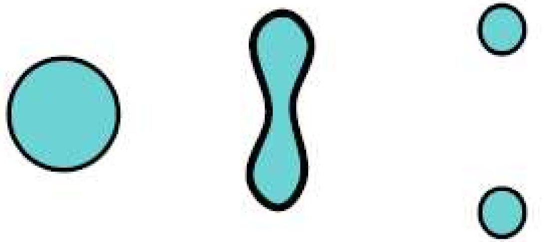
$$Mo = \frac{g {\mu_f}^4 |\rho_f - \rho_b|}{\rho_f^2 \sigma^3}$$

$$Mo = (EoWe\hat{}2)/Re\hat{}4$$

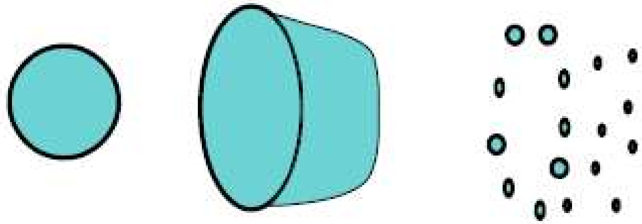


Example: (secondary) breakup of droplets as a function of *We*

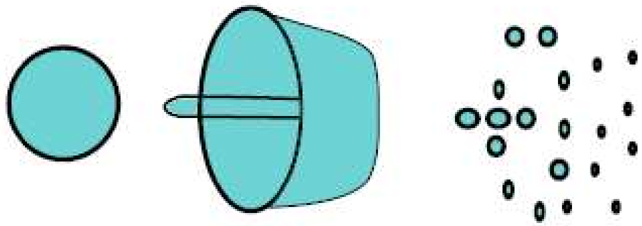
Wierzba, 1990



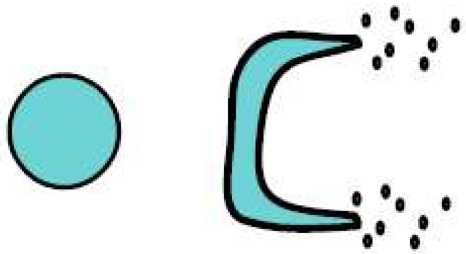
(a) Vibrational breakup, $We_d \approx 12$



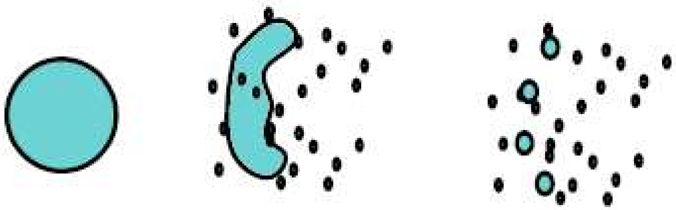
(b) Bag breakup, $We_d < 20$



(c) Bag / streamer breakup, $We_d < 50$

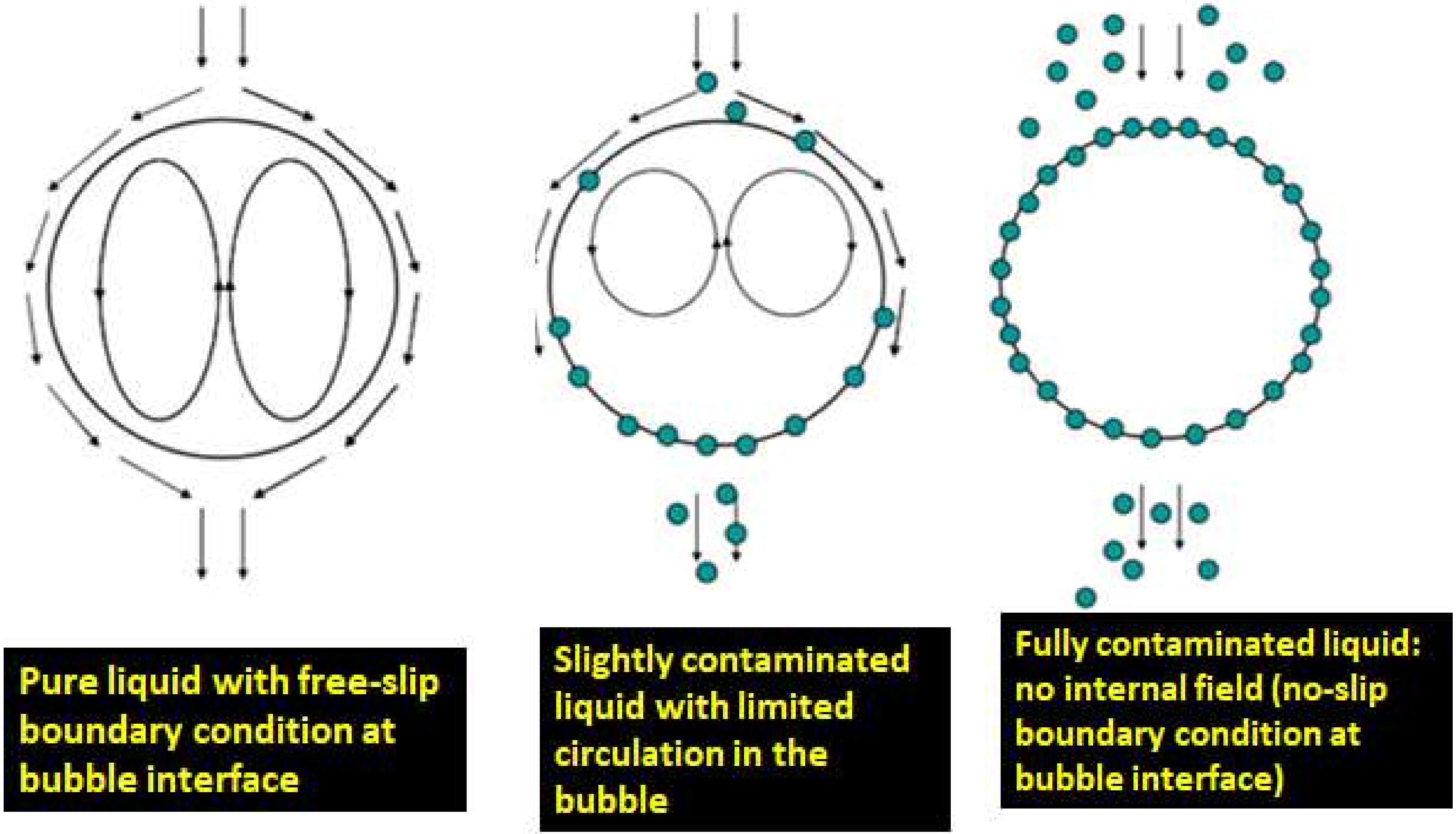


(d) Stripping breakup, $We_d < 100$

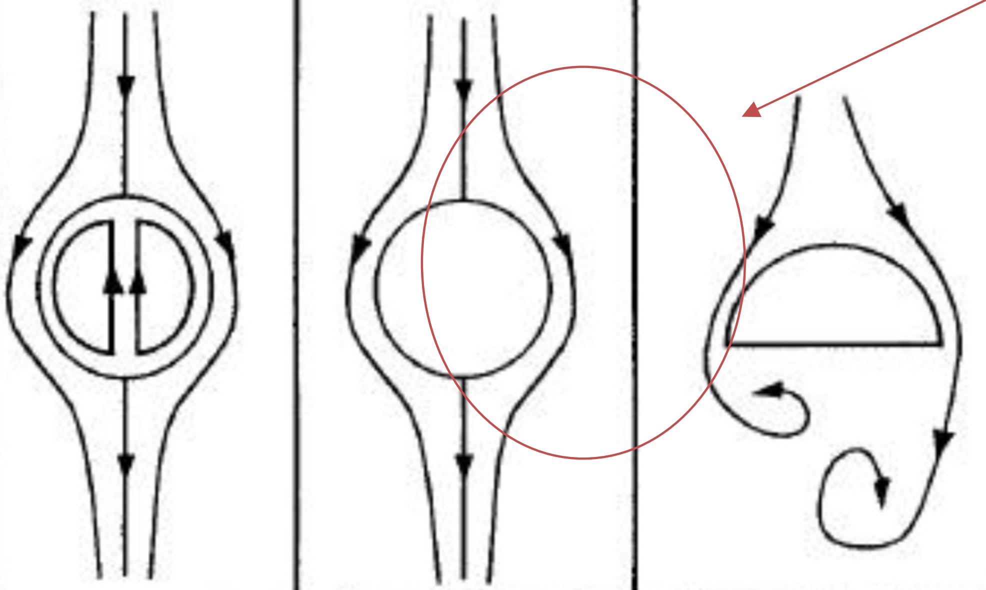


(e) Catastrophic breakup, $We_d > 100$

Drag force as a function of level of contamination



Bubble behaviours - summary

Shape	spherical		non-spherical
Motion	rectilinear		fluctuating
Purity	pure	contaminated	both
Flow pattern			
Governing effects	viscosity	viscosity	surface tension and gravity
Relevant dimension-less number	Re	Re	Eo

Rigid interface re-created?

Tomyama (1998)