

Multiphase flows

Lecture 2

Equation of motion of a single
particle, forces on individual
particles

In this lecture we will:

- Analyze the equation of motion of an individual particle
- Identify and discuss forces acting on individual particles
- Discuss specific features related to bubbles

But first:

Let us (re)visit the concepts of Eulerian and Lagrangian frames of reference – fundamental to understand continuum mechanics (including fluid mechanics)

Lagrangian framework: evolution of properties associated with the i – th particle (X, V, R (radius) – Lagrangian coordinates)

$$\frac{dX_{(i)}}{dt} = V_{(i)} \quad \frac{dV_{(i)}}{dt} = A_{(i)} \quad \frac{dR_{(i)}}{dt} = \Theta_{(i)}$$

A_i – acceleration experienced by the i – th particle

$\Theta_{(i)}$ - rate of change of radius due to interphase mass transfer (e.g. change of phase)

Lagrangian vs. Eulerian reference frames

Eulerian reference frame

$$\vec{u}(\vec{x}, t)$$

Lagrangian reference frame

$$\vec{X}(\overrightarrow{X_0}, t)$$

Eulerian velocity field
at location \vec{x} and time t

Position of element
 $\overrightarrow{X_0}$ at time t

There is something fundamental we have to learn and adopt here

Communication
between Lagrangian and
Eulerian frameworks

Essence of everything we do in continuum mechanics

The property associated with a fluid particle at a point is the same as the (field) property at a point.

The *Lagrangian (or material)* property associated with a fluid particle at a point is the same as the *Eulerian (or spatial)* property evaluated at the point.

Essential: Lagrangian displacement field

$$\vec{x} = \vec{X}(\vec{X}_o, t)$$

X_0 : fluid element

x : "action" that
displaces the fluid
element X_0

How it works: take the density

Lagrangian

Eulerian

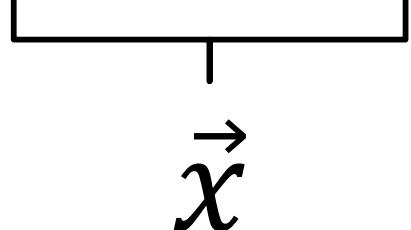
$$R(\vec{X}_o, t) = \rho(\vec{X}[\vec{X}_o, t], t)$$

Density of the fluid
element (fluid particle) X_0

\vec{x}

...or the velocity

Eulerian velocity field

$$u(\overrightarrow{X}(\overrightarrow{X_0}, t), t) = \frac{\partial \overrightarrow{X}}{\partial t}(\overrightarrow{X_0}, t)$$


Now we can study:

Equation of motion of a single particle

- Assumptions
- Some hints on the derivation procedure
- Analysis of the forces
- Differences between different types of particles

Equation of motion of a single particle (Maxey and Riley, 1983)

Index p – particle

Index f - fluid

Action of the fluid on
the particle surface

$$m_P \frac{d\mathbf{u}_P}{dt} = \sum \mathbf{F}_i \quad \longleftrightarrow \quad m_P \frac{d\mathbf{u}_P}{dt} = - \int_{S_P} \sigma_{fij} n_{pj} dS + m_P \mathbf{g}$$

$$m_P \frac{d\mathbf{u}_P}{dt} = \sum \mathbf{F}_i \quad \longleftrightarrow \quad m_P \frac{d\mathbf{u}_P}{dt} = \int_S \underbrace{\left[-p \delta_{ij} + \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] n_j dS}_{\text{Stress tensor of the fluid phase}} + m_P g_i$$

Stress tensor of the fluid phase
(to be evaluated on the surface of the sphere)

Fundamental assumptions in the process of derivation:

- Incompressible flow
- Translational motion
- Unbounded domain - no influence from walls
- Small sphere – smaller than the smallest length scale of the flow
- $Re_p \ll 1$ – creeping flow (defined with the particle Reynolds number)

$$Re_p = \frac{\rho_f (u_f - u_p) d_p}{\mu_f}$$

Some indication on a possible way to do the derivation - Decomposition of the stress tensor

Term 1- Stress tensor on a surface of a particle (if the particle were not present)

Term 2 – Perturbation (due to the presence of a particle)

Result – identification of different forces

- Drag force
- Lift force
- History force
- Added mass force
- Pressure gradient force
- Force due to Brownian motion
- ...

Important to have in mind:

- Analytical solution available only for the Stokes regime (creeping flow)
- Consideration of heat and mass transfer requires solution of two additional PDE (e.g. droplet diameter and droplet temperature)
- What happens for higher particle Re numbers?