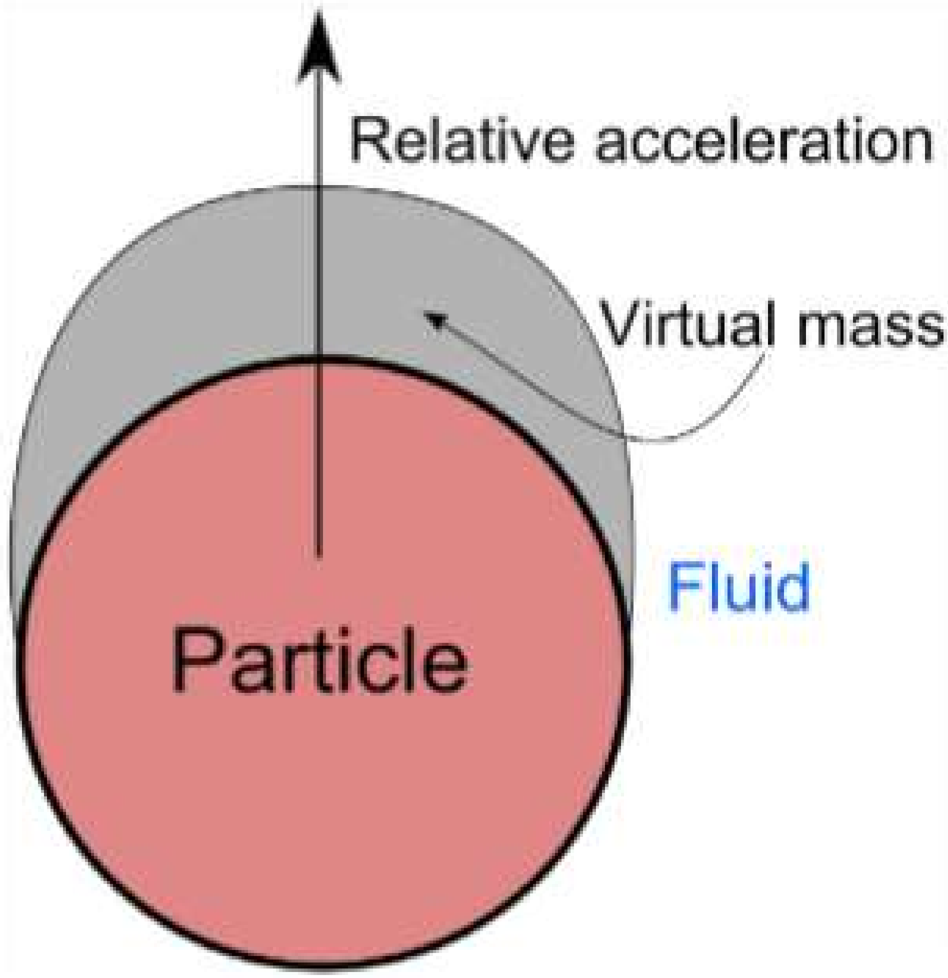
Two important forces will be introduced now:

- Added mass force (virtual mass force)
- History (Basset force)

Sometimes they are referred to as: **unsteady drag** (**acceleration of the relative velocity**)

Added mass force – acceleration of a certain fraction of the surrounding fluid



Inertia added to the system

Crowe, 2012

Added mass force (Virtual mass force)

– Expression and analysis

$$\boldsymbol{F_A} = \frac{1}{2} m_P \frac{\rho_f}{\rho_P} \frac{d}{dt}(\boldsymbol{u_f} - \boldsymbol{u_p})$$

Conclude when (i.e. in what types of flows) can this force be neglected – based on the density ratio

It can be shown: $\boxed{\rho_{virt} = \rho_p + \frac{1}{2}\rho_F}$

History (Basset) force – delay in the boundary layer development with changing relative velocity – hydrodynamic memory

Basset kernel

$$\boldsymbol{F_H} = \sqrt{\pi\rho_f\mu_f}\,\frac{m_P}{\rho_P d_P}\int_0^t {\color{red}K(t-\tau)}\,\frac{d}{dt}\left(\mathbf{u_f} - \boldsymbol{u_p}\right)$$

$$\boldsymbol{F_H} = \sqrt{\pi\rho_f\mu_f}\,\frac{m_P}{\rho_P d_P}\int_0^t \underbrace{{\color{red}\frac{1}{\sqrt{1-\tau}}}\,\frac{d}{dt}}\left(\mathbf{u_f} - \boldsymbol{u_p}\right)$$

Integration along the entire trajectory for each time step of calculation

History (Basset) force – some info:

Inclusion of the history force transforms Newton's second law for the particle from an ODE to an integro-differential equation (not explicit in u_p or du_p/dt)

Effects most pronounced for high frequency unsteady flows when the fluid-to-particle density ratio is high, and for acceleration from rest in a quiescent fluid

Pressure gradient force (presence of a local pressure gradient) and the force due to shear

$$F_P = \frac{m_P}{\rho_P}\left(-\nabla p + \mu_f \nabla \tau_{shear}\right)$$

from the Navier-Stokes equations

$$-\nabla p + \nabla \tau_{shear} = \rho_f \left(\frac{D u_{\boldsymbol{f}}}{D t} - g\right)$$

The total pressure force is:

$$F_P = m_P \frac{\rho_f}{\rho_P}\left(\frac{D\, u_{\mathbf{f}}}{D\ t} - g\right)$$

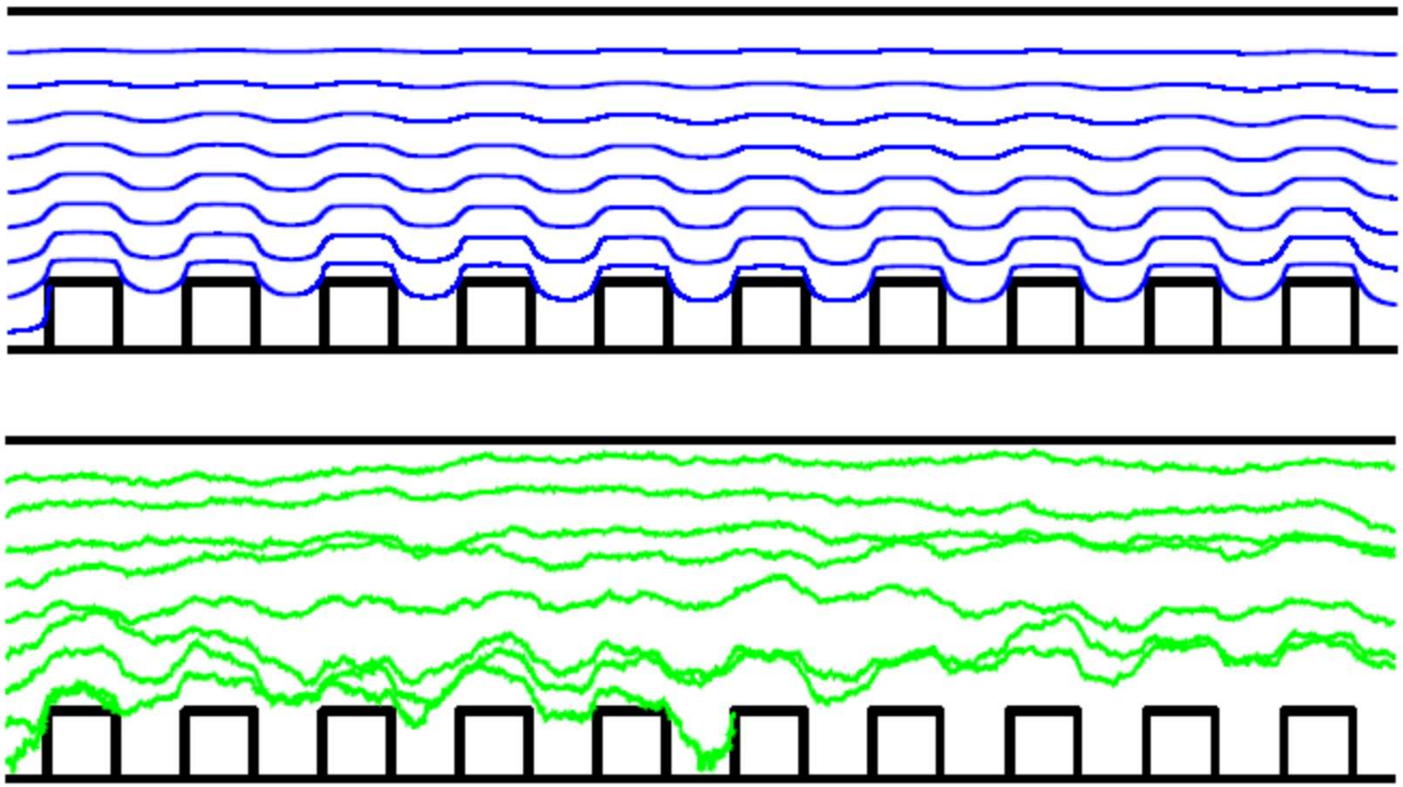
When is this force important?

Pressure gradient force

"For a small sphere, small compared to the scale of the spatial variations of the undisturbed flow, the effect of the undisturbed fluid stresses both from pressure and viscosity is to produce the same net force as would act on a fluid sphere of the same size. This force must equal the product of the fluid mass and local fluid acceleration."

$$F_P = m_P \frac{\rho_f}{\rho_P}\left(\frac{D\,u_{\mathbf{f}}}{D\,t} - g\right)$$

Force due to Brownian motion
(sub-micron particles*)



*: Mind whether particles are in gas or liquid

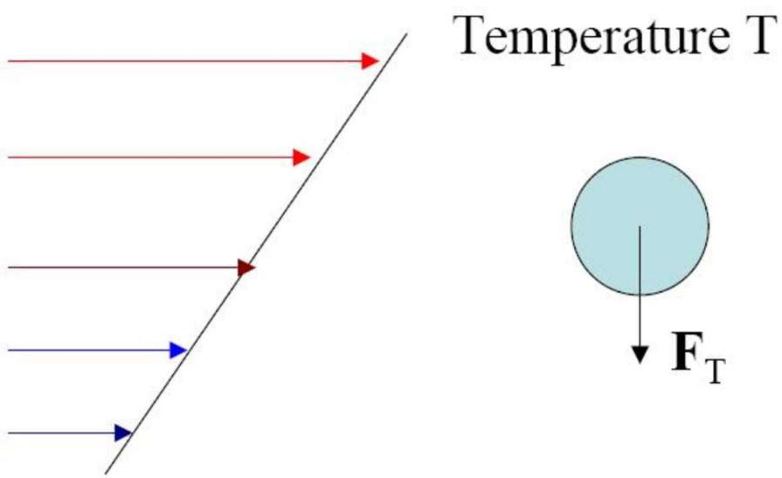
Force due to Brownian motion

$$F_{Brownian} = \xi m_p \sqrt{\frac{216\mu k_B T}{\pi d_p{}^5 \rho_p{}^2 C_c \Delta t}}$$

ξ - random number
k_B – Boltzmann constant
T – temperature
C_c – Cunningham correction

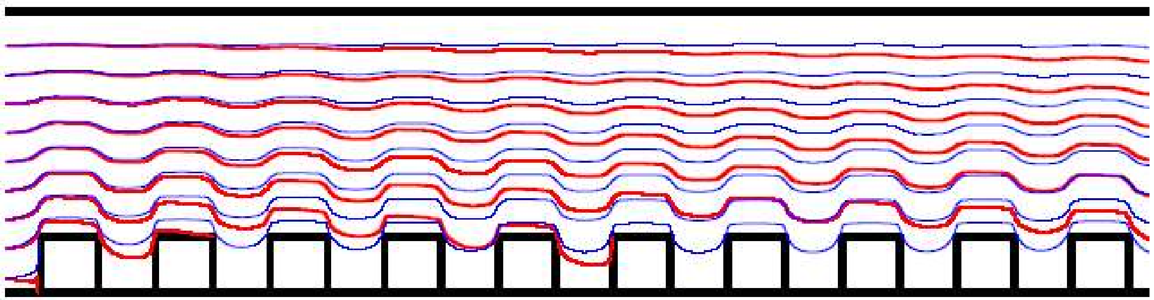
- Can be modelled as a white-noise processes using random numbers
- Transforms Newton's second law for the particle from an ODE to a stochastic differential equation
- Requires very short time steps and many particles

Thermophoretic force

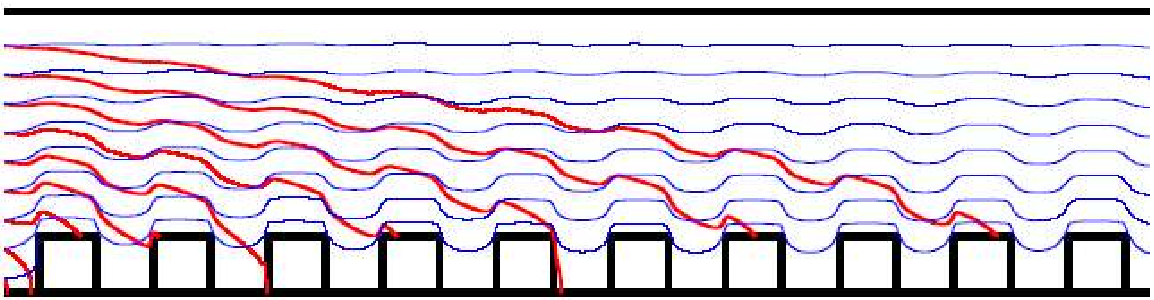


$$F_T = -\frac{6\pi d_p \mu^2 C_s (K + C_t \mathrm{Kn})}{\rho(1+3C_m \mathrm{Kn})(1+2K+2C_t \mathrm{Kn})}\frac{1}{T}\frac{\partial T}{\partial x}$$

Talbot et al., 1980



$$\frac{dT}{dh} = 1 \;\; K/mm$$



$$\frac{dT}{dh} = 10 \;\; K/mm$$

Some relevant questions: how good/general is our equation of motion?

Multiple particles: What is the effect of the neighbouring particles?

What is the effect of the carrier phase flow field (e.g. turbulence)?

What about particle collisions and their effect on the interaction force(s)?

Fundamental question: what happens for $Re_p > 1$ and $Re_p >> 1$

1. The same forces as recognized before, but with coefficients introduced

2. Drag coefficient (C_D), Lift coefficient (C_L), Added Mass coefficients (C_{AM}), History force coefficient (C_H)

3. For the drag force – successful

4. Other forces with varying success

Generalization of the forces (drag)

$$F_D = \frac{1}{2}\rho_f \frac{d_p^2 \pi}{4} C_D |u_f - u_p| (u_f - u_p)$$

Comparison with the Stokes drag

$$C_D = \frac{24}{Re_P}$$

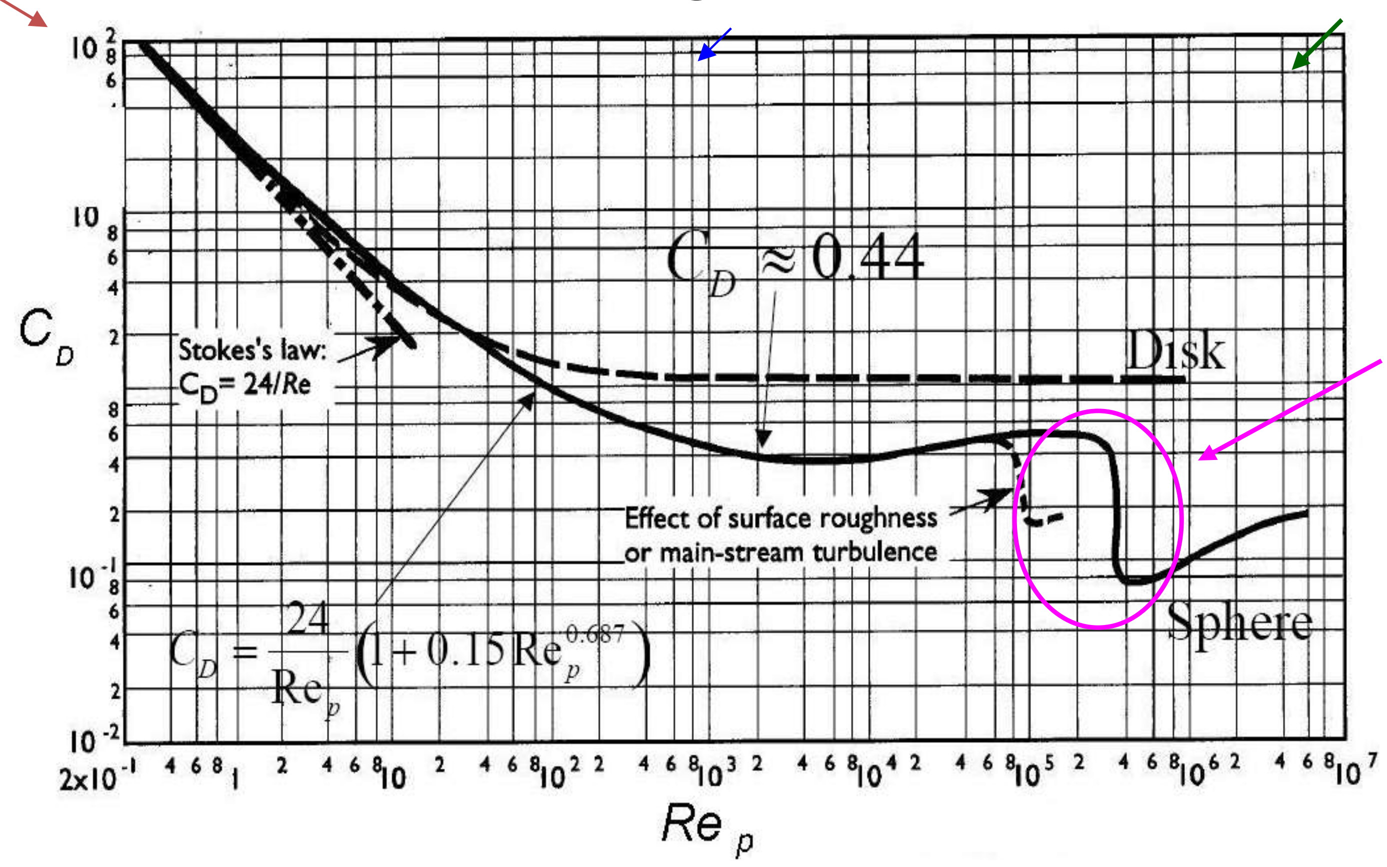
$$Re_P = \frac{\rho_f \, (u_\mathrm{f} - u_P)\, d_\mathrm{p}}{\mu_f}$$

Drag force: friction + form drag

Stokes regime

Transition region

Newton regime



Transition to a turbulent boundary layer (drag crisis)

Particle response time - derivation

We start with $$m_P \frac{du_P}{dt} = \frac{1}{2}\rho_f \frac{d_p^2 \pi}{4} C_D \left|u_f - u_p\right|\left(u_f - u_p\right)$$

$$\frac{du_P}{dt} = \frac{18\mu_f}{\rho_p d_p^2}\frac{C_D \operatorname{Re}_p}{24}\left(u_f - u_p\right) \quad \Longrightarrow \quad \frac{du_P}{dt} = \frac{\left(u_f - u_p\right)}{\tau_p}$$

Stokes response time and correction

$$u_P = u_f\left(1 - e^{-\left(t/\tau_p\right)}\right)$$