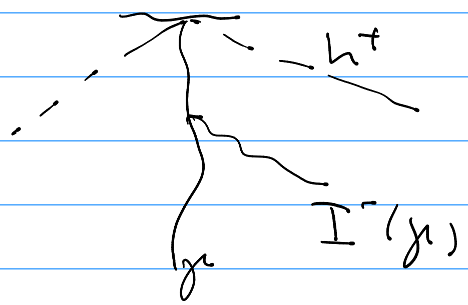
Gravity @ ICTP

Wald

- observer in (M, g_{ab}) is an inextendible timelike curve γ
- let $I^-(\gamma)$ be its chron. past.
- the future horizon is $\partial I^-(\gamma) =: h^+$



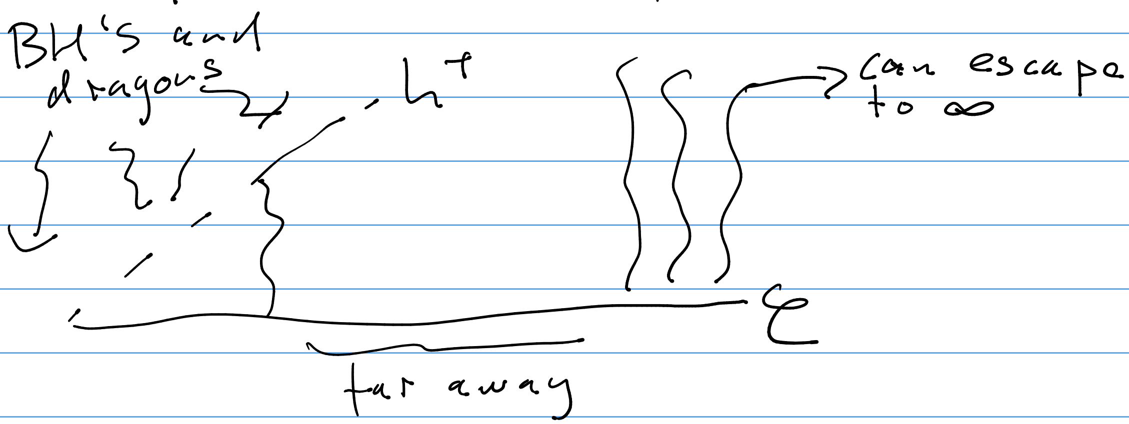
Thm. h^+ is a null sfc with inextendible null geodesics.

- now consider asymptotically flat (M, g_{ab})
- consider family of observers Γ who escape to arb. large distances at late time
  - if $I^-(\Gamma)$ has a horizon, $B := M - \overline{I^-(\Gamma)}$ is a black hole

Def Sfc $\mathcal{C} \subset M$ (M time-orientable) is *Cauchy* if every inextendible timelike γ intersects $\mathcal{C}$ in 1 pt. M is globally hyperbolic if $\exists$ Cauchy sfc $\mathcal{C} \subset M$.
(Its topology is clearly $\mathcal{C} \times \mathbb{R}$)
An asympt. flat (M, g_{ab}) w a BH is said to be *predictable* if $\exists$ region of M containing the entire exterior region and

h^+ which is globally hyperbolic. (no "naked singularities").

Hypothesis (cosmic censorship)
The maximal Cauchy evolution of an AF initial data set (+ suitable matter fields) yields generically an AF spacetime w/ complete null infinity.



- congruence of null-geodesics w/ affine param λ, null tgt k^a. Define expansion $\vartheta = \nabla_a k^a$
- the area A of (infinitesimal) area element transported along the congruence varies as

$$\frac{d \ln A}{d\lambda} = \vartheta$$

- if these generate a null-sfc (e.g. event horizon), $\omega_{ab} = 0$, so the Raychaudhuri eqn gives

$$\frac{d\vartheta}{d\lambda} = -\frac{1}{2}\vartheta^2 - \sigma_{ab}\sigma^{ab} - R_{ab}k^a k^b$$

where σ^{ab} is shear

(and Einstein's eqn)

- provided null-energy cond. holds,

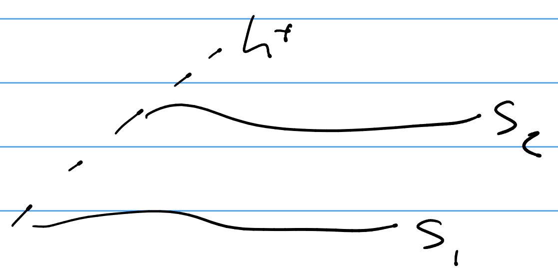
$$\frac{d\vartheta}{d\lambda} \le -\frac{1}{2}\vartheta^2$$

so $$\frac{1}{\vartheta(\lambda)} \le \frac{1}{\vartheta_0} + \frac{1}{2}\lambda$$

so if $\vartheta_0 < 0$, initial convergence, then $\vartheta(\lambda_1) = -\infty$ at $\lambda_1 < \frac{2}{|\vartheta_0|}$ <u>finite</u>

- any horizon h^+ is generated by future inext. geodesics, so we <u>cannot</u> have $\vartheta = -\infty$ anywhere on h^+
- if the generators are complete, we must have $\vartheta \ge 0$
  - this argument can be extended to predictable BH's w/o completeness, by deforming h^+ at pts with $\vartheta < 0$

Thm (Area) For predictable BH w/ $R_{ab}k^a k^b \ge 0$, the sfc. area A of h^+ never decreases with time.



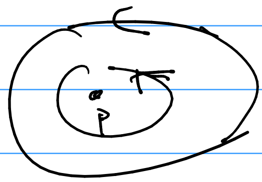
all generators of S_1 are present at S_2, and since $\frac{d \ln A}{d\lambda} = \underline{\vartheta \ge 0}$,

the area $A[h^+ \cap S_2] \ge A[h^+ \cap S_1]$

- recall: for a Killing field ξ^a, let $F_{ab} = \nabla_{[a}\xi_{b]}$. Then ξ^a is determined by values of F_{ab} at some $p \in M$.

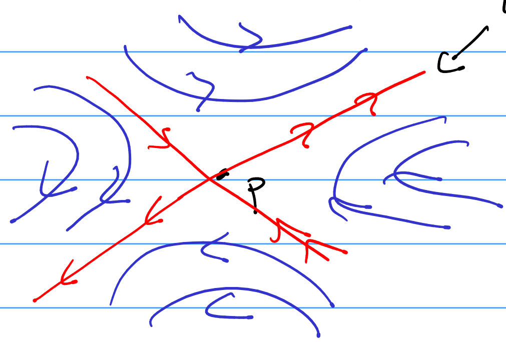
Bifurcate Killing horizons

- 2-dim: supp. $\xi^a|_p = 0$. In 2-dims, $F_{ab} \sim \varepsilon_{ab}$, so ξ^a unique up to scaling
  - in Riemannian case, orbits are



Rotations

  - in Lorentz case,



null Killing fields
→ Killing horizons

Boosts

- similar results hold for sfc Σ in 4-dim Lorentzian M, so we get bifurcate horizons h^A, h^B, for Σ spacelike

0th Law

- let h Killing horizon assoc. w Killing field ξ^a. Let U aff param. of generators of h with k^a tgt
- since ξ^a normal to h, we have by null-ness $\xi^a = f \cdot k^a$, $f = \frac{\partial U}{\partial u}$, where u Killing param along h null-gens.

- define sfc. gravity $\kappa = \xi^a \nabla_a \ln f$
$$= \frac{\partial \ln f}{\partial u}$$

- equivalently, $\xi^b \nabla_b \xi^a = \kappa \xi^a$

- integrate: $U = \exp \kappa u$

- In general, κ can vary on generators
- however:

Zeroth law (1st version) Let h be a (conn.) Killing horizon in (M, g_{ab}) with Einstein eqns and dominant energy matter. Then κ is constant on h
(3rd version) κ constant over bifurcate Killing horizon

- in fact, we may consider any Killing horizon to be a bifurcate one

Thm (Hawking Rigidity) Let (M, g_{ab}) stationary AF sol'n of Einstein eqns with appropriate matter with a BH. Then h^+ is a killing horizon.

- since we know h^+ has null geodesics we see $\exists$ combination of Killing fields which is null on h^+, e.g. $\chi^a = \varphi^a + \Omega t^a$