Kocuma

Batyrev MS & GLSM

- recall X toric if $(\mathbb{C}^*)^r \overset{\curvearrowright}{\hookrightarrow} X$,
  I fans, II polytopes, III functor

- $N_{\mathbb{R}} \supset \sigma$ cone, $M_{\mathbb{R}} \supset \check{\sigma}$ dual cone $= \{m \mid \langle m, v\rangle \geq 0\ \forall v \in \sigma\}$.

- $\mathcal{L}$ line bdl $= \mathcal{O}(D)$, $D = \sum_{\rho=1}^{N} a_\rho D_\rho$ rays in fan Σ
  ⇝ $\Delta_D := \{m \mid \langle m, v_\rho\rangle \geq -a_\rho\ \forall v_\rho \in \Sigma(1)\}$

- considering $\mathbb{C}^N$ with coords x_ρ for each v_ρ
  take $Q_{a\rho}$ as relations $\sum_\rho Q_{a\rho}\vec{v}_\rho = 0$
  and any $\sum m_\rho v_\rho = 0$ regard as lin. comb of such
- $(\mathbb{C}^*)^{N-r} \circlearrowright \mathbb{C}^N$ as $x_\rho \longmapsto \lambda_a^{Q_{a\rho}} x_\rho$ for $\{\lambda_a\}_{a=1}^{N-r} \in (\mathbb{C}^*)^{N-r}$

- $X_\Sigma = \mathbb{C}^N /\!/_{\Sigma} (\mathbb{C}^*)^{N-r} := (\mathbb{C}^N - Z(\Sigma))/(\mathbb{C}^*)^{N-r}$

  for every $v_{\rho_1}, \dots, v_{\rho_n}$ which do not span a cone
  let $S_{\rho_1 \dots \rho_n} := \{x_{\rho_1} = 0\} \cap \dots \cap \{x_{\rho_n} = 0\}$
  and $Z(\Sigma) := \bigcup_{\rho_1 \dots \rho_n} S_{\rho_1 \dots \rho_n}$.

- e.g. we had  $= \mathbb{C}^3 \setminus \{x_1 = 0, x_2 = 0, x_3 = 0\} / \mathbb{C}^*$

- GLSM = gauged lin. σ-model
  - LSM studies maps $\Sigma \to \mathbb{C}^N$
  - $G \circlearrowright \mathbb{C}^N$, but in toric we have $X = (\mathbb{C}^N - Z(\Sigma))/\overset{U(1)^{N-r}}{\overset{\cap}{(\mathbb{C}^*)^{N-r}}}$
  - GLSM has N chiral multiplets $X_\rho(\xi)$ + s partners
    $N - r$ vect. — " — $A_\mu^a(\xi)$ + s partners, belonging to $U(1)^{N-r}$

charge mtrx

$$-D_\mu X_s(\zeta) = \partial_\mu X_s(\zeta) + \sum \underset{\uparrow}{Q_{as}} A_\mu^a(\zeta) X_s(\zeta)$$

$$V(X_s) = \frac{1}{2e^2}\left(\sum_a Q_{as}|X_s(\zeta)|^2 - r_a\right)^2$$