Tanzini.

- TQFT $\rightsquigarrow$ theory of maps $\underset{\text{worldvolume}}{\Sigma_d} \xrightarrow{\varphi} \underset{\text{tgt. space}}{M}$
  - $d=0$ | path int $\leftrightarrow$ ordinary int over M.
  - $d=1$ | p.i $\leftrightarrow$ int. over loop space LM

    t

  - $d=2$ | Σ called world sheet; strings

- to get localization in $d=2$, we need 2d susy

Spinors

- let $V \cong \mathbb{C}^d$ equipped w/ sym. bilinear form g
- define $Spin(V)$ as the $\mathbb{Z}_2$-extension of $SO(V)$, i.e. $0 \to \mathbb{Z}_2 \to Spin(V) \to SO(V) \to 0$

Def. let S be cpx Dirac spinor module of $Spin(V)$, $\dim_{\mathbb{C}} S = 2^{[\frac{d}{2}]}$

- if d odd, S irred
- $S = S^+ \oplus S^-$, S^+, S^- Weyl modules (in $d=2$)

Def. Clifford algebra $\doteq T(V)/\langle v \cdot v - g(v,v) \rangle$

- pick $\{\gamma_\mu\}_{\mu=1\ldots d}$ $2^{[d/2]} \times 2^{[d/2]}$ matrices, provided we pick a basis for V, s.t. $\{\gamma_\mu, \gamma_\nu\} = 2 g_{\mu\nu} \mathbb{1}$ $\to$ module on $Cl(V)$
  - called Dirac matrices

$\mathbb{R}^{2|4}$

- ϑ_α, $\alpha = +,-$ i.e. $\vartheta_\alpha = (\vartheta_+, \vartheta_-)$.
- $\bar{\vartheta}_\alpha := (\vartheta_\alpha)^+$, $\bar{\vartheta}^\alpha := \varepsilon^{\alpha\beta} \bar{\vartheta}_\beta$, $\varepsilon^{+-} = 1$

- $(\gamma^1)_\alpha{}^\beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $(\gamma^2)_\alpha{}^\beta = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\{\gamma^m, \gamma^n\} = 2\delta^{mn}$

- Lorentz: $\vartheta \mapsto \exp\left[\frac{i}{2}\omega^{mn} S_{mn}\right]\vartheta$, $S_{mn} = \frac{i}{4}[\gamma_m, \gamma_n]$

- explicitly, $\vartheta^\pm \mapsto e^{\pm i\frac{\omega}{2}}\vartheta^\pm$, $\bar{\vartheta}^\pm \mapsto e^{\mp i\frac{\omega}{2}}\bar{\vartheta}^\pm$
  for $\omega_{mn} = \begin{pmatrix} 0 & \omega \\ -\omega & 0 \end{pmatrix}$

- superfields: $\varphi(x^m, \vartheta^\pm, \bar{\vartheta}^\pm) = f(x^m) + \vartheta^+ f_+(x^m)$
  $+ \bar{\vartheta}^+ g_+(x^m) + \dots + \vartheta^+\vartheta^-\bar{\vartheta}^+\bar{\vartheta}^- F(x^m)$
  - 2^4 components

- supercharges $\{Q_\alpha, \overline{Q_\beta}\} = 2i\gamma^m{}_{\alpha\beta}\partial_m$
  - realisation: $\begin{cases} Q_\alpha = \frac{\partial}{\partial\vartheta^\alpha} - i\gamma^m{}_{\alpha\beta}\bar{\vartheta}^\beta\partial_m \\ \bar{Q}_\alpha = -\frac{\partial}{\partial\bar{\vartheta}^\alpha} + i\vartheta^\beta\gamma^m{}_{\beta\alpha}\partial_m \end{cases}$

- Weyl modules $\begin{cases} Q_\pm = \frac{\partial}{\partial\vartheta^\pm} + \bar{\vartheta}^\pm\partial_\pm \\ \bar{Q}_\pm = -\frac{\partial}{\partial\bar{\vartheta}^\pm} - \vartheta^\pm\partial_\pm \end{cases}$, $\partial_\pm := \frac{1}{2}(\partial_2 \pm i\partial_1)$ Dolbeault on $\mathbb{R}^2 \cong \mathbb{C}$

  $\{Q_\pm, \bar{Q}_\pm\} = -2\partial_\pm = H \pm P$

  - in $d=1$ we only had $\{Q, \bar{Q}\} \sim H$, since there was no Lorentz group, now $P^m = (H, P)$

- SUSY transformation:

$$\delta\varphi(x^m, \vartheta, \bar{\vartheta}) = \underbrace{\left[\xi_\alpha Q^\alpha + \bar{\xi}^\alpha \bar{Q}_\alpha\right]}_{\xi_+ Q_- - \xi_- Q_+ - \bar{\xi}_+\bar{Q}_- + \bar{\xi}_-\bar{Q}_+}\varphi$$

$\mathcal{N}=(2,2)$ superalgebra

$$\{Q_\pm, \bar{Q}_\pm\} = 2\partial_\pm$$

$$\{\bar{Q}_+, \bar{Q}_-\} = Z \quad , \quad \{Q_+, Q_-\} = Z^*$$
$$\{Q_-, \bar{Q}_+\} = \tilde{Z} \qquad \{Q_+, \bar{Q}_-\} = \tilde{Z}^*$$
$$Q_\pm^2 = \bar{Q}_\pm^2 = 0$$
$$[iM, \overset{\leftrightarrow}{Q}_\pm] = \mp i \overset{\leftrightarrow}{Q}_\pm$$

$$[iF_V, Q_\pm] = -iQ_\pm \quad , \quad [iF_V, \bar{Q}_\pm] = i\bar{Q}_\pm$$
$$[iF_A, Q_\pm] = \mp i Q_\pm \quad , \quad [iF_A, \bar{Q}_\pm] = \pm i \bar{Q}_\pm$$

- vector sym. $\varphi(x^m, \vartheta, \bar{\vartheta}) \mapsto e^{i\alpha q_V}\varphi(x^m, e^{-i\alpha}\vartheta, e^{i\alpha}\bar{\vartheta})$
- axial sym. $\varphi(x^m, \vartheta, \bar{\vartheta}) \mapsto e^{i\alpha q_A}\varphi(x^m, e^{\mp i\beta}\vartheta^\pm, e^{\pm i\beta}\bar{\vartheta}^\pm)$

- superfields are reducible reps.

IRREPS of $\mathcal{N}=(2,2)$

- superspace derivatives $\begin{cases} D_\alpha = \dfrac{\partial}{\partial\vartheta^\alpha} + i\gamma^m_{\alpha\beta}\bar{\vartheta}^\beta\partial_m \\ \bar{D}_\alpha = -\dfrac{\partial}{\partial\bar{\vartheta}^\alpha} - i\vartheta^\beta\gamma^m_{\beta\alpha}\partial_m \end{cases}$

$$\Longrightarrow \begin{cases} D_\pm = \dfrac{\partial}{\partial\vartheta^\pm} - \bar{\vartheta}^\pm\partial_\pm \\ \bar{D}_\pm = -\dfrac{\partial}{\partial\bar{\vartheta}^\pm} + \vartheta^\pm\partial_\pm \end{cases}$$

- chiral superfields $\bar{D}_\pm \varphi = 0$

- can be written as $\varphi(y^\pm, \vartheta^\pm)$ where
  $y^\pm := x^\pm - \vartheta^\pm\bar{\vartheta}^\pm$
- $\bar{D}_\pm y^\pm = \dfrac{\partial}{\partial\bar{\vartheta}^\pm}(\vartheta^\pm\bar{\vartheta}^\pm) + \vartheta^\pm\partial_\pm x^\pm = 0 \quad , \quad \bar{D}_\pm\vartheta^\pm = 0$

$$\varphi(y^\pm, \vartheta^\pm) = \varphi(y^\pm) + \vartheta^+\psi_+ + \vartheta^-\psi_- + \vartheta^+\vartheta^- F(y^\pm)$$

- twisted chiral superfields:

$$\overline{D}_+ U = D_- U = 0$$

- can be written as $U(\tilde{y}^\pm, \vartheta^+, \overline{\vartheta}^-)$
  where $\tilde{y}^\pm := x^\pm \mp \vartheta^\pm \overline{\vartheta}^\pm$

- chiral and twisted chiral superfields get exchanged under mirror sym.!
- and axial $\longleftrightarrow$ vector sym.