Шихомиров

- $P_+F : \mathcal{A}_2 \longrightarrow \Omega^2_+(\mathrm{ad}\,P)_{L^2_1}$, $A \mapsto P_+F_A$
- $(P_+F)_{*,A} : T_A\mathcal{A}_2 \to \Omega^2_+(\mathrm{ad}\,P)_{L^2_1}$

$$T_A\mathcal{A}_2 = \Omega^1(\mathrm{ad}\,P)_{L^2_2}$$

- $F_{A+a} = F_A + \nabla_A a + [a,a]$

  $\longrightarrow \ker (P_+F)_{*,A} = \mathrm{Im}\,\nabla_A$
- P_+A not Fredholm
- $\mathcal{U} \subset \ker \nabla_A^*$ slice, $\Omega^1(\mathrm{ad}\,P)_{L^2_2} = \mathrm{Im}\,\nabla_A \overset{\perp}{\oplus} \ker \nabla_A^*$
  - A – irred $\Rightarrow \mathcal{U} \xhookrightarrow{\text{open}} \mathcal{B}_2(P) = \mathcal{A}_2/\mathcal{G}_3$

    red. $\Rightarrow \mathcal{U}/S^1$

    - for V, $\dim V = m_1 + 2m_2$,

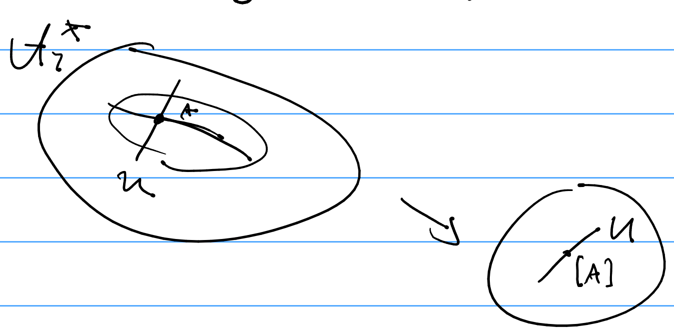
      $V/S^1 \simeq \mathbb{R}^{m_1} \oplus \mathbb{R}^{2m_2}/S^1$

- $\ker (P_+F)_{*,A}\big|_{\ker \nabla_A^*} = \overbrace{\ker \underset{\text{Fredholm}}{D_A}}^{\dim < \infty} \overset{A \text{ is ASD}}{=} H^1(\mathcal{E}^\bullet(A))$

$\widetilde{\mathcal{M}}_2^*(P) := \{A \in \mathcal{A}_2^{\overset{\text{irred}}{*}} \mid A \text{ is ASD}\}$

$\hookrightarrow \mathcal{A}_2^* \xhookrightarrow[\text{dense}]{\text{open}} \mathcal{A}_2$

- assuming $(P_+F)_{*,A}$ surj $\Rightarrow D_A$ surj.



- notice $\widetilde{\mathcal{M}}^* = (P_+F)^{-1}(0)$
- associate to P_+F

$$\begin{array}{c} \mathcal{A}_2^* \times \Omega^2_+(\mathrm{ad}\,P) \\ s \uparrow \downarrow \\ \mathcal{A}_2^* \end{array}$$

- $s(A) = (A, P_+ F_A)$, so $\tilde{\mathcal{M}}^* = s^{-1}(0)$

- action $\mathcal{A}_2^{*} \times \mathcal{G}_3 \to \Omega_+^2(\mathrm{ad}P)_{(2)}$, $(A, g) \mapsto P_+ F_{g^*A}$

$$\begin{tikzcd} E \times \mathcal{G} \arrow[r] \arrow[d, "p \times id"'] & E \arrow[d] \\ X \times \mathcal{G} \arrow[r] & X \end{tikzcd}$$

$$\begin{tikzcd} \hat{\mathcal{E}} = \mathcal{E} = p^*\mathcal{E} \arrow[d] & \mathcal{E} \arrow[d] \\ \mathcal{A}_2^* = X \arrow[r, "p"] & X/\mathcal{G} \end{tikzcd}$$

- does it descend?

- $\mathcal{E} := $ "$\hat{\mathcal{E}}/\mathcal{G}_3$" is vbdl on $\mathcal{B}_2^*$, s a section of $\mathcal{E} \to \mathcal{B}_2^*$

$\mathcal{M}^* = \tilde{\mathcal{M}}/\mathcal{G} = s^{-1}(0)$

$\phi: \mathcal{B}_2^* \xrightarrow{s} \mathcal{E}$, $[A] \mapsto D_A$

$(\phi_*)_{[A]} : \ker D_A^* \to \Omega^2_{+,A}$

$\quad \| $

$D_A|_{\ker D_A^*}$ $\Rightarrow$ A is ASD $\Leftrightarrow$ $[A] \in \phi^{-1}(0) = \mathcal{M}^*$

so $\dim \mathcal{M}^* = \dim \ker D_A|_{\ker D_A^*}$

Thm Let $\phi : \mathcal{H}_1 \supseteq \mathcal{U} \to \mathcal{H}_2 \supseteq \mathcal{V}$ be Fredholm map of Hilbert mflds, $x \in \mathcal{H}_1$ and $\phi_* : T_{x_0}\mathcal{H}_1 \to T_{\phi(x_0)}\mathcal{H}_2$ Fredholm. Then around x (resp. y), $\mathcal{U} = \mathcal{U}_1 \times \mathcal{U}_2$, $\mathcal{V} = \mathcal{V}_1 \times \mathcal{V}_2$ s.t. $\phi(x_1, x_2) = (\phi_1(x_1), \phi_2(x_1, x_2))$ where $\phi_1 : \mathcal{U}_1 \xrightarrow{\sim} \mathcal{V}_1$ is isom., $\phi_2 : \mathcal{U}_1 \times \mathcal{U}_2 \to \mathcal{V}_2$ and $\phi^{-1}(y_0) = \phi_2^{-1}(y_2^0) \cap \{x_1^0\} \times \mathcal{U}_2$, where $y_0 = (y_1^0, y_2^0)$, $x_0 = (x_1^0, x_2^0)$

and $\mathcal{U}_2, \mathcal{V}_2$ are <u>finite</u> dimensional.

$$\begin{tikzcd}
T_{x_1^0}\mathcal{U}_1 \arrow[d, "S\,\phi_1"] \arrow[drr, "\phi_2|_{T_{x_1^0}\mathcal{U}_1}"] & \oplus & T_{x_2^0}\mathcal{U}_2 \arrow[d, "\phi_2|_{T_{x_2^0}\mathcal{U}_2}"] \arrow[d, phantom, shift left=20ex, "\Big\downarrow (\phi_*)_{x_0}"] \\
T_{y_1^0}\mathcal{V}_1 & \oplus & T_{y_2^0}\mathcal{V}_2
\end{tikzcd}$$

- assume $(\phi_*)_{x_0}$ surjects $\Rightarrow \mathcal{V}_2 = \{0\}$