Gauge ⓖ IG&P

Geometric R-matrices for Hilb(S) - N. Arbesfeld

- V: vsp / K, $R(u) \in GL(V \otimes V, K(u))$
- for $V(u_i) := V \otimes K(u_i)$,
  $R(u_1 - u_2) \curvearrowright V(u_1) \otimes V(u_2)$
  and satisfies YBE:

• extending to $R_{12}(u_1-u_2) \curvearrowright \bigotimes_{i=1,2,3} V(u_i)$

$$R_{12}(u_1-u_2) = R(u_1-u_2) \otimes id_{V(u_3)}$$

$$\Rightarrow R_{23}(u_2-u_3) R_{13}(u_1-u_3) R_{12}(u_1-u_2)$$
$$= R_{12}(u_1-u_2) R_{13}(u_1-u_3) R_{23}(u_2-u_3)$$

as elements of $End(\bigotimes V(u_i))$

e.g. $K = \mathbb{C}(\kappa)$, $R(u) = 1 - \frac{\kappa}{u}(1\,2) \curvearrowright \mathbb{C}^2 \otimes \mathbb{C}^2$

$$\{R\text{-matrix}\} \xRightarrow[\text{scattering}]{\text{quantum inv.}} \text{Hopf algs, Yangian } Y \curvearrowright \bigotimes_{i=1}^{n} V(u_i)$$

- generators of Y: coeffs in (comp's of) $R(u)$

- if $\varphi \in End(V)$, $R(\infty) = 1$

$$T_{V,\varphi}(u) = tr_V\left[(\varphi \otimes 1 \otimes \cdots \otimes 1) R_{0n}(u-u_n) \circ \cdots \circ R_{01}(u-u_1)\right]$$

↪ coeffs in $\frac{1}{u}$ expansion

$RTT = TTR$ from YBE

- families of comm subalgebras of Y
  → "Baxter subalg's"
  - given $\varphi \in \mathrm{End}\, V$ s.t. $[\varphi \otimes \varphi, R(u)] = 0$
    then $[T_{V,\varphi}(u), T_{V,\varphi}(u')] = 0$

- Maulik-Okounkov produced R-matrices from Nakajima quiver varieties

e.g. $X =$  $\rightsquigarrow T^* Gr(k,n)$

$n = 1 \to H^*(pt)$

$V = H^*(T^* Gr(0,1) \sqcup T^* Gr(1,1))$

$\cong$

$H^*_{GL(n)}\left(\bigsqcup_{k\geq 0}^{n} T^* Gr(k,n) \right)$

$X =$ framed ADHM $\mathcal{M}(n,r)$

$\mathcal{M}(n,1) = Hilb_n(\mathbb{C}^2)$

$\underbrace{T = (\mathbb{C}^*)^2}_{(t_1 \ t_2)} \circlearrowright \mathcal{M}(n,r)$

$V = H^*_T\left(\bigsqcup_{n\geq 0}^{\infty} Hilb_n(\mathbb{C}^2) \right)$

$\overset{\to}{\bigotimes_{i=1}^{r}} V(u_i) = H^*_{T \times GL(r)}\left(\bigsqcup_{n\geq 0}^{\infty} \mathcal{M}(n,r) \right)$