# QFT.

## Vacuum decay.

$$i S_Q(A) = -\operatorname{tr}\log\frac{(-D_E^2+m^2)}{\mu^2} + \ldots$$
$$= \int_0^\infty \frac{d\beta}{\beta}\operatorname{Tr} e^{-\beta(-D_E^2)}\, e^{\beta m^2} + \ldots$$

$\Longrightarrow Z(\beta) = \operatorname{tr} e^{-\beta H}$, $H = -D_E^2$

- problem mapped to $L = \frac{1}{2}\left(\frac{dq_\mu}{d\tau}\right)^2 - ie\frac{dq_\mu}{d\tau}A_E^\mu(q)$, $A_E^\mu(q) = \frac{1}{2}F_E^{\mu\nu}q_\nu$
- as we did before $q_\mu = $ (Fourier series), due to $\operatorname{tr} e^{-\beta H} = \int_{\text{periodic}} Dq(\tau)\, e^{-\int_0^\beta d\tau L(q;\tau)}$

- $$\int_0^\beta d\tau\, L_E(q(\tau)) = \sum_{n=1}^\infty\left[\frac{m}{2}\left(\frac{2\pi n}{\beta}\right)^2\left(c_{\mu,n}^2 + s_{\mu,n}^2\right) - ie\left(\frac{2\pi n}{\beta}\right)^2 s_{\mu,n}F_{\mu\nu}c_{\nu,n}\right]$$
- $\int dq_{\mu,0} = V_4 = iVT$

$$\to \operatorname{tr} e^{-\beta H} = \int dq_{\mu,0}\prod_{n=1}^\infty dc_{\mu,n}\, ds_{\mu,n}\; e^{-S_E(q;\beta)}$$
$$= iVT\int\prod_{n=1}^\infty dc_{\mu,n}\exp\left[-\sum_{n=1}^\infty\left(\frac{\pi n}{\beta}\right)^2 c_{\mu,n}M_{\mu\nu}c_{\nu,n}\right]\mathcal{N}'$$

$M_{n,\mu\nu} = \delta_{\mu\nu} - \left(\frac{e\beta}{\pi n}\right)^2 F_{E,\mu\sigma}F_{E,\sigma\nu}$

$$= iVT\,\underbrace{\mathcal{N}''}_{(4\pi\beta)^{-2}}\prod_{n=1}^\infty\left(\det M_{n,\mu\nu}\right)^{-1/2}$$

$\to A_4 = -iA_0$, $F_{E,i4} = -iF_{i0} = E_i$, $F_{E,ij} = \varepsilon_{ijk}B$

$\to$ set $B_z = E_y = E_z = 0$

- $F_{\mu\nu}^2$ will decompose in 2 blocks, same determinant

$$\Rightarrow \lambda_\pm = \frac{1}{2}\left(\vec{E}^2 - \vec{B}^2 \pm \sqrt{(\vec{B}^2-\vec{E}^2)^2 + 4(\vec{B}\cdot\vec{E})^2}\right) =: a_\pm^2$$
$$\Rightarrow \det M_{\mu\nu,n} = \left(1 + \left(\frac{e\beta}{\pi n}\right)^2 a_+^2\right)^2\left(1 - \left(\frac{e\beta}{\pi n}\right)^2 a_-^2\right)^2$$

$$\to Z(\beta) = \frac{iVT}{(4\pi\beta)^2}\,\frac{e\beta a_+}{\sin(e\beta a_+)}\,\frac{e\beta a_-}{\operatorname{sh}(e\beta a_-)}$$

$$i S_Q(A) = \int_0^\infty \frac{d\beta}{\beta} e^{-\beta m^2} Z(\beta)$$

- look at the limit $\beta \to 0$ : $Z(\beta) = \frac{iVT}{16\pi^2\beta^2}\left(1 + \frac{e^2\beta^2}{6}(\underbrace{a_+^2 - a_-^2}_{\lambda_+ + \lambda_- = \vec{E}^2 - \vec{B}^2 = \frac{1}{2}F_{\mu\nu}F^{\mu\nu}})^2 + O(\beta^2)\right)$

↳ since we integrated out a scalar, we have vestiges of ~~~~

→ suppose we did the necessary subtractions, ...

$$i S_Q(A) = \frac{iVT}{16\pi^2}\int_0^\infty dv \frac{e^2 a_+ a_-}{\sin\frac{ea_+v}{m^2}\, \mathrm{sh}\frac{ea_-v}{m^2}}$$

→ pole at $v = 0$ independent of gauge field, we can ignore it

→ simple poles of sine? but remember the $\underline{+i\varepsilon}$!

→ recover it by shifting $m^2 \mapsto m^2 - i\varepsilon$

- remembering $\frac{1}{x - i\varepsilon} = P\frac{1}{x} + i\pi\delta(x)$,

put $\frac{1}{\sin(x+i\varepsilon)} = P(\ldots) - i\pi(-)^n\delta(x - \pi n)$, $n \in \mathbb{Z}_+$

$$\mathrm{Im}\, S_Q(A) = \frac{VT}{16\pi^2} e^2 a_+ a_- \sum_{n=1}^\infty \frac{(-)^{n+1}}{\mathrm{sh}\left(\frac{a_-}{a_+}n\pi\right)} e^{-\frac{\pi n m^2}{ea_+}} \cdot \frac{1}{n}$$

- $\vec{E} = 0 \Rightarrow a_+^2 = 0,\ a_-^2 = \vec{B}^2 \Rightarrow \mathrm{Im}\, S_Q(A)$

- $\vec{B} = 0 \Rightarrow a_+^2 = \vec{E}^2,\ a_-^2 = 0 \Rightarrow \mathrm{Im}\, S_Q(A) = \frac{VT}{16\pi^2} e^2 \vec{E}^2 \sum_{n=1}^\infty \frac{(-)^{n+1}}{\pi n^2} e^{-\frac{\pi n m^2}{e|E|}}$

$$\Gamma = \frac{e^2|E|^2}{8\pi^3}\sum_{n=1}^\infty \frac{(-)^{n+1}}{n^2} e^{-\frac{\pi n m^2}{e|E|}}$$