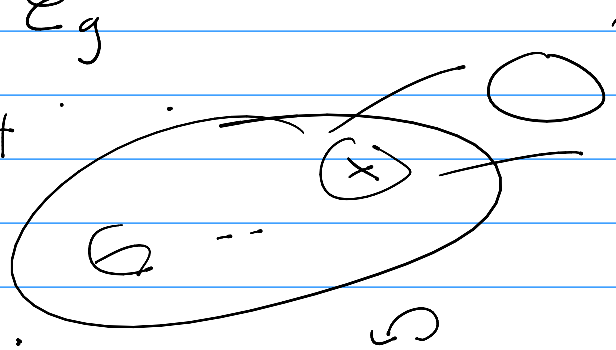
Nadir

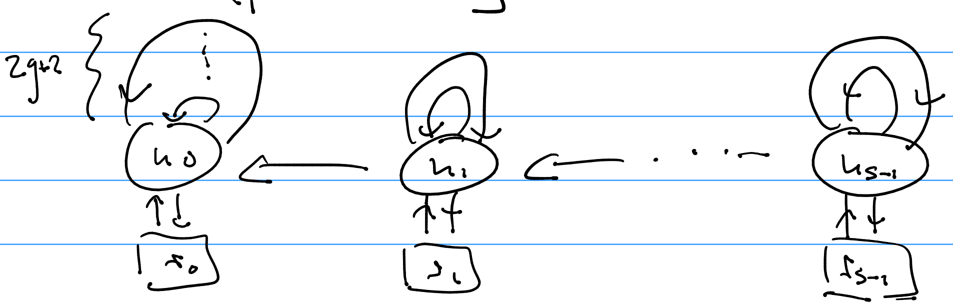
- D3/D7 geom. engineering
- m D3: eff. th. is twisted $d=4$ SYM
- $S = T^2 \times \mathcal{C}_g$
- defect at pt



- described by

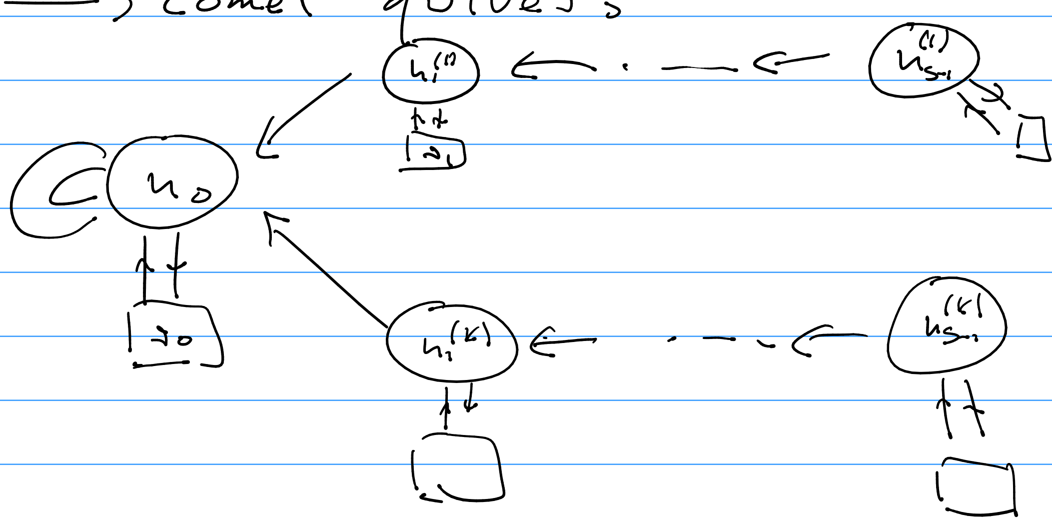
$$n_0 \geq \cdots \geq n_{s-1}$$

and effectively



- now add more defects ($\mathcal{C}_{g,k}$)

⟶ comet quivers:



- study Rep(Q) of

$$\begin{tikzcd} Q: V_{s-1} \arrow[loop above, "B_1^{s-1}"] \arrow[loop below, "B_2^{s-1}"] \arrow[r, shift left, "F_{s-1}"] & \cdots \arrow[l, shift left, "G_{s-1}"] \arrow[r, shift left, "F_2"] & V_1 \arrow[l, shift left, "G_2"] \arrow[loop above, "B_1^1"] \arrow[loop below, "B_2^1"] \arrow[r, shift left, "F_1"] & V_0 \arrow[l, shift left, "G_1"] \arrow[loop above, "B_1^0"] \arrow[loop below, "B_2^0"] \arrow[r, shift left, "J"] & W \arrow[l, shift left, "I"] \end{tikzcd}$$

$r_i = 0 \ \forall i > 0$, $r_0 = r$

we get relations

$$[B_1^0, B_2^0] + IJ = 0$$
$$[B_1^i, B_2^i] = 0, \ i > 0$$
$$B^{i-1}_{1,2} F_i - F_i B^i_{1,2} = 0, \ \forall i$$
$$G_i B^{i-1}_{1,2} - B^i_{1,2} G_i = 0, \ \forall i$$
$$F_i G_i = 0$$
$$J F_1 = 0, \ G_1 I = 0$$

- notions of stability coalesce,

$$\mathcal{M}(r,\underline{n}) = \mathrm{Proj}\Big(\bigoplus_{m \geq 0} [A(\mathbb{X}_0(r,\underline{n}))^{\chi^m}, \xi] \Big)$$

... $\exists$ explicit formulae

- getting back to $g=0$, 1 defect/puncture

$$\boxed{1} \underset{\tau}{\rightleftarrows} u_0 \leftarrow \cdots \leftarrow u_{s-1}$$

$$Z_1^{ell}(S^2; q_0, \ldots, q_{s-1})$$

$$= \sum_{\mu_1 \le \cdots \le \mu_{s-1} \le \mu_0} q_0^{|\mu_0|} q_1^{|\mu_0 - \mu_1|} \cdots q_{s-1}^{|\mu_0 - \mu_{s-1}|} Z^{ell}_{(\mu_0, \ldots, \mu_{s-1})}$$

where

$$Z^{ell}_{(\mu_0, \ldots, \mu_{s-1})} = \mathcal{L}^{ell}_{\mu_0} N^{ell}_{\mu_0} \overline{N}^{ell}_{\mu_0} T^{ell}_{\mu_0 \mu_1} \overline{T}^{ell}_{\mu_0 \mu_1} W_{(\mu_0, \ldots, \mu_{s-1})}$$

$$N^{ell}_{\mu_0} = \prod_{s \in Y_{\mu_0}} \left[\vartheta_1(\tau | E(s)) \, \vartheta_1(\tau | E(s) - \epsilon) \right]^{-1}$$

$$\epsilon = \epsilon_1 + \epsilon_2, \quad E(s) = - \cdots -$$