Dąbrowski

Reconstruction

- D encodes $\begin{cases}\to \text{top.}\\ \to \text{geom.}\end{cases}$, generates K-homology
  $ds = (D + i\varepsilon)^{-1}$
- claim: S.T. + 7 axioms encodes all geom. info of M
  - assuming irreducibility: $\not\exists$ p proj. $\in \mathcal{Z}(A)$ commuting w/ D, J, χ
    - this is slightly stronger than needed
  - was first shown for $A = \mathcal{C}^\infty(M)$ [Connes, '95] where M closed.

Thm (Reconstruction I, for $A = \mathcal{C}^\infty(M)$)

a) $\exists!$ Riemannian metric g on M s.t. the geodesic distance is $d_g(x,y) = d_D(x,y) := \sup\left\{ |a(x) - a(y)| \;\middle|\; a \in A, \|[D,a]\| \le 1 \right\}$

b) g only depends on $[D] = \{ UDU^+ \mid \underset{U \text{ comm. w/ } A, J, \chi}{U \in \mathcal{U}(\mathcal{H})} \}$, which form a fin. coll'n of affine sp.s Ω_σ, labelled by spin str'cts σ on M.

c) the action functional

$$S : D \mapsto W\mathrm{res}(|D|^{2-n}) := \frac{1}{n(2\pi)^n} \int_{S^*M} \mathrm{tr}\, \sigma^{-n}_{|D|^{2-n}}(x,\xi)\, d\xi\, dx$$

is a pos. quad. form on each Ω_σ, with a unique min. attained at the canonical S.T. of (M,g), and its value is equal to $\int_M R \,\mathrm{vol}_g$