Tanzini

A model

- $S_A = \int_{\Sigma_g} \varphi^*(\omega + i B) + \{Q_A, V\}$
- fixed locus of $Q_A \rightsquigarrow \varphi^i, \varphi^{\bar{i}} \in \mathcal{H}ol(M)$
  $\psi^i, \psi^{\bar{i}} \in TM$, $\psi_{\bar{z}}^{i}, \psi_z^{\bar{i}} \in obs$
- vd $\mathcal{M} = (\dim_{\mathbb{C}} M - 3)(1-g) + \int_{\Sigma_g} \varphi^*(c_1(TM)) =: K > 0$
- generic case:
  $\#\ \bar{\psi}$ zero modes $= 0$ i.e. $\ker D_{\bar{z}}^{+} = \{0\}$

- $\dim_{\mathbb{C}} \mathcal{M}_{\Sigma}(M, \beta) = K$

- $O_i(x_i)$ are pullbacks of $\omega_i \in H^\bullet(M)$ via evaluation maps on distinguished pts $\{p_i\}$, $ev_i : \mathcal{M}_{\Sigma_g}(M,\beta) \longrightarrow M$
$$\varphi \longmapsto \varphi(p_i)$$

and the ω_i are Poincaré duals to cycles $D_i \hookrightarrow M$

- alles zusammen gibt es

$$\langle O_1(p_1) \cdots O_n(p_n)\rangle_A = \sum_\beta q^\beta \underbrace{\int_{M_{\Sigma_g}(M,\beta)} ev_1^* \omega_1 \wedge \cdots \wedge ev_n^* \omega_n}_{\equiv N^g_\beta(D_1 \ldots D_n)}$$

so $N^g_\beta(D_1, \ldots, D_n) = \{\#$ of $\varphi \in \mathcal{H}ol(M) \mid \varphi(p_i) \in D_i,\ [\varphi^*(\Sigma_g)] = \beta\ \forall i\}$

$q^\beta = \prod_i e^{-t_i n_i}$
$\omega_i \cdot \beta > 0$, the Kähler form res. to hol. map is pos. semidef

- in large vol limit only $\beta=0$ contribution survives:

$\beta=0 \Rightarrow \varphi$ constant map $\Rightarrow \mathcal{M}=M$

$ev_i \equiv id_M \ \forall i$

$k=\dim_{\mathbb{C}} M$, so only genus 0 allowed

$$\langle \mathcal{O}_1 \cdots \mathcal{O}_n \rangle^{const}_{g=0} = \int_M \omega_1 \wedge \cdots \wedge \omega_n, \text{ classical int. theory}$$

A model chiral ring $\xrightarrow[\varphi=const]{g=0}$ cohomology ring of M

$C_{abc} = \#(D_a \cap D_b \cap D_c)$

$\eta_{ab} = \langle 1\, \mathcal{O}_a \mathcal{O}_b \rangle = \int_M \omega_a \wedge \omega_b$

- nongeneric case, $\ker D_{\bar z}^{+} \neq \{0\}$

$D_z \overline{\psi}_{\bar z \bar i} = D_{\bar z} \overline{\psi}_{z i} = 0$, $\overline{\psi}_{z i} \in H^0(\Sigma, K \otimes \varphi^*(T^*M))$

- let $h^0(\text{previous}) =: \ell$

$\rightarrow \dim \mathcal{M}_{\Sigma_g}(M,\beta) = \underset{\underset{\# \chi \text{ z.m.}}{\uparrow}}{k} + \ell$

- from $\Gamma^i{}_{jk} \partial_{\bar z} \varphi^j$ terms in $\overline{\psi} D_{\bar z} \psi$ we get also for 0-modes

$$S = \int_\Sigma d^2 z\, R_{i\bar i j \bar j}\, \overline{\psi}^{(0)\bar i}_{z}\, \overline{\psi}^{(0)\bar j}_{\bar z}\, \psi^{(0)i}\, \psi^{(0)j}$$

$$- \overline{\psi}_{\bar z \bar j} \psi^k \partial_z \varphi^\ell R^{\bar j i}{}_{\bar k \ell}\, G^{z\bar z}{}_i{}^j\, \chi^k \partial_{\bar z} \varphi^{\ell} R^{n}{}_{jk\bar l}\, \psi_{zn}$$

is $\langle \overline{\psi}, F_v \overline{\psi} \rangle \Rightarrow Pf\, F_v \Rightarrow e(v)$

- put $e(v)$ inside $\int_{M_{\Sigma_g}(M,\beta)} \underbrace{ev_1^* \omega_1 \wedge \cdots \wedge ev_n^* \omega_n}_{(k,k)\text{-form}} \wedge \underbrace{e(v)}_{(\ell,\ell)\text{-form}}$

A twisted nonlin σ model for $\mathbb{P}^1$

- $H^0(\mathbb{P}^1) = H^2(\mathbb{P}^2) = \mathbb{C}$, $\int_{\mathbb{P}^1} H = 1$,
  $1, P \in H^0$, $H, Q \in H^2$
  $$\eta_{\alpha\beta} = \int_{\mathbb{P}^1} \omega_\alpha \wedge \omega_\beta = \begin{cases} 1 & \text{for } (\alpha,\beta) \in \{(P,Q),(Q,P)\} \\ 0 & \text{otherwise} \end{cases}$$

- 3pt function

$$\langle Q\,Q\,Q \rangle = \underbrace{\sum_\beta q^\beta}_{\sum_{n \in \mathbb{N}} q^n \text{ since } \exists\ 1 \text{ class}} \langle Q\,Q\,Q \rangle_\beta$$

$$\dim_{\mathbb{C}} \mathbb{P}^1 (1-g) + \int_{\Sigma_g} \varphi^*(c_1(\mathbb{P}^1)) = 1-g+2\overset{\deg \varphi^*(\Sigma_g)}{n}$$

$g=0$, $\mathbb{P}^1_{w.s.} \to \mathbb{P}^1_{tgt}$, axial Q charge $=2$
so dim of $Q^3 = 6$, must match vd(M)

so $6 = 2(1+2n) \Rightarrow n = 1$

$$\Rightarrow \langle Q\,Q\,Q \rangle = e^{-t}$$

quantum coh. ring of $\mathbb{P}^1$

$C_{PPP} = 0$, $C_{PPQ} = \eta_{PQ} = 1$, $C_{QQQ} = e^{-t}$, $C_{PQQ} = 0$

Exercise. Show $PP = P$, $PQ = Q = QP$, $QQ = e^{-t}P$

- rk 2 vbdl on $\mathbb{P}^1$ s.t. tot. sp. is CY
  - splits $\mathcal{L}_1 \oplus \mathcal{L}_2 \to \mathbb{P}^1$,
    tot sp being $\text{tot}\left(\mathcal{O}(-n_1) \oplus \mathcal{O}(-n_2)\right)_{\mathbb{P}^1}$
  - locally $w' = z^n w$ for $z' = 1/z$
    so if we demand can. bdl to be
    trivial, $dw_1' dw_2' dz' = z^{n_1+n_2-2} dw_1 dw_2 d$
    $\Rightarrow n_1 = n_2 = 1$
  - so $\text{tot}\left(\mathcal{O}(-1)^{\oplus 2}\right)$
    $\to$ conifold $xy - wz = 0$

- let d = degree of $\underset{z=[x:y]}{\overset{\psi}{\mathbb{P}^1}} \to \underset{m=[s:t]}{\overset{\omega}{\mathbb{P}^1}}$

  with explicita $d = 1$, $m = \dfrac{az+b}{cz+d}$

  $d > 1$, $\dfrac{s}{t} = \dfrac{\sum_{i=0}^{d} a_i x^i y^{d-i}}{\cdots b_i \cdots}$

  w mod. sp. $(a_i, b_i) \subset \mathbb{P}^{2d+1}$

- Aspinwall-Morrison [CMP '93],
  $$C_{abc}(t) = [D_a] \cap [D_b] \cap [D_c] + \sum_{k=1}^{\infty} \sum_{d|k} \frac{k^3}{d^3} q^k$$