Gauge ⊂ LCAT

Gukov, cont'd.

- $\hat{Z}\begin{pmatrix}\text{knot}\\ \text{compl.}\end{pmatrix} =: F_K(x;q)$

$\mathrm{Hol}(\text{meridian}) \sim \begin{pmatrix} x & * \\ 0 & x^{-1}\end{pmatrix}$

- $\hat{Z}(q)\big|_{q\to e^{2\pi i/k}} = WRT(k)$ --- closed M_3

$F_K(x;q)\big|_{\substack{q\to e^{2\pi i/k}\\ x=q^n}} = k\text{-}J_n(K;q)$ --- $M_3 = K^{\text{compl.}}$

- relation to Turaev torsion & Alexander polyn.

$$F_K(x;q)\big|_{q\to 1} = \frac{x^{1/2}-x^{-1/2}}{\underbrace{\Delta_K(x)}_{\text{Alex. polyn.}}}$$

$$\hat{Z}_b(q)\big|_{q\to 1} = \frac{1}{\underbrace{T(b)}_{\text{Turaev torsion}}}, \quad b\in \mathrm{coker}\, Q = H_1(M_3)$$

$\mathrm{Spin}^c(M_3)$ is

- relation to $SL(2,\mathbb{C})$ Chern-Simons & A-polynomial

$$\hat{Z}_b(M_3;q) = q^{d_b}(c_0+c_1q+\cdots) \overset{q=e^{\hbar}}{\sim} \exp\left(\frac{1}{\hbar}S_0^{(b)} + S_1^{(b)} + \hbar S_2^{(b)} + \cdots\right)$$

finite-type Vassiliev inv's; Gukov-Witten (of something)

$$\| \quad \text{Donaldson-Thomas}$$

$$\sum_n DT_n\cdot q^n \qquad \frac{N_1,N_2,N_3,\dots}{q=e^{\hbar}} \qquad \| \quad \text{Gromov-Witten}$$

$$\exp\left(\sum_n G_n W_n\cdot \hbar^n\right)$$

$\mathbb{Z}$ $\qquad$ $\mathbb{Q}$

- A-polynomial?

$x, y \in \frac{\mathbb{C}^\times \times \mathbb{C}^\times}{\mathbb{Z}_2}$

- for $M_3 = S^3 \setminus K$, $A_K(x,y) = 0$

spectral curve

- e.g. $A_{3_1}(x,y) = (y-1)(y+x^3)$

red. Coulomb — irred. Higgs

$$\omega = \frac{dx}{x} \wedge \frac{dy}{y}$$

- quantize: $\hat{x}\hat{y} = q\hat{y}\hat{x}$, $q = e^{\hbar}$

and look at

$$\hat{A}(\hat{x},\hat{y}) \exp\left(\frac{1}{\hbar}\sum_{n=0}^{\infty} S_n^{(0)}(x)\cdot\hbar^n\right) = 0$$

$$\exp\left(\frac{1}{\hbar}\int \log y \frac{dx}{x} + \cdots\right)$$

$SL(2,\mathbb{C})$-CS on $S^3 \setminus K$