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Cohomological field theory [Witten 90's, Trieste]

- top. theory of coh. type
  = intersection theory on mod. sp. of solns of PDEs
- moduli space = zero locus of ∞-dim vect bdl
  $$\mathcal{M} = \{ \varphi \in \mathcal{E} \mid s(\varphi) = 0 \} / G$$
- path int = Mathai - Quillen repr of Thom class of the ∞-dim vect bdl
  - morally, $Pf(R)\, e^{-\varepsilon \|s\|^2}$
- equiv. coh wrt G

- physically we have:

  - space of fields
  - action $\leftrightarrow$ (BPS) e.o.m. / choice of section is Q-exact
    $\Rightarrow T_{\mu\nu}$ is Q-exact
- symmetries $\leftrightarrow$ G-equiv coh

- A model fits

Stop ; S g.f. ; we called this $s_{\bar z}$?

$$S_A = \underbrace{\int_\Sigma \varphi^*(\omega + iB)}_{S_{top}} + i \underbrace{\int d^2 z\, Q_A \left[g_{i\bar j} \left(\bar\psi^{i}_{\bar z}\, \partial_z \varphi^{\bar j} + \underbrace{\bar\psi^{\bar j}_{z}\, \partial_{\bar z} \varphi^{i}}_{=0} \right)\right]}_{S_{g.f.}}$$

$$= i \int d^2 z \left[\frac{1}{2} g_{\mu\nu}\, \partial_z \varphi^\mu \partial_{\bar z} \varphi^\nu + g_{i\bar j} \left(\bar\psi^{i}_{\bar z}\, D_z \psi^{\bar j} + \bar\psi^{\bar j}_{z}\, D_{\bar z} \psi^{i} \right) \right.$$

$$\left. - R_{i\bar i j\bar j}\, \bar\psi^{i}_{\bar z}\, \bar\psi^{\bar i}_{z}\, \psi^{j}_{\bar z}\, \psi^{\bar j}_{z} \right]$$

Q_A-fixed pts
$$\partial_z \varphi^{\bar i} = \partial_{\bar z} \varphi^{i} = 0 \quad \rightsquigarrow \mathrm{Hol}(M)$$
$$D_z \psi^{\bar i} = D_{\bar z} \psi^{i} = 0 \quad \rightsquigarrow TM$$
$$D_{\bar z} \bar\psi^{\bar i}_{z} = D_z \bar\psi^{i}_{\bar z} = 0 \quad \rightsquigarrow \text{obstructions}$$

$$\int D\varphi D\psi D\bar{\psi}\, e^{-S_{top}-S_{g.f.}} = \int_{\mathcal{M},\ker D_{\bar z},\ker D_{\bar z}^{\dagger}} dm\, d\psi^{(0)} d\bar{\psi}^{(0)} e^{-S_{top}} \cancel{\frac{\det'\Delta_f}{\det'\Delta_{\bar f}}}^{=1}$$

bosonic part – mod. sp. of hol maps $\Sigma_g \xrightarrow{\varphi} M$

$$\mathcal{M}_g(M,C) = \bigsqcup_d \mathcal{M}_g(M,\beta),$$

where $d \in \mathbb{N}^{b_2(M)}$, $\beta \overset{d}{=} [\varphi_*(\Sigma_g)] \in H_2(M,\mathbb{Z})$

and $\beta = \sum_{i=1}^{b_2(M)} d_i [S_i]$, $[S_i]$ basis of $H_2(M,\mathbb{Z})$

– $e^{-S_{top}} = e^{-\int_{\Sigma_g}\varphi^*(\omega+iB)} = q^{\beta} = \prod_{i=1}^{b_2(M)} q_i^{d_i}$,

where $q_i = e^{-t_i}$, t_i cpx kähler param.

– axial R-sym anomaly:

$\# \psi^{(0)} - \# \bar{\psi}^{(0)} = \dim\ker D_{\bar z} - \dim\ker D_{\bar z}^{\dagger} = \operatorname{ind} D_{\bar z}$

$= \dim H^0(\varphi^*(TM)) - \dim H^1(\varphi^*(TM))$

$= \int_{\Sigma_g} ch(\varphi^*(TM))\, td(T\Sigma)$

$= \dim_{\mathbb{R}} M \cdot (1-g) + \int_{\Sigma_g} \varphi^*(c_1(TM))$

– so we need to insert some observables to offset this, $\langle \prod_k O_k \rangle_A$

– $\sum \mathrm{Axial}(O_k) = 2\operatorname{Ind} D_{\bar z}$

$\rightarrow$ <0 if $g>1$ & (and CY)

$\rightarrow$ solution: integrate over all cpx structures. Something is wrong with rigidity of cpx structs

– topological strings, $\dim \bar{M}_g = 3g-3$, $T M_g = H^1(T\Sigma_g)$

parametrised by Beltrami diffs $\mu^{i}{}_{\bar z}{}^{z}$, $\bar{\mu}^{i}{}_{z}{}^{\bar z}$, $i=1,\dots,3g-3$

$$F_g = \int_{\bar{M}_g} \langle \prod_{i=1}^{3g-3} G_+(\mu_i) \prod_{l=1}^{3g-3} G_-(\bar{\mu}^l) \rangle_A, \text{ with } \begin{array}{l} T_{++} = \{Q, G_+\} \\ T_{--} = \{Q, G_-\} \end{array}$$

- with this in mind, index becomes

$$2(\dim_{\mathbb{R}} M - 3)\cdot(1-g) + 2\int_{\Sigma_g} \varphi^*(c_1(TM))$$

→ = 0 for CY3?

$$F_g = \sum_\beta N_\beta^g q^\beta$$

"number" of hol maps of deg β from Σ to M

⇒ $\in \mathbb{Q}$ due to bdry of $\overline{M}_g$, counts maps up to automorphism

- Gromov-Witten inv