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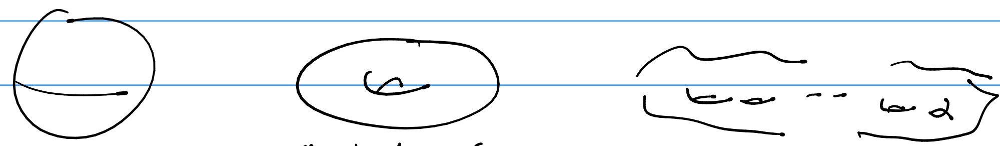
On the classification of alg. varieties.

- birational classif. of alg. varieties
  - e.g. i) quadric $Q \hookrightarrow \mathbb{P}^{n+1} \overset{\text{birat.}}{\longleftrightarrow} \mathbb{P}^n$
    via projection from $q \in Q$ to $\mathbb{P}^1$
    ii) circle $C \overset{\text{birat.}}{\longrightarrow} \mathbb{A}^1$
- dim 1 case:
  - -> cpt smooth curves $C \overset{\text{bir.}}{\sim} C'$ iff isomorphic.

C:



$g=0, \kappa=-\infty$ $g=1, \kappa=0$ $g>1, \kappa=1$

  - -> where κ = Kodaira dimension
- dim 2 case -> nontrivial.
  - -> blow-up a nonsing. pt $P \in S$ to get S'
    - -> replace P with curve C s.t. $S-\{p\} \cong S'-C$
    - -> C is a rational curve, called (-1)-curve
- traditional MMP program in dim 2
  - -> if proj. surf. X has a (-1)-curve, contract it -> repeat it...
    - -> X is then either $\mathbb{P}^1$-bdle or a minimal model
- -> In dim 3, however... contraction can give us a nonproj. variety from a proj. variety
  - -> worse, it can become non-algebraic.
  - -> 1960s-70s: $\exists$ 3-folds w/ no minimal models.

- Mori's question: how do varieties curve?
  - → put a metric on alg. var X
  - → what if X is positively curved everywhere?

- → Frankel's conj.: if a Kähler mfld is pos. curved & $\mathbb{R}^2 < T_{p,X} \Rightarrow X \cong \mathbb{P}^n$.
- → Hartshorne's conj: every proj. mfld w ample tangent bdle is isomorphic to $\mathbb{P}^n$.
  - → proved by Mori
- given curve $C \hookrightarrow X$, the intersection # $(-K_X \cdot C)$ measures the average "curvature" above p.

- key statement: if X pos. curved along any $C \subset X$, X contains a rational curve
  - → originally, there was a char p proof using the Frobenius morphism.