Tanzini

- corr. functions: chiral ring for LG B model
- chiral ring $\mathbb{C}[\varphi^1, \dots, \varphi^n]/\langle \partial_i W \rangle$
- fixed pts: $d\varphi = 0$, $\partial_i W = 0 \rightsquigarrow$ const. maps to crit pts of W
- supposing finite number, label by $\{y_i\}_{i=1,\dots,N}$

$(f_1, \dots, f_s) \mapsto \mathcal{O}_{f_1} \cdots \mathcal{O}_{f_s}$

$$\langle \mathcal{O}_{f_1} \cdots \mathcal{O}_{f_s} \rangle = \sum_{j=1}^{N} \langle \mathcal{O}_{f_1} \cdots \mathcal{O}_{f_s} \rangle \big|_{y_j}$$

$$S = \int d^2 z \Big(g_{i\bar{j}} h^{\alpha\beta} \partial_\alpha \varphi^i \partial_\beta \varphi^{\bar{j}} \sqrt{h} - i g_{i\bar{j}} \psi^{\bar{j}} D_{\bar{z}} \rho_z^i + i g_{i\bar{j}} \psi^{\bar{j}} D_z \rho_{\bar{z}}^{i}$$

$$- \tfrac{1}{2} R_{i\bar{j}k\bar{l}}\, \rho_{\bar{z}}^i \rho_z^k \psi^{\bar{j}} \psi^{\bar{l}}$$

$$+ \tfrac{1}{8} g^{\bar{j}l} \partial_l W \partial_{\bar{j}} \bar{W} + \tfrac{1}{4} D_i \partial_j W \rho_{\bar{z}}^i \rho_z^j + D_{\bar{i}} \partial_{\bar{j}} \bar{W} \psi^{\bar{i}} \psi^{\bar{j}} \Big)$$

- nonconst. modes produce det' factors that cancel
- constant modes:
  1) 1 const. mode for $\varphi^i, \varphi^{\bar{i}}, \psi^i, \psi^{\bar{i}}$
  2) g const. modes for $\rho_z^i, \rho_{\bar{z}}^i$

$$\int d^n\varphi\, d^n\bar{\varphi}\, e^{-1/4\, \varphi^0 \Delta \varphi^0} = |\det \partial_i \partial_j W|^{-2} (y_i)$$

$$\int d^n\psi\, d^n\bar{\psi} \cdots \sim \det \partial_{\bar{i}} \partial_{\bar{j}} \bar{W} (y_i)$$

$$\int d^{n\cdot g}\rho_z\, d^{n\cdot g}\rho_{\bar{z}} \cdots \sim (\det \partial_i \partial_j W)^g (y_i)$$

$$\Rightarrow \langle \mathcal{O}_{f_1} \cdots \mathcal{O}_{f_s} \rangle = \sum_{\{y_j\}} f_1(y_j) \cdots f_s(y_j) (\det \partial_i \partial_j W)^{g-1} (y_j)$$

$$C_{ijk} = \sum_{\{dW=0\}} \frac{f_i f_j f_k}{\det \partial_i \partial_j W}, \quad \eta_{ij} = \sum_{\{dW=0\}} \frac{f_i f_j}{\det \partial_i \partial_j W}$$

- sinh-Gordon model with M = cylinder, $z \in \mathbb{C}^*$
has $W = z + z^{-1} e^{-t}$
$\rightarrow W' = 1 - e^{-t}/z^{*2} = 0 \Rightarrow z^* = \pm e^{-t/2}$

- Hessian: $z \partial_z (z \partial_z W) = z + z^{-1} e^{-t} \Big|_{z^* = \pm e^{-t/2}}$

- chiral ring gen. by $1, z, z^2 = e^{-t}$

$\Rightarrow \langle 1 1 1 \rangle_0 = \frac{1}{2e^{-t/2}} + \frac{1}{-2e^{-t/2}} = 0$

$\langle 1 1 z \rangle_0 = \frac{e^{-t/2}}{2e^{-t/2}} + \frac{-e^{-t/2}}{-2e^{-t/2}} = 1$

$\langle 1 z z \rangle_0 = 0$

$\langle z z z \rangle_0 = \frac{e^{-3t/2}}{2e^{-t/2}} + \frac{-e^{-3t/2}}{-2e^{-t/2}} = e^{-t}$

- same as $\mathbb{P}^1$ maps in A model!

- how to describe using field theory?
- since $\mathbb{CP}^{N-1} \simeq S^{2N-1}/U(1)$, we want to have a 2d susy theory with a quotient tgt mfd
- basically, $U(1)$ bdl:

$L = -\sum_{i=1}^{N} |D_\mu \varphi_i|^2 - U(\varphi)$ where $D_\mu = \partial_\mu + v_\mu$,
$U(\varphi) = \frac{e^2}{2}\left(\sum_{i=1}^{N} |\varphi_i|^2 - r\right)^2$

- v_μ auxiliary, its e.o.m. gives

$$\sum_i (D_\mu \bar{\varphi}^i)\varphi^i - \bar{\varphi}^i (D_\mu \varphi^i) = 0$$

$$\Rightarrow v_\mu = \frac{i}{2} \frac{\sum_{i=1}^N \bar{\varphi}_i \partial_\mu \varphi_i - \partial_\mu \bar{\varphi}_i \varphi_i}{\sum |\varphi_i|^2}$$

- when this is inserted back in the action, we obtain a nonlinear σ-model with a certain induced metric in the IR.
- we call L prior to that a linear σ-model, simply because the metric is flat here
- but eliminating v_μ gives

$$g^{FS} = \frac{\sum_{i=1}^{N-1} |dz_i|^2}{1+\sum |z_i|^2} - \frac{\sum |\bar{z}_i \, dz_i|}{1+\sum |z_i|^2}, \quad z_i := \frac{\varphi_i}{\varphi_N}$$

$\longrightarrow$ NσM on $\mathbb{C}P^{N-1}$ $\quad g_{ij}(\varphi)\partial_\mu \varphi^i \partial^\mu \varphi^j$

- susy?
- here we had local $U(1)$, $\varphi_i(x) \mapsto e^{i\gamma(x)}\varphi_i(x)$

$$L = \int d^2\vartheta \, d^2\bar{\vartheta} \; \bar{\Phi}\Phi$$

$\Phi \mapsto e^{iA}\Phi =: \Phi'$ gives $\bar{\Phi}'\Phi' = \bar{\Phi}\, \underline{e^{-i(\bar{A}-A)}}\,\Phi$

- to absorb it introduce vector superfield V,
$V \mapsto V + i(A - \bar{A})$, s.t. $\underline{\bar{\Phi}' e^{V'} \Phi' = \bar{\Phi} e^{V} \Phi}$.

- V contains $U(1)$ conn., but also scalars, fermions

- supercurvature $\Sigma := \overline{D}_+ D_- V$
  - this is a twisted chiral field since $\overline{D}_+ \Sigma = D_- \Sigma = 0$
  - put $\tilde{y} = x + \overline{\vartheta}^+ \vartheta^-$

$$\Sigma(\tilde{y}) = \sigma(\tilde{y}) + i\vartheta^+ \lambda_+(\tilde{y}) - i\overline{\vartheta}^- \lambda_-(\tilde{y}) + \vartheta^+ \overline{\vartheta}^- \left[D(\tilde{y}) - \mathcal{F}_{12}(\tilde{y}) \right]$$

where $\mathcal{F}_{\mu\nu} = \partial_{[\mu} v_{\nu]}$

$$L_{kin} = \int d^4\vartheta \, \overline{\Phi} e^V \Phi$$

$$L_{gauge} = -\frac{1}{2e^2} \int d^4\vartheta \, \Sigma \overline{\Sigma}$$

$\widetilde{W}_{FI,\vartheta} = -t\Sigma$, $t := r - i\vartheta$ gives

$$L_{FI,\vartheta} = \frac{1}{2}\left(-t \int d^2\tilde{\vartheta} \, \Sigma + c.c. \right) = -r D + \vartheta \mathcal{F}_{12}$$

- in general for $U(1)^k$ we get

$$L = \int d^4\vartheta \left(\sum_{i=1}^{N} \overline{\Phi}_i e^{\sum_{a=1}^{k} Q_{ia} V_a} \Phi_i - \sum_{a,b=1}^{k} \frac{1}{2e^2_{a,b}} \overline{\Sigma}_a \Sigma_b \right) + \frac{1}{2} \int d^2\tilde{\vartheta} \sum_{a=1}^{k} (-t_a \Sigma_a) + c.c.$$

- so Q_{ia} crucially determines tgt space
- this L is invariant under $U(1)_V \times U(1)_A$ unless $\sum_i Q_{ia} \neq 0$

- terms containing D field: $\frac{1}{2e^2}D^2 + D(|\varphi|^2 - r_0)$

- we see that completing the square for Gaussian integration gives us potential for φ

- however, $\langle|\varphi|^2\rangle = \int_{\mu}^{\Lambda_{UV}} \frac{d^2k}{k^2} \sim \log\frac{\Lambda_{UV}}{\mu}$

so effectively $\frac{1}{2e^2}D^2 + D\left(\log\frac{\Lambda_{UV}}{\mu} - r_0\right)$

so $r_0 = r + \log\frac{\Lambda_{UV}}{\mu}$, $r(\mu) \sim \log\frac{\mu}{\Lambda_{UV}}$

- recall that usually the term $2i\overline{\psi}_- D_{\bar{z}}\psi_- + 2i\overline{\psi}_+ D_z\psi_+$ breaks R-sym,

$$D\psi D\overline{\psi} \mapsto e^{-2ik\alpha} D\psi D\overline{\psi}$$

where $k = \operatorname{ind} D_{\bar{z}} = \frac{i}{2\pi}\int c_1(\mathcal{F})$

- but we can negate this by $\vartheta \mapsto \vartheta - 2\alpha$
but this breaks $U(1)_A$ to $\mathbb{Z}_2$

- generally $r_a(\mu) = \sum_i Q_{ia}\log\frac{\mu}{\Lambda}$

$$\vartheta_a \longmapsto \vartheta_a - 2\left(\sum Q_{ia}\right)\alpha$$