Gauge @ IGAP

Top. strings on genus 1 fibred CY3's and string dualities - A. Klemm

- t. string $X : \Sigma_g \to M(\Omega, \omega)$

NS-field   hol (3,0) form   (1,1) kähler

$$Z(\underbrace{B, G}_{\text{background}}) = \int DX\, D\psi - Dh\; e^{S_{(2,2)}(X,\psi,\dots,h,G,B)}$$

- A-twist: $Z_A(\omega_\mathbb{C}) = \exp \sum_g \lambda^{2g-2} F_{g\,\omega_\mathbb{C}}$

$$F_g = \sum_{\beta \in H_2(M,\mathbb{Z})} GW_g^\beta Q^\beta, \quad Q^\beta = e^{\int_{C_\beta} \overbrace{\omega + iB}}$$

- B-twist: $Z_B(\Omega)$
  - depends on periods $X^I = \int_{A^I} \Omega,\ F_I = \int_{B_I} \Omega$
  - homogenous coords: $F_0(t) = \frac{1}{2}(X^0)^{-2} F_I X^I[t], \quad t^i = \frac{X^i}{X^0}$

$g > 0$?

$W = \text{Mirror}(M)$

- $e^{-K} = i\int_W \Omega \wedge \bar{\Omega} = i(X_I \bar{F}^I - \bar{X}_I F^I)$

  $C_{ijk} = \int_W \Omega\, \partial_i \partial_j \partial_k \Omega = X^I \partial^3 F_I - F_I \partial^3 X^I$

HA1 $\quad \partial_a \partial_{\bar a} F_1 = \frac{1}{2} C_{\bar a}{}^{bc} C_{abc} - \left(\frac{\chi}{24} - 1\right) \underset{\partial_a \partial_{\bar a} K}{G_{a\bar a}}$

HA2 $\quad \partial_{\bar a} F_g = \frac{1}{2} C_{\bar a}{}^{bc} \left(D_b D_c F_{g-1} + \sum_{n=1}^{g-1} D_b F_n D_c F_{g-n}\right)$

- sympl. action $\begin{pmatrix} F_I \\ X^I \end{pmatrix} \mapsto S \begin{pmatrix} F_I \\ X^I \end{pmatrix}, \quad S \in Sp(h_3(W), \mathbb{R})$

$\Rightarrow$ Q1) $S \triangleright Z$?

Q2) Invariance of Z under monodromies $M \subset \Gamma_W \subset Sp(h_3(W), \mathbb{Z})$?

$$e^{\sum_g \lambda^{2g-2} F_g}$$
$$\shortparallel$$

A1) • (Witten 93') Z is like a wavefunct.

Ψ on $H_3(W, \mathbb{R})$

• inf. change of polarisation

$\rightsquigarrow$ Bogolyubov transf.:

$$(*) \quad \left[\partial_{\bar{a}} - \left(\frac{\chi}{24} - 1 \right) K_{\bar{a}} + C_{\bar{a}}^{\;ab} \left(\lambda^2 D_a D_b - D_a F_0 D_b \right) \right] \Psi = 0$$

$\Longleftrightarrow$ HA1, HA2 hold

if HA0 holds: $\partial_{\bar{a}} F_g = -\frac{1}{2} C_{\bar{a}}^{\;bc} D_b F_0 D F_c$

- today: use (A1) and use its insight to Q2 to describe Z on $g=1$ fibration

$$\begin{array}{ccc} \Sigma_{g=1} & \longrightarrow & M \\ & & \downarrow \pi \\ & & B_2 \end{array}$$

Ell. curve is $g=1$ curve w/ mkd pt O.
Ell. fibration has 0-sect. defined by O