Percacci.

Homotopy groups.

$\pi_n(M) = [S^n, M]_*$, $n = 1, 2, \dots$ groups

- $S^0 = \mathbb{Z}_2 = \{1, -1\}$, $1 \mapsto$ basepoint,

so $\pi_0(M) = \#$ conn. components, arcwise

Winding number.

M, N cpt. conn. mfds w/o bdry, orientable, $M \xrightarrow{\varphi} N$, ω vol. form on N, $\dim M = \dim N = n$

- winding number $W(\varphi) = \frac{1}{Vol(N)} \int_M \varphi^* \omega$
  - standard results: $y \in N$ regular (inv. Jacobian) if all $x \in \varphi^{-1}(N)$ regular

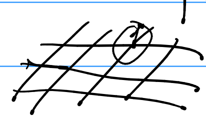
$\to$ $W(\varphi) =$ ($\sum$ pts in φ^{-1}(reg. pt) counted with signs of Jacobians)

Sigma models.

- dimension 2 and $\pi_0(Q) \neq pt$ if we want solitons, $\mathbb{R}^d \xrightarrow{\varphi} N$

$$\to E_S = -\frac{f^2}{2} \int_{\mathbb{R}^d} d^d x \, \partial_\mu \varphi^\alpha \partial^\mu \varphi^\beta \underbrace{h_{\alpha\beta}(\varphi)}_{\text{metric on } N}$$

- we want $E_S < 0$, so $\partial \varphi \to 0$ at ∞
  $\to \varphi \to \varphi_*$
- $Q = \{\text{maps } \mathbb{R}^d \xrightarrow{\varphi} N \mid \varphi \to \varphi_* \text{ at } \infty\}$
  $= \{S^d \xrightarrow{\varphi} N \mid \varphi \to \varphi_*\}$ compactifying
  $\to \pi_0(Q) = \pi_d(N)$

- motivation comes from Heisenberg model in dim $= 2$,  $\Lambda \to S^2$ becomes

$\mathbb{R}^2 \to S^2$ in dense limit

$\to$ at temperature T, density of solitons $\sim e^{-E_S/kT}$

$\to$ no long-range order