Grisonni

- realise RS as 0-locus $\{(w,\lambda)\in\mathbb{C}^2 \mid P(w,\lambda)=0\}$
- in particular hyperelliptic

$$P(w,\lambda) = w^2 - \prod_{j=1}^{2g+2}(\lambda-\lambda_j)$$

- locally
  - $(w,\lambda)\mapsto\lambda$, away from λ_j
  - $(w,\lambda)\mapsto\sqrt{\lambda-\lambda_j}$ at λ_j
  - $(w,\lambda)\mapsto\frac{1}{\lambda}$ at ∞ (compactification)

- and g hol. differentials $\omega_k = \frac{\lambda^{k-1}}{w}d\lambda$, $k=1,\dots,g$

- Def. Torelli marking, i.e. basis for homology $a_1,b_1,\dots,a_g,b_g$ s.t.

$$a_i\# a_j = b_i \# b_j = 0, \quad a_i \# b_j = \delta_{ij}$$

Th. (Riemann bilin.) $\int_X \omega\wedge\eta = \sum_{j=1}^{g}\left(\oint_{a_j}\omega\oint_{b_j}\eta - \oint_{b_j}\omega\oint_{a_j}\eta\right)$

$$\Rightarrow \operatorname{Im}\sum_{j=1}^{g} A_j\overline{B_j} \leq 0.$$

- Torelli matrix ... pick $A=Id$, so B sym $\wedge$ $\operatorname{Im}B > 0$
- Abel map $\mathcal{A}: X\to\mathbb{C}^g$

$$P\mapsto\left\{\int_{P_0}^{P}\omega_1,\dots,\int_{P_0}^{P}\omega_g\right\}$$

- $J(X) = \mathbb{C}^g/(\mathbb{Z}^g + B\mathbb{Z}^g)$ Jacobian variety, so also $\mathcal{A}: X\to J(X)$

- letting τ be $g\times g$ symm & $\operatorname{Im}\tau > 0$, define

$$\vartheta(-,\tau):\mathbb{C}^g\to\mathbb{C}$$

$$z\mapsto\vartheta(z,\tau) = \sum_{n\in\mathbb{Z}^g} e^{i\pi\langle\tau n,n\rangle + 2\pi i\langle z,n\rangle}$$

- let $e \in \mathbb{C}^g$ and define $\vartheta_e : X \to \mathbb{C}$

$$P \mapsto \vartheta(\mathcal{A}(P) - e)$$

- if e is such that $\vartheta_e \equiv 0$, we say e belongs to theta divisor

- $\mathcal{A}(\vartheta_e) = e - \vec{K}$

<u>Prop</u> Let $e = \mathcal{A}(p_1 + \dots + p_{g-1}) + K$

$\Rightarrow F(P, Q) := \vartheta(\mathcal{A}(P+Q) - e)$ is 0

<u>iff</u> $P = Q$ or $P = P_j$, $j = 1, \dots, g-1$

<u>RHP</u> Find $\Psi(\lambda) : \mathbb{C}P \setminus \{\lambda_1, \dots, \lambda_{2g+2}\} \to SL(2, \mathbb{C})$

- $\Psi(\lambda_0) = \mathrm{Id}$ $(\lambda_0 = \infty)$
- Ψ hol. on $\mathbb{C}P \setminus$ br. pts
- monodromy around br. pts. fixed, M_j
- Fuchsian sings. at br. pts

- rephrase as $\Psi' = \sum_{j=1}^{g} \frac{A_j}{\lambda - \lambda_j} \Psi$

if $\frac{dM_j}{d\lambda_i} = 0$, A_j solve Schlesinger eqns.

$$\frac{\partial A_j}{\partial \lambda_i} = \frac{[A_i, A_j]}{\lambda_i - \lambda_j}, \quad \frac{\partial A_i}{\partial \lambda_i} = -\sum_{j \neq i} \frac{[A_i, A_j]}{\lambda_i - \lambda_j}$$

<u>Equiv</u> $\frac{dA_j}{d\lambda_K} = \{H_K, A_j\}$

where $H_K = \operatorname{res}_{\lambda = \lambda_K} \operatorname{Tr} A^2 = -\operatorname{res} \det A$

$$= \sum_{j \neq K} \frac{\operatorname{tr} A_j K}{\lambda_j - \lambda_K}$$

and $\{H_i, H_j\} = 0$

- this also means $\frac{\partial H_j}{\partial \lambda_k} = \frac{\partial H_k}{\partial \lambda_j}$

$\xRightarrow{\text{locally } \exists}$ τ s.t. $\frac{d}{d\lambda_j} \ln \tau = H_j$