Шхомиров

- $\begin{array}{c} P \\ \downarrow \pi \\ B \end{array}$, $G = SU(2)$, $g \in \mathcal{G} = \Gamma(\mathrm{Ad}\,P)$ gauge tr. gp.

- $\mathcal{A}(P) =: \mathcal{A}$ sp. of conn. on P, affine over $\Omega^1(\mathrm{ad}\,P)$
  $\mathcal{A} \times \mathcal{G} \longrightarrow \mathcal{A}$, $(A, g) \mapsto g^* A$
- $\mathcal{A}_{L^p_k}$, $p = 2$, $k = l - 1$, $l \geq 3$

$$\begin{array}{ccc} \mathcal{A} & \longrightarrow & \mathcal{A}/\mathcal{G} =: \mathcal{B} \\ \rotatebox{90}{$\in$} & & \\ A & \mapsto & A \bmod \mathcal{G} =: [A] \end{array}$$

- elaboration on gp action,

$$\begin{tikzcd} \mathrm{Ad}\,P \arrow[r, hook, "\text{cpt}", "\text{subbundle}"'] \arrow[d, "\text{fibre } G"] & P \times_{G, \mathrm{ad}} M_{2\times 2}(\mathbb{R}) =: \xi \arrow[d] \\ B \arrow[r, equal] & B \end{tikzcd}$$

commutes. We have $(A, B) := -\mathrm{tr}(A \cdot \bar{B})$ inner prod.,
$\underline{G} \hookrightarrow M_{2\times 2}(\mathbb{C}) \subset M_{4\times 4}(\mathbb{R})$,

$\mathcal{G}_{L^p_k} \hookrightarrow L^p_k(\Gamma(\xi))$

- $\mathcal{A}_{L^p_k} \times \mathcal{G}_{L^p_{k+1}} \longrightarrow \mathcal{A}_{L^p_k}$ smooth action

- $\mathcal{A}_{L^p_k} / \mathcal{G}_{L^p_{k+1}} = \mathcal{B}_{L^p_k}$

- $\mathcal{A}_{l-1} / \mathcal{G}_l = \mathcal{B}_{l-1}$

Thm $\mathcal{B}_{l-1}$, $l \geq 3$ is Hsdff iff
$\Gamma_{\mathcal{G}_l} := \{(A, B) \mid A = g^* B,\ g \in \mathcal{G}_l\} \overset{\text{closed}}{\subset} \mathcal{A}_{l-1} \times \mathcal{A}_{l-1}$.

- didn't write pf. down

- recall that every pt in $\mathcal{A}_2$ has stabiliser $\mathbb{Z}_2$, $U(1)$ or entire G $\subset SU(2)$
- we call A with $\mathrm{Stab}\, A = U(1)$ reducible
  - $\xi = P \times \mathbb{C}^2$ splits, $\xi = L \oplus L'$
  - if $\mathrm{Stab}\, A = \mathbb{Z}_2$, called irreducible.

- $\mathcal{A}^* := \{ A \in \mathcal{A} \mid A \text{ irred.} \} \xhookrightarrow{\text{open}} \mathcal{A}$

Thm (on slices) Consider action $\mathcal{A}_2 \times \mathcal{G}_3 \to \mathcal{A}_2$, let $A \in \mathcal{A}_2$. $\exists$ a $\mathrm{Stab}\, A$-invariant subset $\mathcal{U} \in \mathcal{A}_2$ s.t. $A \in \mathcal{U}$ and

$$\mathcal{U} \times_{\mathrm{Stab}\, A} \mathcal{G}_3 \underset{\substack{\text{diff. onto}\\ \text{its image}}}{\xrightarrow{\cong}} V \xhookrightarrow{\text{open}} \mathcal{A}.$$

- now, we have inn. prod $(-,-)$ on $L^2_3(\Omega^0(\mathrm{ad}\, P))$ and $L^2_2(\Omega^1(\mathrm{ad}\, P))$

- look at $A \longrightarrow g^* A$, $g \in \tilde{\mathcal{G}} := \mathcal{G}/\mathbb{Z}_2$

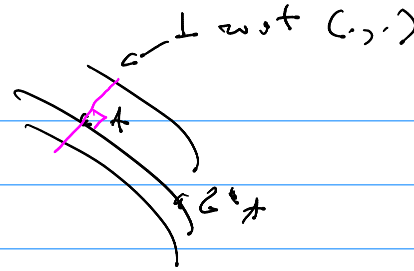
$f : \mathcal{G}_3 \longrightarrow \mathcal{A}_2$, $g \mapsto g^* A$

- the derivative of $A \mapsto g^* A$ is

$$\underset{\substack{\shortparallel \\ T_{id}\mathcal{G}_3}}{\nabla_A : L^2_3(\Omega^0(\mathrm{ad}\, P))} \longrightarrow \underset{\substack{\shortparallel \\ T_A \mathcal{A}_2}}{L^2_2(\Omega^1(\mathrm{ad}\, P))} = (f_*)_{id}$$

$\Rightarrow$ if $\mathrm{Stab}\, A = \mathbb{Z}_2$, $\ker \nabla_A = \{0\} = T_{id}\, \mathrm{Stab}\, A$

- now, we want to look at



- $\chi_A := \{ A + a \mid (a, u) = 0 \quad \forall u \in \mathrm{Im}(\nabla_A)$ and
$u = \nabla_A(v)$ where $v \in L^2_3(\Omega^0(\mathrm{ad} P)) \}$

$= \{ A + a \mid (\nabla_A^* a, v) = 0 \}$

$= \{ A + a \mid \nabla_A^* a = 0 \}$

<u>Coulomb gauge</u>

by nondegeneracy of $(\cdot,\cdot)$,
and $\nabla_A^* : L^2_2(\Omega^1(\mathrm{ad} P)) \longrightarrow L^2_1(\Omega^0(\mathrm{ad} P))$.