GCGT @ old SISSA

Mironov

- Hurwitz p.f. $Z(\beta; p, p') = \sum \chi_R(p)\chi_R(p') e^{\sum \beta_\Delta \varphi_R(\Delta)}$

∞ set of times — Schur polyn.

- $Z_{ov}^{(K)} = \sum H_R^{(K)}(q, A) \cdot \chi_R(p)$, $p_K = \dfrac{A^K - A^{-K}}{q^K - q^{-K}}$

- the system is integrable (KP-hierarchy) if for the exponential we had $e^{\sum \beta_K C_R^{(K)}}$, $C_R^{(K)} = \sum \left[(R_i - i)^K - (-i)^K \right]$

$d_R = \chi_R(p_K = \delta_{K,1})$, $p_\Delta = p_1^{K_1} p_2^{K_2}$, $Z_\Delta = \prod K^{m_K} m_K!$

$\mathrm{Hur}_{g,d}(\{\Delta_i\}_{|\Delta_i| = d}) = \sum d_R^{2-2g} \varphi_R(\Delta_1)\varphi_R(\Delta_2)\cdots\varphi_R(\Delta_n)$

$\varphi_R := \dfrac{\psi_R(\Delta)}{Z_\Delta d_R}$, $\chi_R(p) = \sum_\Delta \dfrac{\psi_R(\Delta)}{Z_\Delta} p_\Delta$

$\hat{W}_\Delta \chi_R = \varphi_R(\Delta) \chi_R$

$|\Delta| < |R|$, $\varphi_R(\Delta) = C_r^{|R| - |\Delta|} \varphi_R(\hat{\Delta})$

$\hat{W} := \dfrac{1}{Z_\Delta} :D_{\delta_1} \cdots D_{\delta_d}:$, $D_K = :\mathrm{Tr}\left(\Pi \dfrac{\partial}{\partial \Pi}\right)^K:$

- claim is $Z = \sum \mathrm{Hur}_0(\Delta, \Delta'; \underbrace{\Delta_1 \ldots \Delta_1}_{K_1}, \underbrace{\Delta_2 \ldots \Delta_2}_{K_2}, \ldots) p_\Delta p_{\Delta'} \dfrac{\beta_1^{K_1}}{K_1!} \dfrac{\beta_2^{K_2}}{K_2!} \cdots$

$= \sum \chi_R(p)\chi_R(p') e^{\sum \beta_{\Delta_K} \varphi_R(\Delta_K)}$

Morozov

- triangular structures

- Kerov functions
- variables X, Y denote Young diags.
- Schur funcs $\chi_R(\{p\})$, $\langle \chi_R | \chi_{R'} \rangle = \delta_{RR'}$
- $\langle p^\Delta | p^{\Delta'} \rangle = z_\Delta \delta_{\Delta\Delta'}$
- $\Delta = [\delta_1 \geq \delta_2 \geq \dots \geq \delta_{\ell_\Delta} \geq 0]$, $p^\Delta = \prod p_{\delta_i}$

- introduce factor $\langle p^\Delta | p^{\Delta'} \rangle = z_\Delta \delta_{\Delta\Delta'} \prod_{i=1}^{\ell_\Delta} g_{\delta_i}$

$\Rightarrow$ orthogonal: $\mathrm{Ker}_R^{(g)}\{p\} = \chi_R\{p\} + \sum_{R'<R} K_{RR'} \chi_{R'}\{p\}$

- depends on ordering choice $(R'<R)$, where $R'<R$ is lexicographical
  - but look at e.g. transposed diagrams w/ lexicographic ordering
    - you get $\widehat{\mathrm{Ker}}$

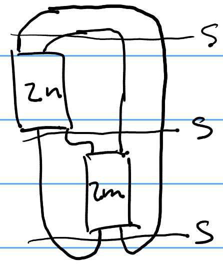
- closer to rep. th. are MacDonald polys with choice $g_i = \frac{\{q^i\}}{\{t^i\}}$, where $\{x\} = x - x^{-1}$

- Schur $\subset$ Macdonald $\subset$ Kerov

- for generalised Macdonalds, easiest to consider them as eigen funcs of some Hamiltonians

$$\mathcal{H} = \oint \frac{dz}{z} e^{\{t_k\} p_k z^k / k} e^{\left(\frac{q}{t}\right)^k \frac{\{q^k\}}{z^k} \frac{\partial}{\partial p_k}}$$

- twist knots $\longleftrightarrow$ (exclusive) Racah matrices S, T



- colored HOMFLY

$$H_R^{(m,n)} = d_R \langle \varphi | \bar S \bar T^{2m} \bar S \bar T^{2n} \bar S | \varphi \rangle$$

- $\bar S$ is associator, $\bar T$ twists.

$$H_R^{(m,n)} \sim \sum_{x,y} c_{x,y} \lambda_x^{2m} \lambda_y^{2n}$$

if you diagonalise $\bar T$ ...

- but usually $\bar S$ is not well known

- rewrite $\langle \varphi | U^\tau U^{-\tau} \bar S \bar T^{2m+2} \bar S U^\tau$
$\cdot U^\tau \bar S \bar T^{-2} \bar S \bar T^{-2} \bar S \bar U^{-1}$
$\cdot \underbrace{U \bar S T^{2n+2} \bar S U^{-1}}_{:= B^{n+2}} U | \varphi \rangle$

- B triangular, $B_{xy} = 0$ if $y > x$
- $U_{x \emptyset} = 1$
- $U_{xy} = \delta_{xy} Z_R^{x} / \Lambda'_x$
- $Z_x = \langle \varphi | \bar S \bar T^2 \bar S \bar T^{-1} \bar S \bar U | \varphi \rangle$

- pentad structure

$$\begin{array}{ccccc} & & B & \longleftrightarrow & \mathcal{E} \text{ triang.} \\ & & \downarrow & & \\ U & \longleftarrow & \bar S & \longrightarrow & S \end{array}$$

$B = \mathcal{E} \bar T^2 \mathcal{E}$, $\bar S^2 = 1$, $\mathcal{E} = U \bar S$, $\bar S = \mathcal{E}^\tau \frac{Z}{\Lambda} \mathcal{E}$

$\bar T \bar S \bar T = S T^{-1} S^{-1}$