Porta.

- $A = \mathbb{M}_\beta^{(\leq N)} \times \{1, \dots, \dim(\mathcal{Y})\}$, $\mathbb{M}_\beta^{(\leq N)} = \{k_0 \in \mathbb{M}_\beta \mid \chi_N(k_0) > 0\}$
- $g_\alpha = \dfrac{\chi_N(u_0)}{-iu_0 + e_\alpha - \mu}$, $Hf_\alpha = e_\alpha f_\alpha$, $\chi_N(u_0) = \chi(\gamma^{-N}|u_0|)$
- $g^{(\leq N)}(z, z') = \dfrac{1}{\beta} \sum_{u_0 \in \mathbb{M}_\beta^{(\leq N)}} \sum_\alpha e^{-iu_0(z_0 - z_0')} \dfrac{f_\alpha(\vec{z}) \overline{f_\alpha(\vec{z}')}}{-iu_0 + e_\alpha - \mu}$

$$P_{\leq N}(d\psi) = \left[\prod_{\alpha \in A} g_\alpha\right] D\psi \exp\left\{-\sum_\alpha \psi_\alpha^+ g_\alpha^{-1} \psi_\alpha^-\right\}.$$

$$\psi_z^+ = \frac{1}{\sqrt{\beta}} \sum_{(u_0, \alpha) \in A} e^{iu_0 z_0} \overline{f_\alpha(\underline{z})} \psi^+_{(u_0, \alpha)}$$

$$\psi_z^- = \frac{1}{\sqrt{\beta}} \sum_{(u_0, \alpha) \in A} e^{-iu_0 z_0} f_\alpha(\underline{z}) \psi^-_{(u_0, \alpha)}$$

$$V(\psi) := \lambda \int dz\, dz'\, \psi_z^+ \psi_{z'}^+ \psi_{z'}^- \psi_z^- v(z - z')$$

$$\langle \vartheta(\psi) \rangle^{(\leq N)}_{\beta, L, \mu} = \int P_{\leq N}(d\psi) e^{-V(\psi)} \vartheta(\psi)$$

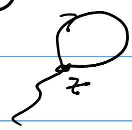
- free energy $f_{\beta,\mu,L,N} = -\dfrac{1}{\beta|\Lambda_L|} \log \int P_{\leq N}(d\psi) e^{-V(\psi)}$

- our model: $\int P_{\leq N}(d\psi) \psi^-_{z_1} \cdots \psi^-_{z_n} \psi^+_{w_1} \cdots \psi^+_{w_n}$
$$= \sum_\pi \operatorname{sign}(\pi) \prod_{i=1}^n g^{(\leq N)}(z_i - w_{\pi(i)}).$$

$\underline{N \to \infty}$? $\lim_{N \to \infty} g^{(\leq N)}(z, z') = g(z, z')$ $(z_0 \neq z_0')$.

- but at coinciding pts:
$$\Delta := \underbrace{\lim_{N \to \infty} g^{(\leq N)}(z, z)}_{\frac{1}{2}\left(g(z,z)\big|_{z_0 - z_0' = 0^+} + g(z,z)\big|_{z_0 - z_0' = 0^-}\right)} - g(z, z) \neq 0$$

$$= \frac{1}{2}\left(g(z,z)\big|_{z_0 - z_0' = 0^+} - g(z,z)\big|_{z_0 - z_0' = 0^-}\right)$$
$$= \frac{1}{2}\left(\langle a^+_{\underline{z}} a^-_{\underline{z}} \rangle - \left(-\langle a^-_{\underline{z}} a^+_{\underline{z}} \rangle\right)\right)$$
$$= \frac{1}{2} \langle \{a^+_{\underline{z}}, a^-_{\underline{z}}\} \rangle = \frac{1}{2}$$

- this discontinuity is only visible in tadpole diagrams 

$\rightsquigarrow g(z,z)$

- "fix" it by adding $-1/2$ to number operat.:

$$V = \lambda \sum_{z,z'} \left(a^+_z a^-_z - 1/2\right) v(z-z') \left(a^+_{z'} a^-_{z'} - 1/2\right)$$

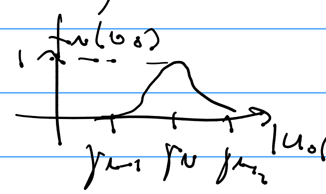
$$= V_{old} + c(v)\,\lambda/2 \cdot N$$ ← just a shift in μ

- multiscale analysis

$$\psi^{(\le N)} = \psi^{(\le N-1)} + \psi^{(N)}$$

cov.

$$g^{(N)}(z,z') = 1/\beta \sum_{k_0 \in \mathbb{M}_\beta^{(\le N)}} e^{-ik_0(z_0-z_0')} \frac{f_N(k_0)}{-ik_0+H-\mu}(\underline{z},\underline{z}')$$

where $f_N(k_0) = \chi_N(k_0) - \chi_{N-1}(k_0)$ 

<u>Claim:</u> $$|g^{(N)}(z,z')| \le \frac{C_M}{1+\big(2^N \underbrace{|z_0-z_0'|_\beta}_{= \min(|z_0-z_0'|,\, |z_0-z_0'-\beta|)}\big)^M} e^{-c\|\underline{z}-\underline{z}'\|} \quad \forall M \in \mathbb{N}$$

UV-multiscale analysis

- goal: $$\int P_{\le N}(d\psi^{(\le N)})\, e^{-V(\psi^{(\le N)})} \overset{\text{notation}}{=\!=} \mathbb{E}_{\le N}\left(e^{-V(\psi^{(\le N)})}\right)$$

$$= \mathbb{E}_{\le N-1}\, \mathbb{E}_N\left(e^{-V(\psi^{(\le N-1)}+\psi^{(N)})}\right)$$

$$= e^{E_{N-1}}\, \mathbb{E}_{\le N-1}\left(e^{-V^{(N-1)}(\psi^{(\le N-1)})}\right)$$

where $E_{N-1} = \log \int P_N(d\psi^{(N)})\, e^{-V(\psi^{(N)})}$

$V^{(N-1)}(\psi^{(\le N-1)}) = -E_{N-1} + \log \int P_N(d\psi^{(\le N)})\, e^{-V(\psi^{(\le N-1)}+\psi^{(N)})}$

Rewrite: $V(\psi) = \Sigma_P \int dz\, W_P^{(N)}(z)\, \psi_z(P)$.

$\psi_z(P) = \prod_{f\in P} \psi_{z(f)}^{\varepsilon(f)}$, $W_P^{(N)} = 0$ unless $\overbrace{|P|=4}^{\text{quartic. int.}}$

$W_P^{(N)} = \lambda\, \delta(z_1-z_2)\,\delta(z_2-z_3)\,\delta(z_3-z_4)$
for $v(z-z') = \lambda\delta(z_0-z_0')\,\delta_{\underline{z}\underline{z}'}$ (of c, $|P|=4$)

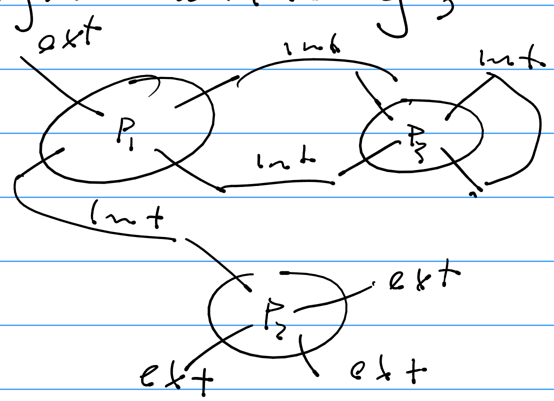
Cumulants: $\mathbb{E}_N^c(A_1(\psi^{(N)}),\dots,A_s(\psi^{(N)}))$
$:= \partial^s_{\varepsilon_1\dots\varepsilon_s} \log \mathbb{E}_N(e^{\Sigma_i \varepsilon_i A_i(\psi)})\big|_{\varepsilon=0}$
- conn. Feyn. diagrams

$\Rightarrow V^{(N-1)}(\psi^{(\leq N-1)}) = \sum_{|P|\geq 2} \int dz\, W_P^{(N-1)}(z)\, \psi_z^{(\leq N-1)}(P)$

where $W_P^{(N-1)}(z) = \sum_{s\geq 1} \frac{1}{s!} \sum_{\substack{P_1\dots P_s \\ \cup_i P_i^{ext}=P \\ |P_i|\geq 2}} \int dz_1^{int}\dots dz_s^{int}$

$\mathbb{E}_N^c(\psi_{z_1^{int}}^{(N)}(P_1^{int}),\dots,\psi_{z_s^{int}}^{(N)}(P_s^{int}))\cdot \prod_{i=1}^s W_{P_i}^{(N)}(z_i)$

diagrammatically,



$P_i = P_i^{ext} \cup P_i^{int}$