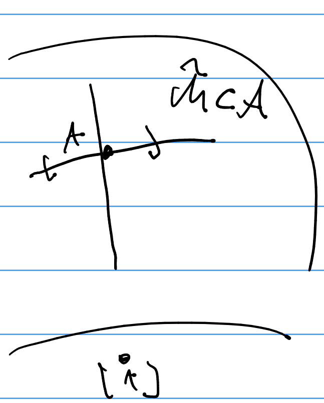
Тихомирова

- $b_2^- \neq 0,\ b_2^+ = 0 \Rightarrow (-,-)_B$ neg. def.
- Uhlenbeck shows: given $([A], g) \in \mathcal{M}(P,g)$, $\exists\, g'$ perturbation of g in $\mathcal{C}$ s.t. $P_{+,g'} \circ \nabla_A$ surjects
- $\underline{\mathcal{M}(P,g)} :=$ closure of $\mathcal{M}^*(P,g)$ in $\mathcal{B}(P)$
- Donaldson: $\mathcal{M}^*(P,g)$ is simply conn $\Rightarrow$ oriented
- in this case $\dim \mathcal{M}(P,g) = 8c_2(P) - 3$, and $c_2(P) = 1 \Rightarrow \dim \mathcal{M}(P,g) = 5$

- $\partial\mathcal{M} := \mathcal{M}(P,g) \setminus \mathcal{M}^*(P,g) = \{[A] \in \mathcal{M}(P,g) \mid A \text{ reducible}\}$
  $\Rightarrow 0 \leq \#\partial\mathcal{M} \leq b_2^-$

- recall A red. $\Leftrightarrow P = Q \times_{S^1} SU(2)$, $c_1(Q) = x \in \underset{\mathbb{Z}^{b_2^-}}{\underset{\shortparallel}{H^2(B,\mathbb{Z})}}$, $(x^2) = -1$

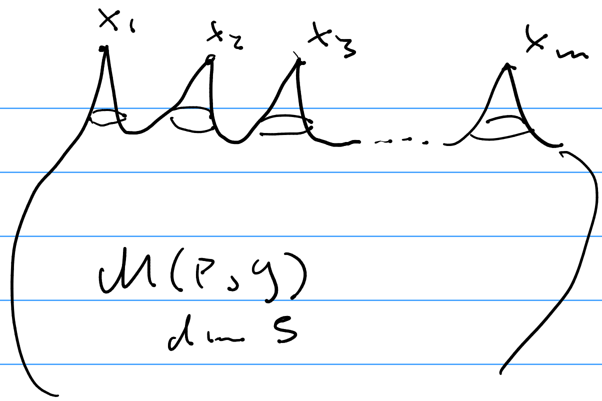
  - $\operatorname{rk}(\ ,\ ) = b_2^-$
- $\exists\, x : (x^2) = -1$, $|\det(\ ,\ )| = 1$
  $\forall\, y \in \mathbb{Z}^{b_2^-} = \mathbb{Z}x \overset{\perp}{\oplus} \mathbb{Z}^{b_2^- - 1}$
  - take $\mathbb{Z}$-basis in $\mathbb{Z}^{b_2^-}$, $x = e_1, \dots, e_{b_2^-}$
  $\Rightarrow y = \alpha e_1 + w$, $w \in (\mathbb{Z}e_1)^\perp$
  $\Rightarrow \alpha = -(y,x)$



$\tilde{\mathcal{M}} \cap$ slice $=$ submfd $\mathcal{U}$ of slice s.t. $T_A\mathcal{U} = \ker D_A \subset \mathbb{R}^6$

Slice $\times_{S^1} \mathfrak{g}_3 = \mathbb{R}^6 \simeq \mathbb{C}^3/S^1$
$=$ cone over $\mathbb{CP}^2$

- nbhd of $[A]$ in $\mathcal{M} \simeq$ slice $\cap\, \tilde{\mathcal{M}} / S^1$



$(x_m^2)_B = -1,\ m \le b_2$

- compact?

Thm (Uhlenbeck) $P \to B$ has $c_2(P) = k > 0$.
Let $\{A_n\}_{n \ge}$ be squc in $\mathcal{A}_2(P)$.
Then

i) $\exists$ SU(2)-bdl $P' \to B$ with $0 \le c_2(P') = k' \le k$ and ASD-conn. A' on P'

ii) $\exists$ pts $x_1, \dots, x_t$ and nonnegative integers $m_1, \dots, m_t$ s.t. $\sum_{i=1}^{t} m_i = k - k'$

iii) $\exists \{g_n \subset \mathcal{G}_3\}_{n \ge 1}$ s.t. for any cpt $K \subset B \setminus \{x_1, \dots, x_t\}$
$$g_n^* A_n|_K \xrightarrow{n \to \infty} A'|_K \text{ in } L^2_2(K)$$

iv) $|F_{A_n}|^2$ weakly converges to
$$|F_{A'}|^2 + 8\pi^2 \sum_{i=1}^{\infty} m_i \delta_{x_i},$$
by which we mean $\forall f \in \mathcal{C}(B)$,
$$\int_B f\, |F_{A'}|^2 dVol \longrightarrow \int_B f\left(|F_{A'}|^2 + 8\pi^2 \sum m_i \delta_{x_i}\right) dVol$$

- so, labelling $P_r \to B$ SU(2)-bdls w/ $c_2(P_r) = r$,
$\mathcal{M}_r := \mathcal{M}(P_r)$, $\mathcal{M}_k \ni [A_n] \xrightarrow{n \to \infty} [A'] \in \mathcal{M}_{k'} \times \mathrm{Sym}^{k-k'}(B)$

$$\hat{\mathcal{M}}_k := \bigsqcup_{0 \le r \le k} \mathcal{M}_r \times \mathrm{Sym}^{k-r} B \supset \mathcal{M}_k$$

$\overline{\mathcal{M}}_k^{UD} :=$ closure of $\mathcal{M}_k$ in $\hat{\mathcal{M}}_k$ ← Uhlenbeck–Donaldson cptification

- $\overline{\mathcal{M}_2}^{uo} = \mathcal{M}_2 \sqcup \underbrace{\mathcal{M}_1 \times \mathbb{C}P^1}_{=\emptyset} \sqcup S^2\mathbb{P}^2 = \mathbb{P}^5$

$\rightarrow$ Kobayashi - Hitchin correspondence

- $\mathcal{M}_2(S^4) = \dfrac{SO(5,1)}{SO(5)}$ $\rightarrow (5+1)(5+1-1)/2$, $\rightarrow 5 \cdot 4/2$

$\hookrightarrow$ conformal gp

$\rightarrow *$ is conf. inv on Ω^2