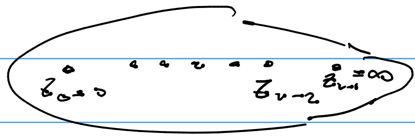
G + P seminar

Fredholm determinants pt. 2.



$$\partial_z \varphi(z) = \varphi(z) \underbrace{\left(\sum_{i=0}^{n-2} \frac{A_i}{z - z_i} \right)}_{A(z)} \qquad (1)$$

- monodromies $\varphi(\gamma_i \cdot z) = M_i \varphi(z)$
- $\partial_{z_i} \varphi(z) = -\varphi(z) \frac{A_i}{z - z_i} \qquad (2)$

$(1) + (11) \Rightarrow \left[\overleftarrow{\partial_z} - A(z), \overleftarrow{\partial_{z_i}} + \frac{A_i}{z - z_i} \right] = 0$

$$\Downarrow$$

$$j \neq i \qquad \partial_{z_i} A_j = - \frac{[A_i, A_j]}{z_i - z_j}$$

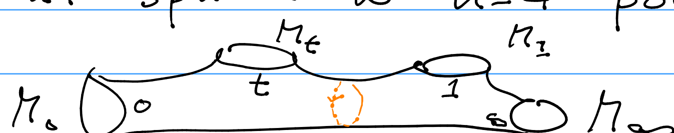
$$j = i \qquad \partial_{z_i} A_i = \sum_{k \neq i} \frac{[A_i, A_k]}{z_i - z_k}$$

- now let $H_i = \frac{1}{2} \operatorname{Res}|_{z = z_i} \operatorname{tr} A(z)^2 dz$ hamiltonian

$\Rightarrow \partial_i H_j = \partial_j H_i$

$\Rightarrow H_i = \partial_i \log \tau$

- look at sphere w $n=4$ punctures



compute monodromy $M_{0t} = M_0 \cdot M_t$

$$\varphi_{in}(e^{2\pi i} z) = \underbrace{M_{0t}}_{e^{2\pi i \hat{\Sigma}_{0t}}} \varphi_{in}(z) \Rightarrow \varphi_{in}(z) = z^{\hat{\Sigma}_{0t}} \underbrace{\psi_{in}(z)}_{\sum \frac{c_i}{z_i}}$$

$$\psi_{out}(z) = \psi_+(z) = \sum z^i \tilde{c}_i$$

$$a(z, z^{-1}) = \frac{\mathbb{1} - \psi_+(z) \psi_+(z')^{-1}}{z - z'}, \quad d(z, z') = \frac{\psi_-(z) \psi_-(z')^{-1} - \mathbb{1}}{z - z'}$$

$$\mathcal{H}_+ = \mathbb{C}[[z]] \otimes \mathbb{C}^N$$
$$\mathcal{H}_- = z^{-1} \mathbb{C}[[z^{-1}]] \otimes \mathbb{C}^N$$

$$(\alpha \cdot f)(z) = \oint \frac{dz'}{2\pi i} a(z, z') f(z')$$

$$\Rightarrow \quad a : \mathcal{H}_- \to \mathcal{H}_+$$
$$d : \mathcal{H}_+ \to \mathcal{H}_-$$

$$\tau(t, \text{monodromies}) = t^{\frac{1}{2}\operatorname{tr}(B_{0t}^2 - A_0^2 - A_t^2)} \times \det\begin{pmatrix} \mathbb{1} & a \\ d & \mathbb{1} \end{pmatrix}$$
$$= t^{\#} \det_{\mathcal{H}_+}(\mathbb{1} - a d)$$

$$P_- f := \frac{1}{2\pi i}\oint_{|z'|<|z|} \frac{dz'}{z - z'} f(z') = (f(z))_- = \sum_{n=1}^{\infty} f_{-n} z^{-n}$$

$$a = -P_+ + \Psi_+ P_+ \Psi_+^{-1}, \quad d = -P_- + \Psi_- P_- \Psi_-^{-1}$$

$$\Rightarrow \hat{\tau} = \det_{\mathcal{H}_+}\left(\mathbb{1} - (\quad)(\quad)\right)$$
$$= \det_{\mathcal{H}_+}\left(\mathbb{1} - \Psi_+ P_+ \Psi_+^{-1} \Psi_- P_- \Psi_-^{-1}\right)$$
$$= \det_{\mathcal{H}_+}\left(\mathbb{1} - P_+ \underbrace{\Psi_+^{-1}\Psi_-}_{J(z)^{-1}} \underbrace{P_-}_{1-P_+} \underbrace{\Psi_-^{-1}\Psi_+}_{J(z)} P_+\right)$$
$$= \det_{\mathcal{H}_+}\left(P_+ J(z)^{-1} P_+ J(z) P_+\right)$$

Widom constant

$$\partial_t \log \hat{\tau} = \operatorname{tr}\oint \frac{dz}{2\pi i} J^{-1} \partial_t J \left((\partial_z \Psi_-)\Psi_-^{-1} + \Psi_+^{-1}\partial_z \Psi_+\right)$$
$$= \underbrace{\operatorname{res}\Big|_{z=t} \frac{1}{2}\operatorname{tr} A(z)^2 dz}_{\partial_t \log \tau} + \underbrace{\operatorname{tr}\frac{\bar{A}_0 \bar{A}_1}{t}}_{\log t \cdot (B_{01}^2 - A_0^2 - A_1^2)}$$