Gauge @ IGAP

Nakajima cont'd

- G cpx reductive gp, $M = N \oplus N^*$ symp. G rep
  $\mathcal{O} = \mathbb{C}[[z]] \subset K = \mathbb{C}((z))$, $D = \operatorname{Spec}\mathcal{O}$, $D^\times = \operatorname{Spec} K$
  $[G(\mathcal{O})\backslash R]$ = mod. st. of G-bdls + N-val sections over "raviolo" $\Omega = D \sqcup_{D^\times} D$
  - note: any G-bdl over D is trivial
- $H_*([G(\mathcal{O})\backslash R]) = H_*^{G(\mathcal{O})}(R)$ comm. ring
- $\mathcal{M}_C := \operatorname{Spec}$ ↖
- $\mathcal{A}_\hbar = H_*^{G(\mathcal{O}) \rtimes \mathbb{C}^\times}(R)$

- $\mathbb{C}^\times \curvearrowright \mathcal{M}_C$ ⇝ extends to (nonhol.) $SU(2)$-action rotating hyperk struct.
- $G(\mathcal{O}) \curvearrowright Gr_G$ orbit ⟷ dominant coweight

  $G = GL(n)$ ... $\lambda(t) = \begin{bmatrix} t^{\lambda_1} & & \\ & \ddots & \\ & & t^{\lambda_n} \end{bmatrix}$, $T \subset G$ max torus
  $\lambda_1 \geq \dots \geq \lambda_n$, $\lambda_i \in \mathbb{Z}$
  - analog of Schubert cell

$\underbrace{Gr_G^\lambda}_{\text{orbit corr. to } \lambda} \longrightarrow G/P_\lambda$ $\left(\text{e.g. } \begin{pmatrix} \llcorner\!\!\urcorner & * \\ 0 & \llcorner\!\!\urcorner \end{pmatrix}\right)$

$$\begin{array}{ccc} R & \longrightarrow & Gr_G \\ \cup & & \\ R^\lambda & \longrightarrow & Gr_G^\lambda \end{array}$$

$$R = \bigsqcup_{\lambda:\ \text{dominant coweight}} R^\lambda$$

$$\Rightarrow P_t(H_*^{G(\mathcal{O})}(R)) = \sum_\lambda P_t(\underbrace{H_*^G(G/P_\lambda)}_{= H^*_{P_\lambda}(pt)}) \cdot t^{\Delta(\lambda)}$$

- now, LHS = $ch_{\mathbb{C}^\times} \mathbb{C}[\mathcal{M}_C]$

RHS = combinatorial

- inspired by Cremonesi-Hanany-Zaffaroni monopole formula

- another grading by fugacity:

$\pi_0(R) = \pi_0(Gr_G) = \pi_0(Map(S^1, G_c)/G_c)$

$= \pi_1(G)$

$\Rightarrow H_*^{G(\mathcal{O})}(R)$: $\pi_1(G)$-graded

$\rightsquigarrow \mathcal{M}_C \curvearrowleft \pi_1(G)^\wedge = Hom_{gp}(\pi_1(G), \mathbb{C}^\times)$ Pontryagin dual

- e.g.

$G = \mathbb{C}^\times \rightsquigarrow \pi_1(G) = \mathbb{Z}$, $\pi_1(G)^\wedge = \mathbb{C}^\times$

$N = 0 \rightsquigarrow \mathcal{M}_C = \mathbb{C} \times \mathbb{C}^\times$

$N = \mathbb{C} \rightsquigarrow \quad = \mathbb{C} \times \mathbb{C}$

Rmk Recall (G, N) defines $\mathcal{N}=2$ 4d Coulomb branch for $\mathbb{R}^3 \times S^1$

- replace $H_*^{G(\mathcal{O})}(R)$ by $K^{G(\mathcal{O})}(R)$, still commuting

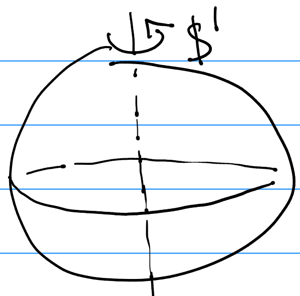
$\mathcal{M}_C^K := Spec\, K^{G(\mathcal{O})}(R)$

- 4d Coulomb br. should be hyperkähler w S^1-action rotating cpx. str.

$\underset{\text{similar}}{\sim}$ mod. sp. of Higgs bdles on a RS Σ

$\parallel$

$Hom(\pi_1(\Sigma), G)/_{conj}$ char. variety



Expectation (Gaiotto)

$\mathcal{M}_C^k$ = Coul. br. of 4d $\mathcal{N}=2$ SYM w/ generic cpx str

→ not very interesting if cpx str generic
→ to do instanton counting, we need the S^1-fixed cpx str, not any odd one!

Higgs branch of class S

- so, its Coulomb branch is Hitchin m.sp. on Σ
- Moore–Tachikawa
- enough to construct on $\mathbb{P}^1$ w/ 0, 1, 2, 3 punctures
- key point:
  use $R \xrightarrow{\pi} Gr_G \xrightarrow{p} pt$
  - can pushforward sheaves to pt
  - $H_*^{G(\mathcal{O})}(R) = H^*_{G(\mathcal{O})}(p_* \pi_* \omega_R)$
    ↑ dualising cpx on R

$$D^{G(\mathcal{O})}_{const}(Gr_G) \supset Perv^{G(\mathcal{O})}(Gr_G) \underset{\text{Satake}}{\overset{\text{geom.}}{=\!=}} Rep\, G^L$$

abelian
$G(\mathcal{O})$-equiv. perverse shvs

monoidal

$\pi_* \omega_R$ is a ring object, commutative, in $D^{G(\mathcal{O})}_{cat.}(Gr_G)$
$\|$
$\mathcal{A}$ $\circ$ m: $\mathcal{A} * \mathcal{A} \to \mathcal{A}$

- so, instead of R, give a comm. ring object $\mathcal{A}$ in $D^{G(\mathcal{O})}_{cont}(Gr_G)$ to define Higgs br as $\operatorname{Spec} H^*_{G(\mathcal{O})}(\mathcal{A})$