QFT.

Effective field theory

- sometimes a good idea to pass to eff. th even if full theory known (e.g. SM)
  -> if theory not known, also good !! (gravity)

$S_{UV}(\varphi_H, \varphi_L)$.

- ordinary approach $e^{i\Gamma_{1PI}(\varphi_H,\varphi_L)} = \int_{1PI} D\tilde{\varphi}_H D\tilde{\varphi}_L e^{iS_{UV}(\varphi_H+\tilde{\varphi}_H, \varphi_L+\tilde{\varphi}_L)}$

- now integrate out, $e^{iS_{IR}(\varphi_L)} = \int_{\substack{1PI \\ L\text{-only}}} D\varphi_H \, e^{iS_{UV}(\varphi_H,\varphi_L)}$

$$e^{i\Gamma_{1PI}(\varphi_L)} = \int_{1PI} D\tilde{\varphi}_L e^{iS_{IR}(\varphi_L+\tilde{\varphi}_L)}$$

-> note that S_{IR} is local (heavy particles, short range, are int. out)

-> $\Gamma_{1PI}(\varphi_H,\varphi_L)$ & $\Gamma_{1PI}(\varphi_L)$ are not

- @ tree lvl $S_{IR}(\varphi_L) = S_{UV}(\varphi_H(\varphi_L), \varphi_L)$, where $\frac{\delta}{\delta\varphi_H} S_{UV} = 0 \Rightarrow \varphi_H = \varphi_H(\varphi_L)$.

-> for quantum case, write most general action S_{IR}, compatible w symm., up to some operator dimension, since full S_{IR} has ∞-ly many terms.
  -> not really a problem, operators get surpressed $\sim \frac{1}{m_H^{n>1}}$

- write $S_{EFT}(\varphi_L) = S_{IR}^{(0)}(\varphi_L) + \Delta S(\varphi_L)$

- we also need MATCHING: $g_{IR} = g_{IR}(g_{UV})$.

not bounded from below, but who cares!?

2 scalars. $\mathcal{L}_{UV} = \frac{1}{2}(\partial H)^2 - \frac{1}{2}M^2 H^2 + \frac{1}{2}(\partial L)^2 - \frac{1}{2}m^2 L^2 - \frac{g}{2} H L^2$, $[g] = 1$ $\sim O(M)$

$m \ll M$

$\Rightarrow \mathcal{L}_{IR} = \frac{1}{2} Z_L (\partial L)^2 - \frac{1}{2}\tilde{m}_L^2 L^2 - \frac{\lambda}{4!} L^4 + \text{h.d.o.}$

$\to$ L's $\mathbb{Z}_2$ symm unbroken.

- matching: $(-\Box - M^2) H - \frac{g}{2} L^2 = 0$

$\Rightarrow H = -\frac{g}{2M^2} L^2$

$\to$ because $H(x) = -\frac{ig}{2}\int d^4y \underbrace{G(x,y)}_{\simeq -\frac{i}{M^2}\delta(x)} L^2(y)$

$\Rightarrow \mathcal{L}_{UV}(\varphi_L) = \frac{1}{2}\frac{g^2}{M^2} L^2 (\partial L)^2 - \frac{1}{2} M^2 \frac{g^2}{4M^2} L^4 + \frac{1}{2}(\partial L)^2 - \frac{1}{2} m^2 L^2 - \frac{g}{2} L^2 \left(-\frac{g}{2M^2} L^2\right)$

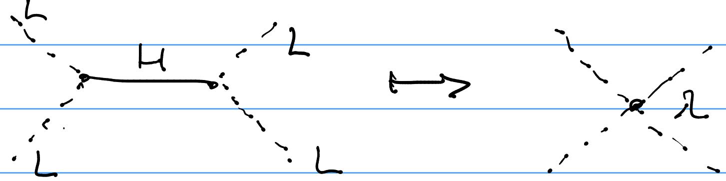
$\Rightarrow \mathcal{L}_{IR}^{(0)}(\varphi_L) = \frac{1}{2}(\partial L)^2 - \frac{1}{2} m^2 L^2 + \frac{g^2}{8M^2} L^4 + \text{h.d.o.}$

$\Rightarrow$ $Z_L = 1 + O(g)$

$\tilde{m}^2 = m^2 + O(g^2)$

$\lambda = -\frac{3g^2}{M^2} + O(g^4)$

- diagramatically, integrating out = shrinking propagators to a pt:



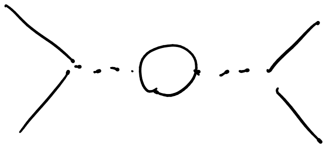
$\Gamma^{(4)}(s,t,u) = -\lambda(\mu) + \frac{\lambda^2(\mu)}{32\pi^2}\int_0^1 dx \log\frac{m^2 - s x(1-x)}{\mu^2} + (s \to t) + (s \to u)$

set $s = \mu$:

$\Gamma^{(4)}(\mu) = -\lambda(\mu) \Rightarrow \lambda^{-1}(\mu) = \lambda^{-1}(\mu_0) - \frac{3}{16\pi^2}\log\frac{\mu}{\mu_0}$, $\mu, \mu_0 > m$

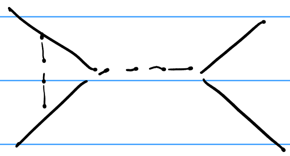
$\lambda(\mu) = -3\frac{g(\mu)^2}{M^2} + \dots$

- let's calculate using diagrams



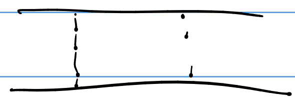
+ perms. $= i\left(\frac{1}{2}\times 3\right)(-ig)^4 \mu^{2\varepsilon}\int \frac{d^dk}{(2\pi)^d}\left(\frac{i}{k^2-m^2}\right)^2\left(\frac{-i}{M^2}\right)^2$

$\overset{MS}{=} \frac{3ig^4}{32\pi^2 M^4}\log\frac{m^2}{\mu^2}$



+ perms. $= 6(-ig)^4\mu^{2\varepsilon}\int\frac{d^dk}{(2\pi)^d}\left(\frac{i}{k^2-m^2}\right)\left(-\frac{i}{M^2}\right)\left(\frac{i}{k^2-m^2}\right)$

$= \frac{3ig^4}{8\pi^2M^4}\left(1+\log\frac{m^2}{\mu^2}\right)$, finite in $d=4$?!



+ perms. $= 6(-ig)^4\mu^{2\varepsilon}\int \frac{d^dk}{(2\pi)^d}\left(\frac{i}{k^2-m^2}\right)\left(\frac{i}{k^2-M^2}\right)$

$= \frac{6ig^4}{8\pi^2M^4}\left(1+\frac{1}{2}\log\frac{m^2}{M^2}\right)$, very finite!

$\Rightarrow \Gamma^{(4)}_{UV}(0) = \frac{3g^2(\mu)}{M^2} + \frac{3g^4}{8\pi^2M^4}\left(3+2\log\frac{m^2}{\mu^2}+\frac{1}{4}\log\frac{m^2}{\mu^2}\right)$

$\Gamma^{(4)}_{IR}(0) = -\lambda(\mu) + \frac{3\lambda^2}{32\pi^2}\log\frac{m^2}{\mu^2}$

$\rightarrow$ match them to get

$$\lambda(\mu) = -\frac{3g^2(\mu)}{M^2} - \frac{3g^4}{32\pi^2M^2}\left(3+2\log\frac{\mu^2}{M^2}\right)$$

- note that $\log\frac{m^2}{-}$ disappeared
- not a coincidence.