Supermoduli @ IGAP

Sgeometry & Smoduli I - D. Hernández Ruipérez

- Kostant model - graded coords. $(\vec{x}, \theta_1, \dots, \theta_n)$

Def. A superscheme is $\mathcal{X} = (X, \mathcal{O}_\mathcal{X})$ where
  1) X is an (ordinary) scheme
  ii) $\mathcal{O}_\mathcal{X}$ is a $\mathbb{Z}_2$-gr. comm. alg, $\mathcal{O}_\mathcal{X} = \mathcal{O}_{\mathcal{X},0} \oplus \mathcal{O}_{\mathcal{X},1}$
  iii) If $\mathcal{J} = (\mathcal{O}_{\mathcal{X},1})^2 \oplus \mathcal{O}_{\mathcal{X},1}$, then
    a) $\mathcal{O}_X = \mathcal{O}_\mathcal{X} / \mathcal{J}$
    b) ...

- a sheaf $\mathcal{M}$ of $\mathbb{Z}_2$-gr. $\mathcal{O}_\mathcal{X}$-modules is loc. free of rk (p, q) if locally isom. to $\mathcal{O}_\mathcal{X}^{\oplus p} \oplus \underbrace{\Pi}_{\text{parity change}} \mathcal{O}_\mathcal{X}^{\oplus q}$

Superstring pert. th using picture changing - A. Sen

- bosonic string
  - P.I. over 2-dim metric (+ other fields)
  - 2d diffeo inv. + Weyl rescalings
    $\rightarrow$ reduces to int. over $\mathcal{M}_{g,n}$ for n-string processes
  - the gauge fix induces F.P ghosts $b, c, \bar{b}, \bar{c}$ but we delegate those to "other fields"
- 2d CFT
  i) state-operator corresp., $|\varphi\rangle = \varphi(0,0)|0\rangle$
  ii) BRST op. Q_B w BRST currents $j_B(z), \bar{j}_B(\bar{z})$
    - $Q_B^2 = 0$
    - $Q_B|\varphi\rangle \xleftrightarrow[\text{op.}]{\text{state}} [Q_B, \varphi\} = \left(\oint_z dw\, j_B(w) + \text{c.c.}\right) \varphi(z, \bar{z})$

- physical states are $H^0(Q_B)$

- amplitudes of $|A_1\rangle, \ldots |A_n\rangle$

$$\mathcal{A}(A_1, \ldots, A_n) = (-2\pi i)^{3g-3+2n} \int_{M_{g,n}} \omega_{6g-6+2n}(A_1, \ldots, A_n)$$

with $\omega_{6g-6+2n} = \left\langle \left\{ \prod_{i=1}^{6g-6+2n} (\eta_i | B) dm^i \right\} A_1 \ldots A_N \right\rangle_{\Sigma_{g,n}}$

$$(\eta_i | B) = \int d^2 z \, \eta_{i\bar{z}}{}^{z} \underbrace{b(z)}_{\text{ghost } b} + \text{c.c.}$$

Beltrami diff., $g_{z\bar{z}} \delta g^{zz} = \sum_{i=1}^{6g-6+2n} \delta m^i \eta_{i\bar{z}}{}^{z}$

moduli

- define p-forms ω_p by

$$\left\langle \exp\left(\sum_i (\eta_i | B) dm^i \right) A_1 \ldots A_n \right\rangle_{\Sigma_{g,n}} = \sum_p \omega_p$$

- we have $\sum_i \omega_p(A_1, \ldots, Q_B A_i, \ldots, A_n)$
$= (-)^p d\omega_{p-1}(A_1, \ldots, A_n)$

- in case of exact $|A_n\rangle = Q_B|\lambda\rangle$, say,
$\omega_{6g-6+2n}(A_1, \ldots, A_{n-1}, \lambda) = d\,\omega_{6g-6+n}(\ldots)$
so $\mathcal{A}(A_1, \ldots, A_{n-1}, Q_B|\lambda\rangle) = 0$ up to bdary terms ($\equiv 0$ in sensible theories)

- now superstrings
  - 2d metric + 2d gravitino χ + matter
  - gauge symm.'s
    - 2d supersym., super Weyl
    - metric $\longrightarrow$ moduli
    - $\chi \longrightarrow$ partly fixed, $\chi = \sum_\alpha \overset{\text{odd}}{\zeta_\alpha} f(z, \bar{z})$

- convenient choice $f_\alpha(z,\bar z) = \delta^{(2)}(z - z_\alpha)$ on $M_{g,n}$
- action $\int d^2z\, \chi(z,\bar z)\, G(z,\bar z)$

$$\Rightarrow \int \prod_\alpha d\zeta_\alpha \exp \prod_\alpha \zeta_\alpha G(z_\alpha, \bar z_\alpha) = \prod_\alpha G_\alpha(z,\bar z)$$

- gauge fixing for χ gives additional <u>even</u> ghosts β, γ

$\Downarrow$

- $(\eta_i | B)$ factors have analogue

$$\delta((\eta_i|B)) = \delta\left(\int d^2z\, f_\alpha(z,\bar z)\beta(z)\right) = \delta(\beta(z_\alpha))$$

- $G(z_\alpha)\,\delta(\beta(z_\alpha)) = X(z_\alpha)$
  - picture changing op.

- employ bosonisation:

$(\beta,\gamma) \mapsto (\underbrace{\xi, \eta}_{odd}, \underbrace{\varphi}_{even})$ s.t. $\beta = \partial\xi e^{-\varphi}$, $\gamma = \eta e^{\varphi}$

- picture #:
  - 1 to ξ
  - (-1) to η
  - q to $e^{q\varphi}$
  - $\#(\beta,\gamma) = 0$

Forms in Supergeometry – S. Noja

- super de Rham cpx not bounded from above, since for local coords $x_1, \dots, x_p | \vartheta_1, \dots, \vartheta_q$, dx_i are odd, but $d\vartheta_j$ are <u>even</u>, so $(d\vartheta_j)^m \neq 0$

Intro 2 SUSY in view of Holography - P. Fré

- susy (rigid or sugra) usally closes only on-shell
  - in some cases, off-shell closure can be obtained by adding auxiliary fields
  - but this is not necessarily bad
    → by demanding closure, we deduce e.o.m
    → we can deform theories, whereas off-shell susy is usually "too unique"
    → can be traced to freedom to choose a section of tgt mfd, if we look at scalar fields as coordinates

- we look at Chevalley cohomology
  - given $[T^A, T^B] = f^{ABC} T^C$, dual MC-forms in $T^*\mathfrak{g}$ $e_A(T^B) = \delta^A{}_B$, then $\delta e_C = \frac{1}{2} f_{CAB}\, e_A \wedge e_B$
  - minimal FDA: if $M^k = F^k \Omega^\bullet$, $dM^k \subset M^k \wedge M^k$ (no $(k+1)$-forms)
  - contractible FDA: $dC^k \subset C^{k+1}$ (sort of trivial)
  - Sullivan-Lem: every FDA is semidir. pr. of min & contr. one

Rmk (Fré): contractible ⇝ curvatures!
  - minimal alg is gauged ...
    → put all "contractibles" = 0 ⇒ consistent, "ungauged" alg
    - curvatures now live everywhere

D=11 sugra (via FDA)

$$T^a = DV^a - i\frac{1}{2}\bar\psi \wedge \Gamma^a \psi$$

$$R^{ab} = d\omega^{ab} - \omega^{ac}\wedge\omega^{cb}$$

$$\rho = D\psi \equiv d\psi - \frac{1}{4}\omega^{ab}\wedge\Gamma_{ab}\psi$$

$$F^{[4]} = dA^{[3]} - \frac{1}{2}\underbrace{\bar\psi\wedge\Gamma_{ab}\psi\wedge V^a\wedge V^b}_{}$$

closed due to 11d Fierz id's, so can be written as $dA^{[3]}$

- also, can be seen by observing that $(\text{Spinor}^{\otimes 4})_{sym} \not\ni$ vector rep

$F^{[7]}$ also exists!

$$= dA^{[6]} - 15F^{[4]}\wedge A^{[3]} - \dots$$

→ $A^{[6]}$ magnetic dual of $A^{[3]}$

→ no need for strings!

- setting all LHS = 0 ⇒ MC eqns

Rmk (to me):

$$\begin{tikzcd}
\text{superspace} \arrow[r, "\text{superdiffeos}"] & \text{superspace} \arrow[d, "\text{restriction}"] \\
\text{spacetime fields} \arrow[u, "\text{rheonomy}"] \arrow[r, "\boxed{\text{SUSY}}"] & \text{spacetime fields}
\end{tikzcd}$$

Fré, cont.

Sen, cont.

- heterotic string
  - fields: $b, c, \bar{b}, \bar{c}, \xi, \eta, \varphi$, matter
  - state-op $|\varphi\rangle \longleftrightarrow \varphi(0)|0\rangle$
  - picture no: $\xi: 1$, $\eta: -1$, $e^{q\varphi}: q$
  - pict. ch. op $\chi(z) = \{Q_B, \xi(z)\}$: p. no. 1
- small Hilb. sp
  - $|\varphi\rangle \in \mathcal{H}_{small}$ iff $\eta_0|\varphi\rangle = 0$
    where $\eta_0 = \oint_0 dw\, \eta(w)$
  - phys. states $\Longleftrightarrow$ Q_B-coh in $\mathcal{H}_{small}$
  - $|\varphi_{phys}\rangle \sim |\varphi_{phys}\rangle + Q_B|\lambda\rangle$
    $\hat{\mathcal{H}}_{sm.}$
- 2 classes of states
  - $\longrightarrow$ NS: pict no -1
  - $\longrightarrow$ R: $-1/2$
  - on Σ_g we need pict. no $2g-2$ to get $\neq 0$ correl. fns
  - insert $2g-2+m+\frac{n}{2}$ PCO's for amplitude w n R and m NS states
  - $\rightarrow \mathcal{M}_{g,m,n} = \{\Sigma_g + {}^{n\,R}_{m\,NS}$ punctures$\}/_{iso}$

$$\mathcal{A}(A_1, \ldots, A_{m+n}) = (-2\pi i)^{\cdots} \int_{\Sigma_{g\,m\,n}} \omega_{top}(A_1, \ldots, A_{m+n})$$

$$\sum \omega_p = \Big\langle \xi(z_0) \exp\Big[\sum_{i=1}^{6g-6+2m+2n} (\eta_i|B)\, dm^i - \sum_{\alpha=1}^{2g-2+m+\frac{n}{2}} \partial\xi(z_\alpha)\, dz_\alpha \frac{1}{\chi(z_\alpha)}\Big] \Big\{\prod_{\alpha=1}^{2g-2+m+\frac{n}{2}} \chi(z_\alpha)\Big\} A_1 \ldots A_{m+n} \Big\rangle_{\Sigma_{g,m,n}}$$

(I) ω_p is z_0-indep if $A_i \in \mathcal{H}_{small}$

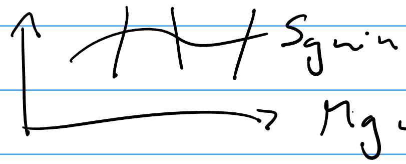
(II) $\omega_p(Q_B A_1, \ldots, A_{m+n}) + \cdots = -(-)^p \, d\omega_{p-1}(A_1, \ldots, A_{m+n})$

$\Rightarrow \mathcal{A} = 0$ if $A_i = Q_B|\lambda\rangle$

-section-indep. $S'_{gmn} - S_{gmn} = \partial R$

-problem: spurious poles

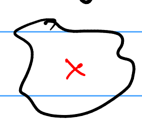
- ω_p has sing's on $\mathrm{codim}_{\mathbb{R}} = 2$ subspaces



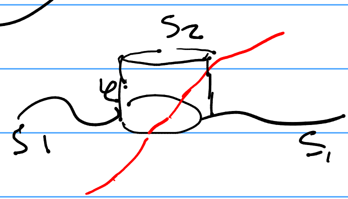
1402.0571
1504.00609

-vertical integration:

- top view



- side view



, $\mathcal{C}$ = cyl.

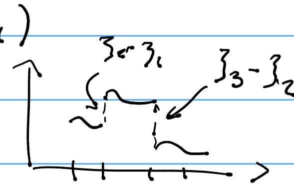
- integrate over $S_1 \cup \mathcal{C} \cup S_2$

-idea similar to path $a \to b$ independence of $\int_a^b \frac{dz}{(z-z_0)^2}$, noting we have $\partial\xi$ in integrand

-for more than one PCO, move them in pairs $(z_1^{(1)}, z_2^{(1)}) \to (z_1^{(2)}, z_2^{(1)}) \to (z_1^{(2)}, z_2^{(2)})$

- so this integration just gives a difference $\xi(z_2) - \xi(z_1)$

-complete perscription:



i) divide M_{gmn} into cells

ii) on each cell choose PCO avoiding spurious poles (connected by vert int $\int \partial\xi$)

H.D-R.

$$\begin{tikzcd} X_0 \arrow[r, hook] \arrow[d] & X \arrow[d, "\pi"] \\ \operatorname{Spec} k \arrow[r, hook, "s_0"] & S \end{tikzcd}$$

deformation. $s_0 \in S$ distinguished point

- infinitesimal if $k = k[\varepsilon_0, \varepsilon_1]$

Tanzini

- quiver gauge theories as effective actions of brane systems
  → study moduli spaces of quivers
- D3-D7 on local sfc S
  - $X_S = \mathrm{tot}(V_S^3)$
    with $V_S^3 = K_S \otimes \det^{-1} V_S^2 \otimes V_S^2$
  - n D3 branes on S, r D7's on total space
  - D3: refined Vafa-Witten th
    (+ chiral fields from D7 bckgrd)
  - D7: (equivariant) Donaldson-Witten
- consider $S = E \times C$
  E → ell. curve
  C → sm. proj. curve w mkd pts $D = \sum_{i=1}^{k} p_i$
- sfc ops in D3-br are $\mathrm{codim}_{\mathbb{R}}\, 2$ defects at $E \times p_i$
  - in small C area limit, defects described by GLSM, flowing to NLSM on $U_{g,r,\underline{\lambda},n,\underline{\mu}}$, moduli sp. of quiver reps

- local descr. of defects

$\mathcal{C}$ curve $\longrightarrow$ orbicurve $\tilde{\mathcal{C}}$,

ws p_i obtained by erasing

Disk p_i, replacing w

$s_i : 1$ cover $(\tilde{z}_i = z_i^{s_i})$

D. H. R