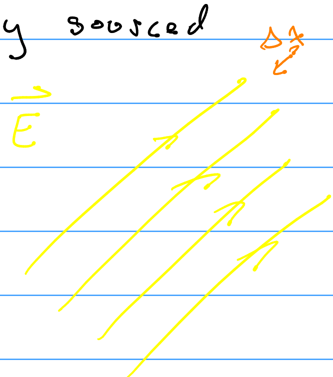
Qft

Vacuum decay in qft.

- consider a very strong uniform electric field, infinitely sourced



- if $\Delta E \sim \Delta x \, e|\vec{E}|_c \sim 2m$, $\Delta x \sim \frac{1}{m}$ de Broglie, then $|\vec{E}|_c = \frac{2m^2}{e}$ is the critical value at which vacuum decays (pair production pairs repel)

$\rightarrow$ we expect a decay $\Gamma \sim e^{-|E|_c/|E|}$

Landau levels in q.m. (review)

$$H = \frac{p^2}{2m} + \frac{(k_y - eBx)^2}{2m}$$ for const. mag. field in z-direction, since $A = \vec{B} \times \vec{e}_y = B\, \vec{e}_z \times \vec{e}_y$

$$\Rightarrow H = \frac{p^2}{2m} + \frac{m}{2}\omega_c (x - x_0)^2, \quad \omega_c = \frac{eB}{m}, \quad x_0 = \frac{k_y}{eB}$$

$\Rightarrow E_n = (n + \frac{1}{2})\omega_c$, but $k_y = \frac{2\pi}{L_y} n_y$.

$\rightarrow$ since $x_0 = \frac{k_y}{eB} = \frac{2\pi}{L_y}\frac{1}{eB} n_y$, we demand $|x_0| \le L_x$, so $n_y \le \frac{eBA}{2\pi}$, $A = L_x L_y$.

$\rightarrow$ the degeneracy is $g_n = \frac{eBA}{2\pi}$

- consider the path integral $\langle \vec{q}_a | e^{-iH(t_a - t_b)} | \vec{q}_b \rangle = \mathcal{N} \int_{\vec{q}(t_{a,b}) = \vec{q}_{a,b}} D\vec{q}\; e^{i\int_{t_a}^{t_b} dt\, L(\vec{q})}$

$$Z(\beta) = \mathrm{tr}\, e^{-\beta H} = \sum g_n e^{-\beta E_n}.$$

- but also, $Z(\beta) = \int d\vec{q}_a \langle \vec{q}_a, \tau = \beta | e^{-\beta H} | \vec{q}_a, \tau = 0 \rangle$

$$= \int_{\vec{q}(0) = \vec{q}(\beta)} D\vec{q}\; e^{-\int_0^\beta d\tau\, L_E(\vec{q})}$$

- we know $L_E = -\frac{m}{2}\dot{q}_E^2 + ie\,\dot{q}_E\cdot\vec{A}$ $\xrightarrow{?}$ $eB\vec{e}_y$ $\Rightarrow$ quadratic

and all periodic $\vec{q}$'s are

$$\vec{q}(\tau) = \vec{q}_0 + \sum_{n=1}^{\infty}\sqrt{\frac{2}{\beta}}\left(\vec{c}_n\cos\frac{2\pi n\tau}{\beta} + \vec{s}_n\sin\frac{2\pi n\tau}{\beta}\right),$$

$$\begin{pmatrix}\vec{c}_n\\ \vec{s}_n\end{pmatrix} = \int_0^\beta d\tau\,\vec{q}(\tau)\sqrt{\frac{2}{\beta}}\begin{pmatrix}\cos\\ \sin\end{pmatrix}\left(\frac{2\pi n\tau}{\beta}\right)$$

$$\Rightarrow D\vec{q} = d\vec{q}_0\prod_{n=1}^{\infty} d\vec{c}_n\, d\vec{s}_n$$

$$\int_0^\beta d\tau\,(\dot{q})^2 = \sum_{n=1}^{\infty}\left(\frac{2\pi n}{\beta}\right)^2\left(c_{x,n}^2 + s_{x,n}^2 + c_{y,n}^2 + s_{y,n}^2\right)$$

$$\int_0^\beta d\tau\left(-ie\frac{dq_y}{d\tau}\underset{\substack{\shortparallel\\ q_x B}}{A_y}\right) = B\sum_{n=1}^{\infty}\left(\frac{2\pi n}{\beta}\right)\left(-c_{x,n}s_{y,n} + c_{y,n}s_{x,n}\right)$$

$$\Rightarrow S_E = \sum\frac{1}{2}\left(c_{y,n}\;\; s_{x,n}\right)M_n^*\begin{pmatrix}c_{y,n}\\ s_{x,n}\end{pmatrix} + \sum\frac{1}{2}\left(c_{x,n}\;\; s_{y,n}\right)M_n\begin{pmatrix}c_{x,n}\\ s_{y,n}\end{pmatrix},$$

$$M_n = \begin{pmatrix} m\left(\frac{2\pi n}{\beta}\right)^2 & ieB\left(\frac{2\pi n}{\beta}\right)\\ ieB\left(\frac{2\pi n}{\beta}\right) & m\left(\frac{2\pi n}{\beta}\right)^2\end{pmatrix}$$

$$\Rightarrow Z(\beta) = \mathcal{N}\underbrace{\int dq_{x,0}\,dq_{y,0}}_{A}\prod_{n=1}^{\infty}(\dots)\,e^{-S_E}$$

$$= \mathcal{N}A\prod_{n=1}^{\infty}\frac{\pi}{\sqrt{\det M_n}}\frac{\pi}{\sqrt{\det M_n^*}}, \quad \det M_n = m^2\left(\frac{2\pi n}{\beta}\right)^4\left[1 + \left(\frac{\beta\omega_c}{2\pi n}\right)^2\right]$$

$$= \mathcal{N}A\prod_{n=1}^{\infty}\frac{\pi^2}{m^2\left(\frac{2\pi n}{\beta}\right)^4}\underbrace{\prod_{n=1}^{\infty}\frac{1}{1+\left(\frac{\beta\omega_c}{2\pi n}\right)^2}}_{\frac{\beta\omega_c/2}{\sinh\beta\omega_c/2}}$$

- for $B=0$, we have $Z(\beta)_{B=0} = \int\frac{d\vec{q}\,d\vec{p}}{(2\pi)^d}e^{-\beta H_0} = A\left(\frac{m}{2\pi\beta}\right)^{d/2}$

$$Z(\beta)_{B=0} = \mathcal{N}A\prod^{\infty}\frac{\pi^2}{m^2\left(\frac{2\pi n}{\beta}\right)^4} \overset{!}{=} A\left(\frac{m}{2\pi\beta}\right)$$

$$\Rightarrow Z(\beta) = A\left(\frac{m}{2\pi\beta}\right)\frac{\beta\omega_c/2}{\sinh\beta\omega_c/2}$$

$$= \frac{Am}{2\pi\beta}\beta\frac{eB}{m}\sum_{n=0}^{\infty}e^{-\beta\omega_c(n+1/2)} = \sum g_n e^{-\beta E_n}$$

$$\Rightarrow E_n = \omega_c(n+1/2),\quad g_n = \frac{AeB}{2\pi}$$

- now do it in qft.

$\rightarrow$ couple a scalar to an external, classical A_μ (const.)

$$\mathcal{L} = |D_\mu \varphi|^2 - m^2 \varphi^+ \varphi$$

$$e^{iS_Q(A)} = \int D\varphi\, D\varphi^+ \, e^{+i\int d^4x\, \mathcal{L}} = \langle 0|0\rangle_A = \langle 0|e^{-iHT}|0\rangle$$

$$\Gamma = \frac{2\,\mathrm{Im}\, S_Q(A)}{VT}.$$

$$e^{iS_Q(A)} = \mathcal{N} \det(-D^2 - m^2)^{-1}, \text{ since } \mathcal{L} = \varphi^+(-D^2 - m^2)\varphi$$

$$= \mathcal{N} \det(D_E^2 - m^2)^{-1} = \mathcal{N}\, e^{-\mathrm{Tr}\log(D_E^2 - m^2)}$$

$$\Rightarrow iS_Q(A) = -\mathrm{Tr}\log(D_E^2 - m^2)/\mu^2$$

- use $\lim\limits_{x\to 0} \int_x^\infty \frac{dt}{t} e^{-t} = -\log x + \text{const}$

$$\Rightarrow \log \frac{D_E^2 - m^2}{\mu^2} = -\int_0^\infty \frac{d\beta}{\beta}\left(e^{-\beta(D_E^2 - m^2)} - e^{-\beta\mu^2}\right)$$

$$iS_Q = -\mathrm{Tr}\,\text{''} = +\int_0^\infty \frac{d\beta}{\beta}\left(\mathrm{Tr}\, e^{-\beta(D_E^2 - m^2)} - \mathrm{Tr}\, e^{-\beta\mu^2}\right) + \text{const.}$$