Gauge @ IG+P

Invariants from 6 dims - Paša Putrov

- Big picture overview

- max dim of interacting QFT w susy (needed for Schwarzian TQFTs): 6
- 6D SCFTs maximal
- classification:

	SC conf. sym.:	Labelled by:
(2,0)	osp(8|4)	simply laced Lie alg. $\mathfrak{g}$ or $\mathfrak{u}(1)$
(1,0)	osp(8|2)	Y → singularity in elliptically fibered CY3 (for (2,0): Y = ADE sing × E)

- not gauge theories
  → morally, contain theory of nonabelian gerbes w SD 3-curvature
- however, if reduced on S^1, can be described in terms of 5d YM
- in particular, 6d $\mathcal{N}$(2,0) $\mathcal{T}_{\mathfrak{g}} / S^1$ ⇒ SYM w $G = \mathrm{Lie}^{-1}(\mathfrak{g})$
- fix a 6d SCFT $\mathcal{T}$ and Σ^{6-d}, a $(6-d)$-mfd, possibly w defects
- do top. twist in remaining d-dims of $\mathcal{T}_{\mathfrak{g}} / \Sigma^{6-d}$ ⇒ CohTQFT $Z_{\mathcal{T}, \Sigma^{6-d}}$:

$$Z_{\mathcal{T}}(M^d \times \Sigma^{6-d}) = Z_{\mathcal{T}, \Sigma^{6-d}}(M^d)$$

$Z_{\mathcal{T},\Sigma^{6-d}}$ is a sym. monoidal functor

$(\mathrm{Bord}^d) \longrightarrow (\mathrm{Vect}_{\mathbb{C}}^{\Gamma})$, Γ grading

Examples

1) $\mathcal{T} = \mathcal{T}_{SU(2)} = \mathcal{H}$

1a) $\Sigma^2 = \mathbb{T}^2$ w/ cpx str τ

$\Rightarrow Z_{\mathcal{T}_{SU(2)},\mathbb{T}^2}(M^4)$ is a gen.

function for Vafa-Witten invariants of M^4 wrt $q = e^{2\pi i \tau}$

1b) $d = 5$, $\Sigma^1 = \mathbb{S}^1$

$Z_{T_{SU(2)},\mathbb{S}^1}(M^4) \in \mathrm{Vect}_{\mathbb{C}}^{\mathbb{Z}}$, $\mathbb{Z} \to$ winding

- V.O.A. structure

1c) $\Sigma^2 = \mathbb{T}^2$ but $Z_{T_{SU(2)},\mathbb{T}^2}(M^3)$

$\to$ "$SL(2,\mathbb{C})$ floer homology"

2) $\mathcal{T} = \mathcal{T}_{\mathfrak{g}}$

2a) $\Sigma^3 = D^2 \times \mathbb{S}^1$, $\partial\Sigma^3 = \mathbb{T}^2$ has cpx str

$Z_{T_{\mathfrak{g}},D^2\times\mathbb{S}^1}(M^3) = Z_{\mathfrak{g}}(M^3) \in \mathbb{Z}[[q]]$

$\xrightarrow[q \to \text{root of unity}]{}$ Witten-Reshetikhin-Turaev

2b) $\Sigma^2 = D^2$. $Z_{T_g, D^2}(M^3) \in \mathrm{Vect}_{\mathbb{C}}^{\mathbb{Z}^2}$

- analog of Khovanov-Rozansky homology for closed 3-mfds
- categorifies Jones polynomials

3)

3a) $g = u(1)$, $\Sigma^2 = S^2$ w 2 mkd pts
$Z_{u(1), \Sigma^2}(M^4) =$ SW inv of M^4

3b) $\Sigma^2 = S^2$ w n ...
=> multimonopole inv.'s

-> interpretation in terms of $Z_{T_g, S^1}(M^4)$ V.O.A.

4) $\mathcal{J}$ is $(1,0)$

$Z_{\mathcal{J}, T^2_\tau}(M^4) \in \mathbb{Z}[[q]]$, $q = e^{2\pi i \tau}$

- K-theoretic "nonab. monopole" inv's.

VOA review

Def. A vertex (super)algebra is given by:

i) vsp $\mathcal{V} = V^{(0)} \oplus V^{(1)}$ w $\mathbb{Z}_2$-grading F
ii) $\mathbb{1} \in \mathcal{V}$, "vacuum", $F(\mathbb{1}) \equiv 0 \mod 2$
iii) $T: \mathcal{V} \to \mathcal{V}$ "translation"
iv) "state-field" correspondence
$Y: \mathcal{V} \to \mathrm{End}\, \mathcal{V}[[z, z^{-1}]]$
or $\mathcal{V} \otimes \mathcal{V} \to \mathcal{V}[[z, z^{-1}]]$

such that the following axioms hold:

A1) $Y(\mathbb{1}, z) = id_V$ z

A2) $[T, Y(u,z)] = \frac{d}{dz} Y(u,z) \quad \forall u \in V$

A3) locality (Jacobi id)

$\forall u, v \in V \ \exists N \in \mathbb{Z}_+$

s.t. $(x-z)^N [Y(u,z), Y(v,x)] = 0$

$\mathbb{Z}$F-graded $[\cdot,\cdot]$

Def. Vertex op. alg is a v.a. w additionally:

- $\omega \in V$ given s.t.

$$Y(\omega, z) =: T(z) = \sum_{n \in \mathbb{Z}} L_n z^{-n-2}$$

where $[L_m, L_n] = (m-n) L_{m+n} + \frac{\delta_{m+n,0}}{12}(m^3 - m) c \cdot id_V$, $c \in \mathbb{C}$

- moreover L_0 induce $\mathbb{Z}$-grading on $V = \bigoplus_{n \in \mathbb{Z}} V_n$, $L_0 V_n = n \cdot V_n$ and $L_{-1} = T$

- modules of VA V are vsp's M equipped w $Y_M : V \to \operatorname{End} M [[z, z^{-1}]]$