QFT.

Exercises.

1) $\mathcal{L} = \sum_i \bar\psi_i i\partial\!\!\!/ \psi + \frac{g^2}{2}(\bar\psi_i \psi_i)^2$

- $[\psi_i] = \frac{d-1}{2} \Rightarrow [g] = d - 4 \cdot \frac{d-1}{2} = 2 - d \geq 0 \Rightarrow d \leq 2$.
- define 2-dim gamma matrices

$$\gamma_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad \gamma_5 = \gamma^0\gamma^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

→ check P-invariance

- $\mathcal{L}_\varphi = \bar\psi(i\partial\!\!\!/ - \varphi)\psi - \frac{1}{2g^2}\varphi^2 = \bar\psi i \partial\!\!\!/ \psi - \frac{1}{2g^2}\underbrace{(\varphi - g^2\bar\psi\psi)^2}_{=:\varphi'^2} + \frac{g^2}{2}(\bar\psi\psi)^2$

$$e^{i\int d^dx\,\mathcal{L}} = \int \mathcal{D}\varphi\, e^{i\int d^d \mathcal{L}_\varphi}$$
$$= \underbrace{\left(\int \mathcal{D}\varphi' e^{-\frac{i}{2g}\varphi'}\right)}_{= \text{const.}} e^{i\int d^d\mathcal{L}}$$

II) $\mathcal{L} = \sum_{i=1}^{N} \bar\psi_i(i\partial\!\!\!/ - \mu^{\frac{\varepsilon}{2}}\varphi)\psi_i - \frac{1}{2g^2}\varphi^2$

$$e^{-iV_d V_{\text{eff}}(\varphi)} = \int \mathcal{D}\bar\psi\mathcal{D}\psi\, e^{i\int d^dx \ldots}$$
$$= e^{-iV_d\frac{1}{2g^2}\varphi^2}\underbrace{[\det(i\partial\!\!\!/ - \mu^{\frac{\varepsilon}{2}}\varphi)]^N}_{\exp N\,\mathrm{Tr}(\ldots)}$$

$$\mathrm{Tr}\log(i\partial\!\!\!/ - \mu^{\frac{\varepsilon}{2}}\varphi) = V_d\int\frac{d^dp}{(2\pi)^d}\frac{1}{2}\log(\underbrace{\det(-p\!\!\!/ - \mu^{\frac{\varepsilon}{2}}\varphi)}_{-p^2 - \mu^\varepsilon\varphi^2}))$$
$$\ldots = -\frac{2}{\varepsilon} + \gamma_e - 1$$

$$\ldots \Rightarrow V_{\text{eff}}^{(0)}(\varphi) = \frac{1}{2g^2}\varphi^2 + \frac{N}{4\pi}\varphi^2\left(\log\frac{\varphi^2}{\mu^2} - 1\right)$$

$$\Rightarrow \langle\varphi\rangle = e^{-\frac{\pi}{g^2(\mu)N}}\mu = m_F$$

→ but fermion mass breaks P-invariance, so it gets spont. broken

→ note that $\langle\varphi\rangle$ is nonperturbative