Gravity @ ICTP

Wald

- recap:
- view stationary BH as body in thermal equilibrium
  - 0^{th} law – κ locally constant
  - 1^{st} law – $\delta M = \frac{1}{8\pi}\kappa\,\delta A + \Omega_H\,\delta J + \Phi_H\,\delta Q$
  - 2^{nd} law – $\delta A \geq 0$

BH and Black Brane stability

- by black brane we mean a $(D+p)$-dim spacetime metric of the form $d\tilde{s}^2_{D+p} = ds^2_D + \sum_{i=1}^{p} dz_i^2$ where ds_0^2 is D-dim BH metric

- we define the "canonical energy" $\mathcal{E}$ of a perturbation γ_{ab} and show its positivity is necessary for stability of BH

- we view entropy as $S = k\ln V$ where V is a volume of phase space on a shell of constant energy and some other parameters $\{X_i\}$
- naturally, starting from a small volume, we evolve to a distinguishable volume which extremises S
  - we could evolve out of it, after a long time, but not generically
- here we consider a "semi ergodic" system where orbits fill the shell s.t. the time spent in a volume is prop. to the volume

- now $\delta E = T\delta S + \sum Y_i \delta x_i$, where this should be viewed as defining δS wrt deformations of the shell
- thermodynamic stability, i.e. entropy being maximum at 2^{nd} order, demands $\delta^2 E - T\delta^2 S - \sum Y_i \delta^2 x_i > 0$ where (E, x_i) are fixed at 1^{st} order
- alternatively, for only extended homogeneous systems, if $H_S = \begin{pmatrix} \partial_E^2 S & \partial_{x_i}\partial_E S \\ \partial_E \partial_{x_i} S & \partial_{x_i}\partial_{x_j} S \end{pmatrix}$ has a positive eigenvalue, the system is unstable, which corresponds to a negative heat capacity
  → this is true for a homogenous, only extended system, but not necessarily for a finite one.

- BH's and branes have

$$\begin{array}{ccc} E & & M \\ S & \longleftrightarrow & A \\ x_i & & J_i, Q_i \end{array}$$

so for $\delta E = \delta x_i = 0$ we want

$$\delta^2 M - \frac{K}{8\pi}\delta^2 A - \sum \Omega_i \delta^2 J_i > 0$$

and since BB's are only extended, H_A needs to lack a pos eigenv.

→ Schwarzschild has $A = 16\pi^2 M^2 \Rightarrow \frac{\partial^2 A}{\partial M^2} > 0$ but it's finite

- but this implies black strings built from Schw. aren't stable (Schw. $\times \mathbb{R}$)