Srni

Deformation theory – Frégier

- quantization: $p \rightsquigarrow \hat{P} : \psi \mapsto -i\hbar \frac{\partial}{\partial x}\psi$
  $x \rightsquigarrow \hat{X} : \psi \mapsto x\psi$

  $\to f *_\hbar g := (\hat{f} \circ \hat{g})^{\wedge -1}$

Thm (Moyal – Grönwald)

$$*_\hbar = * + B_1(-,-)\hbar + B_2(-,-)\hbar^2 + \dots$$

$$\mu_t = \mu_0 + t\mu_1 + \dots$$

$$[a,b]_i := \frac{1}{2}(\mu_i(a,b) - \mu_i(b,a))$$

Prop. $[a,b]_1$ is Poisson.

- converse: given $(\cdot, \{\})$, $\exists ?\ \mu_t$
- a: (Концевич) $\square$ A

- mathematical structures can similarly be deformed
  $\to$ but modulo isomorphisms
- e.g., $GL(V) \circlearrowright L(V \otimes V, V)$
  $$(g \triangleright \mu)(a,b) := g(\mu(g^{-1}a, g^{-1}b))$$

Thm $T_\mu \mathrm{Ass} = H^1_{\mathrm{Hoch}}$

Second lecture

- now $\mathcal{C} = \mathcal{A}$, algebra
- $\mathrm{Der}\,\mathcal{A} = \{\partial \in \mathrm{End}\,\mathcal{A} \mid \partial(ab) = (\partial a)b + (-)^{\partial \cdot a} a(\partial b)\}$
- for $\mathcal{A}$ free, $\mathcal{A} = TV = V^{\oplus n}$
  $\Rightarrow \mathrm{Der}\,TV \longleftrightarrow \{\varphi : V \to TV\}$

$\mathrm{Ass}(V) = MC(\_\ T.\_)$

$\mathrm{Lie}(V) = MC(\_\ S.\_)$

-q: $\mathcal{O}(V) = MC(\_\ ???)$

"all algebras"

- a: Koszul duality
  $\Rightarrow As^! = As$, $Lie^! = Comm$
  $\Rightarrow$ so we need to understand the shriek.

Third lecture.

$(U, \mu) \xrightarrow{\varphi} (V, \nu)$

s.t. $\nu(\varphi(a), \varphi(b)) = \varphi\mu(a, b)$

$\alpha = (\mu, \nu, \varphi)^{\tau} \Rightarrow d\alpha + \frac{1}{2!}\{\alpha, \alpha\} + \frac{1}{3!}\{\alpha, \alpha, \alpha\} + \dots = 0$

- L_∞ structure
- start with Lie alg $(L, [\cdot, \cdot])$ w $a \subset L$, $[a, a] = 0$
  abelian subalg.
  - $P : L \to a$, $P^2 = P$, $[\ker P, \ker P] \subset \ker P$
  - $[\Delta, \Delta] = 0$, $\Delta \in L^1$

Def. a $L_\infty[1]$ algebra has brackets
$\{\cdot \dots \cdot\}_n = [\cdot\cdot[\Delta, \_], \_]\dots[\_]]$

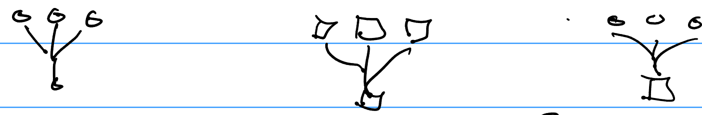
Prop. $L[1] \oplus a$ is $L_\infty[1]$ algebra.

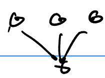
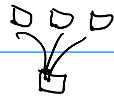
Pf. $d(x[1], a) = (-D(x)[1], P(x + \Delta a))$
where $D = [\Delta, \_]$. $\{x[1], y[1]\} = \{x, y\}[1]$
$\{x_1, \dots x_n\}_n = P\{\dots\{$

- for $L = L^\bullet(U+V, U+V)$

$$= \bigoplus L((U+V)^{\otimes n}, U+V)$$

$$= \bigoplus L(U,U) \oplus L(V,V) \oplus L(U,V) \oplus \text{rest}$$



- now take $L' =$  $+$ , $a =$ 

- for G arbitrary: $L = \mathrm{Der}\, G^!(U+V)$

- for geometry?

$L = (\mathcal{X}^\bullet(M), [-,-]_{\text{Schouten}})$

multivect. fields $= V_L$

$a = \Gamma(\Lambda V_C)$

$P =$ restriction, $\Delta = 0$

<u>Prop</u> $(\pi, \varphi) \in MC(L[1] + a)$

$\Longleftrightarrow$ π Poisson, φ coisotropic

- generalisations (& open issues)
  - → embed an arbitrary diagram (as a small cut) into any other and deform
  - → quantization of alg. varieties and schemes