Srní

Deformation theory – Frégier

- quantization: $p \rightsquigarrow \hat{P} : \psi \mapsto -i\hbar \frac{\partial}{\partial x}\psi$
  $x \rightsquigarrow \hat{X} : \psi \mapsto x\psi$

  $\to f *_\hbar g := (\hat{f} \circ \hat{g})^{-1}$

Thm (Moyal – Grönwald)
$$*_\hbar = * + B_1(-,-)\hbar + B_2(-,-)\hbar^2 + \dots$$

$$\mu_t = \mu_0 + t\mu_1 + \dots$$

$$[a,b]_i := \tfrac{1}{2}\left(\mu_i(a,b) - \mu_i(b,a)\right)$$

Prop. $[a,b]_1$ is Poisson.

- converse: given $(\cdot, \{\,\})$, $\exists?\ \mu_t$
- a: (Концевич) $\square$ A

- mathematical structures can similarly be deformed
  → but modulo isomorphisms
- e.g., $GL(V) \circlearrowright L(V \otimes V, V)$
  $(g \triangleright \mu)(a,b) := g(\mu(g^{-1}a, g^{-1}b))$

Thm $T_\mu \mathrm{Ass} = H^1_{Hoch}$

Second lecture

- now $\mathcal{C} = \mathcal{A}$, algebra
- $\mathrm{Der}\,\mathcal{A} = \{\partial \in \mathrm{End}\,\mathcal{A} \mid \partial(ab) = (\partial a)b + (-)^{\partial \cdot a} a(\partial b)\}$
- for $\mathcal{A}$ free, $\mathcal{A} = TV = V^{\oplus n}$
  $\rightarrow$ $\mathrm{Der}\,TV \longleftrightarrow \{\varphi : V \to TV\}$

$\mathrm{Ass}(V) = MC(\_\ T.\_)$
$\mathrm{Lie}(V) = MC(\_\ S.\_)$
- q: $\mathcal{O}(V) = MC(\_\ ???)$
"all algebras"
- a: Koszul duality
  $\rightarrow$ $As^! = As$, $Lie^! = Comm$
  $\rightarrow$ so we need to understand the shriek.