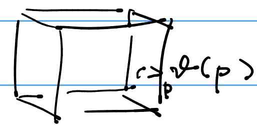
Gravity @ ICTP

Schoen

- we looked at
  1) AF, $n=2,3$
  ii) Ω, $\Sigma = \partial\Omega$ smooth $\Big\}$ $n=2$
  iii) Ω, Σ polyhedral $\Big\}$
- today, ii), iii) for $n=3$
- $n=3$: iii) Σ polyhedron
- (Gromov) cube in $\mathbb{R}^3$, dihedral angle $\vartheta_0 = \frac{\pi}{2}$



- in (M^3, g), $R_g \geq 0$, $\partial\Omega = \Sigma$ cube in M
- assume i) $H \geq 0$ on each face
  ii) $R_g \geq 0$

Thm I Σ cubical. Then $\max_{p\in\Sigma} \vartheta(p) \geq \frac{\pi}{2}$

Thm II Σ tetrahedral. Then $\max_{p\in\Sigma} \vartheta(p) \geq \vartheta_{tet} = \cos^{-1}\left(\frac{1}{3}\right)$

Rmk. thm I v II implies PMT.

- $M^3 \setminus K \simeq \mathbb{R}^3 \setminus \text{Ball}$, $g_{ij} = u(x)^4 \delta_{ij}$
  where $u(x) = 1 + \frac{m}{2|x|} + O(|x|^{-2})$
- assume $m < 0$.
  - then $\exists$ counterex. to thm I
  - draw really big cube s.t. $\vartheta(p) = \frac{\pi}{2}$
  - $m < 0 \Rightarrow H > 0$ on faces $\to$ deform them
    and H remains > 0 $\to$ get $\vartheta_{max}(p) < \frac{\pi}{2}$.