Porta.

- what we did:

*) $\langle\ \rangle_{\beta L \mu}$ analytic for $|\lambda| < \lambda_{\beta,L}$

*) improve? using info on H (like gap)

→ pert. theory. Problem? combinatorics causes $O(n!)$ growth

BBF formula – good if $g(z,z') = \langle A_z, B_{z'} \rangle$

– not really the case due to disc. in T

*) solution? regularized theory $\lim_{N\to\infty} g^{(\le N)} = g$

- however, now $\|A_z^{(\le N)}\| \cdot \|B_z^{(\le N)}\| \le C\, 2^{3N}$

- maybe not an optimal bound. try:

Task: multiscale analysis $g^{(\le N)} = g^{(\le 0)} + \sum_{h>0}^{N} g^{(h)}$ instead of all scales $(\le N)$ at once

- free energy $f_{\beta,L,N} = -\frac{1}{\beta|\Lambda_L|} \log \int P_{\le N}(d\psi^{(\le N)}) e^{-V(\psi^{(\le N)})}$

- now, by fermion magic,

$$\int P_{\le N}(d\psi^{\le N}) e^{-V(\psi^{(\le N)})} = \overbrace{\int P_N(d\psi^N) e^{-V(\psi^{(N)})}}^{e^{E_{N-1}}} \cdot \int P_{\le N-1}(d\psi^{(\le N-1)}) e^{-V^{(N-1)}(\psi^{(\le N-1)})}$$

$$-V^{(N-1)}(\psi^{(\le N-1)}) = \sum_{\substack{|P|\ge 2\\ \text{even}}} \int dz\, W_P^{(N-1)}(z)\, \psi_z^{(\le N-1)}(P)$$

$$\psi_z^{(\le N-1)}(P) = \prod_{g\in P} \psi_{z(g)}^{(\le N-1)\,\varepsilon(g)}$$

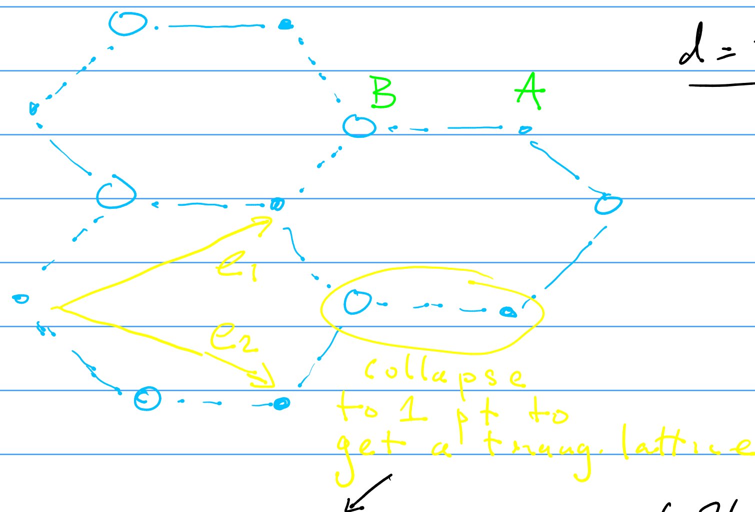
- inductively, $\mathbb{E}_{\le N}(e^{-V(\psi^{(\le N)})}) = e^{\sum_{T=h}^{N-1} E_T}\, \mathbb{E}_{\le h}(e^{-V^{(h)}(\psi^{(\le h)})})$

where $V^{(h)}(\psi^{(\le h)}) = \sum_P \int dz\, W_P^{(h)}(z)\, \psi_z^{(\le h)}(P)$

- some estimates were being made, did not write
- in conclusion, $f_{\beta,L,N} = f_{\beta,L,0} + \sum_{r=0}^{N-1} \frac{\varepsilon_r}{\beta |\Lambda_L|}$
uniform in β, L, N, analytic for $|\lambda| < \lambda'$,
the limit $\beta, L, N \to \infty$ exists
- however, the existance of a gap was crucial
  - serious infrared problems

- let's look at a specific model

Interacting graphene



$\underline{d=2}$, H = Laplacian on honeycomb lattice

$$\Lambda \simeq \mathbb{Z}^2 \times \{A, B\}, \quad \psi(x) = \begin{pmatrix} \psi_A(x) \\ \psi_B(x) \end{pmatrix}$$

$$(H\psi)(x) = \begin{pmatrix} \psi_B(x) + \psi_B(x+e_1) + \psi_B(x+e_2) \\ \psi_A(x) + \psi_A(x-e_1) + \psi_A(x-e_2) \end{pmatrix}$$

A

B

$$(\widehat{H\psi})(\underset{\pi^2}{k}) = \hat{H}(k)\hat{\psi}(k), \quad \hat{H}(k) = \begin{pmatrix} 0 & -t\Omega(k) \\ -t\overline{\Omega(k)} & 0 \end{pmatrix}$$

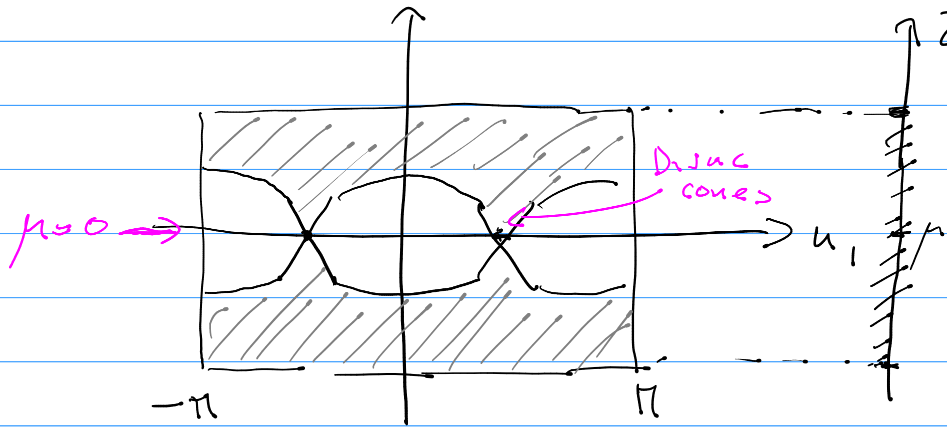
$$\Omega(k) = 1 + e^{-ik\cdot e_1} + e^{-ik\cdot e_2}$$

- eigenvalues $\varepsilon_\pm(k) = \pm t|\Omega(k)|$

$$\Omega(\underline{k}) = 0 \iff \underline{k} = \underline{k}_F^{\omega}, \quad \omega = \pm$$

$$\Omega(\underline{k}' + \underline{k}_F^{\omega}) = \frac{3}{2}\left(i k_1' + \omega k_2'\right) + O(|k'|^2)$$

$$\bigcup_{k_2} \sigma(\hat{H}(k_1, k_2))$$



$\sigma(H)$
''
$\sigma_{a.c.}(H)$