Semi

Quantum flag manifolds - Réamonn Ó Buachalla

- we abstract properties of $C(X)$, $X \in (Top)$

Def. Banach alg $\to$ complete normed alg $(B, \|\cdot\|)$ w sub.-multiplicative norm.
$*$-algebra $\to$ cpt alg w. anti-linear involution $* \circlearrowleft A$.
C^*-algebra $\to$ unital Banach alg $(A, \|\cdot\|)$ with $*$-alg structure s.t. $\|a^* a\| = \|a\|^2$, $\forall a \in A$.

Thm (Gelfand - Naimark '43) Every C^*-alg is isomorphic to $C(X)$ for some cpt. Hausdorff space X.

$\to$ duality of categories!

Rmk. for noncpt. spaces take the C^*-alg to be $C_0(X)$, funcs vanishing at ∞.

- noncommutative topology wants to study noncommutative C^*-algs as if they were "noncommutative function algebras".

- Q: what is a topological group in Gelfand - Naimark terms?
  $\to$ the dual structure in the cat. of comm. C^*-algs is $(A, \Delta, S, \varepsilon)$, with completed topological product $\hat{\otimes}$ $\Rightarrow$ this we can get rid of if we just look at representable functions
  $\to$ Hopf algebra.

- In Leningrad in the '80s physicists working on the inverse scattering problem discovered $\mathcal{U}_q(sl_2)$
- Drinfeld and Jimbo introduced $\mathcal{U}_q(\mathfrak{g})$, quantum groups

- $\mathcal{U}_q(sl_2)$ is the free noncomm. alg gen by E, F, K, K^{-1},

$$KE = q^2 EK$$
$$KF = q^{-2} FK$$
$$[E, F] = \frac{K - K^{-1}}{q - q^{-1}}$$

$\to$ $C(G) \supseteq \mathcal{O}(G)$. But also $\mathcal{O}(G) \times \mathcal{U}(\mathfrak{g}) \to \mathbb{C}$ is a pairing

$C(G)$: Hopf-like structure ($\hat{\otimes}$)

$\mathcal{O}(G)$: genuine Hopf

$\to$ so $\mathcal{U}_q(\mathfrak{g})$ also Hopf alg
- D. & J.: for every semisimple cpx Lie alg $\mathfrak{g}$
  - $\to \exists\ \mathcal{U}_q(\mathfrak{g})$
  - $\to q = 1$, $\mathcal{U}_1(\mathfrak{g})$ rank($\mathfrak{g}$)-fold cover of $\mathcal{U}(\mathfrak{g})$

- while $\mathcal{U}_q(\mathfrak{g})$ is not unique, Kazhdan-Wenzl, Wenzl-Tuba and Liu show that $\mathcal{U}_q(\mathfrak{g})$-Mod is the unique monoidal deformation of $\mathcal{U}(\mathfrak{g})$-Mod.