Geometric correspondences @ old SISSA

Gerald Dunne - Resurgence & phase trans.

- what is the Minkowski path int $\int DA\, e^{+i/\hbar\, S[A]}$?
  - in fin. dim, it is Stokes/Airy paradigm
    - we need cpx analysis and contour deformations to tame these
- we can interpret phase transitions as changes of dominant saddles?
- does this $Z(\hbar) = \int DA\, e^{\frac{i}{\hbar} S[A]}$

$$\text{“}=\text{''} \sum_{\text{thimble}} \mathcal{N}_{th}\, e^{i\varphi_{th}} \int_{th} DA \times (J_{th}) \times e^{\mathrm{Re}[\frac{i}{\hbar} S[A]]}$$

make sense?

- Écalle : resurgent functions closed under all operations
  $\rightarrow$ (Borel transf.) + (analytic cont.) + (Laplace transf)
- basic trans-series (in QM/QFT)

$$f(g^2) = \sum_{p=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=1}^{k-1} \underbrace{c_{k,l,p}\, g^{2p}}_{\text{pert. fluctuations}} \underbrace{\left(\exp\left[-\frac{c}{g^2}\right]\right)^{k}}_{k\text{-instantons}} \underbrace{\left(\ln\left[\pm\frac{1}{g^2}\right]\right)^{l}}_{\text{quasi-zero modes}}$$

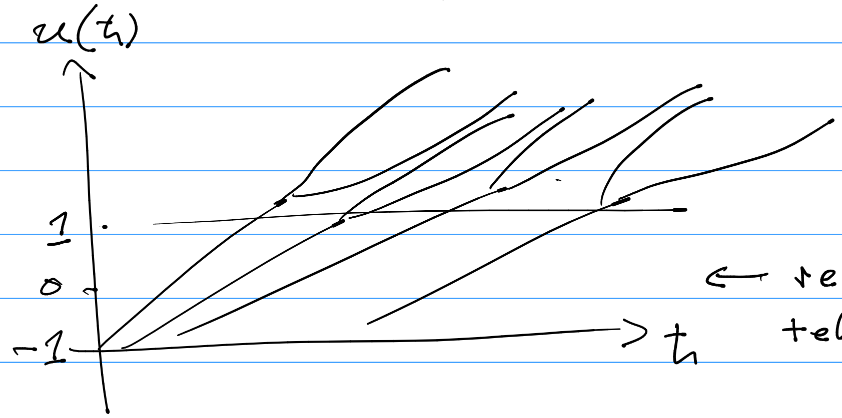
$\rightarrow$ $c_{k,l,p}$ tightly correlated

- all-order steepest descent:

$$I^{(n)}(g^2) = \int_{C_n} dz\, e^{-\frac{1}{g^2} f(z)} = \frac{1}{\sqrt{1/g^2}}\, e^{-\frac{1}{g^2} f_n}\, T^{(n)}(g^2)$$

- where C_n is a contour thru n-th saddle
and $T^{(n)}(g^2)$ goes <u>beyond</u> Gaussian approx
- resurgence <u>relations</u> between saddles

- Mathieu, $-\frac{\hbar^2}{2}\partial_x^2\psi + \cos x \cdot \psi = u\,\psi$



← cpx instantons tell about gaps, otherwise basically constant

← real instantons tell about bands

- $u_\pm(\hbar, N) = u_{pert}(\hbar, N) \pm$ "instantons"
- but these can be given in terms of $u_{pert}(\hbar, N)$!

Kohei Iwaki

Hans Jockers

- 3d $\mathcal{N}=2$ gauge theories w/ $U(1)$ R-sym.
- spectrum: • $G = U(1)$ $\quad V = (A_\mu, \sigma, D, \ldots)$
  • matter $\quad \Phi_A = (\varphi_A, F, \ldots)$
- a_A $U(1)$ charge, Δ_A $U(1)_R$ charge
- action: • superpot. $W = W(\Phi_A)$
  • F.I. params. }
  [• C.S. terms]

- $V_{\mathbb{C}} = \langle\langle \varphi_{\mathbb{C}} \rangle\rangle$ cpx vsp w sympl pairing
- $\mu : V_{\mathbb{C}} \longrightarrow u(1)^* \simeq \mathbb{R}$ moment map

$\Rightarrow X = \mu^{-1}(\xi)/u(1) \cap \{ \text{grad } W = 0 \}$
sympl. quot

- Dim red to $\mathcal{N} = (2,2)$ GLSM w/ tgt X:
  - twisted chiral ring with $\Sigma_A \Sigma_B = C_{AB}{}^{C} (Q = e^{2\pi i t}) \Sigma_C$
  - $\Sigma_A \Longleftrightarrow \omega_A \in H^{ev}(X)$: $\omega_A * \omega_B = C_{AB}{}^{C}(Q)\, \omega_C$
    $= \omega_A \cup \omega_B + \mathcal{O}(Q)$

- in 3d line ops $\mathcal{L}_A \mathcal{L}_B = \tilde{C}_{AB}{}^{C}(Q)\, \mathcal{L}_C$
  - $\mathcal{L}_A \Longleftrightarrow e_A \in K(X)$: $e_A * e_B = e_A \otimes e_B + \mathcal{O}(Q)$

Meng-Chwan Tan

- 4d CS $S = \frac{1}{\hbar} \int_{\Upsilon \times \Sigma} C \wedge \mathrm{Tr}(A\, dA + \frac{2}{3} A^3)$
  - A cpx-valued gauge field, Υ framed 2-mfd, $\Sigma = \mathbb{C}, \mathbb{C}^*$ or $\mathbb{C}/(\mathbb{Z} + \tau\mathbb{Z})$ with a meromorphic 1-form $C = C(z)\, dz$ with no zeroes