Brozzo. Derived fouctors. - suppose F: + -> > left exact. ve have 0->A->A"->B exact seq.

We have 0->FA'->FA->FA"->

->R'FA'->R'FA->R'FA->R'FA->R'FA'-> TRF+1-RF+->R2FA" -> .. long exact (ROFIF) - suppose (K,d) cochain cplx
- uatural notion of cohomology Hh(k,d) = ker d: k-sku+1 - a morphism fiki-stinduces a morphism H(f):H(K) ->H(L), -explicitly, 3 = H(K), 3 = [x] => H(f)(3) = [f(x)] -given a cochain exact seqn. 0-> K+> kf> K"->0, we get a long exact segn 0->H°(K')->H°(K)->H°(K") -> H'(K') -> H'(K") -> ...

where du is the connecting morphism

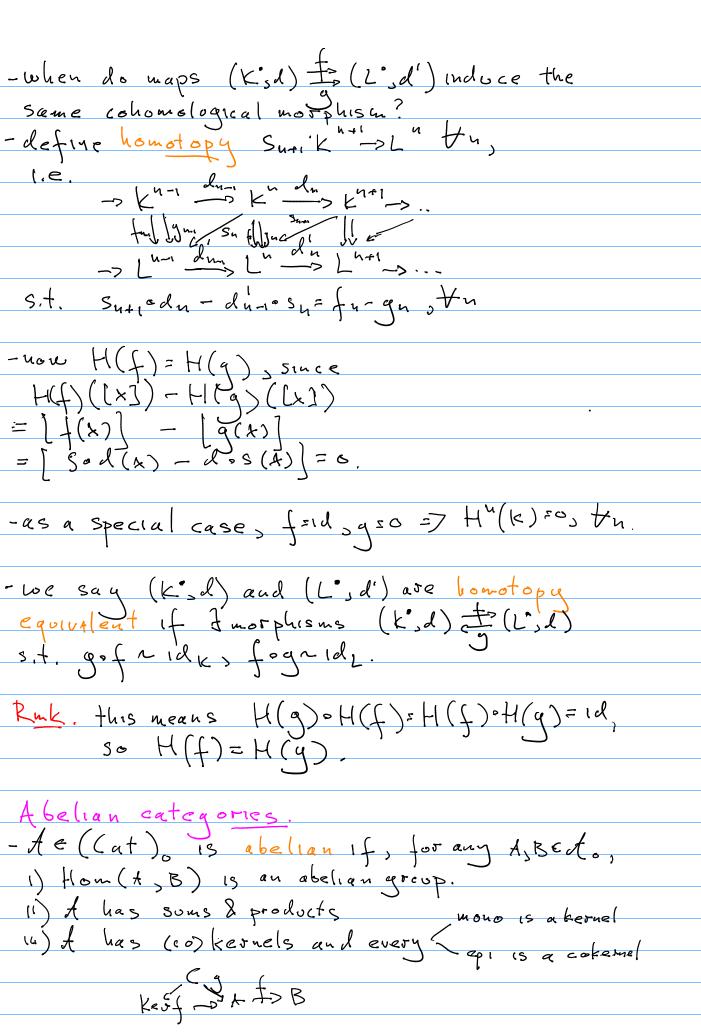
-let's define it.

3eH"(K"), 3geHhai(K')?

-3=[x"], q(x)=x"st. q(dx)=d"q(x)=d"x"=0

-now dx = f(x'), f(d'x')=d^2x = 0 => d'x'=0

 $\Rightarrow \partial_{\mathsf{u}}(\S) = [\chi'].$ 



Def. A resolution of text is a pair ((L,d), E) 2: A -> L° and the sequence

- now take F: A -> B functor of ab. cats, additive & left-exact. - take an injective resolution of texto 0 -> A -> I' -> I' -> I'-> and apply F to get the exact segn.

FA ->+ IP->+ I'->+I'->+ I'->+ I'->+

Def. H"(FI") =: R" + A is the with right derived functor of F.

Pmk, injective resolus aven't unique.

-what is an injective object?

Def. I E A o is injective if Home (-,I)

15 an exact functor.

Ruk, he is only left-exact for any 16 to

-e.g. in (Ab), injectives ore divisible groups,
i.e. Gre(Ab), s.t. tyean Jh s.t. g. nh -> e.g Q

Prop TFAE: 1) I EA is injective (11) every s.e.s. 0-> I -> A -> B-> 0 splits. Def. it has enough injectives if every object can be embeded into an injective object, o-> A ... > T - examples: to, t-mod, Shx, Gx-mod - note that in this case any object has an injective resolu  $0 \longrightarrow A \longrightarrow I_0 \longrightarrow Q_0 \longrightarrow 0$ - note that for 0->t->26->L'->:where I is injective (L'ust necessarily), f "lifts", le fqni L"->T", u=0>1,... These morphisms aren't unique but any 2 such lifts & & g are homotopic.