Qft.

Yukawa theory I

$$\mathcal{L}_{UV} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}M^2\varphi^2 + \bar\psi(i\not\partial - m)\psi - g\varphi\bar\psi\psi$$

$$\mathcal{L}_{IR} = Z_\psi \bar\psi i\not\partial\psi - \tilde m\bar\psi\psi + \frac{\lambda}{2}(\bar\psi\psi)^2 + \text{h.d.o.} \quad (d \geq 5)$$

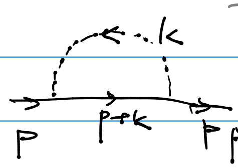
- from the e.o.m. $(-\Box - M^2)\varphi - g\bar\psi\psi = 0$

$$\Rightarrow \varphi = -\frac{g}{M^2}\bar\psi\psi + \dots$$

$$\mathcal{L}_{IR}^{(0)} = \mathcal{L}_{UV}(\varphi(\psi),\psi)$$
$$= -\frac{1}{2}M^2\frac{g^2}{M^4}(\bar\psi\psi)^2 + \bar\psi(i\not\partial - m)\psi - g\bar\psi\psi\left(-\frac{g}{M^2}\bar\psi\psi\right) + \text{h.d.o.}$$
$$= \bar\psi(i\not\partial - m)\psi + \frac{1}{2}\frac{g^2}{M^2}(\bar\psi\psi)^2 + \text{h.d.o.}$$

$$\to \lambda = \frac{g^2}{M^2}, \quad Z_\psi = 1 + O(g^2), \quad \tilde m = m + O(g^2)$$

$\Gamma^{(2)}(p)$ at tree lvl.

- UV: 

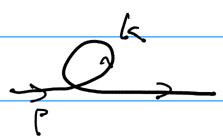
$$= (-ig)^2\mu^\epsilon\int\frac{d^dk}{(2\pi)^d}\frac{i(\not p + \not k - m)}{(p+k)^2 - m^2}\frac{i}{k^2 - M^2}$$
$$\overset{p \ll M}{=} -\frac{g^2}{16\pi^2}\left[\not p A + m B\right]$$

would require higher dim terms like $(\partial_\mu\bar\psi)(\partial^\mu\psi)$, $\Box\bar\psi i\not\partial\psi$ so we cannot match them

$$A = \frac{1}{2}\log\frac{M^2}{\mu^2} - \frac{1}{4} + \frac{m^2}{2M^2} - \frac{p^2}{6M^2} + O(M^{-4})$$

$$B = \log\frac{M^2}{\mu^2} - 1 + \frac{m^2}{M^2}\log\frac{m^2}{M^2} - \frac{p^2}{2M^2} + O(M^{-4})$$

$$\Gamma^{(2)}(p) = \not p - m - \frac{g^2}{16\pi^2}\left[\not p A + m B\right]$$

- IR: 

$$= \dots = -\frac{5i\lambda m^3}{16\pi^2}\left(1 + \log\frac{\mu^2}{m^2}\right)$$

$$\Gamma^{(2)}(p) = Z_\psi\not p - \tilde m - \frac{5i\lambda m^3}{16\pi^2}\left(1 + \log\frac{\mu^2}{m^2}\right)$$

$$\delta Z_\varphi(\mu) = \frac{-g^2}{16\pi^2}\left(\frac{1}{2}\log\frac{M^2}{\mu^2} - \frac{1}{4} + \frac{m^2}{2M^2}\right)$$
$$\delta_m(\mu) = -\frac{m g^2}{16\pi^2}\left(1+\log\frac{\mu^2}{M^2}\right)\left(1+\frac{m^2}{M^2}\right) - \frac{4m^3 g^2}{16\pi^2 M^2}\left(1+\log\frac{\mu^2}{m^2}\right)$$

would be catastrophic when $m \to 0$. thankfully appears w powers of m!

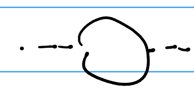
$$m_{phys} = m(\mu)\left(1 - \frac{g^2}{16\pi^2}\left(\frac{5}{4} + \frac{9m^2}{2M^2} + \frac{4m^2}{M^2}\log\frac{\mu^2}{m^2}\right)\right)$$

-now integrate out fermions!

$$\mathcal{L}_{UV} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 + \bar\psi(i\partial\!\!\!/ - M)\psi - g\varphi\bar\psi\psi$$
$$\mathcal{L}_{IR}(\varphi) = \frac{1}{2}Z_\varphi(\partial\varphi)^2 - \frac{1}{2}\tilde m^2\varphi^2 + \frac{\lambda_3}{3!}\varphi^3 + \frac{\lambda_4}{4!}\varphi^4 + \text{h.d.o}$$

-no contribution from fermions @ tree lvl
→ not classical particles
→ come as bilinears from Lorentz sym., so must be contracted to loops.

$\Rightarrow$ $Z_\varphi = 1 + \mathcal{O}(g^2)$, $\lambda_3 = \mathcal{O}(g^3)$, $\lambda_4 = \mathcal{O}(g^4)$, $\tilde m = m + \mathcal{O}(g^2)$

-UV:  $= (-)(-ig)^2\mu^\varepsilon \int \frac{d^dk}{(2\pi)^d}\frac{\mathrm{Tr}[i(k\!\!\!/ + M)\, i(p\!\!\!/ + k\!\!\!/ + M)]}{(k^2-M^2)((p+k)^2-M^2)}$

$$\Gamma^{(2)}_{UV}(p) = p^2 - m^2 - \frac{4g^2}{16\pi^2}\Big[p^2\left(-\frac{1}{2}\log\frac{M^2}{\mu^2} + \frac{4}{3} - \frac{p^2}{20M^2} + \mathcal{O}(M^{-4})\right)$$
$$+ M^2\left(-3\log\frac{M^2}{\mu^2} - 5 + \mathcal{O}(M^{-4})\right)\Big)$$

-IR: no corrections, since $\lambda_n = \mathcal{O}(g^n)$

$$\Gamma^{(2)}_{IR}(p) = Z_\varphi p^2 - \tilde m^2$$

$$m^2_{phys} = m^2(\mu) + \frac{g^2}{16\pi^2}M^2\left(-20 + \frac{16}{3}\frac{m^2}{M^2}\right)$$

- compare

(F) $\to m^2_{phys} = m^2(\mu) - 2\,\frac{m^2(\mu) g^2}{16\pi^2}(\ldots)$

(S) $\to m^2_{phys} = m^2(\mu) + \frac{g^2}{16\pi^2}\,M^2(\ldots)$

- for light scalars, m^2_{phys} should be small
  - → but it depends on an arbitrarily large M^2!
  - → hierarchy problem.
    - → in a sense, separated scales like to intermix ("quantum mechanics")
    - → also possible for $g \to 0$...

Naturalness (t' Hooft)

In any QFT, all dimensionless couplings should be $O(1)$.
Dimensionful couplings should be $O(M^{\#})$, M = max scale.
Small couplings are natural only if a symmetry is restored when they vanish.

- we saw a theory $\mathcal{L}_{UV} \supset -gHL^2$
  and got $g \sim O(M)$, $\lambda \sim \frac{g^2}{M^2} \sim O(1)$.

- in our case, $m^2_{scalar} \sim O(M^2)$ ✓
  but $m^2_{fermion} \sim O(m^2)$?
  - → because $m_\psi \to 0$ restores a $\mathbb{Z}_2$ sym. $\psi_L \to \psi_L$, $\psi_R \to -\psi_R$, $\varphi \to -\varphi$.

- small scalar masses are unnatural.
- IRL, we have Higgs.