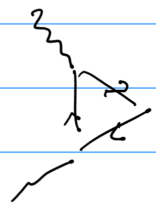
Qft:

$$\langle \partial_\mu J_5^\mu \rangle = -\frac{g^2}{16\pi^2} F_{\mu\nu} F_{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$$ $U(1)_A$ chiral anomaly

$$\Gamma^{\alpha\beta\gamma}_{\mu\nu\rho}\Big|_{ano} = D^{\alpha\beta\gamma}\, \Gamma_{\mu\nu\rho}\Big|_{ano}$$ $G = SU(N)_V \times SU(N)_A \times U(1)_V \times U(1)_A$



$+ \cdots$

$$D^{\alpha\beta\gamma} = \frac{1}{2} \operatorname{tr}\{t^\alpha, t^\beta\} t^\gamma$$

$\rightsquigarrow$ for $SU(N)_V{}^2 \times U(1)_A$, t^γ generates $U(1)_A$

$\Rightarrow D^{\alpha\beta\gamma} \rightsquigarrow \operatorname{tr} t^\alpha t^\beta$

$$\Rightarrow \partial_\mu J_5^\mu = -\frac{g^2}{8\pi^2} \partial_\mu A_\nu^\alpha \partial_\rho A_\sigma^\beta \operatorname{tr} t^\alpha t^\beta \epsilon^{\mu\nu\rho\sigma}$$

$$\longrightarrow -\frac{g^2}{16\pi^2} G^{\mu\nu}_\alpha G^{\rho\sigma,\alpha} \epsilon_{\mu\nu\rho\sigma}$$

Gauge anomalies.

$\mathcal{L} = \bar{\psi}_L i \partial\!\!\!/ \psi_L$. $U(1)$ acts by $\begin{pmatrix} \psi_L \\ \bar{\psi}_L \end{pmatrix} \mapsto \begin{pmatrix} e^{i\alpha}\psi_L \\ \bar{\psi}_L e^{-i\alpha} \end{pmatrix}$

$$J^\mu = \bar{\psi}_L \gamma^\mu \psi_L = \bar{\psi} \gamma^\mu P_L \psi$$

$$\Gamma_{\mu\nu\rho}(k_1, k_2) = \langle J_\mu(-k_1-k_2) J_\nu(k_1) J_\rho(k_2) \rangle$$

$$k^\mu \Gamma_{\mu\nu\rho}(k_1,k_2) = \frac{1}{2}\frac{1}{8\pi^2} \epsilon_{\nu\rho\omega\upsilon}(k_1-k_2+\Delta)^\omega (k_1+k_2)^\upsilon$$

$\rightarrow$ take $\Delta = -\frac{1}{3}(k_1-k_2)$ by Bose symmetry

$$\longrightarrow \langle \partial_\mu J^\mu \rangle = -\frac{1}{2}\frac{g^2}{48\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

- wasn't paying attentn. for a while ...

<u>SM</u> anomaly concellation.

1^{st} gen $\quad (3,2)_{1/6} + (\bar{3},1)_{-2/3} + (\bar{3},1)_{1/3} + (1,2)_{-1/2} + (1,1)_1$

→ anomalies of the kind $SU(N) \times (\overset{\text{not } SU(N)}{\dots})$ are 0 due to tracelessness.

→ possible cases:
$SU(3)^3$
$SU(3)^2 \times U(1)$
$U(1) \times SU(2)^2$
$\cancel{SU(2)}^3$
$U(1)^3$

trivial,
$\mathrm{tr}\,\underbrace{\{\sigma^\alpha, \sigma^\beta\}}_{C\text{-number}}\sigma^\gamma = 0$

→ shorthand $\begin{cases} c \rightsquigarrow SU(3) \text{ indices} \\ Y \rightsquigarrow U(1) \text{ indices} \\ w \rightsquigarrow SU(2) \text{ indices} \end{cases}$

→ from the multiplet structure, $D^{ccc} = 2D^{ccc}_{fund} + 2D^{ccc}_{\overline{fund}} = 0$

$$D^{ccY} = 2 \times \frac{1}{6} D^{ccY}_{fund} + \left(-\frac{2}{3}\right) \overbrace{D^{ccY}_{\overline{fund}}}^{= +D^{ccY}_{fund},\ 2 \text{ generators}} + \overline{D^{ccY}_{\overline{fund}}}\left(\frac{1}{3}\right) = 0$$

$$D^{wwY} = 3D^{wwY}_{u} \times \frac{1}{6} + D^{wwY}_{u} \times \frac{1}{2} = 0$$

$$D^{YYY} = 6\left(\frac{1}{6}\right)^3 + 3\left(-\frac{2}{3}\right)^3 + 3\left(\frac{1}{3}\right)^3 + 2\left(-\frac{1}{2}\right)^3 + 1^3 = \underline{\underline{0}}.$$