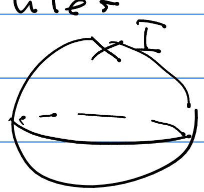
Gauge @ IGAP

Invariants of mfds from non-gauge theories – Du Pei

- why?
  - some cons of gauge theories:
    1) seemingly, no invariants stronger than SW, as most can be related to those ones
    2) won't give full TQFTs, not defined on some mfds

- Pavel's talk: 6d non-gauge th $\Rightarrow$ 4d invs.
- this talk: 4d non-gauge $\Rightarrow$ 3d TQFTs

- 4d $\mathcal{N}=2$ theories have Coulomb branches
  $\mathcal{M}_{\mathcal{T}}$: Coul. br. of theory $\mathcal{T}$ on $S^1 \times \mathbb{R}^3$
  - hyperkähler
    $\Rightarrow$



S^2 of cpx str's, pick I

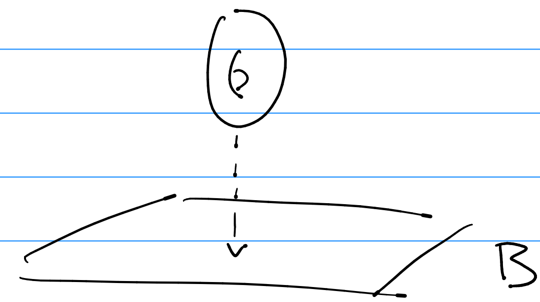
$$\begin{array}{ccc} F & \hookrightarrow & \mathcal{M}_{\mathcal{T}} \\ & & \downarrow \\ & & B_{\mathcal{T}} \end{array}$$

F: cpt., generically abelian var's

$B_{\mathcal{T}}$: Coul. br on $\mathbb{R}^4$ (u-plane)
- special kähler, w/ singularities

- examples: ① $U(1)$ gauge th, $\mathcal{N}=0$, $G = \mathbb{C}^\times$
  $\rightarrow$ labelled by an elliptic curve Σ

$$\mathcal{M}_{\mathcal{T}} = \mathbb{T}^2 \times \mathbb{R}^2 \simeq \mathrm{Jac}(\Sigma) \times \mathbb{C}$$

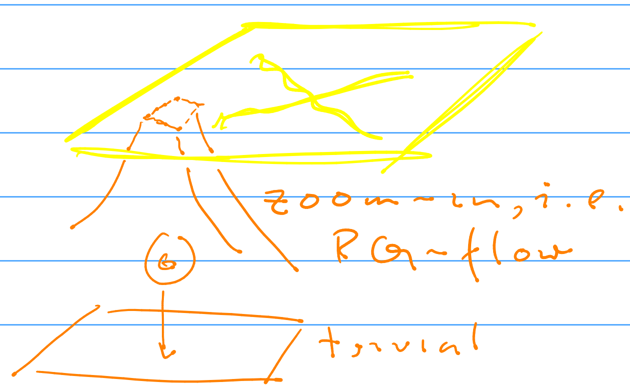


(11) $U(1)^g$ - gauge th, labelled by Σ_g

$$\begin{array}{l} \mathcal{M}_{\mathcal{T}} = (\mathbb{T}^2 \times \mathbb{R}^2)^g = \mathrm{Jac}(\Sigma_g) \times \mathbb{C}^g \\ \quad \downarrow \pi \\ B_{\mathcal{T}} = \mathbb{C}^g \simeq H^1(\Sigma, \mathbb{C}) \end{array}$$

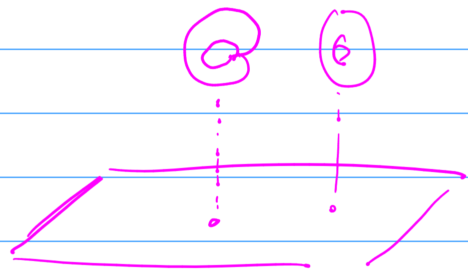
- trivial fibration $\Leftrightarrow$ $\mathcal{T}$ is a "free theory"

$\rightarrow$ but locally, we have triviality, and, physically, $\mathcal{T}$ near a nonsingular point can be approximated by a free $U(1)^g$ gauge theory (low-energy effective th. LEET)



(III) "monopole" theory

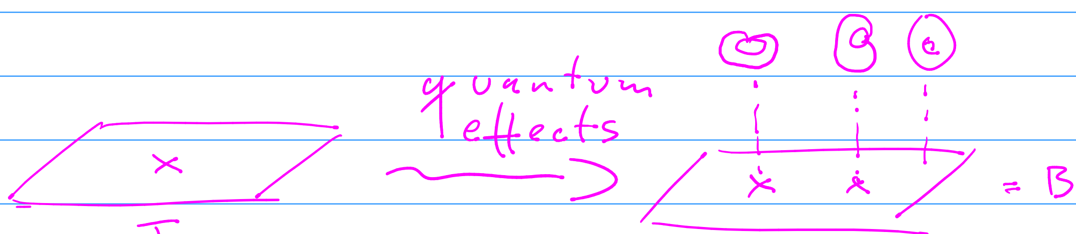
- $G = \mathbb{C}^*$, $N = \mathbb{C}$
- SQED w/ 1 electron

$\mathcal{M}_\gamma =$ 

I_1 singularity
(Kodaira classification)

- loop around I_1 gives $\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix} \in SL_2(\mathbb{Z})$
monodromy

(IV) SU(2) pure SYM, $G = SU(2)$, $N = 0$



I_4 $\quad\quad\quad\quad$ I_1 $\quad$ I_1

$\begin{pmatrix}1 & 4\\ 0 & 1\end{pmatrix}$ $\quad\quad$ $\begin{pmatrix}-1 & 4\\ -1 & 3\end{pmatrix}$ $\begin{pmatrix}1 & 0\\ -1 & 1\end{pmatrix}$

monopole $\quad$ dyon

(III) $\Longrightarrow$ SW invs

(IV) $\Longrightarrow$ Donaldson $\overset{\text{Witten's}}{\underset{\text{conj.}}{=}}$ SW + SW' + u-plane

Rmk • (I), (II), (III) possess S^1-action (rotate B_γ)

→ look like conformal theories

• (IV) $\mathcal{M}_\gamma \cancel{\circlearrowleft} S^1$

→ nonconf. (quantum anomaly of $U(1)_R$-symmetry)

- superconformal theories:

$$\begin{array}{ccc} S^1 \circlearrowright & \mathcal{M}_\gamma & \\ & \Big\downarrow \pi & \qquad \pi \text{ equivariant} \\ \circlearrowright & B_\gamma & \end{array}$$

- $(B_\gamma)^{S^1} = \{pt\}$ superconf. pt. 0

(V) class S, labels: $\mathfrak{g}$: ADE, Σ: RS

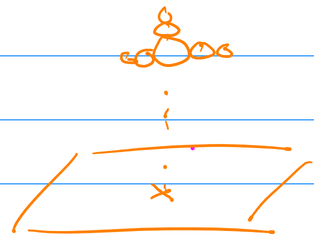
$\mathcal{M}_\gamma$ = m.sp. of G-Higgs bdls on Σ

π: Hitchin fibration

S^1: Hitchin action

- if $\mathfrak{g} = \mathbb{C}$ ⇝ (I), (II)

VI $\mathfrak{g} = sl(3, \mathbb{C})$, $\Sigma = \mathbb{P}^1$ w/ 3 mkd pts

$\mathcal{M}_{\mathcal{T}} =$ 

IV^*

- away from sing.: M.N (E_6)
- can S^1-action help? a little

- Hilbert sp. of $\mathcal{T}$ (top. twisted) on $S^3 \simeq \{$regular functions on $B_{\mathcal{T}}\}$
- partition funct of $\mathcal{T}$ on $S^1 \times S^3 \simeq \dim = \infty$
  $\to$ S^1-equiv. part. funct., though, $\simeq \chi_{S^1}(\mathcal{O}_{B_{\mathcal{T}}})$
- free theory $\rightsquigarrow B_{\mathcal{T}} = \mathbb{C}$
  $\mathcal{H}_{\mathcal{T}}(S^3) \simeq \langle 1, z, z^2, \ldots \rangle$
  $\chi_{S^1} = 1 + t + t^2 + \ldots = \frac{1}{1-t}$
- can we regularize $S^1 \times M^3$ part. function?
  $\rightsquigarrow$ $\mathbb{C}^\times$-family of 3d TQFT's

$\rightsquigarrow$ Impossible.
(Ocneanu rigidity: $\nexists$ 1-param. family of 3d TQFTs)

$\rightsquigarrow$ since S^1 is R-sym, acts on supercharges

- want: discrete flavor sym.,
  won't act on superch.

Rmk. $S^1 \times S^3$ worked, and it does work for $S^1 \times$ Seifert mfd

- often, it is $\mathbb{Z}_N \subset S^1$

  i) $N = 1 \to$ all previous examples,
  all gauge theories

  ii) $N > 1 \to$ non-gauge theories

  $\rightsquigarrow \mathbb{Z}_N^\times$ - family of 3d TQFTs