Gravity @ ICTP

Nicolas Yunes - Black Holes, Binary Systems & Grav. Waves

- syllabus:
  - Kerr BH
  - EMRI
  - comparable-mass binaries
  - BH pert. theory

Kerr BH

- Boyer-Lindquist metric

$$ds^2 = -\left(1-\frac{2Mr}{\rho^2}\right)dt^2 - \frac{4Mar}{\rho^2}\sin^2\theta\, dt\, d\varphi$$

$$+\frac{\Sigma}{\rho^2}\sin^2\theta\, d\varphi^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

where
$$\rho^2 = r^2 + a^2\cos^2\theta$$
$$\Delta = r^2 - 2Mr + a^2$$
$$\Sigma = (r^2+a^2)^2 - a^2\Delta\sin^2\theta$$

- $\exists$ Outgoing Kerr metric, $\exists$ Kerr-Schild which decomposes as $d^2s = \eta_{\alpha\beta}dx^\alpha dx^\beta + \underset{\text{function}}{H}\, l_\alpha l_\beta dx^\alpha dx^\beta$

- props.
  - asympt. flat
  - stationary (not static)
  - $M = M^{ADM} = M^K = M^N$
  - $a = J^{ADM}/M^{ADM}$ Kerr Spin param.
    $a/M \in (-1,1)$

- let $t^\alpha = \frac{\partial x^\alpha}{\partial t}$, $\varphi^\alpha = \frac{\partial x^\alpha}{\partial \phi}$, $K_{\alpha\beta}$ s.t. $K_{(\alpha\beta;\delta)} = 0$

Special observers

- ZAMO ("zero-ang-mom. obs")
  - "rotates with BH"
  - $\frac{dz^\alpha}{d\tau} = u^\alpha$ 4-veloc. s.t. $\tilde{L} \equiv u_\alpha \phi^\alpha = 0$
  - but $\tilde{L} = g_{t\varphi}\dot{t} + g_{\varphi\varphi}\dot{\varphi} = 0$

$$\frac{\dot{\varphi}}{\dot{t}} = \underset{\displaystyle -\frac{g_{t\varphi}}{g_{\varphi\varphi}}}{\underset{\shortparallel}{\frac{d\phi}{dt}}} \equiv \Omega = \frac{2Mar}{\Sigma}$$

- Static obs $u^\alpha = \gamma t^\alpha$
  - $u^\alpha u_\alpha = -1 \Rightarrow \gamma = (g_{tt})^{-1/2}$
  - clearly we require $g_{tt} \neq 0$
  - $\to$ the static observer $\nexists$ in the ERGOSPHERE $r(\vartheta) = M + M\left(1 - \frac{a^2}{M^2}\cos^2\vartheta\right)^{1/2}$

- stationary obs $u^\alpha = \gamma(t^\alpha + \underset{\displaystyle \frac{d\phi}{dt}}{\underset{\shortparallel}{\bar{\Omega}}}\phi^\alpha)$

$$\Rightarrow \gamma = g_{\varphi\varphi}^{-1/2}\left(2\underset{\displaystyle -\frac{g_{t\varphi}}{g_{\varphi\varphi}} = \Omega}{\underset{\shortparallel}{\omega}}\bar{\Omega} - \bar{\Omega}^2 - \frac{g_{tt}}{g_{\varphi\varphi}}\right)$$

$\exists$ if

$\Rightarrow \Omega_- < \bar{\Omega} < \Omega_+$, $\Omega_\pm = \omega \mp \frac{\Delta^{1/2}\rho^2}{\Sigma\sin\vartheta}$

- r_H when $\Omega_- = \Omega_+$, $r_\pm = M \pm M(1 - a^2/M^2)^{1/2}$ (since $\Delta = 0$)
  - 2 horizons $r_H = r_\pm$

- $F = r - r_H = 0$ is a null sfc

- is this a BH?

Kretschmann $= \frac{48M^2}{\rho^2}(r^2 - a^2\cos^2\vartheta)(a^4 - 16a^2r^2\cos^2\vartheta)$

$\rho^2 = r^2 + a^2\cos^2\vartheta$

$\Rightarrow \rho = 0 \Rightarrow r = 0$ and $\vartheta = \frac{\pi}{2}$

$\rightarrow x^2 + y^2 = (r^2 + a^2)\sin^2\vartheta$

$\Rightarrow x^2 + y^2 = a^2, z = 0 \rightarrow$ ring?

Geodesics

$$\frac{d^2 z^\alpha}{d\tau^2} = -\Gamma^\alpha_{\mu\nu} u^\mu u^\nu$$

$\Rightarrow$ $\tilde{E} = -u_\alpha t^\alpha$

$\tilde{L} = u_\alpha \phi^\alpha$ are conserved,

$Q = u_\alpha u_\beta K^{\alpha\beta}$

$u^\alpha u_\alpha = -1$

1st order $\rightarrow$ e.o.m's

$\rho^2 \dot{t} = -a(a\tilde{E}\sin^2\vartheta - \tilde{L}) + (r^2 + a^2)\frac{\tilde{L}}{\Delta}$

$\rho^2 \dot{r} = \_\_$

$\rho^2 \dot{\vartheta} = \_$

$\rho^2 \dot{\varphi} = \ldots$