

B. Carneiro da Cunha

- $y'' + p y' + q y = 0$ Fuchsian 2nd order

$$p(z) = \frac{1-\alpha_0}{z} + \frac{1-\alpha_1}{z-1} + \sum_i \frac{1-\alpha_i}{z-z_i}$$

$$q(z) = \frac{k}{z(z-1)} + \sum_k \frac{k_k}{z(z-1)(z-z_k)}$$

by fixing using conf. tr 3 of the singularities to $z = 0, 1, \infty$.

- related to flat holomorphic conn's

$$\frac{d}{dz} \begin{pmatrix} y \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -q(z) & p(z) \end{pmatrix} \begin{pmatrix} y \\ y' \end{pmatrix} \quad (*)$$

- given sol'n $\phi(z)$, define $A(z) = (\partial_z \phi) \phi^{-1}$

- given such $A(z)$, we can reconstruct $p(z), q(z)$ (an "oper"), but generically we get more singularities than in (*).
if $A_{12} \neq \text{constant}$ \Rightarrow

- flat conn's \Rightarrow observables are holonomies

$$M_i = P \exp \oint_{\gamma_i} A(z) dz$$

$$\text{and } \text{Tr } M_i = 2 \cos \pi \alpha_i, \quad \text{Tr } M_i M_j = 2 \cos \pi \alpha_{ij}$$