G + P seminar.

Fredholm determinants

- probabilities in gapped systems
  $$\to \det\left(1 + \frac{\psi(x)\varphi(y) - \psi(y)\varphi(x)}{x - y}\Big|_{\substack{(t,\infty)\\(0,\infty)}} \right)$$
  where, e.g., $\psi(x) \sim H_{n+1}(x)$, $\varphi(x) \sim H_n(x)$

- Borodin, Deift: $\tau_{BD}(t) = \det(1 - \lambda\cdot(\text{same}))$,
  with $\varphi(x) = x^{z}(1-x)^{\nu_1}\, {}_2F_1\left[\begin{matrix} z+\nu_1+\nu_\infty & z+\nu_1-\nu_\infty \\ 2z & \end{matrix}\Big| x\right]$.
  $\psi(x) = \varphi(x)$ but $z \mapsto z+1$
  $\to$ solves PVI (τ_{BD} has 4 params + 2 initial conds)

- SJM: $\tau = \langle 0 | V_1 \cdots V_n | 0 \rangle$
- GIL: $\tau_{PVI} = \langle 0 | \mathcal{O}_1 \cdots \mathcal{O}_n | 0 \rangle$

  CFT primary fields

  – how to explain this?

- free fermions
  - generated by ideal $\{\psi_I, \psi^*_J\} = \delta_{I,-J}$,
    $\{\psi_I, \psi_J\} = \{\psi^*_I, \psi^*_J\} = 0$
  - $\psi_{I>0}|0\rangle = \psi^*_{I>0}|0\rangle = 0$
  - Fock space $\mathcal{F} = \{\prod \psi^*_{-I_i} \prod \psi_{-J_j} |0\rangle\}$
- group elements $G : \mathcal{F} \to \mathcal{F}$
  $$\begin{cases} G\psi^*_I G^{-1} = \sum_J B(G)_{IJ}\, \psi^*_J \\ G\psi_I G^{-1} = \sum_J B(G)^{-1}_{JI}\, \psi_{-J} \end{cases}$$

- generalised Wick:

$$\langle 0|\psi^*_{I_1}\psi_{J_1}\cdots\psi^*_{I_m}\psi_{J_n}\, G\, \psi^*_{-I_1}\psi_{-J_1}\cdots\psi^*_{-I_n}|0\rangle$$

$$= \langle 0|G|0\rangle \times \det_{\substack{i,j=1,\dots,n\\ p,q=1,\dots,m}} \begin{pmatrix} \left(\dfrac{\langle 0|G\psi^*_{-I_i}\psi_{-J_j}|0\rangle}{\langle 0|G|0\rangle}\right)_{n\times n} & \left(-\dfrac{\langle 0|\psi_{I_p} G\psi^*_{-I_i}|0\rangle}{\langle 0|G|0\rangle}\right)_{n\times m} \\ \left(\dfrac{\langle 0|\psi^*_{I_q} G\psi_{-J_j}|0\rangle}{\langle 0|G|0\rangle}\right)_{m\times n} & \left(\dfrac{\langle 0|\psi^*_{I_q}\psi_{I_p} G|0\rangle}{\langle 0|G|0\rangle}\right)_{m\times m} \end{pmatrix}$$

$$\tau = \langle 0|G_1 G_2|0\rangle \qquad \overset{\shortparallel}{\begin{pmatrix} a & -b \\ c & -d \end{pmatrix}}$$

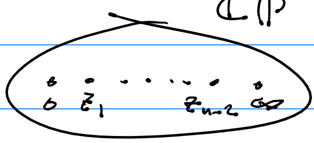
$$= \sum_{\substack{\mathbb{I}=\{I_i\}\\ \mathbb{J}=\{J_j\}\\ |\mathbb{I}|=|\mathbb{J}|}} \langle 0|G_1\psi^*_{-\mathbb{I}}\psi_{-\mathbb{J}}|0\rangle \times \langle 0|\psi^*_{-\mathbb{J}}\psi_{-\mathbb{I}} G_2|0\rangle$$

$$= \sum_k \sum_{|\mathbb{I}|=|\mathbb{J}|=k} \det\left((a_1)^{-\mathbb{I}}_{-\mathbb{J}}\right) \det\left((-d_2)^{\mathbb{J}}_{\mathbb{I}}\right)$$

$$= \sum_k \operatorname{tr} \Lambda^k a \cdot \Lambda^k(-d) = \sum_k \operatorname{tr} \Lambda^k(-ad)$$

$$= \det(1 - ad) = \det\begin{pmatrix} \mathbb{1} & a \\ d & \mathbb{1} \end{pmatrix}$$

$\mathbb{CP}^1$



$$\partial_z \varphi(z) = \overset{n\times n}{\varphi}(z) \sum_{i=0}^{n-1} \frac{A_i}{z - z_i}$$

$$\varphi(\gamma_k \circ z) = M_k\, \varphi(z)$$