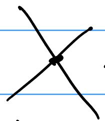
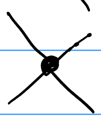
Qft.

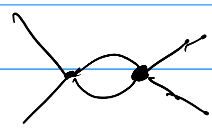
Irrelevance of higher dimensional theories.

- $\mathcal{L} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4!}\varphi^4 + \frac{\hat{\lambda}}{M^2}\varphi^3 \Box \varphi$

redundant?
forget it?

 $\rightsquigarrow -i\lambda$

 $\rightsquigarrow \frac{\hat{\lambda}}{M^2}p^2$

 $\sim \int \frac{d^4q}{(2\pi)^4}\frac{1}{M^2}\frac{q^2\lambda\hat{\lambda}}{(q^2-m^2)((k-q)^2-m^2)}$

$\sim \frac{\lambda^2}{M^2}\lambda\hat{\lambda}\mu^{\varepsilon}\Gamma(\varepsilon)f(p^2, m^2)$

$\to \log\frac{p^2}{\mu^2}$

by dim-reg.

→ no powers of μ^2?

→ but with $\frac{d}{dp^2}\Gamma^{(4)}(s=t=u=-\mu^2) = -\frac{\hat{\lambda}}{M^2}$

$\Rightarrow \Gamma^{(4)} \supset \frac{\lambda\hat{\lambda}}{M^2}\mu^2\log\frac{p^2}{\mu^2}$ as $m \to 0$

→ as we saw, however, MS doesn't have this problem.

Redundant operators.

$$S_{e.f.t.} = S_{d\leq 4} + \sum_{\substack{n \\ \dim O_n > 4}} c_n O_n$$

- however, while off-shell $\Gamma^{(N)}(c_n)$ depends on all c_n's, on-shell some operators will become redundant
  → behind this usually lie physically irrelevant field redefinitions.

Schwinger-Dyson equs.

$$\left\langle \frac{\delta S}{\delta\varphi(x)}\varphi(x_1)\cdots\varphi(x_n)\right\rangle = \sum_{i=1}^{n}\delta^{(4)}(x-x_i)\langle\varphi(x_1)\cdots\hat{\varphi}(x_i)\cdots\varphi(x_n)\rangle$$

→ come from $\int D\varphi \frac{\delta}{\delta\varphi(x)} e^{iS + \int J\varphi} = 0$

→ however, LHS is $\equiv 0$ on-shell

→ classically, $\frac{\delta S}{\delta\varphi} \equiv$ e.o.m. $= 0$

Claim. Operators of the form $\mathcal{O}(x) = F(\varphi(x)) \frac{\delta S}{\delta \varphi(x)}$ are redundant.

Pf. Look at $e^{iW[J,s]} = \int D\varphi\, e^{iS + \int d^4x (s\varphi + J\mathcal{O})}$

Redefine field as $\varphi(x) \mapsto \varphi(x) + J(x) F(\varphi(x))$

note that $S(\varphi - JF) = S(\varphi) - \int d^4x\, J \underbrace{F \frac{\delta S}{\delta \varphi}}_{\equiv \mathcal{O}} + \dots$

$$.. = \int D\varphi \,|\det \tilde{J}|\, e^{iS(\varphi) + i\int d^4x\, s(\varphi - JF)}$$

$\hookrightarrow \tilde{J}(x,y) = \delta(x-y) - \frac{\delta F(\varphi(x))}{\delta \varphi(y)} J(x)$ Jacobian.

Taking derivatives gives

$$i\langle \varphi(x_1)\dots\varphi(x_n)\mathcal{O}(x)\rangle = \frac{\delta}{\delta J(x)} \int D\varphi\, |\det \tilde{J}|\, \overbrace{(\varphi - JF)(x_1)\dots(\varphi - JF)(x_n)}\, e^{iS}$$

$$= -\sum_{i=1}^{n} \delta^{(4)}(x - x_i) \langle \varphi(x_1) \dots F(\varphi(x_i)) \dots \varphi(x_n)\rangle$$

$$- \int d^4x \left\langle \frac{\delta F(\varphi(x))}{\delta \varphi(x)} \varphi(x_1)\dots\varphi(x_n)\right\rangle$$

$\hookrightarrow$ contribution from $\frac{\delta}{\delta J} |\det \tilde{J}|$

$\sim \delta^{(d)}(0) \xrightarrow[\text{dim-reg}]{} 0$ because power-like divergences are ignored

$\to$ let's go back to e.f.t.:

$$\mathcal{L}_{e.f.t.} = \frac{1}{2}(\partial\varphi)^2 - \frac{1}{2} m^2 \varphi^2 - \frac{\lambda}{4!}\varphi^4 + \frac{g_1}{M^2}\varphi^6 + \frac{g_2}{M^2}(\Box\varphi)^2 + \frac{g_3}{M^2}\varphi^3 \Box \varphi + \mathcal{O}\left(\frac{1}{M^4}\right)$$

- we took $\varphi(x) \mapsto -\varphi(x)$ symmetry
- up to total derivatives, no others.

- let's take $m=0$, $\lambda \mapsto 4!\,\lambda$ to make algebra simpler
- e.o.m.: $-\Box\varphi - 4\lambda\varphi^3 = \mathcal{O}\left(\frac{1}{\Lambda^2}\right)$

$$\Rightarrow \mathcal{L}_{e.f.t.} \xmapsto[\substack{\text{leave}\\ \text{kinetic}\\ \text{part alone}}]{\text{e.o.m.}} \mathcal{L}'_{eft} = \frac{1}{2}(\partial\varphi)^2 - \lambda\varphi^4 + \frac{g_1}{\Lambda^2}\varphi^6 + \frac{g_2}{\Lambda^2}16\lambda^2\varphi^6 + \frac{g_3}{\Lambda^2}\varphi^5(-4\lambda) + \mathcal{O}\left(\frac{1}{\Lambda^4}\right)$$

$$= \frac{1}{2}(\partial\varphi)^2 - \lambda\varphi^4 + \frac{\tilde{g}_1}{\Lambda^2}\varphi^6, \quad \tilde{g}_1 = \dots$$

- we could also redefine $\varphi \mapsto \varphi + \frac{a}{\Lambda^2}\Box\varphi + \frac{b}{\Lambda^2}\varphi^3$

$$\Rightarrow \mathcal{L}_{e.f.t.} \xmapsto[\text{redefn.}]{\text{field}} \frac{1}{2}(\partial\varphi)^2 - \lambda\varphi^4 + \frac{\tilde{g}_1}{\Lambda^2}\varphi^6,$$ with a, b chosen s.t. other terms vanish.

$\Rightarrow$ same $\mathcal{L}'_{e.f.t.}$

Important Rmk. Even if we do this, they will reappear as counterterms $\to$ now do it again, and the counterterms will be added to the non-irrelevant ones

$\to$ but this changes the β-funcs.

$\to$ so the „irrelevant" operators are still somehow relevant.