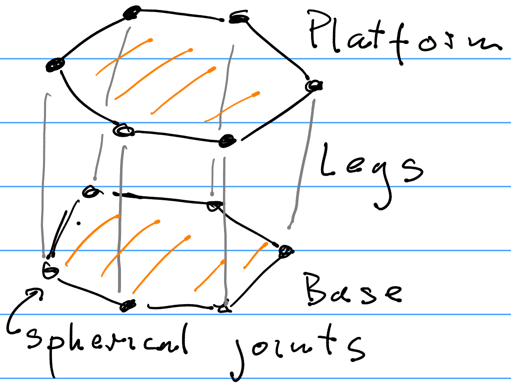
AG seminar - M. Gallet

Alg. geometry for kinematics



← hexapod or Stewart-Gough platform

<u>Goal</u>: classify all <u>mobile</u> hexagons!

- this is an old open problem

<u>Method</u> here: use alg. geom.

<u>Main idea</u>: config. spaces are subset of SE_3 (not cpt.)

→ get a proj. model of SE_3

→ Fobini quadric in $\mathbb{P}^7$ uses doat quaternions to embed it

→ but the equations of the legs turn out to be quadratic

→ should linearize

→ $\varphi: SE_3 \longrightarrow \mathbb{P}^{16}$

$x \cdot M + y \longmapsto (\{m_{ij}\}, \{y_i\}, \{x_i\}, r, h)$

where $x = -M^t y$

$r = \langle x, x\rangle = \langle y, y\rangle$

h homog. parameter

- let $X := \overline{\varphi(SE_3)}$. we get:
  - $\dim X = 6$, $\deg X = 40$, $X = SE_3 \cup B$ where $B = X \cap \{h = 0\}$

→ linear leg conditions also

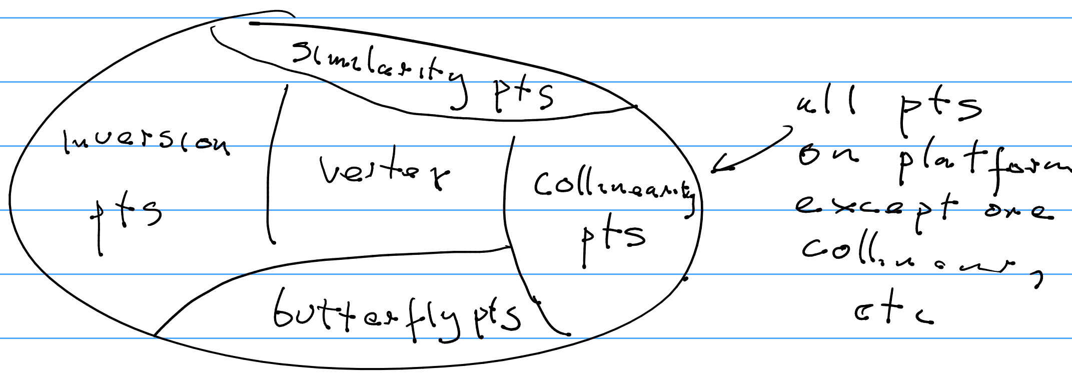
Def. Given hexapod Π, we get lin. space Λ_Π from leg conds. Define $K_\Pi := X \cap \Lambda_\Pi$

Rmk General legs $\Rightarrow K_\Pi = \langle 40$ pts over $\mathbb{C} \rangle$

Main idea: if Π mobile, then K_Π is a curve, furthermore intersects bdry
$\Rightarrow K_\Pi \cap B := B_\Pi$ not empty

- note result: B has only cpx pts, except one real one ("tip of cone"), but that is not a realisable config.

$\rightarrow$ we get a partitions:



- interesting nontrivial case are inversion & similarity
  $\rightarrow$ pts of base are similar or inversions of the pts of platform.

- Möbius photogeometry: $\vec{x}$ 6-tpl of pts in $\mathbb{R}^3$
  $f_{\vec{x}} : S^2 \longrightarrow M_{0,6}$
  S^2 is $\mathbb{P}^1$'s $\{x^2+y^2+z^2=0\}$ in $\mathbb{P}^2$

- to sum up: mobility => Möbius curves of base & pltform intersect
- idea: manufacture Möbius curves w lots of pts in common

Construction:

i) take general 6-tpl. $\vec{A}$ in $\mathbb{R}^3$

ii) let $C := f_{\vec{A}}(\mathbb{P}^1) \subseteq M_{0,6}$

iii) C is a rational sextic curve (turns out) in $M_{0,6} \subseteq \mathbb{P}^4$ ← cubic 3-fold

iv) ideal of C: $I(C) = (\underbrace{\text{cubic of } M_{0,6}, Q_1, Q_2, \ldots}_{\text{zero set } Y})$

v) $Y = C \cup D$

vi) D has degree 6
- liaison theory: $\deg C - \deg D = \alpha(p_a(C) - p_a(D))$ so if D is red, it is rational

vii) $D = f_{\vec{B}}(\mathbb{P}^1)$

viii) $C \cap D = \{14 \text{ pts}\}$

Rmk pts in $C \cap D$ correspond to pts in B_i

ix) pick leg lengths s.t. $K_H \cap B$ with mult ≥ 3

=> we get $|B_H| \geq 3 \cdot 14 = 42 \geq 40$