Буряк.

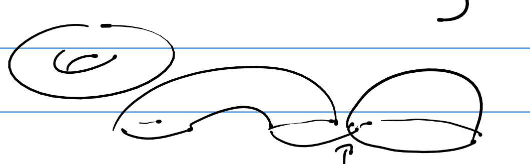
## Moduli of curves

- $\mathcal{M}_{g,n}$ m.s. of stab. curves
  → nodes, $\mathrm{Aut}\{C_g, x_1, \ldots, x_n\}$ finite
- $\overline{\mathcal{M}}_{g,n}$ cpx orbifold of $\dim_{\mathbb{C}} = 3g-3+n$
- the ctg. sp. at each mkd. pt. naturally forms a lbdl $\mathbb{L}_i \to \overline{\mathcal{M}}_{g,n}$
  - let $\psi_i := c_1(\mathbb{L}_i) \in H^2(\overline{\mathcal{M}}_{g,n}, \mathbb{Q})$
- consider $\langle \tau_{d_1} \cdots \tau_{d_n} \rangle_g := \int_{\overline{\mathcal{M}}_{g,n}} \psi_1^{d_1} \cdots \psi_n^{d_n}$

  or $\equiv 0$ if $\sum d_i \neq 3g-3+n$.
- Witten: $w := \varepsilon \dfrac{\partial^2 \mathcal{F}}{\partial t_0^2}$ solves KdV hier.
- Virasoro eqns $L_n e^{\mathcal{F}} = 0,\ n \geq 1$
  - $\underline{n=-1}$ $\partial_0 \mathcal{F} = \sum_{k\geq 0} t_{k+1} \partial_k \mathcal{F} + \frac{\varepsilon^{-2}}{2} t_0^2$
    (string eqn.)

## Moduli of Riem. sfc's w/ bdry.

- nodal R.S w bdry $(C, \partial C)$



bdry node

- introduce "double" s.t. $(C, \partial C)$ "+" "double"
  $= D(C, \partial C)$, which is bdryless
  and def $g(C, \partial C) = g(D(C, \partial C))$.

- we also get $\mathbb{L}_i \to \overline{\mathcal{M}}_{g,k,l}$, $i \in 1, \dots, l$ corresponding to internal mkd pts

- open int. no's depend on bdry conds, so pick $S_i \in \Gamma(\partial\overline{\mathcal{M}}_{g,k,l}, \mathbb{L}_i|_{\partial\overline{\mathcal{M}}_{g,k,l}})$ and, eventually, use $e$ relative Euler classes wrt this choice.

Generalizing Witten's conj.

- $N \geq 1$ rk of theory, $t^\alpha_d$, $\alpha \in 1, \dots, N$, $d \geq 0$
- "total ancestor potentials" $\mathcal{F} = \sum_{g \geq 0} \varepsilon^{2g-2} \mathcal{F}_g(t^*_*, \varepsilon)$