Srní

NC Alg geom based on quantum flag mflds - J. Šťovíček.

Thm (Serre '56) V cpx aff. var.

$\exists$ 1-1 correspondence between

i) alg. vect. bdls $\mathbb{E} \to V$

ii) certain fin. gen. proj $\mathbb{C}[V]$-modules

- all $\mathbb{C}[V]$ modules will correspond to q.c. sheaves, fin. gen to coherent sheaves.

- flag manifolds
  - example: Grassmanians
  - the set $Gr_{n,r}$ of r-dim subspaces of $\mathbb{C}^n$ forms a subset of a proj. space via $z: Gr_{n,r} \longrightarrow \mathbb{P}_{\mathbb{C}}^{\binom{n}{r}}$ embedding $\langle v_1, \dots, v_r \rangle \mapsto \langle v_1 \wedge \dots \wedge v_r \rangle$ where we fix a basis of $\Lambda^r \mathbb{C}^n$ and assign it Plücker coordinates
    $\rightarrow$ it is a zero-set of homogeneous polynomials
  - $Gr_{n,r}$ as flag mflds
    - repres. theoretically, $\Lambda^r \mathbb{C}^n$ is canonically a rep of sl_n, the rth fundamental rep $V(\omega_r)$
    - Im z can be identified with the orbit of $SL_n \cdot v$ of $v \in V(\omega_r)$, with coord. ring given by $S(Gr_{n,r}) \cong \bigoplus_{j=0}^{\infty} V(j\omega_r)^V$.
      $\rightarrow$ this generalises to all flag manifolds $\rightarrow$ cpx. proj. varieties F given by quadratic hom. polynoms w $S(F) \cong \bigoplus_{n=0}^{\infty} V(n\lambda)^V$, $\lambda = \sum$ fund. weights of F

(Part 2)

- $\mathcal{F} \in (QCoh_V)$, $n \in \mathbb{Z}$, $\mathcal{F}(n) := \mathcal{F} \otimes_{\mathcal{O}_V} \mathcal{L}^{\otimes n}$,
  $\mathcal{L}$ c_1 $\mathcal{O}_{\mathbb{P}^n_{\mathbb{K}}}(1)$ ample

- $\Gamma_*(\mathcal{F}) := \bigoplus_{n \in \mathbb{Z}} \Gamma(V, \mathcal{F}(n))$

- e.g. $\Gamma_*(\mathcal{O}_V) \cong S(V)$

- $SL_n/P \cong Gr_{n,r}$ with $P = \left(\begin{array}{c|c} P_r & Q \\ \hline 0 & P_{n-r} \end{array} \right)$,
  $P_r \in M_r(\mathbb{C})$, $P_{n-r} \in M_{n-r}(\mathbb{C})$, $\underset{SL_n}{\overset{\cup}{\uparrow}}$ $P \hookrightarrow h$

$\nabla : P \longrightarrow P \otimes_{\mathcal{O}_q(Gr_{n,r})} \Omega_q$