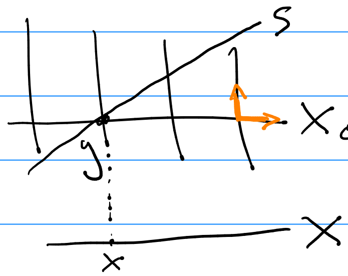
Тихомиров

- $A \in \mathcal{A}_2^*$, $\ker D_A = T_{[A]}\mathcal{M}$.

- first, in general let $s \curvearrowright \begin{array}{c} E \\ \downarrow \\ X \end{array}$ vbdl,

fiber of any rk, with section s



$Z = \{0\text{-set of } s\}$
$= s \cap X_0$

$y = (x, 0) \in Z$

$$\begin{tikzcd} (s_*)_x : T_x M \arrow[r, "s_*"] \arrow[dr] & T_y X_0 \oplus E_x \arrow[d, "pr_2"] \\ & E_x \end{tikzcd}$$

- s transversal on $y \in Z \iff pr_2 \circ s_*$ is epi
- —||— on $Z \implies Z \xhookrightarrow[\text{sbmfd}]{\text{smooth}} X$

$$0 \to \underbrace{TZ \to TX|_Z}_{\text{from } Z \hookrightarrow X} \to N_{Z/X} \to 0$$

- if $Z = (s)_0$, $N_{Z/X} = E|_Z$

- we had $\Omega^1(\mathrm{ad}P)_{L^2_2} \xrightarrow{P_+ \circ D_A \;?} \Omega^2_+(\mathrm{ad}P)_{L^2_1}$

$$\begin{array}{ccc} P_+F : \mathcal{A}_2^* & \to & \Omega^2_+ \\ \uparrow & & \cup \\ ASD = \tilde{\mathcal{M}}^* & \to & \{0\} \end{array} \qquad \tilde{\mathcal{M}}^* = (P_+F)^{-1}(0)$$

$$\begin{tikzcd}
\phi : \mathcal{A}_2^* \arrow[r, "A \mapsto (A, P_+ F_A)"] \arrow[dr, bend right] & \mathcal{A}^* \times \Omega_+^2 \arrow[d] \\
& \mathcal{A}_2^*
\end{tikzcd}$$

$S_* = \phi_* = (id, P_+ \nabla_A)$

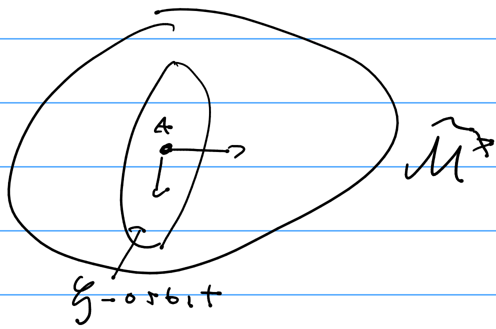
$A \overset{\phi}{\mapsto} y$

$T_A \mathcal{A}_2^* \xrightarrow{P_+ \nabla_A} \Omega_+^2$

$\Rightarrow$ ϕ is transversal at $\tilde{\mathcal{M}}^*$

$\Rightarrow$ $\tilde{\mathcal{M}}^*$ smooth Banach (sub)mfd

$- \ \mathcal{G}_3 \curvearrowright \tilde{\mathcal{M}}^*$



$\mathcal{M}^* = \{A \in \mathcal{A}_2^* \mid ASD\}$

$$\begin{tikzcd}
 & & 0 \arrow[d] & 0 \arrow[d] & \\
\Rightarrow \ 0 \arrow[r] & T_{[A]} \mathcal{G}_3 \arrow[r] \arrow[d, equal] & T_A \tilde{\mathcal{M}}^* \arrow[r] \arrow[d] & T_{[A]} \mathcal{M}^* \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & T_{[A]} \mathcal{G}_3 \arrow[r, "\nabla_A"] & T_A \mathcal{A}_2^* = \Omega_1 \arrow[r] \arrow[d, "P_+ \circ \nabla_A"] & \operatorname{coker} \nabla_A = \ker \nabla_A^* \arrow[r] \arrow[d, "P_+ \circ \nabla_A"] & 0 \\
 & & N_{A, \tilde{\mathcal{M}}^* / \mathcal{A}_2^*} \arrow[r, equal] \arrow[d] & \Omega_2 \arrow[r] \arrow[d] & 0 \\
 & & 0 & 0 &
\end{tikzcd}$$

$T_{[A]} \mathcal{G}_3 = \Omega_0(\mathrm{ad}\, P)_{L^2_3}$

by 5-lemma, or indeed previous considerations
$\rightarrow$ but here everything commutes, more info

$$\Rightarrow \ker\{D_A = (\nabla_A^*, P_+ \circ \nabla_A)\} = \ker(P_+ \circ \nabla_A |_{\ker \nabla_A^*}) = T_{[A]} \mathcal{M}^*$$

- deep result by Uhlenbeck

- now, it's true that $(B, g) \Rightarrow \mathcal{M}^*(P, g)$
  - clear metric dependence
    - can we get rid of it?

$$\begin{array}{ccl} T^*B & F & \text{- frame bdl for } T^*B \\ \downarrow & \downarrow & = GL(4, \mathbb{R})\text{-pbdl} \\ B & B & \text{- it has } \mathcal{G}_F \text{, its gp of gauge tr.} \end{array}$$

- fix metric g_0

$$\{s(x) \mid x \in B\} =: s \in \Gamma(\mathrm{Sym}^2 T^* B)$$

- $\underset{\{\varphi_x \mid x \in B\}}{\varphi} \in \mathcal{G}_F$ acts on s by

$$\varphi(s) = \{\varphi_x \, s(x) \, \varphi_x^{\tau} \mid x \in B\}$$

$$\Rightarrow \mathcal{C} = \{\varphi(g_0) \mid \varphi \in \mathcal{G}_F\}$$

- however, completion of smooth metrics might not give metrics $\Rightarrow$ we lose pos. definiteness
  - UNLESS, pick $g_0 \in \mathcal{C}^k(\mathrm{Sym}^2 T^* B)$, $k < \infty$
    - has canonical norm
      - over cpt B, this is already complete
        - look at cont. funcs over interval, e.g.
    - of course, $\mathcal{G}_F$ should probably $\in \mathcal{C}^{k+1}$

$\mathcal{P}_+ F : \mathcal{A}_2^* \times \mathcal{C} \longrightarrow \Omega^2$

$\quad (A, g) \longmapsto P_{+,g} F_A$

$- (\mathcal{P}_+ F)^{-1}(0) = \bigcup_{g \in \mathcal{C}} \tilde{\mathcal{M}}^*(P, g) =: \tilde{\mathcal{M}}^*(P, \mathcal{C})$

- Uhlenbeck:

  1) 0 is a regular pt of $\mathcal{P}_+ F$

  $\overset{\text{cor.}}{\Longrightarrow} \tilde{\mathcal{M}}^*(P, \mathcal{C})$ is smooth Banach mfd

  ii) $\pi : \tilde{\mathcal{M}}^*(P, \mathcal{C}) \longrightarrow \mathcal{C}$ is a sm. map of Banach mfds and, by Sard-Smale, for almost all $g \in \mathcal{C}$, $\tilde{\mathcal{M}}^*(P, g)$ satisfies

  $\left(\Omega^1(\operatorname{ad} P)_{L^2_2} \xrightarrow{P_+ \circ D_A} \Omega^2_+(\operatorname{ad} P)_{L^2_1} \text{ surjects} \right)$

Rmk $SU(2)$ conn. A is reducible, on P nontrivial, if $\operatorname{Stab}_{\mathcal{G}}(A) = S^1$

- so $\exists$ S^1-pbdl Q s.t.

  $P = Q \times_{S^1} SU(2)$

- in fact, these are the same statement

$\iff \xi = P \times_{SU(2)} \mathbb{C}^2$ decomposes to $L \oplus L^\vee$