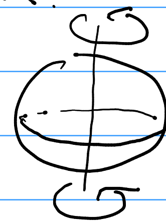
Gauge @ ICTP

Non-gauge invariants, cont'd – Pei

- 4d $\mathcal{N}=2$ superconf. $\mathcal{T}$, $\mathcal{M}_{\mathcal{T}} \circlearrowleft S^1$

  $\Rightarrow$ can regularize $Z_{\mathcal{T}}(S^1\times S^3) \underset{\text{HL-theory}}{=} \dfrac{1}{1-t}$

  but not any old 3-mfd.



- S^1 R-sym $\circlearrowleft$ cpx. str's

$$\underset{\text{cpx. Lagrangian}}{F} \hookrightarrow (\mathcal{M}_{\mathcal{T}}, \overbrace{\Omega_I}^{\text{hol. symp. form}})$$

$$\downarrow \pi$$

$$B_{\mathcal{T}}$$

$\vartheta \in S^1 : \Omega_I \mapsto e^{i\vartheta \cdot N}\, \Omega_I$.

- $\mathbb{Z}_N \subset S^1$ fixes Ω_I

tri-holomorphic

acts on I, J, K

$$\mathbb{Z}_N \hookrightarrow S^1 \longrightarrow U(1)_R$$

no hyperkähler action, no supercharge action → "flavor symmetry"

– symmetry of $\mathcal{T}$
– isometry of $\mathcal{M}_{\mathcal{T}}$

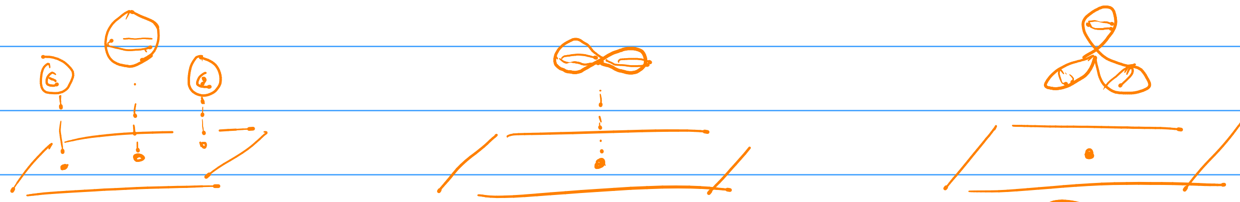
"R-sym", acts on supercharges

- 2 possibilities
  - → $N>1$, gauge theories class S
  - → $N>0$, non-gauge

Examples

① "rank 1 Argyres-Douglas theories"

$\mathcal{J} =$	(A_1, A_2)	(A_1, A_3)	(A_1, D_4)
$\mathcal{M}_{\mathcal{J}} =$	II	III	IV
	$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$
$N =$	5	3	2



- $\mathcal{H}_{\mathcal{J}}(S^3) \simeq \{\text{reg. functions on } B_{\mathcal{J}}\}$

- S^1-character $\chi_{S^1} = \mathrm{Tr}_{\mathcal{H}_{\mathcal{J}}(S^3)}(S^1\text{-action})$

d_i = weights
n_i = multiplicities

$$= \frac{1}{\prod_i (1-t^{d_i})^{n_i}}$$

- is $\chi_{S^1}(t = \zeta_N)$ well defined?

Rmk i) family of Argyres–Douglas theories labelled by (G_1, G_2), $G_{1,2} = ADE$
$\rightsquigarrow CY_3 \subset \mathbb{C}^4$, IIB string on CY_3

ii) another family label $\mathfrak{g} = ADE$
$\Sigma = \mathbb{P}^1$ w/ irregular sing (+ regular sing.)
– M5-brane on  $\rightsquigarrow$ 4d $\mathcal{T}$, N Stokes rays

– condition: $\mathbb{Z}_N$ GB$_{\mathcal{T}}$ only fixes origin

(property $\mathbb{F}$)

iii) all "rk 1 AD theories" have property $\mathbb{F}$
– (A_n, A_m) has $\mathbb{F}$ if $(n+1), (m+1)$ coprime

– $\zeta_N = e^{2\pi i/N}$. $\underbrace{\chi_{S^1}(t = \zeta_N)}_{\text{also } t = \zeta_N^r,\ N, r \text{ coprime}} \in \mathbb{Q}(\zeta_N)$

$\Rightarrow \mathbb{Z}_N^\times$-family of 3d TQFTs

2d TQFT $\leftrightsquigarrow$ comm. Frobenius algebra

$\uparrow S^1$-reduction $\qquad\qquad \uparrow K^0$, "Verlinde ring"

3d TQFT $\leftrightsquigarrow$ modular tensor category

Conjecture $\mathcal{T}$ superconf. w/ $\mathbb{F}$. Then:
(i) $\exists$ family of MTC's $C_{\mathcal{T}}^{\gamma}$, $\gamma \in \mathbb{Z}_N^\times$
(ii) can obtain $C_{\mathcal{T}}^{\gamma=1}$ from $M_{\mathcal{T}}$
(iii) can get $C_{\mathcal{T}}^{\gamma}$ from Galois transf's of $C_N^{\gamma=1}$

- what are MTC's?

example: (A_1, A_2) $N=5$

$$\mathbb{Z}_5^{\times} = \{1, 2, 3, 4\}$$

$$C^{1}_{(A_1,A_2)}, \ldots, C^{4}_{(A_1,A_2)}$$

$$\Lambda = \{1, \varphi\}$$

"Fibonacci fusion rule"

$$\varphi \times \varphi = 1 + \varphi$$

$$\Rightarrow \varphi^n = F_{n-1} 1 + F_n \varphi$$

$r=1$:

$$S = \frac{2}{\sqrt{5}} \begin{pmatrix} -\sin\frac{2\pi}{5} & \sin\frac{2\pi}{5} \\ \sin\frac{\pi}{5} & \sin\frac{2\pi}{5} \end{pmatrix}$$

$$T = \begin{pmatrix} e^{11\pi i/30} & 0 \\ 0 & e^{-i\pi/30} \end{pmatrix}$$

$$F^{\varphi\varphi\varphi}_{\varphi} = \begin{pmatrix} -\varphi & i\varphi^{1/2} \\ -i\varphi^{1/2} & \varphi \end{pmatrix}, \quad \varphi \text{ - golden ratio}$$

non-unitary MTC

Modular Tensor Category:

(i) $\Lambda = \{\underset{\text{unit}}{1}, \ldots\}$ label set for simple objects

(ii) Fusion rule

$$\Lambda \times \Lambda \longrightarrow \mathbb{Z}_{\geq 0} \Lambda$$

(iii) $SL(2, \mathbb{Z}) \curvearrowright \mathbb{C}\Lambda$

- S, T matrices

(iv) F, R-matrices

$C^{r=1}_{(A_1,A_2)}$ = Lee-Yang MTC

$\simeq$ cat. of modules of the Lee-Yang model

(nonunitary $(2,5)$-minimal model)

r	1	2	3	4
$C^{r}_{(A_1,A_1)}$	LY	$(G_2)_1$	$(F_4)_1 \otimes (E_8)_1^{\otimes 2}$	$\overline{LY}$

unitary

non-unitary

$$u u^{\dagger} = 1$$

• Dictionary:

(I) $\Lambda \leftrightsquigarrow (M_{J})^{S^1}$

(II) $S_{\lambda *} \leftrightsquigarrow$ weights of S^1 on normal bdl to λ

(III) $T_{\lambda\lambda} \leftrightsquigarrow$ moment map of S^1-action at λ

- other "translations", e.g. F & R-mtx, not understood.