Tanzini

- $\mathcal{N}=(2,2)$ nonlin σ model
- $S=\int d^2z\, d^4\vartheta\, K(\vartheta^I,\bar\vartheta^J)+\int d^2z\, d\vartheta^+ d\vartheta^-\, W(\varphi^I)$

real func, Kähler potential of tgt mfd M — holom field, superpotential

- top. twist: $U(1)_E' = \begin{cases} \mathrm{diag}(U(1)_E\otimes U(1)_V) & A \\ \mathrm{diag}(U(1)_E\otimes U(1)_A) & B \end{cases}$ models

- basically, if $\psi_- \xmapsto[U(1)_E]{} e^{i\alpha}\psi_-$, $\psi_- \xmapsto[U(1)_V]{} e^{-i\beta}\psi_-$, say, then

$$\psi_- \mapsto \mathrm{diag}(e^{i\alpha}\otimes_{\mathbb{C}} e^{-i\beta})\psi_- = e^{i(\alpha-\alpha)}\psi_-.$$

- noting that $\vartheta_\pm \xmapsto[U(1)_V]{} e^{-i\alpha}\vartheta_\pm$, $\vartheta_\pm \xmapsto[U(1)_A]{} e^{\pm i\beta}\vartheta_\pm$,

to preserve $U(1)_V$, either $W(\varphi^I)\equiv 0$, or is quasihomogeneous of charge 2, because $d\vartheta_+ d\vartheta_- \mapsto e^{-2i\alpha} d\vartheta_+ d\vartheta_-$

Axial anomaly

- toy model $S=\int_{\mathbb{T}^2} d^2z\, i(\bar\psi_+ D_z\psi_+ + \bar\psi_- D_{\bar z}\psi_-)$, $\psi_\pm=\Gamma(\mathbb{T}^2, E\otimes S_\pm)$

$D_z=\partial_z+A_z$, $D_{\bar z}=\partial_{\bar z}+A_{\bar z}$ where $A=A_z dz + A_{\bar z}d\bar z$ Hermitean conn.

- suppose $K:=\int_{\mathbb{T}^2} c_1(E)>0$, then $\mathrm{Ind}\, D_{\bar z}=K>0$, so the measure cannot be invariant under axial symmetry: There is a zero-modes mismatch:

$$\#\psi_- \text{ zero modes} > \#\bar\psi_- \text{ zero modes}$$

- genericity assumption $\ker D_{\bar z} = \emptyset$ gives

$$\text{any correlator} = \int \prod_{a=1}^{k} \underset{\substack{\uparrow \\ \text{not ok}}}{d\psi_-^{(0)}} \int \prod_{n\neq 0} d\psi_-^{(n)} d\bar\psi_-^{(n)} \cdots$$

e.g. $\langle \psi_-(z_1)\cdots\psi_-(z_k)\bar\psi_+(w_1)\cdots\bar\psi_+(w_k)\rangle$
is $U(1)_V$ inv, but gains
$e^{2ik\beta}$ factor under $U(1)_A$

- anomaly breaks $U(1)_A \longrightarrow \mathbb{Z}_k$

- expanding $\psi_\pm, \bar\psi_\mp$ in eigenfuncs of $D_{\bar z}^\dagger D_{\bar z}, D_z^\dagger D_z$

$$\bar\psi_- = \sum_{n\geq 1} b_n \bar\varphi_-^n \qquad \psi_- = \sum_{a=1}^{k} c_{0a}\varphi_-^{0a} + \sum_{n\geq 1} c_n \varphi_-^n$$

$$\psi_+ = \sum_{n\geq 1} \tilde b_n \varphi_+^n \qquad \bar\psi_+ = \sum_{a=1}^{k} \tilde c_{0a}\bar\varphi_+^{0a} + \sum_{n\geq 1} \tilde c_n \bar\varphi_+^n$$

gives

$$D\psi D\bar\psi\, e^{-S} = \prod_{a=1}^{k} dc_{0a}\, d\tilde c_{0a} \prod_{n\geq 1} db_n\, dc_n\, d\tilde b_n\, d\tilde c_n\, e^{-\sum_{n\geq 1}\lambda_n(b_n c_n + \tilde b_n \tilde c_n)}$$

- for $\mathcal{N}=(2,2)$ nonlinear σ model,
$\mathcal{L} = -2i\, g_{I\bar J}\, \bar\psi_-^{\bar J} D_{\bar z} \psi_-^I + \text{conj.}$

but $k = \int_\Sigma c_1(\varphi^*(T^{(1,0)}M)) = \int_\Sigma \varphi^* c_1(M) \neq 0$ in general

$\Rightarrow U(1)_A$ preserved if $c_1(M)=0$
$\Rightarrow (M \text{ Kähler}) \wedge (c_1(M)=0) = (M \text{ is Calabi-Yau})$

- our cases:

	$U(1)_V$	$U(1)_A$	
CY	✓	✓	
$c_1(M)\neq 0$	✓	×	
$W\neq 0$	×	✓	(Landau-Ginzburg)
$W\neq 0$ but quasihom. deg 2	✓	✓	

- to understand A, B models, understand susy fixed pts
- recall

$$\delta\phi = \left[\xi_+ Q_- - \xi_- Q_+ - \bar{\xi}_+ \bar{Q}_- + \bar{\xi}_- \bar{Q}_+\right]\phi$$

for superfield $\phi^I = \varphi^I + \theta^+ \psi_+^I + \bar{\theta}^- \bar{\psi}_-^I + \dots$

gives in components

$$\delta\varphi^I = \xi_+ \psi_-^I - \xi_- \psi_+^I$$
$$\delta\varphi^{\bar{I}} = -\bar{\xi}_+ \bar{\psi}_-^{\bar{I}} + \bar{\xi}_- \bar{\psi}_+^{\bar{I}}$$
$$\delta\psi_+^I = 2\bar{\xi}_- \partial_{\bar{z}} \varphi^I + \xi_+ \left(\Gamma^I{}_{JK} \psi_+^J \psi_-^K - \tfrac{1}{2} g^{I\bar{L}} \partial_{\bar{L}} \bar{W}\right)$$
$$\delta\psi_-^I = -2\bar{\xi}_+ \partial_z \varphi^I + \xi_- \left(\quad — \text{''} — \quad \right)$$
$$\delta\bar{\psi}_+^{\bar{I}} = -2\xi_- \partial_{\bar{z}} \varphi^{\bar{I}} + \bar{\xi}_+ \left(\Gamma^{\bar{I}}{}_{\bar{J}\bar{K}} \bar{\psi}_-^{\bar{J}} \bar{\psi}_+^{\bar{K}} - \tfrac{1}{2} g^{\bar{J}L} \partial_L W\right)$$
$$\delta\bar{\psi}_-^{\bar{I}} = 2\xi_+ \partial_z \bar{\varphi}^{\bar{I}} + \bar{\xi}_- \left(\quad — \text{''} — \quad \right)$$

- for A-model, $Q_A = \bar{Q}_+ + Q_-$, so set

$\xi_+ = \bar{\xi}_- = 1$, $\xi_- = \bar{\xi}_+ = 0$

$Q_A \varphi^I = \chi^I$, $Q_A \varphi^{\bar{I}} = \chi^{\bar{I}}$ ($\psi^I \mapsto \chi^I$ in A model)

$Q_A \chi^I = 0$, $Q_A \chi^{\bar{I}} = 0$

$Q_A \rho_{\bar{z}}^I = 2\partial_{\bar{z}} \varphi^I + \Gamma^I{}_{JK} \rho_{\bar{z}}^J \chi^K$

$Q_A \rho_z^{\bar{I}} = 2\partial_z \varphi^{\bar{I}} + \Gamma^{\bar{I}}{}_{\bar{J}\bar{K}} \rho_z^{\bar{J}} \chi^{\bar{K}}$

- fixed pts? $\chi^I = \chi^{\bar{I}} = 0$, $\partial_{\bar{z}} \varphi^I = \partial_z \varphi^{\bar{I}} = 0$
  $\rightarrow$ holomorphic maps $\varphi : \Sigma \to M$

- B model ($W = 0$), $Q_B = \bar{Q}_+ + \bar{Q}_-$, $\bar{\zeta}_- = 1$, $\bar{\zeta}_+ = -1$, $\zeta_+ = \zeta_- = 0$

$$Q_B \varphi^I = 0, \quad Q_B \varphi^{\bar{I}} = \bar{\eta}^{\bar{I}}$$
$$Q_B \eta^I = 0, \quad Q_B \bar{\eta}^{\bar{I}} = 0$$

$$\left.\begin{array}{l} Q_B \, \rho_z^I = -2 \, \partial_z \varphi^I \\ Q_B \, \rho_{\bar{z}}^{\bar{I}} = -2 \, \partial_{\bar{z}} \varphi^I \end{array}\right\} \quad Q_B \rho^I = -2 \, d\varphi^I$$

- fixed pts? $d\varphi^I = 0$ constant maps