

Qft.

Fierz identities.  $\bar{\psi}_1 \Gamma \psi_2 \bar{\psi}_3 \Gamma \psi_4 \sim \bar{\psi}_1 \Gamma \psi_4 \bar{\psi}_3 \Gamma \psi_2$

- how to prove?

- pick basis  $\{ \delta_{ab}, \gamma_{ab}, \gamma_{ab}^{(12)}, \gamma_{ab}^{(123)}, \gamma_{ab}^{(1234)} \}$
- expand bilinears of the form  $\bar{\psi} \Gamma \psi$ .
- split into chiral reps.

$N=1$  SYM  $\rightarrow d=3, 4, 6, 10$  normed division algebras ( $|xy|=|x||y|$ )  
 $\mathbb{R} \subset \mathbb{H} \subset \mathbb{O}$

$$\text{PSL}(2, \mathbb{H}) \cong \text{SO}_0(3, 1)$$

↑  
Lorentz part (connected to Id).

double cover of Lorentz group modded out by  $\mathbb{Z}_2$

→ SUGRA:  $d=4, 5, 7, 11$  (strings "add a dimension")