Srní

NC Alg geom based on quantum flag mflds - J. Šťovíček.

Thm (Serre '56) V cpx aff. var.
$\exists$ 1-1 correspondence between
i) alg. vect. bdls $E \to V$
ii) certain fin. gen. proj $\mathbb{C}[V]$-modules

- all $\mathbb{C}[V]$ modules will correspond to q.c. sheaves, fin. gen to coherent sheaves.

- flag manifolds
  - example: Grassmanians
  - the set $Gr_{n,r}$ of r-dim subspaces of $\mathbb{C}^n$ forms a subset of a proj. space via $\iota: Gr_{n,r} \longrightarrow \mathbb{P}_{\mathbb{C}}^{\binom{n}{r}}$
    embedding $\langle v_1, \dots, v_r\rangle \mapsto \langle v_1 \wedge \dots \wedge v_r\rangle$
    where we fix a basis of $\Lambda^r \mathbb{C}^n$ and assign it Plücker coordinates
    $\to$ it is a zero-set of homogeneous polynomials
  - $Gr_{n,r}$ as flag mflds
    - repres. theoretically, $\Lambda^r \mathbb{C}^n$ is canonically a rep of sl_n, the rth fundamental rep $V(\omega_r)$
    - $\operatorname{Im}\iota$ can be identified with the orbit of $SL_n \cdot v$ of $v \in V(\omega_r)$, with coord. ring given by $S(Gr_{n,r}) \cong \bigoplus_{j=0}^{\infty} V(j\omega_r)^{\vee}$.
    $\to$ this generalises to all flag manifolds $\to$ cpx. proj. varieties F given by quadratic hom. polynoms w $S(F) \cong \bigoplus_{n=0}^{\infty} V(n\lambda)^{\vee}$, $\lambda = \sum$ fund. weights of F