Tanzini

- we saw how the chiral ring defines a unital $\mathbb{C}$-alg. w Frobenius property
  - A ⇝ twisted chiral, B ⇝ chiral

A model (with $W=0$)

- $\Sigma \xrightarrow{\varphi} M$, $Q_A(\varphi^I, \varphi^{\bar{I}}) = (\chi^I, \chi^{\bar{I}})$, $Q_A(\chi^I, \chi^{\bar{I}}) = 0$

$$Q_A \, S_{\bar{z}}^{I} = \partial_{\bar{z}} \varphi^I + \Gamma^I{}_{JK} \, S_{\bar{z}}^{J} \chi^K$$

$$Q_A \, S_z^{\bar{I}} = \partial_z \varphi^{\bar{I}} + \Gamma^{\bar{I}}{}_{\bar{J}\bar{K}} \, S_z^{\bar{J}} \chi^{\bar{K}}$$

- identifying $\chi^I ⇝ dz^I$, $Q_A ⇝ d = \partial + \bar{\partial}$ we get

$$\underbrace{Q_A \, \omega(\varphi)_{i_1 \cdots i_p \bar{i}_1 \cdots \bar{i}_q} \chi^{i_1} \cdots \chi^{i_p} \chi^{\bar{i}_1} \cdots \chi^{\bar{i}_q} = 0}_{\omega^{(p,q)} \text{ closed}}$$

- fixed locus, $Q_A S_{\bar{z}}^I = \partial_{\bar{z}} \varphi^I = 0, \ldots, \partial_z \varphi^{\bar{I}} = 0$
  Cauchy-Riemann, so $\Sigma \to \varphi_*(\Sigma) \in H_2(M, \mathbb{Z})$

- $\omega \in H^{1,1}(M)$ kähler form,

$$\int_\Sigma \varphi^*(\omega) = \int_\Sigma (-i) \overbrace{\omega_{i\bar{j}}}^{i g_{i\bar{j}}} \left(\partial_z \varphi^i \partial_{\bar{z}} \varphi^{\bar{j}} - \partial_{\bar{z}} \varphi^i \partial_z \varphi^{\bar{j}} \right) d^2 z$$

- but $S = \int_\Sigma g_{i\bar{j}} \left(\partial_z \varphi^i \partial_{\bar{z}} \varphi^{\bar{j}} + \partial_{\bar{z}} \varphi^i \partial_z \varphi^{\bar{j}} \right) d^2 z$

$$= \underbrace{\int_\Sigma g_{i\bar{j}} \, \partial_{\bar{z}} \varphi^i \partial_z \varphi^{\bar{j}}}_{=0 \text{ for holomorphic}} + \int_\Sigma \varphi^*(\omega) \geq \underset{\deg(\varphi)}{\overset{\shortparallel}{K}}$$

- for strings, introduce B-field
  $\Rightarrow \omega_{\mathbb{C}} = \omega + iB$

- on the tgt mfd we look at the Kähler cone, given by cycles C, D, M (in say $d=3$) satisfying $\int_C \omega > 0$, $\int_D \omega^2 > 0$, $\int_M \omega^3 > 0$.

B model

- $Q_B(\varphi^i, \varphi^{\bar{i}}, \theta^i, \eta^{\bar{i}}, \rho^i_z, \rho^{\bar{i}}_{\bar{z}})$
  $\shortparallel$
  $(0, \eta^{\bar{i}}, 0, 0, \partial_z \varphi^i, \partial_{\bar{z}} \varphi^{\bar{i}})$

$\rightarrow \eta^{\bar{i}} \rightsquigarrow dz^{\bar{i}}$, $\theta_i \rightsquigarrow \frac{\partial}{\partial z^i}$

- so $\omega_{\bar{i}_1 \cdots \bar{i}_p}{}^{j_1 \cdots j_q} \eta^{\bar{i}_1} \cdots \eta^{\bar{i}_p} \theta_{j_1} \cdots \theta_{j_q}$
  $\updownarrow$
  $\omega_{\bar{i}_1 \cdots \bar{i}_p}{}^{j_1 \cdots j_q} dz^{\bar{i}_1} \cdots dz^{\bar{i}_p} \partial_{j_1} \cdots \partial_{j_q}$
 has to lie in $H^{0,p}(M, \Lambda^q T^{1,0} M)$

LG model (B model w/ $W \neq 0$)

- change wrt B model: $Q_{LG} \theta^{\bar{i}} = g^{\bar{i}j} \partial_j W$
- here $M = \mathbb{C}^n$
- Q_{LG} cohomology $\simeq \mathbb{C}[\varphi^1 \cdots \varphi^n]/(\partial_j W)$

- for B model, fixed pts = constant maps
- for LG, const. maps to crit. pts of W?

Calabi-Yau moduli space

- CY_n is Kähler with, equivalently:
  - $c_1(CY_n) = 0$
  - holonomy $\subset SU(n)$
  - trivial canonical bdl

- Hodge diamond:
  - $h^{n,0} = 1$
  - $h^{1,0} = h^{0,1} = 0$ by simple connectedness
  - $H^1(\mathcal{T}) \simeq H^2(\mathcal{T} \times K_M)^\vee \Rightarrow h^{2,0} = h^{0,2} = 0$
  - for 3-folds:

$$\begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{2,1} & & h^{1,2} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & & 1 & & & \end{array}$$

  $\to$ by symmetry, only depends on h_{11}, h_{21}

- for cpx structure $\Omega^{3,0}$,

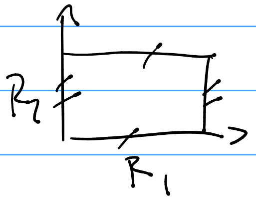
$$\Omega + \delta\Omega = \Omega + \mu^a{}_{\bar{b}}\, \Omega_{abc}\, dz^{\bar{b}} dz^b dz^c,$$

with $\mu \in H^1(M, TM)$

- so cpx deformation moduli dimension is $h_{21} + \underbrace{1}_{\text{rescaling of } \Omega}$

$\Rightarrow$ (Tian-Todorov) no obstructions

- simple example of cpt CY is $\mathbb{T}^2 \simeq \mathbb{C}/\Lambda$



A: kähler modulus $\sim R_1 R_2$

B: cpx modulus $\sim i\, R_2/R_1$

- for $\mathcal{N}=(2,2)$
$$Q_- \longleftrightarrow \overline{Q}$$
$$F_V \longleftrightarrow F_A$$
$$(\, z \longleftrightarrow \tilde{z}\,)$$

- note, you can also twist $\mathcal{N}=(0,2)$ susy, although geometrically still unclear