Qft.

Goldstone thm.

- $\langle \varphi_m \rangle \neq 0$, $\langle \delta \varphi_n^a \rangle \neq 0$, $\delta \varphi_n^a = [Q^a, \varphi_n]$
- $\partial_\mu J^\mu_a(x) = 0$

$\rightarrow$ $\partial^\mu_x \langle 0 | T[J^\mu_a(x)\, \varphi_n(0)] | 0 \rangle$
$= \delta(x^0) \langle 0 | [J^a_0(x), \varphi_n(0)] | 0 \rangle$

$\rightarrow$ write $\langle 0 | T[J^\mu_a(x)\, \varphi_n(0)] | 0 \rangle =: \int \frac{d^4p}{(2\pi)^4} G^\mu_{an}(p) e^{-ip\cdot x}$

$G^\mu_{an}(p) = i p^\mu H_{an}(p^2)$ by covariance

$\rightarrow$ integrate the prior expression:

$$\int d^4x \int \frac{d^4p}{(2\pi)^4} (-ip^\mu)\, i p_\mu H^a_n(p^2)\, e^{-ip\cdot x} = \int dx \langle 0 | [J^a_0(x), \varphi_n(0)] | 0 \rangle$$

$$\int d^4p\, \delta(p)\, p^2 H^a_n(p^2) \qquad\qquad \langle 0 | [Q^a, \varphi_n(0)] | 0 \rangle = \delta \varphi_n^a \neq 0$$

$$\Longrightarrow H_n^a(p^2) = \underbrace{\frac{\delta \varphi_n^a}{p^2}}_{\text{massless particle}} + \ldots \quad \text{as } p^2 \to 0$$

$\rightarrow$ we get a 1-to-1 correspondence between massless scalars w parity same as broken current

$\rightarrow$ we see it is a scalar because $\langle 0 | [J^\mu, \varphi] | 0 \rangle \neq 0$

- Nambu-Goldstone bosons $|NG_a(p)\rangle$

$$\langle 0 | J^a_\mu(x) | NG_b(p)\rangle = \frac{e^{-ipx}\, i p_\mu F_{ab}}{\sqrt{(2\pi)^3 2p_0}}\,, \quad \langle NG_b(p) | \varphi_n(x) | 0\rangle = \frac{Z_{nb}}{\sqrt{\cdots}}$$

- generally, $G^a_{\mu,n}(p) = \int_0^\infty d\varepsilon\, \rho^a_{\mu,n}(p,\varepsilon) \frac{i}{p^2-\varepsilon+i\varepsilon}$

$$\rho^a_{\mu n}(p,\varepsilon) \to \sum_{n\geq 1} \langle 0 | J^a_\mu | n\rangle\langle n | \varphi_n | 0\rangle$$

$$\rightsquigarrow p_\mu \underbrace{\rho^a_{n,NG}(\varepsilon)}_{\delta(\varepsilon)\, i F^a{}_b Z^b{}_n + \cdots}$$

$$\Longrightarrow G^{a,NG}_{\mu,n}(p) = \frac{i}{p^2} F^{ab} Z_{bn} + \cdots$$

- define $\pi^a(x) \rightsquigarrow \langle NG_b(p) | \pi^a(x) | 0\rangle = \frac{\delta^a_b\, e^{-ipx}}{\sqrt{(2\pi)^3 2p^0}}$

$\longrightarrow \varphi_n(x) = (iF_{ab})^{-1} \delta_n\varphi^b\, \pi^a(x) + \ldots$

Rmk. Goldstone thm applies in a weaker sense (losing 1-to-1 correspondence) if symmetries are not internal, $[G^a, \text{Poincaré}] \neq 0$.

Vacuum alignment

- full quantum potential $V(\varphi_n) = \underbrace{V_0(\varphi_n)}_{G\text{-invariant}} + \underbrace{V_1(\varphi_n)}_{\text{explicit sym. breaking}}$

- consider „generic" situation $|V_1(\varphi)| << |V_0(\varphi)|$

$\rightarrow$ $\delta\varphi_n = i\, t^\alpha_{nm} \varepsilon^\alpha \varphi_m$.

$$0 = \delta V_0 = \frac{\partial V_0}{\partial \varphi_n} \delta\varphi_n = \frac{\partial V_0}{\partial \varphi_n} i\, t^\alpha_{nm} \varepsilon^\alpha \varphi_m = 0$$

$$\left.\frac{\partial V_0}{\partial \varphi_n}\right|_{\varphi=\varphi_0} = 0 \Rightarrow \underbrace{\left.\frac{\partial^2 V_0}{\partial\varphi_k \partial\varphi_n}\right|_{\varphi_0}}_{M^2_{kn}} i\underbrace{(t^\alpha_{mn}\varphi^m_0)}_{t^a_{mn}\varphi^m_0} = 0$$

"mass" matrix in general!

for unbroken generators, $t^i \cdot \varphi|0\rangle = 0$ so we only consider broken ones labelled by $\alpha = a$.

$$\rightarrow M^2_{kn} t^{a\ n}_{\ m} \varphi^m_0 = 0$$

(again Goldstone's thm.)

$\rightarrow$ now consider

$$\left.\frac{\partial V}{\partial \varphi_n}\right|_{\varphi = \varphi_0 + \varphi_1} = 0$$

$$\xrightarrow{\cdots} \left.\frac{\partial V_1}{\partial \varphi_n}\right|_{\varphi_0} t^a_{mn}\varphi^m_0 = 0$$

$\rightarrow$ vacuum alignment condition.