Тихомиров

- topologise $\hat{\mathcal{M}}_K \ni a = ([A_a], \{x_1, \dots, x_t\})$
  by picking open geod. ball $B_\varepsilon(x_i)$,
  $K_\varepsilon := B \setminus \bigcup_{i=1}^{t} B_\varepsilon(x_i)$ is cpt.
- $\mathcal{U}_\varepsilon := \{ a' \in \hat{\mathcal{M}}_K \mid$ i) $\exists\, \mathcal{G} : P_{a'}|_{K_\varepsilon} \xrightarrow{\sim} P_a|_{K_\varepsilon}$ s.t. $\|A'_{a'} - \mathcal{G}^* A_a\|_{L^2_l(K_\varepsilon)} < \varepsilon$
  ii) $|\mu_a - \mu_{a'}| < \varepsilon$ in weak-* top
  iii) $\sup\{ |\int_B f_i \mu_a - \int_B f_i \mu_{a'}| \mid i = 1, \dots, m \} < \varepsilon \}$

  $\mathcal{U}_\varepsilon \subset_{\text{open}} \hat{\mathcal{M}}_K$

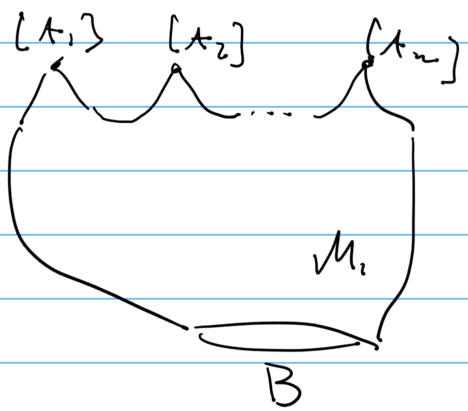
- $k = c_2(P) = 1$, $b_2^+ = 0$, $\dim \mathcal{M}_k = 8k - 3(1 + b_2^+) =: d$

Thm (Donaldson) $\exists$ fund. class $[\overline{\mathcal{M}}_k] \in H_d(\overline{\mathcal{M}}_k, \mathbb{Z})$

- $$\begin{array}{ccc} H_d(\mathcal{M}_k^*, \mathbb{Z}) & \to & H_d(\mathcal{M}_k^*, \mathcal{M}_k - \{*\}, \mathbb{Z}) \\ \cup\!\!\shortmid & & \cup\!\!\shortmid \\ [\overline{\mathcal{M}}_k]|_{\mathcal{M}_k^*} & \overset{\sim}{\longmapsto} & [1] \end{array}$$

- so $\mathcal{M}_k^*$ is oriented, $\mathcal{M}_k^* \subset \mathcal{M}_k \subset \overline{\mathcal{M}}_k$

- $0 \le m \le b_2^-$

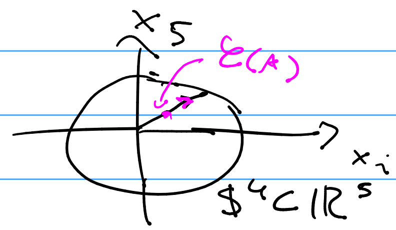


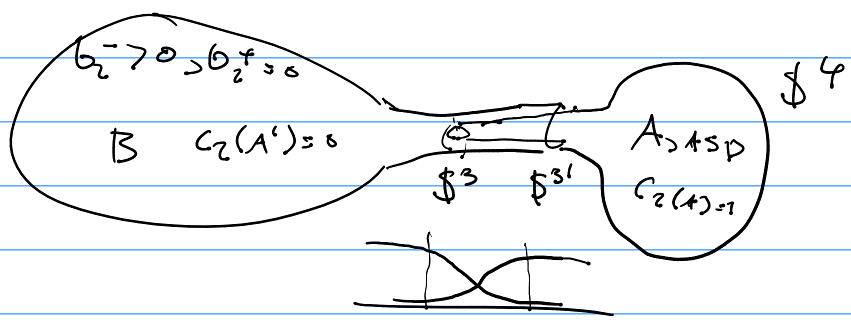
$\overline{\mathcal{M}}_1 - \mathcal{M}_1 = B$

$\hat{\mathcal{M}}_1 = \mathcal{M}_1 \sqcup \underset{\mathcal{M}_0}{\underset{\shortparallel}{\{*\}}} \times B$

- $B = S^4$, $\mathcal{M}_1(S^4) = B^5$, with $S^4 = \partial B^5$
- let $\int_{S^4} x_i |F_A|^2 d\mathrm{vol} = x_i(A)$, $\mathcal{E}(A) := \{x_1(A), \dots, x_5(A)\}$

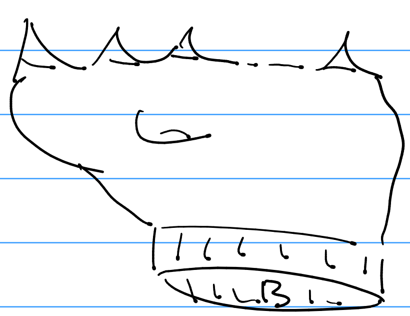
$x \mapsto \lambda x$, $\lambda \to 0$,





$B \# S^4 \overset{\text{diff}}{\cong} B$

- it can be shown that $\exists$ a collar



- $B \cup_i \mathbb{CP}^2_i = \partial(\underbrace{\overline{\mathcal{M}_1} \setminus \text{cones}}_{\text{sm. 5-dim mfd w bdry}})$
- $\sigma(\omega) = \tau_+ - \tau_-$, signature of int. form
- result: $\sigma(B) + \sum_i \sigma(\mathbb{CP}_i^2) = 0$

- $\sigma(\mathbb{CP}^2) = 1 - 0 = 1$, $H^2(\mathbb{CP}^2, \mathbb{Z}) = \mathbb{Z}[\mathbb{CP}^2]$
  $\omega_{\mathbb{CP}^2} = (1)$
- opposite orient., $\sigma(\overline{\mathbb{CP}^2}) = -1$, $\omega_{\overline{\mathbb{CP}^2}} = (-1)$
- $\sigma(B) = \pm b_2^-$

$$\Rightarrow \pm b_2^{-} + \sum_{i=1}^{\ell} (\pm 1) = 0$$

$$\Rightarrow \omega_{B} = (-1) \oplus \cdots \oplus (-1)$$

$$\omega_{\overline{B}} = (1) \oplus \cdots \oplus (1)$$