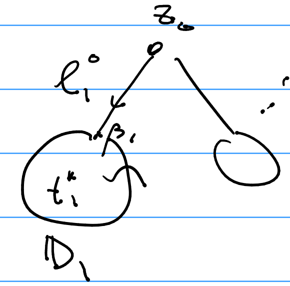
Bertola

Scalar case

- ODE $\psi' = \sum_i \frac{\vartheta_i}{z-t_i}\psi$, $\sum \vartheta_i = 0$



- $\Gamma(z) = \begin{cases} \psi(z) & \text{outside} \\ \psi(z)/(z-t_j)^{\vartheta_j}, & z \in D_j \end{cases}$

- $J = \begin{cases} e^{\mp 2\pi i \vartheta_j} & \text{on } \ell_j^\circ \\ (z-t_j)^{-\vartheta_j} & \text{on } \partial D_j \end{cases}$

- $\omega = \int_\Sigma \Gamma_-^{-1}\Gamma_-' \, \delta J \, J^{-1} = \sum_{j=1}^n \int_\Sigma \frac{\vartheta_j}{z-t_j}\Big(\sum_\ell \Big(2\pi i \, d\vartheta_j \chi_{\ell_j^\circ} + \Big(\frac{\vartheta_\ell \, dt_\ell}{z-t_\ell} - \ln(z-t_\ell)\, d\vartheta_\ell\Big)\Big)$ χ_{ODE}

$$= \dots = \sum_{j=1}^n \Big(\sum_{k\neq j} \frac{\vartheta_j \vartheta_k \, dt_j}{t_j - t_k} + \vartheta_j \, d\vartheta_k \ln(t_k - t_j) - i\pi \vartheta_j \, d\vartheta_j\Big)$$

- $d\omega = \sum_{j,k\neq j} d\vartheta_j \wedge d\vartheta_k \big(\ln(t_k-t_j) - \ln(t_j - t_k)\big)$

$$= i\pi \sum_{j,k\neq j} d\vartheta_j \wedge d\vartheta_k = i\pi \, d\Big(\sum_{j,k<j} \vartheta_j \, d\vartheta_k\Big)$$

- $d\ln\tau = \omega - i\pi \sum_{j,k<j} \vartheta_j \, d\vartheta_k$

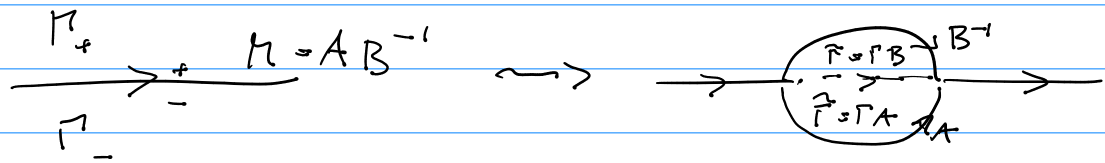
$$\Rightarrow \tau = \prod_{k<\ell} (t_k - t_\ell)^{\vartheta_k \vartheta_j} \prod_{k=1}^n e^{-\frac{i\pi}{2}\vartheta_k}$$

Higher rk

- $\psi' = \sum \frac{A_j}{z-t_j}\psi$, $\psi(z) = G_j\big(\mathbb{1} + O(z-t_j)\big)(z-t_j)^{L_j} C_j^{-1}$
where $M_j = C_j e^{2\pi i L_j} C_j^{-1} = C_j Q_j C_j^{-1}$, $A_j = G_j L_j G_j^{-1}$, $K_\ell := M_1 \cdots M_\ell$

- $\omega = \dots$

- idea: "explode monodromy"



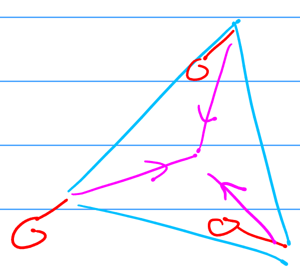
- clearly $\tilde{\Gamma}_+ = \Gamma = \Gamma B B^{-1}$
$= \tilde{\Gamma}_- \cdot B^{-1}$, new RHP

Thm Let $\tilde{\Gamma} = \Gamma P$, $P(z; \underline{Q}, \underline{C})$ pcwise const inv. mtx, $P(z)|_{D_-} = \mathbb{1}$, then $\tilde{\Theta} = \Theta$.

Lemma (Localisation) Supp. Γ ($\tilde{\Gamma}$) solve (outside D_j's) $\Gamma' = A\Gamma$, $\tilde{\Gamma}' = A\tilde{\Gamma}$, then
$$\Theta = \sum_{j=1}^{k} \operatorname{tr} L_j G_j^{-1} dG_j - \sum_{e=1}^{k} H_e dt_e, \quad H_e = \sum_{k \neq e} \frac{\operatorname{tr} A_k A_e}{t_k - t_e}$$

- Fock-Goncharov '06

- "cut off" cherries, pick Δation



$\Sigma = T$ (triangulation)
$+ C$ (herries)
$+ S$ (keleton)

- $V(T) - E(T) + F(T) = 2(-2g)$
- $SL_n(\mathbb{C})$, $\operatorname{tr} A = 0 \Rightarrow \det \varphi = 1$
- edge coords: z_e $\forall e \in \mathcal{E}(T)$
face: $\forall f \in F(T)$, $w_f \in (\mathbb{C}^*)^{(n-1)(n-2)/2}$
vtx: $\forall v \in V(T)$ $s_v \in (\mathbb{C}^*)^{n-1}$

- counting: $V = N$, $E = \frac{3}{2}F$ $\Rightarrow$ $V - E + F = \ldots = N - F/2 = 2$

$\Rightarrow F = 2(N-2)$, $E = 3(N-2)$

- can write: $2\pi i\,\omega_\mu = \sum_{k=1}^{n} \sum_{\ell,\ell' \perp v_k} d\ln z_\ell \wedge d\ln z_{\ell'} + 2\sum_{k=1}^{n} d\ln m_k \wedge d\ln S_k$

Connection w/ Goldman bracket