Fantechi

Det. Let E any ent, x, 7 & 66 E. A product
for X and Y is an object P=X x Y
which represents hx x hy! 20P + Set,
hx x hy (S) = hx (S) x hy (S)

-in other words, to give mot S-2 xx7

15 the same as giving S-2 x and S-27

-exercise: see it in top Grap, Ald,
-exercise: I canonical projs. xxxxxx (use you).

Def C (at, X ∈ G6 €. Pot ht: C → Set, ht/y)= Mor(×, 7). -exercise: define corepresentable functors, state co-Yoneda.

Det. Let E cat, X, Y & 66 C. A copre doct 2 of X and Y corepresents hxxhy.

-exercise : as before, What about commirings?

Prop Let A add. cut satisfying 1) and [11). Then +X, 7 & Obt, a coproduct exists it and only it a product exists. Forther, they are equal. Pt. Assume & product P. Then (idno) - exercise: 1/x = coker ig Yould PixX inscoker TX try and (xes 7) given ZEObC, Hom (P, Z) -> Hom (X,Z) x Hom (7,Z) f 1-> (foixsfoiy) 15 byection, so P direct som. Check converse. Ruk. A abel => A oP abel.

Det Let it abel cat. it (possibly inf.)

sequence of objects and morphisms

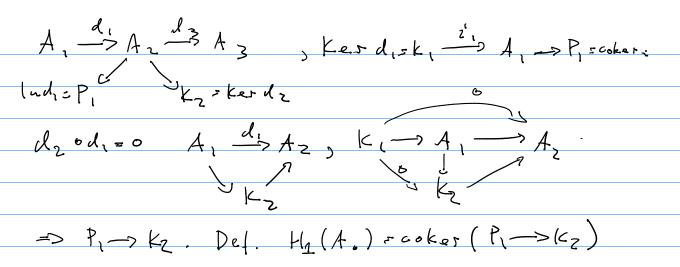
indexed by morphisms indexed by Z

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is called a complex if ti, dixidiso,

if makes sense. Its i-th cohomology

(group) object is ker dixi/ Imdi



Notation we use cohom indexing ti.

Det A cpx A = (..-> Ai - , Ai + (-> .-) (s called exact at i if hi (A*) = 0. It is called exact if thi hi (A*) = 0.

Det. Let A', B' complexes with same index

interval T. A mosphism y'. A' -> B' is the

datum: tic T y: A' -> B' s.t. if i, i + i e I,

then A' di A' commutes.

y: I y': 1

B' d' B' B' + 1

We call y a (co) chain map.

-exercise: show I - completes constitute a cat.

Lemma. (4: A° -> B° mor => h'(4): h'(A) -> h'(B)
whenever i-1, i, i e(E I.

In A and chain maps as morphisms.

-exercise: tiel, hi: C(t) -> A, A. H> hi.,), y w> h (4) 15 an additive functor (respects abelistructure on mos) Det let 4, 4: A -> B & Hos E(A). we say of is homotopic to of if d Vie/ di: Ai -> Bi-1 mor in ot sit. Hiell you grant alition di + de l'odi In this case we call L= (Li)iez a honotopy from y to zy, -, Ai-1 dis Ai-1-> $\frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1$ -philosophical q. & what is the natural structure on C(A)? -higher category theory, due to homotopies. Lemma. Assume 4,78: A -> B homotopic mors in C(A). Then tiez, hi(4)=h2(14). Pf. (We use objects since it is simples. Diagram chasing (AN BE Dowes tho) Let [x] & hi(A), menning x 6 Zi(A) = kerla. h'(4)[x]=[7;(x)]. Now 4:(x) -4:(x) + d:(1:x) + d:(1:-1x)

80 (Px(x)) = (x).

- Det A morphism q in C(A) is called a quasi-isomorphism if tieZ, hi(4) is isom.
- Rmk. Analoguous to weak-equivalence in aly, top.
- Ruk. Like top. spaces, ((t) also has a natural structure; model cati
- why quisos? example:
- Define module of Kähler differentials QB/A

 by saying it represents a functor

 (hodo'P -> (Sets), M -> Der (B, M),

 or by saying B = A[x, -> xn]/(+, -, fr)

 and

 Be. Oxi & B. dx; -> 52B/A -> o exact.
- -the first approach is great, but the 2-1 possibly depends on basis?