Qft.

Spontaneous symmetry breaking.

- only possible with ∞ly many d.o.f.
- unlike the case of explicit sym. breaking ("soft breaking", where the terms w/o symmetry are relevant, or "hard breaking" where they are marginal - in both cases should be "small" wrt the rest of the lagrangian.) the SSB lagrangian is fully invariant

(non-)

Example (QM)

- ordinary QM, $L_{dw.} = \dot{q} - (q-1)^2$.
  - $\rightarrow \mathbb{Z}_2$ symmetry
  - $\rightarrow$ 2 minima $|\pm\rangle$
  - $\rightarrow \hat{S}_{\mathbb{Z}_2} |\pm\rangle = |\mp\rangle$
  - $\rightarrow$ however, the ground state is $\frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$.

(Qft)

- take $|u\rangle, |v\rangle$ discrete ON vacua, A, B local composite operators

$\langle v| A(\vec{x},0) B(\vec{0},0) |u\rangle$

$= \sum_w \langle v| A(0) |w\rangle\langle w| B(0) |u\rangle + \int d^3p\, e^{i\vec{p}\cdot\vec{x}} S(\vec{p})$

$S(p) \doteq \sum_{n\geq 1} \langle v|A(0)|n\rangle \cdot \langle n|B(0)|u\rangle\, \delta^{(3)}(\vec{p}-\vec{q}_n)$

- supposedly Riemann-Lebesgue holds
- take $\|\vec{x}\| \to \infty$ limit:

$\lim \langle v| A(\vec{x}) B(\vec{0}) |u\rangle = \sum_w A_{vw} B_{wu}$

$\rightarrow$ however, microcausality says: $[A(\vec{x}), B(\vec{0})] = 0$

$\cdots = \sum_w \langle v|\underbrace{A(0)|w\rangle}_{a_w|w\rangle} \underbrace{\langle w|B(0)}_{b_w^*|w\rangle}|u\rangle = 0.$

diagonalising basis

Ask: domain bubbles are compact → this forces the v.e.v.s to be dynamical? → extra extended dimensions

→ what about string vacua?

→ compact universes?

Rmk. Infinite volume was essential.
According to prof Serone, it doesn't work on cpt spaces.
A cluster decomp. argument on the Green's function ($G^{(n)}(\vec{x},0) \xrightarrow[|\vec{x}|\to\infty]{} 0$) says we cannot simply superimpose the vacua.

Thm (Goldstone) For each broken "direction", there will $\exists$ a massless (pseudo)scalar.