Tanzini

- recap: looked at $\mathcal{N}=(2,2)$ susy in 2d, nonlin σ models $\Sigma \xrightarrow{\varphi} M$, we required M to be (generalised) kähler
- top. twist $\longrightarrow$ A model, vector R sym
  $\searrow$ B model, axial R sym.
  - for B model we require $c_1(M)=0$ as to not break axial R-sym
- $Q_A = \overline{Q}_+ + Q_-$, $Q_B = \overline{Q}_+ + \overline{Q}_-$ are now nilpotent scalars

Observables.

- recall $\{Q_\pm, \overline{Q}_\pm\} = -2\partial_\pm$, $\{\overline{Q}_+, \overline{Q}_-\} = Z$, $\{Q_-, \overline{Q}_+\} = \tilde{Z}$ with $Z, \tilde{Z}$ central

Rmk. $\tilde{Z}$ breaks $U(1)_A$, Z breaks $U(1)_V$
- so we put $Z = \tilde{Z} = 0$

- since $\langle Q_{A,B} - \rangle = 0$ automatically, $Q_{A,B}$-cohomology gives nontrivial observables

B model.

- $Q_B = \overline{Q}_+ + \overline{Q}_-$
- recall $\overline{Q}_\pm = -\frac{\partial}{\partial \bar{\vartheta}^\pm} - \vartheta^\pm \partial_\pm$, $\overline{D}_\pm = -\frac{\partial}{\partial \bar{\vartheta}^\pm} + \vartheta^\pm \partial_\pm$

- note that first component of chiral superfield is Q_B-closed:
$$\overline{Q}_\pm \varphi = \overline{Q}_\pm \Phi \Big|_{\vartheta^\pm, \bar{\vartheta}^\pm = 0} = \left(\overline{D}_\pm - 2\vartheta^\pm \partial_\pm\right) \Phi \Big|_0 = 0$$

- so B-model observables are functs of φ

A model

- observables are lowest comps of twisted chiral superfields

Chiral ring

- note we can define chiral ring, since $\Phi_1 \cdot \Phi_2$ chiral if Φ_1, Φ_2 are
  - basis $\{\varphi_i\}_{i=1}^{d}$, $\varphi_i \varphi_j = C^k{}_{ij}\varphi_k + (Q\text{-boundary})$
  - since unit $\varphi_0 = 1$ chiral,

    $\varphi_0 \varphi_j = C^k_{0j}\varphi_k \Rightarrow C^k_{0j} = \delta^k{}_j$
  - associativity gives $C^m_{i\ell}\, C^\ell_{jk} = C^\ell_{ij}\, C^m_{\ell k}$
  - Frobenius algebra

Properties

1) Independence from insertion pt on Σ

- if $Q_B \mathcal{O}_B = 0$,

$$-2\partial_{\bar{z}}\mathcal{O}_B = [H+P, \mathcal{O}_B] = [\{Q_+, \bar{Q}_+\}, \mathcal{O}_B]$$

$$\overset{\text{s Jac.}}{=} \{[Q_+, \mathcal{O}_B], \bar{Q}_+\} - \{Q_+, [\bar{Q}_-, \mathcal{O}_B]\}$$

$$\overset{\text{s Jac on 2nd}}{=} \{\bar{Q}_+, [Q_+, \mathcal{O}_B]\} - [\underbrace{\{Q_+, \bar{Q}_-\}}_{=0}, \mathcal{O}_B]$$

$$+ \{\bar{Q}_-, [Q_+, \mathcal{O}_B]\}$$

$$= \{Q_B, [Q_+, \mathcal{O}_B]\}$$

- therefore, $\partial_{\bar{z}_j} \langle \prod \mathcal{O}_i(z_j, \bar{z}_j)\rangle = \langle \mathcal{O}_B \cdots \rangle = 0$

- also note that $[Q_+, \mathcal{O}_B]$ can be thought of as "$\int \mathcal{O}^{(1)}$", with $\mathcal{O}^{(1)}$ a 1-form observable

Descent equations

- for $\mathcal{O}^{(0)}$ Q_B-closed, we get
  $d\mathcal{O}^{(0)} = \{Q, \mathcal{O}^{(1)}\}$ where $\mathcal{O}^{(1)} = \{Q_+ + Q_-, \mathcal{O}^{(0)}\}$
- continue: $d\mathcal{O}^{(1)} = \{Q, \mathcal{O}^{(2)}\}$, $\mathcal{O}^{(2)} = \{Q_+, [Q_-, \mathcal{O}^{(0)}]\}$

- so susy charges $Q_\pm$ act as ladder ops

II) Independence from tgt sp. metric

- we vary F-term since it contains tgt metric as kähler potential

$$\int d^4\vartheta \, \Delta K \overset{\text{Berezin rules}}{=} \frac{\partial}{\partial\vartheta^+}\frac{\partial}{\partial\bar{\vartheta}^-}\frac{\partial}{\partial\vartheta^+}\frac{\partial}{\partial\vartheta^-} \Delta K \Big|_{\vartheta^\pm = \bar{\vartheta}^\pm = 0}$$

$$\sim \{\underbrace{\bar{Q}_+, [\bar{Q}_-}_{\text{add } \bar{Q}_- \text{ for free}}, \int d\vartheta^+ d\vartheta^- \Delta K]\}\Big|_{\vartheta^+ = \vartheta^- = 0} = \{Q_B, \ldots\}$$

III) indep. from twisted chiral deformations

$$\int d^2z \sqrt{h}\, d\vartheta^+ d\bar{\vartheta}^- \Delta\tilde{W} \sim \int d^2z \sqrt{h}\, \{Q_+, [\underbrace{\bar{Q}_-}_{\text{add } \bar{Q}_+}, \Delta W]\}$$

$$\sim \{Q_B, \ldots\}$$

IV) ... antichiral deformations

$$\int d^2z \sqrt{h}\, d^2\bar{\vartheta}\, \Delta\bar{W}(\bar{\varphi}) \sim \int \ldots \{\bar{Q}_+, [\bar{Q}_-, \Delta\bar{W})\}$$

$$\sim \{Q_B, \ldots\}$$

v) dependance on chiral sector

$$\int \sqrt{h}\, d^2 z \int d^2\theta\, \Delta W \sim \int \cdots \underbrace{\{ Q_+ , [Q_- , \Delta W] \}}_{= \Delta W^{(2)}}$$

- for A model, depends on tw. chiral, but not on chiral

- what are struct constants?
- on 2-sphere study $S^2 \to M$

$C_{ijk} := \langle \varphi_i \varphi_j \varphi_k \rangle$, let $C_{ij0} = \langle \varphi_i \varphi_j \rangle \equiv \eta_{ij}$

$$= \langle \varphi_i C^{\ell}_{jk} \varphi_\ell \rangle = C^{\ell}_{jk} \eta_{i\ell}$$

- $\partial_\ell \langle \varphi_i \varphi_j \varphi_k \rangle = \langle \varphi_i \varphi_j \varphi_k \int d^2 z \sqrt{h}\, \mathcal{O}^{(2)}_\ell \rangle$

where $\mathcal{O}^{(2)}_\ell = \{ Q_+ , [Q_- , \varphi_\ell] \}$
- use $PSL(2,\mathbb{C})$ to fix 3 pts

$\Rightarrow \partial_\ell C_{ijk} = \partial_i C_{\ell jk}$ WDVV eqns

$\Rightarrow C_{ijk} = \partial_i \partial_j \partial_k \mathcal{F}$, $\mathcal{F}$ prepotential