Morley J Weston | Graph theory assignment 1

All computation was done using Python, with station lists scraped from Wikipedia. My code can be found here: https://github.com/fgorkhs/miscscripts/blob/master/Graph_theory.py

Part 1:

	Taipei:	**San Francisco:**
e = edges	e = 116	e = 48
v = vertex	v = 108[1]	v = 46
p = subgraphs	p = 1	p = 1

α: Alpha index is the level of connectedness of the "circuitry".

u / (2v-5)

u = e-v+p (number of cycles)

Taipei: 0.043

San Francisco: 0.034

The Taipei metro system, with its higher number of cycles in the downtown area, has a higher alpha index than San Francisco. Taipei's downtown area comprises a dense network of transfer stations, something that San Francisco largely lacks.

β: Beta Index is the number of edges divided by the number of vertices, a simple measure of interconnectedness.

e / v

Taipei: 1.074

San Francisco: 1.043

---

[1] While Taipei Metro websites claims 117 stations, it also says " 117個(西門站、中正紀念堂站、古亭站及東門站等4個轉乘站於不同路線共用站體計為1站，其餘轉乘站計為2站)。" However, the Xiaobitan and Xinbeitou branch lines, I count 108 stations, or 118 including the Danhai light rail. R01 廣慈/奉天宮 has not been included.

The beta index for both metro systems is over 1, which means that neither system is perfectly "treelike." However, Taipei's is slightly higher, owing to the large number of connections in the downtown core. If New Taipei's lines were not included, the number would be much higher, as New Taipei City's lines contain no cycles.

γ: Gamma index is another measure of interconnectivity, comparing the observed links vs the number of possible links.

e / 3(v-2)

Taipei: 0.365

San Francisco: 0.364

Because Taipei has so many branch lines extending into New Taipei City, the gamma index for Taipei and San Francisco is rather similar. While Taipei has many more links than San Francisco, it also has more than double the number of stations.

η: Eta index is the length of a network divided by the number of links.

(L(G)) / e

Taipei:	**San Francisco:**
L(G) = 131.1 km	L(G) = 167 km
η : 1.13	η : 3.479

Taipei's metro stations are much closer together than San Francisco's. With fewer than half the number of links, San Francisco's network reaches farther into the suburbs of the city.

θ: Theta index is the capacity of a network (in tons, individuals, etc.) divided by the number of nodes.

Q(G) / v

Taipei:	**San Francisco:**
Q(G) = 2,100,000 / day	Q(G) = 420,000 / day
θ : 19444.444	θ : 9130.435

A far more widely-used network, Taipei handles more than double the passengers per day than San Francisco.

Part 2:

This part was done by hand, as there was a smaller number of candidates.

Taipei:

Articulation nodes
Beitou
Da'an
Daqiaotou
Guting
Minquan West Rd
Nanjing Fuxing
Qizhang
Ximen

Isthmuses:
Da'an - Technology Building
Da'an - Xinyi Anhe
Guting - Dingxi
Guting - Taipower
Minquan - Daqiaotou
Minquan - Yuanshan
Nanjing Fuxing - Taipei Arena
Ximen - Longshan

Most of these articulation nodes occur near the borders between Taipei and New Taipei city; most nodes within the inner city of Taipei are relatively well-connected. Beitou and Qizhang are exceptions, but these articulation nodes only serve single stations.

The isthmuses follow a similar pattern, except that many branch lines do not have any cycles after them, so the New Taipei lines are excluded.

For **San Francisco**, only transfer stations were counted:

Articulation nodes:
Bay Fair
Coliseum
Daly City
MacArthur
San Bruno
19th St Oakland

Isthmuses:
Coliseum - Bay Fair
Daly City - San Bruno
19th St Oakland - MacArthur

This list comprises every transfer station in the city except San Francisco International Airport. This points to a much less interconnected metro system, presumably because of the metropolitan area's position on a bay acting as a natural barrier. However, with multiple lines running through most stations, it is easier to get from any part of the city to another without changing lines.

Because there are very few cycles in general, there are few isthmuses to count. Taipei has a very tightly connected inner city, but no transfer stations with more than two lines (excluding the airport MRT). San Francisco follows a different pattern, with up to four different lines running through the same station, but fewer areas where routes cross. This makes for more direct journeys, but a less complex network.

Part 3:

Effect of the completion of the Western circular line on Taipei MRT:

Current:	**After Completion:**
$\alpha = 0.043$	$\alpha = 0.052$
$\beta = 1.074$	$\beta = 1.093$
$\gamma = 0.365$	$\gamma = 0.371$

The completion of this section of the circular line will bring three cycles inside New Taipei City, although all of them will be partially completed in Taipei City itself. These cycles will be:
Dapinglin-Jing'an-Guting
Banqiao-Zhongxiao Xinsheng-Jing'an
Banqiao-Touqianzhuang-Zhongxiao Xinsheng, with other possibilities dependent on additional transfers.

This will raise the total number of cycles, and therefore the alpha index, indicating a greater degree of interconnectedness. The beta index will likewise rise slightly, as there are four points where the new line will connect to the existing network. because this new line will be more

interconnected than the average line in Taipei, the gamma index will also rise.

Practically, this new line will ease pressure on Taipei City's metro system, as all transfer stations are currently in the city itself. By being able to go between Xindian, Zhonghe, Yonghe, Banqiao and Xinzhuang directly, New Taipei City will become less dependent on Taipei as a transfer hub. During the 2018 municipal election, New Taipei mayoral candidate Su Tseng-chang ran YouTube ads decrying the lack of New Taipei's identity separate from Taipei City. Perhaps part of that is that New Taipei is so dependent on the inner city for transportation infrastructure, and because mobility is so low.

This is in contrast to the Bay Area, where San Francisco and Oakland maintain separate identities. Although the transport systems are connected, each subnetwork could potentially operate without the other.