

Following Yassin-Kassab's 1998 paper and with the three node diagram shown in the yEd file.

$$\sum_{i \in I_j} NP_{ij} q_{ij} = q_{j0} \quad \forall j \in J \quad (1)$$

where I_j is the set of source nodes supplying node j and J is the set of all demand nodes in the network, and q_{j0} is the outflow at node j .

$$\sum_{i \in I_j} NP_{ij} q_{0i} p_{ij} = q_{j0} \quad \forall j \in J \quad (2)$$

where q_{0i} is the inflow at node i and p_{ij} is the probability that a flow at node i reaches node j .

$$\frac{p_{ij}}{p_{kj}} = \frac{\alpha_i}{\alpha_k} \quad \forall i, k \in I_j; \forall j \in J \quad (3)$$

Substituting this equation back into 2, we arrive at the following equation

$$p_{ij} = \frac{q_{j0} \alpha_i}{\sum_{i \in I_j} NP_{ij} q_{0i} \alpha_i} \quad \forall i, k \in I_j; \forall j \in J \quad (4)$$

where we set $\alpha_1 = 1$.

Now we also have the condition that

$$\sum_{j \in J_i} NP_{ij} p_{ij} = 1 \quad \forall i \in I \quad (5)$$

I indexes the set of source nodes.

Substituting the path flow probabilities into this equation we get a set of equations which we can solve.

Taken our simple subway example

$$\frac{20}{20 + 10\alpha_3} + \frac{10}{20 + 10\alpha_2} = 1 \quad (6)$$

$$\frac{10\alpha_2}{10\alpha_2 + 10\alpha_3} + \frac{10\alpha_2}{20 + 10\alpha_2} = 1 \quad (7)$$

$$\frac{10\alpha_3}{10\alpha_2 + 10\alpha_3} + \frac{20\alpha_3}{20 + 10\alpha_3} = 1 \quad (8)$$

When we solve for this system, we have $\alpha_2 = 1.3146$ and $\alpha_3 = 0.864$ so that

$$p_{1,9} = \frac{20}{20 + 10 \cdot 0.864} = 0.698 \quad (9)$$

$$p_{1,10} = \frac{10}{20 + 10 \cdot 1.3146} = 0.301 \quad (10)$$

$$p_{2,8} = \frac{10 \cdot 1.3146}{10 \cdot 0.864 + 10 \cdot 1.3146} = 0.604 \quad (11)$$

$$p_{2,10} = \frac{10 \cdot 1.3146}{20 + 10 \cdot 1.3146} = 0.396 \quad (12)$$

$$p_{3,8} = \frac{10 \cdot 0.864}{10 \cdot 0.864 + 10 \cdot 1.3146} = 0.396 \quad (13)$$

$$p_{3,9} = \frac{20 \cdot 0.864}{20 + 10 \cdot 0.864} = 0.604 \quad (14)$$

So that we have the flow paths

$$q_{1,9} = 13.96 \quad (15)$$

$$q_{1,10} = 6.04 \quad (16)$$

$$q_{2,8} = 6.04 \quad (17)$$

$$q_{2,10} = 3.96 \quad (18)$$

$$q_{3,8} = 3.96 \quad (19)$$

$$q_{3,9} = 6.04 \quad (20)$$

So that the track segments have this flow

$$t_{1,4} = 20 \quad (21)$$

$$t_{4,5} = 6.04 \quad (22)$$

$$t_{2,5} = 3.96 \quad (23)$$

$$t_{5,10} = 10 \quad (24)$$

$$t_{3,6} = 10 \quad (25)$$

$$t_{6,7} = 3.96 \quad (26)$$

$$t_{2,7} = 6.04 \quad (27)$$

$$t_{7,8} = 10 \quad (28)$$

$$t_{4,9} = 13.96 \quad (29)$$

$$t_{6,9} = 6.04 \quad (30)$$