VP160 Recitation Class

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Notations and Units

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   - Always used to express extremely large or small numbers.

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   - Some commonly-used unit prefixes:

p	n	μ	m	c	k	M	G
10^{-12}	10^{-9}	10^{-6}	10^{-3}	10^{-2}	10^3	10^6	10^9

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3. Basic Units & Derived Units

   [m], [kg], [s], [A], [K], [mol], [cd]   (S.I. Units)

Exercise 1

A simple pendulum consists of a light inextensible string AB with length L, with the end A fixed, and a point mass M attached to B. The pendulum oscillates with a small amplitude, and the period of oscillation is T. It is suggested that T is proportional to the product of powers of M, L and g, where g is the acceleration due to gravity. Use dimensional analysis to find this relationship.

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Solution

$$T \to [s],\, M \to [kg],\, L \to [m],\, g \to [m][s]^{-2}$$

Then we have:

$$T \propto M^0 L^{\frac{1}{2}} g^{-\frac{1}{2}}$$

Uncertainty and Significant Figures

Uncertainty

Because of unavoidavle factors, no measurement can ever be perfect. Its result may therefore only be treated as an estimate of what we call the ”exact value” of a physical quantity. The experiment may both overestimate and underestimate the value of the physical quantity, and it is crucial to provide a measure of the error, or better uncertainty, that a result of the experiment carries.

1. Type-A Uncertainty
2. Type-B Uncertainty

Significant Figures

Experimental uncertainty should almost always be rounded to one significant figure. The only exception is when the uncertainty (if written in scientific notation) has a leading digit of 1 and a second digit should be kept.

Examples

1. 1.7392 (SF=5)
2. 0.0970 (SF=3)
3. 3.7×10^5 (SF=2)
4. (9.8 ± 0.3)g
5. (0.78 ± 0.12)m

Back-of-the-envelop Calculations

Defination

A quick estimation of some physical quantities. You should be able to have an basic idea about the order of magnitude when doing exercises.

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True or False: The power of China Railway High-speed (CRH) is about 500kW.

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True or False: The power of China Railway High-speed (CRH) is about 500kW.

Solution

False

Vectors

Defination

Vectors are quantities that have both magnitude and direction.

Scalar	Vector
Distance	Displacement
Speed	Velocity

Vector in R^n

$$\vec{u} = (u_1, u_2, \ldots, u_n)^T$$

Basic vector operations

- Addition & Subtraction

$$\vec{u} \pm \vec{v} = (u_1 \pm v_1, u_2 \pm v_2, u_3 \pm v_3)$$

- Scalar Multiplication

$$\lambda \vec{u} = (\lambda u_1, \lambda u_2, \lambda u_3)$$

- Dot Product

$$\vec{u} \cdot \vec{v} = |u||v|cos\theta = u_1 v_1 + u_2 v_2 + u_3 v_3$$

- Cross Product
  - ⋆ Magnitude: $|\vec{u} \times \vec{v}| = |u||v|sin\theta$
  - ⋆ Direction: determined by **Right Hand Rule**
  - ⋆ Matrix expression:

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

$$= (u_2 v_3 - u_3 v_2)\hat{i} + (u_3 v_1 - u_1 v_3)\hat{j} + (u_1 v_2 - u_2 v_i)\hat{k}$$

  - ⋆ Note that $(\vec{A} \times \vec{B}) \times \vec{C} \neq \vec{A} \times (\vec{B} \times \vec{C})$. Why?

Exercise 3.1

Consider two vectors $\vec{u} = 3\hat{n}_x + 4\hat{n}_y$, and $\vec{w} = 6\hat{n}_x + 16\hat{n}_y$. Find:
(a) the components of the vector $\vec{w}$ that are parallel and perpendicular to the vector $\vec{u}$,
(b) the angle between $\vec{w}$ and $\vec{u}$.

Solution

(a)

$$\|\vec{\omega_{\|}}\| = \frac{\vec{\omega}\cdot\vec{u}}{\|\vec{u}\|} = \frac{82}{5}$$

$$\vec{\omega_{\|}} = \|\vec{\omega_{//}}\|\cdot\hat{u} = \frac{82}{5}\cdot(\frac{3}{5},\frac{4}{5})$$

$$\vec{\omega_{\perp}} = \vec{\omega} - \vec{\omega_{\|}} = (-\frac{96}{25},\frac{72}{25})$$

(b) 16.3^{o}

Exercise 3.2

Find a vector $\vec{u}$, such that:

$$(2\hat{n}_x - 3\hat{n}_x + 4\hat{n}_x) \times \vec{u} = 4\hat{n}_x + 3\hat{n}_x - \hat{n}_x$$

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Solution

Not exist, since (2,3,4) is not perpendicular to (4,3,-1).

Exercise 3.3

Check that in Cartesian coordinates, the two expression equations of dot product of two vectors $\vec{u}$ and $\vec{v}$ are equivalent:

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$$
$$\vec{u} \cdot \vec{v} = |u||v| cos\theta$$

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Solution

Using cosine law, we have:

$$2\|\vec{u}\|\|\vec{v}\| cos\theta = \|\vec{u}\|^2 + \|\vec{v}\|^2 - \|\vec{u} - \vec{v}\|^2$$

Then we can easily derive the formula.

3D curvilinear coordinate systems

Cartesian Coordinate

1. Coordinates: x, y, z
2. Unit vectors: $\hat{n}_x, \hat{n}_y, \hat{n}_z$
3. Position vector: $\vec{r} = x\hat{n}_x + y\hat{n}_y + z\hat{n}_z$

Cylindrical Coordinate

1. Coordinates: ρ, ϕ, z
2. Unit vectors: $\hat{n}_\rho, \hat{n}_\phi, \hat{n}_z$
3. Position vector: $\vec{r} = \rho\hat{n}_\rho + z\hat{n}_z$
4. Relationship with Cartesian Coordinate:

$$\begin{cases} \rho = \sqrt{x^2 + y^2} \\ \phi = arctan(y/x) \\ z = z \end{cases} \qquad \begin{cases} x = \rho cos(\theta) \\ y = \rho sin(\phi) \\ z = z \end{cases}$$

Spherical Coordinate

1. Coordinates: r, ϕ, θ
2. Unit vectors: $\hat{n}_r, \hat{n}_\phi, \hat{n}_\theta$
3. Position vector: $\vec{r} = r\hat{n}_r$
4. Relationship with Cartesian Coordinate:

$$\begin{cases} r = \sqrt{x^2 + y^2 + z^2} \\ \phi = arctan(y/x) \\ \theta = arctan(\sqrt{x^2 + y^2}/z) \end{cases} \qquad \begin{cases} x = \rho sin(\theta) cos(\phi) \\ y = rsin(\theta) sin(\phi) \\ z = rcos(\theta) \end{cases}$$

Exercise 4

Derive the above relation equations.

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Solution

See lecture note.

1D kinematics

Differential Equations

$$
\begin{aligned}
x &= x(t) \\
v &= v(t) = \frac{d}{dt}x(t) \\
a &= a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}x(t)
\end{aligned}
$$

Relative Motion

$$
x = x_r + x'; \quad v = v_r + v'; \quad a = a_r + a'
$$

Exercise 5.1

A particle moves along a straight line with non-constant acceleration $a_x(t) = -A\omega^2 cos\omega t$, where A and ω are positive constants. At the instant of time $t = 0$ its velocity $v_x(0) = 3$ [m/s] and position $x(0) = 4$ [m].

(a)What are the units of these constants?

(b)Find $v_x(t)$ and $x(t)$ at any instant of time.

(c)Sketch the graphs of $x(t)$, $v_x(t)$, and $a_x(t)$.

(d)What kind of motion may these results describe?

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Solution

(a) $[A] = m, [\omega] = s^{-1}$

(b) $v_x(t) = 3 - A\omega sin(\omega t), \quad x(t) = 3t + 4 + A[cos(\omega t) - 1]$

(c) Try it by yourself!

(d) Simple harmonic motion + uniform linear motion

Exercise 5.2

A particle is moving along a straight line with velocity $v_x(t) = -\beta A\omega e^{-\beta} cos\omega t$, where A, ω, and β are positive constants. Assuming that $x(0) = 5[m]$.

(a)What are the units of these constants?

(b)Find acceleration $a_x(t)$ and position $x(t)$ of the particle.

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Solution

(a) $[\beta] = s^{-1}, [\omega] = s^{-1}, [A] = m \cdot s$

(b) $a_x t = \beta^2 A\omega e^{-\beta t} cos\omega t + \beta A\omega^2 e^{-\beta t} sin\omega t$

$$x(t) = 5 - \frac{\beta A\omega}{\beta^2 + \omega^2}[\beta(1 - e^{-\beta t} cos\omega t) + \omega e^{-\beta t} sin\omega t]$$

(c) Try it by yourself!

(d) Under-damped Oscillation(You will learn it later)

Exercise 5.3

A car is moving in one direction along a straight line. Find the average velocity of the car if:

(a) it travels half of the journey time with velocity v_1 and the other half with velocity v_2,

(b) it covers half of the distance with velocity v_1 and the other one with velocity v_2.

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Solution

(a) $\dfrac{v_1 + v_2}{2}$

(a) $\dfrac{2v_1 v_2}{v_1 + v_2}$

Exercise 5.4

In a certain motion of a particle along a straight line, the acceleration turns out to be related to the position of the particle according to the formula $a_x = \sqrt{kx}$, where $k > 0$ is a constant and $x > 0$. How does the velocity depend on x, if we know that for $v_x(x_0) = v_0$?

Solution

Integration by part:

$$a_x = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v = \sqrt{kx}$$

$$vdv = \sqrt{kx}dx$$

Doing integration on both sides, we have:

$$\frac{1}{2}(v^2 - v_0^2) = \frac{2}{3}\sqrt{k}(x^{2/3} - x_0^{2/3})$$

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Solution

The distance will increase. (Draw a graph to verify the answer!)

Exercise 5.6

A spare paddle drops from a fisherman's canoe. After one hour of paddling the fisherman realizes that the paddle is missing. He turn around and paddles his canoe back to find the paddle. Assume that the fisherman paddles always with the same speed $v = 10km/h$ with respect to the river, the speed of the rivers current is $u = 6km/h$. Find:

(a)the time that the fisherman takes to find the paddle;

(b)the distance between the places where the paddle drops and the fisherman find it.

Can you find the answers in a second?

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Can you find the answers in a second?

Solution

Take the river as the frame of reference. We immediately obtain:
(a) One hour
(b) $D = 2 \times 6 = 12km$