1. $\frac{1}{(2+x)^2} = (2+x)^{-2} = \frac{1}{4}(1+\frac{x}{2})^{-2} = \frac{1}{4}(1-x)$

2. $1-\cos\theta = 1-(1-\frac{1}{2}\theta^2) = \frac{1}{2}\theta^2$

3. $\sin(0.1°) = \sin(\frac{0.1}{180}\pi \text{ rad}) = \frac{0.1\pi}{180}$

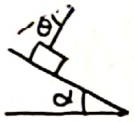
4. $(m-x)^{-2}-(m+x)^{-2} = m^{-2}[1+2\frac{x}{m}-1+2\frac{x}{m}) = \frac{4x}{m^3}$

5. $\ln(1+\frac{x}{m})-\ln 2 = \frac{x}{m}-\ln 2$

Ex. 1:

(1) (a) $\theta = 0$

(b) $\arctan(\frac{a}{g})$.

(2) 

$a_{//} = g\sin\alpha$.

$a_{\perp} = g\cos\alpha$.

$\theta = \alpha$.

Ex. 2:

(a) $a = \frac{M-m}{M+m}g$

(b) $a = \frac{M-m}{M+m}g$.

(c) $f - mg - ma = ma_0$.

$f - Mg = -Ma$.

$Mg - mg - ma = ma_0 + Ma$.

$\therefore a = \frac{(M-m)g - ma_0}{M+m}$.

Ex. 3:

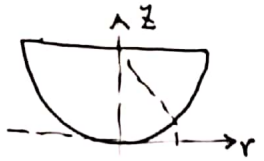
$F = G\frac{Mm}{r^2} = \frac{gR^2m}{(R+x)^2} = mg(1-2\frac{x}{R})$.

$\therefore k = +\frac{2mg}{R}$. $T_0 = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{R}{2g}}$.

$T = \frac{1}{4}T_0 = \frac{\pi}{2}\sqrt{\frac{R}{2g}}$.

$V = W\cdot h = \sqrt{\frac{2g}{R}}h$

Ex. 4.1

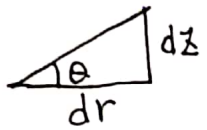


$r^2 + (R-z)^2 = R^2$ $\quad z \ll R.$

$r^2 + R^2\left(1 - \frac{2z}{R}\right) = R^2$

$r^2 = 2zR$

$2r\,dr = 2R\,dz$



$\therefore \frac{dz}{dr} = \frac{r}{R} = \theta$ ↗ k.

$F = -mg\sin\theta = -\frac{mg}{R}r.$

$T = 2\pi\sqrt{\frac{R}{g}}.$

Ex. 4.2.

$\begin{cases} dx = R(d\psi + \cos\psi\, d\psi) = R(1+\cos\psi)\,d\psi \\ dy = R\sin\psi\, d\psi \end{cases}$

$\frac{dy}{dx} = \frac{\sin\psi}{1+\cos\psi} \approx \frac{\psi}{2}.$

$\therefore F = -mg\sin\theta = -\frac{mg}{2}\psi$

Since $x = R(\psi + \sin\psi) = 2R\psi.$

$\therefore F = -\frac{mg}{2}\cdot\frac{1}{2R}x = -\frac{mg}{4R}x.$

$T = 2\pi\sqrt{\frac{4R}{g}} = 4\pi\sqrt{\frac{R}{g}}$