

$tmvar, x, y$ variables

typ, τ	$::=$		types
		nat	Natural numbers
		$\tau_1 \rightarrow \tau_2$	Function types

exp, e	$::=$		expressions
		x	Variables
		z	Zero
		s e	Successor
		rec $e\{\mathbf{z} \rightarrow e_0; \mathbf{s} x \rightarrow e_1\}$	bind x in e_1 Primitive recursion over nats
		$\lambda(x:\tau)e$	bind x in e Functions
		$e_1 e_2$	Application

env, Γ	$::=$		typing environment
		\emptyset	empty
		$\Gamma, x : \tau$	cons

$e \text{ val}$

$\overline{\mathbf{z} \text{ val}}$	VAL_Z
$\frac{e \text{ val}}{\mathbf{s} e \text{ val}}$	VAL_S
$\overline{\lambda(x:\tau)e \text{ val}}$	VAL_ABS

$\Gamma \vdash e : \tau$

$\frac{\text{uniq } \Gamma \quad x : \tau \text{ in } \Gamma}{\Gamma \vdash x : \tau}$	TYPING_VAR
$\frac{\text{uniq } \Gamma}{\Gamma \vdash \mathbf{z} : \mathbf{nat}}$	TYPING_Z
$\frac{\Gamma \vdash e : \mathbf{nat}}{\Gamma \vdash \mathbf{s} e : \mathbf{nat}}$	TYPING_S
$\frac{\Gamma \vdash e : \mathbf{nat} \quad \Gamma \vdash e_0 : \tau \quad \Gamma, x : \mathbf{nat} \vdash e_1 : \tau \rightarrow \tau \quad e_1 \text{ val}}{\Gamma \vdash \mathbf{rec} e\{\mathbf{z} \rightarrow e_0; \mathbf{s} x \rightarrow e_1\} : \tau}$	TYPING_REC
$\frac{\Gamma, x : \tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda(x:\tau_1)e : \tau_1 \rightarrow \tau_2}$	TYPING_ABS
$\frac{\Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 e_2 : \tau_2}$	TYPING_APP

$e \rightsquigarrow e'$

$\frac{e \rightsquigarrow e'}{\mathbf{s} e \rightsquigarrow \mathbf{s} e'}$	EVAL_S
---	--------

$$\begin{array}{c}
\frac{e_1 \rightsquigarrow e'_1}{e_1 e_2 \rightsquigarrow e'_1 e_2} \quad \text{EVAL_APP_LEFT} \\
\\
\frac{e_1 \mathbf{val} \quad e_2 \rightsquigarrow e'_2}{e_1 e_2 \rightsquigarrow e_1 e'_2} \quad \text{EVAL_APP_RIGHT} \\
\\
\frac{e_2 \mathbf{val}}{(\lambda(x:\tau) e_1) e_2 \rightsquigarrow e_1 \{e_2/x\}} \quad \text{EVAL_BETA} \\
\\
\frac{e \rightsquigarrow e'}{\mathbf{rec} \, e \{ \mathbf{z} \rightarrow e_0; \mathbf{s} \, x \rightarrow e_1 \} \rightsquigarrow \mathbf{rec} \, e' \{ \mathbf{z} \rightarrow e_0; \mathbf{s} \, x \rightarrow e_1 \}} \quad \text{EVAL_REC_SCRUT} \\
\\
\frac{}{\mathbf{rec} \, \mathbf{z} \{ \mathbf{z} \rightarrow e_0; \mathbf{s} \, x \rightarrow e_1 \} \rightsquigarrow e_0} \quad \text{EVAL_REC_Z} \\
\\
\frac{(\mathbf{s} \, e) \mathbf{val}}{\mathbf{rec} \, (\mathbf{s} \, e) \{ \mathbf{z} \rightarrow e_0; \mathbf{s} \, x \rightarrow e_1 \} \rightsquigarrow e_1 \{e/x\} (\mathbf{rec} \, e \{ \mathbf{z} \rightarrow e_0; \mathbf{s} \, x \rightarrow e_1 \})} \quad \text{EVAL_REC_S}
\end{array}$$

Definition rules: 16 good 0 bad

Definition rule clauses: 35 good 0 bad