Introduction to MatLab.

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# Chapter 1

# Quick tour MatLab

# I Legal stuff

MatLab is a registered trademark of MathWorks, Inc.

# II What is the use of MatLab?

MatLab (for MATrix LABoratory) aims at delivering quickly some *relatively* inexpensive computations. It shines, as expected from the name, when it involves linear algebra, i.e., operations on matrices.

The main advantage of MatLab compared to language like C/C++ or Fortran are:

- No compilation
- The prompt
- Simplicity
- $\bullet$  portability
- Built-in functions:
  - integration
  - visualization
  - tool-box

The negative points are mostly:

- sub performance
- not open source
- price

# III Equivalent of MatLab

Octave and SciLab are almost identical to MatLab. A MatLab script would work on these two others open-source and free softwares.

Most of the tips can also be applied to python, especially when the packages scipy and numpy (for scientific and engineering computations) are used.

# IV Philosophical idea of these teachings

The objective of these tutorial are to illustrate the ENG 4XXX courses as well at to help you to learn quickly how to numerically solve problems. It will hence give you the first concepts behind MatLab, coding and problem solving. The emphasis here is learning by doing. Therefore, try not to read these documents without a computer close-by.

# V Hello World

# ${ m V}$ a) MatLab as a software

#### V a) i Start MatLab

Click on the icon, duh!

# V a) ii Organization of the window

You can find a few important sections:

• the command windows

This is the command prompt

• current folder

It lists the files

 $\bullet$  the Workspace

It gives details on the objects present in memory

• the editor

this is where you can write a script

• the ribbon

It gives access to properties, functions, etc. Similar in spirit to Words and Excel.

# [[TODO \*\* image opening windows of matlab \*\* ]]

### V b) How to print "hello world"

Click on the Command Window, and type "hello world". You will see:

```
1 >> 'Hello World'
```

- ans =
- 3 Hello World

MatLab printed the 'Hello World', congratulations!

You can see also that the 'Hello World has been assignated to a variable named ans.

efinition

ANS: ans is short for answer.

The results of the command is always stored in the variable ans, except if it is assigned to a given variable. Consequently, the command 1+1 will affect the variable ans, but x=1+1 will not, and 2 will be assign to x.

ans can be re-used in the prompt: x = ans+1. However, a good practice is to assign the result to a user-defined variable.

Now, try without the quotes, and you will see:

- 1 >> Hello World
- 2 Undefined function or variable 'Hello'.

plus some help.

Hello World is understood by MatLab as a function/variable and then an option for this function. MatLab hence thinks that Hello is something that already exists, and it is not the case here. An error follow.

The main reason behind that is that we want to print a string.

finition

STRING: A string is a chain of characters. It is *not* a number.

It has to be between quotes: 'some text' or double quotes: "some text". For example, 'Lorem ipsum dolor sit amet' is a chain or characters.

But it is not that easy. s = '45' is the chain of characters '4' and '5'. But n = 45 is the number 45. s and n are different.

Try now to add a semi colon at the end of the line:

```
1 >> 'Hello World';
2 >>
```

Nothing is printed in the command prompt.

finitio

COMMAND PROMPT: The command prompt is the >> sign.

Command Prompt is a command line interpreter. It is used to execute entered commands. Once enter is hit, MatLab will interpret the command, and send back any results.

One of the most important tip to remember: MatLab will always print the result of a line if it does not have a ";" at the end of the line.

2 2 Do not forget the ";" at the end of lines!

It is not a big deal when dealing with small matrices and small vectors. But when an image is being manipulated, it means that MatLab is manipulated a matrix with dimension around  $1000 \times 1000$ . Forgetting the ";" sign means that MatLab will show around a *million* numbers every line of your script!

#### VI MatLab as a calculator

#### VI a) Algebra

You can use MatLab as a calculator. Click on the prompt:

```
1 >> 4+3
2 ans =
3 7
4 >> 4*3
5 ans =
6 12
```

As expected, MatLab respects the BODMAS (Brackets, Order, Division/Multiplication, Addition/Subtraction). Try a few operations!

```
_{1} >> (4+3)*2
_{2} \text{ ans} =
_{3} 14
```

```
4 >> 4+3*2
5 ans =
6 10
```

## VI b) Variables

ans can be used to store a result, but it will be overwritten every time a command is executed:

```
1 >> 2+2

2 ans =

3 4

4 >> ans+2

5 ans =

6 6

7 >> ans+2

8 ans =

9 8
```

# VI c) Creation and re-assignement

Variable can be easily created and assigned with the sign "=".

```
1 >> x = 2+2

2 x =

3 4

4 >> x+2

5 ans =

6 6

7 >> x*5

8 ans =

9 10
```

Definition

Variable: A variable is essentially a name that is associated with a value. Values can be of several types:

- results, such as string, numbers or matrices: x = 3.
- functions, for instance sin is a built-in function
- complex objects, for instance, a plot

They are usually assigned with the sign "="

#### VI c) i Naming convention

A variable name can be anything, such as goodnameforavariable or GoodNameForAVariable, or  $good_name_for_a\_variable$ . However:

- it cannot start with \_
- it cannot start with a number
- a few names are protected

ro lip

Try to use clever name for variables, it will help to understand the code.

If all the results are named result\_1,result\_2,result\_3, it is hard to know what they should contain.

On the contrary, if you have the variable name\_city, you expect it to be a string and having a proper name.

In a similar way, if you have the variable motor\_freq, you expect it to be a number. Also, if you have the variable price\_pond, the variable price\_dollar and the variable rate\_dollar2pound, the line price\_pound = rate\_dollar2pond \* price\_dollar is pretty explicit. If the variables where instead named x,y,z, then the line x = y\*z is much more cryptic.

The choice of a name is important, for you a, x is good for an unknown, s if you expect its value to be a string, v if it is a vector... More complex names can be used, such as  $x\_problem\_1$ . Try to be consistent thorough the piece of code!

A few tips:

- Use different names for different results
- Use a name that is meaningfull (e.g. str\_name if the variable is assigned with a chain of character that is a name)
- Consequently, avoid unecessary use of index (e.g. result\_1, result\_2 etc.)

Many naming convention exist.

However, you can use the following name convention:

- UpperCamelCase for functions: MyFunction
- CAPITALIZED\_WITH\_UNDERSCORES for constants Pi=3.14
- lowercase\_separated\_by\_underscores for other variables name\_of\_univ = 'BCU'.

#### VI c) ii Reassignement

Updating a variable is handy: you might want to change the variable year from 2017 to 2018. You can easily update a variable, by reassigning a new value to it. It hence uses the sign "=". For instance:

#### VI c) iii Exercices

- 1. Create the variables x,y,z assigned with 1, 2 and 3.
- 2. Create the variable sum\_xyz that is the sum of x,y and z.
- 3. Propose a name for a variable that is assigned as a value 'Birmingham'
- 4. Propose a name for a variable that is assigned as a value 'BCU'
- 5. Try to assign to the variable year the value 2017, and then to 2018!
- 6. Try to assign to the variable girlfriend\_name the value 'Adilah' (using the sign equal, pun totally intended), and to the variable ex\_girlfriend\_name the value 'Marie'. Then, reassign to the variable girlfriend\_name the value 'Kiara', and to the variable ex\_girlfriend\_name the value 'Adilah'.

# VI d) Workspace

When you have created a variable, it is available in the *workspace*. It is the area (usually) on the right. It allows to:

- show what variables are currently known to MatLab
- know what is present in the memory
- indicate what there is in the variables
- eventually modify the content of a variable

# VI e) Entering multiple commands per line

It is possible to enter multiple commands per line. Use commas "," or semicolons ";" for that; the commas will not suppress the outputs.

Try to avoid multiple commands per line.

Most of the time, it makes the code harder to read, especially if there is not a good reason to do so.

Nevertheless, it can make sense write a few commands per line when assigning a variables that are related.

#### VI f) Basic arithmetic

Basic arithmetic operators are pretty classic:

Table 1.1: Arithmetic operators

operation	command	exemple
addition	+	3+4
soustraction	_	3-4
multiplication	*	3*4
division	/	3/4
power	^	2 ^ 4

# VI g) functions

### VI g) i How to find a function or a command?

If you look for something, hit the help button. For instance, if you want to look for the sine function:

# [[TODO \*\* image help sine \*\* ]]

ro Tip

Use the help! It is *very* useful and you will mostly find any function/tool/infos that you need.

Usually, the help contains a few examples. Do not hesitate to read them carefully, and to try them. They will help understanding how to use the functions and properties of MatLab.

There is also a "See Also" section that can be useful when looking for a particular topic.

#### VI g) ii Using a function

Calling a function is relatively easy and intuitive. Let's take the sine function as an illustration.

- $_{1} >> \sin(3.14)$
- $_2$  ans =
- з 0.0016

You ask MatLab to evaluate the function  $\sin$  in  $3.14 \approx \pi$ . For that, you just put the argument in parenthesis.

2

Trigonometric functions in MatLab are in radiant. sin(360) is hence different from 0 but rather close to 0.96.

In a similar spirit, log in MatLab is the natural logarithm, and not the log in base 10.

Typical functions are available with somewhat explicit names, see Tab. 1.2. Similarly, many useful constants for the engineer are implemented in MatLab, see Tab. 1.3.

Table 1.2: A few function names in MatLab. Many others are already implemented in MatLab.

Trigonometry	name	Stats	name	Misc.	name
sine	sin	mean	mean	square root	sqrt
cosine	cos	maximum	max	absolute value	abs
exponential	exp	minimum	min	round up	ceil
natural logarithm	log	standard dev.	std	conjugate	conj

Table 1.3: A few useful constant names in MatLab.

$\pi \approx 3.14$	pi
$i = \sqrt{-1}$	i
$i = \sqrt{-1}$	j
$\infty$	Inf
Not a Number	NaN

# VII Let's solve a real problem

**Problem** Julie's car's odometer reading was 35201km when she last filled the fuel tank. Yesterday she checked it again and it read 35403km. Checking the tank, the car used 11 liters of gas to do so. If her car's gas tank holds 35 liters, how long can she drive before running out of gas?

Solution using MatLab as a calculator How much has she driven?

```
1 >> 35403 - 35201
2 ans =
3 202

How much distance per liter of gas?

1 >> 202/11
2 ans =
3 18.3636

How much gas is left in the tank?

1 >> 35 - 11
2 ans =
3 24

Distance she can drive:

1 >> 24 * 18.3636
2 ans =
3 440.7264
```

**Solution with variables** We can script the same code. It means that we would just have to replace the value in the first three lines to update the whole code.

#### VIII What to remember

MatLab is basically a powerful calculator.

# VIII a) Key example

Let's summarize in a few examples everything we have seen.

```
1 >> year = 2018;
2 >> next_year = year + 1
3 next_year =
4 2019
5 >> angle = pi ;
6 >> ( 2.*cos(angle) + 1.) ^2
7 ans =
8 9.0
```

# Chapter 2

# Basic Linear Algebra

Linear algebra is the fundations of MatLab, and what makes it popular.

In particular, MatLab makes the manipulation of matrices and vectors very easy. This section will show how to do:

- operations involving vectors
- operations involving matrices
- operations involving both arrays and matrices

# I Basics with matrices and vectors

# I a) Creation

Follow the procedure:

- start with a bracket [
- write each element of a column, separated with space or commas
- separate rows with a semi-colon
- end with the bracket ]

For creating a vector:

v is a row vector and w is a column vector.

For creating a matrix, the following lines are equivalent:

The last line shows that a matrix is virtually stacked vectors.

### I b) Manipulation

You can

- add, subtract, multiply matrices if their sizes are compatible
- access to the ith element of matrix M using M(i)
- $\bullet$  access to the (ith,jth) element of matrix M using M(i,j)

To add a constant to a vector, a vector to another vector (it works identically for matrices):

```
v > v = [1 \ 3];
_{2} >> v + 4
  ans =
   5 7
5 >> v * 2.
  ans =
   2 6
  \gg w = [2,8];
  >> v+w
  ans =
11 3 11
w > w/2.
13 ans =
14 1 4
      To access to an element of a vector (it works identically for matrices):
_{1} >> v = [1 \ 3 \ 4 \ -2.5 \ 8] ;
_{2} >> v(1)
  ans =
5 >> v(3)
  ans =
  >> M = [[5, 2]; [9, -1]] ;
9 >> M(1,1)
10
  ans =
  5
_{12} >> M(2,1)
13 ans =
14 9
      To delete remove the ith element, use the operator "[]":
```

It hence reduces the dimension of the vector. It works similarly for a matrix:

The first column has been deleted.

# II More details on vectors

# II a) Creation of a vector

#### II a) i Row vector

Creating a vector  $\mathbf{v}$  is easy. You just put all the components between brackets.

It is also possible to replace the commas with space:

It works fine but makes the code harder to read.

Let's create Cartesian vectors in dimension two.  $e_x = (1,0)$  and  $e_y = (0,1)$ :

#### II a) ii Column vector

Creating a column vector is similar to creating a row vector, except that the element are separated with a semi-colon ";".

#### II a) iii Transpose operator

It is possible to change a column vector to a row vector, and reciprocally, by using the transpose operator "".

```
1 >> ex = [1;0]
2 ex =
3 1
4 0
5 >> ex'
6 ans =
7 1 0
8 >> ey = [0,1]
9 ey =
10 0 1
11 >> ey'
12 ans =
13 0
14 1
```

# II b) Access to the elements of a vector

#### II b) i Access to one element

The first element of a vector v is v(1). The second element is v(2), and so forth.

Accessing the element of a vector is just calling the vector with specifying the desired element:

```
1 >> x = [1 \ 3 \ 4 \ -2.5 \ 8]
2 x =
3 1 \ 3 \ 4 \ -2.5 \ 8
4 >> x(1)
5 ans =
6 1
7 >> x(3)
8 ans =
9 4
10 >> x(4)
11 ans =
12 -2.5
```

The last element of a vector can be called using the argument end: You can also call the ith item from the end using end-i.

```
1 >> x = [1 \ 3 \ 4 \ -2.5 \ 8]
2 x =
3 1.000 \ 3.000 \ 4.000 \ -2.5 \ 8.000
4 >> x(end)
5 ans =
6 8
7 >> x(end-1)
8 ans =
9 -2.5
10 >> x(end-2)
11 ans =
12 4
```

### II b) ii Access to several elements

The operator ":" gives access to all the elements between the first and the last element (included), in a column vector:

Accessing to all the elements between the second and fifth element of v is v(2:5):

```
1 >> v = [1 3 4 -2.5 8 12];

2 >> v(2:5)

3 ans =

4 3

5 4

6 -2.5
```

To access to all the elements between the first and the last element (included), in a column vector, simply use the colon operator:

The colon operator is pivotal in MatLab.

It is noticeably useful to

- generate lists: x= 1:20
- controlling loops (more in a following chapter): for i=1:20
- transform a matrix in a column vector M(:)
- extract sub-parts of vectors/matrices M(2:4,1:2:8)

More details in Sec. IV a) i.

# II c) Basic operations on vectors

#### II c) i Addition/subtraction

It is easy to add or subtract a given value to all the components of a vector, using the signs "+" and "-".

Vectors can be added, as long as their dimensions correspond:

```
1 >> ex = [1 0]; ey = [0,1];
2 >> ex + ey
3 ans =
4 1 1
5 >> ex - ey
6 ans =
7 1 -1
```

Of course, if their dimensions do not correspond, MatLab will send back and error:

<sup>3</sup> Matrix dimensions must agree.

MatLab considers vectors as 1D matrices.

It is important when dealing with multiplication.

If v=[1,2,3]:

• v\*v' is 14 it is multiplying a 1 × 3 matrix with a 3 × 1 matrix.

• v'\*v is [[1,2,3]; [2,4,6]; [3,6,9]] it is multiplying a  $3 \times 1$  matrix with a  $1 \times 3$  matrix.

• v\*v will not work it is multiplying a 1×1 matrix with a 1×3 matrix; dimensions are not compatible.

#### II c) ii Multiplication

II c) ii 1 Multiplication by a scalar It is easy to multiply or divide by a given value to all the components of a vector, using the signs "\*" and "/"

```
1 >> x = [1 \ 3]
2 x =
3 1 3
4 >> x * 4
5 ans =
6 4 12
7 >> y = x / 2.
8 y =
9 0.5 1.5
10 >> z = 3 * [5 \ 10 \ -1 \ 8]
11 z =
12 15 30 -3 24
```

II c) ii 2 dot product It can be done using the function dot.

```
x >> x = [1,0] ; y = [1,0];
_2 \gg \det(x,y)
  ans =
   1
  >> x(1) * y(1) + x(2) * y(2)
  ans =
  \Rightarrow a = [0,0.5,2]; b = [2,0,4];
  \gg dot(a,b)
   ans =
10
11
  \Rightarrow a(1) * b(1) + a(2) * b(2) + a(3) * b(3)
  and =
13
   8
14
```

II c) ii 3 Element-wise multiplication Element-wise multiplication means that each element of a vector is multiply by the corresponding element of the other vector. It is similar to the dot product *except* for the sum. The operator for that is ".\*" (it is read "dot product", which is pretty stupid when you think about it!).

```
1 >> x = [1,0]; y = [1,0];

2 >> x.*y

3 ans =

4 1 0

5 >> [x(1) * y(1), x(2) * y(2)]

6 ans =

7 1 0

8 >> a = [0,0.5,2]; b = [2,0,4];

9 >> a.*b

10 ans =

11 0 0 8

12 >> [a(1) * b(1), a(2) * b(2), a(3) * b(3)]

13 and =

14 0 0 8
```

<u>1</u>9

In MatLab, using "." in front of an operator means that this operator will be applied element-wise (to each element of the vector/matrix).

For instance, v.^2 means that *all* the elements of v will be squared. If v is [2,4,3], then v.^2 is  $[2^2,4^2,3^2]$ , or [4,16,9].

# [[TODO exercices]]

# III Matrices

MatLab sees vectors a line matrices. Building a matrix is the equivalent of stacking lines. For that, MatLab uses the semi colon sign ";". The following lines are equivalent:

```
_{1} >> M = [1, 0 ; 0, 1];
_{2} M =
```

The last line shows that a matrix is virtually stacked vectors.

# ${ m III} { m \ a}) { m \ \ Addition/subtraction}$

It is easy to add or subtract a given value to all the components of a matrix, using the signs "+" and "-".

```
\begin{array}{lll} 1 &>> M = \ [\ [1 & 3\ ]\ ; \ [2\ ,4\ ]\ ; \\ 2 &>> M + \ 4 \\ 3 & ans = \\ 4 & 5 & 7 \\ 5 & 6 & 8 \\ 6 &>> A = M - \ 2 \\ 7 & A = \\ 8 & 3 & 5 \\ 9 & 4 & 6 \end{array}
```

Matrices can be added, as long as their dimensions correspond:

```
\begin{array}{lll} & 1 & >> M = \ [\ [1 & 0\ ]; \ [2\ ,3\ ]; \ A = \ [\ [2\ ,-1\ ]; \ [1\ ,1\ ]]; \\ & 2 & >> M + A \\ & 3 & ans = \\ & 4 & 3 & -1 \\ & 5 & 3 & 4 \\ & 6 & >> M - A \\ & 7 & ans = \\ & 8 & 1 & 1 \\ & 9 & 1 & 2 \end{array}
```

Of course, if their dimensions do not correspond, MatLab will send back and error:

```
\begin{array}{lll} & _{1} >> M \\ & = [\,[\,1\;,0\,]\,;[\,1\;,0\,]\;;\,[\,1\;,0\,] \end{array};\;\; A = \;[\,[\,2\;,-\,1\,]\,;[\,1\;,1\,]\,]\,;\\ & _{2} >> x + y \end{array}
```

3 Matrix dimensions must agree.

#### III a) i Multiplication

It is easy to multiply or divide by a given value to all the components of a vector, using the signs "\*" and "/"

```
1 >> x = [1 3]

2 x =

3 1 3

4 >> x * 4

5 ans =

6 4 12
```

```
7 >> y = x / 2.

8 y =
9 0.5 1.5
10 >> z = 3 * [5 10 -1 8]
11 z =
12 15 30 -3 24
```

What is multiplication for vectors? Two definitions can be proposed.

#### III a) i 1 dot product The first definition is the dot product between two vectors.

Definition

DOT PRODUCT: The dot product (or inner product) of two vectors is the sum of the multiplication of their components. If  $u = (u_i), v = (v_i)$  are *n*-dimensional vectors,  $\langle u, v \rangle = \sum_{i=1}^{n} u_i \times v_i$ .

It can be done using the function dot.

III a) i 2 Element-wise multiplication Another definition could be the element wise multiplication. It means that each element of a vector is multiply by the corresponding element of the other vector. It is similar to the dot product *except* for the sum. The operator for that is ".\*" (it is read "dot product", which is pretty stupid when you think about it!).

# [[TODO exercises]]

Try to create the vector x = (1, 2, 3, 4).

#### III a) ii Accessing and deleting elements

III a) ii 1 Accessing elements It works in a similar way as for vectors, except that the dimensions are separated with a comma. Let's work with M:

To extract the third column of M:

To extract the second row of M:

To extract the first and the second row of M:

```
^{1} >> A = M([1,2],:)

^{2} A =

^{3} 1 2 3

^{4} 4 5 6
```

Following this notation, we can exchange the first rows of M:

```
\begin{array}{lll} {}_{1} &>> A = M(\,[\,2\,\,,1\,]\,\,,:\,) \\ {}_{2} & A = \\ {}_{3} & 4 & 5 & 6 \\ {}_{4} & 1 & 2 & 3 \end{array}
```

Using lists (for instance indices = [1,3,4]) as arguments of matrix is a very powerful methods for altering a matrix or for creating a new one.

Pro Ti

- M(indices,:) will select the rows of M in the order of the elements of indices. They can be copied, if there is repetition in indices, as indices = [1,2,2] They can be rearranged, if the elements in indices are not ranked, as indices = [3,1,2]
- M(:,indices) will select the rows of M in the order of the elements of indices
- M(indices\_row,indices\_col) will extract a sub-matrix of M

In MatLab, using "[]" reinitialize and empty a variable. It is named the *empty vector operator*. When used on a part of a vector or a matrix, it simply deletes this part.

# IV Useful built-in functions for vectors and matrices

# IV a) Generating a vector

### IV a) i The colon operator

One of the most powerful operator in MatLab is the colon operator ":". It allows in particular to generate lists.

We have seen already that v(2:5) gives the elements of v between the 2nd and 5th position. What it does is actually ask MatLab to send back elements of v in position 2, 3, 4, 5. These positions are *generated* by the command 2:5.

```
1 >> 2:5
2 ans =
3 2 3 4 5
```

The command a:b hence generates a vector, starting in a, each element being incremented by 1, until it would be greater than b.

```
1 >> 4:8
2 ans =
3 4 5 6 7 8
4 >> 4.5:8
5 ans =
6 4.5 5.5 6.5 7.5
```

In the second example, the last item is 7.5. 8.5 would be larger than 8 and is hence omitted.

It is possible to force the increment. The command a:da:b generates a vector, starting in a, each element being incremented by da, until it would be greater than b.

```
1 >> 0:.1:1

2 ans =

3 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.
```

#### IV a) ii Other functions

Table 2.1: A few functions that generate vectors.

name	description	illustration
colon operator :	see Sec IV a) i	0:10:.5
linspace(a,b,n)	linearly spaced n-dimensional vector	linspace(0,1,11)
	between a and b	
diag(A)	diagonal of matrix (A)	diag([[1,2];[3,4]])
	between a and b	

### IV b) Generating a matrix

# V Linear Algebra

### VI What to remember

### VI a) Creating a matrix

Follow the procedure:

Table 2.2: A few functions that generate vectors.

name	description	illustration
zeros(m,n)	m, n-dimensional matrix filled with 0	zeros(2,4)
ones(m,n)	m, n-dimensional matrix filled with 1	ones(2,4)
rand(m,n)	m, n-dimensional matrix filled with	rand(2,4)
	random numbers taken between 0 and 1	
eye(n)	n-dimensional identify matrix	eye(10)
diag(v)	matrix filled with 0 with v as diagonal	diag([1,2,3])

Table 2.3: A few functions that are useful.

name	description	illustration
size(M)	number of elements in M	size([1,2,3])
shape(M)	number of elements in each direction of M	shape([2,3,5];[1,2,3]])

- start with a bracket [
- write each element of a column, separated with space or commas
- separate rows with a semi-colon
- end with the bracket ]

# VI b) Manipulating matrices

You can

- add, subtract, multiply matrices if their sizes are compatible
- ullet access to the ith element of matrix M using M(i)
- access to the (ith,jth) element of matrix M using M(i,j)

# VI c) Key example

Let's summarize in a few examples everything we have seen.

- $_{1} >> M = [[1, 2]; [3, 4]]$
- <sub>2</sub> M =
- з 1 2
- 4 3 4

# Bibliography