Dynamics of contagion and immunity thresholds in social environments: an approach from graph systems

Complex Social Systems

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Introduction

Since the outbreak of COVID-19, the world's governments have responded with various policies. Lockdowns, restrictions on public transport, closures of schools and workplaces, and the suspension of mass activities were part of the policy function to counter the epidemic. In all its forms, the preferred instrument of policymakers was population mobility. Lower circulation is associated with a lower level of interactions between people than would be the case in an unrestricted situation. However, fewer interactions between agents are also associated with a lower level of participation by agents in the various markets. This paper aims to study the opportunity costs associated with the decision to combat a situation of epidemiological contagion subject to economic restrictions. So work is encompassed in the following question: How much does economy matter concerning health for an epidemic to become endemic?

The present work lies at the intersection between economics, complex networks, and epidemiology. The study of the spread and immunization of diseases has a long history. (Kermack y McKendrick, 1939) ccontribute to the model known as SIR. It is a *cluster model* that describes a flow between epidemiological classes (susceptible, infected, and recovered) in the population. The fundamental premise is that the infected and the contagion rate of the disease mark the epidemic's progress. This model predicts a peak in infections, followed by the disappearance of the disease. The susceptible ones are fewer and fewer, and the infection does not reach the already immunized recovered ones, ruling out an endemic situation. However, it does not incorporate the spatial particularities or heterogeneities between the links of the population, which would require other types of immunization strategies. Therefore, it is necessary to incorporate the spread of epidemics in complex networks.

(Boguá, Pastor-Satorras y Vespignani, 2003)[1] y (Pastor-Satorras y Vespignani, 2002) study contagion and immunization in a Graph System. The complexity resides in the small average path lengths between any two nodes, also called the small-worlds property, together with different degrees of local clustering within the network. From these properties, some nodes will have more significant influence than others within the structure, or rather, they will face a probability of being connected relative to other nodes in the network. Heterogeneity in connectivity is associated with networks that follow a power law, which implies that in large networks, nodes face a statistically significant probability of having a higher degree of connectivity compared to the average connectivity of the structure that contains them.

[1]Statistical Mechanics of Complex Networks pp 127-147

There are also models of homogeneous networks, where each node has approximately the same number of connections as the average connectivity of the network. In this environment, the epidemic takes place only if the infection rate of infected individuals connected to healthy ones exceeds what is known as the epidemic threshold; If the disease cannot be transmitted faster than the cure time, then the virus is extinct. (Dezső y Barabási, 2002) studied contagion in a heterogeneous network and demonstrated the extinction of the epidemic threshold. Their findings change the incentives for promoting policies to reduce the rate of contagion. The traditional approach of network immunization aims to lower the contagion rate of the virus below the threshold to eradicate the epidemic. With a structure following a power law and a threshold close to zero, a decrease in contagion rate only diminishes the presence of disease. However, there is no guarantee of its extinction. In this sense, the authors show that policies that tend to discriminate between nodes can restore an epidemic threshold and potentially eradicate the virus. Furthermore, the greater the bias of health policy towards nodes with a higher degree, the greater the probability of virus extinction. However, the economic cost involved is also higher.

Assuming the economy as a network of consumers and firms related to each other through perdurable links, each affected node, whether due to a *shock* of productivity, generates damage for itself and its connections through the input-output structure. (Lucas Jr, 1977), studied this and concluded that idiosyncratic shocks dissolve in the aggregate of units. However, (Gabaix, 2011) shows that under the existence of heterogeneity in the size of the firms, the *shocks* of productivity in those that are larger cannot be compensated by the smaller ones, opening the door to fluctuations in macroeconomics of idiosyncratic origin. Along the same lines, (Acemoglu, Akcigit, y Kerr, 2016) finds that the propagation of *shocks* through the input-output network is statistically and economically significant. In this sense, the heterogeneity in the degrees of the network nodes understood as firms or consumers with more connections can represent different levels of influence within a structure. In this way, if a government tries to find an epidemic threshold again by applying health policies biased towards discrimination due to connectivity, the disconnections of nodes with different degrees will generate different economic impacts.

The paper aims to characterize the epidemic threshold of a planner in charge of health policy in an economy characterized by a heterogeneous network. I propose a stylized model where the government decides to discriminate between nodes according to their degree. Depending on the government's preferences, represented by the loss function (1), the composition of welfare will be valued differently. In this way, the evolution of an epidemic will be studied under different shares of health or economy on general well-being through computational experiments.[2].

Model

The experiments take place in a random Erdös–Rényi network. In this model, each node has statistical independence from the rest. Therefore, each new node joins the network with equal probability (Erdős, Rényi, y cols., 1960). An adjacencies matrix is generated that describes the connections of the nodes. The interaction between the network components is beneficial economically, but it is detrimental in health terms during an infection context. Higher connectivity of nodes is associated with a higher persistence of the virus.A disease with a contagion rate p and a recovery rate r enters. Each node is susceptible to being infected, even if it has already contracted the disease in the past. In this way, the model is a variant within the SIS. In terms of the model, contagion unfolds as follows. First, a seed of infection appears. Then, each infected node faces a probability p of infecting each connected neighbor. Herein lies the relevance of discrimination in health policy. Nodes that have more significant influence will infect, on average, more neighbors. The network is associated with a planner, which acts as a central government external to the network and applies a health policy from a given moment.

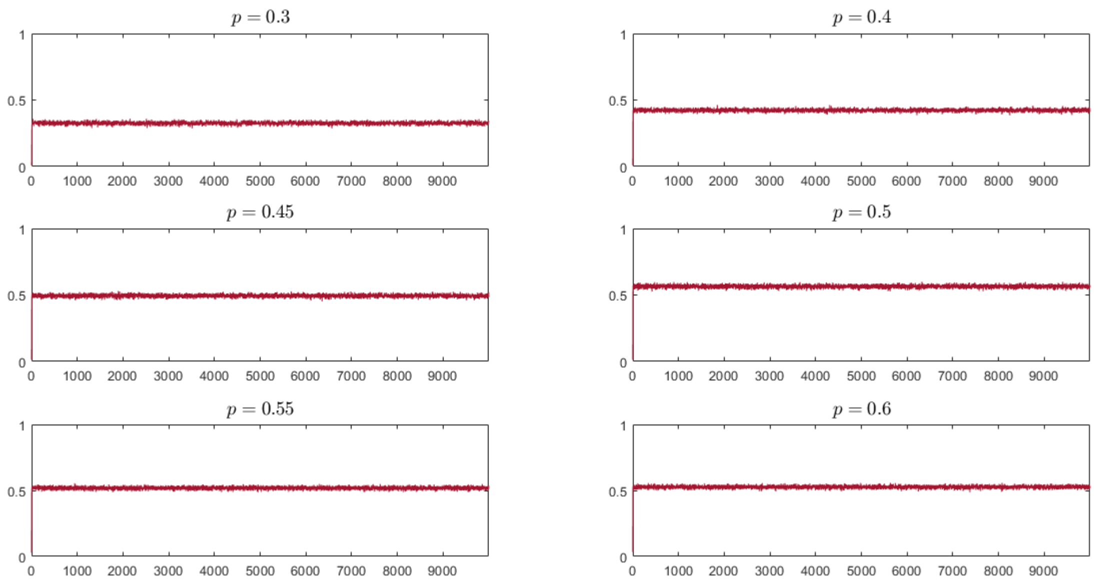

Figura 1: Evolution of disease with different probabilities of contagion p

Figure 1 shows the share of the network that is infected for different probabilities without the planner's intervention considering the following calibration:

[2]Codes are avaliable at request

- Nodes: 300
- Average degree of connectivity: 5.7
- Recovery rate (r): 0.6

Analysis validates the expectation. First, a higher probability of contagion means a higher proportion of infected. The infection reaches an endemic stage in all cases. Figure 2 shows the evolution of the disease for a probability of contagion $p = 0{,}7$, which will be the rate of the experiments. Without a planner, the number of infections rapidly evolves towards a steady state around 53 % of the infected network.

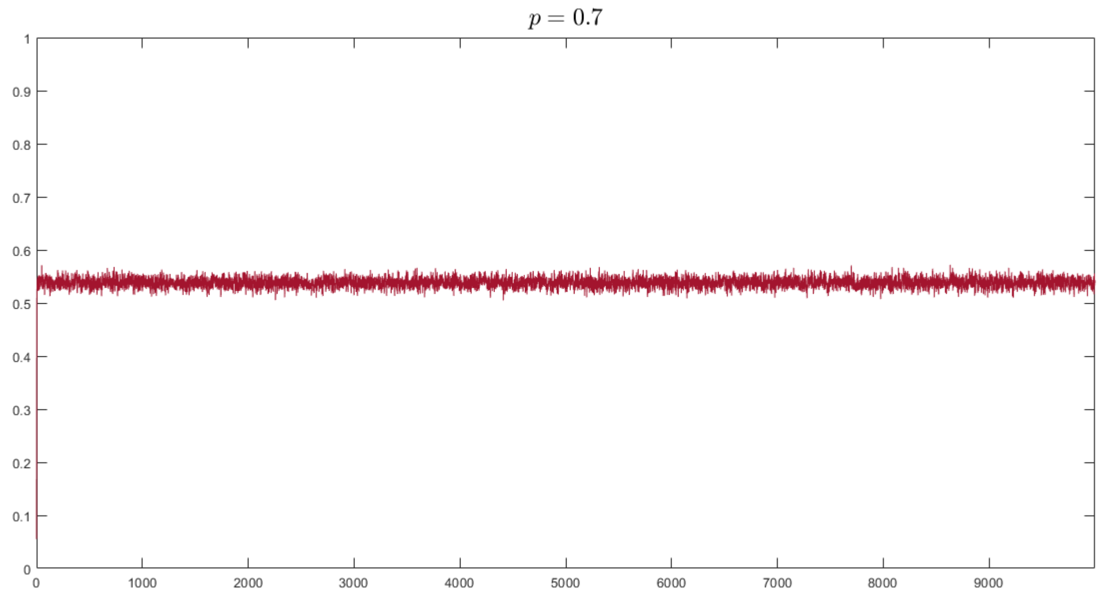

Figura 2: Disease evolution for $p = 0{,}7$

The planner begins to act at tau with a behavior determined by the loss function described by Equation (1). This function punishes the squared deviations to the initial number of nodes weighted by $\beta \in [0, 1]$ and the contagion level of the periods scaled by $(1 - \beta)$. In this way, the economic and health costs are modeled, respectively, in a non-linear way. The weights β and $1-\beta$ play the role of Negishi weights in a social welfare function. In this direction, a value of β close to zero implies that the participation of the losses of economic connections of the economy is not relevant when applying sanitary measures. In contrast, a β close to one implies that only economic costs are significant in the design of the health policies.

The government can restrict a share[3] of nodes for a finite time as a sanitary measure from the moment t_p. It knows the network's adjacency matrix, but it does not have access to the vector of

[3]The persistence of the disease increases with the isolation capacity. In this sense, if the planner did not have

states of the nodes. It runs tests to know the actual state of the nodes. If tests find infection in one, then the planner decides whether to restrict it from the network, cutting off all its connections or not.

$$L_t = \beta \times (links_0 - links_t)^2 + (1 - \beta) \times infected_t^2 \tag{1}$$

Before restricting a specific node i, the planner estimates the economic cost for the network represented by the loss of connections and the healthcare cost of keeping the node connected. Therefore, given the preferences of the central authority, if the economic cost of restricting it is greater than the expected health cost, then the node remains connected even if it is infected. In the case of restriction, the node is reconnected to the network when healthy, reestablishing its original links.

$$\begin{gathered} E\big[L_{t+1}|Don't\ insulates_i\big] = \beta \times (links_0 - links_{t+1|nr_i})^2 + (1-\beta) \times E\big[infected^2_{t+1|nr_i}\big] \\ E\big[L_{t+1}|insulates_i\big] = \beta \times (links_0 - links_{t+1|r_i})^2 + (1-\beta) \times E\big[infected^2_{t+1}|r_i\big] \end{gathered} \tag{2}$$

Donde

$$\begin{gathered} E\big[infected^2_{t+1|nr_i}\big] = (1-H) \times (infected) \times (1 + p \times z) - r \times infected \\ E\big[infected^2_{t+1|r_i}\big] = E\big[infected^2_{t+1|nr_i}\big] - 1 - p \times links_t \end{gathered} \tag{3}$$

Under this, node can belong to this set of categories : [Susceptible, Infected] and [Restricted, Not Restricted].

a limited capacity restricting nodes, the pandemic would immediately die out, and the opportunity costs would disappear. In order to address a question with a certain degree of relevance, the planner will have a restriction on its insulation capacity throughout the entire work. which is shown in Table 1.

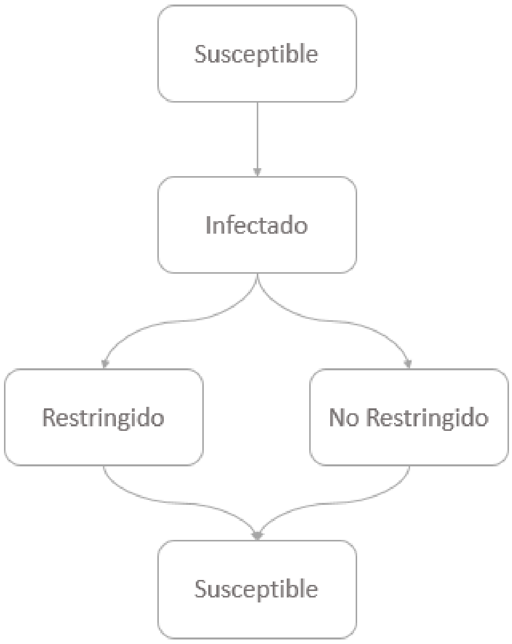

Figura 3: Possible states for node i

Results

This section displays the results of the computational experiments for the following parameter configuration in the model proposed in the previous section.

Total Nodes	300
Average degree of connectivity	5,7
Contagion probability	0,7
Recovery rate	0,6
Total Iterations	10.000
Nave	10
Insulation Capacity	10 % from total.
Iteration until planner acts	2000

Tabla 1: Parameter Calibration

The graphs come for the results of different relative weightings between economy and health [4]. Figure 4 shows the evolution of Infected (red) and Restricted (blue) for different relative weights. For the case in which beta equals one, the scheduler has extreme preferences for the economy

[4] The evolution of the network during the first periods may differ between samples due to randomness associated with the construction of network and determination of the first contagion.

and therefore chooses not to constrain nodes. This case is equivalent to the no-scheduler case, where the epidemic is free-running and settles at its steady-state endemic value of about 53 % of the infected population. As beta approaches zero and health gains relative weight in the loss function, the number of endemic cases decreases while insulated nodes shares increase. At the other extreme, when $\beta = 0$, the authority manages to extinguish the virus quickly.

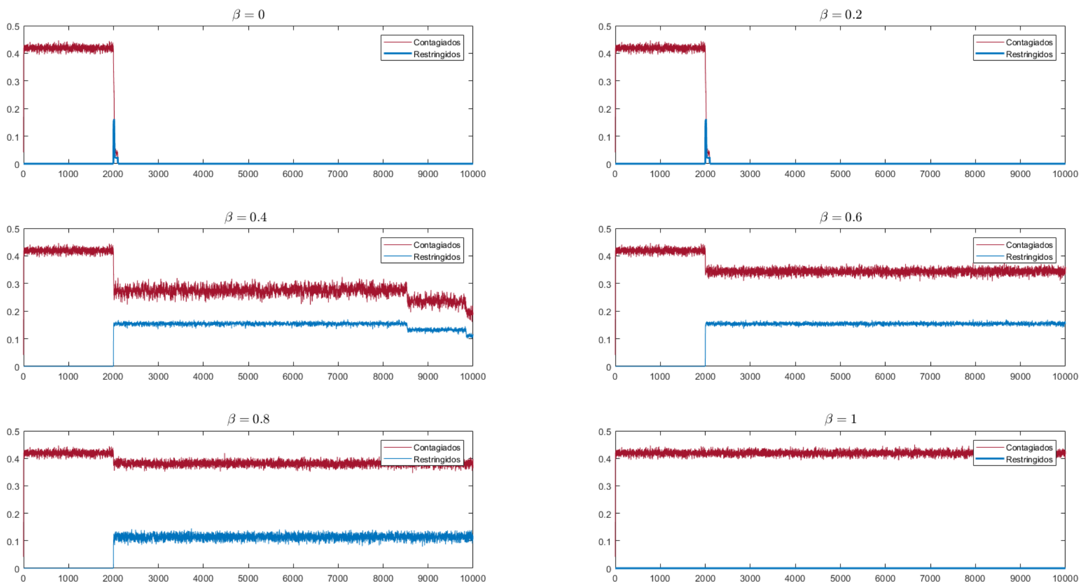

Figura 4: Evolution of disease with different level of β

The following figure shows the *performance* of the economy for the same values of beta, which is a mirror of the epidemiological situation of the network. For all levels of β, the number of connections in the economy fluctuates around a constant. As β gets closer to 1, the constant by which the total number of nodes fluctuates gets larger. This β reflects the planner's preference to take care of network connections despite nodes transmitting the infection

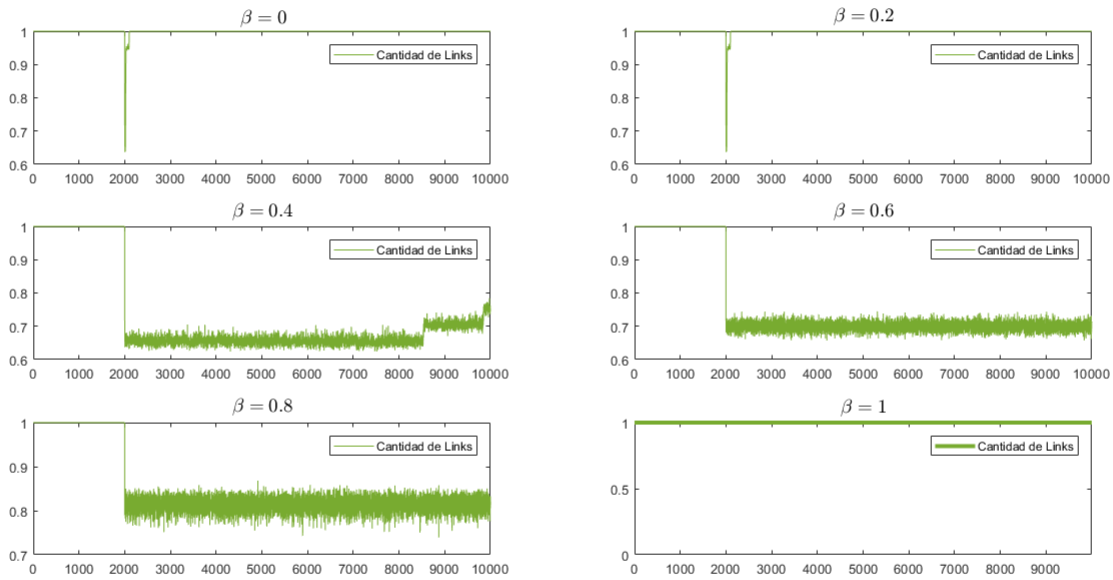

Figura 5: Economic performance due to β

In Figure 4 it can be seen that if $\beta \in [0{,}2; 0{,}6]$ the evolution of the disease changes state. If $\beta = 0{,}2$ the virus is extinct. On the contrary, the epidemic becomes endemic when $\beta = 0{,}6$. Therefore, it is necessary to iterate between these values to find where β is the discontinuity or how relevant the economy has to be for the health cost for the epidemic to persist.

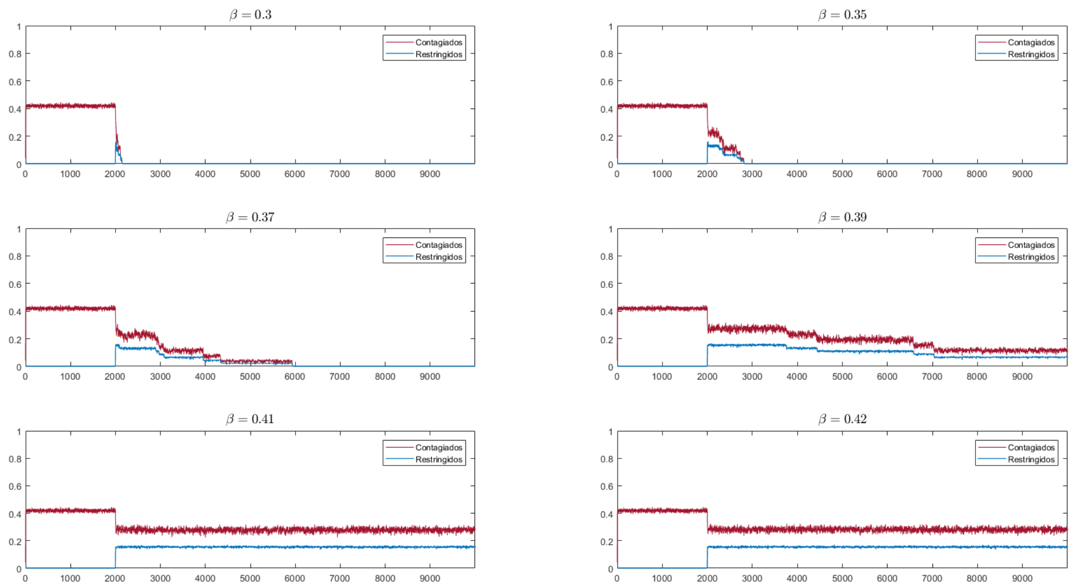

Figura 6: Disease evolution for $\beta \in [0{,}3; 0{,}42]$

Figure 6 shows that if $\beta \geq 0{,}41$, the government considers that the economic costs of extinguishing the pandemic are relatively higher than the epidemiological costs for society. Therefore, the policymaker chooses an endemic epidemic situation.

Additional Results

This section analyzes two variants of the basic model: isolation capacity restrictions and imperfections in the testing technology. Previously the central authority was able to extinguish the infection by applying a policy biased towards the influence of the nodes. For a $\beta \leq 0{,}39$, the number of contagions has a weight relative to the economy such that the average restrictions imposed are sufficient to avoid endemicity. However, these results have to be understood as an *upperbound*. The planner has population data on the vector of states and knows each node's influence. In this section, I propose two scenarios in which the epidemic does not die out despite the extreme preferences of the central authority for the population's health.

Diagnosis's Errors

In the basic model, the planner can know the state vector of the network nodes by testing. This specification does not contemplate the possibility that tests are subject to false positives and false negatives. The following exercise features an imperfection in the testing technology and a level of $\beta = 0$ for carrying out experiments. In other words, I analyzed the evolution of the epidemic for the case in which the planner has extreme preferences for health and, at the same time, faces different levels of diagnostic error.[5].

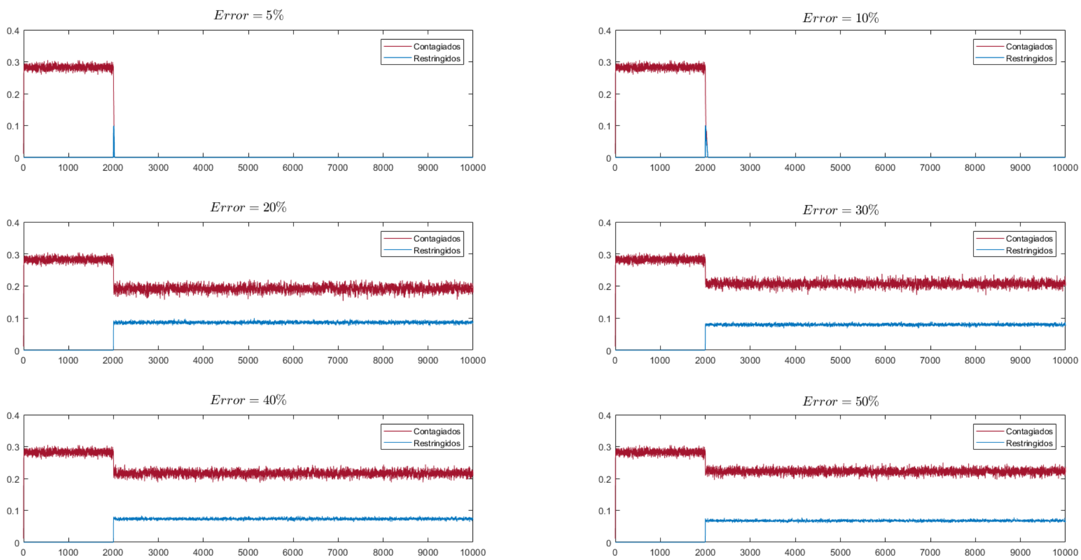

Figura 7: Evolución de la enfermedad para $\beta = 0$ y distintos niveles de error de testeo.

Figure **??** shows that, although the central planner has extreme preferences for health, a

[5]In the absence of limits to the isolation capacity of the planner, this exercise would be trivial. The optimal policy response, given the extreme preferences for health, would be to isolate all nodes regardless of the outcome of the tests.

diagnostic error between 10 % and 20 % is sufficient not to be able to extinguish the epidemic. In particular, Figure 8 shows the threshold error from which the planner is unable to stop the epidemic. A deeper analysis shows that the planner cannot extinguish the virus under a diagnostic error of 18 %, despite extreme health preferences.

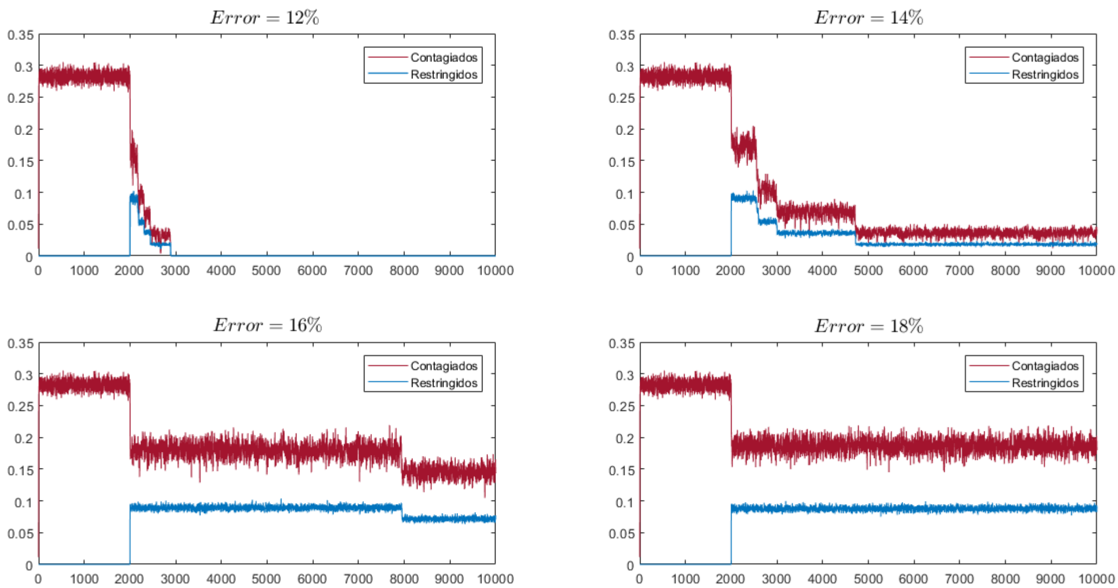

Figura 8: Threshold diagnosis' Error

Limits to the capacity of testing

In the basic model, the planner can know the status of each node through massive testing. This section proposes a variant of the initial model in which the central authority can only evaluate the status of a share of the network nodes. For this exercise, the planner has extreme preferences for health. Figure 9 shows the evolution of the number of infected for different capacities. The analysis found that if the testing capacity is higher than 70 % of the population, Health policies can eradicate the virus. For any minor case, it is only possible to reduce the presence of the virus.

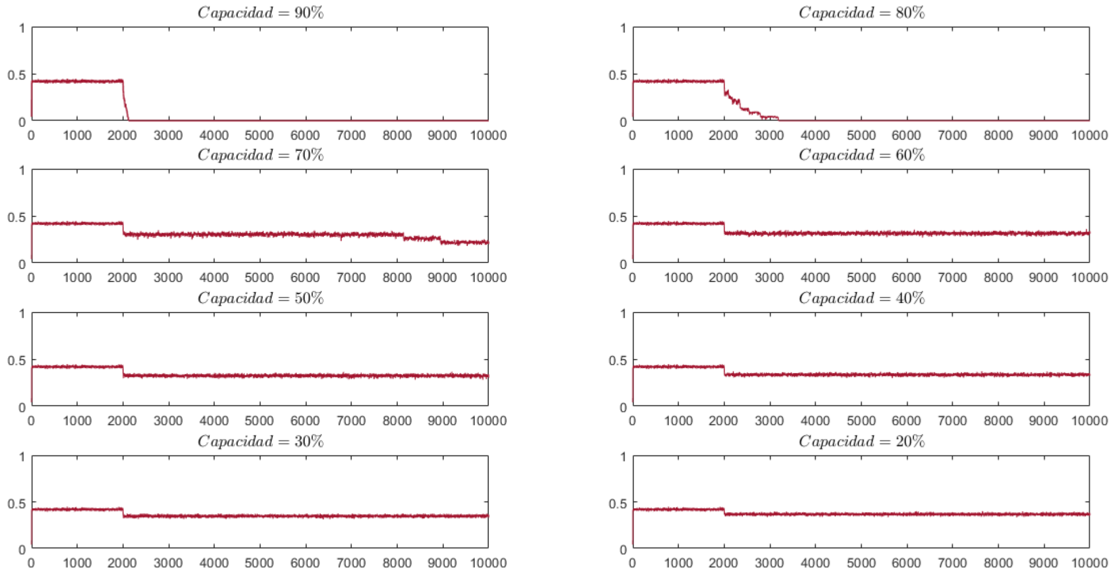

Figura 9: Disease Evolution for different testing capacities

Conclusions and Extensions

The present work intends to study the opportunity costs a central authority faces when choosing an immunization strategy biased towards the nodes with higher connectivity in a heterogeneous network. The model implies a trade-off between general health welfare and the economic cost of applying this policy in terms of connections. The basic model consists of a random network infected by a disease and reaches an endemic level of infection typical of the SIS. A planner is introduced that does not know the state vector of the agents but can test all of them and restrict a fraction. Simulations find in the basic model a health relevance threshold of 0,41. This threshold implies that the central authority does not need to be very aggressive with the structure of network connections and with the economic costs to exterminate the epidemic if it follows a health policy biased towards the most influential nodes.

It is necessary to highlight that as the preferences for health are laxer, the planner's time to extinguish the infection raises. This time calls into question the interpretation of iterations. If each iteration in which the planner chooses to restrict nodes is understood as a cabinet meeting in charge of health policy, then for a β threshold, approximately 250 cabinet meetings would have to be held to exterminate the epidemic. Similarly, it is necessary to highlight an implicit assumption of the model. All nodes share productivity regardless of state. In other words, even if the node x is infected and not restricted, the connections contribute the same in economic terms as any other non-infected node y. A plausible extension is the incorporation of heterogeneity in the productivity of each node. Thus, restricting an infected node with low productivity generates a greater incentive to isolate the node for the same combination of preferences. In the extreme, if an infected node is entirely unproductive, then the opportunity cost disappears, so the optimal policy, both in health and economic terms, is to eradicate the pandemic as soon as possible.

The threshold of preferences fades by adding constraints on testing technology or the ability to test. With a testing error greater than 18 % or a testing capacity of less than 70 % of the population, the epidemic becomes endemic despite applying a health policy biased towards the most influential nodes with a combination of extreme preferences for health. These results become relevant in the current epidemic context where information is incomplete and technology imperfect. Sanitary measures based on the restriction of connections are not enough to extinguish the infection in the presence of mentioned frictions.

These simulations open two paths. First, it is necessary to ask whether, given that the restrictions or isolations are not enough to extinguish the infection, it makes sense or not to face the loss of economic well-being. Second, a compartment not taken into account in the proposed configuration becomes relevant: immunization, because in the presented model, once infected, the node can become susceptible again. If the restriction measures are not enough to extinguish the epidemic, immunization policies, such as vaccination campaigns, become relevant. A possible extension to the basic model would be to incorporate immunization. In this context, new discussions arise, such as priority order in vaccination based on the level of influence of the nodes within the network. Immunizing the nodes with greater relevance in the structure would make the heterogeneity in the degree of the nodes less relevant for contagion. From the work presented, it can be deduced that, as political action, it is not enough to isolate the nodes, no matter how influential they may be. The restriction policy is not enough to exterminate the infection, so it is necessary to complement it with other types of actions to achieve network immunity.

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