State-Space Model Estimation: The Multivariate Filter

1. Examine and write down the following state-space model in Dynare (which is a multivariate filter similar to the one seen in class).

The model consists of four different blocks of equations, each relating to a particular observable variable contained in the vector of observables $\text{obs}_t \equiv (gY_t, U_t, \pi_t, \text{tcr}_t)$.

The first block, which is linked to GDP growth (gY_t^{obs}) is

$$g Y_t^{\text{obs}} = g\bar{Y}_t + y_t - y_{t-1}$$
$$g\bar{Y}_t = G_t + \sigma_{g\bar{Y}} \epsilon_t^{g\bar{Y}}$$
$$G = \theta(G^{SS}/4) + (1-\theta)G_{t-1} + \sigma_G \epsilon_t^G$$
$$y = \phi y_{t-1} + \sigma_y \epsilon_t^y.$$

Then, there is the block corresponding to the unemployment rate (U_t)

$$U_t^{\text{obs}} = u_t + \bar{U}_t$$
$$u_t = -\tau_1 y_t + \tau_2 u_t + \sigma_u \epsilon_t^u$$
$$g\bar{U}_t = (1-\tau_3) g\bar{U}_{t-1} + \sigma_{g\bar{U}} \epsilon_t^{g\bar{U}}$$
$$\bar{U}_t = \tau_4 \bar{U}^{SS} + (1-\tau_4)\bar{U}_{t-1} + g\bar{U}_t + \sigma_{\bar{U}} \epsilon_t^{\bar{U}},$$

and finally, a single equation for the inflation rate (π_t)

$$\pi_t^{\text{obs}} = \frac{\lambda}{1+\beta\lambda}\pi_{t-1}^{\text{obs}} + \frac{\beta}{1+\beta\lambda}\text{E}_t[\pi_{t+1}^{\text{obs}}] + \kappa y_t + \gamma \text{tcr}_t^{\text{obs}} + \epsilon_t^{\pi},$$

and another sole for the real exchange rate (tcr_t)

$$\text{tcr}_t^{\text{obs}} = \tau_5 \text{tcr}_{t-1}^{\text{obs}} + \sigma_{\text{tcr}} \epsilon_t^{\text{tcr}},$$

where $\epsilon_t^i \sim \mathcal{N}(0,1)$ for $i \in (g\bar{Y}, G, y, u, g\bar{U}, \bar{U}, \pi, \text{tcr})$.

2. Define $\psi \equiv (\theta, G^{SS}, \phi, \tau_1, \tau_2, \tau_3, \tau_4, U^{SS}, \lambda, \kappa, \gamma, \tau_5, \sigma_{g\bar{Y}}, \sigma_G, \sigma_{\bar{U}}, \sigma_u, \sigma_{g\bar{U}}, \sigma_{\text{tcr}})$, setup an independent prior for each element in ψ and calibrate a value for β. Using quarterly data of the observed variables from Chile[1] proceed to estimate the model and find the maximum the posteriori. Comment on the output of Dynare.

3. Provide an estimation of the output gap and related variables, along with error bands conditional on the value of the parameters. Additionally, compute the historical decomposition of the variables of interest.

4. (**Optional**). Recover the full posterior distribution of each parameter using the Metropolis-Hastings algorithm and compute and plot the impulse response functions (IRFs) of the model. Provide a credible interval for all the IRFs.

[1] Use the provided `.xlsx` file.