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Oligopoly on a Salop Circle with Centre

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Oligopoly on a Salop Circle with Centre*

Paul Madden and Mario Pezzino

Abstract

We study an oligopolistic market in which consumers located around the perimeter of a Salop circle buy either from firms around this perimeter (providing horizontally differentiated goods) or from a firm located at the centre of the circle (providing a homogeneous good). An entry-pricing game is studied. The market equilibria and social optima indicate various possible market failures, including cases in which the market is served only by perimeter firms whilst central provision would be socially optimal (in this sense, more extreme than the standard Salop excessive product differentiation). Moreover, for some parameters, the standard Salop result might be reversed.

KEYWORDS: differentiated products, homogenous product, social optimum

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1. Introduction

The model presented in this paper studies a market in which differentiated product firms compete in prices with a homogenous product firm. Specifically there is a Salop (1979) circle on whose perimeter is located a continuum of consumers who bear (heterogeneous) transport costs if they buy from one of the firms symmetrically located on the perimeter. However, in addition, there may be a firm located at the centre of the circle[1], with all consumers bearing the same transport cost of buying from the central firm instead of a perimeter firm. Thus the central firm offers a homogeneous product alternative to the horizontally differentiated products of perimeter firms. Throughout, and as in the standard Salop (1979) model, consumers are assumed to be perfectly informed of the available products and their prices. Our focus is on the social (sub)optimality of the market outcome modelled as subgame perfect equilibrium of an entry/pricing game, and how this differs from the standard Salop (1979) result (without the central firm) of excessive market provision of differentiated products (see e.g. Tirole (1988)). In particular, the results of the paper indicate various novel and interesting forms of market failure that may occur when the central location is added to the Salop framework.

There are a number of potential applications of the Salop circle with centre. For instance, competition between geographically dispersed High St. shops selling a particular good (perimeter) and mail order or internet provision of the good (centre) comes to mind immediately – there will be heterogeneity for consumers in the cost of visiting a High St. shop, but access costs to mail order or internet provision are likely to be relatively homogeneous across consumers. Alternatively, we can think of the central location as representing a basic commodity for which consumers in the market have homogenous tastes, whilst the perimeter locations mix the basic commodity with a variety of other ingredients aimed at a smaller niche in the market. Examples include basic pharmaceutical products (e.g. paracetamol) which may be mixed with alternative substances to produce remedies for a variety of specific symptoms, or basic foodstuffs (e.g. corn flakes) which may be packaged with other foods (e.g. nuts, fruit) to target specific tastes of consumers.

In order to provide a consistent motivation for some specific model assumptions made in the text of the paper, we focus on the first of these applications, namely High St. shops versus mail order/internet. The specific assumptions are:

[1] In principle more than one firm could locate at the centre. But the price competition would then produce zero (Bertrand paradox) profits at the centre. There is no loss of generality in assuming at most one firm will enter at the centre.

(a) At stage one of the three stage entry-pricing game, a firm can enter at the centre incurring a fixed cost G. At stage two, (symmetric) perimeter entry occurs at a fixed cost of F per firm, and all firms in the market choose prices simultaneously at stage three. The sequencing fits a scenario where the central firm is considering entry to a market served by High St. shops that can easily enter/exit the market by renting/ceasing to rent shop premises at cost F, while the mail order or internet provider needs to sink costs G on catalogues, warehouses, IT etc. In addition, the sequencing generates unique numbers of entering firms while other sequences do not in general (as we show in the appendix).

(b) There are a large number of two types of potential entrants to the market. One type has the technological knowledge to enter only at the centre at cost G, and the other type can enter only on the perimeter at cost F. The differential fixed costs seem natural. Also it seems reasonable that a potential High St. entrant may not have the know-how to set up a mail order or internet company, and vice versa.

However although we carry (a), (b) and the High St./mail order/internet application throughout the paper, our main results are qualitatively robust to the other possible scenarios. A web appendix addresses the cases where (alternative to (a)) the stage one/two entry sequencing is reversed or made simultaneous, and where (instead of (b)) there is just one type of potential entrant that can choose simultaneously whether to locate centrally or on the perimeter; in a later remark we argue that the latter scenario may fit better the alternative pharmaceutical/foodstuff example.

The paper is not the first to address the Salop circle with centre, but it is the first that focuses on the normative aspects. Also motivated by competition between High St. shops and mail order firms, Bouckaert (2000) has a stage one where a number of the identical potential entrants decide to incur a common fixed cost to enter the market and choose to locate centrally or on the perimeter, with a stage two of simultaneous price announcement. Balasubramanian (1998) studies the entry of a central firm into a market where the number of perimeter firms is fixed at the long run (Salop) equilibrium level. The author focuses on short-run aspects of competition (in terms of the effects on prices and profits) when direct marketers (i.e. the central firm) compete in prices with traditional retailers (i.e. the perimeter firms). In Cheng and Nault (2006) the central location can be taken by either one (of only two) incumbents located on the perimeter, or by a new entrant. The authors show that, if the fixed entry costs are identical for incumbent and entrant, the incumbent occupies the central location first. Otherwise, an entry cost advantage (smaller if the market is not initially covered) is necessary for the new entrant in order to enter the market and locate at the centre. In all these contributions there is no discussion of social optimality of the market outcomes, our main focus[2].

Finally, also related to our analysis is Anderson and De Palma (2000) who extend the standard Salop model in a different direction, introducing instead a second dimension of product differentiation on the circle (they call it *global differentiation*) as well as the spatial, with a parameter capturing the relative importance of these two types of differentiation. When differentiation is predominantly spatial, each firm competes with its immediate neighbours and the standard Salop model describes this *local* competition; at the other extreme there is *global* competition, with all firms competing symmetrically à la Dixit and Stiglitz (1977). The authors study the long run and short run equilibria, and their dependence on the degree of local/global competition. Among other results, they show that the long run number of firms that enter the market is always too high from a social point of view, as in Salop (1979). Anderson and De Palma (2000) is a completely symmetric model. Our model of the circle with centre is asymmetric: using their terminology, on the one hand, perimeter firms compete locally with the central firm and with neighbouring firms on the perimeter; on the other hand, the central firm (the global competitor) faces only global competition. Our results show that quite different kinds of market failure can then emerge.

The comparison described in our paper between the market outcome (the subgame perfect equilibrium of the above three stage game) and the social optimum depends on the transport cost and entry cost parameters, and naturally we replicate the standard Salop result when G is sufficiently large that neither the market nor the social optimum would provide centrally. Elsewhere in the parameter space, some new and quite striking conclusions emerge, including a reversal of the standard Salop excessive product differentiation result, and the possibility that the central firm is forced out of the market even though all perimeter goods are of inferior quality in the vertical differentiation sense.

The remainder of the paper is organized as follows. Section 2 outlines the model, section 3 looks at social optima, section 4 at market equilibria and section 5 compares the two. Section 6 concludes.

[2] Heal (1980) also uses the circle with centre geography, but in a quite different context. The (given) central firm is the sole producer of the good and the perimeter (retail) firms can buy from the centre and sell to consumers. Consumers may buy from the perimeter or directly from the centre. Because of economies of scale in transportation provision via the perimeter may be socially optimal. The market outcome depends on the size of the market, with large markets tending to over-provide retail outlets, and vice versa for small.

2. The model

The model has the following features:

- Consumers are uniformly distributed on the perimeter of a circle of length equal to one with density equal to one, and all consumers demand inelastically one unit of the good (i.e. it is assumed that the reservation price of all consumers, *v*, is very high).
- *N* firms locate symmetrically on the perimeter of the circle, while *c*=0 or 1 firms locate at the centre of the circle; all the *N*+*c* firms sell the same good, differentiated only by location.
- All consumers incur the same transportation cost $\delta \geq 0$ to buy from the centre[3].
- Consumers incur linear transportation costs *ty* to buy the good from a perimeter firm located at perimeter distance *y* from the consumer; without loss of generality let us assume that $t = 1$.
- *N* firms on the perimeter set prices $P_i \geq 0$, *i=1,…,N*.
- If *c=1* the central firm's price is $P_c \geq 0$.
- Production marginal costs are constant and, without loss of generality, normalized to zero.
- In order to enter the market, each perimeter firm incurs a fixed cost equal to *F*>0; a central firm has, instead, to incur the fixed entry cost $G > 0$.

We want to study a three stage game in which, at stage one, *c*=0 or 1 firms decide to enter the market (incurring fixed costs *G*) and locate at the centre of the circle, at stage two *N* perimeter firms decide to enter the market, incurring fixed costs *F*, and locate (by assumption) symmetrically on the circumference of the circle. In the final stage of the game, the *N*+*c* firms compete in prices à la Bertrand. We look for Subgame Perfect Nash Equilibria (SPNE) of this three-stage game as the market outcome (section 4) for comparison with the social optimum (section 3).

3. Social optima

A benevolent central planner must decide to set up either *c*=0 or *c*=1 central firms (*c*>1 will never be socially optimal given the constant marginal costs of production) and a number *N* of (symmetrically located) perimeter firms. For sufficiently large *v* $(\geq G+\delta, F+1/2)$ it will be socially optimal to provide one unit of the good to all consumers (*c*=0, *N*=0 will never be optimal), in which case

[3] We are not necessarily assuming, then, that the distance from the perimeter to a central firm is equal to the radius of the circle. δ represents the transportation costs or the disutility, identical for all consumers, that everybody has to incur buying from the homogeneous firm.

social welfare maximisation requires minimisation of the total cost function (i.e. the sum of the fixed costs of setting up firms and the transport cost of getting consumers to goods). With *c*=0 and *N* perimeter firms the total cost is given by the standard Salop cost formula $SC(N) = NF + 1/(4N)$.

If *c*=1 with *N* perimeter firms, a consumer located at distance *y* from the nearest perimeter firm would be optimally served by that firm rather than the central firm when $y \le \delta$. If $\delta > 1/(2N)$ it follows that all consumers should be served by perimeter firms and the total cost is $G + SC(N)$. On the other hand if $\delta \le 1/(2N)$ it will be optimal for the central firm to serve consumers whose $y \in [\delta, 1/(2N)]$, and the total transport cost of serving all consumers optimally is then:

(1) $$N\left[\delta^2 + \left(\frac{1}{N} - 2\delta\right)\delta\right] = \delta - N\delta^2$$

The first term in the left hand side square bracket is the transport cost incurred by consumers buying from one of the perimeter firms, and the second term is the transport cost for consumers located within distance $1/(2N)$ of that perimeter firm who buy from the central firm.

We can now define precisely the planner's problem, where the feasible set is: $I = \{(c,N) \ne (0,0)\}$.

Definition 1

A social optimum is $(c^s, N^s) \in I$ *such that* $TC(c^s, N^s) \le TC(c,N)$ *for all* $(c,N) \in I$ *where*

$$TC(c,N) = \begin{cases} SC(N) & \text{if } c = 0, N > 0 \\ G + \delta + N(F - \delta^2) & \text{if } c = 1, 0 \le N \le 1/(2\delta) \\ G + SC(N) & \text{if } c = 1, N \ge 1/(2\delta) \end{cases}$$

The minimum of the (strictly convex) function $SC(N)$ occurs at $N = (\sqrt{1/F})/2$ with value $SC = \sqrt{F}$. Obviously there can not be a social optimum with $c = 1, N \ge 1/(2\delta)$ since *G>0*. Thus the social optimum is either the above ($c = 0, N = (\sqrt{1/F})/2$ with $SC = TC = \sqrt{F}$) or at the minimum over

$N \in [0, 1/(2\delta)]$ of $G + \delta + N(F - \delta^2)$ with $c = 1$. If $F \le \delta^2$ the latter minimum value is $G + \delta/2 + F/(2\delta)$ ($N = 1/(2\delta)$ is then a minimizer), but $SC(1/(2\delta)) = \delta/2 + F/(2\delta)$ is smaller, implying that $TC(c, N)$ is minimized by $N = (\sqrt{1/F})/2$ and $c = 0$, i.e. the standard Salop optimum. If $F > \delta^2$ the total cost function is monotonically increasing in N ($N=0$ is then the minimizer) with minimum value equal to $G + \delta$; consequently if $G + \delta \le \sqrt{F}$ then the social optimum will be $c = 1$ and $N = 0$ whereas if $G + \delta > \sqrt{F}$ the social optimum would be given again by $c = 0, N = \sqrt{1/F}/2$. Defining $\varphi(F, \delta) = \sqrt{F} - \delta$, this proves:

Theorem 1
The social optima are:
(a) $c^s = 0, N^s = \frac{1}{2}\sqrt{1/F}$ *either if* $F \le \delta^2$ *or if* $F > \delta^2$ *and* $G \ge \varphi(F, \delta)$*;*
(b) $c^s = 1, N^s = 0$ *if* $F > \delta^2$ *and* $G \le \varphi(F, \delta)$.
Figure 1 illustrates.

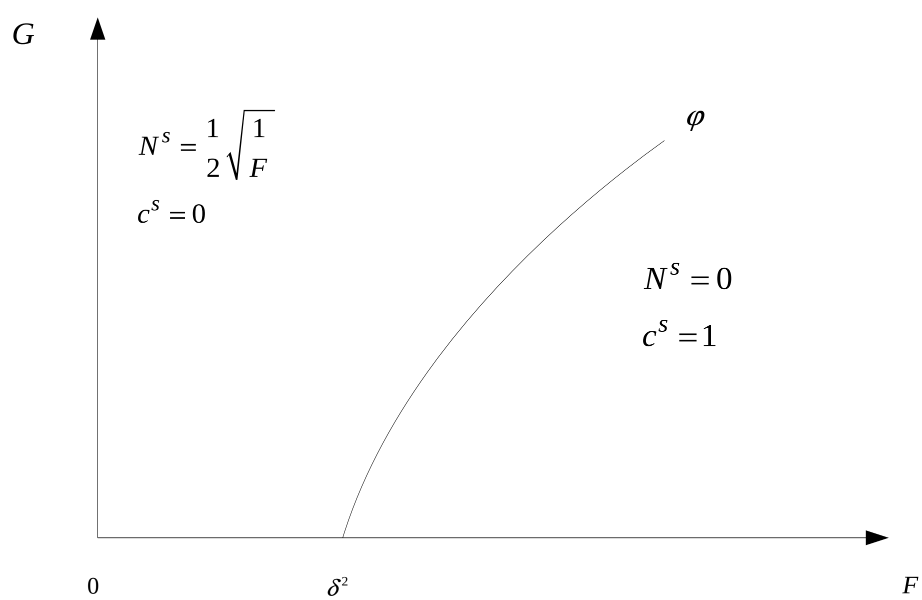

Figure 1: social optima.

The social optimum is dichotomous: the central planner prefers all consumers to be served either by the central firm or by the perimeter firms. The dichotomy of the social optimum does not depend on the assumption of linear transportation costs on the perimeter. In fact, for any strictly convex transportation cost function the optimal portion of the market that should be served by the generic perimeter firm *i* does not depend on *N*, but only on δ. As a result, given the symmetric location of the perimeter firms, *TC(1,N)* is linear in *N* (when $N \leq 1/(2\delta)$) and the dichotomy of the social optimum follows.

4. Market equilibrium

Our ultimate interest is in how the market mechanism performs in relation to the social optimum. We will compare the social optimum with the following market equilibrium:

Definition 2

A market equilibrium is a subgame perfect Nash equilibrium of the following three stage game:

Stage one: c=0 or c=1 central firms enter the market, incurring fixed costs G>0.

Stage two: N perimeter firms enter the market, each incurring fixed costs F>0.

Stage three: the N+c firms choose prices simultaneously ($P_i, i = 1,...,N$ *for the perimeter firms,* P_c *for the central firm when c=1) and meet the demands of consumers wishing to buy from them.*

Our exposition will be abbreviated by assuming that the consumers' reservation price *v* is sufficiently high that all consumers do buy one unit of the good in all subgames[4]. In addition, following previous literature on both the standard Salop model (e.g. Tirole (1988)) and the Salop circle with centre (e.g. Bouckaert (2000)) we assume that a sufficiently high number of perimeter firms is in the market such that the zero profit condition can be used to identify the equilibrium number *N* at stage two[5].

[4] To go beyond this assumption one would need to admit some quite different types of equilibria, including local monopolies and multiple (asymmetric) kinked equilibria. Mérel and Sexton (2010) describe price equilibria in a classical Hotelling duopoly for every possible value of *v* and provide a refinement (i.e. they consider the case of a finite price elasticity of demand) that ensures uniqueness and symmetry of the equilibrium.

[5] We are abstracting from the integer constraint. It can be shown that a discrete treatment of *N* would not change qualitatively the main results of the model.

The backward induction derivation of market equilibria starts with analysis of stage three price subgame equilibria. We restrict attention to equilibria which are perimeter firm symmetric ($P_i = P$, $i = 1,...,N$), and where $N \geq 2$ (to be justified by a later parameter restriction). When c=0 and $N \geq 2$ a consumer at distance x from perimeter firm i faces a full cost of $P_i + x$ of buying from firm i, and comparison of these full costs over i=1,…, N leads to consumer demands and firm payoffs as in the standard Salop model. Specifically the consumer indifferent to buy from either of two neighbouring competitors on the perimeter is located at $x = \left(p_j - p_i\right)/2 + 1/2N$, $i = 1,...,N, j \neq i$, where $p_i + x = p_j + \left(1/N - x\right)$. Firm i's demand and profits would be given respectively by $2x$ and $P_i 2x$. The exact derivation of the following subgame equilibrium, where $\Pi\left(0,N\right)$ denotes the (symmetric) profit of a perimeter firm, can be found in Tirole (1988).

Lemma 1

If c=0 and $N \geq 2$ *the stage three subgame equilibrium prices and profits are* $P = 1/N, i = 1,...,N$ *and* $\Pi\left(0,N\right) = 1/N^2$.

When c=1 and $N \geq 2$ consumers can also buy from a central firm at full cost $P_c + \delta$. Comparison of the full cost for consumers of buying from perimeter or central firms leads to demands, payoffs and the following subgame equilibria, where $\Pi_c\left(1,N\right)$ is the central firm profit and $\Pi\left(1,N\right)$ is the (again symmetric) perimeter firm profit[6]. Specifically the consumer indifferent to buy from the closest perimeter firm (say firm i) and firm c is at a distance from firm i of $z = P_c + \delta - P_i$.

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[6] In fact with $N = 2$ and $\delta \in \left[3/7, 3/4\right]$ there is a continuum of asymmetric equilibria in which $P_c = 0$ and P_1, P_2 differ. This is similar to the asymmetric multiple equilibria described in the Hotelling framework by Mérel and Sexton (2010). Restricting attention to the unique perimeter symmetric stage three equilibria allows us to avoid any equilibrium selection issue that would otherwise arise.

Lemma 2

If c=1 and $N \geq 2$ *the stage three subgame Nash equilibrium prices and profits are:*

(i) Regime 1; if $N < 1/\delta$ *then*

$$P = \frac{1}{6N} + \frac{\delta}{3},\ P_c = \frac{1}{3N} - \frac{\delta}{3}$$
$$\Pi(1,N) = \frac{1}{18N^2}(1+2N\delta)^2,\ \Pi_c(1,N) = \frac{2}{9N}(1-N\delta)^2 \tag{2}$$

(ii) Regime 2; if $1/\delta \leq N \leq 3/(2\delta)$ *then*

$$P = \delta - \frac{1}{2N},\ P_c = 0$$
$$\Pi(1,N) = \frac{\delta}{N} - \frac{1}{2N^2},\ \Pi_c(1,N) = 0 \tag{3}$$

(iii) Regime 3; if $3/(2\delta) < N$ *then*

$$P = \frac{1}{N},\ P_c \in [0,\infty)$$
$$\Pi(1,N) = \frac{1}{N^2},\ \Pi_c(1,N) = 0 \tag{4}$$

Proof

See appendix.

There are thus three types of stage three subgame equilibrium. In regime 1 (small *N*), the central and perimeter firms obtain positive market share and profits. In regimes 2 and 3 all market share goes to perimeter firms; in regime 2 (*N* in a middle range) the presence of a central firm (although it ends up with zero market share offering to sell at marginal cost) still generates competitive pressure on the perimeter firms, producing prices below the standard Salop level. Competition from the centre disappears in regime 3 (large *N*) leading[7] again to the standard Salop outcome of Lemma 1.

[7] Following Bain's (1956) terminology, Lemma 2 describes when the central firm's entry would be deterred (in regime 2 the perimeter firms' price selection does not accommodate strategically the central firm in the market) and blockaded (in regime 3 the presence of a central firm does not affect the perimeter firms' price selection; maximizing their profits, following Salop's (1979) terminology the perimeter firms create an *innocent* entry barrier for the central firm).

For backward induction to the stage two subgames, we restrict attention to the parameter set $S = \left\{ (F,\delta) \middle| 0 \le \delta < 1/2, \frac{1}{18}\left(\frac{1}{2}+2\delta\right)^2 \ge F > 0 \right\}$.

The first inequality rules out the uninteresting case where regime 1 disappears in all stage three continuations; the second inequality ensures $N \ge 2$ perimeter entrants, as remarked earlier. Now it is easy to see that $\Pi(c,N)$ are continuous, strictly decreasing functions of *N*, so the zero profit condition $\Pi(c,N) = F$, $c = 0,1$ defines a unique number of stage two subgame equilibrium entrants, as follows:

(a) if *c=0*, $N = N_0 = \sqrt{1/F}$ (the standard Salop number)

(b) if *c=1*,

(i) regime 1: $N = N_1 = \left(3\sqrt{2F} - 2\delta\right)^{-1}$ if $F > \frac{1}{2}\delta^2$

(ii) regime 2: $N = N_2 = \frac{1}{2F}\left(\delta + \sqrt{\delta^2 - F}\right)$ if $\frac{1}{2}\delta^2 \ge F \ge \frac{4}{9}\delta^2$

(iii) regime 3: $N = N_3 (= N_0) = \sqrt{1/F}$ if $\frac{4}{9}\delta^2 > F$.

Notice that for parameters belonging to set *S* the parameter subsets that ensure the existence of the above regimes are not empty[8].

Anticipating such numbers of perimeter entrants at stage two, the central firm decides at stage one to enter or not, giving:

Theorem 2

For parameters $(F,\delta) \in S$, *the market equilibria are:*

(a) *c=0*, $N = N_0$ *if* $F \le \frac{1}{2}\delta^2$ *or if* $F > \frac{1}{2}\delta^2$ *and* $G \ge \psi(F,\delta)$

(b) *c=1*, $N = N_1$ *if* $F > \frac{1}{2}\delta^2$ *and* $G \le \psi(F,\delta)$

where $\psi(F,\delta) = 2\frac{\left(\sqrt{2F} - \delta\right)^2}{3\sqrt{2F} - 2\delta}$.

Proof

See appendix.

Figure 2 illustrates.

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[8] This is true since $\delta^2/2 < (1/2 + 2\delta)^2/18$ for all $\delta \in [0,1/2]$.

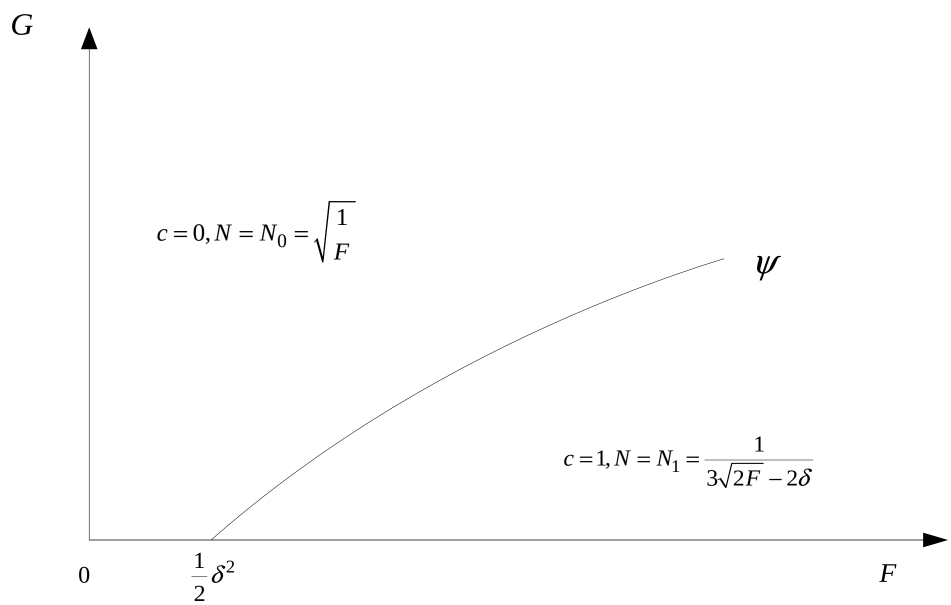

Figure 2: market equilibrium.

Not surprisingly, for sufficiently high fixed costs for the central firm and sufficiently low perimeter firm fixed costs, no central provider would enter the market and the market equilibrium outcome is the same as in the standard Salop model. On the other hand, for sufficiently low *G* and high *F* both types of firm enter in the market equilibrium.

5. Comparisons

We now compare the market equilibria and the social optima and present the main results of the model.

First note that the curves $\varphi(F,\delta)$ and $\psi(F,\delta)$ in Figure 1 and 2 intersect where $F=b=(3+2\sqrt{2})\delta^2$ $(\simeq 5.8\delta^2)$, and that this intersection lies in S only if $\delta<\Delta=(6\sqrt{6+4\sqrt{2}}-4)^{-1}$ $(\simeq 0.06)$. To enrich the set of interesting outcomes we assume the intersection does lie in *S*, and so focus on the parameter set $\hat{S}=\{(F,\delta)\in S \mid \delta<\Delta\}$, in which case we have Figure 3 (the significance of $F=e$ is described later).

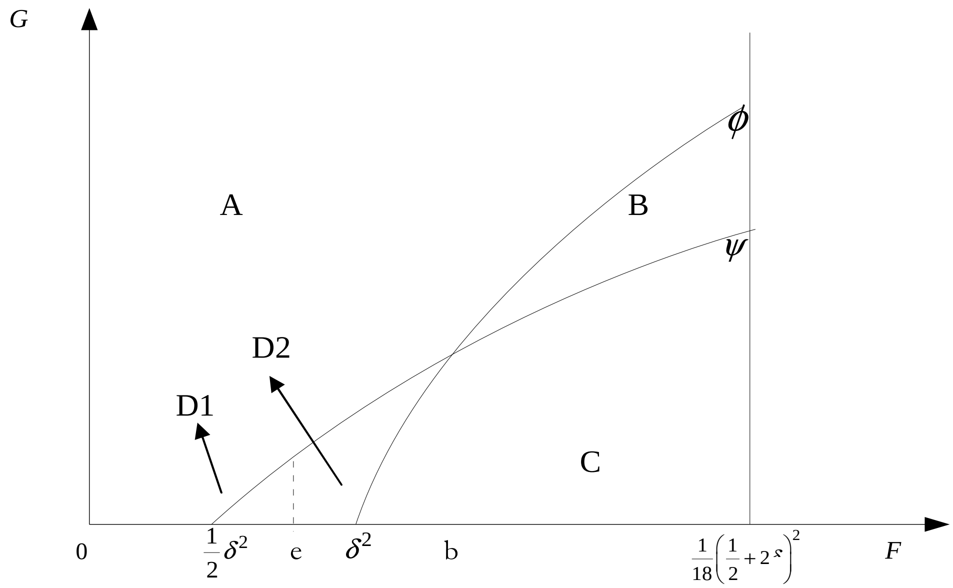

Figure 3: social optima and market equilibria.

In region A (*F* low, *G* high) there is no central firm in either the market equilibrium or the social optimum, and the number of perimeter firms in the market equilibrium is excessive compared to the social optimum. In other words, the model replicates the standard Salop result; the interesting variations are in region *B*, *C* and *D*.

In region *B* (*F* high, *G* intermediate) the market equilibrium involves provision only by perimeter firms, whereas the central firm should be the sole provider in the social optimum. The equilibrium again involves too many perimeter firms, as in region *A*, but now in a most extreme way – the optimal number should be zero. The market mechanism forces out the central, homogenous good, leaving only differentiated products, when it should do the opposite.

Region *C* (*F* high, *G* low) sees the coexistence of both central and perimeter firms in the market equilibrium. Compared with region *B* the lower *G* makes entry to the centre profitable; but like region *B* the social optimum continues to demand only central provision. The market mechanism creates too many differentiated product firms, when again this number should be zero, but now the market forces at least allow some central homogeneous good provision also; in a sense the market failure in region *C* is milder than in *B*.

In region $D = D_1 \cup D_2$ (*F* intermediate, *G* low), like region *C*, both types of firm coexist in the market equilibrium. But the relatively low value of *F* now implies that only perimeter firms should operate for social optimality. Despite the fact that the central firm survives the market forces when it should not, it is still possible that the equilibrium number of perimeter firms may be too high (as it is in the standard Salop model). A straightforward calculation shows that this is the case when F is in region *D1*, namely $F \leq e, e \equiv 4\delta^2 / \left(3\sqrt{2}-2\right)^2$, as shown in Figure 3. On the other hand in region *D2* since $F > e$, the equilibrium number of perimeter firms is too low – there is not enough product differentiation provided by the market, thus reversing the standard Salop model conclusion.

It is interesting to note that in the limit where $\delta = 0$ the homogenous central good dominates in the vertical differentiation sense all differentiated products. In this limit, regions *D1* and *D2* disappear, but *A*, *B* and *C* survive, and perhaps the most striking of our market failures then occurs in region *B*. Here the market mechanism induces the provision of a variety of horizontally differentiated goods which are of uniformly inferior quality (in the vertical differentiation sense) to the homogenous good, which is not provided at all by the market although it would be the only good provided in the social optimum. In all cases in this limit (*A*, *B* and *C*) market provision of the differentiated products is excessive.

Whilst $\delta = 0$ is an idealisation of any real scenario, it is credibly the case where δ is relatively small that provides relevant lessons for our example of internet/mail order shopping (centre) versus High Street shopping (perimeter), since for many nowadays internet access and ordering by mail is a very low cost activity compared to a High Street shopping excursion. It follows that a broad lesson from our analysis is that the nature of the competition between internet/mail order and High Street shops favours the High Street, in the sense that the market allows the survival of an excessive number of such shops compared to the social optimum, and works against the internet/mail order possibly precluding such provision completely when it should be the only source of the commodity, and when it is of uniformly higher quality than any High Street alternative.

Remark

The focus has been on the High Street shops/internet/mail order application, and the resulting (a), (b) assumptions of the introduction. A web appendix shows that the same, novel market failure seen in regions *B*, *C* and *D* above continues to appear also if one changes the (a), (b) assumptions so that there is alternative entry sequencing (either simultaneous, or perimeter at stage one and centre at stage two) or just one type of potential entrant that can choose simultaneously to enter at the centre (cost *G*) or on the perimeter (cost *F*). In particular, the alternative applications (i.e. basic and niched pharmaceuticals or foodstuffs) suggested in the introduction might be better captured by a model of simultaneous entry by just one type of potential entrant. The results of the web appendix show that such a model can generate qualitatively the same market failures as above. In particular, with F high and G not too high (like regions B and C above), the market would produce too many niched products in an extreme way (i.e. when this number should be zero); alternatively, with intermediate costs (like region D_2), the market may lead to too little niche provision.

6. Conclusions

In the geography of a Salop circle with centre, where firms providing horizontally differentiated goods locate around the perimeter of the circle and where a firm providing a homogenous good may locate at the centre of the circle, we have studied and compared the social optima and the market equilibria, the latter as the subgame perfect equilibria of an entry-pricing game. In the limit where the cost of consumer access to the central firm is zero the nature of the market failure is particularly striking. In this case the good provided by the central firm is vertically differentiated and of superior quality to all goods provided on the perimeter. But the market always produces too much product differentiation, and sometimes it leads to provision only by perimeter firms when (from the social point of view) the central firm should be the sole provider.

We have motivated the model with an application to competition between internet/mail order (centre) and High Street (perimeter) shops, in which case the ease of internet/mail order shopping access suggests that the limiting case of the above paragraph may be a reasonable approximation. In this context our results indicate that the market mechanism favours the High Street, always producing too many High Street shops, and sometimes forestalling internet/mail order provision when this should be the only form of provision.

When the cost of consumer access to the central firm is non-zero a further possibility emerges, namely the central firm enters the market when it should not, reducing the number of perimeter firms possibly to a level which is below the socially optimal number. This last possibility reverses the excessive market provision of product differentiation in the standard Salop model (where there is no central firm possibility), whilst the earlier possibilities produce more extreme versions of the standard Salop market failure.

We have suggested further applications to competition between basic pharmaceuticals (e.g. paracetamol) or foodstuffs (e.g. corn flakes) and differentiated variations which mix in further ingredients to serve specific symptoms or tastes. Although the model would be a little different, we have argued that qualitatively similar conclusions emerge. Finally, a potential application which needs further research is to the market for secondary hospital care. Recent literature[9] applies the Salop model to study competition in quality and (sometimes regulated) prices among hospitals located around the perimeter of the circle (where location may represent the specialization of each hospital). This literature could be enriched by allowing the presence in the market of a homogenous alternative, i.e. a general hospital (which, unlike here, may not be a private firm), that provides a full range of medical services. Such an extension of our model would be an interesting topic for future research and would shed some light on competition between general hospitals and speciality clinics, a topical issue currently[10].

Appendix

A. Proofs

Proof of Lemma 2

Let us consider the best price response function of the central firm to $P_i = P$, $i = 1,..,N$. The profit function of c is given by[11]:

$$(5) \qquad \Pi_c = \begin{cases} P_c & if \quad P_c \in [0, P-\delta] \\ P_c\left[1-2N\left(P_c - P + \delta\right)\right] & if \quad P_c \in \left[P-\delta, P-\delta+\dfrac{1}{2N}\right] \\ 0 & if \quad P_c \in \left[P-\delta+\dfrac{1}{2N}, \infty\right) \end{cases}$$

In (5), Π_c is defined as a continuous, piecewise concave function of P_c: linear and increasing for $P_c \in [0, P-\delta]$, strictly concave and quadratic for

[9] See Brekke et al. (2008a,b).

[10] Competition between general hospitals and specialized doctor-owned facilities has been at the centre of a heated debate in USA. See Federal Trade Commission, “Improving health care: a dose of competition”, 17, 2004.

[11] For low levels of P_c (i.e. $P_c \in [0, P-\delta]$) the central firm serves the whole market. For values of P_c in an intermediate range (i.e. $P_c \in \left[P-\delta, P-\delta+1/(2N)\right]$) the central firm shares the market with the rivals on the perimeter and serves a portion of consumers equal to $(1-2Nz)$, where $z = P_c - P + \delta$ as defined in the text. For high levels of P_c (i.e. $P_c \in \left[P-\delta+1/(2N), \infty\right)$) the central firm faces a demand equal to zero.

$P_c \in \left[P-\delta, P-\delta+1/(2N)\right]$ and constant ($\Pi_c = 0$) for $P_c \in \left[P-\delta+1/(2N), \infty\right)$. The stationary point of the quadratic is $P_c = \left(P-\delta+1/(2N)\right)/2$ and will be the global maximizer of Π_c if it belongs to $\left(P-\delta, P-\delta+1/(2N)\right)$, i.e. if $P \in \left(\delta-1/(2N), \delta+1/(2N)\right)$.

If $P \in \left[0, \delta-1/(2N)\right]$ then $\Pi_c = 0$ for all $P_c \geq 0$ and any $P_c \in [0, \infty)$ is a global maximizer. If $P \in \left[\delta+1/(2N), \infty\right)$, the quadratic stationary point belongs to $[0, P-\delta]$ and there is a kink from positive to negative slope in the Π_c graph at $P_c = P-\delta$, which will then be the global maximizer.

Thus the best response mapping for firm c is given by:

$$\text{(6)} \qquad P_c = \begin{cases} P-\delta & \text{if} \quad P \geq \delta + \dfrac{1}{2N} \\ \dfrac{1}{2}\left(P-\delta+\dfrac{1}{2N}\right) & \text{if} \quad \delta - \dfrac{1}{2N} \leq P \leq \delta + \dfrac{1}{2N} \\ [0, \infty) & \text{if} \quad P \leq \delta - \dfrac{1}{2N} \end{cases}$$

Turning to perimeter best responses, we now claim that, given P_c, $P_i = P$, $i = 1, \ldots, N$ will be a best response for each perimeter firm if:

$$\text{(7)} \qquad P = \begin{cases} \dfrac{P_c+\delta}{2} & \text{if} \quad P_c \in \left[0, \dfrac{1}{N}-\delta\right) \\ P_c + \delta - \dfrac{1}{2N} & \text{if} \quad P_c \in \left[\dfrac{1}{N}-\delta, \dfrac{3}{2N}-\delta\right] \\ \dfrac{1}{N} & \text{if} \quad P_c \in \left(\dfrac{3}{2N}-\delta, \infty\right) \end{cases}$$

Let us first start considering the best response to low values of P_c. Suppose that $P_c \in [0, 1/N-\delta)$ and $P_j = (P_c+\delta)/2,\ j \neq i$. If $P_i = P = (P_c+\delta)/2$ we have a price configuration in which both central and perimeter firms obtain a positive market share, i.e. $0 \leq P_c - P + \delta \leq 1/(2N)$.

The profits of firm i are given by $\Pi_i = P_i 2(P_c + \delta - P_i)$ as long as $P_i \in [3(P_c + \delta)/2 - 1/N, P_c + \delta] = [3P - 1/N, 2P]$. Notice that Π_i is strictly concave with stationary point $P_i = (P_c + \delta)/2 = P \in [3(P_c + \delta)/2 - 1/N, P_c + \delta]$. Certainly, firm i would not increase its price beyond $P_c + \delta$ because doing so it would lose all its demand. If instead firm i chooses $P_i \in [0, 3(P_c + \delta)/2 - 1/N]$ it would compete only with the closest rivals on the perimeter and obtain demand equal to $D_i = (P_c + \delta)/2 + [1/(N)] - P_i$ and profits (strictly concave) equal to $\Pi_i = P_i D_i$. Notice that $\partial \Pi_i / \partial P_i \geq 0$ at $P_i = 3(P_c + \delta)/2 - 1/N$ (i.e. the highest price firm i can charge to compete with the closest rivals on the perimeter) if $P_c \leq 6/(5N) - \delta$ (satisfied since $0 \leq P_c \leq 1/N - \delta$). This proves that $P = (P_c + \delta)/2$ is a best response to $P_c \in [0, 1/N - \delta)$.

Let us consider now the best response to intermediate values of P_c. Suppose that $P_c \in [1/N - \delta, 3/(2N) - \delta]$ and $P_j = P_c + \delta - 1/(2N)$, $j \neq i$.

If $P_i = P = P_c + \delta - 1/(2N)$ we have a price configuration in which only the perimeter firms obtain a positive market share, i.e. $P_c - P + \delta = 1/(2N)$. Surely firm i would not choose a price $P_i \geq P_c + \delta$ because doing so it would loose all demand. However, choosing a price $P_i \in (P_c + \delta - 1/(2N), P_c + \delta) =$

$= (P, P + 1/(2N))$ firm i would directly compete only with the central firm, earning demand $D_i = 2(P_c + \delta - P_i)$ and profits (strictly concave) equal to $\Pi_i = P_i D_i$. Notice that $\partial \Pi_i / \partial P_i \leq 0$ at $P_i = P$ since $P_c \geq 1/N - \delta$. Deviating to a price $P_i \in [0, P_c + \delta - 1/(2N)] = [P - 1/N, P]$ firms i competes directly with the closest rivals on the perimeter earning demand $D_i = P_c + \delta - P_i + 1/2N$ and profits (strictly concave) equal to $\Pi_i = P_i D_i$. Notice that $\partial \Pi_i / \partial P_i \geq 0$ at $P_i = P$ since $P_c \leq 3/2N - \delta$. This proves that $P = P_c + \delta - 1/(2N)$ is a best response to $P_c \in [1/N - \delta, 3/(2N) - \delta]$.

Let us finally consider the best response to high values of P_c. Suppose that $P_c \in [3/2N - \delta, \infty)$ and $P_j = 1/N$, $j \neq i$. If $P_i = P = 1/N$ we have again a price configuration in which only perimeter firms obtain a positive market share, i.e. $P_c - P + \delta > 1/(2N)$.

Let us focus first on the case in which in particular $P_c \in \left[3/(2N)-\delta, 2/N-\delta\right]$. Note that for $P_i \in \left[0, 2(P_c+\delta)-2/N\right]$ firm i is competing directly with its closest rivals on the perimeter and earns demand equal to $D_i = 2/N - P_i$ and strictly concave profits $\Pi_i = P_i D_i$ with stationary point $P_i = P = 1/N$.

For $P_i \in \left[2(P_c+\delta)-2/N, P_c+\delta\right]$ firm i is instead competing directly with the central firm and earning demand $D_i = 2(P_c+\delta-P_i)$ and profits (strictly concave) equal to $\Pi_i = P_i D_i$. Notice that $\partial\Pi_i/\partial P_i \leq 0$ at $P_i = 2(P_c+\delta)-2/N$ (i.e. the lowest price that firm i has to choose in order to compete directly with the central firm) if $P_c \geq 4/3N-\delta$ (ensured since $P_c \in \left[3/(2N)-\delta, 2/N-\delta\right]$). Clearly $P_i \geq P_c+\delta$ can not be a best response because in such a case firm i would earn zero profits.

For $P_c \geq 2/N-\delta$ and $P = 1/N$ consumers are never willing to buy from the centre and firm i can only compete with the rival on the perimeter. The profits of firm i are given by $\Pi_i = P_i 2\left(P - P_i + 1/(2N)\right)$ when $P_i \in \left[0, P+1/N\right]$. Notice that Π_i is strictly concave with stationary point $P_i = 1/N = P$.

It is obvious that it can not be profitable for firm i to choose a price $P_i \geq P + 1/N = 2/N$ because it would generate zero profits. This proves that $P = 1/N$ is a best response to $P_c \in \left(3/(2N)-\delta, \infty\right)$.

The best responses described in (6) and (7) are graphically represented in Figures A1-A3. The thin solid lines and the grey area in the figures represent the best response of the central firm for any symmetric price selected on the perimeter (BR_c). The bold solid lines represent the best response of the perimeter firms when they move symmetrically to every price chosen by the central competitor (BR_p).

For $2 \leq N < 1/\delta$ (where BR_c and BR_p are as described in Figure A1) the unique intersection of the best response function of the perimeter firms and the best response mapping of the central firm produces the configuration in which all $N+1$ firms serve a positive portion of the market. We call this case Regime 1 in the statement of Lemma 2.

For $3/2\delta \geq N \geq 1/\delta$ (where BR_c and BR_p are as described in Figure A2) the unique intersection of the best responses produces the price configuration in which the whole market is just covered by the perimeter firms. We call this case Regime 2 in the statement of Lemma 2.

Finally, for $N > 3/2\delta$ (where BR_c and BR_p are as described in Figure A3) the perimeter firms' best response lays on the area of the graph that represents the best response of the central firm. Here we have a multiplicity of best price responses for the central firm, but a unique corresponding payoff ($\Pi_c = 0$). We call this case Regime 3 in the statement of Lemma 2.

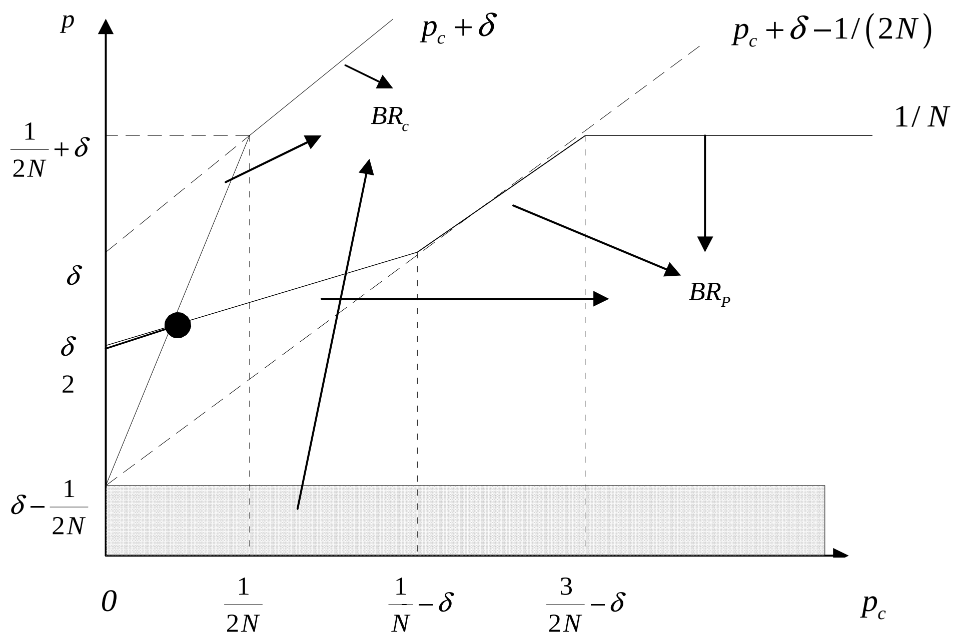

Figure A1: Best Responses when $0 \leq N < 1/\delta$ (Regime 1)

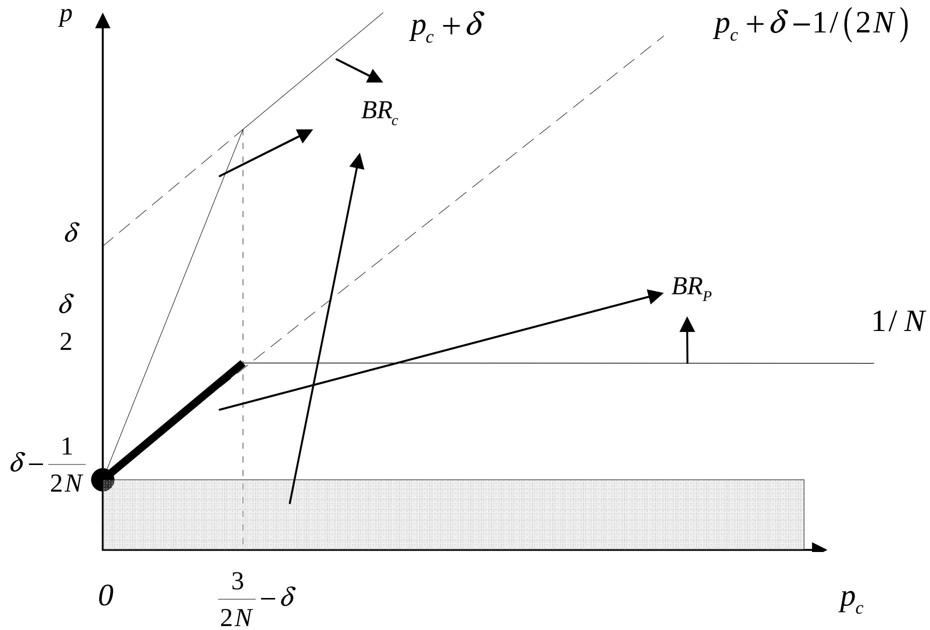

Figure A2: Best Responses when $1/\delta \le N \le 3/(2\delta)$ **(Regime 2)**

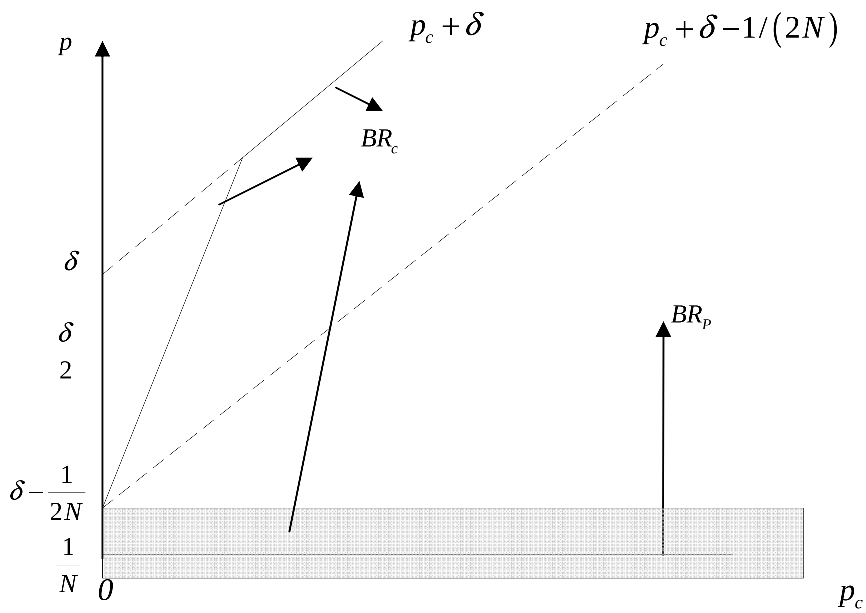

Figure A3: Best Responses when $N > 3/(2\delta)$ **(Regime 3)**

Q.E.D.

Proof of Theorem 2

(a) If *c=0* at stage one then $N = N_0$ in the stage two continuation. If *c=1* at stage one and $F \le \delta^2 / 2$ then the stage two continuation is regime 2 or 3 and $\Pi_c = 0$, so the central firm will not enter at stage one. If c=1 and $F > \delta^2 / 2$ then the stage two continuation is regime 1 and the central firm will not enter iff $\Pi_c = \Pi_c(1, N_1) = 2(1 - N_1\delta)^2 / 9N_1 \le G$ (substituting $N_1 = \left(3\sqrt{2F} - 2\delta\right)^{-1}$) when holds iff $G \ge \psi(F, \delta)$.

(b) If c=1 at stage one and $F > \delta^2 / 2$ then the stage two continuation is regime 1 with $N = N_1$, and $\Pi_c = \Pi_c(1, N_1) \ge G$ (reversing the argument in (a)) iff $G \le \psi(F, \delta)$. So the central firm does enter at stage one then. If $F \le \delta^2 / 2$, as in (a), the central firm does not enter. *Q.E.D.*

B. Extensions

The text makes use of two main assumptions. First sequential entry is considered in which the central firm decides at stage one to enter and the perimeter firms decide upon entry at stage two of the game. Second, it is assumed that firms belong to two different types: one type has the know-how to enter and locate only at the centre, while the other type has the ability to enter and locate only on the perimeter.

The purpose of this section is to show that changes to these main assumptions do not affect qualitatively the results reported in the text. Subsection B1 looks at changes to the sequential entry assumption. Subsection B2 reverts to the assumption that there is just one type of firm that can choose, simultaneously, to locate at the centre or on the perimeter; the remark in the text argues that the Subsection B2 scenario fits best the pharmaceutical/foodstuffs examples.

B1. Alternative entry sequencing

The game with the entry sequencing reversed from that of the text is as follows:

(i) Stage one. *N* perimeter firms, incurring fixed costs $F > 0$, enter the market and locate symmetrically on the perimeter of the circle.

(ii) Stage two. The central firm decides to enter the market ($c = 1$) incurring fixed costs $G > 0$, or not ($c = 0$).

(iii) Stage three. The *N*+*c* firms compete simultaneously in prices.

We solve the game by backward induction. Clearly reversing the entry sequencing does not affect the analysis of the final stage of the game that is still described by Lemmas 1 and 2 in the text. Again, the market produces three types of subgame equilibrium, depending on the number of perimeter firms that entered the market. In particular, only for $N < 1/\delta$ (Regime 1) can the central firm (if it enters) earn a positive market share, as described in the text.

Backward induction to stage two produces the following subgame equilibria:

(A1) $c = 0$ if $N < 1/\delta$ and $2(1-N\delta)^2/9N < G$, or if $N \geq 1/\delta$

(A2) $c = 1$ if $N < 1/\delta$ and $2(1-N\delta)^2/9N \geq G$

Restricting attention[12] again to parameter set $\ddot{S}$, backward induction to stage one produces the following subgame perfect equilibrium:

Proposition B.1

For parameters $(F,\delta) \in S$, the market equilibria are:

(a) $N = N_0, c = 0$ if $F \leq \delta^2$ or if $F > \delta^2$ and $G > \sigma(F,\delta)$

(b) $N = N_1, c = 1$ if $F > \delta^2/2$ and $G \leq \psi(F,\delta)$

where $\sigma(F,\delta) = \dfrac{2}{9}\sqrt{F}\left(1-\delta\sqrt{\dfrac{1}{F}}\right)^2$ *and* $\psi(F,\delta) = 2\dfrac{\left(\sqrt{2F}-\delta\right)^2}{3\sqrt{2F}-2\delta}$.

Proof

(a) If there is to be a subgame perfect (market) equilibrium in which $c = 0$ at stage two, the zero-profit perimeter entry condition at stage one requires that $N = N_0 = \sqrt{1/F}$. From (A1), either $N_0 < 1/\delta$ and $2(1-N_0\delta)^2/9N_0 < G$ (which become $F > \delta^2$ and $G > \sigma(F,\delta)$, respectively), or $N_0 \geq 1/\delta$ (i.e. $F \leq \delta^2$), as stated in (a).

(b) Similar to (a) the market equilibrium with $c = 1$ requirement is now that $N = N_1 = \left(3\sqrt{2F}-2\delta\right)^{-1}$ with, from (A2), $N_1 < 1/\delta$ and $2(1-N_1\delta)^2/9N_1 \geq G$ which are, respectively, $F > \delta^2/2$ and $G \leq \psi(F,\delta)$, as required. Q.E.D.

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[12] Restricting attention to set $\ddot{S}$ ensures the survival of interesting equilibria in which both types of firms enter (with $N \geq 2$) and are active in the market.

Figure B.1 illustrates the market equilibria .

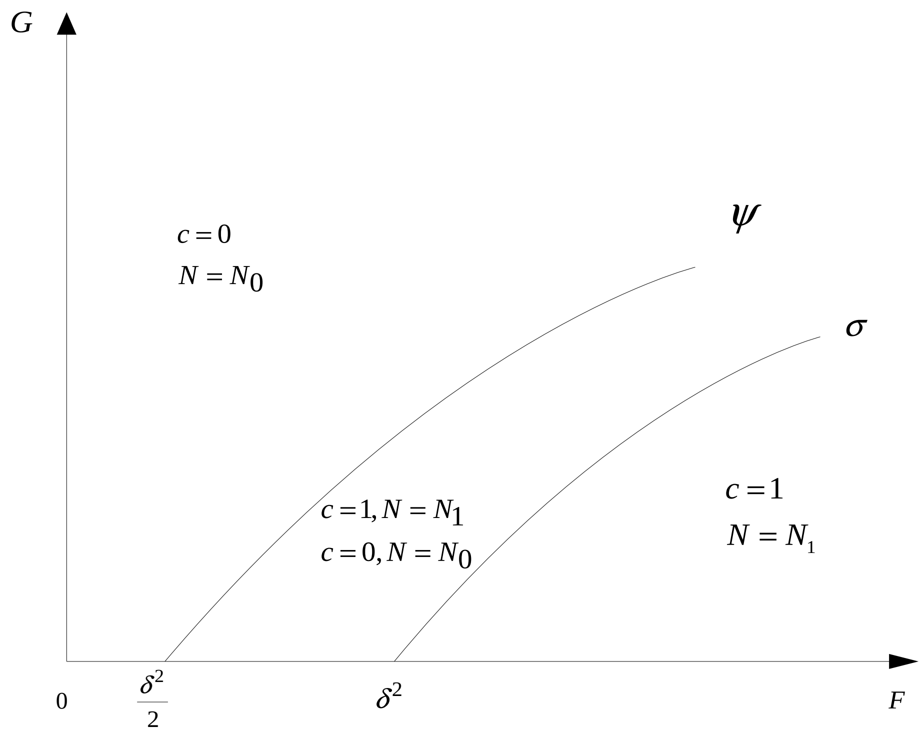

Figure B.1: market equilibrium with reversed entry.

For combinations of fixed costs lying between ψ and σ in Figure B.1 the equilibrium can be given either by $N = N_0, c = 0$ or $N = N_1, c = 1$, depending on the expectation of the perimeter firms at stage one of the game. Thus there are now multiple market equilibria at these parameters. However the market equilibrium which was the unique equilibrium in the text ($c = 1, N = N_1$ – see Figure 2) continues to exist. It follows that the same market equilibrium/social optimum comparisons and the same market failures continue to exist with the reversed entry sequencing. Note that the same equilibrium described by proposition B.1 and hence the same market failures would be generated also if the N+c firms were allowed to enter simultaneously[13].

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[13] Specifically the game with simultaneous entry would be as follows. In stage one N+c firms would enter the market. In stage two the N+c firms compete simultaneously in prices, earning profits as described in Lemmas 1 and 2. Backward induction to stage one produces market equilibrium c=1, N=N_1 if condition A2 above is satisfied (it translates into $G \leq \psi$ and $F > \delta^2/2$). The

B.2. One type of potential entrant

In proposition B.1 and in the text, we assumed that there are two types of firms, i.e. firms with the technology to enter and locate (only) at the centre and firms that the have the knowledge to enter and locate (only) on the perimeter. We now assume that all potential entrants can simultaneously choose between central entry (at cost *G*) or perimeter entry (at cost *F*). Firms' payoffs at the last stage of the game are given again by profit functions from Lemmas 1 and 2:

(B1) $\Pi(1,N)=\frac{1}{18N^2}(1+2N\delta)^2$, $\Pi_c(1,N)=\frac{2}{9N}(1-N\delta)^2$ if $c=1$ and N firms are located on the perimeter.

(B2) $\Pi(0,N)=1/N^2$, $\Pi_C=0$ if $c=0$ and N firms are located on the perimeter.

The configuration in (B1) is a market equilibrium if the zero-profit condition on the perimeter defines the number of perimeter firms (i.e. $N=N_1$) and a firm has the incentive to locate at the centre and not to locate instead on the perimeter. Clearly no perimeter firm would choose to locate at the centre when *c*=1 due to standard Bertrand competition.

Sufficient conditions that ensure that $c=1$ and $N=N_1$ is a market equilibrium are:

(B3) $N_1<1/\delta \Rightarrow F>\delta^2/2$ and $\Pi_C(1,N_1)\geq G$ and

(B4) $\Pi_C(1,N_1)-G\geq 1/(N_1+1)^2-F$.

(B3) is satisfied if $G\leq\psi(F,\delta)$. A sufficient condition that ensures that (B4) is satisfied, at least for $G\leq F$, is $\Pi_C(1,N_1)\geq 1/(N_1+1)^2$, (i.e. the lowest profits a central firm can earn are greater than the highest profits earned if joining N_1 firms already located on the perimeter).

The inequality $\Pi_C(1,N_1)-1/(N_1+1)^2>0$ is satisfied[14] for $F=e$ and $F=b$ if $\delta<\hat{\Delta}=\left[7\left(2+\sqrt{2}\right)/4\right]+\sqrt{17+13\sqrt{2}}\approx 0.026$. If we focus our attention to parameters belonging to set $\hat{S}\subset S$, $\hat{S}=\left\{(F,\delta)\in\hat{S}\,\middle|\,\delta<\hat{\Delta}\right\}$, due to the continuity of the functions involved, it follows that there exists neighbourhoods of *e* and *b* where condition (B4) is satisfied (regions D_3 and C_1 later). Notice in

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market equilibrium *c*=0, *N*=N_0 is generated if condition A1 above is satisfied (it translates into $G>\sigma$ and $F>\delta^2$, or $F\leq\delta^2/2$).

[14] Values for *e* and *b* are reported in the main text.

addition that in these neighbourhoods the type of market failures described in the text about regions D, and C are replicated.

The configuration described in (B2) is an equilibrium if no firm finds it profitable to incur cost G and locate at the centre and the number of perimeter firms is defined by the zero-profit condition (i.e. $N = N_0$). Sufficient conditions to ensure that $c = 0, N = N_0$ is a market equilibrium are either

(B5) $F \le \delta^2$ (ensuring that a new entrant at the centre would produce regimes 2 or 3 in the last stage of the game where the central firm's profits are equal to zero) or

(B6) $F > \delta^2$ and $G > \sigma(F,\delta)$ (ensuring that a new entrant at the centre would produce regime 1 in the last stage of the game, but with $\Pi_C < G$)

AND either

(B7) $N_0 - 1 \ge 1/\delta$ (ensuring that a perimeter switch to the centre would produce regimes 2 or 3 in the last stage of the game) or

(B8) $N_0 - 1 < 1/\delta$, and

$$G > 2\left[1-(N_0-1)\delta\right]^2 / 9(N_0-1) = \frac{2\left[1-\left((1/\sqrt{F})-1\right)\delta\right]^2}{9\left((1/\sqrt{F})-1\right)} \equiv \hat{\sigma}(F,\delta)$$

(ensuring that a perimeter switch to the centre would produce regime 1 in the last stage of the game but $\Pi_C < G$).

Note that conditions (B5) and (B6) correspond to parameters belonging to region A in the text plus the area between function ψ and σ where multiple equilibria appear. In addition (B7) translates into $F \le \delta^2/(1+\delta)^2$, ensuring that there exists a non-empty subset of A (A_1 later) in which $c = 0, N = N_0$ is a market equilibrium.

Note that $\delta^2 > \delta^2/(1+\delta)^2 > \delta^2/2$ for $\delta \in \hat{\hat{S}}$. This means that we need condition (B8) to be satisfied for $c = 0, N = N_0$ to be a market equilibrium for parameters belonging to region B. Note that $\hat{\sigma}\left(F = \delta^2/(1+\delta)^2, \delta\right) = 0$ and that $\partial\hat{\sigma}/\partial F > 0$. In addition note that:

$\hat{\sigma}\left(F = (1/2+2\delta)^2/18, \delta\right) < \psi\left(F = (1/2+2\delta)^2/18, \delta\right)$ for $\delta \in \hat{\hat{S}}$. The result implies that, due to continuity of the functions considered, there exists a non-empty subset of B (B_1 later) in which $c = 0, N = N_0$ is a market equilibrium when there exists only one type of potential entrant in the market and the type of market failure described in the text survives.

We have shown that for parameters belonging at least to open, non-empty subsets of regions A-D in Figure B.2 market equilibria and social optima compare as in the text (see the corresponding A-D regions in Figure 3) when there exist only one type of potential entrant. Specifically, the subsets described above are:

$\hat{S} \subset S$, $\hat{S} = \left\{(F,\delta) \in \hat{S} \,\middle|\, \delta < \hat{\Delta}\right\}$;

$A_1 \subset A$, $A_1 = \left\{(F,G) \in A \,\middle|\, F \le \delta^2\right\}$;

$B_1 \subset B$, $B_1 = \left\{(F,G) \in B \,\middle|\, F \in \left(\left((1/2+2\delta)^2/18\right) - \varepsilon, (1/2+2\delta)^2/18\right]\right\}$ for some small $\varepsilon > 0$;

$C_1 \subset C$, $C_1 = \left\{(F,G) \in C \,\middle|\, G \le F, F \in (b-\varepsilon, b+\varepsilon)\right\}$ for some small $\varepsilon > 0$;

$D_3 \subset D$, $D_3 = \left\{(F,G) \in D \,\middle|\, G \le F, F \in (e-\varepsilon, e+\varepsilon)\right\}$ for some small $\varepsilon > 0$.

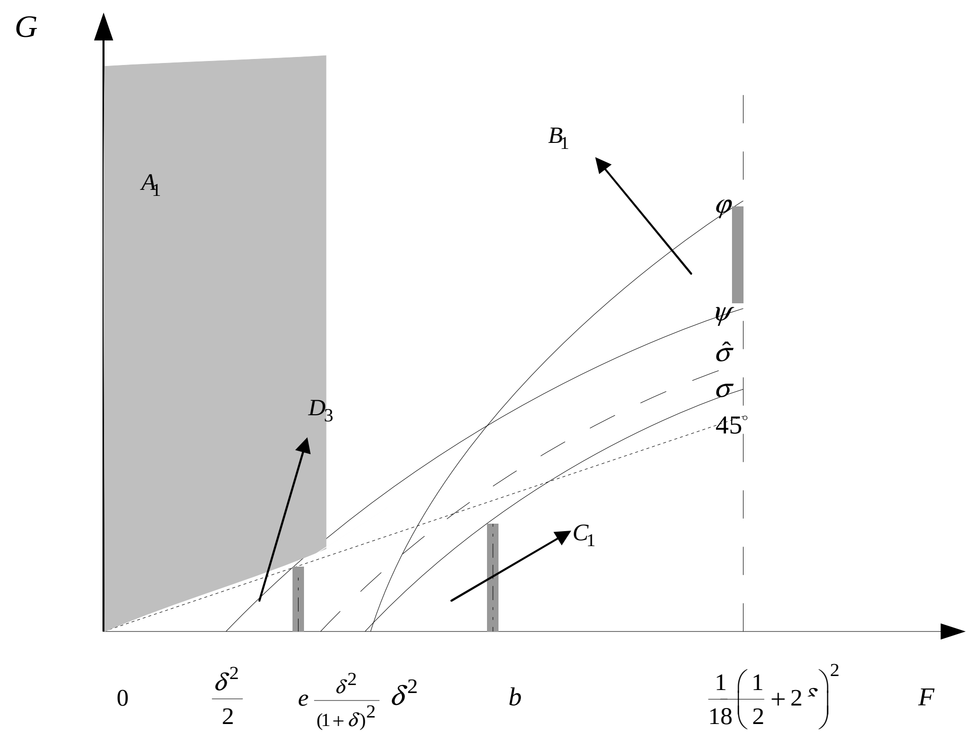

Figure B.2: social optima and market equilibria with one type of entrants.

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