

## OLIGOPOLISTIC COMPETITION FOR THE PROVISION OF HOSPITAL CARE

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# Oligopolistic competition for the provision of hospital care

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## Abstract

Competition in the market for health care has followed different patterns, and some health care systems have opted for mixed markets where public organisations compete alongside private ones. Empirical evidences on these market structures are however mixed. In this article we argue that public hospitals which have different objectives than private ones and faces different constraints, are also perceived differently by patients. For this reason we model the market for hospital care as Salop circle with a centre where the public hospital is located; private providers are located on the circle. We show that, depending on the difference in the productivity advantage, mixed markets may outperform both the benchmark (one public hospital at the centre) and private competition ( $N$  private providers competing along the circle), but the welfare distribution of these improvements should be carefully analysed. In some cases monopoly franchise on the mixed market should be introduced to redistribute these benefits.

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# 1 Introduction

The provision of hospital care has been reshaped in several countries where escalating health care costs, and shrinking public resources have undermined the long run sustainability of public health care. In the quest to reduce cost and enhance quality, competition and privatisation have been introduced in hospital provision.

The process of privatisation has followed different patterns and some health care systems have preferred to create a mixed market where public organisations compete alongside private ones. In Europe, the share of public hospitals is decreasing in most countries (EUROSTAT (2016)), while the performances of these form of competition have been rather mixed (Duggan, 2004; Amirkhanyan, 2008; Gaynor et al., 2015). The empirical literature does in fact agree neither on the estimation of the productivity differential between organizations pursuing different objectives (Chang et al., 2004; Tiemann et al., 2012), nor on the evaluation of quality by users (Gravelle et al., 2014; Gaynor et al., 2015) while from a theoretical point of view these markets seems to have a limited scope (Brekke et al., 2012, 2014; Herr, 2011; Levaggi, 2007).

Most models in this literature assumes that hospitals compete for patients using some forms of spatial competition where providers may be asymmetric in their objectives, but are on the same level as concerns patients' evaluation.

We argue that a more reasonable structure for competition in this market is to consider that private hospitals compete on the same level with a reference supply represented by the public hospital.

Public providers are non profit organisation whose workforce may be intrinsically motivated, i.e. they receive utility from their salary and the output they produce. This positive externality, coupled with their non profit status, may enhance the quality level of the service and can ultimately influence patients' evaluation of the quality of the two providers.

Following Bouckaert (2000); Madden and Pezzino (2011), we model a market for hospital care, where patients are located on the circumference of a Salop circle and may choose between being treated by a public hospital (located at the centre of the circle) or by  $N$  horizontally differentiated private suppliers that chooses their location on the circle.

Due to the nature of the service, the user charge paid by the patient usually covers only a fraction (if any) of the cost, hence the provider has to be subsidised. The technology of production, although universally available, may require fixed, sunk investments, which restricts the number of firms that can acquire it.

Our model shows that if patients perceive public hospitals as different from

private one, a mixed market for hospital care may improve welfare, if some conditions are met. These conditions relate to the productivity differential, the level of intrinsic motivation of public providers, the cost structure of the service involved and patients quality evaluation. From a policy point of view two interesting results are emerging: the first is related to the optimality of a soft budget constraint policy for public hospitals while the second relates to the composition of total welfare. In some cases the welfare improvement brought about by the mixed market is driven by the profit of the private hospital. In this case, the use of monopoly franchise to regulate market entry may be advisable to reduce public deficit and the excessive exploitation of consumers surplus.

The paper will be organised as follows: in Section 2 we present the main features of the model proposed and the most important features of the market structures analyses; in Section 3 we analyse the main determinants of the difference in welfare of the model proposed and in Section 4 we show the lines that a regulator should follow in choosing the preferred market alternative. Finally Section 5 concludes.

## 2 The model

We model the decision process of a regulator wishing to design the architecture of the market for hospital care  $Q$ . The provision can be granted using two types of potential providers: profit-maximizing firms ( $P$ ) and public providers with altruistic preferences ( $A$ ). Productivity level and service quality can be observed by the regulator, but they are not verifiable.  $P$ -s hospitals have a higher incentive to reduce costs compared with  $A$ -s, while the latter pursue different objectives and are more reactive to users' evaluations of quality (Brekke et al., 2012; Levaggi and Levaggi, 2017).

### 2.1 The environment

A community consisting of a mass of patients (normalised to 1 for simplicity) is uniformly distributed on a circumference of unit length. Each individual has an exogenous income  $Y$ , distributed according to a density function  $f$  supported in  $(a, b)$ . We denote the average exogenous income as  $\bar{Y} = \int_a^b Y f(Y) dY$ . They are all entitled to receive one unit of hospital care either from one of the  $N$  private hospitals located along the circumference or from a public hospital located at the

centre.<sup>1</sup> Each unit entitles to a reimbursement  $T$  which is financed using a linear income tax at rate  $\tau$ .  $Q$  is supplied for free, but users incur linear distance costs to acquire it. Individuals are allowed to choose the preferred provider and in doing so they evaluate both the service quality and travel costs.

The objective function of a generic provider  $h$  is:

$$V_h = (1 - \rho_h)\Pi_h + \rho_h \varphi q_h$$

where  $\Pi_h$  is the surplus (i.e. the difference between revenue and costs),  $q_h$  is quality of care,  $\varphi$  is patients evaluation of quality and  $\rho_h$  is a measure of privatisation of the hospital. Public hospitals cannot retain surplus ( $\rho_h = 1$ ) and pursue quality enhancement; as they get privatised ( $0 < \rho_h < 1$ ) they can retain a part of the surplus, but become less interested in quality. Private hospitals retain their surplus ( $\rho_h = 0$ ), but for them quality is simply a means to increase surplus.

The unit cost to produce care with a minimum verifiable level of quality ( $\bar{q} = 0$  for simplicity) is equal to  $\beta$ , but it can be reduced through a non-monetary cost reducing effort  $f_h$ . Costs for quality enhancement depend on the level of privatisation of the hospital. If the staff is intrinsically motivated, the cost to produce quality is non monetary and it is equal to the disutility of such effort.

We assume that intrinsic motivation is inversely related to profit retainment. As the hospital get privatised, workers lose their intrinsic motivation and to increase quality the management has to pay higher salaries, which we measure through a linear cost  $k$ . Finally, we assume the existence of a fixed entry cost  $H$ , which corresponds to the investment in technology which is necessary to run the hospital. With these assumptions, the cost function for a generic hospital  $h$  can be written as:

$$C_h = (\beta - \theta_h f_h) D_h + (1 - \rho_h) k q_h D_h + \frac{1}{2} f_h^2 D_h + \rho_h \frac{\theta_h}{2} q_h^2 + H \quad (1)$$

where the term  $\theta_h$  measure the efficiency of the cost reduction effort. In particular the efficiency of the public hospital will be denoted by  $\theta_c$ .  $D_h$  is the demand for hospital  $h$  and  $1 - \rho_h$  is the fraction of profit that providers are allowed to retain with  $0 \leq \rho_h \leq 1$ . For  $\rho_h = 0$  the hospital is a private organisation while  $\rho_h = 1$  represents a public provider.

The demand  $D_h$  is derived from the choices of patients. The utility of a generic individual choosing hospital  $h$  located at a distance  $x$  as:

$$U_h = Y(1 - \tau) + v + \varphi q_h - mx, \quad (2)$$

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<sup>1</sup> See Levaggi and Levaggi (2011) for an alternative formulation.

where  $Y(1 - \tau)$  is the net income and  $v$  is the intrinsic individual utility of health care which is assumed to be sufficiently high to assure that any user will access the service from some provider. The parameter  $\varphi > 0$  is the evaluation quality, thus  $\varphi q_h$  is the monetary equivalent gain derived from using the service offered by hospital  $h$ . Finally, patients deciding to address the demand to a private provider incur a cost equal proportional to the distance they have to travel, i.e.  $mx$ . Patients that choose to be treated by the public provider located at the centre of the circle incur a fixed travel cost, independent of their position. To simplify notation we denote it by  $\delta$ , so that the utility these patients receive is:

$$U_c = Y(1 - \tau) + v + \varphi q_c - \delta \quad (3)$$

if  $q_c$  is the quality offered by the public provider.

Note that by the hypothesis on  $v$  the total number of users is 1, thus the tax rate is

$$\tau = \frac{T}{\bar{Y}}.$$

Since  $T$  is the unit reimbursement from (1) the surplus of each provider is  $\Pi_h = T D_h - C_h$ . Defining  $S = T - \beta$  we then write the objective function of a generic provider  $h$  as:

$$\begin{aligned} V_h &= (1 - \rho_j)\Pi_h + \rho_h \varphi q_h \\ &= (1 - \rho_h) \left( S + \theta_h f_h - k(1 - \rho_h)q_h - \frac{1}{2}f_h^2 \right) D_h + \rho_h \left( \varphi q_h - \frac{\theta_h}{2}q_h^2 \right) - H \end{aligned} \quad (4)$$

The objective of the regulator is to maximise total welfare, which is generally defined as the aggregation of patients' utility and private providers' profit, net of potential losses of the public hospital.

We develop the model using a three stage game for the problem of a regulator that has to decide whether to open the market for competition. At stage 1 the regulator collects information about the parameters of the environment (e.g.  $v$ ,  $\varphi$ ,  $m$ ), the market (e.g. the costs  $\beta$ ,  $k$  and  $H$ ) and the providers (e.g.  $\theta_c$  and  $\theta_h$ ). In order to take the decision it has to compare the various welfare levels that different types of competition may allow. At stage 2 it thus evaluates the best reply functions of the available providers in different competition settings and compares these alternatives with the status quo represented by a market served by the public hospital located at the centre. In this work three different scenarios are considered:

- benchmark: a public hospital located at the centre of a circle supplies health care to all the patients as a monopolist;

- mixed market:  $N_M$  private suppliers are allowed on the circumference. By assumption, they are symmetric players and their location will therefore be symmetric. The public hospital retains its position at the centre;
- private market:  $N_P$  private hospitals enter the market and decides their position on the circumference. The public hospital is either privatised (hence becoming a private competitor on the circle) or it is closed down.

Finally, at stage 3 the firms compete in quality in the chosen market setting if competition is allowed.

## 2.2 Benchmark

The benchmark represents the status quo, where a public hospital supplies care as a monopolist to the patients around the circle. The optimal levels of quality ( $q_b$ ) and effort ( $f_b$ ) are derived from the maximisation of the objective function of a unique provider with  $\rho_h = 1$  and  $D_h = 1$ . From (4) these quantities solve the following optimisation problem:

$$\max_{q,f} \left( \varphi q - \frac{1}{2} f^2 - \frac{\theta_c}{2} q^2 - H \right)$$

The optimal solution is readily derived using the F.O.C.s:

$$q_b = \frac{\varphi}{\theta_c}, \quad f_b = 0. \quad (5)$$

Given that quality does not depend on the reimbursement, the regulator sets  $T$  to the minimum level that allows the public hospital to be budget balanced, i.e.  $T = \beta + H$ . In the notations of (4) this is equivalent to  $S = H$  and total welfare is then:

$$W_B = \bar{Y} - \beta - H + v + \frac{\varphi^2}{\theta_c} - \delta. \quad (6)$$

## 2.3 Salop competition with a public firm at the centre

In this case the regulator opens to competition and  $N_M$  private providers are allowed to enter the market and decide their location on the circumference. For the sake of clarity from now on we will append the index  $c$  to quantities and parameters related to the public hospital and the index  $i$  to those of the private ones.

Depending on the productivity differential of the providers and travelling costs, three different outcomes of the competition game are possible.

- a. The hospital at the centre supplies very high quality and it gets all the market. This occurs if the quality  $q_c$  provided by the firm at the centre and the quality  $q_i$  offered by hospitals on the circumference are sufficiently different. For a consumer on the boundary of the circle the maximum utility from health care can be obtained when the transport cost is equal to zero. Hence the maximum utility that can be achieved by addressing demand to a private provider is  $v + \phi q_i$ . The cost to travel to the centre is fixed and equal to  $\delta$ , thus the utility for being treated by the public hospital is equal to  $v + \phi q_c - \delta$ . If  $q_c \geq q_i + \frac{\delta}{\phi}$  all the consumers go to the centre.
- b. The hospital at the centre has a very low quality and it gets no market. The public hospital can best attract consumers that are located farther away from the firm on the circumference. In a symmetric model with  $N_M$  firms on the circle the maximum distance that a consumer has to travel to reach the nearest private provider is  $\frac{1}{2N_M}$ . Therefore the minimal net utility for users addressing their demand on the private market is  $v + \phi q_i - \frac{m}{2N_M}$ , while the utility when going to the centre is  $v + \phi q_c - \delta$ . Hence if  $q_c \leq q_i + \frac{\delta}{\phi} - \frac{m}{2N_M\phi}$  all patients choose private hospitals.
- c. The hospital at the centre offers an intermediate quality and is able to attract part of the consumers on the circumference. Considering a representative private hospital  $i$ , the distance  $x$  of the indifferent patient is found by equating the net utilities received from the nearest private hospital and the public one, i.e.  $q_c - \delta = q_i - mx$ . Thus the position of the indifferent patient is

$$x_M = \frac{\phi(q_i - q_c) + \delta}{m}. \quad (7)$$

Therefore each private hospital has a market share equal to

$$D_i^M = 2x_M = 2 \frac{\phi(q_i - q_c) + \delta}{m},$$

while the market share of the public hospital is

$$D_c^M = 1 - 2N_M x_M.$$

We start by examining the equilibrium in the last case and will then state the conditions for the existence of an internal solution. From (4) hospitals on the boundary of the circle maximise the following objective function:

$$V_i = \Pi_i = \left( S + \theta_i f_i - \frac{1}{2} f_i^2 - k q_i \right) 2x_M - H, \quad (8)$$



while for the public hospital the objective function is:

$$\begin{aligned} V_c &= (1 - \rho_c) \left( S + \theta_c f_c - k(1 - \rho_c) q_c - \frac{1}{2} f_c^2 \right) (1 - 2N_M x_M) \\ &\quad + \rho_c \left( \varphi q_c - \frac{\theta_c}{2} q_c^2 \right) - H. \end{aligned} \quad (9)$$

The game is solved in Appendix A.1 for a general value of  $\rho_c$ . Here we concentrate our attention on the specific case of a pure public hospital at the centre, that is to the case  $\rho_c = 1$ . The solution in this situation is:

$$\begin{aligned} f_c^M &= 0, & q_c^M &= \frac{\varphi}{\theta_c}, \\ f_i^M &= \theta_i, & q_i^M &= \frac{\theta_i^2 + 2S}{4k} + \frac{1}{2} \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} \right), \\ x_M &= \frac{\varphi}{2m} \left( \frac{\theta_i^2 + 2S}{2k} + \frac{\delta}{\varphi} - \frac{\varphi}{\theta_c} \right), \\ \Pi_i^M &= 2\varphi \frac{k}{m} \left( \frac{\theta_i^2 + 2S}{4k} + \frac{1}{2} \left( \frac{\delta}{\varphi} - \frac{\varphi}{\theta_c} \right) \right)^2 - H. \end{aligned} \quad (10)$$

The market is feasible if each competitor has a market share and if the profit of the private providers is non negative. Observe that the quality offered by the public hospital is independent of what privates do and is equal to the benchmark, thus the existence of a mixed market depends on the level of efficiency of the private providers. As shown in Appendix A.1 the existence conditions can be translated into the following set of inequalities:

$$H < \frac{mk}{2\varphi}, \quad N_M < \sqrt{\frac{mk}{2H\varphi}}, \quad \theta_i^L \leq \theta_i < \theta_i^H, \quad (11a)$$

$$\theta_i^L := \sqrt{2 \left( k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} \right) + \sqrt{\frac{2Hmk}{\varphi}} - S \right)}, \quad (11b)$$

$$\theta_i^U := \sqrt{2 \left( k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} + \frac{m}{\varphi N_M} \right) - S \right)}. \quad (11c)$$

The conditions on  $\theta_i$  imply that the quality provided is sufficiently homogeneous for both providers to have a market share. The number of competitors on the market affects the range of values for which the mixed market can exist, but there is

not a market clearing condition. The regulator has to decide how many competitors are allowed to enter, thus  $N_M$  is also a strategic variable in this context, but the choice is constrained by the conditions in (11). If  $N$  is the greatest integer for which  $N < \sqrt{\frac{mk}{2H\phi}}$  and  $n \in \{1, \dots, N\}$  we define:

$$\theta_i^U(n) = \sqrt{2 \left( k \left( \frac{\phi}{\theta_c} - \frac{\delta}{\phi} + \frac{m}{\phi n} \right) - S \right)}.$$

Then a mixed market is feasible only if  $\theta_i \in [\theta_i^L, \theta_i^U(1))$  and the number  $N_M$  has to satisfy the following constraints:

$$\theta_i \in [\theta_i^L, \theta_i^U(n)) \Rightarrow 1 \leq N_M \leq n.$$

Therefore on each subinterval of the feasibility set for  $\theta_i$  different choices for  $N_M$  are allowed.

The welfare function in this case comprises the utility of the users, the profits of the private providers and the possible losses of the public provider. If  $S = H$  as in benchmark with the market share  $1 - 2N_M x_M$ , the public hospital is not able to pay for the full cost  $H$ . The ensuing deficit, equal to  $2HN_M x_M$ , has to be covered using public funds. Thus the welfare is:

$$\begin{aligned} W_M &= \bar{Y} - \beta - H + v + 2N_M \int_0^{x_M} (\phi q_i^M - mz) dz + (1 - 2N_M x_M) (\phi q_c - \delta) \\ &\quad - 2HN_M x_M + \Pi_i^M N_M \\ &= \bar{Y} - \beta - H + v + \frac{\phi^2}{\theta_c} - \delta + 2N_M x_M \left( mx_M - \frac{1}{2} mx_M - H \right) + \Pi_i^M N_M \\ &= W_B + N_M (x_M (mx_M - 2kq_i^M + \theta_i^2) - H). \end{aligned} \tag{12}$$

## 2.4 Salop competition among private providers

If the public hospital at the centre is too inefficient or if the regulator decides to privatise it completely, the market is modelled by a standard Salop competition. In this case the solution can be obtained using backward induction starting from stage three. Let us assume that  $N_P$  competitors are located around the circle. Hospitals are identical, are placed at equally spaced locations on the circumference and maximise their profit, as expressed in (8). The market share for each hospital is

determined by the position  $x_P$  of the consumer who is indifferent between hospital  $i$  and hospital  $j$ :

$$x_P = \frac{\varphi(q_i - q_j)}{2m} + \frac{1}{2N_P}.$$

The game is solved in Appendix A.2 and gives the following solution:

$$f_i^P = \theta_i, \quad q_i^P = \frac{\theta_i^2 + 2S}{2k} - \frac{m}{\varphi N_P}, \quad x_P = \frac{1}{2N_P}. \quad (13)$$

The profit of each provider amounts to

$$\Pi_i^P = \frac{mk}{\varphi N_P^2} - H;$$

if market entry is free the number of hospitals is determined by the condition  $\Pi_i^P \geq 0$ , i.e.<sup>2</sup>

$$N_P = \left\lfloor \sqrt{\frac{mk}{H\varphi}} \right\rfloor. \quad (14)$$

Thus, from (13) and (14) the market is feasible if the following conditions are satisfied:

$$\theta_i > \sqrt{\frac{2mk}{\varphi N_P} - 2S}, \quad H \leq \frac{mk}{4\varphi}. \quad (15)$$

For  $S = H$  the welfare function in this case is:

$$\begin{aligned} W^P &= \bar{Y} - \beta - H + v + 2N_P \int_0^{\frac{1}{2N_P}} (\varphi q_i^P - mz) dz + N_P \Pi_i^P \\ &= \bar{Y} - \beta - H + v + \varphi q_i^P - \frac{m}{4N_P} + \left( \frac{mk}{\varphi N_P^2} - H \right) N_P \\ &= \bar{Y} - \beta - H + v + \varphi q_i^P - \frac{m}{4N_P} + \left( \frac{mk}{\varphi N_P^2} - H \right) N_P. \end{aligned} \quad (16)$$

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<sup>2</sup>The standard notation  $\lfloor x \rfloor$  is used to denote the greatest integer that is less than or equal to  $x$ .

		Benchmark	Private market	Mixed Market
centre	effort	0		0
	quality	$\frac{\varphi}{\theta_c}$		$\frac{\varphi}{\theta_c}$
border	effort		$\theta_i$	$\theta_i$
	quality		$\frac{\theta_i^2 + 2H}{2k} - \frac{m}{\varphi N_P}$	$\frac{\theta_i^2 + 2H}{4k} + \frac{1}{2} \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} \right)$
$N$			$N_P = \left\lfloor \sqrt{\frac{mk}{H\varphi}} \right\rfloor$	$N_M < \sqrt{\frac{mk}{2H\varphi}}$ - set by the regulator
demand			$D_i^P = \frac{1}{N_P}$	$D_i^M = \frac{\varphi}{m} \left( \frac{\theta_i^2 + 2H}{2k} + \frac{\delta}{\varphi} - \frac{\varphi}{\theta_c} \right)$ $D_c^M = 1 - N_M D_i^M$
existence			$H \leq \frac{km}{4\varphi}$	$H \leq \frac{km}{2\varphi}$
			$\theta_i > \sqrt{\frac{2mk}{\varphi N_P} - 2H}$	$\sqrt{2 \left( k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} \right) + \sqrt{\frac{2Hmk}{\varphi}} - H \right)} \leq \theta_i$ $\theta_i < \sqrt{2 \left( k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} + \frac{m}{\varphi N_M} \right) - H \right)}$

Table 1: Relevant quantities and existence conditions for the different market structures for  $S = H$ .

### 3 Welfare analysis

In comparing the welfare levels of the various options, several factors such as the productivity of the private firms, the cost structure and the residual profit have to be taken into account. From a policy point of view it is also quite important to determine which of them drives the sign of the welfare differences. For this reason, we propose at first an analysis of the three elements that we think are mostly important for the regulator, namely:

- public expenditure;
- individual net utility;
- profit of the private hospitals.

In what follows we summarise the main technical results, which will then be discussed in Sectionsec:discussion; mathematical details about their derivation can be found in Appendix B. The indices  $B$ ,  $M$  and  $P$  will be used to distinguish between the three different scenarios (benchmark, mixed market and private market).

#### 3.1 Public expenditure

Public expenditure (denoted by  $G$ ) is represented by the cost to provide the service, net of franchise fees that may be asked to private providers entering the market. Since monopoly franchise is not common in hospital care provision, we define public expenditure simply in terms of the cost to provide the service. The welfare comparison will be carried out under the assumption  $S = H$ , that is  $T = H + \beta$ : the provided reimbursement is equal to the one in benchmark. Then

$$\begin{aligned} G_B &= \beta + H, \\ G_M &= \beta + H + 2HN_M x_M \\ G_P &= \beta + H. \end{aligned}$$

Public expenditure increases in the mixed market because the deficit of the public hospital must be repaid. It is interesting to note that  $G_M$  is linear and increasing in  $N_M$ , it depends on  $\theta_i$  through the term  $x_M$  and satisfies the following bounds:

$$HN_M \sqrt{\frac{2H\phi}{mk}} \leq G < H.$$

### 3.2 Patients net utility

Let us examine the money equivalent welfare derived from the service, net of the transport costs, which corresponds to users evaluation of the service (it will be denoted by  $\Phi$ ).

It may be interpreted as an overall quality measure since it depends on the average quality of care net of the private costs to get it. It may be improved either through quality enhancement or by increasing the number of providers.

From (10) and (13) we have:

$$\begin{aligned}\Phi_B &= v + \frac{\varphi^2}{\theta_c} - \delta \\ \Phi_M &= v + \frac{\varphi^2}{\theta_c} - \delta + N_M \frac{\varphi^2}{4m} \left( \frac{\theta_i^2 + 2H}{2k} + \frac{\delta}{\varphi} - \frac{\varphi}{\theta_c} \right)^2 \\ \Phi_P &= v + \varphi \frac{\theta_i^2 + 2H}{2k} - \frac{5}{4} \frac{m}{N_P}.\end{aligned}$$

As expected, in a mixed market patients net utility is higher than in benchmark. Note also that  $\Phi_M$  is increasing in  $N_M$ , hence the highest level of  $\Phi_M$  is achieved by choosing the highest number of private competitors compatible with the existence conditions.

Proposition 1 summarises the results for the comparison between  $\Phi_M$  with the optimal choice of  $N_M$  and  $\Phi_P$ .

**Proposition 1.** *Let the existence conditions in equation (11) and (15) be satisfied. If  $5N_M \geq 3N_P$   $\Phi_M > \Phi_P$  for all  $\theta_i \in [\theta_i^L, \theta_i^U]$ .*

*If*

$$2\sqrt{\frac{2mk}{H\varphi}} - \frac{5mk}{2\varphi N_P H} < N_M < \frac{3}{5}N_P. \quad (17)$$

*there exists  $\theta_i^* \in [\theta_i^L, \theta_i^U]$  such that  $\Phi_M \geq \Phi_P$  for  $\theta_i \in [\theta_i^L, \theta_i^*]$  and  $\Phi_M < \Phi_P$  for higher values of  $\theta_i$ .*

*Otherwise  $\Phi_M < \Phi_P$  for all  $\theta_i \in [\theta_i^L, \theta_i^U]$ .*

Proof: See Appendix B.1.

The above result implies that for  $N_M = 1$  the inequality  $\Phi_M < \Phi_P$  is satisfied for some range of (low) values of  $\theta_i$  only if  $N_P \leq 3$ . For higher numbers of competitors it depends on the ratio between  $N_P$  and  $\sqrt{\frac{mk}{H\varphi}}$ : if the latter is sufficiently high (i.e. profits for private providers are low)  $\Phi_P$  is always higher than  $\Phi_M$  with  $N_M = 1$ .

### 3.3 Profit of private hospitals

In a private market with free entry the number of competitors  $N_P$  is the highest integer compatible with the non negative profit condition, and providers may have a surplus. The total profit  $\Pi_P = N_P \Pi_i^P$  does not depend on  $\theta_i$ , it is a discontinuous function of  $H$  that is zero at each point  $H = \frac{mk}{K^2\phi}$  with  $K$  integer and increases on each interval comprised between two roots. However, as  $H$  tends to zero it becomes negligible (See Appendix B.2 for a formal proof).

The total profit in the mixed market is equal to

$$\Pi_M = N_M \Pi_i^M = N_M((2H + \theta_i^2 - 2kq_i^M)x_M - H);$$

it is increasing and linear in  $N_M$ , and is a quadratic, convex and increasing function of  $\theta_i^2$ . For each fixed  $\theta_i$  the maximum total profit is obtained by choosing the maximum allowed number of competitors  $N_M$  and at the same time, the sequence of maximal values for each  $N_M$  decreases with  $N_M$  (see Appendix B.2).

The difference  $\Pi_P - \Pi_M$  is thus negative for any  $H = \frac{mk}{K^2\phi}$  with  $K$  integer and by continuity it is negative for almost all values of  $\theta_i$  if  $H$  is just above this threshold. As  $H$  increases to the next jump of  $\Pi_P$  the difference  $\Pi_P - \Pi_M$  increases and it is negative only for  $\theta_i$  high enough.

As for “net profits” of the mixed market, i.e  $\Pi_M$  net of the losses of the public hospital, we have:

$$\hat{\Pi}_M := -2HN_M x_M + \Pi_M = N_M((\theta_i^2 - 2kq_i^M)x_M - H).$$

This function is linear in  $N_M$ , quadratic, increasing and convex function in  $\theta_i^2$ . Its minimum is negative, while for  $\theta_i^U$  is positive when  $N_M < \sqrt{\frac{1}{4} + \frac{mk}{2H\phi}} - \frac{1}{2}$ .

### 3.4 Welfare difference: benchmark vs mixed market

The total welfare in a mixed market is given by:

$$W_M = W_B + N_M(x_M(mx_M - 2kq_i^M + \eta_i) - H),$$

where  $W_B$  is the welfare of the benchmark scenario. Thus, if the term  $x_M(mx_M - 2kq_i^M + \eta_i) - H$  is positive, a mixed market will give a higher welfare level than the benchmark and at the same time the highest welfare level is reached by allowing the maximum number of private providers into the market.

**Proposition 2.** *Let the existence conditions in equation (11) be satisfied.*

*If  $H \leq \frac{m\varphi}{8k}$  the inequality  $W_M > W_B$  holds for all  $\theta_i \in [\theta_i^L, \theta_i^U]$ .*

*If*

$$\varphi < 2k \quad \text{and} \quad \frac{m\varphi}{8k} < H < \frac{m(\varphi + 2k)}{4\varphi N_M(N_M + 1)}$$

*there exists  $\hat{\theta}_i \in (\theta_i^L, \theta_i^U)$  such that  $W_M > W_B$  if  $\theta_i > \hat{\theta}_i$  and  $W_M < W_B$  for  $\theta_i < \hat{\theta}_i$ . Otherwise  $W_M < W_B$  for all values of  $\theta_i$  for which a mixed market exists.*

Proof: See appendix

### 3.5 Welfare difference: benchmark vs private market

Let us now turn to the difference  $W_P - W_B$ . In this case we can conclude that

**Proposition 3.** *Let the conditions in equation (15) be satisfied and let  $N_P = \varepsilon \sqrt{\frac{mk}{H\varphi}}$  for some  $0 < \varepsilon \leq 1$ . Then if*

$$\frac{1}{\varepsilon} (1 - \varepsilon^2) \sqrt{\frac{Hmk}{\varphi}} - \frac{1}{4\varepsilon} \sqrt{\frac{Hm\varphi}{k}} - \left( \frac{\varphi^2}{\theta_c} - \delta \right) \geq 0$$

*the inequality  $W_P - W_B > 0$  holds for all admissible values of  $\theta_i$ . Otherwise there exists  $\bar{\theta}_i$  such that  $W_P - W_B > 0$  for  $\theta_i > \bar{\theta}_i$  and  $W_P - W_B < 0$  if  $\theta_i < \bar{\theta}_i$ . Moreover  $W_P - W_B > 0$  for any  $\theta_i \geq \theta_i^L$ .*

Proof: See Appendix B.4.

### 3.6 Welfare difference: mixed vs private market

In the comparison between  $W_P$  and  $W_M$  the ratio  $\frac{\varphi}{k}$  plays a fundamental role. Let us study the behaviour of these two functions wrt  $\eta_i$ . We have:

$$W_P = \Phi_P + \Pi_P, \quad W_M = \Phi_M + \Pi_M - 2HN_M.$$

From the previous analysis  $\Phi_P$  grows linearly in  $\eta_i$  at a rate equal to  $\frac{\varphi}{2k}$ , while  $\Phi_M$  is quadratic and increasing in  $\eta_i$  with a slower growth. The term  $\Pi_P$  does not depend on  $\eta_i$ , while the sum  $\Pi_M - 2HN_M$  is convex and increasing in this variable. The value of its derivative for  $\eta_i = \eta_i^L$  in (25) increases with  $\frac{\varphi}{k}$ , while the value for  $\eta_i = \eta_i^U$  is equal to  $\frac{1}{2} - \frac{1}{2} \frac{\varphi}{k} HN_M$ , therefore it decreases with  $\frac{\varphi}{k}$ . Comparing this



value with  $\frac{\varphi}{2k}$  it turns out that if  $\frac{\varphi}{k} < \frac{1}{1+HN_M}$  there exist values of  $\eta_i$  for which the term  $\Pi_M - 2HN_M$  grows faster than  $\Phi_P$ . As a result if  $\frac{\varphi}{k}$  is low the same behaviour can be shown in comparing  $W_P$  and  $W_M$ . In fact:

$$\begin{aligned}\frac{\partial W_M}{\partial \eta_i} &= \frac{N_M}{4m} \left( \frac{\varphi}{k} \left( \frac{1}{2} \frac{\varphi}{k} + 1 \right) \eta_i + H \frac{\varphi^2}{k^2} - \left( 2 + \frac{\varphi}{k} \right) \left( \frac{\varphi^2}{\theta_c} - \delta \right) \right) \\ &\leq \frac{\partial W_M}{\partial \eta_i} \Big|_{\eta_i = \eta_i^U} = \frac{1}{4} \left( 2 + \frac{\varphi}{k} \right) - N_M \frac{\varphi H}{2km}.\end{aligned}$$

The above quantity is greater than  $\frac{\varphi}{2k}$  whenever  $\frac{\varphi}{k} < \frac{2m}{m+2HN_M}$ , so that if this condition is satisfied the total welfare in the mixed market grows faster than the one in the private market for high values of  $\eta_i$ .

If  $\Pi_P = 0$ , i.e. for  $H = \frac{mk}{N_P^2 \varphi}$ , we can write:

$$\begin{aligned}(W_P - W_M)_{|\eta_i = \eta_i^L} &= \frac{m}{N_P} \left( \sqrt{2} - \frac{5}{4} - \frac{N_M}{2N_P} + \sqrt{2} \frac{N_M}{N_P^2} \frac{\varphi}{k} \right), \\ (W_P - W_M)_{|\eta_i = \eta_i^U} &= m \frac{k}{\varphi} \left( \frac{N_M + 1}{N_P^2} - \frac{1}{2N_M} \right) + \frac{m}{4} \left( \frac{3}{N_M} - \frac{5}{N_P} \right).\end{aligned}$$

## 4 Discussion

At stage 2, the regulator shapes the market by choosing the setting that maximises welfare, given the constraints of the problem. If  $H > \frac{mk}{2\varphi}$ , only a public firm at the centre can be a viable solution. In the other cases, it may be possible for the regulator to choose alternative market settings. In what follows we discuss the best choice of the regulator.

For  $H < \frac{mk}{2\varphi}$  the benchmark may not be the only viable alternative. If this is the case, also  $\theta_i$  and  $N_M$  determine which market organisations are feasible. Figure 1 summarises the available choices. On the horizontal axis we measure the entry costs  $H$ , while on the vertical axis we measure  $\theta_i$ . The red and blue curves delimit the area where a mixed market with  $N_M = 1$  is a viable solution. Higher values of  $N_M$  are feasible only for lower values of  $H$  and for smaller ranges of  $\theta_i$  (between the blue curve and the lower curves). As competition increases market shares shrink and competitive private providers find it easier to push the public provider out of the market. For the combinations of  $H$  and  $\theta_i$  falling below the blue curve or above the red one the comparison can only be done between the benchmark and a private market. In this case if  $\frac{mk}{4\varphi} < H < \frac{mk}{2\varphi}$  a benchmark is the only feasible

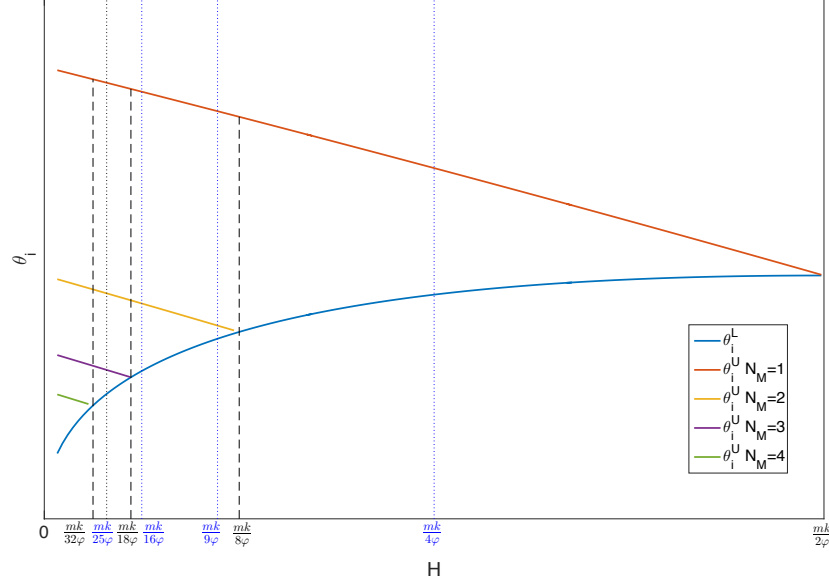


Figure 1: Feasibility areas

alternative. When  $H < \frac{km}{4\varphi}$  also a private market is feasible and in this case the best choice is given by the result in Proposition 3.

In the area delimited by the red and the blue curves the three markets are feasible. From Proposition 3 if both conditions in (11) and (15) are satisfied the welfare level of a private market is higher than in benchmark, hence only the comparison with the best mixed market is needed. From the analysis in Section 3.4 the highest level of welfare in the mixed market is achieved by choosing the greatest possible number of private competitors on the border.

Let us now turn to this comparison. Several countervailing effects contribute to determine the sign of the welfare difference. Given their relevance from a policy point of view, it is important to keep them separate. One of the key parameters is the ratio  $\frac{\varphi}{k}$ . Patients net utility is increasing in this parameter, while the profit of private providers in both markets decreases as this ratio increases. This means that, other things being equal, the composition of the welfare changes according to the value of this ratio. At the same time, as shown in Figures 2-4, the feasible area for the mixed market shrinks as this ratio increases. For this reason, it is not possible to determine necessary or sufficient conditions on  $H$  and  $\theta_i$  for which one solution is superior to the other one.

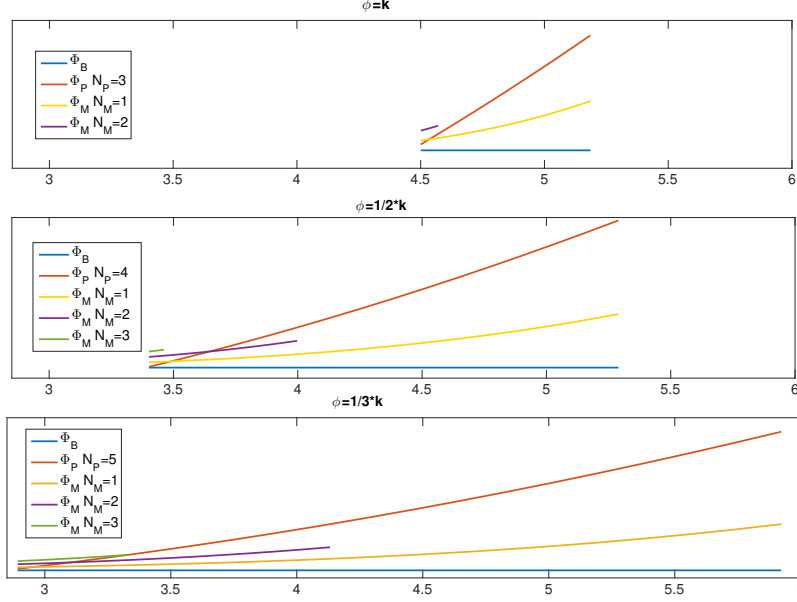


Figure 2: Comparison of the patients net utilities  $\Phi_P$  and  $\Phi_M$ .

However, some conclusions are possible: if  $\frac{\phi}{k}$  is high the market will be shared among a restricted number of providers and  $N_M$  and  $N_P$  are quite close to one another. In this case, from Proposition 1 the patients net utility can be higher in the mixed market, but in these cases, the cost in terms of deficit is higher than this increase and a private market should be chosen (see Figures 2 and 3). As the ratio decreases, more private providers can access the market and total welfare decreases. The gain in terms of patients welfare decreases in both markets, but more rapidly in the mixed one, hence the difference  $\Phi_M - \Phi_P$  may become negative. However, the profit of private providers in the mixed market (where entry is regulated) may increase to the point that total welfare in the mixed market becomes higher than the one in a private setting. In this case the welfare improvement is simply due to a shift of resources from the public to the private sector. This profit could be partially shared with the regulator through a monopoly franchise. However, most real-world system, even when regulating the number of private providers allowed to enter the market, do not seem to use this opportunity to share the rent.

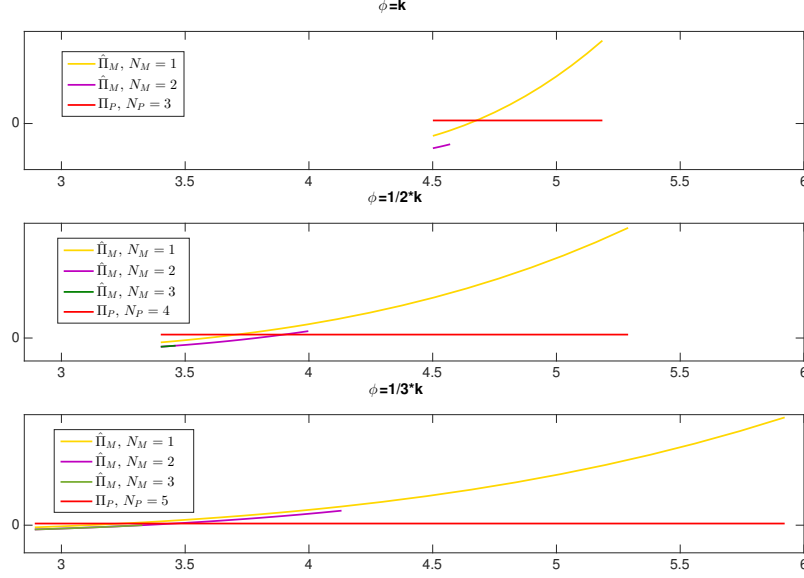


Figure 3: Comparison of total profits  $\Pi_P$  and net total profits  $\hat{\Pi}_M$ .

## 5 Conclusions

Health care provision in the public sector has undergone a process of reform that has introduced competition in diverse ways and with mixed evidences (Gaynor et al. (2015); Gaynor and Vogt (2003); Goddard (2015)). In several countries this process of privatisation has led to the creation of mixed markets for health care where organisations with different objectives and constraints compete. Also in this case a definite answer does not exist. Non profit organisations are usually less efficient and even when they have altruistic objectives their role is not well defined. Brekke et al. (2012) suggest that the reduction in efficiency produced by the presence of profit constraints outweighs the altruistic motivation; on the other hand, Herr (2011) shows that public providers may be better than private one while Levaggi (2007) and Levaggi and Levaggi (2017) show that altruistic providers may be the best choice to maximise welfare, but they should not be made competing with a private providers.

The literature has long recognized the difference in the objectives and constraints of public and private providers, but it usually assumes that the service produced by both types of organisation is perceived as homogeneous by the pa-

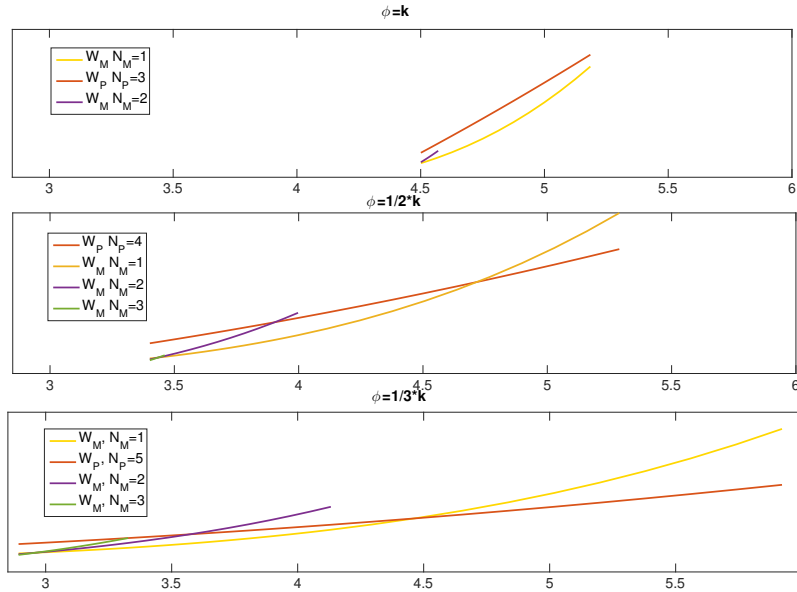


Figure 4: Comparison of total welfares  $W_P$  and  $W_M$ .

tient. In this paper we have taken a different approach. We argue that the patient may perceive the two organisations differently; in particular we argue that the public provider may be perceived by the patient as a sort of reference point. For this reason we have modified the approach usually used by the literature to model competition in the market by assuming that the mixed market develops along the rules of a Salop competition with a competitor at the centre (Bouckaert (2000); Madden and Pezzino (2011)). Our model shows that in this case there might be scope for a mixed oligopoly. Two other interesting conclusions arise from our analysis: the first one is that in a mixed market it may be optimal to set soft budget constraint rules for the public hospital. However, it should be noted that in the long run this may lead to strategic behaviour.

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## A Appendix

### A.1 Salop competition with a public hospital at the centre

Let us start by assuming that an interior solution exists; in this case the indifferent patient is located at  $x_M = \frac{\varphi(q_i - q_c) + \delta}{m}$ . The objective of the firms on the circle is:

$$\max_{f_i, q_i} V_i = \left( S + \theta_i f_i - \frac{1}{2} f_i^2 - k q_i \right) 2x_M - H.$$

From the F.O.C.s the reaction functions are as follows:

$$\begin{aligned} f_i &= \theta_i \\ q_i &= \frac{q_c}{2} + \frac{\varphi \theta_i^2 - 2\delta k + 2\varphi S}{4k\varphi} \end{aligned}$$

The firm at the centre maximises the following function:

$$V_c = (1 - \rho_c) \left( S + \theta_c f_c - k(1 - \rho_c) q_c - \frac{1}{2} f_c^2 \right) (1 - 2N_M x_M) + \rho_c \left( \varphi q_c - \frac{\theta_c}{2} q_c^2 \right) - H.$$

Again, from the F.O.C.s the following reactions functions are derived:

$$\begin{aligned} f_c &= (1 - \rho_c) \theta_c \\ q_c &= \frac{1}{4k(1 - \rho_c)N_M\varphi(1 - \rho_c) + \rho_c m \theta_c} (\varphi \rho m - mk(1 - \rho_c)^2 + 2k(1 - \rho_c)^2 N_M \varphi q_i \\ &\quad + ((\theta_c^2(1 - \rho_c) + 2S)\varphi + 2k\delta(1 - \rho_c))(1 - \rho_c)N_M) \end{aligned}$$

Using the above results the following Nash equilibrium can be derived:

$$\begin{aligned}
f_i &= \theta_i, & f_c &= (1 - \rho_c)\theta_c \\
q_i &= \frac{1}{3k\varphi(1 - \rho_c)^2 N_M + \rho_c \theta_c m} \left( \left( \frac{\theta_i^2 + 2S}{4k} - \frac{\delta}{2\varphi} + \frac{\varphi}{2\theta_c} \right) \rho_c \theta_c m \right. \\
&\quad \left. + (1 - \rho_c) \left( \left( \left( \frac{\theta_c^2}{2} + \theta_i^2 \right) (1 - \rho_c) + 3S - 2S\rho_c \right) \varphi \right. \right. \\
&\quad \left. \left. - k\delta(1 - \rho_c) N_M - \frac{1}{2}mk(1 - \rho_c) \right) \right) \\
q_c &= \frac{\varphi \rho_c m - mk(1 - \rho_c)^2}{3k\varphi(1 - \rho_c)^2 N_M + \rho_c \theta_c m} + \\
&\quad \frac{+(1 - \rho_c) N_M (k\delta(1 - \rho_c) + ((\theta_c^2 + \frac{1}{2}\theta_i^2) (1 - \rho_c) + 3S - S\rho_c) \varphi)}{3k\varphi(1 - \rho_c)^2 N_M + \rho_c \theta_c m}
\end{aligned}$$

so that the position of the indifferent patient is:

$$\begin{aligned}
x_M &= \frac{2\rho_c \theta_c^2 m}{2\theta_c((3(1 - \rho_c)kN_M\varphi + \rho_c \theta_c m))} \left( \frac{(\theta_i^2 + 2S)\varphi}{4mk} + \frac{\delta}{2m} - \frac{\varphi^2}{2m\theta_c} \right) \\
&\quad + (1 - \rho_c) \frac{\varphi N_M (\rho \theta_c^2 - \theta_c^2 + \theta_i^2) \varphi + k(2N_M\delta + m)}{m \cdot 6(1 - \rho_c)kN_M\varphi + 2\rho_c \theta_c m}.
\end{aligned}$$

The above quantities for the case  $\rho_c = 1$  are reported in equation (10) in the text. Observe that for  $\rho_c = 1$  the quality offered by the public provider is not affected by the presence of a private competitor and is equal to the benchmark. However, depending on the level of  $\theta_i$  the above solution may be feasible or not. As discussed in the text, the following conditions have to be satisfied:

$$q_c > q_i + \frac{\delta}{\varphi} - \frac{m}{2N_M\varphi}, \quad q_c < q_i + \frac{\delta}{\varphi}.$$

The first condition is satisfied for all values of  $\theta_i$  whenever  $S > k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} \right)$ , otherwise it is sufficient to have:

$$\theta_i > \sqrt{2 \left( k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} \right) - S \right)}. \quad (18)$$

The second is true if

$$\theta_i < \sqrt{2 \left( k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} + \frac{m}{\varphi N_M} \right) - S \right)} \quad (19)$$



under the condition that the quantity under the square root is positive.

Apart from the market shares, the market is feasible only if the profit of private hospitals entering the market is not negative, that is if:

$$\Pi_i^M = \left( S + \theta_i f_i - \frac{1}{2} f_i^2 - k q_i \right) 2x_M - H \geq 0.$$

Substituting the relevant quantities in the above equation we get:

$$\Pi_i^M = 2\varphi \frac{k}{m} \left( \frac{\theta_i^2 + 2S}{4k} + \frac{1}{2} \left( \frac{\delta}{\varphi} - \frac{\varphi}{\theta_c} \right) \right)^2 - H,$$

which can be translated into a further condition on  $\theta_i$ :

$$\theta_i \geq \sqrt{2 \left( k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} \right) + \sqrt{\frac{2Hmk}{\varphi}} - S \right)}. \quad (20)$$

Obviously condition (20) is more restrictive than condition (18), thus the lowest possible value is the one above. In order for the mixed market to be feasible it is then sufficient that the following inequality is satisfied:

$$\sqrt{2 \left( k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} \right) + \sqrt{\frac{2Hmk}{\varphi}} - S \right)} < \sqrt{2 \left( k \left( \frac{\varphi}{\theta_c} - \frac{\delta}{\varphi} + \frac{m}{\varphi N_M} \right) - S \right)}.$$

The above is equivalent to the following condition on  $N_M$ :

$$N_M < \sqrt{\frac{mk}{2H\varphi}}.$$

The conditions for the market to be feasible are summarised in the text (see equation (11)).

## A.2 Salop competition with private hospitals on the circle

The indifferent patient is located at  $x_P = \frac{\varphi(q_i - q_j)}{2m} + \frac{1}{2N_P}$ . From (8) the objective of hospital  $i$  sitting between hospital  $i - 1$  and hospital  $i + 1$  is:

$$\max_{f_i, q_i} \left( S + \theta_i f_i - \frac{1}{2} f_i^2 - k q_i \right) \left( \frac{1}{N_P} + \frac{\varphi}{2m} (2q_i - q_{i+1} - q_{i-1}) \right)$$

Under the feasibility condition  $\frac{1}{N_P} + \frac{\varphi}{2m}(2q_i - q_{i+1} - q_{i-1}) > 0$  from the F.O.C.s the following reaction functions are easily derived:

$$\begin{aligned} f_i^P &= \theta_i, \\ q_i &= \frac{1}{4}(q_{i+1} + q_{i-1}) + \frac{\theta_i^2 + 2S}{4k} - \frac{m}{2\varphi N_P} \end{aligned}$$

If providers are symmetric it must hold  $q_i = q_j$  for all  $i$  and  $j$ , thus

$$q_i^P = \frac{\theta_i^2 + 2S}{2k} - \frac{m}{\varphi N_P},$$

which is meaningful only if

$$\theta_i > \sqrt{\frac{2mk}{\varphi N_P} - 2S}.$$

Substituting the values of  $f_i^P$  and  $q_i^P$  back into (8) the profit of each provider is found:

$$\Pi_i^P = \frac{mk}{\varphi N_P^2} - H;$$

obviously, the condition  $\Pi_i^P \geq 0$  has to be satisfied, meaning that the number of private providers is  $N_P = \left\lfloor \sqrt{\frac{mk}{H\varphi}} \right\rfloor$ . In order to have  $N_P \geq 2$  the condition  $H \leq \frac{mk}{4\varphi}$  must hold.

## B Welfare comparison

Total welfare is defined as the aggregation of consumers' welfare and private providers' profits, net of the costs deriving from the reimbursement and the possible covering of losses. Apart from a common constant part that considers the average income and the reimbursement costs, the other terms are the net welfare derived from the service and the difference between profits and losses of the providers. In what follows we analyse the case where  $S = H$ , i.e. the reimbursement is equal to the one in benchmark also when competition is allowed. For convenience we define  $\eta_i = \theta_i^2$  and analyse the welfare parts using also this variable. Consistently, we will call  $\eta_i^L = (\theta_i^L)^2$  and  $\eta_i^U = (\theta_i^U)^2$ .

## B.1 Comparison of patients net utility

The first comparison will be made examining only the money equivalent welfare derived from the service, net of the transport costs (in what follows it will implicitly be assumed that the related existence conditions hold). From (10) and (13) in the three examined scenarios (benchmark, mixed and private market) this is equal to:

$$\begin{aligned}
\Phi_B &= v + \frac{\varphi^2}{\theta_c} - \delta \\
\Phi_M &= v + (1 - 2N_M x_M)(q_c \varphi - \delta) + 2N_M x_M \varphi q_i^M - 2m N_M \frac{1}{2} x_M^2 \\
&= v + \frac{\varphi^2}{\theta_c} - \delta + N_M \frac{\varphi^2}{4m} \left( \frac{\eta_i + 2H}{2k} + \frac{\delta}{\varphi} - \frac{\varphi}{\theta_c} \right)^2 \\
\Phi_P &= v + \varphi q_i^P - m N_P x_P^2 \\
&= v + \varphi \frac{\eta_i + 2H}{2k} - \frac{5}{4} \frac{m}{N_P}.
\end{aligned}$$

$\Phi_M$  is increasing in  $N_M$  and is higher than  $\Phi_B$ .

$\Phi_P$  depends linearly on  $\eta_i$ , while  $\Phi_M$  is a quadratic convex function of  $\eta_i$ . Its derivative wrt  $\eta_i$  is:

$$N_M \frac{\varphi}{4mk} \left( \frac{\varphi}{2k} \eta_i + \frac{\varphi}{k} H - \left( \frac{\varphi^2}{\theta_c} - \delta \right) \right).$$

The upper bound of the derivative is found by substituting  $\eta_i^U$  in the above equation. From (11c) this value is  $\frac{\varphi}{4k}$ , while the derivative of  $\Phi_P$  wrt  $\eta_i$  is  $\frac{\varphi}{2k}$ . From Lagrange's theorem this means that the graphs of  $\Phi_P$  and  $\Phi_M$  have at most a common point. Since both functions are increasing, when the value of  $\Phi_M - \Phi_P$  for  $\theta_i = \theta_i^U$  is positive this part of the welfare will be greater in a mixed market for all the values of  $\theta_i$  that allow its existence. Since

$$(\Phi_M - \Phi_P)|_{\eta_i = \eta_i^U} = \frac{m}{4N_M N_P} (5N_M - 3N_P)$$

if  $5N_M \geq 3N_P$  the condition  $\Phi_M > \Phi_P$  will be satisfied for  $\eta_i \in [\eta_i^L, \eta_i^U]$ . The value of the difference for  $\eta_i^L$  is:

$$(\Phi_M - \Phi_P)|_{\eta_i = \eta_i^L} = N_M \frac{H \varphi}{2k} - \sqrt{\frac{2mH \varphi}{k}} + \frac{5}{4} \frac{m}{N_P}. \quad (21)$$

When  $5N_M < 3N_P$  and the above value is negative,  $\Phi_P$  is higher than  $\Phi_M$  for all the admissible values of  $\eta_i$ , if it is positive the graphs of  $\Phi_P$  and  $\Phi_M$  will intersect at some point. Since the rhs of (21) is zero when  $N_M$  is equal to  $2\sqrt{\frac{2mk}{H\phi}} - \frac{5mk}{2\phi N_P H}$ , the intersection point exists if

$$2\sqrt{\frac{2mk}{H\phi}} - \frac{5mk}{2\phi N_P H} < N_M < \frac{3}{5}N_P. \quad (22)$$

Thus the following result proves Proposition 1.

Note that if  $N_M = 1$  the inequality  $\Phi_P > \Phi_M$  is always satisfied if  $\theta_i$  is high enough. Also, by standard algebra the interval in (22) is non empty if  $N_P < \frac{5\sqrt{2}}{6}\sqrt{\frac{mk}{H\phi}}$ , which by (14) is always true. However,  $N_M$  is an integer, therefore whether the condition is true or not depends on the parameters. Since  $\theta_i^U$  decreases with  $N_M$ , for high values of  $\theta_i$  only a mixed market with one private provider is feasible and (22) is valid for  $N_M = 1$  only if  $2\sqrt{\frac{2mk}{H\phi}} - \frac{5mk}{2\phi N_P H} < 1$ . Since  $N_P \leq \sqrt{\frac{mk}{H\phi}}$  the latter equation is verified for any  $N_P \leq 3$ , but for higher numbers of competitors it depends on the ratio between  $N_P$  and  $\sqrt{\frac{mk}{H\phi}}$ : if the latter is sufficiently high (i.e. profits for private providers are low)  $\phi_P$  is always higher than  $\phi_M$  with  $N_M = 1$ .

## B.2 Profits comparison

### Profits in the private market

The total profits of providers on a private market are:

$$N_P \Pi_i^P = \left( \frac{mk}{\phi N_P^2} - H \right) N_P$$

and do not depend on  $\eta_i$ . Since  $N_P = K$  when  $\frac{mk}{(K+1)^2\phi} < H \leq \frac{mk}{K^2\phi}$  the following inequality holds:

$$0 \leq N_P \left( \frac{mk}{N_P^2\phi} - H \right) < N_P \left( \frac{mk}{N_P^2\phi} - \frac{mk}{(N_P+1)^2\phi} \right) = \frac{mk}{\phi} \frac{2N_P+1}{N_P(N_P+1)^2}.$$

Profits are therefore bounded by  $\frac{5mk}{18\phi}$  and tend to zero as  $H \rightarrow 0$ .

### Profits in the mixed market

Total profits of private providers in the mixed market are:

$$\Pi_M = \Pi_i^M N_M = N_M((2H + \eta_i - 2kq_i^M)x_M - H).$$

$\Pi_M$  increases linearly with  $N_M$  and is a quadratic, convex and increasing function of  $\eta_i$ . If  $\frac{mk}{2(N+1)^2} \leq H < \frac{mk}{2N^2}$  for some integer  $N$ , on each interval  $\eta_i^L \leq \eta_i < \eta_i^U(n)$  with  $n \leq N$  the number of private competitors  $N_M$  can be chosen between 1 and  $n$  and for each fixed value of  $\eta_i$  the maximum profit is obtained for  $N_M = n$ . Also, on each interval the value of  $\Pi_M$  is comprised between 0 and  $n\left(\frac{mk}{2\varphi n^2} - H\right)$ , which is a decreasing function of  $n$ .

### Profit difference

From the above analysis the sign of  $\Pi_P - \Pi_M$  depends on several factors: if  $H$  is near to the threshold  $\frac{mk}{K^2\varphi}$  for some integer  $K$  then  $\Pi_P$  is near to zero and the difference will therefore be negative for almost all values of  $\eta_i$  for any choice of  $N_M$ . As  $H$  decreases towards  $\frac{mk}{(K+1)^2\varphi}$  the term  $\Pi_P - \Pi_M$  gradually increases and only for high values of  $\eta_i$  the difference may be negative. In fact for  $\eta_i$  tending to some  $\eta_i^U(n)$  from the left the difference is less than:

$$\frac{mk}{\varphi} \frac{2N_P + 1}{N_P(N_P + 1)^2} - \frac{mk}{2n\varphi} + nH \leq \frac{mk}{\varphi} \left( \frac{2N_P + 1}{N_P(N_P + 1)^2} - \frac{1}{2n} + \frac{n}{N_P^2} \right). \quad (23)$$

By standard algebraic calculations it turns out that if  $n = 1$  the quantity above is negative whenever  $N_P > 2$ , therefore if  $H \leq \frac{mk}{9\varphi}$   $\Pi_M$  is higher than  $\Pi_P$  for values of  $\eta_i$  high enough. More generally, the sign of the rhs of (23) equals that of:

$$-N_P^4 - 2N_P^3 + (2n^2 + 4n - 1)N_P^2 + (4n^2 + 2n)N_P + 2n^2$$

and by Descartes rule the above polynomial has a unique positive root in the variable  $N_P$  whose value increases with  $n$ . This means that as  $H$  decreases the difference  $\Pi_P - \Pi_M$  can be negative for  $\eta_i$  sufficiently high also for values of  $n > 1$ . Note also that from (23) this difference depends linearly on  $\frac{k}{\varphi}$ .

### “Net profit” in the mixed market

In the mixed market the loss of the public hospital equals  $2HN_M$ , which from (10) for each fixed value of  $N_M$  is increasing in  $\eta_i$  from the value  $HN_M\sqrt{\frac{2H\varphi}{mk}}$  to  $H$ . The “net profit” in the mixed market is :

$$-2HN_Mx_M + N_M\Pi_i^M = N_M((\eta_i - 2kq_i^M)x_M - H). \quad (24)$$

From (10) the above quantity can be written as a quadratic, convex function of  $\eta_i$ . Since the its derivative for  $\eta_i = \eta_i^L$  is equal to:

$$\frac{\varphi}{2mk} \left( \sqrt{\frac{2Hmk}{\varphi}} - H \right) > 0 \quad (25)$$

it is also increasing in  $\eta_i$ . Its value for  $\eta_i = \eta_i^L$  is negative, while for  $\eta_i = \eta_i^U$  it is positive only when  $N_M < \sqrt{\frac{1}{4} + \frac{mk}{2H\varphi}} - \frac{1}{2}$ .

### B.3 Welfare comparison: benchmark vs mixed

The total welfare in a mixed market is given by:

$$W_M = W_B + N_M(x_M(mx_M - 2kq_i^M + \eta_i) - H),$$

where  $W_B$  is the welfare of the benchmark scenario. Thus, if the term  $x_M(mx_M - 2kq_i^M + \eta_i) - H$  is positive, a mixed market will give a higher welfare level than the benchmark and at the same time the highest welfare level is reached by allowing the maximum number of private providers into the market. The term  $x_M$  is linear and increasing in  $\eta_i$ , while from (10) it is:

$$mx_M - 2kq_i^M + \eta_i = \left( \frac{\varphi}{4k} + \frac{1}{2} \right) \eta_i + \left( \frac{\varphi}{2} + k \right) \left( \frac{\delta}{\varphi} - \frac{\varphi}{\theta_c} \right) + \frac{H\varphi}{2k} - H. \quad (26)$$

so the same is also true also for this quantity. Its minimum value is achieved for  $\eta_i = \eta_i^L$  and is equal to:

$$\sqrt{\frac{2Hmk}{\varphi}} \left( \frac{\varphi}{2k} + 1 \right) - 2H$$

which is non negative for all values of  $H$  for which the mixed market exists, thus  $W_M$  is increasing in  $\eta_i$ . Substituting the minimum value for  $\eta_i$  we get:

$$x_M(mx_M - 2kq_i^M + \eta_i) \geq \frac{\varphi H}{k} \left( \frac{1}{2} - \frac{1}{m} \sqrt{\frac{2Hmk}{\varphi}} \right)$$

thus if  $H \leq \frac{m\varphi}{8k}$  for all  $\eta_i$  it is  $W_M > W_B$ . This is true for all values of  $H$  that allow the existence of a mixed market if  $\varphi \geq 2k$ .

Since we also have

$$W_M - W_B \leq \frac{m}{4N_M} \left( 1 + \frac{2k}{\varphi} \right) - H(N_M + 1)$$

there exist values of  $\eta_i$  for which  $W_M \geq W_B$  only if  $H < \frac{m(\varphi+2k)}{4\varphi N_M(N_M+1)}$ . The result in Proposition 2 is thus proved.

## B.4 Welfare comparison: benchmark vs private

In the private market setup total welfare is equal to:

$$W_P = \bar{Y} - \beta - H + v + \varphi \frac{\eta_i + 2H}{2k} - \frac{5m}{4N_P} + \left( \frac{mk}{\varphi N_P^2} - H \right) N_P.$$

From (14) if  $\varepsilon = \frac{N_P}{\sqrt{\frac{mk}{H\varphi}}}$  it is

$$\frac{1}{2} \leq 1 - \frac{1}{\sqrt{\frac{mk}{H\varphi}}} < \varepsilon = \frac{\lfloor \sqrt{\frac{mk}{H\varphi}} \rfloor}{\sqrt{\frac{mk}{H\varphi}}} \leq 1, \quad (27)$$

and

$$W_P = \bar{Y} - \beta - H + v + \varphi \frac{\eta_i + 2H}{2k} + \sqrt{\frac{Hmk}{\varphi}} \frac{1}{\varepsilon} \left( 1 - \varepsilon^2 - \frac{5}{4} \frac{\varphi}{k} \right)$$

thus

$$W_P - W_B = \varphi \frac{\eta_i + 2H}{2k} - \left( \frac{\varphi^2}{\theta_c} - \delta \right) + \sqrt{\frac{Hmk}{\varphi}} \frac{1}{\varepsilon} \left( 1 - \varepsilon^2 - \frac{5}{4} \frac{\varphi}{k} \right).$$

The difference is increasing in  $\eta_i$  and from (15) has the following lower limit:

$$\frac{1}{\varepsilon} (1 - \varepsilon^2) \sqrt{\frac{Hmk}{\varphi}} - \frac{1}{4\varepsilon} \sqrt{\frac{Hm\varphi}{k}} - \left( \frac{\varphi^2}{\theta_c} - \delta \right)$$

whose sign depends on the parameters. Since it is reasonable to assume that  $\frac{\varphi^2}{\theta_c} - \delta > 0$ , if the ratio  $\frac{\varphi}{k}$  is sufficiently low there can exist values for  $\varepsilon$  and  $\theta_c$  for which

the above quantity is positive. In these cases  $W_P - W_B > 0$  for all admissible values of  $\eta_i$ . Note however that if for example  $\varepsilon = 1$ , that is if  $N_P = \sqrt{\frac{mk}{H\varphi}}$  and the profit of private providers is zero, the above quantity is negative. In these cases there exists a threshold for  $\eta_i$  below which the welfare in benchmark is higher than in the private market and lower for higher values of this parameter.

Evaluating the difference  $W_P - W_B$  for the lowest threshold that allows the existence of a mixed market we have:

$$(W_P - W_B)_{|\eta_i=\eta_i^L} = \sqrt{\frac{Hmk}{\varphi}} \frac{1}{4k\varepsilon} (4\sqrt{2}\varepsilon\varphi - 4\varepsilon^2k + 4k - 5\varphi).$$

By standard algebra, the last term is positive if  $\varepsilon < \frac{1}{2k}(\sqrt{2}\varphi + \sqrt{4k^2 - 5k\varphi + 2\varphi^2})$ , but this condition is always verified since the quantity on the rhs is greater than 1, therefore the quantity in the above equation is positive. This proves Proposition 3.

## B.5 Welfare comparison: mixed vs private

From Proposition 3 if both conditions in (11) and (15) are satisfied the welfare level of a private market is higher than that of the benchmark solution. From Proposition 2 it is necessary to compare the welfare levels of a private and a mixed market only when  $W_M > W_B$ . Also, the highest level of welfare in the mixed market will in these cases be achieved by choosing the greatest possible number of private competitors on the border.