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Traditional banks, online banks, and number of branches

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Abstract We develop a model of competition between a traditional bank offering its services through physical branches and an Internet bank offering its services exclusively online, and where the customers are heterogeneous in their location in the space as well as in the disutility they sustain when patronizing the Internet bank. We investigate the choice of the traditional bank about the number of branches it decides to operate through. We find the conditions under which the multi-branch model is the preferred option for the traditional bank, and those under which the single-branch model is preferred. We show that the choice of the traditional bank depends on both the characteristics of the demand side and the intensity of the transportation costs.

Keywords Banking competition · Internet banking · Branches

JEL Classification D21 · D43

1 Introduction

Internet banks are a quite recent and increasing phenomenon in the banking industry. The Internet banks (also known as on-line banks, or direct banks) do not have branches with tellers. In contrast, the customers have access to their money in

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several ways, including ATMs, web pages and mobile apps. The essential characteristic that differentiates the Internet banks from the traditional bricks-and-mortar banks is that the depositors have not to walk to physical branches in order to operate financial transactions (i.e. move money between deposit accounts, check accounts, pay bills…).

Internet banks are now relevant players in the banking industry. In a recent study (TNS 2012), it has been shown that the US four major direct banks—Ally Bank, Discover Bank, Capitol One 360, and USAA—have gained share in the past decades among retail customers establishing their primary banking relationships, whereas the traditional banks have decreased their share. The deposits at the US major direct banks have more than doubled from 2008 to 2012, and their growth rate is three times the industry average in the period. Similar figures can be found in Europe (BLUECAP 2013).

On the other hand, traditional banks are still significant actors. Traditional banks operate through physical branches. The branches have been traditionally seen as the principal place of interaction between the bank and its clients, and traditional banks willing to enlarge their customer base typically went through an expansion of the number of branches. While practitioners seem to suggest that the existence of Internet banks should induce a reduction in the number of branches of the traditional banks,[1] the empirical evidence is far from being univocal. For example, in the US, the number of branches increased by a significant 27 % in the period between 1994 and 2006, and it decreased only after that year (Hannan and Hanweck 2008). In France the number of branches increased constantly from 1996 to 2008, while it has remained almost constant after that year. Similar trends can be observed also in Spain and Italy, whereas an opposite trend—with a reduction in the number of the existing branches in the mentioned period—is observed in Germany, Sweden and UK (ECB 2013). That is, the picture is quite complex and it does not seem that the existence of on-line banks has been accompanied everywhere by a reduction of the branches of the traditional banks.[2]

The present paper discusses the optimal strategy of a traditional bank when it has to decide whether to operate with one or two branches and it is faced by an Internet bank that customers may have access to without incurring in transportation costs. In particular, we show that the mixed picture of the trend of the number of branches can be explained by the existence of countervailing incentives of the traditional banks, where the relative strength of these incentives is related to the heterogeneity of customers with regard to the disutility they sustain when accessing the direct bank.

We build on Bouckaert and Degryse (1995) and Degryse (1996), and we develop a parsimonious competition model where the disutility of a depositor choosing the traditional bank depends on the physical distance between the customer's location

[1] For example, DELOITTE (2008) suggests reducing the number of physical branches while increasing the number of the financial services offered to the customers in the remaining branches.

[2] As reported in *The Telegraph*, 14 July 2014: "*more than a half of bank customers use a branch every month despite the rise of Internet and mobile banking*".

and the position of the branch in the space.[3] Therefore, the disutility that a depositor sustains when he deposits in the traditional bank depends on his location in the space and on the location of the branch. On the other hand, if a depositor chooses the Internet bank, he does not pay any transportation cost, as he can operate from everywhere without moving to a branch. At the same time, there is a source of (fixed) disutility when operating through an Internet bank, which is due to the absence of face-to-face interaction with bank's employees. The depositors are heterogeneous with respect to this kind of disutility. For example, for customers who are familiar with the Internet, having no direct interaction with tellers does not imply a significant disutility cost, but the opposite is true for those clients which rarely use the Internet. Moreover, having a personal relationship with a banker may represent a relevant benefit for some people, especially for those customers that want to get personal financial advice,[4] whereas other customers are less worried about the absence of a personal relationship with bank's employees. For these reasons, we distinguish between two types of depositors, type-Y ("young") depositors, which suffer a low fixed disutility when they choose the Internet bank, and type-O ("old") depositors, which suffer a high fixed disutility when they choose the Internet bank. Neither type-Y depositors nor type-O depositors sustain transportation costs when they choose the Internet bank. Therefore, the disutility that a depositor sustains when he deposits his money in the Internet bank does not depend on his location in the space, but it depends on which type of depositor he is.

We consider a two-stage game where in the first stage the traditional bank decides whether to compete against the Internet bank with one or two physical branches, while in the second stage the banks compete through the deposit interest rates. We show that it may be optimal for the traditional bank to operate with one branch instead of two, even if building new branches is costless, depending on how many customers with a strong disutility from accessing the Internet bank (the "old" customers) there are with respect to the customers with a weak disutility from accessing the Internet bank (the "young" customers). In particular, we identify two driving forces behind the decision of the traditional bank about whether or not to open new branches when an Internet bank is a player in the market. On one hand, by opening a new branch, the traditional bank reduces the transportation costs that some customers have to sustain when going to the traditional bank. On the other hand, the Internet bank reacts to this strategy by increasing the deposit interest rate. As the deposit interest rates are strategic complements, the traditional bank also increases its own deposit interest rate, thus paying a higher interest rate to those clients that would go to the traditional bank even in the case of a single branch. The former effect dominates when there are many customers with a strong disutility from using the Internet: in this case, it is better for the traditional bank to have two

[3] Othe competition models applied to banking industry are for example Chiesa (1998, 2001), Niinimaki (2004) and Bolt and Tieman (2004).

[4] Personal preferences may indeed motivate the ongoing existence of physical branches. The following comment of a customer clarifies the point: "*I'm not yet comfortable dealing with an entity that it is entirely electronic, with no people to sit down and see (and possibly go back to if something goes wrong). I assume the direct bank would have customer service reps available by phone, but that doesn't ease my unease!*" (TNS 2012).

branches. In contrast, when there are many customers with a weak disutility from Internet, the latter effect dominates, and the profits of the traditional bank are higher when it operates with only one branch.

The literature on the determinants of the number of bank branches is quite scarce. Avery et al. (1999) and Damar (2007) focus on the impact of mergers between banks on the number of branches existing in a market by adopting an econometric perspective, and find that mergers of banks are associated with a reduction of the branches. Hannan and Hanweck (2008), by using a US banking industry panel data from 1988 to 2006, find that the number of branches is positively associated with the rate of return that the banks are able to obtain from their investments, and inversely related to market concentration. Kim and Vale (2000), by using a data set of Norwegian banks for the period 1988–1995, show that the banks act strategically in their branching decision, as they take into account the reaction of the rival banks to their strategic decision. Rice and Davis (2007) show that the deregulation of previously existing branch restrictions impacts positively on the creation of new branches. None of these papers however considers the role of direct banks in traditional banks' branches proliferation or reduction.

The present paper is also rooted in the literature discussing the applications to banking of spatial competition models.[5] Ali and Greenbaum (1977) generalize the Hotelling model to generate an equilibrium banking structure and an adjustment path reflecting a variety of industry characteristics, and test the predictions of the model by using US banking industry data. Kim et al. (2007) model banking competition through a generalization of the Salop model, where identical traditional banks compete via the deposit interest rate, the loan interest rate and the number of branches, and estimate the model with Spanish data in order to investigate the effects of deregulation of both interest rates and branches on the strategic conduct of the banks. Barros (1999) proposes a spatial model of multi-branch and multi-market competition based on the Salop circle: the theoretical framework is then applied to data of the Portuguese commercial banking industry. One of the main conclusions of the paper is that the measurement of the market power and the explanation of the margins in the banking industry must take into account the local and the spatial nature of the banking industry.

We enlarge the existing theoretical literature on banking competition by including a non-spatial player (direct bank) with no branches that compete with a spatial player (traditional bank) adopting one or two branches. This paper is closely related to Bouckaert and Degryse (1995) and Degryse (1996). As these papers, we take the existence of the banking firms as given and we focus only on the liability side. Moreover, as in Bouckaert and Degryse (1995) and Degryse (1996), we take into consideration technology advancements (Internet banking) that allow reducing the transportation costs for the customers when they obtain financial services. However, in Bouckaert and Degryse (1995) and Degryse (1996) each traditional bank has to decide whether to introduce such technological improvements (like phone-banking, or Internet services) in its own offer. In contrast, we consider a situation where the innovative services are provided by a different bank (the direct

[5] Indeed, as argued by Barros (1999), "*the banking industry is one of the best examples of industries where local market competition matters*" (p. 350).

bank), and we investigate the optimal branch decision of a fully traditional bank that offers its own services entirely through physical branches.

The rest of the paper proceeds as follows. In Sect. 2 we introduce the model. In Sect. 3 we discuss the deposit interest rate equilibrium when the traditional bank has one branch only, whereas in Sect. 4 we consider the case where the traditional bank has two branches. In Sect. 5 we discuss the optimal choice of the traditional bank in terms of number of branches, by finding the conditions under which it is more profitable for the traditional bank to have two branches instead of one, and vice versa. In Sect. 6 we extend the basic model by allowing the transportation costs to be positively or negatively correlated with the customer's type. Section 7 concludes. The proofs are relegated in the Appendix.

2 The model

As in Bouckaert and Degryse (1995) and Degryse (1996), we suppose that the depositors are uniformly distributed in the space. In particular, consider a linear segment of length one, and let us denote with $x \in [0, 1]$ the location of the depositor in the segment. Moreover, let us assume that depositors may be of two types, say "old" (O) and "young" (Y).[6] At each location, there is a share of O (resp. Y) depositors which is given by λ (resp. $1 - \lambda$), where $\lambda \in [0,\ 1]$. Each depositor has one unit of money to be invested in a bank. There are two banks. Bank A is a traditional bank that operates through physical branches. Bank I is an Internet bank which has no branches, and it operates exclusively on-line. The deposit interest rate of Bank A and Bank I is given by r_A and r_I, respectively. Following Bouckaert and Degryse (1995) and Degryse (1996), we assume that the banks invest the proceeding of their deposits and obtain an identical rate of return, say R, per unit of money. Therefore, the profit function for Bank $i = A, I$ is described by the following function:

$$\pi_i = (R - r_i)D_i \tag{1}$$

where D_i is the amount of deposits attracted by Bank i. We assume that R is sufficiently high to guarantee that both banks obtain positive profits in equilibrium. The value attributed by a depositor to a deposit account depends on both his location and type, as well as on the deposit interest rate offered by the bank where the depositor places his money. If a depositor goes to the traditional bank, it pays the transportation costs. Let us assume for the moment that the transportation costs do not depend on whether he is of type Y or type O,[7] but they depend only on the location of the depositor and the branch in the space. Therefore, the value of the deposit account at Bank A for a depositor located at x is:

$$V_{A,x} = v + r_A - t\left|x - b_j\right| \tag{2}$$

where v indicates the reservation value, which is assumed to be large enough such that the market is always covered in equilibrium, $t > 0$ indicates the unit

[6] The model could be easily extended to the case of a continuum of depositor types. Details are available.

[7] This assumption will be removed in Sect. 6.

transportation costs, and b_j is the location of branch $j = \{1, 2\}$. Clearly, in the case of a single branch, branch b_2 does not exist.

On the other hand, if the depositor chooses the Internet bank, he does not pay the transportation costs. However, it pays additional (fixed) costs. For example, these costs may describe the disutility from not having face-to-face interaction with the bank's employees. Such additional disutility costs depend on the depositor's type: they are lower for type-Y depositors, while they are larger for type-O depositors. Therefore, the value of the deposit account at Bank I is as follows:

$$V_{I,Y} = v + r_I - K_Y \tag{3}$$

$$V_{I,O} = v + r_I - K_O \tag{4}$$

for a depositor of type Y and type O, respectively, and where K_O and K_Y indicate the disutility from operating financial operations through the Internet. Let us assume that $K_O > K_Y > 0$.[8]

Finally, let us introduce the following restriction on the parameters of the model. In particular, we assume that:

Assumption 1

$$\max[0, \lambda_O^m] \leq \lambda \leq \min[1, \lambda_{1Y}^m]$$

where $\lambda_O^m \equiv \frac{3K_O - 2K_Y - t}{2(K_O - K_Y)}$ and $\lambda_{1Y}^m \equiv \frac{2K_Y + t}{4(K_O - K_Y)}$. Assumption 1 guarantees interior solutions both in the case of a single branch and in the case of multiple branches.[9]

The timing of the game is the following. In the first stage of the game the traditional bank decides whether to open one branch or two branches. We make the simplifying assumption that opening branches is costless. This allows us focusing only on the strategic determinants of the decision of the traditional bank about whether to operate with one or two branches. In the second stage of the game, the two banks compete in the deposit interest rates. Therefore, depending on the first-stage Bank A's decision, there are two sub-games: in the first, Bank A has only one branch (single-branch model), while in the second Bank A has two branches (multi-branch model). We analyse the two sub-games in turn, and then we shall focus on the first-stage decision of the traditional bank.[10]

[8] Our model allows considering also the case of banks offering financial services of different qualities. Indeed, the quality of the service offered by the Internet bank may be higher than the quality of the service offered by the traditional bank. For example, Internet may allow to make several financial operations simultaneously and not to spend time at the bank desk. This can incorporated in our model by assuming that the utility of depositing in the Internet bank is $V_{I,Z} = v_I + r_I - K_Z$, with $Z = \{Y, O\}$ and $v_I > v$. Clearly, this is the same as assuming that K_Z is lower.

[9] Assumption 1 is removed in the Appendix, where the case of corner solutions is considered. We shall show that the results are qualitatively the same in the case of corner solutions.

[10] As pointed out by an anonymous reviewer, this set-up could be adopted also for competition between on-line shops and bricks-and-mortar shops (see for example Balasubramanian 1998). However, the issue of proliferation of physical branches/stores appears to be a much more relevant problem for traditional banks, as illustrated by the empirical evidence discussed in Sect. 1. On the other hand, traditional shops react to on-line competition mainly through price competition (Nakayama 2009; Forman et al. 2009).

3 Traditional bank with one branch

In this section we consider the case where Bank A in the first stage has decided to operate with one branch only. The superscript "s" shall be used to refer to the single-branch model. We assume that the branch is located at point 0, that is $b_1 = 0$.[11] Therefore, the value of the deposit account in the traditional bank is $V_{A,x} = v + r_A^s - tx$. Given the deposit interest rates of the two banks, there are two indifferent depositors between Bank A and Bank I, depending on the depositor's type. In particular, by equating $V_{A,x}$ and $V_{I,Y}$, we get $x_Y = \frac{K_Y + r_A^s - r_I^s}{t}$, which represents the depositor of type Y which is indifferent between depositing his money in the traditional bank and in the on-line bank. Therefore, type-Y depositors located at the left of x_Y deposit in Bank A, while the depositors located at the right deposit in Bank I. In the same way, by equating $V_{A,x}$ and $V_{I,O}$, we get $x_O = \frac{K_O + r_A^s - r_I^s}{t}$, which represents the indifferent type-O depositor. Clearly, type-O depositors located at the left of x_O deposit in Bank A, while those located at the right deposit in Bank I. As expected, the type-O indifferent depositor is always located to the right of the type-Y indifferent depositor, as type-O depositors are more prone to deposit in Bank A than type-Y depositors. We focus on the case of interior solutions, that is $x_Y(r_A^{s*}, r_I^{s*}) \in [0, 1]$ and $x_O(r_A^{s^*}, r_I^{s*}) \in [0, 1]$, where r_A^{s*} and r_I^{s*} represent the equilibrium deposit interest rates of Bank A and Bank I, respectively, in the case of a single branch for the traditional bank. This situation is represented in Fig. 1, where the grey area indicates the demand of the traditional bank, whereas the white area indicates the demand of the Internet bank.

Therefore, the profits of the two banks can be written as follows:

$$\pi_A^s = \left(R - r_A^s\right)x_Y + \left(R - r_A^s\right)\lambda(x_O - x_Y) = \frac{\left(R - r_A^s\right)\left[K_Y + r_A^s - r_I^s + \lambda(K_O - K_Y)\right]}{t} \tag{5}$$

$$\begin{aligned}\pi_I^s &= \left(R - r_I^s\right)(1 - \lambda)(x_O - x_Y) + \left(R - r_I^s\right)(1 - x_O) \\ &= \frac{\left(R - r_I^s\right)\left[t - K_Y - r_A^s + r_I^s - \lambda(K_O - K_Y)\right]}{t}\end{aligned} \tag{6}$$

Each bank maximizes its profits by choosing the optimal deposit interest rate for any given deposit interest rate set by the rival. The best-reply functions are $b_A\left(r_I^s\right) = \frac{\left[R + r_I^s - (1-\lambda)K_Y - \lambda K_O\right]}{2}$ and $b_I\left(r_A^s\right) = \frac{\left[R + r_A^s - t + (1-\lambda)K_Y + \lambda K_O\right]}{2}$ for Bank A and Bank I, respectively. It can be seen that the deposit interest rates are strategic complements.

[11] Clearly, one may assume a different location of the physical branch. In a model with endogenous choice of the branch location, it is expected that $b_1 = 1/2$ in the case of a single branch and $b_1 = 0$ and $b_2 = 1$ in the case of two branches (see Sect. 4). The results would be qualitatively the same, with only the critical thresholds varying with respect to the case of exogenous locations of branches. Indeed, in the case of endogenous locations of branches, the parameter set under which the traditional bank prefers two branches instead of one would be narrower, as the location of the single branch at point 0 is not optimal. However, the present set-up better describes the situation where a traditional bank has to decide whether or not to add another branch to the existing one.

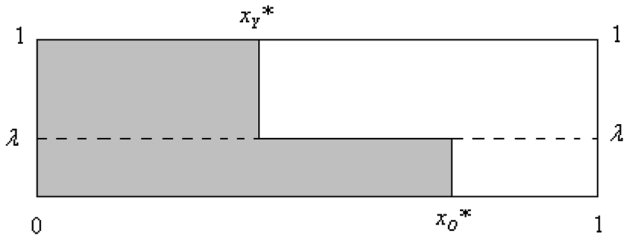

Fig. 1 The single-branch model with interior solutions

By solving the system composed by the best-reply functions, we get the equilibrium deposit interest rates when the traditional bank operates with a unique branch. They are:[12]

$$r_A^{s\,*} = R - \frac{t + K_Y + \lambda(K_O - K_Y)}{3}; \quad r_I^{s*} = R - \frac{2t - K_Y + \lambda(K_O - K_Y)}{3} \tag{7}$$

We have to check now the conditions under which the critical depositors x_O and x_Y are located in the interval [0,1]. It is easy to observe that $x_O{}^* \geq 0$ and $x_O{}^* \geq (\leq)\, 1$ if $\lambda \leq (\geq)\, \lambda_O^s \equiv \frac{3K_O - 2K_Y - 2t}{2(K_O - K_Y)}$, while $x_Y{}^* \geq (\leq)\, 0$ if $\lambda \leq (\geq)\, \lambda_{1Y}^s \equiv \frac{K_Y + t}{2(K_O - K_Y)}$ and $x_Y{}^* \geq (\leq)\, 1$ if $\lambda \leq (\geq)\, \lambda_{2Y}^s \equiv \frac{K_Y - 2t}{2(K_O - K_Y)}$. As $\lambda_O^s \geq \lambda_{2Y}^s$, an interior solution emerges if and only if $\max[0, \lambda_O^s] \leq \lambda \leq \min[1, \lambda_{1Y}^s]$, which is always satisfied under Assumption 1, as $\lambda_O^s \leq \lambda_O^m$ and $\lambda_{1Y}^s \geq \lambda_{1Y}^m$. Allegedly, in the case of interior solutions, the equilibrium profits are:

$$\pi_A^{s*} = \frac{[t + K_Y + \lambda(K_O - K_Y)]^2}{9t}; \quad \pi_I^{s*} = \frac{[2t - K_Y - \lambda(K_O - K_Y)]^2}{9t} \tag{8}$$

As expected, the profits of the traditional bank increase with the percentage of type-O depositors. Indeed, type-O depositors are less prone to on-line banking than type-Y depositors, thus an increase of the fraction of type-O depositors is beneficial for the traditional bank. Clearly, the opposite holds when the percentage of type-Y depositors goes up. Not surprisingly, the profits of the traditional bank increase when either K_Y or K_O go up, as in this case it becomes relatively less costly for the depositors to deposit in the traditional bank. The impact of t on the profits of the traditional bank is interesting. Indeed, π_A^{s*} is U-shape in t.[13] This can be explained as follows. When t goes up, all the depositors pay higher transportation costs to go to the traditional bank. This is detrimental for the profits. On the other hand, when the depositors served by the traditional bank are all close to the location of the branch, Bank A can set a lower deposit interest rate [see Eq. (7)]. This is beneficial for the profits. When t is low the first effect dominates and the profits of Bank A decreases with t, but after a threshold the second effect dominates and the profits of Bank A start to increases with t. Not surprisingly the impact of the parameters on the

[12] The second-order conditions are satisfied everywhere, as $\partial^2 \pi_A^s / \partial r_A^{s^2} = \partial^2 \pi_I^s / \partial r_I^{s^2} = -2/t < 0$.

[13] The profits of the traditional bank are minimum when $t = \lambda K_O + (1 - \lambda) K_Y$.

profits of the on-line bank is the opposite than the impact on the profits of the traditional bank, with the exception of t: indeed, when t goes up, the profits of Bank I strictly increase, as it becomes more convenient for the depositors to choose the on-line bank.

4 Traditional bank with two branches

In this section we consider the sub-game where Bank A has opened two branches. We assume that the two branches are located at 0 and 1, that is $b_1 = 0$ and $b_2 = 1$. Therefore, if a depositor goes to the branch located at 0, the value of the deposit account is still indicated by $V^L_{A,x} = v + r^m_A - tx$, whereas if he goes to the branch located at 1, it is indicated by $V^R_{A,x} = v - r^m_A - t(1 - x)$, where the superscript "L" is used hereafter to indicate the branch in the region at the left, whereas the superscript "R" shall be used to indicate the branch in the region at the right, and the superscript "m" is used to refer to the multi-branch model. Note that we are assuming that the traditional bank cannot apply different interest deposit rates at different branches. Therefore, a depositor located at $x < 1/2$ ($x > 1/2$) never goes to the branch located at point 1 (0), whereas the depositor located at $x = 1/2$ is always indifferent between the two physical branches. Therefore, we have now four indifferent depositors. In region $x \leq 1/2$, the indifferent depositors are still given by x_Y and x_O. Let us denote now these critical depositors as x^L_Y and x^L_O, respectively, to indicate that they are located in the "left" region of the segment. In contrast, in region $x \geq 1/2$ the indifferent depositor of type Y is obtained by equating $V^R_{A,x}$ and $V_{I,Y}$, which yields $x^R_Y = 1 - \frac{K_Y + r^m_A - r^m_I}{t}$, while the indifferent type-O depositor is obtained by equating $V^R_{A,x}$ and $V_{I,O}$, which yields: $x^R_O = 1 - \frac{K_O + r^m_A - r^m_I}{t}$. We focus on the case where each bank receives the money of both types of depositors in equilibrium (i.e. interior solutions). This amounts to require that, in equilibrium, $x^L_Y(r^{m*}_A, r^{m*}_I) \in [0, 1/2]$, $x^L_O(r^{m*}_A, r^{m*}_I) \in [0, 1/2]$, $x^R_Y(r^{m*}_A, r^{m*}_I) \in [1/2, 1]$, and $x^R_O(r^{m*}_A, r^{m*}_I) \in [1/2, 1]$, where r^{m*}_A and r^{m*}_I represent the equilibrium deposit interest rates of Bank A and Bank I respectively in the case of multiple branches of the traditional bank. This situation is represented in Fig. 2.

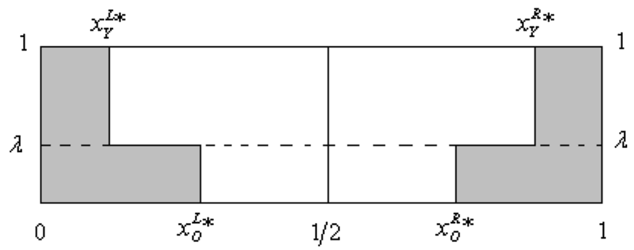

Fig. 2 The multi-branch model with interior solutions

Using x_Y^L, x_O^L, x_Y^R and x_O^R, the profits of the two banks can be written as follows:

$$\pi_A^m = \left(R - r_A^m\right)x_Y^L + \left(R - r_A^m\right)\lambda\left(x_O^L - x_Y^L\right) + \left(R - r_A^m\right)\left(1 - x_Y^R\right) + \left(R - r_A^m\right)\lambda\left(x_Y^R - x_O^R\right)$$
$$= \frac{2\left(R - r_A^m\right)\left[K_Y + r_A^m - r_I^m + \lambda(K_O - K_Y)\right]}{t} \tag{9}$$

$$\pi_I^m = \left(R - r_I^m\right)(1 - \lambda)\left(x_O^L - x_Y^L\right) + \left(R - r_I^m\right)\left(\frac{1}{2} - x_O^L\right)$$
$$+ \left(R - r_I^m\right)(1 - \lambda)\left(x_Y^R - x_O^R\right) + \left(R - r_I^m\right)\left(x_O^R - \frac{1}{2}\right)$$
$$= \frac{\left(R - r_I^m\right)\left[t - 2K_Y - 2r_A^m + 2r_I^m - 2\lambda(K_O - K_Y)\right]}{t} \tag{10}$$

The best-reply functions of the two banks are given by $b_A(r_I^m)$ and $\tilde{b}_I(r_A^m) = \frac{[2R + 2r_A^m - t + 2(1-\lambda)K_Y + 2\lambda K_O]}{4}$. It follows that the equilibrium deposit interest rates set by the two banks are[14]:

$$r_A^{m*} = R - \frac{t + 2K_Y + 2\lambda(K_O - K_Y)}{6}; \quad r_I^{m*} = R - \frac{t - K_Y - \lambda(K_O - K_Y)}{3} \tag{11}$$

We have to check now the conditions under which the interior solutions occur in equilibrium. Before proceeding, note that the model is symmetric. It follows that $x_Y^{L*} + x_Y^{R*} = 1$ and $x_O^{L*} + x_O^{R*} = 1$. As a consequence, $x_O^{L*} \leq (\geq) 1/2 \Leftrightarrow x_O^{R*} \geq (\leq) 1/2$ and $x_Y^{L*} \leq (\geq) 0 \Leftrightarrow x_Y^{R*} \geq (\leq) 1$. Therefore, we can focus on the conditions for the interior solutions in the region $x \leq 1/2$, as they guarantee that also the conditions for the interior solutions in the region $x \geq 1/2$ are satisfied. We have that $x_O^{L*} \geq 0$ and $x_O^{L*} \geq (\leq) 1/2$ if $\lambda \leq (\geq) \lambda_O^m$, while $x_Y^{L*} \geq (\leq) 0$ if $\lambda \leq (\geq) \lambda_{1Y}^m$ and $x_Y^{L*} \geq (\leq) 1/2$ if $\lambda \leq (\geq) \lambda_{2Y}^m \equiv \frac{K_Y - t}{2(K_O - K_Y)}$. As $\lambda_O^m \geq \lambda_{2Y}^m$, an interior solution emerges if $\max[0, \lambda_O^m] \leq \lambda \leq \min[1, \lambda_{1Y}^m]$ (Assumption 1). Allegedly, the equilibrium profits are the following:

$$\pi_A^{m*} = \frac{[t + 2K_Y + 2\lambda(K_O - K_Y)]^2}{18t}; \quad \pi_I^{m*} = \frac{2[t - K_Y - \lambda(K_O - K_Y)]^2}{9t} \tag{12}$$

Note that the impact of the parameters on the equilibrium profits of the traditional bank and of the on-line bank in the case of multiple branches of Bank A replicates the impact in the case of a single branch. Therefore, the profits of Bank A increase with λ, K_O and K_Y, whereas they are U-shape in t.[15] On the other hand, the profits of Bank I decrease with λ, K_O and K_Y, whereas they increase in t.

[14] The second-order conditions are satisfied everywhere, as $\partial^2\pi_A^m \big/ \partial r_A^{m^2} = \partial^2\pi_I^m \big/ \partial r_I^{m^2} = -4/t < 0$.

[15] In the case of two branches, the profits of Bank A are minimum when $t = 2[\lambda K_O + (1 - \lambda)K_Y]$.

5 Single-branch model versus multi-branch model

In this section, we posit the central question of this paper by looking at the optimal strategy of the traditional bank about the number of branches when it is faced by an on-line bank. Therefore, we consider the first-stage decision of the traditional bank.

By comparing π_A^{s*} and π_A^{m*}, we can state the following proposition:

Proposition 1 *The traditional bank decides to open two-branches (one branch) if and only if* $\lambda \geq (\leq)\, \hat{\lambda}$, *where* $\hat{\lambda} = \frac{t\sqrt{2} - 2K_Y}{2(K_O - K_Y)}$.

Proof See the Appendix. □

Therefore, Proposition 1 shows that introducing an additional branch is profitable for the traditional bank if and only if the fraction of type-O depositors is large enough. On the other hand, if the fraction of type-Y depositors is sufficiently large, it is better for the traditional bank to operate only with one branch, even if opening new branches is costless. The intuition is the following. When the traditional bank decides to open two branches, two effects are at work. On one hand, having a new branch located in a different place allows the depositors to sustain lower transportation costs when they go to the traditional bank. In other words, all else being equal, a larger number of depositors prefer the traditional bank to the on-line bank. Therefore, all else being equal, opening an additional branch replicates the demand for one branch in the area of depositors located at $x \geq 1/2$. That is, when there are two branches, ceteris paribus, the demand of the traditional bank is higher with respect to the case of a single branch. This is an incentive to introduce additional branches. We refer to this effect as the *demand effect*. However, there is another effect at work. As we noted above, as a consequence of the creation of the second branch by the traditional bank, the on-line demand loses some depositors. In order to regain competitiveness, the on-line bank increases the deposit interest rate it offers to the depositors. As the deposit interest rates are strategic complements, the traditional bank increases its own deposit interest rate as well. However, by doing so, it pays more also the deposits of those depositors that would have gone to the traditional bank even in the absence of the new branch (i.e. those depositors that in the single-branch model go to the branch located at point 0). This is a disincentive to introduce the second branch. Let us refer to this effect as the *deposit rate effect*. When λ is low, there are few type-O depositors and many type-Y depositors. Therefore, the demand for the traditional bank is low. Allegedly, the demand effect when introducing an additional branch is dominated by the deposit rate effect, and the traditional bank prefers the single-branch model to the multi-branch model. On the other hand, when λ is high, there are many type-O depositors and few type-Y depositors, and the demand for the traditional bank is high. As a consequence, the demand effect dominates the deposit rate effect, and the traditional bank's profits in the multi-branch model are higher than in the single-branch model.

We consider now the impact of parameters K_Y, K_O and t on the profitability of the multi-branch model with respect to the single-branch model. We can state the following proposition:

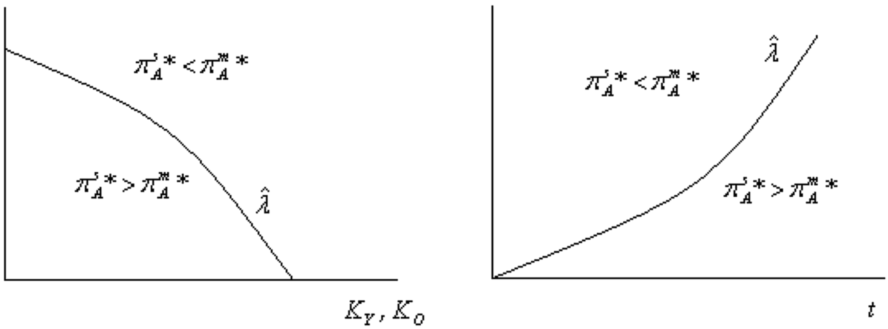

Fig. 3 The impact of the model parameters on the critical threshold

Proposition 2 *All else being equal, when K_Y and K_O increase, the profitability of the multi-branch model with respect to the profitability of the single-branch model increases. All else being equal, when t increases, the profitability of the multi-branch model with respect to the profitability of the single-branch model decreases.*

Proof See the Appendix. □

Figure 3 illustrates Proposition 2. The intuition of Proposition 2 is the following. When K_Y or K_O is high and t is low, the disutility for a depositor when depositing in the traditional bank instead than in the on-line bank is low. Therefore, the demand effect is strong, and it dominates the deposit rate effect. As a consequence, the traditional bank prefers opening an additional branch. At the opposite, when K_Y or K_O is low and t is high, the demand effect is dominated by the deposit rate effect, and the profits of the traditional bank are higher when it owns a single branch.

6 Extension

In this section we consider the following extension of the model. Until now, we have assumed that the depositors are heterogeneous with respect to the disutility that sustain when accessing the Internet bank, and they are also heterogeneous with respect to their location in the space (i.e. with respect to the disutility that sustain when going to the traditional bank). These two heterogeneities have been assumed to be unrelated. However, it may be that a correlation exists between the transportation costs and the disutility when choosing Internet banking. For example, it may be that young customers can move more quickly than old customers to reach the branch. In this case, both the disutility from Internet banking and the transportation costs are lower for type-Y depositors than for type-O depositors, and therefore there is a positive correlation between the disutility from Internet banking and the transportation costs. On the other hand, it may be that the time is more valuable for young customers than for old customers.[16] In this case the

[16] Imagine for example that the young depositors are workers, whereas the old depositors are retired people.

transportation costs are higher for type-Y depositors than for type-O depositors. Therefore, there is negative correlation between the disutility from Internet banking and the transportation costs.

To take into account the possibility of a negative or positive correlation between the disutility from Internet banking and the transportation costs, let us assume that type-Y depositors continue to pay a transportation cost equal to $t\left|x-b_j\right|$ as in (2), whereas type-O depositors pay a transportation cost equal to $\tau\left|x-b_j\right|$, with $\tau>0$. In what follows we focus on the case of interior solutions both in the case of a single branch and in the case of multiple branches.[17] As the analysis proceeds as in Sects. 3 and 4 we simply report the equilibrium deposit interest rates and the equilibrium profits. With a single branch, the equilibrium deposit interest rates are: $\tilde{r}_A^{s*}=R-\frac{K_Y\tau(1-\lambda)+t(\lambda K_O+\tau)}{3(t\lambda+\tau-\tau\lambda)}$ and $\tilde{r}_I^{s*}=R-\frac{2t\tau+K_Y\lambda\tau-tK_O\lambda-\tau K_Y}{3(t\lambda+\tau-\tau\lambda)}$, whereas the equilibrium profits are:

$$\tilde{\pi}_A^{s*}=\frac{\left[K_Y\tau(1-\lambda)+t(\lambda K_O+\tau)\right]^2}{9t\tau(t\lambda+\tau-\tau\lambda)};\quad \tilde{\pi}_I^{s*}=\frac{\left[2t\tau+K_Y\lambda\tau-tK_O\lambda-\tau K_Y\right]^2}{9t\tau(t\lambda+\tau-\tau\lambda)} \tag{13}$$

In the case of two branches, the equilibrium deposit interest rates are: $\tilde{r}_A^{m*}=R-\frac{2K_Y\tau(1-\lambda)+t(2\lambda K_O+\tau)}{6(t\lambda+\tau-\tau\lambda)}$ and $\tilde{r}_I^{m*}=R-\frac{t\tau+K_Y\lambda\tau-tK_O\lambda-\tau K_Y}{3(t\lambda+\tau-\tau\lambda)}$, whereas the equilibrium profits are

$$\tilde{\pi}_A^{m*}=\frac{\left[2K_Y\tau(1-\lambda)+t(2\lambda K_O+\tau)\right]^2}{18t\tau(t\lambda+\tau-\tau\lambda)};\quad \tilde{\pi}_I^{m*}=\frac{2\left[t\tau+K_Y\lambda\tau-tK_O\lambda-\tau K_Y\right]^2}{9t\tau(t\lambda+\tau-\tau\lambda)} \tag{14}$$

We are interested in finding the conditions under which the traditional bank finds it profitable to operate with two branches rather than one. This amounts comparing $\tilde{\pi}_A^{s*}$ and $\tilde{\pi}_A^{m*}$. It emerges that $\tilde{\pi}_A^{m*}\geq(\leq)\,\tilde{\pi}_A^{s*}$ if and only if $\lambda\geq(\leq)\,\tilde{\lambda}$, where:

$$\tilde{\lambda}=\frac{t\tau\sqrt{2}-2\tau K_Y}{2(tK_O-\tau K_Y)} \tag{15}$$

Let us compare $\hat{\lambda}$ and $\tilde{\lambda}$. It is immediate to observe that $\tilde{\lambda}\geq(\leq)\,\hat{\lambda}$ if and only if $t\geq(\leq)\,\tau$. This has an interesting implication. Note that for a depositor of type Y the decision about where to deposit the money, given the deposit interest rates of the banks, depends on the relative strength of its own disutility from Internet banking, K_Y, and the transportation costs, t. Similarly, for a depositor of type O, the decision is driven by the relative strength of K_O and τ. As $K_O>K_Y$ by assumption, when $\tau>t$ the two groups of depositors are more similar. Indeed, the difference in the disutility from using Internet banking between the two depositor types makes the type-Y depositors more prone to depositing in the Internet bank than the type-O depositors, but the difference between the transportation costs acts in the opposite direction, thus making the type-O depositors more prone to depositing in the Internet bank than the type-Y depositors. In contrast, when $\tau<t$, the difference

[17] The description of the parameter set supporting this condition in equilibrium is available upon request.

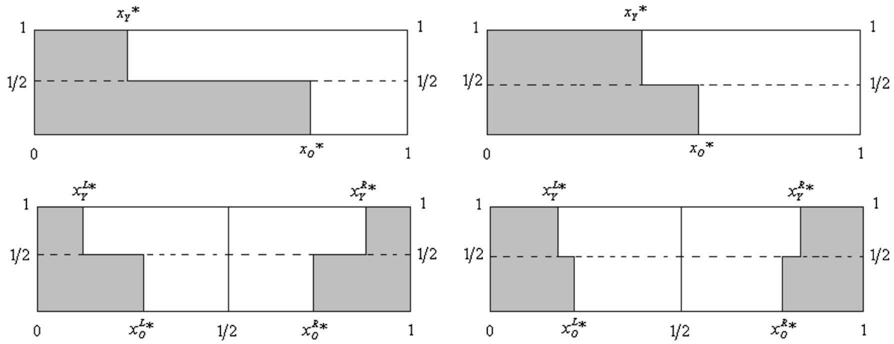

Fig. 4 The impact of depositors' heterogeneity on the traditional bank's choice

between the two groups is stronger, as both the difference in the disutility from using Internet banking and the difference in the transportation costs makes the type-*Y* depositors more prone to depositing in the Internet bank than the type-*O* depositors. This allows us to conclude with the following proposition:

Proposition 3 *When the depositors are more homogenous (heterogeneous), it is more (less) likely that the traditional bank prefers the multi-branch model to the single-branch model.*

Proof See the Appendix. □

The intuition of Proposition 3 is as follows. Consider Fig. 4. It represents two situations. The left side of Fig. 4 represents a situation where the depositors are highly heterogeneous, whereas the right side represents a situation where the depositors are more similar. Note that the demand of Bank *A* is the same in both situations. However, the deposit rate effect is stronger in the case of highly heterogeneous depositors. Indeed, if the Internet bank wants to continue to attract the money of type-*Y* depositors, it has to increase by a larger extent its own deposit interest rate in the case of heterogeneous depositors than in the case of homogenous depositors, as the threshold type-*Y* depositor is closer to Bank *A*'s branch. This implies that also the traditional bank increases the deposit interest rate by a larger extent. Therefore, all else being equal, a greater heterogeneity between depositors induces the traditional bank to prefer the single-branch model.

7 Conclusions

"*Physical branches will become irrelevant*." This is the prediction of Discover Bank's president of consumer banking and operations.[18] However, the empirical evidence of the evolution of the number of branches is mixed: in some countries, it has been observed a decrease in the number of branches, but in other countries the trend has shown a positive sign.

[18] See *American Banker* (1st March 2013).

In this paper we propose a simple model where a traditional bank competing against an Internet bank has to decide whether to adopt two physical branches or one physical branch before choosing the deposit interest rate. We show that both structures—multi-branch model and single-branch model—may be rationale from the point of view of the traditional bank. We argue that the choice depends on countervailing incentives for the traditional bank, and the relative strength of these incentives is related to the heterogeneity of depositors with respect to their disutility when depositing into the direct bank. In particular, the optimal branching decision of the traditional bank depends on two contrasting forces. On one hand, the traditional bank reduces the transportation costs of its own customers when it has two branches instead of one. This is an incentive for expanding the number of branches. On the other hand, the Internet bank sets a higher deposit interest rate when it is faced by a multi-branch traditional bank. As a consequence, also the traditional bank sets a higher deposit interest rate, and this is an incentive to reduce the number of branches. While the former incentive is strong when many customers have strong disutility from accessing the Internet bank, the latter incentive is strong when few customers have strong disutility. Moreover, we show that when the depositors are more homogenous, it is more likely that the traditional bank prefers to adopt two branches instead than one.

We propose a very stylized framework to capture the essential implications of the competition between a traditional bank and an Internet bank. The model can be extended in several directions to adhere more closely to the recent evolution of the banking industry. On one hand, many direct banks are opening physical branches. On the other hand, many traditional banks have started to operate also through the Internet, leaving to customers the choice about which type of platform (physical branch or Internet) to use. This type of hybrid banks has not been considered in the present paper, where the traditional bank is assumed not to operate also on-line. In this way, we aim to highlight the main incentives at work for on-line banks and traditional banks. In the case of a mixed bank (both on-line and off-line), those incentives would be confused. Moreover, including two on-line banks (one independent, and one belonging to the mixed bank) may be hard to tract in the present framework. Indeed, Internet banks have not a physical location. Therefore, the usual differentiation device in the Hotelling framework (i.e. the spatial distance), here does not work. It follows that a Bertrand paradox would apply with regard to the two on-line banks, driving the profits of both of them to zero.

Second, many customers that use Internet banks may still maintain relationships with traditional banks too. Indeed, more than 12 % of US households declare to have a deposit in one of the four major direct banks, but direct banks hold less than 5 % of the primary banking relationship of customers (*American Banker*, 1st March 2013). The possibility that clients deposit their money in several banks is likely to affect the optimal branch decision of the traditional bank, and it is left for future research.

Appendix 1: Proofs of propositions

Proposition 1 *The proof consists in the comparison of* π_A^{s*} *and* π_A^{m*} *in the parameter space defined by Assumption 1. It can be observed that they are equal when* $\lambda = \hat{\lambda}$, *whereas* $\pi_A^{s*} \geq (\leq) \pi_A^{m*}$ *when* $\lambda \leq (\geq) \hat{\lambda}$.

Proposition 2 *The proof consists in checking that* $\partial\hat{\lambda}\big/\partial K_Y \leq 0$, $\partial\hat{\lambda}\big/\partial K_O \leq 0$ *and* $\partial\hat{\lambda}\big/\partial t \geq 0$ *under Assumption 1.*

Proposition 3 *First,* $\tilde{\pi}_A^{m*} \geq \tilde{\pi}_A^{s*}$ *when* $\lambda \geq \tilde{\lambda}$. *Second, as* $\tilde{\lambda} \geq (\leq) \hat{\lambda}$ *if and only if* $t \geq (\leq) \tau$. *Moreover, under* $\tau \leq \sqrt{2}K_O$ *and* $t \geq \sqrt{2}K_Y$, *we have that* $\frac{\partial\tilde{\lambda}}{\partial t} \geq 0$ *and* $\frac{\partial\tilde{\lambda}}{\partial\tau} \leq 0$ *Therefore, the parameter space supporting* $\tilde{\pi}_A^{m*} \geq \tilde{\pi}_A^{s*}$ *is larger (narrower) when* $t \leq (\geq) \tau$.

Appendix 2: Corner solutions

Single branch

Consider the case of corner solutions. Depending on the value of λ with respect to the threshold values λ_O^s, λ_{1Y}^s and λ_{2Y}^s, we may have different situations:

- $\lambda \leq \min[1, \lambda_{2Y}^s]$: all the depositors of both types deposit in Bank A (Fig. 5a).
- $\max[0, \lambda_{2Y}^s] \leq \lambda \leq \min[1, \lambda_O^s, \lambda_{1Y}^s]$: all the type-$O$ depositors deposit in Bank A, while type-Y depositors are shared between Bank A and Bank I (Fig. 5b).
- $\lambda \geq \max[0, \lambda_O^s, \lambda_{1Y}^s]$: all the type-$Y$ depositors deposit in Bank I, while the type-O depositors are shared between Bank A and Bank I (Fig. 5c).
- $\max[0, \lambda_{1Y}^S] \leq \lambda \leq \min[1, \lambda_O^s]$: all the type-$O$ depositors deposit in Bank A, while all the type-Y depositors deposit in Bank I (Fig. 5d).

Note that the case represented in Fig. 5a represents a situation where Bank I has no market in equilibrium. However, it can be excluded by Assumption 1.

Let us consider now the situation represented in Fig. 5d, where Bank A serves all the type-O depositors whereas Bank I serves all the type-Y depositors. The following lemma shows that in this case there is no an equilibrium in pure strategies in the deposit interest rates:

Lemma 1 *Suppose that* $\max[0, \lambda_{1Y}^S] \leq \lambda \leq \min[1, \lambda_O^s]$. *In this case, there is no an equilibrium in pure strategies in the deposit interest rates.*

Proof Bank A serves all type-O depositors if the following inequality holds: $v + r_A^s - t \geq v + r_I^s - K_O$, or $r_A^s \geq r_I^s + t - K_O$, whereas Bank I serves all type-Y depositors if the following inequality holds: $v + r_A^s \geq v + r_I^s - K_Y$, or $r_I^s \geq r_A^s + K_Y$. Suppose that Bank I sets r_I^s. The best reply of Bank A under this demand structure is

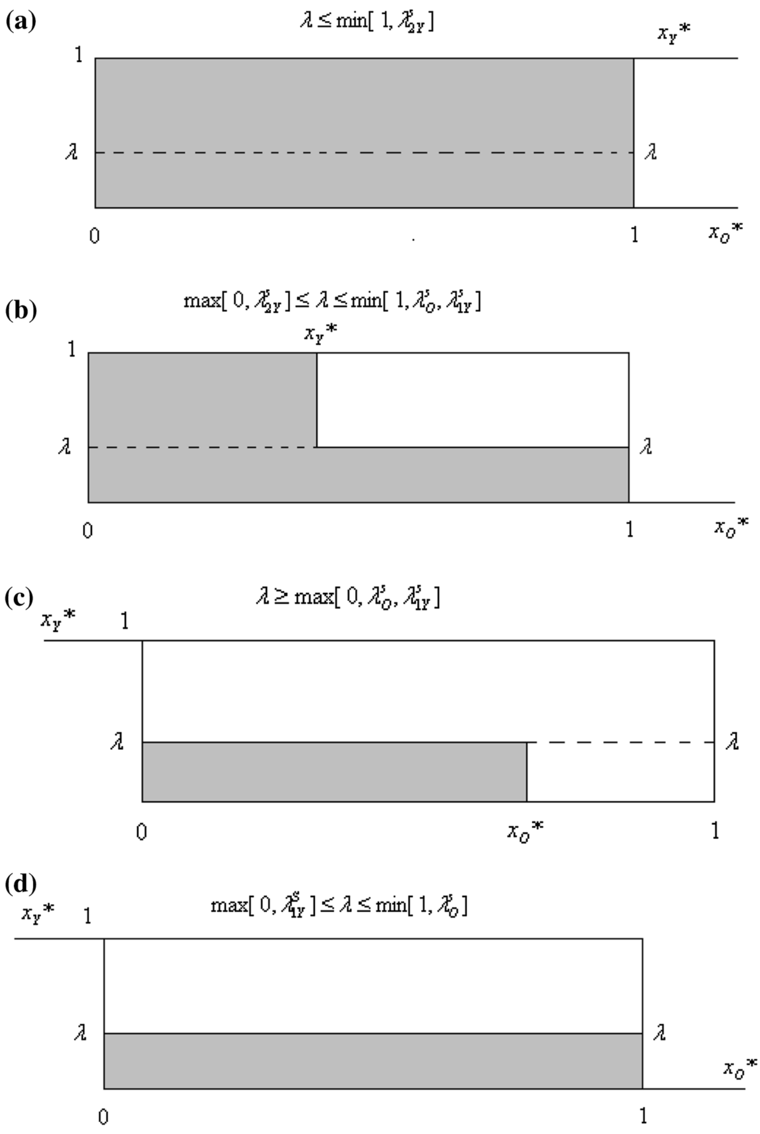

Fig. 5 The single-branch models with corner solutions. Cases **a** only Bank A, **b** all type-O depositors to Bank A and type-Y depositors shared, **c** all type-Y depositors to Bank I and type-O depositors shared, **d** all type-O depositors to Bank A and all type-Y depositors to Bank I

$r^s_A = r^s_I + t - K_O$. On the other hand, if Bank A sets r^s_A, the best reply of Bank I under this demand structure is given by $r^s_I = r^s_A + K_Y$. Therefore, the system of the best-reply functions has no solution, unless $t = K_O - K_Y$, but in this case we have that $\lambda^s_{1Y} = \lambda^s_O$. □

Therefore, in what follows we limit the analysis to the case of "single" corner solutions, that is when one type of depositors is shared in equilibrium by the banks, while the other type is served exclusively by one bank.

Type-O corner solution ($\max[0, \lambda^s_{2Y}] \leq \lambda \leq \min[1, \lambda^s_O, \lambda^s_{1Y}]$):

In this case, all type-O depositors go to the traditional bank, while type-Y depositors are shared in equilibrium between the banks. Therefore, the profits function becomes: $\pi^s_A = (R - r^s_A)x_Y + (R - r^s_A)\lambda(1 - x_Y)$ and $\pi^s_I = (R - r^s_I)(1 - \lambda)(1 - x_Y)$, for Bank A and Bank I, respectively. Standard maximization yields the equilibrium deposit rates: $r^{s*}_{A,\mathbf{corO}} = \frac{(3R-K_Y)(1-\lambda)-t(1+\lambda)}{3(1-\lambda)}$ and $r^{s*}_{I,\mathbf{corO}} = \frac{(3R+K_Y)(1-\lambda)-t(2-\lambda)}{3(1-\lambda)}$. Therefore, the equilibrium profits are $\pi^{s*}_{A,\mathbf{corO}} = \frac{[K_Y(1-\lambda)+t(1+\lambda)]^2}{9t(1-\lambda)}$ and $\pi^{s*}_{I,\mathbf{corO}} = \frac{[K_Y(1-\lambda)-t(2-\lambda)]^2}{9t(1-\lambda)}$.

Type-Y corner solution ($\lambda \geq \max[0, \lambda^s_O, \lambda^s_{1Y}]$):

In this case, all type-Y depositors go to the on-line bank, while type-O depositors are shared in equilibrium between the banks. Therefore, the profits function becomes $\pi^s_A = (R - r^s_A)\lambda x_O$ and $\pi^s_I = (R - r^s_I)(1 - \lambda)x_O + (R - r^s_I)(1 - x_O)$ for Bank A and Bank I, respectively. By maximizing the above profits functions, we get the equilibrium deposit interest rates: $r^{s*}_{A,\mathbf{corY}} = R - \frac{\lambda K_O+t}{3\lambda}$ and $r^{s*}_{I,\mathbf{corY}} = R + \frac{\lambda K_O-6t}{3\lambda}$. It follows that the equilibrium profits are $\pi^{s*}_{A,\mathbf{corY}} = \frac{(t+K_O\lambda)^2}{9t\lambda}$ and $\pi^{s*}_{I,\mathbf{corY}} = \frac{(2t-K_O\lambda)^2}{9t\lambda}$.

Multiple branches

We have that depending on the value of λ with respect to the threshold values λ^m_O, λ^m_{1Y} and λ^m_{2Y}, several situations may arise:

- $\lambda \leq \min[1, \lambda^m_{2Y}]$: all the depositors of both types deposit in Bank A (Fig. 6a).
- $\max[0, \lambda^m_{2Y}] \leq \lambda \leq \min[1, \lambda^m_O, \lambda^m_{1Y}]$: all the type-$O$ depositors deposit in Bank A, while the type-Y depositors are shared between Bank A and Bank I (Fig. 6b).
- $\lambda \geq \max[0, \lambda^m_O, \lambda^m_{1Y}]$: all the type-$Y$ depositors deposit in Bank I, while the type-O depositors are shared between Bank A and Bank I (Fig. 6c).
- $\max[0, \lambda^m_{1Y}] \leq \lambda \leq \min[1, \lambda^m_O]$: all the type-$O$ depositors deposit in Bank A, while all the type-Y depositors deposit in Bank I (Fig. 6d).

As for the case of the single-branch model, the situation represented in Fig. 6a can be excluded by Assumption 1. Furthermore, in the situation represented in Fig. 6d there are no pure strategies equilibria in the deposit interest rates, as indicated in the following lemma:

Lemma 2 :*Suppose that* $\max[0, \lambda^m_{1Y}] \leq \lambda \leq \min[1, \lambda^m_O]$. *In this case, there is no an equilibrium in pure strategies in the deposit interest rates.*

Proof The proof is analogous to the proof of Lemma 1, so it is omitted. □

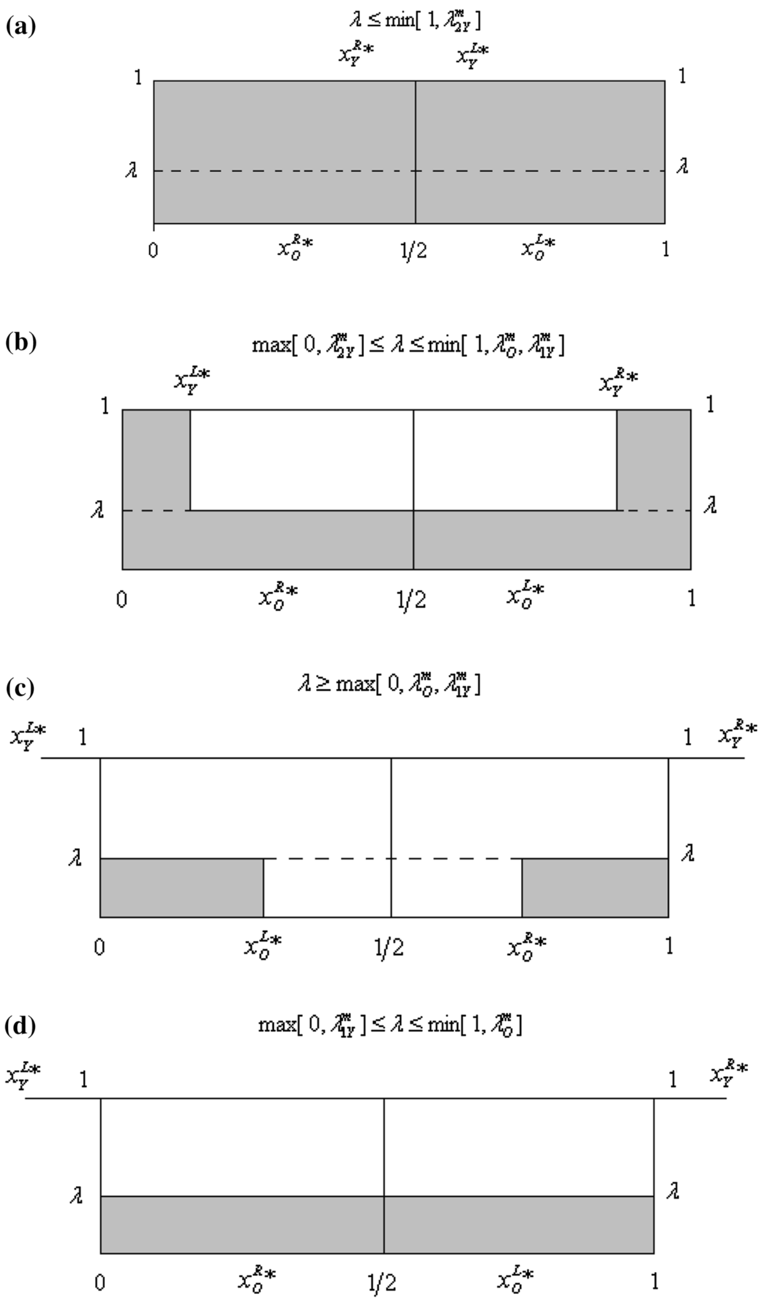

Fig. 6 The multi-branch models with corner solutions. Cases **a** only Bank A, **b** all type-O depositors to Bank A and type-Y depositors shared, **c** all type-Y depositors to Bank I and type-O depositors shared, **d** all type-O depositors to Bank A and all type-Y depositors to Bank I

Therefore, in what follows we focus on "single" corner solutions, where *i*) all the type-*O* depositors go to the traditional bank and the type-*Y* depositors are shared between the two banks, or *ii*) all the type-*Y* depositors go to the Internet bank and the type-*O* depositors are shared between the two banks.

Type-O corner solution $(max[0, \lambda^m_{2Y}] \leq \lambda \leq min[1, \lambda^m_O, \lambda^m_{1Y}])$:

In this case, only the type-*Y* depositors are shared in equilibrium between the banks, while all the type-*O* depositors go to the traditional bank. The profits of Bank *A* are: $\pi^m_A = (R - r^m_A)x^L_Y + (R - r^m_A)\lambda(\frac{1}{2} - x^L_Y) + (R - r^m_A)(1 - x^R_Y) + (R - r^m_A)\lambda(x^R_Y - \frac{1}{2})$, while the profits of Bank *I* are $\pi^m_I = (R - r^m_I)(1 - \lambda)(\frac{1}{2} - x^L_Y) + (R - r^m_I)(1 - \lambda)(x^R_Y - \frac{1}{2})$. From standard maximization of the profits functions, we get the equilibrium deposit interest rates: $r^m_{A,\mathbf{corO}}{}^* = \frac{2(3R - K_Y)(1-\lambda) - t(1+\lambda)}{6(1-\lambda)}$ and $r^m_{I,\mathbf{corO}}{}^* = \frac{2(3R + K_Y)(1-\lambda) - t(2-\lambda)}{6(1-\lambda)}$. The equilibrium profits are $\pi^m_{A,\mathbf{corO}}* = \frac{[2K_Y(1-\lambda) + t(1+\lambda)]^2}{18t(1-\lambda)}$ and $\pi^m_{I,\mathbf{corO}}* = \frac{[2K_Y(1-\lambda) - t(2-\lambda)]^2}{18t(1-\lambda)}$.

Type-Y corner solution $(\lambda \geq max[0, \lambda^m_O, \lambda^m_{1Y}])$:

In this situation, all the type-*Y* depositors posit their deposit in the on-line bank, while the type-*O* depositors are shared in equilibrium between the two banks. The profits functions of Bank *A* and Bank *I* are now: $\pi^m_A = (R - r^m_A)\lambda x^L_O + (R - r^m_A)\lambda(1 - x^R_O)$ and $\pi^m_I = (R - r^m_I)(1 - \lambda)x^L_O + (R - r^m_I)(\frac{1}{2} - x^L_O) + (R - r^m_I)(1 - \lambda)(1 - x^R_O) + (R - r^m_I)(x^R_O - \frac{1}{2})$, respectively. By maximizing the profits functions of the two banks, we get: $r^m_{A,\mathbf{corY}}* = R - \frac{2\lambda K_O + t}{6\lambda}$ and $r^m_{I,\mathbf{corY}}* = R + \frac{\lambda K_O - 3t}{3\lambda}$. Therefore, the equilibrium profits are $\pi^m_{A,\mathbf{corY}}* = \frac{(t + 2K_O\lambda)^2}{18t\lambda}$ and $\pi^m_{I,\mathbf{corY}}* = \frac{2(t - K_O\lambda)^2}{9t\lambda}$.

Comparison

Before proceeding, note that, depending on the parameters values, many situations are possible in principle. Let us denote by (J, W) a generic situation, where *J* refers to the situation in the case of one branch and *W* refers to the situation in the case of two branches. In particular, $J, W = \{\mathbf{int}, \mathbf{corO}, \mathbf{corY}\}$, where "**int**" indicates and interior solution, "**corO**" indicates the type-*O* corner solution, and "**corY**"indicates the type-*Y* corner solution. The following lemma establishes which situations are possible:

Lemma 3 *Only one of the following combinations is possible:* $(\mathbf{int}, \mathbf{int})$, $(\mathbf{int}, \mathbf{corY})$, $(\mathbf{int}, \mathbf{corO})$, $(\mathbf{corO}, \mathbf{corO})$ *and* $(\mathbf{corY}, \mathbf{corY})$.

Proof There are nine possible combinations in theory: $(\mathbf{int}, \mathbf{int})$, $(\mathbf{int}, \mathbf{corY})$, $(\mathbf{int}, \mathbf{corO})$, $(\mathbf{corY}, \mathbf{int})$, $(\mathbf{corY}, \mathbf{corY})$, $(\mathbf{corY}, \mathbf{corO})$, $(\mathbf{corO}, \mathbf{int})$, $(\mathbf{corO}, \mathbf{corY})$ and $(\mathbf{corO}, \mathbf{corO})$. We show now that some of these combinations are never possible. Consider $(\mathbf{corO}, \mathbf{int})$. It requires simultaneously that $\min[1, \lambda^s_0, \lambda^s_{1Y}] \geq \lambda \geq \max[0, \lambda^s_{2Y}]$ and $\min[1, \lambda^m_{1Y}] \geq \lambda \geq \max[0, \lambda^m_0]$, which is impossible as $\lambda^m_0 \geq \min[\lambda^s_0, \lambda^s_{1Y}]$. Consider case $(\mathbf{corO}, \mathbf{corY})$. It requires simultaneously that $\min[1, \lambda^s_0, \lambda^s_{1Y}] \geq \lambda \geq \max[0, \lambda^s_{2Y}]$ and $\lambda \geq \max[0, \lambda^m_0, \lambda^m_{1Y}]$, which is impossible as

$\lambda_0^m \geq \min[0, \lambda_0^s, \lambda_{1Y}^s]$. Consider $(\mathbf{corY}, \mathbf{int})$. It requires simultaneously that $\lambda \geq \max[0, \lambda_0^s, \lambda_{1Y}^s]$ and $\min[1, \lambda_{1Y}^m] \geq \lambda \geq \max[0, \lambda_0^m]$, which is impossible as $\lambda_{1Y}^m \geq \lambda_0^m$ implies $t \geq 2(K_O - K_Y)$, which in turn implies $\lambda_{1Y}^s \geq \max[1, \lambda_0^s]$. Consider $(\mathbf{corY}, \mathbf{corO})$. It requires simultaneously that $\lambda \geq \max[0, \lambda_0^s, \lambda_{1Y}^s]$ and $\min[1, \lambda_0^m, \lambda_{1Y}^m] \geq \lambda \geq \max[0, \lambda_{2Y}^m]$, which is impossible. Indeed, if $t \geq 3(K_O - K_Y)/2$, we have that $\lambda_{1Y}^s \geq \max[\lambda_0^m, \lambda_{1y}^m]$, if $6(K_O - K_Y)/5 \leq t \leq 3(K_O - K_Y)/2$ we have that $\lambda_0^m \geq \lambda_{1Y}^s \geq \lambda_{1y}^m \geq \lambda_0^s$, if $K_O - K_Y \leq t \leq 6(K_O - K_Y)/5$ we have that $\lambda_0^m \geq \lambda_{1Y}^s \geq \lambda_0^s \geq \lambda_{1y}^m$, and if $t \leq K_O - K_Y$ we have that $\lambda_0^m \geq \lambda_0^s \geq \lambda_{1Y}^s \geq \lambda_{1y}^m$. For the remaining cases it is possible to find a parameter space supporting each situation. For example, consider $(\mathbf{int}, \mathbf{int})$, $(\mathbf{int}, \mathbf{corY})$,$(\mathbf{int}, \mathbf{corO})$ and $(\mathbf{corO}, \mathbf{corO})$. If $t \in [K_O, (3K_O - 2K_Y)/2]$, with $K_Y \in [K_O/3, K_O/2]$, we have that $\lambda_{1Y}^s \geq 1 \geq \lambda_{1y}^m \geq \lambda_0^m \geq \lambda_0^s \geq \lambda_{2y}^m \geq \lambda_{2y}^s \geq 0$. Therefore a parameter exists such that $\min[1, \lambda_{1Y}^s] \geq \lambda \geq \max[0, \lambda_0^s]$ and $\min[1, \lambda_{1Y}^m] \geq \lambda \geq \max[0, \lambda_0^m]$; a parameter exists such that $\min[1, \lambda_{1Y}^s] \geq \lambda \geq \max[0, \lambda_0^s]$ and $\lambda \geq \max[0, \lambda_0^m, \lambda_{1y}^m]$; a parameter exists such that $\min[1, \lambda_{1Y}^s] \geq \lambda \geq \max[0, \lambda_0^s]$ and $\min[1, \lambda_0^m, \lambda_{1Y}^m] \geq \lambda \geq \max[0, \lambda_{2Y}^m]$; and a parameter exists such that $\min[1, \lambda_0^s, \lambda_{1Y}^s] \geq \lambda \geq \max[0, \lambda_{2Y}^s]$ and $\min[1, \lambda_0^m, \lambda_{1Y}^m] \geq \lambda \geq \max[0, \lambda_{2Y}^m]$. Finally, consider case $(\mathbf{corY}, \mathbf{corY})$. If $t \in [(3K_O - 3K_Y)/2, (2K_O - 3K_Y)]$, with $K_Y \in [K_O/4, K_O/3]$, we have that $1 \geq \lambda_{1Y}^s \geq \lambda_{1y}^m \geq \lambda_0^m \geq \lambda_0^s \geq \lambda_{2y}^m \geq \lambda_{2y}^s \geq 0$. Therefore a parameter exists such that $\lambda \geq \max[0, \lambda_0^s, \lambda_{1y}^s]$ and $\lambda \geq \max[0, \lambda_0^m, \lambda_{1y}^m]$.[19] □

Let us start with the case $(\mathbf{int}, \mathbf{corO})$.[20] This case emerges when $\max[0, \lambda_0^s, \lambda_{2Y}^m] \leq \lambda \leq \min[1, \lambda_{1Y}^s \lambda_0^m, \lambda_{1Y}^m]$. We state the following proposition:

Proposition 4 *Consider case* $(\mathbf{int}, \mathbf{corO})$. *The traditional bank decides to open two-branches (one branch) if and only if* λ,K_Y *and* t *are sufficiently high (low), and* K_O *is sufficiently low (high).*

Proof The proof consists in the comparison of $\pi_{A,\mathbf{int}}^s*$ and $\pi_{A,\mathbf{corO}}^m*$ in the parameter space supporting the case $(\mathbf{corO}\,,\,\mathbf{corO})$. It can be observed that they are equal when $\lambda = \lambda_{\mathbf{int},\mathbf{corO}}$, whereas $\pi_{A,\mathbf{int}}^s * \geq (\leq)\, \pi_{A,\mathbf{corO}}^m*$ when $\lambda \leq (\geq)\, \lambda_{\mathbf{int},\mathbf{corO}}$.[21] Furthermore, we have that $\partial\lambda_{\mathbf{int},\mathbf{corO}}/\partial K_Y \leq 0$, $\partial\lambda_{\mathbf{int},\mathbf{corO}}/\partial K_O \geq 0$ and $\partial\lambda_{\mathbf{int},\mathbf{corO}}/\partial t \leq 0$ in the parameter space supporting the case $(\mathbf{corO}\,,\,\mathbf{corO})$. □

Now we consider case $(\mathbf{int}, \mathbf{corY})$. The parameter set sustaining this case is $\max[0, \lambda_0^s, \lambda_0^m, \lambda_{1Y}^m] \leq \lambda \leq \min[1, \lambda_{1Y}^s]$. We have the following proposition:

Proposition 5 *Consider case* $(\mathbf{int}, \mathbf{corY})$. *The traditional bank decides to open two-branches (one branch) if and only if* λ,K_Y *and* t *are sufficiently low (high), and* K_O *is sufficiently high (low).*

[19] The complete characterization in terms of the parameters K_Y, K_O and t of the parameter space supporting each case is available upon request.

[20] The case $(\mathbf{int}, \mathbf{int})$ has been discussed in the main text.

[21] The complete expressions of $\lambda_{\mathbf{int},\mathbf{corO}}$ and $\lambda_{\mathbf{int},\mathbf{corY}}$ (see later) are too long for being reported here and they are available upon request.

Proof The proof consists in the comparison of $\pi^{s}_{A,\mathbf{int}}*$ and $\pi^{m}_{A,\mathbf{corY}}*$ in the parameter space supporting the case $(\mathbf{int},\mathbf{corY})$. It can be observed that they are equal when $\lambda=\lambda_{\mathbf{int},\mathbf{corY}}$, whereas $\pi^{s}_{A,\mathbf{int}}*\geq(\leq)\,\pi^{m}_{A,\mathbf{corY}}*$ when $\lambda\geq(\leq)\,\lambda_{\mathbf{int},\mathbf{corY}}$. Furthermore, we have that $\partial\lambda_{\mathbf{int},\mathbf{corY}}/\partial K_Y\leq 0$, $\partial\lambda_{\mathbf{int},\mathbf{corY}}/\partial K_O\geq 0$ and $\partial\lambda_{\mathbf{int},\mathbf{corY}}/\partial t\leq 0$ in the parameter space supporting the case $(\mathbf{int},\mathbf{corY})$. □

Now we consider the case $(\mathbf{corO},\mathbf{corO})$. The parameter set that sustains this case is $\max[0,\lambda^{s}_{2Y},\lambda^{m}_{2Y}]\leq\lambda\leq\min[1,\lambda^{s}_{0},\lambda^{s}_{1Y},\lambda^{m}_{0},\lambda^{m}_{1Y}]$. We have the following proposition[22]:

Proposition 6 *Consider case* $(\mathbf{corY},\mathbf{corY})$. *The traditional bank decides to open two-branches (one branch) if and only if λ and t are sufficiently low (high), and K_Y is sufficiently high (low).*

Proof The proof consists in the comparison of $\pi^{s}_{A,\mathbf{corO}}*$ and $\pi^{m}_{A,\mathbf{corO}}*$ in the parameter space supporting the case $(\mathbf{corO},\mathbf{corO})$. It can be observed that they are equal when $\lambda=\lambda_{\mathbf{corO},\mathbf{corO}}$, where $\lambda_{\mathbf{corO},\mathbf{corO}}\equiv\frac{2K_Y^2+t^2-2tK_Y\sqrt{2}}{2K_Y^2-t^2}$, whereas $\pi^{s}_{A,\mathbf{corO}}*\geq(\leq)\,\pi^{m}_{A,\mathbf{corO}}*$ when $\lambda\geq(\leq)\,\lambda_{\mathbf{corO},\mathbf{corO}}$. Furthermore, we have that $\partial\lambda_{\mathbf{corO},\mathbf{corO}}/\partial K_Y\geq 0$ and $\partial\lambda_{\mathbf{corO},\mathbf{corO}}/\partial t\leq 0$ in the parameter space supporting the case $(\mathbf{corO},\mathbf{corO})$. □

Finally, we consider the case $(\mathbf{corY},\mathbf{corY})$. The parameter set that sustains this case in equilibrium is $\max[0,\lambda^{s}_{0},\lambda^{s}_{1Y},\lambda^{m}_{0},\lambda^{m}_{1Y}]\leq\lambda\leq 1$. We state the following proposition[23]:

Proposition 7 *Consider case* $(\mathbf{corY},\mathbf{corY})$. *The traditional bank decides to open two-branches (one branch) if and only if λ and K_O are sufficiently high (low), and t is sufficiently low (high).*

Proof The proof consists in the comparison of $\pi^{s}_{A,\mathbf{corY}}*$ and $\pi^{m}_{A,\mathbf{corY}}*$ in the parameter space supporting the case $(\mathbf{corY},\mathbf{corY})$. It can be observed that they are equal when $\lambda=\lambda_{\mathbf{corY},\mathbf{corY}}$, where $\lambda_{\mathbf{corY},\mathbf{corY}}\equiv\frac{t}{K_O\sqrt{2}}$, whereas $\pi^{s}_{A,\mathbf{corY}}*\geq(\leq)\,\pi^{m}_{A,\mathbf{corY}}*$ when $\lambda\leq(\geq)\,\lambda_{\mathbf{corY},\mathbf{corY}}$. Furthermore, we have that $\partial\lambda_{\mathbf{corY},\mathbf{corY}}/\partial K_O\leq 0$ and $\partial\lambda_{\mathbf{corY},\mathbf{corY}}/\partial t\geq 0$ in the parameter space supporting the case $(\mathbf{corY},\mathbf{corY})$. □

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[22] Clearly, K_O has no impact in this case, as all type-O depositors go to Bank A both when it has one branch and when it has two branches.

[23] In this case, K_Y has no impact on the choice of the traditional bank, as all type-Y depositors go to Bank I both when Bank A has one branch and when it has two branches.

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