



Chromatic Correlation Clustering



Francesco Bonchi

Aristides Gionis

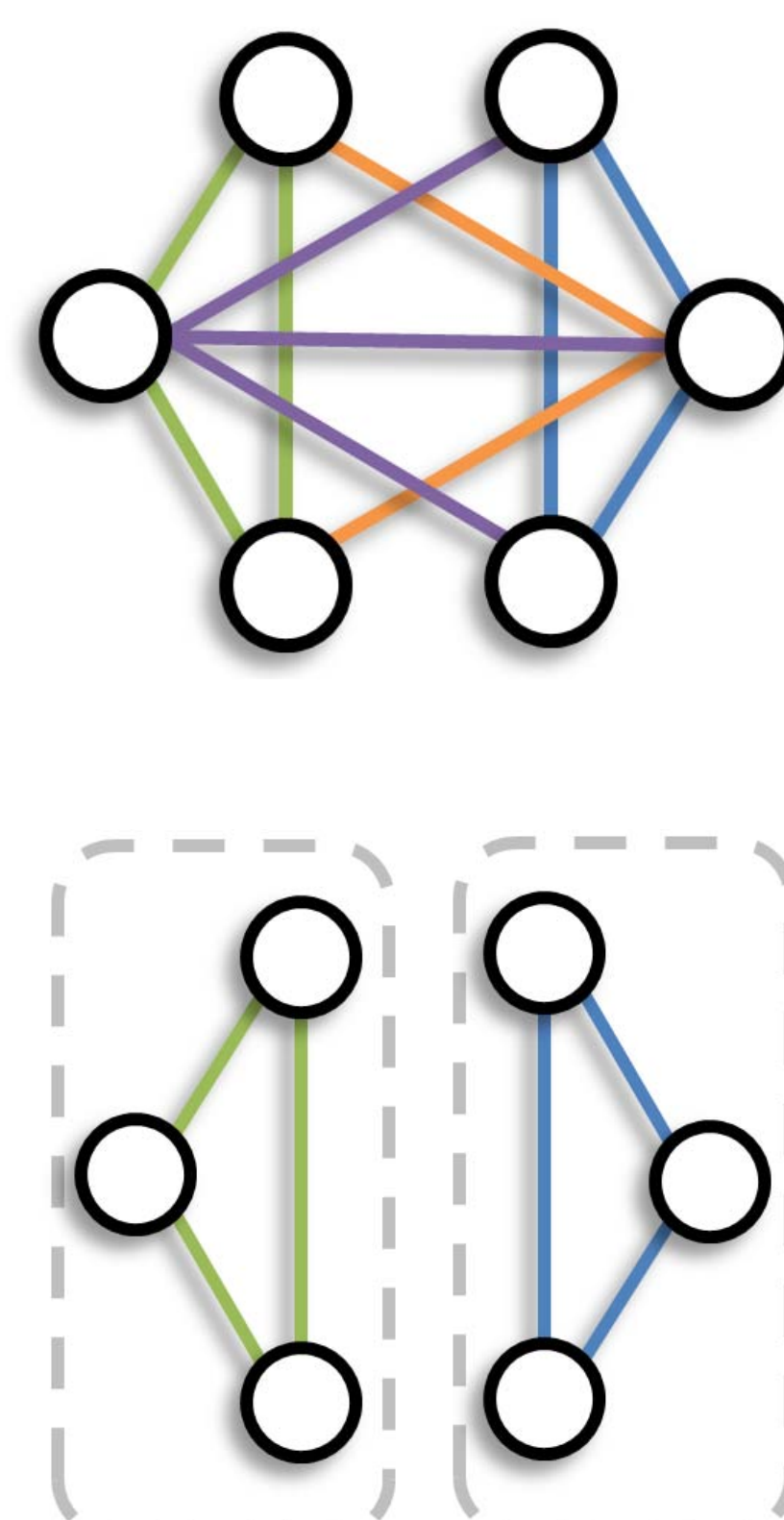
Francesco Gullo

Antti Ukkonen

Yahoo! Research – Barcelona

{bonchi,gionis,gullo,aukkonen}@yahoo-inc.com

- We study a novel clustering problem in which the pairwise relations between objects are **categorical**. This problem can be viewed as clustering the vertices of a graph whose edges have different types (**colors**).
- **Applications**: social networks, protein-to-protein interaction networks, bibliographic networks, and more.
- We define an **objective function** to partition the graph so that the edges in each cluster have, as much as possible, the same color.
- The problem is **NP-hard**. We propose an **approximation algorithm** with provable guarantee, as well as two practical **heuristic algorithms**.
- Experimental evidence on **synthetic** and **real** datasets show that our algorithms outperform a baseline algorithm both in the task of reconstructing a ground-truth clustering and in terms of objective function value.

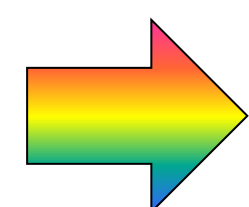


Problem definition:

from **CORRELATION CLUSTERING**...

Given a set of objects V and a pairwise similarity function $\text{sim} : V \times V \rightarrow [0, 1]$, find a clustering $\mathcal{C} : V \rightarrow \mathbb{N}$ that minimizes the cost

$$\text{cost}(\mathcal{C}) = \sum_{\substack{(x,y) \in V \times V \\ \mathcal{C}(x) = \mathcal{C}(y)}} (1 - \text{sim}(x, y)) + \sum_{\substack{(x,y) \in V \times V \\ \mathcal{C}(x) \neq \mathcal{C}(y)}} \text{sim}(x, y).$$



... to **CHROMATIC CORRELATION CLUSTERING**

Given a set V of objects, a set L of labels, a special label l_0 , and a pairwise labeling function $\ell : V \times V \rightarrow L \cup \{l_0\}$, find a clustering $\mathcal{C} : V \rightarrow \mathbb{N}$ and a cluster labeling function $\text{cl} : \mathcal{C}[V] \rightarrow L$ so to minimize the cost

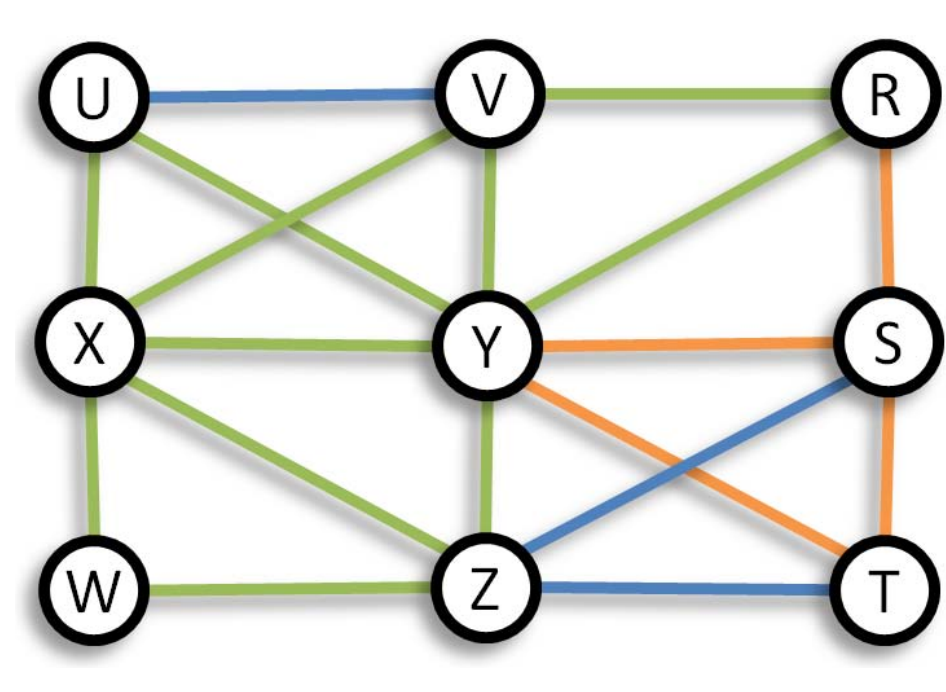
$$\text{cost}(\mathcal{C}, \text{cl}) = \sum_{\substack{(x,y) \in V \times V, \\ \mathcal{C}(x) = \mathcal{C}(y)}} (1 - \mathbb{I}[\ell(x, y) = \text{cl}(\mathcal{C}(x))]) + \sum_{\substack{(x,y) \in V \times V, \\ \mathcal{C}(x) \neq \mathcal{C}(y)}} \mathbb{I}[\ell(x, y) \neq l_0].$$

Solutions:

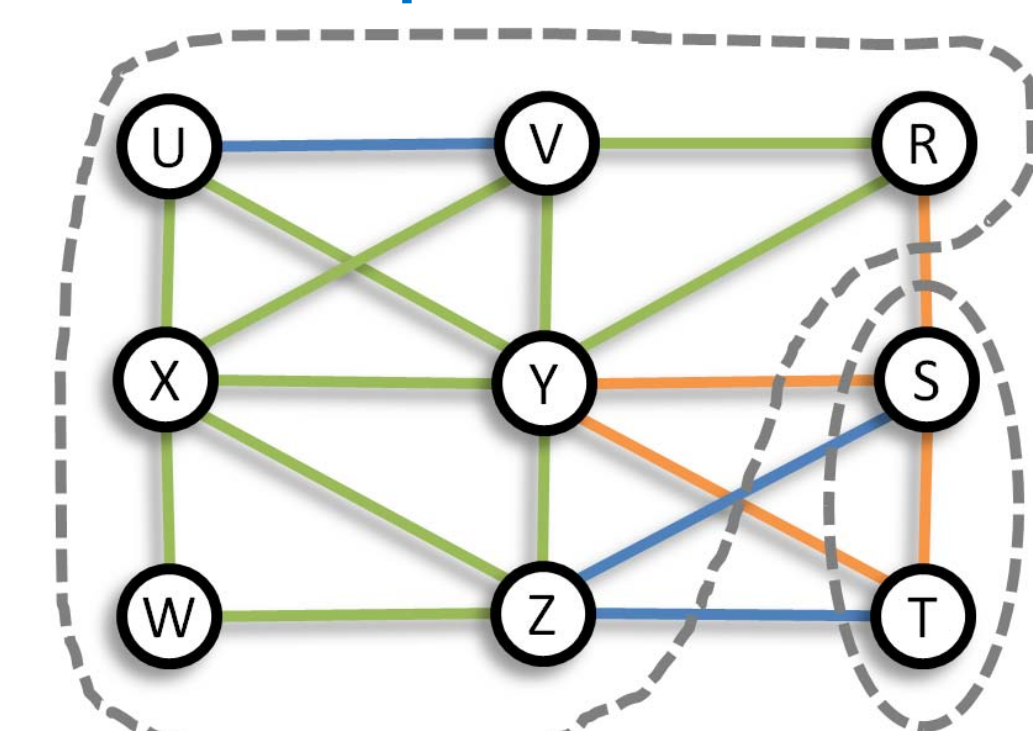
- **Randomized approximation algorithm (CB)** with approximation guarantee proportional to the maximum degree in the graph:

$$r(G) \leq 6(2D_{\max} - 1)$$

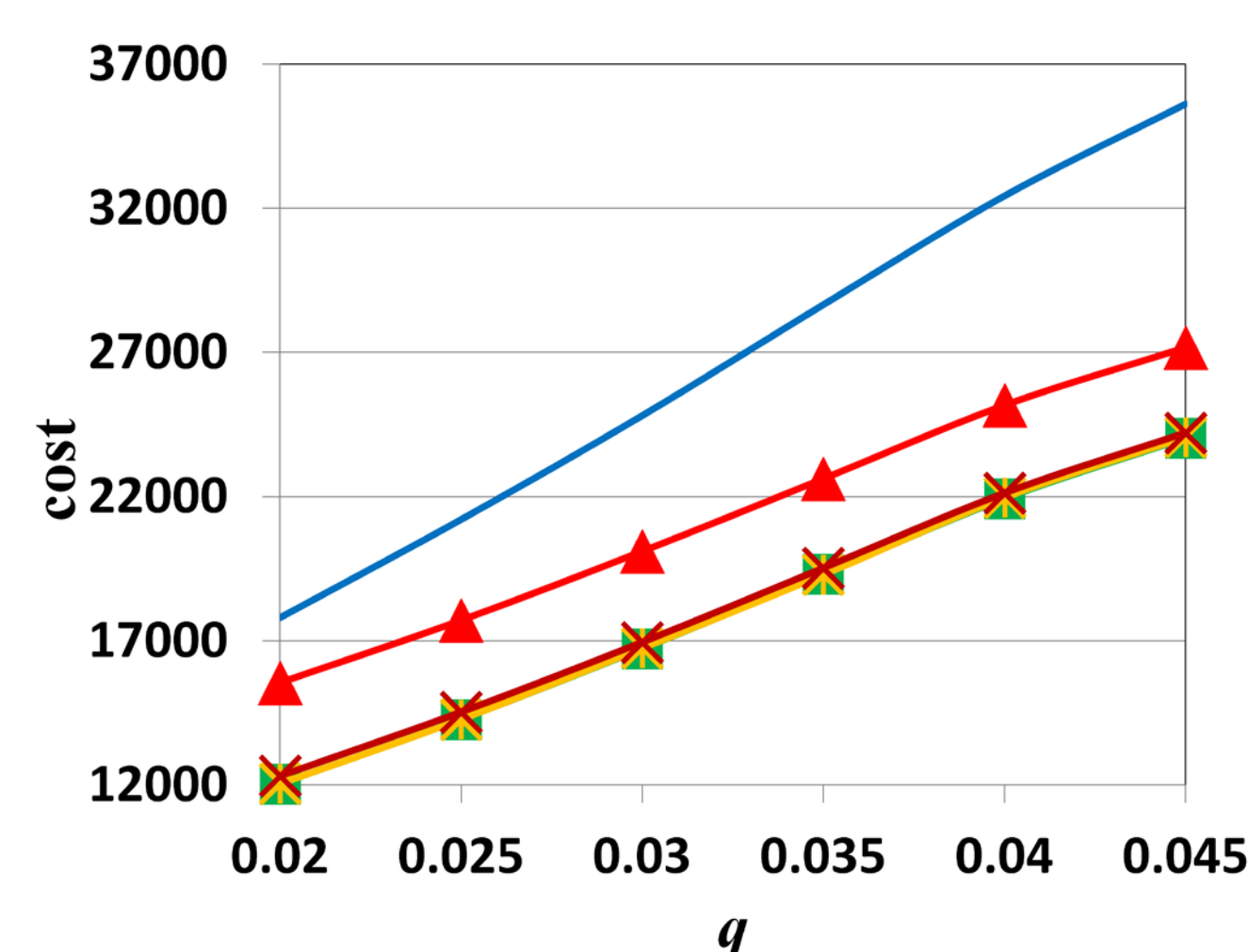
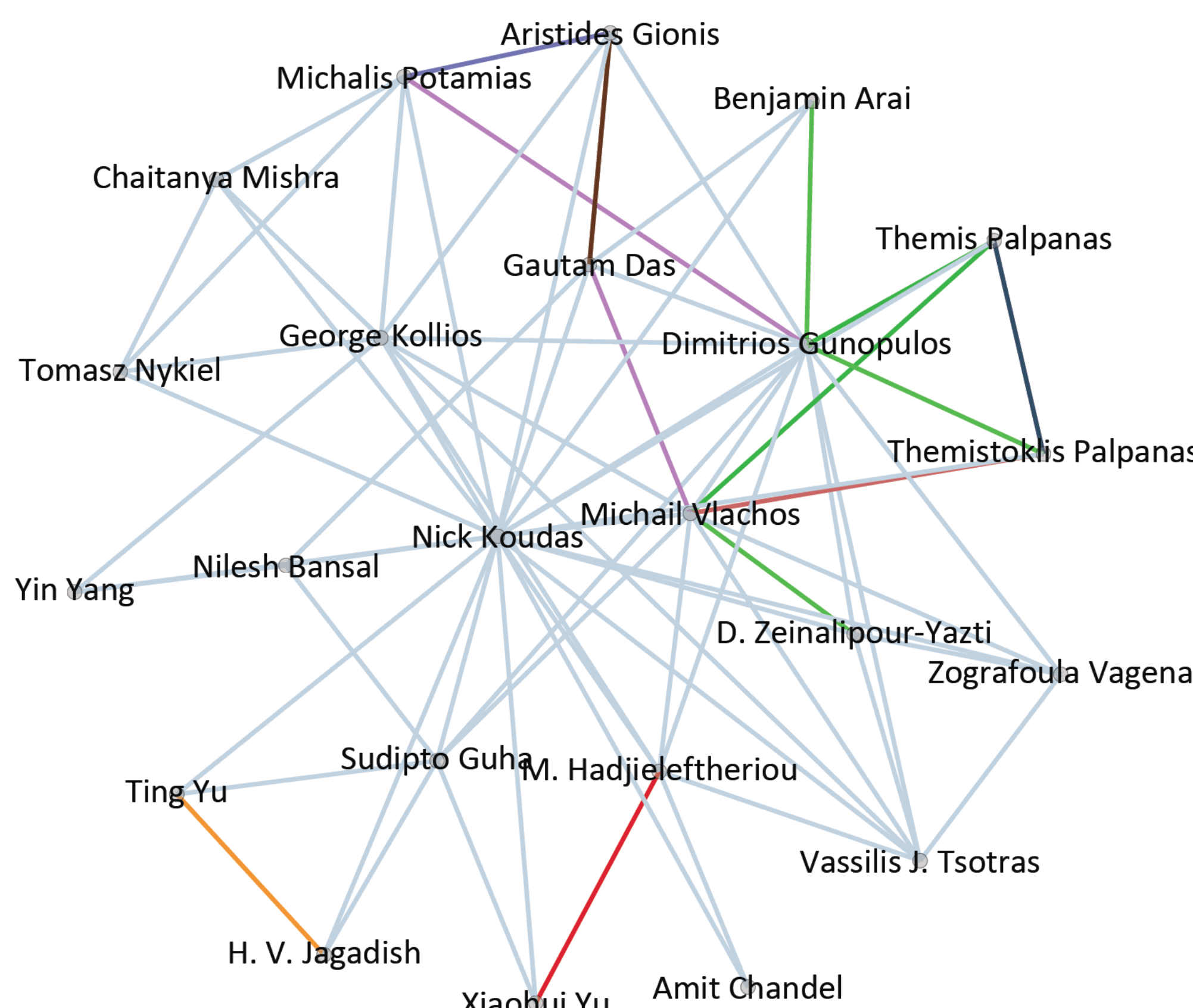
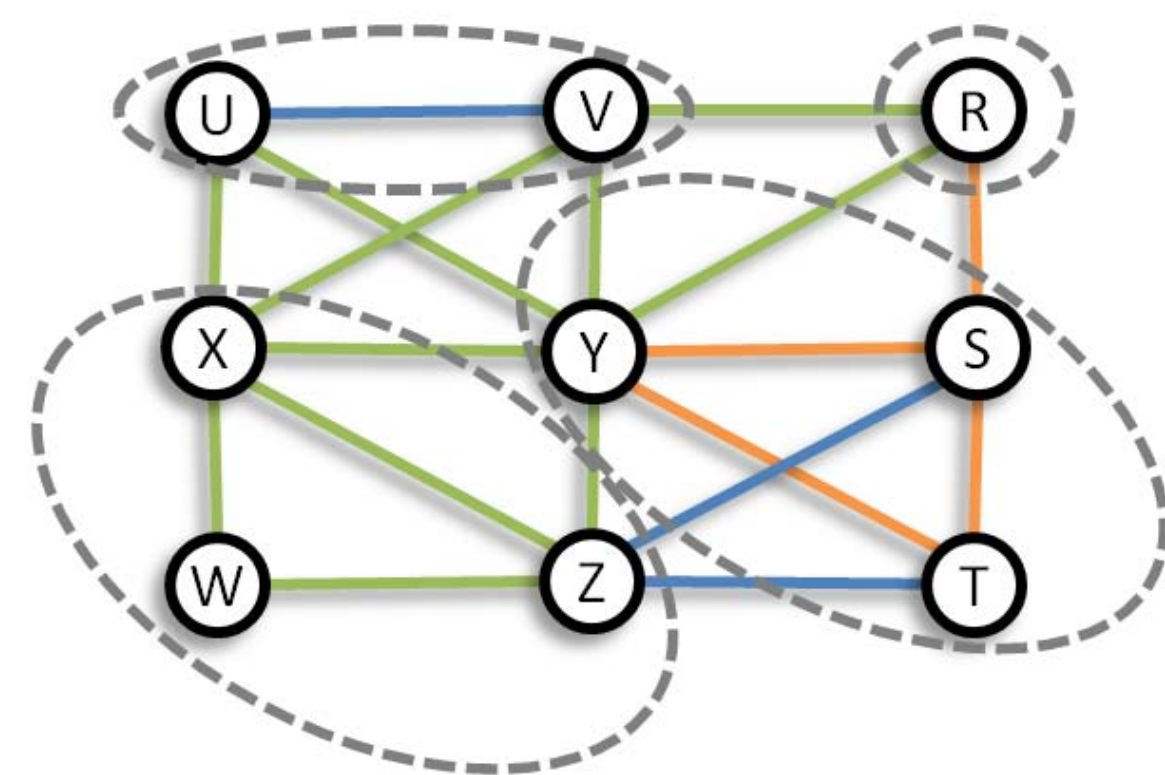
- **Lazy CB (LCB) algorithm**. The random choices are “guided” by heuristic considerations



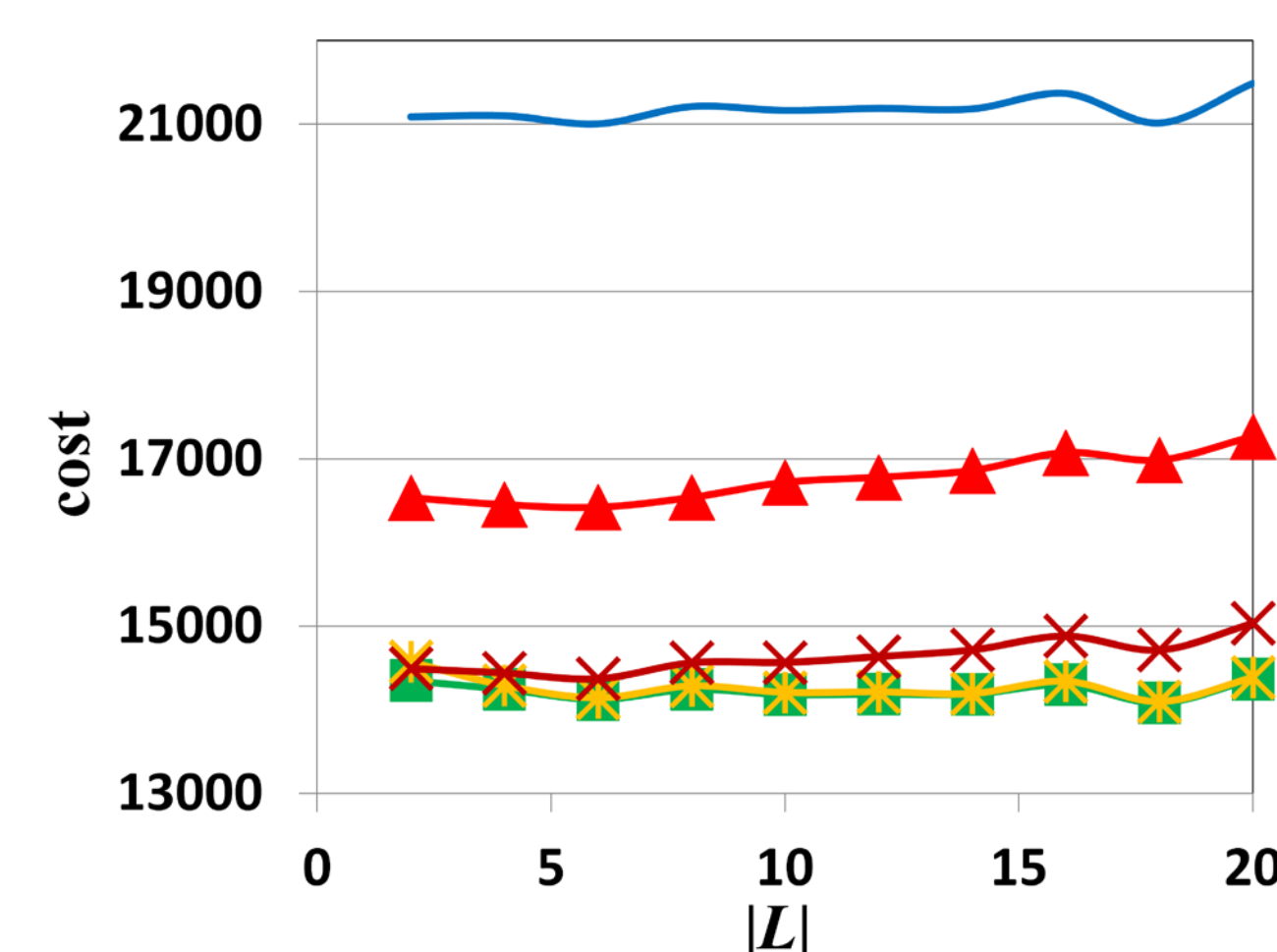
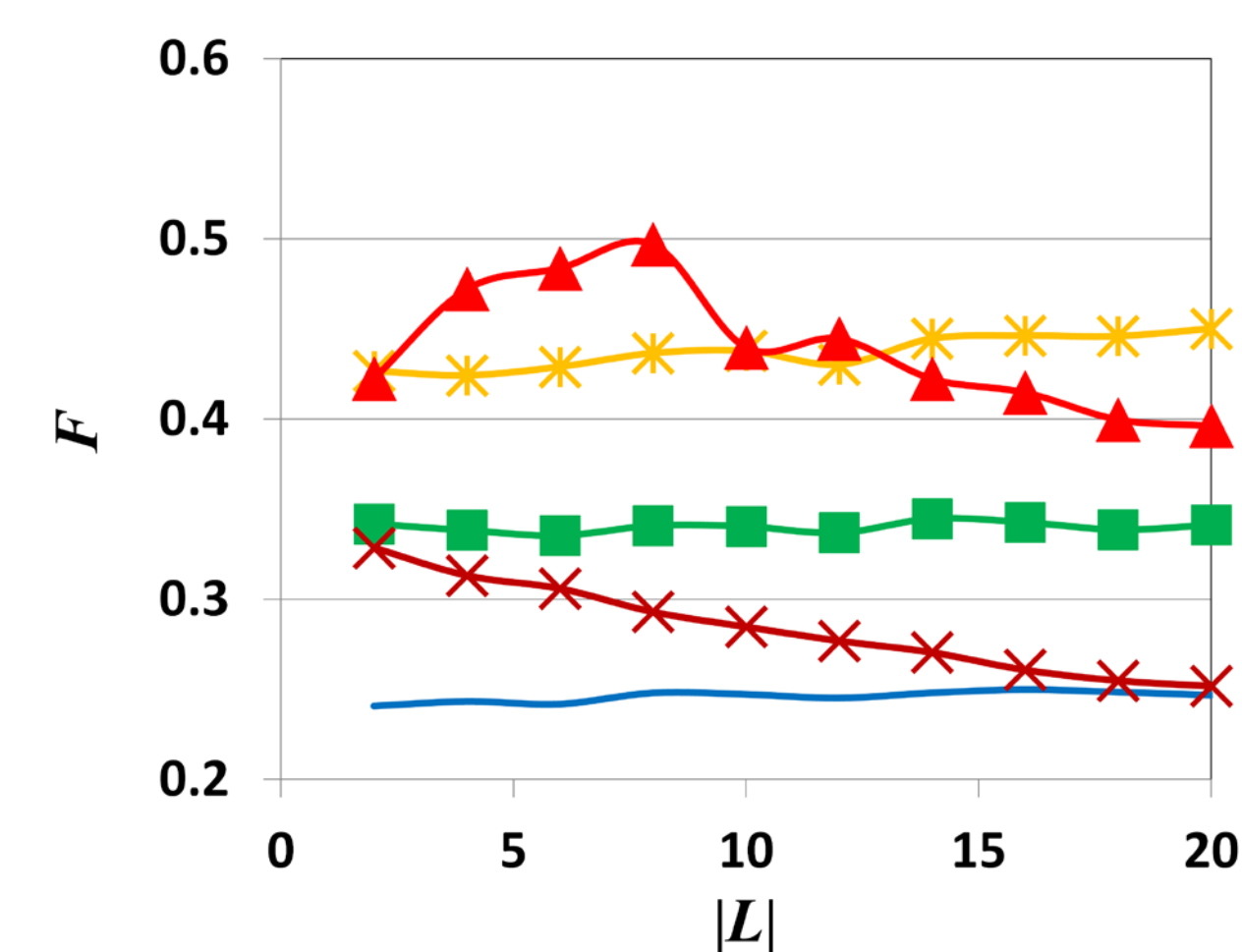
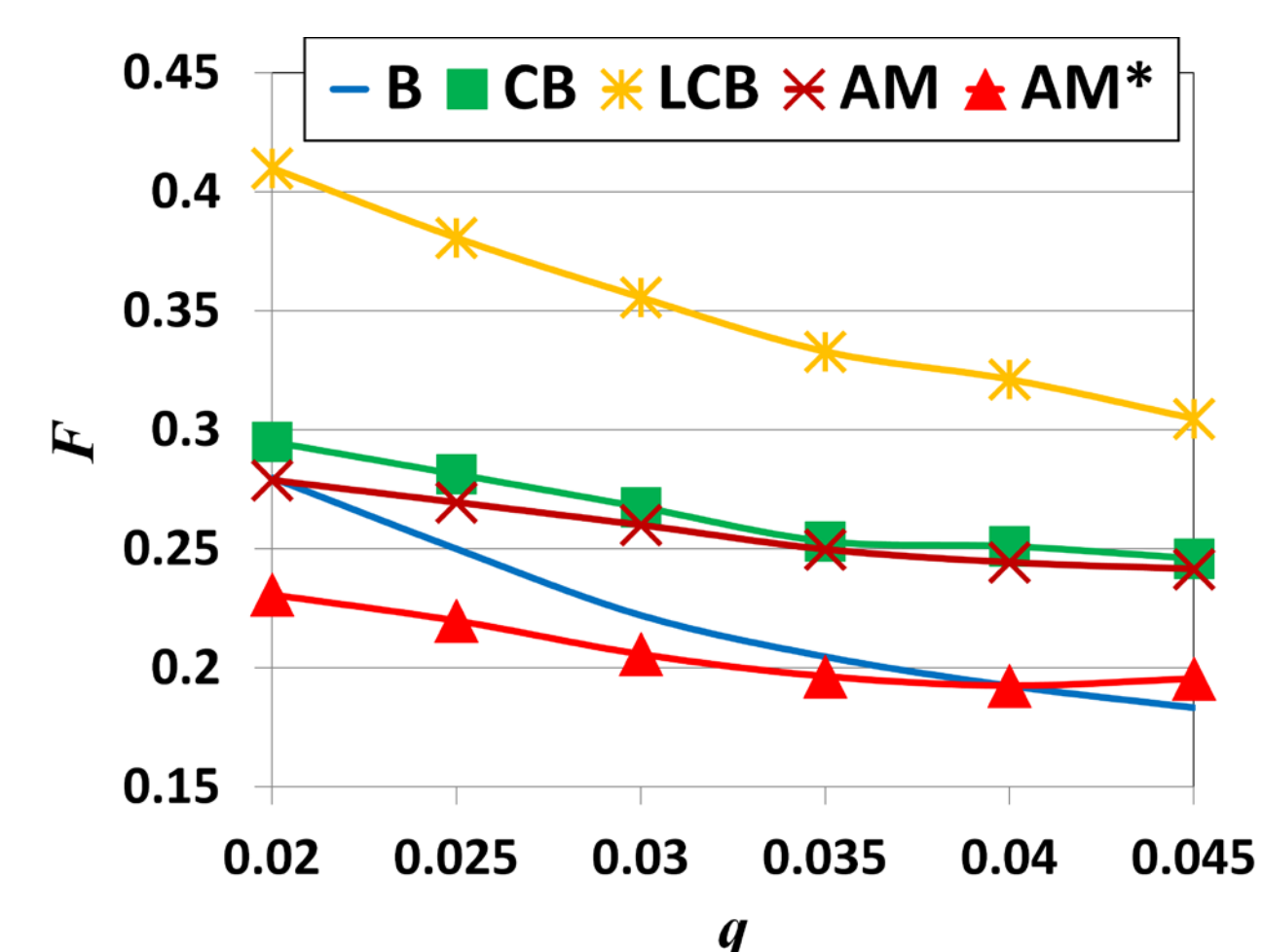
LCB output



CB output



Accuracy on **synthetic datasets** in terms of similarity with respect to ground truth (F) and solution **cost**, by varying the level of noise (q), and the number of labels ($|L|$)



- **AM heuristic algorithm** that allows to choose the number of **output clusters**.

It finds a local optimum of the objective function based on the **alternating minimization** paradigm.

dataset	B	CB	LCB	AM
String	163 305	160 060	155 881	156 976
Youtube	23 550 213	18 956 000	22 644 858	19 670 899
DBLP	2 260 065	1 633 149	1 678 714	2 018 952

Cost of algorithms on **real datasets** in different domains: biological (String), social network (Youtube) and bibliographic (DBLP)