

Pixy formal semantics

A. Finn Hackett, Reed Mullanix

March 19, 2018

1 Term language

$\langle expr \rangle ::= \langle literal \rangle$
| $\langle var \rangle$
| **nil**
| $? \langle expr \rangle$
| **if** $\langle expr \rangle$ **then** $\langle expr \rangle$ **else** $\langle expr \rangle$
| $\langle expr \rangle$ **fby** $\langle expr \rangle$
| $\langle expr \rangle$ **where** $\langle wheredecls \rangle$
| **next** $\langle expr \rangle$
| $\langle var \rangle$ ($\langle opargvals \rangle$)
| $\langle expr \rangle + \langle expr \rangle$
| $\langle expr \rangle - \langle expr \rangle$
| $\langle expr \rangle * \langle expr \rangle$
| $\langle expr \rangle / \langle expr \rangle$

 $\langle opargvals \rangle ::= \langle expr \rangle , \langle opargvals \rangle \mid \langle expr \rangle$

 $\langle oparglist \rangle ::= \langle var \rangle , \langle oparglist \rangle \mid \langle var \rangle$

 $\langle wheredecl \rangle ::= \langle var \rangle = \langle expr \rangle$
| $\langle var \rangle$ ($\langle oparglist \rangle$) = $\langle expr \rangle$

 $\langle wheredecls \rangle ::= \langle wheredecl \rangle ; \langle wheredecls \rangle \mid \langle wheredecl \rangle$

2 Init rules

$$\frac{F \vdash E \overset{init}{\Rightarrow} S}{F \vdash ?E \overset{init}{\Rightarrow} S} [\text{Init} - \text{check}]$$

$$\frac{\begin{array}{c} F \vdash L \overset{init}{\Rightarrow} S_l \\ F \vdash R \overset{init}{\Rightarrow} S_r \end{array}}{F \vdash L \text{ fby } R \overset{init}{\Rightarrow} \text{false}, S_l, S_r} [\text{Init} - \text{fby}]$$

$$\frac{\begin{array}{c} F \vdash C \overset{init}{\Rightarrow} S_c \\ F \vdash T \overset{init}{\Rightarrow} S_t \\ F \vdash F \overset{init}{\Rightarrow} S_f \end{array}}{F \vdash \text{if } C \text{ then } T \text{ else } F \overset{init}{\Rightarrow} S_c, S_t, S_f} [\text{Init} - \text{ite}]$$

$$\frac{F \vdash E \overset{init}{\Rightarrow} S}{F \vdash \text{next } E \overset{init}{\Rightarrow} \text{false}, S} [\text{Init} - \text{next}]$$

$$\frac{\begin{array}{c} F \vdash E \overset{init}{\Rightarrow} s \\ F \vdash E_n \overset{\text{whereinit}}{\Rightarrow} S; F' \end{array}}{F \vdash n = E; E_n \overset{\text{whereinit}}{\Rightarrow} \langle \text{nil}, s \rangle, S; F'} [\text{WhereInit} - \text{v} - \text{decl}]$$

$$\frac{F \vdash E_n \overset{\text{whereinit}}{\Rightarrow} S; F'}{F \vdash f(A) = E; E_n \overset{\text{whereinit}}{\Rightarrow} S; f \rightarrow \langle A, E \rangle, F'} [\text{WhereInit} - \text{fn} - \text{decl}]$$

$$\frac{}{F \vdash \overset{\text{whereinit}}{\Rightarrow} \emptyset; F} [\text{WhereInit} - \text{empty}]$$

$$\frac{\begin{array}{c} F \vdash E \overset{init}{\Rightarrow} S_e \\ F \vdash E_s \overset{\text{whereinit}}{\Rightarrow} S \end{array}}{F \vdash E \text{ where } E_s \overset{init}{\Rightarrow} S_e, S} [\text{Init} - \text{where}]$$

$$\frac{\begin{array}{c} F \vdash A \overset{init}{\Rightarrow} S \\ F \vdash A_n \overset{\text{applyinit}}{\Rightarrow} S_n \end{array}}{F \vdash A, A_n \overset{\text{applyinit}}{\Rightarrow} S, S_n} [\text{ApplyInit} - \text{arg}]$$

$$\frac{}{F \vdash \overset{\text{applyinit}}{\Rightarrow} \emptyset} [\text{ApplyInit} - \text{empty}]$$

$$\begin{array}{c}
F(f) = \langle _, E \rangle \\
F \vdash E \xRightarrow{init} S_e \\
F \vdash A \xRightarrow{apply init} S \\
\hline
F \vdash f(A) \xRightarrow{init} S_e, S \quad [\text{Init} - \text{apply}]
\end{array}$$

$$\frac{}{F \vdash _ \xRightarrow{init} \emptyset} [\text{Init} - \text{literal}]$$

$$\begin{array}{c}
F \vdash L \xRightarrow{init} S_l \\
F \vdash R \xRightarrow{init} S_r \\
\hline
F \vdash L _ R \xRightarrow{init} S_l, S_r \quad [\text{Init} - \text{binop}]
\end{array}$$

3 Evaluation rules

$$\begin{array}{c}
S; F; M \vdash E \Downarrow V; S' \\
V \neq \text{nil} \\
\hline
S; F; M \vdash ?E \Downarrow \text{true}; S' \quad [\text{Eval} - \text{check} - \text{true}]
\end{array}$$

$$\begin{array}{c}
S; F; M \vdash E \Downarrow \text{nil}; S' \\
\hline
S; F; M \vdash ?E \Downarrow \text{false}; S' \quad [\text{Eval} - \text{check} - \text{false}]
\end{array}$$

$$\begin{array}{c}
S_c; F; M \vdash C \Downarrow \text{true}; S'_c \\
S_t; F; M \vdash T \Downarrow V; S'_t \\
S_f; F; M \vdash F \Downarrow \text{nil}; S'_f \\
\hline
S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \Downarrow V; S'_c, S'_t, S'_f \quad [\text{Eval} - \text{ite} - \text{true}]
\end{array}$$

$$\begin{array}{c}
S_c; F; M \vdash C \Downarrow \text{false}; S'_c \\
S_t; F; M \vdash T \Downarrow \text{nil}; S'_t \\
S_f; F; M \vdash F \Downarrow V; S'_f \\
\hline
S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \Downarrow V; S'_c, S'_t, S'_f \quad [\text{Eval} - \text{ite} - \text{false}]
\end{array}$$

$$\begin{array}{c}
S_c; F; M \vdash C \Downarrow \text{nil}; S'_c \\
S_t; F; M \vdash T \Downarrow \text{nil}; S'_t \\
S_f; F; M \vdash F \Downarrow \text{nil}; S'_f \\
\hline
S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \Downarrow \text{nil}; S'_c, S'_t, S'_f \quad [\text{Eval} - \text{ite} - \text{nil}]
\end{array}$$

$$\frac{S_c; F; M \vdash C \not\Downarrow nil; S'_c \quad S_t; F; M \vdash T \not\Downarrow nil; S'_t \quad S_f; F; M \vdash F \not\Downarrow nil; S'_f}{S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \not\Downarrow nil; S'_c, S'_t, S'_f} [\text{Eval} - \text{ite} - \text{C}]$$

$$\frac{S_l; F; M \vdash E_l \Downarrow V; S'_l \quad V \neq nil \quad S_r; F; M \vdash E_r \not\Downarrow nil; S'_r}{false, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow V; true, S'_l, S'_r} [\text{Eval} - \text{fby} - \text{before}]$$

$$\frac{S_l; F; M \vdash E_l \Downarrow nil; S'_l \quad S_r; F; M \vdash E_r \not\Downarrow nil; S'_r}{false, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow nil; false, S'_l, S'_r} [\text{Eval} - \text{fby} - \text{before} - \text{nil}]$$

$$\frac{S_l; F; M \vdash E_l \not\Downarrow nil; S'_l \quad S_r; F; M \vdash E_r \Downarrow V; S'_r}{true, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow V; true, S'_l, S'_r} [\text{Eval} - \text{fby} - \text{after}]$$

$$\frac{S_l; F; M \vdash E_l \not\Downarrow nil; S'_l \quad S_r; F; M \vdash E_r \not\Downarrow nil; S'_r}{c, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow nil; c, S'_l, S'_r} [\text{Eval} - \text{fby} - \text{C}]$$

$$\frac{S_n; F; M \vdash E_n \xRightarrow{\text{names}} F'; M'}{\langle v, _ \rangle, S_n; F; M \vdash n = E; E_n \xRightarrow{\text{names}} F'; n \rightarrow v, M'} [\text{WhereNames} - \text{v} - \text{decl}]$$

$$\frac{S; f \rightarrow \langle A, E \rangle, F; M \vdash E_v \xRightarrow{\text{names}} F'; M'}{S; F; M \vdash f(A) = E; E_n \xRightarrow{\text{names}} F'; M'} [\text{WhereNames} - \text{fn} - \text{decl}]$$

$$\frac{}{\emptyset; F; M \vdash \xRightarrow{\text{names}} F'; M'} [\text{WhereNames} - \text{empty}]$$

$$\frac{s; F; M \vdash E \Downarrow v; s' S_n; F; M \vdash E_n \xRightarrow{\text{values}} S'_n}{\langle _, s \rangle, S_n; F; M \vdash n = E; E_n \xRightarrow{\text{values}} \langle v, s' \rangle, S'_n} [\text{WhereVal} - \text{v} - \text{decl}]$$

$$\frac{S; F; M \vdash E_n \xRightarrow{values} S'}{S; F; M \vdash f(A) = E; E_n \xRightarrow{values} S'} [\text{WhereVal} - \text{fn} - \text{decl}]$$

$$\frac{}{\emptyset; F; M \vdash \xRightarrow{values} \emptyset} [\text{WhereVal} - \text{empty}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E_s \xRightarrow{names} F_i; M_i \\ S; F_i; M_i \vdash E_s \xRightarrow{values} S' \\ S_e; F_i; M_i \vdash E \Downarrow V; S'_e \end{array}}{S_e, S; F; M \vdash E \text{ where } E_s \Downarrow V; S'_e, S'} [\text{Eval} - \text{where}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E_s \xRightarrow{names} F_i; M_i \\ S; F_i; M_i \vdash E_s \xRightarrow{values} S' \\ S_e; F_i; M_i \vdash E \Downarrow \text{nil}; S'_e \end{array}}{S_e, S; F; M \vdash E \text{ where } E_s \Downarrow \text{nil}; S'_e, S'} [\text{Eval} - \text{where} - \text{C}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow V; S' \\ V \neq \text{nil} \end{array}}{\langle \text{false}, \text{nil} \rangle, S; F; M \vdash \text{next } E \Downarrow \text{nil}; \langle \text{true}, V \rangle, S'} [\text{Eval} - \text{next} - \text{before}]$$

$$\frac{S; F; M \vdash E \Downarrow \text{nil}; S'}{\langle \text{false}, \text{nil} \rangle, S; F; M \vdash \text{next } E \Downarrow \text{nil}; \langle \text{false}, \text{nil} \rangle, S'} [\text{Eval} - \text{next} - \text{before} - \text{nil}]$$

$$\frac{\begin{array}{c} v \neq \text{nil} \\ S; F; M \vdash E \Downarrow V; S' \end{array}}{\langle \text{true}, v \rangle, S; F; M \vdash \text{next } E \Downarrow v; \langle \text{true}, V \rangle, S'} [\text{Eval} - \text{next} - \text{after}]$$

$$\frac{S; F; M \vdash E \Downarrow V; S'}{\langle \text{true}, \text{nil} \rangle, S; F; M \vdash \text{next } E \Downarrow V; \langle \text{true}, \text{nil} \rangle, S'} [\text{Eval} - \text{next} - \text{after} - \text{nil}]$$

$$\frac{S; F; M \vdash E \Downarrow \text{nil}; S'}{\langle c, v \rangle, S; F; M \vdash \text{next } E \Downarrow \text{nil}; \langle c, v \rangle, S'} [\text{Eval} - \text{next} - \text{C}]$$

$$\frac{}{\emptyset; F; M \vdash \text{nil} \Downarrow \text{nil}; \emptyset} [\text{Eval} - \text{nil}]$$

$$\frac{}{\emptyset; F; M \vdash nil \Downarrow nil; \emptyset} [\text{Eval} - \text{nil} - \text{C}]$$

$$\frac{N \in \mathbb{R}}{\emptyset; F; M \vdash N \Downarrow N; \emptyset} [\text{Eval} - \text{num}]$$

$$\frac{N \in \mathbb{R}}{\emptyset; F; M \vdash N \Downarrow nil; \emptyset} [\text{Eval} - \text{num} - \text{C}]$$

$$\frac{M(I) = V}{\emptyset; F; M \vdash I \Downarrow V; \emptyset} [\text{Eval} - \text{id}]$$

$$\frac{M(I) = V}{\emptyset; F; M \vdash I \Downarrow nil; \emptyset} [\text{Eval} - \text{id} - \text{C}]$$

$$\frac{E \in \{true, false\}}{\emptyset; F; M \vdash E \Downarrow E; \emptyset} [\text{Eval} - \text{boolean}]$$

$$\frac{E \in \{true, false\}}{\emptyset; F; M \vdash E \Downarrow nil; \emptyset} [\text{Eval} - \text{boolean} - \text{C}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow V; S' \\ A_n; S_n; F; M \vdash E_n \xrightarrow{arg} M_i; S'_n \end{array}}{A, A_n; S, S_n; F; M \vdash E, E_n \xrightarrow{arg} A \rightarrow V, M_i; S', S'_n} [\text{Apply} - \text{arg}]$$

$$\frac{}{\emptyset; \emptyset; F; M \vdash \xrightarrow{arg} \emptyset; \emptyset} [\text{Apply} - \text{arg} - \text{empty}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow nil; S' \\ A_n; S_n; F; M \vdash E_n \xrightarrow{argC} M_i; S'_n \end{array}}{A, A_n; S, S_n; F; M \vdash E, E_n \xrightarrow{argC} A \rightarrow nil, M_i; S', S'_n} [\text{Apply} - \text{arg} - \text{C}]$$

$$\frac{}{\emptyset; \emptyset; F; M \vdash \xrightarrow{argC} \emptyset; \emptyset} [\text{Apply} - \text{arg} - \text{empty} - \text{C}]$$

$$\begin{array}{c}
F(f) = \langle A, E \rangle \\
A; S; F; M \vdash a \xrightarrow{arg} M_i; S' \\
S_e; F; M_i \vdash E \Downarrow V; S'_e \\
\hline
S_e, S; F; M \vdash f(a) \Downarrow V; S'_e, S' \text{ [Eval - apply]}
\end{array}$$

$$\begin{array}{c}
F(f) = \langle A, E \rangle \\
A; S; F; M \vdash a \xrightarrow{arg^C} M_i; S' \\
S_e; F; M_i \vdash E \Downarrow nil; S'_e \\
\hline
S_e, S; F; M \vdash f(a) \Downarrow nil; S'_e, S' \text{ [Eval - apply - C]}
\end{array}$$

$$\begin{array}{c}
L, R \in \mathbb{R} \\
V = L + R \\
\hline
L + R \xrightarrow{binop} V \text{ [Binop - plus]}
\end{array}$$

$$\begin{array}{c}
L, R \in \mathbb{R} \\
V = L - R \\
\hline
L - R \xrightarrow{binop} V \text{ [Binop - minus]}
\end{array}$$

$$\begin{array}{c}
L, R \in \mathbb{R} \\
V = L * R \\
\hline
L * R \xrightarrow{binop} V \text{ [Binop - times]}
\end{array}$$

$$\begin{array}{c}
L, R \in \mathbb{R} \\
V = L / R \\
\hline
L / R \xrightarrow{binop} V \text{ [Binop - divide]}
\end{array}$$

$$\begin{array}{c}
S_l; F; M \vdash L_e \Downarrow L_v; S'_l \\
S_r; F; M \vdash R_e \Downarrow R_v; S'_r \\
L_v \ B \ R_v \xrightarrow{binop} V \\
\hline
S_l, S_r; F; M \vdash L_e \ B \ R_e \Downarrow V; S'_l, S'_r \text{ [Eval - binop]}
\end{array}$$

$$\begin{array}{c}
S_l; F; M \vdash L_e \Downarrow nil; S'_l \\
S_r; F; M \vdash R_e \Downarrow nil; S'_r \\
\hline
S_l, S_r; F; M \vdash L_e \ B \ R_e \Downarrow nil; S'_l, S'_r \text{ [Eval - binop - C]}
\end{array}$$