Pixy formal semantics

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1 Term language

$\langle expr \rangle$ ::= $\langle literal \rangle$
| $\langle var \rangle$
| `nil`
| `?` $\langle expr \rangle$
| `if` $\langle expr \rangle$ `then` $\langle expr \rangle$ `else` $\langle expr \rangle$
| $\langle expr \rangle$ `fby` $\langle expr \rangle$
| $\langle expr \rangle$ `where` $\langle wheredecls \rangle$
| $\langle var \rangle$ `(` $\langle opargvals \rangle$ `)`
| $\langle expr \rangle$ `+` $\langle expr \rangle$
| $\langle expr \rangle$ `-` $\langle expr \rangle$
| $\langle expr \rangle$ `*` $\langle expr \rangle$
| $\langle expr \rangle$ `/` $\langle expr \rangle$

$\langle opargvals \rangle$ ::= $\langle expr \rangle$ `,` $\langle opargvals \rangle$ | $\langle expr \rangle$

$\langle opargslist \rangle$ ::= $\langle var \rangle$ `,` $\langle opargslist \rangle$ | $\langle var \rangle$

$\langle wheredecl \rangle$ ::= $\langle var \rangle$ `=` $\langle expr \rangle$
| $\langle var \rangle$ `(` $\langle opargslist \rangle$ `)` `=` $\langle expr \rangle$

$\langle wheredecls \rangle$ ::= $\langle wheredecl \rangle$; $\langle wheredecls \rangle$ | $\langle wheredecl \rangle$

2 Init rules

$$\frac{F \vdash E \overset{init}{\Rightarrow} S}{F \vdash ?E \overset{init}{\Rightarrow} S}[\texttt{Init} - \texttt{check}]$$

$$\frac{\begin{array}{c} F \vdash L \overset{init}{\Rightarrow} S_l \\ F \vdash R \overset{init}{\Rightarrow} S_r \end{array}}{F \vdash L\,\mathrm{fby}\,R \overset{init}{\Rightarrow} false, S_l, S_r}[\texttt{Init} - \texttt{fby}]$$

$$\frac{F \vdash C \overset{init}{\Rightarrow} S_c \quad F \vdash T \overset{init}{\Rightarrow} S_t \quad F \vdash F \overset{init}{\Rightarrow} S_f}{F \vdash \text{if}\, C \,\text{then}\, T \,\text{else}\, F \overset{init}{\Rightarrow} S_c, S_t, S_f}[\texttt{Init} - \texttt{ite}]$$

$$\frac{F \vdash E \overset{init}{\Rightarrow} S}{F \vdash \text{next}\, E \overset{init}{\Rightarrow} false, S}[\texttt{Init} - \texttt{next}]$$

$$\frac{F \vdash E \overset{init}{\Rightarrow} s \quad F \vdash E_n \overset{whereinit}{\Rightarrow} S; F'}{F \vdash n = E; E_n \overset{whereinit}{\Rightarrow} \langle nil, s \rangle, S; F'}[\texttt{WhereInit} - \texttt{v} - \texttt{decl}]$$

$$\frac{F \vdash E_n \overset{whereinit}{\Rightarrow} S; F'}{F \vdash f(A) = E; E_n \overset{whereinit}{\Rightarrow} S; f \rightarrow \langle A, E \rangle, F'}[\texttt{WhereInit} - \texttt{fn} - \texttt{decl}]$$

$$\frac{}{F \vdash \overset{whereinit}{\Rightarrow} \emptyset; F}[\texttt{WhereInit} - \texttt{empty}]$$

$$\frac{F \vdash E \overset{init}{\Rightarrow} S_e \quad F \vdash E_s \overset{whereinit}{\Rightarrow} S}{F \vdash E \,\text{where}\, E_s \overset{init}{\Rightarrow} S_e, S}[\texttt{Init} - \texttt{where}]$$

$$\frac{F \vdash A \overset{init}{\Rightarrow} S \quad F \vdash A_n \overset{applyinit}{\Rightarrow} S_n}{F \vdash A, A_n \overset{applyinit}{\Rightarrow} S, S_n}[\texttt{ApplyInit} - \texttt{arg}]$$

$$\frac{}{F \vdash \overset{applyinit}{\Rightarrow} \emptyset}[\texttt{ApplyInit} - \texttt{empty}]$$

$$\frac{F(f) = \langle \_, E \rangle \quad F \vdash E \overset{init}{\Rightarrow} S_e \quad F \vdash A \overset{applyinit}{\Rightarrow} S}{F \vdash f(A) \overset{init}{\Rightarrow} S_e, S}[\texttt{Init} - \texttt{apply}]$$

$$\frac{}{F \vdash \_ \overset{init}{\Rightarrow} \emptyset}[\texttt{Init}-\texttt{literal}]$$

$$\frac{\begin{array}{c} F \vdash L \overset{init}{\Rightarrow} S_l \\ F \vdash R \overset{init}{\Rightarrow} S_r \end{array}}{F \vdash L \;\; \_ \;\; R \overset{init}{\Rightarrow} S_l, S_r}[\texttt{Init}-\texttt{binop}]$$

3 Evaluation rules

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow V; S' \\ V \neq nil \end{array}}{S; F; M \vdash ?E \Downarrow true; S'}[\texttt{Eval}-\texttt{check}-\texttt{true}]$$

$$\frac{S; F; M \vdash E \Downarrow nil; S'}{S; F; M \vdash ?E \Downarrow false; S'}[\texttt{Eval}-\texttt{check}-\texttt{false}]$$

$$\frac{\begin{array}{c} S_c; F; M \vdash C \Downarrow true; S'_c \\ S_t; F; M \vdash T \Downarrow V; S'_t \\ S_f; F; M \vdash F \overset{C}{\Downarrow} nil; S'_f \end{array}}{S_c, S_t, S_f; F; M \vdash \text{if}\, C \,\text{then}\, T \,\text{else}\, F \Downarrow V; S'_c, S'_t, S'_f}[\texttt{Eval}-\texttt{ite}-\texttt{true}]$$

$$\frac{\begin{array}{c} S_c; F; M \vdash C \Downarrow false; S'_c \\ S_t; F; M \vdash T \overset{C}{\Downarrow} nil; S'_t \\ S_f; F; M \vdash F \Downarrow V; S'_f \end{array}}{S_c, S_t, S_f; F; M \vdash \text{if}\, C \,\text{then}\, T \,\text{else}\, F \Downarrow V; S'_c, S'_t, S'_f}[\texttt{Eval}-\texttt{ite}-\texttt{false}]$$

$$\frac{\begin{array}{c} S_c; F; M \vdash C \Downarrow nil; S'_c \\ S_t; F; M \vdash T \overset{C}{\Downarrow} nil; S'_t \\ S_f; F; M \vdash F \overset{C}{\Downarrow} nil; S'_f \end{array}}{S_c, S_t, S_f; F; M \vdash \text{if}\, C \,\text{then}\, T \,\text{else}\, F \Downarrow nil; S'_c, S'_t, S'_f}[\texttt{Eval}-\texttt{ite}-\texttt{nil}]$$

$$\frac{\begin{array}{c} S_c; F; M \vdash C \overset{C}{\Downarrow} nil; S'_c \\ S_t; F; M \vdash T \overset{C}{\Downarrow} nil; S'_t \\ S_f; F; M \vdash F \overset{C}{\Downarrow} nil; S'_f \end{array}}{S_c, S_t, S_f; F; M \vdash \text{if}\, C \,\text{then}\, T \,\text{else}\, F \overset{C}{\Downarrow} nil; S'_c, S'_t, S'_f}[\texttt{Eval}-\texttt{ite}-\texttt{C}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash E_l \Downarrow V; S_l' \\ V \neq nil \\ S_r; F; M \vdash E_r \overset{C}{\Downarrow} nil; S_r' \end{array}}{false, S_l, S_r; F; M \vdash E_l \,\mathrm{fby}\, E_r \Downarrow V; true, S_l', S_r'} [\mathtt{Eval-fby-before}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash E_l \Downarrow nil; S_l' \\ S_r; F; M \vdash E_r \overset{C}{\Downarrow} nil; S_r' \end{array}}{false, S_l, S_r; F; M \vdash E_l \,\mathrm{fby}\, E_r \Downarrow nil; false, S_l', S_r'} [\mathtt{Eval-fby-before-nil}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash E_l \overset{C}{\Downarrow} nil; S_l' \\ S_r; F; M \vdash E_r \Downarrow V; S_r' \end{array}}{true, S_l, S_r; F; M \vdash E_l \,\mathrm{fby}\, E_r \Downarrow V; true, S_l', S_r'} [\mathtt{Eval-fby-after}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash E_l \overset{C}{\Downarrow} nil; S_l' \\ S_r; F; M \vdash E_r \overset{C}{\Downarrow} nil; S_r' \end{array}}{c, S_l, S_r; F; M \vdash E_l \,\mathrm{fby}\, E_r \Downarrow nil; c, S_l', S_r'} [\mathtt{Eval-fby-C}]$$

$$\frac{S_n; F; M \vdash E_n \overset{names}{\Rightarrow} F'; M'}{\langle v, \_ \rangle, S_n; F; M \vdash n = E; E_n \overset{names}{\Rightarrow} F'; n \to v, M'} [\mathtt{WhereNames-v-decl}]$$

$$\frac{S; f \to \langle A, E \rangle, F; M \vdash E_v \overset{names}{\Rightarrow} F'; M'}{S; F; M \vdash f(A) = E; E_n \overset{names}{\Rightarrow} F'; M'} [\mathtt{WhereNames-fn-decl}]$$

$$\frac{}{\emptyset; F; M \vdash \overset{names}{\Rightarrow} F'; M'} [\mathtt{WhereNames-empty}]$$

$$\frac{s; F; M \vdash E \Downarrow v; s' S_n; F; M \vdash E_n \overset{values}{\Rightarrow} S_n'}{\langle \_, s \rangle, S_n; F; M \vdash n = E; E_n \overset{values}{\Rightarrow} \langle v, s' \rangle, S_n'} [\mathtt{WhereVal-v-decl}]$$

$$\frac{S; F; M \vdash E_n \overset{values}{\Rightarrow} S'}{S; F; M \vdash f(A) = E; E_n \overset{values}{\Rightarrow} S'} [\mathtt{WhereVal-fn-decl}]$$

$$\frac{}{\emptyset; F; M \vdash \overset{values}{\Rightarrow} \emptyset}[\mathtt{WhereVal-empty}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E_s \overset{names}{\Rightarrow} F_i; M_i \\ S; F_i; M_i \vdash E_s \overset{values}{\Rightarrow} S' \\ S_e; F_i; M_i \vdash E \Downarrow V; S'_e \end{array}}{S_e, S; F; M \vdash E \,\mathrm{where}\, E_s \Downarrow V; S'_e, S'}[\mathtt{Eval-where}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E_s \overset{names}{\Rightarrow} F_i; M_i \\ S; F_i; M_i \vdash E_s \overset{values}{\Rightarrow} S' \\ S_e; F_i; M_i \vdash E \overset{C}{\Downarrow} nil; S'_e \end{array}}{S_e, S; F; M \vdash E \,\mathrm{where}\, E_s \overline{C} \Downarrow nil; S'_e, S'}[\mathtt{Eval-where-C}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow V; S' \\ V \neq nil \end{array}}{\langle false, nil \rangle, S; F; M \vdash \mathrm{next}(E) \Downarrow nil; \langle true, V \rangle, S'}[\mathtt{Eval-next-before}]$$

$$\frac{S; F; M \vdash E \Downarrow nil; S'}{\langle false, nil \rangle, S; F; M \vdash \mathrm{next}(E) \Downarrow nil; \langle false, nil \rangle, S'}[\mathtt{Eval-next-before-nil}]$$

$$\frac{\begin{array}{c} v \neq nil \\ S; F; M \vdash E \Downarrow V; S' \end{array}}{\langle true, v \rangle, S; F; M \vdash \mathrm{next}(E) \Downarrow v; \langle true, V \rangle, S'}[\mathtt{Eval-next-after}]$$

$$\frac{S; F; M \vdash E \Downarrow V; S'}{\langle true, nil \rangle, S; F; M \vdash \mathrm{next}(E) \Downarrow V; \langle true, nil \rangle, S'}[\mathtt{Eval-next-after-nil}]$$

$$\frac{S; F; M \vdash E \overset{C}{\Downarrow} nil; S'}{\langle c, v \rangle, S; F; M \vdash \mathrm{next}(E) \overset{C}{\Downarrow} nil; \langle c, v \rangle, S'}[\mathtt{Eval-next-C}]$$

$$\frac{}{\emptyset; F; M \vdash nil \Downarrow nil; \emptyset}[\mathtt{Eval-nil}]$$

$$\frac{}{\emptyset; F; M \vdash nil \overset{C}{\Downarrow} nil; \emptyset}[\texttt{Eval}-\texttt{nil}-\texttt{C}]$$

$$\frac{N \in \mathbb{R}}{\emptyset; F; M \vdash N \Downarrow N; \emptyset}[\texttt{Eval}-\texttt{num}]$$

$$\frac{N \in \mathbb{R}}{\emptyset; F; M \vdash N \overset{C}{\Downarrow} nil; \emptyset}[\texttt{Eval}-\texttt{num}-\texttt{C}]$$

$$\frac{M(I) = V}{\emptyset; F; M \vdash I \Downarrow V; \emptyset}[\texttt{Eval}-\texttt{id}]$$

$$\frac{M(I) = V}{\emptyset; F; M \vdash I \overset{C}{\Downarrow} nil; \emptyset}[\texttt{Eval}-\texttt{id}-\texttt{C}]$$

$$\frac{E \in \{true, false\}}{\emptyset; F; M \vdash E \Downarrow E; \emptyset}[\texttt{Eval}-\texttt{boolean}]$$

$$\frac{E \in \{true, false\}}{\emptyset; F; M \vdash E \overset{C}{\Downarrow} nil; \emptyset}[\texttt{Eval}-\texttt{boolean}-\texttt{C}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow V; S' \\ A_n; S_n; F; M \vdash E_n \overset{arg}{\Rightarrow} M_i; S'_n \end{array}}{A, A_n; S, S_n; F; M \vdash E, E_n \overset{arg}{\Rightarrow} A \rightarrow V, M_i; S', S'_n}[\texttt{Apply}-\texttt{arg}]$$

$$\frac{}{\emptyset; \emptyset; F; M \vdash \overset{arg}{\Rightarrow} \emptyset; \emptyset}[\texttt{Apply}-\texttt{arg}-\texttt{empty}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \overset{C}{\Downarrow} nil; S' \\ A_n; S_n; F; M \vdash E_n \overset{argC}{\Rightarrow} M_i; S'_n \end{array}}{A, A_n; S, S_n; F; M \vdash E, E_n \overset{argC}{\Rightarrow} A \rightarrow nil, M_i; S', S'_n}[\texttt{Apply}-\texttt{arg}-\texttt{C}]$$

$$\frac{}{\emptyset; \emptyset; F; M \vdash \overset{argC}{\Rightarrow} \emptyset; \emptyset}[\texttt{Apply}-\texttt{arg}-\texttt{empty}-\texttt{C}]$$

$$\frac{\begin{array}{c} F(f) = \langle A, E \rangle \\ A; S; F; M \vdash a \overset{arg}{\Rightarrow} M_i; S' \\ S_e; F; M_i \vdash E \Downarrow V; S'_e \end{array}}{S_e, S; F; M \vdash f(a) \Downarrow V; S'_e, S'} [\texttt{Eval} - \texttt{apply}]$$

$$\frac{\begin{array}{c} F(f) = \langle A, E \rangle \\ A; S; F; M \vdash a \overset{argC}{\Rightarrow} M_i; S' \\ S_e; F; M_i \vdash E \overset{C}{\Downarrow} nil; S'_e \end{array}}{S_e, S; F; M \vdash f(a) \Downarrow nil; S'_e, S'} [\texttt{Eval} - \texttt{apply} - \texttt{C}]$$

$$\frac{\begin{array}{c} L, R \in \mathbb{R} \\ V = L + R \end{array}}{L + R \overset{binop}{\Rightarrow} V} [\texttt{Binop} - \texttt{plus}]$$

$$\frac{\begin{array}{c} L, R \in \mathbb{R} \\ V = L - R \end{array}}{L - R \overset{binop}{\Rightarrow} V} [\texttt{Binop} - \texttt{minus}]$$

$$\frac{\begin{array}{c} L, R \in \mathbb{R} \\ V = L * R \end{array}}{L * R \overset{binop}{\Rightarrow} V} [\texttt{Binop} - \texttt{times}]$$

$$\frac{\begin{array}{c} L, R \in \mathbb{R} \\ V = L / R \end{array}}{L / R \overset{binop}{\Rightarrow} V} [\texttt{Binop} - \texttt{divide}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash L_e \Downarrow L_v; S'_l \\ S_r; F; M \vdash R_e \Downarrow R_v; S'_l \\ L_v \;\; B \;\; R_v \overset{binop}{\Rightarrow} V \end{array}}{S_l, S_r; F; M \vdash L_e \;\; B \;\; R_e \Downarrow V; S'_l, S'_r} [\texttt{Eval} - \texttt{binop}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash L_e \overset{C}{\Downarrow} nil; S'_l \\ S_r; F; M \vdash R_e \overset{C}{\Downarrow} nil; S'_l \end{array}}{S_l, S_r; F; M \vdash L_e \;\; B \;\; R_e \overset{C}{\Downarrow} nil; S'_l, S'_r} [\texttt{Eval} - \texttt{binop} - \texttt{C}]$$