

# Pixy formal semantics

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## 1 Term language

$\langle expr \rangle ::= \langle number \rangle$   
|  $\langle var \rangle$   
|  $\langle bool \rangle$   
| **nil**  
|  $? \langle expr \rangle$   
| **if**  $\langle expr \rangle$  **then**  $\langle expr \rangle$  **else**  $\langle expr \rangle$   
|  $\langle expr \rangle$  **fbv**  $\langle expr \rangle$   
|  $\langle expr \rangle$  **where**  $\langle wheredecls \rangle$   
| **next**  $\langle expr \rangle$   
|  $\langle var \rangle$  (  $\langle exprlist \rangle$  )  
|  $\langle expr \rangle + \langle expr \rangle$   
|  $\langle expr \rangle - \langle expr \rangle$   
|  $\langle expr \rangle * \langle expr \rangle$   
|  $\langle expr \rangle / \langle expr \rangle$   
|  $\langle expr \rangle \dots \langle expr \rangle$   
| (  $\langle exprlist \rangle$  )  
| **len**  $\langle expr \rangle$   
| [  $\langle exprlist \rangle$  ]  $\langle expr \rangle$

$\langle exprlist \rangle ::= \langle expr \rangle , \langle exprlist \rangle \mid \langle expr \rangle$

$\langle varlist \rangle ::= \langle var \rangle , \langle varlist \rangle \mid \langle var \rangle$

$\langle wheredecl \rangle ::= \langle var \rangle = \langle expr \rangle$   
|  $\langle var \rangle$  (  $\langle varlist \rangle$  ) =  $\langle expr \rangle$   
| (  $\langle varlist \rangle$  ) =  $\langle expr \rangle$

$\langle wheredecls \rangle ::= \langle wheredecl \rangle ; \langle wheredecls \rangle \mid \langle wheredecl \rangle$

$\langle bool \rangle ::= \text{true} \mid \text{false}$

## 2 Init rules

$$\frac{F; M \vdash E \xRightarrow{\text{init}} S}{F; M \vdash ?E \xRightarrow{\text{init}} S} [\text{Init} - \text{check}]$$

$$\frac{\begin{array}{c} F; M \vdash L \xRightarrow{\text{init}} S_l \\ F; M \vdash R \xRightarrow{\text{init}} S_r \end{array}}{F; M \vdash L \text{ fby } R \xRightarrow{\text{init}} \text{false}, S_l, S_r} [\text{Init} - \text{fby}]$$

$$\frac{\begin{array}{c} F; M \vdash C \xRightarrow{\text{init}} S_c \\ F; M \vdash T \xRightarrow{\text{init}} S_t \\ F; M \vdash F \xRightarrow{\text{init}} S_f \end{array}}{F; M \vdash \text{if } C \text{ then } T \text{ else } F \xRightarrow{\text{init}} S_c, S_t, S_f} [\text{Init} - \text{ite}]$$

$$\frac{F; M \vdash E \xRightarrow{\text{init}} S}{F; M \vdash \text{next } E \xRightarrow{\text{init}} \text{false}, S} [\text{Init} - \text{next}]$$

$$\frac{F; M; F_f \vdash E_n \xRightarrow{\text{wheredec}} F'; M'}{F; M; F_f \vdash n = E; E_n \xRightarrow{\text{wheredec}} F'; n \rightarrow (), M'} [\text{WhereInit} - \text{v} - \text{decl}]$$

$$\frac{F; M; F_f \vdash E_n \xRightarrow{\text{wheredec}} F'; M'}{F; M; F_f \vdash f(A) = E; E_n \xRightarrow{\text{wheredec}} f \rightarrow \langle A, E, F_f \rangle, F'; M'} [\text{WhereInit} - \text{fn} - \text{decl}]$$

$$\frac{}{F; M; F_f; M_f \vdash \xRightarrow{\text{wheredec}} F; M} [\text{WhereInit} - \text{empty} - \text{decl}]$$

$$\frac{\begin{array}{c} F; M \vdash E \xRightarrow{\text{init}} s \\ F; M \vdash E_n \xRightarrow{\text{whereinit}} S \end{array}}{F; M \vdash n = E; E_n \xRightarrow{\text{whereinit}} \langle \text{nil}, s \rangle, S} [\text{WhereInit} - \text{v} - \text{init}]$$

$$\frac{F; M \vdash E_n \xRightarrow{\text{whereinit}} S}{F; M \vdash f(A) = E; E_n \xRightarrow{\text{whereinit}} S} [\text{WhereInit} - \text{fn} - \text{init}]$$

$$\frac{}{F; M \vdash \overset{whereinit}{\Rightarrow} \emptyset} [\text{WhereInit} - \text{empty} - \text{init}]$$

$$\frac{\begin{array}{c} F; M; F' \vdash E_s \overset{wheredecl}{\Rightarrow} F'' \\ F' = F'' \\ F'; M' \vdash E \overset{init}{\Rightarrow} S_e \\ F'; M' \vdash E_s \overset{whereinit}{\Rightarrow} S \end{array}}{F; M \vdash E \text{ where } E_s \overset{init}{\Rightarrow} S_e, S} [\text{Init} - \text{where}]$$

$$\frac{\begin{array}{c} F; M \vdash E_a \overset{init}{\Rightarrow} S \\ A_n; F; M \vdash E_{a_n} \overset{applyinit}{\Rightarrow} S_n; M_i \end{array}}{A, A_n; F; M \vdash E_a, E_{a_n} \overset{applyinit}{\Rightarrow} S, S_n; A \rightarrow \langle E_a, M \rangle, M_i} [\text{ApplyInit} - \text{arg}]$$

$$\frac{}{F; M \vdash \overset{applyinit}{\Rightarrow} \emptyset; \emptyset} [\text{ApplyInit} - \text{empty}]$$

$$\frac{\begin{array}{c} F(f) = \langle A, E, F_i \rangle \\ A; F; M \vdash E_a \overset{applyinit}{\Rightarrow} S; M_i \\ F_i; M_i \vdash E \overset{init}{\Rightarrow} S_e \end{array}}{F; M \vdash f(E_a) \overset{init}{\Rightarrow} S_e, S} [\text{Init} - \text{apply}]$$

$$\frac{E \in \{true, false\} \vee E \in \mathbb{R}}{F; M \vdash E \overset{init}{\Rightarrow} \emptyset} [\text{Init} - \text{literal}]$$

$$\frac{M(E) = -}{F; M \vdash E \overset{init}{\Rightarrow} \emptyset} [\text{Init} - \text{id}]$$

$$\frac{\begin{array}{c} F; M \vdash L \overset{init}{\Rightarrow} S_l \\ F; M \vdash R \overset{init}{\Rightarrow} S_r \end{array}}{F; M \vdash L \_ R \overset{init}{\Rightarrow} S_l, S_r} [\text{Init} - \text{binop}]$$

### 3 Data type sizing

### 4 Evaluation rules

$$\frac{S; F; M \vdash E \Downarrow V; S' \quad V \neq \text{nil}}{S; F; M \vdash ?E \Downarrow \text{true}; S'} [\text{Eval} - \text{check} - \text{true}]$$

$$\frac{S; F; M \vdash E \Downarrow \text{nil}; S'}{S; F; M \vdash ?E \Downarrow \text{false}; S'} [\text{Eval} - \text{check} - \text{false}]$$

$$\frac{S_c; F; M \vdash C \Downarrow \text{true}; S'_c \quad S_t; F; M \vdash T \Downarrow V; S'_t \quad S_f; F; M \vdash F \Downarrow \text{nil}; S'_f}{S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \Downarrow V; S'_c, S'_t, S'_f} [\text{Eval} - \text{ite} - \text{true}]$$

$$\frac{S_c; F; M \vdash C \Downarrow \text{false}; S'_c \quad S_t; F; M \vdash T \Downarrow \text{nil}; S'_t \quad S_f; F; M \vdash F \Downarrow V; S'_f}{S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \Downarrow V; S'_c, S'_t, S'_f} [\text{Eval} - \text{ite} - \text{false}]$$

$$\frac{S_c; F; M \vdash C \Downarrow \text{nil}; S'_c \quad S_t; F; M \vdash T \Downarrow \text{nil}; S'_t \quad S_f; F; M \vdash F \Downarrow \text{nil}; S'_f}{S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \Downarrow \text{nil}; S'_c, S'_t, S'_f} [\text{Eval} - \text{ite} - \text{nil}]$$

$$\frac{S_c; F; M \vdash C \Downarrow \text{nil}; S'_c \quad S_t; F; M \vdash T \Downarrow \text{nil}; S'_t \quad S_f; F; M \vdash F \Downarrow \text{nil}; S'_f}{S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \Downarrow \text{nil}; S'_c, S'_t, S'_f} [\text{Eval} - \text{ite} - \text{C}]$$

$$\frac{S_l; F; M \vdash E_l \Downarrow V; S'_l \quad V \neq \text{nil} \quad S_r; F; M \vdash E_r \Downarrow \text{nil}; S'_r}{\text{false}, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow V; \text{true}, S'_l, S'_r} [\text{Eval} - \text{fby} - \text{before}]$$

$$\frac{S_l; F; M \vdash E_l \Downarrow \text{nil}; S'_l \quad S_r; F; M \vdash E_r \Downarrow \text{nil}; S'_r}{\text{false}, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow \text{nil}; \text{false}, S'_l, S'_r} [\text{Eval} - \text{fby} - \text{before} - \text{nil}]$$

$$\frac{S_l; F; M \vdash E_l \Downarrow nil; S'_l \quad S_r; F; M \vdash E_r \Downarrow V; S'_r}{true, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow V; true, S'_l, S'_r} [\text{Eval} - \text{fby} - \text{after}]$$

$$\frac{S_l; F; M \vdash E_l \Downarrow nil; S'_l \quad S_r; F; M \vdash E_r \Downarrow nil; S'_r}{c, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow nil; c, S'_l, S'_r} [\text{Eval} - \text{fby} - \text{C}]$$

$$\frac{S_n; F; M; F_f \vdash E_n \xRightarrow{\text{names}} F'; M'}{\langle v, \_ \rangle, S_n; F; M \vdash n = E; E_n \xRightarrow{\text{names}} F'; n \rightarrow v, M'} [\text{WhereNames} - \text{v} - \text{decl}]$$

$$\frac{S; F; M; F_f \vdash E_v \xRightarrow{\text{names}} F'; M'}{S; F; M \vdash f(A) = E; E_n \xRightarrow{\text{names}} f \rightarrow \langle A, E, F_f \rangle, F'; M'} [\text{WhereNames} - \text{fn} - \text{decl}]$$

$$\frac{}{\emptyset; F; M; F_f \vdash \xRightarrow{\text{names}} F'; M'} [\text{WhereNames} - \text{empty}]$$

$$\frac{s; F; M \vdash E \Downarrow v; s' S_n; F; M \vdash E_n \xRightarrow{\text{values}} S'_n}{\langle \_, s \rangle, S_n; F; M \vdash n = E; E_n \xRightarrow{\text{values}} \langle v, s' \rangle, S'_n} [\text{WhereVal} - \text{v} - \text{decl}]$$

$$\frac{S; F; M \vdash E_n \xRightarrow{\text{values}} S'}{S; F; M \vdash f(A) = E; E_n \xRightarrow{\text{values}} S'} [\text{WhereVal} - \text{fn} - \text{decl}]$$

$$\frac{}{\emptyset; F; M \vdash \xRightarrow{\text{values}} \emptyset} [\text{WhereVal} - \text{empty}]$$

$$\frac{S; F; M; F_f \vdash E_s \xRightarrow{\text{names}} F_i; M_i \quad F_f = F_i \quad S; F_i; M_i \vdash E_s \xRightarrow{\text{values}} S' \quad S_e; F_i; M_i \vdash E \Downarrow V; S'_e}{S_e, S; F; M \vdash E \text{ where } E_s \Downarrow V; S'_e, S'} [\text{Eval} - \text{where}]$$

$$\frac{S; F; M; F_f \vdash E_s \xRightarrow{\text{names}} F_i; M_i \quad F_f = F_i \quad S; F_i; M_i \vdash E_s \xRightarrow{\text{values}} S' \quad S_e; F_i; M_i \vdash E \Downarrow nil; S'_e}{S_e, S; F; M \vdash E \text{ where } E_s \Downarrow nil; S'_e, S'} [\text{Eval} - \text{where} - \text{C}]$$

$$\frac{S; F; M \vdash E \Downarrow V; S' \quad V \neq \text{nil}}{\langle \text{false}, \text{nil} \rangle, S; F; M \vdash \text{next } E \Downarrow \text{nil}; \langle \text{true}, V \rangle, S'} [\text{Eval} - \text{next} - \text{before}]$$

$$\frac{S; F; M \vdash E \Downarrow \text{nil}; S'}{\langle \text{false}, \text{nil} \rangle, S; F; M \vdash \text{next } E \Downarrow \text{nil}; \langle \text{false}, \text{nil} \rangle, S'} [\text{Eval} - \text{next} - \text{before} - \text{nil}]$$

$$\frac{v \neq \text{nil} \quad S; F; M \vdash E \Downarrow V; S'}{\langle \text{true}, v \rangle, S; F; M \vdash \text{next } E \Downarrow v; \langle \text{true}, V \rangle, S'} [\text{Eval} - \text{next} - \text{after}]$$

$$\frac{S; F; M \vdash E \Downarrow V; S'}{\langle \text{true}, \text{nil} \rangle, S; F; M \vdash \text{next } E \Downarrow V; \langle \text{true}, \text{nil} \rangle, S'} [\text{Eval} - \text{next} - \text{after} - \text{nil}]$$

$$\frac{S; F; M \vdash E \Downarrow \text{nil}; S'}{\langle c, v \rangle, S; F; M \vdash \text{next } E \Downarrow \text{nil}; \langle c, v \rangle, S'} [\text{Eval} - \text{next} - \text{C}]$$

$$\frac{}{\emptyset; F; M \vdash \text{nil} \Downarrow \text{nil}; \emptyset} [\text{Eval} - \text{nil}]$$

$$\frac{}{\emptyset; F; M \vdash \text{nil} \Downarrow \text{nil}; \emptyset} [\text{Eval} - \text{nil} - \text{C}]$$

$$\frac{N \in \mathbb{R}}{\emptyset; F; M \vdash N \Downarrow N; \emptyset} [\text{Eval} - \text{num}]$$

$$\frac{N \in \mathbb{R}}{\emptyset; F; M \vdash N \Downarrow \text{nil}; \emptyset} [\text{Eval} - \text{num} - \text{C}]$$

$$\frac{M(I) = V}{\emptyset; F; M \vdash I \Downarrow V; \emptyset} [\text{Eval} - \text{id}]$$

$$\frac{M(I) = V}{\emptyset; F; M \vdash I \Downarrow \text{nil}; \emptyset} [\text{Eval} - \text{id} - \text{C}]$$

$$\frac{E \in \{true, false\}}{\emptyset; F; M \vdash E \Downarrow E; \emptyset} [\text{Eval} - \text{boolean}]$$

$$\frac{E \in \{true, false\}}{\emptyset; F; M \vdash E \Downarrow nil; \emptyset} [\text{Eval} - \text{boolean} - \text{C}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow V; S' \\ A_n; S_n; F; M \vdash E_n \xrightarrow{arg} M_i; S'_n \end{array}}{A, A_n; S, S_n; F; M \vdash E, E_n \xrightarrow{arg} A \rightarrow V, M_i; S', S'_n} [\text{Apply} - \text{arg}]$$

$$\frac{}{\emptyset; \emptyset; F; M \vdash \xrightarrow{arg} \emptyset; \emptyset} [\text{Apply} - \text{arg} - \text{empty}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow nil; S' \\ A_n; S_n; F; M \vdash E_n \xrightarrow{argC} M_i; S'_n \end{array}}{A, A_n; S, S_n; F; M \vdash E, E_n \xrightarrow{argC} A \rightarrow nil, M_i; S', S'_n} [\text{Apply} - \text{arg} - \text{C}]$$

$$\frac{}{\emptyset; \emptyset; F; M \vdash \xrightarrow{argC} \emptyset; \emptyset} [\text{Apply} - \text{arg} - \text{empty} - \text{C}]$$

$$\frac{\begin{array}{c} F(f) = \langle A, E, F_i \rangle \\ A; S; F; M \vdash a \xrightarrow{arg} M_i; S' \\ S_e; F_i; M_i \vdash E \Downarrow V; S'_e \end{array}}{S_e, S; F; M \vdash f(a) \Downarrow V; S'_e, S'} [\text{Eval} - \text{apply}]$$

$$\frac{\begin{array}{c} F(f) = \langle A, E, F_i \rangle \\ A; S; F; M \vdash a \xrightarrow{argC} M_i; S' \\ S_e; F_i; M_i \vdash E \Downarrow nil; S'_e \end{array}}{S_e, S; F; M \vdash f(a) \Downarrow nil; S'_e, S'} [\text{Eval} - \text{apply} - \text{C}]$$

$$\frac{\begin{array}{c} L, R \in \mathbb{R} \\ V = L + R \end{array}}{L + R \xrightarrow{binop} V} [\text{Binop} - \text{plus}]$$

$$\frac{L, R \in \mathbb{R} \quad V = L - R}{L - R \xRightarrow{binop} V} [\text{Binop} - \text{minus}]$$

$$\frac{L, R \in \mathbb{R} \quad V = L * R}{L * R \xRightarrow{binop} V} [\text{Binop} - \text{times}]$$

$$\frac{L, R \in \mathbb{R} \quad V = L / R}{L / R \xRightarrow{binop} V} [\text{Binop} - \text{divide}]$$

$$\frac{\begin{array}{l} S_l; F; M \vdash L_e \Downarrow L_v; S'_l \\ S_r; F; M \vdash R_e \Downarrow R_v; S'_r \\ L_v \ B \ R_v \xRightarrow{binop} V \end{array}}{S_l, S_r; F; M \vdash L_e \ B \ R_e \Downarrow V; S'_l, S'_r} [\text{Eval} - \text{binop}]$$

$$\frac{\begin{array}{l} S_l; F; M \vdash L_e \Downarrow \text{nil}; S'_l \\ S_r; F; M \vdash R_e \Downarrow \text{nil}; S'_r \end{array}}{S_l, S_r; F; M \vdash L_e \ B \ R_e \Downarrow \text{nil}; S'_l, S'_r} [\text{Eval} - \text{binop} - \text{C}]$$

$$\frac{a}{\emptyset; F; M \vdash \text{len } E}$$