Pixy formal semantics

A. Finn Hackett, Reed Mullanix

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1 Term language

⟨*expr*⟩ ::= ⟨*number*⟩
  | ⟨*var*⟩
  | ⟨*bool*⟩
  | `nil`
  | `?` ⟨*expr*⟩
  | `if` ⟨*expr*⟩ `then` ⟨*expr*⟩ `else` ⟨*expr*⟩
  | ⟨*expr*⟩ `fby` ⟨*expr*⟩
  | ⟨*expr*⟩ `where` ⟨*wheredecls*⟩
  | `next` ⟨*expr*⟩
  | ⟨*var*⟩ `(` ⟨*exprlist*⟩ `)`
  | ⟨*expr*⟩ `+` ⟨*expr*⟩
  | ⟨*expr*⟩ `-` ⟨*expr*⟩
  | ⟨*expr*⟩ `*` ⟨*expr*⟩
  | ⟨*expr*⟩ `/` ⟨*expr*⟩
  | `(` ⟨*exprlist*⟩ `(` `|` ⟨*expr*⟩ `)`? `)`

⟨*exprlist*⟩ ::= ⟨*expr*⟩ `,` ⟨*exprlist*⟩ | ⟨*expr*⟩

⟨*varlist*⟩ ::= ⟨*var*⟩ `,` ⟨*varlist*⟩ | ⟨*var*⟩

⟨*wheredecl*⟩ ::= ⟨*var*⟩ `=` ⟨*expr*⟩
  | ⟨*var*⟩ `(` ⟨*varlist*⟩ `)` `=` ⟨*expr*⟩
  | `(` ⟨*varlist*⟩ `(` `|` ⟨*var*⟩ `)`? `)` `=` ⟨*expr*⟩

⟨*wheredecls*⟩ ::= ⟨*wheredecl*⟩; ⟨*wheredecls*⟩ | ⟨*wheredecl*⟩

⟨*bool*⟩ ::= `true` | `false`

2 Init rules

$$\frac{F; M \vdash E \overset{init}{\Rightarrow} S}{F; M \vdash ?E \overset{init}{\Rightarrow} S}[\texttt{Init} - \texttt{check}]$$

$$\frac{\begin{array}{c} F; M \vdash L \overset{init}{\Rightarrow} S_l \\ F; M \vdash R \overset{init}{\Rightarrow} S_r \end{array}}{F; M \vdash L \,\text{fby}\, R \overset{init}{\Rightarrow} false, S_l, S_r}[\texttt{Init} - \texttt{fby}]$$

$$\frac{\begin{array}{c} F; M \vdash C \overset{init}{\Rightarrow} S_c \\ F; M \vdash T \overset{init}{\Rightarrow} S_t \\ F; M \vdash F \overset{init}{\Rightarrow} S_f \end{array}}{F; M \vdash \text{if}\, C \,\text{then}\, T \,\text{else}\, F \overset{init}{\Rightarrow} S_c, S_t, S_f}[\texttt{Init} - \texttt{ite}]$$

$$\frac{F; M \vdash E \overset{init}{\Rightarrow} S}{F; M \vdash \text{next}\, E \overset{init}{\Rightarrow} false, S}[\texttt{Init} - \texttt{next}]$$

$$\frac{F; M; F_f \vdash E_n \overset{wheredecl}{\Rightarrow} F'; M'}{F; M; F_f \vdash n = E; E_n \overset{wheredecl}{\Rightarrow} F'; n \to (), M'}[\texttt{WhereInit} - \texttt{v} - \texttt{decl}]$$

$$\frac{n_s \overset{wheredecltuple}{\Rightarrow} M}{n, n_s \overset{wheredecltuple}{\Rightarrow} n \to (), M}[\texttt{WhereInit} - \texttt{tuple} - \texttt{decl} - \texttt{body}]$$

$$\frac{}{\mid\ n \overset{wheredecltuple}{\Rightarrow} n \to ()}[\texttt{WhereInit} - \texttt{tuple} - \texttt{decl} - \texttt{tail}]$$

$$\frac{}{\overset{wheredecltuple}{\Rightarrow} \emptyset}[\texttt{WhereInit} - \texttt{tuple} - \texttt{decl} - \texttt{empty}]$$

$$\frac{\begin{array}{c} n_s \overset{wheredecltuple}{\Rightarrow} M_s \\ F; M; F_f \vdash E_n \overset{wheredecl}{\Rightarrow} F'; M' \end{array}}{F; M; F_f \vdash (n_s) = E; E_n \overset{wheredecl}{\Rightarrow} F'; M_s, M'}[\texttt{WhereInit} - \texttt{tuple} - \texttt{decl}]$$

$$\frac{F; M; F_f \vdash E_n \overset{wheredecl}{\Rightarrow} F'; M'}{F; M; F_f \vdash f(A) = E; E_n \overset{wheredecl}{\Rightarrow} f \to \langle A, E, F_f \rangle, F'; M'}[\texttt{WhereInit} - \texttt{fn} - \texttt{decl}]$$

$$\frac{}{F; M; F_f; M_f \vdash \overset{wheredecl}{\Rightarrow} F; M}[\texttt{WhereInit} - \texttt{empty} - \texttt{decl}]$$

$$\frac{\begin{array}{c} F; M \vdash E \overset{init}{\Rightarrow} s \\ F; M \vdash E_n \overset{whereinit}{\Rightarrow} S \end{array}}{F; M \vdash n = E; E_n \overset{whereinit}{\Rightarrow} \langle nil, s \rangle, S}[\texttt{WhereInit} - \texttt{v} - \texttt{init}]$$

$$\frac{\begin{array}{c} F; M \vdash E \overset{init}{\Rightarrow} s \\ n_s \overset{niltuple}{\Rightarrow} S_s \\ F; M \vdash E_n \overset{whereinit}{\Rightarrow} S \end{array}}{F; M \vdash (n_s) = E; E_n \overset{whereinit}{\Rightarrow} \langle S_s, s \rangle, S}[\texttt{WhereInit} - \texttt{tuple} - \texttt{init}]$$

$$\frac{F; M \vdash E_n \overset{whereinit}{\Rightarrow} S}{F; M \vdash f(A) = E; E_n \overset{whereinit}{\Rightarrow} S}[\texttt{WhereInit} - \texttt{fn} - \texttt{init}]$$

$$\frac{}{F; M \vdash \overset{whereinit}{\Rightarrow} \emptyset}[\texttt{WhereInit} - \texttt{empty} - \texttt{init}]$$

$$\frac{\begin{array}{c} F; M; F' \vdash E_s \overset{wheredecl}{\Rightarrow} F'' \\ F' = F'' \\ F'; M' \vdash E \overset{init}{\Rightarrow} S_e \\ F'; M' \vdash E_s \overset{whereinit}{\Rightarrow} S \end{array}}{F; M \vdash E \text{ where } E_s \overset{init}{\Rightarrow} S_e, S}[\texttt{Init} - \texttt{where}]$$

$$\frac{\begin{array}{c} F; M \vdash E_a \overset{init}{\Rightarrow} S \\ A_n; F; M \vdash E_{a_n} \overset{applyinit}{\Rightarrow} S_n; M_i \end{array}}{A, A_n; F; M \vdash E_a, E_{a_n} \overset{applyinit}{\Rightarrow} S, S_n; A \to \langle E_a, M \rangle, M_i}[\texttt{ApplyInit} - \texttt{arg}]$$

$$\frac{}{F; M \vdash \overset{applyinit}{\Rightarrow} \emptyset; \emptyset}[\texttt{ApplyInit} - \texttt{empty}]$$

$$\frac{\begin{array}{c} F(f) = \langle A, E, F_i \rangle \\ A; F; M \vdash E_a \overset{applyinit}{\Rightarrow} S; M_i \\ F_i; M_i \vdash E \overset{init}{\Rightarrow} S_e \end{array}}{F; M \vdash f(E_a) \overset{init}{\Rightarrow} S_e, S}[\texttt{Init} - \texttt{apply}]$$

$$\frac{E \in \{true, false\} \vee E \in \mathbb{R}}{F; M \vdash E \overset{init}{\Rightarrow} \emptyset}[\texttt{Init} - \texttt{literal}]$$

$$\frac{M(E) = \_}{F; M \vdash E \overset{init}{\Rightarrow} \emptyset}[\texttt{Init} - \texttt{id}]$$

$$\frac{\begin{array}{c} F; M \vdash L \overset{init}{\Rightarrow} S_l \\ F; M \vdash R \overset{init}{\Rightarrow} S_r \end{array}}{F; M \vdash L \ \_ \ R \overset{init}{\Rightarrow} S_l, S_r}[\texttt{Init} - \texttt{binop}]$$

$$\frac{\begin{array}{c} F; M \vdash E \overset{init}{\Rightarrow} S \\ F; M \vdash E_n \overset{inittuple}{\Rightarrow} S_n \end{array}}{F; M \vdash E, E_n \overset{inittuple}{\Rightarrow} S, S_n}[\texttt{Tuple} - \texttt{state}]$$

$$\frac{F; M \vdash E \overset{init}{\Rightarrow} S}{F; M \vdash \mid \ E \overset{inittuple}{\Rightarrow} S}[\texttt{Tuple} - \texttt{state} - \texttt{tail}]$$

$$\frac{}{F; M \vdash \overset{inittuple}{\Rightarrow} \emptyset}[\texttt{Tuple} - \texttt{state} - \texttt{empty}]$$

$$\frac{F; M \vdash E_s \overset{inittuple}{\Rightarrow} S}{F; M \vdash (E_s) \overset{init}{\Rightarrow} S}[\texttt{Init} - \texttt{tuple}]$$

3 Data type sizing

4 Evaluation rules

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow V; S' \\ V \neq nil \end{array}}{S; F; M \vdash ?E \Downarrow true; S'}[\texttt{Eval} - \texttt{check} - \texttt{true}]$$

$$\frac{S; F; M \vdash E \Downarrow nil; S'}{S; F; M \vdash ?E \Downarrow false; S'}[\texttt{Eval} - \texttt{check} - \texttt{false}]$$

$$\frac{\begin{array}{c} S_c; F; M \vdash C \Downarrow true; S'_c \\ S_t; F; M \vdash T \Downarrow V; S'_t \\ S_f; F; M \vdash F \Downarrow\!\!\!\Downarrow nil; S'_f \end{array}}{S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \Downarrow V; S'_c, S'_t, S'_f}[\texttt{Eval} - \texttt{ite} - \texttt{true}]$$

$$\frac{\begin{array}{c} S_c; F; M \vdash C \Downarrow false; S'_c \\ S_t; F; M \vdash T \Downarrow\!\!\!\Downarrow nil; S'_t \\ S_f; F; M \vdash F \Downarrow V; S'_f \end{array}}{S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \Downarrow V; S'_c, S'_t, S'_f}[\texttt{Eval} - \texttt{ite} - \texttt{false}]$$

$$\frac{\begin{array}{c} S_c; F; M \vdash C \Downarrow nil; S'_c \\ S_t; F; M \vdash T \Downarrow\!\!\!\Downarrow nil; S'_t \\ S_f; F; M \vdash F \Downarrow\!\!\!\Downarrow nil; S'_f \end{array}}{S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \Downarrow nil; S'_c, S'_t, S'_f}[\texttt{Eval} - \texttt{ite} - \texttt{nil}]$$

$$\frac{\begin{array}{c} S_c; F; M \vdash C \Downarrow\!\!\!\Downarrow nil; S'_c \\ S_t; F; M \vdash T \Downarrow\!\!\!\Downarrow nil; S'_t \\ S_f; F; M \vdash F \Downarrow\!\!\!\Downarrow nil; S'_f \end{array}}{S_c, S_t, S_f; F; M \vdash \text{if } C \text{ then } T \text{ else } F \Downarrow\!\!\!\Downarrow nil; S'_c, S'_t, S'_f}[\texttt{Eval} - \texttt{ite} - \texttt{C}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash E_l \Downarrow V; S'_l \\ V \neq nil \\ S_r; F; M \vdash E_r \Downarrow\!\!\!\Downarrow nil; S'_r \end{array}}{false, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow V; true, S'_l, S'_r}[\texttt{Eval} - \texttt{fby} - \texttt{before}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash E_l \Downarrow nil; S'_l \\ S_r; F; M \vdash E_r \Downarrow\!\!\!\Downarrow nil; S'_r \end{array}}{false, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow nil; false, S'_l, S'_r}[\texttt{Eval} - \texttt{fby} - \texttt{before} - \texttt{nil}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash E_l \Downarrow\!\!\!\Downarrow nil; S'_l \\ S_r; F; M \vdash E_r \Downarrow V; S'_r \end{array}}{true, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow V; true, S'_l, S'_r}[\texttt{Eval} - \texttt{fby} - \texttt{after}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash E_l \Downarrow nil; S'_l \\ S_r; F; M \vdash E_r \Downarrow nil; S'_r \end{array}}{c, S_l, S_r; F; M \vdash E_l \text{ fby } E_r \Downarrow nil; c, S'_l, S'_r}[\texttt{Eval} - \texttt{fby} - \texttt{C}]$$

$$\frac{S_n; F; M; F_f \vdash E_n \overset{names}{\Rightarrow} F'; M'}{\langle v, \_ \rangle, S_n; F; M \vdash n = E; E_n \overset{names}{\Rightarrow} F'; n \to v, M'}[\texttt{WhereNames} - \texttt{v} - \texttt{decl}]$$

$$\frac{S_n; M \vdash n_n \overset{readtuple}{\Rightarrow} M'}{v, S_n; M \vdash n, n_n \overset{readtuple}{\Rightarrow} n \to v, M'}[\texttt{WhereNames} - \texttt{tuple} - \texttt{decl} - \texttt{read}]$$

$$\frac{}{v_s; M \vdash \mid\ n \overset{readtuple}{\Rightarrow} n \to (v_s), M}[\texttt{WhereNames} - \texttt{tuple} - \texttt{decl} - \texttt{read} - \texttt{tail}]$$

$$\frac{}{\emptyset; M \vdash \overset{readtuple}{\Rightarrow} M}[\texttt{WhereNames} - \texttt{tuple} - \texttt{decl} - \texttt{read} - \texttt{empty}]$$

$$\frac{\begin{array}{c} S; M \vdash n_s \overset{readtuple}{\Rightarrow} M_s \\ S_n; F; M; F_f \vdash E_n \overset{names}{\Rightarrow} F'; M' \end{array}}{\langle S, \_ \rangle, S_n; F; M \vdash (n_s) = E; E_n \overset{names}{\Rightarrow} F'; M_s, M'}[\texttt{WhereNames} - \texttt{tuple} - \texttt{decl}]$$

$$\frac{S; F; M; F_f \vdash E_v \overset{names}{\Rightarrow} F'; M'}{S; F; M \vdash f(A) = E; E_n \overset{names}{\Rightarrow} f \to \langle A, E, F_f \rangle, F'; M'}[\texttt{WhereNames} - \texttt{fn} - \texttt{decl}]$$

$$\frac{}{\emptyset; F; M; F_f \vdash \overset{names}{\Rightarrow} F'; M'}[\texttt{WhereNames} - \texttt{empty}]$$

$$\frac{\begin{array}{c} s; F; M \vdash E \Downarrow v; s' \\ S_n; F; M \vdash E_n \overset{values}{\Rightarrow} S'_n \end{array}}{\langle \_, s \rangle, S_n; F; M \vdash n = E; E_n \overset{values}{\Rightarrow} \langle v, s' \rangle, S'_n}[\texttt{WhereVal} - \texttt{v} - \texttt{decl}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow (V_s); S' \\ S_n; F; M \vdash E_n \overset{values}{\Rightarrow} S'_n \end{array}}{\langle \_, S \rangle, S_n; F; M \vdash (\_) = E; E_n \overset{values}{\Rightarrow} \langle V_s, S' \rangle, S'_n}[\texttt{WhereVal} - \texttt{tuple} - \texttt{decl}]$$

$$\frac{n_n \overset{niltuple}{\Rightarrow} S_n}{n, n_n \overset{niltuple}{\Rightarrow} nil, S_n}[\texttt{NilTuple} - \texttt{body}]$$

$$\frac{}{\mid\ n \overset{niltuple}{\Rightarrow} nil}[\texttt{NilTuple} - \texttt{tail}]$$

$$\frac{}{\overset{niltuple}{\Rightarrow} \emptyset}[\texttt{NilTuple} - \texttt{empty}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow nil; S' \\ N_s \overset{niltuple}{\Rightarrow} V_s \\ S_n; F; M \vdash E_n \overset{values}{\Rightarrow} S'_n \end{array}}{\langle \_, S \rangle, S_n; F; M \vdash (N_s) = E; E_n \overset{values}{\Rightarrow} \langle V_s, S' \rangle, S'_n}[\texttt{WhereVal} - \texttt{tuple} - \texttt{decl} - \texttt{nil}]$$

$$\frac{S; F; M \vdash E_n \overset{values}{\Rightarrow} S'}{S; F; M \vdash f(A) = E; E_n \overset{values}{\Rightarrow} S'}[\texttt{WhereVal} - \texttt{fn} - \texttt{decl}]$$

$$\frac{}{\emptyset; F; M \vdash \overset{values}{\Rightarrow} \emptyset}[\texttt{WhereVal} - \texttt{empty}]$$

$$\frac{\begin{array}{c} S; F; M; F_f \vdash E_s \overset{names}{\Rightarrow} F_i; M_i \\ F_f = F_i \\ S; F_i; M_i \vdash E_s \overset{values}{\Rightarrow} S' \\ S_e; F_i; M_i \vdash E \Downarrow V; S'_e \end{array}}{S_e, S; F; M \vdash E \,\text{where}\, E_s \Downarrow V; S'_e, S'}[\texttt{Eval} - \texttt{where}]$$

$$\frac{\begin{array}{c} S; F; M; F_f \vdash E_s \overset{names}{\Rightarrow} F_i; M_i \\ F_f = F_i \\ S; F_i; M_i \vdash E_s \overset{values}{\Rightarrow} S' \\ S_e; F_i; M_i \vdash E \Downarrow\!\!\Downarrow nil; S'_e \end{array}}{S_e, S; F; M \vdash E \,\text{where}\, E_s \Downarrow\!\!\Downarrow nil; S'_e, S'}[\texttt{Eval} - \texttt{where} - \texttt{C}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow V; S' \\ V \neq nil \end{array}}{\langle false, nil \rangle, S; F; M \vdash \text{next}\, E \Downarrow nil; \langle true, V \rangle, S'}[\texttt{Eval} - \texttt{next} - \texttt{before}]$$

$$\frac{S; F; M \vdash E \Downarrow nil; S'}{\langle false, nil\rangle, S; F; M \vdash \text{next } E \Downarrow nil; \langle false, nil\rangle, S'}[\texttt{Eval} - \texttt{next} - \texttt{before} - \texttt{nil}]$$

$$\frac{\begin{array}{c} v \neq nil \\ S; F; M \vdash E \Downarrow V; S' \end{array}}{\langle true, v\rangle, S; F; M \vdash \text{next } E \Downarrow v; \langle true, V\rangle, S'}[\texttt{Eval} - \texttt{next} - \texttt{after}]$$

$$\frac{S; F; M \vdash E \Downarrow V; S'}{\langle true, nil\rangle, S; F; M \vdash \text{next } E \Downarrow V; \langle true, nil\rangle, S'}[\texttt{Eval} - \texttt{next} - \texttt{after} - \texttt{nil}]$$

$$\frac{S; F; M \vdash E \Cup nil; S'}{\langle c, v\rangle, S; F; M \vdash \text{next } E \Cup nil; \langle c, v\rangle, S'}[\texttt{Eval} - \texttt{next} - \texttt{C}]$$

$$\frac{}{\emptyset; F; M \vdash nil \Downarrow nil; \emptyset}[\texttt{Eval} - \texttt{nil}]$$

$$\frac{}{\emptyset; F; M \vdash nil \Cup nil; \emptyset}[\texttt{Eval} - \texttt{nil} - \texttt{C}]$$

$$\frac{N \in \mathbb{R}}{\emptyset; F; M \vdash N \Downarrow N; \emptyset}[\texttt{Eval} - \texttt{num}]$$

$$\frac{N \in \mathbb{R}}{\emptyset; F; M \vdash N \Cup nil; \emptyset}[\texttt{Eval} - \texttt{num} - \texttt{C}]$$

$$\frac{M(I) = V}{\emptyset; F; M \vdash I \Downarrow V; \emptyset}[\texttt{Eval} - \texttt{id}]$$

$$\frac{M(I) = V}{\emptyset; F; M \vdash I \Cup nil; \emptyset}[\texttt{Eval} - \texttt{id} - \texttt{C}]$$

$$\frac{E \in \{true, false\}}{\emptyset; F; M \vdash E \Downarrow E; \emptyset}[\texttt{Eval} - \texttt{boolean}]$$

$$\frac{E \in \{true, false\}}{\emptyset; F; M \vdash E \Downarrow nil; \emptyset}[\texttt{Eval} - \texttt{boolean} - \texttt{C}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow V; S' \\ A_n; S_n; F; M \vdash E_n \overset{arg}{\Rightarrow} M_i; S'_n \end{array}}{A, A_n; S, S_n; F; M \vdash E, E_n \overset{arg}{\Rightarrow} A \to V, M_i; S', S'_n}[\texttt{Apply} - \texttt{arg}]$$

$$\frac{}{\emptyset; \emptyset; F; M \vdash \overset{arg}{\Rightarrow} \emptyset; \emptyset}[\texttt{Apply} - \texttt{arg} - \texttt{empty}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow nil; S' \\ A_n; S_n; F; M \vdash E_n \overset{argC}{\Rightarrow} M_i; S'_n \end{array}}{A, A_n; S, S_n; F; M \vdash E, E_n \overset{argC}{\Rightarrow} A \to nil, M_i; S', S'_n}[\texttt{Apply} - \texttt{arg} - \texttt{C}]$$

$$\frac{}{\emptyset; \emptyset; F; M \vdash \overset{argC}{\Rightarrow} \emptyset; \emptyset}[\texttt{Apply} - \texttt{arg} - \texttt{empty} - \texttt{C}]$$

$$\frac{\begin{array}{c} F(f) = \langle A, E, F_i \rangle \\ A; S; F; M \vdash a \overset{arg}{\Rightarrow} M_i; S' \\ S_e; F_i; M_i \vdash E \Downarrow V; S'_e \end{array}}{S_e, S; F; M \vdash f(a) \Downarrow V; S'_e, S'}[\texttt{Eval} - \texttt{apply}]$$

$$\frac{\begin{array}{c} F(f) = \langle A, E, F_i \rangle \\ A; S; F; M \vdash a \overset{argC}{\Rightarrow} M_i; S' \\ S_e; F_i; M_i \vdash E \Downarrow nil; S'_e \end{array}}{S_e, S; F; M \vdash f(a) \Downarrow nil; S'_e, S'}[\texttt{Eval} - \texttt{apply} - \texttt{C}]$$

$$\frac{\begin{array}{c} L, R \in \mathbb{R} \\ V = L + R \end{array}}{L + R \overset{binop}{\Rightarrow} V}[\texttt{Binop} - \texttt{plus}]$$

$$\frac{\begin{array}{c} L, R \in \mathbb{R} \\ V = L - R \end{array}}{L - R \overset{binop}{\Rightarrow} V}[\texttt{Binop} - \texttt{minus}]$$

$$\frac{L, R \in \mathbb{R} \quad V = L * R}{L * R \overset{binop}{\Rightarrow} V}[\texttt{Binop} - \texttt{times}]$$

$$\frac{L, R \in \mathbb{R} \quad V = L/R}{L/R \overset{binop}{\Rightarrow} V}[\texttt{Binop} - \texttt{divide}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash L_e \Downarrow L_v; S'_l \\ S_r; F; M \vdash R_e \Downarrow R_v; S'_l \\ L_v \ \ B \ \ R_v \overset{binop}{\Rightarrow} V \end{array}}{S_l, S_r; F; M \vdash L_e \ \ B \ \ R_e \Downarrow V; S'_l, S'_r}[\texttt{Eval} - \texttt{binop}]$$

$$\frac{\begin{array}{c} S_l; F; M \vdash L_e \Downarrow nil; S'_l \\ S_r; F; M \vdash R_e \Downarrow nil; S'_l \end{array}}{S_l, S_r; F; M \vdash L_e \ \ B \ \ R_e \Downarrow nil; S'_l, S'_r}[\texttt{Eval} - \texttt{binop} - \texttt{C}]$$

$$\frac{E_n \overset{len}{\Rightarrow} N \quad C = N + 1}{\_, E_n \overset{len}{\Rightarrow} C}[\texttt{Len} - \texttt{count}]$$

$$\frac{}{\overset{len}{\Rightarrow} 0}[\texttt{Len} - \texttt{count} - 0]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow (V_s); S' \\ V_s \overset{len}{\Rightarrow} N \end{array}}{S; F; M \vdash \text{len}\, E \Downarrow N; S'}[\texttt{Eval} - \texttt{len}]$$

$$\frac{S; F; M \vdash E \Downarrow nil; S'}{S; F; M \vdash \text{len}\, E \Downarrow nil; S'}[\texttt{Eval} - \texttt{len} - \texttt{C}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow V; S' \\ S_n; F; M \vdash E_n \overset{tuplecons}{\Rightarrow} V_n; S'_n \end{array}}{S, S_n; F; M \vdash E, E_n \overset{tuplecons}{\Rightarrow} V, V_n; S', S'_n}[\texttt{Tuple} - \texttt{construct}]$$

$$\frac{}{\emptyset; F; M \vdash \overset{tuplecons}{\Rightarrow} \emptyset; \emptyset}[\texttt{Tuple} - \texttt{construct} - \texttt{empty}]$$

$$\frac{S; F; M \vdash E \overset{tuplecons}{\Rightarrow} (V_s); S'}{S; F; M \vdash \mid E \overset{tuplecons}{\Rightarrow} V_s; S'}[\texttt{Tuple} - \texttt{construct} - \texttt{tail}]$$

$$\frac{S; F; M \vdash E_s \overset{tuplecons}{\Rightarrow} V_s; S'}{S; F; M \vdash (E_s) \Downarrow (V_s); S'}[\texttt{Eval} - \texttt{tuple}]$$

$$\frac{\begin{array}{c} S; F; M \vdash E \Downarrow nil; S' \\ S_n; F; M \vdash E_n \overset{tupleconsC}{\Rightarrow} S'_n \end{array}}{S, S_n; F; M \vdash E, E_n \overset{tupleconsC}{\Rightarrow} S', S'_n}[\texttt{Tuple} - \texttt{construct} - \texttt{C}]$$

$$\frac{}{\emptyset; F; M \vdash \overset{tupleconsC}{\Rightarrow} \emptyset}[\texttt{Tuple} - \texttt{construct} - \texttt{empty} - \texttt{C}]$$

$$\frac{S; F; M \vdash E \Downarrow nil; S'}{S; F; M \vdash \mid E \overset{tupleconsC}{\Rightarrow} S'}[\texttt{Tuple} - \texttt{construct} - \texttt{tail} - \texttt{C}]$$

$$\frac{S; F; M \vdash E_s \overset{tupleconsC}{\Rightarrow} S'}{S; F; M \vdash (E_s) \Downarrow nil; S'}[\texttt{Eval} - \texttt{tuple} - \texttt{C}]$$