Problem_3

September 30, 2021

1 Problem 3

[1]:
```
from IPython.core.pylabtools import figsize
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from random import sample
import seaborn as sns
import scipy.stats as ss
from scipy import optimize
from statsmodels.api import OLS # has better summary stats than sklearn's OLSb
from random import sample
import  arviz as az
```

First load the data.

[2]:
```
penguin = pd.read_csv("penguins.csv")
penguin.describe()
```

[2]:

	culmen_length_mm	culmen_depth_mm	flipper_length_mm	body_mass_g
count	342.000000	342.000000	342.000000	342.000000
mean	43.921930	17.151170	200.915205	4201.754386
std	5.459584	1.974793	14.061714	801.954536
min	32.100000	13.100000	172.000000	2700.000000
25%	39.225000	15.600000	190.000000	3550.000000
50%	44.450000	17.300000	197.000000	4050.000000
75%	48.500000	18.700000	213.000000	4750.000000
max	59.600000	21.500000	231.000000	6300.000000

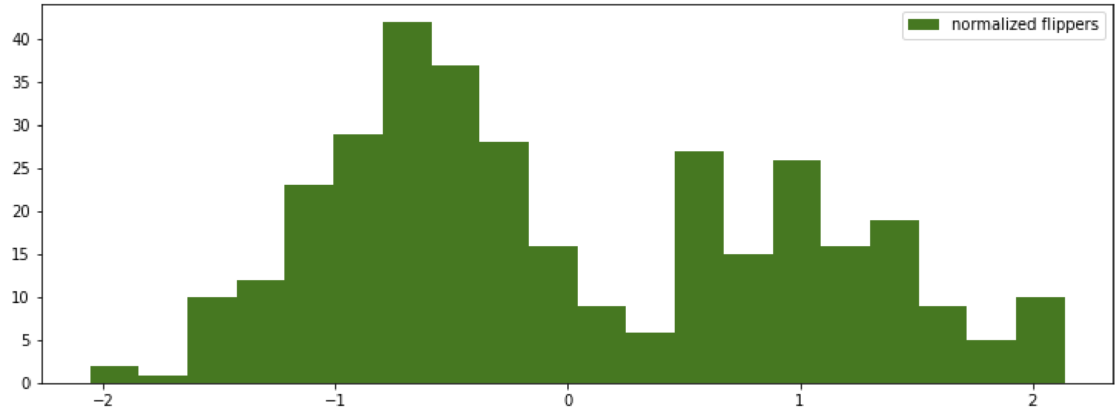
1.1 Part 1

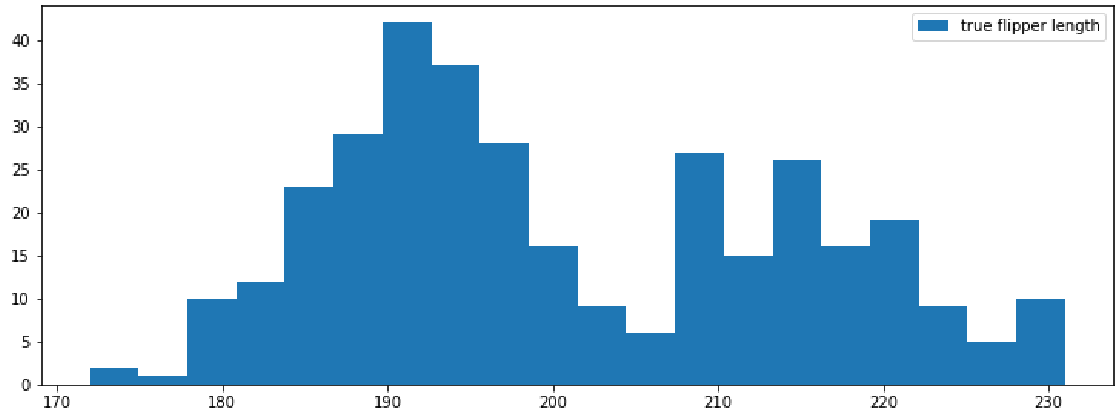
[3]:
```
fl = penguin['flipper_length_mm']
penguin['flipper_length_norm'] = [(i-fl.mean())/fl.std() for i in fl.to_numpy()]
penguin['body_mass_kg'] = penguin['body_mass_g']/1000

print('Normalized flipper data:')
print('\tstd:',penguin['flipper_length_norm'].std())
print('\tmean:',round(penguin['flipper_length_norm'].mean(),10))
```

```
Normalized flipper data:
        std: 0.9999999999999996
        mean: -0.0
```

We can check to make sure the distributions look the same after the normalization.

```
[4]: plt.figure(figsize=(12.5,10))
ax = plt.subplot(211)
plt.hist(penguin.flipper_length_norm.values, bins =20, label="normalized flippers",color="#467821")
plt.legend(loc="upper right");
ax = plt.subplot(212)
plt.hist(penguin.flipper_length_mm.values,bins =20,label="true flipper length")
plt.legend(loc="upper right");
```



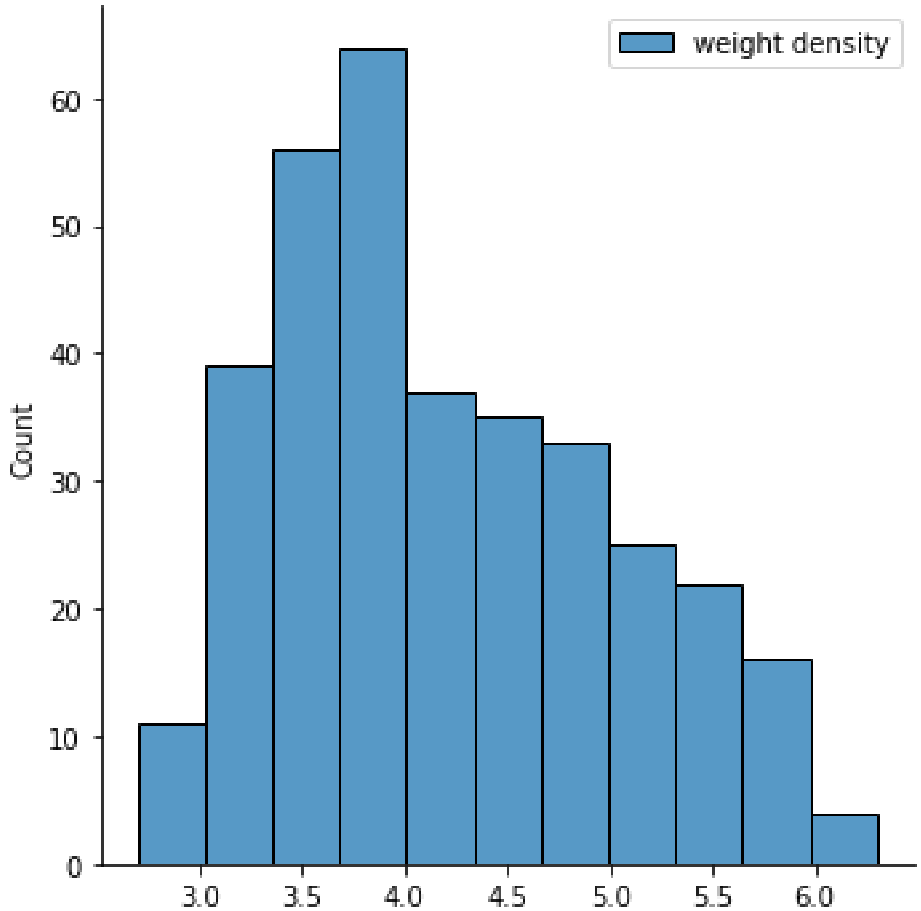


What about the weight distribution of the penguins?

```
[5]: # curious about weight distribution
plt.figure(figsize=(12.5,10))
# plt.hist(penguin.body_mass_kg.values,label="non-normalized",bins = 50)
sns.displot(penguin.body_mass_kg.values, label="weight density")
```

```
plt.legend(loc="upper right")
plt.show();
```

```
<Figure size 900x720 with 0 Axes>
```



1.2 Part 2

We choose normal priors for α and β. The prior for β has a mean of zero, so as not to impose an effect on the linear regression. For the variance, we choose a log-normal distribution.

[22]:
```
def generate_parameters_prior():
    return [ss.norm(5,np.sqrt(0.6)).rvs(), # generates alph
            ss.norm(0,np.sqrt(0.2)).rvs(),  # generates beta
            ss.lognorm(s=1.25).rvs()]  # generates var #lognorm.rvs(s=5)
```

1.3 Part 3

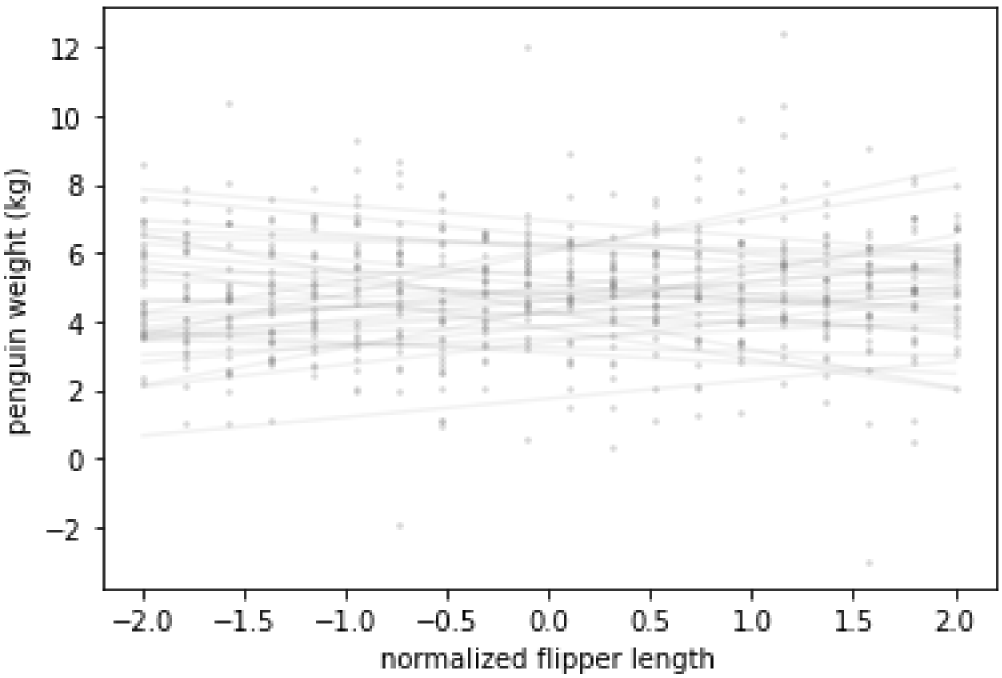
We can start by creating a function that samples X.The priors are shown as normal distributions with the intercept of flipper length being close to 5 and the slope of the mean near and around 0.

We set the variance to be a lognormal because we want have a variance that is non-negative and similar to a normal distribution.

[23]:
```
def sample_from_x(num_samples):
    return sample(list(penguin['flipper_length_norm']),num_samples)
```

[24]:
```
def generate_data(alpha, beta, var, X,num_samples=1):
    return ss.norm(alpha + beta * X, np.sqrt(var)).rvs(num_samples)
```

[25]:
```
import random
random.seed(5)
for X in np.linspace(-2,2,30):

    params = generate_parameters_prior()

    num_samples = 50
    Y0 = generate_data(*params,-2)
    Y1 = generate_data(*params,2)

    x = np.linspace(-2,2,20)
    y = [generate_data(*params,i) for i in x]

    plt.plot([-2,2],[Y0,Y1], c='gray', alpha=0.1)
    plt.scatter(x,y,c='gray',s=3,alpha=0.2)

plt.xlabel('normalized flipper length')
plt.ylabel('penguin weight (kg)')
plt.savefig('prior_predictive.png')
plt.show()
```

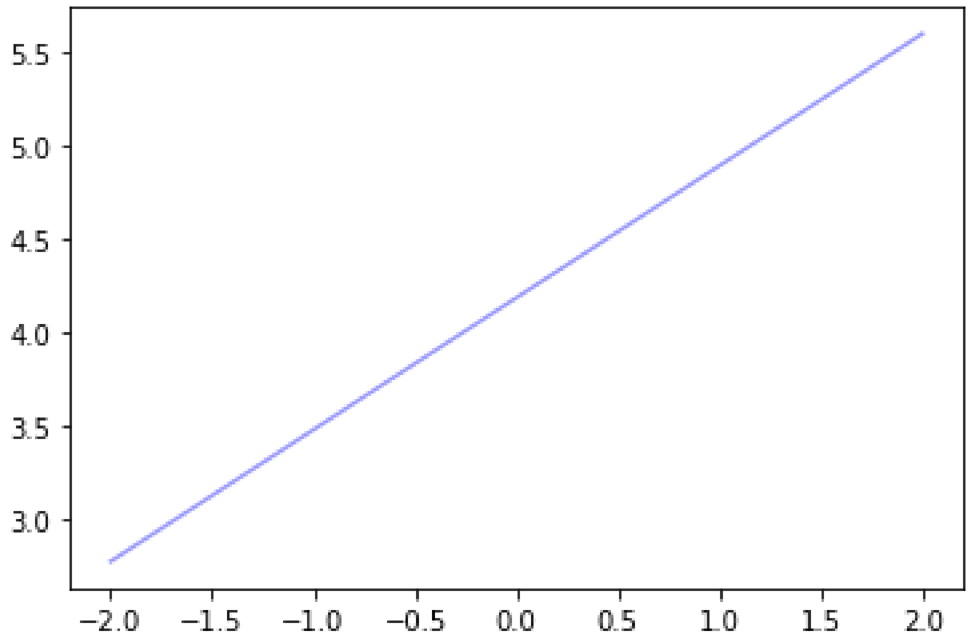


[43]:

```
from statsmodels.api import OLS
import sklearn.linear_model

lm = sklearn.linear_model.LinearRegression()
x = penguin['flipper_length_norm'].values.reshape(len(penguin),1)
y = penguin['body_mass_kg'].values.reshape(len(penguin),1)
intercept = np.full((len(penguin),1),1)
df = pd.DataFrame(intercept)
df['x'] = x
df.rename(columns={0: 'intercept'})
lm = OLS(y, df)
results = lm.fit()
results.summary()

x = np.linspace(-2, 2, 100)
plt.plot(x, params[0] + params[1]*x, c='blue', alpha=0.4,␣
 ↪zorder=-100,label='regression line')
```

[43]:

```
[<matplotlib.lines.Line2D at 0x7fbdbfb8fbd0>]
```



1.4 Part 4

The log posterior expression is given by $\frac{-N \log \sigma^2}{2\sigma^2} \sum(Y_i - \alpha - \beta X_i) - \frac{(\alpha-5)^2}{0.6} - \frac{\beta^2}{0.2} - \frac{(\log \sigma^2)^2}{(1.25)^2} - \sigma^2$

[26]:
```
def log_posterior(alpha, beta, var, Y, X):
    N = len(X)
    if var > 100 or var < 0:
        return -100000000000000000000
    return (-N * np.log(var) / 2 - np.sum((Y - alpha - beta * X) ** 2) / var + ␣
↪   # log_likelihood
        ss.norm(50, np.sqrt(200)).logpdf(alpha) +                          #␣
↪log of the prior on alpha
        ss.norm(0, np.sqrt(100)).logpdf(beta)+ ss.lognorm(s=1.25).logpdf(var) ␣
↪)                            # log of the prior on beta

def minus_log_posterior(theta):
    Y = penguin['body_mass_kg']
    X = penguin['flipper_length_norm']
    return - log_posterior(*theta,Y,X)
```

[27]:
```
fit = optimize.minimize(minus_log_posterior, [50, 0, 10])
```

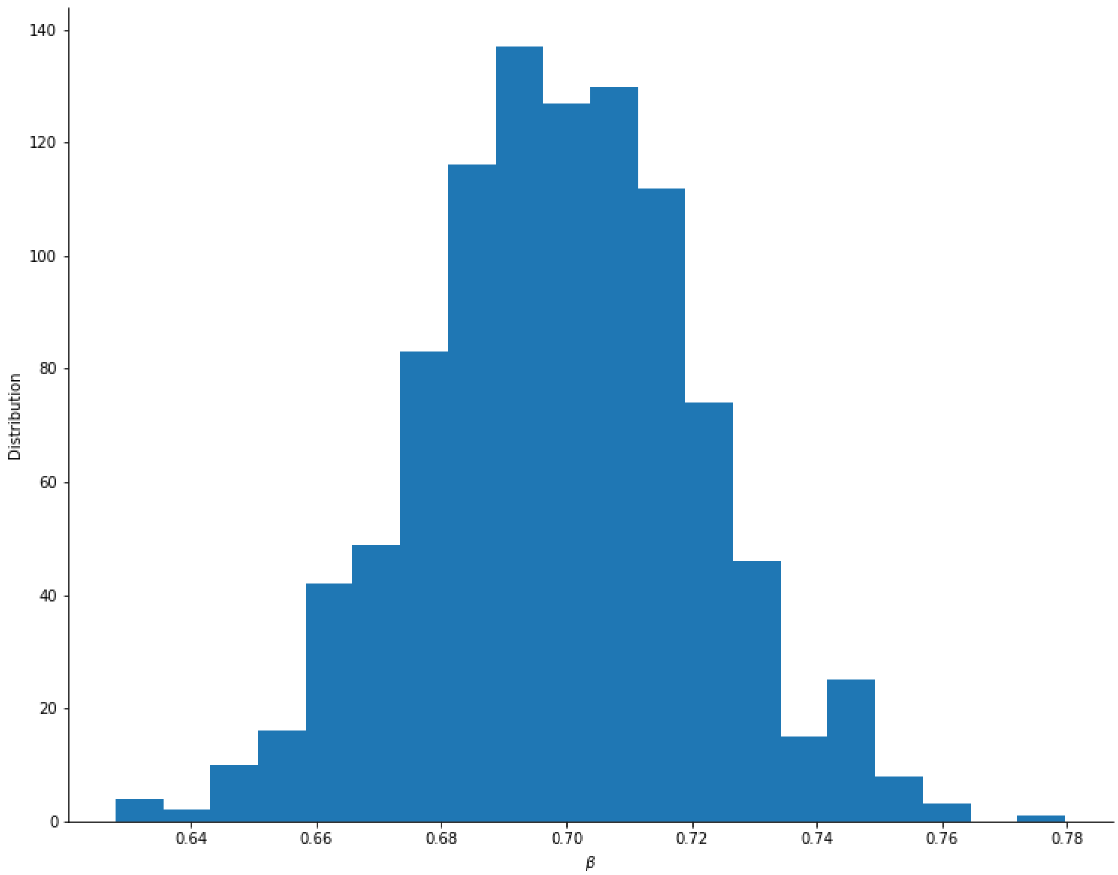
[28]:
```
# fit approx.
MAP = fit['x']
hess_inv  = fit['hess_inv']
```

```
approx = ss.multivariate_normal(MAP, hess_inv)
```

1.5 Part 5

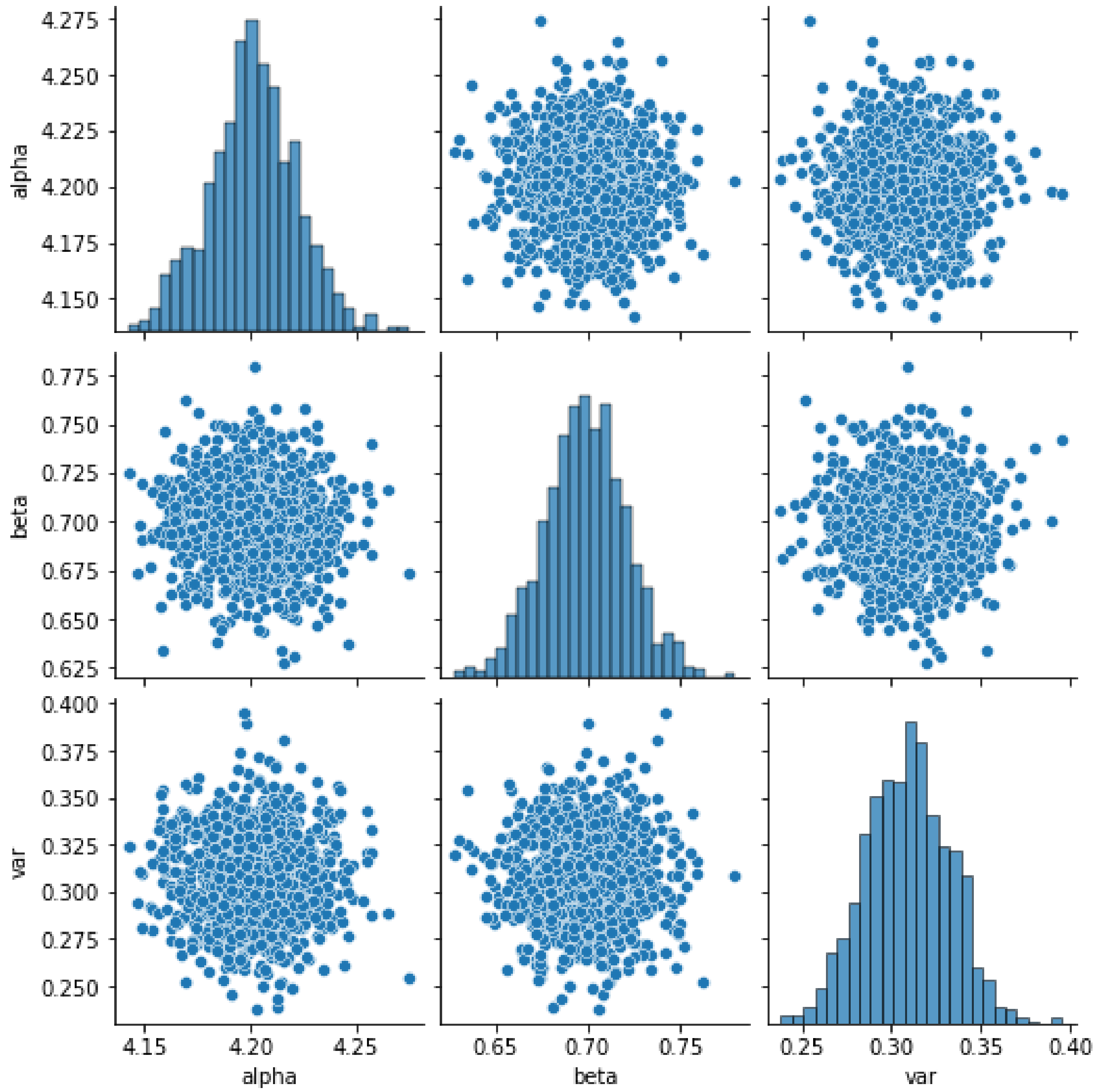
[29]:
```
samples = approx.rvs(1000)
```

[30]:
```
plt.figure(figsize=(12.5,10))
plt.hist(samples[:,1],bins = 20)
plt.ylabel('Distribution')
plt.xlabel(r'$\beta$')
sns.despine()
```



[31]:
```
plt.figure(figsize=(12.5,10))
sns.pairplot(pd.DataFrame(samples, columns=['alpha', 'beta', 'var']))
plt.savefig('pairplot.png')
```

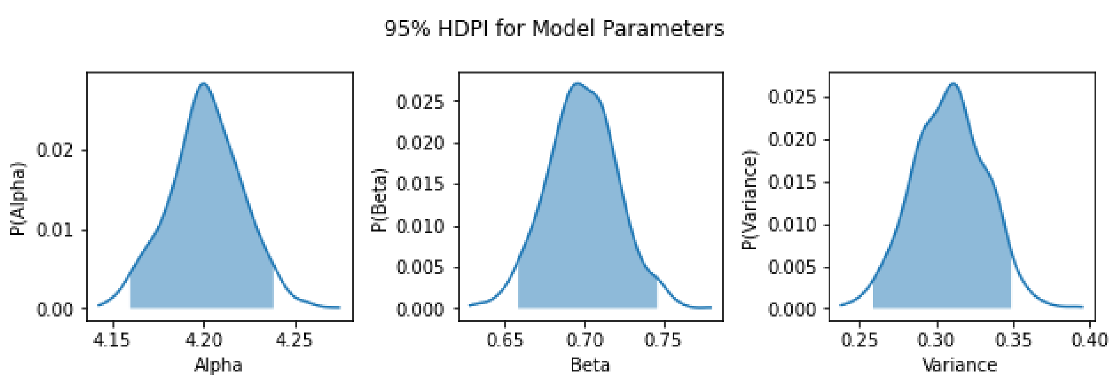
```
<Figure size 900x720 with 0 Axes>
```



We can commute the highest density percentile for 95% inclusion for each of the parameters.

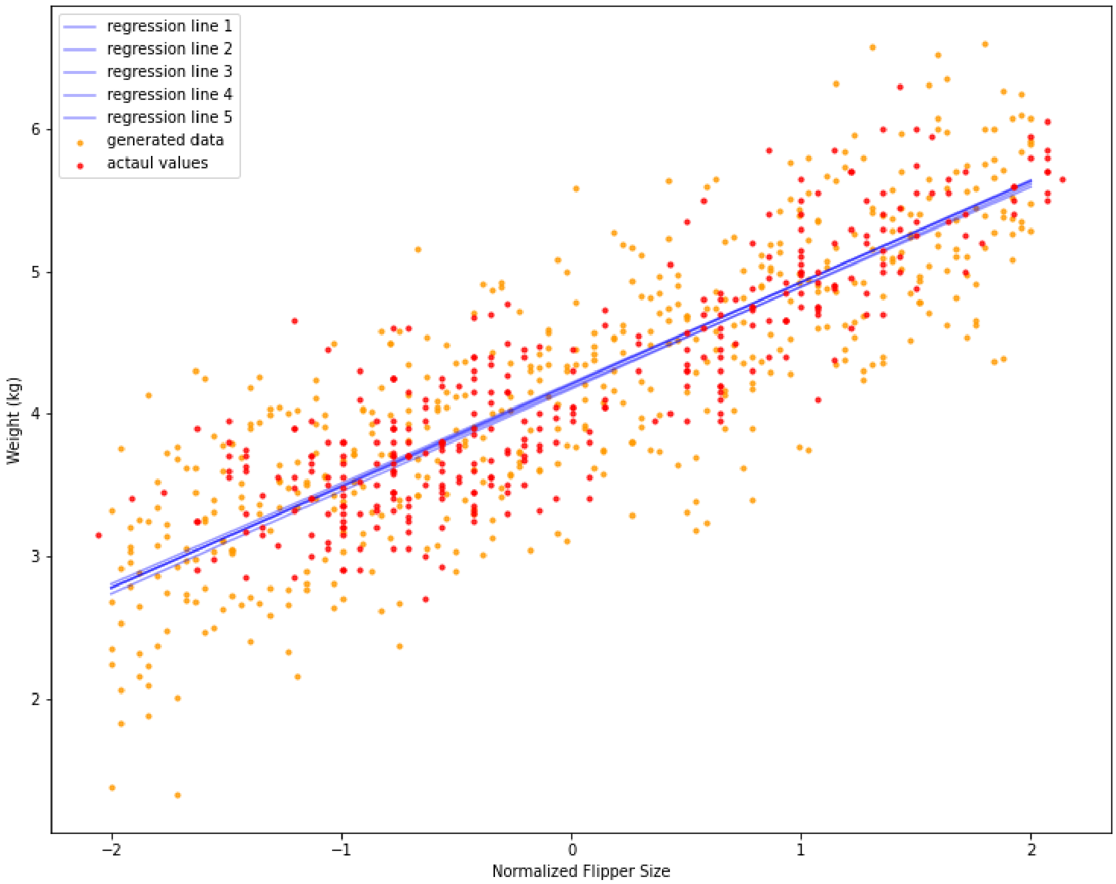
[32]:
```
alphas, betas, variances = list(zip(*samples))

def calculate_kde(data):

    x = np.linspace(min(data),max(data),100)
    y= ss.gaussian_kde(data).pdf(x)

    y = y/sum(y) #normalize

    return x,y
```

```
def calculate_hdi(data):
    return az.hdi(np.array(data),0.95)

fig, ax = plt.subplots(1,3,figsize=(9,3))

x,y = calculate_kde(alphas)
ax[0].plot(x,y)
a,b = calculate_hdi(alphas)
ax[0].fill_between(x,y,where=(x>a)&(x<b),alpha=0.5)
ax[0].set_xlabel('Alpha')
ax[0].set_ylabel('P(Alpha)')

x,y = calculate_kde(betas)
ax[1].plot(x,y)
a,b = calculate_hdi(betas)
ax[1].fill_between(x,y,where=(x>a)&(x<b),alpha=0.5)
ax[1].set_xlabel('Beta')
ax[1].set_ylabel('P(Beta)')

x,y = calculate_kde(variances)
ax[2].plot(x,y)
a,b = calculate_hdi(variances)
ax[2].fill_between(x,y,where=(x>a)&(x<b),alpha=0.5)
ax[2].set_xlabel('Variance')
ax[2].set_ylabel('P(Variance)')

plt.suptitle('95% HDPI for Model Parameters')
plt.tight_layout()
plt.savefig('model_hdpi.png')
plt.show()
```



1.6 Part 6

[33]:
```
# data points and regression lines
x = np.linspace(-2, 2, 100)
plt.figure(figsize=(12.5,10))
appended_data = []
for i in range(1,6):  # 10 data sets

    params = approx.rvs()

    Y0 = generate_data(*params,x,100)  # simulate 100 measurements at 0
    plt.plot(x, params[0] + params[1]*x, c='blue', alpha=0.4,␣
 ↪zorder=-100,label='regression line %s' % i)

    temp_zipped = list(zip(x,Y0))
    df_temp = pd.DataFrame(temp_zipped, columns = ['x', 'y'])
    appended_data.append(df_temp)


appended_data = pd.concat(appended_data)
plt.scatter(appended_data['x'],appended_data['y'], c='#FF9A00',s = 10, alpha=.
 ↪8,zorder=-100, label='generated data')
plt.xlabel('Normalized Flipper Size')
plt.ylabel('Weight (kg)')
plt.scatter(penguin['flipper_length_norm'], penguin['body_mass_kg'],␣
 ↪s=10,alpha=.8, c='red',label="actaul values")
plt.legend()
plt.savefig('generate_sample_data.png')
plt.show()
```



regression line 1
regression line 2
regression line 3
regression line 4
regression line 5
generated data
actaul values
Weight (kg)
Normalized Flipper Size
−2
−1
0
1
2
2
3
4
5
6


```
[ ]:
```