Tiempo: $O(n \log n)$ Ordenamiento "merge-sort"

$$O(n \log_2 n + n - 1) = O(n \log n)$$

$$O(\log_b n) = O(\log n)$$

logaritmo base 10
ln base e
$\log_b$ base b

$f(n) = \log_b n$ $\qquad b^{f(n)} = n$

$\log_b n$ "el valor al cual hay que elevar b para que nos dé n"

$\log_{10} = \log$
Convertir logaritmo base 'b' a base '10'.
de las "leyes de los logaritmos", se sabe que...

$$\frac{\log n}{\log b} = \log_b n$$

$k = \frac{1}{\log b}$ $\qquad \log_b n = k \log n$

$$O(k \log n) = O(\log n)$$

$\subseteq$ $\supseteq$

$f(n) \in O(k \log n) \Rightarrow f(n) \in O(\log n)$
$f(n) \in O(\log n) \Rightarrow f(n) \in O(k \log n)$

1. $f(n) \in O(\underbrace{k \log n}_{g(n)}) \Rightarrow \exists\, c > 0$ y $n_0 > 0$ $\ni$
$0 \leq f(n) \leq c k \log n \quad \forall n \geq n_0$

P.d. $\exists\, c' > 0,\ n_0' > 0 \ni \quad 0 \leq f(n) \leq c' \log n$

$n_0' = n_0 \quad c' = ck \Rightarrow f(n) \in O(\log n)$

2. $f(n) \in O(\log n) \Rightarrow \exists\, c > 0$ y $n_0 > 0$ $\ni$
$0 \leq f(n) \leq c \log n \quad \forall n \geq n_0$

P.d. $\exists\, c' > 0,\ n_0' > 0 \ni \quad 0 \leq f(n) \leq c' k \log n$

$n_0' = n_0,\quad c' = c/k \Rightarrow f(n) \in O(k \log n)$

$\therefore\ O(\log n) = O(\log_b n)$

P.d. $\Theta(k\, g(n)) = \Theta(g(n))$

Tarea

P.d. $f(n) = \Theta(g(n)) \Longleftrightarrow f(n) = O(g(n))$ y
$f(n) = \Omega(g(n))$

P.d. $\max(f(n), g(n)) = \Theta(f(n) + g(n))$

$\max(f(n), g(n)) \in \Theta(f(n) + g(n))$

"Asintóticamente no-negativas" $\begin{cases} f(n) \geq 0, \forall n \geq n_1 \\ g(n) \geq 0, \forall n \geq n_2 \end{cases}$ $\forall n \geq n_0 \ni n_0 = \max\{n_1, n_2\}$

P.d. $\exists\ c_1 > 0,\ c_2 > 0,\ n_0 > 0 \ni\ 0 \leq c_1(f(n) + g(n)) \leq \max\{f(n), g(n)\} \leq c_2(f(n) + g(n)) \quad \forall n \geq n_0$

$$\max\{f(n), g(n)\} \geq f(n)$$
$$\max\{f(n), g(n)\} \geq g(n)$$
$$\overline{2\max\{f(n), g(n)\} \geq f(n) + g(n)}$$
$$\max\{f(n), g(n)\} \geq \frac{1}{2}(f(n) + g(n))$$

Sea $c_1 = \frac{1}{2}$, $\quad c_1(f(n) + g(n)) \geq 0 \quad \forall n \geq n_0$

$$c_1(f(n) + g(n)) \leq \max\{f(n), g(n)\}$$

$\forall n \geq n_0$ $\quad f(n) \geq 0$, $g(n) \geq 0$

$$f(n) + g(n) \geq g(n)$$
$$f(n) + g(n) \geq f(n)$$
$$\Rightarrow$$
$$f(n) + g(n) \geq \max\{f(n), g(n)\}$$
$$1(f(n) + g(n)) \geq \max\{f(n), g(n)\}$$

Sea $c_2 = 1$

$\therefore \max\{f(n), g(n)\} = \Theta(f(n) + g(n))$