The Capacity Region of the Deterministic Interference Channel with a Jammer

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Introduction

- The open nature of wireless communication makes it susceptible to a malicious and intelligent jammer.
- We consider the two-user interference channel in the presence of one adversarial jammer.
- We find the capacity region of the simplified deterministic model for this problem.

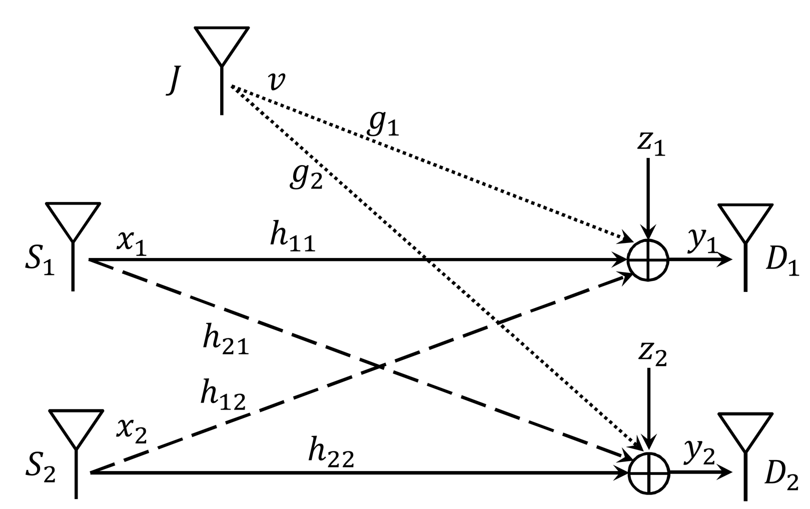
Model for Gaussian Interference Channel with a Jammer

Signal y_i at the receiver D_i:

$$y_1 = h_{11}x_1 + h_{12}x_2 + g_1 v + z_1$$
$$y_2 = h_{21}x_1 + h_{22}x_2 + g_2 v + z_2$$

where

- x_1 and x_2 are N-length vectors representing the user's signal.
- h_{ij}'s and g_i's are channel gains.
- $z_i \sim (0, 1)$ is AWGN independent of x_1, x_2 and v (for $i = 1, 2$).
- v is the adversarial jammer signal.
- Jammer is independent of the users.



Two-user Gaussian IC with a Jammer

Power Constraints

- $\sum_{n=1}^{N} ||x_{i,n}||^2 \leq NP_i$, for $i = 1, 2$
- $||v||^2 \leq NK$

The power-to-noise ratios are defined as:

$$SNR_1 = |h_{11}|^2 P_1,\ SNR_2 = |h_{22}|^2 P_2,\ INR_1 = |h_{21}|^2 P_1, \text{ and } INR_2 = |h_{12}|^2 P_2$$

Deterministic Model for Interference Channel with a Jammer

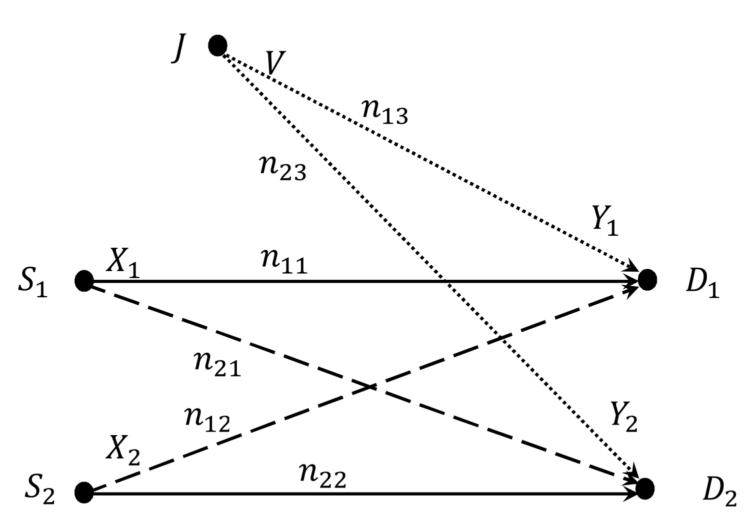
We adopt the deterministic model approach of [1].

- The inputs X_1 and X_2 and the jammer V are written in binary.
- The outputs Y_1 and Y_2 are given by:

$$Y_1 = \lfloor 2^{n_{11}} X_1 \rfloor \oplus \lfloor 2^{n_{12}} X_2 \rfloor \oplus \lfloor 2^{n_{13}} V \rfloor$$
$$Y_2 = \lfloor 2^{n_{21}} X_1 \rfloor \oplus \lfloor 2^{n_{22}} X_2 \rfloor \oplus \lfloor 2^{n_{23}} V \rfloor$$

where

- $\lfloor \cdot \rfloor$ is the integer-part function.
- $\oplus$ denotes module-2 addition.
- $n_{11} = \lfloor \log SNR_1 \rfloor$
- $n_{22} = \lfloor \log SNR_2 \rfloor$
- $n_{12} = \lfloor \log INR_2 \rfloor$
- $n_{21} = \lfloor \log INR_1 \rfloor$
- $n_{13} = \lfloor \log (|g_1|^2 K) \rfloor$
- $n_{23} = \lfloor \log (|g_2|^2 K) \rfloor$



Two-user Deterministic IC with a Jammer

Linear Description of Deterministic Model

The channel outputs at receivers D_1 and D_2:

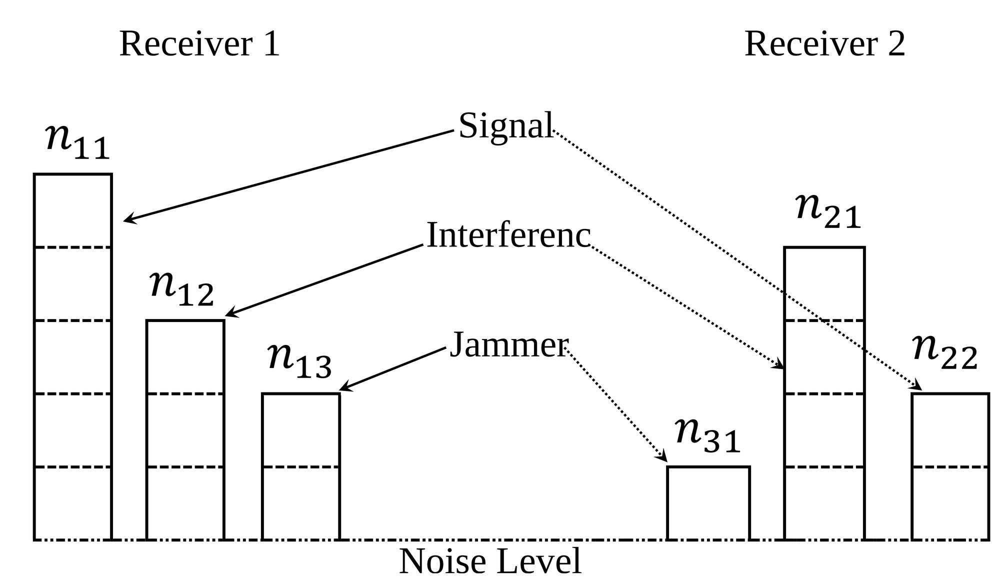
$$Y_1 = S^{q-n_{11}} X_1 + S^{q-n_{12}} X_2 + S^{q-n_{13}} V$$
$$Y_2 = S^{q-n_{21}} X_1 + S^{q-n_{22}} X_2 + S^{q-n_{23}} V$$

- Input vector X_i, jammer vector V and output vector Y_i are from binary field $\mathbb{F}_2^q$.
- $q = \max_{ij} n_{ij}$ for $i = 1, 2$.
- Matrix S is defined by

$$S = \begin{pmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & 1 & 0 \end{pmatrix}.$$

- n_{ij} is the rank of matrix $S^{q-n_{ij}}$.

Noise Level Interpretation
For each signal (legitimate or jamming) the number of bits above noise level is given by its SNR.



Two-user Deterministic IC with a Jammer

Capacity Region of Deterministic Model without jammer

Theorem 1. (proved in [2]) The capacity region of two-user deterministic IC without jammer is the set of all non-negative rates satisfying

$$r_1 \leq n_{11}$$
$$r_2 \leq n_{22}$$
$$r_1 + r_2 \leq (n_{11} - n_{12})^+ + \max(n_{22}, n_{12})$$
$$r_1 + r_2 \leq (n_{22} - n_{21})^+ + \max(n_{11}, n_{21})$$
$$r_1 + r_2 \leq \max(n_{21}, (n_{11} - n_{12})^+) + \max(n_{12}, (n_{22} - n_{21})^+)$$
$$2r_1 + r_2 \leq \max(n_{11}, n_{21}) + (n_{11} - n_{12})^+ + \max(n_{12}, (n_{22} - n_{21})^+)$$
$$r_1 + 2r_2 \leq \max(n_{22}, n_{12}) + (n_{22} - n_{21})^+ + \max(n_{21}, (n_{11} - n_{12})^+)$$

Capacity Region of Deterministic Model with a jammer

Theorem 2. Two-user deterministic interference channel capacity region is given by:

$$r_1 \leq n_{11} - n_{13}$$
$$r_2 \leq n_{22} - n_{23}$$
$$r_1 + r_2 \leq (n_{11} - n_{12})^+ + \max(n_{22} - n_{23}, n_{12} - n_{13})$$
$$r_1 + r_2 \leq (n_{22} - n_{21})^+ + \max(n_{11} - n_{13}, n_{21} - n_{23})$$
$$r_1 + r_2 \leq \max(n_{21} - n_{23}, (n_{11} - n_{12})^+) + \max(n_{12} - n_{13}, (n_{22} - n_{21})^+)$$
$$2r_1 + r_2 \leq \max(n_{11} - n_{13}, n_{21} - n_{23}) + (n_{11} - n_{12})^+ + \max(n_{12} - n_{13}, (n_{22} - n_{21})^+)$$
$$r_1 + 2r_2 \leq \max(n_{22} - n_{23}, n_{12} - n_{13}) + (n_{22} - n_{21})^+ + \max(n_{21} - n_{23}, (n_{11} - n_{12})^+)$$

Achievability Proof

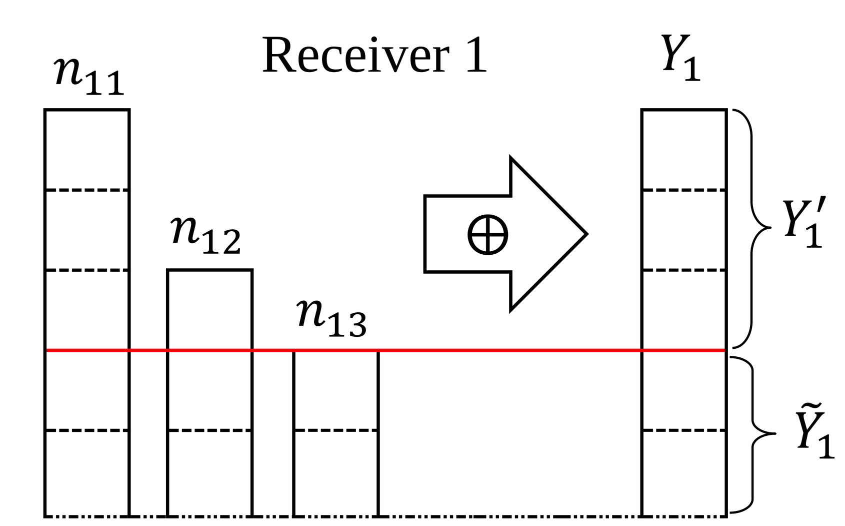
- We assume that (r_1, r_2) satisfy the capacity region.
- Each receiver ignores bits that jammer has access to (below red line).
- Bits above the jammer level behave like a deterministic interference channel with no jammer.
- The proof follows from the achievability proof given in [2].

Converse Proof

- Assume that the rate pair (r_1, r_2) is achievable.
- No matter what the jammer does, each receiver can decode its own message with high probability.
- We may assume the adversary transmits random Bernoulli $(\frac{1}{2})$ bits

Lemma 1. If the jammer transmits Bernoulli $(\frac{1}{2})$ bits independent of X_1^N and X_2^N, then

$$I(X_1^N; Y_1^N) = I(X_1^N; Y_1'^N)$$
$$I(X_2^N; Y_2^N) = I(X_2^N; Y_2'^N)$$



The output Y_1 includes $\acute{Y}_1$ and $\tilde{Y}_1$.

- **Lemma 2.** (proved in [2]) The rate point (r_1, r_2) is achievable if and only if for every $\epsilon > 0$ there exists a block length N and a factorized joint distribution $p(x_1^N)p(x_2^N)$ with

$$r_1 - \epsilon \leq \frac{1}{N} I(x_1^N, y_1^N)$$
$$r_2 - \epsilon \leq \frac{1}{N} I(x_2^N, y_2^N)$$

- From Lemma 1 and Lemma 2, we conclude that the multiletter characterization of the capacity region for the deterministic IC with a jammer is equal to that of the equivalent deterministic IC with the noise level increased by the jammer's signal.

Note: Even if the jammer knows the transmitted signals perfectly, the capacity region does not change because no transmission is more damaging than Bernoulli noise.

References

[1] A. S. Avestimehr, S. N. Diggavi, and D. Tse, Wireless Network Information Flow: A Deterministic Approach, *IEEE Trans. Info. Theory* , vol. 57, no. 4, pp. 1872-1905, Apr. 2011.

[2] G. Bresler, and D. Tse, The two-user Gaussian interference channel: a deterministic view, *Eur. Trans. Telecomms*, vol. 19, pp. 333-354, Apr. 2008.

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