Assignment 2: Evolutionary dynamics in a spatial context

General remarks:
- Deadline 23-11-2016
- Mail your results to Jelena Grujic <jgrujic@vub.ac.be>.
- Provide a single (self-contained PDF) with the Name_Affiliation.PDF file, for example: JelenaGrujic_VUB.pdf
- Put your name and your affiliation (VUB/ULB) both on the document and in the file name.

Problem description

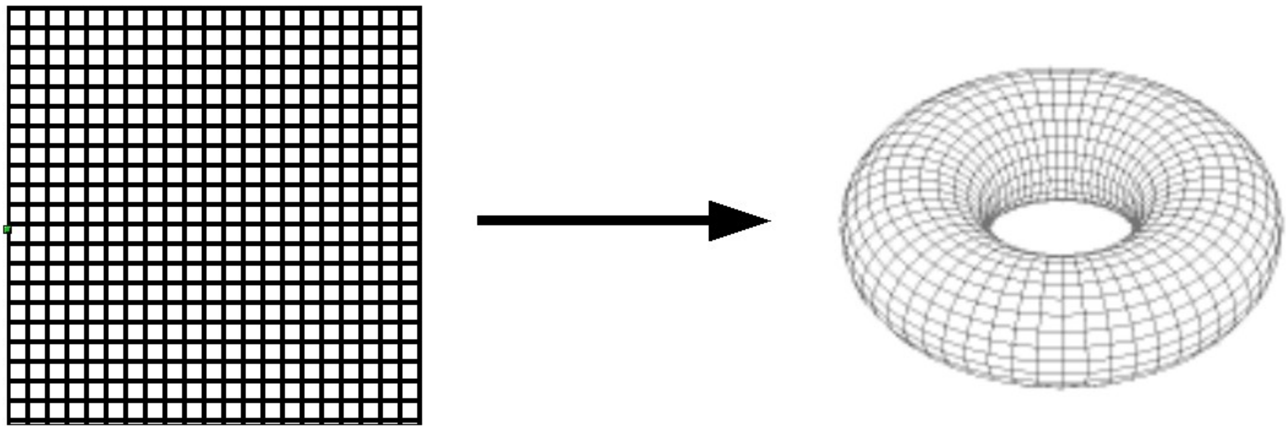
This assignment is based on two scientific contributions in the field of Evolutionary Game Theory:
- The first part:
*Martin Nowak and Robert May in Nature; Nowak, M. & May, R. M. Evolutionary games and spatial chaos. Nature **359**, 826–829 (2002).*
- The second part:
Hauert, Christoph, and Michael Doebeli. "Spatial structure often inhibits the evolution of cooperation in the snowdrift game." Nature 428.6983 (2004): 643-646.

Both parts require that you construct a simulation with individuals playing an iterated game on a lattice and examine whether or not cooperation becomes established depending on the update mechanism used in the simulation, type of lattice and its size.

Part I

Inspired by:
*Nowak, M. & May, R. M. Evolutionary games and spatial chaos. Nature **359**, 826–829 (2002).*

- Construct a lattice size 50x50 wherein each player has 8 neighbors (above, below, left right and both diagonals), which corresponds to the so-called Moore neighborhood (see the image). The lattice should have periodic boundary conditions, which means the above neighbors of the top row are the players of the bottom row, the left neighbors of the first column on the left are the players in the last column on the right etc. as if the players are set on the torus.



Periodic boundary conditions

- Each player is playing a weak Prisoners Dilemma game (T=10, R=7, P=S=0) with each of their neighbors, playing the same action (Cooperate of Defect) with each one of them. The total payoff is calculated as a sum of the payoffs received from playing all eight games. Unlike in the paper, the players will not play with themselves, only with their neighbors.

- In the first round (t=0) , each player is assigned randomly with probability 50% either cooperate or defect.

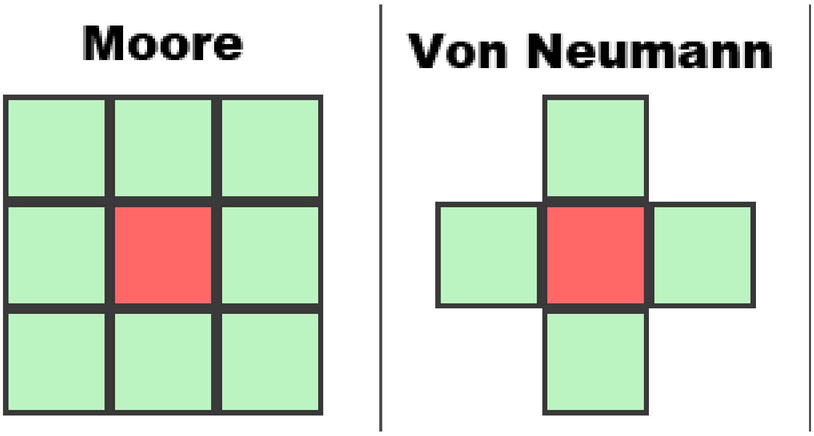
- In every round after the first one (t>0) , they use unconditional imitation to update their actions. They find the player in their neighborhood (eight neighbors plus themselves) that had the highest earnings in the previous round and do copy the action of that player did.

- Plot the cooperation level over time, averaged over 100 simulations. Make sure to run the simulation until you get to the stationary state. Plot the distribution of the final cooperation levels. Is the cooperation level approximately the same in every run?

- Visualize one run where the cooperation level is larger than 0 (using different colors for defectors and cooperators) the full matrix of cooperation for the rounds t=0, t=1, t=5, t=10, t=20, t=50

- How do the results change when you alter the size of the lattice to 4x4, 8x8, 12x12, 20x20? Does the cooperation level change? Explain why?

- Change the number of neighbors to four (no diagonal neighbors, von Neumann neighborhood, check the image bellow). Does the level of cooperation change? Explain why? Keep the size of the network to 50x50.



The difference between the Moore neighborhood (8 neighbors) and Von Neuman (4 neighbors)

Part II

Inspired by:
Hauert, Christoph, and Michael Doebeli. "Spatial structure often inhibits the evolution of cooperation in the snowdrift game." *Nature* 428.6983 (2004): 643-646.

- We do exactly the same analysis as in the previous case, but now we change the game and the update mechanism.

- The game is now the Snowdrift game (T=10, R=7, S=3, P=0)

- The update mechanism should be replicator rule: Each individual *i* chooses randomly one of her closest neighbors *j* and they change their action to the action of that neighbor with a probability P_{ij}, where P_{ij} is defined as:

$$P_{ij} = (1 + [W_j - W_i]/[N^*(max\{P,R,T,S\}-min\{P,R,T,S\})]) / 2$$

where W_i and W_j are the payoffs of the focal player *i* and its neighbor *j respectively*, and N is the number of neighbours in the network (N=4 for von Neuman neigbourhood and N=8 for Moore neighbourhood)

- Explain why would we set up the probability P_{ij} like this? Why does it make sense to update your actions like that?

- Repeat the same analysis as in the part I and answer the same questions. Which results are different in part II and why?

Apart for the two papers above, you can also take a look at this one:
Roca, Carlos P., José A. Cuesta, and Angel Sánchez.
"Evolutionary game theory: Temporal and spatial effects beyond replicator dynamics."
Physics of life reviews 6.4 (2009): 208-249.