

Article

Tensor Modeling and Analysis for Vehicle Traffic

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Keywords: keyword 1; keyword 2; keyword 3 (list three to ten pertinent keywords specific to the article, yet reasonably common within the subject discipline.)

1. Introduction

Content

1. Related work.
2. Contribution.
3. Content.

16 **2. Tensor Algebra****Table 1.** Tensor Algebra Notation Summary.

| | |
|--|---|
| $\mathcal{X}, \mathbf{X}, \mathbf{x}, x$ | Tensor, matrix, vector scalar. |
| $\mathcal{X} \in \mathbb{R}^{I_1 \times \dots \times I_N}$ | A $I_1 \times \dots \times I_N$ tensor. |
| $x_{i_1 \dots i_N}$ | The $(i_1 \dots i_N)$ entry of an N^{th} -order tensor. |
| $\mathbf{X}^{(n)}$ | The n^{th} matrix element from a sequence of matrices. |
| $\mathbf{X}_{(n)}$ | The n-mode matricization of a tensor. |
| \otimes_{outer} | Outer product of two vectors. |
| \otimes_{kron} | Kronecker product of two matrices. |
| \odot | Khatri Rao product of two matrices. |
| $\langle \mathcal{X}, \mathcal{Y} \rangle$ | Inner product of two tensors. |
| $\mathcal{Y} = \mathcal{X} \times_n \mathbf{U}$ | The n-mode product of a tensor \mathcal{X} times a matrix \mathbf{U} along the n dimension. |
| $[[\lambda/\mathcal{G}, \mathbf{U}^{(1)}, \dots, \mathbf{U}^{(N)}]]$ | Simplified form of N^{th} -order tensor decomposition models as factor matrices. |
| $rank_r(\mathcal{X})$ | Decomposition Tensor decomposition/CP rank. |
| $rank_{tr}(\mathcal{X})$ | Multilinear Tensor multilinear/Tucker rank, where $R_n = rank(\mathbf{X}_{(n)})$. |
| $rank_k(\mathcal{X})$ | Tensor Kruskal-rank |
| $\mathcal{X} * \mathcal{Y}$ | t-product of two tensors. |
| $\mathcal{X} *_{\Phi} \mathcal{Y}$ | Φ -product of two tensors. |
| $\mathcal{H}(\cdot)/\mathcal{H}^{-1}(\cdot)$ | Hankelization direct/inverse transformation. |
| $\mathcal{L}(\cdot)/\mathcal{L}^{-1}(\cdot)$ | Löwnerization direct/inverse transformation. |
| \mathcal{V}_{τ} | Video of duration τ , represented as a tensor. |
| \mathcal{B} | Background tensor. |
| \mathcal{F} | Foreground tensor. |
| $\mathcal{Z}\mathcal{T}$ | Vehicle traffic features tensor. |

17 **Content**

- 18 1. Notation.
- 19 2. Basic tensor concepts.
- 20 3. Tensor decompositions (E.G.)
- 21 (a) CANDECOM/PARAFAC Decomposition
- 22 (b) Tucker Decomposition
- 23 (c) Tensor Robust PCA
- 24 (d) Non-negative Tensor Decomposition

25 **3. Problem Statement and Mathematical Definition**26 **Content**

- 27 1. Problem Statement.
- 28 2. Mathematical Definition.

29 **4. Vehicle Traffic Model**30 **Content**

- 31 1. Traffic surveillance video modeling.
- 32 2. Representing vehicle traffic data as an n-way tensor.
- 33 3. Tensor Factorization for vehicle traffic analysis.

5. Experiments

6. Discussion

Authors should discuss the results and how they can be interpreted in perspective of previous studies and of the working hypotheses. The findings and their implications should be discussed in the broadest context possible. Future research directions may also be highlighted.

7. Conclusions

This section is not mandatory, but can be added to the manuscript if the discussion is unusually long or complex.

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Abbreviations

The following abbreviations are used in this manuscript:

| | |
|------|--|
| MDPI | Multidisciplinary Digital Publishing Institute |
| DOAJ | Directory of open access journals |
| TLA | Three letter acronym |
| LD | linear dichroism |

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2. Author2, L. The title of the cited contribution. In *The Book Title*; Editor1, F., Editor2, A., Eds.; Publishing House: City, Country, 2007; pp. 32–58.

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