

Article

Tensor Modeling and Analysis for Vehicle Traffic

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- Abstract: A single paragraph of about 200 words maximum. For research articles, abstracts should
- 2 give a pertinent overview of the work. We strongly encourage authors to use the following style of
- structured abstracts, but without headings: (1) Background: Place the question addressed in a broad
- context and highlight the purpose of the study; (2) Methods: Describe briefly the main methods or
- treatments applied; (3) Results: Summarize the article's main findings; and (4) Conclusion: Indicate
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- should not exaggerate the main conclusions.
- **Keywords:** keyword 1; keyword 2; keyword 3 (list three to ten pertinent keywords specific to the article, yet reasonably common within the subject discipline.)

1. Introduction

- 12 Content
- 1. Related work.
- 2. Contribution.
- 5 3. Content.

2. Tensor Algebra

Table 1. Tensor Algebra Notation Summary.

$\mathcal{X}, \mathcal{X}, \mathcal{X}, \mathcal{X}$ $\widetilde{\mathcal{X}} \in \mathbb{R}^{\widetilde{I}_1 \times \cdots \times I_N}$	Tensor, matrix, vector scalar.
$\mathcal{X} \in \mathbb{R}^{I_1 \times \cdots \times I_N}$	$A I_1 \times \cdots \times I_N$ tensor.
$x_{i_1 \cdots i_N}$	The $(i_1 \cdots i_N)$ entry of an N^{th} -order tensor.
$\mathbf{X}^{(n)}$	The n^{th} matrix element from a sequence of matrices.
$X_{i_{0}\cdots i_{N}}$ $X_{(n)}$ $X_{(n)}$	The n-mode matricization of a tensor.
⊗outer_	Outer product of two vectors.
⊗kron.	Kronecker product of two matrices.
$ \begin{array}{l} \bigcirc \\ \langle \mathcal{X}, \mathcal{Y} \rangle \\ \mathcal{Y} = \mathcal{X} \times_{n} \mathbf{U} \end{array} $	Khatri Rao product of two matrices.
$\langle x, y \rangle$	Inner product of two tensors.
$\mathcal{Y} = \mathcal{X} \times_{n} \mathcal{U}$	The n-mode product of a tensor \mathcal{X} times a matrix \mathbf{U} along the n dimension.
$\llbracket \boldsymbol{\lambda} / \boldsymbol{\mathcal{G}}, \mathbf{U}^{(1)}, \cdots, \mathbf{U}^{(N)} bracket$	Simplified form of N^{th} -order tensor decomposition models as factor matrices.
$rank_t(\mathcal{X})$	Decomposition/CP rank.
$rank_{tc}(\boldsymbol{\mathcal{X}})$	Multilinear/Tucker rank.
~~~	
$\overset{\mathcal{X}}{\underset{\sim}{\times}}\overset{*}{\underset{\sim}{\mathcal{Y}}}$	t-product of two tensors.
$\overbrace{\overset{\sim}{\widetilde{\mathcal{X}}}\overset{\ast}{*}}\overset{\mathcal{Y}}{\mathscr{X}}\overset{\ast}{*}\overset{\circ}{*}\overset{\circ}{\mathscr{Y}}\overset{\circ}{\mathscr{Y}}$	Φ-product of two tensors.
$\overline{\widetilde{\mathcal{H}}(\cdot)}/\mathcal{H}^{-1}(\cdot)$	
	Hankelization direct/inverse transformation.
$\mathcal{L}(\cdot)/\mathcal{L}^{-1}(\cdot)$	Löwnerization direct/inverse transformation.
	Video of denting a second of the second
₩ ₩	Video of duration τ , represented as a tensor.
$\overset{\mathcal{B}}{\widetilde{m{ au}}}$	Background tensor.
<i>≨</i>	Foreground tensor.
_&	Vehicle traffic features tensor.

Content

1. Notation. 18

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- 2. Basic tensor concepts.
- Tensor decompositions (E.G.) 3. 20
 - CANDECOM/PARAFAC Decomposition (a)
 - Tucker Decomposition Tensor Robust PCA (b)
- (c) 23
 - Non-negative Tensor Decomposition (d)

3. Problem Statement and Mathematical Definition

- Content
- 1. Problem Statement.
- Mathematical Definition.

4. Vehicle Traffic Model

- Content
- 1. Traffic surveillance video modeling.
- Representing vehicle traffic data as an n-way tensor. 2.
- Tensor Factorization for vehicle traffic analysis.

34 5. Experiments

₃₅ 6. Discussion

Authors should discuss the results and how they can be interpreted in perspective of previous studies and of the working hypotheses. The findings and their implications should be discussed in the broadest context possible. Future research directions may also be highlighted.

7. Conclusions

This section is not mandatory, but can be added to the manuscript if the discussion is unusually long or complex.

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63 Abbreviations

The following abbreviations are used in this manuscript:

MDPI Multidisciplinary Digital Publishing Institute DOAJ Directory of open access journals

TLA Three letter acronym
LD linear dichroism

References

- Author1, T. The title of the cited article. *Journal Abbreviation* **2008**, 10, 142–149.
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