Resumen segundo parcial

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Conceptos básicos:

Modelo escalar

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \dots + \beta_k X_{ik} + \varepsilon_i$$
$$\varepsilon_i \overset{iid}{\sim} N(0, \sigma^2)$$

k: # covariables; j: Alusión coeficiente $\boxed{j = 1, \dots, k}$

p: # parámetros $\mathbb{R}^p$

$\boxed{p = k+1}$

$\boxed{X_{ij} : i: \text{Observación}\ i = 1, \dots, n}$

Modelo vectorial

$$\begin{cases} Y_1 = \beta_0 + \beta_1 X_{11} + \dots \beta_k X_{1k} + \varepsilon_1 \\ \vdots \\ Y_n = \beta_0 + \beta_1 X_{n1} + \dots + \beta_k X_{nk} + \varepsilon_n \end{cases}$$

$$\boxed{\underline{Y}_n = X\underline{\beta} + \underline{\varepsilon}}$$
$$\boxed{\underline{\varepsilon}_n \sim N_n(\underline{0}_n, \sigma^2 I)}$$

$$\underline{Y}_n = \begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix}_{n\times 1} \quad \underline{\varepsilon}_n = \begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_n \end{bmatrix}_{n\times 1} ; \quad \underline{\beta} = \begin{bmatrix} \beta_0 \\ \vdots \\ \beta_k \end{bmatrix}_{(k+1)\times 1}$$

$$X = \begin{bmatrix} 1 & X_{11} & \cdots & X_{1k} \\ \vdots & & \ddots & \\ 1 & X_{n1} & \cdots & X_{nk} \end{bmatrix}_{n\times p \ \ n\times(k+1)}$$

Intervalo de confianza

$$\boxed{\hat{\beta}_j \pm t_{\alpha/2, n-p} \cdot \sqrt{\widehat{Var}[\hat{\beta}_j]}}$$

Intervalo predicción

$$\begin{cases} \hat{Y}_0 \pm t_{\alpha/2, n-p} \cdot \sqrt{Var[Y_0 - \hat{Y}_0]} \\ \hat{Y}_0 \pm t_{\alpha/2, n-p} \cdot S.E(Y_0 - \hat{Y}_0) \end{cases}$$

$$\boxed{\hat{Y}_0 \pm t_{\alpha/2, n-p} \cdot \sqrt{MSE[1 + X_0(X^T X)^{-1} X_0^T]}}$$

Expresiones útiles

$$\boxed{\hat{\underline{\beta}} = (\underbrace{X^T X}_{})^{-1} X^T \underline{Y}_n}$$

$\boxed{\text{Debe ser invertible}}$

$$\boxed{MSE = \frac{SSE}{n-p} \ \text{(Varianza)}}$$

$$\boxed{\frac{(\underline{Y}_n - \hat{\underline{Y}}_n)^T (\underline{Y}_n - \hat{\underline{Y}}_n)}{n-p} = \frac{(\underline{Y}_n - X\hat{\underline{\beta}})^T (\underline{Y}_n - X\hat{\underline{\beta}})}{n-p}}$$

$$\boxed{H = X(X^T X)^{-1} X^T} \ \text{Matriz hat}$$

Pruebas de hipótesis

Significancia individual

$$\boxed{\begin{matrix} H_0: \beta_j = 0 \\ H_1: \beta_j \neq 0 \end{matrix} ; \ j = 0, \dots, k}$$

$$\boxed{T = \frac{\hat{\beta}_j - 0}{\sqrt{MSE\, C_{jj}}} \sim t_{n-p}}$$

(1). Pval $< \alpha$: Rechazar H_0

(2). $R_c = \{|T_0| > t_{\alpha/2, n-p}\}$

Significancia global

$$\begin{cases} \boxed{H_0: \beta_1 = \beta_2 = \dots = \beta_k = 0} \\ H_1: \text{Algún } \beta_j \neq 0 ; \ j = 1, \dots, k \end{cases}$$

$$\boxed{F = \frac{SSR/k}{SSE/n-p} = \frac{MSR}{MSE} \sim f_{k, n-p}}$$

$\boxed{p = k+1}$ $\boxed{R_c = \{F_0 > f_{\alpha, n-p}\} ; \ P(f_{k, n-p} > F_0)}$

Prueba lineal general

$$\boxed{\begin{cases} H_0: L\underline{\beta} = 0 \\ H_1: L\underline{\beta} \neq 0 \end{cases}} \quad \boxed{F = \frac{MSH}{SSE/n-p} = \frac{SSH/r}{MSE} \sim f_{r, n-p}}$$

$\boxed{R(L) = \text{\# Filas linealmente independientes no-nulas (rango = r)}}$

- $\hat{Y}_0 \pm t_{\alpha/2,n-p} \cdot \sqrt{MSE\left[1 + X_0 (X^T X)^{-1} X_0^T\right]}$

$R(L) =$ # filas linealmente independientes no-nulas (rango = r)

$$L = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \boxed{v = 2}$$

Interpolación oculta

$X_0 (X^T X)^{-1} X_0^T < \max_{1 \le i \le n} h_{ii} \sim$ Diagonal H

$\boxed{H = X(X^T X)^{-1} X^T}$

Suma cuadrados parciales

$$F = \frac{[SSR(MF) - SSR(MR)]/v}{MSE(MF)} \sim F_{v,n-p}$$

$$F = \frac{[SSE(MR) - SSE(MF)]/v}{MSE(MF)} \sim F_{v,n-p};$$

$v =$ # Grados de libertad:

$$v = gl(SSR(MF)) - gl(SSR(MR))$$
$$gl(SSE(MR)) - gl(SSE(MF))$$

Truco: $v =$ # parámetros en la hipótesis nula

Criterios de evaluación

(i). Balanceo: $h_{ii} > 2p/n$; $h_{ii} < 1$

(ii). Atípicos: $|d_i| > 3$; $|r_i| > 3$

(iii). Influenciales:

$D_i > 1$, $|DFBETAS_{j(i)}| > 2/\sqrt{n}$

$|DFFITS_i| > 2\sqrt{p/n}$

- $SSR(MR) = SSR(\beta_0) = SST = \sum_{i=1}^{n} (y_i - \bar{y})^2$

2. Seleccione las expresiones adecuadas que se muestra a continuación, interprételas y corrija las expresiones incorrectas.

a. $d_i = \frac{e_i}{\sqrt{MSE}}$ ✓ b. $r_i = \frac{d_i}{\sqrt{1-h_{ii}}}$
c. $DFBETAS_{j(i)} = \frac{\hat{\beta}_j - \hat{\beta}_{j(i)}}{\sqrt{MSE_{(i)} c_{jj}}}$ d. $DFFITS_i = \frac{\hat{y}_i - \hat{y}_{i(0)}}{\sqrt{MSE_{(i)}}}$ h_{ii}

$$DFBETAS_{j(i)} = \frac{\hat{\beta}_j - \hat{\beta}_{j(i)}}{\sqrt{MSE_{(i)} c_{jj}}}$$ $\sim$ Diagonal principal $(X^T X)^{-1}$