Problem 1

Let X be the minimum of two random throws of a die. Find the pmf of X.

Solution

X takes on a value in the set $\{1, 2, \cdots, 6\}$ each with probability

$$p_X(i) = \frac{2(6-i)+1}{6^2} = \frac{13-2i}{36} \quad \text{for } i \in \{1, 2, \cdots, 6\}$$

since $|\{X = i\}| = 2(6-i)+1$ and $|\Omega| = 6^2$. To see that $|\{X = i\}| = 2(6-i)+1$ consider the following cases:

a) One of the numbers is i and the other is larger than i: There are 2 ways to choose which number is i and $6-i$ ways to choose the other number. So there are $2(6-i)$ such cases.

b) Both numbers are equal to i. There is one such case.

Namely,

$$p_X(1) = \frac{11}{36},$$
$$p_X(2) = \frac{9}{36},$$
$$p_X(3) = \frac{7}{36},$$
$$p_X(4) = \frac{5}{36},$$
$$p_X(5) = \frac{3}{36},$$
$$p_X(6) = \frac{1}{36}.$$

Alternatively, one could write (why?)

$$p_X(i) = \frac{(6-(i-1))^2 - (6-i)^2}{36} = \frac{13-2i}{36} \quad \text{for } i \in \{1, 2, \cdots, 6\}.$$