$X \sim Bin(n, p)$, p is given and n is unknown. Find the ML estimate of n if X is observed.

$$p_x(X) = \binom{n}{X} p^X (1-p)^{n-p}$$

$$\frac{\binom{n}{X}p^{X}(1-p)^{n-X}}{\binom{n-1}{X}p^{X}(1-p)^{n-1-X}} = \frac{n}{n-X}(1-p) \ge 1 \implies n(1-p) \ge n-X \implies np \le X \implies n \le \frac{X}{p}$$

X/p is not an integer: Max n occurs at $n = \lfloor X/p \rfloor$

X/p is an integer: Max n occurs at n=X/p or n=X/p-1