Problem Set 14

Note: It is very important that you solve the problems first and check the solutions afterwards.

Problem 1

Example 4.5.2.

Problem 2

Decide whether X and Y are independent for each of the following three joint pdfs. If they are independent, identify the marginal pdf's f_X and f_Y. If they are not, give a reason.

a) $f_{X,Y}(u,v) = \frac{4}{\pi} e^{-\left(u^2+v^2\right)}$ for $u, v \geq 0$ and 0 elsewhere.

b) $f_{X,Y}(u,v) = -\frac{v^2 \ln u}{21}$ for $0 \leq u \leq 1, 1 \leq v \leq 4$ and 0 elsewhere.

c) $f_{X,Y}(u,v) = 96u^2v^2/\pi$ for $u^2 + v^2 \leq 1$ and 0 elsewhere.

Solution

a) X and Y are independent.

$$f_X(u) = \begin{cases} \frac{2}{\sqrt{\pi}} e^{-u^2} & u \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$f_Y(v) = \begin{cases} \frac{2}{\sqrt{\pi}} e^{-v^2} & v \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

b) X and Y are independent.

$$f_X(u) = \begin{cases} -\int_1^4 \frac{v^2 \ln u}{21} dv = -\ln(u), & 0 \leq u \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

$$f_Y(v) = \begin{cases} \frac{v^2}{21} \int_0^1 (-\ln u)\, du = \frac{v^2}{21}, & 1 \leq v \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

c) X and Y are not independent as the support is not a product set. This can be easily show using the four point test.