Quiz 9

Let X be a random variable with mean $\mu = 12$ and variance $\sigma^2 = 12$. Find the probability of the event $\{X \leq 12 - \sqrt{12}\}$ if

1. X is Normal.

2. X is Uniform.

You may assume $\sqrt{12} = 3.5$.

Solution

1.
$$P\left(X \leq 12 - \sqrt{12}\right) = P\left(\frac{X - 12}{\sqrt{12}} \leq -1\right) = \Phi\left(-1\right) = Q\left(1\right) = 0.1587.$$

2. This part can be solved with the same method as example 3.6.5. Alternatively, one can define the standard uniform random variable: Let $\hat{U} \sim \text{Uni}\,[a, b]$ be the standard uniform RV. We need to find a and b. Since $E\hat{U} = \frac{a+b}{2} = 0$, we have $a = -b$. Furthermore, $Var\left[\hat{U}\right] = \frac{(b-a)^2}{12} = 1$. So

$$\frac{(b - a)^2}{12} = \frac{4b^2}{12} = 1 \Rightarrow b = \sqrt{3}.$$

Hence, $\hat{U} \sim \text{Uni}\left[-\sqrt{3}, \sqrt{3}\right]$. Now

$$P\left(X \leq 12 - \sqrt{12}\right) = P\left(\frac{X - 12}{\sqrt{12}} \leq -1\right) = P\left(\hat{U} \leq -1\right) = \frac{-1 - \left(-\sqrt{3}\right)}{2\sqrt{3}}$$
$$= \frac{\sqrt{3} - 1}{2\sqrt{3}} = \frac{\sqrt{12} - 2}{2\sqrt{12}} = \frac{1.5}{7} = 0.21.$$