Quiz 11

Let $\Theta \sim \mathrm{Uni}[-\pi/2, \pi/2]$. Find the pdf of Y if $Y = \tan \Theta$.

Solution

We have $Y = g(\Theta)$ for $g(\theta) = \tan \theta$. In the interval $\theta \in [-\pi/2, \pi/2]$, the equation $y = g(\theta)$ has only one solution. So, for $y = g(\theta)$,

$$f_Y(y) = \frac{f_\Theta(\theta)}{g'(\theta)} = \frac{1/\pi}{1 + \tan \theta^2} = \frac{1}{\pi(1 + y^2)}.$$

Note that $g'(\theta)$ must be obtained in terms of y. So if $g'(\theta)$ is given in the form of $\frac{1}{\cos^2 \theta}$, one may write

$$\frac{1}{\cos^2 \theta} = \frac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta} = 1 + \tan^2 \theta = 1 + y^2.$$