Bit-Swap

Recursive Bits-Back Coding for Lossless Compression with Hierarchical Latent Variables

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Compression with Hierarchical Latent Variable Models

Abstract

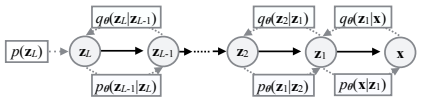
Previous work:
- BB-ANS makes a practical compression scheme with the use of latent variable models.
- Bitrate approx. equal to the ELBO of a VAE.
- Practical for one latent layer
- Impractical for multiple latent layers

Our contribution:
- Making it practical for a hierarchical latent variable model
- Recursively applying the bits-back argument

Hierarchical Latent Variable Model

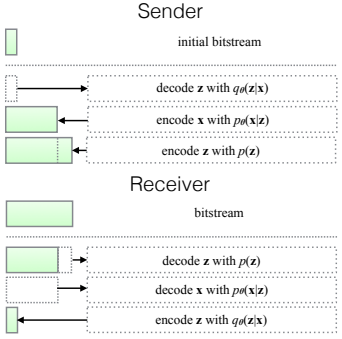
- **VAE** with multiple latent layers
- Sampling process of generative model and inference model obeys **Markov chain**

$$p_{\boldsymbol{\theta}}(\mathbf{x}) = \int p_{\boldsymbol{\theta}}(\mathbf{x}|\mathbf{z}_1)p_{\boldsymbol{\theta}}(\mathbf{z}_1)\mathrm{d}\mathbf{z}_1$$
$$p_{\boldsymbol{\theta}}(\mathbf{z}_1) = \int p_{\boldsymbol{\theta}}(\mathbf{z}_1|\mathbf{z}_2)p_{\boldsymbol{\theta}}(\mathbf{z}_2)\mathrm{d}\mathbf{z}_2$$
$$\vdots$$
$$p_{\boldsymbol{\theta}}(\mathbf{z}_{L-1}) = \int p_{\boldsymbol{\theta}}(\mathbf{z}_{L-1}|\mathbf{z}_L)p(\mathbf{z}_L)\mathrm{d}\mathbf{z}_L,$$



Bits-Back Coding

BB-ANS on **one layer latent variable model**:
- sender uses initial bits and receiver recovers these initial bits
- net bitrate approx. equal to ELBO



Naive: Sampling

Achieves

- A compression rate approx. equal to

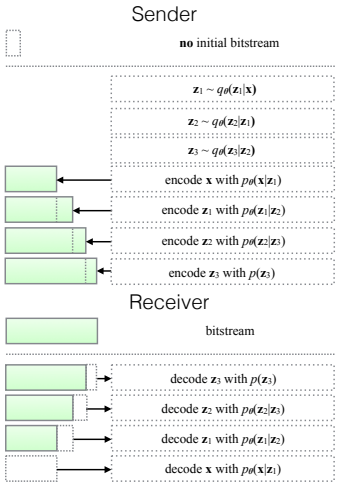
$$N_{\text{net}}^{\text{sampling}} = \mathbb{E}_{q_{\boldsymbol{\theta}}(\mathbf{z}_{1:L}|\mathbf{x})}[-\log p_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{z}_{1:L})]$$

How

- Sender samples $\mathbf{z}_{1:L} \sim q_{\boldsymbol{\theta}}(\mathbf{z}_{1:L}|\mathbf{x})$
- Sender encodes $\mathbf{x}$ using $p_{\boldsymbol{\theta}}(\mathbf{x}|\mathbf{z}_1)$
- Sender encodes $\mathbf{z}_{1:L}$ as well using $p_{\boldsymbol{\theta}}(\mathbf{z}_1|\mathbf{z}_2)$, $p_{\boldsymbol{\theta}}(\mathbf{z}_2|\mathbf{z}_3), \ldots, p(\mathbf{z}_L)$
- Sent the full bitstream over to the receiver.
- Receiver decodes in reverse order

Limitations

After decoding $\mathbf{z}_{1:L}$ and $\mathbf{x}$, the goal of lossless compression is reached: the receiver fully recovered the data $\mathbf{x}$. However, the **additionally recovered latent variables $\mathbf{z}_{1:L}$ are redundant after retaining $\mathbf{x}$.**



Townsend et al. 2019: BB-ANS

Achieves

- A compression rate approx. equal to

$$N_{\text{total}}^{\text{BBANS}} = N_{\text{init}}^{\text{BBANS}} + \mathbb{E}_{q_{\boldsymbol{\theta}}(\mathbf{z}_{1:L}|\mathbf{x})}[\log q_{\boldsymbol{\theta}}(\mathbf{z}_{1:L}|\mathbf{x}) - \log p_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{z}_{1:L})]$$

- **Net number of bits** added to the bitstream is approx. **equal to the negative ELBO**

$$N_{\text{total}}^{\text{BBANS}} - N_{\text{init}}^{\text{BBANS}} = \mathbb{E}_{q_{\boldsymbol{\theta}}(\mathbf{z}_{1:L}|\mathbf{x})}[\log q_{\boldsymbol{\theta}}(\mathbf{z}_{1:L}|\mathbf{x}) - \log p_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{z}_{1:L})] = -\mathcal{L}(\boldsymbol{\theta}) \text{ (ELBO)}$$

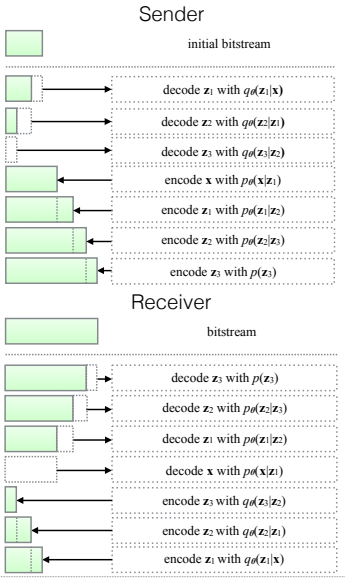
How

Decoding from uniformly random bitstream equals sampling.
- Sender decodes $\mathbf{z}_{1:L}$ from **initial bitstream**
- The receiver later get these "**bits back**".

Limitations

The min. number of  BB-ANS needs **grows linearly with L :**

$$N_{\text{init}}^{\text{BBANS}} = -\log q_{\boldsymbol{\theta}}(\mathbf{z}_{1:L}|\mathbf{x}) = \sum_{i=0}^{L-1} -\log q_{\boldsymbol{\theta}}(\mathbf{z}_{i+1}|\mathbf{z}_i)$$



Our contribution: **Bit-Swap**

Achieves

- Net number of bits added to the bitstream is approx. equal to the negative ELBO:

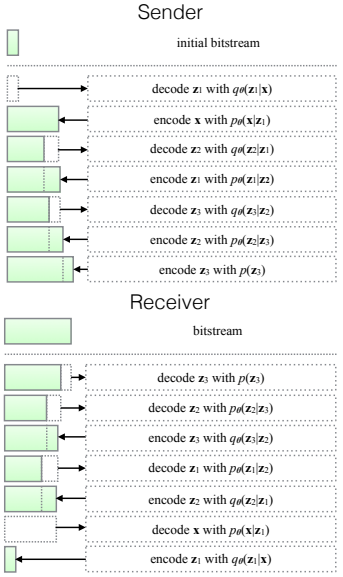
$$N_{\text{total}}^{\text{BitSwap}} - N_{\text{init}}^{\text{BitSwap}} = \mathbb{E}_{q_{\boldsymbol{\theta}}(\mathbf{z}_{1:L}|\mathbf{x})}[\log q_{\boldsymbol{\theta}}(\mathbf{z}_{1:L}|\mathbf{x}) - \log p_{\boldsymbol{\theta}}(\mathbf{x}, \mathbf{z}_{1:L})] = -\mathcal{L}(\boldsymbol{\theta}) \text{ (ELBO)}$$

- Number of **initial bits is bounded:**

$$N_{\text{init}}^{\text{BitSwap}} \leq \sum_{i=0}^{L-1} \max\left(0, \log \frac{p_{\boldsymbol{\theta}}(\mathbf{z}_{i-1}|\mathbf{z}_i)}{q_{\boldsymbol{\theta}}(\mathbf{z}_{i+1}|\mathbf{z}_i)}\right) \leq \sum_{i=0}^{L-1} -\log q_{\boldsymbol{\theta}}(\mathbf{z}_{i+1}|\mathbf{z}_i) = N_{\text{init}}^{\text{BBANS}}$$

How

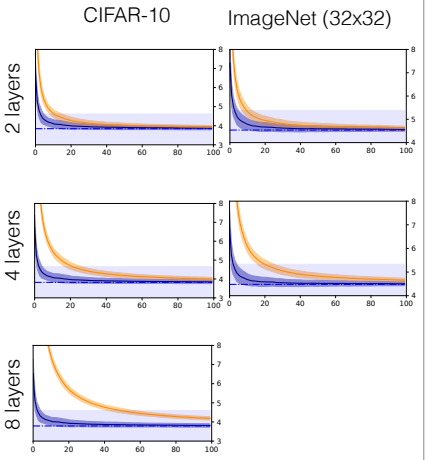
- Interpret HLVM is a multiple nested LVM's
- The "prior" of every layer is again a VAE
- Bit-Swap exploits this nested structure by **recursively applying the bits-back argument**.



Results

Cumulative average compression rate of compressing 100 images in sequence (including initial bits)

= Bit-Swap = BB-ANS



Benchmark compressors

Training: Model trained on random 32x32 patches of Imagenet.
Compression: 100 original ImageNet images cropped to multiples of 32 pixels on each side. Bit-Swap and BB-ANS compressed a grid of 32x32 blocks for each image.

	Compression Rate (bits/dim)
uncompressed	8.00
gzip	5.96
bzip2	5.07
LZMA	5.09
PNG	4.71
WebP	3.66
BB-ANS	3.62
Bit-Swap	**3.51**