Lecture 05, Dynamic analysis:

GPS station in the **West Pamir** recording relative motion to detect strain & hence stress.

From "*Natural laboratory Central Asia – Tracking the tectonic fingerprint of continental collision*"
https://www.gfz-potsdam.de/en/section/lithosphere-dynamics/projects/natural-laboratory-central-asia/

Fossen, sections: 4.1 – 4.7

Concepts are tricky. This lesson will revisit them, but pre-reading is very important – even if you don't "get it" at first.

eModule – Important! : Ch. 4. https://folk.uib.no/nglhe/StructuralGeoBookEmodules2ndEd.html

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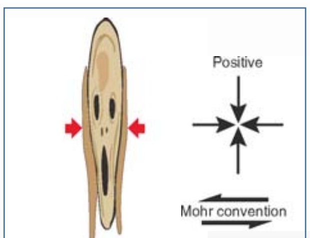
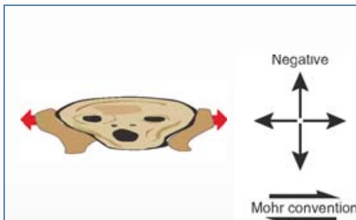
Dynamic Analysis

Interpreting deformation in terms of **forces** and **stresses** responsible.

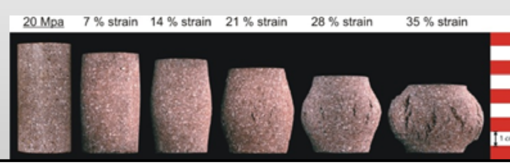


Positive stress



negative stress

Leads to considering strength or *rheology* of the material being deformed.



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Learning Goals

1. Distinguish between a Force and a Stress.
2. Calculate force per unit area (stress) at a given depth assuming a density for overlying rock.
3. Distinguish between surface stress due to force on a single plane versus stress at a point in terms of the stress tensor.
4. Characterize stress ellipsoids for simple principal stress scenarios.
5. Calculate simple stress at depth.
6. Use Mohr circle diagrams to plot stresses and determine preliminary basic parameters.

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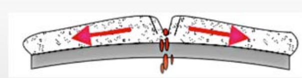
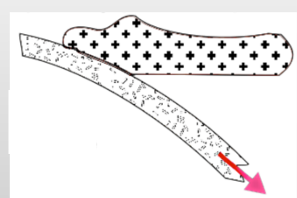
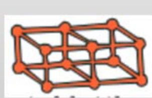
Forces (not ‘stresses’) affecting rock bodies

- Slow regional scale acceleration
  E.g. tectonic plates changing velocity over thousands of years
- Fast short-lived accelerations of relatively small volumes of rock
  E.g. earthquake fault movements
- Body forces: act on body mass independently of other material outside body
  - Gravitational at any scale:
    - crystal settling …
    - tectonic plate ‘ridge push’ and ‘slab pull” → →
  - Electromagnetic
    - Submicroscopic
    - Attractions at the crystal lattice scale.







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Types of **forces** (not ‘stresses’) that can affect rocks

- Contact forces:
  - e.g. across surfaces of contact like faults
- Load:
  - Weight that can be supported to maintain static equilibrium
- Gravitational loading: weight of rock column on any point at depth
  - Generally equals vertical stress.
- Thermal loading:
  - Heating or cooling of confined rock mass,
  - Expansion or contraction effects (eg freeze water in a full, closed container)
- Displacement loading
  - Large scale mechanical disturbance of rocks (plate collisions, intrusions, meteorites)

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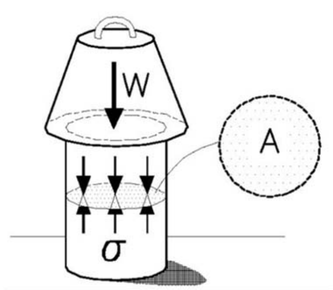
Force and stress



$$\sigma = \frac{Force}{Area}$$

force = mass x acceleration *(F = ma)*
mass (m) = amount of material in a body (kilograms)
volume (V) = space occupied by a given mass (mm^3 to km^3)
density (p) = mass per unit volume (kg/cm^3)

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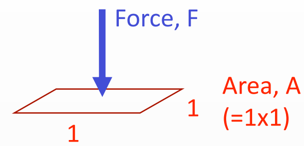
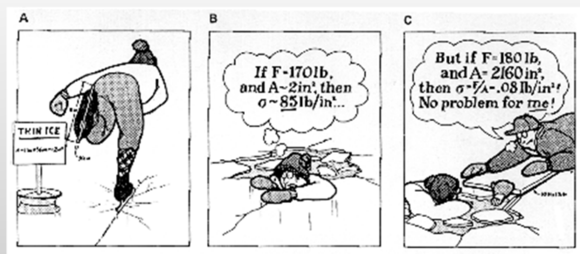
Rock Stress

- Stress on a plane is the FORCE per unit area of the plane (Stress σ=F/A)



- Unit of stress = pascal (Pa): 1 Pa = 1 N/m^2
- Geological stresses
  - kPa (10^3 Pa)
  - MPa (10^6 Pa)
  - GPa (10^9 Pa)
  - 1kbar = 100 MPa



Davis & Reynolds

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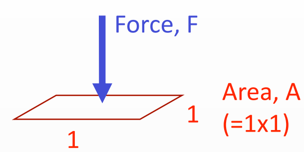
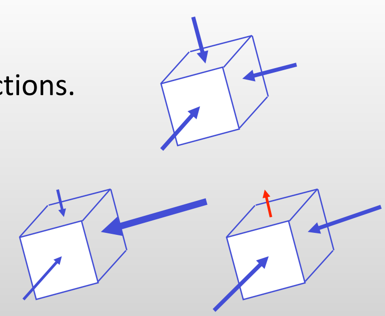
Rock Stress

- Stress on a plane is the FORCE per unit area of the plane (Stress σ=F/A)



- Unit of stress = pascal (Pa): 1 Pa = 1 N/m^2
- Stress is usually not equal in all directions.



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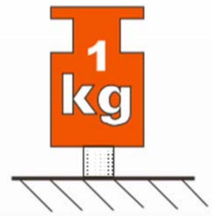
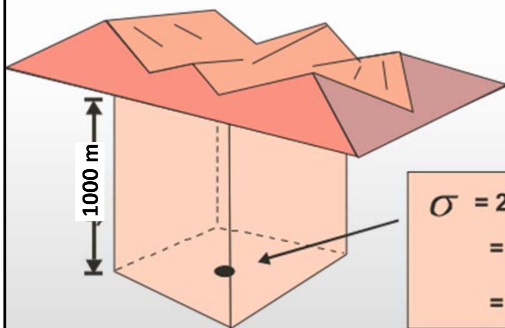
Calculating Stress in Earth's crust

simplest case:

direct load of weight of overlying rock at any depth:

$\sigma = \rho g z$ (rock density x gravity x depth)





ρ for granite = 2.7 gm/cm^3

= 2700 kg/m^3

$$\sigma = 2700\ \text{kg/m}^3 \times 9.8\ \text{m/s}^2 \times 1000\text{m}$$
$$= 26 \times 10^6\ \text{Pa}\ \left(\frac{N}{m^2}\right)$$
$$= 26\ \text{MPa}$$

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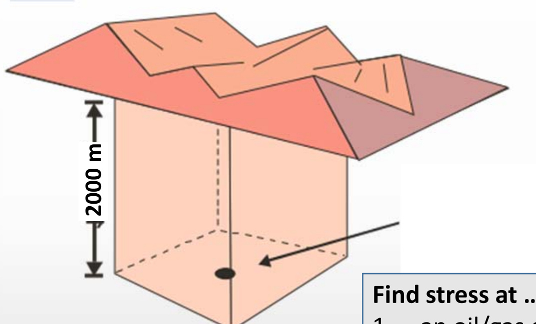
Calculating Stress in Earth's crust

??



Find stress at …

1. an oil/gas deposit that is 2km under impermiable sandstone of 2.7 gm/cc
2. The same oil/gas deposit 2km under a waterfilled drillhole (water is 1.0 gm/cc)
3. What happens if you drill to this reservoir leaving a drill hole filled with only water?

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Stress in rocks; planes, points, normal, shear …

- It’s all in your readings.
- BUT – concepts are challenging.
- Therefore, we will step through the thinking again.
- You are to help us out …

Why is this important?

Example … plane could be a fault, and we may want to know about stresses that could cause faulting (e.g. earthquakes).

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Force / stress on a dipping plane

Fossen (2016), Fig. 4.1

(REPEAT OF Fossen 4.2 and 4.3)

Questions to be asked and answered as a discussion.

a. ?? *What is Stress* on a surface (traction) in terms of force?

b. ?? What are units of *stress*?

Stress on a surface is a vector (σ).

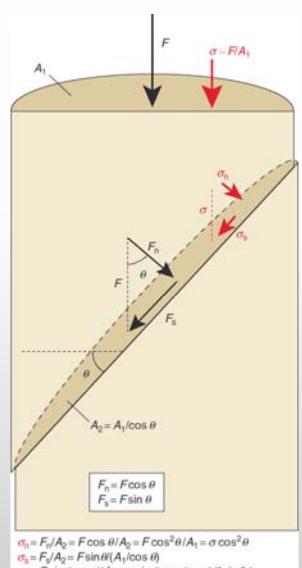
c. ?? What is a vector?

Stress acting on a surface can be split into two components.

d. ?? What are they called?

e. ?? What symbols are used

f. ?? How are relations between force and stress on a dipping plane calculated? (NOT the equations, but the procedure.)



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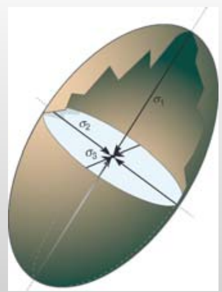
BUT - - - Stress at a point is more useful

A single surface stress relates to a singe plane… not a very informative way to define the state of stress in the whole rock.

Instead….

Define stress at any given point in a body of rock so that we could determine the stress vector for any plane passing through that point



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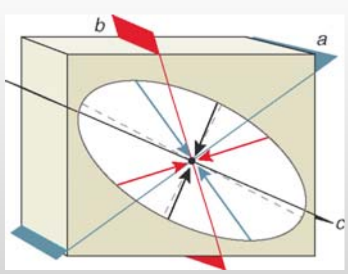
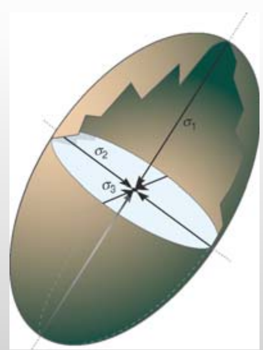
Stress at a point

- 3D state of stress at a point cannot be expressed by a **vector.**
- Stress at a point requires use of a **TENSOR (2nd order).**
- Stress **tensor** is represented graphically as a **stress ellipse** in 2D and as a **stress ellipsoid** in 3D.

Stress at a point – refers to the whole collection of stress vectors acting on each & every orientation passing a single point in a body – this is a 2nd order **tensor**.





Fossen (2016), Figs. 4.4 & 4.5

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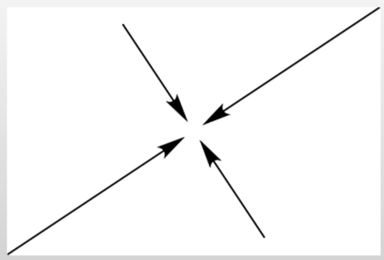
Stress at a point - stress ellipse

- Start with the 2D case
- Imagine that all possible planes are perpendicular to the screen (or paper).
- Also assume that the body of rock is not equally stressed.

HINT: Follow along making sketches in your notes.
Sketching helps improve your memory.



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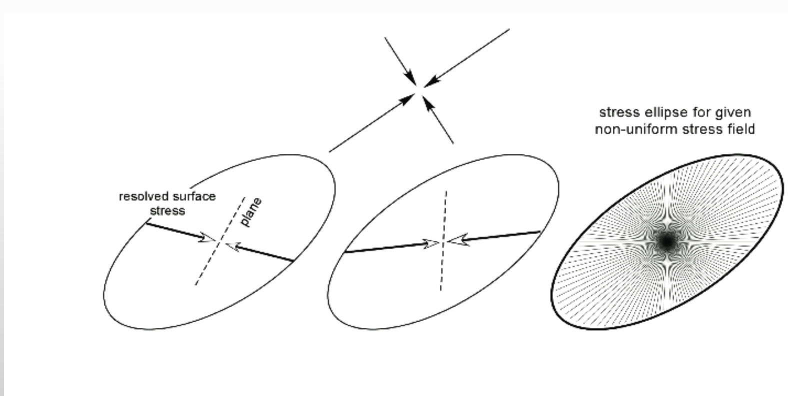
Stress at a point - stress ellipse

- For each plane, the calculated surface stress is a sum of normal and shear stress components
- For each plane we can draw the corresponding stress vector and have its length proportional to magnitude
- An envelope drawn around a finite number of planes defines an ellipse that represents the stress field



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Stress at a point - stress ellipse

- Long axis of the ellipse is parallel to maximum surface stress
- Short axis of the ellipse is parallel to minimum surface stress

- These stresses are **PRINCIPAL STRESSES.**
- They are perpendicular to planes with zero shear stress, called the **PRINCIPAL PLANES** of stress

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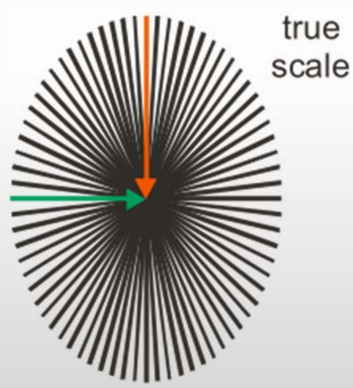
Stress in 2-D

⇒ Each point has an infinite number of intermediate stress directions, with one maximum & one minimum stress direction (σ_1 & σ_3). By convention, the largest stress is called σ_1, so σ_1 is always ≥ σ_3

⇒ All possible stresses to scale for a point = <u>stress</u> ellipse



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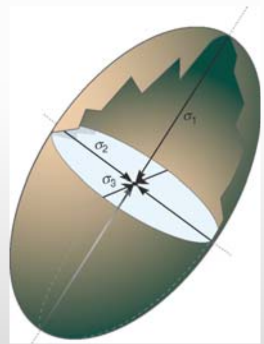
Stress at a point – principle stresses

- In 3D, a stress ellipsoid has 3 **principal stresses.**
- Max, intermediate, min principal stress are: σ_1 , σ_2 , σ_3
- In other words, **by definition**, $\sigma_1 > \sigma_2 > \sigma_3$

Magnitudes and orientations of principal stresses completely define the stress ellipsoid and therefore stress at a point.



From the principal stresses we can calculate **surface stress** on a plane at **any** orientation through that point.

Why? **Example** … plane could be a fault, and we may want to know about stresses that could cause faulting (e.g. earthquakes).

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Stress at a point – principle stresses

Define several specific stress states using **principal stresses**

1. hydrostatic $\sigma_1 = \sigma_2 = \sigma_3$
2. general triaxial stress $\sigma_1 > \sigma_2 > \sigma_3$
3. uniaxial tension $\sigma_1 = \sigma_2 = 0; \sigma_3 < 0$
4. uniaxial compression $\sigma_2 = \sigma_3 = 0; \sigma_1 > 0$

- **??** Sketch the stress ellipsoid for scenarios 1 and 2.
- **??** Why would sketching 3 & 4 look not much like an ellipsoid?

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Relationships BETWEEN stresses:

Three definitions of values that relate the stress:

- differential stress: difference between maximum and minimum.
- deviatoric stress: half the differential.
- mean stress: essentially the “average” value of stress.

These values are used in stress analysis. Here we simply define them.

What are these 3 values if $\sigma_1 = 2.0$, $\sigma_2 = 1.0$, $\sigma_3 = 0.5$?

- differential stress = **??** $\sigma_1 - \sigma_3$
- deviatoric stress = **??** $\sigma_1 - \sigma_3) / 2$
- mean stress = **??** $(\sigma_1 + \sigma_2 + \sigma_3) / 3$

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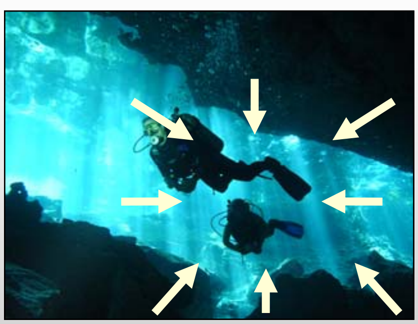
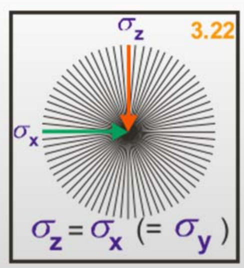
Special case … ??

- What stress situation is depicted here?
- What does 2D ellipse look like? A c______
- What does 3D ellipse look like? A s______
- What value will shear stresses be?
- Will any deformation be expected?





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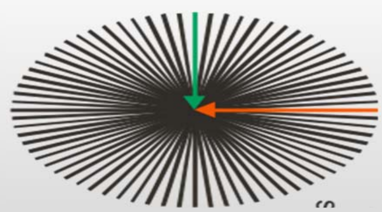
Horizontal Stress ??

1. What force is contributing to vertical stress components?
2. Are there horizontal stresses evident?
3. Do they appear to be more, less or same as vertical stress?
4. What evidence do you see that this is NOT simply a vertical stress that is more than gravity?



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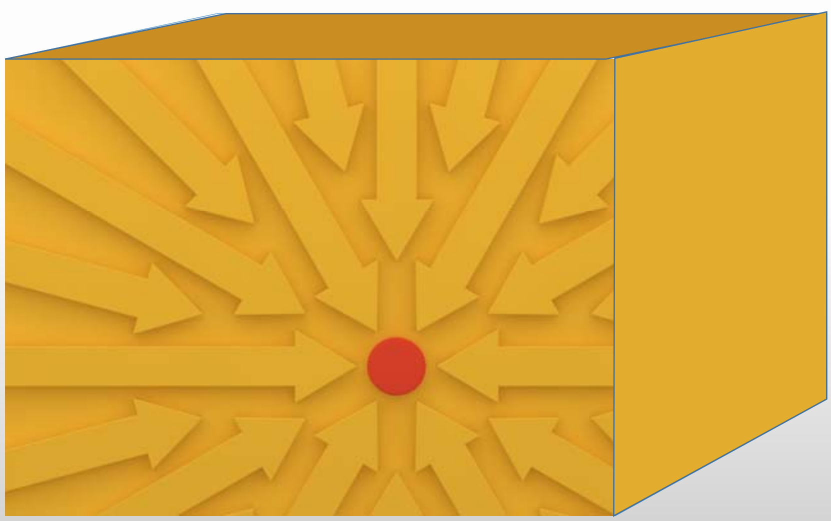
Now - stress at a point in 3D



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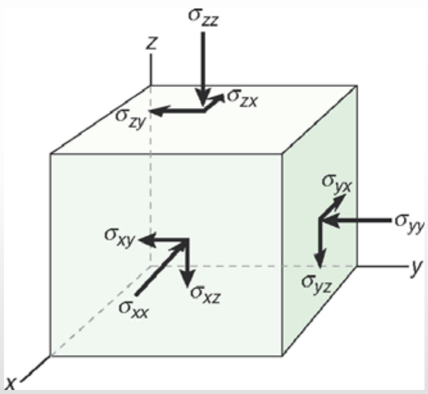
Stress at a point – components

Consider stress at a point by defining stress components (σ_s , σ_n) on an infinitely small cube in a Cartesian XYZ reference frame.



- Sides are perpendicular to the coordinate axes.
- Any surface stress can be resolved into its 3 components
- Stress **normal** to face (σ_{xx} σ_{yy} σ_{zz})
- Stress **parallel** to face (shear) acts along one of the other 2 axes.

Positive stresses are shown.
Equal and opposite stresses act on the negative and inside faces of the cube.

Fossen (2016), Fig. 4.6

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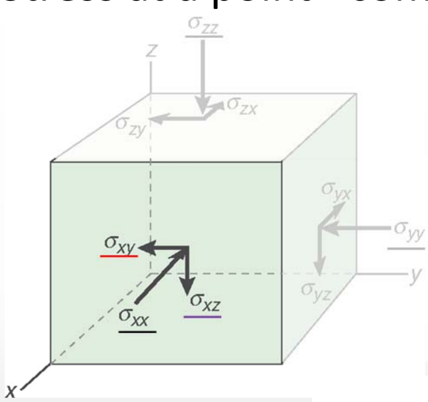
Stress at a point - components



- Consider ONE face only:
- 1 normal stress (black)
- 2 shear stress (red, purple)

	In direction of		
	x:	y:	z:
stress on face normal to x	σ_{xx}	σ_{xy}	σ_{xz}

Fossen (2016), Fig. 4.6

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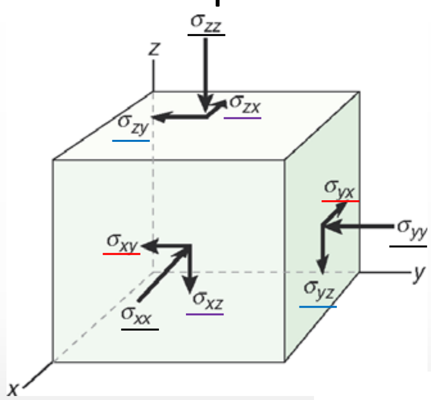
Stress at a point - components



- In total there are 9 stress components
- Stress tensor or stress matrix
- 3 normal (black)
- 6 shear (red, purple, blue)

	In direction of x:	y:	z:
stress on face normal to *x*	σ_{xx}	σ_{xy}	σ_{xz}
stress on face normal to *y*	σ_{xy}	σ_{yy}	σ_{yz}
stress on face normal to *z*	σ_{xz}	σ_{yz}	σ_{zz}

Fossen (2016), Fig. 4.6

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Stress at a point - components

- Cube must be in mechanical equilibrium (at rest) so normal stresses and shear stresses must sum to zero.
- So $\sigma_{xy} = \sigma_{yx}$
- Hence there are only 6 independent components.

	In direction of x:	y:	z:
stress on face normal to *x*	σ_{xx}	σ_{xy}	σ_{xz}
stress on face normal to *y*	σ_{xy}	σ_{yy}	σ_{yz}
stress on face normal to *z*	σ_{xz}	σ_{yz}	σ_{zz}

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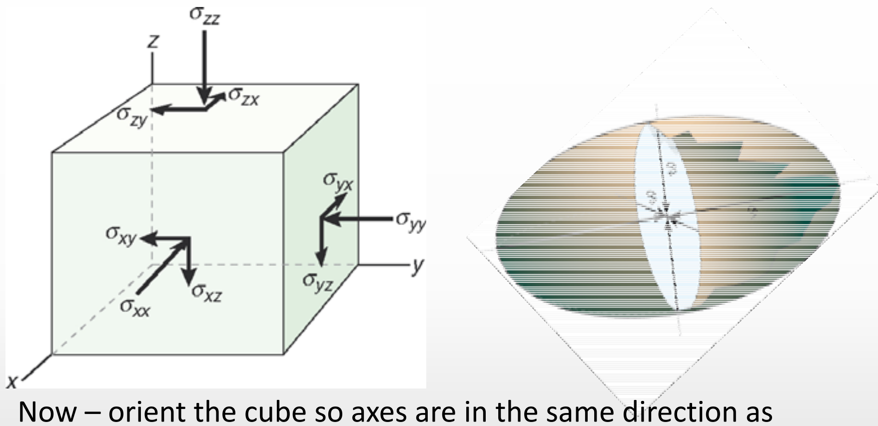
Stress at a point – ellipsoid and cube



Now – orient the cube so axes are in the same direction as principle stresses. Then shear stress = 0

$\sigma_{xx} = \sigma_1$ $\sigma_{yy} = \sigma_2$ $\sigma_{zz} = \sigma_3$

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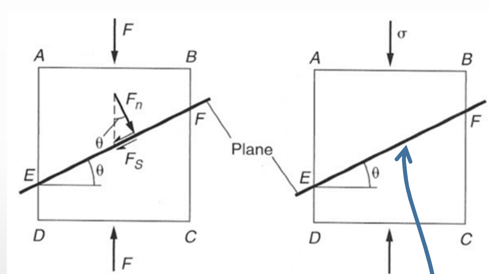
Back up … derive fundamental stress equations with only σ_1

Why?

We are working towards a standard, convenient method of describing and analyzing stresses within rocks.

Example:

- Given stresses in the Earth, will a fault with a known plane or direction be likely to fail?
- What about a fault lying in a different plane?



Fault plane

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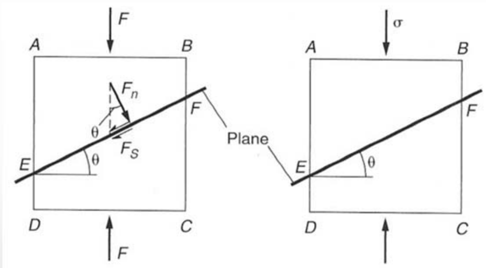
Back up … derive fundamental stress equations with only σ_1

Why? We are working towards standard, convenient method of describing and analyzing stresses within rocks.

- **Recall Fossen Fig. 4.1**
- **??** Force F decomposes into … what?
- Stresses do NOT decompose so easily

?? They require what to be taken into account?



$$\sigma_n = \sigma \bullet \cos^2\theta$$

$$\tau = \sigma \bullet \frac{\sin 2\theta}{2}$$

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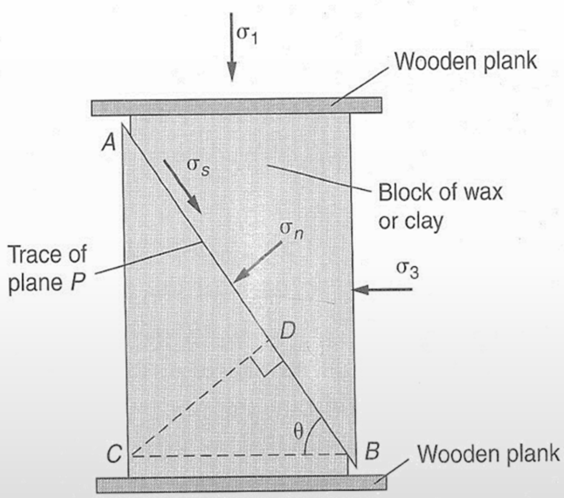
Now, more realistically, σ_1 & σ_3 are both present



Experimental setup for measuring shear behavior under controlled stresses:

- principle stress σ_1

and

- confining stress σ_3

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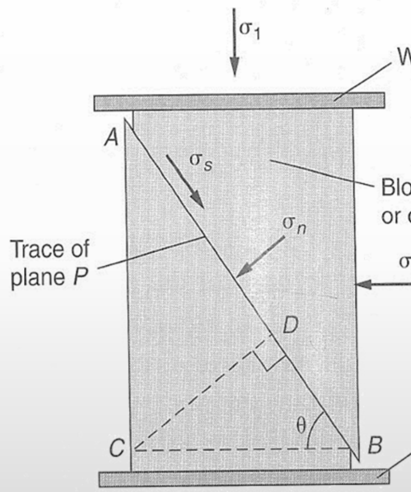
2D Stress - Fundamental Equations



If principal stresses are known (orientation & magnitude)

⇒ σ_n & σ_s can be calculated for *any* plane.

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

$$\sigma_s = \tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

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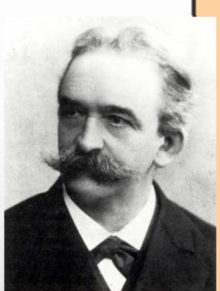
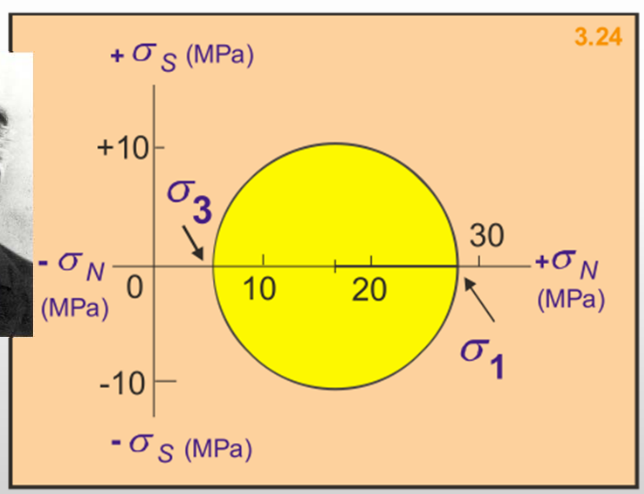
NEXT: A graphical tool for working with stress Mohr circle diagrams



Otto Mohr
(1835-1918)



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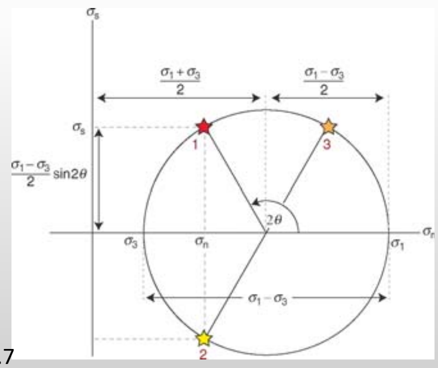
Stress - Mohr diagram

The MOHR DIAGRAM (*Christian Otto Mohr 1835-1918*) is a simple graphical method for determining:

- the state of stress at a point, and
- the values of σ_s , σ_n for any surface through that point
- We will only look at the Mohr diagram for 2D stress, although they can be constructed for stress in 3D as well



Fossen (2016), Fig. 4.7

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Stress - Mohr diagram

Worksheet activity.

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What have we accomplished today?

- ✓ Distinguish between a Force and a Stress.
- ✓ Calculate force per unit area (stress) at a given depth assuming a density for overlying rock.
- ✓ Distinguish between surface stress due to force on a single plane versus stress at a point in terms of the stress tensor.
- ✓ Characterize stress ellipsoids for simple principal stress scenarios.
- ✓ Calculate simple stress at depth.
- ✓ Use Mohr circle diagrams to plot stresses and determine preliminary basic parameters.

Tonight or tomorrow:

Finish the worksheet activity to hand in next lesson for participation.

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