

# Ec140 - Core Concepts from Statistics

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# Housekeeping

- Updated Syllabus
- Unofficial Course Capture!
- What is the weirdest concept you remember from yesterday?
- Switch to finish yesterday's slides

# This Lecture

- Introduction to Data
- Mean and Expectation
- Variance and Standard Deviation

# What Defines a Data Set?

- Data Set is the collection of any type information (of multiple *Datum*)
- In quantitative analysis we focus on *structured* data sets (unlike, for example, unstructured field notes).
- In econometrics the most common way to structure data is in tabular, or rectangular, form.
- A tabular data set is a collection of variables that with information for one or more entities.
- Entities can represent multiple individuals, one individual over time, firms, countries, etc.
- Variables are represented in columns, and observations are represented by rows. (for more on variables [The Effect, Ch3](#))

# Data

# But What Can We Do With Data?

- We summarized it! (see the great [short story by J.L. Borges](#) on why summarizing is essential)
- One of the first thing we do when summarizing data is to look at *some type of average*.
  - Wait? *Type* of average? Isn't there just one average? called *the mean*?

# But What Can We Do With Data?

- We summarized it! (see the great [short story by J.L. Borges](#) on why summarizing is essential)
- One of the first thing we do when summarizing data is to look at *some type of average*.
  - Wait? *Type* of average? Isn't there just one average? called *the mean*?
- These is also referred as measure of central tendency.
- In this course, we will focus primarily on the mean. **From now on in this course**

## average noun



av·er·age | \ 'a-v(ə-)rij \

### Definition of *average* (Entry 1 of 3)

- 1 a** : a single value (such as a mean, mode, or median) that summarizes or represents the general significance of a set of unequal values  
**b** : [MEAN sense 1b](#)
- 2 a** : an estimation of or approximation to an [arithmetic mean](#)  
**b** : a level (as of intelligence) typical of a group, class, or series  
*// above the [average](#)*

# Mean

- The mean is defined by the sum of a set of values divided by the number of values.

Let's look at the mean from the "hang out with a friend" exercise.

- Total over N

$$\textit{Average}(X) = \frac{1 \times 10 + 2 \times 9 + 3 \times 11}{30} = 2.03$$

- One number, **highly informative** for a variable of interest.
- Always important to keep an eye on the units and magnitude (relevant for PS1).



# Mean: Notation (Message to me: draw histogram on the board)

$$\textit{Average}(X) = \frac{1 \times 10 + 2 \times 9 + 3 \times 11}{30} = 2.03$$

$$\textit{Ave}(X) = 1 \times \frac{10}{30} + 2 \times \frac{9}{30} + 3 \times \frac{11}{30} = 2.03$$

# Expected Value

- Let's look at the histogram for the exercise above (drawn in the board) and pretend it is not a sample but the entire population. How can we move from frequencies into probabilities?
- Replace frequencies by probabilities
- The population version of the sample mean is the **expected value**.

# Expected Value: Definition (Discrete)

The expected value of a discrete random variable  $X$  is the weighted average of its  $k$  values  $\{x_1, \dots, x_k\}$  and their associated probabilities:

$$\begin{aligned}\mathbb{E}(X) &= x_1 \mathbb{P}(X = x_1) + x_2 \mathbb{P}(X = x_2) + \dots + x_k \mathbb{P}(X = x_N) \\ &= \sum_x x \mathbb{P}(X = x)\end{aligned}$$

- Also known as the population mean.

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- Also known as the population mean. Compare it to the sample mean:

$$\overline{X}_n = \sum_x \textcolor{brown}{x} \times \textcolor{teal}{prop}_n(x_1)$$

# Expected Value

## Example

Rolling a six-sided die once can take values  $\{1, 2, 3, 4, 5, 6\}$ , each with equal probability. What is the expected value of a roll?

$$\mathbb{E}(\text{Roll}) = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6} = 3.5.$$

- **Note:** The expected value can be a number that isn't a possible outcome of  $X$ .

# Expected Value. Definition (Continuous)

- Compare it to the discrete version
- Continuous

$$\mathbb{E}(X) = \int_{-\infty}^{\infty} x f(x) dx.$$

- Discrete

$$\mathbb{E}(X) = \sum_x x f(x)$$

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This explanation was inspired by  
[this lecture from Eddie Woo](#)

# Expected Value. Definition. One Last Thing 1/2

Let's go back to the mean of our exercise:

$$\overline{X}_n = 1 \times \frac{10}{30} + 2 \times \frac{9}{30} + 3 \times \frac{11}{30} = 2.03$$

But now let's switch the values of the random variables to: 10, 20, 30. How should we compute the mean?

$$\overline{g(X)}_n = 10 \times \frac{10}{30} + 20 \times \frac{9}{30} + 30 \times \frac{11}{30} = 20.33$$



# Expected Value. Definition. One Last Thing 2/2

Hence, we can conclude, that for a random variable  $X$ , any transformation  $g(X)$  has a sample average:

$$\overline{X}_n = \sum_x g(x) \times \text{prop}_n(x_1)$$

And an expectation:

$$\mathbb{E}(X) = \sum_x g(x) f(x)$$

The same idea applies in the case of a continuous random variable

# Expected Value: Rules (or Properties)

## Rule 1

For any constant  $c$ ,  $\mathbb{E}(c) = c$ .

## Not-so-exciting examples

$$\mathbb{E}(5) = 5.$$

$$\mathbb{E}(1) = 1.$$

$$\mathbb{E}(4700) = 4700.$$

# Expected Value

## Rule 2

For any constants  $a$  and  $b$ ,  $\mathbb{E}(aX + b) = a \mathbb{E}(X) + b$ .

## Example

Suppose  $X$  is the high temperature in degrees Celsius in Eugene during August. The long-run average is  $\mathbb{E}(X) = 28$ . If  $Y$  is the temperature in degrees Fahrenheit, then  $Y = 32 + \frac{9}{5}X$ . What is  $\mathbb{E}(Y)$ ?

- $\mathbb{E}(Y) = 32 + \frac{9}{5} \mathbb{E}(X) = 32 + \frac{9}{5} \times 28 = 82.4$ .

# Expected Value

## Rule 3: Linearity

If  $\{a_1, a_2, \dots, a_n\}$  are constants and  $\{X_1, X_2, \dots, X_n\}$  are random variables, then

$$\mathbb{E}(a_1 X_1 + a_2 X_2 + \dots + a_n X_n) = a_1 \mathbb{E}(X_1) + a_2 \mathbb{E}(X_2) + \dots + a_n \mathbb{E}(X_n).$$

In English, the expected value of the sum = the sum of expected values.

# Expected Value

## Rule 3

The expected value of the sum = the sum of expected values.

## Example

Suppose that a coffee shop sells  $X_1$  small,  $X_2$  medium, and  $X_3$  large caffeinated beverages in a day. The quantities sold are random with expected values  $\mathbb{E}(X_1) = 43$ ,  $\mathbb{E}(X_2) = 56$ , and  $\mathbb{E}(X_3) = 21$ . The prices of small, medium, and large beverages are 1.75, 2.50, and 3.25 dollars. What is expected revenue?

$$\begin{aligned}\mathbb{E}(1.75X_1 + 2.50X_2 + 3.35X_n) &= 1.75 \mathbb{E}(X_1) + 2.50 \mathbb{E}(X_2) + 3.25 \mathbb{E}(X_3) \\ &= 1.75(43) + 2.50(56) + 3.25(21) \\ &= 283.5\end{aligned}$$

# Expected Value

## Caution

Previously, we found that the expected value of rolling a six-sided die is  $\mathbb{E}(\text{Roll}) = 3.5$ .

- If we square this number, we get  $[\mathbb{E}(\text{Roll})]^2 = 12.25$ .

**Is  $[\mathbb{E}(\text{Roll})]^2$  the same as  $\mathbb{E}(\text{Roll}^2)$ ?**

**No!**

# Expected Value

## Caution

Except in special cases, the transformation of an expected value **is not** the expected value of a transformed random variable.

For some function  $g(\cdot)$ , it is typically the case that

$$g(\mathbb{E}(X)) \neq \mathbb{E}(g(X)).$$

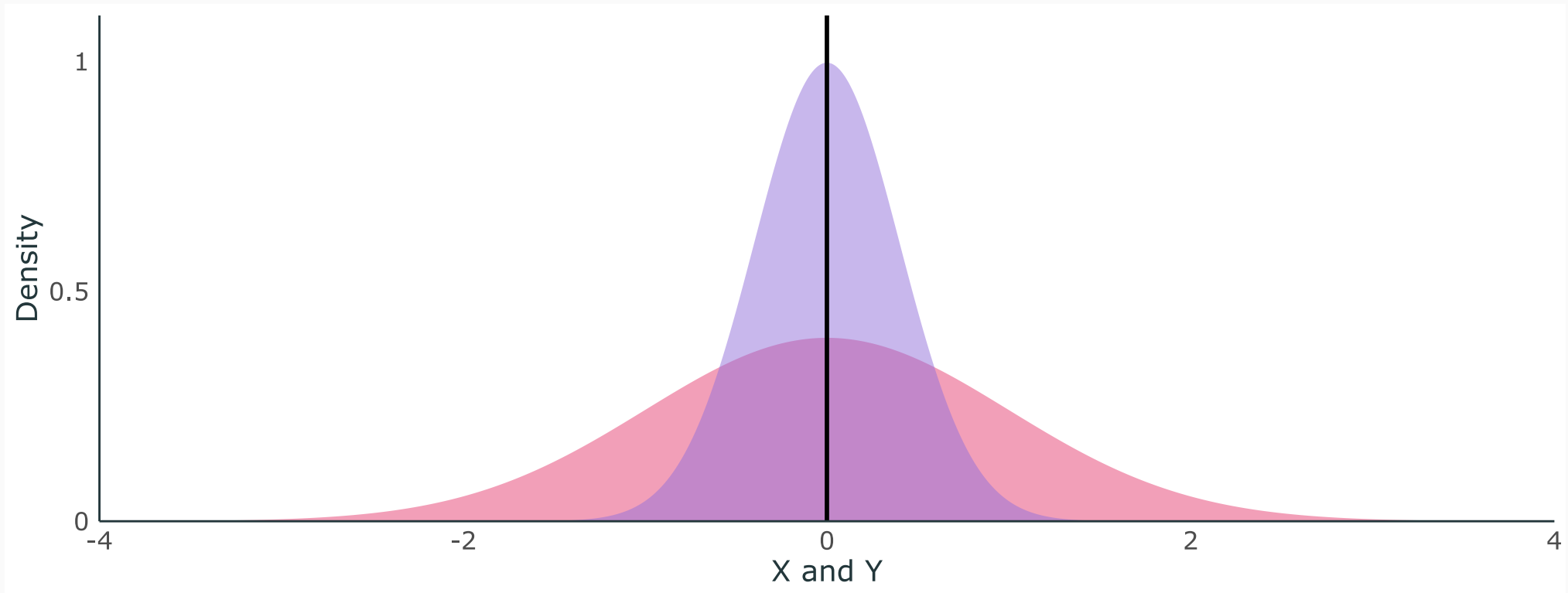
# Activity 1

- Let's watch [another Stat 110's video](#). Then get together in groups of 3 and discuss:
  - Don't worry about the law of large numbers yet
  - How does the random variables becomes continuous?
  - How does linearity help with computations?



# Variance

Random variables  $X$  and  $Y$  share the same population mean, but are distributed differently.



# Variance

How tightly is a random variable distributed about its mean?

- Let  $\mu = \mathbb{E}(X)$ .
- Describe the distance of  $X$  from its population mean  $\mu$  as the squared difference:  $(X - \mu)^2$ .

**Variance** tells us how far  $X$  deviates from  $\mu$ , *on average*:

$$\text{Var}(X) \equiv \mathbb{E}((X - \mu)^2) = \sigma^2$$

- $\sigma^2$  is shorthand for variance.

# Variance

## Rule 1

$\text{Var}(X) = 0 \iff X$  is a constant.

- If a random variable never deviates from its mean, then it has zero variance.
- If a random variable is always equal to its mean, then it's a (not-so-random) constant.

# Variance

## Rule 2

For any constants  $a$  and  $b$ ,  $\text{Var}(aX + b) = a^2 \text{Var}(X)$ .

## Example

Suppose  $X$  is the high temperature in degrees Celsius in Eugene during August. If  $Y$  is the temperature in degrees Fahrenheit, then  $Y = 32 + \frac{9}{5}X$ . What is  $\text{Var}(Y)$ ?

- $\text{Var}(Y) = \left(\frac{9}{5}\right)^2 \text{Var}(X) = \frac{81}{25} \text{Var}(X)$ .

# Standard Deviation

**Standard deviation** is the positive square root of the variance:

$$\text{sd}(X) = +\sqrt{\text{Var}(X)} = \sigma$$

- $\sigma$  is shorthand for standard deviation.

# Standard Deviation

## Rule 1

For any constant  $c$ ,  $\text{sd}(c) = 0$ .

## Rule 2

For any constants  $a$  and  $b$ ,  $\text{sd}(aX + b) = |a| \text{sd}(X)$ .

# Standardizing a Random Variable

When we're working with a random variable  $X$  with an unfamiliar scale, it is useful to **standardize** it by defining a new variable  $Z$ :

$$Z \equiv \frac{X - \mu}{\sigma}.$$

$Z$  has mean **0** and standard deviation **1**. How?

- First, some simple trickery:  $Z = aX + b$ , where  $a \equiv \frac{1}{\sigma}$  and  $b \equiv -\frac{\mu}{\sigma}$ .
- $\mathbb{E}(Z) = a \mathbb{E}(X) + b = \mu \frac{1}{\sigma} - \frac{\mu}{\sigma} = 0$ .
- $\text{Var}(Z) = a^2 \text{Var}(X) = \frac{1}{\sigma^2} \sigma^2 = 1$ .