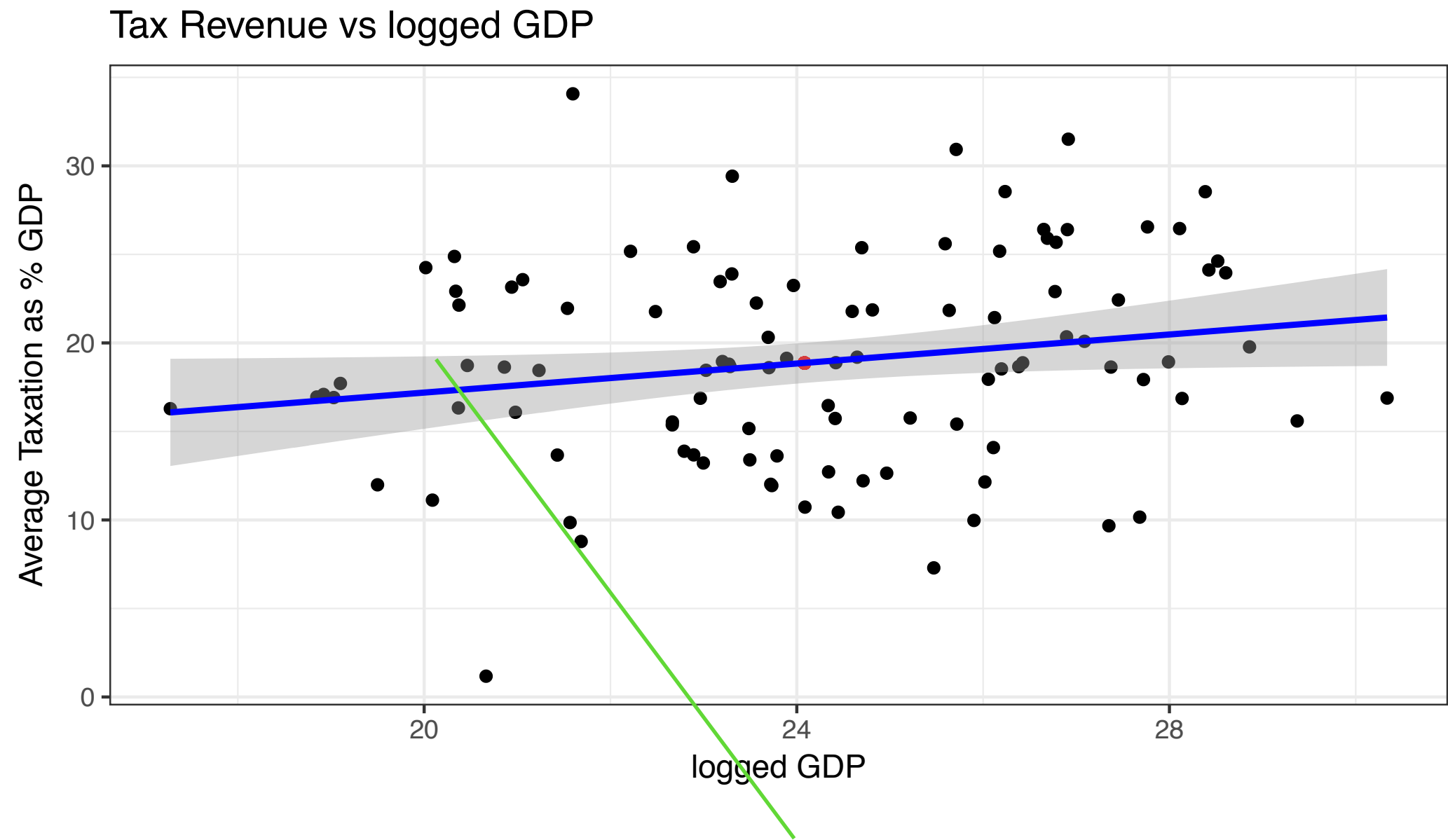
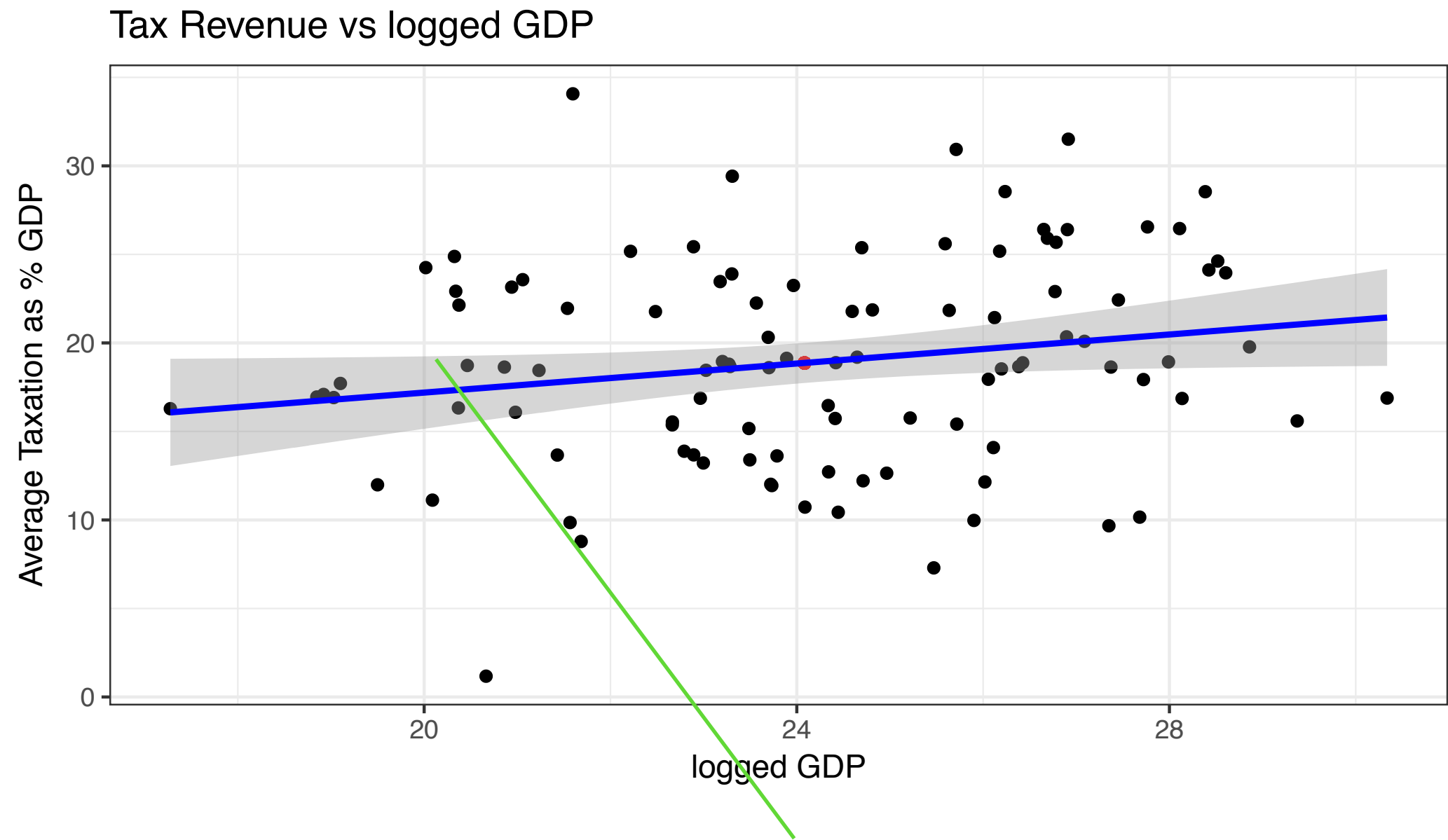


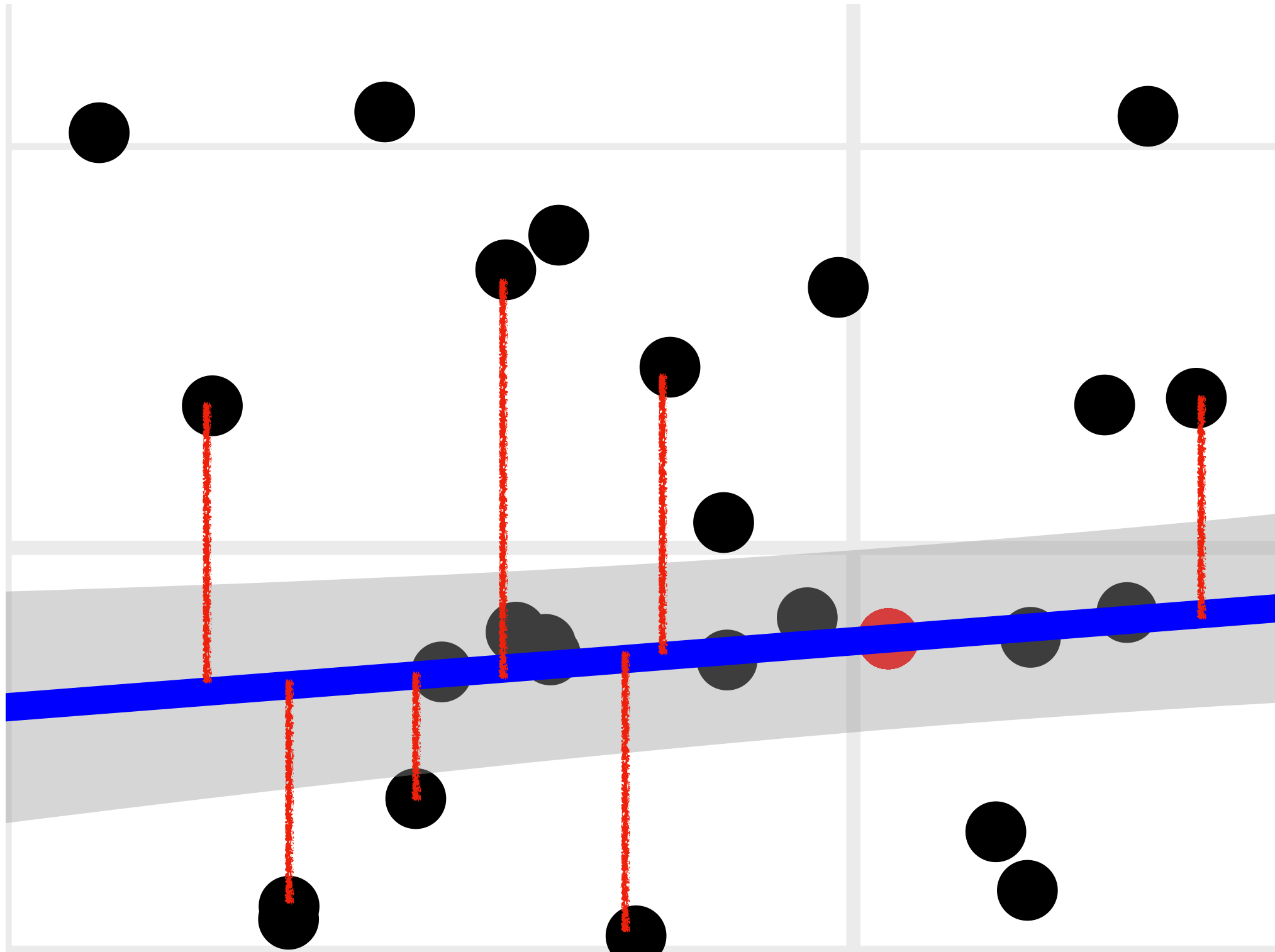
Regression lines minimize distance to all points



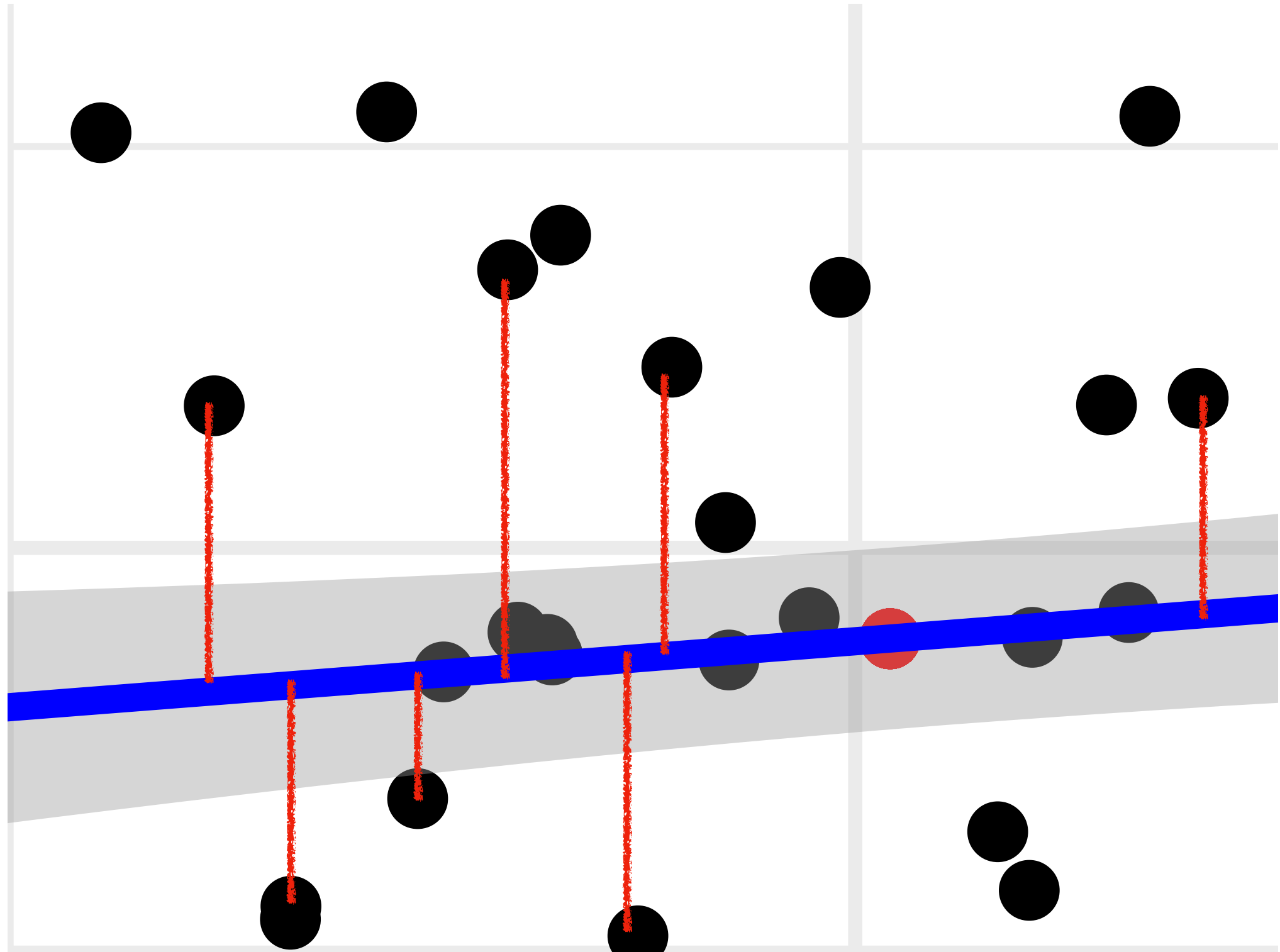
But the line does not go through all points



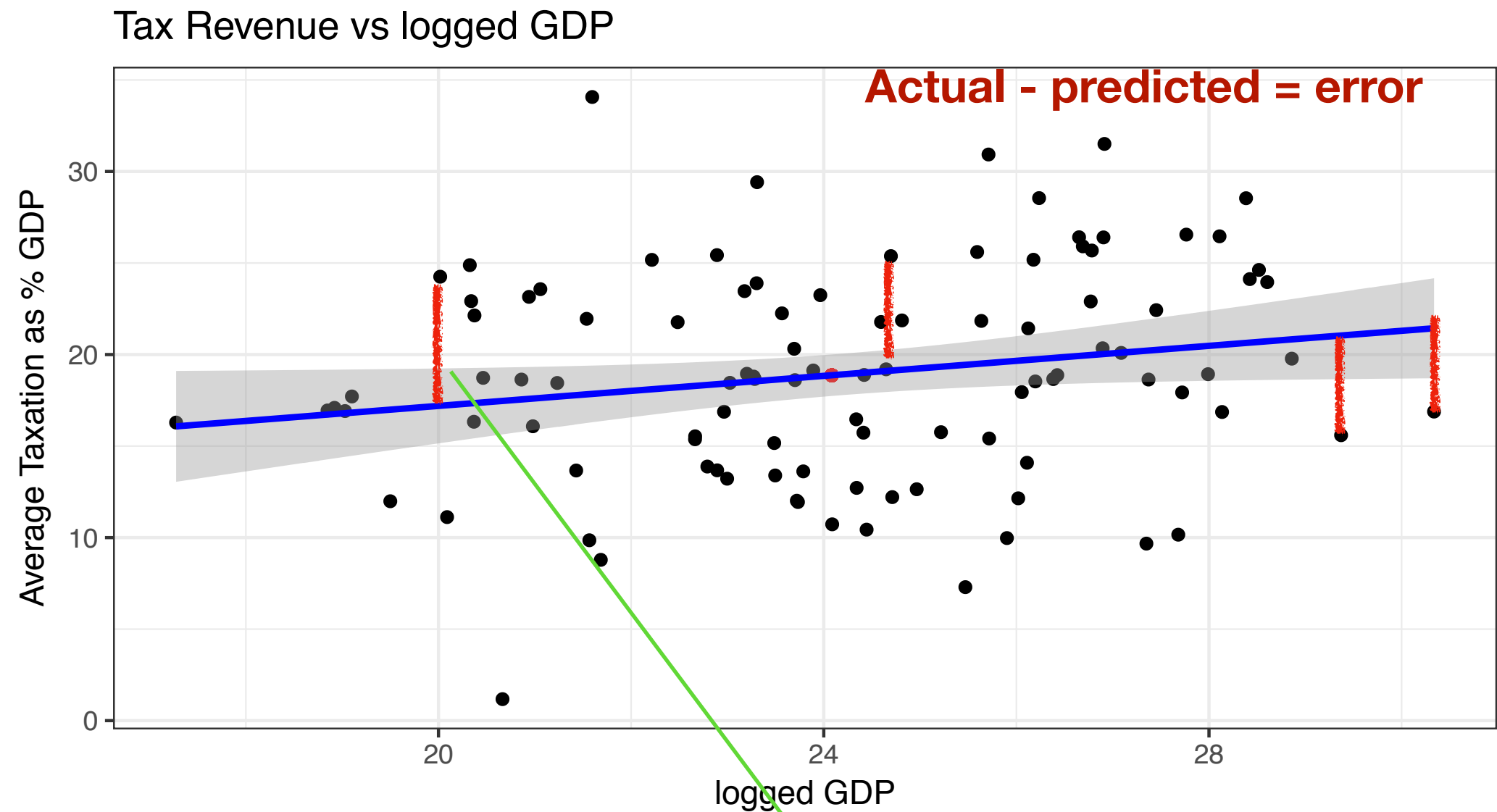
Each point is associated with an error:
prediction at x - actual value of y at x



Error = prediction - actual y

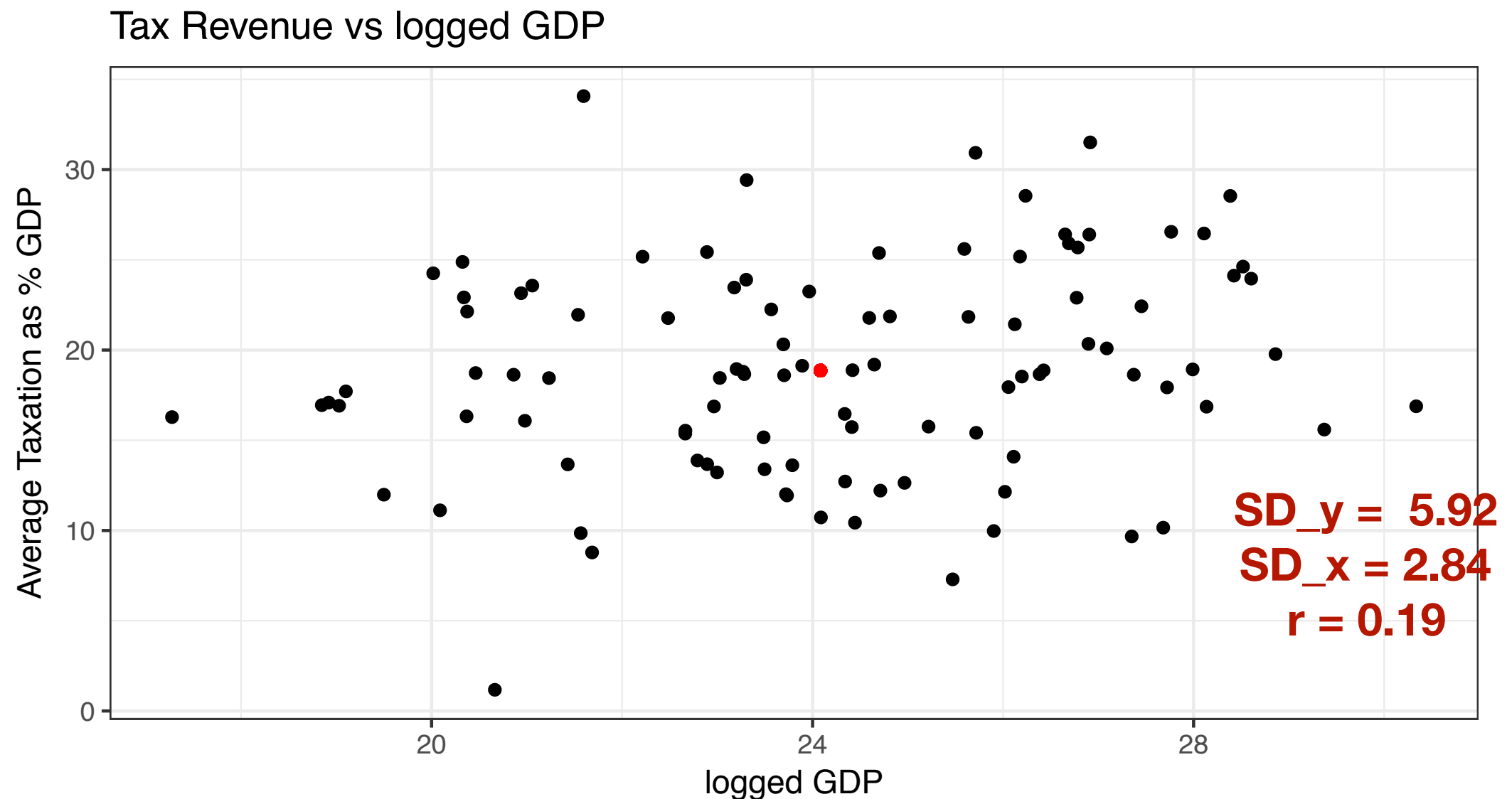


But regression lines are not perfect



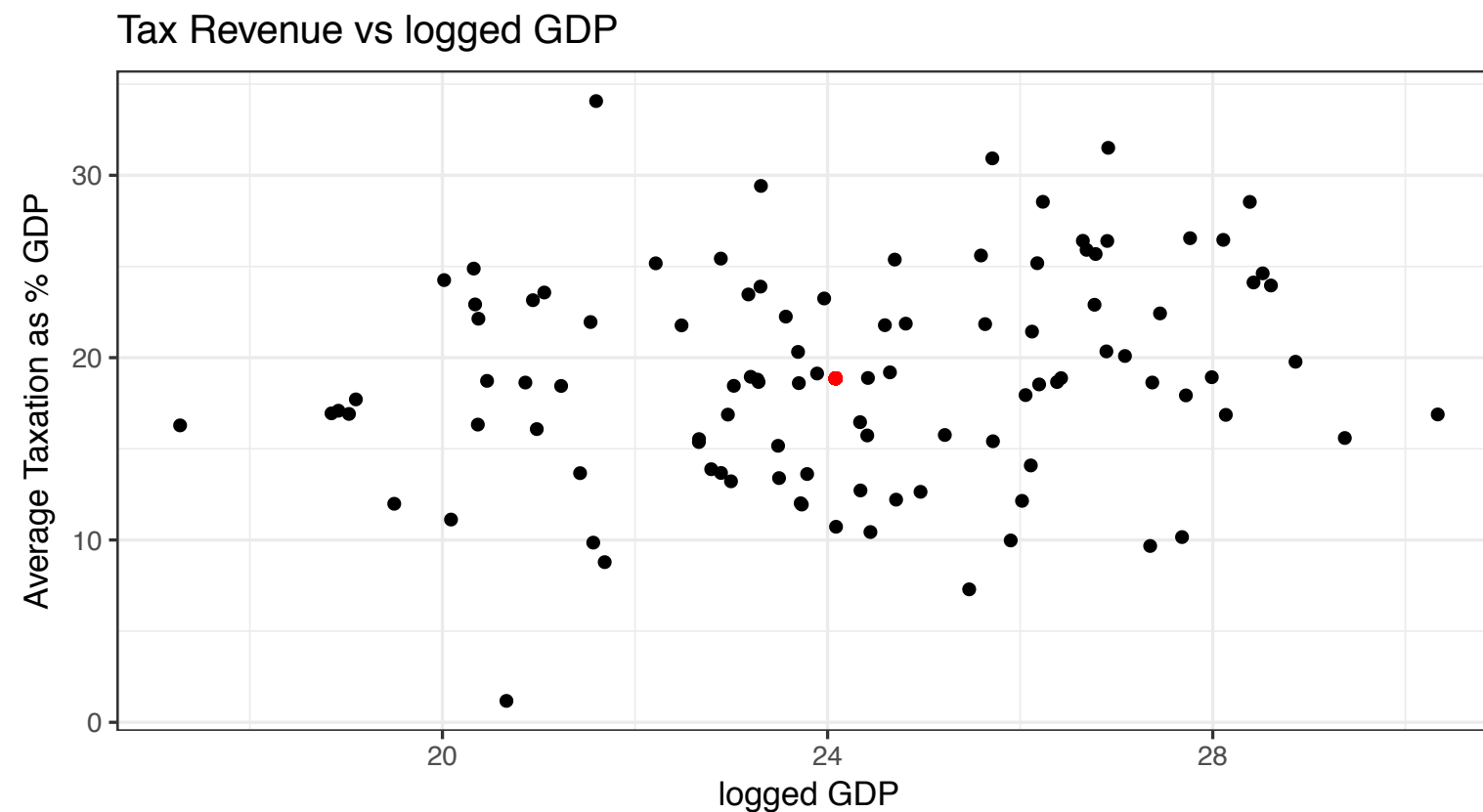
We always measure the error in terms of prediction error in y!
Why?

Example of error calculation



Slope of regression line: $5.92 \cdot 0.19 / 2.84 = 0.4$

Example of error calculation

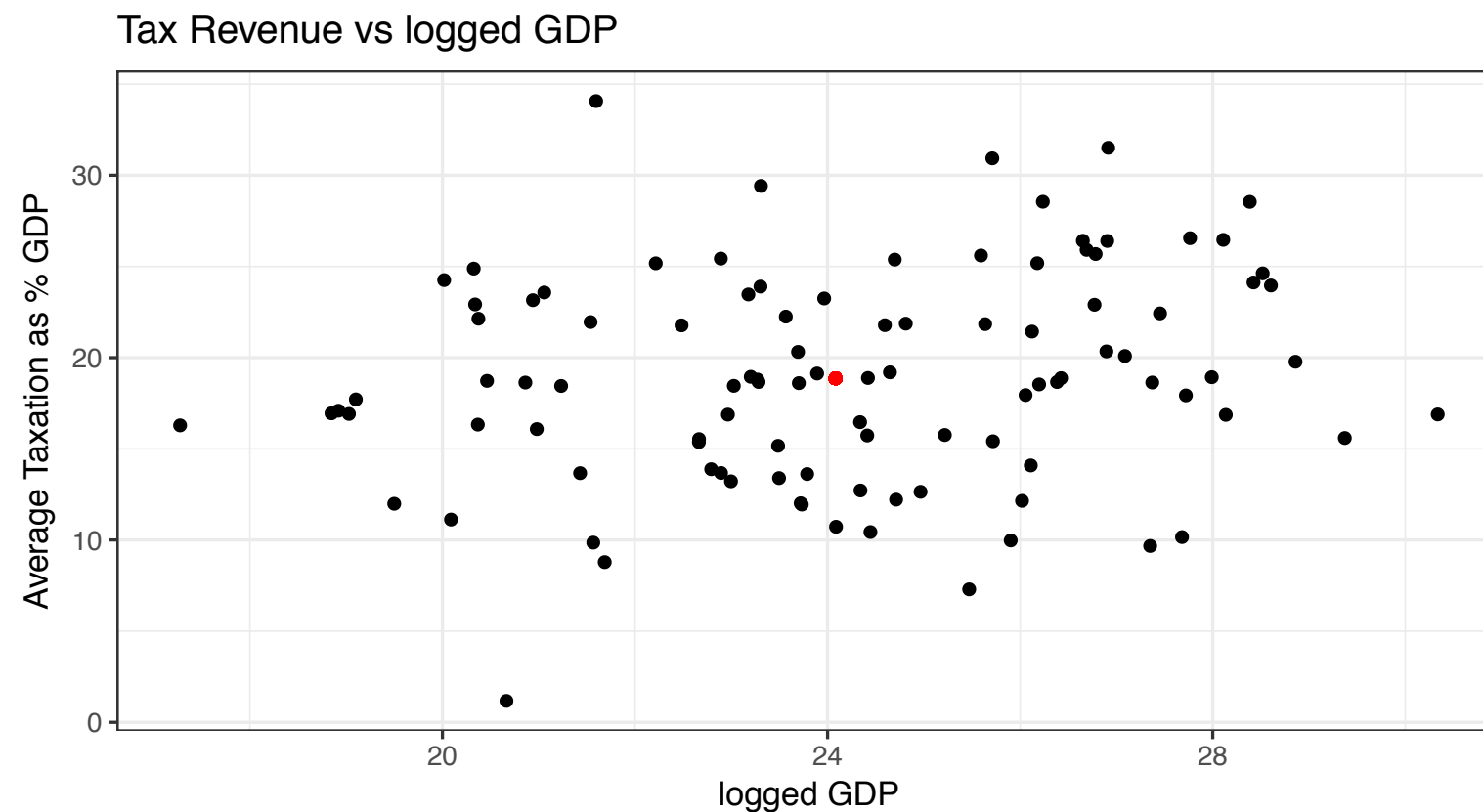


Mean_y = 18.87
Mean_x = 24.09
SD_y = 5.92
SD_x = 2.84
r = 0.19

Slope of regression line: 0.4

What is the prediction for $x = 27.35$????

Example of error calculation

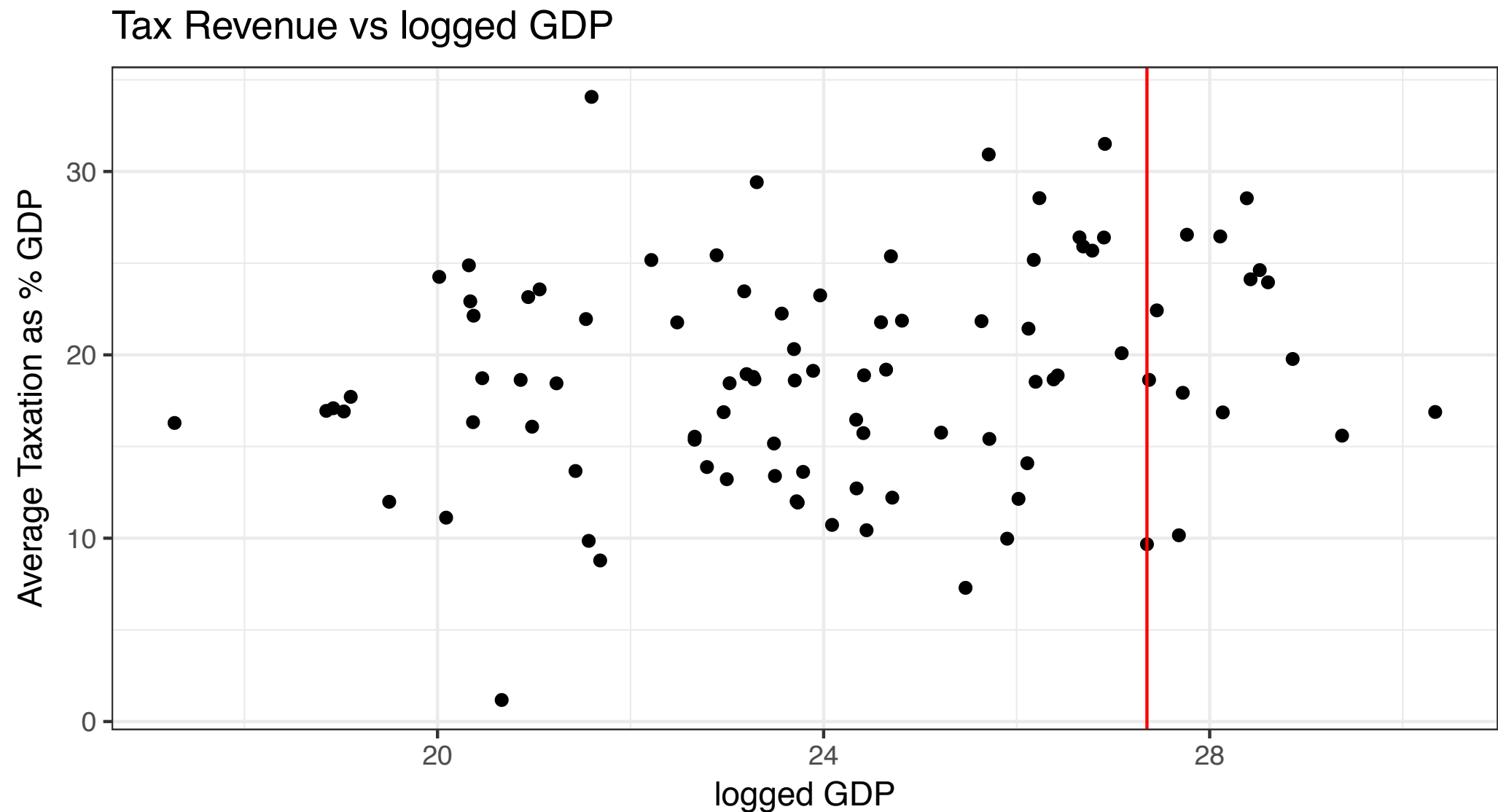


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What is the prediction for $x = 27.35$????

Example of error calculation



What is the prediction for $x = 27.35$???

$y_{\text{pred}} = 20.16$

Actual Y: 9.67

Error = $9.67 - 20.16 = -10.49$

Coach Sumlin asked for a prediction of the number of running plays that the Florida Gators will run on Saturday given that 2 inches of rain are expected. The correlation between rain in inches and number of running plays is 0.6. The average amount of rain in Gainesville is 0.5 inches with a standard deviation of 1. The Florida Gators run 35 running plays on average, with a standard deviation of 8.6. Based on 2 inches of rain, what is your prediction for the number of run plays executed by the Gators on Saturday?

But we had 0 inches of rain. What is the prediction?

32.42 predicted run plays

Actual number of run plays: 42

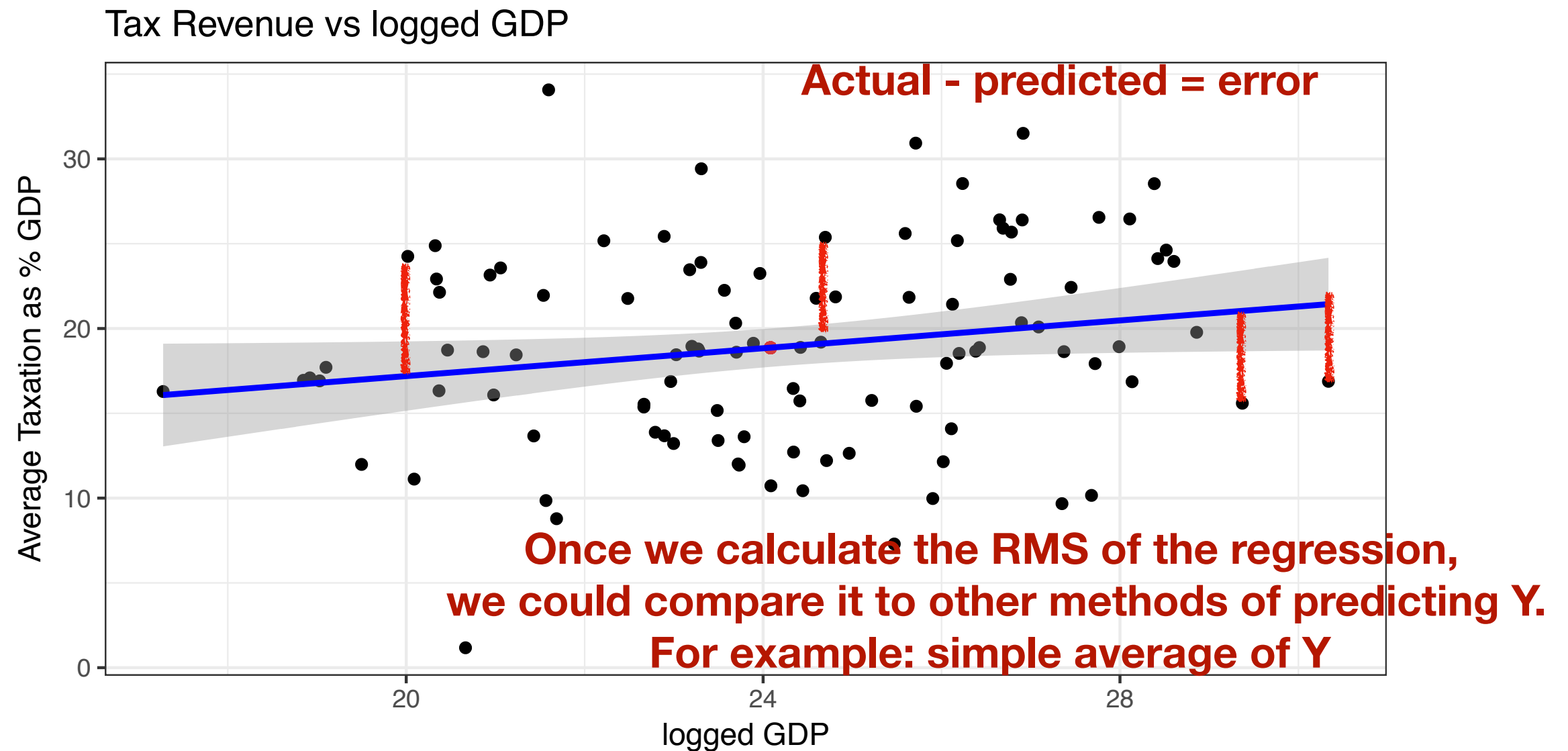
Error: $42 - 32.42 = 9.58$

Recall the root mean squared error

- RMS = square root of the mean of the squared errors
- Approximately equal to the average of how far points are above and below the line
- RMS is always in the unit of the dependent variable (the variable to be predicted - y)
- Why can't we just take the average of the errors?

But regression lines are not perfect

$$\text{RMS} = \sqrt{\text{mean} \left((\text{actual} - \text{predicted})^2 \right)}$$



Recall the root mean squared error

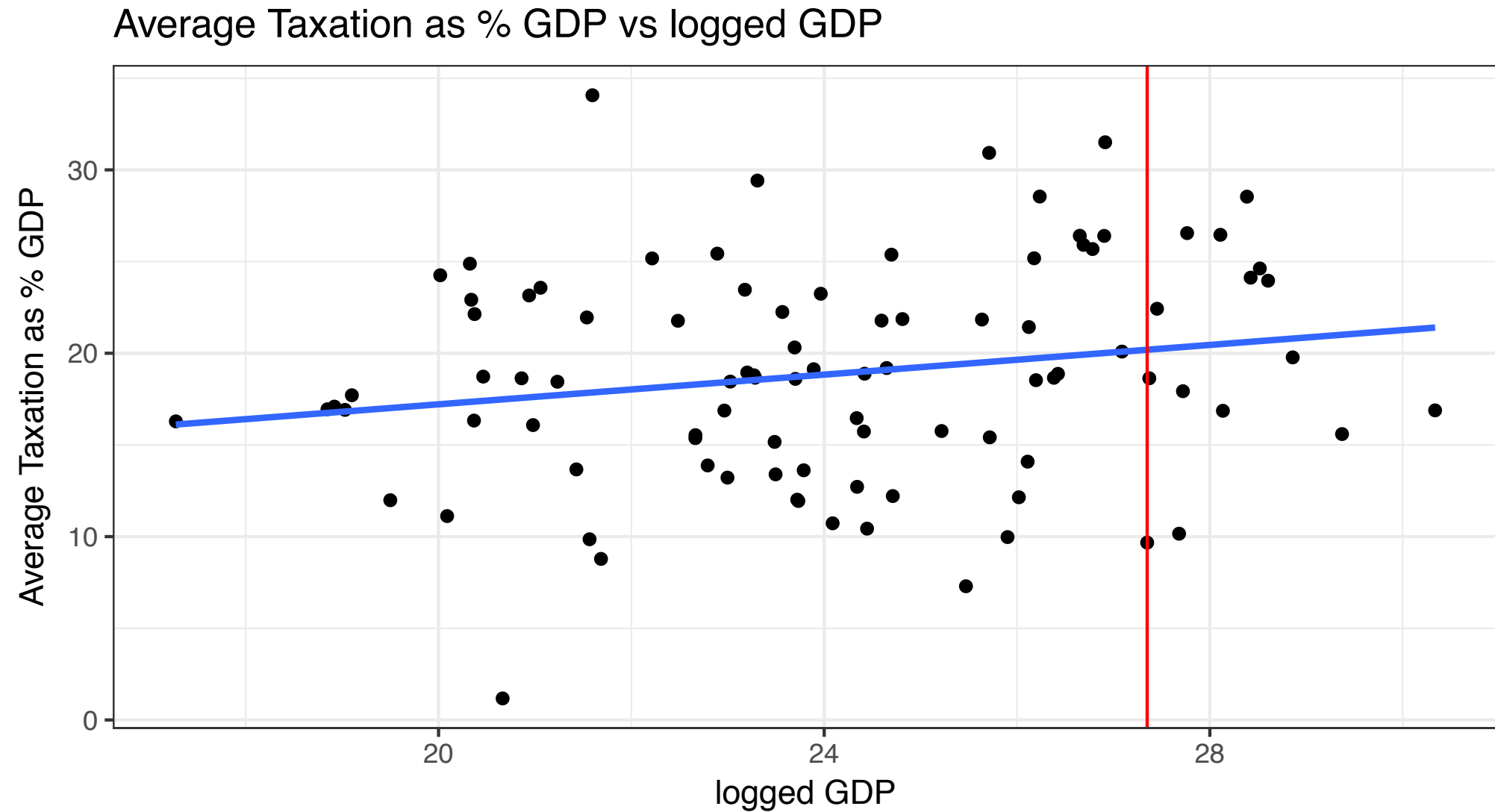
- What is the root mean squared error of using the average of y to predict y ?

Recall the root mean squared error

- What is the root mean squared error of using the average of y to predict y ?
- The standard deviation!

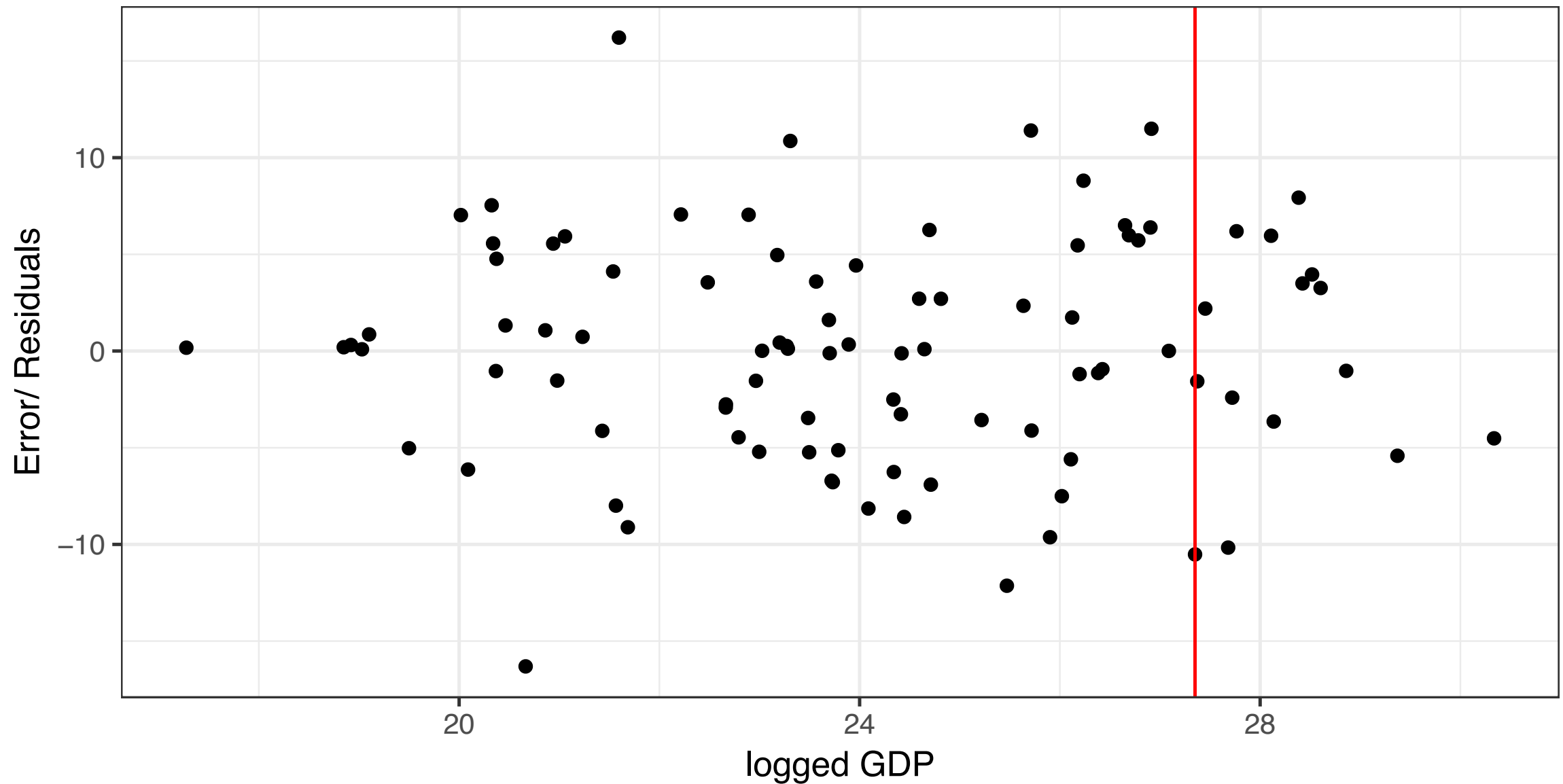
Computing the rms for the regression

- In theory, we could calculate the rms by doing the calculation for every point in our data
- Luckily, we have a formula that makes calculation much simpler: $\text{rms_regression} = \text{SD}_y * \sqrt{1 - r^2}$
- **Again:** rms is in the same units as the dependent variable
- In earlier example, rms would be in tax as % of GDP



Plotting Errors or Residuals

Regression error vs logged GDP



Plotting Errors or Residuals

Often the error is also called the residuals

- We can plot the error/residuals against the x-axis
- The residuals should average out to zero
- Regression line through residuals should be flat
- If residuals look funnel shaped, things are problematic

Homoscedasticity

- Spread around the regression line is similar (the same) along the whole line
- The accuracy of predictions given the regression line should be the same along the whole line
- Football-shaped scatter plot
- If this condition is violated, we say the regression suffers from **heteroscedasticity**

Normal approximation in vertical strips

- What is the new average?
- What is the new SD?
- Everything else stays the same

Exercise

- Law school finds the following relationship btw. LSAT scores and first-year scores:
 - Average LSAT: 162, SD = 6
 - Average first-year score: 68, SD = 10,
 - $R = 0.6$
- A. What is the percentage of students with first-year scores above 75?
- B. Of students who scored 165 on LSAT, what percentage had first-year score greater than 75?

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Exercise

- Correlation in height for 66 boys:
 - Average height at 6, 3 feet and 10 inches, $SD = 1.7$ inches
 - Average height at 18, 5 feet and 10 inches, $SD = 2.5$
 - $R = 0.8$
 - A. RMS for regression predicting height at 18 from height at 6
 - B. RMS for regression predicting height at 6 from height at 18