

# Political Science 209 - Fall 2018

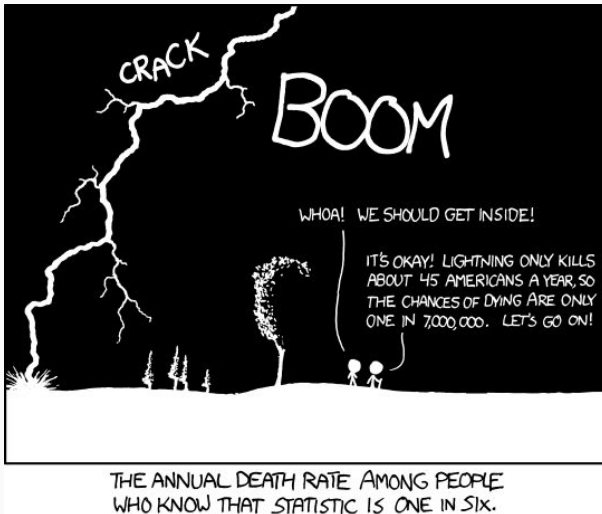
## Probability II

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# Conditional Probability



# Conditional Probability

Sometimes information about one event can help inform us about likelihood of another event

Examples?

# Conditional Probability

Sometimes information about one event can help inform us about likelihood of another event

Examples?

- What is the probability of rolling a 5 and then a 6?
- What is the probability of rolling a 5 and then a 6 given that we rolled a 5 first?

# Conditional Probability

If it is cloudy outside, gives us additional information about likelihood of rain

If we know that one party will win the House, makes it more likely that party will win certain Senate races

# Independence

If the occurrence of one event ( $A$ ) gives us information about likelihood of another event, then the two events are **not** independent.

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If the occurrence of one event ( $A$ ) gives us information about likelihood of another event, then the two events are **not** independent.

**Independence** of two events implies that information about one event does not help us in knowing whether the second event will occur.

For many real world examples, independence does not hold

Knowledge about other events allows us to improve  
guesses/probability calculations



# Independence

When two events are independence, the probability of both happening is equal to the individual probabilities multiplied together

# Conditional Probability

$$P(A \mid B)$$

*Probability of A given/conditional that B has happened*

# Conditional Probability

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

*Probability of A and B happening (joint) divided by probability of B happening (marginal)*

# Conditional Probability

Definitions:

$P(A \text{ and } B)$  - joint probability

$P(A)$  - marginal probability

# Conditional Probability

$$P(\text{rolled 5 then 6}) = ?$$

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$$P(\text{rolled 5 then 6}) = \frac{1}{36}$$

$$P(\text{rolled 5 then 6} \mid 5 \text{ first}) = \frac{P(5 \text{ then } 6)}{P(5)}$$

# Conditional Probability

$$P(\text{rolled 5 then 6}) = ?$$

$$P(\text{rolled 5 then 6}) = \frac{1}{36}$$

$$P(\text{rolled 5 then 6} \mid 5 \text{ first}) = \frac{P(5 \text{ then } 6)}{P(5)}$$

$$\frac{\frac{1}{36}}{\frac{1}{6}} = \frac{1}{6}$$

# Conditional Probability

The probability that it is Friday and that a student is absent is 0.03. What is the probability that student is absent, given that it is Friday?

$$P(\text{absent} \mid \text{Friday}) = ?$$



# Conditional Probability

The probability that it is Friday and that a student is absent is 0.03. What is the probability that student is absent, given that it is Friday?

$$P(\text{absent} \mid \text{Friday}) = ?$$

$$P(\text{absent} \mid \text{Friday}) = \frac{0.03}{0.2} = 0.15$$

# Conditional Probability

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

Also means:

$$P(A \text{ and } B) = P(A \mid B) P(B)$$

If A and B are independent, then

- $P(A \mid B) = P(A)$  &  $P(B \mid A) = P(B)$
- $P(A \text{ and } B) = P(A) \times P(B)$

If  $A|C$  and  $B|C$  are independent, then

- $P(A \text{ and } B \mid C) = P(A \mid C) \times P(B \mid C)$

What is the probability of drawing any card between 2 and 10, or jack, queen, king in any color?

What is the probability of drawing two kings from a full deck of cards?

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$$P(2 \text{ kings}) = \frac{4}{52} \times ?$$

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$$P(2 \text{ kings}) = \frac{4}{52} \times ?$$

$$P(2 \text{ kings}) = \frac{4}{52} \times \frac{3}{51} = \frac{12}{2652} = \frac{1}{221}$$



# Probability Problems

Annual income	Took 209	Took 309	TOTAL
Under \$50,000	30	5	35
\$50,000 to \$100,000	25	6	35
over \$100,000	10	9	19
Total	65	20	

Is the probability of making over \$100,000 and the probability of having taken 309 independent?

# Probability Problems

Annual income	Took 209	Took 309	TOTAL
Under \$50,000	30	5	35
\$50,000 to \$100,000	25	6	35
over \$100,000	10	9	19
Total	65	20	

Is the probability of making over \$100,000 and the probability of having taken 309 independent?

$$P(\text{over \$100k \& 309}) = P(\text{over \$100k}) \times P(309)?$$

# Probability Problems

Annual income	Took 209	Took 309	TOTAL
Under \$50,000	30	5	35
\$50,000 to \$100,000	25	6	35
over \$100,000	10	9	19
Total	65	20	

What is the probability of any student making over \$100,000?

# Probability Problems

Annual income	Took 209	Took 309	TOTAL
Under \$50,000	30	5	35
\$50,000 to \$100,000	25	6	35
over \$100,000	10	9	19
Total	65	20	

What is the probability of a student making over \$100,000, conditional that he took 309?

# Probability Problems

Annual income	Took 209	Took 309	TOTAL
Under \$50,000	30	5	35
\$50,000 to \$100,000	25	6	35
over \$100,000	10	9	19
Total	65	20	

What is the probability of a having taken 309, conditional on making over \$100,000?