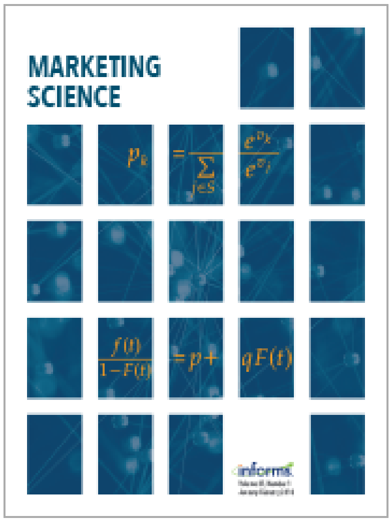

Reward Programs and Tacit Collusion

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Abstract

Reward programs, a promotional tool to develop customer loyalty, offer incentives to consumers on the basis of cumulative purchases of a given product or service from a firm. Reward programs have become increasingly common in many industries. The best-known examples include frequent-flier programs offered by airlines, frequent-guest programs offered by hotels, and frequent-shopper programs offered by supermarkets. Despite the widespread business practice of reward programs, research efforts on reward programs, particularly in marketing, have been scarce. Our paper takes an important step towards understanding the design of reward programs and its implications on pricing strategies.

We study a market that consists of two segments: heavy- and light-user segments. The key distinction between the two segments is that the heavy-user segment purchases in each period and thus is a candidate for the reward programs. In contrast, the light-user segment exits the market after one purchase and is not in a position to exploit reward programs. An important feature of our model is that we allow for different price sensitivity between heavy-user and light-user segments. Our model closely examines the type of rewards. A reward worth a dollar to the consumer might have different cost implications for the offering firm, depending on the type of reward. For example, cash rewards have higher unit reward cost (*inefficient reward*) for the firm than a free product of the firm, such as an airline ticket or long-distance minutes (*efficient reward*). Specifically, we examine an interesting puzzle observed in the marketplace. Several firms offer a cash reward or a product *not* made by the firm, such as jackets, electronic items, etc. These firms could offer their own product as rewards and significantly lower their cost. We examine whether there is any reason for such a seemingly suboptimal practice.

Our analysis shows that reward programs weaken price competition. By offering the incentives for repeat purchases, reward programs increase a firm's cost to attract competing firms' current customers. Because firms gain less from undercutting their prices, equilibrium prices go up. Moreover, as consumers become unwilling to switch because of potential rewards, the firm with a larger market share in the heavy-user segment charges higher prices. Therefore, a low price in the first period, which leads to a larger market share in the heavy-user segment, will always be followed by a high price in the second period. In our model, consumers are rational and can correctly anticipate firms' incentive to offer lower prices initially to enroll them into the reward programs.

Our paper offers an explanation as to why the type and amount of reward may vary across the programs. We identify two determining factors for the selection of rewards: size and relative price sensitivity of the heavy-user segment. We find that in a market with a small heavy-user segment that is also much more price sensitive than the light-user segment, it is optimal for firms to offer the *most inefficient* rewards. The intuition is based on the firms' incentive to exploit the price-insensitive light-user segment. By offering inefficient rewards, firms are able to commit to weaker competition and, therefore, higher prices.

When the heavy-user segment is large or not very price sensitive, when compared to the light-user segment, competing firms should adopt the most efficient rewards to maximize their profit. This may well be the case in a number of real-world situations in which efficient rewards are quite prevalent. We also find that optimal reward amount has a negative relationship with unit reward cost. Because both firms use rewards to attract the heavy users, they tend to offer more when they adopt the more efficient rewards.

Finally, our paper identifies the relationship between market characteristics and the impact of reward programs on firms' profits and consumers' benefits. We find that firms gain from the adoption of reward programs as long as light users are not too price sensitive. When light users are very price sensitive, firms engage in intense price competition, thus benefiting little from the loyalty of heavy users created through rewards. Because reward programs increase market prices, light users, who do not get the reward, earn strictly lower benefit. In contrast, heavy users often stand to gain more from the reward program. In most cases, firms and the heavy users are better off at the expense of light users.
(*Reward Programs; Switching Cost; Price Competition; Game Theory; Loyalty*)

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1. Introduction

Reward programs,[1] a promotional tool to develop customer loyalty, offer incentives to consumers on the basis of cumulative purchases of a given product or service from a firm. The best-known example of such a reward program is the frequent-flier program offered by most of the airlines in the United States. In the past few years, reward programs have become prevalent across a number of diverse industries: cigarette firms such as RJR and Marlboro, credit card companies such as Discover® Card and Visa, telephone companies such as AT&T and MCI, chain stores such as Zellers and Bottom Up, and food manufacturers such as Lean Cuisine® and Pepsi®.

A key purpose of offering reward programs is to create switching cost for consumers (Klemperer 1987a).[2] Switching cost was initially studied by Weizsäcker (1984) and later examined in greater detail by Klemperer and others (for example, Klemperer 1987a and b, Farrel and Shapiro 1988). Although they have studied the impact of switching cost and its implications for competition (see the literature review below for details), they have not examined a key marketing issue in offering reward programs. Specifically, what *type* of reward program should be offered? In the marketplace, we observe a diverse set of offerings, ranging from cash rewards, firm's free products or service, or free products or service of another firm in a different category. For example, airlines offer free tickets, Lean Cuisine® and Discover® Card offer cash rewards, Marlboro and Pepsi® offer leather jackets. As all firms have a range of options to choose from, what type of offer to select becomes an important issue. In this paper, we examine this issue based on a stylized model.

[1]Reward programs are also known as loyalty programs. Both trading stamp and continuity plans are reward programs (Schultz and Robinson 1982). However, other types of promotional tools, such as coupons, cash rebates, and sweepstakes, are not reward programs, because they are redeemed with each purchase and not directly related to the repeat purchase.

[2]In Klemperer's classification (1987a), the switching cost we study here is called artificial switching cost, which arises entirely at firms' discretion.

1.1. A Brief Overview of the Model and the Main Result

We use a game theoretic model to study what type of reward programs firms should select. We differentiate the rewards by their value to the consumers and cost to the firms. We define *unit reward cost* as a firm's cost of offering certain rewards worth one dollar to the consumers. For example, 50 free miles may be worth one dollar to the consumers. The corresponding unit reward cost will then be the firms' marginal cost of producing 50 air miles. From the firms' perspective, we call those rewards with low unit reward cost *efficient rewards* and those with high unit reward cost as *inefficient rewards.* We consider two firms competing for two different segments of customers, namely, heavy- and light-user segments. The key distinction between two segments is that the heavy segment purchases in each period and thus is a candidate for the reward programs. In contrast, the light-user segment exits the market after one purchase. A new cohort of light users enters the market each period. Therefore, this segment does not respond to reward programs.

Our main objective is to show the dependence of reward type and reward amount on market characteristics, in particular, size and relative price sensitivity of the heavy-user segment. We begin the analysis with the impact of reward programs on price competition. First, reward programs create consumers' switching cost and reduce future price competition. When it is more costly for the firms to make consumers switch, firms become less willing to undercut their prices and, therefore, the equilibrium prices go up. Note that the firm with the larger market share in the heavy-user segment is able to charge a higher price, because heavy users face a switching cost in the later period. Therefore, a low price in the first period, which leads to a larger market share in the heavy-user segment, will always be followed by a high price in the second period. In our model, consumers are rational and can correctly anticipate the firms' incentive to offer lower prices initially to enroll them into the reward programs.

The main focus of this paper is on the selection of reward programs, i.e., reward type (what to offer) and reward amount (how much to offer). From a firm's perspective, as noted earlier, cash reward is the most inefficient type because consumers' valuation of the rewards coincides with firm's cost of the rewards. In contrast, free products or the service of the firm may be the most efficient reward, particularly when its marginal cost is low. We find that when the market has a small size of heavy-user segment and it is much more price sensitive than the light-user segment, firms should offer the most inefficient rewards. By offering the most inefficient rewards, firms are able to commit to high price in the first period. Because light users are price insensitive in this case, firms lose little market share with high prices and thus increase the profits from the light-user segment. This is the *benefit of commitment* to high price through inefficient rewards. On the other hand, because heavy users also consider the reward amount when choosing reward programs in which to participate, using inefficient rewards increases the cost to the firms while they are competing for heavy users. Therefore, there is a *cost of commitment* as well. When the light-user segment is dominant, the benefit of commitment to high price dominates the cost of commitment. Therefore, the firm's optimal strategy is to select inefficient rewards.

On the other hand, when the light-user segment is small or it is more price sensitive, the value of commitment to high price through inefficient rewards will be small. In this case, the firms are better off by offering efficient rewards to gain the cost advantage in competing for heavy users. Reward amount has a negative relationship with unit reward cost. Because both firms use rewards to attract the heavy users, they tend to offer more when they adopt the more efficient rewards (smaller unit reward cost).

We also analyze the impact of reward programs on firms' profits and consumers' surplus. With reward programs, heavy users are locked in, and firms primarily compete in the light-user segment. Therefore, adopting reward programs is a profitable strategy unless light users are too price sensitive. Because reward programs help firms reach price collusion, it reduces light users' surplus. Interestingly, it also hurts heavy users when they are extremely price sensitive and firms offer inefficient rewards. Under this scenario, heavy users pay much more with reward programs. At the same time, firms do not hand out enough rewards because the rewards are costly to them.

1.2. Related Literature and Our Contribution

Von Weizsäcker (1984) uses a Hotelling model to show that when consumers randomly change their preferences and firms can commit to the same prices over time, switching cost will increase consumers' price sensitivity and cause the markets to become more competitive. Adding a segment of consumers whose preferences remain the same over time, Klemperer (1987b) shows that consumers' price sensitivity may decrease with switching cost when firms cannot commit to future prices. His explanation is that consumers may rationally expect a price increase after observing a price cut.

Switching cost created by reward programs should be treated as *endogenous decisions,* because firms have direct control over the amount of reward being offered or the amount of switching cost. To the best of our knowledge, there are only two papers studying endogenous reward amount. Banerjee and Summers (1987) investigate a homogeneous market and find that the equilibrium prices coincide with the monopoly prices in both periods. More recently, Caminal and Matutes (1990) study a differentiated duopoly market where consumers randomly change their preferences over time. In both papers, firms *only offer coupon (cash) reward* to consumers who repeat purchase from the same firms. Note that our paper represents the first effort to study the selection of reward types. Moreover, our model is much richer and consistent with marketing literature, because it allows for the different price sensitivities between the heavy users and light users. Furthermore, our paper specifically looks at the issue of firms' credible commitment to higher prices through reward programs and, therefore, also contributes to the literature on tacit price collusion (Salop 1986).

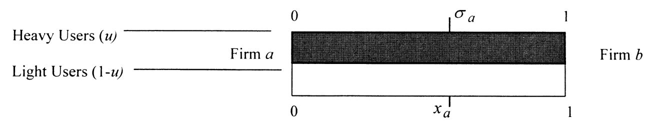
Our paper can also be related to the stream of research that studies the impact of customer loyalty on price promotions. Narasimhan (1988) finds that firms with more loyal customers charge higher average prices. More recently, Raju et al. (1990) find that firms with more customer loyalty have less frequent sales. Although we study the issue in an entirely different context, our results are consistent with theirs. In marketing literature, reward programs are often seen as one type of sales promotional tool (Schultz and Robinson 1984). Whereas some other tools such as coupons (Narasimhan 1984) and trade promotion (Blattberg and Neslin 1990) have been extensively studied, reward programs remain relatively unexplored.

The reminder of the paper is organized as follows: We describe our model in §2. We then analyze price competition and reward decisions in §3. In §4, we discuss the impact of reward programs on firms' profits and consumers' benefits (surplus). We conclude in §5 by identifying avenues for future research efforts.

2. Model

We adopt a two-period model [3] and consider a market that consists of two segments: heavy users and light users. Heavy users consume exactly one unit of product in each period, whereas light users consume one unit of product only in the first period. A new cohort of light users enters the market in the second period, and each purchases one unit in that period.[4] The proportion of heavy users is $u \in (0, 1)$[5] and the proportion of light users is, therefore, $(1 - u)$.[6] Without loss of generality, the size of market is assumed to be one.

Figure 1 Market Segments



To capture consumers' heterogeneous preferences, we use a Hotelling model with horizontal differentiation (Tirole 1990). Consumers of each of the two segments are uniformly distributed in a unit interval with two competing firms (a and b) positioned at the two ends (see Figure 1).[7] Thus, a consumer's net benefit (surplus) by purchasing one unit of product i is $\theta - x_i t - p_i$ if she is a heavy user and $\theta - x_i \alpha t - p_i$ if she is a light user, where x_i is the distance from firm i and p_i is the price she pays to firm i ($i = a$ or b). The base value of the product is denoted by θ, which is the same for the two firms. A consumer's location represents her preference and thus differentiates between the products of the two competing firms. For example, when a consumer is located closer to Firm a, she will buy from Firm a when two firms charge the same price. For Firm b to attract her, it has to charge a price lower than Firm a's price. It is important to note that location affects the evaluation of the firms' products differently for the heavy- and light-user segments. Specifically, for a consumer located at x_i, we let her preference for firm i be $\theta - x_i t$ if she is a heavy user and $\theta - x_i \alpha t$ if she is a light user. Therefore, t and αt indicate the importance (unimportance) of preference (price) in the heavy-users' and light-users' choice decisions, respectively. They can also be interpreted as relative measures of consumers' price *insensitivity.* Because firm i's market share in the standard Hotelling model equals ½ + (p_j

[3]Reward programs often have expiration dates. Therefore, the potential rewards are perishable. This can be one of the empirical justifications for the use of a finite-horizon model. We thank Hasan Pirkul for the comment.

[4]We assume that light users of one period incur sufficiently high transaction costs of purchase in any period other than their entry period. Such high transaction costs are common in the industries in which consumer reward programs are used as prominent promotional tools. For example, light users incur high costs when they take unnecessary flights to receive extra mileage for frequent-flier programs, overstay in hotels simply to obtain points for frequent-guest programs, or eat meals at a particular restaurant just to earn points for frequent-dining programs. Such high transaction cost is the underpinning of effective price discrimination in these markets. In the rest of paper, light users are assumed to face sufficiently high transaction cost to preclude purchases in time periods other than the one they intend to buy.

[5]Both segments must exist for our analytical results to hold. This modeling assumption reflects heterogeneous consumer valuation for extra units of purchases that are widely observed in the market.

[6]The importance of the heavy-user segment was studied in marketing by Twedt (1964). He showed that 20% of consumers account for about 80% of total purchases. This "80/20" law was also studied by Schmittlein et al. (1993).

[7]There are more general models that locate firms inside the interval. Because we do not analyze firms' positioning decisions, we locate firms at the two ends to avoid possible technical difficulties caused by nonexistence of equilibrium in the original Hotelling model (D'Aspremont et al. 1979).

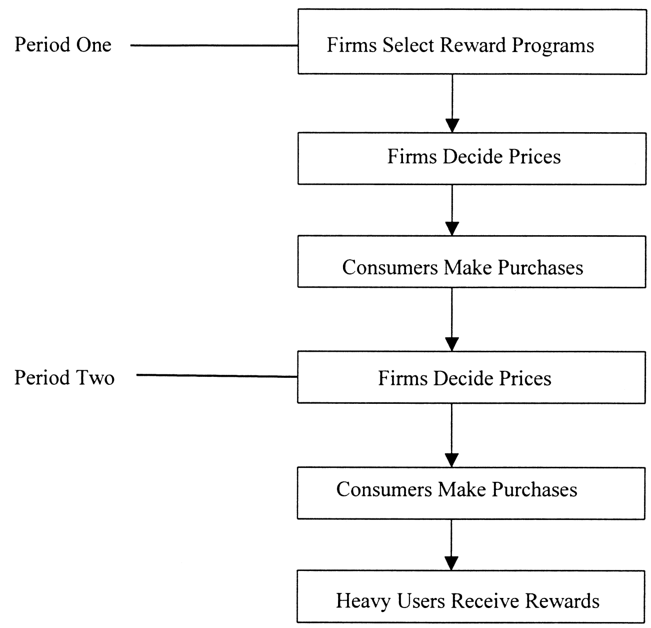
$- p_i)/(2t)$, we denote the heavy-users' and light-users' price sensitivity as $1/(2t)$ and $1/(2\alpha t)$, respectively, for the remainder of the paper.

Firms are assumed to have the same variable production cost, which is normalized to zero for analytical simplicity.[8] Both firms reward those consumers who repeat purchase from them. We define *unit reward* as one dollar's worth of free-good offering to such repeat purchase consumers. Thus, a unit reward can be 50 frequent-flier miles valued by consumers to be worth one dollar or can be an offering of one dollar as a cash reward. On the firms' side, we define *unit reward cost* (c_i) as firm *i*'s cost of offering one unit reward. Thus, it costs $c_i r_i$ for firm i to offer r_i units of reward. Service industries such as telephone companies, airline firms, and hotels, often have high fixed costs, low variable costs, and excess capacities. Hence, the marginal cost of these products or services may be low. Therefore, the cost of rewards such as free tickets or free hotel accommodations is small. In contrast, cash rewards have high unit reward cost, because the value to the consumers is the same as the cost to the firms. Firms may also offer products of other manufacturers as free goods. For example, cigarette manufacturers offer sportswear and jackets as free goods to loyal smokers, based on the volume of purchase over time. The cost of such a reward is likely to lie between the cost of offering own-product (cigarettes) and cash reward. From a strict cost and benefit analysis, we consider the offer of the firms' own products or services as the *most efficient reward* and treat cash as the *most inefficient reward* offer.

We denote the range of rewards available to both firms by a closed interval of unit reward cost $[\underline{c}, \bar{c}]$, in which $\underline{c} > 0$. Because light users purchase the product in only one of the two periods, firms have no incentive to offer them rewards to build loyalty. Thus, the reward offer is relevant only for the heavy-user segment.

Both firms and consumers are assumed to have rational expectations and maximize their total profits or surplus over the entire horizon. For analytical simplicity, we allow the discount factor to be one. *We assume that consumers' reservation prices are sufficiently high and exceed the finite equilibrium prices.* Thus, complete market coverage (all customers buy) is ensured for both segments in both periods. The consumers' decision, therefore, is not whether or not to buy but rather from which firm to buy. Relaxing this assumption will affect our analytical results but not the qualitative implications. Further parametric restrictions necessary to guarantee equilibrium existence will be discussed later in the paper.

Figure 2 **Game Structure**



The structure of our model (given in Figure 2) is as follows: At the beginning of Period 1, both firms announce their reward programs. Specifically, they announce the reward amount (r_a and r_b) and reward type (c_a and c_b). The firms then decide their first-period prices (p_1^a and p_1^b). After observing reward programs and first-period prices, each consumer purchases one unit of product or service from one of the firms. At the end of the first period, all current light users leave and the same number of new light users enter the market. In the second period, after firms announce their prices (p_2^a and p_2^b), consumers make their purchases. Those heavy users who have purchased from the same firms in both periods receive

[8]Assuming variable cost equal to zero does not change our qualitative results. With positive variable cost, the equilibrium prices will be equal to the equilibrium margin plus the variable cost.

Table 1 Notations

c_i: Firm i's unit reward cost ($i = a, b$). (Feasible set $[\underline{c}, \overline{c}]$)
$r_i(r_j, c_i, c_j)$: Firm i's optimal reward amount when firm j offers r_j and their respective unit reward costs are c_i and c_j ($i, j = a, b$ and $i \neq j$).
r_i: Firm i's reward amount offered to its heavy users ($i = a, b$).
p_t^i: Firm i's price at period t ($i = a, b, t = 1, 2$).
p°: Equilibrium price without reward programs.
$1/(2t)$: Heavy users' price sensitivity.
$1/(2\alpha t)$: Light users' price sensitivity.
u: Size of heavy-user segment ($0 < u < 1$, see Appendix 5 for more restriction).
$\sigma_{t,k}^i$: Firm i's market share in segment k at period t ($i = a, b, t = 1, 2, k = H, L$).
σ_{Hi}: Firm i's market share in heavy-user segment ($i = a, b$).
π_t^i: Firm $i\times$'s profit at period t ($i = a, b, t = 1, 2$).
π°: Equilibrium profit without reward program.
$\pi_i(r_i, c_i, r_j, c_j)$: Firm i's profit by offering reward program (r_i, c_i) while the competitor offers reward program (r_j, c_j) ($i, j = a, b$ and $i \neq j$).

rewards. The competing firms always announce their decisions simultaneously. The sequence of moves, first determining reward programs and then prices, is assumed because reward programs are changed less frequently than short-term prices.

In the paper, we study only the pure-strategy Nash equilibrium. We derive the equilibrium by backward induction (Fudenburg and Tirole 1992). Because the two firms are the same in all aspects, we limit our attention to a pure-strategy symmetric equilibrium $(c^{\bullet}, r^{\bullet}, p_1^{\bullet}, p_2^{\bullet})$. All the notations used in this paper are summarized in Table 1 for easy reference.

3. Model Analysis

3.1. Equilibrium Without Reward Programs

As a benchmark, we briefly analyze the price competition without reward programs. In the Hotelling model with horizontal differentiation, there is a marginal consumer located at point x_i who is indifferent as to buying from Firm a or b. All consumers located to the left of this point buy from Firm a and those located to the right of this point buy from Firm b. Therefore, Firm a's market share is determined by $\theta - \sigma_{a,H}t - p_a = \theta - \sigma_{b,h}t - p_b$ in the heavy-user segment and by $\theta - \sigma_{a,L}\alpha t - p_a = \theta - \sigma_{b,L}\alpha t - p_b$ in the light-user segment. With the variable cost set equal to zero, Firm a's profit function is $\pi^a = p_a(u\sigma_{a,H} + (1 - u)\sigma_{a,L})$. Both firms simultaneously maximize their profits with respect to their own prices. We can solve the equilibrium prices for either firm to be:

$$p^\circ = \frac{\alpha t}{\alpha u + (1 - u)} \tag{1}$$

and the profit to be

$$\pi^\circ = \frac{1}{2} \cdot \frac{\alpha t}{\alpha u + (1 - u)}. \tag{2}$$

3.2. Equilibrium with Reward Programs: Price Competition in Period 2

In this section, we analyze the impact of reward programs on price competition. We deploy the standard method of backward induction in our analysis. Therefore, we begin by analyzing the price competition in Period 2 and then move backward to Period 1. In the second period, each heavy user has already made a purchase in the first period. When they continue to buy from the same firm, they receive rewards. However, when they switch to the other firm, they cannot collect a reward from either firm. Consequently, potential loss of the reward becomes the cost of switching for heavy users in the second period. Both firms set their prices to maximize their second-period profits. Each consumer, whether a heavy or light user, buys one unit. Because light users do not incur any switching cost, firms' market shares in the light-user segment depend only on their second-period prices. However, firms' market shares in the heavy-user segment are conditional on their first-period market shares and rewards as well.

We use a three-step procedure to characterize the second-period equilibrium. First, we prove that in any pure-strategy equilibrium both firms sell only to their "own heavy users." (See Appendix 1 for technical details.) Therefore, firms' equilibrium market shares for the heavy-user segment remain the same over the two periods. Second, we then derive the candidate equilibrium from local profit functions where heavy users do not switch. Finally, to ensure that the candidate equilibrium derived in the second step is a Nash equilibrium, in the third step we derive a set of sufficient conditions under which neither firm has the incentive to unilaterally deviate. (See Appendix 2 for technical details.)

The intuition behind the nonswitching result obtained in the first step is rather straightforward. In the second period, a firm can either charge a high price to exploit its customers' switching costs or a much lower price to target its competitor's ''loyal segment.'' A firm prefers to compete for more heavy users only if its first-period market share in the heavy segment is sufficiently small. Because rewards create switching costs for the heavy users, it is much more costly for the firm to attract new heavy users in the second period than in the first period. Therefore, firms are better off by competing for the heavy users in the first period. As a result, neither firm targets its competing firm's ''loyal segment'' in the second-period equilibrium. Rather, they exploit the established ''switching costs.''

The above result provides for any pure-strategy equilibrium with a necessary condition: Equilibrium market shares in the heavy-user segment do not vary across two periods. It thus simplifies our analysis in the sense that the market shares of the heavy-user segment can be treated as invariant across two periods while determining the prices of both periods and the reward structure. We derive the nonswitching second-period equilibrium prices with the following problem (P1).

$$\text{(P1)} \quad \max_{p_2^i} \pi_2^i = (1-u)p_2^i\sigma_{2,L}^i + u(p_2^i - c_i r_i)\sigma_{2,H}^i$$

where

$$\sigma_{2,L}^i = \frac{1}{2}\frac{p_2^j - p_2^i}{2\alpha t} \quad \text{and}$$

$$\sigma_{2,H}^i = \sigma_{1,H}^i \qquad (i, j = a \text{ or } b,\ i \neq j),$$

where $\sigma_{t,k}^i$ is firm i's ($i = a, b$) market share in segment k ($k = H, L$) at time period t ($t = 1, 2$). (Note that the formulation depends on nonswitching in the second period and, therefore a necessary condition is that the reward amount is strictly positive.) Maximizing each firm's profit with respect to their second-period prices, we obtain the following equilibrium prices. (See Appendix 2 for derivation.)

$$p_2^i = \alpha t + \frac{2u}{3(1-u)}(1 + \sigma_{1,H}^i)\alpha t \qquad (i = a \text{ or } b). \quad (3)$$

The above equation shows that *firm i's second-period price increases with its first-period market share in the heavy-user segment* ($\sigma_{1,H}^i$). Note that the heavy-user market share is determined in the first period as a function of first-period prices, reward amounts, and reward cost types. (This can easily be seen from Equation (5), developed later.) Thus, intuitively, the firms compete for the heavy users in the first period through a combination of prices and reward programs. Equation (3) shows that any gain in market share for a given firm in the first period is attractive because it results in a higher second-period price. Thus, when a firm is able to attract more consumers to participate in its reward program, it obtains more ''loyal customers'' and, hence, is able to charge a higher price later on.

Because our equilibrium is symmetric, two competing firms equally share the market. That is, $\sigma_{1,H}^{a*} = \sigma_{1,H}^{b*} = ½$. (While we impose symmetry to find the equilibrium prices of this sub-game, it is important to note that as we proceed to analyze the first-period prices, we use Equation 3 and not the prices obtained from the imposition of symmetry.) According to (3), the second-period, equilibrium price should be $p_2^* = \alpha t/(1-u)$.[9] Comparing this with the price without reward programs given by (1), we find that both firms' prices increase with reward programs. The potential loss of the reward acts as a deterrent for heavy users to switch between firms in the second period. Firms, realizing such a switching cost, are able to charge higher prices. We also find that *the price difference due to the reward program increases with the size of heavy-user segment u.* However, *the price difference decreases with light users' price sensitivity (1/(2αt)).* As heavy users are locked in with their switching costs, firms mainly compete for light users. When a firm increases its price to exploit its heavy users, it risks losing its market share in the light-user segment. This

[9]The possible conjecture that reward amount does not affect equilibrium is erroneous. An analogy will clarify this point. When the market is fully covered, symmetric equilibrium market shares are 0.5 and 0.5. Clearly, market shares are a function of prices charged.

risk becomes smaller when light users are less price sensitive. As a result, market prices go up.

The above discussion leads to our first proposition.

PROPOSITION 1.

1.1. *The second-period market price increases with reward programs.*

1.2. *In the second period, the increase in price with reward programs is positively related to the size of heavy-user segment and negatively related to light users' price sensitivity.*

Proposition 1 indicates that firms reach tacit price collusion when heavy users have switching cost. Similar findings are reported by Banerjee and Summer (1987) and Klemperer (1987b). With reward programs, it becomes more costly for firms to target their competitors' ''loyal segment,'' thus reducing the firms' incentive to lower prices.

Interestingly, reward value and cost do not enter the price function (3). Because firms' market shares in the heavy-user segment do not change over time, reward costs become sunk costs in the second period. Once a firm has developed switching cost for its heavy users, such a cost provides a more captive market, enabling the firm to charge a higher price in the future. Therefore, any attempt to target its competitor's loyal segment through lower prices unnecessarily subsidizes its own loyal segment. In essence, switching costs deter firms from competing for new customers. Implicitly, firms commit to future higher prices by building the switching cost.

As discussed earlier, the first-order solution of Problem (P1) given by Equation (3) satisfies the necessary conditions of the equilibrium. However, first-order conditions may not be sufficient to define a Nash equilibrium. A firm may unilaterally deviate to a much lower price to obtain more heavy users. To ensure that prices defined by Equation (3) are equilibrium solutions, in Appendix 2 we develop a set of sufficient conditions (A2.4 and A2.5.1–A2.6.2), under which neither firm can be better off from unilateral deviation. (See Appendix 2 for more technical details.) We will further discuss parametric restrictions imposed by these conditions later in the paper.

3.3. Equilibrium with Reward Programs: Price Competition in Period 1

Because light users will not buy again in the second period, they buy from the firm that maximizes their current surplus in Period 1. As a result, firms' first-period market shares in the light-user segment depend only on current prices. That is, $\sigma^i_{1,L} = ½ + (p^j_1 - p^i_1)/(2\alpha t)$ with $i, j = a$ or b, $i \neq j$. Heavy users, who will buy in the next period as well, aim to maximize their total surplus from both periods. Therefore, they consider not only the impact of purchase decisions on their current surplus but also the *possible* impact on their second-period surplus while facing a switching cost. In the previous section we showed that heavy users do not switch between two firms over time. Because heavy users can correctly anticipate the equilibrium outcome in the next period, they choose to participate in one of the reward programs that offers higher surplus. Firms' market shares in the heavy-user segment, therefore, are determined by:

$$\theta - (p^a_1 + t\sigma^a_{1,H}) + \theta - (p^a_2 + t\sigma^a_{1,H} - r_a)$$
$$= \theta - (p^b_1 + t\sigma^b_{1,H}) + \theta - (p^b_2 + t\sigma^b_{1,H} - r_b). \quad (4)$$

Each side of Equation (4) represents the surplus of one reward program. Substituting the second-period equilibrium prices given in (3), we can derive firms' market shares in the heavy-user segment as

$$\sigma^i_{1,H} = \frac{1}{2} + \frac{1}{2\beta}[(p^j_1 - p^i_1) + (r_i - r_j)]$$
$$(i, j = a \text{ or } b,\ i \neq j), \quad (5)$$

where

$$\beta = \frac{3 - (3 - \alpha)u}{3(1 - u)} 2t > 2t.$$

Recall that in our model consumers are forward looking. Therefore, they realize that firms have an incentive to increase prices in the second period because of switching costs. Thus, our analysis is a rational expectation equilibrium. Comparing with the benchmark model of no reward program, we find that the reward programs have changed heavy users' price sensitivity from $1/(2t)$ to $1/(2\beta)$. Because $1/(2\beta)$ is smaller than $1/(2t)$, we conclude that *reward pro-*

grams reduce heavy users' first-period price sensitivity as well. Heavy users' *revised price sensitivity* decreases with the size of the heavy-user segment (u) and increases with light users' price sensitivity $1/(2\alpha t)$). After carefully examining the algebra, the intuition turns out to be quite simple but interesting. According to Proposition 1, firms with larger market shares in the heavy-user segment charge higher prices in the second period. Because a firm can increase its market share in the first period with a lower price, there is a negative correlation between the prices in two different periods. That is, *when a firm charges a lower price in the first period, it will take a larger market share of heavy users and then charge a higher price in the second period.* Heavy users, who are able to correctly anticipate this relation, are not "fooled" by low first-period prices. This reduces the incentive for the firms to reduce prices drastically in the first period to achieve higher market share.

Given the demand functions in both consumer segments, the competing firms set their prices to maximize their total profits. The problem for the optimal first-period price is given by Program (P2).

$$\text{(P2)} \qquad \max_{p_1^i} \pi^i = \pi_1^i + \pi_2^i$$

where

$$\pi_1^i = (1-u)p_1^i\left(\frac{1}{2} + \frac{p_1^j - p_1^i}{2\alpha t}\right) + up_1^i\sigma_{1,H}^i,$$

$$\pi_2^i = \frac{1-u}{2\alpha t}\left[\alpha t + \frac{2u}{3(1-u)}(1+\sigma_{1,H}^i)\alpha t\right]^2 - uc_ir_i\sigma_{1,H}^i,$$

$$\sigma_{1,H}^i = \frac{1}{2} + \frac{1}{2\beta}[(p_1^j - p_1^i) + (r_i - r_j)]$$

$$(i, j) = a \text{ or } b,\ i \neq j).$$

Note that in the above program, we have substituted the second-period optimal prices in the profit function, and hence, the maximization problem is a function of first-period prices alone. The second-period equilibrium profit function has been obtained in (A2.3), as shown in Appendix 2. Firms decide their first-period equilibrium prices with a rational expectation of a nonswitching subgame equilibrium in the subsequent period. With each firm's profit function concave with respect to its price, we derive the following closed-form solutions for first-period equilibrium prices (p_1^a, p_1^b) by simultaneously maximizing both firms' total profits with respect to their own prices.

$$p_1^i = \frac{1}{2}\left[\frac{\frac{u}{2\beta}}{\frac{u}{2\beta} + \frac{1-u}{2\alpha t}}(c_ir_i + c_jr_j) + \frac{\left(\frac{u}{\beta} - \frac{2\alpha u^2 t}{9\beta^2(1-u)}\right)(r_i - r_j) + \frac{u}{2\beta}(c_ir_i - c_jr_j)}{3\left(\frac{u}{2\beta} + \frac{1-u}{2\alpha t}\right) - \frac{2\alpha u^2 t}{9\beta^2(1-u)}} + \frac{\frac{2t}{\beta}}{\frac{u}{2\beta} + \frac{1-u}{2\alpha t}}\right] \tag{6}$$

where $i, j = a$ or b, $i \neq j$. (See Appendix 2 for more details.) Note that Equation (6) is derived assuming the existence of a pure-strategy equilibrium. Constraints A2.4 and A2.5.1–A2.6.2 are the sufficient conditions for the existence. We will discuss the joint implications of these constraints and Equation (6) on parametric restrictions at the end of §3.

It is important to note that prices are decided after reward programs. Thus, the Maximization Problem (P2), hence its optimal Solution (6), is conditional on both firms' reward decisions ((r_i, c_i), $i = a, b$). Exploiting the symmetric property, we find the first-period prices in symmetric equilibrium to be

$$p_1^i = \frac{\left(\frac{u}{2\beta}cr + \frac{t}{\beta}\right)}{\left(\frac{u}{2\beta} + \frac{1-u}{2\alpha t}\right)} \qquad (i, j = a \text{ or } b,\ i \neq j).$$

Therefore, equilibrium price is a linear function of reward cost (rc). Because firms offer rewards to their heavy users, they incur reward cost (cr) with each heavy user. Light users, who are charged the same prices as the heavy users, have to share the cost of rewards. Recall that $1/(2\alpha t)$ and $1/(2\beta)$ measure light- and heavy-user price sensitivity. Therefore, the coefficient of cost (cr) increases with size and price sensitivity of heavy users but decreases with size and price sensitivity of light users. Because firms give rewards only to the heavy users, their total reward cost is equal to $(ucr)/2$ in the symmetric case. Hence, their overall profits are equal to $(p_1 + p_2 - ucr)/2$. With Equation (6), we find that the total profit will increase with (cr) if $1/(2\beta) > 1/(2\alpha t)$. That is, a firm's equilibrium profit increases with the cost of rewards when the heavy-user segment is more price sensitive than the light-user segment. Another interesting result is that the intercept of Price Equation (6) proves to be larger than the benchmark price at (1). Therefore, we can conclude that *reward programs increase the first-period prices too.*

PROPOSITION 2.

2.1. *Reward programs decrease heavy users' first-period price sensitivity and increase the first-period market price. Impact of reward programs depends on the size of the heavy-user segment and the difference in price sensitivity between two segments. The larger the heavy-user segment or the more price sensitive it is, the more effective the reward programs will be in reducing heavy users' first-period price sensitivity.*

2.2. *The impact of reward cost on first-period price is positively related to the size of the heavy-user segment and heavy users' relative price sensitivity. When heavy users are more price sensitive than the light users in the first period, firms' equilibrium profits increase with reward costs.*

It must be noted that a higher first-period price need not necessarily be bad for the heavy users. In contrast, the light users are strictly worse off. Firms use the reward programs to discriminate between the light- and heavy-user segments. A higher price in the first period may well be an outcome of a higher reward and in that sense, the heavy-user segment need not be worse off with a higher price in Period 1.

3.4. Reward Decision: Reward Amount and Reward Type

We now analyze firms' decisions on reward programs, i.e., type and amount of reward to be offered. The firms decide reward type and reward amount *simultaneously* as two components of a reward program. The program for optimal reward decisions can therefore be formulated as given by program (P3).

$$\text{(P3)} \qquad \max_{r_i, c_i} \pi^i = \pi_1^i + \pi_2^i$$

where

$$\pi_1^i = (1-u)p_1^i\left(\frac{1}{2} + \frac{p_1^j - p_1^i}{2\alpha t}\right) + up_1^i\sigma_{1,H}^i,$$

$$\pi_2^i = \frac{1-u}{2\alpha t}\left[\alpha t + \frac{2u}{3(1-u)}(1+\sigma_{1,H}^i)\alpha t\right]^2 - uc_i r_i \sigma_{1,H}^i,$$

p_1^i and p_1^j given by Equation (6), $\sigma_{1,H}^i$ given by Equation (5).

In the above program, we substitute the optimal first-period prices given by Equation (6). Again, note that these prices are general expressions without imposition of symmetry. Firms make their reward decisions with rational expectations on subsequent price competition. Because firms compete with pure strategies, they anticipate only the pure-strategy subgame equilibrium. As shown in Appendix 1, such pure-strategy subgame equilibria are always nonswitching. Thus, as in Program (P2), sufficient conditions derived in Appendix 2 to ensure existence of pure-strategy equilibrium apply to Program (P3) as well. We will discuss parameter restrictions resulting from these sufficient conditions at the end of this section. Next, we obtain the first-order conditions for the reward amount and reward type. We solve these first-order conditions simultaneously. Note that this is the final stage of the backward induction and represents the analysis of the entire game, not a subgame. Therefore, we can impose symmetry on the first-order conditions while simultaneously solving for the two elements of the reward program, namely, amount and type.

To solve the equilibrium reward amount, we take the first-order approach (see Appendix 3 for derivation). Both firms' first-order conditions are linear with respect to reward amount. Therefore, we can derive the reaction function $r_i(c_i; r_j, c_j)$, which is firm i's optimal reward amount when firm j offers r_j and their respective unit reward costs are c_i and c_j ($i, j = a, b$

and $i \neq j$). Given reward types offered by both firms, we can solve the equilibrium reward amount from these two linear first-order conditions. In the symmetric equilibrium we have $c_a = c_b = c^*$, and both firms equally share the market. Then the equilibrium reward amount will be:

$$r_i^*(c^*) = f(u, \alpha)/c^* + g(u, \alpha), \tag{7}$$

where

$$f(u, \alpha) = \frac{4\alpha t}{3(1-u)} + \frac{\alpha^2 ut}{3(1-u)} \times \frac{6(1-u) + 3\alpha u}{3(6 - 6u + 5\alpha u)(3 - 3u + \alpha u) - 2u^2\alpha^2} \cdot t, \tag{8}$$

$$g(u, \alpha) = \frac{\alpha - 6(1-u) - 5\alpha u}{3(1-u)} t - (2u^2\alpha^3) \div \{6(1-u)[3(6 - 6u + 5\alpha u)(3 - 3u + \alpha u) - 2u^2\alpha^2]\} t. \tag{9}$$

We derive the equilibrium reward amount through the first-order conditions with the profit functions concave with respect to reward amount, but the profit functions prove to be convex with respect to unit reward cost. We verify this with the positive second derivatives, also as shown in Appendix 3. Recall that each firm can select a unit reward cost only from the closed interval $[\underline{c}, \bar{c}]$. With the profit function convex, when there exists a pure-strategy equilibrium, unit reward cost is *necessary* to be either the upper bound $\bar{c}$ or lower bound $\underline{c}$. In the symmetric equilibrium where two firms equally share the market, we calculate the first derivative

$$\frac{\partial \pi^i}{\partial c_i} = \frac{\frac{u}{2\beta} \frac{1-u}{2\alpha t}}{\frac{u}{2\beta} + \frac{1-u}{2\alpha t}} \cdot g(u, a) r^*,$$

where $g(u, \alpha)$ is given by (9). Clearly the first derivative has the same sign as $g(u, \alpha)$. Therefore, when $g(u, \alpha)$ is negative, the only candidate equilibrium is to offer the most efficient rewards. On the other hand, when $g(u, \alpha)$ is positive, the only candidate equilibrium is to offer the most inefficient rewards.

For this unique candidate equilibrium to be sustained as a Nash equilibrium, it is *sufficient* to show that neither firm has an incentive to deviate to the opposite end of the interval $[\underline{c}, \bar{c}]$. We denote $\pi_i(r_i, c_i, r_j, c_j)$ as firm i's profit by offering reward program (r_i, c_i) while the competitor offers reward program (r_j, c_j). When $g(u, \alpha) < 0$, both firms offer efficient reward $\underline{c}$ and amount $r^*(\underline{c})$ with profit $\pi_i(r^*(\underline{c}), \underline{c})$. When firm i unilaterally switches to inefficient reward $\bar{c}$, its optimal reward amount will be $r_i(\bar{c}; r^*(\underline{c}), \underline{c})$. Then its corresponding profit, which is the maximum profit it can achieve from switching, is $\pi_i(r_i(\bar{c}; r^*(\underline{c}), \underline{c}), \bar{c}; r^*(\underline{c}), \underline{c})$. Therefore, the sufficient condition for $\underline{c}$ to be an equilibrium is $\pi_i(r^*(\underline{c}), \underline{c}) \geq \pi_i(r_i(\bar{c}; r^*(\underline{c}), \underline{c}), \bar{c}; r^*(\underline{c}), \underline{c})$. That is, when one firm offers efficient reward, it is better for another firm also to offer efficient reward, rather than unilaterally switching to inefficient reward. Similarly, when $g(u, \alpha) > 0$, for both firms offering inefficient reward $\bar{c}$ to be an equilibrium, it is sufficient to have profit $\pi_i(r^*(\bar{c}), \bar{c})$ larger than the maximum profit from unilaterally switching to efficient reward $\pi_i(r_i(\underline{c}; r^*(\bar{c}), \bar{c}), \underline{c}; r^*(\bar{c}), \underline{c})$.

Formally, we state the above result in the following proposition.

Proposition 3. *Equilibrium reward type depends on the value of $g(u, \alpha)$ defined in Equation (9) and optimal reward function $r_i(c_i; r_j, c_j)$. More specifically,*

3.1. *If $g(u, \alpha) > 0$ and $\pi_i(r^*(\bar{c}), \bar{c}) \geq \pi_i(r_i(\underline{c}; r^*(\bar{c}), \bar{c}), \underline{c}; r^*(\bar{c}), \bar{c})$, then equilibrium $c^* = \bar{c}$;*

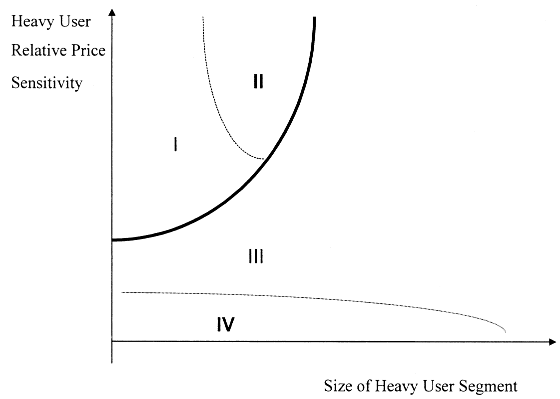
3.2. *If $g(u, \alpha) < 0$ and $\pi_i(r^*(\underline{c}), \underline{c}) \geq \pi_i(r_i(\bar{c}; r^*(\underline{c}), \underline{c}), \bar{c}; r^*(\underline{c}), \underline{c})$, then equilibrium $c^* = \underline{c}$;*

3.3. *Otherwise, there is no pure-strategy equilibrium.*

The above proposition shows that the interval for the available unit reward cost cannot be too wide for a pure-strategy equilibrium to exist. With an extensive simulation, we find that the condition is naturally satisfied in most situations. For additional discussion, please see Appendix 3. Next, we discuss first the equilibrium properties of reward type and then properties of reward amount.

Reward Type: What to Offer. A necessary condition for a positive $g(u, \alpha)$ given in Equation (9) is that

Figure 3 Division of Parameter Space



	Reward Type	Light Users	Heavy Users	Firms
Region I	Inefficient	Loss	Gain	Gain
Region II	Inefficient	Loss	Loss	Gain
Region III	Efficient	Loss	Gain	Gain
Region IV	Efficient	Loss	Gain	Loss

Table 2 Selection of Rewards

Relative Price Sensitivity of Heavy Users	Size of Heavy-User Segment: Large	Size of Heavy-User Segment: Small
High	(A) Efficient Rewards	(D) Inefficient Rewards
Low	(B) Efficient Rewards	(C) Efficient Rewards

$(1 - 5u)\alpha \geq 6(1 - u)$. This requires a small heavy-user segment that is much more price sensitive than the light-user segment. In Figure 3, $g(u, \alpha) > 0$ is shown by the area above the dark curve (Areas I and II), the upper-left corner of the parameter space.

Proposition 3 can then be interpreted as follows: Whether firms should offer efficient or inefficient rewards depends on size and relative price sensitivity of the heavy (light)-user segment. When a market has a small heavy-user segment and that segment is much more price sensitive than the light-user segment, firms offer inefficient rewards. Otherwise, they offer efficient rewards. We illustrate the above result in Table 2.

With heavy users' relative price sensitivity and the size of heavy-user segment as two dimensions, we have a total of four scenarios in the above table. In three of the four possible scenarios, firms offer efficient rewards. However, in Scenario D, they offer inefficient rewards. Notice that this scenario is characterized by a small heavy-user segment that is much more price sensitive than the light-user segment ($g(u, \alpha) > 0$). We glean the following intuition for our result.

In our model, firms set their prices after they announce the reward programs. As we showed earlier in §3.3, there is a positive relationship between the first-period market price and reward cost. For a given reward amount, the total cost of rewards increases with the unit reward cost. Holding all other values constant, when firms adopt inefficient rewards, they will charge higher market prices in the first period because of higher reward cost. On the other hand, when they adopt the efficient rewards, they will charge a lower market price in the first period. Thus, in essence, adoption of inefficient rewards demonstrates firms' willingness or commitment to charge high prices in the first period. Next, we discuss the benefit and cost of this commitment.

Note that inefficient rewards arise in equilibrium only when the heavy-user segment is small and price sensitive, compared to the light-user segment. When firms adopt inefficient rewards, the resulting higher prices in the first period enable them to obtain higher profit from the light-user segment. This is the *benefit of commitment* to high prices. On the other hand, because firms compete for the heavy users through rewards as well, adopting inefficient rewards makes it more costly for them to compete for heavy users. The inefficiency, however, raises the cost of the reward program. This is the *cost of commitment* to achieve high prices in the first period. When the size of the light-user segment is large enough, the benefit of commitment to high prices through inefficient rewards dominates the cost of commitment. Therefore, it becomes optimal for firms to adopt the most inefficient rewards.

Under all other scenarios, we find that firms must adopt efficient rewards. From the above discussion, the intuition is straightforward. When light users are price sensitive, firms will not be able to increase their prices enough to overcome the cost of commitment. Similarly, when light users are a small segment, even when they are price insensitive, the benefit of commitment to high price will always be dominated by the cost of commitment. Therefore, it is optimal for firms to offer the efficient rewards.

To summarize the above discussion, firms want to commit themselves to high prices only when there is strong incentive for them to do so. In our model, this incentive will not be strong enough unless there is a large and price-insensitive light-user segment. However, as shown in Table 2, firms should offer the most efficient rewards in several scenarios. This is consistent with the observation that free products are often offered as rewards.

As noted in the beginning, our work departs from earlier work in the industrial organization literature by explicitly incorporating heterogeneity in price sensitivity. We further endogenize the amount and type of reward decision. By doing so, we find that efficient rewards are indeed optimal outcomes. However, we are also able to demonstrate that under certain conditions, inefficient rewards arise in equilibrium. Thus, our model reconciles with the limited but observed practice of inefficient rewards. Note that in the absence of heterogeneity in price sensitivity, inefficient rewards will never be offered.

Reward Amount: How Much to Offer? In Appendix 3 we show that $f(u, \alpha)$ in Equation (8) is always positive. Thus, we conclude that *reward amount decreases with unit reward cost.* The intuition for this result is quite straightforward. Firms compete for heavy users not only through prices but also through rewards. When they offer the rewards of smaller unit reward cost, it is more efficient for the firms to attract the heavy users through rewards. As a result, the equilibrium reward amount should be higher. Note that the discussion is based on a comparative static analysis, and it is important to remember that the amount and type decisions are made simultaneously.

Equation (8) also indicates that $f(u, \alpha)$ increases with α. That is, with larger α, reward amount becomes more sensitive to unit reward cost. Note that a larger α means a relatively more price-sensitive heavy-user segment and relatively less price-sensitive light-user segment. The intuition is as follows: First, because firms compete for heavy users through both prices and rewards, they should offer a higher reward amount when heavy users are more sensitive to prices. Second, when light users are less price sensitive, firms' market shares in the heavy-user segment become more valuable in the second period. Therefore, in the first period, firms should compete more aggressively for the heavy users by offering a larger reward amount.

Equation (7) can be rewritten as $r^*c^* = f(u, \alpha) + g(u, \alpha) \cdot c^*$, which implies a linear relationship between (r^*c^*) and $c^\bullet$. If $g(u, \alpha) < 0$, then the cost of reward $(rc)^\bullet$ decreases with unit reward cost. However, if $g(u, \alpha) > 0$, then the cost of reward (r^*c^*) increases with unit reward cost.

Our results suggest that firms tend to offer a larger amount of rewards when unit reward cost is lower. Casual empirical observations seem to be consistent with our results. Because airlines' unit reward cost of offering free miles is likely to be low, our results predict that they should offer mileage rewards that are quite valuable to the frequent fliers. The practice in the airline industry appears to be in line with this observation. We also find that when the unit reward cost is low, the reward amount will be sufficiently high that it actually maximizes the total reward cost. According to this proposition, because free mileage is inexpensive for airlines to offer, the amount they hand out is significantly large (''mileage war'') that airlines actually maximize their total reward cost. Finally, because reward amount can take all nonnegative values, profit function has to be concave with respect to reward amount to ensure the equilibrium existence. We provide the details related to this technical issue in Appendix 3. Our extensive numerical simulation shows that the concavity is rarely violated. Therefore, we do not elaborate further on this condition.

We obtain the analytical results from a nonswitching pure-strategy equilibrium. As mentioned earlier, to ensure the existence of such a pure-strategy equilibrium, we develop a set of sufficient conditions in Appendix 2. Under these conditions, neither firm has an incentive to deviate in the second period for either larger or smaller market shares of heavy users. In Appendix 3 we extend these conditions to full game, allowing one firm to unilaterally deviate to any nonnegative rewards while keeping another firm offering the equilibrium strategy. (See Appendix 3 for more technical details.) Under the equilibrium reward decisions, we also derive a set of sufficient conditions under which a pure-strategy equilibrium always exists for *any* nonnegative first-period prices. Finally, at the end of Appendix 3, we combine all of the conditions developed throughout the paper and run a joint simulation to examine the parameter restrictions. We find that it is sufficient (but may not be necessary) to impose a maximum value of μ in order to ensure the equilibrium existence. Intuitively, a smaller heavy-user segment implies less incentive for firms to compete aggressively on heavy users. Thus, for a sufficiently small heavy-user segment, it is not profitable to attempt to switch these heavy users because of loss of rent from the larger light-user segment.

4. Discussion

In this section, we examine the gains or losses to the firms and consumers attributable to reward programs. (See Appendix 4 for technical details). As demonstrated earlier, reward programs increase prices in both periods. Also, note that the reward programs are aimed at only the heavy-user segment, and the light users do not benefit from the programs. Hence, the light-user segment stands to lose when a reward program is offered. The heavy-user segment may gain or lose from the reward program. When the segment gains, such additional benefits are achieved at the cost of the light-user segment or at the expense of both the light-user segment and the competing firms. Although heavy users gain mostly because of subsidy from light users, under certain parameter values, they can be exploited by the competing firms, causing a reduction in their benefit or surplus. Under certain conditions, it can be further established that as long as light users are not excessively price sensitive, the competing firms' profits increase with reward programs. However, when light users are highly price sensitive, profits may decrease with reward programs.

We delineate the parameter space along two dimensions, namely, size of the heavy-user segment and the heavy users' relative price sensitivity. We separate this space into four regions in which the impact of the reward programs on the light- and heavy-user segments, and the competing firms, differ. We graphically present this in Figure 3.

Consider Regions I and II. In these cases, the size of the heavy-user segment is small. In addition, the segment's relative price sensitivity is high. Therefore, firms select inefficient rewards in equilibrium. In Region I, the heavy-user segment is smaller than that in Region II. Suppose there is no reward program offered. Then the competing firms' incentive to compete for the heavy-user segment is mitigated by the small size of the segment. Doing otherwise would mean unnecessarily lowering the price too much for the large light-user segment. Hence, prices without reward programs are only modestly lower. Therefore, reward programs only increase the prices moderately, and heavy-users gain from the reward programs. In Region II, however, the heavy-user segment is larger and, therefore, intense price competition arises without reward programs. This competition, which is beneficial to the heavy-user segment, is moderated by the reward programs. Moreover, with the selection of inefficient reward, firms offer a lower value of reward to the heavy users. Consequently, heavy users achieve lower benefits with the reward programs.

In Regions III and IV, efficient rewards are optimal. Competing firms gain with reward programs in all four regions, except for IV. In Region IV, light users are much more price sensitive when compared to the heavy users. Hence, even with reward programs, the firms compete intensely for the light-user segment. Taken together with the cost of the reward programs, the competing firms earn less profit when compared to the case without reward programs.

Limitations. Reward programs offered in practice are rich and complex. Often, firms offer multiple, or a menu of, rewards. Firms are asymmetric in terms of market share or cost or both. Clearly, firms operate in a multiperiod world. Finally, firms may deploy an array of third-degree price discrimination mechanisms to the heavy- and light-user segments. We have abstracted away from a number of these issues to achieve tractability and gain some insight into the competitive market with reward programs. Doing so is entirely in the spirit of a number of game-theoretic papers in the marketing literature. (For additional discussion, see Moorthy 1993.) More importantly, we are unaware of any published analytical effort in the field on this important marketing issue. In this context, we view our contribution as an initial step, and clearly far removed from the ideal model in which the implications directly translate into managerial practice.

5. Summary and Future Research

In this paper, we develop an analytical model to investigate the effectiveness of reward programs in a competitive market. Our analysis demonstrates that firms achieve tacit price collusion by creating consumers' switching cost through reward programs. Essentially, we find that competitive market prices increase because of the introduction of reward programs. Light users, who do not get the reward, earn strictly lower benefits attributable to reward program offerings. In contrast, heavy users often stand to gain more from the reward program. In most cases, we find that firms and the heavy users are better off, at the expense of light users.

We offer an explanation as to why the type of reward may vary across the programs. We identify two determining factors for the selection of rewards: size and relative price sensitivity of the heavy-user segment. We find that in a market with a small heavy-user segment that is also much more price sensitive than the light-user segment, it is optimal for firms to offer the most inefficient rewards. Our intuition rests on firms' incentive to exploit the price-insensitive light-user segment. By offering inefficient rewards, firms are able to commit to weaker competition and higher prices. When the heavy-user segment is large or not very price sensitive when compared to the light-user segment, competing firms should adopt the most efficient rewards to maximize their profit. This may well be the case in a number of real-world situations where efficient rewards are quite prevalent.

Although our paper contains interesting insights on the implications of the reward programs on competitive prices and selection of the type of reward programs, they must be interpreted within the context of certain limitations. First, if firms use their own products as rewards, rewards may cannibalize their primary demand. We examine this issue in a different setting (Kim et al. 1997). Second, we have limited the model structure to only one type and reward amount. Often, we observe that firms offer multitier or multitype reward programs. We hope to expand our analysis to incorporate a menu of reward offerings by competing firms in the market place.

Appendix 1. Prove That Switching Does Not Occur in Any Pure-Strategy Equilibrium

Switching vs. Nonswitching Equilibrium

In the second period, heavy users receive rewards if purchasing from the same firms. Heavy users will therefore not switch unless another firm offers a much lower price. Firms' second-period market shares in the heavy segment are conditional on their first-period market shares and rewards.

$$\sigma_{2,H}^{i} = \begin{cases} \min\left\{1, \dfrac{1}{2} + \dfrac{p_2^j - p_2^i - r_j}{2t}\right\}, \\ \qquad \text{if } p_2^i + \sigma_{1,H}^i t < p_2^j + \sigma_{1,H}^j t - r_j, & \text{(A1.1.1)} \\ \sigma_{1,H}^i, \quad \text{if } p_2^j + \sigma_{1,H}^j t - r_j \le p_2^i + \sigma_{1,H}^i t \le p_2^j + \sigma_{1,H}^j t + r_i, & \text{(A1.1.2)} \\ \max\left\{0, \dfrac{1}{2} + \dfrac{p_2^j - p_2^i + r_i}{2t}\right\}, \\ \qquad \text{if } p_2^i + \sigma_{1,H}^i t > p_2^j + \sigma_{1,H}^j t + r_i, & \text{(A1.1.3)} \end{cases}$$

where $i, j = a$ or b, $i \neq j$. Equations (A1.1.1–A1.1.3) model three scenarios. In Scenario 1 (A1.1.1), firm i offers a very low price that attracts some of firm j's first-period heavy customers. Opposite to Scenario 1, in Scenario 3 (A1.1.3) firm i charges a very high price, so that some of its heavy customers prefer to switch to firm j. Scenario 2 (A1.1.2) is defined in a medium range of price, where firm i only targets its current customers. Following Equations (A1.1.1–A1.1.3), we classify any pure-strategy equilibrium into one of the two types: switching or nonswitching. In a *switching equilibrium* (equilibrium prices fall in Scenario 1 or 3), some heavy users switch to buy from another firm in the second period. In a *nonswitching equilibrium* (Scenario 2), all heavy users repeat purchase from the same firm in two periods.

Switching Does Not Occur in Any Pure-Strategy Equilibrium

To prove that heavy users do not switch between two firms in any pure-strategy equilibrium, we need to show that pure-strategy equilibrium should exist only in Scenario 2 (A1.1.2). Our proof applies backward induction and contradiction. Suppose $\{(r_a, p_1^a, p_2^a), (r_b, p_1^b, p_2^b)\}$ was a switching equilibrium. Without loss of generality, we let $\sigma_{2,H}^b > \sigma_{1,H}^b$. Note that under the current model framework switching may occur only at one direction. That is, if there is a consumer switching from firm i to firm j, then no one should switch from firm j to firm i, as firm j's offer at the second period is more attractive.

Period 2. In the second period, firm a's profit-maximization problem is as follows:

$$\text{(PA1.1)} \qquad \max_{p_2^a} \pi_2^a = (1-u)p_2^a\sigma_{2,L}^a + u(p_2^a - c_a r_a)\sigma_{2,H}^a,$$

where

$$\sigma_{2,L}^a = \frac{1}{2} + \frac{p_2^b - p_2^a}{2\alpha t} \quad \text{and} \quad \sigma_{2,H}^a = \frac{1}{2} - \frac{p_2^a - p_2^b - r_a}{2t}.$$

Similarly, we formulate firm b's profit-maximization problem as follows:

$$\text{(PA1.2)} \qquad \max_{p_2^b} \pi_2^b = (1-u)p_2^b\sigma_{2,L}^b + u[(p_2^b - c_b r_b)\sigma_{1,H}^b + p_2^b(\sigma_{2,H}^b - \sigma_{1,H}^b)],$$

where

$$\sigma_{2,L}^b = \frac{1}{2} + \frac{p_2^a - p_2^b}{2\alpha t} \quad \text{and} \quad \sigma_{2,H}^b = \frac{1}{2} + \frac{p_2^a - p_2^b - r_a}{2t}.$$

Note that heavy users who switch from Firm a to Firm b do not receive rewards at the end of second period. Solving first-order conditions for both firms simultaneously, we get the equilibrium prices

$$p_2^{a*} = \frac{t + (1+2c_a)ur_a/3}{u + (1-u)/\alpha} \quad \text{and} \quad p_2^{b*} = \frac{t - (1-c_a)ur_a/3}{u + (1-u)/\alpha}. \qquad \text{(A1.2)}$$

Firm b's second-period market share in the heavy-user segment is then

$$\sigma_{2,H}^{b*} = \frac{1}{2} - \frac{(1-c_a)u/3 + (1-u)/\alpha}{u + (1-u)/\alpha}\cdot\frac{r_a}{2t}. \qquad \text{(A1.3)}$$

Since $\sigma_{2,H}^{b*} > \sigma_{1,H}^{b*}$, the following condition should hold:

$$\sigma_{1,H}^{b*} < \frac{1}{2} - \frac{(1-c_a)u/3 + (1-u)/\alpha}{u + (1-u)/\alpha}\cdot\frac{r_a}{2t} < \frac{1}{2}. \qquad \text{(A1.4)}$$

Period 1. As consumers have rational anticipation on the switching in Period 2, firms' market shares at Period 1 are determined by Equation $p_1^a + \sigma_{1,H}^a t = p_1^b + \sigma_{1,H}^b t - r_b$. That is, consumers located at $\sigma_{1,H}^a$ are indifferent from *buying from Firm a first and then switching to Firm b* and *buying from Firm b at both periods.* Therefore, Firm b's market share can be solved by

$$\sigma_{1,H}^b = \frac{1}{2} + \frac{p_1^a - p_1^b + r_b}{2t}. \qquad \text{(A1.5)}$$

We let both firms maximize their total profits from two periods with respect to their own first-period prices. The first-order conditions can be simplified to the following form:

$$2p_1^a - p_1^b = \frac{t - ur_b}{u + (1-u)/\alpha}, \qquad \text{(A1.6)}$$

$$2p_1^b - p_1^a = \frac{t + (1+c_b)ur_b}{u + (1-u)/\alpha}. \qquad \text{(A1.7)}$$

From Equations (A1.6–A1.7), we can solve Firm b's first-period market share in the heavy segment:

$$\sigma_{1,H}^{b*} = \frac{1}{2} + \frac{(1-c_b)u/3 + (1-u)/\alpha}{u + (1-u)/\alpha}\cdot\frac{r_b}{2t} > \frac{1}{2}. \qquad \text{(A1.8)}$$

This contradicts (A1.4), a necessary condition for the occurrence of switching in the equilibrium. Therefore, switching should not occur in the equilibrium. *Intuitively, it is not optimal for firms to be too soft in the first period because it is more difficult to attract consumers in the second period.*

Appendix 2. Equilibrium Price

Second-Period Equilibrium Price

Program (P1) formulates firms' profit-maximization problems with (A1.1.2) as demand functions. We take first-order conditions with respect to price for both firms.

$$\frac{\partial \pi_2^i}{\partial p_2^i} = (1-\mu)\left[\frac{1}{2} + \frac{p_2^j - p_2^i}{2\alpha t} - \frac{p_2^i}{2\alpha t}\right] + \mu\sigma_{1,H}^i = 0$$

$$(i, j = a \text{ or } b,\ i \neq j).$$

Rearrange the first-order conditions,

$$2p_2^i - p_2^j = \frac{\mu}{1-\mu}\sigma_{1,H}^i\cdot 2\alpha t + \alpha t = \alpha t + \frac{2\alpha\mu t}{1-\mu}\sigma_{1,H}^i = 0$$

$$(i, j = a \text{ or } b,\ i \neq j).$$

Solving equilibrium prices from the above first-order conditions,

$$p_2^i = \alpha t + \frac{2\alpha\mu t}{1-\mu}\,\frac{2\sigma_{1,H}^i + \sigma_{1,H}^j}{3} = \alpha t + \frac{2\alpha\mu t}{1-\mu}\,\frac{1+\sigma_{1,H}^i}{3}$$

$$(i, j = a \text{ or } b,\ i \neq j). \qquad \text{(A2.1)}$$

Substituting these prices into market share functions, we solve the resulting second-period market shares in the light-user segment:

$$\sigma_{2,L}^i = \frac{1}{2} + \frac{u}{3(1-u)}(\sigma_{1,H}^j - \sigma_{1,H}^i) \qquad (i, j = a \text{ or } b,\ i \neq j). \qquad \text{(A2.2)}$$

Substituting (A2.1) and (A2.2) into the firm's profit function, we have second-period equilibrium profits.

$$\pi_2^i = \frac{1-u}{2\alpha t}\left[\alpha t + \frac{2u}{3(1-u)}(1+\sigma_{1,H}^i)\alpha t\right]^2 - uc_i r_i \sigma_{1,H}^i$$

$$(i, j = a \text{ or } b,\ i \neq j). \qquad \text{(A2.3)}$$

Sufficient Conditions for Price (A2.1) to Be Equilibrium Prices

To guarantee that (A2.1) defines an equilibrium, we need to ensure that a) it is local optimal in problem (P1); and b) neither firm has an incentive to unilaterally deviate to a price outside the range defined by (A1.1.2).

a) Local Optimality. Equilibrium prices must satisfy (A1.1.2) to ensure local optimality. We substitute equilibrium price (A2.1) into (A1.1.2) and obtain the following condition:

$$\sigma_{1,H}^i - \sigma_{1,H}^j \le \frac{1}{\dfrac{2\alpha u}{3(1-u)} + 1}\frac{r_i}{t} \qquad (i, j = a, b \text{ and } i \ne j). \tag{A2.4}$$

b) No Incentive for Further Deviation. A firm's profit should decrease if the firm *unilaterally deviates from (A2.1)* to a price outside the range in which (A1.1.2.) holds. Without loss of generality, we first fix Firm b's price. We denote $\underline{p_2^a}$ as the lower bound and $\overline{p_2^a}$ as the upper bound for Firm a's price as defined by (A1.1.2). When Firm a lowers its price below $\underline{p_2^a}$ to the range defined by (A1.1.1), its profit-maximization problem:

$$\text{(PA2.1)} \qquad \max_{p_2^a} \pi_2^a = (1-u)p_2^a\sigma_{2,L}^a + up_2^a\sigma_{2,H}^a - uc_ar_a\sigma_{1,H}^a,$$

$$\sigma_{2,L}^a = \frac{1}{2} + \frac{p_2^b - p_2^a}{2\alpha t} \quad \text{and} \quad \sigma_{2,H}^a = \frac{1}{2} + \frac{p_2^b - p_2^a - r_b}{2t},$$

$$p_2^b = \alpha t + \frac{2\alpha u t}{1-u}\frac{1+\sigma_{1,H}^b}{3}.$$

Note that Firm a's market share in the heavy-user segment is now defined by (A1.1.1). To ensure that Firm a will be worse off by reducing its price further below $\underline{p_2^a}$, it is sufficient to impose a positive left-hand derivative at $\underline{p_2^a}$. That is, $(\partial^-\pi_2^a/\partial p_2^a)|(\underline{p_2^a}) \ge 0$. Substituting in profit and market share functions, we can simplify the condition to:

$$\left(\frac{u}{2} + \frac{1-u}{2\alpha}\right)\left(2 + \frac{\alpha u}{3(1-u)}\right)(\sigma_{1,H}^a - \sigma_{1,H}^b) \ge \frac{\alpha u}{2(1-u)} - \left(\frac{u}{2t} + \frac{1-u}{\alpha t}\right)r_b. \tag{A2.5.1}$$

Similarly, when Firm a's price is fixed at (A2.1), we need a sufficient Condition (A2.5.2) to ensure that Firm b cannot benefit from offering a price below $\underline{p_2^b}$.

$$\left(\frac{u}{2} + \frac{1-u}{2\alpha}\right)\left(2 + \frac{\alpha u}{3(1-u)}\right)(\sigma_{1,H}^b - \sigma_{1,H}^a) \ge \frac{\alpha u}{2(1-u)} - \left(\frac{u}{2t} + \frac{1-u}{\alpha t}\right)r_a. \tag{A2.5.2}$$

If Firm a increases its price to a level above $\overline{p_2^a}$, then its profit-maximization problem becomes:

$$\text{(PA2.2)} \qquad \max_{p_2^a} \pi_2^a = (1-u)p_2^a\sigma_{2,L}^a + u(p_2^a - c_ar_a)\sigma_{2,H}^a,$$

$$\sigma_{2,L}^a = \frac{1}{2} + \frac{p_2^b - p_2^a}{2\alpha t} \quad \text{and} \quad \sigma_{2,H}^a = \frac{1}{2} + \frac{p_2^b - p_2^a + r_a}{2t},$$

$$p_2^b = \alpha t + \frac{2\alpha u t}{1-u}\frac{1+\sigma_{1,H}^b}{3}.$$

Now Firm a's market share in the heavy-user segment is defined by (A1.1.3). To ensure that Firm a cannot be better off with further increase in its price beyond $\overline{p_2^a}$, it is sufficient to have a negative right-hand derivative at $\overline{p_2^a}$. That is, $(\partial^+\pi_2^a/\partial p_2^a)|(\overline{p_2^a}) \le 0$. Substituting in profit and market share functions, this condition can be simplified to:

$$\left(\frac{u}{2} + \frac{1-u}{2\alpha}\right)\left(2 + \frac{\alpha u}{3(1-u)}\right)(\sigma_{1,H}^a - \sigma_{1,H}^b) \le \frac{\alpha u}{2(1-u)} + \left(\frac{u}{2t}(1-c_a) + \frac{1-u}{\alpha t}\right)r_a. \tag{A2.6.1}$$

Similarly, when Firm a's price is fixed at (A2.1), we need a sufficient condition (A2.6.2) to ensure that Firm b cannot benefit from offering a price above $\overline{p_2^b}$.

$$\left(\frac{u}{2} + \frac{1-u}{2\alpha}\right)\left(2 + \frac{\alpha u}{3(1-u)}\right)(\sigma_{1,H}^b - \sigma_{1,H}^a) \le \frac{\alpha u}{2(1-u)} + \left(\frac{u}{2t}(1-c_b) + \frac{1-u}{\alpha t}\right)r_b. \tag{A2.6.2}$$

First-Period Equilibrium Price

We solve Problem (P2) of §3.3 through first-order conditions to obtain first-period equilibrium prices. The equilibrium price is conditional on predetermined reward decisions (r_a, c_a; r_b, c_b).

$$\begin{aligned}\frac{\partial \pi^i}{\partial p_1^i} &= \frac{\partial \pi_1^i}{\partial p_1^i} + \frac{\partial \pi_2^i}{\partial p_1^i} \\ &= \frac{1}{2} + \left(\frac{1-u}{2\alpha t} + \frac{u}{2\beta}\right)(p_1^j - 2p_1^i) + \frac{u}{2\beta}(r_i - r_j) - \frac{\alpha u t}{3\beta(1-u)} \\ &\quad - \frac{\alpha u^2 t}{9\beta^2(1-u)}[(p_1^j - p_1^i) + (r_i - r_j)] + \frac{u c_i r_i}{2\beta} = 0.\end{aligned}$$

From the above first-order conditions we can solve equilibrium prices.

$$p_1^i = \frac{1}{2}\left[\frac{\dfrac{u}{2\beta}}{\dfrac{u}{2\beta} + \dfrac{1-u}{2\alpha t}}(c_ir_i + c_jr_j) + \frac{\left(\dfrac{u}{\beta} - \dfrac{2\alpha u^2 t}{9\beta^2(1-u)}\right)(r_i - r_j) + \dfrac{u}{2\beta}(c_ir_i - c_jr_j)}{3\left(\dfrac{u}{2\beta} + \dfrac{1-u}{2\alpha t}\right) - \dfrac{2\alpha u^2 t}{9\beta^2(1-u)}} + \frac{\dfrac{2t}{\beta}}{\dfrac{u}{2\beta} + \dfrac{1-u}{2\alpha t}}\right], \tag{A2.7}$$

where

$$\beta = \frac{3 - 3u + \alpha u}{3 - 3u}2t, \qquad i, j = a \text{ or } b, \qquad i \neq j.$$

Substituting (A2.7) into (5) we can calculate first-period market share in the heavy segment:

$$\sigma_{1,H}^i = \frac{1}{2} + \frac{1}{2\beta}\frac{\left(\frac{u}{2\beta} + \frac{3(1-u)}{2\alpha t}\right)(r_i - r_j) - \frac{u}{2\beta}(c_i r_i - c_j r_j)}{3\left(\frac{u}{2\beta} + \frac{1-u}{2\alpha t}\right) - \frac{2\alpha u^2 t}{9\beta^2(1-u)}}$$

$$(i, j = a \text{ or } b, i \neq j). \tag{A2.8}$$

Appendix 3. Selection of Reward Programs

Reward Amount

1. Reward function. Problem (P3) solves the optimal reward program for firm i ($i = a$ or b):

$$\text{(P3)} \qquad \max_{r_i, c_i} \pi^i = \pi_1^i + \pi_2^i,$$

where

$$\pi_1^i = (1-u)p_1^i\sigma_{1,L}^i + up_1^i\sigma_{1,H}^i,$$

$$\pi_2^i = \frac{1-u}{2\alpha t}\left[\alpha t + \frac{2u}{3(1-u)}(1 + \sigma_{1,H}^i)\alpha t\right]^2 - uc_i r_i \sigma_{1,H}^i, \tag{A2.3}$$

$$p_1^i = \frac{\frac{u}{2\beta}}{\frac{u}{2\beta} + \frac{1-u}{2\alpha t}}\frac{c_i r_i + c_j r_j}{2} + \frac{\left(\frac{u}{2\beta} - \frac{\alpha u^2 t}{9\beta^2(1-u)}\right)(r_i - r_j) + \frac{u}{2\beta}\frac{(c_i r_i - c_j r_j)}{2}}{3\left(\frac{u}{2\beta} + \frac{1-u}{2\alpha t}\right) - \frac{2\alpha u^2 t}{9\beta^2(1-u)}} + \frac{\frac{t}{\beta}}{\frac{u}{2\beta} + \frac{1-u}{2\alpha t}},$$

$$\sigma_{1,L}^i = \frac{1}{2} + \frac{p_1^j - p_1^i}{2\alpha t}, \qquad \sigma_{1,H}^i = \frac{1}{2} + \frac{1}{2\beta}[(p_1^j - p_1^i) + (r_i - r_j)]; \tag{A2.7}$$

with

$$\beta = \frac{3 - (3-\alpha)u}{3(1-u)}2t, \qquad i, j = a \text{ or } b, \qquad i \neq j.$$

We solve the equilibrium reward amount through first-order conditions:

$$\frac{\partial \pi_i}{\partial r_i} = \frac{\partial p_1^i}{\partial r_i}[(1-u)\sigma_{1,L}^i + u\sigma_{1,H}^i] + p_1^i\left[(1-u)\frac{\partial \sigma_{1,L}^i}{\partial r_i} + u\frac{\partial \sigma_{1,H}^i}{\partial r_i}\right] + \frac{1-u}{2\alpha t} \times 2 \times \left[\alpha t + \frac{2u}{3(1-u)}(1 + \sigma_{1,H}^i)\alpha t\right]\frac{2\alpha u t}{3(1-u)}\frac{\partial \sigma_{1,H}^i}{\partial r_i} - uc_i\left(\frac{\partial \sigma_{1,H}^i}{\partial r_i} + \sigma_{1,H}^i\right) = 0. \tag{A3.1}$$

Because we focus on symmetric equilibrium, we apply symmetry ($r_a = r_b = r^*$, $c_a = c_b = c^*$, $\sigma_{1,H}^i = \sigma_{1,L}^a = ½$) to the above first-order condition (A3.1) and obtain the following result:

$$r^* = \left[\frac{5\alpha t}{3(1-u)} - \frac{\left(\frac{1}{2} - \frac{aut}{9\beta(1-u)}\right)\left(1 + \frac{aut}{\beta(1-u)}\right)}{3\left(\frac{1-u}{2\alpha t} + \frac{u}{2\beta}\right) - \frac{2au^2 t}{9\beta^2(1-u)}}\right] \Big/ c^* + \frac{\alpha t(1-2u)}{2(1-u)} - \beta - \frac{\frac{u}{4\beta}\left(\frac{\alpha t}{1-u} + \frac{\beta}{u}\right)}{3\left(\frac{1-u}{2\alpha t} + \frac{u}{2\beta}\right) - \frac{2au^2 t}{9\beta^2(1-u)}}. \tag{A3.2}$$

Substituting β with $3 - (3-\alpha)u/3(1-u)\ 2t$, the above Equation (A3.2) can be rewritten as

$$r_i^*(c^*) = f(u, \alpha)/c^* + g(u, \alpha), \tag{7}$$

where

$$f(u, \alpha) = \frac{4\alpha t}{3(1-u)} + \frac{\alpha^2 ut}{3(1-u)}\frac{6 - 6u + 3\alpha u}{3(6 - 6u + 5\alpha u)(3 - 3u + \alpha u) - 2u^2\alpha^2}, \tag{8}$$

$$g(u, \alpha) = \frac{\alpha - 6(1-u) - 5\alpha u}{3(1-u)}t - \frac{2\alpha^3 u^2}{6(1-u)[3(6 - 6u + 5\alpha u)(3 - 3u + \alpha u) - 2u^2\alpha^2]}t. \tag{9}$$

Note that $f(u, \alpha)$ in Equation (8) can be rewritten as

$$\frac{216(1-u)^2 + 258\alpha u(1-u) + 55\alpha^2 u^2}{54(1-u)^2 + 63\alpha u(1-u) + 13\alpha^2 u^2} \cdot \frac{\alpha t}{3(1-u)},$$

which is always positive, as $\alpha > 0$ and u is between 0 and 1.

2. Equilibrium Existence. Since the profit function is quadratic, to ensure the validity of first-order approach, it is sufficient to have a nonpositive second derivative.

$$\frac{\partial^2 \pi^i}{\partial^2 r_i} = \frac{A_1(u, \alpha)c_i^2 - A_2(u, \alpha)c_i + A_3(u, \alpha)}{\left(3\frac{u}{2\beta} + 3\frac{1-u}{2\alpha t} - \frac{2\alpha u^2 t}{9\beta^2(1-u)}\right)^2} \leq 0, \tag{A3.3}$$

where

$$A_1 = \left(\frac{u}{2\beta}\right)^2\left[2\left(\frac{u}{2\beta} + \frac{1-u}{2\alpha t}\right) - \frac{\alpha u^2 t}{9\beta^2(1-u)}\right],$$

$$A_2 = \frac{u}{2\beta}\left[\left(\frac{u}{2\beta} + \frac{1-u}{2\alpha t}\right)\left(\frac{2u}{\beta} + \frac{9(1-u)}{\alpha t}\right)\right.$$

$$\left. - \frac{\alpha u^2 t}{9\beta^2(1-u)}\left(\frac{u}{\beta} + \frac{8(1-u)}{\alpha t}\right) + \left(\frac{\alpha u^2 t}{9\beta^2(1-u)}\right)^2 \frac{\frac{2(1-u)}{\alpha t}}{\frac{u}{2\beta} + \frac{1-u}{2\alpha t}}\right]$$

$$A_3 = \left(\frac{u}{2\beta} + \frac{1-u}{2\alpha t}\right)\left[\frac{1}{2}\left(\frac{u}{\beta}\right)^2 + \frac{\alpha u^2 t}{9\beta^2(1-u)}\left(-\frac{u}{2\beta} + \frac{9(1-u)}{2\alpha t}\right)\right]$$
$$- \left(\frac{\alpha u^2 t}{9\beta^2(1-u)}\right)^2 \frac{2(1-u)}{\alpha t}.$$

To check Condition (A3.3), we conduct a numerical simulation for u from 0 to 1 with step 0.01, α from 0.2 to 15 with step 0.1, $\underline{c}$ from 0.1 to 1 with step 0.1, and $\bar{c}$ equal to 1. We find that the concavity always holds as long as $\underline{c} > 0.1$. The concavity may not hold only if $\underline{c} \leq 0.1$, $u > 0.36$, and $\alpha > 6$.

Reward Type

1. Convexity.

$$\frac{\partial^2 \pi_i}{\partial c_i^2} = \left(\frac{\frac{u r_a}{2\beta}}{3\left(\frac{1-u}{2\alpha t} + \frac{u}{2\beta}\right) - \frac{2au^2 t}{9\beta^2(1-u)}}\right)^2$$
$$\times \left[2\left(\frac{1-u}{2\alpha t} + \frac{u}{2\beta}\right) - \frac{\alpha u^2 t}{9\beta^2(1-u)}\right] > 0. \qquad \text{(A3.4)}$$

2. Equilibrium Existence. Because profit function is convex with respect to unit reward cost, as stated in Proposition 3, the following conditions should be satisfied to ensure the existence of equilibrium:

a) If $g(u, \alpha) > 0 \qquad (c^\cdot = \bar{c})$,

$$\text{then } \pi_i(r*(\bar{c}), \bar{c}) \geq \pi_i(r_i(\underline{c}; r*(\bar{c}), \bar{c}), c; r*(\bar{c}), \bar{c}). \qquad \text{(A3.5.1)}$$

b) If $g(u, \alpha) < 0 \qquad (c^\cdot = \underline{c})$,

$$\text{then } \pi_i(r*(\underline{c}), \underline{c}) \geq \pi_i(r_i(\bar{c}; r*(\underline{c}), \underline{c}), \bar{c}; r*(\underline{c}), \underline{c}). \qquad \text{(A3.5.2)}$$

We check the above constraints with a numerical simulation for u from 0 to 1 with step 0.01, α from 0.2 to 15 with step 0.1, $\underline{c}$ from 0.1 to 1 with step 0.1, and $\underline{c}$ equal to 1. We find that, first, (A3.5.2) is almost always satisfied. Second, for (A3.5.1), we find that any reasonable value of $\underline{c}$ satisfies the condition when u is small enough.

Combined Parameter Restrictions for Equilibrium Existence

We have derived a set of sufficient conditions for the existence of nonswitching subgame equilibrium in Appendix 2. Now we extend these conditions to full game by allowing firms to unilaterally change their rewards and prices. We will then combine these conditions with (3.3) and (3.5.1–3.5.2) to investigate the parameter restrictions for the existence of nonswitching equilibrium. *It is important to note that these sufficient conditions may not be necessary to ensure the existence of equilibrium.*

Existence of Nonswitching Equilibrium with Any Deviation in Rewards. We first derive conditions under which a nonswitching equilibrium is sustained in second-period competition when unilateral deviation in rewards occurs in the first period. With these conditions we can compute expected profit corresponding to any unilateral deviation in rewards. Recall that we have derived a set of conditions (Condition (A2.4), (A2.5.1–A2.5.2), and (A2.6.1–A2.6.2)) to ensure the candidate equilibrium (A2.1) be a Nash equilibrium. We now substitute (A2.8) into (A2.4), (A2.5.1–A2.5.2) and (A2.6.1–A2.6.2) to establish the conditions for existence.

a) Local Optimality. Condition (A2.4) to ensure local optimality can be rewritten as:

$$\frac{-r_b}{t} \leq \frac{1}{\beta}\frac{\left(\frac{u}{2\beta} + \frac{3(1-u)}{2\alpha t}\right)(r_a - r_b) - \frac{u}{2\beta}(c_a r_a - c_b r_b)}{3\left(\frac{u}{2\beta} + \frac{1-u}{2\alpha t}\right) - \frac{2\alpha u^2 t}{9\beta^2(1-u)}}$$
$$\times \left(1 + \frac{2\alpha u}{3(1-u)}\right) \leq \frac{r_a}{t}. \qquad \text{(A3.6)}$$

To satisfy (A3.6), it is sufficient that

$$\frac{1}{\beta}\frac{\frac{u}{2\beta} + \frac{3(1-u)}{2\alpha t}}{3\left(\frac{u}{2\beta} + \frac{1-u}{2\alpha t}\right) - \frac{2\alpha u^2 t}{9\beta^2(1-u)}}\left(1 + \frac{2\alpha u}{3(1-u)}\right) \leq \frac{1}{t},$$

which can be further simplified to $54(1-u)^2 + 63\alpha u(1-u) + 8\alpha^2 u^2 \geq 0$. *Clearly this condition holds for any positive α and u.*

b) No Incentive for Further Deviation in Price. Conditions (A2.5.1–A2.5.2) ensure that neither firm will have incentive to compete for a larger share of the heavy segment. After substituting (A2.8) into (A2.5.1) and (A2.5.2), these two conditions can be summarized in (A3.7):

$$\frac{1}{\beta}\frac{\left(\frac{u}{2\beta} + \frac{3(1-u)}{2\alpha t}\right)(r_i - r_j) - \frac{u}{2\beta}(c_i r_i - c_j r_j)}{3\left(\frac{u}{2\beta} + \frac{1-u}{2\alpha t}\right) - \frac{2\alpha u^2 t}{9\beta^2(1-u)}}$$
$$\geq \frac{\frac{\alpha u}{2(1-u)} - \left(\frac{u}{2t} + \frac{1-u}{\alpha t}\right) r_j}{\left(\frac{u}{2} + \frac{1-u}{2\alpha}\right)\left(2 + \frac{\alpha u}{3(1-u)}\right)}, \qquad i, j = a \text{ or } b, \qquad i \neq j. \qquad \text{(A3.7)}$$

We reorganize Condition (A3.10) as follows:

$$\frac{1}{\beta}\frac{\frac{u}{2\beta}(1-c_i)+\frac{3(1-u)}{2\alpha t}}{3\left(\frac{u}{2\beta}+\frac{1-u}{2\alpha t}\right)-\frac{2\alpha u^2 t}{9\beta^2(1-u)}}r_i$$
$$+\left[\frac{\left(\frac{u}{2t}+\frac{1-u}{\alpha t}\right)}{\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)}-\frac{1}{\beta}\frac{\frac{u}{2\beta}(1-c_j)+\frac{3(1-u)}{2\alpha t}}{3\left(\frac{u}{2\beta}+\frac{1-u}{2\alpha t}\right)-\frac{2\alpha u^2 t}{9\beta^2(1-u)}}\right]r_j$$
$$-\frac{\frac{\alpha u}{2(1-u)}}{\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)}\geq 0 \qquad (i,\, j = a \text{ or } b,\, i \neq j). \tag{A3.8}$$

Recall that one of the competing firms offer a reward at the equilibrium level (r^*) and another offers any nonnegative amount. If firm i offers equilibrium rewards, then the following two conditions combined are sufficient for (A3.8).

$$\frac{\frac{u}{2t}+\frac{1-u}{\alpha t}}{\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)}\geq\frac{1}{\beta}\frac{\frac{u}{2\beta}(1-\underline{c})+\frac{3(1-u)}{2\alpha t}}{3\left(\frac{u}{2\beta}+\frac{1-u}{2\alpha t}\right)-\frac{2\alpha u^2 t}{9\beta^2(1-u)}}, \tag{A3.9.1}$$

$$r^*\cdot\frac{1}{\beta}\frac{\frac{u}{2\beta}(1-c^*)+\frac{3(1-u)}{2\alpha t}}{3\left(\frac{u}{2\beta}+\frac{1-u}{2\alpha t}\right)-\frac{2\alpha u^2 t}{9\beta^2(1-u)}}\geq\frac{\frac{\alpha u}{2(1-u)}}{\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)}. \tag{A3.9.2}$$

Note that (A3.9.1) ensures a positive coefficient for r_j at any feasible unit reward cost c_j. If firm j offers equilibrium rewards, then the following condition is sufficient for (A3.8).

$$r^*\left[\frac{\frac{u}{2t}+\frac{1-u}{\alpha t}}{\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)}-\frac{1}{\beta}\frac{\frac{u}{2\beta}(1-c^*)+\frac{3(1-u)}{2\alpha t}}{3\left(\frac{u}{2\beta}+\frac{1-u}{2\alpha t}\right)-\frac{2\alpha u^2 t}{9\beta^2(1-u)}}\right]$$
$$\geq\frac{\frac{\alpha u}{2(1-u)}}{\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)}. \tag{A3.9.3}$$

Similarly, Conditions (A2.6.1–A2.6.2) need to be satisfied so that neither firm will have incentive for higher price and smaller share of the heavy segment. After substituting (A2.8) into (A2.6.1–A2.6.2), we obtain the following condition:

$$\frac{1}{\beta}\frac{\left(\frac{u}{2\beta}+\frac{3(1-u)}{2\alpha t}\right)(r_i-r_j)-\frac{u}{2\beta}(c_ir_i-c_jr_j)}{3\left(\frac{u}{2\beta}+\frac{1-u}{2\alpha t}\right)-\frac{2\alpha u^2 t}{9\beta^2(1-u)}}$$
$$\leq\frac{\frac{\alpha u}{2(1-u)}+\left(\frac{u}{2t}(1-c_i)+\frac{1-u}{\alpha t}\right)r_i}{\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)}, \tag{A3.10}$$

where $i, j = a$ or b, $i \neq j$.

We reorganize Condition (A3.10) as follows:

$$\left[\frac{\left(\frac{u}{2t}(1-c_i)+\frac{1-u}{\alpha t}\right)}{\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)}-\frac{1}{\beta}\frac{\frac{u}{2\beta}(1-c_i)+\frac{3(1-u)}{2\alpha t}}{3\left(\frac{u}{2\beta}+\frac{1-u}{2\alpha t}\right)-\frac{2\alpha u^2 t}{9\beta^2(1-u)}}\right]r_i$$
$$+\frac{1}{\beta}\frac{\frac{u}{2\beta}(1-c_j)+\frac{3(1-u)}{2\alpha t}}{3\left(\frac{u}{2\beta}+\frac{1-u}{2\alpha t}\right)-\frac{2\alpha u^2 t}{9\beta^2(1-u)}}r_j+\frac{\frac{\alpha u}{2(1-u)}}{\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)}$$
$$\geq 0. \tag{A3.11}$$

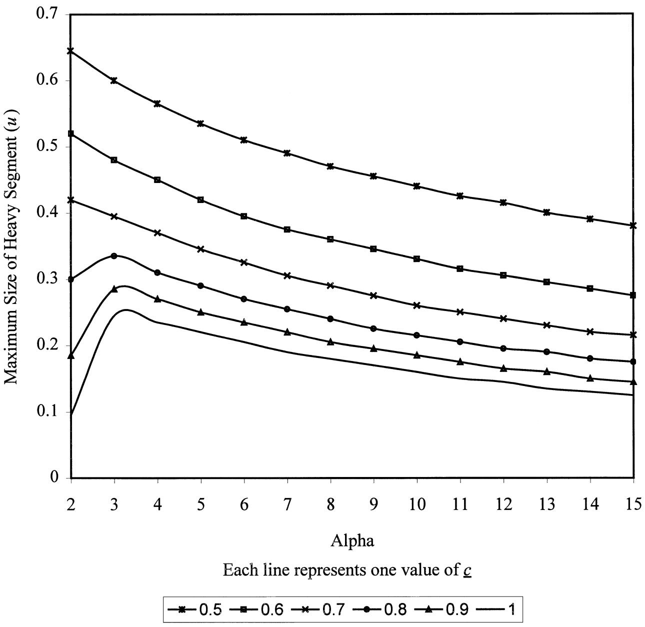
If firm j offers equilibrium reward, Condition (A3.10) (hence (A3.11)) always holds because the left side of Condition (A3.10) becomes negative. (When both firms offer rewards amounts equal to or above equilibrium rewards, Conditions (A3.13)–(A3.15) ensure equilibrium existence.) If firm i offers equilibrium reward, the following condition is sufficient for (A3.11).

$$r^*\left[\frac{\frac{u}{2t}(1-c^*)+\frac{1-u}{\alpha t}}{\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)}-\frac{1}{\beta}\frac{\frac{u}{2\beta}(1-c^*)+\frac{3(1-u)}{2\alpha t}}{3\left(\frac{u}{2\beta}+\frac{1-u}{2\alpha t}\right)-\frac{2\alpha u^2 t}{9\beta^2(1-u)}}\right]$$
$$+\frac{\frac{\alpha u}{2(1-u)}}{\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)}\geq 0. \tag{A3.12}$$

Thus, we have developed a set of sufficient conditions for equilibrium existence under which these existence conditions will not be violated with any unilateral deviation in rewards. These restrictions are given by Equations (A3.9.1–A3.9.3) and (A3.12).

Existence of Nonswitching Equilibrium with Any Deviation in First-Period Price. A firm's unilateral deviation in first-period price will change the firm's first-period market shares, and hence create potential problems for existence Conditions (A2.4), (A2.5.1–A2.5.2) and (A2.6.1–A2.6.2). At this stage, we suppose both firms offer equilibrium rewards (r^*) or above. We next establish a set of sufficient conditions under which these existence conditions will never be violated with any deviation in first-period price. These sufficient conditions are developed with $|\sigma^a_{1,H}-\sigma^b_{1,H}|\leq 1$.

Graph A Maximun μ Satisfying All Sufficient Conditions



First, to satisfy Condition (A2.4) it is sufficient to have

$$\frac{2\alpha u}{3(1-u)} + 1 \le \frac{r^*}{t}. \tag{A3.13}$$

Second, to satisfy (A2.5.1–A2.5.2), it is sufficient that

$$\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)+\frac{\alpha u}{2(1-u)} \le \left(\frac{u}{2t}+\frac{1-u}{\alpha t}\right)r^*. \tag{A3.14}$$

Finally, to satisfy (A2.6.1–A2.6.2), it is sufficient that

$$\left(\frac{u}{2}+\frac{1-u}{2\alpha}\right)\left(2+\frac{\alpha u}{3(1-u)}\right)-\frac{\alpha u}{2(1-u)} \le \left(\frac{u}{2t}(1-c^*)+\frac{1-u}{\alpha t}\right)r^*. \tag{A3.15}$$

Conditions (A3.13), (A3.14), and (A3.15) combined are sufficient to ensure that any unilateral deviation in first-period price will not violate existence conditions developed earlier.

Combined Parameter Restrictions. Combined parameter restrictions (A3.3), (A3.5.1–A3.5.2), (A3.9.1–A3.9.3), and (A3.12)–(A3.15) are too complex to glean insights. Therefore, we conduct a numerical simulation, searching for feasible values of $\{u, \alpha, \underline{c}, \bar{c}\}$ that *jointly* satisfy (A3.3), (A3.5), (A3.9.1–A3.9.3), and (A3.12–A3.15). We choose values for u from 0 to 1 with step 0.01, α from 0.2 to 15 with step 0.1, $\underline{c}$ from 0.1 to 1 with step 0.1, and $\bar{c}$ equal to 1. We find that a maximum value of u needs to be imposed. We plot the maximum u with respect to the value of α and $\underline{c}$ in Graph A. In general, restriction on u becomes more strict with a larger value of α or a larger value of $\underline{c}$. This result is consistent with Klemperer (1987a, b), where a similar restriction was placed on size of the heavy segment. Intuitively, a smaller size of the heavy segment implies less incentive for firms to compete aggressively on heavy users. Therefore, one can find a sufficiently small heavy-user segment that it is not profitable to attempt to switch these heavy users because of loss of rent from the larger light-user segment. Klemperer has relied on this intuition to rule out excessive unilateral deviation to cause a switch. We find that the extensive analysis performed above reinforces this intuition in the context of our model.

Appendix 4. Firms' Profits and Consumers' Benefits

Firms' Profits

With reward programs, each firm's equilibrium profit is $\frac{1}{2}(p_1^* + p_2^* - ur^*c^*)$. Recall that firms' benchmark profits is $\pi^0 = \alpha t/(\alpha u + (1-u))$. Substituting in Equilibrium Prices (3) and (6), we find that firms' profits increase with reward programs if and only if

$$\frac{1}{2}(p_1^* + p_2^* - ur^*c^*) = \frac{1}{2}\left[\frac{\alpha t}{1-u} + \frac{\alpha}{\alpha u + (1-u)\beta}(ur^*c^* + 2t) - ur^*c^*\right]$$
$$\ge \frac{\alpha t}{\alpha u + (1-u)}. \tag{A4.1}$$

i) If $\beta \le \alpha$, then (A4.1) always holds.
ii) If $\beta > \alpha$, (A4.1) holds if

$$c^*r^* \le \frac{\alpha^2 t}{(1-u)(\beta-\alpha)(1-u+\alpha u)}\left[\frac{1}{1-u} + \frac{1/3}{\alpha u + (1-u)\beta}\right]. \tag{A4.2}$$

Note that $\beta > \alpha$ implies $g(u, \alpha) < 0$. That is, if inefficient reward is offered, (A4.1) will always be satisfied. Therefore, firms lose from offering reward programs only if a very small $\underline{c}$ is offered. Intuitively, firms compete heavily on rewards and incur sufficiently high reward costs. Since $\beta > \alpha$, light users are very price sensitive; thus reward programs increase market prices very little. Consequently, firms cannot recover reward costs through price increase and can lose profits with reward programs.

Consumers' Benefits

Since reward programs increase prices in both periods, light users' surplus decrease. Heavy users benefit from reward programs if the reward amount is larger than the increases in prices.

$$p_1^* + p_2^* - r^* = \frac{\alpha t}{1-u} + \frac{\alpha}{\alpha u + (1-u)\beta}(ur^*c^* + 2t) - r^*$$
$$\le \frac{2\alpha t}{\alpha u + (1-u)}. \tag{A4.3}$$

Condition (A4.3) holds in most of parameter space. However, (A4.3) would be violated when $g(u, \alpha) > 0$ and both α and u are relatively large, represented as Area II of Figure 3.

We conducted extensive numerical simulation within the parameter space where all sufficient conditions in Appendix 3 hold, and find that all four scenarios in Figure 3 are nonempty.

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