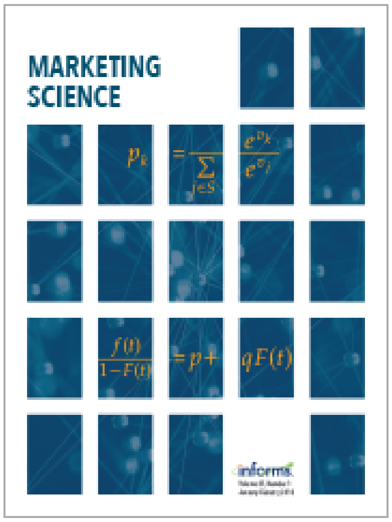

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The Evolution of Influence Through Endogenous Link Formation

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Abstract. Marketing researchers and practitioners are interested in targeting individuals in social networks who may have disproportionately higher levels of influence over others in their network. While the extant literature suggests individual characteristics or network position as proxies for relative influence, our study bridges these two streams by investigating the endogenous acquisition of network position as a function of exogenous individual characteristics. Specifically, do those with higher expertise achieve higher influence when people endogenously choose those to whom they listen? Using an agent-based modeling simulation framework, we model the dynamics of two types of individuals, i.e., independents with exogenous information and imitators. Over the course of multiple diffusions, agents choose whom to "listen to" for information; dropping less useful ties and adding new ones. We find that independents can have less influence (out-degree) than imitators who collect information from multiple sources. Furthermore, this effect is exacerbated by homophily. Noise in communication channels, on the other hand, moderates these effects, yet can increase penetration rates. We show that our results are robust to alternative dynamic network structures. Our research suggests that marketers should consider the environment, community characteristics, communication medium, and product domains when assessing the relative influence of individuals.

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1. Introduction

> "Knowledge is power and it can command obedience. A man of knowledge can make people obey and follow him . . . ."[1]

As the quote above suggests, it has long been assumed that privileged access to information endows one with disproportionate influence over others. The typical argument for this belief is that asymmetric information will serve some instrumental purpose to earn its owner supra-equitable returns in some cooperative or noncooperative interaction. For example, one would expect that the returns in a one-on-one negotiation are increasing in the ownership of unique, relevant information (Akerlof 1970).

Here, however, we ask whether the same intuition is expected to hold in a network setting. Specifically, we examine whether influence, as measured in terms of ties, should be higher for those in a network who "know more" than others. Interpreting the above quote in a 21st century context, should we always expect that those with more knowledge will have more "followers" in a network? On one hand, one might argue that, again, if information is inherently valuable and, if it is costless to follow someone willing to share that information, then more people should follow them. On the other hand, networks are well known to exhibit complex behavior under fairly straightforward assumptions. Our model focuses on the endogenous choices by agents as to whom to (and not to) follow in a network. This decision is, in turn, influenced by two core components of our model. First, and realistically, we believe, there is a limit on how many others one can follow. Second, there may be varying degrees of homophily in tie formation in the sense that some agents may choose to follow only those who are like themselves while others may be less discriminating. With these two assumptions, we find through a series of theory-building simulation studies that, contrary to popular belief, the ownership of proprietary information (i.e., "knowledge") may, in fact, result in lower, not higher, influence in the network. We argue that a primary reason for this is the preference for mediated access to multiple sources of information: Rather than following three "experts," one may prefer to follow just one agent who is adept at finding and linking to the three experts. Given network size constraints, this allows for a more efficient process of tie formation.

We identify several moderating factors in our theory. First, we find that the network structure of those with information has an important impact. Specifically, when independents listen exclusively to other independents, i.e., they exhibit homophily in their network formation, they have less access to information, and can be less influential over generations of ideas than some imitators. In another extension, we characterize the communication channel between agents in the model as being imperfect, occurring only with noise. Here, we find that the redundancy and access benefits one gains by linking to indirect sources is offset by the compounded noise that plagues the information as it spreads through mediated ties in the network. Linking directly to the information source gives one access to "cleaner," albeit perhaps less, information. This result provides an interesting and empirically testable hypothesis: The precision of communication channels decreases the influence of independents relative to others.

Our results provide key insights for academics and practitioners with respect to understanding influence, information, and the diffusion of ideas in a network context. We demonstrate that the relationship among these constructs is complicated. More specifically, high (low) network influence should not be seen as an indication of knowledge or expertise. In fact, we predict an inverse relationship between the two in some contexts. Moreover, we show that the likelihood that those with high knowledge will have lower influence is particularly high when these experts build a network characterized by homophily and when communication is noisy. Finally, from the perspective of a marketer, these results strongly suggest that one should not necessarily focus on recruiting the (possibly, expensive) high-information "experts" to launch viral campaigns. Again, expertise may be a poor proxy for influence. Instead, focusing on those who are one step away from experts may be at least as effective and potentially much more efficient. More generally, the results imply that estimates of *long-run* influence in a network should take into account individual linking decisions and the important relationship between individual characteristics and network position.

The paper is organized as follows. In Section 2, we review the related literature. We present our model in Section 3 and the main results in Section 4. We conclude in Section 5 with a summary of our results, a discussion of the limitations of our analysis, and suggestions for future research.

2. Related Literature

Early researchers defined opinion leaders as "individuals who were likely to influence other persons in their immediate environment" (Katz and Lazarsfeld 1955, p. 3). The strength of word-of-mouth (WOM) communication has been studied in a variety of contexts including online reviews (Chevalier and Mayzlin 2006), restaurant recommendations (Godes and Mayzlin 2009), personal products (Reingen et al. 1984), diffusion of antibiotics (Coleman et al. 1957), women's clothing (Vernette 2004), and cosmetics (Coulter et al. 2002). Researchers have theorized that some users, often called opinion leaders, can disproportionately affect the diffusion of innovations (Rogers 1995, Goldenberg et al. 2001) as well as social learning (Ellison and Fudenberg 1995). More recent studies have investigated the existence and impact of opinion leaders in social networks (Weimann 1991, 1994; Goldenberg et al. 2001, 2006; Watts and Dodds 2007). These studies suggest that, although WOM affects diffusion, there need not be opinion leaders who disproportionately influence others to drive widespread adoption (Watts and Dodds 2007).

Another stream of research has focused on network actors with specific individual-level characteristics such as independent knowledge of a domain. This information may refer, for example, to whether to adopt, where to shop, what to wear, etc. Weimann (1994) finds that opinion leaders can be of either gender, all professions, all social classes, and from all age groups. As a result, he suggests that opinion leaders exhibit a combination of personal and social characteristics.

Recently, researchers have suggested that structural outcomes such as out-degrees are connected to individual or node-level characteristics through a process of link formation. Barabasi and Albert (1999) show that preferential attachment in the World Wide Web leads to a power-law distribution of links. Preferential attachment occurs across a variety of natural and manmade contexts because newer nodes may link to existing nodes with more links. Toubia and Stephen (2009) show that online stores engage in preferential attachment due to node attributes such as product assortment that drive the network to a power-law distribution. In addition, low communication cost can promote endogenous link formation, bringing together agents with similar interests, but separating neighbors who are physically close (Rosenblat and Mobius 2004).

Our investigation extends the endogenous link formation literature by providing a theory about the nature of the relationship between the locus of information and dynamic network structure. To our knowledge, our theory is unique in that it suggests that access to information may not imply more influence, ceteris paribus. In fact, we show that the opposite may be true.

2.1. Theory Building Through Agent-Based Modeling

Our goal is to develop a theory of the dynamics of influence using agent-based modeling (ABM), a form of simulation. Davis et al. (2007, p. 480) outline that "simulation(s) can be a powerful method for sharply specifying and extending extant theory in useful ways."

Rand and Rust (2011) provide an overview of ABM methods in marketing, and a guideline for marketing researchers. They demonstrate that ABM can provide additional insights by exploring more complex models, which is otherwise not possible using analytical or empirical models. Simulation methods such as Markov Chain Monte Carlo (MCMC), system dynamics, NK fitness landscape, genetic algorithms, cellular automata, stochastic processes and ABM (Libai et al. 2013) have been used to build theory and develop causal mechanisms in the business and economics literature. Davis et al. (2007, p. 485) elaborate in their meta study on the value of simulations, arguing that "interactions are often difficult to study with traditional statistical methods or to anticipate with thought processes."

ABM is a common tool for addressing a variety of complex problems including NP-hard problems, large stochastic systems, feedback systems or social network systems where tractable solutions are not feasible or require even greater computational power. ABM is a particularly good fit for our problem, and is becoming a popular methodology for network and diffusion researchers (e.g., Watts and Strogatz 1998, Boccara and Fuks 1999, Goldenberg et al. 2001, Watts et al. 2002, Immorlica et al. 2007, Watts and Dodds 2007, Libai et al. 2009), marketing researchers (Watts and Dodds 2007, Libai et al. 2013) and computational social scientists (Lazer et al. 2009). In addition, Epstein (2006) argue that ABM provides a powerful tool for researchers to study generative processes in social science. As in other ABM studies, while we specify simple rules at the individual level, significant complexity arises from three sources, i.e., (1) the interactions among individuals, (2) the stochastic nature of the diffusion process, and (3) the network dynamics through a link formation process. The ABM approach allows us to investigate the aggregate network effects of individual decisions on adoption and linking. It would be extremely difficult to capture the richness of these interactions in an analytical setting without substantially simplifying the model and, especially, the dynamics.

ABM has been heavily used in mathematics and natural and social sciences. Mathematicians simulate large Markov Chains to find steady states and transition times (e.g., Logofet and Lesnaya 2000). Watts and Strogatz (1998) simulate a small-world network to show the quick spread of information through a network characterized by high clustering. Barabasi and Albert (1999) show that scale-free networks emerge by specifying a model with preferential attachment and that a number of natural and man-made phenomenon may follow such a process. Physicists use ABM to simulate osmosis of water particles to confirm Wilhelm Pfeffer's 1877 experiment (Murad and Powles 1993). Computer scientists frequently use ABM to analyze computer networks including the performance (Hong 2001) and robustness (Albert et al. 2000) of peer-to-peer networks. In the social sciences, evolutionary game theorists simulate large populations to show that norms can evolve (Axelrod 1986). Operation researchers use simulations to derive optimal policies for elevators due to their highly complex and stochastic nature (Siikonen 1993). Finally, marketing academics are using simulations in a variety of theory building approaches, typically involving complex network interactions (Goldenberg et al. 2001; Burks 1971; Ganguly et al. 2003; Boccara and Fuks 1999; Guseo and Guidolin 2009; Libai et al. 2005, 2010; Peres and Van den Bulte 2010).

3. Model

We build our theory by specifying a simple model of the diffusion of a series of innovations through a network. The network evolves dynamically as agents choose those ties that will best position them to learn about and benefit from these innovations. Although our model is similar to that of Watts and Dodds (2007), we add dynamics, repeated diffusions, endogenous network choice, and moderating factors. See Appendix A for a summary of parameters.

3.1. Agents and Adoption

We model a world with K agents who are faced with the decision of whether to adopt an "idea," m. The concept of an idea is meant to capture a broad range of potential adoption decisions including products, services, clothing styles, language idioms, traffic patterns, etc. The idea could be a fashion product, for example, such as bell-bottoms or skinny jeans; a position on a political question or candidate for office; or a media product such as a song or movie. We capture the adoption decision by agent i in period t for idea m by $a_{mit} \in \{0,1\}$. We assume that adoption, if chosen, is costless to the agent, and that the action is observable by others in her network. Because ours is a monopoly model in that there exists a single idea at any given time, we expect that the introduction of prices would have little qualitative impact on the results, serving primarily to slow adoption. All agents who have not yet adopted an idea as of period t choose in that period whether to adopt. Once an idea is adopted, an agent cannot unadopt: $a_{mit'} = 1 \rightarrow a_{mit} = 1, \forall\, t > t'$.

We define two types of agents, $\alpha \in \{0,1\}$ proportion of "independents" and $1-\alpha$ proportion of "imitators." The former stochastically receives information from outside the network and adopts, while the latter only adopts through listening to others. One might imagine that independents acquire this information from a variety of sources including, for example, formal education, research, intuition or even firm-backed marketing efforts such as direct marketing. We might think of independents as highly educated people in some contexts, e.g., doctors, lawyers, academics, or simply those with a high degree of domain-specific knowledge such as fashion-forward people, industry leaders or fans. Clearly, the nature and prevalence of expertise will be a category-specific construct. Our definition of independents and imitators is similar to the notion of "independents" and "pure imitators" (Van den Bulte and Joshi 2007), respectively, in that imitators solely depend on hearing information from others. In the Bass (1969) terminology, independents have a positive innovation term, e.g., $p > 0$, while imitators have a zero term for innovation, e.g., $p = 0$. In our base model, we assume that independents do not listen to others, or that $q = 0$. We then relax this assumption.

3.2. Adoption Dynamics

Adoption differs across agent type. Independents receive information about an idea with probability p_e. When this occurs, they immediately adopt since the precision of the signal is assumed to be perfect in that, conditional on receiving such a signal, the innovation always exists. A value of $p_e = 1$ indicates a world with perfect independents who always receive information about ideas. Conversely, a world where $p_e = 0$ is one in which independents receive no information, and are thus equivalent to imitators.

Imitators become aware of the idea only through observing their peers adopt. They do not, however, know whether their peers' adoption is due to receiving outside information, on one hand, or a consequence of her peer's observational learning, on the other (Reingen et al. 1984, Zhang 2010, Banerjee 1992, Bikhchandani et al. 1992).

We model "complex contagion" (Centola and Macy 2007, Watts and Dodds 2007) whereby agents adopt when the proportion of their peers who have adopted surpasses a fractional threshold. While "simple contagion" occurs as a result of a single tie's adoption, complex contagion implies a higher (and possibly heterogenous) threshold. Centola and Macy (2007) identify the following four underlying mechanisms in complex contagions: (1) *strategic complementarity*: where adoption due to awareness is not sufficient, cost and risk of adoption decreases over time, and some domains require a "critical mass" to gain benefits; (2) *credibility*: where innovations are unproven and adoption from multiple sources acts as confirmation; (3) *legitimacy*: having peers adopt facilitates acceptance, especially for fashion or in risky behavior such as which body parts to pierce; and (4) *emotional contagion*: adoption by peers can facilitate bonding or emotional identity. Furthermore, the threshold rule can account for noise such as agents with imperfect memory; one may not remember whether she observed a tie's adoption.

To determine whether to adopt, the agent may engage in a form of "consensus" voting using a proportional threshold. An advantage of this is that a tie's adoption and nonadoption can affect the agent's decision (Centola and Macy 2007). For example, when choosing whether to adopt bell-bottom pants, the probability does not increase simply as one's network size increases but is based on the relative proportion of adopters and nonadopters (e.g., those who do and do not wear skinny jeans). The proportional threshold requires that more peers adopt as the network increases. By contrast, a simple count threshold ignores nonadopters and tallies only adoption. For example, in diseases, the probability to "adopt" a disease does not depend on one's network size, but on the number of peers to whom the agent is exposed. For our context, complex contagion with proportional thresholds is appropriate for the spread of *ideas* in the marketing setting.

Formally, we define x_{mit} as the proportion of agent i's neighbors who have adopted idea m before period t. We assume that agent i adopts if x_{mit} exceeds a population threshold τ

$$\text{Prob}(a_{mit} = 1 \mid a_{mi(t-1)} = 0) = \begin{cases} 1 & x_{mit} \geq \tau, \\ 0 & x_{mit} < \tau. \end{cases} \tag{1}$$

This simple rule implies that the probability of adoption increases (weakly) monotonically with the number of neighbors who have adopted (Watts and Dodds 2007, Lopez-Pintado and Watts 2008, Morris 2000, Schelling 1973); this seems to capture the salient characteristics of observational learning. Importantly, one may interpret the threshold rule as a "voting process" in which a nonadoption is a like a "no" vote (Centola and Macy 2007). So, imagine that an agent has two friends who have adopted an idea. If the agent has a total of three friends, this is a very strong signal of the quality of the idea (i.e., two yes votes and one no vote). On the other hand, if the agent has 25 friends, then the fact that only two have adopted is a much weaker signal about the idea.

Note that when $p_e = 1$, when independents always adopt new ideas, followers will be better off always listening to independents. That is, if I listen to an independent, I cannot do better, in terms of receiving more information across ideas, by listening instead to imitators. However, when $p_e < 1$, agents may get more information by listening to more than one independent. For example, consider a doctor who is an independent in the medical field and who may be knowledgeable about 80% of conditions through her training. In our model, her probability of adoption is 0.8. By contrast, an imitator who listens to two independents, each with a probability of adopting of 0.8, will adopt new ideas with probability $1 - 0.2^2 = 0.96$. As a result, if I listen to only one of these agents, I will likely prefer the imitator. This simple case illustrates that agents may be better off linking to an imitator than to a single independent agent under some conditions.

3.3. Network

Agents $k \in \mathcal{K}$ are connected via links $e_m^{ij} \in \mathcal{E}_m$ and live in the graph $G(\mathcal{E}_m(\delta), \mathcal{K})$ where $\mathcal{K}$ is the set of nodes (agents) and $\mathcal{E}_m$ is the set of directed links, or edges, for agent j to listen to i for idea m. Note that while in the baseline static model, $\mathcal{E}_m = \mathcal{E}, \forall\, m$, links in subsequent models are idea-specific: This provides the endogenous dynamics on which our model is focused. The set of links is parameterized by δ, which will capture the degree distribution. Although the links are directed in the flow of information, bidirectional relationships are stochastically allowed via reciprocated links. We initialize the graph as an Erdős–Rényi random graph (Erdős and Rényi 1959).

Although for static networks the choice of a graph structure G is an important modeling decision since different structures can lead to different outcomes (Watts and Strogatz 1998, Reingen and Kernan 1986, Brown and Reingen 1987, Bala and Goyal 2000, Abu-Ghazzeh 1999), our study on dynamic networks is less sensitive to the initial graph structure. Thus, a random graph initialization is likely to be the most agnostic starting point since (a) it does not impose any specific structure; (b) multiple iterations will be run over different realizations of the graph; and (c) our endogenous dynamics theoretically allow for the emergence of any long-term structure. Researchers often use random graphs as a base comparison to show increased speed of diffusion in small world networks (Watts and Strogatz 1998) and scale free networks (Barabasi and Albert 1999).[2]

We initialize the random network as follows: For a given agent i, we draw an in-degree, $\rho_i \sim Pois(\delta)$, with population mean δ. For ego agent i, we then randomly draw a set of agents of size ρ_i from the set $\{k \mid k \in \mathcal{K}/i\}$ to represent those agents to whom i listens. Allowing ρ_i to vary at the agent level ensures that the model reflects agent-level heterogeneity such as the amount of resources the agent has to maintain links to others. Our use of the Poisson distribution is also common (Myerson 2000) due to its relatively low variance and high weight on low values. Defining $e_m^{ij} \in \{0,1\}$ as agent i listening to j during idea m, let $E_1^j \equiv \{i\colon e_1^{ij} = 1\}$ and $n_j = |E^j|$. That is, n_j is agent j's out-degree, i.e., the number of people who listen to agent j for information. As discussed in more detail in the next section, we assume that ρ_i, or in-degree, is held constant within agents across ideas, while the out-degree or influence, n_j, varies.

3.4. Link Formation Dynamics

In this section, we describe our endogenous link formation (ELF) process whereby agents form and sever links.

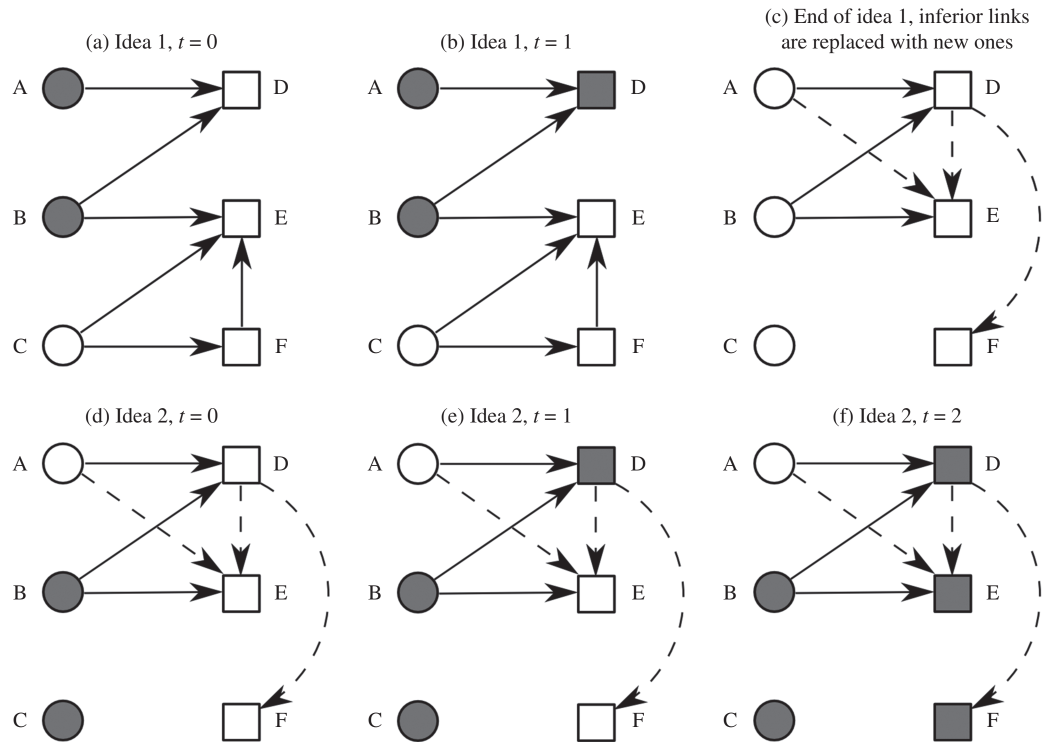
There are two sources of dynamics in our model, i.e., observational learning and endogenous link formation. Agents learn from their peers through observational learning about the existence of an idea and decide whether to adopt it. They also make decisions about whom to include in their network by dropping connections, incoming links, which they deem to be inferior and forming new connections with others that may offer better information. This occurs in between the arrival of new ideas. Let $\sigma_{mi} = 1$ if agent i receives outside information at time $t = 0$ about idea m and $\sigma_{mi} = 0$ otherwise. If $\sigma_{mi} = 1$ then $a_{mi0} = 1$: Agent i adopts at time $t = 0$. Imitators engage in observational learning from $t = 1$ up to period T, when the idea's diffusion ceases. For a given idea m, in each period t, agent i observes how many of her peers adopted the idea in the previous period. If she adopts in period t, her peers will observe her adoption in period $t + 1$ and afterwards. We illustrate these learning dynamics for idea 1 in Figure 1(a) and 1(b), and for idea 2 in Figure 1(d) through 1(f).

The core model defined thus far closely follows that used by Watts and Dodds (2007). Our main point of departure from this earlier work is in our ELF process, which occurs over repeated ideas. Here, we model information seeking agents who maintain links that give them the most potential information on new ideas. Because agents' types are not known, agents build their beliefs of their peers based on historical information from past idea adoption. Specifically, between the completion of the diffusion of idea m and the commencement of the diffusion of idea $m + 1$, agents may drop and add links. Given their information seeking motivation, agents seek links to other agents with better information, and drop those with less information. We adopt the following simple link-formation rule:

$$\text{Prob}[e_m^{ij}=1]=\begin{cases}1 & e_{m-1}^{ij}=1,\ a_{(m-1)iT}=1,\\ (K-\rho_i-1)^{-1} & e_{m-1}^{ij}=0,\\ 0 & e_{m-1}^{ij}=1,\ a_{(m-1)iT}=0.\end{cases} \tag{2}$$

This rule captures the following intuitive process: If my peer adopted idea $m - 1$, then I will continue to maintain her in my network. If she did not adopt, this is a negative signal about her information quality. Therefore, I will delete my link to her. When links are deleted, I replace them with new links to others chosen at random. A key assumption here is that agents maintain the same in-degree, ρ_i: They "listen to" the same number of people, even though those people may change across ideas. Previous researchers have shown that the number of friends one has tends to remain stable (Phan and Airoldi 2015, Bass and Stein 1997, Rapkin and Stein 1989) over time because of resource constraints to sustain relationships. Dunbar (1992) goes so far as to suggest that social circles are constrained by the cognitive processing capabilities of the primate brain size. Goncalves et al. (2011) find evidence that modern humans can entertain around 100–200 sta-

Figure 1. Awareness Dynamics of Independents (Circles) and Imitators (Squares) with a Threshold of $\frac{1}{2}$



Notes. (a) In idea 1, nature stochastically picks independents A and B to receive information, and they adopt at $t = 0$. (b) At $t = 1$, imitator agent D adopts from observing A and B adopt in the previous period. No more agents adopt. (c) At the end of idea 1, imitators drop peers who do not adopt, and listen to new random agents (dashed lines). Agent E drops C and F, and replaces them with agents A and D. Agent F drops ties with C, and listens to agent D. Note that agent D now has two followers despite not being an expert. (d) Stochastically, independents B and C receive information and adopt idea 2 at $t = 0$. (e) At $t = 1$, agent D adopts from observing B adopt. (f) At $t = 2$, agents E and F adopt from observing their peers for an increased penetration rate of $\frac{5}{6}$.

ble relationships using Twitter. This constraint may be driven by the cost to maintain these relationships. In addition, the constraint, which we call "link conservation," enables us to maintain the heterogeneity across agents even as the model dynamics emerge. More specifically, it ensures that those agents who are more "social" or "outgoing," i.e., those with more ties, maintain their distinctiveness throughout the model. We stress that link conservation relates only to in-degree but not out-degree. While each agent i will only listen to ρ_i others throughout the model, the number of others listening to the agent is not exogenously imposed by the researchers. Indeed, it arises endogenously out of the interactions in the model. Thus, if agent i deletes a link, she randomly selects a new link from the $K - \rho_i - 1$ nodes to whom she was not linked during idea m. We interpret deleting the communication link as merely that agent i changes her focus and attention for information from one agent to another; she is not necessarily terminating social ties or other ties outside the domain. Figure 1 demonstrates learning and endogenous linking dynamics across two ideas for six agents.

Our assumption that agents find new sources of information using a random rule is consistent with the theoretical research in network formation and provides a suitable baseline for comparison against other link formation strategies that may require additional assumptions. In Section 4.5 and the online appendix, we compare alternative strategies such as those involving homophilous link formation. Random graphs have been used in graph theory, mathematics (Erdős and Rényi 1959), economics, and marketing as a de facto standard baseline in theoretical models. Roth and Vande Vate (1990) used random matching in a theoretical model to demonstrate that marriage markets converge to a stable outcome. Chen et al. (2016) demonstrate that random matching of buyers and sellers converges to a competitive market. In marketing, Mayzlin and Yoganarasimhan (2012) use random matching to build a theoretical model of readers choosing which blogs to follow. Katona and Sarvary (2009) model web surfers who randomly move from site to site. In the marketing influence literature, Van den Bulte and Joshi (2007) allow imitators to randomly mix with imitators and independents.

Furthermore, the prominence of stochasticity as a driver of link formation is consistent with empirical evidence. In a large study on network formation, Kossinets and Watts (2006) conclude that "it is unclear to what extent individuals are capable of strategically manipulating their positions in a large network, even if that is their intention" (p. 90). Indeed, the empirical evidence in marketing yields similar findings. Iyengar et al. (2011), for example, find there is no significant correlation between status (in-degree) and the doctor's employment at a University Research hospital (see Table 3, p. 205). If we consider the latter to be indicative of "better information," then it might be surprising that it is not associated with a more influential network position. Our model provides an explanation for why this might be the case.

Of course, we acknowledge that there may be observables in many contexts on which a link formation may be based. However, we suggest that our model be considered "conditional" on these observables in some sense. To make this more concrete, consider the following: Imagine there are two "types" of people, i.e., blue and green. Blue (green) people only prefer to connect to other blue (green) people. Note that the inverse could also be true, with the same results. Such preferences might be driven by substantive reasons (e.g., blue information is relevant only for blue people) or for frivolous reasons (I just like how green people look). If the type is completely unobservable, matching would be random and ties would be chosen as we model it. On the other hand, assuming that type only partially predicts the value of the tie, then there would first be sorting on this variable. In such a context, our model could be seen to capture the residual non-type based differences across people following this sorting. That is, blues would first eliminate all greens from consideration and vice versa. Our model would then work exactly as is currently the case within the set of potential ties that have survived the first-stage culling. Via experimentation, the agents would then determine who among their desired type would provide the best information. For this reason, our assumption should not be considered particularly central.

Finally, because our results may be driven by stylized dynamics rules, we extend our study against three variants of our base model. In Section 4, we investigate our core dynamic model and compare it to a static model such as that presented by Watts and Dodds (2007). Furthermore, we study how independents listening to others and their homophily affect their influence. Section 4.2 examines how noisy communication channels affect the independents' influence. In Section 4.3, we incorporate a time discounting factor into our ELF dynamics. Finally, in Section 4.4 we investigate whether our core results hold when agents use a multi-idea ELF rule and can engage in ELF at any period rather than only at the end of an idea.

4. Results

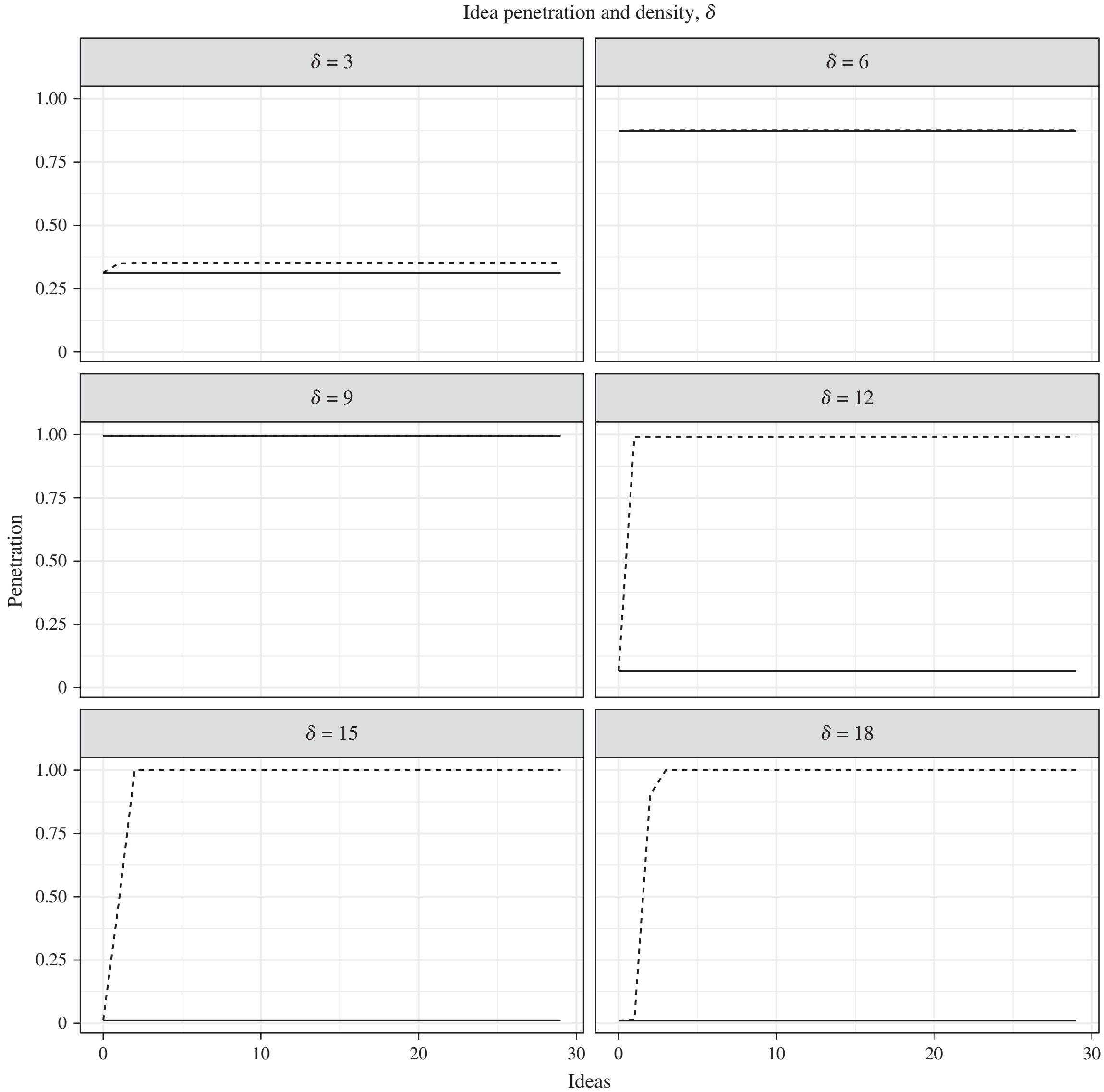
4.1. Base Model—Endogenous Linking Dynamics

We begin by analyzing the base model. We compare the outcomes of our ELF model to that obtained in a static network where ties do not change from one idea to the next (i.e., steps 7 and 8 of the algorithm are not performed, see Appendix B.1). In the latter setting, researchers have established that higher network density, i.e., a higher average number of ties, results in the faster spread of information. However, more ties may also inhibit adoption. This may foster earlier adoption in a threshold model since the higher one's number of connections, the greater the number of peers who would be required to activate an agent beyond her threshold (Watts and Dodds 2007). As a result, in the static model, an idea's penetration rate, i.e., the long-run proportion of adopters, initially increases but then decreases with network density, thus forming an inverted-U shaped pattern (Watts and Dodds 2007). Indeed, the solid lines in Figure 2 show the final penetration rate with the inverted-U shaped pattern as described, for densities of 3, 6, 9, 12, 15, and 18.[3] Note that varying the parameter quadruples, i.e., α, p_e, δ, τ, can yield penetration rates of full or no penetration. However, we focus on the margins and on the relative, rather than absolute, value of the relevant parameters (Watts and Dodds 2007, Goldenberg et al. 2001). We have extensively tested against the range for these values and find the results to be consistent with those reported and with our main effect that independents can lose influence. Thus, as expected, our model yields results equivalent to the previous literature when constrained to a static network structure.[4]

The results, however, are quite different from static results once we allow for endogenous linking decisions. In particular, the final penetration rate monotonically increases with network density in ELF networks. Here, increased network density increases the final penetration rate within a few generations of ideas. In Figure 2, ELF networks start with a penetration rate that is similar to static networks in the first generation, i.e., 0.3, 0.8, 1.0, 0.1, 0.01, and 0 for densities 3, 6, 9, 12, 15, and 18, respectively. However, with each successive idea, the penetration increases. At $\delta = 3$, a fairly low level of network density, the evolving network reaches a maximum penetration of approximately 0.3 within four ideas. At a moderate level of density $\delta = 9$, the ELF network attains a final penetration rate of 1.0 within one generation of ideas. As Watts and Dodds (2007) stipulate, the network is sufficiently dense that imitators have easy access to others with information. However, in a highly dense network such as $\delta = 15$, the ELF model differs from the static network in that it reaches a penetration rate of 1.0 within three ideas from an initial penetration of nearly 0.

By allowing agents to adjust their network ties, a very realistic aspect of a network, we find that ideas travel faster and more broadly. After the initial idea, imitators engage in ELF where they replace links to peers who did not adopt with new randomly selected links. The

Figure 2. Dynamic Networks (Dashed Line) Benefit More from Higher Densities Than Static Networks (Solid Line)



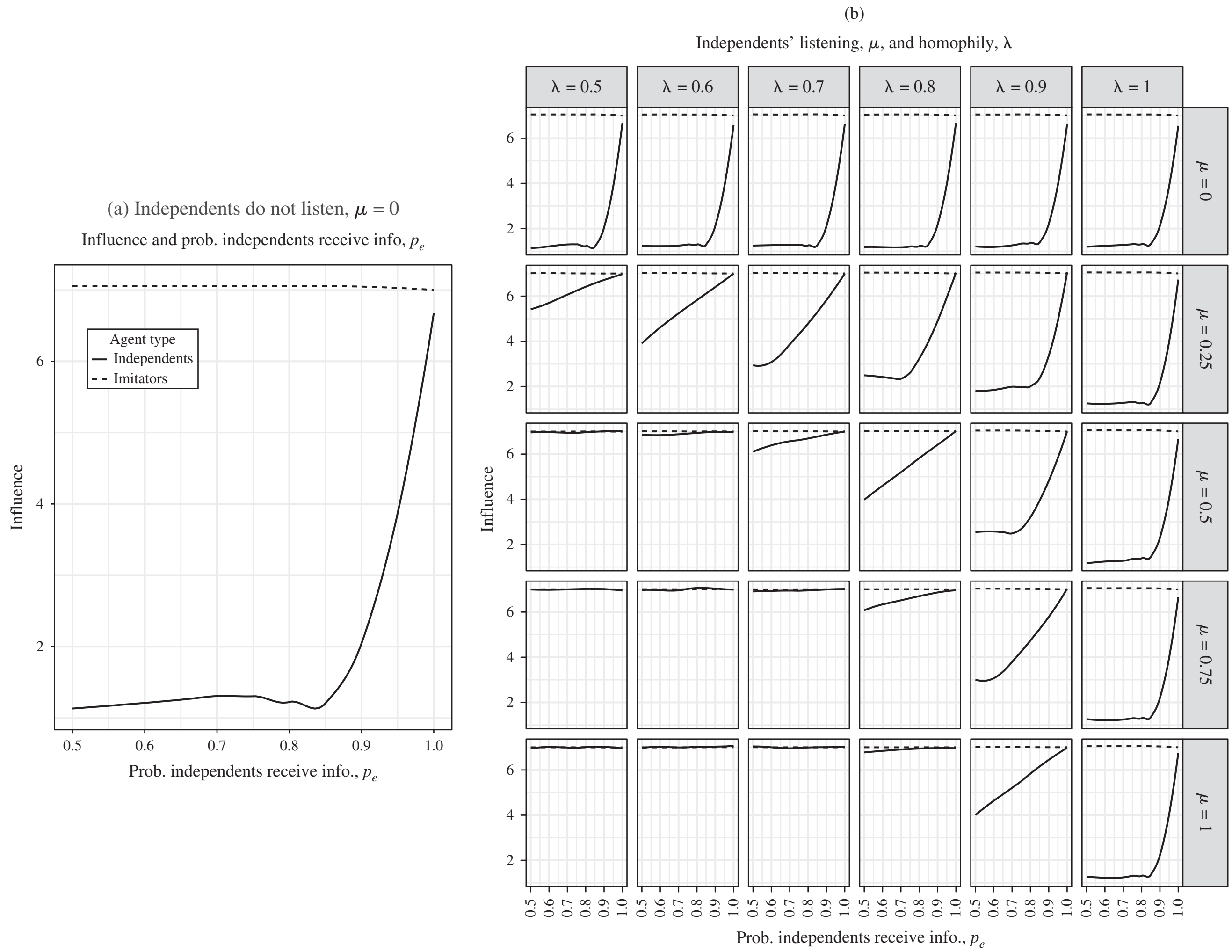
Notes. The final penetration rate for static networks exhibits an inverted U-shape with increasing density, while evolving networks monotonically increase penetration. The full parameters are ($K = 10{,}000$, $T = 50$, $M = 50$, $S = 30$, $p_e = 1.0$, $\alpha = 0.01$, $\tau = 0.18$, $\mu = 1$, $\lambda = 1$).

results show that our model diverges from the Watts and Dodds (2007) static model due to our consideration of endogenous linking in a dynamic setting.

Next, we discuss our research on the impact of access to information on one's influence in a network. We are particularly interested in the number of followers, or "out-degree," as the measure of an agent's influence. Recall that while an agent's in-degree, i.e., the number of people to whom they can listen, is fixed across ideas and varies across individuals, the out-degree can vary greatly as a result of ELF. For example, in a small network of three agents, A, B, and C each listen to one other agent, but may have 0 to a maximum of two followers each based on the quality of their information. Although one could imagine a broad set of alternative metrics for influence,[5] we adopt this measure for the sake of simplicity as a way to measure and demonstrate how influence evolves. Moreover, this is a common metric currently used to evaluate one's influence on- and off-line (Watts and Dodds 2007).

Our main result is presented in Figure 3(a), which shows the long-run influence, as measured in out-degree (y-axis), over multiple diffusions for independents (solid line) and imitators (dashed line) as a function of the "quality" of their information, p_e (x-axis). We find that imitators, i.e., those with less information at the outset, will, on average, have more influence in the long-run dynamic network. Moreover, the difference between the two is decreasing in the information quality, p_e. This is consistent with common sense. However, surprisingly in our model, even when $p_e = 1$, independents seem to be no more influential than the

Figure 3. Independents (Solid Line) Can Have Less Long-Term Influence Than Imitators (Dashed Line) When They Do Not Listen to Others



Notes. All results are at a network density, $\delta = 7$. (a) In the base model as the probability that independents receive information, p_e, increases (x-axis), independents lose less influence; at the outset, $p_e = 1$, independents do not have more influence than imitators. (b) Each subplot illustrates the influence (y-axis) of independents (solid line) and imitators (dashed line) for $p_e \in [0.5, 1]$; each column of subplots represents a fixed level of independent's listening, $\lambda \in [0.5, 1]$; each row of subplots illustrates the level of homophily, $\lambda \in [0, 1]$.

imitators. The intuition is quite clear: Independents may have access to relatively less information than imitators who listen to multiple others. In this baseline model, we assume that independents act only on their private information and do not engage in observational learning. We relax this assumption in subsequent investigations.

Next, we further investigate the effects of relaxing the assumptions on the independents' behavior. Specifically, we consider what happens when independents, those with better, albeit imperfect, information, also engage in observational learning. Before presenting these results, note that the literature is quite clear that the process of information acquisition is very different for independents and imitators. Katz and Lazarsfeld (1955) proposed a two-stage model where information can only flow from independents to imitators, while independents get their data from primary sources. Later findings, however, have suggested that, within the physician communities, for example, independents listen to others and that these communications are characterized by high degrees of homophily: Independents listen to other independents (Coleman et al. 1957). In particular, they find that medical doctors know other doctors through common interests and social interactions such as golf. Furthermore, Van den Bulte and Joshi (2007) analyze an asymmetric two segment world of "influentials" and "imitators." By assuming that the former only listen to other influentials, but imitators listen to influentials, they show that the model is a better fit to historical adoption data than previous models.

Consistent with these results, there is reason to believe that independents may: (1) identify other independents, and (2) exclusively listen to other independents. Independents may form exclusive communities or meet other independents through professional conferences and social gatherings. As a result, we augment the baseline model by allowing independents to "listen to" others (i.e., engage in observational learning) with probability μ. Moreover, reflecting the observed homophily among these network actors, we assume that, conditional on listening to others, the probability that independents listen only to other independents is λ. These results are presented in Figure 3(a) where the upper row is simply a reproduction of Figure 3(b) with $\mu = 0$.

In Figure 3(a), as we increase levels of homophily going down the columns, we find that when independents listen to others, their loss of influence is reduced compared to imitators. That is, while independents may have less influence than imitators who collect information from multiple sources, they can gain more influence by listening to others if they do not receive information. Even with low access to information, e.g., $p_e = 0.5$, independents can have relatively similar levels of influence if they sufficiently listen to any other agents, i.e., imitators or independents (i.e., $\mu \geq 0.5$, $\lambda = 0.5$).

We also find that the greater the homophily by which the independents' communication is characterized, i.e., the greater the extent to which they listen primarily to other independents, the greater the disadvantage compared to when they listen to all other agents. Figure 3 demonstrates that at high levels of homophily, e.g., $\lambda \geq 0.9$, independents have fewer followers than imitators, no matter how much they listen to others. Our results suggest that in dynamic influence games, two-segment markets such as those proposed by Van den Bulte and Joshi (2007), independents may attract fewer followers and gain less influence when their networks are characterized by high levels of homophily as compared to a context in which they interact with a broader cross-section of others.

4.2. Noisy Communication

In the previous sections, we show that independents may be less influential (have fewer followers) than imitators under certain conditions. Moreover, we argue that the primary mechanism behind this is based on the independents' imperfect information: Agents benefit more from linking to imitators who listen to multiple sources than to a single expert. If this is the case, factors that interfere with imitators' ability to collect information should decrease their relative influence and increase that of independents. We test this implication of our theory by introducing noise into the observation process. We expect that, at higher levels of noise, independents should become increasingly influential (relative to imitators) because agents will be less effective at collecting information from their sources and transmitting that information to their ties. We define noise as imperfect observability in the learning process. There can be various reasons for communication noise, including misreading a message, misinterpretation, imperfect recall or vague language (Anand and Shachar 2009). For example, offline WOM may be more noisy due to recall issues as compared to email or online channels where agents can search through their communication history.

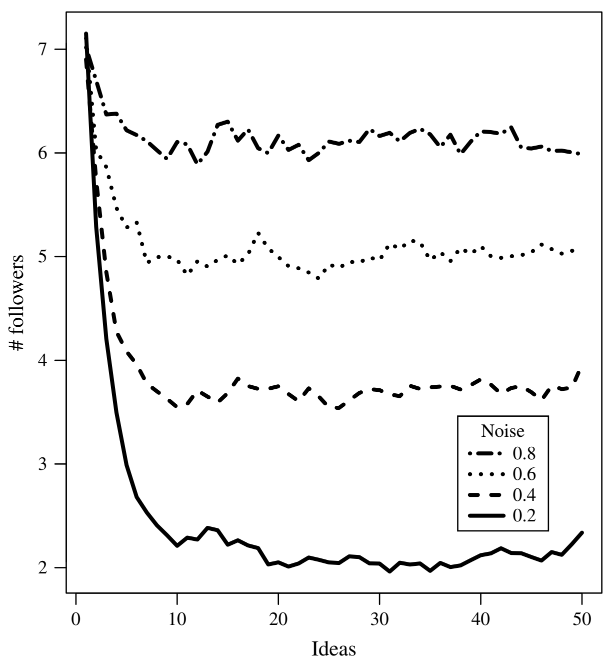
4.2.1. Model. We extend our base model to allow for noise in the observational learning process per communication link and per idea. This relates to steps 5 and 7 in the simulation algorithm.[6] We introduce the parameter ϕ to capture the precision of the communication link. All communication between agents, regardless of whether they are an independent, can be affected by noise. So, with probability ϕ communication link e_m^{ij} between agents i and j for idea m is perfect for the entire idea, and with probability $1-\phi$ link e_m^{ij} is "noisy." When the link is noisy, we randomly "invert" the observation. An inverted communication link will observe an adoption as a nonadoption, and vice versa. For example, assume agent i adopted idea m. With probability ϕ, her tie with agent j, e_m^{ij}, observed her adoption perfectly. With probability $1-\phi$, there is noise. Conditional on the link being noisy, there is a 50% chance that the link is "inverted." Therefore, j observes i's action, ζ_j^i, as summarized in Table 1.[7]

4.2.2. Results. According to our theory, imitators become relatively more influential due to their superior access to information, which they collect from multiple sources. Noisy observation, on the other hand, may interfere with the ability of imitators to collect information as it degrades the quality of their observations and, thus, the informativeness of their adoption. This

Table 1. Summary of Noise and Observation of Actions by Peers

	i's action	
	Adopt, $a_{mi} = 1$	Not adopt, $a_{mi} = 0$
j's observed action of i, ζ_j^i		
i adopted	$\Pr(\zeta_j^i \mid a_{mi} = 1) = \phi + (1-\phi)/2$	$\Pr(\zeta_j^i \mid a_{mi} = 0) = (1-\phi)/2$
i not adopted	$\Pr(\zeta_j^i \mid a_{mi} = 1) = (1-\phi)/2$	$\Pr(\zeta_j^i \mid a_{mi} = 0) = \phi + (1-\phi)/2$

Figure 4. Independents Gain More Followers with Increased Communication Noise, $1-\phi$, at Information Quality, $p_e = 0.7$, and $\delta = 7$



should increase the relative influence of independents because the latter have access to untainted, "perfect" information from exogenous sources. Figure 4 shows that this is the case in our model. We see a positive relationship between noise and the influence of independents. Although imitators still prefer to listen to other imitators, on average, over ideas at every level of noise, the independents' loss in influence declines with the higher noise in the system. The conservation of links suggests that for independents to increase influence, imitator agents must not gain as much influence. We have chosen $p_e = 0.7$ to demonstrate the effects of noise, but have tested a range of different values and obtained equivalent qualitative results.[8] Figure 4 clearly demonstrates the moderating role of communication noise. When one can observe others' adoptions fairly precisely, imitators with ties to multiple independents will be the preferred source of information. On the other hand, when the context is characterized by more noise, the role of independents becomes more prominent. This also demonstrates a core trade-off in forming one's network between the access to higher quantities of information, which one gets via moderated access to independents, and the accuracy of that information, which favors linking directly to independents.

Surprisingly, Figure 5 shows that the total number of adopters (gray) increases with noise. However, fewer imitators who listen to at least one independent, or "1st degree imitators," and second-degree imitators, i.e., those who are separated from an independent by another imitator, adopt with less noise. While greater noise can interfere with information flow and reduce the overall penetration rate in static networks, agents in a dynamic network can adjust their network to better identify direct sources of information. On the other hand, fewer noisy channels such as the Internet and digital communications may reduce penetration because agents are unable to identify original sources of information and signals can be amplified through multipathing: This is called the "echo chamber" effect (Mobius et al. 2017). This result counters the notion that while digital communications can spread quickly and noiselessly, they may hurt the spread of information and adoption overall.

4.3. Time Discounting

Until now, we have assumed that agents equally weigh the time of adoption of their peers. However, in some product categories and domains, agents may give more weight to peers who adopt earlier. For example, for technology and fashion, agents may want to be at the forefront, and hence give more weight to those who adopt earlier. However, in other domains, early adoption may be less important, e.g., taking on risky behavior, engaging in social movements, adopting unproven technology or embarking on risky migration.

4.3.1. Model. We extend our base model to allow for greater weights to be placed on peers who adopt early. We modify our base model to give greater weight for early adopters. For some agent i and peer j who adopt at time $\hat{t}_j$ for idea m, we introduce the time discount factor, $\beta \in [0,1]$, to our link-formation rule, Equation (2), as follows:

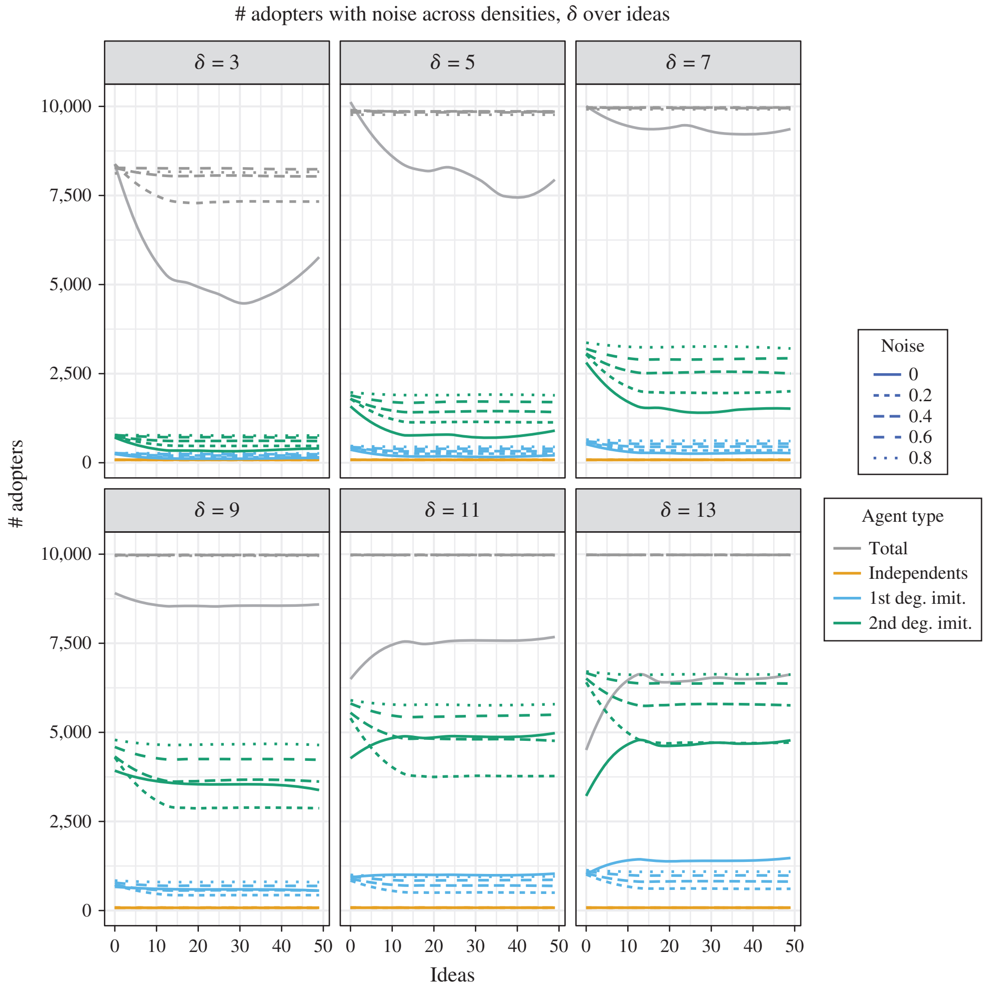
$$\mathrm{Prob}[e_m^{ij}=1]=\begin{cases}\beta^{\hat{t}_j} & e_{m-1}^{ij}=1,\ a_{(m-1)iT}=1,\\ (K-\rho_i-1)^{-1} & e_{m-1}^{ij}=0,\\ 0 & e_{m-1}^{ij}=1,\ a_{(m-1)iT}=0.\end{cases} \quad (3)$$

That is, if my peer is an independent and adopts at $\hat{t}_j = 0$, I will have a probability 1 to retain the link. However, if another peer adopts at $t = 50$, I will keep her with probability β^{50}. If my peer does not adopt, I will drop my link to her.

4.3.2. Results. Figure 6 shows our model results for moderate density, $\delta = 6$, and varying independents do not listen, $\mu = 0$, $\mu = 0.5$, and $\mu = 1$ in panels 6(a), 6(b), and 6(c), respectively. We also present the results for moderate levels of time discount, $\beta \in [0.99, 1]$.

We find that independents have less influence than imitators at $p_e < 1$, and equal or greater influence at $p_e = 1$. When independents always have information, $p_e = 1$, they will always adopt at $t = 0$. As a result, at time discount parameter, $\beta < 1$, where agents give more weight to early adoption (lower values of β), imitators will prefer independents because they always adopt earlier.

Figure 5. (Color online) The Total Number of Adopters (Gray) Increases with Higher Levels of Noise (Dotted Line) Across Ideas with Information Quality, $p_e = 0.7$, and at Different Densities



Notes. Less noise decreases the number of adopters among imitators who are immediately listening to at least one independent. Imitators who are two degrees away from independents are also adversely affected by more noise in communication channels δ.

However, at $p_e < 1$, imitators can prefer listening to other imitators because independents do not always have information and adopt. We also find that giving more weight to early adopters (e.g., Figure 6(a), $\beta = 0.99$, $p_e = 0.9$) mitigates the independents' loss in influence compared to equal weight to early adopters (e.g., $\beta = 0.998$, $p_e = 0.9$). Even if independents listen to others, $\mu = 0.5$ and $\mu = 1$ in panels (b) and (c) in Figure 6, respectively, independents are less influential than imitators.

However, at higher densities, e.g., at $\delta = 12$ in panel (d) in Figure 6, we find that independents are more influential than imitators because the network is sufficiently dense such that it is easy to find and connect to independents. Taking these results together, we find that (1) independents can have less influence than imitators when $p_e < 1$, (2) giving more weight to early adopters can benefit independents, (3) the magnitude of this impact is less than having independents with information, p_e, (4) at the outset, when independents always have information, $p_e = 1$, time discounts can be very beneficial, and (5) higher densities make it easier to find independents. This suggests that in domains where agents place a high value on early adoption, independents can become more influential than imitators.

4.4. Alternative Dynamics

In this section, we investigate the robustness of our results by extending the base model in two ways: (1) by introducing a richer, stochastic, multi-idea linking rule, and (2) by allowing agents to continuously drop links. These alternative dynamic specifications test whether our base results hold under more somewhat realistic conditions.

Our base model assumes that agents sever links with peers who did not adopt by the end of an idea. Although this may be an extreme case, agents maintain relationships over time, and possibly over multi-

Figure 6. At $\delta = 6$ and Different Levels of Independents Listening, $\mu = 0$, $\mu = 0.5$, and $\mu = 1$ in (a), (b), and (c), respectively; Independents (Solid Line) Become Less Influential Than Imitators (Dotted Line) Over Ideas (x-axis) at $p_e < 1$, and Equal or More Influential at the Outset at $p_e = 1$ (Rightmost Column of Each Panel)

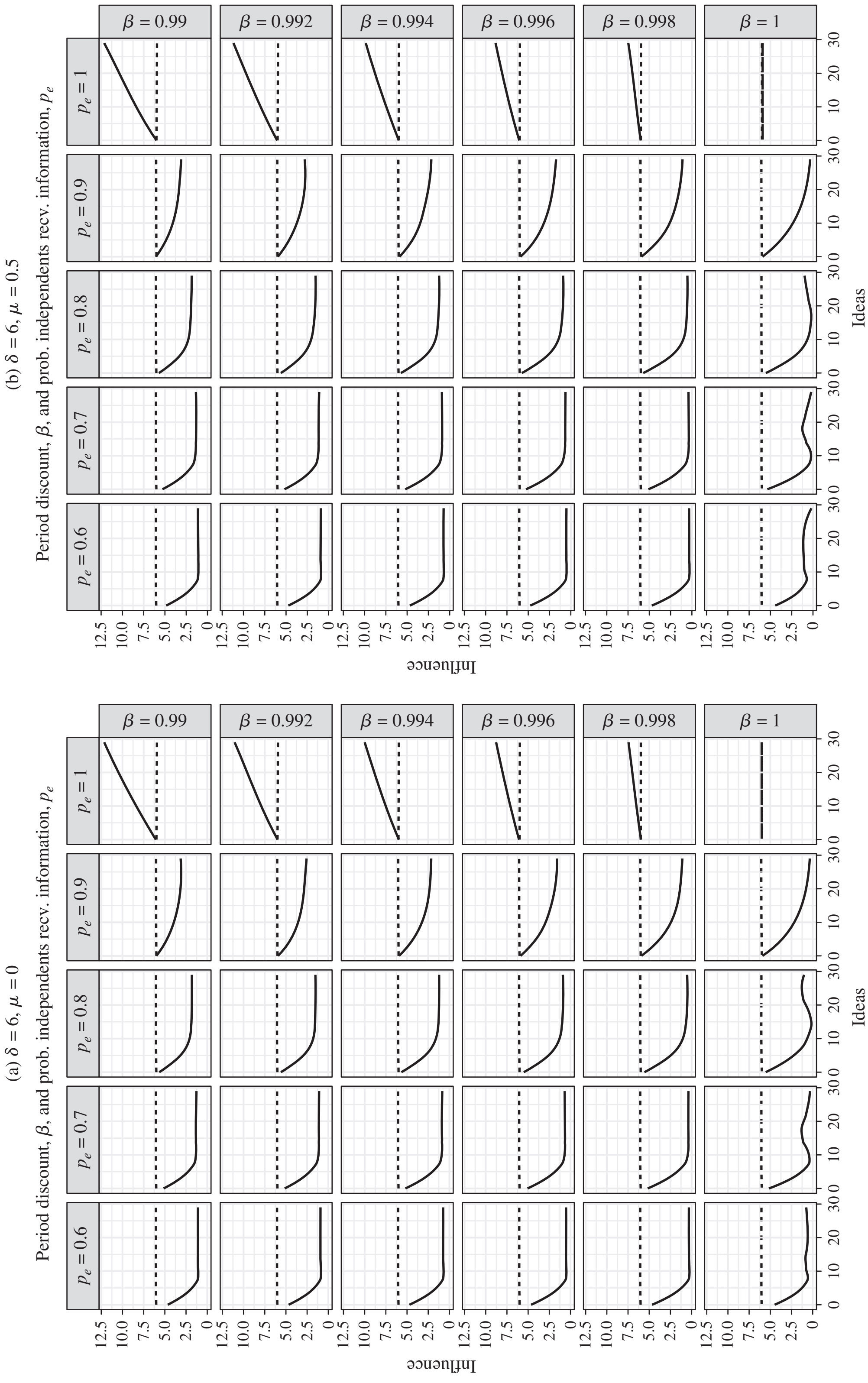
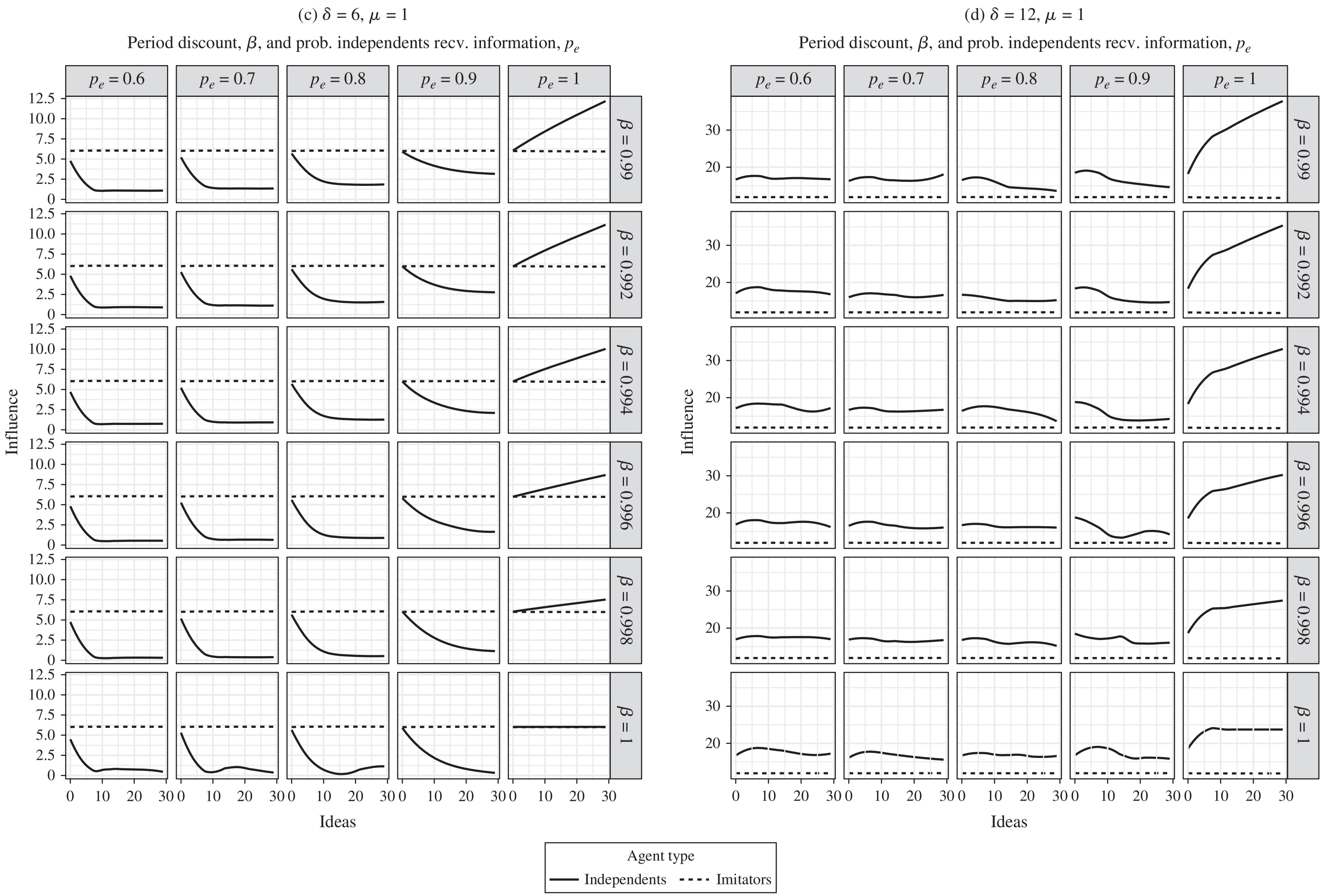


Figure 6. (Continued)



Note. However, in (d), at a higher density, $\delta = 12$, independents have more influence than imitators because the network is sufficiently dense to be found and to connect to independents.

ple ideas. That is, the agent may not know the true informational value of her peer after a single idea, but updates the belief after each observation. We capture this by replacing the single-idea rule in the base model with a stochastic model based on the previous adoption history of her peers.

We also extend the base model by allowing agents to sever and readjust their network in any period rather than only at the end of an idea's diffusion. As a result, our alternative specification is a more realistic behavior model and also addresses potential artifacts in the core model which may have driven our results. This extension allows for a more continuous evolution of networks, and can capture potentially richer phenomenon such as variations in relationship length.

4.4.1. Model. We introduce a multi-idea link dropping rule where agent i keeps her link with peer j for the next idea with probability, $\Pr(e^{ij}_{m+1} = 1) = (\beta_1 + A_{ijm})/(\beta_1 + \beta_2 + M_{ij})$. When a new link forms, we initialize the variables to one, $\beta_1 = \beta_2 = 1$. With the passing of each idea, the ego agent increments the denominator by one, and also increments the numerator by one if the peer adopts. Therefore, A_{ijm} is the number of ideas that i has observed j adopt up to the current idea m, while M_{ijm} is the number of ideas that have diffused in which $e^{ij}_m = 1$. The process resembles the updating of the Beta-Binomial conjugate process.

We also allow for agent i to engage in ELF during any period t with probability $\eta \in [0, 1]$. With probability η, the agent evaluates her network and keeps her links based on the link-dropping rule specified above. When $\eta = 0$, agents only engage in link adjustment at the end of an idea as in the core model. We test our base results using $\eta \in [0, 0.004, 0.008]$.[9] This model also allows for asynchronous link severing among agents, whereas previous models only allowed agents to simultaneously adjust networks. Appendix B.3 summarizes the algorithm.

4.4.2. Results. Even with these changes to the core model, we consistently see that independents can have less influence than imitators. For example, in Figure 7, at $\eta = 0$ in the first column, the model is equivalent to the base model. We observe that they quickly lose followers over a few ideas, even with this new multi-idea dropping rule, which one would expect to imply greater inertia in networks. The results are also consistent at low and higher network densities.[10]

Furthermore, the probability of ELF, η, has similar qualitative effects. That is, as agents engage in more frequent ELF over periods within an idea, independents will continue to lose followers, albeit at a slower rate. This is possible because imitators who listen to others do so fairly quickly and find agents with information within a few periods. Our results are robust and consistent with our base model.

4.5. Robustness

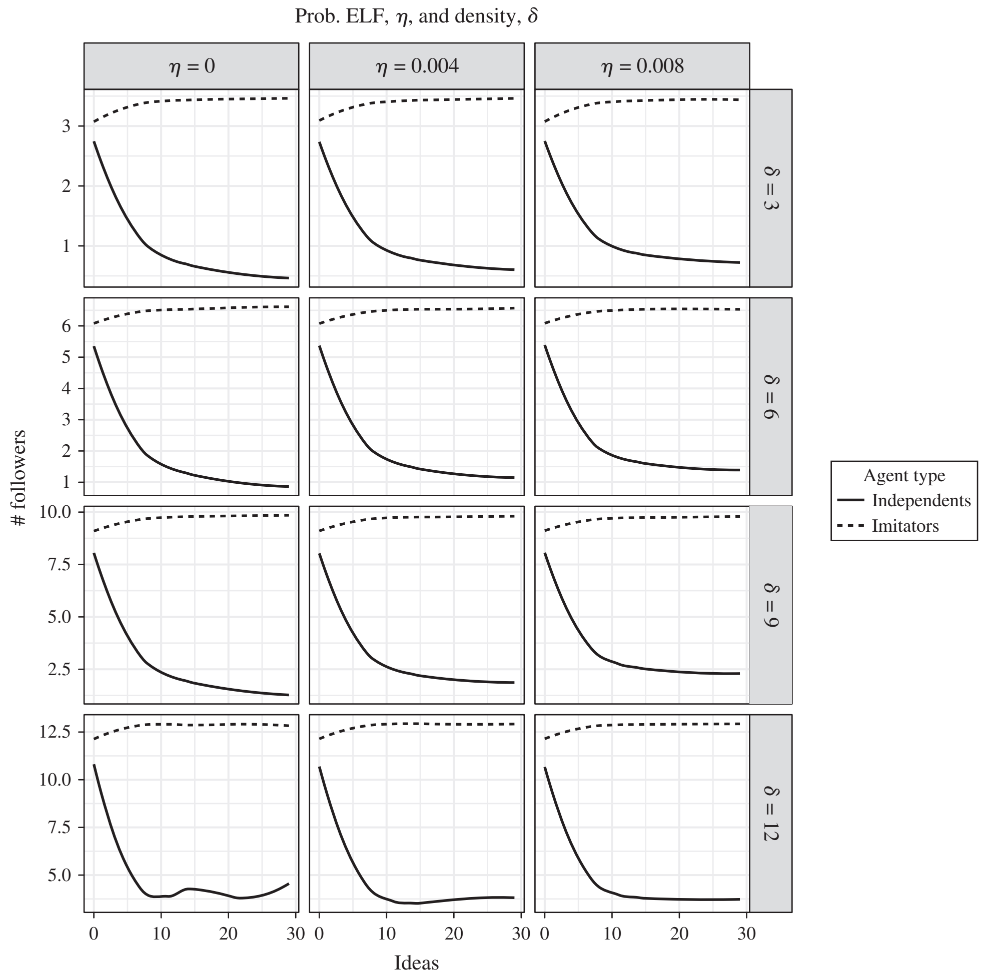
We have also conducted a number of robustness checks in the online appendix to test whether our base result, i.e., that independents can have less influence than imitators, holds under various conditions. Section A1 in the online appendix provides results across wider parameter ranges for independents listening, independents' homophily, the proportion of independents, information quality of independents, and network density. We find that the results are similar to our base results. In Section A2 in the online appendix, we model the link dropping rule where agents prefer to keep peers who adopted an idea earlier. We find similar results to our base model. However, at an extreme case, when independents always have information, $p_e = 1$, we find that they have more influence. Section A3 in the online appendix uses an alternative threshold rule whereby agents adopt based on a count threshold rather than a proportional threshold in our base model. Finally, Section A4 in the online appendix models proportional thresholds that increase (decrease) with the agent's number of influencers, of in-degrees. We find results similar to our base model results that independents can have less influence than imitators under reasonable parameter values.

5. Conclusion

Numerous researchers have noted the importance of WOM marketing in the successful launch and adoption of products and ideas. Practitioners and researchers have been searching for "opinion leaders" who can influence others and contribute to a broader adoption. Our research contributes to the literature on the evolution of network structure through a process of link formation (Toubia and Stephen 2009, Barabasi and Albert 1999, Rosenblat and Mobius 2004) by showing that personal attributes, community characteristics, and environmental factors can impact the network structure to create individuals with greater influence than the norm. Our findings bridge two streams of literature by showing how personal characteristics, such as having exogenous information sources, can drive one's network influence.

First, our baseline study shows that while there is a trade-off between access to information with higher network density and competing signals in static networks, in networks with ELF, penetration is monotonically increasing with network density. We show that, surprisingly, independents, i.e., those with exogenous information outside the network, attain significantly less influence than imitators because imitators are more likely to benefit from listening to multiple sources of information within their peer network. As a result and contrary to common belief, independents with good exogenous information have fewer followers than the average imitator. Second, we show that

Figure 7. The x-axis Within Each Cell Shows the Changing Influence Over Ideas, While the y-axis Gives the Number of Followers for Independents (Solid Line) and Imitators (Dashed Line)



Notes. Each column of cells gives the probability of ELF, $\eta = [0, 0.004, 0.008]$ from left to right. Each row of cells shows the varying density, $\delta = [3, 6, 9, 12]$ from top to bottom. Using a stochastic link dropping rule over multiple ideas, independents lose followers over ideas compared to imitators.

when independents listen to other agents, they gain more influence by providing access to better information. However, when independents' networks are characterized by non-negligible amounts of homophily, i.e., they listen predominantly to other independents, they further decrease their influence relative to imitators because this restricts their access to information.

In Section 4.2, we study how communication noise can moderate the imitators' influence, and help independents to increase their influence. The strength of imitators, which stems from their ability to collect information from others, also works against them by accumulating signal noise in poor communication channels. Less noise allows agents to be further away from the original source, i.e., the independents.

Finally, we show that our baseline results are robust to alternative models. Specifically, we find similar results under two variations: (1) time discounting, which gives more weight to early adopters, and (2) an alternative multi-idea ELF rule where agents take into account past actions within a relationship before severing the tie. With these additional complexities, we show that our baseline model holds, i.e., that independents can lose influence over ideas because imitators can mediate better information by listening to multiple others.

Our studies explain why: (1) dynamic network models can result in substantively different types of "influencers" than static network models; (2) imitators can be more influential than independents when they listen to several sources; (3) although independents can become more influential when they listen to others, if they listen only to other independents, they can lose these added benefits; and (4) independents can become more influential when information exchange occurs through noisy communication channels.

For practitioners who want to disseminate information about their idea or product, our results suggest that they should carefully consider the environment, medium, community, and the communication domain for the WOM message. In particular, they might rethink the assumption that expertise equates to influence in a network. This is particularly true in domains with relatively "cleanly-observed" (i.e., low noise) adoptions.

However, managers should target independents with exogenous information in noisy communication medium. For example, a manager at Apple computers who wishes to promote the latest iPhone will want to directly target well-connected and popular bloggers who collect information around the World Wide Web where it may be less noisy. Managers can also target advertising messages to these imitators with a high number of followers on Facebook or social network sites rather than targeting technology experts who list technical knowledge in their profile. By contrast, the Apple manager will want to target independents in an offline context such as among technology-savvy college students who will bring their device to class and show off the product in their dorm. However, promoting products to college students on the first day of orientation is less effective as the network is nearly random. Targeting independent students later in the year with more established and stable networks, and more senior students will be more effective in spreading ideas from a small initial seed.

Our research also provides interesting theoretical insights for social network researchers and practitioners. Future studies can extend our understanding of influentials and the evolution of networks by empirically testing our findings. Furthermore, our findings raise new questions about how agents weigh signals from different influencers. Do repeated interactions reveal user types, e.g., independents? If so, do risk-averse agents give more weight to signals from independents than imitators? Do influentials emerge in a winner-take-all scenario for competing products? How does network evolution and influence change in various idea contexts and product domains such as shopping for embarrassing products where observational learning may result in a negative social stigma? While we model information-seeking agents who sever links based on the prior adoption history of their peers, link dynamics may be driven by other factors such as social norms, peer ties, and emotional factors. Moreover, they may also be driven by observable characteristics of the individuals in the network. Our model ignored such factors. Future research can also investigate competing diffusion models where firms release similar products in a category.

These questions along with our study open a novel area of research which suggests that the friendship formation process may dictate who becomes influential in social networks and, thereby, the entire diffusion process. To our knowledge, our study provides a first look at influence as a factor of personal characteristics, friendship formation, community characteristics, and environmental factors. Further studies in this area can provide managers with additional insight in effective WOM and social network marketing.

Acknowledgments

The authors thank the editors and referees for the constructive comments and suggestions.

Appendix A. Summary of Parameters

Table A.1. Summary of Indices and Parameters

Parameter	Values	Description
K	$\mathbb{N}$	Number of agents
i	$\{i \in \mathbb{N} \mid 1 \leq i \leq K\}$	Index for an individual agent
$\mathcal{K}$	$\mathcal{K} \in \{k_1, k_2, k_i \ldots k_K\}$	Set of agents
T	$\mathbb{N}$	Number of assumed periods
t	$\{t \in \mathbb{Z} \mid 0 \leq t \leq T\}$	A certain period during an idea
M	$\mathbb{N}$	Assumed number of ideas
m	$\mathbb{N}$	An idea to adopt
a_{mit}	$\{0, 1\}$	Adoption decision of agent i in period t for idea m
α	$\alpha \in (0, 1)$	Proportion of independents
b_e	$b_e = \alpha K \in \mathbb{N}$	Number of independents
b_n	$b_n = \mathbb{N}$	Number of imitators
p_e	$(0, 1]$	Probability that an independent receives the correct exogenous signal
p_n	$p_n = 0$	Probability that imitators receive an exogenous signal
τ	$(0, 1]$	Adoption threshold
x_{mit}	$[0, 1]$	Proportion of agent i's neighbors who adopted idea m prior to period t
e_m^{ij}	$\{0, 1\}$	Agent j listens to agent i's information for idea m
$\mathcal{E}_m$	$e_m^{ij} \in \mathcal{E}_m$	Set of directed links or edges for idea m
$G(\mathcal{E}_m, \mathcal{K})$		Graph with edges E_m and nodes K for idea m
δ	$\{\delta \in \mathbb{R} \mid \delta > 0\}$	Average network density
n_i	$\{n_i \in \mathbb{Z} \mid n_i \geq 0\}$	Number of peers who agent i influences, or her out-degree

Appendix A. (Continued)

Table A.1. (Continued)

Parameter	Values	Description
ρ_i	$\{\rho_i \in \mathbb{Z} \mid \rho_i \geq 0\}$	Number of peers who agent i listens to, or her in-degree
σ_{mi}	$\{0, 1\}$	Agent i received an exogenous signal about idea m
S	$\mathbb{N}$	Number of simulations
$\mathcal{S}$	$s \in \mathcal{S}$	A set of simulations $\mathcal{S}$ with members s
r_m	$[0, 1]$	Penetration rate for idea m
ϕ	$[0, 1]$	Communication channel precision
γ	$[0, 1]$	Information quality metric
ψ	$[0, 1]$	Probability that nature draws a "good" idea
ζ_j^j	$[0, 1]$	Agent j's observed action for agent i
β_1, β_2	$\beta_1 = \beta_2 = 1$	Parameters for the Beta-Binomial distribution
A_j	$\mathbb{N}$	The number of observed adoptions of peer j observed by agent i up to current idea m
μ	$[0, 1]$	Probability independent listens to others in a period
λ	$[0, 1]$	Probability independent listens to other independents in a period
η	$[0, 1]$	Probability of engaging in ELF during period t
r	$[0, 1]$	The time discount factor for early adopters

Appendix B. Summary of Algorithms

B.1. Core Algorithm

Given the stochastic nature of our model, i.e., associated with observation, network structure, and linking, we capture our results as averages over multiple runs of the model. We simulate each model S times where each simulation run $s = 1, \ldots, S$ consists of a randomly chosen initial network, the parameter tuple $G(\mathscr{E}_1, \mathscr{K}, \delta)$; K agents, proportion α, which consists of independents and $1 - \alpha$ imitator agents; N ideas, each idea lasting T periods. The specific algorithm proceeds as follows:

1. Create K agents, a proportion α of whom are independents
2. For each agent i, randomly draw $\rho_i \sim Pois(\delta)$, the agent's in-degree
3. For each agent i, randomly select ρ_i agents whom i listens to
4. For each independent agent and idea m, draw $\sigma_{mi} \sim Bernoulli(p_e)$; for all imitator agents $\sigma_{mi} = 0$ (no signal is received)
5. If $\sigma_{mi} = 1$ then $a_{mi0} = 1$
6. For each $t = 1, \ldots, T$, for each imitator agent i such that $a_{mi(t-1)} = 0$, set $a_{mit} = 1$ if $x_{it} > \tau$. Agents adopt when a proportion greater than τ of their peers adopted in the previous period.
    6.i Independent agent i who has not adopted will listen to others with probability $\mu \in [0, 1]$
    6.ii Given that the independent agent listens to others, with probability $\lambda \in [0, 1]$ at period t she will listen only to peers who are independents. If at least one independent peer adopts, she will adopt. With probability $1 - \lambda$ she will listen to all her peers, and adopt if $x_{it} > \tau$
7. After $t = T$, for each agent j, set $e_{m+1}^{ij} = 1$ if $e_m^{ij} = 1$ and $a_{miT} = 1$, and $e_{m+1}^{ij} = 0$ if $e_m^{ij} = 1$ and $a_{miT} = 0$. To replace dropped links, randomly choose from among the agents with $e_m^{ij} = 0$.
8. Repeat steps 4–7 for M ideas
9. Repeat steps 1–8 for S simulation runs

Our analysis is based on measures averaged over the S simulations. Two metrics of particular importance will be (i) the out-degree of agent i, n_i, and (ii) r_m, the proportion of agents who adopted idea m by time T, calculated as follows:

$$r_m = \frac{1}{KS} \sum_s \sum_i a_{miTs}.$$

We also generate a set of benchmark results from a static network structure model for later comparison. Such a setting is analogous to that studied by Watts and Dodds (2007). In this setting, agents make adoption decisions over repeated ideas on a random network but without endogenous linking decisions. Thus, in this baseline case, we do not perform steps 7 and 8 in the simulation algorithm.

B.2. Noisy Communication

We modify the core algorithm from Appendix B.1 to introduce noisy communication as follows: We define parameter $\phi \in [0, 1]$ as the probability that a communication between two agents is perfect, i.e., the communication is transmitted without noise. Subsequently, a communication is "noisy" with probability $1 - \phi$.

To implement this extension, we amend the core algorithm as follows:

Steps 1 through 5. No change.
Step 5′. For each communication link e_m^{ij}, draw ϕ probability the link is noiseless, and $1 - \phi$ probability that the link is noisy. A noisy link has a 50% chance of being "inverted" for idea m
Step 6. No change.
Step 7′. After $t = T$, for each agent i, form $\tilde{a}_{jT}$ for each tie j as above. Set $e_{m+1}^{ij} = 1$ if $e_m^{ij} = 1$ and $\tilde{a}_{mjT} = 1$, and $e_{m+1}^{ij} = 0$ if $e_m^{ij} = 1$ and $\tilde{a}_{mjT} = 0$. To replace dropped links, randomly choose from among the agents with $e_m^{ij} = 0$.
Steps 8 and 9. No change.

Note that we assume that the channel remains noisy and inverted (not inverted) throughout the entire idea up to period T. That is, the channel cannot be perfect in one period and noisy in the next. We make this assumption to simplify communication about a product by allowing for the channel to be noisy and not the instance of communication within the channel at a particular time t. At the end of the idea, we redraw whether the link is "noisy," and whether it is inverted.

B.3. Alternative Dynamics

We extend our base model as follows: Deletions are in strike-out font, while insertions are in italic.

1. Create K agents, a proportion α of whom are independents
2. For each agent i, randomly draw $\rho_i \sim Pois(\delta)$, the agent's in-degree
3. For each agent i, randomly select ρ_i agents whom i listens to
   (a) *For each ego i and peer j, we initialize the probability to maintain a link as* $\Pr(e_0^{ij}=1)=\beta_1/(\beta_1+\beta_2)$, *where* $\beta_1=1$ *and* $\beta_2=1$.
4. For each independent agent and idea m, draw $\sigma_{mi} \sim Bernoulli(p_e)$; for all imitator agents $\sigma_{mi}=0$ (no signal is received)
5. If $\sigma_{mi}=1$ then $a_{mi0}=1$
6. For each $t=1,\ldots,T$, for each imitator agent i such that $a_{mi(t-1)}=0$, set $a_{mit}=1$ if $x_{it}>\tau$. Agents adopt when a proportion greater than τ of their peers adopted in the previous period.
   (a) *With probability η or $t=T$, the ego agent engages in ELF. For each link between the ego i and friend j, the probability to maintain the link is drawn with probability* $\Pr(e_{m+1}^{ij}=1)=(\beta_1+A_j)/(\beta_1+\beta_2+M_{ij})$, *where A_j is the number of adoptions the ego i has observed j after M_{ij} ideas since the link has formed.*
   (b) *If link e_m^{ij} is severed, agent i randomly selects a new agent, j^*, to listen to, and initializes the probability to maintain the link as* $\Pr(e_0^{ij^*}=1)=\beta_1/(\beta_1+\beta_2)$ *as in step 3a.*
7. ~~After $t=T$, for each agent j, set $e_{m+1}^{ij}=1$ if $e_m^{ij}=1$ and $a_{miT}=1$, and $e_{m+1}^{ij}=0$ if $e_m^{ij}=1$ and $a_{miT}=0$. To replace dropped links, randomly choose from among the agents with $e_m^{ij}=0$.~~
8. Repeat steps 4–7 for M ideas
9. Repeat steps 1–8 for S simulations

Endnotes

[1] The quote appears in Nahj Al-Balagha (literally, "clear path of eloquence"), a compilation of the works of Imam Ali bin Abi Talib. There is some disagreement among scholars as to whether all of the works therein are from Imam Ali himself or whether some may be attributable to the compiler of the anthology, Al-Sayyid Al-Radi.

[2] We have initialized our network using small worlds, scale free, and preferential attachment networks with similar results. We find that the network quickly evolves to its steady state within a few ideas. Results are available from the authors.

[3] For the remainder of this study, we use the following parameters ($K=10{,}000$, $T=50$, $M=50$, $S=30$, $p_e=1.0$, $\alpha=0.01$, $\tau=0.18$, $\mu=1$, $\lambda=1$) unless otherwise stated. Additional robustness results are available from the authors.

[4] See the online appendix for results.

[5] While one might consider, for example, measures of second- and third-generation influence (A's influence on C via her influence on B), recent research questions the magnitude of such influence. In a field study setting, Mobius et al. (2017) suggest that information may decay quickly within social networks. As a result, secondary or tertiary influence may be small and negligible.

[6] See Appendix B.2 for details.

[7] An alternative method to model noise is to randomly draw on an observation per period. That is, a noisy link will result in a random observation on every period. However, this and other similar per-period models may have an adoption bias since these links will adopt with probability $1-(1/2)^t$, which may be higher than the probability of observing an adoption without noise in the system. Furthermore, such models may require additional assumptions such as memory or decay, or heterogeneity in observing different peer weights.

[8] These results are available from the authors.

[9] These values are reasonable since we do not expect agents to change their networks frequently. In expectation, agents will do so at 250 and 125 periods for $\eta=[0.004, 0.008]$, respectively.

[10] These results are available from the authors.

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