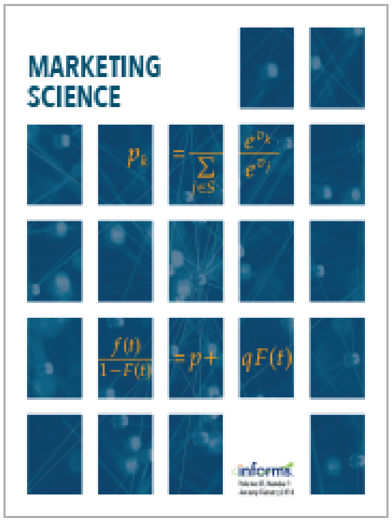


Marketing Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Optimal Category Pricing with Endogenous Store Traffic

Edward J. Fox, Steven Postrel, John H. Semple,

MARKETING SCIENCE
Vol. 28, No. 4, July–August 2009, pp. 709–720
ISSN 0732-2399 | EISSN 1526-548X | 09 | 2804 | 0709

DOI 10.1287/mksc.1080.0442

Optimal Category Pricing with Endogenous Store Traffic

Edward J. Fox
Edwin L. Cox School of Business, Southern Methodist University, Dallas, Texas 75275,
efox@cox.smu.edu

Steven Postrel
Anderson School of Management, University of California at Los Angeles, Los Angeles, California 90095,
steven.postrel@anderson.ucla.edu

John H. Semple
Edwin L. Cox School of Business, Southern Methodist University, Dallas, Texas 75275,
jsemple@cox.smu.edu

We propose a dynamic programming framework for retailers of frequently purchased consumer goods in which the prices affect both the profit per visit in the current period and the number of visitors (i.e., store traffic) in future periods. We show that optimal category prices in the infinite-horizon problem also maximize the closed form sum of a geometric series, allowing us to derive meaningful analytical results.

Modeling the linkage between category prices and future store traffic fundamentally changes optimal pricing policy. Optimal pricing must balance current profits against future traffic; under general conditions, optimal long-run prices are uniformly lower across all categories than those that maximize current profits. This result explains the empirical generalization that category demand in grocery stores is inelastic. Parameterizing profit per visit and store traffic reveals that, as future traffic becomes more sensitive to price, retailers should increasingly lower current prices and sacrifice current profits. We also determine how the burden of drawing future traffic to the store should be distributed across categories; this is the foundation for a new taxonomy of category roles.

Key words: marketing; dynamic programming; pricing; optimization; retailing; store traffic
History: Received: July 2, 2007; accepted: February 19, 2008; processed by David Bell. Published online in *Articles in Advance* November 5, 2008.

1. Introduction

> The basic discounter's idea was to attract customers into the store by pricing these items—toothpaste, mouthwash, headache remedies, soap, shampoo—right down at cost. Those were what early discounters called your "image items." (Walton 1992, p. 44)

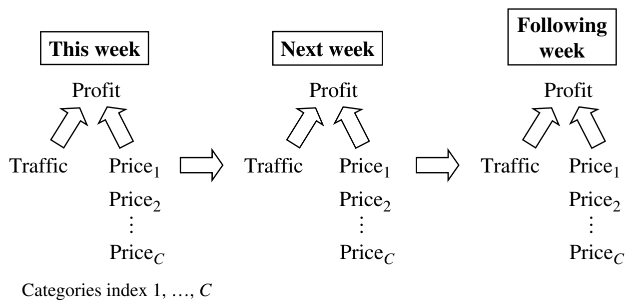
Retailers of frequently purchased consumer goods have long understood that store traffic is essential to their profitability and that prices in some categories affect store traffic more than prices in other categories. This paper introduces a general and tractable dynamic framework that models the impact of category prices on store traffic and, more generally, on retailer profits.

When visiting a store, a shopper incurs travel costs but can purchase products in any category offered by that store. To make purchases at a different store, the shopper would incur additional travel costs, so visiting a store effectively creates an asymmetry in competition for the shopper's purchases. As a result, the store that is visited has an incentive to be opportunistic and charge monopoly or near-monopoly prices. Yet "... the store has a special apprehension. Even though a customer, once in the store, may buy the item at a higher price, will the customer come back?" (Little and Shapiro 1980, p. S199) Moreover, because some categories' prices have a greater effect on customer patronage (toothpaste, mouthwash, headache remedies, soap, and shampoo for discount stores, according to the quote from Sam Walton in 1992), prices in these categories can disproportionately affect sales and profits in other categories.

For retailers of frequently purchased consumer goods (such as grocery, drug, discount, and warehouse club stores), we propose a simple dynamic programming framework to determine the category prices that maximize the infinite-horizon discounted profit.[1] The sole state variable in our dynamic model

[1] Infinite-horizon analysis is particularly appropriate for these retailers because of consumers' shopping frequency and because

Figure 1 **Effect of Category Prices on Retailer Profits and Traffic**



is store traffic, which enters the model in a multiplicative fashion and succinctly captures the effects of current category prices on future traffic and future profits. Figure 1 shows the interrelationship between category prices, profits, and traffic. The multiplicative effect of traffic on profits is an important feature of our model. It allows us to calculate optimal prices for the infinite-horizon problem by solving an unconstrained mathematical programming problem.

This paper makes five main contributions. First, it offers a dynamic framework for analyzing prices that incorporates future store traffic; no such framework currently exists. With this dynamic framework, we explain the empirical generalization that grocery category prices are inelastic (e.g., Neslin and Shoemaker 1983, Bolton 1989) and provide theoretical support for Little and Shapiro's (1980) assertion that grocery store prices reflect a balance of current profit and future traffic considerations. Second, this paper proposes and analyzes different response functions for profit per visit and future traffic, thereby laying the groundwork for empirical implementation. By estimating the proposed response functions, retailers could use our dynamic framework to determine optimal category prices and margins.[2] Third, this paper shows how category-level response parameters affect prices and profits. As store traffic becomes more sensitive to prices, we find that optimal prices, current-period profits, and discounted future profits are all reduced. Thus, higher traffic sensitivity is bad for the retailer. Fourth, we use the relationship of optimal prices across categories to develop a new taxonomy of category roles. Retailers could use these roles, which are based on profit per visit and future traffic response parameters, to refine their intuition and make more fact-based category management decisions. In sum, by adding to the theoretical understanding of optimal category pricing, this paper can help retailers to develop pricing strategies to improve their long-run profitability. Fifth and finally, this paper makes a technical contribution in dynamic analysis by proving that, in models with multiplicative profit functions like ours, the optimal levels of conditioning variables (category prices, in our case) also maximize the sum of a geometric series. This technical result is potentially useful for analyzing other types of problems.

The remainder of the paper is organized as follows. In §2, we review the relevant literature. In §3 we formally introduce the modeling framework and analyze the static case and the multiperiod and infinite-horizon cases. We also introduce functional forms to analyze comparative statics. Section 4 concludes with a discussion of our results and their implications along with ideas for future research.

2. Literature Review

According to standard pricing models, optimal prices are those that maximize the contemporaneous effect of current prices on current profits. Because the impact on future traffic is ignored, we will refer to these prices as *myopic*. Empirical research has consistently found that contemporaneous category sales in grocery stores are price inelastic (see Neslin and Shoemaker 1983, Bolton 1989 for summaries of available studies), implying that retailers consistently price below myopic levels.[3] Thus, despite the consistent empirical findings, "…retailers are hesitant to engage in broad price increases. One possible explanation is that our models are inadequate" (Montgomery 2005, p. 374). Montgomery argues that retailers believe implementing myopic prices would actually hurt their long-term profitability. Our model demonstrates that this belief is correct.

Little and Shapiro (1980) laid the foundation for our dynamic approach to category pricing by introducing the idea that current prices can affect future patronage. In their model, prices are limited by an exogenous utility constraint to ensure that customers return to the store. Determining this constraint, which would almost certainly vary across customers, is quite difficult in practice. Nevertheless, Little and Shapiro's model clearly served as the inspiration for the constrained price optimization models described by Montgomery (2005). A more recent dynamic pricing analysis confirmed that retailers consider future demand when making pricing decisions; however, this study modeled state dependence in demand for brands, not store traffic (Che et al. 2007).

Unlike Little and Shapiro (1980), we model prices at the category level rather than the item level, which

endgame considerations are irrelevant. Note that the pricing of perishable products requires consideration of costs that are not captured in this model.

[2] Parameters relating price to future traffic and current profits could be estimated using weekly point-of-sale and cost data for each category—data that are available to most retailers.

[3] Note that this result does not generalize beyond frequently purchased consumer goods to durables and other hard goods. For example, Mantrala et al. (2006) found subclasses of auto parts to be price elastic.

allows for product substitution within a category. If we were to model individual item prices, we would have to incorporate cross-price effects, which would greatly complicate the analysis. This consideration prompted Simon et al. (2006) to argue that the category is the most appropriate level of analysis for price sensitivity. Our approach also differs from Little and Shapiro's (1980) in that individual shopping decisions are aggregated. Rather than maximizing profits with an individual-level utility constraint, in our dynamic model the retailer maximizes profits with store traffic as a state variable. As a result, the model can accommodate different specifications of individual shopping behavior—we require only that higher prices reduce the probability of purchasing in the current period and returning to the store in the following period. Another benefit of our approach is that store traffic, unlike individual utility, is observable in retail point-of-sale data. Store traffic has been studied previously, but that research focused on the relationship between store traffic and advertised prices (Walters and Rinne 1986, Kumar and Leone 1988, Walters and McKenzie 1988, Grover and Srinivasan 1992). In general, traffic effects from advertised prices were found to be small.

Other studies have considered the mediating role of price image (i.e., customer beliefs about one retailer's prices relative to others) in the relationship between retail prices and store patronage. These studies focused either on how actual prices affect price image (Brown 1969, Desai and Talukdar 2003) or how retailers signal low prices through advertising (Simester 1995, Shin 2005). The clear implication of these studies is that the lower consumers believe a store's prices to be, the more likely they are to visit that store.

Recent studies have also investigated the cross-category effects of pricing, finding both category complementarity (Russell and Petersen 2000, Manchanda et al. 1999) and coincidence because of synchronized purchase timing (Manchanda et al. 1999). However, no cross-category substitution effects have been found (Mulhern and Leone 1991). Other studies have investigated loss leader pricing in which selected items are priced below cost to draw consumers to the store, theoretically (Hess and Gerstner 1987, Lal and Matutes 1994) and empirically (Walters and Rinne 1986, Walters and MacKenzie 1988). The underlying notion applies not only to specific products, but also to entire categories, which can also be used to draw customers to the store. For example, the practice of category management relies on *destination categories* to draw consumers to the store where they contribute to retailer profits primarily through purchases of other categories (Blattberg and Fox 1995).

3. Model

Our model factors retailer profits into store traffic and profit per visit, both of which are readily observable. Store traffic is defined as the number of visits (*not* the number of visitors) to the store over a given period; customers making multiple visits are counted multiple times. The number of visits is usually determined from cash register tickets or from counters at the entrance of the store. Profit per visit is defined as total dollar margin (gross or net) over a given period, divided by the number of visits. Category price is defined as the average price of items in the category, weighted by the item sales.

Our model requires three assumptions: (1) there is a finite optimal price that results in a positive maximum profit; (2) future store traffic is strictly decreasing in current category prices; and (3) traffic cannot grow indefinitely faster than the discount rate. Each assumption will be formalized later in this section as the required notation is introduced.

The first assumption ensures the existence of a finite price solution that rewards the retailer for nonzero traffic (otherwise the retailer would close the business). The second assumption, a downward-sloping future traffic function, is needed to capture the impact of today's prices on future traffic. This assumption is implied by Little and Shapiro's (1980) model, which used a utility constraint to ensure that customers return to the store. The assumption does not depend on a particular model of consumer behavior; it is consistent with many such models. For example, reinforcement learning models and belief-based models in which consumers learn about prices by word of mouth both imply a downward-sloping future traffic function. The assumption does not depend on a particular model of retail competition either. Many models of competitor response and market structure imply a downward-sloping future traffic function, including a monopolist retailer, a dominant retailer facing a competitive fringe, and differentiated Bertrand oligopoly. In fact, one could consider this assumption as incorporating retail competition in reduced form. Our third assumption, which limits the traffic growth rate, is necessary for convergence of the sum of discounted profits in our infinite-horizon analysis. It precludes the possibility that a retailer would be willing to make negative profits indefinitely to grow its customer base. This is consistent with reality in that we do not observe retailers making negative profits in perpetuity. Note that each assumption applies to consumers in aggregate, so our model is robust to heterogeneity in price response across individuals.

We proceed by building intuition about the problem with two static cases, then describe and analyze the full dynamic model. First, we define the profit function and formalize the first assumption.

Definition. Total profit in a period is the product $T \cdot \pi(\mathbf{p})$ where T is the period's store traffic and $\pi(\mathbf{p})$ is the current *profit per unit of traffic* (we will use the more descriptive term profit per visit hereafter) for a vector of category prices, $\mathbf{p}$. $\pi(\mathbf{p})$ can be thought of as the product of *in-store demand* $\mathbf{q}(\mathbf{p})$, where $\mathbf{q}$ is a vector of category quantities purchased, and gross margins $(\mathbf{p}-\mathbf{w})$, where $\mathbf{w}$ is a vector of wholesale category prices.

Assumption 1. *$\pi(\mathbf{p})$ achieves a finite maximum at some price vector $\mathbf{p}^*$ with $\pi(\mathbf{p}^*) > 0$.*

3.1. Myopic Model (Static Model with Exogenous Traffic)

Suppose that traffic T is exogenous, unaffected by the price vector $\mathbf{p}$. We will call this the myopic model. This model, which will later be compared to our full dynamic treatment, reflects the current state of applied pricing models. It requires the retailer to solve

$$\max_{\mathbf{p}} T \cdot \pi(\mathbf{p}).$$

The first-order conditions for this problem are that $\partial\pi^*/\partial p_i = 0$ for each category i. Absent strong cross-category effects, this implies that the price elasticity of in-store demand in each category at a myopic optimum must have an absolute value greater than one. Otherwise, the store could increase its profits by raising prices and reducing the quantity sold (which would simultaneously increase revenue and decrease cost). Empirical studies of retail prices consistently find that this purported optimality condition is violated, and that prices are too low because observed elasticities have absolute values fewer than one (Neslin and Shoemaker 1983, Bolton 1989).

3.2. Static Model with Endogenous Traffic

Now suppose that traffic T is a function of the price vector $\mathbf{p}$. In a static model, this corresponds to customers making their decisions about whether to visit a store with full knowledge of the category prices charged there, i.e., as though all prices were advertised. Although highly stylized, this setup reveals an important intuition about how endogenous traffic changes optimal pricing. The store must solve

$$\max_{\mathbf{p}} T(\mathbf{p}) \cdot \pi(\mathbf{p}).$$

The first-order conditions for this problem require that at the optimum

$$\nabla T(\mathbf{p}^*) \cdot \pi(\mathbf{p}^*) + T(\mathbf{p}^*) \cdot \nabla\pi(\mathbf{p}^*) = \mathbf{0}.$$

Assuming that traffic is strictly decreasing in category prices ($\partial T/\partial p_i < 0$) and that profits are positive at the optimum ($\pi(\mathbf{p}^*) > 0$), $\partial\pi^*/\partial p_i = -\pi(\mathbf{p}^*) \cdot \partial T^*/\partial p_i^* \cdot 1/T(\mathbf{p}^*) > 0$, so that increasing prices raises profit per visit at the optimum. This permits the elasticity of in-store demand at the optimum to have an absolute value fewer than one, consistent with the empirical data. Note that each category's optimum price depends on the overall traffic level.

Dividing any one of these category first-order conditions by another reveals that $(\partial\pi^*/\partial p_i)/(\partial T^*/\partial p_i^*) = (\partial\pi^*/\partial p_j)/(\partial T^*/\partial p_j^*) = -\pi(\mathbf{p}^*)/T(\mathbf{p}^*)$. Optimal prices are set so that the marginal cost/benefit ratio of raising prices is equalized across categories; i.e., the traffic loss incurred by incrementally increasing profit per visit is the same for each category. If this were not true, one could increase total profits by raising prices in categories where the ratio was higher and/or cutting them in categories where the ratio was lower.

3.3. Dynamic Model with Endogenous Traffic

We now capture the idea of endogenous traffic with a richer model in which customers do not observe category prices before visiting the store, but instead use information from their previous visits to make patronage decisions. Time periods are numbered backwards so that period $n+1$ precedes period n. With this in mind, we formalize the second assumption.

Assumption 2. *Traffic T_n in period n is the product of the previous period's traffic T_{n+1} and a traffic growth factor $\phi(\mathbf{p}_{n+1})$, where $\mathbf{p}_{n+1}$ is the previous period's price vector.*[4] *We assume that the traffic growth factor decreases as prices increase; i.e., $\partial\phi/\partial p_i < 0$.*

Assumption 2 implies that traffic is described by the recursive equation $T_n = \phi(\mathbf{p}_{n+1})T_{n+1}$. This expression is consistent with population growth, which is also commonly assumed to follow an exponential process. We do not, however, rule out the possibility that traffic could increase.

Note that ϕ is not indexed by time period; it is assumed to change only with price for the foreseeable future. In practice, traffic growth may vary in the short run because of transient effects (e.g., the opening and closing of stores) that could be captured in our model by allowing the traffic factor to vary with time. While this change would not affect some of our subsequent results, the added complexity would obscure the insights generated by the model. Moreover, by ignoring transient effects, we can focus on the optimal long-run pricing strategy, which is of fundamental interest to retailers.

The development of the dynamic model begins by analyzing the one-period case and then successively expanding the number of periods in the horizon. The one-period model, which is equivalent to the myopic model described earlier, considers the optimal

[4] Our arguments are not affected if traffic is stochastic, provided that expected traffic satisfies $E(T_n) = \phi(\mathbf{p}_{n+1})T_{n+1}$.

profit based on this period's traffic, T_1, and prices, $\mathbf{p}_1$. Because no future periods are considered, the optimal profit for traffic T_1 is

$$f_1(T_1) = \underset{\mathbf{p}_1}{\text{Max}}\, T_1 \cdot \pi(\mathbf{p}_1).$$

We define $K_1 = \text{Max}_{\mathbf{p}_1} \pi(\mathbf{p}_1)$, so the optimal profit function is $f_1(T_1) = K_1 T_1$.

For the two-period horizon, the model is extended to include the impact of current prices on the next period's traffic. Profit from the future period is discounted by the factor γ, $0 < \gamma < 1$, reflecting a preference for current profits. Because we number time periods backwards, period 2 is the first period of the horizon, $\mathbf{p}_2$ is the price vector for period 2, T_2 is the traffic in period 2, and T_1 is the traffic in period 1 (the final period of the horizon). The two-period model is

$$\begin{aligned} f_2(T_2) &= \underset{\mathbf{p}_2}{\text{Max}}\, T_2 \cdot \pi(\mathbf{p}_2) + \gamma \cdot f_1(T_1) \\ &= \underset{\mathbf{p}_2}{\text{Max}} \{\pi(\mathbf{p}_2) T_2 + \gamma \cdot K_1 \phi(\mathbf{p}_2) T_2\} \\ &\qquad\qquad (\text{since } f_1(T_1) = K_1 T_1 \text{ and } T_1 = \phi(\mathbf{p}_2) T_2) \\ &= T_2 \cdot \underset{\mathbf{p}_2}{\text{Max}} \{\pi(\mathbf{p}_2) + \gamma \cdot K_1 \phi(\mathbf{p}_2)\}. \end{aligned}$$

The optimal price vector is $\mathbf{p}_2^* = \arg\max_{\mathbf{p}_2} \{\pi(\mathbf{p}_2) + \gamma \cdot K_1 \phi(\mathbf{p}_2)\}$. Observe that our use of K_1 implicitly assumes that an optimal pricing policy is followed in the last period of the horizon.

Define $K_2 = \text{Max}_{\mathbf{p}_2} \pi(\mathbf{p}_2) + \gamma \cdot K_1 \cdot \phi(\mathbf{p}_2)$. The optimal discounted profit function in the two period model is again linear; $f_2(T_2) = K_2 T_2$. In general, the dynamic n period model is

$$f_n(T_n) = T_n \cdot \left\{ \underset{\mathbf{p}_n}{\text{Max}}\, \pi(\mathbf{p}_n) + \gamma \cdot K_{n-1} \cdot \phi(\mathbf{p}_n) \right\},$$

where $\mathbf{p}_n$ is the vector of prices in period n and T_n is the traffic in period n. The K_n are given recursively, starting with $K_0 = 0$, by

$$K_n = \underset{\mathbf{p}_n}{\text{Max}}\, \pi(\mathbf{p}_n) + \gamma \cdot K_{n-1} \cdot \phi(\mathbf{p}_n). \tag{1}$$

K_n is the discounted sum of optimal profits per visit from period n (the initial period of an n-period horizon) assuming an optimal pricing policy is followed in every period. If the firm knew the ending date on which the market would disappear, i.e., a finite time horizon, Equation (1) would enable calculation of optimal prices for each period, recursively. Market ending dates are rarely known in advance, however, so we analyze the infinite-horizon problem instead. For reasonable time-discount factors, the infinite-horizon problem more closely approximates the retailer's actual situation.

Our strategy for solving the infinite-horizon problem is to establish necessary and sufficient conditions for K_n to converge as n goes to infinity; this is simplified by the following result.

Lemma 1 (Monotonicity). $K_{n+1} > K_n$.

Proof. See Appendix 1.

Given that K_n increases monotonically with n, convergence depends on whether the sequence K_n in Equation (1) is bounded. If the sequence is bounded, there is a value $K^* < \infty$ such that $K_n \uparrow K^*$; if not, $K_n \uparrow \infty$. If $K^* < \infty$, the optimal prices for the infinite-horizon problem are well defined; they are the prices that solve the problem $\text{Max}_{\mathbf{p}} \pi(\mathbf{p}) + \gamma \cdot K^* \cdot \phi(\mathbf{p})$. We now introduce a simple assumption to ensure that the sequence K_n is bounded (more complicated conditions exist that would also ensure that the sequence is bounded).

Assumption 3. $1 - \gamma\phi(\mathbf{p}) > 0$ *for all* $\mathbf{p}$.

Recalling that $\phi(\mathbf{p})$ is a long-run traffic growth factor (and abstracting away from transient effects), Assumption 3 eliminates the possibility of a retailer growing its traffic faster than the discount rate γ indefinitely. If this assumption were violated, the retailer would always be willing to accept negative profits because a bigger one-time profit would be available later. This assumption is similar in spirit to the standard position in economics and finance that *trees don't grow to the sky*—investment opportunities where perpetual negative cash flow is optimal are generally ruled out.

The following proposition implies that the infinite-horizon version of the retailer's problem expressed in (1) can be simplified because the optimal prices also solve $\text{Max}_{\mathbf{p}} \sum_{t=0}^{\infty} \gamma^t \phi(\mathbf{p})^t \pi(\mathbf{p})$, which reduces to

$$V^* = \underset{\mathbf{p}}{\text{Max}}\, \frac{\pi(\mathbf{p})}{1 - \gamma\phi(\mathbf{p})}. \tag{2}$$

Proposition 1 (Convergence of K_n). *Given Assumption* 3, *the K_n in Equation* (1) *are bounded if and only if V^* is finite. If V^* is finite, then* (1) *K_n converges monotonically to V^** (*which is the infinite-horizon discounted profit per visit in the first period*) *and* (2) *the optimal prices $\mathbf{p}^*$ for the infinite-horizon version of Equation* (1) *are identical to the prices that solve Equation* (2).

Proof. See Appendix 1.

Proposition 1 provides a noniterative approach to the infinite-horizon version of Equation (1). One simply chooses the $\mathbf{p}$ that solves Equation (2). In other words, the retailer's infinite-horizon profit maximization problem is equivalent to a closed-form nonrecursive maximization problem with time-independent prices. If the problem has a solution, then that solution is stationary in prices that can be determined without knowledge of K^* and without numerical iteration. Of course, we observe substantial variation in item-level prices over time because of promotions. However, empirical evidence is consistent with more stationary prices at the category level (Fox et al. 2004) because items in the same category are usually promoted sequentially rather than simultaneously.

We characterize the properties of the optimum with the following proposition.

Proposition 2. *In the case where the infinite-horizon profit is finite (as Assumption 3 implies), one can rearrange the first-order optimality conditions to show that the optimal category prices satisfy the following ratio condition:*

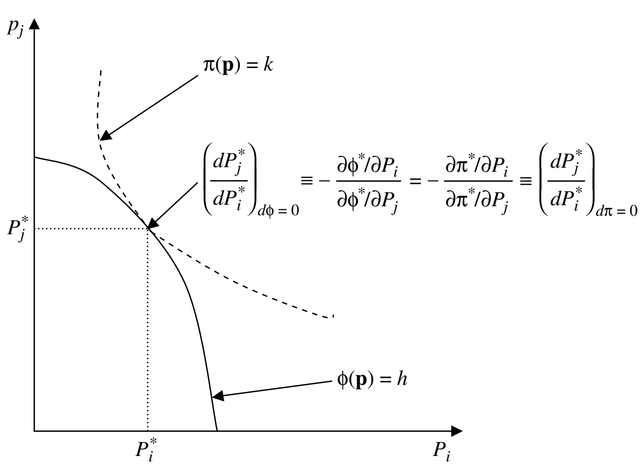
$$\frac{\partial \pi^*/\partial p_i}{\partial \phi^*/\partial p_i} = \frac{\partial \pi^*/\partial p_j}{\partial \phi^*/\partial p_j} = -\gamma \frac{\pi(\mathbf{p}^*)}{1-\gamma\phi(\mathbf{p}^*)} \quad \text{for all categories } i \text{ and } j. \tag{3}$$

Proof. Take the first-order condition of Equation (2) and rearrange terms.

This is the dynamic counterpart of the ratio result for the static model with endogenous traffic. The left-hand side of Equation (3) is the ratio of the marginal profit for category i divided by the marginal traffic for category i. This ratio is the same for all categories, so variation in optimal prices across categories will depend on differences in the relative marginal effect of category prices on both profit per visit and traffic growth.

Figure 2 depicts a typical optimum where the iso-traffic curve $\phi(\mathbf{p}^*) = h$ is just tangent to the iso-profit-per-visit curve $\pi(\mathbf{p}^*) = k$. The mathematical structure of such optima is familiar in the economic theory of the firm or the theory of optimal national production. There, marginal rates of transformation are equated to marginal rates of substitution with quantities of inputs or outputs as the choice variables. Here, we equate marginal rates of traffic generation to marginal rates of profit per visit with category prices as the choice variables. The relationship depicted in Figure 2 does not, by itself, determine optimal prices. There are an infinite number of points in Figure 2 that would satisfy the tangency condition, forming, under typical regularity assumptions, a continuous nonnegatively sloped curve $p_j(p_i)$. To identify which point on this curve is the true optimum, one needs not only equal marginal ratios in each category, but also the right-hand side of Equation (3), which creates an implicit definition of the optimal price vector $\mathbf{p}^*$.

Figure 2 **Optimal Relative Category Prices**



The right-hand side of Equation (3) is the negative long-run profit per visit discounted by one period (including the traffic growth factor in the discount rate). Because the long-run profit is positive, the right-hand side of Equation (3) is negative. Because $\partial\phi/\partial p_i < 0$, the marginal current-period profit per visit must satisfy $\partial\pi/\partial p_i > 0$ at the optimal infinite-horizon price solution. This explains, in a realistic dynamic context, why it appears that retailers are setting prices too low in terms of contemporaneous profit maximization. Because total category-level demand depends on store traffic, retailers must use current prices to drive future traffic and future profits. Later we investigate the impact of future traffic on optimal prices in greater detail. For now, we summarize the directional results of the preceding discussion in the following corollary.

Corollary. *If Assumptions 1, 2, and 3 hold, then the optimal category prices $\mathbf{p}^*$ for the infinite-horizon problem satisfy $\partial\pi(\mathbf{p}^*)/\partial p_i > 0$.*

3.4. Comparative Statics for Traffic, Profits, and Prices

We now characterize the directional impact of changes in traffic sensitivity and profit sensitivity on optimal prices. This requires the introduction of specific functional forms. We consider two cases of the dynamic model analyzed above, the SQ (s-shaped traffic function; quadratic profit per visit function) and LQ (piecewise-linear traffic function; quadratic profit-per-visit function). We assume that the functional relationship between traffic growth and prices is s-shaped or a piece-wise linear approximation to the s-shape because of the following logical requirements: (1) Traffic must approach zero as prices approach infinity, and (2) traffic growth approaches a maximum rate as prices approach zero. Note that the second requirement depends on some distribution of fixed shopping costs across consumers depending on their proximity to the store. These assumptions on traffic growth are natural approximations to more complex functional forms.

3.4.1. Quadratic Profit per Visit. Both the SQ and LQ cases will assume that the profit per visit function has a strictly concave quadratic form $\pi(\mathbf{p}) = 0.5\mathbf{p}^T A\mathbf{p} + \mathbf{c}^T\mathbf{p} + \pi_0$, where A is an $m \times m$ negative definite matrix, $\mathbf{c}$ is an $m\times 1$ vector, and π_0 is a constant.

This assumption on $\pi(\mathbf{p})$ ensures a simple inverse for its gradient, $\nabla\pi(\mathbf{p})$. It is also tantamount to supposing that the in-store demand function $\mathbf{q}(\mathbf{p})$ is linear; i.e., that $q_i = c_i + 0.5\sum_{j=1}^{m} a_{ij}p_j$ for each of the m categories, where q_i is the quantity of category i purchased.[5]

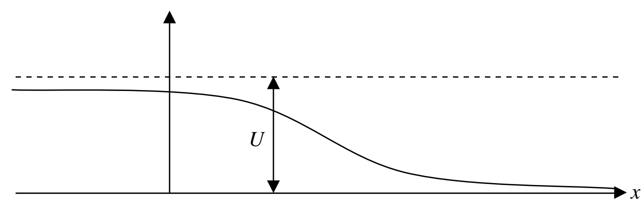
In the myopic profit maximization, future traffic and future profits are ignored and only current period profits $\pi(\mathbf{p})$ are considered. The optimal myopic price with quadratic profits is $\mathbf{p}^{\text{myp}} = -A^{-1}\mathbf{c}$. We use this expression as a benchmark against which to compare the optimal prices from the dynamic model.

3.4.2. ***S*-Shaped Traffic Function.** We now consider the traffic growth function $\phi(\mathbf{p})$ in more detail. Because the optimal price is static, this function represents the sustainable growth (or contraction) in traffic for the price vector $\mathbf{p}$. By sustainable, we mean that traffic would grow (or contract) indefinitely at a compound rate of $\phi(\mathbf{p})$ per period, given fixed prices $\mathbf{p}$. Sustained growth in traffic is common in practice and certainly plausible given growing populations in many retail markets. Issues of store capacity may eventually supervene, but if we interpret traffic growth as applying to all stores of a retail chain in a given area, even that barrier is not absolute. In practice, the discount factor γ implies that periods sufficiently far in the future have little impact on profitability. Thus, indefinitely can be interpreted as lasting until the profit implications are negligible.

Recall that the traffic growth factor is bounded above by $1/\gamma$ (see Assumption 3) and bounded below by 0; a growth factor of 1 represents a constant number of visits. In one dimension, this suggests canonical forms for the traffic growth factor that are s-shaped (see Figure 3). To perform comparative statics analyses on the optimal infinite-horizon price, we must parameterize the degree to which traffic growth is sensitive to prices. We therefore introduce a vector of category-level price response parameters $\boldsymbol{\beta}$ and make traffic growth an s-shaped function of these parameters, $\phi(\mathbf{p}) = s(\boldsymbol{\beta}^T\mathbf{p})$. Assuming that traffic growth follows an s-curve and that profit per visit is quadratic—what we call the *s-quadratic* (SQ) class of models—enables us to prove comparative results about optimal dynamic prices across categories.

3.4.3. Traffic Sensitivity Effects on Prices: The *S*-Quadratic (SQ) Case. In this section, we (1) show that optimal prices for the dynamic model are generally lower than those obtained from myopic profit maximization, and (2) develop intuitive classifications for the roles different categories play in retailer profitability. The first finding provides additional support for retailers' contention that the use of existing models yields prices that are too high (Montgomery 2005, Little and Shapiro 1980). The second finding shows how retailers' intuitions about relative category prices can be informed by actual parameter estimates.

Figure 3 **Canonical Traffic Factor $s(x)$**



Consider a general traffic function $\phi(\mathbf{p}) = s(\boldsymbol{\beta}^T\mathbf{p})$, where $s(x)$ is an s-curve and $\boldsymbol{\beta} > 0$ is an m-vector of traffic sensitivity parameters. Let $K^*(\boldsymbol{\beta})$ represent the optimal infinite-horizon discounted profit per visit given traffic sensitivity parameters $\boldsymbol{\beta}$.

Proposition 3. *If* $A^{-1}\boldsymbol{\beta} < 0$, *then* $\mathbf{p}^* = \mathbf{p}^{\text{myp}} + \theta(\mathbf{p}^*) \cdot A^{-1}\boldsymbol{\beta} < \mathbf{p}^{\text{myp}}$, *where* $\theta(\mathbf{p}^*) \equiv -\gamma \cdot K^*(\boldsymbol{\beta}) \cdot s'(\boldsymbol{\beta}^T\mathbf{p}^*)$.

Proof. See Appendix 1.

In practice, cross-category effects on in-store demand are small. No cross-category substitution of in-store demand has yet been reported (see Mulhern and Leone 1991), and cross-category complementarity of in-store demand has been reported only for cake mix/frosting and detergent/fabric softener; both pairs are obvious complements (Walters 1991, Mulhern and Leone 1991, Manchanda et al. 1999). Because prices in one category generally have little effect on contemporaneous demand in other categories, A will be dominated by its negative diagonal elements and $A^{-1}\boldsymbol{\beta} < 0$. If this were not true, some optimal dynamic prices would be greater than myopic prices (see numerical Example 1 in Appendix 2).[6]

The term $\theta(\mathbf{p}^*) \cdot A^{-1}\boldsymbol{\beta}$ in Proposition 3 is effectively a *discount* to myopic prices so that future traffic is optimally modulated. We will examine this discount across categories by considering the case in which cross-category effects are zero; i.e., $a_{ij} = 0$ for $i \neq j$, $a_{ii} = -d_i$. The discount to the myopic price for category i reduces to $\theta(\mathbf{p}^*) \cdot \beta_i/d_i$. Because the constant $\theta(\mathbf{p}^*)$ is fixed across categories, the relative price

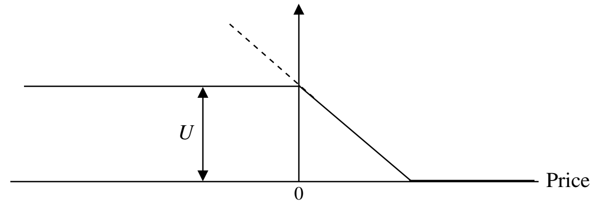
[5] This structure can incorporate wholesale prices in the terms $\mathbf{c}$ and π_0.

[6] For a more qualitative understanding of this last point, consider the extreme (and unrealistic) case in which one category has high traffic sensitivity and another has zero traffic sensitivity, but the two are perfect complements for shoppers in the store (i.e., they are always purchased together). In this case, the optimal price vector would feature a less-than-myopic price for the traffic-sensitive category and a greater-than-myopic price for the traffic-insensitive category. The total price for the bundle would be the sum of the two prices, but because only one category's price affects traffic, the other's price could be raised above the myopic level to soak customers who always purchase the two categories together.

Figure 4 **Category Roles Based on Traffic Sensitivity and In-Store Demand Sensitivity**

	In-store demand sensitivity	
Traffic sensitivity	Low	High
Low	Price-up	No-win
High	Trade-off	Come-on

Figure 5 **Piecewise-Linear *S*-Curve: The Ramp Function,** $r(x)$



discount for category i is β_i/d_i. β_i reflects category i's sensitivity to traffic growth with respect to the price p_i, and d_i reflects the curvature of in-store profit per visit with respect to p_i. Because the slope of the in-store demand function for q_i with respect to its own price p_i is proportional to d_i, high d_i corresponds to highly elastic in-store demand. If d_i is held constant, greater traffic sensitivity increases the price discount. If traffic sensitivity is held constant, then more elastic in-store demand decreases the price discount because elastic in-store demand means that the myopic price is low to begin with. Consider the perfectly elastic case where d_i is infinite and the myopic price approaches marginal cost—there is no room for a discount from the myopic price.

Classifying β_i and d_i in a 2×2 matrix (see Figure 4) leads to a new taxonomy of category roles. Categories with low traffic sensitivity to price and low in-store demand elasticity allow the retailer to price-up; i.e., it is optimal to charge relatively high prices in these categories. Categories with low traffic sensitivity and high in-store demand elasticity are no-win situations for the retailer in that they can neither be used to generate traffic nor to garner high margins from customers who visit the store (it may be useful for the retailer to offer these categories to the customer for one-stop-shopping convenience). Categories with high traffic sensitivity to price and low in-store demand elasticity require retailers to trade off the categories' traffic-generating benefits against their potential to extract high margins from store visitors. Finally, categories with high traffic sensitivity to price and high in-store demand elasticity serve as a come-on; i.e., their primary benefit for the store is in drawing traffic. Most loss-leaders and many advertised specials should come from this group. Obviously, high and low are not well-defined terms, but this heuristic classification scheme may help retailers manage categories consistent with their potential to affect these two aspects of profitability.

3.4.4. Traffic Sensitivity Effects on Profits per Visit: Linear-Quadratic (LQ) Case. We now examine what happens to optimal profit per visit as store traffic becomes more sensitive to prices. While various functional forms generate s-shaped traffic growth, all result in difficult nonlinear optimizations of Equation (2) from which it is hard to obtain closed-form solutions or, when closed-form solutions are obtained, to draw general conclusions. We therefore consider the more restricted case in which traffic growth is locally linear in prices; i.e., the traffic growth function ϕ is a piecewise linear s-shaped curve (henceforth called a ramp function). Together, locally linear traffic growth ϕ and strictly concave quadratic profit per visit π define what we call the LQ class of models.

Even with a piecewise-linear traffic function, we must ensure that the optimal prices occur on the downward-sloping portion of the function. If traffic were too sensitive to prices, the retailer might stop competing for future traffic and choose the myopic prices, effectively liquidating its inventory (see numerical Example 2 in Appendix 2). Alternatively, if traffic were not sensitive enough to prices, the myopic price would permit maximum traffic growth. To ensure a solution on the downward-sloping portion of the function, we impose what amount to regularity conditions for our optimization model (see Proposition 4).

In LQ models, the canonical traffic function is a piecewise-linear ramp function $r(x)$

$$r(x) = U \cdot \min\{1, \max(1-x, 0)\},$$

where U is the maximum flow constant. The ramp function is shown in Figure 5. Note that the range and slope of the ramp are fixed for convenience because, without loss of generality, the price vector and sensitivity parameters can be rescaled. Consistent with our previous characterizations of traffic growth, the m-category traffic function is defined as $\phi(\mathbf{p};\boldsymbol{\beta}) = r(\boldsymbol{\beta}^T\mathbf{p})$ where $\boldsymbol{\beta} > 0$ is again an m-vector of customer sensitivity parameters. As before, the strictly concave quadratic profit function is $\pi(\mathbf{p}) = 0.5\mathbf{p}^T A\mathbf{p} + \mathbf{c}^T\mathbf{p} + \pi_0$, where A is an $m \times m$ negative definite matrix, $\mathbf{c}$ is an $m\times 1$ vector, and π_0 is a constant.

In the following analysis, we treat A, $\mathbf{c}$, and π_0 as fixed parameters and consider perturbations to $\boldsymbol{\beta}$. We think of $\boldsymbol{\beta}$ as having an *initial value* that anchors the problem and a *perturbed value* that represents some hypothetical change. Define our initial sensitivity parameter to be $\boldsymbol{\beta}_L$ (the lower beta) and consider the perturbed value to be $\boldsymbol{\beta}_H$ (the higher beta) so that $\boldsymbol{\beta}_H = \tau\boldsymbol{\beta}_L$ $(\tau \geq 1)$. This corresponds to the situation in which customers with $\boldsymbol{\beta} = \boldsymbol{\beta}_H$ are proportionately more sensitive to price than customers with $\boldsymbol{\beta} = \boldsymbol{\beta}_L$.

Proposition 4. *In the LQ problem* ($\phi(\mathbf{p};\boldsymbol{\beta}) = r(\boldsymbol{\beta}^T\mathbf{p})$), *consider an initial traffic sensitivity parameter* $\boldsymbol{\beta}_L$ *and perturbations* $\boldsymbol{\beta}_H = \tau\boldsymbol{\beta}_L$ *for* $\tau \geq 1$. *In some neighborhood* $\tau \in [1, \delta)$, *assume the following regularity conditions hold*: (i) *the optimal price vector* $\mathbf{p}^*(\tau)$ *satisfies* $\mathbf{p}^*(\tau) \geq 0$ *and* (ii) $1/\gamma > \phi(\mathbf{p}^*(\tau)) > 0$. *Then the optimal current profit per visit and the optimal infinite-horizon discounted profit per visit are both nonincreasing functions of* τ *for* $\tau \in [1, \delta)$.

Proof. See Appendix 1.

This proposition establishes that, under the stated regularity conditions, greater sensitivity of traffic to price implies that the retailer earns less from each store visitor. Holding all else constant, more traffic sensitivity is bad for the retailer and reduces the optimal current and discounted long-run profit per visit.

4. Conclusion, Limitations, and Future Research

Linking category prices to store traffic fundamentally changes the optimal pricing policy. Under very general conditions, optimal prices will not fully exploit the store's market power over customers. Instead, the retailer will trade off profits that could be generated in the current period to stimulate future store traffic. Our model enables the retailer to determine how the burden of stimulating traffic is optimally distributed across categories: The incremental gain in profit per visit divided by the incremental loss in traffic should be the same for all categories; this common benefit/cost ratio should equal the (one-period discounted) maximized value of the store's optimization problem. Applying functional forms to profit per visit and future traffic response yields intuitive comparative statics results: Dynamic optimal prices are always below myopic prices; higher category traffic sensitivity results in lower category prices and profits.

The paper adds to our theoretical understanding of pricing behavior. Most fundamental, it addresses the issue that Little and Shapiro (1980) raised by systematically accounting for store patronage in the determination of optimal retail prices. In so doing, it offers an explanation for the empirical generalization that category demand elasticities at grocery retailers are fewer than one; i.e., retail prices seem generally too low. Under typical conditions, this outcome is a straightforward prediction of our model. It also provides a new analytical basis—the relative price sensitivity of traffic versus in-store demand—for estimating optimal category prices and for classifying and managing categories according to their contribution to retailer profitability. The model is robust to assumptions about individual consumer behavior, including heterogeneity, and to assumptions about competitor response.

Although we have presented a dynamic program with apparent high dimensionality, our model is numerically tractable because the traffic and profit per visit functions collapse this dimensionality into manageable scalars. Assuming one could estimate the necessary traffic and profit-per-visit parameters, optimal prices for each category and all the relevant derivatives could be computed numerically. Thus, the model could serve as a rigorous basis for setting real-world category prices that avoids both the errors attendant on assuming traffic to be exogenous and the ad hoc constraints that concerned Montgomery (2005).

The analytical development in this paper assumes profit per visit in store $\pi(\mathbf{p})$ to be time invariant. If the profit function varies systematically over time, as in the case of seasonality, then the analysis must be changed slightly, but similar propositions still hold. This applies to many retailers that, for example, draw more traffic and higher profit per visit during the holiday season.[7]

An important limitation of our model is that it relates only to retailers for whom repeat business is important, e.g., retailers of frequently purchased consumer goods. Retailers of hard goods may be less concerned that customers return to the store and so face a different pricing problem than the one modeled in this paper. Consequently, our comparative statics results are not entirely consistent with empirical findings for hard goods retailers (Mantrala et al. 2006). Our model also does not address perishable goods because the cost function for these products is time-dependent; this extension is left for future research. Another limitation of our model is that it uses a single category price to reflect the weighted average price of many items in the category. Although incorporating item prices into the model would be more informative, it would also require a much more complex treatment of cross-effects within categories. This extension is therefore left for future research. Finally, like all analytical models, our results are limited by the underlying assumptions. Although we believe our assumptions to be reasonable, they nevertheless await relaxation in future research. In particular, one might select and analyze a particular model of retail competition.

We see other avenues for future research as well. For example, it may be possible to impose more structure on the reduced-form traffic function by applying game theory to model competitive interaction. An interesting adjunct to this could be decomposing the traffic function into different market segments to link category pricing to targeting strategies. It would also be useful to apply mechanism design techniques

[7] The authors can supply details about changing the foregoing analysis to accommodate seasonality on request.

to derive incentives and information that allow the optimal pricing solution to be delegated to decentralized category managers.

Appendix 1. Proofs of Propositions

Lemma 1 (Monotonicity). $K_{n+1} > K_n$.

Proof. The proof is by induction. In the one-period problem, $K_1 > 0 = K_0$. Assume $K_n > K_{n-1}$. Let $\mathbf{p}_n^*$ denote the optimal prices for the beginning period of the n-period horizon; i.e.,

$$\mathbf{p}_n^* = \arg\max_{\mathbf{p}_n} \pi(\mathbf{p}_n) + \gamma \cdot K_{n-1} \cdot \phi(\mathbf{p}_n).$$

Then

$$\begin{aligned} K_{n+1} &= \operatorname*{Max}_{\mathbf{p}_{n+1}} \pi(\mathbf{p}_{n+1}) + \gamma \cdot K_n \cdot \phi(\mathbf{p}_{n+1}) \\ &\geq \pi(\mathbf{p}_n^*) + \gamma \cdot K_n \cdot \phi(\mathbf{p}_n^*) \\ &> \pi(\mathbf{p}_n^*) + \gamma \cdot K_{n-1} \cdot \phi(\mathbf{p}_n^*) \quad \text{(by the induction step)} \\ &= K_n. \quad \square \end{aligned}$$

Proposition 1 (Convergence of K_n). *Assume Assumption* 3. *Then the K_n in Equation* (1) *are bounded if and only if V^* is finite. If V^* is finite, then* (1) *K_n converges monotonically to V^** (*which is the infinite-horizon discounted profit per visit in the first period*) *and* (2) *the optimal prices $\mathbf{p}^*$ for the infinite-horizon version of Equation* (1) *are identical to the optimal prices that solve Equation* (2).

Proof. First, assume that the K_n are bounded. We will show that V^* is finite and (1) and (2) hold. Because the K_n are bounded, they must converge monotonically to a limit $K^* < \infty$ (Proposition 1). This requires $K^* = \operatorname{Max}_{\mathbf{p}} \pi(\mathbf{p}) + \gamma \cdot K^* \cdot \phi(\mathbf{p})$. The optimal infinite-horizon price is defined as $\mathbf{p}^* = \arg\max_{\mathbf{p}} \pi(\mathbf{p}) + \gamma \cdot K^* \cdot \phi(\mathbf{p})$. Then

$$K^* \geq \pi(\mathbf{p}) + \gamma \cdot K^* \cdot \phi(\mathbf{p}) \quad \text{(with equality if } \mathbf{p} = \mathbf{p}^*)$$

$$K^* \geq \frac{\pi(\mathbf{p})}{(1 - \gamma\phi(\mathbf{p}))} \quad \text{(with equality if } \mathbf{p} = \mathbf{p}^*).$$

This implies $V^* = \operatorname{Max}_{\mathbf{p}} \pi(\mathbf{p})/(1 - \gamma\phi(\mathbf{p}))$ is finite with optimal solution $\mathbf{p} = \mathbf{p}^*$ and optimal value K^*.

Now assume V^* is finite. We will show that the K_n are bounded. Define $\mathbf{p}^* = \arg\max_{\mathbf{p}} \pi(\mathbf{p})/(1 - \gamma\phi(\mathbf{p}))$ and denote the (finite) optimal value to (2) by $K^* = \pi(\mathbf{p}^*)/(1 - \gamma\phi(\mathbf{p}^*))$ (i.e., replace V^* with K^* for consistency). It is then straightforward to show that

$$K^* = \operatorname*{Max}_{\mathbf{p}} \pi(\mathbf{p}) + \gamma \cdot K^* \cdot \phi(\mathbf{p}) = \pi(\mathbf{p}^*) + \gamma K^* \phi(\mathbf{p}^*). \quad (4)$$

We will show by induction that the K_n are bounded above by K^*. Observe $0 = K_0 < K^*$. Assume now that $K_n < K^*$. For $n+1$, define $\mathbf{p}_{n+1}^* = \arg\max_{\mathbf{p}_{n+1}} \pi(\mathbf{p}_{n+1}) + \gamma \cdot K_n \cdot \phi(\mathbf{p}_{n+1})$. Then

$$\begin{aligned} K_{n+1} &= \operatorname*{Max}_{\mathbf{p}_{n+1}} \pi(\mathbf{p}_{n+1}) + \gamma \cdot K_n \cdot \phi(\mathbf{p}_{n+1}) \\ &= \pi(\mathbf{p}_{n+1}^*) + \gamma \cdot K_n \cdot \phi(\mathbf{p}_{n+1}^*) \\ &< \pi(\mathbf{p}_{n+1}^*) + \gamma \cdot K^* \cdot \phi(\mathbf{p}_{n+1}^*) \quad \text{(by the induction step)} \\ &\leq \operatorname*{Max}_{\mathbf{p}} \pi(\mathbf{p}) + \gamma \cdot K^* \cdot \phi(\mathbf{p}) \\ &= K^*. \end{aligned}$$

Because the K_n are bounded above by K^*, the first half of the proof requires that the K_n converge monotonically to the optimal value of Equation (2), which is also K^*. The optimal prices for Equation (2) are also optimal infinite-horizon prices as is guaranteed by Equation (4). $\square$

Proposition 3. *If $A^{-1}\boldsymbol{\beta} < 0$, then $\mathbf{p}^* = \mathbf{p}^{\text{myp}} + \theta(\mathbf{p}^*) \cdot A^{-1}\boldsymbol{\beta} < \mathbf{p}^{\text{myp}}$, where $\theta(\mathbf{p}^*) \equiv -\gamma \cdot K^*(\boldsymbol{\beta}) \cdot s'(\boldsymbol{\beta}^T\mathbf{p}^*)$*

Proof. For the dynamic infinite-horizon SQ model, the first-order conditions are $A\mathbf{p} + \mathbf{c} + \gamma K^*(\boldsymbol{\beta}) s'(\boldsymbol{\beta}^T\mathbf{p}) \cdot \boldsymbol{\beta} = \mathbf{0}$, where $K^*(\boldsymbol{\beta})$ is the SQ version of K^* from the general model. So the optimal price vector $\mathbf{p}^*$satisfies the implicit equation (for a given $\boldsymbol{\beta}$)

$$\begin{aligned} \mathbf{p}^* &= -A^{-1}\mathbf{c} - \gamma K^*(\boldsymbol{\beta}) \cdot s'(\boldsymbol{\beta}^T\mathbf{p}^*) \cdot A^{-1}\boldsymbol{\beta} \\ &= \mathbf{p}^{\text{myp}} - \gamma K^*(\boldsymbol{\beta}) \cdot s'(\boldsymbol{\beta}^T\mathbf{p}^*) \cdot A^{-1}\boldsymbol{\beta}. \end{aligned}$$

Because $K^*(\boldsymbol{\beta}) > 0$ and $s'(\boldsymbol{\beta}^T\mathbf{p}^*) < 0$, the term $\theta(\mathbf{p}^*) \equiv -\gamma K^*(\boldsymbol{\beta}) \cdot s'(\boldsymbol{\beta}^T\mathbf{p}^*)$ is positive and $\mathbf{p}^* = \mathbf{p}^{\text{myp}} + \theta(\mathbf{p}^*) \cdot A^{-1}\boldsymbol{\beta}$. In the limiting case where all cross-category effects are zero, we have $A = \operatorname{diag}(-d_1, -d_2, \ldots, -d_n)$ ($d_i > 0$), $A^{-1} = \operatorname{diag}(-1/d_1, -1/d_2, \ldots, -1/d_n)$, and $A^{-1}\boldsymbol{\beta} < 0$. If the cross-category effects are nonzero but sufficiently small, a continuity argument implies that $A^{-1}\boldsymbol{\beta} < 0$ will still hold. Thus, the optimal prices in the infinite-horizon dynamic model are generally lower than the prices that maximize the myopic problem.

Proposition 4. *In the LQ problem ($\phi(\mathbf{p}; \boldsymbol{\beta}) = r(\boldsymbol{\beta}^T\mathbf{p})$), consider an initial traffic sensitivity parameter $\boldsymbol{\beta}_L$ and perturbations $\boldsymbol{\beta}_H = \tau\boldsymbol{\beta}_L$ for $\tau \geq 1$. In some neighborhood $\tau \in [1, \delta)$, assume the following regularity conditions hold*: (1) *the optimal price vector $\mathbf{p}^*(\tau)$ satisfies $\mathbf{p}^*(\tau) \geq 0$, and* (2) *$1/\gamma > \phi(\mathbf{p}^*(\tau)) > 0$. Then the optimal current profit per visit and the optimal infinite-horizon discounted profit per visit are both nonincreasing functions of τ for $\tau \in [1, \delta)$.*

Remark. Condition (2) requires that the optimal price vectors for these perturbations all lie on ϕ's downward slope, i.e., the portion where price influences traffic. This necessarily happens for the more complicated traffic forms outlined earlier. This condition simplifies our optimality conditions and eliminates the case in which the retailer has no incentive to compete for future traffic.

Proof. For each parameter vector $\boldsymbol{\beta}$, define the optimal infinite-horizon discounted profit per visit $K^*(\boldsymbol{\beta})$ by

$$K^*(\boldsymbol{\beta}) = \operatorname*{Max}_{\mathbf{p}} \frac{\pi(\mathbf{p})}{1 - \gamma\phi(\mathbf{p}; \boldsymbol{\beta})}.$$

Let $\mathbf{p}_L$ be an optimal price vector for this problem when $\boldsymbol{\beta} = \boldsymbol{\beta}_L$ and let $\mathbf{p}_H$ be the optimal price vector when $\boldsymbol{\beta} = \boldsymbol{\beta}_H = \tau\boldsymbol{\beta}_L$ for arbitrary $\tau \in [1, \delta)$. Because both optimal price vectors are positive (condition (1)),

$$\begin{aligned} \frac{\pi(\mathbf{p}_L)}{1 - \gamma\phi(\mathbf{p}_L; \boldsymbol{\beta}_H)} &\leq K^*(\boldsymbol{\beta}_H) = \frac{\pi(\mathbf{p}_H)}{1 - \gamma\phi(\mathbf{p}_H; \boldsymbol{\beta}_H)} \leq \frac{\pi(\mathbf{p}_H)}{1 - \gamma\phi(\mathbf{p}_H; \boldsymbol{\beta}_L)} \\ &\leq \frac{\pi(\mathbf{p}_L)}{1 - \gamma\phi(\mathbf{p}_L; \boldsymbol{\beta}_L)} = K^*(\boldsymbol{\beta}_L). \end{aligned}$$

The first inequality is true because $\mathbf{p}_L$ is not necessarily an optimal price vector when $\boldsymbol{\beta} = \boldsymbol{\beta}_H$; the second inequality is true because $\boldsymbol{\beta}_H \geq \boldsymbol{\beta}_L$ and $\mathbf{p}_H \geq 0$ (by condition (1)); the third inequality is true because $\mathbf{p}_H$ is not necessarily the optimal price vector when $\boldsymbol{\beta} = \boldsymbol{\beta}_L$. As a result of these inequalities, $K^*(\boldsymbol{\beta}_H) \le K^*(\boldsymbol{\beta}_L)$, so the optimal infinite-horizon discounted profit per visit is nonincreasing for $\tau \in [1, \delta)$.

Rearranging the preceding inequalities, we also have

$$\frac{1-\gamma\phi(\mathbf{p}_L;\boldsymbol{\beta}_L)}{1-\gamma\phi(\mathbf{p}_L;\boldsymbol{\beta}_H)} \le \frac{K^*(\boldsymbol{\beta}_H)}{K^*(\boldsymbol{\beta}_L)}. \tag{5}$$

Condition (2) implies the first-order optimality conditions can be written as

$$A\mathbf{p} + \mathbf{c} + \gamma U \cdot K^*(\boldsymbol{\beta})(-\boldsymbol{\beta}) = \mathbf{0}.$$

The optimal price vector satisfies the equation

$$\mathbf{p}^* = -A^{-1}\mathbf{c} + \gamma U \cdot K^*(\boldsymbol{\beta}) A^{-1}\boldsymbol{\beta}.$$

When $\boldsymbol{\beta} = \boldsymbol{\beta}_L$, the current period profit per visit is

$$\pi(\mathbf{p}_L) = \pi_0 - 1/2\mathbf{c}^T A^{-1}\mathbf{c} + 1/2[\gamma U \cdot K^*(\boldsymbol{\beta}_L)]^2 \boldsymbol{\beta}_L^T A^{-1}\boldsymbol{\beta}_L.$$

When $\boldsymbol{\beta} = \boldsymbol{\beta}_H$, the current period profit per visit is

$$\pi(\mathbf{p}_H) = \pi_0 - 1/2\mathbf{c}^T A^{-1}\mathbf{c} + 1/2[\gamma U \cdot K^*(\boldsymbol{\beta}_H)]^2 \boldsymbol{\beta}_H^T A^{-1}\boldsymbol{\beta}_H.$$

Therefore, current period profit per visit is nonincreasing (i.e., $\pi(\mathbf{p}_H) \le \pi(\mathbf{p}_L)$) if and only if

$$[K^*(\boldsymbol{\beta}_H)]^2 \boldsymbol{\beta}_H^T A^{-1}\boldsymbol{\beta}_H \le [K^*(\boldsymbol{\beta}_L)]^2 \boldsymbol{\beta}_L^T A^{-1}\boldsymbol{\beta}_L,$$

or, because $\boldsymbol{\beta}^T A^{-1}\boldsymbol{\beta} < 0$, if and only if

$$\frac{[K^*(\boldsymbol{\beta}_H)]^2}{[K^*(\boldsymbol{\beta}_L)]^2} \ge \frac{\boldsymbol{\beta}_L^T A^{-1}\boldsymbol{\beta}_L}{\boldsymbol{\beta}_H^T A^{-1}\boldsymbol{\beta}_H}.$$

Combining this inequality with Equation (5), we observe that a sufficient condition for $\pi(\mathbf{p}_H) \le \pi(\mathbf{p}_L)$ is

$$\left[\frac{1-\gamma\phi(\mathbf{p}_L;\boldsymbol{\beta}_L)}{1-\gamma\phi(\mathbf{p}_L;\boldsymbol{\beta}_H)}\right]^2 \ge \frac{\boldsymbol{\beta}_L^T A^{-1}\boldsymbol{\beta}_L}{\boldsymbol{\beta}_H^T A^{-1}\boldsymbol{\beta}_H}. \tag{6}$$

We show Equation (6) is always true. If $\mathbf{p}_L$ is on the downward-sloping portion of the ramp function $\phi(\mathbf{p}_L;\boldsymbol{\beta}_H)$, then the sufficient condition (6) becomes

$$\left[\frac{1-\gamma U + \gamma U \boldsymbol{\beta}_L^T \mathbf{p}_L}{1-\gamma U + \gamma U \boldsymbol{\beta}_H^T \mathbf{p}_L}\right]^2 \ge \frac{\boldsymbol{\beta}_L^T A^{-1}\boldsymbol{\beta}_L}{\boldsymbol{\beta}_H^T A^{-1}\boldsymbol{\beta}_H},$$

or

$$\left[\frac{1-\gamma U + \gamma U \boldsymbol{\beta}_L^T \mathbf{p}_L}{1-\gamma U + \gamma U \boldsymbol{\beta}_L^T \mathbf{p}_L \cdot \tau}\right]^2 \ge \frac{1}{\tau^2}.$$

Because $1-\gamma U$ and $\gamma U \boldsymbol{\beta}_L^T \mathbf{p}_L$ are both positive (the former by Assumption 3, the latter by condition (1)), this inequality is true for all $\tau \ge 1$. If $\mathbf{p}_L$ is not on the downward-sloping portion of the ramp function $\phi(\mathbf{p}_L;\boldsymbol{\beta}_H)$, then the ramp function must be 0 (i.e., the lowest portion of the ramp function). The sufficient condition Equation (6) is again true because

$$\begin{aligned}\left[\frac{1-\gamma\phi(\mathbf{p}_L;\boldsymbol{\beta}_L)}{1-\gamma\phi(\mathbf{p}_L;\boldsymbol{\beta}_H)}\right]^2 &= \left[\frac{1-\gamma U + \gamma U \boldsymbol{\beta}_L^T \mathbf{p}_L}{1}\right]^2 \\ &\ge \left[\frac{1-\gamma U + \gamma U \boldsymbol{\beta}_L^T \mathbf{p}_L}{1-\gamma U + \gamma U \boldsymbol{\beta}_L^T \mathbf{p}_L \cdot \tau}\right]^2 \\ &\ge \frac{1}{\tau^2} = \frac{\boldsymbol{\beta}_L^T A^{-1}\boldsymbol{\beta}_L}{\boldsymbol{\beta}_H^T A^{-1}\boldsymbol{\beta}_H}.\end{aligned}$$

Therefore, for $\tau \in [1, \delta)$, $K^*(\boldsymbol{\beta}_H) \le K^*(\boldsymbol{\beta}_L)$ and $\pi(\mathbf{p}_H) \le \pi(\mathbf{p}_L)$. This means that the optimal infinite-horizon discounted profit per visit and the current period's optimal profit per visit are both declining as the traffic sensitivity parameter increases, provided the two regularity conditions hold. □

Appendix 2. Numerical Examples

We assume

$$\pi(\mathbf{p}) = 0.5\mathbf{p}^T A\mathbf{p} + \mathbf{c}^T\mathbf{p} + \pi_0 \quad \text{with } \mathbf{p} = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix}$$

in all cases. The traffic factors are linear ramp functions of a weighted sum of prices, with the weights representing the traffic sensitivities of the categories.

Example 1. Significant cross-category effects can lead to some dynamic prices above myopic prices,

$$A = \begin{bmatrix} -1 & -0.5 \\ -0.5 & -1 \end{bmatrix}, \quad c = \begin{bmatrix} 3 \\ 3 \end{bmatrix},$$

$$\pi_0 = -4,$$

$$\phi(p_1, p_2) = 1.05 \cdot \min\{1, \max(1 - 0.0175p_1 - 0.0035p_2, 0)\},$$

and $\gamma = 0.95$.

Then

$$\mathbf{p}^{\text{myp}} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad \mathbf{p}^* = \begin{bmatrix} 0.79 \\ 2.40 \end{bmatrix}, \quad \phi(\mathbf{p}^*) = 1.0267,$$

$$\pi(\mathbf{p}^*) = 1.43, \quad \frac{\pi(\mathbf{p}^*)}{1-\gamma\phi(\mathbf{p}^*)} = 57.96$$

(optimal infinite-horizon discounted profit).

Example 2. Very high traffic sensitivity leads to myopic pricing and drives off all future traffic,

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad c = \begin{bmatrix} 3 \\ 3 \end{bmatrix},$$

$\pi_0 = -4$, $\phi(p_1, p_2) = 1.05 \cdot \min\{1, \max(1 - 0.4p_1 - 0.08p_2, 0)\}$, and $\gamma = 0.95$.

Then

$$\mathbf{p}^{\text{myp}} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad \mathbf{p}^* = \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \quad \phi(\mathbf{p}^*) = 0,$$

$$\pi(\mathbf{p}^*) = 5, \quad \frac{\pi(\mathbf{p}^*)}{1-\gamma\phi(\mathbf{p}^*)} = 5.$$

References

Blattberg, R. C., E. J. Fox. 1995. *Category Management: Getting Started*. Research Department of the Food Marketing Institute, Washington, DC.

Bolton, R. N. 1989. The relationship between market characteristics and promotional price elasticities. *Marketing Sci.* **8**(2) 153–169.

Brown, F. E. 1969. Price image versus price reality. *J. Marketing Res.* **6**(2) 185–191.

Che, H., K. Sudhir, P. B. Seetharaman. 2007. Bounded rationality in pricing under state dependent demand: Do firms look ahead and, if so, how far? *J. Marketing Res.* **44**(3) 434–449.

Desai, K. K., D. Talukdar. 2003. Relationship between product groups' price perceptions, shopper's basket size, and grocery store's overall store price image. *Psych. Marketing* **201**(10) 903–933.

Fox, E. J., A. L. Montgomery, L. M. Lodish. 2004. Consumer shopping and spending across retail formats. *J. Bus.* **77**(2) S25–S60.

Grover, R., V. Srinivasan. 1992. Evaluating the multiple effects of retail promotions on brand loyal and brand switching segments. *J. Marketing Res.* **29**(1) 76–89.

Hess, J. D., E. Gerstner. 1987. Pricing and rain check policy. *Marketing Sci.* **6**(4) 358–374.

Kumar, V., R. P. Leone. 1988. Measuring the effect of retail store promotions on brand and store substitution. *J. Marketing Res.* **25**(2) 178–185.

Lal, R., C. Matutes. 1994. Retail pricing and advertising strategies. *J. Bus.* **67**(3) 345–370.

Little, J. D. C., J. F. Shapiro. 1980. A theory for pricing nonfeatured products in supermarkets. *J. Bus.* **53**(3) S199–S209.

Manchanda, P., A. Ansari, S. Gupta. 1999. The "shopping basket": A model for multicategory purchase incidence decisions. *Marketing Sci.* **18**(2) 95–114.

Mantrala, M. K., P. B. Seetharaman, R. Kaul, S. Gopalakrishna, A. Stam. 2006. Optimal pricing strategies for an automotive aftermarket retailer. *J. Marketing Res.* **43**(4) 588–604.

Montgomery, A. L. 2005. The implementation challenge of pricing decision support systems for retail managers. *Appl. Stochastic Models Bus. Indust.* **21**(4–5) 367–378.

Mulhern, F. J., R. P. Leone. 1991. Implicit price bundling of retail products: A multiproduct approach to maximizing store profitability. *J. Marketing* **55**(4) 63–76.

Neslin, S. A., R. W. Shoemaker. 1983. Using a natural experiment to estimate price elasticity: The 1978 sugar shortage and the ready-to-eat cereal market. *J. Marketing* **47**(1) 44–57.

Russell, G. J., A. Petersen. 2000. Analysis of cross category dependence in market basket selection. *J. Retailing* **76**(3) 367–392.

Shin, J. 2005. The role of selling costs in signaling price image. *J. Marketing Res.* **42**(3) 302–312.

Simester, D. 1995. Signalling price image using advertised prices. *Marketing Sci.* **14**(2) 166–188.

Simon, H., A. von der Gathen, P. W. Daus. 2006. Retail pricing—higher profits through improved pricing processes. M. Krafft, M. K. Mantrala, eds. *Retailing in the 21st Century: Current and Future Trends*. Springer, Berlin, 271–288.

Walters, R. G. 1981. Assessing the impact of retail price promotions on product substitution, complementary purchase, and interstore sales displacement. *J. Marketing* **55**(2) 17–28.

Walters, R. G., S. B. MacKenzie. 1988. A structural equations analysis of the impact of price promotions on store performance. *J. Marketing Res.* **25**(1) 51–63.

Walters, R. G., H. J. Rinne. 1986. An empirical investigation into the impact of price promotions on retail store performance. *J. Retailing* **62**(3) 237–266.

Walton, S. 1992. *Sam Walton: Made in America*. Bantam, New York.