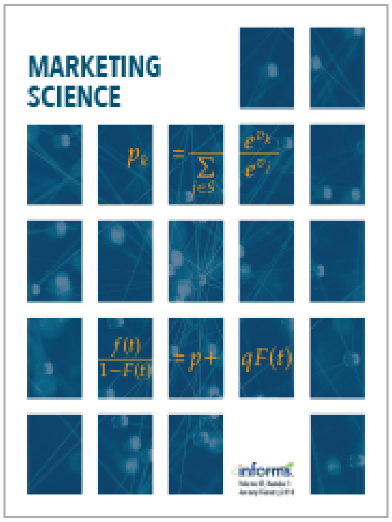

Marketing Science

Delegating Pricing Decisions

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Abstract

An outstanding problem in marketing is why some firms in a competitive market delegate pricing decisions to agents and other firms do not. This paper analyzes the impact of competition on the delegation decision and, in turn, the impact of delegation on prices and incentives. The theory builds on the simplest framework of competition in two dimensions: prices and (sales agents') effort. Specifically, we are interested in answering the following questions: (1) Does competition affect the price-delegation decision and, if yes, why? (2) How do prices vary under price-delegation and no-price-delegation scenarios? (3) Do the incentives to the sales representatives vary under the delegation and no-delegation scenarios?

To address these issues, we build a game-theoretic model that consists of two firms selling through their sales representatives. These representatives are company employees. Sales are a function of prices and selling efforts. The risk-neutral firms decide whether or not to delegate the pricing decision to risk-averse sales representatives. The wages to the sales representatives consist of salary plus a commission on gross margins. The commission on gross margins can be adjusted, either through communicated marginal cost of production, which we call ''virtual'' marginal cost, or directly. The firm and the sales representative are assumed to have the same information about the market, that is, there is no information asymmetry.

With competition in two dimensions, the strategic nature of decision variable *price* depends on the relative intensity of competition. With unobservable contracts and risk-averse sales representatives, firms delegate the pricing decision when price competition is intense. Part of the uncertainty in demand is absorbed by the firm by keeping ''virtual'' marginal cost greater than the marginal cost of production. The competing firm infers this through the risk aversion of the sales representatives. Under price delegation the sales representatives' wages are higher, and they set a price that is higher than what the firms would have set themselves. This leads to softening of price competition that is to the advantage of both firms. When the effort competition is dominant, however, the firms prefer to make the pricing decision themselves, because this reduces the intensity of effort competition among agents.

A single-instrument commission structure in which the firms adjust the ''virtual'' marginal cost is compared to a two-instrument commission structure in which the firms can adjust the commission rate, as well the ''virtual'' marginal cost. Under no price delegation, the two incentive schemes are the same. However, under price delegation the risk premium with the two-instrument scheme is lower. However, the prices and efforts are higher with the single-instrument scheme. When price competition is intense, the increase in risk premium with the single instrument scheme is more than compensated for by the increase in profits. This is the benefit of softening price competition through higher prices.

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1. Introduction[1]

Effort and pricing are involved in virtually all selling situations. Firms that employ sales personnel let them decide on the effort level required for selling the product, because the firm is unable to monitor costlessly the interaction between a sales rep and a client.

Some firms let sales reps decide on the price as well. General Electric and ASEA Brown Boveri delegate pricing decisions to their sales reps in the heavy equipment industry. So do KSB, Inc. and Wallace and Tieman, Inc. in the pumps industry. However, Allen Bradely and Cutler-Hammer in the switchgear industry and Siemens and FluroScan in medical imaging do not delegate their pricing decisions. Why do some firms delegate pricing and others do not?

This paper researches the role of the incentives to delegate prices and analyzes, in turn, the impact of delegation on prices, incentives, and competition. In a model with firms competing through sales representatives, we find that under intense price competition, firms delegate the pricing decision to the sales reps. Similar findings that greater price competition enhances the incentive to delegate price setting have been reported in the survey study by Stephenson et al. (1979). However, they did not provide an explanation for this.

The framework of this paper differs in four ways from that of the existing literature on price delegation. First, it is the first to address the issue of price delegation in a framework with competing firms. Second, delegation is explained by its strategic commitment value, not as a result of agents' superior information, as in Lal (1986). Third, the demand for the products depends on the prices as well as the efforts put in by the agents. Thus, competition is on two dimensions—price and effort—and not on prices alone, as in Sklivas (1987) and Fershtman and Judd (1987). Fourth, the contracts between a firm-rep pair are unobservable to the competitors.

We posit a duopoly model in which the firms sell differentiated products. The demand for the products depends on prices (which may or may not be decided by sales reps) and effort put in by sales reps. Our results indicate that the strategic nature of decision variable *price* depends on the relative intensity of competition in the two dimensions. Even in the region in which price is a strategic complement variable and price competition is intense, firms prefer not to delegate pricing decision when the effort competition is intense. This is contrary to what one would expect when competition is in one dimension, price, in which firms adopt decentralization (Moorthy 1988). In our model, the prices that the rep would set are higher than the prices that the firm sets, which leads to softening of price competition. This is attributable to the role of risk aversion of sales reps and the unobservability of contracts. With our richer framework we get results that cannot be predicted by the research of McGuire and Staelin (1983) in the channels literature.

Commissions on sales volumes and on gross margins are widely observed. When we switch the commission from margins to sales volume, delegation is never optimal. With commissions on sales volume, delegation would have the effect of reps reducing price to increase unit sales and their wages. Thus, delegation leads to increased price competition.[2]

In the next section we review related literature from marketing and economics. We develop our model in §3 and present the results of our analysis in the context of linear demand in §4. In §5 we discuss the effects of nonlinearity in demand. In §6 we discuss the results. The final section contains concluding remarks and directions for future research.

2. Literature Review

The issue of price delegation by a monopolist has been analyzed in the salesforce literature. Under deterministic conditions, the use of a commission that is based on a percentage of gross margin aligns the objectives of the firm and the sales rep. Therefore, price can be delegated, and the sales rep can be expected to set prices that are optimal for the firm (Weinberg 1975). When the demand is stochastic, price delegation is more profitable when the sales

[1]Mathematical details for the paper are in the technical appendixes.

[2]Refer Appendix 2A for details.

rep's information about the selling environment is superior to that of the firm's (Lal 1986). Thus, under monopoly, the incentive to delegating price depends entirely on information asymmetries. However, we analyze a duopoly with competition in two dimensions. An empirical study, Stephenson et al. (1979), finds various market conditions under which price delegation is more likely to be observed, but does not provide an explanation. One of the conditions identified as favoring price delegation is that of intense price competition in the market.

A second set of literature, the channels-of-distribution literature, has analyzed the issue of centralized versus decentralized decision making in intrabrand and interbrand competitive situations (e.g., Katz 1991, McGuire and Staelin 1983).[3] McGuire and Staelin (1983) show that if competing firms' products are highly substitutable in demand, then decentralization is a Nash equilibrium strategy for the firms. However, Moorthy (1988) shows that a necessary condition for this result is that the firms' prices be strategic complements (i.e., when the competitor reduces its price, it is beneficial for the firm to reduce its price as well). In Moorthy (1988), firms compete on one dimension (price), and it can be determined a priori whether the decision variables are strategic substitutes or strategic complements.[4] The strategic nature of the decision variable does not change with the intensity of competition. We find here that when the competition is in two dimensions—price and effort—the strategic nature of the decision variable depends entirely on the relative intensity of competition in these two dimensions. This makes the strategic theory of delegation richer, because no longer can the actions of the firms be determined a priori to be strategic substitutes or strategic complements. Do the results of McGuire and Staelin (1983), as extended by Moorthy (1988), apply when competition is in two dimensions? The answer is no. We find that under intense price and effort competition, even in the region in which price is a strategic complement, no price delegation is the Nash equilibrium.

A third set of literature, strategic delegation literature, focuses primarily on the choice of managerial incentive system (Sklivas 1987, Fershtman and Judd 1987) when the agents make decisions about price (Bertrand competition) or quantity (Cournot competition). It has not addressed the issue of when to and when not to delegate price or quantity decisions. Unlike the principal-agent literature in salesforce, this stream of research assumes that demand does not depend on the effort put in by risk-neutral agents. In our model, price delegation is publicly observable and is, therefore, a commitment to allocate the decision making between the rep and the firm without making public the decisions themselves. With competition in two dimensions—price and effort—our research analyzes when it is beneficial for firms to delegate pricing decisions to their risk-averse agents.

We are addressing the issue of price delegation to sales reps when firms compete in two dimensions. We first show that with competition in two dimensions, the strategic nature of decision-variable price depends on the relative intensity of competition. With unobservable contracts and risk-averse reps, firms delegate the pricing decision when price competition is more intense. Part of the uncertainty in demand is absorbed by the firm by keeping the ''virtual'' marginal cost greater than the marginal cost of production, c. The competing firm can logically infer this through the risk aversion of the reps. Hence, under these circumstances reps would set a higher price. This is the benefit of having risk-averse sales reps.

3. Overview of the Model

We consider two firms selling differentiated products through sales reps. The marginal cost of production, c, is constant. The demand for each firm's product (q_1 and q_2) depends on prices (p_1 and p_2), on efforts (e_1 and e_2) put in by the reps, and on a common random

[3]McGuire and Staelin (1983) find that when product substitutability is high ($0.931 < \theta < 1$), there is no *unique* Nash equilibrium, and for lower values ($0 < \theta < 0.931$) pure integrated system is the *unique* Nash equilibrium. Thus, a decentralized system is never a *unique* Nash equilibrium in their research.

[4]The terms *strategic substitutability* and *strategic complementarity* were introduced by Bulow et al. (1985).

shock, δ.[5] An increase in effort by the competing rep decreases the demand for one's own product. Because effort is not observable by the firm, it is not contractible. We consider two kinds of contracts. The first contract, when the firm delegates pricing decisions, consists of a commission rate on gross margin and a fixed-wage rate. The second contract, in which the firm does not delegate the pricing decision, consists of the price that the rep must charge in the market in addition to a commission rate and a fixed-wage rate.

Structure of the Game

Although the game is modeled as a simultaneous single-shot game in which the players choose optimal strategies, there is an implied order of the play:

Stage 1. The firms simultaneously decide whether or not to delegate price. This delegation decision is observable (to the competitor) before the next stage.

Stage 2. The firms then independently decide on contracts that are not observable.

Stage 3. Reps examine the contract offered and decide whether to accept or reject it.

Stage 4. If the reps accept the contract, then each rep chooses price and effort (or only effort, as the case may be) to maximize expected utility. The prices are observable to both the firms and the sales reps.

Stage 5. The common random shock is realized; the demand for each firm's product is determined; and the commissions are paid.

The model is a Principal-Agent setup in a competitive scenario with conventional assumptions about demand, utility, and cost-of-effort functions. As in Holmstrom and Milgrom (1991), we adopt a linear-quadratic-normal distribution parameterization.

Assumption 1. *The demand for firm i's product is given by*

$$q_i(p_i, e_i, p_j, e_j) = h - p_i + \theta_p p_j + e_i - \theta_s e_j + \delta,$$
$$0 < \theta_p < 1, \quad 0 < \theta_s < 1, \quad (1)$$

where δ is the common random shock and is normally distributed with a mean zero and variance σ_δ^2. The parameters θ_p and θ_s are the marginal impacts on demand of a unit increase in the competitor's price and effort, respectively.

Assumption 2. *The marginal cost c for both firms is constant. The values of the intercept h in the demand function and of marginal cost c are such that the expected demand is always positive in the relevant range of price and effort.*

Assumption 3. *The cost of effort is a convex function given by $G(e_i) = e_i^2$. The cost is expressed in dollars.*

Assumption 4. *The firms are risk neutral. The agents are risk averse with a constant absolute risk aversion (CARA) utility function given by $U(x) = 1 - e^{-rx}$, where x is the sales rep's wealth or earnings and r is the coefficient of risk aversion.*

4. Model Development

General wage structures consisting of salary plus commission on sales or on margins or profitability are commonly observed in the market (Peck 1982, Churchill et al. 1991). The role of these wage structures for eliciting optimal efforts in a *monopoly* have been analyzed by Basu et al. (1985), Lal and Staelin (1986), and Rao (1990) under different assumptions concerning heterogeneity in selling-effort productivity, salesperson's utility function, etc. In our research, we explicitly model competition between firms that employ sales reps. Also, the competition is in two dimensions, prices and efforts. The first step is to see if this competition is any different from competition in prices alone. To address this we examine the strategic nature of price variable when there is competition in two dimensions—prices and selling effort—and the wages are on unit sales or on margins. We then consider the margin-based contracts when determining the Nash equilibrium with and without price delegation.[6]

[5]The qualitative results do not change with uncorrelated random shock. In the common random shock scenario, there is an implicit assumption that a rep cannot be compensated based on the profits or the sales of the competitor. This is based on a realistic consideration that a firm has much better information about its profits and sales than about its competitor's.

[6]It is shown in Appendix 2A that sales-based commissions and price delegation are never optimal.

Price: A Strategic Substitute or a Strategic Complement Variable When Competition Is in Two Dimensions

We consider two kinds of contracts. In the first type of contract the agent is offered a contract of the form $W = \alpha + \beta(q_i)$, where α is the fixed salary and β is the commission on unit sales. The rep makes no pricing decisions. Under this setup, the optimization problem for each firm i is given by

$$\max_{p_i,\beta_i} E\lfloor (p_i - c)q_i(p_i, e_i, p_j, e_j)\rfloor - W \qquad (2a)$$

subject to

$$e_i = \text{argmax}[EU(W, e)]$$

Incentive Compatibility Constraint,

$EU(W, e) \geq \bar{u}$ Participation Constraint.

In the second type of contract each firm offers its sales rep a contract specifying a fixed salary α_i and a commission rate $\beta_i = (p_i - y_i)/(p_i - c)$. The variable y_i can be interpreted as the rep's marginal cost of production (equal to c or higher). This commission structure is equivalent to a standard form of commission on gross margin.[7] Under this contract the optimization problem for each firm is given by

$$\max_{y_i,p_i,\alpha_i} E[(y_i - c)q_i - \alpha_i] \qquad (2b)$$

subject to

$$e_i \in \text{argmax } E\lfloor U((p_i - y_i)q_i + \alpha_i - e_i^2)\rfloor,$$

$$E\lfloor U((p_i - y_i)q_i + \alpha_i - e_i^2)\rfloor \geq 0.$$

LEMMA 1. *Whether price is a strategic substitute or a strategic complement variable is determined by the following equation:*

$$p_i(3 + 8r\sigma_\delta^2) - (2\theta_p + 4\theta_p r\sigma_\delta^2 - \theta_s)p_j$$
$$= 2h + 4hr\sigma_\delta^2 + (1 + \theta_s + 4r\sigma_\delta^2)c. \qquad (3)$$

PROOF (REFER TO APPENDIX 1). Equation (3) allows the characterization of strategic nature, which is endogenous in our model. When the price competition is intense, as compared to effort competition ($\theta_p > \theta_s/(2 + 4r\sigma_\delta^2)$), the reaction to a decrease in price by the rival firm is to decrease its own price. Thus, price is a strategic complement. However, when effort competition is intense ($\theta_p < \theta_s/(2 + 4r\sigma_\delta^2)$), price becomes a strategic substitute. In the strategic substitute region, an increase in price by a competitor results in a decrease in its own demand, to which the best response is a decrease in one's own price (and, similarly, in the strategic complement region an increase in price by a competitor increases its own demand, to which the best response is an increase in one's own price). Thus, in contrast to a scenario in which price is the only decision variable, in our model, where competition is on two dimensions—price and effort—price can be a strategic substitute or strategic complement, depending on the nature of product market competition. In §6 we discuss the importance of this result.

Margin-based Compensation (Plan A)

We now consider the game subsequent to the announcement by the firm of its price-delegation decision and the rep's acceptance of the contract. The equilibrium consists of the contracts offered by the firms and the price and effort levels chosen by the reps. Each firm offers its sales rep a contract specifying a fixed salary α_i and a commission rate β_i. The variable y_i can be interpreted as the rep's marginal cost of production (equal to c or higher). Because the prices are observable, this wage structure is equivalent to commission on margins. By varying this "virtual" marginal cost of production above c, the firm can commit the rep to less aggressive behavior in a market. Because the agent is making the price decision, the firm's decision variable becomes y_i. In §4.7 we consider a more complex remuneration structure and its benefits to the firm.

To determine the equilibrium delegation decision, we use backward induction that ensures that the strategy combination is a subgame perfect equilibrium. Hence, in this section we analyze the contracts between the firms and sales reps in a given delegation situation by studying the subgame subsequent to the announcement of the type of delegation—price dele-

[7]The wages can be rewritten as $\gamma(p_i - c_i)$ (unit sales), where $\gamma = 1 - (x_i)/(p_i - c_i)$.

gation or no price delegation. The unobservability of contracts, commission, and salary in solving for equilibrium implies that in deciding on the contracts with the reps, the firm considers only its effect on its own sales rep's actions while regarding the competitor's contract and actions as fixed.[8]

4.1 Reps Make Effort and Pricing Decisions ("Price Delegation" Subgame)

Each firm, while deciding on the contract, considers the effect of the contract on its rep's actions (again, taking the actions of competing rep as given). The equilibrium contracts, prices, and efforts simultaneously solve the following maximization program for each firm:

$$\operatorname*{Max}_{y_i, \alpha_i} E\lfloor (y_i - c)q_i(p_i, e_i, p_j, e_j) - \alpha_i \rfloor$$

subject to

$$(p_i, e_i) = \operatorname{argmax} E\lfloor U((p_i - y_i)q_i(p_i, e_i, p_j, e_j) + \alpha_i - e_i^2)\rfloor, \tag{4}$$

$$E\lfloor U((p_i - y_i)q_i + \alpha_i - e_i^2)\rfloor \geq 0. \tag{5}$$

The objective function is the firm's profits. Equation (4) is the incentive compatibility constraint, and Equation (5) is the individual rationality constraint for the rep.[9] From Assumption 1, the firm's profit is normally distributed. Because the rep's utility function is exponential, his expected utility can be expressed in certainty equivalent (CE) terms as:

$$CE = \alpha_i + (p_i - y_i)(h - p_i + \theta_p p_j + e_i - \theta_s e_j) - e_i^2 - \frac{1}{2}(p_i - y_i)^2 r\sigma_\delta^2. \tag{6}$$

Thus the agent will, postcontract, choose price and effort levels to maximize CE. The incentive compatibility condition can now be replaced by the first-order conditions given by

$$(p_i - y_i)(-1) + Eq_i - r(p_i - y_i)\sigma_\delta^2 = 0, \tag{7}$$

$$p_i - y_i - 2e_i = 0. \tag{8}$$

Because the salary component α_i does not enter the agent's first-order conditions, the firm can adjust it without affecting the agent's choice of price and effort level. Thus from Equations (5) and (6), the binding participation constraint can be written as

$$\alpha_i + (p_i - y_i)E[q_i(\delta)] - e_i^2 - \frac{1}{2}(p_i - y_i)^2 r\sigma_\delta^2 = 0. \tag{9}$$

The risk premium that the firm needs to give to the rep is the last term on the left-hand side of Equation (9). It increases with the uncertainty (σ_δ^2) in demand and the risk aversion (r) of the sales rep.

LEMMA 2. *A pair of contracts $\{y_i, \alpha_i\}$ and a pair of prices and efforts $\{p_i, e_i\}$, where $i = 1, 2$, constitute an equilibrium if and only if the following are satisfied:*

$$(y_i - c)E\left(\frac{\partial q_i}{\partial y_i}\right) + r\sigma_\delta^2(p_i - y_i) + \frac{\partial p_i}{\partial y_i}\left[-\frac{p_i - y_i}{2} + h - p_i + \theta_p p_j - \theta_s e_j - r\sigma_\delta^2(p_i - y_i)\right] = 0, \tag{10}$$

$$(p_i - y_i)(-1) + Eq_i - r(p_i - y_i)\sigma_\delta^2 = 0, \tag{11}$$

$$p_i - y_i - 2e_i = 0, \tag{12}$$

$$\alpha_i + (p_i - y_i)E[q_i(\delta)] - e_i^2 - \frac{1}{2}(p_i - y_i)^2 r\sigma_\delta^2 = 0. \tag{13}$$

PROOF (REFER TO APPENDIX 1A). With demand uncertainty, the risk-averse agents would set a price that is too low. However, the firm would absorb some of the uncertainty in demand by keeping the ''virtual'' marginal cost of production y_i greater than c (Mathewson and Winter 1984). The competing firm, although it does not know the contract (because it is unobservable), can logically infer through the risk aversion of the reps that the other firm would set y_i greater than c. Under these circumstances the reps would set a higher price (with y_i as cost of production) than that which the firms

[8]The contracts are part of a Nash equilibrium and as such are not observed by each competitor but are rationally inferred as part of the equilibrium concept.

[9]We assume that the reservation utility of the agent is zero.

would set (with c as cost of production). This is the benefit of having risk-averse agents.[10]

PROPOSITION 1. *In the symmetric Nash equilibrium under price delegation, the prices, the effort levels, and the commissions increase as the price competition becomes more intense and decrease as the effort competition becomes more intense.*

PROOF (REFER TO APPENDIX 1B). Prices increase with an increase in cross-price impact and so does y_i. However, the increase in p_i is greater than the increase in y_i, and so the commission income to the rep increases as the price competition becomes more intense. This gives an incentive to the rep to put in more effort or to keep the price higher, and thereby keep the margin higher.

Rewriting the firm's payoff as

$$\underset{y_i}{\text{Max}}\ E[(y_i - c)q_i(\delta) - \alpha_i]$$

$$= (p_i - c)E[q_i] - e_i^2 - \frac{1}{2}r\sigma_\delta^2(p_i - y_i)^2,$$

we can see that the firm's payoff is the payoff when the firm sells directly (i.e., with no sales rep) at the same price and effort, less the risk premium that has to be given to the sales rep. When y_i is set equal to marginal cost c, a small increase in y_i from c has no marginal effect on the firm's profits, but has a negative effect on the risk premium to the agent, which is a gain for the firm. This private incentive, attributable to the reduction in the risk premium, makes it credible for the firm to increase y_i over the marginal cost c. This is an example of the proposition put forth by Katz (1991) that even unobservable contracts can be manipulated for strategic purposes when the agents are risk averse.

4.2. Reps Make Only Effort Decision ("No-Price-Delegation" Subgame)

Now let us consider the case in which the firm sets the retail price and the agent selects the effort level, given price and the demand.

[10]In fact, the condition by Equation 1A.1 (Appendix 1A) shows that y_i is greater than c as long as the expected demand is positive.

PROPOSITION 2. *In the symmetric Nash equilibrium under no-price delegation, the prices, the effort levels, and the commission increase as price competition becomes more intense but decrease as effort competition becomes more intense. In either case the values of prices, the effort levels, and the commission under no-price delegation are lower than the values under price delegation.*

PROOF (REFER TO APPENDIX 1D1). In comparison to the price-delegation contract for the same commission rate, a firm under no price delegation would communicate a higher cost of production. However, if the ''virtual'' marginal cost of production is the same, the firm would set a lower price. This implies that the commission is lower, and that under no price delegation the rep puts in less effort. These observations may not necessarily hold true for the actual equilibrium prices and efforts, because the communicated costs of production are determined endogeneously. However, it does indicate how different types of delegation decisions affect the competition in the market by changing the behavior of firms and sales reps.

The results for the no-delegation case differ from the results for the delegation case through the y_i terms. The firm sets higher values of y_i with increase in cross-effort impact, and so, even though the price increases, the commission to the reps does not increase as much with increase in cross-effort impact.

4.3. Firm 1 Has Price Delegation and Firm 2 Has No Price Delegation ("Asymmetric" Subgame)

For completeness, we must consider the situation in which one firm delegates pricing decisions and the other does not. Intuitively, the earlier analysis can be used here directly, because in finding the equilibrium conditions we did not assume any specific contract offered by the competing firm. Thus, in the equilibrium of this subgame, the contract by Firm 1 (y_i^D, α_i^D; p_i^D, e_i^D) must satisfy the conditions given in Lemma 2, and the contract by Firm 2 (y_j^N, α_j^N, p_j^N; e_j^N) must satisfy the conditions (refer to Appendix 1C) given by Equations (1C.4) to (1C.7), simultaneously. The results and sketch of the proof are given in Appendix 1D2.

Having looked at the equilibrium contracts under various cases, we can comment on the strategic role of price delegation. The firm, while deciding on the contract, must consider the effect of its delegation decision on the price and effort level offered by the competing firm's rep. Choosing a different delegation decision in effect amounts to committing to a different behavior in terms of price, effort, and commission. This commitment influences the competing firm in its contract design for its own sales rep. This is the strategic impact of the delegation decision. To complete the analysis, having worked out the subgames, we turn to the first stage of the game: the delegation decision.

4.4. Equilibrium Delegation Decision

The delegation decision depends on the payoffs resulting from the equilibrium contract under each type of delegation decision, delegate (D) or do not delegate (N). This can be represented as a 2 × 2 matrix shown below.

	Firm 2	
Firm 1	Price Delegation (D)	No-Price Delegation (N)
D	(Π_1^{DD}, Π_2^{DD})	(Π_1^{DN}, Π_2^{DN})
N	(Π_1^{ND}, Π_2^{ND})	(Π_1^{NN}, Π_2^{NN})

The diagonal entries are the profits to each firm under symmetric delegation decisions. Thus Π_1^{DD} and Π_1^{NN} are the profits to each firm ($i = 1, 2$) under price and no-price delegation, respectively. Also (Π_1^{DN}, Π_2^{DN}) represents the profits under mixed equilibrium when Firm 1 decides on price delegation and Firm 2 on no price delegation. To find the equilibrium we first need to calculate the profits.[11] Because the firms and reps are identical in all respects, except for their respective price delegation decisions (for each firm-rep pair), it is expected that the equilibrium will be symmetric. Therefore, to find the Nash equilibrium we now have to compare the profits as given in the 2 × 2 matrix.[12] Price delegation is the dominant strategy for both firms, and (Delegation, Delegation) is the unique equilibrium when $\Pi_1^{DD} > \Pi_1^{ND}$ and $\Pi_1^{DN} > \Pi_1^{NN}$. If these two inequalities hold, then Firm 1 would prefer to delegate the pricing decision, no matter what Firm 2 prefers. Similarly, Firm 2 would prefer price delegation, regardless of Firm 1's choice. When $\Pi_1^{DD} < \Pi_1^{ND}$ and $\Pi_1^{DN} < \Pi_1^{NN}$, no-price delegation is the dominant strategy and (No-Price Delegation, No-Price Delegation) is the unique equilibrium. However, if $\Pi_1^{DD} < \Pi_1^{ND}$ and $\Pi_1^{DN} > \Pi_1^{NN}$, we have an equilibrium in which one firm delegates the pricing decision and the other does not, that is, (D, N) and (N, D) are the equilibria.

To analyze we must turn to graphical characterizations.[13] The only variables in these conditions are θ_p, θ_s, and $r\sigma_\delta^2$. We call the term $r\sigma_\delta^2$ the *utility cost of uncertainty.* As r approaches zero, the rep's risk attitude approaches risk neutrality, and the product $r\sigma_\delta^2$ approaches zero, implying that the rep and the firm bear no cost attributable to uncertainty. As r increases, the product $r\sigma_\delta^2$ also increases, so one can say that the cost due to uncertainty (scaled by the risk attitude of the rep) also increases. To analyze, we fix the value of $r\sigma_\delta^2$ to one, and then examine the contours of $\Pi_1^{DD} - \Pi_1^{ND}$, $\Pi_1^{DN} - \Pi_1^{NN}$, and $\Pi_1^{DD} - \Pi_1^{NN}$. These contours are shown in Figure 1 as curves DD\ND, DN\NN, and DD\NN, respectively.

Hence, in region DD price delegation by both the firms is the equilibrium, and in region NN no-price delegation by both the firms is the equilibrium. In the region marked DN\DD\ND\NN, $\Pi_1^{DD} > \Pi_1^{ND}$, $\Pi_1^{DN} < \Pi_1^{NN}$, and $\Pi_1^{DD} > \Pi_1^{NN}$. As a result, there are multiple Nash equilibria in this region: Both firms either delegate the pricing decision or both firms do not delegate.

From the diagram, given θ_s, the higher the value of θ_p (i.e., the greater the price competition) the more

[11]Refer to Appendix 1E.

[12]The existence of Nash equilibrium has been checked in each subgame for the entire range of cross-price and cross-effort impacts. At the delegation decision stage, existence can be proved using best-response correspondence for the two firms and Kakutani's fixed-point theorem. At the price and effort subgame the strategy sets are convex, nonempty, closed, and bounded. The utility functions are concave and so have quasiconcave contours.

[13]Refer to Appendix 2B for details of analytical solutions.

likely the firms are to pick a price-delegation strategy. Similarly, for a given value of θ_p, the higher the value of θ_s (i.e., the greater the effort competition), the more likely are the firms to pick the no-price-delegation strategy.[14,15]

When $\theta_s > 0.82$, no matter how intense the price competition is, price delegation is never an optimal strategy for the firms to adopt because they want to reduce effort competition. This is achieved by reducing the commission rates to the sales reps and not delegating price.

Proposition 3. *For a firm with a risk-averse sales rep facing an uncertain demand, price delegation is the equilibrium when the price competition is more intense and no price delegation is the equilibrium when the effort competition is more intense.*

The delegation decision that will emerge in equilibrium is the one that is more effective in reducing competition in the dimension in which competition is more intense. Under price delegation, the rep tends to charge a higher price than what the firm would charge under no-price delegation. The firm does this by giving the rep a higher commission on gross margin. Under no-price delegation the rep has a lower commission, which reduces her incentive to provide higher effort.

In region DD\DD\NN\ND, $\Pi_1^{DD} > \Pi_1^{NN}$, so firms collectively prefer price delegation. The firms would like to agree to a price-delegation strategy before making the actual choice. However, the commitment is not credible: Each firm, on its own, would want to deviate to no-price delegation. Hence, we have a prisoners' dilemma situation.

Figure 2 shows the effect of an increase in utility cost of uncertainty ($r\sigma_\delta^2 = 3$) on delegation strategy. The region DD becomes smaller while the region NN increases, i.e., with increase in risk aversion and/or demand uncertainty there is an increased possibility

Figure 1 **Equilibrium Regions** ($r\sigma_\delta^2 = 1$, $h = 100$)

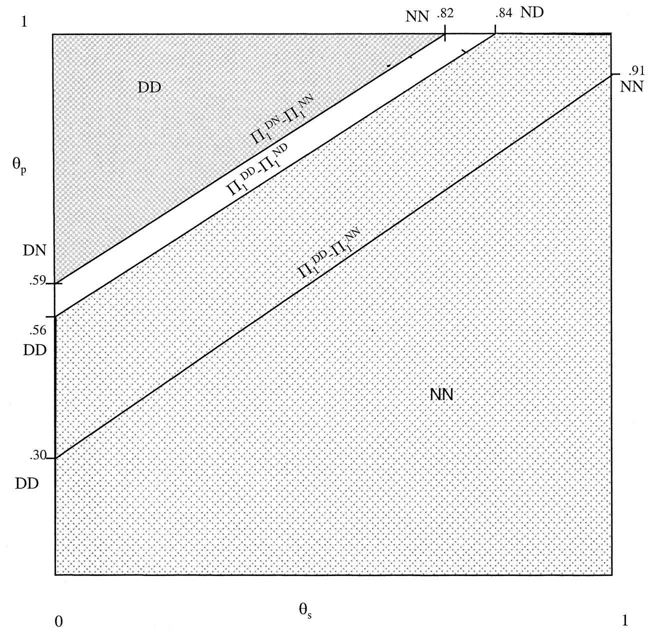
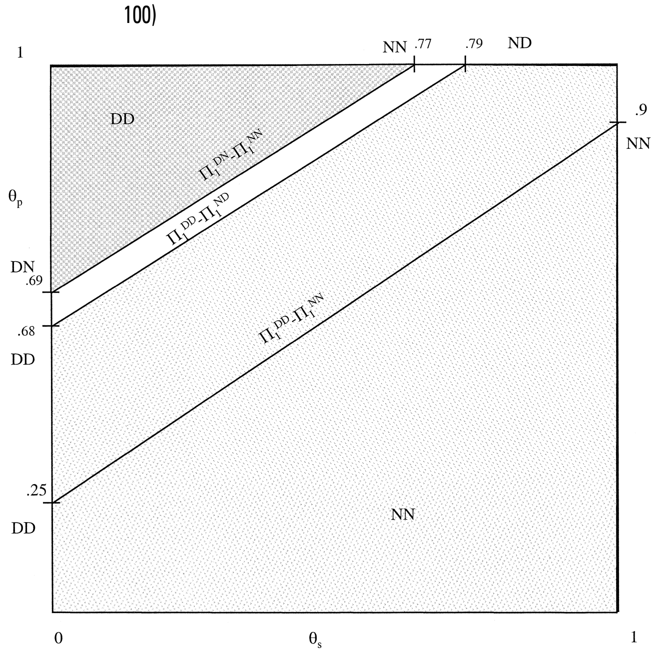


Figure 2 **Effect of Change in Utility Cost of Uncertainty** ($r\sigma_\delta^2 = 3$, $h = 100$)



[14]If θ_p and θ_s are zero, it implies that the demands are independent, so the firms are monopolists. Under these conditions the profit to the monopolist with price delegation is found to be same as the profit from no-price delegation. The result is the same as Lal (1986), under no-information-asymmetry condition.

[15]Specifically, for any θ_s in the range $0 \le \theta_s \le 0.82$, there exists a θ_p^* such that price delegation is optimal if $\theta_p > \theta_p^*$.

that the firm will decide not to delegate pricing, because this will reduce the risk premium to be given to the rep.[16]

4.5. Prices under Price-Delegation and No-Price-Delegation Scenarios

We now turn to the converse issue: How does delegation affect prices and effort? Assuming that c, the marginal cost of production, is zero for both firms and h is large enough to guarantee a nonnegative demand, we plot the prices under price delegation and no-price delegation in Figure 3.

Note that for any given level of effort competition, as price competition increases the price of the product also increases. However, as Figure 3 shows, the price under price delegation is always greater than the price under no-price delegation. When the effort competition is intense, firms prefer not to delegate the pricing decision. This helps in reducing competition in efforts. If there were to be an asymmetric equilibrium, i.e., one firm delegates the pricing decision and the other does not, the prices would vary in a different way. The firm that believes that there is greater effort competition would not delegate price, and the firm that believes that there is higher price competition would delegate the pricing decision. The firm that is not delegating price would set a lower price, and the firm delegating price would be setting a higher price.

4.6. Wages under Price and No-Price Delegation

Again setting $c = 0$, and a high value of h, we can plot the variation in wages to the sales reps with changes in price and effort competition. For any given value of effort competition, it is observed that the wage rate increases as the price competition increases. This is also true under no-price delegation.

We have shown earlier that as effort competition increases the firms move from price delegation to no-price delegation to ensure that the reps do not put in more than the desired level of effort. This is brought about by fixing the commission and not giving the rep the liberty to change the margin. On the other hand, as price competition increases, firms move toward price delegation, hoping to soften price competition. Higher commission rate on gross margins gives greater incentive to the reps to increase price. This, in turn, reduces price competition. The firm, whether adopting price delegation or no-price delegation, has to provide the rep with the same minimum utility to ensure that he works for the firm. The salary component under price delegation is reduced as a result of higher prices and commissions on gross margins. This results in higher salaries under the no-price-delegation scenario. See Figure 4.

Figure 3 **Price Under Price Delegation Higher than Price Under No-Price Delegation**

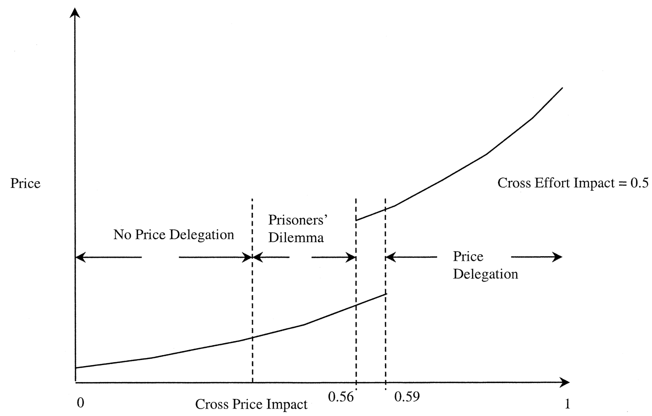
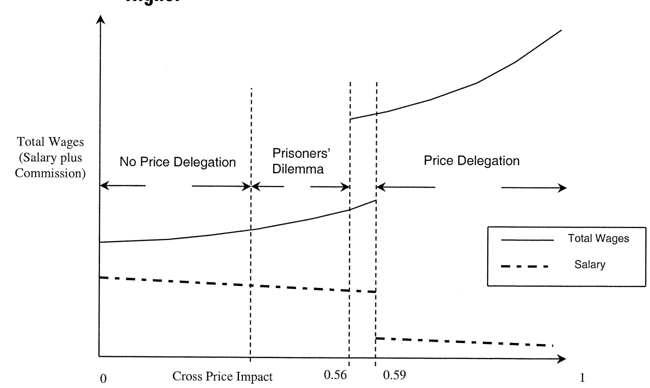


Figure 4 **Salary is Lower Under Price Delegation, but Total Wages are Higher**



[16]When the reps are risk neutral, the firms do not have to pay a risk premium to the reps, so there is no inefficiency due to risk sharing. For all values of cross-price and cross-effort impacts the firms can earn the same profits with price delegation as with no-price delegation, and vice versa. Thus, with risk-neutral reps one can observe an equilibrium where Firm 1 delegates pricing decision and Firm 2 adopts no-price-delegation strategy.

4.7. Margin-Based Compensation (Plan B)

In the earlier analysis we looked at a specific form of commission on gross margins viz., $(p_i - y_i)q_i$, which can be rewritten as

$$\left[\frac{p_i - y_i}{p_i - c}\right](p_i - c)q_i,$$

with the prices observable. This is equivalent to a standard form of commission on margins written as $\beta_i(p_i - c)q_i$. The firms use "virtual" marginal cost, y_i, to elicit the optimal effort by the sales reps.

We now look at margin-based commissions where the firm has two instruments: the "virtual" marginal cost and the commission β_i on the margin $(p_i - y_i)$ instead of on the margin $(p_i - c)$.[17] The wages to the sales rep are $W = \alpha + \beta_i(p_i - y_i)(q_i)$. Under price delegation, the equilibrium contracts, prices, and efforts simultaneously solve the following maximization program for each firm[18]:

$$\operatorname*{Max}_{\beta_i, y_i, \alpha_i} E\lfloor (p_i)q_i(p_i, e_i, p_j, e_j) - \beta_i(p_i - y_i)q_i(p_i, e_i, p_j, e_j) - \alpha_i \rfloor$$

subject to

$$(p_i, e_i) = \operatorname{argmax} E\lfloor U(\beta_i(p_i - y_i)q_i(p_i, e_i, p_j, e_j) + \alpha_i - e_i^2)\rfloor, \quad (14)$$

$$E[U(\beta_i(p_i - y_i)q_i + \alpha_i - e_i^2)] \geq 0. \quad (15)$$

The objective function is the firm's profits. Equation (14) is the incentive compatibility constraint, and Equation (15) is the individual rationality constraint for the rep.[19] From Assumption 1, the firm's profit is normally distributed. Because the rep's utility function is exponential, his expected utility can be expressed in certainty equivalent terms as

$$C = \alpha_i + \beta_i(p_i - y_i)(h - p_i + \theta_p p_j + e_i - \theta_s e_j) - e_i^2 - \frac{r}{2}\beta_i^2(p_i - y_i)^2\sigma_\delta^2. \quad (16)$$

The incentive compatibility constraints can now be replaced by the first-order conditions:

$$(p_i - y_i)(-1) + Eq_i - r\beta_i(p_i - y_i)\sigma_\delta^2 = 0, \quad (17)$$

$$\beta_i(p_i - y_i) - 2e_i = 0. \quad (18)$$

From Equations (17) and (18) we get

$$p_i = \frac{2y_i + 2h + 2\theta_p p_j - \beta_i y_i - 2\theta_s e_j + 2\beta_i y_i r\sigma^2}{4 - \beta_i + 2r\sigma^2\beta_i} \quad \text{and}$$

$$\frac{\partial p_i}{\partial y_i} > 0,$$

$$e_i = \frac{(-y_i + h + p_j\theta_p - \theta_s e_j)\beta_i}{4 - \beta_i + 2r\sigma^2\beta_i} \quad \text{and} \quad \frac{\partial e_i}{\partial y_i} < 0.$$

As the "virtual" marginal cost increases, the price set by the rep increases to maintain the margin. All else being equal, as the marginal cost increases the rep has a lesser incentive to work, and hence the effort decreases. Because the salary component α_i, does not enter the agent's first-order conditions, the firm can adjust it without affecting the agent's choice of price and effort level. Thus, from Equations (15) and (16) the binding participation constraint can be written as

$$\alpha_i + (p_i - y_i)E[q_i(\delta)] - e_i^2 - \frac{1}{2}\beta_i^2(p_i - y_i)^2 r\sigma_\delta^2 = 0. \quad (19)$$

The firm's maximization problem can now be expressed as

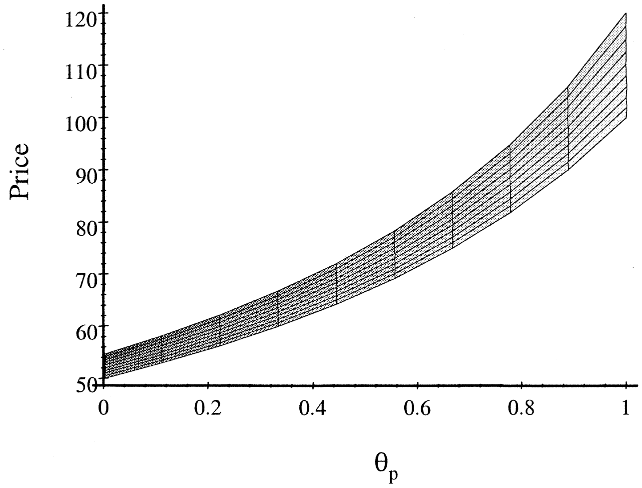
$$\operatorname*{Max}_{\beta_i, y_i} E[p_i q_i - \beta_i(p_i - y_i)q_i - \alpha_i].$$

Because the only difference between this plan and Plan A is the commission rate β_i, we need to see how commission affects the other variables. We find that $\partial p_i^D/\partial\beta_i > 0$, $\partial e_i^D/\partial\beta_i > 0$, $\partial y_i^D/\partial\beta_i > 0$ (refer to Appendix 3 for details). As the commission on margin increases, the rep puts in more effort and also has an incentive to increase the price, which results in reduced price competition. Also, as price competition increases, the optimal level of price also increases (Figure 5). This helps in reducing price competition.

Consider now the no-price-delegation situation. The equilibrium contracts, prices, and efforts simultaneously solve the following maximization program for each firm:

[17] We thank the area editor for this interesting extension.
[18] The true marginal cost of production, c, is assumed to be zero.
[19] We assume that the reservation utility of the agent is zero.

Figure 5 Prices Under Delegation Increasing with Price Competition



$$\underset{p_i,\beta_i,y_i,\alpha_i}{\text{Max}}\ E\lfloor(p_i)q_i(p_i, e_i, p_j, e_j) - \beta_i(p_i - y_i)q_i(p_i, e_i, p_j, e_j) - \alpha_i\rfloor$$

subject to

$$e_i = \text{argmax}\ E\lfloor U(\beta_i(p_i - y_i)q_i(p_i, e_i, p_j, e_j) + \alpha_i - e_i^2)\rfloor, \quad (20)$$

$$E\lfloor U(\beta_i(p_i - y_i)q_i + \alpha_i - e_i^2)\rfloor \geq 0. \quad (21)$$

The incentive compatibility constraint for the sales rep can be replaced by the first-order condition:

$$\beta_i(p_i - y_i) - 2e_i = 0.$$

From the first-order conditions, after substituting for α_i and e_i, we can write equations for price, effort, and the "virtual" marginal cost as

$$p_i = \frac{h + 2\theta_p p_j - \beta_i y_i - 2\theta_s e_j + \beta_i^2 y_i + 2r\beta_i^2 y_i \sigma_\delta^2}{4 - 2\beta_i + \beta_i^2 + 2r\beta_i^2\sigma_\delta^2},$$

$$e_i = \frac{.5\beta_i(h + 2\theta_p p_j + \beta_i y_i - 2\theta_s e_j - 4y_i)}{4 - 2\beta_i + \beta_i^2 + 2r\beta_i^2\sigma_\delta^2},$$

$$y_i = \frac{200(\beta_i + 2r\sigma_\delta^2\beta_i - 1)}{\beta_i(3 + \theta_s + 8r\sigma_\delta^2 - 2\theta_p + 4\theta_p r\sigma_\delta^2)}.$$

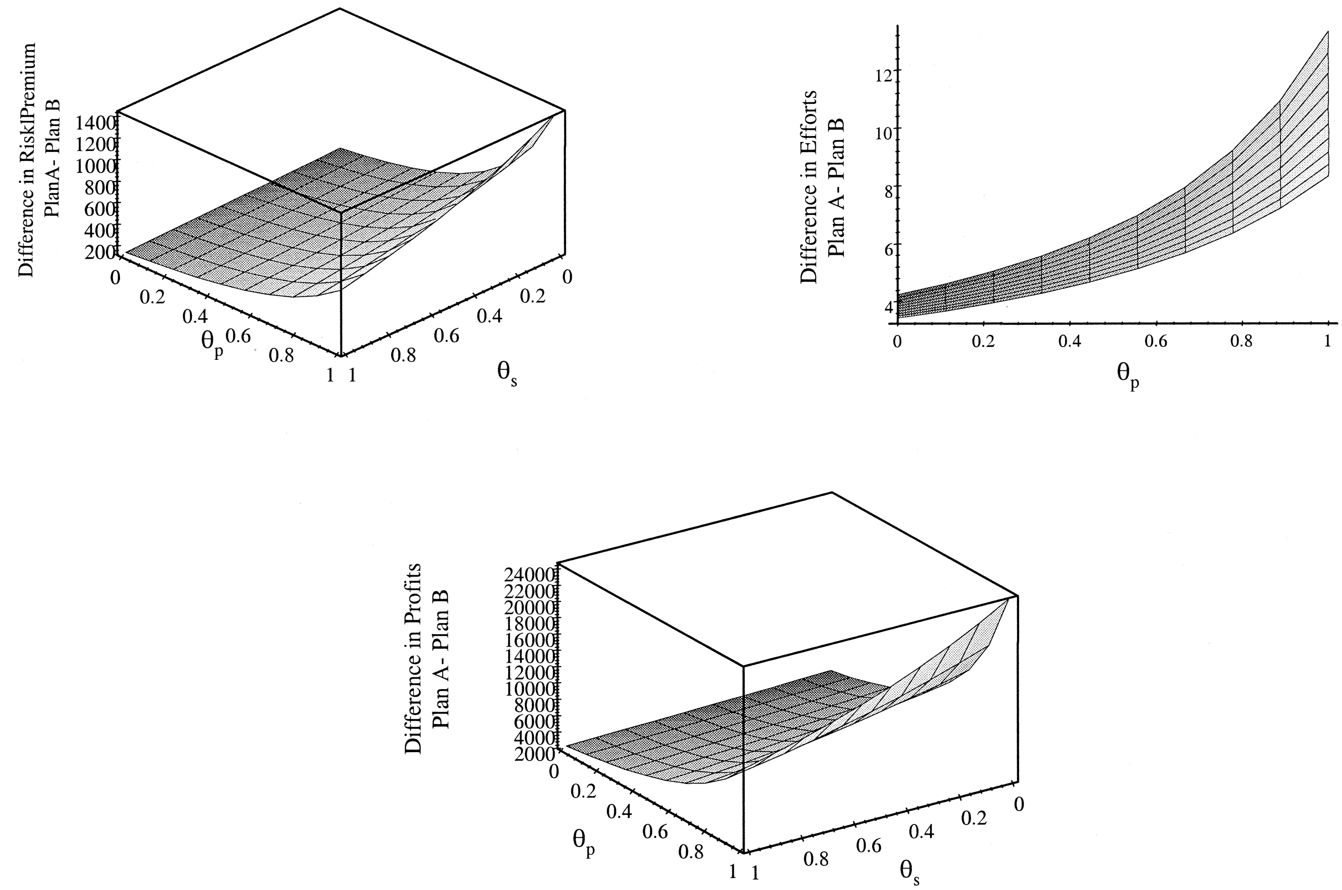
Calculating the optimal values of price, "virtual" marginal cost, and effort as a function of commission rate, we find that $\partial p_i/\partial\beta_i = 0$, $\partial e_i/\partial\beta_i > 0$, and $\partial y_i/\partial\beta_i > 0$ (refer to Appendix 3 for details). Table 1 shows the comparative statics under the two types of plans.

Table 1 Margin-based Compensation: Plans A and B

	Plan B $\beta(p - y)q$	Plan A $(p - y)q$
Delegation		
$\frac{\partial p_i}{\partial y_i}$	Positive	Positive
$\frac{\partial e_i}{\partial y_i}$	Negative	Negative
$\frac{\partial p_i}{\partial \beta_i}$	Positive	N/A
$\frac{\partial e_i}{\partial \beta_i}$	Positive	N/A
No Delegation		
$\frac{\partial p_i}{\partial y_i}$	Positive	Positive
$\frac{\partial e_i}{\partial y_i}$	Negative	Negative
$\frac{\partial p_i}{\partial \beta_i}$	No Effect	N/A
$\frac{\partial e_i}{\partial \beta_i}$	Positive	N/A

With Plan B, under no delegation and all else being equal, an increase in the commission rate is accompanied by a decrease in price and an increase in "virtual" marginal cost. However, the total derivative of price with respect to commission rate is zero; that is, the commission rate has no effect on pricing decision when the firm sets the price. This is because the firm can adjust the wages to the rep and its own profits by adjusting "virtual" marginal cost. Thus, under no-price delegation the two plans are the same. This is to be expected because what the firm can do with $\beta(p - y)q$ it can also do with $(p - y)q$, with suitable recalibration of y. The salesperson is paid a dollar amount for each unit sold under either plan, and once the firm sets the price, any amount can be chosen using the simple margin Plan A. Under Plan A the firms can decrease the effort by increasing the "virtual" marginal cost. Thus, if the firms wanted to reduce the effort competition between reps it could do so by decreasing β_i under Plan B or by increasing y_i under Plan A.

Figure 6 **Differences Between the Two Plans When Both Firms Adopt Price Delegation** ($r\sigma_\delta^2 = 1$, $h = 100$)



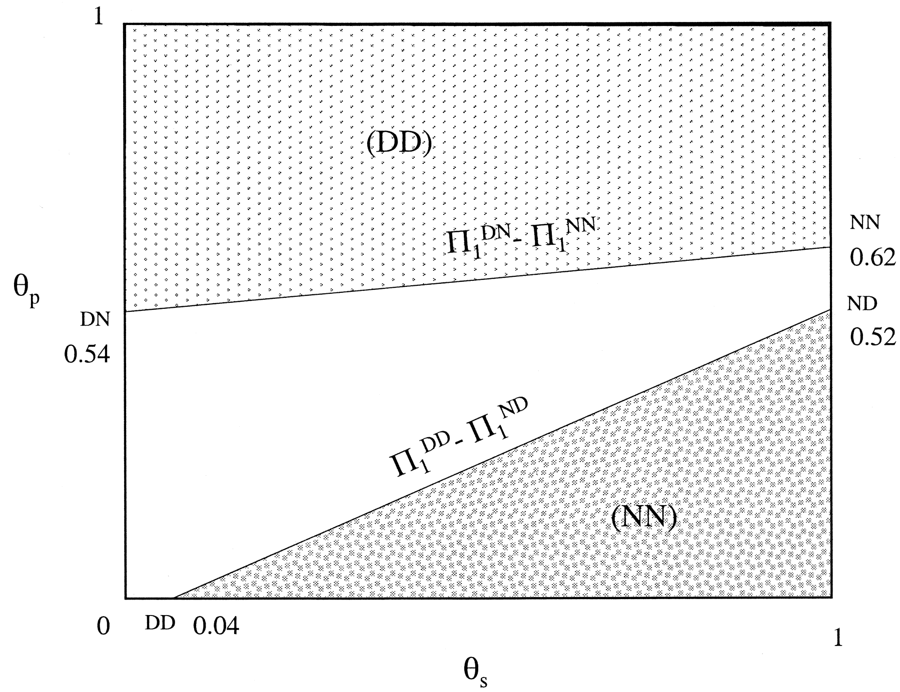
Under no-price delegation by firms, the risk premium under Plan A and Plan B is the same. However, with price delegation, the risk premium under Plan A is higher than under Plan B. However, prices and efforts under delegation are higher with Plan A than under Plan B. Thus when price competition is higher, the increase in risk premium with Plan A is more than compensated for by the decrease in competition through higher prices and higher efforts. Hence, the profits from Plan A are greater than the profits from Plan B (Figure 6). Plan A is better when keeping prices higher is important.

The delegation decision depends on the payoffs resulting from the equilibrium contract under each type of delegation decision, delegate (D) or do not delegate (N). This can be represented as the 2 × 2 matrix shown below:

	Firm 2	
Firm 1	Price Delegation (D)	No-Price Delegation (N)
D	(Π_1^{DD}, Π_2^{DD})	(Π_1^{DN}, Π_2^{DN})
N	(Π_1^{ND}, Π_2^{ND})	(Π_1^{NN}, Π_2^{NN})

The diagonal entries are the profits to each firm under symmetric delegation decisions. Profits (Π_1^{DN}, Π_2^{DN}) represent the profits under mixed equilibrium when Firm 1 decides on price delegation and Firm 2 on no-price delegation. Once again, price delegation is the dominant strategy for both the firms and (Delegation, Delegation) is the unique equilibrium when $\Pi_1^{DD} > \Pi_1^{ND}$ and $\Pi_1^{DN} > \Pi_1^{NN}$. No-price delegation is the dominant strategy when both the inequalities are reversed. We examine the contours of $\Pi_1^{DD} - \Pi_1^{ND}$and

Figure 7 **Equilibrium Regions** ($r\sigma_\delta^2 = 1$, $h = 100$, $\beta_i = 0.5$)



$\Pi_1^{DN} - \Pi_1^{NN}$ for the case $h = 100$, $\beta_i = 0.5$, and $r\sigma_\delta^2 = 1$. These contours are shown in Figure 7. In the Region DD, price delegation by both the firms is the equilibrium, and in region NN no-price delegation by both firms is the equilibrium. In the region marked DN\DD\ND\NN there are multiple equilibria: Both firms either delegate the pricing decision or both firms do not delegate.

5. Nonlinear Demand Function

In many marketing equilibrium analyses, linear demand functions are popular because of mathematical tractability in providing valuable analytical results. Yet one can expect nonlinearity in demand functions in several settings. In this section we analyze the issue of delegating the pricing decision to the sales reps when the demands faced by the firms are nonlinear. We begin with a general setup and then apply a specific functional form. Because of the functional complexity with two decision variables—price and (selling) effort—it is at times extremely difficult to obtain analytical solutions. When analytical solutions are hard to get, we use numerical techniques for the analysis. The mathematical software MATLAB is used for simulations.

Let the expected quantity sold by firm i be represented by the general form

$$E(q_i) = h + f_1(p_i) + \theta_p p_j + f_2(e_i) - \theta_s e_j, \quad (22)$$

where

$$f_1'(\cdot) < 0, \quad f_1''(\cdot) > 0 \quad \text{and} \quad f_2'(\cdot) > 0, \quad f_2''(\cdot) < 0.$$

The quantity sold decreases with an increase in its own price. The impact of a unit change in its own price on quantity sold is more at lower prices than at higher prices. Also, the quantity sold increases with the effort put in by its own rep. However, at higher levels of effort a unit change in its own rep's effort has a smaller effect on quantity, i.e., the demand equation reflects diminishing marginal returns to effort. The cross-price and cross-selling effort-demand derivatives are constant for any state of the world. This means that the sensitivity of demand for one product with respect to the other product's price and the other rep's effort is not a function of the level of demand (Coughlan 1985). Because we are modeling a duopoly with similar firms, the prices and efforts by the sales reps are equal in the equilibrium. Thus, the cross-price and cross-selling effort-demand derivatives do not change with the level of equilibrium prices and efforts. We do not consider the Cobb-Douglas function because this class of constant elasticity functions is known to result in price strategies that are independent of the competitor's strategy (Moorthy 1988, Choi 1991). Our interest is in studying the strategic role of price delegation and the role it plays in softening price competition.

Setting up the price-delegation maximization problem similar to the linear demand case, and solving, gives us the following first-order conditions (refer to details in Appendix 2C):

$$\text{(i)}\quad p_i\colon\quad (p_i - y_i)\frac{\partial f_1}{\partial p_i} + E[q_i] - r(p_i - y_i)\sigma_\delta^2 = 0,$$

$$\text{(ii)}\quad e_i\colon\quad (p_i - y_i)\frac{\partial f_2}{\partial e_i} - 2e_i = 0,$$

$$\text{(iii)}\quad y_i\colon\quad (y_i - c)E\left(\frac{\partial f_1}{\partial y_i}\right) + (y_i - c)E\left(\frac{\partial f_2}{\partial y_i}\right) + r(p_i - y_i)\sigma_\delta^2 = 0.$$

Thus Equations (i), (ii), and (iii) are the implicit equations that solve the maximization problem. Under the no-price-delegation scenario the incentive compatibility constraint can be replaced by

$$\text{(iv)}\quad (p_i - y_i)\frac{\partial f_2}{\partial e_i} - 2e_i = 0.$$

and the first-order condition with respect to y_i is

$$\text{(v)}\quad (y_i - c)E\left(\frac{\partial f_1}{\partial y_i} + \frac{\partial f_2}{\partial y_i}\right) + r(p_i - y_i)\sigma_\delta^2 = 0.$$

Taking a specific functional form, we can substitute for $\partial f_2/\partial e_i$, α_i, and $E(q_i)$ in the firm's maximization problem and take the derivative with respect to p_i. We consider the following specific functional form for the expected demand:

$$E(q_i) = h + p_i^{-2} + \theta_p p_j + e_i^{1/2} - \theta_s e_j. \qquad (23)$$

The above function is nonlinear in price and effort with the following derivatives:

$$\frac{\partial q_i}{\partial p_i} < 0,\qquad \frac{\partial^2 q_i}{\partial p_i^2} > 0,\qquad \frac{\partial q_i}{\partial e_i} > 0,\quad \text{and}\quad \frac{\partial^2 q_i}{\partial e_i^2} < 0.$$

Thus, the quantity sold decreases as price increases, but this effect is smaller at higher prices. Setting up the profit-maximization problems for the firms with the participation and the incentive compatibility constraints for the sales reps, we get these first-order conditions.

Delegation

$$\text{(iii)}\quad p_i^D - \frac{(100 + (p_i^D)^{-2} + \theta_p p_i^D + (e_i^D)^{1/2} - \theta_s e_i^D)}{2(p_i^D)^{-3} + 1} - y_i^D = 0,$$

$$\text{(iv)}\quad e_i^D = \left(\frac{p_i^D - y_i^D}{4}\right)^{2/3}.$$

No Delegation

$$\text{(v)}\quad \frac{(p_i^N - y_i^N)^{5/3}}{4^{2/3}} + \frac{(p_i^N - y_i^N)}{12} - \frac{p_i^N}{12} = 0,$$

$$\begin{aligned}\text{(vi)}\quad & (100 + (p_i^N)^{-2} + \theta_p p_i^N)\left(\frac{p_i^N - y_i^N}{4}\right)^{2/3} \\ & + 2\left(\frac{p_i^N - y_i^N}{12}\right) - \theta_s\left(\frac{p_i^N - y_i^N}{4}\right)^{4/3} \\ & - 2(p_i^N)^{-2}\left(\frac{p_i^N - y_i^N}{4}\right)^{2/3} + \frac{p_i^N}{12} \\ & = \frac{(p_i^N - y_i^N)^{5/3}}{4^{2/3}},\end{aligned}$$

$$\text{(vii)}\quad e_i^N = \left(\frac{p_i^N - y_i^N}{4}\right)^{2/3}.$$

Substituting for effort in the price equation, we solve for y_i, which is a quadratic function of a transformed variable $x = [(p_i - y_i)/4]^{1/3}$. We solve for the positive roots of the equation and solve for y_i. Simulations are run in MATLAB to identify the regions in $\theta_p\theta_s$ square in which profits from delegation are higher than the profits from no price delegation. For higher prices (>2,000), the profit from price delegation is almost linear (Figure 8). The same is true for profits with no-price delegation.

Figure 8

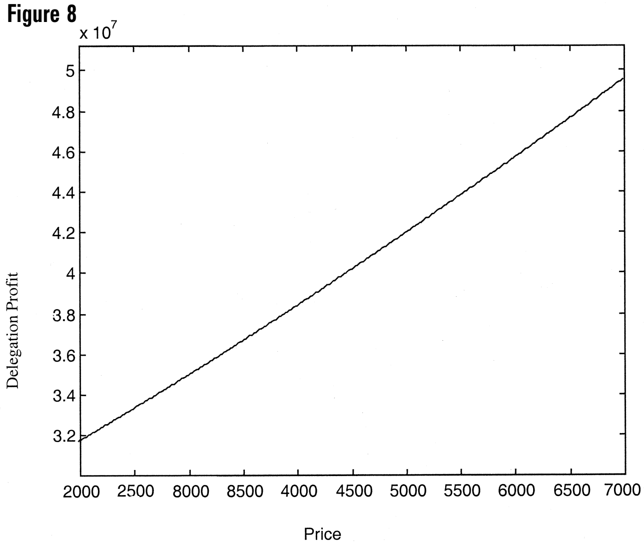
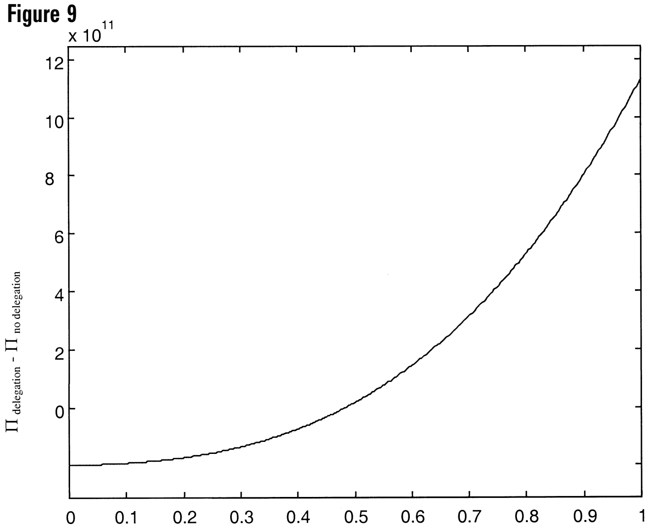


Figure 9



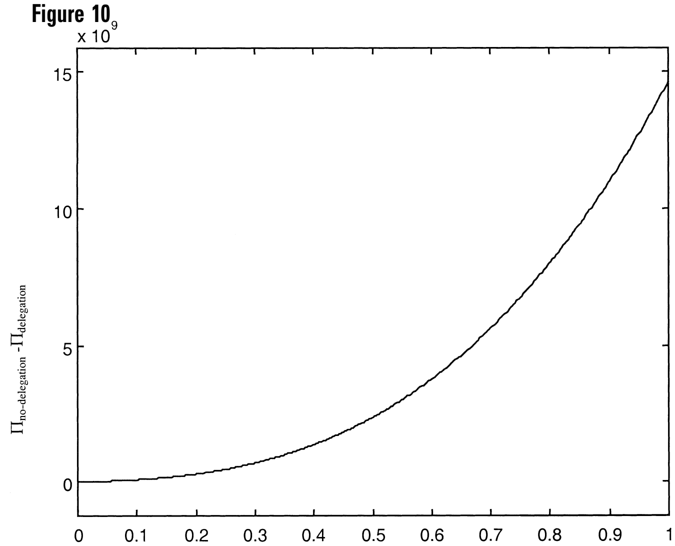
Cross Price Impact θ_p

θs = 0.2 and Price = 1000

We therefore concentrate on values of prices that are below 2,000. Figure 9 shows that for a given low value of cross-selling effort impact ($\theta_s = 0.2$), the difference in profits from price delegation and no-price delegation increases as the cross-price impact increases. However, for θ_p below 0.5, the profit from no price delegation is higher.

Also, for a given high value of cross-selling effort impact ($\theta_s = 0.9$) the profit from no-price delegation

Figure 10



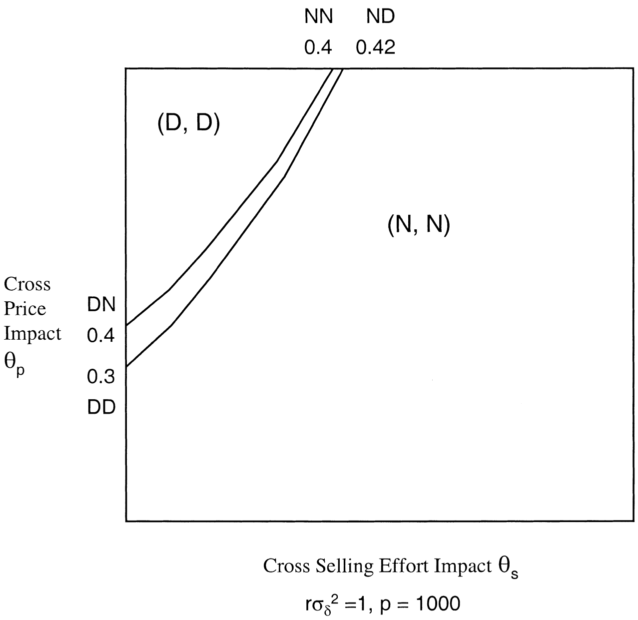
Cross Price Impact θp

θs =0.9, Price = 1000

dominates the profit from price delegation (Figure 10). For a high value of cross-price impact and a low value of cross-selling effort impact, the delegation profit increases with price. For the same values of cross-price and cross-selling effort impacts the profit from no-price delegation decreases as the price increases.

To identify the equilibrium strategies we once again look at the contours $\Pi_1^{DN} - \Pi_1^{NN}$ and $\Pi_1^{DD} - \Pi_1^{ND}$. These contours are shown as DN\NN and DD\ND in Figure 11 for $r\sigma_\delta^2 = 1$ and $h = 100$. Above the curve DN\NN, the profit from price delegation is higher than the profit from no-price delegation. In the region marked (D,D) both firms prefer to delegate the pricing decision to the sales reps because this leads to softening of price competition. Below the curve DD\ND, the profit from no-price delegation is higher than the profit from price delegation. Hence, in the region marked (N, N) both firms prefer not to delegate the pricing decision to the sales reps. For low values of cross-selling effort impact and high values of cross-price impact it is optimal for the firms to delegate the pricing decision to the sales reps. The sales reps, because they now have a higher commission on gross margin, set a higher price than what the firm would have set. For higher values of prices,

Figure 11 **Equilibrium Strategies: Nonlinear in Price and Effort**



the delegation region expands to the right on the θ_s axis and shifts up on θ_p axis (becoming closer to the linear case). For lower levels of prices, the price delegation region shifts up.

The nonlinear results, though not analytical, lend support to the linear demand function result that price delegation is the optimal strategy for the firms when the price competition is intense. Adopting the price-delegation strategy when cross-price impacts are high helps in softening the price competition in the market.

6. Discussion

We have addressed the issue of price delegation in a duopoly with competition in two dimensions: price and effort. Under linear and nonlinear demand functions it is shown that the firms prefer price delegation when the price competition is intense. This helps in reducing the price competition because the prices set by the reps are higher. The firms give their reps higher wages on margins that provide an incentive to set higher prices. It is important to keep in mind the unobservability of contract to the competing firm-rep pair. With risk-averse agents, the firm absorbs some of the uncertainty in demand by keeping the "virtual" marginal cost of production greater than c. The competing firm, although it does not know the competitor's contract, can logically infer through the risk aversion of the reps that the other firm would set y_i greater than c. This leads to reps setting a higher price, and is the benefit of risk aversion. It is the risk aversion of reps that provides the commitment to positive commissions, which are risk sharing because they transfer risk back to the principal. This, in turn, implies a commitment to soften price competition even though the contract is not directly observed by the competitor.

When effort competition is more intense, the firms tend not to delegate. This keeps the prices lower and helps in reducing the competition in the effort dimension. In the linear demand case, when price competition is much lower than effort competition, price is a strategic substitute variable. Hence, if the competitor were to increase the price, it would result in greater effort put in by the competitor's rep, which in turn would reduce his own demand. Thus, the firm reacts by reducing its own price. This leads to a reduction in effort by its own rep, the reaction to which is lower desired effort by the competitor, and lower prices. With no delegation, the firms can raise the commissions without concern over excessive price. Thus, commission is freed up as an instrument to elicit effort. Here, unlike the decentralization in channels under intense price competition and strategic complementarity (Moorthy 1988), in the region in which prices are strategic complement variables, price delegation is not necessarily the equilibrium even when price competition is at its maximum ($\theta_p = 1$). Said differently, for every level of price competition there is a level of effort competition for which the equilibrium is no-price delegation.

Under the linear demand function case with both firms adopting price delegation, profits under Plan A are higher than the profits under commission on margin Plan B. Although the prices under both plans and delegation increase as price competition increases, the prices under Plan A are higher than prices under Plan B, but the risk premium under Plan A is higher. When both firms adopt price delegation, the profits to each are higher under Plan A in spite of a higher risk premium. This is because the higher prices soften price competition and the efforts by reps are higher under Plan A. When firms do not delegate prices, the two contracts are equivalent.

As the utility cost of uncertainty increases, the region of price delegation decreases because of increase in risk premium. Risk aversion and uncertainty affect the profits in two ways: directly through risk premium, and indirectly through prices and effort. The direct (negative) effect of an increase in $r\sigma^2$ will be greater when prices are higher and/or the ''virtual'' marginal cost is lower. In this respect, no-price delegation can be said to be more effective in providing insurance for the sales rep because it incurs a smaller cost. In the standard agency models, with no competition, providing insurance for the rep is often in conflict with providing proper incentives. However, when there is competition, firms may take advantage of this conflict to affect the strategic interaction, e.g., facilitate implicit collusion between the reps of the firms.

7. Conclusion

In this research we have analyzed the impact of price delegation in a duopoly with competition in two dimensions: price and effort. With this richer characterization of the real world, it cannot be determined a priori whether price is a strategic substitute or a strategic complement. Under intense price competition, prices are delegated and the reps set a higher price. It is the risk aversion of reps that provides the commitment to positive commissions and the transfer of risk back to firms.

There are several interesting directions for future research. Firms need to make numerous marketing decisions besides price—for example, advertising, promotions, co-branding, product redesign, etc. In this situation, which, if any, decisions should the firms delegate to their sales reps? In a market-entry scenario, can an incumbent firm deter entry through delegation? Firms also sell multiple products that may have interrelated demand. In a multiple-product scenario how do the intrafirm and interfirm demand interdependencies affect the delegation decision? Another very interesting issue is the role of multiple reps in a firm in which decisions are made based on a bargaining process. The reps of a firm bargain (in a Nash bargaining sense) to set the optimal values of decision variables. It would be valuable to explore whether the firms can achieve a higher profit through delegation in which the reps compromise first through bargaining. Also, what incentives should the firms provide, for example, a pure profit-maximizing or a mix of profits and sales-maximizing, to affect the outcome of the bargaining process? Empirically testing the results of the analytical model would be meaningful. Higher price competition is to be expected in categories where products are perceived as close substitutes. However, there can be categories in which products are perceived to be very similar, but in which either price is not important or service provided by the sales reps is critical in making a choice. After identifying some of these categories a survey method may be employed to empirically test the results of the model.

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Appendix 1

Proof of Lemma 1. Because we are using a CARA function, the participation constraint could be written in terms of certainty equivalent *CE*. The optimization problem, assuming reservation utility to be zero, is simplified to

$$\operatorname*{Max}_{p_i,\beta} E[(p_i - c)q_i - \alpha - \beta q_i]$$

subject to

$$e_i \in \operatorname{argmax} \alpha + \beta E(q_i) - e_i^2 - \frac{1}{2}\beta^2 r\sigma_\delta^2,$$

$$\alpha + \beta E(q_i) - e_i^2 - \frac{1}{2}\beta^2 r\sigma_\delta^2 = 0.$$

The first-order conditions for the firm's optimization problem, after substituting for salary α and effort e_i, are

$$(1) \quad (p_i - c) = (h - p_i + \theta_p p_j + e_i - \theta_s e_j),$$

$$(2) \quad (p_i - c) = 2e_i + 4e_i r\sigma_\delta^2.$$

Simplifying the above conditions gives

$$p_i(3 + 8r\sigma_\delta^2) - (2\theta_p + 4\theta_p r\sigma_\delta^2 - \theta_s)p_j = 2h + 4hr\sigma_\delta^2 + (1 + \theta_s + 4r\sigma_\delta^2)c.$$

Under margin-based contracts, each firm will simultaneously solve the following maximization program:

$$\operatorname*{Max}_{y_i, p_i, \alpha_i} E[(y_i - c)q_i - \alpha_i]$$

subject to

$$e_i \in \operatorname{argmax} E\lfloor U((p_i - y_i)q_i + \alpha_i - e_i^2)\rfloor,$$

$$E\lfloor U((p_i - y_i)q_i + \alpha_i - e_i^2)\rfloor \geq 0.$$

The first-order conditions for the optimization problem are

$$(y_i - c)(-1/2) + r(p_i - y_i)\sigma_\delta^2 = 0, \tag{a}$$

$$(p_i - y_i)(1 + 2r\sigma_\delta^2) - E[q_i] = 0, \tag{b}$$

$$p_i - y_i = 2e_i. \tag{c}$$

Substituting for y_i in (b) and (c) and for e_i in (b) gives

$$p_i - c = h - p_i + \theta_p p_j + \frac{p_i - c}{2(1 + 2r\sigma_\delta^2)} - \theta_s \frac{p_j - c}{2(1 + 2r\sigma_\delta^2)}.$$

Simplifying the above expression gives

$$p_i(3 + 8r\sigma_\delta^2) - (2\theta_p + 4\theta_p r\sigma_\delta^2 - \theta_s)p_j = 2h + 4hr\sigma_\delta^2 + (1 + \theta_s + 4r\sigma_\delta^2)c. \quad \square$$

Appendix 1A

Proof of Lemma 2. The firm's maximization problem can be written as

$$\operatorname*{Max}_{y_i} E[(y_i - c)q_i(p_i(y_i), e_i(y_i)) - \alpha_i(y_i)].$$

Because the ICC and the IR for the agent define p_i, e_i, and α_i as functions of y_i, we can write the firm's maximization problem as

$$\operatorname*{Max}_{y_i} (y_i - c)q_i + (p_i - y_i)[h - p_i + \theta_p p_j + e_i - \theta_s e_j] - e_i^2 - \frac{1}{2}r\sigma_\delta^2(p_i - y_i)^2$$

$$\Rightarrow (y_i - c)q_i + (p_i - y_i)\left[h - p_i + \theta_p p_j + \frac{p_i - y_i}{2} - \theta_s e_j\right] - \left(\frac{p_i - y_i}{2}\right)^2 - \frac{1}{2}r\sigma_\delta^2(p_i - y_i)^2.$$

The first-order condition for the above maximization is

$$\begin{aligned} &q_i + (y_i - c)E\left(\frac{\partial q_i}{\partial y_i}\right) + (p_i - y_i)\left[-\frac{\partial p_i}{\partial y_i} + \frac{1}{2}\frac{\partial p_i}{\partial y_i} - \frac{1}{2}\right] \\ &\quad + \left[h - p_i + \theta_p p_j + \frac{p_i - y_i}{2} - \theta_s e_j\right]\left[\frac{\partial p_i}{\partial y_i} - 1\right] - 2\left(\frac{p_i - y_i}{2}\right)\left[\frac{1}{2}\frac{\partial p_i}{\partial y_i} - \frac{1}{2}\right] \\ &\quad - \frac{2}{2}r\sigma_\delta^2(p_i - y_i)\left[\frac{\partial p_i}{\partial y_i} - 1\right] = 0 \\ &\quad \Rightarrow q_i + (y_i - c)E\left(\frac{\partial q_i}{\partial y_i}\right) \\ &\qquad + \frac{\partial p_i}{\partial y_i}\left[-(p_i - y_i) + \frac{p_i - y_i}{2} + q_i - \frac{p_i - y_i}{2} - r\sigma_\delta^2(p_i - y_i)\right] \\ &\qquad - \frac{p_i - y_i}{2} - q_i + \frac{p_i - y_i}{2} + r\sigma_\delta^2(p_i - y_i) = 0 \\ &\quad \Rightarrow (y_i - c)E\left(\frac{\partial q_i}{\partial y_i}\right) + r\sigma_\delta^2(p_i - y_i) \\ &\qquad + \frac{\partial p_i}{\partial y_i}[-(p_i - y_i) + h - p_i + \theta_p p_j + e_i - \theta_s e_j - r\sigma_\delta^2(p_i - y_i)] \\ &\qquad = 0 \end{aligned}$$

$$(y_i - c)E\left(\frac{\partial q_i}{\partial y_i}\right) + r\sigma_\delta^2(p_i - y_i) + \frac{\partial p_i}{\partial y_i}\left[-\frac{p_i - y_i}{2} + h - p_i + \theta_p p_j - \theta_s e_j - r\sigma_\delta^2(p_i - y_i)\right] = 0. \tag{1A.1}$$

The expected value of demand, q_i, is

$$\begin{aligned} E(q_i) = h &- \frac{1}{\frac{3}{2} + r\sigma_\delta^2}\left[h + \theta_p p_j + y_i\left(\frac{1}{2} + r\sigma_\delta^2\right) - \theta_s e_j\right] + \theta_p p_j \\ &+ \frac{1}{2\left(\frac{3}{2} + r\sigma_\delta^2\right)}\left[h + \theta_p p_j + y_i\left(\frac{1}{2} + r\sigma_\delta^2\right) - \theta_s e_j - y_i\left(\frac{3}{2} + r\sigma_\delta^2\right)\right] \\ &- \theta_s e_j. \end{aligned}$$

Therefore, the expected value of the derivative of q_i with respect to y_i is

$$\begin{aligned} E\left[\frac{\partial q_i}{\partial y_i}\right] &= \frac{-\left(\frac{1}{2} + r\sigma_\delta^2\right)}{\frac{3}{2} + r\sigma_\delta^2} + \frac{\left(\frac{1}{2} + r\sigma_\delta^2\right)}{2\left(\frac{3}{2} + r\sigma_\delta^2\right)} - \frac{\left(\frac{3}{2} + r\sigma_\delta^2\right)}{2\left(\frac{3}{2} + r\sigma_\delta^2\right)} \\ &= \frac{-\frac{1}{2} - r\sigma_\delta^2 + \frac{1}{4} + \frac{1}{2}r\sigma_\delta^2 - \frac{3}{4} - \frac{1}{2}r\sigma_\delta^2}{\frac{3}{2} + r\sigma_\delta^2} = \frac{-(1 + r\sigma_\delta^2)}{\frac{3}{2} + r\sigma_\delta^2}. \end{aligned}$$

Also,

$$p_i = \frac{1}{\frac{3}{2} + r\sigma_\delta^2}\left[h + \theta_p p_j + y_i\left(\frac{1}{2} + r\sigma_\delta^2\right) - \theta_s e_j\right]$$

and therefore,

$$\frac{\partial p_i}{\partial y_i} = \frac{\frac{1}{2} + r\sigma_\delta^2}{\frac{3}{2} + r\sigma_\delta^2}.$$

The derivative q_i with respect to y_i is always negative, indicating that as less incentive is left with the rep (because of increase in y_i), the quantity sold also reduces. We can apply the envelope theorem to the first-order condition or write it as Equation (1A.1). The contract $\{y_i, \alpha_i\}$ and the prices and efforts $\{p_i, e_i\}$ for $i = 1, 2$ constitute an equilibrium if and only if they satisfy the conditions given in the Lemma.

Appendix 1B

Proof of Proposition 1. The symmetric Nash equilibrium in prices (p_i^{*D}) and effort levels (e_i^{*D}) calculated using Equations (7) and (8) is

$$(p_i - y_i)(-1) + \left(h - p_i + \theta_p p_i + \frac{p_i}{2} - \frac{y_i}{2} - \frac{\theta_s p_i}{2} + \frac{\theta_s y_i}{2}\right)$$
$$- r(p_i - y_i)\sigma_\delta^2 = 0$$
$$\Rightarrow p_i^{*D} = \frac{1}{3 + 2r\sigma_\delta^2 - 2\theta_p + \theta_s}[2h + (1 + \theta_s + 2r\sigma_\delta^2)y_i],$$
$$e = \frac{p_i - y_i}{2}$$
$$\Rightarrow e_i^{*D} = \frac{1}{3 + 2r\sigma_\delta^2 - 2\theta_p + \theta_s}[h + (\theta_p - 1)y_i].$$

Now substituting the value of p_i^{*D} in Equation (10) we get

$$y_i\left(-\frac{1 + r\sigma_\delta^2}{1.5 + r\sigma_\delta^2}\right) + r\sigma_\delta^2\left[\frac{2h + (1 + \theta_s + 2r\sigma_\delta^2)}{3 + 2r\sigma_\delta^2 - 2\theta_p + \theta_s} - y_i\right] + \left(-\frac{.5 + r\sigma_\delta^2}{1.5 + r\sigma_\delta^2}\right)$$
$$\times\left[\frac{2h + (1 + \theta_s + 2r\sigma_\delta^2)}{3 + 2r\sigma_\delta^2 - 2\theta_p + \theta_s}(-1.5 + \theta_p - .5\theta_s) + y_i(.5 + .5\theta_s) + h\right.$$
$$\left. - r\sigma_\delta^2\left(\frac{2h + (1 + \theta_s + 2r\sigma_\delta^2)}{3 + 2r\sigma_\delta^2 - 2\theta_p + \theta_s} - y_i\right)\right] = 0.$$

Simplifying, we get the value for y_i^{*D}, for $c = 0$ as

$$y_i^{*D} = \frac{h\lfloor 3 + 2r\sigma_\delta^2\rfloor r\sigma_\delta^2}{3 - 2\theta_p + \theta_s + 8r\sigma_\delta^2 + 4r^2\sigma_\delta^4 - 5\theta_p r\sigma_\delta^2 - 2\theta_p r^2\sigma_\delta^4 + \theta_s r\sigma_\delta^2}.$$

Taking partial derivatives of the equation for price with respect to θ_p and θ_s, we get

$$\frac{\partial P_i^{*D}}{\partial\theta_p} = \frac{(4 + 20r\sigma_\delta^2 + 33r^2\sigma_\delta^4 + 20r^3\sigma_\delta^6 + 4r^4\sigma_\delta^8)h}{W^2} > 0 \quad \text{and}$$
$$\frac{\partial P_i^{*D}}{\partial\theta_s} = \frac{-(2 + 7r\sigma_\delta^2 + 7r^2\sigma_\delta^4 + 2r^3\sigma_\delta^6)h}{W^2} < 0,$$

where

$$W = -3 + 2\theta_p - \theta_s - 8r\sigma_\delta^2 - 4r^2\sigma_\delta^4 + 5\theta_p r\sigma_\delta^2 + 2\theta_p r^2\sigma_\delta^4 - \theta_s r\sigma_\delta^2.$$

Similar analysis for effort and commission shows that both effort and commission increase with price competition and decrease with effort competition.

Appendix 1C

Proof of Equilibrium Contract under No Price Delegation. Each firm will simultaneously solve the following maximization program:

$$\underset{y_i, p_i, \alpha_i}{\text{Max}}\ E[(y_i - c)q_i - \alpha_i]$$

subject to

$$e_i \in \operatorname{argmax} E\lfloor U((p_i - y_i)q_i + \alpha_i - e_i^2)\rfloor,$$
$$E\lfloor U((p_i - y_i)q_i + \alpha_i - e_i^2)\rfloor \geq 0.$$

Because the utility function of the agent is assumed to be exponential, she or he maximizes the certainty equivalent:

$$\underset{e_i}{\text{Max}}\ CE = \alpha_i + (p_i - y_i)(h - p_i + \theta_p p_j + e_i - \theta_s e_j)$$
$$- e_i^2 - \frac{1}{2}(p_i - y_i)^2 r\sigma_\delta^2,$$

$$\text{F.O.C.:} \qquad (p_i - y_i) - 2e_i = 0.$$

The firm therefore maximizes

$$\underset{y_i, p_i, \alpha_i, e_i}{\text{Max}}\ E[(y_i - c)q_i - \alpha_i] \tag{1C.1}$$

subject to

$$p_i - y_i = 2e_i,$$
$$-\alpha_i = (p_i - y_i)(h - p_i + \theta_p p_j + e_i - \theta_s e_j) - e_i^2 - \frac{1}{2}(p_i - y_i)^2 r\sigma_\delta^2.$$

The first-order condition with respect to y_i is

$$(y_i - c)(-1/2) + r(p_i - y_i)\sigma_\delta^2 = 0. \tag{1C.2}$$

We substitute in the firm's maximization program the values for α_i, e_i, and $E[q_i]$, which gives

$$\text{Max}(y_i - c)\left[h - p_i + \theta_p p_j + \frac{p_i}{2} - \frac{y_i}{2} - \theta_s e_j\right] - \frac{(p_i - y_i)^2}{4}$$
$$- \frac{1}{2}(p_i - y_i)^2 r\sigma_\delta^2 + (p_i - y_i)\left[h - p_i + \theta_p p_j + \frac{p_i}{2} - \frac{y_i}{2} - \theta_s e_j\right].$$

Using the above function to obtain the first-order condition with respect to p_i, we get

$$(y_i - c)(-1/2) - (p_i - y_i) - (p_i - y_i)r\sigma_\delta^2 + E[q_i] = 0.$$

Using Equation (1C.2) to simplify, we get

$$(p_i - y_i)(1 + 2r\sigma_\delta^2) - E[q_i] = 0. \tag{1C.3}$$

Thus, we get the following conditions for an equilibrium under no-price delegation:

$$(y_i - c)(-1/2) + r(p_i - y_i)\sigma_\delta^2 = 0, \tag{1C.4}$$

$$(p_i - y_i)(1 + 2r\sigma_\delta^2) - E[q_i] = 0, \tag{1C.5}$$

$$p_i - y_i = 2e_i, \tag{1C.6}$$

$$-\alpha_i = (p_i - y_i)(h - p_i + \theta_p p_j + e_i - \theta_s e_j) - e_i^2 - \frac{1}{2}(p_i - y_i)^2 r\sigma_\delta^2. \tag{1C.7}$$

In comparison to the price-delegation contract, we note that on one hand Equation (1C.4) implies that a firm under no price delegation would communicate a higher cost of production given the same commission rate. On the other hand, Equation (1C.5) implies that the firm would set a lower price given that the communicated cost of production is the same. Given these two conditions, Equation (1C.6) implies that under no price delegation the rep puts in less effort. These observations may not necessarily hold true for the actual equilibrium prices and efforts because the communicated costs of productions are determined endogeneously. However, it does indicate how different types of delegation decisions affect the competition in the market by changing the behavior of firms and sales reps.

Appendix 1D1

Proof of Proposition 2. The symmetric Nash equilibrium in prices (p_i^{*N}) is obtained using Equation (1C.3), in effort levels (e_i^{*N}) using Equation (1C.1), and in margins (y_i^{*N}) using the first-order condition given by Equation (1C.2). The equilibrium is

$$(p_i - y_i)(1 + 2r\sigma_\delta^2) - (h - p_i + \theta_p p_i + e_i - \theta_s e_i) = 0$$

$$\Rightarrow p_i^{*N} = \frac{1}{3 + 4r\sigma_\delta^2 - 2\theta_p + \theta_s}[2h + (1 + \theta_s + 4r\sigma_\delta^2)y_i],$$

$$e_i = (p_i - y_i)$$

$$\Rightarrow e_i^{*N} = \frac{1}{2[3 + 4r\sigma_\delta^2 - 2\theta_p + \theta_s]}[2h + (2\theta_p - 2)y_i], \quad \text{and}$$

$$y_i^{*N} = \frac{[h + \theta_p c - c]4r\sigma_\delta^2}{3 - 2\theta_p + \theta_s + 8r\sigma_\delta^2 - 4\theta_p r\sigma_\delta^2} + c,$$

$$\beta_i^{*N} = \frac{2h + (1 + \theta_s + 4r\sigma_\delta^2)\left(\dfrac{4(h + \theta_p c - c)r\sigma_\delta^2}{W} + c\right)}{2[3 + 4r\sigma_\delta^2 - 2\theta_p + \theta_s]} - \frac{4(h + \theta_p c - c)r\sigma_\delta^2}{W} - c,$$

where

$$W = 3 - 2\theta_p + \theta_s + 8r\sigma_\delta^2 - 4\theta_p r\sigma_\delta^2.$$

Taking partial derivatives of the equation for price with respect to θ_p and θ_s, we get

$$\frac{\partial P_i^{*N}}{\partial \theta_p} = \frac{4(1 + 4r\sigma_\delta^2 + 4r^2\sigma_\delta^4)h}{M^2} > 0 \quad \text{and}$$

$$\frac{\partial P_i^{*N}}{\partial \theta_s} = \frac{-2(1 + 2r\sigma_\delta^2)h}{M^2} < 0,$$

where

$$M = -3 + 2\theta_p - \theta_s - 8r\sigma_\delta^2 + 4\theta_p r\sigma_\delta^2.$$

Similar analysis for effort and commission rate shows that both effort and commission rate increase with price competition and decrease with effort competition. To show that the values of price, effort, and commission rate under no-price delegation are less than the values under price delegation, we take the differences of these variables and plot the difference on the $\theta_p\theta_s$ square.

Appendix 1D2

In the Nash equilibrium when one firm delegates pricing decision and the other does not, the prices, effort levels, and commission rates are given by

$$p_i^D = \frac{2h\lfloor(3 + 4r\sigma_\delta^2) + 2\theta_p - \theta_s\rfloor + y_i^D\lfloor(3 + 4r\sigma_\delta^2)(1 + 2r\sigma_\delta^2) + (2\theta_p - \theta_s)\theta_s\rfloor + y_j^N[(3 + 4r\sigma_\delta^2)(\theta_s - 1) + 2\theta_p(1 + 4r\sigma_\delta^2) - \theta_s]}{(3 + 4r\sigma_\delta^2)(3 + 2r\sigma_\delta^2) - (2\theta_p - \theta_s)^2},$$

$$p_j^N = \frac{2h\lfloor(3 + 4r\sigma_\delta^2)(3 + 2r\sigma_\delta^2) - (2\theta_p - \theta_s)^2\rfloor + y_i^D\lfloor(3 + 2r\sigma_\delta^2)\theta_s + (2\theta_p - \theta_s)(1 + 2r\sigma_\delta^2)\rfloor + y_j^N[(1 + 4r\sigma_\delta^2)(3 + 2r\sigma_\delta^2) + (2\theta_p - \theta_s)\theta_s]}{(3 + 4r\sigma_\delta^2)(3 + 2r\sigma_\delta^2) - (2\theta_p - \theta_s)^2},$$

$$\begin{aligned} y_i^D = {} & [(8\theta_p cr^3 + 8h\theta_p r^3 + 16hr^3 + 16cr^3)\sigma_\delta^6 + (46cr^2 + 16h\theta_p r^2 + 30hr^2 + 14c\theta_p r^2 - 2h\theta_s r^2 - 8c\theta_p^2 r^2 + 4c\theta_s\theta_p r^2 + 2c\theta_s r^2)\sigma_\delta^4 \\ & + (-rc\theta_s^2 + 3rc\theta_s + 39rc + 3rc\theta_p + 9rh - 12rc\theta_p^2 - 3rh\theta_s + 8rc\theta_p\theta_s + 6rh\theta_p)\sigma_\delta^2 - \theta_s^2 c + 9c - 4c\theta_p^2 + 4c\theta_s\theta_p] \\ & \div [(32r^3 - 8r^3\theta_p^2)\sigma_\delta^6 + (-24r^2\theta_p^2 + 76r^2 + 6r^2\theta_p\theta_s)\sigma_\delta^4 + (-18r\theta_p^2 - r\theta_s^2 + 48r + 11r\theta_p\theta_s)\sigma_\delta^2 - \theta_s^2 + 4\theta_p\theta_s + 9 - 4\theta_p^2], \end{aligned}$$

$$\begin{aligned} y_j^N = {} & [(8\theta_p cr^3 + 8h\theta_p r^3 + 16hr^3 + 16cr^3)\sigma_\delta^6 + (44cr^2 + 20h\theta_p r^2 + 32hr^2 + 12c\theta_p r^2 - 4h\theta_s r^2 - 4c\theta_p^2 r^2 + 2c\theta_s\theta_p r^2 + 4c\theta_s r^2)\sigma_\delta^4 \\ & + (-rc\theta_s^2 + 4rc\theta_s + 36rc + 4rc\theta_p + 12rh - 10rc\theta_p^2 - 4rh\theta_s + 7rc\theta_p\theta_s + 8rh\theta_p)\sigma_\delta^2 - \theta_s^2 c + 9c - 4c\theta_p^2 + 4c\theta_s\theta_p] \\ & \div [(32r^3 - 8r^3\theta_p^2)\sigma_\delta^6 + (-24r^2\theta_p^2 + 76r^2 + 6r^2\theta_p\theta_s)\sigma_\delta^4 + (-18r\theta_p^2 - r\theta_s^2 + 48r + 11r\theta_p\theta_s)\sigma_\delta^2 - \theta_s^2 + 4\theta_p\theta_s + 9 - 4\theta_p^2], \end{aligned}$$

$$e_i^D = \frac{p_i^D - y_i^D}{2}, \qquad e_j^N = \frac{p_j^N - y_j^N}{2}.$$

Sketch of Proof

Assume that Firm 1 follows price delegation and Firm 2 follows no price delegation. For Firm 1, the contract and the rep's price and effort levels satisfy Equations (10) to (13) in Lemma 2. For Firm 2, the contract and the rep's price and effort levels satisfy Equations (1C.4) to (1C.7) in Appendix 1C. We have from Equation (11)

$$(p_i^D - y_i^D)(-1) + h + \theta_p p_j^N - p_i^D + e_i^D - \theta_s e_j^N - r(p_i^D - y_i^D)\sigma_\delta^2 = 0, \tag{1D.1}$$

where terms with superscript D are for Firm 1 (price delegation) and terms with superscript N are for Firm 2 (no-price delegation). From Equations (1C.4) and(1C.5), we get

$$p_j^N = \frac{1}{\left(\frac{3}{2} + 2r\sigma_\delta^2\right)}\left[h + y_j^N\left(\frac{1}{2} + 2r\sigma_\delta^2\right) + \theta_p p_i^D - \theta_s e_i^D\right]. \tag{1D.2}$$

Substituting in (1D.1) from (1D.2) and solving for p_i^D

$$\begin{aligned} p_i^D = \{&2h\lfloor(3 + 4r\sigma_\delta^2) + 2\theta_p - \theta_s\rfloor \\ &+ y_i^D\lfloor(3 + 4r\sigma_\delta^2)(1 + 2r\sigma_\delta^2) + \theta_s(2\theta_p - \theta_s)\rfloor \\ &+ y_j^N[(3 + 4r\sigma_\delta^2)(\theta_s - 1) + 2\theta_p(1 + 4r\sigma_\delta^2) - \theta_s^2]\} \\ &\div [(3 + 4r\sigma_\delta^2)(3 + 2r\sigma_\delta^2) - (2\theta_p - \theta_s)^2]. \end{aligned} \tag{1D.3}$$

Once again, from Equation (11) we get

$$p_i^D = \frac{1}{\left(\frac{3}{2} + r\sigma_\delta^2\right)}\left[y_i^D\left(\frac{1}{2} + r\sigma_\delta^2\right) + h - \frac{\theta_s p_j^N}{2} + \theta_p p_j^N + \frac{\theta_s y_j^N}{2}\right]. \tag{1D.4}$$

Substituting for p_i^D in Equation (1C.5), we have

$$\begin{aligned} p_j^N = \{&2h\lfloor 3 + 2r\sigma_\delta^2 + 2\theta_p - \theta_s\rfloor \\ &+ y_j^N\lfloor(1 + 4r\sigma_\delta^2)(3 + 2r\sigma_\delta^2) + (2\theta_p - \theta_s)\theta_s\rfloor \\ &+ y_i^D[\theta_s(3 + 2r\sigma_\delta^2) + (2\theta_p - \theta_s)(1 + 2r\sigma_\delta^2)]\} \\ &\div [(3 + 2r\sigma_\delta^2)(3 + 4r\sigma_\delta^2) - (2\theta_p - \theta_s)^2]. \end{aligned} \tag{1D.5}$$

Substituting for p_i^D in Equation (10) and collecting terms, we get y_i^D as a function of y_j^N. Similarly, from Equation (1C.4) we get y_j^N as a function of y_i^D. Solving the two equations in two unknowns gives y_i^D and y_j^N.

Appendix 1E

LEMMA. *The profit to a firm that decides to delegate pricing decision, regardless of what the other firm decides, is given by*

$$\Pi^D = (p_i - y_i)^2\left[\frac{3 + 2r\sigma_\delta^2}{4}\right][1 + 2r\sigma_\delta^2].$$

The profit to a firm that decides not to delegate pricing decision, regardless of what the other firm decides, is given by

$$\Pi^N = (p_i - y_i)^2\left[\frac{3 + 8r\sigma_\delta^2}{4}\right][1 + 2r\sigma_\delta^2].$$

PROOF. From Equations (10) and (11) we have

$$(y_i - c)\left[-\frac{1 + r\sigma_\delta^2}{\frac{3}{2} + r\sigma_\delta^2}\right] + r\sigma_\delta^2(p_i - y_i) + \left(\frac{\frac{1}{2} + r\sigma_\delta^2}{\frac{3}{2} + r\sigma_\delta^2}\right)[-(p_i - y_i) + E(q_i) - r\sigma_\delta^2(p_i - y_i)] = 0,$$

$$E(q_i) = (p_i - y_i) + r\sigma_\delta^2(p_i - y_i). \tag{1E.1}$$

Substituting for q_i in (1E.1) we get

$$(y_i - c)\left[-\frac{1 + r\sigma_\delta^2}{\frac{3}{2} + r\sigma_\delta^2}\right] + r\sigma_\delta^2(p_i - y_i) = 0.$$

Profit to the firm that chooses to delegate pricing decision is

$$\Pi^D = (y_i - c)E(q_i) - \alpha_i = (p_i - c)E(q_i) - \frac{(p_i - y_i)^2}{4} - \frac{1}{2}r\sigma_\delta^2(p_i - y_i)^2$$

after substituting for α_i

$$\begin{aligned} &= (p_i - y_i + y_i - c)E(q_i) - \frac{(p_i - y_i)^2}{4} - \frac{1}{2}r\sigma_\delta^2(p_i - y_i)^2 \\ &= (p_i - y_i)[(p_i - y_i) + r\sigma_\delta^2(p_i - y_i)] + \left[r\sigma_\delta^2(p_i - y_i)\left(\frac{\frac{3}{2} + r\sigma_\delta^2}{1 + R\sigma_\delta^2}\right)\right] \\ &\quad \times [(p_i - y_i) + r\sigma_\delta^2(p_i - y_i)] - \frac{(p_i - y_i)^2}{4} - \frac{1}{2}r\sigma_\delta^2(p_i - y_i)^2 \\ &= (p_i - y_i)^2(1 + r\sigma_\delta^2)\left[1 + r\sigma_\delta^2\frac{3 + 2r\sigma_\delta^2}{2 + 2r\sigma_\delta^2}\right] - (p_i - y_i)^2\left[\frac{1 + 2r\sigma_\delta^2}{4}\right] \\ &= \frac{(p_i - y_i)^2}{4}[3 + 8r\sigma_\delta^2 + 4r^2\sigma_\delta^2] = (p_i - y_i)^2\left[\frac{(3 + 2r\sigma_\delta^2)(1 + 2r\sigma_\delta^2)}{4}\right]. \end{aligned}$$

Similarly, using Equations (1C.4) and (1C.5), the profit to the firm choosing not to delegate the pricing decision is determined. Thus, the profit Π^D would vary under (D, D) and (D, N) case only through the component $p_i - y_i$. Similarly, Π^N would vary under the (N, N) and (N, D) case only through the component $p_i - y_I$ because r and σ_δ^2 are exogeneous.

The next step is to compute the difference $p_i - y_i$ by using the equilibrium outcomes for the case in which both firms decide on price delegation (D, D) and the case when Firm 1 decides on price delegation and Firm 2 on no price delegation (D, ND).

From the equilibrium values for Proposition 2 we can calculate $p_i - y_i$ for (D, D) case to be

$$p_i^D - y_i^D = \frac{2(1 + r\sigma_\delta^2)(h - c + \theta_p c)}{8r\sigma_\delta^2 + 4r^2\sigma_\delta^4 + \theta_s r\sigma_\delta^2 - 2\theta_p r^2\sigma_\delta^4 - 2\theta_p + 3 + \theta_s - 5r\theta_p\sigma_\delta^2}.$$

From the equilibrium values in the proposition (given in Appendix 1D.2) we can calculate $p_i - y_i$ for (D, N) case to be

$$\begin{aligned} p_i^D - y_i^D = {} & [2(1 + r\sigma_\delta^2)(h - c + \theta_p c)(4r\theta_p\sigma_\delta^2 + 2\theta_p + 8r\sigma_\delta^2 + 3 - \theta_s)] \\ & \div [4\theta_p\theta_s + 11\theta_p\theta_s r\sigma_\delta^2 + 32r^3\sigma_\delta^6 + 48r\sigma_\delta^2 + 6\theta_p\theta_s r^2\sigma_\delta^4 \\ & + 76r^2\sigma_\delta^4 + 9 - \theta_s - 18r\theta_p^2\sigma_\delta^2 - \theta_s^2 r\sigma_\delta^2 - 4\theta_p^2 \\ & - 24\theta_p^2 r^2\sigma_\delta^4 - 8\theta_p^2 r^3\sigma_\delta^6]. \end{aligned}$$

From the equilibrium values for Proposition 3 we can calculate $p_i - y_i$ for (N, N) case to be

$$p_i^N - y_i^N = \frac{2(h - c + \theta_p c)}{8r\sigma_\delta^2 + 3 + \theta_s - 2\theta_p - 4\theta_p r\sigma_\delta^2}.$$

Similarly, from the Proposition given in Appendix 1D2, the value of $p_i - y_i$ for the (N, D) case is

$$\begin{aligned} p_i^N - y_i^N & \\ = {} & [2(h - c + \theta_p c) \\ & \times (4r^2\sigma_\delta^4 + 3 + 8r\sigma_\delta^2 - \theta_s r\sigma_\delta^2 + 5\theta_p\sigma_\delta^2 + 2\theta_p - \theta_s + 2\theta_p r^2\sigma_\delta^4)] \\ & \div [4\theta_p\theta_s + 11\theta_p\theta_s r\sigma_\delta^2 + 32r^3\sigma_\delta^6 + 48r\sigma_\delta^2 + 6\theta_p\theta_s r^2\sigma_\delta^4 + 76r^2\sigma_\delta^4 \\ & + 9 - \theta_s - 18r\theta_p^2\sigma_\delta^2 - \theta_s^2 r\sigma_\delta^2 - 4\theta_p^2 - 24\theta_p^2 r^2\sigma_\delta^4 - 8\theta_p^2 r^3\sigma_\delta^6]. \end{aligned}$$

Substituting the relevant difference $(p_i - y_i)$ in the profit equations gives us the analytical solutions for the profits as a function of exogeneous variables.

Appendix 2A

Sales-Based Commissions

When both firms decide to delegate the pricing decision to their sales reps, the firms' maximization problem can be written as

$$\operatorname*{Max}_{\beta_i} E[(p_i - c)q_i] - W \tag{2A.1}$$

subject to

$$\begin{aligned} & e_i, p_i \in \operatorname{argmax}[EU(W, e)] && \text{Incentive Compatibility Constraint,} \\ & EU(W, e) \geq \bar{u} && \text{Participation Constraint.} \end{aligned}$$

The above problem may be rewritten as

$$\operatorname*{Max}_{\beta} E[(p_i - c)q_i - \alpha - \beta q_i] \tag{2A.2}$$

subject to

$$e_i, p_i \in \operatorname{argmax} \alpha + \beta(h - p_i + \theta_p p_j + e_i - \theta_s e_j) - e_i^2 - \frac{1}{2}\beta^2 r\sigma_\delta^2,$$

$$\alpha + \beta E(q_i) - e_i^2 - \frac{1}{2}\beta^2 r\sigma_\delta^2 = 0.$$

From Equation (2A.2) the rep maximizes his compensation by setting the price as low as possible ($p_i = 0$) because this would increase the quantity sold, and hence his earning from the commissions. The first-order condition with respect to effort is $e_i = \beta/2$. Substituting these in the firm's maximization problem and taking the first-order condition with respect to commission rate gives $\beta = 0$. If the firm did delegate the pricing decision to the sales rep, it would set a commission rate of zero and a wage rate that would provide the reservation utility to the sales rep. The sales rep would put in zero effort. With a fixed wage contract under moral hazard, the rep has no incentive to put in effort.

Proposition. *In the symmetric Nash equilibrium under price delegation and commission on unit sales, the firm sets a commission rate of zero and the sales rep puts in zero effort.*

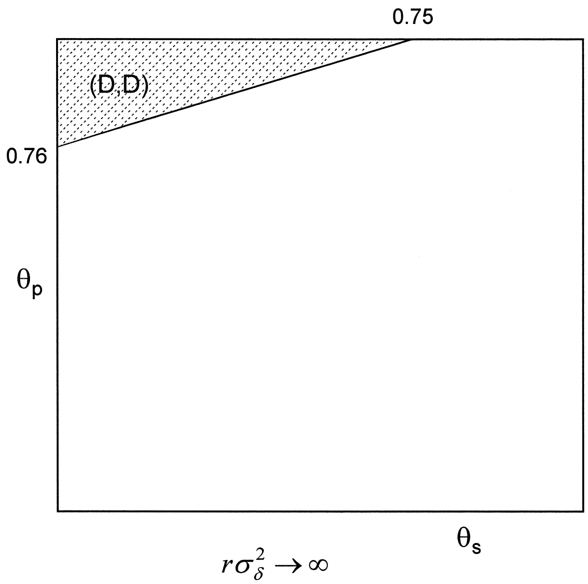
With a pure wage rate contract, the sales reps put in zero effort and thus the competition is in only one dimension. If the firms were to give a greater than zero commission rate with price delegation, the reps would continue to drop the price to increase sales volume. This would only lead to increased price competition.

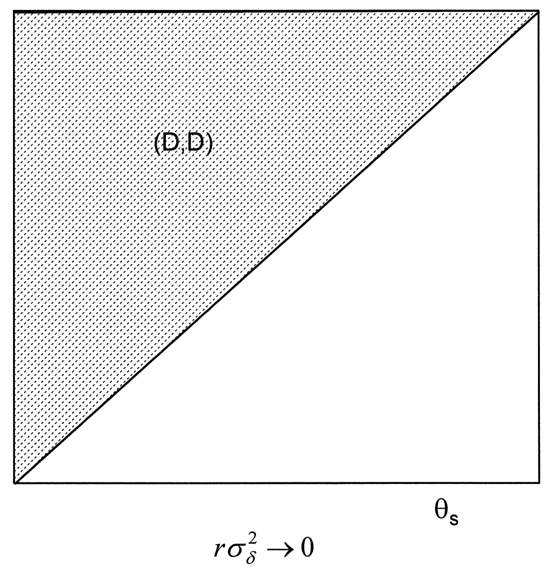
Appendix 2B

In the graphical solution, for certain values of $r\sigma_\delta^2$, we solve for regions in $\theta_p - \theta_s$ square where price delegation is the equilibrium. Here we show that for $0 \leq \theta_p, \theta_s \leq 1$ there are regions where (D, D) is the equilibrium when $r\sigma_\delta^2 \to \infty$.

Step 1. For $\Pi_1^{DD} - \Pi_1^{ND}$ and $\theta_p = 1$ we solve for θ_s given the constraint $0 \leq \theta_s \leq 1$. The value $\text{limit}_{r\sigma_\delta^2 \to \infty}\, \theta_s = 0.75$. For $\Pi_1^{DD} - \Pi_1^{ND}$ and $\theta_s = 0$ we solve for θ_p given the constraint $0 \leq \theta_p \leq 1$. The value $\text{limit}_{r\sigma_\delta^2 \to \infty}\, \theta_p = 0.7559$.

Step 2. A similar calculation for $\Pi_1^{DN} - \Pi_1^{NN}$ gives $\lfloor \theta_p = 1, \theta_s = 0.75 \rfloor$ and $\lfloor \theta_p = 0, \theta_s = 0.7559 \rfloor$ for $r\sigma_\delta^2 \to \infty$. Plotting these gives us the region where delegation by both firms is the unique equilibrium. A similar region for $r\sigma_\delta^2 \to 0$ can be calculated. We have thus shown that results similar to Proposition 3 are obtained over the entire range of $r\sigma_\delta^2$.





Appendix 2C (Nonlinear Demand Function)

Under the price-delegation scenario the firm maximizes expected profits

$$\max_{y_i, \alpha_i} E[(y_i - c)q_i - \alpha_i]$$

subject to

$$(1) \quad (p_i, e_i) \in \operatorname{argmax} E[U((p_i - y_i)q_i + \alpha_i - e_i^2)]$$

Incentive Compatibility Constraint,

$$(2) \quad EU \geq \bar{U} \quad \text{Participation Constraint,}$$

where $\bar{U}$ is the reservation utility of the sales rep.
The certainty equivalent is

$$CE = \alpha_i + (p_i - y_i)q_i - e_i^2 - \frac{1}{2}(p_i - y_i)^2 r\sigma_\delta^2.$$

ICC can be replaced by first-order conditions.

$$\text{(i)} \quad (p_i - y_i)\frac{\partial f_1}{\partial p_i} + E[q_i] - r(p_i - y_i)\sigma_\delta^2 = 0,$$

$$\text{(ii)} \quad (p_i - y_i)\frac{\partial f_2}{\partial e_i} - 2e_i = 0.$$

The binding participation constraint is

$$\alpha_i + (p_i - y_i)q_i - e_i^2 - \frac{1}{2}(p_i - y_i)^2 r\sigma_\delta^2 = 0.$$

Thus the firm will maximize

$$\max_{y_i} E\left[(y_i - c)q_i + (p_i - y_i)q_i - e_i^2 - \frac{1}{2}(p_i - y_i)^2 r\sigma_\delta^2\right].$$

Because p_i, e_i, and α_i are functions of y_i we get the first-order condition as

$$\text{(viii)} \quad (y_i - c)E\left(\frac{\partial f_1}{\partial y_i}\right) + (y_i - c)E\left(\frac{\partial f_2}{\partial y_i}\right) + r(p_i - y_i)\sigma_\delta^2 = 0.$$

Thus Equations (i), (ii), and (iii) are the implicit equations that solve the maximization problem. Under the no-price-delegation scenario the firm will maximize

$$\max_{y_i, \alpha_i, p_i} E[(y_i - c)q_i - \alpha_i]$$

subject to

$$(1) \quad e_i \in \operatorname{argmax} E[U((p_i - y_i)q_i + \alpha_i - e_i^2)]$$

Incentive Compatibility Constraint,

$$(2) \quad EU \geq \bar{U} \quad \text{Participation Constraint.}$$

The ICC can be replaced by

$$\text{(ix)} \quad (p_i - y_i)\frac{\partial f_2}{\partial e_i} - 2e_i = 0.$$

The binding participation constraint is

$$\alpha_i + (p_i - y_i)q_i - e_i^2 - \frac{1}{2}(p_i - y_i)^2 r\sigma_\delta^2 = 0.$$

The first-order condition with respect to y_i is

$$\text{(x)} \quad (y_i - c)E\left(\frac{\partial f_1}{\partial y_i} + \frac{\partial f_2}{\partial y_i}\right) + r(p_i - y_i)\sigma_\delta^2 = 0.$$

Appendix 3

Case 1. Delegation by Both Firms. The optimal values for price, effort, and marginal cost as a function of commission rate are (for $r\sigma_\delta^2 = 1$)

$$p_i^D = -1(-100\theta_s^2\beta_i^2 + \theta_s^2\beta_i^2 h - 200\theta_s\beta_i^2 - 100\beta_i^2 - 7\beta_i^2 h + 6\beta_i^2 h\theta_p + 2\theta_s\beta_i h - 400\theta_s\beta_i - 400\beta_i - 2\beta_i h - 400)/Z,$$

$$y_i^D = -1[-200\theta_s\theta_p\beta_i + 600\beta_i - 8h + 800 + 100\beta_i^2 - 400\theta_p - 6\beta_i^2 h\theta_p - 6\theta_s\beta_i h + 600\theta_s\beta_i - 4\beta_i h + 100\theta_s^2\beta_i^2 + 7\beta_i^2 h + 200\theta_s\beta_i^2 + 4\beta_i h\theta_p - 200\beta_i\theta_p + 2\theta_s\beta_i h\theta_p - 1\theta_s^2\beta_i^2 h + 4\theta_p h] \div [-Z], \qquad e_i^D = \frac{\beta_i(p_i - y_i)}{2},$$

where

$$Z = 6\beta_i^2\theta_p^2 - 4\theta_p - 2\theta_s\beta_i\theta_p + 10\beta_i^2 + \theta_s^2\beta_i^2 - 15\beta_i^2\theta_p - 3\theta_s\beta_i^2\theta_p + 8 + 5\theta_s\beta_i^2 + 12\beta_i - 8\beta_i\theta_p + 6\theta_s\beta_i.$$

Taking the derivative of price with respect to commission ($r\sigma_\delta^2 = 1$, $h = 100$), we see

$$\frac{\partial p_i^D}{\partial \beta_i} = -200\frac{[-4\theta_s\beta_i\theta_p - 24\beta_i - 18\beta_i^2 + 4\theta_s - 4\theta_p - \theta_p\theta_s^2\beta_i^2 - 6\beta_i^2\theta_p^2 + 4\theta_s\beta_i + 2\theta_s^2\beta_i^2 - 11\theta_s\beta_i^2 + 10\theta_s\beta_i^2\theta_p + 20\beta_i\theta_p + 23\beta_i^2\theta_p + 4\theta_s^2\beta_i + \theta_s^3\beta_i^2]}{(Z)^2} > 0,$$

$$\frac{\partial y_i^D}{\partial \beta_i} = 2\frac{-800\theta_s\beta_i\theta_p + 6500\beta_i + 800 + 3800\beta_i^2 + 900\theta_p^2\beta_i^2 + 400\theta_p - 300\theta_p\theta_s^2\beta_i^2 + 3300\beta_i^2\theta_p^2 + 2400\beta_i\theta_p^2 + J}{(-Z)^2} > 0,$$

where

$$J = 1600\theta_s\beta_i + 500\theta_s^2\beta_i^2 + 3100\theta_s\beta_i^2 - 3600\theta_s\beta_i^2\theta_p - 8000\beta_i\theta_p - 400\theta_p^2 - 600\beta_i^2\theta_p^3 - 6300\beta_i^2\theta_p$$

$$\frac{\partial e_i^D}{\partial \beta_i} = -200\frac{[4\theta_s\beta_i\theta_p - 16\beta_i - 2\beta_i^2 - \theta_p^2\theta_s\beta_i^2 + 4\theta_p + \theta_p\theta_s^2\beta_i^2 + 2\beta_i^2\theta_p^2 - 4\beta_i\theta_p^2 - 8\theta_s\beta_i - 2\theta_s^2\beta_i^2 - 7\theta_s\beta_i^2 + 6\theta_s\beta_i^2\theta_p + 16\beta_i\theta_p - 8 - \beta_i^2\theta_p]}{(Z)^2} > 0.$$

The optimal values of price, ''virtual'' marginal cost, and commission as a function of parameters are

$$p_i^D = \frac{2h[1 + 2r\sigma_\delta^2]}{8r\sigma_\delta^2 + \theta_s + 3 - 4\theta_p r\sigma_\delta^2 - \theta_p},$$

$$y_i^D = (1000r\sigma_\delta^2 - 400\theta_p r\sigma_\delta^2 + 4hr\sigma_\delta^2\theta_p - 8hr\sigma_\delta^2 + 2h\theta_p - 200\theta_p - h\theta_s - 3h + 300 + 100\theta_s)/(8r\sigma_\delta^2 + \theta_s + 3 - 4\theta_p r\sigma_\delta^2 - 2\theta_p),$$

$$\beta_i^D = 200/(h\theta_s + 3h + 8hr\sigma_\delta^2 + 200\theta_p + 400\theta_p r\sigma_\delta^2 - 100\theta_s - 100 - 600r\sigma_\delta^2 - 2h\theta_p - 4hr\sigma_\delta^2\theta_p).$$

Note that once again as price competition increases the optimal level of prices that helps in reducing price competition also increases.

Case 2. No Delegation by Both Firms. The first-order conditions for the firm's maximization, after substituting for α_i and e_i, are

$$p_i{:}\quad h - 2p_i + \theta_p p_j + p_i\beta_i - .5p_i y_i - \theta_s e_j - .5\beta_i^2 p_i + .5\beta_i^2 y_i - r\beta_i^2 p_i\sigma_\delta^2 + r\beta_i^2 y_i\sigma_\delta^2 = 0,$$

$$y_i{:}\quad -.5p_i\beta_i + .5p_i\beta_i^2 - .5y_i\beta_i^2 + r\beta_i^2 p_i\sigma_\delta^2 - ry_i\beta_i^2\sigma_\delta^2 = 0,$$

$$\beta_i{:}\quad .5p_i^2 - .5p_i y_i - .5\beta_i p_i^2 + \beta_i p_i y_i - .5\beta_i y_i^2 - r\beta_i\sigma_\delta^2 p_i^2 + 2rp_i y_i\beta_i\sigma_\delta^2 - r\beta_i\sigma_\delta^2 y_i^2 = 0.$$

Solving the above equations for the symmetric firms we get (for $r\sigma_\delta^2 = 1$, $h = 100$)

$$p_i^N = \frac{600}{11 + \theta_s - 6\theta_p}, \qquad y_i^N = \frac{200(3\beta_i - 1)}{\beta_i(11 + \theta_s - 6\theta_p)}, \qquad e_i^N = \frac{\beta_i(p_i^N - y_i^N)}{2}.$$

Also,

$$\frac{\partial p_i^N}{\partial \beta_i} = 0, \qquad \frac{\partial y_i^N}{\partial \beta_i} = \frac{200}{\beta_i^2(11 + \theta_s - \theta_p)} > 0,$$

and, the total derivative of effort is

$$\frac{de_i^N}{d\beta_i} = \frac{\partial e_i^N}{\partial \beta_i} + \frac{\partial e_i^N}{\partial y_i}\frac{dy_i^N}{d\beta_i} + \frac{\partial e_i^N}{\partial p_i}\frac{dp_i^N}{d\beta_i} = \frac{0.5(200 - \theta_s\beta_i^2 - 11\beta_i^2 + 6\theta_p\beta_i^2)}{\beta(11 + \theta_s - 6\theta_p)} > 0.$$

We see that price is independent of commission rate when the firm is setting both price and marginal cost. Any increase in the commission rate leads to increase in the marginal cost y_i that reduces the wages paid to the rep. Note that for $y_i^N = 0$, the commission is $\beta_i = 0.33$ and the price is

$$p_i^N = \frac{200}{4 - 2\beta_i + \theta_s\beta_i + \beta_i^2 + 2\beta_i^2 - 2\theta_p}.$$

Case 3. Firm 1 Delegating (D) and Firm 2 Not Delegating (N). For Firm 1, the contract and the rep's price and effort levels satisfy first-order conditions of Case 1 above. For Firm 2, the contract and the effort levels satisfy focs of Case 2.

In the N case substituting for y_2 from the first-order condition for y_2 and solving for p_2 gives

$$p_2 = 2[100 + 200r\sigma_\delta^2 + \theta_p p_1 + 2\theta_p p_1 r\sigma_\delta^2 - .5\theta_s\beta_1(p_1 - y_1) - \theta_s\beta_1(p_1 - y_1)r\sigma_\delta^2]/(3 + 8r\sigma_\delta^2).$$

In the first-order condition for p_1 substituting for p_2 and solving we get

$$\begin{aligned} p_1 = {} & [3\theta_s\beta_2 y_2 + 400\theta_p + 6y_1 - 200\theta_s\beta_2 + 16hr\sigma_\delta^2 + 16y_1 r\sigma_\delta^2 \\ & - 3\beta_1 y_1 + 6h + 2\theta_p\theta_s\beta_1 y_1 + 4\theta_s\beta_1 r\sigma_\delta^2 y_1 - \theta_s^2\beta_2\beta_1 y_1 \\ & - 2\theta_s^2\beta_2\beta_1 r\sigma_\delta^2 y_1 - 2r\sigma_\delta^2\beta_1 y_1 + 16(r\sigma_\delta^2)^2\beta_1 y_1 - 400\theta_s\beta_2 r\sigma_\delta^2 \\ & + 8\theta_s r\sigma_\delta^2\beta_2 y_2 + 800\theta_p r\sigma_\delta^2] \\ & \div [12 + 32r\sigma_\delta^2 - 3\beta_1 + 2\theta_p\theta_s\beta_1 + 4\theta_p\theta_s\beta_1 r\sigma_\delta^2 - \theta_s^2\beta_2\beta_1 \\ & - 2\theta_s^2\beta_2\beta_1 r\sigma_\delta^2 - 2r\sigma_\delta^2\beta_1 - 4\theta_p^2 - 8\theta_p^2 r\sigma_\delta^2 + 2\theta_s\theta_p\beta_2 \\ & + 4\theta_s\theta_p\beta_2 r\sigma_\delta^2 + 16\beta_1(r\sigma_\delta^2)^2]. \end{aligned}$$

Similarly, solving for p_2 we get

$$\begin{aligned} p_2 = {} & [\beta_2^2 y_2\beta_1 + 4\theta_p y_1 + 4h\theta_p - 2\theta_p\beta_1 y_1 - 200\beta_1 + \beta_2 y_2\beta_1 \\ & - 2\theta_s h\beta_1 + 2\theta_s\beta_1 y_1 + 4\beta_2^2 y_2 + 8r\sigma_\delta^2\beta_2^2 y_2 - 4\beta_2 y_2 \\ & + 4(r\sigma_\delta^2)^2\beta_2^2\beta_1 y_2 - \theta_s^2\beta_1\beta_2 y_2 + 2\theta_p\theta_s\beta_2 y_2 - 2\beta_2 y_2\beta_1 r\sigma_\delta^2 \\ & + 4\theta_p\beta_1 y_1 r\sigma_\delta^2 + 400r\sigma_\delta^2\beta_1 + 800] \\ & \div [-4\beta_1 - 4\theta_p^2 + 16 + 2\beta_2\beta_1 - \beta_1\beta_2^2 - 8\beta_2 + 8\beta_2^2 r\sigma_\delta^2 + 4\beta_2^2 \\ & + 4(r\sigma_\delta^2)^2\beta_2^2\beta_1 + 2\theta_p\theta_s\beta_1 - \theta_s^2\beta_2\beta_1 - 4\beta_2\beta_1 r\sigma_\delta^2 + 2\theta_s\theta_p\beta_2 \\ & + 8\beta_1 r\sigma_\delta^2]. \end{aligned}$$

Substituting for p_1 and p_2 in the first-order conditions for y_1 and y_2 and solving, we get values for "virtual" marginal costs as a function of commission rates. Substituting these back into equations for prices, we get prices as a function of commission rates. From these we can then calculate profits to the delegating and not-delegating firms, respectively.

Profits under different price-delegation choices for $r\sigma_\delta^2 = 1$, $h = 100$ are

$$\Pi_1^{D,D} = \frac{3.3 \times 10^5}{(6\theta_p - \theta_s - 11)^2}, \qquad \Pi_1^{N,N} = \frac{600\theta_s + 3.29 \times 10^5}{(6\theta_p - \theta_s - 11)^2},$$

$$\begin{aligned} \Pi_1^{N,D} = {} & [(365630\theta_s^2 + 5.5688 \times 10^5\theta_p + 534380\theta_s^2\theta_p + 56250\theta_s^3 \\ & + 2.413 \times 10^6 + 1.575 \times 10^5\theta_s\theta_p^2 + 2.925 \times 10^5\theta_p\theta_s \\ & + 1.35 \times 10^5\theta_p^3 + 3.7125 \times 10^5\theta_s - 5.85 \times 10^5\theta_p^2) \\ & \times (3.87 \times 10^6\theta_p^3)(-5 \times 10^3) + A] \\ & \div (-6.478 \times 10^5\theta_s^2 + 56250\theta_s^4 + 5.4 \times 10^5\theta_p^4 \\ & + 4.2333 \times 10^6\theta_p\theta_s + 3.7125 \times 10^5\theta_p^2\theta_s^2 \\ & - 7.65 \times 10^5\theta_p^3\theta_s - 759380\theta_s^3\theta_p - 6.5681 \times 10^6\theta_p^2 \\ & + 1.8098 \times 10^7)^2, \end{aligned}$$

$$\begin{aligned} \Pi_1^{D,N} = {} & (4.5563 \times 10^{14}\theta_s^4\theta_p^2 - 1.2457 \times 10^{17}\theta_s + 2.3227 \times 10^{15}\theta_s^3\theta_p \\ & + 7.2894 \times 10^{16}\theta_s^2\theta_p^4 + 5.6936 \times 10^{15}\theta_s^4\theta_p + B + C + D) \\ & \div (-6.478 \times 10^5\theta_s^2 + 56250\theta_s^4 + 5.4 \times 10^5\theta_p^4 \\ & + 4.2333 \times 10^6\theta_p\theta_s - 3.7125 \times 10^5\theta_p^2\theta_s^2 - 7.65 \times 10^5\theta_p^3\theta_s \\ & - 759380\theta_s^3\theta_p - 6.5681 \times 10^6\theta_p^2 - 1.8098 \times 10^7)^2, \end{aligned}$$

where

$$\begin{aligned} A = {} & 4\theta_s^3\theta_p + 1.0481 \times 10^6\theta_s^2 + 1.677 \times 10^7\theta_p^2 - 1.6126 \times 10^5\theta_s^3 \\ & + \theta_s^4 - 8.3852 \times 10^6\theta_p\theta_s - 1.1137 \times 10^7\theta_s + 1.5319 \times 10^6\theta_s^2\theta_p \\ & - 1.6706 \times 10^7\theta_p - 4.515 \times 10^6\theta_s\theta_p^2 - 7.2391 \times 10^7, \\ B = {} & 1.8014 \times 10^{17}\theta_s\theta_p^2 - 3.61 \times 10^{16}\theta_p^4\theta_s - 4.56 \times 10^{13}\theta_s^5\theta_p \\ & - 2.43 \times 10^{15}\theta_s^3\theta_s^3\theta_p^3 + 2.88 \times 10^{16}\theta_p^5, \\ C = {} & -2.86 \times 10^{13}\theta_s^5 - 2.4 \times 10^{17}\theta_p^3 + 3.5 \times 10^{18}\theta_p^2 + 3.77 \times 10^{16}\theta_s^3 \\ & - 3.78 \times 10^{16}\theta_p^4 + 7.78 \times 10^{15}\theta_p^6 + 1.9 \times 10^{12}\theta_s^6 - 5.8 \times 10^{15}\theta_s^2, \\ D = {} & -1.44 \times 10^{14}\theta_s^4 + 4.65 \times 10^{17} + 1.8 \times 10^{16}\theta_p^3\theta_s^2 \\ & + 4.55 \times 10^{16}\theta_s\theta_p + 3.76 \times 10^{16}\theta_p^3\theta_s - 4.83 \times 10^{17}\theta_s^2\theta_p \\ & - 1.4 \times 10^{16}\theta_s^2\theta_p^2 - 1.17 \times 10^{16}\theta_s\theta_p^5 - 4.54 \times 10^{15}\theta_s^3\theta_p^2. \end{aligned}$$

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