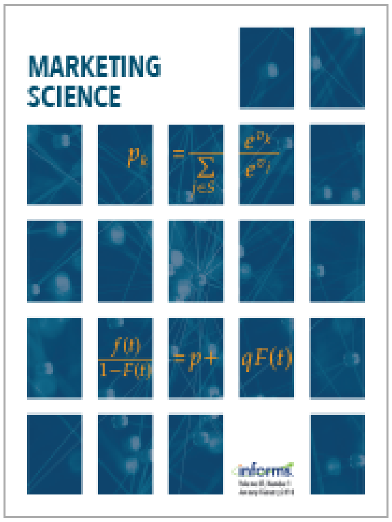

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Designing Optimal Sales Contests: A Theoretical Perspective

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Abstract

Sales contests are commonly used by firms as a short-term motivational device to increase salespeople's efforts. Conceptually, sales contests and piece-rate schemes, such as salary, commission, or quotas, differ in that in sales contests payment to salespeople is based on relative rather than absolute sales levels. Using the agency theoretic framework where the firm is risk neutral and the salespeople are risk averse, we examine how a firm should design an optimal contest to maximize its profit through stimulating salespeople's efforts. Specifically, we investigate how many salespeople should be given awards and how the reward should be allocated between the winners. Three commonly used sales contest formats are studied. In the first format, termed as Rank-Order Tournament, there are many winners and the amount of reward is based on relative rank achieved, with larger amounts awarded to higher ranks. We also examine two special cases of Rank-Ordered Tournament: a Multiple-Winners format, where the reward is shared equally, and a Winner-Take-All format, where a single winner gets the entire reward.

We model salespeople's behavior by considering utility of the reward from achieving one of the winning ranks in the contest and assessing incremental chances of winning by exerting more effort. The analysis was done for two situations based on whether the total reward is large enough for salespeople to participate in the effort-maximizing sales contest or not. The analysis shows that factors impacting contest design include the salespeople's degree of risk aversion, number of salespeople competing in the contest, and degree of sales uncertainty (which reflects strength of the sales-effort relationship). The results show that salespeople exert lower effort when there are larger numbers of participants or when sales uncertainty is high. We find that the Rank-Order Tournament is superior to the Multiple-Winners contest format. In a Multiple-Winners format, the salesperson whose performance is just sufficient to win is better off than any of the other winners as he exerts the least effort to win but obtains as high a reward as any other winners.

Specific recommendations on contest designs are obtained assuming that sales follow either a logistic or uniform distribution. Assuming that sales outcome is logistically distributed and the contest budget is high enough to ensure participation, our analysis shows that the total number of winners in a sales contest should not exceed half the number of the contestants. This result is due to the symmetric nature of the logistic distribution. Our analysis also indicates that the total number of winners should be increased and the spread decreased when salespeople are more risk averse. When salespeople are more risk averse, their marginal values for higher rewards become smaller. The spread should increase with ranks when rate of risk tolerance is high and decrease with ranks when the rate of risk tolerance is lower. In the extreme case of risk-neutral salespeople, the optimal design is a Winner-Take-All format. We also conclude that since the probability of winning the contest decreases with number of contestants, the optimal number of winners should increase and interrank spread decrease when there are a larger number of participants. If the firm does not allocate a large enough budget for salespeople to participate in the effort-maximizing sales contest, then the firm may increase the number of winners to more than half the salesforce. Increasing the number of winners and decreasing the spread are required to encourage the salespeople to participate, particularly when there are many participants who are risk averse. A counterintuitive result is that the number of winners should be reduced and the spread increased when sales uncertainty is high. Increasing sales uncertainty leads to lower equilibrium effort levels while keeping the expected utility of the contest rewards the same. Therefore, increased uncertainty results in higher participation incentive. The firm should thus relatively reduce the number of winners in high-uncertainty situations.

Under the assumption of uniformly distributed sales, the recommendation is that a Winner-Take-All contest induces maximum efforts regardless of the level of risk aversion, number of players, or the degree of uncertainty. When the Winner-Take-All format does not meet the participation constraint, our analysis recommends offering a big reward to the top salesperson and a small reward to many other salespeople. The small reward should be just sufficient to ensure that all salespeople participate. Consistent with logistic distribution, the spread should decrease when salespeople are more risk averse or there are more players but should increase when sales uncertainty is larger. These results highlight that some of the conclusions drawn may be sensitive to distributional assumptions.
(*Agency Theory; Sales Contests; Salesforce Compensation*)

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1. Introduction

A sales contest is a short-term incentive program used by firms to increase sales volume (Churchill et al. 1993). The use of sales contests as an incentive tool is widespread and growing. The total expenditure for sales contests increased from $1.6 billion in 1971 to $8.0 billion in the late 1980s (Chrapek 1989). A review study conducted by Murphy and Sohi (1995) reports that in the several surveys that have been conducted, between 75% and 90% of the surveyed firms were found to use sales contests. Sales contests are used for a variety of objectives, including increasing overall sales, increasing market penetration, introducing new products, overcoming seasonal slumps, and easing unfavorable inventory situations. Anecdotal evidence suggests that sales managers view sales contests as an important component of the salesforce compensation package and spend considerable time designing the contests (e.g., Murphy and Dacin 1998).

Sales contests are designed in several formats. In a contest conducted by the computer system division of Toshiba, the top two sales representatives were given $40,000. The second prize of $5,000 was given to 60 employees. Another 100 employees received an award of $500 (*Business Wire* 1997). The American Express establishment service awards their top 75 employees with a ''lavish jaunt'' including cash (*Sales and Marketing Management* 1998). Merrill Lynch awards their top 100 brokers with trips to London; the second award of new computers is given to 95 brokers; and 175 brokers are awarded the third prize of $1,000 (*Wall Street Letter* 1995). These anecdotes reveal puzzling differences both in the number of winners that are given rewards and in the variations in the amounts awarded between ranks achieved in the contest. These stylized facts raise the question of what the optimal sales contest design should be to induce maximum effort from the salesforce. In particular, how many employees should be awarded? How should the total reward be allocated? How much difference should there be in the awards allocated between the ranks obtained in the contest? The objective of this paper is to provide guidelines on these contest design issues.

Existing literature on salesforce compensation provides very few recommendations on how contests should be designed. Churchill et al. (1993) provide similar guidelines for both contests and quotas, recommending that ''every member of the sales force should have a reasonable chance of winning an award.'' Similarly, Kotler (1997) suggests that ''the contest should present a reasonable opportunity for enough salespeople to win.'' An article in *Sales and Marketing Management* (1989) advises that ''sales contests should not be designed as Winner-Take-All.'' In practice, however, a wide variety of formats are observed. Churchill (1993) reports a survey that found that 35% of the firms use contests in which the odds of being a winner are 1 in 5 or less; 31% of the firms design contests in which the odds are 2 in 5. The odds are 3 in 5 in 21% of the firms; 8% of the firms use contests where the odds are 4 in 5; and in 5% of the firms the odds are more than 4 in 5.

The key conceptual difference between a contest and a piece-rate scheme (e.g., salary, commissions, and quotas) is that in contests, the compensation is based on rank order of performance or performance relative to other employees, whereas in the piece-rate schemes the compensation is based on absolute output. While the economics literature on contests has conceptually viewed contests to be a compensation scheme based on rank-order of performance, the definition in marketing texts and the trade press has been broader. For example, Churchill et al. (1993) interpret schemes where salespeople earn awards for meeting or exceeding quota levels as contests where individuals compete against themselves. In this paper, we focus only on cases where the compensation structure is based on relative performance levels.

The seminal papers on contests (Lazear and Rosen 1981, Green and Stokey 1983, Nalebuff and Stiglitz 1983) focus on investigating the conditions under which contest-tournament-based schemes are more appropriate than the traditional piece-rate incentive approaches. As is standard in the agency literature, they assume a stochastic relationship between the participants' effort and the outcome achieved. They distinguish between two types of uncertainty. One type of uncertainty is idiosyncratic to each salesperson. The second type of uncertainty is common to all the players in the tournament (for example, macroeconomic conditions or competitive entry). They demonstrate that contest-based compensation plans perform better than piece rates when the common uncertainty is larger than the individual uncertainty. The intuition behind this result can be seen by comparing the amount of risk premium that firms incur under these two types of compensation schemes. According to the salesforce-compensation literature, a risk-neutral firm needs to compensate a risk-averse salesperson with a risk premium. This premium increases with the uncertainty related to sales outcome. Under a rank-order-based incentive scheme, a firm compensates its salespeople based on their relative performance, thereby eliminating the impact of common uncertainties. As a result, the firm needs to pay the salespeople much less risk premium under sales contests than under piece-rate schemes (salary, commission, or quota-based compensation), which depend on the salesperson's own output. To view the intuition from another perspective, note that sales contests allow the firm to relate outcome (ranking under sales contest) with selling effort with smaller noise than is possible with independent schemes. When sales contests are used, a salesperson's performance is evaluated against a random reference point. The reference point is random because it is based on the output levels of other contestants. Because the common uncertainty impacts all contestants equally, relative ranking as a measure of sales outcome filters away all common noises. Therefore, if the common uncertainty is relatively large, the relative ranking provides much more precise information about the unobservable effort to the firm. Lazear and Rosen (1981) also show that, under some conditions, contests are equivalent to piece rates in performance when agents are risk neutral. In such situations, when it is cheaper to monitor ranks than individual outputs, they argue that contests may dominate piece rates (Lazear 1989).

Sales contests are just one example of incentive schemes being based on rank order of performance. Other common examples of contests are sporting events, promoting managers to limited upper management positions, the Teacher of the Year award, and the selection of *Business Week* Top 25 schools. Several other reasons have been advanced about the benefits of contests over piece-rate schemes (O'Keeffe et al. 1984). First, merely observing a contest may provide utility for both the participants and viewers (e.g., game shows). Second, contests are useful when the awards are indivisible. Third, monitoring costs are lower when only the rank-order of the participants needs to be monitored (e.g., in many situations, it is much harder to rate participants than to rank them). Fourth, relative to quotas or piece rates, the costs of the employee compensation are certain. Finally, the utility derived from a contest can include nonmonetary aspects (e.g., recognition).

Although there is limited theoretical work on sales contests in the marketing literature (see Gaba and Kalra 1999 for an exception), the economics literature has investigated issues pertaining to the conditions under which contests are efficient, as well as on some issues about how they should be designed. Lazear and Rosen (1981) show that heterogenous/disadvantaged players contaminate the efficiency of a contest. Nalebuff and Stiglitz (1983) demonstrate that negative penalties (e.g., firing) may be superior to positive rewards. They also show that contests become more efficient when a firm assigns a higher rank only if there is a sufficient gap between the performances. O'Keefe et al. (1984) show that to maximize social efficiency, the spread between ranks in a two-person game should be higher when the monitoring capacity of a firm is low. Rosen (1986) demonstrates that the spread should be increasing in elimination games (e.g., tennis). Assuming that effort levels are fixed, Gaba and Kalra (1999) demonstrate that employees will opt for prospects of higher risk when the contest has increasingly fewer winners. Experimental tests of tournament theory (e.g., Bull et al. 1987, Weigelt et al. 1989), as well as empirical tests, have been conducted (e.g., Knoeber and Thurman 1994, Ehrenberg and Bognanno 1990).

Motivated by the prevalence of sales contests as an incentive device and the paucity of theoretical guidelines for aiding design decisions, we examine how many winners there should be in a contest and how the total reward should be allocated between the winners. Using an agency theory framework, the authors analyze how a risk-neutral firm should design the contest to maximize profits through increasing salespeople effort levels. Three commonly used sales contest formats are studied. In the first format termed as Rank-Order Tournament, there are many winners and the amount of reward is based on the relative rank achieved, with larger amounts rewarded to higher ranks. We also examine two special cases of the Rank-Order Tournament: a Multiple-Winners format where the reward is shared equally and a Winner-Take-All format where a single winner gets the entire reward. The salespeople's behavior in the contest is modeled as maximizing their expected utility when the trade-off is between the anticipated utility of achieving a specific winning rank and the incremental probability of obtaining the rank from extra effort. The optimal contest designs are examined for situations in which the reward is sufficiently high so that all salespeople participate, as well as for situations in which the amount of expected reward is not enough to ensure participation. We show that the factors influencing the contest design are the risk averseness of the salespeople, the number of salespeople participating in the contest, and the level of sales uncertainty. The contest design is analyzed assuming that the sales follow two distributions: (i) logistic and (ii) uniform. The results indicate that the number of winners should increase and the spread between winners decrease when salespeople are risk averse and when there are a large number of players in the contest. The level of sales uncertainty does not impact the design of the contest when the reward is high enough to obtain salespeople participation. A nonintuitive result is that when the rewards are not high enough, increasing sales uncertainty implies that the optimal contest should be designed with fewer winners and larger spreads. The analysis also shows that assumptions about the sales distributions imply some differences in contest designs.

2. Model

Designing a sales contest often involves a number of decisions. For example, components of the decisions include the number of contenders that should be awarded, how the total amount of reward should be distributed, whether the contest should be individual versus team-based, the type of awards (money versus travel awards), and the duration of the contest. The design issue that we mainly focus on is the number of contenders to be awarded and the allocation of reward between the winners. In our model the salespeople generate sales with their own rather than team effort. We assume that the reward of the contest will be in the form of money, which is the most frequently adopted reward format compared to others, such as travel rewards. We do not explicitly address the duration of sales contest, assuming that it follows standard selling cycles. We further assume that the conditions favoring a contest exist, i.e., the firm is at or above the frontier at which the common shock and/or the number of salespeople are sufficiently large.

2.1. Salespeople and Their Decisions

Consider a firm that employs a group of N salespeople (denoted by $i = 1, 2, \ldots, N$). For convenience of exposition, we assume that the firm assigns each salesperson to one of N identical territories. We make this assumption only to highlight two points: First, each salesperson's sales volume is independent of other salespeople's effort levels. Therefore, salespeople will not sabotage others' performances. Second, we do not allow for the possibility that the salespeople can collude. A salesperson (i) spends effort (e_i) selling the firm's products or services and then receives compensation of an amount r_i from the firm. A salesperson's utility function is separable in the effort exerted and the payment received:

$$U(r_i, e_i) = u(r_i) - c(e_i). \tag{1}$$

We assume that all salespeople share the same preferences over income and effort levels. Also, $u(r_i)$ is increasing and concave with respect to reward r_i, and $u(0) = 0$. The cost function $c(e_i)$ is strictly increasing and convex with respect to e_i. As is normally assumed, the salespeople value income positively with decreasing marginal utility. The salespeople incur costs by expending selling effort with increasing marginal cost.

Consistent with the literature, the salespeople's effort levels are not observable to the firm. The sales outcome in each territory, x_i, is determined stochastically by the selling effort of the salesperson within the territory. In particular,

$$x_i(e_i) = s(e_i) + \epsilon_i + \eta. \tag{2}$$

The sales response function in (2) follows Green and Stokey (1983). The right side of the equation consists of a deterministic term, $s(e_i)$, and uncertainty variables. Here $s(e_i)$ represents the expected sales, given selling effort e_i. As is also standard, $s(e_i)$ is a strictly increasing and concave function of e_i. Note that $s(\cdot)$ is same for all the N territories as salespeople have equal productivity and work in identical territories. Following Green and Stokey (1983), we separate the uncertainties into those unique to each territory, ϵ_i, and those common to all territories η. The ϵ_is are identically and independently distributed across the territories, with mean equal to zero and variance σ_ϵ^2. We denote its cumulative distribution function by $F(\cdot)$ and density function by $f(\cdot)$. Note that ϵ_is have zero mean, as the sales function $s(e_i)$ has captured the mean effect. The common random variable, η, captures the impact from the actions of competitors, the market's acceptance of a new product, the impact of weather, and the macroeconomic environment. We denote its cumulative distribution function by $G(\cdot)$. Similar to the ϵ_is, η has a mean equal to zero and a variance denoted by σ_η^2.

Each salesperson decides the level of selling effort to maximize her expected utility:

$$e_i^* = \underset{e_i}{\operatorname{argmax}}\ \mathrm{E}U(r_i, e_i). \tag{3}$$

Equation (3) is often referred to as the condition of incentive compatibility (IC). The condition indicates that each salesperson trades off effort for the potential reward from the firm. Therefore, the amount of effort expended depends on the design of the sales compensation plan.

We further assume that if a salesperson does not participate in the sales contest, the salesperson will receive a fixed utility equal to $\bar{u}$. Thus, a sales contest is required to provide an expected utility sufficiently large so that all salespeople find it rational to participate. This condition is referred to as the condition of individual rationality (IR),

$$\mathrm{E}U(r_i, e_i) \geq \bar{u}. \tag{4}$$

2.2. Firm's Decision

Before the salespeople make decisions, the firm moves first to offer a compensation scheme, correctly anticipating its impact on salespeople's behavior. The firm is risk neutral and seeks to maximize its expected profit, the total outputs from the salespeople net of total compensation payments to the salespeople. The firm's expected profit from all N salespeople is

$$\pi = \delta \sum_{i=1}^{N} Ex_i - \sum_{i=1}^{N} r_i = \delta \sum_{i=1}^{N} s(e_i) - \sum_{i=1}^{N} r_i, \tag{5}$$

where δ is profit margin, $\Sigma_{i=1}^N s(e_i)$ is expected total revenue, and $\Sigma_{i=1}^N r_i$ is total compensation to salespeople. As is standard in the salesforce-compensation literature, we assume that the firm can only observe the output level of each territory, x_i, but the firm cannot directly observe either the amount of effort from the salespeople or the realization of the random variables.

3. Model Analysis

3.1. Salespeople's Behavior under Sales Contests

In this section we examine the salespeople's effort decisions under a rank-order sales contest. We consider a reward structure denoted by $\{R_1, R_2, \ldots, R_N\}$, where R_j is the amount awarded to the salesperson ranked at the jth position by sales volume/dollars. As it is the relative ranking and not the absolute amount of sales that is of concern, a salesperson's compensation not only depends on her own effort, but also on the other salespeople's effort levels. Because all the salespeople are identical, we examine the case of symmetric equilibrium. Each salesperson chooses a level of selling effort to maximize her utility. A salesperson's expected utility (Equation (1)) can thus be computed as

$$\mathrm{E}U(r_i, e_i) = \sum_{j=1}^{N} u(R_j)\mathrm{Prob}(r_i = R_j) - c(e_i), \tag{6}$$

where $\text{Prob}(r_i = R_j)$ denotes the probability that salesperson i's sales level ranks at the jth position among all N salespeople participating in the contest. This occurs when her sales volume is lower than the sales in $(j - 1)$ territories but higher than the rest of $(N - j)$ territories. Let the salesperson's effort be e_i. Assuming that all the other salespeople expend effort e^*, the probability that salesperson i achieves the rank of jth position is equal to:[1]

$$\text{Prob}(r_i = R_j) = \int_{\epsilon_i} \binom{N-1}{j-1} [1 - F(y)]^{j-1} F^{N-j}(y) f(\epsilon_i)\, d\epsilon_i, \quad (7)$$

where $y = s(e_i) - s(e^*) + \epsilon_i$.

Note that the common uncertainty factor η that impacts all the salespeople cancels out. Because common uncertainty does not affect salespeople's rankings, the salespeople's utilities, and hence the design of the sales contest, will be independent of the common uncertainty. In the rest of the paper, uncertainty refers to the idiosyncratic territory-specific uncertainty. Substituting the expected Utility Function (6) into Equation (3), we can solve for the optimal selling effort using the first-order approach. The first-order condition, assuming that all salespeople expend equal selling effort e^*, is given by

$$\sum_{j=1}^{N} u(R_j) \frac{\partial\, \text{Prob}(r_i = R_j)}{\partial e_i}(e_i = e^*) - c'(e^*) = 0. \quad (8)$$

Substituting Probability Equation (7) into (8), we obtain the following symmetric first-order condition for optimal level of effort:

$$\left(\sum_{j=1}^{N} u(R_j)\psi_j \right) s'(e^*) - c'(e^*) = 0, \quad (9)$$

where

$$\psi_1 = \int_{\epsilon_i} (N-1) F^{N-2}(\epsilon_i) f^2(\epsilon_i)\, d\epsilon_i;$$

$$\psi_j = \int_{\epsilon_i} \binom{N-1}{j-1} [1 - F(\epsilon_i)]^{j-2} F^{N-j-1}(\epsilon_i) f^2(\epsilon_i) \times [(N-j) - (N-1)F(\epsilon_i)]\, d\epsilon_i$$

$$\text{for } 2 \le j \le N-1; \quad \text{and}$$

$$\psi_N = -\int_{\epsilon_i} (N-1)[1 - F(\epsilon_i)]^{N-2} f^2(\epsilon_i)\, d\epsilon_i.$$

In Equation (9), $c'(e^*)$ is the marginal cost and $s'(e^*)$ is the marginal productivity of selling effort. The obvious implication is that the equilibrium effort expended by the salesperson will increase with the effectiveness of the effort in generating sales and that the effort will decrease with the increase in the salesperson's marginal cost of expending extra effort.

Because marginal cost $c'(e)$ increases and marginal sales $s'(e)$ decreases with effort e, from Equation (9) we can conclude that optimal effort increases with $\Sigma_{j=1}^{n} u(R_j)\psi_j$, the summation of marginal changes in expected utility through all ranks. The marginal change in expected utility through each rank contains two components: the utility of the reward $u(R_j)$ and the marginal change in probability of achieving a specific rank (ψ_j). Given other salespeople's efforts unchanged, extra effort increases the chance to obtain the top prize ($\psi_1 > 0$) and decreases the chance to be ranked at the bottom ($\psi_N < 0$). The value of ψ_j for an intermediate level can be either positive, zero, or negative, depending on the rank and sales distribution function. The salespeople attach a positive utility $u(R_j)$ to every position with positive rewards. Thus, Equation (9) captures the relation between the salespeople's effort and expected utility.

The general implication of Equation (9) is that the equilibrium effort levels of the contest participants depend on (i) the probability distribution function of local sales uncertainty (F), (ii) the number of salespeople in the contest (N), and (iii) the combination of the salesperson's utility function $u(\cdot)$ and the design of the sales contest. We will elaborate on these factors later.

[1]Distributions of order statistics are available in Arnold et al. (1992).

3.2. Types of Sales Contests

We consider a set of sales contests with total amount of reward equal to R. The rewards are distributed to m highest-ranked contestants out of N participants. Therefore, $\Sigma_{j=1}^{N} R_j = R$, $R_j > 0$ for $j = 1, 2, \ldots, m$ and $R_j = 0$ for $j = m + 1, m + 2, \ldots, N$. The most general form of a contest is that in which there are many winners and the amount of reward is based on the relative rank achieved, with larger amounts rewarded to higher ranks. We refer to this as the Rank-Order Tournament (ROT). In a Rank-Order Tournament, the spread is denoted by $d_j = R_j - R_{j+1}$, where $j = 1, 2, \ldots, m - 1$. In practice, some commonly used special cases of ROT are Winner-Take-All (WTA) in which the entire reward R is awarded to the highest-ranked contestant (that is, $m = 1$), and Multiple-Winners (MW) in which the top m ($m > 1$) winners share the reward equally (R/m). Note that in the MW format, the spread is zero ($d_1 = d_2 = \cdots = d_{m-1} = 0$).

3.3. Optimal Sales Contest

We now discuss the key factors of salespeople is behavior to be considered in designing optimal contests. We first derive the relation between the salesperson's effort and distribution of contest rewards. We consider a decrease of R_k by dR_k (where $R_k > 0$) and an increase of R_j by dR_j ($j \neq k$), while keeping rewards of all other ranks the same. The resulting change in salespeople's efforts (de^*) can then be derived from Equation (9):

$$\left(\sum_{j=1}^{N} u(R_j)\psi_j\right)s''(e^*)de^* + [u'(R_j)\psi_j dR_j + u'(R_k)\psi_k dR_k]\cdot s'(e^*) - c''(e^*)de^* = 0. \tag{10}$$

Because $\Sigma_{j=1}^{N} R_j = R$, it should satisfy $dR_k = -dR_j$. Substituting this into Equation (10), we can solve

$$\frac{de^*}{dR_j} = \frac{[u'(R_j)\psi_j - u'(R_k)\psi_k]\cdot s'(e^*)}{c''(e^*) - \left(\sum_{j=1}^{N} u(R_j)\psi_j\right)s''(e^*)}. \tag{11}$$

Equation (11) shows that, given other rewards remaining the same, the marginal change in salespeople's efforts resulting from a unit increase in the jth reward (R_j) and a unit decrease in the kth reward is positively related to $u'(R_j)\psi_j - u'(R_k)\psi_k$ (all other items are not rank-specific).

As described by Equation (4), the design of the sales contest also has to satisfy the individual rationality (IR) condition. That is, the sales contest is required to provide each salesperson with an expected utility that is sufficiently large so that the salesperson finds it rational to participate. In the symmetric equilibrium all salespeople spend the same amount of effort e^* and expect an equal probability ($1/N$) of reaching any of the N ranks. In equilibrium, the IR Condition (4) becomes

$$\frac{1}{N}\sum_{j=1}^{N} u(R_j) - c(e^*) \geq \bar{u}. \tag{12}$$

Now we examine the important issues relevant to the design of sales contests by comparing the resulting profit levels under various scenarios. The firm offers a sales contest to maximize its expected profit, knowing that its salespeople will spend the amount of effort that maximizes their own utilities (IC condition given by (9)), and expect a sufficient amount of incentive to participate in the sales contest (IR Condition (12)). Then the firm's total revenue from all N territories becomes $\Sigma_{i=1}^{N} s(e_i^*) = N\cdot s(e^*)$. We next formally state the firm's problem in (P1).

$$\underset{\{R_1,R_2,\ldots,R_N\}}{\text{Max}} \quad \pi = N\delta\cdot s(e^*) - R \tag{P1}$$

$$\text{s.t.} \quad \left(\sum_{j=1}^{N} u(R_j)\psi_j\right)s'(e^*) - c'(e^*) = 0; \tag{9}$$

$$\frac{1}{N}\sum_{j=1}^{N} u(R_j) - c(e^*) \geq \bar{u}; \tag{12}$$

$$\sum_{j=1}^{N} R_j = R; \tag{13.1}$$

$$R_j \geq 0. \tag{13.2}$$

In Program (P1), Equation (13.1) imposes a constraint on the total amount of budget allocated to prizes of different ranks. Condition (13.2) requires that the firm offer only nonnegative prizes. Suppose the optimal sales contest $\{R_1^*, R_2^*, \ldots, R_N^*\}$ rewards the top m

salespeople; that is, $R_j^* > 0$ for any $j \le m$, and $R_j^* = 0$ for any $j > m$. To derive the necessary conditions for the optimal sales contest design, we construct Lagrange

$$L = N\delta \cdot s(e^*) - R + \mu\left[\left(\sum_{j=1}^{N} u(R_j)\psi_j\right)s'(e^*) - c'(e^*)\right] + \lambda\left[\frac{1}{N}\sum_{j=1}^{N} u(R_j) - c(e^*) - \bar{u}\right], \tag{14}$$

where μ and λ are Lagrange multipliers. For any two ranks j and k such that $R_j^* > 0$, $R_k^* > 0$, and $j \neq k$, consider a decrease of R_k by dR_k and an increase of R_j by dR_j ($j \neq k$), while keeping other rewards the same. Such an allocation should not alter the total reward budget and thus require $dR_k = -dR_j$. Because $R_j^* > 0$, it is necessary that $dL/dR_j = 0$. With Lagrange given in (14),

$$\frac{dL}{dR_j} = N\delta \cdot s'(e^*)\frac{de^*}{dR_j} + \lambda\left[\frac{1}{N}u'(R_j^*) - \frac{1}{N}u'(R_k^*) - c'(e^*)\frac{de^*}{dR_j}\right] = 0. \tag{15}$$

The term μ does not appear in Equation (15) because it multiplies by zero according to Equation (10). Equation (11) gives de^*/dR_j, the marginal change in equilibrium effort with respect to R_j.

Equation (15) models two critical factors that a firm has to consider in designing an optimal sales contest. First, as indicated by the first term ($N\delta \cdot s'(e^*)de^*/dR_j$), a sales contest should be designed to maximize salespeople's efforts and hence maximize the firm's profit within the budget constraint (hereafter we will refer to such a contest design as the *effort-maximizing sales contest*). Second, a sales contest must be feasible by offering an expected utility large enough for salespeople to actively participate. The Lagrange multiplier λ measures the marginal increase in equilibrium profit due to a unit decrease in opportunity utility $\bar{u}$. If the effort-maximizing contest satisfies the participation constraint, then $\lambda = 0$ because any small change in $\bar{u}$ will not affect the equilibrium result. When $\lambda = 0$ the effort-maximizing sales contest is a feasible as well as the optimal solution to (P1). We now substitute $\lambda = 0$ into Equation (15) to derive the necessary conditions for such an effort-maximizing sales contest:

$$N\delta \cdot s'(e^*)\frac{de^*}{dR_j} = 0. \tag{16}$$

Equation (16) shows that reallocating a small unit of reward between R_j^* and R_k^* changes the level of effort by de^*/dR_j, which leads to a marginal change in sales $s'(e^*)$ at each territory and a total of $Ns'(e^*)$ from all N territories. At the effort-maximizing sales contest, the marginal profit from such a reallocation of rewards between jth rank and kth rank should be equal to zero.

After substituting Equation (11) into Equation (16), we can derive the following condition that is necessary for all positive prizes:

$$u'(R_j^*)\psi_j = u'(R_k^*)\psi_k \quad \text{for any } j, k \quad \text{with } R_j^*, R_k^* > 0. \tag{17}$$

Participation Incentive. When a sales contest provides an expected utility at or above $\bar{u}$, salespeople will expend e^* amount of effort as predicted by the incentive compatible condition. Under conditions where the sales contest budget (R) is sufficiently low and/or the required threshold utility ($\bar{u}$) is sufficiently large ($\bar{u}$), the salespeople will not participate in the sales contest (i.e., expend zero effort). If the equilibrium $\lambda > 0$, that is, the firm benefits from a unit decrease in opportunity utility $\bar{u}$, then the firm has to deviate from the effort-maximizing sales contest to provide enough incentives of participation. The firm's goal is to sacrifice the salespeople's efforts as little as possible. Naturally, the amount of deviation from the effort-maximizing sales contest will be smaller when salespeople's opportunity utility $\bar{u}$ is lower.

The firm can appropriately redesign the reward structure to increase the salesperson's incentive to participate in the contest. The last term of Equation (15) that is multiplied by λ measures the marginal change in the salesperson's expected utility from a small increase in reward R_j and a small decrease in R_k. We substitute in Equation (11) and obtain the following expression:

$$u'(R_j)\left(\frac{1}{N} - \frac{\psi_j \cdot s'(e^*)c'(e^*)}{c''(e^*) - \left(\sum_{j=1}^{N} u(R_j)\psi_j\right)s''(e^*)}\right) - u'(R_k)\left(\frac{1}{N} - \frac{\psi_k \cdot s'(e^*)c'(e^*)}{c''(e^*) - \left(\sum_{j=1}^{N} u(R_j)\psi_j\right)s''(e^*)}\right). \quad (18)$$

Equation (18) implies that it is more effective to increase the expected value of participation through the lower ranks and ranks with smaller ψ_j. First, a salesperson's utility of achieving a particular rank increases with the reward associated with the rank. The marginal increase in the utility of achieving the rank is bigger at lower ranks because smaller rewards are provided to lower ranks. (For any $R_j < R_k$, $u'(R_j) > u'(R_k)$.) Therefore, allocating a small unit of reward from a higher rank (k) to a lower rank (j) increases the expected utility from rewards ($1/N\ \Sigma_{j=1}^{N}\ u(R_j)$). Second, the increase in R_j raises the salesperson's incentive to reach jth rank, resulting in a change in the salesperson's selling effort proportional to ψ_j. With smaller ψ_j, a unit of extra effort leads to a smaller increase in the salesperson's chance to reach jth rank. Then the salesperson will add less effort. Consequently, the salesperson's cost of effort will be lower, and the salesperson will find higher incremental value of participation from the increase in R_j.

3.4. Contest Design Under Logistic Distribution

We develop our insights on the optimal number of winners and optimal interrank spread under two distributions: (a) logistic and (b) uniform. We adopt the logistic distribution as a good representative of bell-shaped distributions that are frequently used in the literature. Like the frequently used normal distribution, the logistic distribution is symmetric and displays a central tendency in density. The logistic distribution is also characterized by a mean parameter (0 in this paper) and a variance parameter (β). Thus, the results can be easily generalized across all bell-shaped distributions. Next, we also consider the case of uniform distribution. The uniform distribution has also been used earlier in the literature (e.g., Bull et al. 1987, Weigelt et al. 1989) but has different characteristics than bell-shaped distributions. Therefore, it is important to examine the similarities and differences in the results across different distributions.

The probability distribution function for a logistic distribution with zero mean is

$$f(x) = \frac{1}{\beta}\frac{\exp(-x/\beta)}{[1 + \exp(-x/\beta)]^2},$$

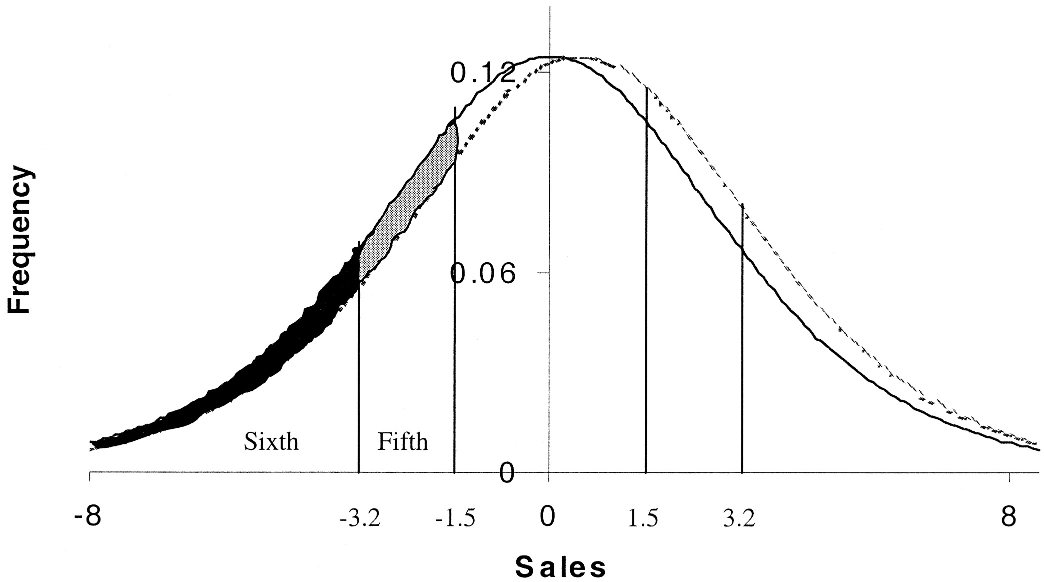
where variance is $\pi^2\beta^2/3$. Substituting the distribution function in the first-order condition (Equation (9)), we obtain the following relationship between salespeople's efforts (e^*) and specification of the sales contest (see Appendix 1 for the derivation):

$$s'(e^*) \sum_{j=1}^{N} u(R_j)\frac{N - 2j + 1}{N(N + 1)}\frac{1}{\beta} - c'(e^*) = 0. \quad (19)$$

The first term in the summation in Equation (19) represents the utility of achieving rank j. The second term indicates the marginal change in probability of obtaining the jth rank. Note that this term decreases with the rank j. The absolute value of this term is maximum for the top rank ($j = 1$), moves closer to zero at the intermediate ranks ($(N + 1)/2$) and increases again at the lower ranks. Therefore, holding other salespeople's efforts as equal and constant, a salesperson's extra effort will increase her chance of achieving above-midpoint ranks and decrease her chance of achieving below-midpoint ranks. Note that due to the central tendency of the logistic distribution, marginal increases in effort are more likely in the contestant achieving changes in rank closer to the midpoint rather than at the extreme highest or lowest rank. Moreover, compared with the probability of reaching the midpoint ranks, a salesperson's chance of reaching top or bottom ranks is more sensitive to extra effort.

To explain this observation, we graphically illustrate the intuition. Consider a group of six salespeople. When all six salespeople expend the same amount of effort, their sales volume is identically distributed. The probability that a salesperson reaches a specific rank is equal to the probability that the salesperson's sales volume falls into a corresponding interval. Specifically, the chance of reaching first rank is equal to the probability that a salesperson's sales

Figure 1 **Increasing Effort Under Logistic Distribution ($\beta = 2$)**



volume is above the 83.33th percentile. Similarly, the chance of reaching second rank is equal to the probability that sales volume falls between the 66.67th percentile and the 83.33th percentile, and so on (see Figure 1).

Now suppose that sales volume satisfies a logistic distribution ($\beta = 2$). The solid curve represents the frequency distribution of sales when a salesperson expends the same amount of effort as the rest of the salesforce. Under this distribution, -3.2 is the 16.67th percentile, -1.5 is the 33.33th percentile, 0 is the 50th percentile, 1.5 is the 66.67th percentile, and 3.2 is the 83.33th percentile. Therefore, the probability that a salesperson obtains the first rank is the same as the probability that the salesperson's sales volume is larger than 3.2. Similarly, the probability that a salesperson reaches second rank is equal to the probability that the salesperson's sales volume falls in the interval [1.5, 3.2], and so on. Now if a salesperson unilaterally increases selling effort, the frequency distribution of her sales volume will shift toward the right (dotted curve). The new distribution (first order) stochastically dominates the original sales distribution. Because the probability of sales falling within an interval is represented by the area under the frequency distribution curve, extra effort decreases the probability that the salesperson will obtain bottom ranks while increasing the probability of achieving top ranks. The area between these two distribution curves measures the resulting change of probability that the salesperson will achieve the corresponding ranking. For example, in Figure 1 the decrease in the probability of achieving the sixth rank is represented by the dark shaded area, while the decrease in the probability of achieving the fifth rank is represented by the light shaded area. From the graph it is easy to see that the dark shaded area is larger than the light shaded area. Figure 1 clearly shows that the salesperson's extra effort will induce a larger change in the probability of achieving the bottom and top ranks than the change in the probability of achieving intermediate ranks. Consistent with our analytical results, extra effort increases the probability of obtaining the top-half ranks and decreases the probability of achieving the bottom-half ranks. With bell-shaped distributions such as logistic and normal distribution, density is concentrated on the intermediate ranges of sales. Consequently, intervals of sales volume corresponding to intermediate ranks are very narrow. Therefore, changes in the probability of sales falling into these intervals (light shaded area) are smaller. In contrast, intervals of sales volume corresponding to top and bottom ranks are much wider. Therefore, changes in probability of sales belonging to these intervals are much larger.

Another implication from Equation (19) is that equilibrium effort decreases with sales uncertainty (β) and number of participants (N). Note that the central tendency of a logistic distribution decreases with variance and number of participants. Therefore, as confirmed by Equation (19), the marginal change in probability of achieving rank j decreases with the size of sales uncertainty (β). This indicates a weaker relationship between effort levels and the expected rank achieved when variance (β) is high. As a result, the salespeople are likely to opt for lower levels of effort with increases in size of uncertainty. Similarly, when the number of sales contestants (N) increases, the marginal change in probability of achieving any ranks also decreases. The increase in N reduces the marginal change in probability of achieving a higher rank (ψ_j for small j). Consequently, with increases in the pool of players, the firm will be less likely to induce higher effort levels.

Finally, the salespeople's efforts also depend on the design of the sales contest. A firm designs a contest by determining the specific values of R_j ($j = 1, \ldots, N$), which can take a wide variety of forms. Next, we discuss alternative design formats and their effectiveness in inducing salespeople's effort levels. The equilibrium format of a sales contest needs to satisfy Equilibrium Condition (15). Our discussion will proceed by examining the characteristics of an effort-maximizing sales contest characterized by (17) and then by also discussing the implications of the participation constraint on the equilibrium sales contest design.

Substituting ψ_j into Equation (17), we obtain the following necessary condition for all positive prizes:

$$u'(R_j^*)(N - 2j + 1) = u'(R_k^*)(N - 2k + 1) \quad \text{for any } R_j^*, R_k^* > 0. \tag{20}$$

Similarly, by substituting ψ_j into Equation (18), we obtain the marginal impact of reward allocation from kth to jth rank on a salesperson's expected utility of participation:

$$\frac{u'(R_j) - u'(R_k)}{N} - \frac{(N - 2j + 1)u'(R_j) - (N - 2k + 1)u'(R_k)}{N(N + 1)\beta} \times \frac{s'(e^*)c'(e^*)}{c''(e^*) - \left(\sum_{j=1}^{N} u(R_j)\psi_j\right)s''(e^*)}. \tag{21}$$

Our objective is to determine the characteristics of the sales contest design that induces the highest level of profit. We will analyze Equations (20) and (21) to address the following design issues. First, is there any upper bound for the number of winners? Second, what is the optimal number of winners, and how should the interrank spread be optimally structured?

Maximum Number of Winners. Equilibrium Condition (20) requires that the optimal number of winners (m^*) should satisfy $N - 2m^* + 1 > 0$. For any ranks with positive rewards, ψ_j must be positive; otherwise, Condition (20) will not be satisfied. Thus, in the effort-maximizing sales contest design, positive rewards should be allocated to rank j only if $N - 2j + 1 > 0$, which implies that $j < (N + 1)/2$. In other words, no more than half the participants should receive rewards. The intuition for the above results is as follows. When sales uncertainty follows a logistic distribution (given all other salespeople's efforts to be equal and constant), if a salesperson increases her effort, the probability of this salesperson being ranked from 1 to INT[$(N + 1)/2$] increases and the probability of being ranked below INT[$(N + 1)/2$] decreases.[2] Therefore, rewards given to any ranks below rank INT[$(N + 1)/2$] will reduce the salesperson's incentive to work. To induce the maximal effort from the salesperson, the firm should therefore allocate rewards to only ranks above INT[$(N + 1)/2$]. Note that the specific result of rewarding the prizes to no more than half the number of participants arises due to the symmetric nature of the logistic distribution.

[2] INT is defined as the largest integer that is less than or equal to $(N + 1)/2$.

With a budget (R) large enough for the contest and/or small opportunity cost ($\bar{u}$), the effort-maximizing sales contest provides enough incentive for the salespeople to participate and thus is the equilibrium sales contest design as well. However, if the effort-maximizing sales contest does not provide enough incentive for salespeople to participate, then the firm has to deviate by allocating rewards from the high ranks to the low ranks to increase expected utility and decrease cost of effort. As a result, the number of winners has to surpass more than one-half of the contestants. To see the conditions under which the optimal number of winners can be more than INT[$(N + 1)/2$], imagine a MW contest with $m_N =$ INT[$(N + 1)/2$] winners. This contest provides salespeople with the highest value of participation compared to any other that has the same or fewer winners. When such a sales contest does not provide enough incentive to participate, then the optimal number of winners has to exceed INT[$(N + 1)/2$]. A salesperson's value of participating in such a MW sales contest is

$$\frac{m_N}{N}u\left(\frac{R}{m_N}\right) - c(e^*),$$

where the equilibrium effort

$$c(e^*) = s(e^*)\left(\frac{m_N}{N}u\left(\frac{R}{m_N}\right)\frac{N - m_N}{(N + 1)\beta}\right).$$

Thus, the value of participation is higher with lower risk aversion (larger $(m_N/N)u(R/m_n)$), fewer number of participants (N), and larger sales uncertainty (β). When risk aversion is lower, the relative utility of higher rewards is more, and therefore the contest should have fewer winners. When there is a larger number of contestants, the probability of winning is lower, which in turn reduces the expected utility of the rewards. Correspondingly, the incentive to participate is lowered. In this situation, the firm should offer rewards to more salespeople. Finally, when uncertainty is high, the salespeople lower effort levels. Note that this occurs regardless of the contest design. Consider any contest at a given level of uncertainty and assume that the contest does not offer enough participation incentive at the equilibrium effort level. When the uncertainty increases, the equilibrium effort level decreases, also leading to a decrease in the cost of effort but not a decrease in the utility of the rewards. Therefore, the incentive to participate in the contest increases with increasing uncertainty. With increasing uncertainty, any contest comes closer to an effort-maximizing contest in which fewer winners are required to induce more effort. We summarize the above discussion in Proposition 1.

Proposition 1.

1.1. *If sales revenue is distributed logistically and the participation constraint is met, then the number of winners in the effort-maximizing sales contest should be no more than* INT[$(N + 1)/2$].

1.2. *If the effort-maximizing sales contest does not provide enough incentive for salespeople to participate, then the number of winners may exceed half the salesforce. Further, the number of winners in this case must increase with risk aversion and number of salespeople but decrease with sales uncertainty.*

Risk-Neutral Salespeople. To address the issue of which contest design format a firm should select, we begin with the case where salespeople are risk neutral, followed later with a section on the implications of risk aversion. When salespeople are risk neutral, marginal utility becomes constant and identical for any level of awards ($u'(R_j) = u'(R_k)$). This implies that the necessary Condition (20) can never be satisfied between any two ranks. Therefore, Winner-Take-All will be the effort-maximizing sales contest design. To contrast the Winner-Take-All format against the formats with more winners, recall that the key considerations facing the salespeople are the utility of achieving a specific rank and the incremental probability of doing so from extra effort. When the number of winners is one, it is apparent that the entire incentive is based on the chance to obtain the highest rank. In earlier discussion, we have shown that when all other salespeople's efforts are fixed, a salesperson's incremental chance to achieve the top rank is higher than for any other ranks ($\psi_1 >$ other ψ_js).

To ensure salespeople's participation in the Winner-

Take-All sales contest, it is necessary that either budget R is sufficiently large or $\bar{u}$ is sufficiently small. When the condition is not satisfied, the firm has to allocate the contest rewards among more winners. Because salespeople are risk neutral, their expected utilities from rewards are constant under any prize allocations. The firm can increase salespeople's incentives of participation only through the decrease in the cost of effort. The firm's optimal strategy is to allocate part of the top reward to intermediate levels to achieve an effort that satisfies $c(e^*) = (1/N)u(R) - \bar{u}$, the highest effort that a salesperson is willing to spend given total rewards R and N contestants.

We also can conclude that the Winner-Take-All design is more likely to be the optimal sales contest with larger sales uncertainty (β) and/or smaller number of participants (N). According to Equation (19), selling effort (e^*) decreases with β. When sales outcomes are more uncertain and less dependent on selling effort, salespeople will lower (effort) costs. As the utility from the reward remains the same and the costs of effort decrease, the expected value of participation becomes larger, and hence the sales contest is more likely to satisfy the IR condition. On the other hand, the Winner-Take-All contest is less likely to be the optimal sales contest with a larger number of participants (N). Note that the expected value of participation in a sales contest has a common factor ($1/N$), implying that a large number of participants reduces the incentive to participate in any sales contest. Intuitively, given a fixed budget, a sales contest will become less attractive if more people are competing for the rewards. We summarize these results in Proposition 2.

PROPOSITION 2.

2.1. *When salespeople are risk neutral, the effort-maximizing sales contest is Winner-Take-All. The Winner-Take-All contest is more likely to be the optimal contest with higher sales uncertainty and smaller number of participants.*

2.2. *If the Winner-Take-All contest does not meet the participation constraint, then the optimal contest must include more than one winner.*

Risk-Averse Salespeople. When salespeople are risk averse, having more than one winner is the design that stimulates more selling effort. With risk-averse salespeople, marginal utility from achieving higher ranks is smaller than the marginal utility from achieving lower ranks ($u'(R_j) < u'(R_{j+1})$ with $R_j > R_{j+1}$). As the marginal incentive to achieve higher ranks diminishes, it is optimal to allocate rewards to more winners. In contrast to the case of risk-neutral salespeople, the effort-maximizing Condition (20) may hold for many ranks. We now discuss the impact of the degree of risk aversion, sales uncertainty, and number of participants on the design of an optimal sales contest when salespeople are risk averse.

Degree of Risk Aversion. According to Equation (20), for any two consecutive ranks, the equilibrium condition for an effort-maximizing sales contest requires that $u'(R_j^*)/u'(R_{j+1}^*) = (N - 2j - 1)/(N - 2j + 1)$. When salespeople are more risk averse, it takes a smaller interrank spread $d_j = R_j - R_{j+1}$ to reach a given ratio of $u'(R_j)/u'(R_{j+1})$. With smaller interrank spreads, when the total reward budget is fixed at R, the number of winners would be larger. In essence, risk aversion reduces salespeople's marginal valuation of large rewards, therefore making the large rewards less effective in inducing salespeople's efforts. As a result, it is better to distribute rewards more evenly across the ranks.

Such sales contests are also more likely to provide enough incentives for salespeople to participate. According to Equation (21), an increase in reward R_j (decrease in R_k) leads to a marginal change in the incentive to participate in a contest proportional to the marginal utility $u'(R_j)(u'(R_k))$. Therefore, when salespeople are more risk averse, allocating rewards to top ranks becomes less effective in encouraging salespeople's participation. It is much more efficient to increase salespeople's expected value of participation by adding rewards to relatively lower ranks. Thus, the participation constraint also implies a more even distribution of rewards among the ranks for salespeople who are risk averse. We summarize the results in Proposition 3.

PROPOSITION 3. *With risk-averse salespeople, the optimal number of winners must exceed one. Regardless of whether the participation constraint is met or not, the optimal num-*

ber of winners increases and the size of interrank spread decreases with the degree of risk aversion.

Sales Uncertainty. Note that the indicator of sales uncertainty (β) does not appear in Condition (20). The reason behind this result is that both ψ_j and ψ_k (the marginal probability to reach a rank resulting from an increase in reward allocated to the rank) are proportional to $1/\beta$. The uncertainty parameter β then cancels out in the ratio of ψ_j and ψ_k. This term therefore does not have an impact when we compare the rewards allocated to different ranks. As a result, the characteristics of an effort-maximizing sales contest are independent of the size of the sales uncertainty.

The role that sales uncertainty (β) plays in the design of a sales contest is through the salesperson's incentive to participate. According to Equation (19), selling effort will be lower with greater sales uncertainty. All else being equal, greater sales uncertainty means a lower effort e^* and hence a lower cost of participation $c(e^*)$ being incurred. Therefore, greater sales uncertainty makes the salespeople more likely to find enough incentive to participate in the effort-maximizing sales contest. Moreover, according to Equation (21), with greater sales uncertainty the difference in marginal incentive to participate between different ranks (j and k) becomes smaller. As a result, larger sales uncertainty β reduces the firm's incentive to allocate more rewards to lower ranks to increase the incentive of participation. In this case, it is more profitable for the firm to allocate larger rewards to higher ranks and increase the interrank spread. Given the fixed total budget, this also implies fewer winners. We summarize the above discussion in Proposition 4.

Proposition 4.

4.1. *When the effort-maximizing contest design meets the participation constraint, the level of sales uncertainty does not affect the contest design.*

4.2. *When the effort-maximizing contest design does not satisfy the participation constraint, the optimal number of winners decreases and the interrank spread increases with the level of sales uncertainty.*

Number of Participants. According to Equation (20), when the number of salespeople is larger (N), the equilibrium ratio of marginal utilities between two consecutive ranks, $u'(R_j)/u'(R_{j+1}) = (N - 2j - 1)/(N - 2j + 1)$, becomes larger and closer to one. Given a fixed degree of risk aversion, the interrank spread will be smaller, and hence larger number of ranks will be awarded. The intuition is that when there are more contestants, both the probabilities and marginal probabilities to win any ranks become lower. The difference in the marginal probabilities of winning any consecutive ranks also becomes smaller. In other words, effort plays a smaller role in determining a salesperson's rank.

The implications for the design of sales contests from Equation (21) follow the same line of reasoning as above. According to Equation (21), the marginal increase in the incentive of participation resulting from a reallocation of unit reward from rank j to lower rank k ($R_j > R_k$) decreases with the number of participants (N). Consequently, it is more difficult to provide sufficient incentives to participate. For the same effectiveness of producing the expected value of participation, given R_j, it requires an allocation to larger $u'(R_k)$ and hence a smaller R_k. Intuitively, with a larger number of participants and a smaller chance to reach top ranks, it becomes increasingly more difficult to satisfy salespeople's participation constraint if rewards are limited to the top ranks. To ensure that salespeople will participate in the sales contest, the firm will have to allocate more toward lower ranks. As a result, the interrank spread will be small, and more salespeople will have the chance to win a prize. We summarize our results in Proposition 5.

Proposition 5. *Regardless of whether the effort-maximizing sales contest meets the participation constraint or not, the optimal number of winners increases and the size of interrank spread decreases with greater number of contestants.*

Interrank Spreads. So far, the discussion of the factor influencing the optimal interrank spreads has focused on whether the spread should be small or large. Another important question is whether the interrank spread should be larger with high ranks or lower ranks. For example, if the firm was allocating \$10,000 between three top ranks, which of the follow-

Table 1 Risk Tolerance and Rewards Distribution

Utility Function	$u(x)$	$T(x)$	$T'(x)$	Distribution of Rewards
Power function	$x^\rho/\rho^{(0<\rho<1)}$	$x/(1-\rho)$	$1/(1-\rho) > 1$	Convex
Log function	Log(x)	X	1	Linear
Exponential function	$a - be^{-\gamma x}$ $(a, b, \gamma > 0)$	$1/\gamma$	$0 < 1$	Concave

ing three designs should be used: {R_1 = \$7,000, R_2 = \$2,000, R_3 = \$1,000}, {R_1 = \$4,333, R_2 = \$3,333, R_3 = \$2,333}, or {R_1 = \$5,000, R_2 = \$4,000, R_3 = \$1,000}? Note that the spread between the top two ranks in the first alternative is \$5,000 and then decreases to \$1,000, is the same \$1,000 in the second alternative, while in the third alternative, the spread increases from \$1,000 to \$3,000. When interrank spread is larger, equal, or smaller with higher ranks, the distribution of rewards among winning salespeople will be either convex, linear, or concave as the marginal increase of rewards between ranks become greater, equal, or smaller at higher ranks, respectively.

We find that the distribution of rewards (convex, linear, or concave) depends on salespeople's degree of risk aversion. Following the approach adopted by Basu et al. (1985), we define the degree of risk aversion by the salesperson's risk tolerance, which is the inverse of Arrow-Pratt's definition of risk aversion. More specifically, suppose risk tolerance is $T(x)$ and degree of risk aversion is $\tau(x)$ when income is x, then

$$T(x) = \frac{1}{\tau(x)} = -\frac{u''(x)}{u'(x)}. \tag{22}$$

We investigate the implications of risk tolerance on distribution of rewards through effort-maximizing Condition (20). We find that the rewards distribution of an effort-maximizing sales contest should be convex, linear, or concave across the ranks when $T'(x)$, the rate of risk tolerance, is greater, equal to, or less than unity, respectively. The intuition is that when salespeople have higher marginal risk tolerance, they value larger rewards more, and hence the firm is able to induce more selling effort by using larger prizes for top ranks and larger interrank spread. In Table 1, we provide one example of utility function specification for convex, linear, and concave distribution, respectively. For example, when salespeople's utility function follows a log-function ($u(x) = \log(x)$), the rate of change in risk tolerance is equal to one. Then the effort-maximizing sales contest as given by (20) has equal interrank spread ($d_1 = d_2 = \cdots = d_{m-1}$). When salespeople's utility function follows exponential function (constant risk aversion), rate of change in risk tolerance is zero, and hence is smaller than unity. Then we find that Condition (20) requires smaller interrank spread for higher ranks ($d_1 < d_2 < \cdots < d_{m-1}$). Finally, when salespeople's utility function follows power function, the rate of change in risk tolerance is larger than unity. Then we find that Condition (20) requires larger interrank spread for higher ranks ($d_1 > d_2 > \cdots > d_{m-1}$). See Appendix 2 for details of the analysis. Intuitively, compared with higher risk tolerance, salespeople with smaller risk tolerance prefer smaller variance in income and hence prefer the concave reward distributions leading to higher levels of effort. We summarize these results in the following proposition.

PROPOSITION 6. *The optimal interrank spread decreases with rank when rate of risk tolerance is greater than 1,is constant across the ranks when rate of risk tolerance is 1, and increases with rank when rate of risk tolerance is lower than 1.*

Multiple-Winners Design. According to Equation (20), an effort-maximizing sales contest requires positive interrank spreads. Note that for any two consecutive ranks j and $j + 1$, ψ_j is always strictly larger than ψ_{j+1}. To satisfy Equation (20), it is then necessary that $u'(R_j) < u'(R_{j+1})$ which implies interrank spread $d_j = R_j - R_{j+1} > 0$. To see the intuition, consider the case of a contestant in the MW format. The contestant who obtains the highest utility is the one who spends the amount of effort just enough to achieve the mth rank because any higher rank does not offer extra utility. When there is a spread, however, this is not the case as there are higher rewards associated with higher ranks. If salespeople are more risk averse, the benefit of the extra reward diminishes, leading to the MW format effort-inducing ability to be more similar to a design with a spread.

Theoretically, the discussion above recommends that when there is more than one winner, sales contests should have positive interrank spreads. This leads to the question of why the Multiple-Winners design is so prevalent. We speculate that from the practical standpoint, having the contests with identical prizes for all winners (MW) is easier to implement than contests with spreads. Multiple-Winner formats may be logistically easier to manage, easily communicable, and may be perceived as more equitable. Also, when the rewards are merchandise or travel, the cost of rewards for the firm may decrease while keeping the value to the winners the same. By adopting MW rather than a design with positive spreads, a firm may be able to reduce the implementation costs. If a firm decides to adopt a MW format, the interrank spread equals zero, and the number of winners (m) thus becomes the only decision. The optimal number of winners that induces maximal amount of effort should therefore be

$$\mathrm{Max}\left\{1,\ m^* \text{ such that } (N - 2m^*)u\left(\frac{R}{m^*}\right) - \frac{R}{m^*}u'\left(\frac{R}{m^*}\right)(N - m^*) = 0\right\}. \tag{23}$$

See Appendix 3 for the derivation. The optimal solution to the number of winners given by (23) increases with the degree of risk aversion and the number of participants. Similar to the logic discussed earlier, when salespeople are more risk averse, marginal utility decreases faster. Therefore, it requires a large m^* to satisfy the equilibrium condition. From Equation (23), we can also see the impact of N. When the number of salespeople participating in the contest increases, the ratio of $(N - 2m)/(N - m)$ becomes larger and closer to one. Given the same risk aversion, Equation (23) then requires a bigger ratio between $u(R/m^*)$ and $u'(R/m^*)R/m^*$, which in turn requires a large m^* to satisfy the equilibrium condition. Intuitively, a larger number of participants implies a weaker relationship between effort and outcomes. Therefore, it becomes more risky to invest selling effort in increasing sales.

Finally, according to Equation (23), the optimal number of winners in inducing selling efforts does not depend on sales uncertainty. However, more uncertainty leads to lower amount of effort due to the weaker connection between sales and ranking. As a result, the participation constraint is more likely to be satisfied with any given m. Consequently, the effort-maximizing number of winners (m^*) is more likely to be the optimal solution as larger sales uncertainty implies lower equilibrium efforts. We state the results of the above discussion in the next proposition.

Proposition 7. *The optimal number of winners in the Multiple-Winners design increases with salespeople's risk aversion and the number of participants, but decreases with sales uncertainty.*

Numerical Example. We conclude the discussion with a numerical example to illustrate our results. We let $c(e) = \frac{1}{2}e^2$, $s(e) = e$, and $\bar{u} = 0$. We consider a contest with a total reward budget R equal to \$100 and number of salespeople N equal to 6 and do the analysis for both the power function as well as the exponential utility function (with $a = b = 1$) shown in Table 1. We solve the constrained nonlinear program problem (P1) using simulation and present some of the results for the optimal sales contests in Table 2. The parameters ρ and γ represent risk aversion that is decreasing from left to right. We assume that the sales uncertainty parameter β takes the value of 1 or 0.84, where the value of 0.84 indicates smaller sales uncertainty. The numbers in the parenthesis represent the optimal amount of rewards that should be given to each rank. To illustrate, assuming the power utility function, when $\beta = 1$ and there are 6 risk-averse salespeople with $\rho = 0.5$, the optimal sales contest is one in which the salesperson with the highest rank is awarded \$71, the second-ranked salesperson gets \$26, and the third-ranked salesperson is given \$3. The results show that only the top three salespeople should be rewarded.

First, consistent with Proposition 1, the numerical example shows that the number of winners is no more than three (half the number of participants).

Table 2 Optimal Sales Contest Design (R_1, R_2, R_3, R_4, R_5, R_6)

		Power Utility Function (ρ)		
β	N	0.5	0.8	1
1	6	(71, 26, 3, 0, 0, 0)	(93, 7, 0, 0, 0, 0)	(100, 0, 0, 0, 0, 0)
1	5	(80, 20, 0, 0, 0)	(97, 3, 0, 0, 0)	(100, 0, 0, 0, 0)
0.84	6	(71, 26, 3, 0, 0, 0)	(93, 7, 0, 0, 0, 0)	(99, 1, 0, 0, 0, 0)
		Exponential Utility Function (γ)		
β	N	0.1	0.01	0.001
1	6	(41, 36, 24, 0, 0, 0)	(76, 24, 0, 0, 0, 0)	(100, 0, 0, 0, 0, 0)

Second, when salespeople are risk neutral ($\rho = 1$), the optimal sales contest is Winner-Take-All. Third, when salespeople are risk averse ($\rho = 0.8$ or 0.5), it is optimal to have multiple winners. Moreover, when ρ decreases from 0.8 to 0.5, the number of winners increases from 2 to 3, and interrank spread d_1 decreases from 86 to 45. Fourth, when the sales-uncertainty parameter β decreases from 1 to 0.84, the optimal sales contest does not change for $\rho = 0.8$ or 0.5. However, when the salespeople are risk neutral, the participation constraint is not satisfied. Then, consistent with Proposition 4.2, the number of winners increases and interrank spread decreases. Fifth, when the number of contestants decreases from 6 to 5, the optimal number of winners decreases and interrank spread between the rewards increases. For example, for $\rho = 0.5$, the number of winners decreases from 3 to 2 and d_1 increases from 45 to 60. Finally, consistent with Proposition 6, the distribution of positive rewards is convex with the power utility function (for example, when $\rho = 0.5$, $d_1 = 45 > d_2 = 23$) and concave with the exponential function (for example, when $\gamma = 0.1$, $d_1 = 5 < d_2 = 12$).

3.5. Contest Design Under Uniform Distribution

We now examine the results for optimal contest designs by assuming a uniform sales distribution. The density function of a uniformly distributed random variable in the interval of $[-V, V]$ is given by

$$f(\epsilon) = \begin{cases} \dfrac{1}{2V} & \text{if } \epsilon \in [-V, V] \\ 0 & \text{otherwise.} \end{cases}$$

With uniform distribution, the marginal change in the probability of achieving rank j is

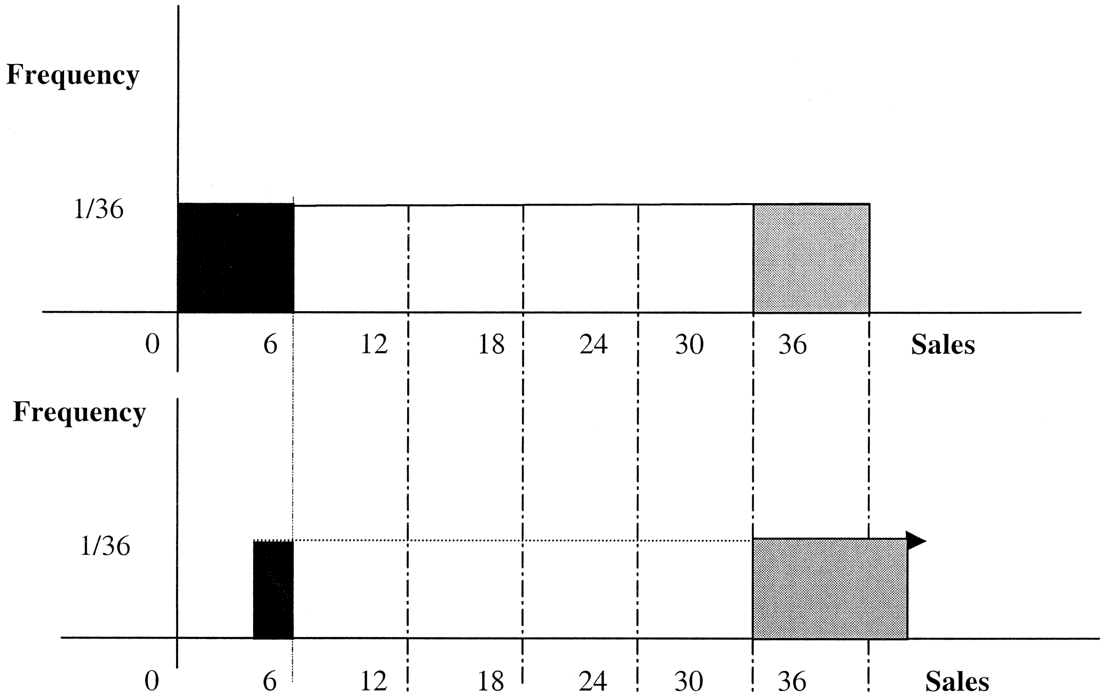
$$\frac{\partial \text{ prob}(r_i = R_j)}{\partial e_i}(e^*) = \begin{cases} \dfrac{s'(e)}{(2V)} & \text{if } j = 1 \\ -\dfrac{s'(e)}{(2V)} & \text{if } j = N \\ 0 & \text{otherwise.} \end{cases} \tag{24}$$

Equation (24) shows an important distinction from the case of the logistic distribution. Assuming that all other salespeople's efforts are fixed, if a salesperson unilaterally changes the level of effort, only the salesperson's probability of achieving the first or the last rank is altered. The chance that a salesperson moves into any rank between 2 and $N - 1$ is exactly the same.

To illustrate the intuition, suppose that each salesperson's sales volume is distributed uniformly from 0 to 36 (see the solid line in Figure 2).

We divide the interval [0, 36] into six equal-size smaller intervals, terminal points 6, 12, 18, 24, and 30 being the 16.67th, 33.33th, 50th, 66.67th, and 83.33th percentiles, respectively. Then the probability that a salesperson obtains the first rank is equal to the probability that a salesperson's sales volume falls into the interval [30, 36]. Similarly, the probability of being ranked second is equal to the probability of sales being within the interval [24, 30], and so on. Now if one of the salespeople *unilaterally* increases her selling effort by a small amount, the salesperson's sales distribution will shift slightly toward the right (the dotted line in the bottom panel). As a result, the probability that the salesperson's sales amount falls into interval [30, 36] increases, while the probability of sales falling into interval [0, 6] decreases by the same amount. However, the chance of the sales volume falling into all intermediate intervals (second, third, fourth, and fifth) does not change. As the distribution shifts toward the right, the chance of moving in and moving out of any of these interior intervals is identical. Therefore, only the probability of achieving the top and bottom ranks will be affected by a salesperson's marginal increase in effort. The probability of being ranked at any position except first and last does not

Figure 2 Increasing Effort Under Uniform Distribution



change with incremental effort. Substituting (24) into Equation (9) we obtain

$$\frac{s'(e^*)}{2V}(u(R_1) - u(R_N)) - c'(e^*) = 0. \tag{25}$$

Equation (25) indicates that when the uncertainty is uniformly distributed, equilibrium selling effort increases with the gap between top (R_1) and bottom reward (R_N). Rewards allocated to other ranks do not affect the equilibrium selling efforts. This characteristic is unique to the case with uniformly distributed sales. In contrast to the logistic distribution where awards to intermediate ranks influence effort levels, in the uniform distribution they do not. In Equation (25), rewards affect the salespeople's selling efforts through the difference in valuation of the top ($u(R_1)$) and bottom rewards ($u(R_N)$) only. Given a sales contest budget R, the value of the difference becomes maximum for risk-neutral salespeople and minimum for risk-averse salespeople. Therefore, the equilibrium selling effort decreases with salespeople's risk aversion. Finally, selling effort decreases with V, which measures the variance of the distribution.

With the effort-prizes relationship given by Equation (25), the effort-maximizing sales contest should be Winner-Take-All. From Equation (25), it is easy to see that only the awards offered to the top-ranked salesperson induce positive selling effort. Any award allocated to other ranks will either not change selling effort or decrease selling effort.

As in the case of logistically distributed sales, we now discuss the implications of the participation constraint on the sales contest design. We first study the factors that may affect the value that salespeople expect from participating in the Winner-Take-All contest. From Equation (25) we can infer that effort decreases with risk aversion and sales uncertainty. Because the expected utilities from rewards is $u(R)/N$, clearly the value of participation, and hence WTA, is more likely to work with a smaller number of contestants, lower risk aversion, or larger sales uncertainty. When the Winner-Take-All sales contest does not satisfy the participation constraint, we find that the optimal design will contain a large prize R_1^* and many small prizes ($R_2^* = R_3^* = \cdots = R_{N-1}^*$), the amount of small rewards being such that is just enough to satisfy the participation constraint:

$$\frac{u(R_1^*) + (N-2)u\left(\dfrac{R - R_1^*}{N-2}\right)}{N} - c(e^* | R_1^*) = \bar{u}. \quad (26)$$

(See Appendix 4 for proof.) Note that R_1 is the only reward that contributes to salespeople's efforts. Any R_1 below R_1^* given by (26) is not optimal because the firm can increase its profit with higher R_1^*.

As in the case of logistic distribution, we examine the impact of salespeople's risk preference, number of sales contest participants (N), and variance (V). First, regardless of salespeople's risk preference, the effort-maximizing sales contest design is always to have the winner take all. However, when salespeople are more risk averse, to satisfy the participation constraint it takes a more even (concave) distribution of prizes, and hence a smaller spread between large (top) and small (second) prizes, to satisfy the participation constraint.

Second, according to Equation (25), the equilibrium amount of effort decreases with V (uncertainty). That is, when uncertainty plays a larger role in determining sales, the relation between effort and sales becomes weaker. In this situation, salespeople will expend a smaller amount of effort. The implication is that, all else being equal, greater uncertainty implies a smaller amount of effort, the expected value increases with V, and hence salespeople are more likely to participate. Therefore, more uncertainty implies that the firm is more likely to offer WTA. If a contest with multiple winners using spreads between ranks is necessary, then larger V implies a bigger spread.

Third, according to Equation (25), a salesperson's equilibrium effort does not change with the number of salespeople participating in the contest. However, the chance to win any prize ($1/N$), and hence the expected utility from participation, will be lower with a larger number of participants. The implication for the design is that Winner-Take-All is still the effort-maximizing sales contest. However, given the contest design and hence the resulting effort, it is less likely to satisfy the participation condition when the number of salespeople increases. Thus, when the number of salespeople increases, the optimal design of the sales contest is more likely to have multiple winners. Moreover, the interrank spread between top and second place will also be smaller with larger number of participants. We state these results in the next proposition.

Proposition 8. *When the sales are uniformly distributed,*

8.1. The Winner-Take-All design is the optimal format if it satisfies the participation constraint.

8.2. If the Winner-Take-All design does not satisfy the participation constraint, the optimal design contains a big prize and many small prizes of the same amount. The spread between top and second prize decreases with the number of participants in the sales contest and risk aversion, but increases with sales uncertainty.

To summarize, when employees are risk neutral, the design implications from both the distributions are the same. However, when employees are risk averse, the implications for contest design do not converge. Uniform distribution recommends a single-winner (WTA) design format regardless of the extent of risk aversion, whereas logistic distribution recommends a single winner when salespeople are risk neutral, but multiple winners when they are risk averse.

4. Discussion and Conclusion

Firms are increasingly using sales contests as a motivating mechanism to induce greater effort from their salespeople. The objective of this paper is to provide guidelines for the optimal design of sales contests. Specifically, guidelines on the number of salespeople that should win the contest and the spread between reward levels are obtained. The contests that are examined are what we term the Rank-Order Tournament, where the winners are given rewards based on the rank order of sales performance achieved. Two special cases of Rank-Order Tournament are also investigated: a Multiple-Winner format where a prespecified number of winners share the reward equally, and a Winner-Take-All where the top-ranked

Table 3 Summary of Results

	Sales Uncertainty Distribution			
	Logistic Distribution		Uniform Distribution	
	Participation Constraint Satisfied	Participation Constraint Not Satisfied	Participation Constraint Satisfied	Participation Constraint Not Satisfied
Number of winners				
Maximum number of winners	Not more than half the sales force	May exceed half the sales force	Winner-Take-All	One big prize and many small prizes
Factors impacting number of winners				
Increasing risk aversion	Increase	Increase	No impact	No impact
Increasing number of contestants	Increase	Increase	No impact	No impact
Increasing sales uncertainty	No impact	Decrease	No impact	No impact
Factors impacting spread				
Increasing risk aversion	Decrease	Decrease	No impact	Decrease
Increasing number of contestants	Decrease	Decrease	No impact	Decrease
Increasing uncertainty	No impact	Increase	No impact	Increase

salesperson with the best performance gets the entire reward. In contest situations, we model salespeople as considering the utility of the reward from achieving one of the winning ranks in the contest and assessing the incremental chances of winning by exerting more effort. The analysis was done for two situations: where the reward is large enough for salespeople to participate in the effort-maximizing contest, and where the reward is insufficient to motivate participation.

The analysis shows that the factors impacting the design of the contest include the degree of risk aversion of the salespeople, the number of salespeople for whom the contest is conducted, and the degree of sales uncertainty in the environment (i.e., the strength of the relationship between effort and sales realized). In general, salespeople exert less effort when there are a larger number of participants and when the sales uncertainty is high. We find that the Rank-Order Tournament is superior to the Multiple-Winner contest format. In a Multiple-Winner format, the salesperson whose performance is just sufficient to win is better off than any of the other winners as he exerts the least effort to win, but obtains as high a reward as any other winners. Therefore, salespeople exert less effort in such a contest. Specific recommendations on contest design are obtained assuming that the sales follow either a logistic or uniform distribution. The results of the analysis are summarized in Table 3.

The analysis indicates that the total number of winners should be increased and the spread decreased when salespeople are increasingly risk averse. When salespeople are risk averse, they value higher rewards less. The spread should increase with ranks when rate of risk tolerance is high, and decrease with ranks when the rate of risk tolerance is lower. In the extreme case of risk neutrality, the optimal design is a Winner-Take-All format. We also conclude that because the probability of winning the contest decreases with an increasing number of contestants, the optimal number of winners should increase and the interrank spread decrease when there are a greater number of players.

If the firm does not allocate a large enough budget for salespeople to participate in an effort-maximizing sales contest, then increasing the number of winners and decreasing the spread is required to motivate the salespeople to participate, particularly when they are more risk averse or when there are many players. A counterintuitive result is that the number of winners should be reduced and the spread increased when sales uncertainty is high. The reason is that in a situation of high uncertainty, the salespeople exert less effort regardless of the type of contest.

Under the assumption of uniformly distributed sales, the recommendation is that (for a fixed amount of reward) a Winner-Take-All contest induces maximum efforts regardless of the level of risk aversion, number of players, or the degree of uncertainty. Due to the nature of the uniformly distributed sales, increasing effort leads to changes only in the probabilities of obtaining the first or last rank. Therefore, the entire incentive is based on the size of the difference between the reward given to the highest and lowest rank.

The recommendations for risk-neutral salespeople are essentially the same for both the logistic and uniform distribution assumptions. However, the results differ in the case in which the salesforce is risk averse. Under an assumption of uniformly distributed sales, the recommendation is that (for a fixed amount of reward) there should be as few winners as possible, whereas the logistic distribution recommends more winners. These results highlight that some of the conclusions drawn may be sensitive to the distributional assumptions.

The model guidelines are contingent on the firm allocating a fixed amount for the contest reward. The implications when the reward amount is changed are straightforward. If the firm increases the reward budget, then the effort-maximizing sales contests are more likely to provide enough incentives for participation and therefore, relatively, the number of winners can be decreased and the spread increased (for a given level of risk aversion and for a fixed number of contestants).

A major assumption made in the analysis is that the salesforce is homogeneous. Heterogeneity in a workforce can arise because of several reasons that can either factor in with the cost of effort differing for high-/low-ability workers, or because the sales response varies because of territorial or customers' differences. Heterogeneity in the salesforce is likely to make the contest less effective, as some of the players may elect not to participate or may exert less than optimal effort. For instance, the advantaged players will expend less effort, as their probability of winning will be high, and the players with a disadvantage may elect not to participate as their chances to win the contest will be low. If possible to implement, the optimal option for the firm is to conduct contests for homogeneous subsets of salespeople. For example, Merrill Lynch designs separate contests for brokers who have more than six years of experience and for those who have five or less (*Wall Street Letter* 1995). If conducting a contest for homogeneous players is not possible, then the firm needs to design the contest based on whether the objective of the contest is to motivate the high-ability/high-advantage players or to motivate the low-ability/low-advantage salespeople. If the objective is to motivate the advantaged and/or high-ability players, then the contest should have relatively fewer winners and larger spreads. On the other hand, when the emphasis is on maximizing effort levels from the lower-ability and/or disadvantaged players, the contest should be designed with more winners and lower spreads. If the objective of the contest is to equally motivate the entire salesforce, then we speculate that the number of winners should be increased and the spread reduced with increasing levels of heterogeneity.

An attractive aspect of the model developed in this paper is that the guidelines developed can easily be put into practice. For our model to be implemented, managers need information on the key factors that impact the contest design. First, managers need to assess the level of risk aversion of its employees. Risk aversion can be measured using scales that have been developed in the literature (e.g., Johnson and Schkade 1989). Second, managers need to know the number of employees participating in the contest. Third, the sales-uncertainty distribution needs to be determined. We believe that experienced managers will have a good feel for this aspect based on the specific industry in which they operate. For example, when salespeople are in an industry where average order size is small and the number of customers very large, then the sales are likely to follow a symmetric distribution like the logistic distribution. On the other hand, when a small number of units are sold to a very few customers (e.g., expensive industrial equipment), then the sales uncertainty is more likely to be uniformly distributed. It is important to note that while the model discussed was in the context of increasing sales, contests are routinely conducted by services-oriented firms where the rewards are based not only on sales, but also on the average quality or satisfaction or quality ratings. In these situations, we expect that the sales uncertainty follows a symmetric distribution. Finally, before implementation, managers need to determine whether the reward amount is sufficiently attractive to ensure participation. Again, managerial experience is required to assess whether this condition is met. Factors such as the relative size of the reward amount compared to other components of the compensation, such as salary or commission, are likely to influence the perceptions of whether the reward amount is large enough.

There are two issues regarding the empirical testing of the model. The two-person models developed in the literature (e.g., Bull et al. 1987, Weigelt et al. 1989) have been subjected to experimental testing. Given the several confounds likely in any field test, we believe that experimental methodology is the appropriate approach. The face validity of our model can be easily determined through experimental procedures.[3] Another important empirical issue is to compare the normative predictions of the model with the contest designs used in practice. Such empirical tests can be conducted following the procedures used by other researchers (e.g., Anderson 1985, Lal et al. 1994).

This paper, of course, has not examined all possible forms of sales contests. A commonly observed format is where the ROT design is modified to include multiple winners. In this form, the top reward is given to multiple winners, followed by a second award that is of lower value but also is given to multiple winners (e.g., in Mary Kay Cosmetics, multiple winners win the highest award of a pink Cadillac, and there are several second- and third-place winners). This format is probably practiced due to implementation issues. Although theoretically a ROT design with higher awards for better ranks is preferred, the implementation costs of such a contest are also likely to be high. Managing a ROT design with different award levels for each rank would lead to both higher costs (particularly if awards are indivisible) as well as increased problems in measuring performance levels. In addition, the incentive scheme may be difficult to communicate to the salespeople. Another particular form not considered is a variation of a WTA where employees are awarded entries to a lottery. The number of entries the salesperson gets increases with sales levels, leading to a higher probability of winning. Recently, a salesperson who had 1,900 entries out of a possible 1.8 million won \$1 million in a contest conducted by American Investors Life Insurance (*The Cincinnati Enquirer* 1997). We leave these other designs for future research.

[3]Interested readers can contact the authors for details on the experimental tests of some aspects of the model.

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Appendix 1: Derivation of Equation (19)

The probability and cumulative distribution functions of a logistically distributed random variable with mean at 0 are

$$f(x) = \frac{1}{\beta}\frac{\exp(-x/\beta)}{[1 + \exp(-x/\beta)]^2} \quad \text{and} \quad F(x) = \frac{1}{1 + \exp(-x/\beta)},$$

respectively. For any integer $m \geq 2$, $n \geq 2$, and $m > n$, the distribution functions satisfy:

$$\int_x [1 + \exp(-x/\beta)]^{-m}[\exp(-x/\beta)]^n \, dx$$
$$= \frac{n-1}{m-1}\int_x [1 + \exp(-x/\beta)]^{-(m-1)}[\exp(-x/\beta)]^{(n-1)} \, dx. \quad \text{(A1)}$$

Then, for any $k = 2, 3, \ldots, N$, by repeating the above transformation, we obtain

$$\int_x [1 + \exp(-x/\beta)]^{-(N+2)}[\exp(-x/\beta)]^k \, dx = \frac{(k-1)!(N-k+1)!}{(N+1)!}\beta. \qquad \text{(A2)}$$

Now we are able to simplify ψ_j in Equation (9) as follows:

$$\begin{aligned}
\psi_j &= \int_\epsilon \binom{N-1}{j-1}[1-F(\epsilon)]^{j-2}F^{N-j-1}(\epsilon)f^2(\epsilon)[(N-j)-(N-1)F(\epsilon)]\, d\epsilon \\
&= \binom{N-1}{j-1}(N-j)\int_\epsilon [1-F(\epsilon)]^{j-2}F^{N-j-1}(\epsilon)f^2(\epsilon)\, d\epsilon \\
&\quad - \binom{N-1}{j-1}(N-1)\int_\epsilon [1-F(\epsilon)]^{j-2}F^{N-j}(\epsilon)f^2(\epsilon)\, d\epsilon \\
&= \binom{N-1}{j-1}\left[\frac{(N-j)}{\beta^2}\int_\epsilon\left[1+\exp\left(-\frac{\epsilon}{\beta}\right)\right]^{N+1}\left[\exp\left(-\frac{\epsilon}{\beta}\right)\right]^j d\epsilon \right.\\
&\qquad \left. - \frac{(N-1)}{\beta^2}\int_\epsilon\left[1+\exp\left(-\frac{\epsilon}{\beta}\right)\right]^{N+2}\left[\exp\left(-\frac{\epsilon}{\beta}\right)\right]^j d\epsilon\right] \\
&= \binom{N-1}{j-1} \\
&\quad \times \left[\frac{(N-j)}{\beta}\frac{(j-1)!(N-j)!}{N!} - \frac{(N-1)}{\beta}\frac{(j-1)!(N-j+1)!}{(N+1)!}\right] \\
&= \frac{1}{\beta}\frac{N-2j+1}{N(N+1)}. \qquad \text{(A3)}
\end{aligned}$$

Substituting ψ_j in Equation (9) with (A3), we have

$$s'(e^*)\sum_{j=1}^{N} u(R_j)\frac{N-2j+1}{N(N+1)}\frac{1}{\beta} - c'(e^*) = 0. \qquad \text{(A4)}$$

Appendix 2: Interrank Spread and Risk Tolerance

Power function ($u(x) = 1/\rho x^\rho$ $(0 < \rho < 1)$ and $u'(x) = x^{\rho-1}$).
For two consecutive ranks, Equation (20) becomes

$$\frac{(R_j)^{\rho-1}}{(R_{j+1})^{\rho-1}} = \frac{N-2j-1}{N-2j+1} \quad \text{and} \quad \frac{(R_{j-1})^{\rho-1}}{(R_j)^{\rho-1}} = \frac{N-2j+1}{N-2j+3}.$$

From these two equations we can derive interrank spreads

$$R_j - R_{j+1} = \left[1 - \left(\frac{N-2j+1}{N-2j-1}\right)^{1/(\rho-1)}\right]R_j \quad \text{and}$$
$$R_{j-1} - R_j = \left[\left(\frac{N-2j+1}{N-2j+3}\right)^{1/(\rho-1)} - 1\right]R_j.$$

Because $0 < \rho < 1$, we have $d_j < d_{j-1}$.

Log function ($u(x) = \text{Log}(x)$ and $u'(x) = 1/x$).
For two consecutive ranks, Equation (20) becomes

$$\frac{R_{j+1}}{R_j} = \frac{N-2j-1}{N-2j+1} \quad \text{and} \quad \frac{R_j}{R_{j-1}} = \frac{N-2j+1}{N-2j+3}.$$

We can rewrite these two equations as

$$\frac{(R_j - R_{j+1})}{R_j} = \frac{2}{N-2j+1} \quad \text{and} \quad \frac{(R_{j-1} - R_j)}{R_j} = \frac{2}{N-2j+1},$$

respectively. From these two equations we can conclude that

$$R_j - R_{j+1} = R_{j-1} - R_j = \frac{2R_j}{N-2j+1}.$$

Thus the interrank spread $d_j = d_{j-1}$.

Exponential function ($u(x) = a - be^{-\gamma x}$ $(a, b, \gamma > 0)$ and $u'(x) = b\gamma e^{-\gamma x}$).
For two consecutive ranks, Equation (20) becomes

$$\frac{\gamma b e^{-\gamma R_j}}{\gamma b e^{-\gamma R_{j+1}}} = \frac{N-2j-1}{N-2j+1}.$$

which is equivalent to

$$R_j - R_{j+1} = \frac{1}{\gamma}\ln\left(1 + \frac{2}{N-2j-1}\right),$$

which increases with j. Thus interrank spread $d_j > d_{j-1}$.

Appendix 3. Optimal Multiple-Winner Sales Contest

Corresponding to Problem (P1) for a rank-order tournament, we formulate Problem (P2) for MW contests.

$$\max_m \pi = N\delta \cdot s(e^*) - R \qquad \text{(P2)}$$

$$\text{s.t.} \quad \left(\sum_{j=1}^{m}\psi_j\right)u\left(\frac{R}{m}\right)s'(e^*) - c'(e^*) = 0, \qquad \text{(A5)}$$

$$\frac{m}{N}u\left(\frac{R}{m}\right) - c(e^*) \geq \bar{u}, \qquad \text{(A6)}$$

$$R_j \geq 0. \qquad \text{(A7)}$$

With sales following logistic distribution, we have

$$\sum_{j=1}^{m}\psi_j = \sum_{j=1}^{m}\frac{N-2j+1}{N(N+1)\beta} = \frac{m(N-m)}{N(N+1)\beta}.$$

We can first analyze the optimal number of winners in the effort-maximizing sales contests. Taking the first-order condition of Equation (A5) with respect to m, we get

$$\frac{\partial e^*}{\partial m} = \frac{\dfrac{s'(e^*)}{N(N+1)\beta}\left[(N-2m)u\left(\dfrac{R}{m}\right) - \dfrac{R}{m}u'(R_j)(N-m)\right]}{c''(e^*) - \dfrac{m(N-m)}{N(N+1)\beta}u\left(\dfrac{R}{m}\right)s''(e^*)}. \qquad \text{(A8)}$$

The optimal number of winners that induces maximal amount of effort should therefore be

$$\text{Max}\left\{1,\ m^* \text{ such that } (N-2m^*)u\left(\frac{R}{m^*}\right) - \frac{R}{m^*}u'\left(\frac{R}{m}\right)(N-m^*) = 0\right\}. \qquad \text{(A9)}$$

Appendix 4. Optimal Sales Contest Under Uniformly Distributed Sales

If the Winner-Take-All design does not meet the participation constraint, the Lagrange multiplier λ of Equation (15) is greater than zero. We now analyze equilibrium Condition (15), allocating reward dR_j from top prize R_1 to R_j ($dR_1 + dR_j = 0$, $j = 2, 3, \ldots, N-1$):

$$\frac{dL}{dR_j} = N\delta \cdot s'(e^*)\frac{de^*}{dR_j} + \lambda\left[\frac{1}{N}u'(R_j^*) - \frac{1}{N}u'(R_1^*) - c'(e^*)\frac{de^*}{dR_j}\right] = 0, \quad \text{(A10)}$$

where de^*/dR_j is independent of R_j according to (25). Since the first-order condition (A10) is identical for all ranks from 2nd to ($N-1$)th, the optimal design of sales contest should offer the same amount of rewards to all these levels; that is, $R_2^* = R_3^* = \cdots = R_{N-1}^*$.

References

Anderson, E. 1985. The salesperson as outside agent or employee: A transaction cost analysis. *Marketing Sci.* **4**(1) 234–254.

Arnold, B. C., N. Balakrishnan, H. N. Nagaraja. 1992. *A First Course in Order Statistics.* John Wiley & Sons, Inc., New York.

Basu, A. K., R. Lal, V. Srinivasan, R. Staelin. 1985. Salesforce compensation plans: An agency theoretic perspective. *Marketing Sci.* **4**(4) 267–291.

Bull, C., A. Schotter, K. Weigelt. 1987. Tournaments and piece rates: An experimental study. *J. Political Econom.* **95**(1) 1–33.

Business Wire. 1997 Toshiba awards top resellers with $40,000 cash and other prizes in wavelength reseller contest. (November 10).

Chrapek, D. 1989. Sales incentives expenditures top $7.8 billion mark in 1988. *Premium Incentive Bus.* (June 3.)

Churchill, G. A., N. M. Ford, O. Walker. 1993. *Sales Force Management: Planning, Implementation and Control.* Richard D. Irwin, Homewood, IL.

Ehrenberg, R. G., M. L. Bognanno. 1990. Do tournaments have incentive effects? *J. Political Econom.* **98**(6) 1307–1324.

Freedman, Robert J. 1989. For more profitable sales, look beyond volume. *Sales Marketing Management* **141**(9) 50–53.

Gaba, A., A. Kalra. 1999. Risk behavior in response to quotas and contests. *Marketing Sci.* **18**(3) 417–434.

Green, J. R., N. L. Stokey. 1983. A comparison of tournaments and contracts. *J. Political Econom.* **91**(3) 349–364.

Johnson, E. J., D. A. Schkade. 1989. Bias in utility assessments: Further evidence and explanation. *Management Sci.* **35**(4) 406–424.

Kotler, P. 1997. *Marketing Management: Analysis, Planning, Implementation and Control,* 9th ed. Prentice-Hall.

Knoeber, C. R., W. N. Thurman. 1994. Testing the theory and tournaments: An empirical analysis of broiler production. *J. Labor Econom.* **12**(2) 155–179.

Lal, R., D. Outland, R. Staelin. 1994. Salesforce compensation plans: An individual-level analysis. *Marketing Lett.* **6**(1) 117–130.

Lazear, E. P. 1989. Pay equality and industrial politics. *J. Political Econom.* **97**(3) 561–580.

——, S. Rosen. 1981. Rank-order tournaments as optimum labor contracts. *J. Political Econom.* **89** 843–864.

Marchetti, Michele. 1998. Master motivators. *Sales Marketing Management.* **150**(4) 38–46.

Murphy, W. H., R. S. Sohi. 1995. Salespersons perceptions about sales contests: Towards a greater understanding. *Eur. J. Marketing* **29**(13) 42–66.

——, P. A. Dacin. 1998. Sales contests: A research agenda. *J. Personal Selling Sales Management* **XVIII**(1) 1–16.

Nalebuff, B., J. E. Stiglitz. 1983. Prizes and incentives: Toward a general theory of compensation and competition. *Bell J. Econom.* **14** 21–43.

O'Keeffe, M., W. K. Viscusi, R. J. Zeckhauser. 1984 Economic contests: Comparative reward schemes. *J. Labor Econom.* **2** 27–56.

Rosen, S. 1986. Prizes and incentives in elimination tournaments. *Amer. Econom. Rev.* **76**(4) 701–714.

Stanton, W. J., R. H. Buskirk, R. L. Spiro. 1991. *Management of a Sales Force.* Irwin, Burr Ridge, IL.

The Cincinnati Enquirer. 1997. Tristate business summary, p. C12.

Wall Street Letter. 1995. Merrill launches sales contest. November 6.

Weigelt, K., J. Dukerich, A. Schotter. 1989. Reactions to discrimination in an incentive pay compensation scheme: A game-theoretic approach. *Organ. Behavior Human Decision Processes* **44** 26–44.

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