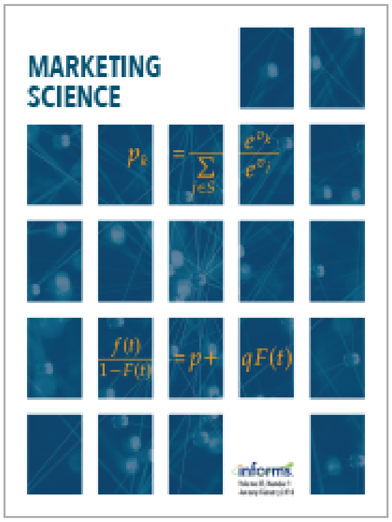


Marketing Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

The Strategic Role of Exchange Promotions

Preyas S. Desai, Devavrat Purohit, Bo Zhou

MARKETING SCIENCE
Vol. 35, No. 1, January–February 2016, pp. 93–112
ISSN 0732-2399 (print) | ISSN 1526-548X (online)

http://dx.doi.org/10.1287/mksc.2015.0955

The Strategic Role of Exchange Promotions

Preyas S. Desai
Fuqua School of Business, Duke University, Durham, North Carolina 27708, desai@duke.edu

Devavrat Purohit
Fuqua School of Business, Duke University, Durham, North Carolina 27708, purohit@duke.edu

Bo Zhou
Robert H. Smith School of Business, University of Maryland, College Park, Maryland 20742, bzhou@rhsmith.umd.edu

An exchange promotion allows consumers to turn in an old good and receive a discount toward the purchase of a new product. The old good that is turned in can either be within the same category as the new good or it may be in a different category. For example, one can turn in an old CD player to count toward a new CD player (a within-category exchange or traditional trade-in) or toward a new television (a cross-category exchange). This paper studies both within-category and multicategory exchange promotions and analyzes their similarities and differences. In a competitive setting with two firms, we model exchange promotions and establish the equilibrium outcomes. We find that categories in which consumers have a high level of waste aversion are more likely to have multicategory exchange promotions rather than within-category or no promotions. Multicategory exchange promotions can increase both consumers' replacement purchases and their new purchases. Interestingly, we also find that strategic considerations can lead to a prisoner's dilemma outcome in which neither firm offers any kind of exchange promotion. However, waste aversion and multicategory exchange promotions can give firms stronger incentives to get out of the prisoner's dilemma outcome.

Keywords: behavioral economics; durable goods; emerging markets; marketing strategy; waste aversion; promotion; price discrimination; trade-in; segmentation

History: Received: December 26, 2012; accepted: July 6, 2015; Jagmohan Raju served as the guest editor-in-chief and John Zhang served as associate editor for this article.

1. Introduction

Marketing researchers have long pointed to the important role of promotions as an effective means of price discrimination. For example, consumers who are willing to incur the cost of using coupons pay a lower price than consumers who do not like to use coupons. Recently, we saw the emergence of a new form of promotion that allows consumers to exchange an old item and receive a discount toward a new item. There are two ways in which this type of promotion is novel. First, the old item may still be in use, and second, the old and new items may be in different categories. Although there has been considerable research on within-category exchange promotions (or traditional trade-ins), the idea of cross-category exchange promotions remains to be explored. In this paper, we model both within- and cross-category exchange promotions and analyze their strategic roles. The central question we address is, in a competitive market, is it optimal for firms to offer an exchange promotion, and if so, what type of exchange promotion? We address this question and establish conditions under which an exchange promotion is likely to be observed.

Although exchange promotions have been common in China and India for several years, they are only now becoming more popular in the United States. In 2009, China had a large-scale exchange promotion that allowed consumers to exchange old televisions, laundry machines, personal computers, air conditioners, and refrigerators for new goods, where the old and new goods did not have to be in the same category. In India, the Big Bazaar Super Center allows consumers to exchange unwanted old products (e.g., electronics, utensils, furniture, plasticware, newspapers) for new products sold at the store. And in the United States, HP has an exchange promotion that allows consumers to turn in their old electronics, where the old and new items need not be in the same category.[1] Similarly, Staples and Best Buy have their own multicategory trade-in and recycling promotions that allow consumers to exchange old electronics for store gift cards.

Given the growing popularity of exchange promotions, it is important to understand the theoretical

[1] See http://tech.sina.com.cn/e/focus/jdyjhx09/, http://big-bazaar.co.in/big-bazaar-promotional-schemes/, and http://www.hp.com/united-states/tradein/faq.html (accessed April 2015).

rationale for their use by a firm in a competitive market. For starters, note that exchange promotions are typically associated with durable goods that consumers can trade in or exchange, e.g., cars and consumer electronics. If the firm allows exchanges, then consumers who trade in a product that they are currently using have to trade off the costs and benefits of turning in their old good. On the other hand, if the firm does not offer an exchange promotion, then consumers have to trade off the costs and benefits of placing functional items in storage. In making these trade-offs, we argue that consumers display a degree of waste aversion that plays out in two ways. First, waste aversion leads many consumers to store the old item at home instead of throwing it away. By storing, consumers are not wasting the old product even though they may still feel a bit wasteful because the item is not being utilized (Arkes and Blumer 1985, Arkes 1996). Second, if consumers are subsequently able to utilize this stored product in a meaningful way, they can experience a gain in utility. In this paper, we specifically model these aspects of waste aversion to understand its impact on optimal firm strategies.

Most of the research on exchange promotions has focused on traditional trade-ins from the economic perspective of price discrimination and the psychological perspective of turning in an old good. In the former case, the focus is on understanding how trade-ins can be used for intertemporal and third-degree price discrimination (e.g., Van Ackere and Reyniers 1993, 1995). More broadly, a trade-in policy can be seen as an intervention by the firm in the used goods market, which reduces inefficiencies arising from the "lemons" problem (e.g., Rao et al. 2009). From a psychological perspective, because of the potential for giving up the old good, trade-ins are influenced by consumers' mental accounting of the transaction (Thaler 1985, 1999). In particular, because trade-ins force the consumer to play the role of buyer as well as seller, the overall satisfaction is heavily influenced by the trade-in offer (Purohit 1995). Similarly, the cost of the trade-in can be economic as well as psychological because of the "loss" associated with retiring, possibly prematurely, and giving up the old good (Okada 2001). This attachment to the old good can have important economic effects such that consumers who trade in a product reveal a higher willingness to pay for the new product than do consumers who do not have a product to trade in (e.g., Zhu et al. 2008, Kim et al. 2011). Our paper differs from the trade-in literature in that we model not only within-category or traditional trade-in but also the broader cross-category exchange, whereby consumers turn in old products from a different category from the new product for some store credit. This key difference turns out to play an important role on firms' optimal pricing strategy.

Because an exchange is a special promotion that allows firms to price discriminate, our paper is also related to other instruments for price discrimination, mainly coupons. Essentially, coupons serve as an effective price discrimination mechanism because they lead to a lower price only for consumers who are willing to incur the cost of using coupons (e.g., Narasimhan 1984). On the surface, an exchange promotion is like a coupon promotion—consumers who are willing to go through the hassle of bringing in an old good can receive a lower price than consumers who are not willing to bring in an old good. However, an important difference from a coupon is that the item being exchanged may currently be in use and consumers may still value the item they are exchanging. Moreover, when competing firms use coupons, research has established that these promotions can result in a less profitable prisoner's dilemma outcome, especially when the coupons compete for a segment of switchers who are indifferent between the firms (e.g., Shaffer and Zhang 1995, 2002; Raju et al. 1994; Krishnan and Rao 1995). On the other hand, our paper identifies an exchange promotion as a new instrument of price discrimination and characterizes situations in which different types of exchange promotions can be profitable and not the outcome of a prisoner's dilemma.

We model two competing firms located at opposite ends of a Hotelling line. A specific proportion of consumers has a high valuation for the product, and the remaining proportion has a low valuation for the product. Consumers also differ in whether they own old goods that can potentially be used for an exchange—a fraction of consumers have these goods, and the remaining fraction do not have any goods for exchange. We find that when waste aversion is high, it is optimal for the firm to offer multicategory exchange promotions. When waste aversion is not that high, it is optimal to offer within-category exchange promotions, and when waste aversion is low, it is optimal not to offer any exchange promotion. Interestingly, we find that not offering an exchange promotion can be a prisoner's dilemma outcome.

The reminder of this paper is organized as follows. Section 2 lays out the assumptions of our model and the rules of the game. In §3, we present profits, prices, and, if applicable, the exchange discounts for three symmetric subgames. Section 4 discusses the equilibrium promotion outcomes, further characterizes equilibrium pricing, explores the implications of relaxing some of the main assumptions, and discusses other rationales for exchange promotions. Section 5 concludes.

2. Model

In this section, we detail our assumptions about the firms and the consumers.

2.1. Firms

We consider a market in which two firms, labeled A and B, are located at opposite ends of a linear city of unit length (Hotelling 1929). Because we do not model the coordination issues between channel members, these two firms can be either manufacturers or retailers. In all other respects, the firms are symmetric, and each sells a horizontally differentiated product, X. Without further loss of generality, we assume that the marginal cost of producing X is zero. Each firm i ($i = \text{A, B}$) has to choose a price and whether to offer a promotion. If firm i does not offer any promotion, then it simply chooses an optimal retail price P_i. However, if it offers a promotion, it chooses a retail price as well as the specifics of the promotion.

The promotion on which we focus is an exchange in which a consumer can turn in an "old" product and receive a specified discount toward the new item. The firm decides what type of old products it will accept and the level of the discount it will offer on the new purchase. In particular, we define two types of exchange promotions:

1. *Within-category exchange* (W): This is a promotion in which a consumer who is currently using an old product X turns it in to receive a discount of e_{iX} toward a new product X. This type of exchange is the equivalent of a traditional trade-in.

2. *Multicategory exchange* (M): In this case, consumers can use multiple categories—either an old X or another old product, but not both—and receive a discount for a new X. If they use an old X, it is a within-category exchange, and if they use another product, it is a cross-category exchange. For convenience, we refer to the other product as Y. In practice, there are many possible Y goods, and each could be associated with a unique discount. Here we take Y as a single representative product from a category other than X. Within multicategory exchanges, consumers who exchange an old X receive a discount of e_{iX}, and those who turn in an old Y receive a discount of e_{iY}.

Finally, to focus on demand-side issues, we assume that the firms' salvage value and disposal costs associated with used items are zero. If either of these were positive, they would have a straightforward effect—a positive salvage value imposes a lower "cost" of an exchange and the firm passes some of these savings to consumers in terms of a larger discount; similarly, a positive disposal cost (equivalent to a lower salvage value) leads to a smaller discount.[2]

2.2. Consumers

Below, we detail our assumptions about consumers, their aversion to waste, and their valuations for old and new goods, and then highlight the strategies available to them under various promotional choices by the firms. We begin with a discussion of waste aversion, a novel feature of our model.

2.2.1. Waste Aversion. Anecdotal evidence suggests that consumers have many items, especially electronics, that are no longer being used but are still stored at home. This propensity to store may arise from a general aversion to see themselves as wasteful (Zultan et al. 2010, Bolton and Alba 2012), but also from an underlying trait that leads them to retain consumption-related old items (Haws et al. 2012). Consumers can throw away products they are no longer using, but the fact that they choose to store some of them implies that for these products, storing dominates throwing. However, even though storage is the better option, it is only a temporary solution for dealing with the old products—the whole idea behind storage is that the product may ultimately be used by someone in a meaningful manner. Meaningful use can be the consumer beginning to use the product again, giving it away to someone who may find it useful, or, in our case, using it in an exchange promotion. Clearly, selling the product is also a meaningful use of the product; however, our focus is on products that are not easy to sell because potential buyers/users are hard to find. Consumers may still keep their old products in the hope that at some point in the future they will find a meaningful use for them. Once consumers can use the stored item in a meaningful manner, they feel that they are finally putting the old item to a good, productive use, and therefore they experience a psychological "gain." Note that this psychological gain occurs because the consumers find a *final* resolution for the problem of handling an old product.

We model this psychological gain by a *waste aversion* parameter, $a \geq 0$. Specifically, if consumers participate in an exchange promotion, then in addition to receiving a discount, e_i, from the exchange, they also experience a psychological gain of a. On the other hand, if the old product is not exchanged but sits idle at home, consumers feel that they have not resolved the disposal problem, so they do not experience the gain of a. Note that consumers could also get the psychological gain of a if they put the old product to any other meaningful use, e.g., giving it to a friend or sibling. Because a is a relative change of utility between two states, we operationalize this by labeling the gain from exchange to be a and labeling the loss from keeping the old product idle at zero (any other linear operationalization would lead to rescaling and the same equilibrium results). In other words, consumers who participate in an exchange promotion get a gross benefit of $(e_i + a)$; consumers who purchase a new product and move their old product into storage do not get the gross

[2] Additional details about these cases are available from the authors on request.

benefit from above.[3] Finally, note that a arises from an aversion to wasting products that are already in the endowment. If $a = 0$, there is no waste aversion, and consumers do not get any psychological gain from exchanging the product, whereas a higher a means that consumers are more waste averse, and they experience a bigger gain when they put the product to good use.

2.2.2. Product Valuations and Costs. Because of our interest in studying exchange promotions as a means of price discrimination, we develop a model in which consumers differ not only in terms of their valuations for the old and new goods in the market but also in terms of their incentives and abilities to participate in an exchange promotion. In terms of the new good, we assume that a $\lambda \in (0, 1)$ fraction of consumers have a "low" valuation of θ_L for the product, and the remaining fraction have a "high" valuation of θ_H ($\theta_H > \theta_L > 0$). Consumers within each segment are uniformly distributed on the $[0, 1]$ Hotelling line segment described earlier and incur the same transportation cost of $t > 0$ per unit length traveled. In §3, we detail further assumptions about how these segments are covered in equilibrium.

Now consider the valuation of old goods. If consumers have an old X that they are currently using, they derive a utility of $g\theta_j$, where θ_j is the valuation of the new good for consumer j ($j = H, L$), and $0 < g < 1$ represents the value retained by the old good and incorporates the good's physical deterioration as well as obsolescence.[4] By contrast, an old Y is not currently being used and provides no utility from product usage, and consumers' valuation comes simply from having the product in their endowment. Furthermore, because the old product is stored at home, consumers also incur a storage cost. Thus, consumers' net valuations for an old Y, v_j, are defined by $v_j = \bar{v}_j - k_j$, which reflects the base valuation, $\bar{v}_j$, less a storage cost, k_j. The net valuation captures the mental value consumers assign to the products in their endowment after accounting for the storage costs. We use the modifier "mental" to highlight the fact that this value may include subjective individual valuations and is not restricted to a market price for the used good.

As defined in §2.1, the distinction between the two exchange promotions is principally around whether the old product is currently in use (i.e., an old X) or is simply being stored in a drawer or a garage (i.e., an old Y). Importantly, the former currently provides utility from product usage, whereas the latter does not. This distinction also implies the following: if the consumer purchases a new X and moves the old X to storage, the consumer's valuation of the old X decreases from $g\theta_j$ to v_j, where $g\theta_j > v_j$.

If consumers participate in an exchange promotion, they incur a transaction cost, c_j ($j = H, L$, $c_H > c_L$).[5] This transaction cost reflects the hassle of finding and taking the old item to the store, the potential embarrassment from carting around old goods, etc. Because we are studying the potential price discrimination advantages of exchange promotions, it is necessary that any exchange promotion differentially affects high- and low-valuation consumers. We assume that the high-valuation consumers' transactions costs, c_H, are prohibitively high so that they will not find it in their interests to participate in the exchange.[6] This assumption is identical to the idea that high transaction cost consumers do not clip coupons (e.g., Shaffer and Zhang 1995), visit multiple stores (Lal and Rao 1997), or haggle over price (Desai and Purohit 2004). As a result, if an exchange promotion is offered, the H segment consumers will choose not to participate, whereas the L segment consumers can participate.[7] For simplicity and without any loss of generality, we set $c_L = 0$ in the following analysis.

Low-valuation consumers' abilities to participate in an exchange promotion depend on the old products they currently own. This leads to four subsegments of L consumers: those who have an old product X (segment L_X), an old product Y (segment L_Y), both an old X and an old Y (segment L_{XY}), or neither (segment L_N). To treat within-category and multicategory promotions symmetrically, we assume that an f fraction of consumers have an old X and an f fraction have an old Y. We assume that ownership of an old X and an old Y is independent so that an f^2 fraction have an old X and an old Y, and an $(1-f)^2$ fraction have no old product to exchange.[8]

In summary, consumers' participation in an exchange promotion depends on their ownership of a qualifying old good and their incentives to participate, the latter of which are determined by their valuations of the old

[3] We have also allowed a to vary with the product being exchanged and found that our central results remain unchanged but the algebra becomes more messy.

[4] We use $i = A, B$ to denote firms and $j = H, L$ to denote consumer segments throughout this paper.

[5] It is possible that consumers could always sell their old goods through a private sale. In this case, consumers would still experience transaction costs, which would now be even higher because of the difficulty of finding and verifying legitimate buyers. Allowing for a private sale does not affect our central results and is analyzed further in the appendix.

[6] Equivalently, we can assume that H-segment consumers' valuations for their old products are sufficiently high.

[7] In terms of exogenous parameters, this means

$$c_H \geq \frac{1}{2}(a - (1-g)\theta_L + \frac{2(2(1-f)^2\theta_L\lambda + (1-\lambda)t)}{1+\lambda(3-4f-4f(1-f)))} - v_H \quad \text{and}$$

$$c_L \leq \frac{1}{2}(a - (1-g)\theta_L + \frac{2(2(1-f)\theta_L\lambda + (1-\lambda)t)}{1+\lambda(3-4f))} - v_L.$$

[8] If we allow the proportions to vary for products X and Y, respectively, f_X and f_Y, we find that the algebra gets more complicated, but the central results and the equilibria do not change.

goods, applicable discounts, and transaction costs of participating in the promotion.

2.3. Strategies and Utilities

Conditional on the firms' promotional offerings, we now describe the strategies that consumers can follow and the utilities associated with these strategies. Because high-valuation consumers do not participate in an exchange promotion, we mainly need to clarify the options available to low-valuation consumers who may have old goods in their endowment. In this case, based on low-valuation consumers' ownership of old goods, there are four segments (L_X, L_Y, L_{XY}, and L_N), and each firm has three promotional strategies (no exchange, within-category, and multicategory). For ease of exposition, we focus our attention on low-valuation consumers who buy from firm A and outline their entire set of strategies and corresponding utilities (in terms of our full model, the consumer also chooses which firm to use). For example, if firm A offers only a within-category promotion, then owning an old Y does not affect consumers' decisions in any way; conversely, consumers who own an old X have to decide whether to participate in the promotion. Furthermore, if the firm offers a multicategory promotion, consumers who own both old X and old Y will also have to choose which old product to exchange.

Figure 1 lays out all of the strategies that consumers who purchase from firm A can potentially face. As an illustrative example, consider the case where the firms offer a multicategory promotion and low-valuation consumers who own old products X and Y face a potential replacement purchase decision. In this case, the low-valuation consumers' options related to purchasing from firm A are as follows:

1. Purchase a new product without participating in the exchange by keeping the old X and Y. Consumers' total utility of choosing this option is $u_L = \theta_L - P_A - t \cdot x + 2v_L$. In this case, they get utility from the new X of $(\theta_L - P_A - t \cdot x)$. Furthermore, the old X moves into storage and is no longer used, so its utility decreases from $g\theta_L$ to v_L. Therefore, the utility from the old X and old Y in storage yields another $2v_L$.
2. Participate in the exchange promotion by exchanging one of the old products, yielding a utility of $u_L = \theta_L - P_A - t \cdot x + e_{AX}$ (or e_{AY}) $+ a + v_L$. In this case, note that there is always one item left in storage, which contributes v_L utility, so the only difference in total utilities of using either the old X or the old Y in the promotion is simply due to the exchange discount: e_{AX} or e_{AY}.
3. Do not purchase the new unit; that is, use the old X and keep the old Y, yielding a total utility of $g\theta_L + v_L$.

In the example described above, the optimal end state for the consumer will be based on a comparison of several end-state utilities associated with buying from either firm A or B. For the consumer to purchase firm A's product by participating in this multicategory exchange promotion by using the old X, that strategy needs to dominate all of the other options detailed in Figure 1 and equivalent strategies for purchasing firm B's product. All of these trade-offs are detailed in the appendix.

Figure 1 Low-Valuation Consumers' Total Utilities in Each End State from Products X and Y Related to Purchasing from Firm A

Segment L_X	Buy new X and exchange old X	$\theta_L - P_A - t \cdot \bar{x} + e_{AX} + a$
	Buy new X and do not exchange old X	$\theta_L - P_A - t \cdot \bar{x} + v_L$
	Do not buy new X	$g\theta_L$
Segment L_Y	Buy new X and exchange old Y	$\theta_L - P_A - t \cdot \bar{x} + e_{AY} + a$
	Buy new X and do not exchange old Y	$\theta_L - P_A - t \cdot \bar{x} + v_L$
	Do not buy new X	v_L
Segment L_{XY}	Buy new X and exchange old X	$\theta_L - P_A - t \cdot \bar{x} + e_{AX} + a + v_L$
	Buy new X and exchange old Y	$\theta_L - P_A - t \cdot \bar{x} + e_{AY} + a + v_L$
	Buy new X and do not exchange	$\theta_L - P_A - t \cdot \bar{x} + 2v_L$
	Do not buy new X	$g\theta_L + v_L$
Segment L_N	Buy new X	$\theta_L - P_A - \bar{x}$
	Do not buy new X	0

Table 1 Firms' Strategies and Profits

Firm A/Firm B	No promotion	Within exchange	Multiexchange
No promotion	Case NN (Π^{NN}, Π^{NN})	Case NW (Π^{NW}, Π^{WN})	Case NM (Π^{NM}, Π^{MN})
Within exchange	Case WN (Π^{WN}, Π^{NW})	Case WW (Π^{WW}, Π^{WW})	Case WM (Π^{WM}, Π^{MW})
Multiexchange	Case MN (Π^{MN}, Π^{NM})	Case MW (Π^{MW}, Π^{WM})	Case MM (Π^{MM}, Π^{MM})

3. Firms' Strategies

In this section, we begin our analysis by laying out all of the possible outcomes in the game between firms A and B. When it comes to the promotion decisions, the options available to each firm are not to offer any promotion, offer an exchange promotion on old X (within-category), and offer an exchange promotion on old X and old Y (multicategory). As shown in Table 1, there can be six possible outcomes: neither firm offers any promotion (denoted as NN), both firms offer within-category exchange promotions (denoted as WW), both firms offer multicategory exchange promotions (denoted as MM), one firm offers within-category exchange and the other does not offer promotion (denoted as WN or NW), one firm offers multicategory exchange and the other does not offer any promotion (denoted as MN or NM), and one firm offers within-category exchange and the other offers multicategory exchange (denoted as WM or MW). After solving each of these six subgames, in the subsequent section, we analyze whether and when each of them is an equilibrium.

To capture the competition between the two firms and yet allow demand to be sensitive to price, we assume that the market is fully covered in the high-valuation segment and partially covered in the low-valuation segment.[9] This structure is appealing because it does not assume a fixed market size, and it also allows for the possibility that the promotion can enhance demand by pulling in "new" consumers from the low-valuation segment who would not have purchased in the absence of the promotion. In terms of consumer demand, it also allows a firm's own price effect to be stronger than the cross-price effect. Finally, because the parameters are such that only low-valuation consumers will participate in the exchange promotions, for simplifying the exposition, we drop the subscript L in valuation v_L in the subsequent analysis. We now present the analysis of three symmetric cases, NN, WW, and MM (refer to the appendix for details). The analysis of asymmetric cases is given in the appendix.

3.1. Case NN: Neither Firm Offers Promotion

Without promotional discounts, all of the H consumers and some of the L consumers purchase the new product. Firms A and B maximize their profits by simultaneously choosing their optimal prices, yielding

$$P_A^{NN} = P_B^{NN} = \frac{2\lambda((1-fg)\theta_L + fv) + t(1-\lambda)}{1+3\lambda}. \quad (1)$$

Although none of the consumers has an opportunity to exchange any old good in their possession, the functional value of the old X, $g\theta_L$, and the value of the old X after being put into storage, v, still affect retail prices. The reason is that consumers who have an old X in their possession have the option of replacing it with a new X unit from either firm. In equilibrium, some consumers engage in the replacement purchase, and put their old X to storage. Each firm's profit is given by

$$\Pi_i^{NN} = \frac{(1+\lambda)(t - t\lambda + 2(\theta_L + f(v - g\theta_L))\lambda)^2}{2t(1+3\lambda)^2}. \quad (2)$$

3.2. Case WW: Both Firms Offer Within-Category Exchange

When both firms offer within-category exchange promotions, only the low-valuation consumers who have the old product X can potentially trade in their old unit while purchasing a new unit. In this case, only the L_X and L_{XY} segments, which make up a total of f fraction of the L segment, can claim the exchange discount e_{iX}, and will do so if it is in their interest. These consumers pay an effective price of $(P_i - e_{iX})$, whereas all of the other consumers pay the full price P_i. The equilibrium prices and exchange discounts are as follows:

$$P_i^{WW} = \frac{t(1-\lambda) + 2(1-f)\theta_L\lambda}{1+(3-4f)\lambda}, \quad (3)$$

$$e_{iX}^{WW} = \frac{t(1-\lambda) + 2(1-f)\theta_L\lambda}{1+(3-4f)\lambda} - \frac{(1-g)\theta_L + a}{2}. \quad (4)$$

Though we discuss the details later in §4.1.3, we note here that because of price discrimination enabled by the exchange promotion, each firm charges a higher price in this case than in the previous case: $P_i^{WW} > P_i^{NN}$. In addition, because the old X is exchanged instead of being put into storage as in the case of NN, price P_i^{WW} is no longer a function of v. Recall that consumers who trade in their old units to participate in the exchange promotion represent replacement demand for the products. Because of the exchange discounts, the replacement demand from these consumers is higher compared to

[9] These assumptions require that $\theta_H > [((1-\lambda)t + 2\lambda(1-f)^2\theta_L)/(1+\lambda(3-4f(2-f))) + t - v_H]/(1-g)$ and $(1-\lambda)t/(1+\lambda(1-2f(2-f))) < \theta_L < t - a + v$.

their demand without any exchange promotion. By contrast, because of the higher price, the new demand from the remaining L consumers without the old X is lower compared to their demand when the firms do not offer a promotion.

Because the retail price increases with exchange discounts, this means that high-valuation consumers as well as low-valuation consumers without old X goods (i.e., segments L_N and L_Y) end up paying a higher price and thus subsidizing consumers who get promotional discounts. Interestingly, this highlights an inefficiency of within-category promotions in the sense that one group of low-valuation consumers end up subsidizing another group of low-valuation consumers.

Each firm's profit in this case is

$$\Pi_i^{WW} = \frac{f((1-g)\theta_L+a)^2\lambda}{4t} + \frac{(1+(1-2f)\lambda)(t(1-\lambda)+2(1-f)\theta_L\lambda)^2}{2t(1+(3-4f)\lambda)^2}. \quad (5)$$

3.3. Case MM: Both Firms Offer Multicategory Exchange

In this subsection, we look at the case where both firms offer multicategory exchange promotions. The two firms simultaneously choose their prices and exchange discounts for old X and old Y. This leads to the following prices and discounts:

$$P_i^{MM} = \frac{t(1-\lambda)+2(1-f)^2\theta_L\lambda}{1+(3-4(2-f)f)\lambda}, \quad (6)$$

$$e_{iX}^{MM} = \frac{t(1-\lambda)+2(1-f)^2\theta_L\lambda}{1+(3-4(2-f)f)\lambda} - \frac{(1-g)\theta_L+a}{2}, \quad (7)$$

$$e_{iY}^{MM} = \frac{t(1-\lambda)+2(1-f)^2\theta_L\lambda}{1+(3-4(2-f)f)\lambda} - \frac{\theta_L+a-v}{2}. \quad (8)$$

Note that among low-valuation consumers, only consumers in L_N segment do not have any old units and hence cannot participate in any exchange promotion offered by the firms. In addition, consumers who own both old X and old Y can potentially exchange either of them; in equilibrium, we see that it is optimal for L_{XY} consumers to exchange old X, not old Y, because $e_{iX}^{MM} > e_{iY}^{MM}$.

We have deliberately developed a model in which old goods within the same category (old X goods) and those from other categories (old Y goods) are symmetrically treated. Therefore, as in the WW case, consumers who do not participate in the exchange promotion, the L_N segment, end up paying the full retail price and thus subsidize the low-valuation consumers from L_X, L_Y, and L_{XY} segments who participate in exchange promotions. Note that even though not all consumers from L_X, L_Y, and L_{XY} segments purchase a new good, those who do also find it optimal to exchange an old unit. Because some consumers who do not have old X units could have old Y units, a greater number of consumers can potentially participate in multicategory as opposed to within-category exchange promotions. As a result, holding all else constant, a smaller number of low-valuation consumers pay full retail price in multicategory exchange promotions than in within-category exchange promotions. In other words, the promotional inefficiency is reduced.

Another interesting difference between the two types of exchange promotions is that within-category promotions work by increasing replacement demand, whereas multicategory promotions increase replacement as well as new demand. Because there are two types of exchange discounts in a multicategory exchange promotion, firms can fine-tune their segmentation better and choose promotion discounts depending on the types of old products consumers possess. In addition, a larger number of consumers get exchange discounts so the end result is that the retail price is now higher, $P_i^{MM} > P_i^{WW}$, as is the exchange discount for old X, $e_{iX}^{MM} > e_{iX}^{WW}$.

Each firm's profit in this case is

$$\Pi_i^{MM} = ((1-f)f(a-v+\theta_L)^2\lambda+f(a+(1-g)\theta_L)^2\lambda)/(4t) + \frac{(1+(1-2(2-f)f)\lambda)(t-t\lambda+2(1-f)^2\theta_L\lambda)^2}{2t(1+(3-4(2-f)f)\lambda)^2}. \quad (9)$$

In summary, we note the following key differences between multicategory and within-category promotions:

- More consumers participate in multicategory promotions, resulting in a lower promotional inefficiency.
- Multicategory promotions increase new as well as replacement demand, whereas within-category promotions increase only replacement demand.
- In multicategory promotions, firms can tailor exchange discounts based on the type of old goods exchanged.
- Retail price is higher in multicategory promotions than in within-category promotions.

4. Promotion Equilibria

In the previous section, we laid out six possible outcomes and highlighted the cases where the firms followed symmetric strategies. In this section, we determine whether any of the six strategies leads to an equilibrium. Across all of the possible outcomes, it is clear that there are no asymmetric equilibria. Furthermore, depending on parameter values, one of the three symmetric strategies, WW, MM, and NN, can be observed in equilibrium. From the previous section, we see that consumers' waste aversion and valuations for their old products affect the prices and the relative profitability of firms' promotion strategies. In §§4.1–4.3, we detail the specific conditions under which each type of equilibrium arises.

4.1. Equilibria with Waste Aversion

4.1.1. Impact of Waste Aversion. The equilibrium outcome is determined by the benefits of price discrimination which, in turn, are partly influenced (moderated) by the number of consumers who qualify for exchange discounts and the level of waste aversion among consumers. Since we have already discussed the impact of the number of consumers who qualify for exchange discounts, we now describe the impact of waste aversion.

Proposition 1. *When waste aversion, a, is above a threshold, i.e., $a > \max\{a_{M3}, a_{WM2}, a_{WM3}\}$, the unique equilibrium is one in which both firms offer multicategory exchange promotions. When waste aversion is within an intermediate range, i.e., $a \in (a_{W3}, \min\{a_{WM2}, a_{WM3}\})$, both firms offering within-category exchange promotions is the unique equilibrium. When waste aversion is sufficiently low, i.e, $a < \min\{a_{W2}, a_{M2}\}$, the unique equilibrium is both firms not offering any exchange promotion.*[10]

Note that Proposition 1 applies to positive values of waste aversion parameter, a, and the thresholds are functions of other parameters in the model. When these thresholds are positive, the three conditions specified in Proposition 1 can be interpreted respectively as high, moderate, and low waste aversion. In both types of exchanges, a higher level of waste aversion implies that consumers have a greater psychological gain from participating in the exchange promotion. As a result, the firm needs a smaller exchange discount to induce consumers to participate in the promotion. In other words, each firm's equilibrium exchange discount is decreasing in a.

Proposition 1 shows that consumers' waste aversion affects the strategic benefits and relative profitability of the two types of exchange promotions. To understand this result, we focus on the MM equilibrium and discuss how the effects outlined above collectively determine the firms' incentives to deviate from the MM equilibrium (the intuition for WW equilibrium is similar and is not repeated here). First, consider the case when firm A deviates to the no-promotion strategy while firm B is conducting the multicategory exchange promotion. In this case, by deviating to a strategy with no promotions, firm A gives up price discrimination and charges a uniform price to all consumers. Thus, relative to the MM case, firm A charges a lower price to the high-valuation segment and the L_N segment (low-valuation consumers without any old goods), and a higher price to L_X, L_Y, and L_{XY} consumers (who get exchange discounts in the MM case). Clearly, as the potential loss from L_X, L_Y, and L_{XY} segments increases, firm A has less incentives to deviate. Holding all else constant, the latter sets of consumers derive more benefits from the exchange promotion as their waste aversion increases. Therefore, firm A's loss from these segments due to the deviation is greater when waste aversion is higher. For sufficiently high values of waste aversion, the deviation from MM to NM is not profitable.

Now consider the case when firm A deviates to the within-category exchange promotion strategy (W) when firm B is conducting the multicategory promotion (M). In this case, firm A offers a promotion that is narrower in scope because it allows only the exchange of old X, whereas firm B allows the exchange of old X and old Y. Relative to the MM case, firm A adopts a milder form of price discrimination with a lower regular price, and it gives no discount to consumers who have an old Y. Thus, the potential loss of deviating to the within-category exchange comes from lower demand from L_Y consumers (who only own an old Y that cannot be exchanged) as well as a lower margin from the H segment. This means that firm A loses customers who own old Y products because they have to pay too high a price without an exchange promotion. When waste aversion is beyond a threshold, this effect is even more significant, and the loss for firm A from deviation to W increases as waste aversion increases.

Thus, we have shown how a multicategory exchange equilibrium is more likely to exist when waste aversion is beyond a threshold. It is easy to see that the trade-off between WW and NN is also about relative profitability of price discrimination, which is affected by waste aversion level. As waste aversion increases (decreases), price discrimination becomes more (less) profitable for each firm.

Conventional wisdom in marketing is that although promotions can increase demand, they sometimes can lead to a prisoner's dilemma in a competitive setting. Therefore, we investigate whether that is the case for exchange promotions.

Proposition 2. *There are two types of prisoner's dilemma equilibria:* (1) *when $a_{W1} \le a \le \min\{a_{W2}, a_{M2}\}$, neither firm offers any exchange promotion, and* (2) *when $\max\{a_{WM1}, a_{W3}\} \le a \le \min\{a_{WM2}, a_{WM3}\}$, both firms offer a within-category exchange promotion. An equilibrium in which both firms offer multicategory exchanges is never a prisoner's dilemma outcome.*

Proposition 2 applies to a subset of equilibria described in Proposition 1, and there are two novel results in Proposition 2. First, we find that no promotion can be a prisoner's dilemma outcome, which is usually not the case in promotion games. Second, when WW is a prisoner's dilemma, it arises due to deviations from

[10] All of the cutoff thresholds are defined in the appendix.

the MM case and not the NN case. As we discussed before, by deviating from an M to a W strategy, a firm reduces its regular price and gains market share in the segments that do not get exchange discounts under the W strategy, namely, H, L_Y, and L_N segments. Under some conditions, the gains in market share are sufficient to create incentives for each firm to deviate unilaterally, but the ensuing price reductions lead to a prisoner's dilemma outcome. By a similar reasoning, when a firm deviates from a W to a N strategy, it can lower the regular price and offer a lower exchange discount for old X goods. This allows it to gain market share in H and L_N segments that pay regular prices under both M and W strategies. Here again, these gains in market share give each firm incentives to deviate unilaterally, but could also result in a prisoner's dilemma when the price cut is too high. It is important to note that these results are not due to the costs of running or administering exchange promotions, but rather to strategic considerations arising out of each firm's incentives to compete aggressively for the high-valuation segment that pays regular prices.

An implication of the above result is that multicategory exchanges have the potential to bring firms out of prisoner's dilemma situations that involve extreme price cuts and result in no exchange promotions being offered. With multicategory promotions, firms may end up with a milder prisoner's dilemma of offering only a within-category promotion. However, with only within-category promotions, firms are more likely to end up in prisoner's dilemmas with large price cuts and no promotions being offered.

Other Equilibria. Finally, we note that there are cases when we can have two equilibria involving MM and WW—both exchange promotions. In all of these instances, the MM equilibrium is more profitable than the WW equilibrium, so the problem is for the firms to coordinate. Figures 2 and 3 summarize the equilibria by displaying pairwise comparisons between NN and MM and between WW and MM. It is clear that when waste aversion is sufficiently high, choosing a multicategory exchange promotion is a dominant

Figure 2 **Equilibrium Outcomes Between NN and MM**

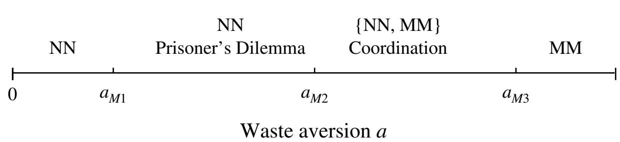
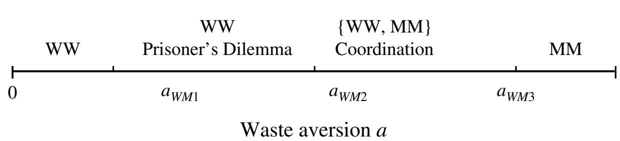


Figure 3 **Equilibrium Outcomes Between WW and MM**



strategy. Furthermore, a multicategory exchange is more profitable even when there are multiple equilibria, thus suggesting firms have an incentive to coordinate. When waste aversion is low, we see how the exchange promotion can be broken and lead to the prisoner's dilemma outcomes highlighted in Proposition 2.

4.1.2. Impact of Market Structure. In §1, we asked how characteristics of the market affect the equilibrium outcomes. In particular, if low-valuation consumers represent a large fraction of consumers, such as in emerging markets, it is important to understand the impact of this consumer segment. We address this issue in the next proposition.

Proposition 3. *As the proportion of the low-valuation segment, λ, increases, the NN equilibrium becomes more likely.*

At first blush, it may seem that when there are more low-valuation consumers, there are stronger reasons to engage in price discrimination. However, Proposition 3 shows that as the low-valuation segment becomes larger, both types of exchange promotions become less likely, whereas the no-promotion equilibrium becomes more likely. This result arises for two reasons. First, as the low-valuation segment becomes larger, the number of consumers receiving a discount increases. Thus, the "cost" of promotion—be it within-category or multicategory—increases. In addition, the number of consumers paying the higher, nondiscounted price decreases. Therefore, firms' ability to possibly make up for the cost of discounts through higher prices to the high-valuation segment is reduced. The second reason is that, holding all else constant, the exchange promotion becomes more inefficient as the fraction of the low-valuation segment increases. As we discussed in §§3.2 and 3.3, a limitation of exchange promotions is that even some of the low-valuation consumers end up paying the nondiscounted price. In particular, low-valuation consumers (L_N) who do not have old goods to exchange cannot get the promotional discount. Holding f constant, as the size of the low-valuation segment increases, the size of this segment without old products, $\lambda(1-f)^2$, increases. As a result, discrimination benefits of exchange promotions are reduced.

An interesting aspect of Proposition 3 is that the results hold for the entire feasible range of parameters and not merely for specific cases involving very high or very low values of λ, the size of the low-valuation consumer segment. When the segment of low-valuation consumers is very large, neither multicategory nor within-category exchange promotions will be observed in equilibrium. In other words, the effect described in Proposition 3 continues until NN is the only equilibrium that survives.

4.1.3. Equilibrium Prices and Discounts. Equilibrium prices and discounts under different symmetric strategies were detailed in §3. In comparing these prices across strategies, we see that when the firm does not offer an exchange promotion, the retail price depends on g, the fraction of value retailed by the old good X. Because consumers who own an old X always have their old product as an outside option, it is straightforward to see that as the outside option improves, consumers have a lower incentive to purchase the new product. As a result, to induce consumers to replace their old product, the firm has to charge a lower price, $\partial P^{NN}/\partial g < 0$. On the other hand, when the firm offers an exchange promotion (either within-category or multicategory exchange), then it can better isolate the effect of ownership of the old X on prices for the new X; now, g has no effect on the retail price ($\partial P^{WW}/\partial g = \partial P^{MM}/\partial g = 0$). In particular, we find that regular prices are highest when the firms offer multicategory promotions and lowest when they offer no promotions: $P^{MM} > P^{WW} > P^{NN}$. This is straightforward when interpreted within the context of price discrimination. A multicategory exchange results in three different net prices, P^{MM}, $(P^{MM} - e_X^{MM})$, and $(P^{MM} - e_Y^{MM})$, such that consumers who do not participate in an exchange end up paying the highest price. Similarly, when there is no exchange promotion, the single, uniform price falls to the lowest level.

In terms of the optimal exchange discounts, we see a pattern that applies across both within-category and multicategory promotions. In particular, we have the following proposition:

Proposition 4. *When the firms offer exchange promotions, the equilibrium exchange discount decreases in waste aversion, a, and the size of the L segment, λ, but increases in the fraction of qualified customers, f, and the valuation of the old goods, g and v.*

In both types of promotions, we find that the optimal discount decreases in the level of waste aversion, a, and the size of the L segment, λ. Intuitively, when a is higher, the firm needs to offer less of an incentive for consumers to exchange; similarly, when λ is higher, more consumers are available to participate in the promotion and the firm lowers the discount. On the other hand, we see that the exchange discount increases in the fraction of qualified customers, f, and in consumers' valuations of their old goods, g and v. When consumers' valuation of their old good is higher, they need more of an incentive to participate in the exchange.

It may seem that as more consumers qualify for a discount, firms should decrease the amount of discount to avoid giving up too much surplus to consumers. Furthermore, one may conjecture that the impact of f and λ on the equilibrium discount e^* would be similar because an increase in either size means more low-valuation consumers will join the exchange promotion and get the exchange discount. However, Proposition 4 states otherwise.

We explain the key intuition for this proposition by using a within-category exchange. (A similar logic applies to a multicategory exchange.) A promotion is effective when it separates high- and low-valuation consumer segments. With the exchange promotion as a discrimination instrument, a firm can effectively identify the f fraction of L consumers whose net utility from participating in the exchange is $(1-g)\theta_L + a$, where the second component comes from exchanging their old product and not feeling wasteful. Therefore, the firm can charge the following net optimal monopoly price to the f fraction of L consumers: $P^* - e_X^* = ((1-g)\theta_L + a)/2$. As f increases, $\lambda(1-f)$ decreases, and there are fewer L consumers who are not qualified for exchange, thus making this particular segment even less attractive to the firm. As a result, the firm will raise its retail price to better extract surplus from the H segment and at the same time, increase the exchange discount to the same extent and maintain its optimal monopoly price on the f fraction of L consumers. On the other hand, as λ increases, there are more low-valuation consumers overall. In particular, the size of the nonqualified low-valuation consumer segment, $\lambda(1-f)$, increases. As a result, the firm needs to decrease the retail price, and subsequently reduce the exchange discount e^* to achieve optimal pricing for the f fraction of the L segment.

4.2. Equilibria Without Waste Aversion

Given the foregoing discussion, a natural question is whether the existence of waste aversion is necessary to sustain either MM or WW equilibrium. We find that even when there is no waste aversion, i.e., $a = 0$, then, depending on the parameters, we can still observe WW, MM, and NN equilibria. In Proposition 1, we characterized conditions for various equilibria in terms of the waste aversion parameter, a. In Proposition 5, we characterize them in terms of the valuation for the old good, g.

Proposition 5. *If there is no waste aversion, i.e., $a = 0$, then the equilibria are as follows:*

1. *If valuation for old X is sufficiently high, i.e., $g > \max\{g_{W2}, g_{W3}, g_{M2}, g_{M3}\}$, then either within-category or multicategory exchange is the equilibrium outcome. In this case, if valuation for old Y is low, i.e., $v < \min\{v_2, v_3\}$, then multicategory exchange is the unique equilibrium. If valuation for old Y is high, i.e., $v > \max\{v_2, v_3\}$, then within-category exchange is the unique equilibrium.*
2. *If valuation for old X is sufficiently low, i.e., $g < \min\{g_{W2}, g_{W3}, g_{M2}, g_{M3}\}$, no promotion is the equilibrium outcome.*

3. *If valuation for old X is at a moderate level, i.e., g takes values other than above, there exist multiple symmetric equilibria*: {*NN*, *WW*}, {*NN*, *MM*}, *and* {*WW*, *MM*}.

This proposition emphasizes that even when there is no waste aversion, i.e., $a = 0$, the firms may still choose to offer exchange promotions. Although a moderates the strategic benefits of price discrimination and the relative profitability of each type of exchange promotion, firms have reasons to offer each type of exchange promotion even without waste aversion. In particular, compared to NN, a WW exchange can induce current owners of X to replace their old goods. When g is high, old X goods are valued more and consumers are reluctant to upgrade to the new X. However, an exchange promotion can induce these consumers to replace their old goods. Compared to WW, MM can extend the benefits of exchange discount to a larger pool of consumers. When consumers' valuation for old Y is sufficiently low ($v < \min\{v_2, v_3\}$), it is not costly to attract these owners to give up their old Y. As a result, firms end up offering a multicategory exchange and giving exchange discounts to a larger pool of consumers. On the flip side, each type of promotion comes at the cost of lower prices to L_X, L_Y, and L_{XY} segments as well as lower sales from the L_N segment that cannot benefit from the exchange. When g is at a moderate level, whether a firm has incentives to engage in price discrimination depends on the other firm's strategy, leading to multiple symmetric equilibria.

Thus, Proposition 5 shows the presence of waste aversion, $a > 0$, is not necessary to obtain the general pattern of results. On the other hand, by ignoring waste aversion, we would be ignoring its strong effect on price and demand. Holding all else constant, we note that consumers are more likely to participate in an exchange promotion when they also have an aversion to wasting their old goods. Thus, if a firm offers an exchange promotion, not only do consumers have an added incentive to "use" their old good and get a gain of a, but the firm recognizes this and offers a smaller discount. In general, as waste aversion increases, firms' discounts decrease. However, because the exchange promotion applies only to the low-valuation consumer, the regular price is unaffected by the level of waste aversion in the market. This leads to the interesting result that when there is more waste aversion, prisoner's dilemma outcomes are less likely because firms now obtain higher profits from low-valuation consumers who participate in exchanges.

With an exchange promotion, we see that replacement demand is higher when there is a positive level of waste aversion, but nonreplacement demand is not affected by the level of waste aversion. With the caveat that our model is static, if we still were to draw long-term implications, our analysis suggests that the repurchase frequency with exchange promotions would be higher as a increases. The result is ironic because when $a > 0$, consumers are less likely to throw away their old products; however, this is precisely why the exchange promotion can be so effective. This seemingly counterintuitive result can also be understood by noting that when $a > 0$, the replacement demand would be lower without exchange promotions.

4.3. Other Motivations for Exchange Promotions

In this section, we elaborate on other possible motivations for firms to offer exchange promotions. Although it is clear that firms are increasing their use of exchange promotions, it is important to note that there may be other reasons why exchange promotions are becoming more popular. Although these reasons do not detract from the analysis in this paper and they have been incorporated in the main model, it is still useful to highlight their roles in our analysis.

Clearly consumers incur some costs associated with storing their old items, and these costs can vary from low (e.g., an old cell phone in a drawer) to high (e.g., an extra dishwasher). One may argue that exchange promotions are beneficial because they relieve consumers of the cost of storing old items, and consumers would be especially happy to get rid of items with high storage costs. From a utility standpoint, even though consumers may be irked by storage costs, the fact that they choose to keep the old items in their possession suggests that their net utility from storage is still positive. Thus, the driver to get consumers to participate in the exchange is their waste aversion and not the storage cost. In particular, consumers' net utility is captured by the parameter v, and as storage costs, k, increase, v decreases. If storage costs increase so that v becomes negative, consumers will get rid of the item, for example, by throwing it away. Interestingly, from the firm's perspective, as consumers' storage costs increase, the firm decreases the exchange discount—higher storage costs mean that consumers have *more* of an incentive to participate in the exchange so the firm can give them less of a financial incentive through the discount.

In addition, it is important to note that storage costs should be considered at an incremental level, i.e., they apply to stored items that are not in use and not to items in use. For example, if a consumer replaces an old refrigerator that is currently being used with a new refrigerator, there is no incremental storage cost involved. However, if the new refrigerator is purchased and the old one is stored, then there is an incremental storage cost. In our model, for the within-category exchange promotion, there is no storage cost involved; for the cross-category exchange, there is a storage cost involved.

Another possible argument for why we observe exchange promotions is that firms want to increase the rate of replacement purchases. This argument suggests that when the proportion of consumers with old goods is high, firms may offer an exchange promotion to induce some of these consumers to upgrade to newer versions of the product. To evaluate this argument, it is useful to distinguish between a *replacement exchange* (within-category) and a *pure exchange* (cross-category) decision. In considering the purchase of a new X, consumers who are using an old X face a replacement decision, whereas those who do not own X face a pure exchange decision. As highlighted in Figure 1, the outside options for these two sets of consumers are different. We find that as the value of the old X increases, the firm has to offer a *larger* discount to replacement consumers because they have less of an incentive to trade up to the new product. On the other hand, this increase in the value of the old X has no effect on the discount offered to consumers who participate in a cross-category exchange with their old Y, and it has no impact on the base price offered to consumers who do not participate in the exchange promotion. Importantly, even when we hold fixed the value of the old product, we show that it is not always profitable for the firms to offer an exchange promotion, i.e., it is not always profitable to accelerate replacement purchases.

5. Conclusion

Since the mid-2000s, we have seen an increase in the use of exchange promotions as a promotional tool. In this paper, we present a general model of exchange promotions in a competitive setting and identify conditions under which they would be offered. Our main finding is that an exchange promotion can play an important role not only in price discriminating between low- and high-valuation consumers but also in having a strong effect on replacement and new demand. Our analysis also provides interesting insights about the differences between within-category and multicategory exchanges.

A cost of within-category promotions (that is applicable across many real-world situations) is that low-valuation consumers who do not have old units to exchange end up paying the full retail price for the product. Because this retail price increases with exchange discounts, it means that in addition to high-valuation consumers, low-valuation consumers without old goods also subsidize the consumers who get promotional discounts. This makes the promotion partly inefficient in the sense that one group of low-valuation consumers end up subsidizing another group of low-valuation consumers. When multicategory exchanges are allowed by firms, this inefficiency is reduced because there are fewer low-valuation consumers who pay the full retail price. Interestingly, both new and replacement demands increase when firms offer multicategory exchange programs, whereas within-category exchange programs only increase replacement demand. Finally, another consequence of allowing multicategory promotions is that firms now have weaker incentives to move away from offering discounts by trying to get more of their profits from the high-valuation segment. In this way, when multicategory promotions are allowed, the competition for the high-valuation segment gets less intense (i.e., retail prices can be higher). As a result, multicategory promotions provide firms with stronger incentives to get out of prisoner's dilemma situations and thus soften the competition between the firms.

A novel feature of this research is that it presents a formal analysis of the impact of consumers' waste aversion on firm strategies. We find that holding all else constant, as waste aversion increases, firms will need to offer a lower discount for exchange programs. In addition, with exchange promotions, replacement demand is higher as consumer waste aversion increases. Although our model is static, if we were to try and draw longer-term implications, this suggests that the repurchase frequency with exchange programs is higher as waste aversion increases. The result is ironic in a way because with higher waste aversion, consumers are less likely to throw away old products, but this is precisely why the exchange program can be so effective. This seemingly counterintuitive result can be understood by noting that with waste aversion, the replacement demand would be (as expected) lower without exchange programs. An appealing aspect of this setup is that the new (nonreplacement) demand is not affected by the level of waste aversion.

Although the presence of many low-valuation consumers suggests ideal conditions for exchange promotions, we show that in such cases, the cost of providing discounts to a large number of consumers can undermine some of the benefits of price discrimination. We also find that under such conditions, firms may not offer any promotion in spite of differences in consumer valuations. The reasons here are not due to the costs of administering such promotions, but arise from strategic incentives of competing firms. When the two firms have incentives to compete aggressively for the high-valuation segment, exchange promotions cannot be sustained in equilibrium. In fact, we show that such incentives can lead to the no-promotion equilibrium being a prisoner's dilemma outcome. This is in contrast to the received wisdom that running promotions is often a prisoner's dilemma situation. Our analysis reveals that firms may often find themselves in coordination equilibria situations such that two equilibria can be sustained, but the specific outcome will depend on firms' ability to coordinate their actions. The possibility of the above prisoner's dilemma as well as coordination equilibria points to a role for third-party interventions. For example, if multicategory exchange promotions can improve the total welfare but cannot be accomplished, a government can intervene and help firms come to the multicategory exchange equilibrium. Some of these ideas played out in the Chinese government's large-scale exchange program and the U.S. government's cash-for-clunkers program.

Although this paper formalizes the competitive implications of exchange promotions, it is important to explore the empirically testable implications of our work, especially in terms of understanding waste aversion across cultures and in understanding the effectiveness of exchange promotions across different product categories. At the outset, we note that exchange promotions tend to be more common in China and India than they are in the United States. This suggests that collectivist cultures tend to have higher levels of waste aversion compared to individualistic cultures. Although direct measures of waste aversion are not readily available, the World Values Survey measures the importance of "thriftiness," which can be considered as a proxy for waste aversion. In particular, thriftiness was mentioned as being important by respondents in China (62%), India (55%), and Japan (52%), whereas respondents in the United States (30%), Canada (27%), and Great Britain (25%) rated thriftiness as considerably less important (World Values Survey Wave 5 2014). This suggests that differences in waste aversion across countries can influence firms' choices about exchange promotions. When put within the context of within- and cross-category promotions, our work suggests that product markets that have less scope for additional new demand will be more likely to see within-category exchange promotions, whereas product markets where there is a greater scope for new demand will be more likely to see cross-category exchange promotions. In the same vein, markets that are more price competitive are more likely to have cross-category promotions. Exploring these ideas in an empirical context will help us better understand the manner in which exchange promotions can be used.

As exchange promotions become more popular, it is important to increase our understanding of how they work at a conceptual level. In particular, as consumers begin to expect these promotions to exist in multiple categories or third parties begin to handle exchanges, we need to develop more elaborate models. For example, if consumers are forward looking and form expectations about future exchange values, then extending this work to a dynamic setting would be a nice extension. A dynamic model in this context has a flavor of durable goods models where consumers form expectations of future resale prices, but it also brings in the interesting question of the role of competitive exchanges, i.e., should a firm accept its competitor's old product as part of its own exchange promotion? In addition, it is important to understand other ways in which exchange promotions can have an impact on the market. For example, not only can they fit within the idea of being green—both for consumers and the firm—but they can also play an important role in planning how to market refurbished goods. Finally, as data on these promotions become more available, it would be interesting to do an empirical analysis of within-category versus multicategory exchange promotions.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mksc.2015.0955.

Acknowledgments

The order of authors merely reflects alphabetic sequence. The authors are grateful for comments from seminar participants at Johns Hopkins University, University of Washington, Seattle, UT Dallas FORMS conference, Marketing Science Conference, and the Marketing Science Emerging Markets Conference at Wharton. The authors also thank the editor, the associate editor, and two anonymous reviewers for their helpful suggestions.

Editor's Note

The paper was submitted for the special section on emerging markets. Special section co-editors Kannan Srinivasan and K. Sudhir requested Jagmohan Raju to serve as guest Editor-in-Chief for the article.

Appendix

Analysis of Symmetric Strategies of Firms

In this subsection, we present the analysis of three symmetric cases: NN, WW, and MM.

Case NN: Neither Firm Offers Promotion. When neither firm offers any promotion, the demand for firm A from the H and L segments is given by

$$\bar{x}_{AH} = \frac{P_B - P_A + t}{2t},$$
$$\bar{x}_{AL_X} = \bar{x}_{AL_{XY}} = \frac{(1-g)\theta_L - P_A + v}{t} \in \left(0, \frac{1}{2}\right),$$
$$\bar{x}_{AL_Y} = \bar{x}_{AL_N} = \frac{\theta_L - P_A}{t} \in \left(0, \frac{1}{2}\right).$$

Similarly, firm B's demand is

$$\bar{x}_{BH} = 1 - \bar{x}_{AH},$$
$$\bar{x}_{BL_X} = \bar{x}_{BL_{XY}} = \frac{(1-g)\theta_L - P_B + v}{t}, \quad \text{and}$$
$$\bar{x}_{BL_Y} = \bar{x}_{BL_N} = \frac{\theta_L - P_B}{t}.$$

The problem of firm A can now be stated (and that of firm B is symmetrically defined) as $\Pi_A = \max P_A((1-\lambda)\cdot \bar{x}_{AH} + f\lambda\bar{x}_{AL_X} + (1-f)\lambda\bar{x}_{AL_Y})$, by choosing the optimal price. This optimization yields

$$P_A^{NN} = P_B^{NN} = \frac{2\lambda((1-fg)\theta_L + fv) + t(1-\lambda)}{1+3\lambda}, \quad x_H^{NN} = \frac{1}{2},$$

$$x_{L_X}^{NN} = x_{L_{XY}}^{NN} = \frac{\left\{\begin{array}{l}(1+3\lambda-2f\lambda)v - t(1-\lambda)\\ \quad +\theta_L(1-g+\lambda-3g\lambda+2fg\lambda)\end{array}\right\}}{t(1+3\lambda)},$$

$$x_{L_Y}^{NN} = x_{L_N}^{NN} = \frac{(1+\lambda+2fg\lambda)\theta_L - t(1-\lambda) - 2fv\lambda}{t+3t\lambda},$$

$$\Pi^{NN} = \frac{(1+\lambda)(t-t\lambda+2(\theta_L+f(v-g\theta_L))\lambda)^2}{2t(1+3\lambda)^2}.$$

Case WW: Both Firms Offer Within-Category Exchange. When both firms decide to adopt a within-category exchange policy, the demand of the H and L segments for firm A is

$$\bar{x}_{AH} = \frac{P_B - P_A + t}{2t},$$

$$\bar{x}_{AL_X} = \bar{x}_{AL_{XY}} = \frac{(1-g)\theta_L - P_A + e_{AX} + a}{t} \in \left(0, \frac{1}{2}\right),$$

$$\bar{x}_{AL_Y} = \bar{x}_{AL_N} = \frac{\theta_L - P_A}{t} \in \left(0, \frac{1}{2}\right).$$

(That of firm B is symmetrically defined.)

The problem of firm A can now be stated as $\Pi_A = \max P_A(1-\lambda)\bar{x}_{AH} + (P_A - e_{AX})\lambda f \bar{x}_{AL_X} + P_A\lambda(1-f)\bar{x}_{AL_Y}$, by choosing the optimal price and exchange discount. (That of firm B is symmetrically defined.) This optimization yields

$$P_A^{WW} = P_B^{WW} = \frac{t(1-\lambda)+2(1-f)\theta_L\lambda}{1+(3-4f)\lambda},$$

$$e_{AX}^{WW} = e_{BX}^{WW} = \frac{t(1-\lambda)+2(1-f)\theta_L\lambda}{1+(3-4f)\lambda} - \frac{(1-g)\theta_L + a}{2},$$

$$\Pi^{WW} = \frac{f((1-g)\theta_L+a)^2\lambda}{4t} + \frac{(1+(1-2f)\lambda)(t(1-\lambda)+2(1-f)\theta_L\lambda)^2}{2t(1+(3-4f)\lambda)^2}.$$

Case MM: Both Firms Offer Multicategory Exchange. When both firms decide to offer multicategory exchange promotions, the demand from different groups of consumers for firm A is given by

$$\bar{x}_{AH} = \frac{P_B - P_A + t}{2t},$$

$$\bar{x}_{AL_N} = \frac{\theta_L - P_A}{t} \in \left(0, \frac{1}{2}\right),$$

$$\bar{x}_{AL_X} = \frac{(1-g)\theta_L - P_A + e_{AX} + a}{t} \in \left(0, \frac{1}{2}\right),$$

$$\bar{x}_{AL_Y} = \frac{\theta_L - P_A + e_{AY} + a - v}{t} \in \left(0, \frac{1}{2}\right),$$

$$\bar{x}_{AL_{XY}} = \frac{(1-g)\theta_L - P_A + e_{AX} + a}{t} \in \left(0, \frac{1}{2}\right).$$

(That of firm B is symmetrically defined.)

Each firm maximizes its profits, $\Pi_A = \max P_A((1-\lambda)\bar{x}_{AH} + (1-f)^2\lambda\bar{x}_{AL_N}) + (P_A - e_{AX})f\lambda\bar{x}_{AL_X} + (P_A - e_{AY})(1-f)f\lambda\bar{x}_{AL_Y}$, by choosing the optimal price and two exchange discounts. This optimization yields

$$P_A^{MM} = P_B^{MM} = \frac{t(1-\lambda)+2(1-f)^2\theta_L\lambda}{1+(3-4(2-f)f)\lambda},$$

$$e_{AX}^{MM} = e_{BX}^{MM} = \frac{t(1-\lambda)+2(1-f)^2\theta_L\lambda}{1+(3-4(2-f)f)\lambda} - \frac{(1-g)\theta_L + a}{2},$$

$$e_{AY}^{MM} = e_{BY}^{MM} = \frac{t(1-\lambda)+2(1-f)^2\theta_L\lambda}{1+(3-4(2-f)f)\lambda} - \frac{\theta_L + a - v}{2}, \quad \text{and}$$

$$\Pi^{MM} = \frac{(1-f)f(a-v+\theta_L)^2\lambda + f(a+(1-g)\theta_L)^2\lambda}{4t} + \frac{(1+(1-2(2-f)f)\lambda)(t-t\lambda+2(1-f)^2\theta_L\lambda)^2}{2t(1+(3-4(2-f)f)\lambda)^2}.$$

Analysis of Asymmetric Strategies of Firms

There are three asymmetric cases: First, one firm offers within-category exchange and the other does not offer a promotion (denoted as WN for A and NW for B). Second, one firm offers multicategory exchange and the other does not offer a promotion (denoted as MN for A and NM for B). Third, one firm offers within-category exchange and the other offers multicategory exchange (denoted as WM for A and MW for B).

Analysis of WN. Suppose firm A offers a within-category exchange program and firm B does not offer any promotion. The demand from different groups of consumers for firms A and B is

$$\bar{x}_{AH} = \frac{P_B - P_A + t}{2t}, \bar{x}_{BH} = \frac{P_A - P_B + t}{2t},$$

$$\bar{x}_{AL_X} = \bar{x}_{AL_{XY}} = \frac{(1-g)\theta_L - P_A + e_A + a}{t} \in \left(0, \frac{1}{2}\right),$$

$$\bar{x}_{AL_Y} = \bar{x}_{AL_N} = \frac{\theta_L - P_A}{t} \in \left(0, \frac{1}{2}\right),$$

$$\bar{x}_{BL_X} = \bar{x}_{BL_{XY}} = \frac{(1-g)\theta_L - P_B + v}{t} \in \left(0, \frac{1}{2}\right),$$

$$\bar{x}_{BL_Y} = \bar{x}_{BL_N} = \frac{\theta_L - P_B}{t} \in \left(0, \frac{1}{2}\right).$$

Each firm's problem is to maximize profits, $\Pi_A = \max P_A(1-\lambda)\bar{x}_{AH} + (P_A - e_{AX})\lambda f \bar{x}_{AL_X} + P_A\lambda(1-f)\bar{x}_{AL_Y}$ and $\Pi_B = \max P_B((1-\lambda)(1-\bar{x}_{AH}) + f\lambda\bar{x}_{BL_X} + (1-f)\lambda\bar{x}_{BL_Y})$, by choosing the optimal prices and exchange discount. This yields

$$P_A^{WN} = \frac{\left\{\begin{array}{l} t(1-\lambda)(3+\lambda)+2fv\lambda(1-\lambda)\\ \quad +2\theta_L\lambda(3+\lambda-f(2+2\lambda+g(1-\lambda)))\end{array}\right\}}{3+\lambda(10+3\lambda-8f(1+\lambda))},$$

$$e_{AX}^{WN} = P_A^{WN} - \frac{1}{2}(a+(1-g)\theta_L),$$

$$P_B^{NW} = \frac{\left\{\begin{array}{r} 2\theta_L\lambda(3+\lambda+4f^2g\lambda - f(1+3\lambda+2g(1+\lambda)))\\ -2\lambda(4f^2v\lambda - 2fv(1+\lambda))\\ +t(1-\lambda)(3+(1-4f)\lambda)\end{array}\right\}}{3+\lambda(10+3\lambda-8f(1+\lambda))},$$

$$\Pi^{WN} = \frac{1}{4t}f(a+(1-g)\theta_L)^2 + \frac{\left\{\begin{array}{l}(1+(1-2f)\lambda)(t(1-\lambda)(3+\lambda)+2\lambda(\theta_L(3+\lambda)\\ \quad +f(v(1-\lambda)-\theta_L(2+g+2\lambda-g\lambda))))^2\end{array}\right\}}{2t(3+\lambda(10+3\lambda-8f(1+\lambda)))^2},$$

and

$$\Pi^{NW} = \frac{\left\{\begin{array}{r}(1+\lambda)(t(1-\lambda)(3+(1-4f)\lambda)+2\lambda(\theta_L(3+\lambda)\\ -4f^2(v-g\theta_L)\lambda+f(2v(1+\lambda)\\ -\theta_L(1+3\lambda+2g(1+\lambda)))))^2\end{array}\right\}}{2t(3+\lambda(10+3\lambda-8f(1+\lambda)))^2}.$$

Analysis of MN. Next suppose firm A offers multicategory exchange and firm B does not offer any promotion. The demand of the two segments for firms A and B is

$$\bar{x}_{AH} = \frac{P_B-P_A+t}{2t},\ \bar{x}_{BH}=1-\bar{x}_{AH},$$
$$\bar{x}_{AL_X} = \frac{(1-g)\theta_L-P_A+e_{AX}+a}{t}\in\left(0,\frac{1}{2}\right),$$
$$\bar{x}_{AL_Y} = \frac{\theta_L-P_A+e_{AY}+a-v}{t}\in\left(0,\frac{1}{2}\right),$$
$$\bar{x}_{AL_{XY}} = \frac{(1-g)\theta_L-P_A+e_{AX}+a}{t}\in\left(0,\frac{1}{2}\right),$$
$$\bar{x}_{BL_X} = \bar{x}_{BL_{XY}} = \frac{(1-g)\theta_L-P_B+v}{t}\in\left(0,\frac{1}{2}\right),$$
$$\bar{x}_{AL_N} = \frac{\theta_L-P_A}{t}\in\left(0,\frac{1}{2}\right),$$
$$\bar{x}_{BL_Y} = \bar{x}_{BL_N} = \frac{\theta_L-P_B}{t}\in\left(0,\frac{1}{2}\right).$$

Each firm's problem is to maximize profits, $\Pi_A = \max P_A((1-\lambda)\bar{x}_{AH}+(1-f)^2\lambda\bar{x}_{AL_N})+(P_A-e_{AX})f\lambda\bar{x}_{AL_X}+(P_A-e_{AY})(1-f)f\lambda\bar{x}_{AL_Y}$ and $\Pi_B=\max P_B((1-\lambda)(1-\bar{x}_{AH})+f\lambda\bar{x}_{BL_X}+(1-f)\lambda\bar{x}_{BL_Y})$, by choosing the optimal prices and exchange discounts. This optimization yields

$$P_A^{MN} = \frac{\left\{\begin{array}{r}t(1-\lambda)(3+\lambda)+2\lambda(2f^2\theta_L(1+\lambda)+\theta_L(3+\lambda)\\ +f(v-(4+g)\theta_L-v\lambda-(4-g)\theta_L\lambda))\end{array}\right\}}{3+\lambda(10+3\lambda-8(2-f)f(1+\lambda))},$$

$$e_{AX}^{MN} = P_A^{MN}-\frac{a+(1-g)\theta_L}{2},\ e_{AY}^{MN}=P_A^{MN}-\frac{a+\theta_L-v}{2},$$

$$P_B^{NM} = \frac{\left\{\begin{array}{r}t(1-\lambda)(3+(1-4(2-f)f)\lambda)+2\lambda(2fv+3\theta_L\\ +f(f-2(1+g))\theta_L+(2f(1-2(2-f)f)v\\ +\theta_L+f(f(3+4(2-f)g)-2(3+g))\theta_L)\lambda)\end{array}\right\}}{3+\lambda(10+3\lambda-8(2-f)f(1+\lambda))},$$

$$\Pi^{MN} = \frac{\left\{\begin{array}{r}(1+(1-2(2-f)f)\lambda)(t(1-\lambda)(3+\lambda)\\ +2\lambda(2f^2\theta_L(1+\lambda)+\theta_L(3+\lambda)\\ +f(v-(4+g)\theta_L-v\lambda-(4-g)\theta_L\lambda)))^2\end{array}\right\}}{2t(3+\lambda(10+3\lambda-8(2-f)f(1+\lambda)))^2} + \frac{(1-f)f(a-v+\theta_L)^2\lambda+f(a+\theta_L-g\theta_L)^2\lambda}{4t},$$

$$\Pi^{NM} = \frac{\left\{\begin{array}{l}(1+\lambda)(-t(1-\lambda)(3+(1-4(2-f)f)\lambda)\\ +2\lambda(-3\theta_L+f(-2v+(2-f+2g)\theta_L)\\ -(2f(1-2(2-f)f)v+\theta_L\\ \quad +f(f(3+4(2-f)g)-2(3+g))\theta_L)\lambda))^2\end{array}\right\}}{2t(3+\lambda(10+3\lambda-8(2-f)f(1+\lambda)))^2}.$$

Analysis of MW. Finally, assume firm A offers multicategory exchange and firm B offers within-category exchange. The demand of the two segments for firms A and B is

$$\bar{x}_{AH} = \frac{P_B-P_A+t}{2t},\ \bar{x}_{BH}=1-\bar{x}_{AH},$$
$$\bar{x}_{AL_X} = \frac{(1-g)\theta_L-P_A+e_{AX}+a}{t}\in\left(0,\frac{1}{2}\right),$$
$$\bar{x}_{AL_Y} = \frac{\theta_L-P_A+e_{AY}+a-v}{t}\in\left(0,\frac{1}{2}\right),$$
$$\bar{x}_{AL_{XY}} = \frac{(1-g)\theta_L-P_A+e_{AX}+a}{t}\in\left(0,\frac{1}{2}\right),$$
$$\bar{x}_{BL_X} = \bar{x}_{BL_{XY}} = \frac{(1-g)\theta_L-P_B+e_{BX}+a}{t}\in\left(0,\frac{1}{2}\right),$$
$$\bar{x}_{AL_N} = \frac{\theta_L-P_A}{t}\in\left(0,\frac{1}{2}\right),\ \bar{x}_{BL_Y}=\bar{x}_{BL_N}=\frac{\theta_L-P_B}{t}\in\left(0,\frac{1}{2}\right).$$

Each firm's problem is to maximize profits, $\Pi_A = \max P_A((1-\lambda)\bar{x}_{AH}+(1-f)^2\lambda\bar{x}_{AL_N})+(P_A-e_{AX})f\lambda\bar{x}_{AL_X}+(P_A-e_{AY})(1-f)f\lambda\bar{x}_{AL_Y}$ and $\Pi_B=\max P_B(1-\lambda)\bar{x}_{BH}+(P_B-e_{BX})\lambda f\bar{x}_{BL_X}+P_B\lambda(1-f)\bar{x}_{BL_Y}$, by choosing the optimal prices and exchange discounts. This optimization yields

$$P_A^{MW} = \frac{\left\{\begin{array}{r}t(1-\lambda)(3+(1-4f)\lambda)\\ +2(1-f)\theta_L\lambda(3+\lambda-2f(1+(3-2f)\lambda))\end{array}\right\}}{3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda))},$$

$$e_{AX}^{MW} = P^{MW}-\frac{a+(1-g)\theta_L}{2},\ e_{AY}^{MW}=P^{MW}-\frac{a+\theta_L-v}{2},$$

$$P_B^{WM} = \frac{\left\{\begin{array}{r}t(1-\lambda)(3+(1-4(2-f)f)\lambda)\\ +2(1-f)\theta_L\lambda(3+\lambda-f(1+(7-4f)\lambda))\end{array}\right\}}{(3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda))},$$

$$e_{BX}^{WM} = P^{WM}-\frac{a+(1-g)\theta_L}{2},$$

$$\Pi^{MW} = \frac{(1-f)f(a-v+\theta_L)^2\lambda+f(a+\theta_L-g\theta_L)^2\lambda}{4t} + \frac{\left\{\begin{array}{r}(1+(1-2(2-f)f)\lambda)(t(1-\lambda)(3+(1-4f)\lambda)\\ +2(1-f)\theta_L\lambda(3+\lambda-2f(1+(3-2f)\lambda)))^2\end{array}\right\}}{2t(3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda)))^2},$$

$$\Pi^{WM} = \frac{f(a+\theta_L-g\theta_L)^2\lambda}{4t} + \frac{\left\{\begin{array}{r}(1+(1-2f)\lambda)(t(1-\lambda)(3+(1-4(2-f)f)\lambda)\\ +2(1-f)\theta_L\lambda(3+\lambda-f(1+(7-4f)\lambda)))^2\end{array}\right\}}{2t(3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda)))^2}.$$

Proofs of Propositions

Proof of Proposition 1. First, we consider firms' trade-offs between WW and NN. Conditional on the other firm not offering any promotion, the focal firm's profit difference between offering within-category exchange and no promotion is

$$\Pi^{WN}-\Pi^{NN} = \frac{\lambda f(a+(1-g)\theta_L)^2}{4t} - \frac{(1+\lambda)(t-t\lambda+2(\theta_L+f(v-g\theta_L))\lambda)^2}{2t(1+3\lambda)^2} + \frac{\left\{\begin{array}{r}(1+(1-2f)\lambda)(-t(1-\lambda)(3+\lambda)\\ +2\lambda(f(\theta_L(2+g+2\lambda-g\lambda)\\ -v(1-\lambda))-\theta_L(3+\lambda)))^2\end{array}\right\}}{2t(3+\lambda(10+3\lambda-8f(1+\lambda)))^2}.$$

This profit difference is positive when $a > a_{W3} = \sqrt{A_{W3}} - (1-g)\theta_L$, where

$$A_{W3} = \frac{2(1+\lambda)(t-t\lambda+2(\theta_L+f(v-g\theta_L))\lambda)^2}{\lambda f(1+3\lambda)^2} - \frac{\left\{\begin{array}{r} 2(1+(1-2f)\lambda)(-t(1-\lambda)(3+\lambda) \\ +2\lambda(f(\theta_L(2+g+2\lambda-g\lambda) \\ -v(1-\lambda))-\theta_L(3+\lambda)))^2 \end{array}\right\}}{\lambda f(3+\lambda(10+3\lambda-8f(1+\lambda)))^2}.$$

Conditional on the other firm offering a within-category exchange program, the focal firm's profit difference between offering within-category exchange and no promotion is

$$\Pi^{WW} - \Pi^{NW} = \frac{\lambda f(a+(1-g)\theta_L)^2}{4t} + \frac{((1+(1-2f)\lambda)(t(1-\lambda)+2(1-f)\theta_L\lambda)^2}{2t(1+(3-4f)\lambda)^2} - (1+\lambda)\frac{\left\{\begin{array}{r} (t(1-\lambda)(3+(1-4f)\lambda)+2\lambda(\theta_L(3+\lambda) \\ -4f^2(v-g\theta_L)\lambda+f(2v(1+\lambda) \\ -\theta_L(1+3\lambda+2g(1+\lambda)))))^2 \end{array}\right\}}{2t(3+\lambda(10+3\lambda-8f(1+\lambda)))^2}.$$

This profit difference is positive when $a > a_{W2} = \sqrt{A_{W2}} - (1-g)\theta_L$, where

$$A_{W2} = -\frac{2(1+(1-2f)\lambda)(t(1-\lambda)+2(1-f)\theta_L\lambda)^2}{\lambda f(1+(3-4f)\lambda)^2} + (1+\lambda)\frac{\left\{\begin{array}{r} 2(t(1-\lambda)(3+(1-4f)\lambda)+2\lambda(\theta_L(3+\lambda) \\ -4f^2(v-g\theta_L)\lambda+f(2v(1+\lambda) \\ -\theta_L(1+3\lambda+2g(1+\lambda)))))^2 \end{array}\right\}}{\lambda f(3+\lambda(10+3\lambda-8f(1+\lambda)))^2}.$$

The profit difference between Case WW and Case NN is given as follows:

$$\Pi^{WW} - \Pi^{NN} = \frac{\lambda f(a+(1-g)\theta_L)^2}{4t} - \frac{(1+\lambda)(t-t\lambda+2(\theta_L+f(v-g\theta_L))\lambda)^2}{2t(1+3\lambda)^2} + \frac{(1+(1-2f)\lambda)(t(1-\lambda)+2(1-f)\theta_L\lambda)^2}{2t(1+(3-4f)\lambda)^2}.$$

When $a > a_{W1} = \sqrt{A_{W1}} - (1-g)\theta_L$, where $A_{W1} = (2(1+\lambda)\cdot(t-t\lambda+2(\theta_L+f(v-g\theta_L))\lambda)^2)/(\lambda f(1+3\lambda)^2) - (2(1+(1-2f)\lambda)\cdot(t(1-\lambda)+2(1-f)\theta_L\lambda)^2)/(\lambda f(1+(3-4f)\lambda)^2)$, both firms are better off by adopting the within-category exchange policy.

Next, we present firms' trade-off between MM and WW. Conditional on the other firm offering within-category exchange, the focal firm's profit difference between offering multicategory exchange and within-category exchange is

$$\Pi^{MW} - \Pi^{WW} = \frac{(1-f)f(a-v+\theta_L)^2\lambda}{4t} - \frac{(1+(1-2f)\lambda)(t(1-\lambda)+2(1-f)\theta_L\lambda)^2}{2t(1+(3-4f)\lambda)^2} + \frac{\left\{\begin{array}{r} (1+(1-2(2-f)f)\lambda)(t(1-\lambda)(3+(1-4f)\lambda) \\ +2(1-f)\theta_L\lambda(3+\lambda-2f(1+(3-2f)\lambda)))^2 \end{array}\right\}}{2t(3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda)))^2}.$$

This profit difference is positive when $a > a_{WM3} = \sqrt{A_{WM3}} - \theta_L + v$, where

$$A_{WM3} = \frac{2(1+(1-2f)\lambda)(t(1-\lambda)+2(1-f)\theta_L\lambda)^2}{(1-f)f\lambda(1+(3-4f)\lambda)^2} - \frac{\left\{\begin{array}{r} 2(1+(1-2(2-f)f)\lambda)(t(1-\lambda)(3+(1-4f)\lambda) \\ +2(1-f)\theta_L\lambda(3+\lambda-2f(1+(3-2f)\lambda)))^2 \end{array}\right\}}{(1-f)f\lambda(3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda)))^2}.$$

Conditional on the other firm offering a multicategory exchange program, the focal firm's profit difference between offering multicategory exchange and within-category exchange is

$$\Pi^{MM} - \Pi^{WM} = \frac{(1-f)f(a-v+\theta_L)^2\lambda}{4t} + \frac{(1+(1-2(2-f)f)\lambda)(t-t\lambda+2(1-f)^2\theta_L\lambda)^2}{2t(1+(3-4(2-f)f)\lambda)^2} - \frac{\left\{\begin{array}{r} (1+(1-2f)\lambda)(t(1-\lambda)(3+(1-4(2-f)f)\lambda) \\ +2(1-f)\theta_L\lambda(3+\lambda-f(1+(7-4f)\lambda)))^2 \end{array}\right\}}{2t(3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda)))^2}.$$

This profit comparison is positive when $a > a_{WM2} = \sqrt{A_{WM2}} - \theta_L + v$, where

$$A_{WM2} = \frac{\left\{\begin{array}{r} 2(1+(1-2f)\lambda)(t(1-\lambda)(3+(1-4(2-f)f)\lambda) \\ +2(1-f)\theta_L\lambda(3+\lambda-f(1+(7-4f)\lambda)))^2 \end{array}\right\}}{(1-f)f\lambda(3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda)))^2} - \frac{2(1+(1-2(2-f)f)\lambda)(t-t\lambda+2(1-f)^2\theta_L\lambda)^2}{(1-f)f\lambda(1+(3-4(2-f)f)\lambda)^2}.$$

Furthermore, when $a > a_{WM1} = \sqrt{A_{WM1}} - \theta_L + v$, where

$$A_{WM1} = \frac{2(1+(1-2f)\lambda)(t(1-\lambda)+2(1-f)\theta_L\lambda)^2}{(1-f)f\lambda(1+(3-4f)\lambda)^2} - \frac{2(1+(1-2(2-f)f)\lambda)(t-t\lambda+2(1-f)^2\theta_L\lambda)^2}{(1-f)f\lambda(1+(3-4(2-f)f)\lambda)^2},$$

MM is more profitable than WW based on the following profit difference:

$$\Pi^{MM} - \Pi^{WW} = \frac{(1-f)f(a-v+\theta_L)^2\lambda}{4t} + \frac{(1+(1-2(2-f)f)\lambda)(t-t\lambda+2(1-f)^2\theta_L\lambda)^2}{2t(1+(3-4(2-f)f)\lambda)^2} - \frac{(1+(1-2f)\lambda)(t(1-\lambda)+2(1-f)\theta_L\lambda)^2}{2t(1+(3-4f)\lambda)^2}.$$

Last, we present firms' trade-off between MM and NN. Conditional on the other firm offering no promotion, the focal firm's profit difference between offering multicategory exchange and no promotion is

$$\Pi^{MN} - \Pi^{NN} = \frac{(1-f)f(a-v+\theta_L)^2\lambda}{4t} + \frac{f(a+\theta_L-g\theta_L)^2\lambda}{4t}$$

$$-\frac{(1+\lambda)(t-t\lambda+2(\theta_L+f(v-g\theta_L))\lambda)^2}{2t(1+3\lambda)^2}$$
$$+\frac{\left\{\begin{array}{r}(1+(1-2(2-f)f)\lambda)(t(1-\lambda)(3+\lambda)\\+2\lambda(2f^2\theta_L(1+\lambda)+\theta_L(3+\lambda)\\+f(v-(4+g)\theta_L-v\lambda-(4-g)\theta_L\lambda)))^2\end{array}\right\}}{2t(3+\lambda(10+3\lambda-8(2-f)f(1+\lambda)))^2}.$$

This profit difference is positive when

$$a>a_{M3}=\frac{\sqrt{A_1^2-4A_0B_{M3}}-A_1}{2A_0},$$

where

$$A_0=\frac{(2-f)f\lambda}{4t},\ A_1=\frac{f((2-f-g)\theta_L-(1-f)v)\lambda}{2t},$$

$$B_{M1}=\frac{(1-f)f(v-\theta_L)^2\lambda+f(\theta_L-g\theta_L)^2\lambda}{4t}+\frac{(1+(1-2(2-f)f)\lambda)(t-t\lambda+2(1-f)^2\theta_L\lambda)^2}{2t(1+(3-4(2-f)f)\lambda)^2}-\frac{(1+\lambda)(t-t\lambda+2(\theta_L+f(v-g\theta_L))\lambda)^2}{2t(1+3\lambda)^2},$$

$$B_{M2}=\frac{(1-f)f(v-\theta_L)^2\lambda+f(\theta_L-g\theta_L)^2\lambda}{4t}+\frac{(1+(1-2(2-f)f)\lambda)(t-t\lambda+2(1-f)^2\theta_L\lambda)^2}{2t(1+(3-4(2-f)f)\lambda)^2}$$
$$-(1+\lambda)\frac{\left\{\begin{array}{r}t(1-\lambda)(3+(1-4(2-f)f)\lambda)\\+2\lambda(2fv+3\theta_L+f(f-2(1+g))\theta_L\\+(2f(1-2(2-f)f)v+\theta_L-f(2(3+g)\\-f(3+4(2-f)g))\theta_L)\lambda))^2\end{array}\right\}}{2t(3+\lambda(10+3\lambda-8(2-f)f(1+\lambda)))^2},$$

$$B_{M3}=\frac{(1-f)f(v-\theta_L)^2\lambda+f(\theta_L-g\theta_L)^2\lambda}{4t}-\frac{(1+\lambda)(t-t\lambda+2(\theta_L+f(v-g\theta_L))\lambda)^2}{2t(1+3\lambda)^2}$$
$$+\frac{\left\{\begin{array}{r}(1+(1-2(2-f)f)\lambda)(t(1-\lambda)(3+\lambda)\\+2\lambda(2f^2\theta_L(1+\lambda)+\theta_L(3+\lambda)\\+f(v-(4+g)\theta_L-v\lambda-(4-g)\theta_L\lambda)))^2\end{array}\right\}}{2t(3+\lambda(10+3\lambda-8(2-f)f(1+\lambda)))^2}.$$

Conditional on the other firm not offering any promotion, the focal firm's profit difference between offering multicategory exchange and no promotion is

$$\Pi^{MM}-\Pi^{NM}=\frac{(1-f)f(a-v+\theta_L)^2\lambda+f(a+(1-g)\theta_L)^2\lambda}{4t}+\frac{(1+(1-2(2-f)f)\lambda)(t-t\lambda+2(1-f)^2\theta_L\lambda)^2}{2t(1+(3-4(2-f)f)\lambda)^2}$$
$$-(1+\lambda)\frac{\left\{\begin{array}{r}t(\lambda-1)(3+(1-4(2-f)f)\lambda)\\+2\lambda(-3\theta_L+f(-2v+(2-f+2g)\theta_L)\\-(2f(1-2(2-f)f)v+\theta_L\\+f(f(3+4(2-f)g)-2(3+g))\theta_L)\lambda))^2\end{array}\right\}}{2t(3+\lambda(10+3\lambda-8(2-f)f(1+\lambda)))^2}.$$

This profit difference is positive when

$$a>a_{M2}=\frac{\sqrt{A_1^2-4A_0B_{M2}}-A_1}{2A_0}.$$

Finally, the profit difference between Case MM and Case NN is given as follows:

$$\Pi^{MM}-\Pi^{NN}=\frac{(1-f)f(a-v+\theta_L)^2\lambda+f(a+(1-g)\theta_L)^2\lambda}{4t}+\frac{(1+(1-2(2-f)f)\lambda}{2t(1+(3-4(2-f)f)\lambda)^2}(t-t\lambda+2(1-f)^2\theta_L\lambda)^2-\frac{(1+\lambda)(t-t\lambda+2(\theta_L+f(v-g\theta_L))\lambda)^2}{2t(1+3\lambda)^2}.$$

This profit difference is positive when

$$a>a_{M1}=\frac{\sqrt{A_1^2-4A_0B_{M1}}-A_1}{2A_0}.$$

To summarize, when waste aversion is above a threshold, i.e., $a\ge\max\{a_{M3},a_{WM2},a_{WM3}\}$, both firms offer multicategory exchange programs in equilibrium. When waste aversion is moderate, i.e., when $a_{W3}\le a\le\min\{a_{WM2},a_{WM3}\}$, both firms offer within-category exchange programs in equilibrium. When waste aversion is below a threshold, i.e., $a\le\min\{a_{W2},a_{M2}\}$, neither firm promotes in equilibrium. □

In the online appendix (available as supplemental material at http://dx.doi.org/10.1287/mksc.2015.0955), we show that $a_{W1}<a_{W2}<a_{W3}$, $a_{M1}<a_{M2}<a_{M3}$, and $a_{WM1}<\min\{a_{WM2},a_{WM3}\}$.

Proof of Proposition 2. When $a<a_{M2}$, $\Pi^{NN}>\Pi^{MN}$ and $\Pi^{NM}>\Pi^{MM}$, so MM is never the equilibrium. When $a_{W1}\le a\le a_{W2}$, $\Pi^{WW}>\Pi^{NN}$ but $\Pi^{WW}<\Pi^{NW}$, $\Pi^{WN}<\Pi^{NN}$. Therefore, NN is a prisoner's dilemma outcome in this parameter range: $a_{W1}\le a\le\min\{a_{W2},a_{M2}\}$.

When $a>a_{W3}$, $\Pi^{WN}>\Pi^{NN}$ and $\Pi^{WW}>\Pi^{NW}$, so NN is never the equilibrium. When $a_{WM1}\le a\le\min\{a_{WM2},a_{WM3}\}$, $\Pi^{MM}>\Pi^{WW}$ but $\Pi^{MM}<\Pi^{WM}$, $\Pi^{MW}<\Pi^{WW}$. Therefore, WW is a prisoner's dilemma outcome in this parameter range: $\max\{a_{WM1},a_{W3}\}\le a\le\min\{a_{WM2},a_{WM3}\}$. □

Proof of Proposition 3. We now show comparative statics of the relevant cutoff thresholds a^*'s with respect to λ:

$$\frac{\partial a_{WM2}}{\partial\lambda}>0,\quad\frac{\partial a_{WM3}}{\partial\lambda}>0,\quad\frac{\partial a_{W3}}{\partial\lambda}>0.$$

$$sign\left[\frac{\partial a_{WM2}}{\partial\lambda}\right]=C_1(\lambda,f)t-C_2(\lambda,f)\theta_L,$$

where $C_1(\lambda,f)$ and $C_2(\lambda,f)$ are continuous functions of λ and f. We have $C_1(\lambda,f)>0$, $C_2(\lambda,f)>0$ when $\lambda\in(0,1)$, $f\in(0,1)$. Because $\theta_L<2t$, $C_1(\lambda,f)t-C_2(\lambda,f)\theta_L>t(C_1(\lambda,f)+2C_2(\lambda,f))$.

$$\begin{aligned}C_1(\lambda,f)+2C_2(\lambda,f)&=(1-\lambda)(1+(3-4(2-f)f)\lambda)\\&\quad\times(7-5f+2(13-f(43-4(9-2f)f))\lambda\\&\quad+(15-f(101-8f(27-22f+6f^2)))\lambda^2)\\&\quad\times(3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda)))>0,\end{aligned}$$

when $\lambda\in(0,1)$, $f\in(0,1)$. Hence, $\partial a_{WM2}/\partial\lambda>0$.

$$sign\left[\frac{\partial a_{WM3}}{\partial\lambda}\right]=C_3(\lambda,f)t-C_4(\lambda,f)\theta_L,$$

where $C_3(\lambda, f)$ and $C_4(\lambda, f)$ are continuous functions of λ and f. We have $C_3(\lambda, f) > 0$, $C_4(\lambda, f) > 0$ when $\lambda \in (0,1)$, $f \in (0,1)$. Because $\theta_L < 2t$, $C_3(\lambda, f)t - C_4(\lambda, f)\theta_L > t(C_3(\lambda, f) + 2C_4(\lambda, f))$.

$$\begin{aligned} &C_3(\lambda, f) + 2C_4(\lambda, f)\\ &\quad = (1-\lambda)(1+(3-4f)\lambda)\\ &\quad\quad \times \big(7-2f+2(13-6f(5-2f))\lambda\\ &\quad\quad\quad - (3-2f)(5-24(1-f)f)\lambda^2\big)\\ &\quad\quad \times (3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda))) > 0, \end{aligned}$$

when $\lambda \in (0,1)$, $f \in (0,1)$. Hence, $\partial a_{WM3}/\partial\lambda > 0$.

$$\begin{aligned} &sign\left[\frac{\partial a_{W3}}{\partial \lambda}\right]\\ &\quad = (1-\lambda)(1+3\lambda)(3+\lambda(10+3\lambda-8f(1+\lambda)))\\ &\quad\quad \times \big(4(1+\lambda)(1+3\lambda)^2+16f^2\lambda^2(3+5\lambda)\\ &\quad\quad\quad - f(1+3\lambda)(-1+5\lambda(6+7\lambda))\big) > 0, \end{aligned}$$

when $\lambda \in (0,1)$, $f \in (0,1)$. Hence, $\partial a_{W3}/\partial\lambda > 0$.

The first two inequalities, $\partial a_{WM2}/\partial\lambda > 0$ and $\partial a_{WM3}/\partial\lambda > 0$, mean that an increase in λ leads to a smaller likelihood of MM equilibrium compared to WW. The third inequality, $\partial a_{W3}/\partial\lambda > 0$, means that an increase in λ leads to a smaller likelihood of WW equilibrium compared to NN. The combination of the two results above has proved Proposition 3. □

Proof of Proposition 4. We now show comparative statics of exchange discounts with respect to different parameters

$$\frac{\partial e_X^{MM}}{\partial a} = -1/2, \quad \frac{\partial e_X^{MM}}{\partial g} = \theta_L/2, \quad \frac{\partial e_X^{MM}}{\partial v} = 0,$$

$$\frac{\partial e_X^{MM}}{\partial f} = \frac{4(1-f)(2t-\theta_L)(1-\lambda)\lambda}{(1+(3-4(2-f)f)\lambda)^2} > 0,$$

$$\frac{\partial e_X^{MM}}{\partial \lambda} = -\frac{2(1-f)^2(2t-\theta_L)}{(1+(3-4(2-f)f)\lambda)^2} < 0;$$

$$\frac{\partial e_Y^{MM}}{\partial a} = -\frac{1}{2}, \quad \frac{\partial e_Y^{MM}}{\partial g} = 0, \quad \frac{\partial e_Y^{MM}}{\partial v} = \frac{1}{2},$$

$$\frac{\partial e_Y^{MM}}{\partial f} = \frac{4(1-f)(2t-\theta_L)(1-\lambda)\lambda}{(1+(3-4(2-f)f)\lambda)^2} > 0,$$

$$\frac{\partial e_Y^{MM}}{\partial \lambda} = -\frac{2(1-f)^2(2t-\theta_L)}{(1+(3-4(2-f)f)\lambda)^2} < 0;$$

$$\frac{\partial e_X^{WW}}{\partial f} = \frac{2(2t-\theta_L)(1-\lambda)\lambda}{(1+(3-4f)\lambda)^2} > 0,$$

$$\frac{\partial e_X^{WW}}{\partial \lambda} = -\frac{(2(1-f)(2t-\theta_L))}{(1+(3-4f)\lambda)^2} < 0. \quad \square$$

Proof of Proposition 5. Before we present the proof, we list all of the cutoffs of g's. First, we define the following constants:

$$A_{W1} = f\theta_L^2\lambda\left(1-\frac{8f\lambda(\lambda+1)}{(3\lambda+1)^2}\right),$$

$$B_{W1} = \frac{8f\theta_L\lambda(\lambda+1)(2\lambda(fv+\theta_L)-\lambda t+t)}{(3\lambda+1)^2} - 2f\theta_L\lambda(a+\theta_L),$$

$$\begin{aligned} C_{W1} &= f\lambda(a+\theta_L)^2 - \frac{2((2f-1)\lambda-1)(2(f-1)\theta_L\lambda+(\lambda-1)t)^2}{((4f-3)\lambda-1)^2}\\ &\quad - \frac{2(\lambda+1)(2\lambda(fv+\theta_L)-\lambda t+t)^2}{(3\lambda+1)^2}; \end{aligned}$$

$$A_{W2} = \lambda\left(f\theta_L^2 - \frac{8\lambda(\lambda+1)(4f^2\theta_L\lambda-2f\theta_L(\lambda+1))^2}{(\lambda(-8f(\lambda+1)+3\lambda+10)+3)^2}\right),$$

$$\begin{aligned} B_{W2} &= -2f\theta_L\lambda(a+(8(\lambda+1)((2f-1)\lambda-1)\\ &\quad\quad \cdot(2\lambda(-4f^2\lambda v+f(2(\lambda+1)v-\theta_L(3\lambda+1))\\ &\quad\quad +\theta_L(\lambda+3))+(\lambda-1)t((4f-1)\lambda-3)))\\ &\quad\quad \cdot((\lambda(-8f(\lambda+1)+3\lambda+10)+3)^2)^{-1}+\theta_L), \end{aligned}$$

$$\begin{aligned} C_{W2} &= f\lambda(a+\theta_L)^2-(2(\lambda+1)(2\lambda(-4f^2\lambda v+f(2(\lambda+1)v\\ &\quad -\theta_L(3\lambda+1))+\theta_L(\lambda+3))+(\lambda-1)t((4f-1)\lambda-3))^2)\\ &\quad \cdot((\lambda(-8f(\lambda+1)+3\lambda+10)+3)^2)^{-1}\\ &\quad - \frac{2((2f-1)\lambda-1)(2(f-1)\theta_L\lambda+(\lambda-1)t)^2}{((4f-3)\lambda-1)^2}; \end{aligned}$$

$$A_{W3} = f\theta_L^2\lambda\left(-\frac{8f\lambda(\lambda-1)^2((2f-1)\lambda-1)}{(\lambda(8f(\lambda+1)-3\lambda-10)-3)^2} - \frac{8f\lambda(\lambda+1)}{(3\lambda+1)^2}+1\right),$$

$$\begin{aligned} B_{W3} &= 2f\theta_L\lambda\bigg(-a+\frac{4(\lambda+1)(2\lambda(fv+\theta_L)-\lambda t+t)}{(3\lambda+1)^2}\\ &\quad +(4(\lambda-1)((2f-1)\lambda-1)(2\lambda(f(2\theta_L(\lambda+1)\\ &\quad +(\lambda-1)v)-\theta_L(\lambda+3))+(\lambda-1)(\lambda+3)t))\\ &\quad \cdot((\lambda(8f(\lambda+1)-3\lambda-10)-3)^2)^{-1}-\theta_L\bigg), \end{aligned}$$

$$\begin{aligned} C_{W3} &= f\lambda(a+\theta_L)^2 - \frac{2(\lambda+1)(2\lambda(fv+\theta_L)-\lambda t+t)^2}{(3\lambda+1)^2}\\ &\quad -(2((2f-1)\lambda-1)(2\lambda(f(2\theta_L(\lambda+1)+(\lambda-1)v)\\ &\quad -\theta_L(\lambda+3))+(\lambda-1)(\lambda+3)t)^2)\\ &\quad \cdot((\lambda(8f(\lambda+1)-3\lambda-10)-3)^2)^{-1}; \end{aligned}$$

$$A_{M1} = f\theta_L^2\lambda\left(1-\frac{8f\lambda(\lambda+1)}{(3\lambda+1)^2}\right),$$

$$B_{M1} = \frac{8f\theta_L\lambda(\lambda+1)(2\lambda(fv+\theta_L)-\lambda t+t)}{(3\lambda+1)^2} - 2f\theta_L\lambda(a+\theta_L),$$

$$\begin{aligned} C_{M1} &= f\lambda(a+\theta_L)^2-(f-1)f\lambda(a+\theta_L-v)^2\\ &\quad + \frac{2((2(f-2)f+1)\lambda+1)(2(f-1)^2\theta_L\lambda-\lambda t+t)^2}{((4(f-2)f+3)\lambda+1)^2}\\ &\quad - \frac{2(\lambda+1)(2\lambda(fv+\theta_L)-\lambda t+t)^2}{(3\lambda+1)^2}; \end{aligned}$$

$$A_{M2} = f\theta_L^2\lambda\left(1-\frac{32f\lambda(\lambda+1)((2(f-2)f+1)\lambda+1)^2}{(\lambda(8(f-2)f(\lambda+1)+3\lambda+10)+3)^2}\right),$$

$$\begin{aligned} B_{M2} &= -2f\theta_L\lambda(a+(8(\lambda+1)((2(f-2)f+1)\lambda+1)\\ &\quad \cdot((\lambda-1)t((4(f-2)f+1)\lambda+3)-2\lambda((f-2)f\theta_L\\ &\quad +\lambda(f(3(f-2)\theta_L+(4(f-2)f+2)v)+\theta_L)+2fv+3\theta_L)))\\ &\quad \cdot((\lambda(8(f-2)f(\lambda+1)+3\lambda+10)+3)^2)^{-1}+\theta_L), \end{aligned}$$

$$C_{M2} = f\lambda(a+\theta_L)^2-(f-1)f\lambda(a+\theta_L-v)^2$$

$$
\begin{aligned}
&+\frac{2((2(f-2)f+1)\lambda+1)(2(f-1)^2\theta_L\lambda-\lambda t+t)^2}{((4(f-2)f+3)\lambda+1)^2}\\
&-(2(\lambda+1)((\lambda-1)t((4(f-2)f+1)\lambda+3)-2\lambda((f-2)f\theta_L\\
&+\lambda(f(3(f-2)\theta_L+(4(f-2)f+2)v)+\theta_L)\\
&+2fv+3\theta_L))^2)\cdot((\lambda(8(f-2)f(\lambda+1)+3\lambda+10)+3)^2)^{-1};
\end{aligned}
$$

$$
\begin{aligned}
A_{M3} &= f\theta_L^2\lambda\biggl(\frac{8f\lambda(\lambda-1)^2((2(f-2)f+1)\lambda+1)}{(\lambda(8(f-2)f(\lambda+1)+3\lambda+10)+3)^2}\\
&\qquad-\frac{8f\lambda(\lambda+1)}{(3\lambda+1)^2}+1\biggr),\\
B_{M3} &= 2f\lambda\biggl(-\theta_L(a+\theta_L)-(4\theta_L(\lambda-1)((2(f-2)f+1)\lambda+1)\\
&\qquad\cdot((\lambda-1)(\lambda+3)t-2\lambda(2f^2\theta_L(\lambda+1)\\
&\qquad+f(-4\theta_L(\lambda+1)-\lambda v+v)+\theta_L(\lambda+3))))\\
&\qquad\cdot((\lambda(8(f-2)f(\lambda+1)+3\lambda+10)+3)^2)^{-1}\\
&\qquad+\frac{4\theta_L(\lambda+1)(2\lambda(fv+\theta_L)-\lambda t+t)}{(3\lambda+1)^2}\biggr),\\
C_{M3} &= f\lambda(a+\theta_L)^2-(f-1)f\lambda(a+\theta_L-v)^2\\
&\quad+(2((2(f-2)f+1)\lambda+1)((\lambda-1)(\lambda+3)t\\
&\quad-2\lambda(2f^2\theta_L(\lambda+1)+f(-4\theta_L(\lambda+1)-\lambda v+v)\\
&\quad+\theta_L(\lambda+3)))^2)\cdot((\lambda(8(f-2)f(\lambda+1)+3\lambda+10)+3)^2)^{-1}\\
&\quad-\frac{2(\lambda+1)(2\lambda(fv+\theta_L)-\lambda t+t)^2}{(3\lambda+1)^2};
\end{aligned}
$$

$$
g_{W1}=\frac{\sqrt{B_{W1}^2-4A_{W1}C_{W1}}-B_{W1}}{2A_{W1}},
$$

$$
g_{W2}=\frac{\sqrt{B_{W2}^2-4A_{W2}C_{W2}}-B_{W2}}{2A_{W2}},
$$

$$
g_{W3}=\frac{\sqrt{B_{W3}^2-4A_{W3}C_{W3}}-B_{W3}}{2A_{W3}},
$$

$$
g_{M1}=\frac{\sqrt{B_{M1}^2-4A_{M1}C_{M1}}-B_{M1}}{2A_{M1}},
$$

$$
g_{M2}=\frac{\sqrt{B_{M2}^2-4A_{M2}C_{M2}}-B_{M2}}{2A_{M2}},
$$

$$
g_{M3}=\frac{\sqrt{B_{M3}^2-4A_{M3}C_{M3}}-B_{M3}}{2A_{M3}}.
$$

Finally, we list the following cutoffs based on v:

$$
\begin{aligned}
A &= (1-f)f\lambda,\quad B=-2(1-f)f(a+\theta_L)\lambda,\\
C_1 &= (1-f)f(a+\theta_L)^2\lambda\\
&\quad-\frac{2(1+(1-2f)\lambda)(t(1-\lambda)+2(1-f)\theta_L\lambda)^2}{(1+(3-4f)\lambda)^2}\\
&\quad+\frac{2(1+(1-2(2-f)f)\lambda)(t-t\lambda+2(1-f)^2\theta_L\lambda)^2}{(1+(3-4(2-f)f)\lambda)^2};\\
C_2 &= (1-f)f(a+\theta_L)^2\lambda\\
&\quad+\frac{2(1+(1-2(2-f)f)\lambda)(t-t\lambda+2(1-f)^2\theta_L\lambda)^2}{(1+(3-4(2-f)f)\lambda)^2}\\
&\quad-(2(1+(1-2f)\lambda)(t(1-\lambda)(3+(1-4(2-f)f)\lambda)\\
&\qquad+2(1-f)\theta_L\lambda(3+\lambda-f(1+(7-4f)\lambda)))^2)\\
&\quad\cdot((3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda)))^2)^{-1};\\
C_3 &= (1-f)f(a+\theta_L)^2\lambda\\
&\quad-\frac{2(1+(1-2f)\lambda)(t(1-\lambda)+2(1-f)\theta_L\lambda)^2}{(1+(3-4f)\lambda)^2}\\
&\quad+(2(1+(1-2(2-f)f)\lambda)(t(1-\lambda)(3+(1-4f)\lambda)\\
&\qquad+2(1-f)\theta_L\lambda(3+\lambda-2f(1+(3-2f)\lambda)))^2)\\
&\quad\cdot((3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda)))^2)^{-1};\\
v_1 &= \frac{-\sqrt{B^2-4AC_1}-B}{2A},\quad v_2=\frac{-\sqrt{B^2-4AC_2}-B}{2A},\\
v_3 &= \frac{-\sqrt{B^2-4AC_3}-B}{2A}.
\end{aligned}
$$

When $a=0$, then the following inequalities hold:

1. When $g>\max\{g_{W2},g_{W3}\}$, then $\Pi^{WW}>\Pi^{NW}$, $\Pi^{WN}>\Pi^{NN}$, implying WW is the equilibrium outcome compared to NN. When $g>\max\{g_{M2},g_{M3}\}$, then $\Pi^{MM}>\Pi^{NM}$, $\Pi^{MN}>\Pi^{NN}$, implying MM is the equilibrium outcome compared to NN. Therefore, when $g>\max\{g_{W2},g_{W3},g_{M2},g_{M3}\}$, then either within-category or multicategory exchange is the equilibrium outcome. Furthermore, when $v<\min\{v_2,v_3\}$, $\Pi^{MM}>\Pi^{WM}$, $\Pi^{MW}>\Pi^{WW}$, implying that under these conditions, multicategory exchange is the unique equilibrium. By contrast, when $v>\max\{v_2,v_3\}$, $\Pi^{MM}<\Pi^{WM}$, $\Pi^{MW}<\Pi^{WW}$, implying that within-category exchange is the unique equilibrium.
2. By contrast, when $g<\min\{g_{W2},g_{W3},g_{M2},g_{M3}\}$, we observe $\Pi^{WW}<\Pi^{NW}$, $\Pi^{WN}<\Pi^{NN}$ and $\Pi^{MM}<\Pi^{NM}$, $\Pi^{MN}<\Pi^{NN}$. Therefore, no promotion is the equilibrium outcome.
3. When g takes values other than the above, coordination equilibria such as {WW, MM} exist. □

Boundary Conditions on Waste Aversion

The following conditions ensure that all exchange discounts across the six unique subgames are weakly positive: $e_k^{SS'}\geq 0$, where $S,S'\in\{N,W,M\}$, $k\in\{X,Y\}$;

$$
a\leq a_X^{WN}=\frac{\left\{\begin{array}{c}4fv(1-\lambda)\lambda+2t(1-\lambda)(3+\lambda)\\+\theta_L(3+\lambda)(1-g-(1+3g-4fg)\lambda)\end{array}\right\}}{3+\lambda(10+3\lambda-8f(1+\lambda))};
$$

$$
a\leq a_Y^{MM}=\frac{v+2t(1-\lambda)-\theta_L(1-\lambda)+(3-4(2-f)f)v\lambda}{1+(3-4(2-f)f)\lambda};
$$

$$
a\leq a_Y^{MW}=v+\frac{(2t-\theta_L)(1-\lambda)(3+(1-4f)\lambda)}{3+\lambda(10+3\lambda-8f(3-f+(1-f)(3-2f)\lambda))};
$$

$$
a\leq a_Y^{MN}=\frac{\left\{\begin{array}{c}2t(1-\lambda)(3+\lambda)-\theta_L(1-\lambda)(3+\lambda+4fg\lambda)\\+v(3+\lambda(10+3\lambda-4f(3+5\lambda-2f(1+\lambda))))\end{array}\right\}}{3+\lambda(10+3\lambda-8(2-f)f(1+\lambda))}.
$$

Consumers' Private Sale of Old Goods

Assume that the private sale prices for the old products X and Y are P_X and P_Y. Similar to the case of exchange, assume that consumers get a utility increase of a if they engage in a private sale.

When the benefits from a private sale are lower than consumers' valuations for their old products for all consumers, i.e., $v > P_X + a$ and $v > P_Y + a$, it is clear that no consumer will engage in a private sale. For example, consumers may find the transaction costs of engaging in private sales such as finding a buyer, inviting strangers to their home to inspect the good, etc., to be prohibitively high. In such instances, consumers will simply store the goods rather than try to sell them. In these scenarios, all of the results of our analysis still apply.

Another possibility is that the best prices that any consumer can get from private sale are lower than the exchange discounts, i.e., $e_X > P_X$ and $e_Y > P_Y$. In this case, all consumers will prefer exchange to private sale, and all of our exchange equilibrium results stay the same. In this scenario, the only time that consumers' private sales can affect our analysis is when the equilibrium outcome results in neither firm offering an exchange promotion, i.e., the NN equilibrium. In this case, consumers who have old products may privately sell their old products, and some of these consumers may in turn buy new X.

The only time that private sale prices affect our equilibrium results is when private sales prices are moderately high, i.e., $\theta_L > P_X > g\theta_L$, $\theta_L > P_Y > v$, and they are higher than the previous exchange discounts given in §3. In this case, firms' new optimal exchange discounts are

$$e_X^{WW} = \frac{t(1-\lambda)+2\theta_L\lambda(1-f)}{1+3\lambda-4f\lambda} - \frac{\theta_L - P_X}{2},$$

$$e_X^{MM} = \frac{t(1-\lambda)+2\theta_L\lambda(1-f)^2}{1+3\lambda-8f\lambda+4f^2\lambda} - \frac{\theta_L - P_X}{2}, \quad \text{and}$$

$$e_X^{MM} = \frac{t(1-\lambda)+2\theta_L\lambda(1-f)^2}{1+3\lambda-8f\lambda+4f^2\lambda} - \frac{\theta_L - P_Y}{2}.$$

As seen from the above expressions, exchange discounts increase in P_X and P_Y, and they no longer depend on consumers' waste aversion a. This happens because both private sales and exchange promotions provide consumers the utility gain of a, so it cancels out from consumers' trade-off between their best options. Note that WW and MM exchange promotions can still be equilibrium outcomes because of price discrimination reasons. As long as it is not too costly to offer exchange promotions, i.e., when P_X and P_Y are not too high, firms offer exchanges in the equilibrium. On the other hand, when P_X and P_Y are very high, we expect NN equilibrium to prevail.

Given the existence of exchange discounts, we believe that our analysis captures the situation in some markets where the private sale opportunities are not attractive, as to support only the NN equilibrium. It is easy to see that consumers may differ in terms of private sale opportunities open to them and their abilities to engage in private sales. This consumer heterogeneity may or may not be correlated with consumers' valuations and ownership of old goods. Even when these heterogeneities are correlated, the correlations may be positive or negative. Thus, the impact of private sale options on firm strategies would in fact be more complex (and its analysis more tedious).

References

Arkes H (1996) The psychology of waste. *J. Behav. Decision Making* 9(3):213–224.

Arkes H, Blumer C (1985) The psychology of sunk cost. *Organ. Behav. Human Performance* 35:124–140.

Bolton LE, Alba JW (2012) When less is more: Consumer aversion to unused utility. *J. Consumer Psych.* 22(3):369–383.

Desai P, Purohit D (2004) Let me talk to my manager: Haggling in a competitive environment. *Marketing Sci.* 23(2):219–233.

Haws KL, Naylor RW, Coulter RA, Bearden WO (2012) Keeping it all without being buried alive: Understanding product retention tendency. *J. Consumer Psych.* 22(2):224–236.

Hotelling H (1929) Stability in competition. *Econom. J.* 39(153):41–57.

Kim J, Rao RS, Kim K, Rao AR (2011) More or less: A model and empirical evidence on preferences for under- and overpayment in trade-in transactions. *J. Marketing Res.* 48(1):157–171.

Krishnan TV, Rao R (1995) Double couponing and retail pricing in a couponed product category. *J. Marketing Res.* 32(4):419–432.

Lal R, Rao R (1997) Supermarket competition: The case of every day low pricing. *Marketing Sci.* 16(1):60–80.

Narasimhan C (1984) A price discrimination theory of coupons. *Marketing Sci.* 3(2):128–147.

Okada EM (2001) Trade-ins, mental accounting, and product replacement decisions. *J. Consumer Res.* 27(4):433–446.

Purohit D (1995) Playing the role of buyer and seller: The mental accounting of trade-ins. *Marketing Lett.* 6(2):101–110.

Raju JS, Dhar SK, Morrison DG (1994) The effect of package coupons on brand choice. *Marketing Sci.* 13(2):145–164.

Rao RS, Narasimhan O, John G (2009) Understanding the role of trade-ins in durable goods markets: Theory and evidence. *Marketing Sci.* 28(5):950–967.

Shaffer G, Zhang ZJ (1995) Competitive coupon targeting. *Marketing Sci.* 14(4):395–416.

Shaffer G, Zhang ZJ (2002) Competitive one-to-one promotions. *Management Sci.* 48(9):1143–1160.

Thaler R (1985) Mental accounting and consumer choice. *Marketing Sci.* 4(3):199–214.

Thaler R (1999) Mental accounting matters. *J. Behav. Decision Making* 12(3):183–206.

Van Ackere A, Reyniers DJ (1993) A rationale for trade-ins. *J. Econom. Bus.* 45(1):1–16.

World Values Survey Wave 5 (2014) 2005–2008 Official Aggregate v.20140429. World Values Survey Association, http://www.worldvaluessurvey.org.

Van Ackere A, Reyniers DJ (1995) Trade-ins and introductory offers in a monopoly. *RAND J. Econom.* 26(1):58–74.

Zhu R, Chen X, Dasgupta S (2008) Can trade-ins hurt you? Exploring the effect of a trade-in on consumers' willingness to pay for a new product. *J. Marketing Res.* 45(2):159–170.

Zultan R, Bar-Hillel M, Guy N (2010) When being wasteful appears better than feeling wasteful. *J. Behav. Decision Making* 5(7): 489–496.