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Social Learning and Dynamic Pricing of Durable Goods

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We analyze the impacts of social learning (SL) on the dynamic pricing and consumer adoption of durable goods in a two-period monopoly. Consumers can make either early, uninformed purchases or late but potentially informed purchases as a result of social learning. Several results are derived. First, we identify the market conditions under which ex ante homogeneous consumers may choose to purchase at different times. Second, equilibrium adoption may demonstrate inertia (where all adopt late) or frenzy (where all adopt early). In particular, adoption inertia appears when SL intensity is reasonably high but may vanish when SL intensity exceeds a certain threshold. Third, firm profits and social welfare first weakly decrease in SL intensity and may then jump up by a lump-sum amount at the threshold SL intensity level mentioned above. Last, we show that the firm potentially benefits from informative advertising or investing to cultivate more social learning.

Key words: durable goods; dynamic pricing; social learning

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1. Introduction

Social learning refers to the general phenomenon that individual social beings often can learn from each other about an object of interest. Besides humans, social learning is well observed among animals and insects. For instance, ants, when faced with two identical food sources, tend to concentrate more on one and only occasionally switch their attention to the other (Deneubourg et al. 1987). In this paper, we investigate how social learning among peer consumers drives the dynamic pricing and adoption of durable experience goods. People frequently exchange information about and experience with an array of products such as books, movies, electronic devices, and even automobiles. More recently, the widespread adoption of the Internet and online social media (such as online communities) has dramatically magnified the influence of word of mouth, as a product review online is easily viewed by tens of thousands of prospective buyers.¹

For consumable goods with relatively low prices and high repurchase frequency, purchase itself is a viable way of consumer learning (Bergemann and Välimäki 2006; Villas-Boas 2004, 2006). However, purchase is often not feasible for learning about durables,

which, on average, carry higher prices and are not repurchased frequently. The lack of this alternative means of consumer learning makes social learning particularly important in durable goods markets.

We develop a simple model of social learning in a two-period monopoly of some durable good. The finite horizon reflects that the product has a limited market window, possibly because of advances in technology or changes in consumer tastes. The durable is an experience good in that ex ante consumers are uncertain about their match with the product. The product is launched at the beginning of period 1, and all purchases in this period are uninformed because of a lack of consumer learning. If some consumers purchase in period 1, however, by period 2, the remaining consumers are likely to discover their true valuation via social learning. We assume that the probability that a consumer deferring purchase becomes informed by period 2 depends on two factors: (1) the product's social learning (SL) intensity, which reflects its tendency to be discussed via word of mouth; and (2) the product's installed base developed in period 1. At the beginning of each period, the firm chooses a price for that period, and each consumer who has not yet purchased then decides whether to purchase. The firm maximizes its discounted profits, and consumers maximize their expected discounted utility. By manipulating the period 1 installed base via price, the firm effectively controls the pace of dissemination of product match information.

¹ Besides online or off-line word of mouth, another channel of social learning about durable goods is momentary use of others' products. For example, playing with a friend's cell phone or video game console helps reveal how one likes or dislikes that phone or console.

The consumers face the following trade-off in period 1: *buying now* means an uninformed purchase but sooner gratification, whereas *deferring adoption* delays gratification but renders a potentially informed purchase. To capture this trade-off, we allow mixed consumer strategies: if a consumer is indifferent between buying and not buying in period 1, she chooses a probability with which to purchase. At the core of the analysis lies the positive externality of social learning (which we call the “learning externality”): the late buyers potentially gain from the product information generated by the early buyers. However, when the period 1 price is sufficiently lower than the expected period 2 price, the consumers will be indifferent to adoption timing, and some will indeed purchase early.

Several results are reached. First, we identify the market conditions under which *ex ante* homogeneous consumers may adopt at different times. The existing literature on the diffusion of innovation (e.g., Rogers 1962, Bass 1969) often attributes dispersed adoption to behavioral heterogeneity among consumers (the familiar innovator-imitator dichotomy). In contrast, it is the learning externality that drives dispersed adoption in our model. Consumers are naturally inclined to postpone adoption to make potentially informed purchases through social learning. This tendency forces the firm to lower the period 1 price so that early adoption becomes equally attractive.

Second, the equilibrium may exhibit *adoption frenzy*, where all consumers purchase early although uninformed, or *adoption inertia*, where all consumers defer purchase to the second period. Adoption frenzy occurs when weak SL intensity or severe discounting dissipates the benefit of delaying adoption. When SL intensity is reasonably high, consumers’ propensity to exploit each other’s early consumption experience can be overwhelming so that they all postpone purchase. When adoption inertia is obtained, firm profits are at their lowest level, and consumers receive zero expected surplus. Interestingly, adoption inertia may vanish when SL intensity exceeds a certain threshold (defined below). This is because when SL intensity exceeds this threshold and the period 1 installed base is sufficiently large, the firm will raise the period 2 price from the mean consumer valuation to the full information price, causing a lump-sum drop in the expected utility of deferring adoption. Anticipating such an outcome, consumers will modify their purchase decision, preventing adoption inertia.

Third, when the unit cost exceeds consumers’ mean valuation, firm profits and social welfare weakly increase in SL intensity. Otherwise, they both first (weakly) decrease in SL intensity and may jump up by a discrete amount at the threshold level alluded to

above. Below this threshold, stronger social learning may encourage delayed adoption. Discounting and more heterogeneous consumer valuation in period 2 then undermine firm profits and social welfare. When SL intensity exceeds this threshold, the firm may switch to a higher period 2 price, which helps promote early adoption and increase firm profits and social welfare.

Last, we extend the model in several directions. When SL intensity is below the threshold level, we show that the firm potentially benefits from informative advertising or investing to cultivate more social learning about its product. The spirits of our key results are preserved when SL intensity is endogenous to the realized valuation of the early adopters or when consumers have heterogeneous prior valuation.

This paper is related to several streams of literature: durable goods monopolists, Bayesian and empirical models of social learning, diffusion of innovations, and experience goods. Coase (1972) and many subsequent works (e.g., Bulow 1982, Conlisk et al. 1984, Sobel 1991, Stokey 1981) argue that a monopolist producing a durable good may lose its market power if it is unable to commit to future price or production. In a durable goods monopoly, Desai and Purohit (1998) analyze the optimal portfolio of leasing and selling, and Bulow (1986) and Levinthal and Purohit (1989) examine the effect of product obsolescence. In these models, *ex ante* consumers know their valuation of the product, and the monopolist’s market power is undermined by the inherent time inconsistency instead of by consumers’ uncertainty over product value (as in ours).

There exist several Bayesian models of social learning, where each consumer observes a signal and can potentially use it to update her belief about product value. McFadden and Train (1996) develop a three-period model of self versus social learning of consumable (rather than durable) goods. In Banerjee (1992) and Bikhchandani et al. (1992), consumers make decisions in a predetermined sequence. Each consumer receives a private signal and also observes the choices made by previous consumers. The equilibrium in both models is herding; i.e., people choose what others have chosen rather than use their private information. In Ellison and Fudenberg (1993, 1995), nonstrategic consumers repeatedly choose between two technologies and follow exogenously specified behavior rules. They find that the naive decision rules can lead to fairly efficient choices in the long run.

In these Bayesian models, pricing is not explicitly addressed, and the pace of social learning is exogenously specified. In contrast, we analyze how the firm’s dynamic pricing policy interacts with the adoption decision of rational consumers. The pace of social learning is thus endogenous. In our

model, social learning is informative to some of the late adopters and uninformative to the others. Thus, consumers do not invoke Bayesian updating. Several studies empirically examine the effects of social learning (e.g., Chevalier and Mayzlin 2006, Godes and Mayzlin 2004, Manchanda et al. 2008, Sorensen 2006, Villanueva et al. 2008). Chevalier and Mayzlin (2006) show that negative online reviews have greater impacts on book sales than positive reviews. Using data from the University of California system, Sorensen (2006) identifies a significant social effect in an employee's choice of health plans.

Diffusion theory (e.g., Bass 1969, Rogers 1962) builds on an important behavioral assumption: some consumers (called innovators) make adoption decisions independently, whereas the adoption decision of the remaining consumers (called imitators) is influenced by the number of other buyers.² Adoption is thus driven by both innovation and imitation effects. The Bass (1969) model lacks marketing-mix variables. Other researchers extend the Bass model by incorporating price (e.g., Robinson and Lakhani 1975, Kalish 1985, Horsky 1990, Krishnan et al. 1999) and advertising (e.g., Dodson and Muller 1978, Mahajan et al. 1995), among others. Assuming experience curve effects, these models show that either skimming or penetration pricing can be optimal under different circumstances. When the unit cost remains constant, we show that penetration pricing is the optimal strategy.

Both the diffusion theory and our current model highlight the importance of social forces in the adoption of a new product. However, we note a few crucial differences. First, the diffusion models focus on the *persuasive* role of social contagion and ignore learning about product match, because the imitation effect says that the larger the installed base, the more likely the remaining consumers will adopt. In contrast, we concentrate on the *informative* role of social learning: the larger the installed base, the more likely the remaining consumers will discover their product match. The informed consumers with low valuation then will not buy. Therefore, stronger social learning does not always lead to more adoption. Second, the diffusion models mostly examine aggregate market behavior, whereas we investigate the adoption decision of individual consumers. Our focus at the individual level helps uncover the learning externality, which underlies dispersed adoption and adoption inertia. Third, unlike the diffusion models, ours does not require exogenous heterogeneity in consumer behavior.

The present paper is also related to several models of dynamic pricing of experience goods (Shapiro 1983; Villas-Boas 2004, 2006; Bergemann and

Välimäki 2006). In Shapiro (1983), myopic consumers attempt to discover product quality. Villas-Boas (2004, 2006) considers price competition in a duopoly and finds that the skewness of consumer value distribution is the primary factor driving brand loyalty and hence the nature of price rivalry. Bergemann and Välimäki (2006) present a dynamic-monopoly model of consumer learning about horizontal match. However, they examine self-learning about consumable goods through repeat purchase, whereas we consider social learning about a durable good. They show that when the full information price exceeds (is below) the mean consumer valuation, penetration pricing (skimming) is the firm's optimal strategy. In our model, penetration pricing is always optimal. Besides, the equilibria of adoption frenzy and inertia are unique to social learning in a durable good context but do not arise in their model.

The rest of this paper proceeds as follows. Section 2 presents the model. Section 3 analyzes the firm's optimal pricing strategy, the adoption equilibrium, and the profit and welfare implications of social learning. Section 4 explores several extensions. Section 5 concludes. The proofs are collected in the appendices.

2. A Model of Social Learning

In this market, a monopolist releases a durable good at time 0. The durable has a market window of two time periods, beyond which it becomes obsolete because of technological advances or evolution of consumer tastes. A continuum of consumers with unit mass each demand at most one unit over the entire horizon. Consumers have idiosyncratic valuation of the durable. Their prior valuation v follows the same distribution F , which has a positive density f and an increasing hazard rate over $[0, +\infty)$.³ The durable is an experience good (as in Nelson 1970), and no consumer knows her true valuation when it is first introduced to the market. F is common knowledge, and the firm cannot observe individual consumers' true types. Production has a constant marginal cost c and zero fixed costs.

If some consumers have purchased in period 1, however, by period 2 the remaining consumers are likely to discover their true valuation through social learning. Information about a product's design and functional features and its owners' experience may spread via online or off-line word of mouth. For example, when a consumer plans to buy a car, she may ask her friends about their experience with certain brands or models. If a product review evaluates the performance of the key attributes of the

² The percentage of innovators in the consumer population is 2.5% in Rogers (1962) and ranges from 0.2% to 2.8% in Bass (1969).

³ An extension in §4 considers the case of ex ante heterogeneous consumer types.

durable, it readily reveals the product's match to each reader who knows her relative preference over these attributes. For example, a recent consumer review of the Toro e-Cycler 20360 electric lawn mower on Amazon.com states, "Pros: (1) Metal construction with adjustable heights on all four wheels; (2) Quiet and odor-free; (3) Toro quality, local service. Cons: (1) The mower is 77lbs and not self-propelled. On hills, it takes some effort; (2) Battery is not quickly removable. This isn't the mower for you if you need two batteries to mow your lawn." Clearly, the e-Cycler 20360 is ideally suited for small, flat lawns but less so with very large or hilly lawns. Alternatively, if the review provides the utility scores of individual attributes, the reader can compute the product's total utility for her based on the weights she places on these attributes.

The informative nature of social learning in our model is somewhat analogous to that of advertising as described in Anderson and Renault (2009), Grossman and Shapiro (1984), and Meurer and Stahl (1994). In these models, the advertising message transmits the relevant product characteristics, and the consumers receiving the message discover their respective match values. Furthermore, the firms cannot target their ads only to those consumers with sufficiently high match values. Similarly, in our model an informed late adopter's true valuation is a random draw from distribution F .

Let α ($0 \leq \alpha \leq 1$) denote the population of consumers who purchase in period 1, i.e., the period 1 installed base. We assume that by period 2, a consumer who did not purchase in the previous period discovers her true valuation with probability αs , where s ($0 \leq s \leq 1$) measures the product's inherent social strength and is called its SL intensity. Because $s \leq 1$, even if all other consumers adopt in period 1, the probability that a prospective buyer becomes informed via social learning does not exceed 1. A product's SL intensity reflects the propensity that its attribute information disseminates among peer consumers via social forces. More specifically, it is the tendency of the product's attributes and functions being discussed through online or off-line word of mouth. On average, a product with more chat-worthy features may have a higher SL intensity. Our notion of SL intensity corresponds to the intensity of word-of-mouth recommendation in Easingwood et al. (1983) and Horsky and Mate (1988).

Note that the larger the period 1 installed base, the more likely a late adopter becomes informed. A virtually identical assumption also appears in Banerjee (1993) and Bundgaard-Neilson (1976). In the market of a new technology, Bundgaard-Neilson (1976) argues that with more information the late adopters are in a better position to assess the new technology and hence may adopt faster than earlier ones. To ease

exposition, we assume that the probability of a late adopter becoming informed increases linearly in α , which amounts to treating each early adopter as an equal source of word of mouth. However, we later show (in §4) that allowing SL intensity to vary with the realized value of each early adopter does not alter our key results. The assumption that the late adopters become informed with equal probability also appears in the diffusion models mentioned above and is perhaps better suited to online markets, where the members of a community or the visitors to a commercial website are equally likely to heed the reviews posted there by others.⁴ The multiplicative form (αs) captures the complementarity between α and s , as the marginal effect of each tends to increase in the value of the other (see also Bass 1969, Horsky and Mate 1988).

To better focus on social learning, we abstract from other types of learning.⁵ Our two-period setting then aims to capture the following. In the initial period immediately after the product is released, all purchases are uninformed because of a lack of consumer learning. After an installed base has developed, however, the early adopters can function as a source of social learning so that some of the potential buyers will become informed by period 2.

For simplicity, we assume that the firm and consumers face a common discount factor, β ($0 < \beta \leq 1$). The firm maximizes expected (discounted) profits and consumers' expected (discounted) utility. At the beginning of each period i ($i = 1, 2$), the firm announces a unit price, p_i . After observing p_i , each consumer decides whether to purchase in that period. We allow mixed strategies regarding period 1 purchases: if buying and not buying in period 1 yield equal (discounted) expected utility, each consumer chooses a probability α with which to purchase. Because consumers are ex ante identical, following convention we assume that they choose the same probability to adopt in period 1. If $\alpha = 1$ ($\alpha = 0$), all consumers will buy in period 1 (2). Any α between 0 and 1 represents each consumer's propensity to buy in period 1, and a larger α means that consumers are more inclined to buy early. Because consumers are infinitesimal, α is also the population of early adopters. An uninformed consumer bases her purchase decision on expected valuation. In period 2, an informed consumer decides whether to buy based

⁴ In off-line markets, a consumer who is less socially connected often gains less from social learning and thus may have less incentive to delay adoption. Allowing heterogeneous connectedness complicates the analysis and is left as a direction for future investigation. Katona and Sarvary (2008) provide a useful framework along this direction.

⁵ Section 4 shows that the basic model can readily accommodate informative advertising, the firm's efforts to "engineer" social learning, and third-party product reviews.

on her true valuation. For tractability, we rule out resale, either because transaction costs are too high or because used goods have limited appeal in this market.

Last, let p^* denote the full information price in a static monopoly. That is, p^* is the unique solution to

$$p^* - c = \frac{1 - F(p^*)}{f(p^*)}, \quad (1)$$

and \bar{v} is the mean consumer valuation; i.e.,

$$\bar{v} \equiv \int_0^\infty v dF(v). \quad (2)$$

These two values will prove pivotal in the analysis below. Note that $p^* > c$ by construction.

3. Equilibrium Analysis

The analysis has three cases, depending on the relative magnitudes of p^* , \bar{v} , and the unit cost c . Here, we only present the cases where $c \geq \bar{v}$ and $p^* \geq \bar{v} > c$, respectively. The remaining case where $p^* < \bar{v}$ does not add new insights and is relegated to Appendix C.

3.1. The Case in Which $c \geq \bar{v}$

In this case, for trade to occur, some consumers must purchase in period 1. Because we assume social learning is the only means of consumer learning, if no one buys in period 1, then all consumers will remain uninformed in period 2 and hold expected valuation $\bar{v} \leq c$. Consequently, no trade occurs because the firm has no incentive to price below cost in the final period.

As Lemma 1 (in Appendix B) shows, the firm's period 1 problem of choosing price is equivalent to choosing consumers' probability of early adoption. The unconstrained maximizer of the firm's period 1 objective function is

$$\alpha_1 = \frac{\beta s (p^* - c)(1 - F(p^*)) - (c - \bar{v})}{2\beta s \int_{p^*}^\infty v - c dF(v)}. \quad (3)$$

When $c \geq \bar{v}$, we can readily verify that α_1 increases in s and that $\alpha_1 < \frac{1}{2}$.⁶ Proposition 1 describes the unique equilibrium.

PROPOSITION 1. *Suppose $c \geq \bar{v}$. Then, (1) there is no trade when $s \leq (c - \bar{v})/(\beta(p^* - c)(1 - F(p^*)))$. (2) Otherwise, the consumers' equilibrium probability of adopting in period 1 is $\alpha^* = \alpha_1$, the optimal prices are $p_1^* = \bar{v} - \beta\alpha_1 s \int_{p^*}^\infty v - p^* dF(v)$ and $p_2^* = p^*$, and the discounted firm profits and expected consumer and social welfare all increase in s .*

⁶ Note that $\int_{p^*}^\infty v - c dF(v) > \int_{p^*}^\infty p^* - c dF(v) = (p^* - c)(1 - F(p^*))$.

Without social learning, trade would not be viable because uninformed consumers are willing to pay no more than their mean valuation, which is below the unit cost. With social learning, to activate trade the firm has to subsidize period 1 transactions so that some consumers purchase early. By period 2, some of the remaining consumers become informed via social learning. The firm can then potentially profit from selling to those informed consumers with sufficiently high valuation.

In period 2, the firm prices above cost. Because $c \geq \bar{v}$, only the informed consumers (if any) will purchase. The optimal period 2 price is thus p^* . When SL intensity is relatively weak ($s \leq (c - \bar{v})/(\beta(p^* - c)(1 - F(p^*)))$), in period 2 the informed population, and hence the profits will be too small to justify any period 1 subsidy. Thus, no trade occurs in this case. Otherwise, the firm will indeed exploit social learning by subsidizing early adoption and make positive overall profits. In this case, the period 1 sales serve as a loss leader, and social learning helps revive trade. We can verify that the discounted firm profits increase in SL intensity. Consumers receive positive expected surplus, whether they buy early or defer purchase.⁷ There is a loss of social welfare in period 1, but the welfare gain in period 2 dominates the period 1 loss. The overall social welfare increases with SL intensity.

3.2. The Case in Which $p^* \geq \bar{v} > c$

We proceed backwards and first examine the second period. Let

$$\hat{s} \equiv \frac{(\bar{v} - c)}{(p^* - c)(1 - F(p^*)) + (\bar{v} - c)F(\bar{v})} \quad \text{and} \quad \hat{\alpha} \equiv \frac{\hat{s}}{s}. \quad (4)$$

We can verify that $0 < \hat{s} = \hat{\alpha}s < 1$.⁸ Proposition 2 characterizes the optimal period 2 price.

PROPOSITION 2 (OPTIMAL PERIOD 2 PRICING). *Suppose α ($0 \leq \alpha \leq 1$) consumers have purchased in period 1. The optimal period 2 price is $p_2^* = \bar{v}$ when $\alpha \leq \hat{\alpha}$ and is $p_2^* = p^*$ otherwise.*

Period 2 pricing follows a threshold-type policy. In period 2, the remaining consumers divide into two segments, the informed and the uninformed. The optimal price depends on their relative sizes, which in turn depend on the period 1 installed base. Intuitively, the period 1 installed base is the basis for social learning. When it is small ($\alpha \leq \hat{\alpha}$), by period 2 the proportion

⁷ The expected consumer welfare is $\beta\alpha_1 s \int_{p^*}^\infty v - p^* dF(v)$, because each early adopter receives the same expected surplus as one who defers purchase.

⁸ By construction, p^* maximizes the profits in a static, full-information monopoly. Therefore,

$$(p^* - c)(1 - F(p^*)) > (\bar{v} - c)(1 - F(\bar{v})).$$

The denominator of \hat{s} is thus greater than $\bar{v} - c$.

of uninformed consumers in the market $(1 - \alpha s)$ is relatively large. Therefore, the firm will charge a lower price \bar{v} to capture the entire uninformed segment and some informed consumers. Otherwise, the relatively small uninformed segment loses its appeal, and the firm will charge a higher price p^* to serve exclusively the informed consumers.

Next, we seek the period 1 equilibrium. The following definition is conducive to the ensuing exposition.

DEFINITION. The unit cost is said to be LOW if $\int_0^{\bar{v}} v - c dF(v) \geq 0$ or, equivalently, $c \leq \int_0^{\bar{v}} v dF(v)/F(\bar{v})$; it is said to be HIGH otherwise.

The period 1 solution turns out to depend on the magnitudes of SL intensity and the unit cost. Proposition 3 below states the equilibrium when $s \leq \hat{s}$ (which implies $\hat{\alpha} \geq 1$). Let

$$\alpha_2 = \frac{[1 - \beta(1 + sF(\bar{v}))](\bar{v} - c)}{-2\beta s \int_0^{\bar{v}} v - c dF(v)}. \quad (5)$$

PROPOSITION 3 (EQUILIBRIUM WITH LOW SL INTENSITY). Suppose $0 \leq s \leq \hat{s}$. Then, (1) when the unit cost is high, $\alpha^* = 1$ (adoption frenzy) if $s \leq s_1$, $\alpha^* = \alpha_2$ (dispersed adoption) if $s_1 < s \leq s_2$, $\alpha^* = 0$ (adoption inertia) if $s \geq s_2$. (2) When the unit cost is low, $\alpha^* = 1$ if $s \leq s_3$, and $\alpha^* = 0$ otherwise. (3) The optimal prices in periods 1 and 2 are $p_1^* = \bar{v} - \beta\alpha^*s \int_0^{\bar{v}} v - \bar{v} dF(v)$ and $p_2^* = \bar{v}$, respectively (see Appendix A for the thresholds s_1 – s_3).

Figures 1 and 2 depict consumers' equilibrium probability of early adoption (α^*) as a function of s for high and low unit costs, respectively. When $s \leq \hat{s}$, a noteworthy property of the equilibrium is that all consumers adopt early when s is low and adopt late when s is relatively high. In the presence of discounting, weak social learning dissipates the benefit of delayed adoption, prompting all consumers to adopt early. When this happens, we say that the market exhibits *adoption frenzy*. In adoption frenzy, there is no active social learning, but the firm fails

Figure 1 Optimal Probability of Early Adoption: High Unit Cost

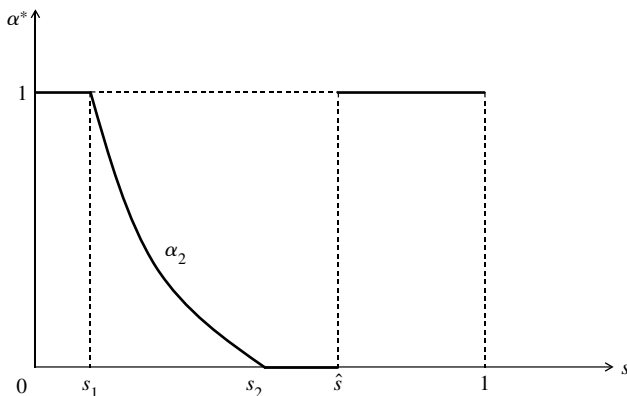
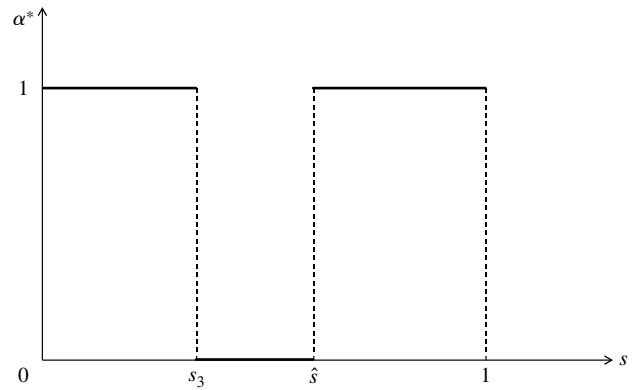


Figure 2 Optimal Probability of Early Adoption: Low Unit Cost



to fully extract consumers' expected utility, because to adopt early the consumers must receive a commensurate amount of surplus as deferring purchase confers. When SL intensity is high enough, the consumers are tempted to exploit social learning, which entails deferring purchase. In fact, this tendency to rely on others' early adoption experience can be so overwhelming that every consumer defers adoption. We call such a market outcome *adoption inertia*. When adoption inertia occurs, social learning is killed off as the early installed base is nil. In contrast to adoption frenzy, here, the period 2 price extracts the entire expected valuation of each consumer.

Adoption inertia constitutes the worst scenario for both the firm and the consumers. To see this, the firm can always set the period 1 price at above consumers' mean valuation so that no one buys early. Without an early installed base, no social learning occurs, and the firm then sets the period 2 price at \bar{v} , extracting the entire (expected) consumer surplus.

Social learning creates potential positive externality among consumers. When consumers differ in the timing of their purchase, the late adopters potentially discover their product match through social learning and make informed purchases. Meanwhile, the late adopters also extend an indirect benefit to the early adopters. To induce some consumers to buy early, the firm must lower the period 1 price so that early and delayed adoption yields equal expected utility. The externality caused by social learning is purely informational and distinguished from the usual network externality (e.g., Farrell and Saloner 1986, Katz and Shapiro 1985), which refers to the value of a product or service increasing in its user base (as with telephones or software). Our likely adoption frenzy or inertia also differs from the herd phenomenon in Banerjee (1992), where by ignoring her private information and joining the herd, each consumer inflicts a negative externality on the ensuing consumers. In our model, an informed consumer always acts on her true valuation. We call the externality as a result of social learning the *learning externality*. It is this

learning externality that potentially leads to dispersed adoption and adoption inertia.

PROPOSITION 4. *The discounted firm profits weakly decrease in s over $[0, \hat{s}]$.*

The business press often suggests that firms benefit from more buzz (or positive word of mouth) about their products. When the experience attribute is quality, more buzz may help increase the perceived quality. In a market with competing brands, the one generating more buzz likely attracts greater attention and market share. When consumers care about horizontal match, however, Proposition 4 shows that a higher SL intensity even drives down monopoly profits when $s \leq \hat{s}$.

The intuition for Proposition 4 is that stronger social learning generally leads to more heterogeneous consumer valuation, which undermines the firm's rent extraction. In the absence of social learning ($s = 0$), consumers lack the incentive to delay adoption. They all will purchase upon product release and face a price equal to the mean valuation. When social learning is present but not too strong ($s \leq \hat{s}$), in period 2 the relatively small informed segment also limits price to the mean valuation. Social learning does not lead to a higher price but causes incomplete market coverage, as the informed consumers with valuation below \bar{v} drop out. This is why greater SL intensity makes the firm worse off when $s \leq \hat{s}$. In Figures 3 and 4, the solid curves depict firm profits as a function of s .

PROPOSITION 5. *When s increases over $[0, \hat{s}]$, expected consumer welfare may increase, decrease, or stay constant, and expected social welfare (weakly) decreases.*

In Figures 3 and 4, the dark dotted curves depict expected social welfare (SW). Intuitively, the learning externality deteriorates coordination among self-interested consumers, who as a group may fail to benefit from stronger SL intensity. This then causes the society at large to become weakly worse off. In adoption frenzy, the period 1 price decreases in s (Proposition 3) and expected consumer welfare increases, but social welfare remains constant (at $\bar{v} - c$)

Figure 3 Firm Profits and Social Welfare: High Unit Cost

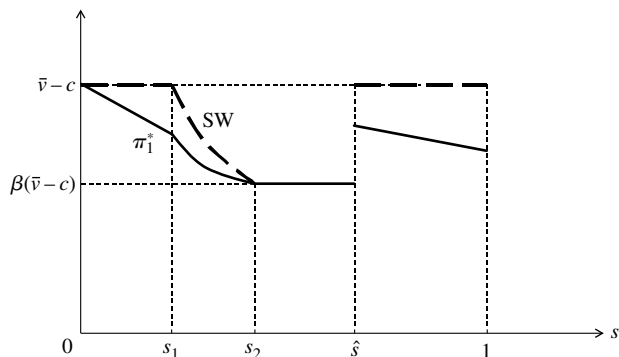
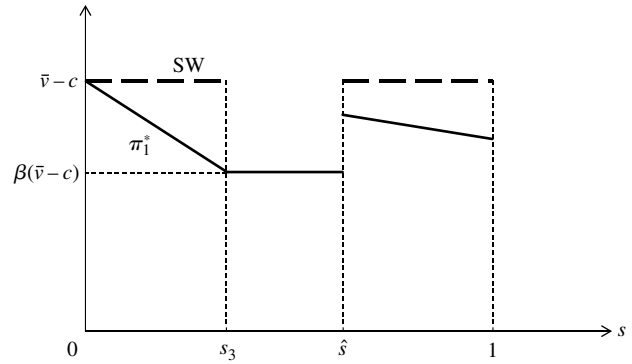


Figure 4 Firm Profits and Social Welfare: Low Unit Cost



as s increases. In adoption inertia, consumer welfare is 0 and, as s increases, social welfare (equal to firm profits) remains at $\beta(\bar{v} - c)$. When the unit cost is high and $s_1 < s \leq s_2$, we can verify that expected consumer and social welfare both decrease in s .

The reason that social welfare decreases over $[s_1, s_2]$ is as follows. In this region, some consumers purchase early and others postpone their adoption. Each early purchase yields expected social surplus $\bar{v} - c$. Because of discounting, each late, uninformed purchase generates lower social surplus $\beta(\bar{v} - c)$. When SL intensity increases over $[s_1, s_2]$, the number of early buyers (α_2) and the probability of a late adopter becoming informed ($\alpha_2 s$) both decrease. Consequently, the number of uninformed period 2 buyers $((1 - \alpha_2) \cdot (1 - \alpha_2 s))$ increases. Even though an informed purchase in period 2 potentially helps increase social welfare, the informed population does not monotonically increase over $[s_1, s_2]$. It turns out that the welfare loss as a result of the uninformed late purchases dominates any potential welfare gain as a result of the informed late purchases, causing social welfare to decrease over this interval.

Propositions 6 and 7 characterize the equilibrium when $\hat{s} < s \leq 1$.

PROPOSITION 6 (EQUILIBRIUM WITH HIGH SL INTENSITY AND HIGH UNIT COST). *Suppose $\hat{s} \leq s \leq 1$ and unit costs are high. Then, (1) $\alpha^* = 0$ (adoption inertia) when $\max\{s_2, s_5\} \leq s \leq s_6$, and $\alpha^* = 1$ (adoption frenzy) when $s_2 \leq s \leq \min\{s_5, s_6\}$. (2) When $s \leq \min\{s_4, s_6\}$, $\alpha^* = 1$. (3) When $s_4 < s < \min\{s_2, s_6\}$, $\alpha^* = \alpha_2$ (dispersed adoption) if $\Psi_2(\alpha_2) > \Psi_1(1)$ and is $\alpha^* = 1$ otherwise; and (4) the optimal period 1 price is $p_1^* = \bar{v} - \beta\alpha^*s \int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$ for $\alpha^* \leq \hat{\alpha}$ and is $p_1^* = \bar{v} - \beta\alpha^*s \int_{p^*}^{\infty} v - p^* dF(v)$ otherwise. (See Appendices A and B for thresholds s_4 through s_6 and Ψ_1 and Ψ_2 .)*

For strong SL intensity ($s \geq \hat{s}$) and high unit costs, the equilibrium adoption pattern is contingent on s , and adoption frenzy, adoption inertia, or staggered adoption may appear. The rationale underlying adoption frenzy and inertia is similar to that discussed earlier. We illustrate Proposition 6 with case (2). When β

is sufficiently low, both $s_4 \geq 1$ and $s_6 \geq 1$ hold so that case (2) applies.⁹ Over $[\hat{s}, 1]$, adoption frenzy appears, and the firm makes profits $\pi_1^* = \bar{v} - c - \beta s \int_{p^*}^{\infty} v - p^* dF(v)$. For this case, Figure 1 depicts α^* , and Figure 3 shows firm profits and social welfare as functions of SL intensity.

PROPOSITION 7 (EQUILIBRIUM WITH HIGH SL INTENSITY AND LOW UNIT COST). Suppose $\hat{s} \leq s \leq 1$ and unit costs are low. Then, (1) $\alpha^* = 1$ (adoption frenzy) when $s \leq \min\{s_5, s_6\}$, and $\alpha^* = 0$ (adoption inertia) if $s_5 < s \leq s_6$. (2) The optimal period 1 price is $p_1^* = \bar{v} - \beta \alpha^* s \int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$ for $\alpha^* \leq \hat{\alpha}$ and is $p_1^* = \bar{v} - \beta \alpha^* s \int_{p^*}^{\infty} v - p^* dF(v)$ otherwise.

For strong SL intensity ($s \geq \hat{s}$) and low unit cost, adoption frenzy or inertia may again arise depending on specific market conditions. For example, when discounting is severe enough, we have $s_5 > 1$ and $s_6 > 1$, and thus adoption frenzy prevails over $[\hat{s}, 1]$.¹⁰ In this case, Figure 2 illustrates the adoption pattern, and Figure 4 shows firm profits and expected social welfare.

3.3. Discussion

The equilibrium in §3.2 is worth some further comments. First, adoption inertia may occur only for intermediate values of SL intensity but vanish when $s > \hat{s}$ (see Propositions 6 and 7 and Figures 1 and 2). This is intriguing, because intuition lets one expect stronger social learning to reinforce adoption inertia. Here, the reasoning is as follows. When $s \leq \hat{s}$ (so that $\hat{\alpha} \geq 1$), the period 2 price is the mean consumer valuation (Proposition 2). When $s > \hat{s}$ and $\hat{\alpha}$ or more consumers buy early, in period 2 the firm will charge the (higher) full information price, penalizing late adoption. Therefore, at \hat{s} the expected utility of delaying adoption drops by a lump-sum amount. This explains why the equilibrium rate of early adoption may jump upwards at \hat{s} .

⁹ We have that

$$s_4 \geq 1 \Leftrightarrow \beta \leq \beta_1 \equiv \frac{1}{F(\bar{v}) + 1 - A},$$

where

$$A \equiv \frac{2 \int_0^{\bar{v}} v - c dF(v)}{(p^* - c)(1 - F(p^*)) + (\bar{v} - c)F(\bar{v})}.$$

Because p^* is the optimal full information price, $(p^* - c)(1 - F(p^*)) \geq (\bar{v} - c)(1 - F(\bar{v}))$. We then have

$$A \leq \frac{2 \int_0^{\bar{v}} \bar{v} - c dF(v)}{(\bar{v} - c)(1 - F(\bar{v})) + (\bar{v} - c)F(\bar{v})} = 2F(\bar{v}),$$

and thus $F(\bar{v}) + 1 - A \geq 1 - F(\bar{v}) > 0$. Therefore, $\beta_1 > 0$.

¹⁰ Here,

$$s_5 \geq 1 \Leftrightarrow \beta \leq \frac{\bar{v} - c}{\int_{p^*}^{\infty} v - p^* dF(v) + (\bar{v} - c)},$$

and

$$s_6 \geq 1 \Leftrightarrow \beta \leq \frac{\bar{v} - c}{\int_{p^*}^{\infty} \bar{v} - c dF(v) + \int_{p^*}^{\infty} v - p^* dF(v)}.$$

Second, social learning about horizontal match frequently makes the monopolist worse off, and the discounted firm profits first weakly decrease in s and may then jump upwards at \hat{s} (see Figures 3 and 4). When the period 2 price switches from the mean consumer valuation to the full information price at \hat{s} , the corresponding period 1 price also jumps upwards, causing firm profits to increase.

That social learning makes the firm worse off is not too surprising. The underlying intuition is seen by comparing the static monopolies where each consumer is informed and uninformed about her valuation, respectively. When each consumer knows her preference, the monopoly profits are $(p^* - c)(1 - F(p^*))$. With uninformed consumers, the monopoly profits are $\bar{v} - c$. Imperfect information restricts profit margin but allows full participation. The former profits exceed the latter when unit cost is sufficiently high. When consumers differ in the timing of purchase, social learning informs some of the late buyers about their preferences. Our model can thus be viewed as a dynamic hybrid between the two canonical paradigms. However, to induce early adoption the firm must price low initially and forgo enough consumer surplus. Consequently, firm profits are lower than those under imperfect information.

Even though social learning about match makes the firm worse off, it can hardly block or restrict spontaneous communication among peer consumers. As we shall show in the next section, however, the firm may improve its profits via informative advertising or even cultivating more social learning.

Third, consumer and social welfare is not monotone in SL intensity over $[0, 1]$. The expected consumer welfare increases and decreases in s in adoption frenzy and staggered adoption, respectively, and it remains at zero in adoption inertia. The expected social welfare remains constant in adoption frenzy or inertia and decreases in SL intensity during staggered adoption. In particular, when equilibrium adoption switches from inertia to frenzy at \hat{s} , both consumer and social welfare jump upwards.

Last, our analysis provides a theory of penetration pricing for introducing durable experience goods. In equilibrium, the period 1 price is always below consumers' mean valuation, and the period 2 price exceeds their mean valuation. The lower introductory price is essential for gaining market acceptance and spawning word of mouth, and the higher period 2 price helps extract surplus from the late adopters.

4. Extensions

4.1. Firm-Generated Social Learning

We so far have assumed that SL intensity is exogenously given. The exogenous SL intensity generally

captures spontaneous communication among consumers. In practice, however, a firm may manipulate the SL intensity of its product through advertising in social media (Edgar 2009, Needleman 2009) and sponsoring brand community events (Muniz and O'Guinn 2001) and referral programs (Biyalogorsky et al. 2001). Walt Disney Studios, for example, recently used "cliffhanger" screening to market its new movie *Toy Story 3* to college students, and it succeeded in getting them to communicate on Twitter and Facebook about the movie: "I love the combination of a classic marketing effort—a flier with rip-off tabs—and combining it with the most modern of connections, which is Facebook and the idea of viral content," said Rich Ross, Disney's studio chairman" (Barnes 2010).

In the case where $p^* \geq \bar{v} > c$, the firm's equilibrium profits may have an upward jump at \hat{s} (see Figures 3 and 4). When the exogenous SL intensity (s) is below \hat{s} and $\pi_1^*(s) < \pi_1^*(\hat{s})$, the firm may benefit from investing in social learning to increase the SL intensity to \hat{s} . Let $h(x | s)$ be the firm's costs of raising SL intensity from s to a new level x ($x \geq s$). Propositions 3, 6, and 7 jointly imply the following result.

PROPOSITION 8. *Suppose $p^* \geq \bar{v} > c$. When $s < \hat{s}$ and $h(\hat{s} | s) < \pi_1^*(\hat{s}) - \pi_1^*(s)$, the firm benefits from investing in social learning to increase SL intensity to \hat{s} .*

When SL intensity increases to \hat{s} , §3 shows that the firm may switch to a different pricing strategy, which causes its profits to increase by a discrete amount. When the profit gain from increasing SL intensity to \hat{s} exceeds the associated costs, such firm-generated social learning indeed becomes profitable.

4.2. Informative Advertising

Next, we show that the firm may also benefit from intervening in social learning with informative advertising. We retain the setup of the basic model. In addition, the firm may choose to advertise its product features during period 1. Because the episodes of advertisement may be released over the entire course of period 1, for simplicity we suppose that advertising does not lead to informed period 1 purchases but helps some consumers become informed by period 2. Let t ($0 \leq t \leq 1$) denote the probability that a late adopter becomes informed through advertisements, and let $g(t)$ denote the firm's advertising costs. That is, t represents the reach or penetration rate of advertising. We assume that firm advertising and social learning are independent means of consumer learning. Therefore, by period 2 a late adopter discovers her valuation with probability $t + (1 - t)\alpha s$, where α is the period 1 installed base and s is the SL intensity. When $t = 0$, this probability reduces to αs . At the beginning of period 1, the firm announces price p_1

and advertising level t , and the (uninformed) consumers decide whether to buy now. At the beginning of period 2, the firm sets price p_2 , and any remaining consumers decide whether to buy then.

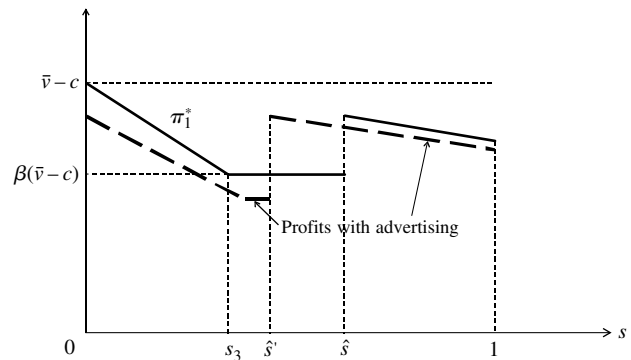
We focus on the case where $p^* \geq \bar{v} > c$. For a given level of advertising t , we can readily derive the subgame-perfect market equilibrium (see Appendix D). As s increases, firm profits first weakly decrease but may then jump upwards at $\hat{s}' \equiv (\hat{s} - t)/(1 - t)$. Recall that without advertising, the profit jump may only occur at \hat{s} . Since $\hat{s}' \leq \hat{s}$, advertising induces the discrete profit increase even when SL intensity is below \hat{s} . Comparing the profits with and without advertising readily leads to the next proposition.

PROPOSITION 9. *Suppose $p^* \geq \bar{v} > c$. When $s < \hat{s}$ and $g((\hat{s} - s)/(1 - s)) < \pi_1^*(\hat{s}) - \pi_1^*(s)$, the firm can increase its profits by investing in informative advertising at level $(\hat{s} - s)/(1 - s)$.*

The intuition behind the proposition is as follows. Informative advertising increases consumers' probability of being informed by period 2 and thus encourages them to postpone purchase. In period 2, the larger fraction of informed consumers may induce the firm to raise the price from the mean valuation to the full information price at SL intensity \hat{s}' . The higher period 2 price then enables a higher period 1 price. The shift of pricing strategy increases firm profits over $[\hat{s}', \hat{s})$.

For low unit costs, the solid curve in Figure 5 depicts the firm's profits without advertising, and the dark dotted curve depicts its profits when advertising at level t . Informative advertising reduces firm profits except over $[\hat{s}', \hat{s})$, because more consumers now postpone adoption and make informed purchases. For high unit costs, advertising has a similar impact on firm profits. From the above analysis, it is clear that our basic model equally accommodates third-party product reviews (such as *Consumer Reports*) as an additional learning channel. Because incorporating

Figure 5 Impacts of Advertising on Firm Profits: Low Unit Cost



third-party reviews does not generate substantially new results, the expanded treatment is omitted.

4.3. Endogenous SL Intensity

So far, SL intensity s has been treated as a constant. This amounts to assuming that the early adopters are equally effective sources of social learning. In practice, however, the likelihood that an early adopter generates word of mouth often depends on her realized valuation.¹¹ We now allow the SL intensity of each early buyer to vary with her valuation. Specifically, we let the probability that each early adopter educates a prospective buyer about the product be a function of the former's valuation v , $\phi(v)$ ($0 \leq \phi(v) \leq 1$). Let $\bar{s} \equiv \int_0^\infty \phi(v) dF(v)$. Because consumers' prior valuation follows distribution F , \bar{s} is essentially the mean SL intensity. When the period 1 installed base is α , a late adopter becomes informed by period 2 with probability $\alpha\bar{s}$. Therefore, \bar{s} now plays the role of s in the basic model. The preceding analysis thus remains valid after replacing s with \bar{s} .

4.4. Ex Ante Heterogeneous Consumers

We have assumed that consumers are ex ante homogeneous as their prior values follow the same distribution. We now show that allowing ex ante heterogeneous consumers does not significantly alter the spirit of our analysis. Suppose for simplicity that consumers hold two possible values for the product, v_1 and v_2 , with $v_1 < c < v_2$, where c is the firm's (constant) unit cost. Ex ante, there are two consumer types, H and L. An H (L) consumer has the value v_2 with probability γ_H (γ_L) and the value v_1 with probability $1 - \gamma_H$ ($1 - \gamma_L$), where $0 \leq \gamma_L \leq \gamma_H \leq 1$. The mean values of the H and L consumers are thus $\bar{v}_H \equiv \gamma_H v_2 + (1 - \gamma_H)v_1$ and $\bar{v}_L \equiv \gamma_L v_2 + (1 - \gamma_L)v_1$, respectively. The total consumer population is normalized to 1, the H population is θ ($0 \leq \theta \leq 1$), and the L population $1 - \theta$. Following §4.3, we assume that an early adopter with value v_i ($i = 1, 2$) has SL intensity s_i , with $0 \leq s_1 \leq s_2 \leq 1$. That is, a consumer with a higher realized value is more likely to contribute word of mouth. If there are α_2 (α_1) period 1 buyers with value v_2 (v_1), the probability that a consumer deferring purchase becomes informed by period 2 is $\alpha_2 s_2 + \alpha_1 s_1$. The consumer-type distribution is common knowledge. Ex ante, each consumer knows her type but not her value, and the firm does not observe the type or value of any consumer.

To ease exposition, we focus on the case in which $\bar{v}_L < c < \bar{v}_H$, and θ is large enough so that it is not profitable for the firm to subsidize (uninformed)

type L purchases in period 1. Let α_H denote the number of H consumers who purchase in period 1. We can show that the optimal period 2 price is v_2 if $\alpha_H \geq \hat{\alpha}_H$ and is \bar{v}_H otherwise (see Appendix E for $\hat{\alpha}_H$ and a detailed analysis). Let $s_H \equiv \gamma_H s_2 + (1 - \gamma_H)s_1$; i.e., s_H is the "weighted" SL intensity of the H consumers.

The adoption equilibrium (see Appendix E) is reminiscent of that in the basic model and has three cases: (1) adoption inertia, where no consumer buys early and only the (uninformed) H consumers buy in period 2; (2) adoption frenzy in the H segment, where all H consumers buy early and only the informed L consumers with high valuation adopt in period 2; and (3) staggered adoption in the H segment, where only some of the H consumers adopt early and the informed remaining (H or L) consumers with high valuation adopt in the second period. The rationale underlying each case parallels that discussed in §3.

5. Conclusion

We have analyzed the effects of social learning on the dynamic pricing and consumer adoption of a durable good. The monopolist firm pursues penetration pricing, and the equilibrium involves dispersed adoption, adoption frenzy, and inertia under different market conditions. At the core of the analysis lies the learning externality of social learning, which causes possible dispersed adoption and adoption inertia. We thus explain staggered adoption without resorting to exogenous heterogeneity in consumer behavior. Under certain conditions, the firm benefits from informative advertising or investing to enhance the extent of social learning around its product. The welfare implications of social learning are also explored. A salient property of the model is that the equilibrium adoption pattern, firm profits, and social welfare are all likely to be discontinuous at the threshold SL intensity level (\hat{s}). The reason is that the optimal period 2 price is contingent on the number of early adopters (Proposition 2), and hence the expected utility of deferring purchase is discontinuous at \hat{s} .

We end with a discussion of some directions for future research. First, the two-period setting effectively captures the learning externality and consumers' trade-off between an early, uninformed purchase and a late but possibly informed purchase. Extending to multiple periods need not generate additional insights because the basic driver remains unchanged. Second, we have confined our attention to a monopoly. In a Bertrand duopoly where consumers have identical and independent valuations about the brands, any symmetric (subgame-perfect) equilibrium involves an equal number of consumers buying each brand in period 1. The equilibrium maintains the property that these early buyers are indifferent

¹¹ We thank the editor and an anonymous reviewer for this important observation. In §4.4, we assume that an early adopter with a higher realized value has a higher SL intensity.

between adopting early and deferring adoption. It then follows from Villas-Boas (2004, 2006) that the skewness of the consumer value distribution also plays a role here. Last, our model features horizontal match as the relevant attribute. A worthy direction is to study social learning about the quality of a durable good. When the true quality exceeds consumers' perceived quality, two opposing forces determine the optimal pricing. On one hand, the firm wishes to price low in period 1 to promote early adoption and positive word of mouth. On the other hand, the firm will price reasonably high to prevent too many consumers from adopting in period 1. If too many consumers adopt in period 1, few would remain to take advantage of social learning and pay a higher price according to the revealed true quality. When the true quality falls below the perceived quality, in period 1 the firm will price relatively high to contain early adoption and dissemination of negative word of mouth.

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Appendix A. Threshold Values of SL Intensity

$$\begin{aligned} s_1 &\equiv \frac{(1-\beta)(\bar{v}-c)}{\beta[(\bar{v}-c)F(\bar{v})-2\int_0^{\bar{v}} v-c dF(v)]}, & s_2 &\equiv \frac{1-\beta}{\beta F(\bar{v})}, \\ s_3 &\equiv \frac{(1-\beta)(\bar{v}-c)}{\beta \int_0^{\bar{v}} \bar{v}-v dF(v)}, \\ s_4 &\equiv \left(\frac{2\int_0^{\bar{v}} v-c dF(v)}{(p^*-c)(1-F(p^*))+(\bar{v}-c)F(\bar{v})} + \frac{1-\beta}{\beta} \right) \frac{1}{F(\bar{v})}, \\ s_5 &\equiv \frac{(1-\beta)(\bar{v}-c)}{\beta \int_{p^*}^{\infty} v-p^* dF(v)}, \quad \text{and} \\ s_6 &\equiv \frac{\bar{v}-c}{\beta(\int_{p^*}^{\infty} v-c dF(v)+\int_{p^*}^{\infty} v-p^* dF(v))}. \end{aligned}$$

Appendix B. Proofs of Lemmas 1–3 and Propositions 1–7

LEMMA 1. Suppose $c \geq \bar{v}$. (1) The optimal period 2 price is $p_2^* = p^*$. The period 1 price that induces α ($0 \leq \alpha \leq 1$) consumers to purchase early is $p_1 = \bar{v} - \beta\alpha s \int_{p^*}^{\infty} v - p^* dF(v)$. (2) The total discounted firm profits as a function of α are $\pi_1 = \Psi_1(\alpha)$, where

$$\Psi_1(\alpha) \equiv -\alpha^2 \beta s \int_{p^*}^{\infty} v - c dF(v) + \alpha [\bar{v} - c + \beta s (1 - F(p^*)) (p^* - c)].$$

PROOF. We first examine the period 2 market. Suppose α ($0 \leq \alpha \leq 1$) consumers have purchased in period 1. Then, by the second period, $1 - \alpha$ consumers remain in the market. Among these, $(1 - \alpha)\alpha s$ consumers have discovered their true valuation via social learning. At any price $p_2 > c$, the informed consumers who value the product at or above p_2 will buy. The optimal period 2 price is thus $p_2^* = p^*$.

We now turn to period 1. When α consumers purchase in period 1, an individual consumer not buying in period 1 expects to receive discounted expected utility $u_0 = \beta\alpha s \int_{p^*}^{\infty} v - p^* dF(v)$, as she will buy in period 2 if and only if she is informed (with probability αs) and values the product higher than p^* . Inducing α consumers to buy in period 1 requires $\bar{v} - p_1 = u_0$, which yields the corresponding period 1 price, $p_1 = \bar{v} - \beta\alpha s \int_{p^*}^{\infty} v - p^* dF(v)$. From the firm's perspective, therefore, choosing price p_1 amounts to choosing the consumers' probability of buying early (α). The firm's total discounted profits are

$$\begin{aligned} \pi_1(\alpha) &= \Psi_1(\alpha) \equiv \alpha \left[\bar{v} - \beta\alpha s \int_{p^*}^{\infty} v - p^* dF(v) - c \right] \\ &\quad + \beta(1 - \alpha)s(1 - F(p^*))(p^* - c). \end{aligned}$$

Here, the first and second terms are the (discounted) profits from periods 1 and 2, respectively. Equivalently, we may rewrite $\Psi_1(\alpha)$ as given in the lemma. Q.E.D.

PROOF OF PROPOSITION 1. First, because $\int_{p^*}^{\infty} v - c dF(v) > \int_{p^*}^{\infty} p^* - c dF(v) = (p^* - c)(1 - F(p^*))$, the denominator of α_1 exceeds $2\beta s(p^* - c)(1 - F(p^*))$. Hence, $\alpha_1 < \frac{1}{2}$. Next, note that $\alpha_1 > 0$ if and only if $s > (c - \bar{v})/(\beta(p^* - c)(1 - F(p^*)))$. The equilibrium probability of adopting in period 1 (α^*) then follows. The optimal prices, p_1^* and p_2^* , follow directly from Lemma 1. When $s \geq (c - \bar{v})/(\beta(p^* - c)(1 - F(p^*)))$, the discounted firm profits are

$$\Psi_1(\alpha_1) = \frac{[\bar{v} - c + \beta s(p^* - c)(1 - F(p^*))]^2}{4\beta s \int_{p^*}^{\infty} v - c dF(v)},$$

which increases in s . The expected consumer welfare is $\beta\alpha_1 s \int_{p^*}^{\infty} v - p^* dF(v)$, which also increases in s . Therefore, the expected social welfare must also increase in s . Q.E.D.

PROOF OF PROPOSITION 2. First, we observe that the optimal period 2 price takes two possible values, p^* and \bar{v} . Suppose α ($0 \leq \alpha \leq 1$) consumers have adopted in period 1. In period 2, $1 - \alpha$ consumers remain. Among these, $(1 - \alpha)\alpha s$ consumers are informed about their valuation via social learning, and $(1 - \alpha)(1 - \alpha s)$ consumers are uninformed. At any price $p_2 \leq \bar{v}$, all uninformed consumers and those informed consumers valuing the product more than p_2 will buy. The period 2 profits from the uninformed consumers are $(1 - \alpha)(1 - \alpha s)(p_2 - c)$. The period 2 profits from the informed segment are $(1 - \alpha)\alpha s(1 - F(p_2))(p_2 - c)$, whose derivative with respect to p_2 is positive when $p_2 < p^*$. Therefore, $p_2 = \bar{v}$ strictly dominates any price below \bar{v} . At any price $p_2 > \bar{v}$, only the informed consumers with valuation above p_2 will buy. Therefore, the optimal period 2 price above \bar{v} is p^* .

Next, we identify the conditions for each period 2 price to be optimal. When $p_2 = \bar{v}$, in period 2 only the informed consumers with valuation below \bar{v} (in the population of $(1 - \alpha)\alpha s F(\bar{v})$) will not buy. The firm's period 2 profits are $\pi_2(\bar{v}) = (1 - \alpha)(1 - \alpha s F(\bar{v}))(\bar{v} - c)$. When $p_2 = p^*$, in period 2 only the informed consumers with valuation above p^* (in the population of $(1 - \alpha)\alpha s(1 - F(p^*))$) will buy. The period 2 profits are $\pi_2(p^*) = (1 - \alpha)\alpha s(1 - F(p^*))(p^* - c)$. Comparing $\pi_2(\bar{v})$ and $\pi_2(p^*)$ yields the optimal period 2 price, $p_2^* = \bar{v}$ when $\alpha \leq \hat{\alpha}$, and $p_2^* = p^*$ otherwise. Q.E.D.

When $p^* \geq \bar{v} > c$, Lemma 2 below shows that the firm's period 1 problem of choosing price is equivalent to choosing consumers' probability of early adoption (α) and identifies its objective function. Lemma 2 is pivotal to proving Propositions 3, 6, and 7.

LEMMA 2. Suppose $p^* \geq \bar{v} > c$. (1) The period 1 price that induces α ($0 \leq \alpha \leq 1$) consumers to purchase in period 1 is $p_1 = \bar{v} - \beta\alpha s \int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$ for $\alpha \leq \hat{\alpha}$, and it is $p_1 = \bar{v} - \beta\alpha s \int_{p^*}^{\infty} v - p^* dF(v)$ for $\hat{\alpha} < \alpha \leq 1$. (2) The firm's total discounted profits as a function of α are

$$\pi_1(\alpha) = \begin{cases} \Psi_2(\alpha) & \text{for } 0 \leq \alpha \leq \hat{\alpha}, \\ \Psi_1(\alpha) & \text{for } \hat{\alpha} < \alpha \leq 1, \end{cases}$$

where Ψ_1 is as given in Lemma 1 and

$$\begin{aligned} \Psi_2(\alpha) &\equiv \alpha^2 \beta s \int_0^{\bar{v}} v - c dF(v) \\ &\quad + \alpha(\bar{v} - c)[1 - \beta(1 + sF(\bar{v}))] + \beta(\bar{v} - c). \end{aligned}$$

PROOF. First, suppose $\alpha \leq \hat{\alpha}$. By Proposition 2, $p_2^* = \bar{v}$. In period 2, an informed buyer receives expected utility $\int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$, and an uninformed buyer receives zero utility. When α consumers purchase in period 1, the discounted expected utility of a consumer who does not buy in period 1 is therefore $u_0 = \beta\alpha s \int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$. To induce α consumers to buy in period 1, they each must receive expected utility u_0 , i.e., $\bar{v} - p_1 = u_0$, which gives the corresponding period 1 price $p_1 = \bar{v} - \beta\alpha s \int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$. The firm's choice of price p_1 thus reduces to choosing the consumers' probability of adopting early, α . Since $p_2^* = \bar{v}$, in period 2, all remaining consumers except the informed ones with valuation below \bar{v} will buy. The firm's total discounted profits as a function of α are

$$\begin{aligned} \pi_1(\alpha) &= \Psi_2(\alpha) \equiv \alpha \left[\bar{v} - \beta\alpha s \int_{\bar{v}}^{\infty} v - \bar{v} dF(v) - c \right] \\ &\quad + \beta(1 - \alpha)(1 - \alpha s F(\bar{v}))(\bar{v} - c), \end{aligned}$$

where the first and second terms are the (discounted) expected profits from periods 1 and 2, respectively. The function $\Psi_2(\alpha)$ can be rewritten as in Lemma 2.

Second, suppose $\alpha > \hat{\alpha}$. By Proposition 2, $p_2^* = p^*$. In period 2, an informed buyer receives expected utility $\int_{p^*}^{\infty} v - p^* dF(v)$, and an uninformed buyer does not buy. When α consumers purchase in period 1, by deferring adoption an individual receives discounted expected utility $u_0 = \beta\alpha s \int_{p^*}^{\infty} v - p^* dF(v)$. To induce α consumers to buy in period 1 requires $\bar{v} - p_1 = u_0$, which gives the corresponding period 1 price $p_1 = \bar{v} - \beta\alpha s \int_{p^*}^{\infty} v - p^* dF(v)$. In this case, its discounted profits are thus $\Psi_1(\alpha)$ (as given in Lemma 1). Q.E.D.

PROOF OF PROPOSITION 3. When $s \leq \hat{s}$, we have $\hat{\alpha} \geq 1$. By Lemma 2, the firm's period 1 objective function is $\pi_1(\alpha) = \Psi_2(\alpha)$. When $\int_0^{\bar{v}} v - c dF(v) < 0$ (i.e., the unit cost is high), $\Psi_2(\alpha)$ is concave, and its unconstrained maximizer is α_2 . When $\int_0^{\bar{v}} v - c dF(v) \geq 0$ (i.e., the unit cost is low), $\Psi_2(\alpha)$ is convex, and its maximum attains at either $\alpha = 0$ or $\alpha = 1$. When $\int_0^{\bar{v}} v - c dF(v) < 0$, we can readily verify that $\alpha_2 \leq 1 \Leftrightarrow s \geq s_1$ and that $\alpha_2 \geq 0 \Leftrightarrow s \leq s_2$. When

$\int_0^{\bar{v}} v - c dF(v) \geq 0$, we have $\Psi_2(0) \geq \Psi_2(1) \Leftrightarrow s \geq s_3$. Parts (1) and (2) of the Proposition then follow. The optimal pricing strategy (part (3)) immediately follows from Proposition 2 and Lemma 2. Q.E.D.

PROOF OF PROPOSITION 4. From Proposition 3, we can readily derive the discounted firm profits. When $\int_0^{\bar{v}} v - c dF(v) < 0$, the discounted firm profits are $\pi_1 = \bar{v} - c - \beta s \int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$ when $0 < s \leq s_1$,

$$\pi_1 = \beta(\bar{v} - c) + \frac{[1 - \beta(1 + sF(\bar{v}))]^2 (\bar{v} - c)^2}{-4\beta s \int_0^{\bar{v}} v - c dF(v)} \quad \text{when } s_1 < s \leq s_2,$$

and $\pi_1 = \beta(\bar{v} - c)$ when $s_2 \leq s \leq \hat{s}$. Here, we show that

$$\frac{[1 - \beta(1 + sF(\bar{v}))]^2 (\bar{v} - c)^2}{-4\beta s \int_0^{\bar{v}} v - c dF(v)}$$

decreases in s for $s_1 < s \leq s_2$. Since $\int_0^{\bar{v}} v - c dF(v) < 0$ by assumption, we only need to show that $(A - Bs)^2/s$ decreases in s over $(s_1, s_2]$, where $A \equiv 1 - \beta$ and $B \equiv \beta F(\bar{v})$. The first derivative of $(A - Bs)^2/s$ with respect to s is $-(A - Bs)(A + Bs)/s^2$, which is negative when $s \leq s_2$. When $\int_0^{\bar{v}} v - c dF(v) \geq 0$, firm profits are $\pi_1 = \bar{v} - c - \beta s \int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$ when $0 < s \leq s_3$ and $\pi_1 = \beta(\bar{v} - c)$ when $s_3 \leq s \leq \hat{s}$. In this case, firm profits also weakly decrease in s over $[0, \hat{s}]$. Q.E.D.

PROOF OF PROPOSITION 5. Suppose that α ($0 \leq \alpha \leq 1$) consumers purchase in period 1. When $s \leq \hat{s}$, in period 2 the price is \bar{v} (by Proposition 3), an informed consumer receives expected utility $\int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$, and an uninformed consumer receives zero expected utility. In equilibrium, each early adopter receives the same (discounted) expected utility as a late adopter. Therefore, when α consumers purchase in period 1, the (discounted) consumer welfare is $\beta\alpha s \int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$. We can then readily verify (from Proposition 3) that expected consumer welfare increases in s over $[0, \hat{s}]$ when $\{\int_0^{\bar{v}} v - c dF(v) < 0 \text{ and } s \leq s_1\}$ or $\{\int_0^{\bar{v}} v - c dF(v) \geq 0 \text{ and } s \leq s_3\}$, decreases in s when $\{\int_0^{\bar{v}} v - c dF(v) < 0 \text{ and } s_1 < s \leq s_2\}$, and remains constant at 0 otherwise. It is then easy to verify that expected social welfare (the sum of expected consumer welfare and firm profits) weakly decreases in s over $[0, \hat{s}]$. Q.E.D.

Lemma 3 below is needed to prove Propositions 6 and 7.

LEMMA 3. (1) $\Psi_2(\hat{\alpha}) \leq \Psi_1(\hat{\alpha})$; (2) Ψ_1 and Ψ_2 intersect only once in $(0, \hat{\alpha})$; and (3) $\Psi_2(\alpha) < \Psi_1(\alpha)$ over $(\hat{\alpha}, 1]$.

PROOF. (1) By construction, Ψ_1 and Ψ_2 are the total discounted profits when the period 2 price is p^* and \bar{v} , respectively (see the proofs of Lemmas 1 and 2 above). By construction of $\hat{\alpha}$, the period 2 profits of charging \bar{v} and p^* are equal when $\alpha = \hat{\alpha}$. We then have $\Psi_2(\hat{\alpha}) - \Psi_1(\hat{\alpha}) = \hat{\alpha}^2 \beta s (\int_{p^*}^{\infty} v - p^* dF(v) - \int_{\bar{v}}^{\infty} v - \bar{v} dF(v)) \leq 0$, because $\int_x^{\infty} v - x dF(v)$ decreases in x and $p^* \geq \bar{v}$ by assumption.

(2) We can verify that $\Psi_2(0) = \beta(\bar{v} - c) > \Psi_1(0) = 0$. Part (1) has shown that $\Psi_2(\hat{\alpha}) \leq \Psi_1(\hat{\alpha})$. Because Ψ_2 and Ψ_1 are continuous in α , they must intersect in $(0, \hat{\alpha})$. Because Ψ_2 and Ψ_1 are both quadratic functions of α , they intersect at most twice. Thus, they intersect once or twice in $(0, \hat{\alpha})$. However, if they intersect twice in $(0, \hat{\alpha})$, we then

have $\Psi_1(\hat{\alpha}) \leq \Psi_2(\hat{\alpha})$, contradicting part (1) of the lemma. Therefore, Ψ_1 and Ψ_2 intersect once in $(0, \hat{\alpha})$.

(3) If Ψ_2 and Ψ_1 also intersect once in $[\hat{\alpha}, 1]$, we then have $\Psi_2(1) > \Psi_1(1)$. However, we can verify that $\Psi_2(1) = \bar{v} - c - \beta s \int_{\bar{v}}^{\infty} v - \bar{v} dF(v) < \Psi_1(1) = \bar{v} - c - \beta s \int_{p^*}^{\infty} v - p^* dF(v)$. Again, it is a contradiction. Therefore, Ψ_2 and Ψ_1 cannot intersect in $(\hat{\alpha}, 1]$. We have shown above that $\Psi_2(\hat{\alpha}) \leq \Psi_1(\hat{\alpha})$. The statement of part (3) immediately follows. Q.E.D.

PROOF OF PROPOSITION 6. By Lemma 2, the firm's period 1 objective is $\Psi_2(\alpha)$ when $\alpha \leq \hat{\alpha}$, and it is $\Psi_1(\alpha)$ otherwise. Their unconstrained maximizers are α_2 and α_1 , respectively, as identified in §3.

Part (1) When $s \geq s_2$, we have $\alpha_2 \leq 0$. When $s \leq s_6$, $\alpha_1 \geq 1$. Therefore, α^* is either 0 or 1, depending on the comparison between $\Psi_2(0)$ and $\Psi_1(1)$. We have $\Psi_2(0) = \beta(\bar{v} - c)$ and $\Psi_1(1) = -\beta s \int_{p^*}^{\infty} v - c dF(v) + \bar{v} - c + \beta s(1 - F(p^*))(p^* - c)$. Note that $\Psi_2(0) \geq \Psi_1(1) \Leftrightarrow s \geq s_5$. Part (1) then readily follows.

Part (2) When $s \leq s_4$, $\alpha_2 \geq \hat{\alpha}$. When $s \leq s_6$, $\alpha_1 \geq 1$. By Lemma 3, $\Psi_2(\alpha) \leq \Psi_1(\alpha)$ over $[\hat{\alpha}, 1]$. Therefore, $\alpha^* = 1$.

Part (3) When $s_4 < s < s_2$, we have $0 < \alpha_2 < \hat{\alpha}$. When $s \leq s_6$, $\alpha_1 \geq 1$. Therefore, α^* is either α_2 or 1. When $\Psi_2(\alpha_2) \geq \Psi_1(1)$, $\alpha^* = \alpha_2$. Otherwise, $\alpha^* = 1$.

Part (4) The optimal period 1 price immediately follows from Proposition 2 and Lemma 2. Q.E.D.

PROOF OF PROPOSITION 7. When $\hat{s} < s \leq 1$, we have $0 < \hat{\alpha} < 1$. By Lemma 2, the firm's profit function is $\Psi_2(\alpha)$ for $0 \leq \alpha < \hat{\alpha}$, and it is $\Psi_1(\alpha)$ for $\hat{\alpha} \leq \alpha \leq 1$. When $\int_0^{\bar{v}} v - c dF(v) > 0$, $\Psi_2(\alpha)$ is convex. By Lemma 3, $\Psi_2(\hat{\alpha}) \leq \Psi_1(\hat{\alpha})$. When $s \leq s_6$, $\alpha_1 \geq 1$. Therefore, if $\Psi_2(0) \leq \Psi_1(1)$ or equivalently if $s \leq s_5$, the firm's profits are maximized at $\alpha^* = 1$. When $s_5 < s \leq s_6$, firm profits are maximized at $\alpha^* = 0$. Q.E.D.

Appendix C. Analysis of the Case in Which $p^* < \bar{v}$
We first characterize the optimal price in period 2.

PROPOSITION 10. Suppose α ($0 \leq \alpha \leq 1$) consumers have purchased in period 1. The optimal period 2 price is $p_2^* = \bar{v}$ if $\alpha s[F(p_2) + f(p_2)(p_2 - c)] < 1$ for all $p_2 \in [p^*, \bar{v}]$, and it is the unique solution to $p_2^* - c = (1/\alpha s - F(p_2^*))/f(p_2^*)$ otherwise.

PROOF. First, we note that the optimal period 2 price must satisfy $p^* \leq p_2 \leq \bar{v}$. If $p_2 < p^*$, then all but those informed consumers who value the product at below p_2 will buy. But raising p_2 slightly while keeping it below p^* increases the period 2 profits from both the informed and uninformed consumers. Thus, $p_2 \geq p^*$. If $p_2 > \bar{v}$, then only the informed consumers whose valuation exceeds p_2 will buy. However, the optimal price for informed consumers is $p^* < \bar{v}$. This is a contradiction. Thus, $p_2 \leq \bar{v}$.

Suppose α ($0 \leq \alpha \leq 1$) consumers have adopted in period 1. By period 2, $(1 - \alpha)\alpha s$ consumers are informed, and $(1 - \alpha)(1 - \alpha s)$ consumers remain uninformed. When $p^* \leq p_2 \leq \bar{v}$, only the informed consumers with valuation below p_2 will not buy. The firm's period 2 profit function is thus $\pi_2(p_2) = (1 - \alpha)(1 - \alpha s F(p_2))(p_2 - c)$, whose first derivative is

$$\frac{d\pi_2}{dp_2} = (1 - \alpha)\{1 - \alpha s[F(p_2) + f(p_2)(p_2 - c)]\}.$$

When $\alpha s[F(p_2) + f(p_2)(p_2 - c)] < 1$ for all $p_2 \in [p^*, \bar{v}]$, we have $d\pi_2/dp_2 > 0$ over $[p^*, \bar{v}]$, and hence $p_2^* = \bar{v}$. Otherwise, the optimal period 2 price is uniquely given by $d\pi_2/dp_2 = 0$. Q.E.D.

We now turn to the period 1 market. When α consumers buy in period 1, by deferring adoption a consumer receives discounted expected utility

$$u_0 = \beta \left\{ \alpha s \int_{p_2^*}^{\infty} v - p_2^* dF(v) + (1 - \alpha s)(\bar{v} - p_2^*) \right\},$$

or equivalently, $u_0 = \beta \{ \alpha s \int_0^{p_2^*} p_2^* - v dF(v) + \bar{v} - p_2^* \}$. To induce α consumers to buy in period 1, we must have $\bar{v} - p_1 = u_0$ or $p_1 = \bar{v} - \beta \{ \alpha s \int_0^{p_2^*} p_2^* - v dF(v) + \bar{v} - p_2^* \}$. The firm's period 1 objective function is

$$\pi_1(\alpha) = \alpha \left[\bar{v} - c - \beta \left(\alpha s \int_0^{p_2^*} p_2^* - v dF(v) + \bar{v} - p_2^* \right) \right] + \beta(1 - \alpha)(1 - \alpha s F(p_2^*))(p_2^* - c).$$

The partial derivative of π_1 with respect to α is (after consolidating terms)

$$\frac{\partial \pi_1}{\partial \alpha} = (1 - \beta)(\bar{v} - c) - \beta s(p_2^* - c)F(p_2^*) + 2\alpha\beta s \int_0^{p_2^*} v - c dF(v).$$

PROPOSITION 11. *Case (A).* When $\partial \pi_1/\partial \alpha|_{p_2^*=\bar{v}} < 0$ for all $\alpha \in [0, 1]$, $\alpha^* = 0$ and $p_2^* = \bar{v}$.

Case (B). When $s[F(p_2) + f(p_2)(p_2 - c)] < 1$ for all $p_2 \in [p^*, \bar{v}]$ and $\partial \pi_1/\partial \alpha|_{p_2^*=\bar{v}} > 0$ for all $\alpha \in [0, 1]$, $\alpha^* = 1$ and $p_2^* = \bar{v}$.

Case (C). When $\partial \pi_1/\partial \alpha|_{p_2^*=\bar{v}} = 0$ for some $\alpha \in [0, 1]$,

$$p_2^* = \bar{v} \quad \text{and} \quad \alpha^* = \frac{(\bar{v} - c)[F(\bar{v}) - (1 - \beta)/\beta s]}{2 \int_0^{\bar{v}} v - c dF(v)},$$

provided $\alpha^* s[F(p_2) + f(p_2)(p_2 - c)] < 1$ for all $p_2 \in [p^*, \bar{v}]$.

Case (D). Otherwise, α^* and p_2^* are jointly given by

$$p_2^* - c = \frac{1/\alpha^* s - F(p_2^*)}{f(p_2^*)} \quad \text{and} \quad \alpha^* = \frac{(p_2^* - c)[F(p_2^*) - (1 - \beta)/\beta s]}{2 \int_0^{p_2^*} v - c dF(v)}.$$

PROOF. (A) When $\partial \pi_1/\partial \alpha|_{p_2^*=\bar{v}} < 0$ for all $\alpha \in [0, 1]$, we have $\alpha^* = 0$. Since $\alpha^* s[F(p_2) + f(p_2)(p_2 - c)] = 0 < 1$ for all $p_2 \in [p^*, \bar{v}]$, by Proposition 10 we indeed have $p_2^* = \bar{v}$. (B) When $\partial \pi_1/\partial \alpha|_{p_2^*=\bar{v}} > 0$ for all $\alpha \in [0, 1]$, clearly, $\alpha^* = 1$. By Proposition 10, when $s[F(p_2) + f(p_2)(p_2 - c)] < 1$ for all $p_2 \in [p^*, \bar{v}]$, we have $p_2^* = \bar{v}$. (C) When $\partial \pi_1/\partial \alpha|_{p_2^*=\bar{v}} = 0$ for some $\alpha \in [0, 1]$, then α^* is given by this first-order condition; i.e.,

$$\alpha^* = \frac{(\bar{v} - c)[F(\bar{v}) - (1 - \beta)/\beta s]}{2 \int_0^{\bar{v}} v - c dF(v)}.$$

By Proposition 10, if $\alpha^* s[F(p_2) + f(p_2)(p_2 - c)] < 1$ for all $p_2 \in [p^*, \bar{v}]$, we have $p_2^* = \bar{v}$. (D) Otherwise, α^* and p_2^* are clearly the solutions to $\partial \pi_1/\partial \alpha^* = 0$ and $p_2^* - c = 1/\alpha^* s - F(p_2^*)/f(p_2^*)$. Q.E.D.

Cases (A) and (B) reflect adoption inertia and frenzy, and (C) and (D) reflect staggered adoption.

Appendix D. Equilibrium with Informative Advertising

We first introduce some necessary notation. Let $\hat{s}' \equiv (\hat{s} - t)/(1 - t)$, and let $\hat{\alpha}' \equiv (\hat{\alpha} - t)/(1 - t)s$. Let Φ_1 and Φ_2 be

the respective counterparts of Ψ_1 and Ψ_2 after replacing α with $t + (1-t)\alpha s$. Define

$$\begin{aligned}\alpha'_1 &= \frac{\bar{v} - c - \beta t \int_{p^*}^{\infty} v - p^* dF(v) + \beta(p^* - c)(1 - F(p^*))[-t + (1-t)s]}{2\beta s(1-t) \int_{p^*}^{\infty} v - c dF(v)}, \\ \alpha'_2 &= \frac{\beta t \int_0^{\bar{v}} v - c dF(v) + [1 - \beta - \beta(1-t)sF(\bar{v})](\bar{v} - c)}{-2\beta s(1-t) \int_0^{\bar{v}} v - c dF(v)}, \\ s'_1 &= \frac{\beta t \int_0^{\bar{v}} v - c dF(v) + (1 - \beta)(\bar{v} - c)}{\beta(1-t)[(\bar{v} - c)F(\bar{v}) - 2 \int_0^{\bar{v}} v - c dF(v)]}, \\ s'_2 &= \frac{(1 - \beta)(\bar{v} - c) + \beta t \int_0^{\bar{v}} v - c dF(v)}{(1-t)\beta F(\bar{v})(\bar{v} - c)}, \\ s'_3 &= \frac{\beta t \int_0^{\bar{v}} v - c dF(v) + (1 - \beta)(\bar{v} - c)}{(1-t)\beta \int_0^{\bar{v}} \bar{v} - v dF(v)}, \\ s'_4 &= \frac{\beta(2\hat{s} - t) \int_0^{\bar{v}} v - c dF(v) + (1 - \beta)(\bar{v} - c)}{(1-t)\beta F(\bar{v})(\bar{v} - c)}, \\ s'_5 &= \frac{1}{1-t} \left\{ \frac{[1 - \beta(1-tF(\bar{v}))](\bar{v} - c)}{\beta \int_{p^*}^{\infty} v - p^* dF(v)} - t \right\},\end{aligned}$$

and

$$s'_6 = \frac{\bar{v} - c - \beta t \int_{p^*}^{\infty} v - c dF(v)}{\beta(1-t) \left(\int_{p^*}^{\infty} v - c dF(v) + \int_{p^*}^{\infty} v - p^* dF(v) \right)}.$$

Here, α'_i ($i = 1, 2$) is the counterpart of α_i , and s'_i ($i = 1, \dots, 6$) is the counterpart of s_i . It is straightforward to verify that Propositions 2, 3, 6, and 7 hold if we replace \hat{s} with \hat{s}' , $\hat{\alpha}$ with $\hat{\alpha}'$, α_i with α'_i , and s_i with s'_i . In Proposition 3, $p_1^* = \bar{v} - \beta[t + (1-t)\alpha^*s] \int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$. In Propositions 6 and 7, $p_1^* = \bar{v} - \beta[t + (1-t)\alpha^*s] \int_{\bar{v}}^{\infty} v - \bar{v} dF(v)$ for $\alpha^* \leq \hat{\alpha}'$, and $p_1^* = \bar{v} - \beta[t + (1-t)\alpha^*s] \int_{p^*}^{\infty} v - p^* dF(v)$ otherwise.

Appendix E. Ex Ante Heterogeneous Consumers

We first derive the optimal period 2 price. When α_H ($0 \leq \alpha_H \leq \theta$) H consumers buy in period 1, a consumer who defers purchase becomes informed by period 2 with probability $\alpha_H s_H$. Because $\bar{v}_L < c < \bar{v}_H$ by assumption, in period 2, the firm prices at either \bar{v}_H or v_2 . When $p_2 = v_2$, only the informed (H or L) consumers with value v_2 will buy, and the firm makes period 2 profits $\pi_2(v_2) = [(\theta - \alpha_H)\gamma_H + (1 - \theta)\gamma_L]\alpha_H s_H(v_2 - c)$, where $(\theta - \alpha_H)\gamma_H$ and $(1 - \theta)\gamma_L$ are the remaining H and L consumers with value v_2 , respectively. When $p_2 = \bar{v}_H$, the informed (H or L) consumers with value v_2 , and the uninformed H consumers will buy. The firm's period 2 profits are

$$\begin{aligned}\pi_2(\bar{v}_H) &= [(\theta - \alpha_H)\alpha_H s_H \gamma_H + (1 - \theta)\alpha_H s_H \gamma_L \\ &\quad + (\theta - \alpha_H)(1 - \alpha_H s_H)](\bar{v}_H - c),\end{aligned}$$

where $(\theta - \alpha_H)\alpha_H s_H \gamma_H$ and $(1 - \theta)\alpha_H s_H \gamma_L$ are the informed H and L consumers with value v_2 , respectively, and $(\theta - \alpha_H)(1 - \alpha_H s_H)$ is the uninformed H consumers. Comparing $\pi_2(v_2)$ and $\pi_2(\bar{v}_H)$ shows that the firm prices at $p_2^* = v_2$ if $\alpha_H \geq \hat{\alpha}_H \equiv (-B + \sqrt{B^2 - 4AC})/2A$, where $A \equiv -s_H[(v_2 - \bar{v}_H)\gamma_H + (\bar{v}_H - c)]$, $B \equiv s_H(v_2 - \bar{v}_H)[\theta\gamma_H + (1 - \theta)\gamma_L] + (\bar{v}_H - c)(1 + \theta s_H)$, and $C \equiv -(\bar{v}_H - c)\theta$. Otherwise, $p_2^* = \bar{v}_H$. By construction, $0 < \hat{\alpha}_H < \theta$.

We now seek the period 1 solution. When $\alpha_H < \hat{\alpha}_H$, by deferring purchase an H consumer receives discounted expected surplus $\beta\alpha_H s_H \gamma_H(v_2 - \bar{v}_H)$. To induce α_H H consumers to adopt early, the period 1 price must satisfy $\bar{v}_H - p_1 = \beta\alpha_H s_H \gamma_H(v_2 - \bar{v}_H)$ or $p_1 = \bar{v}_H - \beta\alpha_H s_H \gamma_H(v_2 - \bar{v}_H)$. When $\alpha_H < \hat{\alpha}_H$, the firm's total discounted profits are $\pi_1 = G_2(\alpha_H) \equiv \alpha_H[\bar{v}_H - \beta\alpha_H s_H \gamma_H(v_2 - \bar{v}_H) - c] + \beta\pi_2(\bar{v}_H)$, whose (unconstrained) maximizer is

$$\alpha_{H2} = \frac{(\bar{v}_H - c)\{1 - \beta + \beta s_H[(1 - \theta)\gamma_L - \theta(1 - \gamma_H)]\}}{2\beta s_H(1 - \gamma_H)(c - v_1)}.$$

When $\alpha_H \geq \hat{\alpha}_H$, by deferring purchase, an H consumer receives zero expected surplus. The optimal period 1 price is thus $p_1 = \bar{v}_H$. The firm's total profits are $\pi_1 = G_1(\alpha_H) \equiv \alpha_H(\bar{v}_H - c) + \beta\pi_2(v_2)$, whose (unconstrained) maximizer is

$$\alpha_{H1} = \frac{(\bar{v}_H - c) + \beta s_H(v_2 - c)[\theta\gamma_H + (1 - \theta)\gamma_L]}{2\beta\gamma_H s_H(v_2 - c)}.$$

The proposition below identifies adoption equilibrium.

PROPOSITION 12. (1) When $\alpha_{H2} \leq 0$ and $\alpha_{H1} \leq \hat{\alpha}_H$, $\alpha_H^* = 0$ if $G_1(\hat{\alpha}_H) < \beta\theta(\bar{v}_H - c)$ and $\alpha_H^* = \hat{\alpha}_H$ otherwise. (2) When $\alpha_{H2} \leq 0$ and $\alpha_{H1} \geq \theta$, $\alpha_H^* = 0$ if $G_1(\theta) < \beta\theta(\bar{v}_H - c)$ and $\alpha_H^* = \theta$ otherwise. (3) When $\alpha_{H2} \geq \hat{\alpha}_H$, $\alpha_H^* = \hat{\alpha}_H$ if $\alpha_{H1} \leq \hat{\alpha}_H$, $\alpha_H^* = \theta$ if $\alpha_{H1} \geq \theta$, and $\alpha_H^* = \alpha_{H1}$ otherwise.

PROOF. (1) When $\alpha_{H2} \leq 0$ and $\alpha_{H1} \leq \hat{\alpha}_H$, α_H^* is either 0 or $\hat{\alpha}_H$ by the construction of α_{H2} and α_{H1} . Part (1) then follows from $G_2(0) = \beta\theta(\bar{v}_H - c)$. (2). When $\alpha_{H2} \leq 0$ and $\alpha_{H1} \geq \theta$, α_H^* is either 0 or θ , depending on the comparison between $G_2(0)$ and $G_1(\theta)$. (3). When $\alpha_{H2} \geq \hat{\alpha}_H$, $G_2(\alpha_H)$ increases over $[0, \hat{\alpha}_H]$. By the construction of $\hat{\alpha}_H$, the period 2 profits of charging \bar{v}_H and v_2 are equal when $\alpha_H = \hat{\alpha}_H$. We then have $G_2(\hat{\alpha}_H) - G_1(\hat{\alpha}_H) = -\hat{\alpha}_H^2 \beta s_H \gamma_H(v_2 - \bar{v}_H) \leq 0$. Therefore, $G_2(\hat{\alpha}_H) \leq G_1(\hat{\alpha}_H)$. Part (3) then follows. Q.E.D.

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