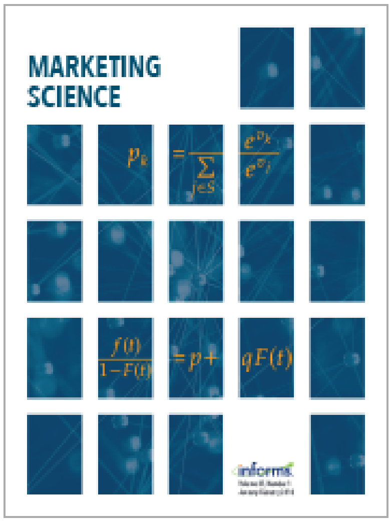

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Equilibrium Price Communication and Unadvertised Specials by Competing Supermarkets

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Abstract

This paper is concerned with how retailers, supermarkets in particular, communicate price discounts and use unadvertised specials. A common practice for supermarkets is to communicate price deals on some products through newspaper advertisements, while communicating discounts on other products through in-store mechanisms such as shelf-talkers. This raises the question: So far as store choice is concerned, how might consumers take into account not only advertised prices at competing stores, but also expected prices of unadvertised goods? It also begs the question of why stores have unadvertised specials since their effect on store choice is not quite the same as the advertised discounts. Further, competing supermarkets advertise the same products part of the time, and different products at other times. They also tend to sometimes advertise a product in consecutive weeks, but sometimes not. Can these actions be part of a strategy? We formulate a game-theoretic model of retail competition by first extending the work of Lal and Matutes (1994) and then developing an alternative framework to answer these questions. Our model has two retailers, each of whom carries two goods. To simplify exposition, we assume that the stores are symmetric, the two goods are symmetric in their reservation prices, and are neither substitutes nor complements. Consumers are identical in their preferences and consumer heterogeneity is in the convenience that each store presents to a representative consumer. The stores may advertise the price of one good, reflecting the reality that stores do not advertise their whole assortment. They compete through advertising and prices to maximize profits. We thus recognize the strategic role of advertised prices and furthermore, we investigate the strategic role of unadvertised prices in retail competition. For this model, we derive a Rational Expectations Nash equilibrium in which each store randomly advertises the price of one good following a mixed strategy. Consumer expectations of the prices of the unadvertised goods are rational. We obtain three kinds of results. First, unadvertised specials occur in equilibrium, and induce temporal and cross-sectional variation in the identity of advertised goods, consistent with casual observation. In this equilibrium, the two stores advertise the same good part of the time and different goods at other times. When they advertise the same good they do not offer any unadvertised discount on the other good. However, when they advertise different goods, they offer an unadvertised discount on the good that they do not advertise. Intuitively, unadvertised discounts come about because stores randomize the identity of the advertised good in the mixed strategy equilibrium. If retailers were to advertise the same good at all times, they would have to compete intensely for store traffic and therefore discount the advertised good very deeply. And, having done so, they would find it optimal to set the unadvertised good at the reservation price and offer no discount on it. However, if stores randomize the advertised good as shown in this paper, both stores advertise the same good some of the time and at other times they advertise different goods. Because they advertise different goods some of the time, they do not fight intensely for store traffic on just one good, but rather they find it optimal to offer a discount on the unadvertised good also. As a result, an implication of our equilibrium for consumer choice is that unadvertised discounts affect store choice, and in equilibrium some consumers may shop around. Second, we obtain managerial insights into the role of unadvertised specials. They affect store choice, prevent consumer shopping around either fully or partly, and reduce head-to-head competition on the price of the advertised good. The most salient strategic implication of retailers' offering unadvertised discounts is to reduce competition among stores, and this is again due to the randomization strategy of the stores. In fact, stores can reduce head-to-head competition further by increasing the number of products in their assortment and randomizing on the advertised good from this assortment. Third, we provide a resolution of the Diamond (1971) paradox, which says that prices at competing stores approach the monopoly price. In our equilibrium, expected prices of both advertised and unadvertised goods are always below the monopoly price.

(*Retailing; Supermarkets; Advertising; Pricing; Unadvertised Specials; Competition; Game Theory; Consumer Choice; Rational Expectations; Diamond Paradox*)

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1. Introduction

In this paper we analyze the strategies of two competing supermarkets that must decide how to communicate prices of the goods they carry and what the prices should be. In particular, we treat not only prices, but also their communication as strategic choices of the firms. Further, we view consumers as strategic agents who take into account the prices that are communicated, as well as prices of other goods which they may purchase once in the store. From a practical point of view it is necessary to explicitly consider the fact that most interactions between consumers and supermarkets involve multiproduct purchases, incomplete knowledge of prices on the consumer's part before visiting a supermarket, and a choice among supermarkets. Supermarkets can affect demand for their goods in two obvious ways. First, by advertising prices of some products they can influence consumers' store choice, and thus the number of consumers visiting the store, and second, by their choice of prices of unadvertised products they can affect what consumers buy once they have arrived at the store. Less obvious is whether, and to what extent, prices of unadvertised products can be used to affect store choice because these prices are not known to consumers before they visit a store. The problem of consumers choosing a retail establishment under incomplete knowledge of prices has been addressed by Diamond (1971). Pricing and communication of prices by multiproduct firms such as supermarkets have been examined by Lal and Matutes (1994), primarily with a view to understanding loss-leader pricing and the role of advertising as a commitment device to attract consumers to the store. Our paper builds on these prior works to shed light on some unresolved research questions that arise from an observation of pricing and advertising practice.

1.1. Research Questions

At a broad level, retailers are seen to engage in large advertising expenditures designed to communicate prices. For example, in 1997 Sears Roebuck spent $588 million on advertising, while Federated Department Stores, J.C. Penney, Dayton Hudson, and K-Mart spent $405 million, $305 million, $270 million, and $241 million respectively. Among supermarkets, Kroger spent $302 million, $281 million, and $250 million for 1996, 1995, and 1994, respectively. While these figures point to the large advertising budgets of retailers, not all of this represents advertising that conveys price information. If we restrict our attention to advertising in newspapers, we find that Federated Department Stores, for example, spent $340 million in 1996. In a similar vein, we find that the newspaper advertising expenditure by Circuit City Stores was $250 million in 1996, while spot TV expenditures by Circuit City Stores and Best Buy were to the tune of $105 million and $70 million, respectively, in 1996. It would be safe to surmise that retailers are spending considerable amounts on conveying price information. Despite these large advertising outlays to attract consumers by informing them of prices, we find, interestingly, that supermarkets often have some goods whose prices have been discounted in the store, but not advertised in the newspaper. This can be seen in Table 1, which displays for representative product categories the specials for a given week. We refer to these as *unadvertised specials.* These unadvertised specials are highlighted through shelf talkers and other means inside the store, suggesting that they are being communicated, but not necessarily to affect store choice.[1] When a supermarket cuts the price of a product temporarily in the form of a promotion, it would seem natural for this to be communicated to consumers. Indeed, empirical evidence suggests that consum-

Table 1 Advertised and Unadvertised Products by a Given Supermarket

Product (Examples)	Advertised Discounts	Unadvertised Discounts
Cheese	Kraft Regular	Kraft Deluxe, Borden, Food Club
Laundry detergent	None	Fab Ultra, Top Crest, All Ultra, Wisk, Wisk Bleach, Ajax, Surf Ultra
Paper towels	Brawn	Brawn, Sparkle
Frozen orange juice	Food Club	Food Club, Minute Maid, Seneca, Welch, Texsun, Bacardi, Hawaii's Own
Potato chips	Ruffles	Ruffles, Remarkable BBQ, Remarkable Regular

[1]They are communicated as "cents off" relative to the "regular price".

er response to price cuts can be increased substantially by advertising the price cut. Moreover, from the consumer's perspective, his (her) choice of store to visit would have been made without fully knowing which products carry low prices, and at which store. Given this, two questions arise. How do consumers make store choice in the presence of unadvertised specials? What is more interesting: Can unadvertised specials be part of an equilibrium strategy of competing supermarkets?

Most retailers do not advertise the prices of their entire assortment of products on any given occasion. The advertisements do not convey product prices in a matter-of-fact way, but usually attempt to inform consumers about price specials, or temporary price reductions. In many instances, retailers also communicate the savings associated with each purchase through their advertisements. Newspaper advertisements by some supermarkets even announce savings associated with purchases of certain products in conjunction with the use of a ''loyalty'' card. Finally, consumers are informed by these advertisements of prices relative to competitive stores. Despite intense competition, we often see that competing stores do not always advertise the same good. In fact, they advertise different goods at any given time. In a typical chosen week we found that, of the goods whose prices were advertised through newspaper inserts by two major grocery chains in a metropolitan area, only 10% were common and 90% were different. Similarly, in a typical week we found that, of the goods whose prices were advertised through newspaper inserts by two major electronics retailers in the same metropolitan area, 60% were different products and 40% were common. This situation is thus similar to supermarkets, though the differences between the percentages of same and different advertised goods appear to be less pronounced. Table 2 shows goods advertised by two supermarkets during the same week in a metropolitan area. It illustrates the fact that retail competition can involve advertising the same products as well as different products. Lal and Matutes (1994) have identified an equilibrium in which competing stores advertise the same good. The question that comes to mind then is, can it be part of an equilibrium strategy of competing supermarkets to advertise different goods?

Table 2 Products Advertised by Competing Supermarkets

Product (Examples)	Only Store A Advertises	Only Store B Advertises	Both Advertise
Raisin Bran cereal	Post	Janet Lee	
Orange juice	Dole	Janet Lee	Tropicana
Cheese	Jarlsberg	Athenos	President Brie
Frozen lowfat waffles	X	Aunt Jemima	X
Yogurt	X	Yoplait	X
Frozen fish sticks	X	Fisherboy	X
Frozen quiche	Store brand	X	X
Frozen potatoes	Food Club	X	X
French crepes	Melissa's	X	X
Total # products	**78**	**32**	**13**

Table 3 Products Advertised by a Supermarket in Consecutive Weeks

Product (Examples)	Advertised Only in Week 1	Advertised Only in Week 2	Advertised in Both Weeks
Orange juice	X	X	Tropicana
Wine	X	X	Kendell Jackson
Cooked ham	X	X	Hormel
Fire logs	Top Crest	X	X
Lobsters	Store brand	X	X
Corn dogs	Foster Farms	X	X
Pork ribs	X	Store brand	X
Olive oil	X	President's Choice	X
Tanning lotion	X	Ocean Potion	X

A final interesting element of retail advertising is the fact that the products being advertised are not the same over time. There could be many reasons for this. For example, at the end of a fashion season department stores may advertise certain goods to clear inventory, an effect attributable to uncertainty associated with demand. Similarly, at different times of the year stores may consider it appropriate to advertise different goods, which can be thought of as an effect due to seasonality. If we restrict our attention to supermarkets, the effects of uncertain demand and seasonality are unlikely to be significant. However, we see that even supermarkets change the goods they advertise over time. Table 3 identifies products whose prices were advertised in consecutive weeks by a supermarket. We see that the same products are not always advertised in consecutive weeks, and neither is it the case that advertised products always change. While some products continue to be advertised in the following week, many products advertised in the second week were not advertised in the previous week, while others were discontinued from the previous week. In other words, not only might products advertised in a given week differ across supermarkets, as shown in Table 2, but the advertised products within a supermarket might also differ across time. Thus, we see that there is a temporal variation of products advertised by a given store. We could regard this as due to the retailer changing his policies because of exogenous factors such as trade promotions.[2] Or could it be that this variation in the products being advertised is endogenous and is in fact an equilibrium strategy for competing supermarkets?

1.2. Brief Review of Literature

Feichtinger et al. (1988) analyze a monopolist retailer's problem of pricing and advertising which influence both the store's image and the demand. They do not explicitly consider consumers' knowledge of prices. Several other researchers have studied retail pricing involving some goods being priced relatively low to attract consumers to the store, and others being priced high to consumers visiting the store. An extreme form of this practice is *loss-leader pricing* in which goods that are used to attract consumers to the store are priced below (marginal) cost. The main reason for this is the fact that retailers sell many products, and consumers buy several products on a given visit to the store. In other words, not only do we have a multiproduct firm (seller) but we also have a consumer (buyer) whose store choice is based on considerations over several products. Thus we have a multiproduct-marketing situation. An early paper by Bliss (1988) recognized that the markup on one or more of the goods in a multiproduct situation would depend on, among other things, the crossprice effects across products. These crossprice effects could lead to loss-leader pricing. Bliss did not model consumers' imperfect knowledge of prices. Hess and Gerstner (1987) assume consumers to be informed of prices of one good (a shopping good) across stores but uninformed of prices of other goods (impulse goods). They find that competing stores price the shopping good low to benefit from selling impulse goods. However, they do not model the identity of shopping and impulse goods, or consumer knowledge of prices. Simester (1995) studies retail pricing under asymmetric costs by analyzing a signaling model of pricing assuming that a store's cost is hidden from its competitor and consumers. He asks how stores carrying two goods would price one good that they advertise, knowing that consumers will buy the other unadvertised good in addition after visiting the store. He shows that the unadvertised good would be priced at the monopoly level, but its price at a low-cost store would be lower than that at a high-cost store. The price of the advertised good at the low-cost store could either be lower than that at the competitor store or equal to it. When it is lower, consumers can infer the price of the unadvertised good from the price of the advertised good.

Lal and Matutes (1994) focus on retail pricing and advertising in a multistage game framework treating all decisions of both stores and consumers as endogenous. Each firm carries two goods and can advertise prices of both goods, one good, or neither. Competition between firms is on two dimensions: what prices to charge and which prices to communicate. Consumer store choice depends on advertised prices and rational expectations of unadvertised prices. The idea of consumers using price expectations to make store choices has also been addressed by Diamond (1971). Diamond has consumers visiting one store randomly and proceeding sequentially, learning the prices at visited stores and having price expectations of other stores. Consumers thus face a search problem in need of a stopping rule. In Lal and Matutes, consumers make an initial store choice with incomplete information, and then sequentially decide on visiting more than one store. Diamond's analysis is for a single-product situation (like that of a durable good), while Lal and Matutes focus on multiproduct purchases in supermarkets. Diamond finds that even with com-

[2]In §3.4 we discuss the effect of trade promotions on supermarket advertising strategies.

petition the equilibrium price is, surprisingly, the joint profit-maximizing price. This result is often referred to as the Diamond paradox. Kalai and Satterthwaite (1986), who develop the game-theoretic arguments, have obtained a result similar to Diamond's. For the multiproduct case, Lal and Matutes show that consumers expect the price to be at the reservation price for an unadvertised good. In their equilibrium both firms communicate the price of only one good, and of the same good. In this equilibrium both stores set the price of the unadvertised good at the reservation price, a result that can be seen as analogous to Diamond's. Lal and Matutes also find that the equilibrium price of the advertised good can be below cost, thus demonstrating loss-leader pricing in multiproduct situations.

Walters and McKenzie (1988) find empirically that loss-leader pricing does not significantly affect store traffic or profits. Their results are consistent with Arnold et al. (1983), that store traffic depends on locational convenience and overall price perceptions rather than merely on advertised low prices. In other words, consumers seem to be taking into account unadvertised prices they would find in the store. Lal (1990) finds empirically that a store accepts trade promotions from different brands thus helping national brands to avoid head-to-head competition. Rao et al. (1995) find that the prices across stores of the same brand, when on discount, are uncorrelated, a fact they take to be consistent with mixed strategies across stores.

We analyze retail competition using the Lal and Matutes (1994) framework and show that in equilibrium retailers will advertise the same good sometimes, but different goods at other times. In the latter case they will use unadvertised discounts to prevent shopping around by consumers. Further, unadvertised specials affect store choice of consumers, consistent with Arnold et al.'s (1983) findings on the importance of overall price perceptions of a store. An important managerial finding is that using unadvertised specials results in higher profits than not using them. This happens because of reduced head-to-head competition on the advertised good. Finally, in our equilibrium expected prices of both advertised and unadvertised goods are below the monopoly price, thus resolving the Diamond paradox in the case of multiproduct sellers.

The paper is organized as follows. In §2 we present our assumptions. In §3 we present our model and characterize the rational expectations Nash equilibrium pricing and communication strategies of retailers and discuss our results. In §4 we present our conclusions.

2. Model Assumptions

Our model closely follows that of Lal and Matutes (1994). We next state the assumptions with respect to retailers and consumers.

2.1. Retailers

In this model there are two retailers, indexed by j, $j = A, B$, who compete with each other. The stores are located at either end of a straight line of length 1, with A at the left end and B at the right end. In keeping with our focus on supermarkets, we assume that each store carries more than one good. In particular, we assume that each carries two goods, indexed by i, $i = 1, 2$. Again, for supermarkets the assumption that both stores carry the same goods is reasonable. The restriction to two goods simplifies the exposition considerably without affecting our results. In §3.4 we extend our model to more than two goods. These two goods are assumed to be neither complements nor substitutes. This reflects the reality that supermarkets carry goods such as shampoo and cake mix. We assume that the marginal costs of the two goods are constant and identical, and without loss of generality assume them to be zero. As in Lal and Rao (1997), we assume that the cost of advertising one good is zero. While we focus on the case in which each store advertises only one good in equilibrium, we allow them to advertise both goods, and find conditions under which they advertise only one good in equilibrium.

2.2. Consumers

Consumers in our model are identical in their preferences, and are in the market for both goods. In particular, they wish to buy one unit of each good, provided the price of the good does not exceed their reservation price for it, R. Note that the reservation price of each good is set to be identical, as in Lal and Matutes (1994) and Lal and Rao (1997). Consumer heterogeneity is in the convenience that each store presents to a representative consumer. This is captured by assuming that the consumers are distributed uniformly along the straight line, AB, joining the two stores. A consumer, denoted by x, $0 \leq x \leq 1$, is located at a distance x from Store A and $(1 - x)$ from Store B. Each consumer is assumed to incur a unit transportation cost $c/2$. Thus, consumer x incurs a cost cx for a round-trip visit to Store A and $c(1 - x)$ for a round-trip visit to Store B. We will assume that $R > c$. This assumption says that the reservation price R is not too low relative to the transportation cost c, thus assuring that all consumers in the market are served. In addition, for the analysis in §3 we assume that $2c > R$. This assures that firms can make positive profits. Finally, we assume that consumers are uniformly distributed along AB.

2.3. Consumer's Decision Problem

All consumers are assumed to have identical expectations of prices of each good at each store. Denote by P_{ij}^E the expected price of good i at store j. A consumer x is assumed to maximize his (her) surplus by choosing which store(s) to visit and what good(s) to buy at each store. The consumer's choice, based on expected prices, can be thought of as a choice of visiting no store (and so not buying any good); visiting only Store A (or B); and visiting both stores. Thus, the consumer does not visit either store if the maximum surplus from visiting either store is less than zero. Mathematically,

$$\max\{-cx + \max\{R - P_{1A}^E, R - P_{2A}^E, 2R - P_{1A}^E - P_{2A}^E\}, \\ -c(1 - x) + \max\{R - P_{1B}^E, R - P_{2B}^E, \\ 2R - P_{1B}^E - P_{2B}^E\}\} < 0.$$

The first term on the left-hand side is the consumer's surplus from visiting Store A, incurring a transportation cost of cx, and buying one or more goods in that store. The second term is the surplus from visiting Store B by incurring transportation cost of $c(1 - x)$, and buying one or more goods in that store. Note that in each term there is a maximization operation over buying only Good *1* or only Good *2* or both goods. If none of these choices yields a nonnegative surplus, then consumer x will visit neither store. Consumer x visits only Store A if the surplus from visiting Store A is nonnegative and exceeds that from visiting only Store B or visiting both stores. Mathematically,

$$-cx + \max\{R - P_{1A}^E, R - P_{2A}^E, 2R - P_{1A}^E - P_{2A}^E\} \\ > \max\{0, \max\{-c(1 - x) + \max\{R - P_{1B}^E, R - P_{2B}^E, 2R - P_{1B}^E - P_{2B}^E\}, \\ -c + 2R - \min\{P_{1A}^E, P_{1B}^E\} - \min\{P_{2A}^E, P_{2B}^E\}\}\}.$$

The first term represents the surplus from shopping at Store A, incurring transportation cost cx, and the consumer buying only one good or both goods, depending on which is better. The second term consists of two subterms. The first, zero, is the surplus from not shopping at all. The second is the maximum surplus from either shopping only at Store B, incurring transportation cost of $c(1 - x)$, and buying one or both goods at Store B, or shopping at both stores, incurring a transportation cost of $c = cx + c(1 - x)$, and buying one good at each store. Note that they buy both goods in this last case and obtain a gross surplus of $2R$ by buying each good at the store in which it is lower priced. A similar inequality can be written to describe the condition for consumer x to shop only at Store B. Finally, consumer x plans to visit both stores if the surplus from buying one good at each store is nonnegative and exceeds that from visiting only one of the two stores. Mathematically,

$$
\begin{aligned}
&-c + 2R - \min\{P^E_{1A}, P^E_{1B}\} - \min\{P^E_{2A}, P^E_{2B}\} \\
&\quad > \max\{0, -cx + \max\{R - P^E_{1A}, R - P^E_{2A}, 2R - P^E_{1A} - P^E_{2A}\}, \\
&\qquad -c(1 - x) + \max\{R - P^E_{1B}, R - P^E_{2B}, 2R - P^E_{1B} - P^E_{2B}\}\}.
\end{aligned}
$$

The LHS of the above inequality is the surplus from shopping at both stores, incurring total transportation cost of $c = cx + c(1 - x)$, and buying one good at each store, described earlier. The RHS consists of three alternatives: getting a surplus of zero from not visiting either store, visiting only Store A, or visiting only Store B.

Note that in order for the consumer to make this store choice(s) he must have expectations of prices of each good at each store. For the sake of convenience we shall represent the price set by the store in terms of a discount relative to the reservation price R.[3] Discounts may be either advertised or unadvertised. Let D_{ij} be the discount that store j advertises on good i, and u_{ij} be the unadvertised discount, if any, that store j offers on good i. First consider the case in which store j advertises good i. Then, the consumer is assumed to know the discount on this good, and therefore its price. Further, the store is assumed not to renege on the advertised price. In this case, the expected price must be equal to the advertised price. We therefore have $P^E_{ij} = P_{ij} = R - D_{ij}$, where P_{ij} is the price set and advertised on good i by store j. Next consider the case in which store j does not advertise good i. Then the consumer does not know the price of this good. As a result, unadvertised specials in our paper have substantive force on consumer behavior, and so, stores' strategies. We assume that the consumer expects an unadvertised discount of u^E_{ij}, and so $P^E_{ij} = R - u^E_{ij}$. One question that comes to mind is what restrictions should be placed on u^E_{ij}. Following Lal and Matutes (1994) and Lal and Rao (1997), we assume that in equilibrium consumers have rational expectations so that the actual price chosen by the firm P_{ij} satisfies $P_{ij} = P^E_{ij} = R - u^E_{ij}$. In other words, $u_{ij} = u^E_{ij}$. This naturally leads us to the question of how the stores choose prices.

[3]We follow prior research in defining discount as the difference between the reservation price and store price. Our definition is appropriate for another reason also. If a store were to price a good below the reservation price, by D, with probability 1 the exact interpretation of D would be unclear. In the main result of this paper stores charge the reservation price with a probability that lies strictly between 0 and 1, and so the interpretation of D as a discount is appropriate.

2.4. Stores' Decision Problem

Recall from §2.1 that in our model each store carries two goods and may advertise neither good, only one good, or both goods. For expositional simplicity we ignore the case of advertising both goods and, in §3.3, characterize the equilibrium in which both firms advertise only one good. We then check, in §3.4, whether this equilibrium continues to hold if the stores could advertise both goods. Thus, each store must decide which good (if any) it should advertise, what the price of the advertised good should be, and whether it should offer any unadvertised discount on the other good. We assume that stores make these decisions to maximize profit.

Before proceeding to the analysis it is useful to eliminate the possibility that there will be no advertising by either store. Lal and Matutes (1994) have shown that if a store does not advertise any good, consumers will rationally expect that when they arrive at the store prices of both goods at that store will be set at the reservation price R. This, in turn, will yield consumers a negative surplus from shopping since they have to bear the transportation cost to make the trip to the store. In other words, in equilibrium there cannot be a situation in which neither store advertises. The intuition for this is simple. Retailers' advertised prices play the role of a commitment of positive surplus once the consumer arrives at the store, and this surplus can offset the transportation cost. Because we are using the Lal-Matutes framework, stores in our model also will advertise at least one good in equilibrium, provided that the cost of advertising one good is sufficiently small.

Three possibilities arise: both stores advertise

Good *1*, both stores advertise Good *2* or one advertises Good *1* while the other advertises Good *2*. Because the two goods are essentially identical we need to consider only two cases: Both stores advertise the same good or both advertise opposite goods. The case of same-good advertising can be visualized as follows:

	Store A	Store B
Product *1*	$R - D_{1A}$	$R - D_{1B}$
Product *2*	$R - u_{2A}$	$R - u_{2B}$

A similar situation exists if both advertise Good *2*. Without loss of generality, we will only consider the case of both stores advertising Good *1*. Next, the case of opposite-good advertising is as follows:

	Store A	Store B
Product *1*	$R - D_{1A}$	$R - u_{1B}$
Product *2*	$R - u_{2A}$	$R - D_{2B}$

We have shown Store A advertising Good *1* and Store B advertising Good *2*. A similar situation exists if the stores and goods are interchanged. Again, without loss of generality, we consider only the case in which Store A advertises Good *1* and Store B advertises Good *2*.

3. Pricing and Advertising Equilibrium

The interaction between the two stores and the consumers can be viewed as a multistage game. In the first stage, the two stores decide which good to advertise and the price of that good. In the second stage, the prices of the advertised goods are revealed to consumers and the competitor. In this stage stores determine the discount, if any, on the unadvertised good; consumers form expectations of prices of unadvertised goods and make a shopping plan consisting of whether to visit Store *A*, Store *B*, both stores, or neither store. As described in §2.3, consumer expectations of prices of advertised goods are equal to the advertised prices, while the expectations of prices of unadvertised goods must satisfy the condition of rational expectations, and are therefore endogenous to the analysis in the second stage. An alternative way of modeling the pricing of the unadvertised good is to make those prices first-stage decisions rather than second-stage decisions. This corresponds to the situation in which each store must determine the prices without knowing its competitors' prices even when the prices are not advertised. We will address this in §3.7. Finally, in the third stage, consumers arrive at one store and learn of the unadvertised price at that store. They may then revise their shopping plans. In equilibrium, because consumers have rational expectations, this never occurs, although to analyze off-equilibrium outcomes we must incorporate a third stage.

We derive the subgame perfect equilibrium to this multistage game by considering the three cases: same-good advertising, opposite-good advertising, and mixing across the two. For each case, we first derive the second-stage decisions of the consumer shopping choice and the stores' choice of unadvertised discount. We then proceed to solve for the first-stage decision of the advertised discount. Finally, we verify whether or not the advertising strategy of same-good and opposite-good advertising is part of a Nash equilibrium. We find in §3.1 that there exists a pure strategy Nash equilibrium in the same-good advertising case, as has been shown by Lal and Matutes (1994). We also find in §3.2 (Lemma 3) that there does not exist a pure strategy Nash equilibrium involving opposite-good advertising. Finally, we show the existence of a mixed-strategy Nash equilibrium that involves both same-good advertising and opposite-good advertising. This is our central result of §3, and it is contained in Theorem 1.

3.1. Same-Good Case

Consider the case in which both stores advertise the same good, Good *1* (or 2) with probability one. This can be thought of as a pure strategy by the stores with respect to which good to advertise. We need to verify that such a pure strategy can indeed be an equilibrium of the game. As shown in Proposition 3 of Lal and Matutes (1994, p. 357), there indeed exists a subgame perfect Nash equilibrium in which the two stores advertise the same good, implying that neither has an incentive to deviate by advertising the opposite good.[4] Also, as shown in Proposition 2 (p. 355) of the same paper, in the second stage of such an equilibrium, consumers expect the unadvertised discount to be zero. In a rational expectations equilibrium both the stores offer a discount of zero on the unadvertised good. For the sake of completeness we derive the Lal-Matutes solution for the advertised discount in this situation. Now, the marginal consumer who is indifferent between shopping at Store A and Store B is located at m, where m solves

$$cm + 2R - D_A = c(1 - m) + 2R - D_B.$$

We have dispensed with the subscript denoting the product on which the discount is given because both stores advertise and discount the same good. With this value of m the profit for Store A is

$$\max_{D_A} (2R - D_A)\left(\frac{D_A - D_B + c}{2c}\right).$$

The first-order condition for profit maximization yields

$$D_A + D_B - c + 2R - D_A = 0.$$

A similar condition is obtained from Store B's profit-maximization problem. Imposing symmetry $D_A = D_B = D$, the optimal advertised discount is $D^* = 2R - c$. As the unadvertised discount is zero in the same-good advertising equilibrium, the optimal discounts are $D^* = 2R - c$ and $u^* = 0$. Lal and Matutes have shown that when the two stores advertise the same good, neither has an incentive to switch to advertising the opposite good in the first stage. Thus, this constitutes a subgame perfect Nash equilibrium. In this equilibrium the stores share the market equally and there is no shopping around in equilibrium. The equilibrium profit of each store is $c/2$.

3.2. Opposite-Good Case

Consider the case in which the two stores advertise Good *1* (or *2*) with probability 1, with Store A advertising Good *1* (or *2*) and Store B Good *2* (or *1*). This can also be thought of as a pure strategy by the stores with respect to which good to advertise. We later examine (in Lemma 3) whether such a pure strategy can indeed be part of a subgame perfect Nash equilibrium of the game. As in §3.1, we are interested in situations in which *all* consumers buy at least one good in equilibrium. For ease of exposition we proceed as though advertised prices and expectations of unadvertised prices are such that this will hold. Later we verify that this is indeed the case. Thus, in the second stage, the consumers' decision is to formulate a plan to visit one or both stores, depending on the advertised prices. If they plan to visit only one store they must further plan on buying one or both goods at that store. Note that if they plan to visit both stores it must be the case that they plan on buying both goods.

We develop the analysis in §3.2 as follows. Given advertised prices, we ask what expectation of unadvertised prices must consumers have such that all consumers plan to buy two goods, and buy both of them at one store. In other words, what expectations will result in a plan of store choice that does not include shopping around even though all consumers buy both goods. We will denote these as *basket expectations* to mean that these expectations result in all consumers buying baskets (of two goods) at one store. After deriving the basket expectations, we identify the conditions under which they are rational (in Lemmas 1 and 2). By this we mean, given these expectations and the consequent arrival of consumers at each store, when is it best for each store to set prices of unadvertised goods equal to the basket expectations? In this section we do not consider the case in which some consumers buy one good at each store.[5] We also ignore the case in which some consumers buy only one good, because in the equilibrium we identify that all consumers buy both goods at one store. Having identified the basket expectations, we solve for the discounts on the advertised and unadvertised goods corresponding to a Nash equilibrium when stores advertise opposite goods as part of a

[4] Lal and Matutes (1994) require that $4R > 3c$, a condition that does not contradict the conditions imposed by us.

[5] In fact, in Stage 2 an equilibrium in which consumers shop around, buying one good at each store, does not exist in this case.

pure strategy (Lemma 2). Finally, in Lemma 3 we show that stores advertising opposite goods as part of a pure strategy cannot be a part of a subgame perfect Nash equilibrium.

Basket Expectations. Recall that consumers know the price of Good *1* at Store A to be $R - D_{1A}$ and Good *2* at Store B to be $R - D_{2B}$. Further, all consumers have identical expectations of prices of unadvertised goods to be $R - u_{2A}^E$ for Good *2* at Store A and $R - u_{1B}^E$ for Good *1* at Store B. A consumer located at x would therefore prefer to buy both goods at Store A iff

$$cx + 2R - D_{1A} - u_{2A}^E < c(1 - x) + 2R - D_{2B} - u_{1B}^E.$$

This inequality says that the net cost (negative of consumer surplus) of buying both goods at Store A should be less than that at Store B. Further, though suppressed here, it should be less than or equal to zero, yielding a nonnegative surplus. A consumer located at x would prefer to buy both goods at Store B iff

$$cx + 2R - D_{1A} - u_{2A}^E > c(1 - x) + 2R - D_{2B} - u_{1B}^E.$$

Further, the RHS of the above inequality should be less than zero to assure the consumer of nonnegative surplus. Denote the marginal consumer as the one located at m, $0 \leq m \leq 1$, such that she or he is indifferent to buying at either store. Mathematically, this implies

$$cm + 2R - D_{1A} - u_{2A}^E = c(1 - m) + 2R - D_{2B} - u_{1B}^E.$$

The foregoing equation can be used to solve for m as:

$$m = \max\{0, \min\{(D_{1A} - D_{2B} + u_{2A}^E - u_{1B}^E + c)/2c, 1\}\}.$$

This gives the market share for Store A, and we note that it should lie between zero and one. A full characterization of the solution for all possible values of the discounts requires us to consider the corner solutions for m. For now we restrict our attention to the interior solution, $0 < m < 1$, and verify that this is part of an equilibrium.

A basket expectation not only determines from which store to buy both goods, but also rules out the possibility of buying one good from each store. To characterize the basket expectation we must impose this condition. We do this next. A consumer anticipates what will happen when she or he arrives at Store A. For him (her) not to want to shop around, his (her) expectation of the unadvertised discount at Store A should satisfy

$$c(1 - x) + R - D_{2B} > R - u_{2A}^E.$$

The inequality says that if this consumer were to arrive at Store A, she or he would find it more costly to go to Store B to buy the Good *2* at the price advertised by Store B because of the cost of going to Store B. Furthermore, she or he would have to trade this off against the (expected) unadvertised discount on that good at Store A. The inequality can be rewritten as

$$u_{2A}^E \geq D_{2B} - c(1 - x).$$

The foregoing inequality, and a similar one arrived at by considering a consumer who visits Store B first, yield two relationships between expected unadvertised discounts, u_{ij}^E, and advertised discounts D_{ij}. However, our assumption of rational expectations requires that the firms should choose unadvertised discounts to be equal to the expected discounts. The next question is: How will Store A (and Store B) choose u_{2A} (u_{1B})?

Firms' Choice of Unadvertised Discounts u_{2A} and u_{1B}. Consider Store A's decision with respect to the unadvertised discount on Good *2*. Its market share is m. Consider a consumer x, $0 \leq x \leq m$, who has arrived at Store A. Faced with a value of unadvertised discount of u_{2A}, she or he may choose to visit Store B to buy Good *2* only if

$$c(1 - x) + R - D_{2B} \leq R - u_{2A}.$$

Store A's profits are therefore given by

$$m(2R - D_{1A} - u_{2A}) \quad \text{if } u_{2A} \geq D_{2B} - c(1 - m) \quad \text{and}$$

$$m(2R - D_{1A}) + m_A(R - u_{2A}),$$

$$m_A = \max\{0, 1 - (D_{2B} - u_{2A})/c\}$$
$$\text{if } 0 \leq u_{2A} \leq D_{2B} - c(1 - m).$$

Note that m is the location of the consumer who is indifferent between visiting the two stores at the out-set based on the basket expectations. Once a consumer has visited Store A, if the actual unadvertised discount exceeds the basket expectation she or he will not modify the initial store choice. If, on the other hand, the unadvertised discount is less than the basket expectation, consumers in (m_A, m) will find it optimal to go to Store B. There they will buy Good 2 based on its advertised price at Store B. Store A must choose u_{2A} to maximize the profits from sales of Good 2 to consumers who visit it. In other words, u_{2A} must solve the following problem:[6]

$$\max_{u_{2A}} (R - u_{2A})m_A.$$

This yields, for an interior solution,

$$u_{2A} = (R + D_{2B} - c)/2.$$

However, Store A cannot gain market share or retain additional customers by choosing u_{2A} to be greater than the basket expectation, which is given by $D_{2B} - c(1 - m)$. In other words, we have the constraint that $u_{2A} < D_{2B} - c(1 - m)$. Moreover, obviously it would also not choose the unadvertised discount to be greater than R. The optimal choice of the unadvertised discount u^*_{2A} for the constrained maximization problem is then given by

$$u^*_{2A} = \min\{(R + D_{2B} - c)/2, D_{2B} - c(1 - m), R\}.$$

In the brackets on the RHS there are three terms. The first corresponds to a situation in which the store does not retain all consumers who visit the store and is an interior solution to the problem of choosing the unadvertised discount. The second corresponds to choosing the unadvertised discount so as to retain all consumers who visit the store and is a corner solution. The third is a consistency condition that will not be part of an equilibrium. Note that when the first condition holds, an equilibrium may consist of expectations that are not basket expectations, while the second condition corresponds to expectations that are basket expectations. Following an analysis similar to that for Store A, we can obtain the optimal unadvertised discount for Store B on Good 1 as

$$u^*_{1B} = \min\{(R + D_{1A} - c)/2, D_{1A} - cm, R\}.$$

[6]We should also require that $u_{2A} > 0$, which in turn requires $4R > 3c$. We suppress that here for ease of exposition. It does not have any force in equilibrium because the interior solution is not part of an equilibrium.

Rationality of Basket Expectations. We are now in a position to identify the conditions under which basket expectations are also rational expectations. For this we need two conditions. First, we need stores' choices must be equal to expectations, i.e., $u^*_{2A} = u^E_{2A}$ and $u^*_{1B} = u^E_{1B}$. Next, the stores' choice of unadvertised discount should correspond to basket expectations and therefore be a corner solution.

In Appendix 1 we prove, for the sake of completeness, the obvious result that a rational expectations equilibrium with basket expectations cannot exist with interior solutions for u^*_{2A} and u^*_{1B} (Lemma 1). This result is proved by contradiction. We next show that a rational expectations equilibrium with basket expectations is possible with corner solutions for u^*_{2A} and u^*_{1B} (Lemma 2). This is proved by construction.

LEMMA 2. *Let* 2c > R > c. *Suppose stores commit to advertising opposite goods in the first stage. If consumers have basket expectations* u^E_{2A} *and* u^E_{1B} *for the unadvertised goods at Stores* A *and* B *respectively, then the stores' optimal choices in the second stage can be a corner solution to their maximizing problems. Moreover, the Nash equilibrium discounts are given by:*

the discount on the advertised good, in stage 1, is R − c; *and*

the discount on the unadvertised good, if it is nonzero, is R − *1.5*c.

PROOF. See Appendix 2.

Lemma 2 is a mathematical result demonstrating the existence of a second-stage equilibrium and also allows us to solve for the equilibrium. It also says that the optimal unadvertised discount is equal to the price at the competing store plus the transportation cost of the marginal consumer. If consumers expect unadvertised discounts at each store to be such that all consumers plan on buying both goods at one store, then we can solve for the second-stage decisions by holding the constraint binding in the constrained optimization problem of the optimal discounts given the advertised discounts.

Advertising Opposite Goods as Part of a Pure Strategy. We next show that a commitment on the part of the competing stores to advertising opposite goods cannot be part of a subgame perfect Nash equilibrium. This is done by demonstrating that a store has an incentive to deviate profitably. This is Lemma 3.

LEMMA 3. *An outcome of opposite goods advertising resulting from a pure strategy by stores with respect to which good to advertise in Stage 1 cannot be part of a subgame perfect Nash equilibrium.*

PROOF. See Appendix 3.

The economic intuition behind Lemma 3 becomes clear if we ask what a store should do if it knows that following an opposite-good advertising strategy in the first stage the only equilibrium in the second stage would be to retain all of the customers visiting the store. It would realize that it might as well have offered the identical advertised discount on the same good. This would attract the same fraction of customers to the store as before, and also retain all customers, but without having to offer a positive discount on the unadvertised good. Thus, opposite-good advertising in the first stage cannot be sustained as an equilibrium.

3.3. Mixed Strategy Advertising Equilibrium

We have seen that an outcome of opposite-goods advertising as a result of stores' pure strategy of which good to advertise cannot be part of a subgame perfect Nash equilibrium. In this section we show that an outcome of opposite-goods advertising can indeed be part of a subgame perfect Nash equilibrium as a result of stores' mixed strategies. It is useful to elaborate on exactly what the mixed strategy consists of. Firm A can advertise the price (discount) of either Good *1* or *2*, and similarly, Store B can do the same. A mixed strategy, for Store A, then consists of advertising Good *1* with probability $\mu_A > 0$, and Good *2* with probability $1 - \mu_A$. Similarly Store B, can advertise Good *1* with probability $\mu_B > 0$, and Good *2* with probability $1 - \mu_B$. Note that this mixed strategy is at Stage 1. The stores must also decide in Stage 1 the discount to offer on the advertised good. In Stage 2 both stores and the consumers know which good is being advertised by each store, as well as the discount on that good. In Stage 2 stores will decide on the unadvertised discount, if any, on the second good. The mixed-strategy equilibrium is developed in Theorem 1. The proof of the theorem is by construction. Following the theorem we characterize some of the properties of this equilibrium.

THEOREM 1. *Let* $2c > R > c$. *Then, there exists a symmetric Nash equilibrium to our model of retail competition of the following kind:*

- *Each store advertises each good with probability* $\mu = 0.5$ *in Stage 1.*
- *The discount on the advertised good, in Stage 1, is* $1.5R - c$. *In other words, the price of the advertised good will equal the transportation cost* c *less half the reservation price* R.
- *If stores advertise the same good in Stage 1 there is no discount on the unadvertised good in Stage 2.*
- *The discount on the unadvertised good, if it is nonzero, is* $1.5R - 1.5c$. *In other words, the price of the unadvertised good will equal 1.5 times the transportation cost* c *less half the reservation price* R.
- *Consumers have rational expectations and all consumers buy both goods at the same store.*
- *The marginal consumer is located at* $m = 0.5$.
- *Both stores make positive expected profits. Further, the equilibrium profits of the firms under this mixed-strategy are higher than the equilibrium profits under the pure strategy in which both the firms advertise the same good.*

PROOF. See Appendix 4.

Discussion of Theorem 1. It is useful to contrast our Theorem 1 with the results of Lal-Matutes (1994). Recall that they established a pure strategy equilibrium in which each store advertises the same good, and further, the price of the unadvertised good is set at the reservation price R. In contrast, in our equilibrium the price of the unadvertised good is less than R half the time. In this way we are able to offer an explanation for unadvertised specials in equilibrium. A second aspect of our equilibrium is that it is in mixed strategies, while that of Lal-Matutes is in pure strategies.[7] Their equilibrium implies that competing stores must advertise the same good. Our equilibrium encompasses both outcomes of competing stores advertising opposite goods or the same good. Thus, we are able to reconcile the observed practice of stores advertising the same good some of the time and opposite goods at other times. Finally, the fact that our equilibrium is in mixed strategies means that its implications can be consistent with temporal patterns in which stores sometimes do not change the good advertised in consecutive time periods, while at other times they do. The pure strategy equilibrium of Lal-Matutes could also accommodate temporal changes in a store's strategy, but the two stores' strategies would have to be perfectly correlated.

Another interesting aspect of our equilibrium is that it leads to greater profits than the Lal-Matutes equilibrium. The reason is that the Lal-Matutes equilibrium extracts maximum consumer surplus on the unadvertised good and shifts all competition to a single good to generate store traffic. The mixed-strategy equilibrium identified by us reduces the intensity of competition for generating store traffic by making the competitor uncertain about which good is being advertised. However, this uncertainty also leads to the need for unadvertised specials when opposite goods are advertised. The unadvertised special, designed to help retain customers who visit a store, is in the nature of price matching attenuated by consumer transportation cost. The intensity of competition is therefore low on this dimension. Note that if the stores could commit to advertising opposite goods they would make even higher profits even though they would also have unadvertised discounts with probability 1. In fact, any mechanism that allows stores to coordinate on opposite-good advertising would be profitable. In our equilibrium, in the absence of a coordinating device, a mixed strategy that maximizes the probability of opposite-good advertising is the only alternative. This is our key result. In §3.5 we show that increasing product assortment could increase the probability of opposite-good advertising further. Another coordinating device could consist of manufacturers strategically offering trade promotions to retailers, as has been suggested by Lal (1990).

Finally, given the potential outcome of unadvertised specials in equilibrium, stores offer smaller advertised discounts in Stage 1. In fact, the advertised discount is such that there is never any loss-leader pricing in equilibrium. Thus, while Lal-Matutes (1994) equilibrium can comprehend loss-leader pricing, our equilibrium cannot. Another implication of our equilibrium is that the unadvertised special at one store is perfectly correlated with the advertised good at the other store. This is a consequence of the essential argument of Diamond (1971) that is also present in the Lal-Matutes equilibrium. We next ask if the unadvertised special can exist if we relax our assumption that stores can advertise only one good.

3.4. Advertising Both Goods

We now consider the possibility that stores can advertise two goods in the first stage as opposed to only one good. Our goal is to see if unadvertised specials would occur even if stores have no constraint on the number of goods they can advertise.[8] We assume that the additional cost of advertising the second good to be $F \geq 0$. We will derive the conditions under which the equilibrium identified in Theorem 1 continues to hold, thus showing that unadvertised specials do not necessarily depend on whether stores can advertise one or both goods. In fact, our finding is that unadvertised specials result from strategic considerations on the part of stores.

Corollary 1. *Suppose the following condition holds:*

$$F > (1/8c)(9c^2 - 7Rc + R^2).$$

Then the equilibrium in Theorem 1 holds.

Proof. We briefly outline the strategy of proof here, with the details in Appendix 5. We proceed by

[7]Lal and Matutes (1994) restricted their attention to pure strategies. Their analysis is therefore not complete in a mathematical sense. From a substantive point of view their analysis misses the strategic considerations that might influence unadvertised specials, as we see here.

[8]We thank the area editor for pointing out that the equilibrium in Theorem 1 seems to depend on the assumption that stores are limited in the number of goods they can advertise, and quite possibly unadvertised specials are merely a consequence of that assumption.

fixing one store's strategy in the first stage, corresponding to the equilibrium in Theorem 1, as advertising each good with probability 0.5, and choosing the advertised discount to be $1.5R - c$. We then ask if it would be profitable for the other store to "deviate" from the equilibrium by advertising both goods in the first stage, choosing advertised discounts on each and having the first store choose price of its unadvertised good in the second stage. As before, consumers make decisions in the third stage. Based on this analysis we find the "optimal deviation" results in profits of

$$(1/8c)(4c - R)^2.$$

Recall that the equilibrium profits in Theorem 1 are given by $(1/8)(7c - R)$. The theorem follows from a comparison of the two profits. It is important to note that even in the extreme case of $F = 0$, there is a range of parameters, $2c > R > (7 - \sqrt{13})c/2$, for which the equilibrium includes unadvertised specials.[9]

The economic intuition behind this corollary is that advertising both goods does not maximize the probability of opposite-good advertising. As discussed earlier, profits are increasing in the probability of opposite-good advertising. Note that even if $F = 0$, if c is sufficiently high the magnitude of the unadvertised discount is not so high, and so opposite-good advertising continues to be profitable. Obviously, if F is large, stores do not wish to incur the additional cost of advertising and face lower probability of opposite-good advertising.

3.5. Trade Promotions

Newspaper advertisements of prices of goods carried by supermarkets are often part of trade promotion activity. In many instances manufacturers bear some or all the cost of advertisements, thus unadvertised specials may correspond to a lack of trade promotions.[10] Trade promotions are offered to all stores in a trading area. If both stores accept trade promotions on a good and advertise that good as a condition of the promotion, they would move away from advertising opposite goods to a situation of advertising the same good. We know that this would result in lower profits. This is in light of the strategic reasoning behind our equilibrium being that of coordination, to the extent possible, on maximizing the probability of advertising opposite goods. We could regard the decision to advertise a particular good in the first stage, in our model, as one of whether to accept or reject a trade promotion. In this case, stores would not want to accept trade promotions with probability 1 so as to avoid head-to-head competition on the same good. Indeed, stores do reject some trade promotions. Thus, while the reimbursement of advertising costs would influence stores to accept trade promotions, strategic considerations have the opposite effect. In other words, even if all advertisements are a result of trade promotions, our Theorem 1 provides a basis for understanding why competing stores may choose to accept trade promotions on different goods rather than on the same good. We are still left with the question of whether unadvertised specials occur because there is no trade promotion or because of rejected trade promotions. In other words, should the price of the unadvertised good be reduced if the store has rejected a trade promotion? Theorem 1 provides an answer to this, and shows that if a store's competitor accepts trade promotion on a good, then the store would have an incentive to offer an unadvertised special after rejecting the trade promotion. In this way our model can be interpreted in the context of trade promotions, and unadvertised specials could result not from a lack of trade promotions but from strategic considerations following the rejection of a trade promotion. Krishnan and Rao (1995) find empirical support for a situation similar to this. They find that if a manufacturer drops coupons, thus rendering a store with a "double couponing" policy more competitive than a store without such a policy, the latter offers an unadvertised price cut on the couponed item.

3.6. More than Two Goods

Suppose now that the two stores carry n products labeled *1, 2, . . . , n.* Again consider the mixed strategy

[9]Given the strict inequalities, even if $F < 0$, i.e., there were to be a promotional incentive to the store, there would still be an equilibrium of the type described in Theorem 1. We thank the editor for pointing this out.

[10]We thank a reviewer for suggesting this alternative possibility for unadvertised specials.

in which the randomization is only on the good on which stores advertise the discount. To keep results comparable we assume that the reservation price of each of the n goods is $2R/n$. As before, suppose that Store B advertises a discount on Good *1*. Firm A's profit function under the mixed strategy is

$$\max_{D_A} \left\{ \frac{1}{n}(2R - D_A)\left(\frac{D_A - D_B + c}{2c}\right) + \left(\frac{n-1}{n}\right) \times \left(2R - \frac{5}{4}D_A - \frac{3}{4}D_B + \frac{c}{2}\right)\left(\frac{D_A - D_B + 2c}{4c}\right) \right\}.$$

As before, the first-order condition with $D_A = D_B = D$ gives the optimal discount as

$$D^* = (n + 1)R/n - c.$$

Again, as before, to ensure that the corner solutions to unadvertised specials hold, we require $D < R$. This just reduces to the condition $R < nc$ which is satisfied for $n \geq 2$.

The equilibrium profit of firm A under the mixed strategy is

$$\pi_A^* = \frac{1}{2n}\left[\frac{(5n - 3)c}{2} - \frac{(n - 1)R}{n}\right].$$

Taking the first partial above with respect to n we get

$$\frac{\partial \pi_A^*}{\partial n} = \frac{12c}{16n^2} + \left(\frac{n-2}{2n^3}\right)R.$$

The term on the right-hand side of inequality (5) is positive if $n \geq 2$. As we are dealing with more than two goods, this implies that $\partial\pi_A/\partial n > 0$. This shows that the equilibrium profit increases in the number of products that the stores carry. Moreover, the limiting profit of a firm is $\lim_{n\to\infty} \pi_A^* = 5c/4$. Let us compare this to the profits if both stores can commit to advertising opposite goods. In this case, we know from Lemma 2 that the optimal discounts would be $D^* = R - c$, and $u^* = R - 3c/2$. With these values of the optimal discounts, the profit of Store A, given functionally by $0.5(2R - D^* - u^*)$, would be $5c/4$. Advertising opposite goods reduces head-to-head competition that occurs whenever they advertise the same good. Therefore, the profit of $5c/4$ from advertising opposite goods with probability 1 is an upper limit on the equilibrium profit that each store can earn. In our model this plays the role of the profit from the joint profit-maximizing prices in Diamond's (1971) sense. Now, as we already noted above, such a strategy of advertising opposite goods with probability 1 cannot be a Nash equilibrium. However, as the number of products that stores carry grows very large, it is possible for them to approach this upper limit under the mixed-strategy equilibrium.[11] We have thus identified a mechanism by which the joint profit-maximizing prices, in Diamond's sense, can be attained in equilibrium in a model of retail competition with advertising to generate store traffic. A managerial implication of this is that stores can increase profits by increasing their assortment, which in turn reduces head-to-head competition on the price of the advertised good and maximizes the probability of opposite-good advertising.

3.7. Role of Price Matching

In our model stores determine the prices of unadvertised goods in the second stage, after knowing the competitor's first-stage decision of which good to advertise and at what price. Stores in effect are able to match prices of competitor's advertised good, attenuated by the transportation cost. If stores are unable to adjust prices after observing competitor's advertisement, how would the prices differ and what consequence would there be for consumer store choice?

It is possible to analyze the competition assuming that stores must choose prices of unadvertised goods in the first stage without knowing the competitor's choice of which good to advertise and at what price. This game differs from Lal-Matutes' (1994) framework. The equilibrium to this game can be derived using arguments similar to those in Theorem 1. We state the resulting equilibrium in Theorem 2 without proof.[12]

THEOREM 2. *Let* R > c. *Suppose that prices of unadvertised goods must be chosen by stores in the first stage. Then there exists a symmetric Nash equilibrium to our model of retail competition of the following kind:*

[11] The argument would be valid as long as the number of advertised products is small in relation to the total number of products carried by the stores.

[12] See Rao and Syam (1999) for proof.

- *Each store advertises each good with probability $\mu = 0.5$ in Stage 1.*
- *The discount on the advertised good, in Stage 1, is* R $-$ *19c/42. In other words, the price of the advertised good will equal (19/42) times the transportation cost* c.
- *The discount on the unadvertised good, in Stage 1, is* R $-$ *41c/42. In other words, the price of the advertised good will equal (41/42) times the transportation cost* c.
- *In Stage 2 consumers form rational expectations such that* $u_A^{12} = R - 41c/42 = u_B^E$ *and* $m^E = 0.5$.
- *All consumers buy both goods at the same store when the two stores advertise the same good. However, when stores advertise opposite goods, some consumers in the interval (10/21, 11/21) shop around. They buy the advertised good at each store. Others buy both goods at the same store, with those in the interval (0, 10/21) buying at Store* A *and those in the interval (11/21, 1) buying in Store* B.
- *Both stores make positive expected profits.*

First, note that equilibrium prices are below the reservation price for the advertised and unadvertised goods. Consumers take into account prices of both goods and derive surplus from both goods. Finally, some consumers shop around.

Second, in this equilibrium there is not a perfect correlation between the advertised good at one store and discount on the unadvertised good at the other store. Such a correlation was implicit in the equilibrium of Theorem 1, resulting from the fact that, in that case, stores made a decision about the unadvertised discount in the second stage, *after* they had observed each other's first-stage decision about the advertised discounts.

Finally, store profits are higher in the case in which the unadvertised prices are chosen in the first stage if $2c > R > 1.336c$, and lower if $1.336c > R > c$. In other words, for relatively smaller values of c the first-stage choice yields higher profits. This is because in Theorem 1, store strategies prevent consumers from shopping around. In contrast, in Theorem 2 store strategies permit some consumers to shop around. When the transportation cost c is high, competition for store traffic is less intense and preventing shopping around is not necessarily as profitable.

4. Conclusions and Directions for Future Work

In this paper we have addressed the question of the role of the unadvertised special in retailing, particularly for grocery products retailing. The practice of unadvertised specials is quite common, and we have provided a sample of the widely prevalent evidence. An important feature of our paper is a rational expectations model of consumer behavior.

We have shown that in equilibrium, stores follow a mixed strategy of advertising one or the other good. Further, stores offer a discount on the other, unadvertised, good. Consumers have rational expectations of this unadvertised discount, and base their store choice on it in addition to the advertised discount. Our results reconcile several features of observed pricing practice. First, we have shown why it might make sense for supermarkets to offer unadvertised specials. Essentially, these specials help retain customers who visit the store. Second, our equilibrium implies that competitive stores may advertise the same good or opposite goods. The prior work of Lal and Matutes (1994) had identified an equilibrium in which competing stores advertise the same good. Thus, we extend prior work and reconcile the observed practice of supermarkets not always advertising the same good. Third, we show that not only competing stores might advertise different goods, but also a given supermarket will advertise different goods over time.

Our mixed-strategy equilibrium yields higher profits than the Lal-Matutes equilibrium. The reason is that while the Lal-Matutes equilibrium extracts maximum consumer surplus on the unadvertised good, it also focuses competition on a single good insofar as generating store traffic is concerned. In contrast, the mixed-strategy equilibrium reduces the intensity of competition for generating store traffic by making the competitor uncertain about which good is being advertised. This reduction in competition at this level does not necessarily lead to intense competition for the unadvertised good because of the possibility of price matching and the attenuation of competition by the cost of visiting another store. Thus, we have identified a reason why supermarkets should keep competitors uncertain about their strategy of price advertising. It is important to recognize the fact that unadvertised specials come about because of rational consumer expectations. Thus, consumers can expect prices for one of the goods to be lower than the reservation price even without the commitment of advertising for that good.

4.1. Managerial Implications

There are several managerial implications of our results. First, unadvertised specials affect consumer behavior in two ways. They influence store choice because consumers have rational expectations. They also reduce shopping around by consumers when opposite goods are advertised by competing supermarkets. Second, by using unadvertised specials as part of a mixed strategy in advertising, stores are able to reduce the intensity of price competition on the advertised good. Note that mixed-strategy advertising is important because it keeps the competitor uncertain about the good being advertised, and so reduces price competition on the advertised good. Given this uncertainty, stores find it profitable to affect store choice through unadvertised specials. Third, by employing unadvertised specials as part of their strategy, stores are able to obtain higher profits than they would without them. Indeed, in the pure strategy equilibrium proposed by Lal and Matutes (1994) there are no unadvertised specials and the equilibrium profits are lower than in the mixed-strategy equilibria that we propose.

4.2. The Diamond Paradox

Diamond (1971) showed that, even with store competition, equilibrium price approaches the monopoly price. This is because once a customer is in a store, that store can charge a price higher than the competitor's by an amount equal to the cost of visiting another store. By induction, each store's price will be higher than the other stores' until the equilibrium price approaches the reservation (monopoly) price.

Lal and Matutes (1994) relaxed two implicit assumptions in the Diamond model. They let stores advertise prices so a consumer's choice of the first store to visit is endogenous, in contrast to Diamond's model. Further, they let each store carry two products. In the Lal-Matutes equilibrium one of the two goods is priced below the reservation (monopoly) price but the other is priced at the reservation price. This can be thought of as a weakening of Diamond's result. Other approaches to resolving the paradox center on the role of advertising and consumer search, but all are concerned with single-good situations, for example, Stahl (1989) and Robert and Stahl (1993).

We show that in our mixed-strategy equilibrium, neither good is priced at the reservation price half the time. In other words, the expected price of each good is below the reservation price. We believe that this resolves the Diamond paradox. Essentially, if stores can advertise prices to compete for store traffic and consumers have rational expectations of unadvertised prices, Diamond's result of equilibrium prices approaching monopoly prices does not obtain unless the number of goods in the stores' assortment is large relative to the number of advertised items.

4.3. Directions for Future Work

There are several directions for future work. The most interesting one would be to integrate the manufacturer trade promotion decision, retailer acceptance decision, and retail pricing and advertising under competition. We have provided a beginning for this in §3.5. Another direction would be to model explicitly consumer stockpiling to help characterize differences across products exhibiting different levels of ability to be held in inventory. Another interesting possibility is the analysis of differing formats such as everyday low pricing (EDLP) and high-low supermarkets. The game theoretic model we have developed could also be the foundation for analyzing retail competition among department stores or electronics retailers. Finally, our model provides a basis for designing empirical studies of retail price communication behavior.[13]

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Appendix 1

LEMMA 1. *Suppose stores commit to advertising opposite goods in the first stage. If consumers have basket expectations* u^E_{2A} *and* u^E_{1B} *for the unadvertised goods at Stores* A *and* B *respectively, then the stores' optimal choices are not an interior solution to their maximizing problems.*

PROOF OF LEMMA 1. (By contradiction) Suppose stores choose $u_{2A} = (R + D_{2B} - c)/2 = u^E_{2A}$ and $u_{1B} = (R + D_{1A} - c)/2 = u^E_{1B}$, corresponding to an interior solution.

Recall that for the marginal consumer located at m to be indifferent to the two stores, we need

$$cm + 2R - D_{1A} - u^E_{2A} = c(1 - m) + 2R - D_{2B} - u^E_{1B}.$$

Substituting for u^E_{2A} and u^E_{1B} into the above, and solving for m, we get

$$m = (2c + D_{1A} - D_{2B})/4c.$$

Then for D_{1A} to be part of a Nash equilibrium we require that it solves the following maximization problem:

$$\max_{D_{1A}} (2R - D_{1A} - (R + D_{2B} - c)/2)(2c + D_{1A} - D_{2B})/4c.$$

There is a similar problem for Store B. Solving the two maximization problems simultaneously for interior solutions we get

$$D_{1A} = D_{2B} = R - c.$$

Now substituting these we can obtain the unadvertised discount at Store B for Good *1* as

$$u_{1B} = R - c.$$

We will now see that this value for u_{1B} is not a solution to the problem of optimal choice of unadvertised discount. Recall that the optimal choice of unadvertised discount is given by

$$u^*_{1B} = \min\{(R + D_{1A} - c)/2, D_{1A} - cm, R\}.$$

Substituting for D_{1A} and D_{2B} into the above, we obtain

$$u^*_{1B} = \min\{(R - c), R - 3c/2, R\} = R - 3c/2.$$

This is different from the value obtained as an interior solution. □

Appendix 2

PROOF OF LEMMA 2. (By construction) We will show that there exist ranges of parameters such that the solutions to the stores' maximization problem can support the corner solutions to unadvertised discounts.

Suppose that both u_{1B} and u_{2A} are corner points. A corner solution must satisfy

$$D_{2B} - c(1 - m) = u_{2A} < \frac{D_{2B} + R - c}{2},$$

$$D_{1A} - cm = u_{1B} < \frac{D_{1A} + R - c}{2}.$$

These inequalities say that u^* is less than the value corresponding to the interior solution. The inequalities can be combined to yield

$$\frac{D_{1A} - R + c}{2c} < m < \frac{-D_{2B} + R + c}{2c}. \quad (1)$$

In other words, given D_{1A} and D_{2B}, any m satisfying the system of inequalities in (1) is admissible. For convenience, let us call the lower limit of m in the above as m_L and the upper limit as m_U.

Let us determine the marginal consumer located at m who is indifferent between going to Store A and Store B as far as his second-stage store choice is concerned. The consumer located at m_L knows that the true value of m is at least m_L. Therefore, there are two scenarios for the equilibrium m: $m = m_L$ or $m > m_L$. If $m = m_L$, then he is exactly indifferent between Stores A and B. In this case, if he goes to Store A, his total surplus is zero because the discount on the unadvertised good at that store will be just sufficient to induce him to shop for his basket at Store A. However, if the equilibrium m is such that $m > m_L$, then the consumer at m_L will get a positive surplus by going to Store A. This happens because Store A will offer enough discount on the unadvertised good to induce the consumer at m to make his basket purchase at Store A. This is more than the discount required to make the consumer at m_L buy both goods at Store A. Thus, the dominant strategy for the consumer located at m_L is to visit Store A. Now consider the case of the consumer located at $m_L + \epsilon$ for some small $\epsilon > 0$. We can assume that all consumers located to the left of $m_L + \epsilon$ visit Store A. The question is: What will the consumer located at $m_L + \epsilon$ do? This consumer knows that the equilibrium m is at least $m_L + \epsilon$. Thus there are two scenarios: $m = m_L + \epsilon$ or $m > m_L + \epsilon$. If $m = m_L + \epsilon$, then consumer at $m_L + \epsilon$ gets a surplus of exactly zero if he visits Store A by the same reasoning as above. However, if $m > m_L + \epsilon$, then that consumer gets positive surplus if he visits Store A. Hence, for the consumer located at $m_L + \epsilon$, visiting Store A is a dominant strategy. Similarly, we can see that the dominant strategy for the consumer located at m_U is to visit Store B. Given that, the dominant strategy for the consumer located at $m_U - \epsilon$ ($\epsilon > 0$) is to visit Store B also. Finally, consider the consumer located at $m = (m_L + m_U)/2$. From what we have said in the foregoing, the dominant strategy for all consumers to the left of him is to visit Store A, whereas the dominant strategy for all consumers to the right of him is to visit Store B. Thus, the consumer located at exactly $(m_L + m_U)/2$ is indifferent between visiting Store A or Store B. Hence, the marginal consumer is located at $m = (m_L + m_U)/2$.

To show that there indeed exist values of parameters which support the corner solutions to unadvertised discounts, we solve the firms' profit-maximization problem given the above m and the corner values for unadvertised discounts. We write below the profit-maximization problems for Store A and Store B respectively, after setting $u^*_{2A} = D_{2B} - c(1 - m)$, $u^*_{1B} = D_{1A} - cm$ and $m = (m_L + m_U)/2$:

$$\max_{D_{1A}} (2R - D_{1A} - D_{2B} + c(2c - D_{1A} + D_{2B})/4c)(2c + D_{1A} - D_{2B})/4c$$

for Store A, and

$$\max_{D_{2B}} (2R - D_{1A} - D_{2B} + c(2c + D_{1A} - D_{2B})/4c)(2c - D_{1A} + D_{2B})/4c$$

for Store B.

From the first-order condition for profit maximization for Store A we obtain

$$8R - 10D_{1A} + 2D_{2B} - 8c = 0.$$

Similarly, from the first-order condition for Firm B's profit maximization we get the symmetric condition

$$8R - 10D_{2B} + 2D_{1A} - 8c = 0.$$

Solving the two equations simultaneously we obtain $D_{1A} = D_{2B} = R - c$. From these we can solve for the unadvertised discounts at the two stores, as $u_{1B} = u_{2A} = R - 3c/2$.

Also, as is easily seen from the above, $m = 0.5$. Now we are in a position to check that there exist values of parameters for which the corner solutions hold. Substituting $D_{1A} = D_{2B} = R - c$ and $m = 0.5$ into Inequality (1) we get $D < R$, or equivalently, $c > 0$.

Thus we have shown that there exist ranges of parameters such that the solutions to the stores' maximization problem can support the corner solutions for the unadvertised discounts. □

Appendix 3

Proof of Lemma 3. (By contradiction) Assume, to the contrary, that the stores can commit to advertising opposite goods as part of a Nash equilibrium. Suppose that in this equilibrium, in Stage 1, Firm A advertises Good *1* and Firm B advertises Good *2*. Now consider a deviation by Firm B in which it deviates by advertising Good *1* also. In such a case the marginal consumer who is indifferent between shopping at Store A and at Store B is located at m where m is given by

$$cm + R - D_{1A} + R - u_{2A} = c(1 - m) + R - D_{1B} + R - u_{2B}.$$

Moreover, when both stores advertise the same good, we already saw that the unadvertised discount in Stage 2 is zero. Setting $u_{2A} = u_{2B} = 0$ in the above equation and solving for m we get

$$m = \frac{D_{1A} - D_{1B} + c}{2c}.$$

The Stage 1 optimization problem for B's deviation is then

$$\max_{D_{1B}} (2R - D_{1B})\left(1 - \frac{D_{1A} - D_{1B} + c}{2c}\right).$$

The first-order condition for a maximum gives $2R - 2D_{1B} - c + D_{1A} = 0$. Recalling that the supposed equilibrium requires $D_{1A} = R - c$, and solving for D_{1B}, gives the optimal (deviation) discount for Firm B as $D_{2B} = 1.5R - c$. Corresponding to this the maximum profit from deviation is $(c/2)(R/2c + 1)^2$. If Store B were to be at the opposite-good equilibrium, the profit would be $3c/4$. Under the assumption $2c > R > c$, the profit from deviation exceeds the profit from opposite-good equilibrium. In other words, we cannot have a pure strategy Nash equilibrium in which the two firms commit to advertising opposite goods in Stage 1. □

Appendix 4

Proof of Theorem 1. (By construction) To see that Store A's strategy is optimal, first let us suppose that Store B advertises Goods *1* and *2*, each with probability 0.5. Then, if Store A adopts a pure strategy of advertising Good *1*, there are two equally likely scenarios. These are displayed below:

SCENARIO 1	Store A	Store B
Product *1*	$R - D_A$	$R - D_B$
Product *2*	R	R

SCENARIO 2	Store A	Store B
Product *1*	$R - D_A$	$R - u_B$
Product *2*	$R - u_A$	$R - D_B$

Note that in Scenario 1, the Stage 2 decision for both stores is to offer no discount, following Lal and Matutes (1994). In Scenario 2, following Lemma 2, we know that the unadvertised discounts are $u_A = D_B - 0.5c$ and are $u_B = D_A - 0.5c$. The two scenarios represent same-good advertising with probability 0.5, and opposite-good advertising with probability 0.5. If Store A were to adopt a pure strategy of advertising Good *2*, once again we will have two scenarios with same-good advertising with probability 0.5, and opposite-good advertising with probability 0.5. Therefore, *any* mixed strategy by Store A in Stage 1 will also give two identical scenarios with same-good advertising with probability 0.5, and opposite-good advertising with probability 0.5. We can now write the maximizing problem for Store A, after some simplification, as:

$$\max_{D_A} \left\{\frac{1}{2}(2R - D_A)\left(\frac{D_A - D_B + c}{2c}\right) + \frac{1}{2}\left(2R - \frac{5D_A}{4} - \frac{3D_B}{4} + \frac{c}{2}\right)\left(\frac{D_A - D_B + 2c}{4c}\right)\right\}. \quad (2)$$

The first-order condition for profit maximization yields

$$3R - \frac{13}{4}D_A + \frac{5}{4}D_B - 2c = 0.$$

We also have a similar optimizing problem for Store B under the mixed strategy. It is given by the following:

$$\max_{D_B} \left\{\frac{1}{2}(2R - D_B)\left(1 - \frac{D_A - D_B + c}{2c}\right) + \frac{1}{2}\left(2R - \frac{5D_B}{4} - \frac{3D_A}{4} + \frac{c}{2}\right)\left(\frac{D_B - D_A + 2c}{4c}\right)\right\}.$$

The first-order condition from this yields

$$3R - \frac{13}{4}D_B + \frac{5}{4}D_A - 2c = 0.$$

Solving the two first-order conditions simultaneously we get the optimal discounts under the mixed strategy as

$$D_A^* = D_B^* = 1.5R - c.$$

Denote this common discount by D^*.

We next show that all consumers buy both goods. By symmetry, the most distant consumer for either store is located at a distance of 0.5 and bears a transportation cost of $0.5c$. Consider first the case of Store A. This consumer is guaranteed to get a surplus of D^*. It is sufficient if $D^* > 0.5c$. This translates to

$$R > c.$$

By assumption this inequality is satisfied.

Next we note, following Lemma 2, that the value of the unadvertised discount is $u^* = D^* - 0.5c = 1.5R - 1.5c$. The expected discount at each store is then $D^* + 0.5(1.5R - 1.5c) = 9R/4 - 7c/4$. For the stores to make nonnegative profits we require that the expected margin be greater than zero. In other words, we need

$$2R > 9R/4 - 7c/4$$

or

$$7c > R.$$

By assumption this inequality holds.

Now we will show that the stores' equilibrium profit under this mixed strategy is higher than their profit under the pure strategy in which they always advertise the same good. The equilibrium profit of Store A under the mixed strategy is

$$\Pi_A^* = 0.25(3.5c - 0.5R).$$

Profit of Firm A when both the firms advertise discounts on the same good is $0.5c$. This is because in this case $m = 0.5$, discount on the advertised good is $2R - c$, and the discount on the unadvertised good is 0. Clearly $0.25(3.5c - 0.5R) > 0.5c$ if $3c > R$. Again, by assumption this is true.

Lastly, we need to check that the corner solutions to unadvertised specials are feasible with the admissible range of parameters. For the corner solution to hold, the optimal discounts should satisfy Inequality (1) in Appendix 1. Using the symmetry of the stores and $m = 0.5$, Inequality (1) reduces to $D < R$. From the optimal value of D obtained above, this condition is $2c > R$. Again by assumption, this is true.

We have now fully characterized the equilibrium and the theorem is proved. □

Appendix 5

Proof of Corollary 1. Fix Store A's strategy in the first stage as advertising each good with probability 0.5, and choosing the advertised discount to be $1.5R - c$. This corresponds to the equilibrium in Theorem 1. Suppose that Store B chooses advertised discounts of D_1 and D_2 on Goods *1* and *2* in Stage 1. As before, Store A must determine the discount D on the advertised good. As in Theorem 1, we first have to determine the unadvertised discount to be given by the Store A in the second stage. As Store A randomizes the advertised good in the first stage with probability 0.5 each, and Store B advertises both goods, we will have both stores advertising Good *1* half the time and Good *2* at other times. We start by addressing the consumers' store choice problem under these two scenarios.

Both stores advertise good 1. As in Lemma 2, we assume that Store A's choice corresponds to the corner solution for the unadvertised discount in the second stage. We will determine this corner solution and verify that it is part of the equilibrium. Let the unadvertised discount when the two stores advertise Good *1* be u^1.

The consumer located at m is indifferent between buying both goods at Store A and buying both at Store B if

$$-mc + D + u^1 = -(1 - m)c + D_1 + D_2,$$

which gives

$$m = \frac{D + u^1 - D_1 - D_2 + c}{2c}.$$

By analogy with §3.2, the corner solution to the stores choice of unadvertised discount is given by $u^1 = -(1 - m)\,c + D_2$. Substituting for m from above we get

$$u^1 = D + D_2 - D_1 - c.$$

Similarly, when both stores advertise Good *2*, the unadvertised discount u^2 is given by

$$u^2 = D + D_1 - D_2 - c.$$

It is easy verify that these corner solutions are indeed part of equilibrium. Store B's maximization problem under deviation is therefore

$$\max_{D_1, D_2} \left\{ \left[0.5\left(1 - \frac{D + u^1 - D_1 - D_2 + c}{2c}\right) + 0.5\left(1 - \frac{D + u^2 - D_1 - D_2 + c}{2c}\right)\right](2R - D_1 - D_2)\right\}.$$

Putting in the values of u^1 and u^2 from above, using $D = 1.5R - c$ and solving the first-order conditions, we get the optimal deviation discounts of Store B as

$$D_1^* = D_2^* = \frac{5R}{4} - c.$$

The profits from optimal deviation for Store B is therefore $(1/8c)(4c - R)^2 - F$, as it now incurs an additional cost F of advertising the second good. If Store B were not to deviate from its equilibrium strategy of Theorem 1, its profit would be $(1/8)(7c - R)$. Deviation is unprofitable if $F > (1/8c)(9c^2 - 7Rc + R^2)$.

Let us now assume that there is no additional cost of advertising the second good, i.e., $F = 0$. Deviation is still unprofitable if $(1/8)(7c - R) > (1/8c)(4c - R)^2$. There exists a range of parameters given by $2c > R > (7 - \sqrt{13})c/2$, for which there will be no deviation, and the equilibrium in Theorem 1 holds. □

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