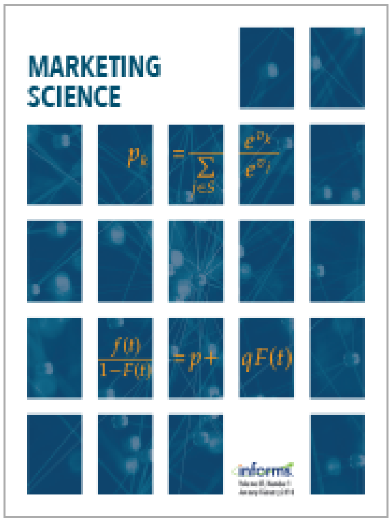

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List Price and Discount in a Stochastic Selling Process

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Abstract. From B2B sales to AI-powered ecommerce, one common pricing mechanism is "list price–discount": The seller first publishes a (committed) list price, then during interactions with a buyer, offers (noncommitted) discounts off of the list price. Some B2B sellers never sell at their list prices. In such cases, what role does a list price play and how to choose the optimal list price and discounts? In this paper, I study a stochastic sales process in which a buyer and a seller discover their match value sequentially. The seller can adjust its price offers over time, and the buyer decides whether to accept each offer. I show that this discovery process creates a hold-up problem for the buyer that results in inefficient no-trades. The seller alleviates this problem by committing to an upper bound in the form of a list price. But in equilibrium players always reach agreement at a discount. I show that the seller prefers this mechanism to having no list price or committing not to discount. When the cost of selling is high, the seller's ability to offer discount is necessary for trade to happen. There exists reverse price discrimination when buyers are heterogeneous, and list price can serve as a signaling device when sellers are heterogeneous. Extensions with alternative pricing or matching mechanisms are discussed.

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1. Introduction

In many sales contexts, the final pricing agreement has two components: a list price and a discount. The seller first sets its list price internally, then after interacting with a buyer, quotes a discount. This is common in business-to-business transactions such as purchasing of industrial equipment or enterprise software, big-ticket consumer items such as real estate or automobile, or AI-powered e-commerce where sellers offer personalized discounts based on buyers' browsing behaviors. These two types of prices are different in their timing and commitment power. In the above examples, list price is set by the seller long before a buyer arrives, and the same list price is used regardless of who the buyer is. From a buyer's perspective, the list price is visible (either through public information or direct inquiry with the seller), and will remain constant during their interactions. On the other hand, the seller chooses discounts only after interacting with the buyer through activities such as sales pitch and product demonstration. The size of the discount depends on the information that seller acquires during these interactions, and can vary from buyer to buyer. From a buyer's perspective, the seller makes no commitment on whether, when, or what discount will be offered. Even after a discount is offered, the quote often has a short deadline, after which the seller can retract the discount or offer a bigger one. Thus, the list price promises an upper bound, while the seller maintains the flexibility to offer any price below it through dynamic discounting.

Articles from management consulting firms suggest that this is a common B2B practice. In an article by Boston Consulting Group, Schurmann et al. (2015) state that "[i]n most B2B industries, discounts represent a company's largest marketing investment, often amount to 30% or more of list-price sales." A Mckinsey article on B2B pricing suggests that firms should focus on transaction pricing, which is "for each transaction—starting with the list price and determining which discounts . . .should be applied" (Marn et al. 2003). PricewaterhouseCoopers states that "discounting off of the 'list' price of as much as 100% is common" in the software industry (PwC Technology Institute 2013). L.E.K. Consulting states that "[d]iscounts are a universal feature of any buying process" (Pierre and Ossmann 2017), with similar statements found in a Bain article (Mewborn et al. 2014). Furthermore, Bain and McKinsey each reveals data on final prices as percentages of list prices for one of their clients (Kermisch and Burns 2018, Marn et al. 2003). Interestingly, both of these firms offer discounts to 100%

of their customers, so no client pays the full list price. Other anecdotal evidence suggests that these two components of pricing perform more than just their sum. Bain documents a case in which a firm's strategy of setting high list price and quoting deep discounts hurt sales, "even though the final prices (after discounts) would have been very competitive" (Burns and Murphy 2018).

This "list price–discount" mechanism has not received sufficient academic attention. In dynamic bargaining or pricing models where players can adjust offers over time, there lacks an explanation for why a seller wants to self-restrain its future offers by establishing a list price ex-ante. For example, if one allows the seller to set an upper bound in the repeated-offers model of Fudenberg et al. (1985) and Gul et al. (1986), then the upper bound is optimal as long as it is higher than the valuation of the highest type. Thus, providing the seller with the ability to set a list price does not affect the equilibrium outcome, making "list price" and "discount" meaningless terms in such a model. One may also view the use of list price and discount as a method of price discrimination, where the list price is the highest price tier. However, this cannot account for cases in which no buyer pays the list price, as documented in Kermisch and Burns (2018) and Marn et al. (2003). If the highest price tier receives no demand, then neither its existence nor its size has any impact. One may also hypothesize that the never-used list price serves as a signaling tool to separate sellers of different qualities. But for there to be separation in equilibrium, there needs to be an adverse effect of increasing list price, otherwise all sellers can mimic the highest quality seller. The micro foundation for such adverse effect is unclear.

If nobody pays the list price, what purpose does this upper bound serve? If the seller changes this list price, how does this affect the equilibrium price that players agree to? What factors should be considered when selecting the optimal list price? How does this mechanism compare to ones where seller does not use a list price or commits to never offer a discount? This paper studies these questions in a setting where the total surplus from trade between a buyer and a seller is uncertain ex-ante due to heterogeneity, and players spend time discovering the size of this surplus prior to transaction. In many industries, sellers differ in attributes of the products/services that they offer, and customers differ in the attributes that they need. A buyer may not know how well the seller can address his needs and has to acquire information to guide his purchasing decision.

But the buyer is not the only party that learns. In practice, many B2B firms have an explicit "discovery" step in their sales processes, in which the salesperson inquires about the buyer's situations and needs.[1] Industry studies find that a well-executed discovery plan plays an important role in securing sales (Zarges 2017). The information about the buyer's needs helps the seller to fine-tune her selling strategy, such as whether to continue pursuing a buyer or what price to quote. For example, firms often delegate some pricing power to salespersons, because the salespersons know more about a client's willingness-to-pay than the firm does, through their communications with the client (see Mantrala et al. 2010 and Coughlan and Joseph 2011 for surveys on the literature of price delegation). A survey of sales forces by Hansen et al. (2008) find that 72% of companies give their salespersons pricing authority. The number rises to 88% for industrial-goods companies. The two parties discover how well they match with each other, which determines their total surplus from trade. This view is consistent with writings from practitioners (see, e.g., Nick 2017 and Mehring 2017).

In e-commerce, this "list price–discovery–discount" phenomenon begins to emerge as well. Using AI-powered tools, online retailers can rate a customer's purchase intent in real time and intervene with discount offers. One company providing this technology is Appier, whose product AiDeal allows retailers to track a customer's likelihood of purchase through "real-time activities across the site, such as how they tap and swipe on their phones, the cursor position and the amount of scrolling." Retailers can then choose the right moment to offer time-limited deals during a browsing session to trigger purchases based on the inferred likelihood of purchase (Appier 2020). Research has shown that a user's behavior during a browsing session can be used to predict her purchase intent. For example, a user's clickstream on a site contains information about her purchase horizon (Moe 2003). On computers, cursor movements correlate with eye gaze, which highlights user attention (Guo and Agichtein 2010). Guo et al. (2019) use data from Toubao that include users' mobile taps and swipes to predict a user's purchasing intent in real time. They show that the use of touch-interactive data significantly improves prediction accuracy. Their A/B testing during "Double 11," the Chinese shopping holiday, shows that a coupon allocation strategy that targets users whose real-time purchase intent falls into an intermediate range outperforms a nontargeted strategy by about 40% in GMV. A simple use of real-time intervention that is common in e-commerce already is exit-intent popup. When a user moves cursor toward certain parts of the browser, or highlight product names, the website can infer from these movements that the user is about the leave, and intervene with a popup offering discount (Webtrends Optimize 2020). As the underlying technology matures, the practice of real-time personalized discounting based on predicted purchase intent may become more widely adopted.

This paper analyzes its effect on equilibrium prices and welfare.

We study a dynamic model where a buyer and a seller discover how well they match with each other over time while negotiating on price. The seller chooses a list price ex-ante, and can offer dynamic discounts over time as players receive gradual information on their level of match. The buyer decides whether to accept an offer or continue the process, and both players can choose to leave because waiting is costly for both players.[2]

In this model, discovery and pricing are interdependent. Information on product fit affects pricing strategy and vice versa. For example, intuitively, a good match benefits the buyer. But if the seller observes that the match is good, the seller may end up charging a higher price (with a smaller discount), which reduces the buyer's incentive to discover their match in the first place if doing so is costly. This presents a hold-up problem, where the expectation of a future "bargaining" outcome affects the players' current choice of "investment" (whether to continue or leave). Another important feature of the model is that we allow the seller to make offers at any moment during the discovery process, hence endogenizing the amount of learning and the timing of agreement.

I show that list price acts as an instrument to reduce the buyer's concern for hold-up. If the list price is too high (or there is no list price), then the buyer leaves immediately. The buyer does not expect to get surplus ex-ante because the seller can charge a high price once the good match is revealed. In order to encourage the buyer to gather information, the seller needs to set a low enough list price to limit the impact of such hold-up. On the other side, a lower list price decreases the seller's ability to exploit the buyer's discovery of high match value, because it increases the buyer's continuation value. The optimal list price thus has to balance these two effects. More importantly, I find that the parties always trade below the list price, regardless of what the list price is.[3] The reason is that it is not efficient for the seller to wait until the buyer is willing to pay the full list price. Instead, the seller prefers to close the sale earlier by offering a price discount. The model provides a rational explanation for the list price–discount pattern that is observed in practice.

Should the seller commit to not offer a discount? I find that using a fixed price leads to a higher transaction price and a lower ex-ante probability of trade. Having the ability to discount always benefits the seller and also increases overall welfare, but its effect on the buyer's utility depends on the ratio of the players' costs. When the seller's cost of selling is low, buyers are better off under fixed price. But, when the seller's cost is high, having the ability to discount is necessary for trade. The flexibility in price improves overall efficiency by saving time and cost and by increasing the ex-ante success rate. In such a case, the ability to discount is welfare-enhancing for both parties. Comparisons to other mechanism to reduce hold-up, including nonretractable offers and direct subsidy, are explored.

The model extends to cases with asymmetric information. When buyers have private outside options, I show the existence of reverse price discrimination, where the buyer with a higher valuation for the product pays a lower price. If sellers have private information in their quality, the model finds that a high type seller can set a higher-than-optimal list price as a way to signal. Thus, a list price can have signaling power even though it is never traded in equilibrium, through its indirect impact on equilibrium price. The main results can also be replicated in a 3-period model with asymmetric learning, where a seller only observes noisy signals of a buyer's preferences.

Overall, the model provides novel insights on the role of list price and price discounts in facilitating information discovery. The analyses give implications on pricing and selling strategies in a stochastic environment. On the theoretical side, the paper expands existing repeated-offers literature by introducing a hold-up problem to the stochastic bargaining framework.

1.1. Literature Review

The paper is related to the literature on sequential information acquisition. Similar to Roberts and Weitzman (1981), Moscarini and Smith (2001), Branco et al. (2012), and Ke et al. (2016), this paper uses a Brownian motion to capture the effect of gradual arrival of information on product value. Although previous papers focus on single-agent decision making, this paper features a two-player game and allows price to be bargained. Kruse and Strack (2015) look at a problem in which a principal tries to influence an agent's stopping decision through a transfer. The price discount in this paper can be seen as a transfer, but unlike the principal in Kruse and Strack (2015), the seller in this paper cannot commit to future transfers.

The paper follows Fudenberg et al. (1985) and Gul et al. (1986) in studying a repeated-offers model where one party makes all the offers, which can be interpreted as either bargaining or dynamic pricing of a durable goods monopoly. Other papers have studied repeated-offers models with stochastic payoffs. Daley and Green (2020) look at a repeated-offers game with asymmetric information and gradual signals. One party knows the true quality of the product, and the other party receives noisy signals of the quality over time. In contrast, both players acquire information in the present paper. Ortner (2017) studies dynamic pricing of a durable goods monopoly whose marginal cost changes over time. Fuchs and Skrzypacz (2010) look at bargaining with a stochastic arrival of events that can end the game. Ishii et al. (2018) studies wage bargaining with both sequential learning and stochastic arrival of competitor.

Merlo and Wilson (1995) present a general framework for stochastic bargaining in which the total surplus fluctuate over time and the order of proposing is random. However, they do not allow players to quit for outside options, as in this paper. Rubinstein and Wolinsky (1985), Gale (1987), and Binmore and Herrero (1988) expand the alternating-offers model of Rubinstein (1982) with a different type of stochastic matching. In these papers, buyers and sellers have fixed values for the product, but can only bargain when matched with a player of the opposite type. The matching process is exogenous, so players are automatically in search of new partners as long as they are in the market. This literature explores the difference in equilibrium prices between centralized and decentralized markets. In comparison, the present model focuses on the hold-up problem created by the combination of stochastic surplus and endogenous quitting, both absent in the aforementioned papers.

The idea that list price can reduce hold-up relates to consumer search literature which shows that sellers can use published price to encourage search. Wernerfelt (1994b) shows that, when product quality is uncertain and requires search/inspection, the seller wants to inform the buyer about the price before search. Similarly, to encourage visits, multiproduct retailers advertise the prices of loss-leaders to put an upper-bound on the total price of a basket (Lal and Matutes 1994). However, in these papers, buyers learn all information in one shot while sellers do not learn. Sellers cannot, and do not have incentives to, give discounts after buyers inspect the products. Thus, they do not capture the "list price–discount" pattern studied in this paper. The current model bridges this gap by adding stochastic arrival of information that is observed by both parties, which allows the seller to offer dynamic discounts. We show that setting a list price reduces hold-up even if the seller always offers discounts in equilibrium. Other papers have discussed the use of list price in different contexts. Xu and Dukes (2020) show that the list price can be used to convince uninformed buyers of their types when the seller has superior information. Yavas and Yang (1995) and Haurin et al. (2010) discuss the signaling role of list price in the real estate market. Shin (2005) shows that a noncommittal price can signal true prices when the sales process is costly.

The paper also relates to earlier works that study the role of sales in providing information to consumers, such as Wernerfelt (1994a) and Bhardwaj et al. (2008). Shin (2007) studies a firm's decision to provide presales service that reveals the match between a customer and the product, when competitor can free-ride on such service. This paper also studies a firm's decision to delegate pricing authority, but with a different approach from the principal-agent models of Lal (1986), Bhardwaj (2001), and Joseph (2001).

The rest of the paper is organized as follows. Section 2 presents the baseline model. Section 3 solves the equilibrium outcome. Sections 4 to 6 presents various extensions to the model. Concluding remarks are offered in Section 7.

2. The Model

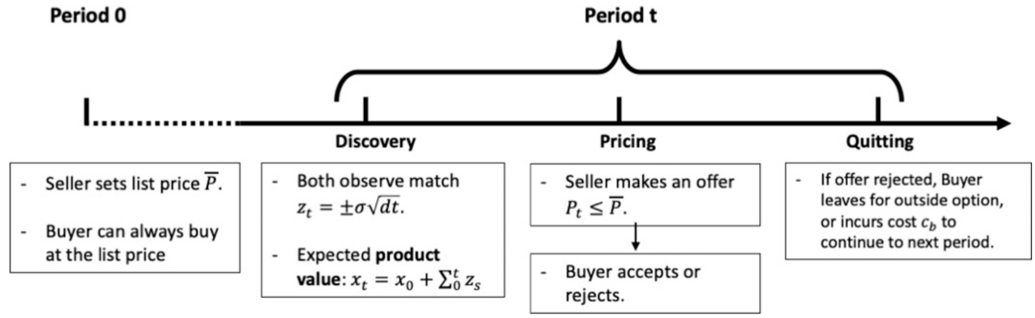
We first describe the model in discrete time, then present the solution to the continuous-time limit of the game.

2.1. Description of the Model

Consider two players, a Buyer and a Seller. Seller has a product with T attributes. Each attribute is worth $v_h\sqrt{dt}$ to Buyer if it matches Buyer's needs or $v_l\sqrt{dt}$ if it does not, where dt measures the size of each attribute. The total value of the product is the sum of its attributes. For simplicity, I assume that the probability of match for each attributes is $\frac{1}{2}$ and independent across attributes.[4] Outside options and the cost of producing the product are normalized to 0 without loss of generality. Note that this means that the value of the product can be negative if it is less than the sum of the outside options and the production cost.

The game is illustrated in Figure 1.

Figure 1. Illustration of the Game



2.1.1. Discovery of Match. Each period, players simultaneously discover their match on the next attribute. The expected value of the product after observing t attributes can be written as $x_t = x_0 + \sum_0^t z_s$, where x_0 is the expected value of the product at time 0, and z_t is the new information from period t. Define $\sigma = \frac{v_h - v_l}{2}$, then we have $\mathbb{E}[z_t] = 0$ and $Var[z_t] = \sigma^2 dt$.

2.1.2. Pricing. Before the game starts, Seller can set a list price $\overline{P}$. Buyer can always buy the product at $\overline{P}$ in any future period. Denote $\overline{P} = \infty$ if Seller choose not to set a list price. Each period, Seller can offer a discounted price P_t with no future guarantee. Buyer decides whether to accept the offer. The game ends if Buyer accepts. Notice that Seller cannot effectively offer any price above $\overline{P}$. Also, if Seller does not offer a discount, Buyer can still buy at $\overline{P}$, so there is a standing offer at each period.

2.1.3. Quitting. Buyer incurs flow cost each period and can choose to quit at the end of each period. If Buyer quits, the game ends and players receive their outside options. In Section 4, I study the case where Seller has flow cost and can quit too.

2.1.4. Stationary Baseline. As the length of each period (or size of each attribute) approaches 0, the game approaches continuous time. As the mass of total attributes T approach infinity, the product value x_t becomes a Brownian motion. I examine this limiting case in the baseline for its tractability. Nonstationary extensions are discussed in Section 6 and Online Appendix to check that results from the baseline model are robust.[5]

Figure 2 presents a sample evolution of the product value in the stationary baseline. Number of attributes discovered, t, is on the horizontal axis, and the expected product value, x, is on the vertical axis. First assume that Seller never offers a discount. This is Buyer's single agent optimization problem as he decides between discovering and buying. In the beginning, Buyer optimally chooses to wait and find out more about the product. Waiting is optimal even for some x above the list price because of the option value of learning. Buyer's optimal strategy is to buy when product value reaches a threshold. However, Seller may prefer to close the sale earlier at a discounted price. Trading earlier saves time and cost, while also reducing the possibility of Buyer leaving upon discovering a bad match. This is shown in Figure 3.

Solving the equilibrium in Figure 3 is difficult. Because Seller cannot commit to future discounts, Buyer's decision depends on Seller's entire pricing strategy P_t in all future states. Conversely, Seller's optimal offer depends on what Buyer is willing to pay in all future states. An equilibrium then should be the solution to a two-sided stopping problem. Buyer's strategy gives him the optimal stopping time given what Seller is willing to offer in the future, and Seller's pricing strategy gives her the optimal stopping time given what Buyer is willing to accept in the future.

2.2. Formal Definitions

The game has an infinite horizon. Product value x_t is observable to both players and follows $dx_t = \sigma dW_t$ with initial position x_0, for some Wiener process W_t.

2.2.1. Equilibrium. I look for stationary SPE with pure strategy, henceforth referred to as "equilibrium."[6] An equilibrium strategy profile $\theta(x_t) = (P(x_t), a(x_t, P_t), q(x_t))$ maps each state x_t to Seller's price offer, $P_t = P(x_t)$, Buyer's acceptance decision for each price offer, $a_t = a(x_t, P_t) \in \{0, 1\}$, and Buyer's quitting decision, $q_t = q(x_t) \in \{0, 1\}$. I focus on stationary behavior because product value x_t is stationary and time is payoff-irrelevant conditional on x_t. An equilibrium outcome can be described by a quadruple $(A, Q, U(x), V(x))$, where $A = \{x | a(x, P(x)) = 1\}$ is the region where players reach agreement, $Q = \{x | q(x) = 1\}$ is the region where Buyer quits, $U(x)$ is Seller's equilibrium value function, and $V(x)$ is Buyer's equilibrium value function.

2.2.2. Utility. The game ends when Buyer buys or quits. Let $\tau_\theta = \inf\{t \mid a_t + q_t > 0\}$ denote the stopping time of the game given a strategy θ. Buyer incurs flow cost c_b during the game. Players are risk neutral and

Figure 2. (Color online) Example with a Fixed Price

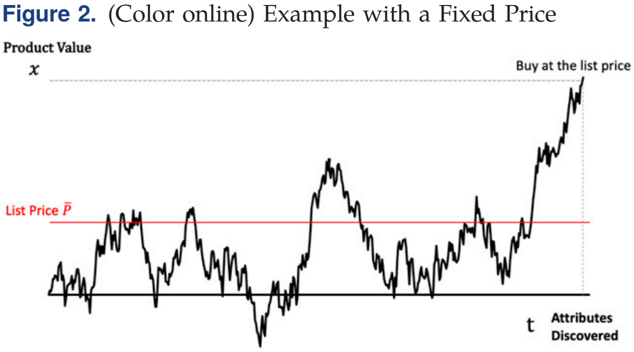


Figure 3. (Color online) Example with Discounts

perfectly patient.[7] If Buyer and Seller agree to trade at time τ, Buyer receives utility $x_\tau - P_\tau$ and Seller receives P_τ. If Buyer quits at time τ, then players get their outside options, normalized to 0 without loss of generality. Thus, Seller's utility at time t from a strategy θ is defined as:

$$u_t = \mathbb{E}_{\tau_\theta}\left[P_{\tau_\theta}\mathbb{1}\{a_{\tau_\theta} = 1\}\right].$$

And Buyer's utility at time t from a strategy θ is defined as:

$$v_t = \mathbb{E}_{\tau_\theta}\left[\overbrace{-c_b(\tau_\theta - t)}^{\text{Buyer's cost}} + \overbrace{(X_{\tau_\theta} - P_{\tau_\theta})\mathbb{1}\{a_{\tau_\theta} = 1\}}^{\text{utility from purchase}}\right].$$

2.2.3. Key Assumptions and Extensions. This stylized model contains a number of features that help to simplify the problem. In various extensions, I explore what happens if these features are altered. For example, in the baseline model, product value x_t is assumed to be fully observable by both players. Seller observes Buyer's preference and Buyer observes each attribute accurately. In reality, potential buyers may hide their preferences in order to barter for a lower price.[8] In Section 5, I extend the model by giving players private information. Buyer can have private information on his outside option, Seller can have private information on her quality, and Seller may learn about Buyer's preference with noise. The full list of extensions are summarized in Table 1.

3. Equilibrium Outcome

When Buyer and Seller reach agreement, Buyer's utility should be equal to his continuation value, otherwise Seller should raise the price offer. Because the product is always available at the list price $\overline{P}$, Buyer cannot have a lower continuation value than if Seller never gives a discount. Thus, solving for Buyer's value function facing a fixed price of $\overline{P}$ provides a lower bound on Buyer's continuation value in equilibrium. This is simply Buyer's single-agent optimal stopping problem. Each moment, Buyer decides between buying at $\overline{P}$, continuing, or quitting. Such problems have been studied for investment under uncertainty (e.g., Dixit 1993), R&D funding (Roberts and Weitzman 1981), experimentation (Moscarini and Smith 2001), and consumer search (e.g., Branco et al. 2012).

For states in which players trade, Seller receives $U(x) = P(x)$ and Buyer receives $V(x) = x - P(x)$. For states in which no agreement is reached and a player quits, $U(x) = V(x) = 0$. For a state x such that players choose to continue negotiating, we can write recursively:

$$\begin{aligned} U(x,t) &= -c_s dt + \mathbb{E}U(x+dx, t+dt) \\ V(x,t) &= -c_b dt + \mathbb{E}V(x+dx, t+dt). \end{aligned} \tag{1}$$

Under stationarity, and by taking Taylor expansion and applying Ito's Lemma on $\mathbb{E}U$ and $\mathbb{E}V$ terms, these expressions can be reduced to the following equations:

$$\begin{aligned} 0 &= -c_s + \frac{\sigma^2}{2}U''(x) \\ 0 &= -c_b + \frac{\sigma^2}{2}V''(x). \end{aligned} \tag{2}$$

The solutions to the equations must be of the form:

$$\begin{aligned} U(x) &= \frac{c_s}{\sigma^2}\left(x - \overline{P}\right)^2 + A_s\left(x - \overline{P}\right) + B_s \\ V(x) &= \frac{c_b}{\sigma^2}\left(x - \overline{P}\right)^2 + A_b\left(x - \overline{P}\right) + B_b. \end{aligned} \tag{3}$$

Let $\underline{V}(x)$ denote the Buyer's value function facing a fixed price of $\overline{P}$. Let $\overline{\overline{x}}$ denote the Buyer's buying threshold, and let $\underline{\underline{x}}$ denote the Buyer's quitting threshold. By Equation (3), Buyer's value function must be of the form $\underline{V}(x) = \frac{c_b}{\sigma^2}(x - \overline{P})^2 + \alpha(x - \overline{P}) + \beta$ for some coefficients α and β. The function has to satisfy the following boundary conditions:

$$\begin{cases} \underline{V}(\overline{\overline{x}}) = \overline{\overline{x}} - \overline{P} & \text{and} \quad \underline{V}\left(\underline{\underline{x}}\right) = 0 \\ V'(\overline{\overline{x}}) = 1 & \text{and} \quad V'\left(\underline{\underline{x}}\right) = 0. \end{cases} \tag{4}$$

The first two conditions ensure that the value function must match the stopping value when Buyer buys or quits. The last two conditions, often referred to as

Table 1. List of Extensions

Baseline model	Extensions	Section
Seller has no flow cost	Seller has flow cost and can quit	4
	Seller can subsidize Buyer's cost	4.1
Symmetric information	Heterogeneous Buyers and price discrimination	5.1
	Heterogeneous Sellers and signaling	5.2
	Seller observes match with noise	5.3
Seller can retract offers	Seller can never retract past offers	6
Seller makes offers	Buyer makes offers	6
Infinite horizon	Finite horizon	6
Additive attributes	Bayesian signals on the true value	6

"smooth-pasting" conditions, ensure that the stopping time is optimal. See Dixit (1993, pg. 34–37) for more details.

Solving this system of equations shows that Buyer buys at

$$\overline{\overline{x}} = \overline{P} + \frac{\sigma^2}{4c_b} \tag{5}$$

and quits at

$$\underline{\underline{x}} = \overline{P} - \frac{\sigma^2}{4c_b}. \tag{6}$$

Buyer's value function of facing a fixed price of $\overline{P}$ is

$$\underline{V}(x|\overline{P}) = \begin{cases} 0, & \text{if } x - \pi_b \le \overline{P} - \frac{\sigma^2}{4c_b} \\ \frac{c_b}{\sigma^2}\left(x - \overline{P}\right)^2 + \frac{1}{2}\left(x - \overline{P}\right) + \frac{\sigma^2}{16c_b}, & \\ & \text{if } x \in \left(\overline{P} - \frac{\sigma^2}{4c_b}, \overline{P} + \frac{\sigma^2}{4c_b}\right). \\ x - \overline{P}, & \text{if } x - \pi_b \ge \overline{P} + \frac{\sigma^2}{4c_b} \end{cases} \tag{7}$$

Lemma 1 shows that there exists an equilibrium where Buyer's continuation value is $\underline{V}(x)$, and all equilibrium in which Buyer receives $\underline{V}(x)$ must yield the same outcome. In this equilibrium outcome, Buyer acts as if future prices are fixed at $\overline{P}$. He accepts an offer if the price makes him indifferent between continuing or stopping, and rejects the offer if the price is higher. In Online Appendix, we show that if one solves the discrete-time game described in Section 2.1, then Buyer's equilibrium value functions must converge to $\underline{V}(x)$ as the game approaches continuous time. As a result, the equilibrium outcome with $V(x) = \underline{V}(x)$ represents the continuous-time limit of the discrete-time equilibrium outcomes. For this reason, the equilibrium outcome with $\underline{V}(x)$ is our outcome of interest.

Lemma 1. *There exists an equilibrium in which $V(x) = \underline{V}(x)$. If $V(x) = \underline{V}(x)$, then there are unique thresholds $\overline{x}$ and $\underline{x}$ such that*

If $x \ge \overline{x}$, then Seller offers $P(x) = x - \underline{V}(x)$, and Buyer accepts.

If $x < \overline{x}$, then Seller offers $P(x) > x - \underline{V}(x)$, and Buyer rejects.

If $x \le \underline{x}$, then Buyer quits.

Even though Seller cannot commit to future prices, she can still enforce Buyer's continuation value to be $\underline{V}(x)$. Consider the following strategy: Seller offers price $P(x) = x - \underline{V}(x)$ if she wants to trade, and offers $P(x) > x - \underline{V}(x)$ if she does not want to trade. If Seller follows this strategy, Buyer never receives more than $\underline{V}(x)$ utility. Buyer's optimal response then is to buy if $P(x) = x - \underline{V}(x)$, which is the price that makes Buyer indifferent between buying now and continuing. If $P(x) > x - \underline{V}(x)$, Buyer rejects because he gets less utility than he gets from continuing, because continuation value must be at least $\underline{V}(x)$. Thus, by following this pricing strategy, Seller can control when they trade. In the proof we show that $\overline{x} < \overline{\overline{x}}$, so that the solution is interior.

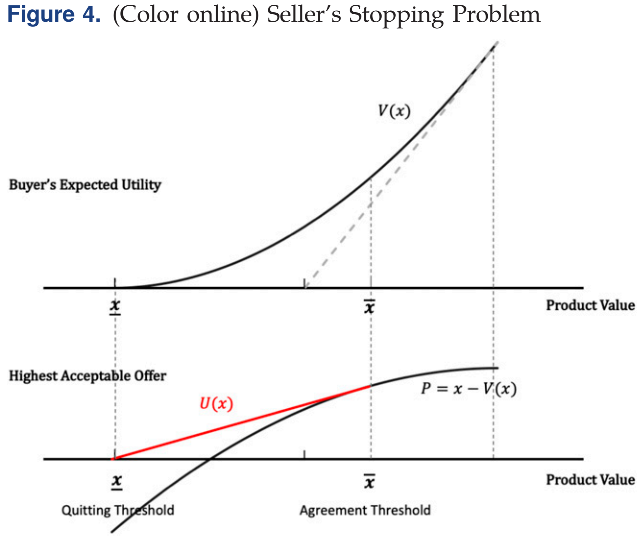
Intuitively, at every moment in time, Seller is choosing between two options. She can continue the sales process, or she can close the sale right now by offering a discount. If Seller chooses to close the sale, she offers a discounted price of $P(x) = x - \underline{V}(x)$. This transforms the game into Seller's optimal stopping problem. Solving Seller's optimal stopping problem also solves the equilibrium, because Buyer's optimal action is to comply with Seller's stopping decision. The questions become when Seller should make the final offer, and what offer Seller should make.[9]

Figure 4 illustrates Seller's problem graphically. Product value x is on the horizontal axis, and shifts stochastically as the games continues. Buyer's expected value from continuing the sales process given product value x, $V(x) = \underline{V}(x)$, is shown on top. The bottom graph shows $P(x) = x - V(x)$, which is the highest offer that Buyer is willing to accept given x. Seller's value function $U(x)$ is the straight line (due to zero flow cost) that hits 0 at $\underline{x}$ when Buyer quits, and hits $P(x) = x - \underline{V}(x)$ when they trade at $\overline{x}$. To maximize $U(x)$, the agreement threshold $\overline{x}$ must make $U(x)$ tangent to $P(x)$. Otherwise, Seller can profitably deviate by making the offer earlier or later.

Note that Buyer's continuation value, $V(x)$, increases in product value x. However, this does not mean that Buyer has stronger incentive to delay purchase when product value increases. Because product value x increases faster than $V(x)$, Buyer is willing to pay a higher price, $P = x - V(x)$, or accept a lower discount, when x is higher.

Proposition 1 gives the closed-form solution of the equilibrium outcome given a list price $\overline{P}$.

Figure 4. (Color online) Seller's Stopping Problem



Proposition 1. *Buyer and Seller trade at* $\overline{x} = \overline{P} + \frac{\sigma^2}{c_b}\left[\sqrt{\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}} - \frac{1}{4}\right]$ *at price* $P(\overline{x}) = \frac{\sigma^2}{c_b}\left[\sqrt{\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}} - 2\left(\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}\right)\right]$, *Buyer quits at* $\underline{x} = \overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b}$, *and players continue for* $\underline{x} < x < \overline{x}$. *The size of the price discount is strictly positive.*

Seller's ex-ante utility is

$$U(x_0|\overline{P}) = \begin{cases} 0 & \text{if } x_0 < \underline{x} \\ \left(\frac{1}{4}\frac{\sigma^2}{c_b} + x_0 - \overline{P}\right)\left(1 - 2\sqrt{\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}}\right) & \\ & \text{if } \underline{x} \le x_0 \le \overline{x} \\ \overline{P} & \text{if } x_0 > \overline{x}. \end{cases} \tag{8}$$

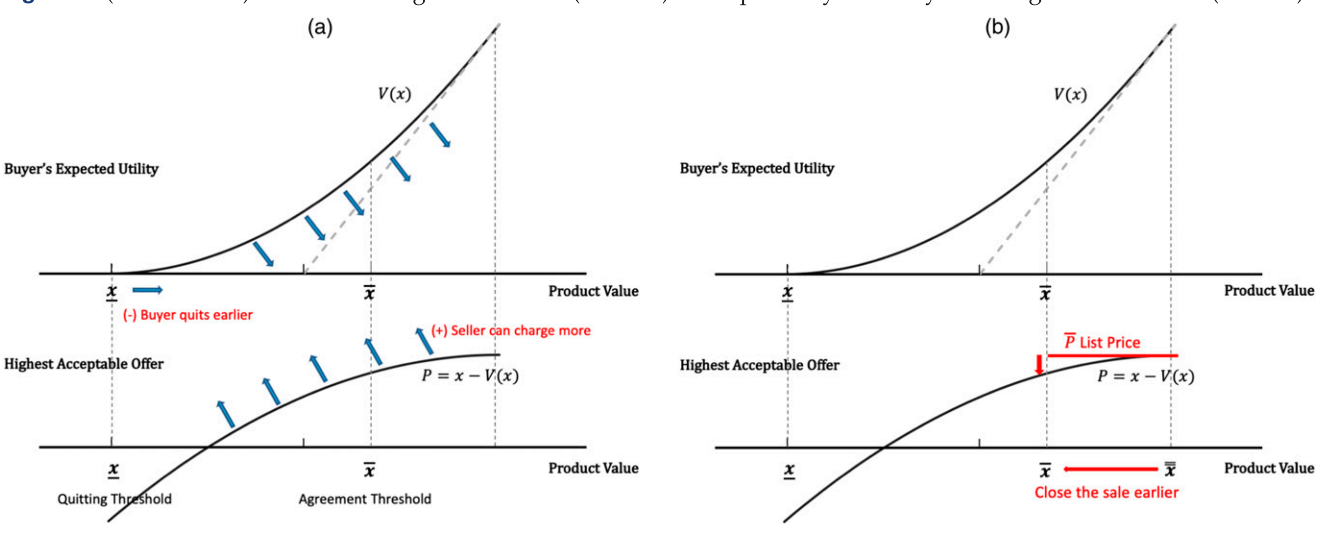
Proposition 1 shows that the list price plays a crucial role in facilitating discovery, and there exists an optimal list price for Seller. A higher list price discourages Buyer from discovering but allows Seller to extract a bigger share of the pie. Figure 5(a) illustrates this point.

As $\overline{P}$ increases, $\underline{x} = \overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b}$ increases, which means that Buyer leaves the sales process earlier when he receives unfavorable information about product match. Note that any list price above $\underline{x} + \frac{1}{4}\frac{\sigma^2}{c_b}$, including a list price of ∞, leads to immediate breakdown of the process. Buyer wants to quit at time 0 because $x_0 < \underline{x}$. Its expected gain from future trade is not enough to justify the cost of staying in the sales process. This is conceptually similar to the hold-up problem in Wernerfelt (1994b). It is costly for Buyer to find out about product match (or quality), and Seller can hold Buyer up by charging a price equal to the product value after Buyer incur the cost. This gives Buyer negative utility ex-ante, and as a result, Buyer chooses to not spend any effort in the first place. Thus, in order to encourage Buyer to participate, Seller has to impose a low enough list price to raise the option value of discovery for Buyer. If the product fit is bad, Buyer does not have to buy the product, but if the product fit is good, Buyer is guaranteed to pay no more than the list price. On the downside, a lower list price restricts Seller's ability to extract rent. Buyer's higher continuation value means that Seller has to offer lower price in order to close the sale, because $P(x) = x - \underline{V}(x)$ decreases as $\overline{P}$ increases. The list price can be viewed as an instrument that balances Seller's two incentives: inducing Buyer to discover and exploiting surplus post discovery.

The second finding is that the final trading price is always lower than the list price, regardless of what the list price is. Thus, Proposition 1 predicts that the sale must come at a discount.[10] Figure 5(b) illustrates why it is never optimal for Seller to sell at the list price. If Seller never offers a discount, Buyer will continue until product value reaches $\overline{\overline{x}} = \overline{P} + \frac{\sigma^2}{4c_b}$. As Buyer gets close to this threshold, Buyer's value function $\underline{V}(x)$ becomes increasingly tangent to $x - \overline{P}$. As a result, $P(x)$ becomes increasingly tangent to $\overline{P}$. So, Buyer is willing to accept offers with discounts that approach 0. Seller can close the deal earlier by sacrificing very little on price. Closing the sale earlier is beneficial because it increases the ex-ante success rate and saves cost (in this case, only Buyer's cost is saved, but Seller can extract Buyer's savings through price). The amount of discount required to close the sale increases at a faster rate as product value decreases, so it is not optimal to close the deal too early. In other words, having some discovery of product match is valuable to Seller, because discovery has the possibility of increasing total surplus by resolving uncertainty.

Proposition 2 provides the optimal list price as a function of the initial value, x_0.[11]

Figure 5. (Color online) Effect of Raising the List Price (Panel A) and Optimality of Always Offering Some Discount (Panel B)



Proposition 2. *(Optimal List Price)*

For intermediate values of x_0 ($-\frac{1}{4}\frac{\sigma^2}{c_b} < x_0 < \frac{1}{16}\frac{\sigma^2}{c_b}$), the optimal list price is $\frac{1}{3}x_0 - \frac{1}{18}\frac{\sigma^2}{c_b}\sqrt{1-12x_0\frac{c_b}{\sigma^2}} + \frac{7}{36}\frac{\sigma^2}{c_b}$, and the game continues beyond $t = 0$.

For $x_0 \geq \frac{1}{16}\frac{\sigma^2}{c_b}$, any list price above $x_0 + \frac{\sigma^2}{4c_b}$ (including no list price) is optimal. Players trade at price $P = x_0$ at $t = 0$.

For lower x_0, any nonnegative list price (including no list price) is optimal. Buyer quits at $t = 0$.

Seller's ex-ante utility at optimal list price is

$$U(x_0) = \begin{cases} x & \text{if } x_0 > \frac{1}{16}\frac{\sigma^2}{c_b} \\ \frac{2}{3}x_0 + \frac{1}{54}\frac{\sigma^2}{c_b} + \left(\frac{1}{54}\frac{\sigma^2}{c_b} - \frac{2}{9}x_0\right)\sqrt{1-12x_0\frac{c_b}{\sigma^2}} & \\ \qquad \text{if } -\frac{1}{4}\frac{\sigma^2}{c_b} \leq x_0 \leq \frac{1}{16}\frac{\sigma^2}{c_b} & \\ 0 & \text{if } x_0 < -\frac{1}{4}\frac{\sigma^2}{c_b}. \end{cases} \tag{9}$$

Proposition 2 shows that the selling process only takes place if the initial surplus is not too big or too small. If the initial surplus is big enough ($x_0 \geq \frac{1}{16}\frac{\sigma^2}{c_b}$), then Seller does not want Buyer to learn anything about the product. Seller publishes a list price high enough to deter Buyer from discovering, and offers a monopoly spot price that extracts all existing surplus. On the other hand, if the initial surplus is too low ($x_0 \leq -\frac{1}{4}\frac{\sigma^2}{c_b}$), matching is socially inefficient. Buyer will leave the game even if the list price is set to 0.[12]

When coming to the market, should Seller commit to offering a fixed price? The following corollary compares the equilibrium outcome to the outcome under an optimal fixed price. In the context of e-commerce, this comparison provides a prediction on what would happen if more retailers can monitor customers' browsing behaviors and make dynamic offers based on their real-time purchase intent. We find that this would improve profit, reduce consumer welfare even though final prices are lower, and reduce the amount of information discovered as retailers use discounts to shorten sales cycle.

Corollary 1 (Comparison with Fixed Price). *Social welfare and Seller's utility are higher with discounting. Buyer's utility is lower with discounting. The optimal list price under discounting is higher than the optimal fixed price, and the final price after discount is lower than the optimal fixed price. Expected length of the game is shorter if discount is allowed.*

Proof. Let T denote the total length of the game. Buyer's ex-ante utility must satisfy $V(x_0) = \frac{x_0 - \underline{x}}{\overline{x} - \underline{x}}(\overline{x} - P(x)) - c_b\mathbb{E}[T]$, where $\frac{x_0 - \underline{x}}{\overline{x} - \underline{x}}$ is the ex-ante probability that players reach agreement. Expected length of the sales process is thus calculated as $\mathbb{E}[T] = \frac{1}{c_b}[\frac{x_0 - \underline{x}}{\overline{x} - \underline{x}}(\overline{x} - P(x)) - V(x_{n,0})]$. The comparisons are straightforward. □

Figure 3 from Section 2 illustrates the value of Seller's ability to offer dynamic discounts. A small discount significantly decreases Buyer's buying threshold from $\overline{P} + \frac{\sigma^2}{4c_b}$ to $\overline{x}$. Seller's ability to offer discount significantly decreases the length of the sales process. The cost savings are captured by Seller through price and increases her ex-ante utility. If players subsequently discover that the product match is bad, then the game ends without a trade. Thus, lowering trading threshold increases the ex-ante success rate.[13]

Corollary 2 (Comparative Statics with respect to c_b and x_0 for $-\frac{1}{4}\frac{\sigma^2}{c_b} < x_0 < \frac{1}{16}\frac{\sigma^2}{c_b}$). *Optimal list price and final price decrease in Buyer's cost c_b; size of price discount decreases in Buyer's cost c_b; ex-ante probability of trade is unaffected by Buyer's cost c_b; expected length of the game decreases in Buyer's cost c_b, and has inverse U-shape in initial value x_0.*

Interestingly, the expected length of the game is nonmonotonic in initial position x_0. Note that the initial position is normalized with respect to outside options and cost of production. Intuitively, one would expect that worse outside options make players more interested in matching with each other. However, as outside options become increasingly poor, surplus from trade gets higher. As a result, Seller is more inclined to close the deal early. Thus, the length of the sales process is shorter for both very good and very bad outside options.

Another finding is that the ex-ante probability of trade is unaffected by Buyer's cost. For a given list price, the probability of trade decreases in c_b. However, a higher cost for Buyer makes the hold-up problem more severe, which then pushes Seller to set a lower list price. In equilibrium, these two effects negate each other. Note that if discount is not allowed, one can show that the ex-ante probability of trade is monotonically decreasing in Buyer's cost, even if Seller set the price optimally.

Furthermore, when Buyer's cost of continuing the sales process increases, the price discount he receives actually decreases. The reason is that with a higher cost for Buyer, Seller has to set a lower list price to begin with. Buyer still receives a lower final price even though the size of the discount is smaller. In the context of e-commerce, this means that as online shopping and information search become more convenient, we may observe higher list prices but with deeper personalized discounts.

4. Costly Selling

Selling can be a costly effort so that Seller may want to quit before Buyer does. Following notation from the last section, let indicator functions $q_b(x)$ and $q_s(x)$ denote Buyer and Seller's quitting decisions and $\underline{x} = \sup\{x | \sum_i q_i(x) > 0\}$ denote the threshold where the "earliest" quitting happens. There are many trivial equilibria in which both players quit simultaneously. If the opponent quits at x, then a player is indifferent between quitting and not quitting at x. As a result, quitting at any state x can be supported in an equilibrium by having both players quit simultaneously.[14] To avoid this triviality, I restrict attention to equilibrium outcomes that satisfy the following condition.

Condition 1. *Either $U(\underline{x}) = U'(\underline{x}) = 0$ or $V(\underline{x}) = V'(\underline{x}) = 0$.*

The condition implies that the quitting decision is optimal for at least one of the players. In a single-agent optimal stopping problem, the quitting threshold must satisfy the value-matching condition, $u_i(\underline{x}) = 0$, and smooth-pasting condition, $u_i'(\underline{x}) = 0$. The value-matching condition ensures that the player does not quit if continuation value is positive, and the smooth-pasting condition ensures that the timing of quitting is optimal (Dixit 1993, pg. 34–37). This condition is satisfied for Buyer's quitting threshold in Section 2. In Online Appendix, I show that if one requires the players to quit if and only if quitting is strictly preferred, then the limit of discrete-time equilibrium outcomes satisfies Condition 1. Thus Condition 1 eliminates the simultaneous quitting triviality.

As in Section 3, I first derive the lower bound on Buyer's equilibrium value function. One can solve for Buyer's value function facing a fixed price of $\overline{P}$, subject to a game-ending state at $\underline{x}$. This gives a lower bound on the Buyer's equilibrium value function $\underline{x}$. Denote this lower bound as $\underline{V}(x|\underline{x})$. The closed-form solution for this lower bound is presented in Appendix A.2. Then one can construct equilibrium strategies that give such payoff to Buyer and satisfy Condition 1. As before, this lower bound payoff is the limit of Buyer's equilibrium payoff in the discrete-time game, so this is the outcome of interest in this paper.

Lemma 2. *There exists a unique equilibrium outcome that has $V(x) = \underline{V}(x|\underline{x})$ and satisfies Condition 1.*

It is unclear which player quits first in equilibrium.[15] Intuitively, the player with a higher flow cost is more likely to quit earlier. However, Proposition 3 shows that this is not the case. If Seller chooses the list price optimally, then Buyer always quits before Seller does.

Proposition 3. *If Seller quits earlier than Buyer does, that is, $\sup\{x|q_s(x) = 1\} > \sup\{x|q_b(x) = 1\}$, then the list price $\overline{P}$ is suboptimal (too low) for the equilibrium outcome in Lemma 2.*

Consider the effect of raising the list price. As Figure 5(a) shows, increasing the list price has both positive and negative effects for Seller. The negative effect is that Buyer's lower continuation value pushes him to quit earlier. However, if Seller is quitting earlier than Buyer does anyway, then pushing Buyer to quit earlier has no effect. Thus, raising the list price is strictly beneficial to the Seller. The original list price must be suboptimal.

Proposition 3 implies that, to find the outcome under optimal list price, one can ignore Seller's quitting decision, then maximizing $U(x_0)$ over $\overline{P}$, and finally verifying that Seller does not want to quit for $x > \underline{x}$. For tractability, I restrict attention to the case of $x_0 = 0$.

Proposition 4 (Costly Selling with $x_0 = 0$). *Let $k = \frac{c_s}{c_b}$. The optimal list price is $\overline{P} = \frac{\sigma^2}{c_b}(\frac{1}{4} - \frac{18+10k-6\sqrt{9+10k+k^2}}{16k^2})$. Buyer and Seller trade at $\overline{x} = \overline{P} + \frac{\sigma^2}{c_b}[\frac{3-\sqrt{\frac{9+k}{1+k}}}{4k} - \frac{1}{4}]$ at price $P(\overline{x}) = \overline{P} - \frac{\sigma^2}{c_b}(\frac{3-\sqrt{\frac{9+k}{1+k}}}{4k} - \frac{1}{2})^2$. Buyer quits at $\underline{x} = \overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b}$.*

The ex-ante utility of Seller in equilibrium is:

$$U(x_0) = \frac{\sigma^2}{c_b}\left[kz^2 + z - 2\sqrt{1+k}z^{3/2}\right]$$
$$\text{where} \quad z = \frac{18 + 10k - 6\sqrt{1+k}\sqrt{9+k}}{16k^2}. \tag{10}$$

If Seller has the option to commit to a fixed price, should Seller do it? Corollary 3 shows that, in some cases, having the ability to discount is necessary for the sales process to take place. Under fixed price, when Seller' cost is high, Seller needs to set a high price to make up for her expected cost of selling. But a higher fixed price pushes Buyer to wait longer before buying, which makes the process even more costly for Seller, who in turn has to charge an even higher price. If Seller charges too high a price, Buyer quits immediately. Thus, there does not exist a fixed price such that both find it worthwhile to "sit down" at time 0. This problem is avoided if Seller can offer discounts. The flexibility in price lowers both player's expected costs from engaging in the sales process. This flexibility is particularly beneficial when Seller's flow cost is high. When Seller's cost is low, Buyer prefers fixed price, but when the ratio $k = \frac{c_s}{c_b}$ exceeds a certain threshold, both Buyer and Seller gain from Seller's ability to discount.

Corollary 3 (Importance of Discounting). *If $k(= \frac{c_s}{c_b}) \geq x_0\frac{4c_b}{\sigma^2} + 1$, then the game ends at $t > 0$ with positive probability of trade if and only if price can be discounted. Seller's ex-ante utility is higher under discounting for all k. Buyer's ex-ante utility is higher under discounting if $k > \underline{k}$, for some $\underline{k} \leq x_0\frac{4c_b}{\sigma^2} + 1$.*

This result helps to explain the prevalence of list price–discount in many B2B industries. Firms have to incur significant cost in employing and training salespersons, and salespersons often spend a significant amount of resources on each client. Corollary 3 shows that, if this cost is high relative to the customer's cost of participating in the sales process, then the firm must be open to discounting. Otherwise, trade cannot take place.

In practice, a firm that wants to offer individual discounts has to delegate pricing authority to its salespersons. The question of whether to delegate has been studied under principal-agent models. See, for example, Lal (1986), Bhardwaj (2001), and Joseph (2001). The principal-agent models used in these papers highlight the disadvantage of delegating pricing authority when selling cost is high. Salespersons have the incentive to give customers too much discount so they can shirk on selling efforts. This problem is intensified when the selling efforts are more costly. On the other hand, the information acquisition approach of this paper emphasizes the advantage of delegating pricing authority when selling cost is high. Using a survey of 270 companies from different industries, Hansen et al. (2008) find that firms that need more calls to close a sale are more likely to delegate pricing authority to salespersons. Corollary 3 provides one explanation for their observation.

Corollary 4 (Comparative Statics w.r.t c_s and c_b at $x_0 = 0$)**.** *Optimal list price decreases in Buyer's cost, c_b, and increases in Seller's cost, c_s; final price decreases in both players' costs, c_b and c_s; size of price discount decreases in Buyer's cost, c_b, and increases in Seller's cost, c_s; and expected length of the game decreases in both player's costs, c_b and c_s.*

A higher c_s leads to a higher $\overline{P}$ but a lower $P(\overline{x})$. When Seller's cost increases, Seller posts a higher list price, but gives a bigger discount quickly and sells at a lower price than before. The lower final price helps to reduce the length of the game and saves cost. Note that, if Seller is not able to give a discount (as under a fixed price), Seller would want to charge a higher price when cost increases. This eventually leads to the no-trade result from Corollary 3.

4.1. Subsidy to Buyer

In the previous analysis, Seller's cost is exogenous. In practice, this cost may be endogenous as sellers often subsidizes buyers' costs by providing certain perks such as meals, resort conferences, gifts, and coupons. These activities are intended to encourage buyers to participate and stay.

Assume Seller's flow cost without subsidy to be 0, and Buyer's cost without subsidy to be c. For simplicity, let the initial value $x_0 = 0$. At time 0, Seller promises a "per-period" subsidy, g, to be paid to Buyer. This transfer of utility is not free. I assume that Buyer only receives θg, where $\theta \le 1$. This is a parsimonious way to capture the idea that the use of subsidy involves efficiency loss. Typically, sellers cannot directly pay buyers monetarily. Instead, sellers provide perks to buyers through hosted events or free meals. These activities require administrative costs, and buyers may not value these perks at their monetary face value. These losses make up $1-\theta$. For a period of dt, Seller incurs cost $g\,dt$ while Buyer incurs cost $(c-\theta g)dt$.

No shirking is allowed. Seller cannot change per-period subsidy after time 0, and Buyer cannot receive subsidy g but avoid paying cost c. Note that if shirking is allowed, both players have incentives to shirk. A more realistic model may consider letting Seller promise future subsidy with a finite duration. Such model is beyond the scope of this paper.

As before, $k = \frac{g}{c-\theta g}$ denotes the ratio between Seller's cost and Buyer's cost. One can solve the equilibrium with subsidy by calculating Seller's ex-ante utility for a given subsidy g, then maximizes with respect to g, to find the optimal size of subsidy. From Proposition 4, Seller's ex-ante utility in equilibrium must be:

$$U_0 = \frac{\sigma^2}{c}(1+\theta k)\left[kz^2 + z - 2\sqrt{1+k}z^{3/2}\right] - F(g), \quad (11)$$

$$\text{where } z = \frac{1}{4} - \overline{P}\frac{c-\theta g}{\sigma^2} = \frac{18+10k-6\sqrt{1+k}\sqrt{9+k}}{16k^2}. \quad (12)$$

The first order derivative with respect to k gives:

$$\frac{dU_0}{dk} = \left(\overbrace{\frac{\partial U_0}{\partial z}}^{0}\frac{dz}{dk} + \frac{\partial U_0}{\partial k}\right) \quad (13)$$

$$= \frac{\sigma^2}{c}z^{3/2}\left[\sqrt{z} - \frac{\theta-1}{\theta}\frac{1}{\sqrt{1+k}}\right] \quad (14)$$

$$= \frac{\sigma^2}{c}z^{3/2}\left[\frac{3\sqrt{1+k}-\sqrt{9+k}}{4k} - \frac{\theta-1}{\theta}\frac{1}{\sqrt{1+k}}\right]. \quad (15)$$

All interior solutions to $\frac{dU_0}{dk} = 0$ have $\frac{d^2U_0}{dk^2} > 0$. As a result, there is no intermediate size of subsidy that maximizes Seller's utility. In equilibrium, Seller either chooses to not provide subsidy at all, or subsidizes away all of Buyer's cost.

What happens if Seller chooses to subsidize all of Buyer's cost? Buyer will always have a utility of 0 and be indifferent between quitting and not quitting. If we assume that Buyer does not quit, then Seller acts like a social planner as it incurs all cost and receives all welfare. Solving Seller's stopping problem with a cost of c/θ and a stopping value of x gives Seller an ex-ante utility of $U_0 = \theta\frac{\sigma^2}{16c}$. Seller does not need a list price here, as a full subsidy eliminates the hold-up problem.

Compare this to Seller's ex-ante utility without subsidy from Proposition 2, $\frac{1}{27}\frac{\sigma^2}{c}$, one can see that Seller prefers to subsidize Buyer's cost when $\theta > \frac{16}{27}$. Thus, if the efficiency loss from subsidy is high, Seller prefers to not subsidize and use a list price to deter Buyer from quitting. If the efficiency loss is low, Seller prefers to subsidize away Buyer's cost, rather than using a list price.

Note that in this example, there's no interior solution for the optimal subsidy because the cost of subsidizing is linear. If there is increasing cost of subsidizing, or if there's a fixed cost that is increasing in the size of subsidy, Seller may only subsidize part of Buyer's cost. However, such equilibrium would give the same qualitative findings as having exogenous costs. Subsidy lessens Buyer's concern of hold-up but does not eliminate it. Given the optimal subsidy, the equilibrium list price and discount must still follow Proposition 4. The list price, though never executed, provides a lower bound on Buyer's utility and encourages discovery. Given a list price, Seller always prefers the ability to discount rather than committing to a fixed price. Corollary 3 also has a new interpretation in such a case. Seller is restricted to providing low subsidy if she commits to a fixed price. The flexibility to offer discount allows Seller to offer higher subsidy.

5. Asymmetric Information

5.1. Heterogeneous Buyers and (Reverse) Price Discrimination

The previous Sections assume that Buyer's expected value for the product, x_t, is fully observable to Seller. In many settings, though, the buyer may have private information regarding his willingness-to-pay. Also, even if the seller observes the buyer's preference for each attribute, the seller may not know what the buyer's outside option is. This section expands the model to incorporate this scenario.

There are two types of Buyer, H and L, with different outside options. This is Buyer's private information. L type has a **better** outside option than H type. A better outside option decreases Buyer's maximum willingness-to-play. If we normalize Buyer's outside option into his product value, then at each moment, Seller is facing a distribution of Buyers with different levels of net product values. Seller observes how this distribution shifts over time, but does not observe where Buyer falls within this distribution.

Nature draws H type with probability λ and L type with probability $1-\lambda$. Define ϵ as the different in outside options between the two types. Let x_H and x_L denote the H type and L type's product values minus their respective outside options. Denote $x = \frac{1}{2}(x_H + x_L)$ so that $x_H = x + \frac{\epsilon}{2}$ and $x_L = x - \frac{\epsilon}{2}$. Both H type and L type incur flow cost $c_b > 0$. Seller is assumed to have no cost and never quits.

I look for stationary PBE with pure strategies. Equilibrium utilities and strategies now depend on two state variables, x and μ, where $\mu \in [0,1]$ denote Seller's belief that Buyer is of type H. Type i's value function is $V_i(x,\mu)$. Note that with pure strategies, Seller's belief μ can only take 3 values: 0, 1, or λ. If $\mu = 0$ or $\mu = 1$, then there is no private information. This happens on the path of a separating equilibrium. I assume that Seller does not update her belief off the equilibrium path.

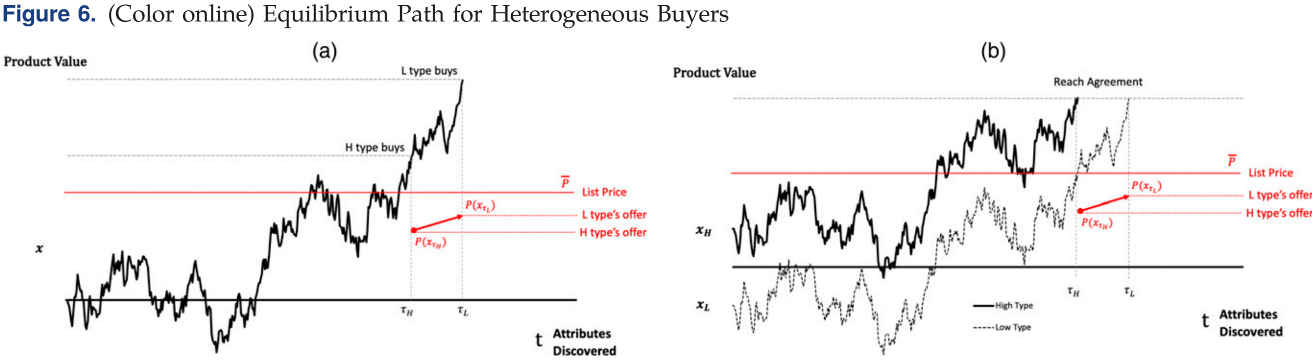
Similar to earlier sections, I first look for the lower bound on Buyer's equilibrium utility. Let $\underline{V}_i(x,\mu)$ denote the lower bound on type i's equilibrium value function in state x and under belief μ. If only one type of Buyer remains in the game ($\mu = 0$ or $\mu = 1$), then the lower bound must be $\underline{V}(x)$ from Section 3. One can show that L type's lower bound is $\underline{V}(x)$ even when both types are still in the game. This then pins down H type's lower bound. The proof and closed-form solutions for $\underline{V}_i(x,\mu)$ are presented in Appendix A.3.

Seller then faces the new optimal stopping problem regarding when to sell to H type and at what price. Proposition 5 describes the equilibrium outcome in which each type of Buyer receives the lower bound of his equilibrium value functions. We only need to solve for the case of $\overline{P} < x_0 - \frac{\epsilon}{2} + \frac{\sigma^2}{4c_b}$, otherwise types unravel immediately.[16]

Proposition 5 (For $\overline{P} < x_0 - \frac{\epsilon}{2} + \frac{\sigma^2}{4c_b}$)**.** *Buyer with type $i \in \{H,L\}$ quits at $\underline{x}_i = \overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b}$, and buys at $\overline{x}_i = \overline{P} + \frac{\sigma^2}{c_b}\left[\sqrt{\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}} - \frac{1}{4}\right]$. H type buys earlier and pays less than L type.*

Figure 6(a) presents a sample equilibrium path. H type buys earlier, and pays a lower price. Price increases from time τ_H to τ_L. Figure 6(b) shows x_H and x_L separately. Both types of Buyers buy when their net product value reach $\overline{x}$. H type reaches the threshold

Figure 6. (Color online) Equilibrium Path for Heterogeneous Buyers



earlier, because his worse outside option translates to a higher net value from buying the product.

The finding that H type pays less than L type runs counter to previous repeated-offers models, in which the seller makes declining offers to screen through buyers' reservation prices, so types that buy earlier pay higher prices. Why does L type not take H type's offer, if waiting is costly and price is rising? The reason is that, when Seller makes an offer to H type, L type's net product value is lower by ϵ, and this difference makes L type strictly prefer to wait and collect more information. On the flip side, Seller has to offer H type a lower price because H type can pretend to be L type. H type can wait until x_L reaches the buying threshold, and then pay the same price as L type. However, Seller prefers to close the sale with H type earlier, which saves cost as well as locking in the surplus. Intuitively, if a Buyer already comes into the sales process with a good intention to buy, then discovery is less valuable, and closing the sale earlier is more efficient. This creates a binding incentive compatibility constraint that forces the seller to concede more utility to H type. Thus, in this game, Buyers self-sort into processes of different lengths: a shorter process for H type and a longer process for L type. The price cut for H type represents H type's information rent.[17]

5.2. Heterogeneous Seller and List Price Signaling

If sellers have private information about the quality of their products or services, they can publish different list prices as a way of signaling. In this extension, I show that there exists separating equilibria in which the high quality firm sets a higher-than-optimal list price to reveal its type.

There are two types of sellers, H and L, who are different in their qualities and in theirs costs of delivering the product. I assume that quality is a vertical component of product utility that is additive to the match value x_t. Conditional on the same level of match, purchasing from H type Seller delivers q more utils than purchasing from L type. In the model, this translates into different starting positions. Let $x_0 = 0$ for L type, then $x_0 = q$ for H type. Consumers observe the incremental changes of x_t, but has to form belief about the starting position from the list price. L type's cost of delivering the product is normalized to 0, and H type's cost of delivering the product is $K > 0$. I look for separating PBE with stationary strategies. If L type Seller mimics H type's list price, it may still have incentives to deviate from H type's subsequent discounting strategy. Thus, on off-path nodes, I assume that Buyer infers that Seller is L type. That is, if L type mimics H type's list price but engages in different discounting behavior afterward, Buyer believes that the Seller is actually a L type upon seeing such deviation.

Let $\overline{P}_H$ and $\overline{P}_L$ denote H type and L type's list prices, respectively. Also, let $\overline{P}_H^*$ and $\overline{P}_L^*$ denote H type and L type's optimal list prices if quality is public information. In a separating equilibrium, Buyer correctly infers Seller's type at $t = 0$ upon observing the list price. That means L type must set its list price at $\overline{P}_L = \overline{P}_L^* = \frac{5}{36}\frac{\sigma^2}{c_b}$ from Proposition 2.

From Section 3, we know that Seller's ex-ante utility for a given list price, with the cost of product normalized to 0 is:

$$U(x_0|\overline{P}) = \left(\frac{1}{4}\frac{\sigma^2}{c_b} + x_0 - \overline{P}\right)\left(1 - 2\sqrt{\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}}\right). \tag{16}$$

Then L type's equilibrium utility must be $U(0|\frac{5}{36}\frac{\sigma^2}{c_b}) = \frac{1}{27}\frac{\sigma^2}{c_b}$. H type's equilibrium utility can be written as $U(q - K|\overline{P}_H - K)$. If L pretends to be H type by using H's list price, then L has to subsequently follow through with H's offer strategy, otherwise Buyer infers that Seller is actually L type. But comparing to H type, L type saves the cost of product K when a sale is made. Thus, L's deviating utility is $U(q - K|\overline{P}_H - K) + K * \Pr(q - K|\overline{P}_H - K)$, where $\Pr(q - K|\overline{P}_H - K)$ is the ex-ante probability of a sale. If H pretends to be L by using L's list price, H does not have to follow through with L's offering strategy, because Buyer already believes Seller to be L type. H's optimal offering strategy given Buyer's belief gives a deviating utility of $U(-K|\overline{P}_L^* - K)$. Combining these, we get the following incentive-compatibility constraints:

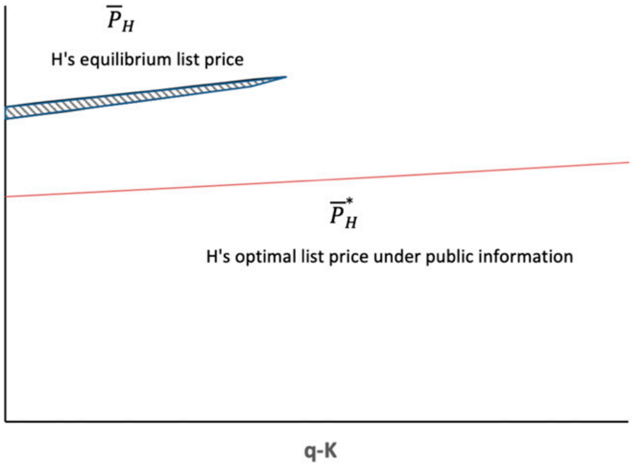
$$(IC_L) \qquad \frac{1}{27}\frac{\sigma^2}{c_b} \geq U(q - K|\overline{P}_H - K) + K * \Pr(q - K|\overline{P}_H - K)$$

$$(IC_H) \qquad U(q - K|\overline{P}_H - K) \geq U\left(-K|\overline{P}_L^* - K\right).$$

H type's list price $\overline{P}_H$ is the only unknown in these equations. Any $\overline{P}_H$ that satisfies these two conditions constitute a separating equilibrium. There is no analytical solution to $\overline{P}_H$, but given any q and K, one can solve for $\overline{P}_H$ numerically. Figure 7 shows the range of $\overline{P}_H$ and H type optimal list price $\overline{P}_H^*$ under public information as functions of $q - K$.

There are a few observations to make. First, H type's list price is higher than its list price under public information. A higher list price decreases the ex-ante probability of sale, which hurts L type more than H type because L type's cost advantage. So, a higher-than-optimal list price is used by H type to deter L type from imitating. Second, separating equilibria exist if $q - K$ is not too high. If H type's quality advantage is much higher than its cost, then H cannot stop L from imitating. Third, the IC condition for L type implies that L type must earn higher profit than H type in a separating equilibrium, because L type can

Figure 7. (Color online) H Type's List Price in Separating Equilibria with $K = \frac{1}{20}\frac{\sigma^2}{c_b}$



set its list price optimally, whereas H type must set a higher-than-optimal list price to deter imitation.

Other papers, including Yavas and Yang (1995), Shin (2005), Haurin et al. (2010), and Xu and Dukes (2020), have discussed how list price can signal the seller's private information. The current model is different from previous papers in several way. In Xu and Dukes (2020), some consumers buy at the list price, whereas all trades happen below list price in the current model. Both Yavas and Yang (1995) and Haurin et al. (2010) directly assume that a higher list price leads to less matched customers. The current model provides a micro foundation for their assumption. In Shin (2005), actual price cannot be changed over time, as is studied here.

5.3. Noisy Learning by Seller

In the baseline model, the discovery process is symmetric. Buyer observes its match with Seller's attributes, and Seller learns Buyer's preferences on attributes perfectly. However, in reality, one can imagine the discovery process to be asymmetric. Seller may not know exactly how Buyer feels about each attribute, but only get a noisy signal of Buyer's preference.

Fully allowing asymmetric learning in the baseline model with continuous time and continuous state space is intractable. Instead, to capture the idea, I study the discrete model motivated in Section 2.1 with three periods (0, 1, and 2). The product is consisted of two attributes. Depending on the fit, each attribute delivers v_l or v_h utility to Buyer, and the value of the product is the sum of the two attributes. As in the baseline model, the ex-ante probability of match for each attribute is $\frac{1}{2}$ and independent across attributes. Starting in $t = 0$, Buyer incurs a small cost of c for moving to the next period, and learns the value of one attribute in $t = 1$ and the other in $t = 2$. When Buyer learns, Seller receives a signal on the match with an accuracy of θ. That is, if Buyer learns that attribute 1 is a good match, Seller observes that the match is good with probability θ and observes that the match is bad with probability $1 - \theta$.

Let $\Delta v = v_h - v_l$. Outside options and production cost are normalized to 0. Let v_0 denote the expected value of the product at $t = 0$. To make learning socially relevant, we assume $v_0 - \Delta v < 0$. This means that a product with two mismatched attributes has less value than its cost (production cost plus the opportunity cost of outside options).

One can solve SPE of this game using backward induction. For θ not too high, the equilibrium outcome is summarized as follows:

Proposition 6 (For $\theta \le \frac{\Delta v - 6c}{\frac{3}{2}\Delta v - 8c}$).

For intermediate v_0 $(6c - \Delta v < v_0 < \frac{1}{2}\Delta v - 2c)$, *Seller sets a list price of* $v_0 + \Delta v - 6c$. *At* $t = 1$, *Seller offers a discounted price of* $v_0 + \frac{1}{2}\Delta v - 2c$. *Buyer buys at* $t = 1$ *if the first attribute is a match, and leaves at* $t = 1$ *if the first attribute is a mismatch.*

For lower v_0, *Buyer leaves at* $t = 0$.

For higher v_0, *Seller does not set a list price and charges* $P_0 = v_0$ *at* $t = 0$. *Buyer buys immediately.*

Proposition 6 closely resembles Propositions 1 and 2. The list price–discount pattern replicates. If the ex-ante value of the product is too high or too low, games end at time 0. But for an intermediate range, the Seller sets a list price to encourage Buyer to learn, but always close the sale with a discount before all information is revealed.

One may be interested in a model with two-sided noise. However, such models are generally difficult to solve due to the explosion of states and possibility of signaling. Past models of repeated-offers with stochastic value, such as Daley and Green (2020), focus on one-sided learning in which only one player updates his belief about the value. I argue that it is important to understand what happens if both parties learn. The baseline model explores the special case in which the arrival of information is symmetric. This three-period extension shows that the findings can replicate even when Seller does not observe Buyer's preferences perfectly.

6. Other Extensions

This section discuss key results from other extensions of the model. Details can be found in Online Appendix.

6.1. Nonretractable Offers

In the baseline model, an offer expires instantly if Buyer does not accept it. Seller makes no promise regarding future discounts and can actually raise prices over time. However, one may imagine circumstances in which social norms prevent Seller from

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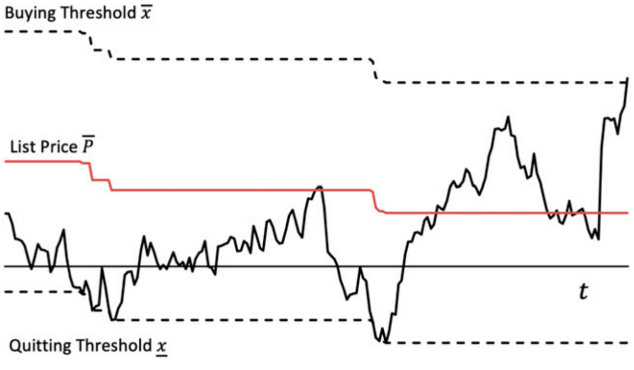
doing so. If Buyer can recall all previous price offers, then each new lowest offer becomes the new list price, so the standing list price can only decrease over time. We can show that Seller prefers to maintain the highest price possible, so the only time that Seller decreases list price is when Buyer is about to leave. Figure 8 gives an example of the equilibrium path. Furthermore, there exists a threshold, $\tilde{x}$, such that for $x_0 < \tilde{x}$, Seller prefers to use nonretractable offers, and for $x_0 > \tilde{x}$, Seller prefers the offers to be retractable. The intuition is that, using nonretractable offers allow Seller to lower list price over time to prevent Buyer from quitting. This is beneficial when ex-ante product value is low. The disadvantage of nonretractable offers is that it takes away Seller's ability to close sale with discount. Any new discount lowers the list price, which increases Buyer's continuation value. This limits Seller's ability to extract utility when product value is high.

6.2. Finite Mass of Attributes

The baseline model assumes an infinite mass of attributes. In reality, a product may have only a limited number of features, or consumers may care about only a finite subset of a product's features. As a result, players exhaust all the information they can learn from the other party if the sales process continues long enough. Suppose that the product has a finite mass of attributes, T. The product value x_t follows a Brownian motion between time 0 and time T. One can solve the equilibrium numerically. Figure 10 presents the solution for the case of $T = 1$, $c_b = 0.25$, $\sigma^2 = 1$, and $x_0 = 0$. We recover the list price–discount pattern. List price cannot be too high, otherwise Buyer quits at time 0. Also, players always reach agreement at a discount. The difference between finite horizon and infinite horizon is that all three thresholds shrink over time. This happens because as t approaches T, the remaining game is just a smaller version of the game started at $t = 0$. Also note that the thresholds collapse before T, so players do not wait until they discover all attributes.

Figure 8. (Color online) Nonretractable Offers



6.3. Bayesian Learning

Instead of matching on attribute, players observe signals on the true value of the product and update their beliefs using Bayes' rule. Suppose that, before the negotiation begins, Buyer and Seller have a common prior on the true value x^* that follows a normal distribution with mean ν and variance e^2. Let S_t denote the cumulative signal up to time t. The signal is assumed to follow the process $dS_t = x^* dt + \eta dW$. Bayesian updating on the normal prior implies that the expected value of the product after observing t signals, x_t, can be written as

$$x_t = \frac{\nu/e^2 + S_t/\eta^2}{1/e^2 + t/\eta^2}. \tag{17}$$

Figure 9 illustrates the numerical solution for the case of $c_b = 0.25$, $\nu = 0$, $e^2 = 1$, $\eta^2 = 1$, at a list price of 0.5.

Similar to the previous extension to finite attributes, we replicate the list price–discount pattern, and the game must end before Buyer's quitting threshold and agreement threshold collapse at 0.

Note that if Buyer has evolving preferences so that the true value of x_t^* evolves with a random walk of variance ρ^2, then the posterior variance follows $\frac{d\sigma_t}{dt} = -\sigma_t^2/\eta^2 + \rho^2$. If $\sigma_0 = \rho\eta$, then $\sigma_t = \sigma_0$ remains constant for all t. Thus, the baseline model can be interpreted as a Bayesian learning model with an evolving preference.

6.4. Buyer Offers and Price Floor

If Buyer, instead of Seller, makes all the offers, list price becomes useless. In a subgame-perfect equilibrium, Buyer never offers any price above 0. A price ceiling in the form of list price cannot prevent Seller from being held-up, and if Seller has any cost, Seller would quit immediately. Instead, Seller may benefit from a price floor that bound offers from below.

Figure 9. (Color online) Finite Mass of Attributes

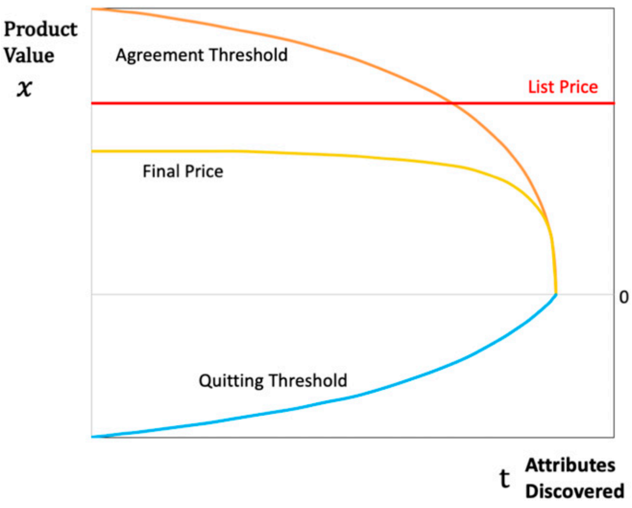
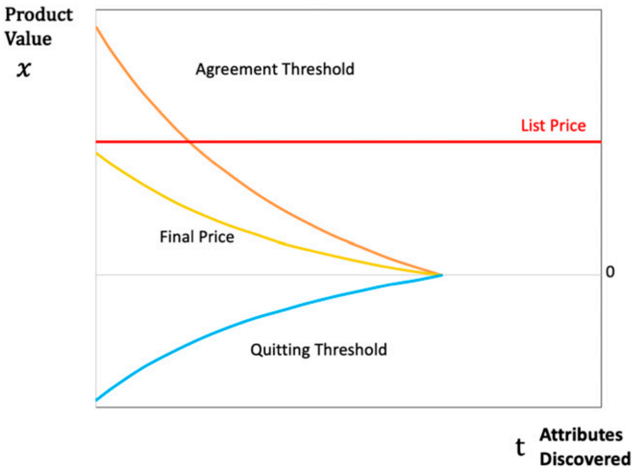


Figure 10. (Color online) Bayesian Learning



The model is modified as follows: (1) at time 0, Seller chooses a price floor, $\underline{P}$, such that all future offers below $\underline{P}$ are automatically rejected; (2) Buyer makes offers, and Seller decides whether to accept or reject. The optimal lower bound is $\underline{P} = \frac{\sigma^2}{8c_b}$. Buyer offers $\underline{P}$ when x_t reaches $\underline{P} + \frac{\sigma^2}{4c_b}$ and quits when x_t reaches $\underline{P} - \frac{\sigma^2}{4c_b}$.

One can compare the equilibrium outcome with Buyer offers to the equilibrium outcome with Seller offers. Final price is actually higher when Buyer makes offers. The sales process is expected to be longer when Buyer makes offers. Buyer's utility increases and Seller's utility decreases when Buyer makes offers. However, social welfare is lower when Buyer makes offers.

7. Conclusion

This paper studies the pricing mechanism where a seller sets a list price upfront, then decides what discounts to offer during interaction with a buyer. Such mechanism is common in B2B sales but also begins to emerge in AI-powered e-commerce. A given pair of buyer and seller do not know their value from trading with each other ex-ante due to heterogeneity, but they can learn about their match over time. The seller chooses a list price ex-ante that acts as a price ceiling but can adjust discounts dynamically as the two parties learn. The matching process causes the expected total surplus to be stochastic. The process is costly so that the parties have incentives to quit.

The model shows that the combination of stochastic surplus and endogenous quitting creates a hold-up problem for the buyer. A list price that puts an upper bound on seller's future offers serves as an instrument that reduces the buyer's concern for being held-up after spending efforts to match. The optimal choice of list price has to be low enough to encourage the buyer to stay, without being too restrictive on the seller's ability to extract surplus later. Not setting a list price leads to an immediate breakdown of trade as the buyer becomes unwilling to engage in the process. On the other hand, the model shows that trade always happen strictly below the list price. The seller always prefers to shorten the sales cycle by offering discounts. This result provides a rational explanation for the list price–discount pattern observed in practice, especially in industries where buyers rarely pay the list prices.

Sellers always prefer this mechanism to committing to a fixed price. When the seller's cost of matching is high relative to the buyer's, having the ability to discount is necessary for both parties to participate in the sales process, which explains the prevalence of price negotiation in B2B sales. On the other hand, under this mechanism, comparing with fixed price, buyers face higher list prices, receive bigger discounts that lead to net price drops, spend less time discovering product information, and receive lower utility. We may observe these changes in e-commerce if more retailers use AI to track buyers' real-time purchase intentions.

List price and discounts play additional roles when firms have private information. If sellers have private information in their quality, a high type seller can set a higher-than-optimal list price as a way to signal, even though the list price is never used. When buyers have private information on their outside options, the paper finds a case of reverse price discrimination, where the seller screens different types of buyer by reducing discounts over time.

The model highlights how a nonbinding price ceiling can facilitate information acquisition and trade. The analyses provide implications on selling and pricing strategies in a stochastic environment. However, the paper has several important limitations. It ignores important aspects of sales, such as manager-salespersons relationship. The interaction between agency and hold-up can bring additional insights. The model also does not fully explore the effects of asymmetric learning and alternating offers due to tractability concerns. They remain as important topics for future research.

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Appendix A. Proofs

A.1. Section 3

Proof of Lemma 1. First we assume the existence of an equilibrium with $V(x) = \underline{V}(x)$, and show that the equilibrium outcome must have the stated properties.

Suppose $V(x) = \underline{V}(x)$. Buyer's quitting strategies must be the same as if price is fixed at $\overline{P}$. So, Buyer quits for all $x < \underline{x} = \underline{x}$. Given the continuation value $\underline{V}(x)$, Buyer buys at x if and only if $P(x) \le x - \underline{V}(x)$. So, the maximum price that Seller can extract is $x - \underline{V}(x)$. This is Seller's optimal stopping problem. Seller can stop and receive a payoff of $x - \underline{V}(x)$, or continue. If Seller wants to continue, she must charge $P(x) > x - \underline{V}(x)$. The solution to Seller's stopping problem is a threshold $\bar{x}$, such that Seller stops for all $x \ge \bar{x}$.

Next we prove that, in equilibrium, Buyer buys at all $x \ge \bar{x}$, so that $A = [\bar{x}, \infty]$. Suppose there exists $\tilde{x} > \bar{x}$ such that $a(x, P(x)) = 0$ for an open neighborhood around $\tilde{x}$. Let $x_{left} = \sup\{x | x \in A \ \&\ x < \tilde{x}\}$, and $x_{right} = \inf\{x | x \in A \ \& \ x > \tilde{x}\}$. Because there is no trade between x_{left} and x_{right}, $U(\tilde{x}_n)$ is on a strictly convex function connecting $U(x_{left})$ and $U(x_{right})$. But Buyer is willing to accept $P(x) = x - \underline{V}(x)$, which is a weakly concave function connecting $U(x_{left})$ and $U(x_{right})$, so we must have $U(\tilde{x}) < \tilde{x} - \underline{V}(\tilde{x}, P(\tilde{x}))$. Thus, Seller should stop at $\tilde{x}$. If there is no open neighborhood around $\tilde{x}$, then when $x_t = \tilde{x}$, the games stops at $\tau = 0$ and pays utility of 0 by definition of utilities in Section 2. Thus, Seller should stop at $\tilde{x}$, a contradiction.

Finally, we confirm that the outcome above is indeed an equilibrium. Buyer's continuation value is $\underline{V}(x)$ for all x. Given the continuation value, buying at x if and only if $P(x) \le x - \underline{V}(x)$ is optimal. Seller does not have a profitable deviation either, because A is the solution to the optimal stopping problem with stopping payoff of $x - \underline{V}(x)$.

Note that by Equation (7) from Section 3, we know that $\underline{V}(x)$ is a convex C^1 function. Thus, Buyer's quitting threshold $\underline{x}$ is internal and satisfies smooth-pasting condition. Seller' stopping value $x - \underline{V}(x)$ is then a concave C^1 function, which equals to a constant, $\overline{P}$, for all $x \ge \bar{\bar{x}}$. This insures that Seller's optimal stopping threshold $\bar{x}$ must be strictly lower than $\bar{\bar{x}}$. Thus, both equilibrium thresholds must be internal solutions and satisfy smooth-pasting conditions.

Proof of Proposition 1 and 2. Take any $\overline{P} \le x_0 + \frac{1}{4}\frac{\sigma^2}{c_b}$. Given Lemma 1, $V(x) = \underline{V}(x)$ from Equation (7).

The trading threshold $\bar{x}$ must solve Seller's optimal stopping problem with stopping value $x - \underline{V}(x, \overline{P})$. By Taylor expansion and Ito's Lemma, Seller's value function satisfies $rU(x) = -c_s + \frac{\sigma^2}{2}U''(x)$. Having $r = 0$ and $c_s = 0$ imply a linear value function for the Seller $U(x) = \alpha_s(x - \overline{P}) + \beta_s$. We have three boundary conditions. The first two conditions match values at quitting and trading points. The third condition is a first-order smooth-pasting condition ensuring the stopping time is optimal (Dixit 1993).

$$\begin{cases} \alpha_s\left(-\frac{1}{4}\frac{\sigma^2}{c_b}\right) + \beta_s & = 0 \\ \alpha_s(\bar{x} - \overline{P}) + \beta_s & = \bar{x} - \frac{c_b}{\sigma^2}\left(\bar{x} - \overline{P}\right)^2 - \frac{1}{2}\left(\bar{x} - \overline{P}\right) - \frac{\sigma^2}{16c_b} \\ \alpha_s & = 1 - \frac{2c_b}{\sigma^2}\left(\bar{x} - \overline{P}\right) + 1/2 \end{cases} \quad (18)$$

Solving the system of equations, we get the trading threshold

$$\bar{x} = \overline{P} + \frac{\sigma^2}{c_b}\left[\sqrt{\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}} - \frac{1}{4}\right]$$

and trading price

$$P(\bar{x}) = \bar{x} - \underline{V}(\bar{x}, \overline{P}) = \frac{\sigma^2}{c_b}\left[\sqrt{\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}} - 2\left(\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}\right)\right].$$

The optimal list price is found by

$$\begin{aligned} \arg\max_{\overline{P}} U(x_0) &= \arg\max_{\overline{P}} \alpha_b(x_0 - \overline{P}) + \beta_b \\ &= \arg\max_{\overline{P}} \left(\frac{1}{4}\frac{\sigma^2}{c_b} - \overline{P}\right)\left(1 - 2\sqrt{\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}}\right). \end{aligned}$$

If $\overline{P} > \frac{1}{4}\frac{\sigma^2}{c_b}$, then $\underline{x} = \overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b} > 0$. At $\underline{x}$, Seller would offer $P(\underline{x}) = \underline{x}$ and Buyer buys. The only difference now is that Seller receives utility of $\underline{x}$ instead of 0 at Buyer's quitting threshold. We can re-solve the system of equations by swapping out the first condition of Equation (18) with

$$\alpha_b\left(-\frac{1}{4}\frac{\sigma^2}{c_b}\right) + \beta_b = \overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b}, \quad (19)$$

and we get $\bar{x} = \underline{x} = \overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b}$. Thus, players trade immediately for $x \ge \underline{x}$. Furthermore, for $0 < x \le \underline{x}$, Seller would offer $P(x) = x$ and Buyer accepts, otherwise Buyer would quit. For $x \le 0$, Buyer quits immediately. Thus, the game must end immediately for all $x \in \mathbb{R}$.

A.2. Section 4

Lower Bound on Buyer's Value Function We calculate the lower bound on Buyer's value function for a given list price $\overline{P}$ and a given quitting threshold $\underline{x} = \sup\{x | \sum_i q_i(x) > 0\}$.

Treating $\underline{x}$ as an exogenous stopping point, let $\underline{V}(x, \underline{x})$ denote the lower bound. By Ito's Lemma, $\underline{V}(x, \underline{x}) = -c_b + \frac{\sigma^2}{2}\frac{d}{dx^2}\underline{V}(x, \underline{x}|\overline{P})$, which implies $\underline{V}(x, \underline{x}|\overline{P}) = \frac{c_b}{\sigma^2}(x - \overline{P})^2 + \alpha(x - \overline{P}) + \beta$ for some α and β. Let $\bar{\bar{x}}$ denote the point at which Buyer chooses to buy if price is fixed at $\overline{P}$. Then we have the following value-matching and smooth-pasting conditions:

$$\begin{cases} \frac{c_b}{\sigma^2}\left(\bar{\bar{x}} - \overline{P}\right)^2 + \alpha\left(\bar{\bar{x}} - \overline{P}\right) + \beta & = \bar{\bar{x}} - \overline{P} \\ \frac{2c_b}{\sigma^2}\left(\bar{\bar{x}} - \overline{P}\right) + \alpha & = 1 \\ \frac{c_b}{\sigma^2}\left(\underline{x} - \overline{P}\right)^2 + \alpha\left(\underline{x} - \overline{P}\right) + \beta & = 0. \end{cases} \quad (20)$$

Solving these three conditions and combining with the fact that $\underline{V}(x) = 0 \quad \forall x \le \underline{x}$, we get:

$$\underline{V}(x, \underline{x}) = \begin{cases} 0, & x \le \underline{x} \\ \frac{c_b}{\sigma^2}\left(x + \eta - \overline{P}\right)^2 + \frac{1}{2}\left(x + \eta - \overline{P}\right) + \frac{\sigma^2}{16c_b}, & x \in \left(\underline{x}, \bar{\bar{x}}\right) \\ x - \overline{P}, & x \ge \bar{\bar{x}}, \end{cases} \quad (21)$$

where $\eta = \frac{1}{4}\frac{\sigma^2}{c_b} - (\underline{x} - \overline{P}) - \sqrt{-\frac{\sigma^2}{c_b}(\underline{x} - \overline{P})}$ and $\bar{\bar{x}} = \overline{P} + 1 + \sqrt{1 - (\underline{x} - \overline{P})^2 - \frac{\sigma^2}{c_b}(\underline{x} - \overline{P})}$.

Note that the Costless Seller lower bound in Equation (7) is a special case of this, where $\eta = 0$ and his value function does not depend on Seller's cost. Buyer's quitting threshold $\underline{x}_b$ is smaller than $\overline{P}$, so $V(x,\underline{x}) = 0$ for $x \le \underline{x}$. If Seller quits first, then η is positive and Buyer's value function is shifted down and left from a positive η.

Proof of Lemma 2. First, we claim that for any x, there must exist $x' < x$ such that some player quits at x'. Suppose not, then players' utilities must approach $-\infty$ as $x \to -\infty$, and players should quit, a contradiction. Thus, $\underline{x} = \sup\{x | \sum_i q_i(x) > 0\}$ exists.

Suppose $V(\underline{x}) = V'(\underline{x}) = 0$, then the case is identical to the case of a costless seller. So, we can use the proof directly from Lemma 1.

Suppose $U(\underline{x}) = U'(\underline{x}) = 0$ and $V'(\underline{x}) \neq 0$, first we see that in this case, we must have $\underline{x} > \overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b}$, otherwise, $\underline{V}(x,\underline{x}) < 0$ for $x < \overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b}$, and Buyer would quit at $\overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b}$, a contradiction. An equilibrium outcome should be Seller's optimal stopping solution, with stopping payoff of $x - \underline{V}(x,\underline{x})$.

Let $\overline{\overline{x}}$ denote the point where Buyer would buy if price is fixed at $\overline{P}$, and let $\overline{x}$ denote the point where trade happens in equilibrium. From earlier analysis, we know that Buyer's value function is in the form of $V(x) = \frac{c_b}{\sigma^2}(x-\overline{P})^2 + \alpha_b(x-\overline{P}) + \beta_b$ and Seller's value function is in the form of $U(x) = \frac{c_s}{\sigma^2}(x-\overline{P})^2 + \alpha_s(x-\overline{P}) + \beta_s$. Then $\overline{\overline{x}}$, $\overline{x}$, and $\underline{x}$ must satisfy the following seven value-matching and smooth-pasting boundary conditions—the first three for the Buyer and the last four for the Seller:

$$\begin{cases} \frac{c_b}{\sigma^2}\left(\overline{\overline{x}}-\overline{P}\right)^2 + \alpha_b\left(\overline{\overline{x}}-\overline{P}\right) + \beta_b & = \overline{\overline{x}} - \overline{P} \\ 2\frac{c_b}{\sigma^2}\left(\overline{\overline{x}}-\overline{P}\right) + \alpha_b & = 1 \\ \frac{c_b}{\sigma^2}\left(\underline{x}-\overline{P}\right)^2 + \alpha_b\left(\underline{x}-\overline{P}\right) + \beta_b & = 0 \\ \frac{c_s}{\sigma^2}\left(\underline{x}-\overline{P}\right)^2 + \alpha_s\left(\underline{x}-\overline{P}\right) + \beta_s & = 0 \\ 2\frac{c_s}{\sigma^2}\left(\underline{x}-\overline{P}\right) + \alpha_s & = 0 \\ \frac{c_s}{\sigma^2}\left(\overline{x}-\overline{P}\right)^2 + \alpha_s\left(\overline{x}-\overline{P}\right) + \beta_s & = \overline{x} - V(x) \\ 2\frac{c_s}{\sigma^2}\left(\overline{x}-\overline{P}\right) + \alpha_s & = 1 - \frac{d}{dx}V(x). \end{cases} \tag{22}$$

Solving for $\overline{x}$, $\underline{x}$, α_b, β_b, α_s, and β_s then generates the equilibrium outcome.

Lastly, we show that the game cannot simultaneously have an equilibrium outcome with $V'(\underline{x}) = 0$ and an outcome with $U'(\underline{x}) = 0$ and $V'(\underline{x}) \neq 0$. Using the last six conditions from the system of equations, we can derive $\underline{x} = -\frac{c_b}{c_s}\overline{P}$. One can then compare this threshold to the quitting threshold for $V'(\underline{x}) = 0$, which is $\underline{x} = \overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b}$. Only the higher of the two thresholds can be supported in an equilibrium. If quitting happens at the lower threshold, then in the region between the two thresholds, one of the players must have a negative continuation value. That player can profitably deviate by quitting at those states.

Proof of Proposition 3. We prove by showing that, if Seller quits earlier than Buyer, then there's a higher $\overline{P}$ such that the equilibrium outcome in Lemma 2 generates a higher ex-ante utility for Seller. From the proof of Lemma 2, we know that $\underline{x} = -\frac{c_s}{c_b}\overline{P} > \overline{P} - \frac{1}{4}\frac{\sigma^2}{c_b}$, otherwise Buyer's quitting threshold must be higher than $\underline{x}$, a contradiction. Note that $\underline{x} = -\frac{c_s}{c_b}\overline{P}$ decreases in $\overline{P}$.

Increasing $\overline{P}$ by $\Delta P = \underline{x} + \frac{1}{4}\frac{\sigma^2}{c_b} - \overline{P}$ is strictly better for the Seller ex-ante, because (1) $\underline{x}$ is unchanged and (2) $V(x,\underline{x})$ is strictly lower. Fixing an $\underline{x}$ in equilibrium, we can think of Seller as facing an optimal stopping problem regarding when to sell, with a lower boundary at $\underline{x}$. Her utility from stopping is $P(x) = x - V(x,\underline{x}) = x - \underline{V}(x+\eta,\overline{P}) + \eta$. Now suppose the list price is increased from $\overline{P}$ to $\overline{P} + \Delta P$, where $\Delta P = \underline{x} + \frac{1}{4}\frac{\sigma^2}{c_b} - \overline{P}$. Seller's quitting threshold is decreased as argued above, and Buyer's quitting threshold increases to $\overline{P} + \Delta P - \frac{1}{4}\frac{\sigma^2}{c_b} = \underline{x}$, so the lower boundary of the game, $\underline{x} = \max\{\underline{x}_b, \underline{x}_s\}$, is unchanged. With the new list price $\overline{P} + \Delta P$, η becomes 0, and Seller's payoff from trading is $x - \underline{V}(x, \overline{P} + \Delta P)$, which is strictly higher than her selling price under $\overline{P}$: $x - \underline{V}(x+\eta,\overline{P}) + \eta$, for all $x > \underline{x}$. Thus, Seller is facing the same lower boundary under the two list prices, and a stopping utility under $\overline{P} + \Delta P$ that strictly dominates $\overline{P}$. Thus, regardless of where she wants to trade, Seller's ex-ante utility is strictly higher under $\overline{P} + \Delta P$ than under $\overline{P}$. Thus, $\overline{P}$ is suboptimal.

Proof of Proposition 4. From Proposition 3, we know that Buyer quits first in equilibrium. Thus, given $\overline{P}$, Buyer's value function must be

$$V(x) = \underline{V}(x) = \frac{c_b}{\sigma^2}\left(x-\overline{P}\right)^2 + \frac{1}{2}\left(x-\overline{P}\right) + \frac{\sigma^2}{16c_b}.$$

and Buyer's quitting threshold is $\underline{x} = \overline{P} - \frac{\sigma^2}{4c_b}$.

Because $c_s > 0$, by $U''(x) = \frac{2c_s}{\sigma^2}$, we know that $U(x) = \frac{c_s}{\sigma^2}(x-\overline{P})^2 + A_s(x-\overline{P}) + B_s$ for some coefficients A_s and B_s. At $\underline{x}$, we must have (1) $U(\underline{x}) = 0$. Because $\overline{x}$ is Seller's optimal stopping points, $\overline{x}$ must satisfy: (2) $U(\overline{x}) = P(\overline{x}) = \overline{x} - V(\overline{x})$, and (3) $U'(\overline{x}) = 1 - V'(\overline{x})$. From these three conditions, we can solve $\overline{x}$, A_s, and B_s by solving the following system of equations:

$$\begin{cases} \frac{c_s}{\sigma^2}\left(-\frac{\sigma^2}{4c_b}\right)^2 + A_s\left(-\frac{\sigma^2}{4c_b}\right) + B_s & = 0 \\ \frac{c_s}{\sigma^2}\left(\overline{x}-\overline{P}\right)^2 + A_s\left(\overline{x}-\overline{P}\right) + B_s & = \overline{x} - \frac{c_b}{\sigma^2}\left(\overline{x}-\overline{P}\right)^2 - \frac{1}{2}\left(\overline{x}-\overline{P}\right) - \frac{\sigma^2}{16c_b} \\ 2\frac{c_s}{\sigma^2}\left(\overline{x}-\overline{P}\right) + A_s & = 1 - \frac{2c_b}{\sigma^2}\left(\overline{x}-\overline{P}\right) - \frac{1}{2} \end{cases} \tag{23}$$

from which we get

$$\begin{cases} \overline{x} & = \overline{P} - \frac{\sigma^2}{4c_b} + \frac{\sigma^2}{c_b}\frac{1}{\sqrt{1+k}}\sqrt{\frac{1}{4} - r} \\ A_s & = \frac{2+k}{2} - 2\sqrt{1+k}\sqrt{\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}} \\ B_s & = \left(\frac{1}{4} + \frac{k}{16}\right)\frac{\sigma^2}{c_b} - \frac{\sqrt{1+k}}{2}\frac{\sigma^2}{c_b}\sqrt{\frac{1}{4} - \overline{P}\frac{c_b}{\sigma^2}}. \end{cases} \tag{24}$$

Plugging A_s and B_s into $U(x_0)$, and letting $r = \overline{P}\frac{c_b}{\sigma^2}$, we get

$$U(x_0 = 0) = \frac{\sigma^2}{c_b}\left[k\left(\frac{1}{4} - r\right)^2 + \left(\frac{1}{4} - r\right) - 2\sqrt{1+k}\left(\frac{1}{4} - r\right)^{3/2}\right]. \tag{25}$$

Maximize $U(0)$ with respect to r to get the optimal r^* and $\overline{P}^* = r^*\frac{\sigma^2}{c_b}$. For $x_0 = 0$, we get $r^* = \frac{1}{4} - \frac{18+10k-6\sqrt{9+10k+k^2}}{16k^2}$. Plugging $\overline{P}$ into $\overline{x} = \overline{P} - \frac{\sigma^2}{4c_b} + \frac{\sigma^2}{c_b}\frac{1}{\sqrt{1+k}}\sqrt{\frac{1}{4} - r}$ produces $\overline{x}$. Plugging $\overline{P}$ into $P(\overline{x}) = \overline{x} - V(\overline{x})$ produces $P(\overline{x})$. It is easy to check that $\lim_{x \to \underline{x}^+} U'(x) > 0$ and $U(x) > 0\ \forall x > \underline{x}$, which confirms that Seller does not want to quit before Buyer does.

Proof of Corollary 3 and 4. To make the comparison, we need to first solve for the equilibrium under a fixed price. Let P^* be the optimal fixed price. We solve separately for the cases of (1) Buyer quits first and (2) Seller quits first.

Suppose Buyer quits first. We know that Buyer's value function is $V(x) = \frac{c_b}{\sigma^2}(x-P^*)^2 + \frac{1}{2}(x-P^*) + \frac{\sigma^2}{16c_b}$, and Seller's value function is $U(x) = \frac{c_s}{\sigma^2}(x-P^*)^2 + A_s(x-P^*) + B_s$ for some coefficients A_s and B_s. Let $\overline{x}$ and $\underline{x}$ denote Buyer's buying and quitting thresholds, respectively. Because both buying and quitting are Buyer's decision, we get $\overline{x} = P^* + \frac{\sigma^2}{4c_b}$ and $\underline{x} = P^* - \frac{\sigma^2}{4c_b}$ from Section 3. Solving $U(\overline{x}) = P^*$ and $U(\underline{x}) = 0$ simultaneously give us $A_s = P^*\frac{2c_b}{\sigma^2}$ and $B_s = -k\frac{\sigma^2}{16c_b} + \frac{1}{2}P^*$. Thus,

$$U(x_0) = \frac{kc_b}{\sigma^2}\left(x_0 - P^*\right)^2 + P^*\frac{2c_b}{\sigma^2}\left(x_0 - P^*\right) + -k\frac{\sigma^2}{16c_b} + \frac{1}{2}P^*. \quad (26)$$

Taking derivative with respect to P^* and setting to zero, we get $P^* = \frac{1}{2-k}\frac{\sigma^2}{4c_b} + \frac{1-k}{2-k}x_0$. To make sure $\underline{x} < x_0$, we need $k < x_0\frac{4c_b}{\sigma^2} + 1$, otherwise Buyer quits immediately.

Now suppose Seller quits first. Because now quitting is Seller's decision, we do not know coefficients A_b and B_b in Buyer's value function $V(x) = \frac{c_b}{\sigma^2}(x-P^*)^2 + A_b(x-P^*) + B_b$. We still have $U(x) = \frac{c_s}{\sigma^2}(x-P^*)^2 + A_s(x-P^*) + B_s$. To solve A_b, B_b, A_s, B_s, $\overline{x}$, and $\underline{x}$, we solve the following system of equations:

$$\begin{cases} U(\overline{x}) = P^* & \text{and} \quad U(\underline{x}) = 0 \\ V(\overline{x}) = \overline{x} - P^* & \text{and} \quad V(\underline{x}) = 0 \\ U'(\underline{x}) = 0 & \text{and} \quad V'(\overline{x}) = 1. \end{cases} \quad (27)$$

This gives $\underline{x} = (1 - \frac{1}{k})P^*$, and $U(x_0) = \frac{kc_b}{\sigma^2}(x_0 - P^*)^2 + \frac{2c_b}{\sigma^2}P^*(x_0 - P^*) + \frac{kc_b}{\sigma^2}(\frac{P^*}{k})^2$. Then maximize $U(x_0)$ with respect to P^*. If $k \le x_0\frac{4c_b}{\sigma^2} + 1$, then $\frac{d}{dP}U > 0$ for all P. This implies Seller will raise the price till Buyer quits first, that is, $P^* - \frac{\sigma^2}{4c_b} \ge (1-\frac{1}{k})P^*$. If $k > x_0\frac{4c_b}{\sigma^2} + 1$, we get $P^* = \frac{k}{k-1}x_0 = \underline{x}$. Thus, Seller quits immediately. Thus, there does not exist an equilibrium with positive length such that Seller quits strictly before Buyer. For $k < x_0\frac{4c_b}{\sigma^2} + 1$, the length of the game is positive and Buyer quits first. For $k >= x_0\frac{4c_b}{\sigma^2} + 1$, one player stops immediately regardless of the price.

Now we can compare the equilibrium outcome under bargaining to the outcome under a fixed price. Particularly, for $x_0 = 0$, we can compare $\overline{P} = \frac{\sigma^2}{c_b}[\frac{1}{4} - \frac{18+10k-6\sqrt{9+10k+k^2}}{16k^2}]$ with $P^* = \frac{1}{2-k}\frac{\sigma^*}{4c_b}$. There exists a $\underline{k} < 1$ such that $\overline{P} > P^*$ for $k < \underline{k}$ and $\overline{P} < P^*$ for $k > \underline{k}$.

A.3. Section 5

Lower Bound on Buyer's Value Function Let $\underline{V}_i(x, \mu)$ denote type i's lower bound in state x with Seller's belief μ.

First, if $\mu = 0$ or $\mu = 1$, then Buyer's type is revealed. Only one type of Buyer remains and the problem is identical to the costless seller model in Section 3. Thus, the lower bound is the same as in Equation (7). Thus, we must have $\underline{V}_i(x, \mu_i) = \underline{V}(x_i)$ from Equation (7).

It remains to solve for $\underline{V}_i(x, \lambda)$. The following Lemma proves that, if type L receives $\underline{V}(x_L)$ in equilibrium when his type is revealed, then H type must buy (weakly) earlier than L type when the type has not been revealed, and the existence of H type does not change L type's lower bound.

Lemma 3. If $V_L(x, 0) = \underline{V}(x_L)$ from Equation (7), then H type must buy (weakly) earlier than L type does, and there exists an equilibrium outcome in which $V_L(x, \lambda) = \underline{V}(x_L)$.

Proof. Suppose both types are still in the game. Because L type cannot be worse than if price is fixed at $\overline{P}$, we must have $V_L(x, \lambda) \ge \underline{V}(x_L)$, where $\underline{V}$ is from Equation (7). If L type buys at state x, then we must have $P(x, \lambda) \le x_L - \underline{V}(x_L)$. If H types also buys, he gets utility of at least $x_H - (x_L - \underline{V}(x_L)) = \underline{V}(x_L) + \epsilon$. If H type does not buy at x, then he will be the only type remaining after L type buys, and will get continuation value $V_H(x, 1) = \underline{V}(x_H) = \underline{V}(x_L + \epsilon)$. It is straightforward to show that $\underline{V}(x_L) + \epsilon \ge \underline{V}(x_L + \epsilon)$; hence, H type must buy no later than L type does.

Suppose that H type buys strictly earlier ($\overline{x}_H < \overline{x}_L + \epsilon$), then we have a separating equilibrium. After H type buys, L type buys immediately if $x_L \ge \overline{x}_L$. Thus, H type must buy strictly before x_H reaches $\overline{x}_L + \epsilon$, otherwise there cannot be separation. After H buys, L type is revealed, so the outcome must follow Proposition 1. The existence of H type does not affect L type's equilibrium payoff.

Suppose instead we have a pooling equilibrium where H and L types buy together ($\overline{x}_H = \overline{x}_L + \epsilon$). If Seller cannot separate the two types, then it is as if she is only dealing with L type only. Seller can charge up to $P(x, \mu) = P(x_L + \frac{\epsilon}{2}, \mu) = x_L - \underline{V}(x_L)$ at the point of trade. The optimal price and thresholds follow Proposition 1. ◻

By Lemma 3, L type buys and quits at the same thresholds as in the case without private information. The existence of H type does not affect L type's lower bound value function, nor does it affect L type's equilibrium strategies. Thus, we must have:

$$\underline{V}_L(x, \lambda) = \underline{V}(x_L) = \frac{c_b}{\sigma^2}\left(x_L - \overline{P}\right)^2 + \frac{1}{2}\left(x_L - \overline{P}\right) + \frac{\sigma^2}{16c_b}, \quad x_L \in \left(\overline{P} - \frac{\sigma^2}{4c_b}, \overline{P} + \frac{\sigma^2}{4c_b}\right). \quad (28)$$

The lower bound on H type's value function when $\mu = \lambda$ must be higher than $\underline{V}(x)$ from Equation (7). Seller cannot "threaten" to never make discount, because H type rationally expects Seller to make L type an offer when x_L reaches $\overline{x}_L$.

When x drops to $\underline{x}_L + \frac{\epsilon}{2}$, L type quits and H type is revealed with product value x_H. When x increases to $\overline{x}_L + \frac{\epsilon}{2}$, L type buys, and by Lemma 3, H type buys too. When L quits at $x_L = \underline{x}_L$, H's utility becomes $\underline{V}(\underline{x}_L + \epsilon)$. When L buys, H's utility becomes $\underline{V}(\overline{x}_L) + \epsilon$. These two conditions bound H type's value function from below. By Taylor's expansion and Ito's lemma, we have

$$V_H(x, \lambda) = \frac{c_b}{\sigma^2}\left(x_H - \frac{\epsilon}{2} - \overline{P}\right)^2 + \alpha_H\left(x_H - \frac{\epsilon}{2} - \overline{P}\right) + \beta_H \quad \forall x_H \in \left[\underline{x}_L + \epsilon, \overline{x}_L + \epsilon\right]. \quad (29)$$

The two conditions translate to:

$$\begin{cases} \frac{c_b}{\sigma^2}\left(\frac{\epsilon}{2}-\frac{\sigma^2}{4c_b}\right)^2+\alpha_H\left(\frac{\epsilon}{2}-\frac{\sigma^2}{4c_b}\right)+\beta_H \\ \quad =\frac{c_b}{\sigma^2}\left(\epsilon-\frac{\sigma^2}{4c_b}\right)^2+\frac{1}{2}\left(\epsilon-\frac{\sigma^2}{4c_b}\right)+\frac{\sigma^2}{16c_b} \\ \frac{c_b}{\sigma^2}\left(\overline{x}_L+\frac{\epsilon}{2}-\overline{P}\right)^2+\alpha_H\left(\overline{x}_L+\frac{\epsilon}{2}-\overline{P}\right)+\beta_H \\ \quad =\frac{c_b}{\sigma^2}\left(\overline{x}_L-\overline{P}\right)^2+\frac{1}{2}\left(\overline{x}_L-\overline{P}\right)+\frac{\sigma^2}{16c_b}+\epsilon. \end{cases} \tag{30}$$

Solving these two equations produces α_H, β_H, and consequently $V_H(x,\lambda)$. The solution is:

$$\alpha_H=\frac{1}{2}+\frac{\frac{\epsilon}{2}-\frac{c_b}{\sigma^2}\epsilon^2-\epsilon\sqrt{\frac{1}{4}-\overline{P}\frac{c_b}{\sigma^2}}}{\frac{\sigma^2}{c_b}\sqrt{\frac{1}{4}-\overline{P}\frac{c_b}{\sigma^2}}}$$

$$\beta_H=\frac{3c_b}{4\sigma^2}\epsilon^2+\frac{\sigma^2}{16c_b}-\frac{\frac{\epsilon}{2}-\frac{c_b}{\sigma^2}\epsilon^2-\epsilon\sqrt{\frac{1}{4}-\overline{P}\frac{c_b}{\sigma^2}}}{\frac{\sigma^2}{c_b}\sqrt{\frac{1}{4}-\overline{P}\frac{c_b}{\sigma^2}}}\left(\frac{\epsilon}{2}-\frac{\sigma^2}{4c_b}\right). \tag{31}$$

Proof of Proposition 5. The only case we need to prove is when Buyer is H type and Seller's belief is $\mu=\lambda$. Given H type's lower bound $\underline{V}_H(x,\lambda)$, which is calculated in Appendix A.3, it is easy to check that $\underline{V}_H(x,\lambda)>\underline{V}_L(x+\epsilon,\lambda)$ for $\underline{x}_L+\frac{\epsilon}{2}<x<\overline{x}_L+\frac{\epsilon}{2}$. Thus, H type has a higher utility than L type in every state, even after compensating L type for the difference in outside options, ϵ. It is also easy to check that $\underline{V}_H(x,\lambda)-\epsilon<\underline{V}_L(x+\epsilon,\lambda)$ for $\underline{x}_L+\frac{\epsilon}{2}<x<\overline{x}_L+\frac{\epsilon}{2}$; thus, L type would not buy when Seller makes an offer to the H type. Now, given $V_H(x,\lambda)=\underline{V}_H(x,\lambda)$, Seller receives $P(x,\lambda)=x_H-V_H(x,\lambda)$ when she trades with H type. Seller's decision of when to trade with H type must solve the following optimal stopping problem, with two value matching boundary conditions and one smooth-pasting boundary condition:

$$\begin{cases} \alpha_s\left(\overline{x}_H-\frac{\epsilon}{2}\right)+\beta_s=-\left(\overline{x}_H-\frac{\epsilon}{2}-\overline{P}\right)^2+(1-\alpha_H)\left(\overline{x}_H-\frac{\epsilon}{2}-\overline{P}\right) \\ \quad +\overline{P}-\beta_H \\ \alpha_s=-2\left(\overline{x}_H-\frac{\epsilon}{2}-\overline{P}\right)+(1-\alpha_H) \\ \alpha_s\left(\frac{\epsilon}{2}-\frac{\sigma^2}{4c_b}\right)+\beta_s=\left(1-2\sqrt{\frac{1}{4}-\overline{P}\frac{c_b}{\sigma^2}}\right)\epsilon. \end{cases} \tag{32}$$

where α_H and β_H are solutions to Equation (30). Using the system of equations, we can derive the following condition

$$\left(\overline{x}_H-\overline{P}-\frac{\sigma^2}{4c_b}\right)^2+\overline{P}-\frac{\sigma^2}{4c_b}=0$$

which implies that

$$\overline{x}_H=\overline{x}_L=\overline{P}+\frac{\sigma^2}{c_b}\left[\sqrt{\frac{1}{4}-\overline{P}\frac{c_b}{\sigma^2}}-\frac{1}{4}\right].$$

Note that even though they buy at the same threshold, H type arrives at the threshold earlier than L type does because $x_H=x_L+\epsilon$. Also, this common threshold is the same trading threshold as when there is no private information.

When L type quits, H type's value function becomes $\underline{V}(\underline{x}_L+\epsilon)>0$; thus, H type does not quit as long as L is present. After L quits, H has the same quitting threshold as in Proposition 1. So

$$\underline{x}_i=\underline{x}=\overline{P}-\frac{1}{4}\frac{\sigma^2}{c_b}.$$

At their respective times of trade, L type pays price

$$P_L=\overline{x}-V_L\left(\overline{x}+\frac{\epsilon}{2}\right)$$

and H type pays price

$$P_H=\overline{x}-V_H\left(\overline{x}-\frac{\epsilon}{2}\right).$$

Thus, $P_L-P_H=V_H(\overline{x}-\frac{\epsilon}{2})-V_L(\overline{x}+\frac{\epsilon}{2})$. Because we showed that $V_H(x)>V_L(x+\epsilon)$, this proves that $P_L>P_H$.

Proof of Proposition 6. First we show that Proposition 6 gives the unique equilibrium outcome conditional on $\overline{P}$. If there is no list price, then Seller charges $v_0+\Delta v$ at period 2, and charges $v_0+\frac{1}{2}\Delta v$ at period 1. Buyer's continuation value is then negative in all periods, so it must buy or quit immediately. If $\overline{P}=v_0+\Delta v-6c$, then, at period 2, given $\theta\le\frac{\Delta v-6c}{\frac{3}{2}\Delta v-8c}$, Seller's optimal price is $\overline{P}=v_0+\Delta v-6c$ regardless of its signal. This implies that at period 1, Buyer's continuation value is $\frac{1}{2}(v_0+\Delta v-\overline{P})-c=2c$ if the first attribute is a match, and is $-c$ if the first attribute is a mismatch. Seller's optimal response in period 1 is to charge $v_0+\frac{1}{2}\Delta v-2c$, which makes Buyer with a matched attribute indifferent between buying and continuing, and let Buyer with a mismatched attribute quit. Buyer's continuation value is 0 at period 0. Seller can either sell at $t=0$ with price v_0, or wait till period 1 for an expected profit of $\frac{1}{2}(v_0+\frac{1}{2}\Delta v-2c)$. Seller prefers to wait for $v_0\le\frac{1}{2}\Delta v-2c$. So, Proposition 6 gives the unique equilibrium conditional on $\overline{P}$.

Now consider alternative $\overline{P}$. For higher $\overline{P}$, Buyer's continuation value at time 0 is negative, so it must buy or leave immediately. For lower $\overline{P}$, Buyer continuation value must be weakly higher in all history. In either case, Seller cannot be more profitable.

Endnotes

[1] See, for example, HubSpot's sales process in Skok (2012) and Talview's sales process in Jose (2017) with more details. Both companies offer Software-as-a-Service to other businesses.

[2] The buyer incurs cost from processing information or opportunity cost when dedicating employees to talk to the salesperson, whereas seller's cost can come from the salesperson's salary and product demonstration cost.

[3] This is before considering consumer heterogeneity in costs.

[4] Using other probabilities increase the analytic complexity without providing additional insights. If attributes are correlated, then attributes that are discovered first reveal more information about the total surplus, whereas the additional information revealed by subsequent attributes must still be independent from information revealed before. This case is discussed in Online Appendix.

[5] One qualitative implication of infinite horizon is that there is always more information to discover so that players can never finish learning

before trade. In reality, though the amount of information is not infinite, there is often too much information available so that buyers cannot feasibly learn everything before making a decision. Toman et al. (2017) observe that B2B buyers face "open-ended learning loops by the deluge of information. With each iteration they work harder to ensure that they fully understand the requirements and the alternatives. More information begets more questions." For consumer products, there is often a wealth of consumer reviews or third-party reviews on the internet so that buyers often do not exhaust all information before making a purchase.

[6] As shown by Simon and Stinchcombe (1989), a strategy profile may not produce a well-fined outcome in continuous time. Subsequent utility functions are well-defined if and only if τ_θ is a measurable function. Thus, when considering profitable deviations, only strategies that produce a measurable stopping time τ_θ is allowed. In the Online Appendix, I construct an alternative equilibrium concept that does not restrict the strategy space.

[7] In Online Appendix, I discuss how using time discounting instead of flow costs does not affect results qualitatively, as long as outside options are positive so that the players have an incentive to quit. The paper uses flow cost to illustrate the "investment effort" in the hold-up problem more directly.

[8] We can motivate truthful revelation in a simple model. Suppose that Buyer can choose whether to reveal his preference for each attribute. Buyer always chooses to reveal if he does not like the attribute. Then Seller can infer Buyer's preference when he does not reveal. Thus, Buyer's preference becomes unraveled.

[9] Note that Seller can only control the stopping decision for states between thresholds $\overline{\overline{x}}$ and $\underline{x}$, which are Buyer's stopping thresholds facing a fixed price of $\overline{P}$. At $\underline{x}$, Buyer quits and ends the game. At $\overline{\overline{x}}$, Buyer buys the product even if there is no discount, so Seller cannot delay trade beyond these thresholds.

[10] If there are different types of buyers with different costs and starting positions, then some buyers could buy at the list price in equilibrium.

[11] Buyers in this model arrive with the same x_0 and face the same list price. Suppose Buyers arrive with different x_0 drawn from a distribution, and Sellers observe x_0 after setting $\overline{P}$. Proposition 1 shows that, given a list price, equilibrium thresholds do not depend on initial position. But final price can be different. If a buyer's initial position is already above $\overline{x}$, then Seller's price offer, $P = x - V(x)$, must be higher than $\overline{x} - V(\overline{x})$. Price for $x > \overline{x}$ is shown in Figure 5(b). One can solve the optimal list price by maximizing

$$U_0 = \max_{\overline{P}} \int U(x_0|\overline{P})dF(x_0),$$

where $U(x_0|\overline{P})$ is given in Equation (8) and $F(x_0)$ is the CDF of x_0. If Seller does not observe x_0, then Buyers may receive different discounts in equilibrium. This latter case is explored in Section 5.1.

[12] Because only Buyer has cost, Buyer's action is socially optimal when the list price is 0.

[13] Note that the graph does not show the full price path. There are infinite price paths leading up to time τ that produce the same equilibrium outcome. Price strategies before time τ only need to be high enough so that Buyer does not want to buy. Keeping the price at the list price before time τ, for example, would work.

[14] Due to the nature of continuous time, a strategy profile can achieve the effect of simultaneous quitting without having players quitting at the same x. For example, suppose Seller quits in a set B except a single point x'. Then on x', Buyer cannot extend the game whether he quits or not. The game ends immediately even if Buyer does not quit, making him indifferent between quitting at x' or not. This example illustrates that it is not sufficient to simply restricting players to not quit at the same x.

[15] That is, whether $\underline{x}$ is the optimal quitting threshold for Buyer or for Seller. If $\underline{x}$ is optimal for Buyer (Seller), then we must have $V(\underline{x}) = V'(\underline{x}) = 0$ ($U(\underline{x}) = U'(\underline{x}) = 0$), due to the discussion.

[16] By Lemma 3 from Appendix A.3, L type should act the same way as in Proposition 1. Thus, when $\overline{P} \geq x_0 - \frac{\epsilon}{2} + \frac{\sigma^2}{4c_b}$, L type buys immediately if $x_0 - \frac{\epsilon}{2} > 0$ or quits immediately if $x_0 - \frac{\epsilon}{2} \leq 0$. By Lemma 3, H type should buy immediately if L type buys immediately. If L type quits immediately and H type stays, then there is only a single type of Buyer remaining, which is again solved in Proposition 1. So, we do not need to solve for the case of $\overline{P} \geq x_0 - \frac{\epsilon}{2} + \frac{\sigma^2}{4c_b}$.

[17] Some factors ignored in this model can cause a downward trend in price. For example, if Seller can commit to each price offer for a short period of time and if players have time discounting, then Seller can use declining prices to screen different types. This is common in the repeated-offers bargaining literature. Section 6 shows a few extensions in which price can decrease over time—for example, if the product has a finite mass of attributes, or if players observe signals of the true match value. However, the core intuition should still remain. The fact that H type has a worse outside option makes the product more appealing ex-ante and decreases the value of information acquisition. This encourages Seller to trade with him earlier, which puts downward pressure on H type's price in order for the offer to be incentive compatible.

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