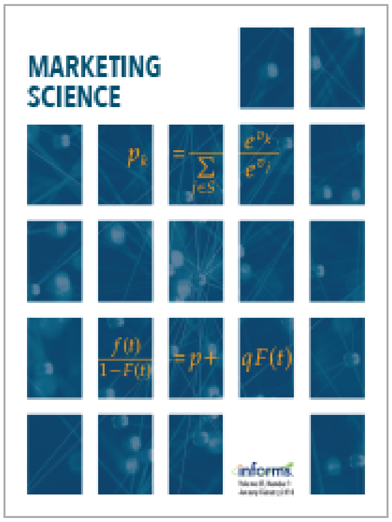

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Search Advertising: Budget Allocation Across Search Engines

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Abstract. In this paper, we investigate advertisers' budgeting and bidding strategies across multiple search platforms. We develop a model with two platforms and budget-limited advertisers that compete for advertising slots across platforms. When platform reserve prices are low and exogenous, we find that symmetric advertisers pursue asymmetric budget allocation strategies and *partially differentiate*: one advertiser allocates a share of its budget to platform A *higher* than A's share of user traffic and a share of its budget to platform B *lower* than B's share of user traffic, whereas the second advertiser does the reverse. This partial differentiation balances two forces: a demand force arising from a desire to be present on both platforms to obtain more clicks and a strategic force driven by a desire to be budget dominant on at least one platform to obtain clicks at a lower cost. We then show that the benefit from differentiation for advertisers diminishes if platforms strategically increase their reserve prices. At reserve prices that maximize platform revenues, advertisers allocate their budgets *proportional* to each platform's share of user traffic, and platforms fully appropriate these budgets.

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Keywords: search advertising • advertising budgets • differentiation • competitive strategy • auctions • bid jamming • game theory

1. Introduction

Search advertising has become one of the most prevalent types of online advertising. There are multiple platforms, such as Google, Bing, Yahoo, Facebook, Twitter, Snapchat, and Amazon, through which advertisers can reach their customers. Most of these platforms use variations of second price auction to allocate advertisers to available ad spaces. An important decision that advertisers should make is the budget allocated to each platform. This naturally raises several questions. For example, should a limited budget be allocated all to one platform or split across platforms? If it is to be split, what fraction of the budget should be allocated to each platform? Moreover, does it matter what a competing advertiser does? In this paper, we derive equilibrium budget allocations by competing advertisers when multiple search engines are available for them to use. Our goal is to obtain results that provide normative guidelines to managers.

Understanding the advertisers' budgeting and bidding behavior across multiple platforms can enable platforms to strategically maximize their revenues. Consider a platform, Bing for example, that would like to optimize the reserve price of its second price ad auctions. On one hand, a higher reserve price can increase Bing's revenue owing to higher earnings per click on the last advertiser's link. On the other hand, a higher reserve price may depress advertisers' incentives to invest in Bing, motivating them to allocate more budget to Google or other advertising platforms. Thus, in a multiplatform environment, platforms need to take into account how their strategic decisions might influence advertisers' budgeting and bidding behavior at other platforms in the market.

Of course, the presence of two search engines leads to an allocation problem for advertisers only if their budgets are limited. Every day, millions of internet users use general search engines, such as Google, Yahoo, and Bing, to find their desired products and services.[1] As a result, the number of clicks that these platforms are able to generate can potentially be very high. Based on Google AdWords keyword planner tool, there are about 1 million monthly searches for the single keyword "flowers" in the United States. This tool also predicts that, if an advertiser puts in a bid of \$5 for this keyword, it receives 57,571 daily impressions and 672 clicks, resulting in an average daily cost of \$2,360. Likewise, Bing Ads keyword planner tool predicts that, with the same bid of \$5, an advertiser receives 9,259 impressions and 135 clicks, resulting in an average daily cost of \$229. These advertising costs are way beyond the amount that most advertisers in the flower industry are able or willing to spend on their online advertising campaigns on a daily basis. Consequently, in practice, advertisers are *budget constrained*: they have a limited amount to spend on search advertising. In turn, this means that managers must decide how to allocate their limited budget across major search engines.

It turns out that search engines, while keen on attracting advertisers, also want them to control their costs that likely weigh heavily on managers' minds. Even with pay per click, managers can measure costs more precisely than returns if for no other reason than the challenging task of attributing revenues to one marketing instrument when consumers are certainly influenced by multiple marketing activities. Regardless of their motivations, it is a fact that all major platforms require advertisers to set a daily budget before starting their advertising campaigns. The daily budget serves as an upper bound on the amount that an advertiser would pay to a search engine for clicks that it receives on a particular day.[2]

The budgets affect bidding strategies and through that, profits for both advertisers and search engines. The intuition is that, after the advertiser fully exhausts its budget, its ad is not displayed anymore, and this, in turn, provides an opportunity for other remaining advertisers to move up and obtain better positions potentially at lower costs. This implies that budget restrictions of advertisers can have strategic effects on their bidding behavior and profits, even in the presence of only a single search engine. Starting from a (off-equilibrium) bid level, a lower-ranked rival (with a lower bid) may have an incentive to increase its bid strategically to raise the cost of the advertiser just above it. A lower-ranked advertiser, by raising its bid, can cause the higher-ranked advertiser's budget to be exhausted quickly. As a result, the lower-ranked advertiser can move up to obtain a better position and receive more clicks without paying a higher cost per click (CPC). This is because in second price auctions, the cost that an advertiser incurs for each click is the bid of the advertiser just below it. With two search engines, there is an additional strategic element. By moving some of the budget from one search engine to the other, an advertiser can increase its profit in the latter platform but decrease it in the former. The total profit *across* two platforms, however, may increase or decrease depending on the rival's allocation of budget and bidding behavior. Thus, the allocation decision is part of the competitive strategy for each advertiser.

1.1. Related Research

Our research falls into the broad stream of work on search engine advertising. Previous research has studied several strategic issues related to search advertising, such as advertisers' bidding behavior (Edelman et al. 2007, Varian 2007), interaction between sponsored and organic links (Katona and Sarvary 2010), search engine optimization (Berman and Katona 2013), buying competitors' keywords (Desai et al. 2014, Sayedi et al. 2014), hybrid auctions (Zhu and Wilbur 2011), first-page bid estimates (Amaldoss et al. 2015), click frauds (Wilbur and Zhu 2009), and contextual advertising (Zhang and Katona 2012). What all of these works have in common is that they consider a single-platform environment. We are interested in the problem of search advertising strategies when advertisers can use more than one platform. By deriving the equilibrium budget allocations across search engines, we add to extant work, thus obtaining new insights into advertisers' strategies.

Prior research in the auction literature has also examined different auction formats with budget-constrained bidders (Che and Gale 1998, 2000; Benoit and Krishna 2001; Borgs et al. 2005; Bhattacharya et al. 2010; Dobzinski et al. 2012). The common assumption in this stream of literature is that, if a bidder wins the item but cannot afford to pay for the item, it will incur a very high negative utility. The results of these works do not inform search engine advertising, because it is possible for an advertiser to bid high and win a high slot to enjoy clicks to the point that its budget is exhausted. Ashlagi et al. (2010) extend the interesting concept of Generalized English Auction (GEA) developed by Edelman et al. (2007) to incorporate advertisers' budget constraints and show that there exists a unique equilibrium. Although theoretically appealing, GEA is different from practice, and thus, their results are not readily applicable to real-world search engine auctions with budget-limited advertisers.

Turning to budget-limited advertisers in position auctions, our work is closest to the works of Koh (2013), Lu et al. (2015), and Shin (2015). Their analyses show that the equilibrium bidding outcome could result in what has come to be known as bid jamming: one advertiser bids just below its rival with the intention to exhaust the rival's budget quickly. Some interesting counterintuitive results emerge from these analyses: for example, an advertiser may bid higher than its valuation (Shin 2015), advertiser's and search engine's profits could be decreasing in budgets (Lu et al. 2015), and a search engine's revenue with budget-constrained advertisers may be higher than its revenue without budget constraints (Koh 2013). Although all of these papers model a single-platform environment, our paper contributes to this literature by examining new forces that are in place in a multiplatform search advertising market. We find that budgeting, bidding, and ordering of advertisers in a platform can be influenced by the presence of a second platform. Moreover, we show that advertisers can increase their profits by considering strategic forces across different platforms rather than making optimal decisions in each platform separately.

Inasmuch as our research focuses on advertisers' budgeting and bidding strategies in the presence of multiple platforms, it is also related to the stream of literature studying competing parallel auctions and the competition among sellers in the design of auction procedures (McAfee 1993, Peters and Severinov 1997, Burguet and Sákovics 1999, Haruvy et al. 2008, Gavious 2009, Ashlagi et al. 2013, Taylor 2013). This literature investigates how the design of an auction can influence bidders' choices of participation in the auctions. In the context of search engine advertising, Ashlagi et al. (2011) consider a model with two simultaneous Vickrey–Clarke–Groves (VCG) advertising auctions with different click-through rates (CTR), where each advertiser chooses a single auction to participate. Liu et al. (2008) consider second price auctions with different quality score mechanisms and show that the auction with a more favorable policy for less efficient bidders tends to attract more of these bidders. In our paper, we abstract away from auction design and fix the search engine's allocation and payment rule as it is commonly practiced.

1.2. Preview of Results

We analyze a full game with two search engines that strategically set their reserve prices and two budget-limited advertisers that compete to obtain advertising positions on these platforms. Our analysis confirms bid jamming to be an equilibrium strategy with limited budgets. We also obtain interesting results on advertisers' bidding and ranking outcomes. We find that the equilibrium bid is (weakly) increasing in advertising budgets. Moreover, the equilibrium bid is such that a low-budget advertiser is indifferent between bidding just above the high-budget rival and just below it. We also find that advertisers' equilibrium profits are increasing in own budget but decreasing in the rival's budget.

What is even more interesting is our finding on budget allocation strategies. When platform reserve prices are exogenous and sufficiently low, we find that advertisers pursue *asymmetric* allocation strategies across platforms. In other words, advertisers *partially differentiate*. This differentiation results in an equilibrium such that one advertiser allocates a higher share of its budget to one of the platforms, and the other advertiser allocates the same higher share of its budget to the other platform when platforms are also symmetric. The intuition behind partial differentiation in budget allocation strategies is that it balances two forces: (1) a demand force, arising from a desire to obtain a higher number of clicks, pulls advertisers toward each other and (2) a strategic force, driven by a desire to obtain clicks at a lower cost, creates the differentiation in allocation strategies.

We extend our symmetric model to incorporate the real-world environment of asymmetry in platform traffic or reserve prices as well as advertisers' budgets or valuations. We show that advertisers allocate a higher share of their budgets to the platform that generates higher traffic for them or has a lower reserve price while still differentiating across the two platforms.[3] Moreover, our analysis shows that the partial differentiation strategy does not obtain if the asymmetry in advertisers' budgets is sufficiently high. In this case, the unique equilibrium for the advertisers is to allocate their budgets proportional to the traffic of each platform. Taken together, our analysis informs an advertiser how to tailor its strategy depending on the rival's budget and valuation relative to its own.

Our work offers insights into how the presence of a second search engine can help advertisers to improve their profits. For symmetric advertisers, we show that their profits in a two-platform world are strictly higher than their profits in a world with a single-platform producing the equivalent traffic with the two platforms combined into one. Although the number of clicks that they receive does not change in these two scenarios, advertisers on average pay less for each click in the duopoly platform setting. In fact, partial differentiation helps advertisers to mitigate bid competition, and hence, they bid less aggressively compared with the single-platform case.

The benefit of this partial differentiation for advertisers, however, decreases if platforms strategically increase their reserve prices. We show that a platform always has an incentive to increase its reserve price, even though this increase in reserve price could result in advertisers shifting some portion of their budgets to the other platform. When platforms set their reserve prices to maximize their revenues in the first stage of the game, we find that they raise their reserve prices to fully exhaust advertisers' budgets. In this case, advertisers' optimal strategies are to split their budgets proportional to each platform's share of traffic. Thus, in the multiplatform search advertising environment, whereas advertisers strive for partial differentiation to mitigate competition and decrease their average cost per clicks, platforms leverage their reserve prices to better monetize clicks, in turn resulting in an allocation of budgets proportional to each platforms' click volume.

The rest of this paper is organized as follows. We describe our general model and its subcomponents in Section 2. To understand the main forces that are in place, we first analyze a fully symmetric model with exogenous reserve prices in Section 3. We then investigate the role of asymmetries in platform traffics and their reserve prices as well as advertiser total budgets in Section 4. In Section 5, we endogenize platforms' choice of reserve prices and examine how it affects advertisers' optimal bidding and budgeting strategies. Finally, we conclude in Section 6 with a managerial discussion of our results and suggest directions for future research. Proof of all propositions, corollaries, and lemmas as well as several variations and extensions of the main model are in the online appendix.

2. The Model

We consider a market with two search engines, each of which is a platform for search advertising.[4] Denote the two search engines by SE_j, $j \in \{1,2\}$. Throughout the paper, we refer to them as "Search Engine j" (SE_j) and "Platform j" interchangeably. To keep the model tractable, we assume that each platform offers only one advertising slot. This means that only one of the advertisers is able to advertise at a given time and that customers observe only one ad. This single-slot assumption helps us to capture the main forces in place without needlessly complicating the model.[5]

Search engines can generate clicks for the advertisers that obtain the advertising slot. We define c_j to be the *click volume* or *traffic* of SE_j. The click volume is the potential number of clicks that the ad slot can receive on a daily basis. This number usually depends on the size of the customer base of each platform. For example, Google has a bigger user base and higher incoming traffic than the Bing network, and thus, it can generate more clicks for an advertiser (keeping all other thing equal). Moreover, we denote the ratio $c_j/(c_1 + c_2)$ to be SE_j's *market share*.

We assume that there are two advertisers denoted by A_i, $i \in \{1, 2\}$ competing for the advertising slots. Advertiser i is characterized by two dimensions: valuation v_i and total budget T_i. The valuation v_i is the A_i's expected value for each click. This value can be thought of as the expected net margin from a purchase, taking into account the purchase probability. For the rest of the paper, we assume that $v_1 = v_2 \triangleq v$, and we relegate the analysis of the extended model with heterogeneous valuations ($v_1 \neq v_2$) to Online Appendix, Section II.[6]

The second dimension is the "total" budget T_i, which is the maximum amount of money that A_i is able to spend for search advertising over two platforms on a daily basis. An advertiser is said to have *limited* (or *exhaustible* or *constrained*) total budget if its total budget T_i satisfies $T_i < c_j v$ for $j \in \{1,2\}$. These inequalities imply that an advertiser with a limited budget is not able to pay for all potential clicks in a day at a price equal to its valuation for a click. If both of these inequalities do not hold, the advertiser's total budget is said to be *unlimited* (or *inexhaustible* or *unconstrained*). We assume that advertisers' total budgets (T_i) are exogenous and common knowledge, and we model the strategic interactions between advertisers as a game of perfect information.[7]

We assume that search engines use the second price auction with reserve price r_j to assign ad slots to the advertisers. At the beginning of the day, the advertiser with the higher bid wins the slot and starts receiving clicks.[8] For each click, it pays an amount equal to the other advertiser's bid. Depending on its budget and the bids, it is possible that the advertiser exhausts its budget before receiving all clicks in the day. According to the common practice in the industry, if the advertiser runs out of its budget, it cannot participate in the auction for the remaining traffic in the day. Therefore, the other advertiser takes over the ad slot, starts receiving the remaining clicks in the day, and pays the reserve price r_j for each click that it receives. This implies that the low bidder might be able to enjoy the ad slot at a lower price whenever the high bidder runs out of its budget. The following example illustrates how limited budgets affect the advertisers' rankings and profits.

Example 1. Suppose that SE_j can generate $c_j = 120$ daily clicks for its ad slot. Suppose that each advertiser has a valuation $v = 1$ for each click. Moreover, assume that Advertisers 1 and 2 allocated budgets of 50 and 40, respectively, to this platform. Finally, suppose that advertisers' bids are $b_1 = 1$ and $b_2 = 0.5$, and the reserve price is $r_j = 0.1$.

Because $b_1 > b_2$, A_1 gets ad slot and starts receiving clicks. For each click that it receives, A_1 pays $b_2 = 0.5$ to the platform. Clearly, A_1's budget of 50 is depleted after receiving 100 clicks. At this time, A_2 takes over the slot, starts receiving the "remaining" 20 clicks, and pays $r_j = 0.1$ per click. Thus, advertisers' profits are $\pi_1 = 100(1 - 0.5) = 50$ and $\pi_2 = 20(1 - 0.1) = 18$.

The example makes clear how both the budgets and the bids together determine the advertisers' profits. Therefore, a strategic decision for the advertisers is how their total budget T_i is split across the two search engines. Denote δ_i, $0 \le \delta_i \le 1$, to be the fraction of the total budget T_i that A_i allocates to SE_1. This implies that A_i allocates $B_i^1 \triangleq \delta_i T_i$ to SE_1 and $B_i^2 \triangleq (1 - \delta_i) T_i$ to SE_2.[9] We call δ_i the A_i's *allocation strategy*. Therefore, A_i's decisions consist of δ_i and b_i^j, where the latter is the A_i's bid on platform j. Advertisers' objectives are to maximize their total profit summed over the two search engines.

The timeline of the full game is as follows. First, search engines simultaneously choose their reserve prices. Second, given the reserve prices, advertisers decide how to allocate their limited budgets across the two platforms. We call this stage the *allocation stage*, where advertisers simultaneously choose their allocation strategy δ_i. Third, advertisers bid for the ad slot in each of the search engines. We call this stage the *bidding stage*, where bids b_i^j are chosen by advertisers in each platform. Fourth, users visit the platforms and click on the ad link, and profits are realized. In each stage, we seek a Nash equilibrium (NE) and impose subgame perfectness. In the next two sections (Sections 3 and 4), we solve the subgame by taking the reserve prices as exogenous, and then, in Section 5, we take up the full model in which search engines choose reserve prices.[10] Table 1 summarizes the model notations.

3. A Symmetric Model

In this section, we analyze a model with symmetric, nonstrategic platforms and symmetric advertisers. In other words, we assume that $T_1 = T_2 \triangleq T$, $c_1 = c_2 \triangleq c$, and $r_1 = r_2 \triangleq r$. This fully symmetric model provides key insights in a transparent way. We then consider asymmetric advertisers and/or search engines in Section 4. Moreover, we let platforms choose reserve prices in Section 5. For our analysis, we proceed backward by first finding the equilibrium for bidding stage given the allocation decisions.

3.1. Bidding-Stage Equilibrium

In this stage, advertisers bid for the ad slot in each search engine conditioned on their budget and their rival's budget allocated to each platform in the first stage of the game. In light of symmetry, the analysis would be similar for both search engines. After budgets have been chosen in the allocation stage, bids in one search engine do not affect the bids on the other. Therefore, we can analyze the bidding stage as though there is just one platform. Therefore, in this section, we drop the superscript j referring to platforms and thus, denote advertiser bids and budgets by b_i and B_i.

To determine the equilibrium bids, first consider the case where the sum of advertisers' budgets is small enough such that $B_1 + B_2 < cr$. In this situation, advertisers' equilibrium bids must be equal to the reserve price r. At $b_1^* = b_2^* = r$, each advertiser has equal chance of getting the slot in the beginning of the day. Suppose that A_1 gets the slot first. It then receives B_1/r number of clicks, exhausts its budget, and leaves remaining $c - B_1/r$ number of clicks for A_2. Advertiser 2 then gets the ad slot, but it also cannot afford to pay for all of the remaining clicks, because $c - B_1/r > B_2/r$. As a result, when bids are at the reserve price level, A_i receives B_i/r number of clicks regardless of whether it gets the slot first or second. Therefore, deviating to higher bids does not increase an advertiser's profit. This is because both number of clicks that it receives, B_i/r, and the margin on each click, $v - r$, remain unchanged.

Consider now the case when the reserve price is relatively small: that is, $r < (B_1 + B_2)/c$. In this case, bidding at reserve price cannot be NE, because the advertiser that gets the slot second will be left with some extra budget at the end of the day. Therefore, it will have an incentive to increase its bid from reserve price in order to be the first advertiser that is assigned to the ad slot. Of course, if an advertiser deviates to a bid slightly higher than r, its rival could respond by also increasing its bid. As a result, one could conjecture that the equilibrium bids will be higher than the reserve price. We construct these bids in two steps.

First, we note that it is always a weakly dominant strategy for the advertiser that bids lower to bid one penny below its rival's bid. By doing so, the lower-bid advertiser increases the cost for its rival. Consequently, higher-bid advertiser's budget will be depleted faster, and the lower-bid advertiser will receive more remaining clicks. This type of strategic bidding to exhaust the rival's budget has been referred to in the literature as *bid jamming* or *aggressive bidding*. For expository purposes, henceforth we call the advertiser that has the lower bid just below the bid of the higher-bid advertiser the *jammer*. The higher-bid advertiser that bids higher and gets the advertising slot initially but is jammed by the jammer is denoted as *jammee*.

Second, we show that the high-budget advertiser jams the low-budget one in equilibrium. To see this, it is useful to consider an advertiser's revenue and cost separately. Regarding revenue, both advertisers' incentives are identical. In fact, the *difference* between an advertiser's revenues when being a jammer versus a jammee does *not* depend on the advertiser's type (high or low budget). The difference in costs, however, does. A jammer pays a smaller cost to the platform, because reserve price is lower, whereas a jammee exhausts its budget fully. Consequently, a high-budget advertiser has greater incentive to be the jammer, because it pays less compared with the low-budget advertiser. Given these two facts, we can compute the Nash equilibrium bid level. We summarize our results in the following lemma.

Lemma 1. *Suppose that advertisers have limited budgets (B_1, $B_2 < cv$), and let* $B_H = \mathrm{Max}(B_1, B_2)$ and $B_L = \mathrm{Min}(B_1, B_2)$.

Table 1. Notation

Notation	Explanation
SE_j	Search engine (platform) j, $j \in \{1, 2\}$
c_j	Search engine j's click volume
r_j	Search engine j's reserve price
A_i	Advertiser i, $i \in \{1, 2\}$
v	Advertisers' valuation for a click
T_i	Advertiser i's total budget
b_i^j	Advertiser i's bid in search engine j
δ_i	Advertiser i's allocation strategy
B_i^j	Advertiser i's allocated budget to search engine j

i. *If reserve price is relatively high, $r > (B_1 + B_2)/c$, then equilibrium bids and profits are $b_i^* = r$ and $\pi_i^* = \tilde{\pi}(B_i)$, where $\tilde{\pi}(B_i) \triangleq B_i(v - r)/r$ for $i \in \{1, 2\}$.*

ii. *If reserve price is relatively low, $r \leq (B_1 + B_2)/c$, then equilibrium bids are $b_L^* = b^*$, $b_H^* = b^* - \epsilon$, where $\epsilon > 0$ is one smallest discrete unit of payment, and equilibrium profits are $\pi_L^* = \underline{\pi}$ and $\pi_H^* = \overline{\pi}$, where*

$$b^*(B_1, B_2) \triangleq \frac{(B_H + B_L)v - B_H r}{c(v - r) + B_L},$$

$$\underline{\pi}(B_1, B_2) \triangleq \frac{B_L(cv - B_H)(v - r)}{(B_H + B_L)v - rB_H},$$

and

$$\overline{\pi}(B_1, B_2) \triangleq \frac{\left(cvB_H - B_L^2 - cr(B_H - B_L)\right)(v - r)}{(B_H + B_L)v - rB_H}.$$

Lemma 1 fully characterizes the equilibrium bids and profits of advertisers given their allocated budgets to a platform. One interpretation of a search engine's reserve price r can be the level of outside competition. In other words, there might be other advertisers that are not strategic, and their bids for the ad slot are always fixed at r. If the level of outside competition is high and therefore, the reserve price r is high, then advertisers' bids will also be r. In this case, each advertiser's profit is a linear function of its budget and does not depend on the rival's budget. However, when r is relatively low, both equilibrium bids and profits are influenced by the advertisers' budgets. In particular, the high-budget advertiser jams the low-budget advertiser, and the bid level decreases with budgets. In fact, advertisers shade more (from bidding their valuations) when their budgets become more constrained. Furthermore, advertiser profits are increasing in their own budget but decreasing in the rival's budget: that is, budgets are strategic substitutes.

From Lemma 1, we can see that the jammee's budget is fully spent, whereas the jammer's budget has slack. Therefore, one might argue that the jammer can increase its profit by *overstating* its budget to the search engine. We interpret hard budgets to mean that the advertisers must take into account the possibility that the search engine's second price algorithm could mistakenly assign them to the ad slot first, and then, the hard budget could be violated. Indeed, if the jammer is mistakenly assigned to ad slot first and charged its equilibrium bid (b^*) per click (minus one smallest discrete unit of payment), then it will fully exhaust its budget. Thus, if the probability of mistake by search engine's algorithm is positive,[11] then advertisers have no incentive to overstate their budgets.[12]

It is also useful to see how search engine's revenue is affected by different parameters. First, platform revenue is increasing in advertiser's budgets. Higher budgets increase the advertisers' incentives to bid and thus, the platform's revenue. Second, platform's revenue is weakly increasing in its traffic and reserve price. When reserve price is relatively high, $r > (B_1 + B_2)/c$, then advertisers exhaust their budgets. Thus, platform's revenue is equal to the sum of the budgets. In this case, platform revenue does not depend on its click volume or reserve price. However, if $r \leq (B_1 + B_2)/c$, then search engine's revenue is equal to $\pi_{SE} = B_L + (c - B_L/b^*)r$. The first part is the revenue generated from clicks received by jammee at the bid price. This is equal to the smaller budget, because the low-budget advertiser exhausts its budget. The second part is the revenue generated from clicks received by jammer at the reserve price. This second part is increasing in both c and r. With higher c, the positive effect (thorough click volume) dominates the negative effect (through lower bid level). With higher r, again the positive effect of higher price per click dominates the negative effect of fewer clicks. We summarize these results in the following corollary.

Corollary 1. *In a single-platform world, search engine's revenue is strictly increasing in advertiser budgets and weakly increasing in reserve price and platform traffic.*

The analysis of this section demonstrates that the budget in a search engine determines both equilibrium bid and advertisers' profits in that search engine. In particular, an advertiser's allocation of budget across search platforms has strategic effects on its own and the rival's profit. By moving the budget from one platform to the other, its effect on the profits across search engines is in opposite directions, and therefore, the net effect is not obvious but must be analyzed explicitly. We do this next by working backward to characterize the equilibrium allocation strategies.

3.2. Allocation-Stage Equilibrium

Denote advertisers' equilibrium allocation strategies by δ_1^* and δ_2^*. Recall that, in the fully symmetric model, the total budgets of the two advertisers are equal, $T_1 = T_2 = T$. Moreover, A_i allocates $B_i^1 \triangleq \delta_i T$ to SE_1 and $B_i^2 \triangleq (1 - \delta_i)T$ to SE_2. To obtain equilibrium allocation strategies, we characterize $\rho_i(\delta)$, the A_i's best response to competitor's allocation strategy δ. The point of intersection of advertisers' best response functions corresponds to a Nash equilibrium.

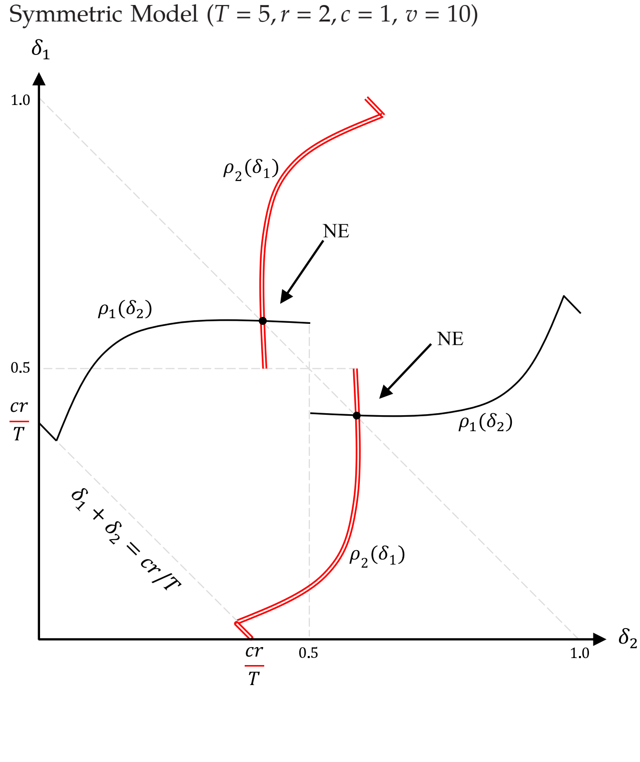
In deriving best response functions, we exploit two types of symmetry that exist in the fully symmetric model to simplify exposition. First, advertisers' best response functions should be identical, because advertisers are symmetric. Therefore, we derive only A_1's best response to A_2's choice of δ_2. Second, platforms are also fully symmetric. In essence, we can swap the platform names, because they are identical.

Thus, it is sufficient for us to derive the best response function only for $0 \le \delta \le 0.5$. The second part of the graph would be a mirror image of the first part.

The explicit formula for the best response function $\rho_1(\delta)$ and the details of the derivation can be found in the online appendix. We briefly outline our procedure for deriving $\rho_1(\delta)$. First, A_1's total profit over both platforms $\pi_1(\delta_1, \delta_2)$ takes on different functional forms depending on the values of δ_1, δ_2. For example, if both δ_1, δ_2 were both small, then the sum of budgets would be small in SE_1 and large in SE_2. Then, according to Lemma 1, advertisers' profits in SE_1 will be of $\tilde{\pi}(.)$ form and the form of $\overline{\pi}(.,.)$ or $\underline{\pi}(.,.)$ in SE_2. We identify five different cases covering the entire $\delta_1 - \delta_2$ space, with $\pi_1(\delta_1, \delta_2)$ differing in each case. Second, we characterize the effect of A_1's allocation strategy δ_1 on $\pi_1(\delta_1, \delta_2)$ in each of the five cases. We then construct the best response function by comparing profit maximizing allocation strategy across cases. Advertisers' best response functions and their intersections (NE) are illustrated in Figure 1.

Let us now discuss the rationale and intuition behind A_1's best response function. Suppose that A_2 has assigned its entire budget to SE_2: that is, $\delta_2 = 0$. One may think that A_1's best response then could be to differentiate maximally by assigning its entire budget to SE_1. By doing so, each advertiser can get all of the clicks in their platform at the cheap reserve price. However, this will leave some part of the budget unused when reserve price is not too high. Therefore, A_1 can do better by moving the unused portion of its budget to SE_2 and earn more profit. Consequently, the best response to $\delta_2 = 0$ will be to allocate just enough budget to SE_1 to obtain all of the potential clicks (at reserve price) and allocate the remaining budget to SE_2. This implies that $\rho_1(0) = cr/T$ as shown in Figure 1.

Figure 1. (Color online) Best Response Functions in Fully Symmetric Model ($T = 5, r = 2, c = 1, v = 10$)



When $\delta_2 > 0$ and A_2 allocates some part of its budget to SE_1, Advertiser 1's best response will be such as to keep the price at its minimum in SE_1. Recall that, per Lemma 1, bid level will be at reserve price if and only if the sum of budgets is not larger than cr. Thus, A_1 decreases its budget in SE_1 so that the sum of advertisers' budget in SE_1 does not exceed this threshold. Because A_2 has allocated most of its budget to SE_2, the marginal benefit of a budget increase for A_1 is lower in that platform. Consequently, A_1 wants to put its budget in SE_1 as long as the price is at the reserve price. This implies that $\rho_1(\delta_2) = cr/T - \delta_2$, which is the negatively sloped line shown in Figure 1.

With even higher δ_2, keeping the bid at reserve price requires a greater reduction in the budget allocated to SE_1 by A_1. This makes advertisers' budgets closer to each other in *both* platforms, making competition fiercer. Thus, A_1's response function starts to *increase* in δ_2 at some point. This is the *turning point* (kink point) in $\rho_1(\delta)$. At this point, the bid level in SE_1 starts to rise from its reserve price minimum. Moreover, A_1 now becomes the jammer in SE_1 (and the jammee in SE_2). This is another reason for A_1 to increase its allocation to SE_1 when δ_2 increases.

Finally, consider $\delta_2 = 0.5$ when A_2 splits its budget equally across platforms. In the online appendix, we show that $\rho_1(\delta_2 = 0.5) \neq 0.5$, and there is a discontinuity at this point (as shown in Figure 1). Thus, A_1's best response is not to split its budget equally across platforms. Intuitively, advertisers are willing to differentiate, at least to some degree, in order to mitigate competition. Therefore, A_1 has an incentive to move part of its budget from one platform to the other. This is because the marginal benefit of becoming a jammer in one platform outweighs the marginal loss of becoming a jammee in the other platform. This partial differentiation equilibrium is a key result of our study, and it is summarized in the following proposition.

Proposition 1. *In the fully symmetric model (symmetric platforms and symmetric advertisers) with $cr < T < cv$, there is a unique, up to renaming the advertisers, asymmetric pure strategy Nash equilibrium in which advertisers partially differentiate; one advertiser allocates $\delta^* > 1/2$ fraction of its budget to Platform 1, whereas the other allocates $\delta^* > 1/2$ fraction of its budget to Platform 2, where*

$$\delta^* = \frac{1}{2} + \frac{1}{4T} \times \left\{2cv - cr - \sqrt{4(cv)^2 - 4T^2 + cr(cr + 4T - 4cv)}\right\}.$$

Proposition 1 shows how symmetric advertisers allocate their limited budgets across symmetric platforms. In particular, advertisers differentiate by focusing more on different platforms. One advertiser allocates a higher share of its budget to Platform 1, and the other advertiser allocates a higher share to Platform 2. However, they do not fully specialize. We can think of this as partial differentiation by platform. Because the outside competition is not too high ($cr < T$), full specialization ($\delta^* = 0$ or 1) cannot be a Nash equilibrium, because the extra remaining budget can be allocated profitably to the other platform. This can be thought of as a *demand force*. Advertisers are willing to allocate their budget equally across symmetric platforms to obtain a higher total number of clicks. What is interesting is that $\delta^* = 0.5$ is not equilibrium. This is because advertisers can benefit from differentiation and mitigate competition by moving a fraction of their budget from one platform to the other. The advertiser's tendency to differentiate is a *strategic force*. Differentiation decreases the average price per click and thus, increases the advertiser's total profit. In fact, the total adverting expenditure is minimized when one advertiser allocates its entire budget to Platform 1 and the other advertiser allocates none of its budget to Platform 1. The strategic force motivates advertisers to differentiate, whereas the demand force incentivizes them to spread their budget more evenly across platforms. The outcome is partial differentiation. This is analogous to the classical differentiation literature, where sellers benefit from differentiation by being able to increase price and mitigate competition. The critical difference, however, is that, in classical differentiation, sellers rely on asymmetry in customer valuation or preference. However, the differentiation in our context arises through advertisers' bidding and budgeting mechanisms.

It also important to see how the *degree of differentiation* (D) defined as the difference between advertisers' equilibrium allocation strategies,

$$\begin{aligned} D &= |\delta_1^* - \delta_2^*| \\ &= \frac{1}{2T}\left\{2cv - cr - \sqrt{4(cv)^2 - 4T^2 + cr(cr + 4T - 4cv)}\right\}, \end{aligned}$$

changes with reserve price r, traffic c, and total budgets T. Corollary 2 establishes this important result.

Corollary 2. *When $cr < T$, the degree of differentiation $D = |\delta_1^* - \delta_2^*|$ is increasing in advertiser budgets T and decreasing in platforms' reserve price r and traffic c.*

Thus, when reserve prices are at their lowest level, $r = 0$, the maximum differentiation occurs in equilibrium: that is, $D = \frac{1}{2T}(2cv - \sqrt{4(cv)^2 - 4T^2})$. We note that this maximum differentiation is not "full differentiation" (that is, $D < 1$), because each advertiser still invests a strictly positive portion of its budget in each of the platforms. As reserve prices increase from this lowest level, the benefit of partial differentiation decreases, and thus, the degree of differentiation (D) becomes smaller. When reserve prices approach T/c, D approaches zero. In other words, the benefit of differentiation, as reflected in the difference between equilibrium bid level and reserve price, disappears when reserve prices become sufficiently large. This implies that the partial differentiation derived in Proposition 1 relies on the assumption that the reserve prices are sufficiently low ($r < T/c$). When the reserve price is larger than T/c, advertisers' allocation strategies are such that the equilibrium bid level remains equal to the reserve price r in both search engines. In this case, proportional allocation by advertiser i, $\delta_i^* = \frac{c_i}{c_1 + c_2} = \frac{1}{2}$ (because $c_1 = c_2$), keeps the total budgets allocated to each search engine below cr, resulting in bids equal to reserve price in both platforms.[13]

As mentioned earlier, one interpretation of the reserve price r is the level of outside competition (that is, the maximum bid among other not strategic advertisers). Thus, our results indicate that the two strategic advertisers partially differentiate when outside competition is low, and they allocate budgets proportional to platform's market share when it is sufficiently high. When r is interpreted as the reserve price that is actually set by the platforms, anecdotal evidence suggests that search engines, such as Google and Bing, may not be imposing a high reserve price on advertisers, which is consistent with our assumption required for partial differentiation. One reason could be the fact that search platforms consider many diverse factors when optimally setting their reserve prices. For example, they may set r such that it is applicable to all keywords in a category, not only one specific keyword (advertisers' valuations for different keywords might be different). In fact, according to one of the Google employees,[14] "Theoretically, if no other advertisers were bidding on a particular keyword for which we [Google] were showing ads, your CPC [which would be equal to r] would be \$0.01." As another example, one of the Inside AdWords crew explains that CPCs as low as \$0.01 can be achieved by creating highly relevant ads.[15] Furthermore, some AdWords users reported that they indeed achieved CPCs in the range \$0.01 to \$0.05.[16] Although these pieces of anecdotal evidence suggest that the reserve prices across major search platforms can be as low as a few cents, the aggregate data from Google's AdWords and Bing's Adcenter suggest that top-ranked advertisers pay, on average, a much higher cost per click.[17] As a result, there seems to be an opportunity for advertisers to implement partial differentiation as a profitable strategy to allocate their budgets across search engines. That said, in Section 5, we ask how search engines should choose their reserve prices strategically to maximize their revenues and how that affects the partial differentiation strategy of advertisers.[18]

To better understand the critical role of multiple platforms, we compare advertisers' bids and profits in the main model with an alternative model when there is only one search engine. To make the comparison meaningful, we assume that there exists a single search engine that can produce $2c$ $(= c + c)$ number of clicks for the advertisers. This assumption helps us to fix the effect of the number of clicks on bids and profits and instead, concentrate on how the existence of the second platform changes the bid levels and profits. The following proposition summarizes these results.

Proposition 2. *Advertisers' profits are strictly higher when there are two platforms compared with the case when there is a single platform producing the same combined amount of traffic.*

Proposition 2 is an important result on multiplatform search advertising. Advertisers can exploit multiple search platforms strategically to mitigate competition and obtain a higher share of the total surplus. When there is only one search engine for two competing symmetric advertisers, each advertiser will obtain the ad slot half the time. Therefore, each advertiser will be a jammer or jammee with equal probability, and neither can dominate the other. However, with two search engines, advertisers allocate their budgets across platforms so that each becomes the jammer in one platform. Neither advertiser can bid jam on both search engines, and this leads to differentiated strategies that prevent the search engines from maximally extracting surplus even with a generalized second price auction. The strategy results in a lower average cost per click and therefore, higher profits. Thus, it is the expanded possibility in bidding strategies that multiple search engines offer that is responsible for advertisers increasing their profits. Of course, in practice, search engines may also offer advertisers other incentives to allocate more of their budget to them. What we have shown is that, even without that sort of explicit search engine competition, advertisers can benefit when there is more than one search engine.

4. Asymmetric Platforms or Advertisers

4.1. Asymmetry in Search Engine Click Volume

What is the effect of asymmetry in search engine click volumes ($c_1 \neq c_2$) on advertisers' allocation strategies? Remember that we denote the ratio $c_j/(c_1 + c_2)$ to be the SE_j's *attractiveness* or *relative market share*. This ratio reflects the ability of SE_j to produce leads for advertisers relative to the other search engine. It is not clear a priori that a platform with higher attractiveness will attract a higher share of advertisers' limited budgets. This is because more allocation of budgets to a search engine might intensify competition and increase bid level. This, in turn, might cause advertisers to move their budgets away from the attractive search engine. Furthermore, advertiser valuation for a click (v) is constant, and it does not depend on the search engine from which the click is received. Consequently, one might argue that the platform attractiveness should not have any effect on the allocation strategies of the advertisers.

Similar to the base model, we assume that advertiser budgets are limited: that is, $T < c_j v$ for $j \in \{1, 2\}$. From Lemma 1, we know that, for a single platform, the equilibrium bid is $b^* = \frac{(B_H + B_L)v - B_H r}{c(v-r) + B_L}$, which is decreasing in c. Higher c effectively means lower budgets, which leads to lower equilibrium bid. Advertisers' profits π_L^* and π_H^*, however, are increasing in c. Intuitively, low budget's profit increases, because it pays less for each click, and therefore, it affords more clicks. High budget's profit increases, because it receives more of the remaining clicks. The following proposition summarizes our results on the allocation of budget across platforms with asymmetric click volumes. For the sake of exposition and tractability, we assume that reserve prices are zero.

Proposition 3. (i) *Advertisers together allocate a higher share of budgets to the platform with higher attractiveness (that is, $\delta_1^* + \delta_2^* >=< 1$) if and only if $c_1 >=< c_2$.* (ii) *Advertisers partially differentiate; one advertiser allocates a share of its budget to SE_1 that is greater than the attractiveness of SE_1 and a share of its budget to SE_2 that is less than the attractiveness of SE_2, whereas the other advertiser does the opposite. Mathematically, allocation strategies δ_1^*, δ_2^* are not equal and satisfy $\delta_1^* < \frac{c_1}{c_1+c_2} < \delta_2^*$.* (iii) *Allocation strategies δ_1^*, δ_2^* solve the simultaneous equations:*

$$\left(\frac{\delta_1^* + \delta_2^*}{2 - \delta_1^* - \delta_2^*}\right)^2 = \frac{(c_1 v + \delta_2^* T)\delta_2^*}{(c_2 v - (1 - \delta_2^*)T)(1 - \delta_2^*)} = \frac{(c_1 v - \delta_1^* T)\delta_1^*}{(c_2 v + (1 - \delta_1^*)T)(1 - \delta_1^*)}.$$

Proposition 3 illustrates how platform asymmetry in the ability to produce clicks influences advertisers' budget allocations. First, higher budgets are allocated to the platform that generates more traffic. Intuitively, other things being equal, the higher number of clicks decreases the bid price in a platform, incentivizing advertisers to move their budget to that platform. This is a demand force that incentivizes advertisers to move their budgets toward the platform that generates more click volume. However, advertisers can still benefit by deviating from the proportional (symmetric) allocations. This is because of the strategic force. Partial differentiation, allocating a relatively lower share of the budget to one platform and a higher share to the other, helps advertisers to benefit from dampened price competition and reduced average cost per clicks. The following example illustrates this partial differentiation.

Example 2. Suppose that $c_1 = 3, c_2 = 1, T = 5, r = 0$, $v = 10$. Therefore, SE_1's and SE_2's attractiveness values are 75% and 25%, respectively. The solution of the equations in Proposition 3 yields $\delta_1^* = 0.71$, $\delta_2^* = 0.8$. Therefore, one of the advertisers allocates 80% to SE_1 (higher than SE_1's attractiveness), whereas the other allocates 71% to SE_1 (lower than SE_1's attractiveness). Moreover, bids are $b^1 = 2.25$ in SE_1 and $b^2 = 2.22$ in SE_2. Therefore, the bid level is higher in SE_1, the more attractive search engine.

Another interesting question is how a platform's attractiveness affects the bid level in each platform and platform revenues. Suppose that SE_1's traffic (c_1) increases. The bid level in SE_1 can decrease, because the jammee finds it attractive to decrease its bid. However, the bid level might as well increase, because advertisers allocate more budgets to SE_1. Hence, the net effect of an increase in SE_1's traffic on the bid level is not obvious. Our analysis shows that the equilibrium bid level is higher in the platform that is more attractive (that is, has higher traffic). We note that a platform's revenue is equal to the budget of the jammee.[19] Therefore, Proposition 3 predicts that a platform's revenue increases when its traffic increases, whereas its rival loses revenue. We summarize these results in Corollary 3.

Corollary 3. *When advertisers allocate budgets across platforms, bid level and platform revenue are both increasing in a platform's click volume.*

Corollary 3 has interesting insights about platforms' revenues and bid levels in a multiplatform environment. Recall that, in Corollary 1, we showed that the bid level is decreasing in a single-platform market. The reason is that advertisers' budgets were fixed, and higher click volume dampened the bidding competition. However, with two platforms, there is an important second effect: higher click volume draws more budget investments into a platform. This, in turn, increases both revenue and bid competition in a platform. This is consistent with what we often see in the real world: bid levels are higher in larger platforms, such as Google, than smaller ones, such as Bing.

So far, in this section we assumed reserve prices to be zero. With a positive, sufficiently low reserve price, the results stated in Proposition 3 would not change qualitatively. Similar to the symmetric model in Section 3, increasing reserve price decreases the benefits of differentiation. Figure 2 shows the advertisers' allocation strategies as a function of reserve price ($r_1 = r_2 = r$) using parameters of Example 2. As can be seen, equilibrium allocation strategies in Platform 1, δ_1^* and δ_2^*, become closer to each other and closer to the market share of Platform 1 ($\frac{c_1}{c_1+c_2}$) as reserve price r increases. When reserve price approaches $T/\bar{c}$ form below, where $\bar{c} = \frac{c_1+c_2}{2}$ is the average click volume across platforms, the sequence of equilibrium allocation strategies converges to the proportional allocation (that is, $\delta_1^* = \delta_2^* = \frac{c_1}{c_1+c_2}$).

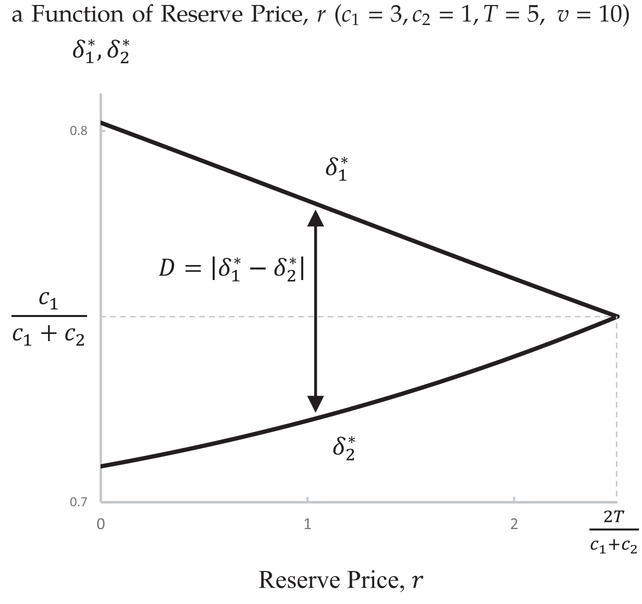
4.2. Asymmetry in Platform Reserve Prices

In Section 3, we assumed that platforms are symmetric in their reserve prices. In this section, we relax this assumption and ask how the asymmetry in platforms' reserve prices can influence advertisers' allocation of budget to each platform. Because the profits of advertisers in each platform are decreasing in the platform's reserve price, we expect that advertisers are less willing to allocate their budgets to the platform that has a higher reserve price. We prove and summarize this result in the following proposition.

Proposition 4 (Asymmetry in Platform Reserve Price $r_1 \neq r_2$)**.** *When r_1, r_2 are low,* (i) *advertisers together allocate a higher share of budgets to the platform with a lower reserve price. Mathematically, $\delta_1^* + \delta_2^* >=< 1$ if and only if $r_1 <=> r_2$.* (ii) *Advertisers partially differentiate; their allocation strategies δ_1^*, δ_2^* are obtained from the joint equations*:

$$\left(\frac{\delta_1^*(v-r_1)+\delta_2^* v}{(1-\delta_1^*)v+(1-\delta_2^*)(v-r_2)}\,\frac{v-r_2}{v-r_1}\right)^2 = \frac{\delta_2^*\big(c(v-r_1)+\delta_2^* T\big)}{(1-\delta_2^*)\big(cv-(1-\delta_2^*)T\big)} = \frac{\delta_1^*\big(cv-\delta_1^* T\big)}{(1-\delta_1^*)\big(c(v-r_2)+(1-\delta_1^*)T\big)}.$$

Figure 2. Advertisers' Allocation Strategies, δ_1^* and δ_2^*, as a Function of Reserve Price, r ($c_1 = 3, c_2 = 1, T = 5,\ v = 10$)



Thus, a platform with lower reserve price can attract a higher portion of advertisers' budgets in equilibrium. This happens, because advertisers' profits increase when reserve price goes down. However, a lower reserve price does not attract the advertisers' budgets fully. In fact, differentiation across platforms can still improve profits by mitigating the bidding competition and lowering the average bid price.

Proposition 4 characterizes the allocation strategies for advertisers when reserve prices are not too high. Let us now consider the situation where one or both reserve prices are high relative to the advertisers' budget per click, T/c. When one of the reserve prices (for example, r_1) becomes much larger than the other platform's reserve prices ($r_1 \gg r_2$), advertisers decrease and finally, stop allocating budgets to Platform 1. This occurs, because the higher reserve price r_1 can become larger than the equilibrium bid level in the rival platform (b_2^*). In this case, it is not beneficial to allocate any amount of budget to Platform 1. Therefore, Platform 1 does not receive any investment. When both reserve prices increase together, the benefit of differentiation shrinks as was illustrated in Figure 2. When reserve prices become sufficiently large ($T/c < r_1, r_2$), the bid level in each platform becomes equal to the reserve price in that platform, and advertiser budgets are fully exhausted in both platforms. Of course, the platform with the lower reserve price, Platform 2, will get a higher share of the budgets, just enough to pay for all of the clicks at the reserve price: that is, $\delta_1^* T + \delta_2^* T = cr_2$. Platform 1 will receive the rest of the budgets, which would not be enough to pay for every click: that is, $(1-\delta_1^*)T + (1-\delta_2^*)T < cr_1$.

The fact that advertisers put more money where reserve price is lower gives rise to a natural question: what would be the optimal reserve price policy for the platforms that want to maximize their revenues? We investigate this in Section 5.

4.3. Asymmetry in Advertisers' Total Budgets

In practice, advertisers' budgets may not be equal. The total amount of budget assigned for online advertising and in particular, search engine advertising might vary depending on the firm's total sales or profit or other considerations. Therefore, we ask the following: how does asymmetry in advertisers' total budgets affect their allocation strategies in equilibrium?

To understand the effect of budget asymmetry, we start from the single-platform results in Lemma 1. In particular, from the profit functions of jammee and jammer, one could observe that the sum of the advertisers' profits ($\pi_L^* + \pi_H^*$) is increasing in *budget asymmetry*. To be more specific, suppose that $B_H = B + \epsilon$ and $B_L = B - \epsilon$, where the ϵ captures the budget asymmetry. If ϵ is zero, budgets are equal. The budget asymmetry increases when ϵ increases. In the single-platform setting with zero reserve price, we have $\pi_H^* + \pi_L^* = cv - B_L$, which increases when ϵ (asymmetry) increases. In fact, this is the main reason that symmetric allocation is not a Nash equilibrium in the full symmetric setting, noting that a small deviation from the symmetric allocation increases both advertisers' profits across two platforms.

We now return to the allocation strategies across platforms when advertisers' total budgets are asymmetric. In particular, suppose that $T_H = T + \epsilon$ and $T_L = T - \epsilon$. When the asymmetry (ϵ) is small, the symmetric strategies will result in advertiser budgets close to each other in each platform ($B_1^j \approx B_2^j$). In this situation, a reallocation of the total budget, which would result in one advertiser becoming a jammee in one platform and a jammer in the other platform, can benefit both advertisers. This deviation increases the asymmetry *within* each platform, resulting in Pareto superior outcomes. However, when the asymmetry in total budgets (ϵ) is high, this may not be the case. Proposition 4 characterizes the full picture. For the sake of exposition, we have assumed that $r = 0$.

Proposition 5. (i) *When advertisers' total budgets are sufficiently close, advertisers partially differentiate: $\delta_H^* \neq \delta_L^*$; equilibrium allocation strategies, δ_L^* and δ_H^*, solve the system of equations*

$$\left(\frac{\delta_H^* T_H + \delta_L^* T_L}{(1-\delta_H^*)T_H + (1-\delta_L^*)T_L}\right)^2 = \frac{\delta_L^*(c_1 v \mp \delta_L^* T_L)}{(1-\delta_L^*)(c_2 v \pm (1-\delta_L^*)T_L)} = \frac{\delta_H^*(c_1 v \pm \delta_H^* T_H)}{(1-\delta_H^*)(c_2 v \mp (1-\delta_H^*)T_H)}.$$

(ii) *When advertisers' total budgets are sufficiently asymmetric, both advertisers split their budgets proportional to each platform's attractiveness: that is, $\delta_H^* = \delta_L^* = \frac{c_1}{c_1+c_2}$.*

First, when T_L and T_H are sufficiently close, partial differentiation happens in equilibrium. Similar to the base model, one advertiser becomes a jammer in SE_1 and a jammee in SE_2. Strategic and demand forces resulting in these allocation strategies are similar to the symmetric case.

Second, when the budget asymmetry is high enough, advertisers allocate their budget proportional to the platform's click volume. In this case, the high-budget advertiser becomes the jammer in both platforms and enjoys the clicks at the low reserve price. In fact, the strategic force that we described earlier is absent here. The reason is that the profits cannot be improved by reallocation of the budgets, because the low-budget advertiser remains the jammee in both platforms. As a result, the demand force will be the only driver of equilibrium strategies, and hence, advertisers split their budgets proportional to each platform's attractiveness. Figure 3 illustrates the regions of partial differentiation and proportional allocation. The boundary of these regions, $f(T_i, T_j)$, has been characterized in the proof of Proposition 5 in the online appendix.

5. Endogenous Reserve Prices

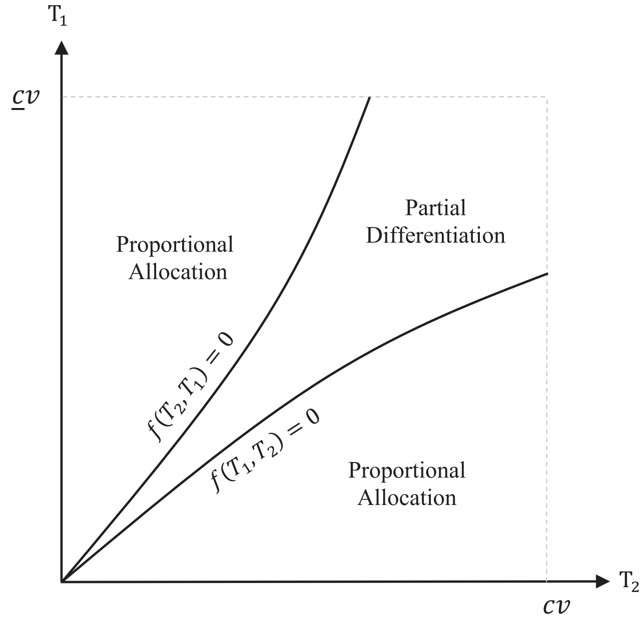
In our analysis so far, we assumed that search engines' reserve prices are exogenous. Moreover, partial differentiation in budget allocation allows advertisers to take advantage of low reserve price. Therefore, a natural question that comes to mind is the following: how should search engines choose their reserve prices strategically to maximize their revenues? To answer this, we analyze the first stage of our model in which platforms choose reserve prices strategically.

Reserve prices affect platform revenues in different ways. On one hand, a higher reserve price increases revenue by helping the platform to make more money on each click that the last advertiser (the jammer) receives. On the other hand, increasing reserve price can decrease revenue, because advertisers might reallocate their budgets away from the platform with a higher reserve price toward the platform with a lower reserve price. Furthermore, higher reserve price decreases advertisers' incentives to invest in search advertising. Faced with a higher minimum CPC, advertisers might prefer to promote their brands using other types of advertising instead of search advertising.[20] To analyze these effects, we assume that platforms set their reserve prices simultaneously in the first stage of the game. Then, symmetric advertisers allocate their budgets (T) across platforms. Finally, advertisers bid in each platform, are assigned to the ad slots, and receive user clicks. The following proposition describes the equilibrium of this full game.

Proposition 6 (Platforms Choose Reserve Prices)**.** *There is an equilibrium in which search engines set reserve prices* $r_1^* = r_2^* = r^* = \frac{2T}{c_1+c_2}$ *and advertisers allocate their budgets proportional to each platform's market share: that is,* $\delta_1^* = \delta_2^* = \frac{c_1}{c_1+c_2}$.

Proposition 6 sheds light on the strategic interactions of search platforms and advertisers in the multiplatform search advertising environment. It states that strategic platforms optimally set their reserve prices at $r_1^* = r_2^* = r^* = \frac{2T}{c_1+c_2}$, the level of reserve prices at which budgets are fully appropriated and the degree of differentiation reaches zero. At any reserve price below the equilibrium level r^*, the marginal increase in platform's revenue owing to increase in earnings per click (direct effect) outweighs the marginal decrease of revenue owing to loss of the total allocated budgets (indirect effect). Moreover, any reserve price above equilibrium r^* would not increase a platform's revenue, because the platform is already fully capturing the advertisers invested budgets.

Figure 3. Regions of Partial Differentiation and Proportional Allocation ($r = 0$, $\underline{c} = \mathrm{Min}(c_1, c_2)$)



What is the effect of endogenous reserve prices on budget allocation strategies by advertisers? Remember that, in previous sections, we showed how symmetric advertisers could exploit partially differentiated allocation strategies to mitigate bid competition when reserve prices were sufficiently low. Figure 4 shows that this differentiation decreases and finally disappears when reserve prices increase. In fact, when $r_1^* = r_2^* = r^*$, there are multiple equilibria in allocation strategies specified by $\delta_1^* + \delta_2^* = \frac{2c_1}{c_1+c_2}$. This implies that any pair of advertisers' allocation strategies that make the total allocated budgets to each platform proportional to its click volume is an equilibrium outcome. This includes not only the partial differentiation type of allocation strategies (that is, $0 < \delta_1^*, \delta_2^* < 1$, and $\delta_1^* \neq \delta_2^*$) but also, proportional allocation (that is, $\delta_1^* = \delta_2^* = \frac{c_1}{c_1+c_2}$).[21] Interestingly, proportional allocation is the equilibrium that survives the trembling hand perfection condition. Consider advertisers' strategies when reserve prices are sufficiently low. In this case, we know that partial differentiation can help advertisers to earn clicks at lower bid prices. Moreover, degree of differentiation, $|\delta_1^* - \delta_2^*|$, is decreasing in reserve prices (see Figure 4). As reserve prices approach their equilibrium level specified in Proposition 6 ($r_1^* = r_2^* = \frac{2T}{c_1+c_2}$), degree of differentiation approaches zero. We, therefore, conclude that the unique trembling hand-perfect equilibrium is proportional allocation (that is, $\delta_1^* = \delta_2^* = \frac{c_1}{c_1+c_2}$), because it remains robust to possible trembles in reserve prices.

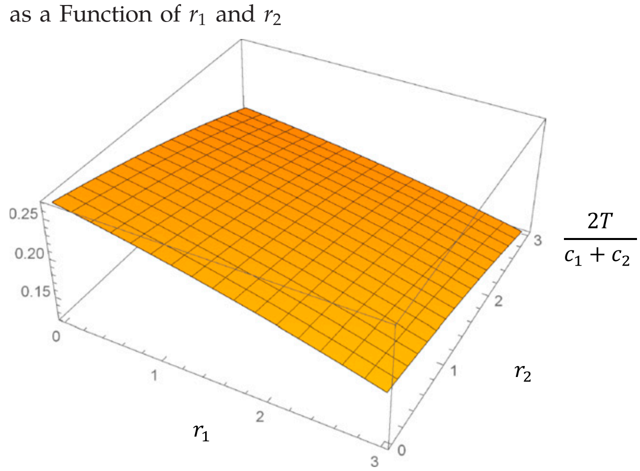
Thus, Proposition 6 tells us that, if search engine reserve prices are chosen to maximize their revenues, then advertisers should allocate their budgets proportional to search engine traffic. In other words, partial differentiation is not an equilibrium outcome under endogenous reserve prices. It is interesting to recall that proportional allocation is the equilibrium outcome not only if search engines are strategic but also, if one of the advertisers has a much larger budget than the other. Finally, our analysis tells search engines to choose a relatively high reserve price, fully exploit the advertisers' allocated budgets, and eliminate their incentives to differentiate across platforms.

6. Managerial Insights, Conclusions, and Future Research

By focusing on a multiplatform environment as well as advertisers' bidding and budgeting decisions, we have analyzed an aspect of search engine advertising that has received relatively less attention in prior work. Our analysis is based on a game-theoretic model of advertisers that may be asymmetric in their budgets or valuations for a click. Advertisers must allocate their budgets across platforms that may be asymmetric in their traffic or reserve prices. Our work provides insights for advertisers and search engines in a multiplatform industry.

To understand the essential forces, we analyzed a fully symmetric model with similar advertisers and similar nonstrategic platforms. We find a novel result; when reserve prices are low, advertisers partially differentiate: one advertiser allocates more of its budget to one platform, whereas the other advertiser focuses on the other platform. This partial differentiation helps reduce the bid competition, lowering cost of search advertising and in turn, boosting profits. In addition, we find that bid jamming occurs in each platform; a high-budget advertiser bids just below a low-budget advertiser to exhaust the rival's budget, move up, and earn more clicks at the lowest cost per click. We show that advertisers' partial differentiation in allocation of budgets remains robust even after accounting for real-world asymmetries in platforms. In particular, a platform that can produce more clicks (or has a lower reserve price) attracts more advertiser budgets. Nevertheless, advertisers still differentiate by allocating unequal portions of their total budgets to this platform.

Figure 4. (Color online) Degree of Differentiation, $|\delta_1^* - \delta_2^*|$, as a Function of r_1 and r_2



When we incorporate advertiser asymmetries in their total budgets or valuations, we find that large asymmetry reduces the benefits of differentiation. In particular, if advertiser budgets (or valuations) are sufficiently different, then advertisers follow a symmetric allocation strategy; each advertiser allocates to each platform a portion of budget equal to the market share of that platform. In fact, the dominance of a high-type advertiser in budget prevents the low-type advertiser from obtaining clicks at the lowest cost. In the absence of strategic force, the demand force leads advertisers to allocate their budgets proportional to each platform's click volume. Interestingly, the same propositional allocation strategies are obtained when platforms are strategic and set their reserve prices to maximize revenues. We show that strategic platforms fully extract advertisers' budgets and eliminate the benefits of differentiation for advertisers.

Our results provide guidance to managers that are concerned with the rising cost of search advertising. Our finding is that a firm can potentially benefit from allocating its budget across multiple search engines. However, managers should pay attention to platforms' reserve prices and click volumes when making these decisions. When reserve prices are low, we show that an advertiser should consider concentrating more resources in one search engine while its rival concentrates on the other search engine. With higher reserve prices, an allocation that is proportional to each platforms' click volume is more appropriate. In addition, a firm that has a large budget advantage should allocate its budget proportional to each platform's attractiveness. Moreover, this allocation should be followed up by a not too aggressive bidding strategy. However, a firm that has a disadvantage in its budget should also allocate its budget proportional to the platform's attractiveness but follow it up with a more aggressive bidding strategy.

Our results provide insights for strategic platforms by helping them better understand advertisers' bidding and budgeting behavior in a multiplatform environment. First, platforms might be able to increase their click volumes by expanding their user bases or improving the attractiveness of their websites. Our analysis reveals that a higher click volume has an indirect and positive effect on a platform's revenue, which is absent in the single-platform world;

it can move the allocation of advertiser budgets away from the rival platforms. As a result, a higher click volume can lead to a higher allocation of budget to a platform, which in turn, can drive the bid levels up in that platform. Second, we provide insight into how setting the right reserve prices can help platforms. When increasing its reserve price, a platform should not only consider the positive effect that a higher reserve price can have on its revenue from each click but also, consider the negative effect that a higher reserve price can have on advertisers' budgets allocated to that platform. We show that platforms can set their reserve prices to fully deplete advertisers' budgets, diminishing the benefits of partial differentiation for advertisers. In setting their optimal reserve prices, platforms need to take into account not only the advertiser's valuation for clicks but also, competing platform's ability to generate clicks for the advertisers.

Our paper could inform empirical work. First, we expect budget-constrained advertisers to bid close to each other. Second, we expect advertisers' differentiation strategy across platforms to be negatively correlated with the degree of asymmetry in their budget-to-valuation ratio. We believe that these predictions could be tested with appropriate data, although we also realize the challenges involved in that, because many factors in addition to budgets, valuations, and platform attractiveness may influence advertisers' bidding and budgeting decisions.

Our work can be extended in different ways. Although we have assumed only two advertisers, in practice there are many advertisers for each keyword. Moreover, there are more than 2 ad spots, sometimes approaching as many as 10. Therefore, a more elaborate model could be appropriate depending on the goal of the research. Another interesting aspect of search engine advertising is the variation in the auctions used, such as automatic bidding, targeted ads, quality score mechanism, and hybrid auctions to name a few. In addition, future research can examine the role of information and timing of the decisions on equilibrium outcomes of search advertising. Finally, little is known about the customers' search and visit behavior across multiple platforms, which in turn, contributes to each platform's ability to generate clicks for advertisers, something that we treat as an exogenous parameter in our paper.

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Endnotes

[1] Many other *specialized* platforms, such as Amazon, Expedia, Priceline, and Facebook, have also adopted search-based advertising to monetize their traffic. This paper's implications will also apply to those platforms.

[2] One may wonder why platforms "force" advertisers to set the budget. Quite possibly, not doing so can cause platforms ill will. There is the possibility that advertisers complain that "we did not want to pay this much" or that "clicks are fraudulent or repetitive." Therefore, by forcing advertisers to set budgets, platforms can offer "peace of mind" to advertisers, assuring them that they would not pay more than what they really intend. Therefore, in practice, the use of a budget constraint along with generalized second price auction is something that, as Edelman et al. (2007) state, "emerged in the wild" (p. 253).

[3] This is under the assumption that platform reserve prices are exogenous and sufficiently low.

[4] This is a plausible assumption, because in the United States, search engines constitute a duopoly, with Google and Y!Bing network holding approximately 65% and 20% market shares, respectively. It is also a duopoly in other countries, such as China (Baidu has 55% and Yahoo 360 has 28%), Russia (Yandex has 58% and Google has 34%), and Japan (Google has 57% and Yahoo has 40%; source: http://goo.gl/YKkbdq, accessed August 8, 2019).

[5] We relax this assumption in Online Appendix, Section II, and we show that our main result is robust when advertisers' total budgets are not too high.

[6] In short, we find that budget and valuation are two opposite sides of the same coin; advertisers budget-to-valuation (T_i/v_i) ratios determine their equilibrium budget allocation and bidding strategies.

[7] The exogenous advertising budgets are a common assumption in the literature (Sayedi et al. 2014, Lu et al. 2015, Shin 2015). Moreover, with numerous online tools for keyword research, firms can obtain information on the amount of money that their competitors spend on search advertising and infer their budgets. For example, www.spyfu.com claims that it provides competitors' keywords, bids, and daily spending. In addition, in Online Appendix, Section II, we analyzed three variations of the model—endogenous budgets, incomplete information, and simultaneous bidding/budgeting setting—to provide support for the robustness of our results with respect to these assumptions.

[8] In practice, search engines weight advertisers' bids by their *quality scores*, which are a measure of advertiser's ad relevance, landing page quality, and expected click-through rate. We abstract away from the quality score mechanism for simplicity, because it does not affect our result.

[9] Note that we use T to refer to *Total* budget, whereas capital B refers to the allocated budget to each platform (and hence, $T_i = B_i^1 + B_i^2$). We retain this notation consistently throughout the paper.

[10] We thank the review team for suggesting that we extend our analysis and endogenize platform reserve prices to explore the strategic interactions between advertisers and search engines.

[11] If the jammer pays the reserve price for each click after a mistake, then it exhausts its budget if and only if $B_H < cr$. The spirit of the "possibility of mistake" argument is that of *trembling hand perfection* in the sense that the hard budget commitment should remain a best response even if the search engine makes a mistake.

[12] We thank an anonymous reviewer for suggesting this point.

[13] When $cr > T$, there are multiple equilibria in allocation strategies: any δ_1^*, δ_2^* satisfying $2 - \frac{cr}{T} < \delta_1^* + \delta_2^* < \frac{cr}{T}$. All of these equilibria are

payoff equivalent for advertisers and platforms. The "proportional allocation," $\delta_1^* = \delta_2^* = 0.5$, is always an equilibrium when $cr > T$, and it is the unique equilibrium when $cr = T$.

[14] Kelly Williams, senior account manager at Google (https://goo.gl/frFvTJ; accessed March 8, 2019).

[15] See https://goo.gl/rMpoqy (accessed March 8, 2019).

[16] See comments by "Michelle" and "Ydwer" at https://goo.gl/k3yGMX (accessed March 8, 2019).

[17] For example, Bing's "Suggested Bid" for keywords "Flowers Deliver," "Car Insurance," "Web Hosting," "Online Degree," and "Plumber Near Me" are $4.83, $8.87, $7.05, $4.61, and $6.63, respectively.

[18] We thank the SE for suggesting that we include the real-world casual evidence on bids and reserve prices.

[19] In this section, we assumed zero reserve prices. Therefore, the search engine revenue is equal to the budget of jammee. With positive reserve price, the results in Corollary 3 would remain unchanged. In fact, higher click volume can increase the platform revenue even more, because now, there is a second source of revenue via remaining clicks received by the jammer at reserve price.

[20] To capture this last effect, we further endogenized the total budget decisions of advertisers by assuming that there is an opportunity cost of λT^2 for setting aside a budget of T. We find that advertisers set budgets $T_1^* = T_2^* = T^* = \frac{(c_1+c_2)r^*}{2}$ and that platforms set $r_1^* = r_2^* = r^*$, where $r^* = \frac{\sqrt{1+4\lambda v(c_1+c_2)}-1}{2\lambda(c_1+c_2)}$.

[21] The full differentiation allocation strategy (that is, $\min(\delta_1^*, \delta_2^*) = 0$ and $\max(\delta_1^*, \delta_2^*) = 1$) can be an equilibrium only if $c_1 = c_2$, and it is not a trembling hand-perfect equilibrium.

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