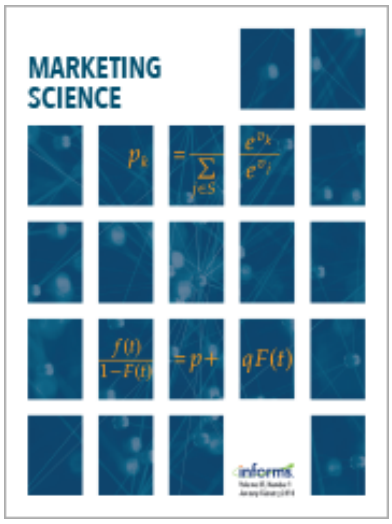

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Limited Edition Products: When and When Not to Offer Them

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Many brands today introduce limited edition (LE) products as part of their product line. However, little is known about the conditions under which a brand should introduce an LE product or the competitive implications of doing so. We investigate this issue using a game theoretic model of a market where two brands compete for consumers who desire exclusivity. Our analysis shows that adding an LE product has a positive direct effect on brand profits through the increased willingness of consumers to pay for such a product, but also has a negative strategic effect by increasing price competition between brands. These effects result in different conclusions depending on the nature of brand differentiation. When brands differ in quality, we show that only the high-quality brand may gain in comparison to a scenario where there are no LE products. Although a low-quality brand may offer an LE product as a defensive strategy, its profits are lower than would be in a world without LE products because of the negative strategic effect. When we consider brands that are differentiated on a horizontal attribute such as taste, we find that the negative strategic effects cause lower equilibrium profits if both brands introduce LE products. Yet brands cannot avoid introducing LE products because they face a *prisoners' dilemma*.

Key words: game theory; marketing strategy; product management; pricing research; limited edition products
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1. Introduction

Many brands today are introducing limited edition (LE) products as part of their product line. Steinway's LE pianos are classic examples of this strategy. In 1993, on its 140th anniversary, Steinway offered an LE piano with a special mahogany finish. Interestingly, production was limited to only 140 grand pianos, which sold out to dealers within hours of availability (*Steinway & Sons* 1999). In 2004, Steinway offered the Roger Williams Limited Edition Gold Piano to honor renowned pianist Roger Williams. Only eight of these pianos were built. Steinway has a long history of offering LE pianos. However, in 2006 their competitor, Yamaha, also offered an LE piano: the Elton John Limited Edition Signature Series Red Piano, autographed by the pop music star. Most notably, Yamaha announced that only 30 pianos would be produced initially and that no more than 50 would be sold (*The Australian* 2006).

LE products have also been used in other industries such as automobiles (e.g., Jaguar XKR Silverstone Coupe; see *Automotive News* 2000) and in fashion goods (e.g., Prada's $750 boots; see *Globe and Mail* 1999). However, there has been little discussion of the rationale for such products in the academic literature. A motivation for LE products that is often discussed in the business press is to engender scarcity by limiting the quantity of a product (*Fortune* 2005). Scarcity provides a sense of exclusivity to the customer (Brown 2001). However, the competitive implications of offering such LE products have not been analyzed. In this paper, we evaluate the competitive implications of offering LE products as part of a brand's product line in a setting where customers value exclusivity. In particular, we address the following research questions: If brands are differentiated in terms of quality or desirability, should LE products be offered by the high-quality brand, the low-quality brand, or both? If the difference between the brands is one of taste rather than quality or desirability, should LE products be offered by the brands? What are the profit implications of LE products under the above scenarios?

We consider a model of two brands competing for consumers in a product category. We assume that consumers value exclusivity in the product. We model the preference for exclusivity by assuming that a consumer incurs a utility loss if he encounters another purchaser of the identical product (where product

refers to a particular branded item). Furthermore, we assume that a brand can sell at most two versions of the product, a regular version and an LE version. The LE product is operationalized by assuming that the offering brand makes a credible quantity commitment for this product. The regular version is, however, available to all consumers who are willing to pay the price. After choosing their respective product lines, the brands compete on prices. Our principal findings are as follows. When a brand offers an LE product as part of its product line, it surprisingly aggravates price competition. The increase in price competition depresses profits, but the brand can gain because of the LE product's price premium and because it sells a larger overall quantity. These effects lead to different implications depending on how the competing brands are differentiated. We first consider the case when the two brands differ in quality. In this case, both brands may introduce an LE product in equilibrium, although it is only the high-quality brand that may gain in comparison to a scenario where there are no LE products. Thus, the low-quality brand would prefer that neither competitor introduce an LE product, but is forced to respond with one of its own because of the anticipated introduction of an LE product by the high-quality brand. In other words, the more intense price competition resulting from LE products has a greater adverse effect on the low-quality brand. In contrast, our results show that a monopoly brand, irrespective of quality, gains by offering an LE product in addition to its regular product. In doing so, the monopoly brand taps into the additional value created as each of its products becomes more exclusive, but does not suffer from the strategic effects induced by competition.

In other findings on the competition between quality-differentiated brands, we show that if there is a cost to introduce an LE product, and if this cost is sufficiently high, only the high-quality brand introduces an LE product.

Next, we consider the case where the perceived difference between the brands is one of taste rather than quality. In this case, we find, interestingly, that introduction of LE products results in lower profits for both brands because of the escalation of price competition.

While LE products have not been considered specifically by past research, there are a few papers that show why a firm may limit the quantity of its product and induce a situation of excess demand (scarcity) for the product. DeGraba (1995) suggests that a firm makes a product scarce to induce a buying frenzy among consumers who, to avoid being rationed, rush to buy before being fully informed about their valuation of the product. He shows that by doing so, the firm can sell the product at consumers' expected valuation, which allows it to obtain more of the consumer surplus and maximize profits. On the other hand, Stock and Balachander (2005) show that a firm may induce scarcity of a product to signal its high quality to uninformed consumers. Observing the scarcity, uninformed consumers infer that informed consumers are buying the product implying that the product must be of high quality.[1] In another related paper, Becker (1991) shows that tables for customers may be scarce at a restaurant, and yet the restaurant would not raise prices when there are positive demand externalities for social reasons. However, Becker's result assumes that capacity is exogenous.

Our paper differs from the above literature in the following ways. First, the above papers do not assume that consumers value exclusivity. Their focus is on identifying reasons other than exclusivity for producing a limited quantity of a product.[2] On the other hand, we study the impact of offering products in limited quantities when consumers value exclusivity. Second, and more important, we examine the competitive implications of LE products while the above papers consider a monopoly setting. Third, we examine the implications of an LE product as part of a brand's product line while the above papers consider a single product. Many firms offer LE products as part of their product line consistent with our model assumption. Last, in contrast to the above papers, our study shows that limiting production of a product can be an optimal strategy for a brand, even without an objective of creating a buying frenzy or of signaling quality. Furthermore, unlike Becker (1991), an LE strategy can be optimal without the presence of positive demand externalities. Indeed, our model assumes negative demand externalities by assuming that consumers value exclusivity.

Another related literature is one that pertains to social effects on consumption and its effect on firms' strategies. Pesendorfer (1995) considers a monopolist who sells a fashion product to consumers who use the product to signal their type to other consumers. As the monopolist reduces the price of a fashion over time to sell to new consumers, the fashion loses its exclusivity and signaling power. This leads the monopolist to periodically create new designs resulting in fashion cycles. Amaldoss and Jain (2005a) also study social effects and show that in a market consisting of consumers who value exclusivity or

[1] In an empirical study, Balachander et al. (2008) test the buying frenzy versus the signaling explanation for scarcity strategies with data from the U.S. car market and find more support for the signaling explanation in this market.

[2] Indeed, a firm can make its product exclusive simply by charging a high price, and this could be more profitable than making it scarce. See also Stock and Balachander (2005) on this point.

uniqueness (snobs) and other consumers who value conformity (followers), the demand curve for the former can be upward-sloping consistent with the idea of conspicuous consumption. Amaldoss and Jain (2005b) extend this model to a competitive setting and find that competitive prices and profits increase with the degree of snobbishness and decrease with the level of conformity. The papers by Pesendorfer (1995) and Amaldoss and Jain (2005a, b), however, do not consider the effect of limiting the quantity of a product, as occurs in the case of LE products.

While the introduction of LE products in our analysis is driven by consumers' desire for exclusivity and the firms' desire to influence price competition, there may be other motivations for LE products that are not considered by our analysis. For example, LE versions of candies and soft drinks are introduced to test demand and to cater to the needs of small segments of consumers with unique tastes (*Wall Street Journal* 2003, *Boston Globe* 2005). In these cases, the products are available for a limited time because of the niche nature of the demand but there appears to be no limiting of the quantity by design. Furthermore, if a product is successful in generating a lot of sales, it is often added to the regular product line. In other cases, an LE product may be simply a subbrand designation to denote a product with additional features, its quantity may be limited by demand rather than by design. This appears to be the case for some LE automobile products such as the Ford Explorer Limited Edition (*Automotive News* 1998). Thus, the key characteristic of an LE product in this paper is that the brand has committed to producing no more than a specific quantity of the LE product.

The rest of the paper is organized as follows: In §2, we introduce the modeling framework. We begin our analysis by considering the case of a monopoly brand introducing an LE product in §3. We then consider the competitive scenarios of vertical and horizontal brand differentiation in §§4 and 5, respectively. We close the paper with a discussion of the results and limitations as well as avenues for further research (§6). We present key proofs in the appendix. Other proofs and additional details are available in an online Technical Appendix.[3]

2. Model

2.1. Brands and Consumers

We consider a product category consisting of two brands, A and B. Each brand can have up to two distinct products (brand items) in its product line as discussed later. The potential market consists of N consumers, each of whom purchases at most one unit in the product category. All consumers value exclusivity in the product; i.e., they attach value to owning a product not possessed by others. To model the consumer's value for exclusivity, we set up a socialization or dating scenario somewhat similar to Pesendorfer (1995). Specifically, each consumer meets exactly one other consumer at random from among the N consumers in the market, and if this second consumer has the same brand item as the first, the product's value for both consumers would decline by k, $k > 0$, where k is a measure of the consumer's value for exclusivity.[4] If the second consumer does not have the same brand item, the value of the product does not decline for either consumer. Thus, the expected value to a consumer from purchasing a brand item depends on the quantity of the brand item sold. If a consumer purchasing a brand item expects a quantity q of this item to be sold, the probability with which this consumer expects to meet another consumer with the same product is $(q-1)/(N-1)$. Suppose this consumer has a value or reservation price v for the brand item provided he is the only purchaser (we refer to v as the base value of a product). Then, if the consumer expects a quantity q of the brand item to be sold, his value for the brand item now becomes equal to $v - k(q-1)/(N-1)$.

It could be argued that the above formulation captures only a portion of potential exclusivity effects. In particular, the proposed utility function may capture what could be called first-order effects stemming from sameness because consumers experience a disutility only when encountering another consumer with the same brand item. However, one can conceive of additional second-order exclusivity effects that go beyond sameness. For example, a consumer with a less unique brand item (a brand item that is sold in larger quantities) may experience a disutility when he meets another consumer with a more unique brand item, while the latter consumer may experience a positive utility from the same encounter. We term this factor the second-order uniqueness effect. Another kind of second-order effect, the second-order price effect, may arise out of differences in price paid for the products. For example, a consumer possessing a brand item with a lower price may experience a disutility when encountering another consumer who possesses a higher-priced brand item with the latter consumer gaining a positive utility increment from the meeting. Note that with our sameness formulation, prices are higher for more unique products in equilibrium, ceteris paribus, as our analysis below

[3] The Technical Appendix can be found at http://mktsci.pubs.informs.org.

[4] We note that the exclusivity may mainly be relevant for products that are used in more public settings such as automobiles, pianos, and apparel.

will show. Thus, the second-order price effect captures exclusivity factors other than sameness (such as brand quality) that affect price differences across products. Note also that the use of the term, "second-order effects," to characterize exclusivity effects other than sameness does not imply that these effects are less significant than the first-order sameness effect captured in our utility function. In sum, the implication of the above discussion of the sources of exclusivity effects is that our model formulation may not capture all kinds of exclusivity effects. However, an advantage of our model is that it is parsimonious. Following our analysis, we discuss how our results may change when incorporating the second-order effects discussed above, but we leave the modeling of these second-order effects to future research. Modeling these additional effects may also benefit from a more structural approach in which brand items are used as signals by consumers (e.g., Pesendorfer 1995, Kuksov 2007) in contrast to the reduced-form approach used in the paper.

With the above-specified basic model of consumer's utility from exclusivity, we consider two cases of differentiation of the brands A and B: (a) vertical differentiation, and (b) horizontal differentiation.

2.2. Vertical Differentiation Case

Here, the brands are assumed to be differentiated on a vertical attribute, i.e., all consumers prefer a brand with more of this attribute to another with less, at equal prices. An example of a vertical attribute commonly analyzed in the modeling literature is quality (Shaked and Sutton 1982; Moorthy 1988; Tirole 1988, p. 296). We will follow the literature in referring to the vertical attribute differentiating the brands as quality. We use a model similar to Shaked and Sutton (1982) except that the qualities of the two brands are given exogenously. Because the two brands are differentiated in quality in the vertical differentiation case, we use the notation H and L (instead of A and B) to refer to the high- and low-quality brands, respectively, in this case. The exogenous qualities of the two brands H and L are given by s_H and s_L, respectively, with $s_H > s_L$. For simplicity, we assume $s_L = 1$ and $s_H = 1 + s$ where s denotes the quality difference between the high- and low-quality products. A consumer's base value (value when the consumer is the only purchaser of the branded item) for a brand of quality $\bar{s}$ is given by $v = \theta\bar{s}$, where θ is a parameter that captures the consumer's value for a unit of quality. Similar to Shaked and Sutton (1982) and Tirole (1988), we assume that consumers' taste for quality is heterogeneous with θ being uniformly distributed across consumers between $\underline{\theta}$ and $\bar{\theta}$, and $\bar{\theta} > \underline{\theta} > 0$.

2.3. Horizontal Differentiation Case

In this case, we use the Hotelling (1929) model with the two brands A and B located at the end points 0 and 1, respectively, of a unit interval $[0, 1]$ representing potential ideal values to consumers of a "horizontal" product attribute. As in the Hotelling model, we assume that the consumers' ideal values of this attribute are uniformly distributed in the interval $[0, 1]$. A consumer's base value for either brand is given by $v = u - xt$, where u is a constant, x is the distance of the consumer's ideal value from the location of the brand under consideration, and t is a "transportation cost" or disutility experienced by the consumer per unit length of deviation of a brand from his ideal value.

With the base value v of the product thus defined in the case of vertical and horizontal differentiation of the two brands, the value of a branded item whose expected quantity sold is q, becomes equal to $v - k(q-1)/(N-1)$, after incorporating the effect of exclusivity as discussed above. We regard the vertical and horizontal differentiation cases as two polar cases of brand differentiation in the real world that are interesting to analyze. In addition, it appears that in many categories one type of differentiation is more important than the other. For example, quality (vertical) differentiation appears to be more relevant for piano and automobile brands while taste (horizontal) differentiation may be more important for brands of jeans. We now describe our assumptions about the product line offered by each brand.

2.4. Brands' Product Lines

Each brand can offer one or two products (referred to as brand items in the paper) as part of its product line. We operationalize an LE product as one for which the brand credibly commits to a production quantity, q. Such an ability to credibly commit to a production quantity seems to be common for brands that offer LE products (e.g., *Steinway & Sons* 1999). Credibility is typically based on a brand's reputation, which serves as a bond that prevents the brand from reneging on the committed quantity of LE product. In many cases, brands carefully cultivate such a reputation through procedures that can assure consumers that the quantity commitment is being met. For example, Steinway currently offers the Henry Z. Steinway Limited Edition pianos with an announced quantity of only 91 units. Each of the pianos will have a brass medallion that displays the serial number, e.g., 1 of 91 (*American Music Teacher* 2007, Steinway 2007). Similarly, Rotary, a brand of designer watches, offers only 500 pieces worldwide of a LE watch called the Rotary VI. Each LE watch carries a serial number between 1 and 500 on the back of its case and comes with a certificate of authenticity (*Retail Jeweller* 2006).

Whether a brand offers an LE product, we assume that the brand always offers a regular version of the product. Unlike the LE product, the regular product has no preannounced quantity and is available to all consumers who are willing to pay the price. Thus, each brand has the following two choices for its product line: (1) the regular version alone, or (2) both the regular and LE versions. Consistent with our assumption that a brand always offers a regular product, note that we do not consider the case where it offers only an LE version. Offering only an LE version amounts to a brand committing not to offer a regular product. For a brand with an established regular product, such a commitment would require withdrawing the regular product, which may be difficult because existing production lines would need to be shut down or because the established regular product may be the basis of the brand's image or differentiation. Thus, with the above product-line assumptions, our model may be interpreted as analyzing the strategic effect of offering an LE product for a brand that has an established regular product. Note that we also assume that a brand offers no more than one LE product. We make this assumption for reasons of analytical tractability. However, later we discuss the implication of relaxing this assumption.

We assume that a consumer's utilities for both the LE product and the regular product carry a penalty—$k(q-1)/(N-1)$—for loss of exclusivity, as discussed above. Whereas consumers use the announced quantity of the LE product to compute the nonexclusivity penalty for this product, they form rational expectations about the quantity sold of the regular version in computing the nonexclusivity penalty for that product (cf. Amaldoss and Jain 2005). Note that a consumer experiences a loss of exclusivity only if the consumer that he meets has the identical brand item. The consumer purchases the brand item that maximizes his surplus, calculated as the item's utility less its price. If a consumer is unable to obtain the most preferred product because of limited availability, we assume that the consumer chooses among the products that are available. We also make the tie-breaking assumption that if a consumer is indifferent between a brand's LE and regular product, he purchases either product with equal probability subject to product availability.

2.5. Product Features and Costs

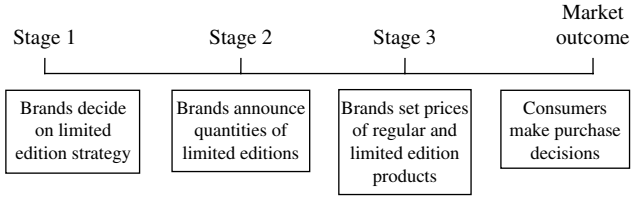
In most of our analysis, we assume that a brand's LE product is identical to the regular version except for the preannounced commitment to quantity in the former case. This assumption allows us to focus on the effect of quantity limitation of the LE product. We subsequently discuss the effect of adding additional features to the LE product. Given the assumption that a brand's LE product and regular version are identical, we assume that both these products have identical marginal costs. We also assume that both brands have identical marginal costs (which we set to zero without loss of generality) in both the horizontal and vertical differentiation cases. This assumption may be restrictive in the vertical differentiation case when brands differ in quality. However, similar to the previous literature on vertical differentiation (Shaked and Sutton 1982, Tirole 1988), this assumption allows the analysis of the demand-side effects of introducing an LE product.

We assume that a brand incurs a fixed cost F ($F > 0$) for introducing an LE product. (Because we assume that a brand always offers a regular version, the cost for introducing this product is irrelevant for our analysis.) Such a fixed cost for introducing a product may include production, product development, and promotion costs (e.g., Netessine and Taylor 2007).

2.6. Game Structure

The sequence of moves involves three stages (see Figure 1). In stage 1, the two brands simultaneously announce whether they will introduce an LE product. Subsequently, in stage 2, the brands simultaneously announce the quantity of their respective LE product, if one is being introduced. Finally, in stage 3, the brands set the prices for their regular and LE product (if applicable). Consumers form rational expectations about the quantity of each brand's regular product that will be sold and make purchase decisions that will maximize their surplus. We solve for the subgame perfect equilibrium of the game using backward induction. As discussed earlier, we do not consider the case where one or both brands offer an LE product alone in stage 1 (commit not to offer a regular product). Note, however, that, in stage 3, we consider the possibility of brands effectively offering only the LE product or the regular product by pricing the other product sufficiently high. This consideration ensures the validity of our stage 3 equilibrium results. Before we analyze brand competition in the presence of LE products, we first consider the case of a monopoly brand to provide a benchmark for the competitive analysis. We subsequently analyze the case of vertically differentiated competing brands, followed by the horizontal differentiation case.

Figure 1 Sequence of the Game



3. Monopoly Case

In this section, we consider the case of a monopoly brand M introducing an LE product, for comparison with the subsequent competitive analysis. For our monopoly framework, we use the Hotelling model outlined in §2, with brand M alone in the market, where it is located at the endpoint 0 of the interval $[0, 1]$. All other assumptions remain the same as in the competitive case. Note that the assumed location of brand M at the endpoint does not affect the generality of the results.[5] Although we use the Hotelling model with two brands in the competitive case to represent consumer taste differences for a brand's horizontal attribute, the same model with one brand can also capture consumer differences in willingness to pay for a brand's quality. To see this, substitute $u = \bar{\theta} s_M$, $t = 1$, and $x = (\bar{\theta} - \theta)s_M$ in the expression for the base value, $v = u - xt$, where s_M is the quality of the monopoly brand. Then, we have a market of consumers whose base value for brand M is given by $v = \theta s_M$, with θ being uniformly distributed in the interval $[\bar{\theta} - (1/s_M), \bar{\theta}]$. In the following analysis, we consider the case where u is sufficiently low that the market is not fully served by the brand at the optimum solution; this allows us to work with interior solutions. The results are similar when the brand serves all consumers.

We use l_M and q_M to denote, respectively, the announced quantity of the LE product, and the quantity sold of the regular product. Furthermore, we define p_{lM} and p_M as the prices of the LE and regular products. We consider two cases, one in which brand M offers only a regular product and a second case where it considers offering an LE product in addition to the regular product. The following proposition summarizes the main results of the monopoly case.

Proposition 1. (i) *When brand M offers only a regular product and the market is not covered, the product's price increases with k, while the quantity sold and profit decrease with k.*

(ii) *Brand M introduces the LE product if and only if $F \leq \widehat{F}$, selling equal quantities of the LE and regular products in this case. $\widehat{F}$ is given in the proof.*

(iii) *Brand M's profit before introduction costs increases when offering both the LE and regular product in comparison to the case where it offers only the regular product.*

The proposition shows that the monopoly brand offering only the regular product would charge a higher price and reduce the quantity sold when consumers desire exclusivity or their desire for exclusivity increases (higher k).[6] Intuitively, the result follows because demand becomes less elastic when consumers' desire for exclusivity increases. Interestingly, the brand's profit decreases with an increase in k (except when the optimal quantity sold equals one, in which case it is unaffected by k). This result reflects the lower value for the product when k increases, provided $q_M > 1$. The optimal quantity sold exceeds one unless k is very large.[7]

The proposition also shows that the brand increases its profit before introduction cost by offering an LE product in addition to the regular product. The rationale can be best understood by starting from the optimal solution to the problem where the brand only sells the regular product. If the brand now substitutes one unit of the regular product with that of the LE product, the marginal profit from the more exclusive LE product exceeds the marginal profit lost by selling less of the regular product. Thus, the brand increases profit before product introduction costs by offering the LE product. Indeed, if the brand sells unequal quantities of both products, it always gains by substituting one unit of the product sold in larger quantities with a unit of the product sold in smaller quantities. This explains why the monopoly brand, when offering an LE product ($F \leq \widehat{F}$), sells equal quantities of the LE and regular products and sells each product for an identical price. Thus, the LE product is indistinguishable from a regular product in terms of price or quantity sold in the monopoly case. This is to be expected because consumers internalize the quantity sold of the regular product in making their purchase decisions by forming rational expectations. Furthermore, without competition, there are no strategic effects from announcing the quantity of the LE product.

Note that by extension of the above results, if product introduction costs are sufficiently small, a monopolist would gain by offering more than two brand items to the point where only one unit of each brand item is sold. Thus, by restricting to no more than two items for the brand, we are assuming that product introduction costs are sufficiently large. We also make this assumption for reasons of tractability in the competitive case.

[5] If brand M is located more towards the center of the interval $[0, 1]$, its profit would increase, but the implication of introducing LE products is unchanged.

[6] When the market is covered, the optimal price decreases with k while the quantity sold is unchanged.

[7] Note that with rational consumer expectations about quantity sold, the results for the monopoly brand with one product are the same whether or not the quantity made available is preannounced. This is because there are no strategic effects from announcing quantity, in the absence of competition. Thus, a regular product is indistinguishable from an LE product (as defined by our paper) in the monopoly case.

Figure 2 Subgames in Vertical Differentiation Model

	Low-quality brand	
High-quality brand	Only regular	Regular & LE
Only regular	S_{11}	S_{12}
Regular & LE	S_{21}	S_{22}

4. Vertical Brand Differentiation

In this section we consider the case where the brands are vertically differentiated. We assume that the market is covered in equilibrium.[8] We further assume that $\bar{\theta} > 2\underline{\theta}$ which ensures that the low-quality brand has a positive market share in equilibrium for all $k > 0$ (see also Tirole 1988). This condition implies that the heterogeneity in consumers' taste for quality is sufficiently large. To introduce some notation, let l_H and l_L denote the quantity of the LE product announced by brand H and L, respectively, where $l_H \in (0, N]$ and $l_L \in (0, N]$. We use p_{lH} and p_H (p_{lL} and p_L) to denote the prices of the LE and regular products, respectively, of brand H (brand L). We denote the quantity of the regular product of brand H by q_H (q_L for brand L).

We have four subgames S_{ij} with $i, j \in [1, 2]$ depending on which brand, if any, chooses to offer an LE product (see Figure 2). We solve these subgames using backward induction by first solving for stage 3 equilibrium prices of the products, and then solving for the equilibrium quantities of any LE products in stage 2. The equilibrium profits of the two brands for these subgames are presented in Table 1, the equilibrium prices are shown in Table 2, and the equilibrium quantities of the different brand items are given in Table 3. The proofs are presented in the appendix.

Before presenting the subgame perfect equilibrium of the game, we discuss the results of the subgame, S_{11}, in which neither brand offers an LE product (both brands offer only the regular version). The result for this subgame serves as a useful benchmark for comparison with the results from the other subgames in which one or both brands offer an LE product in addition to the regular product.

4.1. Subgame with Regular Product Only

The equilibrium results reveal that the profit, price, and quantity are greater for the high-quality brand than for the low-quality brand. This result is similar to that obtained in the standard vertical differentiation model with no exclusivity effects (Tirole 1988). The more interesting question relates to the effect of the exclusivity parameter k on the equilibrium quantities. The following proposition presents this result.

Proposition 2. *An increase in k decreases the quantity sold by the high-quality brand while increasing the quantity sold by the low-quality brand. The prices and profits for both brands increase at the same rate with an increase in k.*

When consumers' value for exclusivity increases (k increases), they become less willing to switch to a competing brand that offers a price cut because they expect the competing brand to become less exclusive as a result. Consequently, the demand becomes less elastic causing the price reaction functions of competing brands to shift upward consistent with a softening of price competition.[9] The result is higher prices and profits for the competing brands, and moreover, the rate of increase in these entities is symmetric for both brands. Because the high-quality brand sells more than the low-quality brand, an increase in the exclusivity parameter k makes the former less attractive resulting in a quantity gain for the low-quality brand at the expense of the high-quality brand. In contrast to the above results, note that a monopoly brand's profit decreases with an increase in k because of the absence of strategic effects on price competition.

In a model without exclusivity, the low-quality brand is shut out of the market if the consumer heterogeneity in taste for quality is low, $\bar{\theta} < 2\underline{\theta}$. However, if consumers value exclusivity, a low-quality brand may have a positive market share even under such a condition as long as k is sufficiently high. Thus, product categories whose purchases may be influenced by exclusivity considerations may accommodate more brands than otherwise. In particular, newer brands (with lower perceived quality or "brand equity") may be better able to compete in such categories. This may explain the frequent emergence of trendy new brands in apparel categories where uniqueness is valued by consumers. We now present the equilibrium results of the overall game.

4.2. Stage 1 Results: LE Product or Not?

The following proposition outlines the subgame perfect equilibrium of the game as a function of the introduction cost, F, of an LE product.

Proposition 3. *In the unique subgame perfect equilibrium, for $F \le F^*$, both brands offer an LE product; for $F^* < F \le F^{**}$, only the high-quality brand offers an LE product, and for $F > F^{**}$, neither brand offers an LE product. F^* and F^{**} are given in the proof.*

The subgame profit comparisons that drive the above result are as follows. It is obvious that neither brand introduces an LE product if the product introduction cost is sufficiently high (exceeds F^{**}). Therefore, let us consider lower product introduction

[8] See the Technical Appendix, found at http://mktsci.pubs.informs.org, for the market coverage condition.

[9] Amaldoss and Jain (2005b) obtain a similar result with an increase in the degree of snobbishness of consumers.

Table 1 Vertical Differentiation Model: Brands' Profits in Subgames

	Low-quality brand	
High-quality brand	Only regular	Regular & LE
Only regular	$\begin{pmatrix} \frac{N(3kN+(N-1)s(2\bar{\theta}-\underline{\theta}))^2}{9(N-1)(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{N(3kN+(N-1)s(\bar{\theta}-2\underline{\theta}))^2}{9(N-1)(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))} \end{pmatrix}$	$\begin{pmatrix} \frac{N(7kN+3(N-1)s(2\bar{\theta}-\underline{\theta}))^2(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))}{(N-1)(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))^2} \\ \frac{N(3kN+(N-1)s(\bar{\theta}-2\underline{\theta}))^2}{(N-1)(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \end{pmatrix}$
Regular & LE	$\begin{pmatrix} \frac{N(3kN+(N-1)s(2\bar{\theta}-\underline{\theta}))^2}{(N-1)(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{N(7kN+3(N-1)s(\bar{\theta}-2\underline{\theta}))^2(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))}{(N-1)(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))^2} \end{pmatrix}$	$\begin{pmatrix} \frac{N(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))(7kN+3(N-1)s(2\bar{\theta}-\underline{\theta}))^2}{9(N-1)(14kN+9(N-1)s(\bar{\theta}-\underline{\theta}))^2} \\ \frac{N(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))(7kN+3(N-1)s(\bar{\theta}-2\underline{\theta}))^2}{9(N-1)(14kN+9(N-1)s(\bar{\theta}-\underline{\theta}))^2} \end{pmatrix}$

Note. The column vector in each cell contains (from top to bottom) Π_H^* and Π_L^*.

costs, $F \leq F^{**}$. If either brand unilaterally introduces an LE product, that brand increases its profit before introduction costs in comparison to the benchmark case where neither brand introduces an LE product. On the other hand, the brand not introducing an LE product loses profit in comparison to the benchmark case. However, this brand may then find it optimal to introduce an LE product of its own, if F is not too high. In particular, if unilateral introduction of an LE product is profitable for a low-quality brand after product introduction costs, it is also profitable for a high-quality brand to follow suit. On the other hand, if $F > F^*$, a low-quality brand does not want to follow suit by introducing its own LE product in response to a similar introduction by the high-quality brand. Thus, it appears that a high-quality brand gains more from introducing an LE product than a low-quality brand so that we do not have an equilibrium where the low-quality brand alone introduces an LE product.[10] These results appear to be consistent with anecdotal evidence in the piano and luxury car markets where it is typically the high-quality brands that introduce LE products and where such introduction of an LE product may be costly.

4.2.1. Characteristics of the Equilibrium. Before we discuss the intuition for the equilibrium results of stage 1, it is useful to understand the characteristics of the equilibrium solutions. Consider, first, the subgame perfect equilibrium where only brand H offers a LE product ($F^* < F \leq F^{**}$). In this equilibrium, the LE product of brand H sells at a price premium over its regular product. However, both products of brand H sell at a higher price than brand L's regular product. Similar to the base case, brand H sells a higher combined quantity than brand L. On the other hand, the quantity of LE product sold by brand H is smaller than that of its regular product. The intuition for this last result is discussed later. With respect to consumer segmentation, there are two primary consumer segments in the market in the equilibrium for this subgame (see Figure 3), as indicated in the derivation of the equilibrium results. Consumers in the interval $[\underline{\theta}, \theta_2]$ purchase brand L's regular product, while those in the region $[\theta_2, \bar{\theta}]$ buy either the regular or the LE product of the brand H. Indeed, all consumers in the region $[\theta_2, \bar{\theta}]$ have an identical preference ordering between brand H's regular and LE product. The intuition for this result is as follows. Because the only differentiator between the two products is the difference in exclusivity benefit (recall that the products are otherwise identical), and because the value for this benefit is independent of the consumer's location in $[\theta_2, \bar{\theta}]$, all consumers in this region have a similar preference ordering between the two products.[11] Thus, in any equilibrium in which brand H sells positive quantities of both products, consumers should derive at least as much surplus from the LE product as they do from the regular product—otherwise no consumer buys the LE product. However, brand H can maximize its profit by pricing the LE product high enough so that consumers are indifferent between it and the regular product. Because brand H sells fewer units of the LE product than the regular product in equilibrium, the LE product has a greater exclusivity benefit and thus carries a price premium over brand H's regular product. Such a price premium for an LE product is consistent with casual observation. For example, the 2001 model of the Jaguar XKR Silverstone two-door convertible was offered for a price premium of \$11,750

[10] Because this result reflects the higher market share of the high-quality brand (under the assumption of equal marginal costs), it is possible that the result may be reversed if the marginal cost of a high-quality product is sufficiently high so that the low-quality brand has the higher equilibrium market share.

[11] Mathematically speaking, the utility component stemming from the quality in both products of brand H is $\theta(1+s)$. Thus, the relative preference between the two products is independent of θ.

Table 2 Vertical Differentiation Model: Brands' Prices in Subgames

	Low-quality brand	
High-quality brand	Only regular	Regular & LE
Only regular	$\begin{pmatrix} — \\ \frac{3kN+(N-1)s(2\bar{\theta}-\underline{\theta})}{3(N-1)} \\ — \\ \frac{3kN+(N-1)s(\bar{\theta}-2\underline{\theta})}{3(N-1)} \end{pmatrix}$	$\begin{pmatrix} — \\ \frac{(7kN+3(N-1)s(2\bar{\theta}-\underline{\theta}))(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))}{(N-1)(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{3(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))(3kN+(N-1)s(\bar{\theta}-2\underline{\theta}))}{(N-1)(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{(5kN+3(N-1)s(\bar{\theta}-\underline{\theta}))(3kN+(N-1)s(\bar{\theta}-2\underline{\theta}))}{(N-1)(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \end{pmatrix}$
Regular & LE	$\begin{pmatrix} \frac{3(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))(3kN+(N-1)s(2\bar{\theta}-\underline{\theta}))}{(N-1)(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{(5kN+3(N-1)s(\bar{\theta}-\underline{\theta}))(3kN+(N-1)s(2\bar{\theta}-\underline{\theta}))}{(N-1)(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ — \\ \frac{(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))(7kN+3(N-1)s(\bar{\theta}-2\underline{\theta}))}{(N-1)(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \end{pmatrix}$	$\begin{pmatrix} \frac{(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))(7kN+3(N-1)s(2\bar{\theta}-\underline{\theta}))}{(N-1)(14kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{(5kN+3(N-1)s(\bar{\theta}-\underline{\theta}))(7kN+3(N-1)s(2\bar{\theta}-\underline{\theta}))}{3(N-1)(14kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))(7kN+3(N-1)s(\bar{\theta}-2\underline{\theta}))}{(N-1)(14kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{(5kN+3(N-1)s(\bar{\theta}-\underline{\theta}))(7kN+3(N-1)s(\bar{\theta}-2\underline{\theta}))}{3(N-1)(14kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \end{pmatrix}$

Note. The column vector in each cell contains (from top to bottom) p_{lH}^*, p_H^*, p_{lL}^*, and p_L^*.

U.S. dollars above the MSRP of the regular version of the same car. At the same time only 500 Jaguar XKR Silverstones were sold compared to a total of 5,137 units of Jaguar XKRs (*Ward's Automotive Yearbook* 2001).

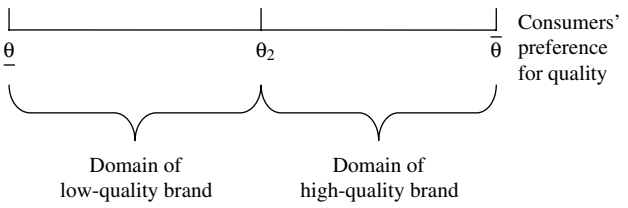
Because consumers in the region $[\theta_2, \bar{\theta}]$ expect equal surplus from the LE and regular product of brand H, half of them would opt for the LE product. However, because fewer LE products than regular products are sold in equilibrium, this implies that not all consumers opting for the LE product are able to purchase it. Thus, the LE product has to be assigned to these consumers by means other than price. This result may be consistent with LE products typically

Table 3 Vertical Differentiation Model: Brands' Quantities in Subgames

	Low-quality brand	
High-quality brand	Only regular	Regular & LE
Only regular	$\begin{pmatrix} — \\ \frac{N(3kN+(N-1)s(2\bar{\theta}-\underline{\theta}))}{3(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))} \\ — \\ \frac{N(3kN+(N-1)s(\bar{\theta}-2\underline{\theta}))}{3(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))} \end{pmatrix}$	$\begin{pmatrix} — \\ \frac{N(7kN+3(N-1)s(2\bar{\theta}-\underline{\theta}))}{(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{N(3kN+(N-1)s(\bar{\theta}-2\underline{\theta}))}{(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{2N(3kN+(N-1)s(\bar{\theta}-2\underline{\theta}))}{(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \end{pmatrix}$
Regular & LE	$\begin{pmatrix} \frac{N(3kN+(N-1)s(2\bar{\theta}-\underline{\theta}))}{(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{2N(3kN+(N-1)s(2\bar{\theta}-\underline{\theta}))}{(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ — \\ \frac{N(7kN+3(N-1)s(\bar{\theta}-2\underline{\theta}))}{(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \end{pmatrix}$	$\begin{pmatrix} \frac{N(7kN+3(N-1)s(2\bar{\theta}-\underline{\theta}))}{3(14kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{2N(7kN+3(N-1)s(2\bar{\theta}-\underline{\theta}))}{3(14kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{N(7kN+3(N-1)s(\bar{\theta}-2\underline{\theta}))}{3(14kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \\ \frac{2N(7kN+3(N-1)s(\bar{\theta}-2\underline{\theta}))}{3(14kN+9(N-1)s(\bar{\theta}-\underline{\theta}))} \end{pmatrix}$

Note. The column vector in each cell contains (from top to bottom) l_H^*, q_H^*, l_L^*, and q_L^*.

Figure 3 Vertical Differentiation Model: Equilibrium Consumer Segmentation



being sold on a first-come, first-serve basis sometimes using a waiting list. For example, consumers had to "get in line for the privilege of paying $750 for fall's most-wanted Prada boots," (*Globe and Mail* 1999).[12] It should be noted that the above result is sensitive to the assumption that both LE and regular products are otherwise identical except for the preannounced quantity for the LE product. This may be a valid assumption for some products such as handbags or sneakers where the LE products may offer little scope for additional differentiation beyond cosmetic differences. However, in other cases, a firm may provide in the LE product an additional quality or feature that is valued by all consumers. In an extension of this model, we consider such a scenario and find that such an LE product's price is such that it appeals only to those customers with the highest valuation of quality (highest θ).[13] To summarize, the LE product of a vertically differentiated brand is assigned to consumers by means other than price if there are limited opportunities to incorporate additional quality into the LE product or if such additional quality is very costly.

Now consider the characteristics of the subgame perfect equilibrium when $F \leq F^*$. In this case as well, the LE products of both brands sell at a price premium over their respective regular products. Furthermore, the prices of the LE and regular products of brand H are higher than the prices of the corresponding products of brand L. There are again two primary segments (see Figure 3) except that the consumer segment, $[\underline{\theta}, \theta_2]$, is now indifferent between purchasing brand L's regular and LE product. Thus, the LE products of both brands are assigned to consumers by means other than by price. Both brands sell fewer units of their LE product, although the quantity of LE product sold by brand H is higher.

4.2.2. Intuition of the Subgame Perfect Equilibrium Results. To understand the intuition behind the subgame perfect equilibrium results, we examine the strategic effect of introducing an LE product by either competitor. Consider, for example, the subgame S_{21} in which the high-quality brand alone introduces an LE product. As discussed above, there are two primary consumer segments in the market in equilibrium (see Figure 3) with the consumer at θ_2 being indifferent between buying the regular products of the low-quality brand and the high-quality brand. Consider now the effect of brand H increasing the quantity l_H of its LE product from zero. Because the LE product is produced in a lower quantity than brand H's regular product, it commands a price premium over the latter as discussed above. Thus, although there is some cannibalization of brand H's regular product by its LE product, the "direct effect" of increasing the quantity of LE product on brand H's overall profits is positive for low values of l_H. This direct effect is also observed in the monopoly case where the marginal profit from selling a second, LE product is seen to be higher than that obtained by selling additional units of the regular product. However, in the competitive case, an increase in l_H also has a negative strategic effect on brand H's profits by fostering an increase in price competition. The intuition for the strategic effect is as follows. With an increase in l_H, fewer consumers in the region $[\theta_2, \bar{\theta}]$ now buy the regular product, as they choose between the two products with equal probability (because both products yield the same surplus). Consequently, the exclusivity value of brand H's regular product increases, so that the consumer at θ_2 now perceives a higher value for this product. Thus, the location of the consumer indifferent between products from the two brands shifts left towards $\underline{\theta}$ implying that an increase in l_H decreases the quantity sold of the regular product of brand L. The latter factor causes brand L to price its regular product more aggressively resulting in a decrease in the price levels of both brands. Thus, the strategic effect of an increase in l_H is to increase price competition thereby negatively affecting profits of both brands. For small values of l_H, however, the direct effect of increasing l_H overwhelms the strategic effect so that brand H gains by increasing l_H. However, with continued increase in l_H, the direct effect on profit from an increase in l_H declines because of declining price premiums and consequent price-volume trade-offs. Thus, an optimum value of l_H results when the direct and strategic effects are in balance. Overall, negative strategic effects induce brand H to sell fewer LE products than regular products, unlike a monopoly brand that sells equal quantities of both.

For the reasons presented, if brand H alone offers an LE product, it increases its profit in comparison to the benchmark case, while brand L loses profit in relation to the benchmark. However, if product introduction costs are low, brand L can also gain from

[12] Note that all consumers in $[\theta_2, \bar{\theta}]$ cannot choose to avoid the waiting list by opting for the regular product (which would presumably yield the same surplus). In such a scenario, no consumer would be buying the LE product. However, a consumer can gain surplus by switching to it from the regular product.

[13] The analysis is available from the authors on request.

the direct effect of introducing an LE product of its own, although the strategic effect of such an introduction results in more price competition. Because brand L gains less from an LE product in comparison to brand H, because of the latter's superior quality positioning, brand L introduces an LE product only when product introduction costs are sufficiently low.

It may be interesting to compare the subgame perfect equilibrium results with LE products against the equilibrium results of the benchmark case where no LE products are offered. Consistent with the strategic effect of increasing price competition stemming from the introduction of LE products, prices of regular products are lower when one or both brands introduce LE products. While unilateral introduction of an LE product by either brand helps it to improve its profit before introduction costs, brand L does worse in comparison with the benchmark case when H also introduces a LE product. Because brand H finds it optimal to introduce an LE product of its own whenever it is profitable for brand L to do so, brand L never gains from introducing an LE product, and would prefer competition that does not involve such products. On the other hand, even if brand L counters with an LE product of its own, brand H can do better in comparison with the benchmark case when its quality advantage s over brand L is sufficiently high (provided F is sufficiently small). Under these conditions, the beneficial direct effect to brand H from offering an LE product outweighs the negative strategic effect stemming from an increase in price competition. This finding suggests that a very high-quality brand can benefit from an LE product even when a competing low-quality brand introduces an LE product of its own. In contrast to these results, note that a monopoly brand always increases profit before introduction cost when adding an LE product to its product line because the strategic effect is absent. The following proposition summarizes the comparison of equilibrium profits with the benchmark case.

Proposition 4. *In the subgame perfect equilibrium in which both brands offer LE products, brand L always makes a smaller profit in comparison with the benchmark case. Conversely, brand H can increase its profit in comparison with the benchmark case, if s is sufficiently large and F is sufficiently small.*

4.2.3. Comparative Statics of the Subgame Perfect Equilibrium. The equilibrium results lead to some interesting comparative statics. The next proposition relates the changes in equilibrium quantities sold by the brands to changes in the exclusivity parameter k.

Proposition 5. *In the subgame perfect equilibrium in which both brands offer LE products, when k increases,* (i) *the overall quantity sold by brand H* (*brand L*) *decreases* (*increases*), *while* (ii) *the ratio of the quantity sold of LE product to regular product for each brand stays constant.*

Similar to the result in the base case, an increase in the exclusivity parameter k adversely affects the value of brand H because it sells a higher quantity. Thus, brand H sells a lower total quantity in equilibrium when k increases, while the total quantity sold by brand L correspondingly increases. However, the product mix for either brand, as indicated by the proportion of LE and regular product sold, is unaffected by k. As might be expected, the price premium for the LE product increases with the exclusivity parameter, k, as established by the following proposition.

Proposition 6. *In equilibrium, the price premium for the LE product increases with the desire for exclusivity, k.*

The price premium for the LE product reflects the greater exclusivity (smaller quantity) of this product. Naturally, the price premium increases with k (see also Equation (1) in the appendix). Next, we discuss conditions that result in brand H alone offering an LE product.

Proposition 7. *For a given F, brand L does not offer an LE product in equilibrium for sufficiently high s and sufficiently low k.*

Thus, if the quality difference between the brands is sufficiently high or if consumers' value for exclusivity is low, brand L does not find it optimal to respond to brand H's offer of an LE product. The implication is that these conditions result in a lower payoff to brand L from adding an LE product to its product line.

5. Horizontal Brand Differentiation

In this section, we consider a market scenario where the brands are horizontally differentiated according to the Hotelling model described in §2. We assume that the reservation price u for all consumers is high enough to purchase the product such that the market is covered in equilibrium. We denote l_A and l_B as the quantities of the LE product announced by brand A and B, respectively, with $l_A \in (0, N]$ and $l_B \in (0, N]$. We define p_{lA} and p_A (p_{lB} and p_B) as the prices of the LE and regular products of brand A and brand B, respectively. Furthermore, we denote the quantity of the regular product of brand A by q_A (q_B for brand B). For the horizontal differentiation model we consider three subgames: subgame S_{RR} where neither brand introduces an LE product, subgame S_{LR} where only brand A introduces an LE product, and subgame S_{LL} where both brands introduce an

Table 4 Horizontal Differentiation Model: Equilibrium Results for Subgames

LE product subgame	Neither brands S_{RR}	Only brand A S_{LR}	Both brands S_{LL}
P_{li}^*	—	$\frac{9(kN+(N-1)t)^2}{(N-1)(8kN+9(N-1)t)}$	$\frac{kN}{N-1}+t$
P_A^*	$\frac{kN}{N-1}+t$	$\frac{kN(15kN+17(N-1)t)}{2(N-1)(8kN+9(N-1)t)}+t$	$\frac{5kN}{6(N-1)}+t$
P_B^*	$\frac{kN}{N-1}+t$	$\frac{kN(7kN+8(N-1)t)}{(N-1)(8kN+9(N-1)t)}+t$	$\frac{5kN}{6(N-1)}+t$
l_i^*	—	$\frac{3N(kN+(N-1)t)}{2(8kN+9(N-1)t)}$	$\frac{N}{6}$
q_A^*	$\frac{N}{2}$	$\frac{3N(kN+(N-1)t)}{(8kN+9(N-1)t)}$	$\frac{N}{3}$
q_B^*	$\frac{N}{2}$	$\frac{N(7kN+9(N-1)t)}{2(8kN+9(N-1)t)}$	$\frac{N}{3}$
Π_A^*	$\frac{Nt}{2}+\frac{kN^2}{2(N-1)}$	$\frac{9N(kN+(N-1)t)^2}{2(N-1)(8kN+9(N-1)t)}$	$\frac{1}{18}N\left(\frac{8kN}{N-1}+9t\right)$
Π_B^*	$\frac{Nt}{2}+\frac{kN^2}{2(N-1)}$	$\frac{N(kN+(N-1)t)(7kN+9(N-1)t)^2}{2(N-1)(8kN+9(N-1)t)^2}$	$\frac{1}{18}N\left(\frac{8kN}{N-1}+9t\right)$

LE product.[14] Once again, we solve these subgames using backward induction by first solving for stage 3 equilibrium prices of the products, and then solving for the equilibrium quantities of any LE products in stage 2. The equilibrium profits, prices, and quantities for these subgames are presented in Table 4. The proofs are given in the appendix. It is instructive to begin by considering the benchmark subgame, S_{RR}, in which neither brand offers an LE product (both brands offer only the regular version). A key issue is the effect of consumers' need for exclusivity and how this changes the results from a standard Hotelling model. The following proposition summarizes the result.

Proposition 8. *In the subgame with no LE products, the prices and profits for both brands increase with k.*

Thus, the effect of introducing exclusivity into the standard Hotelling model is to soften price competition leading to an increase in the prices charged by the competing brands and higher profits. The intuition for this result is similar to that in the case of vertical competition in that the demand becomes less elastic as consumers' value for exclusivity increases. We now present the subgame perfect equilibrium of the overall game involving introduction of LE products.

5.1. Stage 1 Results: LE Product or Not?

The following proposition outlines the subgame perfect equilibrium of the game as a function of the introduction cost, F, of an LE product.

[14] The subgame where only brand B introduces the LE product is identical to subgame S_{LR} because of symmetry.

Proposition 9. *In the unique subgame perfect equilibrium, for $F \le \underline{F}$, both brands offer an LE product; for $\underline{F} < F \le \overline{F}$, only one brand offers an LE product and for $F > \overline{F}$ neither brand offers an LE product. $\underline{F}$ and $\overline{F}$ are given in the proof.*

The rationale for the equilibrium results is as follows. When product introduction costs are sufficiently low, namely, $F \le \underline{F}$, a brand can increase its profit if it alone introduces an LE product. However, the brand not introducing an LE product suffers a profit loss in this case and can do better by introducing an LE product of its own. In the subgame perfect equilibrium in which both brands introduce LE products, each brand sells a smaller quantity of the LE product in comparison to its regular product. The LE product consequently enjoys a price premium over the regular product of each brand because of the former's better exclusivity value. As in the vertical differentiation case, the quantity of the LE product produced represents a balance between its direct effect on profit because of selling a more exclusive product and its strategic effect in increasing the intensity of price competition. The intuition for the strategic effect of LE products on price competition is similar to that in the vertical differentiation case. An increase in quantity of a brand's LE product cuts into the sales of its regular product, making the latter appear more exclusive and valuable in the eyes of consumers. Because the quantity of the brand's regular product is not limited, sales of the competing brand are adversely affected triggering aggressive price reaction from the competing brand. The resulting drop in price levels is

damaging to profits for both brands. Thus, the price of the regular product decreases in comparison with the benchmark case when neither brand offers an LE product (see Table 4). For intermediate product introduction costs, i.e., $\underline{F} < F \leq \overline{F}$, only one of the brands will offer an LE product. In that case it is not profitable for the competitor to also introduce an LE product because it leads to more intense price competition. However, the brand that introduces the LE product will make a greater profit after introduction costs than the brand without an LE product. This suggests that if the brands were to make decisions sequentially, the brand moving first can gain a first-mover advantage by offering an LE product in the horizontal differentiation case, for intermediate product introduction costs. Eventually, if the product introduction cost is very high, namely $F > \overline{F}$, it is not profitable for either brand to add an LE product to its product line.

We saw earlier that a monopoly brand increases profit before product introduction costs when it offers an LE product in addition to its regular product. A pertinent related question is whether competing horizontally differentiated brands are better off when they both introduce LE products, in comparison to the benchmark case. Interestingly, the answer is that they are not better off, even if product introduction costs are zero. Thus, the option of introducing LE products leads to a prisoners' dilemma with horizontally differentiated brands, if F is not too high. Therefore, both brands would prefer a scenario where no brand introduces an LE product, but such a scenario is not sustainable as a brand can gain by unilaterally offering an LE product. Thus, we have the following proposition.

Proposition 10. *In the subgame perfect equilibrium in which both brands introduce LE products, their profits are lower in comparison with the benchmark case where neither brand introduces an LE product. Thus, the equilibrium is a case of prisoners' dilemma.*

Comparative statics analysis reveals that for a given level of introduction costs, lower levels of differentiation between the brands (lower values of t) or higher values of k induce both brands (rather than one) to introduce an LE product in equilibrium. Further analysis shows that a brand matching a competitor's offer of an LE product gains from the price premium on its LE product and from increasing its overall market share. These gains are more pronounced when brands are less differentiated or when consumers' value for exclusivity is high. Conversely, a brand matching a competitor's offer of an LE product charges a lower price for its regular product than otherwise because of the increase in price competition stemming from the strategic effect of offering an LE product. This downward price pressure on the regular product is more pronounced when the brands are more differentiated or when k is high. Overall, the above effects tend to increase a brand's profit from countering with an LE product as brands become less differentiated, while the positive effects on profit predominate when k becomes higher. Thus, low values of t or high values of k promote the equilibrium with both brands offering an LE product.

Likewise, if F is very high that neither brand offers an LE product, a decrease in t or an increase in k make it attractive for one of the brands to move towards offering an LE product. The intuition is somewhat similar in this case. The brand that unilaterally offers an LE product gains from the larger quantity sold and from the higher price charged for the LE product and these gains are higher for lower values of t and higher values of k. However, it loses because of the lower price on the regular product as a result of increased price competition from offering the LE product. This reduction in price of the regular product is higher at lower values of t and higher values of k. However, the positive effects increase more than the negative price competition effect for lower values of t and higher values of k.

We considered an extension of our horizontal differentiation model in which a brand may offer an additional (vertical) feature in its LE product which is valued by all consumers. An example of an additional feature in an LE product is an LE watch made of rose gold (*Times* 2006). If the consumers' value for the added feature is small, we find that the prisoners' dilemma equilibrium continues to obtain when both brands introduce LE products. However, as might be expected, if the additional feature is sufficiently more valuable to consumers than its marginal cost, both brands can be better off in a market with LE products because of the additional profit from the added feature.

6. Conclusion

6.1. Summary and Discussion

LE products are an important part of the marketing mix in product categories such as pianos, cars, handbags, apparel, and fashion items. Conventional wisdom suggests that LE products ought to help a brand improve its profits because these products help to make a brand's offerings more unique and differentiated. We observe such an effect in our analysis as the LE product sells at a price premium over a brand's regular product. We call this the direct effect of introducing an LE product. However, our analysis of the competitive implications of LE products uncovers a surprising strategic effect, which results in greater price competition and can lead, counterintuitively, to lower profits for a brand introducing LE

products. In contrast to the results for the competitive case, our analysis for a monopoly brand conforms more to conventional wisdom in the sense that the brand increases its profit by adding an LE product to its product line. The absence of a negative strategic effect explains the different result obtained for a monopoly brand.

Our analysis further delineates the condition under which introducing LE products can be profitable to a brand under competition. We first consider the scenario where there are quality differences between competing brands. Here, quality can be interpreted broadly to include objective attributes such as reliability or performance as well as subjective attributes such as brand equity or brand appeal. The key requirement for an attribute difference between brands to be considered a quality difference is that all consumers agree on which brand has the higher quality based on the particular attribute. When there are quality differences between brands, we find that a high-quality brand introduces an LE product provided product introduction costs are not too high. On the other hand, a competing low-quality brand introduces an LE product under more restricted conditions—when product introduction costs are small. The greater propensity for the LE product to be offered by the high-quality brand is consistent with casual observation in many markets. For example, Steinway, a high-end piano brand, appears to offer LE products more frequently than Yamaha, whose quality image is not as high as Steinway's. Similarly, luxury car makers such as Jaguar appear to offer LE cars more often than nonluxury brands.

The LE product can enable the high-quality brand to improve its profit and market share in comparison to the case when LE products are not offered by either competitor. Thus, introduction of an LE product can be a good offensive marketing strategy for a high-quality brand competing against a low-quality one. On the other hand, the low-quality brand finds it optimal to respond with an LE product of its own if product introduction costs are low. However, the low-quality brand then enjoys a lower profit before product introduction cost in comparison with the case when neither brand introduces an LE product. Thus, the use of an LE product by a low-quality brand can be interpreted as a defensive strategy. In other words, the low-quality brand would prefer a situation where brands do not compete on the basis of LE products to one in which there is such competition.

When a brand introduces an LE product in equilibrium, it chooses the quantity of LE product that balances the positive direct effect on profits because of the higher margins of the LE product, and the negative strategic effect on profits stemming from increasing price competition. Increased price competition from the introduction of an LE product drives down the price of the regular product to a level lower than the equilibrium values obtained when neither brand introduces an LE product. Our comparative statics analysis shows that an increase in consumer's desire for exclusivity increases the price premium for the LE product as expected while also inducing the high-quality brand to be less aggressive on overall market share. A characteristic of the equilibrium outcome is that the LE products have to be allocated to consumers by means other than price. This is consistent with anecdotal evidence that LE products are often sold on a first-come, first-serve basis sometimes using a waiting list. Our explanation hinges on the fact that the only differentiating feature between the LE product and the regular product of a brand is that the quantity of the former is limited. In other words, the result applies when there are no significant quality differences between the LE and regular products, perhaps because of limited opportunities to introduce such differences. Such may be the case for LE products in categories such as handbags or sneakers where the differences between regular and LE products of brand-name manufacturers may be primarily cosmetic. In these categories, LE products are often sold on a first-come, first-serve basis and may involve waiting lists.

It is possible that the brands in some categories are of similar quality so that quality differences are less significant. In this case, it may be more appropriate to consider the brands to be competing on horizontal attributes such as style or design for which consumers may have diverse preference ordering across brands. We consider such a model and find that the introduction of LE products entails direct and strategic effects as in the case of quality competition. However, when product introduction costs are low enough that both competing brands introduce an LE product in equilibrium, their profits are smaller than when neither brand introduces an LE product. Thus, introduction of LE products leads to a prisoners' dilemma. Only when product introduction costs are sufficiently high, do we have only one brand introducing an LE product. In this case, the brand introducing the LE product gains in comparison with the case when neither brand introduces an LE product while the competing brand loses. Thus, in general, we may conclude that introduction of LE products may decrease industry profits when quality differences between brands are less significant. In such cases, brands may do well to avoid competing on the basis of LE products. This may explain why LE products are relatively rare for casual apparel such as jeans where the quality difference between brands appears to be small.

We analyzed an extended horizontal differentiation model in which the brands could add an extra feature or quality to the LE product. For example, in the watch category, the Chiffre Rouge I01 Irreductible by Dior had a production limit of only 300 units, each with a single diamond set into the case (*Financial Times* 2004). As might be expected, the prisoners' dilemma result persists if consumers' value for the added feature is sufficiently small in relation to its cost. However, not surprisingly, both brands are better off when introducing LE products if the feature's value significantly exceeds its cost.

To summarize, our results suggest that managers should consider both the direct and strategic effects of introducing LE products in a competitive environment. It is also important for managers to understand the nature of product differentiation of their brand with respect to the competition i.e., whether brand differences are vertical, as in quality difference, or horizontal, as in style difference. As discussed earlier, the implications are quite different for these types of differences. Furthermore, our comparative statics results suggest that introduction of LE products may be more justifiable when the quality or style difference between brands is low or when consumers' desire for exclusivity is high. We mentioned in the introduction that there may be other motivations for LE products such as test marketing, niche marketing, subbranding or generating word of mouth. These other motivations are not considered by this study but if pertinent, should be considered by managers in conjunction with the direct and strategic effects of LE products identified here.

6.2. Limitations and Future Research Directions

We now discuss limitations of our analysis arising out of our model assumptions. As noted in §2, our model of the consumer's utility from exclusivity considers only the first-order effects because of sameness. In that section, we describe additional second-order exclusivity effects that our model does not incorporate, namely the second-order uniqueness effect and the second-order price effect. We conjecture that these second-order effects are likely to increase the price premium for more unique products but are unlikely to change the basic conclusion of our analysis that introduction of LE products causes negative strategic effects. The rationale is that by increasing the quantity of the LE product, a brand would still induce a strategic effect on price competition by making the regular product more valuable. In the case of vertically differentiated brands, we speculate that the second-order price effect may put the low-quality brand at a further disadvantage because of its lower price, thus making LE products even less profitable for the low-quality brand. However, future research is needed to rigorously examine the impact of the second-order exclusivity effects on our findings. It is also possible that the different exclusivity effects could be more efficiently modeled using a structural approach in which consumers' utility for exclusivity is derived from their need to signal to other consumers (e.g., Pesendorfer 1995). In contrast, our model uses a reduced-form formulation to capture exclusivity effects.

Another limitation of our model is that we allow brands to offer only one LE product. Conversely, many brands such as Steinway offer several LE products at any time. As noted in our monopoly analysis, if product introduction costs are sufficiently low, a brand may increase profit by offering multiple LE products, as each LE product can be priced higher because of greater exclusivity. In the competitive case, an analysis allowing for multiple LE products may become less tractable, although we are likely to find that offering multiple LE products increases profit in comparison to offering a single LE product. It is then possible that profits from multiple LE products may be sufficient to overcome negative strategic effects to the extent that a brand may be better off offering multiple LE products as opposed to none, even in the horizontal differentiation case. Thus, an extension of our model to multiple LE products is a fruitful avenue for further research. Another limitation of our analysis is that we assume that brands cannot offer only LE products at the product-line selection stage (stage 1). This is an additional issue that may merit further research.

We note that our model assumes that consumers are homogeneous in their value for exclusivity. We examined the sensitivity of our analysis to this assumption by extending our horizontal differentiation model to include a second consumer segment that does not care about exclusivity. Our preliminary analysis of this problem finds that the equilibrium with both brands introducing LE products continues to be a prisoners' dilemma, if the exclusivity-seeking segment is sufficiently numerous. In this equilibrium, some consumers in the exclusivity-seeking segment purchase the regular product in equilibrium, after being unable to obtain the LE product. However, we conjecture that if the exclusivity-seeking segment is less numerous, consumers may be perfectly sorted in equilibrium, with the exclusivity-seeking segment purchasing LE products, while the segment that does not care about exclusivity purchases regular products. In this case, we expect that the negative strategic effect may not come into play because competition between the brands' regular products would occur solely in the segment not seeking exclusivity. Thus, both brands' profits may be higher in comparison to the case where they do not offer LE products.

The analysis for this case is quite complicated as it involves corner solutions. Therefore, we leave a complete analysis of this problem for further research while noting that heterogeneity in consumers' value for exclusivity may modify our conclusions. Another possible related extension could examine the effect on LE strategies when some consumers seek exclusivity while others seek conformity as in Amaldoss and Jain (2005a, b).

We conclude with some additional suggestions for further research. An extension of our analysis that suggests itself is a situation where there is both horizontal and vertical differentiation between brands. We suspect that the results in this case will lie between the outcomes obtained for the polar cases of horizontal and vertical differentiation. It may also be useful to consider how introduction of LE products as part of the product line may signal the quality of the brand. Based on our results that high-quality brands stand to gain more from LE products; we suspect that introduction of LE products may signal high quality of the brand itself (and not just that of the LE product as shown in Stock and Balachander 2005). It will be useful to verify this conjecture formally with the help of a model. Finally, given the paucity of research on LE products, it may be useful to carry out empirical research comparing the use of LE products across industries and the differing motivations, if any, of offering such products.

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Appendix
In this appendix we provide sketches of the most important derivations and proofs. More details and all the remaining derivations can be found in the Technical Appendix, found at http://mktsci.pubs.informs.org.

Vertical Differentiation Model: Derivation for Price Subgames

Subgame S_{11}
We analyze the case where the market is fully covered in equilibrium. In any equilibrium, in which both brands have positive sales, the market is divided into two contiguous segments of consumers, with the consumer segment with higher values of θ buying from the high-quality brand. This is because if a consumer θ' prefers the high-quality brand, so do all consumers with $\theta > \theta'$. Thus, the consumer θ who is indifferent between purchasing the high- and the low-quality products can be computed as

$$\theta - k\frac{(q_L - 1)}{N-1} - p_L = (1+s)\theta - k\frac{(q_H - 1)}{N-1} - p_H.$$

Note that the consumer's utility function for both brands incorporates an exclusivity penalty that depends on anticipated q_H and q_L. Assuming rational expectations, we substitute $q_H = N(\bar{\theta} - \theta)/(\bar{\theta} - \underline{\theta})$ and $q_L = N(\theta - \underline{\theta})/(\bar{\theta} - \underline{\theta})$ to obtain

$$\theta = \frac{(N-1)(p_H - p_L)(\bar{\theta} - \underline{\theta}) + kN(\bar{\theta} + \underline{\theta})}{(N-1)s(\bar{\theta} - \underline{\theta}) + 2kN}.$$

The profit functions of the low- and high-quality sellers are $\Pi_H(p_H) = p_H N(\bar{\theta} - \theta)/(\bar{\theta} - \underline{\theta})$ and $\Pi_L(p_L) = p_L N(\theta - \underline{\theta})/(\bar{\theta} - \underline{\theta})$, respectively. By solving the first-order conditions we obtain the equilibrium prices presented in Table 2. The second-order sufficient conditions for a maximum are satisfied at these prices. Our assumption that $\bar{\theta} > 2\underline{\theta}$ is sufficient for $q_L^* > 0$. Moreover, it can be shown with the help of some algebra that $k < k_1$ is sufficient for the market to be covered in equilibrium for the overall game, where k_1 is given as follows:

$$k_1 = \frac{3(N-1)(2N\underline{\theta} + s(\bar{\theta} - 2N\bar{\theta} + (3N-1)\underline{\theta}))}{6N(3N-2)} + \frac{\sqrt{3}}{6N(3N-2)} * \big((N-1)^2((3-4N)s^2\bar{\theta}^2 + 6(N-1)s(2N+s)\bar{\theta}\underline{\theta} + (2N(6-s)s + 3s^2 + 3N^2(4+s^2))\underline{\theta}^2)\big)^{1/2}.$$

Subgames S_{12}, S_{21}, and S_{22}
In this section we derive the equilibrium for subgame S_{22}. The equilibrium results of subgames S_{12} and S_{21} can be derived in a similar fashion.[15] To derive the equilibrium for subgame S_{22}, we note first that a consumer's preference between a brand's LE product and regular product is independent of his type θ since both these products have the same quality. Thus, if a brand makes positive sales of its LE product, the surplus to a consumer θ from the LE product cannot be less than his surplus from the brand's regular product. Thus, we have the following condition for brand H:

$$(1+s)\theta - k\frac{(l_H - 1)}{N-1} - p_{lH} \geq (1+s)\theta - k\frac{(q_H - 1)}{N-1} - p_H$$
$$\Longrightarrow\ p_{lH} \leq p_H + \frac{k(q_H - l_H)}{N-1}.$$

A similar condition results for positive sales of LE product in the case of brand L. In any stage 3 equilibrium in which a brand makes positive sales of both its LE and regular product, there are two possibilities for consumer preferences of the brand's products in equilibrium: (a) all consumers strictly prefer the LE product of the brand to its regular product, or (b) all consumers are indifferent between the regular and LE products of the brand. In any stage 3 equilibrium in which all consumers strictly prefer the LE product of brand H to its regular product, a brand will charge the maximum possible for its LE product. Thus, we should have for $\varepsilon > 0$ and arbitrarily small,

$$p_{lH} = p_H + \frac{k(q_H - l_H)}{N-1} - \varepsilon. \quad (1)$$

Similarly, we compute the following relationship between the prices for the LE product and the regular product for brand L, for $\varepsilon > 0$ and arbitrarily small,

$$p_{lL} = p_L + \frac{k(q_L - l_L)}{N-1} - \varepsilon. \quad (2)$$

[15] Details are available from the authors.

Therefore, note that when $l_i \le q_i$, $i \in \{H, L\}$, both cases (a) and (b) result in identical profits to brand i since ε is arbitrarily small ($\varepsilon \to 0$) in Equations (4) and (5).

Next, we claim that $l_i < q_i$, $i \in \{H, L\}$, in the unique subgame perfect equilibrium of S_{22}. We establish this claim in three steps. First, we identify the necessary conditions for an equilibrium with $l_i < q_i$, $i \in \{H, L\}$. These necessary conditions allow us to compute the equilibrium strategies. Second, we show that no brand can improve its profit by deviating in either stage 3 (pricing subgame) or in stage 2. Last, we show that no other equilibrium exists.

Necessary Conditions for the Claimed Equilibrium

Since $l_i < q_i$, $i \in \{H, L\}$ in the claimed equilibrium, the equilibrium profits are identical whether we assume case (b) or case (a) with $\varepsilon \to 0$. We will assume case (a) with $\varepsilon \to 0$, since the necessary conditions are easier to work with under this assumption. Because consumer preference between a brand's LE product and regular product in the claimed equilibrium is independent of the consumer's location, the market is composed of two contiguous segments with the consumer segment closer to $\bar{\theta}$ ($\underline{\theta}$) purchasing the LE product of brand H (brand L) if it is available, and settling for the same brand's regular product otherwise. To derive each brand's demand, we solve for the consumer θ_2 who is indifferent between brand H's and brand L's products as follows (in the rest of the proof, we ignore the ε for ease of exposition):

$$(1+s)\theta_2 - k\frac{(q_H-1)}{N-1} - p_H = \theta_2 - k\frac{(q_L-1)}{N-1} - p_L$$

$$\Longrightarrow \quad \theta_2 = \frac{(N-1)(p_H-p_L)+k(q_H-q_L)}{(N-1)s}.$$

Assuming rational expectations, we substitute $q_H = N((\bar{\theta}-\theta_2)/(\bar{\theta}-\underline{\theta})) - l_H$ and $q_L = N((\theta_2-\underline{\theta})/(\bar{\theta}-\underline{\theta})) - l_L$ in the above expression for θ_2 to obtain

$$\theta_2 = \frac{(N-1)(p_H-p_L)(\bar{\theta}-\underline{\theta}) + k((l_L-l_H)(\bar{\theta}-\underline{\theta}) + N(\bar{\theta}+\underline{\theta}))}{(N-1)s(\bar{\theta}-\underline{\theta}) + 2kN}. \tag{3}$$

The profit functions for the high- and low-quality sellers are $\Pi_H(p_H, l_H) = p_H q_H + p_{lH} l_H$ and $\Pi_L(p_L, l_L) = p_L q_L + p_{lL} l_L$, respectively, where p_{lH} and p_{lL} are given by Equations (1) and (2), respectively. Furthermore, we substitute $q_H = N((\bar{\theta}-\theta_2)/(\bar{\theta}-\underline{\theta})) - l_H$ and $q_L = N((\theta_2-\underline{\theta})/(\bar{\theta}-\underline{\theta})) - l_L$, where θ_2 is given by Equation (3). A necessary condition for equilibrium is that each brand's prices in stage 3 be a best response to the other brand's prices, assuming both brands follow the proposed equilibrium strategy. Therefore, we solve the first-order conditions in prices for p_H and p_L as follows:

$$p_H^* = \frac{k(3N - l_H - 2l_L) + (N-1)s(2\bar{\theta} - \underline{\theta})}{3(N-1)}$$

$$p_L^* = \frac{k(3N - 2l_H - l_L) + (N-1)s(\bar{\theta} - 2\underline{\theta})}{3(N-1)}.$$

The second-order sufficient conditions for a local maximum are satisfied. Now, considering stage 2, a necessary condition for equilibrium is that l_H and l_L are best responses to each other. Therefore, we solve the first-order conditions with respect to l_H and l_L to yield the equilibrium values of l_H^* and l_L^* given in Table 3. It can be shown that $l_i^* < q_i^*$, $i \in \{H, L\}$, as claimed.

Check for Deviations from Equilibrium Strategy

In stage 3, we consider two possibilities for a brand deviating from the equilibrium prices: (1) deviation prices that result in all consumers strictly preferring the regular product of the brand to its LE product (so that the brand sells no LE product), and (2) deviation prices that result in a brand selling only the LE product (because the price of the regular product is so high that the consumer who cannot obtain the LE product either chooses not to purchase or purchases a product from the competing brand). It can be shown that either of these price deviations by a brand is unprofitable or infeasible. The intuition in both cases is that an offering of two products results in greater exclusivity value. Now, consider a brand's deviations by a brand in stage 2 to a quantity of LE product (chosen from $(0, N]$) other than the equilibrium quantity. Once again, it can be shown that no such deviation is profitable.[16]

Checking for Other Equilibria

In general, equilibrium may entail one of three possibilities for consumer preference with respect to a brand's products: (1) All consumers strictly prefer the LE product of the brand to its regular product, (2) all consumers are indifferent between the regular and LE products of the brand, and (3) all consumers strictly prefer the regular product of the brand to its LE product. With two brands, we thus have a total of 3^2 or nine possibilities of potential equilibria. The claimed equilibrium presents one of the nine possibilities, with consumer preferences for both brands consistent with (1). Note that consumer preferences in this equilibrium are also consistent with (2) because $l_i^* < q_i^*$, $i \in \{H, L\}$. With some lengthy algebra, it can be shown that none of the other eight cases constitute an equilibrium of the subgame S_{22}. We also consider the possible existence of an equilibrium in which one or both brands price in stage 3 such that they sell only the LE product (by pricing the regular product very high). However, we find that such an equilibrium cannot exist. Thus, the claimed equilibrium is unique. □

Proof of Proposition 3. We use the notation Π_{bij}^* to denote a brand's equilibrium profit in a subgame, where the subscript b denotes the brand and $i, j \in [1, 2]$ refer to the subscript of the subgame. For example, Π_{H22}^* refers to the profit of brand H in subgame S_{22}. Define $F^* = \Pi_{L22}^* - \Pi_{L21}^*$. F^* simplifies to the following:

$$F^* = \frac{2kN^2\left(1 + \frac{2kN(44kN+27(N-1)s(\bar{\theta}-\underline{\theta}))}{(14kN+9(N-1)s(\bar{\theta}-\underline{\theta}))^2}\right)(7kN+3(N-1)s(\bar{\theta}-2\underline{\theta}))^2}{9(N-1)(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))^2}.$$

Since we can show that $\Pi_{H22}^* - \Pi_{H12}^* > F^*$, we have a subgame perfect equilibrium with both brands offering a LE product when $F \le F^*$. Now, define $F^{**} = \Pi_{H21}^* - \Pi_{H11}^*$. F^{**} simplifies to the following:

$$F^{**} = \frac{2kN^2(3kN+(N-1)s(2\bar{\theta}-\underline{\theta}))^2}{9(N-1)(2kN+(N-1)s(\bar{\theta}-\underline{\theta}))(16kN+9(N-1)s(\bar{\theta}-\underline{\theta}))}.$$

We can show that $F^{**} > F^*$ for $\bar{\theta} > 2\underline{\theta}$. Thus, if $F^* < F \le F^{**}$, we have a subgame perfect equilibrium in which

[16] When brand H deviates to certain values of l_H, there can be multiple equilibria. However, the deviation is unprofitable, no matter which equilibrium is played.

only brand H offers an LE product. It follows that if $F > F^{**}$, we have a subgame perfect equilibrium in which neither brand introduces an LE product. Finally, we can show that $\Pi^*_{H22} - \Pi^*_{H12} > \Pi^*_{L12} - \Pi^*_{L11}$ for $\bar{\theta} > 2\underline{\theta}$. Thus, if $\Pi^*_{L12} - \Pi^*_{L11} > F$, then $\Pi^*_{H22} - \Pi^*_{H12}$ will also exceed F. Thus, we cannot have a subgame perfect equilibrium in which only brand L introduces an LE product. Because each of the subgame perfect equilibria identified exist for different values of F, the equilibria are unique. □

Horizontal Differentiation Model: Derivation for Price Subgames

Subgame S_{RR}

We assume that the reservation price u is sufficiently high that the market is fully covered in equilibrium. If both brands enjoy positive sales in equilibrium, there exists a consumer $z \in [0, 1]$ who is indifferent between the brands, with consumers located to the left and right of z purchasing brands A and B, respectively. Given the indifference of consumer z between the brands, we have

$$u - k\frac{q_A - 1}{N-1} - p_A - tz = u - k\frac{q_B - 1}{N-1} - p_B - t(1-z).$$

Note that the exclusivity penalty in the consumer's utility function depends on anticipated quantities, q_A and q_B. Assuming rational expectations we substitute $q_A = Nz$ and $q_B = N(1-z)$ to obtain

$$z = \frac{kN - (N-1)(p_A - p_B - t)}{2Nk + 2t(N-1)}.$$

The profit functions of brands A and B are $\Pi_A(p_A) = p_A Nz$ and $\Pi_B(p_B) = p_B N(1-z)$, respectively. By solving the first-order conditions we obtain the equilibrium prices presented in Table 4. The second-order sufficient conditions for a local maximum are fulfilled at these prices.

Subgames S_{LL} and S_{LR}

We derive the equilibrium for subgame S_{LL} in this section. The equilibrium results of subgame S_{LR} can be similarly obtained.[17] When deriving the equilibrium for subgame S_{LL} we note that a consumer's preference between a brand's LE product and regular product is independent of his location relative to the brand because both these products have the same horizontal product characteristic. Thus, if a brand enjoys positive sales of its LE product, the surplus from the brand's LE product to a consumer at x cannot be less than his surplus from the brand's regular product. Hence, we have the following necessary condition for positive sales of the LE product for brand A:

$$u - k\frac{l_A - 1}{N-1} - p_{lA} - tx \geq u - k\frac{q_A - 1}{N-1} - p_A - tx$$
$$p_{lA} \leq p_A + \frac{k(q_A - l_A)}{N-1}.$$

A similar condition is obtained for positive sales of an LE product in the case of brand B. In any stage 3 equilibrium in which a brand makes positive sales of both its LE and regular product, there are two possibilities for consumer preferences of the brand's products in equilibrium: (a) all consumers strictly prefer the LE product of the brand to its regular product, or (b) all consumers are indifferent between the regular and LE products of the brand. In any stage 3 equilibrium in which all consumers strictly prefer the LE product of brand A to its regular product, a brand will charge the maximum possible for its LE product. Thus, we should have for $\varepsilon > 0$ and arbitrarily small,

$$p_{lA} = p_A + \frac{k(q_A - l_A)}{N-1} - \varepsilon. \tag{4}$$

Similarly, we compute the following relationship between the prices for the LE product and the regular product for brand B, for $\varepsilon > 0$ and arbitrarily small,

$$p_{lB} = p_B + \frac{k(q_B - l_B)}{N-1} - \varepsilon. \tag{5}$$

Therefore, note that when $l_i \leq q_i$, $i \in \{A, B\}$, both cases (a) and (b) result in identical profits to brand i since ε is arbitrarily small ($\varepsilon \to 0$) in Equations (4) and (5).

Next, we claim that $l_i < q_i$, $i \in \{A, B\}$, in the unique subgame perfect equilibrium of S_{LL}. We establish this claim in three steps. First, we identify the necessary conditions for an equilibrium with $l_i < q_i$, $i \in \{A, B\}$. These necessary conditions allows us to compute the equilibrium strategies. Second, we show that no brand can improve its profit by deviating in either stage 3 (pricing subgame) or in stage 2. Last, we show that no other equilibrium exists.

Necessary Conditions for the Claimed Equilibrium

Since $l_i < q_i$, $i \in \{A, B\}$ in the claimed equilibrium, the equilibrium profits are identical whether we assume case (b) or case (a) with $\varepsilon \to 0$. We will assume case (a) with $\varepsilon \to 0$, since the necessary conditions are easier to work with under this assumption. Because consumer preference between a brand's LE product and regular product in the claimed equilibrium is independent of the consumer's location, the market is composed of two contiguous segments with the consumer segment closer to 0 (1) purchasing the LE product of brand A (brand B) if it is available, and settling for the same brand's regular product otherwise. We solve for the consumer z who is indifferent between brand A's and brand B's products as follows (in the rest of the proof, we ignore the ε for ease of exposition since $\varepsilon \to 0$):

$$u - k\frac{q_A - 1}{N-1} - tz - p_A = u - k\frac{q_B - 1}{N-1} - t(1-z) - p_B$$
$$\Longrightarrow\ z = \frac{k(q_B - q_A) - (N-1)(p_A - p_B) + (N-1)t}{2(N-1)t}.$$

Assuming rational expectations, we substitute $q_A = Nz - l_A$ and $q_B = N(1-z) - l_B$ in the above expression for z to obtain

$$z = \frac{k(l_A - l_B + N) - (N-1)(p_A - p_B - t)}{-2t + 2N(k+t)}. \tag{6}$$

The profit functions for brands A and B are $\Pi_A(p_A, l_A) = p_A q_A + p_{lA} l_A$ and $\Pi_B(p_B, l_B) = p_B q_B + p_{lB} l_B$, respectively, where p_{lA} and p_{lB} are given by Equations (4) and (5), respectively.

[17] Details are available from the authors.

Moreover, we substitute $q_A = Nz - l_A$ and $q_B = N(1-z) - l_B$ where z is given by Equation (6). A necessary condition for equilibrium is that each brand's prices in stage 3 be a best response to the other brand's prices, assuming both brands follow the proposed equilibrium strategy. Therefore, solving the first-order conditions in prices for p_A and p_B as follows:

$$p_A^* = \frac{k(3N - l_A - 2l_B)}{3(N-1)} + t$$

$$p_B^* = \frac{k(3N - 2l_A - l_B)}{3(N-1)} + t.$$

The second-order sufficient conditions for a local maximum are satisfied. Now, considering stage 2, a necessary condition for equilibrium is that l_A and l_B are best responses to each other. Therefore, we solve the first order conditions with respect to l_A and l_B to yield the equilibrium values of l_A^* and l_B^* given in Table 4. It can be shown that $l_i^* < q_i^*$, $i \in \{A, B\}$, as claimed.

Check for Deviations from Equilibrium Strategy

In stage 3, we need to consider two possibilities for a brand deviating from the equilibrium prices: (1) deviation prices that result in all consumers strictly preferring the regular product of the brand to its LE product (so that the brand sells no LE products), and (2) deviation prices that result in a brand selling only the LE product (because the regular product is so high that the consumer who cannot obtain the LE product either chooses not to purchase or purchases a product from the competing brand). It can be shown that either of these price deviations by a brand is unprofitable or infeasible. The intuition in the case of (1) is that an offering of two products results in greater exclusivity value. This intuition also applies to case (2), and we also need u to be sufficiently high that a brand loses by having a high price for the regular product. Now, consider a brand's deviations by a brand in stage 2 to a quantity of LE products (chosen from $(0, N]$) other than the equilibrium quantity. Once again, it can be shown that no such deviation is profitable.[18]

Checking for Other Equilibria

In general, equilibrium may entail one of three possibilities for consumer preference with respect to a brand's products: (1) all consumers strictly prefer the LE product of the brand to its regular product, (2) all consumers are indifferent to the regular and LE products of the brand, and (3) all consumers strictly prefer the regular product of the brand to its LE product. With two brands, we thus have a total of 3^2 or nine possibilities of potential equilibria. Because of the symmetry between the brands, the number of unique cases reduces to six. The claimed equilibrium represents one of the six possibilities, with consumer preferences for both brands consistent with (1). Note that consumer preferences in this equilibrium are also consistent with (2) because $l_i^* < q_i^*$, $i \in \{A, B\}$. With some tedious algebra, it can be shown that none of the other five cases constitute an equilibrium of the subgame S_{LL}. We also consider the possible existence of an equilibrium in which one or both brands price in stage 3 such that they sell only the LE product (by pricing the regular product very high). However, we find that such an equilibrium cannot exist. Thus, the claimed equilibrium is unique. □

[18] When a brand deviates to a sufficiently high quantity of LE products, there can be multiple equilibria. However, the deviation is unprofitable, no matter which equilibrium is played.

Proof of Proposition 9. We use the notation Π_{bij}^* to denote a brand's equilibrium profit in a subgame, where the subscript b denotes the brand and $i, j \in [1, 2]$ refer to the subscript of the subgame. For example, Π_{ALL}^* refers to the profit of brand A in subgame S_{LL}. Define $\underline{F} = \Pi_{BLL}^* - \Pi_{BLR}^*$. $\underline{F}$ simplifies to the following:

$$\underline{F} = \frac{kN^2(71k^2N^2 + 153k(N-1)Nt + 81(N-1)^2t^2)}{18(N-1)(8kN + 9(N-1)t)^2}.$$

Thus, we have a subgame perfect equilibrium with both brands offering a LE product when $F \le \underline{F}$. Now, define $\overline{F} = \Pi_{ALR}^* - \Pi_{ARR}^*$. We compute

$$\overline{F} = \frac{kN^2(kN + (N-1)t)}{2(N-1)(8kN + 9(N-1)t)}.$$

We can show that $\overline{F} > \underline{F}$ for $k > 0$. Thus, if $\underline{F} < F \le \overline{F}$, we have a subgame perfect equilibrium in which only one brand offers a LE product. It follows that if $F > \overline{F}$, we have a subgame perfect equilibrium in which neither brand introduces a LE product. The subgame perfect equilibria identified above for $F \le \underline{F}$ and $F > \overline{F}$ are unique. With $\underline{F} < F \le \overline{F}$, we have two subgame perfect equilibria, with a different brand offering a LE product in each equilibrium. □

Proof of Proposition 10.

$$\Pi_{ALL}^* - \Pi_{ARR}^* = \Pi_{BLL}^* - \Pi_{BRR}^* = -\frac{kN^2}{18(N-1)} < 0. \quad \square$$

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