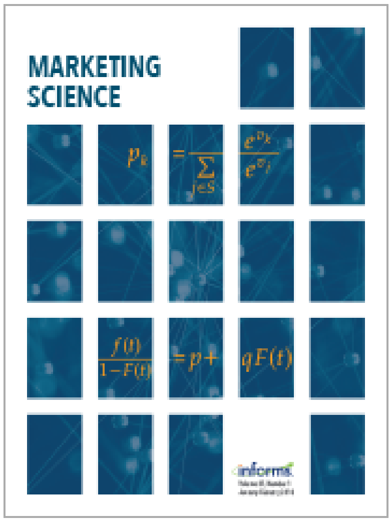

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A Model of Brand Architecture Choice: A House of Brands vs. A Branded House

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Abstract. Some firms that operate in multiple product markets use the same brand in different markets, whereas others use different brands in different markets. This research investigates in which product markets a firm should use the same or different brands and how this decision depends on the relatedness of product markets. To answer this question, I propose a framework of market relatedness that characterizes the relationships among distinct product markets from the supply side (e.g., shared production technology) and demand side (e.g., correlated customer preferences). This framework is applied to a model of reputation in which a multiproduct firm's product quality is jointly determined by its hidden capability type (i.e., adverse selection) and hidden choice of effort level (i.e., moral hazard) in each product market. Consumers obtain noisy information about the firm by observing its track record, that is, product quality produced in the past. Umbrella branding allows consumers to pool the firm's track record across different product markets and form expectations about the product quality based on market relatedness. The analysis shows that umbrella branding is optimal if supply-side relatedness is high and demand-side relatedness is not too high. However, if the product markets are closely related in both dimensions, then independent branding may be optimal because, as an umbrella brand, the firm faces a temptation to exploit positive information spillover across product markets through its shared brand name. By using different brand names, a firm can credibly commit to investing in all product markets and thereby earn higher profits. Finally, this paper provides implications for an umbrella brand's customer relationship management strategy whether to serve the same or distinct customer segments with its products.

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1. Introduction

Many firms operate in multiple product markets while seeking opportunities to further expand their product portfolio. When a firm enters a new product category, one of the major challenges it faces is the decision of whether to use an existing brand or a different brand. A brand is a firm's most valuable intangible asset, so a firm with a reputable brand will want to leverage that asset by using its existing brand. For example, Honda first established its brand with motorcycles and later used the same brand to enter other markets, such as automobiles, lawnmowers, and several different categories of power equipment. Chanel sells a variety of products under its brand, such as apparel, accessories, cosmetics, and fragrances. This practice of housing different products under the same brand leads to a brand architecture called *a branded house* or, more commonly, *umbrella branding*. Through the common brand name, consumers can easily associate the firm's different products with one another. However, some other firms use new brands when they enter new product markets, even though they possess strong existing brands. For instance, Procter & Gamble (P&G) has several brands in different product markets, such as Tide for laundry detergent, Crest for toothpaste, and Pampers for baby diapers. Similarly, Unilever has several distinct brands, such as Dove, Lipton, and Hellmann's. This branding strategy results in *a house of brands* or, alternatively, *independent branding*. Because different brands are used in various product markets, most lay customers of these product brands are not aware that they are produced by the same firm. Given the significance of successful brand management, firms must choose between umbrella branding and independent branding optimally.[1]

Umbrella branding is different from independent branding in that, in the former, the shared brand name establishes an association between different products produced by the same firm. As it is easy for them to recognize the association between products, consumers can draw inferences about a product quality based on the brand's performance in other product markets. Through consumers' inferences across markets, an umbrella brand induces reputation spillovers. However, if the firm adopts different brands for its products, many consumers will be unaware of the association between the products. Therefore, consumers will be much less likely to draw inferences across product markets.[2] For example, in the wake of the Volkswagen scandal, in which the company was found to have cheated on carbon emissions tests for diesel engine models, sales of not only its diesel but also its nondiesel models suffered.[3] On the other hand, models of the Audi brand, also owned by Volkswagen, fared better than Volkswagen models. Therefore, these key trade-offs through reputation spillover induced by umbrella branding should be analyzed carefully to understand a firm's optimal branding decision.

These trade-offs of umbrella branding have been studied in the literature (e.g., Wernerfelt 1988, Cabral 2000, Miklós-Thal 2012, Moorthy 2012), and a main takeaway is that a firm's mere adoption of umbrella branding can credibly signal high quality, because if a firm were not of high quality, then it would not risk its brand assets by placing the same brand name on multiple products, each of which may fail and induce negative reputation spillovers. Thus, only a high-quality firm would benefit from using umbrella branding. However, in reality, some high-quality firms, such as P&G, deliberately choose independent branding, a puzzle for which the signaling story does not offer a satisfactory explanation.

This paper tries to provide an explanation for this puzzle and offer managerial implications for a firm's optimal branding decision by investigating how reputation spillover depends on the relatedness between product markets. The extent of positive and negative reputation spillover critically depends on market relatedness. For instance, a consumer looking to purchase a camcorder may rely on her experience with the same brand's camera, as a camcorder and a camera have some shared features, such as lenses, motion detection, and light adjustment. On the other hand, the same buyer will likely find it irrelevant to know the quality of the brand's unrelated products, such as handbags.

To conceptualize the relatedness between product markets, I propose a parsimonious framework of market relatedness, which captures stylized patterns. First, distinct product markets may be related through supply-side factors, which is called *supply-side relatedness*. For example, Honda has applied its key technologies in making engines in different product categories that require engines. The more production technologies are shared, the more correlated the firm's capabilities in producing high-quality products in different markets are. Therefore, for a firm using an umbrella brand, consumers will draw more inferences about the firm's ability to produce a high-quality product in one market based on its track record in another market. Thus, the extent of reputation spillover is determined by supply-side relatedness. However, if the firm uses different brands, then consumers will be unaware of the association between different products, and therefore unable to draw inferences across markets.

Second, firms such as Chanel may serve similar target segments across different product markets, perhaps because their target segment has correlated preferences for different products in fashion and beauty. Therefore, Chanel, which has a profound understanding of the preferences of its target customers, finds it profitable to cater to the needs of its target segment in various product markets, such as apparel, handbags, sunglasses, jewelry, and even perfume. Therefore, these products may be related through the shared customer bases, which is denoted as *demand-side relatedness*. The higher the demand-side relatedness is, the more likely consumers are to interact with the firm in multiple product markets. Through their interactions, they will have access to more information about the firm.

Based on these stylized patterns, the relationship between two product markets is characterized in two dimensions: the supply side and demand side. This framework of market relatedness, which fundamentally influences how consumers observe and infer from information about the firm, is applied to a model of reputation in which product quality is jointly determined by the firm's hidden characteristic (i.e., how capable the firm is) and hidden actions (i.e., how much effort it exerts). This modeling integrates two distinct approaches in the extant literature on umbrella branding in which quality is determined by either the firm's hidden characteristic or hidden action. This approach is intended to capture how product quality is determined in reality. Even companies with great capabilities, such as Volkswagen, are tempted to save costs at the expense of sacrificing product quality. When a firm's great capability is complemented by self-discipline to exert its best efforts, the firm can continue to produce high-quality products and maintain a good reputation.

Key ingredients of the model are as follows. In each product market, a monopolist can be either competent or incompetent. In each period of an infinite-time-horizon game, a competent type can, unlike an incompetent one, make a costly investment in quality to increase the probability of producing a high-quality product. In the beginning of each period, one short-lived consumer who prefers a high-quality product arrives but does not observe the realized quality in the same period. Instead, the consumer observes the firm's realized quality in the previous period and updates the consumer's beliefs about the firm's hidden capability types, as well as the current realized quality. Thus, a competent type can differentiate itself from an incompetent one by investing in quality and producing high-quality products. If consumers believe that competent types invest in quality, then they will make positive inferences from a firm's good track record about its capability type and reward the firm by paying a greater price for its product. This upfront reward will, in turn, provide incentives for the competent type to continue to invest in quality if this reward is greater than the investment cost. If the brand names in two product markets are the same, then consumers can pool the firm's track record across product markets.

Given the crucial effect of branding decisions on consumers' beliefs, this paper investigates a firm's optimal branding decision by comparing the best feasible profits under independent branding and umbrella branding. This comparison boils down to whether each branding regime can sustain the most profitable equilibrium, which is referred to as the *high-effort equilibrium* (*HE*). In this equilibrium, the competent type always invests in quality, and therefore maximally differentiates itself from the incompetent type. Accordingly, consumers pay the highest prices in this equilibrium.

The analysis shows that umbrella branding is optimal for high supply-side relatedness if demand-side relatedness is low. However, the result can be different if demand-side relatedness is high; that is, independent branding can be optimal if both demand-side relatedness and supply-side relatedness are high. The intuition behind this finding is as follows. An umbrella brand used for highly related product markets can induce an excessive amount of information spillover across product markets. In particular, consumers who observe the firm's good track record in one market will make positive inferences about the firm in another market. Then, the firm is tempted to exploit this positive information spillover by investing in only one product market instead of exerting its best efforts in both markets.[4] Given this temptation, consumers do not trust the firm to invest in quality in both markets. Accordingly, they are no longer willing to pay as high a price, which negatively affects the firm's profits. The firm can resist the temptation to exploit the positive reputation spillover by adopting independent brands, which will eliminate reputation spillover across product markets. Therefore, through independent branding, the firm can credibly communicate to consumers that it will exert its best efforts in both product markets.

The results provide new insights into how firms should choose between independent branding and umbrella branding on the basis of how related the product markets are. Interestingly, the benefit of independent branding as a disciplinary mechanism can be the greatest when the two product markets are closely related on both supply side and demand side. Moreover, the findings of this paper provide an explanation for why some high-quality firms, such as P&G and Unilever, may avoid umbrella branding and still manage to maintain several successful brands; the decision to use different brands may contribute to the continued success of brands by providing additional incentives for firms to invest in each brand.

The rest of this paper is organized as follows. The next section discusses related literature. Section 3 presents the model setup and framework of market relatedness. Section 4 analyzes the model and investigates a firm's optimal branding decision. Section 5 explores an umbrella brand's optimal customer relationship management (CRM) strategy for whether to target the same or distinct customer segments with its different products. Section 6 concludes the paper. The proofs of all results are presented in the appendix and the online appendix.

2. Related Literature

Umbrella branding has become a productive area of marketing and economics research. The approach in the present paper draws on a prevalent economic view of branding as a vehicle of information about a firm. In the existing literature, depending on which type of information is communicated through branding, there are two distinct approaches to umbrella branding.

In one approach, a brand can be used to communicate a firm's hidden characteristic, its quality. A monopolist is given an exogenous quality type, which determines the product quality. Then, consumers' expectations of a brand correspond to their posterior beliefs about the firm's quality type.[5] Analyzing adverse selection models, this stream of research has documented that the mere decision to adopt umbrella branding can be a signal of high quality. Wernerfelt (1988) was the first to show that an umbrella brand can signal high quality in all product markets. This result relies on two key assumptions: the first assumption is that introducing a new product as an umbrella brand costs more than introducing it as an independent brand, and the second assumption is that the most pessimistic off-equilibrium beliefs are imposed on consumers who believe a firm to be of low quality for both products if any one product turns out to be bad.[6] However, subsequent research finds that the signaling effect of umbrella branding may not be robust if these assumptions are relaxed. In particular, Moorthy (2012) shows that such a result can only hold under a very stringent set of conditions. According to Miklós-Thal (2012), umbrella branding can signal a positive correlation in the quality of its different products so that a firm whose products are both high quality or both low quality chooses umbrella branding.

In another approach (e.g., Andersson 2002, Hakenes and Peitz 2008, Cabral 2009), a brand conveys information about the firm's hidden action, that is, its choice of an effort level, either high (costly) or low (costless). More specifically, product quality is determined by the firm's effort choice unobserved by consumers. In such a moral hazard model, a brand serves as an implicit contract in which consumers believe the firm to choose high effort if it has always produced good outcomes. In turn, consumers' beliefs provide the right incentives for the firm to always choose high effort, as long as the cost of investment is sufficiently low and the discount factor is sufficiently large.[7] In these models, consumers' beliefs about the firm's choice of effort level are arbitrarily imposed by researchers, and the scope of the implicit contract is sensitive to which beliefs are imposed.[8] Moreover, the firm cannot gradually improve or exploit consumers' beliefs through its choice of effort level. Instead, the best possible reputation can arise or be stripped away altogether, depending on the imposed beliefs.[9]

This paper integrates two existing approaches to umbrella branding by analyzing a model of reputation with both a firm's hidden type (exogenous capability type) and hidden action (endogenous effort decision) in the spirit of Mailath and Samuelson (2001). Based on the firm's record of past quality, consumers form expectations about both of the firm's unobservables which jointly determine the quality of the present. In turn, the firm decides whether to exert effort or not given the consumers' expectations. A more capable type can build up its reputation by making a costly investment or squander it by not investing. This paper extends the model of Mailath and Samuelson (2001), who analyze a model for a monopolist in a single product market, to investigate a firm's reputation in multiple product markets. A prominent theme in this stream of research is whether the efficient equilibrium (or the HE in this paper) in which the competent type always exerts high effort can be sustained. Over an infinite time horizon, as consumers accumulate near-perfect information about the firm's type, the marginal return to the firm from an additional costly effort is very small such that such an equilibrium unravels. However, in Mailath and Samuelson (2001), an HE exists if the firm ownership is replaced with a positive probability, where the replacement is unknown to consumers. Hörner (2002) shows that in a duopoly market, competition can also discipline each firm's behavior and sustain the equilibrium. Liu and Skrzypacz (2014) analyze a model with limited consumer memory and demonstrate that a firm's reputation can fluctuate over time. In a study related to this paper, Neeman et al. (2019) explore a model with limited memory and show that forming a collective brand with another firm can help sustain the HE. This paper also assumes consumers' limited memory and investigates the effects of the firm's branding decision and market relatedness on the HE.[10]

Previous research experimentally documents that relatedness between an existing and a new product is an important basis for consumers' evaluation of the new product (e.g., Aaker and Keller 1990, Boush and Loken 1991). Park et al. (1991) find that, depending on the perception of brands—for example, functional (Timex) versus prestigious (Rolex)—there could be different bases of market relatedness through which a brand could be extended more successfully. Miklós-Thal et al. (2018) propose a different micromodel of supply-side relatedness that captures an idea similar to the one explored in this paper, that is, a firm's capabilities in distinct markets may be correlated. Their focus is on the firm's optimal product strategy. This paper provides a microfoundation for market relatedness on both the supply side and demand side and investigates how market relatedness affects a firm's decision of whether to adopt umbrella branding.

3. Model and Equilibrium

3.1. Model Setup

A long-lived monopolist sells two experience goods, one in each of the two product markets A and B, over a discrete infinite time horizon ($t = 1, 2, \ldots$). The product quality in each period is a random variable, the realization of which can be either high quality (H) or low quality (L). In each market $i \in \{A, B\}$, the firm is either persistently competent ($\theta^i = C$) or incompetent ($\theta^i = I$), depending on whether it has the capability to produce a high-quality product. In each period, a competent type can invest in quality at cost $c > 0$ to produce a high-quality product (H) with probability $\Delta > 0$. Otherwise, if it does not invest in quality, the firm will always produce a low-quality product (L). On the other hand, an incompetent type is incapable of producing a high-quality product regardless of whether it incurs an investment cost or not.[11] Thus, realized quality is a joint and noisy outcome of the firm's capability type and its investment decision. The distribution of the firm's capability types in the two product markets $\theta = (\theta^A, \theta^B)$ is represented by $\Pr(\theta^A, \theta^B)$, which yields the marginal distribution of the firm's type in each product market $\Pr(\theta^i = C) = \mu_0 \in (0, 1)$.

In every period, one short-lived representative consumer is drawn from a pool of consumers and interacts with the firm in that period. The consumer (sometimes referred to as she) may or may not have a unit demand in each of the two product markets, each event occurring with probability 1/2. In each market in which she has a unit demand, the consumer receives zero utility from a low-quality product (L) and positive utility from a high-quality product (H) normalized to one. She does not participate in the market in which she has no unit demand. Because the firm sells experience goods, the consumer does not know the realized quality of the product before purchase. She knows neither the firm's persistent capability type in each market (θ^i) nor its investment decisions. Instead, the consumer in period t observes the realized quality experienced by a consumer in the previous period, denoted by $\mathbf{h}_{t-1} = (h^A_{t-1}, h^B_{t-1})$.[12] If the previous consumer makes a purchase in market i, then the consumer in the subsequent period will observe a track record of either a high-quality (H) or low-quality (L) product. However, if no product is purchased, then the track record will be $\emptyset$. Thus, the firm's track record in each market available in period t is $h^i_{t-1} \in \mathcal{H} := \{H, L, \emptyset\}$.

Whether the firm uses umbrella branding or independent branding, denoted by $br \in \{umb, ind\}$, has a profound influence on how consumers observe and infer from the firm's track record. If the brand names in the two markets are the same ($br = umb$), then consumers can readily recognize that the two products are produced by the same firm. Therefore, they are able to use the information in both product markets h^A_{t-1} and h^B_{t-1} and make a purchase decision in each market. However, if the brand names are different ($br = ind$), then it will be costly for consumers to establish the association between the two products. To capture this fundamental distinction between independent branding and umbrella branding, consumers are assumed to be unaware of an association between two products if their brand names are different. Consequently, a consumer facing an independent brand will interact with the firm in each product market independently; that is, only the firm's track record in the same market is relevant.[13]

The consumer arriving in each period observes the firm's track record and whether the brand names are the same or different. As the consumer is short lived, she behaves myopically. If she has a unit demand in product market i, it is assumed that she pays her expected utility.[14] It is assumed that $\frac{\Delta}{2} > c$, which implies that an investment in quality is an efficient action because each consumer has a unit demand in each market with probability 1/2.

The sequence of the game is as follows. At $t = 0$, the firm's persistent capability types $\theta = (\theta^A, \theta^B) \in \{C, I\}^2$ in the two product markets are realized. In each period $t \geq 1$, the firm decides whether to make a costly investment in quality in each product market. Then, one short-lived consumer arrives and interacts with the firm in each product market in which she has a unit demand.[15] The consumer observes the realized quality experienced by the consumer in the previous period $\mathbf{h}_{t-1} \in \mathcal{H}^2$ and makes purchasing decisions. The quality of each purchased product $\mathbf{h}_t$ is publicly realized. Then, the consumer leaves and the period ends.

3.2. Strategy and Equilibrium Concept

The price that a consumer pays is determined by her posterior beliefs about the firm's unobserved capability type, which is determined by the firm's track record. Thus, Markovian pure strategies in which the firm's investment strategy depends only on the payoff-relevant state are considered.[16] More specifically, an investment strategy of each type $\theta = (\theta^A, \theta^B) \in \{C, I\}^2$ is a mapping from the firm's track record $\mathbf{h} = (h^A, h^B) \in \mathcal{H}^2$ to a decision whether to invest in each product market i, denoted by $\sigma^i_\theta(\mathbf{h}) \in \{0, 1\}$. An incompetent type has a trivial strategy to never invest in quality because its costly investment does not lead to a better outcome. Thus, the firm has a nontrivial investment decision to make only if it is competent in at least one product market. The (C, C)-type firm makes an investment decision in each market, corresponding to the strategies denoted by $\sigma^A_{C,C}(\mathbf{h})$ and $\sigma^B_{C,C}(\mathbf{h})$. If the firm is competent in only one market, then it has only one decision to make in each period: $\sigma^A_{C,I}(\mathbf{h})$ and $\sigma^B_{I,C}(\mathbf{h})$.

In period t, the consumer updates her beliefs about the firm's capability types in each product market based on the firm's track record $\mathbf{h}_{t-1}$ and branding regime $br \in \{ind, umb\}$. Given the firm's track record, the posterior distribution of the firm's capability types is $\Pr(\theta^A, \theta^B | \mathbf{h}_{t-1}; br)$.[17] From this posterior distribution, the consumer's posterior beliefs about the firm's capability types in both product markets can be computed. For instance, the consumer believes the firm to be competent in product market A with probability

$$\mu^A(\mathbf{h}_{t-1}; br) = \Pr(C, C|\mathbf{h}_{t-1}; br) + \Pr(C, I|\mathbf{h}_{t-1}; br). \quad (1)$$

Similarly, the consumer's posterior belief about the firm in product market B is $\mu^B(\mathbf{h}_{t-1}; br) = \Pr(C, C|\mathbf{h}_{t-1}; br) + \Pr(I, C|\mathbf{h}_{t-1}; br)$.

If the consumer has a unit demand in product market $i \in \{A, B\}$, then she pays her expected utility, which coincides with the expected probability of receiving a high-quality product. Only a competent type can produce a high-quality product with probability $\Delta > 0$ if it chooses to make an investment in quality. Thus, if the consumer in period t has a unit demand in market i and observes a track record $\mathbf{h}_{t-1}$, then she pays the following price:

$$p^i(\mathbf{h}_{t-1}; br) = \sum_{\theta^A, \theta^B \in \{C, I\}} \Big(\Pr(\theta^A, \theta^B | \mathbf{h}_{t-1}; br) \cdot \sigma^i_{\theta^A, \theta^B}(\mathbf{h}_{t-1}) \Big) \cdot \Delta. \tag{2}$$

Otherwise, if she does not have a unit demand, then she does not pay a nonnegative price, and no transaction occurs. Therefore, it is as if $p^i(\cdot) = 0$, and no outcome (Ø) is generated.[18]

The firm's per-period profit in period t is $\pi_t = \sum_{i \in \{A,B\}} p^i_t - c \cdot d^i_t$, where $d^i_t \in \{0, 1\}$ denotes whether the firm makes an investment in quality ($d^i_t = 1$) or not ($d^i_t = 0$) in period t and market i. Note that the firm's current investment decision d^i_t is not observed by the current consumer and, therefore, affects the incurred investment cost and not the price. The firm's payoff in the future is discounted by $\delta \in (0, 1)$, so its present discounted payoff in period t is $V_t = \sum_{s \geq t} \delta^{s-t} \cdot \pi_s$.

The equilibrium of this game is defined as follows: (i) given the strategy of all other types, an investment strategy of each type θ, $\sigma^A_\theta(\mathbf{h}_{t-1})$ and $\sigma^B_\theta(\mathbf{h}_{t-1})$, maximizes the firm's present discounted payoff at each track record $\mathbf{h}_{t-1} \in \mathcal{H} \times \mathcal{H}$; (ii) each consumer pays $p^i_t(\mathbf{h}_{t-1}; br)$ whenever she has a unit demand in market i; and (iii) the consumer's beliefs are consistent with the firm's equilibrium strategy and updated using Bayes' rule. Note that an equilibrium of this game is a perfect Bayesian equilibrium with Markovian refinement.

The game admits multiple equilibria. For instance, the firm's strategy to never invest ($\sigma^i_\theta(\cdot) \equiv 0$) and consumers' beliefs consistent with this strategy are always part of an equilibrium for any $c > 0$. In this equilibrium, which is referred to as the *low-effort equilibrium* (*LE*), only low-quality products are produced. Thus, consumers always pay a price of zero, which, in turn, eliminates the firm's incentives to invest in quality. Therefore, the profit of a firm of any type θ is zero in this equilibrium.

An efficient equilibrium is dubbed the HE, in which the firm always invests in each market where it is competent. The investment strategy in this equilibrium is represented by $\sigma^i_\theta(\cdot) \equiv 1$ whenever $\theta^i = C$. By making costly investments, the competent type can produce a high-quality product with a positive probability (Δ). The firm's track record will be observed by the consumer arriving in the next period, so the competent type can differentiate itself from the incompetent type. Then, each consumer will pay a higher price to a firm with a better track record based on her rational expectation that a competent type will, in fact, invest in quality. Expecting this future reward, the firm may find it optimal to indeed invest in quality even though its current investment decision is unobserved by the consumer. A consumer pays the highest price in this equilibrium for each history compared with any other equilibrium, which makes it the most profitable equilibrium unless the investment cost is prohibitively high.[19]

This paper compares the best feasible equilibrium in independent branding and umbrella branding and identifies the optimal branding regime. The analysis shows that this comparison hinges on which of the two branding regimes can sustain the HE. Accordingly, Section 4.1 first establishes conditions under which the HE exists under each branding regime by analyzing the two regimes separately. Then, Section 4.2 more formally examines the firm's optimal branding decision by analyzing an extended version of the current game in which the firm makes a branding decision at $t = -1$ before the current game begins.

This paper abstracts away from signaling through a firm's branding decision, a topic that has already been extensively studied in the extant literature. This means that consumers in the current model do not draw information from the mere fact that the firm chose a particular branding. The purpose of this approach is to preclude complications from potential signaling through branding decisions, which allows the paper to focus on the firm's track record as the main channel through which the firm can convey information about quality and thereby manage its own reputation.

3.3. Belief-Updating Process

A consumer in each period makes inferences about the firm's private capability types on the basis of its track record in the two markets, depending on the consumer's prior beliefs and her rational expectations about the firm's equilibrium investment strategy. Importantly, consumers' belief updating also depends on the firm's branding regime (whether the brand names are the same or different) and market relatedness (how related the product markets are on the supply side and the demand side).

3.3.1. Branding Regime and Market Relatedness. If the firm uses the same brand, then the extent of information pooling across product markets is determined by the market relatedness. Based on observed patterns of multiproduct firms, a conceptual framework of market relatedness is proposed.

One pattern is that a firm sometimes applies its key supply-side technologies to multiple product markets. For example, Honda makes a range of products, such as motorcycles, automobiles, lawn mowers, and air jets, that all require engines. Canon's lens-making technology is used for its cameras, camcorders, scanners, projectors, and digital radiography devices. In such cases, the firm's distinct products are related through their shared technologies or key product attributes. This supply-side relatedness implies that a firm's capability to produce high-quality products in different markets will be positively correlated. The supply-side relatedness between product markets is captured through a joint distribution $\Pr(\theta^A, \theta^B)$ satisfying two properties: first, the marginal distribution of the firm's type in each market is $\Pr(\theta^i = C) = \mu_0$, and second, the firm's capability types in two product markets can be positively correlated, as represented by a parameter $\rho \in [0, 1]$. Accordingly, the types are distributed as follows:

$$\begin{aligned} \Pr(C, C) &= \mu_0(\rho + (1-\rho)\mu_0), \\ \Pr(I, I) &= (1-\mu_0)(\rho + (1-\rho)(1-\mu_0)) \\ \Pr(C, I) &= \Pr(I, C) = \mu_0(1-\mu_0)(1-\rho). \end{aligned} \tag{3}$$

For example, if $\rho = 0$, then the firm's types in the two markets are independent, whereas if $\rho = 1$, then they are perfectly positively correlated. Note that $\Pr(C, C)$ and $\Pr(I, I)$ increase in ρ, whereas $\Pr(C, I) = \Pr(I, C)$ decreases. Thus, as ρ increases, the firm's capability types in two markets are more likely to be the same. If the firm uses the same brand, then consumers account for the correlation and accordingly make inferences across product markets. Otherwise, if the brand names are different, then consumers are unaware of the association between the two products and are thus unable to account for the correlation. Therefore, for an independent brand, the effective ρ perceived by consumers is zero.[20]

Another observed pattern is that firms sometimes serve similar target segments in distinct product markets, which is widely practiced by fashion or lifestyle brands such as Chanel and Ralph Lauren. These companies have a profound understanding of their target segments to whom they find it valuable to sell multiple products, ranging from clothes and handbags to sunglasses and perfumes. Then, a firm's different products that may seem unrelated in terms of production technology are, in fact, related through a shared customer base. This demand-side relatedness is captured by parameter $\alpha \in [0, 1]$, which represents the extent of the shared customer base across two product markets.

More specifically, in each period, one consumer is randomly drawn from a unit mass of consumers and interacts with the firm. With probability $\frac{\alpha}{2}$, the consumer has a unit demand in both markets. The consumer has a unit demand only in product market A with probability $\frac{1-\alpha}{2}$ and only in market B with the same probability. The consumer has no demand with the remaining probability of $\frac{\alpha}{2}$. Thus, the demand-side relatedness determines the distribution of the consumer's unit demand and amount of information about the firm generated. If it is high (or if α is large), then each consumer is more likely to make a purchase in both product markets or neither product market. In the former, information about the firm will be realized in both markets, whereas in the latter, no information will be realized. This formulation of demand-side relatedness satisfies two features. One, the mass of consumers interested in both products is increasing in α. Two, the total expected demand is one in each period, 1/2 in each product market, and thus independent of α.[21]

3.3.2. Posterior Beliefs. In the current period, a consumer observes the firm's track record $\mathbf{h}_{t-1} = (h^A_{t-1}, h^B_{t-1}) \in \mathcal{H} \times \mathcal{H}$, which was experienced by the consumer in the previous period. Based on the firm's track record, the consumer updates her beliefs about the firm's private capability types. In doing so, the consumer also accounts for her rational expectations about the firm's equilibrium investment strategy. Given this paper's focus on the HE, the posterior beliefs within this equilibrium are presented.[22] In the HE, a firm's investment strategy in each product market is determined by the firm's capability type in that market. Therefore, $\mu^i(\mathbf{h}_{t-1}; br)$ defined in (1) is sufficient to characterize the consumers' posterior beliefs.[23]

Belief Updating for an Independent Brand. Under independent branding, consumers' posterior beliefs about the firm's type in market i depend only on the information in the same market, h^i, and are independent of the information in the other market, h^j ($j \neq i$). For example, the posterior beliefs in product market A given for any $h^B_{t-1} \in \mathcal{H}$ are as follows:

$$\begin{aligned} \mu^A(H, h^B_{t-1}; ind) &= \frac{\mu_0 \Delta}{\mu_0 \Delta + (1-\mu_0) \cdot 0} = 1, \\ \mu^A(L, h^B_{t-1}; ind) &= \frac{\mu_0(1-\Delta)}{\mu_0(1-\Delta) + 1 - \mu_0}, \end{aligned} \tag{4}$$

and $\mu^A(\varnothing, h^B_{t-1}; ind) = \mu_0$. Only a competent type can produce a high-quality product. Therefore, upon observing a firm's track record of a high-quality product, consumers learn that the firm is competent in that market, that is, $\mu^A(H, h^B_{t-1}) = 1$. A bad track record can be caused by a competent type with probability $1 - \Delta$ or by an incompetent type with probability 1. Therefore, consumers account for the distribution of competent and incompetent types in the market and compute the posterior probability that the firm is competent. If there is no track record available for the firm in the market, then the posterior beliefs are the same as the prior.

Belief Updating for an Umbrella Brand. Under umbrella branding, even if the firm has no track record in a product market, consumers can use its record in the other market to update beliefs about the firm's unobserved capability type. If the observed track record is good, then consumers draw more optimistic inferences. Similarly, if the track record is bad, this leads to more pessimistic posterior beliefs about the product in the other market. The extent of cross-market inferences is determined by the supply-side relatedness (ρ), which enters through the prior joint probability distribution $\Pr(\theta^A, \theta^B)$. For example, upon observing histories $(h^A, h^B) = (\varnothing, H)$, $(\varnothing, L)$, and $(\varnothing, \varnothing)$, the respective posterior beliefs are as follows:

$$\mu^A(\varnothing, H; umb) = \frac{\Pr(C,C)\cdot\Delta + \Pr(C,I)\cdot 0}{\Pr(C,C)\cdot\Delta + \Pr(C,I)\cdot 0 + \Pr(I,C)\cdot\Delta + \Pr(I,I)\cdot 0},$$

$$\mu^A(\varnothing, L; umb) = \frac{\Pr(C,C)\cdot(1-\Delta) + \Pr(C,I)\cdot 1}{(\Pr(C,C)+\Pr(I,C))\cdot(1-\Delta) + (\Pr(C,I)+\Pr(I,I))\cdot 1}.$$

Clearly, $\mu^A(\varnothing,\varnothing; umb) = \mu_0$. A track record $(\varnothing, H)$ could have been produced by two types of firms—(C, C) and (I, C)—because the firm has to be competent in market B to produce a high-quality product. Between these two types, only the (C, C)-type firm is also competent in market A. Therefore, the posterior probability of the firm being competent in market A is obtained by dividing the probability that the (C, C)-type firm produces a track record $(\varnothing, H)$ by the total probability of realizing the same track record.

The discussions thus far on the consumer's posterior beliefs under independent branding and umbrella branding can be summarized as follows:[24]

Lemma 1 (Belief Updating)**.** *For any h^A, $h^B \in \mathcal{H}$, the consumer's posterior beliefs about the firm's capability type in market A satisfy the following properties:*

i. *Direct information effect:* $\mu^A(H,h^B;br)=1\geq\mu^A(\varnothing, h^B;br)\geq \mu^A(L,h^B;br)$.

ii. *No information spillover for an independent brand:* $\mu^A(h^A,h^B;ind)$ *is independent of* h^B, *that is,* $\mu^A(h^A,H; ind) = \mu^A(h^A,\varnothing;ind) = \mu^A(h^A,L;ind)$.

iii. *Information spillover within an umbrella brand:* $\mu^A(h^A,H;umb)\geq\mu^A(h^A,\varnothing;umb)\geq\mu^A(h^A,L;umb)$, *where* $\mu^A(h^A, H;umb)$ *is increasing, and* $\mu^A(h^A,L;umb)$ *decreasing, in* ρ.

The lemma illustrates the effect of the firm's track record, good or bad, on the customer's posterior beliefs in the same market and in the other market. If the firm's track record in a market is good, then the firm is believed to be competent in the same market with probability 1, that is, $\mu^A(H,h^B;umb) = 1$. This belief will lead the consumer to pay the maximal price. On the other hand, even if the firm's track record in a market is not good, a good outcome in another market $\mu^A(h^A,H;umb)$ may spill over, and the extent of which is determined by the supply-side relatedness. Accordingly, the consumer may pay a greater price in both markets than she would have otherwise.

The demand-side relatedness (α) determines the distribution of available information about the brand. If this relatedness is high, then each consumer is more likely to have a unit demand in both product markets. Therefore, it is more likely that there is information available about the firm in both product markets, in which case there is more information to be pooled across markets. At the same time, the probability that the consumer buys neither product also increases, so it becomes more likely that no information about the firm is available in both markets. However, if α is small, then it becomes more likely that a consumer buys either product, so information about the firm will be available in only one of the two product markets.

The posterior beliefs affect the price that each consumer pays, as shown in (2). In the HE, $\sigma^i_\theta(\mathbf{h}) = 1$ whenever $\theta^i = C$, so $p^i(\mathbf{h}; br) = \mu^i(\mathbf{h}; br)\cdot\Delta$. The price is increasing in the consumer's posterior beliefs, which indicates that a competent type firm has an incentive to invest in quality; a costly investment may generate a good track record, resulting in greater prices in the future.

4. The Optimal Brand Architecture

This section compares the best feasible profits in the model of independent branding and umbrella branding. This comparison turns out to depend on each branding regime's scope of the HE, the most profitable equilibrium. Section 4.1 characterizes the conditions under which the HE exists for each branding regime. Section 4.2 analyzes a game in which a firm makes its branding decision at $t = -1$ before the current game begins. In particular, the section characterizes all pure-strategy equilibria and identifies the equilibrium branding decision.

4.1. The High-Effort Equilibrium

4.1.1. Independent Branding (House of Brands). If the brand names are different in different markets, consumers are not able to associate the firm's two product markets, so the analysis boils down to a market-by-market investigation. In the HE, the firm always invests in quality if it is competent in the market. The firm's investment decision determines the distribution of realized quality, which is observed by the consumer in the next period and affects the continuation payoff. Given a track record $\mathbf{h}_{t-1} = (h^A_{t-1}, h^B_{t-1})$, a firm of type $\theta = (\theta^A, \theta^B)$ expects a per-period profit $\frac{p^A(\mathbf{h}_{t-1};ind)+p^B(\mathbf{h}_{t-1};ind)}{2} - c\cdot(\sigma^A_\theta(\mathbf{h}_{t-1}) + \sigma^B_\theta(\mathbf{h}_{t-1}))$. Depending on the firm's investment decisions in both product markets, the new outcomes $\mathbf{h}_t = (h^A_t, h^B_t)$ will be realized, the distribution of which is denoted by $\Pr(\mathbf{h}_t|\theta)$.[25]

Given the new outcomes, the continuation payoff is $V_\theta(\mathbf{h}_t; ind)$. Therefore, the firm's value function must satisfy

$$V_\theta(\mathbf{h}_{t-1}; ind) = \frac{p^A(\mathbf{h}_{t-1}; ind) + p^B(\mathbf{h}_{t-1}; ind)}{2} - c \cdot \left(\sigma_\theta^A(\mathbf{h}_{t-1}) + \sigma_\theta^B(\mathbf{h}_{t-1})\right) + \delta \cdot \sum_{\mathbf{h}_t \in \mathcal{H}^2} \Pr(\mathbf{h}_t|\theta) \cdot V_\theta(\mathbf{h}_t; ind). \quad (5)$$

The HE exists if and only if the firm has no profitable deviation from the equilibrium strategy. According to the *one-shot deviation principle*, it is necessary and sufficient to rule out any profitable one-shot deviation in which the firm deviates in one period and thereafter returns forever to the equilibrium strategy. If a firm is competent in only one market ($\theta = (C, I)$ or (I, C)), then it can deviate in only that market. A firm that is competent in both markets ($\theta = (C, C)$) can deviate in one or both markets. However, the firm's considerations for these two types of deviations are identical in independent branding because the firm's decision in one market has no effect on its decision in the other market. In other words, the firm will deviate in both product markets if and only if it is optimal to deviate in each market independently.

For a deviation in one market, without loss of generality, a deviation in market A is examined, which implies that $\theta^A = C$, so $\theta = (C, \theta^B)$. Then, the payoff from the deviation is denoted and defined by

$$V_\theta\left(\mathbf{h}_{t-1}; ind, d_t^A = 0\right) = \frac{p^A(\mathbf{h}_{t-1}; ind) + p^B(\mathbf{h}_{t-1}; ind)}{2} - c \cdot \sigma_\theta^B(\mathbf{h}_{t-1}) + \delta \cdot \sum_{\mathbf{h}_t \in \mathcal{H}^2} \Pr\left(\mathbf{h}_t|\theta, d_t^A = 0\right) \cdot V_\theta(\mathbf{h}_t; ind)$$

where $\Pr(\mathbf{h}_t|\theta, d_t^A = 0)$ is the distribution of realized quality in period t as a result of the firm's deviation in market A. Compared with the equilibrium payoff in (5), the firm can save its investment cost in market A. However, the lack of an investment results in a bad outcome in market A (i.e., $h_t^A = L$), which leads to a worse probability distribution of new outcomes, reflected in $\Pr(\mathbf{h}_t|\theta, d_t^A = 0)$. Thus, the firm expects a lower continuation payoff in this deviation than on the equilibrium path. Therefore, this deviation is not profitable, that is, $V_\theta(\mathbf{h}_{t-1}; ind) \geq V_\theta(\mathbf{h}_{t-1}; ind, d_t^A = 0)$, if and only if the cost of investment is less than the gain in the continuation payoff:

$$c < \bar{c}_\theta^{ind} := \delta \cdot \sum_{\mathbf{h}_t \in \mathcal{H}^2} \left(\Pr(\mathbf{h}_t|\theta) - \Pr\left(\mathbf{h}_t|\theta, d_t^A = 0\right)\right) \cdot V_\theta(\mathbf{h}_t; ind), \quad (6)$$

where $\theta \in \{(C, C), (C, I)\}$. The right-hand side is the marginal effect of the firm's investment in market A on the continuation payoff, which leads to the following remark.

Remark 1. The firm's decision of whether to invest in a product market in period t is independent of the track record of the firm ($\boldsymbol{h}_{t-1}$).

This independence is because the consumer in the current period does not observe the realized quality. Thus, the firm's investment decision does not affect the current-period prices but, rather, affects only the continuation payoff. The HE exists if (6) holds for $\theta \in \{(C, C), (C, I)\}$.

Proposition 1. *For independent branding, the HE exists if and only if $c \leq \bar{c}^{ind}$, where*

$$\bar{c}^{ind} = \frac{\delta \cdot \Delta}{2} \cdot \frac{p^A\left(H, h_t^B; ind\right) - p^A\left(L, h_t^B; ind\right)}{2} = \frac{\delta \cdot \Delta}{4} \cdot \frac{(1 - \mu_0)\Delta}{1 - \mu_0 \cdot \Delta}, \quad (7)$$

for any track record $h_t^B \in \mathcal{H}$. The existence of the HE does not depend on the two-dimensional market-relatedness parameters α and ρ.

The proposition shows that the necessary and sufficient condition for the existence of the HE is a cutoff rule. The larger the threshold cost level $\bar{c}^{ind}$ for independent branding is, the greater the scope of the HE. The proposition illustrates that the threshold can be expressed as a weighted sum of the price premium that the firm can enjoy in the future as a result of its current investment in quality. This price premium is captured by $p^A(H, h_t^B; ind) - p^A(L, h_t^B; ind)$. Thus, in return for a costly investment, the firm can produce a high-quality product, which will be observed by the consumer in the next period. This consumer will update her beliefs accordingly, so the firm can expect a greater price $p^A(H, h_t^B; ind)$ in the next period instead of $p^A(L, h_t^B; ind)$.

Given that consumers facing independent brands cannot associate the firm's different products, it is straightforward that the threshold level does not depend on the two-dimensional relatedness parameters. By adopting different brands, the firm can effectively mute the effects of market relatedness, ensuring that consumers treat each product independently.

The threshold cost level is increasing in Δ, the success probability of a competent type's investment in quality, which is intuitive because if Δ is large, then each investment in quality is more effective in generating a good outcome, thus directly amplifying the firm's investment incentives. This fact also implies that each outcome is more informative about the firm's capability types. Therefore, a large Δ indirectly increases $\bar{c}^{ind}$ through the consumer's belief ($\frac{(1-\mu_0)\Delta}{1-\mu_0 \cdot \Delta}$). Moreover,

$\overline{c}^{ind}$ decreases in μ_0, and vanishes at $\mu_0 = 1$. In this model, a good outcome reveals the firm to be competent, whereas the effect of a bad outcome on the consumer's posterior beliefs is not as extreme. If the consumer's prior beliefs about the firm's unobserved capability type are already optimistic (or, μ_0 is large), then the firm receives less marginal benefits from an investment in quality.[26]

4.1.2. Umbrella Branding (Branded House). If the firm uses the same brand for the different product markets, then its investment in quality in one product market can affect the future payoffs in both product markets because the consumer arriving in the next period will observe the outcome of the investment and make inferences across product markets. The above fact implies that, unlike in the case of independent branding, a market-by-market analysis is not appropriate for the case of umbrella branding.

Aside from these fundamental distinctions, the analysis for umbrella branding is similar to the analysis for independent branding. The HE in which a competent type always invests in quality exists if and only if there is no profitable deviation. This condition can be shown to be true if and only if $c < \overline{c}^{umb} := \min_\theta \overline{c}_\theta^{umb}$, $\theta \in \{(C, C), (C, I), (I, C)\}$. The (C, C)-type firm can deviate in both product markets. However, intuitively, this deviation is too costly for the firm because it can only produce a low-quality product in both product markets, which will negatively affect the firm's continuation payoff. Instead, the firm finds it more tempting to deviate in just one product market, for example A, while investing in the other market B. This deviation will generate the threshold for the HE, $\overline{c}^{umb}$.

Proposition 2. *For an umbrella brand, the HE exists if and only if $c < \overline{c}^{umb}$, where the cutoff level is obtained by the (C, C) type's deviation in a single product market, B. The cutoff cost $\overline{c}^{umb}$ is characterized as an expected premium in price the firm may realize in the next period as a result of the firm's current investment in product market A:*

$$\overline{c}^{umb} = \frac{\delta\Delta}{2} \cdot \sum_{h_t^B \in \mathcal{H}} w(h_t^B) \cdot \left(\underbrace{\frac{p^A(H, h_t^B; umb) - p^A(L, h_t^B; umb)}{2}}_{\text{expected premium in market } A} + \underbrace{\frac{p^B(H, h_t^B; umb) - p^B(L, h_t^B; umb)}{2}}_{\text{spillover in market } B} \right),$$

where $w(h_t^B)$ is the probability that h_t^B is produced in market B, satisfying $\sum_{h_t^B \in \mathcal{H}} w(h_t^B) = 1$.

The threshold cost $\overline{c}^{umb}$ corresponds to the marginal benefit of the firm's investment in product market A in the continuation payoff. The first part of the proposition identifies the deviation that characterizes $\overline{c}^{umb}$, where the (C, C)-type deviates in product market A. This is because the (C, C) type firm invests in product market B, which may result in a good outcome from which the consumer draws positive inferences in both product markets. As a result, the firm is tempted to deviate in product market A and instead exploit the positive reputation spillover from product market B. On the other hand, the (C, I) type does not face such a temptation because there is no positive spillover from market B to be exploited.

Clearly, the (C, C) type's deviation in market A will lead to a low-quality product, which will induce some negative spillover to market B. The fear of this negative spillover reduces, but cannot entirely offset, the firm's incentive to deviate, because a good outcome is a clear signal that the firm is competent, whereas a bad outcome is only a noisy signal that it may be incompetent. Therefore, the magnitude of positive spillover from market B to A will dominate the negative spillover in the opposite direction.

The extent of the reputation spillover depends on the supply-side relatedness (ρ), as well as other model parameters, such as the demand-side relatedness (α), the success probability of a competent type's investment (Δ), and consumers' prior beliefs that the firm is competent in each market (μ_0).

Proposition 3. *For umbrella branding,*

i. *if the success probability of a competent type's investment is low ($\Delta \le \frac{1}{2}$), then $\frac{\partial \overline{c}^{umb}}{\partial \rho} > 0$;*

ii. *if $\Delta > \frac{1}{2}$, then*

a. *$\frac{\partial \overline{c}^{umb}}{\partial \rho} \ge 0$ if demand-side relatedness is low ($\alpha \le \underline{\alpha}$) or the prior probability is large ($\mu_0 \ge \overline{\mu}$), so $\overline{c}^{umb}$ is maximized at $\rho = 1$;*

b. *$\frac{\partial \overline{c}^{umb}}{\partial \rho} \le 0$ if demand-side relatedness is high ($\alpha \ge \overline{\alpha}$) and the prior probability is low ($\mu_0 \le \underline{\mu}$), so $\overline{c}^{umb}$ is maximized at $\rho = 0$; and*

c. *$\frac{\partial \overline{c}^{umb}}{\partial \rho} \ge 0$ for $\rho \le \hat{\rho}$ and $\frac{\partial \overline{c}^{umb}}{\partial \rho} < 0$ for $\rho > \hat{\rho}$ for $\hat{\rho} \in (0, 1)$ if $\alpha > \underline{\alpha}$ and $\underline{\mu} < \mu_0 < \overline{\mu}$, or $\underline{\alpha} < \alpha < \overline{\alpha}$ and $\mu_0 < \overline{\mu}$, and thus $\overline{c}^{umb}$ attains its maximum at $\hat{\rho}$.*

The proposition shows how the scope of the HE for umbrella branding, proxied by $\overline{c}^{umb}$, depends on the supply-side relatedness ρ. Whether $\overline{c}^{umb}$ is monotonically increasing, decreasing, or nonmonotonic in ρ depends on model parameters Δ, μ_0, and, importantly, α.

The first part of the proposition states that if Δ is small ($< \frac{1}{2}$), then a closer supply-side relatedness helps sustain the HE. Because the success probability of a competent type's investment is low, competent and incompetent types are less distinguishable in terms of the distribution over realized outcomes. This fact implies that the realized outcomes are less informative of the firm's unobserved capability types.[27] Then, the firm is able to provide more information about its competence, especially if the supply-side relatedness is high.

On the one hand, if $\Delta > \frac{1}{2}$, the case depicted in Figure 1, then the realized outcome carries more information about the firm's capability types. Thus, there can be substantial information spillover across product markets. The proposition shows that, in general, if the demand-side relatedness is low ($\alpha < \underline{\alpha}$), then higher supply-side relatedness helps sustain the HE. This case corresponds to the lightest gray region on the leftmost part of the x-axis, demand-side relatedness. On the other hand, if the relatedness is high on the demand-side ($\alpha > \overline{\alpha}$), then high supply-side relatedness makes it more difficult to sustain the HE because an umbrella brand induces excessive information spillover across product markets. Therefore, the firm has a significant temptation to exploit the positive spillover across product markets instead of investing in both markets. In this case, a reduction in supply-side relatedness can provide more commitment power by limiting information spillover. Thus, a smaller ρ will expand the scope of the HE. The darkest gray region on the bottom right corner of Figure 1 represents this case. Based on this interaction between the supply-side relatedness and demand-side relatedness, $\overline{c}^{umb}$ can be nonmonotonic in ρ if α is intermediate.

As discussed before, a firm's deviation in one product market will generate negative spillover, which may discipline the firm's temptation to deviate. The consumers' prior μ_0 dictates which of the two spillovers—positive and negative—will dominate, as graphically illustrated in Figure 2. If μ_0 is large, then there is little room to improve the consumer's beliefs about the firm's capability type. On the other hand, the beliefs can be updated more pessimistically if a bad track record is observed. As a result, the disciplinary force induced by negative spillover dominates the temptation generated by positive spillover. Given that the firm's behavior is disciplined, it is better to generate more information through a closer supply-side relatedness. Therefore, as demonstrated by the bottom curve of Figure 2 $\overline{c}^{umb}$ increases in ρ. By contrast, if μ_0 is small, then the firm's good track record can affect the consumer's beliefs more optimistically. However, a bad track record cannot worsen the beliefs as much. Thus, the positive spillover dominates the negative spillover. Given that the firm's behavior is not disciplined, more information spillover will only increase the temptation to exploit the positive spillover, thus reducing the threshold for the HE. Consequently, as shown in the top curve of Figure 2, $\overline{c}^{umb}$ decreases in ρ. Finally, based on the two opposing forces that μ_0 imposes on the spillovers, if μ_0 is intermediate, then $\overline{c}^{umb}$ is nonmonotonic in ρ.

Figure 1. The HE for Umbrella Branding and Supply-Side Relatedness: $\frac{\partial \overline{c}^{umb}}{\partial \rho}$ ($\Delta > \frac{1}{2}$)

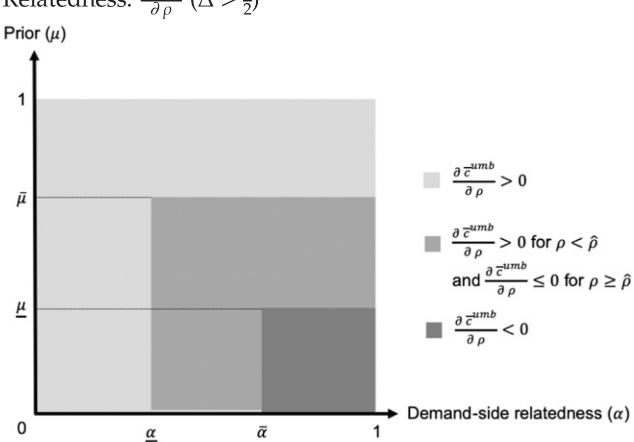
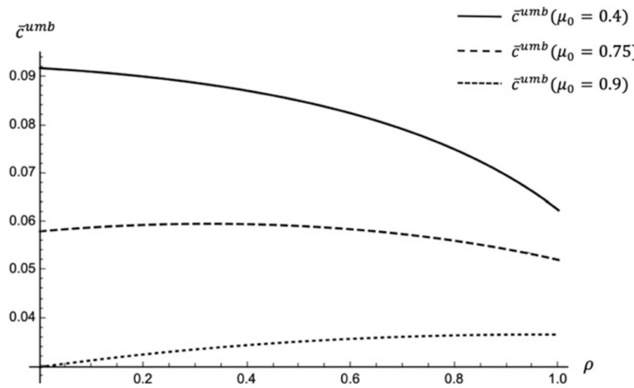


Figure 2. The HE for Umbrella Branding and the Prior μ_0 ($\alpha = 1$, $\Delta = 0.7$, $\delta = 0.9$)



4.2. The Optimal Branding Regime

This paper has thus far analyzed which of the two branding regimes can sustain the most profitable equilibrium, the HE. This section further investigates a firm's optimal branding decision between independent and umbrella branding by analyzing a game that includes the game defined in Section 3.1 as a subgame. In period $t = -1$, the firm chooses a branding decision between the two branding regimes. Then, in period $t = 0$, its type $\theta \in \{C, I\}^2$ is realized. The subgame starting at $t \geq 1$ is the same as the game defined previously.[28]

An equilibrium of this game consists of an equilibrium of the subgame and a firm's optimal branding decision given the equilibrium of the subgame. So, for instance, (ind, HE) will indicate an equilibrium in which the firm chooses independent branding at $t = -1$ and, subsequently, both the firm and consumers play the HE. Consumers' beliefs in this game are the same as the beliefs in the subgame because a firm's branding decision is made prior to realizing its private capability types and, therefore, does not influence consumers' beliefs about the firm's type. Importantly, we maintain the assumption that consumers are still unaware of the association between the two products if the firm chooses to use different brand names, that is, adopt independent branding.[29] To identify a firm's optimal branding decision at $t = -1$, the best feasible profits under the two branding regimes must be identified and compared. This requires an analysis of alternative equilibria of the subgame besides the HE and the LE.

In any stationary equilibrium of the subgame, a firm's profit can be summarized by the expected per-period profit, which is a weighted sum over the firm's profit in all possible track records. The stationary distribution over the track records $\Pr(h^A, h^B; \theta)$ is determined by the firm's capability types $\theta = (\theta^A, \theta^B)$. Thus, the expected per-period profit in an equilibrium characterized by strategies $\sigma_\theta^{*A}(\mathbf{h})$ and $\sigma_\theta^{*B}(\mathbf{h})$ is

$$E\pi_\theta^*(br) = \sum_{\mathbf{h}\in\mathcal{H}^2} \Pr{}^*(\mathbf{h};\theta)\cdot\left(\frac{p^A(h^A,h^B;br)+p^B(h^A,h^B;br)}{2} - c\left(\sigma_\theta^{*A}(\mathbf{h})+\sigma_\theta^{*B}(\mathbf{h})\right)\right). \quad (8)$$

For instance, for the HE, $\sigma_\theta^{*i}(\mathbf{h}) = 1$ whenever $\theta^i = C$. Thus, for $\theta = (C, C)$, the probability distribution is as follows: $\Pr^{*HE}(H,H;C,C) = \frac{\alpha}{2}\cdot\Delta^2$, $\Pr^{*HE}(H,\varnothing;C,C) = \Pr^{*HE}(\varnothing,H;C,C) = \frac{1-\alpha}{2}\cdot\Delta$, $\Pr^{*HE}(\varnothing,\varnothing;C,C) = \frac{\alpha}{2}\cdot\Delta$, and zero otherwise, that is, $\Pr^{*HE}(L,\varnothing;\ C,C) = \Pr^{*HE}(\varnothing,L;\ C,C) = \Pr^{*HE}(L,L;C,C) = 0$. For the investment decision, $\sigma_{C,C}^{*A}(\cdot) = \sigma_{C,C}^{*B}(\cdot) = 1$. Distinct branding regimes yield different per-period profits because the consumer's beliefs are different, which leads to distinct prices.

Further analysis shows that, besides the HE and LE, the subgame can admit only two additional equilibria, which are dubbed the medium-effort equilibria (ME). Here, a competent type plays the HE in one market (for instance, $\sigma_\theta^A(\cdot) \equiv 1$ whenever $\theta^A = C$) and the LE in the other market ($\sigma_\theta^B(\cdot) \equiv 0$). By symmetry, another ME exists where the firm plays the HE in product market B and the LE in product market A.

Proposition 4. *The subgame admits four equilibria only: the HE, the LE, and two symmetric MEs. No other equilibria exist for the subgame.*

Recall that the firm's strategy in each product market does not depend on its track record, as noted in Remark 1.[30] The (C, C)-type firm has a binary investment decision to make in two product markets, and the (C, I)-type and (I, C)-type firms have one decision to make. Therefore, there are $2^4 = 16$ possible equilibria, only four of which can exist: the HE, LE, and two symmetric MEs. In particular, the following can never be true: In equilibrium, the (C, C)-type firm invests in product market A, but the (C, I)-type firm does not, and vice versa.[31] Roughly speaking, this is because, given that the (C, C)-type firm finds it optimal to invest, the (C, I)-type firm has a profitable deviation by investing, that is, mimicking the (C, C)-type firm's strategy. Therefore, a competent type in product market A, whether it belongs to type (C, C) or (C, I), will either invest in the market together or not invest together. Thus, in each product market, there can only be the HE or the LE. Then, combining the two product markets, there are only four possibilities, all of which have been identified.

Given that the ME plays the HE in only one product market and the LE in the other, each type's profit in the ME is only a fraction of its profit under the HE for independent branding.[32] Therefore, the HE is further shown to be the most profitable equilibrium whenever it exists.

Proposition 5. *If the HE exists for a subgame in a branding regime $br \in \{ind,\ umb\}$, that is, $c < \bar{c}^{br}$, then it is the most profitable equilibrium of the subgame.*[33]

The ME cannot make a branding regime optimal. If $\bar{c}^{ind} < c < \bar{c}^{umb}$, then only the LE exists for independent branding, whereas the HE exists for umbrella branding, which makes it the optimal branding regime. Otherwise, if $\bar{c}^{umb} < c < \bar{c}^{ind}$, then the ME exists for umbrella branding (as well as for independent branding). However, it is dominated by the HE for independent branding. In both cases, the optimal branding regime sustains the HE.

However, if the investment cost is sufficiently large, that is, $c > \max\{\bar{c}^{ind}, \bar{c}^{umb}\}$, then neither independent branding nor umbrella branding sustains the HE, and the ME does not exist. Therefore, only the LE exists for both branding regimes, so both types of firms are indifferent between the two branding regimes. Therefore, both branding regimes can be an equilibrium.

Otherwise, if c is sufficiently low so that the HE exists for both branding regimes, that is, $c < \min\{\bar{c}^{ind}, \bar{c}^{umb}\}$, then the expected profit of the firm is the same. This is because the HE is an efficient equilibrium and the consumer surplus is zero, which implies that the entire surplus goes to the firm. Therefore, at $t = -1$ prior to drawing its capability type $\theta \in \{C, I\}^2$, the firm is indifferent between the two branding regimes.[34]

The threshold costs $\bar{c}^{ind}$ and $\bar{c}^{umb}$ are functions of key model parameters. Therefore, a comparison between these thresholds and c, which determines the optimal branding regime, boils down to a set of conditions on the model parameters. An equilibrium of the entire game is characterized by the firm's optimal branding decision $br \in \{ind, umb\}$ and the equilibrium of the subgame, which can be either *HE* or *LE*.

Proposition 6. *Equilibria of the game including the firm's branding decision at $t = -1$ are characterized as follows:*

i. *Suppose that $\alpha < \underline{\alpha}$. If ρ is sufficiently large, then (umb, HE) is the unique equilibrium. Otherwise, if ρ is not large, then the firm is indifferent between two equilibria: (ind, LE) and (umb, LE).*

ii. *Suppose that $\Delta > \frac{1}{2}$ and $\mu_0 < \underline{\mu}$. If $\alpha > \overline{\alpha}$, then (ind, HE) is the unique equilibrium if and only if ρ is sufficiently large. If ρ is not large, then the firm is indifferent between two equilibria: (ind, HE) and (umb, HE).*

iii. *Suppose that $\Delta > \frac{1}{2}$ and $\underline{\mu} < \mu_0 < \overline{\mu}$, and $\alpha > \underline{\alpha}$ and $\mu > \overline{\mu}$. Then, (umb, HE) is the unique equilibrium if and only if ρ is in an intermediate range. Otherwise, if ρ is sufficiently small or sufficiently large, then the firm is indifferent between two equilibria: (ind, LE) and (umb, LE).*

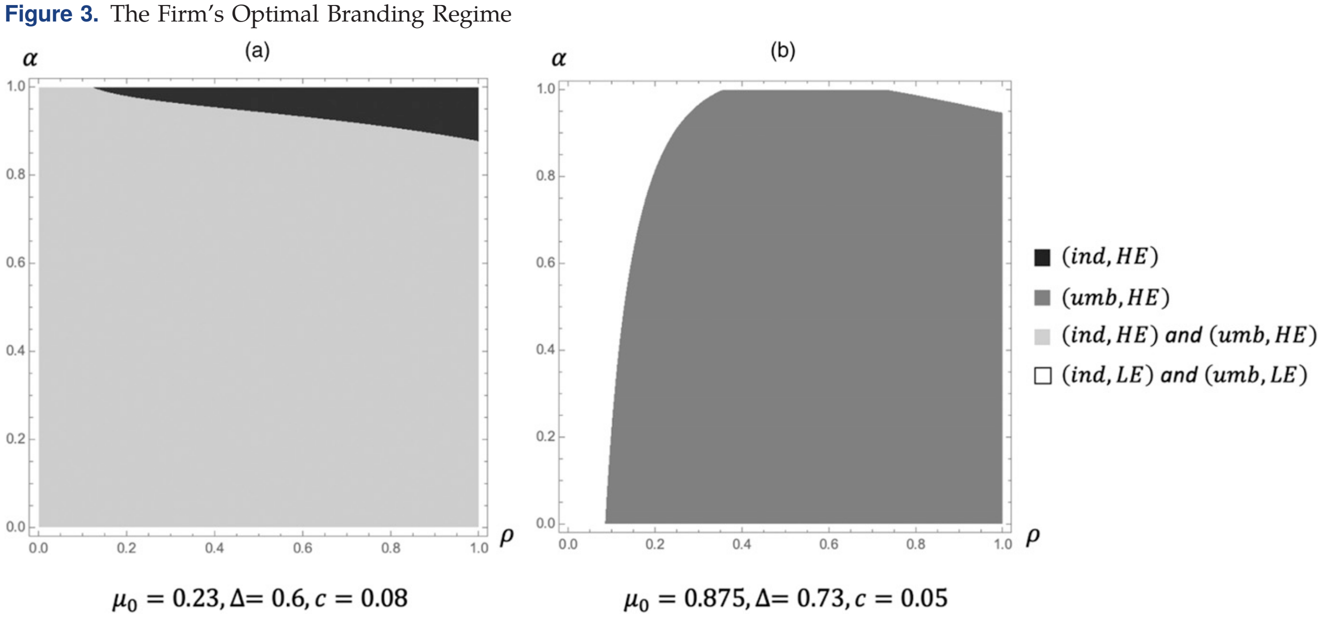
Part i of the proposition states that if the demand-side relatedness is low ($\alpha < \underline{\alpha}$), then umbrella branding is optimal under sufficiently high supply-side relatedness. Given the limited overlap in the customer bases of the two product markets, an insufficient amount of information is available about the brand. Therefore, having an umbrella brand can effectively increase the amount of information available to consumers, as they can pool the information across product markets. This benefit from umbrella branding increases as the supply-side relatedness becomes higher. Therefore, if ρ is sufficiently large, then umbrella branding can sustain the HE, that is, $\overline{c}^{umb} > c$. Otherwise, if ρ is not large, then both independent branding and umbrella branding can sustain only the LE, and therefore, the firm is indifferent between the two branding regimes. This part of the proposition is consistent with findings in the literature on umbrella branding. By using an adverse selection model with supply-side relatedness, Miklós-Thal (2012) and Moorthy (2012) find that umbrella branding is more likely to hold signaling value if the supply-side relatedness is high.

However, part ii shows that sometimes the result may not hold; that is, independent branding can be optimal if both the demand-side ($\alpha > \overline{\alpha}$) and supply-side relatedness are high. Under these circumstances, as discussed in Proposition 2, an umbrella brand induces excessive positive information spillover, which the firm is tempted to exploit. As a result, an umbrella brand is less capable of sustaining the HE, especially if ρ is large. The firm can avoid this temptation by adopting independent brands, and thus better sustain the HE. Therefore, if ρ is sufficiently large, then only independent branding can sustain the HE, which makes it the optimal branding choice. This part of the result highlights the role of independent branding as a disciplinary mechanism.

Given the distinct results in the first two parts, the results in the intermediate case of part iii are intuitive. If the demand-side relatedness is not too small or not too large, then $\overline{c}^{umb}$ can be nonmonotonic in ρ. Therefore, the HE may exist only for umbrella branding in an intermediate range of ρ and not for independent branding.

Figure 3 depicts the firm's optimal branding regime as a function of the supply-side relatedness (on the x–axis) and the demand-side relatedness (on the y–axis). The left panel describes part ii for a smaller $\mu_0 (= 0.23)$. Independent branding is optimal at the top right corner where both supply- and demand-side relatedness are high; otherwise, both branding regimes

Figure 3. The Firm's Optimal Branding Regime



can support the HE, in which case the firm is indifferent. The right panel describes parts i and iii. For a large $\mu_0(= 0.875)$, if the demand-side relatedness α is small, umbrella branding is optimal if the supply-side relatedness ρ is sufficiently large. However, as α approaches 1, umbrella branding is optimal in an intermediate range of ρ, and otherwise the firm is indifferent between the two branding regimes, as they can support only the LE.

5. CRM for an Umbrella Brand

This paper has thus far investigated a two-product firm's branding decision between independent branding and umbrella branding for a given level of two-dimensional market relatedness. However, firms sometimes decide which markets to enter conditional on their predetermined branding choice. In particular, given a decision to use umbrella branding, an important CRM question arises: For distinct products under an umbrella brand, should the firm target the same customers or acquire a new and distinct set of customers?[35] To address this question, the effect of the demand-side relatedness on $\bar{c}^{umb}$ is analyzed. For example, if $\alpha = 1$, then all consumers who make any purchase will buy both products, so the firm will be targeting the same customers for different products. However, if $\alpha = 0$, then each customer makes a purchase in only one product market, thereby representing the case in which the firm targets distinct sets of customers in different product markets.

There are two opposing forces emerging from demand-side relatedness that affect the firm's investment incentives. On the one hand, for a higher α, there is more information to be pooled across product markets, so the firm will find it more tempting to invest in only one of the product markets. Therefore, $\bar{c}^{umb}$ can decrease in α. On the other hand, there are more consumers who make purchases in both product markets. Therefore, if they observe any bad outcome in the firm's track record, then they may punish the firm in both product markets by paying lower prices. This fear can discipline the firm's behavior and amplify the incentives for investments. If the supply-side relatedness is high, the former dominates, and $\bar{c}^{umb}$ is maximized at $\alpha = 0$.

Proposition 7. *If the supply-side relatedness is sufficiently high, then the scope of the HE decreases in demand-side relatedness, so the maximum of* $\bar{c}^{umb}$ *is attained at* $\alpha = 0$.

Thus, if an umbrella brand's products share similar production technologies, then the firm will want to target customers with different preferences and prevent excessive information pooling across product markets. This result provides an important implication for CRM: the firm needs to consider trade-offs between providing more information to consumers and granting them greater purchasing power to punish the firm.

6. Conclusions

This paper explores how a firm should manage its brand when entering new product markets. Should it leverage its existing brand or create a new brand? This classic question in marketing has received substantial attention from academics and practitioners alike since the 1980s. Although some may believe that the topic has become less relevant over time, it is now just as relevant as before. Some classic example firms that had insisted on a house of brands strategy, such as P&G and Coca-Cola, have made efforts to shift toward a branded house strategy.[36] Facebook initially adopted a house of brands strategy for the acquired firms like Instagram and WhatsApp by keeping them as independent brands. However, more recently, Facebook is considering including the Facebook name on the acquired platforms and redesign its brand architecture.[37]

This paper contributes to the literature on branding by providing an analytical framework of market relatedness and offering managerial implications for how a multiproduct firm should make the branding decision based on how related the product markets are. This paper finds that if demand-side relatedness is low, then umbrella branding (a branded house) becomes increasingly more attractive as supply-side relatedness increases. This result seems consistent with example companies, such as Honda and Canon. However, if the product markets are closely related on both the supply and demand sides, then the firm may prefer independent branding (a house of brands). The reason is that an umbrella brand can induce too much positive information spillover that the firm is tempted to exploit, instead of investing in quality in both product markets. Therefore, independent branding can be a disciplinary mechanism that eliminates information spillover, thus allowing the firm to commit to investing in both product markets.

This result provides new insight into why high-quality firms, such as P&G, may want to avoid using their existing brand to enter a new product market.[38] The firm's well-known house of brands strategy may have acted as a disciplinary mechanism and contributed to the continued success of all of the company's brands.

This new insight was made possible because this paper analyzes a model of reputation with the features of both adverse selection and moral hazard. The model of adverse selection, which has been extensively studied in the extant literature, seems to indicate that closer market relatedness should expand the scope of the signaling role of umbrella branding. In the model of moral hazard without adverse selection, consumers will not make rational inferences across product markets about unobserved firm types. Instead, information spillover will be captured through some arbitrary beliefs imposed by the researcher, which is neither appealing nor realistic.

This paper treats the firm-generated track record as a medium of information transfer between the firm and consumers. However, this study can be applied to more general modes of information transmission, such as advertising (e.g., Mayzlin and Shin 2011), word of mouth (e.g., Mayzlin 2006), and consumer search (e.g., Wolinsky 1986). Two main features are common to all of these settings: first, firms have some control over the information conveyed to consumers, whether it is through investment in quality or advertising, and second, umbrella branding will invite consumers to pool information across different product markets. Thus, the reputation spillover in this paper can be interpreted in other settings as the spillover in advertising or word of mouth, both of which are sometimes integral parts of a firm's branding strategy.[39]

In this paper, consumers gain utility from the quality of a product, and therefore, which branding regime can better convey information about quality is an important issue. However, consumers may sometimes be more invested in finding a product that will match their horizontal preferences. In such a setting, a firm may use branding as a tool to communicate information about its horizontal positioning and provide value to consumers by mitigating their uncertainty about their preference matching (e.g., Sappington and Wernerfelt 1985). This important issue of brand positioning remains to be further explored in future research.

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Appendix. Proofs

Proof for Section 3

Proof of Lemma 1. Part 2 of the lemma directly follows from (4). Parts 1 and 3 require a proof. The posterior beliefs for all nine possible track records of the firm in the HE are as follows:

$$\mu^A(L,H;umb) = \frac{\Pr(C,C)\cdot(1-\Delta)\Delta+\Pr(C,I)\cdot 0}{\Pr(C,C)\cdot(1-\Delta)\Delta+\Pr(C,I)\cdot 0+\Pr(I,C)\cdot\Delta+\Pr(I,I)\cdot 0},$$

$$\mu^A(L,\varnothing;umb) = \frac{\Pr(C,C)\cdot(1-\Delta)+\Pr(C,I)\cdot(1-\Delta)}{\Pr(C,C)\cdot(1-\Delta)+\Pr(C,I)\cdot(1-\Delta)+\Pr(I,C)+\Pr(I,I)},$$

$$\mu^A(L,L;umb) = \frac{\Pr(C,C)\cdot(1-\Delta)^2+\Pr(C,I)\cdot(1-\Delta)}{\Pr(C,C)\cdot(1-\Delta)^2+(\Pr(C,I)+\Pr(I,C))\cdot(1-\Delta)+\Pr(I,I)},$$

$$\mu^A(\varnothing,H;umb) = \frac{\Pr(C,C)\cdot\Delta+\Pr(C,I)\cdot 0}{\Pr(C,C)\cdot\Delta+\Pr(C,I)\cdot 0+\Pr(I,C)\cdot\Delta+\Pr(I,I)\cdot 0},$$

$$\mu^A(\varnothing,L;umb) = \frac{\Pr(C,C)\cdot(1-\Delta)+\Pr(C,I)}{\Pr(C,C)\cdot(1-\Delta)+\Pr(C,I)+\Pr(I,C)\cdot(1-\Delta)+\Pr(I,I)},$$

$$\mu^A(H,H;umb) = \frac{\Pr(C,C)\cdot\Delta^2+\Pr(C,I)\cdot\Delta\cdot 0}{\Pr(C,C)\cdot\Delta^2+\Pr(C,I)\cdot\Delta\cdot 0+\Pr(I,C)\cdot\Delta\cdot 0+\Pr(I,I)\cdot 0\cdot 0}=1,$$

$$\mu^A(H,\varnothing;umb) = \frac{\Pr(C,C)\cdot\Delta+\Pr(C,I)\cdot\Delta}{\Pr(C,C)\cdot\Delta+\Pr(C,I)\cdot\Delta+\Pr(I,C)\cdot 0+\Pr(I,I)\cdot 0}=1,$$

$$\mu^A(H,L;umb) = \frac{\Pr(C,C)\cdot\Delta(1-\Delta)+\Pr(C,I)\cdot\Delta}{\Pr(C,C)\cdot\Delta(1-\Delta)+\Pr(C,I)\cdot\Delta+\Pr(I,C)\cdot 0\cdot+\Pr(I,I)\cdot 0}=1,$$

and finally, $\mu^A(\varnothing,\varnothing;umb) = \mu_0$.

The posterior beliefs for independent branding are defined in (4). These beliefs can be obtained by plugging $\rho = 0$ into the distribution $\Pr_\Theta(\theta)$ and computing the beliefs for umbrella branding.

Part 1. The direct effect is straightforward for both independent branding and umbrella branding; $h^B = \varnothing$ because there is no information spillover, and the case is identical to the independent branding. Moreover, $\mu^A(H,h^B;umb)=1\geq\mu^A(\varnothing, h^B;umb)$ is obvious. Thus, proofs are required for the case of umbrella branding and $h^B\in\{L,H\}$; that is, $\mu^A(\varnothing, H;umb)\geq\mu^A(L,H;umb)$ and $\mu^A(\varnothing,L;umb)\geq\mu^A(L,L;umb)$ need to be shown.

First, if $h^B=H$, then $\mu^A(\varnothing, H; umb) - \mu^A(L, H; umb) = \frac{\Delta(1-\mu_0)(1-\rho)(\rho+(1-\rho)\mu_0)}{1-\Delta(\rho+(1-\rho)\mu_0)} \geq 0$.

Second, if $h^B = L$, then the difference $\mu^A(\emptyset, L; umb) - \mu^A(L, L; umb)$ is positive because in the combined fraction, the denominator is by definition positive, and the numerator is $(\Pr(C,C)(1-\Delta)+\Pr(C,I))\times(\Pr(C,C)(1-\Delta)^2+\Pr(C,I)(1-\Delta)+\Pr(I,C)(1-\Delta)+\Pr(I,I))-(\Pr(C,C)(1-\Delta)^2+\Pr(C,I)(1-\Delta))\times(\Pr(C,C)(1-\Delta)+\Pr(C,I)+\Pr(I,C)(1-\Delta)+\Pr(I,I))=\Delta(\Pr(C,C)\times\Pr(I,C)(1-\Delta)^2+\Pr(C,I)\Pr(I,C)(1-\Delta)+\Pr(C,C)\Pr(I,I)(1-\Delta)+\Pr(C,I)\Pr(I,I))\geq 0$.

Part 3. For $h^A = H$, $\mu^A(H, h^B) = 1$ for all $h^B \in \mathcal{H}$, so the condition trivially holds.

For $h^A = \emptyset$, $\mu^A(\emptyset,H;umb)=\mu_0+\rho(1-\mu_0)$, $\mu^A(\emptyset,\emptyset;umb) = \mu_0$, and $\mu^A(\emptyset,L;umb) = \frac{\Pr(C,C)\cdot(1-\Delta)+\Pr(C,I)}{\mu_0\cdot(1-\Delta)+1-\mu_0} = \mu_0 - \frac{\mu_0(1-\mu_0)\cdot\Delta\cdot\rho}{\mu_0\cdot(1-\Delta)+1-\mu_0}$. Clearly, $\mu^A(\emptyset,H;umb)\geq\mu^A(\emptyset,\emptyset;umb)$. Additionally, $\mu^A(\emptyset,\emptyset;umb)\geq\mu^A(\emptyset,L;umb)$. It is straightforward that $\mu^A(\emptyset,H;umb)$, $\mu^A(\emptyset,\emptyset;umb)$, and $\mu^A(\emptyset,L;umb)$ are increasing in, independent of, and decreasing in ρ, respectively.

Finally, for $h^A = L$, $\mu^A(L,H;umb) = \frac{(\mu_0+(1-\mu_0)\rho)(1-\Delta)}{(\mu_0+(1-\mu_0)\rho)(1-\Delta)+(1-\mu_0)(1-\rho)}$, $\mu^A(L,\emptyset;umb) = \frac{\mu_0(1-\Delta)}{\mu_0(1-\Delta)+1-\mu_0}$, and $\mu^A(L,L;umb) = \frac{\mu_0(1-\Delta)[(\mu_0+(1-\mu_0)\rho)(1-\Delta)+(1-\mu_0)(1-\rho)]}{\mu_0(\mu_0+(1-\mu_0)\rho)\cdot(1-\Delta)^2+2\mu_0(1-\mu_0)(1-\rho)(1-\Delta)+(1-\mu_0)(1-\mu_0+\mu_0\rho)}$. Therefore, $\mu^A(L,H;umb)\geq\mu^A(L,\emptyset;umb)$ if and only if $\frac{\mu_0+(1-\mu_0)\rho}{(\mu_0+(1-\mu_0)\rho)(1-\Delta)+(1-\mu_0)(1-\rho)} \geq \frac{\mu_0}{\mu_0(1-\Delta)+1-\mu_0}$. This can be simplified to $\mu_0+(1-\mu_0)\rho\geq\mu_0(1-\rho)$, which always holds.

We have $\mu^A(L,\emptyset;umb)\geq\mu^A(L,L;umb)$ if and only if $\frac{\mu_0}{\mu_0(1-\Delta)+1-\mu_0} \geq \frac{\mu_0(1-\Delta)[(\mu_0+(1-\mu_0)\rho)\cdot(1-\Delta)+(1-\mu_0)(1-\rho)]}{\mu_0(\mu_0+(1-\mu_0)\rho)\cdot(1-\Delta)^2+2\mu_0(1-\mu_0)(1-\rho)(1-\Delta)+(1-\mu_0)(1-\mu_0+\mu_0\rho)}$ holds. This can be simplified to $\mu_0(1-\mu_0)\Delta(1-\Delta)\rho\geq 0$, which always holds. Therefore, $\mu^A(L,H;umb)\geq\mu^A(L,\emptyset;umb)\geq\mu^A(L,L;umb)$.

Taking the first derivative of each posterior belief with respect to ρ, it is straightforward that $\mu^A(L,H;umb)$, $\mu^A(L,\emptyset;umb)$, and $\mu^A(L,L;umb)$ are increasing in, independent of, and decreasing in ρ, respectively. □

Proofs for Section 4

Proof of Proposition 1. The threshold level for a deviation in one product market is computed first, and then the threshold for a deviation in both product markets is identified. For the former case, without loss of generality, assume that the firm is competent in product market A, without assuming its type in market B, that is, $\theta = (C, \theta^B)$. For the latter case, the firm has to be competent in both product markets, that is, $\theta = (C, C)$. Then, $\bar{c}^{ind}_{C,\theta^B}$ defined in (6) is $\bar{c}^{ind}_{C,\theta^B} = \delta\cdot[\frac{\alpha}{2}(x\cdot\Delta\cdot V_{C,\theta^B}(H,H;ind)+(1-x)\Delta\cdot V_{C,\theta^B}(H,L;ind)-x\Delta\cdot V_{C,\theta^B}(L,H;ind)-(1-x)\Delta\cdot V_{C,\theta^B}(L,L;ind))+\frac{1-\alpha}{2}\Delta(V_{C,\theta^B}(H,\emptyset;ind)-V_{C,\theta^B}(L,\emptyset;ind))]=\delta\Delta\cdot[\frac{\alpha}{2}(x(V_{C,\theta^B}(H,H;ind)-V_{C,\theta^B}(L,H;ind))+(1-x)(V_{C,\theta^B}(H,L;ind)-V_{C,\theta^B}(L,L;ind)))+\frac{1-\alpha}{2}(V_{C,\theta^B}(H,\emptyset;ind)-V_{C,\theta^B}(L,\emptyset;ind))]$, where $x=\Delta$ if $\theta^B = C$ and $x = 0$ if $\theta^B = I$. This cutoff level is a weighted sum over the difference in payoff functions of the form $V_{C,\theta^B}(H,h^B_t;br)-V_{C,\theta^B}(L,h^B_t;br)$, where $h^B_t\in\mathcal{H}$. Then, it follows that this difference equals the difference in prices, that is,

$$V_{C,\theta^B}(H,h^B_t;ind)-V_{C,\theta^B}(L,h^B_t;ind) = \frac{p^A(H,h^B_t;ind)+p^B(H,h^B_t;ind)}{2} - \frac{p^A(L,h^B_t;ind)+p^B(L,h^B_t;ind)}{2}. \tag{A.1}$$

This is because in (5), the payoff function is the sum of the expected per-period profit and the continuation payoff, the latter of which is independent of the endowed track record of the firm. Therefore, in $V_{C,\theta^B}(H,h^B_t;ind)-V_{C,\theta^B}(L,h^B_t;ind)$, the continuation payoff cancels out, and only the difference in per-period profit survives.

Therefore,

$$\bar{c}^{ind}_{C,\theta^B} = \frac{\delta\Delta}{2}\cdot\sum_{h^B_t\in\mathcal{H}} w_{\theta^B}(h^B_t)\cdot\left(\underbrace{\frac{p^A(H,h^B_t;ind)-p^A(L,h^B_t;ind)}{2}}_{\text{Premium in market } A} + \underbrace{\frac{p^B(H,h^B_t;ind)-p^B(L,h^B_t;ind)}{2}}_{\text{Premium in market } B=0}\right), \tag{A.2}$$

where the weights are as follows: $w_{\theta^B}(H) = \alpha x$, $w_{\theta^B}(L) = \alpha(1-x)$, and $w_{\theta^B}(\emptyset) = 1-\alpha$. Therefore, the sum of these weights is $\sum_{h^B_t\in\mathcal{H}} w_{\theta^B}(h^B_t) = 1$. The premium in market B vanishes for any $h^B_t\in\mathcal{H}$ because of Lemma 1. The firm's track record improves in market A, from which the consumer cannot make inferences about the realized quality in market B under independent branding. Additionally, because of the lemma, $p^A(H,h^B_t;ind)$ and $p^A(L,h^B_t;ind)$ do not depend on h^B_t. Therefore, $\bar{c}^{ind}$ simplifies to $\bar{c}^{ind} = \frac{\delta\Delta}{2}\cdot\frac{p^A(H,h^B_t;ind)-p^A(L,h^B_t;ind)}{2} = \frac{\delta\Delta^2}{4}\cdot\frac{1-\mu_0}{\mu_0(1-\Delta)+1-\mu_0}$.

Because $\bar{c}^{ind}_{\theta^B}$ does not depend on θ^B, the notation for the type is dropped and simply written as $\bar{c}^{ind}$. The cutoff level is independent of the firm's capability type in market B because under independent branding, the firm's performance in the two markets is observed independently. □

Proof of Proposition 2. The threshold cost level for the umbrella brand's HE is characterized as in the case of independent branding. Two key distinctions are as follows: (1) information spillovers are induced through consumers' beliefs, and (2) the (C,C) type may deviate in both product markets. Therefore, the overall threshold cost level for an umbrella brand's HE is the minimum of three levels: a single deviation by (C,I) and (C,C) and a double deviation by (C,C). The threshold cost level from the double deviation is denoted by $\tilde{c}^{umb}_{C,C}$.

The analysis of the firm's deviation in a single product market is exactly the same as the analysis in the independent branding case defined in (A.2). We have $\bar{c}^{umb}_{C,\theta^B} = \frac{\delta\Delta}{2}\cdot\sum_{h^B_t\in\mathcal{H}} w_{\theta^B}(h^B_t)\cdot(\frac{p^A(H,h^B_t;umb)-p^A(L,h^B_t;umb)}{2}+\frac{p^B(H,h^B_t;umb)-p^B(L,h^B_t;umb)}{2})$, where the premium in market B, $p^B(H,h^B_t;umb)-p^B(L,h^B_t;umb)$, does not vanish. The term $w_{\theta^B}(h^B_t)$ is the probability with which outcome h^B_t is produced in market B, given that the firm is of type θ^B. If $\theta^B = C$, then $w_C(H) = \alpha\Delta$, $w_C(L) = \alpha(1-\Delta)$, and $w_C(\emptyset) = 1-\alpha$. However, if $\theta^B = I$, then $w_I(H) = 0$, $w_I(L) = \alpha$, and $w_I(\emptyset) = 1-\alpha$.

Therefore, which of $\bar{c}^{umb}_{C,C}$ and $\bar{c}^{umb}_{C,I}$ is smaller is determined by which of the two distributions $w_C(\cdot)$ and $w_I(\cdot)$ places a greater weight on a smaller price premiums. More precisely, the difference between two thresholds is $\bar{c}^{umb}_{C,C}-\bar{c}^{umb}_{C,I} = \frac{\delta\Delta}{2}\sum(w_C(h^B_t)-w_I(h^B_t))(\frac{p^A(H,h^B_t;umb)-p^A(L,h^B_t;umb)}{2}+\frac{p^B(H,h^B_t;umb)-p^B(L,h^B_t;umb)}{2})$, which is equal to $\frac{\delta\Delta^2\alpha}{2}\cdot(p^A(H,H;umb)-p^A(H,L;umb)-(p^A(L,H;umb)-p^A(L,L;umb)))$. Therefore, $\bar{c}^{umb}_{C,C}\leq\bar{c}^{umb}_{C,I}$ if and only if $p^A(H,H;umb)-p^A(H,L;umb)\leq p^A(L,H;umb)-p^A(L,L;umb)$.

The left-hand side vanishes because $p^A(H,H;umb) = p^A(H,L;umb) = \Delta$, and the right-hand side is positive. Thus, $\bar{c}^{umb}_{C,C} \le \bar{c}^{umb}_{C,I}$.

The cutoff level regarding the (C,C) type's deviation in both product markets, $\tilde{c}^{umb}_{C,C}$, remains to be characterized. Under this deviation, the firm is able to save the total investment cost $2c$ and, in return, produce a bad outcome in both product markets, that is, $\mathbf{h}_t = (L,L)$ with probability 1. For the HE to exist, the equilibrium payoff (similarly defined for independent branding in (5)) must be greater than the payoff from this particular deviation, $V_{C,C}(\mathbf{h}_{t-1};umb,d^A = d^B = 0) = \frac{p^A(\mathbf{h}_{t-1};umb)+p^B(\mathbf{h}_{t-1};umb)}{2} + \delta\cdot\sum \Pr(\mathbf{h}_t|\theta,d^A = d^B = 0)\cdot V_{C,C}(\mathbf{h}_t;umb)$, where $\Pr(L,\varnothing|\theta,\ d^A = d^B = 0) = \Pr(\varnothing,L|\theta,d^A = d^B = 0) = \frac{1-\alpha}{2}$ and $\Pr(L,L|\theta,d^A = d^B = 0) = \frac{\alpha}{2}$, and zero otherwise.

This deviation is not profitable if and only if $c \le \tilde{c}^{umb}_{C,C} = \frac{\delta}{2}\cdot[\frac{\alpha}{2}((\Delta^2-0^2)V_{C,C}(H,H;umb) + (\Delta(1-\Delta)-0(1-0))(V_{C,C}(L,H;umb)+V_{C,C}(H,L;;umb)) + ((1-\Delta)^2-(1-0)^2)V_{C,C}(L,L;umb)) + \frac{1-\alpha}{2}((\Delta-0)(V_{C,C}(H,\varnothing;umb) + V_{C,C}(\varnothing,H;umb)) + (1-\Delta-(1-0))\ (V_{C,C}(L,\varnothing;umb)+V_{C,C}(\varnothing,L;umb)))]$. Reorganizing the expression results in $\tilde{c}^{umb}_{C,C} = \frac{\delta\Delta}{2}\cdot\sum_{h^B_t\in\mathcal{H}} w_{C,C}(h^B_t;d^A = d^B = 0)\cdot(\frac{p^A(H,h^B_t;umb)-p^A(L,h^B_t;umb)}{2} + \frac{p^B(H,h^B_t;umb)-p^B(L,h^B_t;umb)}{2})$, where the weights $w^{br}_{C,C}(h^B_t;d^A = d^B = 0)$ for each $h^B_t \in \mathcal{H}$ are defined as follows: $w^{br}_{C,C}(H;d^A = d^B = 0) = \frac{\alpha\Delta}{2}$, $w^{br}_{C,C}(L;d^A = d^B = 0) = \frac{\alpha(2-\Delta)}{2}$, and $w^{br}_{C,C}(\varnothing;d^A = d^B = 0) = 1-\alpha$; thus, the sum of these weights is $\sum_{h^B_t\in\mathcal{H}} w^{br}_{C,C}(h^B_t;d^A = d^B = 0) = 1$.

Repeating the exercise of comparing $\bar{c}^{umb}_{C,C}$ and $\bar{c}^{umb}_{C,I}$ above, it is straightforward to show that $\bar{c}^{umb}_{C,C} \le \tilde{c}^{umb}_{C,C}$. Therefore, $\bar{c}^{umb} = \bar{c}^{umb}_{C,C}$. □

Proof of Proposition 3. By using the properties in Lemma 1, $p^A(H,h^B;umb) = p^B(h^A,H;umb)$ because the posterior beliefs are 1 in these cases. By plugging the prices into $\bar{c}^{umb} = \bar{c}^{umb}_{C,C}$, we have $\bar{c}^{umb} = \frac{\delta\Delta}{2}\cdot[\frac{\alpha\cdot\Delta}{2}(p^A(H,H;umb) - p^A(L,H;umb) + p^B(H,H;umb) - p^B(L,H;umb)) + \frac{\alpha\cdot(1-\Delta)}{2}(p^A(H,L;umb) - p^A(L,L;umb) + p^B(H,L;umb) - p^B(L,L;umb)) + \frac{1-\alpha}{2}(p^A(H,\varnothing;umb) - p^A(L,\varnothing;umb) + p^B(H,\varnothing;umb) - p^B(L,\varnothing;umb))]$. Further plugging in $p^A(H,h^B;umb) = \Delta$, $p^A(H,h) - p^A(H,\tilde{h}) = 0$, and $p^B(h,H) - p^B(\tilde{h},H) = 0$ for any $h,\tilde{h} \in \{H,L,\varnothing\}$ results in

$$\bar{c}^{umb} = \frac{\delta\Delta}{2}\cdot\left[\frac{\alpha}{2}\left(\Delta + (1-2\Delta)\underbrace{p^A(L,H;umb)}_{\text{increasing in }\rho} - 2(1-\Delta)\cdot\underbrace{p^A(L,L;umb)}_{\text{decreasing in }\rho}\right) + \frac{1-\alpha}{2}\left(\Delta - p^A(L,\varnothing;umb) + \underbrace{p^B(H,\varnothing;umb) - p^B(L,\varnothing;umb)}_{\text{increasing in }\rho}\right)\right].$$

In this equation, the only term that can be decreasing in ρ is $(1-2\Delta)p^A(L,H;umb)$ if and only if $\Delta > 1/2$. Therefore, if $\Delta \le \frac{1}{2}$, then each term is increasing in ρ, so the maximum of $\bar{c}^{umb}$ is attained at $\rho = 1$. Additionally, irrespective of the value of Δ, if α is sufficiently small (close to 0), then the last line of the equation above dominates, which increases in ρ. Thus, $\bar{c}^{umb}$ is again maximized at $\rho = 1$.

Now, suppose that $\Delta > \frac{1}{2}$. Then, the second derivative of $\bar{c}^{umb}$ with respect to ρ is

$$\frac{4}{\delta\Delta}\cdot\frac{\partial^2\bar{c}^{umb}}{\partial\rho^2} = \underbrace{2(1-\mu_0)^2\Delta^2(1-\Delta)}_{>0}\left[\underbrace{\frac{1-2\Delta}{\left(1-\mu_0\Delta-(1-\mu_0)\Delta\rho\right)^3}}_{<0\text{ because }\Delta>1/2} - \underbrace{\frac{2\mu_0^2\Delta^2(1-\Delta)(1-\mu_0\Delta)}{\left(1-\mu_0\Delta(2-\mu_0\Delta-(1-\mu_0)\Delta\rho)\right)^3}}_{>0}\right].$$

The first term in the brackets is negative because $\Delta > \frac{1}{2}$, and the second term is always positive. Therefore, $\frac{\partial^2\bar{c}^{umb}}{\partial\rho^2} < 0$.

Because the second derivative is negative, there are three possible cases in which the value of ρ maximizes the cutoff level $\bar{c}^{umb}$. First, if $\frac{\partial\bar{c}^{umb}}{\partial\rho}|_{\rho=0} < 0$, then $\bar{c}^{umb}$ is decreasing in ρ in the entire space of ρ, so the maximum of $\bar{c}^{umb}$ is attained at $\rho = 0$. Second, if $\frac{\partial\bar{c}^{umb}}{\partial\rho}|_{\rho=1} > 0$, then $\bar{c}^{umb}$ is increasing in ρ everywhere, so its maximum is at $\rho = 1$. Finally, if $\frac{\partial\bar{c}^{umb}}{\partial\rho}|_{\rho=0} > 0$ and $\frac{\partial\bar{c}^{umb}}{\partial\rho}|_{\rho=1} < 1$, then, according to the mean value theorem, the maximum of $\bar{c}^{umb}$ is attained at an intermediate $\hat{\rho} \in (0,1)$. These conditions for $\frac{\partial\bar{c}^{umb}}{\partial\rho}$ for $\rho = 0$ and $\rho = 1$ remain to be checked. The second derivative is $\frac{2}{\delta\Delta}\cdot\frac{\partial\bar{c}^{umb}}{\partial\rho} = (1-\mu_0)\Delta(\frac{\alpha}{2}(\frac{(1-\Delta)(1-2\Delta)}{(1-\mu_0\Delta-(1-\mu_0)\Delta\rho)^2} + \frac{2\mu_0\Delta(1-\Delta)^2(1-\mu_0\Delta)}{(1-\mu_0\Delta(2-\mu_0\Delta-(1-\mu_0)\Delta\rho))^2}) + \frac{1-\alpha}{2}\frac{1}{1-\mu_0\Delta})$.

Case 1. First, if $\rho = 0$, then the expression above is equal to $(1-\mu_0)\Delta(\frac{\alpha}{2}(\frac{(1-\Delta)(1-2\Delta)}{(1-\mu_0\Delta)^2} + \frac{2\mu_0\Delta(1-\Delta)^2}{(1-\mu_0\Delta)^3}) + \frac{1-\alpha}{2}\frac{1}{1-\mu_0\Delta})$. This is linear in α. Thus, $\frac{\partial\bar{c}^{umb}}{\partial\rho}|_{\rho=0} < 0$ if and only if $\mu_0 < \underline{\mu} := \frac{2\Delta-1}{\Delta}$ and α is sufficiently large, that is, $\alpha > \overline{\alpha} := \frac{(1-\mu_0\Delta)^2}{(1-\mu_0)\Delta(2(1-\Delta)+1-\mu_0\Delta)}$.

Case 2. Second, $\bar{c}^{umb}$ attains its maximum at $\rho=1$ if $\frac{\partial\bar{c}^{umb}}{\partial\rho}|_{\rho=1} > 0$, where $\frac{2}{\delta\Delta}\cdot\frac{\partial\bar{c}^{umb}}{\partial\rho}|_{\rho=1} = (1-\mu_0)\Delta(\frac{\alpha}{2}(-\frac{2\Delta-1}{1-\Delta} + \frac{2\Delta(1-\Delta)^2\mu_0(1-\mu_0\Delta)}{(1-2\mu_0\Delta+\mu_0\Delta^2)^2}) + \frac{1-\alpha}{2}\frac{1}{1-\mu_0\Delta})$. This is again linear in α, and it is positive if and only if either $\mu_0 \ge \overline{\mu} := \frac{1-2\Delta-\Delta^2+\Delta^3-(1-\Delta)^2\sqrt{2\Delta-1+\Delta^2}}{\Delta^2(2-6\Delta+3\Delta^2)^2}$ or $\mu_0 < \overline{\mu}$ and α is sufficiently small, that is, $\alpha < \underline{\alpha} = \frac{(1-\Delta)(1-2\mu_0\Delta+\mu_0\Delta^2)^2}{\Delta(1-\mu_0+2\mu_0\Delta)(1-2\mu_0\Delta+\mu_0\Delta^2)^2-2\mu_0\Delta(1-\Delta)^3(1-\mu_0\Delta)^2}$.
Note that $\overline{\alpha} > \underline{\alpha}$ and $\overline{\mu} > \underline{\mu}$.

Case 3. Finally, the maximum of $\bar{c}^{umb}$ is attained at an intermediate value of $\rho \in (0,1)$ if $\frac{\partial\bar{c}^{umb}}{\partial\rho}|_{\rho=0} > 0$ and $\frac{\partial\bar{c}^{umb}}{\partial\rho}|_{\rho=1} < 0$ simultaneously hold. Based on the necessary and sufficient conditions identified in the previous two cases, the following conditions must be satisfied: (1) $\mu_0 > \underline{\mu}$ or $\alpha < \overline{\alpha}$ and (2) $\mu_0 < \overline{\mu}$ and $\alpha > \underline{\alpha}$. These conditions can be summarized as follows: $\underline{\mu} < \mu_0 < \overline{\mu}$ and $\alpha > \underline{\alpha}$, or $\mu_0 < \underline{\mu}$ and $\underline{\alpha} < \alpha < \overline{\alpha}$.

Combining all the identified conditions proves the proposition. □

Proof of Proposition 4. It must be shown that there are no other equilibria of the model in addition to the HE, LE, and ME. The proposition is proven in two steps. First, it is shown that the firm's optimal investment decision in any equilibrium does not depend on its track record; therefore, in equilibrium, the firm should either always invest or never invest in any given product market. It is still possible that the firm always invests in one product market and never invests in the other product market, as in the ME. Second, each equilibrium that satisfies this condition is considered (in addition to the HE, LE, and ME), and its existence is ruled out. The proof of the second step is relegated to the online appendix.

Claim A.1. *In equilibrium, the firm either always invests or never invests in any given product market.*

Proof. For any given track record of the firm, $\mathbf{h}_{t-1}$, the payoff from investing in quality in product market A is denoted by $V_\theta(\mathbf{h}_{t-1}; br, d_t^A = 1)$, where $d_t^A = 1$ represents the firm's decision to incur an investment cost. This payoff function is

$$\begin{aligned} V_\theta(\mathbf{h}_{t-1}; br, d_t^A = 1) &= \frac{p^A(\mathbf{h}_{t-1}; br) + p^B(\mathbf{h}_{t-1}; br)}{2} - c - c \cdot d_t^B \\ &+ \delta \cdot \bigg(\frac{\alpha}{2}\big(\Delta \ \Pr(h_t^B = H) V\theta(H, H; br) + \Delta\big(1 - \Pr(h_t^B = H)\big) \\ &\times V_\theta(H, L; br) + (1 - \Delta)\Pr(h_t^B = H) V_\theta(L, H; br) \\ &+ (1 - \Delta)\big(1 - \Pr(h_t^B = H)\big) V_\theta(L, L; br) + V_\theta(\varnothing, \varnothing; br)\big) \\ &+ \frac{1-\alpha}{2}\big(\Delta V_\theta(H, \varnothing; br) + (1 - \Delta) V_\theta(L, \varnothing; br) \\ &+ \Pr(h_t^B = H) V_\theta(\varnothing, H; br) + \big(1 - \Pr(h_t^B = H)\big) \\ &\times V_\theta(\varnothing, L; br)\big)\bigg), \end{aligned}$$

where $\Pr(h_t^B = H)$ is the probability with which the outcome $h_t^B = H$ is produced, given that the firm is of type θ^B in market B. Thus, $\Pr(h_t^B = H) = \Delta$ if $\theta^B = C$ and $d^B = 1$ and zero otherwise.

On the other hand, if the firm decides not to invest in quality, denoted by $d^A = 0$, then the payoff function is

$$\begin{aligned} V_\theta(\mathbf{h}_{t-1}; br, d^A = 0) &= \frac{p^A(\mathbf{h}_{t-1}; br) + p^B(\mathbf{h}_{t-1})}{2} \\ &- c \cdot d^B + \delta \cdot \bigg(\frac{\alpha}{2}\big(0 \cdot \Pr(h_t^B = H) \\ &\times V_\theta(H, H; br) + 0 \cdot \big(1 - \Pr(h_t^B = H)\big) V_\theta(H, L; br) \\ &+ \Pr(h_t^B = H) V_\theta(L, H; br) + \big(1 - \Pr(h_t^B = H)\big) \\ &\times V_\theta(L, L; br) + V_\theta(\varnothing, \varnothing; br)\big) + \frac{1-\alpha}{2}\big(\Delta V_\theta(H, \varnothing; br) \\ &+ (1 - \Delta) V_\theta(L, \varnothing; br) + \Pr(h_t^B = H) V_\theta(\varnothing, H; br) \\ &+ \big(1 - \Pr(h_t^B = H)\big) V_\theta(\varnothing, L; br)\big)\bigg). \end{aligned}$$

Then, in an arbitrary equilibrium, it is optimal for the firm to invest in product market A if and only if $V_\theta(\mathbf{h}_{t-1}; br, d^A = 1) > V_\theta(\mathbf{h}_{t-1}; br, d^A = 0)$, which holds if and only if $c < \bar{c}_\theta^*$, where $\bar{c}_\theta^* := \delta\Delta \cdot [\frac{\alpha}{2}(\Pr(h_t^B = H)(V_\theta(H, H; br) - V_\theta(L, H; br)) + (1 - \Pr(h_t^B = H))(V_\theta(H, L; br) - V_\theta(L, L; br))) + \frac{1-\alpha}{2}(V_\theta(H, \varnothing; br) - V_\theta(L, \varnothing; br))]$. Note that $\bar{c}_\theta^*$ is independent of the track record of the firm, implying that this condition either holds for all track records or no track records. In other words, for each type of firm $\theta \in \{C, I\}^2$, the firm either always invests or never invests. □

Claim A.2. *In addition to the HE, LE, and ME, no other equilibria can exist.*

Proof. The proof is relegated to the online appendix. □

Corollary A.1. *An independent brand's HE is more profitable than both brandings' ME.*

Proof. Given the definition of the ME, the firm types expected profit in the equilibrium are $\pi_{C,C}^{*ME}(br) = \pi_{C,I}^{*ME}(br) = \frac{\pi_{C,C}^{*HE}(ind)}{2}$ and $\pi_{I,C}^{*ME}(br) = \pi_{I,I}^{*ME}(br) = 0$. □

Proof of Proposition 5. The firm's expected profit is positive if and only if the expected per-period profit defined in (8) is positive. In the HE, the firm makes an investment whenever it is competent in a product market, that is, $\sigma_\theta^i(\mathbf{h}) = 1$ if $\theta^i = C$. Given the investment strategy, the distribution over realized quality depends on the firm type. First, for $\theta = (C, C)$, $\Pr(H, H|\theta) = \frac{\alpha}{2} \cdot \Delta^2$, $\Pr(H, L|\theta) = \Pr(L, H|\theta) = \frac{\alpha}{2} \cdot \Delta(1 - \Delta)$, $\Pr(L, L|\theta) = \frac{\alpha}{2} \cdot (1 - \Delta)^2$, $\Pr(H, \varnothing|\theta) = \Pr(\varnothing, H|\theta) = \frac{1-\alpha}{2} \cdot \Delta$, $\Pr(L, \varnothing|\theta) = \Pr(\varnothing, L|\theta) = \frac{1-\alpha}{2} \cdot (1 - \Delta)$, and, finally, $\Pr(\varnothing, \varnothing|\theta) = \frac{1-\alpha}{2}$.

Then, using the previous results on prices, under independent branding, the (C, C)-type firm expects the profit

$$\begin{aligned} \pi_{C,C}(br) = -2c &+ \frac{\alpha}{2}\big(\Delta^2 p^A(H, H; br) + \Delta(1 - \Delta) \\ &\times \big(p^A(H, L; br) + p^B(H, L; br)\big) + (1 - \Delta)^2 p^A(L, L; br)\big) \\ &+ \frac{1-\alpha}{2}\big(\Delta\big(p^A(H, \varnothing; br) + p^B(H, \varnothing; br)\big) \\ &+ (1 - \Delta)\big(p^A(L, \varnothing; br) + p^B(L, \varnothing; br)\big)\big) \\ &+ \frac{\alpha}{2} p^A(\varnothing, \varnothing; br). \end{aligned} \tag{A.4}$$

The expected profit in each period for (C, I)-type and (I, C)-type firms is

$$\begin{aligned} \pi_{C,I}(br) = -c &+ \frac{\alpha}{2}\bigg(\frac{\Delta}{2}\big(p^A(H, L; br) + p^B(H, L; br)\big) \\ &+ (1 - \Delta) p^A(L, L; br)\bigg) \\ &+ \frac{1-\alpha}{2}\bigg(\frac{\Delta}{2}\big(p^A(H, \varnothing; br) + p^B(H, \varnothing; br)\big) \\ &+ \frac{2-\Delta}{2}\big(p^A(L, \varnothing; br) + p^B(L, \varnothing; br)\big)\bigg) \\ &+ \frac{\alpha}{2} p^A(\varnothing, \varnothing; br). \end{aligned} \tag{A.5}$$

Finally, the profit of the (I, I)-type firm is

$$\begin{aligned} \pi_{I,I}(br) = \frac{\alpha}{2} p^A(L, L; br) &+ \frac{1-\alpha}{2}\big(p^A(L, \varnothing; br) + p^B(L, \varnothing; br)\big) \\ &+ \frac{\alpha}{2} p^A(\varnothing, \varnothing; br). \end{aligned} \tag{A.6}$$

For independent branding ($br = ind$), $\pi_{C,C} \geq 0$ if and only if $-2c + \frac{\alpha}{2}(\Delta \cdot p^A(H, h^B; ind) + (1 - \Delta) \cdot p^A(L, h^B; ind)) + \frac{1-\alpha}{2}(\Delta \cdot p^A(H, h^B; ind) + (1 - \Delta) p^A(L, h^B; ind) + p^A(\varnothing, h^B; ind)) + \frac{\alpha}{2} \cdot p^A(\varnothing, h^B; ind) \geq 0$.

For the (C, I)-type firm, $\pi_{C,I} \geq 0$ if and only if $-c + \frac{\Delta}{2}\frac{1}{2} + \frac{1}{2} p^A(\varnothing, h^B; ind) + (\frac{\alpha}{2}\frac{2-\Delta}{2} + \frac{1-\alpha}{2}\frac{2-\Delta}{2}) p^A(L, h^B; ind) \geq 0$. Also, $\pi_{I,I} \geq 0$ because it never incurs investment costs.

Therefore, an independent brand of any type makes a nonnegative profit in the HE if and only if $c \leq \hat{c}^{ind} := \min\{\frac{1}{4}(\Delta \cdot p(H, h^B; ind) + (1 - \Delta) \cdot p(L, h^B; ind) + p(\varnothing, h^B; ind)), \frac{1}{4}(\Delta \cdot p^A(H, h^B;$

$ind) + (2-\Delta)\cdot p^A(L,h^B;ind) + 2p^A(\varnothing,h^B;ind))\}$, so $\hat{c}^{ind} = \frac{\Delta}{4}(\Delta + \frac{\mu_0(1-\Delta)^2}{1-\mu_0\Delta} + \mu_0)$. This cutoff level is greater than $\overline{c}^{ind}$. Therefore, if the HE exists, that is, $c < \overline{c}^{ind}$, then $c < \hat{c}^{ind}$ must also hold, that is, the expected profit of all types must be positive.

For umbrella branding ($br = umb$), repeating similar computations, it is straightforward to show that the condition $\pi_{C,C}(umb) \geq 0$ holds whenever $c < \overline{c}^{umb}$. Similarly, $\pi_{C,I}(umb) \geq 0$ holds whenever $c < \overline{c}^{umb}$. Therefore, if the HE exists for umbrella branding, the firm is expected to earn a positive profit, regardless of its type.

This completes a proof that for each branding regime the HE is the most profitable equilibrium whenever it exists. □

Proof of Proposition 6. The threshold levels in the two branding regimes, $\overline{c}^{ind}$ and $\overline{c}^{umb}$, are compared. Recall from (7) that for independent branding, $\overline{c}^{ind} = \frac{\delta\Delta}{4}\cdot\frac{\Delta(1-\mu_0)}{1-\mu_0\Delta}$, and for umbrella branding, from (A.4), $\overline{c}^{umb} = \frac{\delta\Delta}{4}\cdot[\Delta - \alpha\cdot((2\Delta-1)p^A(L,H;umb) + 2(1-\Delta)p^A(L,L;umb)) - (1-\alpha)\cdot(p^A(L,\varnothing;umb) - p^B(H,\varnothing;umb) + p^B(L,\varnothing;umb))]$.

Case 1 ($\Delta > \frac{1}{2}$, $\mu_0 < \underline{\mu}$, and $\alpha > \overline{\alpha}$). The term $\overline{c}^{umb}$ monotonically decreases in ρ. Because $\overline{c}^{umb}$ evaluated at $\rho = 0$ coincides with $\overline{c}^{ind}$, $\overline{c}^{ind} \geq \overline{c}^{umb}$.

Case 2 ($\Delta < \frac{1}{2}$, or ($\Delta > \frac{1}{2}$ and $\mu_0 > \overline{\mu}$), or ($\Delta > \frac{1}{2}$ and $\alpha < \underline{\alpha}$). The term $\overline{c}^{umb}$ monotonically increases in ρ. Thus, $\overline{c}^{umb} \geq \overline{c}^{ind}$.

Case 3 ($\Delta > \frac{1}{2}$ and ($\alpha > \overline{\alpha}$ and $\underline{\mu} < \mu_0 < \overline{\mu}$), or ($\underline{\alpha} < \alpha < \overline{\alpha}$ and $\mu_0 < \overline{\mu}$). The term $\overline{c}^{umb}$ is nonmonotonic and concave in ρ. If $\overline{c}^{umb}$ evaluated at $\rho = 1$ is greater than $\overline{c}^{ind}$, then $\overline{c}^{umb} \geq \overline{c}^{ind}$. Otherwise, if $\overline{c}^{umb}$ evaluated at $\rho = 1$ is less than $\overline{c}^{ind}$, then $\overline{c}^{ind} \geq \overline{c}^{umb}$ if and only if ρ is sufficiently large. At $\rho = 1$, $\overline{c}^{umb}|_{\rho=1} = \frac{\delta\cdot\Delta(1-\mu_0)}{2}\cdot\frac{(1-2\mu_0\Delta+\mu_0\Delta^2-\alpha(1-\mu_0)\Delta)}{(1-\mu_0\Delta)(1-2\mu_0\Delta+\mu_0\Delta^2)}$, which is less than $\overline{c}^{ind}$ if and only if $\mu_0 < \hat{\mu} := \frac{2\Delta-1}{\Delta^2}$ and $\alpha > \hat{\alpha} := \frac{1-2\mu_0\Delta+\mu_0\Delta^2}{2(1-\mu_0)\Delta}$. Note that $\hat{\mu} \in (\underline{\mu}, \overline{\mu})$ and $\hat{\alpha} \in (\underline{\alpha}, \overline{\alpha})$. Therefore, $\overline{c}^{ind}|_{\rho=1} > \overline{c}^{umb}|_{\rho=1}$ if either $\alpha \in (\hat{\alpha}, 1)$ and $\mu_0 \in (\underline{\mu}, \hat{\mu})$, or $\alpha \in (\hat{\alpha}, \overline{\alpha})$ and $\mu_0 \in (0, \underline{\mu})$. In this case, $\overline{c}^{ind} > \overline{c}^{umb}$ if and only if $\rho > \hat{\rho}$ for some $\hat{\rho} \in (0, 1)$. □

Proof of Proposition 7. If $\rho = 1$, then $p^A(H, h^B; umb) = p^B(H, h; umb) = \Delta$ for any $h^B \in \mathscr{H}$. By plugging this into $\overline{c}^{umb}$ in (A.3) and differentiating with respect to α, $\frac{d\overline{c}^{umb}}{d\alpha}|_{\rho=1} = -\frac{\delta\Delta^3}{2}\cdot\frac{(1-\mu_0)^2}{(\mu_0(1-\Delta)^2+(1-\mu_0))(\mu_0(1-\Delta)+1-\mu_0)} \leq 0$. □

Endnotes

[1] In practice, some firms have adopted a hybrid brand architecture between these two extreme strategies. This paper focuses on the trade-offs between the two extreme strategies. The findings provide insights into why some firms use an intermediate approach in the whole spectrum of brand architectures.

[2] For these reasons, most previous research on umbrella branding has assumed that consumers are aware of the association between a firm's different products only when the firm uses the same brand. This paper makes the same assumption, which is discussed in Section 3.3.

[3] For more information on the Volkswagen scandal, see http://www.bbc.com/news/business-34324772 (last accessed August 12, 2020).

[4] When exploiting this positive spillover, the firm is afraid of the negative spillover that the firm's lack of investment will generate. This fear can discipline the firm's behavior. Therefore, a complete intuition depends on which of the two spillovers is stronger, which is determined by consumers' prior beliefs about the firm's competence type. For more details, see Section 4.2.

[5] This view is based on the seminal works of Milgrom and Roberts (1982) and Kreps and Wilson (1982).

[6] The assumption about the cost disadvantage of umbrella branding is contrary to an oft-cited anecdotal evidence (e.g., Tauber 1981).

[7] This mechanism was first laid out in the seminal works by Klein and Leffler (1981) and Shapiro (1983).

[8] For example, Cabral (2009) considers two assumptions: one in which a failure of one product induces consumers' pessimistic beliefs about the quality of both products and another in which failures of both products are required to induce such beliefs. Depending on the imposed beliefs, different equilibria can be sustained.

[9] Choi (1998) is an exception in that he analyzes a model of both adverse selection and moral hazard in a setting in which an opportunity to launch a new product arrives randomly over an infinite time horizon. However, the underlying mechanism corresponds to one for moral hazard models. The difference in his work is that the firm posts future opportunities to introduce new products (as opposed to future profits of existing products) as a bond.

[10] The demand-side relatedness in this paper affects the amount of information consumers observe, so greater demand-side relatedness can be interpreted as a longer consumer memory. Consistent with findings in research with finite consumer memory, this paper finds that a longer consumer memory (or higher demand-side relatedness) can reduce the scope of the efficient equilibrium.

[11] An earlier version of the paper relaxes this assumption, and an incompetent type can also produce a high-quality product with a positive probability less than Δ. The main results of the current version are robust.

[12] This is a simple way to capture the firm's reputation concerns by connecting its track record to future payoffs. This assumption can be relaxed so that a consumer observes a longer track record of a firm. However, this will add analytical complexity and provide little additional insight. Importantly, this assumption helps reduce the state space and hence the number of equilibria.

[13] A rational consumer might be aware of the possibility that even distinct brands could belong to the same firm and engage in a costly search to establish the association on their own. Taken literally, this assumption implies that this cost is prohibitively high. The results in this paper will be robust if there is a small fraction of consumers who are able to establish the link between the same firm's products with different brand names.

[14] This is a common assumption in the branding literature, which can be motivated by having two consumers with the same unit demand arrive in each period and by the firm having the capacity to serve only one consumer in each market. Given this assumption, this paper abstracts away from pricing issues, especially price signaling, and instead focuses on the reputation established by the firm's performance record as the main channel through which information about quality is communicated.

[15] This particular sequence is mathematically equivalent (after scaling the investment cost by a constant) to an alternative sequence in which the consumer arrives first and then the firm makes investment decisions. In the former, an investment in quality can be wasted because a consumer may not have a unit demand. This situation would not occur in the latter.

[16] The analysis focuses on pure strategies, but Endnote 33 discusses mixed strategies.

[17] The posterior beliefs are discussed in Section 3.3.

[18] Therefore, the case in which a consumer has no unit demand and the case in which she has a unit demand and pays a price of zero are not the same. The effective price she pays in both cases is zero.

However, in the former, no outcome is generated, whereas in the latter, either H or L will be the outcome.

19 This is more formally stated in Proposition 5.

20 Thus, conceptually, independent branding is a special case of umbrella branding with $\rho = 0$, which implies that consumers do not make any inferences across product markets.

21 The total expected demand is $2 \cdot \frac{\alpha}{2} + 2 \cdot \frac{1-\alpha}{2} + 0 \cdot \frac{\alpha}{2} = 1$.

22 Posterior beliefs under general investment strategies can be computed similarly using Bayes' rule.

23 In an alternative equilibrium, this need not always be true. For instance, if the (C, C)-type firm invests in product market A and the (C, I)-type firm does not, then $\mu^i(\mathbf{h}_{t-1}; br)$ is not sufficient. Rather, consumers' posterior distribution over the type space in two product markets $\{C, I\}^2$ must be considered. However, Proposition 4 shows that such strategies cannot be part of an equilibrium. Thus, for all equilibria of the model, dealing with the posterior belief in each product market $\mu^i(\mathbf{h}_{t-1}; br)$ is indeed sufficient.

24 By symmetry, the consumer's posterior beliefs about θ^B, $\mu^B(h^A, h^B; br)$, are updated similarly and have the same properties.

25 This distribution of new outcomes also depends on demand-side relatedness. For instance, $\Pr(H, L|C, C) = \frac{\alpha}{2}\Delta(1-\Delta)$ because the consumer experiences the realized quality in both markets with probability $\frac{\alpha}{2}$, and the (C, C)-type firm will produce an outcome (H, L) with probability $\Delta(1-\Delta)$.

26 If incompetent firms can also produce a high-quality good with a positive probability (less than Δ), then $\bar{c}^{ind}$ will vanish at $\mu_0 = 0$ and $\mu_0 = 1$.

27 It is still true that a track record of high-quality products reveals the firm's competence. However, given a smaller Δ, such a situation is less likely to occur. Moreover, a track record of low-quality products is less informative about the firm's incompetence.

28 By assuming that the firm chooses its branding decision before realizing its type, this game precludes potential signaling through the firm's branding choice.

29 This assumption requires a further justification because if consumers know that a firm is deciding whether to use the same or different brands in distinct product markets, then consumers should be aware of the association between the two products regardless of the firm's branding decision. To address this issue, Cabral (2000) assumes that there is a continuum of firms with a basic product in which only firms of measure zero are endowed with another product and make the branding decision. Then, if consumers face different brands in different product markets, then each product belongs to a distinct firm with probability one, and therefore, consumers treat them as unrelated.

30 Other equilibria in which the firm's investment decision is different for different track records can exist if the consumer can observe a longer history of the firm. However, the result that the HE is the most profitable if $c > 0$ is robust. Neeman et al. (2019) explore this issue.

31 The same is true about product market B; that is, it can never be true that the (C, C)-type firm invests in market B but the (I, C)-type firm does not, and vice versa.

32 This result is stated as Corollary A.1 in the appendix.

33 Although this paper analyzes pure-strategy equilibria, intuitively, the results in Proposition 5 should be robust to mixed strategies given this paper's focus on the most profitable equilibrium. Suppose there is a mixed-strategy equilibrium in which a firm of type θ invests in a product market with a positive probability. An indifference condition implies that the investment cost c equals the marginal benefit of an investment through the continuation payoff given the mixed strategy. This benefit is only a fraction of the threshold under an alternative equilibrium in which the firm adopts a pure strategy to always invest, for two reasons: one, the firm is less likely to produce a high-quality product (a fraction of Δ), and two, consumers are paying lower prices for each of the firm's track records. Consequently, the mixed-strategy equilibrium will be dominated by a more profitable pure-strategy equilibrium.

34 After the type realization at $t = 0$, different firm types may prefer different branding regimes. Provided that the HE exists, the (C, C)-type firm always prefers umbrella branding, whereas the (I, I) type wants to separate out its record in two markets by adopting independent branding. However, (C, I) and (I, C) sometimes prefers one branding to the other. The online appendix provides an analysis of each firm type's preferred branding regime.

35 The literature on behavior-based pricing (e.g., Villas-Boas 1999, Fudenberg and Tirole 2000, Shin and Sudhir 2010) investigates whether a firm should reward its own and its competitor's customers in a single product market. In this paper, a different CRM problem is examined for a multiproduct firm.

36 P&G released its Super Bowl 2020 advertisement featuring not only its product brands but also the corporate brand.

37 See https://www.ideasbig.com/blog/facebook-rebrands-its-sub-brands/.

38 In fact, the company has a number of products that are highly related on both the demand and supply sides. For instance, P&G sells a laundry detergent (Tide), a dishwashing soap (Dawn), a fabric softener (Downy), and an all-purpose cleaner (Mr. Clean), which serve similar consumer needs to clean and disinfect various objects in the house. Moreover, all these products are liquid cleaning products, which share common ingredients.

39 For example, research has shown that firms can benefit from advertising spillover by using an umbrella brand (e.g., Smith 1992, Smith and Park 1992, Erdem and Sun 2002).

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