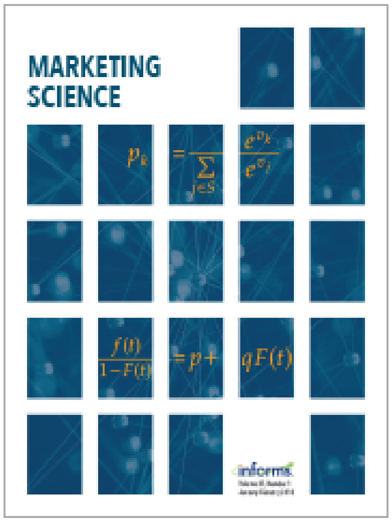


Marketing Science

Modeling Seasonality in New Product Diffusion

Yuri Peers, Dennis Fok, Philip Hans Franses,

MARKETING SCIENCE
Vol. 31, No. 2, March–April 2012, pp. 351–364
ISSN 0732-2399 (print) | ISSN 1526-548X (online)
http://dx.doi.org/10.1287/mksc.1110.0696

Modeling Seasonality in New Product Diffusion

Yuri Peers
Waikato Management School, University of Waikato, Hamilton 3240, New Zealand,
ypeers@waikato.ac.nz

Dennis Fok, Philip Hans Franses
Econometric Institute, Erasmus University Rotterdam, 3000 DR Rotterdam, The Netherlands
{dfok@ese.eur.nl, franses@ese.eur.nl}

We propose a method to include seasonality in any diffusion model that has a closed-form solution. The resulting diffusion model captures seasonality in a way that naturally matches the original diffusion model's pattern. The method assumes that additional sales at seasonal peaks are drawn from previous or future periods. This implies that the seasonal pattern does not influence the underlying diffusion pattern. The model is compared with alternative approaches through simulations and empirical examples. As alternatives, we consider the standard Generalized Bass Model (GBM) and the basic Bass Model, which ignores seasonality. One of the main findings is that modeling seasonality in a GBM generates good predictions but gives biased estimates. In particular, the market potential parameter is underestimated. Ignoring seasonality in cases where data of the entire diffusion period are available gives unbiased parameter estimates in most relevant scenarios. However, ignoring seasonality leads to biased parameter estimates and predictions when only part of the diffusion period is available. We demonstrate that our model gives correct estimates and predictions even if the full diffusion process is not yet available.

Key words: diffusion models; seasonality; forecasting
History: Received: July 7, 2010; accepted: November 2, 2011; Eric Bradlow and then Preyas Desai served as the editor-in-chief and Eitan Muller served as associate editor for this article.

1. Introduction

Sales of new products typically follow a diffusion process that has an S-shaped pattern for cumulative sales, whereas the corresponding pattern for sales is hump-shaped. There is a variety of models that can capture such a diffusion pattern. In marketing, the Bass (1969) Model is most often used. The main application of diffusion models concerns forecasting sales. For new products, one can use the parameter estimates based on data of similar products. And, after having observed the diffusion of a product for a while, one can also use these observations to estimate the relevant parameters and forecast the remainder of the diffusion pattern.

Most diffusion models are set in a continuous-time context and assume a smooth development of sales. This smooth development often matches well with observed diffusion data at a yearly frequency. However, at a higher frequency, the sales development tends to be less smooth. For example, within a year, sales data are likely to show a strong seasonal pattern. Seasonality systematically generates periods with higher sales followed by periods with lower sales. For example, Christmas sales usually generate a sales spike in the month of December. In this paper, we present a model that allows for such seasonality while preserving the basic diffusion pattern.

The importance of having a diffusion model that incorporates seasonality is amplified by the increasing availability of high-frequency data. Although high-frequency diffusion data are nowadays often available, researchers usually opt to aggregate such data to the yearly level. For example, Venkatesan et al. (2004) mention this practice of aggregating to annual data to get rid of seasonal fluctuations. Although this aggregation reduces or even removes seasonality, it comes with a loss of information. Using only a small number of (annual) data points leads to the so-called ill-conditioning problem, which, in turn, leads to biased estimates; see Van den Bulte and Lilien (1997) and Bemmaor and Lee (2002). Putsis (1996) and Non et al. (2003) find that the use of quarterly or monthly data significantly improves estimates of diffusion model parameters compared to only using annual data. The main reason for this improvement is the reduction in the data-interval bias that originates from the discrete-time approximation of the underlying continuous-time diffusion model. Neither study, however, explicitly covers seasonality for monthly or quarterly data.

From a managerial point of view, seasonal patterns also contain valuable information. This information can be used to predict short-term demand as well as to support inventory management. Hence, filtering out seasonal effects, which is common practice in the literature on financial and macroeconomic time series, is not a preferable solution in case of diffusion models.

The conclusion is that seasonality must somehow be incorporated in a diffusion model. In this paper, we propose a natural way to do this for any diffusion model that has a closed-form solution. We build on the Bass Model for expository purposes but indicate that many other models can be considered. The main idea that underlies our approach is that seasonality in a peak period is a result of consumers who delay or speed up their purchases. In other words, the model captures the pattern of intertemporal demand shifts that cause seasonal peaks. We treat the underlying diffusion model as the appropriate model for time-aggregated sales and study how these aggregates are distributed over, say, the months of a year.

We contrast our model with other approaches on theoretical grounds and using empirical examples. The first alternative approach we consider is to include seasonal dummies in a way that matches with the Generalized Bass Model (GBM) (Bass et al. 1994). We show that for this model, the estimates for the diffusion parameters are biased; the market potential parameter especially gets underestimated. Estimates based on our model are intuitively more appealing and do not lead to a bias. The second alternative approach is the traditional Bass Model, which ignores seasonality even when it is present. Our results give reassuring outcomes for the common practice of "guessing by analogy," because for the case where the full diffusion series is available, we demonstrate that the traditional Bass estimates are not biased.[1] However, if seasonality is ignored when the diffusion process is before its saturation level, the estimates as well as the forecasts are biased.

Next to our model, we also propose another variation to the Generalized Bass Model with seasonality. In many practically relevant cases, this variation has the same nice statistical features as our model; that is, the parameter estimates are unbiased. Contrary to our model, this variation of the GBM is not based on intertemporal demand shifts. This results in biased estimates if the diffusion is fast.

The outline for the rest of our paper is as follows. In §2 we start by showing typical diffusion data, where we consider the monthly sales of flat-screen television sets (LCD and plasma). In §3 we propose our model and theorize why the alternative approaches are less useful when seasonality is present. In §4 we return to actual sales data and demonstrate that the new seasonal diffusion model fits naturally to these data and that it gives plausible forecasts. In §5 we conclude with some suggestions for further research.

2. An Example of Seasonality in Diffusion Data

Before we start modeling seasonality in diffusion models, we first take a look at a typical example of seasonality in diffusion data. In particular, we have available monthly sales figures (in millions) of flat-screen televisions for 10 countries in Europe.[2] We obtained the diffusion data from a European consumer electronics firm that operates around the globe. This firm bought these data from a market research company. The period covered by the data ranges from February 2004 to December 2009 or January 2010. This means that we have 71 or 72 monthly observations per country, implying the presence of six seasonal cycles.

In Table 1, we present some summary statistics on the diffusion series. The second column gives the average sales over all monthly observations. Table 1 also presents the averages over a particular month of the year relative to the overall average. These figures give a rough impression of the seasonal pattern in the different countries. For example, the sales in December tend to be above the overall average for all countries. This is an indication that there is a seasonal spike in sales because of Christmas. The month with the lowest sales seems to be April. Next, in the period from May to September, there are large differences across countries. Note that Table 1 can only be used as a first indication of the seasonal patterns as the presented statistics completely ignore the curvature in the diffusion series. Also, for some countries, we do not have data for January 2010. This results in a relatively lower average for January, as this last year is around the moment of peak sales.

To give a graphical example of the feature we study in this paper, we present in Figure 1 monthly sales of flat-screen televisions in 2 of the 10 countries: the United Kingdom and the Netherlands.

From Table 1 and Figure 1, we conclude that the series clearly show systematic seasonal patterns. Furthermore, the graphs suggest that the seasonal pattern is proportional to the speed and position of the

[1] *Guessing by analogy* is a popular method among researchers and managers to predict the diffusion parameters of a new product based on the diffusion parameters of earlier introduced similar products; see Ofek (2005) and Lilien et al. (2000). Thus, if published or obtained estimates are biased by ignoring seasonality, this would affect the prediction of the diffusion of the new product as well.

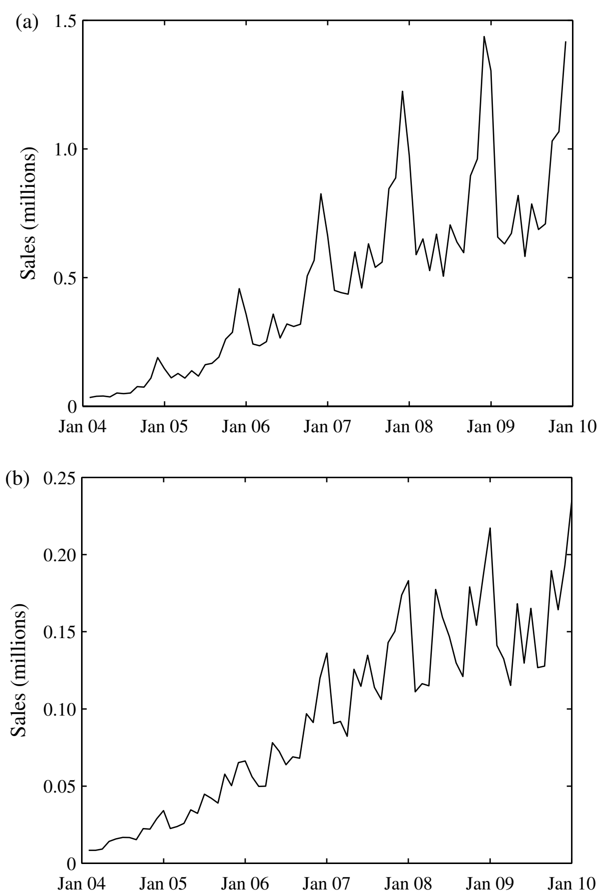
[2] Austria, Belgium, France, Germany, Italy, the Netherlands, Portugal, Scandinavia, Spain, and the United Kingdom. The data concern aggregated sales figures for the Scandinavian countries. For simplicity, we refer to Scandinavia as a "country."

Table 1 Summary Statistics on Monthly Sales of Flat-Screen Televisions Across Various Countries

Country	Overall avg. (millions)	Difference of the average per month with overall average (%)											
		Jan.	Feb.	March	April	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
United Kingdom	0.491	40.4	−29.2	−27.8	−30.9	−11.0	−32.7	−9.9	−18.7	−16.7	22.8	31.8	88.6
France	0.323	65.8	−28.8	−31.3	−36.5	−14.6	−23.6	−11.0	−20.0	−11.4	12.9	12.9	85.5
Germany	0.355	46.7	−24.9	−20.2	−30.7	−15.9	−21.7	−15.3	−19.9	−10.6	22.1	29.4	61.0
The Netherlands	0.096	52.1	−25.1	−26.3	−30.7	4.3	−8.6	−0.2	−13.1	−16.8	20.1	10.3	33.9
Italy	0.265	22.8	−30.8	−31.3	−36.9	−12.7	−23.0	−16.0	−20.5	2.1	33.7	25.9	86.6
Spain	0.225	62.3	−24.6	−27.9	−28.4	−3.2	−15.7	3.1	−19.9	−13.4	17.8	6.0	43.7
Austria	0.035	20.4	−21.7	−22.5	−30.2	−12.2	−20.2	−9.4	−20.4	−8.2	20.4	25.4	82.1
Belgium	0.046	82.1	−21.9	−25.7	−28.2	−10.3	−19.1	21.4	−21.4	−17.2	6.4	−3.1	37.2
Portugal	0.034	4.5	−26.9	−30.8	−27.2	−7.7	−24.6	−9.0	−15.3	−5.6	18.9	21.4	103.1
Scandinavia	0.125	34.1	−23.6	−18.9	−28.8	−21.6	−16.7	−6.6	−7.5	−3.4	13.0	2.9	77.3

diffusion process; that is, the seasonal peaks become larger closer to the moment of peak sales. Next, the seasonal patterns are not similar across countries. For example, in the Netherlands, the January peak is larger than the December peak, whereas in the United Kingdom, it is the other way around. This shows that we need to allow for different seasonal structures across countries.

Figure 1 Flat-Screen Television Sales Data of (a) the United Kingdom and (b) the Netherlands



3. A Diffusion Model with Seasonality

Although seasonality is a major issue for many market processes, the literature on seasonality in marketing is small. Shugan and Radas (1999) give an overview of seasonality-related issues in the context of services marketing. They consider how to overcome these issues and how managers should react to them. Fok et al. (2007) look at weekly seasonality in sales in a panel of fast-moving consumer goods. There are, to our knowledge, only two papers that focus on modeling seasonality in diffusion processes: Radas and Shugan (1998) and Einav (2007). Both papers consider the movie industry. Einav (2007) uses a structural model to distinguish between seasonal demand and seasonal supply effects. Sales could be higher because more people go to the movies in holiday seasons or because better movies are screened during these periods. The view on seasonality discussed in Radas and Shugan (1998) comes closest to our setting. These authors interpret seasonality as a time transformation process; that is, it is as if the service or product ages more quickly along its life cycle in peak seasons. In off-peak seasons, it is as if the product ages more slowly. The resulting diffusion model is to some extent in line with the Generalized Bass Model (Bass et al. 1994). However, the seasonal structure—that is, the set of periods that corresponds to seasonal peaks and troughs—is imposed rather than estimated in their model. Our model and the benchmark seasonal models we consider allow for a selection of the seasonal structure based on the observed diffusion process. In the model presented in §3.1.2, we even make the intertemporal demand shift pattern endogenous; that is, we estimate the proportion in which one month contributes to the seasonal peak in another month. Finally, the paper of Radas and Shugan (1998) does not consider the impact of seasonality on the estimation and interpretation of the standard diffusion parameters.

In the following subsection, we build up to our proposed methodology, where we make a distinction between a *given fixed* intertemporal demand shift pattern (§3.1.1) and an *estimated flexible* intertemporal demand shift pattern (§3.1.2). Next, in §3.2, we discuss the alternative approaches and indicate why these approaches are less satisfactory. Further, we consider the consequences of ignoring seasonality. In the last subsection, we consider model selection; that is, we suggest a procedure that leads to a parsimonious model specification for the seasonal pattern.

3.1. Our Proposed Model

Our aim is to create a seasonal model that is consistent with any (closed-form) diffusion model.[3] To emphasize this, we first start with a general closed-form function for the cumulative diffusion curve, $F(t)$. $F(t)$ specifies the cumulative fraction of adopters relative to the market potential at time t. At the end of this subsection, we will use the Bass (1969) Model as the leading example for the functional form of $F(t)$.

We first discuss two general features of diffusion modeling: linking $F(t)$ to observed sales and heteroscedasticity. Concerning the former, we adopt the nonlinear estimation technique of Srinivasan and Mason (1986). In their paper, the authors measure sales as the difference in the cumulative adopters between period t and $t-1$, multiplied by the eventual number of adopters. Although Srinivasan and Mason (1986) do this specifically for the Bass Model, this estimation technique can be used for other functional forms for $F(t)$ as well.

The second issue is heteroscedasticity. Recently, Boswijk and Franses (2005) addressed the possibility that the observed diffusion deviates from the underlying S shape. They introduced a specification that contains a heteroscedastic error process and a tendency for the diffusion to return toward its equilibrium growth path. The heteroscedasticity implies that larger fluctuations are more likely to occur around the moment of peak sales. In the (seasonal) models introduced in this paper, we take this feature into account. Although the models can also be specified without heteroscedasticity, we believe that heteroscedasticity occurs in almost every empirical diffusion process. Heteroscedasticity also affects the estimation of the seasonal structure. In particular, it helps to disentangle seasonality from random shocks.

The dependent variable in all models in this paper is the monthly sales[4] of a new product at month t, denoted by S_t. The basis for our model, as well as for the alternatives presented later, is

$$S_t = m(F(t) - F(t-1)) + \varepsilon_t \quad \varepsilon_t \sim \mathrm{N}(0, f(t)^2\sigma^2), \quad (1)$$

where $F(t)$ is the fraction of cumulative adopters at time t, and $f(t)$ is the fraction of current adopters. $F(t)$ and $f(t)$ are the solutions of the differential equation associated with a continuous-time diffusion model. The m is the parameter capturing the market potential—that is, the ceiling of the typical S-shaped sales curve. The variance of the error term is proportional to $f(t)^2$. This variance specification is slightly different from that proposed by Boswijk and Franses (2005), as they scale the variance with the square of the sales of the previous period. The advantage of our specification is that it leads to a smoother pattern for the variance, especially in case of seasonality.

To model seasonal peaks and troughs, we need to increase or decrease the sales in some months relative to the general specification in (1). The seasonal effect should be proportional to the speed and position of the diffusion process (see also Figure 1). This proportionally additional effect of seasonality can be represented as

$$\begin{gathered} S_t = m(F(t) - F(t-1))\left(1 + \sum_{k\in K} \delta_k D_{kt}^{01}\right) + \varepsilon_t, \\ \varepsilon_t \sim \mathrm{N}(0, f(t)^2\sigma^2), \end{gathered} \quad (2)$$

where D_{kt}^{01} represents a $0/1$ dummy for each month k in the set K, where K can consist of one month or more. To put it more formally,

$$D_{kt}^{01} = \begin{cases} 1 & \text{if observation } t \text{ is in month } k\text{, that is,} \\ & \quad \kappa(t) = k, \\ 0 & \text{otherwise,} \end{cases} \quad (3)$$

where $\kappa(t)$ gives the month number corresponding to observation t. In this formulation, there is a maximum of 11 months that can be included in the set K, because otherwise the model parameters would not be identified.

In (2), the summation of the seasonal effects over a year is not necessarily equal to zero. This affects the interpretation of the diffusion parameters, especially m. As we will show in §3.2, model (2) introduces a bias in the parameter estimates.

[3] In cases where there is no closed-form solution, the differential equation for the diffusion model can be solved numerically. The numerical solution can then be used instead of the closed-form solution. In these cases, the seasonal method we present in this paper can also be used. In the rest of the paper, however, we focus on the cases with a closed-form solution.

[4] In this paper, we take months as the frequency of the observed data, as the empirical data we use also have a monthly frequency. Furthermore, in the diffusion literature, a month is the most often used data interval for which seasonality is relevant. Of course, our model can be considered for any other data interval.

To avoid this bias, we need to introduce the seasonal pattern in such a way that it does not interfere with the underlying S shape. Thus, the added seasonal effect should have mean zero. This means that the additional sales at a seasonal peak should be compensated in other months. For monthly data, we can define dummies such that the effect in the focal month is still 1, whereas the effect in the other months is minus 1/11. Over an entire year, this results in a dummy that has mean zero. This zero-mean dummy is formally defined as

$$D_{kt}^{\mathrm{ZM}} = \begin{cases} 1 & \text{if } \kappa(t) = k, \\ -\dfrac{1}{11} & \text{otherwise.} \end{cases} \tag{4}$$

This zero-mean dummy (D^{ZM}) can replace the 0/1 dummy (D^{01}) in (2). In §3.2, we will show that the resulting model has some preferable features. However, there is a counterintuitive feature as well. Consider again the case of a seasonal peak at some period t corresponding to month k. The additional sales equals $m\delta_k(F(t) - F(t-1))$. In other words, it is a fraction of the sales predicted by the underlying diffusion model. The "compensating" decrease in sales in the next month is $(1/11)m\delta_k(F(t+1) - F(t))$. The counterintuitive aspect here is that the compensation is associated with the "predicted" sales for the *next* period. Intuitively, it would be more appealing if the compensation equals $(1/11)m\delta_k(F(t) - F(t-1))$, that is, a fraction of the current increase itself. Stated differently, although the dummies have mean zero, the seasonal effect itself, $m(F(t) - F(t-1))\delta_k D_{kt}^{\mathrm{ZM}}$, does not have mean zero. If the diffusion is relatively fast, this may have a substantial impact, as we will also show in §3.2.

In our final model, we want to correct for this above-mentioned counterintuitive feature. As a result, we impose that a seasonal peak originates from consumers delaying or speeding up their purchases. In this case, the additional sales during a seasonal peak is the summation of the postponed and accelerated purchases of the other months. Hence, the seasonal peak equals the sum of a fraction of the underlying adoption curve of all the months influencing the focal month. The extent to which a month contributes to the seasonal peak in another month is referred to as the *intertemporal demand shift pattern*, or seasonal structure for short. This seasonal structure can be fixed from the outset (the fixed version of our model), or it can be estimated (the flexible version of our model).

3.1.1. Fixed Seasonal Structure. First, we define the set of months that influence a focal month k. We denote this set as H_k; the number of elements in H_k is denoted by $|H_k|$. For example, if $H_k = \{-3, -2, -1, 1, 2\}$, a fraction of the sales from up to three months before the focal month is delayed to the focal month, and a fraction of the sales from up to two months after is accelerated toward the focal month, and $|H_k| = 5$. For monthly data, one may also consider all the other months; for example, $H_k = \{-6, \ldots, -1, 1, \ldots, 5\}$.

The seasonal diffusion model now becomes

$$\begin{aligned} S_t &= m\Bigg[F(t) - F(t-1) + \sum_{k \in K} \delta_k |H_k|^{-1} \\ &\qquad \cdot \Bigg(D_{kt}^{01} \sum_{h \in H_k} f(t+h) - D_{kt}^{OM} f(t)\Bigg)\Bigg] + \varepsilon_t, \end{aligned} \tag{5}$$

where still $\varepsilon_t \sim \mathrm{N}(0, f(t)^2\sigma^2)$. The first dummy, D_{kt}^{01}, is a 0/1 dummy as used in Equation (3). The second dummy is defined as

$$D_{kt}^{OM} = \begin{cases} 1 & \text{if period } t \text{ influences month } k \text{, that is,} \\ & \quad (\kappa(t) - k) \in H_k, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

The corresponding part of the specification concerns the decrease in sales at time t because of individuals delaying or speeding up their purchase. The first dummy, D_{kt}^{01} in (5), makes sure that the delayed and accelerated sales are added to the sales of the focal month. The summation $\sum_{h \in H_k} f(t+h)$ sums the sales from the months influenced by the focal period. The seasonal effect parameter (δ_k) is divided by the number of elements in H_k—that is, $|H_k|$. This ensures that δ_k has a similar interpretation as in the previous models. More specifically, no matter which other months (hence the superscript OM) influence a seasonal peak, the resulting δ_k parameter is comparable to that in the other models.

An additional advantage of model (5) is that the parameters for all months are identified. In case the monthly effects are not strong, a model with 12 monthly dummies is, of course, not advisable, but the intuition behind our formulation is in this case still preferable.

3.1.2. Flexible Seasonal Structure. Above, we assumed that the pattern of purchase acceleration or postponement is given. Although the model allowed to specify the moments of seasonal peaks through the set K and allowed to specify the origin of seasonal peaks through the sets H_k, $k = 1, \ldots, K$, we assumed that the additional sales in the seasonal peak is proportionally drawn from all months in the set H_k. We will now relax this assumption and propose a method to estimate the pattern of the intertemporal shifts.

We suggest a functional form for the intertemporal demand shift pattern that is flexible and parsimonious. The two main features to capture are (i) the percentage of sales as a result of purchase acceleration versus postponement and (ii) the extent to which specific months contribute to the sales of a focal month.

The specification we propose introduces three additional parameters. The parameter θ, $0 \leq \theta \leq 1$, indicates the relative influence of the months before on the focal month (purchase postponement). The relative influence of the months after the focal month (purchase acceleration) is given by $1 - \theta$. For the period before the focal month, the relative influence of l months prior to the focal month is λ_1^l. For l months after the focal month, the relative influence is λ_2^l. Both parameters are restricted to the $(0, 1]$ interval. Finally, we normalize the weights by considering the relevant months before, denoted by the set H_k^-, and after the seasonal month, denoted by H_k^+. For example, $H_k^- = \{-2, -1\}$ indicates that the two months just before the seasonal peak contribute to the seasonal sales. Note that the setting $\theta = |H_k^-|/(|H_k^-| + |H_k^+|)$ and $\lambda_1 = \lambda_2 = 1$ corresponds to the model specification we had before, that is, equal weights for all months in $H_k = H_k^- \cup H_k^+$. The combination of the weights gives the function

$$g_k^{\text{flex}}(t) = \begin{cases} \theta \dfrac{\lambda_1^{-(\kappa(t)-k)}}{\sum_{h\in H_k^-} \lambda_1^{-h}} & \text{if } t \text{ is before the focal month } k\text{, that is, } (\kappa(t)-k) \in H_k^-, \\ (1-\theta)\dfrac{\lambda_2^{(\kappa(t)-k)}}{\sum_{h\in H_k^+} \lambda_2^{h}} & \text{if } t \text{ is after the focal month } k\text{, that is, } (\kappa(t)-k) \in H_k^+, \\ 0 & \text{otherwise.} \end{cases} \tag{7}$$

This function replaces the dummy D^{OM} in our proposed model (5), and at the same time, we drop the factor $|H_k|$ as this factor is now contained in the definition of $g_k^{\text{flex}}()$. The parameter θ indicates the relative influence of the period before the focal month in determining the seasonal peak. If $\theta = 1$, only the period before the focal month determines the peak. Note that if θ equals 0 or 1, one of the two λ parameters is not identified. The fact that $0 < \lambda_i \leq 1$, $i = 1, 2$, makes sure that the effect for months further from the focal month does not increase. In Figure 2, we show the resulting seasonal pattern for different parameter settings, where we arbitrarily choose the focal month to be December. In these examples, as well as in the empirical section, we include all 11 months before and after this focal month in the set H_k. Note that this does not necessarily mean that all these months influence the focal month, as this depends on the shape of $g_k^{\text{flex}}(t)$. The figure shows that we can deal with a wide variety of patterns. The figure also shows that smaller λ parameters result in more mass in the months close to the focal month.

The model with a flexible seasonal structure is now as follows:

$$S_t = m\Bigg[F(t) - F(t-1) + \sum_{k\in K} \delta_k \Bigg(D_{kt}^{01} \sum_{h\in H_k} g_k^{\text{flex}}(t+h) f(t+h) - g_k^{\text{flex}}(t) f(t)\Bigg)\Bigg] + \varepsilon_t, \tag{8}$$

where, as before, the seasonal peak in the focal month is the summation of "lost" sales in the months in H_k, and where $\varepsilon_t \sim \mathrm{N}(0, f(t)^2\sigma^2)$.

As argued above, for certain combinations of θ, λ_1, and λ_2, the flexible version of our model is equivalent to the fixed version. An advantage of the flexible version over the fixed version is that it can be used for a wider variety of intertemporal demand structures. In particular, the flexible model is helpful if one does not have a well-defined idea about the intertemporal structure. However, a disadvantage is that the flexible model asks more of the data, as one needs to estimate three additional parameters. This might become tedious when there are only a few seasonal cycles in the data set.

Models (5) and (8) can both be applied to any diffusion model with a closed-form solution. In Table 2, we give an overview of possibly relevant diffusion models.

For expository purposes, we use the Bass Model in the rest of the paper. The closed-form solution of the Bass (1969) Model is

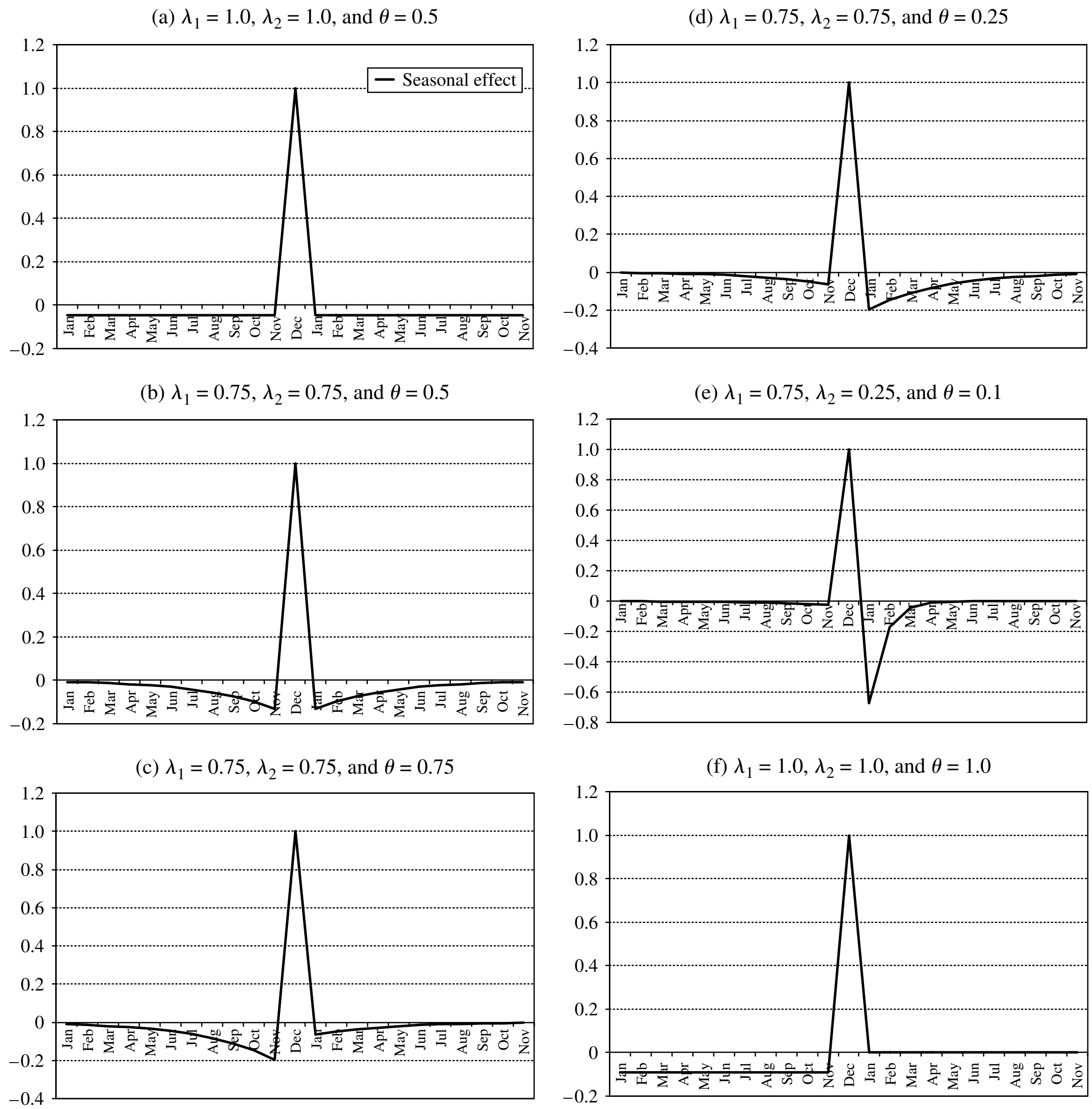
$$F(t) = \frac{1 - \exp\{-(p+q)t\}}{1 + (q/p)\exp\{-(p+q)t\}} \quad \text{and}$$
$$f(t) = \frac{((p+q)^2/p)\exp\{-(p+q)t\}}{(1 + (q/p)\exp\{-(p+q)t\})^2}, \tag{9}$$

where p and q are the traditional Bass parameters capturing innovation and imitation, respectively.

3.2. Alternative Approaches

In this subsection, we briefly discuss some alternative approaches to our model. All alternatives will be compared to the fixed version of our model (5). A comparison with the flexible model is not straightforward, as this model depends on three additional parameters. Note though that all advantages of the fixed version of our model over the alternatives also apply to the flexible version. As alternatives we consider (i) the standard Bass Model without seasonality, (ii) the Bass Model with additive dummies, (iii) the Generalized Bass Model with 0/1 seasonal dummies, and (iv) the Generalized Bass Model with zero-mean seasonal dummies. In this section, we give arguments for the strengths and weaknesses of these alternative approaches. All features of the alternative models discussed in this subsection are validated by a simulation

Figure 2 Several Intertemporal Patterns



Notes. λ_1 determines the pattern before the focal month, λ_2 determines the pattern after the focal month, and θ determines the relative importance of these parts on the focal month. Note that smaller λ parameters result in more mass in the months near the focal month.

study, which we include in an electronic companion (at http://mktsci.journal.informs.org/).

Of course, a first thought may be to ignore seasonality altogether. In general, if seasonality is not properly dealt with, the parameter estimates are biased. However, it turns out that if the full diffusion process is used for parameter estimation, this problem disappears to a large extent for almost all relevant cases. The seasonal fluctuations will then simply be seen as (large) errors. The symmetry in the diffusion curve helps to identify the underlying diffusion curve. Therefore, fitting a basic nonseasonal Bass curve to a completed diffusion series—that is, where sales have become zero at the end—yields sensible results. However, if the diffusion process is before its saturation level and seasonality is not explicitly modeled, parameter estimates are likely to be strongly biased.

One could also extend the Bass Model with additive dummies; that is,

$$S_t = m(F(t) - F(t-1)) + \sum_{k \in K} \delta_k D_{kt}^{01} + \varepsilon_t. \tag{10}$$

However, in such a model, the size of the seasonal effect is the same throughout the diffusion process. Furthermore, seasonal peaks will keep occurring even

Table 2 Diffusion Models with a Closed-Form Solution of the Differential Equation

Model	Differential equation	Closed-form solution	Explanation parameters
Bass (1969)	$\frac{dF}{dt} = (p+qF)(1-F)$	$F = \frac{1-\exp\{-(p+q)t\}}{1+(q/p)\exp\{-(p+q)t\}}$	p = innovation parameter q = imitation parameter
Gompertz curve (Hendry 1972, Dixon 1980)	$\frac{dF}{dt} = qF\ln\left(\frac{1}{F}\right)$	$F = \exp\{-\exp\{-(c+qt)\}\}$	c = constant q = imitation parameter
Mansfield (1961)	$\frac{dF}{dt} = qF(1-F)$	$F = \frac{1}{1+\exp\{-(c+qt)\}}$	c = constant q = imitation parameter
Nelder (1962) (see also McGowan 1986)	$\frac{dF}{dt} = qF(1-F^{\Phi})$	$F = \frac{1}{[1+\Phi\exp\{-(c+qt)\}]^{1/\Phi}}$	c = constant q = imitation parameter Φ = shape parameter
von Bertalanffy (1957) (see also Richards 1959)	$\frac{dF}{dt} = \frac{q}{1-\theta}F^{\theta}(1-F^{\theta})$	$F = [1-\exp\{-(c+qt)\}]^{1/(1-\theta)}$	c = constant q = imitation parameter θ = shape parameter
Stanford Research Institute (e.g., Teotia and Raju 1986)	$\frac{dF}{dt} = \frac{q}{t}F(1-F)$	$F = \frac{1}{1+(T^*/t)^q}$	q = imitation parameter T^* = moment of peak sales
Flexible logistic growth (Bewley and Fiebig 1988)	$\frac{dF}{dt} = q[(1+kt)^{1/k}]^{\mu-k}$	$F = \frac{1}{1+\exp\{-(c+qt(\mu,k))\}}$	c = constant q = imitation parameter $t(\mu,k)$ = based on μ and k; this model has different shapes
Gamma/shifted Gompertz curve (Bemmaor 1994)	—	$F = \frac{1-\exp\{-(p+q)t\}}{[1+(q/p)\exp\{-(p+q)t\}]^{\alpha}}$	p = innovation parameter q = imitation parameter α = shape parameter

Note. This overview is based on earlier overviews of Mahajan et al. (1990, 1993), complemented with the Gamma/shifted Gompertz curve from Bemmaor (1994).

as $t \to \infty$. These two aspects rule out the practical use of this specification. We will therefore only consider models where the seasonality appears in a multiplicative form.

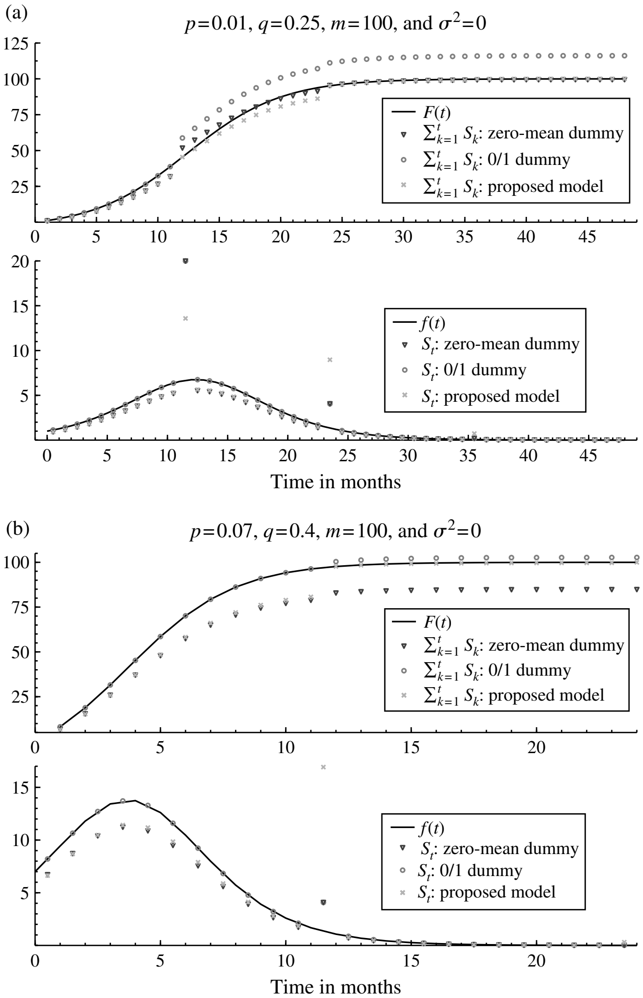
We compare our model with two seasonal alternatives. Both these alternatives are inspired by the Generalized Bass Model (Bass et al. 1994), where, instead of marketing mix variables, we use seasonal dummies as explanatory variables. The first seasonal alternative is the GBM with a 0/1 dummy, which we presented in Equation (2), and which we call the SGBM_{01}. The second seasonal alternative is the GBM model with zero-mean dummies. This model is the same model as the SGBM_{01} but now with a zero-mean dummy (4) instead of the 0/1 dummy. This model we call the SGBM_{ZM}. Both seasonal models assume that seasonal effects are largest around the moment of peak sales.

In Figure 3, we give two examples to illustrate the main differences between our model and the two seasonal GBMs. The top two graphs show the (cumulative) diffusion curves for the case $p = 0.01$, $q = 0.25$, and $m = 100$. The other two graphs correspond to a speedy adoption; that is, $p = 0.07$, $q = 0.4$, and $m = 100$. In both cases, a seasonal peak occurs every 12th period, say, each December. We set $\sigma^2 = 0$. For our model, we specify the seasonal structure such that the additional sales in December are a result of postponed sales in earlier months of the same calendar year. Overall, a model should follow the basic shape that is implied by the Bass parameters. The figures therefore also show both $F(t)$ and $f(t)$.

From Figure 3(a), it is clear that SGBM_{01} is not appropriate. In this model, the sales are equal to $m(F(t) - F(t-1)) \approx mf(t)$, but for each 12th month, the sales are higher. Therefore, the cumulative sales in this model eventually exceed m. In other words, the market potential parameter in this model does not have its natural interpretation. Note that this is a different kind of bias than the "traditional" estimation bias. In fact, if one generates data with the SGBM_{01} and estimates parameters using the SGBM_{01} itself, one will find the true parameter values as in the data-generating process. In this sense, there is no traditional bias. However, the actual ceiling of the cumulative sales will be higher than the market potential estimate, where this estimate should represent the ceiling. Hence, the SGBM_{01} model finds an inappropriate estimate of the market potential.

The difference between the SGBM_{ZM} and our proposed model Figure 3(a) is more subtle. As we assume that the additional sales are a result of postponed sales, the cumulative sales at the end of every last month of the year are exactly equal to $mF(t)$. This is not the case for the SGBM_{ZM}. The seasonal peaks also

Figure 3 **Two Examples of Differences Between the SGBM with 0/1 Dummies, the SGBM with Zero-Mean Dummies, and Our Proposed Model** (OM_{fixed})



differ, which is also because in our model, seasonal peaks are a result of postponed sales. If consumers postpone sales to the seasonal peak, seasonal peaks before the inflection point must be small relative to seasonal peaks after this point. Both curves in the end attain m, and in that sense, the differences between the two models are not extremely large.

Figure 3(b) shows the case where the seasonal peak occurs after the inflection point of the underlying diffusion curve. As the zero-mean dummy is positioned relative to the current adoptions, the resulting seasonal peak in $SGBM_{ZM}$ is too low relative to the postponed sales. This implies that the cumulative sales do not reach m. In this situation, the $SGBM_{ZM}$ will therefore also give misleading parameter estimates.

In Figure 3, we assumed that the seasonal peak only consists of postponed sales. This clearly shows the differences between our proposed model and the $SGBM_{ZM}$. If the seasonal peak originates from consumers delaying *and* speeding up their purchases, the differences between these two models, including the bias in $SGBM_{ZM}$, become smaller. Note that the problem in $SGBM_{01}$ does not depend on the intertemporal pattern.

Summarizing, in this subsection we discussed four alternatives to our proposed seasonal model. The model with an additive dummy clearly is not a good option. Furthermore, the standard Bass Model, ignoring the seasonal fluctuations, gives biased results unless the full diffusion process is used. The graphs in Figure 3 show that the $SGBM_{01}$ is theoretically not appropriate. Furthermore, there are potentially substantive differences between $SGBM_{ZM}$ and our model. However, in some cases, the overall shape of the diffusion process for $SGBM_{ZM}$ closely resembles that generated by our model and the real data.

3.3. Deciding on the Seasonal Structure

For our model, as well as for the seasonal alternatives we consider, one needs to decide on which seasonal dummies to include. On top of that, we need to decide from which months the corresponding months may draw sales, that is, the pattern of the intertemporal demand shifts. In §2 we showed that different countries can have different seasonal patterns. The researcher can use the data to decide on both issues. We now propose guidelines for doing so.

First, note that trying all combinations of dummies and sets H_k for each country is a cumbersome and unnecessary task. Our suggestion is to select a limited number of seasonal structures and then to use information criteria to select the best-fitting one, correcting for the number of parameters. The first step in limiting the number of structures is based on visual inspection. For example, for our data, there seems to be a seasonal peak in January and December. So all seasonal structures should at least include dummies for these months. Next, one sometimes can limit the number of seasonal dummies by excluding some months from further consideration, again based on visual inspection. These two steps will substantially reduce the number of possible models. As a next step, one should estimate the relevant parameters for various seasonal structures. From all these models, some can be rejected, for example, when the estimation routine does not converge or when there are improbable parameter estimates. The final step is to choose the best seasonal structure from the remaining set. For this step, we recommend to use the Bayesian information criterion (BIC), because it penalizes the number of parameters more than other information criteria.

4. Empirical Illustration

In this section, we consider the performance of various models for the actual diffusion data, which we

Table 3 The Moment of Peak Sales in Months (T^*) and Market Potential in Millions per Country (m), as Estimated by the Different Models

	T^*					Market potential (m)				
Country	OM_{fixed}	OM_{flex}	$SGBM_{ZM}$	$SGBM_{01}$	BM	OM_{fixed}	OM_{flex}	$SGBM_{ZM}$	$SGBM_{01}$	BM
United Kingdom	111.48	110.91	111.33	111.39	111.98	49.81	49.23	49.68	40.39	50.90
France	117.87	117.33	117.70	117.74	119.74	38.20	37.79	38.05	31.31	40.94
Germany	117.24	116.72	117.09	117.13	118.35	41.11	40.80	40.98	35.12	43.02
The Netherlands	100.16	99.69	100.02	100.08	100.94	10.00	9.95	9.98	8.22	10.30
Italy	124.02	123.57	123.86	123.89	125.40	37.90	37.56	37.78	32.30	40.17
Spain	114.98	114.50	114.88	114.90	115.96	24.55	24.32	24.50	22.11	25.53
Austria	91.81	91.40	91.64	91.68	92.41	3.96	3.93	3.95	3.40	4.08
Belgium	111.44	111.06	111.34	111.37	112.22	4.60	4.56	4.59	4.04	4.76
Portugal	105.26	104.64	105.07	105.11	106.99	3.92	3.86	3.90	3.28	4.20
Scandinavia	106.50	106.28	106.43	106.46	107.20	11.41	11.38	11.40	9.99	11.66

presented in §2. The data concern the sales figures (in millions) of flat-screen televisions for 10 countries in Europe and cover February 2004 until January 2010 (or in some cases, December 2009). The first months of the diffusion process are not available. This has no major consequences for the estimation procedures, which is one of the benefits of the Srinivasan and Mason (1986) approach. With this approach, it is only necessary to know the number of months since the introduction (Jiang et al. 2006).

The models we compare are the two seasonal GBMs ($SGBM_{ZM}$ and $SGBM_{01}$), the standard Bass Model (BM), and the two versions of our proposed model (to be labeled as OM_{fixed} and OM_{flex}). The OM_{fixed} has the given intertemporal demand shift pattern as in the previous section; that is, $H_k = \{-6, \ldots, -1, 1, \ldots, 5\}$. In OM_{flex} we estimate the intertemporal demand shift pattern using the model in (8). The seasonal structure of each model for each country is selected based on the method described in §3.3.

The results are summarized in Tables 3–5 and in Figure 4. Table 3 gives the estimated moment of peak sales ($T^* = (\log(q/p)/(p+q))$ and market potential based on the different models per country. Table 4 gives the in-sample performance of the models based on information criteria. To be more precise, this table gives the Akaike information criterion (AIC) and BIC values for our flexible model, whereas the other values represent the relative performance of the other models. A positive percentage indicates a higher information criterion and thus a worse in-sample fit. In Figure 4, we give a graphical insight in this in-sample fit for the United Kingdom and the Netherlands. Finally, in Table 5, a more complete overview of the results for the same two countries is shown.[5]

The results in Table 3 show that the Bass Model finds similar moments of peak sales; that is, the difference with the seasonal models is only one or two months. For the market potential, the difference is often between 3% and 7%. If we look more closely at the estimation results for the Netherlands and the United Kingdom (see Table 5), we see that parameters from the Bass Model have larger confidence intervals compared to the seasonal models. Based on in-sample fit (see Table 4), the Bass Diffusion Model is outperformed by the seasonal models by a large margin.

If we now evaluate the seasonal models, we first find that the major difference between the seasonal models concerns the market potential (see Table 3), where our proposed two models and $SGBM_{ZM}$ give a higher estimate than $SGBM_{01}$. This matches with our discussion in §3.2. Another difference is that the seasonal parameters are estimated to be higher for the $SGBM_{01}$, which is partly because this model uses a different (lower) underlying diffusion curve. For OM_{flex}, we have sometimes selected a slightly different set of seasonal effects. The latter is partly explained by the difference across the models in the months influenced by a seasonal peak. To get a similar seasonal peak, the magnitude of the seasonal effect can be slightly different in OM_{flex} versus OM_{fixed}.

Based on the in-sample performance (see Table 4), our flexible model outperforms the other seasonal models in 8 of the 10 countries, as the other models have higher values of the information criteria. For the other three seasonal models, this fit is similar. Figure 4 shows that the seasonal models fit the actual in-sample data well. The standard Bass Model only manages to roughly fit the underlying curve. The fitted curve matches neither the seasonal months nor the nonseasonal months.

Table 5 and Figure 4 also show that seasonal patterns differ across countries. These differences are present in the seasonal components included in the model, as well as in the levels of the seasonal parameters. For example, the Netherlands have

[5] The tables and figures of the other eight countries are excluded from the paper but can be obtained from the authors upon request.

Table 4 In-Sample Performance (AIC and BIC) of Diffusion Models for Flat-Screen Televisions in Europe

	OM_{flex} (Absolute)		OM_{fixed}		$SGBM_{ZM}$		$SGBM_{01}$		BM	
Country	AIC	BIC	AIC (%)	BIC (%)	AIC (%)	BIC (%)	AIC (%)	BIC (%)	AIC (%)	BIC (%)
United Kingdom	−230.7	−221.7	0.5	0.5	0.5	0.5	0.4	0.5	57.5	58.8
France	−272.8	−263.7	0.2	1.0	1.2	1.2	1.2	1.2	42.0	42.6
Germany	−252.5	−241.1	−1.3	−2.3	−1.4	−2.4	−1.4	−2.4	29.4	28.9
The Netherlands	−492.4	−476.5	0.9	1.0	0.8	0.8	0.7	0.7	20.3	19.0
Italy	−280.0	−270.9	0.7	0.7	0.6	0.6	0.6	0.6	29.1	29.3
Spain	−298.7	−291.9	1.7	1.8	1.6	1.6	1.6	1.6	16.4	16.8
Austria	−604.4	−590.9	−0.2	−1.0	−0.3	−1.1	−0.3	−1.1	16.3	15.5
Belgium	−522.1	−515.3	1.1	1.1	0.9	0.9	0.9	0.9	12.4	12.5
Portugal	−592.2	−583.2	0.3	0.7	0.2	0.5	0.2	0.5	16.6	16.5
Scandinavia	−343.4	−338.9	1.2	1.9	1.2	1.9	1.2	1.9	17.8	18.7

Notes. The results for our proposed model with flexible intertemporal demand (OM_{flex}) are the absolute values of the information criteria; for the other models, the results are given relative to the flexible model. A positive percentage represents a higher AIC or BIC. For example, the in-sample performance for the United Kingdom is best for the flexible model, because the information criteria are higher for the other models.

many seasonal fluctuations compared to the United Kingdom—respectively, seven and four peaks. However, the peaks in the Netherlands are relatively small compared to the United Kingdom. The seasonal models can clearly capture these different seasonal patterns across countries. Additionally, from Figure 4, it seems that the flexible model we propose captures the periods between the peaks better than the other seasonal models.

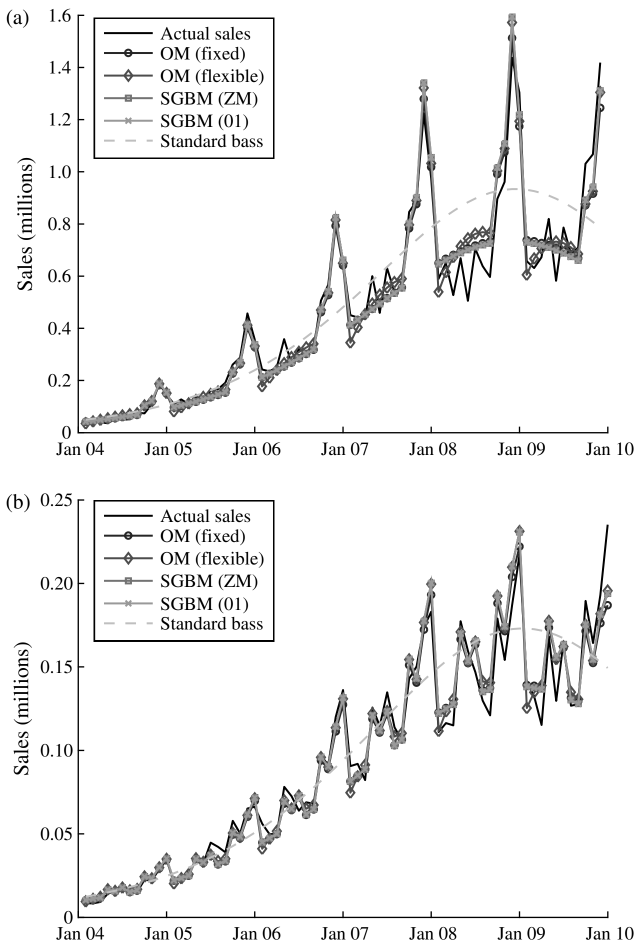
Furthermore, there are differences in the intertemporal demand shift parameters (mainly λ_2). In comparing the seasonal components, we note that the selected seasonal dummies can differ across the models we considered. However, for the United Kingdom

Table 5 Results of the Diffusion Model for Flat-Screen Televisions in the United Kingdom and the Netherlands

	United Kingdom					The Netherlands				
	OM_{fixed}	OM_{flex}	$SGBM_{ZM}$	$SGBM_{01}$	BM	OM_{fixed}	OM_{flex}	$SGBM_{ZM}$	$SGBM_{01}$	BM
p	2.2E−05	2.2E−05	2.2E−05	2.2E−05	2.0E−05	7.9E−05	8.1E−05	8.1E−05	8.0E−05	7.6E−05
	(2.1E−06)	(2.2E−06)	(2.3E−06)	(2.3E−06)	(5.1E−06)	(5.1E−06)	(5.9E−06)	(5.4E−06)	(5.4E−06)	(1.0E−05)
q	0.073	0.073	0.073	0.073	0.073	0.067	0.067	0.067	0.067	0.067
	(0.001)	(0.002)	(0.001)	(0.001)	(0.004)	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)
m	49.81	49.23	49.68	40.39	50.90	10.00	9.95	9.98	8.22	10.30
	(1.49)	(5.12)	(1.34)	(1.30)	(3.94)	(0.26)	(0.94)	(0.23)	(0.25)	(0.54)
Jan.	0.48	0.61	0.50	0.67	—	0.49	0.56	0.50	0.67	—
	(0.06)	(0.21)	(0.06)	(0.08)	—	(0.05)	(0.21)	(0.05)	(0.06)	—
May	—	—	—	—	—	0.23	0.20	0.23	0.31	—
	—	—	—	—	—	(0.05)	(0.06)	(0.04)	(0.06)	—
June	—	—	—	—	—	0.12	0.09	0.12	0.16	—
	—	—	—	—	—	(0.04)	(0.05)	(0.04)	(0.06)	—
July	—	—	—	—	—	0.18	0.13	0.18	0.24	—
	—	—	—	—	—	(0.04)	(0.06)	(0.04)	(0.06)	—
Oct.	0.28	0.26	0.29	0.39	—	0.30	0.29	0.30	0.40	—
	(0.06)	(0.10)	(0.05)	(0.07)	—	(0.05)	(0.12)	(0.04)	(0.06)	—
Nov.	0.37	0.39	0.38	0.51	—	0.19	0.23	0.20	0.26	—
	(0.06)	(0.12)	(0.05)	(0.07)	—	(0.05)	(0.12)	(0.04)	(0.06)	—
Dec.	0.86	0.94	0.87	1.17	—	0.39	0.44	0.39	0.52	—
	(0.07)	(0.26)	(0.06)	(0.08)	—	(0.05)	(0.16)	(0.04)	(0.06)	—
λ_1	—	0.73	—	—	—	—	0.70	—	—	—
	—	(0.10)	—	—	—	—	(0.12)	—	—	—
λ_2	—	0.78	—	—	—	—	0.58	—	—	—
	—	(0.12)	—	—	—	—	(0.12)	—	—	—
θ	—	0.68	—	—	—	—	0.69	—	—	—
	—	(0.26)	—	—	—	—	(0.16)	—	—	—
RMSE	0.069	0.072	0.069	0.069	0.189	0.010	0.011	0.010	0.010	0.024
AIC	−229.6	−230.7	−229.7	-229.7	−98.1	−487.9	−492.4	−488.7	−488.9	−392.7
BIC	−220.6	−221.7	−220.6	−220.7	−91.4	−471.9	−476.5	−472.8	−472.9	−385.8

Notes. Standard errors are in parentheses. RMSE, root mean square error.

Figure 4 **The In-Sample Fit for Two Countries: (a) United Kingdom and (b) the Netherlands**



and the Netherlands, the optimal structure was similar across models. Concerning the shape of the intertemporal demand shift, the λ parameters are between 0.57 and 0.85 for all countries. This means that the effect on the focal month becomes negligible after five to eight months from the focal month. Furthermore, θ across countries is between 0.67 and 0.76, which means that more consumers postpone than speed up their purchase, which is an interesting and managerially relevant insight.

In Table 6, the models are compared on their out-of-sample performance relative to our flexible model. In this table, we compare the root mean squared prediction error (RMSPE) for up to three or six months ahead out-of-sample forecasts. The table gives the RMSPE for our flexible model. The values for the other models are relative to this model, where a higher percentage represents a higher RMSPE and thus worse out-of-sample performance. To obtain these RMSPE values, we reestimated the models, after leaving out the last three or six months of the sales figures. From Table 6, we find that in terms of out-of-sample forecasting, the seasonal models OM_{fixed}, SGBM_{ZM}, and SGBM_{01} perform almost equally well. This suggests that for short-term prediction, all seasonal models can be useful. The Bass Model is outperformed by all seasonal models.

Note that the seasonal models, including our model, do not solve all issues common to diffusion models. In particular, if the moment of peak sales is not in the data, the predictions are downward biased. However, by distinguishing the seasonal fluctuations from the random fluctuations, the seasonal models do outperform the standard diffusion models.

Our flexible model outperforms most models on the three-month-ahead prediction horizon. However, for six-month-ahead forecasting, our flexible model performs worse, as it only outperforms the other seasonal models in 5 out of 10 cases. The reason for this might be that for this longer horizon, there are less data left for estimation, and our flexible model by definition asks more from the data. This is supported by the fact that in these cases, the flexible model also performs worse in-sample. This shows that we can only accurately estimate the intertemporal pattern if there

Table 6 **Forecasting Performance of Diffusion Models for Flat-Screen Televisions in Europe, for up to Three-Month- or Six-Month-Ahead Forecasts**

	OM_{flex} (Absolute)		OM_{fixed}		SGBM_{ZM}		SGBM_{01}		BM	
Country	Three	Six	Three (%)	Six (%)	Three (%)	Six (%)	Three (%)	Six (%)	Three (%)	Six (%)
United Kingdom	26.18	23.98	12.26	17.54	9.98	16.33	8.66	17.30	139.95	93.10
France	41.36	29.38	−34.10	−3.81	−35.06	−5.83	−35.37	−6.23	31.84	51.93
Germany	18.44	22.08	10.59	3.40	6.14	1.44	6.65	0.82	132.82	69.79
The Netherlands	4.19	4.43	17.98	8.00	16.82	7.75	15.52	8.06	89.95	53.61
Italy	20.21	32.01	10.46	−0.94	8.99	−1.89	8.27	−1.97	98.99	31.62
Spain	21.11	19.74	5.97	−7.06	5.05	−8.76	5.49	−7.89	44.26	22.23
Austria	2.33	2.04	13.65	14.91	10.19	13.78	9.88	14.23	107.51	75.13
Belgium	5.10	4.23	−10.85	−5.35	−12.13	−5.15	−11.66	−5.45	25.80	16.62
Portugal	2.51	2.67	9.93	12.59	4.17	11.61	4.83	12.44	117.84	59.43
Scandinavia	8.93	11.09	7.97	−0.30	7.20	−0.49	7.53	−0.56	50.22	17.12

Notes. The results for the proposed with flexible intertemporal demand (OM_{flex}) are the absolute RMSPEs; for the other models, the results are given relative to the flexible model. A positive percentage represents a higher RMSPE. For example, the out-of-sample performance for three months ahead in the United Kingdom are best for the flexible model, because all other models have a higher RMSPE. The RMSPEs are multiplied by 100 for convenience.

are enough data. The results show that in these cases, our model with a fixed pattern is more suitable, as it outperforms the other models on fit.

Next to the increased fit and the better short-term forecasting, our model also allows to study the seasonal structure across countries. Of course, the quality of generalizing conclusions is limited by the number of countries in our empirical case. To give an example of differences across countries, we compare the seasonal peaks in December and January. In the Netherlands and Belgium, the seasonal peaks are higher in January compared to December, where the reverse is true for the United Kingdom and other countries. This relatively higher January peak in the Netherlands and Belgium is probably because in the Netherlands and Belgium, the prices of flat-screen televisions are often reduced in January, as product line extensions are often introduced in February. Another likely factor to influence seasonal peaks is the timing of additional income, such as new year's or vacation bonus, as consumers often use these for large expenditures such as a flat-screen television.

5. Conclusion

In this paper, we addressed seasonality in diffusion models. The current availability of high-frequency data makes this a subject of increasing importance. Using high-frequency diffusion data can prove to be very helpful in academic research and in practice. This has actually been acknowledged in the literature, both implicitly and explicitly. To incorporate seasonality in diffusion models, we developed a seasonal structure that can be used in combination with standard diffusion models. We based our models on the classic Bass diffusion model using monthly data, but our seasonal structure works with any closed-form diffusion model. Also, extensions of diffusion models can be used in combination with our method (for example, cross-country diffusion models, generational diffusion models, and multilevel diffusion models), which shows the relevance of this paper for future academic research. Concerning the data frequency, our method works for any interval as long as there is periodicity in the peaks. Furthermore, because estimated diffusion parameters are often used for the practice of "guessing by analogy," it is important that the seasonal component in the model does not influence the estimates and interpretation of the underlying diffusion pattern.

Through a detailed empirical case, we showed that our proposed model lives up to all these goals. In contrast, the use of the Generalized Bass Model with seasonal dummies, which may seem a straightforward way to take seasonality into account, produces biased estimates. In particular, the market potential is biased. Next to our model, we also have put forward a variation of this Generalized Bass Model, which uses a zero-mean dummy. This variation seems to give results similar to our model in most practical cases, with the additional benefit that it can be used more straightforwardly in standard statistical software packages.

Our model allows to estimate the seasonal pattern. The basic premise is that the seasonal peak in a focal month consists of sales drawn from the months around it. In our proposed model, we made a distinction between a flexible, estimated, intertemporal demand pattern and a fixed pattern. The flexible pattern allowed to study intertemporal demand shifts, and in most cases, the corresponding model outperformed the other models on in-sample fit and short-term forecasting. However, with less data the flexible pattern is less suitable, and in these cases, it is safer to set a fixed pattern. In this paper, we showed that the given pattern, six months before the focal months and five months after influence a seasonal peak, works well for the empirical cases in our paper, but in practice, other underlying structures are possible as well. For example, it may be true that a focal month is influenced only by the months in the quarter surrounding it. Such structures are all possible in our setup.

An additional advantage of our proposed model is that it can be used to give managers a tool to handle practical challenges concerning seasonal fluctuations, such as inventory management. Also, the shape parameter (λ) and balance parameter (θ) of the intertemporal pattern hold useful information. In our set of countries, however, the differences are relatively small. In future research, it would be interesting to compare more countries and products.

It is obvious that ignoring seasonality is not suitable for estimating and predicting seasonal peaks. However, for the estimation of the basic diffusion parameters the current practice is often to ignore seasonality. In this paper, we found the reassuring result that if the completed diffusion series is available, this practice indeed finds the underlying diffusion pattern. However, if the series is truncated, this does not hold anymore.

The implications for future empirical analysis of seasonal diffusion data are threefold. First, if the goal is to find a model that can be used for short-term forecasting, all three seasonal models described in this paper can be used. Second, if the interest is only to elicit the underlying diffusion, ignoring seasonality seems tempting. However, this only works for completed diffusion series. The seasonal GBM with 0/1 dummies definitely does not work. Finally, if the interest lies with both the correct estimation as well as short-term forecasting, our model is the way to go.

Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mktsci.journal.informs.org/.

Acknowledgments

This paper was written while the first author was affiliated with the Erasmus University of Rotterdam. The authors thank the editor, an area editor, and two anonymous referees for their detailed and helpful comments.

References

Bass, F. M. 1969. A new product growth model for consumer durables. *Management Sci.* **15**(5) 215–227.

Bass, F. M., T. V. Krishnan, D. C. Jain. 1994. Why the Bass Model fits without decision variables. *Marketing Sci.* **13**(3) 203–223.

Bemmaor, A. C. 1994. Modeling the diffusion of new durable goods: Word-of-mouth effect versus consumer heterogeneity. G. Laurent, G. L. Lilien, B. Pras, eds. *Research Traditions in Marketing*. Kluwer Academic Publishers, Boston, 201–223.

Bemmaor, A. C., J. Lee. 2002. The impact of heterogeneity and ill-conditioning on diffusion model parameter estimates. *Marketing Sci.* **21**(2) 209–220.

Bewley, R., D. G. Fiebig. 1988. A flexible logistic growth model with applications in telecommunications. *Internat. J. Forecasting* **4**(2) 177–192.

Boswijk, H. P., P. H. Franses. 2005. On the econometrics of the Bass diffusion model. *J. Bus. Econom. Statist.* **25**(3) 255–268.

Dixon, R. 1980. Hybrid corn revisited. *Econometrica* **48**(6) 1451–1461.

Einav, L. 2007. Seasonality in the U.S. motion picture industry. *RAND J. Econom.* **38**(1) 127–145.

Fok, D., P. H. Franses, R. Paap. 2007. Seasonality and non-linear price effects in scanner-data-based market-response models. *J. Econometrics* **138**(1) 231–251.

Hendry, I. C. 1972. The three-parameter approach to long range forecasting. *Long Range Planning* **5**(1) 40–45.

Jiang, Z., F. M. Bass, P. I. Bass. 2006. Virtual Bass model and the left-hand data-truncation bias in diffusion of innovation studies. *Internat. J. Res. Marketing* **23**(1) 93–106.

Lilien, G. L., A. Rangaswamy, C. Van den Bulte. 2000. Diffusion models: Managerial applications and software. V. Mahajan, E. Muller, Y. Wind, eds. *New-Product Diffusion Models*. Springer-Verlag, New York, 295–332.

Mahajan, V., E. Muller, F. M. Bass. 1990. New product diffusion models in marketing: A review and directions for research. *J. Marketing* **54**(1) 1–26.

Mahajan, V., S. Sharma, R. D. Buzzell. 1993. Assessing the impact of competitive entry on market expansion and incumbent sales. *J. Marketing* **57**(3) 39–52.

Mansfield, E. 1961. Technical change and the rate of imitation. *Econometrica* **29**(4) 741–766.

McGowan, I. 1986. The use of growth curves in forecasting market development. *J. Forecasting* **5**(1) 69–71.

Nelder, J. A. 1962. An alternative form of generalized logistic equation. *Biometrics* **18**(4) 614–616.

Non, M., P. H. Franses, C. Laheij, T. Rokers. 2003. Yet another look at temporal aggregation in diffusion models of first-time purchase. *Tech. Forecasting Soc. Change* **70**(5) 467–471.

Ofek, E. 2005. Forecasting the adoption of a new product. HBS Case 9-505-062, Harvard Business School, Boston.

Putsis, W. P., Jr. 1996. Temporal aggregation in diffusion models of first-time purchase: Does choice of frequency matter? *Tech. Forecasting Soc. Change* **51**(3) 265–279.

Radas, S., S. M. Shugan. 1998. Seasonal marketing and timing new product introduction. *J. Marketing Res.* **35**(3) 296–315.

Richards, F. J. 1959. A flexible growth function for empirical use. *J. Experiment. Botany* **10**(2) 290–300.

Shugan, S. M., S. Radas. 1999. Services and seasonal demand. T. A. Swartz, D. Iacobucci, eds. *Handbook of Services, Marketing and Management*. Sage Publications, London, 147–170.

Srinivasan, V., C. H. Mason. 1986. Nonlinear least squares estimation of new product diffusion models. *Marketing Sci.* **5**(2) 169–178.

Teotia, A. P. S., P. S. Raju. 1986. Forecasting the market penetration of new technologies using a combination of economic costs and diffusion models. *J. Product Innovation Management* **3**(4) 225–237.

Van den Bulte, C., G. L. Lilien. 1997. Bias and systematic change in the parameter estimates of macro-level diffusion models. *Marketing Sci.* **16**(4) 338–353.

Venkatesan, R., T. V. Krishnan, V. Kumar. 2004. Evolutionary estimation of macro-level diffusion models using genetic algorithms: An alternative to nonlinear least squares. *Marketing Sci.* **23**(3) 451–464.

Von Bertalanffy, L. 1957. Quantitative laws in metabolism and growth. *Quart. Rev. Biol.* **32**(3) 217–231.