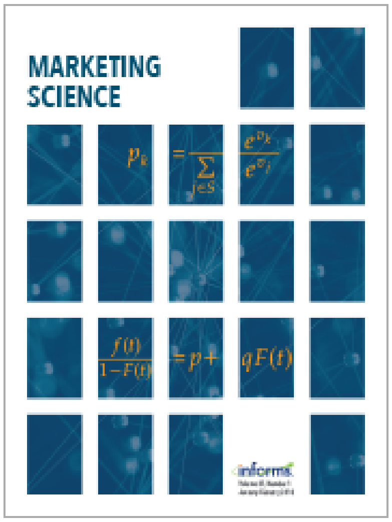


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# Robust New Product Pricing

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We study the pricing decision for a monopolist launching a new innovation. At the time of launch, we assume that the monopolist has incomplete information about the true demand curve. Despite the lack of objective information the firm must set a retail price to maximize total profits. To model this environment, we develop a novel two-period non-Bayesian framework where the monopolist sets the price in each period based only on a *nonparametric set of all feasible demand curves*. Optimal prices are dynamic as prices in any period allow the firm to learn about demand and improve future pricing decisions. Our main results show that the direction of dynamic introductory prices (versus static prices) depends on the type of heterogeneity in the market. We find that (1) when consumers have homogeneous preferences, introductory dynamic price is higher than the static price; (2) when consumers have heterogeneous preferences and the monopolist has no ex ante information, the introductory dynamic price is the same as the static price; and (3) when consumers have heterogeneous preferences and the monopolist has ex ante information, the introductory dynamic price is lower than the static price. Furthermore, the degree of this initial reduction increases with the amount of heterogeneity in the ex ante information.

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*A Bayesian analysis may be 'rational' in the weak axiomatic sense, yet be terrible in a practical sense if an inappropriate prior distribution is used.*
—Berger (1985) (Statistical Decision Theory and Bayesian Analysis, p. 121)

## 1. Introduction

### 1.1. Overview

Consider the pricing decision of a monopolist launching a new nonstorable product or technology with unit per-period demand. To understand the demand curve, the manager conducts market research. This research could be in the form of concept testing (Schwartz 1987), experiments (e.g., Green and Srinivasan 1978) or surveys (Dolan 1993) to assess consumer valuations. Such data provides the manager with some information about the underlying demand curve for their new product. However, it is unrealistic for the manager to have complete information about the demand curve at the time of launch (e.g., Lodish 1980, Besbes and Zeevi 2009, Kahn 2010, Harrison et al. 2012). In this paper we investigate how a monopolist should set new product prices with limited pre-launch information.

This question relates to a large literature that studies new product introduction. Most papers in this literature assume that firms have complete information about the underlying demand curve (e.g., Robinson and Lakhani 1975, Wernerfelt 1986, Liu and Zhang 2013; please see the online appendix (available as supplemental material at http://dx.doi.org/10.1287/mksc.2015.0914) for a more complete list of papers), and derive the optimal firm policy.[1] Researchers have recognized that firms are uncertain about the underlying demand curve for new products (e.g., Rothschild 1974, Lodish 1980, Braden and Oren 1994, Desai et al. 2007, Bonatti 2011, Hitsch 2006, Biyalogorsky and Koenigsberg 2014; please see the online appendix for a more complete list of papers). However, in solving for optimal firm policy, these papers assume that the firm has a prior (represented as full knowledge or a prior distribution) over the uncertainty in the demand curve. For example, Desai et al. (2007) and Biyalogorsky and Koenigsberg (2014) assume that demand can be in one of two states and that firms know precisely the probability of each state. Alternatively, Braden and Oren (1994) and Hitsch (2006)

[1] In this literature the dynamics in new product pricing can be optimal due to dynamics in demand (e.g., evolving consumer preferences and preference heterogeneity) or supply (e.g., inventory concerns and competition).

assume that firms have a prior over the possible uncertainty. With this information, the manager can use Bayesian decision theory to choose the policy that maximizes expected profits.

The practical reliability of a Bayesian decision theory analysis depends critically on prior assumption used (Manski 2005, Berger 1985). A critical input to new product pricing decisions is an accurate demand forecast. Survey studies that have tracked the accuracy of forecasts suggest very large forecasting errors for new products and technologies. Gartner and Thomas (1993) report new product forecast errors vary from −2,900% to +1,500% with a mean of −46.9%; and Kahn (2002) report a forecast accuracy of 40% for new to market products. This suggests that firms may not always have appropriate priors. The literature on judgment and decision making suggests that one reason for this is that managers may have behavioral biases when selecting priors for new products (Tyebjee 1987, Forlani et al. 2002, Bolton 2003, Bolton and Reed 2004, Schwartz and Cohen 2004, Lawrence et al. 2006). Kahn (2010, p. 184) summarizes "it is very unlikely that new-product forecasting will be free of all biases."

Motivated by the quote by Berger (1985) to start this paper, we develop and investigate a novel dynamic non-Bayesian pricing methodology. Despite the lack of objective information the firm must set a retail price to maximize total profits. To achieve this the retail price must (i) consider current profits, and (ii) allow the firm to learn about demand to extract higher profits in the future (Rothschild 1974, Grossman et al. 1977, Mirman et al. 1993). This is more generally labeled as learning by doing (Arrow 1962). Our framework is robust in the sense that the monopolist's price does not depend on subjective information. Instead, at each point in time the monopolist bases her pricing decision only on the *set* of all feasible demand curves.[2] This kind of uncertainty is also known as *Knightian uncertainty* (Knight 2012) or *ambiguity*. Our proposed methodology suggests prices that can be used by firms who may be unwilling to make a prior assumption or, alternatively, that can provide managers with information about the implications of subjective assumptions on pricing decisions.

We present a two-period model where the monopolist prices to a unit mass of consumers. Within this framework, we assume that each consumer has unit demand with a constant product valuation over time. We consider two environments where consumers have homogeneous or heterogeneous preferences. We assume that, at the time of purchase, the monopolist does not have the information to price discriminate and set a single price in each period. For example, consider pricing at a retail store. These assumptions imply that consumers are *nonstrategic* in the sense that they do not have an incentive to misrepresent their valuation to obtain lower future prices. Of particular importance, if the monopolist has full information about consumer valuations, these demand assumptions would not predict dynamic prices. For the heterogeneous preference model, we make one additional distinction as to ex ante information available to the firm. We assume that firms have access to some known consumer characteristics (e.g., segmentation, location, age, or income) based on which they can group consumers. Furthermore, based on the prelaunch market research, firms have partial preference (i.e., ex ante) information for each group.[3] Aggregating this information across groups of consumers allows the firm to partially identify the set of feasible demand curves (see Handel et al. 2013 for econometric identification and estimation methodology).

### 1.2. Robust Pricing

To study the firm's pricing decisions under ambiguity we develop a novel dynamic non-Bayesian framework that simultaneously considers current profitability and the value of learning. The monopolist's objective function is to maximize aggregate profits. However, she only observes the *set* of all feasible demand curves. Without a subjective prior, the manager cannot integrate over this set to calculate the maximum expected profits. Instead, we assume that the monopolist selects a price in each period using *dynamic minimax regret*, a decision criterion that compares prices based strictly on the set of feasible demand curves. This dynamic decision rule is based on the minimax regret criterion (introduced by Wald 1950) and has been axiomatized by Milnor (1954) and Stoye (2011).[4] We discuss alternatives to the dynamic minimax regret criterion in §3.2.4.

Minimax regret has been used to characterize robust decision making in a variety of social choice settings (see, e.g., Manski 2005) as well as in firm decisions (see Bergemann and Schlag 2008, 2011; Handel et al. 2013 in economics; Perakis and Roels 2010, Ball and Queyranne 2009, Besbes and Zeevi 2009, 2011 in operations). In the operations literature, Perakis and Roels (2010) find that the minimax regret approach

[2] For example, if the information available to the firm is that 50% of consumer purchase at \$5 and 10% purchase at \$10, then any downward sloping demand curve that is consistent with these data is feasible.

[3] Our notion of observable ex ante heterogeneity permits a range of pricing environments. Specifically, if all consumers are individually identifiable our setting is equivalent to one in which the firm has panel data.

[4] As summarized by Schlag (2006, p. 3) "Minimax Regret is the unique criterion that satisfies Ordering, Symmetry, Strong Domination, Continuity, Column Duplication, Convexity, INABA [IIA] and S [Strategic] Independence."

outperforms traditional heuristics used in the literature. Moreover, the notion of minimax regret is the foundational building block of popular computer science models such as multiarm bandit problems and machine learning (see Lai 1987, Bubeck and Cesa-Bianchi 2012). Minimax regret centers around the notion of *regret* defined in our environment as the profits foregone by the monopolist from not charging the optimal price for the true demand curve. This notion of regret from the statistical decision literature (e.g., Savage 1951) is completely distinct from the notion of regret discussed in the psychology literature (Janis and Mann 1977). Specifically, in the absence of ambiguity, the objective function under minimax regret is exactly the same as the standard expected profit maximization.

Our research objective in dynamic robust pricing is similar to that considered in operations literature (Besbes and Zeevi 2009) and computer science literature (Kleinberg and Leighton 2003). Both areas consider a continuous time model where consumers enter with a random (Poisson) rate wherein firms can continuously change prices. The proposed solution in operations (Besbes and Zeevi 2009) is an algorithm with an experimental stage where the firm can learn about demand and then use minimax regret to select a price. Here the experimental stage is divorced from the profit stage, whereas in our set-up, we consider the potential profit from the learning stages. In the computer science literature, Kleinberg and Leighton (2003) propose a multiarm bandit solution (i.e., parametric approximations to the minimax regret problem) and show that such an algorithm will converge to the first-best solution in finite time.

### 1.3. Contributions

To our knowledge, this is the first work on dynamic pricing from microeconomic foundations that accounts for both (i) setting prices without subjective Bayesian information, and (ii) learning about demand through pricing. Our main results show that the monopolist can offer a lower, unchanged or higher introductory price in a dynamic environment (as compared to a static environment) depending on the type of heterogeneity in the market. We find that (1) when consumers have homogeneous preferences, the introductory dynamic price is higher than the static price; (2) when consumers have heterogeneous preferences and the monopolist has no ex ante information, the introductory dynamic price is the same as the static price; and (3) when consumers have heterogeneous preferences and the monopolist has ex ante information, the introductory dynamic price is lower than the static price. Furthermore, the degree of this initial reduction increases with the amount of heterogeneity in the ex ante information. The difference in results between the homogeneous and heterogeneous preference models is driven by the fact that the homogeneous preference model restricts the set of feasible demand functions to mass points. Here increasing initial price is attractive as consumer valuations must be above or below the higher price. If we increase price and consumers do not purchase, then reducing future prices significantly decreases future regret. In the heterogeneous preference model, this is not the case; a worst-case demand would have consumers with valuations above and below the first-period price. Moreover, under the worst-case demand consumers who do not purchase will have lower ex ante valuations than consumers who purchase. Here decreasing price allows the firm to bound the maximum regret from these ex ante lower value consumers and potentially target the ex ante higher valuation consumers with an increasing price in period 2. This result depends critically on ex ante information. When the firm has no ex ante information, lowering price is no longer valuable.

The remainder of the paper is organized as follows. Section 2 describes the static model of robust firm pricing. Section 3 presents our main results for the dynamic model of robust firm pricing. Section 4 summarizes this work, discusses its limitations, and suggests avenues for future research.

## 2. Static Monopoly Pricing Under Ambiguity

Before studying the monopolist's dynamic decision problem, we review the benchmark static model for monopoly pricing under ambiguity. This section closely follows the previous work of Bergemann and Schlag (2008) and Bergemann and Schlag (2011) on robust monopoly pricing without a subjective prior.[5]

We begin with an overview of minimax regret in a static setting. We then solve for the optimal static minimax regret prices in an environment where consumers are homogeneous in their valuations. We then consider a model where consumer preferences are heterogeneous. Here we will also allow for heterogeneity in ex ante information for the firm, whereby firms can have different ex ante partially identified preferences for consumers.

[5] Bergemann and Schlag (2008, 2011) study a case wherein one consumer is drawn from any feasible probability distribution of consumers over a known support of potential valuations. The monopolist has no subjective information concerning the relative likelihood of these feasible distributions, and selects a random pricing rule to minimize her maximum expected regret over this space of uncertainty. We present a simplified version of this model, where the simplification results in a deterministic pricing rule.

### 2.1. Overview of Static Minimax Regret

In a static setting where the monopolist knows that true distribution of consumer valuations $F$ lies within a set of feasible demand curves $\Psi$, regret for any chosen price $p$ is defined with respect to each $F \in \Psi$ as

$$R(p, F) = \text{First Best Profit}(F) - \text{Actual Profit}(p, F).$$

Under the First Best Profits, the firm has complete information and will charge each consumer her valuation.[6] Regret here can be interpreted as the measure of the consumer surplus that the firm is unable to capture. For any $(p, F)$ this regret will result from overpricing (underpricing) corresponding to the consumers with valuations below (above) $p$. In the Bayesian set-up, this overpricing and underpricing for each $(p, F)$ pair is weighted by a subjective Bayesian prior over $\Psi$. Regret minimization with respect to this weighting is equivalent to expected profit maximization. With no Bayesian prior to weight the space of feasible demand curves, minimax regret evaluates each possible price by its *maximum* regret over the set $\Psi$. For any $p$, maximum regret occurs at a given $F_{wc}(p) \in \Psi$ where actual profits under $p$ are farthest away from the first-best profits under $F_{wc}(p)$. Intuitively, maximum regret is the worst possible case of foregone profits over the set of feasible demand curves, given $p$. After maximum regret is determined for each price, the monopolist solves her problem by selecting the price that minimizes this maximum regret. Thus, minimax regret trades off losses from overpricing with losses from underpricing in a manner that is robust to subjective uncertainty over the set of feasible demand curves. Under the interpretation that regret represents the consumer surplus the firm is unable to capture (or lost consumer surplus), we can interpret the minimax regret price as that which minimizes the lost surplus for any distributions of valuations.

### 2.2. Model Setup and Key Informational Assumptions

In our model, we assume that the monopolist sets prices to maximize profits for a zero marginal cost product. In this section we discuss the information sets and decision variables we assume for consumers and the monopolist.

The monopolist faces a unit mass of consumers with unit per period demand. We assume that each consumer has a stable valuation $v_i$ which is known privately only to the consumer. In any purchase occasion, the consumer observes the price set by the firm $p$ and derives a utility $u_i = v_i - p$. She decides to purchase the good if $u_i \geq 0$ or equivalently if $v_i \geq p$.

We assume that the firm conducts pre-launch market research that provides *partial information about* $v_i$. In particular, the only information available to the firm is that $v_i$ lies between $v_{iL}$ and $v_{iH}$, i.e., the firm knows $v_{iL}$ and $v_{iH}$ and has no prior information about the likelihoods of valuations in this range. In a related paper, Handel et al. (2013) provide an econometric framework that the monopolist can use to estimate $v_{iL}$ and $v_{iH}$ from pre-launch market research data. For notational convenience, we assume that $v_{iL} = v_L < v_{iH}/4$ for all consumers.[7] Therefore, the information available to the firm about consumer valuations can be summarized as $v_i \in [v_L, v_{iH}] \equiv \delta_i$ for every consumer $i$. The firm can aggregate these $\delta_i$ across consumers to define the set of possible distribution of consumer valuations. We allow for fully nonparametric heterogeneity in preferences across consumers. For example, consider a case where the firm is facing two consumers ($A$ and $B$) and knows $v_A \in [v_L, v_{AH}]$ and $v_B \in [v_L, v_{BH}]$. The set of feasible aggregate valuations is defined as $\{(v_1, v_2) \mid v_1 \in [v_L, v_{AH}], v_2 \in [v_L, v_{BH}]\}$.

The type of uncertainty where the decision maker does not have information to place a subjective distribution over the support of possible outcomes is defined as *ambiguity*. Under ambiguity, it is impossible to calculate expected profits. Instead, the monopolist must make a decision based strictly on knowledge of the support of potential valuations. In this paper, we derive the optimal prices assuming that the firm will use the minimax regret criterion. Here, regret is a statistical characterization of the trade-off between potential losses from overpricing and potential losses from underpricing.

### 2.3. Static Minimax Regret with Homogeneous Preferences

In this section, we assume that all consumers are homogeneous in their preferences (valuations). Under this assumption, we can equivalently consider the monopolist selling to one representative consumer. The firm knows that the consumer has valuation $v \in [v_L, v_H]$ but no prior information about the likelihoods of these different feasible valuations. The consumer knows her valuation, and will purchase the product if $v \geq p$.

In our framework, the monopolist trades off the losses from not making a sale with the losses from underpricing by choosing a price that minimizes her

[6] Note that in the first-best solution we assume that the firm has complete information about $F(\cdot)$ and also that the firm observes the consumers' type at the point of purchase. Therefore under the first-best solution the firm does price discrimination.

[7] Note that our results are robust to this assumption, which allows us to simplify notation (see Lemma 1). In our main results (see Lemmas 1 and 2, Theorems 1–3), we find that the optimal minimax regret price does not depend on $v_L$.

maximum regret. The monopolist's regret is her *first-best* profit, denoted as $\pi^*$ given the resolution of ambiguity minus the profit actually earned in that scenario under the chosen minimax regret price. In our setting, the monopolist's first-best profit given true valuation $v$, is $v$. The actual profit earned, denoted by $\pi(p, v)$ is $p$, if $v \geq p$, and 0 otherwise. Thus, regret conditional on $p$ and true valuation $v$ is

$$R(p, v) = \begin{cases} v - p, & \text{if } v \geq p \\ v, & \text{if } v < p \end{cases} = v - p\mathbf{1}(v \geq p).$$

If the monopolist sells the product at $p$, regret is how much more she could have earned if charging the consumer's true value. This is the regret of underpricing. If the monopolist does not sell the product, her regret equals the consumer's value $v$. The regret of overpricing is the difference between the zero profit she earned and the maximum amount that she could have earned. The monopolist trades off these two different types of losses by choosing $p$ to minimize her maximum regret over the entire space of ambiguity. She solves the general problem

$$\begin{aligned} \text{MMR} &= \min_p \max_{v \in [v_L, v_H]} R(p, v) \\ &= \min_p \max_{v \in [v_L, v_H]} \{v - p\mathbf{1}(v \geq p)\}. \end{aligned}$$

The following lemma restates the static optimal pricing rule found in Bergemann and Schlag (2011) under deterministic pricing:

Lemma 1. *The monopolist's static minimax regret price is* $p^* = v_H/2$.

Proof. The regret from underpricing $(v - p; v \geq p)$ is maximized when $v = v_H$. Therefore the maximum regret is $v_H - p$. Whereas regret from overpricing $(v; v < p)$ is maximized[8] at $v = p - \epsilon$ for $\epsilon \to 0$. Therefore the maximum regret for overpricing is $p$. Thus, for a given $p$, the maximum regret, $\text{MR}(p) = \max[v_H - p, p]$. Because $v_H - p$ is decreasing in $p$, this implies that minimax regret is attained when the regret from overpricing with that from underpricing are equal, i.e., when $v_H - p = p$ or $p^* = v_H/2$.

*Note.* If we remove the assumption that $v_L \leq v_H/4$, then the monopolist's solution, $p^* = \max[v_H/2, v_L]$. □

### 2.4. Static Minimax Regret with Heterogeneous Preferences

The firm in this setting prices to a continuum of consumers, each of whom is known to have a valuation $v_i \in [v_L, v_{iH}]$. Define $v_{H+} = \max(v_{iH})$, or the highest upper bound across consumers. Define $v_{H-} = \min(v_{iH})$ to be the lowest upper bound across consumers. To simplify exposition, we assume that $v_{H-} > v_{H+}/2$. We also assume that the distribution of $v_{iH}$ can be described by the continuously differentiable distribution $G(v_H)$ with bounded density $g(v_H)$. Because we assume that the firm knows $v_{iH}$ for every consumer $i$, it follows that the firm knows $G(v_H)$ perfectly.

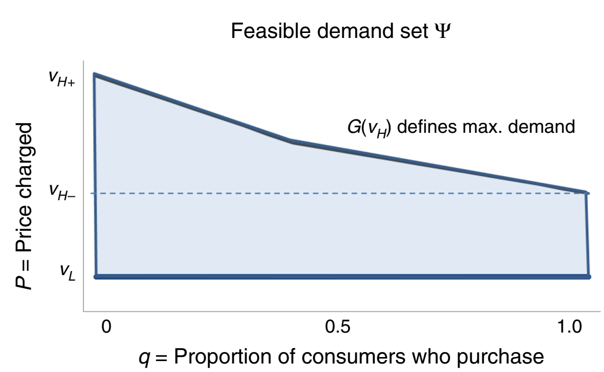
We interpret the ex ante heterogeneity in preference bounds as arising from a situation wherein the monopolist has some demographic information to characterize the consumer population but only partial information about preferences conditional on known demographic information. We assume that at the time of purchase the firm cannot distinguish between consumers and therefore does not have the required information to price discriminate. To solve its pricing problem, the firm must contend with ambiguity over the set of feasible demand curves, which can be derived from knowledge about possible sets of valuations for each consumer. Each potential demand curve, $F(p)$ describes the proportion of buyers who will buy the product at a given price, $p$. Each $F(p)$ is a weakly decreasing function, mapping the space of feasible valuations to $[0, 1]$. We define the set of feasible demand curves, $\Psi$, as the set of weakly decreasing functions that satisfy the following restrictions:

$$\Psi \equiv F(p)\colon \begin{cases} F(p) = 1, & \text{if } p < v_L, \\ F(p) \leq 1 - G(p), & \text{if } v_L \leq p \leq v_{H+}, \\ F(p) = 0, & \text{if } p > v_{H+}. \end{cases}$$

The set $\Psi$, is the set of all possible true demand curves that the monopolist could be facing given her knowledge of the distribution of consumer valuation supports. For each price, $p$, as many as $1 - G(p)$ consumers could have actual values more than $p$, whereas it is possible for up to all consumers to have a value less than $p$ so long as $p > v_L$. The set $\Psi$ is depicted in Figure 1.

Now, the monopolist chooses $p$ to minimize her maximum regret relative to the set of feasible demand

**Figure 1** **(Color online) A Representation of the Set $\Psi$**




*Note.* The set of feasible demand curves ($\Psi$) is the set of all weakly downward sloping demand curves that lie in the blue region.

[8] Note that while this is labeled a maximum, this is more formally a supremum. We follow this convention for the remainder of the paper.

distributions. Define $\pi^*(F)$ as the first-best profit, given complete information about $F(\cdot)$. This equates to charging all consumers their valuations as follows:

$$\pi^*(F) = \int_{v_L}^{v_{H+}} v\, dF(v).$$

The monopolist's actual profit given price $p$ and demand distribution $F(\cdot)$ is

$$\pi(p, F) = pF(p).$$

Thus, the monopolist's regret for charging price $p$ with true demand $F(\cdot)$ is

$$\begin{aligned} R(p, F) &= \pi^*(F) - \pi(p, F) \\ &= \int_{v_L}^{v_{H+}} v\, dF(v) - pF(p). \end{aligned}$$

For a given price, $\hat{p}$, we define the monopolist's *worst case demand* as the potential demand curve $F(\cdot) \in \Psi$ that will yield maximum regret. For each price $\hat{p}$ charged, a different $F(\cdot)$ can yield maximum possible regret, making worst case regret. More formally

$$F_{wc}(\hat{p}) = \arg\max_{F(\cdot)\in\Psi} R(\hat{p}, F),$$

$$\mathrm{MR}(\hat{p}) = R(\hat{p}, F_{wc}(\hat{p})).$$

The price that the monopolist chooses under ambiguity given the set of feasible demand curves $\Psi$ solves the minimax regret problem

$$\min_p \max_\Psi R(p, F) \equiv \min_{\hat{p}} R(\hat{p}, F_{wc}(\hat{p})).$$

The monopolist's minimax regret problem is equivalent to the problem wherein the monopolist minimizes regret under $F_{wc}(\hat{p})$ with respect to $\hat{p}$. Intuitively, for each possible price the monopolist could charge, she considers the worst case outcome conditional on that price within the set of feasible demand functions. The minimax regret price is the price that minimizes regret under its worst case demand. Given the constant lower bound $v_L$ for all $\delta_i$, $F_{wc}(\hat{p})$ can be found using a threshold value assignment rule that depends on $v_{iH}$. Define $v_{wc}(v_{iH}, \hat{p})$ as the worst-case valuation for specific consumer $i$ given price $\hat{p}$. The following lemma shows 2(a) how we can determine $v_{wc}$ as a function of $p$, and 2(b) the solution to the monopolist's minimax regret problem $\min_p \mathrm{MR}(p, v_{wc})$:

Lemma 2. (a) *For $\hat{p} \le v_{H-}$; $F_{wc}(\hat{p})$ is composed from potential individual valuations using the cut-off rule*:

$$v_{wc}(v_{iH}, \hat{p}) = \begin{cases} \hat{p} - \epsilon & \text{if } v_{iH} < 2\hat{p} \\ v_{iH} & \text{if } v_{iH} \ge 2\hat{p}. \end{cases}$$

(b) *The monopolist's static regret minimizing price is* $\mathrm{med}(v_H)/2$.

Proof. The proof is in the appendix. □

# 3. Dynamic Monopoly Pricing Under Ambiguity

To study the monopolist's multiperiod pricing problem we use a dynamic version of minimax regret that extends the static criterion to account for the value of learning (see Hayashi 2011 for axiomatic foundations). We structure the learning dynamics assuming that the monopolist is forward-looking and non-Bayesian. Specifically, the monopolist understands how her set of feasible demand curves could be narrowed in each period conditional on (i) the price that she charges, and (ii) the possible purchase quantities she could observe given that price.

In each period, the monopolist evaluates multiperiod regret by computing foregone profits over *all* remaining periods for a given price, feasible demand curve, *and* future price.[9] The future price the monopolist considers depends on her decision-making dynamics. We structure decision-making dynamics with the assumption that the monopolist is *sequentially rational*. In our context, sequential rationality means that in each period the monopolist (i) dynamically minimizes maximum regret given her current information set, and (ii) knows that in all subsequent periods she will do the same.[10] This assumption corresponds to the scenario wherein the monopolist's management team meets each period to determine current prices and cannot credibly commit to future prices.[11] Thus, when the monopolist computes multiperiod maximum regret for a given current price, she endogenizes her future pricing behavior at each possible contingent information set. After computing multiperiod maximum regret in this way for each possible price, the monopolist selects the price that minimizes this object.

The dynamic minimax regret solution for an $N$ period problem (or $N$ period continuation problem) can be found recursively using backwards induction. In the final period, this criterion reduces to static minimax regret. In the two-period framework, the monopolist solves her static minimax regret pricing problem

[9] Though the monopolist may learn over time that an initially feasible demand curve is not true demand, from the current perspective the regret calculation with respect to any feasible demand curve presumes that that curve *is* true demand and, as a result, does not need to consider removal of that demand function over time.

[10] Sequential rationality in this single agent problem is similar to the concept of subgame perfection in dynamic multiplayer games.

[11] If the firm commits to a full sequence of prices it will be unable to take advantage of its ability to learn as it will evaluate the entire sequence of prices with respect to only the initially available information. Specifically, it will minimize multiperiod maximum regret in period 1, but, in period 2, will be forcing itself to choose a potentially suboptimal price. As a result, we view the assumption that the firm does not commit, and acts in a sequentially rational manner, as an appropriate way to set up dynamic decision making and learning with the minimax regret criterion.

for all potential second-period information sets, and incorporates this information into her dynamic problem in the first period. We assume that there is no discount factor, i.e., both period profits are weighted equally in period one. The analytical challenge in analyzing this situation is that the worst-case distribution (i.e., the demand curve that causes the most regret) must be consistent with decision making in each time period.

We will start with the analysis of a homogeneous preference model and then consider the heterogeneous preference model.

### 3.1. Dynamic Minimax Regret with Homogeneous Preferences

We introduce dynamic minimax regret in a two-period model wherein consumers have homogeneous preferences. Here we consider the monopolist selling to one representative consumer. The consumer's first-period purchase decision provides information to the monopolist that she will incorporate into her second-period pricing rule. As in the static problem, we assume at the beginning of the first period that the monopolist knows that the consumer has valuation $v \in [v_L, v_H]$. We denote the firm's first-period information set as $\delta_1$. After the monopolist sets a first-period price, $p_1$, she observes whether the consumer purchases, and updates her range of possible valuations. If the consumer purchases, the monopolist knows that the consumer's valuation must be at least $p_1$. If the consumer does not purchase, the monopolist knows that the consumer's valuation is lower than $p_1$. The monopolist's second-period information sets, $\delta_2$, under these contingencies are

$$\delta_2 = \begin{cases} [v_L, p_1], & \text{if } v < p_1, \\ [p_1, v_H], & \text{if } v \geq p_1. \end{cases}$$

Once the monopolist has an updated information $\delta_2$ at the beginning of the second period, she chooses the second-period price $p_2^*(\delta_2)$ that minimizes maximum regret as described in the static model in §2.3. In the first period the monopolist minimizes maximum regret over a state space composed of all possible valuations $\delta_1$ *and* incorporates the way she will price in the second period conditional on the information set she must have at that point, contingent on $v \in \delta_1$ and $p_1$. For any given $v \in \delta_1$ and price, $p_1$, there is *only one purchase history* that could be consistent with $v$. In other words, if $v$ is the *true* valuation the monopolist knows in the first period, then she knows the information set she will have in period two. The monopolist's multiperiod regret will be the difference between her ideal profit (twice the consumer's actual valuation) and her actual profit summed over both periods. However, the space of uncertainty with which she is concerned in the first period will include only feasible second-period behavior/information if $v$ is, in fact, the true valuation. More formally, the firm's multiperiod regret as a function of $p_1$ and true valuation $v$ is

$$R(p_1, v) = 2v - p_1 \mathbf{1}[v \geq p_1] - p_2(\delta_2(p_1, v))\mathbf{1}[v \geq p_2^*(\delta_2(p_1, v))].$$

The ability to learn enters this formulation through the ability of the monopolist to impact the second-period information set with its choice of $p_1$. When choosing $p_1$ the monopolist considers the impact that this choice will have on $\delta_2$, $p_2(\delta_2)$, and on first-period regret. The sequential optimality assumption implies that the monopolist cannot commit to $p_2$ in period one but does know what she will choose in that period conditional on her information set.

Maximum regret as a function of $p_1$ is

$$\begin{aligned} \mathrm{MR}(p_1) &= \max_{v \in \delta_1} R(p_1, v) \\ &= \max_{v \in \delta_1} \{2v - p_1 \mathbf{1}[v \geq p_1] \\ &\qquad - p_2^*(\delta_2(p_1, v))\mathbf{1}[v \geq p_2^*(\delta_2(p_1, v))]\}. \end{aligned}$$

Given maximum regret conditional on $p_1$, the monopolist selects the first-period price $p_1^*$ that minimizes this maximum regret

$$\begin{aligned} p_1^* &= \arg\min_{p_1} \max_{\delta_1} R(p_1, v) \\ &= \arg\min_{p_1} \max_{\delta_1} \{2v - p_1 \mathbf{1}[v \geq p_1] \\ &\qquad - p_2^*(\delta_2(p_1, v))\mathbf{1}[v \geq p_2^*(\delta_2(p_1, v))]\}. \end{aligned}$$

Given this setting we can solve for the monopolist's optimal solution. Our main result is stated in Theorem 1, where we find that learning will lead the monopolist to increase her introductory price.

THEOREM 1 (HIGHER INTRODUCTORY PRICES WHEN CONSUMERS HAVE HOMOGENEOUS PREFERENCES). *The monopolist facing consumers with homogeneous preferences will set a higher introductory price in a dynamic setting relative to the static setting.* $p_1^* = 4v_H/7 > p^* = v_H/2$; *with* $\mathrm{MR}(p_1^*) = 6v_H/7 = \frac{6}{7}\mathrm{MR}(p^*)$.

PROOF. The proof is in the appendix. □

When we make the model dynamic to incorporate the value of learning under ambiguity, in the one consumer model the monopolist always chooses a first-period price that is higher than the optimal price in the static model. If the consumer purchases, the price remains at the higher level in the second period; if the consumer does not purchase, the price is lowered. In the dynamic framework, the monopolist's minimax regret is $\frac{1}{7}$ less than it would be applying static minimax regret in a multiperiod setting, and incorporating learning in both settings.

A firm lowering its initial price would want to price in that direction only if information learned from the consumer purchasing could later reduce regret coming from high valuation consumers (otherwise aggregate regret could not be lower than repeating the static solution twice). However, in this case, lowering initial price provides *no information to change* the second-period regret. Consider the case wherein the consumer has a valuation $v = v_H$. If $p_1^* < v_H/2$ then the consumer will purchase, and $\delta_2 \equiv [p_1^*, v_H]$. By Lemma 1 $p_2^*(\delta_2) = p^*(\delta_1) = v_H/2$, the static minimax regret pricing rule. The regret in the second period will be $v_H - p_2^*(\delta_2) = v_H/2$. This is exactly the same as the static maximum regret.

Conversely, when the firm increases initial price, it learns valuable information that it can use if the consumer has a valuation just below the price charged.[12] The ability to obtain this information gives the firm flexibility to extract more of the profit if the consumer has a high valuation. Because it knows if the consumer has a low valuation, it can re-optimize in the second period and significantly lower regret. In effect, this is because there is less total value to be lost from the consumer having a low valuation. Here the firm can first try to extract profits assuming that the consumer has a high valuation. If that fails, the firm can lower the second-period price.

### 3.2. Dynamic Minimax Regret with Heterogeneous Preferences

In this section, we extend the analysis to consider dynamic minimax regret to the case wherein consumers have heterogeneous preferences. Here each consumer's first-period purchase decision provides information to the monopolist that she will incorporate into her second-period pricing rule. As in the static problem, we assume at the beginning of the first period that the monopolist knows that each consumer, $i$, has valuation $v_i \in [v_L, v_{iH}]$. We define $\delta_{i1} \equiv [v_L, v_{iH}]$ as the identified set for consumer $i$ in time period 1. We describe the first-period purchase decisions of each consumer with the binary variable $b_i = \mathbf{1}[v_i \geq p_1]$. The monopolist's second-period information on each consumer, $\delta_{i2}$, is obtained by narrowing $\delta_{i1}$ for each consumer after the first period conditional on $b_i$

$$\delta_{i2}(p_1, b_i) \equiv \begin{cases} [v_L, p_1], & \text{if } b_i = 0, \\ [p_1, v_{iH}], & \text{if } b_i = 1. \end{cases}$$

[12] In the static case, the regret from overpricing is highest when the consumer's valuation is just below the price charged.

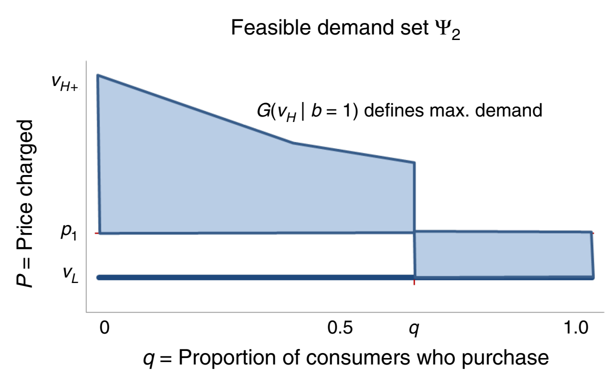
Once the monopolist has updated information $\delta_{i2}$ for each $i$, she determines the space of feasible demand curves in period two and sets $p_2$ as described in the static model of pricing under ambiguity. To derive the set of feasible demand curves contingent on consumer first-period purchase decisions, we introduce some additional notation. Denote by $q$ the proportion of consumers that purchased the product in the first period. Define $G(v_H \mid b=1)$ as the distribution of valuation upper bounds in the second period conditional on consumers having purchased in the first period. For consumers that purchase, their (a) lower bound of preferences will be $p_1$, and (b) upper bound will remain the same in period two (when $p_1 < v_{H-}$, this will be true from the assumption that $v_{H-} > v_{H+}/2$). Because consumers with different $v_{iH}$ are distinguishable from one another, the monopolist derives the distribution $G(\cdot \mid b=1)$ as follows:

$$G(v_H \mid b=1) = \frac{\int_{v_{H-}}^{v_H} g(s \mid b=1)\, ds}{q}.$$

A decision not to purchase the product in the first period reduces the upper bound of feasible valuations for consumer $i$ from $v_{iH}$ to $p_1$, implying that $G(v_H \mid b=0)$ is degenerate with all mass at value $p_1$. The firm will maintain the lower bound of $v_L$ for these consumers. Using these properties, we derive the set of feasible second-period demand curves, $\Psi_2$, as the set of weakly decreasing functions satisfying the following restrictions (shown in Figure 2):

$$\Psi_2 \equiv F_2(p)\colon \begin{cases} F_2(p) = 1, & \text{if } p < v_L, \\ F_2(p) \geq q, & \text{if } v_L \leq p < p_1, \\ F_2(p) = q, & \text{if } p = p_1, \\ F_2(p) \leq q(1 - G(p \mid b=1)), & \text{if } p_1 < p \leq v_{H+}, \\ F_2(p) = 0, & \text{if } p > v_{H+}. \end{cases}$$

**Figure 2** **(Color online) A Representation of $\Psi_2$**




*Notes.* The shaded region represents the space in which all downward sloping demand curves are feasible in period 2. Note that all demand curves must go through point $(p_1, q)$, which is the exact price and quantity sold in period one.

In addition to describing the set of feasible demand curves in the second period, note that $\Psi_2$ implicitly describes consumers' first-period purchase history. That is, knowledge of $\Psi_2$ implies knowledge of $q$ and $G(v_H \mid b=1)$.

In the heterogeneous preference setting the structure of the second-period information set is different from that in the static model. Therefore, with dynamic consistency we solve this in two steps. In §3.2.1 we study the second-period pricing problem conditional on the information structure, $\Psi_2$, and use the results from that to study the first-period pricing problem in §3.2.2, where we will derive our main results.

Before presenting our main results we add two regularity assumptions on the distribution $G(v_H)$. These assumptions are conservative and ensure that there are no sections of $G(v_H)$ where there is low density $g(v_H)$ relative to $1-G(v_H)$, similar to a monotone likelihood ratio assumption. These relationships for the median of $v_H$ of truncated distributions of $G(v_H)$ hold for all standard distributions that occur on a bounded interval $G(\cdot)$ (truncated normal, uniform, etc.). Condition (I) states that the median of successive truncations of $G$ does not change too quickly as $q$ changes. Condition (II) states that the median of successive truncations of $G$ does not change too quickly as the truncation threshold $x$ changes.[13]

Assumption 1 (Median Regularity).

$$\text{(I)}\quad \frac{\partial \mathrm{med}(v_H \mid v_H > Q_{1-q})}{\partial q} > -\frac{\mathrm{med}(v_H \mid v_H > Q_{1-q})}{q}, \quad \textit{for } q>0,$$

$$\text{(II)}\quad \frac{\partial \mathrm{med}(v_H \mid v_H > x)}{\partial x} < 1,$$

*where $Q_x$ denotes the $x$ quantile of $G(v_H)$.*

**3.2.1. Second-Period Pricing.** The second-period monopoly pricing problem is conceptually identical to the static pricing problem under ambiguity described in §2.4. However, learning from the prior period gives the space of feasible demand curves $\Psi_2$ a different structure in the second-period problem. We solve the monopolist's second-period problem considering the two cases of selecting $p_2 > p_1$ or $p_2 \le p_1$. This is a useful framework because, conditional on choosing $p_2 > p_1$, the monopolist essentially ignores consumers who did not purchase in the first period, as the maximum regret for consumers who did not purchase in the first period will be $p_1$, which does not depend on $p_2$. Therefore the prices in the second period are set to minimize maximum second-period regret among only consumers who purchased in the first period. Conversely, if $p_2 < p_1$, the monopolist sells definitely to all first-period purchasers in the second period. However the regret from first-period purchasers increases relative to the first period as a result of the reduced price. On the other hand, lowering $p_2$ can only reduce regret for consumers who did not purchase in the first period.

Lemma 3 describes the monopolist's second-period solution, conditional on $p_2$ being larger or smaller than $p_1$. For each feasible $\Psi_2$ and corresponding $q$ that could arise conditional on $p_1$, the monopolist will choose $p_2$ to minimize maximum regret across the restricted solutions described in Lemma 3. Specifically, if the minimax regret solution conditional on $p_2 > p_1$ yields lower (higher) maximum regret than the solution conditional on $p_2 \le p_1$, the monopolist will choose the $p_2^*$ and face the maximum regret in the solution restricting $p_2 > p_1$ ($p_2 \le p_1$). In the solution to the full model, we use these second-period solution properties as input into finding the dynamic minimax regret solution.

Lemma 3. (I) *Conditional on selecting second-period price, $p_2 > p_1$, the monopolist's second-period minimax regret price $p_2^*$ and maximum regret given that price are*

$$p_2^* = \frac{\mathrm{med}(v_H \mid b=1)}{2},$$

$$\mathrm{MR}_2 = (1-q)p_1 q\left(p_2 G(2p_2 \mid b=1) + \int_{2p_2^*}^{v_{H+}} (v_H - p_2)\, dG(v_H \mid b=1)\right).$$

(II) *Conditional on selecting second-period price, $p_2 \le p_1$, the monopolist's second-period minimax regret price $p_2^*$ and maximum regret given that price are*

$$p_2^* = \begin{cases} p_1/2 & \text{if } q < 0.5 \\ \in [p_1/2, p_1] & \text{if } q = 0.5 \\ p_1 & \text{if } q > 0.5, \end{cases}$$

$$\mathrm{MR}_2 = (1-q)p_2^* + \left[\int_{p_1}^{v_{H+}} (v_H - p_2)\, dG(v_H \mid b=1)\right] q.$$

**3.2.2. First-Period Pricing.** When setting a dynamic minimax regret first-period price, the monopolist knows what second-period price she will charge contingent on any information set $\Psi_2$. For each $p_1$ the monopolist could charge, she knows which $\Psi_2$ are *possible* in the second period. Furthermore, the monopolist knows that the demand function $F(\cdot)$ is the same for both periods. This has two main implications for our solution. First, since the worst-case

[13] These two assumptions can be mapped directly into one another given a specified relationship between values and quantiles of $G$. Instead of unifying these assumptions with such a mapping, for exposition and clarity we state both. Last, while condition (II) states that the change in the median relative to changes in the truncation value cannot be too large in absolute value, condition (I) ensures that this derivative with respect to quantiles does not jump too quickly relative to the actual median normalized by the fraction of individuals purchasing.

demand function must be the same for both periods, the two periods need to be solved jointly. Second, when evaluating dynamic regret from a given $F(\cdot) \in \Psi_1$ for a specific $p_1$, the monopolist knows exactly what consumer purchase decisions in the first period she will observe under this demand function.

The monopolist's first-period problem dynamic minimax regret problem is

$$\min_{p_1} \max_{F(\cdot)\in\Psi_1} \{2\pi^* - \pi_1(p_1, F) - \pi_2(p_2^*(\Psi_2(p_1, F)), F)\}.$$

The monopolist's ideal profit $2\pi^*$ is still the single-period first-best profit given known demand $F(\cdot)$ earned in each period. The first-period profit $\pi_1$ depends on the first-period price and true $F(\cdot)$. The second-period profit $\pi_2$ depends on the second-period price, set contingent on the information learned from first-period purchase decisions, and true demand. The monopolist minimizes maximum regret dynamically by selecting the price that yields the lowest possible maximum aggregate regret over all feasible demand curves. The knowledge of *how* she will set the second-period price impacts the multiperiod maximum regret calculation for each first-period price. For instance, if the monopolist sets a high initial price and nobody purchases, she knows that she will respond by setting a much lower price in the second period. Therefore from a *first-period perspective* maximum multiperiod regret from setting a high price may not come from a feasible demand curve that leads to no consumers initially purchasing because second-period maximum regret will be low in this contingency.

To solve the monopolist's first-period dynamic minimax regret pricing problem, we need to determine what feasible demand curve will yield the highest maximum regret for a given $p_1$, conditional on sequentially optimal behavior in the second period. We define worst-case demand in period $t$ conditional on $p_t$

$$F_{wc}(p_t) = \arg\max_{F(\cdot)\in\Psi_t} R(p_t, F).$$

In §3.2.1 we studied the monopolist's second-period solution, which depends on $F_{wc}(p_2)$. Here, $F_{wc}(p_2)$ is determined within the context of the second-period static minimax regret problem. In this section, we characterize properties of $F_{wc}(p_1)$ to find the dynamic minimax regret pricing solution $p_1^*$. Determining $F_{wc}(p_1)$ is much more challenging than $F_{wc}(p_2)$ because it must account for the dynamic price and outcome path engendered by $p_1$, not just static outcomes. To analyze $F_{wc}(p_1)$ we must know (i) $F_{wc}(p_2)$ for each feasible $\Psi_2$ and $p_2$, (ii) $p_2^*$ conditional on each value of $(p_1, \Psi_1, F)$, and (iii) what the multiperiod maximum regret will be for $p_1$ for each $F(\cdot)$ given the learning and second-period pricing that will occur. Once we know $F_{wc}(p_1)$ for each $p_1$, the monopolist selects the first-period price $p_1$ that minimizes multiperiod maximum regret with respect to $F_{wc}(p_1)$. In §3.2.1 we discussed how to characterize (i) and (ii), while this section uses those results to address (iii) and characterize the dynamic minimax regret first-period solution.

We prove our main results in the following steps:

1. *Characterize the worst-case demand function*. We identify a subset of demand curves within $\Psi_1$ that could be worst case demand, conditional on $p_1$. We consider the two cases $p_1 < p_2^*$ and $p_1 \geq p_2^*$ separately and for each consider what could be the worst case demand functions. This step simplifies our problem by determining the effect of the two possible sequential minimax regret pricing rules on $F_{wc}(p_1)$. Restricting $p_2$ relative to $p_1$ allows us to narrow the space of feasible $F_{wc}(p_1)$ to two demand curves conditional on $p_1$, which we use to determine which $p_2$ yields maximum regret. Specifically, here we establish that the worst-case demand curve does not have any weight on valuations (i.e., consumers) who purchase in only one period. Our result is formally characterized by the following claim:

Claim 1. (I) *When $p_2^*(\Psi_2(F_{wc}(p_1), p_1)) > p_1^*$, $F_{wc}(p_1)$ does not contain consumers who purchase only in the first period*. (II) *When $p_2^*(\Psi_2(F_{wc}(p_1), p_1)) \leq p_1^*$, there exists $F_{wc}(p_1)$ that does not contain consumers who purchase only in the second period*.

Proof. The proof is in the online appendix. □

2. *Showing dynamic consistency*. Using the results from §3.2.1, we show dynamic consistency along the dynamic minimax regret price path. This shows that, along the optimal price path, the monopolist prefers the same $p_2^*$ (i.e., same pricing direction relative to $p_1$) before and after learning the worst case demand in period one. Our result is formally characterized by the following lemma.

Lemma 4. *Conditional on $p_1$ and on $\Psi_2$ consistent with $F_{wc}(p_1)$ occurring, along the dynamic minimax regret price path, the monopolist would choose $p_2 = p_2(\Psi_2(F_{wc}(p_1), p_1))$ ex ante.*

Proof. The proof is in the appendix. □

3. *Optimal pricing*. We use these results as input into Theorem 2 where we solve for optimal first-period prices.

Theorem 2. *When $G$ is nondegenerate $p_1^* < p^*$, i.e., the monopolist will lower the introductory price in a dynamic setting relative to the static setting. Furthermore, for any $\Psi_1$, $p_2^*(\Psi_2(F_{wc}(p_1), p_1)) > p_1^*$ along the dynamic minimax regret price, i.e., if a purchase history consistent with worst-case demand is observed, the monopolist will price upwards over time. When $G$ is degenerate, $p_1^* = p^* = p_2(\Psi_2(F_{wc}(p_1, p), p_1))$.*

Proof. The proof is in the appendix. □

This result characterizes two important features of the monopolist's dynamic minimax regret pricing rule in a setting with arbitrary heterogeneity in preferences across consumers. First, the ability to learn about demand causes the monopolist to lower the introductory price relative to a static setting. Second, after the introductory price, if the worst possible partial realization of uncertainty occurs through learning from first-period purchases, the monopolist will increase her price in the second period.

Intuitively, both these results are true because of the way information obtained through first-period purchases impacts dynamic maximum regret in an environment with preference heterogeneity. For any $p_1$, the monopolist's worst case demand will have some consumers who do not purchase with values just below $p_1$, and some consumers who do purchase and have values equal to their maximum valuations. Crucially, under worst-case demand, consumers who do not purchase will have low $v_H$ relative to those who do purchase. Setting $p_2 > p_1$ allows the monopolist to establish low maximum regret levels from consumers who do not purchase over both periods (low $v_H$), which lets the monopolist then focus exclusively on minimizing maximum regret for high valuation consumers in the second period. Setting $p_2 < p_1$ does not similarly bound maximum regret from consumers who purchase. When the monopolist lowers price over time she can restrict the valuation of someone who purchases to above $p_1$, but when lowering price deterministically increases maximum regret from this set of consumers by $p_1 - p_2$ for each consumer, this result depends critically on ex ante information heterogeneity. We have shown that lower initial prices are valuable when ex ante heterogeneity information exists in a dynamic setting with ambiguity and that the monopolist will decrease initial prices. However, where there is no ex ante heterogeneity information ($G$ is degenerate), lowering initial prices is not valuable because there are no consumers with relatively low $v_H$ to target with a lower first-period price.

We derive a dynamic minimax regret result with uniform ex ante heterogeneity information in §§3.2.3–3.2.5.

**3.2.3. Dynamic Pricing with Uniform Ex Ante Information Heterogeneity.** We illustrate the solution to the heterogeneous preference model by explicitly solving the monopolist's dynamic minimax regret problem when pricing to a mass of consumers with heterogeneous preferences ($v_i$). The firm has ex ante information to bound each consumer's preferences ($v_i \in [v_L, v_{iH}]$). In this section we add the assumption that the distribution of $v_{Hi}$ ($G(v_H)$) is uniform, or $G(v_H) \to U[v_{H-}, v_{H+}]$. This form of demand ambiguity could arise in a setting wherein the monopolist observes a uniform distribution of an important demographic variable in its target population, and believes that the most someone could value their product is linked directly to that variable.

Given the result of Theorem 2, we know that the monopolist will charge a lower introductory price when consumers have heterogeneous preferences and that, under a first-period purchase history consistent with worst-case demand, the monopolist will choose a higher second-period price. The following theorem solves explicitly for the monopolist's solution:

Theorem 3. *The dynamic minimax regret first-period pricing rule for the monopolist with uniform ex ante heterogeneity information is* $p_1^* = (23v_{H+} + 49v_{H-})/144 < (v_{H-} + v_{H+})/4 = p^*$. $p_2(\Psi_2(F_{wc}(p_1^*, p), p_1^*)) = (714v_{H+} + 294v_{H-})/2{,}016 > p_{\text{MMR}}$.

Proof. The proof is in the appendix. □

Corollary 1 describes how the extent of introductory price reduction relates to the degree of ex ante information heterogeneity in the population. We find as the distribution of ex ante information heterogeneity becomes more dispersed, the monopolist's introductory price decreases. Likewise, as the distribution of ex ante information heterogeneity becomes less disperse, the monopolist's introductory price increases. In the limiting case, as $v_{H+} - v_{H-} \to 0$, we find $p_1^* \to p^*$ from below. This is consistent with the ex ante homogenous case (no ex ante information, or $G(\cdot)$ is degenerate), where the first-period dynamic price ($p_1^*$) is exactly equal to the static price ($p^*$) (see Theorem 2).

Corollary 1. *For* $G(\cdot) \to U[v_{H-}, v_{H+}]$ *and* $G'(\cdot) \to U[v_{H-} - \alpha, v_{H+} + \alpha]$, $p_1^*(\Psi_1(G)) > p_1^*(\Psi_1(G'))$ *for* $\alpha > 0$.

Proof. Follows directly from

$$p_1^* = (23v_{H+} + 49v_{H-})/144. \quad \square$$

**3.2.4. Alternatives to Minimax Regret with Uniform Ex Ante Information Heterogeneity.** An alternative to minimax regret in the decision theory literature is to use the maxmin criterion, or to set a price that maximizes the firm's minimum possible payoff over the range of possible valuations. Formally, $p^{\text{MM}} = \arg\max_p \min_{F(\cdot)\in\Psi} \pi(p, F)$. In our setting the maxmin price is set to $v_L$ for the static or dynamic first- and second-period cases. To see this, consider a distribution wherein all consumers are of type $v_L$ (a feasible distribution). Any price larger than $v_L$ will yield zero profit. Therefore the minimum profits for any price higher than $v_L$ is 0, whereas the minimum profits for a price $v_L$ is $v_L$. We believe the maxmin criterion is less appealing that minimax regret in a dynamic pricing context for two reasons. First, maxmin focuses only on the potential losses from not selling the product (i.e., overpricing) and does not consider the potential losses from foregone profits by selling at a price far lower than the consumer's valuation (i.e., underpricing). By contrast, minimax regret explicitly considers the trade-off between underpricing and overpricing. Second, unlike dynamic minimax regret, there is no price experimentation or learning with dynamic max min.

In §A.8 in the online appendix, we derive the optimal Bayesian price assuming an uninformative prior. Explicitly here the decision maker assumes, based on a *subjective prior*, that each consumer's valuations $v_i \sim U[v_L, v_{Hi}]$. In a dynamic setting the Bayesian decision maker can update her information set based on the price charged ($p$) and the purchase decision of the consumer. If consumer $i$ purchases at a price $p$, the monopolist will update her information to $v_i \sim U[p, v_{Hi}]$. If consumer $i$ does not purchase at a price $p$, the monopolist will update her information to $v_i \sim U[v_L, p]$. We derive the uninformative prior Bayesian price as

$$p^B = \frac{1}{2}\left(v_L + \frac{v_{H+} - v_{H-}}{\log(v_{H+} - v_L) - \log(v_{H-} - v_L)}\right).$$

We find that the optimal Bayesian price depends on $v_L$, $v_{H+}$, and $v_{H-}$, whereas the dynamic minimax regret price depends only on $v_{H+}$ and $v_{H-}$.[14] Moreover, we show that $\partial p^B/\partial v_L < 0$ (§A.8). Therefore if the firm obtains new data that increases $v_L$ for all consumers, the Bayesian price derived $p^B$ will decrease (until $p^B = v_L$). Consider the two time period pricing decisions for a decision maker using Bayesian updating. The implication here is that if the firm sets a price in the first time period ($p_1^B$) and all consumers purchase, the second-period price ($p_2^B$) will be equal to $p_1^B$. This is unlike the dynamic minimax regret solution where $p_2^{\mathrm{MMR}}$ will be higher than $p_1^{\mathrm{MMR}}$ if all consumers purchase in the first time period.

Next, we evaluate the implications of these decision criterion for firm profits.

**3.2.5. Numerical Example with Uniform Ex Ante Information Heterogeneity.** We present a numerical example that illustrates the solution with uniform ex ante information heterogeneity. Consider a new product launch for a nonstorable good with considerable ambiguity. Assume that the monopolist knows that all consumers could have the same minimal value $v_L = 2$ and that the highest possible valuations $v_{iH}$ are distributed $U[10, 18]$ ($v_{H-} = 10$, $v_{H+} = 18$). The static minimax regret price ($p^*_{\mathrm{MMR}}$), the first-period dynamic minimax regret price ($p_1^*$), the maxmin price ($p^{\mathrm{MM}}$), and Bayesian price with an uninformative prior ($p^B$) are given below

$$p^{\mathrm{MMR}} = \frac{\mathrm{med}(v_H)}{2} = \frac{v_{H+} + v_{H-}}{4} = 7.0,$$
$$p_1^{\mathrm{MMR}} = \frac{23v_{H+} + 49v_{H-}}{144} = 6.3,$$
$$p^{\mathrm{MM}} = v_L = 2.0,$$
$$p^B = \frac{1}{2}\left(v_L + \frac{v_{H+} - v_{H-}}{\log(v_{H+} - v_L) - \log(v_{H-} - v_L)}\right) = 6.8.$$

To understand the profit implications of the minimax regret price, we simulate true preferences from a parametric distribution and compute the resultant firm profits. Of particular importance and consistent with our model, we assume that the firm only knows that $v_i \in [v_L, v_{Hi}]$, and that it does not know the distribution within this set. We generate consumers' true valuations $v_i$ from a Beta distribution with parameters $\alpha$ and $\beta$ between $v_L$ and $v_{Hi}$.[15] We compare the profit results for the following six prices: (1) static minimax regret price ($p^{\mathrm{MMR}} = 7.0$); (2) first-period dynamic minimax regret price ($p_1^{\mathrm{MMR}} = 6.3$); (3) second-period dynamic minimax regret price ($p_2^{\mathrm{MMR}}$, derived numerically as a function of $p_1^{\mathrm{MMR}}$ and first-period purchase decisions); (4) maxmin price, wherein there is a single solution for the static and both periods in the dynamic model ($p^{\mathrm{MM}} = 2.0$); (5) static Bayesian price assuming an uninformative prior ($p^B = 6.8$); and (6) second-period Bayesian updating with an uninformative prior ($p_2^B$ derived numerically as a function of $p^B$ and first-period purchase decisions). We simulate outcomes for three sets of parameters ($\alpha$ and $\beta$) to see how the shape of the true preferences impacts the resultant profits. For each set of parameters, we simulate preferences for 500,000 consumers.

The results of the simulation are shown in Figure 3. The rows of the figure represent the three simulations (labeled A though C) with different parameter $(\alpha, \beta)$ values to generate the true preferences ($v_i \sim \mathrm{Beta}(\alpha, \beta)$ between $v_L$ and $v_{Hi}$).[16] The first column of the figure plots the distribution of valuations across the population. The second column plots the prices charged based on the different pricing methods described above. The third column plots the realized profits. We will discuss the results for each simulation in detail.

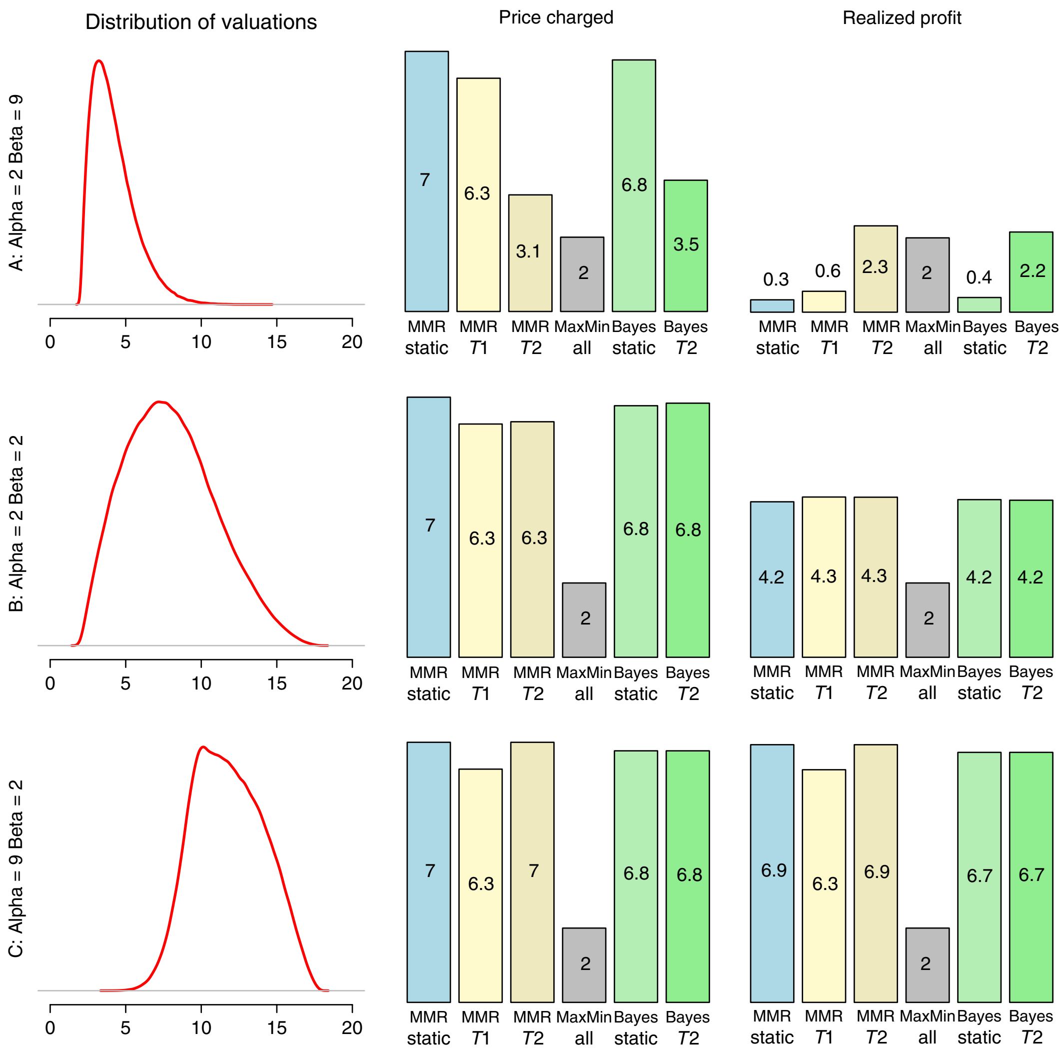
- In simulation A ($\alpha = 2$, $\beta = 9$), true preferences are skewed to the left. In a static setting, the maxmin

[14] One of the implications here is that adding heterogeneity in $v_L$ across consumers will impact the Bayesian price but not the minimax regret price.

[15] We thank the associate editor for suggesting this simulation.

[16] In addition to the simulations discussed below we added a simulation with U-shaped preferences ($\alpha = 0.5$, $\beta = 0.5$). The results are similar to the simulations shown here.

**Figure 3** **Simulation Experiment with the Following Setting** $v_L = 2$; $v_{Hi} \sim U[10, 18]$; $v_i \sim \text{Beta}(\alpha, \beta)$ **Between** $v_L$ **and** $v_{Hi}$




*Notes.* We consider three sets of parameters ($\alpha$ and $\beta$) labeled A through C. In each case, we simulate the true preferences for 500,000 consumers from the assumed distribution. The charts in the first column represent the true distribution of preferences for the simulation. The charts in the second and third columns represent the prices charged and the corresponding realized profits.

price ($p^{\text{MM}} = 2.0$) ensures that all consumers purchase and results in higher profits than both the minimax regret price ($p^{\text{MMR}}$) and the uninformed Bayesian price ($p^B$). In a dynamic setting, under minimax regret the firm observes that only 9% of consumers purchase in the first period and lowers the price in the second period ($p_2^{\text{MMR}} = p_1^{\text{MMR}}/2 = 3.1$, as in Lemma 3, part II under condition $q < 0.5$). We find that this results in higher profits under the second-period dynamic minimax regret price than under the maxmin price. With Bayesian updating, the firm will also lower the price in the second period ($p_2^B$). We find that the dynamic minimax regret prices result in higher profits than the Bayesian updating prices in both time periods.

- In simulation B ($\alpha = 2$, $\beta = 2$), true preferences are symmetric. In a static setting, both the minimax regret price ($p^{\text{MMR}}$) and the uninformed Bayesian price ($p^B$) result in higher profits than the conservative maxmin price ($p^{\text{MM}}$). In the dynamic setting, under minimax regret the firm observes that 69% of consumers purchase in the first period. Thus the firm will not change the price in the second period ($p_2^{\text{MMR}} = p_1^{\text{MMR}} = 6.3$, as in Lemma 3, part II under the condition $q > 0.5$). With Bayesian updating, the firm will also maintain the same price in the second period ($p_2^B$). We find that dynamic minimax regret prices and Bayesian updating prices result in similar profits in both time periods.
- In simulation C ($\alpha = 9$, $\beta = 2$), true preferences are skewed to the right. In a static setting, the minimax regret price ($p^{\text{MMR}}$) results in higher profits than the uninformed Bayesian price ($p^B$) and the conservative maxmin price ($p^{\text{MM}}$). In the dynamic setting, under minimax regret, the firm observes that

nearly all consumers purchase in the first period. Thus the firm will increase the price in the second period ($p_2^{\text{MMR}} = \text{med}(v_H \mid b = 1)/2 = 7.0$, as in Lemma 3, part I). This is unlike the case with Bayesian updating, where despite high demand, the firm will maintain the same price in the second period ($p_2^B$) (as discussed in §3.2.4). We find that the dynamic minimax regret second-period price results in higher profits than the Bayesian updating price.

In summary, these simulations highlight the way dynamic minimax regret helps the monopolist to learn about demand from the first period and to adjust prices in the second period. Furthermore, these simulations illustrate that the ability to learn results in comparable or higher second-period profits.

### 3.3. Summary of Results

Overall our results suggest that the difference between the first-period dynamic price and the static price depends critically on consumer heterogeneity. The main results are characterized in Table 1. The results show that the role of learning information varies with heterogeneity in preferences and ex ante information heterogeneity. In a setting where all consumers have homogeneous preferences, the monopolist will set high period 1 prices (compared to the static model), and will lower the price in period 2 if consumers do not purchase. By contrast, in a setting with heterogeneity in preferences and ex ante information, the monopolist will set low period 1 prices (compared to the static model), and if the worst case demand is realized will then increase prices in period 2. The main difference lies in the fact that the homogeneous preference model restricts the set of feasible demand functions as mass points, which alters the intuition in the heterogeneous preference model. In the homogeneous consumers model, increasing the initial price is attractive as consumer valuations must be above or below the higher price. If we increase price and consumers do not purchase, then reducing future prices significantly decreases future regret. In the heterogeneous preference model, this would not be the case because worst-case demand would have consumers with valuations above and below $p_1$. Moreover, under the worst-case demand, consumers who do not purchase will have lower ex ante valuations than consumers who do purchase. Here decreasing price allows the firm to bound the maximum regret from these ex ante lower value consumers and to potentially target the ex ante higher valuation consumers with increasing prices in period 2. This result depends critically on ex ante information heterogeneity. When the firm does not have information about ex ante heterogeneity, lowering the price is no longer valuable.

**Table 1** Main Results for Robust New Products Prices

| Assumption about heterogeneity | | Main result | | |
|---|---|---|---|---|
| | | | Dynamic | |
| Preferences | Ex ante information | Static $p^*$ | $p_1^*$ | $p_2^*$ (under $F_{wc}$) |
| No | | $\frac{v_H}{2}$ | $> p^*$ | $p_1^*$ or $\frac{p_1^*}{2}$ |
| Yes | No | $\frac{\text{med}(v_H)}{2}$ | $= p^*$ | $= p_1^*$ |
| Yes | Yes | $\frac{\text{med}(v_H)}{2}$ | $< p^*$ | $> p_1^*$ |

## 4. Conclusion

This paper focuses on how monopolists will price when they face ambiguity about demand, but how they can reduce that ambiguity over time as they acquire information from consumer purchase decisions. The manager will make pricing decisions in a manner that optimally trades off information learned about demand with current profits in an environment. We show that incorporating learning causes the monopolist to reduce introductory prices and then to adaptively price based on the information that is learned. When the first-period purchase outcomes are as bad as possible from the monopolist's perspective, she will price upwards over time. We present a novel non-Bayesian framework for studying dynamic decision making. Marketers introducing new products face significant ambiguity when pricing. The current literature assumes that the manager uses a Bayesian prior to price in early periods. By contrast, we assert that the manager knows only that each consumer's valuation lies within a range of possible valuations and has no subjective information on which of these are more likely. An alternative view of our results is to understand the implications of subjective beliefs on pricing decisions.

Our main result shows that the monopolists can offer a lower, unchanged or higher introductory price in a dynamic environment (as compared to a static environment) depending on the type of heterogeneity in the market. We find that (1) When consumers have homogeneous preferences, introductory dynamic price is higher than the static price; (2) When consumers have heterogeneous preferences and the monopolist has no ex ante information, the introductory dynamic price is the same as the static price; and (3) When consumers have heterogeneous preferences and the monopolist has ex ante information, the introductory dynamic price is lower than the static price. Furthermore, the degree of this initial reduction increases with the amount of heterogeneity in the ex ante information. The extant literature shows that dynamics in pricing are optimal due to dynamics in demand (e.g., evolving preference) or supply (e.g., inventory constraints). We add a demand learning objective for the firm that can also lead to dynamic optimal prices.

In this paper, we acknowledge that we study a stylized problem with three main limitations. First, while we study a monopoly pricing problem, situations where pricing under ambiguity is relevant might involve multiple firms pricing strategically. Second, our model incorporates firm-side learning, while consumer valuations are static over time. In new product settings, consumers might also exhibit preference formation or learning (e.g., Erdem and Keane 1996). Third, we are restricted to nonstorable goods. Considering storable goods, future research might focus on goods where consumers have incentives to stockpile (Hendel and Nevo 2006) and goods (e.g., technology) where firms might consider temporal price discrimination (e.g., Nair 2007).

Whereas the focus of our paper is pricing, the framework presented here can be applied to other marketing mix instruments (e.g., advertising). In many settings, marketers have incomplete information about consumer preferences and responses to marketing mix instruments. Formally analyzing decisions under ambiguity provides an alternative for managers unwilling to make subjective prior assumptions. We hope future empirical and analytical marketing research will explore robust marketing mix decisions.

### Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mksc.2015.0914.

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## Appendix

### A.1. Proof of Lemma 2

Proof. Sketch of the proof provided here. See the online appendix for the complete proof.

First, consider the regret from prices $p < v_{H-}$.

As in the representative consumer model, consider the regret from overpricing and underpricing for each consumer. The regret from overpricing is maximized by solving $\max_v v$, subject to $v < p$, which is solved at $v = p - \epsilon$ for $\epsilon \to 0$. The regret from underpricing is maximized by solving $\max_v\{v - p\}$, subject to $v > p$, which is solved at $v = v_{iH}$. The first kind of regret is greater than the second if $\hat{p} - \epsilon > v_{iH} - \hat{p}$ or $2\hat{p} > v_{iH} + \epsilon$ ($\epsilon \to 0$). This gives the worst-case valuations for each consumer. In our model, we allow for full nonparametric heterogeneity in true valuations across consumers. Therefore, $F_{wc}(\hat{p})$ can be found by aggregating the worst-case valuations for each consumer.

The monopolist's maximum regret for each price is

$$\mathrm{MR}(\hat{p}) = \hat{p}G(2\hat{p}) + \int_{2\hat{p}}^{v_{H+}} v_H\, dG(v_H) - \hat{p}[1 - G(2\hat{p})]$$
$$\Rightarrow\ p^* = \frac{\mathrm{med}(v_H)}{2}.$$

In the online appendix we show that the regret from $p = \mathrm{med}(v_H)/2$ is lower than the regret from $p \geq v_{H-}$. □

### A.2. Proof of Theorem 1

Proof. Sketch of the proof provided here. See the online appendix for the complete proof.

We begin by proving in the following steps:

1. The monopolist's optimal first-period price $p_1^* \geq v_H/2$. We show that $p_1 = v_H/2$ has lower regret than any $p_1 < v_H/2$. The intuition for this result is as follows: The static maximum regret from charging a price is $v_H/2$, therefore maximum multiperiod regret from charging $p_1 = p_2 = v_H/2$ is $v_H$. If $p_1 < v_H/2$, maximum regret occurs when the consumer has valuation $v_H$. Here, regret in the first period is $v_H - p_1$ and regret in the second period is $v_H - v_H/2$ (by Lemma 1). Therefore the multiperiod regret is $\frac{3}{2}v_H - p_1 > v_H$.

2. Given that $p_1^* \geq v_H/2$, the maximum regret under a minimax regret first-period price rule for the monopolist, conditional on a first-period purchase is equal to $2(v_H - p_1^*)$. When the consumer does not purchase, maximum regret equals $3p_1^*/2$.

3. Because dynamic maximum regret conditional on no first-period purchase is increasing in $p_1$ and dynamic maximum regret conditional on a first-period purchase is decreasing in $p_1$, actual dynamic minimax regret is achieved by setting $p_1$ to where these two quantities are equal. Given the results of the previous claims, this occurs when

$$\frac{3}{2}p_1 = 2(v_H - p_1)$$
$$\Rightarrow\ p_1^* = \frac{4v_H}{7}.$$

The dynamic minimax regret value, given this optimal pricing rule, equals $2(v_H - p_1^*) = 6v_H/7$, which occurs when the consumer has true valuation of $v_H$ or $4v_H/7 - \epsilon$. This is $\frac{6}{7}$ of what maximum regret would be if naively applying static minimax regret twice ($\mathrm{MR}(p^*) = v_H$). □

### A.3. Proof of Lemma 3

Proof. Sketch of the proof provided here. See the online appendix for the complete proof.

For notational convenience we partition $\Psi_2$ into sets of feasible demand curves conditional on whether ($\Psi_2^{\mathrm{P}}$) or ($\Psi_2^{\mathrm{NP}}$) a consumer purchased the product in the first period (see the online appendix for the mathematical formulation).

If $p_2 > p_1$, then maximum regret for the proportion $1 - q$ of the population that did not purchase in the first period is fixed for any $p_2$ and equal to $(1 - q)p_1$. As a result, the monopolist only considers feasible demand curves for the population that purchased in the first period when selecting a second-period price. The monopolist's problem is equivalent to the static pricing problem with heterogeneity as described in §2 with $\Psi_2^{\mathrm{P}}$ as the state space. The solution to Lemma 2 then applies to this setting, implying $p_2^* = \mathrm{med}(v_H \mid b=1)/2$. Maximum regret for the second period as a function of $p_2$ is the fixed maximum regret for those consumers who did not purchase in the first period $(1-q)p_1$, plus the maximum regret for the static problem with heterogeneity, given $\Psi_2^{\mathrm{P}}$, weighted by $q$.

If $p_2 < p_1$, then maximum regret for the proportion $q$ of the population that did purchase will be $E[v_H \mid b=1] - p_2$ because the monopolist knows that each consumer who bought in the first period will buy in the second period and maximum regret for each such buyers occurs when their valuation is as high as possible. Thus, the monopolist's maximum regret restricting $p_2 < p_1$ for this portion of the population is $[\int_{p_1}^{v_{H+}} (v_H - p_2)\, dG(v_H \mid b=1)]q$. The monopolist solves the second-period minimax regret problem considering $\Psi_2^{\mathrm{NP}}$ along with the additional maximum regret for the high value consumers as a function of $p_2$.

If $q > 0.5$, then $p_2^* = p_1$: Lowering price will increase maximum regret to high value consumers by $q\Delta p_2$ whereas the change in price can only decrease maximum regret for low value consumers by $(1-q)\Delta p_2$. If $q < 0.5$, then lowering $p_2$ decreases maximum regret to low value consumers by $q\Delta p_2$ until $p_2^* = p_1/2$, after which lowering $p_2$ has no additional impact on lowering maximum regret because all low value consumers could have value $p_1 - \epsilon$. Because $(1-q)\Delta p_2 > q\Delta p_2$ for $q < 0.5$, $p_2^* = p_1/2$. If $q = 0.5$ then $(1-q)\Delta p_2 = q\Delta p_2$ and any price in between $p_1/2$ and $p_1$ minimizes maximum regret for $p_2 \le p_1$. The formula for maximum regret follows. □

### A.4. Proof of Claim 1

See the online appendix for the proof.

### A.5. Proof of Lemma 4

Proof.[17] Sketch of the proof provided here. See the online appendix for the complete proof.

Consider the multiperiod regret

$$\begin{aligned} R(p_1, p_2, F_{wc}(p_1, p \mid q)) \\ = R(p_1, F_{wc}(p_1, p \mid q)) + R(p_2, F_{wc}(p_1, p \mid q)). \end{aligned}$$

Conditional on $p_1$ and $q_{wc}(p_1)$ occurring, $R(p_1, F_{wc}(p_1, p \mid q))$ is independent of $p_2$. Then, picking the $p_2$ that minimizes maximum $R(p_2, F_{wc}(p_1, p \mid q))$ conditional on $q$, $p_1$, and purchase history consistent with $F_{wc}(p_1, p \mid q)$ is identical to picking $p_2$ to minimize maximum $R(p_1, p_2, F_{wc}(p_1, p \mid q))$ ex ante. □

### A.6. Proof of Theorem 2

Proof. Outline of the main steps and results of the proof provided here. See the online appendix for the complete proof.

[17] Note in the proofs that we use $F_{wc}(p_1, p)$ and $F_{wc}(p_1)$ interchangeably. This represents the worst-case demand curve under a price $p_1$ evaluated at all prices $p$.

The first five steps focus on the case wherein $G$ is nondegenerate, whereas step 6 focuses on the degenerate case.

*Step* 1. *Determining* $F_{wc}(p_1, p \mid q)$. This allows us to determine the maximum regret from upward or downward pricing in the second period. We derive

$$\begin{aligned} \mathrm{MR}(p_1, p_2^U \mid q) &= 2p_1(1-q) + [2E[v_H \mid v_H > Q_{1-q}] - p_1 - p_2^U]q, \\ \mathrm{MR}(p_1, p_2^D \mid q) &= \frac{3p_1}{2}(1-q) + \left[2E[v_H \mid v_H > Q_{1-q}] - \frac{3p_1}{2}\right]q. \end{aligned}$$

*Step* 2. *Pricing direction over time given q and* $p_1$. Define $q^*$ as the $q$ such that $\mathrm{MR}(p_1, p_2^D \mid q) = \mathrm{MR}(p_1, p_2^U \mid q)$. We find that $\forall\, q > q^*$, $\mathrm{MR}(p_1, p_2^U \mid q) < \mathrm{MR}(p_1, p_2^D \mid q)$ or the monopolist will increase price in the second time period; and $\forall\, q < q^*$, $\mathrm{MR}(p_1, p_2^U \mid q) > \mathrm{MR}(p_1, p_2^D \mid q)$ or the monopolist will increase price in the second time period.

*Step* 3. $\mathrm{MR}(p_1, p_2^D \mid q)$ *is increasing over the range* $[0, q^*]$ *for each* $p_1$. We show that for any $p_1$ the maximum regret over the range $[0, q^*]$, where the monopolist will price downwards over time under worst case demand (see Step 2), will occur at $q^*$

*Step* 4. $\exists\, q > q^*$ *such that* $\mathrm{MR}(p_1, p_2^U \mid q) > \mathrm{MR}(p_1, p_2^U \mid q^*)$, *implying that* $p_1^* < p_2^*(\Psi_2(F_{wc}(p_1^*, p), p_1^*))$ *along the dynamic minimax regret price*. We show that $\exists\, q > q^*$ such that $\mathrm{MR}(p_1, p_2^U \mid q) > \mathrm{MR}(p_1, p_2^U \mid q^*) = \mathrm{MR}(p_1, p_2^D \mid q^*)$. This will imply that under the dynamic minimax regret first-period solution $p_1^*$, $p_2^*(\Psi_2(F_{wc}(p_1^*, p), p_1^*)) > p_1^*$, i.e., the monopolist will always price upwards when a purchase history consistent with $F_{wc}(p_1, p)$ is observed in period one.

*Step* 5. $p_1^* < p^*$, *The monopolist price experiments downward*. Given the result in Step 4, we find that the ability to learn by increasing the price in the second-period results in a monopolist decreasing price in the first period.

*Step* 6. *When G is degenerate,*

$$p_1^* = p^* = p_2(\Psi_2(F_{wc}(p_1^*, p), p_1^*)).$$

This shows that our result depends critically on ex ante heterogeneity. In the absence of ex ante heterogeneity the firm will not price experiment. □

### A.7. Proof of Theorem 3

Proof. Sketch of the proof provided here. See the online appendix for the complete proof.

Claim 1 implies that under worst-case demand for dynamic minimax regret price $p_1$ there will be two groups of consumers, those who purchase the product in both periods and those who never purchase. The threshold to determine who buys and who does not in the first period under worst case demand for a given $p_1$ solves

$$v_H^* = \frac{3p_1 + p_2^*(\Psi_2(\Psi_1, p_1))}{2} = \frac{12p_1 + v_{H+}}{7}. \tag{1}$$

This implies that the maximum regret conditional on a given first-period price $p_1$ is

$$\mathrm{MR}(p_1, \cdot) = \frac{144p_1^2/7 - 46p_1 v_{H+}/7 - 14p_1 v_{H-}}{7(v_{H+} - v_{H-})}.$$

The first-period dynamic minimax regret price can then be determined by solving

$$\min p_1 \mathrm{MR}(p_1) \quad \Rightarrow \quad p_1^* = \frac{23v_{H+} + 49v_{H-}}{144}.$$

Because $v_{H+} > v_{H-}$, $(23v_{H+} + 49v_{H-})/144 < (v_{H-} + v_{H+})/4 = p^*$.

$p_2(\Psi_2(F_{wc}(p_1^*, p), p_1^*))$ can be found by inserting the value of $v_H^*$ determined as a function of $p_1^*$ into the formula for contingently optimal pricing under Lemma 3

$$v_H^* = \frac{12p_1 + v_{H+}}{7} = \frac{420v_{H+} + 588v_{H-}}{1{,}008},$$

$$p_2(\Psi_2(F_{wc}(p_1^*, p), p_1^*)) = \frac{v_{H+} + v_H^*}{4} = \frac{714v_{H+} + 294v_{H-}}{2{,}016} > p^*. \quad \square$$

### A.8. Analysis of a Uninformed Bayesian Decision Maker with Uniform Ex Ante Heterogeneity

In this section, we consider the model with uniform ex ante heterogeneity, or $v_{Hi} \sim U[v_{H-}, v_{H+}]$. We derive the optimal Bayesian price assuming an uninformative prior. Explicitly here the decision maker assumes, based on a *subjective prior*, that each consumer's valuations $v_i \sim U[v_L, v_{Hi}]$. The lemma below derives the optimal Bayesian price assuming an uninformed prior.

Lemma. *If the monopolist assumes a flat subjective uniform prior for each consumer i between $[v_L, v_{iH}]$, the optimal price that maximizes expected profits for the heterogeneous consumers model is given by*

$$p_B^* = \left(\frac{1}{2}v_L + \frac{v_{H+} - v_{H-}}{\log(v_{H+} - v_L) - \log(v_{H-} - v_L)}\right).$$

Proof. Sketch of the proof provided here. See the online appendix for the complete proof.

$$\pi_B = \int_{v_{H-}}^{v_{H+}} \frac{v_H - p}{v_H - v_L} p \, dv_H.$$

The first order conditions give us

$$2p \int_{v_{H-}}^{v_{H+}} \frac{1}{v_H - v_L} dv_H = \int_{v_{H-}}^{v_{H+}} \frac{v_H}{v_H - v_L} dv_H$$

$$\Rightarrow \quad p = \frac{1}{2}\left(v_L + \frac{v_{H+} - v_{H-}}{\log(v_{H+} - v_L) - \log(v_{H-} - v_L)}\right). \quad \square$$

Corollary. *In the heterogeneous consumer model, the optimal Bayesian solution is strictly decreasing in $v_L$, or $\partial p_B^*/\partial v_L < 0$.*

Proof. See the online appendix for the proof. $\square$

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