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Going Where the Ad Leads You: On High Advertised Prices and Searching Where to Buy

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An important role of informative advertising is to inform consumers of the simple fact that the shop that advertises sells a particular product. This information may help consumers to save on their search activities: instead of wandering around, a consumer can simply visit the shop that has advertised, knowing that there he can find the commodity he is looking for. The implications of this simple fact have not been studied before. Using game theoretic reasoning in a model that combines consumer search and firms' advertising we show that firms may find it optimal to advertise prices that are higher than nonadvertised prices. The important mechanism underlying this result is that advertising lowers the expected search cost for consumers. Through this analysis we provide a new insight into the role of informative advertising.

Key words: consumer search; informative advertising; pricing strategy

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1. Introduction

Imagine yourself attending a conference in a foreign country. You sit with your laptop in a hotel room and realize that when your battery expires you cannot charge it because of the different electricity outlet system. You decide to go out in town to search for an electricity converter and enter a first electronics shop, where the charming sales representative tells you that they unfortunately do not carry such an item in their store. The same story repeats in a number of stores, after which you disappointingly go back to the hotel. In a last desperate attempt you ask at the hotel lobby whether they, by any chance, would know a shop where they carry the item you are looking for. Triumphant, the clerk at the desk tells you that a firm has left an advertisement behind informing people that they carry all different types of electronic converters one may ever wish to use (possibly with the prices at which they sell). You are very happy for this piece of information, immediately go to the shop, and are prepared to buy at any (somewhat reasonable) price.

This story contains an element that we believe is important in many markets, not just when hanging around in far away destinations: namely, that part of the search activities of people is not about "searching for firms with the lowest price," but rather about "searching for firms that sell the product." This distinction has not been made before. The typical search

model only considers situations where all firms in the market carry the product and the only reason for consumers to search (further) is to look for a price-quality combination that better fits the individual's preferences. That is, the literature on consumer search is not about the "real" search activity of consumers when they are uncertain about which firms carry the product.

Another important aspect of the story above is that a potentially important role of advertising is simply to inform consumers about the fact that the advertising firm carries the product, thereby helping the consumer to save on expected search costs. If the uncertainty about which firm carries the product is very large, then the reduction in "real" search costs may be quite significant. This, so the present paper argues, may lead to advertised prices being higher than nonadvertised prices. This is contrary to conventional wisdom expressed in the literature on informative advertising, according to which informative price advertising leads to better-informed consumers and therefore to more competition and lower prices (see, e.g., Farris and Albion 1980; Tirole 1998, §7.3). Thus, this paper contributes to the strategic literature on advertising by arguing that in the presence of uncertainty about which firms sell the product, informative price advertising may lead to higher prices compared

to the nonadvertised prices. This insight is also important in empirical work on advertising. It shows that one should be careful to conclude from an observed positive correlation between advertising and prices that advertising is persuasive; see, e.g., Boulding et al. (1994) and Clark (2007).

The mechanism we uncover is potentially also important in understanding emerging markets that use multimedia technologies. The first issue in such markets concerns discussions one can find in the popular press and in online discussion groups on personalized advertising.¹ Google CEO Eric Schmidt has claimed that over the next few years he wants to be able to send personalized ads on GPS-based in-car communication systems to consumers to help them find the shops they are looking for. One interpretation of this would be that personalized advertising helps consumers to get products that are really close to their tastes, i.e., in a world where product heterogeneity is important. Another interpretation of personalized advertising is one where these ads help consumers to economize on their search costs by directing their search activities on the firm(s) from which they received an ad. This second interpretation comes very close to what we analyze in this paper.

A second aspect of these markets relates to the commercial advertisements one sees when using websites, like <http://www.allbookstores.com/>, where book prices of different online bookshops are quoted. These websites offer the option to search for the lowest price or to buy directly at one particular online bookseller. Apparently, this bookseller is willing to pay for this link, suggesting that a significant fraction of consumers does not continue the search, but simply buys immediately at the firm where they know they can buy the book. Something similar happens at well-known search engines like Google and Yahoo, where firms pay to get their firm's website listed, knowing that (a fraction of) consumers are likely to click on one of the first-listed sites first.² Firms may exploit this search behavior by charging higher prices to compensate for the advertising expenditures.³

This paper studies "searching for the product" and "high prices through informative advertising" in relation to each other. To this end, we develop a sim-

ple three-stage model.⁴ In the first stage, firms decide whether or not they want to allocate shelf space to a particular type of product. Doing so has an opportunity cost of not using that space for having some other commodity on display. We call all firms who decide to carry the product "active firms." In the second stage, active firms decide on their price and on whether they will advertise the fact that they carry the product (and the price at which they sell it) by sending an advertisement to consumers. In the third stage, after potentially having received some ads, consumers decide on whether or not to search for a firm that carries the product, with a potentially lower price than the firms that advertised. However, if the firm has not advertised, the consumer does not know whether or not the firm carries the product in the first place.

The simplest search model we can imagine that makes the point that advertised prices can be higher than nonadvertised prices, even under informative price advertisement, has two types of consumers: low-valuation consumers having low search costs c_L and high-valuation consumers having higher search costs c_H . For simplicity, we normalize c_L to be equal to zero. One may think of the high-demand consumers as having high income from demanding jobs to justify the positive correlation between the size of consumers' search cost and their willingness to pay. In previous literature this positive relation between a consumer's willingness to pay and her search costs has been used by, e.g., Iyer (1998) and Coughlan and Soberman (2005).

In such a model, there may be many different types of equilibria, depending on the parameter configurations. We focus on equilibria where advertised prices are higher than nonadvertised prices.⁵ The simplest such equilibrium has the following structure. Low-valuation consumers search all firms and then buy at the lowest-priced firm. As soon as high-valuation consumers receive an advertisement, they buy at the advertising firm. High-demand consumers who get no advertisements do not search for an active firm as they find the probability that these firms do not carry the product too high compared to the search costs they have to make. Nonadvertising firms therefore completely concentrate on the low-valuation consumers, whereas advertising firms concentrate on high-valuation consumers. The probability that firms

¹ See, e.g., <http://blogs.zdnet.com/micro-markets/index.php?p=131>.

² In this sense, our paper is related to the modeling of page views on the Internet (see, e.g., Danaher 2007).

³ The main difference between this example and our paper is that prices are typically not mentioned in these advertisements. Without price advertisement it is, however, easier to see that advertising firms may set higher prices (see §6 for more details on this), and the present paper basically makes the additional point that even if prices were advertised, they still can be higher and attract consumers.

⁴ The model we develop abstracts from some of the features that are relevant to the examples mentioned above. For example, we abstract from product heterogeneity (that probably is important even in the book example when we consider that the book sale comes with a certain level of additional services) and also from nonprice advertisement (see also the concluding remarks).

⁵ In this context, the paper may contribute to an understanding of performance regimes (as in Pauwels and Hanssens 2007) or the implications of different advertising themes (cf. Bass et al. 2007).

are active and the intensity with which active firms advertise are determined endogenously in such a way that firms are indifferent between being inactive, advertising high prices, and not-advertising and setting low prices. We show for which parameter values such an equilibrium exists, and in addition, we show for which subset of these parameter values the equilibrium is unique. The main result is thus best interpreted as a screening result: high-valuation consumers buy at the advertising firms (if any) at high prices and the low-valuation consumers buy at the nonadvertising firms (if any) at low prices.

The paper is, of course, related to the large literature on consumer search and advertising (see, e.g., the seminal papers by Stigler 1961; Diamond 1971; Stahl 1989, 1994; Butters 1977). The main difference with the consumer search literature is, as we mentioned, the uncertainty consumers face of not finding the product at a shop they visit. As Diamond (1971) has shown, the price uncertainty in search models can lead to monopoly prices. Rao and Syam (2001) obtain the same type of result in an advertising model. Here, uncertain prices in the sense of unadvertised prices are above advertised prices. Our results are exactly the reverse: known, advertised prices are higher than unknown, unadvertised prices. One of the reasons why the Diamond effect does not play a role in our model is that there is a fraction of consumers with zero search costs.

In the advertising literature the main distinction is between persuasive and informative advertising. As indicated above, the main role of advertising in our model is to inform consumers of the availability of the product at the firm. In this way, advertising creates value to consumers and this value is evaluated differently by different consumers. The value that is created is different from that studied in the literature on persuasive advertising (see, e.g., Dixit and Norman 1978), however. This persuasive advertising literature basically argues that advertising changes the preferences of individuals and the demand effect that emerges is mainly dependent on psychological factors (exogenous to the model) determining how much people's preferences have been affected. In our paper, preferences of individuals are unaffected and the only effect of advertising, namely, the reduction in expected search costs, is endogenously determined.

In the informative advertising literature (see Meurer and Stahl 1994, Soberman 2004), price differentiation sometimes arises from horizontal product differentiation. Informative advertising plays two roles in this context. First, it creates awareness of products so that consumers know what best fits their tastes. This strengthens the product differentiation aspect, giving firms an incentive to raise prices. On the other hand, it also leads to more consumers

with full information giving firms an incentive to reduce prices. When product differentiation is important enough, the first aspect dominates the second so that advertising can lead to price increases. In our model, however, all firms are *ex ante* identical and any form of differentiation is thus endogenously created.

The literature that combines consumer search and advertising is much more limited.⁶ Robert and Stahl (1993) is the first paper where consumers' ignorance about prices can be resolved by consumers searching for prices or by firms informing consumers about the prices they charge through advertising. Following on their work, Stahl (2000) and Janssen and Non (2008) check the robustness of the model by investigating the properties of different modeling assumptions. Janssen and Non (2008) show that in some equilibria advertised prices may be higher than nonadvertised prices. This result in their model is driven by the assumption that less-informed consumers can buy at firms that advertise without incurring search cost, giving advertising firms an advantage above nonadvertising firms that have to be searched for. In the present paper we formally explain how this difference in search cost to buy from an advertising and a nonadvertising firm can emerge out of the uncertainty consumers face when they visit a shop that did not advertise.

Finally, this version of the paper—with the assumption of low-valuation consumers having zero search cost—is close in spirit to papers understanding the nature of online price competition, modeling the importance of shopbots and search engines (see, e.g., Chen and Sudhir 2004, Iyer and Pazgal 2003, Janssen et al. 2007, and most notably He and Chen 2006). He and Chen (2006) analyze behavior of e-marketplaces where consumers have the possibility to use the embedded search engine or they can shop directly at a featured store. They show that prices at the featured stores can be higher, but that consumers with relatively high search cost may still want to shop there as by doing so they economize on their time. Insofar as advertising is similar in nature to featuring, the results of the present paper reinforce the insight obtained by He and Chen (2006). Our paper shows that this insight is not confined to e-marketplaces, and that it may hold under a very different set of assumptions. For example, we employ sequential instead of

⁶ Apart from the literature mentioned here, there is also a recent paper by Stivers and Tremblay (2005). Their model is, however, very different from the standard search models as they model search costs as the wedge between producer prices and consumer prices, very much like the analysis in traditional tax studies. Moreover, they assume that advertising lowers the search costs of consumers. In such a world, they show that it is possible that advertising raises the price the firms ask, while at the same time decreasing the price (including search costs) that consumers have to pay.

simultaneous search, we use price advertising instead of nonprice advertising, and consumers in our model are not loyal to any of the firms. Moreover, advertising in our model is a signal of product availability, an aspect that is absent in previous models. Since the gains of buying from an advertising firm depend on the product availability in nonadvertising firms and product availability is endogenously determined, in our model the gains of buying from an advertising firm are endogenous. In previous models these gains are exogenously given. Finally, in the working paper on which this paper is based (Janssen and Non 2006), we show that even if both groups of consumers have positive search costs, the result that advertised equilibrium prices are higher than nonadvertised prices does hold.

The rest of this paper is organized as follows. Section 2 presents the model. In §3 we analyze a simplified version of the model where we show that the result of high advertised prices does not depend on price uncertainty but instead is caused by uncertainty about whether the firm carries the product. Section 4 formally characterizes one equilibrium where advertised prices are higher than nonadvertised prices. In §5 we will elaborate on the existence of other equilibria and we will show that the equilibrium with high advertised prices is unique in some parameter region. Section 6 concludes.

2. The Model

Consider a homogeneous goods market where (at most) two firms produce without incurring production costs.⁷ The only cost relevant for our analysis is the opportunity cost, S , firms face for shelving the product. The decisions firms take are modeled as a three-stage game. In the first stage, firms decide on whether or not to carry the product. We will denote the probability of a firm being active by β . If a firm decides to be inactive, it makes no profits or losses.

In the second stage of the model, firms decide simultaneously on their advertising strategy and price. Firms do not know the outcome of the first stage (the decision to be active), and therefore, it is just as if firms play these two stages simultaneously. An active firm can decide to advertise that it sells the product and at which price. Note here that advertising is purely informative: an advertisement informs about existence and price. Advertising is an “all-or-nothing” decision; that is, a firm either advertises to the complete market or does not advertise at all. The cost of advertising is A . We will denote the probability

with which a firm advertises by α . The pricing strategy depends on whether a shop advertises or not. We will therefore specify a pricing strategy conditional on advertising and a pricing strategy conditional on not advertising. Denote by $F_1(p)$ the price distribution conditional on advertising and let $F_0(p)$ denote the price distribution conditional on not advertising. We use \bar{p}_1 to indicate the highest price and p_1 to indicate the lowest price in the support of $F_1(p)$. Similarly, \bar{p}_0 and p_0 denote the highest and lowest price in the support of $F_0(p)$.⁸

The “all-or-nothing” advertising technology used in the model may seem somewhat unrealistic at first sight. Many other advertising models (e.g., Butters 1977, Stahl 1994, Robert and Stahl 1993) assume that firms choose an advertising reach, indicating the fraction of consumers who are informed by an advertisement. This advertising intensity generally depends on the price chosen, and so a firm’s strategy in this context can be denoted by a price distribution $F(p)$ and an advertising function $\kappa(p)$, indicating the advertising reach conditional on a price p . When the advertising costs are linear and given by $A\kappa(p)$ it can be shown⁹ that the two formulations are equivalent, meaning that if in the “all-or-nothing” model α , $F_0(p)$ and $F_1(p)$ are part of an equilibrium, then $F(p) = (1 - \alpha)F_0(p) + \alpha F_1(p)$ and $\kappa(p) = \alpha f_1(p) / ((1 - \alpha)f_0(p) + \alpha f_1(p))$ form an equilibrium in the “advertising reach” model. On the other hand, when $F(p)$ and $\kappa(p)$ are part of an equilibrium in the “advertising reach” model, then $\alpha = \int_p^{\bar{p}} \kappa(p) dF(p)$, $F_0(p) = \int_p^{\bar{p}} 1 - \kappa(p) dF(p) / (1 - \alpha)$, and $F_1(p) = \int_p^{\bar{p}} \kappa(p) dF(p) / \alpha$ form an equilibrium in the “all-or-nothing” model. Since the two formulations are equivalent and the “all-or-nothing” model is easier to analyze, we will use this formulation throughout the paper.

In the third stage of the game, consumers receive the advertisements that are sent and decide on their search strategy. There is a unit mass of consumers with unit demand. Consumers come into two types. A fraction γ has a low-valuation θ_L for the product and zero search costs.¹⁰ A fraction $1 - \gamma$ has a

⁷ Extending the model to more than two firms does not change the results but only complicates the analysis. Production costs can be easily introduced, but they are simply normalized to be equal to 0.

⁸ Note that the price distributions $F_0(p)$ and $F_1(p)$ can describe randomized price strategies as well as pure strategies. For instance, if advertising firms use the pure strategy of always setting price p^* , then $F_1(p) = 0$ for $p < p^*$ and $F_1(p) = 1$ for $p \geq p^*$. It is, however, easy to see that in equilibrium advertising firms always use a randomized price strategy. For an advertising firm there is a strictly positive probability $\alpha\beta$ that the competitor advertises as well, and so a standard undercutting argument can be used to show there are no atoms in $F_1(p)$.

⁹ Details can be found in the Technical Appendix, available at <http://mktsci.pubs.informs.org>.

¹⁰ Note that because of the zero search costs these consumers know all active firms and prices. Firms compete for these consumers and

high-valuation θ_H for the product and strictly positive search costs c . We assume $\theta_H - c > \theta_L$. Consumers search sequentially, conditional on the advertisements that are sent. Sequential search means that the consumers first look at the advertisements they have received. They then decide on whether to visit one additional firm, to buy immediately from the cheapest advertising firm, or not to buy at all. After searching one firm they again decide on whether to visit a second firm, to buy from the cheapest known firm, or not to buy at all. Note that consumers only search nonadvertising firms, since they already know that advertising firms are active and the ad also tells at which price the active firm sells. We assume that for the high-valuation consumers every first visit to a firm costs c .¹¹ Visiting an advertising firm, a consumer has to bear the search costs, but is sure to find the product. Visiting a nonadvertising firm, a consumer incurs the same search costs, but in this case is not sure to find the product. This implies that the “real” search cost for searching a nonadvertising firm is higher than the search cost for searching an advertising firm.

We will solve this model for a symmetric perfect Bayesian equilibrium, the standard equilibrium notion for games with asymmetric information. Symmetry implies that all *ex ante* identical players play the same strategy. Bayesian updating plays a role when consumers form expectations about the probability a firm is active given that it did not advertise.

3. Equilibrium Under Price Certainty

For didactical purposes, and to highlight that not price uncertainty, but uncertainty about product availability, is the key element driving the result mentioned in the introduction, we first consider a version of the model where firms do not freely choose prices, but instead choose between an exogenously given high price p_H and low price p_L , with $\theta_H - c \geq p_H > \theta_L > p_L$. The question we address is whether there exists an equilibrium with the same features as the equilibrium mentioned in the introduction: the high price is advertised, the low price is not advertised and high-valuation consumers do not search. Such an equilibrium has price certainty for searching consumers as all firms that do not advertise set the same price. We concentrate on a symmetric equilibrium and

in this case that means that both firms are indifferent between choosing the high price p_H and advertising it and choosing the low price and not advertising it. It is clear that for a symmetric equilibrium to exist, it must be the case that the probability with which a firm is active is strictly smaller than one as otherwise a consumer who did not receive any advertisements or received only one advertisement will infer that the nonadvertising firm(s) is (are) active and charge(s) a low price, making it profitable to search. Thus, in such an equilibrium a firm has to be indifferent between being active, and not being active, implying that the equilibrium payoff should equal zero.

We pose that there exists a symmetric equilibrium where advertising firms charge price p_H and nonadvertising firms charge a price p_L , where high-valuation consumers buy at an advertising firm without searching and where high-valuation consumers who get no advertisements do not search at all. Low-valuation consumers observe all prices and only buy if a firm sells at p_L .

We now have to check that this is indeed an equilibrium, i.e., that none of the players has an incentive to deviate. We first look at consumer behavior and note that if it is not optimal to search after not receiving any advertisements, then it certainly is not optimal to search after receiving one advertisement as the utility of following the ad is positive. We therefore concentrate on the case where high-valuation consumers did not receive any advertisements. If their first search leads to an active firm, they will stop searching since a lower price cannot be found. If they search once and do not find an active firm, the utility of a second search is $-c + ((1 - \alpha)\beta/(1 - \alpha\beta))(\theta_H - p_L)$.¹² If this expression is below zero, the utility of not searching, consumers will not search a second time and the utility of the first search also is $-c + ((1 - \alpha)\beta/(1 - \alpha\beta))(\theta_H - p_L) < 0$. Therefore if this inequality holds, consumers will not search at all.

Given the strategy of the high-valuation consumers, the payoffs to a firm choosing p_H and advertising are given by

$$\pi_H = p_H(1 - \gamma)(1 - \frac{1}{2}\beta\alpha) - A - S$$

and the payoffs to a firm choosing p_L and not advertising are given by

$$\pi_L = p_L\gamma(1 - \frac{1}{2}\beta(1 - \alpha)) - S.$$

As $\pi_H = \pi_L = 0$ in the equilibrium we are looking for, we can determine α and β endogenously. It follows

therefore an undercutting argument shows there are no atoms in $F_0(p)$ for $p \leq \theta_L$. In an earlier version, we analyzed a model where the low-valuation consumers had a strictly positive search cost. The equilibrium with high advertised prices is also an equilibrium in this modified model, but we have not been able to prove uniqueness.

¹¹ For simplicity, we follow the search literature in assuming “free-recall”; i.e., consumers do not bear any costs for return visits to firms they already visited once.

¹² Note that consumers only search nonadvertising firms and therefore the probability of finding an active firm is given by $(1 - \alpha)\beta/(1 - \alpha\beta)$.

that $\beta = 4 - 2(A + S)/(p_H(1 - \gamma)) - 2S/(p_L\gamma)$ and $\alpha = (2/\beta) - (2/\beta)((A + S)/(p_H(1 - \gamma)))$. As $0 < \alpha < 1$ and $0 < \beta < 1$ this gives restrictions

$$\frac{A + S}{p_H(1 - \gamma)} < 1,$$

$$\frac{S}{p_L\gamma} < 1,$$

and

$$3 - 2\frac{A + S}{p_H(1 - \gamma)} - 2\frac{S}{p_L\gamma} < 0.$$

Firms should also not have an incentive to deviate. If a firm would set p_H and not advertise, its profits are $-S$, which is clearly below the equilibrium profits of zero. If it sets a price p_L and advertises this price, its profits are given by

$$p_L(1 - \gamma) + p_L\gamma(1 - \frac{1}{2}\beta(1 - \alpha)) - A - S.$$

Using the expressions for α and β that are given above we can rewrite these profits as $p_L(1 - \gamma) - A$, and so for this deviation not to be profitable it should be that $p_L(1 - \gamma) < A$.

It is easy to find parameter values such that all restrictions are satisfied. For example, take $\theta_L = 1$, $p_L = 0.9$, $\gamma = 0.5$, $A = 0.75$, $S = 0.375$, and $p_H = 2.5$. Then if we choose θ_H and the search cost parameter c such that $2.5 + c < \theta_H < 0.9 + 2.4c$, all conditions are satisfied.

4. Existence of an Equilibrium with High Advertised Prices

The previous section has shown that the result of high advertised prices is not driven by price uncertainty but instead by uncertainty about the availability of the product. In this section we will construct a symmetric equilibrium of the full model specified in §2 where firms are free to choose any price. In this equilibrium, again, advertising firms set higher prices than non-advertising firms and high-valuation consumers do not search. The equilibrium is symmetric in the sense that both firms choose the same (ex ante) strategy. Their ex post behavior (whether or not to advertise and which price to choose) may be different because of different realizations of the mixed strategy chosen.

Intuitively, when high-valuation consumers do not search the nonadvertising firms compete for the low-valuation consumers. Advertising firms, however, also sell to the high-valuation consumers and therefore have an incentive to ask higher prices. Even in such a case high-valuation consumers do not want to search for the lower nonadvertised prices as long as the probability of a firm being active, β , is low enough.

PROPOSITION 4.1. *If the following four conditions are satisfied:*

1. $(1 - \gamma)\theta_L < A < (1 - \gamma)(\theta_H - c) - S$;
2. $S < \gamma\theta_L$;
3. $1 - S/(\gamma\theta_L) - (A + S)/((1 - \gamma)(\theta_H - c)) < 0$;
4. $\theta_H(1 - S/(\gamma\theta_L)) - S/\gamma \ln(\gamma\theta_L/S) < (A + S)c/((1 - \gamma)(\theta_H - c))$;

then there exists a symmetric equilibrium with $p_1 > \bar{p}_0$. In this equilibrium consumer search behavior is characterized by

(i) *Low-valuation consumers buy at the cheapest active firm out of all firms, provided the price is not above θ_L .*

(ii) *High-valuation consumers who receive at least one advertisement buy immediately at the the advertising firm with the lowest price, provided the price is not above $\theta_H - c$.*

(iii) *High-valuation consumers who do not receive an ad do not search.*

Firm behavior is characterized by

(i) *Firms are active with probability $\beta = 2 - S/(\theta_L\gamma) - (A + S)/((1 - \gamma)(\theta_H - c))$.*

(ii) *Active firms advertise with probability*

$$\alpha = \frac{1 - (A + S)/((1 - \gamma)(\theta_H - c))}{(2 - S/(\theta_L\gamma) - (A + S)/((1 - \gamma)(\theta_H - c)))}.$$

(iii) *Nonadvertising firms choose prices according to distribution*

$$F_0(p) = 1 - \frac{(\theta_L - p)(1 - \beta(1 - \alpha))}{p\beta(1 - \alpha)},$$

with $\bar{p}_0 = \theta_L$ and $p_0 = \theta_L(1 - \beta(1 - \alpha))$.

(iv) *Advertising firms choose prices according to distribution*

$$F_1(p) = 1 - \frac{(\theta_H - c - p)(1 - \beta\alpha)}{p\beta\alpha},$$

with $\bar{p}_1 = \theta_H - c$ and $p_1 = (\theta_H - c)(1 - \beta\alpha)$.

The proof can be found in the appendix and basically shows that under the four conditions specified in the proposition no player has an incentive to deviate from his strategy.

The interpretation of the four restrictions used in Proposition 4.1 is fairly straightforward. The first part of the first restriction tells us that the advertising costs A should be so high that advertising a price at or below θ_L is not profitable. This makes sure that advertising firms concentrate completely on the high-valuation consumers and enables our screening result. The second part of the first restriction states that the advertising costs A should be low enough to make advertising profitable¹³ while the second restriction states that the shelving costs S should be low enough

¹³Note that the profits of an advertising firm are equal to $(\theta_H - c)(1 - \gamma)(1 - \alpha\beta) - A - S$ and that when $A + S$ is close to $(\theta_H - c)(1 - \gamma)$, α is close to 0.

to make shelving the product a profitable strategy. On the other hand, the third restriction states that both advertising and shelving costs should be high enough to guarantee that firms do not always want to be active, which in turn is needed as otherwise high-valuation consumers want to continue searching after receiving one advertisement with a relatively high price. Finally, the last restriction basically gives a lower bound on the search costs c such that consumers do not want to search.

The first restriction implies that $\theta_H - c > \theta_L$, which can be interpreted as that the difference in valuations between the two types of consumers should be high enough. This restriction is necessary for any equilibrium where advertised prices are higher. To make this clear, suppose to the contrary that $\theta_H - c < \theta_L$. The maximum price that an advertising firm will then ask is still $\theta_H - c$. This is because if an advertising firm sets a price between $\theta_H - c$ and θ_L it would only sell to low-valuation consumers and so there would be no reason to advertise. For nonadvertising firms it could, however, be profitable to set a price above $\theta_H - c$. There are two reasons for this. First, low-valuation consumers are willing to pay θ_L and so a nonadvertising firm setting a price θ_L has a probability of $1 - \beta$ to be the only firm in the market and to sell to the low-valuation consumers. Second, searching high-valuation consumers are willing to pay θ_H when they find an active nonadvertising firm. This difference in willingness to pay between an advertising firm and a nonadvertising firm comes from the search costs being sunk when visiting a nonadvertising firm. It can be shown that when $\theta_H - c < \theta_L$, it is indeed always profitable for a nonadvertising firm to ask a higher price than an advertising firm and so an equilibrium where advertising firms charge higher prices than nonadvertising firms is not feasible anymore.

We now want to check whether all four restrictions can jointly hold, that is, whether there really exists a parameter region where the above equilibrium exists. To do so we set, without loss of generality, $\theta_L = 1$.¹⁴ It is easily checked that the four restrictions can jointly hold when, for instance, $\gamma = 0.1$, $S = 0.099$, $A = 1$, $c = 0.04$, and $1.3 < \theta_H < 2.5$. Moreover, we would like to establish the following result, arguing that the existence conditions become easier to satisfy when both θ_H and c are relatively large.

PROPOSITION 4.2. *If for some values A^* , S^* , θ_H^* , c^* , and γ^* the equilibrium of Proposition 4.1 exists, then the equilibrium also exist for values A^* , S^* , $\theta_H^* + x$, $c^* + x$, and γ^* , where x is an arbitrary positive number.*

¹⁴ Looking at the equilibrium shows that if the parameters θ_L , θ_H , A , S , and c are multiplied by a scalar x , the same equilibrium emerges, except that the prices are multiplied by x . To ease the analysis we decided to normalize the parameters such that $\theta_L = 1$.

The proof can be found in the appendix. Below we provide an intuitive explanation for the result. The expressions for the strategy of the firms, $(F_0(p), F_1(p), \alpha, \beta)$, given in Proposition 4.1 do not depend on θ_H or c in isolation, but instead only depend on $\theta_H - c$, since this is the maximum price that advertising firms can ask. As the first three restrictions are imposed to guarantee that α and β are in between 0 and 1, these restrictions remain unaffected as long as $\theta_H - c$ is unaffected.

The fourth restriction ensures that consumers do not want to search. Consumers who do not receive any advertisement have to make a trade-off between incurring search costs c and having a probability of finding an active firm, which would give an uncertain payoff of $\theta_H - p$, where p is the uncertain price being found. Suppose that for some parameters θ_H^* and c^* it is optimal not to search, so the search costs are above the expected payoff. Then if both c^* and θ_H^* increase by x , the expected payoff, given by $(\beta(1 - \alpha)/(1 - \alpha\beta))(\theta_H - p)$, with $\beta(1 - \alpha)/(1 - \alpha\beta) < 1$, also increases but with less than x . Therefore, if both c^* and θ_H^* increase by x , it is still not optimal to search and the fourth restriction is still satisfied.

5. Other Equilibria

The model we specified in §2 has many possible equilibria. It is natural to inquire whether for some parameter values the equilibrium we have identified in the previous section is the unique symmetric perfect Bayesian equilibrium. If the equilibrium is unique, we can legitimately argue that there is a parameter region where advertised prices are higher than nonadvertised prices. If such a parameter region does not exist, then there is a possibility that advertised prices are higher, but this cannot be guaranteed. In this section, we do three things. First, we provide some observations on how the reservation price characterizing consumer search behavior is determined. This reservation price is needed in the remainder of this section and the exact derivation of it can be found in the Technical Appendix, available at <http://mktsci.pubs.informs.org>. Next, we illustrate by means of an example that there are parameter values where the equilibrium described in the previous section exists, but it is not unique. Finally, we prove that when θ_H and c are relatively large, the equilibrium described in the previous section is unique.

As in any consumer search model, we can define a reservation price r by

$$\int_{p_0}^{r+c} F_0(p) dp = \frac{(1 - \alpha\beta)}{(1 - \alpha)\beta} c,$$

where $\int_{p_0}^{r+c} F_0(p) dp$ is the benefit of an additional search provided the firm found is active and did not

advertise; $((1 - \alpha)\beta)/(1 - \alpha\beta)$ is the probability a firm is active conditional on that it does not advertise. When high-valuation consumers receive an advertisement, they buy immediately when the lowest advertised price is at or below r , while for higher prices they search until they find an active nonadvertising firm asking a price at or below $r + c$ or until all firms have been searched. When high-valuation consumers have not received any advertisement, they search when $r + c < \theta_H$, they do not search when $r + c > \theta_H$, and they are indifferent between searching and not searching if $r + c = \theta_H$. When they search, they continue to search until they find an active firm asking a price at or below $r + c$ or until all firms have been searched. The derivation of this result follows the usual lines, except that we need to take into account that consumers have to pay search costs when visiting any firm, independent of whether it advertises or not. Moreover, there is a difference between advertising and nonadvertising firms in the sense that when a consumer observes the price set by a nonadvertising firm, its search costs are already sunk, while it still has to make the search costs just after having observed the advertised price.

EXAMPLE (PURE CONSUMER SEARCH). In the previous section we noted that the equilibrium with high advertised prices exists for $\gamma = 0.1$, $S = 0.099$, $A = 1$, $c = 0.04$, and $1.3 < \theta_H < 2.5$. We now show that for these very same parameter values a “pure search equilibrium” exists where firms do not advertise ($\alpha = 0$) and where all firms are active ($\beta = 1$). In this example we look at the case where $r + c < \theta_L < \theta_H - c$, implying that all high-valuation consumers search. A traditional undercutting argument shows that when $p > r + c$, there are no atoms and so $\pi_0(\bar{p}_0) = -S$ for any $\bar{p}_0 > r + c$. On the other hand, \bar{p}_0 cannot be smaller than $r + c$ either, since deviation to $r + c$ would be profitable. For any price $p \leq \bar{p}_0 = r + c$ profits are given by

$$\pi_0(p) = p\gamma(1 - F_0(p)) + p(1 - \gamma)\frac{1}{2} - S.$$

Equating this to $\pi_0(r + c) = \frac{1}{2}(1 - \gamma)(r + c) - S$ gives

$$F_0(p) = 1 - \frac{(1 - \gamma)(r + c - p)}{2p\gamma},$$

and $p_0 = (\frac{1}{2}(1 - \gamma)/(\gamma + \frac{1}{2}(1 - \gamma)))(r + c)$. Furthermore, $\int_{p_0}^{r+c} F_0(p) dp = c$ gives $r = c((1 - \kappa)/\kappa)$, with $\kappa = 1 - (\frac{1}{2}(1 - \gamma)/\gamma) \ln(\frac{1}{2}(1 - \gamma) + \gamma/(\frac{1}{2}(1 - \gamma)))$. This equilibrium holds whenever $\pi_0 > 0$, $\pi_1 < \pi_0$, and $r + c < \theta_L = 1$. Note that the first restriction holds if, and only if, $\pi_0 = \frac{1}{2}(r + c)(1 - \gamma) - S > 0$, while the second holds if $\frac{1}{2}(1 - \gamma)r < A$. Substituting the parameter values that are given above shows that these three restrictions do hold for these values. \square

The intuition behind the co-existence of multiple equilibria is as follows. For the given parameter values, S is very close to γ . In the equilibrium with high advertised prices described in §4 the maximum profits from not advertising, realized when the competitor advertises its high price or is not active at all, are $\gamma - S$. When γ is close to S these maximum profits are low and to make not advertising attractive the probability of obtaining these profits should be high. This implies that β has to be low. A low value of β also means that high-valuation consumers have no incentives to search, even when the search costs c are low. The equilibrium with high advertised prices can therefore also exist for low values of c . For such low values of c it is, however, also possible to have equilibria where consumers search (as in the above example). If consumers continue to search, it is not profit maximizing to advertise and a standard consumer search equilibrium emerges as discussed in the example.

We note that in the example above firm profits are strictly positive while in the equilibrium with high advertised prices profits equal zero. The intuition behind this is the difference in search behavior of consumers. In the example, all high-valuation consumers search and firms set prices in such a way that these consumers buy at the first firm they find. This implies that firms have monopoly power over the high-valuation consumers while they compete for the low-valuation consumers. Since in our numerical example the fraction of high-valuation consumers, $1 - \gamma$, is relatively high, firms can extract high profits from these consumers. In contrast, in the equilibrium with high advertised prices high-valuation consumers do not search. This means that nonadvertising firms only sell to the low-valuation consumers and in the best case realize (low) profits $\gamma - S$, while advertising firms have to pay relative high advertising costs to be able to sell to the high-valuation consumers, and consequently realize low profits as well. Another way to explain the difference in profits is to note that in the equilibrium with high advertised prices, firms are indifferent between being active and not being active. This is needed to ensure that high-valuation consumers do not want to search. Since being inactive leads to zero profits, firms that are active also should earn no profits at all. In the example above high-valuation consumers search and all firms are active, which means their profits can be strictly positive.

In the example above not only are firm profits higher than in the equilibrium with high advertised prices, but consumer welfare is also higher. We note that there are two types of consumers, with low valuations and with high valuations, and that for both types welfare is higher. The reason for this is the same for both consumer types and is twofold. First, in the example the prices are lower. In the numerical

example the maximum price is 0.41, while in the equilibrium with high advertised prices the maximum nonadvertised price is 1 and the maximum advertised price varies between 1.26 and 2.46, depending on the value of θ_H . The second reason for the difference in consumer welfare is the difference in the level of activity of the firms. In the example above, all firms are active and consumers search so in the end all consumers buy. In the equilibrium with high advertised prices firms are active with probability β and therefore there is a probability that no firm is active at all and consumers cannot buy. Furthermore, there also is a strictly positive probability that none of the active firms advertises and in that case only the low-valuation consumers buy. We note that in the numerical example β is fairly low,¹⁵ and so the difference in consumer welfare can be quite high. To give an example, the low-valuation consumers obtain a welfare of 0.0001 in the equilibrium with high advertised prices while welfare is 0.64 in the example above. The welfare for high-valuation consumers depends on θ_H and, e.g., for $\theta_H = 2$ welfare equals 0.28 and 1.59 respectively.

These observations on firm profits and consumer welfare show that the equilibrium with searching consumers Pareto dominates the equilibrium with high advertised prices. If one uses Pareto dominance as an equilibrium selection criterion the equilibrium with high advertised prices would therefore be rejected in favor of the equilibrium with searching consumers. In the remainder of this section we will show, however, that the coexistence argument crucially depends on the assumed small value of c . The next proposition shows that when θ_H and c are relatively large, the equilibrium of Proposition 4.1 is unique.

PROPOSITION 5.1. *Fix parameter values θ_H^* , c^* , S^* , γ^* , and A^* such that the equilibrium described in Proposition 4.1 exists. Then there exists a positive real number \hat{x} such that for all $x > \hat{x}$ the equilibrium described in Proposition 4.1 exists and is unique for all parameter values $\theta_H^* + x$, $c^* + x$, S^* , γ^* , and A^* .*

The existence part of this proposition has already been shown in §4. The proof of the uniqueness part can be found in the appendix. Here, we will provide an intuitive explanation. The equilibrium we defined in Proposition 4.1 has high-valuation consumers not searching in case they did not receive an advertisement. Other equilibria where consumers do not search are shown not to overlap with the equilibrium with high advertised prices. This is because in these equilibria the consumers behave in the same way, and therefore there is enough continuity in the firms' pricing and advertising decision problem to

prevent overlap in the equilibrium regions. Overlap can occur between equilibria with different consumer behavior. For example, as we showed above, the high advertised price equilibrium with no consumer search (partly) overlaps with an equilibrium where high-valuation consumers do search. Therefore it is necessary to rule out the coexistence of equilibria where consumers (partly) search, i.e., equilibria with $\beta > 0$ and $\alpha < 1$.¹⁶ We show in the proof that these equilibria, except for one equilibrium, do not exist for x high enough. The equilibrium that does exist even when x gets infinitely large is shown not to overlap for x high enough.

The two main arguments used to obtain this result are as follows. First, as argued before, consumers who did not receive any advertisements need to compare the search costs c with the possible gains from searching, $(\beta(1 - \alpha)/(1 - \alpha\beta))(\theta_H - p)$. When a constant x is added to both the search costs and θ_H , the search costs generally increase more than the gains from searching. Therefore, the higher the constant x , the more difficult it is to have an equilibrium where consumers search. Second, when high-valuation consumers search, nonadvertising firms may decide to concentrate on high-valuation consumers, and since the costs of searching a nonadvertising firm are sunk at the moment the consumer has arrived at the shop, these firms can ask a price equal to the maximum advertised price plus the search costs c without losing any customers. When the search costs increase, this option of not advertising and setting a high price becomes increasingly more attractive. This pricing strategy is, however, detrimental for consumer search since consumers who expect a high price will not search. So, this second argument exploits the fact that firms that advertise are committed to the price they offer before consumers search, whereas nonadvertising firms are not committed. The two arguments together rule out other equilibria for the case where θ_H and c are relatively large.

6. Conclusion and Extensions

The core of the argument developed in this paper centers around the uncertainty consumers face concerning the shops that carry the product they are looking for: some firms do have the product; others do not. This uncertainty is important in explaining consumer search behavior, but so far this type of uncertainty has not been considered in the large literature on consumer search. An important role of advertising in such a situation is to inform consumers that the advertising firm indeed sells the product. Advertising therefore can lower consumers' expected search

¹⁵ β is between 0.04 and 0.51, depending on the value of θ_H .

¹⁶ When $\beta = 0$, no firm is active and searching is not profitable.

costs. Since visiting an advertising firm comes with lower expected search costs than finding the product in a nonadvertising firm, advertising firms have an advantage above nonadvertising firms. In this paper we show that advertising firms can use this advantage to set higher prices. We have argued that the mechanism we uncover may be important in understanding recent developments in emerging markets using multimedia technologies.

We analyzed the case where advertisements contain both information on the availability of the product and on the price the advertising firm asks. As we noted in the introduction, however, in many cases where our argument applies, advertisements may not contain price information. It therefore would be interesting to analyze a model where advertisements only contain information on product availability. The analysis of such a model is not straightforward because of complications arising in consumer search behavior. Without price advertisement, consumers not only have to decide on whether they will search, but also on where to search: an advertising firm or a nonadvertising firm. Another complicating factor is that when prices are not advertised, advertising firms may exploit the fact that search costs are sunk at the time the price is revealed to consumers. Still, the argument used in this paper suggests that an equilibrium exists with high-valuation consumers visiting and buying at an advertising firm and not searching in case they did not receive an advertisement. Such an equilibrium with advertising firms setting high prices could hold as long as the probability of finding an active firm is low enough. The development of such a model is an interesting area for further research. Future research may also relax some of the restrictive assumptions we have employed, most notably the assumption that advertising costs are linear.

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Appendix

PROOF OF PROPOSITION 4.1. The proof consists of two parts. First, we show that the proposed strategy is indeed in the strategy space of the players, and second, we show that none of the players has an incentive to deviate from his proposed strategy.

To show that the proposed strategy is indeed in the strategy space of the players we need to show that $0 \leq \alpha \leq 1$, $0 \leq \beta \leq 1$ and that $F_0(p)$ and $F_1(p)$ are proper cdfs. To show that $0 \leq \alpha \leq 1$, we note that

$$\alpha = (1 - (A + S)/((1 - \gamma)(\theta_H - c)))/\beta$$

and so when $0 < \beta \leq 1$, $\alpha > 0$ whenever $1 - (A + S)/((1 - \gamma)(\theta_H - c)) > 0$, which is the second part of restriction 1. Restriction 2 ensures that $\alpha < 1$. The second part of restrictions 1 and 2 together also ensure that $\beta > 0$: restriction 2 gives $\beta > 1 - (A + S)/((1 - \gamma)(\theta_H - c))$ and the second part of restriction 1 gives $1 - (A + S)/((1 - \gamma)(\theta_H - c)) > 0$. Restriction 3 finally ensures that $\beta < 1$.

To show that $F_0(p)$ and $F_1(p)$ are proper cdfs, note that both $F_0(p)$ and $F_1(p)$ are increasing in p , $F_0(p_0)$ and $F_1(p_1)$ equal 0, and $F_0(\bar{p}_0)$ and $F_1(\bar{p}_1)$ are equal to 1. We also note that for $p_1 > \bar{p}_0 = \theta_L$ to hold, $A + S$ should be larger than $(1 - \gamma)\theta_L$. The first part of condition 1 ensures that this is indeed the case.

We now show that under the conditions specified, none of the players has an incentive to deviate. We first consider the search behavior of consumers. It is clear that since low-valuation consumers have no search costs they will search all firms and so know all active firms and their prices. For them it is optimal to buy at the cheapest of these firms, provided the price is not above the valuation for the product. High-valuation consumers who get two advertisements know that both firms are active and also know both prices. Again, it is optimal to buy at the cheapest of the two firms.

Next, consider the case where a high-valuation consumer did not receive any advertisements. Suppose such a consumer has searched already once and found an inactive firm so that the consumer has to decide whether to search once more. He will not search for a second time when the utility from searching is smaller than the utility from not searching, which gives

$$-c + \frac{\beta(1 - \alpha)}{1 - \alpha\beta} \int_{p_0}^{\theta_H} (\theta_H - p) f_0(p) dp < 0. \quad (1)$$

Integrating by parts and rearranging terms gives that searching a second time is not profitable when $\int_{p_0}^{\theta_H} F_0(p) dp < ((1 - \alpha\beta)/\beta(1 - \alpha))c$. This is the case if

$$\theta_H \left(1 - \frac{S}{\theta_L \gamma}\right) - \frac{S}{\gamma} \ln\left(\theta_L \frac{\gamma}{S}\right) < \frac{A + S}{(1 - \gamma)(\theta_H - c)} c,$$

which is condition 4.

Now suppose the consumer has searched once and found an active firm asking price $p^* \leq \theta_H$. The utility from searching a second firm is given by

$$\begin{aligned} -c + \frac{\beta(1 - \alpha)}{1 - \alpha\beta} \left[\int_{p_0}^{p^*} (\theta_H - p) f_0(p) dp + (1 - F_0(p^*))(\theta_H - p^*) \right] \\ + \left(1 - \frac{\beta(1 - \alpha)}{1 - \alpha\beta}\right) (\theta_H - p^*). \end{aligned}$$

Integrating by parts and rearranging terms gives

$$-c + \theta_H - p^* + \frac{\beta(1 - \alpha)}{1 - \alpha\beta} \int_{p_0}^{p^*} F_0(p) dp.$$

Since the utility from not searching is $\theta_H - p^*$ and $p^* \leq \theta_H$, it is easy to see that under condition 4 searching a second time is also, in this case, not profitable.

We conclude that if a consumer searches one time, he will (under condition 4) certainly not search a second time. However, this implies that the consideration of whether or

not to search the first time is exactly identical to the consideration of searching a second time after having found an inactive firm. Therefore, under condition 4 it is indeed not optimal to search at all if no advertisement was received.

If a high-valuation consumer receives a single advertisement with price $p^* \leq \theta_H - c$, his utility from buying at the advertising firm is $\theta_H - p^* - c$ (remember that a consumer who visits an advertising firm incurs search costs c). The utility of searching the nonadvertising firm is

$$-c + \frac{\beta(1-\alpha)}{1-\alpha\beta} \cdot \left[\int_{p_0}^{p^*+c} (\theta_H - p) f_0(p) dp + (1 - F_0(p^* + c))(\theta_H - p^* - c) \right] + \left(1 - \frac{\beta(1-\alpha)}{1-\alpha\beta} \right) (\theta_H - p^* - c).$$

Integrating in parts and comparing the two utilities shows that again under condition 4 searching is not profitable. This shows that consumers have no incentives to deviate from the strategy outlined in the proposition.

We next consider the behavior of firms. Let $\pi_0(p)$ denote the profits from not advertising and setting a price p and let $\pi_1(p)$ denote the profits from advertising a price p . Given the consumer behavior specified in the proposition we have for $p \leq \theta_L$,

$$\pi_0(p) = p\gamma(1 - \beta(1 - \alpha)F_0(p)) - S.$$

Substituting α , β , and $F_0(p)$ gives that $\pi_0(p) = 0$ for all $p_0 \leq p \leq \theta_L$. For $p < p_0$ we have that $\pi_0(p) = p\gamma - S < p_0\gamma - S = 0$ and for $p > \theta_L$ the firm does not sell anything so that $\pi_0(p) = -S < 0$. This shows that for a nonadvertising firm it is indeed optimal to choose a price between p_0 and \bar{p}_0 .

An advertising firm setting a price between θ_L and $\theta_H - c$ makes a profit of

$$\pi_1(p) = p(1 - \gamma)(1 - \beta\alpha F_1(p)) - S - A.$$

For prices above $\theta_H - c$, it does not generate any sales so that

$$\pi_1(p) = -S - A$$

and for prices below θ_L we have that

$$\pi_1(p) = p\gamma(1 - \beta(1 - \alpha)F_0(p)) + p(1 - \gamma) - A - S.$$

When we substitute the values for α , β , $F_0(p)$, and $F_1(p)$ we see that for $p_1 \leq p \leq \bar{p}_1$, $\pi_1(p) = 0$. Moreover, $\pi_1(p) < 0$ for $p > \bar{p}_1$. For $\theta_L < p \leq p_1$, $\pi_1(p) = p(1 - \gamma) - S - A \leq p_1(1 - \gamma) - S - A = 0$. For $p_0 < p \leq \theta_L$, $\pi_1(p) = p(1 - \gamma) - A \leq \theta_L(1 - \gamma) - A < 0$ where in the last inequality we use the first part of restriction 1. And lastly, for $p \leq p_0$, $\pi(p) = p - A - S \leq p_0 - A - S = S/\gamma - A - S = S(1/\gamma - 1) - A < \theta_L(1 - \gamma) - A < 0$, where the second inequality comes from restriction 2 and the last inequality from the first part of restriction 1. We conclude that for an advertising firm deviating from $F_1(p)$ is not profitable.

Since the profits from advertising and the profits from not advertising equal each other, there are no gains from deviating from the proposed advertising probability α , and since both profits equal 0 deviating from the proposed probability of being active β also does not lead to more profits.

We conclude that firms have no incentives to deviate from the firm strategy outlined in the proposition.

PROOF OF PROPOSITION 4.2. Assume that for A^* , S^* , θ_H^* , c^* , and γ^* the equilibrium exists, and so all four restrictions hold. The first three restrictions do not depend on θ_H or c in isolation but instead only depend on $\theta_H - c$. Since this value does not change when x is added to both θ_H^* and c^* , these three restrictions still hold. The fourth restriction depends on θ_H and c in isolation. When x is added to both θ_H^* and c^* , this restriction changes to

$$\theta_H^* \left(1 - \frac{S^*}{\gamma^*} \right) + x \left(1 - \frac{S^*}{\gamma^*} \right) - \frac{S^*}{\gamma^*} \ln \frac{\gamma^*}{S^*} < \frac{A^* + S^*}{(1 - \gamma^*)(\theta_H^* - c^*)} c^* + \frac{A^* + S^*}{(1 - \gamma^*)(\theta_H^* - c^*)} x. \quad (2)$$

Since (restriction 4 of Proposition 4.1)

$$\theta_H^* \left(1 - \frac{S^*}{\gamma^*} \right) - \frac{S^*}{\gamma^*} \ln \frac{\gamma^*}{S^*} < \frac{A^* + S^*}{(1 - \gamma^*)(\theta_H^* - c^*)} c^*$$

and (due to restriction 3)

$$x \left(1 - \frac{S^*}{\gamma^*} \right) < \frac{A^* + S^*}{(1 - \gamma^*)(\theta_H^* - c^*)} x,$$

inequality (2) holds, and so when x is added to θ_H^* and c^* , all restrictions are still satisfied and the equilibrium exists.

PROOF OF PROPOSITION 5.1. To prove Proposition 5.1 we need to check all possible equilibria of the model and show that neither of these overlaps with the equilibrium of Proposition 4.1 for x large enough. To this end, we classify the possible equilibria by the probabilities α and β . In the equilibrium of Proposition 4.1 these probabilities are strictly between 0 and 1. To save space in this appendix we only look at equilibria for which it is easy to show that they do not overlap with the equilibrium of Proposition 4.1 at all. The proof for the other equilibria can be found in the Technical Appendix, available at <http://mktsci.pubs.informs.org>.

(i) An equilibrium where $\beta = 0$ implies that profits of being active should be below zero. If a firm deviates and chooses not to advertise and sets a price $\theta_L = 1$, its profits are $\gamma - S$. Hence, an equilibrium where $\beta = 0$ only holds for $\gamma < S$, while condition (2) of Proposition 4.1 stipulates that $\gamma > S$.

(ii) In an equilibrium with $\alpha = 1$ and $0 < \beta < 1$ we should have that $\pi_1 = 0$. First suppose $p_1 > \theta_L$. If a firm deviates to not advertising a price $\theta_L = 1$, profits are at least equal to $\gamma - S$. As deviating should not be profitable, this equilibrium only holds for $\gamma < S$. Now suppose $p_1 \leq \theta_L$. It is easy to show that in this case θ_L is in the support of $F_1(p)$ and so $\pi_1(\theta_L) = 1 - \beta + \beta(1 - F_1(\theta_L)) - A - S = 0$, or $1 - \beta + \beta(1 - F_1(\theta_L)) = A + S$. Deviating to not advertising a price θ_L gives a profit that is at least as large as

$$\pi_0(\theta_L) = \gamma(1 - \beta + \beta(1 - F_1(\theta_L))) - S = \gamma(A + S) - S.$$

As deviating should not be profitable it follows that $\gamma(A + S) - S$ has to be smaller than 0. This restriction leads to a parameter region that does not overlap with a region defined by $S < \gamma$ and $A > 1 - \gamma$.

(iii) $\alpha = 1$ and $\beta = 1$. In this case an equilibrium does not exist. When every firm is active and advertises, we get Bertrand competition and equilibrium prices equalling 0. This leads to negative profits $-A - S$.

Note that the equilibria that are left to analyze are equilibria with $\alpha < 1$ and $\beta > 0$. For these equilibria we cannot universally show that they do not overlap with our equilibrium and in fact some of these equilibria partially overlap. As we show in the Technical Appendix, available at <http://mktsci.pubs.informs.org>, these equilibria, however, only overlap for relatively small values of θ_H and c .

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