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# Sale or Lease? Durable-Goods Monopoly with Network Effects

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**T**his paper studies the pricing problem of a durable-goods monopolist. It finds that contrary to the existing literature, profits from selling durable goods might be higher than from leasing when the products exhibit network effects. Under the influence of network effects, there exist multiple self-fulfilling equilibria that would sustain different network sizes at the same price. By using the assumption that consumers are cautious about network growth, we find that consumption externalities among heterogeneous groups of consumers generate a discontinuous demand function, which requires a lessor to offer a low price if she wants to reach the mass market. In contrast, a seller enjoys a relative advantage in that she can build a customer base by setting a lower initial price and raise the price later in the mass market. Our finding that selling can be more profitable than leasing holds when consumers are more cautious about the prospect of the product's success, which might be the case if, for example, the technology or manufacturer is relatively unknown.

*Key words:* selling and leasing; penetration pricing; network externality; introductory pricing; product life cycles

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## 1. Introduction

When the market for personal computers started to flourish in the early 1980s, Microsoft aggressively promoted its operating system by reducing the license fees of MS-DOS by half for a brief period of time.<sup>1</sup> Yet as soon as Microsoft prevailed in both operating system and application software for personal computers, it adopted a new pricing plan in which business users paid annual *subscription* fees.<sup>2</sup> The path of Microsoft's pricing strategies defied the conventional wisdom that dictates the relative advantage of leasing over selling for durable goods.

This paper proposes that Microsoft's pricing strategies might be driven by the network effects of its product. That is, to take advantage of network effects the firm wants to establish an installed user base in the early stage of a product's life cycle and selling proves to be a more effective strategy than leasing. The situation shifts when the market matures and the concern for building the network starts to fade.

Therefore, at a later stage the business strategy would resemble that of a conventional firm, which opts for leasing rather than selling a durable product.

The idea of network effects used here draws on the work of Rohlfs (1974), which holds that the more total buyers there are in the market, the higher each individual's willingness to pay. This attribute accurately describes a distinctive market characteristic for many products in the digital economy. In our paper, we predict the use of preemptive strategies such as the deep discount in the example of MS-DOS. Our goal is to provide a model that describes the two-staged pricing path during which the firm would switch from selling to leasing when it has sufficiently penetrated the market of a network product.

The impetus of our analysis comes from the observation that adopting an emerging technology takes time. Normally, the better-informed professional group adopts it first; then the information spreads gradually, leading to the adoption by less-informed, less professional groups, etc.<sup>3</sup> Rohlfs (1974) and Katz

<sup>1</sup> See Manes and Andrews (1993). Cabral et al. (1999) discuss other examples in computer software.

<sup>2</sup> Evidently, this new pricing strategy was so much in favor of Microsoft that its deployment provoked strong opposition from the customers. Microsoft eventually suspended the Office XP subscription plan for the U.S. market because of the pressure.

<sup>3</sup> Although Waldman (1993) considers a similar setup with different groups of consumers joining the market sequentially, consumers' willingness to pay in his model is assumed to be identical within each group. Therefore, pricing strategy is irrelevant in the monopolist's decision.

and Shapiro (1986) have pointed out that market expansion coupled with network effects could result in multiple market equilibria. In other words, the potential buyers' expectations could sustain different network sizes for the same price. Therefore, equilibrium selection plays a crucial role when determining the firm's pricing strategy.

By assuming that consumers can coordinate their actions at the most efficient outcome, the conventional analysis of network effects is to select the equilibrium that maximizes consumers' total surplus.<sup>4</sup> In this paper, we envision a situation where consumers are more conservative about their expectations of network growth. Accordingly, we select the stable equilibrium with a different network size. Our selection criterion results in a possibly discontinuous demand function—a small decrease in price at the critical point generates a discrete jump in sales. Thus, when the market expands, the monopolist should exercise penetration pricing by “offering a low price to invade another market” (Shapiro and Varian 1999, p. 288).<sup>5</sup> In effect, the demand discontinuity places a restriction on the monopolist's choice of quantities, a unique phenomenon that we call *penetration-pricing constraint*. Under this constraint, the firm can either access the niche market (small quantity) with a high price, or reach the mass market (large quantity) with a low price. The effects will depend on whether the monopolist sells or leases the product. In the first possible scenario, the seller has the option to establish a larger customer base in the beginning, which changes the residual demand and enables the seller to raise the price later and take full advantage of the network growth.<sup>6</sup> Such an option is not feasible to the lessor because she cannot demand renters committing to continuous renting.<sup>7</sup> Essentially, the seller's optimal strategy is to sacrifice her profits in the early

stage to capitalize the gains later. In contrast, the lessor extracts maximal profits early on, but her pricing choice later is limited by the penetration-pricing constraint.

We further show that a selling scenario might generate higher profits than leasing, depending on the market conditions. When the market is burgeoning, the monopolist is likely to prefer selling the product. However, when the market matures, resorting to a rental-based pricing model might be optimal. Thus, our model fully describes the life cycle of a network product, and it fits well with the evolution of pricing strategies illustrated by the example of Microsoft.

In sum, this paper establishes that the existence of network effects will lead a monopolist to favor selling her durable products over leasing under certain market conditions. This finding is in stark contrast to the conventional wisdom in the literature of durable-goods monopoly as well as that of network effects, many of which focus on the selling scenario while assuming that leasing is not feasible. In comparison, the decision of selling or leasing in our model is endogenous, and therefore the results are more robust.

Our work draws on two streams of research. The first studies the implications of network effects, while the second examines a durable-goods monopolist's choice between selling and leasing. Economics literature has analyzed various aspects of network effects, such as the issues of standardization and compatibility and their welfare implications (Katz and Shapiro 1985, 1986; Farrell and Saloner 1985). Meanwhile, marketing research on network effects has mainly focused on the strategic implications for individual firms, such as pricing, choices of product features, product upgrade, etc. For instance, Dhebar and Oren (1985) demonstrate that introductory pricing is optimal for a monopolist to market a product that exhibits network effects. Xie and Sirbu (1995) extend this finding to the scenario with a duopoly. Sun et al. (2004) explore the market conditions under which different product strategies such as technology licensing and product-line extension can be more profitable. Chen and Xie (2007) demonstrate that asymmetry in customer loyalty can turn a first-mover advantage into a disadvantage in the presence of cross-market network effects. Hauser et al. (2006) provide an excellent survey and identify challenges for research on network effects.

The leasing-selling decision has been analyzed in both economics and marketing literature. Bulow (1982) and Stokey (1981) demonstrate that leasing avoids the problem of time inconsistency and hence is more profitable than selling. Other authors have proposed various circumstances under which the assertion no longer holds. These include entry of new

<sup>4</sup> See, for example, Economides and Himmelberg (1995). Note, however, that some authors have analyzed various processes of coordination by consumers (e.g., Farrell and Katz 2005).

<sup>5</sup> Lee and O'Connor (2003) also assert that a penetration-pricing strategy during product launch performs better than a skimming pricing strategy for network effects products.

<sup>6</sup> A seller cannot always commit customers to continue using her product. For instance, an owner of e-book reader devices can abandon his current reader at any time and adopt a new device that supports different formats of e-books. Such switching effectively curtails network externalities for a particular format, and thereby limits the seller's benefits from her commitment power. We thank a referee for pointing out the caveat and suggesting this example.

<sup>7</sup> When there are switching costs, the monopolist could lock in some early renters and establish a customer base in rental market. In effect, the presence of switching costs changes the demand of rental units in period 2 by shifting inverse demand upward for all  $x < x_{1l}$ , where  $x_{1l}$  is the period-1 rental quantity. Thus, the producer still encounters a discontinuous demand; the disadvantage of leasing due to the penetration-pricing constraint does not vanish.

customers that leads to price cycles (Conlisk et al. 1984, Sobel 1991), entry deterrence of new competitors (Bulow 1986, Bucovetsky and Chilton 1986), increasing marginal costs (Kahn 1986), depreciation of goods (Bond and Samuelson 1987, Desai and Purohit 1998), discrete demand (Bagnoli et al. 1989), competitiveness of the market and reliability of the product (Desai and Purohit 1999). Our analysis adds to the literature by pointing out that network externalities have similar effects that induce a durable-goods monopolist to favor selling over leasing.

The rest of this paper is arranged as follows. In §2, we present the basic model that characterizes network effects. By using this model, we provide a theoretical foundation for inverted U-shaped demands that are commonly seen in the network effect literature. In §3, we provide a numerical example of dynamic pricing to illustrate our point. We then formulate and solve the general dynamic-pricing problems faced by a monopolist in §4. The final section discusses the results and presents our conclusions.

## 2. Network Effect: A Characterization

### 2.1. Consumer Preferences

We consider a durable good that exhibits network externalities. The good is sold by a monopoly and lasts two periods. For simplicity, we assume that the monopolist's discount factor is zero. The monopolist's goal is to maximize the aggregate revenues. There are two heterogeneous groups of consumers in the economy. The first consists of early adopters. These consumers arrive at the market in period 1, and hence have the opportunity to enjoy the product early on. The second group of consumers are the majority, who join the market only in period 2.

Within group  $g$  ( $g = 1, 2$ ), there is a continuum of consumers; each is indexed by a parameter  $x_g$ , which is uniformly distributed on  $[0, m_g]$ . Following Rohlfs (2001, p. 209), we assume that an individual's value for the good in question is composed of two parts: One is the generic value, and the other is the magnification of the network size. To facilitate comparison with the literature, we consider the linear form of generic valuation. To simplify the algebra, we further assume that the network effects for both groups are represented by the expected network size. These assumptions yield a specific demand structure as follows. For an early adopter  $x_1$ , her consumption value in period 1 is  $(A_1 - a_1 x_1)n_{1,1}$ .<sup>8</sup> The term in parentheses

represents the generic part of valuation, while  $n_{1,1}$  is the expected network size in period 1.<sup>9</sup> For the same consumer, her consumption value in period 2 will be  $(A_1 - a_1 x_1)(n_{1,2} + n_2)$ . Likewise, a group 2 consumer  $x_2$  has a consumption value of  $(A_2 - a_2 x_2)(n_{1,2} + n_2)$  in period 2.

Following Bulow (1982), we assume that a perfect second-hand market exists.<sup>10</sup> This eliminates the possibility of price discrimination. Assuming there is no income effect for any individual, we focus on the equilibrium analysis of the market in the presence of network effects. In the Technical Appendix,<sup>11</sup> we discuss extension of our linear model to a more general demand specification.

### 2.2. Inverted U-Shaped Demands and the Critical Mass

In conventional analysis, a demand function associates any given price with a desired quantity of the commodity. Nonetheless, with network effects, a price might correspond to multiple quantities. To derive the demand correspondences in our model, note that if a consumer indexed  $x_g$  is willing to buy the object, anyone in the same group with a lower index  $x < x_g$  will also demand it. Consequently, the total demand in the economy is determined by the marginal consumers.

In period 1 only early adopters appear in the market, and the inverse demand is simply

$$p_1(x) = (A_1 - a_1 x)x. \quad (1)$$

The diagram in Figure 1(a) illustrates the demand correspondence. For any  $p < A_1^2/4a_1$ ,  $p$  associates with three possible equilibria:  $x = 0$  or the roots of the quadratic function  $(A_1 - a_1 x)x = p$ .<sup>12</sup> As is well known, the larger root  $x_{\max}$  in the downward-sloping part of the parabola is a stable equilibrium (see the appendix of Rohlfs 2001), so is  $x = 0$ . Meanwhile, the smaller root  $x_{\min}$  on the upward slope is an unstable equilibrium.  $x_{\min}$  usually represents a critical mass, which the seller has to overcome to reach the stable and more profitable equilibrium.

In period 2, the product's appeal extends to the second group. For any price  $p$ , let  $x_1$  and  $x_2$  be

<sup>8</sup> By construction, individuals in each group are ordered decreasingly in terms of their generic willingness to pay, represented by a decreasing function  $q_g(x_g) = A_g - a_g x_g$ . This particular ordering enables us to interpret  $q_g(x_g)$  as an inverse generic demand, and thus facilitates our analysis.

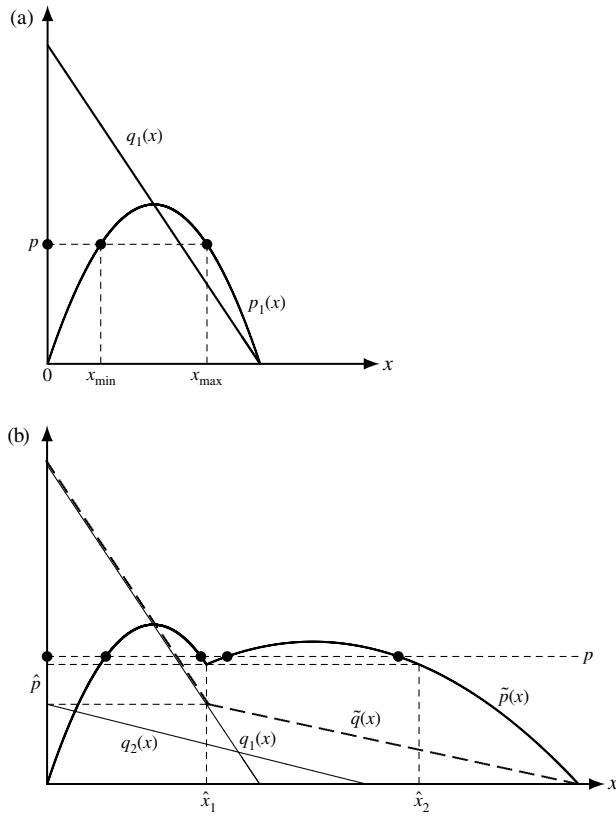
<sup>9</sup> Specifically,  $n_{1,i} = |X_{1,i}|$  for  $i = 1, 2$ , where  $X_{1,i} \subset [0, m_1]$  is a subset of group 1 and consists of those who are expected to consume the product in period  $i$ . In addition, we implicitly assume that an individual with the highest index  $m_g$  has zero value for consumption, regardless of network sizes, which implies  $m_g = A_g/a_g$ .

<sup>10</sup> For instance, Internet marketplaces such as Half.com or Amazon.com allow people to trade used computer software.

<sup>11</sup> The Technical Appendix can be found at <http://mktsci.pubs.informs.org>.

<sup>12</sup> Because the network effect is multiplicative, the willingness to pay by *any* consumer would be zero if he expects no one else to buy the product. Therefore,  $x = 0$  is an equilibrium.

**Figure 1** Single- and Double-Hump Demand Curves in the First and Second Periods



the marginal consumers for groups 1 and 2, respectively.  $x_1$  and  $x_2$  must satisfy the demand correspondences:  $p = (A_1 - a_1x_1)(x_1 + x_2)$  and  $p = (A_2 - a_2x_2) \cdot (x_1 + x_2)$ . Alternatively, one can first derive the aggregate inverse generic demand as  $\tilde{q}(x)$  by summing  $q_1(x) = A_1 - a_1x$  and  $q_2(x) = A_2 - a_2x$  horizontally. The aggregate inverse demand is then given by  $\tilde{p}(x) = \tilde{q}(x)x$ . In Figure 1(b),  $\tilde{q}(x)$  is the dashed line, while  $\tilde{p}(x)$  is the double-hump shaped curve.

For  $x \leq \hat{x}_1$ ,  $\tilde{p}(x)$  is equal to  $A_1x - a_1x^2$ ; otherwise,

$$\tilde{p}(x) = \tilde{A}x - \tilde{a}x^2 = \frac{a_2A_1 + a_1A_2}{a_1 + a_2}x - \frac{a_1a_2}{a_1 + a_2}x^2, \quad (2)$$

where the critical value  $\hat{x}_1 \equiv (A_1 - A_2)/a_1$  is determined by  $p_1(\hat{x}_1) = \tilde{p}(\hat{x}_1)$ . From Figure 1(b), we observe that  $\tilde{p}(x)$  has two humps if and only if  $A_1/2a_1 < \hat{x}_1 < \tilde{A}/2\tilde{a}$ .<sup>13</sup> The double-hump shape of demand is similar to the one drawn in Rohlfs (2001, p. 219), except

<sup>13</sup> To be precise, a *hump* is a local maximum of the  $\tilde{p}(\cdot)$  curve. In Figure 1(ii),  $\tilde{p}(x)$  reaches local maxima  $A_1^2/4a_1$  and  $\tilde{A}^2/4\tilde{a}$  at  $x = A_1/2a_1$  and  $\tilde{A}/2\tilde{a}$ , respectively. However, if  $\hat{x}_1 \leq A_1/2a_1$ ,  $\tilde{p}(x)$  is increasing  $\forall x \in [0, \hat{x}_1]$ , and thus has only one local maximum at  $x = \tilde{A}/2\tilde{a}$ . The argument for the case with  $\hat{x}_1 \geq \tilde{A}/2\tilde{a}$  is analogous. Note that  $A_1/2a_1 < \tilde{A}/2\tilde{a} = A_1/2a_1 + A_2/2a_2$ .

that we provide a theoretical justification for it here.<sup>14</sup> According to Rohlfs (2001) and Shapiro and Varian (1999), the monopolist can *penetrate* the section of  $\tilde{p}(x)$  beyond  $\hat{x}_1$  in the second period only if the price is set lower than or equal to  $\hat{p}$ . This is the *penetration-pricing constraint* referred to in the literature.

Due to the double-hump shape of demand, for any price  $p$  between  $\hat{p} \equiv \tilde{p}(\hat{x}_1)$  and  $\tilde{A}^2/4\tilde{a}$ ,  $p$  associates with five equilibria (see the bullets in Figure 1(b)), two (the third and the fifth from the left) of which are stable with positive quantities. In the presence of multiple stable equilibria, the criterion for equilibrium selection proves to be critical in our analysis of a monopolist's market strategy.

### 2.3. The Penetration-Pricing Constraint and Equilibrium Selection

It has long been recognized that a market price can associate with multiple network sizes in equilibrium (Oren and Smith 1981, Oren et al. 1982, Katz and Shapiro 1986). Following Katz and Shapiro (1986), the conventional approach is to select the equilibrium that is Pareto-preferred by consumers.<sup>15</sup> As seen from Figure 1(b), this selection criterion implies a different demand curve faced by the monopolist lessor so that she can choose to lease any quantity from  $[\tilde{A}/2\tilde{a}, \tilde{A}/\tilde{a}]$  in period 2. In this paper, however, we assume that consumers are more conservative in terms of their expectations of network growth. Consequently, we focus on the stable equilibrium with the smallest network size for any given price. The resulting demand corresponding with the price range  $[\hat{p}, \tilde{A}^2/4\tilde{a}]$  is  $[x'_1, \hat{x}_1]$  instead of  $[\tilde{A}/2\tilde{a}, \hat{x}_2]$ , where  $x'_1$  is the larger root of  $p_1(x) = \tilde{A}^2/4\tilde{a}$  and  $\hat{x}_2 \equiv A_2/\tilde{a}$  satisfies  $\tilde{p}(\hat{x}_2) = \tilde{p}(\hat{x}_1)$ .

One way to justify our criterion for equilibrium selection is that it validates the dichotomy of pricing strategies when marketing a new product, as proposed by Dean (1976). The first segment of demand for  $x \in [A_1/2a_1, \hat{x}_1]$  represents the strategy of *skimming pricing* with which the monopolist targets the elite group of consumers; the second segment of demand for  $x \in (\hat{x}_2, \tilde{A}/\tilde{a}]$  corresponds to the strategy of *penetration pricing* that directs at the general public. In this paper, we focus on the situation under which penetration pricing dominates skimming pricing.

<sup>14</sup> Rohlfs (2001, p. 215) assumed  $q(x) = cx^\beta$  and  $f(n) = kn$ . However, this will not generate the inverted-U figure he suggested. For under his specification,  $p(x) = q(x)f(x) = c k x^{\beta+1}$ , which (depending on the size of  $\beta$ ) is either increasing or decreasing in  $x$ . Thus, the willingness to pay by the marginal consumer would be increasing or decreasing in the *whole range* of  $x$ .

<sup>15</sup> Other authors such as Oren et al. (1982) simply assumed a single-peaked willingness-to-pay function, which eliminates the possibility of multiple equilibria.

### 3. Dynamic Pricing: A Numerical Example

In this section, we consider a specific set of parameters and demonstrate that selling is more profitable than leasing under this particular demand specification. Let  $x_{il}$  be the quantity produced by the monopolist lessor up to the  $i$ th period, and  $x_{is}$  the quantity produced by the monopolist seller up to the  $i$ th period.<sup>16</sup>

**EXAMPLE.** Suppose  $A_1 = 4$ ,  $a_1 = 2$ ,  $A_2 = 1$ , and  $a_2 = \frac{1}{4}$ . One obtains  $\tilde{A} = \frac{4}{3}$  and  $\tilde{a} = \frac{2}{9}$ . Assuming zero production costs, the optimal leasing quantities are given by  $x_{1l} = \frac{4}{3}$  and  $x_{2l} = \frac{9}{2}$ , while the optimal sales are  $x_{1s} = \frac{3}{2}$  and  $x_{2s} - x_{1s} \approx 2.8$ . The pricing strategies yield a profit below 9.12 for leasing the product, which is lower than the 9.23 yield from selling it. Below we explain this.

If the monopolist wants to lease the product, she maximizes  $\pi_l = p_1(x_{1l}) \cdot x_{1l} + \tilde{p}(x_{2l}) \cdot x_{2l}$  by choosing  $x_{1l}$  and  $x_{2l}$ . It is readily verified that the first-best prices are  $p_1 = \frac{16}{9}$  ( $x_{1l} = \frac{4}{3}$ ) and  $p_2 = \frac{16}{9}$  ( $x_{2l} = 4$ ). Nonetheless, as we have discussed in §2.3, the penetration-pricing constraint limits the range of feasible  $x_{2l}$ s. In particular,  $(x_{2l}, p_2) = (4, \frac{16}{9})$  is not an achievable strategy because the second-group demand cannot be penetrated by the price  $p_2 = \frac{16}{9}$ .

In the presence of the penetration-pricing constraint, the lessor can select only  $x_{2l}$  from  $[1, \frac{3}{2}]$  and  $(\frac{9}{2}, 6]$ . With this discontinuous demand curve, the optimal strategy under leasing is to set  $p_1 = \frac{16}{9}$  and  $p_2$  slightly below  $\frac{3}{2}$  so that  $x_{1l} = \frac{4}{3}$  and  $x_{2l}$  slightly above  $\frac{9}{2}$ . As such, the highest achievable two-period profits under leasing are bounded from above by  $\pi_l = \frac{16}{9} \cdot \frac{4}{3} + \frac{3}{2} \cdot \frac{9}{2} \approx 9.12$ .

Now consider the selling strategy. The monopolist's second-period strategy is still restricted by the need to penetrate the market. However, being able to build a customer base in the first period alleviates the constraint for the seller. In our example, if  $x_{1s} < \frac{3}{2}$ , the corresponding range for feasible  $x_{2s}$ 's is  $(x_{1s}, \frac{3}{2}] \cup (\frac{9}{2}, 6]$ , and the penetration-pricing constraint is similar to that in the leasing scenario. On the contrary, if  $x_{1s} \geq \frac{3}{2}$ , the seller can select any  $x_{2s}$  from  $[3, 6]$ .

The total profits under the selling regime are  $\pi_s = (p_1(x_{1s}) + \tilde{p}(x_{2s})) \cdot x_{1s} + \tilde{p}(x_{2s})(x_{2s} - x_{1s})$ . Let  $x_2^p(x_{1s})$  denote the solution to the last optimization problem subject to the penetration-pricing constraint. It is straightforward to verify that the constraint is binding when  $x_{1s} < \frac{3}{2}$  and nonbinding otherwise.

Therefore, the seller's problem in the example is given by

$$\begin{aligned} \max_{x_{1s}, x_{2s}} \quad & (p_1(x_{1s}) + \tilde{p}(x_2^p(x_{1s}))) \cdot x_{1s} + \tilde{p}(x_{2s})(x_{2s} - x_{1s}), \\ \text{s.t.} \quad & x_{2s} \in \left(x_{1s}, \frac{3}{2}\right] \cup \left(\frac{9}{2}, 6\right] \quad \text{for } x_{1s} < \frac{3}{2}, \quad \text{or} \\ & x_{2s} \in [3, 6] \quad \text{for } x_{1s} \geq \frac{3}{2}. \end{aligned}$$

The optimal sales that solve the above problem are  $x_{1s} = \frac{3}{2}$  and  $x_{2s} \approx 4.3$ . It follows that the highest profits available to the seller are  $\pi_s \approx \frac{3}{2} \cdot \frac{3}{2} + 1.62 \cdot 4.3 \approx 9.23$ , which are greater than the leasing profits.

### 4. The General Analysis

The previous section shows that leasing might be less profitable than selling. In this section, we identify the general conditions under which the comparison holds true. To highlight the implications from imposing the penetration-pricing constraint, we start with the benchmark case where the monopolist is not bounded by the constraint.

#### 4.1. The Case Without Penetration-Pricing Constraints

An important characteristic that sets our model apart is the assumption of a growing market. In this subsection we show that this property alone does not lead to our findings. Without the penetration-pricing constraint, the analysis and conclusion are similar to those in Bulow (1982), even though the latter assumes an identical market in both periods.

The monopolist lessor's aggregate profits are  $\pi_l = (p_1(x_{1l}) - c) \cdot x_{1l} + \tilde{p}(x_{2l}) \cdot x_{2l} - c \cdot (x_{2l} - x_{1l})$ , where  $c$  is the marginal cost of production.<sup>17</sup> Let  $MR_1(x_1) \equiv p_1(x_1) + p'_1(x_1) \cdot x_1$  and  $MR_2(x_2) \equiv \tilde{p}(x_2) + \tilde{p}'(x_2) \cdot x_2$  be the marginal revenues for period-1 and aggregate production, respectively. The first-order conditions can then be written as  $MR_1(x_{1l}) = 0$  and  $MR_2(x_{2l}) = c$ . Solving these equations yields the interior solution as follows:

$$x_{1l}^* = \frac{2}{3} \frac{A_1}{a_1}, \quad x_{2l}^* = \frac{\tilde{A}}{3\tilde{a}} + \frac{1}{3} \sqrt{\left(\frac{\tilde{A}}{\tilde{a}}\right)^2 - \frac{3c}{\tilde{a}}}. \quad (3)$$

For (3) to solve the monopolist lessor's problem, we need the following regularity assumptions:

**ASSUMPTION 1.**  $c < \tilde{A}^2/4\tilde{a}$ .

**ASSUMPTION 2.**  $\pi_l(x_{1l}^*, x_{2l}^*) > \max_x (2p_1(x) - c) \cdot x$ .

<sup>16</sup> We adopt the notations so that  $x_{2l}$  or  $x_{2s}$  represents the total number of units on the period 2 market. Period 2 production is therefore  $x_{2l} - x_{1l}$  for  $j = l$  or  $S$ .

<sup>17</sup> It is possible that  $x_{2l} < x_{1l}$ , in which case the last term shall be zero. Nonetheless, one can show that  $x_{2l} > x_{1l}$  in equilibrium, and thus we adopt the current formulation for simplicity.

Assumption 1 is equivalent to requiring either  $x_{2l}^* > \tilde{A}/2\tilde{a}$  or  $\tilde{p}(x_{2l}^*) > c$ . Therefore, the assumption guarantees that  $x_{2l}^*$  lies on the downward sloped part of  $\tilde{p}(x)$  and yields positive profits. Throughout the paper, we assume that Assumption 1 always holds. Note that the monopolist has a simple strategy, where she ignores group-2 consumers in period 2 and focuses on leasing to group 1 in both periods. Assumption 2 ensures that this alternative strategy is less profitable than  $(x_{1l}^*, x_{2l}^*)$ .<sup>18</sup>

**PROPOSITION 1.** *Suppose the growth of market is significant for the lessor in the sense of Assumption 2. Then,  $(x_{1l}^*, x_{2l}^*)$  in (3) maximizes the monopoly lessor's profits in the absence of penetration-pricing constraints.*

We now consider the seller's problem. Let  $x_2^*(x_{1s}) \equiv \arg \max_{x_{2s}} (\tilde{p}(x_{2s}) - c)(x_{2s} - x_{1s})$  be the solution to the second-stage optimization problem. Realizing her own choice in period 1 restricts her action in period 2, the seller's problem is to maximize  $\pi_s = (p_1(x_{1s}) + \tilde{p}(x_2^*(x_{1s})) - c)x_{1s} + (\tilde{p}(x_{2s}) - c)(x_{2s} - x_{1s})$ . Let  $(x_{1s}^*, x_{2s}^*)$  denote the solution. The maximization program is different from that in the leasing scenario in the term  $\tilde{p}(x_2^*(x_{1s}))$ , which represents the consumers' rational expectation. The first-order conditions are

$$\begin{aligned} MR_1(x_{1s}) &= -\frac{\partial \tilde{p}(x_2^*(x_{1s}))}{\partial x_{1s}} x_{1s}, \\ MR_2(x_{2s}) &= c + \tilde{p}'(x_{2s})x_{1s}. \end{aligned} \quad (4)$$

The next assumption parallels Assumption 2. It ensures that the monopolist finds it more profitable selling to group-2 consumers, which implies that the solution to (4) attains the optimum.

**ASSUMPTION 3.** *The profits attained by the solution to (4) are higher than those attained by maximizing  $\pi_s$  while excluding group-2 consumers.*

The mathematical expression for Assumption 3 is not provided here. Essentially, it requires either a high  $A_2$  (high willingness to pay) or a low  $a_2$  (a large population).

**PROPOSITION 2.** *Suppose the growth of market is significant for the seller in the sense of Assumption 3. The solution to (4) maximizes the monopolist seller's profits in the absence of penetration-pricing constraints.*

<sup>18</sup> In the absence of a growing market, the right-hand side represents the lessor's profits attained by leasing the same amount of product in both periods. Intuitively, Assumption 2 holds when the addition of group-2 consumers contributes significantly to the monopolist's profits. Roughly speaking, it is true when either their willingness to pay is high enough ( $A_2$  is high), or the population is large enough ( $A_2/a_2$  is high). We discuss such specific conditions in the Technical Appendix, found at <http://mktsci.pubs.informs.org>.

It is straightforward to compare the equilibrium profits: the strategy  $(x_{1s}^*, x_{2s}^*)$  is accessible to the lessor, and thus her maximal profits must be weakly higher. The details for comparing the equilibrium quantities can be found in the Technical Appendix (at <http://mktsci.pubs.informs.org>). The next proposition summarizes our findings. It is important to emphasize that these findings are consistent with Bulow's (1982).

**PROPOSITION 3.** *Suppose the growth of market is significant so that Assumptions 2 and 3 hold. In the absence of penetration-pricing constraints, the seller produces less than the lessor does in period 1, but the seller's total quantity in period 2 is higher; that is,  $x_{1s}^* < x_{1l}^*$  and  $x_{2s}^* > x_{2l}^*$ . Moreover, leasing is more profitable than selling.*

#### 4.2. The Case with a Penetration-Pricing Constraint

The monopolist's pricing strategy will be much different when she takes into account the penetration-pricing constraint. We start by reiterating some notations illustrated in Figure 1(b):  $\hat{x}_1 \equiv (A_1 - A_2)/a_1$  is the critical value at which  $p_1(\hat{x}_1) = \tilde{p}(\hat{x}_1)$ ; while  $\hat{x}_2 \equiv A_2/\tilde{a}$  leads to the same critical price with  $\tilde{p}(\hat{x}_2) = \tilde{p}(\hat{x}_1)$ . For the monopolist lessor, the range for feasible  $x_{2l}^*$  is  $[A_1/2a_1, \hat{x}_1] \cup (\hat{x}_2, \tilde{A}/\tilde{a}]$ . In other words, the lessor cannot penetrate the period-2 market unless the price is lower than  $\hat{p} \equiv p_1(\hat{x}_1)$ .<sup>19</sup> Therefore, the lessor's problem is to maximize  $\pi_l = (p_1(x_{1l}) + \tilde{p}(x_{2l}) - c) \cdot x_{1l} + (\tilde{p}(x_{2l}) - c)(x_{2l} - x_{1l})$ , subject to  $x_{2l} \in [A_1/2a_1, \hat{x}_1] \cup (\hat{x}_2, \tilde{A}/\tilde{a}]$ . Let  $(x_{1l}^p, x_{2l}^p)$  be the solution to this problem.

We are more interested in the case where the penetration-pricing constraint is binding, which requires the unconstrained solution  $x_{2l}^*$  be lower than  $\hat{x}_2$ .

**ASSUMPTION 4.**  $x_{2l}^* < \hat{x}_2$ .

As in §4.1, we further assume that the market enjoys significant growth so that the monopolist finds it more profitable leasing to some of the group-2 members.

**ASSUMPTION 5.**  $\pi_l(x_{1l}^*, \hat{x}_2) > \max_x (2p_1(x) - c) \cdot x$ .

Because Assumption 4 implies that  $\pi_l$  is decreasing in  $x_{2l}$  for  $x_{2l} \in (\hat{x}_2, \tilde{A}/\tilde{a}]$ , we know that  $\pi_l$  is maximized at  $x_{2l}^p$  slightly above  $\hat{x}_2$ .

**PROPOSITION 4.** *Suppose the penetration-pricing constraint is binding for the lessor (Assumptions 4) and that the market is growing in the sense of Assumption 5. The monopolist's optimal leasing strategy is  $x_{1l}^p = x_{1l}^*$  and  $x_{2l}^p \approx \hat{x}_2$ .*

For the monopolist seller, the restrictions imposed by the penetration-pricing constraint depend on how

<sup>19</sup> For the parameters assumed in the example, one can verify that  $A_1/2a_1 = 1$ ,  $\hat{x}_1 = 3/2$ ,  $\hat{x}_2 = 9/2$ , and  $\tilde{A}/\tilde{a} = 6$ .

much she sells in period 1. If  $x_{1S}$  is lower than the critical amount  $\hat{x}_1$ , the seller faces a constraint similar to the lessor's, so that the feasible  $x'_{2S}$ s are limited to  $(x_{1S}, \hat{x}_1] \cup (\hat{x}_2, \tilde{A}/\tilde{a}]$ . In other words, when  $x_{1S} < \hat{x}_1$ , the seller has two options in the next period: She can either charge a high price ( $p_2 \geq \hat{p}$ ) that appeals only to group-1 consumers, or a low price ( $p_2 < \hat{p}$ ) that reaches group-2 consumers. These options correspond to the intervals  $(x_{1S}, \hat{x}_1]$  and  $(\hat{x}_2, \tilde{A}/\tilde{a}]$ , respectively. On the contrary, if  $x_{1S}$  is greater than  $\hat{x}_1$ , the seller has built a customer base large enough to penetrate the market in period 2, so that she can select any  $x_{2S}$  from  $[\tilde{A}/2\tilde{a}, \tilde{A}/\tilde{a}]$ . The penetration-pricing constraint has no effect in this case. For the constraint to be binding in equilibrium, we make the following assumption.

ASSUMPTION 6.  $x_{1I}^* < \hat{x}_1$ ;  $x_2^*(\hat{x}_1) < \hat{x}_2$ .

Given the time-consistency constraint, for any  $x_{1S}$ , the seller maximizes  $(\tilde{p}(x_{2S}) - c)(x_{2S} - x_{1S})$  subject to the penetration-pricing constraint corresponding to  $x_{1S}$ . Let  $x_2^p(x_{1S})$  be the solution to the seller's constrained problem in period 2. Along with Proposition 3 ( $x_{1S}^* < x_{1I}^*$ ), the first part of Assumption 6 implies that  $x_{1S}^* < \hat{x}_1$ . Because we know that  $x_2^*(\cdot)$  is an increasing function, the second part implies  $x_2^*(x_{1S}) < \hat{x}_2 \forall x_{1S} < \hat{x}_1$ . As such, one concludes that the unconstrained solution  $x_{2S}^*$  ( $=x_2^*(x_{1S}^*)$ ) in §4.1 is lower than  $\hat{x}_2$ . In short, Assumption 6 implies a binding penetration-pricing constraint so that  $(x_{1S}^*, x_{2S}^*)$  is not achievable.

$x_2^p(x_{1S})$  can now be derived as follows. For  $x_{1S} < \hat{x}_1$ , the interior solution  $x_2^*(x_{1S})$  is not achievable, and thus  $x_2^p(x_{1S}) = x'_2(x_{1S})$  or  $\approx \hat{x}_2$ .<sup>20</sup> For  $x_{1S} \geq \hat{x}_1$ , the seller is able to penetrate the market in period 2, and  $x_2^p(x_{1S}) = x_2^*(x_{1S})$ . Given  $x_2^p(x_{1S})$ , the seller's optimization problem is

$$\begin{aligned} \max_{x_{1S}, x_{2S}} \quad & \pi_S^p = (p_1(x_{1S}) + \tilde{p}(x_2^p(x_{1S})) - c)x_{1S} \\ & + (\tilde{p}(x_{2S}) - c)(x_{2S} - x_{1S}), \\ \text{s.t.} \quad & x_{2S} \in (x_{1S}, \hat{x}_1] \cup \left(\hat{x}_2, \frac{\tilde{A}}{\tilde{a}}\right] \quad \text{for } x_{1S} < \hat{x}_1, \quad \text{or} \quad (5) \\ & x_{2S} \in \left[\frac{\tilde{A}}{2\tilde{a}}, \frac{\tilde{A}}{\tilde{a}}\right] \quad \text{for } x_{1S} \geq \hat{x}_1. \end{aligned}$$

To solve (5), we maximize  $\pi_S^p$  separately over  $x_{1S} < \hat{x}_1$  and  $x_{1S} \geq \hat{x}_1$ . In the former case,  $x_2^p(x_{1S}) = x'_2(x_{1S})$  or  $\approx \hat{x}_2$ . Provided Assumption 5 holds,  $\hat{x}_2$  yields a higher profit than  $x'_2(x_{1S})$ . The local maximizer in this scenario is thus  $(x_{1I}^*, \hat{x}_2)$ . In the latter case,  $x_2^p(x_{1S}) = x_2^*(x_{1S})$ . Given the assumption that  $x_{1S}^* < \hat{x}_1$

and that  $\partial^2 \pi_S / \partial x_{1S}^2 < 0$ , the objective function  $\pi_S^p$  is decreasing in  $x_{1S}$  for the relevant range and hence is maximized at  $x_{1S} = \hat{x}_1$ . Combining these scenarios, we need to compare  $\pi_S^p(x_{1I}^*, \hat{x}_2)$  against  $\pi_S^p(\hat{x}_1, x_2^*(\hat{x}_1))$  to determine the solution to (5).

PROPOSITION 5. Suppose the market is growing and the penetration-pricing constraint is binding for the seller (Assumptions 5 and 6). The monopolist's optimal selling strategy,  $(x_{1S}^p, x_{2S}^p)$ , is either  $(x_{1I}^*, \hat{x}_2)$  or  $(\hat{x}_1, x_2^*(\hat{x}_1))$ .

When comparing  $(x_{1I}^*, \hat{x}_2)$  and  $(\hat{x}_1, x_2^*(\hat{x}_1))$ , the seller trades off her profits in two periods. In the former strategy, the seller is bounded in period 2 by the penetration-pricing constraint, but she is able to maximize her period-1 profits. In the latter strategy, the seller sacrifices her period-1 profits to penetrate the market and capture bigger gains in period 2. If the period-2 market is lucrative enough, as we assume in this paper, the latter strategy will be optimal.

From Proposition 5, we observe that the leasing solution  $(x_{1I}^*, \hat{x}_2)$  is feasible in the selling regime. It follows immediately that leasing is no longer more profitable than selling. The leasing profits are reduced because the lessor has to lower the period-2 price to reach group-2 consumers. In contrast, the seller's loss occurs mostly in period 1. Thus, the constraint affects the lessor's and the seller's profits differently.

COROLLARY 6. Following Proposition 5, one obtains  $\pi_I^p \leq \pi_S^p$  in equilibrium. Therefore, selling is more profitable than leasing under the penetration-pricing constraint.

The inequality in the corollary holds strictly when  $A_2$  is large enough or  $a_2$  is small enough. Recall that  $A_2/2$  is the average willingness to pay by group-2 consumers, and  $A_2/a_2$  characterizes the size of population. Therefore, Corollary 6 implies that when the monopolist expects a significant growth of the market, she will prefer selling rather than leasing the product.

Our analysis used a particular form of demand specification. In addition, we also made certain regularity assumptions to ensure that penetration-pricing constraints are binding. It can be shown that these presumptions do not place severe restrictions on our findings. A detailed mathematical analysis of these assumptions can be found in the Technical Appendix, found at <http://mktsci.pubs.informs.org>.

## 5. Concluding Remarks

In this paper, we study the pricing dynamics of a monopolist who produces durable goods with network effects. We demonstrate that introductory pricing constitutes the monopolist seller's optimal strategy and that selling might be more profitable than leasing under certain market conditions.

<sup>20</sup>  $x'_2(x_{1S}) \equiv \arg \max_{x_{2S}} (p_1(x_{2S}) - c)(x_{2S} - x_{1S})$  maximizes the seller's period-2 profits when she excludes group-2 consumers; see (A10) in the Technical Appendix, found at <http://mktsci.pubs.informs.org>.



Our analysis relies on the specific criterion of equilibrium selection. That is, in face of multiple equilibria, we select the stable equilibrium that yields the smallest network size for any given price. This criterion presumes that consumers are generally cautious about the prospect of network growth. Such presumption is plausible in reality if, for instance, the product/technology is novel and unfamiliar, or the firm is relatively unknown so that its success is in doubt. In contrast, for a product like iPod by Apple, Inc., a potential consumer will likely anticipate a very optimistic growth of its user network. Our criterion will hence be inapplicable in the latter scenario.

This finding helps us to interpret the relative advantage of seller over lessor as predicated in our model, a result different from that of Bulow (1982). The advantage of the seller derives from her ability to establish a large installed base of committed users. In contrast, the lessor cannot demand renters commit to continuous renting because in each period the renters will always make independent leasing decisions. The seller's strategic flexibility thus leads to higher profits. Note, however, that our conclusion conflicts with past findings. Future empirical tests are needed to determine the correct theory.

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