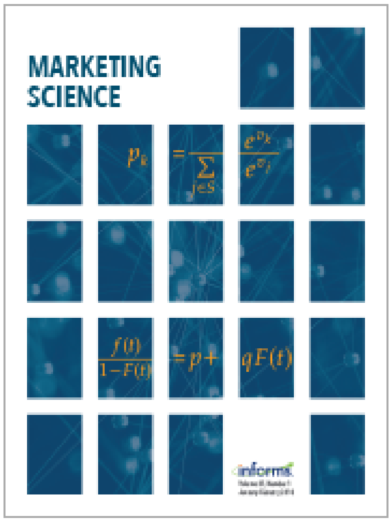

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A Dynamic Model of Entry and Exit in a Growing Industry

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The potential demand in a new industry evolves over time. Demand is initially low, but advertising by the industry's early entrants can speed up demand growth. However, there is intrinsic uncertainty of the demand level in each period and uncertainty of the demand evolution path, which can be affected by the underlying economic environment. We construct a dynamic model that features the stochastically and endogenously expanding demand of a new industry, and we investigate the optimal entry and exit behavior of firms as the industry evolves. We find that firms' incentive to enter early depends critically on the cost that early entrants have to pay in developing the market. When the cost is high and the benefit spills over to potential entrants, firms have an incentive to wait, and the probability of entry can increase with the number of incumbents under certain circumstances. Firms' entry strategy is also influenced by the transition of economic states. Firms are more likely to enter under a state that shows the prospect of demand taking off soon. We also find that, in the early stage of an industry, higher demand uncertainty can induce faster entry.

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1. Introduction

Whether and when to enter a new industry or market is a critical decision for firms. This decision is deeply tied to the endogenously evolving market structure and the exogenously changing economic environment. Should a firm enter at the introductory stage of an industry to claim a space in the market or delay entry and free ride on other firms' effort in cultivating the demand? Does a good economic state always attract more firms to enter than does a bad state? Will higher uncertainty in demand or cost lead to faster or slower diffusion of firms? In this paper, we attempt to answer these questions by investigating firms' optimal entry and exit strategies and the resulting diffusion pattern in a dynamic model that features industry evolution.

Unlike a mature industry in which consumers have an established preference for a product, a new industry is characterized by the fact that consumer preference evolves with uncertainty. To capture such characteristics, we assume that potential demand stochastically grows over time and that industry advertising can positively affect demand growth. We also assume that incumbents have to spend on advertising before demand reaches its long-term potential. Furthermore, firms need to spend more on advertising when the demand is at a low stage. It creates a trade-off in the timing of entry. Early entry guarantees the firm a space in the market where a limited number of firms can be supported, but potential entrants can free ride on other firms' market expansion efforts by entering late. Our analysis shows that firms' incentive to enter early depends critically on the cost in developing the market. When the cost is high, it creates first-mover disadvantage and discourages early entry. With endogenously expanding demand, there is also a "bandwagon effect" that more firms enter when other firms enter until the competition effect dominates.

We also assume that the demand shock in each period not only affects the demand state of the period but also has a persistent effect that shapes the demand evolution path. Consecutive positive demand shocks can speed up demand diffusion, whereas a series of negative demand shocks in the early stage will delay demand takeoff. In our model, the distribution of demand shocks is governed by the underlying economic state, which is assumed to be exogenously evolving. Demand shocks are more likely to be favorable under a good economic state than under a bad state. We find that firms are more likely to enter in a good economic state, but the difference in entry probability under different economic states becomes smaller as the industry matures. Entry strategy also depends on the stickiness of the states. Moreover, when the discount factor is linked to the economic environment through interest rate, with a higher interest rate in a booming economy and a lower rate in a recession, firms are not necessarily more likely to favor good economic states for entry.

In addition to the uncertainty of the demand evolution path, we also investigate the uncertainty of demand in a given period as characterized by the variance of demand shocks. Given that potential demand is intrinsically evolving from low to high, we find that the effect of period demand uncertainty on firm entry depends on the stage of industry development. Specifically, higher uncertainty can increase the expected value of entry and attract more firms when the demand is at a low stage. However, higher uncertainty results in fewer firms when the demand approaches its long-run potential. Uncertainty about the production cost has a similar effect on firm diffusion.

In our model, individual firms make their entry and exit decisions based on the current industry state and their expectation of future evolution. The expectation not only involves demand and cost forecasts but also the actions of other competitors (potential or active). By solving the equilibrium of the dynamic game, we provide insights about firms' optimal entry and exit decisions when facing important trade-offs and uncertainties in an endogenously and stochastically expanding industry.

This paper is related to the literature on industry evolution. A stream of literature focuses on empirically documenting and characterizing the regularities in the evolution of new industries (e.g., Gort and Klepper 1982, Dunne et al. 1988, Klepper and Graddy 1990, Klepper 1996, Agarwal and Bayus 2002). Alternatively, Jovanovic (1982), Hopenhayn (1992), and Jovanovic and MacDonald (1994) use dynamic stochastic models to study firm dynamics and industry equilibrium in a perfectly competitive industry. Related to these studies, this paper uses a dynamic model of entry and exit that endogenously determines the evolution of an industry. Instead of assuming an infinite number of firms and abstracting from strategic interaction among firms, this paper considers the case of imperfect strategic competition. Our model builds on the framework proposed by Ericson and Pakes (1995).[1] To characterize the nature of industry development, our model allows the potential demand to stochastically grow over time until reaching its long-run maximum. We also consider the case that potential demand can be endogenously expanded with the number of firms operating in the industry, which can be an important feature in new industry development. The model allows for a large number of firms and characterizes the diffusion of firms as the equilibrium path.

The paper focuses on investigating the optimal entry and exit strategies and firm diffusion as the industry endogenously evolves.[2] In a related paper, Shen and Villas-Boas (2010) examine the strategic entry of firms with exogenously expanding demand. They find that firms being forward-looking and competition can lead to firms entering the market before demand takeoff. We consider endogenously and stochastically expanding demand in the model and focus on the trade-off of entering early versus free riding on other firms' market expansion effort and the uncertainty in demand evolution. The paper can also be seen as related to the literature on entry or investment under uncertainty (e.g., Dixit 1989, Pindyck 1991). We investigate not only the uncertainty of demand or cost of a period but also the uncertainty of the demand evolution path. Furthermore, we discuss how these effects may change in different stages of industry evolution.

The remainder of the paper is organized as follows. Section 2 develops the dynamic model of entry and exit that characterizes demand evolution. In §3, we analyze the impact of the key demand side factors on firm diffusion by numerically solving the equilibrium of the dynamic game. We briefly discuss the cost-side factors on firm strategy in §4 and conclude the paper in §5.

2. The Theory Model

2.1. Basic Setup

The timeline starts with the beginning of a new industry and goes to infinity. Assume that there is a total of $\bar{E}$ firms interested in the industry. The potential demand is at the minimum level at the beginning of the new industry. Each period, potential entrants independently get a draw of entry cost from a commonly known distribution and simultaneously decide whether to enter. If an entrant enters, it pays the entry cost this period and becomes active in the next period.

Meanwhile, each incumbent receives independently a draw of sell-off value from a commonly known distribution and decides whether to exit. If the incumbent decides to exit, it produces in the current period and leaves the industry at the beginning of the next period. Simultaneous moves of the potential entrants and incumbents determine the evolution path of the number of firms. Incumbents' profit at each period depends on the competition among the active firms as well as the demand and cost conditions in that period. Production cost is assumed to stochastically decrease over time. Equilibrium prices and quantities are determined based on the assumption that firms engage in price competition.

[1] There is a growing literature that employs this framework to empirically study industry dynamics. Examples include Collard-Wexler (2013), who studies plant turnover under demand fluctuation, and Ryan (2012), who inspects the impact of entry cost shifting as a result of regulation change on firms' entry and exit decisions.

[2] Related to the timing of industry entry, Joshi et al. (2009) study the optimal timing of entry to a new market when there exists cross-market influences between the original and the second market.

In contrast to a mature market where consumer preference is relatively stable, the potential demand for a new product evolves over time with uncertainty. In addition to the intrinsic growth of potential demand, we model two forces that can potentially affect the evolution path of consumers' preference. One is the total advertising at the industry level and the other is the economic state. Industry advertising, which depends on the number of incumbents, can speed up the growth of potential demand. A good economic state can boost demand, whereas an economic downturn can delay the takeoff of demand.

In addition to affecting potential demand, these two factors also directly affect a firm's payoff. First, incumbents need to pay for advertising each period. Moreover, the advertising amount is a function of the demand state, and a firm needs to spend more on advertising when the potential demand is low. Second, the economic state affects the interest rate and therefore affects the discount factor, which scales the present value of future payoff from entering (staying in) the industry relative to the entry cost (sell-off value). Firms strategically decide the timing of entry and exit by taking into consideration the trade-offs and expected industry evolution.

Following Maskin and Tirole (1988), we focus on the Markov perfect equilibrium of the game. It means that firms' dynamic entry and exit decisions and incumbents' pricing decisions only depend on payoff-relevant state variables. Equilibrium occurs when each firm's expectation of other firms' actions are consistent with their optimal policies.

2.2. Product Demand

Consumers' intrinsic preference or demand for a new product evolves over time. How fast the potential demand would grow may depend on the characteristics of the product itself, new product advertising, as well as the economic environment.[3] Let F_t be consumers' intrinsic preference for the new product or potential demand at period t:

$$F_t = F_{t-1} + (\bar{F} - F_{t-1}) \cdot (a_1 + a_2 \cdot F_{t-1}) + \alpha D_t + \xi_t(g_t). \quad (1)$$

There are three essential elements in this specification. First, the current preference is a function of consumers' preference in the last period, F_{t-1}. The trajectory of the evolution is flexibly determined by the value of the parameters a_1 and a_2.[4] The parameter $\bar{F}$ can be considered as consumers' intrinsic preference to the product in the long term.

Second, the potential demand can be affected by the total industry advertising in the period, D_t, which is endogenously determined by the number of active firms. Industry advertising D_t approaches 0 as the potential demand approaches $\bar{F}$. We discuss in more detail about advertising when introducing firms' decision problems. The parameter α captures the effectiveness of advertising in driving consumer demand. Note that the cumulative effect of advertising on potential demand is factored into F_{t-1}.

Finally, consumer preference is subject to random shocks ξ_t. We assume that the distribution of ξ_t depends on the state of the economy g at period t,

$$\xi_t(g) \sim \mathrm{N}(\mu_g, \sigma_\xi^2). \quad (2)$$

For simplicity, we assume that the economic state can be either "good" ($g = 1$) or "bad" ($g = 0$), and $\mu_1 \geq \mu_0$. In other words, demand shocks are more likely to be favorable under a good economy than under a bad one. The transition of economic state is exogenously given as

$$T_g = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix}, \quad (3)$$

where $p = \Pr[g = 1 \mid g = 1]$ and $q = \Pr(g = 0 \mid g = 0)$. The parameters p and q capture the stickiness of economic states. The demand shock ξ_t has a persistent effect in the preference evolution path as it affects future preference F_{t+1} through F_t. A consecutive series of negative demand shocks will dampen the growth of preference, and positive demand shocks will speed up the new product acceptance process.

Although the specification of F_t is of reduced form, it captures the most important features in the evolution of the potential demand for new products. Consumers' intrinsic preference to a new product generally follows an upward trend, and the advertising from the early entrants may speed up this process. However, in addition to the uncertainty of demand in a given period induced by demand shock ξ_t, there is also inherent uncertainty in terms of when the new product will take off as the demand shocks also affect the evolution path of F. Persistent negative demand shocks (e.g., economic downturn) would delay the acceptance of the product by the market.

We assume that consumers make purchase decisions in each period and use a nested logit model to describe the choice problem. For consumer h, the net utility of purchasing product j at period t is[5]

$$U_{hjt} = F_t - bP_{jt} + \varepsilon_{hjt}, \quad j = 1, 2, \ldots, n_t, \quad (4)$$

[3] See Mahajan et al. (1990) for a review of new product diffusion models.

[4] Consider the deterministic part involving F_{t-1}. If $a_1 = a_2 = 0$, then preference does not increase. If $0 < a_1 < 1$ and $a_2 = 0$, F_t becomes a first-order autoregressive (AR(1)) process. When $0 < a_1 < 1$ and $0 < a_2 < 1$, the deterministic part permits an S-shaped evolution path. The diffusion parameters a_1 and a_2 may depend on the product and market characteristics.

[5] The paper does not model the dynamic effects with the possibility of consumers being forward looking. Allowing for consumers' forward-looking behavior and the strategic interactions between

where P_{jt} is the price of the product offered by firm j at period t and ε_{hjt} is consumer h's idiosyncratic demand shock. The deterministic utility of the outside option (no purchase) is normalized to be 0. The vector of idiosyncratic demand shocks follows a generalized extreme value distribution in which the marginal distribution of ε_{hjt} is univariate extreme value distribution. While ε_{hjt} $(j > 0)$ is assumed to be independent of ε_{h0t}, the demand shock associated with the outside option, the terms ε_{hjt} $(j > 0)$, are allowed to be correlated. The degree of correlation is captured by a parameter $1-\rho$ $(0 < \rho \leq 1)$.

Let n_t be the total number of producers at period t. Then the sales for product j could be derived as

$$q_{jt} = M\left(\frac{\exp[(F_t - bP_{jt})/\rho]\cdot\{\sum_{i=1}^{n_t}\exp[(F_t - bP_{it})/\rho]\}^{\rho-1}}{1+\{\sum_{i=1}^{n_t}\exp[(F_t - bP_t)/\rho]\}^{\rho}}\right), \tag{5}$$

where M is the total market size.

2.3. Profit per Period

We assume that active firms in the industry need to pay for advertising in each period.[6] Furthermore, the advertising expense is endogenous to the demand state. Assuming symmetric firms, an incumbent in period t pays A_t for advertising,

$$A_t = \gamma_d\left(\frac{\bar{F} - F_{t-1}}{\bar{F}}\right), \tag{6}$$

which is a function of the demand growth potential. The parameter γ_d can be interpreted as the unit cost of advertising and $(\bar{F} - F_{t-1})/\bar{F}$ the amount of advertising. The functional form of advertising amount suggests that firms need to spend more on advertising at the early stage of an industry when the potential demand F_t is low. When the potential demand approaches the long-term limit $\bar{F}$, advertising expense approaches 0. It also implies that early incumbents in the industry bear more cost in advertising the new product to consumers than late entrants do. The total industry advertising at period t is therefore

$$D_t = n_t\left(\frac{\bar{F} - F_{t-1}}{\bar{F}}\right). \tag{7}$$

firms and consumers could be important in some markets. See, for example, Desai and Purohit (1999) and Villas-Boas (2004). Erdem et al. (2003), Nair (2007), and Gordon (2009) examine demand models with forward-looking consumers. Che et al. (2007) look at dynamic competition with consumer state dependence. In this paper we focus on the dynamics on the firm side.

[6] To simplify the problem and focus on firms' entry and exit decisions, we do not allow advertising being a decision variable. For studies of dynamic advertising policies, see, e.g., Villas-Boas (1993) and Dubé et al. (2005).

We assume that firms compete on price and set the optimal price to maximize the period profit (gross of advertising):

$$\tilde{\pi}_t = \max_{P_t}(P_t - c_t)q_t, \tag{8}$$

where q_t depends on the price and the industry state as shown in Equation (5); c_t is the cost state that follows an AR(1) process:

$$c_t = \gamma_0 + \gamma_1 \cdot c_{t-1} + u_t, \quad u_t \sim \mathrm{N}(0, \omega^2), \tag{9}$$

where u_t is the independent and identically distributed (i.i.d.) random cost shock that follows normal distribution with standard error ω. At the beginning of a new industry, the production cost is typically high but decreases as the industry evolves. The AR(1) process captures the first-order cost evolution effect with $\gamma_1 < 1$.

Before the realization of the current demand and cost shocks, the expected profit in period t is

$$\pi(\mathbf{s}_t) = \int\int \tilde{\pi}(\mathbf{s}_t, \xi_t, u_t) f_\xi^g(\xi) f_u(u)\, d\xi\, du - \gamma_d\left(\frac{\bar{F} - F_{t-1}}{\bar{F}}\right), \tag{10}$$

where $\mathbf{s}_t = (F_{t-1}, c_{t-1}, g_t, n_t)$ is the state vector that summarizes the demand and cost condition at the beginning of the period (before the current shocks are realized), the economic state, and the number of active firms in the period.

2.4. Entry and Exit Decisions

Potential entrants and incumbents make entry and exit decisions simultaneously in each period based on the industry state $\mathbf{s}_t$. Potential entrants that decide to enter pay the entry cost now and start to produce in the next period. Incumbents that decide to exit produce in the current period and exit permanently from the next period. Therefore, the change in the number of active firms is realized in the beginning of the following period, $n_{t+1} = n_t + n_t^e - n_t^x$, where n_t^e and n_t^x represent the number of firms who decide to enter and exit in the current period, respectively. In the following, we examine an individual firm's entry or exit decision.

2.4.1. Potential Entrants' Decision. With the start of the industry, potential entrants have the opportunity to enter in every period. The question is whether and when to enter. In each period, a potential entrant's problem can be formulated as choosing between entering now or waiting:

$$V^{\text{out}}(\mathbf{s}, k) = \max\{-k + \beta_g E[V^{\text{in}}(\mathbf{s}', \varphi') \mid \mathbf{s}], \beta_g E[V^{\text{out}}(\mathbf{s}', k') \mid \mathbf{s}]\}. \tag{11}$$

If the firm chooses to enter, it pays the entry cost k in this period, which is a random draw from a uniform distribution $[\underline{K}, \bar{K}]$. The discount factor β_g depends on the interest rate r_g, $\beta_g = 1/(1+r_g)$, which further depends on the state of economy g. The interest rate is typically higher in a booming economy than in a recession. We therefore assume that $r_1 \geq r_0$, or $\beta_1 \leq \beta_0$. Recall that the economic state also governs the distribution of demand shocks. More favorable demand shocks are likely to occur under a good economy than under a bad one, suggesting a higher expected profit when the economy is good. Meanwhile, a growing economy is accompanied by a higher interest rate, suggesting that future returns may be discounted further. It creates a trade-off of entry in a different state of the economy. The term $E[V^{\text{in}}(\mathbf{s}', \varphi') \mid \mathbf{s}]$ corresponds to the expected value of being active in the industry from the following period, conditional on the current industry state and the decision to enter. The expectation is taken over the state variables as well as the draw of exit value φ' as an incumbent. If the potential entrant decides not to enter, it remains in the pool of potential entrants in the next period and would receive a new draw of entry cost k' and undergo the same decision process. The term $E[V^{\text{out}}(\mathbf{s}', k') \mid \mathbf{s}]$ is the expected value of choosing to wait and behave optimally thereafter, conditional on the current information set. It represents the option value of waiting.

A potential entrant would choose to enter the industry immediately if the discounted expected value of entry net of entry cost is greater than the discounted expected value of staying out, or

$$k < \beta_g \{E[V^{\text{in}}(\mathbf{s}', \varphi') \mid \mathbf{s}] - E[V^{\text{out}}(\mathbf{s}', k') \mid \mathbf{s}]\}. \tag{12}$$

The entry threshold depends on the state of the industry. Because of the option value of entering later, the potential entrant may choose to delay entry even if the discounted expected value of entry, $\beta_g E[V^{\text{in}}(\mathbf{s}', \varphi') \mid \mathbf{s}]$, outweighs the entry cost k.

This entry rule implies that, without knowing the private information on one's entry cost, the probability of a potential entrant to enter the industry under state $\mathbf{s}$ is

$$p^e(\mathbf{s}) = \frac{\beta_g \{E[V^{\text{in}}(\mathbf{s}', \varphi') \mid \mathbf{s}] - E[V^{\text{out}}(\mathbf{s}', k') \mid \mathbf{s}]\} - \underline{K}}{\bar{K} - \underline{K}}. \tag{13}$$

Let $\bar{V}(\mathbf{s})$ be the integrated value function or ex ante value function before the private information on entry cost or sell-off value is observed. The integrated value function for a potential entrant is

$$\begin{aligned} \bar{V}^{\text{out}}(\mathbf{s}) = {} & p^e(\mathbf{s})\{-E[k \mid k < \beta_g E[\bar{V}^{\text{in}}(\mathbf{s}') - \bar{V}^{\text{out}}(\mathbf{s}') \mid \mathbf{s}] \\ & + \beta_g E[\bar{V}^{\text{in}}(\mathbf{s}') \mid \mathbf{s}]\} \\ & + (1 - p^e(\mathbf{s}))\beta_g E[\bar{V}^{\text{out}}(\mathbf{s}') \mid \mathbf{s}]. \end{aligned} \tag{14}$$

The integrated value function for incumbents, $\bar{V}^{\text{in}}(\mathbf{s})$, is derived below.[7]

2.4.2. Incumbents' Decision. At the beginning of each period, each incumbent privately receives the signal of its sell-off value and decides whether to accept it and exit in the next period or remain in the industry. The Bellman equation for an incumbent firm can be written as

$$V^{\text{in}}(\mathbf{s}, \varphi) = \pi(\mathbf{s}) + \beta_g \max\{E[V^{\text{in}}(\mathbf{s}', \varphi') \mid \mathbf{s}], \varphi\}, \tag{15}$$

where $\pi(\mathbf{s})$ is the expected current-period profit determined by Equation (10), and φ is the firm's current draw of sell-off value from a uniform distribution $[\underline{R}, \bar{R}]$. If the firm chooses to stay, $E[V^{\text{in}}(\mathbf{s}', \varphi') \mid \mathbf{s}]$ is the expected value of continuation conditional on the current industry state. The expected value of continuation involves the expectation of the industry state in the next period and the expected sell-off value. Alternatively, if the incumbent decides to exit, it collects the sell-off value φ in the next period and exits the industry forever.[8]

An incumbent would choose to exit the industry if the draw of sell-off value is greater than the expected value of staying in the industry,

$$\varphi > E[V^{\text{in}}(\mathbf{s}', \varphi') \mid \mathbf{s}]. \tag{16}$$

It implies that the exit probability of an incumbent under industry state $\mathbf{s}$ is

$$p^x(\mathbf{s}) = \frac{\bar{R} - E[V^{\text{in}}(\mathbf{s}', \varphi') \mid \mathbf{s}]}{\bar{R} - \underline{R}}. \tag{17}$$

Integrating out the private information on sell-off value, the value function for an incumbent can be expressed as

$$\begin{aligned} \bar{V}^{\text{in}}(\mathbf{s}) = {} & \pi(\mathbf{s}) + \beta_g [1 - p^x(\mathbf{s})] E[\bar{V}^{\text{in}}(\mathbf{s}') \mid \mathbf{s}] \\ & + \beta_g p^x(\mathbf{s}) E[\varphi \mid \varphi > E[\bar{V}^{\text{in}}(\mathbf{s}') \mid \mathbf{s}]]. \end{aligned} \tag{18}$$

2.5. Expectation and Equilibrium

The above subsections introduce the decision rules of the potential entrants and incumbents, but a critical component that deserves further discussion is how firms form the expectation of the future state $\mathbf{s} = [F, c, g, n]$ conditional on their current information set. The transition of F_t, the state of potential demand or consumers' intrinsic preference to the product, is characterized by Equation (1). Given that the demand shock follows i.i.d. normal distribution, the transition

[7] See the appendix for the full derivation of the integrated value functions.

[8] We assume that the total number of firms (incumbents and potential entrants) is constant over time. Therefore, if one firm exits, then the pool of potential entrants will increase by one.

density of F conditional on the current state s can be expressed as

$$f^F(F' \mid \mathbf{s}) = \phi\left(\frac{F' - F - (\bar{F} - F)(a_1 + a_2 F + a_3 n) - \mu_g}{\sigma}\right), \quad (19)$$

where $a_3 = \alpha/\bar{F}$ and $\phi(\cdot)$ is the density function of the standard normal distribution. Notice that as F depends on industry advertising, which in turn depends on the number of incumbents, F is endogenously evolving over time when advertising effect is positive ($\alpha > 0$).

The evolution of the cost state c follows Equation (9). The cost states are exogenously evolving with transition density:

$$f^c(c' \mid \mathbf{s}) = \phi\left(\frac{c' - \gamma_0 - \gamma_1 c}{\omega}\right). \quad (20)$$

The transition of the economic states is characterized by matrix T_g of Equation (3).

We now characterize how firms form expectations about the evolution of the number of firms. A firm does not observe the private signals of entry cost or sell-off value of other firms, and the rational expectation of other firms' entry and exit probabilities can be characterized by Equations (13) and (17) respectively. Then the probability of having a number of n^e new entrants given the current state can be expressed as[9]

$$p(n^e \mid \mathbf{s}) = \binom{\bar{E} - n}{n^e} \cdot p^e(\mathbf{s})^{n^e} \cdot [1 - p^e(\mathbf{s})]^{\bar{E} - n - n^e}, \quad (21)$$

which is the probability that among all the potential entrants, n^e of them draw a favorable entry cost and enter now while the rest wait for future opportunities. Similarly, one can derive the probability of having a number of n^x incumbents to exit under state $\mathbf{s}$:

$$p(n^x \mid \mathbf{s}) = \binom{n}{n^x} \cdot p^x(\mathbf{s})^{n^x} \cdot [1 - p^x(\mathbf{s})]^{n - n^x}. \quad (22)$$

The simultaneous entry and exit decisions in the current period lead to a change in the number of active firms of the next period. Therefore, the probability of having n' firms operating in the industry conditional on the current state is

$$p(n' \mid \mathbf{s}) = \sum_{(n^e,\, n^x) \in B} p(n^e \mid \mathbf{s}) p(n^x \mid \mathbf{s}), \quad (23)$$

where B is the complete set of (n^e, n^x) that satisfies the following conditions: $n + n^e - n^x = n'$, $n^x \leq n$, and $n^e \leq \bar{E} - n$.

[9] Note that the expectations are taken over the choices of *other* firms; therefore $p(n^e \mid \mathbf{s})$ and $p(n^x \mid \mathbf{s})$ are slightly different for different firms depending on whether a firm is a potential entrant or an incumbent. A more detailed discussion is provided in the appendix.

The expectation of the evolving industry state determines a firm's expected value of being in or out of the industry. The expected value of different options drives a firm's entry and exit decisions, as shown in Equations (13) and (17). The entry and exit probabilities in turn seed in the expected value of being in or out of the industry, as seen from Equations (14) and (18). Therefore, finding an equilibrium of the game can be considered as to find a vector of choice probabilities that satisfies the value functions and the choice rules simultaneously. The details of solving for the equilibrium in the choice probability space are provided in the appendix.

3. Model Analysis

In this section, we characterize firms' optimal behavior and diffusion pattern by numerically solving the dynamic model introduced in the last section. In particular, we are interested in the effect of advertising, the transition of the economic states, and the demand and cost uncertainty of firms' entry and exit decisions.

3.1. Advertising Effect

Advertising can be important in opening up the market for a new product. Whereas the cost of advertising is paid by incumbents, the benefit of advertising in increasing the potential demand is shared by late entrants.[10] It creates a trade-off in the timing of entry. On one hand, firms have an incentive to enter early to occupy a space in the market, as the industry can only support a limited number of firms. Bresnahan and Reiss (1990) show that, conditional on demand state, a firm's probability of entry decreases with the number of incumbents in the same market. Shen and Villas-Boas (2010) show in a dynamic setting that such a strategic interaction among firms can induce faster entry relative to demand growth when firms are forward looking and facing competition. On the other hand, potential entrants may have an incentive to delay entry to free ride on the market development effort by the early entrants. Also recall that the advertising expense A_t is decreasing in F_t, the demand state. In other words, advertising expenditure is lower as the demand approaches the long-term potential.

To investigate such trade-offs in firms' entry behavior, we simulate different cases by varying the effectiveness of advertising α and the unit cost of advertising γ_d, which reflects the benefit of advertising that is collectively shared by the industry relative to the marginal cost of advertising that is bared by an individual firm. The baseline model assumes away the role of advertising, i.e., $\alpha = 0$ and $\gamma_d = 0$, in which case the

[10] More generally, early entrants of a new industry may bear more cost in cultivating consumers or establishing infrastructure, which has positive externality on industry development.

Table 1 Model Parameter

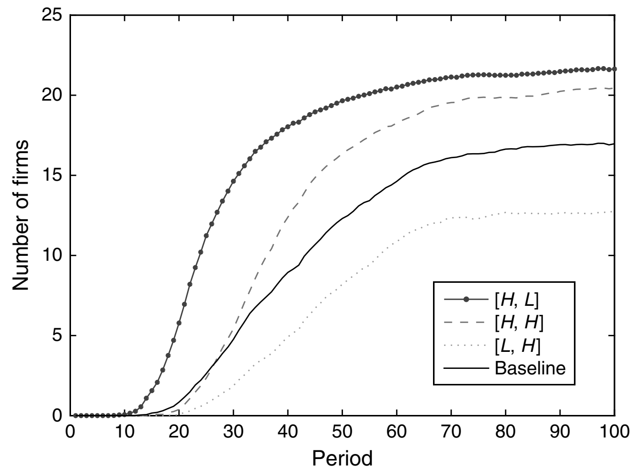
Parameter	Baseline model	Description
$\bar{F}$	5	Long-term preference (steady state)
a_1, a_2	(0.02, 0.01)	Demand growth parameter
α	0	Advertising effectiveness
b	1	Price coefficient
μ_g	0	Mean of demand shocks under economic state g
σ_ξ	0.2	Standard deviation of demand shock ξ
ρ	0.3	Error correlation parameter in nested logit model
M	5	Total market size
γ_0, γ_1	(0.5, 0.9)	Cost evolution parameter
γ_d	0	Advertising cost parameter
ω	0.3	Standard deviation of cost shock
β_g	0.95	Discount factor under economic state g
k	$k \sim [2, 6]$	Entry cost which is uniformly distributed $[\underline{\kappa}, \bar{\kappa}]$
φ	$\varphi \sim [1, 2]$	Sell-off value which is uniformly distributed $[\underline{R}, \bar{R}]$
$\bar{E}$	25	Total number of firms

demand is exogenously evolving.[11] We then simulate four cases with different combinations of high and low advertising effectiveness and high and low cost while keeping other parameters constant across the cases. In each case, given the set of parameters, we solve for the equilibrium of the dynamic entry game and then simulate the industry evolution paths 100 times and take the average.[12]

Figure 1 depicts the diffusion of firms under each case. When advertising is highly effective in driving the potential demand and the advertising cost is low ($[H, L]$), firms enter at a faster rate than in the baseline case. When the advertising effectiveness is low but advertising cost is high ($[L, H]$), we find the opposite—that the number of firms diffuses at a slower rate than in the baseline case. When both adverting effect and advertising cost are low ($[L, L]$), the firm diffusion pattern is similar to the baseline case.

The more interesting case is when both advertising effectiveness and cost are high ($[H, H]$). Compared with the baseline case, entry rate is initially lower but the number of firms increases at a faster rate in the later stage. To further compare the entry strategies under the baseline case and the $[H, H]$ case, we plot the conditional probability of entry as a function of the number of incumbents in Figure 2. The left and right panels show the equilibrium entry probability when the potential demand is at low and median states, respectively, holding other state variables constant. Whereas the entry probability under the baseline case is monotonically decreasing with the number of incumbents

Figure 1 Evolution of Firms with Varying Advertising Effects



Notes. The solid line represents the average evolution of the number of firms under the baseline specification, where $\alpha = 0$ and $\gamma_d = 0$. For other cases, the legend means [level of advertising effectiveness, level of advertising unit cost], where $\alpha^H = 0.2$, $\alpha^L = 0.01$, $\gamma_d^H = 0.5$, and $\gamma_d^L = 0$. The $[L, L]$ case is omitted from the figure for clear display (close to baseline).

given an industry state, the entry probability in the $[H, H]$ case shows a nonmonotonic pattern depending on the demand state.

When the potential demand is low (left panel of Figure 2), the entry probability under $[H, H]$ displays an inverse U shape that first increases with the number of firms and then decreases as the number of incumbents becomes large.[13] Note that the initial entry probability under $[H, H]$ is lower than that in the baseline case, suggesting that when the cost of advertising is high and the potential demand is still at a low state, potential entrants have an incentive to wait and free ride on other firms' advertising in driving the potential demand. The more that firms enter, the more likely that demand will expand and take off. Thus previous entry by other firms in the early stage makes the industry more attractive to the potential entrants and increases the probability of entry.[14] However, the competition effect ultimately dominates if the number of incumbents further increases, which generates the downward part of the response function.

When the potential demand reaches median level but still has significant space to grow (right panel of Figure 2), the entry probability in the $[H, H]$ case decreases with the number of firms, but at a slower rate than in the baseline case. This is because both the competition effect and market expansion effect

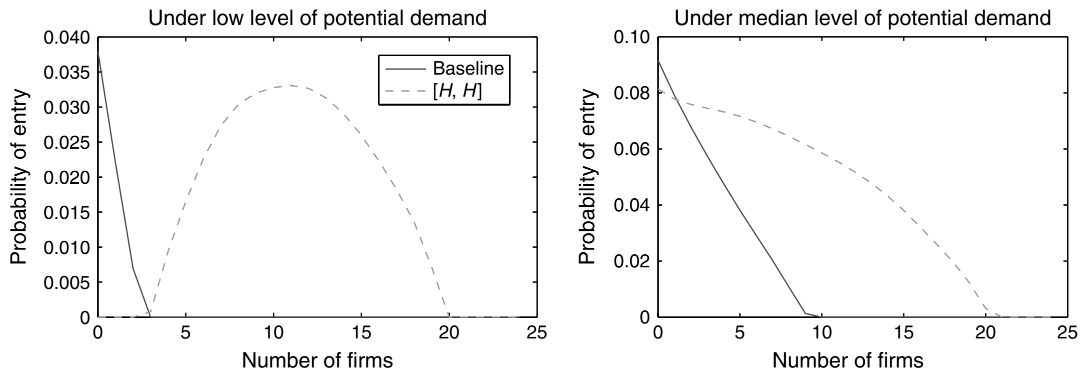
[11] The description of each model parameter along with the value in the baseline model is in Table 1. The parameter values are determined such that the industry stabilizes within 100 periods and the expected profit of entry with a maximum number of incumbents is lower than the lower bound of entry cost.

[12] Please refer to the appendix for computational details.

[13] To check for multiple equilibria in this case of endogenous demand expansion, we solve the game with different starting values of the policy function. All converges to the same solution, which may alleviate the concern of multiple equilibria.

[14] Debruyne and Reibstein (2005) find, using data from the retail brokerage industry, that firms are more likely to enter a new market when similar firms enter the market.

Figure 2 **Entry Probability**



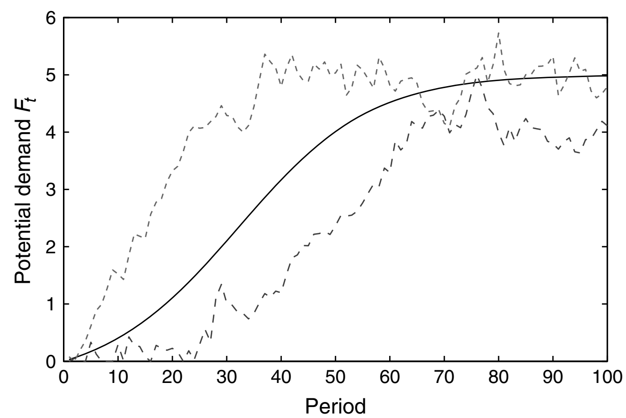
exist by having more competitors in the $[H, H]$ case. How the entry probability changes with the number of incumbents depends on the relative magnitude of the two. As the potential demand further approaches the long-term maximum, the competition effect is the dominating force, and the entry probability under $[H, H]$ and the baseline case becomes closer.

The entry of firms in the early stage of a new industry is likely to increase consumer awareness of the product, promote consumer learning, and expand the potential demand (Agarwal and Bayus 2002). Our analysis shows that with endogenously expanding demand, firms' incentive to enter early critically depends on the cost of being entrepreneurs in a new industry. When the cost is high and early entrants cannot fully appropriate the benefit of market development, it creates first-mover disadvantage and discourages early entry. When firm entry can endogenously expand demand, there is also a bandwagon effect such that more firms enter when other firms enter until the competition effect dominates the positive spillover in demand expansion.

3.2. Economic State

The state of economy affects both consumers and firms. In the demand model, we assume that the economic state affects potential demand through the distribution of demand shocks. A good economic state is associated with more favorable demand shocks and a bad economic state is associated with more negative demand shocks. Demand shocks lead to uncertainty in demand growth. Figure 3 illustrates the evolution paths of potential demand F_t under two series of realization of demand shocks (represented by the two dotted lines), along with the deterministic evolution path without demand shocks (represented by the solid line). The upper dotted line, which shows faster growth from the beginning, represents the case where the first 25 periods are mostly in a good economic state; the lower dotted line with delayed growth represents the case where a bad economic state dominates during the same periods. Different demand evolution paths

Figure 3 **Evolution Paths of Potential Demand**

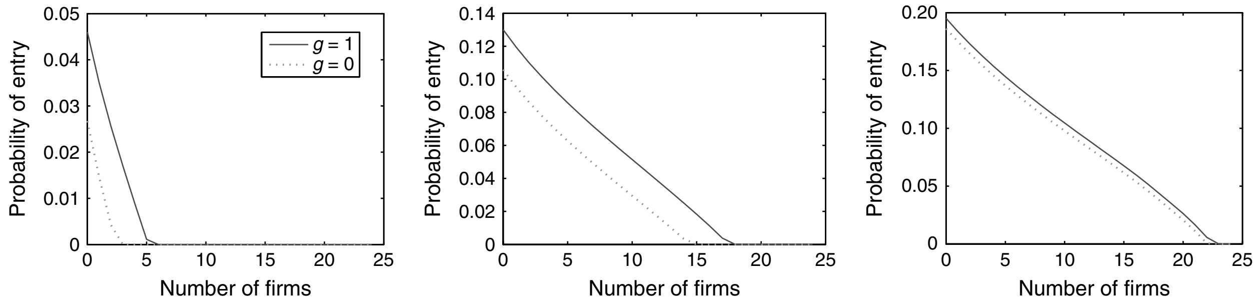


can occur depending on the economic state and the realization of demand shocks.

Intuitively, firms should be more likely to enter under a good economic state, as the expected demand of the period $E(F_t)$ is higher in a good economy, holding everything else equal. However, does this effect remain constant over the industry evolution process? We solve the equilibrium of the dynamic game with $\mu_1 > \mu_0$ and inspect the optimal entry strategy at different demand stages.[15] Figure 4 shows the equilibrium entry probability under either state as a function of the number of incumbents when the potential demand is low (left panel), median (middle panel), or high (right panel), holding the cost state constant. Regardless of the demand state, firms are more likely to enter under a good economic state ($g = 1$) than under a bad economic state ($g = 0$). However, we also find that the *relative* difference in entry propensity under different economic states becomes increasingly smaller as the potential demand reaches its long-term maximum.

[15] Specifically, we assume that $\mu_1 = 0.2$ and $\mu_0 = -0.2$, and the variance of demand shocks σ_ξ is assumed to be 0.2 under either state. The transition probability of economic states is set at $p = q = 0.8$, which implies sticky states. Other parameters are the same as the baseline case in Table 1.

Figure 4 **Entry Probability Under Different Economic States: Low (Left), Median (Middle), and High (Right)**



The difference is most pronounced when the potential demand is at a low level. It suggests that the decision to enter is more sensitive to the economic state at the initial stage of a new industry. This is because in the early stage, when the potential demand is low, the economic environment can significantly affect the timing of takeoff, as illustrated by Figure 3, and lead to a different prospect of the future. After demand takeoff and as the industry approaches the mature stage, the impact of economic environment on entry becomes small. The comparison of the equilibrium exit probability under good versus bad economic states shows a consistent pattern. Firms are less likely to exit when facing a good economy than when facing an economic downfall. If the transition of the economic states can be viewed as a business cycle, then the entry and exit patterns in a new industry reinforce the business cycle of the economy.

A related question is whether the "stickiness" of the economic states, as captured by the transition probability p and q in Equation (3), would also have an impact on firm diffusion. When the states are stickier (large p and q), the discrepancy between the expected demand evolution path starting in a good economy versus that in a bad economy is increased. Therefore, we find that when demand is low, the contrast in entry probability under different economic states is bigger when the states are less likely to switch.

Another moderating factor of the entry incentive under different economic states is the interest rate. In the above simulation, we assume that the interest rate, and therefore the discount factor, is the same in all the states. However, a good (bad) economic state is typically associated with a higher (lower) interest rate. As the discount factor β_g is an inverse function of interest rate, raising the interest rate lowers the present value of future profit. We find that when $r_1 > r_0$, the gap between the equilibrium entry probability under a good versus bad economic state shrinks compared with the case above where $r_1 = r_0$ and can even flip if the interest rate becomes too high under a good economy. In other words, the tendency to enter in a good economic state is curbed by a high interest rate and the entry in a bad economic state is encouraged by a lower interest rate. It implies that policy intervention on interest rates, or the cost of entry under different economic states, can counter the business cycle to some extent.

3.3. Uncertainty

Firms make entry and exit decisions in an uncertain environment. With the presence of the demand shocks, the evolution path of potential demand is not deterministic, so too is the demand level in a particular period before the realization of these shocks. The presence of the cost shock induces uncertainty in the cost evolution and contributes to the uncertainty in price. How does the uncertainty of demand and cost affect the evolution of the number of firms?

We first focus on the effect of demand uncertainty by comparing the equilibrium paths of firm diffusion with different levels of demand uncertainty, as captured by the variance of demand shocks, σ_ξ^2. Figure 5 shows the evolution of the number of firms when σ_ξ is 0.2 versus 0.4, keeping all the other parameters constant.[16] We find that higher demand uncertainty leads to fewer firms in the long-run equilibrium when the industry stabilizes. However, higher uncertainty induces faster takeoff of firms in the early stage when the demand level is low. The intuition behind it is that when the demand level F_t is low, greater demand uncertainty increases the chance that F transits to a high level in the next period, which can be inferred from the transition density of F in Equation (19). Therefore higher demand uncertainty raises the expected value of entry when demand is at a low level. When the potential demand approaches the long-run maximum $\bar{F}$, the effect is opposite: greater uncertainty discourages entry.

This is evident from Figure 6, where we plot the expected value of entry relative to staying out ($EV^{\text{in}} - EV^{\text{out}}$) under the two levels of demand uncertainty as a function of the demand state F, holding the number of incumbents and the cost state constant. First note that the value function is convex in the lower range of

[16] Other parameters are the same as in the baseline model. We do not consider the advertising effect and the difference between economic states in this case so as to focus on demand uncertainty.

Figure 5 Evolution of Firms Under Different Demand Uncertainty

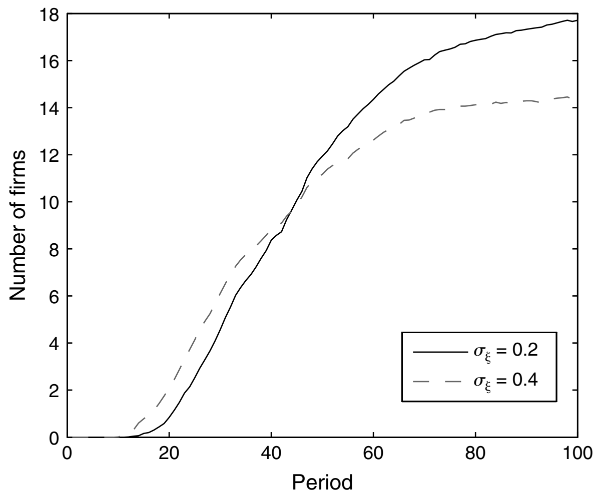
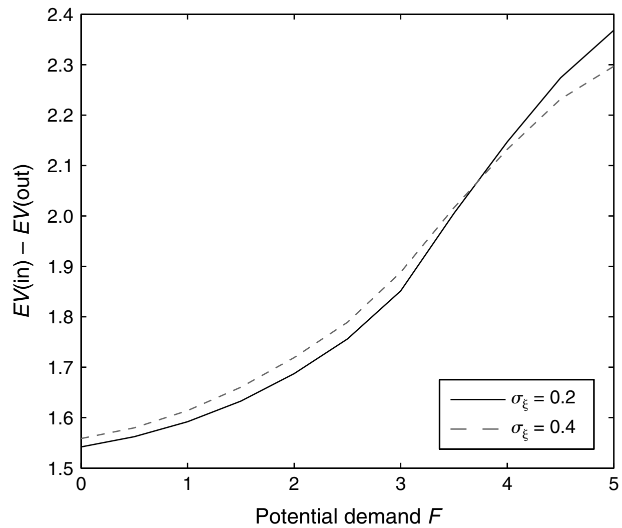


Figure 6 Expected Value of Entry

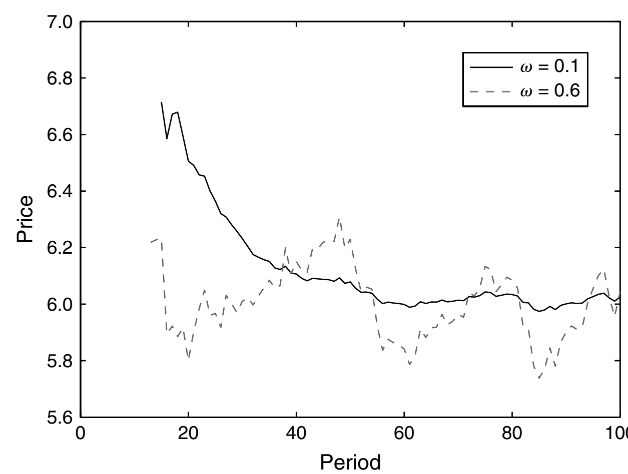


F values and concave toward $\bar{F}$. The variability of F in the convex part results in a higher average value of entry. However, variation of F in the concave part leads to the opposite. The shape of the value function is different from that of the period profit function, which is convex in demand level.[17] It implies that the effect of uncertainty on entry in a dynamic setting is different from that in a static case. Second, we compare the two value functions, which cross over at a demand state close to the maximum. As a higher expected value of entry relative to staying out suggests a higher probability of entry, as shown by Equation (12), we can infer from the figure that higher demand uncertainty leads to more entries when the potential demand is low and fewer entries when the demand is close to the long-term potential or reaches the mature phase.

Note that the demand uncertainty discussed here is intrinsic to the system and not induced by lack of information. It is different from the case where firms are uncertain about the size of the ultimate market ($\bar{F}$) and can learn about it through entry. We do not consider learning in the model. If potential entrants can reduce the uncertainty of market size through the entry of other firms, then one may observe lower entry rate in the early stage of industry development when the demand uncertainty and the benefit of learning is high.[18]

The uncertainty of cost has a similar effect on firm diffusion. Higher cost variance results in fewer firms in the long-run equilibrium but more entries in the early stage, similar to the pattern shown in Figure 5. In Figure 7, we compare the price evolution under two levels of cost variation, $\omega = 0.1$ and $\omega = 0.6$. Higher variation in cost leads to higher variation in price and a fewer number of firms as the industry approaches the mature stage.[19] Yet in the early stage of the industry, cost variation could encourage more entries.

Figure 7 Price Evolution Under Different Cost Uncertainty



4. Further Discussion

We have focused our discussion mainly on firms' entry behavior in response to demand-side considerations, including endogenous demand expansion through advertising and uncertainty in demand level

[17] Oi (1961) shows in a static case that higher uncertainty can result in a greater profit under perfect competition because of the convexity of profit function.

[18] Horvath et al. (2001) consider a model where firms are uncertain about the profitability of the industry and can resolve the uncertainty through learning from the incumbents' performance. They find that firms delay entry as information accumulation to support the entry decision takes time.

[19] Ghosal (1996) empirically shows that there is a negative relationship between price uncertainty and the number of firms in an industry using cross-sectional data of U.S. manufacturing industries.

and growth path. We now briefly discuss the implications of entry cost and production cost on firm diffusion.

Entry cost is an important factor that can affect entry decision. The entry rule (Equation (13)) suggests that entry probability decreases with the spread of the possible entry cost $\bar{K} - \underline{K}$ or, more generally, the variance of the entry cost distribution. This is because with higher variation in the possible realization of entry cost, firms have more incentive to wait to draw a new entry cost in the next period if the current draw of entry cost is large.

Another related issue is how the entry cost is determined. Currently, we assume that each potential entrant independently draws an entry cost from a commonly known distribution in each period. This assumption ensures a symmetric entry policy among firms and makes the model tractable with a large number of firms. Alternatively, one can think of the case that entry cost is drawn once and fixed for the rest of the time. In this latter case, firms are no longer symmetric even before entry, and their decisions to enter or wait provides some information to their competitors about their private draw of entry cost.[20] Firms with a low entry cost enter early. Compared with the case with an independent draw of entry cost across time, the fixed entry cost is likely to yield a lower total number of firms in the steady state. The intuition is that firms with a high entry cost have no chance to draw a lower entry cost now and are therefore less likely to enter. On the other hand, firms with an entry cost below the threshold in each period have less incentive to wait compared with the case where they can possibly draw an even lower entry cost in the next period.

For simplicity, the model assumes a symmetric production cost among the firms. One may consider the case where incumbents that have operated in the industry for longer periods enjoy a lower production cost than the new entrants because of experience accumulation.[21] In this case, firms have an incentive to enter early to gain a cost advantage in future competition and endogenously erect entry barriers. Similarly, allowing other forms of first-mover advantage will prompt firms to enter early.

5. Conclusion

In this paper, we construct a dynamic model that features stochastically and endogenously expanding demand of a new industry. The model allows industry advertising by the incumbents to positively affect demand growth. There is demand shock in each period related to the underlying economic environment that not only introduces uncertainty to the demand level of the period but also leads to uncertainty in the demand evolution path. We characterize firms' optimal entry and exit behavior when facing such trade-offs and uncertainties as the industry evolves.

We find that when the cost of market development (advertising in this case) is high and the benefit spills over to the potential entrants, firms have an incentive to wait. The probability of entry can increase with the number of incumbents when the demand is low, which results in a fast increase in the number of firms when some start to enter. We also find that higher uncertainty in demand as measured by the variance of demand shocks can induce faster entry in the initial stage of the industry. The chance that demand will immediately reach a high level raises the expected value of being in the industry and attracts entry. The transition of economic environment can also affect a firm's entry decision. Because a good economic state is more likely to boost demand and raise the expected profit, it is favored by the potential entrants, especially when the industry is at the initial stage when demand has not yet taken off. However, if a higher interest rate is associated with a good economy or the economic states switch easily, then the difference in entry tendencies between the economic states becomes negligible.

The paper also shows that many of the effects are dynamically changing with the stage of industry development. Endogenous demand expansion can result in entry probability increasing with the number of firms, but it only happens when the demand is low and the competition is not severe. As the industry grows, the competition effect eventually dominates the positive spillover, which restores the negative relationship between entry probability and the number of incumbents. The difference in entry probability under good versus bad economic states is most pronounced in the early stage and gradually diminishes as the demand approaches the long-term potential. Higher demand or cost uncertainty prompts entry when demand is low and cost is high but discourages entry when the industry reaches the mature stage.

The dynamics of a new industry development are complex. In this paper, we try to capture some important features on the demand side in a parsimonious way. There are many questions awaiting further research. One important area is uncertainty and learning. For extremely new innovations, there could be fundamental uncertainty about whether there is a market for the product. Although we capture the uncertainty in the takeoff timing by allowing demand shocks to change demand evolution path, we do not model the

[20] Shen and Villas-Boas (2010) discuss the implications of fixed entry cost on the probability of entry in a duopoly entry game. Fudenberg and Tirole (1986) study the case that duopoly firms enter the market knowing its own production cost but not that of the opponent. Remaining active for long periods signals that a firm has a low cost. The discouraged rival then exits.

[21] Jovanovic and Lach (1989) consider an opposite case where production cost is vintage-specific and later entrants enjoy a lower cost structure.

uncertainty of market existence. It would be interesting to further model such uncertainty and allow firms to update their belief of market potential through learning. In this case, demand not taking off in a long period of time may suggest that the market does not exist for the new product. Therefore, entry probability can decrease over time conditional on demand not taking off.

Another avenue for future research is to investigate firm diffusion allowing heterogeneous firm belief and strategy. In our model, firms are rational, strategic, and forward looking. The diffusion path is the equilibrium of a dynamic game in which firms' beliefs are mutually consistent. However, firms may have different beliefs about the demand growth, with some being overly optimistic and some being pessimistic. Firms may also have different levels of strategic thinking that some players do not fully foresee the competitive response of others (Camerer et al. 2004, Goldfarb and Xiao 2011). Firms may also differ in their patience for future returns. It would be interesting to investigate the impact of such a mix of firm beliefs and strategies on industry dynamics.

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Appendix

A.1. Incumbents' vs. Potential Entrants' Expectations
When making entry or exit decisions, firms form expectations about the actions of *other* active or potential competitors. As a result, the probabilities calculated by a potential entrant or an incumbent are slightly different.

The probability of having a number of n^e new entrants given state S from the incumbents' perspective is simply

$$p(n^e \mid \mathbf{s}) = \binom{\bar{E}-n}{n^e} \cdot [p^e(\mathbf{s})]^{n^e} \cdot [1-P^e(\mathbf{s})]^{\bar{E}-n-n^e}; \qquad (24)$$

from a potential entrant's perspective, it is

$$p(n^e \mid \mathbf{s}) = \binom{\bar{E}-n-1}{n^e} \cdot [p^e(\mathbf{s})]^{n^e} \cdot [1-P^e(\mathbf{s})]^{\bar{E}-n-n^e-1}.$$

The difference is that the number of other potential entrants is $\bar{E}-n$ for incumbents and is $\bar{E}-n-1$ for a potential entrant. A further note is that n^e is the number of new entrants not including the focal potential entrant that may decide to enter.

Similarly, from the perspective of potential entrants, the probability of having n^x (other) firms exit is

$$p(n^x \mid \mathbf{s}) = \binom{n}{n^x} \cdot [1-P^x(\mathbf{s})]^{n-n^x} \cdot P^x(\mathbf{s})^{n^x};$$

from the perspective of incumbents, it is

$$p(n^x \mid \mathbf{s}) = \binom{n-1}{n^x} \cdot [1-P^x(\mathbf{s})]^{n-1-n^x} \cdot P^x(\mathbf{s})^{n^x}.$$

A.2. Numerical Algorithm to Solve the Equilibrium
We use policy iteration to find the equilibrium of the game. We first derive the integrated value function as a function of the entry and exit probabilities.

Following Equation (18) and given the uniform distribution of entry cost, the value function for incumbents integrated over the private signal of sell-off value can be expressed as

$$\begin{aligned}\bar{V}^{\text{in}}(\mathbf{s}) &= \pi(\mathbf{s}) + \beta_g(1-p^x)E[\bar{V}^{\text{in}}(\mathbf{s}') \mid \mathbf{s}] \\ &\quad + \beta_g p^x E[\varphi \mid \varphi > E[\bar{V}^{\text{in}}(\mathbf{s}') \mid \mathbf{s}]] \\ &= \pi(\mathbf{s}) + \beta_g(1-p^x)E[\bar{V}^{\text{in}}(\mathbf{s}') \mid \mathbf{s}] \\ &\quad + \beta_g p^x \left(\frac{E[\bar{V}^{\text{in}}(\mathbf{s}') \mid \mathbf{s}] + \bar{\phi}}{2}\right),\end{aligned}$$

where $E[\bar{V}^{\text{in}}(\mathbf{s}' \mid \mathbf{s})]$ depends on the transition matrix. Let $\mathbf{T}$ be the transition matrix with each element being the transition probability from state s to s'. The transition of each variable in s is described in §2.5. By collecting terms, we now have the integrated value function expressed as a function of firms' entry and exit policies:

$$\bar{V}^{\text{in}}(\mathbf{s}) = \left[I - \beta_g\left(1-\frac{1}{2}p^x\right)\mathbf{T}\right]^{-1}\left[\pi(\mathbf{s}) + \frac{1}{2}\beta_g p^x \bar{\phi}\right]. \qquad (25)$$

Similarly, following Equation (14) and the uniform distribution of exit value, the integrated value function of potential entrants can be expressed as

$$\begin{aligned}\bar{V}^{\text{out}}(\mathbf{s}) &= p^e(\mathbf{s})\{-E[k \mid k < \beta_g E\bar{V}^{\text{in}} - \beta_g E\bar{V}^{\text{out}}\} \\ &\quad + \beta_g E[\bar{V}^{\text{in}}(\mathbf{s}') \mid \mathbf{s}]\} + (1-p^e(\mathbf{s}))\beta_g E[\bar{V}^{\text{out}}(\mathbf{s}') \mid \mathbf{s}] \\ &= -p^e(\mathbf{s})\frac{\beta_g E\bar{V}^{\text{in}} - \beta_g E\bar{V}^{\text{out}} + \underline{K}}{2} + p^e(\mathbf{s})\beta_g E[\bar{V}^{\text{in}}(\mathbf{s}') \mid \mathbf{s}] \\ &\quad + (1-p^e(\mathbf{s}))\beta_g E[\bar{V}^{\text{out}}(\mathbf{s}') \mid \mathbf{s}] \\ &= \left[I - \beta_g\left(1-\frac{1}{2}p^e\right)\mathbf{T}\right]^{-1} \\ &\quad \cdot \left[\frac{\beta_g p^e E[\bar{V}^{\text{in}}(\mathbf{s}) \mid \mathbf{s}] - p^e \underline{K}}{2}\right]. \end{aligned} \qquad (26)$$

The state space is four-dimensional; $s = [F, c, n, g]$. The number of firms n and the economic state $g = \{0, 1\}$ are discrete. The continuous demand and cost states, F and c, are discretized into N_d and N_c points, respectively. Therefore, the total number of states is $R = N_d \times N_c \times (\bar{E}+1) \times 2$. Let $\mathbf{S} = \{S_1, S_2, \ldots, S_R\}$ be the set of discretized states. Provide an initial guess of the entry probabilities of potential entrants and the exit probabilities of incumbents under each state $[p_e^0(S_r), p_x^0(S_r)]$. The iteration process proceeds as follows, given a set of model parameters:

Step 0. Compute the per-period expected profit under each state. We use Gauss–Hermite quadrature to integrate over the demand and cost shock to obtain the expected profit

(Equation (10)). Moreover, compute the conditional transition probability f^F and f^c given Equations (19) and (20).

Step 1. In iteration ι, given the current guess of choice probabilities $[p_e^\iota(S_r), p_x^\iota(S_r)]$, compute the transition matrix for the number of active firms using Equation (23). Together with the transition matrix of other state variables, obtain the full transition matrix $\mathbf{T}^\iota$.

Step 2. Given the transition matrix and the policy functions, calculate the integrated value function for incumbents using Equation (25) above. Compute the integrated value function for potential entrants using Equation (26).

Step 3. Update firms' optimal entry and exit probabilities $[p_e^*(S_r), p_x^*(S_r)]$ under each state using Equations (13) and (17).

Step 4. Check convergence. If the following condition is met, then we obtain an equilibrium:

$$\max\{d[p_e^*(S_r), p_e^\iota(S_r)], d[p_x^*(S_r), p_x^\iota(S_r)]\} < \epsilon,$$

where $d[\cdot, \cdot]$ is a distance function and ϵ is a prespecified tolerance parameter. If the convergence criterion is not met, then update firms' entry and exit probabilities as follows:

$$p_e^{\iota+1}(S) = wp_e^\iota(S) + (1-w)p_e^*(S),$$
$$p_{x.\ell}^{\iota+1}(S) = wp_x^\iota(S) + (1-w)p_x^*(S),$$

where $w \in (0, 1)$ is a smoothing parameter. Repeat Steps 1–4 until convergence is achieved.

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