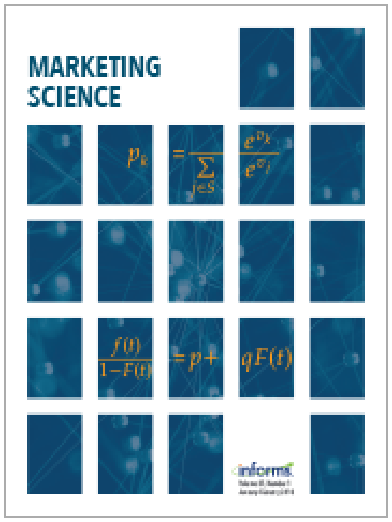

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Research Note

On the Profitability of Firms in a Differentiated Industry

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In a model of vertical differentiation, the principal concern of this paper is to identify sufficient conditions for producing a higher- or lower-quality good to be more profitable (in terms of profits and profit margin). Our basic model considers a *short-run* scenario where the firms' quality levels are fixed and they engage in price competition. Here, our results are three. First, we develop the notion of *relative cost efficiency* and show that its (increasing or decreasing) monotonicity in product quality implies that of firm profitability in equilibrium. A firm's relative cost efficiency refers to its quality-adjusted cost (dis)advantage relative to its immediate competitors, for a given distribution of consumer tastes. Second, selling a higher-quality good is more profitable when *absolute cost efficiency* (defined as the ratio between a firm's quality and unit cost) is increasing in quality. Third, we also establish a set of lower and upper bounds on each firm's profitability.

The basic model is then extended in two directions. We examine unit cost functions that demonstrate monotone profitability, even when both quality and price are endogenous variables. We also show that the spirit of relative cost efficiency and its associated sufficient condition hold valid for a logconcave consumer distribution.

Key words: vertical differentiation; profitability; Bertrand competition; game theory
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1. Introduction

Consider a market where a number of firms each offer a product of distinct quality and consumers differ in their willingness to pay for quality. The firms have constant, but likely different, unit variable costs, and they compete through setting prices simultaneously. In this vertically differentiated industry, we are mainly concerned with the following question: When is producing a higher-quality product more (less) profitable at equilibrium? To better illuminate the nature of this question, it is helpful to first revisit two familiar benchmarks. If the firms differ only in product quality, but have the same unit variable cost, then offering a higher-quality product is more profitable (Shaked and Sutton 1982). If the firms offer a homogeneous product, but differ in unit variable costs, then the firm with the lowest cost is more profitable by the standard Bertrand argument. However, the picture becomes much less clear if the firms differ in both quality and unit costs. By tackling this central question, this paper aims to fulfill two purposes. First, our results can provide a firm with useful guidelines on how to better approach its product-positioning strategies by taking into account its product and process know-how relative to that of its competitors. Second, we all observe that in some industries it is the firms offering superior product qualities that dominate, while in some others it is those with the highest operating efficiency that rule. It seems intuitive that the former emerges when the costs of the highest-quality firms are not substantially higher, and the latter emerges when the qualities of the lowest-cost firms are not substantially lower than those of the other firms. We make this intuition precise by pointing out the exact industry conditions for each scenario to arise.

Besides, this paper also makes a technical contribution. For the above setup with multiple firms and a general quality-cost structure, expressions of the equilibrium prices are too complex to allow a direct manipulation of firm profitability.[1] Instead of seeking an explicit price equilibrium, this paper introduces an inductive method for ranking the firms'

[1] For a glimpse of the intractability in solving the general pricing game in a vertical setting, see page 332 of Gabszewicz and Thisse (1980), where the product qualities are assumed to be equally spaced and the unit variable costs are suppressed to zero. Price expressions for a general quality-cost structure (as we examine) are much more complex, prohibiting any meaningful comparison of the derived profitability measures.

various performance measures (profit margin, profits, and market share) based on the system of first-order conditions. This method appears to be rather effective for tasks that do not require precise knowledge of the equilibrium prices (such as establishing the ranking or bounds of the firms' performance measures; cf. Propositions 1, 2, 4, 7, and A1 in the technical appendix) and may not be effective for the other tasks (such as determining the equilibrium quality location of the firms for a given unit cost function).

Our main model assumes that the firms' product qualities and unit costs are fixed exogenously, and that consumer types are uniformly distributed. Here, our major result is to introduce the concept of relative cost efficiency, whose monotonicity in quality is shown to ensure that of firm profitability. A firm's *relative cost efficiency* is a comprehensive measure that captures the unit costs and qualities of *both* the firm itself *and* its immediate competitors, for a given distribution of consumer types. We also demonstrate how the results may be useful in (re)positioning a new (old) product. In particular, we show when a firm with an inferior product may outperform its competitors with superior offerings. Besides, a firm selling a higher-quality good is more profitable if it has a higher quality/cost ratio (which we call its *absolute cost efficiency*). A set of lower and upper bounds on each firm's profitability is also given.

The main model is then extended in two directions. First, we explore the unit cost functions demonstrating monotone profitability, when the firms compete by locating their products according to the same cost function and setting prices. We also show that the notion of relative cost efficiency and our induction-based analytical approach hold valid for a logconcave distribution of consumer types.[2]

The present paper is related to the literature on vertical differentiation in economics and marketing (e.g., Mussa and Rosen 1978; Gabszewicz and Thisse 1980; Shaked and Sutton 1982, 1983; Itoh 1983; Moorthy 1984, 1988; Champsaur and Rochet 1989). Mussa and Rosen (1978), Itoh (1983), and Moorthy (1984) deal with how a monopolist firm should price discriminate against its consumers. In a duopoly setting, Shaked and Sutton (1982) and Moorthy (1988) characterize the subgame perfect equilibria in qualities and prices. Gabszewicz and Thisse (1980) and Shaked and Sutton (1983) show that under certain conditions on the cost function, the market can only sustain a limited number of firms at a Nash price equilibrium, even in the absence of any fixed costs of entry. Therefore, comparing the profitability of firms is not the focus of these papers. In addition to these Bertrand models, there also exist Cournot models of vertical differentiation, such as Gal-Or (1985), Moorthy (1985) and, more recently, De Fraja (1996) and Johnson and Myatt (2003). The article by Johnson and Myatt (2003) examines when a multiproduct monopolist may expand or contract its product line in response to entry.

This paper is also related to the marketing literature on product-line design and pricing (e.g., Moorthy 1984; Oren et al. 1984; Reibstein and Gatignon 1984; Shugan 1984; Dobson and Kalish 1988; Balachander and Srinivasan 1994; Randall et al. 1998; Villas-Boas 1998, 2004; Desai 2001; Tyagi 2004). However, most of these papers are monopoly models and thus do not address the issue of comparing the firms' profitability. For example, Villas-Boas (1998) shows that a manufacturer selling through an intermediary should design its product line differently than when selling directly to end consumers. Randall et al. (1998) provide an empirical examination on the relationship between the length of a vertical product line and brand equity. In Desai (2001), consumers' sensitivity to horizontal features is shown to affect the cannibalization between products of different qualities.[3]

The main model is presented in §2. Section 3 shows how monotonicity of relative cost efficiencies implies that of firm profit and profit margin in equilibrium and discusses its managerial implications. We then derive further results involving absolute cost efficiency and profitability bounds. Section 4 studies cost functions embodying monotone profitability and extends the main model to a logconcave distribution of consumer types. Section 5 concludes the paper. The proofs of the propositions in the text are given in the appendix. A technical appendix at http://mktsci.pubs.informs.org contains expanded material referred to in the paper.

2. Main Model

The main model considers a *short-run* scenario in which both the number of firms and their quality and cost levels are fixed. In this market, there are K firms with constant unit variable costs. Specifically, firm k produces a good of quality v_k at a constant unit cost c_k, and there are no fixed costs. The firms compete through setting prices simultaneously. We assume that the quality levels of the firms are all distinct to avoid trivialities due to the Bertrand argument. Without loss of generality, the firms are indexed such that $0 < v_1 < v_2 < \cdots < v_K$. Each firm is thus represented by a point (v_k, c_k) in the quality-cost space.

[2] Consumer distributions with logconcave densities play an important role in the theory of imperfect competition (cf. Caplin and Nalebuff 1991 and Anderson et al. 1995).

[3] Recently, frameworks of product differentiation are used by Liu et al. (2004), Shaffer and Zettelmeyer (2004), Sun et al. (2004), and Syam et al. (2005) to examine various issues such as customization and advertising.

Connecting each pair of adjacent points (v_{k-1}, c_{k-1}) and (v_k, c_k) with a straight line would lead to the cost trajectory of this oligopoly. Define

$$d_k = \frac{c_k - c_{k-1}}{v_k - v_{k-1}}, \quad \text{for } k = 1, \dots, K, \tag{1}$$

where $v_0 = c_0 = 0$. Here a convention is to view v_0 as a virtual product with zero quality and no cost. Each d_k represents the slope of the cost trajectory between v_{k-1} and v_k. Because no restrictions are imposed on the firms' costs (other than $c_k < v_k$; due to our specification of consumer demand below, any product with $c_k \geq v_k$ cannot generate positive sales, even when priced at cost), d_k need not be monotonic in k. In other words, there may be zigzags in the cost trajectory.

Each consumer either purchases precisely one unit from the products offered or does not purchase. Assume that a consumer's utility is multiplicatively separable in product quality and her own type characteristic. Without further loss of generality, we adopt the familiar Mussa-Rosen utility function.[4] A consumer of type θ obtains net utility $\theta v - p$ from a product of quality v priced at p. Thus, consumer type represents her constant marginal willingness to pay for quality. To ease exposition, we assume that θ is uniformly distributed[5] on $[0, 1]$.

Denote the price of firm k as p_k. Connecting each pair of adjacent points (v_{k-1}, p_{k-1}) and (v_k, p_k) in the quality-price space with a straight line, one would obtain the price trajectory of this oligopoly. Let

$$\theta_k = \frac{p_k - p_{k-1}}{v_k - v_{k-1}}, \quad \text{for } k = 1, \dots, K, \tag{2}$$

where $v_0 = p_0 = 0$, i.e., v_0 is a free product with zero utility. Selecting v_0 amounts to not purchasing at all.

Here θ_k is the slope of the price trajectory between firms $k-1$ and k. A consumer of type θ_k obtains the same level of net utility from buying v_{k-1} and v_k, and she is therefore indifferent between these two quality levels. A familiar implication of the Mussa-Rosen utility function is that consumers between θ_k and θ_{k+1} (with $\theta_{K+1} = 1$) will purchase product k. Therefore, if the prices are such that θ_k is nondecreasing in k, the demand and profit of firm k are

$$s_k = \theta_{k+1} - \theta_k \tag{3}$$

and

$$\pi_k = (p_k - c_k)(\theta_{k+1} - \theta_k), \tag{4}$$

respectively. The firms set prices simultaneously to maximize their own profits.

A Nash equilibrium exists in this pricing game, as shown by Shaked and Sutton (1983) and Caplin and Nalebuff (1991). Apparently, the firms have no incentive to price below cost. Because we focus on the situation where all K firms are active at equilibrium, in the sense that each attracts strictly positive demand at a price at or above its unit cost, the equilibrium price trajectory is increasing and convex. Let $r_k = 1/(v_k - v_{k-1})$ denote the degree of product differentiation between firms k and $k-1$. The first-order conditions (FOC) are

$$\theta_{k+1} - \theta_k = (p_k - c_k)(r_{k+1} + r_k), \quad \text{for firm } k \leq K-1, \tag{5}$$

$$1 - \theta_K = (p_K - c_K) r_K, \quad \text{for firm } K. \tag{6}$$

Firm k's FOC equates the marginal benefit to the marginal cost of a unit increase in its price p_k: The left-hand side (LHS) equals firm k's demand and represents its profit gain from the unit price increase. A unit increase in p_k ($k < K$) also lowers θ_{k+1} by r_{k+1} and raises θ_k by r_k, thus driving away original purchasers of v_k in the number of $r_{k+1} + r_k$. Likewise, a unit increase in p_K will cause r_K of firm K's consumers to switch to firm $K-1$. The right-hand side (RHS) thus stands for firm k's profit loss due to the unit price increase.

3. Analysis

In this section, we introduce a set of production efficiency measures and show that their monotonicity in quality ensures that of firm profit and per-unit margin, respectively, in a Nash price equilibrium.[6] A firm with a strategic orientation is concerned about not only its absolute profit level but also its competitiveness relative to its rivals. Our analysis can aid such a strategic firm in selecting and calibrating its product strategy.

3.1. Relative Cost Efficiencies and Monotone Profitability

Because profit maximization is often imputed to be a firm's highest objective, we shall first investigate the conditions under which offering a higher- or lower-quality product generates a higher profit. Our first step is to translate the FOCs into expressions of firms' profits, instead of prices. From (4), (5), and (6), at equilibrium firm k's profit is $\pi_k = (p_k - c_k)^2(r_{k+1} + r_k)$. Therefore,

$$p_k = c_k + \sqrt{\pi_k/(r_{k+1} + r_k)}, \quad \text{where } r_{K+1} = 0. \tag{7}$$

[4] A multiplicatively separable utility function can be transformed into the Mussa-Rosen form through rescaling (cf. Itoh 1983 and Johnson and Myatt 2003).

[5] This assumption is relaxed in §4.2.

[6] Proposition A1 in the technical appendix at http://mktsci.pubs.informs.org shows that the monotonicity of relative cost efficiency indicates that of equilibrium market share.

With (7), we can then rewrite (5) and (6) as:

$$r_{k+1}\sqrt{\pi_{k+1}/(r_{k+2}+r_{k+1})} = 2\sqrt{(r_{k+1}+r_k)\pi_k} - r_k\sqrt{\pi_{k-1}/(r_k+r_{k-1})} - (d_{k+1}-d_k), \tag{8}$$

for $k \le K-1$, where $\pi_0 = r_0 = 0$, and

$$0 = 2\sqrt{r_K\pi_K} - r_K\sqrt{\pi_{K-1}/(r_K+r_{K-1})} - (1-d_K), \quad \text{for firm } K. \tag{9}$$

Equations (8)–(9) allow for direct manipulation of the firms' profits. Let

$$t_1^\pi \equiv 2\sqrt{r_2+r_1} - r_2\big/\sqrt{r_3+r_2},$$
$$t_K^\pi \equiv 2\sqrt{r_K} - r_K\big/\sqrt{r_K+r_{K-1}},$$

and for $1 < k < K$,

$$t_k^\pi \equiv 2\sqrt{r_{k+1}+r_k} - r_k\big/\sqrt{r_k+r_{k-1}} - r_{k+1}\big/\sqrt{r_{k+2}+r_{k+1}}, \quad \text{where } r_{K+1} = 0.$$

Because each $r_k > 0$, we have $t_k > 0$. Just like r_k, each t_k still measures the degree of differentiation between firm k's product v_k and those of its neighboring firms, except that t_k takes into account the qualities of up to two (instead of one) firms below and above v_k.

Definition 1. Firm k's *relative cost efficiency w.r.t. profit* is defined as $E_k^\pi \equiv (d_{k+1}-d_k)/t_k^\pi$, where $1 \le k \le K$ and $d_{K+1} = 1$.

E_k^π incorporates the costs and qualities of both firm k itself and those offering adjacent quality levels. Conceptually, it is a quality-weighted measure of firm k's production efficiency relative to its immediate competitors.

Proposition 1. *If $E_1^\pi \le \cdots \le E_K^\pi$ ($E_1^\pi \ge \cdots \ge E_K^\pi$), then $\pi_1 \le \cdots \le \pi_K$ ($\pi_1 \ge \cdots \ge \pi_K$) in equilibrium. Further, if in equilibrium $\pi_1 = \cdots = \pi_K$, then $E_1^\pi = \cdots = E_K^\pi$.*

When a firm offering a higher-quality product is more (less) efficient according to E_k^π, it also obtains greater (lower) profit at equilibrium. By Proposition 1, a necessary and sufficient condition for the firms to obtain equal profits is when they possess the same relative cost efficiency.

Adding up the K FOCs in (8)–(9) yields

$$\frac{2r_1+r_2}{\sqrt{r_1+r_2}}\sqrt{\pi_1} + \sum_2^K \sqrt{(r_k+r_{k+1})\pi_k} = 1-d_1. \tag{10}$$

Thus, in equilibrium the weighted sum of $\sqrt{\pi_k}$ in the industry depends only on the unit cost of the lowest-quality product but not on that of any other product. If $\pi_1 = \cdots = \pi_K$, then (10) readily gives the common profit level.

We now turn to profit margin, which measures a firm's per-unit profitability. Because unit variable costs are constant by assumption, we may alternatively view firm k's profit margin $m_k = p_k - c_k$ as its decision variable, and rewriting the FOCs ((5)–(6)) in terms of m_k greatly simplifies the analysis. Again, the notion of relative cost efficiency plays a central role in deriving the next result.

Definition 2. Firm k's *relative cost efficiency w.r.t. profit margin* is defined as $E_1^m \equiv (d_2-d_1)/(r_2+2r_1)$ for Firm 1 and $E_k^m \equiv (d_{k+1}-d_k)/(r_{k+1}+r_k)$ for firm $k > 1$, where $d_{K+1} = 1$ and $r_{K+1} = 0$, respectively.

Compared with E_k^π, E_k^m allows a more concrete interpretation. For $1 < k < K$, E_k^m can be rewritten as $E_k^m = (r_{k+1}c_{k+1} + r_kc_{k-1})/(r_{k+1}+r_k) - c_k$, where the first term is the quality-weighted average cost of firms $k-1$ and $k+1$. Therefore, E_k^m represents the quality-weighted cost (dis)advantage of firm k relative to the two firms with which it directly competes. Likewise, E_1^m (E_K^m) reflects the quality-weighted cost (dis)advantage of Firm 1 (K) relative to Firm 2 ($K-1$).

Proposition 2. *If $E_1^m \le \cdots \le E_K^m$ ($E_1^m \ge \cdots \ge E_K^m$), then $m_1 \le \cdots \le m_K$ ($m_1 \ge \cdots \ge m_K$) in equilibrium. Further, if in equilibrium $m_1 = \cdots = m_K$, then $E_1^m = \cdots = E_K^m$.*

Its proof is given in the technical appendix at http://mktsci.pubs.informs.org. Proposition 2 thus parallels Proposition 1 in spirit: The monotonicity of relative cost efficiency E_k^m implies that of profit margin. When the firms obtain equal profit margin, i.e., $m_1 = \cdots = m_K = m^*$, summation of the K FOCs yields $m^* = (1-d_1)/(2\sum_1^K r_k)$. Not surprisingly, m^* decreases when the products become less differentiated (i.e., the r_ks are larger) or when the number of firms K increases.

The conditions in Propositions 1 and 2 may at first give an impression of being complex. However, such an impression is somewhat alleviated, considering that they enable us to compare the firms' equilibrium profitability without solving the pricing game, which is much more daunting, as noted earlier.[7] Further, these propositions become much easier to implement for many frequently studied production technologies.

Example 1 (The Linear Technology (e.g., Gabszewicz and Thisse 1980, Shaked and Sutton 1982, Ronnen 1991, Lehmann-Grube 1997)). When $d_k = a < 1$ for all k, Propositions 1 and 2 predict that profitability is increasing in product quality.

Example 2 (The Quadratic Technology (e.g., Moorthy 1988)). When $c(v) = v^2$, we have $d_k = v_k + v_{k-1}$ for all k, which further simplifies the relative cost efficiencies. For instance, now $E_1^m = v_2/(r_2+2r_1)$,

[7] In §4.1, we also explore cost functions demonstrating monotone profitability even with *equilibrium* quality choice, where the conditions are very simple to check.

$E_k^m = (v_{k+1} - v_k)(v_k - v_{k-1})$ for $1 < k < K$, and $E_K^m = (1 - v_K - v_{K-1})(v_K - v_{K-1})$.

Example 3 (The Technology Embodying Fixed-Quality Increments (e.g., Gabszewicz and Thisse 1980)).[8] Suppose the product qualities are spaced as follows: $v_1 = 2$, $v_k - v_{k-1} = 1$, for $2 \le k \le K$, which implies $2r_1 = r_2 = \cdots = r_K = 1$. The condition in Proposition 2 then reduces to $d_2 - d_1 \le \cdots \le d_{K+1} - d_K$ or $d_2 - d_1 \ge \cdots \ge d_{K+1} - d_K$; the condition in Proposition 1 is also greatly simplified.

Besides, the conditions in the propositions will also simplify as the number of firms decreases. In particular, Propositions 1 and 2 have the following corollary.

Corollary 1. *Suppose* $K = 2$. *Then*:

(1) $\pi_1 \lesseqqgtr \pi_2$ *if* $(v_1c_2 - v_2c_1)/((v_2 - v_1) - (c_2 - c_1)) \lesseqqgtr \sqrt{v_1 v_2}$.

(2) $m_1 \lesseqqgtr m_2$ *if* $(v_1c_2 - v_2c_1)/((v_2 - v_1) - (c_2 - c_1)) \lesseqqgtr 2v_2 - v_1$.

3.2. Managerial Implications

The above analysis focuses on the trade-off between product differentiation and production efficiency. In some markets (such as high-tech and pharmaceuticals), product differentiation plays a more prominent role that overshadows production efficiency, and the most lucrative firms are those offering cutting-edge product qualities. In markets lacking product differentiation, however, the most successful firms are often those operating with extreme cost efficiencies. Our analysis above may serve as a theoretical foundation for when either case is more likely to emerge and has further managerial implications.

(1) *"Going cheap" may not be a bad strategy*. Perhaps the more interesting result above relates to when a firm with an inferior product may outperform its rivals with superior offerings. While it is natural for firms to covet the highest quality and lowest cost in the industry, in reality the two are often conflicting objectives. Because a better-quality good demands higher consumer valuation, a customary misconception is that the high-end market is more lucrative than the low-end market and that quality outweighs cost as a critical success factor. Consequently, the cost side and the low-end market often do not receive enough managerial attention. Propositions 1 and 2 help make it clear that efficiency in production and distribution matters as much as product quality. In particular, *a firm with the lowest quality can be the most profitable if it has the highest relative cost efficiency*. Climbing the quality ladder pays only if the incremental quality does not entail an excessive increase in unit cost.

According to *Business Week* (2005), in the fall of 2004 French automaker Renault rolled out the no-frills Logan, a mid-sized sedan to sell at as little as 5,000 euros (or \$6,000) in European markets, with an estimated production cost of \$1,089, less than half the \$2,468 estimate for an equivalent Western auto. According to Kenneth Melville, head of the Logan design team, the concept of Logan was "Reliable engineering without a lot of electronics, cheap to build and easy to maintain and repair." In the 1990s, Dell emerged as a leader in the computer industry, mainly due to effective cost control through selling directly to customers and integrating its supply chain (Magretta 1998).

(2) *Entry*. Positioning and pricing a new brand is an important decision frequently made by firms. Suppose that a potential entrant knows its own unit cost at each potential quality level as well as the quality and cost levels of the incumbents.[9] Following entry, the incumbent firms can readily adjust prices but usually cannot rapidly adjust qualities, as relocating a product can be a costly and time-consuming process. Anticipating this, among its feasible choices the foresighted entrant will select an entry point according to its profitability in the *postentry* price equilibrium.

Proposition 1 may help facilitate a more informed entry decision, as our analysis reveals information about the profitability ranking associated with each candidate entry point. To simplify illustration, consider a market currently controlled by a monopolist operating at $(5, 2)$ in the quality-cost space (i.e., quality 5 and unit cost 2). Suppose a potential entrant is deliberating between two possible entry points, A: $(3, 0.6)$ and B: $(6, 3)$. Then, Corollary 1 indicates that entering at A (B) makes it earn strictly higher (lower) profit than the incumbent.

(3) *Repositioning a product*. Even though relocating a product does not occur instantly and costlessly, over time firms may have the incentive to reposition their products as technology, consumer taste, or competition evolves. For example, on July 11, 2005 Hewlett-Packard (HP) launched a low-cost inkjet printing technology whose printing quality and speed can match typical laser printers, which gave HP a huge manufacturing cost advantage (Associated Press). Release of this technology was widely viewed as a strategic move by HP to regain its dominance in inkjet printing. An incumbent's defensive response to an encroaching entrant is the focus of Hauser and Shugan (1983), who find that the optimal defensive strategies often involve lowering price and improving product quality. Johnson and Myatt (2003)

[8] This type of technology appears to suit well with how processor memory size (such as RAM and Cache) is determined in the semiconductor industry.

[9] Alternatively, if the entrant observes the quality and price levels of the incumbents prior to entry, then it can infer their unit cost levels.

study when an incumbent monopolist should scale its product-line length up or down in response to low-end entry.

Our Proposition 1 may also help a firm reposition its current product. Continuing from our previous numerical example, suppose the entrant has already entered at A: $(3, 0.6)$. Through R&D explorations, the incumbent (initially at $(5, 2)$) discovers that it is also capable of producing at two new positions: $(5.5, 2)$ and $(6, 3)$. Then Corollary 1 tells that moving to $(5.5, 2)$ would make it outperform its competitor.

3.3. Further Results

We have shown above how relative cost efficiencies may determine firms' profitability rankings. We now look at a new production efficiency measure involving a firm's own quality and cost characteristics. Specifically, we call the ratio v_k/c_k firm k's *absolute cost efficiency* in quality provision.

Proposition 3. *If* $v_i/c_i \le v_j/c_j$ *for* $1 \le i < j \le K$, *then* $m_i < m_j$ *and* $\pi_i < \pi_j$.

A firm with a higher quality is more profitable if it has a higher absolute cost efficiency. Here the intuition is that when $v_i/c_i \le v_j/c_j$ (or equivalently $c_i/v_i \ge c_j/v_j$), *all* consumers would strictly prefer v_j over v_i if they were made available at unit costs. Because the spirit of this argument does not rely on the nature of consumer distribution, Proposition 3 holds for any type of distribution. Further, note that the two firms compared in Proposition 3 need not be directly competing, in the sense that there may be other firm(s) offering qualities between those offered by these two firms.

Corollary 2. *If* $v_1/c_1 \le \cdots \le v_K/c_K$, *then* $m_1 < \cdots < m_K$ *and* $\pi_1 < \cdots < \pi_K$.

The next proposition gives a set of lower and upper bounds on the equilibrium profit margins. For $1 \le k \le K$, define

$$l_k \equiv \frac{1}{2}(d_1 + d_k) \quad \text{and} \quad u_k \equiv \frac{1}{2^{K-k+1}} + \sum_{i=k}^{K} \frac{1}{2^{i-k+1}} d_i.$$

As shown by Lemma 1 in the appendix, the consumer type indifferent between qualities v_k and v_{k-1} satisfies $l_k < \theta_k < u_k$ at equilibrium.

Proposition 4. *Let* $r_{K+1} = 0$ *and* $u_{K+1} = 1$. *Then at equilibrium,*

$$\max\left(0, \frac{d_{k+1} - d_k}{2(r_{k+1} + r_k)}\right) < m_k < \frac{u_{k+1} - l_k}{r_k}.$$

Proposition 4 then immediately leads to the corresponding bounds on equilibrium profits, because $\pi_k = (m_k)^2(r_{k+1} + r_k)$. These bounds delineate a possible range of profitability without computing the explicit price equilibrium. Proposition 4 thus can help a new entrant or incumbent assess the attractiveness of its potential product locations, given the positions of the remaining firms.

4. Extensions to the Main Model

4.1. Cost Functions Demonstrating Monotone Profitability

Our analysis above has focused on a *short-run* scenario in which the firms' quality and unit cost levels are assumed to be fixed exogenously. While this treatment admits possible idiosyncrasies in the firms' technological know-how, the following question also arises naturally: If the firms face some common production technology (as represented by a unit variable cost function) and endogenize their quality decisions according to it, then what properties of the technology ensure monotone profitability?[10] We now explore answers to this question.

Suppose the relevant quality space is $[\underline{v}, \bar{v}]$. Here the upper bound $\bar{v}$ represents the state-of-the-art product quality in the industry, and the lower bound $\underline{v}$ may stand for some minimum quality as expected by consumers or imposed by regulation.[11] Let $c(\cdot)$ be the unit cost function faced by all firms. First, we examine unit cost functions that are either weakly concave or rise very rapidly in the quality domain.

Proposition 5. (1) *Suppose* $c(0) = 0$, $c'(v) > 0$, *and* $c''(v) \le 0$ *for* $v \le \bar{v}$. *Then for any* $\underline{v} \le v_1 < \cdots < v_K \le \bar{v}$, $m_k > 2m_{k-1}$ *and* $\pi_k > \pi_{k-1}$, *for* $2 \le k \le K$ *in the Nash price equilibrium*. (2) *Suppose* $c(v) < v$ *and* $c'(v) > 1$ *on* $[\underline{v}, \bar{v}]$. *Then for any* $\underline{v} \le v_1 < \cdots < v_K \le \bar{v}$, $m_k < m_{k-1}$ *and* $\pi_k < \pi_{k-1}$, $2 \le k \le K$, *in the Nash price equilibrium*.

Because Proposition 5 holds for *any* quality configuration, the identified sufficient conditions are stronger than those required for an equilibrium quality location. The rationale behind part (1) is as follows. For a weakly concave unit cost function, a consumer's marginal valuation for quality (θ) remains constant, but the marginal cost of quality improvement ($c'(v)$) decreases as quality increases, which gives the higher-quality product a competitive advantage. Note that for a weakly concave cost function, any quality configuration demonstrates increasing absolute cost efficiency and, hence, $\pi_k > \pi_{k-1}$ follows from Corollary 2.

[10] We thank an anonymous reviewer (Reviewer 2) for suggesting this direction.

[11] For examples of minimum quality standards due to regulation, see Ronnen (1991).

However, Proposition 5(1) gives a much stronger result regarding profit margins: The profit margin of firm k is more than twice that of firm $k-1$. In some industries (e.g., semiconductors and pharmaceuticals), quality improvement primarily entails expenditures of fixed costs in R&D, but the unit variable costs may rise at a decreasing rate as quality improves. Proposition 5(1) predicts that in such industries firms with better-quality products are more profitable. Such a prediction appears to be largely coherent with casual empirical impressions.

The condition in Proposition 5(2) is essentially the *finiteness* condition of Shaked and Sutton (1983). Unlike horizontal differentiation, a vertically differentiated market may only sustain a finite number of firms in equilibrium (even in the absence of fixed costs of entry) (Gabszewicz and Thisse 1980, Shaked and Sutton 1983). The latter further gives a necessary and sufficient condition for this finiteness property, whose essence is that, if the products were offered for sale at costs, all consumers would agree in their utility ranking (in either increasing or decreasing order) of the products. In our current model, this finiteness condition simply reduces to $c'(v) > 1$ on $[\underline{v}, \bar{v}]$, which ensures monotone decreasing profitability.[12] Because consumer types are confined to $[0, 1]$ in our model, when $c'(v) > 1$ on $[\underline{v}, \bar{v}]$, all consumers would rank the utilities of the products in strictly decreasing order of quality if they were made available at unit variable costs. Intuitively, if the production technology is such that the marginal cost of quality improvement exceeds the maximal marginal consumer valuation over the quality space, offering a lower-quality product is strictly more profitable.

Next, we consider a convex unit cost function. For tractability, we confine our attention to a duopoly and assume that the two firms choose both quality and price simultaneously.[13] Also in a duopoly setting, Lehmann-Grube (1997) shows that when the *fixed cost* of quality development is convex, the firm choosing a higher quality makes higher profits in equilibrium. However, he assumes that unit variable costs are zero at any quality.

[12] Two differences between Shaked and Sutton (1983) and our current model are noteworthy. First, they adopt a Hicksian *composite utility* function (instead of the Mussa-Rosen type), where a consumer with income t obtains a net utility of $v(t-p)$ by purchasing a product of quality v at price p. Second, they assume that consumers are uniformly distributed on $[a, b]$, where $a > 0$, i.e., consumer tastes are bounded strictly above zero. It is readily verified that, under the Mussa-Rosen utility function, the finiteness condition of Shaked and Sutton (1983) simplifies to $c'(v) < a$ or $c'(v) > b$, for $v \in [a, b]$. Because $a = 0$ and $b = 1$ in our model, it suffices to address only the case in which $c'(v) > 1$.

[13] Proposition A2 in the technical appendix at http://mktsci.pubs.informs.org deals with the case in which the two firms first choose qualities simultaneously, and once their quality choices are known, they then choose prices simultaneously.

Proposition 6. *Suppose that $c(0) = 0$, $c'(v) > 0$, and $c''(v) > 0$ for $v \in [\underline{v}, \bar{v}]$ and that the two firms choose quality and price simultaneously.*

(1) *If $\sqrt{v_1 v_2}(c'(v_2) - c'(v_1))/(v_2 - v_1) + c'(v_2) \lesseqgtr 1$ for any $\underline{v} \le v_1 < v_2 \le \bar{v}$, then $\pi_1 \lesseqgtr \pi_2$ in equilibrium.*

(2) *Further suppose $c'''(v) > 0$ for $v \in [\underline{v}, \bar{v}]$. If $\bar{v}c''(\bar{v}) + c'(\bar{v}) \le 1$ ($\underline{v}c''(\underline{v}) + c'(\underline{v}) \ge 1$), then $\pi_1 < \pi_2$ ($\pi_1 > \pi_2$) in equilibrium.*

Conceptually, the LHS of the condition in Proposition 6(1) measures the "steepness" of the cost curve in the quality domain. If this steepness measure is less than the highest consumer type, then the firm choosing a higher quality is more profitable. The converse holds otherwise. In some sense, Proposition 6 complements Proposition 5(2).

To make Proposition 6 more transparent, we provide two examples involving polynomial and exponential cost functions, respectively. First, consider $c(v) = av^2$, where $a > 0$. Then $c'(v) = 2av$, and the LHS of the condition in Proposition 6(1) becomes $2a\sqrt{v_2}(\sqrt{v_1} + \sqrt{v_2})$. Because $2a\sqrt{v_2}(\sqrt{v_1} + \sqrt{v_2}) < 4a\bar{v}$, $\pi_1 < \pi_2$ if $a \le 1/4\bar{v}$. Since $2a\sqrt{v_2}(\sqrt{v_1} + \sqrt{v_2}) > 4a\underline{v}$, $\pi_1 > \pi_2$ if $a \ge 1/4\underline{v}$. Now suppose $c(v) = av^3$, for $a > 0$. Then Proposition 6(2) implies that if $a \le 1/9\bar{v}^2$ ($a \ge 1/9\underline{v}^2$), then $\pi_1 < \pi_2$ ($\pi_1 > \pi_2$). Second, consider $c(v) = a(\exp[v] - 1)$, where $a > 0$. Then $c'(v) = c''(v) = a\exp[v]$, and Proposition 6(2) becomes:

$$\text{If } a \le \frac{1}{(\bar{v}+1)\exp[\bar{v}]} \quad \left(a \ge \frac{1}{(\underline{v}+1)\exp[\underline{v}]}\right),$$
$$\pi_1 < \pi_2 \quad (\pi_1 > \pi_2).$$

4.2. A Logconcave Distribution of Consumer Types

We now relax the assumption of a uniform consumer distribution. Assume instead that θ is distributed on $[0, 1]$ according to a density function $f(\theta)$, which is logconcave and twice differentiable on $(0, 1)$. Let $F(\theta) = \int_0^\theta f(x)\,dx$. A logconcave density of consumers is an important regularity condition that ensures the existence of a Nash price equilibrium in models of imperfect competition (cf. Caplin and Nalebuff 1991, Anderson et al. 1995). Many frequently used distributions have logconcave densities, e.g., the beta, exponential, normal, uniform, and Weibull distributions, among others.

To proceed, it is necessary to introduce two functions. For $0 < y < x < 1$, let

$$G(x, y \mid \alpha, \beta) \equiv \frac{F(x) - F(y)}{\alpha f(x) + \beta f(y)} \quad \text{and}$$

$$H(y \mid \beta) \equiv \frac{1 - F(y)}{\beta f(y)}, \quad \text{where } \alpha, \beta > 0.$$

When $f(\theta)$ is logconcave and twice differentiable, $G(x, y \mid \alpha, \beta)$ is increasing in x and decreasing in y, and $H(y \mid \beta)$ is nonincreasing in y (see Lemma 2 in the appendix). These two functions play a pivotal role in the next result.

Proposition 7. *Suppose the density of consumer types, $f(\theta)$, is logconcave and twice differentiable on $(0, 1)$. If $m_1' \gtreqless G(d_3, d_2 \mid r_3, r_2) \gtreqless \cdots \gtreqless G(d_K, d_{K-1} \mid r_K, r_{K-1}) \gtreqless H(d_K \mid r_K)$, where m_1' is the unique solution to $m_1' = G(d_2, d_1 + m_1' r_1 \mid r_2, r_1)$, then $m_1 \gtreqless \cdots \gtreqless m_K$ in equilibrium.*

Note that $m_1' \geq 0$ if and only if $d_2 \geq d_1$. To see the rationale behind Proposition 7, we first interpret each term in its condition. The kth intermediate term ($1 < k < K$) is $[F(d_{k+1}) - F(d_k)]/[f(d_{k+1})r_{k+1} + f(d_k)r_k]$. To spell out its economic meaning, we focus on the case when $d_{k+1} \geq d_k$. The numerator of this ratio is firm k's demand if all firms were to price at their unit variable costs, and the denominator the number of purchasing consumers firm k would lose after raising its price by one unit. For $1 < k \leq K$, let η_k denote firm k's elasticity of demand when all K firms price at costs, i.e.,

$$\eta_k = -[f(d_{k+1})r_{k+1} + f(d_k)r_k]\frac{c_k}{F(d_{k+1}) - F(d_k)},$$

where $d_{K+1} = 1$ and $r_{K+1} = 0$. We may then rewrite Proposition 7 in terms of η_k:

$$\text{If } m_1' \lesseqgtr -\frac{c_2}{\eta_2} \lesseqgtr \cdots \lesseqgtr -\frac{c_K}{\eta_K}, \quad \text{then } m_1 \lesseqgtr \cdots \lesseqgtr m_K.$$

Since the kth term in the condition of Proposition 7 contains the unit costs and quality levels of both firm k and its immediate competitors, it can still be viewed as a measure of firm k's *relative cost efficiency* for a given distribution $F(\cdot)$. Proposition 7 is thus the counterpart of Proposition 2: *When the relative cost efficiency is monotone* (*increasing or decreasing*) *in product quality, so are the equilibrium profit margins*. To some extent, therefore, this indicates that the key spirits of relative cost efficiency and our inductive method are robust for a much wider class of distributions.

5. Conclusion

This paper has identified conditions for producing a higher- or lower-quality good to be more profitable, as well as a set of profitability bounds. Proposition A1 in the technical appendix at http://mktsci.pubs.informs.org also gives a sufficient condition for market share to be monotone in quality. These results can help marketing managers (and consultants) understand the competitive landscape in their industries and facilitate the framing of effective product strategies. Specifically, they can help assess the profitability of a potential brand location. Besides, this paper also makes a technical contribution: Our inductive method avoids computing an explicit price equilibrium, which is often intractable.

Our main model takes as fixed the quality location of the firms. At the core of our analysis are the notions of relative and absolute cost efficiencies. A firm's relative cost efficiency refers to its quality-adjusted cost (dis)advantage relative to its immediate competitors and is thus a "localized" efficiency measure. As competition itself is localized in a vertical setting, this measure proves sufficient for establishing the order of firm performance. An ideal property of relative cost efficiency is that its monotonicity in quality indicates that of profitability. A firm's absolute cost efficiency, defined as the ratio between its quality and its unit cost, captures its quality output from each dollar of input in the production process and thus does not reflect on any other firm. Note that these two measures have different domains of applicability. For example, absolute cost efficiency cannot help predict when producing a lower-quality good is more profitable.

The current paper has its limitations. As in many previous studies, we have also assumed that each consumer demands zero or one unit of the product. A durable good is a commonly used justification for the unit-demand assumption. Perhaps our most critical assumption is the Mussa-Rosen utility function. This is admittedly our key limitation, but its multiplicative form allows a precise characterization of the firms' demand and profits, which leads to a tractable analysis. Gal-Or (1983), Moorthy (1984), and Champsaur and Rochet (1989) are exemplary papers that employ more general utility functions. Extending the current analysis along their tradition is left for future work. For the most part, we have used the uniform distribution of consumers to ease exposition, but Proposition 7 suggests that the spirit of relative cost efficiency and our inductive method hold valid for a logconcave consumer distribution. In Propositions 5 and 6, we have also explored the set of cost functions that demonstrate monotone profitability, even with endogenous quality location.

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Appendix

This appendix contains a summary of notation, and the proofs of Propositions 1 and 3–7. The remaining proofs are given in the technical appendix at http://mktsci.pubs.informs.org.

Table of Notation:

k: The index of a firm from 1 to K.

v_k: The quality of firm k's product.

c_k: The (constant) unit variable cost of firm k.

p_k: The price of firm k.

d_k: The marginal cost of quality improvement between firms $k-1$ and k. Defined as $d_k = (c_k - c_{k-1})/(v_k - v_{k-1})$ and $d_{K+1} = 1$.

θ_k: The consumer type indifferent between the products of firms $k-1$ and k. Defined as $\theta_k = (p_k - p_{k-1})/(v_k - v_{k-1})$ and $\theta_{K+1} = 1$.

m_k: The profit margin of firm k, i.e., $m_k = p_k - c_k$.

s_k: The demand of firm k. $s_k = \theta_{k+1} - \theta_k$ for a uniform consumer distribution.

π_k: The profit of firm k.

r_k: The extent of differentiation between v_k and v_{k-1}. Defined as $r_k = 1/(v_k - v_{k-1})$ and $r_{K+1} = 0$.

t_k^π: A combined measure of differentiation between the products of firm k and its adjacent firms (up to two firms above and below k). For its definition, see §3.1.

E_k^π: Firm k's relative cost efficiency w.r.t. profit. Defined as $E_k^\pi = (d_{k+1} - d_k)/t_k^\pi$.

E_k^m: Firm k's relative cost efficiency w.r.t. profit margin. Defined as $E_1^m = (d_2 - d_1)/(r_2 + 2r_1)$ and $E_k^m = (d_{k+1} - d_k)/(r_{k+1} + r_k)$ for $k > 1$.

l_k: The lower bound of θ_k in the price equilibrium.

u_k: The upper bound of θ_k in the price equilibrium.

$c(s)$: The unit cost function (or production technology) faced by the firms in §4.1.

F: The c.d.f. of consumer tastes in §4.2. Its density function is f.

G: A bivariate function defined as $G(x, y \mid \alpha, \beta) = [F(x) - F(y)]/[\alpha f(x) + \beta f(y)]$, where $\alpha, \beta > 0$. In particular, $G(d_{k+1}, d_k \mid r_{k+1}, r_k)$ reflects firm k's $(1 < k < K)$ relative cost efficiency w.r.t. profit margin for a consumer distribution F.

H: A univariate function defined as $H(y \mid \beta) = [1 - F(y)]/[\beta f(y)]$, where $\beta > 0$. In particular, $H(d_K \mid r_K)$ reflects firm K's relative cost efficiency w.r.t. profit margin for a consumer distribution F.

Proof of Proposition 1. To prove the first statement, we only consider the case of increasing relative cost efficiencies, the other case being analogous. The proof is by induction and has two steps.

Step 1. We wish to show $\pi_1 \le \pi_2$ when $E_1^\pi \le \cdots \le E_K^\pi$.

Suppose $\pi_1 > \pi_2$. Then, (8) gives

$$\sqrt{\pi_1} < E_1^\pi. \tag{11}$$

If $d_2 \le d_1$, then $E_1^\pi \le 0$ and (11) is already a contradiction, and thus $\pi_1 \le \pi_2$ must hold.

Otherwise, we aim to show $\pi_K < \cdots < \pi_2 < \pi_1$ when $\pi_1 > \pi_2$. Suppose $\pi_3 \ge \pi_2$. Then from (8) we have

$$\begin{aligned} r_3\sqrt{\pi_2/(r_4 + r_3)} &\le r_3\sqrt{\pi_3/(r_4 + r_3)} \\ &= 2\sqrt{(r_3 + r_2)\pi_2} - r_2\sqrt{\pi_1/(r_2 + r_1)} - (d_3 - d_2) \\ &< 2\sqrt{(r_3 + r_2)\pi_2} - r_2\sqrt{\pi_2/(r_2 + r_1)} - (d_3 - d_2), \end{aligned}$$

which leads to $\sqrt{\pi_2} > E_2^\pi$. This and (11) jointly imply $E_2^\pi < E_1^\pi$, contradicting the condition of the proposition. Therefore, we must have $\pi_3 < \pi_2$.

Similarly, when $\pi_1 > \pi_2$, we can show $\pi_k < \pi_{k-1}$ for successively higher k. That is, $\sqrt{\pi_K} < \cdots < \sqrt{\pi_1} < E_1^\pi$ holds under the supposition that $\pi_1 > \pi_2$. Note that $\pi_K < \pi_{K-1}$ and (9) jointly imply $1 - d_K < (2\sqrt{r_K} - r_K/\sqrt{r_K + r_{K-1}})\sqrt{\pi_K}$ or, equivalently, $\sqrt{\pi_K} > E_K^\pi$. We thus have $E_K^\pi < E_1^\pi$, a contradiction to the condition of the proposition. Therefore, $\pi_1 > \pi_2$ cannot hold. This establishes $\pi_1 \le \pi_2$, completing Step 1.

Step 2. Suppose $\pi_{k-1} \le \pi_k$ $(k < K)$ holds, and we wish to show $\pi_k \le \pi_{k+1}$ when $E_1^\pi \le \cdots \le E_K^\pi$.

Suppose $\pi_k > \pi_{k+1}$. Then from (8) we have

$$\begin{aligned} &r_{k+1}\sqrt{\pi_k/(r_{k+2} + r_{k+1})} \\ &\quad > r_{k+1}\sqrt{\pi_{k+1}/(r_{k+2} + r_{k+1})} \\ &\quad = 2\sqrt{(r_{k+1} + r_k)\pi_k} - r_k\sqrt{\pi_{k-1}/(r_k + r_{k-1})} - (d_{k+1} - d_k) \\ &\quad \ge 2\sqrt{(r_{k+1} + r_k)\pi_k} - r_k\sqrt{\pi_k/(r_k + r_{k-1})} - (d_{k+1} - d_k), \end{aligned}$$

which gives

$$\sqrt{\pi_k} < E_k^\pi. \tag{12}$$

If $d_{k+1} \le d_k$, then $E_k^\pi \le 0$ and the last inequality is already a contradiction, and thus $\pi_k \le \pi_{k+1}$ must hold. Otherwise, we aim to show that $\pi_K < \cdots < \pi_k$ when $\pi_k > \pi_{k+1}$. If $\pi_{k+2} \ge \pi_{k+1}$, then it follows from (8) that

$$\begin{aligned} &r_{k+2}\sqrt{\pi_{k+1}/(r_{k+3} + r_{k+2})} \\ &\quad \le r_{k+2}\sqrt{\pi_{k+2}/(r_{k+3} + r_{k+2})} \\ &\quad = 2\sqrt{(r_{k+2} + r_{k+1})\pi_{k+1}} - r_{k+1}\sqrt{\pi_k/(r_{k+1} + r_k)} - (d_{k+2} - d_{k+1}) \\ &\quad < 2\sqrt{(r_{k+2} + r_{k+1})\pi_{k+1}} - r_{k+1}\sqrt{\pi_{k+1}/(r_{k+1} + r_k)} - (d_{k+2} - d_{k+1}), \end{aligned}$$

and thus $\sqrt{\pi_{k+1}} > E_{k+1}^\pi$. This and (12) above jointly imply $E_{k+1}^\pi < E_k^\pi$, a contradiction to the condition of the proposition. Therefore, when $\pi_k > \pi_{k+1}$, we must have $\pi_{k+2} < \pi_{k+1}$.

Similarly, we can show that when $\pi_k > \pi_{k+1}$, $\sqrt{\pi_K} < \cdots < \sqrt{\pi_k} < E_k^\pi$ holds. However, $\pi_K < \pi_{K-1}$ and (9) jointly imply $\sqrt{\pi_K} > E_K^\pi$ and, hence, $E_K^\pi < E_k^\pi$, a contradiction to the condition of the proposition. Therefore, $\pi_k > \pi_{k+1}$ cannot hold. We have shown that $\pi_{k-1} \le \pi_k$ $(k < K)$ implies $\pi_k \le \pi_{k+1}$, completing Step 2. This proves the first statement of the proposition. The second statement follows directly from the FOCs in (8) and (9). □

Proof of Proposition 3. Suppose $(p_1, \ldots, p_K)$ is the unique price equilibrium. This means that when the remaining $K-1$ firms price at $(p_1, \ldots, p_{j-1}, p_{j+1}, \ldots, p_K)$, firm j's best choice is p_j. Recall that when all K firms are active, the equilibrium price trajectory is a piecewise linear, increasing, and convex function of quality, i.e., $\theta_k < \theta_{k+1}$, for $k < K$. This implies $(p_j - p_i)/(v_j - v_i) > p_i/v_i$.

Note that the condition of the proposition is equivalent to $c_i/v_i \ge (c_j - c_i)/(v_j - v_i)$. Because $p_i/v_i > c_i/v_i$, we have $(p_j - p_i)/(v_j - v_i) > (c_j - c_i)/(v_j - v_i)$, or equivalently, $m_i < m_j$. Note that firm j has the option to lower its price to $p_j' = c_j + m_i$. Because $(p_j' - p_i)/(v_j - v_i) = (c_j - c_i)/(v_j - v_i) \le c_i/v_i < p_i/v_i$, at price p_j' firm j would take over the entire demand of firms $i, \ldots, j-1$ (plus possibly the demand of some other firms).

Therefore, at price p'_j firm j's profit would clearly exceed firm i's equilibrium profit π_i. However, p'_j is a price firm j does not choose. Therefore, we must have $\pi_i < \pi_j$. □

Proof of Proposition 4. The following lemma is critical for establishing Proposition 4.

Lemma 1. *At equilibrium, the indifferent consumers θ_k ($1 \le k \le K$) satisfy $l_k < \theta_k < u_k$.*

Proof of Lemma 1. The proof is divided into two parts. In Part I (II), we establish the lower (upper) bounds on m_k.

Part I: Lower bounds. For $1 \le k \le K-1$, (5) states that $\theta_{k+1} - \theta_k = (p_k - c_k)(r_{k+1} + r_k)$, or equivalently,

$$\begin{aligned}[(p_{k+1} - c_k) - (p_k - c_k)]r_{k+1} - [(p_k - c_k) + (c_k - p_{k-1})]r_k \\ = (p_k - c_k)(r_{k+1} + r_k).\end{aligned}$$

Rearranging terms, we have $(p_k - c_k)(r_{k+1} + r_k) = (1/2) \cdot [(p_{k+1} - c_k)r_{k+1} - (c_k - p_{k-1})r_k]$. Therefore, $\theta_{k+1} - \theta_k = (1/2)[(p_{k+1} - c_k)r_{k+1} - (c_k - p_{k-1})r_k] > (1/2)[(c_{k+1} - c_k)r_{k+1} - (c_k - c_{k-1})r_k] = (1/2)(d_{k+1} - d_k)$, where the inequality is due to $p_i > c_i$. This gives $\theta_{k+1} > \theta_k + (1/2)(d_{k+1} - d_k)$. Because $\theta_1 > l_1 \equiv d_1$ holds trivially (due to $p_1 > c_1$), the lower bounds of the remaining θ_ks are then derived by induction for successively higher k: $\theta_k > l_k \equiv (1/2)(d_1 + d_k)$, for $2 \le k \le K$.

Part II: Upper bounds. From (6), we have

$$\begin{aligned}1 - \theta_K &= (p_K - c_K)r_K = [(p_K - p_{K-1}) + (p_{K-1} - c_K)]r_K \\ &> \theta_K + (c_{K-1} - c_K)r_K = \theta_K - d_K.\end{aligned}$$

Therefore, $\theta_K < u_K \equiv (1/2)(1 + d_K)$. Similarly, for $1 \le k \le K-1$, we have (from (5))

$$\begin{aligned}\theta_{k+1} - \theta_k &= (p_k - c_k)(r_{k+1} + r_k) > (p_k - p_{k-1} + p_{k-1} - c_k)r_k \\ &> \theta_k - d_k,\end{aligned}$$

from which the upper bounds of the remaining θ_ks are established recursively for successively lower k: $\theta_k < (1/2) \cdot (d_k + \theta_{k+1}) < u_k \equiv (1/2)(d_k + u_{k+1})$, where u_k may be rewritten as

$$u_k \equiv \frac{1}{2^{K-k+1}} + \sum_{i=k}^{K} \frac{1}{2^{i-k+1}}\, d_i.$$

This completes the proof of Lemma 1. The rest of the proof uses the fact that at equilibrium, $m_k(r_{k+1} + r_k) = \theta_{k+1} - \theta_k$, where $r_{K+1} = 0$ and $\theta_{K+1} = 1$ (from (5) and (6)). The upper bound of m_k then follows directly from Lemma 1. Part I of the proof of Lemma 1 also shows that $\theta_{k+1} - \theta_k > (1/2)(d_{k+1} - d_k)$. This leads to the lower bound of m_k. □

Proof of Proposition 5. Part (1). Under the specified concave unit cost function, the absolute cost efficiency is nondecreasing in quality, and from Corollary 2, $\pi_k > \pi_{k-1}$ follows. We now show that $m_k > 2m_{k-1}$. Note that $d_{k+1} \le d_k$ under the concave cost function. The rest of the proof has two steps. First, we show that $m_2 > 2m_1$. Rewriting the FOC in (5) with $m_k = p_k - c_k$, we have $(m_2 - m_1)r_2 = m_1(r_2 + 2r_1) - (d_2 - d_1) > m_1(r_2 + 2r_1)$ (since $d_2 - d_1 \le 0$), which implies $m_2 > 2m_1(r_1 + r_2)/r_2 > 2m_1$.

Second, suppose $m_k > 2m_{k-1}$ holds, and we show that $m_{k+1} > 2m_k$. The FOC of firm k (with respect to m_k) implies $(m_{k+1} - m_k)r_{k+1} = (m_k - m_{k-1})r_k + m_k(r_{k+1} + r_k) - (d_{k+1} - d_k) > m_k(r_{k+1} + r_k)$, which leads to $m_{k+1} > 2m_k$.

Part (2). We first show that $m_1 > \cdots > m_K$ under the stated condition. Consider any two firms offering adjacent quality levels v_{k-1} and v_k, where $k \ge 2$. Note that $c'(v) > 1$ implies

$$d_k = \frac{c_k - c_{k-1}}{v_k - v_{k-1}} > 1.$$

Suppose $m_{k-1} \le m_k$ in equilibrium. Then

$$\theta_k = \frac{p_k - p_{k-1}}{v_k - v_{k-1}} = d_k + \frac{m_k - m_{k-1}}{v_k - v_{k-1}} > 1,$$

i.e., the consumer type indifferent between purchasing v_{k-1} and v_k exceeds 1, the upper bound of the consumer type distribution, a contradiction. Therefore, the desired statement holds under the condition of the proposition.

We now show that $\pi_1 > \cdots > \pi_K$. For $k \ge 2$, $m_{k-1} > m_k$ holds, as just shown above. Note that firm $k-1$ maintains the option of lowering its equilibrium profit margin from m_{k-1} to m'_{k-1}, such that

$$\theta'_{k-1} = d_k + \frac{m_k - m'_{k-1}}{v_k - v_{k-1}} = 1.$$

In other words, with such a profit margin m'_{k-1}, firm $k-1$ can take over the entire demand of all upstream competitors, firms $k, \ldots, K$. Because $d_k > 1$, $m'_{k-1} > m_k$ must hold. Therefore, with the new profit margin m'_{k-1}, firm $k-1$ can make a profit strictly greater than firm k's equilibrium profit. However, this is an option firm $k-1$ does not choose, which implies that $\pi_{k-1} > \pi_k$ holds at the price equilibrium. □

Proof of Proposition 6. Part (1). Recall that each firm's profit function is $\pi_k = (p_k - c(v_k))(\theta_{k+1} - \theta_k)$ (from (4)). When the two firms choose quality and price simultaneously, the FOCs are

$$\begin{aligned}\frac{\partial \pi_k}{\partial v_k}&:\; -c'(v_k)(\theta_{k+1} - \theta_k) \\ &\quad + (p_k - c(v_k))(r_{k+1}\theta_{k+1} + r_k\theta_k) = 0, \qquad (13)\end{aligned}$$

$$\frac{\partial \pi_k}{\partial p_k}:\; \theta_{k+1} - \theta_k = (p_k - c(v_k))(r_{k+1} + r_k), \qquad (14)$$

where $1 \le k \le 2$, $\theta_3 = 1$ and $r_3 = 0$.

From (4) and (14) we have $\pi_k = (\theta_{k+1} - \theta_k)^2/(r_{k+1} + r_k)$, $k = 1, 2$. Substituting (14) into (13) gives $r_{k+1}\theta_{k+1} + r_k\theta_k = (r_{k+1} + r_k)c'(v_k)$, $k = 1, 2$. That is, $r_2\theta_2 + r_1\theta_1 = (r_2 + r_1)c'(v_1)$ and $\theta_2 = c'(v_2)$. Combining these two equations gives $\theta_1 = c'(v_1) - (r_2/r_1)[c'(v_2) - c'(v_1)]$. Therefore, the equilibrium profits are $\pi_1 = [c'(v_2) - c'(v_1)]^2(r_1 + r_2)/(r_1)^2$ and $\pi_2 = [1 - c'(v_2)]^2/r_2$. It is readily verified that $\pi_1 \lesseqgtr \pi_2 \Leftrightarrow v_1v_2[(c'(v_2) - c'(v_1))/(v_2 - v_1)]^2 \lesseqgtr [1 - c'(v_2)]^2$. This proves part (1) of the proposition.

Part (2). When $c'''(v) > 0$, we have $c''(v_1) < (c'(v_2) - c'(v_1))/(v_2 - v_1) < c''(v_2)$. This implies that

$$\begin{aligned}\sqrt{v_1v_2}\frac{c'(v_2) - c'(v_1)}{v_2 - v_1} + c'(v_2) &< v_2c''(v_2) + c'(v_2) \\ &< \bar{v}c''(\bar{v}) + c'(\bar{v}),\end{aligned}$$

and

$$\begin{aligned}\sqrt{v_1v_2}\frac{c'(v_2) - c'(v_1)}{v_2 - v_1} + c'(v_2) &> v_1c''(v_1) + c'(v_1) \\ &> \underline{v}c''(\underline{v}) + c'(\underline{v}).\end{aligned}$$

The desired statements then follow from part (1). □

Proof of Proposition 7. We start with formulating the firms' profit functions and characterizing the price equilibrium. Because $m_k = p_k - c_k$, the profit function of firm k is now

$$\pi_k = m_k(F(\theta_{k+1}) - F(\theta_k)), \tag{15}$$

where $\theta_{K+1} = 1$. The first-order conditions (FOC) are:

$$F(\theta_{k+1}) - F(\theta_k) = m_k[f(\theta_{k+1})r_{k+1} + f(\theta_k)r_k], \quad \text{for } k < K, \tag{16}$$

$$1 - F(\theta_K) = m_K f(\theta_K) r_K. \tag{17}$$

The following lemma is critical to the proof of the proposition.

Lemma 2. *Suppose $f(\theta)$ is logconcave and twice differentiable on $(0,1)$. We have: (1) $G(x, y \mid \alpha, \beta)$ is increasing in x and decreasing in y, for $0 < y < x < 1$; and (2) $H(y \mid \beta)$ is nonincreasing in y, for $0 < y < 1$.*

Proof of Lemma 2. Under the condition stated in the lemma, $f(x) > 0$ and $f'(x)/f(x)$ is nonincreasing on $(0,1)$. A strictly positive density on the interior of its support implies that $F(x)$ is strictly increasing on $[0,1]$.

(1) It is straightforward to verify that

$$\begin{aligned} &\frac{\partial}{\partial x} G(x, y \mid \alpha, \beta) > 0 \\ &\quad\Longleftrightarrow f(x)[\alpha f(x) + \beta f(y)] - \alpha f'(x)[F(x) - F(y)] > 0 \\ &\quad\Longleftrightarrow \frac{f'(x)}{f(x)} < \frac{\alpha f(x) + \beta f(y)}{\alpha[F(x) - F(y)]} \end{aligned}$$

and that

$$\frac{\alpha f(x) + \beta f(y)}{\alpha[F(x) - F(y)]} > \frac{f(x)}{F(x)}.$$

Therefore, to show $G(x, y \mid \alpha, \beta)$ is increasing in x, we only need to show $f'(x)/f(x) \le f(x)/F(x)$, which holds because

$$\frac{f(x)}{F(x)} = \frac{\int_0^x f'(z)\,dz}{\int_0^x f(z)\,dz} = \frac{\int_0^x [f'(z)/f(z)] f(z)\,dz}{\int_0^x f(z)\,dz} \ge \frac{f'(x)}{f(x)}.$$

We can also verify that

$$\begin{aligned} &\frac{\partial}{\partial y} G(x, y \mid \alpha, \beta) < 0 \\ &\quad\Longleftrightarrow -f(y)[\alpha f(x) + \beta f(y)] - \beta f'(y)[F(x) - F(y)] < 0 \\ &\quad\Longleftrightarrow \frac{f'(y)}{f(y)} > -\frac{\alpha f(x) + \beta f(y)}{\beta[F(x) - F(y)]} \end{aligned}$$

and that

$$-\frac{\alpha f(x) + \beta f(y)}{\beta[F(x) - F(y)]} < \frac{-f(y)}{F(x) - F(y)} < \frac{-f(y)}{1 - F(y)}.$$

Therefore, $G(x, y \mid \alpha, \beta)$ is decreasing in y if $f'(y)/f(y) \ge -f(y)/(1 - F(y))$, which follows from

$$\begin{aligned} \frac{-f(y)}{1 - F(y)} &\le \frac{f(1) - f(y)}{1 - F(y)} = \frac{\int_y^1 f'(z)\,dz}{\int_y^1 f(z)\,dz} \\ &= \frac{\int_y^1 [f'(z)/f(z)] f(z)\,dz}{\int_y^1 f(z)\,dz} \le \frac{f'(y)}{f(y)}. \end{aligned}$$

(2) We can verify that $H'(y \mid \beta) \le 0$ if and only if $-f^2(y) - f'(y)[1 - F(y)] \le 0$, or equivalently $f'(y)/f(y) \ge -f(y)/(1 - F(y))$, which holds as just shown above. □

This completes the proof of Lemma 2. To facilitate the ensuing proof, we express the FOCs in (16) and (17) in terms of profit margins: For firms $k < K$,

$$\begin{aligned} &F[(m_{k+1} - m_k)r_{k+1} + d_{k+1}] - F[(m_k - m_{k-1})r_k + d_k] \\ &\quad = m_k\{f[(m_{k+1} - m_k)r_{k+1} + d_{k+1}]r_{k+1} \\ &\qquad + f[(m_k - m_{k-1})r_k + d_k]r_k\}, \end{aligned} \tag{18}$$

where $m_0 = v_0 = 0$, and for firm K,

$$\begin{aligned} &1 - F[(m_K - m_{K-1})r_K + d_K] \\ &\quad = m_K f[(m_K - m_{K-1})r_K + d_K]r_K. \end{aligned} \tag{19}$$

Proof of the Proposition. First, we show the uniqueness of m_1'. Let $W(m_1) \equiv m_1$. Clearly, $W(m_1)$ is strictly increasing. From Lemma 2, $G(d_2, d_1 + m_1 r_1 \mid r_2, r_1)$ is strictly decreasing in m_1. The remaining argument to establish the uniqueness of m_1' splits into three cases. Case (1): when $d_2 > d_1$. At $m_1 = 0$, $W(m_1) = 0$ and $G(d_2, d_1 + m_1 r_1 \mid r_2, r_1) > 0$. At $m_1 = (d_2 - d_1)v_1$, $W(m_1) > 0$ and $G(d_2, d_1 + m_1 r_1 \mid r_2, r_1) = 0$. Therefore, $m_1' > 0$ must be unique. Case (2): when $d_2 < d_1$. At $m_1 = 0$, $W(m_1) = 0$ and $G(d_2, d_1 + m_1 r_1 \mid r_2, r_1) < 0$. At $m_1 = (d_2 - d_1)v_1$, $W(m_1) < 0$ and $G(d_2, d_1 + m_1 r_1 \mid r_2, r_1) = 0$. Therefore, $m_1' < 0$ must be unique. Case (3): when $d_2 = d_1$. In this case, at $m_1 > 0$, $W(m_1) > 0$ but $G(d_2, d_1 + m_1 r_1 \mid r_2, r_1) < 0$. At $m_1 < 0$, $W(m_1) < 0$, but $G(d_2, d_1 + m_1 r_1 \mid r_2, r_1) > 0$. Therefore $m_1' = 0$.

The rest of the proof is by induction and has two steps. We only show that increasing relative cost efficiencies ensure increasing profit margins, the other case being analogous.

Step 1. We wish to show $m_1 \le m_2$ under the condition of the proposition. Suppose $m_1 > m_2$. Then from (16) we have $m_1 = G(d_2 + (m_2 - m_1)r_2, d_1 + m_1 r_1 \mid r_2, r_1) < G(d_2, d_1 + m_1 r_1 \mid r_2, r_1)$ (by Lemma 2), which implies $m_1 < m_1'$.

If $d_2 \le d_1$, the last inequality implies $m_1 < 0$. A contradiction to $m_1 > 0$ for Firm 1 to be active at equilibrium, and thus $m_1 \le m_2$ must hold.

If $d_2 > d_1$, setting $k = 2$ in (16) gives $m_2 = G(d_3 + (m_3 - m_2)r_3, d_2 + (m_2 - m_1)r_2 \mid r_3, r_2) > G(d_3 + (m_3 - m_2)r_3, d_2 \mid r_3, r_2)$ (by Lemma 2). Therefore, we have $G(d_3 + (m_3 - m_2) \cdot r_3, d_2 \mid r_3, r_2) < m_2 < m_1 < m_1' = G(d_2, d_1 + m_1' r_1 \mid r_2, r_1) \le G(d_3, d_2 \mid r_3, r_2)$ (by assumption), or equivalently, $G(d_3 + (m_3 - m_2)r_3, d_2 \mid r_3, r_2) < G(d_3, d_2 \mid r_3, r_2)$. By Lemma 2, the last inequality implies $m_3 < m_2$. Similarly, we can show that when $m_1 > m_2$, $m_k < m_{k-1}$ for all successively higher k. That is, $m_K < \cdots < m_1 < m_1'$.

However, $m_K < m_{K-1}$ and (17) jointly imply $m_K = H(d_K + (m_K - m_{K-1})r_K \mid r_K) \ge H(d_K \mid r_K)$ (by Lemma 2). We thus have $H(d_K \mid r_K) < m_1'$, a contradiction to the condition of the proposition. This proves $m_1 \le m_2$, completing Step 1.

Step 2. Suppose $m_{k-1} \le m_k$ $(k < K)$ holds, and we wish to show $m_k \le m_{k+1}$ under the condition of the proposition. This step is somewhat analogous to Step 1.

Suppose $m_k > m_{k+1}$ instead. From (16), we have

$$\begin{aligned} m_k &= G(d_{k+1} + (m_{k+1} - m_k)r_{k+1}, d_k + (m_k - m_{k-1})r_k \mid r_{k+1}, r_k) \\ &< G(d_{k+1}, d_k \mid r_{k+1}, r_k) \quad \text{(by Lemma 2).} \end{aligned}$$

If $d_k \geq d_{k+1}$, the above inequality is already a contradiction to $m_k > 0$, and thus $m_k \leq m_{k+1}$ must hold. Otherwise, the FOC of firm $k+1$ (replacing k with $k+1$ in (16)) gives

$$\begin{aligned} m_{k+1} &= G(d_{k+2} + (m_{k+2} - m_{k+1})r_{k+2}, d_{k+1} \\ &\qquad + (m_{k+1} - m_k)r_{k+1} \mid r_{k+2}, r_{k+1}) \\ &> G(d_{k+2} + (m_{k+2} - m_{k+1})r_{k+2}, d_{k+1} \mid r_{k+2}, r_{k+1}) \\ &\qquad\qquad \text{(by Lemma 2).} \end{aligned}$$

We then have

$$\begin{aligned} &G(d_{k+2} + (m_{k+2} - m_{k+1})r_{k+2}, d_{k+1} \mid r_{k+2}, r_{k+1}) \\ &\quad < m_{k+1} < m_k < G(d_{k+1}, d_k \mid r_{k+1}, r_k) \\ &\quad \leq G(d_{k+2}, d_{k+1} \mid r_{k+2}, r_{k+1}), \end{aligned}$$

where the last inequality follows from the condition of the proposition.

Thus, $G(d_{k+2} + (m_{k+2} - m_{k+1})r_{k+2}, d_{k+1} \mid r_{k+2}, r_{k+1}) < G(d_{k+2}, d_{k+1} \mid r_{k+2}, r_{k+1})$, which implies $m_{k+1} > m_{k+2}$ by Lemma 2. Similarly, we can show that when $m_k > m_{k+1}$, $m_K < \cdots < m_k < G(d_{k+1}, d_k \mid r_{k+1}, r_k)$. Again, $m_K < m_{K-1}$ and (17) jointly imply $m_K = H(d_K + (m_K - m_{K-1})r_K \mid r_K) \geq H(d_K \mid r_K)$. We thus have $H(d_K \mid r_K) < G(d_{k+1}, d_k \mid r_{k+1}, r_k)$, a contradiction to the condition of the proposition. This shows that when $m_{k-1} \leq m_k$ $(k < K)$, $m_k > m_{k+1}$ can never hold, and therefore, we must have $m_k \leq m_{k+1}$ under the condition of the proposition. This completes Step 2 of the proof. □

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