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This study develops and estimates a dynamic model of consumer choice behavior in markets for seasonal goods, where products are sold over a finite season and availability is limited. In these markets, retailers often use dynamic markdown policies in which an initial retail price is announced at the beginning of the season and the price is subsequently marked down as the season progresses. Strategic consumers face a trade-off between purchasing early in the season, when prices are higher but goods are available, and purchasing later, when prices are lower but the stockout risk is higher. If the good starts providing utility as soon as it is purchased (e.g., apparel), consumers purchasing earlier in the season can also get more use from the product compared to those purchasing later.

Our structural model incorporates three features essential for modeling the demand for seasonal goods: changing prices, limited availability, and possible dependence of total consumption utility on the time of purchase. In this model, heterogeneous consumers have expectations about future prices and product availability, and they strategically time their purchases. We estimate the model using aggregate sales and inventory data from a fashion goods retailer.

The results indicate that, in the fashion goods context, ignoring consumers' expectations about future availability or the change in total consumption utility over the season can lead to biased demand estimates. We find that strategic consumers delay their purchases to take advantage of markdowns and that these strategic delays hurt the retailer's revenues. Retailer revenues facing strategic consumers are 9% lower than they would have been facing myopic consumers. Limited availability, on the other hand, reduces the extent of strategic delays by motivating consumers to purchase earlier. We find that the impact of strategic delays on retailer revenues would have been as high as 35% if there were no stockout risk. By means of counterfactual experiments, we show that the highest retailer profits are achieved by offering small markdowns early in the season. On the other hand, given current markdown percentages, the retailer can improve profits by carrying less stock as consumers accelerate purchases and purchase at higher prices when they anticipate scarcity in future periods. As long as the reduction in availability is not great, the profit gain from earlier higher-priced sales can overcome the loss resulting from the reduction in overall sales.

Key words: pricing research; choice models; forward-looking behavior; limited availability; short life cycle; fashion; retailing; revenue management
History: Received: June 4, 2010; accepted: October 18, 2011; Preyas Desai served as the editor-in-chief and Scott Neslin served as associate editor for this article.

1. Introduction

The goal of this paper is to develop and estimate a dynamic model of consumer choice behavior in markets for seasonal goods,[1] where products are sold over a finite season and availability is limited.

Some examples of seasonal goods are fashion apparel, holiday merchandise, and concert and airline tickets. The empirical context for our analysis is the fashion apparel market. Seasonal goods exhibit unique demand characteristics when compared with consumer packaged goods or durable goods. First, there is a well-defined, finite selling horizon; goods

are introduced into the market, sold over a (usually short) season, and are then discontinued. Second, products provide utility either over a finite season (e.g., fashion goods or holiday merchandise) or as a lump sum all at once (e.g., airline or concert tickets). Also, in the case where the product provides utility over a finite season, consumers' total utility from consumption could depend on the time of purchase within the season. For example, a consumer in the market for a swimsuit would get more use out of the product if she purchases it earlier in the season.

These demand characteristics, coupled with two important supply-side considerations, create challenges for the seasonal goods retailer in pricing

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[1] Also called short life-cycle goods or perishable goods.

and inventory management. First, replenishment lead times are much longer in some seasonal goods industries (e.g., fashion apparel) compared with the length of the selling season. This limits the retailer's opportunity to replenish the inventory during the season (capacity is fixed well in advance for other seasonal goods such as airline tickets, concert tickets, or hotel rooms, and it can only be increased at a substantially high marginal cost). Second, end-of-season salvage value is very low; i.e., products are perishable. So the seasonal goods retailer faces the challenge of maximizing profits from ordering a fixed amount of inventory before the selling season and selling it over a finite horizon.

After setting an initial stock level, seasonal goods retailers often resort to intertemporal (dynamic) pricing policies, and prices for seasonal goods exhibit substantial variation within the season. In the fashion goods industry, for example, it is common practice to employ markdown pricing. Every new product line is introduced at a "retail price," and prices are marked down a number of times until the inventory is cleared or the selling season ends. In the rest of this paper, we will talk about the fashion apparel market, but our methodology and results would extend to any market where the selling horizon is well defined and availability is limited.

Intertemporal pricing can help the retailer in two ways. First, it enables the retailer to segment the market and take advantage of differences in consumers' willingness to pay and willingness to wait for the products. If some consumers prefer to buy early in the season at the expense of paying higher prices while others choose to wait to purchase at lower prices, the retailer can serve the first group at higher prices and can lower the price to serve the second group once the first group purchases and exits the market. Second, it helps the retailer respond to a decrease in his option value from holding on to inventory as the season progresses. If the retailer has overstocked in the initial period, for example, he can reduce prices later in the season to clear the shelves before the end of the season (i.e., before the inventory perishes).

Advances in information technology and marketing research have increased the ability of retailers to collect and analyze consumer data, making it easier for them to employ complex pricing strategies. On the other hand, consumers have been getting increasingly more sophisticated as well. In making a purchase, a strategic consumer weighs the benefits of purchasing today against the benefits of waiting and purchasing in the future at a lower price. A *Wall Street Journal* (2002) article reported that it is now possible for the consumers to "crack the retailer's pricing code" and "not pay retail."

In contrast to durable goods markets, where prices also decline over a product's life cycle, the game between the retailers and the consumers has an interesting dimension in the fashion goods market (and in all markets where capacity is fixed in the short term). Because supply is limited, a consumer cannot wait for a sale without taking into account the risk of stockout. So consumers need to trade off decreasing prices against the possibility of not being able to find the product later in the season. The retailer also faces a trade-off. Having too much in stock might increase his chances of meeting the demand, but limiting the stock, on the other hand, might motivate strategic consumers to buy earlier at higher prices. Zara, a large Spanish producer and retailer of fashion goods, for example, is well known for its success in implementing a deliberate limited stock strategy.

This study develops a structural model of the dynamic decision process on the consumer (demand) side and uses the estimates from this model to investigate retailer pricing and inventory policies. We model strategic consumer behavior where consumers form expectations about future prices and availability and take stockout risk into account when timing their purchases. Our contribution can be summarized under three headings. First, we empirically investigate the impact of consumers' product availability expectations on purchase behavior. An important strategic consideration is that consumers may accelerate purchases (even at the expense of paying higher prices) if they anticipate that the product may be unavailable in the future. We are not aware of prior empirical work that models consumers' expectations about product availability. Previous work on the role of price expectations in consumer purchase behavior assumes unlimited capacity, so that consumers would have the option of buying the product again next period with certainty if they choose to delay their purchases (Erdem et al. 2003, Song and Chintagunta 2003, Nair 2007). Through a counterfactual experiment, we empirically illustrate that anticipated scarcity can lead to purchase acceleration and higher retailer profits. Second, we explicitly model the dependence of the total utility consumers get from the consumption of a product on the time of purchase. Consumer uncertainty about future product availability and change in total consumption utility over time are two factors that rationalize early purchases despite declining prices over a short season. Ignoring these factors would result in biased demand estimates in the fashion goods context. Third, we add to the vast theoretical literature on revenue management by providing empirical evidence on seasonal goods demand and pricing using a realistic demand model and counterfactual experiments.

Our demand model enables a seasonal goods retailer to decompose the effects of different factors that contribute to change in demand over time. Understanding the separate effects of decreasing total consumption utility, increasing stockout risk, changing market size, and market composition on demand and market responsiveness is important because each of these factors has different implications for the retailer's pricing and inventory strategy. Our structural approach allows us to obtain behavioral predictions that are invariant to the effects of policy changes and allows us to simulate various pricing/stocking policies and study their profit implications. A seasonal goods retailer can use our demand model in jointly determining optimal initial inventory levels, retail prices, and the timing and depth of markdowns.

Our focus is on permanent markdowns as opposed to temporary promotions (e.g., Labor Day sales) where prices are reduced for a limited time and go back up again. In our model, we assume that there is a separate market for each product. Every period (week), each consumer in the market for a specific product decides to buy the product or wait until the next period. Consumers are strategic and heterogeneous with respect to their response parameters. Consumers have expectations regarding the likelihood of future states such as prices and availability and choose to buy the product and exit the market if the expected discounted sum of utilities from buying in that period exceeds that from waiting.

The data used to estimate our model come from a specialty apparel retailer that sells private-label fashions. Aggregate weekly sales, inventory, and cost data are available for 105 stock-keeping units (SKUs) (which we aggregate into 61 products, as discussed in §5) from the women's coats category for a period of two years. Each product is introduced, sold over a finite season, and is discontinued. The season length varies from 11 to 30 weeks. There is significant variation in sales and prices across products. Each product is marked down at least once during its season.

To preview our results, we note that our model produces a good fit to the data. Our estimates imply that the market is composed of two distinct consumer segments. The first segment consists of low price sensitivity consumers who account for 80% of the total market at the start of the season. The second segment consists of high price sensitivity consumers who start purchasing late in the season and account for an important percentage of the end-of-season sales.

We compare our model to a benchmark model that reflects the current state of the art. The benchmark model, similar to Nair (2007), accounts for heterogeneity in response parameters and for the price expectations of strategic consumers, but it does not take consumers' availability expectations into

account and assumes that the total consumption utility does not depend on the time of purchase. This model produces unreasonable demand estimates (e.g., the price sensitivity estimate is positive and significant for the first segment), and the likelihood ratio test shows that our model outperforms the benchmark model.

We find that pricing decisions in the early periods are very critical because early markdowns can significantly increase sales but might affect total revenue very negatively if not timed optimally. Given the estimated demand model, we run three counterfactual experiments to investigate different elements of a retailer's markdown and inventory policy. The first experiment provides insights about the retailer's trade-off between the timing and depth of markdowns. This experiment shows that, under a uniform single markdown policy, the highest retailer profits are achieved by offering early and small markdowns. Late markdowns do not have very favorable profit outcomes, and early and deep markdowns are very detrimental to retailer profits. The second experiment examines how limiting availability impacts the retailer's performance and shows that given the current markdown percentages and timing, the retailer can improve his profits by carrying less stock. We find that a slight decrease in the initial stock offered can encourage strategic consumers to purchase earlier at higher prices as a result of increased stockout risk. Despite the negative effect on the total quantity sold, sales at higher prices compensate for lost sales. In the third experiment, we show that strategic consumers who delay their purchases to take advantage of lower prices contribute to a 9% reduction in the retailer's revenue. However, decreasing product availability reduces consumers' incentive to wait for lower prices. We show that if consumers had not taken stockout risk into account when timing their purchases, strategic delays would have been more pronounced, and the loss in revenue as a result of strategic behavior would have been larger (35%).

The rest of this paper is organized as follows: Section 2 introduces the related literature. Section 3 presents the model, and §4 outlines the estimation strategy. Section 5 introduces the data used in the empirical application. Section 6 presents demand estimates and the counterfactual experiments and discusses pricing and inventory management implications. Section 7 concludes with a discussion of the results and future directions.

2. Related Literature
Our study is closely related to three main streams of literature. First is the economics literature on

intertemporal price discrimination. Second is the economics and operations literature on revenue management, and third is the recent marketing literature on structural models of strategic consumer behavior. We will briefly discuss each research stream, highlighting ideas and articles closely related to our study.

Interest in intertemporal demand considerations and strategic consumer behavior in the economics literature started in the area of durable goods monopoly pricing (Coase 1972, Stokey 1979, Besanko and Winston 1990). This literature is rich in theoretical work investigating demand-side considerations but does not study supply-side considerations such as limited availability.

Clearance sales have received some attention in the economics literature. Pashigian (1988) provides empirical evidence on sales offered by department stores. He gives the growing importance of "fashion" (variety) as an explanation for the changes in markdowns over time and between merchandise groups. Pashigian and Bowen (1991) provide further empirical evidence and offer demand uncertainty and price discrimination as two alternative hypotheses to explain the observed pricing practices.

The second research stream of interest is the vast operations literature on revenue management; the reader is referred to Elmaghraby and Keskinocak (2003) for an extensive review. Gallego and van Ryzin (1994), Bitran and Mondschein (1997), and Bitran et al. (1998) all study analytical dynamic pricing models where a retailer sells a fixed inventory over a finite horizon. These studies derive two structural properties of the optimal policy that state that at any given time, the optimal price decreases in the number of items left, and for any given number of items, the optimal price decreases over time. Research in revenue management has largely focused on analytical models incorporating supply-side considerations. Strategic consumer behavior has been considered more recently in the literature on seasonal goods pricing (Su 2007, Aviv and Pazgal 2008, Dasu and Tong 2008, Cho et al. 2009). These models show analytically that heterogeneity in willingness to pay, change in consumer utility over time, and limited and uncertain product availability may all be factors that can rationalize early purchasing by some consumers even though prices are expected to fall. In this study, we empirically model and investigate the potential role of these factors on consumer purchase decisions. Liu and van Ryzin (2008), using a stylized two-period model, show that under certain conditions, it might be profitable for a monopolist seller facing strategic consumers to create rationing risk by intentionally understocking products. We provide empirical evidence for a seller's incentive to deliberately limit stock

to induce earlier purchases at higher prices. This literature, however, lacks empirical research with realistic demand models. To our knowledge, Heching et al. (2002) is the only recent empirical study in this area. They estimate a simple demand model using data from a specialty apparel retailer and obtain estimates of revenues under various pricing policies. The analysis, consistent with our findings, suggests that the firm would have increased its revenue if it had smaller markdowns earlier in the season.

There has been some recent theoretical (McAfee and te Velde 2007) and empirical (Sweeting 2008) work investigating dynamic pricing of seasonal goods in the economics literature. Sweeting (2008), in the context of Major League Baseball ticket prices, shows that a significant decline in ticket prices as the game approaches is mostly due to declining option values of the sellers rather than changes in elasticity of demand. Sweeting shows that many cases of observed early buying can be rationalized by uncertainty about future prices and availability as well as return to market costs (hassle costs associated with returning to the *Internet* store at a later date). In the case of baseball tickets (and airline tickets, concert tickets, etc.), consumption utility from the product is received as a lump sum all at once and does not depend on the time of purchase. On the other hand, when goods start to provide utility as soon as they are purchased, as in the case of fashion goods (and holiday merchandise, etc.), there is another interesting dynamic. Waiting decreases the lifetime utility from the product and the likelihood of purchase. We show that the decrease in lifetime utility over time is another important dynamic that rationalizes early purchases in the presence of decreasing prices over a short season. Future researchers could incorporate return-to-market costs into our model as another factor that explains early purchases to study goods that have the dynamics captured by Sweeting (2008) as well as the dependence of total consumption utility on the time of purchase. There might also be other costs involved in waiting to buy, as pointed out by Sweeting (2008). One such cost could be complementary investments such as booking hotels or transportation tickets for a sporting event or finding complements for apparel items—e.g., a shirt that matches a pair of pants. Consistent with this, we should observe people living farther from the stadium and people who have hard-to-find sizes purchasing earlier. This effect and its interaction with uncertainty about future availability can also be incorporated into the model.

There has been increasing interest in studying the impact of product availability and stockouts on consumer demand. Gönül and Srinivasan (1996) and Erdem et al. (2003) model household product availability and consumers' consideration of in-house

inventory and stockout risk in a utility maximization framework. Bruno and Vilcassim (2008) incorporate information on store product availability in a structural demand model and show that neglecting the effects of stockouts leads to biases in the demand estimates. Musalem et al. (2010) develop a structural demand model using store-level sales and inventory data to endogenously capture the effect of stockouts and provide insights on the lost sales and financial consequences resulting from stockouts. We add to this literature by explicitly modeling forward-looking consumers' consideration of expected future stockouts in making purchase decisions. In the presence of scarcity risk in future periods, we show that consumers would be willing to purchase earlier even at the expense of paying higher prices. This finding highlights the importance of product availability as a strategic decision variable for the retailer. Impact of product availability on demand is not limited to lost sales or product substitution resulting from stockouts. We also show that limited availability does not always lead to negative financial consequences.

The third literature stream is the recent economics and marketing literature on structural models of strategic consumer behavior. A large number of papers have studied dynamic models of consumer decision making where there is uncertainty about product quality, future prices, promotions, or product introductions. Erdem and Keane (1996) present a structural dynamic choice model where "forward-looking" consumers are uncertain about attributes of a set of brands and learn about these brands through advertising exposure and usage experience. They find that the forward-looking model fits the data statistically better than myopic models. Erdem et al. (2003) and Hendel and Nevo (2006) study demand models for frequently purchased storable goods that are subject to stochastic price fluctuations. Both studies show that price expectations have important effects on demand elasticities, and long-run cross-price elasticities (allowing for the effect of price cut on future expected prices) are much larger than the short-run cross-price elasticities (holding expectations fixed). All these models are constructed for frequently purchased consumer goods and are estimated on scanner panel data.

High-tech durables markets are similar to the seasonal goods markets in the sense that prices exhibit a declining pattern over the life cycle of a product creating an incentive for consumers to delay purchases, and repeat purchases are rare. Our model is similar to discrete choice models of durable goods adoption developed in the high-tech durables context. Melnikov (2000) models strategic consumers' adoption behavior using data from the computer printer market but does not allow for consumer heterogeneity. Song and Chintagunta (2003) analyze the impact of price expectations on the diffusion patterns of new high-tech products using aggregate data but do not allow for econometric errors in the demand function. Our model is closest to Nair (2007), who empirically estimates a dynamic structural demand model in the video game industry using aggregate data, incorporating forward-looking consumer behavior (where consumers have expectations about future prices) in addition to allowing for consumer heterogeneity and econometric error terms. He also numerically computes the optimal dynamic sequence of prices and shows that strategic purchase timing by consumers who face declining prices over time has a significant effect on optimal pricing of video games, that profit losses from ignoring consumer forward-looking behavior are large, and that market research that provides information regarding the extent of discounting by consumers would be valuable to firms. We not only apply the durable goods adoption model to the seasonal goods markets but also extend this model to allow for consumer's consideration of their expectations about future product availability (in addition to future prices) as well as accounting for the dependence of total consumption utility on the time of adoption. We also show that although strategic consumer behavior impacts firm profits negatively, consumers' consideration of future stockout risk helps to dampen this negative impact. Our study lies on the interface of the recent economics and marketing literature on dynamic structural discrete choice demand models and the vast operations literature on revenue management.

3. Model

3.1. Overview
We develop a dynamic structural model of demand in markets for seasonal goods where consumers are strategic and heterogeneous. We treat each product as a separate market in our model. Every period, a consumer in the market for a specific product decides to buy the item and exit the market or wait until the next period and make a decision again. Consumers choose to buy the product if the expected discounted sum of utilities from buying in that period exceeds that from waiting. Consumers also have the option of not buying the item at the end of the season.[2] We assume that consumers do not buy multiples of the same

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[2] This assumption does not imply that if the consumer does not purchase the focal coat by the last period that the customer does not buy any coat. In our model, each coat is a separate market, and if the customer is in the market for multiple coats, her decisions are independent across coats.

product (i.e., our model is an incidence model, not a volume model). When calculating expected future utilities, consumers take into account their expectations about future prices as well as their expectations about future availability (i.e., stockout risk).

Our model captures three important characteristics of seasonal goods demand. First, consumers' responsiveness to prices and other marketing variables change through the season because of the decrease in the number of time periods left for consumption and increase in stockout risk. Second, because the product is a durable, consumers purchasing the product exit the market and the potential market for a specific item shrinks through the season. Third, the composition of the market changes over time (as long as there is heterogeneity in the consumer population). For example, if consumers have different price sensitivities and face declining prices, less price-sensitive consumers will purchase the product in the earlier periods and exit the market, and the proportion of more price-sensitive consumers in the remaining market will increase over time.

Capturing the effect of dependence of the total consumption utility on the time of purchase as well as consumers' consideration of future stockout risk is important. An empirical regularity in the data is that, except for a brief period early in the season, sales of a specific product decline over time at a given price. Sales increase in the periods where prices are marked down but decrease immediately after the markdown period. Decreasing total utility from consumption and increasing stockout risk both reduce a consumer's incentive to wait and contribute to the decrease in sales over time. Estimates from a demand model would be biased if one does not account for these two effects. Ignoring these effects would result in a downward bias in the price sensitivity parameter and/or an upward bias in the markdown sensitivity parameter.

It is also important to account for strategic consumer behavior. A number of studies have found that, in the consumer packaged goods (CPG) industry, consumers form expectations about product quality (Erdem and Keane 1996), coupon availability (Gönül and Srinivasan 1996), and future prices (Erdem et al. 2003). These studies have shown that strategic models fit the data better than myopic models. In the fashion apparel market, consumers have even higher incentives to behave strategically in timing their purchases as they face significant reduction in prices over a short season and are also subject to availability risk. The retailer needs to account for strategic behavior because it affects the shape of the aggregate sales curve and induces price dynamics in the market (Song and Chintagunta 2003). Facing strategic consumers, the retailer needs to take intertemporal substitution into account because a price reduction

today will influence sales in future periods. Besanko and Winston (1990) showed that reduction in profit by assuming that the consumers are myopic when, in fact, they are strategic could be rather significant for a retailer.

It is also important to account for consumer heterogeneity. Allowing for heterogeneity provides a flexible pattern for the aggregate sales curve (Song and Chintagunta 2003). Understanding heterogeneity enables the retailer to take advantage of differences in the consumer population by adjusting prices dynamically through the sales season. We incorporate consumer heterogeneity through an aggregate analog to the latent-class models used with household purchase data (Kamakura and Russel 1989). We assume that each consumer belongs to one of a finite number of segments and each segment is characterized by its own response parameters.

Finally, our structural approach allows us to obtain behavioral predictions that are invariant to the effects of policy changes and allow us to simulate various pricing/stocking policies and study their profit implications. The interested reader is referred to Chintagunta et al. (2005) for a discussion comparing structural and reduced-form modeling approaches.

3.2. The Utility Specification

Consumer i's conditional indirect utility from purchasing product j in period t is defined as

$$U_{ijt} = \alpha_j + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + \beta_{ip} p_{jt} + \beta_{im} d_{jt} + \beta_{is} s_t + \xi_{jt} + \varepsilon_{ijt}, \quad (1)$$

where p_{jt} is the price of product j in period t, d_{jt} is the markdown dummy, and s_t is a seasonal dummy. α_j is the intrinsic preference for product j, c is the postpurchase per-period consumption utility, T_j is the length of the sales season for product j, and γ is the discount factor. β_{ip} is price sensitivity, β_{im} is markdown sensitivity, and β_{is} is the seasonality parameter. ξ_{jt} is a product- and time-specific demand shock, and ε_{ijt} is a mean-zero stochastic term.

U_{ijt} is defined as the total utility the consumer gets from purchasing the product and includes not only the instantaneous (current-period) utility but also the discounted sum of all future utilities the consumer will get from using this product within the finite season. As we discussed earlier, the total utility a consumer gets from a product changes over the season depending on the time of purchase. A consumer in the market for a winter coat, for example, would be able to wear it for a longer period if she purchases the coat earlier in the season.

In the utility specification, α_j controls for taste for product j relative to the outside option that is constant over time and c is the postpurchase per-period consumption utility. It would be more realistic to make

the consumption utility parameter c product-specific as well, but we face the limitation that α_j and c_j are not separately identified. Our particular specification could be justified in the context of fashion goods, where product differences stem mostly from hedonic attributes such as color, texture, or design rather than utilitarian attributes such as warmth or durability. At the time of purchase, an overall liking for the product driven by hedonic attributes may drive the decision. This is captured by the product-specific parameter α_j. In the consumption of the product, utilitarian attributes such as warmth and durability may matter more. This effect is captured by the post-purchase consumption utility parameter c. Support for this view comes from behavioral research that shows that hedonic products or attributes are likely to be evaluated spontaneously on the basis of the liking or disliking that they evoke (Kahneman and Frederick 2002, Schwarz and Clore 1983), whereas utilitarian products or attributes are likely to be evaluated based on analytical rules and calculations. Furthermore, hedonic products or attributes are associated with immediate pleasure, whereas utilitarian products and attributes are associated with delayed instrumentality (Khan et al. 2005). At the same time, however, wearing the "in" coat could also provide hedonic consumption utility, which argues for a product-specific c_j parameter. We do not want to take this argument much further. An alternative specification would be to replace α_j and c with a single parameter, c_j, as in Sun (2005). We prefer our particular specification, because with the alternative specification, the c_j's would perform double duty and control not only for the reduction in total consumption utility from a product over time but also the differences in mean level of sales across products. The main disadvantage of this specification would be in interpretation. Because we have an outside option (and its utility is normalized to 0), inevitably, some of these c_j's would turn out to be negative.

$\sum_{\tau=t}^{T_j} \gamma^{\tau-t} c$ represents the discounted present value of the total utility from consumption of the product through the finite season if a purchase takes place in period t. We assume that the product provides utility only over the finite season and that there is no salvage value. The change in the remaining number of periods that the product can provide utility over the season allows us to capture the change in total consumption utility as the season advances. As we discuss later in §4.3, both α_j and c are assumed to be common across segments (to reduce the number of nonlinear parameters to a manageable number), but because the price parameter is heterogeneous, it allows us to capture the differences in willingness to pay across segments. The discount factor is also assumed to be common across segments following the literature.

The markdown dummy, d_{jt}, is set to 0 in the earlier periods when the product is sold at retail (full) price and is set to 1 as soon as the product is marked down and stays at 1 until the end of the season. The markdown dummy captures the "mere markdown" effect—the possibility that consumers might get extra (or less) utility from purchasing on sale. The impact of the depth of the markdown on utility, on the other hand, is captured by the price parameter as the price variable reflects the actual markdown price. The markdown dummy also captures any merchandising effort used to support markdown products, which is uniform across products. In our empirical application, markdown products are moved to a special section in the store, and if this treatment makes these products more noticeable, this effect will be captured by the markdown dummy.

The seasonal dummy, s_t, is included to capture the possibility that utility from a product might be higher (lower) during peak or low seasonal periods. A close examination of the seasonality patterns in the data reveals a strong demand peak in the six-week "holiday shopping period" that starts after Thanksgiving and ends after Christmas. A regression of overall sales on relative prices and a set of dummies for all possible seasonal periods (e.g., Mother's Day, Labor Day, holiday shopping period) reveals that the holiday shopping period is the only period that has a significant effect on the overall demand. In our application, s_t is set to 1 for the holiday shopping period and 0 for all other periods.

Note that the ξ_{jt} is a product- and time-specific demand shock, and ε_{ijt} is a mean-zero stochastic term, observed by consumers but not by the econometrician. ξ_{jt} controls for any additional product- and time-specific factors consumers observe and take into account when making a purchase decision but the econometrician does not observe. In the fashion apparel context, the ξ_{jt}'s could correspond to demand shifters such as a specific product appearing in an advertisement or a TV show in a specific week. The ξ_{jt}'s also serve as the econometric error term in the estimation of demand. The instantaneous utility from not buying product j in period t (utility from waiting) is normalized to ε_{ij0t}; that is, $U_{ij0t} = \varepsilon_{ij0t}$.

3.3. Availability
In a limited stock environment, a consumer is likely to face a stockout in any period. In a consumer packaged goods context and a multistore environment, Bruno and Vilcassim (2008) operationalize availability as the probability of finding the product in a store in a given shopping trip. For the purposes of this study, we resort to a similar definition and define availability of a specific item in a time period as the probability that a consumer visiting a store in that period finds the item in stock.

It is well accepted that consumers value high availability. However, accounting for the effect of availability on demand is a challenge. To account for the effect of availability on individual consumer purchase decisions in a multistore retail environment, one would need real-time data on individual consumer store visits, purchases, and real-time inventory data at the store and SKU level. Practitioners and researchers, though, typically only have access to data where sales and inventory are aggregated across time and/or stores. The data set we use in our empirical application comes from a multistore retailer. In our main data set, sales and inventory information is aggregated across stores, and total sales and opening inventory levels for each (active) SKU are reported for 104 weeks.

In the absence of detailed real-time data, "retail distribution" has been used as a proxy for availability (Bruno and Vilcassim 2008). Retail distribution is defined, in its simplest form, as the number of outlets carrying a product (has the product in stock) as a percentage of total outlets. In this study, we follow a similar approach and use retail distribution as our availability measure. To calculate retail distribution, we supplement our main data set with weekly individual store-level inventory data over the 104-week period. In the supplementary data set, we have store-level inventory data for 57 of the 105 SKUs in our main data set.

Calculation of retail distribution levels is straightforward for these 57 SKUs. Define D_{jt} as the store-level inventory vector for product j and week t as $D_{jt} = (i_{j1t}, i_{j2t}, \ldots, i_{jSt})$, where i_{jst} represents the inventory level for product j in store s in period t. Next, define $I_{jst} \in \{0, 1\}$ as the indicator of the event "item j is in stock at store s in period t." For each product and time period, we need to set the indicator I_{jst} to 1 for those stores that have positive inventory of product j and to 0 for other stores. The retail distribution λ_{jt} is then computed by summing I_{jst} across all stores and dividing by the total number of stores S; i.e., $\lambda_{jt} = \sum_{s=1}^{S} I_{jst}/S$. So if the retailer has 100 stores, and 65 stores have positive inventory of product j in period t, I_{jst} is set to 1 for these 65 stores and to 0 for all other stores, and retail distribution is computed as $\lambda_{jt} = 65/100 = 0.65$.

At the data preparation stage, we calculate retail distribution levels observed in the store-level data for each product–week combination for these 57 SKUs. For the remaining 48 SKUs in our sample, we observe aggregate weekly inventory levels but not the store-level weekly inventory levels. Inv_{jt} corresponds to the aggregate inventory level for product j and period t; i.e., $Inv_{jt} = \sum_{s=1}^{S} i_{jst}$. To predict the retail distribution levels for these 48 SKUs, we use the relation $\lambda_{jt} = a_1 \ln(Inv_{jt}) + a_2 Inv_{jt} + a_3 Inv_{jt} + \varepsilon_{jt}$.

Since we observe both Inv_{jt} and λ_{jt} for the 57 SKUs, we estimate the specified relation between retail distribution λ_{jt} and aggregate inventory Inv_{jt} with ordinary least squares (OLS) using data for these SKUs ($R^2 = 0.98$, $N = 1{,}556$). Then we use the parameter estimates to predict the retail distribution levels (λ_{jt}) for each period for the 48 SKUs for which we observe only the aggregate inventory levels (Inv_{jt}).

3.4. Expectations

Consistent with the majority of studies in the dynamic choice models literature (e.g., Song and Chintagunta 2003, Erdem et al. 2003), we assume that consumers have rational expectations about the future values of state variables. Rational expectations assumptions have been questioned by Manski (2004) because these assumptions may be intrinsically implausible in some contexts. It would be ideal to collect data on individual consumers' expectations and incorporate this information into the dynamic choice model. Erdem et al. (2005), for example, relax the rational expectations assumption by using survey data on self-reported consumer price expectations. We do not, however, have access to similar data on consumer expectations. In the absence of such data, we use the rational expectations assumption to provide a reasonable approximation to consumer expectations.

It is well accepted that consumers rely on past experience and other signals to predict future states of the world such as future prices and availability. In modeling consumers' price and availability expectations, we assume that consumers observe the current price and availability every period and compute expected future prices and availability relying on past experience.

3.4.1. Price Process. To approximate consumers' expectations of future prices, we need to specify a realistic price process that captures key characteristics of seasonal goods pricing: (1) each item is introduced at a "retail" (full) price; (2) prices are constant for several weeks, followed by permanent markdowns; and (3) the probability and the magnitude of markdowns depend on the retail price, time in the season, and whether the product has been previously marked down. One important observation is that products with higher initial prices are typically marked down earlier and deeper. Also, the time between the introduction of the product and the announcement of the first markdown is typically longer than the time between two successive markdowns, and the first markdown is typically deeper than the later markdowns in dollar terms.

To capture these important characteristics, we specify a multivariate jump process (see Erdem et al. 2003 for a similar specification in the CPG context).

We assume that the price for product j that was introduced in period 1 is marked down in period t with probability φ_{jt} and stays constant from period $(t-1)$ to period t with probability $\varphi_{0jt} = (1 - \varphi_{jt})$. If we define P_{jt} as the price of product j in period t and MD_{jt} as the markdown depth for product j in period t, we have

$$P_{jt} = P_{j,t-1} - MD_{jt} \quad \text{with probability } \varphi_{jt}, \quad \text{and}$$

$$P_{jt} = P_{j,t-1} \quad \text{with probability } \varphi_{0jt}.$$

Define P_{j1} as the retail (initial) price of product j and $time_{jt}$ as the number of weeks since the introduction of the product (time in the season). To accommodate the possibility that the price process may differ before and after the first markdown, we specify and estimate the markdown probability process separately for these two cases. We specify the markdown probability conditional on no previous markdowns as

$$(\varphi_{jt} \mid P_{j,t-1} = P_{j1})$$

$$= \frac{\exp[a_0 + a_1 P_{j1} + a_2 time_{jt}]}{1 + \exp[a_0 + a_1 P_{j1} + a_2 time_{jt}]} \quad t = 2, \ldots, T. \quad (2)$$

We specify the markdown probability conditional on a previous markdown as

$$(\varphi_{jt} \mid P_{j,t-1} < P_{j1})$$

$$= \frac{\exp[b_0 + b_1 P_{j1} + b_2 time_{jt}]}{1 + \exp[b_0 + b_1 P_{j1} + b_2 time_{jt}]} \quad t = 3, \ldots, T. \quad (3)$$

In the case of a markdown for product j in period t, the markdown depth (in log-dollars) is specified as a function of log-retail price and a markdown dummy variable $mdum_{jt}$ (that is set to 1 if the product was previously marked down and to 0 otherwise) as follows:

$$\ln(MD_{jt}) = \theta_0 + \theta_1 \ln(P_{j1}) + \theta_2 mdum_{jt} + \varepsilon_{jt}, \quad (4)$$

Where the price shocks are assumed to be independent and identically distributed (i.i.d.) normal across products and time periods; $\varepsilon_{jt} \sim \mathrm{N}(0, \sigma_{MD}^2)$.

We tested numerous alternative specifications for markdown probabilities and markdown depth (e.g., inclusion of the current period price instead of the retail price, time since last price change instead of time since introduction), none of which improved the predictive power significantly. We believe our specifications capture important characteristics of the seasonal goods price process in a parsimonious way.

3.4.2. Availability Process. To approximate consumers' expectations of future product availability, we specify the following linear process that links current

availability to past availability, product's retail price, and time in the season:

$$\lambda_{j,t+1} = \gamma_0 \lambda_{jt} + \gamma_1 P_{j1} + \gamma_2 time_{jt} + e_{jt}, \quad \text{where}$$

$$e_{jt} \sim \mathrm{N}(\mu_\lambda, \sigma_\lambda^2), \quad (5)$$

where λ_{jt} is the availability of item j at time t, P_{j1} is the retail (initial) price for item j and $time_{jt}$ is the number of periods since the beginning of the season for item j at time t.[3]

The price and availability process parameters are estimated in a first stage using data from our full data set and are reported in §6. In the demand estimation stage, we assume that consumers know and use these parameters to form their estimates of prices and availability for future periods. A similar strategy has been employed to model consumers' price expectations in a CPG context by Erdem et al. (2003). Note that the specified processes are adaptive. In other words, consumers are assumed to observe actual realizations of prices and availability every period and then update their expectations for future periods.

3.4.3. Evolution of States. The consumer's optimal purchase timing problem can be described by the solution to a finite-horizon dynamic programming problem with time, price, availability, markdown status, seasonality, demand shocks, and the error terms as state variables. We define the state vector, S_t, as a vector of all these variables that influence a consumer's purchase decision at time t.

We assume that the unobservable (by the retailer) error terms ($\varepsilon_{ijt}, \varepsilon_{ij0t}$) evolve independently from the other state variables. Partitioning S_t into Z_t and $\varepsilon_t = (\varepsilon_{ijt}, \varepsilon_{ij0t})$, where Z_t represents all state variables except the unobservable error terms, the transition probabilities have the following form:

$$P(S_{t+1} \mid S_t) = P(Z_{t+1}, \varepsilon_{t+1} \mid Z_t, \varepsilon_t) = P(Z_{t+1} \mid Z_t) P(\varepsilon_{t+1} \mid \varepsilon_t).$$

This is the well-known conditional independence assumption widely used in the literature (Rust 1994). We further assume that the unobservable error terms are i.i.d. extreme value distributed. Among the state variables, time and seasonality evolve naturally. Time- and product-specific demand shocks are assumed to

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[3] Note that the availability measure lies between 0 and 1, but we chose a linear specification with a normal error term. In our empirical application, because availability is closely related to last period's availability and price, this model works very well prediction-wise, and the fitted λ's stay between 0 and 1. In an alternative specification, one can transform the dependent variable to $\ln((1 - \lambda_{j,t})/\lambda_{j,t})$. This way, $\lambda_{jt} = 1/(\exp(Z) + 1)$ where $Z = \gamma_0 + \gamma_1 \lambda_{j,t-1} + \gamma_2 P_{j1} + \gamma_3 time_{j,t} + e_{jt}$ and is constrained to be between 0 and 1, where $e_{jt} \sim \mathrm{N}(0, \sigma_\lambda^2)$. The λ's from the alternative and the original specifications are highly correlated (0.96). We decided to retain the linear model for its simplicity and smaller standard error.

be distributed i.i.d. normal; $\xi_{jt} \sim N(0, \sigma_\xi^2)$. The price process we specified in §3.4.1 describes the evolution of prices and the markdown status, and the availability process we defined in §3.4.2 describes the evolution of availability. Every period, consumers observe actual realizations of price and availability for the specific product, and then update their expectations for future period prices and availability levels. Consumers also observe time, seasonality, current period demand shocks, and error terms before making a purchase decision. The retailer, on the other hand, is assumed to observe time, prices, availability, markdown status, seasonality, and the product- and time-specific demand shocks (ξ_{jt}) for the current period but not the error terms (ε_{ijt}) before making pricing decisions.

3.5. Consumer's Decision Rule and Dynamic Optimization Problem

$U_{ijt}(S_t)$ represents the utility consumer i gets from purchasing item j in period t when the state of the world is S_t, and $U_{ij0t}(S_t)$ is the instantaneous utility from the "no-purchase" option under the same conditions. Then, the value of buying product j at time t for a strategic consumer i is given by

$$V_{ijt}(S_t) = U_{ijt}(S_t).$$

The value of the no-purchase option (waiting) at time t for a strategic consumer i is the value from delaying the purchase. The value of the no-purchase option in period t is modeled as the sum of (a) the discounted expected value that a consumer can get at time $t+1$ and (b) the instantaneous utility the consumer can get from the no-purchase option. With the no-purchase option the consumer gets to choose again next period between purchasing and waiting. Therefore, her expected next-period value is the maximum of the value from choosing to wait and the value from choosing to buy. One important point to note here is that the consumer will make this choice only if the product is available next period. If the product is not available, she will get zero utility. So a strategic consumer i calculates the expected value from waiting in period t, taking expected availability and prices into account as follows:

$$V_{ij0t}(S_t) = \gamma E[\lambda_{j,t+1} \max\{V_{ij,t+1}(S_{t+1}), V_{ij0,t+1}(S_{t+1})\}$$
$$+ (1 - \lambda_{j,t+1}) \times 0 \mid S_t] + U_{ij0t}(S_t),$$

where γ is the discount factor and $\lambda_{j,t+1}$ is availability in period $t+1$. If we normalize $U_{ij0t}(S_t)$ to ε_{ij0t}, the expression simplifies as follows:

$$V_{ij0t}(S_t) = \gamma E[\lambda_{j,t+1} \max\{V_{ij,t+1}(S_{t+1}),$$
$$V_{ij0,t+1}(S_{t+1})\} \mid S_t] + \varepsilon_{ij0t}. \quad (6)$$

The individual consumer's decision rule is such that consumer i buys item j and exits the market in period t only if her value from buying in period t exceeds her value from waiting and she had chosen to wait in all previous periods:

$$V_{ijt} \geq V_{ij0t} \quad \text{and} \quad V_{ij\tau} < V_{ij0\tau} \quad \text{for all } \tau < t.$$

On the other hand, the consumer does not buy product j in period t and stays in the market if her value from waiting in period t exceeds that from buying and she had chosen to wait in all previous periods:

$$V_{ijt} < V_{ij0t} \quad \text{and} \quad V_{ij\tau} < V_{ij0\tau} \quad \text{for all } \tau < t.$$

4. Estimation

4.1. Overview

We have described the consumer's decision process in §3. In this section we describe the estimation of the model parameters. We start by discussing the computation of the unconditional purchase probabilities under distributional assumptions about the stochastic term ε_{ijt}. Then we discuss the computation of the market shares by aggregating these probabilities across heterogeneous consumers for each product and time period. Finally, we present the maximum likelihood estimation (MLE) strategy used in our dynamic setting to estimate the model parameters. This estimation strategy allows for efficient estimation of a large number of parameters.

4.2. Calculation of the Purchase Probabilities

Recall the specification of the value function for the purchase and no-purchase options, respectively:

$$V_{ijt}(S_t) = \alpha_j + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + \beta_{ip} p_{jt}$$
$$+ \beta_{im} d_{jt} + \beta_{is} s_t + \xi_{jt} + \varepsilon_{ijt}, \quad (7)$$

$$V_{ij0t}(S_t) = \gamma E[\lambda_{j,t+1} \max\{V_{ij,t+1}(S_{t+1}),$$
$$V_{ij0,t+1}(S_{t+1})\} \mid S_t] + \varepsilon_{ij0t}. \quad (8)$$

The expectation in (8) is taken with respect to the distribution of future variables unknown to the consumer conditional on the current information.

Remember that we have assumed that the unobservable error terms ($\varepsilon_{ijt}, \varepsilon_{ij0t}$) evolve independently from the other state variables. We further assume that the unobservable error terms are i.i.d. extreme value distributed.

We define W_{ijt} and W_{ij0t} as the observable (by the retailer) part of the value functions for the purchase and no-purchase options, respectively:

$$V_{ijt} = W_{ijt} + \varepsilon_{ijt} \quad \text{and} \quad V_{ij0t} = W_{ij0t} + \varepsilon_{ij0t}.$$

We can define W_{ijt} and W_{ij0t} as a function of state variables as follows:

$$W_{ijt}(S_t) = \alpha_j + \sum_{\tau=t}^{T_j} \gamma^{\tau-t}c + \beta_{ip}p_{jt} + \beta_{im}d_{jt} + \beta_{is}s_t + \xi_{jt}, \quad (9)$$

$$W_{ij0t}(S_t) = \gamma E[\lambda_{j,t+1} \max\{V_{ij,t+1}(S_{t+1}),$$
$$V_{ij0,t+1}(S_{t+1})\} \mid S_t]. \quad (10)$$

Note that in Equation (10), following Rust (1987), the calculation of the expectation with respect to the distribution of future variables unknown to the consumer can be simplified. The integration with respect to the extreme value errors can be done analytically, and $W_{ij0t}(Z_t)$ can be expressed by the following equation:

$$W_{ij0t}(Z_t) = \gamma \int \lambda_{j,t+1} \ln \{ \exp[W_{ij,t+1}(Z_{t+1})]$$
$$+ \exp[W_{ij0,t+1}(Z_{t+1})]\} dF(Z_{t+1} \mid Z_t). \quad (11)$$

Equations (8)–(11) define W_{ijt} and W_{ij0t} as a function of state variables. Because we have assumed that the unobserved error terms in Equations (7) and (8) follow an i.i.d. extreme value distribution, the individual-level unconditional purchase probabilities have the following logit form:

$$P_{ijt} = \frac{\exp(W_{ijt})}{\exp(W_{ij0t}) + \exp(W_{ijt})}. \quad (12)$$

The aggregate purchase probability (market share) for each product j and period t is calculated by integrating P_{ijt} over the consumer heterogeneity distribution. Before specifying the aggregate purchase probabilities, we will discuss our approach in modeling consumer heterogeneity and the evolution of heterogeneity.

4.3. Consumer Heterogeneity and Market Shares

We model consumer heterogeneity using a random coefficients approach. We use a discrete approximation to the parameter distribution, and our method is an aggregate analog of the latent-class models widely used for individual-level data (Kamakura and Russel 1989). We assume there are K segments in the population and consumers in segment k ($k = 1, \ldots, K$) share the common parameters β_k, where β_k is a vector consisting of the price sensitivity parameter (β_{kp}), the markdown sensitivity parameter (β_{km}), and the seasonality parameter (β_{ks}); that is, $\beta_k = (\beta_{kp}, \beta_{km}, \beta_{ks})$. These parameters are common across all products. The initial size of segment k, i.e., the proportion of consumers who belong to segment k in the potential market, is represented by π_k and $\sum_{k=1}^{K} \pi_k = 1$. As segments are allowed to be heterogeneous in their

response parameters, they would exhibit different adoption patterns, and segment sizes would change over time within the season. Segments with lower price sensitivities would adopt earlier, and the proportion of consumers belonging to these segments in the remaining market would fall over time.

Let M_{j0} be equal to the market size for product j, i.e., the number of potential consumers that are in the market for product j. Define N_{jkt} to be the number of remaining consumers from segment k in the market for product j at any period t. N_{kjt} is determined by the total market size M_{j0}, segment size π_k, and the proportion of consumers from segment k who have not bought the item at any period before t. Then, the evolution of N_{kjt} in the market is given by

$$N_{kjt} = N_{kj,t-1}(1 - P_{kj,t-1}) \quad \text{or} \quad N_{kjt} = M_{j0}\pi_k \prod_{l=1}^{t-1}(1 - P_{kjl}).$$

If we define θ_{kjt} as the size of segment k in the market for product j at time period t, θ_{kjt} can be calculated as follows:

$$\theta_{kjt} = \frac{N_{kjt}}{\sum_{m=1}^{K} N_{mjt}} = \frac{\pi_k \prod_{l=1}^{t-1}(1 - P_{kjl})}{\sum_{m=1}^{K} \pi_m \prod_{l=1}^{t-1}(1 - P_{mjl})}.$$

Aggregating over the heterogeneity distribution, market share for product j at time t, MS_{jt} can be calculated as follows:

$$MS_{jt} = \sum_{k=1}^{K} \theta_{kjt}P_{kjt} = \sum_{k=1}^{K} \theta_{kjt} \frac{\exp(W_{kjt})}{\exp(W_{kj0t} + W_{kjt})}. \quad (13)$$

Now that we have explained the calculation of market shares, we will next discuss the strategy employed to estimate the model parameters.

4.4. Estimation Strategy

The model parameters to be estimated consist of product fixed effects represented by the vector α, $\alpha = (\alpha_1, \ldots, \alpha_J)$; the per-period consumption parameter c; segment-specific parameters represented by the vector β, $\beta = (\beta_1, \ldots, \beta_K)$, where $\beta_k = (\beta_{kp}, \beta_{km}, \beta_{ks})$; standard deviation of the mean zero normally distributed demand shocks σ_ξ; and the initial segment sizes $\pi = (\pi_2, \ldots, \pi_K)$. Call the set of all model parameters Ω, where $\Omega = (\alpha, c, \beta, \pi, \sigma_\xi)$.

We do not estimate the discount factor γ. It has been noted that it is extremely difficult to identify the discount factor in dynamic decision models (Gowrisankaran and Rysman 2011, Magnac and Thesmar 2002), and it is common practice to set the discount factor to a reasonable predetermined value. Following Gowrisankaran and Rysman (2011) and Song and Chintagunta (2003), who set the monthly discount factor to 0.99, we set the weekly discount factor to 0.9975, which is equivalent to a monthly discount of 0.99. Define $X_{jt} = (p_{jt}, d_{jt}, s_t)$ as the set

of covariates. Let $\delta_{jt} = \alpha_j + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + X_{jt}\beta_1 + \xi_{jt}$ denote segment 1's mean utility for product j at time t. Also, let $\bar{\beta}_k = (\beta_k - \beta_1)$ denote segment k's parameter difference relative to segment 1 for $k = 2, \ldots, K$. Using this notation, we can now rewrite the share Equation (13) as

$$MS_{jt} = \theta_{1jt} \frac{\exp(\delta_{jt})}{\exp(W_{1j0t}) + \exp(\delta_{jt})}$$
$$+ \sum_{k=2}^{K} \theta_{kjt} \frac{\exp(X_{jt}\bar{\beta}_k + \delta_{jt})}{\exp(W_{kj0t}) + \exp(X_{jt}\bar{\beta}_k + \delta_{jt})}. \quad (14)$$

W_{kj0t}, the observable part of the value from waiting for segment k, product j, and time period t, is a function of $(\delta_{j,t+1}, \ldots, \delta_{jT})$, observed covariates, and model parameters.

4.4.1. Calculation of Value from Waiting. Next, we will discuss how one can compute the value from waiting in period t, W_{kj0t}, starting from period T_j and working backwards for $t = T_j - 1, T_j - 2, \ldots, 1$ and $k = 2, \ldots, K$. Note that $\bar{\beta}_k = 0$ for segment 1. Consumers are uncertain about the evolution of prices, availability levels, and the time- and period-specific demand shocks through the season conditional on the current information. Therefore, integration will be performed over the distribution of future prices, availability levels, and demand shocks given the current state of the world. We can rewrite Equation (11) as

$$W_{kj0t} = \gamma \int \lambda_{j,t+1} \ln \big\{ \exp[\delta_{j,t+1} + X_{j,t+1}\bar{\beta}_k]$$
$$+ \exp[W_{kj0,t+1}(Z_{t+1})] \big\} dF(Z_{t+1} \mid Z_t). \quad (15)$$

The value from waiting is calculated by simulated integration of (15). Recall that we have earlier defined the process consumers use to form their expectations about future prices and availability given the current state of the world. We also assume that consumers know the distribution of demand shocks.

To integrate over price expectations, we simulate price paths conditional on the current state. Let us first define an expected markdown path for product j in period t as $\Delta_{jt} = (Y_{t+1}, Y_{t+2}, \ldots, Y_T)$, where Y_t equals 1 if a new markdown takes place in period t and 0 otherwise. If we are two periods away from the last period, for example, there are four expected markdown paths. We might face another markdown the next period $(1, 0)$ or in the last period $(0, 1)$, we might face one markdown in each period $(1, 1)$, or we might not face a markdown at all $(0, 0)$. Suppose there are M such possible paths, Δ_{jt}^m, $m = 1, \ldots M$, in period t.[4] Using Equations (2) and (3), we can

calculate the probability associated with each path conditional on the current state of the world, $P(\Delta_{jt}^m \mid S_t)$. Given all possible future markdown paths, we draw N random vectors, E_n, $n = 1 \ldots N$, each consisting of $(T_j - t)$ error terms corresponding to Equation (4) that defines markdown depth for a possible markdown in each period until the end of the season; $E_n = (\varepsilon_{j,t+1}^n, \varepsilon_{j,t+2}^n, \ldots, \varepsilon_{jT}^n)$. Error terms in these random vectors are i.i.d. normal with mean zero and variance σ_{MD}^2. As mentioned in §3.4.1, σ_{MD}^2 is estimated from the data together with other coefficients in Equation (4) in the first stage. For each random vector, there is a corresponding markdown depth vector, $MD_{jt}^n(S_t) = (MD_{j,t+1}^n, MD_{j,t+2}^n, \ldots, MD_{j,T_j}^n)$. The combination of a possible markdown path and a simulated vector of markdown depths corresponds to a simulated price path.

To integrate over availability expectations, on the other hand, we simulate availability paths conditional on the current state. For each time period t and product j, we draw L random vectors consisting of $(T_j - t)$ error terms, $\Psi_l = (e_{j,t}^l, e_{j,t+1}^l, \ldots, e_{j,T_j-1}^l)$. Error terms in these random vectors are i.i.d. normal with mean zero and variance σ_λ^2. As mentioned in §3.4.2, σ_λ^2 is estimated from data together with other coefficients in Equation (5) in the first stage. For each random vector, there is a corresponding future availability vector $\Lambda_{jt}^l(Z_t) = (\lambda_{j,t+1}^l, \lambda_{j,t+2}^l, \ldots, \lambda_{j,T}^l)$. To integrate over the time- and product-specific demand shocks, ξ_{jt}, on the other hand, we use the estimate of σ_ξ^2 from the latest iteration and simulate from the distibution of the demand shocks. Remember we have assumed that the demand shocks are distributed i.i.d. normal; $\xi_{jt} \sim N(0, \sigma_\xi^2)$. For integration, we draw R random demand shock vectors. W_{kj0t} is calculated by averaging the value of the integrand over N random availability vectors, L random markdown depth vectors, R random demand shock vectors, and M markdown paths in two steps. In the first step, we calculate the value from waiting corresponding to an availability vector, a markdown depth vector, and a demand shock vector as can be seen in Equation (16) below. To calculate the value from waiting in any period t, we make use of the finite horizon nature of the problem, start at the final period, period T_j, and calculate the value from waiting at each time period backwards recursively. The value from waiting in the last period is normalized to zero, $W_{kj0T_j}^{lmnr} = 0$:

$$W_{kj0t}^{\ln r}(Z_t) = \gamma \sum_{m=1}^{M} P(\Delta_{jt}^m)[\lambda_{j,t+1}^l \ln \big\{ \exp[\delta_{j,t+1}^{mn} + X_{j,t+1}^{mn}\bar{\beta}_k]$$
$$+ \exp[W_{kj0,t+1}^{lmnr}] \big\} \mid \Lambda_{jt}^l(Z_t), MD_{jt}^n(Z_t), \xi_{jt}^r, Z_t].$$
$$(16)$$

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[4] To reduce the dimensionality of the problem, we assume a maximum of two markdowns for each product during a season. This is a reasonable assumption because 99.4% of all sale revenue comes from sales before the third markdown.

In the second step, we compute the value from waiting for each product and time period by averaging the values from the first step for the L markdown depth, N availability, and R demand shock vectors:

$$W_{kj0t}(S_t) = \frac{1}{L}\frac{1}{N}\frac{1}{R}\sum_{l=1}^{L}\sum_{n=1}^{N}\sum_{r=1}^{R} W_{kj0t}^{\ln r}(Z_t). \quad (17)$$

4.4.2. Estimation Algorithm. Now that we have completed the discussion of how we compute the market shares for each product and each period, we will next discuss the estimation algorithm. Our estimation algorithm nests a finite horizon dynamic programming solution to the optimal stopping problem within a Berry (1994)-style fixed-point calculation for demand estimation. Remember that we assume time- and product-specific demand shocks are distributed i.i.d. normal; $\xi_{jt} \sim N(0, \sigma_\xi^2)$.

The estimation algorithm can be summarized in four levels of loops:

1. In the outermost loop, we search over nonlinear parameter values ($\bar{\beta}_k$, parameter differences of other segments relative to segment 1; and π_k, relative segment sizes, $k = (2, \ldots, K)$). The objective function we optimize over is explained in the third step.

2. In the second loop, given a value of the unknown nonlinear parameters $\bar{\beta}_k$ and π_k, mean valuations δ_{jt} that equate the observed market shares to the predicted market shares (these come from the fourth loop) are computed by inverting (14). Inversion is made using the contraction-mapping algorithm suggested by Berry (1994), since the function cannot be inverted analytically. Output of this loop is the corresponding mean valuations δ_{jt}, given a fixed value of the nonlinear parameters.

3. In the third loop, given mean valuations δ_{jt}, linear parameters (β_1, α_j, c) (response parameters of the first segment, product fixed effects, and per-period consumption parameter, respectively) are estimated, and the implied demand shocks ξ_{jt} are calculated using the equality $\delta_{jt} = \alpha_j + \sum_{\tau=t}^{T_j} \gamma^{\tau-t} c + X_{jt}\beta_1 + \xi_{jt}$. Then we maximize the likelihood for the observed data generated by the demand distribution implied by the distribution of the demand shocks ξ_{jt}.[5] Outputs of this loop are (β_1, α_j, c) and the objective function value given a fixed value of nonlinear parameters and the corresponding mean valuations.

4. In the innermost (fourth) loop, given a value of the nonlinear parameters and the corresponding mean valuations δ_{jt}, the predicted market shares are calculated by solving for the consumers' dynamic optimization problem. As discussed in §4.4.1, this step requires the calculation of value from waiting for each segment for each remaining time period starting from the last period and working backward. Given the value from waiting in each time period, calculation of predicted market shares follows the discussion in §4.3. Output of this loop is the predicted market shares given a fixed value of the nonlinear parameters and the corresponding mean valuation.

Our estimation strategy is similar to those used in the recent literature for estimating aggregate discrete choice demand models of differentiated goods. We include dynamics by nesting the innermost loop that is used for solving the consumers' dynamic optimization problem in a finite horizon setting. We also allow for a discrete heterogeneity distribution. Interested readers are referred to Berry et al. (1995) and Nevo (2000) for a more detailed discussion of a related estimation strategy using the generalized method of moments (GMM) framework in a static setting.

4.5. Market Size
To estimate demand, we need to have a measure of the initial (potential) market size for each SKU. Knowledge of the initial market size allows us to calculate the observed market share of purchasers (and nonpurchasers) using sales data every time period. The previous literature has reported two important considerations in defining the initial market size (Nevo 2000). The first consideration is to allow for a nonzero share of the outside good, and the second consideration is to check the sensitivity of the results to the market definition. Unlike the retailer, who has information on the market potential of each product, the researcher does not have data on market potential and needs to infer the size of the potential market from the data. Previous studies define the market size by choosing a variable to which the market size is proportional and choosing the proportionality factor. Nevo (2000) calculates the size of the market for ready-to-eat cereal to be one serving of cereal per capita per day. Bresnahan et al. (1997) define the market for computers to be the total number of office-based employees.

In our case, because the retailer places his order for a specific product before the product is introduced, this order size provides us information on the retailer's beliefs about the sales potential for that product. We chose the order size as the variable to which the market potential is proportional and selected a proportionality factor of 1.25 to allow for a

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[5] The likelihood as a function of demand parameters is $l(\beta) = \prod_{t=1}^{T} f_\xi(D_t^{-1}(q_t; \Omega))\|J\|$, where D_t is the aggregate demand function, f_ξ is the probability density function for ξ, and $\|J\|$ is the Jacobian; $\|J\| = \|\partial D_t^{-1}(q_t; \Omega)/\partial q_t\| = |\partial \xi_{jt}/\partial q_{jt}|$. We can define $G = \sum_{k=1}^{K} N_{kjt}P_{kjt}(q_t; \Omega) - q_{jt} = 0$ and calculate the Jacobian using the implicit function theorem $\|J\| = |-(\partial G/\partial q_{jt})/(\partial G/\partial \xi_{jt})| = |-1/\sum_{k=1}^{K} N_{kjt}P_{kjt}(q_t; \Omega)[1 - P_{kjt}(q_t; \Omega)]|$. The derivation follows a similar derivation by Nair (2007).

nonzero share of the outside good. Our demand estimates are not sensitive to the proportionality factor. When we increase the proportionality factor, the only change that we observe is an increase in the size of the more price-sensitive segment, and the increase in the size of this segment comes from additional customers who do not purchase the product by the end of the season.

4.6. Identification
Our model is a dynamic logit model estimated with aggregate data using the Berry, Levinsohn, and Pakes contraction-mapping approach (BLP; see Berry et al. 1995) combined with MLE. The reader is referred to Nevo (2000) for a detailed discussion of the identification of the random coefficients multinomial logit model in a static setting and to Gowrisankaran and Rysman (2011) for the identification of the same model in a dynamic setting. Because we use a binomial logit specification, the identification of our model directly follows that of Nair (2007), who discusses identification of the binomial logit model in a dynamic setting.

We can summarize the identification of our model parameters under three headings. Product fixed effects are identified from the variation in mean level of sales across different coats. Price, per-period consumption, markdown, and seasonality parameters are identified from the within-coat variation in these characteristics over the coat's season. The change in market share of product j associated with a change in a characteristic of j (e.g., price) over time identifies the (first segment's) parameter associated with that characteristic. Heterogeneity, on the other hand, is identified from the deviations from the standard logit implied elasticities. Without heterogeneity, our model is a standard binomial logit model which implies own elasticities are proportional to the outside good's market share. If our consumer population was homogeneous—i.e., we had only one segment—when the elasticity varies over time, it would do so in a way that it stays proportional to the outside good's market share.[6] A number of studies have also provided

simulation-based evidence on identification of heterogeneity from aggregate data with logit demand models in static and dynamic settings (Chintagunta 2003, Song and Chintagunta 2003).

We find that a two-segment specification fits the data best. This results in an estimation of a total of 70 parameters. Remember that we represent the set of all model parameters with Ω, where $\Omega = (\alpha, c, \beta, \pi, \sigma_\xi)$. First, we have a set of 61 fixed effect parameters, one for each product (the α_j).[7] Second, we have the per-period consumption parameter, c. Third, we have β. We estimate a price sensitivity parameter, a markdown sensitivity, and a seasonality parameter for each segment, totaling six parameters. Last, we estimate a segment size parameter that informs us about the relative size of the first segment in the initial potential market, π, and the standard deviation of the mean zero normal distributed demand shocks, σ_ξ.

4.7. Competition and Endogeneity
In this section, we discuss three key modeling assumptions and address potential concerns related to these assumptions. The first assumption concerns cross-demand effects from products within the same category, the second one relates to the impact of competition from retailers selling similar products, and the third relates to the concern for price endogeneity.

This study treats each product as a separate market and does not consider demand effects across different coats. We resort to this assumption because of the computational complexity of including cross-product demand effects in a dynamic model that already incorporates the consideration of future price and availability expectations by forward-looking consumers. One might expect substitution effects not to be large in "fashion" categories such as women's coats that are associated with greater use of colors, prints, and unique designs (Pashigian 1988). However, there is still concern for competition between different colors and or models of coats within a store. To address these concerns, we resort to aggregating across similar SKUs that are likely to compete with each other. To be more specific, we aggregate SKUs for different colors of the same style and SKUs for different styles of coats that are sold at the same price within a season. Details of this product aggregation are provided in §5. This aggregation across similar SKUs allows for substitution between close alternatives. Aggregation reduces the number of products analyzed from 105 to 61.

Our model also does not explicitly account for the effect of competitors' (other coat sellers') within-season pricing decisions on demand. One

[6] Consider two periods, t_1 and t_2, with price levels p_1 and p_2 and 1% change in price in both periods, holding everything else constant. If our consumer population was homogeneous, these changes would have resulted in percentage changes in demand proportional to the outside good's market share. Let the outside good's market share in period $t1$ and $t2$ be OS1 and OS2, and let the percentage change in demand be PCD1 and PCD2. We should have the equality $PCD1/PCD2 = (p1 * OS1)/(p2 * OS2)$. But instead, if we observe a change in demand different from the change implied by the logit elasticity, this would indicate the existence of a second consumer segment that is different from the first in price sensitivity, and the extent of the deviation helps to identify the extent of the difference in the two segments' price sensitivities. See Nair (2007) for a similar example on how rate of change in market shares helps identify the relative sizes of the segments.

[7] Remember that we aggregated 105 SKUs to 61 products, as explained in §4.7.

big limitation we face is data availability. Whereas company-level data on sales and prices are more readily available, data from multiple competitors are hard to obtain, not only for academicians but also for practitioners. There are no data vendors who track clothing sales such as Nielsen or IRI for grocery products or IMS for pharmaceutical products. Not being able to account for the impact of interstore competition in markdown prices on demand is a limitation of this study. If competitors respond to markdowns with price decreases on similar products, normalizing utility from the outside option to zero over the season might lead to underestimation of the price sensitivity parameter and/or the markdown sensitivity parameter depending on the timing of the competitive markdowns.[8] Within the context of this study, though, one might expect competitive effects not to be very pronounced. While retail prices and temporary promotions (e.g., Mother's Day sale) are set before the season starts taking competitors' strategy into account, decisions regarding the timing and depth of markdowns are often operational made dynamically during the short season, taking into account the inventory and time left until the end of the season (Bitran and Mondschein 1997). Also, because different products are discounted at different points in time, markdowns are rarely advertised (Bitran and Mondschein 1997), and competitive reactions seem less likely for unadvertised markdowns (Smith and Achabal 1998).

There might also be concern for potential endogeneity between product- and time-specific demand shocks and prices. This concern would arise if the retailer observes the product- and time-specific demand shocks (ξ_{jt}) and takes them into consideration in his pricing decisions. In the absence of appropriate instruments, we leave handling of endogeneity to future research.[9]

5. The Data

Main distribution channels for the apparel industry in the United States are specialty apparel stores and department stores. Specialty apparel retailers sell their own brands through multiple stores across the country mostly located within shopping malls. In 2007, specialty store sales for clothing and clothing accessories added up to $222 billion, and department store sales accounted for $209 billion (U.S. Census Bureau 2011). In the market for apparel, specialty retailers have been gaining market share from the department stores. Some examples of prominent specialty apparel retailers are Polo Ralph Lauren, Liz Claiborne, Ann Taylor, Chico's, the Limited, Gap, Banana Republic, Old Navy, Gymboree, and the Children's Place. In addition to a brand name to communicate to customers, specialty apparel retailers have the advantages of specialization and closer customer contact. Specialization results in more effective targeting and positioning, whereas close contact with the consumers enables the retailer to respond faster to consumer needs. Specialty apparel retailers also often offer superior service levels compared with department stores.

The data used in our analysis come from a leading specialty apparel retailer that sells its own private label fashion apparel across hundreds of stores throughout the United States. The data consist of weekly sales, revenue, and starting inventory levels, as well as unit acquisition costs at the SKU level. The data cover a two-year period including the years 2003 and 2004.

We estimate the model using data from the "Women's coats" category. We believe this category is suitable for the purposes of this study because the products in this category are sold typically over a season, it is a high involvement category, consumer purchase frequency is low, and repeat purchases from the same consumer, especially for the same product, are unlikely.

We excluded data for about 30 SKUs for which we did not observe the whole sales cycle and ended up with 105 SKUs from this category. Most coat styles are offered in a number of different colors (two to four colors). Different colors of the same style are separate SKUs but share the same retail price and are put on shelves within a few weeks of each other. Also, markdowns for different colors are usually bunched together; i.e., they are marked down at the same period and the markdown magnitude is the same. This is almost always true for the first markdown. So price differences across colors are negligible over the season in most cases. Furthermore, there are also some instances where two different styles are introduced within a few weeks of each other; they are sold at the same retail price and are marked down to the same price. Because it is likely that different colors of the same style and styles offered at the same retail price and marked down together (to the same price) compete with each other, we decided to

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[8] This is because the observed demand increase in response to a markdown would be smaller in the presence of competitive markdowns. If we fail to account for competitive markdowns when they are present, we would attribute the small demand response to low price sensitivity and/or markdown sensitivity.

[9] An earlier version of this model was estimated in a GMM framework using average men's coats prices from the same period as instruments for women's coats prices. The instruments produced an incorrect negative sign for the markdown sensitivity parameter in one of the segments. The problem is that men's coat prices may not be very appropriate instruments becauuse men's coat prices and women's coat prices are likely to be subject to some common demand factors. The model implications, however, with and without the instruments were very similar. So we removed the instruments in light of the feedback from the reviewers.

aggregate across such SKUs. This aggregation allows us to lessen the concern for intrastore competition in our model.[10] To calculate the price of the aggregate product, we average prices across all aggregated SKUs.[11] To calculate the initial market size of the aggregate product, we add the initial market sizes of the aggregated SKUs (as explained in §4.5, we basically multiply the retailer's initial order size by the proportionality factor, 1.25, to calculate the market size for each SKU). The size of the remaining potential market is calculated each period by subtracting the sum of sales for all aggregated SKUs from the aggregate remaining potential market size at the end of the previous period. To calculate the aggregate availability level, on the other hand, we take the maximum of availability levels across the aggregated SKUs. The reason is that not all colors are ordered in equal amounts, but the colors with less stock are distributed to the stores that carry the more popular colors. So the distribution of the aggregate product is successfully captured by the highest distribution observed across SKUs. There might be some exceptions at the very end of the sales period, when only very few items are left in the stores, but sales at the end of the season are very small, so it should not impact our results. This aggregation reduces the number of products analyzed from 105 to 61. After aggregation, we have, on average, nine products sold together at the same time by the retailer (reduced from 15), and there is substantial variation in these products in retail price levels (e.g., a $99 coat, a $199 coat, and a $269 coat) and in markdown price levels.

We observe each product through its season (life cycle). Products were introduced and discontinued at different times during the two-year observation period. So different products have different seasons and different season lengths. The season length varies from 11 to 30 weeks, with a median season length of 19 weeks. Each observation corresponds to a product–week combination, and we have a total of 1,178 observations.

One important observation is that there is significant variation in total sales across products as well as the number and the depth of markdowns the product faces during its sales cycle. Total sales range from 834 units to 43,222 units, and the median is 8,166 units. Retail (initial) price range from $100 to $350, and the median retail price is $200. In our sample, all products face at least one markdown, the maximum number of markdowns is five, and the median

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Table 1   Total Revenue and Quantity Sold at Different Relative Price Points

Relative price (%)	Revenue (%)	Quantity sold (%)
100	57	43
80–99	2	2
60–79	16	16
40–59	20	29
<40	5	9
Total	100	100

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number of markdowns is three. The average first markdown is 38% of the retail price, and the average second markdown is an additional 21% of the retail price. Table 1 summarizes total revenue and quantity sold at relative price points (price as a percentage of the retail price). We can see that in our sample, only 43% of the quantity sold and 57% of the revenue from sales was at full (retail) price over the two-year period; 45% of the quantity sold and 36% of the revenue correspond to sales that took place when the price relative to retail price was in the range of 40% to 80%.

Looking at the prices and sales over time, one can easily see important patterns. Figure 1 plots unit sales and prices over time for a sample product. This product consists of a single SKU and is offered for sale in period 1 at a retail price of $200. Sales start around 350 units and fall down quickly until the first markdown. The first markdown occurs in period 12 and is around 60%, and sales make a significant jump at that period, increasing 15-fold compared to the period before the markdown. Following the first markdown, sales fall down even more quickly until the second markdown in period 19. The second markdown is around an extra 15% of the retail price, and this time sales increase by 80%. After the second markdown, sales decrease quickly for a few periods and then die slowly.

We observe significant response to price changes in the early periods, but sales drop very quickly over time at a given price. The drop in sales over time at a given price can be the result of decrease in useful life of the product (i.e., consumers prefer to purchase earlier than later at a given price) and/or shrinkage in the potential market size as well as limited availability. The spikes, on the other hand, could be the result of dramatic promotion response, strategic consumer waiting, or capturing different segments of customers. Let us focus on the period before the first markdown. There might be at least two different explanations behind the sales decrease before the first markdown. One explanation might be that strategic consumers are familiar with the retailer's discounting pattern and are delaying their purchases to take advantage of lower prices. This is similar to the

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[10] We thank an anonymous reviewer for this suggestion of aggregating across SKUs to handle intrastore competition

[11] Notice that although the shelf prices are the same across aggregated SKUs, there may be minor differences in observed average selling prices (because of individual-level coupon use, etc.). We average across the observed average selling prices.

Figure 1   Prices and Sales for a Sample Product
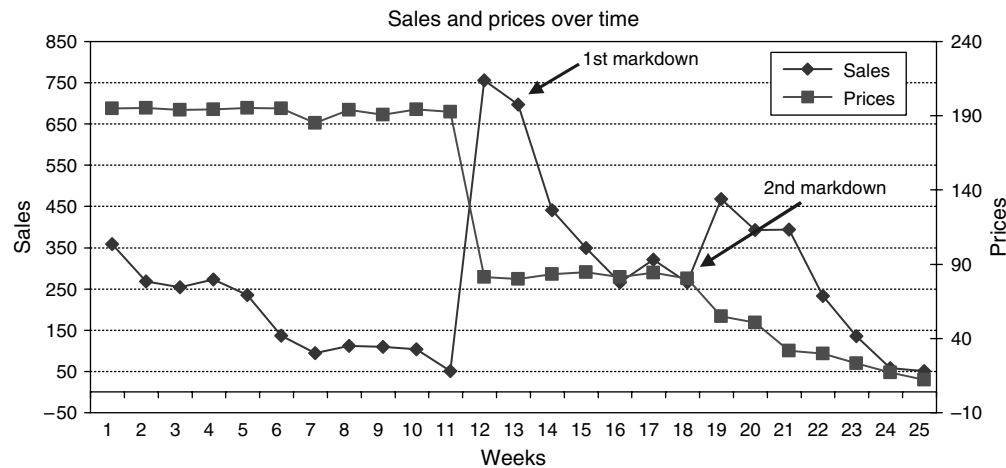
prepromotion dip documented in the marketing literature in the CPG context (e.g., Van Heerde et al. 2000). Another explanation might be that the retailer faces different consumer segments with different levels of price sensitivities. This first drop in sales might mean that the retailer has only a small segment of low price sensitivity consumers, and this segment is saturated early in the season and the retailer needs to lower prices to capture demand from more price-sensitive consumers. This emphasizes the importance of a model like ours, because understanding the reasons behind these patterns of observed demand is very important for the retailer's policy. So the retailer wants to know: Do consumers strategically wait for markdowns? To what extent does strategic waiting explain the observed demand peak? And can limiting availability help to dampen the effect of strategic waiting by creating urgency in consumers?

6. Empirical Results

6.1. Parameter Estimates

6.1.1. Price Process Parameter Estimates.
Table 2a reports the maximum likelihood estimates of the parameters of the markdown probability model (conditional on no previous markdown) specified in Equation (2).

Table 2b, on the other hand, reports the markdown probability model parameters (conditional on

Table 2a   Markdown Probability Model (Conditional on No Previous MD) Parameter Estimates

Parameter	Symbol	Estimate	S.E.
Constant	$a0$	−6.2187	0.5809
Retail price	$a1$	0.0035	0.0020
Time in season (week)	$a2$	0.4315	0.0398

Note. Number of observations = 1,054; log likelihood = −256.4.

Table 2b   Markdown Probability Process (Conditional on Previous MD) Parameter Estimates

Parameter	Symbol	Estimate	S.E.
Constant	$b0$	−3.2683	0.6151
Retail price	$b1$	0.0039	0.0017
Time in season (week)	$b2$	0.0590	0.0300

Note. Number of observations = 799; log likelihood = −347.74.

Table 2c   Markdown Depth Process Parameter Estimates

Parameter	Symbol	Estimate	Standard error
Intercept	$\theta0$	−0.6102	0.4554
ln(Retail price)	$\theta1$	0.9201	0.0868
Markdown dummy	$\theta2$	−0.7225	0.0566

Notes. R^2 = 0.535. Number of observations = 233; mean squared error = 0.184.

a previous markdown) specified in Equation (3). The estimates indicate that the probability of markdown conditional on a previous markdown is positively related to the product's retail price. The probability of markdown is also positively related to the time in the season (number of weeks since product j has been introduced) both conditional on no previous markdown and conditional on a previous markdown.

Table 2c reports the OLS estimates of the parameters of the markdown depth model specified in Equation (4). The estimates indicate that the natural logarithm of markdown depth is positively related to the natural logarithm of the product's retail price and negatively related to the markdown dummy. Products with higher retail prices face deeper markdowns compared to products with lower retail prices and first markdowns are deeper than later markdowns.

6.1.2. Availability Process Parameter Estimates.
Table 3 reports the OLS estimates of the parameters of the availability expectations process specified in Equation (5). The estimates indicate that availability

Table 3 Availability Expectations Process Parameter Estimates

Parameter	Symbol	Estimate	S.E.
Current period availability	γ_0	0.9360	0.0078
Retail price	γ_1	0.0003	0.0000
Time in season	γ_2	−0.0039	0.0003

Notes. $R^2 = 0.947$. Number of observations $= 1,827$; mean squared error $= 0.012$.

in the next period is closely related to the availability in the current period. The retail price parameter is significant and has a positive sign, meaning the next-period availability is high when the retail price is high, and vice versa; i.e., products with higher retail prices are sold at a slower rate. Time in season (number of periods since the beginning of the season) is significant and has a negative sign, meaning availability falls over time.

6.1.3. Demand Model Parameter Estimates. Table 4 reports the MLE estimates of demand model parameters for a two-segment specification. Following Besanko et al. (2003), we determine the number of segments by adding segments until one of the segment size parameter estimates is not statistically different from zero. The estimates for the three-segment specification yield an insignificant segment size parameter for the third segment. Thus the data identify two segments. As discussed in §4, we estimate the demand parameters for the first segment and deviations of the second segment's parameters from those of the first segment. Segment 2 parameters reported in Table 4 are calculated using these estimates, and the standard errors are adjusted accordingly. We also estimate 61 product fixed effects that are not reported here. Product fixed effect estimates lie in the range $(-6.3, -2.8)$, with an average of -5.0. Our demand estimates reflect a larger, less price-sensitive segment, segment 1, and a smaller, more price-sensitive segment, segment 2. We call the first segment the *fashion-sensitive* segment and the second segment the *price-sensitive* segment. Price sensitivity parameters for both segments have the expected negative sign (not significant for segment 1). Segment 1 corresponds to 80% of the total potential market at the beginning of the season, and the estimated price sensitivity parameter for this segment is -0.001. The estimated price sensitivity parameter for segment 2 is -0.027. Mere markdown effect (markdown sensitivity) is positive and significant for both segment 1 and segment 2 (2.095 and 1.903, respectively). The seasonality parameter is positive and significant for both segments (1.199 for segment 1 and 1.140 for segment 2). This indicates that both segments get extra utility from purchases in the six-week holiday shopping period.

Table 4 Demand Estimates

	Segment 1		Segment 2	
Parameter estimates	Estimate	S.E.	Estimate	S.E.
Price sensitivity	−0.001	0.001	−0.027	0.001
Per-period consumption utility	0.148	0.008	0.148	0.008
Markdown sensitivity	2.095	0.108	1.903	0.108
Seasonality	1.199	0.072	1.140	0.072
Segment size parameter $(\ln((1-\theta)/\theta))$	−1.414	0.008		
Standard deviation of demand shocks	0.994	0.034		

Notes. Number of observations $= 1,178$; log likelihood $= -5,751.1$. Size of segment 1 $= 80.4\%$.

We have seen that the fashion-sensitive segment accounts for a significant portion of the potential market at the beginning of the season. As consumers in this segment represent a relatively less price-sensitive group, they make purchases and exit the market early in the season. As a result, relative sizes of the two segments in the remaining potential market change dramatically over time. Averaging across all products, size of the fashion-sensitive segment reduces from 80% in period 1 to 37% in period 25.

To understand the purchase behavior of the two segments, we investigate the simulated sales for a sample product. Figure 2 represents the sales simulated using the demand estimates from our model for both segments across time for the sample product. The fashion-sensitive segment (segment 1) consumers start purchasing early in the season, and some consumers of this segment take advantage of the markdowns. The price-sensitive segment (segment 2) consumers, on the other hand, start purchasing later in the season. Simulated sales for the two segments show that although segment 2 customers account for an important portion of sales at the end of the season and are important in clearing the retailer's shelves for the next season, they do not start buying until late in the season.

6.2. Model Comparison

To demonstrate the importance of accounting for consumers' availability expectations and the change in total utility from consumption over time, we present estimates from a benchmark model and compare our model to this model. The benchmark model is the state-of-the-art model in the current literature. Similar to Nair (2007), it accounts for forward-looking behavior (where consumers have expectations about future prices) and consumer heterogeneity in response parameters, but it does not account for consumers' availability expectations and does not consider the change in total utility from consumption over the season as a result of the reduction in the product's

Figure 2   Simulated Sales for a Sample Product

Simulated sales for a sample SKU
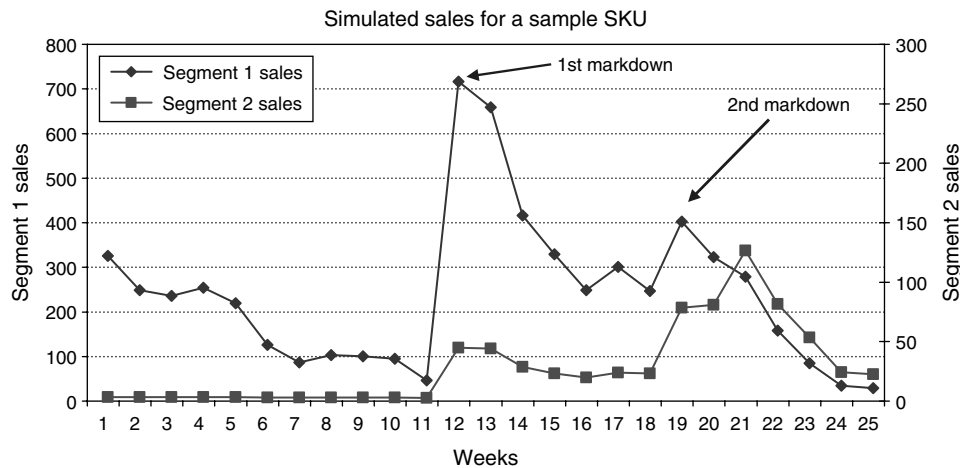
remaining useful life. Our model, on the other hand, takes both of these considerations into account.

In the benchmark model, consumers do not take stockout risk into account, so $\hat{\lambda}_{j,t+1}$ in Equation (8) is set to 1 for all products and time periods. The benchmark model also assumes that total utility from consumption of the product does not depend on the time of purchase, so the per-period consumption utility c is set to 0 for all segments and time periods, and total discounted consumption utility is captured by the product fixed effect α_j. Table 5 presents the parameter estimates from this restricted model. This model produces a positive and significant price coefficient for segment 1 and a lower price sensitivity coefficient for segment 2. This is because the model ignores the impact of availability expectations and decreasing total consumption utility over time and attributes consumers' incentive to accelerate purchases (to avoid stockouts and to get the most use out of the product) to lower price sensitivity.

A likelihood ratio test comparing the benchmark model log likelihood of $-7{,}021.4$ to our model's log likelihood of $-5{,}751.1$ (see Table 4) produces a chi-square test statistic of 1,270.3, which is significant.

Table 5   Demand Estimates for the Benchmark Model

Parameter estimates	Segment 1		Segment 2	
	Estimate	S.E.	Estimate	S.E.
Price sensitivity	0.036	0.001	−0.008	0.001
Markdown sensitivity	2.190	0.124	2.183	0.124
Seasonality	0.332	0.085	0.023	0.085
Segment size parameter $(\ln((1-\theta)/\theta))$	−1.734	0.003		
Standard deviation of demand shocks	0.857	0.061		

Notes. Number of observations = 1,178. Log likelihood = −7,021.4. Size of segment 1 = 85%.

6.3.   Price Elasticities of Demand

We calculate price elasticities by first simulating the predicted sales using the observed prices, discounting the prices in a specific period by a small amount for each product, simulating the predicted sales once again for the new prices, and computing the change in sales. To isolate the price effects, we hold consumers' availability expectations constant throughout the simulations and exclude the mere markdown effect, because the price decrease in our simulation is temporary. Throughout the season, the retailer faces a less elastic demand curve from segment 1 consumers (the fashion-sensitive segment; mean price elasticity of −0.085 across all products and time periods) than from segment 2 consumers (the price-sensitive segment; mean price elasticity of −3.18). Price elasticities of demand indicate that demand from both segments becomes less responsive to price changes throughout the season. This clearly indicates that correct timing of the markdowns is very critical for the retailer. Earlier markdowns can significantly accelerate demand, as demand is more sensitive to price changes in the earlier periods. On the other hand, a higher-than-necessary early markdown can have very negative profit implications.
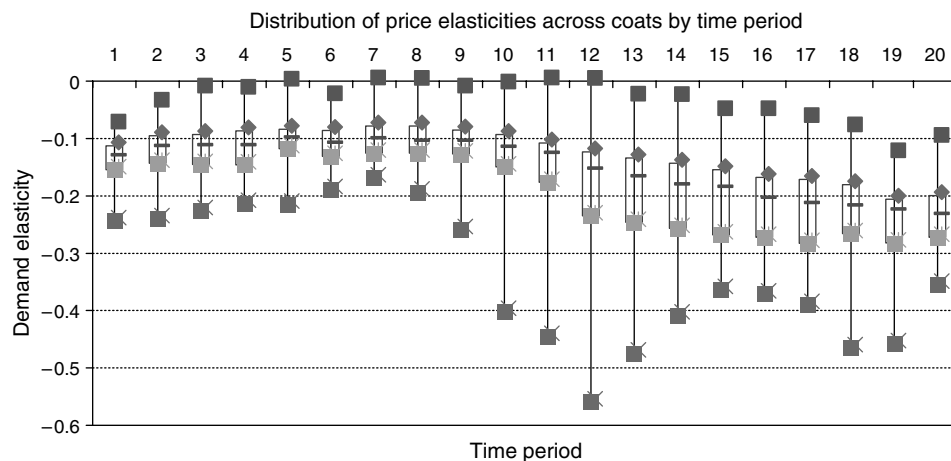
Although current period price elasticities for both segments decrease (in absolute value) over time, the overall price elasticity increases over time because the relative size of the price-sensitive segment in the remaining potential market increases over time. However, because the price-sensitive segment accounts for only a small portion of the sales, the overall price elasticity is small. A number of studies have documented the impact of changing market composition on demand and overall price elasticity (Ching 2000, Nair 2007).

Price elasticities exhibit significant variation across products and time periods. The most price-elastic product has an average price elasticity of −0.45 over

Figure 3   Distribution of Overall Price Elasticities Across Coats Over Time

Distribution of price elasticities across coats by time period

its life cycle, whereas the least price-elastic product has an average price elasticity of -0.13 over its life cycle.[12]

7.   Counterfactuals

An important strength of a structural demand model is that we can forecast how consumer behavior will change in response to fundamental changes in pricing and inventory management policy. In this section we investigate effects of two such policy changes through policy experiments and conduct a third experiment to quantify the impact of strategic consumer behavior on retailer revenues. The first experiment investigates the retailer's trade-off between the timing and depth of markdowns through a uniform single markdown policy. This experiment shows that the highest retailer profits are achieved by small markdowns offered early in the season. The second experiment studies the effect of limiting availability throughout the season and shows that, counter to intuition, the retailer can improve profits by stocking less. The third experiment quantifies the impact of strategic consumer behavior on retailer revenues. This experiment shows that the retailer's revenue is much lower under strategic consumer behavior than it would have been under myopic consumer behavior and that limited availability considerably helps to dampen the effect of strategic behavior.

7.1.   Uniform Single Markdown Policy

In this experiment we investigate the retailer's trade-off between the timing and depth of markdowns

when setting a uniform single markdown across all products. We focus on a single markdown because 82% of all sales at markdown prices take place at the first markdown price for the retailer in our application. In this policy, the seller changes the price only once during the season and sets the same percentage markdown for all products. To investigate the sales and revenue impacts of different uniform single markdown policies, we keep the initial prices and initial inventories fixed and vary the timing and depth of the markdown. We vary the depth of the markdown between 5% and 50% in 5% increments and vary the timing of the markdown between periods 1 and 25 in one-period increments. For each depth–timing combination, we simulate sales and calculate resulting total revenue. Note that revenue is the relevant metric here because the entire inventory is purchased at the beginning of the season and salvage value is zero. Table 6 summarizes revenue outcomes of each time–depth combination. Because of confidentiality concerns, results are rescaled so that the maximum table entry is 100. After calculating and normalizing the total revenues, we divide the table entries into four regions where region 1 corresponds to the top (fourth) quartile of all table entries, region 4 corresponds to the bottom (first) quartile, and so on. Each region is represented by a different shade.

Results indicate that early and deep markdowns as well as late markdowns (region 4) result in the lowest revenues. Because the market consists of mostly lower price sensitivity customers at the start of the season, marking down the prices too early and too deep does not have a big impact on sales, but the revenue loss as a result of sales at lower prices is significant. Also, because the consumers' interest in the product reduces significantly by the end of the season (because of the diminished opportunity to use the product), very late markdowns also do not produce favorable revenue outcomes. Early and small markdowns (region 1) have the most favorable

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[12] Note that the overall demand effect of the first markdown is captured both by the price parameter and the markdown parameter in our model. But in calculating price elasticities here, we exclude the mere markdown effect. Calculated price elasticities would be larger if one includes the mere markdown effect.

Table 6   Revenues from a Uniform Single Markdown Policy

Time period	Markdown percentage									
	5	10	15	20	25	30	35	40	45	50
1	100.00	95.56	90.91	86.17	81.43	76.69	71.96	67.22	62.45	57.62
2	99.97	95.65	91.08	86.41	81.70	76.99	72.28	67.56	62.81	57.99
3	99.93	95.81	91.40	86.84	82.22	77.58	72.93	68.26	63.56	58.79
4	99.83	95.97	91.75	87.35	82.85	78.31	73.75	69.17	64.55	59.86
5	99.62	96.09	92.12	87.92	83.59	79.20	74.77	70.30	65.79	61.21
6	99.26	96.14	92.49	88.53	84.38	80.12	75.80	71.43	67.00	62.51
7	98.69	96.09	92.87	89.25	85.39	81.36	77.24	73.04	68.77	64.41
8	97.87	95.83	93.11	89.91	86.41	82.70	78.84	74.89	70.84	66.70
9	96.28	94.96	92.92	90.34	87.36	84.12	80.71	77.17	73.53	69.81
10	92.98	92.49	91.32	89.56	87.30	84.65	81.73	78.61	75.33	71.91
11	89.66	89.65	89.07	87.96	86.40	84.45	82.17	79.64	76.88	73.96
12	86.79	87.08	86.88	86.25	85.22	83.83	82.13	80.15	77.92	75.49
13	84.58	85.09	85.18	84.87	84.20	83.20	81.90	80.34	78.53	76.51
14	82.04	82.65	82.89	82.78	82.34	81.60	80.62	79.41	78.01	76.41
15	79.03	79.73	80.18	80.36	80.25	79.88	79.25	78.40	77.34	76.08
16	76.84	77.52	78.04	78.34	78.43	78.30	77.94	77.38	76.63	75.70
17	75.24	75.84	76.32	76.66	76.84	76.86	76.70	76.37	75.89	75.25
18	73.79	74.34	74.80	75.16	75.40	75.52	75.49	75.34	75.04	74.60
19	72.07	72.57	73.04	73.44	73.75	73.95	74.04	74.00	73.83	73.53
20	70.85	71.26	71.66	72.03	72.36	72.62	72.80	72.88	72.87	72.74
21	69.95	70.25	70.55	70.84	71.11	71.33	71.51	71.61	71.64	71.58
22	69.37	69.59	69.83	70.06	70.29	70.50	70.68	70.82	70.90	70.92
23	68.92	69.07	69.24	69.41	69.58	69.74	69.89	70.02	70.11	70.17
24	68.50	68.60	68.69	68.80	68.90	69.01	69.11	69.19	69.27	69.32
25	68.24	68.29	68.34	68.40	68.46	68.52	68.58	68.64	68.69	68.74

revenue outcomes. The retailer in our empirical application, under a uniform single markdown strategy, should offer markdowns earlier in the season before consumers lose interest in the product. Smaller markdowns result in higher revenues, but if the business conditions and rules indicate a certain markdown percentage, markdown time should be set carefully as marking down too early as well as marking down too late can result in lower revenues.

7.2.   Change in Availability

In this section, we investigate the results of a change in the inventory policy. The initial stock ordered by the retailer at the beginning of the season is an important part of the retailer's pricing and stocking strategy. In this experiment, we simulate sales for both segments using a 5% to 30% reduction in the initial stock offered, varying the reduction in 5% increments.[13]

[13] The retailer would limit availability throughout the season by reducing the initial stock ordered because the retailer can order only once before the season starts as lead times are longer than season length. We assume customers observe availability throughout the season and form expectations about future availability relying on past experience. Also, it is very likely that store personnel inform consumers about availability.

In doing the simulations, we hold the pricing strategy constant and allow for consumers' availability expectations to adjust in accordance with the change in the initial period availability and the changes in sales in all periods. Table 7 reports the resulting change in sales, retailer revenue, and retailer profits relative to the original policy. Note that the retailer profit is the relevant measure here because the policy change involves a reduction in the initial stock ordered and changes the total cost of acquisition.

Results show that although reducing availability has a negative effect on the total quantity sold, a 5% decrease in the initial stock offered can improve retailer's profits. A slight decrease in the availability increases stockout risk and motivates consumers to

Table 7   Effects of a Change in the Initial Stock

Change compared to current policy (%)	Change in total sales (%)	Change in total revenues (%)	Change in total profits (%)
Limit initial stock by 5	−2.63	0.10	4.56
Limit initial stock by 10	−7.60	−2.83	3.45
Limit initial stock by 15	−12.71	−6.33	1.26
Limit initial stock by 20	−17.85	−10.15	−1.54
Limit initial stock by 25	−22.98	−14.20	−4.76

buy earlier at higher prices, and the profit gain from earlier sales overcomes the reduction as a result of lost sales. Reducing availability further, on the other hand, results in lower profits (compared to 5%), and a reduction of more than 15% would result in lower profits compared to the current situation.

7.3. Impact of Strategic Consumer Behavior on Retailer Revenue

We have seen that although the less price-sensitive segment accounts for a large portion of the potential market, almost half of the sales take place at markdown prices. Under the current pricing and inventory policy, some less price-sensitive consumers strategically delay their purchases to take advantage of lower prices. But how significant are these strategic purchase delays, and what is the impact of strategic consumer behavior on retailer revenue?

To quantify this effect, we keep the retailer's current price schedule and initial stock levels for all the products, use the demand estimates from our model, and simulate sales and resulting revenues assuming consumers are myopic. Myopic consumers maximize immediate utility and buy when purchase utility exceeds utility from the outside option (which is normalized to 0). Under myopic consumer behavior, we observe earlier sales at higher prices, and the resulting total revenue when compared to retailer's current revenue helps us quantify the effect of strategic consumer behavior on the retailer's revenues. As shown in Figure 4, under strategic consumer behavior, the retailer's revenue is 8.8% lower than it would have been under myopic consumer behavior.

Another interesting question is whether limited product availability is helpful in dampening the effect of strategic consumer behavior on retailer revenue. Limited product availability within the season creates stockout risk that is increasing over time and reduces the strategic consumers' incentive to delay their purchases and wait for lower prices. To quantify the extent to which limited availability (stockout risk) dampens the impact of strategic behavior on retailer revenue, we once again keep the retailer's initial stock levels and pricing schedule fixed, use the demand estimates, and simulate sales under strategic consumer behavior, but we assume that consumers do not discount future utilities because of stockout risk. As there is no future stockout risk, consumers are more likely to wait for lower prices, and we observe further delays in purchases, which results in lower retailer revenues. As summarized in Figure 4, this analysis shows that if the availability was not limited, strategic consumer behavior would have resulted in a 34.6% (8.8% + 25.8%) reduction in retailer revenue, and stockout risk considerably helps to dampen the effect of strategic behavior on retailer revenue.

Figure 4 Impact of Strategic Behavior and Limited Availability on Retailer Revenue
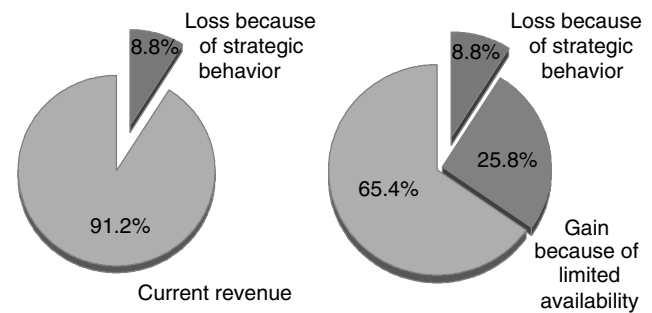
8. Discussion

In this paper, we estimate a dynamic structural model of consumer choice behavior in a market for seasonal goods. Our model accounts for features essential to modeling seasonal goods demand: decreasing prices, limited availability, and change in total utility from consumption through the season. In our model, heterogeneous consumers have expectations about future prices and availability levels, and they strategically time their purchases. The results indicate that ignoring consumers' expectations about future product availability and change in total utility from consumption can result in incorrect demand estimates. We find that a model that ignores these two effects produces lower price sensitivity parameters for both segments and a positive price sensitivity parameter for the first segment.

Our analysis shows that the retailer in the empirical application faces a large, less price-sensitive segment and a much smaller, more price-sensitive segment. Although the more price-sensitive segment is essential for clearing up the excess inventory at the end of the season, its share of total sales and revenue is quite small. Calculated price elasticities suggest that the responsiveness of demand to price changes decreases for both segments over time. This finding highlights the importance of price changes in the earlier periods on the retailer's sales and revenues. Through counterfactual experiments we show that the highest retailer profits are achieved by offering small and early markdowns. Late markdowns do not have very favorable profit outcomes, and early and deep markdowns are very detrimental to retailer profits. Given the current markdown schedule, on the other hand, the retailer can improve profit by carrying less stock. When the retailer limits the initial stock, an increase in stockout risk in later periods forces the customers to buy earlier at higher prices. As long as the reduction in availability is not large, profit gain from earlier sales can overcome the loss as a result of the reduction in overall sales. We also show that strategic consumers who delay their purchases to take advantage of lower prices contribute to a 9% reduction in

retailer's revenue. However, increasing stockout risk later in the season motivates consumers to purchase earlier at higher prices. We show that if the consumers had not taken stockout risk into account when timing their purchases, strategic delays would have been more pronounced, and the loss in revenue as a result of strategic behavior would have been much larger (35%).

This study contributes to the current literature on both methodological and substantive grounds. With regard to the methodological dimension, we develop an estimable structural model of strategic consumer choice in the presence of stockout risk that accounts for change in total consumption utility depending on the time of purchase. With regard to the substantive dimension, we demonstrate that the limited availability and the change in total utility from consumption over the season can affect the aggregate sales curve, and we show that our model can effectively explain interesting regularities in the data, such as big sales spikes at the markdown periods and rapid decrease in sales over time at a given price. An important empirical finding is that anticipated scarcity leads to purchase acceleration, and under limited supply, a retailer might benefit from reducing availability. Our demand model enables the retailer to understand the different factors resulting in change in demand over time: change in total utility from consumption, reduced availability over time, shrinking potential market, and changing consumer mix over time. Accounting for each of these factors separately gives the retailer the opportunity to set optimal initial stock levels and dynamically set optimal prices over the course of the season for different products.

In this study we use counterfactual experiments to investigate the performance implications of changes in the retailer's pricing and inventory policy rather than solving the retailer's optimization problem. One possible extension of this study is formulating and solving the retailer's optimal dynamic pricing and initial inventory-level determination problem. This would require incorporating retailer's uncertainty about demand before the season starts and learning about demand as the season progresses into the dynamic pricing problem. Modeling retailer's demand uncertainty and demand discovery though is beyond the scope of this study, and we leave the modeling of the retailer's optimal pricing problem for future research.

Allowing for within- and cross-category demand effects between products is another important extension to this study. These effects would be important for a seasonal goods retailer in pricing products from complementary categories (e.g., shirts and ties) or substitute products in categories with lower fashion elements (e.g., men's dress shirts). On the other hand,

accounting for these effects at the SKU level in the fashion apparel context brings a computational challenge as a result of the large number of SKUs simultaneously offered for sale. In this study, we resort to combining similar SKUs to lessen the concern for substitution effects and leave the structural treatment of substitution across SKUs to future research. One observation in the data is that markdowns for products introduced together (e.g., all winter coats introduced in October) are usually announced at the same time. As a consequence, the reason behind substitution across products is usually low availability rather than a price change. For this reason, in the case of counterfactual analysis that investigates changes in pricing policy (i.e., single uniform markdown policy), we expect the qualitative nature of the results from our model to be fairly similar to those from a model that allows for intrastore competition. In the case of counterfactual analysis that investigates changes in the inventory policy, on the other hand, if substitution effects are significant, we expect to find a smaller impact of stockout risk on purchase acceleration in a model with intrastore competition compared with our model.

We have taken the initial steps in developing a realistic demand model for seasonal goods products that accounts for limited availability and the impact of purchase timing on overall consumption utility as well as strategic consumer behavior and consumer heterogeneity. Future research can benefit from richer data on instruments, consumer expectations, availability, and competitors' prices.

Acknowledgments

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