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# **Endogenous Evaluation and Sequential Search**

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Abstract. Consumers may not be perfectly informed of the availability and attributes of competitive offerings (price, match value). We examine equilibrium outcomes when consumers can search among potential options and invest evaluation efforts to resolve product value uncertainty by endogenizing the information structure in the canonical model of sequential search with horizontal differentiation. We show that consumers' joint decisions on search and evaluation interact in an asymmetric manner. That is, greater evaluation pushes up the search-terminating threshold, whereas a higher stopping threshold first increases and then decreases the incentive for evaluation. As a result, changes in the search/evaluation cost may exert unusual influences on equilibrium behaviors and payoffs. Both the impact of the search cost on optimal evaluation and that of the evaluation cost on expected number of searched products exhibit an inverted U pattern. Because of endogenous evaluation, the equilibrium price and profit can vary nonmonotonically with the search/evaluation cost. This implies that we may observe the coexistence of consumers searching fewer products and firms charging lower prices as sampling becomes more costly. Moreover, consumers can benefit from more costly sampling. We discuss how these findings can shed light on strategic design of shopping environment, user acquisition/retention, and empirical research.

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Keywords: search • evaluation • information acquisition • pricing • monopolistic competition

#### 1. Introduction

Normally, consumers are not perfectly informed about competitive product offerings (i.e., goods or services) in the marketplace. They may not know which firms are providing relevant solutions for their needs and how much is being charged by each firm. To figure out the availability and prices, consumers can invest time and effort to search sequentially among potential providers. For example, they may visit multiple retailers or sales agencies, travel around store aisles, and move eyes across shelves to consider different alternatives. Consumers can also use search engines (e.g., Google) to construct their option set and then to explore these options by clicking through the obtained links one by one. For shopping in online platforms (e.g., Amazon, eBay, Taobao), consumers are usually offered a tremendous number of similar varieties in a category, and they need to navigate across the website to land onto the purchase page of each listing.

Moreover, consumers may be uncertain about the extent to which an offering may satisfy their idiosyncratic needs. Many categories (e.g., automobiles, books, cosmetics, electronics, food, financial products,

games, toys) are characterized by a significant number of attributes (e.g., color, design, genre, image, quality, style, size). For instance, a cell phone may include tons of specifications, and an insurance plan may be accompanied by hundreds of clauses. As the number and complexity of attributes increase, it would become difficult for consumers to know ex ante which specific attributes are contained in an offering, whether contractual terms are incomplete or ambiguous, or how technical details should be interpreted, for example. In addition, a consumer may not be able to tell immediately how he or she should weigh different attributes, how a bundle of product characteristics may interact with each other, or how the overall performance may vary across individualized consumption scenarios (e.g., habits). Therefore, consumers may desire to evaluate the searched options carefully to partially resolve their value uncertainty before making a purchase. They can inspect physical attributes, process detailed product information (e.g., descriptions, pictures, videos), and contemplate interpretations/implications of product features. They can engage in showrooming/webrooming activities across shopping channels (Jing 2018, Kuksov and Liao 2018). Consumers can also gather relevant information by reading reviews/ratings in online platforms or mobile devices, consulting friends or relatives, and paying for expert advice.

It is usually costly and time/effort consuming to search across products and to evaluate individual ones. Store visits involve expenses and take time. Browsing the internet and navigating across and within shopping/review platforms can deplete cognitive resources and lead to fatigue. It is not rare for buyers to pay for valuable information (e.g., record check for automobiles and properties). Nevertheless, the search/evaluation costs can differ substantially across categories, channels, or markets. Many other factors may also influence the extent to which product sampling costs vary across regions, over time, and/or between search and evaluation. Importantly, it is becoming increasingly prevalent for online sellers/ intermediaries to strategically manipulate consumers' search/evaluation costs (Dukes and Liu 2016, Zhong 2018). For instance, firms can affect search costs through the selection/design of a store location, retail layout, web structure, and online shopping tools (e.g., search engines, attribute screenings/refinements, recommendation systems), whereas evaluation costs can be influenced by the management of free trials, commercials, demonstrations, off-line experiences, consumer reviews, three-dimensional virtual presentations, etc.

These observations motivate us to address the following questions in this research:

- How should consumers make joint decisions on search and evaluation? Put differently, as a consumer invests more evaluation efforts to resolve value uncertainty, should he or she have a higher incentive to search more options, and vice versa?
- How should sampling (i.e., search and evaluation) costs influence equilibrium behaviors (i.e., consumer search and evaluation, firm pricing) and payoffs (i.e., profits, consumer surplus)?
- How should the equilibrium impacts of the search cost differ from those of the evaluation cost?

These questions are important both theoretically and practically. Characterizing consumers' optimal search and evaluation strategies helps firms decide how to set their prices, and it serves as the first step for us to derive equilibrium market outcomes. Investigating the impacts of sampling costs on equilibrium behaviors can improve our understanding of many competitive markets and, as will be discussed, can shed light on empirical research as well. Moreover, analyzing the differential effects of the search/evaluation costs on profits and consumer surplus can provide insights into firm strategies on shopping environment design (e.g., store location/layout, web design, search-refining technologies, shopping

assistants) and on consumer acquisition/retention, respectively.

Our model extends the classical setup of sequential search under monopolistic competition (Wolinsky 1986). Each consumer in the market is initially uninformed of the price and product value for each of a continuum of firms. A consumer can pay a search cost to sample sequentially and randomly across the firms. The key innovation we introduce is that, for each search, a consumer can decide how much effort to invest to evaluate the sampled product. Greater effort involves higher evaluation cost but can generate a more informative signal about idiosyncratic product value. We consider a general information structure for the signal-generating process—that is, the rotation order (Johnson and Myatt 2006). Consumers' posterior estimates of product values are independent across firms and endogenously determined by evaluation choices.

We characterize the consumers' joint sampling decisions. We show that the relationship is asymmetric between optimal search and evaluation strategies. A consumer's optimal search decision is characterized by some stopping threshold (i.e., the reservation value), which is raised as the consumer invests more evaluation efforts. Conversely, the impact of increasing the stopping threshold on optimal evaluation exhibits an inverted U pattern. This is because the value of product information is endogenously determined by the uncertainty about whether search should be continued (i.e., the intermediacy of the stopping threshold). Therefore, the impact of a higher search/ evaluation cost on optimal sampling is generally negative, except that of the search cost on optimal evaluation, which is nonmonotonic.

Changes in the sampling costs lead to interesting consequences on the equilibrium outcomes through their impacts on the optimal stopping threshold (i.e., the search effect) and on optimal evaluation (i.e., the evaluation effect). We demonstrate that, across several distributions, although the evaluation effect of a higher search cost always reinforces its search effect to decrease the equilibrium expected number of products a consumer may search, these two effects for the evaluation cost can be in the same or opposite direction. As a result, the evaluation cost can nonmonotonically influence the equilibrium search breadth. Interestingly, as evaluation becomes more costly, the equilibrium search process can be lengthened despite the stopping threshold being diminished, because less information would be acquired, and thus more posterior values below the stopping threshold may be generated endogenously.

It is also because of endogenous evaluation that increasing the search/evaluation cost may exert a nonmonotonic influence on the equilibrium price (and profit). The equilibrium price can be reduced as sampling becomes more costly, because that may suppress optimal evaluation to increase consumers' price sensitivity and reduce product differentiation. Nevertheless, the evaluation effect per se is sufficient to yield the nonmonotonic relationship between the search cost and the equilibrium price/profit, whereas it is the interaction of the search and the evaluation effects that sustains such pattern for the evaluation cost. In addition, because of the evaluation effect, a higher search/evaluation cost can lead us to observe both the consumers searching fewer products and the firms charging lower equilibrium prices at the same time. Moreover, endogenous evaluation can make the consumers benefit from more costly sampling by inducing lower market prices.

Our research is built on the literature of consumer search.<sup>2</sup> It belongs to the branch of sequential-search models with (ex post) horizontal differentiation. Early contributions include Weitzman (1979), Wolinsky (1986), and Anderson and Renault (1999). Some recent developments identify mechanisms under which a higher search cost may lead to lower equilibrium prices. For example, Kuksov (2004) shows that when product design is endogenous, smaller search cost may induce firms to increase product differentiation to sustain higher prices and profits. Similarly, firms may respond to a lower search cost by expanding the size of their assortments, thereby improving their profitability by increasing consumers' willingness to pay for the best product among the carried assortments (Cachon et al. 2008). Another mechanism to yield a negative equilibrium relationship between search cost and prices is due to the joint search effect: consumers can pay a constant cost to search for multiple products from one single firm (Zhou 2014). We complement these studies by introducing consumer evaluation to endogenously generate the nonmonotonic impact of the search/evaluation cost on the equilibrium price.

There is an emerging research stream on the implications of varying search environment. One consideration is on the order of search. For instance, Armstrong et al. (2009) investigate a model where consumers may search a prominent firm before randomly sampling all other firms. Another issue is about changes in the distribution of product values. Eliaz and Spiegler (2011) consider a setup where a search engine can display to consumers only those products with a probability of match (i.e., quality) above a certain threshold. Similarly, Zhong (2018) examines a shopping platform's incentive to improve search precision by setting the minimum match value for the pool of searched products. Our work shares with these two papers the common feature that the distribution of product values is not fixed, but with two critical differences. They consider a third party's role in increasing the value distribution in the sense of first-order stochastic dominance (which is taken as given for the product market), whereas we focus on consumers' endogenous information acquisition in the sense of mean-preserving spread. In this regard, this paper is closely related to Dukes and Liu (2016), who study consumers' joint decisions on search and evaluation as well. However, they consider simultaneous search, and bundle the search and the evaluation effects, under a specific distribution. We will elaborate, throughout the paper, how our setting differs fundamentally from theirs in both the underlying mechanism and the results.

Our work is connected to the literature on consumer information acquisition. Kuksov and Villas-Boas (2010) show that too many options may lead to less information collection. Branco et al. (2012) consider gradual learning where consumers can decide when to stop receiving attribute information that arrives as a Brownian process. Ke et al. (2016) further extend the analysis to a multiple-option setting. Consumers can also acquire information about preferences (i.e., deliberation) even when product attributes are perfectly known (e.g., Guo and Zhang 2012, Guo 2016, and Guo and Wu 2016). Another recent work is by Doval (2018), who extends Weitzman (1979) to consider sequential information acquisition and choice among multiple options where search is unnecessary for choice. In general, this literature typically abstracts away from search among competitive offerings.

The next section presents our model and lays out the benchmark case with exogenous evaluation. In Section 3 we characterize equilibrium strategies and identify the search and the evaluation effects. We investigate in Section 4 the influences of the sampling costs on equilibrium behaviors and payoffs. The last section presents implications of our findings and some directions for future research.

### 2. Setup

Our model builds on the canonical framework of sequential consumer search and choice (e.g., Wolinsky 1986 and Anderson and Renault 1999). The market consists of a continuum of firms, each selling one single product. The number of firms is infinite. Each firm's fixed and marginal costs of production and supply are constant and normalized to 0. There is a continuum of consumers in the market, each demanding at most one unit of the product from one of the firms. The measure of consumers per firm is normalized to 1. All firms and consumers are risk neutral.

Each firm's product value is idiosyncratic to the consumers. Let  $u_i$  be the product value of firm i for a representative consumer. The surplus of buying from firm i is  $u_i - p_i$ , where  $p_i$  is the price of firm i. The utility

of not buying from any firm is normalized to 0. When all product values and prices are known, a consumer would choose the product that yields the highest positive surplus but may leave the market without any purchase if  $u_i - p_i < 0$  for all firms. Initially, the consumers are informed of neither their idiosyncratic product values nor the price of any firm. The prior belief is that  $u_i$  is identically and independently drawn, across all firms and consumers, from a distribution F(u) on  $[\underline{u}, \overline{u}]$ , with a strictly positive density f(u) and a finite mean  $\hat{u} > 0$ . As a standard regularity condition, suppose 1 - F is log concave, which implies that the hazard rate function  $\frac{f}{1-F}$  is increasing.

Each consumer can engage in a sequential search process to gather purchase-relevant information. By incurring a cost s > 0 to search product i, a consumer can become informed of  $p_i$ . A firm's product cannot be purchased if it is not searched. Search is random among all to-be-sampled products. In addition, the search process is without replacement and with costless recall: returning to buy from a previously sampled product involves no extra cost. In the standard models (e.g., Wolinsky 1986 and Anderson and Renault 1999), search would reveal the product value as well. That is, once a consumer searches product *i*, he or she would know perfectly the surplus  $u_i - p_i$  and would then determine whether to buy this product, to return to buy a previously searched product, or to continue the search process by incurring s to sample one of the remaining products.

We relax the assumption of perfect revelation and separate the search decision from that of product evaluation. In particular, when a product *i* is sampled, a consumer acquires a signal  $w_i$  that only imperfectly uncovers  $u_i$ . The consumer's belief about  $u_i$  is updated according to the Bayes' rule by taking into account the prior belief  $F(\cdot)$ , the acquired signal  $w_i$ , and the signalgenerating process that is indexed by  $\alpha_i \in (0,1)$ . Because consumer utility is linear in product value, it is without loss of generality to summarize all relevant information contained in  $w_i$  about  $u_i$  by the posterior mean  $v_i \equiv E[u_i|w_i]$ , where the expectation is taken over the posterior distribution of  $u_i$  conditional on  $w_i$ . The posterior mean  $v_i$  is a random variable from the ex ante perspective (prior to obtaining the signal  $w_i$ ). We assume that  $v_i$  is independently drawn, across all firms and consumers, from a continuous and differentiable distribution  $H(v, \alpha_i)$  on  $[\underline{v}, \overline{v}]$  with a strictly positive density  $h(v, \alpha_i)$ . As  $\alpha_i \to 0$ , no new information would be acquired over the prior, and hence the density of  $v_i$  would concentrate on the prior mean  $\hat{u}$ . Conversely, as  $\alpha_i \rightarrow 1$ , evaluation would be perfect, and  $H(v, \alpha_i)$  would converge to the prior distribution F. In addition, a higher  $\alpha_i$  represents a more informative evaluation in the sense of the rotation order (Johnson and Myatt 2006).

**Definition 1** (Rotation Order). The family of distributions  $H(v, \alpha)$  are rotation ordered if there exists a rotation point  $v^+ \in (\underline{v}, \overline{v})$  such that

$$\frac{\partial H(v,\alpha)}{\partial \alpha} > 0 \text{ if } v < v^+ \text{ and } \frac{\partial H(v,\alpha)}{\partial \alpha} < 0 \text{ if } v > v^+$$
 for all  $\alpha \in (0,1)$ .

For any  $\alpha > \alpha'$ ,  $H(v,\alpha)$  crosses  $H(v,\alpha')$  only once from above at the rotation point  $v=v^+$ . Equivalently, the density  $h(v,\alpha)$  is more spread out than the density  $h(v,\alpha')$ . In addition, Bayesian updating implies that the mean of  $H(v,\alpha)$  for any  $\alpha$  is constant and equal to  $\hat{u}$ . This yields

$$\int_{\underline{v}}^{v} \frac{\partial H(x,\alpha)}{\partial \alpha} dx > 0 \text{ for all } v \in (\underline{v}, \overline{v}), \text{ and}$$

$$\int_{\underline{v}}^{\overline{v}} \frac{\partial H(x,\alpha)}{\partial \alpha} dx = 0. \tag{1}$$

The rotation order represents a general structure to endogenize product evaluation. It accommodates two well-known cases of information acquisition, which will be the focus in Section 3.2.

Case 1 (Gaussian Learning). A firm's product value  $u_i$  follows a normal distribution with mean  $\hat{u}$  and variance  $1/\beta$ :  $u_i \sim N(\hat{u}, 1/\beta)$ . A consumer can acquire a signal about  $u_i$ :  $w_i = u_i + \varepsilon_i$ , where  $\varepsilon_i$  is independent of  $u_i$  and normally distributed with mean 0 and variance  $1/\tilde{\alpha}_i$ , and  $\tilde{\alpha}_i \in (0, +\infty)$  is increasing in  $\alpha_i$ . Conditional on  $w_i$ , the posterior distribution of  $u_i$  is normal with mean  $\frac{\tilde{\alpha}_i w_i + \beta \hat{u}}{\tilde{\alpha}_i + \beta}$  and variance  $\frac{1}{\tilde{\alpha}_i + \beta}$ . The distribution of the posterior mean,  $H(v, \alpha_i)$ , is rotation ordered with the rotation point  $v^+ = \hat{u}$ .

**Case 2** (Truth-or-Noise). A firm's product value  $u_i$  is drawn from a distribution  $F(\cdot)$  with mean  $\hat{u}$ . A consumer can obtain a signal  $w_i$  about  $u_i$ . With probability  $\alpha_i$ ,  $w_i = u_i$ , and with probability  $1 - \alpha_i$ ,  $w_i$  is independently drawn from  $F(\cdot)$ . Conditional on  $w_i$ , the posterior mean of  $u_i$  is  $v_i = \alpha_i w_i + (1 - \alpha_i) \hat{u}$ . The distribution of the posterior mean is then  $H(v, \alpha_i) = F(\frac{v - (1 - \alpha_i) \hat{u}}{\alpha_i})$ . Therefore,  $\frac{\partial H(v, \alpha_i)}{\partial \alpha_i} = -\frac{v - \hat{u}}{\alpha_i} h(v, \alpha_i)$ ; that is,  $H(v, \alpha_i)$  is rotation ordered with the rotation point  $v^+ = \hat{u}$ .

The sequence of actions is as follows. First, the firms simultaneously and independently set their prices. Second, the consumers independently decide how to sequentially search and evaluate the firms' products. A consumer needs to pay a search cost s > 0 to know a firm's price  $p_i$  and to be able to buy from it. Conditional on the consumer deciding to sample product i, he or she also makes the joint decision about how much costly effort to invest to evaluate this product by choosing  $\alpha_i \in [\underline{\alpha}, \bar{\alpha}] \subset (0, 1)$ . The evaluation choice  $\alpha_i$  determines the informativeness of the

signal-generating process (i.e., the distribution  $H(v,\alpha_i)$ ) according to the rotation order. Note that we do not presume constant evaluation across products. The evaluation cost  $c(\alpha_i)$  is increasing in  $\alpha_i$ . We set  $\underline{c} \equiv c(\underline{\alpha}) = 0$  and  $\overline{c} \equiv c(\overline{\alpha}) > 0$ . After the product i is sampled, both  $v_i$  and  $p_i$  will be revealed. The consumer then decides whether to make a purchase (among the sampled products) or to continue the search process and sample another product (and, conditional on search, sets the level of evaluation effort for the next-sampled product).

In our model of sequential search with endogenous evaluation, the consumers are ex ante homogeneous, whereas their posterior expected product values can be heterogeneous and varying across firms (even if the evaluation choices are the same across firms and consumers). Therefore, the firms can become ex post horizontally differentiated, even though they are ex ante identical from the consumers' perspective (prior to search). These features can be seen in standard models of sequential search where the common distribution for product values is exogenous (e.g., Wolinsky 1986).

Our solution concept is a perfect Bayesian equilibrium. That is, each firm seeks to maximize expected profit in price setting, in anticipation of the other firms' pricing strategies as well as the consumers' optimal search and evaluation decisions. The consumers maximize their expected surplus at each point of the sequential search/evaluation process, given their expectation about the firms' pricing strategies. Moreover, we adopt the conventional assumption that, in case a consumer observes that a firm charges an off-equilibrium price, his or her equilibrium belief about the to-be-sampled products' prices remains unchanged.

As a benchmark, we consider sequential search with exogenous and fixed evaluation for all firms. Suppose evaluation is costless such that the consumers can perfectly be informed of the value of each searched product. The common distribution for the posterior mean of product values would be the prior  $F(\cdot)$ . Let  $p^b$  be the price charged by all firms under the symmetric equilibrium.<sup>5</sup> Given that all prices are expected to be equal, consumers' optimal search is characterized by a stationary stopping rule (e.g., Wolinsky 1986 and Anderson and Renault 1999). That is, they would stop the search process and buy the current product immediately if and only if the product's surplus  $v_i - p_i$  is higher than the expected surplus of continuing search,  $z^b - p^b$ , where the optimal stopping threshold  $z^b$  (i.e., reservation value) is obtained by solving

$$\int_{z}^{\bar{v}} (v - z) \ dF(v) = s. \tag{2}$$

This condition says that the optimal search strategy should equalize the marginal benefit and the marginal cost of one more search. It is evident that the left-hand side in (2) is decreasing in z. Therefore, a larger s decreases the optimal stopping threshold  $z^b$ , making the consumers less choosy, and they discontinue the search process sooner. Actually, a higher search cost would decrease the expected number of products a consumer may search in equilibrium,  $ENS^b = \frac{1}{1-F(z^b)}$ .

Anticipating the optimal search strategy, a consumer searching firm i with  $p_i$  would generate an expected demand of  $1 - F(p_i + z^b - p^b)$  for this firm. The firm's expected profit is thus equal to  $p_i[1 - F(p_i + z^b - p^b)]/[1 - F(z^b)]$ , given that consumers are ex ante symmetric, search is random, and the average measure of consumers per firm is one. The first-order condition yields  $1 - F(p_i + z^b - p^b) - p_i f(p_i + z^b - p^b) = 0$ . Given that the hazard rate is an increasing function, the first-order condition gives rise to the optimal price. By imposing the symmetry condition (i.e.,  $p_i = p^b$  in equilibrium), we can then obtain the equilibrium price:

$$p^{b} = \frac{1 - F(z^{b})}{f(z^{b})}. (3)$$

It follows that the equilibrium price  $p^b$  is decreasing in  $z^b$  and is thus increasing in s. This captures the intuition that a higher search cost can soften price competition by demotivating consumer search. The equilibrium profit for each firm,  $\pi^b = p^b$ , is increasing in s. Conversely, the equilibrium consumer surplus (of participating in the market)  $V^b = z^b - p^b$  is decreasing in the search cost. For  $V^b$  to be nonnegative to sustain consumer participation and search, the search cost s cannot be too high. Moreover, the equilibrium social welfare  $SW^b = z^b$  is decreasing in s.

These findings are standard in the literature. We will show subsequently that qualitatively different results may arise when we consider endogenous evaluation under sequential search.

### 3. Equilibrium Strategies

In this section we characterize the consumers' and firms' equilibrium strategies when product evaluation is endogenous. As in the benchmark, we focus on symmetric equilibrium where all firms charge the same price  $p^*$ . We will analyze how the consumers' search and evaluation decisions are simultaneously made. We will show that the relationship is asymmetric between the consumers' search and evaluation incentives. As a result, the impacts of the search cost on optimal consumer strategies can be qualitatively different from those of the evaluation cost. We will also identify the differential effects of search and evaluation on the equilibrium price.

### 3.1. Optimal Consumer Strategies

Given that all equilibrium prices are constant, consumers' optimal search/evaluation strategies are stationary, and the optimal stopping strategy continues to be characterized by a threshold. These are standard features in sequential-search models with ex ante symmetric and unlimited options, which do not change even in our setting with endogenous evaluation.6 Let the optimal stopping threshold and the optimal evaluation level be  $z^*$  and  $\alpha^*$ , respectively. Similarly, let  $V^*$  be the expected consumer surplus (of starting/continuing search) under the optimal strategies and given the expected price  $p^*$ . Given that market participation is desirable, the expected consumer surplus can be obtained by solving the following dynamic optimization problem:

$$V^* = \max_{\{z,\alpha\}} \left\{ -s - c(\alpha) + \int_z^{\bar{v}} (v - p^*) dH(v, \alpha) + H(z, \alpha)V^* \right\}. \tag{4}$$

The reservation value z and the evaluation effort  $\alpha$  are chosen to maximize the expected payoff of sampling another product. One extra sampling costs  $s + c(\alpha)$ . If the realized posterior value v is above the stopping threshold z, the sampled product will be purchased immediately (at the expected price  $p^*$ ). If otherwise (with probability  $H(z, \alpha)$ ), another product will be sampled that yields the expected surplus  $V^*$ . To derive the optimal sampling strategies, let us define  $B(z,\alpha) \equiv \int_{z}^{v} (v-z) dH(v,\alpha)$  as the expected gain of drawing a posterior v that is above the stopping threshold and  $NB(z, \alpha) \equiv B(z, \alpha) - c(\alpha)$  as the expected net gain of evaluation.

**Lemma 1.** Optimal consumer strategies  $(z^*, \alpha^*)$  are jointly determined by

$$B(z,\alpha) = s + c(\alpha),\tag{5}$$

$$B(z,\alpha) = s + c(\alpha), \tag{5}$$
  

$$\alpha(z) = \arg\max_{\alpha} NB(z,\alpha). \tag{6}$$

Under the optimal sampling strategies, the marginal benefit of search/evaluation should be equal to the respective marginal cost. It is important to note that although the marginal benefit of search is  $B(z, \alpha)$ , the marginal benefit of evaluation is  $\frac{\partial B(z,\alpha)}{\partial \alpha}$ . The condition (5) to determine the optimal stopping threshold is akin to that in (2), except that the marginal benefit  $B(z, \alpha)$  is now conditional on the evaluation choice  $\alpha$ and that the per-sampling costs also include the evaluation cost  $c(\alpha)$ . In addition, as in (6), the marginal benefit of evaluation,  $\frac{\partial B(z,\alpha)}{\partial \alpha}$ , represents the impact of increasing the informativeness of evaluation on the expected surplus of continuing search.

We investigate how consumers' optimal search and evaluation strategies are interactively determined. First, the marginal benefit of search as in (5) is decreasing in z and increasing in  $\alpha$ . To see this, we can integrate by parts to rewrite  $B(z, \alpha)$  as  $\int_{z}^{v} [1 - H(v, \alpha)] dv$ , which is increasing in  $\alpha$  because a higher  $\alpha$  implies a mean-preserving spread in  $H(v, \alpha)$ . Therefore, a more informative evaluation would increase the consumers' incentive to set a higher z. Intuitively, increasing the informativeness of evaluation raises the chance to draw a high v from a new search, thus making the consumers more picky and less likely to terminate the search process (the chance to draw a low v is increased as well, but that does not matter because products with low posterior values will not be bought anyway). In other words, in the absence of cost consideration, the response of optimal search to greater evaluation is positive.

What about the impact of the stopping threshold on the incentive for evaluation? Let us take the derivative of the marginal benefit of evaluation (i.e.,  $\frac{\partial B(z,\alpha)}{\partial \alpha}$ ) with respect to z:

$$\frac{\partial^2 B(z,\alpha)}{\partial z \partial \alpha} = \frac{\partial H(z,\alpha)}{\partial \alpha}.$$
 (7)

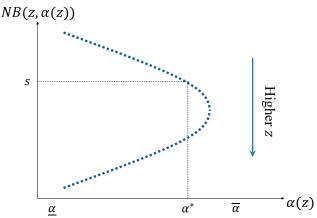
It follows from the definition of rotation order that  $\frac{\partial H(z,\alpha)}{\partial \alpha} > 0$  for  $z < v^+$  and  $\frac{\partial H(z,\alpha)}{\partial \alpha} < 0$  for  $z > v^+$ . Interestingly, the impact of the stopping threshold on the marginal benefit of evaluation is not monotonic. In particular, when the stopping threshold is relatively low, it positively influences the consumers' incentive to acquire more information about product values. Conversely, when the stopping threshold becomes relatively high, it decreases the consumers' incentive for information acquisition. The marginal benefit of evaluation is the highest when the stopping threshold z is intermediate and closer to the rotation point  $v^+$ . In other words, an increase in *z* exerts an inverted U influence on the marginal benefit of evaluation. Intuitively, when the reservation value is either too low or too high, the consumers can readily decide whether to make an immediate purchase or to sample another product, even without knowing much about actual product values. They will be eager to accept a sampled product when z is low and will be too picky for most searched products when z is high. In either case, they will find it unwise to delve too much into the evaluation of each individual product. It is when z is intermediate that they will benefit the most from being better informed of the values of sampled products. This is why the consumers' incentive for improving evaluation first increases and then decreases with the stopping threshold.

Therefore, the relationship between optimal search and evaluation is asymmetric. Although greater evaluation induces the consumers to raise the stopping threshold, a choosier search strategy may influence optimal evaluation in a nonmonotonic manner. The former result is similar to, whereas the latter one contrasts sharply with, those in Dukes and Liu (2016). They show that the interaction between search and evaluation is mutually complementary: the marginal benefit of search breadth increases with evaluation depth, and vice versa. This is because they consider simultaneous sampling across products, whereas sampling is sequential in our setting. In either search scheme, the value of evaluation stems from the reduction of uncertainty about the most preferred option among the considered products. Preference uncertainty would necessarily be higher under simultaneous search, as the pool of sampled products expands. However, the value of evaluation under sequential search is assessed at the margin between the current product and the next drawn one. As a result, preference uncertainty, and hence the marginal benefit of evaluation, increases with the stopping threshold if z is relatively low but decreases with it if z is relatively high.

The joint determination of optimal search and evaluation is illustrated in Figure 1. Each point corresponds to a stopping threshold: the horizontal axis represents optimal evaluation (i.e.,  $\alpha(z)$ ), and the vertical axis the optimal expected net gain of evaluation (i.e.,  $NB(z, \alpha(z)) = \max_{\alpha} NB(z, \alpha)$ ), both of which are conditional on z. A point located further to the "south" constitutes a higher z than a point to the "north." Therefore,  $NB(z, \alpha(z))$  becomes lower as zincreases, which follows from applying the envelope theorem to the optimization problem for evaluation. By contrast, increasing the value of z traces out a nonmonotonic curve for  $\alpha(z)$ . Equalizing  $NB(z,\alpha(z))$ with the search cost s would then give rise to the (unconditionally) optimal evaluation  $\alpha^*$ , as well as the optimal reservation value  $z^*$ .

Next, we examine how the search/evaluation cost may influence optimal sampling strategies. To this end, we consider two evaluation cost functions where

**Figure 1.** (Color online) Joint Determination of Optimal Search and Evaluation Strategies



 $c_1(\alpha) < c_2(\alpha)$  and  $\frac{\partial c_1(\alpha)}{\partial \alpha} < \frac{\partial c_2(\alpha)}{\partial \alpha}$  for all  $\alpha$ . Let the corresponding optimal strategies be  $(z_i^*, \alpha_j^*), j = 1, 2$ .

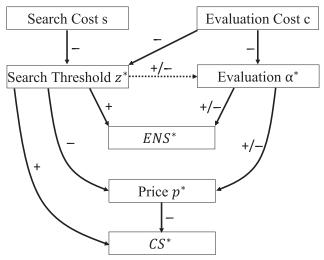
**Proposition 1.** Both optimal stopping threshold and optimal evaluation decrease with the cost of evaluation (i.e.,  $z_1^* > z_2^*$  and  $\alpha_1^* > \alpha_2^*$ ). Optimal stopping threshold  $z^*$  decreases with the search cost s, and optimal evaluation  $\alpha^*$  first increases and then decreases with s.

As evaluation becomes more costly, it is not surprising that consumers will spend less in their evaluation effort. Search will become less attractive as well, because the overall per-sampling costs are raised. This is consistent with the complementary impact of evaluation on the marginal benefit of search. In addition, a higher search cost decreases the optimal stopping threshold  $z^*$ , as in the benchmark. However, a higher s need not always induce the consumers to invest less in evaluation. Proposition 1 shows that optimal evaluation  $\alpha^*$  can vary nonmonotonically with the search cost. Optimal evaluation would be positively (negatively) influenced by the search cost when s is relatively small (large). The intuition for this interesting result is the following. Note that the search cost does not influence the benefit/cost of evaluation directly: *s* does not appear in (6). Nevertheless, by influencing the optimal stopping threshold  $z^*$ , changes in the search cost can affect optimal evaluation  $\alpha^*$  indirectly. As *s* increases from a low level,  $z^*$ would decrease toward the rotation point  $v^+$  and hence enhance the marginal benefit of evaluation. When *s* becomes sufficiently high, a further increase in the search cost would drive down the optimal stopping threshold away from  $v^+$ , thus suppressing consumers' incentive for evaluation. Therefore, the impact of s on optimal evaluation exhibits an inverted U pattern, which follows from the negative effect of s on  $z^*$  and in turn from the inverted U effect of  $z^*$  on the marginal benefit of evaluation.

The nonmonotonic effect of s on  $\alpha^*$  can be demonstrated in Figure 1. As we move up s, the equalizing point between  $NB(z,\alpha(z))$  and s would yield the optimal evaluation level that first increases (along the northeast direction) and then decreases (along the northwest direction). This pattern would not arise if search is simultaneous across products: if Dukes and Liu (2016) separated the cost of search from that of evaluation, they would find that the impacts of the search cost on optimal search and evaluation are both negative (and so are the impacts of the evaluation cost).

The impacts of the search/evaluation cost on optimal sampling strategies are summarized in Figure 2. The dashed line captures the nonmonotonic relationship, from the stopping threshold to optimal evaluation, that mediates the indirect inverted U impact of s on  $\alpha^*$ . We term the impact of the search/evaluation cost via

**Figure 2.** Summary of the Impacts of the Search/Evaluation Cost via the Search/Evaluation Effects



the optimal stopping threshold  $z^*$  as the *search effect* and that via the optimal evaluation level  $\alpha^*$  as the *evaluation effect*. We will subsequently investigate how changes in the sampling costs may influence equilibrium outcomes through the interaction of these alternative channels on optimal consumer strategies.

### 3.2. Equilibrium Price

The firms' pricing problem under endogenous evaluation is similar to that in the benchmark. This is because consumers are still ex ante homogeneous, their sampling strategies are stationary, and search continues to be random. The first-order condition for the symmetric equilibrium price is

$$p^* = \frac{1 - H(z^*, \alpha^*)}{h(z^*, \alpha^*)}.$$
 (8)

This would yield the equilibrium price if the hazard rate function is increasing (at  $\alpha^*$ ). It is satisfied for all  $\alpha$  for Gaussian learning (Case 1) as well as for the truthor-noise setup (Case 2) given that 1-F is log concave and the posterior mean v is linear in the signal w. We will focus on these two cases to investigate how the equilibrium price can be differentially influenced by the search or the evaluation effect (i.e.,  $z^*$  or  $\alpha^*$ ).

It follows from the increasing hazard rate that the equilibrium price  $p^*$  is decreasing in the consumers' optimal stopping threshold  $z^*$ . We have seen this (negative) search effect in the benchmark. By contrast, the impact of changing  $a^*$  on the equilibrium price is less straightforward. To see this, we can simplify (8) for the Gaussian learning case as

$$p^* = \sigma^* \frac{1 - \Phi\left(\frac{z^* - \hat{u}}{\sigma^*}\right)}{\phi\left(\frac{z^* - \hat{u}}{\sigma^*}\right)},\tag{9}$$

where  $\Phi(\cdot)$  is the standard normal distribution,  $\phi(\cdot)$  is the standard normal density, and  $\sigma^{2*} = \frac{\tilde{\alpha}^*}{\beta(\tilde{\alpha}^* + \beta)}$  is the variance of posterior mean and increasing in  $\alpha^*$ . Similarly, for Case 2 we have

$$p^* = \alpha^* \frac{1 - F\left(\frac{z^* - \hat{u}}{\alpha^*} + \hat{u}\right)}{f\left(\frac{z^* - \hat{u}}{\alpha^*} + \hat{u}\right)}.$$
 (10)

A higher evaluation  $\alpha^*$  can influence the equilibrium price in two ways. First, as the consumers invest effort to gain more information about product values, they will be less sensitive to (off-equilibrium) price changes, which would be a force to soften price competition. This is the differentiation effect. Second, evaluation can influence the consumers' search outcomes, even with fixed stopping threshold  $z^*$ . That is, all else being equal, changes in the posterior distribution can affect the consumers' purchase likelihood at each firm by influencing the chance to draw a sufficiently satisfied product. As a higher evaluation increases the dispersion of the consumers' posterior belief, the probability of continuing search can be either reduced or increased. The sign of this search moderation effect depends on whether the stopping threshold  $z^*$  is above or below the prior mean  $\hat{u}$ . In particular, when the search effect is relatively strong (i.e.,  $z^* > \hat{u}$ ), it would be mitigated by greater evaluation because of the increasing probability to draw high product values. This is then a force to increase the equilibrium price. Conversely, when the search effect is relatively weak (i.e.,  $z^* < \hat{u}$ ), it would be strengthened by a higher  $\alpha^*$  because low product values become more likely (as well), which tends to reduce the equilibrium price.

**Proposition 2.** (i) When  $z^* > \hat{u}$ , the equilibrium price  $p^*$  for Cases 1 and 2 increases with  $\alpha^*$ . (ii) When  $z^* < \hat{u}$ , the equilibrium price  $p^*$  for Case 2 is either monotonic in  $\alpha^*$  or first decreases and then increases with  $\alpha^*$  if the hazard rate f/(1-F) is log concave.

Part (i) of Proposition 2 demonstrates that when the differentiation and the search moderation effects reinforce each other, greater evaluation would unambiguously increase the equilibrium price. However, when  $z^* < \hat{u}$ , these two effects would be in opposite direction, and their interaction is not straightforward. Part (ii) presents a sufficient condition to help us identify a clearer pattern about the net impact of  $\alpha^*$  on  $p^*$  for Case 2. It shows that if the hazard rate is log concave, the equilibrium price is quasiconvex in  $\alpha^*$ . As a result,  $p^*$  would be increasing (decreasing) in  $\alpha^*$  if the differentiation (the search moderation) effect is more dominant; alternatively, the net impact of  $\alpha^*$  on  $p^*$  would exhibit a U-shaped pattern if neither effect dominates the other one.

It follows from Proposition 1 that, as in the benchmark, the search cost s has a positive effect on the equilibrium price through reducing the consumer's incentive to search aggressively. Similarly, an increase in the cost of evaluation can lower the consumers' optimal stopping threshold  $z^*$  to induce the firms to charge a higher equilibrium price. In addition, the search/evaluation cost can influence the equilibrium price  $p^*$  through the evaluation effect (i.e.,  $\alpha^*$ ) as well. However, as indicated in Figure 2, the sign of this channel of influence can be ambiguous. The first ambiguity results from the nonmonotonic impact of the search cost on optimal evaluation (Proposition 1). The second ambiguity is about how greater evaluation may influence the equilibrium price (Proposition 2).

The search and the evaluation effects may reinforce or offset each other in influencing the equilibrium price. When they have opposite signs, the net impact of changing the search/evaluation cost on the equilibrium price would depend on which effect is more dominant. As we will show subsequently (in some specific distributions), the evaluation effect can represent a countervailing and more influential force such that some commonly seen results (in the benchmark) can be reversed.

### 4. Comparative Statics

We now examine how the sampling costs may influence the equilibrium outcomes. To facilitate tractability, we consider the truth-or-noise setup for product evaluation (i.e.,  $H(v,\alpha) = F(\frac{v-(1-\alpha)\hat{u}}{\alpha})$ ). Our main focus is on the uniform distribution for  $F(\cdot)$ . We also analyze the case of exponential distribution, which allows us to demonstrate the robustness of our major results and to differentiate the underlying mechanism from that of simultaneous search. We will consider linear evaluation cost  $c(\alpha) = \frac{\alpha-\alpha}{\bar{a}-\alpha}\bar{c}$ , which allows us to derive reduced-form solutions.

### 4.1. Uniform Distribution

Consider the case where the prior distribution for product values is F(u) = u, with support [0,1] and mean  $\hat{u} = 1/2$ . It can be readily verified that the distribution for the posterior mean is  $H(v,\alpha) = \frac{2v-1+\alpha}{2\alpha}$ , with  $h(v,\alpha) = \frac{1}{\alpha}$ , for  $v \in [\frac{1-\alpha}{2}, \frac{1+\alpha}{2}]$ . The marginal benefit of search is then

$$B(z,\alpha) = \begin{cases} \max\{1/2 - z, 0\} & \text{for } \alpha \le |1 - 2z|, \\ \frac{((1+\alpha)/2 - z)^2}{2\alpha} & \text{for } \alpha \ge |1 - 2z|. \end{cases}$$

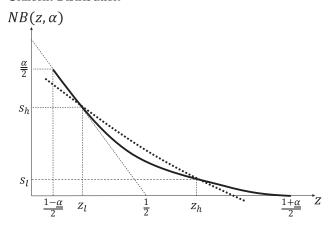
**Lemma 2.** Consider uniform distribution for product values. If  $\bar{c} \leq (\bar{\alpha} - \underline{\alpha})/8$ , there exist  $0 < s_l < \underline{\alpha}/8 < s_h < \bar{\alpha}/2$  such that optimal search and evaluation strategies are given by  $(z^*, \alpha^*) = (\frac{1+\alpha}{2} - \sqrt{2\alpha s}, \underline{\alpha})$  when  $s < s_l$  or  $s_h < s < \underline{\alpha}/2$ , and  $(z^*, \alpha^*) = (\frac{1+\alpha}{2} - \sqrt{2\bar{\alpha}(\bar{c}+s)}, \bar{\alpha})$  when  $s_l \leq s \leq s_h$ .

Because the marginal benefit of evaluation (i.e.,  $\frac{\partial B(z,\alpha)}{\partial \alpha}$ ) is increasing in  $\alpha$  while the marginal cost is constant, optimal evaluation would involve a corner solution. That is, the consumers would optimally choose either  $\alpha(z) = \underline{\alpha}$  or  $\alpha(z) = \bar{\alpha}$ . This suggests some kind of polarization in the consumers' optimal evaluation behavior: they may either barely inspect or attend to product features, or they may delve deeply to collect as much available information as possible.

In addition, the difference of the optimal expected net gain of evaluation between high and low evaluation levels (i.e.,  $NB(z,\bar{\alpha}) - NB(z,\underline{\alpha})$ ) first increases and then decreases in z. As a result, the minimum-evaluation strategy ( $\alpha^* = \underline{\alpha}$ ) is favored when the search cost s is either sufficiently low or high, whereas maximum evaluation ( $\alpha^* = \bar{\alpha}$ ) is optimal when s falls into an intermediate range. This is illustrated in Figure 3, where the solid and the dotted curves represent  $NB(z,\underline{\alpha})$  and  $NB(z,\bar{\alpha})$ , respectively. Note that as s increases, there is a discontinuous jump in optimal evaluation at the point  $s = s_l$  and a sudden drop at  $s = s_h$ . This demonstrates the nonmonotonic effect of the search cost on optimal evaluation (Proposition 1).

We start by investigating how the sampling costs may influence observable consumer behaviors in equilibrium. We have shown (in Proposition 1 and Lemma 2) that the impact of the evaluation cost on optimal evaluation  $\alpha^*$  is necessarily negative, whereas that of the search cost is first positive and then negative. In other words, the consumers spend less effort in evaluation as it becomes more costly, although they may actually evaluate each product more when it is more costly to search across products. We have also shown that the consumers set a lower stopping threshold  $z^*$  as the search/evaluation cost increases. Does this mean that the equilibrium expected number of searched products (for each consumer),

**Figure 3.** Optimal Search and Evaluation Strategies under Uniform Distribution



 $ENS^* = \frac{1}{1 - H(z^*, \alpha^*)'}$  is smaller when sampling becomes more costly?

**Proposition 3.** Consider uniform distribution for product values. ENS\* decreases with s and that ENS\* decreases with  $\bar{c}$  for small  $\bar{c}$ , but it may increase discontinuously for large  $\bar{c}$  if s is small.

An increase in the search/evaluation cost may exert two influences on the expected number of products a consumer may search (see Figure 2). The first is to make the consumers less picky in deciding whether to accept a sampled product:  $z^*$  would be lower, and all else being equal, the consumers would be less likely to engage in an extra search (i.e., the search effect). This is the standard force that we have identified in the benchmark. There is another impact on the number of products we expect the consumers to sample: the consumers can respond to changes in the sampling costs by modifying their evaluation strategy  $\alpha^*$ , which may influence the distribution of the posterior product values (i.e., the evaluation effect). That is, the probability  $H(z^*, \alpha^*)$  that a consumer may continue the search process is influenced not only by the equilibrium stopping threshold  $z^*$  but also by the endogenous likelihood to draw a sufficiently high posterior value v above the threshold.

It turns out that these two effects are in the same direction for changes in the search cost. When s is relatively small, the optimal stopping threshold  $z^*$  is above the rotation point  $v^+$ , and a higher search cost may increase optimal evaluation  $\alpha^*$ . This would increase the likelihood to draw high posterior product values above  $z^*$ . Conversely, when s becomes sufficiently large, the optimal stopping threshold  $z^*$  is below  $v^+$ , and a higher search cost may decrease optimal evaluation. This implies that the likelihood to draw low posterior product values below *z*\* would be lowered. In other words, as s increases, the consumers may acquire more product information to counteract their strong inclination for search when  $z^*$  is high, or they may acquire less information to further undermine their weak inclination for search when  $z^*$  is low. Therefore, the evaluation effect of increasing the search cost always yields a lower likelihood of search.

However, the evaluation effect of increasing the evaluation cost  $\bar{c}$  can be positive or negative in influencing the equilibrium probability of search  $H(z^*, \alpha^*)$ . Unlike the search cost, a higher evaluation cost can only decrease but never increase optimal evaluation (Proposition 1 or Lemma 2). Nevertheless, as already discussed, a lower  $\alpha^*$  may either increase or decrease the likelihood of search, depending on whether s is relatively small to raise the optimal stopping threshold  $z^*$  above the rotation point  $v^+$  or high enough to drive down  $z^*$  below  $v^+$ . In the latter scenario, as  $\bar{c}$ 

increases, the consumers may acquire less product information to further erode their weak incentive for search. By contrast, with a sufficiently small s, a higher  $\bar{c}$  may induce the consumers to acquire less product information and thus generate more posterior values that are below the stopping threshold  $z^*$ . The evaluation effect can then represent a positive force to lengthen the search process. In addition, the evaluation effect can dominate the search effect when optimal evaluation  $\alpha^*$  decreases considerably, whereas the change in  $z^*$  is negligible. This means that a higher evaluation cost can unambiguously increase the expected number of searched products.

Therefore, changes in the sampling costs yield qualitatively different consequences regarding the consumers' equilibrium behaviors. A higher search cost always shortens the consumers' expected search breadth but can nonmonotonically influence the amount of evaluation the consumers spend. Analogously, a higher evaluation cost diminishes equilibrium evaluation depth all the time but may either increase or decrease the expected number of searched products. Both nonmonotonic results are driven by the asymmetric relationship between the consumers' search and evaluation incentives. These findings are absent in Dukes and Liu (2016), because they consider simultaneous sampling such that the interaction between search and evaluation is symmetrically complementary.

We then turn to the equilibrium price and profit. It can be readily obtained that  $\pi^* = p^* = \frac{1+\alpha^*}{2} - z^*$ . As expected, a higher stopping threshold tends to decrease the equilibrium price. Nevertheless, greater evaluation can raise the equilibrium price by reducing the consumers' price sensitivity: the differentiation effect is always stronger than the search moderation effect for uniform distribution. These two forces on the equilibrium price/profit work in an additive manner here.

**Proposition 4.** Consider uniform distribution for product values. The equilibrium price and profit increase with s for  $s < s_h$  but drop discontinuously at  $s = s_h$ . For a given  $s \in (0, \underline{\alpha}/2)$ , the equilibrium price and profit increase with  $\bar{c}$  for small  $\bar{c}$ , but it may drop discontinuously for large  $\bar{c}$ .

A higher search cost can bring up the usual priceincreasing search effect by reducing optimal stopping threshold  $z^*$ . Changes in the search cost can also affect optimal evaluation to influence the equilibrium price, which can be either positive or negative (Figure 2). This ambiguity is due to the nonmonotonic impact of s on optimal evaluation  $\alpha^*$ , whereas here, greater evaluation always positively influence the equilibrium price. The overall impact of the search cost can be illustrated from Figure 3. As s increases across the point  $s_l$ , optimal evaluation  $\alpha^*$  jumps upward to reinforce the search effect. However, as s becomes sufficiently high and crosses the point  $s_h$ , the consumers will endogenously reduce evaluation from  $\bar{\alpha}$  to  $\underline{\alpha}$ , thus exerting a negative pressure on the equilibrium price. It turns out that the countervailing evaluation effect is more powerful than the search effect in a small neighborhood of  $s = s_h$ , because the change in  $z^*$  is negligible while  $\alpha^*$  becomes dramatically lower. Therefore, the firms' equilibrium price/profit may decrease with the search cost.

Proposition 4 also shows that the evaluation cost can nonmonotonically influence the equilibrium price. When  $\bar{c}$  is relatively small, the consumers will spend maximum effort on product evaluation (i.e.,  $\alpha^* = \bar{\alpha}$ ). An increase in  $\bar{c}$  will then lead the consumers to be less choosy in search ( $z^* = \frac{1+\bar{\alpha}}{2} - \sqrt{2\bar{\alpha}(\bar{c}+s)}$ ) is decreasing in  $\bar{c}$ ), which, in turn, induces the firms to charge a higher equilibrium price. However, when  $\bar{c}$  becomes sufficiently high, it will be more preferable to reduce the level of evaluation ( $s_h$  is decreasing and  $s_l$  is increasing in  $\bar{c}$ ). It follows that the evaluation effect can dominate the search effect to yield a lower equilibrium price as  $\bar{c}$  increases, which is similar to that of sufficiently increasing the search cost s.

Recall from Proposition 3 that  $ENS^*$  decreases with the search cost. This means that as s increases, in equilibrium, the consumers search fewer products while the firms may charge lower market prices. A similar pattern can also be seen for an increase in the evaluation cost  $\bar{c}$ . These results are against our common sense that less search should soften competition to yield higher prices. This is because of the endogeneity of evaluation that may enhance the consumers' price sensitivity as the search/evaluation cost becomes higher.

Next, we examine how the sampling costs may affect the equilibrium consumer surplus  $V^* = z^* - p^* = 2z^* - \frac{1+\alpha^*}{2}$ .

**Proposition 5.** Consider uniform distribution for product values. The equilibrium consumer surplus  $V^*$  decreases with s for  $s < s_h$ , but it increases discontinuously at  $s = s_h$ . For a given  $s \in (0, \underline{\alpha}/2)$ , the equilibrium consumer surplus decreases with  $\bar{c}$  for small  $\bar{c}$ , but it may increase discontinuously for large  $\bar{c}$ .

This proposition demonstrates that changes in the sampling costs may nonmonotonically influence the consumers' equilibrium surplus. As in the benchmark, the general impact of increasing the search/evaluation cost is to make it less attractive for the consumers to search more products, which is harmful for the consumers' surplus. However, interestingly, the consumers can be better off as search or evaluation becomes more costly. These counterintuitive results may arise when higher sampling costs reduce optimal

evaluation to induce lower market prices (see Figure 2). Moreover, the consumers can benefit from more costly sampling even when they are observed to search fewer products, which is analogous to the coexistence of lower equilibrium prices and smaller expected number of searched products. Actually, in the current setting with uniform distribution, impacts of the sampling costs on the equilibrium consumer surplus are qualitative reflections of those on the equilibrium price/profit (Proposition 4).<sup>12</sup>

### 4.2. Exponential Distribution

The prior distribution for product values is  $F(u) = 1 - e^{-u}$  for  $u \ge 0$ , with mean  $\hat{u} = 1$ . The distribution for the posterior mean is  $H(v, \alpha) = F(\frac{v-1+\alpha}{\alpha})$ , with  $h(v, \alpha) = f(\frac{v-1+\alpha}{\alpha})/\alpha$ , over  $v \in [1-\alpha, \infty]$ . The marginal benefit of search is

$$B(z,\alpha) = \begin{cases} 1-z & \text{for } \alpha \le 1-z, \\ \alpha e^{\frac{1-\alpha-z}{\alpha}} & \text{for } \alpha \ge 1-z. \end{cases}$$

It can be readily verified that  $B(z,\alpha)$  is increasing and convex in  $\alpha$  and that  $\frac{\partial B(z,\alpha)}{\partial \alpha}$  is increasing in z for  $z < v^+ = 1$  and decreasing in z for z > 1. Therefore, as in the case of uniform distribution, the consumers choose minimum evaluation when the search cost s is either sufficiently low or high and maximum evaluation when s is intermediate. In particular, if  $\bar{c} \leq (\bar{\alpha} - \underline{\alpha})/e$ , there exist  $0 < s_l < \underline{\alpha}/e < s_h < \bar{\alpha}$  such that optimal search and evaluation strategies are  $(z^*, \alpha^*) = (1 - \underline{\alpha} - \underline{\alpha} \ln (s/\underline{\alpha}), \underline{\alpha})$  when  $s < s_l$  or  $s_h < s < \underline{\alpha}$  and  $(z^*, \alpha^*) = (1 - \bar{\alpha} - \underline{\alpha} \ln (\bar{c} + s)/\bar{\alpha}], \bar{\alpha})$  when  $s_l \leq s \leq s_h$ .

It is evident that the impact of the evaluation  $\cos \bar{c}$  on optimal evaluation  $\alpha^*$  is negative and that of the search cost is nonmonotonic:  $\alpha^*$  jumps discontinuously at  $s = s_l$  and drops suddenly at  $s = s_h$ . Moreover, it can be shown that the evaluation effect of increasing the search cost on the equilibrium probability of search,  $H(z^*, \alpha^*)$ , is always of the same sign as that of the search effect (i.e., negative), whereas that of increasing the evaluation cost can be positive and stronger than the corresponding search effect if s is small such that  $z^*$  is above  $v^+$ . Therefore, the impacts of the sampling costs on the equilibrium expected number of searched products,  $ENS^*$ , are qualitatively similar to those in Proposition 3.

The equilibrium price/profit is  $\pi^* = p^* = \alpha^*$ , which is independent of the consumers' search behavior because the hazard rate of the prior is constant (i.e., equal to 1) under the exponential distribution. So the sampling costs influence the firms' equilibrium price/profit only through the differentiation effect of evaluation: both the search effect  $z^*$  and the search moderation effect of evaluation are null here (see Figure 2). It follows that a higher search cost s first increases and then decreases the equilibrium

price/profit, and an increase in the evaluation  $\cot \bar{c}$  can only decrease  $p^*$ . As in Proposition 4, these results demonstrate that higher sampling costs need not always benefit the firms. It is worthwhile to note that here it is endogenous evaluation but not consumer search that underlies the nonmonotonic impact of the search cost on firm competition.

Critically, the exponential case allows us to pinpoint the importance to consider sequential sampling and to separate the search cost from the evaluation cost. If we considered simultaneous search for the exponential distribution, we would obtain that the impact of increasing the search/evaluation cost on the equilibrium price is negative: the sampling costs would always negatively influence optimal evaluation, which would, in turn, positively affect the equilibrium price. This means that the main finding in Dukes and Liu (2016) regarding the nonmonotonic impact of the sampling costs on the equilibrium price is not robust to the exponential distribution. Put differently, for the sampling costs to generate the nonmonotonic result under simultaneous search, it may be necessary to bundle the search and the evaluation effects. By contrast, we show that the evaluation effect itself is sufficient to yield nonmonotonicity under the sequential-search setup for changes in the search cost but not in the evaluation cost. This would be the case even when the search moderation effect of evaluation is absent such that greater evaluation unambiguously increases the equilibrium price (see Proposition 2). Fundamentally, this is because the relationship between search and evaluation is symmetrically complementary under simultaneous sampling such that the evaluation effect of the search/ evaluation cost is always negative, whereas the relationship is asymmetric under sequential sampling such that the impact of the evaluation cost on optimal evaluation is negative but that of the search cost can be of either sign.

Finally, as in Proposition 5, the interaction of the search and the evaluation effects may yield either lower or higher equilibrium consumer surplus as the search/evaluation cost increases. The consumers can benefit from more costly sampling, because that may undermine optimal evaluation, which in turn induces the firms to charge lower prices.

### 5. Concluding Remarks

We endogenize the information structure in the canonical model of sequential search with horizontal differentiation (Wolinsky 1986). We do this by expanding the consumers' decisions to include costly evaluation. At each search, a consumer can decide whether to sample another product and how much information to acquire about the product's match value. Our setting generates results that are new to the

sequential-search literature. We show that optimal search and evaluation interact with each other in an asymmetric manner. Greater evaluation raises the search-stopping threshold, whereas optimal evaluation first increases and then decreases with the stopping threshold.

As a result, changes in the search/evaluation cost can yield interesting impacts on equilibrium behaviors and payoffs. As the search cost becomes higher, the consumers' evaluation effort first increases and then decreases. Conversely, the evaluation cost can nonmonotonically influence the expected search breadth. This implies that more costly sampling need not undermine sampling efforts. In addition, because of endogenous evaluation, the search/evaluation cost can exert a nonmonotonic effect on the equilibrium price. Therefore, we may observe that, in response to an increase in the search/evaluation cost, the consumers search fewer products, but the firms charge lower prices. Moreover, more costly sampling may hurt the firms while benefiting the consumers.

Our work provides managerial insights into the strategic design of a shopping environment. It suggests that, contrary to common sense, firms should not always seek to increase consumers' search costs. For instance, retail stores need not locate too far away from each other, and in-store layout and shelves can be appropriately designed to moderate (but not minimize) navigation costs. Similarly, online platforms can design their website and shopping tools (e.g., search engines, screening/refining functions, recommendation systems) to strategically influence the costs consumers need to incur to search across alternatives. We suggest that firms need not minimize (or maximize) consumers' costs of evaluating product values either. The sampling costs can exhibit nonmonotonic influences on firm profitability. Moreover, to acquire or retain users of their platform, online shopping intermediaries should not always minimize the search/evaluation costs. Essentially, firms should exercise more care to investigate how their actions may influence consumers' search as well as evaluation incentives and to distinguish the impacts of search costs from those of evaluation costs.

This paper has profound implications for empirical research as well. It is a typical approach in the literature (e.g., Kim et al. 2010 and Chen and Yao 2015) to measure the size/distribution of search costs from observational data on consumer search behavior (e.g., size/composition of consideration set). This reverse-engineering method is valid to the extent that there is a one-to-one relationship between the search cost and the expected number of products a consumer may search. However, our model suggests that endogenous evaluation can be another channel through which the search cost negatively influences the expected

search breadth. This implies that consumers' actual search behavior can be more responsive to changes in the search cost than what can be inferred from a model with only the search effect. In other words, if the evaluation effect were ignored (as in the literature), we might overestimate high search costs but underestimate low search costs. Similarly, failing to consider the evaluation effect may result in qualitatively different inferences that are based on observed pricing behavior (e.g., Hong and Shum 2006). This is because, as shown in this paper, equilibrium prices may be nonmonotonic in the search cost. One solution would be to collect additional data on consumer evaluation (e.g., time spent on inspecting product attributes), which is increasingly feasible for online shopping behavior.

Our research can be extended in several directions. The first is to consider other timings between search and evaluation. These two consumer decisions for a given product are concurrent in the current model. Alternatively, we can consider postsearch evaluation (i.e., deliberation as in Guo and Zhang (2012) and Guo (2016)). That is, consumers can decide how much information to acquire about their value/preference after searching a given product. Of course, consumers may engage in presearch evaluation and, conditional on the information acquired for a product, decide whether to search that product. Consumers would then become endogenously heterogeneous before they make the search decision. One well-known issue for such a setting is that consumers may be ex post held up, because firms know that only those consumers with a sufficiently high match value may search. To ensure that the market does not collapse in equilibrium, we need to augment the model with additional features (e.g., free search for some consumers, postsearch information).

The second direction is to consider consumer evaluation under other search settings. For example, search may not be random, and consumers may have some predetermined search order. Some firms may be more prominent than others such that they are more likely to be sampled first (Armstrong et al. 2009). Or consumers can use search-refinement tools to rank products according to some attributes (e.g., price) and then sample them sequentially. The third direction is to investigate an alternative setup for consumer information acquisition. For instance, information acquisition can be dynamic and gradual (Branco et al. 2012, Ke et al. 2016). We have assumed in the current paper that evaluation is static in the sense that only one signal about a product's value can be acquired. This allows us to consider a general information structure, although at the cost of some realism. To consider dynamic or gradual learning about product value, we may have to focus on a simpler information structure (e.g., two-point signals). Overall, we hope

that tractable models can be developed in future research to enrich our setup along these challenging but fruitful avenues.

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### **Appendix**

**Proof of Lemma 1.** Let us formally prove the optimality of stationary search/evaluation strategies under unlimited options. Consider a consumer who has just sampled a collection I of products and decides whether to search another product and (if so) how much evaluation to invest. Let the realized best surplus so far be  $v_I = \max_{j \in I} (v_j - p_j)$ , which is the only relevant information about I (because of one-unit demand and independence across products). Let us show that the expected surplus of continuing search (if preferable), as well as the optimal search/evaluation strategies, is independent of the history (i.e., unconditional on  $v_I$ ). This would be the case if the current best product  $j_I = \arg \max_{i \in I} (v_i - p_i)$ will never be recalled for purchase beyond the current point, given that any to-be-made evaluation decision would only influence the distribution of the expected value of futuresampled product. Suppose otherwise that after sampling another set K of products, the consumer may find that purchasing the product  $j_I$  is the best decision at that point. This would mean that  $j_I$  is better than any sampled product in K (i.e.,  $j_I = \arg \max_{i \in I \cup K} (v_i - p_i)$ ). It follows that the expected surplus of continuing search does not change across the two points when either I or  $I \cup K$  has been sampled, because the realized best surplus remains  $v_I$ , and the set of unsampled products is still unlimited (and hence the same). In other words, the decision problems are basically the same at these two points. As a result, it is contradictory that the product  $j_I$  is not chosen after sampling I but is chosen after sampling  $I \cup K$ .

The preceding argument holds for any I. Therefore, a sampled product is either purchased immediately or never purchased in the future (even with free recall). In addition, given constant prices for all unsampled products, the optimal search/evaluation strategies and the expected surplus of search are all stationary. The stationarity of continuation surplus implies that the optimal search strategy must be characterized by a stopping threshold. In particular, the optimal search and evaluation strategies can be obtained by taking the first-order derivatives of (4) with respect to  $(z,\alpha)$  (assuming that second-order conditions are satisfied). The first-order condition with respect to z is  $-(z-p^*)h(z,\alpha)+h(z,\alpha)V^*=0$ . This yields  $V^*=z-p^*$ . Substituting this into (4) yields

$$z - p^* = -s - c(\alpha) + \int_{z}^{\bar{v}} (v - p^*) dH(v, \alpha) + H(z, \alpha)(z - p^*)$$

$$= -s - c(\alpha) + \int_{z}^{\bar{v}} (v - z) dH(v, \alpha) + [1 - H(z, \alpha)](z - p^*)$$

$$+ H(z, \alpha)(z - p^*)$$

$$= -s - c(\alpha) + \int_{z}^{\bar{v}} (v - z) dH(v, \alpha) + z - p^*.$$

This gives rise to the optimal stopping threshold  $z^*$  as in (5), as well as the optimal evaluation level  $\alpha^*$  as in (6).

**Proof of Proposition 1.** It is straightforward that as the marginal cost of evaluation increases, the optimal evaluation would be smaller for any z. This implies that  $\alpha_1^* > \alpha_2^*$ . In addition, as the cost of evaluation increases from  $c_1(\alpha)$  to  $c_2(\alpha)$ , the optimal expected net gain of evaluation (i.e.,  $NB(z,\alpha(z))$ ) would become smaller, according to the envelope theorem (i.e., the indirect impact of increasing the evaluation cost through optimal evaluation  $\alpha(z)$  is negligible). It then follows from (5) that the optimal stopping threshold would be lower (i.e.,  $z_1^* > z_2^*$ ), because  $NB(z,\alpha(z))$  is decreasing in z.

Similarly, it follows from (5) that the optimal stopping threshold  $z^*$  decreases with s, because  $NB(z,\alpha(z))$  is decreasing in z. As shown in the paper (i.e., from (7) and the rotation order), the marginal benefit of evaluation  $\frac{\partial B(z,\alpha)}{\partial \alpha}$  is increasing in z for  $z < v^+$  and decreasing for  $z > v^+$ . Therefore, according to Topkis's theorem, optimal evaluation  $\alpha(z)$  is increasing in z for  $z < v^+$  and decreasing for  $z > v^+$ . Note that this holds even if the objective function  $NB(z,\alpha)$  is not concave in  $\alpha$  for all z, which can be the case, as shown in Section 4. It then follows that the optimal evaluation  $\alpha^*$  first increases and then decreases with s (note from (6) that s does not directly influence the optimization problem for  $\alpha$ ).

**Proof of Proposition 2.** Part (i) follows directly from the discussion in the text. To prove part (ii), by changing the variable from  $\alpha^*$  to  $\gamma = \frac{z^* - \hat{u}}{\alpha^*} + \hat{u}$  in (10), we have  $p^* = \frac{\hat{u} - z^*}{\hat{u} - \gamma} \frac{1 - F(\gamma)}{f(\gamma)}$ . Note that when  $z^* < \hat{u}$ ,  $\gamma$  is increasing in  $\alpha^*$  and  $\frac{\hat{u} - z^*}{\hat{u} - \gamma}$  is convex in  $\gamma$ . It follows from the latter point that  $p^*$  is log convex in  $\gamma$  if f/(1 - F) is log concave. Therefore,  $p^*$  is quasiconvex in  $\alpha^*$ . This implies that  $p^*$  either is monotonic in  $\alpha^*$  or first decreases and then increases with  $\alpha^*$  for all  $\alpha^* \in [\alpha, \bar{\alpha}]$ .

**Proof of Lemma 2.** Consider first the optimization of evaluation for a given stopping threshold z. Note that  $B(z,\alpha)$  is constant for  $\alpha \le |1-2z|$ . In addition,  $B(z,\alpha)$  is increasing and convex in  $\alpha$  for  $\alpha \ge |1-2z|$ . This implies that, given the linear evaluation cost, the optimization of evaluation must involve a corner solution; that is,  $\alpha(z) = \alpha$  or  $\alpha(z) = \bar{\alpha}$ .

It can be verified that  $B(z,\bar{\alpha})-B(z,\underline{\alpha})$  first increases and then decreases in z. In addition,  $B(z,\bar{\alpha})-B(z,\underline{\alpha})=0$  for  $z\leq (1-\bar{\alpha})/2$  or  $z\geq (1+\bar{\alpha})/2$ , and  $B(z,\bar{\alpha})-B(z,\underline{\alpha})$  achieves the highest value  $(\bar{\alpha}-\underline{\alpha})/8$  when z=1/2. Note also that  $NB(z,\bar{\alpha})-NB(z,\underline{\alpha})=B(z,\bar{\alpha})-\bar{c}-B(z,\underline{\alpha})$ . This means that  $NB(z,\bar{\alpha})-NB(z,\underline{\alpha})<0$  when  $z\leq (1-\bar{\alpha})/2$  or  $z\geq (1+\bar{\alpha})/2$  and that, given  $\bar{c}\leq (\bar{\alpha}-\underline{\alpha})/8$ ,  $NB(z,\bar{\alpha})-NB(z,\underline{\alpha})>0$  when z=1/2. Therefore, there must exist  $z_l\in ((1-\bar{\alpha})/2,1/2)$  and  $z_h\in (1/2,(1+\bar{\alpha})/2)$  such that  $\alpha(z)=\underline{\alpha}$  for  $z\leq z_l$  or  $z\geq z_h$  and  $\alpha(z)=\bar{\alpha}$  for  $z\in [z_l,z_h]$ .

Let  $s_l \equiv NB(z_h, \bar{\alpha}) = NB(z_h, \underline{\alpha})$ , and let  $s_h \equiv NB(z_l, \bar{\alpha}) = NB(z_l, \underline{\alpha}) < \bar{\alpha}/2$ . Equalizing  $NB(z, \alpha(z))$  and s would yield the optimal sampling strategies:  $(z^*, \alpha^*) = (\frac{1+\alpha}{2} - \sqrt{2\alpha}s, \underline{\alpha})$  when  $s < s_l$  or  $s_h < s < \underline{\alpha}/2$ , and  $(z^*, \alpha^*) = (\frac{1+\bar{\alpha}}{2} - \sqrt{2\bar{\alpha}(\bar{c}+s)}, \bar{\alpha})$  when  $s_l \le s \le s_h$ . Note that for  $z^*$  to be an interior solution in the equilibrium support  $[\frac{1-\alpha^*}{2}, \frac{1+\alpha^*}{2}]$ , we need  $s < \max\{s_h, \underline{\alpha}/2\}$ .

**Proof of Proposition 3.** The equilibrium expected number of searched products,  $ENS^*$ , is increasing in  $H(z^*, \alpha^*) = \frac{2z^*-1+\alpha^*}{2\alpha^*}$ . Note first that  $H(z^*, \alpha^*)$  is increasing in  $z^*$  for a given  $\alpha^*$ . So it follows from Proposition 1 or Lemma 2 that a higher s or  $\bar{c}$  decreases  $H(z^*, \alpha^*)$  when optimal evaluation  $\alpha^*$  remains unchanged.

According to Proposition 1 or Lemma 2, when s is sufficiently small such that  $z^*$  is above the rotation point  $v^+=1/2$ , a higher s may increase  $\alpha^*$  (from  $\underline{\alpha}$  to  $\bar{\alpha}$ ), and when s is sufficiently high such that  $z^* < v^+ = 1/2$ , a higher s may decrease  $\alpha^*$  (from  $\bar{\alpha}$  to  $\underline{\alpha}$ ). In addition, it follows from the definition of rotation order that  $\frac{\partial H(z^*,\alpha^*)}{\partial \alpha}>0$  for  $z^* < v^+$  and that  $\frac{\partial H(z^*,\alpha^*)}{\partial \alpha}<0$  for  $z^*>v^+$ . This means that the impact of s on  $H(z^*,\alpha^*)$  through the influence on  $\alpha^*$  (as well as through the influence on  $z^*$ ) is always negative.

When  $\bar{c}$  is sufficiently small (for a given s), we would have  $\alpha^* = \bar{\alpha}$ . Note that  $H(z^*, \alpha^*)$  would be increasing in  $z^*$  and hence decreasing in  $\bar{c}$  if  $\alpha^*$  remains to be  $\bar{\alpha}$ . As  $\bar{c}$  increases, optimal evaluation may change from  $\bar{\alpha}$  to  $\underline{\alpha}$ . When this happens, if s is sufficiently small such that  $z^* > v^+$ , we would have  $\frac{\partial H(z^*, \alpha^*)}{\partial \alpha} < 0$ , which implies that a higher  $\bar{c}$  would increase  $H(z^*, \alpha^*)$ . If instead s is sufficiently large such that  $z^* < v^+$ , because  $\frac{\partial H(z^*, \alpha^*)}{\partial \alpha} > 0$ , the reduction in  $\alpha^*$  because of the increase in  $\bar{c}$  would yield a lower  $H(z^*, \alpha^*)$ .

#### **Endnotes**

<sup>1</sup>According to a survey by Zillow Mortgage Marketplace in 2010, consumers in the United States spend significant amount of time prior to purchases (e.g., on average, 40 hours for a new home) (Zillow.com 2010).

<sup>2</sup> An extensive stream of studies (e.g., Hong and Shum 2006, Kim et al. 2010, Santos et al. 2012, Honka 2014, Chen and Yao 2015, and Honka and Chintagunta 2016) exploits data on consumer search and/or price dispersion to make inferences about primitives of consumers (e.g., search cost, search methods, and preferences).

<sup>3</sup>With equal means, the rotation order between two distributions implies mean-preserving spread and the convex order. Thus, a higher  $\alpha$  yields more informative signal in that sense that the ex ante expected surplus increases (respectively, decreases) in  $\alpha$  if the interim utility is convex (respectively, concave) in v.

<sup>4</sup>The timing between a firm's pricing and the consumers' search and evaluation decisions is de facto simultaneous, because without search/evaluation, a consumer is uninformed of the firm's price.

<sup>5</sup> As is standard in search models, there always exist equilibria where all firms charge sufficiently high prices and no consumer participates in the market, which will not be considered throughout the paper.

<sup>6</sup>This is because, as elaborated in the appendix, irrespective of whether evaluation is endogenous, the recall option would never be exercised (even though it is costless), and the optimal search/evaluation strategies at each point are determined *solely* by expectations on the next product, which always remain the same given an infinite number of ex ante symmetric products. Regarding the latter point of history independence, note that the realized best surplus among the already sampled products determines whether to continue the search (by comparing with the expected continuation surplus), but it does not influence the threshold-based strategy per se, the optimal stopping threshold, or subsequent evaluation. However, for finite products, a previously sampled product may be recalled for purchase (e.g., after all products are sampled). As a result, the realized best surplus so far may influence future evaluation and hence

- the current stopping threshold, which would then be history dependent and not be stationary anymore. Therefore, it would be intractable, even with ex ante symmetry, to extend standard sequential-search problems with finite options (e.g., Weitzman 1979) to allow for endogenous evaluation as in our setting.
- <sup>7</sup> This does not apply to Gaussian learning because the hazard rate for normal distribution is not log concave.
- <sup>8</sup> If evaluation involves binary levels, most results in this section will hold for the Gaussian learning case and for any prior  $F(\cdot)$  under the truth-or-noise setup.
- <sup>9</sup> We have obtained similar results in reduced-form solutions for other distributions: the logistic distribution where the prior is  $F(u) = \frac{1}{1+e^{-(u-i)}}$  for u defined on the real line and  $\hat{u} > 0$  and the modified Pareto distribution with the prior  $F(u) = 1 (1-u)^{\beta}$  for  $u \in [0,1]$ ,  $\beta > 0$ , and mean  $\hat{u} = \frac{1}{1+\beta}$ .
- <sup>10</sup> The results are exactly the same whether  $c(\alpha)$  is linear or concave because the objective function for evaluation,  $NB(z,\alpha)$ , is convex in  $\alpha$  for the distributions we consider. If  $c(\alpha)$  is convex,  $NB(z,\alpha)$  will not be well behaved, and the analysis will become complicated without generating new insights.
- <sup>11</sup> For the optimal stopping threshold to be an interior solution (i.e.,  $z^* \in (\frac{1-\alpha^*}{2}, \frac{1+\alpha^*}{2})$ ), the search cost cannot be too high:  $s < \max\{s_h, \underline{\alpha}/2\}$ . This implies that the scenario  $s_h < s < \underline{\alpha}/2$  in Lemma 2 cannot arise if the cost of evaluation is too low (i.e.,  $\overline{c} < \frac{(\bar{\alpha}-\underline{\alpha})^2}{8\bar{\alpha}}$ ) such that  $s_h > \underline{\alpha}/2$ .
- <sup>12</sup> It follows from Proposition 1 that, similar to the benchmark, the equilibrium social welfare  $SW^* = z^*$  is decreasing in the search/evaluation cost.
- <sup>13</sup> For  $z^*$  to be interior (i.e., greater than  $1 \alpha^*$ ), we must have  $s < \max\{s_h, \underline{\alpha}\}$ . Therefore, the scenario  $s_h < s < \underline{\alpha}$  is null if  $\bar{c} < \bar{\alpha}e^{\frac{\alpha^{-\bar{a}}}{\bar{a}}} \underline{\alpha}$  such that  $s_h > \alpha$ .

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