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The Economics of Buyer Uncertainty: Advance Selling vs. Probabilistic Selling

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Although advance selling and probabilistic selling differ in both motivation and implementation, we argue that they share a common characteristic—both offer consumers a choice involving buyer uncertainty. We develop a formal model to examine the general economics of purchase options that involve buyer uncertainty, explore the differences in buyer uncertainty created via these two strategies, and derive conditions under which one dominates the other. We show that the seller can address unobservable buyer heterogeneity by inducing sales involving buyer uncertainty via two different mechanisms: (1) homogenizing heterogeneous consumers and (2) separating heterogeneous consumers. Offering advance sales encourages customers to purchase while they are uncertain about their consumption states (more homogeneous), but offering probabilistic goods encourages customers to reveal their heterogeneity via self-selecting whether or not to purchase the uncertain product. The relative attractiveness of these two selling strategies depends on the degree of two types of buyer heterogeneity: (1) *Max_Value-Heterogeneity*, which is the variation in consumers' valuations for their preferred good, and (2) *Strength-Heterogeneity*, which is the variation in the strength of consumers' preferences. Neither strategy is advantageous unless the market exhibits sufficient *Max_Value-Heterogeneity*. However, whereas *Strength-Heterogeneity* can destroy the profit advantage of advance selling, a mid-range of *Strength-Heterogeneity* is necessary for probabilistic selling to be advantageous.

Key words: advance selling; probabilistic selling; demand uncertainty; consumer heterogeneity; pricing

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1. Introduction

Technological advances have facilitated the adoption of innovative selling strategies. In this paper, we focus on two selling strategies that have recently drawn the attention of practitioners and the marketing literature—advance selling (AS) and probabilistic selling (PS). The term *advance selling* refers to a marketing practice in which the seller offers buyers opportunities to make purchases before the time of consumption. For instance, consumers often book air travel, hotels, and cruises weeks or months prior to their travel dates. Similarly, entertainment (e.g., concert tickets, Broadway shows, and theme park admission) is frequently purchased in advance. Following these common applications of advance selling, much of the previous research on advance selling has focused on travel-related industries (e.g., airlines) and was motivated by price discrimination and yield management (e.g., Gale and Holmes 1992, Chatwin 1999, Biyalogorsky et al. 1999, Biyalogorsky and Gerstner 2004). However, recent developments in advance selling theory (e.g., Shugan and Xie 2000, 2005; Xie and Shugan 2001) have illustrated that the conditions

required for a profit advantage from advance selling are far more general than previously thought. Specifically, the profit advantage of advance selling does not require specific industry structures such as capacity constraints and the existence of particular segments (i.e., leisure customers who are price sensitive and arrive early, and business customers who are price insensitive and arrive late) that we often observe in travel-related industries. These developments bring our attention to a fundamental but underexplored consumer phenomenon existing in almost all markets: a consumer's utility from a given product or service is not fixed but is instead affected by various personal factors such as health, mood, finances, work schedule, and family situation.¹ For example, a Saturday night concert would be less valuable when one suffers from a headache or is facing a deadline at work, and the value of a seafood buffet dinner on a given evening

¹ Other research has considered the impact of consumption state uncertainty on a seller's refund policy (Guo 2009, Xie and Gerstner 2007) and on a consumer's proclivity to purchase multiple products to maintain consumption flexibility (Guo 2006).

depends on how much the individual craves seafood and the degree of her hunger. These personal factors are often known to the buyer but are unknown to the seller at the time of consumption and are unknown to both in advance. Recent research (Shugan and Xie 2000, 2005; Xie and Shugan 2001) has shown that advance selling can improve profit in many markets simply because it allows the seller to transact with buyers at a time when it does not encounter an information disadvantage (i.e., in the advance period).

The term *probabilistic selling*, as defined by Fay and Xie (2008), refers to a marketing practice in which a multi-item seller offers buyers an additional purchase choice, that of buying a *probabilistic good*, which is an offer involving a probability of getting any one of a set of multiple distinct items. Several examples of sellers of probabilistic goods include (1) Priceline.com, Hotwire.com, and LastMinuteTravel.com, all of which offer probabilistic travel services where consumers can purchase hotel rooms, airline tickets, or car rentals for which some specific attributes of the service (e.g., the itinerary of the flight, the location of the hotel, or the identity of the car rental company) are not revealed until after payment; (2) KidSurplus.com, which offers children's clothing at a discounted price if the buyer lets the seller pick the print from the listed options; (3) SwimOutlet.com, which offers a discounted "grab bag" of swimsuits where patterns and styles are chosen randomly by the website; and (4) Fairgrab.com, which offers apparel and shoes where consumers receive a discount if they let the seller randomly select which color they will get. Although most of these current offerings of probabilistic goods appear to be primarily motivated by retailers' desire to rid themselves of distressed goods, research on probabilistic selling (Fay and Xie 2008) has illustrated that the profit advantage of probabilistic selling is fundamentally driven by an underexplored type of buyer heterogeneity existing in almost all markets: consumers often differ in the *strength* of their product preferences (e.g., some consumers love one color of sweater but hate another color, whereas other consumers may only slightly prefer one over the other or like both equally). Specifically, introducing a discounted probabilistic good allows the seller to segment markets based on buyer product preference strength and to expand the market to reach previously unserved consumers. Like advance selling, the profit advantage of probabilistic selling does not require specific industry characteristics such as capacity constraints, a perishable product, an intermediary channel structure, or pricing via a reverse price auction (which are characteristics of Priceline.com, the most prominent seller of probabilistic goods). Notice that these findings, that advance selling and probabilistic selling have broad potential applications, suggest that in many diverse environments sellers may be

able to increase their profits by introducing advance sales or probabilistic products.

Although advance selling and probabilistic selling are motivated by fundamentally different market phenomena and are implemented via completely different market mechanisms, we argue that these two strategies share a common characteristic—both offer consumers a choice that involves *buyer uncertainty*. Specifically, advance selling offers consumers a choice to buy before their consumption utility is known, and probabilistic selling offers consumers a choice to buy a product that can turn out to be any of a set of multiple items. This common characteristic raises several important and interesting research questions: What are the general economic forces behind consumer choices involving uncertainty? How, from a seller's perspective, does the buyer uncertainty utilized by advance selling differ from that created via probabilistic selling? Are the two types of buyer uncertainty substitutable under some conditions (i.e., can the seller achieve the same profit improvement by either offering advance sales or introducing a probabilistic good)? Under what conditions can one type of buyer uncertainty help the seller more than the other type can and why? Answers to these questions will advance our theoretical understanding of the relationship between buyer uncertainty and marketing strategies.

Beyond providing theoretical insights, answering these questions has important practical implications. The literature has underscored the general applicability of both advance selling and probabilistic selling because neither requires specific industry characteristics to achieve its profit advantage, which implies that there are many situations where the seller can potentially benefit from adopting either strategy. A key question becomes the following: Which selling strategy yields the greatest potential benefit? For example, consider cable companies that sell pay-per-view movies. The current selling procedure is that customers examine the movies that are currently offered at any given time and then order (via the telephone, Internet, or TV remote control) the specific movie that they want to watch. Once the order is placed, the movie begins. Cable companies would be very interested in developing ways to generate greater revenue from pay-per-view movies. Probabilistic selling may be useful in this setting. By offering consumers the option either to purchase their preferred movie at a "high" price or to purchase a probabilistic movie (e.g., a random draw of either *17 Again* or *Confessions of a Shopaholic*) at a discount, the cable company may be able to increase its revenue because more pay-per-view movies will be purchased (because of the discounted price for the probabilistic movie offerings) and/or higher prices can be charged to

consumers with strong preferences to watch a specific movie. Note that introducing probabilistic movies would involve significant investments in additional infrastructure (e.g., making revisions to the ordering interfaces and writing computer code to make the random movie selection) as well as an informational campaign to make customers aware that this additional purchase option exists and what it entails. Advance selling is another approach that might be useful to a cable company. In particular, consumers could be offered the option to order their movie well before the actual viewing. Obviously, consumers would need to be offered a sufficient discount, relative to the spot price, to induce them to purchase in advance because they may not know what their schedule would be ahead of time or the type of movie they would be in the mood to see. Despite the lower selling price, selling movies in advance could increase the cable company's revenue if there is a sufficient increase in the number of movies purchased. Again, note that introducing advance purchase options would require the cable company to invest in infrastructure to facilitate this additional purchase option (e.g., updated interfaces, an automated pricing algorithm to reflect the discount for advance purchases, and informational campaigns to alert customers to this purchase option). Practically speaking, the cable company would be very interested in knowing whether it would be more advantageous to introduce probabilistic products or the advance purchase option.

It is important to note that our research questions cannot be answered by existing advance selling and probabilistic selling theory because the extant models are not compatible and thus do not allow for direct comparisons between the two selling strategies. In particular, the advance selling models developed recently (Shugan and Xie 2000, 2005; Xie and Shugan 2001) focus on a single-product market, not multiproduct markets, which is necessary for there to be buyer heterogeneity in the strength of their product preferences. On the other hand, models of probabilistic selling (Fay 2008, Fay and Xie 2008), although focusing on buyer heterogeneity in the strength of their product preferences in multiproduct markets, only consider a static environment where an individual's valuations are known; i.e., they do not allow for the possibility of an advance period in which valuations are unknown to the buyer.

To address our research questions, we develop in this paper a formal model that is general enough to capture the important characteristics of a multiproduct market in which both advance selling and probabilistic selling are viable strategies. Figure 1 summarizes the positioning of this paper. In particular, although previous papers have separately considered the impact of buyer uncertainty about consumption states and the

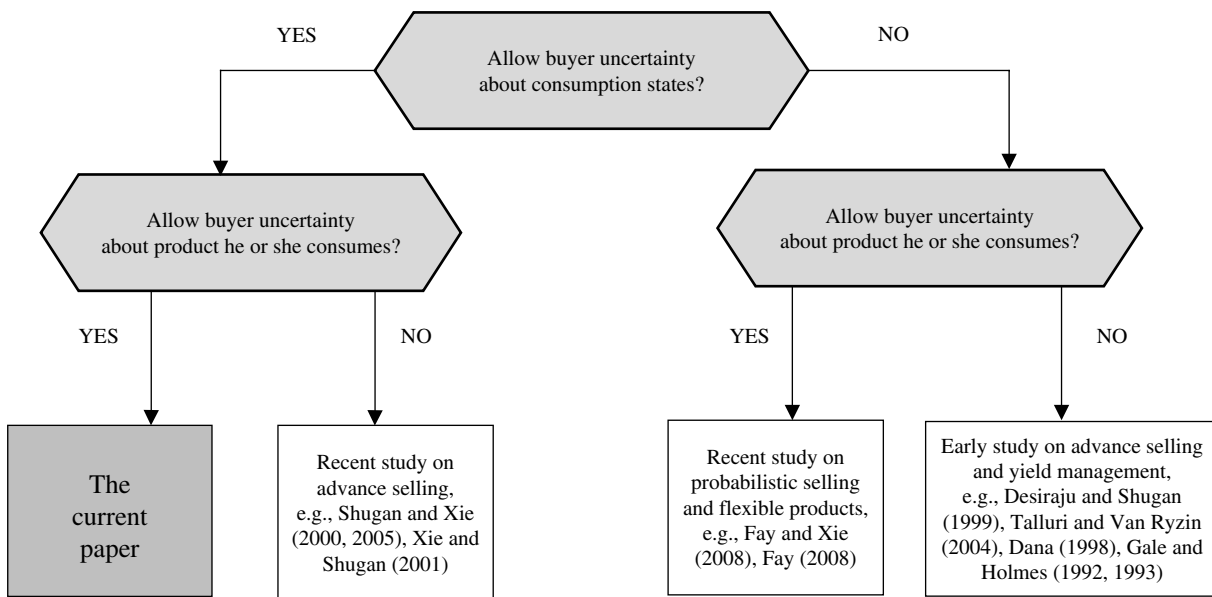
impact of buyer uncertainty about the product he or she consumes, ours is the first to examine both types of buyer uncertainty in a single model. In doing so, we are able to develop a more integrative understanding of buyer uncertainty. Furthermore, to the best of our knowledge, the current paper is the first to compare the advance selling and probabilistic selling strategies.

The analysis of our analytical model reveals that a general benefit of offering consumers choices involving uncertainty is that doing so eases the seller's difficulty in addressing buyer heterogeneity. Buyer heterogeneity exists in all markets. For example, some customers may prefer traditional over contemporary style furniture, or a yoga class over tai chi, or a Friday over a Saturday concert, whereas others may have the opposite preferences. Even those who have the same preferred product often differ as to the strength of their preferences. Some would be willing to pay considerably more for their preferred option than for a less preferred option (e.g., an Eastern Caribbean versus a Western Caribbean cruise), but others may be willing to pay only a slightly higher price for the former than for the latter. Because an individual consumer's valuations are often unobservable to the seller, such buyer heterogeneity generally reduces profit.

Our analysis identifies two conceptually different mechanisms that use consumer choices involving uncertainty to help the seller deal with unobservable buyer heterogeneity: (1) homogenizing heterogeneous consumers and (2) separating heterogeneous consumers. We use advance selling and probabilistic selling as two excellent examples to illustrate these two different mechanisms. Under advance selling, the seller uses advance sales to encourage consumers to make decisions before their consumption states are known, i.e., their decisions are based on their *expected* rather than *realized* valuations, or when their valuations are more homogeneous than they are in the spot period. As a result, by offering consumers a choice involving uncertainty about their future consumption states, advance selling motivates buyers to make advance purchases, which effectively homogenizes heterogeneous consumers. Under probabilistic selling, the seller uses a probabilistic good to induce consumers to reveal their "type" via self-selection. As a result, probabilistic selling, by offering consumers a choice involving uncertainty about their product assignment, motivates those with weak product preferences to choose the uncertain option, which effectively separates heterogeneous consumers.

Because the underlying motivation of the two mechanisms differs substantially, their profit advantages depend on the characteristics of buyer heterogeneity of the underlying market. In particular, we characterize a two-product market via two forms of heterogeneity: (1) *Max_Value-Heterogeneity*, which is the variation

Figure 1 Positioning of the Current Paper



across consumers in the valuations of their preferred goods, and (2) *Strength-Heterogeneity*, which is the variation across consumers in the strengths of their preferences, i.e., how much more the preferred good is valued over the less preferred good. For example, consider a family entertainment center that offers roller skating and miniature golf. Consumers may differ in their preferred activity and how much they are willing to pay for it. *Max_Value-Heterogeneity* captures the variation across consumers in how much a preferred product is valued. Furthermore, consumers may also vary in how strongly they prefer one over the other (i.e., how much more they would be willing to pay for their preferred activity compared with that for their less preferred). *Strength-Heterogeneity* refers to the difference in relative strength between the consumer with the strongest and that with the weakest preference. Often, neither type of buyer heterogeneity is observable to the seller, which implies a potential profit disadvantage, as discussed earlier. We find that the relative profit advantage of exploiting these two different types of buyer uncertainty depends on the relative strength of these two types of buyer heterogeneity. In particular, our key findings are the following.

First, we find that *Max_Value-Heterogeneity* contributes to the value of both advance selling and probabilistic selling. As the variation in consumers' valuations for their preferred products grows, the difference between traditional selling and first-degree price discrimination also grows because the practice of charging all consumers the same price captures a smaller percentage of the available potential surplus in a market. Such an expansion in untapped surplus creates greater opportunities for alternative selling

strategies, such as advance selling and probabilistic selling.

Second, we find that *Strength-Heterogeneity* undermines the value of advance selling. Under advance selling, a consumer must commit to consuming a certain item, which may turn out to be her less preferred product. As consumers' preferences become stronger, such a commitment imposes a greater cost on consumers, and thus the seller has to charge a lower price in the advance period. Notice that the minimum possible strength preference is zero, i.e., a consumer who is indifferent when given a choice between two products. Thus, larger *Strength-Heterogeneity* implies that some consumers' preferences are stronger, and as a result the value of advance selling decreases.

Third, we find that probabilistic selling is most profitable when *Strength-Heterogeneity* is moderate. Probabilistic selling operates by price discriminating according to the strength of consumers' preferences, e.g., charging a higher price to consumers with stronger preferences. Thus, it is not particularly surprising that when there is very little heterogeneity in preference strength, probabilistic selling is unsuccessful at capturing much additional surplus in a market. What may be more interesting is that probabilistic selling will be most effective in markets where *Strength-Heterogeneity* is not too large. Note that the probabilistic good presents consumers with the possibility of consuming either of two products, and thus the price consumers are willing to pay for the probabilistic product will depend on their valuations for each of the products. As *Strength-Heterogeneity* rises, consumers, on average, have stronger preferences that, as noted in the previous paragraph, increase

Table 1 Examples of Customer Heterogeneity and Optimal Strategy

Assumptions and definition:

- A seller with two component products ($j = 1, 2$) faces three potential customers ($i = A, B, C$).
- The seller has no marginal costs, capacity constraints, or fixed costs to offering either advance sales or probabilistic products.

	Market 1 A(3,0), B(3,3), C(0,3)	Market 2 A(1,1), B(2,2), C(4,4)	Market 3 A(3,1), B(2,2), C(1,3)	Market 4 A(4,3), B(2,2), C(3,4)																																								
Profit	<table><tr><td rowspan="4">Profit</td><td rowspan="4">{</td><td>1st degree PD</td><td>\$9</td></tr><tr><td>TS</td><td>\$9 (0% loss)</td></tr><tr><td>AS</td><td>\$6 (33% loss)</td></tr><tr><td>PS</td><td>\$9 (0% loss)</td></tr></table> $\Pi^{TS} = \Pi^{PS} > \Pi^{AS}$	Profit	{	1st degree PD	\$9	TS	\$9 (0% loss)	AS	\$6 (33% loss)	PS	\$9 (0% loss)	<table><tr><td rowspan="4">Profit</td><td rowspan="4">{</td><td>1st degree PD</td><td>\$7</td></tr><tr><td>TS</td><td>\$4 (43% loss)</td></tr><tr><td>AS</td><td>\$7 (0% loss)</td></tr><tr><td>PS</td><td>\$4 (43% loss)</td></tr></table> $\Pi^{AS} > \Pi^{TS} = \Pi^{PS}$ $\frac{\Pi^{AS} - \Pi^{TS}}{\Pi^{TS}} = 75\%$	Profit	{	1st degree PD	\$7	TS	\$4 (43% loss)	AS	\$7 (0% loss)	PS	\$4 (43% loss)	<table><tr><td rowspan="4">Profit</td><td rowspan="4">{</td><td>1st degree PD</td><td>\$8</td></tr><tr><td>TS</td><td>\$7 (12.5% loss)</td></tr><tr><td>AS</td><td>\$6 (25% loss)</td></tr><tr><td>PS</td><td>\$8 (0% loss)</td></tr></table> $\Pi^{PS} > \Pi^{TS} > \Pi^{AS}$ $\frac{\Pi^{PS} - \Pi^{TS}}{\Pi^{TS}} = 14.3\%$	Profit	{	1st degree PD	\$8	TS	\$7 (12.5% loss)	AS	\$6 (25% loss)	PS	\$8 (0% loss)	<table><tr><td rowspan="4">Profit</td><td rowspan="4">{</td><td>1st degree PD</td><td>\$10</td></tr><tr><td>TS</td><td>\$8 (20% loss)</td></tr><tr><td>AS</td><td>\$9 (10% loss)</td></tr><tr><td>PS</td><td>\$8 (20% loss)</td></tr></table> $\Pi^{AS} > \Pi^{TS} = \Pi^{PS}$ $\frac{\Pi^{AS} - \Pi^{TS}}{\Pi^{TS}} = 12.5\%$	Profit	{	1st degree PD	\$10	TS	\$8 (20% loss)	AS	\$9 (10% loss)	PS	\$8 (20% loss)
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		PS	\$8 (20% loss)																																									
(Δ_S, Δ_V)	(3, 0)	(0, 3)	(2, 1)	(1, 2)																																								
$\Delta = \Delta_S + \Delta_V$	3	3	3	3																																								
$\gamma = \frac{\Delta_S}{\Delta_S + \Delta_V}$	$\gamma = 1$	$\gamma = 0$	$\gamma = 2/3$	$\gamma = 1/3$																																								

the gap between what a consumer would be willing to pay for her preferred product and what she would pay for an unknown product. Thus, greater *Strength-Heterogeneity* reduces the potential revenue from sales of the probabilistic good. Therefore, probabilistic selling is most advantageous when *Strength-Heterogeneity* is large enough to enable segmentation on the basis of preference strengths, but not so large that too few consumers are willing to purchase the probabilistic product.

The remainder of this paper is organized as follows. In §2, we present a series of examples to illustrate the relative advantages of advance selling and probabilistic selling. In §3, we introduce a model of demand that is sufficiently flexible to capture several of the most prominent models in the extant literature as sub-cases. In §4, we examine the profits when the firm only sells traditional products in the spot period and then calculate and compare the profit when the seller offers the additional option of either a probabilistic good or purchasing in advance. In §5, we summarize the insights from this analysis, offer concluding remarks, and suggest areas for future research. The appendix contains the analytical details.

2. Buyer Heterogeneity and Three Selling Strategies: An Illustration

In this section, we identify buyer heterogeneities in a multiproduct market and use a series of examples to illustrate that the relative attractiveness of three selling strategies—advance selling, probabilistic selling, and traditional selling—can be predicted based on the characteristics of buyer heterogeneity we define.

Consider the four hypothetical markets given in Table 1. In each of these markets, a seller offers two products and faces three potential segments of customers (A, B, C), each of equal size.² Each consumer will purchase at most one product, and a customer's valuations for the two products are v_1 and v_2 , respectively. We assume consumers are risk neutral. However, in §5, we discuss the implications of allowing for risk-averse consumers. As in real markets, the seller is unable to observe each customer's realized valuations but may know the overall distribution of consumers based on historical sales data.

² We need at least three consumers to maintain symmetry and illustrate the main results of this paper.

The four markets differ in the distribution of the consumers.

For each market, we calculate four types of profits: (1) under first-degree price discrimination (PD) (each consumer pays a price that equals her willingness to pay), (2) under traditional selling (no buyer uncertainty), (3) under advance selling (offering a choice involving buyer uncertainty about personal consumption states), and (4) under probabilistic selling (offering a choice involving buyer uncertainty about which product he or she will receive).³ As shown in Table 1, traditional selling suffers from a profit decrease in most of these markets (0%, 43%, 12.5%, and 20%, respectively). When buyer uncertainty is exploited, such a profit decrease can be fully eliminated (advance selling in market 2, probabilistic selling in market 3), partially reduced (advance selling in market 4), unaffected (probabilistic selling in markets 2 and 4), or even worsened (advance selling in market 3). Moreover, the optimal strategy varies across these examples: traditional selling in market 1, advance selling in markets 2 and 4, and probabilistic selling in markets 1 and 3. Can these seemingly patternless results be explained systematically and be predicted based on the characteristics of buyer heterogeneity of these markets?

To answer this question, we need first to identify variables that are capable of characterizing buyer heterogeneity in a multiple-product market and capturing the differences in buyer heterogeneity across different markets. As shown in Table 1, in each market, consumers differ in their preferred products, their willingness to pay for their preferred product, and their willingness to pay for their less preferred product. Let $v_{\theta H}$ be consumer θ 's value for her preferred product, and let $v_{\theta L}$ be consumer θ 's value for her less preferred product, where $v_{\theta L} \leq v_{\theta H} \forall \theta$ (i.e., each consumer has a higher valuation for her preferred product). We are able to systemically characterize the consumers' differences across markets by defining the two variables: (1) *Max_Value-Heterogeneity* (Δ_v), which is the variation across consumers in the values for their preferred product, and (2) *Strength-Heterogeneity* (Δ_s), which is the variation across consumers in the strength of their preferences (i.e., how much more the preferred good is valued over the less preferred good).

Formally,⁴

$$\begin{aligned} \text{Max_Value-Heterogeneity:} \\ \Delta_v &= \max_{\theta} [v_{\theta H}] - \min_{\theta} [v_{\theta H}], \\ \text{Strength-Heterogeneity:} \\ \Delta_s &= \max_{\theta} [v_{\theta H} - v_{\theta L}] - \min_{\theta} [v_{\theta H} - v_{\theta L}]. \end{aligned} \quad (1)$$

To characterize the *magnitude* of total buyer heterogeneity and the *relative importance* of each type of buyer heterogeneity in a given market, we define two additional variables: (1) *Total Heterogeneity* (Δ), which measures the total amount of buyer heterogeneity in a given market, and (2) *Relative Heterogeneity Ratio* (γ), which measures the proportion of buyer heterogeneity that is due to variation across consumers in the strength of their preferences (i.e., $1 - \gamma$ measures the proportion of buyer heterogeneity that is due to variation across consumers in the value for their preferred good). Formally,

$$\begin{aligned} \text{Total Heterogeneity: } \Delta &= \Delta_v + \Delta_s, \\ \text{Relative Heterogeneity Ratio: } \gamma &= \frac{\Delta_s}{\Delta}. \end{aligned} \quad (2)$$

With these measures, we now can quantify the consumer heterogeneity presented in the four markets in Table 1. For instance, in market 1, *Strength-Heterogeneity* (Δ_s) is 3, *Max_Value-Heterogeneity* (Δ_v) is 0, *Total Heterogeneity* (Δ) is 3, and *Relative Heterogeneity Ratio* (γ) is 1. Table 1 presents the values of $\{\Delta_s, \Delta_v, \Delta, \gamma\}$ for all four markets. Note that all four markets exhibit the same level of *Total Heterogeneity*, $\Delta = 3$; i.e., they only differ in the *Relative Heterogeneity Ratio*, γ .

As shown in Table 1, market 1 presents a case where consumers vary only in the strength of their preferences and do not differ in the valuation for their preferred product (i.e., $\Delta_s = 3, \Delta_v = 0, \gamma = 1$). That is, the market lacks *Max_Value-Heterogeneity* but is high on *Strength-Heterogeneity*. Market 2 presents a case where consumers vary only in their valuation for the preferred product but not in the strength of their preferences; i.e., $\Delta_s = 0, \Delta_v = 3, \gamma = 0$. That is, the market lacks *Strength-Heterogeneity* but is high on *Max_Value-Heterogeneity*. Although market 3 ($\Delta_s = 2, \Delta_v = 1$) and market 4 ($\Delta_s = 1, \Delta_v = 2$) present variation in both dimensions, the variation in preference strengths

³ Consumers are assumed to have the equal probability to be in each of the three segments. The distribution is known to both the buyer and the seller in the advance period. See the appendix for the derivation of these profits.

⁴ We have chosen the Max – Min functions for analytical convenience. In the formal model introduced in the next section, we assume valuations are uniformly distributed over a continuous interval. Thus, the max-min construct is perfectly correlated with variance, and either measure of variation yields identical results. For more general valuation functions, constructing appropriate measures of variation would be important to calculate the cutoff for which advance selling or probabilistic selling would be preferred. However, the intuition behind our results should continue to be valid.

is significantly larger than the variation in maximum valuations in market 3 ($\gamma = 2/3$), but the opposite holds in market 4 ($\gamma = 1/3$).

Examining the characteristics of buyer heterogeneity and the optimal strategy shown in Table 1, we are able to identify a systematic pattern, which is stated in the following finding.

FINDING 1 (BASED ON EXAMPLES IN TABLE 1). *Under the same high level of Total Heterogeneity, the optimal strategy is as follows:*

(a) *Advance selling when γ is low (market 2, which has strong Max_Value-Heterogeneity but weak Strength-Heterogeneity);*

(b) *Probabilistic selling when γ is in a mid-range (markets 3 and 4, where both types of heterogeneity are important and neither significantly dominates the other);*

(c) *Traditional selling when γ is high (market 1, which has weak Max_Value-Heterogeneity but strong Strength-Heterogeneity).*

Finding 1 illustrates that using the two types of consumer heterogeneity in a multiple-product market defined in this research, we are able to predict the relative attractiveness of the three strategies for the four markets given in Table 1. In the next two sections, we model consumer heterogeneity in a general model and illustrate that the patterns stated in Finding 1 are generally applicable (Proposition 3).

3. Modeling Buyer Heterogeneity in a Multiproduct Market

In this section, we present a multiproduct market that captures the two types of heterogeneities introduced in the preceding sections. To focus on the basic concepts and economic intuition, we consider a market with two symmetric products, $j = 1, 2$; i.e., product j is the preferred product for a given consumer with a probability of $1/2$. We assume a cost c is incurred to produce a unit of either product. To examine how buyer heterogeneities may affect the seller in a rather general setting, we construct the valuation functions, $v_{\theta H}$ and $v_{\theta L}$, so that they are capable of capturing the various buyer heterogeneities defined above yet are flexible enough to be consistent with some prominent models of two-good markets proposed in extant studies. Specifically, we consider the following three two-product models suggested in the literature.

(1) *The Model of Common Reservation Values:* Gale and Holmes (1992) present a two-product preference model where the values of one's preferred good are the same for all consumers (i.e., $v_{\theta H} = v_H$, $\forall \theta$) and consumers differ only in their less preferred good. Thus, all buyer heterogeneity is in the form of variation in the strength of preferences. Thus, for this model,

$$v_{\theta H} = v_H, \quad \Delta_v = 0, \quad \Delta = \Delta_s, \quad \gamma = 1. \quad (3)$$

(2) *The Model of Perfect Substitutes:* The single-product model presented by Xie and Shugan (2001) can be viewed as a two-product market with perfect substitution (i.e., consumers have the same valuation for both products, $v_{\theta H} = v_{\theta L}$), and all buyer heterogeneity is in the form of variation in the value of the preferred good. Thus, for this model,

$$v_{\theta H} = v_{\theta L}, \quad \Delta_s = 0, \quad \Delta = \Delta_v, \quad \gamma = 0. \quad (4)$$

(3) *Standard Hotelling Model:* In this standard two-good model, consumers share a common reservation value for their ideal product but have differing tastes. In particular, consumers are located uniformly along a linear segment that stretches between the two products located at the two ends of the line $(0, 1)$. In this model, consumer θ 's valuation for her preferred product is $v_{\theta H} = 1 - v_{\theta L}$.⁵ Note that although this model allows consumers to differ both in the maximum value that can be obtained from one of the two existing products and in how strongly they prefer one product over the other, it is a specific case of markets with the two types of heterogeneity (where $\Delta_v = 1 - 0.5 = 0.5$, $\Delta_s = 1$). Thus, for this model,

$$v_{\theta H} = 1 - v_{\theta L}, \quad \Delta_v = 0.5, \quad \Delta_s = 1, \\ \Delta = 1.5, \quad \gamma = \frac{2}{3}. \quad (5)$$

Each of these three models focuses on a special situation in markets with buyer heterogeneity. To examine the related issues more broadly, we present a more general model below, which can be reduced to each of the three two-product models discussed above.

Specifically, let R denote the lowest valuation for one's preferred product among all consumers; i.e., $R = \min_{\theta} [v_{\theta H}]$, where $\theta \sim U[0, \Delta]$. We consider the following value functions for a given consumer θ :

$$v_{\theta H} = R + (1 - \gamma)\theta, \\ v_{\theta L} = R + (1 - 2\gamma)\theta. \quad (6)$$

First, the value functions given in (6), although simple, are quite general, such that the three existing analytical models of two products discussed above can be viewed as special cases by varying a single parameter, γ . It is easy to see that the value functions given in (6) reduce to

- *The Model of Common Reservation Values* when letting $\gamma = 1$ (i.e., $v_{\theta H} = R \forall \theta$);
- *The Model of Perfect Substitutes* when letting $\gamma = 0$ (i.e., $v_{\theta H} = v_{\theta L} \forall \theta$);
- *Standard Hotelling Model* when letting $\gamma = \frac{2}{3}$ (i.e., when $R = \frac{1}{2}$, $v_{\theta H} = 1 - v_{\theta L}$, $\forall \theta$).

⁵ The valuations for the preferred and nonpreferred products are $v_{\theta H} = 1 - \min[\theta, 1 - \theta]$ and $v_{\theta L} = 1 - \max[\theta, 1 - \theta]$, respectively.

Second, the value functions given in (6) can characterize markets that differ in the relative significance of the two types of buyer heterogeneity, *Max_Value-Heterogeneity* and *Strength-Heterogeneity*. Specifically, individual consumer θ 's valuation for her preferred product (i.e., $v_{\theta H} = R + (1 - \gamma)\theta$) decreases with γ , but the difference in value between her preferred and less preferred product (i.e., $v_{\theta H} - v_{\theta L} = \gamma\theta$) increases with γ . This suggests that a small γ represents markets with a large variation in the value of the preferred product but a small variation in consumers' preference strengths, whereas a large γ represents markets where buyer heterogeneity is largely reflected in the strength of buyer preference but not in their valuations of the preferred product.

It is important to note that consumers are often clearer about their product preference and valuation at the time of consumption (the "spot" period) but face greater uncertainty when the consumption is in the distant future (the "advance" period). Following the literature on advance selling, we consider two periods: (1) a spot period in which consumers are fully aware of their consumption states and thus also aware of their product preference and valuation, and (2) an advance period in which consumers are aware of the distribution of their consumption states but are uncertain about their future realized product valuation. In terms of the model, consumers only know the distribution of θ in the advance period but learn their own realized value of θ in the spot period.

4. Three Selling Strategies

In this section, we examine how the characteristics of buyer heterogeneity of a given market affect the relative profit of three selling strategies: (1) traditional selling (TS), (2) advance selling (AS), and (3) probabilistic selling (PS). These strategies differ in *when* and *how* the products are offered for sale. The firm first decides which selling strategy to utilize (which involves making an investment, in the case of AS and PS). Then the firm selects its prices. Finally, consumers choose the purchase option that yields the highest expected surplus.

Specifically, under TS, the seller offers each specified product for sale only in the spot period. Under AS, the seller offers consumers the additional option of buying the specified products in the advance period. This strategy requires an investment of $F_{AS} \geq 0$, which reflects the implementation costs associated with providing a mechanism for taking advance orders and for fulfilling these advance orders. Under PS, the seller offers consumers a different additional option, that of buying a "probabilistic product"—a new type of product offering defined by Fay and Xie (2008) in which the product to be consumed is determined via a random

draw of a set of distinct items and hence is unknown to the buyer before payment. This strategy requires an investment, $F_{PS} \geq 0$, which reflects the cost to develop the infrastructure to create such goods and to display them to consumers. To ensure both AS and PS are viable strategies, we assume that the costs (e.g., c , F_{AS} , and F_{PS}) are sufficiently small so that at least under some conditions, adoption of AS or PS can improve profit.⁶

Note that, mathematically, TS can be viewed as a special case of AS (i.e., when advance sales are zero) or a special case of PS (i.e., when sales for the probabilistic good are zero).⁷ To facilitate comparisons, we conceptually separate these strategies by requiring a positive advance demand for the advance selling strategy and a positive demand for the probabilistic product for the probabilistic selling strategy. Precisely, the three selling strategies are defined as follows.

1. Traditional selling (TS), under which the seller offers the specified products in the spot period only.
2. Advance selling (AS), under which the seller offers the specified products in both the advance and the spot period and is subject to a positive demand for advance sales.
3. Probabilistic selling (PS), under which the seller offers each specified product and a probabilistic good in the spot period and is subject to a positive demand for the probabilistic product.

It is interesting to recognize that from a consumer's perspective, the three strategies differ in terms of whether one is facing a choice with uncertainty and, if so, what type of uncertain choice is offered. Under the TS strategy, buyers do not face uncertainty: they know which product they will be consuming and they know their valuations for each product. Under the AS strategy, buyers are given a choice to buy in advance before they learn their product preferences and valuations, which exploits buyers' uncertainty about their own consumption states. Under the PS strategy, buyers are given a choice to buy the probabilistic good that can be any one of a number of specified products, which introduces buyers' uncertainty about which product they will actually receive.

In the following exposition, we assume that all consumers are willing to consider the option of purchasing in advance (but will do so only if such a purchase maximizes their expected surplus). Our objective is to derive the key economic insights with a basic model without introducing unnecessary mathematical complexity.

⁶ See the appendix for formal expressions of these specific conditions.

⁷ Thus, if there were no implementation costs, both PS and AS must weakly dominate TS.

4.1. Traditional Selling

Under the TS strategy, the firm only sells specified products in the spot period (at prices P_j^{TS} , $j = 1, 2$). Each consumer purchases the good that yields the highest surplus, provided that her surplus is nonnegative. Given symmetrical demand for the two goods, the optimal prices are also symmetric: $P_1^{\text{TS}} = P_2^{\text{TS}} \equiv P^{\text{TS}}$. Therefore, all consumers for whom $v_{\theta H} \geq P^{\text{TS}}$ will purchase their preferred good. Hence, for any given price, P^{TS} , the seller's profit can be calculated by (7):

$$\Pi^{\text{TS}} = (P^{\text{TS}} - c) \int_{v_{\theta H} = P^{\text{TS}}}^{R + \Delta_v} \frac{1}{\Delta_v + \Delta_s} dv_{\theta H}. \quad (7)$$

As shown in the appendix, for sufficiently low costs ($c \leq R - \Delta_v$), the optimal price under TS is $P^{\text{TS}*} = R$, under which the seller earns a maximum profit, $\Pi^{\text{TS}*} = R - c$. For the remainder of this paper, we assume $c \leq R - \Delta_v$. This condition implies that the seller is able to achieve full market coverage under TS (because among all consumers, R is the lowest valuation for one's preferred product). This setting rules out any potential demand disadvantage of TS. That is, if we find AS or PS to be more advantageous than is TS, such an advantage has to come from a source other than simple market expansion.

4.2. Advance Selling: Homogenizing Heterogeneous Consumers

In this section, we show why and when offering consumers an option to purchase in advance can help a multi-item seller to improve profit. We first derive the optimal price and profit under AS and then compare them with those under TS.

4.2.1. AS. Under the AS strategy, the seller offers consumers the option to purchase specified products both in the advance and spot periods. In the advance period, consumers are uncertain about their future consumption states. Hence, they have to make their advance purchase decisions based on their expected rather than realized valuations. Given the valuation function specified in (6), consumers' expected value of each product is

$$EV = \int_{\theta=0}^{\Delta_v + \Delta_s} \frac{v_{\theta H} + v_{\theta L}}{2(\Delta_v + \Delta_s)} d\theta = R + \frac{2\Delta_v - \Delta_s}{4} \quad \forall \theta. \quad (8)$$

Consumers will buy in the advance period if they receive an equal or higher surplus from making an advance purchase than from waiting for the spot period, which clearly depends on buyer expectations about the spot price. The advance selling literature (e.g., Xie and Shugan 2001, Shugan and Xie 2005) suggests that a seller is capable of committing in advance to announced spot prices if the latter are observable at the time when customers are making advance purchases. Note that this applies to most advance selling markets, because sellers often offer "advance price"

and "gate price" simultaneously (e.g., tickets for concerts, sport games, amusement parks, and exhibitions, or registration fees for conferences, recreation activities, and professional training classes). Following this reasoning, we allow such seller credibility. When all consumers arrive in the advance period, the most profitable way to generate positive advance sales is to set sufficiently high spot prices ($> \max[V_{\theta H}]$), which eliminates any incentive for consumers to delay purchasing until the spot period (see the appendix for further details). Consequently, consumers will purchase in advance if the advance price is no larger than the expected value of a given product. Hence, the profit under AS is

$$\Pi^{\text{AS}} = \begin{cases} \text{Min}[P_1^{\text{AS}}, P_2^{\text{AS}}] - c - F_{\text{AS}} & \text{if } \text{Min}[P_1^{\text{AS}}, P_2^{\text{AS}}] \leq R + \frac{2\Delta_v - \Delta_s}{4}, \\ -F_{\text{AS}} & \text{if } \text{Min}[P_1^{\text{AS}}, P_2^{\text{AS}}] > R + \frac{2\Delta_v - \Delta_s}{4}. \end{cases} \quad (9)$$

As shown in Lemma A1, the optimal advance price is $P_j^{\text{AS}} = EV = R + (2\Delta_v - \Delta_s)/4$.

4.2.2. Why and When Advance Selling Can Help.

Proposition 1 summarizes the key results from comparing the AS strategy to the TS strategy.

PROPOSITION 1 (ADVANCE SELLING VS. TRADITIONAL SELLING). (a) *Offering consumers the choice to purchase in advance allows a multiproduct seller to homogenize heterogeneous consumers.*

(b) *Such homogenization creates a profit advantage over traditional selling if the market possesses sufficient Max_Value-Heterogeneity.*

(c) *A higher level of Strength-Heterogeneity makes it less likely for advance selling to be advantageous.*

Formally, $\Pi^{\text{AS}} > \Pi^{\text{TS}}$, iff $\Delta_v > \hat{\Delta}_v$, where $\partial \hat{\Delta}_v / \Delta_s > 0$ and $\hat{\Delta}_v$ is given in the appendix.

Previous studies (Shugan and Xie 2000, Xie and Shugan 2001) have illustrated that in a single-product market, AS can improve profit because offering advance sales motivates consumers to transact with the seller in the advance period when they are more homogeneous (i.e., before they realize their individual consumption states for each product because of their own idiosyncratic preferences). Our model extends this analysis to a multiple-product setting. Unlike the case of a single product, however, in a multiproduct market, consumers have to decide not only *when* to buy but also *which* product to buy. Also, when facing multiple alternatives, consumers differ not only in their valuation for any given product but also in the strength of their product preferences. These differences lead to some intriguing new findings about advance selling. Specifically, Proposition 1 reveals that a multiproduct seller can benefit from homogenizing demand via offering advance sales only in markets

where buyers significantly differ in their valuation for their preferred products (i.e., a high *Max_Value-Heterogeneity*) but do not vary greatly in the strength of their product preferences.

First, a sufficiently high level of *Max_Value-Heterogeneity* ($\Delta_v > \hat{\Delta}_v$) is necessary for the profit advantage of advance selling. Large *Max_Value-Heterogeneity* implies that unobservable buyer heterogeneity significantly undermines profits under TS. Thus, there exists a sufficient potential for profit increase when the seller uses advance selling to homogenize demand. Specifically, under TS, the seller achieves full market coverage by setting a price equal to the value of the consumer with the lowest valuation for her preferred product ($P^{\text{TS}} = R$); i.e., all consumers pay the same low price, although many have higher valuations. The larger the variation in consumers' maximum willingness to pay, the more money would be left on the table under TS, and the more the seller can gain by transacting with consumers before such buyer heterogeneity is realized. Second, an increase in *Strength-Heterogeneity* makes it harder to achieve a profit advantage from AS ($\partial \hat{\Delta}_v / \Delta_s > 0$) because higher *Strength-Heterogeneity* implies a higher cost for the seller to induce advance sales. Under AS, the seller induces advance sales by setting a price equal to the expected valuation ($P_j^{\text{AS}} = \text{EV}$; see the proof in the appendix). Unlike a single product market, however, in a multiproduct market, an advance sale not only requires a commitment to purchase, but it also requires a commitment to purchase a certain product. A sufficiently high Δ_s implies that consumers differ significantly as to how much more they value their preferred product compared with their less preferred product. Hence, making a purchase without knowing their future consumption states involves a high opportunity cost because some advance buyers will be committing to consume a product that may turn out to have a much lower value than does their preferred product. The expectation that one may receive a very low-valued product diminishes consumers' willingness to purchase in advance (i.e., EV in (8) decreases with γ) and thus increases the cost of inducing advance purchase.

Taking the two conditions together, the multiproduct seller benefits from the demand homogenization function of AS only when the seller suffers a significant profit loss under TS (i.e., Δ_v is large) but is not subject to a significant cost of inducing advance sales (i.e., Δ_s is small).

4.3. Probabilistic Selling: Separating Heterogeneous Consumers

In this section, we show why and when offering consumers an additional option to buy a probabilistic good can help a multi-item seller to improve profit.

4.3.1. PS. Under this selling strategy, the seller gives consumers the option to purchase a probabilistic good. Thus, in the spot period, the firm sells both specified products (at P_1^{PS}) and a probabilistic, or "opaque," good (at P_o^{PS}), for which a buyer will be randomly assigned product 1 or product 2. Each consumer chooses whether to purchase product 1, product 2, or the probabilistic good depending on which option yields the highest expected surplus net of the purchase price. A consumer chooses not to purchase if all three of these purchase options yield negative surpluses. Fay and Xie (2008) provide extensive analyses to demonstrate that a seller typically finds an equal probability of assignments optimal under various demand conditions. We allow consumers to expect such an optimal decision by the seller. Thus, the expected value to consumer θ for the opaque good equals the average value of the two specified products, $v_{\theta o}$:

$$v_{\theta o} = \frac{v_{\theta H} + v_{\theta L}}{2} = R + \frac{\theta(\Delta_s - 2\Delta_v)}{2(\Delta_v + \Delta_s)}. \quad (10)$$

Given the symmetric demand function, the optimal prices for the specified products are symmetric: $P_1^{\text{PS}} = P_2^{\text{PS}} \equiv P^{\text{PS}}$. Let $D_s(P^{\text{PS}}, P_o^{\text{PS}})$ denote the total demand for the specified products, which includes consumers who are willing to buy their preferred product rather than the probabilistic good at a lower price (i.e., $v_{\theta H} \geq P^{\text{PS}}$, $v_{\theta H} - P^{\text{PS}} \geq v_{\theta o} - P_o^{\text{PS}}$). Let $D_o(P^{\text{PS}}, P_o^{\text{PS}})$ denote the demand for the probabilistic good, which includes consumers who receive a higher surplus from the uncertainty choice than from buying their preferred product (i.e., $v_{\theta o} \geq P_o^{\text{PS}}$, $v_{\theta o} - P_o^{\text{PS}} > v_{\theta H} - P^{\text{PS}}$). The seller's profit under the PS strategy is

$$\Pi^{\text{PS}} = (P^{\text{PS}} - c)D_s(P^{\text{PS}}, P_o^{\text{PS}}) + (P_o^{\text{PS}} - c)D_o(P^{\text{PS}}, P_o^{\text{PS}}) - F_{\text{PS}}. \quad (11)$$

Lemma A1 in the appendix presents the optimal prices and profit under PS. From Lemma A1, we calculate

$$\Pi^{\text{PS}} - \Pi^{\text{TS}} = \begin{cases} \frac{(\Delta_v)^2}{2\Delta_s} - F_{\text{PS}} & \text{if } \Delta_s \geq 2\Delta_v, \\ \frac{\Delta_s}{8} - F_{\text{PS}} & \text{if } \Delta_s < 2\Delta_v. \end{cases} \quad (12)$$

4.3.2. Why and When Probabilistic Selling Can Help. We now compare the PS and TS strategies. Proposition 2 summarizes the key results.

PROPOSITION 2 (PROBABILISTIC SELLING VS. TRADITIONAL SELLING). (a) *Offering consumers the choice to purchase a probabilistic good allows a multiproduct seller to separate heterogeneous consumers.*

(b) *Such separation increases profit if the market possesses (i) sufficient Max_Value-Heterogeneity and (ii) a mid-level of Strength-Heterogeneity.*

Formally, $\Pi^{\text{PS}} > \Pi^{\text{TS}}$ iff $\Delta_v > \bar{\Delta}_v$ and $\hat{\Delta}_s \leq \Delta_s < \bar{\Delta}_s$, where $\bar{\Delta}_v$, $\hat{\Delta}_s$, and $\bar{\Delta}_s$ are given in the appendix.

First, Proposition 2 reveals that as opposed to AS, which aggregates demand, PS separates demand. Like TS, under PS, the seller transacts with buyers after they realize their own idiosyncratic preferences (which are unobserved by the seller). Different from TS, however, PS allows the seller to separate heterogeneous consumers without requiring additional information. This unique separation function of PS is accomplished by offering consumers an additional purchase option, that of buying a probabilistic good. Under this strategy, the specified products at a high price ($P^{\text{PS}} > R$) attract high valuation consumers who have relatively strong preferences and thus are willing to pay a premium to secure their preferred products, and the discounted probabilistic product ($P_0^{\text{PS}} \leq R$) attracts consumers with weak preferences who are willing to accept the uncertain product assignment in exchange for a discounted product price. This separation function of PS offers the seller an opportunity to implement price discrimination, which is not possible under TS.

Second, Proposition 2 shows that such a separation function of PS is beneficial compared with TS only in markets with two characteristics. First, as in the case of AS, the profit advantage of PS requires a sufficiently high level of *Max_Value-Heterogeneity* ($\Delta_v > \bar{\Delta}_v$) because a high level of buyer variation in maximum valuations increases the seller's information disadvantage under TS, thus creating the potential for PS to improve profit. Second, unlike the case of AS, in which *Strength-Heterogeneity* makes AS less likely to be advantageous, a mid-level of *Strength-Heterogeneity* is required for PS to be advantageous ($\hat{\Delta}_s \leq \Delta_s < \bar{\Delta}_s$). This is because an increase in *Strength-Heterogeneity* creates two opposite effects on the profit of PS: it (1) enhances the benefit of market segmentation and (2) reduces profit from the sales of the probabilistic good. As discussed earlier, PS separates heterogeneous consumers based on the strength of their product preferences, i.e., selling the specified goods at full price to those with strong preferences and selling the probabilistic good at a discounted price to those with weak preferences. On the one hand, such a segmentation benefit increases with *Strength-Heterogeneity* because a higher level of buyer variation in the strengths of their product preferences increases the seller's information disadvantage under TS, thus increasing the potential for PS to improve profit. On the other hand, for any given level of *Max_Value-Heterogeneity*, an increase in *Strength-Heterogeneity* implies an increase in buyer product value variation for their less preferred product. Because the price of the probabilistic good must be low enough to attract the marginal consumer who

is indifferent between purchasing the probabilistic good or the full price product, too much buyer value variation for the less preferred product implies either a low price or low sales for the probabilistic good.

The finding that too much heterogeneity in product preference strength may actually destroy the profit advantage of PS is not intuitive, especially given that the profit advantage of PS is fundamentally motivated by the existence of such heterogeneity in preference strength. We are able to uncover this new and important feature of PS because our model explicitly characterizes the buyer heterogeneity of a given market in two different dimensions.

4.4. Homogenization or Separation:

Advance Selling vs. Probabilistic Selling

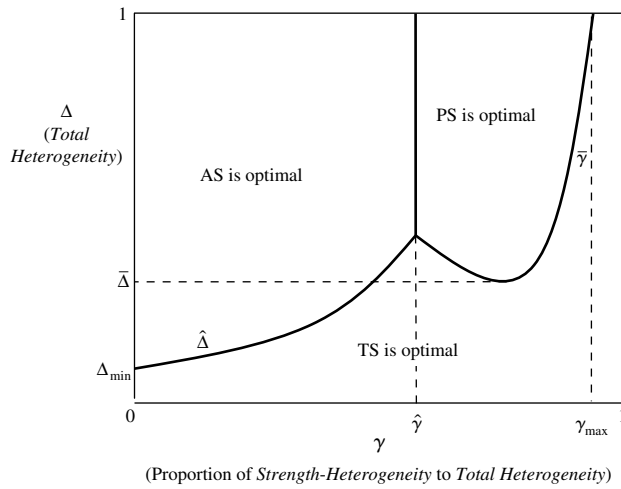
The preceding analysis illustrates that although both AS and PS create buyer uncertainty, these two types of uncertainty help the seller via fundamentally different economic mechanisms: AS homogenizes heterogeneous consumers by motivating them to purchase before their heterogeneity is realized, whereas PS separates heterogeneous consumers by motivating them to reveal their heterogeneity. Next, we derive conditions under which homogenization helps the seller more than does separation, where neither is advantageous and where the two strategies can be perfect substitutes. Proposition 3 and Corollary 1 summarize our main findings.

PROPOSITION 3 (THE OPTIMAL SELLING STRATEGY). *The two strategies, homogenizing or separating heterogeneous consumers via offering consumers an additional choice involving uncertainty, can help or hurt the seller depending on (a) the total amount of buyer heterogeneity and (b) the relative importance of the two types of buyer heterogeneity. The conditions under which each of the strategies dominates the others are given in the table below.*

Optimal strategy	Conditions required	
	Total buyer heterogeneity ($\Delta = \Delta_v + \Delta_s$)	Relative importance of Strength-Heterogeneity vs. Max_Value-Heterogeneity ($\gamma = \Delta_s / \Delta$)
Advance selling	High	Low \Rightarrow { Strong Max_Value-Heterogeneity Weak Strength-Heterogeneity
Probabilistic selling	High	Mid-range \Rightarrow { Both types of heterogeneity are important Neither significantly dominates the other
Traditional selling	High	High \Rightarrow { Weak Max_Value-Heterogeneity Strong Strength-Heterogeneity
	Low	Does not matter

Formally, AS is optimal if $\Delta > \bar{\Delta}$ and $\gamma < \hat{\gamma}$; PS is optimal if $\Delta > \bar{\Delta}$ and $\hat{\gamma} < \gamma < \bar{\gamma}$; otherwise, TS is optimal. Closed-form expressions for $\bar{\Delta}$, $\hat{\gamma}$, and $\bar{\gamma}$ are provided in the appendix.

Figure 2 Optimal Price and Profit



COROLLARY 1 (EQUALITY OF AS AND PS). *The seller can achieve the same profit improvement by homogenizing or separating heterogeneous consumers in markets with sufficient total buyer heterogeneity and a moderate amount of both types of buyer heterogeneity. Formally, $\Pi^{PS} = \Pi^{AS} > \Pi^{TS}$ iff $\Delta > \Delta^*$ and $\gamma = \hat{\gamma}$.*

Proposition 3 reveals that the optimal strategy critically depends on two variables that characterize buyer heterogeneity in the market: (a) the total amount of buyer heterogeneity that exists (Δ) and (b) the relative importance of the two types of buyer heterogeneity (γ). We illustrate the results of Proposition 3 in Figure 2.

As shown in Figure 2, with too little total variation across consumers ($\Delta \leq \Delta_{\min}$), TS is optimal regardless of the relative importance of the two types of buyer heterogeneity ($0 \leq \gamma \leq 1$). When there is sufficient total buyer heterogeneity ($\Delta > \Delta_{\min}$), however, the seller may benefit from offering uncertainty choices to consumers, but the best type of uncertainty depends critically on the source of the consumer heterogeneity. Specifically, in markets where buyers differ substantially in their valuations for their preferred product rather than in the strengths of their product preference ($\gamma < \hat{\gamma}$), then the seller should introduce the advance purchase option. In markets where both types of buyer heterogeneity are sufficiently high ($\hat{\gamma} < \gamma < \bar{\gamma}$), it is best to introduce the probabilistic good option. Finally, in markets where *Strength-Heterogeneity* overwhelmingly dominates *Max_Value-Heterogeneity* ($\gamma > \bar{\gamma}$, where $\bar{\gamma} \leq \gamma_{\max}$), the seller fails to gain from either AS or PS.

As discussed earlier, with sufficient total variation across consumers (a large Δ), the seller suffers significantly from uncertainty under TS, which creates opportunities for profit improvement via AS or PS. However, these two strategies are most beneficial in very diverse situations. AS is more effective

than PS at addressing *Max_Value-Heterogeneity*. However, PS is better than AS at capitalizing on *Strength-Heterogeneity*. Under AS, the seller forces consumers to commit to purchase prior to learning their individual product valuations. Because consumers do not know their spot valuations, they decide whether to purchase based on expected, or average, values, and thus the impact of *Max_Value-Heterogeneity* is minimized. On the other hand, under PS, purchase decisions are made after valuations are realized. PS enables the firm to segment customers according to the strength of their preferences; i.e., consumers self-select into purchasing the probabilistic product or the specified products. Such price discrimination is not feasible under AS because purchase decisions are made prior to consumers knowing the strength of their preferences.

An intriguing result of our analysis is that it is possible for the seller to benefit equally from offering consumers either an option to purchase in advance or an option to purchase a probabilistic product (Corollary 1), even though the two product offerings work very differently, i.e., homogenizing demand under AS and separating demand under PS. In effect, there is a trade-off between addressing *Max_Value-Heterogeneity* (at which AS is better) and addressing *Strength-Heterogeneity* (which is only possible through PS). With a moderate amount of both types of buyer heterogeneity, these two effects balance out, and thus the seller equally benefits from introducing either additional product offering. When $\Delta = \Delta^*$, this equivalent advantage is equal to the investment cost in either AS or PS. Thus, investing in either AS or PS is optimal as long as the *Total Heterogeneity* is sufficiently large; i.e., $\Delta > \Delta^*$.

5. Discussion and Conclusion

In this paper, we study two selling strategies, advance selling and probabilistic selling, that have the common characteristic of helping the seller address buyer heterogeneity by inducing sales involving buyer uncertainty. In this section, we first discuss some of the important insights derived from our analysis. Then, we conclude the paper by offering suggestions for future research.

5.1. New Insights

5.1.1. Impact of Buyer Heterogeneity: Homogenizing or Separating Heterogeneous Consumers. Our analysis demonstrates that a seller may be able to improve profit by addressing buyer heterogeneity in profoundly different ways. In particular, the AS strategy offers consumers a choice of buying in an advance period while they are uncertain about their future consumption states, which allows the seller to aggregate consumers who would be differentiated at a

later point in time. In contrast, under the PS strategy, the seller offers consumers an additional purchase option where the product assignment is unknown to them. This strategy allows the seller to separate consumers into two segments—one group that strongly prefers one product over the other, and one group that only has weak preferences between the two specified products. Various other mechanisms, such as coupons (Narasimhan 1984, Gerstner et al. 1994), quantity discounts (Oi 1971, Dolan 1987), and versioning (Varian 2000), rely on a similar approach (but utilize different sources of buyer heterogeneity) to segment consumers. Importantly, our analysis, especially Proposition 3, provides insight into the conditions under which aggregating is more profitable than separating, and vice versa.

A parallel can be drawn between these results and those from the bundling literature. Pure bundling is a method of aggregating consumers. Such aggregation is beneficial to the seller if demand for the bundle is more homogeneous than is demand for each individual item, as would be the case if an individual's valuations for the items are negatively correlated (Adams and Yellen 1976), or if the bundle consists of a very large number of items and each valuation is drawn independently from a common distribution (Bakos and Brynjolfsson 1999). On the other hand, mixed bundling induces consumers to self-select whether to purchase the bundle or individual items, thus allowing the seller to separate customers according to unobserved heterogeneity. Such separation is advantageous when values for the bundle and individual items vary significantly across consumers, as would be the case if item valuations are not negatively correlated (McAfee et al. 1989) or consumers tend to be knowledgeable about what items they value, which is only a small subset of the full bundle (Basu and Vitharana 2009). Similar to mixed bundling, a menu of bundles can be used to separate heterogeneous consumers (Kolay and Shaffer 2003).

5.1.2. Dealing with a Seller's Uncertainty: Reducing or Revealing Buyer Heterogeneity. It is also interesting to note that the AS and PS strategies represent very different ways in which a seller can deal with its information disadvantage, i.e., the fact that consumers know their valuations and product preferences in the spot period, but the seller does not. One approach is to try to eliminate this informational asymmetry. When sales are made in advance, the seller and the consumers have the same information; i.e., they both only know the distribution of preferences, not an individual's spot valuation. A second approach to dealing with an information advantage is to try to minimize the effects of this information disadvantage. In particular, offering a probabilistic good encourages consumers to reveal the strength of their preferences, thus

allowing the seller to charge differentiated prices to consumers who have unobservable heterogeneities. Our results indicate that reducing heterogeneity is advantageous in markets where buyer heterogeneity is largely due to *Max_Value-Heterogeneity* but not to *Strength-Heterogeneity*. However, inducing consumers to reveal their differences is optimal in markets where the two types of heterogeneity are more balanced. Finally, neither strategy can enhance profit in markets where *Max_Value-Heterogeneity* is not sufficiently large.

5.2. Future Research

Probabilistic selling is a new marketing strategy that to date has received only limited attention. Therefore, many interesting questions remain. For instance, it would be interesting to incorporate risk aversion into future analysis. Attitudes toward the probabilistic good and to purchasing in advance may depend on one's disposition towards risk. Note that the number of consumers who experience uncertainty at the time of purchase varies across the three selling strategies. Under TS, no consumers face uncertainty because they all know their values for the products at the time of purchase. Under AS, all consumers face uncertainty because all consumers purchase in the advance period, i.e., prior to learning their valuations. Under PS, only consumers who purchase the probabilistic good will experience uncertainty. Furthermore, the consumers who choose to purchase the probabilistic good are the ones that have the weakest preferences and thus experience a lower magnitude of uncertainty. Thus, we expect risk aversion to have a greater negative impact upon the advance selling strategy than upon the probabilistic selling strategy.

As a preliminary test of this conjecture, we consider how allowing for risk averse consumers would affect the four markets that we previously analyzed in Table 1. In particular, we model risk aversion by assuming consumers have the following concave utility function: $U = \sqrt{W + V_i - P_i}$. The utility from choosing not to purchase, U_0 , would be captured by setting $V_i = P_i = 0$. In Table 2, we compare the profit for each of the selling strategies depending on whether consumers are risk neutral or risk averse (for these calculations we assume $W = 5$). For instance, consider market 1. If a consumer purchases good 1 in advance at a price P_A , she will have a value of $V_i = 3$ with a probability of $2/3$ and a value of $V_i = 0$ with a probability of $1/3$. Thus, under risk aversion, $U = (2\sqrt{8 - P_A} + \sqrt{5 - P_A})/3$. A consumer would purchase in advance only if $U \geq U_0$, or $P_A \leq 1.89$. Under AS, the firm will set $P_A = 1.89$, sell three units, and earn a profit of \$5.67. Notice that in all four markets, risk aversion decreases the profit of AS but does not affect PS or TS. Furthermore, the rankings of the selling strategies are unaffected. In these examples, PS

Table 2 The Impact of Risk Aversion

	Market 1 A(3, 0), B(3, 3), C(0, 3)	Market 2 A(1, 1), B(2, 2), C(4, 4)	Market 3 A(3, 1), B(2, 2), C(1, 3)	Market 4 A(4, 3), B(2, 2), C(3, 4)
Profit	$\text{Profit} \begin{cases} \text{TS} & \$9 \\ \text{AS} & \$6 \\ \text{PS} & \$9 \end{cases}$	$\text{Profit} \begin{cases} \text{TS} & \$4 \\ \text{AS} & \$7 \\ \text{PS} & \$4 \end{cases}$	$\text{Profit} \begin{cases} \text{TS} & \$7 \\ \text{AS} & \$6 \\ \text{PS} & \$8 \end{cases}$	$\text{Profit} \begin{cases} \text{TS} & \$8 \\ \text{AS} & \$9 \\ \text{PS} & \$8 \end{cases}$
Risk-neutral consumers	$\Pi^{\text{TS}} = \Pi^{\text{PS}} > \Pi^{\text{AS}}$	$\Pi^{\text{AS}} > \Pi^{\text{TS}} = \Pi^{\text{PS}}$	$\Pi^{\text{PS}} > \Pi^{\text{TS}} > \Pi^{\text{AS}}$	$\Pi^{\text{AS}} > \Pi^{\text{TS}} = \Pi^{\text{PS}}$
Profit	$\text{Profit} \begin{cases} \text{TS} & \$9.00 \\ \text{AS} & \$5.67 \\ \text{PS} & \$9.00 \end{cases}$	$\text{Profit} \begin{cases} \text{TS} & \$4.00 \\ \text{AS} & \$6.78 \\ \text{PS} & \$4.00 \end{cases}$	$\text{Profit} \begin{cases} \text{TS} & \$7.00 \\ \text{AS} & \$5.90 \\ \text{PS} & \$8.00 \end{cases}$	$\text{Profit} \begin{cases} \text{TS} & \$8.00 \\ \text{AS} & \$8.90 \\ \text{PS} & \$8.00 \end{cases}$
Risk-averse consumers	$\Pi^{\text{TS}} = \Pi^{\text{PS}} > \Pi^{\text{AS}}$	$\Pi^{\text{AS}} > \Pi^{\text{TS}} = \Pi^{\text{PS}}$	$\Pi^{\text{PS}} > \Pi^{\text{TS}} > \Pi^{\text{AS}}$	$\Pi^{\text{AS}} > \Pi^{\text{TS}} = \Pi^{\text{PS}}$

is unaffected by risk aversion because the only consumers who choose to purchase the probabilistic good (e.g., consumer B in market 3) have equal valuations for the two goods. If purchasers of the probabilistic good have weak preferences (rather than being completely indifferent between the two goods), then the price of the probabilistic good would need to be discounted, and thus profit under PS would decline (relative to the case of risk-neutral consumers). However, for the reasons outlined above, we expect this decline in profits to be smaller than the decline in profits that occurs under AS. Clearly, additional study regarding the impact of risk aversion of probabilistic selling and on advance selling is needed. Future research is especially needed to account for heterogeneity in risk aversion. For instance, probabilistic selling may enable the seller to discriminate according to variation in risk aversion.

Another possible direction for future research is to consider how advance selling and probabilistic selling differ in terms of their ability to enhance capacity utilization. For instance, previous research has shown that probabilistic selling (Fay and Xie 2008) and advance selling (Gale and Holmes 1992, 1993) separately can be useful tools for a seller facing demand uncertainty and capacity constraints. However, it would be interesting to consider the conditions under which one selling strategy is preferred over the other. Finally, it would be interesting to see whether there are settings where advance selling and probabilistic selling might be complementary strategies rather than substitutes. Note that in the model developed in this paper in which all consumers are a priori identical, there is no advantage for a firm to implement both advance selling and probabilistic selling. However, in a more complex model, such as one where consumers resolve their uncertainty at different times or where consumers' expected product

valuations are drawn from different distributions, we suspect that it may be optimal for a firm to both sell probabilistic goods and sell in advance. For instance, future research may be able to identify situations under which it would be beneficial for a cable company to sell both specified pay-per-view movies in advance and probabilistic movies in the spot market.

Appendix

Derivation of Profit for the Four Markets in Figure 1

In Table A.1, we report the prices and profits under traditional selling (TS), advance selling (AS), and probabilistic selling (PS) for the four examples discussed in §1.

Under first-degree price discrimination, the seller provides each customer with her preferred product at a price equal to the valuation of that product. Thus, the total revenue obtained from the three customers equals

$$\text{Max}[v_{1A}, v_{2A}] + \text{Max}[v_{1B}, v_{2B}] + \text{Max}[v_{1C}, v_{2C}].$$

Under TS, a consumer purchases product 1 if $v_1 \geq P_1^{\text{TS}}$ and $v_1 - P_1^{\text{TS}} \geq v_2 - P_2^{\text{TS}}$, purchases product 2 if $v_2 \geq P_2^{\text{TS}}$ and $v_1 - P_1^{\text{TS}} < v_2 - P_2^{\text{TS}}$, and purchases nothing if $v_1 < P_1^{\text{TS}}$ and $v_2 < P_2^{\text{TS}}$. The seller chooses its prices ($P_1^{\text{TS}}, P_2^{\text{TS}}$) to maximize its profit given this demand.

Under AS, in the advance period, the consumer does not know which type (A, B, or C) she will be or which product will be preferred. Thus, she will purchase in advance only if $P_j^{\text{AS}} \leq \text{EV} - \text{EV}_{\text{waiting}}$, where $\text{EV} = (v_{1A} + v_{2A} + v_{1B} + v_{2B} + v_{1C} + v_{2C})/6$, and $\text{EV}_{\text{waiting}}$ is the expected consumer surplus from waiting to purchase until the spot period (in which valuations will be known). Notice that each consumer has the same expectations in the advance period. Thus, either all customers wish to purchase in the advance period or no customers do. AS is only distinct from TS if purchases occur in the advance period. Thus, we report in Table A.1 the profit for AS, assuming the price is set such that customers prefer to purchase in advance. Under AS, the seller's profit is $\text{Min}[P_1^{\text{AS}}, P_2^{\text{AS}}]$. Thus, the maximum profit will be obtained by setting $P_1^{\text{AS}} = P_2^{\text{AS}} = \text{EV}$ and choosing large enough spot prices such that $\text{EV}_{\text{waiting}} = 0$ (i.e., spot prices in excess of $\text{Max}_{i=[A, B, C]}[v_{ji}]$).

Table A.1 Analysis of Examples

Assumptions and definition:												
<ul style="list-style-type: none"> A seller with two component products ($j = 1, 2$) faces three potential customers ($i = A, B, C$). The seller has no marginal costs, capacity constraints, or fixed costs to offering either advance sales or probabilistic products. Let P_j^s, $j = 1, 2$; $s = \{TS, AS, PS\}$ denote the price for product j under strategy s and P_o^{PS} denote the price for the probabilistic good (which is equally likely to be either product 1 or product 2). Π_s^* denotes the optimal profit under strategy s. 												
Consumer (v_1, v_2)	Market 1			Market 2			Market 3			Market 4		
	A	B	C	A	B	C	A	B	C	A	B	C
	(3, 0)	(3, 3)	(0, 3)	(1, 1)	(2, 2)	(4, 4)	(3, 1)	(2, 2)	(1, 3)	(4, 3)	(2, 2)	(3, 4)
First-degree price discrimination	3 + 3 + 3 = 9			1 + 2 + 4 = 7			3 + 2 + 3 = 8			4 + 2 + 4 = 10		
Prices	$P_j^{TS} = 3$			$P_j^{TS} = 2$			$P_1^{TS} = 3, P_2^{TS} = 2$			$P_j^{TS} = 4$		
	$P_j^{AS} = 2$			$P_j^{AS} = 2.33$			$P_j^{AS} = 2$			$P_j^{AS} = 3$		
	$P_j^{PS} = P_o^{PS} = 3$			$P_j^{PS} = P_o^{PS} = 2$			$P_j^{PS} = 3; P_o^{PS} = 2$			$P_j^{PS} = P_o^{PS} = 4$		
Profit	$\pi_{TS}^* = 3 * 3 = 9$			$\pi_{TS}^* = 2 * 2 = 4$			$\pi_{TS}^* = 3 * 1 + 2 * 2 = 7$			$\pi_{TS}^* = 4 * 2 = 8$		
	$\pi_{AS}^* = 2 * 3 = 6$			$\pi_{AS}^* = 2.33 * 3 = 7$			$\pi_{AS}^* = 2 * 3 = 6$			$\pi_{AS}^* = 3 * 3 = 9$		
	$\pi_{PS}^* = 3 * 3 = 9$			$\pi_{PS}^* = 2 * 2 = 4$			$\pi_{PS}^* = 3 * 2 + 2 * 1 = 8$			$\pi_{PS}^* = 4 * 2 = 8$		

Under PS, the expected value for the probabilistic good equals $(v_1 + v_2)/2$. Each consumer chooses the purchase option that yields the highest expected profit. Specifically, a consumer purchases product 1 if $v_1 \geq P_1^{PS}$, $v_1 - P_1^{PS} \geq v_2 - P_2^{PS}$, and $v_1 - P_1^{PS} \geq (v_1 + v_2)/2 - P_o^{PS}$; purchases product 2 if $v_2 \geq P_2^{PS}$, $v_1 - P_1^{PS} < v_2 - P_2^{PS}$, and $v_2 - P_2^{PS} \geq (v_1 + v_2)/2 - P_o^{PS}$; purchases the probabilistic good if $(v_1 + v_2)/2 \geq P_o^{PS}$, $v_1 - P_1^{PS} < (v_1 + v_2)/2 - P_o^{PS}$, and $v_2 - P_2^{PS} < (v_1 + v_2)/2 - P_o^{PS}$; and purchases nothing if $v_1 < P_1^{PS}$, $v_2 < P_2^{PS}$, and $(v_1 + v_2)/2 < P_o^{PS}$. The seller chooses its prices (P_1^{PS} , P_2^{PS} , P_o^{PS}) to maximize its profit given this demand. Table A.1 reports the optimal prices and resulting profit.

Traditional Selling, Advance Selling, and Probabilistic Selling

Under TS, the seller chooses P^{TS} to maximize the profit given in Equation (8). Taking the derivative of profit w.r.t. P^{TS} and setting equal to zero, we get $P^{TS} = (R + \Delta_v + c)/2$. Notice that the lower bound on the integral in (8) is only valid if $P^{TS} \geq R$. For smaller values of P^{TS} , we would be at a corner solution where the market is completely covered. To focus on the cases where AS or PS can improve profit (relative to TS) without relying on a market expansion effect, we limit consideration to the parameter region in which $P^{TS} = R$, which will be optimal when condition (13) is met:⁸

$$c \leq R - \Delta_v. \quad (13)$$

Under AS, profit is given by Equation (10). Profit is maximized by choosing the highest advance price that induces

purchases in the advance period: $P_j^{AS} = R + (\Delta(2 - 3\gamma))/4$.⁹ All consumers purchase in advance. Thus, the seller earns a profit of $\Pi^{AS} = R + (\Delta(2 - 3\gamma))/4 - c - F_{AS}$.

Under PS, the expected value to consumer θ for the opaque good equals $v_{\theta o} = (v_{\theta H} + v_{\theta L})/2 = R + (\theta(2 - 3\gamma))/2$. Notice that $v_{\theta o}$ is increasing in θ for $\gamma < \frac{2}{3}$ but decreasing in θ for $\gamma > \frac{2}{3}$. Thus, we divide the analysis of the optimal prices under PS into two parts, when $\gamma < \frac{2}{3}$ and when $\gamma > \frac{2}{3}$. First consider $\gamma < \frac{2}{3}$. As θ increases, valuations for the opaque good rise and valuations for one's preferred good rise even faster: $v_{\theta H} - v_{\theta o} = \gamma\theta/2$; $\partial[v_{\theta H} - v_{\theta o}]/\partial\theta = \gamma/2 > 0$. Thus, the market can be divided into three segments: those that purchase their preferred good ($\theta \geq \hat{\theta}$), those that purchase the opaque good ($\theta_L \leq \theta < \hat{\theta}$), and those that purchase nothing ($\theta < \theta_L$). The seller's profit is

$$\Pi^{PS} = (P^{PS} - c) \left(\frac{\Delta - \hat{\theta}}{\Delta} \right) + \left(P_o^{PS} - c \right) \left(\frac{\hat{\theta} - \theta_L}{\Delta} \right) - F_{PS},$$

where $\hat{\theta} \geq \theta_L$ and $\theta_L \geq 0$. (14)

θ_L is defined as the smallest θ that will purchase the opaque good: $v_{\theta L o} - P_o^{PS} = 0 \Rightarrow P_o^{PS} = R + (\theta_L(2 - 3\gamma))/2$. $\hat{\theta}$ is defined as the θ who is indifferent between purchasing her preferred good and purchasing the opaque good: $v_{\hat{\theta} o} - P_o^{PS} = v_{\hat{\theta} H} - P^{PS} \Rightarrow P^{PS} = \gamma\hat{\theta}/2 + P_o^{PS}$. Substituting these formulas for prices into (14), the seller maximizes profit with respect to (w.r.t.) θ_L and $\hat{\theta}$. Taking the derivative of profit θ_L and setting it equal to zero, we find that $\theta_L = -([2(R - c) - \Delta(2 - 3\gamma)]/(2(2 - 3\gamma)))$, which is less than that zero if condition (15) is met:

$$c \leq R - \Delta. \quad (15)$$

⁸ Other papers have considered the impact of even larger marginal costs on the advance selling strategy (Shugan and Xie 2000, Xie and Shugan 2001) and on the probabilistic selling strategy (Fay and Xie 2008). The key finding is that if costs are too large, the seller will not benefit from allowing advance purchases or introducing probabilistic products.

⁹ As shown in the preceding section, the seller maximizes profit by choosing large enough spot prices such that $EV_{\text{waiting}} = 0$ (i.e., spot prices in excess of $\text{Max}_{\theta}[v_{\theta H}]$ so that consumers do not have any incentive to wait until the spot period to purchase).

Because we are focusing on the cases where the differences between selling strategies cannot be explained by market expansion effects, we assume (15) holds (which is a more restrictive assumption than (13)). Here, we are at a corner solution in which the market is completely covered; i.e., $\theta_L = 0$. This implies $P_o^{\text{PS}} = R$. Taking the derivative of (14) w.r.t. $\hat{\theta}$ and setting it equal to zero, we find that the maximum profit is obtained at $\hat{\theta} = \Delta/2$. The optimal prices and resulting profit are reported in Lemma A1.

Now we turn to the case where $\gamma > \frac{2}{3}$. Here the valuations for the opaque good are decreasing in θ . Thus, the market can be divided into three segments: those that purchase their preferred good ($\theta \geq \theta_H$), those that purchase the opaque good ($0 \leq \theta \leq \theta_L$), and those that purchase nothing ($\theta_L < \theta < \theta_H$). The seller's profit is

$$\Pi^{\text{PS}} = (P^{\text{PS}} - c) \left(\frac{\Delta - \theta_H}{\Delta} \right) + (P_o^{\text{PS}} - c) \left(\frac{\theta_L}{\Delta} \right) - F_{\text{PS}},$$

where $\theta_L \geq 0$, $\theta_H \leq \Delta$, and $\theta_L \leq \theta_H$. (16)

θ_L is defined as the largest θ that will purchase the opaque good: $v_{\theta_L, o} - P_o^{\text{PS}} = 0 \Rightarrow P_o^{\text{PS}} = R + (\theta_L(2 - 3\gamma))/2$. θ_H is defined as the smallest θ who is willing to purchase her preferred good: $v_{\theta_H, H} - P^{\text{PS}} = 0 \Rightarrow P^{\text{PS}} = R + (1 - \gamma)\theta_H$. Substituting these formulas for prices into (14), the seller maximizes profit with respect to θ_L and θ_H . Taking the derivative of profit w.r.t. θ_H and setting it equal to zero, we find that $\theta_H = -([R - c - \Delta(1 - \gamma)]/(2(1 - \gamma)))$, which is less than zero (from condition (15)). Thus, we are at a corner solution in which the market is completely covered; i.e., $\theta_L = \theta_H = 0$. Therefore, profit is given by $\Pi^{\text{PS}} = P^{\text{PS}}((\Delta - \hat{\theta})/\Delta) + P_o^{\text{PS}}(\hat{\theta}/\Delta) - c - F_{\text{PS}}$, where the consumer at $\hat{\theta}$ obtains zero surplus from purchasing her preferred product or from purchasing the opaque good: $v_{\hat{\theta}, o} - P_o^{\text{PS}} = 0 \Rightarrow P_o^{\text{PS}} = R + (\hat{\theta}(2 - 3\gamma))/2$; $v_{\hat{\theta}, H} - P^{\text{PS}} = 0 \Rightarrow P^{\text{PS}} = R + \hat{\theta}(1 - \gamma)$. Substituting these prices into the profit function and then taking the derivative of profit w.r.t. $\hat{\theta}$ and setting it equal to zero, we find that the maximum profit is obtained at $\hat{\theta} = (\Delta(1 - \gamma))/\gamma$. The optimal prices and resulting profit are reported in Lemma A1.

Lemma A1 records the optimal prices and the resulting profit for these three selling strategies.

LEMMA A1 (OPTIMAL PRICE AND PROFIT). *The seller's optimal prices and profit under the three selling strategies are given below.*

Strategy	Price	Total profit
Traditional selling (TS)	$P^{\text{TS}} = R$	$\Pi^{\text{TS}} = R - c$
Advance selling (AS)	$P_j^{\text{AS}} = R + \frac{\Delta(2 - 3\gamma)}{4}$	$\Pi^{\text{AS}} = R + \frac{\Delta(2 - 3\gamma)}{4} - c - F_{\text{AS}}$
Probabilistic selling (PS)	$P_o^{\text{PS}} = \begin{cases} R - \frac{\Delta(1 - \gamma)(3\gamma - 2)}{2\gamma} & \text{if } \gamma \geq \gamma_2, \\ R & \text{if } \gamma < \gamma_2 \end{cases}$ $P^{\text{PS}} = \begin{cases} R + \frac{(1 - \gamma)^2 \Delta}{\gamma} & \text{if } \gamma \geq \gamma_2, \\ R + \frac{\gamma \Delta}{4} & \text{if } \gamma < \gamma_2 \end{cases}$	$\Pi^{\text{PS}} = \begin{cases} R + \frac{(1 - \gamma)^2 \Delta}{2\gamma} - c - F_{\text{PS}} & \text{if } \gamma \geq \gamma_2, \\ R + \frac{\gamma \Delta}{8} - c - F_{\text{PS}} & \text{if } \gamma < \gamma_2 \end{cases}$

where $\gamma_2 = 2/3$

PROOF OF PROPOSITION 1. Using Lemma A1, we can compare the profit under AS and TS:

$$\Pi^{\text{AS}} - \Pi^{\text{TS}} = \frac{\Delta(2 - 3\gamma)}{4} - F_{\text{AS}} = \frac{2\Delta_v - \Delta_s}{4} - F_{\text{AS}}. \quad (17)$$

Thus,

$$\Pi^{\text{AS}} > \Pi^{\text{TS}} \text{ if and only if } \gamma < \gamma_{\text{AS}}(\Delta),$$

where $\gamma_{\text{AS}}(\Delta) = \frac{2}{3} - \frac{4F_{\text{AS}}}{3\Delta}$, (18)

or in terms of Δ_v and Δ_s ,

$$\Pi^{\text{AS}} > \Pi^{\text{TS}} \text{ if and only if } \Delta_v > \hat{\Delta}_v,$$

where $\hat{\Delta}_v = \frac{4F_{\text{AS}} + \Delta_s}{2}$. (19)

Taking the derivative of $\hat{\Delta}_v$ w.r.t. Δ_s ,

$$\frac{\partial \hat{\Delta}_v}{\partial \Delta_s} = \frac{1}{2} > 0. \quad (20)$$

Furthermore, notice that $\Pi^{\text{AS}} - \Pi^{\text{TS}}$ reaches its maximum when $\Delta_s = 0$ and $\Delta_v = \Delta$ at a value of $\Delta/2 - F_{\text{AS}}$. We assume $F_{\text{AS}} < \Delta/2$ so that it is possible for the benefit of introducing advance sales to exceed the implementation cost of offering products in advance. Q.E.D.

PROOF OF PROPOSITION 2. Using Lemma A1, we can compare the profit under PS and TS:

$$\Pi^{\text{PS}} - \Pi^{\text{TS}} = \begin{cases} \frac{(1 - \gamma)^2 \Delta}{2\gamma} - F_{\text{PS}} & \text{if } \gamma \geq \frac{2}{3}, \\ \frac{\gamma \Delta}{8} - F_{\text{PS}} & \text{if } \gamma < \frac{2}{3}. \end{cases} \quad (21)$$

Thus,

$$\Pi^{\text{PS}} > \Pi^{\text{TS}} \text{ if and only if } \gamma_{\text{PS}}^L(\Delta) < \gamma < \gamma_{\text{PS}}^H(\Delta),$$

$$\text{where } \gamma_{\text{PS}}^L(\Delta) = \frac{8F_{\text{PS}}}{\Delta}$$

$$\text{and } \gamma_{\text{PS}}^H(\Delta) = \frac{\Delta + F_{\text{PS}} - \sqrt{F_{\text{PS}}(2\Delta + F_{\text{PS}})}}{\Delta}. \quad (22)$$

Also notice from Equation (20) that $(\Pi^{\text{PS}} - \Pi^{\text{TS}})$ reaches its maximum at $\gamma = \frac{2}{3}$ because $\partial[\Pi^{\text{PS}} - \Pi^{\text{TS}}]/\partial\gamma > 0$ if $\gamma < \frac{2}{3}$ and $\partial[\Pi^{\text{PS}} - \Pi^{\text{TS}}]/\partial\gamma < 0$ if $\gamma > \frac{2}{3}$. At this point, the profit advantage of PS is $\Pi^{\text{PS}} - \Pi^{\text{TS}} = \Delta/12 - F_{\text{PS}}$. We assume $F_{\text{PS}} < \Delta/12$ so that it is possible for the benefit of introducing a probabilistic good to exceed the implementation cost of offering such a product.

It is also possible to present these results in terms of Δ_v and Δ_s (as is done in Proposition 2). In particular, using (21) and substituting for γ and Δ :

$$\Pi^{\text{PS}} - \Pi^{\text{TS}} = \begin{cases} \frac{(\Delta_v)^2}{2\Delta_s} - F_{\text{PS}} & \text{if } \Delta_v \leq \frac{\Delta_s}{2}, \\ \frac{\Delta_s}{8} - F_{\text{PS}} & \text{if } \Delta_v > \frac{\Delta_s}{2}. \end{cases} \quad (23)$$

Thus,

$$\Pi^{\text{PS}} > \Pi^{\text{TS}} \quad \text{if and only if} \quad \hat{\Delta}_s < \Delta_s < \bar{\Delta}_s$$

$$\text{where} \quad \hat{\Delta}_s = 8F_{\text{PS}} \quad \text{and} \quad \bar{\Delta}_s = \frac{(\Delta_v)^2}{2F_{\text{PS}}}. \quad (24)$$

Furthermore, $\hat{\Delta}_s < \bar{\Delta}_s$ only if $\Delta_v > 4F_{\text{PS}}$. Thus, a necessary condition for $\Pi^{\text{PS}} > \Pi^{\text{TS}}$ is $\Delta_v > \bar{\Delta}_v$, where $\bar{\Delta}_v = 4F_{\text{PS}}$. Q.E.D.

PROOF OF PROPOSITION 3.

When the Probabilistic Selling Strategy Is Optimal. Equation (20) provides the condition under which PS outperforms TS. For PS to be optimal, it must also yield higher profits than does AS. Using Lemma A1, we can compare these profits:

$$\Pi^{\text{PS}} - \Pi^{\text{AS}} = \begin{cases} \frac{\Delta(2 - \gamma(6 - 5\gamma))}{4\gamma} + F_{\text{AS}} - F_{\text{PS}} & \text{if } \gamma \geq \frac{2}{3}, \\ \frac{\Delta(7\gamma - 4)}{8} + F_{\text{AS}} - F_{\text{PS}} & \text{if } \gamma < \frac{2}{3}. \end{cases} \quad (25)$$

Notice that $\Pi^{\text{PS}} - \Pi^{\text{AS}}$ is strictly increasing in γ . We define the cutoff $\hat{\gamma}(\Delta)$ such that $\Pi^{\text{PS}} - \Pi^{\text{AS}} = 0$ if $\gamma = \hat{\gamma}(\Delta)$ and for any higher γ , $\Pi^{\text{PS}} - \Pi^{\text{AS}} > 0$:

$$\hat{\gamma}(\Delta) = \begin{cases} 1 & \text{if } \Delta < \Delta_1, \\ \frac{6\Delta - 4(F_{\text{AS}} - F_{\text{PS}}) + \sqrt{[6\Delta - 4(F_{\text{AS}} - F_{\text{PS}})]^2 - 40\Delta^2}}{10\Delta} & \text{if } \Delta_1 < \Delta < \Delta_2, \\ \frac{4(\Delta - 2(F_{\text{AS}} - F_{\text{PS}}))}{7\Delta} & \text{if } \Delta \geq \Delta_2, \end{cases} \quad (26)$$

where $\Delta_1 = 4(F_{\text{PS}} - F_{\text{AS}})$ and $\Delta_2 = 12(F_{\text{PS}} - F_{\text{AS}})$.

Thus, we have $\Pi^{\text{PS}} > \text{Max}[\Pi^{\text{TS}}, \Pi^{\text{AS}}]$ if

$$\text{Max}[\hat{\gamma}(\Delta), \gamma_{\text{PS}}^{\text{L}}(\Delta)] \equiv \hat{\gamma} < \gamma < \gamma_{\text{PS}}^{\text{H}}(\Delta) \equiv \bar{\gamma},$$

where $\gamma_{\text{PS}}^{\text{L}}(\Delta)$, $\gamma_{\text{PS}}^{\text{H}}(\Delta)$, and $\hat{\gamma}(\Delta)$ are given in Equations (22) and (26), respectively.

When the Advance Selling Strategy Is Optimal. Equation (18) provides the condition under which AS outperforms TS. For AS to be optimal, it must also yield higher profits than does PS. This profit comparison is made in (25). Thus, $\Pi^{\text{AS}} > \Pi^{\text{PS}}$ if $\gamma < \hat{\gamma}(\Delta)$ and $\Pi^{\text{AS}} > \text{Max}[\Pi^{\text{TS}}, \Pi^{\text{PS}}]$ if $\gamma < \text{Min}[\hat{\gamma}(\Delta), \gamma_{\text{AS}}(\Delta)]$, where $\gamma_{\text{AS}}(\Delta)$ and $\hat{\gamma}(\Delta)$ are given in (18) and (26), respectively.

When the Traditional Selling Strategy Is Optimal. TS is optimal whenever neither AS nor PS is advantageous:

$$\Pi^{\text{TS}} \geq \text{Max}[\Pi^{\text{AS}}, \Pi^{\text{PS}}] \quad \text{if } \gamma_{\text{AS}}(\Delta) \leq \gamma \leq \gamma_{\text{PS}}^{\text{L}}(\Delta)$$

$$\gamma \geq \text{Max}[\gamma_{\text{AS}}(\Delta), \gamma_{\text{PS}}^{\text{H}}(\Delta)]. \quad (27)$$

For example, the lower region of Figure 2 illustrates the parameters under which TS is optimal when $F_{\text{PS}} = F_{\text{AS}} = \frac{1}{36}$. In this case, the following table summarizes the conditions under which TS will be optimal.

Conditions required for traditional selling to be optimal

Total amount of buyer heterogeneity	Required condition on the relative importance of heterogeneity in preference strength
$\Delta \leq \frac{1}{18}$	Any $\gamma \in [0, 1]$
$\frac{1}{18} < \Delta \leq \frac{1}{3}$	$\gamma \geq \frac{18\Delta - 1}{27\Delta}$
$\frac{1}{3} < \Delta \leq \frac{7}{18}$	$\frac{18\Delta - 1}{27\Delta} \leq \gamma \leq \frac{2}{9\Delta}$ or $\gamma \geq \frac{1 + 36\Delta - \sqrt{1 + 72\Delta}}{36\Delta}$
$\Delta > \frac{7}{18}$	$\gamma \geq \frac{1 + 36\Delta - \sqrt{1 + 72\Delta}}{36\Delta}$

Interestingly, regardless of the proportion of heterogeneity that is due to variation in preference strengths relative to variation in maximum valuations, neither the advance selling strategy nor the probabilistic selling strategy is advantageous if there is a low magnitude of total heterogeneity. Furthermore, TS will be optimal for any level of total heterogeneity if that heterogeneity is almost entirely in the form of *Strength-Heterogeneity*. For example, using the above parameters, for any level of Δ , TS is optimal if $\gamma \geq 0.79044 \equiv \gamma_{\text{max}}$, and for any level of γ , TS is optimal if $\Delta \leq 1/18 \equiv \Delta_{\text{min}}$. Q.E.D.

References

- Adams, W. J. A., J. L. Yellen. 1976. Commodity bundling and the burden of monopoly. *Quart. J. Econom.* **90**(3) 475–498.
- Bakos, Y., E. Brynjolfsson. 1999. Bundling information goods: Pricing, profits, and efficiency. *Management Sci.* **45**(12) 1613–1630.
- Basu, A., P. Vitharana. 2009. Impact of customer knowledge heterogeneity on bundling strategy. *Marketing Sci.* **28**(4) 792–801.
- Biyalogorsky, E., E. Gerstner. 2004. Contingent pricing to reduce price risks. *Marketing Sci.* **23**(1) 146–155.
- Biyalogorsky, E., Z. Carmon, G. E. Fruchter, E. Gerstner. 1999. Research note: Overselling with opportunistic cancellations. *Marketing Sci.* **18**(4) 605–610.
- Chatwin, R. E. 1999. Continuous-time airline overbooking with time-dependent fares and refunds. *Transportation Sci.* **33**(2) 182–191.
- Dana, J. D., Jr. 1998. Advance-purchase discounts and price discrimination in competitive markets. *J. Political Econom.* **106**(2) 395–422.
- Desiraju, R., S. M. Shugan. 1999. Strategic service pricing and yield management. *J. Marketing* **63**(1) 44–56.
- Dolan, R. J. 1987. Quantity discounts: Managerial issues and research opportunities. *Marketing Sci.* **6**(1) 1–22.
- Fay, S. 2008. Selling an opaque product through an intermediary: The case of disguising one's product. *J. Retailing* **84**(1) 59–75.
- Fay, S., J. Xie. 2008. Probabilistic goods: A creative way of selling products and services. *Marketing Sci.* **27**(4) 674–690.
- Gale, I. L., T. J. Holmes. 1992. The efficiency of advance-purchase discounts in the presence of aggregate demand uncertainty. *Internat. J. Indust. Organ.* **10**(3) 413–437.

- Gale, I. L., T. J. Holmes. 1993. Advance-purchase discounts and monopoly allocation of capacity. *Amer. Econom. Rev.* **83**(1) 135–146.
- Gerstner, E., J. D. Hess, D. M. Holthausen. 1994. Price discrimination through a distribution channel: Theory and evidence. *Amer. Econom. Rev.* **84**(5) 1437–1445.
- Godes, D., D. Mayzlin. 2009. Firm-created word-of-mouth communication: Evidence from a field test. *Marketing Sci.* **28**(4) 721–739.
- Guo, L. 2006. Consumption flexibility, product configuration, and market competition. *Marketing Sci.* **25**(2) 116–130.
- Guo, L. 2009. Service cancellation and competitive refund policy. *Marketing Sci.* **28**(5) 901–917.
- Kolay, S., G. Shaffer. 2003. Bundling and menus of two-part tariffs. *J. Indust. Econom.* **51**(3) 383–403.
- McAfee, R. P., J. McMillan, M. D. Whinston 1989. Multiproduct monopoly, commodity bundling and correlation of values. *Quart. J. Econom.* **104**(2) 371–383.
- Narasimhan, C. 1984. A price discrimination theory of coupons. *Marketing Sci.* **3**(2) 128–147.
- Oi, W. Y. 1971. A Disneyland dilemma: Two-part tariffs for a Mickey Mouse monopoly. *Quart. J. Econom.* **85**(1) 77–96.
- Shugan, S. M., J. Xie. 2000. Advance pricing of services and other implications of separating purchase and consumption. *J. Service Res.* **2**(3) 227–239.
- Shugan, S. M., J. Xie. 2005. Advance-selling as a competitive marketing tool. *Internat. J. Res. Marketing* **22**(3) 351–373.
- Talluri, K. T., G. J. van Ryzin. 2004. *The Theory and Practice of Revenue Management*. International Series in Operations Research and Management Science. Springer Science and Business, New York.
- Varian, H. R. 2000. Versioning information goods. B. Kahin, H. R. Varian, eds. *Internet Publishing and Beyond: The Economics of Digital Information and Intellectual Property*. MIT Press, Cambridge, MA, 190–202.
- Xie, J., E. Gerstner. 2007. Service escape: Profiting from customer cancellations. *Marketing Sci.* **26**(1) 18–30.
- Xie, J., S. M. Shugan. 2001. Electronic tickets, smart cards, and online prepayments: When and how to advance sell. *Marketing Sci.* **20**(3) 219–243.