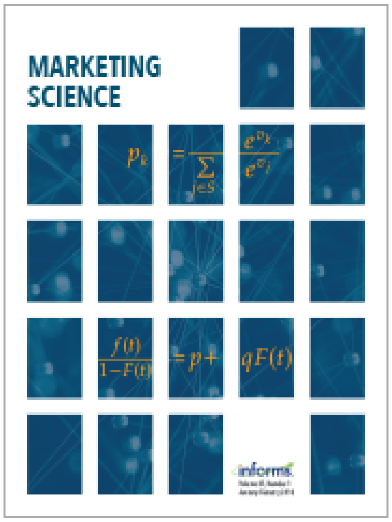

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Probabilistic Selling for Vertically Differentiated Products with Salient Thinkers

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Abstract. This paper studies probabilistic selling for vertically differentiated products, whereby consumers do not know the exact identity of a product until after making the purchase. An important feature of probabilistic selling overlooked by previous literature is that it changes the product line, which often determines consumers' choice context. Our work discovers the crucial role of context effects taking into account consumers' salient thinking behavior: consumers focus their limited attention on and hence overweight the salient attribute of a product in their perception, leading to context-dependent preferences. We show that probabilistic selling can improve the seller's profit with salient thinkers even when this strategy does not emerge with rational consumers. With salient thinking, the probabilistic product enables the seller to transform the consumers' choice context favorably and direct their attention to quality. Our findings demonstrate the importance of exploiting consumers' salient thinking behavior and suggest that probabilistic selling, as a context management tool, can be more beneficial than previously shown.

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Keywords: probabilistic selling • vertically differentiated products • context effect • salience • decoy • behavioral pricing

> Our mind has a useful capability to focus on whatever is odd, different or unusual. —Kahneman (2011, p. 324)

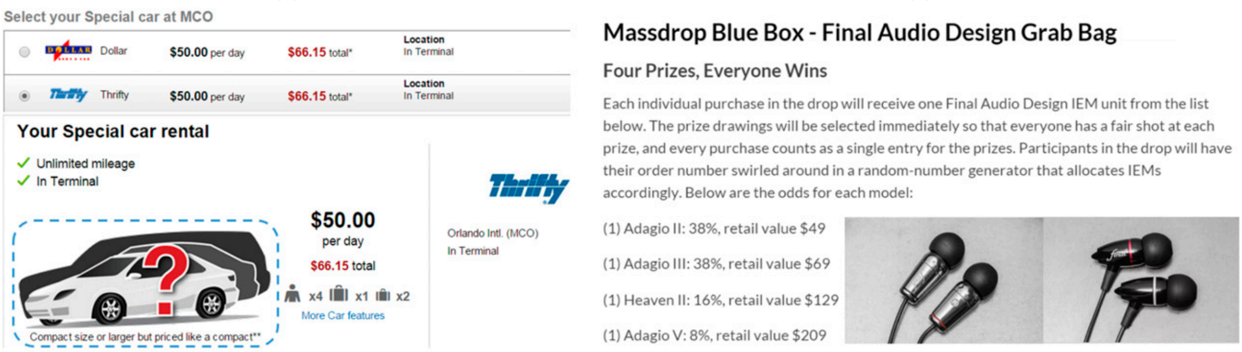
1. Introduction

Recently, many companies are utilizing a unique selling strategy, which assigns consumers a product in a probabilistic manner. For example, Thrifty has one "Smart Bet, Wild Card" program.[1] With this program, consumers can obtain a compact car or a larger one at the *same* rate charged for the compact car, advertised as a "special car" on Hotwire (Figure 1(a)). For online retailing, Massdrop.com offers a grab bag containing different versions of earphones, whereby each consumer receives a product randomly assigned by the seller on the basis of a preannounced probability at the *same* price of the lowest-end earphone (Figure 1(b)).[2] In the hotel industry, a deeply discounted Run of House room is a virtual room whereby the customer books a generic or unspecified room within a hotel with the possibility of being assigned to any room type (e.g., ocean-view or not).[3] In the internet broadband service industry, a provider offers two levels of service, Gold and Palladium. The Gold service has a guaranteed download speed of 50 Mbps with a price of $59.95, whereas the Palladium service with a price of $49.95 offers a download speed varying between 20 Mbps and 50 Mbps (Zhang et al. 2015).

In the above examples, these products exhibit characteristics of a vertically differentiated market; that is, all consumers prefer a high-quality product (e.g., ocean-view room) to a low-quality product given the same prices. These firms also offer a synthetic product, known as *probabilistic product*, which consists of a lottery among different quality-differentiated products.[4] That is, consumers do not know which component product (either high or low-quality product) they will receive before the nonrefundable payment is made. Following the term coined by Fay and Xie (2008), we refer to the practice of offering the probabilistic product as *probabilistic selling*. An important feature of probabilistic selling is that the introduction of probabilistic products can change consumers' local choice context. Behavioral researchers have documented through experiments that when facing choices with multiple attributes, consumers evaluate each option by considering both its absolute utility and its relative standing in the choice

Figure 1. Leading Examples of Probabilistic Products

(a) (b)



Special Car on Hotwire.com Grab Bag on Massdrop.com

set, leading to context-dependent preferences (e.g., Huber et al. 1982, Simonson 1989, Drolet et al. 2000, Kivetz et al. 2004). A growing body of literature further shows that firms change their strategies significantly when consumers exhibit context-dependent preferences (e.g., Chen and Turut 2013, Narasimhan and Turut 2013, Azar 2014, Hedgcock et al. 2016). Given a product line, probabilistic selling enriches consumers' choice set by adding a synthetic product, and choice reversals may occur across contexts.

Interestingly, both Massdrop.com and Thrifty set the price of the probabilistic product the same as the low-quality product, and Run of House rooms have the cheapest rate. Why do the firms price the probabilistic products so differently? What role does the context-dependent behavior play in probabilistic selling? Indeed, to incorporate context-dependent behavior into probabilistic selling seems especially pertinent from both a theoretical and an applied perspective.

From the theoretical perspective, a deeper understanding of the mechanism behind probabilistic selling in vertical markets is necessary to further explain this nascent strategy. Although others have focused on probabilistic goods in horizontally differentiated markets (e.g., Jiang 2007, Fay and Xie 2008, 2010, Jerath et al. 2010, Fay and Xie 2015, Fay et al. 2015), our work examines probabilistic selling for vertically differentiated products à la Mussa and Rosen (1978) and is most closely related to Fay and Xie (2008) and Zhang et al. (2015). However, Fay and Xie (2008) sustain negative correlation in the values for the vertically differentiated products as in a horizontal market, which is different from the traditional literature on vertical differentiation. Indeed, negative correlation is the driving force behind probabilistic selling, which cannot arise under positive correlation. Zhang et al. (2015) consider two types of consumers, and thus the demand jumps at certain values of the prices and remains constant otherwise. They find that probabilistic selling can be a way to profitably dispose excess capacity of high-quality products, which cannot explain why Thrifty adopts this strategy even in a short supply season. Additionally, in a market with a large population of heterogeneous consumers (i.e., a continuous valuation distribution), probabilistic selling never emerges regardless of the capacity constraints. Furthermore, all of the existing theories, including two recent behavioral papers (Huang and Yu 2014, Chao et al. 2016), cannot adequately account for this contemporary pricing strategy in practice: the compact cars offered by Thrifty seem trivial given that they are priced the same as the special cars, which could be a better car.[5]

From the applied perspective, it remains unclear how a firm should design and price the probabilistic product in the presence of context-dependent behavior. The extant theoretical literature has explored the optimal product line design when consumers exhibit context-dependent preferences (e.g., Orhun 2009, Dahremöller and Fels 2015). However, in revenue management applications and in the retail industry, the product line is fixed, and thereby sellers do not have much freedom to exploit context-dependent preferences. Because a probabilistic product is a convex combination of the high- and low-quality product, the seller can change the choice context by endogenously determining the "expected quality" of the probabilistic product, which is between the high and low quality. The positioning of the probabilistic product should take into account both roles of price discrimination and context management. Although this strategy is appropriate for a seller with limited access to alternate product variants, it is not readily apparent how the probability associated with the probabilistic product and corresponding prices should be optimized, given different possible combinations of the high- and low-quality product.

To model context-dependent behavior, we draw on recent work in the area of multiattribute choice and notably the concept of "salient thinking" as advanced in Bordalo et al. (2013). It has been suggested that consumers focus limited attention on and hence overweight the salient attribute in their perception (DellaVigna 2009, Fehr and Rangel 2011). The salience mechanism has explained a plethora of experimental evidence on context-dependent behavior of consumers, including well-established decoy effects (Huber et al. 1982) and compromise effects (Simonson 1989). Decoy effects pertain specifically to our motivation examples: the decoy (either an inferior or extreme option) plays an important role in influencing consumers' salient thinking, because its presence directs consumers' attention to the relative advantage of the dominating or superior product.

We build a stylized model of vertical differentiation with a continuous valuation distribution. Consumers may overweight the quality or price of a product when making comparisons, depending on whether its price or quality differs more compared with the respective market average. Absent salient thinking, the rational model advocates that the seller should *not* adopt the probabilistic selling strategy regardless of the supply capacity constraints, which departs sharply from the work of Fay and Xie (2008) and Zhang et al. (2015) (Section 3). The distinction lies in the assumptions of value correlation and consumer distribution. Specifically, negative correlation drives the emergence of probabilistic selling in Fay and Xie (2008), and the cannibalization effect is underestimated under a binary distribution in Zhang et al. (2015). By contrast, our salient model indicates that probabilistic selling can be used as an important context management tool to improve the seller's profit when the low-quality product has a higher cost/quality ratio than the high-quality product (Section 4). Our work contributes to the literature on probabilistic selling for vertically differentiated products by discovering the pivotal role of consumers' salient thinking behavior and demonstrating that probabilistic selling might be more attractive than previously shown.

Further, we characterize the optimal probabilistic selling strategy in the presence of salient thinking. The seller meticulously designs the probabilistic product and sets prices so as to establish either the high- or low-quality product as the decoy with sufficiently high price/quality ratios and zero demand. We find that, when the cost of the high-quality product is relatively low, the seller offers all three products in the assortment. However, the low-quality product only plays a decoy role such that the high-quality and probabilistic products are quality salient and used for price discrimination. When the cost of the high-quality product is in an intermediate range all three products are still included in the assortment, but the seller instead sets the high-quality product as the decoy. In this case, quality is salient only for the probabilistic product. When the high-quality product has a relatively high cost, it should be excluded from the assortment. The seller would use the low-quality product as the decoy to boost the sales of the probabilistic product. These results have managerial implications on the design of probabilistic products for an existing product line in the presence of context-dependent behavior.

Recall the earphone example at the beginning of the paper. The grab bag is sold at the same price of the lowest quality earphone ($49), yet consumers obtain a higher-quality product with a probability of 62%, which seems to be a good deal for consumers, as discussed on the website.[6] Thrifty uses a similar pricing scheme that the "special car" is priced at the same rate of a compact car. With aforementioned pricing schemes, consumers' attention is distracted from price as quality becomes more salient. These are examples in which firms profit from probabilistic selling, thereby creating a virtual product with quality salience for consumers. Note that probabilistic selling is particularly valuable when providing an additional midrange quality product is too costly or logistically impossible, because its implementation cost is negligible for online sellers. In particular, if the cost structure of the component products varies with advancements in technology or cost shocks in the short run, then the seller can easily adjust the associated probabilities and prices.

Our paper also belongs to the growing literature on context-dependent behavior, or broadly behavioral pricing. We refer readers to Özer and Zheng (2012) for an excellent review on behavioral pricing and Bordalo et al. (2013) for a discussion about alternate approaches for context effects. We herein highlight the differences between the loss aversion model and the salience theory. Several models rationalize context-dependent choice by incorporating loss aversion relative to a reference point (see Tversky and Kahneman 1991, Tversky and Simonson 1993, Bodner and Prelec 1994). However, these models cannot reconcile their predictions with some experimental evidence concerning context effects, as reviewed in Bordalo et al. (2013). For example, because consumers avoid losses across all attributes, one prediction is a bias toward middle-of-the-road options, in contrast to the evidence that consumers do choose extreme options in many situations. As such, the formalization of context dependence based on loss aversion cannot give rise to the asymmetry of decoy effects (Heath and Chatterjee 1995): adding an appropriate decoy typically boosts the demand for high-quality products but cannot work for low-quality products. As opposed to disappointment in the loss aversion model, salience captures the idea that choice is driven by attention. One would expect that the decoy induces consumers to focus more on the hitherto neglected upside of the target product instead of the painful disappointment of losing the target product that they expect to own. Because of its ability to accommodate a slew of disparate evidence and appealing psychological intuitions, we adopt the salience theory à la Bordalo et al. (2013) in our setting. Based on the salience mechanism, our work contributes to the literature on behavioral pricing by substantiating an emerging practice in the internet age: probabilistic selling for vertically differentiated products, which is difficult to elucidate through a rational model.[7]

The rest of our paper is organized as follows. Model setup is in Section 2. We present the results for the rational and salient model in Sections 3 and 4, respectively. Finally, Section 5 concludes our work and provides directions for future research. All proofs are presented in the Appendix.

2. Model Setup

Consider a monopolist seller offering two vertically differentiated products, h and l, representing high- and low-quality products, respectively. He also has an option of selling the probabilistic product, in which the consumer receives the high-quality product h with a *preannounced* probability of $\phi \in (0,1)$ and the low-quality product l with probability $1-\phi$, as in Zhang et al. (2015) and real-life examples such as Massdrop.com. The focus of this paper is to examine whether and how the probabilistic product can be used as a context management tool. Consequently, we treat the design of probabilistic products (i.e., ϕ) as a strategic decision variable for the seller rather than determined by the game interaction between the seller and consumers as in Fay and Xie (2008, 2015) and Jerath et al. (2010).

We use the variable p to denote the probabilistic product. Let $c_i \ge 0$ and $q_i > 0$ denote the variable cost and the quality of product i, $i \in \{h,p,l\}$, respectively, and $q_h > q_l$. We do not restrict our attention to $c_h \ge c_l$, and our model applies to damaged goods ($c_h < c_l$) as well. The expected variable cost of the probabilistic product is $c_p = \phi c_h + (1-\phi)c_l$. We use r_i to denote the selling price for product $i \in \{h,p,l\}$. We assume that the seller does not incur any fixed cost of offering a product and $\min\{c_l/q_l, c_h/q_h\} \le 1$ to ensure that at least one product is offered in the optimal strategy, as shown below. To ease interpretation, we define relative versus absolute strength, specifically that product i is *relatively* stronger than another product j if $c_i/q_i < c_j/q_j$, and product i is *absolutely* stronger if $q_i - c_i > q_j - c_j$ (Inderst and Obradovits 2016).

We normalize the utility of no purchase (denoted by $\varnothing$) and the market size to zero and one, respectively, without loss of generality. Absent salience distortions, a consumer with *valuation* θ obtains a net utility $u_i(\theta) = \theta q_i - r_i$ from product i, where $\theta \in [0,1]$ denotes the consumer's heterogeneous willingness to pay for the quality. The seller cannot identify the θ value associated with each customer but knows the valuation distribution $F(\theta)$ with density $f(\theta)$. Define $\psi(\theta) \equiv \theta - (1-F(\theta))/f(\theta)$, also known as virtual valuation in the literature on mechanism design. We assume that the distribution $F(\theta)$ is *regular*, namely, $\partial\phi(\theta)/\partial\theta > 0$. This assumption holds for many common unimodal distributions, such as the uniform, exponential, normal, logistic, and χ^2 distribution. Consumers are risk-neutral and the utility from the probabilistic product is expected to be $u_p(\theta) = \phi\theta q_h + (1-\phi)\theta q_l - r_p = \theta(\phi q_h + (1-\phi)q_l) - r_p$. We can define $q_p \equiv \phi q_h + (1-\phi)q_l$, which is a convex combination of q_h and q_l. It is independent of θ and can be seen as the (expected) quality of the probabilistic product.

Following Bordalo et al. (2013), we introduce a "reference good," whose attributes are the market average, denoted by $(\bar{q}, \bar{r})$.[8] Thus, the reference good is endogenously determined by the context (i.e., the selling strategy). According to proposition 1 in Bordalo et al. (2013), which attribute is salient essentially depends on the relative difference of attributes, compared with the market average. Specifically, if $q_i > \bar{q}$ and $r_i > \bar{r}$ for product i, quality is salient when $r_i/q_i < \bar{r}/\bar{q}$, and price is salient otherwise; if $q_i < \bar{q}$ and $r_i < \bar{r}$, then price is salient when $r_i/q_i < \bar{r}/\bar{q}$, and quality is salient when the converse holds strictly.

Because salient consumers overweight the most salient attribute when evaluating a product, we specify that the nonsalient attribute is discounted by some scale factor. In other words, quality salience induces consumers to become less price sensitive (i.e., overvalue the product), whereas price salience makes consumers undervalue the product. Formally, the perceived utility from (q_i, r_i) is

$$u_i^s(\theta) = \begin{cases} \theta q_i - \eta r_i & \text{if quality is salient;} \\ \theta\eta q_i - r_i & \text{if price is salient;} \\ \theta q_i - r_i & \text{if equally salient,} \end{cases} \tag{1}$$

where the superscript s reflects the distortion by salience, and the parameter $\eta \in (0,1]$ captures the severity of salient thinking.[9] A lower value of η indicates more severe distortion by salience. When $\eta = 1$, consumers behave rationally. Consistent with Bordalo et al. (2012, 2013), we assume that all the consumers have the same level of salient thinking for simplicity.

Let $D_i(S)$ denote the *demand proportion* for product $i \in S$ and $S \subseteq \{h,p,l\}$, which is the proportion of consumers who purchase product i given the assortment S offered. A salient consumer with valuation θ chooses product i if $u_i^s(\theta) = \max_{j\in S} u_j^s(\theta)$ and $u_i^s(\theta) > 0$. In such a framework, demand proportions can be characterized by the valuation of indifferent consumers (e.g., Pan and Honhon 2012, Chen et al. 2013). Let θ_i be the indifferent consumer between product i and $i-1$, where i has a higher perceived quality than $i-1$. Then any consumer with $\theta > \theta_i$ would strictly prefer product i to $i-1$ because $u_i^s(\theta) > u_{i-1}^s(\theta)$. We can obtain $D_i(S)$ as follows.

$$D_i(S) = \begin{cases} 1 - F(\theta_h) & \text{for } i = h; \\ F(\theta_h) - F(\theta_p) & \text{for } i = p; \\ F(\theta_p) - F(\theta_l) & \text{for } i = l; \\ F(\theta_l) & \text{for } i = \emptyset. \end{cases}$$

The seller's profit is then $\pi(S) = \sum_{i\in S}(r_i - c_i)D_i(S)$ for a given set of products S. The seller's objective is to find S^* and the optimal selling prices r_i^* for $i \in S^*$ such that the profit is maximized. Because there is a one-to-one correspondence between $\vec{r} = (r_l, r_p, r_h)$ and $\vec{\theta} = (\theta_l, \theta_p, \theta_h)$, we can also optimize S and $\vec{\theta}$. Note that S^* includes the decision of ϕ if the probabilistic product is present. The notation is summarized in Table A.1 in the Appendix.

3. Rational Model: $\eta = 1$

When $\eta = 1$, consumers' perceived utility for one product does not depend on the other products offered by the seller (no context effects). Removing product i from consideration in the assortment is equivalent to preserving it but setting a sufficiently high price r_i such that no consumer purchases it. Therefore, the seller can regard the retail prices $\vec{r} = (r_l, r_p, r_h)$ as the only decision variables when consumers are rational (i.e., nonsalient) thinkers. In this regard, S^* contains those products that have positive demands corresponding to optimal $\vec{r^*}$.

Without loss of generality, we assume that prices $\vec{r^*}$ are set such that $0 \le \theta_l \le \theta_p \le \theta_h \le 1$. As a direct result of the one-to-one correspondence between $\vec{r} = (r_l, r_p, r_h)$ and $\vec{\theta} = (\theta_l, \theta_p, \theta_h)$, given any ϕ, we can rewrite the profit function as a function of $\vec{\theta}$ only, $\pi(\vec{\theta};\phi)$.

$$\begin{aligned}\pi(\vec{\theta};\phi) = &\ (1 - F(\theta_h))[(\theta_h(q_h - q_p) - (c_h - c_p)] \\ &+ (1 - F(\theta_p))[\theta_p(q_p - q_l) - (c_p - c_l)] \\ &+ (1 - F(\theta_l))(\theta_l q_l - c_l) \qquad (2)\end{aligned}$$

Hence, the seller's profit maximization problem is

$$\begin{aligned}&\max_{\vec{\theta}} \ \pi(\vec{\theta};\phi) \\ &\text{s.t.} \ \ 0 \le \theta_l \le \theta_p \le \theta_h \le 1. \qquad (3)\end{aligned}$$

Obviously, if no probabilistic products are offered for any given ϕ in (3), probabilistic selling should not be adopted. Surprisingly, in contrast to the literature, we find that with rational consumers, probabilistic selling does not improve the seller's profit for vertically differentiated products, as shown in Proposition 1.

Proposition 1. *When consumers are rational, probabilistic selling is never optimal, and the optimal strategy is as follows:*

$$S^* = \begin{cases} \{h\} & \text{if } \dfrac{c_h}{q_h} \le \dfrac{c_l}{q_l}; \\ \{l\} & \text{if } \dfrac{c_l}{q_l} < 1 \le \dfrac{c_h - c_l}{q_h - q_l}; \\ \{h, l\} & \text{if } \dfrac{c_l}{q_l} < \dfrac{c_h - c_l}{q_h - q_l} < 1. \end{cases} \qquad (4)$$

On the one hand, the probabilistic product enhances price discrimination by segmenting consumers more effectively (*price discrimination effect*). On the other hand, the availability of a probabilistic product might encourage some consumers who were planning to purchase a high-quality product to save money by purchasing the probabilistic product instead. Specifically, the introduction of the probabilistic product would cannibalize the high profit margin that the seller can obtain from the high-quality product (*cannibalization effect*). Because $c_p = \phi c_h + (1-\phi)c_l$ and $q_p = \phi q_h + (1-\phi)q_l$, we have $\frac{c_h - c_p}{q_h - q_p} = \frac{c_p - c_l}{q_p - q_l} = \frac{c_h - c_l}{q_h - q_l}$, which implies that the probabilistic product does not have a cost/quality advantage over the high- or low-quality products. Therefore, the introduction of the probabilistic product cannot improve the total profit because the cannibalization effect dominates the price discrimination effect. Interestingly, under abundant supply, Proposition 1 implies that the optimal selling strategy depends only on the cost and quality of both products but does not depend on how the consumer valuation of quality is distributed: offering the high-quality product only when the high-quality product is relatively stronger than the low-quality product, the low-quality product only when the low-quality product is absolutely stronger than the high-quality product, and both products otherwise. Note that this result holds for *any given quality levels*, so that the absence of probabilistic products cannot be attributed to the assumption of exogenously given quality levels.

Moreover, as shown in the proof of Proposition 1, the qualitative result that probabilistic selling is never optimal still holds for any level of supply. This robust result is in stark contrast to the findings of Zhang et al. (2015), where the probabilistic product is offered along with the high- (and low-) quality products in the optimal strategy; that is, $S^* = \{h, p, l\}$ or $\{h, p\}$. They assume, however, two discrete types of consumer in the market: high valuation and low valuation. The demand of each product changes (jumps) at the price that equals to the value of the discrete consumer valuation of the high- or low-quality products only, and is constant otherwise.

A necessary condition in their paper for probabilistic selling to emerge is that the seller has excess capacity of the high-quality product. The seller can dispose of excess capacity by creating probabilistic products that only target low-valuation consumers. The introduction of the probabilistic product does not affect the high-valuation consumers' demand for the high-quality product because only low-valuation consumers would purchase the probabilistic product as long as the incentive compatibility constraints are satisfied. That is, the cannibalization effect is limited. In our model, consumer valuation follows a continuous distribution, and the demand of each product continuously responds to a price change. Consequently, some consumers who were going to purchase the high-quality product switch to the probabilistic product if it is offered, which entails significant cannibalization so that the seller would *not* offer the probabilistic product.

The insights from Zhang et al. (2015) are applicable when the valuation distribution is bimodal; for example, consumers come from two separating sources (e.g., home and business sectors). Then, probabilistic selling could improve the profit by disposing the excess supply of the high-quality products. In a market with a large population of heterogeneous consumers, however, the demand continuously responds to any gradual change in price, and thus their insights are inapplicable. We find that probabilistic selling cannot improve the seller's profit when the valuation of fully heterogeneous consumers follows a continuous distribution.

Fay and Xie (2008) focus on horizontally differentiated products and show that probabilistic selling allows the seller to segment a market. Consequently, the seller can raise the selling prices of the component products to extract high surplus from consumers with strong preferences and offer a discounted probabilistic product to attract consumers with weaker preferences. In addition, they find that this advantage of probabilistic selling holds qualitatively in a quality-differentiated market defined in their work. Similar to the classic model of vertical differentiation (Mussa and Rosen 1978, Maskin and Riley 1984), their model also highlights that consumers always prefer the high-quality product over the low-quality one if they are offered at the same price. However, a key distinction between Fay and Xie (2008) and the classic models lies in the characterization of the underlying preferences. More specifically, Fay and Xie (2008) avail themselves of a Hotelling style model, and under this assumption probabilistic selling can still improve the profit in the vertically differentiated market in their work owing to the nature of negative correlation in the values for the component products. In contrast, we adopt the classic model, whereby the values for the quality-differentiated products are positively correlated. We show that the qualitative results concerning the advantage of probabilistic selling shown in Fay and Xie (2008) no longer hold when utilizing this canonical model of vertical differentiation.[10]

In summary, this rational benchmark demonstrates that the extant driving forces (Fay and Xie 2008, Zhang et al. 2015) for probabilistic selling would not arise in a standard model of vertical differentiation, which calls for a new theory.

4. Salient Model: $\eta < 1$

In this section, we incorporate the consumers' salient thinking behavior into the model. First, we consider a benchmark without the probabilistic product to identify a fundamental tension in a small assortment. Then, we show how to meticulously design the probabilistic product and determine the assortment so as to optimize the profit. Essentially, the seller has more flexibility to manipulate consumers' attention through endogenously determined salience by including the probabilistic product in the assortment.

4.1. Benchmark Without Probabilistic Products

The reference good associated with the assortment $S = \{h, l\}$ has attributes $\bar{q} = (q_l + q_h)/2$ and $\bar{r} = (r_l + r_h)/2$. The seller can change consumers' reference good by removing or adding a product in the assortment. In the rational model presented in Section 3, if a product has zero demand, preserving or removing it in the assortment does not change the results and implications. In contrast, in the salient model, offering the zero demand product could change consumers' reference good, possibly engendering different selling strategies. Before we present the optimal selling strategies, we first establish an important property concerning different salience relationships in Lemma 1.

Lemma 1. *In the assortment $S = \{h, l\}$, we have the following homogenous salience relationship for **both** products:*

i. *quality is salient if and only if $\frac{r_h}{q_h} < \frac{r_l}{q_l}$;*

ii. *price is salient if and only if $\frac{r_h}{q_h} > \frac{r_l}{q_l}$;*

iii. *price and quality are equally salient if and only if $\frac{r_h}{q_h} = \frac{r_l}{q_l}$.*

By definition, price and quality are equally salient for any assortment containing only one product. Nevertheless, for any assortment with two products, Lemma 1 states that both products have the same salient attribute (i.e., they possess a homogeneous salient feature). The seller can offer either a single product ($\{h\}$ or $\{l\}$) or both products ($\{h, l\}$). Note that we highlight the decoy product with zero demand in **bold**. Additionally, the assortment $\{h, l\}$ could have three salience relationships depending on different prices. Through excluding dominated strategies, we can reduce the number of potential optimal selling strategies as summarized in Lemma A.1 in the Appendix: $\{h, \boldsymbol{l}\}$, $\{h, l\}$, and $\{l\}$, which stand for the strategy of offering both products with quality salience but the low-quality product serving as the decoy, offering both products with price salience and positive demands on both products, and offering only the low-quality product with equal salience, respectively. The corresponding optimal profits and prices are

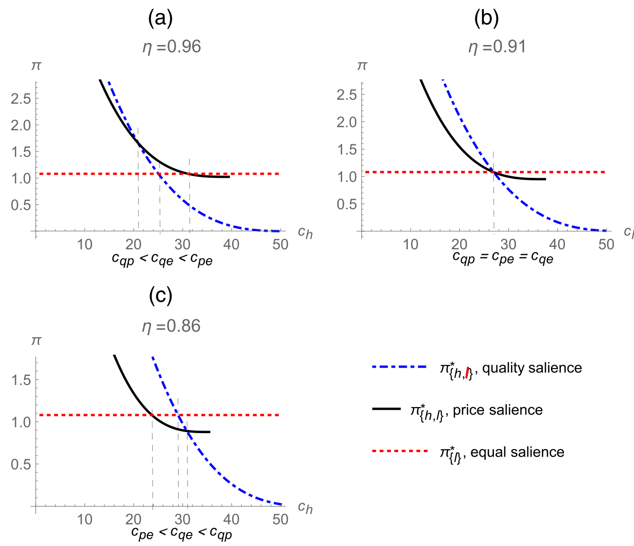
$$\pi^*_{\{h,\boldsymbol{l}\}} = (1 - F(\theta^*_h))\left(\frac{\theta^*_h q_h}{\eta} - c_h\right)$$
$$\text{where } \theta^*_h = \psi^{-1}\left(\frac{\eta c_h}{q_h}\right), r^*_h = \frac{\theta^*_h q_h}{\eta},$$
$$\text{and } \frac{\theta^*_h q_l}{\eta} < r^*_l \le r^*_h,$$
$$\pi^*_{\{h,l\}} = (1 - F(\theta^*_h))[\eta\theta^*_h(q_h - q_l) - (c_h - c_l)] + (1 - F(\theta^*_l))(\eta\theta^*_l q_l - c_l)$$
$$\text{where } \theta^*_l = \psi^{-1}\left(\frac{c_l}{\eta q_l}\right), \theta^*_h = \psi^{-1}\left(\frac{c_h - c_l}{\eta(q_h - q_l)}\right),$$
$$r^*_l = \eta\theta^*_l q_l, \text{ and } r^*_h = r^*_l + \eta\theta^*_h(q_h - q_l),$$
$$\pi^*_{\{l\}} = (1 - F(\theta^*_l))(\theta^*_l q_l - c_l) \text{ where } \theta^*_l = \psi^{-1}\left(\frac{c_l}{q_l}\right),$$
$$\text{and } r^*_l = \theta^*_l q_l.$$

Because all the prices $\frac{\theta^*_h q_l}{\eta} < r^*_l \le r^*_h$ give rise to the same outcome, we specify $r^*_l = r^*_h$ hereafter for strategy $\{h, \boldsymbol{l}\}$. It is obvious that $\pi^*_{\{h,\boldsymbol{l}\}}$ and $\pi^*_{\{h,l\}}$ decrease in c_h, whereas $\pi^*_{\{l\}}$ does not change with c_h. Through the comparative statics of the optimal profits with respect to c_h, we can identify the optimal strategy. To facilitate the analysis, for a given η, we define c_{qe}, c_{pe}, and c_{qp} for c_h such that $\pi^*_{\{h,\boldsymbol{l}\}}(c_{qe}) = \pi^*_{\{l\}}$, $\pi^*_{\{h,l\}}(c_{pe}) = \pi^*_{\{l\}}$, and $\pi^*_{\{h,\boldsymbol{l}\}}(c_{qp}) = \pi^*_{\{h,l\}}(c_{qp})$, respectively. We further define $\hat{\eta}$ such that $c_{qe}(\hat{\eta}) = c_{pe}(\hat{\eta})$ and use Example 1 for further illustration.

Example 1. Assume $F(\theta) = 1 - (1 - \theta)^2$, which means that there are more consumers with low valuations as compared with the uniform distribution. The parameters are $q_h = 50, q_l = 10$ and $c_l = 1$. The optimal profits are graphed in Figure 2. We have $\hat{\eta} \approx 0.91$ such that $c_{pe} = c_{qe}$ as shown in Figure 2(b). Figure 2(a) and 2(c) show how the profits change with c_h when $\eta = 0.96 > \hat{\eta}$, where $c_{pe} > c_{qe} > c_{qp}$ and when $\eta = 0.86 < \hat{\eta}$, where $c_{pe} < c_{qe} < c_{qp}$, respectively.

Note that $\pi^*_{\{l\}}$ is independent of c_h and η. Ceteris paribus, as η decreases, consumers' preferences are distorted more severely, impacting the profit under quality salience ($\pi^*_{\{h,\boldsymbol{l}\}}$) and price salience ($\pi^*_{\{h,l\}}$) in an opposite manner. Specifically, under quality salience, consumers overvalue the high-quality product more as η decreases (positive effect). Thus the

Figure 2. Profit as a Function of c_h with $\eta = 0.96$, 0.91, and 0.86 in Example 1



profit improvement from consumers' salient thinking is more pronounced and c_{qe} increases as η decreases (see the formal proof of Lemma A.2 in the Appendix). However, under price salience, the seller adopts a price discrimination strategy but consumers become more sensitive to price as η decreases (negative effect), leading to a deeper profit decrease; consequently, c_{pe} decreases as η decreases. From Figure 2, we observe that as the salience effect increases (i.e., η decreases), the range of c_h where price discrimination occurs ($\pi^*_{\{h,l\}}$) disappears. Essentially, the optimal strategy depends on the relationship among c_{qe}, c_{pe}, and c_{qp}, which are functions of η. Proposition 2 summarizes the results formally.

Proposition 2. *There exists a threshold value $\hat{\eta}$ such that (see Figure 3)*

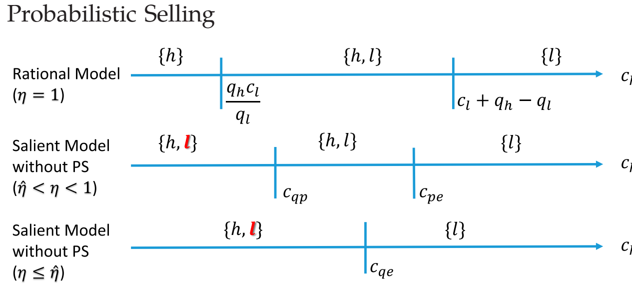
i. *for $\hat{\eta} < \eta < 1$, we have $\frac{q_h c_l}{q_l} < c_{qp} < c_{qe} < c_{pe} < c_l + \eta(q_h - q_l)$. At optimality, ceteris paribus,*

a. *when c_h is low ($< c_{qp}$), strategy $\{h, \boldsymbol{l}\}$ is optimal;*

b. *when c_h is in the intermediate range ($[c_{qp}, c_{pe}]$), strategy $\{h, l\}$ is optimal, i.e., the seller uses the second-degree price discrimination;*

c. *when c_h is high ($> c_{pe}$), strategy $\{l\}$ is optimal.*

Figure 3. Summary of Optimal Strategies Without Probabilistic Selling



Note. When η is small enough, it is possible that $c_{qe} > c_l + q_h - q_l$.

ii. *for $\eta \leq \hat{\eta}$, we have $c_{qp} \geq c_{qe} \geq c_{pe}$. At optimality, ceteris paribus,*

a. *when c_h is low ($< c_{qe}$), strategy $\{h, \boldsymbol{l}\}$ is optimal;*

b. *when c_h is high ($> c_{qe}$), strategy $\{l\}$ is optimal.*

Owing to the homogeneous salience relationship between two products, price salience arises from the second-degree price discrimination. Intuitively, whenever the firm screens consumers who self-select their own favorite product, price has to be the salient attribute so that the low-quality product can indeed attract low-valuation consumers. A lower value of η ($\leq \hat{\eta}$) indicates a stronger price salience effect such that the price discrimination effect is dominated and the seller would not adopt a price discrimination strategy. When the cost of the high-quality product is relatively low ($< c_{qe}$), the seller should offer the high-quality product along with the low-quality product, which serves as a decoy to induce consumers to overvalue the high-quality product; otherwise, only the low-quality product is offered. This analysis provides a new explanation for damaged goods besides the traditional screening explanation (Deneckere and McAfee 1996): Offering damaged goods makes the consumers overvalue the superior product and thus boosts its sales.

Instead, when $\hat{\eta} < \eta < 1$, the price salience effect is not strong, so the seller would adopt a price discrimination strategy for the intermediate range of c_h ($[c_{qp}, c_{pe}]$). Yet, this range of c_h shrinks compared with the rational counterpart ($\left[\frac{q_h c_l}{q_l}, c_l + q_h - q_l\right]$) because the price salience and price discrimination effects work in the opposite direction. Hence, the price salience effect constrains the seller's ability to implement a second-degree price discrimination strategy.

The above analysis reveals that, in the assortment with two products, salient thinking turns out to be a double-edged sword, in that it not only enables the seller to exploit quality salience when the cost of the high-quality product is relatively low but also leads to less profitable price discrimination otherwise. Under price discrimination, the incentive compatibility constraints cause the price attribute to become more pronounced, making it less profitable in comparison with the rational counterpart. In other words, the coexistence of quality salience and price discrimination is impossible, a fundamental tension with only two products. Typically, retailers do not have control over a product line but choose what products to offer from a given set of variants developed by manufacturers. Probabilistic selling avails the seller to alleviate this tension by creating a virtual product, expanding the assortment, and further manipulating consumers' preferences so that quality salience and price discrimination could coexist, as shown in the next section.

4.2. Consideration of Probabilistic Products

On the basis of the benchmark obtained in Section 4.1, we investigate whether probabilistic selling can arise as the optimal strategy. By the same token, the reference good of the assortment $S = \{h, p, l\}$ is $(\bar{q}, \bar{r})$ with $\bar{q} = (q_l + q_p + q_h)/3$ and $\bar{r} = (r_l + r_p + r_h)/3$. With the option of offering the probabilistic product, the seller has more flexibility to control the salient attribute for each product by setting different values of the price r_p and the probability ϕ.

In contrast to the homogeneous salience relationship in the absence of the probabilistic product shown in Lemma 1, the salience relationship for the assortment $S = \{h, p, l\}$ can be heterogeneous, as illustrated in Example 2.

Lemma 2. *It is possible for the salience relationship for $S = \{h, p, l\}$ to be heterogeneous.*

Example 2. Suppose $r_p > \frac{r_l + r_h}{2}$ and $\phi > \frac{1}{2}$, namely, $r_h > r_p > \bar{r} > r_l$ and $q_h > q_p > \bar{q} > q_l$. When $\frac{r_l}{q_l} < \frac{r_p}{q_p} < \frac{\bar{r}}{\bar{q}} < \frac{r_h}{q_h}$, we have the situation in which quality is salient for the probabilistic product but price is salient for both the high- and low-quality products.

Observing the change of the salience relationship, we next examine certain conditions under which the probabilistic strategy is not optimal.

Proposition 3. *When the high-quality product is relatively stronger than the low-quality product (i.e., $\frac{c_h}{q_h} < \frac{c_l}{q_l}$), a probabilistic selling strategy is never optimal and $S^* = \{h, \boldsymbol{l}\}$ where no consumers demand the low-quality product.*

Absent the probabilistic product, Proposition 2 shows that the seller offers both the low and high-quality products (i.e., $S^* = \{h, \boldsymbol{l}\}$) when the high-quality product is relatively stronger. In this situation, quality is salient for the high-quality product owing to the decoy l in the assortment. Further, because the high-quality product has a lower cost/quality ratio than the low-quality product (i.e., $\frac{c_h}{q_h} < \frac{c_l}{q_l}$), price discrimination is unprofitable in the same vein as Proposition 1. Therefore, offering a probabilistic product cannot further improve the profit through either enhancing price discrimination or inducing quality salience. This result implies that the probabilistic selling strategy is not effective for information goods, consistent with the observation in practice.

However, when the relative strength between the high- and low-quality product reverses, probabilistic selling improves the profit, as shown in Proposition 4. Let the notation $\{h, p, \boldsymbol{l}\}$ represent a strategy whereby the low-quality product serves as a decoy, leading to quality salience for all products. Denote $\{\boldsymbol{h}, p, l\}$ as a strategy whereby the high-quality product serves as a decoy such that the probabilistic product is quality salient whereas both the high- and low-quality products are price salient. Finally, let $\{p, \boldsymbol{l}\}$ represent a strategy whereby the low-quality product serves as a decoy such that quality is salient for both products. As shown in the proof of Proposition 4 in the Appendix, only the above three probabilistic selling strategies can be optimal. Following Herweg et al. (2017a), we refer to such a decoy described above as an *appropriate decoy good*.[11] To formally illustrate how the appropriate decoy good works, we next present the formulation for strategy $\{h, p, \boldsymbol{l}\}$ as an example.

$$\max_{\vec{r},\phi} \pi(r_h, r_p, \phi) = (1 - F(\theta_h))[(\theta_h(q_h - q_p)/\eta - (c_h - c_p)] + (1 - F(\theta_p))(\theta_p q_p/\eta - c_p) \tag{5}$$

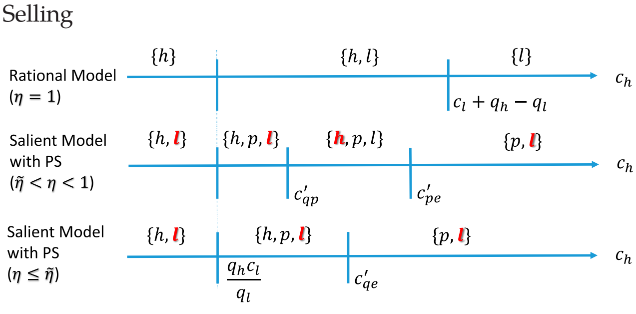
$$\text{s.t. } \frac{r_p}{q_p} < \frac{r_h}{q_h} < \frac{\bar{r}}{\bar{q}} < \frac{r_l}{q_l} \text{ and } \phi > \frac{1}{2} \tag{6}$$

$$\frac{r_h - r_p}{q_h - q_p} < 1 \tag{7}$$

In constraint (6), $\frac{r_h}{q_h} < \frac{\bar{r}}{\bar{q}}$ and $\frac{r_p}{q_p} < \frac{\bar{r}}{\bar{q}}$ (together with $\phi > \frac{1}{2}$) ensure that quality is salient for the high-quality and probabilistic product, respectively; $\frac{r_l}{q_l} > \frac{\bar{r}}{\bar{q}}$ ensures that no consumer indeed buys the decoy good (i.e., the low-quality product) when all three products are offered. Constraints (7) and $\frac{r_p}{q_p} < \frac{r_h}{q_h}$ ensure that both the probabilistic and high-quality products have positive demands. We refer the readers to the proof of Proposition 4 in the Appendix for more details. Note that the appropriate decoy good receiving zero demand does not influence the objective function directly but instead indirectly impacts the salient attribute of other products. Solving the maximization problem with this simplified constraint gives us the optimal prices and probability $(r_l^*, r_p^*, r_h^*, \phi^*)$ for strategy $\{h, p, \boldsymbol{l}\}$.

Similar to the benchmark in the absence of the probabilistic product, for any given η, we define c'_{qe}, c'_{pe}, and c'_{qp} for c_h such that $\pi^*_{\{h,p,\boldsymbol{l}\}}(c'_{qe}) = \pi^*_{\{p,\boldsymbol{l}\}}(c'_{qe})$, $\pi^*_{\{\boldsymbol{h},p,l\}}(c'_{pe}) = \pi^*_{\{p,\boldsymbol{l}\}}(c'_{pe})$, and $\pi^*_{\{h,p,\boldsymbol{l}\}}(c'_{qp}) = \pi^*_{\{\boldsymbol{h},p,l\}}(c'_{qp})$, respectively. The optimal probabilistic selling strategy depends on the relationship among c'_{qe}, c'_{pe} and c'_{qp}, which

Figure 4. Summary of Optimal Strategies with Probabilistic Selling



are functions of η, as summarized in Proposition 4. Analogously, we define $\tilde{\eta}$ such that $c'_{qe}(\tilde{\eta}) = c'_{pe}(\tilde{\eta})$.

Proposition 4. *When the low-quality product is relatively stronger than the high-quality product (i.e., $\frac{c_h}{q_h} \ge \frac{c_l}{q_l}$), by exploiting consumers' salient thinking behavior, the seller is able to adopt the following optimal probabilistic selling strategy to improve the profit (see Figure 4):*

i. *For $\tilde{\eta} < \eta < 1$, we have $\frac{q_h c_l}{q_l} < c'_{qp} < c'_{qe} < c'_{pe}$. At optimality, ceteris paribus,*

a. *when c_h is relatively low (i.e., $[\frac{q_h c_l}{q_l}, c'_{qp})$), the optimal strategy is $\{h, p, \boldsymbol{l}\}$ with $r_l^* = 2r_p^* - r_h^*$ and $\phi^* \in (\frac{1}{2}, 1)$, where r_p^* and r_h^* maximize (5);*

b. *when c_h is in the intermediate range (i.e., $[c'_{qp}, c'_{pe})$), the optimal strategy is $\{\boldsymbol{h}, p, l\}$ with $r_h^* = 2r_p^* - r_l^*$ and $\phi^* \to 1^-$, where r_l^* and r_p^* maximize (27);*

c. *when c_h is high (i.e., $[c'_{pe}, \infty)$), the optimal strategy is $\{p, \boldsymbol{l}\}$, $r_l^* = r_p^*$, and $\phi^* \to 0^+$, where r_p^* maximizes (22).*[12]

ii. *For $\eta \le \tilde{\eta}$, we have $c'_{qp} \ge c'_{qe} \ge c'_{pe}$. At optimality, ceteris paribus,*

a. *when c_h is relatively low (i.e., $[\frac{q_h c_l}{q_l}, c'_{qe})$), strategy $\{h, p, \boldsymbol{l}\}$ is optimal;*

b. *when c_h is high (i.e., $[c'_{qe}, \infty)$), strategy $\{p, \boldsymbol{l}\}$ is optimal.*

The price/quality ratio, r_i/q_i, plays a central role in shaping salience and determining price discrimination. Comparing with the reference good ($\bar{r}/\bar{q}$) and other products (r_j/q_j) respectively, we can determine which attribute is salient and which product may have positive demand accordingly. Because the reference good is endogenously determined by the selling strategy, any change in r_i and ϕ would potentially reverse the salience relationship. With more products in the assortment, the seller has more flexibility to select *the appropriate decoy with zero demand* ingeniously to configure the price/quality ratio of the market average and thus change the salience relationship to his favor. Meanwhile, the seller can optimize the price/quality ratios of the nondecoys to implement price discrimination under certain conditions. In other words, the products in the enlarged assortment are partitioned into two parts: the decoy with zero demand to influence the reference good and thus the salience relationship, and the nondecoys with positive demand to achieve price discrimination. Therefore, probabilistic selling extends the seller's capability to capitalize on both quality salience and price discrimination. We next discuss each specific region with probabilistic products when $c_h \ge \frac{q_h c_l}{q_l}$ and $\eta > \tilde{\eta}$.

When the cost of the high-quality product is relatively low (i.e., $\frac{q_h c_l}{q_l} < c_h < c'_{qp}$), as in the case of $c_h < \frac{q_h c_l}{q_l}$, the low-quality product still serves as the appropriate decoy with a sufficiently high price/quality ratio (i.e., $\frac{\bar{r}}{\bar{q}} < \frac{r_l}{q_l}$) such that quality is salient for both the high-quality and probabilistic product (i.e., $\frac{r_p}{q_p} < \frac{\bar{r}}{\bar{q}}$ and $\frac{r_h}{q_h} < \frac{\bar{r}}{\bar{q}}$). In addition to quality salience, probabilistic selling allows the seller to implement second-degree price discrimination on the probabilistic and high-quality products by optimizing their price/quality ratios (i.e., $\frac{r_p}{q_p} < \frac{r_h}{q_h}$). At optimum, the price of the probabilistic product is the average price of the high- and low-quality products, but the probability is greater than half (i.e., $\frac{1}{2} < \phi < 1$).

As the cost increases (i.e., in the intermediate range $c'_{qp} \le c_h < c'_{pe}$), the high-quality product is no longer effective when offered alongside the probabilistic product but can serve as the appropriate decoy with a sufficiently high price/quality ratio (i.e., $\frac{\bar{r}}{\bar{q}} < \frac{r_h}{q_h}$). All three products are still offered, but unlike the previous case, we have a heterogeneous salience relationship: quality remains salient for the probabilistic product, whereas price becomes salient for the low- and high-quality products (i.e., $\frac{r_p}{q_p} < \frac{\bar{r}}{\bar{q}}$, $\frac{r_l}{q_l} < \frac{\bar{r}}{\bar{q}}$, and $\frac{\bar{r}}{\bar{q}} < \frac{r_h}{q_h}$). Compared with strategy $\{h, l\}$ in the benchmark without probabilistic selling, price discrimination is still sustained (i.e., $\frac{r_l}{q_l} < \frac{r_p}{q_p}$), but we create a decoy effect driving more consumers to buy the probabilistic product with quality salience. It is interesting to observe that the price of the probabilistic product is still the average at optimum as in the previous case, but the associated probability is approaching one now. Another interpretation of this strategy is *misleading sales* as in Bordalo et al. (2013): the original price of a product on sale acts as a decoy to increase the salience of its quality. At optimum, because the probabilistic and high-quality products have almost the same quality, the probabilistic product can be regarded as the sale version of the high-quality product. Misleading sales boost demand for the high-quality product at the expense of demand for the low-quality product, consistent with the salience relationship here.

When the high-quality product cost is relatively high ($c_h \ge c'_{pe}$), it should be excluded from the assortment because it cannot act effectively as a decoy or for price discrimination. Compared with the benchmark in which only the low-quality product is offered, the probabilistic product turns equal salience into quality salience. Specifically, the low-quality product is now offered as the decoy to render the quality salient for the probabilistic product (i.e., $\frac{r_l}{q_l} < \frac{\bar{r}}{\bar{q}} < \frac{r_p}{q_p}$). At optimum, the seller should set $r_p = r_l$ and $\phi \approx 0$. Of course, the extremely high or low probability associated with the probabilistic product in theory can be implemented as, for example, 95% or 5% in practice. Note that although the high-quality product is not available for purchase individually, its specification information should still be present to consumers so that they can perceive the quality of the probabilistic product.

We next proceed to the scenario with severe salience distortion (i.e., $\eta < \tilde{\eta}$). The major distinction lies in the profitability of strategy $\{\boldsymbol{h}, p, l\}$ when $c'_{qp} \le c_h < c'_{pe}$. Recall that the probabilistic product is quality salient but the low-quality product is price salient when the high-quality product serves as the decoy. With salient thinking being more severe (i.e., η is lower), the negative effect of price salience would overrun the benefit of price discrimination and quality salience, making the strategy $\{\boldsymbol{h}, p, l\}$ suboptimal. Therefore, whether the high-quality product can serve as a decoy at optimum depends on the level of consumers' salient thinking. Put differently, when $\eta < \tilde{\eta}$, the only strategy with a heterogeneous salience relationship (i.e., $\{\boldsymbol{h}, p, l\}$) cannot be optimal, and thereby at optimum, the seller always sustains a homogeneous salience relationship, namely, quality salience no matter what products are offered. Interestingly, we observe that, regardless of the strength of salient thinking and cost structure, the optimal assortment always includes the low-quality product, which may not be purchased by any consumer.

Lemma 3. *The low-quality product is always offered (i.e., $l \in S^*$).*

Conventional wisdom in assortment planning suggests that the seller should not provide the lowest-quality product when it does not own any cost/quality advantage relative to other products (for example, when $\frac{c_h}{q_h} \le \frac{c_l}{q_l}$ as shown in Proposition 1). By contrast, in the presence of salient consumers, the lowest-quality product should always be included in the assortment. Apart from the role of price discrimination under certain conditions, the lowest-quality product is most effective to serve as a decoy. Further, from Proposition 4, we have the following lemma concerning the probabilistic product.

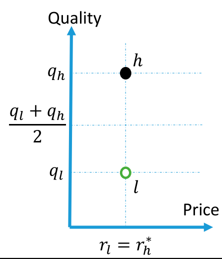
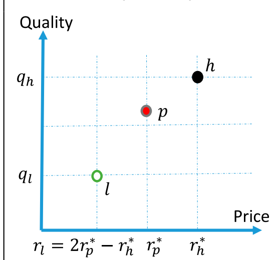
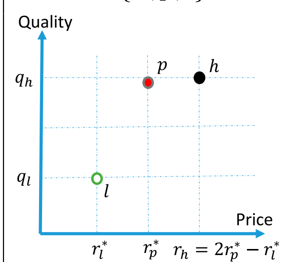
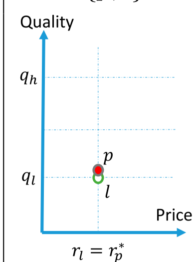
Lemma 4. *The probabilistic product never serves as the decoy. Moreover, quality is always salient for the probabilistic product.*

Lemma 4 indicates that the probabilistic product, as a virtual product with intermediate quality, is actually not effective to serve as a decoy. Instead, the seller should tilt consumers' attention to the quality attribute of the probabilistic product, which then is perceived as a "good deal" relative to the decoy.

Example 3 (Continued from Example 1). Suppose $\eta = 0.92$. We consider the optimal strategy for four instances corresponding to $c_h = 4, 8, 27, 40$, respectively. Table 1 lists the optimal solutions in the presence and absence (benchmark) of the probabilistic product and the magnitude of the profit advantage from probabilistic selling.

In this example, we have $\frac{q_h c_l}{q_l} = 5 < c'_{qp} = 8.88 < c'_{qe} = 26.00 < c'_{pe} = 30.95$ as $\eta = 0.92 > \tilde{\eta} = 0.7$. For instance (1),

Table 1. (Color online) Optimal Strategy in Example 3

Instances		1	2	3	4
c_h		4	8	27	40
Strategy (S^*)		$\{h, \boldsymbol{l}\}$ 	$\{h, p, \boldsymbol{l}\}$ 	$\{\boldsymbol{h}, p, l\}$ 	$\{p, \boldsymbol{l}\}$ 
Prob. (ϕ^*)		-	51.4%	≈ 1	≈ 0
Prices ($\vec{r^*}$)		$r_h^* = 20.78$ $r_l^* = 20.78$	$r_p^* = 14.13, r_h^* = 23.45$ $r_l^* = 4.82$	$r_l^* = 3.77, r_p^* = 35.99$ $r_h^* = 68.22$	$r_p^* = 4.29$ $r_l^* = 4.29$
Profit (π^*)		6.40	5.00	1.46	1.21
Benchmark	S	$\{h, \boldsymbol{l}\}$	$\{h, \boldsymbol{l}\}$	$\{h, l\}$	$\{l\}$
	$\vec{r}$	$r_l^* = r_h^* = 20.78$	$r_l^* = r_h^* = 23.45$	$r_l^* = 3.73, r_h^* = 33.33$	$r_l^* = 4$
	π	6.40	4.99	1.10	1.08
Increase (%)		0	0.05	31.98	11.62

$c_h = 4 < \frac{q_h c_l}{q_l}$ and probabilistic selling cannot improve the profit such that strategy $\{h, \boldsymbol{l}\}$ remains optimal, consistent with Proposition 3. In the other three instances, c_h falls in the three intervals shown in Proposition 4(i) (i.e., $\frac{q_h c_l}{q_l} < c_h = 8 < c'_{qp}$, $c'_{qp} < c_h = 27 < c'_{pe}$, and $c_h = 40 > c'_{pe}$, respectively). Compared with the benchmark, probabilistic selling enables the seller to take an additional advantage of price discrimination in instance (2) and turn price and equal salience into quality salience in instances (3) and (4), respectively. The decoy effect raises the seller's profit significantly by 31.98% in instance (3) and 11.62% in instance (4).

4.3. Discussion

A few remarks are in order. First, these results concerning the importance of consumers' salient thinking behavior on probabilistic selling extends beyond those immediately shown here. In Proposition 4, we demonstrate the impact of salience based on regular distributions of consumer valuation. As a matter of fact, the salience mechanism also applies to discrete distributions as adopted in Zhang et al. (2015). A necessary condition for the probabilistic product to emerge in their work is that the supply of the high-quality product is abundant. Our salience theory does not impose any conditions on supply. In essence, probabilistic selling expands the set of candidate products and enables the seller to manipulate consumers' preference structure favorably by tilting their attention to quality without weakening the ability to implement price discrimination. This manipulation only depends on the choice context (i.e., the assortment) facing the salient thinkers rather than any operational conditions. Therefore, our noteworthy findings apply to settings in which a seller faces uncertain demand, capacity constraints, endogenous quality, or a general case with more than two component products. It is imperative for practitioners to re-evaluate the implications of adopting a probabilistic selling strategy in light of these salience effects.

Second, salient thinking provides different implications on the marketing mix decisions for probabilistic selling compared with other theories (Fay and Xie 2008, Zhang et al. 2015). In our paper, whether the seller should offer the probabilistic product is contingent only on the relative strength between the low- and high-quality products. In addition, in stark contrast to Zhang et al. (2015), the low-quality product is always included in the assortment regardless of the cost structure in our model. At optimum, the probabilistic product is sold either at the average price (with a probability greater than half) or at the price of the low-quality product (with an extremely low probability). As shown in practice, this product mix strategy can be implemented easily, providing sellers a context management tool to exploit consumers' salient thinking behavior.

Last, it is worthwhile to point out the difference between the salience model and reference dependent models that incorporate loss aversion. An important implication of the loss aversion model is a bias toward middle-of-the-road options (e.g., a midquality probabilistic product), whereas the salience theory may predict that consumers prefer extreme options (e.g., a nondecoy). Consider a representative consumer with $\theta = 1$ choosing from the following three products of (q_i, r_i): $(50, \$39), (41, \$31), (30, \$20)$. Our salience definition suggests that price is salient for all three products. A rational consumer would choose product 1, yet the loss aversion and salience models would predict that this consumer chooses product 2 and 3, respectively, for most parameters.[13] Therefore, re-examining the results on the basis of the loss aversion model in different settings, such as product line design, will be interesting for future research.

5. Conclusion

We study whether probabilistic selling improves the profit of a seller offering vertically differentiated products. Interestingly, we first show that adding the probabilistic product in the assortment cannibalizes the existing sales such that the profit decreases for any level of supply when consumers are rational. Next, we extend the model by incorporating consumers' salient thinking behavior. The seller can include the probabilistic product to manipulate the consumer's attention to a particular attribute. Specifically, when the low-quality product is relatively stronger than the high-quality product, the introduction of the probabilistic product leads to a higher profit through either enhancing price discrimination with an enlarged product line or mitigating the negative effect of price salience by transforming the salient attribute to quality for some products. Our findings provide a new rationale for this novel selling strategy: consumers' salient thinking. We show that probabilistic selling could be more attractive than previously exhibited in the literature when considering the effects of salience.

Our results also have important implications for the design of probabilistic products and assortment planning. The seller can establish either the high- or low-quality product as the decoy with sufficiently high price/quality ratios such that its demand is driven to zero by meticulously designing the probabilistic product. Specifically, when the cost of the high-quality product is relatively low or high, the low-quality product would serve as the decoy and quality is salient for the products in the assortment. When the high-quality product has an intermediate cost, it instead acts as the decoy such that quality becomes salient only for the probabilistic product, resembling a misleading sales policy. Moreover, these results are especially useful to a seller who does not control the product design decisions. A key benefit of the probabilistic product is that no supplementary fixed costs and lead time are incurred with the design and physical production of the probabilistic goods.

For future study, we provide several interesting directions. First, in some business situations (e.g., car rental and hotel industry), the probability is not prespecified, and the seller assigns the component product according to the product availability at the consumption time. An analysis of this characteristic would shed light on revenue management techniques appropriate for addressing dynamic capacity issues. Another extension could address the possibility of endogenizing the choice of the quality levels in the presence of salient thinking. For a market with homogeneous consumers, Herweg et al. (2017b) show that a manufacturer always under-invests quality when competing against a competitive fringe. In a rational model à la Mussa and Rosen (1978) with two types of consumers (i.e., $\theta \in \{\theta_h, \theta_l\}$), it is well known that the monopolist either offers the standard screening contract or targets on consumer segments of high valuation θ_h only. In the salient thinking model, Herweg et al. (2017b) show that the monopolist may also offer a pooling contract whereby all the consumers are served with a single product when the high-type consumer segment is not large enough. Moreover, together with a single target product having positive sales, the monopolist always offers a decoy good so that consumers would overvalue the quality and have a higher willingness to pay. It would be interesting to investigate product line design with salient thinking behavior in a market with fully heterogeneous consumers.

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Appendix. Notation and Proofs

For ease of illustration in the subsequent optimization problem, we introduce additional notation of salience function.

Definition A.1. Salience function $s(\cdot,\cdot)$ is a symmetric and continuous salience function satisfying the following properties.

Ordering: $s(x,y) < s(x',y')$ if $[x,y] \subset [x',y']$ for any $x, y, x', y' \in \mathbb{R}^+$;

Homogeneity of degree zero: $s(\lambda x, \lambda y) = s(x,y)$ for any $\lambda > 0$ with $s(0,0) = 0$.

The two properties imply diminishing sensitivity and further induce proposition 1 in Bordalo et al. (2013), a central result in the salience mechanism. We refer readers to their seminal paper for psychological motivations. Therefore, the salience of quality and price for a product i is given by $s(r_i,\bar{r})$ and $s(q_i,\bar{q})$, respectively. Utilizing this nomenclature, we state that quality is salient for product i if and only if $s(r_i,\bar{r}) < s(q_i,\bar{q})$, price is salient for product i if and only if $s(r_i,\bar{r}) > s(q_i,\bar{q})$, and price and quality are equally salient if and only if $s(r_i,\bar{r}) = s(q_i,\bar{q})$.

Proof of Proposition 1. We prove the result for a general case with any level of supply. The corresponding model is

$$\begin{aligned}
\max_{\vec{\theta}} \quad & \pi(\vec{\theta};\phi) \\
\text{s.t.} \quad & 0 \le \theta_l \le \theta_p \le \theta_h \le 1 \\
& D_h + \phi D_p \le k_h \\
& D_l + (1-\phi) D_p \le k_l,
\end{aligned}$$

where k_h and k_l are the capacity of the high- and low-quality products, respectively. The last two constraints ensure that the demand for the each quality product does not exceed the corresponding supply. Note that $k_i = 1$ corresponds to the unlimited supply case because the demand for each product is bounded by one.

The seller maximizes the profit function (2), subject to the constraints:

$$1 - F(\theta_h) + \phi(F(\theta_h) - F(\theta_p)) \le k_h \tag{A.1}$$

$$(1-\phi)(F(\theta_h) - F(\theta_p)) + F(\theta_p) - F(\theta_l) \le k_l \tag{A.2}$$

$$\theta_p - \theta_h \le 0 \tag{A.3}$$

$$\theta_l - \theta_p \le 0 \tag{A.4}$$

$$\theta_h \le 1. \tag{A.5}$$

Define $\hat{\theta}_{l1}$, $\hat{\theta}_{l2}$, $\hat{\theta}_{l3}$, $\hat{\theta}_{h1}$, $\hat{\theta}_{h2}$, $\hat{\theta}_{h3}$ and the Lagrangian as follows,

$$\psi(\hat{\theta}_{l1}) = \frac{c_l}{q_l}, \psi(\hat{\theta}_{l2}) = \frac{c_h - (q_h - q_l)}{q_l},$$

$$\psi(\hat{\theta}_{h1}) = \frac{c_h}{q_h}, \psi(\hat{\theta}_{h2}) = \frac{c_h - c_l}{q_h - q_l},$$

$$\psi(\hat{\theta}_{h3}) = \frac{c_h - q_l\psi(\hat{\theta}_{l3})}{q_h - q_l}, F(\hat{\theta}_{h3}) - F(\hat{\theta}_{l3}) = k_l,$$

$$\begin{aligned}
\mathcal{L} = {} & (1 - F(\theta_h))(\theta_h(q_h - q_p) - (c_h - c_p)) \\
& + (1 - F(\theta_p))(\theta_p(q_p - q_l) - (c_p - c_l)) \\
& + (1 - F(\theta_l))(\theta_l q_l - c_l) \\
& - \lambda_1\big[1 - F(\theta_h) + \phi(F(\theta_h) - F(\theta_p)) - k_h\big] \\
& - \lambda_2\big[(1-\phi)(F(\theta_h) - F(\theta_p)) + F(\theta_p) - F(\theta_l) - k_l\big] \\
& - \lambda_3(\theta_p - \theta_h) - \lambda_4(\theta_l - \theta_p) - \lambda_5(\theta_h - 1),
\end{aligned}$$

where λ_1 to λ_5 correspond to constraints (8) to (12). Solving Karush–Kuhn–Tucker (KKT) conditions gives the optimal solutions in Tables A.2–A.4 for any supply level, which suggests that offering a probabilistic product is never optimal because $\theta_h^* = \theta_p^*$ always holds. The detailed proof is available from the author upon request.

Table A.1. Notation

Symbol	Definition	Symbol	Definition
h	High quality product	u^s	Perceived utility distorted by salience
l	Low quality product	$s(\cdot,\cdot)$	Salience function
p	Probabilistic product	π_S	Profit under assortment (strategy) S
$\varnothing$	Decision of no purchase	c'_{qp}	Cutoff of c_h such that $\pi_{\{h,p,l\}} = \pi_{\{\mathbf{h},\mathbf{p},\mathbf{l}\}}$
i	Product index, $i \in \{h,p,l\}$	c'_{qe}	Cutoff of c_h such that $\pi_{\{\mathbf{h},\mathbf{p},\mathbf{l}\}} = \pi_{\{p,\mathbf{l}\}}$
D_i	Demand proportion of product i	c'_{pe}	Cutoff of c_h such that $\pi_{\{\mathbf{h},p,\mathbf{l}\}} = \pi_{\{p,\mathbf{l}\}}$
q_i	Quality level of product i	c_{qp}	Cutoff of c_h such that $\pi_{\{h,\mathbf{l}\}} = \pi_{\{h,l\}}$
k_i	Supply for product $i \in \{h,l\}$	c_{qe}	Cutoff of c_h such that $\pi_{\{h,\mathbf{l}\}} = \pi_{\{l\}}$
c_i	Variable cost of product i	c_{pe}	Cutoff of c_h such that $\pi_{\{h,\mathbf{l}\}} = \pi_{\{l\}}$
η	Salience parameter		
$\hat{\eta}$	Cutoff of η such that $c_{pe} = c_{qe}$	Decision variables	
$\tilde{\eta}$	Cutoff of η such that $c'_{pe} = c'_{qe}$	r_i	Selling price of product i
$\bar{q}, \bar{r}$	Market average	θ_i	Indifferent consumer btw $i-1$ and i
θ	Consumer valuation, $\theta \in [0,1]$	ϕ	Probability of receiving product h
$f(\theta)$	Probability density function of θ	S	Assortment (set of products offered)
$F(\theta)$	Cumulative density function of θ	$\vec{\theta}$	Indifferent consumer vector
$\psi(\theta)$	Virtual valuation	$\vec{r}$	Selling price vector

Table A.2. Solutions for $\frac{c_h}{q_h} < \frac{c_l}{q_l} < 1$ Where $\hat{\theta}_{h1} < \hat{\theta}_{l1}$

		$F(\hat{\theta}_{h1}) < 1 - k_h$		
	$F(\hat{\theta}_{h1}) \geq 1 - k_h$	$F(\hat{\theta}_{l1}) \geq 1 - k_h$	$1 - k_h - k_l < F(\hat{\theta}_{l1}) < 1 - k_h$	$F(\hat{\theta}_{l1}) \leq 1 - k_h - k_l$
θ_l^*	$\hat{\theta}_{h1}$	$F^{-1}(1 - k_h)$	$\hat{\theta}_{l1}$	$F^{-1}(1 - k_h - k_l)$
$\theta_h^* = \theta_p^*$	$\hat{\theta}_{h1}$	$F^{-1}(1 - k_h)$	$F^{-1}(1 - k_h)$	$F^{-1}(1 - k_h)$

Table A.3. Solutions for $\frac{c_l}{q_l} < 1 < \frac{c_h - c_l}{q_h - q_l}$ Where $\hat{\theta}_{l1} < \hat{\theta}_{h2}$

		$F(\hat{\theta}_{l1}) < 1 - k_l$		
			$F(\hat{\theta}_{l2}) < 1 - k_l$	
	$F(\hat{\theta}_{l1}) \geq 1 - k_l$	$F(\hat{\theta}_{l2}) \geq 1 - k_l$	$F(\hat{\theta}_{h3}) \geq 1 - k_h$	$F(\hat{\theta}_{h3}) < 1 - k_h$
θ_l^*	$\hat{\theta}_{l1}$	$F^{-1}(1 - k_l)$	$\hat{\theta}_{l3}$	$F^{-1}(1 - k_h - k_l)$
$\theta_h^* = \theta_p^*$	1	1	$\hat{\theta}_{h3}$	$F^{-1}(1 - k_h)$

Table A.4. Solutions for $\frac{c_l}{q_l} \leq \frac{c_h - c_l}{q_h - q_l} \leq 1$ Where $\hat{\theta}_{l1} < \hat{\theta}_{h2}$

	$F(\hat{\theta}_{h2}) > 1 - k_h$		$F(\hat{\theta}_{h2}) \leq 1 - k_h$	
	$F(\hat{\theta}_{h2}) - F(\hat{\theta}_{l1}) < k_l$	$F(\hat{\theta}_{h2}) - F(\hat{\theta}_{l1}) \geq k_l$	$F(\hat{\theta}_{l1}) > 1 - k_h - k_l$	$F(\hat{\theta}_{l1}) \leq 1 - k_h - k_l$
θ_l^*	$\hat{\theta}_{l1}$	$\hat{\theta}_{l3}$	$\hat{\theta}_{l1}$	$F^{-1}(1 - k_h - k_l)$
$\theta_h^* = \theta_p^*$	$\hat{\theta}_{h2}$	$\hat{\theta}_{h3}$	$F^{-1}(1 - k_h)$	$F^{-1}(1 - k_h)$

For the case with $k_h = k_l = 1$ (i.e., under abundant supply), we obtain the optimal solutions as shown in (4). The simplified KKT conditions imply that the products in the optimal assortment must have strictly increasing ratios of cost differential to quality differential in the quality levels, consistent with lemma 5 in Pan and Honhon (2012). Moreover, the seller's optimal strategy depends on the cost/quality ratios only. □

Proof of Lemma 1. For part (i), when $\frac{r_h}{q_h} < \frac{r_l}{q_l}$, we have $\frac{r_h}{q_h} < \frac{\bar{r}}{\bar{q}} < \frac{r_l}{q_l}$. Therefore, quality is salient for both products. In a similar vein, we can obtain part (ii) and (iii). □

Lemma A.1.

i. *For strategy* $S = \{h, l\}$,

a. *the seller can maximize the profit by optimally setting the prices such that quality is salient and no consumers desire the low-quality product for any* η *(i.e.,* $\{h, \mathbf{l}\}$*) and*

$$\psi(\theta_h^*) = \frac{\eta c_h}{q_h} \text{ and } \theta_l^* > \theta_h^*,$$

$$\pi_{\{h,\mathbf{l}\}}^* = (1 - F(\theta_h^*))\left(\frac{\theta_h^* q_h}{\eta} - c_h\right) \text{ and } \frac{\partial \pi_{\{h,\mathbf{l}\}}^*}{\partial c_h} < 0;$$

b. *the seller can maximize the profit by optimally setting the prices such that price is salient and both products have a positive demand if* $\frac{c_l}{q_l} < \frac{c_h - c_l}{q_h - q_l} < \eta$ *(i.e.,* $\{h, l\}$*), and*

$$\psi(\theta_l^*) = \frac{c_l}{\eta q_l} \text{ and } \psi(\theta_h^*) = \frac{c_h - c_l}{\eta(q_h - q_l)},$$

$$\pi_{\{h,l\}}^* = (1 - F(\theta_h^*))[\eta\theta_h^*(q_h - q_l) - (c_h - c_l)] + (1 - F(\theta_l^*))(\eta\theta_l^* q_l - c_l) \text{ and } \frac{\partial \pi_{\{h,l\}}^*}{\partial c_h} < 0.$$

c. *this strategy is never optimal when price and quality are equally salient.*

ii. *Strategy* $S = \{h\}$ *with equal salience is never optimal.*

iii. *For strategy* $S = \{l\}$ *with equal salience, the seller can maximize the profit by optimally setting the price of the low-quality product such that*

$$\psi(\theta_l^*) = \frac{c_l}{q_l},$$

$$\pi_{\{l\}}^* = (1 - F(\theta_l^*))(\theta_l^* q_l - c_l) \text{ and } \frac{\partial \pi_{\{l\}}^*}{\partial c_h} = 0.$$

Proof of Lemma A.1. The proof is organized according to different salience relationships.

1. When quality is salient (i.a), from Lemma 1, we have $\frac{r_h}{q_h} < \frac{r_l}{q_l}$ and thus $\frac{\eta r_h}{q_h} < \frac{\eta r_l}{q_l}$; that is, $\theta_l > \theta_h$. Therefore, no consumers demand the low-quality product, but it is still offered along with the high-quality product because it renders the quality salient for the high-quality product and consumers overvalue the quality $\frac{q_h}{\eta}$. Consequently, the seller can charge a higher price on the high-quality product compared with the case without the low-quality product. The seller maximizes the following problem:

$$\pi_{\{h,\mathbf{l}\}}(\vec{\theta}) = (1 - F(\theta_h))\left(\frac{\theta_h q_h}{\eta} - c_h\right),$$

subject to the constraints: $\theta_l > \theta_h$. Then we have

$$\psi(\theta_h^*) = \frac{\eta c_h}{q_h} \text{ and } \theta_l^* > \theta_h^*.$$

By the envelope theorem, $\frac{\partial \pi_{\{h,\mathbf{l}\}}^*}{\partial c_h} = -(1 - F(\theta_h^*)) < 0$.

2. When price is salient (i.b), from Lemma 1, we have $\frac{r_h}{q_h} > \frac{r_l}{q_l}$ and thus $\frac{r_h - r_l}{q_h - q_l} > \frac{r_l}{q_l}$. When $\frac{r_h - r_l}{q_h - q_l} < \eta$, $\theta_l = \frac{r_l}{\eta q_l} < \theta_h = \frac{r_h - r_l}{\eta(q_h - q_l)} < 1$. Price discrimination is used, and the seller maximizes the following problem:

$$\pi_{\{h,l\}}(\vec{\theta}) = (1 - F(\theta_h))[\eta\theta_h(q_h - q_l) - (c_h - c_l)] + (1 - F(\theta_l))(\eta\theta_l q_l - c_l),$$

subject to the constraints: $\theta_l < \theta_h < 1$. Then we have

$$\psi(\theta_l^*) = \frac{c_l}{\eta q_l} \text{ and } \psi(\theta_h^*) = \frac{c_h - c_l}{\eta(q_h - q_l)} < 1.$$

The above solution is optimal in the range $\frac{c_l}{q_l} < \frac{c_h - c_l}{q_h - q_l} < \eta$. By the envelope theorem, $\frac{\partial \pi^*_{\{h,l\}}}{\partial c_h} = -(1 - F(\theta_h^*)) < 0$. However, when $\frac{r_h - r_l}{q_h - q_l} > \eta > \frac{r_l}{q_l}$, $\theta_l = \frac{r_l}{\eta q_l} < 1 < \theta_h = \frac{r_h - r_l}{\eta(q_h - q_l)}$, that is, no consumers purchase the high-quality product but price is still salient. Then we have the following problem:

$$\pi_{\{h,l\}}(\vec{\theta}) = (1 - F(\theta_l))(\eta\theta_l q_l - c_l).$$

The optimal solution is

$$\psi(\theta_l^*) = \frac{c_l}{\eta q_l} \text{ and } \psi(\theta_h^*) > 1.$$

In summary, we can set the prices such that:

$$\psi(\theta_l^*) = \frac{c_l}{\eta q_l} \text{ and } \psi(\theta_h^*) = \frac{c_h - c_l}{\eta(q_h - q_l)},$$

and the corresponding profits are

$$\pi^*_{\{h,l\}} = \begin{cases} (1 - F(\theta_h^*))[\eta\theta_h^*(q_h - q_l) - (c_h - c_l)] \\ \quad + (1 - F(\theta_l^*))(\eta\theta_l^* q_l - c_l) & \text{if } \dfrac{c_l}{q_l} < \dfrac{c_h - c_l}{q_h - q_l} < \eta, \\ (1 - F(\theta_l^*))(\eta\theta_l^* q_l - c_l) & \text{if } \dfrac{c_l}{q_l} < \eta \leq \dfrac{c_h - c_l}{q_h - q_l}. \end{cases}$$

However, when $\frac{c_l}{q_l} < \eta < \frac{c_h - c_l}{q_h - q_l}$, $\theta_h^* > 1$ and no consumers desire the high-quality product. In this case, trimming off the high-quality product would prompt more sales of the low-quality products and thereby enhance the profit. In other words, it is dominated by offering the low-quality product with equal salience (iii) and thus can be excluded. Note that when $\eta < \frac{c_l}{q_l}$, price salience is never optimal because no consumers purchase the high- or low-quality product.

3. When price and quality are equally salient, we have three possible situations: $\{h, l\}$, $\{l\}$ and $\{h\}$. If $S = \{h, l\}$ (i.c), according to Lemma 1, we have $\frac{r_h}{q_h} = \frac{r_l}{q_l}$ and thus only the high-quality product has positive demand ($\theta_h = \theta_l$). No one would purchase the low-quality product, and thereby it achieves the same profit as offering the high-quality product only (ii). Therefore, the equal salience for $S = \{h, l\}$ is excluded. Moreover, offering the high-quality product only (ii) is dominated by offering both products with quality salience (i.a) because consumers overvalue the quality and thus more consumers would purchase the high-quality product.

After excluding the dominated equal salience with $\{h, l\}$ and $\{h\}$, we only need to consider the case of offering the low-quality product only (i.e., $S = \{l\}$) (iii). The seller maximizes the following problem:

$$\pi_{\{l\}}(\vec{\theta}) = (1 - F(\theta_l))(\theta_l q_l - c_l).$$

Then we have

$$\psi(\theta_l^*) = \frac{c_l}{q_l}.$$

Obviously, $\frac{\partial \pi^*_{\{l\}}}{\partial c_h} = 0$ for $S = \{l\}$ by the envelope theorem. □

Lemma A.2. The cost thresholds changes with η in the opposite direction: $\frac{\partial c_{qe}}{\partial \eta} < 0$ *and* $\frac{\partial c_{pe}}{\partial \eta} > 0$.

Proof of Lemma A.2. $c_{qe}(\eta)$ is the solution to the implicit function $\pi^*_{\{h,\mathbf{l}\}}(c_{qe}; \eta) = \pi^*_{\{l\}}$. By the implicit function theorem, we have $\frac{\partial \pi^*_{\{h,\mathbf{l}\}}}{\partial c_{qe}} \cdot \frac{\partial c_{qe}}{\partial \eta} + \frac{\partial \pi^*_{\{h,\mathbf{l}\}}}{\partial \eta} = 0$. Because $\frac{\partial \pi^*_{\{h,\mathbf{l}\}}}{\partial c_{qe}} < 0$ from Lemma A.1 and $\frac{\partial \pi^*_{\{h,\mathbf{l}\}}}{\partial \eta} = -\frac{(1 - F(\theta_h^*))\theta_h^* q_h}{\eta^2} < 0$ by the envelope theorem, we have $\frac{\partial c_{qe}}{\partial \eta} < 0$. In a similar vein, we can obtain $\frac{\partial c_{pe}}{\partial \eta} > 0$. □

Proof of Proposition 2. In the range $c_h \in \left[\frac{q_h c_l}{q_l}, c_l + \eta(q_h - q_l)\right]$, we have $\pi^*_{\{h,\mathbf{l}\}}(c_h) > \pi^*_{\{h,l\}}(c_h)$ when $c_h = \frac{q_h c_l}{q_l}$. Moreover, $\frac{\partial \pi^*_{\{h,\mathbf{l}\}}}{\partial c_h} < \frac{\partial \pi^*_{\{h,l\}}}{\partial c_h} < 0$ because $\frac{\partial \pi^*_{\{h,\mathbf{l}\}}}{\partial c_h} = -(1 - F(\theta_h^*))$ where $\psi(\theta_h^*) = \frac{\eta c_h}{q_h}$, $\frac{\partial \pi^*_{\{h,l\}}}{\partial c_h} = -(1 - F(\theta_h^*))$ where $\psi(\theta_h^*) = \frac{c_h - c_l}{\eta(q_h - q_l)}$, and $\frac{\eta c_h}{q_h} < \frac{c_h - c_l}{\eta(q_h - q_l)}$. If $c_{pe} < c_{qe}$, then there is no intersection point between $\pi^*_{\{h,\mathbf{l}\}}(c_h)$ and $\pi^*_{\{h,l\}}(c_h)$ in that range; if $c_{pe} > c_{qe}$, then $\pi^*_{\{h,\mathbf{l}\}}(c_h)$ and $\pi^*_{\{h,l\}}(c_h)$ intersect only once in that range at c_{qp}, that is, $\pi^*_{\{h,\mathbf{l}\}}(c_{qp}) = \pi^*_{\{h,l\}}(c_{qp})$. Because $c_{pe}(\hat{\eta}) = c_{qe}(\hat{\eta})$, according to Lemma A.2, if $\eta > \hat{\eta}$, $c_{pe} > c_{qe}$ and then $\frac{q_h c_l}{q_l} < c_{qp} < c_{pe} < c_l + \eta(q_h - q_l)$, otherwise $c_{qp} \geq c_{pe}$. In summary (see Figure 3),

i. If $\eta \leq \hat{\eta}$,

a. when $c_h < c_{qe}$, $\pi^*_{\{h,\mathbf{l}\}} > \max\{\pi^*_{\{h,l\}}, \pi^*_{\{l\}}\}$ and hence $S^* = \{h, \mathbf{l}\}$ with $\psi\left(\frac{\eta r_h^*}{q_h}\right) = \frac{\eta c_h}{q_h}$ and $r_l^* > \frac{q_l r_h^*}{q_h}$;

b. when $c_h \geq c_{qe}$, $\pi^*_{\{l\}} > \max\{\pi^*_{\{h,\mathbf{l}\}}, \pi^*_{\{h,l\}}\}$ and hence $S^* = \{l\}$ with $\psi\left(\frac{r_l^*}{q_l}\right) = \frac{c_l}{q_l}$.

ii. If $\eta > \hat{\eta}$,

a. when $c_h < c_{qp}$, $\pi^*_{\{h,\mathbf{l}\}} > \max\{\pi^*_{\{h,l\}}, \pi^*_{\{l\}}\}$ and hence $S^* = \{h, \mathbf{l}\}$ with $\psi\left(\frac{\eta r_h^*}{q_h}\right) = \frac{\eta c_h}{q_h}$ and $r_l^* > \frac{q_l r_h^*}{q_h}$;

b. when $c_{qp} \leq c_h < c_{pe}$, $\pi^*_{\{h,l\}} > \max\{\pi^*_{\{h,\mathbf{l}\}}, \pi^*_{\{l\}}\}$ and hence $S^* = \{h, l\}$ with $\psi(\theta_l^*) = \frac{c_l}{\eta q_l}$ and $\psi(\theta_h^*) = \frac{c_h - c_l}{\eta(q_h - q_l)}$;

c. when $c_h \geq c_{pe}$, $\pi^*_{\{l\}} > \max\{\pi^*_{\{h,\mathbf{l}\}}, \pi^*_{\{h,l\}}\}$ and hence $S^* = \{l\}$ with $\psi\left(\frac{r_l^*}{q_l}\right) = \frac{c_l}{q_l}$. □

Proof of Lemma 2. The result follows directly from Example 2. □

Proof of Proposition 3. In the base model without the option of probabilistic product, when $c_h < \frac{q_h c_l}{q_l}$, the optimal assortment is $S^* = \{h, \mathbf{l}\}$ where $\psi(\theta_h^*) = \frac{\eta c_h}{q_h}$ and $\theta_l^* > \theta_h^*$. With the probabilistic product, we have $\frac{c_h}{q_h} < \frac{\phi c_h + (1-\phi)c_l}{\phi q_h + (1-\phi)q_l} = \frac{c_p}{q_p}$ because $\frac{c_h}{q_h} < \frac{c_l}{q_l}$. Obviously, any single product assortment cannot do better than the above optimal solution because the low-quality product makes the quality salient and consumers overvalue the quality. According to the proof of Proposition 1, the price discrimination on any two or three products is not implementable because $\frac{c_h}{q_h} < \frac{c_p}{q_p} < \frac{c_l}{q_l}$, and thus offering all the three products even with one product as a decoy cannot improve the profit. □

Proof of Proposition 4. We first characterize the locally optimal probabilistic selling strategy when the low-quality, high-quality, and probabilistic product serve as the decoy, respectively, and then identify the globally optimal strategy. Note that the subsequent analysis focuses on $c_h > \frac{q_h c_l}{q_l}$.

i. The low-quality product serves as a decoy. We have two probabilistic selling strategies: (a) $\{h, p, l\}$, both the probabilistic and high-quality products have positive demands, and (b) $\{p, l\}$, only the probabilistic product has a positive demand.

a. The profit maximization problem with both the probabilistic and high-quality products having positive demands ($\{h, p, l\}$) is stated as follows.

$$\max_{\vec{r},\phi} \quad \pi(r_h, r_p, \phi) = (1 - F(\theta_h))[(\theta_h(q_h - q_p)/\eta - (c_h - c_p)] + (1 - F(\theta_p))(\theta_p q_p/\eta - c_p) \tag{A.6}$$

$$\text{s.t.} \quad s(q_h, \bar{q}) > s(r_h, \bar{r}) \tag{A.7}$$
$$s(q_p, \bar{q}) > s(r_p, \bar{r}) \tag{A.8}$$
$$\max\{u_h^s(\theta), u_p^s(\theta)\} \geq u_l^s(\theta), \forall \theta \in [0,1] \tag{A.9}$$
$$0 < \theta_p < \theta_h < 1 \tag{A.10}$$

Constraints (A.7) and (A.8) ensure that quality is salient for the high-quality and probabilistic product, respectively. Constraint (A.9) ensures that no consumer indeed buys the decoy good (i.e., the low-quality product) when all three products are offered. Constraint (A.10) ensures that both the probabilistic and high-quality products have positive demands.

As suggested in Herweg et al. (2017a), we observe a defining feature in the optimization problem, which helps to solve it without using the traditional KKT conditions: the decision variable (i.e., r_l) associated with the decoy (i.e., product l) only plays a role in the constraints, and thereby we can manipulate it so as to extend the feasible region of other decision variables as large as possible. The salience relationship defined by the price/quality ratio enables us to reformulate constraints (A.7) to (A.10) as follows, respectively.

$$\frac{r_h}{q_h} < \frac{\bar{r}}{\bar{q}},$$
$$\frac{r_p}{q_p} < \frac{\bar{r}}{\bar{q}} \text{ with } \phi > \frac{1}{2} \text{ and } r_p > \frac{r_l + r_h}{2},$$
$$\text{or } \frac{r_p}{q_p} > \frac{\bar{r}}{\bar{q}} \text{ with } \phi < \frac{1}{2} \text{ and } r_p < \frac{r_l + r_h}{2},$$
$$\max\left\{\frac{r_p}{q_p}, \frac{r_h}{q_h}\right\} < \frac{r_l}{q_l},$$
$$\frac{r_p}{q_p} < \frac{r_h}{q_h},$$

which is equivalent to

$$\frac{r_p}{q_p} < \frac{r_h}{q_h} < \frac{\bar{r}}{\bar{q}} < \frac{r_l}{q_l} \text{ and } \phi > \frac{1}{2}. \tag{A.11}$$

As we can see from condition (A.11), quality is also salient for the low-quality products, and we have $r_h > r_p > \bar{r} > r_l$ and $q_h > q_p > \bar{q} > q_l$. For an appropriate decoy product l, we can manipulate r_l, which should be as large as possible (i.e., raising $\frac{\bar{r}}{\bar{q}}$) but still bounded by $r_l \leq 2r_p - r_h$ as $\bar{r} \leq r_p$. Therefore, we can set $r_l = 2r_p - r_h$ and then $\bar{r} = r_p$. Condition (18) reduces to

$$\frac{r_h}{q_h} < \frac{r_p}{\bar{q}} < \frac{2r_p - r_h}{q_l}, \text{ i.e., } \frac{r_p q_l}{2r_p - r_h} < \bar{q} < \frac{r_p q_h}{r_h}.$$

Therefore, we need to have

$$\phi \in \left(\frac{3r_p q_l - (2r_p - r_h)(q_h + 2q_l)}{(2r_p - r_h)(q_h - q_l)}, \frac{3r_p q_h - r_h(q_h + 2q_l)}{r_h(q_h - q_l)}\right) \cap \left(\frac{1}{2}, 1\right). \tag{A.12}$$

The first order conditions (FOCs) are

$$\frac{\partial \pi(\phi, r_p, r_h)}{\partial r_h} = (1 - F(\theta_h))\frac{q_h - q_p}{\eta} - f(\theta_h)\left(\theta_h \frac{q_h - q_p}{\eta} - (c_h - c_p)\right)$$
$$\frac{\partial \pi(\phi, r_p, r_h)}{\partial r_p} = (1 - F(\theta_p))\frac{q_h - q_p}{\eta} - f(\theta_p)\left(\theta_p \frac{q_p}{\eta} - c_p\right)$$
$$\frac{\partial \pi(\phi, r_p, r_h)}{\partial \phi} = (F(\theta_p) - F(\theta_h))\frac{c_h - c_l}{q_h - q_l} + \frac{1}{\eta}(\theta_p(1 - F(\theta_p)) - \theta_h(1 - F(\theta_h))) < 0.$$

The inequality is from $F(\theta_p) < F(\theta_h)$ and $\theta_p(1 - F(\theta_p)) < \theta_p(1 - F(\theta_p))$ as $F(\theta)$ is *regular* (i.e., $\frac{\partial \psi(\theta)}{\partial \theta} > 0$). Therefore, ϕ^* should be the minimum value such that the above interval of ϕ is not empty. For ϕ^*, the corresponding FOCs give the following optimal solutions.

$$\psi(\theta_p^*) = \frac{\eta c_p}{q_p} = \frac{\eta(\phi^* c_h + (1 - \phi^*)c_l)}{\phi^* q_h + (1 - \phi^*)q_l} \text{ and } \psi(\theta_h^*) = \frac{\eta(c_h - c_l)}{q_h - q_l},$$

and hence

$$r_p^*(\phi^*) = \frac{\theta_p^* q_p}{\eta} = \frac{\theta_p^*(\phi^* q_h + (1 - \phi^*)q_l)}{\eta} \tag{A.13}$$
$$r_h^*(\phi^*) = \frac{\theta_p^* q_p}{\eta} + \frac{\theta_h^*(q_h - q_p)}{\eta} = \frac{\theta_p^*(\phi^* q_h + (1 - \phi^*)q_l)}{\eta} + \frac{\theta_h^*(1 - \phi^*)(q_h - q_l)}{\eta} \tag{A.14}$$

Constraint (17) requires $\frac{\eta c_p}{q_p} < \frac{\eta(c_h - c_l)}{q_h - q_l} < 1$ (i.e., $\frac{c_l}{q_l} < \frac{c_h - c_l}{q_h - q_l} < \frac{1}{\eta}$). The first inequality is always satisfied for $c_h > \frac{q_h c_l}{q_l}$. When the second inequality is not satisfied (i.e., $\frac{c_h - c_l}{q_h - q_l} > \frac{1}{\eta}$), this probabilistic selling strategy is not feasible, and it will be degenerated to strategy $\{p, l\}$, namely, the high-quality product should not be offered along with the other two products. Therefore, $\frac{r_p}{q_p} < \frac{r_h}{q_h}$ in (A.11) is automatically satisfied at optimum. Plugging expressions (A.13) and (A.14) into the interval, we obtain

$$\frac{3r_p q_l - (2r_p - r_h)(q_h + 2q_l)}{(2r_p - r_h)(q_h - q_l)} = \frac{3q_l}{q_h - q_l}\frac{r_h - r_p}{2r_p - r_h} - 1$$
$$= \frac{3q_l}{q_h - q_l}\frac{\theta_h^*(1 - \phi^*)(q_h - q_l)}{\theta_p^*(\phi^* q_h + (1 - \phi^*)q_l) - \theta_h^*(1 - \phi^*)(q_h - q_l)} - 1$$
$$= \frac{3\theta_h^*(1 - \phi^*)q_l}{\theta_p^*(\phi^* q_h + (1 - \phi^*)q_l) - \theta_h^*(1 - \phi^*)(q_h - q_l)} - 1,$$

which is less than $\frac{1}{2}$ if $\theta_p^*(\phi^* q_h + (1-\phi^*)q_l) > \theta_h^*(1-\phi^*)(q_h+q_l)$.

$$\begin{aligned}
&\frac{3r_p q_h - r_h(q_h+2q_l)}{r_h(q_h-q_l)}\\
&= \frac{3}{q_h-q_l}\frac{r_p q_h - r_h q_l}{r_h} - 1\\
&= \frac{3}{q_h-q_l}\frac{\theta_p^* q_h q_p - q_l(\theta_p^* q_p - \theta_h^*(1-\phi^*)(q_h-q_l))}{\theta_p^* q_p + \theta_h^*(1-\phi^*)(q_h-q_l)} - 1\\
&= \frac{3(\theta_p^* q_p - \theta_h^*(1-\phi)q_l)}{\theta_p^* q_p + \theta_h^*(1-\phi)(q_h-q_l)} - 1,
\end{aligned}$$

which is greater than $\frac{1}{2}$ if $\theta_p^* q_p > \theta_h^*(1-\phi)(q_h+q_l)$. Note that when $\phi = \frac{1}{2}$, $\theta_p^* q_p < \theta_h^*(1-\phi)(q_h+q_l)$ as $\theta_p^* < \theta_h^*$ at optimum. Therefore, to ensure the nonempty interval of ϕ, we should have $\theta_p^* q_p > \theta_h^*(1-\phi)(q_h+q_l)$. That is, $\phi > \tilde{\phi}$ $(> \frac{1}{2})$, where $\tilde{\phi}$ is the solution to $\theta_p^* q_p = \theta_h^*(1-\phi)(q_h+q_l)$, and further $\tilde{\phi} < \phi < \frac{3r_p q_h - r_h(q_h+2q_l)}{r_h(q_h-q_l)}$. The lowest value of ϕ (i.e., ϕ^*), satisfying the above constraint is the solution to $\tilde{\phi} = \frac{3r_p q_h - r_h(q_h+2q_l)}{r_h(q_h-q_l)}$.

In addition, for $\frac{q_h c_l}{q_l} < c_h < c_{qp}$, the probabilistic selling strategy $\{h, p, \boldsymbol{l}\}$ indeed improves the profit vis-à-vis strategy $\{h, \boldsymbol{l}\}$, because price discrimination is added and the quality remains salient for the high-quality product.

b. The profit maximization problem for strategy $\{p, \boldsymbol{l}\}$ where only the probabilistic product has a positive demand is as follows.

$$\max_{\vec{r},\phi} \quad \pi(r_p, \phi) = (1 - F(\theta_p))(\theta_p q_p/\eta - c_p) \tag{A.15}$$
$$\text{s.t.} \quad s(q_l, \bar{q}) > s(r_l, \bar{r}) \tag{A.16}$$
$$s(q_p, \bar{q}) > s(r_p, \bar{r}) \tag{A.17}$$
$$u_p^s(\theta) \geq u_l^s(\theta), \forall \theta \in [0,1] \tag{A.18}$$
$$0 < \theta_p < 1. \tag{A.19}$$

Constraints (A.16) and (A.17) ensure that quality is salient for the low-quality and probabilistic products, respectively. Constraint (A.18) makes sure that only the probabilistic product has a positive demand when both the low-quality and probabilistic products are offered. Constraint (A.19) ensures that some consumers desire the probabilistic products. From the first-order condition, we obtain the following optimal solution:

$$\psi(\theta_p^*) = \frac{\eta c_p}{q_p} \text{ and } r_p^*(\phi) = \frac{\theta_p^* q_p}{\eta}.$$

Constraint (A.19) requires $\frac{\eta c_p}{q_p} < 1$, that is,

$$\phi \leq \bar{\phi} = \begin{cases} 1 & \text{if } c_h < c_l + \dfrac{q_h - q_l}{\eta}, \\ \dfrac{q_l - \eta c_l}{\eta(c_h - c_l) - (q_h - q_l)} & \text{otherwise.} \end{cases}$$

Taking the derivative with respect to ϕ on both sides of $\psi(\theta_p^*) = \frac{\eta c_p}{q_p}$, we have $\frac{\partial \psi}{\partial \theta_p^*}\frac{\partial \theta_p^*}{\partial \phi} = \frac{\eta(c_h q_l - c_l q_h)}{c_p^2} > 0$ because of $\frac{c_l}{q_l} < \frac{c_h}{q_h}$. Thus, $\frac{\partial \theta_p^*}{\partial \phi} > 0$ as $\frac{\partial \psi}{\partial \theta_p^*} > 0$. The envelope theorem implies that $\frac{\partial \pi^*}{\partial \phi} = (1 - F(\theta_p^*))\left(\frac{\theta_p^*}{\eta}(q_h - q_l) - (c_h - c_l)\right)$, which is always less than 0 when $c_h > c_l + \frac{q_h - q_l}{\eta}$ or increases from a negative value to a positive one as ϕ goes up otherwise. Therefore, π^* reaches the maximum at $\phi \approx 0$ in the former case. When $c_{pe} < c_h < c_l + \frac{q_h - q_l}{\eta}$, π^* reaches the maximum at either $\phi \approx 0$ or $\phi \approx 1$. However, from the benchmark in Section 4.1, we know strategy $\{l\}$ outperforms $\{h, l\}$ when $c_{pe} < c_h < c_l + \frac{q_h - q_l}{\eta}$, and thus π^* reaches the maximum at $\phi \approx 0$. Therefore, the optimal strategy is $\{p, \boldsymbol{l}\}$ with $\phi \approx 0$. Moreover, strategy $\{p, \boldsymbol{l}\}$ outperforms strategy $\{l\}$ as consumers overvalue the product quality now. To ensure quality salience (i.e., $\frac{r_l}{q_l} > \frac{\bar{r}}{\bar{q}}$ and $\frac{r_p}{q_p} < \frac{\bar{r}}{\bar{q}}$ from the salience constraints), we can derive $r_l^* = r_p^*$ as $\phi \approx 0$. Note that r_l here has a unique solution, whereas r_l has multiple solutions in strategy $\{h, \boldsymbol{l}\}$ where $r_l^* = r_h^*$ can be easily implemented.

ii. The high-quality product serves as a decoy. The possible probabilistic selling strategies are $\{\boldsymbol{h}, p, l\}$ and $\{\boldsymbol{h}, p\}$. For strategy $\{\boldsymbol{h}, p\}$, because the high-quality product is a decoy and no consumers purchase it, we have $\frac{r_p}{q_p} < \frac{r_h}{q_h}$, and the probabilistic product is price salient. This strategy is dominated by strategy $\{p, \boldsymbol{l}\}$ because the probabilistic product is quality salient, and thereby it is excluded from further consideration. For strategy $\{\boldsymbol{h}, p, l\}$, we have either $\frac{r_l}{q_l} < \frac{r_p}{q_p} < \frac{\bar{r}}{\bar{q}} < \frac{r_h}{q_h}$ or $\frac{r_l}{q_l} < \frac{\bar{r}}{\bar{q}} < \frac{r_p}{q_p} < \frac{r_h}{q_h}$. Therefore, we know either (i) both the low-quality and probabilistic products are price salient or (ii) the probabilistic product is quality salient and the low-quality product is price salient. According to the proof of Proposition 2, any price discrimination with price salience on both products (including (i)) is dominated by $S = \{h, l\}$. Therefore, we only need to solve the following profit maximization problem when the high-quality product serves as a decoy.

$$\max_{\vec{r},\phi} \quad \pi(r_p, r_l, \phi) = (1 - F(\theta_p))[\theta_p(q_p/\eta - q_l) + \theta_l q_l - c_p] + (F(\theta_p) - F(\theta_l))(\eta\theta_l q_l - c_l) \tag{A.20}$$
$$\text{s.t.} \quad s(q_p, \bar{q}) > s(r_p, \bar{r}) \tag{A.21}$$
$$s(q_l, \bar{q}) < s(r_l, \bar{r}) \tag{A.22}$$
$$\max\{u_p^s(\theta), u_l^s(\theta)\} \geq u_h^s(\theta), \forall \theta \in [0,1] \tag{A.23}$$
$$0 < \theta_l < \theta_p < 1. \tag{A.24}$$

Constraint (A.21) ((A.22)) ensures that quality (price) is salient for the probabilistic (low-quality) product. Constraint (A.23) makes sure that no consumer indeed buys the decoy good (i.e., the high-quality product) when all three products are offered. Constraint (A.24) implies that both the low-quality and probabilistic products have positive demands. Using the first order conditions, we can obtain the optimal solution $r_p^*(\phi)$ and $r_l^*(\phi)$ for any given value of ϕ and then the optimal ϕ^* by examining the change of $\pi^*(\phi)$ with ϕ. In the absence of the probabilistic product, strategy $\{h, l\}$ is optimal for $[c_{qp}, c_{pe}]$, and thus $\pi^*(\phi)$ is increasing in ϕ. Indeed, we can show that the upper bound of feasible ϕ approaches one.

To this end, we reformulate constraints (A.22) and (A.21) as follows.

$$\frac{r_p}{q_p} < \frac{\bar{r}}{\bar{q}} \text{ with } \phi > \frac{1}{2} \text{ and } r_p > \frac{r_l + r_h}{2},$$
$$\text{or } \frac{r_p}{q_p} > \frac{\bar{r}}{\bar{q}} \text{ with } \phi < \frac{1}{2} \text{ and } r_p < \frac{r_l + r_h}{2},$$
$$\frac{r_l}{q_l} < \frac{\bar{r}}{\bar{q}}.$$

We only need to consider the case with $\phi > \frac{1}{2}$ and the above two conditions reduce to

$$\frac{r_l}{q_l} < \frac{r_p}{q_p} < \frac{\bar{r}}{\bar{q}} < \frac{r_h}{q_h}. \tag{A.25}$$

Given any ϕ, $\frac{r_l}{q_l} < \frac{r_p}{q_p}$ always holds for the optimal solution, that is, constraint (29) is redundant. For an appropriate decoy good h, we should raise r_h to increase $\frac{\bar{r}}{\bar{q}}$ but meeting condition $r_h \le 2r_p - r_l$ as $\bar{r} \le r_p$. Therefore, condition (A.25) holds under the strategy $\{\boldsymbol{h}, p, l\}$ with $\phi \approx 1$ and $r_h = 2r_p - r_l$ as $\bar{r} = r_p$. Moreover, the probabilistic product dominates the high-quality product because the probabilistic product is quality salient with $q_p/\eta > q_h$ and $r_p < r_h$, and thus no consumers purchase the high-quality product, satisfying condition (A.23). In summary, we can set $r_h = 2r_p - r_l$ for the decoy good h and $\phi \approx 1$ for the probabilistic product, which can be regarded as an upgraded high-quality product with an approximate quality of q_h/η. To show that this optimal selling strategy improves the profit over $\pi^*_{\{h,l\}}$, we consider the following suboptimal solution.

$$\psi(\theta_l) = \frac{c_l}{\eta q_l} \text{ and } \psi(\theta_p) = \frac{c_h - c_l}{\eta(q_h - q_l)}.$$

Note that this solution automatically makes $\frac{r_l}{q_l} < \frac{r_p}{q_p}$ satisfied in the parameter range $c_{qp} \le c_h < c_{pe}$. Compared with $\pi^*_{\{h,l\}}$ in Lemma A.1, that is,

$$(1 - F(\theta_h))[\eta\theta_h(q_h - q_l) + \eta\theta_{lq_l} - c_h] \\ + (F(\theta_h) - F(\theta_l))(\eta\theta_l q_l - c_l),$$

where $\psi(\theta_l) = \frac{c_l}{\eta q_l}$ and $\psi(\theta_h) = \frac{c_h - c_l}{\eta(q_h - q_l)}$, we see that the demand for both products and the unit profit of the low-quality product are the same, but we have a higher unit profit for the probabilistic product because $\theta_p(q_p/\eta - q_l) + \theta_l q_l - c_p > \eta\theta_h(q_h - q_l) + \eta\theta_{lq_l} - c_h$. Therefore, the optimal probabilistic selling strategy improves the profit when $c_{qp} \le c_h < c_{pe}$.

iii. The probabilistic product serves a decoy. We next show that the strategy is dominated by the above three strategies: $\{h, p, \boldsymbol{l}\}$, $\{\boldsymbol{h}, p, l\}$, and $\{p, \boldsymbol{l}\}$, and will be excluded for further consideration. For the strategies with the probabilistic product as the decoy and without price discrimination, the probabilistic product is offered only to make the high- or low-quality product quality salient. Having the high- or low-quality product as a decoy can achieve the same profit. On the other hand, for the strategies with price discrimination, we must have $\frac{r_l}{q_l} < \frac{r_h}{q_h}$. To have an improved strategy, we need to set r_p and q_p such that $\frac{r_l}{q_l} < \frac{r_h}{q_h} < \frac{\bar{r}}{\bar{q}} < \frac{r_p}{q_p}$. With this appropriate decoy, price is salient for the low-quality product, whereas quality is salient for the high-quality product. However, this strategy does not yield a higher profit than case ii, where the high-quality product is the appropriate decoy.

The above analysis shows that strategies $\{h, p, \boldsymbol{l}\}$, $\{\boldsymbol{h}, p, l\}$, and $\{p, \boldsymbol{l}\}$ outperform $\{h, \boldsymbol{l}\}$, $\{h, l\}$, and $\{l\}$ for $\frac{q_h c_l}{q_l} \le c_h < c_{qp}$, $c_{qp} \le c_h < c_{pe}$, and $c_h \ge c_{pe}$, respectively. Similar to the proof of Proposition 2, we can show that all the three optimal profits are nonincreasing in c_h from the envelope theorem, and the profit achieved by strategy $\{h, p, \boldsymbol{l}\}$ is the highest, whereas the profit under strategy $\{p, \boldsymbol{l}\}$ is the lowest at $c_h = \frac{q_h c_l}{q_l}$. Let c'_{qp} and c'_{pe} denote the new threshold among these three strategies such that $\{h, p, \boldsymbol{l}\}$, $\{\boldsymbol{h}, p, l\}$, and $\{p, \boldsymbol{l}\}$ are the global optimal strategy for $\frac{q_h c_l}{q_l} \le c_h < c'_{qp}$, $c'_{qp} \le c_h < c'_{pe}$, and $c_h \ge c'_{pe}$, respectively. □

Proof of Lemmas 3 and 4. The results follow directly from Proposition 4. □

Endnotes

[1] http://www.thrifty.com/OurCars/WildCard.aspx, accessed on June 15, 2018.

[2] https://www.massdrop.com/search/grab-bag, accessed on June 15, 2018.

[3] https://docs.oracle.com/cd/E14004_01/books/Hospitality/Hospitality_InvenManage48.html, and http://blog.chaukhat.com/2009/07/run-of-house-hotel-rooms.html, accessed on June 15, 2018.

[4] We use terms "product" and "service" interchangeably throughout the paper.

[5] Huang and Yu (2014) assume that the probability of obtaining component products when purchasing a probabilistic product is unknown to consumers who have to utilize anecdotal reasoning to infer these probabilities. Chao et al. (2016) consider a duopoly model in which consumers would experience the potential postpurchase regret because of the possibly assigned inferior products.

[6] https://www.massdrop.com/buy/massdrop-blue-box-final-audio-design-grab-bag/talk, accessed on June 15, 2018.

[7] See Herweg et al. (2017b) for a comprehensive survey on more applications of salience theory in industrial organization.

[8] Note that we put zero weight on the outside option and abstract it away from the market average. This assumption does not change our major results qualitatively (Bordalo et al. 2013, 2016).

[9] As in Bordalo et al. (2016) and Herweg et al. (2017a), in the utility function (1), we omit the normalization factor $2/(1+\eta)$ in Bordalo et al. (2013) for ease of exposition.

[10] This conclusion is robust to a general utility function $u(\theta, q)$ satisfying the single crossing property $\frac{d^2u}{d\theta dq} > 0$. The proof is available upon request.

[11] We appreciate one referee pointing out this paper.

[12] Owing to the salience constraints (i.e., strict nonequality) or practical restrictions (i.e., $0 < \phi < 1$), the optimal ϕ can only be close to a specific value.

[13] This result holds when the sensitivity to losses with respect to the reference point (i.e., (40.33, \$30)) is greater than 0.125 (the sensitivity to gains is normalized to zero) and $\eta < 0.95$.

References

Azar OH (2014) Optimal strategy of multi-product retailers with relative thinking and reference prices. *Internat. J. Indust. Organ.* 37(November):130–140.

Bodner R, Prelec D (1994) The centroid model of context-dependent choice. Working paper, Massachusetts Institute of Technology, Cambridge, MA.

Bordalo P, Gennaioli N, Shleifer A (2012) Salience theory of choice under risk. *Quart. J. Econom.* 127(3):1243–1285.

Bordalo P, Gennaioli N, Shleifer A (2013) Sailence and customer choice. *J. Political Econom.* 121(5):803–843.

Bordalo P, Gennaioli N, Shleifer A (2016) Competition for attention. *Rev. Econom. Stud.* 83(2):481–513.

Chao Y, Liu L, Zhan D (2016) Vertical probabilistic selling under competition: The role of consumer anticipated regret. Working paper, University of Louisville, Louisville, KY.

Chen Y, Turut Ö (2013) Context-dependent preferences and innovation strategy. *Management Sci.* 59(12):2747–2765.

Chen Y, Tomlin B, Wang Y (2013) Coproduct technologies: Product line design and process innovation. *Management Sci.* 59(12): 2772–2789.

Dahremöller C, Fels M (2015) Product lines, product design, and limited attention. *J. Econom. Behav. Organ.* 119(November):437–456.

DellaVigna S (2009) Psychology and economics: Evidence from the field. *J. Econom. Literature* 47(2):315–372.

Deneckere RJ, McAfee RP (1996) Damaged goods. *J. Econom. Management Strategy* 5(2):149–174.

Drolet A, Simonson I, Tversky A (2000) Indifference curves that travel with the choice set. *Marketing Lett.* 11(3):199–209.

Fay S, Xie JH (2008) Probabilistic goods: A creative way of selling products and services. *Marketing Sci.* 27(4):674–690.

Fay S, Xie JH (2010) The economics of buyer uncertainty: Advance selling vs. probabilistic selling. *Marketing Sci.* 29(6):1040–1057.

Fay S, Xie JH (2015) Timing of product allocation: Using probabilistic selling to enhance inventory management. *Management Sci.* 61(2):474–484.

Fay S, Xie J, Feng C (2015) The effect of probabilistic selling on the optimal product mix. *J. Retailing* 91(3):451–467.

Fehr E, Rangel A (2011) Neuroeconomic foundations of economic choicerecent advances. *J. Econom. Perspect.* 25(4):3–30.

Heath TB, Chatterjee S (1995) Asymmetric decoy effects on lower-quality versus higher-quality brands: Meta-analytic and experimental evidence. *J. Consumer Res.* 22(3):268–284.

Hedgcock WM, Rao RS, Chen HA (2016) Choosing to choose: The effects of decoys and prior choice on deferral. *Management Sci.* 62(10):2952–2976.

Herweg F, Müller D, Weinschenk P (2017a) Salience, competition, and decoy goods. *Econom. Lett.* 153(April):28–31.

Herweg F, Müller D, Weinschenk P (2017b) Salience in markets. Technical report, Mimeo, University of Bayreuth, Bayreuth, Germany.

Huang TL, Yu YM (2014) Sell probabilistic goods? A behavioral explanation for opaque selling. *Marketing Sci.* 33(5):743–759.

Huber J, Payne JW, Puto C (1982) Adding asymmetrically dominated alternatives: Violations of regularity and the similarity hypothesis. *J. Consumer Res.* 9(1):90–98.

Inderst R, Obradovits M (2016) Competitive strategies when consumers are relative thinkers: Implications for pricing, promotions, and product choice. Working paper, Goethe University, Frankfurt, Germany.

Jerath K, Netessine S, Veeraraghavan SK (2010) Revenue management with strategic customers: Last-minute selling and opaque selling. *Management Sci.* 56(3):430–448.

Jiang Y (2007) Price discrimination with opaque products. *J. Revenue Pricing Management* 6(2):118–134.

Kahneman D (2011) *Thinking, Fast and Slow* (Farrar, Straus and Giroux, New York).

Kivetz R, Netzer O, Srinivasan V (2004) Extending compromise effect models to complex buying situations and other context effects. *J. Marketing Res.* 41(3):262–268.

Maskin E, Riley J (1984) Monopoly with incomplete information. *RAND J. Econom.* 15(2):171–196.

Mussa M, Rosen S (1978) Monopoly and product quality. *J. Econom. Theory* 18(2):301–317.

Narasimhan C, Turut Ö (2013) Differentiate or imitate? The role of context-dependent preferences. *Marketing Sci.* 32(3):393–410.

Orhun AY (2009) Optimal product line design when consumers exhibit choice set-dependent preferences. *Marketing Sci.* 28(5): 868–886.

Özer Ö, Zheng Y (2012) Behavioral issues in pricing management. Özer Ö, Phillips R, eds. *The Oxford Handbook of Pricing Management* (Oxford University Press, New York), 415–461.

Pan XA, Honhon D (2012) Assortment planning for vertically differentiated products. *Production Oper. Management* 21(2):253–275.

Simonson I (1989) Choice based on reasons: The case of attraction and compromise effects. *J. Consumer Res.* 16(2):158–174.

Tversky A, Kahneman D (1991) Loss aversion in riskless choice: A reference-dependent model. *Quart. J. Econom.* 106(4):1039–1061.

Tversky A, Simonson I (1993) Context-dependent preferences. *Management Sci.* 39(10):1179–1189.

Zhang Z, Joseph K, Subramaniam R (2015) Probabilistic selling in quality-differentiated markets. *Management Sci.* 61(8):1959–1977.