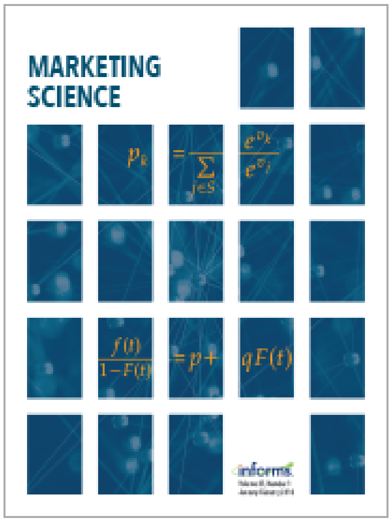

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Research Note

A Theoretical Investigation of the Effects of Similarity on Brand Choice Using the Elimination-by-Tree Model

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IIA (independence of irrelevant alternatives) axiom states that the ratio of choice probabilities of any two brands will depend only on the utilities of the brands. However, even if the utilities of brands are assumed to be fixed, their choice probabilities will be affected by the similarity between them. This study and several other previous studies show that a more preferred (higher utility) brand benefits more in a high similarity situation than a less preferred (lower utility) brand, which is called the asymmetric similarity effect or simply *the asymmetric effect* in this study. This study expands on the asymmetric effect that has been reported by many previous empirical studies and implied in choice modeling literature, by giving it an explicit mathematical formulation based on the analysis of the elimination-by-tree (EBT) model (Tversky and Sattath 1979). This study also provides an integrative theoretical summary showing how the asymmetric effect is related to the similarity effect, dominance effect, and IIA condition.

Key words: brand choice model; asymmetric effect; elimination-by-tree model; similarity effect; IIA
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Introduction

The theory of rational choice assumes that the ratio of market shares between options does not depend on the presence or absence of other options. This principle, called *independence of irrelevant alternatives* (IIA) (Luce 1959), is essentially equivalent to the value maximization (VM) principle which is based on single scale (utility) value assumption (cf. Tversky and Simonson 1993). According to the single scalability assumption, the ratio of the choice probabilities of any two brands has a constant value as long as the utilities of the brands do not change. Let $P_X(A)$ denote the probability that alternative A will be chosen when set X is the set of available alternatives and $u(A)$ the utility of alternative A. If we assume that there is another alternative A', with the same utility as A ($u(A) = u(A')$), Luce's IIA axiom predicts that

$$\frac{P_X(A)}{P_X(B)} = \frac{P_Y(A')}{P_Y(B)}, \tag{1}$$

for any choice set $A, B \in X$, and $A', B \in Y$.

Does Equation (1) hold true even when the similarity between A and B is different from the similarity between A' and B? Past studies, especially those based on parametric choice models, revealed that brand-specific utilities and interbrand similarities are the two most important dimensions in analyzing brand choice probabilities (Luce 1959; McFadden 1973, 1978; Tversky 1972; Tversky and Sattath 1979; Batsell and Polking 1985). This paper harmonizes previous findings on similarity, asymmetry, and dominance effects in consumer choice in the underlying elimination-by-tree (EBT) model allowing for similarity between alternatives. The results of this study show that if the similarity between A' and B is greater than that between A and B (i.e., $s(A', B) > s(A, B)$),

$$\frac{P_X(A)}{P_X(B)} < \frac{P_Y(A')}{P_Y(B)}, \quad \text{if } u(A) = u(A') > u(B) \tag{2}$$

and

$$\frac{P_X(A)}{P_X(B)} > \frac{P_Y(A')}{P_Y(B)}, \quad \text{if } u(A) = u(A') < u(B). \tag{3}$$

The direction and the magnitude of the effect of differential similarities on choice probabilities differ depending on the relative preferences of the compared brands. This study refers to such an interaction effect of the relative preference and similarity on choice probability as the asymmetric similarity effect (or simply the asymmetric effect), because the more preferred brand will benefit when it is perceived as being similar to the less preferred brand. This study provides a clear mathematical formulation of the asymmetric effect which has been shown to exist empirically by previous studies, thus makes it possible to analyze the relationships among the asymmetric effect, similarity effect (Debreu 1960, Tversky 1972), dominance effects (Huber and Sewall 1978) and IIA property (Luce 1959). This investigation will be based on Tversky and Sattath's (1979) elimination-by-tree model and Tversky's (1977) contrast model of similarity.

Asymmetric Effect in Binary Choice Cases

I investigate how different levels of similarities between A and B affect $P(A)/P(B)$, or $P(A) - P(B)$, while the utilities of A and B remain constant. Tversky (1977) expresses the similarity between alternatives as a linear combination of the measures of their common and distinctive features. Let's assume that there are only two alternatives A and B. Let α be the total set of features (attributes) of alternative A, β be that of B, and θ be the set of features both A and B have. According to the feature matching model (contrast model) of similarity (Tversky 1977), the similarity between A and B, $s(A, B)$, is expressed as a function F of three arguments: θ (common features of A and B), $\alpha - \theta$ (the distinctive features of A) and $\beta - \theta$ (the distinctive features of B).

$$s(A, B) = F(\theta, \alpha - \theta, \beta - \theta). \tag{4}$$

If it is assumed that the values of α and β are fixed, both $\alpha - \theta$ and $\beta - \theta$ become functions of θ. Thus, the similarity between A and B can be represented simply as a monotonic increasing function of θ.

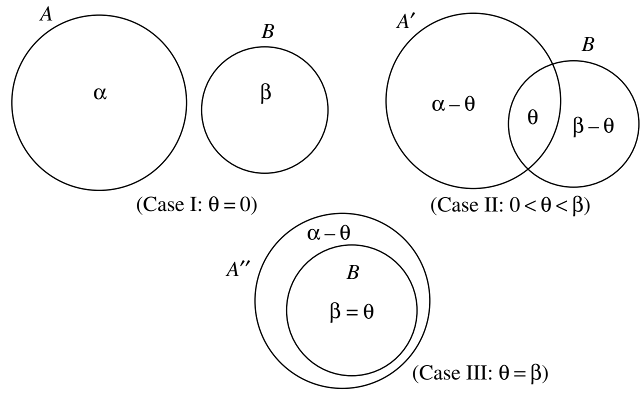
Let $u(A)$ and $u(B)$ denote the overall utilities of A and B, respectively. I use α, β, and θ as representing the features as well as the utilities of the features as in Tversky and Sattath's (1979) work. Thus, $u(A) = \alpha$, $u(B) = \beta$. It is assumed that A is preferred to B ($\alpha > \beta$), and the range of θ is between 0 and β ($0 \le \theta \le \beta < \alpha$). The greater the value of θ, the more similar A and B are to each other (Tversky 1977). When $\theta = 0$, A and B are completely dissimilar and share no common features. When A and B become as similar as possible ($\theta = \beta$), α encloses β. In this case, the relative preference relationship becomes the dominance relationship as shown in Figure 1 (Case III).

According to the EBT model (Tversky and Sattath 1979), the choice probabilities of A and B are

$$P(A) = \frac{\alpha - \theta}{\alpha + \beta - 2\theta} \quad \text{and} \quad P(B) = \frac{\beta - \theta}{\alpha + \beta - 2\theta}. \tag{5}$$

Note that the notations used in this study are slightly different from those in Tverksy and Sattath (1979), or Tversky (1972). $\alpha - \theta$ and $\beta - \theta$ (rather than single parameters) are used to represent distinctive features of A and B in order to incorporate the situation where the similarity values θ are different

Figure 1 **Similarity Relationships in a Binary Choice Case**

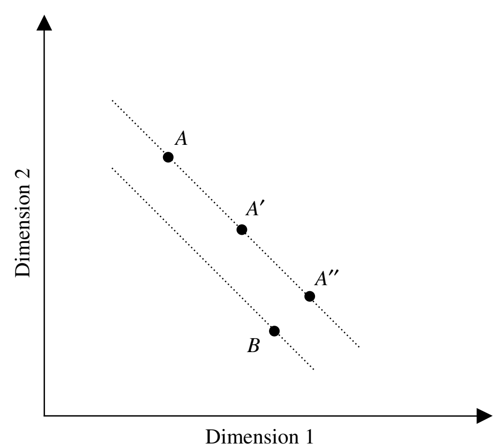


while the overall utilities of A and B, i.e., α and β are fixed. We assume that there are many different brands which have the same level of utilities but have differential similarities with a certain other brand as represented in Figure 1. The inter brands relationships in Figure 1 can be represented equivalently in two-dimensional (attribute) space as in Figure 2 (cf. Huber et al. 1982).

As represented by the iso-utility lines in Figure 2, it is assumed that $u(A) = u(A') = u(A'') = \alpha$ and $u(B) = \beta$. Brand B is perceived to be more similar to Brand A″ than to Brand A. B is even dominated by A''. One can intuitively infer that $P(A'')/P(B)$ would be greater than $P(A)/P(B)$ (cf. Huber et al. 1982). In order to investigate the effect of the differential degree of similarity (θ) on $P(A)/P(B)$, I calculate the first and second derivatives of the choice ratio derived from the EBT model with respect to θ. Let's denote $P(A)/P(B)$ as a function of θ, $f(\theta)$.

$$f(\theta) = \frac{P(A)}{P(B)} = \frac{\alpha - \theta}{\beta - \theta}. \tag{6}$$

Figure 2 **Similarity Relationships in a Binary Choice Case Represented in Two-Dimensional Space**



If one calculates the first and second derivative of $f(\theta)$ for the range $0 < \theta < \beta$, then

$$\frac{df(\theta)}{d\theta} = \frac{\alpha-\beta}{(\beta-\theta)^2} > 0, \quad \text{(asymmetric effect)} \tag{7}$$

$$\frac{d^2 f(\theta)}{d\theta^2} = \frac{2(\alpha-\beta)}{(\beta-\theta)^3} > 0. \tag{8}$$

The positive first and second derivative values imply that the share ratio $P(A)/P(B)$ is a monotonic increasing, convex function of θ. The first derivative being positive (Equation 7) represents the asymmetric effect (type 1). The greater the similarity between A and B, the greater the advantage that the more preferred brand (Brand A) has over the less preferred brand (Brand B). It can be derived that

$$\lim_{\theta\to 0} \frac{P(A)}{P(B)} = \frac{\alpha}{\beta}$$

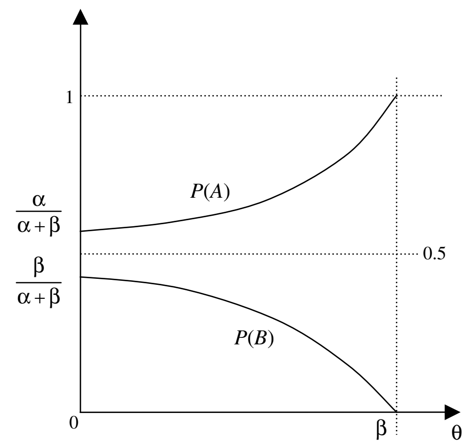
(lowest similarity case; IIA condition), while

$$\lim_{\theta\to \beta} \frac{P(A)}{P(B)} = \infty$$

(highest similarity case; dominance case).

There are many empirical studies manifesting the similar implications. Carpenter and Nakamoto (1989) show that similarities between two brands ("me-too" and "pioneer") are likely to emphasize the advantages of the dominant brand (the pioneer). It is also shown that similarity along one attribute tends to enhance the difference on other attributes (Tversky and Russo 1969). Chernev (1997) shows that the addition of common features (thus increased similarity) can (*but not always*) increase the likelihood of choosing the brand with the best value on the primary attribute, thus leading to a divergence of the choice shares of the alternatives. Kahn et al. (1987) have shown that it is advantageous for the more preferred brand to be paired with the less preferred similar brand, whereas the less preferred similar brand gets more damage from being paired with the preferred brand. This phenomenon has been called the asymmetric effect by Glazer et al. (1991) and I adopt their terminology to refer to the similar phenomenon discussed in this study. MacCrimmon (1973) points out that if a decision maker neglects to include in his choice set an option that would "dominate" a second option, the probability that the second option will be chosen is substantially increased. In summary, an external constraint that positions a strong brand (Brand A) close to a weaker, similar brand (Brand B) helps the strong brand and hurts the weak brand, making the value of $P(A)/P(B)$ (or $P(A) - P(B)$) greater.

Figure 3 **Changes of $P(A)$ and $P(B)$ with Respect to θ**



One can investigate the changes of $P(A)$ and $P(B)$ independently with a varying degree of θ.

$$\frac{dP(A)}{d\theta} = \frac{\alpha-\beta}{(\alpha+\beta-2\theta)^2} > 0, \qquad \frac{d^2P(A)}{d\theta^2} = \frac{4(\alpha-\beta)}{(\alpha+\beta-2\theta)^3} > 0. \tag{9}$$

$$\frac{dP(B)}{d\theta} = \frac{\beta-\alpha}{(\alpha+\beta-2\theta)^2} < 0, \qquad \frac{d^2P(B)}{d\theta^2} = \frac{4(\beta-\alpha)}{(\alpha+\beta-2\theta)^3} < 0. \tag{10}$$

The graphs of $P(A)$ and $P(B)$ in Figure 3 show that the increased similarity enhances the polarization of consumer preferences, leading to a divergence of relative shares.

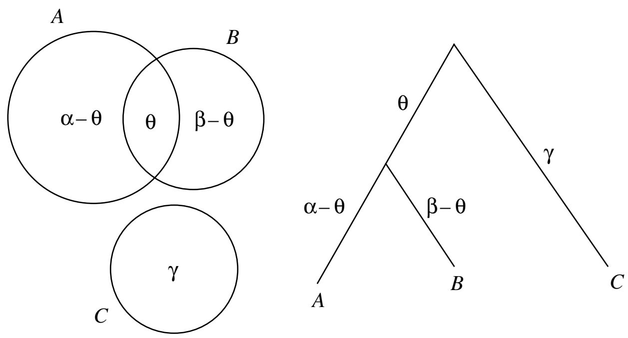
If θ increases to β, the maximum possible value of θ, Brand A *dominates* Brand B and takes all the share from B. The dominance effect (Huber and Sewall 1978) can be considered as an extreme case of the asymmetric effect.

$$\lim_{\theta\to\beta} P(A) = 1, \qquad \lim_{\theta\to\beta} P(B) = 0 \quad \text{(dominance effect)}. \tag{11}$$

One of the most well known examples of complete dominance would be Debreu's (1960) example of choice between a trip to Paris and a trip to Paris plus a 1 dollar bonus. Complete overlapping of the features between options makes the option with even the slightest added feature dominate the other. On the other hand, if the similarity value θ becomes 0, the choice probabilities are just as the Luce model (Luce 1959) predicts, thus

$$P(A) = \frac{\alpha}{\alpha+\beta} \quad \text{and} \quad P(B) = \frac{\beta}{\alpha+\beta}.$$

Figure 4 Similarity Relationships among Brands in a Venn Diagram and a Grouped Tree



Thus, the IIA condition can be considered as the other extreme case of the asymmetric effect.

The relative advantage can be represented not only as a ratio but also as the difference between $P(A)$ and $P(B)$. Let's denote $P(A)-P(B)$ as $f_\Delta(\theta)$. Then

$$\frac{df_\Delta(\theta)}{d\theta}=\frac{2(\alpha-\beta)}{(\alpha+\beta-2\theta)^2}>0 \quad \text{(asymmetric effect: type 2)} \tag{12}$$

$$\frac{d^2f_\Delta(\theta)}{d\theta^2}=\frac{8(\alpha-\beta)}{(\alpha+\beta-2\theta)^3}>0 \tag{13}$$

$f_\Delta(\theta)$ is a convex, increasing function of θ. Also note that

$$\lim_{\theta\to 0}\{P(A)-P(B)\}=\frac{\alpha-\beta}{\alpha+\beta} \quad \text{and} \quad \lim_{\theta\to\beta}\{P(A)-P(B)\}=1.$$

Asymmetric Effect and Other Non-IIA Choice Behaviors in a Three-Alternative Choice Set Case

Let us assume a case where a third alternative, Brand C, is added to the two-alternative choice set $\{A, B\}$. It will be shown that the asymmetric effect is a general phenomenon that occurs independent to irrelevant third options. I also show how the asymmetric effect is related to the similarity effect. Brand C is assumed to be dissimilar to both A and B (Figure 4). This case is equivalent to other cases where many irrelevant alternatives are added to the focal pair of brands. Let the utilities of the alternatives be $u(A)=\alpha$, $u(B)=\beta$, and $u(C)=\gamma$ ($\alpha>\beta$ and $\alpha,\beta,\gamma>0$). The similarity structure of the three alternatives can be represented by either a Venn diagram or a tree equivalently.

Based on the EBT model, the probabilities of choosing A, B, and C from the choice set $\{A, B, C\}$ can be calculated as follows.

$$P(A)=\frac{(\alpha-\theta)+\theta\cdot\frac{\alpha-\theta}{\alpha+\beta-2\theta}}{\alpha+\beta-\theta+\gamma}=\frac{(\alpha+\beta-\theta)(\alpha-\theta)}{(\alpha+\beta-\theta+\gamma)(\alpha+\beta-2\theta)}$$

$$P(B)=\frac{(\beta-\theta)+\theta\cdot\frac{\beta-\theta}{\alpha+\beta-2\theta}}{\alpha+\beta-\theta+\gamma}=\frac{(\alpha+\beta-\theta)(\beta-\theta)}{(\alpha+\beta-\theta+\gamma)(\alpha+\beta-2\theta)}$$

$$P(C)=\frac{\gamma}{\alpha+\beta-\theta+\gamma}. \tag{14}$$

Calculations of the first and second derivatives of $f(\theta)$ $(=P(A)/P(B))$ show the same results as the binary choice case (Mathematica 4.1 Software is used for the derivatives calculations throughout the article).

$$\frac{df(\theta)}{d\theta}=\frac{\alpha-\beta}{(\beta-\theta)^2}>0 \tag{15}$$

$$\frac{d^2f(\theta)}{d\theta^2}=\frac{2(\alpha-\beta)}{(\beta-\theta)^3}>0. \tag{16}$$

It can also be derived that $df_\Delta(\theta)/d\theta>0$, where $f_\Delta(\theta)=P(A)-P(B)$ (proof omitted). When θ becomes zero, $P(A)=\alpha/(\alpha+\beta+\gamma)$, $P(B)=\beta/(\alpha+\beta+\gamma)$, and $P(C)=\gamma/(\alpha+\beta+\gamma)$, just as the Luce model predicts. When the features of Brand A and Brand B completely overlap ($\beta=\theta$), the dominating Brand A prohibits Brand B from having any of the total share at all. Equations (17) and (18) represent the dominance effect in a three-alternative case.

$$\lim_{\theta\to\beta}P(A)=\frac{\alpha}{\alpha+\gamma} \tag{17}$$

$$\lim_{\theta\to\beta}P(B)=0 \quad \text{(dominance effect)} \tag{18}$$

$$\lim_{\theta\to\beta}P(C)=\frac{\gamma}{\alpha+\gamma}. \tag{19}$$

As θ, the similarity between A and B, increases, the share of the third alternative (C) increases as well. This is due to the similarity effect, which states that similar brands take a proportionally greater share from each other than from a dissimilar brand (Debreu 1960, Tversky 1972). Calculations show that

$$\frac{dP(C)}{d\theta}=-\frac{d}{d\theta}\{P(A)+P(B)\}=\frac{\gamma}{(\alpha+\beta-\theta+\gamma)^2}>0 \quad \text{(similarity effect).} \tag{20}$$

The combination of the asymmetric effect and similarity effect can lead to another important rule in choice behavior. Since $(d/d\theta)\{P(A)-P(B)\}>0$ and $(d/d\theta)\{P(A)+P(B)\}<0$,

$$\frac{dP(B)}{d\theta}<0. \tag{21}$$

Equation (21) shows that the share of the less preferred similar brand, $P(B)$, always decreases with increasing θ. Kahn et al. (1987) have shown that the probability of choosing a less preferred similar alternative always increases as the similarity with the more preferred alternative decreases (Hypothesis 2a). Equation (21) is a derivation from both the similarity and asymmetric effects, but this study gives it a separate name, the *inferior option effect*. The inferior option effect refers to the fact that increasing similarity between two brands *always decreases* the share of the less preferred (inferior) option, *regardless* of the existence of third brands or the preference levels of the brands. It is quite a robust principle strongly supported by past empirical studies.

The sign of $dP(A)/d\theta$ is indeterminate. Thus $P(A)$ can be either increased by or decreased by the increasing level of θ (proof based on derivative values is omitted). It can be understood in the following way.

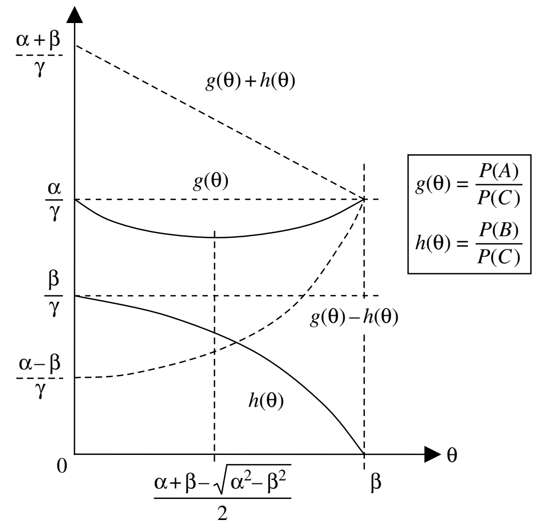
$$P(A) = \{P(A)+P(B)\} \cdot \frac{P(A)}{\{P(A)+P(B)\}}. \qquad (22)$$

As the similarity between A and B, θ, increases, $P(A)+P(B)$ decreases due to the similarity effect and $P(A)/\{P(A)+P(B)\}$ increases due to the asymmetric effect. Thus, depending on which of the two factors becomes more prominent, $P(A)$ can either increase or decrease. As to B, since both $\{P(A)+P(B)\}$ and $P(B)/\{P(A)+P(B)\}$ decrease, $P(B)$ always decreases with an increasing θ. However, $P(A)-P(B)$ always increases with θ and it is both theoretically derived and empirically shown by Kahn et al. (1987, H4). Their notation $\Delta(x \mid E_x) - \Delta(y \mid E_y) > 0$ is a different version of my expression of the asymmetric effect (type 2), $(d/d\theta)\{P(A)-P(B)\} > 0$. The derivative values of $P(A)$, $P(B)$, and $P(C)$ are not presented here because they are too complicated.

In order to analyze the asymmetric effect and similarity effect from a different perspective, I will investigate how $P(A)/P(C)$ and $P(B)/P(C)$ change as θ changes. Analysis of $P(A)/P(C)$ $(=g(\theta))$ and $P(B)/P(C)$ $(=h(\theta))$, instead of $P(A)$ and $P(B)$, simplifies the calculation results and thus makes it easier to represent the discussed effects in a three alternative case. The calculations are provided in Appendix I and the results are shown in Figure 5. The graph of $g(\theta)+h(\theta)$ represents the similarity effect, $g(\theta)-h(\theta)$ the asymmetric effect, and $h(\theta)$ the inferior option effect (a type of asymmetric effect).

The similarity effect, asymmetric effect, and dominance effect in addition to the IIA principle comprise all phenomena which can be theoretically derived from the EBT model. Table 1 summarizes the three non-IIA effects and the IIA property with some examples of the related empirical studies. The similarity effect and asymmetric effect are the phenomena which arise within the zone between the two extreme points where the IIA principle applies ($\theta = 0$) at one

Figure 5 **Graphs of $g(\theta)$, $h(\theta)$, $g(\theta)+h(\theta)$, and $g(\theta)-h(\theta)$**



end and where the dominance effect ($\theta = \beta$) applies at the other end. Regularity violation cannot be represented by the choice models which assume fixed utilities at the alternative level (parametric models), such as the EBA (elimination-by-aspect) model (Tversky 1972), the EBT model, or the Luce model. Note that all the formulations developed from the EBT model in this study are identical to those of the EBA model.

Analyses of consumer choice behaviors based on the multinomial logit model (McFadden 1973) or any other related choice models, even those incorporating

Table 1 **IIA and Non-IIA Choice Behaviors Implied by the EBT Model (Three-Alternative Case)**

Choice behaviors	Mathematical representation	Relevant empirical studies
Similarity effect ($0<\theta<\beta$)	$\frac{d}{d\theta}\{P(A)+P(B)\} = -\frac{dP(C)}{d\theta} < 0$ $\frac{d}{d\theta}\left\{\frac{P(A)}{P(C)}+\frac{P(B)}{P(C)}\right\} < 0$	Debreu (1960), Tversky (1972), Tversky and Sattath (1979)
Asymmetric effect ($0<\theta<\beta$)	$\frac{d}{d\theta}\{P(A)-P(B)\} > 0$ $\frac{d}{d\theta}\left\{\frac{P(A)}{P(B)}\right\} > 0$ $\frac{d}{d\theta}\left\{\frac{P(A)}{P(C)}-\frac{P(B)}{P(C)}\right\} > 0$ $\frac{dP(B)}{d\theta} < 0,\ \frac{d}{d\theta}\left\{\frac{P(B)}{P(C)}\right\} < 0$	Carpenter and Nakamoto (1989), Kahn et al. (1987), Glazer et al. (1991), Chernev (1997)
Dominance effect ($\theta=\beta$)	$\lim_{\theta\to\beta} P(A) = \frac{\alpha}{\alpha+\gamma}$ $\lim_{\theta\to\beta} P(B) = 0$ $\lim_{\theta\to\beta} P(C) = \frac{\gamma}{\alpha+\gamma}$	Huber et al. (1982), Huber and Sewall (1978)
Luce's IIA axiom ($\theta=0$)	$\lim_{\theta\to 0} P(A) = \frac{\alpha}{\alpha+\beta+\gamma}$ $\lim_{\theta\to 0} P(B) = \frac{\beta}{\alpha+\beta+\gamma}$ $\lim_{\theta\to 0} P(C) = \frac{\gamma}{\alpha+\beta+\gamma}$	Luce (1959), cf. McFadden (1973)

consumer heterogeneity (cf. Zhang 2006), with the assumption of uncorrelated utilities among alternatives, can be considered as belonging to the category of IIA axiom, in the proposed classification (Table 1).

Implications and Conclusion

The asymmetric effect is a very important market phenomenon which has been empirically validated. The asymmetric effect can explain many marketing phenomena but has not been given enough explicit attention. The asymmetric effect can directly lead to the attraction effect (Huber et al. 1982) if the lone alternative effect (Kahn et al. 1987) is assumed to exist. However, the present study is based on the theory of rational choice. Thus, choice behaviors considered to belong to the realm of the regularity violation (Huber et al. 1982, Simonson 1989, Gourville and Soman 2005), or to the no choice option effect (Dhar 1997, Dhar and Simonson 2003) are beyond the scope of this study.

The asymmetric effect is in fact a very general phenomenon that can be derived from many different perspectives of modeling consumers' choice behaviors, such as random utility maximization models (Appendix II). The asymmetric effect can be directly derived from many choice models based on random utility theory (RUT) incorporating the similarity effect (cf., McFadden 1978, Currim 1982). However, in either the random or non-random utility approaches of choice modeling, the asymmetric effect has not been explicitly mentioned nor has it received separate attention. This study provides an explicit formulation of the asymmetric effect and an integrative summary of the asymmetric effect, similarity effect, dominance effect, and IIA condition. The mathematical formulation presented in this study does not give completely new practical implications but does make it possible to develop an integrative and exhaustive mathematical summary of all possible implications of parametric choice models. It is noteworthy that the economic-theory-based summary of choice behaviors presented in this study may not have significant meaning in the context of reason-based choice (Simonson 1989) or semiparametric choice modeling approaches (cf. Cui and Curry 2005) where there is no assumption of a particular model structure.

The asymmetric effect can be applied in firms' marketing practices effectively and trigger further academic research as well. Interbrand similarity often determines which brand is considered as a reference brand in the focal brand's evaluation. The focal brand's market share is affected by which competing brand is compared to it (Hsee and Leclerc 1998, Dholakia and Simonson 2005). Due to the loss aversion effect (Kahneman and Tversky 1979), direct comparison makes the comparative disadvantage receive greater weight than the comparative advantages. If we assume that the greater similarity between two alternatives (ex, alternatives being in the same product category) implies the greater likelihood of their direct comparison (Leclerc et al. 2005), previous studies on comparison effect lead to the same conclusion as the asymmetric effect; increased similarity causing more damage to the less preferred alternative whereas favoring the preferred alternative. Thus, it would be interesting to explore the conceptual and mathematical relationship between the EBT/EBA models and the prospect theory (Kahneman and Tversky 1979) from the perspective that the level of similarity can play a critical role in determination of a referent.

Appendix I

From Equation (14),

$$\frac{P(A)}{P(C)} = \frac{\alpha - \theta + \theta\left(\frac{\alpha-\theta}{\alpha+\beta-2\theta}\right)}{\gamma} \tag{A1}$$

$$\lim_{\theta\to 0}\frac{P(A)}{P(C)} = \lim_{\theta\to\beta}\frac{P(A)}{P(C)} = \frac{\alpha}{\gamma}. \tag{A2}$$

Let's denote $P(A)/P(C)$ as a function of θ, $g(\theta)$. If we calculate the first and the second derivatives of $g(\theta)$ for $0 < \theta < \beta$,

$$\frac{dg(\theta)}{d\theta} = \frac{-\beta^2 - 2\theta^2 - \alpha\beta + 2\alpha\theta + 2\beta\theta}{\gamma(\alpha+\beta-2\theta)^2} \quad \text{and} \tag{A3}$$

$$\frac{d^2g(\theta)}{d\theta^2} = \frac{2(\alpha^2-\beta^2)}{\gamma(\alpha+\beta-2\theta)^3} > 0. \tag{A4}$$

The sign of $dg(\theta)/d\theta$ is indeterminate and that of $d^2g(\theta)/d\theta^2$ is positive. The general shape of the function $g(\theta)$ is shown in Figure 5. $g(\theta)$ has the lowest value when $dg(\theta)/d\theta = 0$. Thus, $g(\theta)$ has the lowest value when $\theta = (\alpha+\beta-\sqrt{\alpha^2-\beta^2})/2$. Likewise, from Equation (14),

$$\frac{P(B)}{P(C)} = \frac{\beta - \theta + \theta\cdot\frac{\beta-\theta}{\alpha+\beta-2\theta}}{\gamma}. \tag{A5}$$

Let's denote $P(B)/P(C)$ as $h(\theta)$, then the first and second derivatives of $h(\theta)$, for $0 < \theta < \beta$ are

$$\frac{dh(\theta)}{d\theta} = \frac{-\alpha^2 - 2\theta^2 - \alpha\beta + 2\alpha\theta + 2\beta\theta}{\gamma(\alpha+\beta-2\theta)^2} < 0 \quad \text{and} \tag{A6}$$

$$\frac{d^2h(\theta)}{d\theta^2} = -\frac{2(\alpha^2-\beta^2)}{\gamma(\alpha+\beta-2\theta)^3} < 0. \tag{A7}$$

$h(\theta)$ is a concave, monotonic decreasing function of θ. Equation (A6) shows that a strong brand (A) draws a proportionally greater share from the closer competitor (B) than from the distant competitor (C) as θ increases, thus it manifests the similarity effect as well as the asymmetric effect.

$$\lim_{\theta\to\beta}\frac{P(B)}{P(C)} = 0, \qquad \lim_{\theta\to 0}\frac{P(B)}{P(C)} = \frac{\beta}{\gamma}. \tag{A8}$$

The graph of $h(\theta)$ is shown in Figure 5. The difference in shapes between the graphs of $g(\theta)$ and $h(\theta)$ also reflects the asymmetric effect. In Figure 5, β/γ can be either greater than, equal to, or less than $(\alpha-\beta)/\gamma$ depending on the relative values of α, β, but the general shapes of the graphs do not change. The similarity effect can be represented in a different way.

$$g(\theta)+h(\theta)=\frac{\alpha+\beta-\theta}{\gamma} \tag{A9}$$

$$\frac{d}{d\theta}\{g(\theta)+h(\theta)\}=\frac{-1}{\gamma}<0 \quad \text{(similarity effect: type 2)} \tag{A10}$$

$$g(\theta)-h(\theta)=\frac{\alpha-\beta+\theta\cdot\frac{\alpha-\beta}{\alpha+\beta-2\theta}}{\gamma} \tag{A11}$$

$$\frac{d}{d\theta}\{g(\theta)-h(\theta)\}=\frac{\alpha^2-\beta^2}{\gamma(\alpha+\beta-2\theta)^2}>0 \quad \text{(asymmetric effect: type 3)} \tag{A12}$$

$$\frac{d^2}{d\theta^2}\{g(\theta)-h(\theta)\}=\frac{4(\alpha^2-\beta^2)}{\gamma(\alpha+\beta-2\theta)^3}>0. \tag{A13}$$

Appendix II

Let's assume that utilities of alternatives are not fixed values but random variables with stochastic distribution. Let's assume a binary choice situation that utility of A consists of a deterministic component α and a random component ε_A and utility of B consists of β and ε_B $(\alpha>\beta>0)$.

$$u(A)=\alpha+\varepsilon_A$$
$$u(B)=\beta+\varepsilon_B.$$

According to random utility maximization principle, the probability of choosing A from choice set $\{A, B\}$ is

$$\begin{aligned}P(A)&=\Pr\{u(A)>u(B)\}\\&=\Pr\{\alpha+\varepsilon_A-\beta-\varepsilon_B>0\}=\Pr\{\varepsilon_A-\varepsilon_B>\beta-\alpha\}.\end{aligned} \tag{A14}$$

If we denote the random variable $\varepsilon_A-\varepsilon_B$ as ε, which has a density function of $f(\varepsilon)$ and distribution function of $F(\varepsilon)$. The density function $f(\varepsilon)$ is assumed to be symmetrically distributed around the mean value of 0.

$$P(A)=\int_{\beta-\alpha}^{\infty}f(\varepsilon)\,d\varepsilon=1-F(\beta-\alpha)=F(\alpha-\beta). \tag{A15}$$

If it is assumed that ε follows a logistic distribution (for the ease of calculation) (cf. McFadden 1973), which is a symmetric, bell-shaped distribution just like a normal distribution,

$$\begin{aligned}P(A)&=\frac{1}{1+\exp[-(\alpha-\beta)\pi/\sqrt{3}\sigma_{AB}]}\\P(B)&=\frac{1}{1+\exp[-(\beta-\alpha)\pi/\sqrt{3}\sigma_{AB}]}\end{aligned} \tag{A16}$$

where

$$\sigma_{AB}^2=\operatorname{var}(\varepsilon)=\operatorname{var}(\varepsilon_A)+\operatorname{var}(\varepsilon_B)-2\rho\sqrt{\operatorname{var}(\varepsilon_A)\operatorname{var}(\varepsilon_B)},$$
$$\rho=\operatorname{cor}(\varepsilon_A,\varepsilon_B).$$

Differential similarities among products lead to correlated errors (Allenby and Ginter 1995, Bhat 1995), thus similarity between a pair of brands are often represented by the correlation, ρ, of the random utilities of the two brands (McFadden 1978, Currim 1982).

$$\ln\left\{\frac{P(A)}{P(B)}\right\}=\frac{\pi}{\sqrt{3}\sigma_{AB}}(\alpha-\beta) \tag{A17}$$

$$\frac{d}{d\sigma_{AB}}\ln\left\{\frac{P(A)}{P(B)}\right\}<0, \qquad \frac{d}{d\sigma_{AB}}\left\{\frac{P(A)}{P(B)}\right\}<0 \tag{A18}$$

$$\frac{d}{d\rho}\left\{\frac{P(A)}{P(B)}\right\}>0 \quad \text{(asymmetric effect)} \tag{A19}$$

$$\frac{dP(A)}{d\rho}>0, \qquad \frac{dP(B)}{d\rho}<0. \tag{A20}$$

The asymmetric effect holds true even in the random utility context. As the value of ρ gets larger, σ_{AB} gets smaller, thus distribution of ε becomes more concentrated around the mean 0. Thus, a larger ρ, in turn, polarizes the options' choice shares. If we assume that $\operatorname{var}(\varepsilon_A)=\operatorname{var}(\varepsilon_B)$ (McFadden 1978), as ρ approaches 1, σ_{AB} approaches 0. Thus, $\lim_{\rho\to1}\sigma_{AB}=0$, $\lim_{\rho\to1}P(A)=1$, and $\lim_{\rho\to1}P(B)=0$ (dominance effect). The random utility counterpart of the EBT model would be the nested logit model (Domencich and McFadden 1975) and the generalized extreme value model (McFadden 1978). The analysis of these random-utility-based models provides the same results as the EBT model.

References

Allenby, G. M., J. L. Ginter. 1995. The effects of in-store displays and feature advertising on consideration sets. *Internat. J. Res. Marketing* **12** 67–80.

Batsell, R. R., J. C. Polking. 1985. A new class of market share models. *Marketing Sci.* **4**(Summer) 177–198.

Bhat, C. R. 1995. A heteroscedastic extreme value model of intercity travel mode choice. *Transportation Res.* **29**(6) 471–483.

Carpenter, G. S., K. Nakamoto. 1989. Consumer preference formation and pioneering advantage. *J. Marketing Res.* **26**(August) 285–298.

Chernev, A. 1997. The effect of common features on brand choice: moderating role of attribute importance. *J. Consumer Res.* **23**(4) 304–311.

Cui, D., D. Curry. 2005. Prediction in marketing using the support vector machine. *Marketing Sci.* **24**(4, Fall) 595–615.

Currim, I. S. 1982. Predictive testing of consumer choice models not subject to independent of irrelevant alternatives. *J. Marketing Res.* **19**(May) 208–222.

Debreu, G. 1960. Review of R. D. Luce, individual choice behavior: a theoretical analysis. *Amer. Econom. Rev.* **50** 186–188.

Dhar, R. 1997. Consumer preference for a no-choice option. *J. Consumer Res.* **24**(September) 215–231.

Dhar, R., I. Simonson. 2003. The effect of forced choice on choice. *J. Marketing Res.* **40**(2) 146–161.

Dholakia, U. M., I. Simonson. 2005. The effect of explicit reference points on consumer choice and online bidding behavior. *Marketing Sci.* **24**(2, Spring) 206–217.

Domencich, T., D. McFadden. 1975. *Urban Travel Demand: a Behavioral Analysis*. North Holland, Amsterdam.

Glazer, R., B. Kahn, W. L. Moore. 1991. The influence of external constraints on brand choice: the lone-alternative effect. *J. Consumer Res.* **18**(June) 119–127.

Gourville, J. T., D. Soman. 2005. Overchoice and assortment type: when and why variety backfires. *Marketing Sci.* **24**(3) 382–395.

Hsee, C. K., F. Leclerc. 1998. Will product look more attractive when presented separately or together. *J. Consumer Res.* **25**(September) 175–186.

Huber, J., M. A. Sewall. 1978. Covariance bias of Thurstone V Scaling as applied to consumer preferences and purchase intentions. *Advances in Consumer Research*, Vol. VI. Association for Consumer Research, Chicago, IL, 578–581.

Huber, J., J. W. Payne, C. Puto. 1982. Adding asymmetrically dominated alternatives: violation of regularity and similarity hypothesis. *J. Consumer Res.* **9**(June) 90–98.

Kahn, B., W. L. Moore, R. Glazer. 1987. Experiments in constrained choice. *J. Consumer Res.* **14**(June) 96–114.

Kahneman, D., A. Tversky. 1979. Prospect theory: an analysis of decision under risk. *Econometrica* **47**(March) 263–291.

Leclerc, F., C. Hsee, J. C. Nunes. 2005. Narrow focusing: why the relative position of a good in its category matters more than it should. *Marketing Sci.* **24**(2) 194–205.

Luce, R. D. 1959. *Individual Choice Behavior: a Theoretical Analysis*. John Wiley & Sons, New York.

MacCrimmon, K. R. 1973. An overview of multiple objective decision making. J. L. Cochrane, M. Zeleny, eds. *Multiple Criteria Decision Making*. University of South Carolina Press, Columbia.

McFadden, D. 1973. Conditional logit analysis of qualitative choice behavior. P. Zarembka, ed. *Frontiers in Econometrics Applications*. MIT Press, Cambridge, MA.

McFadden, D. 1978. *Modeling the Choice of Residential Location in Spatial Interaction Theory and Residential Location*. A. Karlgvist et al. eds. North Holland, Amsterdam, 75–96.

Simonson, I. 1989. Choice based on reasons: the case of attraction and compromise effects. *J. Consumer Res.* **16**(2) 158–174.

Tversky, A. 1972. Elimination by aspects: a theory of choice. *Psych. Rev.* **79**(4) 281–299.

Tversky, A. 1977. Features of similarity. *Psych. Rev.* **84**(4, July) 327–352.

Tversky, A., J. Russo. 1969. Substitutability and similarity in binary choice. *J. Math. Psych.* **6** 1–12.

Tversky, A., S. Sattath. 1979. Preference trees. *Psych. Rev.* **86**(6) 542–573.

Tversky, A., I. Simonson. 1993. Context-dependent preferences. *Management Sci.* **39**(10) 1179–1189.

Zhang, J. 2006. An integrated choice model incorporating alternative mechanisms for consumers' reactions to in-store display and feature advertising. *Marketing Sci.* **25**(3) 278–290.