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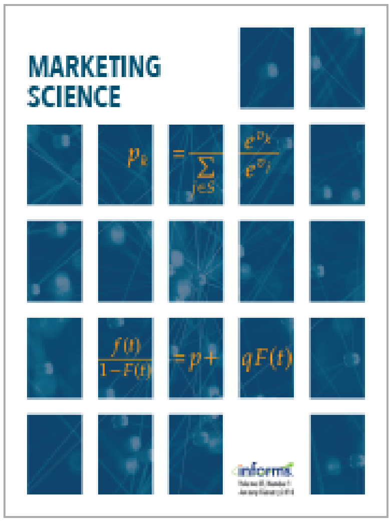

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Deconstructing Each Item's Category Contribution

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Retailers and manufacturers believe that the mere presence of certain items in a retail assortment increases the sales volume of the whole assortment. This paper provides an empirical study of the role of every item in an assortment. Our results show that many items affect category sales over and above their own sales volume. After deconstructing the role of a stockout of individual items into three effects—lost own sales, substitution to other items, and the category sales impact—we find that the category impact has the largest magnitude. Interestingly, the disproportionate impact of individual items on category sales is not restricted to top-selling items, for almost every single individual item affects category sales. It seems that variety is indeed the price of entry in retailing. Our results support recent findings that more frequently purchased categories are less adversely affected by reductions in assortment. We also find that the assortment appears to gain attractiveness when certain items are out of stock, a result that is consistent with the discussion in the literature concerning category clutter.

Key words: apparel retailing; retail assortment; category management; out-of-stocks; substitution; key items; hierarchical Bayes; COM-Poisson
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1. Introduction

In retailing, one of the most important decisions is the selection of items to inventory. Items not only generate profits directly by their own sales, but items also are interrelated within categories and indirectly contribute to category volume. For instance, individual items can serve as buffer inventory (purchased as substitutes when preferred items are temporarily out of stock) or as alternatives for variety seekers, and the collection of inventoried items helps craft the visual presentation of the retailer and affects the store image (e.g., a wide assortment category killer). Therefore, retail inventory decisions involve more than sales projections of individual items.

Considering the contribution of an item to category sales volume, the presence or absence of any individual item in a category can contribute in one of three ways. First, an item contributes its own sales volume to the category. Second, the absence of an item can affect sales of other items, in that remaining in-stock items can serve as alternatives when a more preferred item is temporarily out of stock (A. C. Nielsen 2005, Anupindi et al. 1998). Third, the presence of an individual item may impact sales of the entire category over and above the sales of the item itself, possibly by drawing attention to the category, improving the presentation, and stimulating the category sales overall (Berlyne 1960, Koopmans 1964, Kahn and Lehmann 1991, Kahn 1995).

The contribution of this research is a model that deconstructs every item's category contribution, using out-of-stocks in an apparel category. In apparel categories, items within a collection are typically from the same manufacturer and are priced identically within a time period. The lack of price variation prohibits use of typical utility models, because these models mine variation in prices to infer consumer preferences. However, out-of-stocks induce variation in the assortment, and the model that we develop utilizes the variation in product assortment to infer consumer preferences. The out-of-stocks can be viewed as natural experiments, and they can (1) result in lost sales, (2) lead to substitution to other items, and (3) impact the sales of the entire category due to a temporary change in the available selection. A key contribution of this research is the simultaneous modeling of all three effects for every item at the store level, using data with assortment variation but no price variation. The state space of out-of-stocks can be complex, and our model provides a parsimonious approach compared to utility-theoretic approaches (Kim et al. 2002, Kalyanam and Putler 1997).

One of our major findings is that out-of-stocks of many items, in addition to affecting own sales, significantly affect the sales of the entire set of remaining items in the category. A few important items such as navy-large have large impacts on category sales. Additionally, items with relatively large category impacts are not necessarily top sellers. The item tan-extra large ranks as one of the top five items in terms of overall category impact, but it only ranks a 10 in terms of sales. Our findings suggest that retailers should look beyond marketing tactics such as category advertising (Bass et al. 2005) to the role of key items in stimulating category demand.

A simulation analysis based on our model estimates shows that although a few key items have a very high impact on category sales, the out-of-stocks of many other items also depress category sales. It seems that after all, as speculated upon by others (Hoch et al. 1999), variety is indeed the price of entry in retailing despite the fact that only a few items are best sellers. A second set of findings is with regard to substitution. Although substitution proportional to share is a common assumption in the literature, our results show that substitution is not widespread, is limited to only a few colors, and can be asymmetric. Consistent with intuition, there is very little substitution across sizes.

There are theoretical foundations for why the absence of any item can increase or decrease the demand for the entire category over and above the item's own sales. First, each individual item might contribute to the variety within the assortment (Hoch et al. 1999), and a loss of variety would reduce the desirability of the assortment for consumers with uncertain preferences (Koopmans 1964, Reibstein et al. 1975, Kreps 1979, Kahn and Lehmann 1991) and for those who tend to seek variety (Berlyne 1960, Helson 1964, McAlister and Pessemier 1982, Kahn 1995). Second, retailers work to maintain aesthetically pleasing displays of categories in order to attract consumers; out-of-stocks may erode the aesthetic value of the category presentation, which is the so-called "broken assortment" effect (Smith and Achabal 1997). Both of these reasons support the conclusion that out-of-stocks of an item would lessen category demand over and above own sales.

There is also a theoretical foundation for a result in which some out-of-stocks would increase category demand. Reasons why purchase amounts would increase after deletion of nonfavorite items include the elimination of clutter (Boatwright and Nunes 2001, Borle et al. 2005). In addition, Iyengar and Lepper (2000) demonstrated that choosing among too many items is demotivating and can reduce sales. In light of the relationship of individual items and category sales, retailing textbooks (Levy and Weitz 2004, p. 420) advise readers to take into account the impact of an item on the overall assortment, although these texts do not provide any guidance on the magnitude of these effects or how they should be identified.

The empirical study of out-of-stock data requires careful attention to model specification. In retail sales data, concurrent stockouts of multiple items can occur in periods of high demand. The simultaneity of stockouts and periods of high demand can lead to a positive stockout coefficient implying that a stockout can actually increase sales of the item! We control for demand shocks and concurrent stockouts using a simultaneous equations approach, estimating the joint distribution of sales and stockouts in a hierarchical Bayes model. Manchanda et al. (2005) present a similar modeling approach in a different context. We find that a naïve model, one that does not account for this joint distribution, yields coefficients with the wrong signs on the stockout variable.

Another characteristic of retail sales data for apparel categories is that unit sales for each item at the store level are sparse. For example, in our data set, 3.6 units of each item were sold per store in each time period. In addition, stockouts occur in any time period with probability less than 0.03, leaving little information at the store level to estimate item- and store-specific parameters. We therefore utilize a hierarchical model that shares information across stores. Additionally, our model allows for over- or underdispersion relative to the common Poisson sales model. We also model substitution in a flexible manner, without imposing the independence of irrelevant alternatives (IIA) restriction of switching proportional to share.

Our research contributes to the existing research on the effects of out-of-stocks. Anupindi et al. (1998) developed a model that used information on stockouts to infer item demand and substitution patterns; they found significant differences between demand rates and observed sales of products even for items that were rarely out of stock. We extend their work by estimating not only substitution and lost sales, but by also estimating which items, if any, affect sales of the entire category. Anupindi et al. (1998) contains a thorough discussion of prior models that incorporate stockouts, which did not measure substitution effects or quantify the impact of individual items on category sales.

Our paper also contributes to the growing research on how consumers make choices within assortments and respond to changes in assortments. Lattin and Roberts (1992) measured how the structure of a category affects product selection. We consider the opposite directional effect, how the presence of individual products affects purchase in the category. Kim et al. (2002) use price variation to infer item utilities and substitution, whereas we use the variation in the in-stock items to deconstruct each item's absence into own sales, substitution, and a category effect.

Borle et al. (2005) assessed the impact of a large-scale reduction in assortment on store sales, and found a decline in store sales. Prior work had found the sales impact of assortment reductions to be small (Drèze et al. 1994, Broniarczyk et al. 1998, Boatwright and Nunes 2001). These previous studies all measured the impact of permanent and simultaneous elimination of multiple items from frequently purchased grocery categories, using experimental data or household data. Our paper examines these effects in the context of an infrequently purchased and discretionary apparel category. We use store-level data with out-of-stock information, a type of data that is more readily accessible to retailers.

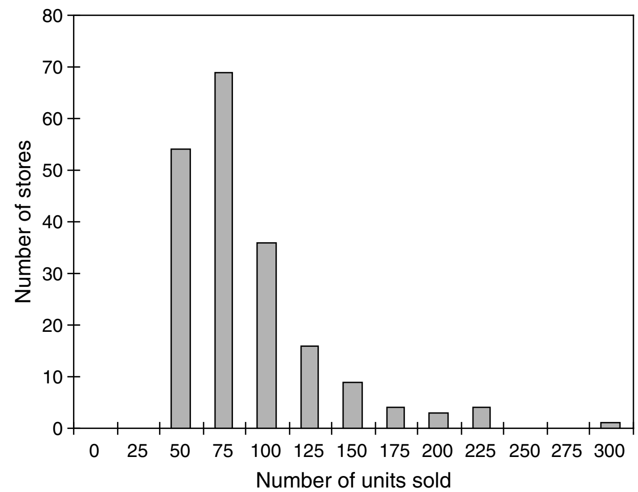
Our research also builds on and contributes to the literature on measuring the variety of an assortment. Category assortment is known to affect store visits (Borle et al. 2005), an issue even more critical to supermarkets in light of the growth of Walmart (Singh et al. 2006). Hoch et al. (1999) used the information structure of a category to provide models of perceived variety. We empirically infer the contribution of each item on the attractiveness of an assortment, showing how the temporary absence of even small-share products disproportionately affects category sales. We formulate an assortment attractiveness index that changes when individual items are out of stock. The assortment attractiveness index allows us to study the differential effects of colors and sizes on the attractiveness of the assortment using actual sales, as opposed to perceptions of variety.

The rest of this paper is organized as follows: In the next section (§2) we describe the data, in §3 we introduce the hierarchical demand model, and in §4 we discuss the estimates of the model. In §5, we deconstruct the net impact of a stockout with a simulation analysis. We conclude with a discussion of our key findings and directions for future research.

2. The Data

The data come from a national retailer based in the United States. The data pertain to a two-year time period and consist of quarterly[1] sales and stockout information of the "men's work T-shirts" category in 196 stores spread across the United States. These are pocketless, round neck, short-sleeve T-shirts and come in eight colors and four sizes (a total of 32

Figure 1(a) **Average Quarterly Sales Per Store**



items).[2] All items are made by the same manufacturer, and the retailer is not considered to be a trend setter in this category. Figure 1(a) is a histogram plot of the average quarterly sales of these T-shirts (sum of all 32 items) across the 196 stores. More than 80% of the stores sell, on average, 50 to 100 T-shirts per quarter, although there are some stores (five stores in total) that sell in excess of 200 T-shirts.

Table 1(a) gives the average quarterly sales per store for each of these 32 items. As seen from Table 1(a), on average a store sells only 3.62 units of a particular item in a quarter. Further, viewing sales in terms of colors and sizes (which in turn can be viewed as attributes of the item), we see that the "grey" color followed by "navy" and then "black" are the largest sellers, while "tan" sells the least. Amongst the sizes, "large" sells the most and "medium" sells the least. The item "grey-large" is the largest-selling item and "tan-XXL" is the slowest seller (with sales of a mere 1.11 units per store per quarter). The two interesting facets of Table 1(a) are, firstly, the "low" sales observed and, secondly, "significant" sales differences across colors and sizes.

Table 1(b) provides stockout information for these 32 color-size combinations. The stockout data available were the number of weeks in a quarter the particular item was out of stock. This is the type of data available for planning and replenishing apparel products on a seasonal basis at this retailer.

A striking feature of Table 1(b) is the large range vis-à-vis the small averages. On average, a particular item may have stocked out a small percentage of weeks, but in some stores (during some time periods) the extent of stockouts can be significant. On average, the color "white" stocked out the most amongst

[1] This retailer operates its merchandise planning in this category on a quarterly basis.

[2] The price and promotions in this category were very infrequent and were not distinguishable from seasonal effects.

Table 1(a) Average Quarterly Sales Per Store Per item

Color	Size = M	Size = L	Size = XL	Size = XXL	
Black	2.39	5.03	4.87	2.46	3.69
Grey	3.99	9.78	9.68	4.56	7.00
Jade	1.52	3.41	3.80	1.69	2.61
Lt. Blue	1.83	4.01	3.70	1.74	2.82
Navy	2.61	6.37	6.35	2.95	4.57
Royal	1.86	4.43	4.40	2.01	3.18
Tan	1.18	2.54	2.47	1.11	1.83
White	2.24	4.59	4.34	1.86	3.26
	2.20	5.02	4.95	2.30	3.62

Table 1(b) The Range and Average % of Stockouts**

Color	Size = M (%)	Size = L (%)	Size = XL (%)	Size = XXL (%)	(%)
Black	0 to 41.67 *2.35*	0 to 46.15 *1.83*	0 to 50.00 *1.71*	0 to 61.54 *3.67*	2.39
Grey	0 to 61.54 *2.21*	0 to 30.77 *1.15*	0 to 53.85 *1.41*	0 to 53.85 *2.00*	1.69
Jade	0 to 46.15 *2.51*	0 to 38.46 *2.19*	0 to 46.15 *2.76*	0 to 69.23 *4.09*	2.89
Lt. Blue	0 to 53.85 *3.51*	0 to 76.92 *3.67*	0 to 76.92 *4.74*	0 to 61.54 *4.26*	4.05
Navy	0 to 46.15 *3.03*	0 to 69.23 *2.07*	0 to 46.15 *2.74*	0 to 61.54 *4.36*	3.05
Royal	0 to 53.85 *4.05*	0 to 53.85 *3.77*	0 to 61.54 *4.05*	0 to 46.15 *4.64*	4.13
Tan	0 to 69.23 *4.57*	0 to 61.54 *3.75*	0 to 61.54 *3.84*	0 to 61.54 *4.05*	4.05
White	0 to 69.23 *6.45*	0 to 46.15 *3.59*	0 to 69.23 *4.03*	0 to 46.15 *4.80*	4.72
	3.59	2.75	3.16	3.98	3.37

**The first entry in a cell gives the *range* and the second entry the *average*.

all the colors, an average out of stock of 4.72% of weeks in a quarter as compared to the color "grey," which stocked out the least (an *average* of 1.69% of weeks in a quarter). The variation in average percentage of weeks stocked out across sizes is much less as compared to the variation across colors. Stockouts, when they occur, not only induce lost sales (the loss of potential sales of the stocked out item), but also cause changes in demand for other items in the category and induce temporary changes in the selection available to shoppers in the category. In summary, the data provide a nice setting to investigate the impact of retail sales and stockouts in the apparel category. In the next section, we introduce the demand model.

3. The Demand Model

3.1. Overview of Model Specification

For ease of exposition, we introduce the demand model in stages. We begin with a "basic" demand model, which assumes no stockouts (§3.1.1). The "own" effects[3] of a stockout are subsequently introduced (§3.1.2). Next, we incorporate the "cross" item effects[4] of a stockout (§3.1.3) and the impact on overall category demand (§3.1.4). Finally, we introduce how we control for demand shocks (§3.1.5).

3.1.1. The Basic Demand Model. Let the demand for a T-shirt with color c $(c = 1, 2, \ldots, 8)$ size g $(g = 1, 2, 3, 4)$ in store s $(s = 1, 2, \ldots, 196)$ for time t $(t = 1, 2, \ldots, 8)$[5] be given by demand_{scgt} units. Let SALES_{scgt} be the sales of T-shirt of color c size g in time t in store s. In the *absence* of stockouts, observed sales and demand are equal. We assume demand to be distributed COM-Poisson:

$$\text{demand}_{scgt} = \text{SALES}_{scgt} \sim \text{ComP}(\lambda^0_{scgt}, \nu), \qquad (1)$$

wherein the parameters (λ^0_{scgt}, ν) specify the distribution of demand.

The probability mass function of the COM-Poisson is as follows,

$$P_z(\lambda, \nu) = \left[\sum_{k=0}^{\infty} \frac{\lambda^k}{(k!)^\nu}\right]^{-1} \frac{\lambda^z}{(z!)^\nu}, \quad z = 0, 1, \ldots. \qquad (1a)$$

This formulation allows for a nonlinear change in ratios of successive probabilities in the form

$$P(Z = z+1)/P(Z = z) = \lambda/(z+1)^\nu,$$

and the distribution itself is a generalization of some well-known discrete distributions (Shmueli et al. 2005, Boatwright et al. 2003, Conway and Maxwell 1961). Depending on values of ν (the decay parameter), the distribution has thicker or thinner tails relative to the Poisson. The Poisson, the geometric, and the Bernoulli distributions are special cases of the COM-Poisson (with $\nu = 1$, $\nu = 0$, and $\nu = \infty$, respectively). For $\nu = 1$ the infinite sum series in Equation (1a) can be represented as e^λ, thus resulting in a Poisson distribution, for $\nu = 0$ (and $\lambda < 1$)[6] the sum series is a geometric sum $1/(1-\lambda)$, resulting in a geometric distribution and as $\nu \to \infty$ the sum series converges to $(1+\lambda)$ and the distribution itself converges to a Bernoulli distribution with $P(Z = 1) = \lambda/(1+\lambda)$. Thus, values of $\nu > 1$ are helpful in characterizing "underdispersed" data while values of $\nu < 1$ characterize "overdispersed" data relative to the Poisson. The distribution is defined over positive integers and is flexible in representing a variety of shapes.

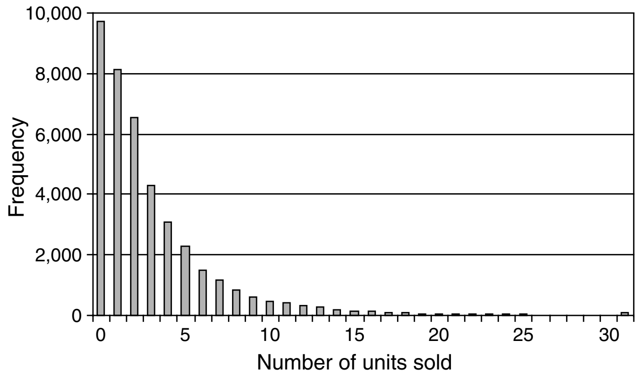
[3] The observed sales for an item will be less than the true demand for that item when a stockout for that item occurs.

[4] Demand substitution to other items and the impact of item stockout on the overall category sales due to a temporary change in the assortment variety.

[5] The quarterly data span two years.

[6] The distribution is undefined for $\nu = 0$ (and $\lambda \geq 1$).

Figure 1(b) Quarterly Sales of T-Shirts (32 Color-Size Combinations) Across the 196 Stores



Although the mean and variance of this distribution do not have closed-form expressions, modern-day computing overcomes this inconvenience, allowing us to utilize the advantages of that this distribution entails relative to other simpler discrete distributions.

Figure 1(b) shows a histogram plot of the quarterly sales of T-shirts (32 color-size combinations) across the 196 stores. The data is overdispersed relative to a Poisson distribution, so a Poisson may not approximate the data-generating process as well as the COM-Poisson.

The parameter λ^0_{scgt} in Equation (1) (given ν) can be interpreted as a measure of central tendency.[7] We specify this parameter as follows,

$$\log \lambda^0_{scgt} = \beta^{\text{color}}_c + \beta^{\text{size}}_g + \alpha \log \text{SQFT}_s + \gamma_{st} \qquad (2)$$

where β^{color}_c, β^{size}_g are intercepts decomposed into color and size effects,[8] and the variable SQFT_s is the square footage of store s. The parameter α is the impact of the physical size of a store[9] on demand of various items (the impact being the same for all items in the category). The last term in Equation (2), γ_{st}, can be viewed as a store- and time period-specific demand shock that affects all the items in the assortment. Thus, a higher value for this parameter would imply a higher demand across all items in the assortment in that particular store during that particular time-period. The parameter captures both time-specific effects such as seasonality, and exogenous demand shocks. Section 3.1.5 describes this parameter in detail.

[7] From the probability mass function of the COM-Poisson distribution.

[8] For purposes of model identification, all coefficients pertaining to the "Medium" size ($g = 1$) are set to zero.

[9] The store size in terms of square footage is used as a proxy for the "store volume class." This proxy should capture some, but not all, of the macro differences in populations, in that the company would utilize information on the local population for its store specifications.

3.1.2. The Own Effects of a Stockout. When there are no stockouts, the observed sales reflect the demand for that particular color-size combination in a store in a quarter. Equations (1) and (2) can model demand. However, in the presence of a stockout of color-size combination (c, g), the observed sales SALES_{scgt} is a truncated (and hence biased downwards) representation of the demand. The observed sales need to be adjusted upwards to SALES'_{scgt} so that it is a closer representation of the true demand for that particular color-size combination.

We do so by way of an imputation procedure. We observe the truncated sales SALES_{scgt} whenever there is a stockout. The imputed sales SALES'_{scgt} is a number drawn from the right tail of the demand distribution (Equation (1)) such that $\text{SALES}'_{scgt} \geq \text{SALES}_{scgt}$.[10] The imputation procedure provides an upward adjustment to the observed sales of color-size combination (c, g), in store s for quarter t when that particular color-size combination is stocked out for some period in that store during that particular time. The demand Equation (1) is accordingly modified as follows,

$$\text{SALES}'_{scgt} \sim \text{ComP}(\lambda^0_{scgt}, \nu) \quad \text{where}$$
$$\text{SALES}'_{scgt} = \text{SALES}_{scgt} \text{ when there is no stockout.} \qquad (3)$$

3.1.3. The Cross-Item Effects of a Stockout. The demand Equation (3) incorporates the own effects of a stockout; i.e., it provides an upward adjustment to the observed sales when there is a stockout for a particular item in a particular store in a particular time period. However, when stockouts occur it is likely that there are cross-item effects. For example, the unmet demand for the stocked-out item may spill over to other items, thus increasing their sales.

We incorporate these effects by modifying demand Equation (3) as follows,

$$\text{SALES}'_{scgt} \sim \text{ComP}(\lambda'_{scgt}, \nu) \qquad (4)$$

where,

$$\lambda'_{scgt} = \left\{ \prod_{i=1}^{N_c} \prod_{j=1}^{N_g} (\delta^{\text{color}}_{ic} \delta^{\text{size}}_{jg})^{\text{ratio}_{sijt}} \right\} \lambda^0_{scgt}. \qquad (5)$$

In the above equations (λ'_{scgt}, ν) are the parameters of the COM-Poisson distribution and $\delta^{\text{color}}_{ic}$, $i, c = 1, 2, \ldots, N_c$, are elements of the $N_c \times N_c$ cross-impact matrix $\boldsymbol{\delta}^{\text{color}}$, where N_c is the number of colors

[10] Performed at the beginning of every MCMC cycle, whenever in any particular store s, during any time period t, if the color-size combination (c, g) stocks out. Use the current value of λ^0_{scgt} and ν and draw a random number from the truncated COM-Poisson truncated at the observed sales SALES_{scgt} such that the draw $\text{SALES}'_{scgt} \geq \text{SALES}_{scgt}$. This procedure is an application of data augmentation (see Tanner and Wong 1987).

observed in the data (in our case, eight colors). Similarly, $\delta^{\text{size}}_{jg} j$, $g = 1, 2, \ldots, N_g$, are elements of the $N_g \times N_g$ cross-impact matrix $\boldsymbol{\delta}^{\text{size}}$, where N_g is the number of sizes observed in the data (in our case, four sizes). If size substitution is not relevant in certain applications, the model can be simplified by omitting the cross-impact matrix $\boldsymbol{\delta}^{\text{size}}$.

The variable ratio_{sijt} is the proportion of weeks that a T-shirt of color i, size j, in store s, during quarter t was stocked out.[11] It is a measure of the extent of stockout of the particular item in the store in that quarter. Thus, if during a time period a particular color-size combination never stocked out in a particular store, then the value of the corresponding *ratio* variable is zero. The expression $\{\prod_{i=1}^{N_c}\prod_{j=1}^{N_g}(\delta^{\text{color}}_{ic}\delta^{\text{size}}_{jg})^{\text{ratio}_{sijt}}\}$ thus specifies the cross-item impact on color-size combination (c, g) on account of stockouts in any of the other items. Further, this cross-item impact is decomposed across the item attributes viz. color and size with the 8×8 matrix $\boldsymbol{\delta}^{\text{color}}$ specifying the parameters for the demand changes on account of the color attribute and the 4×4 matrix $\boldsymbol{\delta}^{\text{size}}$ specifying the parameters for the impact on account of the size attribute. By definition, $\delta^{\text{color}}_{ic} = 1$ when $i = c$; similarly $\delta^{\text{size}}_{jg} = 1$ when $j = g$. In line with past empirical work in retail stockouts (Anupindi et al. 1998) and for tractability of estimation, we restrict the cross-item impact to be positive; i.e., the elements of the two matrices $\boldsymbol{\delta}^{\text{color}}$ and $\boldsymbol{\delta}^{\text{size}}$ (Equation (5)) are restricted to be ≥ 1.

Equation (5) also has a desirable property with respect to how it handles concurrent stockouts. Suppose two items, a "focal" item whose demand is being modeled in Equation (4) and another called a "comparison" item, stock out concurrently. Our implementation of Equation (5) first conditions on the cross effect of the comparison item, and then it imputes sales of the focal item. This avoids underestimation of the cross effects and of the sales of the focal item due to concurrent stockouts.

Utility-theoretic models implemented with household-level data (Kim et al. 2002, Kalyanam and Putler 1997) provide a parsimonious approach to modeling cross effects. However, when these models are implemented with aggregate data, they need to be modified to reflect the state space of stockouts.[12] This is a nontrivial problem that introduces lots of additional parameters. Equation (5) provides a more parsimonious approach.

3.1.4. Category Impact of a Stockout: An Assortment Attractiveness Index. The parameter λ^0_{scgt} in Equation (2) (and (5)) is a measure of the core demand for color c, size g, in store s, for quarter t. This core demand parameter can be modified to reflect the impact of a stockout on the entire category as follows,

$$\log \lambda^0_{scgt} = \beta^{\text{color}}_c + \beta^{\text{size}}_g + \alpha \log \text{SQFT}_s + \underbrace{\left[\sum_{\substack{i=1\\ \{i,j\}\neq\{c,g\}}}^{N_c}\sum_{j=1}^{N_g} \text{ratio}_{sijt}(\theta^{\text{color}}_i + \theta^{\text{size}}_j)\right]}_{\text{AAI}_{st}\text{ (Assortment Attractiveness Index)}} + \gamma_{st} \qquad (6)$$

where, as in Equation (2), β^{color}_c and β^{size}_g are the intercepts decomposed into color and size effects, and the variable SQFT_s is the square footage of store s. As before, the variable ratio_{sijt} is the proportion of weeks that color-size combination (i, j) was stocked out in store s during quarter t; N_c and N_g are the number of colors and sizes (8 and 4, respectively).

The expression in square brackets (in Equation (6)) is an attribute-based measure of the attractiveness of an assortment. This could be due to the variety of the assortment of T-shirts (Herpen and Pieters 2002). We call it the assortment attractiveness index (AAI) for the assortment in store s during time t. The θ^{color}_i and θ^{size}_j parameters, then, are the relative weights of various colors and sizes in the AAI. The magnitudes and signs of these coefficients would be indicative of the importance of that particular attribute (color/size) on demand for the entire category.[13] When there are no stockouts in a category in a time period, the AAI term is equal to zero.

Thus, Equations (4), (5), and (6) specify a model where the demand for an item decomposes into a core demand as specified by the parameter λ^0_{scgt} and the cross-item impact as specified by the expression in curly brackets in Equation (5). The core demand in turn is influenced by changes in the attractiveness of the assortment (the AAI term in Equation (6)) due to stockouts. These effects (the core demand, the cross-item impact, and the AAI effect at the category level) are further decomposed into color and size effects.

[11] Thus, $\text{ratio}_{sijt} = \text{week}_{sijt}/\text{week}_t$, where week_{sijt} is the number of weeks the T-shirt with color i, size j, in store s during quarter t was stocked out, while week_t is the number of weeks in the quarter t.

[12] For example, the Kim et al. model would be modified as follows

$$\text{Max}\, \text{E}[U(x)] = \left[\iiint_{OOS} \sum_j \pi(oos)_j \psi_j (x_j + \gamma_j)^{\alpha_j}\, d(oos)\right] - \lambda * (B - P_j * x_j).$$

The parsimony of this model depends on $\pi(OOS)$. Our product category has 32 SKUs. Therefore, $\pi(OOS)$ is a 32-dimensional vector with 32×32 covariance terms. A fully Bayesian analysis would require the estimation of this joint density for each time period in each store, requiring many more model parameters.

[13] For the purpose of model identification, all coefficients pertaining to the medium size $(g = 1)$ as well as the first quarter $(t = 1)$ are set to zero. Also, color category effects θ_i can be separately identified from cross effects $\delta^{\text{color}}_{ic}$ because the former are common across all other items in the assortment, whereas the latter are asymmetric pairwise effects. The size parameters are similarly identified.

Thus, a stockout of an item can impact the sales of another item in two ways; the *first* is through the cross-item impact, where unfulfilled demand of the stocked-out item spills over to other items; or more generally, where one item's absence affects demand patterns for other individual items. The *second* is through the impact that the stocked-out item has on the attractiveness of the whole assortment—a category-level effect. The relative impacts of these two phenomena can critically inform a manager's assortment and inventory planning decisions, and hence is of substantive interest.

3.1.5. Controlling for Demand Shocks. A potential concern with the model, as specified in Equations (4) through (6), is that the stockout patterns across items in the same time period may not be random; i.e., for example, a surge in demand may lead to stockouts of multiple items. Thus, stockouts and demand surges may have a high correlation; this phenomenon, if not controlled for, may lead to erroneous conclusions from the model (Sun 2005). We attempt to control for this by allowing stockouts to be a function of the store/time-specific demand shock parameter γ_{st} (Equation (6)). We do so by allowing a binomial distribution for week_{scgt} (the number of weeks the color-size combination c, g was out of stock in store s during quarter t) as follows,

$$\text{week}_{scgt} \sim \text{Binomial}\left(\text{weeks}_t, \frac{\exp(\gamma_{st})}{\exp(\gamma_{st})+\exp(\omega)}\right), \quad (7)$$

where weeks_t[14] is the number of weeks in quarter t, γ_{st} is the store/time-specific demand shock parameter (Equation (6)) and ω is a parameter that along with γ_{st} specifies the binomial probability of number of weeks of stockout given weeks_t weeks in the quarter. Thus, Equation (7) allows the stockouts themselves to be a function of demand, addressing the concern of "nonrandom" stockouts. A hierarchy is also allowed on the store/time-specific demand shocks γ_{st} as follows,

$$\gamma_{st} \sim \text{Normal}(\bar{\gamma}, \tau^2), \quad (8)$$

where $\bar{\gamma}$ and τ^2 are the mean and variance parameters of the normal distribution.

3.2. Model Estimation

A hierarchical Bayesian framework is used to estimate the demand model as specified by Equations (4) through (8). The Bayesian specification is completed by assigning appropriate prior distributions on the parameters to be estimated. Appendix 1 lays out the prior distributions used in the analysis. The model is estimated using an MCMC sampling algorithm.[15]

[14] Recollect that the variable ratio_{scgt} used in Equations (5) and (6) is given by $\text{week}_{scgt}/\text{week}_t$, where week_{scgt} is the number of weeks the T-shirt with color c, size g, in store s during quarter t was stocked out, whereas weeks_t is the number of weeks in the quarter t.

[15] Details of which can be obtained from the authors on request.

Table 2 Estimates of the "Core" Demand Parameters (Equation 6)[1]

Color	β^{color}	Size	β^{size}
Black	−3.1059* (0.01686)	M	0.0000 0.00000
Grey	−2.8998* (0.01737)	L	0.4049* (0.00959)
Jade	−3.3710* (0.01943)	XL	0.4066* (0.00932)
Lt. Blue	−3.3005* (0.01856)	XXL	0.0403* (0.00971)
Navy	−2.9265* (0.01665)		
Royal	−3.2363* (0.01864)		
Tan	−3.3613* (0.01778)		
White	−3.2107* (0.01781)		

[1]The significant estimates have been marked with an asterisk, where estimates are deemed significant when the 95% posterior interval does not contain zero.

4. The Estimated Coefficients

The estimation result is posterior distributions for each of the parameters. These are summarized by their posterior means and standard errors. Tables 2 through 4 in the following text report these estimates.

4.1. Basic Model Parameters

We first report the estimates of the basic model parameters. The posterior mean (s.e.) of ν (Equation (4)) is 0.4140 (0.0053); a ν less than one indicates that the data exhibits a greater degree of dispersion than the Poisson allows. The posterior mean (s.e.) of α (Equation (7)) is 0.2952 (0.0011); its positive sign reflects that stores that are physically larger have greater sales volumes in this category, a result that could occur due to category sales being correlated with store square footage. The parameters $\bar{\gamma}$ and τ^y (Equation (8)) measure the mean and variance (respectively) in the (log of the) demand shocks.[16] The estimated values for these parameters are 0.1592 (0.0155) and 0.3335 (0.0143), respectively. The posterior means of γ_{st} (Equation (6)) range from −1.25 to 2.61; this suggests that the spike in category sales due to seasonality and demand shocks is substantial.

4.2. Core Demand Parameters

In Table 2, we report the estimates of $\boldsymbol{\beta}^{\text{color}}$ and $\boldsymbol{\beta}^{\text{size}}$ (Equation (6)). These parameters reflect the relative importance of item attributes. As expected, these estimates correlate with the (log of) item sales reported

[16] Recollect that the γ_{st} in Equation (6), (7) can be viewed as store-specific, time period-specific demand shocks that affect all the items in the store.

Table 3(a) Estimates of the Cross-Item Impact for the Color Attribute[1] (The δ^{color}_{ic} Parameters in Equation (5))

	Black	Grey	Jade	Lt. Blue	Navy	Royal	Tan	White
Black	1.0000 0.00000	1.1319* (0.05992)	1.1128 (0.06999)	1.1217 (0.06801)	1.0591 (0.03560)	1.2348* (0.08706)	1.0394 (0.03078)	1.2453* (0.08458)
Grey	1.1600* (0.06246)	1.0000 0.00000	1.1265 (0.07481)	1.1114 (0.06619)	1.0646 (0.04489)	1.0595 (0.04682)	1.1008 (0.06711)	1.0869 (0.05691)
Jade	1.2038* (0.05570)	1.0436 (0.03147)	1.0000 0.00000	1.1049 (0.05578)	1.1406* (0.04926)	1.1266* (0.06086)	1.0437 (0.03419)	1.0395 (0.03113)
Lt. Blue	1.1365* (0.04292)	1.0735* (0.03236)	1.1132* (0.05073)	1.0000 0.00000	1.0825* (0.03766)	1.0964* (0.04609)	1.0172 (0.01537)	1.0307 (0.02399)
Navy	1.0058 (0.00550)	1.3693* (0.07062)	1.4300* (0.10031)	1.3887* (0.10029)	1.0000 0.00000	1.5260* (0.10016)	1.4304* (0.06750)	1.5163* (0.09892)
Royal	1.1987* (0.04560)	1.0748* (0.03485)	1.1527* (0.05682)	1.2045* (0.06009)	1.0799* (0.03624)	1.0000 0.00000	1.0211 (0.01852)	1.0304 (0.02462)
Tan	1.0054 (0.00516)	1.2492* (0.03829)	1.2521* (0.05407)	1.3050* (0.05734)	1.0367 (0.02366)	1.2034* (0.05247)	1.0000 0.00000	1.2906* (0.05147)
White	1.3296* (0.04523)	1.0381 (0.02494)	1.0348 (0.02780)	1.0371 (0.02671)	1.0729* (0.03275)	1.0484 (0.03028)	1.0165 (0.01443)	1.0000 0.00000

Notes. The (row, column) entry indicates the parameter $\delta^{color}_{row,\,column}$. The cell entry is impact on the column color when the row color stocks out.

[1]For visual clarity, the significant estimates have also been shaded, where we define estimates as significant when the posterior mean is more than two standard deviations above 1.0.

Table 3(b) Estimates of the Cross-Item Impact for the Size Attribute[1] (The δ^{size}_{jg} Parameters in Equation (5))

	M	L	XL	XXL
M	1.0000 0.00000	1.0372* (0.01418)	1.0030 (0.00278)	1.0051 (0.00474)
L	1.1045* (0.02458)	1.0000 0.00000	1.0058 (0.00487)	1.0156 (0.01317)
XL	1.0096 (0.00852)	1.0118 (0.00893)	1.0000 0.00000	1.0510* (0.02045)
XXL	1.0054 (0.00516)	1.0057 (0.00489)	1.0272 (0.01372)	1.0000 0.00000

Notes. The (row, column) entry indicates the parameter $\delta^{size}_{row,\,column}$ for the impact on the column size when the row size stocks out.

[1]For visual clarity, the large estimates have also been shaded, where we define estimates as large when the posterior mean is more than two standard errors above 1.0.

in Table 1(a), with correlation of 0.95. In spite of the high correlation, there are some large discrepancies between estimated demand (based on the $\boldsymbol{\beta}^{color}$ and $\boldsymbol{\beta}^{size}$ coefficients) and observed sales, which is impacted by stockouts. Figure 2 plots observed sales share plotted versus estimated demand share to illustrate these differences. If observed sales and demand share were comparable, all points would fall on the diagonal (the dashed line). Instead, some points are far from the diagonal. For instance, grey L and grey XL had a sales share of around 8.4%, whereas the estimated demand share for these items is less than 5%.

4.3. Cross-Item Impacts

Tables 3(a) and 3(b) contain the parameter estimates that specify the cross-item demand impact of one item when another stocks out. This impact is further segregated into color and size attributes. The interpretation of these tables is as follows: In Table 3(a), when the row color stocks out, the parameters in the table reveal the impact on the column colors. For instance, when jade stocks out, the demand for black, navy, and royal increases. Note that all of the parameters in this table are greater than 1.0 by assumption; the shaded entries are those that reflect significantly positive coefficients.[17]

As one would expect, the magnitudes of cross-item impacts between sizes are smaller in general than those between colors, because body sizes are inflexible in the short term. The substitution across sizes is limited to medium size to large size (and vice versa) and substitution to XXL size when XL stocks out. Mass-produced clothes are not exact in terms of their fit, probably by design so there is some flexibility in accommodating different body sizes. We would expect some substitution around adjacent sizes (M to L and vice versa) but little across nonadjacent ones (example M to XL).

As for cross-color demand impacts, Table 3(a) reveals that not all colors are equal. Black and grey are in some ways symmetric, in that when black stocks out, demand for grey increases (so does the demand for royal and white). When grey stocks out,

[17] Due to our assumptions, the entire mass of the posterior distribution exceeds 1.0, so a formal test of the hypothesis that a parameter equals 1 is not meaningful. Even so, given the familiarity of classical hypothesis tests based on asymptotic normality, we here report results in this classical format to reflect the precision of the posterior point estimates.

Table 4 Estimates of the Category Assortment Attractiveness Index (AAI) Parameters (Equation (6))[1]

Color	θ^{color}	Size	θ^{size}
Black	−0.4461* (0.04716)	M	0.0000 0.00000
Grey	−0.3895* (0.06218)	L	0.0034 (0.03901)
Jade	−0.1019* (0.05529)	XL	−0.2047* (0.03658)
Lt. Blue	0.0384 (0.03758)	XXL	−0.0950* (0.03304)
Navy	−1.5610* (0.05081)		
Royal	−0.3361* (0.05178)		
Tan	−0.6629* (0.03604)		
White	−0.1421* (0.03898)		

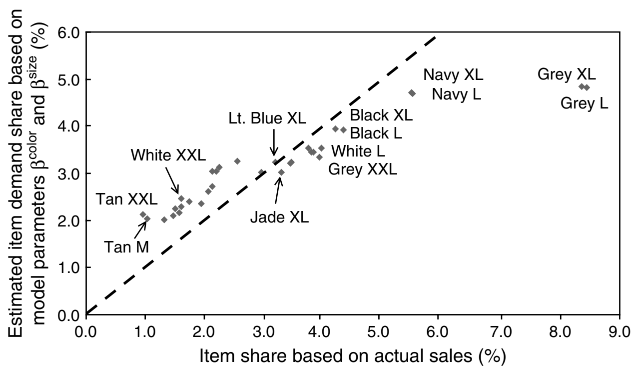
[1]The significant estimates have been marked with an asterisk, where estimates are deemed significant when the 95% posterior interval does not contain 0.

demand for black increases. The color grey is notable in that it has an impact only on black color. However, it receives substitution from five other colors (black, lt. blue, navy, royal, tan). The color navy is notable in that its stockout affects six of the remaining seven colors. The color tan is notable in that it is only affected by the stockout of one color, navy. At the other extreme, black, grey, and royal are affected by the stockouts of five of the remaining seven colors.

4.4. Assortment Attractiveness Index Parameters

Table 4 presents the estimates of the $\boldsymbol{\theta}^{\text{color}}$ and $\boldsymbol{\theta}^{\text{size}}$, showing how out-of-stock items affect demand for the entire assortment. Almost all of these coefficients are significantly negative, showing that stockouts of items with these attributes depress demand for the entire assortment. Navy's coefficient is much larger than the others in magnitude, more than twice the magnitude of the next tier, consisting of black and tan. Navy's presence in the assortment is clearly important for sales of the rest of the items. In spite of its low sales share, tan's absence is also deleterious to sales of remaining colors. In contrast, stockouts of lt. blue (medium and large) do not harm, but actually boost, the sales of the assortment because these coefficients are positive. Differential results for the colors could potentially occur due to presentation effects and consumer desire for variety.

Figure 2 Item Category Sales Share vs. Item Category Demand Share



4.5. Robustness Check and Predictive Analysis

Our model imputes sales (when there is a stockout) using the tail of a COM-Poisson distribution. To assess the robustness of the estimated parameters, one needs to test the sensitivity of the model to various tail specifications as well as the sensitivity towards the imputation procedure itself. To this end, we estimated four alternate model specifications. Model 1 is based on the Poisson and Model 2 on the geometric distribution.[18] Model 3 is based on a COM-Poisson distribution without an imputation procedure. Instead, the observed sales (when there is a stockout) were treated as censored observations and the likelihood modified accordingly. Model 4 was a simple baseline model assuming observed sales to follow a normal distribution and incorporating the stockout information as a covariate in the model. The details of these four alternate models are given in Appendix 2. For the sake of brevity, we do not report the tables of estimated coefficients from these comparison models.[19] The coefficients of Model 1 are directly comparable and the estimates are markedly similar to those of the proposed model. The estimates of Models 2 and 3 indicate that the results are robust to varying assumptions on the tail mass of the COM-Poisson. Model 4 reveals the inadequacy of a naïve regression model, which we discuss in more detail below.

We also conducted a predictive analysis using our proposed model and these four alternate models. For the purpose of predictive validation, we randomly selected one quarter (out of the eight quarters in our data set) for each of the 196 stores and then held out all the observations for that store in that particular quarter. In other words, we randomly held out 1/8th of our data and estimated our model along with the four comparison models (discussed in Appendix 2) using the remaining 7/8ths of the data. We then predicted sales for the held-out quarters for each store conditional on the actual pattern of stockouts observed

[18] The COM-Poisson nests the geometric and Poisson distributions.

[19] The tables of estimated coefficients can be obtained from the authors upon request.

Table 5 Predictive Analysis Across Models

	MAD* in unit sales
Model 0 (The proposed model)	2.57
Model 1 (Poisson distribution)	2.53
Model 2 (geometric distribution)	2.64
Model 3 (COM-Poisson with censored likelihood)	2.57
Model 4 (A baseline normal distribution)	3.48
Model 5 (rule-of-thumb approach)	2.95

*Mean absolute deviation.

in those stores.[20] We also used a "rule-of-thumb approach" as the 5th comparison model, wherein we used the mean SKU sales (across the remaining seven quarters) as the predicted sales for the held-out quarter. The Mean Absolute Deviation (MAD) from the actual sales was used to compare predictive ability across models. Table 5 reports results of the predictive analysis.

We find that our proposed model (with imputation or with a censored likelihood) does reasonably well in a hold-out analysis. This lends some face validity to the model and its estimated coefficients. The model with normal distribution for sales (Model 4, see Appendix 2 for details) performs the worst in a hold-out analysis. This model assumes a normal distribution for sales (as has been typically done in modeling apparel sales, Smith et al. 1994) and uses the "in-stock" position of a SKU directly as a covariate in the model. The model is parsimonious and does not have any cross effects or category effects. However, it is interesting that the coefficient of the in-stock variable (ϕ in Equation (7) of Appendix 2) is estimated to be negative; the posterior mean (s.e.) being -1.6123 (0.2372)—the interpretation being, stockouts lead to more sales. This is why we need to control for demand shocks as we do in our proposed model (see §3.1.5). Stockouts and demand surges are likely to have a high correlation and this phenomenon, unless controlled for, will give us interpretations of the kind we get in Model 4 (the baseline normal distribution sales model).

We also performed a robustness check of the relative importance of the category effects compared to the cross-item effects. We compared the performance of a model with only δs (setting all θs to zero) to a model with only θs (setting all δs to zero), which results in a comparison of a model with the core demand parameters and only the substitution matrix to another model with the core demand parameters and only the category effects. We find that the model with category effects outperforms the model with only the substitution effects (a mean absolute deviation of 2.59 compared to a mean absolute deviation of 3.42, respectively). These results reconfirm the relative importance of the category effects.

4.6. Discussion

We propose a model that mines natural variation in store assortments due to out-of-stocks. This natural variation in assortment allows us to study each item's role in the consumer demand for the category. Our model decomposes the impact of an out-of-stock item into three components—lost sales, cross-item impact, and category demand impact.

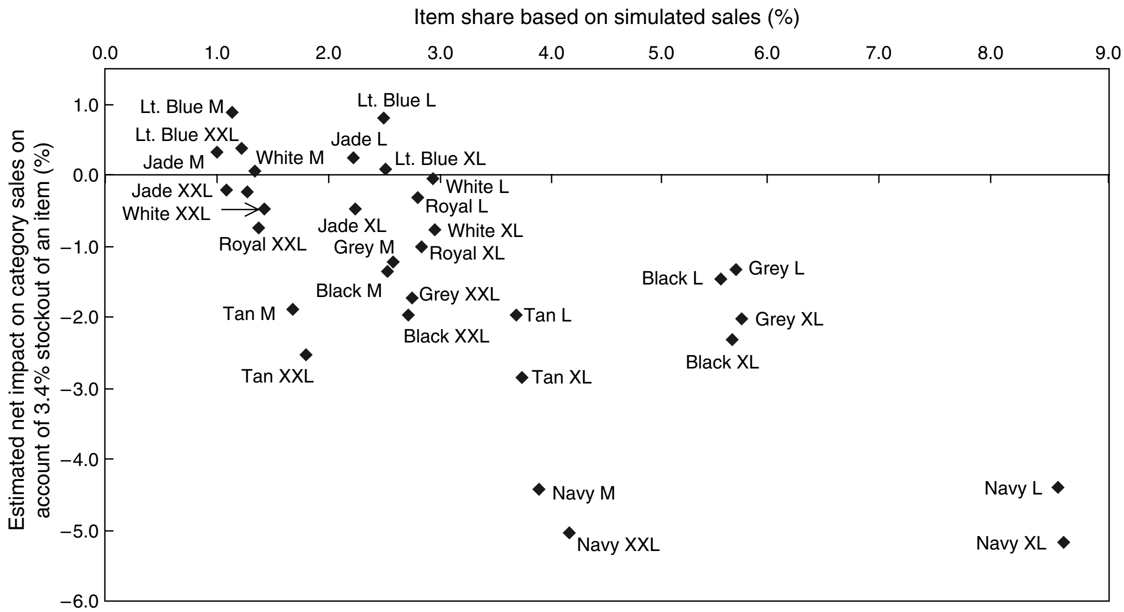
Extant empirical research on stocked-out items explores the balance of lost sales and transferred demand and the bias of using sales as a proxy for demand (Anupindi et al. 1998). Like the extant literature, we document discrepancies between observed sales and demand, discrepancies which are important to retailers. Such discrepancies are important because the equivocation of sales and demand perpetuates errors. Consider an inventory analyst who allocates inventory based on sales. She would look at last year's sales (not demand) and allocate inventory based on past sales performance. This makes the best sellers from last year even more available in the current year and the weak sellers from last year (probably due to stockouts) even less available in the current year. This action by the analyst will create a big difference in the core demand parameters and the net impact numbers for the weaker sellers because these items would be more susceptible to stocking out. We see in our scatter plot (Figure 2) a result consistent with this story. The top two or three items in terms of sales are also the top two or three in terms of being off the diagonal. Confounding due to analyst action is a possible explanation for this observed result.[21]

Extant studies have found a nontrivial relationship between individual items, that stockouts of individual items have significantly increased demand for certain remaining in-stock items (Anupindi et al. 1998). Our results confirm that the cross effects are asymmetric, e.g., that navy's effect on tan is much greater than tan's effect on navy (Table 3(a)). One reason for asymmetry would be that the cross effects are correlated to share, in that stockouts of higher-selling items would have proportionally greater impacts on sales of remaining items. On average, our results do support the theory that cross effects are correlated to share, as we show in the next subsection (where we provide a simulation exercise). However, the cross effects are not proportional to share, e.g., Navy XXL has a much larger impact than its share would suggest.

[20] The γ_{st} parameter in Equation (6) for the held-out store quarter is set to the average value of γ_{st} across the remaining seven quarters.

[21] As suggested by an anonymous reviewer, shifts in fashion trends would serve as an additional bias in using previous sales to predict future sales.

Figure 3 Item Share vs. Net Stockout Impact on Category Sales



In addition to measuring these aspects of the impact of out-of-stocks, we document that the absence of individual items adversely impacts demand for the entire category. Our analysis of the category details each and every item's individual role in the category. In the category that we examined, almost every item in the category is important to category demand, in that an out-of-stock of practically any single item would reduce category sales.[22] This suggests a somewhat different perspective from the extant empirical literature on assortment that indicates that categories can be pared with little to no loss in sales, because our results indicate that variety with a broad assortment is indeed important in apparel retailing. However, the variety appears to be simply the price of entry, in that consumers concentrate their purchases on a few items.

5. Deconstructing the Net Impact of Stockout: A Simulation Analysis

To assess the relative magnitudes of the various effects of out-of-stock items, we conducted a simulation analysis. In the simulation, we assumed that an individual item stocks out at the level of 3.4%,[23] and then used our model to estimate the own sales loss, substitution, and category sales impact. Figure 3 shows the results.

The horizontal axis in Figure 3 is the percent share of an item in the simulated category sales (if there were no stockouts at all), and the vertical axis is the net impact of a 3.4% stockout of an item on the entire category sales. Taking the example of a particular item, say navy XL, in the simulation, we assume that navy XL stocked out 3.4% (and no other item stocked out), and we use our model to estimate the demand of all items in the category under this condition. Therefore, the "v" corresponding to navy XL shows on the horizontal axis the share of navy XL of category sales (if there are no stockouts), and on the vertical axis the corresponding figure is the net impact of a 3.4% stockout of navy XL on the entire category sales. Navy XL's own sales account for around 8.6% of total category sales (horizontal axis of Figure 3), but a 3.4% stockout of navy XL (the assumption of the simulation) would cause category sales to decrease by nearly 5.2% (vertical axis of Figure 3).

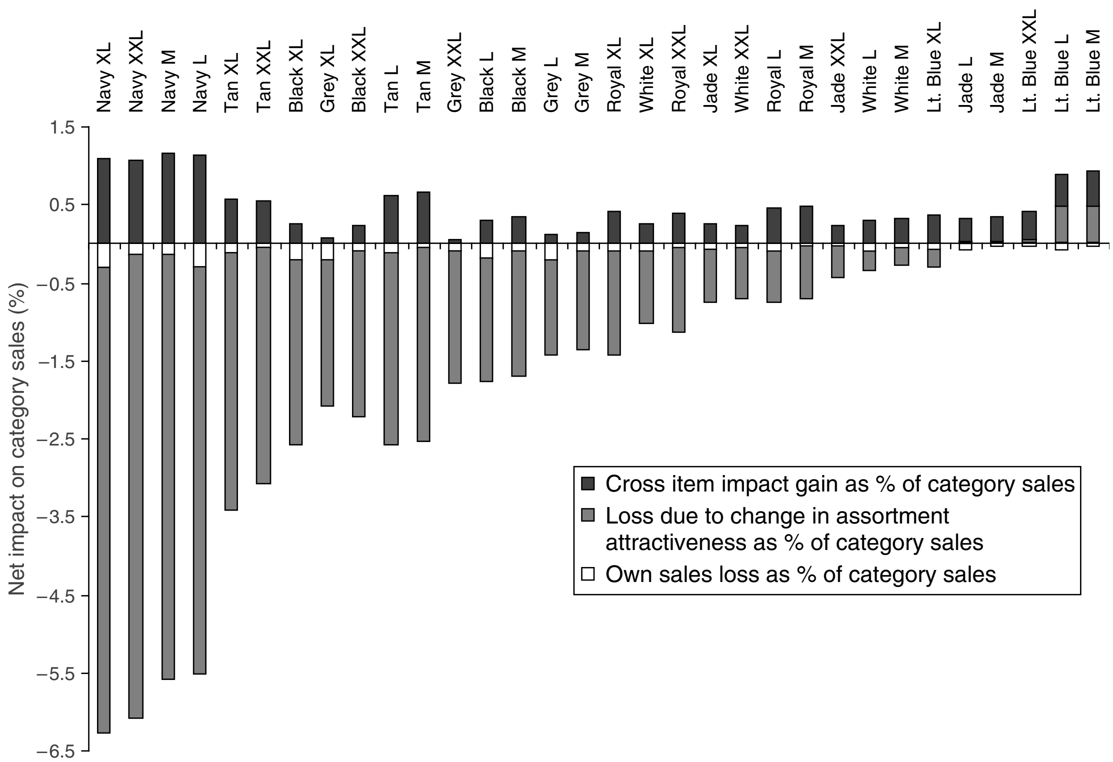
This same 5.2% is further deconstructed in the first column of Figure 4. The own sales loss as a percent of the quarter's category sales is 0.3%, the impact on category sales on account of a change in the "assortment attractiveness" (the AAI index) is a negative 5.9%, and the gain due to substitution to other items, the uppermost portion of the column, is about 1.0% of the total category sales. If this last portion of the bar graph, the substitution gain, is subtracted from the other components, the total loss in category sales ends up to be 5.2%.

Now that we have defined Figures 3 and 4 using navy XL as an example, we next consider the results

[22] Except for two items, lt. blue medium, and lt. blue large. Reasons for purchase amounts increasing in the absence of certain items could be due to the simplification of the category choices (Boatwright and Nunes 2001, Borle et al. 2005, Iyengar and Lepper 2000).

[23] That is, it stocks out during 3.4% weeks in a quarter. In other words, the variable ratio$_{scgt}$ in Equation (5), (6) is set at 0.034 for all stores and all time periods for that particular item. The number 3.4% was chosen because the average stockout percent observed in our data across all SKUs is 3.37% (Table 1(b)).

Figure 4 Deconstruction of the Net Category Impact



of all items. On average, the absence of the greatest selling items reduces category sales the most. Figure 3 shows this with a positive correlation between the SKU share of category sales and the net impact on category sales of the stockout of an individual item. Navy is one of the best sellers (especially, the sizes L and XL) and has the greatest impact on category sales. However, the correlation is far from perfect. Note that the L and XL size of Black and Grey have far less impact on category sales than Navy M and XXL in spite of their larger share of category sales.

Figure 4 deconstructs the impact of an individual item on category sales into individual components: the loss in own sales, the aggregate gain in sales due to individual cross-item effects, and the effect of assortment attractiveness. In general, the loss due to own sales is a trivial portion of the total decline in category sales, in that the middle portions of the bars in Figure 4 are all trivial relative to the total bar heights. Similarly, the cross item effects are also small. By far the largest portion of total category loss is due to the change in assortment attractiveness. As an example, the impact of a 3.4% stockout of navy XL on the assortment attractiveness contributes a 6.0% loss in category sales.[24, 25] This result reveals that almost every single item in this category is integral to the success of the category; that consumers do care about the variety within this assortment, supporting retailers' insistence on large assortments even when some individual items exhibit quite low sales. This result, that practically every item is integral to the success of the category, is the opposite of results in the extant literature on assortment reductions (Drèze et al. 1994, Broniarczyk et al. 1998, Boatwright and Nunes 2001, 2004), where even large-scale cuts in product assortments have at most a modest impact on category sales. We point out that while the extant literature has focused on grocery items, where categories tend to be frequently purchased and nondiscretionary, our study uses an infrequently purchased discretionary apparel category. Our speculation about purchase frequency is consistent with the findings in Borle et al. (2005), that more frequently purchased categories are less adversely affected by reductions in assortment.

Note also that assortment appears to gain attractiveness when certain items are out of stock. The increased attractiveness actually leads to a gain in category sales. For example, when medium lt. blue is out

[24] This, when combined with the own sales loss and the gain due to cross item impact, results in a net category loss of 5.2%.

[25] The overall category impact of a cut in assortment in our simulation exercise is large especially for some items (for example, the navy color). In addition to checking the robustness of our results by using alternative models in §4.5, we examined the empirical distribution of stockouts across various colors/sizes across various stores to see if certain outlier behavior is driving our results. No such outliers were discernable. We thank an anonymous reviewer for pointing us in this direction.

of stock, net category sales increase 0.9%. As noted earlier, this is consistent with the arguments in the literature regarding elimination of clutter.

6. Conclusions

We discussed the conventional wisdom and the theoretical foundations for the idea that the presence of an item in an assortment would impact sales of that assortment or category over and above its own sales. We empirically document this direct impact of individual items on category sales by using natural variation in the assortment due to stockouts. We find that this category effect is much larger than the cross-item effects or the loss in own sales. Our empirical study also validates the conventional wisdom that a few key items have a big impact. However, we find that the absence of each item has a significant effect on category sales. To put it another way, we confirm a schizophrenia in retailing, that retailers need a broad assortment to sell the category, but after the fact only a few items are major sellers.

Although best-selling items are typically the ones considered as "key items" in a category, our results show that best sellers are not necessarily the items with the largest impact on sales volume of the category. More importantly, we found that items impact assortment sales in more than one way. Certain items, like black and jade in our data, soak up a portion of sales from out-of-stock items, sales that might otherwise be lost. Hence, it is important that they are in stock. Other items such as navy XL (high sales share) and tan M (low sales shares) should also remain in stock for a different reason. Their absence has a deleterious effect on sales of all the remaining items. One avenue for future research would be to understand which product attributes have a higher impact on category sales and why. Another potential avenue for future research could be to assess how these impacts vary by store area demographics and competitive factors.

As one managerial implication of the results of our research, consider the calculation of levels of inventory that stores must retain to avoid out-of-stocks. Typically, the inventory allocations are calculated for each item independently of the remaining items (Fisher et al. 2001). At best, such calculations take cross-item effects into account (Smith and Agrawal 2000) with specific restrictions such as switching proportional to share. Our results show that, at least for the category studied here, the impact of an item goes beyond own sales and cross effects on other items and has large implications for the sales of the whole category.

We have made some assumptions in framing our model. Note that Equation (6) accounts for store heterogeneity only through the variation in square footage of stores, whereas store sales may vary due to many other factors. We have also evaluated a model that allowed for unobserved store-level heterogeneity, finding the results reported here to be robust with respect to store heterogeneity.

We also chose to use a highly flexible model with respect to cross-item impact. An alternative approach would be to calculate the discrepancy between sales and demand of a stocked-out item and to constrain lost sales and substitutionary sales to be equal to the discrepancy. To allow for the effect of assortments on customer behavior, we specifically sought to allow disproportionate effects.

Finally, we note the importance of the use of simultaneous equations to assess the effect of stocked-out items. At the same time that individual item stockouts reduce sales of those items, the stockouts themselves occur in periods of high sales (unexpected demand shocks). If one were to ignore Equation (7), the estimates of the thetas in an equation similar to (6) would be inflated to account for the positive correlation of category sales and stockouts. (The naïve interpretation of such a model would be that stockouts increase sales.)

Although not all retailers retain information on out-of-stocks in their data warehouses, our study shows that stockouts provide an empirical context in which to study issues concerning product assortments. The variability due to stockouts is very useful because in many apparel categories items with different attributes are priced identically, and hence lack price variability. Our study shows that even when average out-of-stocks are quite low, variation in out-of-stocks can provide the richness needed for empirical research (see Table 1(b) to see these statistics for our data).

One avenue for future research would be a utility model that addresses the contribution of individual items to the category, using aggregate data as done here. In an economic analysis, the effects of price changes lead to substitution and income effects (that are related to sales loss, intracategory effects, and category loss), which are conceptually related to those addressed in this paper. In traditional utility models, the consumer utility is maximized over a given set of product alternatives. For aggregate data with out-of-stocks, the utility model would need to account for the uncertainty in the product assortment at the time that products were purchased.

Appendix 1

We use a hierarchical Bayes approach using an MCMC sampler (Casella and George 1992, Gelfand and Smith 1990) to

Table A.1 The Priors Used in the Estimation

Parameter	Priors
ν	Gamma(2,1)
$\delta_{ic}^{\text{color}}$, $\delta_{jg}^{\text{size}}$ $i, c = 1 \ldots N_c$, $j, g = 1 \ldots N_g$	$1.0 + \text{Gamma}(1.5, 0.1)$
β_c^{color}, β_g^{size} $c = 1 \ldots N_c$, $g = 1 \ldots N_g$	Normal(0, 100)
α	Normal(0, 100)
θ_c^{color}, θ_g^{size} $c = 1 \ldots N_c$, $g = 1 \ldots N_g$	Normal(0, 100)
ω	Normal(0, 100)
$\bar{\gamma}$	Normal(0, 100)
τ^2	Inverse gamma(2.5, 2.5)

estimate the model specified by Equations (4) through (8). The full conditional distributions used in the estimation can be obtained from the authors on request; the following appendix lays out the prior parameter specifications associated with the model.

A1.1. Prior Specifications

Table A.1 provides the priors used in the estimation. ν is the decay parameter of the COM-Poisson and is defined over the positive real line, values of $\nu > 1$ indicate underdispersion, and values of $\nu < 1$ indicate overdispersion relative to the Poisson. Looking at the empirical distribution of quarterly sales one might suspect overdispersion in the data. However, a priori, there is little information on the range of values ν can take; very high values of ν, though, seem unlikely given the dispersion observed in our data. The prior distribution on ν, a gamma(2, 1) with a mode at 1, is a reasonable representation of our belief on the values ν can take.

$\delta_{ic}^{\text{color}}$, $i, c = 1, 2, \ldots, N_c$ (Equation (5)) are parameters of the $N_c \times N_c$ cross-impact matrix $\boldsymbol{\delta}^{\text{color}}$, where N_c is the number of colors observed in the data (in our case, eight colors). Similarly, $\delta_{jg}^{\text{size}}$ $j, g = 1, 2, \ldots, N_g$ (Equation (5)) are parameters of the $N_g \times N_g$ cross-impact matrix $\boldsymbol{\delta}^{\text{size}}$, where N_g is the number of sizes observed in the data (in our case, four sizes). The expression $\{\prod_{i=1}^{N_c} \prod_{j=1}^{N_g} (\delta_{ic}^{\text{color}} \delta_{jg}^{\text{size}})_{sijt}^{\text{ratio}}\}$ specifies the cross-item impact on color-size combination (c, g), on account of stockouts in any of the other items. It multiplicatively impacts the "core demand" parameter λ_{scgt}^0 (Equation (5)). We restrict the cross-item impact to be positive, i.e., the parameters $\delta_{ic}^{\text{color}}$ and $\delta_{jg}^{\text{size}}$ are restricted to be ≥ 1. Accordingly, we put a prior of $1.0 + \text{Gamma}(1.5, 0.1)$ on these parameters, which is consistent with our prior beliefs on the range of possible values that they can take.

The parameter α (Equation (6)) is the impact of the physical size of the store on demand of various items. A prior of Normal(0, 100) allows a wide range of prior values for this parameter. The θ_i^{color} and θ_j^{size} parameters (Equation (6)) are the relative weights of various colors and sizes in the AAI (Assortment Attractive Index). The magnitudes and signs of these coefficients are indicative of the importance of that particular attribute (color-size) on the demand for the entire category. A priori there is little information on either the sign or magnitudes of these parameters, and we feel a prior of Normal(0, 100) is consistent with this prior belief.

The ω parameter (Equation (7)) along with γ_{st} (Equations (6), (7)) specifies the binomial probability of number of weeks of stockout given weeks$_t$, weeks in the quarter. Again, a relatively diffuse prior of Normal(0, 100) is in line with our not-so-sharp prior beliefs on the range of values for this parameter. Similar considerations were followed in setting the normal prior distribution for $\bar{\gamma}$ and the conjugate inverse gamma prior distribution for τ^2 (Equation (8)).

Appendix 2. Alternate Model Specifications

A2.1. Model 1 (Poisson)

In this model, the demand SALES'_{scgt} follows a Poisson distribution:

$$\text{SALES}'_{scgt} \sim \text{Poisson}(\lambda'_{scgt}) \tag{9}$$

where $\text{SALES}'_{scgt} = \text{SALES}_{scgt}$ (the observed sales) when there is no stockout. When there *is* a stockout, we impute SALES'_{scgt} from the right tail of a Poisson distribution, conditioning on the observed sales SALES_{scgt}. All other aspects of the model remain unchanged from our proposed model (Equations (5) through (8) in the main text).

A2.2. Model 2 (Geometric)

In this model, the demand SALES'_{scgt} follows a geometric distribution:

$$\text{SALES}'_{scgt} \sim \text{geometric}\left(\frac{\lambda'_{scgt}}{1 + \lambda'_{scgt}}\right), \tag{10}$$

where $\text{SALES}'_{scgt} = \text{SALES}_{scgt}$ when there is no stockout. When there *is* a stockout, we impute SALES'_{scgt} from the right tail of a geometric distribution conditional on the observed sales SALES_{scgt}. All other aspects of the model remain unchanged from our proposed model (Equations (5) through (8) in the main text).

A2.3. Model 3 (COM-Poisson without Imputation)

In this model, the demand SALES'_{scgt} follows a COM-Poisson distribution:

$$\text{SALES}'_{scgt} \sim \text{ComP}(\lambda'_{scgt}, \nu). \tag{11}$$

However, instead of imputing from the tail when there is a stockout, we treat the observed sales for that SKU as a censored observation and modify our likelihood function accordingly, as follows:

We observe SALES_{scgt}, which is the sales of color-size combination (c, g) in store s in quarter t.

When there is *no* stockout,

$$\text{LIKELIHOOD}(\text{SALES}_{scgt}) = \text{pdf of ComP}(\lambda'_{scgt}, \nu). \tag{12}$$

When there *is* a stockout,

$$\begin{aligned} &\text{LIKELIHOOD}(\text{SALES}_{scgt}) \\ &\quad = 1 - [\text{cdf of ComP}(\lambda'_{scgt}, \nu), \\ &\qquad\qquad \text{evaluated at } (\text{SALES}_{scgt} - 1)]. \end{aligned} \tag{13}$$

All other aspects of the model remain unchanged from our proposed model (Equations (5) through (8) in the main text).

A2.4. Model 4 (A Baseline Model, Normal Distribution) In this model, observed sales SALES_{scgt} follow a Normal distribution:

$$\text{SALES}_{scgt} \sim \text{Normal}(\lambda'_{scgt}, \nu); \tag{14}$$

further,

$$\lambda'_{scgt} = \beta_c^{\text{color}} + \beta_g^{\text{size}} + \alpha \log \text{SQFT}_s + \phi \log(1 - \text{ratio}_{scgt}), \tag{15}$$

where SQFT_s is the store size is square feet and ratio_{scgt} is the percent of weeks that a color-size combination (c, g) was *out of stock* in store s in time period t. Thus, $(1 - \text{ratio}_{scgt})$ is a measure of in-stock position for color-size combination (c, g) in store s, quarter t. The coefficient ϕ specifies the impact of in-stock position on observed sales. A key aspect of this model is that it incorporates the stockout information directly as a covariate.

To keep the model simple so that it serves as a baseline reference to our proposed model, we do not incorporate any cross effects or category effects of a stockout in this model.

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