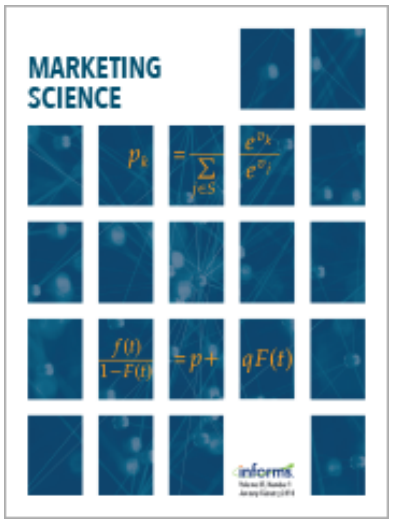


Marketing Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Dynamic Allocation of Pharmaceutical Detailing and Sampling for Long-Term Profitability

Ricardo Montoya, Oded Netzer, Kamel Jedidi,

To cite this article:
Ricardo Montoya, Oded Netzer, Kamel Jedidi, (2010) Dynamic Allocation of Pharmaceutical Detailing and Sampling for Long-Term Profitability. Marketing Science 29(5):909-924. https://doi.org/10.1287/mksc.1100.0570

MARKETING SCIENCE
Vol. 29, No. 5, September–October 2010, pp. 909–924
ISSN 0732-2399 | EISSN 1526-548X | 10 | 2905 | 0909

informs

DOI 10.1287/mksc.1100.0570

Dynamic Allocation of Pharmaceutical Detailing and Sampling for Long-Term Profitability

Ricardo Montoya
Industrial Engineering Department, University of Chile, Santiago, Chile, rmontoya@dii.uchile.cl

Oded Netzer, Kamel Jedidi
Marketing Division, Columbia Business School, Columbia University, New York, New York 10027
{on2110@columbia.edu, kj7@columbia.edu}

The U.S. pharmaceutical industry spent upwards of $18 billion on marketing drugs in 2005; detailing and drug sampling activities accounted for the bulk of this spending. To stay competitive, pharmaceutical managers need to maximize the return on these marketing investments by determining *which* physicians to target as well as *when* and *how* to target them.

In this paper, we present a two-stage approach for dynamically allocating detailing and sampling activities across physicians to maximize long-run profitability. In the first stage, we estimate a hierarchical Bayesian, nonhomogeneous hidden Markov model to assess the short- and long-term effects of pharmaceutical marketing activities. The model captures physicians' heterogeneity and dynamics in prescription behavior. In the second stage, we formulate a partially observable Markov decision process that integrates over the posterior distribution of the hidden Markov model parameters to derive a dynamic marketing resource allocation policy across physicians.

We apply the proposed approach in the context of a new drug introduction by a major pharmaceutical firm. We identify three prescription-behavior states, a high degree of physicians' dynamics, and substantial long-term effects for detailing and sampling. We find that detailing is most effective as an acquisition tool, whereas sampling is most effective as a retention tool. The optimization results suggest that the firm could increase its profits substantially while decreasing its marketing spending. Our suggested framework provides important implications for dynamically managing customers and maximizing long-run profitability.

Key words: pharmaceutical marketing; marketing resource allocation; long-term effect of marketing activities; hidden Markov model; Bayesian estimation; dynamic programming
History: Received: December 17, 2007; accepted: February 12, 2010; processed by Duncan Simester. Published online in *Articles in Advance* May 27, 2010.

1. Introduction

> The pharmaceutical industry is under significant pressure to consider its costs very carefully.... Currently, much budget is spent despite marketers being unable to identify which combination of activities has the greatest growth potential, and without knowing what specific effect individual activities are having on physicians over time. *Managing Director of Campbell Belman Europe Andrée Bates* (2006)

Marketing is essential to company growth. The U.S. pharmaceutical industry spent upwards of $18 billion on marketing drugs in 2005 (Donohue et al. 2007), representing approximately 6% of industry sales revenues. Detailing and drug sampling activities account for the bulk of this spending. To stay competitive, pharmaceutical marketing managers need to optimally allocate these resources and ensure that they achieve the highest possible return on investment for the firm.

Marketing resource allocation decisions are complex. Pharmaceutical firms need to determine *which* physicians to target as well as *when* and *how* to target them. Optimizing these decisions requires insights into (i) physicians' heterogeneity in prescription behavior and their responsiveness to marketing activities, (ii) the evolution of physicians' preferences over time, and (iii) the short- and long-term impact of marketing activities on prescription behavior. Perhaps because of these complexities, there is evidence that pharmaceutical firms do not allocate their marketing budgets optimally (Manchanda and Chintagunta 2004, Narayanan et al. 2005). This research offers a first step in providing pharmaceutical marketing managers with a state-of-the-art model and optimization procedure for dynamically targeting marketing activities to individual physicians.

Previous research suggests that physicians are heterogenous and may exhibit *dynamic* prescription behavior, particularly for a new drug (e.g., Janakiraman et al. 2008, Narayanan et al. 2005). This research stream has also shown that pharmaceutical marketing actions can have both *short- and long-term*

effects (Manchanda and Chintagunta 2004, Mizik and Jacobson 2004, Narayanan et al. 2005). Thus, accounting for physicians' heterogeneity and dynamics in prescription behavior and the enduring impact of marketing activities are critical for optimizing marketing decisions. Ignoring physicians' dynamic behavior can result in misleading inferences regarding the temporal pattern of elasticities. Similarly, a myopic firm is likely to underallocate marketing resources with primarily long-term effects.

In this paper, we present an integrative approach for dynamically targeting and allocating marketing activities to physicians. We first model the dynamics in physicians' prescription behavior while accounting for the short- and long-term effects of marketing actions and physicians' heterogeneity. We then use the estimation results to derive an optimal[1] dynamic marketing resource allocation policy. Specifically, we use a nonhomogeneous hidden Markov model (HMM) that accounts for the dynamics in prescription behavior and the enduring effect of marketing actions. We capture physicians' dynamic behavior by allowing them to transition over time among a set of latent states of prescription behavior. To model the long-term impact of marketing actions, we allow the nonhomogeneous transition matrix to be dynamically affected by these actions. Finally, integrating over the posterior distribution of the physician-level, time-varying parameter estimates, we implement a partially observable Markov decision process (POMDP) to dynamically allocate pharmaceutical marketing resources across physicians. Although implemented within a pharmaceutical context, our approach can be readily used in other domains where firms have access to longitudinal, customer-level data, such as retailing, telecommunication, and financial services firms.

We demonstrate the managerial value of the proposed approach using data from a major pharmaceutical company. In this application, we find a high degree of physicians' heterogeneity and dynamics and substantial long-term effects for detailing and sampling. Specifically, we find that detailing is most effective as an acquisition tool, whereas sampling is most effective as a retention tool. The optimization results suggest that the firm could increase its profits substantially while decreasing its marketing efforts by as much as 20%.

The rest of this paper is organized as follows. Section 2 reviews the relevant literature. Section 3 describes the pharmaceutical data we use in our empirical application. Section 4 presents the modeling approach. Section 5 reports the empirical results. Section 6 presents the optimization procedure, and §7 discusses the derived resource allocation policy. Section 8 concludes this paper and discusses limitations and future research directions.

[1] Throughout this paper, we use the term "optimal" to refer to our approximate solution to the optimization problem.

2. Literature Review

In this section, we briefly review the pharmaceutical literature and other work related to the different components of our approach: dynamics in physician prescription behavior, the long-term effect of marketing activities, and marketing resource allocation.

The pharmaceutical marketing literature shows that physicians can be dynamic in their prescription behavior. Such dynamic behavior can arise from internal factors such as state dependence (Janakiraman et al. 2008, Manchanda et al. 2004) and learning (Narayanan and Manchanda 2009, Narayanan et al. 2005), or from the long-term effect of marketing actions such as detailing and sampling (Gönül et al. 2001, Janakiraman et al. 2008, Manchanda and Chintagunta 2004, Mizik and Jacobson 2004, Narayanan et al. 2005, Narayanan and Manchanda 2009). Erdem and Sun (2001) demonstrate that dynamics in consumer behavior and the long-term effect of marketing actions need to be accounted for simultaneously to properly quantify their marginal effects. In a pharmaceutical context, Janakiraman et al. (2008) show that ignoring physicians' habit persistence may bias the estimates of the effectiveness of marketing actions.

In this paper, we account for physician dynamics through a nonhomogeneous HMM (Netzer et al. 2008), in which the states are defined by both physician behavior and external factors such as marketing activities. From a methodological point of view, our paper belongs to the small but growing number of HMM applications in marketing. HMMs have been used to study the dynamics in consumer attentions (Liechty et al. 2003), Web search behavior (Montgomery et al. 2004), competitive environment (Moon et al. 2007), customer relationships (Netzer et al. 2008), and service portfolio choice (Schweidel et al. 2010). The nonhomogeneous HMM simultaneously captures physicians' dynamics, physicians' heterogeneity, and the short- and long-term effects of marketing activities. It captures dynamics by allowing physicians to dynamically transition among a set of prescription states. The long-term effects of detailing and sampling are often captured in the literature by the exponential decay or the cumulative detailing stock (e.g., Gönül et al. 2001, Manchanda and Chintagunta 2004) approaches. In our HMM, marketing actions can have a "regime shift" effect on physicians' behavior (i.e., they affect the physician transition to a different state of behavior), thus providing a more flexible approach for capturing their long-term effect.

Despite the rich body of research investigating physicians' responses to detailing and sampling, little work has been devoted to the optimal dynamic allocation of these marketing activities. In this research, we formulate a POMDP (see Littman 2009 for a review; Aviv and Pazgal 2005, Knox 2006, and Hauser et al. 2009 for marketing applications) that uses the posterior distribution of the HMM parameters as input to dynamically allocate marketing resources across physicians and maximize long-run profitability. Several papers in the marketing literature have used such a two-step approach (i.e., estimation followed by optimization) to optimize advertising effort (e.g., Dubé et al. 2005, Hitsch 2006), catalog mailing (Simester et al. 2006), and pricing (e.g., Nair 2007, Dubé et al. 2009). Our optimization approach advances the marketing resource allocation literature (e.g., Jedidi et al. 1999, Lewis 2005, Naik et al. 2005) by accounting for the short- and long-term effects of marketing activities as well as physicians' heterogeneity and latent dynamics when allocating detailing and sampling to physicians over time.

3. Data

Our data comprise physician-level new prescriptions as well as detailing and sampling activities received over a 24-month period after the launch of a new drug used to treat a medical condition in postmenopausal women. Monthly *new* prescriptions are measured for both the new drug and the total category.[2] Detailing activity corresponds to the monthly number of face-to-face meetings in which pharmaceutical representatives present information about the drugs to physicians. Sampling activity corresponds to the monthly number of free drug samples offered to physicians by the pharmaceutical representatives.[3] Our sample consists of 300 physicians who have received at least one detail and one sample during the first 12 months of the data. These data are compiled from internal company records and pharmacy audits.

Table 1 presents descriptive statistics of the data. On average, a physician writes 22.5 new prescriptions in the category per month, 1.62 of which correspond to the new drug. Each physician receives an average of 2.18 details and 9.07 samples of the new drug per month. Furthermore, an average physician was detailed in 87% of the months, suggesting a relatively nontargeted detailing allocation by the pharmaceutical firm. Finally, there is variability in prescription behavior across physicians as well as in the number of details and samples received.

[2] Throughout this paper, "prescriptions" refer to only new prescriptions made by the physician, excluding refills.

[3] The sponsoring pharmaceutical firm did not use direct-to-consumer advertising for marketing the new drug.

Table 1 Descriptive Statistics

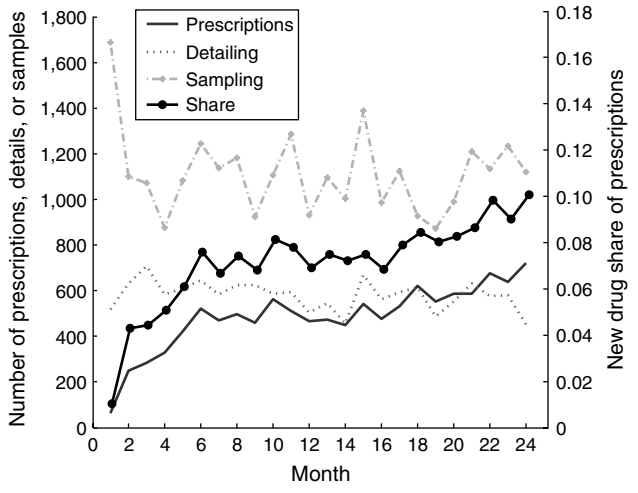
	Mean	Std. dev.	Lower 5%	Upper 95%
New drug prescriptions	1.62	1.35	0.54	3.21
Number of details	2.18	0.63	1.22	3.71
Number of samples	9.07	3.30	4.17	16.33
Months detailed (%)	0.87	0.15	0.35	1.00
Category prescriptions	22.50	13.05	10.10	37.79
New drug share	0.08	0.06	0.03	0.14

Note. Average monthly values computed for each physician across the sample of 300 physicians.

Figure 1 shows the monthly evolution of the total volume of new drug prescriptions, details, samples, and share of new prescriptions for the 24-month span of our data. The figure suggests an increasing trend in the level of new prescriptions of the new drug but relatively stable detailing and sampling activities by the firm. In addition, the share of the new drug increases from almost 0% in the first month to about 10% in the last month, closely following the increase in prescriptions of the new drug. Thus, the increase in the volume of prescriptions for the new drug cannot be attributed to category expansion. Furthermore, because the new drug reaches only 10% share by month 24, it is evident that demand for the new drug has not reached saturation by the end of our observation period.

Several questions arise from Figure 1. (i) How did the marketing actions (detailing and sampling) influence physicians' prescribing behavior? (ii) Do these marketing activities have primarily short-term or enduring effects? (iii) Could the firm have implemented a better targeting policy? We address these and other questions in the following sections.

Figure 1 Total Number of New Drug Prescriptions, Details, Samples, and Share of New Prescriptions per Month



4. The Nonhomogeneous Hidden Markov Model

In this section, we describe the use of a nonhomogeneous HMM to capture physicians' dynamics in prescription behavior and the short- and long-term effects of marketing actions.

An HMM is a Markov process with unobserved states. In our application, the hidden states represent a finite set of prescription-behavior states. For instance, assume two prescription-behavior states. At the low prescription-behavior state, physicians make only a few prescriptions for the new drug, possibly because of the need to acquire information about the drug. Consequently, physicians in this state may be responsive to information-based marketing initiatives (e.g., journal advertising). In contrast, physicians at the higher prescription-behavior state are likely to be affected by retention-type marketing initiatives (e.g., sampling).

Physicians stochastically transition among these states through a Markovian first-order process. The transitions between states are functions of marketing activities and physicians' intrinsic propensities to switch. For example, detailing may educate physicians about the drug and move them from the low prescription-behavior state to the higher one. Thus, the HMM model can capture the long-term effect of marketing activities through their impact on the transition probabilities. Marketing initiatives also affect physician behavior in the short term. We capture this effect by relating the marketing variables to the observed prescription behavior through a state-dependent component.

Let Y_{it} be the number of prescriptions of the new drug written by physician i in month t. In the HMM, the joint probability of a sequence of decisions up to time t $\{Y_{i1} = y_{i1}, \ldots, Y_{it} = y_{it}\}$ is a function of three main components: (1) the initial hidden states membership probabilities ($\boldsymbol{\pi}_i$), (2) a sequence of transition probabilities among the prescription-behavior states ($\mathbf{Q}_{it}$), and (3) a set of prescription probabilities conditioned on the prescription-behavior states ($\mathbf{M}_{it}$). We describe our formulation of each of these components next.

4.1. Initial State Membership Probabilities

Let s denote a prescription-behavior state ($s = 1, \ldots, S$). Let π_{is} be the probability that physician i is initially in state s, where $\pi_{is} \geq 0$ and $\sum_{s=1}^{S} \pi_{is} = 1$. Such a probability can depend, for example, on the physician's prior exposure to detailing and sampling activities for other drugs made by the pharmaceutical firm. That is, physicians with higher levels of exposure to marketing activities prior to the launch of the new drug are likely to be in more favorable states of prescription behavior initially. Because we do not have access to such information and because our application involves a new drug, we assume that all physicians start at state 1, which corresponds to the lowest prescription-behavior state, in the first month.[4] Thus,

$$\boldsymbol{\pi}_i' = [\pi_{i1}, \pi_{i2}, \ldots, \pi_{iS}] = [1, 0, \ldots, 0]. \tag{1}$$

4.2. The Markov Chain Transition Matrices

The transition matrix $\mathbf{Q}_{it}$ governs physician i's transitions among the states after period 1. We model $\mathbf{Q}_{it}$ as a function of detailing and sampling activities. Let $\mathbf{z}_{it} = [f(Detailing_{it}), f(Sampling_{it})]$ be the vector of marketing actions where $Detailing_{it}$ and $Sampling_{it}$ correspond to the number of details and samples that physician i receives in month t, respectively, and $f(x) = (\ln(x+1) - \mu)/\sigma$, where $\mu = \text{mean}(\ln(x+1))$ and $\sigma = \text{std}(\ln(x+1))$. We log-transform detailing and sampling to capture the potentially diminishing returns of their effectiveness (Manchanda and Chintagunta 2004). We normalize these variables to ensure a proper identification of the prescription-behavior states.

Let $X_{it} \in \{1, \ldots, S\}$ denote physician i's state membership at time t. Then each element of the transition matrix, corresponding to the probability that physician i switches from state s' to s in period t, can be written as

$$q_{is'st} = P(X_{it} = s \mid X_{it-1} = s', \mathbf{z}_{it-1}), \tag{2}$$

where $q_{is'st} \geq 0$, $\sum_{s=1}^{S} q_{is'st} = 1$. Thus, the propensity to transition from one state to another is a function of unobserved factors that can be captured by a transition random-effect coefficient and a set of marketing actions $\mathbf{z}_{it-1}$ in period $t-1$. Note that we use $\mathbf{z}_{it-1}$ to ensure temporal precedence of detailing and sampling to the physician's transition among states between period $t-1$ and period t. In contrast, one should use $\mathbf{z}_{it}$ in situations where marketing actions can transition customers in the same period (i.e., in-store promotions). Alternatively, the customer transition could also depend on the cumulative past exposure to marketing activities, such as stock variables for detailing and sampling ($\sum_{l=1}^{t} \mathbf{z}_{il}$). However, using stock variables would substantially complicate the formulation of the POMDP resource allocation problem (see §6). In our empirical analysis, we have tested these specifications and found that a model with $\mathbf{z}_{it-1}$ fits the data best.

We follow Netzer et al. (2008) in parametrizing the nonhomogeneous hidden-state transitions as an

[4] A more general specification of estimating the vector $\boldsymbol{\pi}_i$ did not provide significant improvement in fit.

ordered *logit* model. Thus, the transition probabilities in Equation (2) are given by

$$q_{is1t} = \frac{\exp(\hat{\tau}_{is1} - \boldsymbol{\rho}'_{is} \cdot \mathbf{z}_{it-1})}{1+\exp(\hat{\tau}_{is1} - \boldsymbol{\rho}'_{is} \cdot \mathbf{z}_{it-1})},$$

$$q_{is2t} = \frac{\exp(\hat{\tau}_{is2} - \boldsymbol{\rho}'_{is} \cdot \mathbf{z}_{it-1})}{1+\exp(\hat{\tau}_{is2} - \boldsymbol{\rho}'_{is} \cdot \mathbf{z}_{it-1})} - \frac{\exp(\hat{\tau}_{is1} - \boldsymbol{\rho}'_{is} \cdot \mathbf{z}_{it-1})}{1+\exp(\hat{\tau}_{is1} - \boldsymbol{\rho}'_{is} \cdot \mathbf{z}_{it-1})},$$

$$\vdots$$

$$q_{isSt} = 1 - \frac{\exp(\hat{\tau}_{isS-1} - \boldsymbol{\rho}'_{is} \cdot \mathbf{z}_{it-1})}{1+\exp(\hat{\tau}_{isS-1} - \boldsymbol{\rho}'_{is} \cdot \mathbf{z}_{it-1})}, \quad (3)$$

where $\{\hat{\tau}_{iss'}, s' = 1, \ldots, S-1\}$ is a set of ordered logit thresholds parameters specific to state s that delineates the regions of switching, and $\boldsymbol{\rho}_{is}$ is a vector of regression weights intended to capture the effect of marketing activities on the propensity of physician i to transition from state s to other states. To constrain the ordering of the thresholds, we set $\hat{\tau}_{is1} = \tau_{is1}$, $\hat{\tau}_{iss'} = \hat{\tau}_{iss'-1} + \exp(\tau_{iss'})\ \forall\, i, s,$ and $s' = 2, \ldots, S-1$, such that $\hat{\tau}_{is1} \le \hat{\tau}_{is2} \le \cdots \le \hat{\tau}_{isS-1}$.

4.3. Conditional Prescription Behavior

Conditional on being in state s in month t, we assume that the number of new prescriptions of the new drug written by physician i, Y_{it}, follows a binomial distribution with parameters W_{it} and p_{ist}; that is,

$$P_{ist}(Y_{it} = y_{it} \mid X_{it} = s, \mathbf{z}_{it}) = \binom{W_{it}}{y_{it}} p_{ist}^{y_{it}} (1-p_{ist})^{W_{it}-y_{it}}, \quad (4)$$

where W_{it} is the total number of new prescriptions in the category written by physician i in month t. Because the new drug is prescribed for a very specific disease that needs to be medically treated, category prescription is not likely to be affected by the introduction of the new drug and its associated marketing efforts. Accordingly, we treat W_{it} as exogenous.[5]

To capture the short-term impact of marketing actions, we reparametrize p_{ist}, the probability of physician i to prescribe the new drug in month t, as

$$p_{ist} = \frac{\exp(\hat{\alpha}^0_s + \boldsymbol{\alpha}'_{is}\mathbf{z}_{it})}{1+\exp(\hat{\alpha}^0_s + \boldsymbol{\alpha}'_{is}\mathbf{z}_{it})}, \quad (5)$$

where $\hat{\alpha}^0_s$ is the intrinsic probability of prescribing given state s and $\mathbf{z}_{it}$ includes the transformed *Detailing*$_{it}$ and *Sampling*$_{it}$ variables. To ensure identification of the states, we impose the restriction that the choice probabilities are nondecreasing in the behavioral states. That is, $\hat{\alpha}^0_1 \le \cdots \le \hat{\alpha}^0_S$ is imposed by setting $\hat{\alpha}^0_1 = \alpha^0_1$; $\hat{\alpha}^0_s = \hat{\alpha}^0_{s-1} + \exp(\alpha^0_s)\ \forall\, s = 2, \ldots, S$ at the mean of the vector of covariates, $\mathbf{z}_{it}$.

There are several advantages for using the binomial distribution in the current application. First, accounting for category prescriptions allows us to control for variation in patients' category demand. For example, consider a physician who experiences a sudden increase of patients in a particular month for nonmarketing reasons (e.g., practice expansion). In such a case and in contrast to the binomial model, a prescription volume model (e.g., Poisson) would attribute the change in new prescriptions to marketing activities. Second, category prescriptions help to control for seasonal or time-specific effects that may affect the market or the specific physician. Third, the binomial distribution can easily handle extreme values of share of new prescriptions observed in our data (0 and 1).

Following standard notation in HMMs (McDonald and Zucchini 1997), we write the vector of state dependent probabilities as a diagonal matrix $\mathbf{M}_{it}$.

To summarize, the nonhomogeneous HMM captures the dynamics in physician prescription behavior and allows marketing activities to have both short- and long-term effects. To capture dynamics, we allow physicians to stochastically transition among latent prescription-behavior states. The marketing actions are included in the state-dependent decision ($\mathbf{z}_{it}$ in Equation (5)) to capture their short-term effect. This means that conditional on a physician's current state, marketing interventions may have an immediate effect on prescription behavior. Additionally, the marketing actions are included in the transition probabilities ($\mathbf{z}_{it-1}$ in Equation (2)) to capture their long-term effect. This means that marketing interventions can move a physician from one state to another, possibly more favorable, state. Such a regime shift may have a long-term impact on the physician's decisions depending on the stickiness of the states.

4.4. Model Estimation

Let $(Y_{i1}, \ldots, Y_{it}, \ldots, Y_{iT_i})$ denote a sequence of T_i drug prescription observations for physician i ($i = 1, \ldots, N$). Given the HMM structure, the likelihood function for a set of N physicians can be succinctly written as

$$L = \prod_{i=1}^{N} P(Y_{i1}, \ldots, Y_{it}, \ldots, Y_{iT_i}) = \prod_{i=1}^{N} \boldsymbol{\pi}'_i \mathbf{M}_{i1} \prod_{t=2}^{T_i} \mathbf{Q}_{it}\mathbf{M}_{it}\mathbf{1}, \quad (6)$$

where $\mathbf{1}$ is an S × 1 vector of ones. To ensure that cross-individual heterogeneity is distinguished from time dynamics, we specify the HMM parameters $\boldsymbol{\theta}_i = \{\tau_{is1}, \ldots, \tau_{isS-1}, \boldsymbol{\rho}_{is}, \boldsymbol{\alpha}_{is}\}_{s=1}^{S}$ at the individual level and use a hierarchical Bayesian Markov

[5] To test if W_{it} is indeed exogenous to the marketing activities, we calculated for each physician the correlations between the number of details and samples he or she received for the new drug and his or her category prescriptions (W_{it}) across the 24 months. The average correlations between category demand (W_{it}) and detailing and sampling are 0.026 and 0.023, respectively; both correlations are statistically insignificant. This analysis suggests that in the context of our empirical application, category demand may not be affected by marketing efforts.

chain Monte Carlo (MCMC) procedure for parameter estimation (see the electronic companion, available as part of the online version that can be found at http://mktsci.pubs.informs.org, for the complete specification of the prior and the full conditional distributions). To be able to interpret and compare physicians' behavior across states, we set the intrinsic probabilities of prescribing $\Phi = \{\alpha_s^0\}_{s=1}^S$ in Equation (5) to be common across physicians.

HMMs are shown to be parametrically and nonparametrically identified under mild identification restrictions (Ephraim and Merhav 2002, Henry et al. 2009, Ryden 1994). Our model specification, however, differs from extant HMMs because it allows both the state-dependent vector and the transition matrix to be a function of covariates in a heterogeneous random-effect framework. Accordingly, we ran several simulations to demonstrate the ability of our estimation algorithm to recover the model's parameters and the number of states under different modeling and data scenarios. Using the context of our pharmaceutical study to simulate data, we find that our estimation procedure does well in recovering the model's parameters and the number of states, even for smaller sample size and number of time periods than the ones observed in our data. Specifically, we could fully recover the true number of states and all the model parameters for a sample size $N \geq 100$ and number of time periods $T \geq 8$. Further details of the simulation analyses are available upon request from the authors.

5. Empirical Results

In this section, we report the results of estimating the nonhomogeneous HMM model using the pharmaceutical data set described in §3. We use the first 20 months of the data to calibrate the model and the last four months for validation. We ran the hierarchical Bayes estimation for 300,000 iterations. The first 200,000 iterations were used as a "burn-in" period, and the last 100,000 iterations were used to estimate the conditional posterior distributions. The HMM exhibits a high degree of autocorrelation between successive MCMC draws. We therefore use the adaptive Metropolis–Hastings procedure (Atchadé 2006) to improve convergence and mixing properties. Convergence was assessed by running multiple parallel chains following Gelman and Rubin's (1992) criterion.

5.1. Model Selection

To infer the number of states that best represents our data, we estimated the HMM for varying number of states. Based on the log-marginal density (LMD), the log Bayes factor (Log BF), the deviance information criterion (DIC) value, and the validation log-likelihood for the validation periods (periods 21–24) criteria, we selected a *three*-state model (see Table 2).[6]

Table 2 Selecting the Number of States

States	LMD	DIC/2	Log BF	Validation log-likelihood
1	−10,854	11,001	—	−2,842
2	−9,037	9,207	1,817	−2,240
3	**−8,468**	**8,791**	**568**	**−2,171**
4	−8,489	8,859	−21	−2,179

Note. The best model in each column is in bold.

5.2. Predictive Validity

We compare the predictive validity of the selected three-state HMM relative to four benchmark models: two nested versions of the HMM; a latent class (LC) model; and a recency, frequency, and monetary value (RFM) model. The last two are commonly used in the literature to capture heterogeneity and dynamics in buying behavior. In all models, we assume that prescriptions follow a binomial distribution and use an MCMC approach to estimate the models' parameters.

Nested HMM. We estimate two nested versions of our full three-state HMM (full HMM-3). The first is a fixed-parameter, three-state HMM, where the parameters do not vary across physicians (fixed-parameter HMM-3). Comparing the full HMM to this model allows us to assess the magnitude of heterogeneity among physicians in the sample. The second is a three-state HMM with a stationary transition matrix. In this model the marketing activities are included only in the conditional choice component (**M**), allowing for only short-term effect of marketing actions (stationary HMM-3). Comparing the full HMM to this model allows us to assess the value of capturing the long-term effect of detailing and sampling.

LC Model. The latent class model of Kamakura and Russell (1989) captures heterogeneity in customer behavior through a set of latent segments (or states). However, unlike the HMM, the LC model cannot capture dynamics because customers cannot transition among segments. Thus, the LC model can be viewed as a special case of an HMM. We estimate this model for three segments to emphasize the differences between a model that accounts for heterogeneity only and a model that accounts for both heterogeneity and dynamics.

[6] We also tested a four-state model with an absorbing no-prescription ("defected") state. The fit criteria for this model are LMD = −8517, DIC/2 = 8840, and validation log-likelihood = −2195. Therefore, a model where physicians can move to a defected state is rejected in favor of a model with three or four states.

Table 3 Predictive Validity

Model	LMD	Validation log-likelihood	RMSE
Full HMM-3	**−8,468**	**−2,171**	**0.075**
Stationary HMM-3	−8,597	−2,177	0.077
Fixed-parameter HMM-3	−9,334	−2,232	0.089
RFM	−9,084	−2,261	0.075
Latent class	−10,495	−2,357	0.087

Note. The best model in each column is in bold.

RFM Model. One of the models most commonly used to capture dynamics and manage the firm's customer base is the RFM model (Pfeifer and Carraway 2000). We construct the RFM variables as follows. Recency corresponds to the number of months since the last new prescription. Frequency corresponds to the average incidence of the new prescription up to the current time period. Monetary value is measured by the average number of monthly new prescriptions of the new drug up to the current time period. These variables are defined at the physician level and are updated every month. Additionally, we include the transformed detailing and sampling variables to account for the effect of marketing activities.

Based on the validation log-likelihood and the RMSE criteria (see Table 3), the selected three-state HMM (full HMM-3) predicted the holdout prescription data best. Comparing the fit and predictive ability of the full HMM-3 and the stationary HMM-3, we conclude that by incorporating the effect of detailing and sampling in both the transition and conditional choice matrices, we not only capture and disentangle the short- and long-term effects of detailing and sampling but also better represent the physicians' behavior. The relatively poor predictive ability of the latent class model suggests a high degree of dynamics in the physicians' prescription behavior. Finally, the RFM model shows a relatively good predictive ability, albeit slightly worse than the full HMM-3. Indeed, models that include lagged dependent variables as covariates, such as the RFM or the Guadagni and Little (1983) models, tend to have good predictive ability. However, these models provide little insight into how the observed-state dynamics can be used to assess the enduring effects of marketing activities and to dynamically allocate these activities across physicians, which is the main objective of our research.

5.3. The HMM's Parameter Estimates

We now discuss the parameter estimates for the three-state HMM (full HMM-3). In Table 4 we report the posterior means and posterior standard deviations of the parameters, as well as the 95% heterogeneity intervals. We then use the parameter estimates to (i) interpret the three HMM states, (ii) investigate physicians' dynamics, and (iii) disentangle the short- and long-term effects of detailing and sampling.

Table 4 Posterior Means, Standard Deviations, and 95% Heterogeneity Intervals

	Parameter label	Posterior mean	Posterior std. dev.	Heterogeneity interval 2.5%	Heterogeneity interval 97.5%
Transition matrix					
Threshold parameters					
Low threshold−state 1	τ_{11}	0.36	0.12	−0.34	1.53
High threshold−state 1	τ_{12}	1.67	0.13	1.10	2.05
Low threshold−state 2	τ_{21}	−1.80	0.14	−2.79	−0.72
High threshold−state 2	τ_{22}	1.53	0.11	0.52	2.31
Low threshold−state 3	τ_{31}	−1.98	0.23	−3.45	−1.16
High threshold−state 3	τ_{32}	0.77	0.19	0.07	1.30
Long term					
Marketing effects					
Detailing−state 1	ρ_1^d	0.31	0.12	−0.36	0.88
Detailing−state 2	ρ_2^d	0.02	0.15	−0.56	0.47
Detailing−state 3	ρ_3^d	0.02	0.17	−0.64	0.44
Sampling−state 1	ρ_1^s	0.18	0.11	−0.43	0.78
Sampling−state 2	ρ_2^s	0.21	0.13	−0.26	0.82
Sampling−state 3	ρ_3^s	0.28	0.17	−0.19	0.58
Conditional choice					
State-specific effects					
Intercept−state 1	α_1^0	−4.98	0.11		
Intercept−state 2	α_2^0	0.83	0.05		
Intercept−state 3	α_3^0	0.19	0.04		
Short term					
Marketing effects					
Detailing−state 1	α_1^d	0.27	0.11	−0.72	1.05
Detailing−state 2	α_2^d	0.00	0.05	−0.61	0.51
Detailing−state 3	α_3^d	−0.04	0.09	−0.42	0.46
Sampling−state 1	α_1^s	0.09	0.11	−0.73	0.98
Sampling−state 2	α_2^s	0.06	0.05	−0.43	0.60
Sampling−state 3	α_3^s	0.02	0.08	−0.42	0.51

Note. The 95% heterogeneity interval indicates that 95% of the physicians have a posterior mean that falls within that interval.

5.3.1. Interpreting the States. To characterize the three states, we convert the intercept parameters ($\alpha_1^0, \alpha_2^0, \alpha_3^0$) in Table 4 into prescription probabilities (i.e., shares of new prescriptions) conditional on being in each state (Equation (5)) with and without detailing and sampling. The results in Table 5 suggest that, on

Table 5 State-Specific Share of New Prescription Estimates With and Without Sampling and Detailing

States		No marketing activities	Detailing only	Sampling only
Inactive	p_1	0.004	0.007	0.004
Infrequent	p_2	0.062	0.062	0.067
Frequent	p_3	0.196	0.184	0.201

Note. The three columns "No marketing activities," "Detailing only," and "Sampling only" represent the mean share of new prescriptions with no detailing or sampling, with the average number of detailing, and with the average number of sampling, respectively.

average, the share of new prescriptions of physicians in the first state is very close to zero. Accordingly, we call this the "inactive" state. In the second state, physicians present a somewhat more favorable prescription behavior towards the new drug—prescribing to the new drug 6% of their total volume of new prescriptions in the category. Thus, we call this the "infrequent" state. In the third state, physicians frequently prescribe the drug to their patients, with the new prescription share nearing 20%. Consequently, we label this state as the "frequent" state. Note that even in the frequent state, the share of new prescriptions reaches only 20%. This result suggests that, even at the frequent state, physicians do not "run out" of patients for whom to prescribe the drug. This result is typical for a new drug. To further characterize the states, we now focus on the prescription dynamics as physicians transition among the three states.

5.3.2. Physicians' Dynamics. Similar to Table 5, Table 6 converts the mean posterior transition matrix parameters to probability transition matrices, with and without the effect of marketing activities.

Examining the left-hand side matrix in Table 6, which represents the mean transition matrix with no detailing or sampling, we observe a high degree of stickiness in the inactive and infrequent states. That is, on average, physicians in these states are very likely to remain in the same state in the next period. In contrast, in the frequent state, physicians are more likely to drop to the infrequent state than they are to stay in the frequent state. Thus, consistent with Janakiraman et al. (2008), who find a high degree of physician persistence, we find that physicians are reluctant to fully adopt the new drug as they stick to the inactive and infrequent states. It should be noted that we estimate random-effect parameters for the transition matrix (Equation (3)). Thus, a separate set of transition matrices similar to the ones presented in Table 6 can be obtained for each physician.

Dynamics in State Membership. We further examined physicians' dynamics by calculating the state membership distribution across physicians over time. We use the *filtering* approach (McDonald and Zucchini 1997) to calculate the probability that physician i is in state s at period t. The filtering probability is given by

$$P(X_{it} = s \mid Y_{i1}, \ldots, Y_{it}) = \boldsymbol{\pi}_i \mathbf{M}_{i1} \prod_{\tau=2}^{t} \mathbf{Q}_{i\tau} \mathbf{M}_{i\tau}^{s} / L_{it}, \qquad (7)$$

where $\mathbf{M}_{i\tau}^{s}$ is the sth column of the matrix $\mathbf{M}_{i\tau}$, and L_{it} is the likelihood of the observed sequence of physician i's decisions up to time t, which is given by $L_{it} = \boldsymbol{\pi}_i' \mathbf{M}_{i1} \prod_{l=2}^{t} \mathbf{Q}_{il} \mathbf{M}_{il} \mathbf{1}$.

Table 6 Posterior Means of the Transition Matrix Probabilities Across Physicians

No marketing activities			Detailing only			Sampling only		
0.75	0.25	0.00	0.62	0.38	0.00	0.70	0.30	0.00
0.17	0.78	0.05	0.16	0.79	0.05	0.13	0.81	0.06
0.15	0.46	0.39	0.15	0.45	0.40	0.10	0.41	0.49

Note. The detailing and sampling matrices are calculated assuming the firm allocates the average number of details and samples to each physician.

Figure 2 Distribution of Physicians' State Membership Over Time

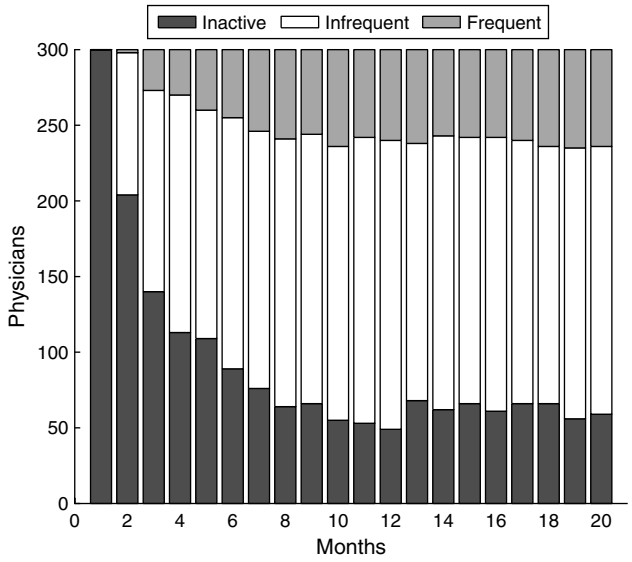


Figure 2 shows that the majority of physicians quickly moved from the inactive state to the infrequent state and to a lesser extent to the frequent state. It took approximately 10 months for the aggregate distribution of physicians' state membership to stabilize at approximately 28%, 51%, and 21% in the inactive, infrequent, and frequent states, respectively.

5.3.3. Disentangling the Short- and Long-Term Effects of Detailing and Sampling. The nonhomogeneous HMM allows us to disentangle the total effect of marketing activities into two components: immediate effects and enduring effects. The immediate impact of detailing and sampling can be assessed by the effect that these activities have on the share of new prescriptions, conditional on being in a particular state (see Equation (5) and Table 5). On the other hand, the enduring effect of detailing and sampling can be assessed by their effect on the transitions between the states (see Equations (2) and (3) and Table 6).

The results in Tables 4 and 5 show that, on average, detailing and sampling have a relatively small short-term effects. This result is consistent with the finding of Mizik and Jacobson (2004). Furthermore, the

short-term impact of detailing is strongest for physicians in the inactive state; that is, consistent with prior research (e.g., Narayanan et al. 2005), we find that detailing primarily plays a role in affecting product adoption. In contrast, when physicians are in the frequent state, we find an average small negative (although statistically insignificant) short-term effect for detailing. This result is consistent with the finding of Manchanda and Chintagunta (2004), who suggest that physicians may be conscious about the pressure being put on them by the companies' sales force and the possible physicians' backlash as a result of excessive marketing exposure.[7]

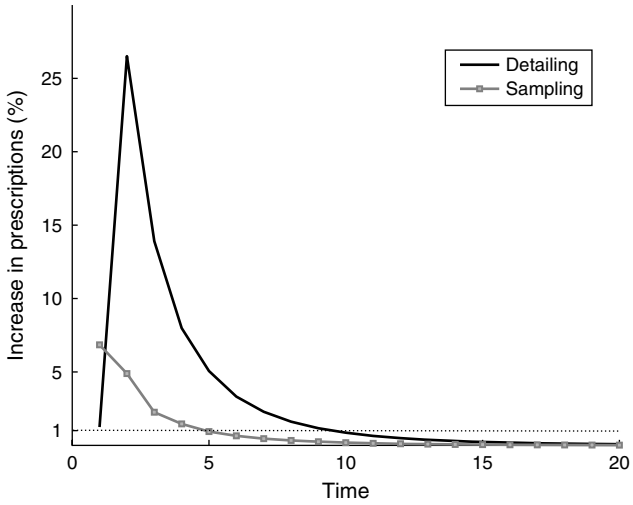
In contrast to their relatively small short-term effects, detailing and sampling have, on average, strong enduring effects. The results in Table 4 and in the center and right matrices in Table 6 show that detailing and sampling have substantial effects in switching physicians from lower states to higher ones. By comparing the transition matrices without marketing interventions (left side of Table 6), with detailing only (center of Table 6), and with sampling only (right side of Table 6), we can see that detailing has a strong effect in moving physicians away from the inactive state and sampling is most effective in keeping them in the frequent state. Thus, whereas detailing may be more useful as an acquisition tool, sampling is more useful as a retention tool. A possible explanation for this result is that when physicians are in the inactive state, they are more receptive to new information about the drug. Then, as they move to the frequent state and are familiar with the drug, physicians can primarily benefit from receiving free samples to encourage them to keep prescribing the drug. We take advantage of this behavior when optimizing the detailing and sampling allocation.

Magnitude and Duration of the Marketing Actions Effects. To assess the magnitude and duration of the impact of detailing and sampling, we use the individual-level parameter estimates from the HMM to simulate the effect of targeting one additional detail or sample to each physician on the number of prescriptions in the first month (short-term) or the following 19 months (long-term) after targeting.

Figure 3 shows the percentage increase in prescriptions over time. Relative to a no-detailing and no-sampling policy, we find that in the long run, one detail (sample) produces 0.55 (0.11) additional prescriptions. We observe that in the long run the effect of detailing is both stronger in magnitude and longer

[7] Another explanation that cannot be ruled out is that competitors may be aware of the favorable behavior of physicians in the frequent state and can consequently increase their marketing efforts to those physicians. However, there is no direct evidence of pharmaceutical companies reacting in such a way.

Figure 3 Duration of the Effect of Marketing Actions



in duration relative to the effect of sampling. In particular, the effect of detailing in the long term is about five times the effect of sampling. However, the short-term (one month) effect of sampling is 5.5 times that of detailing. The duration of the effectiveness of detailing and sampling is 10 and 5 months, respectively, after which the increase in prescriptions as a result of the marketing action is less than 1%. Moreover, only 25% (35%) of the total effect of detailing (sampling) occurs in the first two months. Thus, a model that ignores the long-term effect of detailing and sampling is likely to severely underestimate their effectiveness.

Similarly, the short-term elasticities for detailing and sampling are much smaller compared with their long-term elasticities (Table 7). Furthermore, in the short term, sampling has a stronger effect relative to detailing; in the long term, detailing has a stronger effect. These elasticities are consistent in magnitude with the elasticities reported by Manchanda and Chintagunta (2004) and Mizik and Jacobson (2004).

5.4. Endogeneity

Pharmaceutical marketing resources are often targeted based on physicians' *category* prescription volume (Manchanda and Chintagunta 2004). This targeting approach may lead to an endogenous relationship between prescription behavior and marketing efforts. We used the approach proposed by Manchanda et al. (2004) to check for the presence of endogeneity in our study. Specifically, we estimated a

Table 7 Detailing and Sampling Elasticities

Marketing action	Short term	Long term	Total
Detailing	0.002	0.652	0.654
Sampling	0.021	0.232	0.253

simultaneous system of equations where the share of new prescriptions is modeled using a one-state HMM. Detailing is modeled as a Poisson process, where the rate parameter is specified as a function of the physician-level intercept and the detailing response parameters from the share of new prescription model. The system of equations can be written as

$$Y_{it} \sim \text{Binomial}(W_{it}, p_{it}), \tag{8}$$

$$p_{it} = \exp(\beta_{0i} + \beta_{1i} \cdot d_{it})/(1 + \exp(\beta_{0i} + \beta_{1i} \cdot d_{it})), \tag{9}$$

$$d_{it} \sim \text{Poisson}(\lambda_{it}), \tag{10}$$

$$\lambda_{it} = \exp(\gamma_{0i} + \gamma_1 \cdot \beta_{0i} + \gamma_2 \cdot \beta_{1i}), \tag{11}$$

where d_{it} is the number of details received by physician i in month t. Endogeneity in detailing exists if the parameters relating the number of details to the physician's propensity to prescribe the new drug (γ_1) and responsiveness to detailing (γ_2) are significantly different from zero. Estimating the full system of Equations (8)–(11) using our data, we find $\hat{\gamma}_1 = 0.029$ and $\hat{\gamma}_2 = -0.085$. Both coefficients are not significantly different from zero. The 95% posterior confidence intervals are, respectively, $[-0.167, 0.137]$ and $[-0.690, 0.472]$. Therefore, we fail to reject the hypothesis that endogeneity is not present in our study. Full details of this endogeneity analysis are available in the electronic companion.

There are several reasons why this result runs counter to previous findings in the literature. First, our application involves a new product. Thus, endogeneity is less likely in the earlier stages of the drug's diffusion, prior to observing actual response to the marketing efforts for the drug. Second, unlike previous research, we model the share of new prescriptions rather than number of new prescriptions. The former variable is less likely to be endogenous because the share of new prescriptions "controls" for the physician's category prescription volume.

Next, we discuss how to use the parameter estimates from the nonhomogeneous HMM to dynamically allocate marketing resources across physicians.

6. The POMDP Procedure for Resource Allocation

In this section we outline the formulation of a POMDP and the optimization procedure we use to derive the resource allocation policy.

Two aspects of the HMM that make the dynamic optimization difficult are that the firm has uncertainty regarding (i) the physicians' state at any period t and (ii) how the physicians may evolve over time through the prescription-behavior states. In other words, unlike most dynamic programming (DP) problems that use observed state variables (e.g., past purchases), the state variable X_{it} in our model is only probabilistically observed. To address the state uncertainty, we formulate the dynamic optimization problem as a POMDP. A POMDP is a sequential decision problem, pertaining to a dynamic Markovian setting, where the information concerning the state of the system is incomplete. Thus, the POMDP approach is well suited for handling control problems of HMMs (see Lovejoy 1991a for a survey).

In the POMDP approach, the first step is to define the firm's beliefs about physicians' latent-state membership. We define $b_{it}(s)$ as the firm's belief that physician i is in state s at time t. After observing the physician's decision (y_{it}) and its own marketing intervention decision ($\mathbf{z}_{it}$), the firm can update its beliefs in a Bayesian manner. Specifically, using the Bayes' rule, the transition probability estimates ($q_{is's t}$) from Equation (3) and the conditional choice probabilities (P_{ist}) from Equation (5), the firm's beliefs about the physician's state can be updated from period t to $t+1$ as

$$b_{it+1}(s \mid \mathbf{B}_{it}, y_{it}, \mathbf{z}_{it-1}, \mathbf{z}_{it}) = \frac{\sum_{s'=1}^{S} b_{it}(s') q_{is's t} P_{ist}}{\sum_{s'=1}^{S} \sum_{l=1}^{S} b_{it}(s') q_{is'l t} P_{ilt}}, \tag{12}$$

where $\sum_{s=1}^{S} b_{it}(s) = 1$ and $\mathbf{B}_{it} = (b_{it}(1), \ldots, b_{it}(S))'$.

We model the pharmaceutical firm's resource allocation as a DP problem under state uncertainty. The objective of the firm is to determine, for each period, the optimal allocation of detailing and sampling so as to maximize the sum of discounted expected future profits over an infinite planning horizon. The optimal resource allocation is the solution to the following problem:

$$\max_{z_{it}} E\left\{\sum_{\tau=t}^{\infty} \delta^{\tau-t} R_{i\tau}\right\}, \tag{13}$$

where $\delta \in [0, 1]$ is the discount rate, $E[R_{it}] = \sum_{s=1}^{S} b_{it}(s) r_{ist}$, and r_{ist} is the expected profit the firm earns at period t if physician i is in state s and given marketing intervention $\mathbf{z}_{it}$.

The firm's optimal scheduling of marketing interventions is the solution to the dynamic program from that time forward, and it needs to satisfy the Bellman optimality equation (Bertsekas 2007):

$$\begin{aligned} V_i^*(\mathbf{B}_{it}) &= \max_{z_{it}} E\left\{\sum_{\tau=t}^{\infty} \delta^{\tau-t} R_{i\tau}\right\} \\ &= \max_{z_{it}} \left\{\sum_{s=1}^{S} b_{it}(s) \cdot r_{ist} \right. \\ &\quad \left. + \delta \sum_{s=1}^{S} \sum_{y_{it} \in D} b_{it}(s) P(y_{it} \mid b_{it}(s), \mathbf{z}_{it}) [V_i^*(\mathbf{B}_{it+1})]\right\}, \end{aligned} \tag{14}$$

where $V_i^*(\mathbf{B}_{it})$ denotes the maximum discounted expected profits that can be obtained for physician i given the current beliefs $\mathbf{B}_{it}$. The optimal allocation is thus

$$z_{it}^*(\mathbf{B}_{it}) = \arg\max_{z_{it}} \left\{ \sum_{s=1}^{S} b_{it}(s) \cdot r_{ist} + \delta \sum_{s=1}^{S} \sum_{y_{it} \in D} b_{it}(s) P(y_{it} \mid b_{it}(s), \mathbf{z}_{it}) [V_i^*(\mathbf{B}_{it+1})] \right\}. \quad (15)$$

The Bellman optimality equation can be rewritten as $V_i^* = \Gamma(V_i^*)$. That is, the problem reduces to find a fixed point of the mapping Γ. For the POMDP case, Γ has been shown to be a contraction mapping; thus, the problem has a unique fixed-point solution (Lovejoy 1991b).

The *exact* solution to POMDPs involves complex computations and can be found and confirmed only for problems of low computational complexity (Littman 2009). The complexity arises mainly from the use of a continuous space of beliefs to represent the uncertainty under the partial observability of the Markov decision process. Exact algorithms like the enumeration algorithm (Monahan 1982) or the witness algorithm (Kaelbling et al. 1998) are not practical for solving our pharmaceutical problem because the marketing decision variables (detailing and sampling) and the outcome variable (physician's prescriptions) can take on a large number of values. Similarly, other exact algorithms like linear support (Cheng 1988) or incremental pruning (Cassandra et al. 1997) are not possible because they lead to a large number of optimization problems to be solved (Hauskrecht 2000). Consequently, we use the approximation algorithm proposed by Lovejoy (1991b) to solve our infinite-horizon POMDP and find a closed-loop policy.

Lovejoy's method combines two approximation approaches: (i) *value iteration*, and (ii) *value function interpolation*. Value iteration (Bellman 1957) has been used extensively to solve infinite-horizon discounted DP problems in general and POMDPs in particular (Lovejoy 1991a). The value function interpolation (Hauskrecht 2000, Lovejoy 1991b) procedure has been used to approximate the *continuous* state space of beliefs using a grid of belief points and then interpolating for other points in the state space. We adopt the fixed-grid interpolation based on Freudenthal triangulation (Lovejoy 1991b). The procedure constructs a piecewise-linear function by evaluating only the vertices of the triangulation. Sondik (1978) demonstrates that the value function of a POMDP can be approximated arbitrarily closely by a convex piecewise-linear function. Lovejoy (1991b) shows that his proposed approximation algorithm is a contraction mapping, and thus, any stationary point found will be the unique fixed point, "in equilibrium" (Bertsekas 2007).[8] Thus, in what follows, we use "optimal" to refer to the approximate solution to the optimization problem.

One of the advantages of the Bayesian estimation procedure is that it provides a full posterior distribution for each individual-level parameter. These distributions reflect the uncertainty in the estimation. We incorporate this uncertainty in the optimization procedure by integrating out the parameters' distribution over the MCMC draws (Ansari and Mela 2003) when solving the individual-level POMDP. Specifically, given the estimation results, the value function in Equation (14) is calculated by

$$V_i^*(\mathbf{B}_{it}) = \int_{\mathbf{\Phi}} \int_{\theta_i} V_i^*(\mathbf{B}_{it} \mid \mathbf{\Phi}, \theta_i) f(\mathbf{\Phi}) f(\theta_i) \, d\mathbf{\Phi} d\theta_i \approx \frac{1}{K} \sum_{k=1}^{K} V_i^*(\mathbf{B}_{it} \mid \mathbf{\Phi}_k, \theta_{ik}), \quad (16)$$

where K is the number of retained MCMC draws, $\mathbf{\Phi}_k$ is the set of fixed-effect parameters, and θ_{ik} is the set of random-effect parameters from the kth draw of the MCMC.

Next, we use the POMDP procedure and the posterior distribution of the physician-level HMM parameter estimates to dynamically allocate detailing and sampling for each physician in our sample.

7. Optimization Results

We obtain an optimal *forward-looking dynamic* (FL dynamic) policy for an infinite time horizon by solving the POMDP problem described previously. We compare the performance of the proposed policy to the performances obtained by three competing policies: (i) a no-marketing policy, (ii) a myopic static policy, and (iii) a myopic dynamic policy. The no-marketing policy does not allocate detailing or sampling for the entire time horizon. Both myopic policies consider only the short-term effects of detailing and sampling and can be seen as a special case of Equation (14) when $\delta = 0$. The myopic static policy neglects physicians' dynamics in the planning horizon, whereas the myopic dynamic policy considers only the short-term effect of detailing and sampling in updating the firm's beliefs in each period.

[8] To further examine the accuracy of our approximation approach, we applied our optimization procedure to Sondik's (1978) POMDP problem, for which the optimal solution is known. In addition, we used a pharmaceutical problem that is similar to our empirical application to compare the approximate solution to an infinite-horizon POMDP to a finite-horizon solution that can be solved numerically. In both cases, our approach was able to recover the optimal policy and accurately approximate the true value function. Further details are available in the electronic companion.

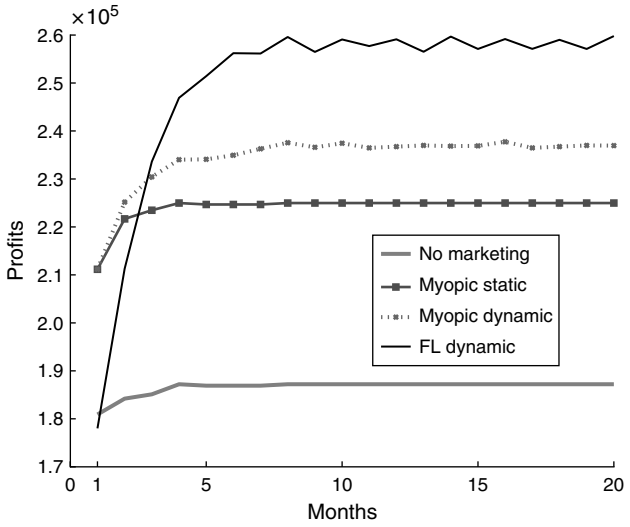
In other words, the myopic static policy identifies a unique set of physicians based on their initial state-membership probabilities and short-term responsiveness to detailing and sampling and repetitively targets them during each period of the planning horizon. The myopic dynamic policy does the same except that the set of physicians to target, and the amounts of detailing and sampling to allocate, are optimized each month given the physicians' updated state-membership probabilities.

In solving the optimization problem, we make the following assumptions:[9] the retail price of a prescription (including refills) p is \$300, the cost of one detail c_d is \$80, the cost of one sample c_s is \$30, and δ is 0.985 (i.e., 0.9 yearly). The profit r_{ist} in Equation (14) can be specified as $r_{ist} = pW_{it}p_{ist} - c_d Detailing_{it} - c_s Sampling_{it}$, where W_{it} is the total number of new prescriptions in the category written by physician i in month t, and p_{ist} is the share of prescriptions of the new drug allocated by physician i in month t (see Equation (5)). Additionally, we impose the constraint that each physician needs to be detailed in order to receive a sample. This constraint is common in the industry (Manchanda et al. 2004) and was observed in our data. To visualize the performance of each policy, we depict in Figure 4 the effect of applying each policy on physicians' behavior for the first 20 months of the infinite planning horizon used to solve the DP problem.

In Figure 4, the proposed FL dynamic policy performs worse than the myopic policies during the first couple of months as it invests in moving the physicians base to higher states. However, within three to four months the FL dynamic policy substantially outperforms the alternative policies, demonstrating the importance of accounting for dynamics in prescription behavior and the long-term effectiveness of detailing and sampling. Furthermore, the superior profit performance of the myopic dynamic policy over the static one emphasizes the importance of dynamically allocating resources to physicians, even if only short-term effects are considered. Finally, the results in Figure 4 suggest that after six to eight months, the profits from all policies stabilize.

Table 8 compares the resource allocation and profits of the alternative policies over the first 20 periods

Figure 4 Infinite Horizon Policy Performance Comparison Over 20 Months



of the infinite planning horizon. The FL dynamic policy resulted in a 33% return on investment (ROI) for detailing and sampling. This is substantially higher than the ROI obtained from the myopic static and myopic dynamic policies (20% and 25%, respectively). These differences in ROI performance can be attributed to the failure of the two myopic policies to capture the long-term effect of detailing and sampling. Accordingly, both policies allocate fewer details and samples relative to the FL dynamic policy. The 8% (33%–25%) improvement in ROI for the FL dynamic policy relative to the myopic dynamic policy stresses the importance of accounting for the long-term effect of detailing and sampling and the firm's forward-looking behavior. The 5% (25%–20%) improvement in ROI for the dynamic myopic policy relative to the myopic static policy emphasizes the importance of dynamically allocating resources as physicians' behavioral states change over time.

In summary, our results highlight the possibly substantial financial implications from simultaneously accounting for the dynamics in consumer behavior and the long-term effect of marketing actions when allocating marketing resources.

7.1. Comparison with the Current Resource Allocation Policy

We compare the FL dynamic policy to the policy currently applied by the company during the last four months in our data (months 21–24). This analysis provides several insights. First, we find that the pharmaceutical firm is currently overspending on detailing and sampling. Under the proposed FL dynamic policy, the firm should cut its overall spending by 20%. This result is directionally consistent with the finding of Mizik and Jacobson (2004) and the industry

[9] These estimates were determined based on discussions with the data provider and industry standards. The cost of one detail considers that three drugs are discussed during a 10- to 15-minute visit. Sampling costs include the cost of the drug itself and packaging, shipping, and storing costs. Based on treatment specifications for this condition, we assume that one new prescription corresponds to a treatment of three months on average. That is, an average patient needs to obtain two additional refills of the drug. The procedure presented in this section could be easily modified given an alternative cost structure.

Table 8 Comparison of the Resource Allocation Policies

Policy	Prescriptions	Details	Samples	Discounted budget ($)	Discounted profits ($)	Profits increase[a] (%)
No marketing	12,439	0	0	0	3,243,450	—
Myopic static	16,715	4,080	6,940	464,859	3,893,091	20
Myopic dynamic	17,567	4,322	7,930	507,243	4,069,117	25
FL dynamic	20,761	8,583	18,356	1,077,750	4,319,797	33

[a]The percentage increase in profits for each policy is relative to a no-marketing policy.

cut on detailing and sampling efforts since the data period. Second, despite the 20% cut in spending, the FL dynamic policy allows the firm to increase its prescriptions by 61.9%, generating an additional $412 in profits per physician per month.

Our targeting approach requires first estimating the HMM parameters for *every* physician in the potential target market and then using the POMDP procedure to optimize the allocation of detailing and sampling for each of these *same* physicians. Our optimization approach accounts for uncertainty in the parameter estimates by integrating over the posterior distribution of the parameters. If one wishes instead to estimate the HMM only for a sample of physicians and then use the resulting posterior distributions of the parameters to optimize and target the allocation of detailing and sampling to the *full* physician base, the percentage improvement of the policies in Table 8 relative to the no-marketing policy (and the improvement of the FL dynamic policy over the current policy) may be overstated (Mannor et al. 2007). As demonstrated by Mannor et al., because model parameters are estimated with error on a specific sample, value functions estimated for optimal policies using the same sample are on average positively biased.

We adapted the cross-validation approach of Mannor et al. (2007) to explore the extent to which the POMDP procedure we employ overstates the profit performance in such a context. First, we randomly divided our sample of physicians into two subsamples, a calibration sample and a validation sample, and separately estimated our HMM for each subsample. Second, because our estimation produces full posterior distribution for each physician, we followed Ansari et al. (2000) and used the population distribution of the parameters from the calibration sample to infer the posterior distribution of the parameters of each physician in the validation sample. We then used these latter estimates to derive an optimal policy for each physician in the validation sample. We used this "calibration-sample optimal policy" to calculate the value function of each physician in the validation sample. Third, we used the validation sample parameters to derive the optimal policy for each physician in the validation sample. We used the "validation-sample optimal policy" to calculate the value function of each physician in the validation sample. The difference between the value function calculated in the second step and the value function calculated in the third step provides an estimate for the bias.

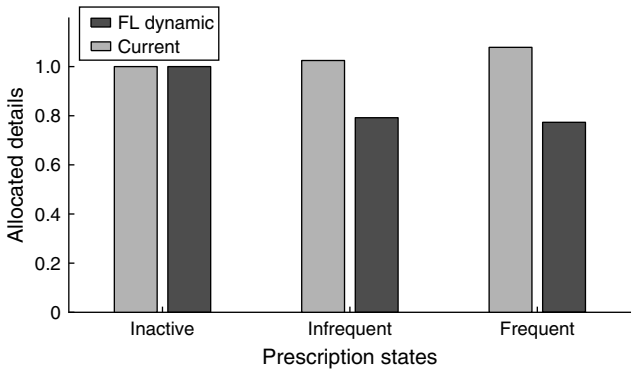
Consistent with Mannor et al. (2007), we find that the optimal FL dynamic policy and the myopic dynamic policy resulted in value functions that are biased upward by 10.2% and 8.2%, respectively (see the electronic companion for more details about this analysis). However, it is clear from Table 8 that even after correcting for the bias suggested by Mannor et al., the improvement of the FL dynamic policy over the current and myopic policies remains substantial. As noted in Mannor et al., biases that arise from functional form and distribution assumptions are not corrected by the approach proposed above.

As we show below, the superior performance of the FL dynamic policy relative to the current policy is due to better decisions on *which physicians* to target, *when* to target them, and *how many* details and samples each physician receives.

7.1.1. Who Is Being Targeted. We compare the physicians targeted by the current policy with those targeted by the FL dynamic policy in terms of their responsiveness (elasticities) to marketing actions. Specifically, we simulate the effect of allocating one detail or one sample in the first period of the planning horizon (period 21) and compute the average elasticities over the first four periods of the infinite planning horizon.[10] We find that the FL dynamic policy targets physicians with substantially higher responsiveness to detailing and sampling relative to the current policy. The average detailing elasticities for the physicians targeted by the current and FL dynamic policies were 0.1 and 0.57, respectively. For sampling, the elasticities were 0.07 and 0.22 for the current and FL dynamic policies, respectively. Furthermore, the elasticities corresponding to the current targeting policy did not differ substantially from those corresponding to a random targeting policy (calculated by randomly shuffling the identity of the targeted physicians). Specifically, we find that the current policy is targeting physicians with low (and sometimes even

[10] We cannot use a longer time horizon because there are only four holdout periods for the current policy.

Figure 5 Detailing Allocation per Prescription State



negative) detailing and sampling coefficients. This result is consistent with the findings of Manchanda and Chintagunta (2004).

7.1.2. When to Target Resources. Because of the dynamic structure of our model, the proposed dynamic policy is capable of optimizing the *timing* of the detailing and sampling allocation to each physician. Figure 5 shows the distribution of detailing, allocated per state. We can observe that whereas the current policy allocates details almost uniformly across states, the FL dynamic policy allocates more details to physicians who are in the inactive state. Once these physicians have transitioned to the higher state, the allocation of details decreases. These results suggest that detailing should be used primarily as an acquisition tool.

7.1.3. Detailing Depth vs. Breadth. We compared the physicians targeted by the FL dynamic policy with those targeted by the current policy in terms of breadth and depth of detailing over time. Overall, we find that the current policy targets almost twice as many physicians as the FL dynamic policy but with fewer details. Specifically, over the four-month period, the current and FL dynamic policies have targeted, at least once, 85% and 44% of the physicians every month, with an average of 2.20 and 3.43 details per month, respectively. Thus, it appears that the pharmaceutical company is pursuing a "shot gun" approach to targeting, indiscriminately allocating details to most physicians. In contrast, the FL dynamic policy appears to suggest a "rifle" targeting approach, detailing fewer physicians more intensively. These latter physicians have higher responsiveness to detailing and are more likely to be in the inactive state (state 1).

The emphasis of the FL dynamic policy on depth over breadth is in line with our finding that detailing is to be used primarily as an acquisition tool (see also Narayanan et al. 2005). Thus, for a successful physician acquisition, detailers need to educate potential adopters "in depth" about the uses and benefits of the new drug.

In summary, the improved profitability of the proposed dynamic policy relative to the current policy can be attributed to (i) the targeting of physicians with higher responsiveness to marketing resources, (ii) the timing of these resources depending on the physicians' behavioral states, and (iii) a better trade-off between breadth and depth of resource allocation.

8. Conclusions

This paper presents an integrative nonhomogeneous HMM model and a POMDP dynamic programming approach to dynamically target and allocate detailing and sampling across physicians. The HMM model accounts for physicians' heterogeneity and captures the dynamics in physicians' behavior and the long-term effect of marketing activities.

The application of our modeling framework in the context of a new pharmaceutical drug introduction reveals several insights. First, we find three latent prescription-behavior states that characterize physicians' dynamic prescription behavior. Second, for the particular drug studied, both detailing and sampling have long-term impact on physicians' prescription behavior. Third, detailing is particularly effective as an acquisition tool, moving physicians from the inactive state, whereas sampling is mostly effective as a retention tool, keeping physicians in a high prescription-behavior state. Fourth, sampling has a stronger short-term effect than detailing, but detailing has a stronger long-term effect. Fifth, we demonstrate that ignoring the dynamics in physician buying behavior and the long-term effects of marketing activities leads to suboptimal allocation of marketing interventions. Specifically, using a counterfactual analysis, we demonstrate that a dynamic policy can lead to a substantial increase in profitability relative to the current and myopic policies and that the firm should cut its marketing spending by 20% relative to the current policy. The optimal dynamic allocation of sampling and detailing involves first moving physicians away from the inactive state to the frequent state and then retaining these physicians in the frequent state.

We highlight several limitations and directions that future research could explore. First, in our empirical application, we find no evidence of endogeneity in the detailing and sampling of the new drug. In general, if endogeneity is present, one could integrate into our modeling approach a targeting process equation along the lines of Manchanda et al. (2004). Second, an alternative source of dynamics not considered in this research comes from the belief that physicians have foresight regarding their prescription-behavior evolution and the firm's marketing resource allocation. One could extend our modeling framework by formulating a structural model of state dependence with

forward-looking behavior (Erdem and Keane 1996). Third, our optimization procedure did not consider geographical, multiphysician practices and intertemporal constraints on the sales-force allocation. Such constraints can be added to the optimization procedure. Generally, though, because salespeople detail multiple drugs, the firm often has flexibility in the detailing allocation for any particular drug. Fourth, recent studies have suggested that social interactions among physicians can influence their prescription behavior (Nair et al. 2010). Future research could extend our modeling approach to account for such effects. Finally, although we have applied our model in a pharmaceutical setting, our approach can be readily used in other application areas where firms individually target multiple marketing activities and possess longitudinal, customer-level transaction data.

9. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mktsci.pubs.informs.org/.

Acknowledgments

The first author gratefully acknowledges partial funding by FONDEF (Project D06I1015) and the Millennium Institute on Complex Engineering Systems. The authors are also grateful for the support of the Marketing Science Institute through the Alden G. Clayton Dissertation Competition. They thank Asim Ansari, Brett Gordon, Sunil Gupta, Raghu Iyengar, Rajeev Kohli, three anonymous reviewers, the area editor, and the editor in chief for their helpful comments and suggestions. This paper will also appear in the *Festschrift to Honor John D. C. Little*.

References

Ansari, A., C. F. Mela. 2003. E-customization. *J. Marketing Res.* **40**(2) 131–145.

Ansari, A., S. Essegaier, R. Kohli. 2000. Internet recommendation systems. *J. Marketing Res.* **37**(3) 363–375.

Atchadé, Y. F. 2006. An adaptive version for the Metropolis adjusted Langevin algorithm with a truncated drift. *Methodology and Comput. Appl. Probab.* **8**(2) 235–254.

Aviv, Y., A. Pazgal. 2005. A partially observed Markov decision process for dynamic pricing. *Management Sci.* **51**(9) 1400–1416.

Bates, A. 2006. Using ROI data for effective decision making in pharmaceutical marketing. *KeywordPharma Expert Rev.* (January 26), http://www.keywordpharma.com/prods/bates.asp#top.

Bellman, R. 1957. *Dynamic Programming*. Princeton University Press, Princeton, NJ.

Bertsekas, D. 2007. *Dynamic Programming and Optimal Control*, Vols. I and II. Athena Scientific, Belmont, MA.

Cassandra, A., M. L. Littman, N. L. Zhang. 1997. Incremental pruning: A simple, fast, exact method for partially observable Markov decision processes. *Proc. 13th Annual Conf. Uncertainty Artificial Intelligence* (*UAI-97*), Morgan Kaufmann, San Francisco, 54–61.

Cheng, H.-T. 1988. Algorithms for partially observable Markov decision processes. Ph.D. thesis, University of British Columbia, Vancouver.

Donohue, J. M., M. Cevasco, M. B. Rosenthal. 2007. A decade of direct-to-consumer advertising of prescription drugs. *New England J. Med.* **357**(7) 673–681.

Dubé, J.-P., G. Hitsch, P. Manchanda. 2005. An empirical model of advertising dynamics. *Quant. Marketing Econom.* **3**(2) 107–144.

Dubé, J.-P., G. Hitsch, P. E. Rossi. 2009. Do switching costs make markets less competitive? *J. Marketing Res.* **46**(4) 435–445.

Ephraim, Y., N. Merhav. 2002. Hidden Markov processes. *IEEE Trans. Inform. Theory* **48**(6) 1518–1569.

Erdem, T., M. P. Keane. 1996. Decision-making under uncertainty: Capturing dynamic brand choice processes in turbulent consumer goods markets. *Marketing Sci.* **15**(1) 1–20.

Erdem, T., B. Sun. 2001. Testing for choice dynamics in panel data. *J. Bus. Econom. Statist.* **19**(2) 142–152.

Gelman, A., D. B. Rubin. 1992. Inference from iterative simulation using multiple sequences. *Statist. Sci.* **7**(4) 457–472.

Gönül, F. F., F. Carter, E. Petrova, K. Srinivasan. 2001. Promotion of prescription drugs and its impact on physicians' choice behavior. *J. Marketing* **65**(3) 79–90.

Guadagni, P. M., J. D. C. Little. 1983. A logit model of brand choice calibrated on scanner data. *Marketing Sci.* **2**(3) 203–238.

Hauser, J. R., G. L. Urban, G. Liberali, M. Braun. 2009. Website morphing. *Marketing Sci.* **28**(2) 202–223.

Hauskrecht, M. 2000. Value-function approximations for partially observable Markov decision processes. *J. Artificial Intelligence Res.* **13**(1) 33–94.

Henry, M., Y. Kitamura, B. Salanie. 2009. Identifying finite mixtures in econometric models. Working paper, Columbia University, New York.

Hitsch, G. J. 2006. An empirical model of optimal dynamic product launch and exit under demand uncertainty. *Marketing Sci.* **25**(1) 25–50.

Janakiraman, R., S. Dutta, C. Sismeiro, P. Stern. 2008. Physicians' persistence and its implications for their response to promotion of prescription drugs. *Management Sci.* **54**(6) 1080–1093.

Jedidi, K., C. F. Mela, S. Gupta. 1999. Managing advertising and promotion for long-run profitability. *Marketing Sci.* **18**(1) 1–22.

Kaelbling, L. P., M. L. Littman, A. R. Cassandra. 1998. Planning and acting in partially observable stochastic domains. *Artificial Intelligence* **101**(1–2) 99–134.

Kamakura, W. A., G. J. Russell. 1989. A probabilistic choice model for market segmentation and elasticity structure. *J. Marketing Res.* **26**(4) 379–390.

Knox, G. 2006. Modeling and managing customers in a multichannel setting. Ph.D. dissertation, The Wharton School, University of Pennsylvania, Philadelphia.

Lewis, M. 2005. A dynamic programming approach to customer relationship pricing. *Management Sci.* **51**(6) 986–994.

Liechty, J., M. Wedel, R. Pieters. 2003. Global and local covert visual attention: Evidence from a Bayesian hidden Markov model. *Psychometrika* **68**(4) 519–541.

Littman, M. L. 2009. A tutorial on partially observable Markov decision processes. *J. Math. Psych.* **53**(3) 119–125.

Lovejoy, W. S. 1991a. A survey of algorithmic methods for partially observed Markov decision processes. *Ann. Oper. Res.* **28**(1) 47–65.

Lovejoy, W. S. 1991b. Computationally feasible bounds for partially observed Markov decision processes. *Oper. Res.* **39**(1) 162–175.

Manchanda, P., P. K. Chintagunta. 2004. Responsiveness of physician prescription behavior to salesforce effort: An individual level analysis. *Marketing Lett.* **15**(2–3) 129–145.

Manchanda, P., P. Rossi, P. Chintagunta. 2004. Response modeling with nonrandom marketing-mix variables. *J. Marketing Res.* **41**(4) 467–478.

Mannor, S., D. Simester, P. Sun, J. N. Tsitsiklis. 2007. Bias and variance approximation in value function estimates. *Management Sci.* **53**(2) 308–322.

McDonald, I., W. Zucchini. 1997. *Hidden Markov and Other Models for Discrete-Valued Time Series*. Chapman & Hall, London.

Mizik, N., R. Jacobson. 2004. Are physicians "easy marks"? Quantifying the effects of detailing and sampling on new prescriptions. *Management Sci.* **50**(12) 1704–1715.

Monahan, G. E. 1982. A survey of partially observable Markov decision processes: Theory, models, and algorithms. *Management Sci.* **28**(1) 1–16.

Montgomery, A. L., S. Li, K. Srinivasan, J. C. Liechty. 2004. Modeling online browsing and path analysis using clickstream data. *Marketing Sci.* **23**(4) 579–595.

Moon, S., W. A. Kamakura, J. Ledolter. 2007. Estimating promotion response when competitive promotions are unobservable. *J. Marketing Res.* **44**(3) 503–515.

Naik, P. A., K. Raman, R. S. Winer. 2005. Planning marketing-mix strategies in the presence of interaction effects. *Marketing Sci.* **24**(1) 25–34.

Nair, H. S. 2007. Intertemporal price discrimination with forward-looking consumers: Application to the U.S. market for console video-games. *Quant. Marketing Econom.* **5**(3) 239–292.

Nair, H. S., P. Manchanda, T. Bhatia. 2010. Asymmetric social interactions in physician prescription behavior: The role of opinion leaders. *J. Marketing Res.* Forthcoming.

Narayanan, S., P. Manchanda. 2009. Heterogeneous learning and the targeting of marketing communications for new products. *Marketing Sci.* **28**(3) 424–441.

Narayanan, S., P. Manchanda, P. K. Chintagunta. 2005. Temporal differences in the role of marketing communication in new product categories. *J. Marketing Res.* **42**(3) 278–290.

Netzer, O., J. M. Lattin, V. Srinivasan. 2008. A hidden Markov model of customer relationship dynamics. *Marketing Sci.* **27**(2) 185–204.

Pfeifer, P. E., R. L. Carraway. 2000. Modeling customer relationships as Markov chains. *J. Interactive Marketing* **14**(2) 43–55.

Ryden, T. 1994. Consistent and asymptotically normal parameter estimates for hidden Markov models. *Ann. Statist.* **22**(4) 1884–1895.

Schweidel, D., E. Bradlow, P. Fader. 2010. Portfolio dynamics for customers of a multi-service provider. Working paper, The Wharton School, University of Pennsylvania, Philadelphia.

Simester, D. I., P. Sun, J. N. Tsitsiklis. 2006. Dynamic catalog mailing policies. *Management Sci.* **52**(5) 683–696.

Sondik, E. J. 1978. The optimal control of partially observable Markov processes over the infinite horizon: Discounted costs. *Oper. Res.* **26**(2) 282–304.