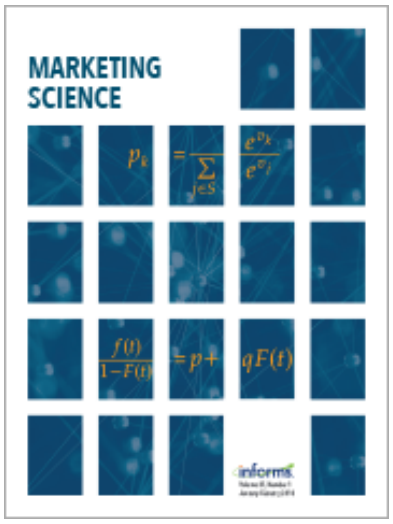

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Cheap-Talk Advertising and Misrepresentation in Vertically Differentiated Markets

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I consider a cheap-talk model in which a firm has a chance to communicate its product quality to consumers. The model describes how advertising can be both informative to consumers and profitable for the firm through its content in a vertically differentiated market. I find that advertising content may be effective in inducing search even if incentives for misrepresentation exist. In particular, a firm with an undesirable (low-quality) product is able to attract consumers who would have not incurred a search cost had they known its true quality. In this case, a semiseparating equilibrium occurs where the lowest firm types pool upward in order to increase the expected product quality while simultaneously signaling that the product is affordable. Although consumers always benefit from truth in advertising, total welfare may decrease if an undesirable firm is required to reveal its type. Finally, I show that the extent to which misrepresentation can take place increases with the cost of advertising coverage.

Key words: analytic models; advertising; targeting; marketing strategy
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1. Introduction

Advertising is paramount for most firms. In vertically differentiated contexts, firms often position themselves in the quality/price spectrum so as to create a stable image that is attractive to consumers. For example, a new automotive brand or a new supermarket may want to position itself in the "premium" category to attract high-value customers or in the lower-end market to attract customers looking for more affordable options. The goal of this positioning effort is to match products to buyers: an advertisement provides some information that is relevant to consumers before they have a chance to verify the product or service for themselves. This verification, or search, can take very different forms depending on the context of analysis: it may consist of a visit to a physical or online store in order to find more about a product or service, a tour of an open house while looking for a new home, or a test drive while looking for a new car.

Despite the informative value of advertising communication, a well-known tension arises: because the source of the information is an interested party, advertisements can be biased so as to cater to the interests of the firm. The automotive manufacturer may boast about the stability and driving comfort of a new model, the supermarket may claim to have the best prices or the best produce in town, and a real estate agent may highlight photos of the exquisite art nouveau handrails, digitally enhanced to look appropriately exquisite.

This paper investigates the conditions under which credible advertising exists in vertically differentiated markets when claims about quality can be verified by consumers through search. The result of advertising acting as a matching mechanism between products and buyers is extended to vertically differentiated markets with incentives for misrepresentation. Credibility arises because an advertisement can be attractive to a customer segment while unattractive to another one, leading the firm to make trade-offs across messages. For example, a claim of "high quality" is credible to higher-income customers (who have a high willingness to pay for quality) precisely because that statement simultaneously detracts other customers (in this case, lower-income ones). In this sense, high quality is not always "good news" for all consumers. Because the firm has an incentive to misrepresent itself, the question of what the optimal policy of advertising should be naturally arises. I find that the socially optimal advertising policy allows for a limited amount of misrepresentation by the firm.

A model of communication between a firm and consumers is analyzed in the context of a vertically differentiated market. The main advertising decision for the firm is the *content* of the message conveyed to consumers. The model is applied to a market for a "search good," where the goal of advertising is to

induce consumers to become interested in the product to the point of spending some type of effort to find out more about it, much in the spirit of search goods as defined by Nelson (1970). Hence, the search cost can be thought of as any cost that consumers may incur to gain knowledge about the product.[1]

The model is first applied to a setting where quality can be either high or low. In this case I find that advertising can play a role because the incentives of the firm and of the consumers match: the firm tells the truth about its quality, and consumers respond appropriately. A necessary condition for this outcome to hold is that the uncertainty over quality has to be high enough for the content of advertising to be credible.

The case of continuous quality is also analyzed. First, the outcome where the incentives of the firm and of consumers are aligned is replicated. Second, a new case where the firm may have an incentive to mask its true quality so as to induce consumer search is found: a semiseparating equilibrium takes place where some firm types are able to attract consumers through advertising even though the latter would be unwilling to search had they known the true product quality. This result is in contrast with the "bait-and-switch" practice (cf. Lazear 1995) where the firm lures the unsuspecting (naïve) customer with an offer, only to trade the consumer up (in terms of quality and price) after the search cost has been sunk.[2] In contrast, I find that a low-type firm lures the forward-looking (strategic) consumer by communicating that product quality and price are moderate and then trades the consumer down toward a less desirable quality level (as well as lower price).

An important assumption in the current model is that different types of consumers have access to different outside options.[3] This assumption follows from the interpretation that the marginal utility of quality (parameter "θ" in some vertical differentiation models) is proportional to the inverse of the consumers' marginal utility for income (cf. Tirole 1988, pp. 96–97). In that case, higher consumer types (in θ) hold higher incomes and may have access to different outside alternatives than those accessible to lower consumer types. This assumption implies that only the firm holding a product of high enough quality prefers to serve the high-end consumers, because it competes with a high outside option. It follows that whenever significant demand heterogeneity exists (i.e., marginal valuations for quality differ significantly in the market), low-type consumers may not always want to search if they believe that the quality of the product is high. Instead, they will anticipate that the firm selling such a product is focusing on high-type consumers and setting a high price. In this sense, high quality is not always good news for the lower type consumers.[4] This setting reveals a matching problem between the firm and the consumers. The firm makes a strategic choice with its advertising message, which may or may not be truthful. Consumers are not required to believe the claim of the firm. Instead, they update their beliefs about the quality and the price of the product and decide whether to undergo search.

The present analysis describes how advertising can be credible because of its content (i.e., its message). It differs from the classical approach considered by Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986), where the observable cost of advertising serves as a credible signal for consumers.[5] The cheap-talk literature (mainly Crawford and Sobel 1982) provides the underlying framework for costless (or cheap) communication between agents. The current work endogenizes the "sender bias" presented by Crawford and Sobel (1982) by considering that the firm may want to misreport its product quality so as to attract different consumers and increase profits. The result that communication is found to be informative as long as the bias for misrepresentation is not too high holds in the current analysis.

The advertising literature includes only a small number of references that focus on the content of advertising. Bagwell and Ramey (1993) initiate this work by proposing that in certain conditions firms may prefer to advertise truthfully if they share similar interests with consumers. In their paper, firms have preferences over consumer types because of a quantity/margin trade-off. Credibility occurs when the preferences of the firms and of the consumers match. For example, high-type consumers may prefer a firm with high quality, but low-type consumers may become discouraged because of price. In addition, high-type firms would rather attract high-type consumers who buy a few units at a high price, whereas low-type firms may prefer to attract low-type

[1] Increasingly, advertising follows (online) search through advertising search auctions. I discuss this point in §6.

[2] See also Gerstner and Hess (1990), Wilkie et al. (1998), and Hess and Gerstner (1998) for a discussion of the effects of bait-and-switch on consumer welfare.

[3] The assumption that different types of consumers may have access to different outside options has been widely used in the literature. See Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995), Villas-Boas and Schmidt-Mohr (1999), Jullien (2000), Laffont and Martimort (2002), and Stole (2007).

[4] A similar case where apparent good news can be bad for consumers in a search good setting is provided by Villas-Boas (2009), where by observing a dense product line consumers infer that the firm has done a good job in segmenting the market, and they expect a higher price. Ultimately, this deters some consumers from searching.

[5] See Kirmani and Rao (2000) for a review of other credible signals in the context of adverse selection models in vertically differentiated models.

consumers and sell more units at a lower margin. A firm undergoing misrepresentation would attract undesirable customers and would thus become worse off.[6] Bagwell and Ramey (1993) show that "costless" communication may not always suffice to match products to buyers; in this case firms have to resort to dissipative advertising when the incentives of consumers and of the firms' are not aligned: "Incentives for misrepresentation may exist [...] and, thus, costless quality claims will not always suffice to communicate quality." In contrast, this paper finds that costless quality claims can be informative despite the existence of incentives for misrepresentation. It considers the case of unattractive firm types, which may be better off from misrepresentation. The rationale is as follows. Although customers understand that there is a likelihood of the firm to be unattractive, they may be willing to search if they are not fully certain of its true quality. Misrepresentation is profitable for the firm first and foremost because it conveys that the product is affordable. Second, such a firm benefits even if consumers "regret" their decision of visiting the store. At that point, the search cost has been sunk, and consumers may prefer to purchase after all. The model predicts that unattractive firm types induce consumer search by associating with higher types, but not too high, and the information conveyed by advertising becomes sparser as quality decreases because of lower-type firms pooling upward. Finally, a very low-quality firm would not like to attract the high-end customers because it will compete with a higher outside option.

Chakraborty and Harbaugh (2010) consider the case of unverifiable advertising of products with multidimensional attributes. They show that when consumers are uncertain about both horizontal and vertical characteristics of the product, the firm can benefit from communication by emphasizing one of such dimensions (but not both). In particular, the firm partitions the message space so as to become indifferent between messages and attain credibility. Chakraborty and Harbaugh (2013) apply this result to advertising in the context of discrete choice models and also extend the analysis to comparative advertising. Unlike the applications by Chakraborty and Harbaugh, the current model focuses on a market for a "search good" in which consumers have a chance to gather information about the product before buying. Communication requires only one dimension of uncertainty, and informativeness is attained because of the trade-offs that the firm has to face due to demand heterogeneity and because of endogenous pricing.

Anderson and Renault (2006) investigate advertising strategies in horizontally differentiated settings. They find that when the firm can disclose the product fit to the consumer through advertising, forcing the firm to reveal its type may be socially harmful. I find a similar result holds in a vertically differentiated market under cheap-talk communication. Anderson and Renault (2013) extend the analysis to a vertically differentiated setting. However, advertising credibility depends on the firm not being able to misrepresent itself.

Anand and Shachar (2007) provide a model of advertising in which firms decide how much to invest in the precision of their messages, which may or may not be interpreted correctly by consumers. Mayzlin and Shin (2011) consider the advertising disclosure decisions of firms and show that they may choose uninformative advertising in the context of bandwidth-limited communication. A review of research in advertising in its role of matching products to buyers is provided by Bagwell (2007).[7]

The rest of the paper is organized as follows. Section 2 presents the model and defines the equilibrium concept. Section 3 presents and discusses the main results. Section 4 analyzes the welfare effects of allowing misrepresentation, and §5 considers the case of costly advertising coverage. Section 6 provides a discussion and concludes.

2. A Model of Advertising and Search

Consider the case of a monopolist firm endowed with a product of quality $q \sim F(\cdot)$, where $F(\cdot)$ is a cumulative distribution function with a discrete or continuous bounded support Q. The product quality is the firm's own private information. After the firm observes its product quality, it decides its price and its advertising strategy. Its message space coincides with Q, and its advertising message is observed

[6] In particular, credibility takes place when the fixed costs of both types of firms are relatively similar, and so the expected number of customers served by each firm is approximately the same. Whenever the fixed costs are very different, there exists a profitable deviation through misrepresentation: because the firms with low entry costs have little ability to attract their preferred customers because of competition, they may be better off by changing their advertising message and instead attract a higher number of customers of their least desirable type. In the current model, consumers buy only one unit of the good and costs are constant across types, so no quantity/margin trade-off occurs. Instead, the preferences of the firm over different consumer segments are driven by the fact that consumers have access to distinct outside options. In this case, a low-type quality firm may prefer to attract low-type consumers so as to compete with a lower outside option.

[7] Some additional work related to cheap talk and advertising has been considered by the marketing field. See, for example, Wernerfelt (1990) and Kuksov et al. (2013).

by consumers prior search. Production costs are normalized to 0 and assumed independent from quality.[8] Two segments of consumers exist (denoted by subscript $S \in \{L, H\}$) each of size 1, who differ in their marginal valuations for quality $\{\theta_H, \theta_L\}$, such that $\theta_H > \theta_L \geq 0$, and in their outside options $\{t_H, t_L\}$, where $t_H > t_L \geq 0$. Consumers observe the advertising strategy of the firm and decide whether to search or not by incurring a search cost $k > 0$. Their utility from buying is given by $\theta_S q - p + \varepsilon_i$, where q is the product quality, ε_i is a preference shock distributed $U(-\gamma, \gamma)$, and p is the price of the product. Consumers learn each of these elements in case they decide to search.[9] If consumers decide not to search or decide not to buy after the search cost has been incurred, they get some utility t_S from their outside option. The search cost is assumed to be relevant in the sense that it is not too low that consumers would always like to search or too high that advertising could not change their search decisions; γ is assumed to be large relative to the remaining parameters so that the pricing analysis relies on a linear demand function where the firm chooses interior prices. The assumption that ε_i is uniformly distributed with large support (i.e., γ is large) is not necessary, but it greatly simplifies the analysis.[10] The support of the distribution function $F(\cdot)$ is assumed to be bounded such that the seller is willing to cater to all customer segments in case all of them search.[11] All proofs are presented in the appendix.

Two comments about the basic setup of the model are in order. First, I assume prices are unobservable to consumers prior to search, and second, I limit the message space to the quality space Q. Both assumptions are also present in Bagwell and Ramey (1993). The first assumption is consistent with the fact that final prices are often unavailable to consumers before search. Even when the final prices are available, firms often offer extra conditions, warranties, and add-ons that are unavailable to consumers until search is realized. Other times, extra negotiations take place during the transaction phase. This is the norm for some products such as real estate and automobiles. It is possible, however, to consider more complex models where prices could also be used as signals of quality together with the advertising message. Second, limiting the communication space to the quality space Q is assumed for simplification purposes. Consider the case where price is also communicated. If the message $m = \{q, p\}$ is internally consistent (i.e., $p = \arg\max \pi_q(p)$), then the quality component is a sufficient indicator for the price (consumers will have correct beliefs over prices given the communication). On the other hand, if the message is not internally consistent (i.e., p is not profit maximizing given the quality component q), then consumers would have to construct reasonable beliefs to account for such contradictory messages. In the current model, consumers interpret message $m \in Q$ as a claim of a pair $\{q, p^*(q)\}$, expecting to see high (low)-quality claims associated with high (low) prices according to the nature of the firm's profit maximization problem.

Finally, the requirement about the amount of information that consumers learn upon searching can be easily relaxed. For example, the results do not depend on the ability of customers to separately identify the quality level from the preference shock in $\theta_S q + \varepsilon_i$. In fact, the results hold even if there is minimal learning about the composite $\theta_S q + \varepsilon_i$. The underlying intuition is that even a little learning is enough for consumers to care about which type of firm they decide to search and, consequently, for the firm to care about the type of consumers it will attract. The model results are also robust to consumers learning with some strictly positive probability. In this case search affects consumers' information on average, and the firm's communication and pricing incentives are driven by the mass of consumers who end up learning after searching.

The timing of the model is presented in Figure 1.

2.1. Purchase and Search Decisions

In this section I describe the stages of the model that are relevant for the purchase decisions of consumers, conditional on their search decisions. If the consumers of segment S decide to search, each consumer will then decide to buy if and only if $\theta_S q - p + \varepsilon_i \geq t_S$. Consider first the case where product quality q is known to customers before visiting, and price p is anticipated correctly. After searching, consumers decide whether to buy the product or not. For the ones who decide to buy, their expected surplus becomes

$$E_\varepsilon[\theta_S q - p + \varepsilon_i \mid \varepsilon_i \geq -(\theta_S q - p - t_S)] = \tfrac{1}{2}(\theta_S q - p + t_S + \gamma). \quad (1)$$

Consumers search if and only if the ex ante consumer surplus unconditional on buying is higher than their

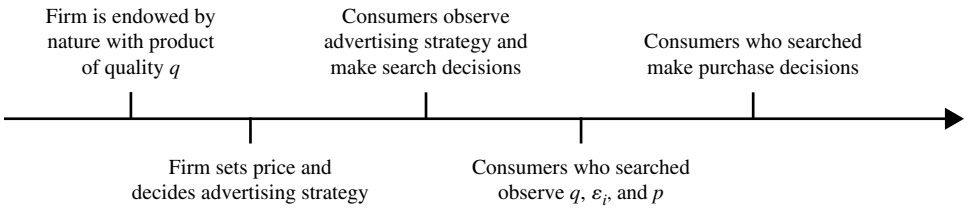
[8] This ensures that the nature of the outcomes is driven by demand incentives rather than by cost heterogeneity.

[9] Later in this section, I discuss the fact that only mild learning requirements are needed for advertising to be informative.

[10] A linear demand function at each quality level results. In addition, the assumption ensures that some consumers earn a strictly positive surplus after searching, thus avoiding the Diamond paradox (cf. Diamond 1971). Some additional assumptions are discussed in the appendix: the firm is assumed to want to cater to both consumer segments (if it can attract both) and the search cost is neither too low or too high such that advertising would be rendered useless.

[11] Expressions for the upper and lower bounds for quality are presented in the appendix.

Figure 1 Timing of the Model



outside option plus the search cost, which is equivalent to the condition

$$E_\varepsilon(u_{i\in S} \mid q) \geq k + t_S$$
$$\Leftrightarrow \quad \Pr(Buy \mid q)E(u_S \mid Buy, q) + \Pr(Not\,Buy \mid q)E(u_S \mid Not\,Buy, q) \geq k + t_S,$$

which becomes

$$CS_S(q) = \frac{(\theta_S q - p + \gamma - t_S)^2}{4\gamma} \geq k, \tag{2}$$

where the left-hand side of the inequality is the ex ante expected consumer surplus net of the outside option t_S and assumes the notation $CS_S(q)$ going forward. When consumers hold beliefs $\mathcal{B} = q \in Q_1$ about quality, the expected net consumer surplus of segment S is given by $E_q[CS_S(q) \mid \mathcal{B}]$.

2.2. Pricing Decision and Comparative Statics

Consider first the case where only one segment of consumers S visits the firm. The only candidate equilibrium price, p^*, maximizes the expected profit:

$$\pi_q(S) = p\,\frac{\theta_S q - p + \gamma - t_S}{2\gamma}, \tag{3}$$

where $\pi_q(S)$ denotes the profit of the firm from serving solely consumer segment S. In this case the firm faces a linear demand function. The equilibrium price depends on the quality level of the firm and on the consumer segment that decides to search. The only candidate equilibrium price consistent with only customer segment S visiting is given by

$$p_S^*(q) = \tfrac{1}{2}(\theta_S q + \gamma - t_S). \tag{4}$$

The following propositions follow from the consumer and firm-side behavior and are valid independent of quality being discrete or continuous.

Proposition 1 (Ex Ante Consumer Surplus Comparative Statics). *A segment's ex ante equilibrium surplus from searching conditional on quality level q is increasing in both q and θ_S, and it is decreasing in t_S.*

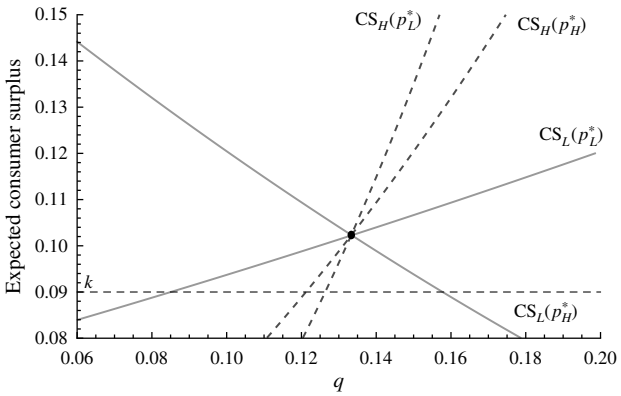
Proposition 1 makes intuitive sense: the product of the firm becomes more attractive as quality increases; in addition, as the outside option t_S increases, the search option loses value because both the probability of purchase and the expected surplus from purchase decrease. Finally, the expected surplus is not necessarily increasing with type S (because θ_S increases with S but so does t_S), which means that depending on the product quality, the low-type consumers may be interested in searching but the high types may not be, and vice versa.

Propositon 2 (Ex Ante Consumer Surplus Comparative Statics for Nonparticipating Consumers). *When the equilibrium price targets the low-type consumer segment, the ex ante surplus of the high-type consumers conditional on quality level q is increasing in q. However, when the equilibrium price targets the high-type consumers, the ex ante surplus of the low-type consumers conditional on quality level q is decreasing in q for consumers of the low segment.*

The intuition for the second part of Proposition 2 is that the firm draws a substantial amount of profit from the high-type consumers as quality increases. If the firm sets a high price, the benefits from quality increase slower than price for the low-type consumers and ultimately leads them to avoid a product with too much quality.

The helix-shaped graph in Figure 2 summarizes the findings presented until this point. The consumer surplus for the high-type consumers is always

Figure 2 Comsumer Surplus from Searching, Conditional on Quality Level



Notes. $CS_S(p)$ denotes the ex ante expected consumer surplus of customer segment S from searching, net of the outside option, when facing price p. Parameter values are $\theta_H = 8$, $\theta_L = 2$, $t_H = 1$, $t_L = 0.2$, $\gamma = 1.5$, and $k = 0.09$ and will be used henceforth.

increasing in quality, independently of price. However, the surplus for low-type consumers decreases in quality if the firm has set price level p_H^*. All consumer surplus curves intersect at quality threshold $\tilde{q} = (t_H - t_L)/(\theta_H - \theta_l)$. This occurs because $\tilde{q}$ equates the benefits of quality and opportunity costs across consumer segments. When $q < \tilde{q}$, the firm prefers to attract the low-type consumers. This happens because it would have to set a very low price (p_H^*) in order to profitably sell its product to the high-type consumers. Similarly, when quality is high ($q > \tilde{q}$), the high-type consumers are relatively more attractive to the firm.

Because of firm incentives, notice that $p_H^* < p_L^*$ whenever $q < \tilde{q}$ and $p_H^* > p_L^*$ whenever $q > \tilde{q}$. This is primarily because different consumer segments have access to different outside options.

2.3. Equilibrium Concept and Its Implications

The equilibrium concept I use is that of perfect Bayesian equilibrium. I focus on equilibria in which advertising is informative and decisive. *Informative* means that the consumers' prior beliefs are updated by the existence of communication; *decisive* means that communication induces search for at least one consumer segment that was previously unwilling to do so. An important point is that there cannot be an informative and decisive equilibrium in which both consumer segments want to visit when exposed to a given advertising message. In this case all firm types would like to use the attractive message, and credibility would break down.

An implication of the equilibrium concept is that the set of candidate equilibrium prices is reduced to the ones that maximize profits when only one segment of consumers has visited, either L or H.[12]

3. Market Outcome

This section outlines the market outcome under two possible scenarios. The first describes the case where quality can take only two values and serves as an introduction to the incentives of agents in the market. This case replicates the finding of matching products to buyers proposed by Bagwell and Ramey (1993), in which the interests of consumers and of the firm match. The second scenario considers the case where quality is continuous and uncovers the possibility of the firm gaining from misrepresentation. In both cases equilibrium can be characterized by a set of participation and incentive compatibility constraints for the firm and consumers. The following property must hold for informative and decisive equilibria.

Proposition 3 (Two Regions in Equilibrium). *Any informative and decisive equilibrium is composed of two regions $\mathcal{Q}_L$ and $\mathcal{Q}_H$, such that only the low-type consumers are willing to search if they believe that $q \in \mathcal{Q}_L$ and only the high-type consumers are willing to search if they believe that $q \in \mathcal{Q}_H$.*

Proposition 3 follows from the fact that if a region that is attractive to both consumer segments in equilibrium exists—say, $\mathcal{Q}_{LH}$—then all firm types would like to claim to be of that region and increase profits. Similarly, if an unattractive region $\mathcal{Q}_\varnothing$ existed in an outcome, the firm type in that region could do better by changing its message. Hence, the union of $\mathcal{Q}_L$ and $\mathcal{Q}_H$ spans the entire quality space Q.

An equilibrium outcome satisfies

$$E[CS_L^*(q) \mid q \in \mathcal{Q}_L] \geq k, \tag{5}$$

$$E[CS_H^*(q) \mid q \in \mathcal{Q}_H] \geq k, \tag{6}$$

$$E[CS_L^*(q) \mid q \in \mathcal{Q}_H] < k, \tag{7}$$

$$E[CS_H^*(q) \mid q \in \mathcal{Q}_L] < k, \tag{8}$$

where conditions (5) and (6) are participation constraints and conditions (7) and (8) are incentive compatibility constraints. The incentive compatibility constraints of the firm are given by

$$\pi_{q \in \mathcal{Q}_L}(\hat{\mathcal{Q}}_L) \geq \pi_{q \in \mathcal{Q}_L}(\hat{\mathcal{Q}}_H), \tag{9}$$

$$\pi_{q \in \mathcal{Q}_H}(\hat{\mathcal{Q}}_H) \geq \pi_{q \in \mathcal{Q}_H}(\hat{\mathcal{Q}}_L), \tag{10}$$

where $\pi_{q \in \mathcal{Q}_L}(\hat{\mathcal{Q}}_H)$ is the profit of a firm type whose actual quality is in $\mathcal{Q}_L$ when it is believed to be in region $\mathcal{Q}_H$. Finally, the following conditions must be satisfied:

$$\mathcal{Q}_S \neq Q, \quad S \in \{L, H\}, \tag{11}$$

$$\exists S\colon E_q[CS_S^*(q)] < k, \tag{12}$$

for informative and decisiveness to hold, respectively. Below I outline the case where quality is discrete with two possible levels ($q \in \{q_L, q_H\}$), and the case where quality is continuous ($q \in [q_L, q_H]$).

3.1. Discrete Quality Case: Two Levels

Let quality assume two values, q_L and q_H, with equal probability, and consider the quality thresholds $q_{(1)}$ to $q_{(3)}$ (explicit expressions are provided in the appendix):

$$CS_L(q_{(1)} \mid p_L^*) = k, \tag{13}$$

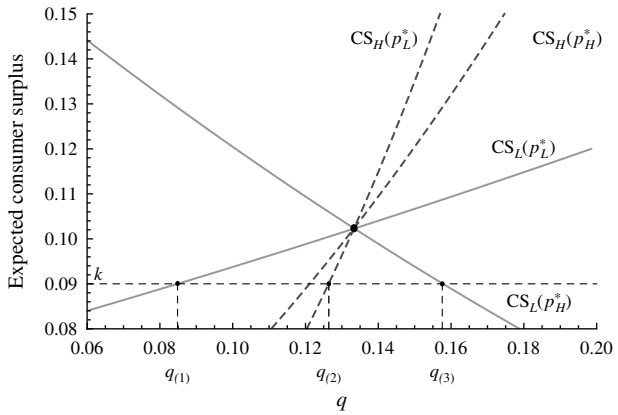
$$CS_H(q_{(2)} \mid p_L^*) = k, \tag{14}$$

$$CS_L(q_{(3)} \mid p_H^*) = k, \tag{15}$$

as depicted in Figure 3.

[12] I ignore equilibria where consumers expect an arbitrarily high price that leads them not to visit. The absence of demand would ultimately lead the firm to be indifferent among any price level, including the price that was expected by consumers in the first place. I also ignore "babbling" equilibria in which consumers decide to ignore the message of the firm, and the firm becomes indifferent as to which message to send.

Figure 3 Quality Thresholds Characterize Areas with Different Search Incentives for Consumers



The thresholds in Figure 3 denote quality levels at which consumers change their decision of searching. For example, when quality is believed to be lower than $q_{(1)}$ and the firm is believed to practice price p_L^*, no consumer is willing to search. However, the low-type consumers are willing to search if they believe that quality is higher than $q_{(1)}$ and the firm is practicing price p_L^*.

Note that no informative and decisive equilibria can exist in the two-type case if quality falls between $(q_{(2)}, q_{(3)})$. In that range the quality/price relation is attractive to both consumer segments prior search, and hence both firm types would like to claim to be in that interval. Another case where no informative and decisive equilibria occur is when $q_L < q_{(1)}$. The fact that such a firm would like to claim to be of quality q_H in order to attract customers would destroy informativeness and decisiveness.

Consider the condition for the candidate equilibrium of truthful communication, referred to as C_1:

$$C_1 \equiv \begin{cases} q_L \in (q_{(1)}, q_{(2)}), \\ q_H > q_{(3)}. \end{cases} \tag{16}$$

Under the profile above, each type of firm sets prices and advertising to target its preferred consumer segment. In this case the incentives of the firm and of consumers align, and advertising credibility takes place.

Proposition 4 (Equilibrium with Matching Interests). *Under C_1, there is an informative and decisive equilibrium where the interests of the firm and of consumers are aligned. Firm q_L sets price p_L^*, and firm q_H sets price p_H^*. Firms announce their types, and beliefs are* $\Pr(q_L \mid q_L^m) = \Pr(q_H \mid q_H^m) = 1$.

The intuition for this result is that the possible levels of quality match those sought by different consumer segments: low-type consumers are willing to visit a low-quality firm, and high-type consumers are willing to visit the high-quality firm. Advertising brings value to the market by inducing one or both consumer segments to search. The low-type consumers are now informed that the product is affordable and the high-type consumers are informed that the quality of the product is high enough. In fact, the overall existence of the market may depend on the capacity of the firm to advertise to consumers.

3.2. Continuous Quality Case

It is worth considering the case where quality is continuous—in particular, $q \sim \mathrm{U}(q_L, q_H)$.[13]

The message space is assumed to be the same as the support of quality, $Q = [q_L, q_H]$. Suppose a firm type is believed to belong to some quality space $Q_0 \subset Q$. As in Crawford and Sobel (1982), I look for equilibria where the firm randomizes its advertising message uniformly within the space $Q_0 = [a, b)$, $b > a$ and beliefs are such that consumers believe the claim. In this case, it follows that in equilibrium $\mathcal{Q}_L = [q_L, \tilde{q})$ and $\mathcal{Q}_H = [\tilde{q}, q_H]$, because firm types to the left of $\tilde{q}$ prefer to attract the low-type segment, and firm types to the right of $\tilde{q}$ prefer to attract the high-type segment.[14] Given these beliefs, it is also clear that all informative and decisive equilibria can only occur when $q_L < q_{(2)}$ and $q_H > q_{(3)}$. Otherwise, there would be regions where a firm type would effectively attract both consumer segments, and informative advertising would break down.

Focusing on the case of mixing over messages introduces some complexity to the analysis. Although it is not necessary, it ensures that all messages are sent in equilibrium and there is no need to specify beliefs off the equilibrium path. It also carries a straightforward interpretation: a firm may provide signals of a larger or smaller support so as to induce more or less uncertainty about its precise quality level. Other approaches are also possible and often preserve the informational and matching nature of the case of message mixing. One such approach would allow only one message per interval of quality cutoffs to mean that the firm belongs to that interval, and all remaining messages would mean the lowest possible quality, q_L. In this case the matching and informational results would be similar to the case of mixing over messages; however, the interpretation of the advertising technology would differ. Such a model would suggest an interpretation of

[13] The uniform distribution is particularly useful to characterize equilibria. Other distributions with continuous support may augment or shrink the parameter space of where informative and decisive equilibria exist, but they would not predict any other types of equilibria than the ones discussed here.

[14] Instead, suppose that $\mathcal{Q}_L = [q_L, q_1) \cup [q_2, \tilde{q})$, where $q_2 > q_1$. Then, firm types in the interval $[q_1, q_2)$ would have a profitable deviation by claiming they are of type $q \in \mathcal{Q}_L$.

message clustering, as only a few messages would be used in equilibrium. For example, a firm wanting to pool with a certain type could do so by simply imitating its advertising message. Yet another approach is to restrict the number of messages that the firm can use. In particular, it is possible to show that constraining the message space to two elements (low and high quality) yields the same matching and informational results as those provided by advertising mixing. The resulting advertising messages could be interpreted as a result of constraints related to language not being able to partition the quality space finely enough or consumers/firms having bandwidth-limited communication.

3.2.1. Equilibrium with Matching Interests. An equilibrium with matching interests occurs when quality is continuous under conditions C_1.

Proposition 5 (Equilibrium with Matching Interests). *Under C_1, there exists an informative and decisive equilibrium where the interests of the firm and of the consumers are aligned. Firm types with quality $q \in \mathcal{Q}_L$ set price $p_L^*(q)$, and firm types with quality $q \in \mathcal{Q}_H$ set price $p_H^*(q)$. Firm types with quality $q \in Q_0$ mix among messages $m \in Q_0$, and consumers believe $q \in Q_0$ with uniform probability when they get a message $m \in Q_0$.*

As in the case of two quality levels, an informative and decisive equilibrium with matching interests also exists when quality is continuous. The firm types below the quality threshold $\tilde{q}$ are attractive for the low-type consumers, and the firm types above it are attractive to high-type consumers. In this case advertising leads one or both consumer segments to search and increases the ex ante welfare of the firm and of consumers. Although there exist multiple possible configurations of advertising messages, the regions $\mathcal{Q}_L$ and $\mathcal{Q}_H$ are unique. For example, one can consider the case where firm types with lower-quality levels pool only with nearby types or do not pool at all. However, all such arrangements yield the same profits for the firm types and the same search decisions for the consumers, independently of the particular advertising configuration.

3.2.2. Equilibrium with Misaligned Interests. Suppose that the lowest level of quality is such that $q_L < q_{(1)}$. In this case there exist firm types that the low-type consumer segment would be better off not searching. However, these firm types may induce search by associating their advertising message with higher ones, $(q_{(1)}, \tilde{q})$.[15] In this case consumers learn two things through advertising. First, there is a chance they will "regret" visiting the store, but despite this, the firm is still willing to attract these consumers because at that point all search costs are sunk and they may be willing to buy. Second, advertising can be decisive because even though there is a chance the product is unattractive, it still informs consumers that the quality (and price) are not very high (i.e., quality is not higher than $q_{(3)}$, which would be bad news for the low-type consumers). An equilibrium with misaligned interests occurs when quality is continuous under conditions C_2:

$$C_2 \equiv \begin{cases} q_L < q_{(1)}, \\ q_H > q_{(3)}. \end{cases} \tag{17}$$

Under the conditions above, advertising leads consumers to search even if the quality of the firm is unattractive. Consumers cannot tell firm types with low quality apart through their message. However, when exposed to a lower quality claim, they understand that the product is affordable.

Proposition 6 (Equilibrium with Misaligned Interests). *Under C_2, there is an informative and decisive equilibrium where the interests of the firm and of the consumers may not be aligned. Firm types with quality $q \in \mathcal{Q}_L$ set price $p_L^*(q)$, and firm types with quality $q \in \mathcal{Q}_H$ set price $p_H^*(q)$. Firm types such that $q \in Q_0$ mix between messages $m \in Q_0$, and consumers believe $q \in Q_0$ with uniform probability when they get a message $m \in Q_0$.*

In this case advertising leads firm types of very low quality to be able to be in business: by associating their advertising message to those of moderate quality, they inform consumers that their quality and price are either low or moderate, but not too high. Notice that in this case firm types in the interval $(q_{(1)}, q_{(2)})$ have no incentive to "burn money" to signal their type, because they would have no gain from separation. Finally, the overall welfare effects of allowing misrepresentation are not clear: there is a chance low-type consumers search a product they may not like because some firm types with very low quality are now able to enter the market.

4. Welfare Analysis

This section analyzes the welfare implications of allowing a firm with very low quality to attract consumers through advertising. In particular, I compare the scenario of the firm with quality less than $q_{(1)}$ revealing its type (and being excluded from the market) with the scenario where it pools with types of higher quality and induces search. The surplus of the low-consumer segment unambiguously decreases when the firm is allowed to misrepresent its type. However, the fact that these firm types can now enter the market generates profits, which may outset the consumer loss. Let $B = q \in (q_L, q_{(1)})$ be the fact that product quality is unattractive. Table 1 describes the

[15] It suffices that they associate their message with enough types in $(q_{(1)}, \tilde{q})$ so as to increase the expected consumer surplus above the search cost.

Table 1 Ex Ante Consumer Surplus for Different Advertising Policies

	Firm is known to be B	Firm is not known to be B
Expected utility	(1) No search: t_L	(2) Search: $E_q(CS_L^* \mid B) - k$
Expected profits	(3) Leave market: 0	(4) Serve market: $E_q(\pi \mid B)$

ex ante utility of the low consumer segment when the firm is of undesirable quality. If consumers know this, they decide not to search and get the outside option t_L. However, if they do not know the firm's type, advertising will induce them to search, and they will get some utility from the firm. The second row displays the expected profits of the undesirable firm when its type is known (profits $= 0$) and when it is not known (profits > 0) to be of very low quality.

The gains in profits from misrepresentation outweigh the losses in consumer surplus if (4) is greater than (1) and (2). It is straightforward to show that $CS_L^*(q) = \frac{1}{2}\pi^*(q)$ in this case, and so the increase in profits from advertising outweigh the loss in consumer welfare when

$$E_q(CS_L^* \mid B) > \tfrac{1}{3}(t_L + k), \tag{18}$$

$$\int_{q_L}^{q_{(1)}} \frac{(\theta_L q + \gamma - t_L)^2}{16\gamma} \frac{1}{q_{(1)} - q_L}\, dq > \frac{1}{3}(t_L + k), \tag{19}$$

which yields the following result.

Proposition 7 (Net Benefit of Prohibiting Misrepresentation). *The impact of allowing an unattractive firm to induce search through advertising on total surplus is positive if q_L or k is high enough or if t_L is low enough.*

The result above reveals that advertising may have a positive effect on total surplus even when it enables an unattractive firm to induce search. This happens as long as the quality or the search costs are high enough or as long as the utility from the outside option is low enough. A policy implication for advertising regulation is that it may be optimal to allow unattractive firms to use advertising in order to induce consumer search. However, this should be allowed only to an extent: when there is a chance firms are of very low quality, it is best to implement a "truth-in-advertising" policy that forces them to reveal their true quality.

5. Costly Advertising Coverage

It is natural to consider how the investment in advertising coverage interacts with the content of the advertising message and the capacity of unattractive firm types to enter the market.[16] Denote α as the advertising intensity required to reach a proportion α of the market (equally distributed across types of consumers) at cost $c(\alpha^2/2) \geq 0$ for the firm, so that reaching more consumers is increasingly costly.[17] Suppose first that consumers exposed to the advertisement also observe the advertising intensity α directly. A firm that has no incentives for misrepresentation nor for distorting its advertising coverage is able to attract α consumers of segment S through solving the problem

$$\max_{\alpha,\, p} \pi_q(S) = p\alpha \frac{\theta_S q - p + \gamma - t_S}{2\gamma} - c\frac{\alpha^2}{2} \tag{20}$$

with solution

$$p_S^*(q) = \tfrac{1}{2}(\theta_S q + \gamma - t_S), \tag{21}$$

$$\alpha^*(q) = \frac{(\theta_S q + \gamma - t_S)^2}{8c\gamma}, \quad S = \begin{cases} L & \text{if } q < \tilde{q}, \\ H & \text{if } q > \tilde{q}. \end{cases} \tag{22}$$

Expression (22) shows that, keeping the consumer segment constant, advertising intensity is increasing on the quality level. It follows that unattractive firm types wanting to pool with higher ones must increase advertising coverage accordingly. It is easy to show that when the cost of coverage c is high, they may be prevented from doing so and misrepresentation may be eliminated altogether, in the spirit of the results by Kihlstrom and Riordan (1984) and Milgrom and Roberts (1986).

More surprisingly, when advertising coverage is not public information, the chances of misrepresentation occurring become higher. In this case the firm has no incentive to manipulate its advertising intensity to change consumer beliefs, as coverage is not directly observable by consumers. However, the fact that consumers receive an advertisement may be informative in itself. Because advertising intensity is associated with higher levels of quality, receiving an advertisement shifts consumers' beliefs about quality upward. This gives rise to the following result.

Proposition 8 (Scope for Misrepresentation Increases with Advertising Costs). *As the cost of advertising coverage increases, the lower bound for quality q_L supporting an informative and decisive equilibrium with mismatch of interests decreases.*

This follows from the fact that firm types with higher quality levels spend more on advertising coverage. Two results follow: First, a firm with very low quality will cover only a small fraction of the market. Second, because the odds of being advertised by

[16] This section focuses on the case where advertising coverage is costly and is required for awareness of the product to exist. Hence, one needs not take into account the beliefs of consumers who did not receive the advertising message. Although an interesting analysis, it would also be rather complex.

[17] The case where the firm can target consumers in different proportions (for example, through different media channels) is not analyzed here. Some related work is provided by Iyer et al. (2005) and Anand and Shachar (2009).

a firm with very low quality are lower than being advertised by one with higher quality, consumers shift their beliefs about quality upward upon being covered by advertising, and thus the scope for misrepresentation increases.

6. Concluding Remarks

This paper describes how advertising can be informative through its content in the context of a vertically differentiated market, even when incentives for misrepresentation from the firm exist. The fact that different firm types may have different preferences over consumer segments in a vertically differentiated market is central for credibility, and no money burning is required for informative communication to take place. In the present setting, a firm with a low-quality product may not want to exaggerate its claims considerably, because doing so can simultaneously scare away potential consumers (who now expect high prices) and attract mostly consumers who would be disappointed with the quality upon search and could decide not to buy. Similarly, a firm with a high-quality product may not want to understate its true quality level because that statement would attract consumers unwilling to pay enough for quality while discouraging others willing to pay more.

When incentives to misreport exist, effective advertising communication can still take place. In this case the firm with very low quality will associate its advertising message with others right "above it." This has two effects for consumers. First, they cannot tell how low the quality of the firm is. Second, they still learn that the quality is not too high, and so the product is likely to be affordable.

Allowing misrepresentation in advertising has varying effects on welfare. Although misrepresentation always hurts consumers, it may benefit the firm to the extent that the net effects may be positive. This is especially true if the lowest possible quality level is high enough when compared with the outside option of the low-type consumers or if the search cost is high enough. Surprisingly, I find that the existence of unobservable investments in advertising coverage can lead to existence of a higher degree of misrepresentation, albeit to fewer customers.

Although the current model is static, the same results apply to the case of repeated interaction. In that case it suffices to verify the incentives for search in the first period, because the option value of searching decreases over time. When deciding whether to search or not, consumers evaluate the scenario of learning about quality today and the probability of purchasing in each of the subsequent periods versus the utility of the outside option over time. The case of a two-period model is presented in the appendix.

The model is also applicable to online search advertising. For example, a consumer may search online for "fine pens" or "golf clubs." Then, an online search advertisement may provide her some information and induce further search online through a click, or further search off-line through a store visit. For example, the customer can click the ad and then buy a fine pen after inspecting its quality and fit or go through further off-line search and visit the pro shop to try out the set of golf clubs.

Whereas the analysis above considers the case of a single advertising channel, the case of competition may yield interesting results. Another avenue for future work is the interaction of the advertising message and the availability of multiple advertising channels: the firm may signal its preference over multiple consumer types through its observable allocation of advertising budget across channels. In this case media choice may interact with the role of the advertising message. For example, the use of different media may increase the advertising appeal because of increased fit, but it may also hurt credibility because consumers understand that the firm has an incentive to customize its message according to the advertising channel.

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Appendix

As discussed in §2, a product quality range is imposed such that the firm is better off serving both types of consumers rather than just serving one and that all price equilibria are interior. Let such a range be $[\underline{q}, \bar{q}]$. I equate the profits of serving the low (high) customer segment to those of serving both customers in order to find some preliminary bounds on quality, $\underline{q}'$ ($\bar{q}'$) while assuming that the optimal prices are interior at such quality levels. I then check for the conditions for interior price solutions to exist. Throughout this analysis, it is assumed that $\gamma > t_H$ such that consumers are willing to buy the product at zero cost as long as they get the highest possible utility draw. This condition is not required, but when it is assumed it greatly simplifies the relevant parameter space for q.

Quality Range

The auxiliary lower bound for product quality $\underline{q}'$ solves the expression

$$\pi_{\underline{q}'}(L) = \pi_{\underline{q}'}(L, H)$$

$$\Leftrightarrow \frac{(\theta_L \underline{q}' + \gamma - t_L)^2}{8\gamma} = \frac{((\theta_H + \theta_L)\underline{q}' - t_H - t_L + 2\gamma)^2}{16\gamma} \quad (23)$$

$$\Leftrightarrow \underline{q}' = \frac{t_H - (\sqrt{2}-1)t_L - (2-\sqrt{2})\gamma}{\theta_H - (\sqrt{2}-1)\theta_L}. \quad (24)$$

Similarly, the auxiliary upper bound for quality level $\bar{q}'$ solves

$$\pi_{\bar{q}'}(H) = \pi_{\bar{q}'}(L,H)$$

$$\Leftrightarrow \frac{(\theta_H\bar{q}'+\gamma-t_H)^2}{8\gamma} = \frac{((\theta_H+\theta_L)\bar{q}'-t_H-t_L+2\gamma)^2}{16\gamma} \quad (25)$$

$$\Leftrightarrow \bar{q}' = \frac{(\sqrt{2}-1)t_H - t_L + (2-\sqrt{2})\gamma}{(\sqrt{2}-1)\theta_H - \theta_L}, \quad (26)$$

where the profit expressions come directly from substituting the optimal price from Equation (4) into (3), and the analogue procedure is done for the case where the firm serves both consumer segments. When

$$\theta_H > (1+\sqrt{2})\theta_L, \quad (27)$$

$\bar{q}'$ is positive such that over this quality threshold, the firm prefers to price the consumers of low type out of the market.

Interior Profit Maximization

It is easy to see that prices are interior upon search by one or two consumer segments as long as γ is large.

Single-Customer Segment. By taking the first-order condition of Equation (3), in the scenario of interior solutions, and solving it with respect to price, we get that p^* is interior for each of the consumer segments if and only if

$$0 \le \frac{\theta_S q - t_S + \gamma}{4\gamma} \le 1 \quad (28)$$

for $\forall S \in \{L,H\}$. Given the nonnegativity conditions on the remaining parameters and the assumption $\gamma > t_H$, the inequality conditions above can be reduced to

$$q \le \frac{3\gamma + t_H}{\theta_H}. \quad (29)$$

Two-Customer Segments. Let $p_L^*(p_H^*)$ maximize the firm's profits when only the low (high)-type consumers visit. Then, to ensure that all outcomes are interior for when consumers deviate (i.e., both consumers visit), one needs

$$0 \le \frac{(2\theta_L - \theta_H)q + \gamma + t_H - 2t_L}{4\gamma} \le 1, \quad (30)$$

$$0 \le \frac{(2\theta_H - \theta_L)q + \gamma + t_L - 2t_H}{4\gamma} \le 1. \quad (31)$$

Notice that all the conditions are satisfied by allowing for a large enough γ.

Assumptions on Parameters. As described above, it is sufficient to assume that $\theta_H > (1+\sqrt{2})\theta_L$ so that there exists a firm type that prefers to price the low-type consumers out of the market and to assume that γ is large so that prices are interior and the firm faces a linear demand firm at each degenerate belief over quality. A range for the search cost k is discussed below.

Bounds on Search Costs. For effective communication to take place, the search cost k must be sufficiently low so that advertising can provide decisive information to consumers. The upper bound for the search cost $\bar{k}$ is defined as $CS_S(\tilde{q} \mid p_{S'}^*)$, where $S, S' \in \{L,H\}$. Hence,

$$k < \bar{k} = \frac{(\gamma(\theta_H - \theta_L) - (\theta_H t_H - \theta_L t_L))^2}{16\gamma(\theta_H - \theta_L)^2}. \quad (32)$$

To see this, notice that if this were not the case, any firm with a product quality $q < \tilde{q}$ would not be worth visiting for any consumer in equilibrium (refer to Figure 3). Thus, if $k > \bar{k}$, then all firm types would try to behave so as to be believed they were above $\tilde{q}$ (and target the same consumer segment), leading decisiveness to break down.

Finally, for misrepresentation to occur, one needs the probability of finding an undesirable product to be positive (i.e., $q_{(1)} > 0$), which is equivalent to search costs not being too low:

$$k > \underline{k} = \frac{(\gamma - t_L)^2}{16\gamma}. \quad (33)$$

Expressions for Quality Cutoffs. The expressions for quality cutoffs $q_{(1)}$, $q_{(2)}$, $q_{(3)}$ are as follows:

$$q_{(1)} = \frac{t_L - \gamma + 4\sqrt{k\gamma}}{\theta_L}, \quad (34)$$

$$q_{(2)} = \frac{2t_H - t_L - \gamma + 4\sqrt{k\gamma}}{2\theta_H - \theta_L}, \quad (35)$$

$$q_{(3)} = \frac{t_H - 2t_L + \gamma - 4\sqrt{k\gamma}}{\theta_H - 2\theta_L}. \quad (36)$$

It is possible to show that $q_{(3)} > \tilde{q} > q_{(2)} > q_{(1)}$ follows from (32) and the remaining assumptions.

Proofs

Proof of Proposition 1. Expressions for ex ante consumer surplus (2) and optimal price (4) yield the following net ex ante expected consumer surplus of searching, conditional on quality level q:

$$CS_S(q \mid p_S^*) \equiv \frac{(\theta_S q - t_S + \gamma)^2}{16\gamma} - k, \quad (37)$$

where p_S^* is defined as the profit-maximizing price for the firm with quality q facing only consumers of type S. It follows that

$$\frac{\partial CS_S(q \mid p_S^*)}{\partial q} = \frac{\theta_S(\theta_S q - t_S + \gamma)}{8\gamma} > 0 \quad (38)$$

by $\gamma > t_H$. In addition, the comparative statics on θ_S and t_S become

$$\frac{\partial CS_S(q \mid p_S^*)}{\partial \theta_S} = \frac{q(\theta_S q - t_S + \gamma)}{8\gamma} > 0, \quad (39)$$

$$\frac{\partial CS_S(q \mid p_S^*)}{\partial t_S} = -\frac{\theta_S q - t_S + \gamma}{8\gamma} < 0, \quad (40)$$

Proof of Proposition 2. The ex ante consumer surplus of the high-type customers given that they face p_L^* (where p_L^* maximizes the firm's profits when only the low-type consumers visit) is

$$CS_H(q \mid p_L^*) = \frac{[(2\theta_H - \theta_L)q + \gamma - 2t_H + t_L]^2}{16\gamma}, \quad (41)$$

and the analogous consumer surplus for the low-type consumers is

$$CS_L(q \mid p_H^*) = \frac{[(2\theta_L - \theta_H)q + \gamma - 2t_L + t_H]^2}{16\gamma}. \quad (42)$$

It is easy to verify that

$$\frac{\partial(CS_H(p_L^*) - k)}{\partial q} > 0 \quad (43)$$

and

$$\frac{\partial(CS_L(p_H^*) - k)}{\partial q} < 0 \quad (44)$$

under the assumptions above.

Proof of Proposition 3. The proof for this proposition is laid out in the text.

Proof of Proposition 4. Decisiveness implies that at least one consumer segment is unwilling to search under no information; i.e.,

$$\frac{1}{2}\frac{(\theta_L q_L - t_L + \gamma)^2}{16\gamma} + \frac{1}{2}\frac{[(2\theta_L - \theta_H)q_H + \gamma - 2t_L + t_H]^2}{16\gamma} < k \quad (45)$$

or

$$\frac{1}{2}\frac{(\theta_H q_H - t_H + \gamma)^2}{16\gamma} + \frac{1}{2}\frac{[(2\theta_H - \theta_L)q_L + \gamma - 2t_H + t_L]^2}{16\gamma} < k \quad (46)$$

and that each is willing to search after being exposed to advertising:

$$\frac{(\theta_L q_L - t_L + \gamma)^2}{16\gamma} > k \quad (47)$$

and

$$\frac{(\theta_H q_H - t_H + \gamma)^2}{16\gamma} > k. \quad (48)$$

It is easiest to show that all conditions required above are not mutually exclusive by inspection. For example, $\theta_H = 8$, $\theta_L = 2$, $t_H = 1$, $t_L = 0.2$, $\gamma = 1.5$, and $k = 0.09$ constitutes an informative and decisive equilibrium when $q_L = 0.1$ and $q_H = 0.14$.

Proof of Proposition 5. The necessary conditions are analogous to those of Proposition 4, adapted to a continuum $q \in Q$:

$$\frac{1}{16\gamma(q_H - q_L)}\left(\int_{q_L}^{\tilde{q}} (\theta_L q - t_L + \gamma)^2\, dq + \int_{\tilde{q}}^{q_H} [(2\theta_L - \theta_H)q + \gamma - 2t_L + t_H]^2\, dq\right) < k, \quad (49)$$

or

$$\frac{1}{16\gamma(q_H - q_L)}\left(\int_{\tilde{q}}^{q_H} (\theta_H q - t_H + \gamma)^2\, dq + \int_{q_L}^{\tilde{q}} [(2\theta_H - \theta_L)q + \gamma - 2t_H + t_L]^2\, dq\right) < k, \quad (50)$$

and

$$\frac{1}{16\gamma(\tilde{q} - q_L)}\int_{q_L}^{\tilde{q}} (\theta_L q - t_L + \gamma)^2\, dq > k, \quad (51)$$

and

$$\frac{1}{16\gamma(q_H - \tilde{q})}\int_{\tilde{q}}^{q_H} (\theta_H q - t_H + \gamma)^2\, dq > k, \quad (52)$$

where $q_L \in (q_{(1)}, q_{(2)})$ and $q_H > q_{(3)}$. In particular, the values used in Proposition 4 also provide an informative and decisive equilibrium.

Proof of Proposition 6. The same conditions as those required by Proposition 5 need to be satisfied, but here, $q_L < q_{(1)}$ and $q_H > q_{(3)}$. For example, the existence of an informative and decisive equilibrium can be verified by setting $q_L = 0.08$.

Proof of Proposition 7. The net impact of allowing misrepresentation is given by

$$Surplus = \int_{q_L}^{q_{(1)}} \frac{(\theta_L q + \gamma - t_L)^2}{16\gamma}\frac{1}{q_{(1)} - q_L}\, dq - \frac{1}{3}(t_L + k). \quad (53)$$

Replacing $q_{(1)}$ and taking derivatives with respect to (w.r.t.) q_L, k, and t_L, one gets

$$\frac{\partial\, Surplus}{\partial q_L} = \frac{\theta_L(\theta_L q_L + \gamma - t_L + 2\sqrt{k\gamma})}{24\gamma} > 0, \quad (54)$$

$$\frac{\partial\, Surplus}{\partial k} = \frac{\theta_L q_L + \gamma - t_L}{24\sqrt{k\gamma}} > 0, \quad (55)$$

$$\frac{\partial\, Surplus}{\partial t_L} = -\frac{\theta_L q_L + 9\gamma - t_L + 2\sqrt{k\gamma}}{24\gamma} < 0. \quad (56)$$

It remains to show that the total welfare change can attain positive and negative values within the parameter range. For example, the values used above with $q_L = 0.08$ provide a net loss from misrepresentation, and decreasing t_L to 0.1 generates a net gain from misrepresentation.

Proof of Proposition 8. Consider the case where misrepresentation can occur; i.e., $q_L < q_{(1)}$. As shown in Proposition 6, the advertising content separates $q < \tilde{q}$ from $q > \tilde{q}$. In addition, the advertising coverage has further implications. Let us focus on the case where the firm draws a product with quality $q < \tilde{q}$ and $q_L < q_{(1)}$. Misrepresentation occurs if quality level q is very low. Consumers use the fact that they are covered by advertising as an informative signal. Define $Covered = u < \alpha$, $u \sim \mathrm{U}(0, 1)$ such that α is the probability of covering a consumer. Upon being exposed by the advertisement, covered consumers form belief $E[q \mid Covered]$. For the low segment, being covered is equivalent to

$$Covered = u < \alpha^*(q) = q > \frac{2\sqrt{2c\gamma u} - (\gamma - t_L)}{\theta_L} \quad (57)$$

such that the consumer belief after being covered by advertising becomes

$$E\left[q \,\middle|\, q > \frac{2\sqrt{2c\gamma u} - (\gamma - t_L)}{\theta_L}\right]. \quad (58)$$

Because $\hat{q} \sim \mathrm{U}(q_L, \tilde{q})$ and $u \sim \mathrm{U}(0, 1)$, it is easy to show that

$$\frac{\partial}{\partial c} E\left[q \,\middle|\, q > \frac{2\sqrt{2c\gamma u} - (\gamma - t_L)}{\theta_L}\right] > 0, \quad (59)$$

because increasing c is equivalent to stating that quality is above a higher (probabilistic) threshold. Because higher

quality is always good news in this region, as the cost of advertising increases, so too does the expected surplus of the consumer. It follows that an informative and decisive equilibrium with misrepresentation can admit a wider support for quality (i.e., lower q_L) as the cost of advertising increases. Welfare effects are not clear, however, because as the cost of advertising increases, the number of consumers exposed to advertisements of the unattractive firm decreases.

Two-Period Interaction

Suppose a two-period setting where consumers can decide when to search. Once they do, they learn the product quality q. Idiosyncratic shocks are assumed to be independent over time. Because the option value of search is higher in the first period (because it can affect decisions in both periods), one only needs to inspect the case of searching in the first period. When q and p are known, the expected utility from searching w.r.t. independent shocks ε_{i1} and ε_{i2} is given by

$$\begin{aligned}
&\Pr(Buy\,2\,units)E(2\,units \mid Buy\,2) + 2\Pr(Buy\,1\,unit)\\
&\quad\cdot E(1\,unit \mid Buy\,1) + \Pr(Not\,Buy)E(Not\,Buy)\\
&= \frac{(\theta_S q - p + \gamma - t_S)^2}{4\gamma^2}(\theta_S q - p + \gamma + t_S)\\
&\quad + 2\left(\frac{\theta_S q - p + \gamma - t_S}{2\gamma}\,\frac{\gamma - (\theta_S q - p - t_S)}{2\gamma}\right.\\
&\qquad\left.\cdot\left(\frac{1}{2}(\theta_S q - p + \gamma + t_S) + t_S\right)\right)\\
&\quad + \left(1 - \frac{(\theta_S q - p + \gamma - t_S)^2}{4\gamma^2}\right)t_S > 2t_S + k\\
&= \frac{(\theta_S q - p + \gamma - t_S)^2}{2\gamma} > k. \qquad (60)
\end{aligned}$$

The two-period case yields the same surplus inequality condition for consumers as the one-period case. Although the search decision provides the option value of buying over two periods instead of one, now the consumer trades this benefit to purchasing two units of the outside good or, alternatively, one unit of the good and one unit of the outside good. The simultaneous increase in both options provides the equivalence between the single-period case and the case with multiple periods.

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