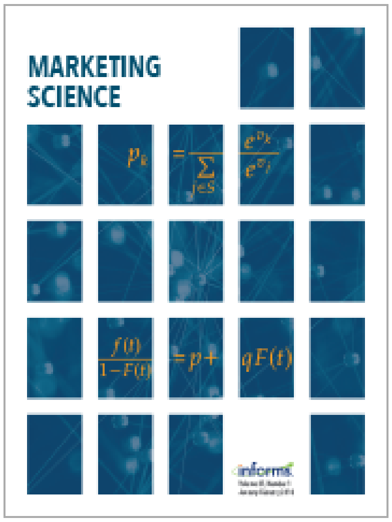

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Periodic Advertising Pulsing in a Competitive Market

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The question as to the optimality of advertising pulsing has attracted many researchers over the last half-century. In this paper we specify a market share model in which there are two advertising-setting firms as well as a no-purchase option. The framework is that of a first-order Markov process with three states. The objective of both firms is to maximize profits. We are able to demonstrate, for a diminishing returns advertising function, that the optimal advertising strategy is pulsing. The frequency of the advertising pulse is shown to depend on the magnitude of the market share retention rate (state dependence); the higher it is, the less frequent the advertising. We further find that the optimal advertising budgets do not remain the same when the frequency of pulsing changes. Finally, we show that it is optimal for both firms to advertise in phase.

Key words: advertising; pulsing; dynamic models; game theory
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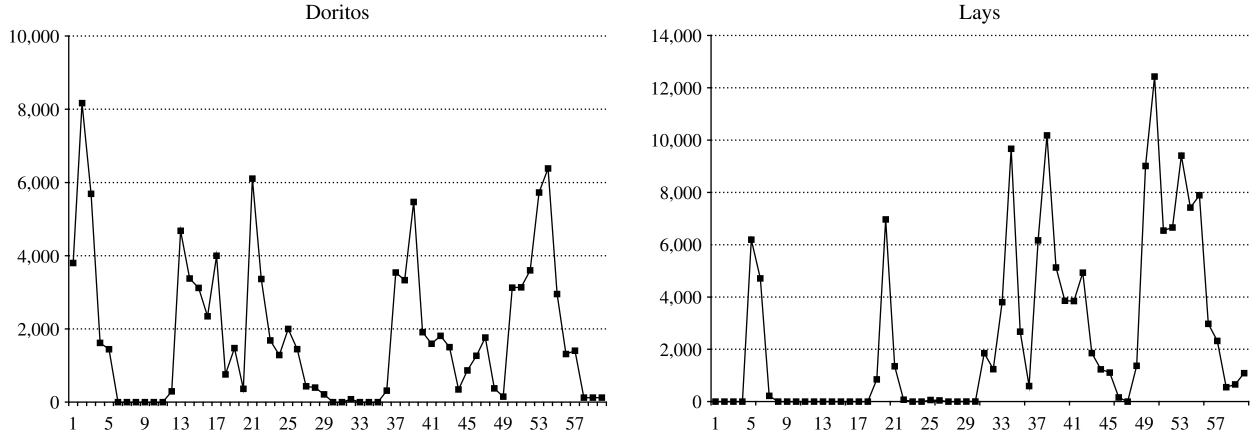
Introduction

Established competitive brands in frequently purchased consumer categories often exhibit pulsing policies in their advertising behaviors. They advertise for a couple of months and then stop for the same or even longer periods. Evidence for such behavior can be observed in Figure 1 for the national advertising expenditures by the two largest brands of snacks as reported in the IRI data set (Bronnenberg et al. 2008). The IRI data set includes advertising information for brands in two product categories: snacks and beer. Interestingly, none of the brands in those two categories exhibits constant advertising behavior. A similar pattern of advertising pulsing is documented in Figure 1 in Dubé et al. (2005), who study the advertising of major frozen entrée brands. The authors point out that the low nonzero levels of advertising that sometimes follow the high levels are not planned by the firms but rather are a result of the "make-good" practice in which the agencies make up for not delivering the contracted-for gross rating points. Casual examination of these types of data sets seems to also indicate that the competing brands advertise and cease to advertise more or less in phase. It is further evident in examining the corresponding sales data that in periods when brands do not advertise, their sales do not drop to zero. This indicates that carryover effects of some kind must exist: either state dependence to prior purchases or lagged effects of advertising.

The question of whether brands should pulse or use even-level advertising has been extensively examined in the marketing literature since first being posed by Vidale and Wolfe (1957). Studies of a monopolistic nature have been conducted by Mahajan and Muller (1986), Bronnenberg (1998), and Feinberg (1992, 2001), among others. These studies use the Vidale and Wolfe (V-W) state dependence framework. Studies that allow for competition have been conducted by Villas-Boas (1993) and Dubé et al. (2005). These latter studies use the goodwill (lagged effects of advertising) framework. Both sets of advertising pulsing studies focus on a particular type of advertising response function. The monopolistic studies assumed an S-shaped response-to-advertising function; the competitive studies further assumed the existence of an advertising threshold level below which advertising has no effect at all.

Some noteworthy comments need to be made about the assumptions that underlie the above-mentioned studies. In empirical market share studies, when state dependence is allowed for, it is always found to have a significant impact. In fact, in attempting to untangle the effects of purchase reinforcement and advertising carryover, Givon and Horsky (1990) find that state dependence is a much "stronger" phenomenon than lagged effects of advertising. That is, individuals are much more impacted by their consumption

Figure 1 Monthly Advertising Expenditure (in \$000s)



experience with the brands than by retention of past advertising. Similarly, in an individual-level study using scanner panel data, Seetharaman (2004) finds that state dependence effects are much stronger than carryover effects of marketing variables. Turning to the issue of the nature of the advertising response function, Hanssens et al. (1990) survey the empirical advertising research that was done mostly in the 1970s and 1980s, before scanner panel data created an emphasis on price promotion effects. They state (see pp. 178–180) that the preponderance of empirical evidence favors the strictly concave sales response function and that there is very little empirical support for S-shaped sales response functions or the existence of a threshold effect. Moreover, the use of S-shaped response functions in explaining why firms use advertising pulsing is rather problematic. It is easy to see this in an example with no dynamic carryover effects in the form of state dependence or lagged effects of advertising. If sales have an immediate S-shaped reaction to advertising, then advertising has first an increasing and then a decreasing marginal return. So the response function is first convex and, after an inflection point, becomes concave. If the firm wants to maximize its multiperiod profits and has a multiperiod advertising budget constraint, then it is possible to show the following. If the multiperiod budget constraint only allows for even-level, every-period advertising below the inflection point, it might be optimal to pulse. However, one may logically ask why a manager in this scenario would set the budget constraint at a range in which there are increasing returns each period to advertising in the first place. A profit-maximizing manager will, in all likelihood, operate in the diminishing returns part of the S-shaped function.

The above discussion suggests that when investigating optimal advertising policies, in the more realistic and general setting that allows for competition, a more meaningful result would be one where pulsing is identified as an optimal strategy while allowing for state dependence and a concave advertising response function. Thus, in this paper we extend the state dependence framework of the V-W model to a duopolistic setting. We then address a set of questions that arise from the above-cited studies and actual firms' behavior. Can established competing brands obtain higher profits with a pulsing strategy than with even-level advertising? Can we show pulsing to be optimal without resorting to an S-shaped advertising response function, but rather by using a diminishing returns function? Why do some brands, after a pulse, stop advertising for several periods before pulsing again? Is advertising in phase more profitable then out-of-phase advertising?

In the next section, we develop a discrete duopolistic V-W type market share model. The framework is that of a first-order Markov process with three states. It incorporates two advertising-setting brands as well as a no-purchase option. Many of the V-W type models examined in the literature, both monopolistic and duopolistic, are shown to be special cases of the proposed model. We further specify an increasing, but at a decreasing rate, effective advertising response function. In the section that follows, we outline the long-term advertising strategies to be evaluated. We then investigate the competitive profit-maximizing equilibrium associated with even-level advertising and with in-phase advertising pulsing at different frequencies: for example, every two periods, every three periods, and so on. Out-of-phase advertising is the focus of the section after that. In those two latter optimization sections, we attempt to answer all the questions we have posed here.

A Model of Advertising Competition

The monopolistic Vidale and Wolfe (1957) sales-response-to-advertising model that is at the foundation of many of the studies on this topic was specified originally as

$$\dot{S} = rA(t)\frac{M-S}{M} - \lambda S, \tag{1}$$

where S is the firm's sales, M is the market potential, $A(t)$ is the advertising expenditure, r is the effectiveness of advertising, and λ is the sales decay constant. We can specify this differential equation more generally in market share terms as

$$\dot{x} = v(t)(1-x) - \lambda x, \tag{2}$$

where x is the market share and $v(t)$ is the ad effectiveness associated with an ad budget $A(t)$.

Advertising pulsing—advertising in one period and then not advertising for a period or two—is by its very nature a discrete type of managerial policy. We therefore prefer to investigate this phenomenon through a discrete time model and optimization procedure. Horsky (1977) points to the fact that the continuous-time V-W model, in its discrete version, can be cast as a Markov process. The discrete analog of the V-W model, where $\delta = 1 - \lambda$, is a market share model governed by the transition matrix of a first-order Markov process with two states: (i) buying the brand and (ii) not buying the brand:

$$[x_t \quad 1-x_t] = [x_{t-1} \quad 1-x_{t-1}] \begin{bmatrix} \delta & 1-\delta \\ v_t & 1-v_t \end{bmatrix}. \tag{3}$$

Equation (3) is a *deterministic* equation for the evolution of the market share of the brand even though it is written using a stochastic matrix. The discrete time market share V-W model is then

$$x_t = \delta x_{t-1} + v_t(1-x_{t-1}). \tag{4}$$

It is evident from model (4) that a fraction δ of the previous-period brand buyers, as a result of their positive experiences, will repeat buy, and that advertising in this model is informative and impacts the consumers who did not buy the brand. The interpretation of the consumers who did not buy the brand, $1 - x_{t-1}$, in model (4) (and also $M - S$ in the V-W model (1)) is somewhat vague. They could be individuals who bought from another major brand or individuals who did not buy at all in the product class. In our extension of the V-W model, we will separate these two populations.

The Duopolistic Model

In extending the V-W model to a duopolistic setting, we assume that there are two competing brands that compete through advertising in addition to a third nonstrategic, nonadvertising brand. That last brand may be a house or local brand or, in general, a no-purchase option. The market shares in this setting are governed by a three-state transition matrix:

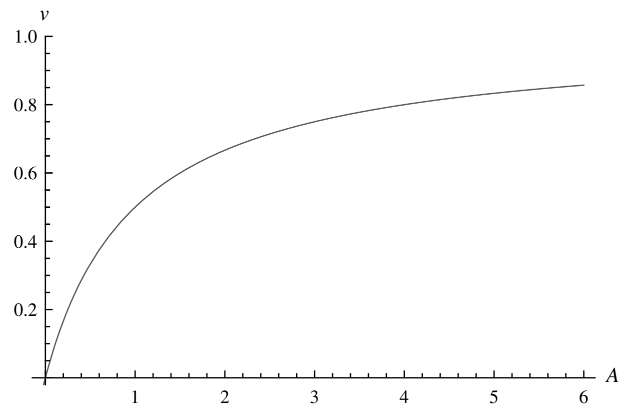
$$\begin{aligned}
[x_{1t} \quad x_{2t} \quad 1-x_{1t}-x_{2t}] &= [x_{1,t-1} \quad x_{2,t-1} \quad 1-x_{1,t-1}-x_{2,t-1}] \\
&\quad \cdot \begin{bmatrix} \delta - v_2 & v_2 & 1-\delta \\ v_1 & \delta - v_1 & 1-\delta \\ v_1 & v_2 & 1-v_1-v_2 \end{bmatrix},
\end{aligned} \tag{5}$$

where v_1 and v_2 are the ad effectiveness of brand 1 and brand 2, respectively. In model (5), just like in the V-W model, the brand's advertising affects the consumers who did not buy it. Its advertising is informative and causes switches from consumers who bought the competing brand and from consumers who did not buy at all. A fraction δ of the previous-period brand buyers wish to repeat. However, the brand's share retention rate is reduced due to the impact of the competing brand's advertising.

The duopolistic model (5) has two special cases that have appeared frequently in the literature. If we eliminate the second brand and, in the transition matrix, erase the second row and column and set $v_2 = 0$, the above model becomes the monopolistic V-W model (3). If, instead, we eliminate the no-purchase state and, in the transition matrix, erase the third row and column and set $\delta = 1$, we get the competitive two-state Markov model used in empirical works and optimization studies (Horsky 1977, Chintagunta and Vilcassim 1992, Erickson 1985, Fruchter and Kalish 1997). The first two studies also provide empirical support for this two-state model (a general review of this "us" versus "them" model is provided in Little 1979). All of these papers examine long-term optimal competitive advertising strategies based on the continuous-time version of this two-state model, often referred to as the Lanchester model (Kimball 1957), using optimal control and differential games, but they do not examine the optimality of pulsing.

It is important to note that having the third, no-purchase state is consistent with the outside good option used currently in all econometric applications of the logit model. For example, Horsky et al. (2006), Che et al. (2007), and Dubé et al. (2010) allow for an outside good in their state dependence models. In the context of using Markov models, the idea of the no-purchase state was recently advanced by Freimer and Horsky (2008) in their study on the optimality of price promotions. One advantage of having such a state in the context of model (5) is that in the monopolistic V-W model (2), when a brand stops advertising, it does not get new consumers, and its existing share is gradually depleted; here, if both competing brands stop advertising, their shares will be gradually depleted. This is a realistic consequence that does not happen in the absence of the third state.

Figure 2 **Effective Advertising as a Function of Advertising Expenditure**



Functional Form of Advertising

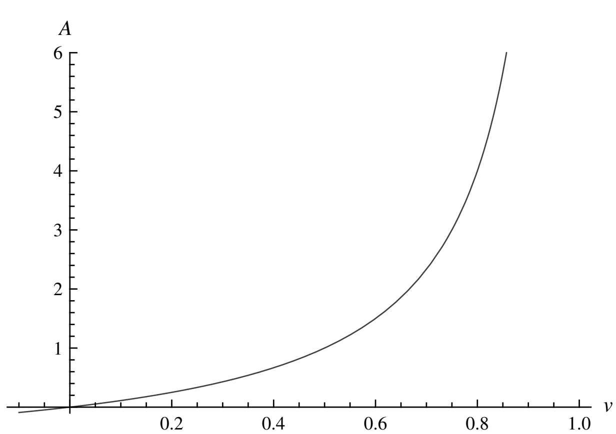
A key question in all studies of optimal advertising strategies is the functional form of the advertising effectiveness as a function of advertising expenditure, $v(A)$. The monopolistic and competitive studies cited in the introduction have tried to justify pulsing by specifying the effectiveness function as an S-shaped curve or as a function with a threshold. One of our objectives is to try to justify pulsing without resorting to these types of functions. Rather, we justify pulsing by using a regular increasing with diminishing returns function.

We will specify the advertising effectiveness function as $\nu(A) = A/(A+1)$. This function, which is illustrated in Figure 2, is bounded between zero and one, a desirable property in our context because it enters the cells in the transition matrix.

Advertising Costs

The advertising expenditure needed to provide a certain level of advertising effectiveness is then $A = v/(1-v)$. This function, which is illustrated in Figure 3, is an increasing marginal cost function.

Figure 3 **Advertising Expenditure as a Function of Effective Advertising**



It should be noted that using a diminishing returns advertising effectiveness function and an increasing marginal advertising costs function is common in the literature. The studies that have used differential games to find optimal advertising strategies for duopolists using the continuous-time version of the two-state Markov model, such as Deal (1979) and Fruchter and Kalish (1997), have used a linear term for advertising effectiveness and a quadratic term for advertising costs, whereas Erickson (1985) and Chintagunta and Vilcassim (1992) use the square root and the linear function for the same purpose, respectively.

The Advertising Strategies

Having formulated a model of advertising competition and having specified the functions of advertising effectiveness and cost, we are set to investigate the optimal competitive advertising strategies.

Comparison to Existing Contributions

The number of papers investigating optimal advertising strategies in the context of monopolistic and duopolistic V-W–type models is very large. A comprehensive review of the optimization research in this area, which included around a hundred papers, was provided by Feichtinger et al. (1994). Additional papers have been published since. However, if we focus on pulsing by duopolists, it is worthwhile to contrast our upcoming analysis with a noteworthy contribution made by Park and Hahn (1991). In their paper they specify a discrete duopolistic two-state V-W model. They investigate the optimal two-period cycle policy of the first brand when the second one is "passive." That is, the second brand uses either even-level advertising over the two periods or pulses; in both cases, however, the first brand knows with certainty the policy of the second. The authors further assume that the first brand has a fixed advertising budget and can use the money for either even-level advertising or pulsing.[1] This simplifies the problem: if for any policy the cost is the same, the profit maximization problem simplifies to being the maximization of the sum of market shares over the two-period cycles. The authors show that pulsing can be optimal without resorting to an S-shaped function. In our analysis, aside from allowing for a third, no-purchase state, we do not assume that the second brand is passive; we actually search for a competitive equilibrium. Moreover, we do not assume that the advertising budget is fixed regardless of the nature of the policy, but rather, we search for the optimal profit-maximizing

[1] Comparing pulsing to even-level advertising under a fixed budget constraint is common in the monopolistic advertising pulsing papers cited previously.

advertising budget associated with each policy. In addition, we try to identify the optimal frequency of advertising pulsing by not limiting our analysis to a two-period cycle.

The Advertising Strategy Space

Advertising pulsing, as discussed earlier, is practiced by established competitive national brands. As a result, our focus will be on the advertising strategies of such brands and not, for example, on the advertising strategies that accompany the market entry of new brands. As such, our analysis will center on the long-term, steady-state behavior of established brands.

The pulsing patterns observed, advertising for a month and then not advertising for a couple of months, are clearly an outcome of a yearly planning process in which the yearly advertising budget and its monthly allocation are predetermined. We assume that each of the brands is limited, as a result of additional costs, to either even-level advertising or pulsing at a fixed frequency. These are institutional and operational costs that arise when a firm implements an irregular pattern of advertising. Apart from direct managerial time investments in continuously modifying the yearly plan, there would be difficulty in coordinating such irregular advertising activities with retailers' promotional calendars. Moreover, the way advertising agencies buy national media for brands (for example, by precommitting to particular TV programs in up-front media markets) may make constant tinkering with yearly plans prohibitively expensive.

We thus assume that each of these national brands can employ the type of long-term advertising policies that are evident in firms' market behavior and have been discussed in the literature:

a. Both brands advertise in each period at an even level.

b. Both brands pulse in phase with the same frequency of once in two periods, three periods, etc.

c. Both brands pulse out of phase with the same cycle of once in two periods, three periods, etc.

We further assume that, within each strategy, if the brand advertises, it advertises at the same level in the different periods. We do not assume that, within a strategy, the competing brands advertise at the same level, nor do we assume that the level of advertising is identical across strategies. In all strategies the optimal advertising levels for the brands will be sought by taking into account the repeated nature of the game—the fact that the established national brands compete repeatedly, albeit cyclically, through their advertising policies.

Once the optimal levels are identified, we compare the profitability associated with each strategy and choose the best one. It is at that point that we try to ascertain whether the profits associated with pulsing are higher than those associated with even-level advertising and whether in-phase pulsing dominates out-of-phase pulsing.

In-Phase Advertising Pulsing

In this section we examine whether established competitive brands should advertise every period at an even rate of advertising, which corresponds to their steady-state shares, or whether they actually achieve higher profits by employing in-phase advertising pulsing.

Advertising Every Period

If the two competing brands advertise every period at an effectiveness level of v_1 and v_2, respectively, their steady-state shares, x_{11} and x_{21}, where the first subscript indicates the brand and the second the time period, would satisfy

$$\begin{aligned}
&[x_{11} \quad x_{21} \quad 1-x_{11}-x_{21}] \\
&\quad = [x_{11} \quad x_{21} \quad 1-x_{11}-x_{21}] \\
&\qquad \cdot \begin{bmatrix} \delta - v_2 & v_2 & 1-\delta \\ v_1 & \delta - v_1 & 1-\delta \\ v_1 & v_2 & 1-v_1-v_2 \end{bmatrix}
\end{aligned} \tag{6}$$

and could thus be derived as

$$x_{11} = \frac{v_1}{v_1+v_2-\delta+1} \quad \text{and} \quad x_{21} = \frac{v_2}{v_1+v_2-\delta+1}. \tag{7}$$

Later in this section we will compare the profitability of this even-level advertising policy with the profitability of a pulsing policy; it should be noted here that Equation (7) can also be viewed as the average revenue per period per potential consumer of each of the brands.

Advertising In Phase Every Second Period

Our first case of pulsing is when the two brands advertise in phase every second period. The advertising effectiveness of the first brand over time would follow $v_1, 0, v_1, 0, \ldots$, and the second brand would follow $v_2, 0, v_2, 0, \ldots$. In steady state, each of the brands would alternate between a high share when advertising is used and a low share when no advertising is used. The notion and theory of alternating steady-state market shares is advanced in Appendix A of Freimer and Horsky (2008) to describe a situation in which the state variable converges to alternating repeated values as a result of alternating repeated control variable values. Each of the brands would then alternate between two steady-state market shares. The first brand would follow $x_{11}, x_{12}, x_{11}, x_{12}, \ldots$, and the second would follow

$x_{21}, x_{22}, x_{21}, x_{22}, \ldots$. These alternating steady states satisfy

$$[x_{12} \quad x_{22} \quad 1-x_{12}-x_{22}] = [x_{11} \quad x_{21} \quad 1-x_{11}-x_{21}] \cdot \begin{bmatrix} \delta - v_2 & v_2 & 1-\delta \\ v_1 & \delta - v_1 & 1-\delta \\ v_1 & v_2 & 1-v_1-v_2 \end{bmatrix}, \tag{8}$$

$$[x_{11} \quad x_{21} \quad 1-x_{11}-x_{21}] = [x_{12} \quad x_{22} \quad 1-x_{12}-x_{22}] \cdot \begin{bmatrix} \delta & 0 & 1-\delta \\ 0 & \delta & 1-\delta \\ 0 & 0 & 1 \end{bmatrix}.$$

When solved, Equations (8) lead to the average revenue per period per potential consumer for brand 1:

$$\frac{1}{2}\cdot(x_{11}+x_{12}) = \frac{1}{2}\cdot\frac{v_1\cdot(\delta+1)}{\delta\cdot v_1+\delta\cdot v_2-\delta^2+1}. \tag{9}$$

A similar term can be derived for brand 2.

Advertising In Phase Every Third Period

If the two competitive brands advertise in phase every third period, then the first brand follows $v_1, 0, 0, v_1, \ldots$, and the second follows $v_2, 0, 0, v_2, \ldots$. Their respective steady-state market shares would alternate between three levels: $x_{11}, x_{12}, x_{13}, x_{11}, \ldots$ and $x_{21}, x_{22}, x_{23}, x_{21}, \ldots$. These alternating steady states satisfy

$$[x_{12} \quad x_{22} \quad 1-x_{12}-x_{22}] = [x_{11} \quad x_{21} \quad 1-x_{11}-x_{21}] \cdot \begin{bmatrix} \delta - v_2 & v_2 & 1-\delta \\ v_1 & \delta - v_1 & 1-\delta \\ v_1 & v_2 & 1-v_1-v_2 \end{bmatrix},$$

$$[x_{13} \quad x_{23} \quad 1-x_{13}-x_{23}] = [x_{12} \quad x_{22} \quad 1-x_{12}-x_{22}] \cdot \begin{bmatrix} \delta & 0 & 1-\delta \\ 0 & \delta & 1-\delta \\ 0 & 0 & 1 \end{bmatrix},$$

$$[x_{11} \quad x_{21} \quad 1-x_{11}-x_{21}] = [x_{13} \quad x_{23} \quad 1-x_{13}-x_{23}] \cdot \begin{bmatrix} \delta & 0 & 1-\delta \\ 0 & \delta & 1-\delta \\ 0 & 0 & 1 \end{bmatrix}. \tag{10}$$

When solved, Equations (10) lead to the average revenue per period per potential consumer for brand 1:

$$\frac{1}{3}\cdot(x_{11}+x_{12}+x_{13}) = \frac{1}{3}\cdot\frac{v_1\cdot(\delta^2+\delta+1)}{\delta^2\cdot v_1+\delta^2\cdot v_2-\delta^3+1}. \tag{11}$$

Profits as a Result of Advertising In Phase Every *n*th Period

When one considers Equations (7), (9), and (11), it becomes apparent that the average revenue per period per potential consumer follows a pattern. In fact, by induction we can specify it as a function of v_1, v_2, δ, and n.

When each of the brands advertises every nth period, then for a cycle of n periods, their advertising expenditures are $v_1/(1-v_1)$ and $v_2/(1-v_2)$, respectively. It is then possible to specify the average per-period profit formula for brand 1 as a function of v_1, v_2, δ, and n:

$$\frac{1}{n}\cdot\left(\frac{v_1\cdot((1-\delta^n)/(1-\delta))}{\delta^{n-1}\cdot v_1+\delta^{n-1}\cdot v_2-\delta^n+1}-\frac{v_1}{1-v_1}\right). \tag{12}$$

For brand 2,

$$\frac{1}{n}\cdot\left(\frac{v_2\cdot((1-\delta^n)/(1-\delta))}{\delta^{n-1}\cdot v_2+\delta^{n-1}\cdot v_1-\delta^n+1}-\frac{v_2}{1-v_2}\right). \tag{13}$$

Having the profit functions of the competing brands, we can now determine whether pulsing is superior to even-level advertising and, if so, look for the optimal pulsing frequency. To gain some intuition, we start by doing so for the case where we assume that the market share retention rate is $\delta = 0.3$. We then solve for the general δ case.

Reaction Functions When Advertising Every Period

Based on Equation (12), when we set $n = 1$ and $\delta = 0.3$, the best response for brand 1 conditional on the advertising of brand 2 can be derived from the first-order condition:

$$\frac{d}{dv_1}\left[\frac{1}{1}\cdot\left[\frac{v_1\cdot((1-0.3^1)/(1-0.3))}{0.3^{1-1}\cdot v_1+0.3^{1-1}\cdot v_2-0.3^1+1}-\frac{v_1}{1-v_1}\right]\right]$$
$$=\frac{10\cdot v_1^2\cdot(10\cdot v_2-3)-40\cdot v_1\cdot(10\cdot v_2+7)+(3-10\cdot v_2)\cdot(10\cdot v_2+7)}{(v_1-1)^2\cdot(10\cdot v_1+10\cdot v_2+7)^2}$$
$$=0.$$

Since the denominator is positive, the reaction function $v_1(v_2)$ for brand 1 satisfies

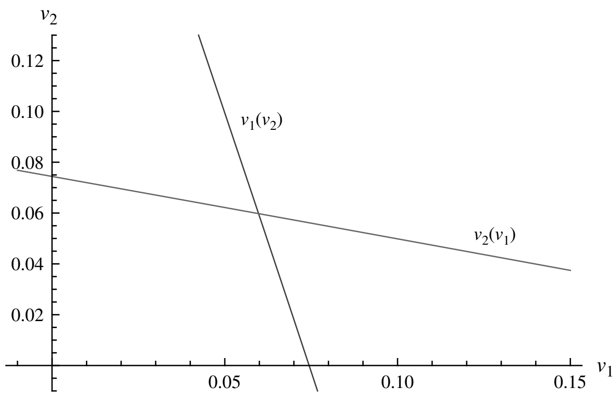
$$10\cdot v_1^2\cdot(10\cdot v_2-3)-40\cdot v_1\cdot(10\cdot v_2+7) + (3-10\cdot v_2)\cdot(10\cdot v_2+7)=0. \tag{14}$$

In a similar manner, the reaction function $v_2(v_1)$ for brand 2 can be derived as follows:

$$\frac{d}{dv_2}\left[\frac{1}{1}\cdot\left[\frac{v_2\cdot((1-0.3^1)/(1-0.3))}{0.3^{1-1}\cdot v_2+0.3^{1-1}\cdot v_1-0.3^1+1}-\frac{v_2}{1-v_2}\right]\right]$$
$$=\frac{10\cdot v_2^2\cdot(10\cdot v_1-3)-40\cdot v_2\cdot(10\cdot v_1+7)+(3-10\cdot v_1)\cdot(10\cdot v_1+7)}{(v_2-1)^2\cdot(10\cdot v_2+10\cdot v_1+7)^2}$$
$$=0,$$

$$10\cdot v_2^2\cdot(10\cdot v_1-3)-40\cdot v_2\cdot(10\cdot v_1+7) + (3-10\cdot v_1)\cdot(10\cdot v_1+7)=0. \tag{15}$$

Figure 4 Reaction Functions and Equilibrium When $n = 1$ and $\delta = 0.3$



Equilibrium When $n = 1$

The reaction functions Equations (14) and (15) for this $n = 1$, $\delta = 0.3$ in-phase game can be drawn (Figure 4) and are seen to have a unique point of intersection in the feasible region $0 \leq v_1, v_2 \leq \delta$.

The competitive equilibrium is thus found when $v_1 = v_2 = v$ is substituted into Equation (14) or (15), and the following solvable cubic function results:

$$10 \cdot v^2 \cdot (10 \cdot v - 3) - 40 \cdot v \cdot (10 \cdot v + 7) + (3 - 10 \cdot v) \cdot (10 \cdot v + 7) = 0. \qquad (16)$$

It occurs when both brands set their advertising effectiveness at a level of $v = 0.0597740$.

Advertising Every Second or Third Period

In a similar manner to the case of advertising every period, we can now derive the reaction functions for the case of $n = 2$, $\delta = 0.3$. The reaction functions can then be used to identify the competitive equilibrium for this case, which occurs at $\nu = 0.116065$. For the case of $n = 3$, $\delta = 0.3$, when each brand advertises every third period, the competitive equilibrium can be identified at $\nu = 0.146436$.

Equilibrium Advertising Expenditure and Profits When $\delta = 0.3$

Having derived the optimal level of effective advertising for the cases of $n = 1$, $n = 2$, and $n = 3$, we can now calculate the average per-period advertising expenditure per potential customer associated with each of these cases based on $(1/n)A = 1/n(v/(1 - v))$. The corresponding profit can be calculated based on Equation (12). The profit values provided in Table 1 indicate that the optimal frequency of pulsing when $\delta = 0.3$ is $n = 2$ (pulse and advertise every second period). The average per-period advertising expenditures provided in Table 1 indicate that in the $\delta = 0.3$ case, the highest average per-period expenditure is associated with the largest average per period profit when each brand advertises every second period ($n = 2$).

Table 1 Average Per-Period Advertising Expenditure and Profits

n	Average advertising expenditure	Profits
1	0.0635740	0.00936124
2	0.0656524	0.0113578
3	0.0571861	0.0107061

It is important to note that the optimal average per-period advertising expenditure varies with the length of the pulsing cycle. This fact indicates that comparing pulsing to even-level advertising under a fixed budget constraint, which allows one to evaluate the sales results rather than the profit ones, is inappropriate. The optimal profit-maximizing advertising budget is different for even-level advertising and pulsing every second or third period.

Solving the General δ Case

We have just completed the process of identifying the optimal pulsing frequency for the case of $\delta = 0.3$. It would be interesting to know whether at other levels of share retention, δ, advertising at an even level, or pulsing every three periods, becomes optimal. We will thus turn to identifying the optimal pulsing frequency for any value of δ. The per-period profits of brands 1 and 2 were shown for the general δ case in Equations (12) and (13), respectively. To solve the general δ case, we follow similar steps to the ones we have just taken. We differentiate Equations (12) and (13) with respect to v_1 and v_2, respectively. The resulting equations, which are similar to Equations (14) and (15), can be graphed. As in Figure 4, we find a unique point of intersection (v_1, v_2); in this unique solution, the values of v_1 and v_2 are equal. Returning to the equations, when we set $v_1 = v_2 = v$ and solve, we get a solution for v that corresponds to the competitive equilibrium in terms of δ and n. The solution is substituted back into the profit formula. It provides us the in-phase equilibrium profits as a function of δ and n.

The per-period profits per potential customer are plotted in Figure 5 for the values of $\delta = 0.3, 0.5$, and 0.7.

For graphical purposes, the values of n were entered into the calculations on a continuous basis, but only the integer values of n and the corresponding profit values should be considered. In Figure 6 we plot the corresponding per-period advertising expenditures per potential customer.

What can be seen is that as the market share retention δ gets larger, so too does the optimal per-period advertising expenditure and the corresponding profits. It stands to reason that the brands are more profitable when they can retain a larger fraction of their previous-period purchasers. Looking at the lines

Figure 5 **Optimal Per-Period Profits**

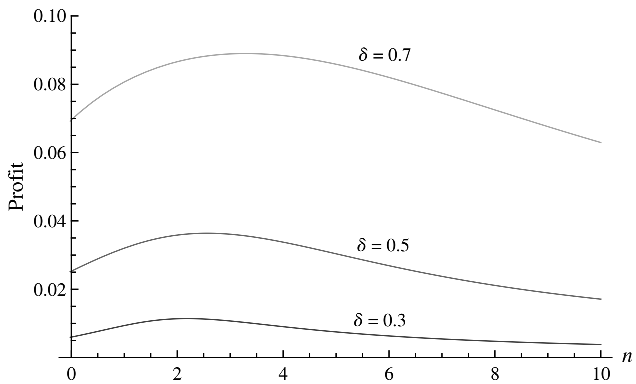
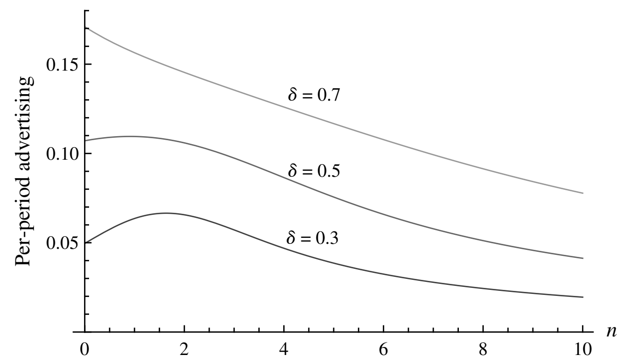


Figure 6 **Optimal Per-Period Advertising Expenditures**



that correspond to $\delta = 0.3$, we can see that the profit function is maximized at $n = 2$ and that the advertising expenditure is the highest there, consistent with our earlier calculations. When δ gets larger, the profit lines in Figure 5 indicate that the optimal frequency of pulsing, n, gets larger. We now investigate this issue more precisely.

Optimal In-Phase Pulsing Frequency as a Function of δ

To identify the values of δ for which even-level advertising is better or worse than pulsing every second period, we can subtract the profits associated with $n = 2$ from those associated with $n = 1$ and see which are bigger for which value of δ. After subtracting the profits of $n = 2$ from those of $n = 1$, we find that they are negative for all positive values of δ, and therefore as long as some retention exists, it is never optimal to advertise every period, and a pulsing policy is always optimal.[2] By subtracting the profits of $n = 3$ from those of $n = 2$, we get the lower curve in Figure 7. It indicates that it is optimal to pulse every second period when $\delta < 0.46$. In a similar manner, we derive the upper curve in Figure 7, which indicates that it is optimal to pulse every third period when $0.46 < \delta < 0.73$.

The values of $0.3 < \delta < 0.7$ are the ones commonly identified empirically as share retention values. For example, in his survey article, Clarke (1976) reports the value of the coefficient of lagged share in studies of monthly data to be, on average, 0.44. In this relevant range, the optimal pulsing frequency is either every two periods or every three. Another very intuitive result of our analysis is that the pulsing frequency depends on the market share retention rate δ. The higher it is, the less frequent the need to have another advertising pulse in order to boost the brands' shares.

[2] Harsanyi (1992) calls this "payoff dominant," but we use the term "optimal."

It should be pointed out that the result above that advertising every second period dominates advertising every period only holds for values of δ greater than zero. For those positive values, the share retention acts as a form of goodwill, which is carried to the following periods. It gets depleted over time and needs to be replenished through an advertising pulse. When the share retention value δ is zero—or in other words, when there is no state dependence—the Markovian market share model we use should be replaced with one in which the current market shares are dependent only on current advertising. Then, obviously, to maximize current profits, advertising every period would be optimal.

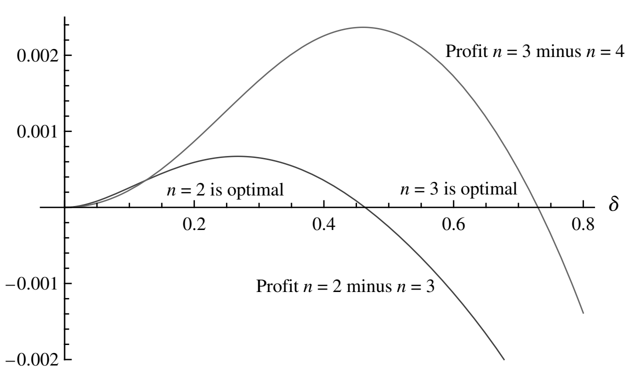
Out-of-Phase Pulsing

So far we have examined the strategies of even-level, every-period advertising and in-phase pulsing. We now examine the cases of out-of-phase pulsing to see whether it results in higher profitability than in-phase pulsing.

Out-of-Phase, Every-Second-Period Pulsing

If the two competing brands advertise out of phase every second period, each would alternate between

Figure 7 **Optimum Pulsing Frequency**



two steady-state market shares:

$$
\begin{aligned}
[x_{12} \quad x_{22} \quad 1-x_{12}-x_{22}] &= [x_{11} \quad x_{21} \quad 1-x_{11}-x_{21}] \\
&\quad \cdot \begin{bmatrix} \delta & 0 & 1-\delta \\ v_1 & \delta-v_1 & 1-\delta \\ v_1 & 0 & 1-v_1 \end{bmatrix}, \\
[x_{11} \quad x_{21} \quad 1-x_{11}-x_{21}] &= [x_{12} \quad x_{22} \quad 1-x_{12}-x_{22}] \\
&\quad \cdot \begin{bmatrix} \delta-v_2 & v_2 & 1-\delta \\ 0 & \delta & 1-\delta \\ 0 & v_2 & 1-v_2 \end{bmatrix}.
\end{aligned}
\tag{17}
$$

The average revenue per period per potential consumer for brand 1 is

$$
\frac{1}{2}\cdot(x_{11}+x_{12}) = \frac{1}{2}\cdot\frac{v_1\cdot(v_2-\delta-1)}{v_1\cdot(v_2-\delta)-\delta\cdot v_2+\delta^2-1}. \tag{18}
$$

A similar function would hold for the average revenue of brand 2.

Reaction Functions and Equilibrium for Every-Second-Period, Out-of-Phase Pulsing

In steps similar to the ones taken for in-phase pulsing, we can obtain the reaction functions and equilibrium for every-second-period, out-of-phase pulsing in the case of $\delta = 0.3$ (see Figure 8). The equilibrium level of effective advertising is now $v = 0.0975832$, which is lower than the corresponding level of $v = 0.116065$ that we found in the two-period in-phase case.

Comparison of Equilibrium Profits of In-Phase vs. Out-of-Phase, Every-Second-Period Pulsing

We now compare, at each level of δ, the optimal profits associated with in-phase versus out-of-phase, every-second-period pulsing. The values are provided in Figure 9. Based on the figure, regardless of the value of δ, in-phase pulsing leads to higher profits than those associated with out-of-phase pulsing. To uncover the reason for this result, we examined the numerical computations of the shares, revenues, and profits of the two competing brands when they advertise in phase and out of phase. We found that if the brands advertise out of phase in the first period, when the first brand takes its turn and advertises, its share and revenues are slightly higher than they would have been had both brands advertised in that period. However, in the second period, when the first brand does not advertise and the second one does, the share and revenues of the first brand are smaller than they would have been under the in-phase strategies. Overall, the loss in revenues and profits in the second period is greater than the slight gain in the first period.

Figure 8 **Reaction Functions and Equilibrium When $n = 2$ and $\delta = 0.3$ (Out of Phase)**

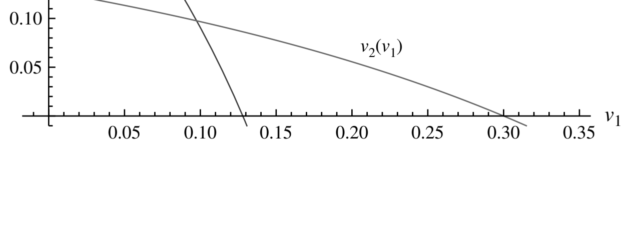
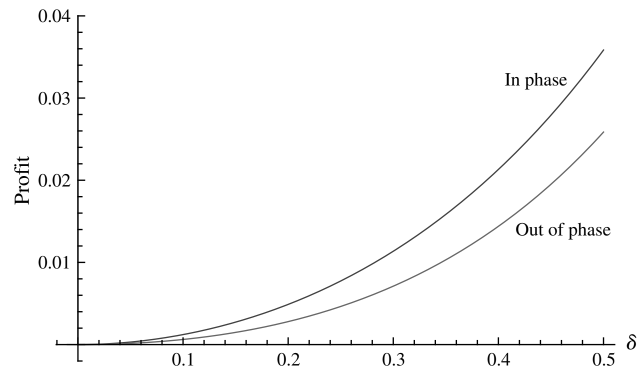


Figure 9 **Comparison of Optimal Profits When $n = 2$**

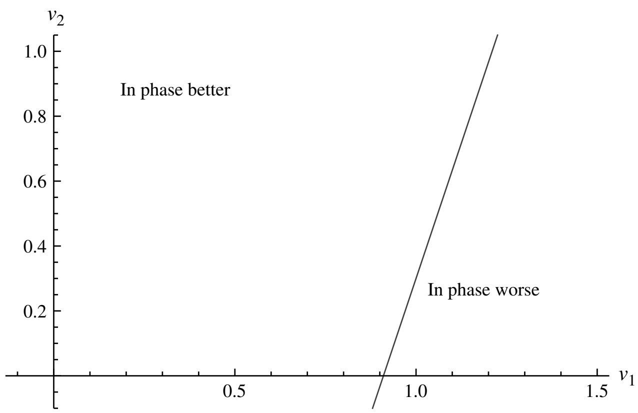


The Generality of the Dominance of In-Phase Pulsing When $n = 2$

So far in this paper, we have used a particular function for advertising effectiveness. It turns out that the dominance of the in-phase strategy, when $n = 2$, is more general and can be shown to be independent of the choice of an advertising response function. Let us say that we use any reasonable function and identify the optimal level of v_1 and v_2 for the out-of-phase case. We can then use the exact same values of v_1 and v_2 for the in-phase case. When v_1 and v_2 are thus held constant, the in-phase versus out-of-phase profit comparison turns to comparing revenues as the cost terms cancel out. The condition for the in-phase and out-of-phase revenues, Equations (9) and (18), to be equal is

$$
\begin{aligned}
&\frac{1}{2}\cdot\frac{v_1\cdot(\delta+1)}{\delta\cdot v_1+\delta\cdot v_2-\delta^2+1} \\
&\quad = \frac{1}{2}\cdot\frac{v_1\cdot(v_2-\delta-1)}{v_1\cdot(v_2-\delta)-\delta\cdot v_2+\delta^2-1}.
\end{aligned}
\tag{19}
$$

Figure 10 Comparison of Revenues When $\delta = 0.3$



Equation (19) turns out to specify a line that is drawn from the perspective of brand 1, for the case of $\delta = 0.3$, in Figure 10.

Based on Figure 10, out-of-phase revenues will be higher for brand 1 only when it advertises at very high levels and brand 2 at very low ones (the lower right corner). However, the optimal advertising levels of our two symmetric brands lie on the 45° line, and the brands have no reason to choose that corner region. Moreover, in examining the cells of the transition matrices of Equations (8) and (17), we see that v_1 has to be smaller than δ, and thus the corner region is not in an admissible range of v_1 values. The same scenario repeats itself when $\delta = 0.46$. We can thus conclude that in-phase revenues dominate when $n = 2$, regardless of the advertising response function.

Out-of-Phase, Every-Third-Period Pulsing

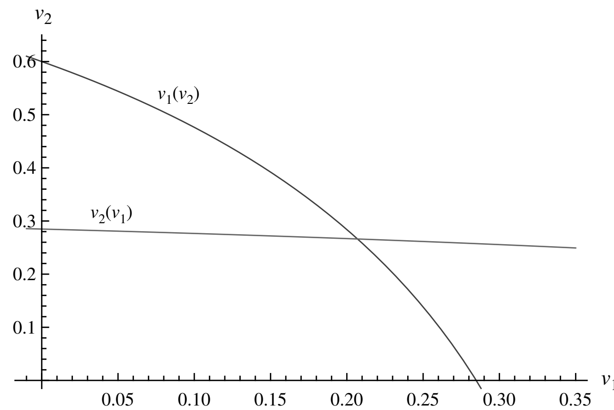
If the two competitive brands advertise out of phase every third period, with the second brand always advertising in the period just after the first brand has advertised, the first brand follows $v_1, 0, 0, v_1, 0, 0, \ldots$, and the second follows $0, v_2, 0, 0, v_2, 0, \ldots$. Their respective steady state market shares would alternate between three levels: $x_{11}, x_{12}, x_{13}, x_{11}, \ldots$ and $x_{21}, x_{22}, x_{23}, x_{21}, \ldots$. The equations that these alternating steady states satisfy can be obtained by modifying Equations (17) in the same way that the in-phase Equations (10) were a modification of Equations (8). The average revenues per period per potential consumer for the two brands are

$$\frac{x_{11}+x_{12}+x_{13}}{3} = \frac{v_1 \cdot (v_2 \cdot (\delta+1) - \delta^2 - \delta - 1)}{3 \cdot (\delta \cdot v_1 \cdot (v_2 - \delta) - \delta^2 \cdot v_2 + \delta^3 - 1)}, \quad (20)$$

$$\frac{x_{21}+x_{22}+x_{23}}{3} = \frac{v_2 \cdot (\delta \cdot v_1 - \delta^2 - \delta - 1)}{3 \cdot (\delta \cdot v_1 \cdot (v_2 - \delta) - \delta^2 \cdot v_2 + \delta^3 - 1)}. \quad (21)$$

In similar steps to the ones taken for in-phase pulsing, we can obtain the reaction functions and equilibrium for every-third-period, out-of-phase pulsing in the case of $\delta = 0.6$ (see Figure 11).

Figure 11 Reaction Functions and Equilibrium When $n = 3$ and $\delta = 0.6$ (Out of Phase)



The optimal levels of effective advertising are now $v_1 = 0.207018$ and $v_1 = 0.266154$. The optimal levels are no longer equal, so our earlier device of setting $v_1 = v_2 = v$ in the response functions to get closed-form solutions as functions of δ is inappropriate. Instead, for each fixed value of δ, we must solve numerically for v_1 and v_2. When we did this for various values of δ in the range $0.46 < \delta < 0.73$, we found that the profits were always lower than those obtained by advertising in phase every third period.

Nash Equilibrium

In the last two sections, we identified the optimal long-term steady-state strategies of the competing established national brands given the advertising strategy space specified earlier. When both brands are pursuing the same strategy, in the relevant range of δ we have been able to show that pulsing in phase dominated both even-level (every-period advertising) and out-of-phase pulsing. In Figure 7, we showed that when $0 < \delta < 0.46$, pulsing every second period $n = 2$ is the optimal in-phase pulsing frequency, and when $0.46 < \delta < 0.73$, $n = 3$ is optimal. We now further investigate whether these optimal in-phase pulsing strategies are indeed Nash equilibria. To do this we must consider what happens when the two brands are using optimal in-phase pulsing and one of the brands decides to change its advertising strategy and move to a different long-term strategy. Details of our investigation into this matter are provided in an online appendix (at http://dx.doi.org/10.1287/mksc.1120.0712).

What we have been able to show is the following. First, when both brands are using optimal in-phase pulsing every second period, in the full relevant range $0 < \delta < 0.7$, the manager of brand 2 has no incentive to deviate and either advertises every period or advertises less frequently (i.e., every three or more periods). We can thus conclude that, given the long-term steady-state advertising strategy space, optimal in-phase pulsing every second period is Nash for all $\delta < 0.70$, which includes the full range of empirically identified share retention values. Interestingly, we also find for the full range of δ that if both brands are using optimal constant-level advertising, a brand that switches to advertising less frequently (i.e., every two or more periods) would suffer a net decrease in profit. From this it follows that optimal constant-level advertising is also Nash. Of course, it is still payoff dominated by optimal in-phase pulsing. This result in itself establishes that a pulsing policy is superior to an even-level advertising policy, which resolves the major research question addressed in this paper.

We then show that when the two brands are using optimal in-phase pulsing every third period, in the range of $0.46 < \delta < 0.5649$ the advertising manager of brand 2 has no incentive to deviate and starts advertising every period, or pulse every two periods, or pulse every four or more periods. For the range of $0.5649 < \delta < 0.7$, we cannot, however, rule out the possibility that the advertising manager of brand 2 will resort to advertising every period.

Contributions and Extensions

To better assess the contribution of our theoretical results regarding advertising pulsing, it is best to contrast them with the results of previous research in this area.

The studies that have addressed advertising pulsing differ in their assumptions related to two key modeling aspects: the nature of the dynamic carryover effects and the shape of the advertising effectiveness function. In terms of carryover effects, some studies use the V-W share retention (state dependence) approach, whereas others assume a goodwill created by the retention of prior advertising. In terms of the advertising effectiveness (in response to advertising expenditure), most of the prior studies assume an S-shaped function.

The first set of studies on this topic examined the optimal policy of a monopolist, using the V-W share retention model and the continuous-time optimal control methodology. Feinberg (2001) revisits this stream of research and concludes that in the continuous-time framework, even an assumption of an S-shaped advertising response function cannot lead to pulsing being the optimal policy. Subsequently, the focus has turned to the competitive scenario. Two studies that deal with competition use lagged effects of advertising instead of the V-W share retention approach. Villas-Boas (1993) assumes an advertising response function that is an extreme case of an S-curve. Up to a certain critical level of advertising expenditure, there is zero response, and then there is a discrete jump to maximal response. There is also partial memory of the previous-period advertising. His competitive scenario is more general and includes price promotions. The brands operate in a two-period cycle and offer price promotions every second period out of phase. He concludes that, in that scenario, advertising pulsing is optimal and should be out of phase with the price promotions and, consequently, with the advertising of the other brand. Dubé et al. (2005) use an advertising goodwill function that is embedded in a logit framework. Their goodwill production function has a threshold level, and its response to current advertising expenditure is S-shaped. They conclude that pulsing is optimal and, in fact, in their Appendix A show that if the advertising goodwill response to advertising expenditure is concave, pulsing cannot be justified.

As discussed in our introduction, Hanssens et al. (1990) cast some doubt on the existence of S-shaped advertising response functions and threshold levels. Given their views and the fact that the empirical existence of state dependence is universally agreed upon, it is noteworthy that we are able to show in a framework that assumes state dependence that pulsing can be justified for a concave advertising response function. Another noteworthy contribution of our analysis is the result that the higher the share retention, the longer the time duration before another pulse in advertising is needed. This result is consistent with those reported by Dubé et al. (2005) and Villas-Boas and Villas-Boas (2008).

There are several potential extensions of our framework that are worth addressing. One is the inclusion of price promotions in addition to advertising. We believe that it is a worthy topic of analysis, but we do not feel it is critical in terms of the real-world robustness of our results. In examining the IRI data set, we noticed that whereas advertising pulsing is detectible in the national expenditure data, price promotions in the form of regular prices interrupted by periods of price drops cannot be detected in the aggregate national data. It is only at the level of a particular stock-keeping unit in a particular retail store that one observes this pricing behavior in the data. This is consistent with the fact that whereas advertising is clearly decided at the national level, price promotions that are usually initiated by the manufacturer have to be negotiated, passed through, and executed by the different retailers. In a casual inspection of the data, the timing of price promotions, their duration, and their cycle length did not correspond to the national advertising policies of the brands. Other types of extensions, such as allowing for asymmetrical brands in terms of share size, retention, and effectiveness of advertising (which would map into share size and ad budget differences), would be interesting, but they would have to be based on numerical examples that are less general than our current analytical results. Finally, it would be interesting to extend our results to the multiple-brand case; we leave that to future research.

Summary

It is clear from examining the monthly advertising behavior of established national brands in frequently purchased product categories that they often employ pulsing strategies. Our objective in this paper has been to provide a possible theoretical rationale to this type of behavior.

We extended the state dependence sales retention monopolistic model of V-W to a duopolistic setting while allowing for a no-purchase option. We were able to show that in this setting, a competitive advertising pulsing strategy can be justified without resorting to an S-curve assumption. Rather, a simple diminishing returns advertising function was assumed. The frequency of pulsing was found to depend on the market share retention magnitude—the higher it is, the less frequent the need for a pulse. The magnitude of the pulse and the corresponding advertising expenditure did not remain the same when the frequency of pulses changes. We further found that in-phase pulsing is more profitable than out-of-phase pulsing for values of share retention consistent with those found in empirical studies.

In conclusion, our results indicate that existence of market share retention rate, or consumers' state dependence/brand loyalty, may well be the underlying force that leads brands to use advertising pulsing. This is consistent with the findings in Freimer and Horsky (2008), that the practice of competitive price promotion is likely to have its roots in the same consumer phenomena.

Electronic Companion

An electronic companion to this paper is available as part of the online version at http://dx.doi.org/10.1287/mksc.1120.0712.

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