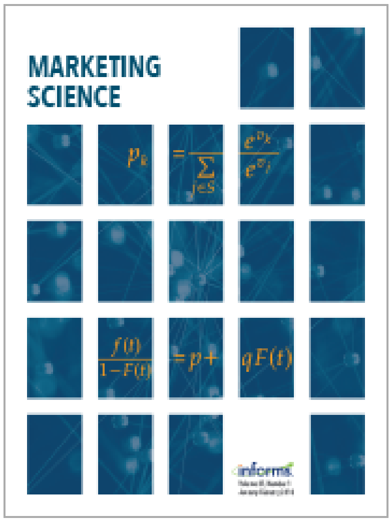

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Investigating Consumers' Purchase Incidence and Brand Choice Decisions Across Multiple Product Categories: A Theoretical and Empirical Analysis

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We propose a framework to investigate consumers' brand choice and purchase incidence decisions across multiple categories, where both decisions are modeled as an outcome of a consumer's basket utility maximization. We build the model from first principles by theoretically explicating a general model of basket utility maximization and then examining the reasonable restrictions that can be placed to make the solution tractable without sacrificing its flexibility. Comparing with prior models, we show why prior multicategory purchase incidence models overemphasize the role of the cross effects of a market mix of brands in other categories on the purchase incidence decision of a given category. Additionally, we show that prior single-category models are a special case of the proposed model when further restrictions are placed on the basket utility structure.

We estimate the model on household basket data for the laundry family of categories. We show (i) why prior single-category and multicategory models would systematically bias the estimates of the own- and cross-price/promotional purchase incidence elasticities; and (ii) how the market mix of each brand in each category affects the purchases across all categories, which can help retailers make promotional decisions across a portfolio of products.

Key words: multicategory brand choice and purchase incidence decision making; microeconomic theory of demand; basket utility maximization; simulated maximum likelihood
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1. Introduction

When consumers visit a store, they typically purchase a basket of items that consists of brands from a set of product categories in the store. Such typical store visits involve two sets of decisions by consumers: first, which product categories to purchase in the store, and second, within the purchased categories, which brand to choose. An important question that arises in this context from a managerial and academic perspective is: How are these two sets of decisions made, and how are they influenced by the market-mix variables in the store?

To answer this question, we first need to understand the process through which consumers make the joint category purchase and brand-choice decisions. If we assume consumers to be utility-maximizing agents, it follows that their joint purchase decisions across all categories should be an outcome of their maximizing utility over their entire basket. Thus, a proper approach requires considering both brand-choice and category purchase decisions simultaneously across all categories in the store. Such an approach is theoretically appealing, and would also yield proper measures of impact of market-mix variables on consumers' purchase behavior. However, a drawback of this "proper" approach is that it is fairly intractable even for a moderate number of brands and categories.

Prior research, by and large, has followed one of two tracks. It has either addressed both brand-choice and category purchase incidence decisions in a single-category context (Chiang 1991, Chintagunta 1993) or addressed one aspect of the decision process in a multicategory context, which is brand choice (Ainslie and Rossi 1998, Erdem 1998) or category purchase incidence (Manchanda et al. 1999, Chib et al. 2002). However, a drawback of single-category models is that they ignore cross-category interrelationships by implicitly assuming that the basket utility maximization reduces to independently maximizing subutilities over each single category. Similarly, the implicit assumption in prior multicategory models is that the basket utility maximization reduces to independently maximizing utilities over brand-choice and category purchase incidence decisions. These assumptions, however, are not always well founded, and can lead to suboptimal solutions as well as biases in estimates of price and promotional elasticities.

The above discussion highlights the trade-off between optimality and tractability. To obtain a microeconomic theoretic specification that balances these imperatives, one needs to (a) start with a completely general basket utility maximization model from first principles, and (b) articulate the reasonable restrictions that can be imposed on such a model, while allowing flexibility in modeling the interrelationships between categories. Such an approach offers the following advantages. First, it will allow us to pin down the effects of market-mix variables of brands in any category on purchases across other categories. Second, it will yield significantly better estimates of these effects, which has significant implications for both retailers and manufacturers. Third, it will shed insight on the nature of *additional* restrictions that are required for the basket utility maximization to reduce to independent subutility maximizations as assumed in prior models. Articulating these additional restrictions will help us understand the nature of biases that can potentially exist in prior models.

1.1. Research Objectives

Our *first* objective is to build a tractable yet flexible model of multicategory purchase incidence and brand choice behavior. We start with a general framework of *direct* basket utility maximization that allows for all possible interactions among all brands in all categories inside the store. We discuss the factors that can be relaxed to get tractable solutions for brand choice and purchase incidence decisions. Specifically, we first show that under relatively mild assumptions of "weak separability" and "aggregation across unrelated categories," it is sufficient to consider only a family of categories that can possibly be used together during consumption. Second, we use the principle of "duality" to show how the joint purchase decisions can also be derived from an *indirect* basket utility. This result is crucial for empirical analysis because flexible functional forms lie mainly in the realm of indirect utilities, which allow for much greater flexibility in modeling the interrelationships between categories, while maintaining tractability (Pollak and Wales 1992).

Our *second* objective is to compare the specifications of our joint category purchase incidence conditions with those used in prior single-category brand choice purchase incidence models (Chiang 1991, Chintagunta 1993) and multicategory purchase incidence models (Manchanda et al. 1999). We show how prior multicategory purchase incidence models tend to overemphasize the role of cross effects of market mix of brands in other categories on the purchase incidence decision of a given category. We also show that prior single-category models are a special case of the proposed model when further restrictions are placed on the structure of the basket utility.

Our *third* objective is to empirically test the proposed model on household-level basket data. We choose the laundry family of categories—that includes detergents, washer and dryer softeners. We use a flexible functional form for the *indirect* basket utility to characterize the stochastic specification of the purchase decisions. We compare the proposed specification with (i) a nested specification that has no cross-category effects (that is, when the multicategory model reduces to multiple single-category models), and (ii) the statistical specification of Manchanda et al. (1999) that only models the purchase incidence decisions across categories. We show that our specification performs significantly better than the competing specifications, and why competing specifications systematically bias the estimates of the own- and cross-price/promotional elasticities. Furthermore, we show how market-mix variables of each brand affect purchases across all categories, estimates of which can help retailers make promotional decisions across a portfolio of products.

1.2. Related Literature

There are four streams of research relevant to this paper. Table 1 provides an overview of these four streams and the relative positioning of our work. The first stream consists of papers that have investigated brand choice and purchase incidence decisions in a single-category context (e.g., Chiang 1991, Chintagunta 1993). The second stream consists of papers that have investigated only brand choice decisions across categories (e.g., Ainslie and Rossi 1998, Seetharaman et al. 1999). The third steam consists of papers that have investigated only purchase incidence decisions across categories (e.g., Manchanda et al. 1999, Russell and Petersen 2000, Chib et al. 2002).

Apart from the methodological limitations discussed before, the usefulness of these three streams is also limited from a retailer's or multiproduct manufacturer's perspective—whose interest lies in understanding how the prices and promotions of each brand in a given category affect both brand choice and category purchase incidence decisions, not only in that category but also across other categories. For instance, the single-category models in the first stream do not model the cross-category effects of a brand's market mix. Similarly, the multicategory brand choice models in the second stream do not model the impact of each brand's market mix on category purchase incidence. Finally, the multicategory purchase incidence models in the third stream cannot pin down the effect of market mix at the brand level.

Table 1 Review of Prior Literature

	Single category	Multiple categories
Single decision (brand choice only)	Guadagni and Little (1983), etc.	Ainslie and Rossi (1998); Erdem (1998); Seetharaman, Ainslie, and Chintagunta (1999); Kim, Srinivasan, and Wilcox (1999); Iyengar, Ansari, and Gupta (2003); Singh, Hansen, and Gupta (2004)
Single decision (category purchase incidence only)	Jain and Vilcassim (1991), Seetharaman (2004), etc.	Chintagunta and Haldar (1998); Manchanda, Ansari, and Gupta (1999); Russell and Petersen (2000); Chib, Seetharaman, and Strijnev (2001); Deepak, Ansari, and Gupta (2004); Borle, Boatwright, Kadane, Nunes, and Shmueli (2005)
Multiple decisions (brand choice and category purchase incidence)	Gupta (1988); Bucklin and Lattin (1991); Chiang (1991); Bucklin and Gupta (1992); Chintagunta (1993); Arora, Allenby, and Ginter (1998); Chib, Seetharaman, and Strijnev (2004); Zhang and Krishnamurthi (2004)	Bell and Lattin (1998); Song and Chintagunta (2006); Chib, Seetharaman, and Strijnev (2005); Mehta (2007)

The fourth stream consists of papers that have modeled brand choice and purchase incidence decisions across categories: There are two recent papers in this stream. The first, by Song and Chintagunta (2006), uses a basket utility maximization approach to estimate the two decisions (along with the quantity decision) across categories. However, they start by choosing a specific functional form of the *direct* basket utility, which results in the purchase incidence probability of any category to be independent of the market mix in other categories. We show that their model is similar to a nested version of our model when further restrictions of "indirect additivity" are imposed on the utility. The second paper, by Chib et al. (2005), uses a statistical approach to specify the two decisions. A benefit of their approach is that it is not bound by theoretical restrictions, which allows more flexibility in modeling the interrelationships between categories.

We extend the four streams by building a model of basket utility maximization from the first principles that simultaneously specifies the purchase incidence and brand choice decisions across multiple categories, and properly accounts for the role of cross effects in the decisions. The rest of the paper is arranged as follows. In §2, we discuss the theoretical specification of the joint purchase conditions based on a general specification of the basket utility. In §3, we discuss the stochastic specification. In §4, we discuss the data used for estimating the model, report the parameter estimates, and discuss the implications. In §5, we conclude, and offer directions for future research.

2. Model Development: Theoretical Specification

In §2.1, we propose a general framework of deriving the joint purchase decisions based on *direct* basket utility maximization. Following that, we discuss the assumptions that can be imposed on the structure of the direct basket utility that would yield tractable solutions for the joint purchase decisions. In §2.2, we derive the general specification of the joint purchase decisions after incorporating the assumptions discussed in §2.1. Using the solution laid out in §2.2, we show how the joint category purchase decisions can be derived from an *indirect* basket utility in §2.3, the results of which will be used to characterize the stochastic specifications of the joint purchase decisions in §3.

2.1. General Model of Basket Utility Maximization[1]

Consider a consumer who can possibly make a purchase in $1, \ldots, M$ product categories in a store on a shopping trip. We define product category l as a set of brands, $1, \ldots, J_l$. Let the *observed* covariates on the consumer's shopping trip be: the total basket expenditure in dollars, y; unit price of a brand j in category l, $p_{l,j}$; and the perceived quality of brand j in category l, $\Psi_{l,j}$.

Given the observed covariates on a shopping trip, the consumer's problem is to choose which categories to purchase and, among the purchased categories, which brands to choose. In order to specify the solution to this problem, we define the consumer's *direct* basket utility as a function of her perceived qualities, $\{\psi_{l,j}\}$, and purchased quantities, $\{x_{l,j}\}$, of all brands $j = 1, \ldots, J_l$ in all categories $l = 1, \ldots, M$. We start with a very general specification of the direct utility, which allows for all possible interactions between brands across all categories,[2]

$$u \equiv U(\{X_{l,1}, X_{l,2}, \ldots, X_{l,J_l}\}_{l=1}^{M}), \tag{2.1}$$

[1] For expositional ease, we suppress the indices for shopping trips and consumers. We introduce these indices in the estimation section of the paper.

[2] We are referring to two types of interactions between brands/categories: First is the interaction in the purchase quantities of any two brands in the direct basket utility. This measures the change in marginal utility of one brand with a change in quantity of the other—which after controlling for inventory, can intuitively be interpreted as their extent of consumption complementarity/substitutability. The second is the extent of purchase

where $X_{l,j}$ is a function of consumer's perceived quality and purchased quantity of brand j in category l. Thus, the consumer's problem can be written as

$$\{x^o_{l,j}\} = \underset{\{x_{l,j}\}}{\arg\max}\, U(\{X_{l,1}, X_{l,2}, \ldots, X_{l,J_l}\}_{l=1}^{M})$$

subject to

$$\sum_{j=1}^{J_1} p_{1,j}x_{1,j} + \sum_{j=1}^{J_2} p_{2,j}x_{2,j} + \cdots + \sum_{j=1}^{J_M} p_{M,j}x_{M,j} = y \quad \text{(budget constraint)} \tag{2.2}$$

$$x_{l,j} \geq 0 \quad \text{for all } l, j \quad \text{(nonnegativity constraints)}.$$

The utility maximization in (2.2) is subject to the budget and nonnegativity constraints. The nonnegativity constraints are critical—they show how the *optimal* solutions of quantities of all brands across all categories, $\{x^o_{l,j}\}$, are related to category purchase incidence and brand choice decisions. They imply that if the optimal solution is such that: (a) $x^o_{l,j} = 0$ for all brands $j = 1, \ldots, J_l$ in category l, then category l will not be purchased; (b) for any brand k in category l, $x^o_{l,k} > 0$, then category l will be purchased and brand k will be chosen in category l.

The derivation of the joint purchase conditions entails simultaneously solving the first-order conditions that follow from the utility maximization.[3] Because the direct basket utility in Equation (2.1) allows for all possible interactions between all brands across all categories, the first-order condition for any brand will be a function of the quantities of all brands across all categories. Note that such an approach is analytically and empirically intractable. As the number of brands across all categories increases, simultaneously solving the first-order conditions across all brands and all categories becomes analytically intractable; additionally, it leads to an explosion in the number of parameters that capture the pairwise interactions between all brands, thus making it empirically intractable. Therefore, we will next make some reasonable assumptions on the structure of the utility that will yield relatively more tractable solutions to the brand choice and category incidence decisions.

Assumption 1 (Weak Separability Among All M Categories). *Weak separability is the least restrictive way to group brands in one product category and separate them from brands in other categories* (*Pollak and Wales* 1992). *This is done by simplifying the basket utility in Equation* (2.1) *in terms of subutilities for each category as*

$$\begin{aligned} u = U(&v_1(X_{1,1}, X_{1,2}, \ldots, X_{1,J_1}),\\ &v_2(X_{2,1}, X_{2,2}, \ldots, X_{2,J_2}), \ldots,\\ &v_M(X_{M,1}, X_{M,2}, \ldots, X_{M,J_M})), \end{aligned} \tag{2.3}$$

where $v_l(\{X_{l,j}\}_{j=1}^{J_l})$ is category l's subutility, which is a function of the qualities and quantities of all brands in category l only. Equation (2.3) *simplifies the utility maximization problem by allowing the interactions between brands across different categories to occur through a common channel—the interactions in their respective category subutilities. Thus, instead of taking into account all pairwise interactions between brands across different categories, we only need to consider the interactions between the subutilities of different categories.*

Assumption 2 (Perfect Substitutes Specification of the Subutilities). *We assume that in any product category, at most one brand can be purchased by a consumer on a given purchase occasion. We thus specify the subutility for any category l as* (*Hanemann* 1984) *as*

$$v_l(\{X_{l,j}\}_{j=1}^{J_l}) = \sum_{j=1}^{J_l} \Psi_{l,j} x_{l,j}. \tag{2.4}$$

Although it is a reasonable assumption for categories such as detergents, softeners, ketchup, etc., in which consumers typically purchase at most one brand on an occasion, it may not hold true for categories like soft drinks, in which consumers do purchase more than one brand. For such cases, in a spirit similar to Bell and Lattin (1998) and Chib et al. (2005), we can further divide such a category into two or more categories such that in each of the subcategories consumers typically purchase at most one brand. By doing so, we can specify a perfect substitute subutility for each subcategory, as in (2.4).[4]

Assumption 3 (Aggregation over Unrelated Product Categories). *If we are only interested in studying the purchase decisions for a subset $\{1, \ldots, L\}$ of the M categories, then under certain conditions we can assume that the remaining categories, $L+1, \ldots, M$ can be aggregated into a composite category. Such aggregation implies that* (i) *the unit prices of all brands in categories $L+1, \ldots, M$ can be summarized in terms of an aggregate unit price p_z,* (ii) *the purchased quantities of categories $L+1, \ldots, M$ can be summarized as $z = z(\{x_{L+1,j}\}_{j=1}^{J_{L+1}}, \ldots, \{x_{M,j}\}_{j=1}^{J_M})$,* (iii) *the subutilities of all categories $L+1, \ldots, M$ can be represented in terms of an aggregate subutility $v_z(z) \equiv \Psi_z z$, where Ψ_z is the quality index associated with the composite category.*

As a result of aggregation, the number of categories that one needs to consider in (2.3) reduces from M

complementarity/substitutability, which is measured by the cross effects of market-mix variables of one brand on the purchase probability of the other brand. These two concepts are not the same. We will discuss the relationship between these two concepts in §2.2.

[3] The details of this derivation are discussed in §1 of the technical appendix.

[4] Alternatively, instead of subdividing such a category, we can also use the "imperfect substitute" specification for the subutility for the category as proposed by Kim et al. (2002), Dubé (2004), or Chan (2006).

to $L+1$, which makes the model a lot more tractable. An important condition that needs to be satisfied (Varian 1992) for aggregation is that preferences over purchase quantities in the set of categories $L+1, \ldots, M$ should be independent of preferences over purchase quantities in categories $1, \ldots, L$. In other words, this condition would be violated if any category l in the group $l \in \{L+1, \ldots, M\}$ is a consumption complement or substitute of any category in the other group $i \in \{1, \ldots, L\}$. In such a case, the utility maximization in (2.2) would suffer from aggregation biases. Thus, to minimize such biases, the group $1, \ldots, L$ should consist of all categories that close consumption complements or substitutes of each other.[5]

2.2. Solution to the Basket Utility Maximization Problem

In this section, we provide an intuitive proof of solution to the utility maximization in (2.2) for a simple case of two categories ($L=2$) after incorporating the three assumptions. A formal proof for the general case of L categories is given in §2 of the technical appendix.

We first represent the two categories by the indices $l = 1, 2$. Next, we reparameterize the utility by letting $x^*_{l,j} = \Psi_{l,j} x_{l,j}$ and $p^*_{l,j} = p_{l,j}/\Psi_{l,j}$. The variable $x^*_{l,j}$ can be interpreted as the *quality-adjusted quantity* and $p^*_{l,j}$ as the *quality-adjusted price*. Thus, the utility maximization can be written as

$$\begin{aligned} &\max_{\{x^*_{l,j}\}, z^*} \; U\left(\sum_{j=1}^{J_1} x^*_{1,j}, \sum_{j=1}^{J_2} x^*_{2,j}, z^*\right) \\ &\text{subject to } \sum_{j=1}^{J_1} p^*_{1,j} x^*_{1,j} + \sum_{j=1}^{J_L} p^*_{2,j} x^*_{2,j} + p^*_z z^* = y \\ &\qquad\qquad\qquad \text{(budget constraint)} \\ &\qquad z^* \geq 0, \quad x^*_{l,j} \geq 0 \\ &\qquad\quad \text{for all } j \in (1, \ldots, J_l),\ l \in \{1, 2\} \\ &\qquad\qquad\qquad \text{(nonnegativity constraints).} \end{aligned} \tag{2.5}$$

Given the problem in (2.5), we next specify the solutions for the brand choice decision (conditional on purchase in the category) and the joint purchase incidence decisions.

Conditional Brand Choice Decision. If brand k is chosen in a purchased category l, then the brand choice decision in category l, $d_{l,k}$, would be

$$d_{l,k} = 1 \quad \text{if } p^*_{l,k} \leq p^*_{l,j} \quad \forall j \in \{1, \ldots, J_l\}. \tag{2.6}$$

Two implications follow from (2.6): First, the conditional brand choice decision is independent of the specification of the direct utility U and the number of categories, L; and second, the chosen brand should have the lowest quality-adjusted price among all brands in that category. These implications can be understood by observing that the subutility of any category l, $\sum x^*_{l,j}$, in Equation (2.5) is additive and symmetric with respect to the quality-adjusted quantities of all brands $j = 1, \ldots, J_l$ in that category. Thus, the only differentiating factor among the brands in category l are their quality-adjusted prices $\{p^*_{l,j}\}_{j=1}^{J_l}$. This implies that the choice of the brand with the lowest quality-adjusted price in the category is optimal, and is independent of the specification of direct utility U and the number of categories L.

Joint Category Purchase Incidence Decisions. Because at most one brand can be purchased in a category (the brand with the lowest quality-adjusted price), a category will be (not) purchased if the demand for the brand in that category with lowest quality-adjusted price is (zero) positive. Thus, consider the case where the brand *indexed by* "k" has the lowest quality-adjusted price in each of the two categories. Because all brands other than k in both categories will not be chosen regardless of whether the categories are purchased or not, we set the demand for such brands to zero. Thus, the utility maximization in (2.5) reduces to

$$\begin{aligned} &\max_{\{x^*_{l,k}\}_{l=1}^{L}, z^*} \; U(x^*_{1,k}, x^*_{2k}, z^*) \\ &\text{subject to } p^*_{1,k} x^*_{1,k} + p^*_{2,k} x^*_{2,k} + p^*_z z^* = y \\ &\qquad\qquad\qquad \text{(budget constraint)} \\ &\qquad x^*_{1,k} \geq 0, \quad x^*_{2,k} \geq 0, \quad z^* \geq 0 \\ &\qquad\qquad\qquad \text{(nonnegativity constraints).} \end{aligned} \tag{2.7}$$

Given the utility maximization in (2.7), we next discuss the theoretical specification of the joint incidence conditions for three cases: First when both categories are purchased, second when only Category 1 is purchased, and third when none of the two categories are purchased.

[5] We assume that the consumer simultaneously allocates the total basket expenditure, y, among all the categories. Thus, if the L categories are the laundry family of categories, the composite commodity will contain all other categories that are unrelated by consumption to the laundry categories. If, however, the consumer's decision rule was a two-stage process (in the first stage, the consumer a priori decides, without looking at the prices, how to allocate the total basket expenditure among a broad group of categories, such as cleaning products that include the laundry categories, food, etc.; and in the second stage, conditional on the expenditure on the cleaning categories, she decides how much money to allocate within the laundry categories), our model would capture the second stage of the decision process, except that the total basket expenditure, y, would be replaced by the expenditure on the cleaning products and the composite commodity would instead consist of those categories in the cleaning group of categories that are unrelated by consumption to the laundry categories.

Case 1. Both categories are purchased: In this case, the utility maximization in (2.7) should yield positive interior solutions for quality-adjusted quantities for both categories; that is, $x_{1,k}^* > 0$, $x_{2,k}^* > 0$. The interior solutions for both categories can be written as functions of the covariates in (2.7) as $x_{1,k}^* = x_1(p_{1,k}^*, p_{2,k}^*, p_z^*, y)$ and $x_{2,k}^* = x_2(p_{1,k}^*, p_{2,k}^*, p_z^*, y)$. Because brand k has the lowest quality-adjusted price in its category, we substitute $p_{l,k}^* = \min_j p_{l,j}^*$ into the demand functions of both categories to get the joint purchase incidence conditions as

$$\begin{aligned} &\text{Category 1:} \quad x_1(\min_j p_{1,j}^*, \min_j p_{2,j}^*, p_z^*, y) > 0;\\ &\text{Category 2:} \quad x_2(\min_j p_{1,j}^*, \min_j p_{2,j}^*, p_z^*, y) > 0. \end{aligned} \tag{2.8}$$

Case 2. Only Category 1 is purchased: For Category 1, the utility maximization in (2.7) should yield positive interior solution for the quality-adjusted quantities for brand k. Thus, if we can characterize the interior solution for the quality-adjusted quantity of brand k in Category 1, we can specify the incidence condition for Category 1 as $x_{1,k}^* > 0$. For Category 2, the utility maximization in (2.7) yields a boundary solution at zero for the quality-adjusted quantity of brand k (that is, $x_{2,k}^* = 0$). This implies that as long as the purchase quantity of brand k in Category 2 is zero, any local variation in its quality-adjusted price, $p_{2,k}^*$, will not change the solution to the utility maximization. Thus, a question that follows is: To what threshold level, R_2^*, can the quality-adjusted price of brand k in Category 2 be decreased such that the demand for that brand k is still zero and the solution to the maximization remains unchanged? This threshold price R_2^* will be such that if $p_{2,k}^* > R_2^*$, then $x_{2,k}^* = 0$, which is a boundary solution; if $p_{2,k}^* = R_2^*$, then $x_{2,k}^* = 0$, which is an interior solution; and if $p_{2,k}^* < R_2^*$, then $x_{2,k}^* > 0$, which is an interior solution. Thus, if we can characterize the threshold quality-adjusted price for Category 2, we can specify its incidence condition as $p_{2,k}^* \geq R_2^*$.

Next, we modify the utility maximization in (2.7) that simultaneously yields the specifications of $x_{1,k}^*$ and R_2^*. Consider the case where brand k in Category 1 were to charge its quality-adjusted price, $p_{1,k}^*$, while brand k in Category 2 were to instead charge the threshold quality-adjusted price R_2^*. For such a case, we will get the same solution as in (2.7), except that the nonpurchase solution for brand k in Category 2 will be an interior solution. The modified utility maximization will be

$$\begin{aligned} &\max_{\{x_{l,k}^*\}_{l=1}^2 z^*} \quad U(x_{1,k}^*, x_{2,k}^*, z^*),\\ &\text{subject to} \quad p_{1,k}^* x_{1,k}^* + R_2^* x_{2,k}^* + p_z^* z^* = y. \end{aligned} \tag{2.9}$$

The interior solutions for the quality-adjusted quantities of brands k in both categories will be functions of covariates in (2.9) as $x_1^* = x_1(p_{1,k}^*, R_2^*, p_z^*, y)$ and $x_2^* = x_2(p_{1,k}^*, R_2^*, p_z^*, y)$. Because Category 2 is not purchased, we set its quantity to zero; that is, $x_2(p_{1,k}^*, R_2^*, p_z^*, y) = 0$. Inverting it, we get the specification of the threshold quality-adjusted price of Category 2 as

$$R_2^* = R_2(p_{1,k}^*, p_z^*, y). \tag{2.10a}$$

Next, we plug in the expression of R_2^* in (2.10a) into the solution for the quality-adjusted quantity of Category 1 to get

$$x_1^* = x_1(p_{1,k}^*, R_2(p_{1,k}^*, p_z^*, y), p_z^*, y). \tag{2.10b}$$

Thus, we have the specifications of the threshold quality-adjusted prices for Category 2, R_2^*, in (2.10a) and the quality-adjusted quantity for Category 1, $x_{1,k}^*$, in (2.10b). Substituting $p_{l,k}^* = \min_j p_{l,j}^*$ into (2.10a) and (2.10b), we get the joint purchase incidence conditions as

$$\begin{aligned} &\text{Category 1:} \; x_1\Big(\min_j p_{1,j}^*, R_2\Big(\min_j p_{1,j}^*, p_z^*, y\Big), p_z^*, y\Big) > 0;\\ &\text{Category 2:} \; R_2\Big(\min_j p_{1,j}^*, p_z^*, y\Big) \leq \min_j p_{2,j}^*. \end{aligned} \tag{2.11}$$

Case 3. None of the categories are purchased: For this case, the utility maximization in (2.7) yields boundary solutions at zero for quality-adjusted quantities of brands k in both categories. Thus, similar to Case 2, we modify the utility maximization in (2.7) that simultaneously yields specifications of threshold quality-adjusted prices $\{R_l^*\}_{l=1}^2$ for both categories—and given these specifications, the incidence conditions for both categories will be $p_{1,k}^* \geq R_1^*$ and $p_{2,k}^* \geq R_2^*$. Thus, consider the modified utility maximization in which brands k in both categories were instead to charge their respective threshold prices, $\{R_l^*\}_{l=1}^2$:

$$\begin{aligned} &\max_{\{x_{l,k}^*\}_{l=1}^2, z^*} \quad U(x_{1,k}^*, x_{2,k}^*, z^*),\\ &\text{subject to} \quad R_1^* x_{1,k}^* + R_2^* x_{2,k}^* + p_z^* z^* = y. \end{aligned} \tag{2.12}$$

The utility maximization in (2.12) will yield the same solution as in (2.7) except that the nonpurchase solutions for both categories will be interior solutions. The interior solutions for the quality-adjusted quantities of brands k in both categories will be functions of covariates in (2.12) as $x_1^* = x_1(R_1^*, R_2^*, p_z^*, y)$ and $x_2^* = x_2(R_1^*, R_2^*, p_z^*, y)$. Setting these two interior solutions to zero (because both categories are not purchased) and inverting the two equations yields the threshold quality-adjusted prices of both categories as functions of the remaining variables as $R_l^* = R_l(p_z^*, y)\ \forall\, l = \{1, 2\}$. Thus, the joint purchase incidence conditions are

$$\begin{aligned} &\text{Category 1:} \quad R_1(p_z^*, y) \le \min_j p_{1,j}^*;\\ &\text{Category 2:} \quad R_2(p_z^*, y) \le \min_j p_{2,j}^*. \end{aligned} \tag{2.13}$$

In summary, Equations (2.8), (2.11), and (2.13) represent the joint incidence conditions for the three different menus of purchases. We next discuss the implications of these conditions:

(i) *Cross Effects in Purchase Incidence Conditions.* Notice in (2.8) and (2.11) that the incidence condition for any category is a function of the quality-adjusted prices of brands in that category (the own effects) and the quality-adjusted prices of brands in the other *purchased* category (the cross effects). These cross effects (which we hereby refer to as *purchase* complementarity/substitutability) from the other purchased category stem from two sources: (i) interactions in subutilities of the two categories (which is a measure of their *consumption* complementarity/substitutability) in the *direct* utility in (2.5), and (ii) the budget constraint.

The roles played by the two sources can be understood by considering two consumption complements—softeners and detergents, where detergent is one of the purchased categories. A decrease in the price of detergents will increase its demand, which will lead to the following two effects: (i) it will increase the marginal utility of softeners (as a result of consumption complementarity), which will increase the purchase probability of softeners. Thus, the first factor (consumption complementarity) will lead to purchase complementarity between the two categories. (ii) Depending on the elasticity of demand for detergents, the budget allocated to all other categories (including softeners) can decrease if demand elasticity for detergents is greater than one (and vice versa if demand elasticity is less than one). A decrease in the budget allocated to all other categories will decrease the purchase probability of softeners. Thus the second factor (budget constraint) will lead to purchase substitutability between the two categories. The overall extent of purchase complementarity/substitutability will depend on the relative magnitudes of the two factors.

Equations (2.11) and (2.13) also show that the incidence condition for any category does not depend on the qualities or prices of brands in other nonpurchased categories. This stands in contrast to the prior reduced-form specifications (e.g., Manchanda et al. 1999), in which the incidence condition of any category depends on the market-mix variables of all other categories regardless of whether the other categories are purchased or not.

(ii) *Generalizing to L categories*: Consider the case where only categories $1, \ldots, m$ are purchased (and categories $m+1, \ldots, L$ are not). As discussed in the two-category case, for a category to be purchased, its quality-adjusted quantity should be positive; and for a category not to be purchased, the quality-adjusted prices of all of its brands should be greater than its threshold quality-adjusted price. The process of characterizing the quality-adjusted quantities of all purchased categories and threshold quality-adjusted prices of all nonpurchased categories is similar to the two-category case. We first modify the utility maximization such that there is one brand k per category (brand with lowest quality-adjusted price in that category), each purchased category $i = 1, \ldots, m$ is assumed to charge its quality-adjusted price $p_{i,k}^*$, and each nonpurchased category $i = m+1, \ldots, L$ is assumed to charge its threshold quality-adjusted price $R_{i,k}^*$. This yields the interior solutions for the quality-adjusted quantities of all L categories as $\{x_l(\{p_{i,k}^*\}_{i=1}^m, \{R_i^*\}_{i=m+1}^L, p_z^*, y)\}_{l=1}^L$. Next, we set demand of all nonpurchased categories to zero:

$$x_l(\{p_{i,k}^*\}_{i=1}^m, \{R_i^*\}_{i=m+1}^L, p_z^*, y) = 0 \quad \text{for all } l \in \{m+1, \ldots, L\}. \tag{2.14}$$

Solving the $L-m$ equations in (2.14) yields the threshold quality-adjusted prices of nonpurchased categories, $\{R_l^*\}_{l=m+1}^L$, as functions of the remaining variables in (2.14) as

$$R_l^* = R_l(\{p_{i,k}^*\}_{i=1}^m, p_z^*, y) \quad \text{for all } l \in \{m+1, \ldots, L\}. \tag{2.15}$$

Next, given the expressions of $\{R_l^*\}_{l=m+1}^L$ in (2.15), we plug them into the interior solutions for the quality-adjusted quantities of all purchased categories $l = 1, \ldots, m$ to get

$$x_l^* = x_l(\{p_{i,k}^*\}_{i=1}^m, \{R_i(.)\}_{i=m+1}^L, p_z^*, y) \quad \forall\, l \in \{1, \ldots, m\}. \tag{2.16}$$

Thus, we have the specifications of $\{R_l^*\}_{l=m+1}^L$ in (2.15) and $\{x_{l,k}^*\}_{l=1}^m$ in (2.16). Substituting $p_{i,k}^* = \min_j p_{i,j}^*$ into (2.15) and (2.16), we get the joint purchase incidence conditions as

Purchased Categories:

$$x_l\left(\left\{\min_j p_{i,j}^*\right\}_{i=1}^m, \{R_i(.)\}_{i=m+1}^L, p_z^*, y\right) > 0 \quad \text{for all } l \in \{1, \ldots, m\};$$

Nonpurchased Categories:

$$R_l\left(\left\{\min_j p_{i,j}^*\right\}_{i=1}^m, p_z^*, y\right) \le \min_j p_{l,j}^* \quad \text{for all } l \in \{m+1, \ldots, L\} \tag{2.17}$$

2.3. Deriving the Joint Purchase Incidence Conditions from an Indirect Basket Utility

In §2.2, we have shown that the joint category purchase incidence conditions *only* requires the specifications of the demand functions, $\{x_l(\{q_i\}_{i=1}^L, p_z^*, y)\}_{l=1}^L$ for all L categories, in which (i) there is only one brand per category; (ii) a purchased category i is assumed to charge the lowest quality-adjusted price in that category, that is, $q_i = \min_j p_{i,j}^*$; (iii) a nonpurchased category i is assumed to charge the threshold quality-adjusted price $q_i = R_i^*$.

This result is crucial because it obviates the need to derive the joint purchase incidence conditions using the first-order conditions that follow from direct utility maximization. It shows that as long as we are given a set of "well-behaved" demand functions $\{x_l(\{q_i\}_{i=1}^L, p_z^*, y)\}_{l=1}^L$ for each of the L categories, we can use the procedure detailed in §2.2 to derive the joint category purchase incidence conditions. A straightforward way to get such a set of L demand functions is to derive them from an indirect utility using the Roy's Identity. Doing so will not only be less burdensome, but would also yield tractable demand functions that allow for a great deal of flexibility in terms of the interactions among the categories (Deaton and Muellbauer 1980). On the other hand, if we were to derive tractable category purchase incidence conditions using a direct utility, we would have to restrict ourselves to simple functional forms of direct utilities like the linear expenditure system, Cobb-Douglas, or constent expenditure system. Using these functional forms is impractical for cross-category analysis because they place strong restrictions on the nature of interrelationships between categories (Samuelson 1965) that can lead to significant biases in estimates of cross-price elasticities (Deaton 1974).

As we show, the specification of the purchase incidence conditions from an indirect utility becomes even simpler—it only requires the specification of the numerator of the Roy's Identity. The proof of this result is given in §2 of the technical appendix.

Joint Purchase Incidence Conditions. Consider an indirect basket utility $V \equiv V(\{q_i\}_{i=1}^L, p_z^*, y)$, in which (i) there are $L+1$ categories and each category has only one brand; (ii) the total basket expenditure is y; (iii) the unit prices of the L categories are $\{q_i\}_{i=1}^L$; and (iv) the price of the composite commodity ($(L+1)$th category) is the quality-adjusted price p_z^*. The joint incidence conditions can be derived by first specifying a function s_l (which is the numerator of the Roy's Identity) for each category $l = 1, \ldots, L$ as

$$s_l(\{q_i\}_{i=1}^L, p_z^*, y) \equiv -\partial V(\{q_i\}_{i=1}^L, p_z^*, y)/\partial q_l. \qquad (2.18)$$

Given the functions $\{s_l(\{q_i\}_{i=1}^L, p_z^*, y)\}_{l=1}^L$ for L categories, the procedure for deriving the joint incidence conditions is discussed in the appendix—it is essentially identical to that discussed in §2.2, except that instead of using demand functions, $\{x_l(.)\}_{l=1}^L$, we use the functions $\{s_l(.)\}_{l=1}^L$ in Equations (2.14)–(2.17). Because the knowledge of the specifications of numerators of the Roy's Identity for all categories is sufficient to derive the purchase incidence conditions, in the analysis that follows, we refer to s_l as the "incidence function" for category l.

The results so far allow us to immediately derive the conditions under which the joint incidence conditions for L categories reduce to L single-category incidence conditions. This would happen when the "incidence function" for any category l does not depend on prices of other $L-1$ categories, that is, $s_l = s_l(q_l, p_z^*, y)$. This is only possible if the indirect utility V is additive in L categories; that is, $V(\{q_l\}_{l=1}^L, p_z^*, y) = \sum_{l=1}^L V_l(q_l, p_z^*, y)$. Such a specification yields the incidence function for any category l as $s_l = -\partial V(q_l, p_z^*, y)/\partial q_l$, which is independent of prices in other $L-1$ categories. In §4.3, we discuss the implications of this "indirectly additive model" and show how it can potentially bias the estimates.

3. Econometric Specification of the Proposed Model

In §3.1, we specify the functional form of quality indices of brands and the *indirect* basket utility to get the stochastic specification of brand choice and joint purchase incidence conditions. In §3.2 we compare the stochastic specification of the proposed model with a nested specification that assumes indirect additivity.

3.1. Specification of the Functional Forms for Quality Indices and the Indirect Utility Specification of the Quality Indices

We specify the consumer's quality index of brand j in category l, similar to that used by Nair et al. (2005), as

$$\Psi_{l,j} = \exp((\beta_l' F_{l,j} + \varepsilon_{l,j})/\mu_l). \qquad (3.1)$$

In Equation (3.1), the explanatory variables in the vector $F_{l,j}$ impact the preference for brand j and the subutility of category l. The econometrician's error, $\varepsilon_{l,j}$, is assumed to be extreme valued, that is, independent across all brands, categories, consumers, and shopping trips. The parameters β_l' represent the sensitivities of explanatory variables on the preference for the brand, and the parameter μ_l is the inverse of consumer's quality sensitivity in category l.

The explanatory variables $F_{l,j}$. Recall that while the conditional brand choice decision depends on the relative valuations of quality indices among all brands in the category, the purchase incidence decisions depend on the absolute levels of quality indices of the brands in the category. Thus, if the explanatory variable is specific to brand j in category l, it would not only impact brand j's conditional choice probability, but also the joint purchase incidence probabilities. On the other hand, if the explanatory variable is category specific, it would only impact the joint purchase incidence probabilities. We thus specify two sets of explanatory variables: The first set consists of brand-specific variables and the second set consists of category-specific variables.

The brand-specific variables that we include in the first set are: (i) the brand dummies, (ii) presence of promotions on the brand (that includes features and displays), and (iii) state dependence or brand loyalty. The rationale for choosing promotional variables follows from the findings of prior literature, which show that brand promotions can significantly impact the conditional brand choice probabilities, the purchase incidence probability for that category, and the purchase incidence probabilities of other categories (Chiang 1991, Chintagunta 1993, Manchanda et al. 1999). Similarly, the rationale for choosing brand loyalty follows from findings of prior literature, which show that it has a significant impact on both brand choice and purchase incidence decisions (Chiang 1991, Chintagunta 1993). The category-specific variable we include in the second set is inventory of the category. Because the inventory of category l is the same across all brands in category l, it will only play a role in the purchase incidence decisions of the categories. Thus the vector of the explanatory variables in Equation (3.1) can be represented as

$$\beta_l' F_{l,j} = \alpha_{l,j} + \beta_{l,\text{pro}} \text{Pro}_{l,j} + \beta_{l,\text{loy}} \text{GL}_{l,j} + \beta_{l,\text{inv}} \ln(\text{Inv}_l + 1), \tag{3.2}$$

where $\alpha_{l,j}$, is the dummy for brand j in category l, $\text{Pro}_{l,j}$ indicates presence of promotions for brand j in category l, $\text{GL}_{l,j}$ represents the Guadagni and Little's (GL) brand loyalty, Inv_l denotes the household's inventory of category l on the given purchase occasion, and $\{\beta_{l,\text{pro}}, \beta_{l,\text{loy}}, \beta_{l,\text{inv}}\}$ are the sensitivity parameters that are assumed to be category specific. Next, we specify the quality-adjusted price of the composite commodity as

$$p_z^* = \exp(-\tau - \varepsilon_z \mu_z), \tag{3.3}$$

where τ is the deterministic component and $\varepsilon_z \mu_z$ is the random shock in the quality-adjusted price of the composite commodity z. ε_z is assumed to be an extreme-valued random variable that is independent across all consumers and all purchase occasions and also independent of the errors $\varepsilon_{l,j}$. The term μ_z is the factor by which the shocks ε_z are scaled. Because the composite commodity comprises all brands in all categories in the store (excluding categories under consideration, $1, \ldots, L$), $\varepsilon_z \mu_z$ represents the aggregate measure of the unobserved variation in the preferences of all brands in the composite commodity.

Specification of the Indirect Basket Utility and the Incidence Functions. As discussed in §2.2, we can specify the incidence conditions by specifying the indirect utility, assuming that there is only one brand in each category. We choose a flexible form of the indirect utility in the Translog family, which is the Basic Translog (BTL) Indirect Utility (Christensen et al. 1975, Pollak and Wales 1992). The BTL indirect utility for the $L+1$ categories is given as

$$V = -\sum_{l=1}^{L+1} \hat{a}_l \ln \frac{q_l}{y} + \frac{1}{2} \sum_{l=1}^{L+1} \sum_{i=1}^{L+1} b_{l,i} \ln \frac{q_i}{y} \ln \frac{q_l}{y}, \tag{3.4}$$

where $\{q_i\}_{i=1}^{L}$ are the unit prices of the L categories and $q_{L+1} = p_z^*$ is the quality-adjusted price of the composite commodity z ($(L+1)$th category). In the BTL indirect utility, there are two sets of parameter restrictions. The first is that of symmetry of the $(L+1) \times (L+1)$ parameter matrix $B \equiv \{b_{l,i}\}$, that is $b_{i,l} = b_{l,i}$. The second set of restrictions is that the diagonal terms $b_{l,l}$ of the parameter matrix B should be strictly positive and B should be positive definite. Using Equations (3.4) and (2.18), the incidence function for category l can be derived as

$$s_l(\{q_i\}_{i=1}^{L}, p_z^*, y) = \hat{a}_l - \sum_{l=1}^{L} b_{l,i} \ln q_i - b_{l,z} \ln p_z^* + \left(\sum_{l=1}^{L} b_{l,i} + b_{l,z} \right) \ln y. \tag{3.5a}$$

Substituting $\ln p_z^* = -(\tau + \varepsilon_z \mu_z)$ into Equation (3.5a) and reparameterizing Equation (3.5a) by defining $a_l = \hat{a}_l + b_{l,z}\tau$, we get the incidence function of category l as

$$s_l = a_l - \sum_{i=1}^{L} b_{l,i} \ln q_i + b_{l,z} \mu_z \varepsilon_z + \left(\sum_{i=1}^{L} b_{l,i} + b_{l,z} \right) \ln y. \tag{3.5b}$$

Given the incidence function in (3.5b), we next explain its constituents:

(i) The term a_l is the intercept in the incidence function for category l. It can be interpreted as the consumer's intrinsic preference to purchase category l.

(ii) The term $b_{l,l}$ in the $(L+1) \times (L+1)$ matrix $B \equiv \{b_{l,i}\}$ represents the own effects of category l's prices on its purchase incidence. The parameter $b_{l,i}$ represents the cross effects of category i on the purchase incidence of category l—if $b_{l,i} > 0$, the purchase incidence probability of category l increases if the price of category i decreases (and vice versa if $b_{l,i} < 0$). This implies that if $b_{l,i} > 0$, categories i and l will be purchase complements, if $b_{l,i} < 0$, categories i and l will be purchase substitutes. Thus, in the analysis that follows, we refer to the matrix B as the "own and cross effects matrix."

(iii) The term $b_{l,z}\mu_z\varepsilon_z$ represents the impact of unobserved shocks in the preference for the composite good on the incidence function of category l. Because the aggregate shocks, ε_z, are common across the incidence functions of all categories, they result in correlations in the joint incidence conditions, which can be interpreted as the coincidence effects (Manchanda et al. 1999).

(iv) The set of terms $\sum_{i=1}^{L} b_{l,i} + b_{l,z}$ denotes the impact of the total basket size y on the incidence function of category l. If $\sum_{i=1}^{L} b_{l,i} + b_{l,z} > 0$, it implies that the purchase probability of the category increases with an increase in the total basket expenditure.

The aforementioned points show that the joint purchases of any two categories l and i are influenced by consumer heterogeneity, extent of purchase complementarity/substitutability between the two categories, and the coincidence effects. In other words, we will observe a high incidence of joint purchases in both categories if: (i) the consumer's intrinsic preferences for both categories, a_l and a_i, is high; or (ii) cross effect between categories i and l, $b_{l,i}$, is positive, implying both categories are purchase complements or (iii) the coefficients of the aggregate shocks for the two categories, $\mu_z b_{l,z}$ and $\mu_z b_{i,z}$, are large and are of the same sign, implying positive correlations due to the coincidence effects.

We next discuss the stochastic specifications of the conditional brand choice and the joint category purchase incidence decisions that follow from the specifications of the quality indices and the incidence functions. The stochastic specification of the joint purchase conditions for a general case of L categories is derived in §3 of the technical appendix. For expositional ease, we only discuss its result for a two-category case, when $L=2$.

Brand Choice Decision. If brand k is the optimal brand in category l, it follows from Equations (2.11) and (3.1) that the brand choice condition conditional on $I_l = 1$ will be

$$\begin{aligned}&\alpha_{l,k} + \beta_{l,\text{pro}}\text{Pro}_{l,k} + \beta_{l,\text{loy}}\text{GL}_{l,k} - \mu_l \ln p_{l,k} + \varepsilon_{l,k}\\ &\quad \geq \alpha_{l,j} + \beta_{l,\text{pro}}\text{Pro}_{l,j} + \beta_{l,\text{loy}}\text{GL}_{l,j} - \mu_l \ln p_{l,j} + \varepsilon_{l,j}.\end{aligned} \tag{3.6}$$

Because the error terms, $\varepsilon_{l,j}$, are i.i.d. extreme value distributed, the brand choice probability conditional on incidence in the category will be

$$\begin{aligned}&\Pr(d_{l,k}=1 \mid I_l = 1)\\ &\quad = \frac{\exp(\alpha_{l,k} + \beta_{l,\text{pro}}\text{Pro}_{l,k} + \beta_{l,\text{loy}}\text{GL}_{l,k} - \mu_l \ln p_{l,k})}{\sum_{j=1}^{J_l} \exp(\alpha_{l,j} + \beta_{l,\text{pro}}\text{Pro}_{l,j} + \beta_{l,\text{loy}}\text{GL}_{l,j} - \mu_l \ln p_{l,j})}.\end{aligned} \tag{3.7}$$

Note that Equations (3.6)–(3.7) restrict the brand choice probabilities to be independent across categories at the *observational level*. Prior research has relaxed this restriction at the *consumer level* (e.g., Ainslie and Rossi 1998) by allowing for correlated heterogeneity in brand choice parameters across categories.[6] In a similar vein, we will allow for cross-category correlated heterogeneity in the inverse of the quality sensitivities (or the price sensitivities in brand choice decisions), $\{\mu_l\}_{l=1}^{L}$. The implication of doing so is that if $\{\mu_l\}_{l=1}^{L}$ are positively correlated across categories, then a consumer's choice of premium brand in one category would positively affect her choice probability of premium brands in other categories.

Joint Purchase Incidence Conditions of Categories. Given the specification of the incidence function in Equation (3.5b), we can derive the joint category purchase incidence conditions using the procedure outlined in the appendix. We discuss the joint incidence conditions for three cases: first when both categories are purchased, second when only Category 1 is purchased, and third where none are purchased. For all cases, we will represent the purchase incidence condition for any category l in terms of its inclusive value, W_l, which is a function of the quality indices and prices of all brands in that category l,

$$\begin{aligned}W_l \equiv \ln\bigg(\sum_{j=1}^{J_l} &\exp\big(\alpha_{l,j} + \beta_{l,\text{pro}}\text{Pro}_{l,j} + \beta_{l,\text{loy}}\text{GL}_{l,j}\\ &- \mu_l \ln p_{l,j} + \beta_{l,\text{inv}} \ln(\text{Inv}_l + 1)\big)\bigg),\end{aligned} \tag{3.8}$$

such that category l will be purchased (not purchased) if the inclusive value of category l exceeds (is less than) the threshold category inclusive value for that category[7] W_{IR}. The difference across the three cases will be that the specification of W_{IR} would change

[6] It is noteworthy to mention that a recent work by Chib et al. (2005) relaxes this restriction at the observational level by allowing the errors in the utilities of brands to be correlated across categories. If we were to translate their approach, we would need to allow the errors $\varepsilon_{l,j}$ (in Equation (3.6)) to be correlated across categories (instead of assuming that they are iid). However, doing so would substantially complicate the stochastic specifications of the joint purchase incidence conditions in our proposed model—because the brand choice and purchase incidence decisions are structurally related to each other. On the other hand, Chib et al. (2005) are still able to maintain tractability because they use a statistical approach in which the brand choice and purchase incidence decisions are not structurally related.

[7] Because the stochastic specification of joint incidence conditions is given in terms of category inclusive values (which is different from the theoretical specification of joint incidence conditions in §2.2, which was given in terms of category quality-adjusted prices), it is more intuitive to use the concept of threshold inclusive values rather than threshold quality-adjusted prices (that were used in §2.2).

depending on the menu of categories that are purchased.

Case 1. Both Categories 1 and 2 Are Purchased. The joint purchase incidence conditions can be shown to be

$$\{I_1 = 1,\ I_2 = 1\} \quad \text{if} \begin{cases} W_1 + \varepsilon_1 > W_{1R}, \\ W_2 + \varepsilon_2 > W_{2R}, \end{cases} \tag{3.9}$$

where ε_l is an extreme-valued error associated with the inclusive value of category l, which is independent across all categories, all consumers, and all purchase occasions. The threshold inclusive value of any of the two categories l, W_{lR}, is a function of that category's intrinsic preference a_l, the aggregate shock ε_z, the total basket expenditure y, and the *cross effects* from the inclusive value of the other category as

$$\begin{aligned} W_{1R} &= -((a_1 + b_{1,z}\varepsilon_z\mu_z + (b_{1,1} + b_{1,2} + b_{1,z})\ln y \\ &\qquad + b_{1,2}\mu_2^{-1}(W_2 + \varepsilon_2))/b_{1,1})\mu_1, \\ W_{2R} &= -((a_2 + b_{2,z}\varepsilon_z\mu_z + (b_{2,2} + b_{1,2} + b_{2,z})\ln y \\ &\qquad + b_{1,2}\mu_1^{-1}(W_1 + \varepsilon_1))/b_{2,2})\mu_2. \end{aligned} \tag{3.10}$$

From Equations (3.8)–(3.10), the following observations can be made regarding the roles of own, cross and coincidence effects. Regarding the own effects, the prices and promotions of all brands in any category l and the impact of inventories on preference for category l influence the purchase of category l through the category inclusive value W_l. Regarding the cross effects, prices and promotions of all brands in category i influence the threshold inclusive value of the other category $l \neq i$ through its category inclusive value W_i. Finally, notice that the common error terms in the incidence conditions of categories are the aggregate shocks ε_z, the errors in inclusive values of both categories: ε_1 and ε_2. Thus, the correlations in purchase conditions for the categories stem from both coincidence (that results from common aggregate shocks, ε_z) and the extent of purchase complementarity/substitutability between the categories (that results from the common errors ε_1 and ε_2).[8]

Case 2. Only Category 1 Is Purchased. The joint category purchase incidence conditions are

$$\{I_1 = 1,\ I_2 = 0\} \quad \text{if} \begin{cases} W_1 + \varepsilon_1 > W_{1R}, \\ W_2 + \varepsilon_2 \le W_{2R}, \end{cases} \tag{3.11}$$

where the threshold inclusive values of the two categories are given as

$$\begin{aligned} W_{1R} &= -((a_1 b_{2,2} - b_{1,2}a_2 + \varepsilon_z\mu(b_{2,2}b_{1,z} - b_{1,2}b_{2,z}) \\ &\qquad + \ln y(b_{1,1}b_{2,2} - b_{1,2}^2 + b_{2,2}b_{1,z} - b_{1,2}b_{2,z})) \\ &\qquad \cdot (b_{1,1}b_{2,2} - b_{1,2}^2)^{-1})\mu_1, \end{aligned} \tag{3.12}$$

$$\begin{aligned} W_{2R} &= -((a_2 + b_{2,z}\varepsilon_z\mu_z + (b_{2,2} + b_{1,2} + b_{2,z})\ln y \\ &\qquad + b_{1,2}\mu_1^{-1}(W_1 + \varepsilon_1)) \cdot (b_{2,2})^{-1})\mu_2. \end{aligned}$$

Note that the threshold inclusive value of the nonpurchased Category 2, W_{2R}, is identical in Cases 1 and 2. However, the threshold inclusive value of the purchased Category 1, W_{1R}, in Case 2 is different from that in Case 1—it is not a function of the market-mix variables in Category 2. This shows that while there would be cross effects of Category 1's market-mix variables on the incidence condition of Category 2, there would be no cross effects of Category 2's market-mix variables on the incidence condition of Category 1.[9] Further, unlike Case 1, the common error terms in the incidence conditions of the two categories for Case 2 in (3.11) are the aggregate shock ε_z and the first category's error ε_1. This suggests that the correlation in incidence conditions of the categories changes from Case 1 to Case 2.

Case 3. None of the Two Categories Are Purchased. The joint category purchase incidence conditions for this case are

$$\{I_1 = 0,\ I_2 = 0\} \quad \text{if} \begin{cases} W_1 + \varepsilon_1 \le W_{1R}, \\ W_2 + \varepsilon_2 \le W_{2R}, \end{cases} \tag{3.13}$$

where the threshold inclusive values of the two categories are given as

$$\begin{aligned} W_{1R} &= -((a_1 b_{2,2} - b_{1,2}a_2 + \varepsilon_z\mu_z(b_{2,2}b_{1,z} - b_{1,2}b_{2,z}) \\ &\qquad + \ln y(b_{1,1}b_{2,2} - b_{1,2}^2 + b_{2,2}b_{1,z} - b_{1,2}b_{2,z})) \\ &\qquad \cdot (b_{1,1}b_{2,2} - b_{1,2}^2)^{-1})\mu_1, \\ W_{2R} &= -((a_2 b_{1,1} - b_{1,2}a_2 + \varepsilon_z\mu_z(b_{1,1}b_{2,z} - b_{1,2}b_{1,z}) \\ &\qquad + \ln y(b_{1,1}b_{2,2} - b_{1,2}^2 + b_{1,1}b_{2,z} - b_{1,2}b_{1,z})) \\ &\qquad \cdot (b_{1,1}b_{2,2} - b_{1,2}^2)^{-1})\mu_2. \end{aligned} \tag{3.14}$$

Note that the threshold inclusive value of Category 1, W_{1R}, is identical in Cases 2 and 3. However, the threshold inclusive value of purchased Category 2, W_{2R}, is different from its specification in Case 2—it is not a function of the market-mix variables in Category 1. Thus, Equation (3.14) shows that when none of the categories are purchased, there are no cross effects in the incidence conditions of the categories. Further,

[8] Note that if there were no cross effects between the two categories, that is, if $b_{1,2} = 0$, then the incidence conditions of the two categories would not have the common error terms ε_1 and ε_2.

[9] This result follows from our discussion in §2.2, where we showed that the incidence condition for any category *only* depends on the quality-adjusted prices of brands in that category and those in other *purchased* categories.

the only common error term in incidence conditions of the two categories is the aggregate shock ε_z. Therefore, unlike Cases 1 and 2, the correlations in incidence conditions in Case 3 only stem from *coincidence effects*.

Cases 1–3 show that the specifications of the threshold inclusive values of a category depends on whether or not the other category is purchased. As a result, the cross and coincidence effects vary depending on the menu of the purchased categories. Comparing these specifications with those used in prior cross-category purchase incidence models (e.g., Manchanda et al. 1999), we see that in prior models, (i) the specification of correlations in purchase incidence conditions of all categories are assumed to be the same regardless of which categories are purchased, and (ii) the purchase incidence condition of any category i is assumed to depend on the cross effects of all other categories regardless of whether the other categories are purchased or not. This difference highlights the importance of specifying the cross and coincidence effects by building a model from first principles.

We next turn our attention to the computation of the joint incidence probabilities. Because the joint incidence probabilities do not have a closed-form expression, they have to be computed using Monte Carlo simulations. Given the complicated structure of the errors, $\{\varepsilon_1, \varepsilon_2, \varepsilon_z\}$, simulating their draws such that they conform to the truncated state space defined by the purchase incidence conditions is a challenging task. However, as we show in §4 of the technical appendix, the computation of joint category purchase incidence probabilities becomes greatly simplified by using an efficient importance sampler for simulating the random errors. Specifically, we derive a *smooth recursive conditioning simulator* in the technical appendix (Supan and Hajivassiliou 1993, Geweke et al. 1994) that makes the computation of the joint incidence probabilities remarkably efficient.

Having characterized the stochastic specifications for the brand choice and the joint category incidence conditions, we will next compare the properties of these stochastic specifications with those of the (indirectly) additive model, in which the joint purchase incidence conditions reduce to multiple single-category purchase incidence conditions.

3.2. Comparison with the (Indirectly) Additive Specification

Recall from §2.2 that the proposed L category model reduces to L single-category models when restrictions of additivity are imposed on the indirect utility. For the indirect BTL utility in Equation (3.3), the restriction of additivity implies that the cross terms $b_{i,l} = 0 \ \forall i = 1, \ldots, L, \ l = 1, \ldots, L$ and $i \neq l$. For the additive BTL model, the specification of the conditional brand choice decision remains the same as in Equations (3.6)–(3.7) for the proposed model. However, the purchase incidence condition for any category l reduces to

$$\begin{aligned} I_l &= 1 \quad \text{if } W_l + \varepsilon_l > W_{lR}, \\ &= 0 \quad \text{if } W_l + \varepsilon_l \leq W_{lR}, \end{aligned} \tag{3.15}$$

where the threshold inclusive value of category l is given as

$$W_{lR} = -\frac{a_l + b_{l,z}\mu_z\varepsilon_z + (b_{l,l} + b_{l,z})\ln y}{b_{l,l}}\mu_l. \tag{3.16}$$

Equations (3.15)–(3.16) show that the purchase incidence condition of any category l does not depend on the market-mix variables in other categories $i \neq l$. Therefore, if we are only interested in the incidence probability of a single category unconditional on the purchase of other categories (as in Chiang 1991, Chintagunta 1993), it will suffice to study each category separately—thus implying that the single-category framework follows from the additive model. Equations (3.15)–(3.16) also show that the only common stochastic terms in the incidence conditions of all categories are the aggregate shocks ε_z. Thus, if we are interested in characterizing the joint purchase incidence probabilities of all L categories, we need to take into account the correlations that result from the common aggregate shocks ε_z. Such characterization of joint category incidence conditions would be similar to that of Song and Chintagunta (2006). Although Song and Chintagunta (2006) derive the purchase incidence conditions using a different functional form of the direct basket utility, their direct basket utility assumes "indirect additivity," which results in the specification of their joint category purchase incidence conditions, similar to those of the additive BTL model.

4. Data, Analysis, and Discussion

4.1. Data

We use the Stanford basket data to calibrate the proposed model. For our analysis we choose the laundry family of categories, which consists of liquid detergents, washer softeners, and dryer softeners. To reduce the estimation burden, we do not consider the powder detergent category.[10] Thus, to correct for any bias that might result from not considering powder detergents, we choose only those households in our analysis that have *never* purchased powder detergents during their observed purchase history.

[10] With three categories (liquid detergents, washer softeners, and dryer softeners), the proposed model has a total of 68 parameters. However, if we add the powder detergent category, which has eight brands, the total number of parameters explodes to 100. Thus, in order to reduce the number of parameters to be estimated, we consider only three categories.

Table 2 Joint Purchase Frequencies in the Data Across Liquid Detergents, Dryer Softeners, and Washer Softeners

Basket constituents	Purchase frequencies
Liquid detergents only	1,038
Dryer softeners only	132
Washer softeners only	185
Liquid detergents + dryer softeners only	76
Liquid detergents + washer softeners only	212
Dryer softeners + washer softeners only	20
Liquid detergents + dryer softeners + washer softeners	12
No purchase	8,063
Total	9,738

The total sample consists of the purchase activities of 150 households over two years, and the households are chosen such that all have purchased at least once in the laundry family of categories. Following the precedence set in the prior literature (Chib et al. 2002, 2004), we consider only those purchase observations in the sample in which the household's total basket expenditure was greater than 20 dollars. The total sample consists of 9,729 purchase observations. The frequencies of joint and single purchases for all three categories are reported in Table 2. The liquid detergent data set consists of eight brands, the dryer softener data set consists of five brands, and the washer softener data set consists of four brands. The descriptive statistics for all brands across the three categories are reported in Tables 3A–3C.

For each purchase observation, we have two sets of dependent variables: (i) purchase incidence in each category, and (ii) conditional on purchase in the category, brand choice. The explanatory variables for each purchase observation are: (i) per-ounce prices for each brand, (ii) presence of promotions for each brand, (iii) GL loyalty for each brand, (iii) total basket expenditure, and (iv) inventories of the three categories. We randomly split the total sample into two parts: estimation and hold-out samples. The estimation sample consists of 100 households with 6,545 purchase observations and the hold-out sample consists of 50 households with 3,184 observations.

4.2. Model Parameters

The parameters that we have introduced so far in the model are: (i) intercepts for all brands $j = 1, \ldots, J_l$ in all categories $l = 1, \ldots, L$ in the quality indices, $\{\{\alpha_{l,j}\}_{j=1}^{J_l}\}_{l=1}^{L}$; (ii) promotional sensitivities in quality indices for all categories, $\{\beta_{l,\text{pro}}\}_{l=1}^{L}$; (iii) state-dependence sensitivity in quality indices of all categories, $\{\beta_{l,\text{loy}}\}_{l=1}^{L}$; (iv) impact of each category's inventory on its preference, $\{\beta_{l,\text{inv}}\}_{l=1}^{L}$; (v) inverse of quality sensitivities in brand choice decisions across all categories

Table 3A Descriptive Statistics for Liquid Detergent Data

Brand index/name	Share in the category (%)	Price per oz. in cents (std dev.)	Promotional frequency
1. All	12.11	4.64 (0.84)	0.24
2. Cheer	5.83	6.93 (0.79)	0.14
3. Purex	7.92	4.05 (0.60)	0.22
4. Surf	6.28	6.43 (1.45)	0.17
5. Tide	22.87	6.73 (1.01)	0.26
6. Wisk	14.13	6.63 (1.35)	0.31
7. Yes	10.61	4.38 (0.35)	0.13
8. Others	20.25	4.79 (0.80)	0.10

Table 3B Descriptive Statistics for Dryer Softener Data

Brand index/name	Share in the category (%)	Price per oz. in cents (std dev.)	Promotional frequency
1. Private Label	11.69	4.12 (0.61)	0.02
2. Bounce	43.29	5.79 (0.59)	0.14
3. Downy	10.39	6.06 (0.47)	0.04
4. Snuggle	22.51	5.71 (0.48)	0.06
5. Others	12.12	4.70 (0.56)	0.04

Table 3C Descriptive Statistics for Washer Softener Data

Brand index/name	Share in the category (%)	Price per oz. in cents (std dev.)	Promotional frequency
1. Downy	55.71	7.15 (2.70)	0.21
2. Touch	4.76	3.75 (0.49)	0.01
3. Snuggle	30.95	6.11 (3.05)	0.14
4. Others	8.57	3.19 (1.84)	0.03

$\{\mu_l\}_{l=1}^{L}$; (vi) intercepts in incidence functions for all categories, $\{a_l\}_{l=1}^{L}$; (vii) scale parameter μ_z for the aggregate shocks ε_z in incidence functions of all categories; (viii) parameters in first L rows[11] of the $(L+1) \times (L+1)$ matrix $B \equiv \{b_{l,i}\}$ in incidence functions for all categories. For identification purposes, we set the diagonal elements of first L rows of the $(L+1) \times (L+1)$ matrix B as unity. Because B is symmetric, it implies there will be $L(L+1)/2$ parameters in first L rows of the matrix B (which are $\{b_{l,z}\}_{l=1}^{L}$, $\{b_{l,i}\}_{l,i=1,\ldots,L,\,i\neq l}$).

Controlling for Unobserved Heterogeneity. In order to control for unobserved heterogeneity in the parameters, we assume:

(i) The intercept of brand j in category l, $\alpha_{l,j}$ to be normally distributed across the consumer population as $\alpha_{l,j} \sim N(\bar{\alpha}_{l,j}, \sigma^2_{\alpha_{l,j}})$. For identification, we set the mean and the variance of the intercepts of one of the brands in all three categories to zero.

(ii) The consumer's promotional sensitivity in category l to be independently normally distributed across all categories as $\beta_{l,\text{pro}} \sim N(\bar{\beta}_{l,\text{pro}}, \sigma^2_{\beta_{i,\text{pro}}})$.

[11] The last row of the $(L+1) \times (L+1)$ matrix B corresponds to the incidence function of the composite commodity. Because the composite commodity is always purchased, the elements in the last row are not required in the estimations.

(iii) The consumer's sensitivity to state dependence in category l to be independently normally distributed across all categories as $\beta_{l,\text{loy}} \sim N(\bar{\beta}_{l,\text{loy}}, \sigma^2_{\beta_{l,\text{loy}}})$.

(iv) The vector of price sensitivities (or the inverse of the quality sensitivities) in brand choice decisions in the L categories, $\mu = [\mu_1, \mu_2, \ldots, \mu_L]^T$, to be log-normally distributed with a mean $\bar{\mu} = [\bar{\mu}_1, \bar{\mu}_2, \ldots, \bar{\mu}_L]^T$ and covariance Ω^μ as $\ln \mu \sim N(\ln \bar{\mu}, \Omega^\mu)$. We assume a full covariance matrix Ω^μ with $L(L+1)/2$ parameters to generate cross-category correlations in the quality sensitivities at the population level.

(v) The vector of intercepts in the incidence functions of all L categories, $A = [a_1, a_2, \ldots, a_L]^T$ to be multivariate normally distributed with mean $\bar{A} = [\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_L]^T$ and covariance Ω^a as $A \sim N(\bar{A}, \Omega^a)$. We assume a full covariance matrix Ω^a with $L(L+1)/2$ parameters to generate cross-category correlations in the intercepts at the population level.

Summing up, we have a total of $2\sum_{l=1}^{L} J_l + (3/2)L^2 + (13/2)L + 1$ parameters in the proposed model. Because we have three product categories (that is, $L = 3$) with eight, five, and four brands each (that is, $J_1 = 8$, $J_2 = 5$, and $J_3 = 4$), we have 68 parameters. We use the method of simulated maximum likelihood to simultaneously estimate all 68 parameters. The estimation program is coded in MATLAB, and its methodology is detailed in §5 of the technical appendix.

4.3. Comparison with Competing Specifications

For the same data set, we compare the predictive power, goodness of fit, and parameter estimates of the proposed specification with those of the (i) additive specification, and (ii) specification of Manchanda et al. (1999), which only models the joint category incidence decisions (referred to as MAG henceforth). The additive specification has a total of 65 parameters and the MAG has a total of 54 parameters.[12] To reiterate the differences in the proposed and competing specifications, recall that in the proposed model, the joint purchases of any two categories i and l at the observational level are explained by two factors: (i) cross effects in incidence conditions of the two categories, where category i's market mix influences the purchase incidence condition of category l *only if* category i is purchased; (ii) correlation in the incidence conditions of the two categories that stems from coincidence. As compared to the proposed model, the additive model assumes no cross effects in incidence conditions, that is, $b_{l,i} = 0 \ \forall\, l \neq i$ and $l, i \in \{1, \ldots, L\}$. On the other hand, MAG overemphasizes the role of cross effects in the joint purchase incidence conditions by assuming the incidence condition of category l to be a function of cross effects from category i, irrespective of whether category i is purchased or not.

[12] The details of the MAG model are given in §7 of the technical supplement.

Table 4A Goodness-of-Fit Tests for Proposed vs. Additive Specifications

	Proposed model	Additive model
Estimation sample		
−Log likelihood	5,180.1	5,194.3
Hit rate (%)	87.84	85.23
Hold out sample		
−Log likelihood	2,765.9	2,666.2
Hit rate (%)	85.11	83.98

Table 4B Goodness-of-Fit Tests for Proposed vs. MAG Specifications

	Proposed model*	Additive model
Estimation sample		
−Log likelihood	3,833.3	3,913.9
Hit rate (%)	87.84	83.09
Hold out sample		
−Log likelihood	1,639.3	1,684.5
Hit rate (%)	85.11	82.25

*The log likelihoods reported for the proposed model in Table 4B is only for the purchase incidence decisions.

Goodness-of-fit tests: Tables 4A and 4B summarize the log-likelihoods and joint category purchase incidence hit rates of the proposed and competing models for both estimation and hold-out samples.[13] Looking at the predicted log-likelihoods and the joint purchase incidence hit rates for the three models in the estimation and hold-out samples, we see that the proposed model outperforms the two competing models. Because the additive model is a nested version of the proposed model, we use the likelihood ratio (LR) test for the estimation sample to test whether the proposed model significantly outperforms the additive model. As opposed to the proposed model, which has $L(L+1)/2$ parameters in the first L (where $L = 3$ is the number of categories) rows of the own- and cross-effects matrix B, the additive model only has L parameters (which are $\{b_{l,z}\}_{l=1}^{L}$). Thus, the additive model has $L(L-1)/2 = 3$ fewer parameters as compared to the proposed model, which implies that the LR statistic will be χ^2 distributed with three degrees of freedom. From Table 4A, we get the value of the LR statistic as 28.4, which indicates that the proposed model outperforms the additive model with $p < 0.01$.

In the next section, we discuss the parameter estimates of the proposed model and compare them with the estimates of the additive and MAG models. Before

[13] Note that likelihood of MAG represents the likelihood of the joint purchase incidence decisions only, while the likelihood of the proposed model represents the likelihood of the joint purchase incidence decisions and the brand choice decisions. Thus, in order to meaningfully compare MAG and the proposed model, we report their purchase incidence hit rates and the log likelihoods of the joint category purchase incidence decisions only.

we do so, we will conjecture on the potential biases that can exist in the estimates of competing models.

(i) *Biases in correlations in the purchase incidence conditions of any two categories i and l that result from coincidence*. Consider the case where the categories i and l are purchase complements (substitutes). Since positive (negative) interactions between the two categories that result from purchase complementarity (substitutability) are ignored in the additive model, these will be subsumed in the coincidence effects. Thus, in the additive model, the coincidence effects between two categories will be overestimated (underestimated) if they are purchase complements (substitutes).

(ii) *Biases in the own effects of categories* (*or category's own purchase incidence elasticity*). Omission of cross effects in the additive model can lead to systematic biases in the estimates of incidence price/promotional elasticities in the additive model. The extent of such biases would depend on (a) the extent of correlation in the inclusive values of category l, W_l, and other categories $i \neq l$, W_i (or the extent of correlation in the prices/promotional variables in the two categories l and i) across all purchase observations; (b) the extent of *purchase* complementarity or substitutability between the two categories. If prices/promotions in category l and other categories $i \neq l$ are positively correlated across purchase observations, then a promotion/price cut in category l will imply promotions/price cuts in the other category i also. In such a case, if the two categories are purchase complements, on any given purchase occasion both own and cross effects in the incidence condition of category l will impact its purchase probability in the same direction. However, if we force cross effects to be zero (as in the additive model), the true own and cross effects will be subsumed in the own effects. Because the true own and cross effects act in the same directions, it implies that the own effects of category l in the additive model will be overestimated. Similar arguments can be made if the prices and promotions covary positively/negatively across the categories l and i, and if both categories are purchase substitutes/complements. On the other hand, if we over emphasize the role of cross effects (as in MAG), the relative value of cross effects (effect of market mix of category i on purchase of category l) to own effects (effect of market mix of category l on purchase of category l itself) will be overestimated.[14]

[14] Because MAG uses weighted-share category prices in the purchase decisions (as opposed to the category-inclusive values in the proposed model), it is difficult to compare the absolute magnitudes of the own- and cross-price effects as predicted by both models. Thus, we comment only on the relative values of the cross effects to the own effects across the two models.

4.4. Results and Discussion of the Estimates of the Proposed Model

The estimates of all three models are given in §§6 and 7 of the technical appendix. The parameters are by and large statistically significant. We summarize the discussion of the estimates of the proposed model in two parts: first, we discuss the category-level parameter estimates. Following that, we discuss the impact of market-mix variables at the brand level.

4.4.1. Category-Level Analysis.

Category Specific Effects in Purchase Incidence Decisions. The category-specific effects are captured by the intercepts $\{a_l\}_{l=1}^{L}$ in the incidence functions of categories. Because these intercepts are assumed to be multivariate normally distributed across the consumer population, the correlations in the category-specific intercepts represent the extent of similarity in consumers' preferences across the three categories. For the proposed model, we get the correlation for the liquid detergent-dryer softener pair as 0.154, for the liquid detergent-washer softener pair as 0.545, and for the dryer softener-washer softener pair as −0.330. The positive correlation for the liquid detergent-washer softener pair indicates that consumers who have a high preference for purchasing liquid detergents also have a high preference for purchasing washer softeners. The same suggestion holds true for the liquid detergents-dryer softener pair, although the correlation is relatively weaker. Also, the negative correlation for the washer softener-dryer softener pair suggests that between washer and dryer softeners, a majority of consumers only purchase one or the other over their purchase histories.

Own and Cross Effects in Purchase Incidence of Categories. The cross effects of categories' market-mix variables in purchase incidence decisions are captured by the nondiagonal parameters in the matrix $B \equiv \{b_{l,i}\}$. There are two points to note regarding the estimates of nondiagonal interaction terms, $b_{l,i}$ $\forall i \neq l$. First, recall that the value of $b_{l,i}$ lies between −1 and 1. A positive (negative) value indicates purchase complementarity (substitutability) between two categories i and l. Second, the estimate of $b_{l,i}$ is only identified for those consumers who would have purchased both during their purchase history (not necessarily on the same purchase occasion). Moreover, $b_{l,i}$ does not capture the interaction between categories for those consumers who have never made a purchase in one of the two categories. For such consumers, the interactions between categories are identified by a negative correlation in their intrinsic preferences for the two categories, a_i and a_l.

From the parameter estimates, we find the cross effects between liquid detergents and dryer softeners to be $b_{\mathrm{ld,ds}} = 0.063$, between liquid detergents and washer softeners to be $b_{\mathrm{ld,\,ws}} = 0.336$, and between dryer and washer softeners to be $b_{\mathrm{ds,\,ws}} = 0.176$. The positive signs of cross-effect parameters for liquid detergent-dryer softener and liquid detergent-washer softener pairs suggest that liquid detergents share a much greater purchase complementarity with washer softeners as compared to dryer softeners. Next, the cross effects between washer and dryer softeners suggest that washer and dryer softeners are also weak purchase complements. The result for weak purchase complementarity between washer and dryer softeners can be understood by recalling that the extent of purchase complementarity between any two categories depends on two factors: (i) impact of the budget constraint and (ii) extent of *consumption* substitutability/complementarity between the two categories. Regarding the second factor, it is not clear whether washer and dryer softeners are consumption substitutes—although both soften fabrics (which implies consumption substitutability), using them together does enhance softness in fabrics (which implies consumption complementarity). Because budget constraint effects are typically small, we can conjecture that both are consumption substitutes for those consumers who only purchase one instead of the other, and complements for those who have purchased both over their purchase histories.

Next, we discuss the own and cross effects in purchase incidence decisions of categories in terms of their own- and cross-incidence elasticities across the consumer population. We define the purchase incidence elasticity between any two categories, l and i, E_l^i, as the percentage change in the purchase incidence probability of category i with respect to 1% changes in prices across *all* brands (simultaneously) $j = 1, \ldots, J_l$ in category l. The own- and cross-incidence price elasticities of the three categories for the proposed model are reported in Table 5A. The own-incidence price elasticities for liquid detergents, dryer softeners, and washer softeners are $E_{\mathrm{ld}}^{\mathrm{ld}} = -0.641$, $E_{\mathrm{ds}}^{\mathrm{ds}} = -0.049$, and $E_{\mathrm{ws}}^{\mathrm{ws}} = -0.161$, respectively.

The cross-incidence elasticity between two categories i and l at the population level depends on the magnitude and sign of the interaction terms $b_{l,\,i}$, and population-level correlations in intrinsic preferences for the categories, a_i and a_l. The greater the magnitude of $b_{l,\,i}$, the greater will be the magnitude of the cross-price elasticity. Similarly, the higher the correlation in the intrinsic preferences, the lower will be the fraction of consumers who only purchase one of the two categories—thus, the greater will be the population level cross-category price elasticity. The estimates of the cross-category incidence price elasticities across for the proposed model are given in Table 5A, which shows the following pattern of results:

Table 5A Own- and Cross-Category Incidence Price Elasticities for the Proposed Model**

	Purchase incidence probability of LD	Purchase incidence probability of DS	Purchase incidence probability of WS
Price of liquid detergents	−0.6411	−0.000266	−0.0430
Price of dryer softeners	−0.00018	−0.0428	−0.000433
Price of washer softeners	−0.0013	−0.00095	−0.1611

**In Table 5A, the own-category elasticities and the cross-category elasticities of the price of liquid detergents on washer softeners are statistically significant at 95% confidence.

Table 5B Own- and Cross-Category Incidence Price Elasticities for the Additive Model

	Purchase incidence probability of LD	Purchase incidence probability of DS	Purchase incidence probability of WS
Price of liquid detergents	−0.6897	0	0
Price of dryer softeners	0	−0.0612	0
Price of washer softeners	0	0	−0.1848

Table 5C Own- and Cross-Category Incidence Price Elasticities for MAG Model

	Purchase incidence probability of LD	Purchase incidence probability of DS	Purchase incidence probability of WS
Price of liquid detergents	−0.9893	−0.1129	−0.1416
Price of dryer softeners	−0.0697	−0.2080	−0.1095
Price of washer softeners	−0.0695	−0.0630	−0.3317

Table 5D Own- and Cross-Category Incidence Promotional Elasticities for the Proposed Model

	Purchase incidence probability of LD	Purchase incidence probability of DS	Purchase incidence probability of WS
Promotions in liquid detergents	0.0124	0.00002	0.0016
Promotions in dryer softeners	0.00001	0.0022	0.00003
Promotions in washer softeners	0.0003	0.00004	0.0066

(i) For all three categories, the cross-category incidence elasticities are smaller than the own-category incidence elasticities;

(ii) There are weak cross-price effects in liquid detergent-dryer softener and washer softener-dryer softener pairs, and relatively stronger cross-price effects in liquid detergent-washer softener pairs. The reason for such small cross-price effects between liquid detergents-dryer softeners is that the interaction term ($b_{\mathrm{ld,\,ds}} = 0.063$) is small in magnitude. Similarly, the reason for small cross-price effects between washer and dryer softeners is that the intrinsic preferences for the two categories are negatively correlated across the consumer population (correlation in a_{ds} and $a_{\mathrm{ws}} = -0.330$).

(iii) For the liquid detergent-washer softener pair, the cross elasticities are asymmetric—that is, price cuts in liquid detergents have a much higher impact on the purchases of washer softeners than vice versa ($E_{\mathrm{ld}}^{\mathrm{ws}} = -0.043$ and $E_{\mathrm{ws}}^{\mathrm{ld}} = -0.00135$).

(iv) Although the cross-category effect of liquid detergents on washer softeners is modest in magnitude, it is comparable to the own-category effect of dryer softeners ($E_{\mathrm{ds}}^{\mathrm{ds}} = -0.049$) and is roughly 27% of the own-category effect of washer softeners ($E_{\mathrm{ws}}^{\mathrm{ws}} = -0.139$).

Next, we compare the own- and cross-category incidence elasticities of the proposed model with those of the competing models. The estimates of own purchase incidence price elasticities for the additive model are reported in Table 5C. For the additive model, the own purchase incidence price elasticities for liquid detergents, dryer softeners, and washer softeners are $E_{\mathrm{ld}}^{\mathrm{ld}} = -0.689$, $E_{\mathrm{ds}}^{\mathrm{ds}} = -0.061$, and $E_{\mathrm{ws}}^{\mathrm{ws}} = -0.185$, respectively. Notice that as compared to the proposed model, the own incidence elasticities for all categories are higher for the additive model. As discussed in §4.3, this can be explained on the basis of two factors: purchase complementarity and the correlations in prices/promotions across the pairs of categories. First note that all three categories are purchase complements. Next, given the estimates of the proposed model, we compute the inclusive values of all three categories for all purchase observations in the data. Following that, we compute correlations in the inclusive values for each pair of categories across all purchase observations. We get the pairwise correlation in the inclusive values of liquid detergents and dryer softeners across all purchase observations as[15] 0.214, for liquid detergents and washer softeners as 0.289, and for washer and dryer softeners as 0.197. The positive signs of the correlations in inclusive values of the three categories imply positive correlations in prices/promotions across the three pairs. Thus, given purchase complementarity and positive correlations in prices/promotions across all pairs of categories, it follows that the magnitudes of own-category incidence price elasticities for all three categories will be overestimated in the additive model.

[15] Because the inclusive values of the three categories are correlated, it implies that the manufacturers and the retailers are coordinating marketing activities across the three categories. This can potentially lead to endogeneity issues in the estimations. We refer an interested reader to the recent work by Ma et al. (2004), which deals with such endogeneity issues.

The own- and cross-purchase incidence price elasticities for MAG are reported in Table 5C. Notice that the relative values of the cross incidence elasticities (effect of price of category i on the purchase of another category l) with respect to the own incidence elasticity (effect of price of category l on the purchase of category l itself) are higher in MAG. This can be understood by first noting that the number of purchase observations where none of the three categories are purchased is around 82.7%. Because MAG allows for cross-price effects in the purchase decisions for such observations, it implies that as compared to the proposed model, MAG would greatly overemphasize the role of cross effects. Thus, given that all three categories are purchase complements and their market mixes are positively correlated, the rationale for high cross effects in MAG follows from the discussion in §4.3. We also report the own- and cross-category incidence promotional elasticities as predicted by the proposed model in Table 5D. Although promotional elasticities are smaller in magnitude compared to price elasticities, they indicate the same pattern of results.

Similarities in Consumers' Sensitivities to Prices in Brand Choice Decisions Across Categories. The price sensitivities are captured by the parameters $\{\mu_l\}_{l=1}^{L}$. Because these sensitivities are assumed to be multivariate normally distributed across consumers, the correlations represent the extent of similarity in consumers' price sensitivities across the three categories. We get the pairwise correlation for liquid detergents-dryer softeners as 0.640, for liquid detergents-washer softeners as 0.251, and for washer softeners-dryer softeners as 0.329 (these correlations are statistically significant at 95% confidence level). Because these correlations are positive, it implies that consumers who are price sensitive in brand choice decisions in one category are also price sensitive in other categories.

Correlations in the Error Structure of the Incidence Conditions of Categories. As discussed in §2.24, the correlations in the joint incidence conditions of any two categories in the proposed model stem from two sources: the common aggregate shocks ε_z (or coincidence) and the common unobserved category-specific errors, $\{\varepsilon_l\}_{l=1}^{L}$ (which result from the extent of purchase complementarity between the categories). Therefore, to understand the relative contribution of the two sources, we separately calculate the pairwise correlations in the incidence conditions for any two categories i and l that result from (i) common aggregate shocks and (ii) from the extent of purchase complementarity between categories. Further, recall from §2.24 that correlations in incidence conditions of categories vary depending on the menu of purchased categories. Therefore, rather than reporting pairwise correlations between all pairs of categories for all possible menus of purchased categories, we only report average pairwise correlations in incidence conditions. This average pairwise correlation in the incidence conditions of any two categories i and l (from coincidence or purchase complementarity) is computed by taking the weighted sum of correlations in the error structures between the two categories (from coincidence or purchase complementarity) across each menu of purchased categories, where the weights are the joint incidence probabilities of each menu of purchased and nonpurchased categories.

For the proposed model, the average correlations in incidence conditions of the three categories that stem from purchase complementarity are reported in Table 6A and those that stem from coincidence are reported in Table 6B. As expected, the correlations that stem from purchase complementarity/substitutability between all pairs of categories are positive (because they are purchase complements). However, they are small in magnitude. This is because these are weighted-average correlations between categories. Because 82.7% of the weight is attached to the case where none of the categories are purchased (which is the probability that none of the categories are purchased), the average correlations due to complementarity become very small (recall from §3.1 that the correlations in the errors due to complementarity are zero when none of the categories are purchased). Further, the correlations that stem from coincidence are all positive, which implies positive correlations due to coincidence.

Impact of Basket Size on Purchase Incidence. Recall from Equation (3.5) that the impact of total basket expenditure on the purchase probability of any category l is given by $\theta_{l,y} \equiv (\sum_{i=1}^{L} b_{l,i} + b_{l,z})$. From our parameter estimates, we get the impact of basket size on the purchase probability of liquid detergents as $\theta_{\mathrm{ld},y} = 0.651$, on dryer softeners as $\theta_{\mathrm{ds},y} = 0.518$, and on washer softeners as $\theta_{\mathrm{ws},y} = 0.757$. Because the estimates are positive for all categories, it implies that increasing basket size increases the purchase probabilities of all three categories.

Next, we discuss the impact of the total basket expenditure on category purchase incidence price elasticities. In Table 7, we report the estimates of the own-incidence price elasticities for the three categories and only the cross-price elasticity between liquid detergents and washer softeners (because other cross elasticities are not significant) for different values of the total basket expenditure. Observe that the greater the total basket expenditure, the smaller are the own-purchase incidence and the cross-category purchase incidence elasticities.

Table 6A **Proposed Model: Average Correlations in the Error Structure of the Purchase Incidence Conditions of the Three Categories Due to Coincidence**

	Liquid detergents	Dryer softeners	Washer softeners
Liquid detergents	1	0.461	0.365
Dryer softeners	0.461	1	0.359
Washer softeners	0.365	0.359	1

Table 6B **Proposed Model: Average Correlations in the Error Structure of the Purchase Incidence Conditions of the Three Categories Due to Complementarity**

	Liquid detergents	Dryer softeners	Washer softeners
Liquid detergents	1	0.0128	0.0314
Dryer softeners	0.0128	1	0.0116
Washer softeners	0.0314	0.0116	1

Table 6C **Additive Model: Correlations in the Error Structure of the Purchase Incidence Conditions of the Three Categories Due to Coincidence**

	Liquid detergents	Dryer softeners	Washer softeners
Liquid detergents	1	0.554	0.469
Dryer softeners	0.554	1	0.373
Washer softeners	0.469	0.373	1

4.4.2. Brand-Level Analysis. In this section we discuss the impact of market mix of each brand on its own conditional brand choice probability and the purchase incidence probabilities of all categories. Accordingly, we define (i) the category (own- and cross-) incidence elasticity $\mathrm{EB}_{j,l}^{i}$ as the percent of change in the purchase incidence probability of category i with respect to 1% change in the price of brand j in category l; (ii) own brand choice elasticity of brand j in category l, $\mathrm{BCE}_{j,l}$, as the percent of change in the conditional brand choice probability of brand j in category l with respect to 1% change in the price of the same brand.

We report the brand choice, own-category and cross-category, purchase incidence elasticities for each brand j in liquid detergents in Table 8A, in washer softeners in Table 8B, and in dryer softeners in Table 8C. The following observations can be made from these elasticities:

(i) All brands have a significant impact on the purchase incidence of their own categories;

(ii) In all categories, brands that have a high brand choice elasticity also have high own-category and cross-category incidence elasticities. In liquid detergents, *All* ranks the highest on the three elasticities, followed by *Tide* and *Wisk* (excluding the "other" brand); in washer softeners, *Downy* ranks the highest,

Table 7 Impact of Total Basket Expenditure on the Purchase Incidence Elasticities of All Categories

Total basket expenditure, y	Own-category incidence elasticity for liquid detergents	Own-category incidence elasticity for dryer softeners	Own-category incidence elasticity for washer softeners	Cross-category incidence elasticity—price of liquid detergents on purchase of washer softeners
$y = \$30.00$	−0.833	−0.059	−0.215	−0.061
$y = \$40.00$	−0.741	−0.054	−0.189	−0.052
$y = \$50.00$	−0.638	−0.049	−0.161	−0.043
$y = \$60.00$	−0.547	−0.043	−0.131	−0.032
$y = \$70.00$	−0.469	−0.037	−0.099	−0.021

followed by *Snuggle*; and in dryer softeners, *Bounce* ranks the highest, followed by *Snuggle*. Thus, for a retailer, whose objective is to maximize the purchases across all categories, it is most beneficial to promote the aforementioned brands;

Table 8A Proposed Model: Conditional Brand Choice, Own-Category Purchase Incidence and Cross-Category Incidence Elasticities for Brands in the Liquid Detergent Data Set***

	Own-brand choice conditional on purchase in LD	Purchase incidence of LD	Purchase incidence of DS	Purchase incidence of WS
Price of All (LD)	−0.882	−0.157	−0.0001	−0.0107
Price of Cheer (LD)	−0.208	−0.039	0	−0.0023
Price of Purex (LD)	−0.448	−0.069	0	−0.0049
Price of Surf (LD)	−0.183	−0.029	0	−0.0017
Price of Tide (LD)	−0.759	−0.128	0	−0.0101
Price of Wisk (LD)	−0.529	−0.080	0	−0.0054
Price of Yes (LD)	−0.475	−0.066	0	−0.0053
Price of Others (LD)	−1.010	−0.211	−0.0001	−0.0109

Table 8B Conditional Brand Choice, Own-Category Purchase Incidence and Cross-Category Incidence Elasticities for Brands in the Washer Softener Data Set***

	Own-brand choice conditional on purchase in WS	Purchase incidence of LD	Purchase incidence of DS	Purchase incidence of WS
Price of Downy (WS)	−0.504	−0.0010	−0.0003	−0.0774
Price of Touch (WS)	−0.115	0	0	−0.0099
Price of Snuggle (WS)	−0.451	−0.0004	−0.0001	−0.0663
Price of Others (WS)	−0.163	−0.0002	0	−0.0108

Table 8C Conditional Brand Choice, Own-Category Purchase Incidence and Cross-Category Incidence Elasticities for Brands in the Dryer Softener Data Set***

	Own-brand choice conditional on purchase in DS	Purchase incidence of LD	Purchase incidence of DS	Purchase incidence of WS
Price of private label (DS)	−0.2511	0	−0.010	−0.0001
Price of Bounce (DS)	−0.3688	0	−0.019	−0.0004
Price of Downy (DS)	−0.0933	0	−0.004	0
Price of Snuggle (DS)	−0.2760	0	−0.016	−0.0002
Price of Others (DS)	−0.2038	0	−0.007	−0.0001

***All values of the elasticities less than 10^{-4} are reported as zero in Tables 8A, 8B, and 8C.

(iii) All brands in washer and dryer softener categories have negligible cross-price effects on purchases in other categories. Additionally, all brands in liquid detergents have a negligible cross-price effect on the purchases in the dryer softener category. However, brands in liquid detergents have a modest impact on the purchases in washer softeners. The cross-category impact of *All*, *Tide*, and *Wisk* in the liquid detergent category on the purchase of washer softeners is comparable to the own-category purchase incidence elasticities of *Touch* and *Others* in the washer softener category. This implies that for a retailer, it is much better to have price discounts for *All* and *Tide* in the liquid detergent category instead of having price discounts for *Touch* and *Others* in the washer softener category.

5. Conclusions and Limitations

In this paper, we propose a framework to investigate consumers' brand choice and purchase incidence decisions across multiple categories, where both decisions are modeled as an outcome of the consumer's basket utility maximization. We first explore the theoretical properties of a model that would achieve a balance between tractability and flexibility. We do so by explicating a completely general model of basket utility maximization. We then examine reasonable restrictions that need to be placed on the structure of the basket utility to make the solution tractable. Finally, we use "duality" to show how the purchase conditions can be derived from an indirect basket utility.

We estimate this framework on a data set consisting of the laundry family of categories. We use a flexible functional form of the indirect basket utility to get stochastic specifications of the joint purchase conditions. In our analysis, we compare the proposed specification with prior single-category brand choice-purchase incidence and prior multicategory purchase incidence models. We show that the proposed specification performs significantly better than these specifications, and why the prior specifications systematically bias the estimates of the own, cross and coincidence effects. Furthermore, we show how market-mix variables of each brand affect purchases of brands across all categories, estimates of which can help retailers make promotional decisions across a portfolio of products.

Our paper suffers from some limitations. First, we do not model the purchase quantity decision in the paper. However, the modeling and estimation of quantity choice would not change the specification and estimation of the brand choice and purchase incidence decisions. This is because both modeling and estimation of the quantity choice decision is done conditional on the specification and the parameter estimates of the brand choice-purchase incidence decisions (as in Chiang 1991, Chintagunta 1993). Second, unlike the recent work by Chib et al. (2005), our model assumes the brand-specific errors in the brand choice decisions to be independently distributed across categories, which restricts the umbrella branding effects at the observational level. Although we can potentially relax this restriction, doing so will substantially complicate the specification of the joint purchase incidence conditions because our model structurally relates the brand choice decision to the purchase incidence decisions. Finally, although not a focus of this paper, our model can be extended to study how retailers should set optimal levels of promotions/prices for brands across categories (similar to the analysis done in a single-category context by Zhang and Krishnamurthi 2004) and also to predict the effect of brand deletion/assortment reduction in a category on the sales of brands across all categories in the store (similar to Borle et al. 2005).

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Appendix

Procedure for Deriving the Joint Category Purchase Incidence Conditions from an Indirect Basket Utility for a General Case of L Categories

The procedure for specifying the joint category incidence conditions for the case where "categories $1, \ldots, m$ are purchased and categories $m+1, \ldots, L$ are not purchased" is detailed as follows. Consider an indirect basket utility, $V \equiv V(\{q_i\}_{i=1}^{L}, p_z^*, y)$, with $L+1$ categories, where (i) there is one brand per category and each category $l = 1, \ldots, L$ is associated with a unit price q_l; (ii) the price of the composite commodity ($(L+1)$th category) is its quality adjusted price p_z^*; (iii) total basket expenditure is y. We first specify the incidence functions s_l (the numerator of the Roy's Identity) for each category $l = 1, \ldots, L$ as

$$s_l(\{q_i\}_{i=1}^{L}, p_z^*, y) \equiv -\partial V(\{q_i\}_{i=1}^{L}, p_z^*, y)/\partial q_l \qquad \forall\, l \in \{1, \ldots, L\}. \tag{A1}$$

Next, we take the per-unit price of each purchased category $i \in \{1, \ldots, m\}$ as the lowest quality-adjusted price among all brands in that category i, that is, $q_i \equiv \min_j p_{i,j}^*$; and the per-unit price of each nonpurchased category $i \in \{m+1, \ldots, L\}$ as the threshold quality-adjusted price of that category i as $q_i \equiv R_i^*$. Substituting them into (A1), we get the incidence functions for each category $l = 1, \ldots, L$ as

$$s_l = s_l\Big(\Big\{\min_j p_{i,j}^*\Big\}_{i=1}^{m}, \{R_i^*\}_{i=m+1}^{L}, p_z^*, y\Big) \quad \forall\, l \in \{1, \ldots, L\}. \tag{A2}$$

Next, we set the incidence functions for all nonpurchased categories $l = m+1, \ldots, L$ to zero. This yields the following system of $L-m$ equations in which there are $L-m$ unknowns, which are the threshold quality-adjusted prices of all nonpurchased categories, $\{R_l^*\}_{l=m+1}^{L}$,

$$s_l\Big(\Big\{\min_j p_{i,j}^*\Big\}_{i=1}^{m}, \{R_i^*\}_{i=m+1}^{L}, p_z^*, y\Big) = 0 \qquad \forall\, l \in \{m+1, \ldots, L\}. \tag{A3}$$

Solving the $L-m$ equations in (A3) simultaneously yields the specifications of the threshold quality-adjusted prices of the nonpurchased categories, $\{R_l^*\}_{l=m+1}^{L}$, as functions of the other remaining variables, $(\{\min_j p_{i,j}^*\}_{i=1}^{m}, p_z^*, y)$ in (A3) as

$$R_l^* = R_l\Big(\Big\{\min_j p_{i,j}^*\Big\}_{i=1}^{m}, p_z^*, y\Big) \quad \forall\, l \in \{m+1, \ldots, L\}. \tag{A4}$$

Setting the quality-adjusted prices of all brands $j = 1, \ldots, J_l$ in each nonpurchased category $l = m+1, \ldots, L$ to be less than the threshold quality-adjusted price of that category l, R_l^* (as given in A4) gives us the incidence conditions for a nonpurchased category as

$$R_l\Big(\Big\{\min_j p_{i,j}^*\Big\}_{i=1}^{m}, p_z^*, y\Big) \le \min_j p_{l,j}^* \quad \forall\, l \in \{m+1, \ldots, L\}. \tag{A5}$$

Next, we plug the expressions for R_l of all nonpurchased categories $l = m+1, \ldots, L$ as given in (A4) into the incidence functions for the purchased categories $l = 1, \ldots, m$ in (A2) to get

$$s_l = s_l\Big(\Big\{\min_j p_{i,j}^*\Big\}_{i=1}^{m}, \Big\{R_i\Big(\Big\{\min_j p_{i,j}^*\Big\}_{i=1}^{m}, p_z^*, y\Big)\Big\}_{i=m+1}^{L}, p_z^*, y\Big) \qquad \forall\, l \in \{1, \ldots, m\}. \tag{A6}$$

Next, we get the incidence conditions for any purchased category $l = 1, \ldots, m$ by setting the incidence functions of that purchased category l as given in (A6) to be positive

$$s_l\Big(\Big\{\min_j p_{i,j}^*\Big\}_{i=1}^{m}, \Big\{R_i\Big(\Big\{\min_j p_{i,j}^*\Big\}_{i=1}^{m}, p_z^*, y\Big)\Big\}_{i=m+1}^{L}, p_z^*, y\Big) > 0 \qquad \forall\, l \in \{1, \ldots, m\}. \tag{A7}$$

Note that the incidence function s_l is a decreasing function of the lowest quality-adjusted price in that category l, $\min_j p^*_{l,j}$ (this is because the indirect utility is convex in the prices). This implies that for category l to be purchased, the lowest quality-adjusted price in that category l, $\min_j p^*_{l,j}$, should be less than some threshold value. Thus, (A7) can be inverted such that we can write the incidence condition for a purchased category $l = 1, \ldots, m$ in terms of the lowest quality-adjusted price in that category l, $\min_j p^*_{l,j}$, and a threshold category quality-adjusted price R_l for that purchased category as

$$R_l\Big(\Big\{\min_j p^*_{i,j}\Big\}_{i=1, i\neq l}^{m}, p^*_z, y\Big) > \min_j p^*_{l,j} \quad \forall\, l \in \{1, \ldots, m\}. \quad \text{(A8)}$$

Thus, Equations (A5) and (A8) yield the joint purchase incidence conditions as

Nonpurchased Categories.

$$R_l\Big(\Big\{\min_j p^*_{i,j}\Big\}_{i=1}^{m}, p^*_z, y\Big) \leq \min_j p^*_{l,j} \quad \forall\, l \in \{m+1, \ldots, L\}. \quad \text{(A9)}$$

Purchased Categories.

$$R_l\Big(\Big\{\min_j p^*_{i,j}\Big\}_{i=1, i\neq l}^{m}, p^*_z, y\Big) > \min_j p^*_{l,j} \quad \forall\, l \in \{1, \ldots, m\}. \quad \text{(A10)}$$

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