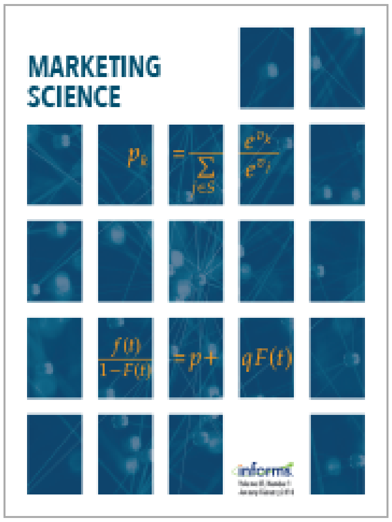

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Cartel Formation Through Strategic Information Leakage in a Distribution Channel

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Abstract. This paper studies the ability of competing retailers to form a cartel by sharing information with their mutual manufacturer. In a market characterized by demand uncertainty, colluding retailers wish to share information about the potential market demand to coordinate on the optimal collusive retail price. However, in light of potential exposure to antitrust investigations and possible sanctions, the retailers search for mechanisms to exchange information while avoiding the risks of scrutiny by the antitrust authorities. This paper examines such a mechanism: each retailer shares his private information with the mutual manufacturer; the wholesale price set by the latter is thereafter used by the retailers to infer the market condition and coordinate on the cartel's price. Although a cartel at the retail level limits the manufacturer's sold quantity, under certain conditions the manufacturer is better off accepting the retailers' private information, thereby assisting the cartel formation. Moreover, vertical information sharing between the retailers and their mutual manufacturer can result in lower consumer surplus than that would have occurred had the retailers been permitted to collude directly.

History: Ganesh Iyer served as the senior editor and Anthony Dukes served as associate editor for this article.

Keywords: information sharing • channel coordination • collusion • regulation

> People of the same trade seldom meet together, even for merriment and diversion, but the conversation ends in a conspiracy against the public, or in some contrivance to raise prices.
>
> —Adam Smith, *The Wealth of Nations*, Book IV, Chapter VIII, p. 145

1. Introduction

In late 2003, the UK competition authority (the Office of Fair Trading (OFT)) concluded in two cases (Replica Kit[1] and Hasbro[2]) that information that was exchanged between retailers via a mutual partner that operates at a different level of the distribution channel constitutes a violation of the competition law, and is equivalent to a horizontal price-fixing agreement between the retailers themselves. In this paper, I explore this phenomenon and study the way retailers who wish to form a cartel and coordinate on a monopoly price can do so successfully by sharing information vertically with their mutual manufacturer—a mechanism that is being adopted instead of sharing information directly between the retailers and being exposed to antitrust scrutiny.

Being aware of the negative welfare implications of collusion among competing firms, regulators attempt to fight and deter such behavior. In the United States, as well as in most other jurisdictions, price-fixing agreements are illegal. Under the Sherman Act of 1890, parties to a collusive agreement are generally subject to harsh penalties, which may consist of fines and prison terms. However, while the rule against collusion and price fixing may be "the least controversial prohibition in competition law" (Kaplow 2011a, p. 343), the questions regarding the precise definition of these practices and the ways they should be detected are still being debated (Kovacic 1993, Hay 2006, and Kaplow 2011b). Because an agreement to collude can rarely be demonstrated, courts have ruled that to prove the existence of a conspiracy, an agreement under the Sherman Act can be proved by circumstantial evidence, such as the exchange of private information between otherwise competing firms (DeSanti and Nagata 1994, Kühn 2001).

Conscious of the fact that direct information sharing may expose them to investigations by the antitrust authorities with potentially heavy penalties, competing firms constantly seek alternative schemes to exchange information and collude, while minimizing the risks of detection and persecution. Sometimes third parties are used to achieve this goal. For instance, Asker (2010) describes a 1990s bidding cartel of stamp collectors that, to limit the number of meetings between the cartel members and thus reduce the risk of exposure, employed a taxi driver to collect the internal bids from the cartel members and determine the winner of each lot.

While scholars have highlighted the possible collusive effect of horizontal information exchange, academic research has generally overlooked the ability to form a horizontal cartel by sharing information in a vertical manner. In fact, study of vertical exchange of information has mostly evaluated the benefits of sharing information in a distribution channel and the incentives of firms to exchange such information (Cachon and Lariviere 2001; Özer and Wei 2006; Gal-Or et al. 2007, 2008; He et al. 2008; Guo and Iyer 2010; Dukes et al. 2011).

Li (2002) was among the first researchers to study the incentives for vertical information exchange in a distribution channel with competing retailers. He used the term "information leakage" to describe the way a retailer can infer the private information of his competitor based on the actions of the mutual manufacturer. Li (2002) concluded that because of this leakage, retailers may refrain from sharing information vertically with their manufacturer. Anand and Goyal (2009) also examined the issue of information leakage to show that a manufacturer may deliberately leak private information of a retailer to intensify the competition level at the retailers' level. In spite of the conclusion in these papers that, under competitive settings, information leakage reduces the incentives for information exchange, I demonstrate in this paper how a group of retailers can use information leakage to their benefit, when they wish to share information directly, but cannot do so because of antitrust concerns.

I focus on the way vertical information sharing between a group of retailers and their mutual manufacturer and the phenomenon of information leakage can assist the retailers to form a cartel. This scenario differs from a simple indirect horizontal information exchange where competing firms avoid engaging in face-to-face communications and use an outside messenger as a mere vessel that conveys the contents of the information received from one firm to its competitor. Contrary to this scheme, in the scenario analyzed in this paper, the vertically related firm—the manufacturer—does not forward the content of the data received from one retailer to another; rather, she uses this information for her own benefit. However, although the information is not forwarded to the retailers, I demonstrate that such indirect communication can be used as a signaling device to coordinate price fixing among the competing retailers.

To be more specific, I analyze a distribution channel comprised of price-setting retailers, sourcing their products from a single manufacturer, who sets the wholesale price strategically. The retailers interact repeatedly, and the market demand during each period is assumed to fluctuate in a similar manner as in the model of Rotemberg and Saloner (1986). At the beginning of the selling season, each retailer can observe a signal about the current market status. To increase their profit and reduce the competition level, the retailers attempt to form a cartel. Endowed with private information, and attempting to form a cartel, the retailers face the challenge of coordinating on the appropriate monopoly price. Absent any antitrust concerns, the efficient outcome, from the retailers' perspective, is achieved by aggregating their private information and setting a monopoly price based on their cumulative knowledge. However, when direct exchange of information is not possible due to antitrust concerns, I analyze the alternative in which each retailer shares information in a confidential manner with the mutual manufacturer. Based on the wholesale price, determined strategically by the manufacturer, the retailers are able to form a cartel and coordinate on a monopoly price.

Although the manufacturer is not part of the cartel, and a cartel limits the sold quantity in the market compared with the competitive outcome, there are cases in which the manufacturer is willing to receive the retailers' information and, thus, assist the retailers to form the cartel by setting the wholesale price based on the retailers' private information. This occurs, for example, when the market volatility is high; in this case, the manufacturer prefers to receive the private forecast information of the retailers to set the wholesale price in a manner consistent with the actual demand, even when understanding that by doing so, she assists the cartel formation.

The main contributions of this paper are twofold. From a policy perspective, this paper highlights the collusive role of vertical information exchange in a distribution channel. In addition to its policy implications, this paper adds another layer of understanding to the phenomenon of information leakage in distribution channels. It shows that contrary to previous research, information leakage can actually promote vertical information exchange. In the current settings, retailers exchange information with their manufacturer because of their expectation that this information will later be leaked to their competitors. In this setting, it is the information leakage that incentivizes the retailers to vertically exchange information.

2. Literature Review

This paper contributes primarily to two streams of research: the first studies the role of communication and information sharing in cartel formation; the second examines the incentives to share and acquire information in a distribution channel.

2.1. Communication and Cartel Formation

Since direct communication is considered a possible signal for collusion, researchers have evaluated the scope of collusion when firms are not allowed to share information directly. In a series of papers, Athey and Bagwell (2001) and Athey et al. (2004) examined cartel formation when firms have asymmetric cost information. Gerlach (2009) demonstrated how communication improves coordination by avoiding undercutting prices by poorly informed firms. Hanazono and Yang (2007) studied the ability of competing firms to establish a cartel when they face fluctuating demand and each firm observes a signal about the market condition. The current paper differs from the economic models outlined above by extending the environment in which the colluding retailers operate. The economic models assume that if the retailers cannot share information horizontally, they must collude by using sophisticated signaling games or by ignoring their private information. In this work, I assume that the retailers source their product from a mutual manufacturer and pose the question as to whether or not the retailers can collude using the manufacturer's wholesale price as a coordination device.

Harrington (2004) studied the pricing strategy of a cartel in the presence of antitrust authority. The concern of the cartel in Harrington's (2004) work is that the pricing strategy can reveal the existence of the cartel, and consequently the cartel can either gradually increase the pricing level or decrease the price. Nocke and White (2007) showed that vertical integration increases the ability of upstream firms to collude. While in the case of Nocke and White (2007), there is full integration between an upstream firm and a downstream one, in the current paper there is no pricing cooperation between the retailers and the manufacturer; each firm in the market chooses strategically whether to share or receive the demand information and determines the price independently.

2.2. Information Sharing and Information Leakage in Distribution Channels

The second stream of research relevant to this work studies the motivation for vertical information exchange in a market with competing firms. Villas-Boas (1994) examine whether competing firms should share the same advertising agency, under the risk that the shared information can become available to the competitors. Li (2002) studies the incentives of firms to share information in a model with one supplier and n symmetric competing retailers. He shows that by observing the wholesale price, retailers who choose not to share their private information with the manufacturer can infer the information of those who did share their information. Li (2002) refers to this phenomenon as "information leakage" and concludes that no information is shared in equilibrium—a result that is undesired by the manufacturer. Zhang (2002) studies a similar model to that of Li (2002) and adds an analysis of Bertrand competition between the retailers. Li and Zhang (2008) explore the issue of information leakage in distribution channels with one manufacturer and Bertrand competing firms. Anand and Goyal (2009) analyze the incentives of an incumbent retailer, facing possible entry to his market, to acquire new demand information when this information can be leaked to an entrant by the mutual manufacturer. They demonstrate that the manufacturer prefers to leak demand information to the entrant to promote competition at the retailers' level. In a later work, Kong et al. (2013) demonstrate that the problem of information leakage can be resolved using revenue sharing contracts between the incumbent retailer and the manufacturer. Chen and Ozer (2016) recommend using downside-protection contracts and upside-protection contracts to prevent leakage. An excellent survey of the recent literature about information leakage can be found in Kong et al. (2016).

The models described above emphasize the negative effects of information leakage on the incentives of firms to share information in distribution channels. In contrast with this view, this work demonstrates that the incentives of the retailers to share their demand information with the manufacturer are driven by the retailers' expectation that their private information will be leaked to their competitors, thus enabling them to coordinate on the monopoly price.

This paper is also related to the growing literature on vertical information sharing in distribution channels and the effect of information exchange on the pricing decisions of the firms in the distribution channels. Shin and Tunca (2010) discuss the use of auctions to achieve information sharing. Ha and Tong (2008) and Ha et al. (2011) explore information sharing in a market with two competing distribution channels, and Shang et al. (2016) analyze information sharing in a distribution channel comprised of one common retailer and two competing manufacturers. Guo and Iyer (2010) examine the interaction between a manufacturer's optimal information acquisition and sharing strategies in a vertical relationship. He et al. (2008) study a market in which firms have asymmetric information about the demand volatility. They examine the potential benefits of sharing information and the contracts that facilitate such cooperation. Li (2005) considers the role of "cheap talk" in channel communication. He shows that communication credibility can be achieved in a vertical relationship even without any signaling costs. Gal-Or et al. (2008) study the way information sharing affects wholesale pricing. These authors highlight the inference effect, which results in a manufacturer setting a low wholesale price when the retailer is uninformed. Guo (2009) highlights two effects of a retailer's information acquisition in a vertical relationship: the efficiency effect that improves the retail pricing decision in a market with uncertain demand and the strategic effect of disclosing the acquired private information to influence the manufacturer's wholesale price. Mittendorf et al. (2013) study the incentives of a retailer to share information with his manufacturer, when the latter can exploit this information to her benefit. In their work, Mittendorf et al. (2013) weigh the trade-off between the retailer's fear of being exploited and the ability of the manufacturer to increase the retailer's demand.

The current work contributes to this fruitful research stream by introducing a new motivation to share information between a retailer and his manufacturer—as a means to facilitate collusion. This paper also highlights two pricing effects of this information exchange. The first, the direct effect, results in the manufacturer setting the wholesale price in a manner consistent with the realized demand. This outcome hurts the retailers, as it provides the manufacturer with additional power in the distribution channel. The second effect is the coordination and inference effect. Based on the wholesale price, the retailers are able to coordinate on the monopoly price and form a cartel.

3. The Model

This section describes the model. I start with a description of the events in a single period and then embed the single-period model in an infinite repeated game.

3.1. The One-Period Model

Consider n retailers (I refer to each retailer as *he*) operate in a market characterized by demand uncertainty. The retailers interact over an infinite horizon, and they source their product from a common manufacturer (*she*) for a cost of w, which is determined at the beginning of each period by the manufacturer. The demand during each period is a function of the market price $\underline{p}$ (explained below) and an independent and identically distributed random shock

$$Q(A_\theta, p) = (A_\theta - \underline{p})^+, \quad \text{where } \theta \in \{L, H\},$$

with $A_L < A_H$ and $(A_\theta - \underline{p})^+ = \max(A_\theta - \underline{p}, 0)$.[3] The prior probability that demand is in state $\theta = H$ is given by $\Pr(A_H) = \mu \in (0,1)$, and it is common knowledge. The profit of a monopoly at market condition A_θ setting a retail price of p and having a constant marginal cost of w is given by

$$\pi^m(A_\theta, p, w) \equiv Q(A_\theta, p)(p - w). \tag{1}$$

I denote the price a monopoly would set at market state $A_\theta \in \{A_L, A_H\}$, and with a marginal cost of w by $p^m(A_\theta, w) \equiv \arg\max_p \pi^m(A_\theta, p, w)$.

At the beginning of each selling period, each retailer has an opportunity to observe a signal $Y_i \in \{H, \phi\}$ about the market condition. The probability of observing the signal $Y_i = H$ is given by

$$\Pr(Y_i = H) = \begin{cases} \rho & \text{if } \theta = H, \\ 0 & \text{if } \theta = L, \end{cases} \tag{2}$$

and the probability of observing the signal $Y_i = \phi$ is given by

$$\Pr(Y_i = \phi) = \begin{cases} 1 - \rho & \text{if } \theta = H, \\ 1 & \text{if } \theta = L. \end{cases}$$

When the market condition is $\theta = H$, each retailer has a probability of ρ to learn the true market condition and with the complement probability a retailer observes the signal ϕ. When the market condition is $\theta = L$, a retailer can observe only the less informative signal $Y_i = \phi$.[4] Upon observing the signal Y_i, each retailer updates the probability that the market condition is $\theta = H$ in a Bayesian fashion.

This specific signal structure, in which upon observing the signal $Y_i = H$ a retailer knows with probability 1 the market condition, was chosen to highlight the role of information sharing in coordinating the cartel. Under this information structure, a retailer who observes the informative signal does not have an incentive to share information to reduce his level of uncertainty in the market; thus, his only incentive to share information is to ensure that all other cartel members set the appropriate price. Muting the role of information sharing in reducing the uncertainty level allows me to focus on the role of information sharing in achieving coordination among the retailers.[5]

Following the information-sharing stage (to be explained shortly), the manufacturer sets the wholesale price w based on her available information. At the next stage, based on their information and the wholesale price, the retailers simultaneously set the retail prices p_i, $i = 1, \ldots, n$. After the retailers set their prices, the market price $\underline{p}(\mathbf{p}) = \min\{\mathbf{p}\}$ is determined based on the lowest price quoted by the retailers, where $\mathbf{p} = (p_1, \ldots, p_n)$ denotes the vector of prices set by the retailers. The market share m_i of retailer i is given by

$$m_i(\mathbf{p}) = \begin{cases} 0 & \text{if } p_i > \underline{p}, \\ \dfrac{1}{k(\underline{p})} & \text{if } p_i = \underline{p}, \end{cases} \tag{3}$$

where $k(\underline{p})$ denotes the number of retailers setting the lowest price. I adopt the standard assumption that the retailers share the market equally if they set the same market price $\underline{p}$. The one-period profit of retailer i is thus given by

$$\pi_i(A_\theta, \mathbf{p}, w) = Q(A_\theta, \underline{p}(\mathbf{p}))(p_i - w)m_i(\mathbf{p}). \tag{4}$$

Since the retailers have identical marginal costs, the standard Bertrand game with a homogenous product suggests that the stage game has a unique Nash equilibrium: all retailers charge their marginal costs regardless of the realized signals and earn zero profit in every period.

After observing their own private signals, the retailers have an opportunity to share information. To study the effect of communication on the ability of the retailers to collude, I analyze a few different settings of information sharing. In the first scenario, which serves as a benchmark, I assume that communication between the retailers is allowed and I analyze the scope of collusion and the incentives to share information when the retailers can exchange information. This scenario is denoted by S1. I then assume that the retailers cannot share information horizontally, since such behavior results in the collusion being exposed by the antitrust authorities. I examine two alternative collusion mechanisms for the retailers: in the first (denoted by S2), each retailer shares his private information vertically, i.e., solely with the manufacturer, and no retailer is exposed to the information shared between his competitors and the manufacturer. To evaluate the attractiveness of this collusion method, I also study an alternative collusive method (denoted by S3) in which the retailers collude without any information sharing.

When n retailers share their private information, the posterior belief that the demand is high is updated in the following Bayesian manner:

$$\mu(\mathbf{Y}_n) = \Pr(A_H \mid \mathbf{Y}_n) = \begin{cases} 1 & \text{if } \exists\, i,\ Y_i = H, \\ \dfrac{\mu(1-\rho)^n}{\mu(1-\rho)^n + 1 - \mu} & \text{if } \forall i,\ Y_i = \phi, \end{cases} \tag{5}$$

where $\mathbf{Y}_n = (Y_1, Y_2, \ldots, Y_n)$ denotes the vector of the n observed signals. I summarize the information in the market using the information set I. I denote the state in which at least one retailer observes an informative signal by I_H, and the state in which all retailers observe a noisy signal by I_ϕ. I further use the notation $A_\mu = E[A_\theta] = \mu A_H + (1-\mu)A_L$ to denote the ex ante value of the demand intercept, and the notation $\tilde{A}_L = E[A_\theta \mid Y_1 = \cdots Y_n = \phi]$ to denote the expected value of the demand intercept given that all of the retailers observe the noisy signal. I also make the assumption that $(1-\mu)$ is high enough that the retailers wish to sell also when the realized demand is low. Alternatively, the retailers decide to always set a high retail price and sell only when demand is high; in this case, the problem of pricing coordination does not arise.

Another notation I use is ψ, which is defined in the following way:

$$\psi = \frac{A_H}{\tilde{A}_L}. \tag{6}$$

The parameter ψ can be viewed as a proxy for the level of demand uncertainty in the market given the set of observed signals, as it measures the ratio between the demand intercept during high demand periods and the expected demand intercept during the periods in which all retailers observe the noisy signal. A similar measure of demand uncertainty was used by Anand and Goyal (2009), Shamir and Shin (2016), and Chu et al. (2016).

Table 1. Information Available to the Supply Chain Participants in the Different Settings

	Retailer's information	Manufacturer's information
Setting S1	$\mathbf{Y}_n$	None
Setting S2	Y_i and $w(\mathbf{Y})$	$\mathbf{Y}_n$
Setting S3	Y_i	None

Table 1 summarizes the information available to the parties in the distribution channel under the different information-exchange settings. In settings S1 and S3, the manufacturer has no information about the market demand and she sets the wholesale price based on her prior belief, whereas in scenario S2, she receives messages from all of the retailers. In scenario S2, although the retailers do not exchange information horizontally, they observe the wholesale price w, which can convey some information about the market demand.

3.2. The Repeated Game

This paper studies the ability of a group of retailers to collude. In a setting of collusion, the standard tool used to capture the trade-off between the incentives to increase the immediate profit and the prospect of future profit that stems from cooperation is repeated games (Fudenberg and Tirole 1991). Therefore, I embed the single-period model in an infinitely repeated game. I assume that the market demand in each period is independent of the market demand in any other period. In each period t, the complete history of interactions between the retailers and the manufacturer, $h_t = ((w_1, \mathbf{p}_1), (w_2, \mathbf{p}_2), \ldots, (w_{t-1}, \mathbf{p}_{t-1}))$, is observable to both the retailers and the manufacturer. In addition, the complete history of shared information is available to the recipients of the information. Let H_t be the set of all possible period t histories. A repeated game strategy σ consists of a mapping from every possible history of actions, h_t, to the retailers' and manufacturer's one-period strategy. The payoff to each retailer is the sum of the payoffs during each period discounted using a common discount factor $\delta \in (0,1)$. I denote the expected discounted profit of retailer i, starting from period t, by $V_{i,t}(h_t)$, and the expected discounted profit of the manufacturer, starting at period t, by $V_{M,t}(h_t)$. I also use the notation $V_{i,t}(h_t \mid Y_i)$ to denote the stream of discounted profit of retailer i given the information Y_i the retailer has about the current period t. I summarize the notation in this paper in Table 2.

Table 2. Model Notation

Notation	Explanation
n	Number of retailers in the market
w	Wholesale price
$Q(A_\theta, p)$	Quantity sold in the market, given state A_θ and price p
A_L	Low demand state
A_H	High demand state
μ	Prior probability of high demand state
π^m	Monopoly profit in the market
p^m	Monopoly retail price
Y_i	Private information observed by retailer i
ρ	Probability of observing the signal Y_H given high demand state
$\underline{p}$	Market price
$\mathbf{p}$	Vector of retail prices
m_i	Market share of retailer i
$k(\underline{p})$	Number of retailers setting the market price $\underline{p}$
π_i	One-period profit of retailer i
ψ	Ratio between the two possible values of the demand intercept
$V_{i,t}$	Stream of discounted profit of retailer i starting at period t
$V_{M,t}$	Stream of discounted profit of the manufacturer starting at period t
h_t	History of retail and wholesale prices until period t
$V_{i,t}(h_t \mid Y_i)$	Retailer's discounted profit starting at period t and conditioned on private information Y_i

Infinitely repeated games typically have many equilibria (Fudenberg and Tirole 1991). As a solution concept I use the public perfect equilibria (PPE; Fudenberg and Tirole 1991, Definition 5.3, p. 188). A strategy profile is a perfect public equilibrium if the strategy played by each player depends on the public information (rather than on private information available to a specific player) and at each date t and history h_t the strategies are Nash equilibrium from that point on (i.e., no firm has a one shot profitable deviation at any point in time). The use of this equilibrium concept has been popular due to a set of tools developed by Abreu et al. (1986, 1990). A few examples of the use of this equilibrium concept include the work of Green and Porter (1984), who studied the scope of collusion with unobservable prices; Athey et al. (2004), who examined cartel formation without communication; and Levin (2003), who analyzed the performance of relational contracts.

4. Collusion with Horizontal Information Sharing (Setting S1)

As a benchmark, and to develop the intuition for the main results, I first analyze the collusive agreement between the retailers when they are able to exchange information directly. Although the manufacturer is aware of the information exchange between the retailers, she is not exposed to the content of the shared information if the retailers choose not to share information with her.

When the retailers exchange information, they are all in the same information set and need to determine the set of collusive prices $\mathbf{p}^{S1} = (p_\phi^{S1}, p_H^{S1})$ for a given marginal cost w and a discount factor δ. I adopt here the assumption that when information is shared, it is verifiable and, thus, shared truthfully. In Appendix B, I relax this assumption and show that the results hold even when the retailers exchange "soft information" that cannot be verified.

For a vector of prices (p_ϕ^{S1}, p_H^{S1}) to qualify as a PPE, an individual firm should have no incentive to deviate from the current-period collusive price. In the case of complete information, each deviation from the collusive scheme is immediately detected and punished with the harshest possible punishment—repeatedly playing the one-period Nash equilibrium and earning zero profit. Hence, the cartel's price vector (p_ϕ^{S1}, p_H^{S1}) can be sustained if the following incentive constraints are satisfied:

$$\pi_i(A_H, \mathbf{p}_H^{S1}, w) + \delta V_i \geq \pi_i(A_H, \tilde{p}; \mathbf{p}_H^{S1}, w)$$
$$\text{for any } \tilde{p} \neq p_H^{S1} \text{ and } i; \quad \text{(IC-H)}$$

$$E_\theta[\pi_i(A_\theta, \mathbf{p}_\phi^{S1}, w) \mid I_\phi] + \delta V_i \geq E_\theta[\pi_i(A_\theta, \tilde{p}; \mathbf{p}_\phi^{S1}, w) \mid I_\phi]$$
$$\text{for any } \tilde{p} \neq p_\phi^{S1} \text{ and } i. \quad \text{(IC-}\phi\text{)}$$

The constraint IC-H suggests that a retailer is better off adhering to the cartel's price at the information set I_H. The left-hand side (LHS) denotes the expected discounted profit for a retailer setting the collusive price, given that all other retailers adhere to the cartel plan as well. The right-hand side (RHS) denotes the profit of deviating from the cartel's scheme and setting the price $\tilde{p}$. Naturally, the best deviation is to cut the cartel's price slightly and capture the entire market during the current period. However, such deviation triggers a price war and results in zero profit in all future periods. The constraint IC-ϕ is similar and ensures that a retailer does not deviate from the cartel's plan during the information set I_ϕ.

When firms are patient enough, the fear of future punishment can outweigh the incentive to deviate and cooperation can be achieved. The following lemma formulizes this intuition.

Lemma 1. *When information can be horizontally shared and* $\delta \geq \delta^{S1} = (n-1)/(n-(1-[\mu(1-(1-\rho)^n)]))$,

(a) $p_H = p^m(A_H, w)$, $p_\phi = p^m(\tilde{A}_L, w)$;

(b) *no information is shared with the manufacturer.*

The result demonstrates that if the discount factor is high enough, the retailers are able to set the monopoly price during each information set. Furthermore, for a discount factor higher than δ^{S1}, the manufacturer sets the wholesale price as if she sells the product to a single monopoly.

The second part of the lemma studies the incentives of the retailers to share information with the manufacturer when they are able to communicate horizontally. The lemma suggests that in this case, the retailers do not have an incentive to share information with the manufacturer. Under the settings of S1, the retailers do not choose to voluntarily share information with the manufacturer since it gives the latter additional power in setting the wholesale price. This result is used to demonstrate that when the retailers cannot share information horizontally (setting S2), their only incentive to share information with the manufacturer is to establish the cartel. Since the focus of this paper is the role of information sharing in achieving collusion, in the reminder of this paper I assume that the retailers are patient (i.e., have a high discount factor), so a collusion according to Lemma 1 is possible.[6] In Section 7.3, I relax this assumption and explore the minimum discount factor that supports each equilibrium.

5. Collusion with Vertical Information Sharing (Setting S2)

In this section, I study the scope of collusion when the retailers cannot share information horizontally, and thus share their private information with the manufacturer. By sharing their private information with the manufacturer, the model is transformed from the setting in which the retailers have the superior information in the distribution channel into a model in which the manufacturer has the cumulative knowledge about the observed signals. Such a setting raises a few important issues such as the way the demand information affects the wholesale price and, subsequently, the way the wholesale price is used by the retailers as an inference tool.

I focus on a separating equilibrium that allows the retailers to infer the private information of the manufacturer based on the wholesale price. In such an equilibrium, although the retailers are not exposed to the private information of their competitors, they can infer this information by observing the wholesale price w. The way each retailer interprets I depends on the belief system of the retailers about the relationship between the wholesale price w and the market condition. In the subsequent analysis, I show that such a pair of prices w_H and w_ϕ exists, such that when observing w_H, the retailers infer that at least one retailer has received an informative signal, and when observing w_ϕ, they infer that all of the retailers have received the noisy signal ϕ. In addition, based on the belief system of the retailers regarding the relationship between the wholesale price and the state of the demand, the manufacturer finds it in her best interest to set the wholesale price according to the correct state of the demand. It is important to emphasize that in this setting, although information is exchanged in a vertical manner, there is no cooperation between the retailers and the manufacturer—the manufacturer is not part of the cartel; thus, she sets the wholesale while taking the independent pricing behavior of the retailers into account. In addition, the retailers cannot condition their decision of whether to share information with the manufacturer on the latter setting a specific wholesale price. Consequently, when information is shared, the manufacturer uses this information to maximize her per-period profit.

Under this equilibrium, before the first selling season, the retailers and the manufacturer take the necessary measures to transfer the demand information during every selling season. I assume that information is transferred truthfully between each retailer and the manufacturer. Appendix B relaxes this assumption and evaluates the validity of the results presented in this section when information can also be manipulated.

In a separating equilibrium, the manufacturer maximizes her profit given the information she receives from the retailers. In such an equilibrium, the following incentive constraints must be satisfied:

$$\max_{w_H^{S2},\, w_\phi^{S2}} V_M$$

s.t.

$$\pi_M(A_H, p_H^{S2}, w_H^{S2}) \geq \pi_M(A_H, p(\tilde{w}), \tilde{w}), \quad \text{for any } \tilde{w} \neq w_H^{S2}; \qquad \text{(M-IC-H)}$$

$$E_\theta[\pi_M(A_\theta, p_\phi^{S2}, w_\phi^{S2}) \mid I_\phi] \geq E_\theta[\pi_M(A_\theta, p(\tilde{w}), \tilde{w}) \mid I_\phi], \quad \text{for any } \tilde{w} \notin w_\phi^{S2}. \qquad \text{(M-IC-}\phi\text{)}$$

In addition, the following set of incentive constraints of the retailers must also be satisfied in equilibrium:

$$\max_{p_H^{S2},\, p_\phi^{S2}} V_i$$

s.t.

$$\pi_i(A_H, \mathbf{p}_H^{S2}, w_H^{S2}) + \delta V_i \geq \pi_i(A_H, \tilde{p}, \mathbf{p}_H^{S2}, w_H^{S2}), \quad \text{for any } \tilde{p} \neq p_H^{S2} \text{ and every } i; \qquad \text{(IC-H)}$$

$$E[\pi_i(A_\theta, \mathbf{p}_\phi^{S2}, w_\phi^{S2}) \mid I_\phi] + \delta V_i \geq E[\pi_i(A_\theta, \tilde{p}, \mathbf{p}_\phi^{S2}, w_\phi^{S2}) \mid I_\phi], \quad \text{for any } \tilde{p} \neq p_\phi^{S2} \text{ and every } i. \qquad \text{(IC-}\phi\text{)}$$

When the manufacturer receives all of the signals from the retailers, in a separating equilibrium, the set of wholesale prices $\{w_\phi^{S2}, w_H^{S2}\}$ is chosen such that it reveals information to the retailers while maximizing the manufacturer's profit. Constraint M-IC-H states that on receiving an informative signal, the manufacturer is better off setting the wholesale price of w_H^{S2} and anticipating that the retailers set the retail price of p_H^{S2}. The LHS of this constraint characterizes the profit of the manufacturer when she sets the wholesale price of w_H^{S2}. The RHS of this constraint characterizes the profit of the manufacturer when she chooses any wholesale price that differs from w_H^{S2} under the information set of I_H. In a similar way, the constraint M-IC-ϕ states that on receiving only the noisy signals, the manufacturer is better off setting the wholesale price of w_ϕ^{S2} over any other possible wholesale price $\tilde{w}$. Since there is no pricing cooperation between the retailers and the manufacturer, the latter uses the information she receives from the retailers to maximize her profit during every selling period. The manufacturer cannot condition her pricing decisions on future cooperation or retaliation of the retailers. As a result, the equilibrium wholesale prices characterized below (Proposition 1) are independent of the manufacturer's discount factor. By contrast, when the retailers determine the collusive prices, they take into account the prospects of future cooperation or retaliation.

The next proposition characterizes the separating equilibrium in the vertical information sharing scenario.

Proposition 1. *There exists a separating equilibrium in which*

(a) *the manufacturer sets the wholesale price*

$$w_H = \frac{A_H}{2},$$

if she receives at least one informative signal,

$$w_\phi = \begin{cases} \dfrac{\mu(1-\rho)^n A_H + A_L(1-\mu)}{2[\mu(1-\rho)^n + (1-\mu)]}, & \text{if } I_\phi \text{ and } \psi \ge 3, \\ \hat{w}_\phi, & \text{if } I_\phi \text{ and } \psi < 3, \end{cases}$$

where

$$\hat{w}_\phi = \frac{2A_H - \tilde{A}_L - \sqrt{3A_H^2 - 4A_H\tilde{A}_L + (\tilde{A}_L)^2}}{2};$$

(b) *the retailers set the prices*

$$p_H = \tfrac{3}{4}A_H, \quad \text{if } w = w_H,$$

$$p_\phi = \begin{cases} \tfrac{3}{4}\tilde{A}_L, & \text{if } w = w_\phi \text{ and } \psi \ge 3, \\ \hat{p}_\phi, & \text{if } w = w_\phi \text{ and } \psi < 3, \end{cases}$$

where

$$\hat{p}_\phi = \frac{\tilde{A}_L}{2} + \frac{2A_H - \tilde{A}_L - \sqrt{3A_H^2 - 4A_H\tilde{A}_L + (\tilde{A}_L)^2}}{4}$$

with off the equilibrium path prices of

$$p = \begin{cases} p^m(A_H, w), & \text{if } w \ne w_H \text{ and } w > w_\phi, \\ p^m(\tilde{A}_L, w), & \text{if } w < w_\phi \end{cases}$$

and consistent with the belief system that

$$\mu(w) = \Pr(A_H \mid w) = \begin{cases} 1, & \text{if } w > w_\phi, \\ \dfrac{\mu(1-\rho)^n}{\mu(1-\rho)^n + 1 - \mu}, & \text{if } w \le w_\phi. \end{cases}$$

Proposition 1 characterizes the following equilibrium: each retailer shares his observed signal with the manufacturer during every selling period. Upon observing that all retailers received the noisy signal $Y_i = \phi$, the manufacturer sets the wholesale price of w_ϕ^{S2}. If the manufacturer receives at least one informative signal, she sets the price to be w_H^{S2}.

To better understand the results of Proposition 1, it is beneficial to compare them to a setting of complete information. In a complete information scenario, the manufacturer and the retailers know whether the current state is I_H or I_ϕ. In this case, the wholesale price has the operational role of maximizing the manufacturer's profit. In a complete information setting, the manufacturer would set the wholesale price to $w_H = A_H/2$ during the state I_H and to $w_\phi = (\mu(1-\rho)^n A_H + A_L(1-\mu))/(2[\mu(1-\rho)^n + (1-\mu)])$ during the state I_ϕ.

Under asymmetric information, when the manufacturer has the superior information, the wholesale price plays a dual role: the first role is the operational role of maximizing the manufacturer's profit. The additional role, which is the result of asymmetric information, is to signal information about the demand state to the retailers. Note that the manufacturer has the incentive to induce the retailers to set a low retail price to increase the sold quantity. In a separating equilibrium, the set of wholesale prices must be determined in such a way that the retailers infer correctly whether the current state is I_H or I_ϕ. According to Proposition 1, in the information set I_H, the manufacturer sets the same wholesale price as in the complete information scenario since this wholesale price signals to the retailers correctly the state of demand—the manufacturer does not have an incentive to try and mislead the retailers to believe that demand is high when this is not the case.

On observing the wholesale price of w_ϕ^{S2}, the retailers should infer that the current state is I_ϕ. When ψ is high ($\psi \ge 3$), the manufacturer is able to set the same wholesale price as in the complete information setting. In this case, the wholesale price of

$$w_\phi = \frac{\mu(1-\rho)^n A_H + A_L(1-\mu)}{2[\mu(1-\rho)^n + (1-\mu)]}$$

conveys to the retailers, in a credible manner, that the manufacturer did not receive any informative signal. The wide gap between w_ϕ^{S2} and w_H^{S2} deters the manufacturer from setting the wholesale price w_ϕ^{S2} when receiving an informative signal. However, if ψ is not high (measured by $\psi < 3$), setting the complete information wholesale price cannot convey to the retailers that indeed the manufacturer did not receive any informative signal. In this case, even if the manufacturer observed an informative signal, she has an incentive to set the price $w_\phi = (\mu(1-\rho)^n A_H + A_L(1-\mu))/(2[\mu(1-\rho)^n + (1-\mu)])$ to induce the retailers to lower their retail prices, thereby increasing the sold quantity and the manufacturer's profit. Anticipating such behavior, the retailers ignore the informative role of the wholesale price. As a result, to achieve a separating equilibrium, the manufacturer must design a price schedule that will convey the true state of demand to the retailers based on the wholesale price. This is achieved by lowering the wholesale price during periods in which she does not receive any informative signal—in this way, when demand is high, the manufacturer does not have an incentive to set the low wholesale price. Proposition 1 characterizes the optimal wholesale price, from the manufacturer's perspective, that achieves this accountability when there is no cooperation between the retailers and the manufacturer. As in many signaling games, a separating equilibrium is not unique; other possible separating equilibria are ruled out by imposing the intuitive criterion (Cho and Kreps 1987) that the equilibrium in Proposition 1 satisfies.

Gal-Or et al. (2008) also observe price distortion under settings of asymmetric information. They refer to this distortion as the *inference effect*. In their work, they show that the inference effect results in a lower wholesale price compared with the case of complete information. Proposition 1 is aligned with the findings of Gal-Or et al. (2008), showing that under settings of asymmetric information, the manufacturer may need to decrease the wholesale price to be accountable for the shared information the wholesale price signals to the retailers.

6. Collusion Without Information Sharing (Setting S3)

The Folk Theorem (Fudenberg and Tirole 1991) suggests that the equilibrium studied in setting S2 is not unique, and scholars in the area of antitrust have showed that even in the absence of communication, the retailers can still collude without any information exchange. Therefore, the natural question is how the equilibrium characterized in setting S2 is preferable relative to other mechanisms that are available to the retailers. To answer this question, I briefly analyze two additional equilibria that are available to the retailers and that also do not require any horizontal information exchange: the responsive pricing strategy and the rigid pricing strategy. In the responsive pricing equilibrium, each retailer sets a price based only on his observed signal, whereas in the rigid pricing strategy, the retailers ignore their private information and set a uniform price during all selling periods. I choose to focus on these two alternative mechanisms for cartel formation since the literature has concentrated so far on such equilibria (see, for example, Athey and Bagwell 2001, Athey et al. 2004, Gerlach 2009), and I wish to demonstrate that the equilibrium analyzed in setting S2 is preferable to these alternative equilibria for the retailers and the manufacturer.

6.1. Responsive Pricing Strategy

In this option, which I refer to as a *responsive pricing strategy*, the retailers set different prices when they observe distinctive signals; i.e., the pricing strategy of the retailers is responsive to the signal they observe.[7]

Denote the probability that, given a high demand state, k retailers observe the noisy signal out of n possible retailers by

$$P_k^n \triangleq \binom{n}{k}(1-\rho)^k \rho^{n-k}. \tag{7}$$

The expected profit during the current period of a retailer observing the informative signal $Y_i = H$ and setting the price p_H is

$$\Pi_i(p_H \mid Y_i = H) = \rho^{n-1}\pi_i(A_H, \mathbf{p}_H, w), \tag{8}$$

and the expected profit during the current period of a retailer observing the noisy signal $Y_i = \phi$ and setting the price p_ϕ is

$$\begin{aligned}&\Pi_i(p_\phi \mid Y_i = \phi)\\ &= \frac{1-\mu}{1-\mu+\mu(1-\rho)}\pi_i(A_L, p_\phi, w)\\ &\quad + \frac{\mu(1-\rho)}{1-\mu+\mu(1-\rho)}\sum_{k=0}^{n-1} P_k^{n-1}\pi_i(A_H, p_\phi^{k+1}, p_H^{n-k-1}, w). \end{aligned}\tag{9}$$

Equation (8) denotes the current-period profit of a retailer observing the informative signal. When a retailer observes the informative signal, he infers that demand is high in the current period. However, setting the collusive price p_H results in strictly positive profit only if all other $n-1$ retailers observe the informative signal as well. With probability $1-\rho^{n-1}$, at least one retailer observes a noisy signal. In the latter case, the retailer who observes the noisy signal sets the price p_ϕ and leaves zero profit to all of the retailers who observe an informative signal. This problem is unique to the responsive pricing strategy: In setting S1, a retailer observing the informative signal was able to share this information with the other cartel members, thus ensuring that all of the retailers set the appropriate price. In setting S2, a retailer observing the informative signal was also able to share this information with the other retailers in an indirect manner by using the wholesale price mechanism. However, in setting S3, when no information sharing is permitted, a retailer who sets a high retail price based on his private information is exposed to the risk of being left without any sales due to another retailer observing the noisy signal and setting a lower retail price. This problem can result in a motivation for a retailer, who observes the informative signal, to act as if he observed the noisy signal and to set a lower retail price. Note that such a deviation, of setting a low retail price instead of the high price, cannot be detected by the other retailers since even when demand is high there is a positive probability of $1-\rho$ of observing the noisy signal ϕ.

Equation (9) denotes the profit of a retailer who is observing the noisy signal. With probability $(1-\mu)/(1-\mu+\mu(1-\rho))$, the demand during the current selling season is low; hence, all other retailers would observe the noisy signal as well. With the probability $\mu(1-\rho)/(1-\mu+\mu(1-\rho))$, the demand is actually high, and the retailer shares the market with all other retailers observing the noisy signal.

The following proposition provides some of the properties of the optimal pricing strategy of the cartel when information cannot be shared directly between the retailers.

Proposition 2. (a) *The retailers' pricing strategy* (*denoted by* p_H^*, p_ϕ^*) *is given by the set of equations*

$$p_H^* = p^m(A_H, w) = \frac{A_H + w}{2},$$
$$p_\phi^* = p^m(E[A_\theta \mid Y_i = \phi], w) = \frac{1}{2}E[A_\theta \mid Y_i = \phi] + \frac{w}{2},$$

if and only if the following condition holds:

$$\rho^{n-1}\pi_i(A_H, p_H^*, w) \geq \sum_{k=0}^{n-1} P_k^{n-1}\pi_i(A_H, p_\phi^*, w). \tag{10}$$

(b) *In all other cases, the cartel chooses a price* $p_\phi < p_\phi^*$, *and price wars can be observed on the equilibrium path.*

Proposition 2 characterizes the optimal pricing strategy of the colluding retailers when the condition given in part (a) is satisfied. This condition suggests that a retailer observing the high signal is better off setting the price p_H^* than mimicking a retailer observing the low signal. The condition shows that if the difference between the expected profit obtained by setting the price p_H^* and the expected profit obtained by setting the price p_ϕ^* is high enough, a retailer prefers to set the price p_H^* after observing the informative signal. Note that the profit of the retailers when they are able to set the prices (p_H^*, p_ϕ^*) serves as an upper bound on the attainable cartel's profit; the equilibrium in which the retailers are able to set the prices (p_H^*, p_ϕ^*) Pareto dominates any other equilibrium that does not include information exchange. When the retailers cannot set the prices (p_H^*, p_ϕ^*), they must adopt another mechanism that induces a retailer observing the informative signal to set the appropriate price and to choose not to set the price designed for a retailer observing the noisy signal. Part (b) of the proposition suggests that such a mechanism includes downward distortion of the price p_ϕ^* to make it less compelling for a retailer observing the informative signal to set this low retail price. In addition, the retailers can choose to initiate price wars on the equilibrium path when the realized demand is high but the retailers set the price p_ϕ.[8]

6.2. Rigid Pricing Scheme

As an alternative to the separating pricing scheme, the retailers can choose rigid pricing. In this case, the retailers set the same price during all selling periods, regardless of the observed signal. When the retailers set a rigid price, each deviation from this price is immediately observed and can be punished. Hence, for a high discount factor, a retailer does not find it beneficial to deviate from this strategy.

The following lemma characterizes the pricing strategy and the profit of the different firms in the rigid pricing scheme.

Lemma 2. *When the retailers choose a rigid pricing scheme*

$$p = \frac{3}{4}A_\mu, \quad w = \frac{1}{2}A_\mu, \quad \Pi_i = \frac{A_\mu^2}{16}, \quad \Pi_M = \frac{A_\mu^2}{8}.$$

7. Discussion

7.1. Equilibrium Selection

Following the analysis of Sections 5 and 6, both of which allow the retailers to collude without direct information sharing, the natural question is, what is the preferred mechanism from the retailers' perspective? In this section, I compare the option of the retailers to collude without communication (setting S3) and the scenario in which the retailers share information with the manufacturer and infer the market condition using the wholesale price. In addition, setting S2 is plausible only if the manufacturer also prefers this equilibrium over setting S3. Thus, setting S2 is a viable option only if this equilibrium Pareto dominates setting S3 for both the retailers and the manufacturer. I start by examining some of the properties of each setting:

Information efficiency. In setting S3, under the responsive pricing strategy, with probability $\mu\sum_{k=1}^{n-1}P_k^n$, the cartel suffers from information inefficiency. Although at least one member in the cartel observes an informative signal, there is also at least one retailer who observes the noisy signal. In this case, the market price is set to be lower than the efficient price. By contrast, in setting S2, if at least one retailer observes an informative signal, all of the other retailers can infer that demand is high through the wholesale price. Clearly, if the cartel adopts the rigid pricing scheme, there is also information inefficiency, since the retailers ignore the private information of its members. The difference in the information efficiency can also lead to implications regarding information acquisition: assuming information acquisition is costly, when the cartel members cannot use their information efficiently, they may prefer not to acquire such information.

Cartel stability. The second difference between scenario S2 and scenario S3 is the stability of the cartel. While in scenario S2 the cartel can operate over an infinite horizon, in scenario S3, when the retailers choose the responsive pricing scheme, the cartel may adopt to use price wars on the equilibrium path.

Market knowledge. The third interesting difference is related to whether the manufacturer or the retailers have better information regarding the future demand. In setting S3, when the retailers use the rigid pricing strategy, both the retailers and the manufacturer do not use any information about the future demand as they determine the retail and wholesale prices. In setting S3, when the retailers adopt the responsive pricing strategy, each retailer uses his private information as he determines the retail prices, but the manufacturer uses only the prior belief as she sets the wholesale price. In this case, the retailers have better information regarding the demand than the manufacturer has. In setting S2, the manufacturer receives the private information from all of the retailers; following this information exchange, she is endowed with better information than the retailers. In this setting, the retailers use the wholesale price to make inference regarding the state of the demand. As a result, when the pricing decisions are made, both the manufacturer and the retailers have the same information regarding the future demand.

I now return to the issue of whether the retailers will choose to form a cartel by sharing information vertically with their manufacturer. The retailers prefer to share their private information with the manufacturer if their discounted profit in scenario S2 is higher than their discounted profit in scenario S3. Analogously, the manufacturer is willing to act as the information aggregator for the retailers if her profit in scenario S2 is higher than the one she can obtain in scenario S3.

In this section I adopt the ex ante perspective when comparing between the performance of each equilibrium. The analysis in Sections 4–6 studies the ex post incentives of the firms in the distribution channel to act according to their observed signal. In this section, I study the choice of the firms in a specific equilibrium, and thus I assume that this decision is made prior to the first selling season. Another way to view this choice is to assume that if the retailers and the manufacturer wish to exchange information, they must install dedicated hardware and software for this purpose, and thus the decision to install this hardware must be made before the beginning of the first selling season.

Let V_i^{S2} and V_M^{S2} be the discounted profits of the typical retailer and the manufacturer, respectively, in setting S2. Similarly, a retailer's discounted profit in setting S3 is given by V_i^{S3} and the manufacturer's profit by V_M^{S3}. The participants in the distribution channel prefer setting S2 over setting S3 if the following conditions are satisfied:

$$\begin{aligned} V_i^{S2} &\geq V_i^{S3}, \\ V_M^{S2} &\geq V_M^{S3}. \end{aligned} \tag{11}$$

The next proposition demonstrates that there are cases in which the constraints given in Equation (11) are satisfied.

Proposition 3. *For $\psi \geq 3$, there are $\tilde{n}$ and $\tilde{\rho}$ such that for $n > \tilde{n}$ and $\rho < \tilde{\rho}$*

$$\begin{aligned} V_i^{S2} &\geq V_i^{S3}, \\ V_M^{S2} &\geq V_M^{S3}. \end{aligned}$$

Proposition 3, which is an important result in this paper, suggests that there are cases in which the manufacturer and the retailers are better off in setting S2 compared with any other mechanism for collusion studied in this paper. When the number of retailers operating in the market is relatively high, adopting the responsive pricing strategy is challenging. In this case, the probability that at least one retailer will observe the noisy signal while another retailer observes the informative signal is high. As a result, to satisfy the incentive constraints of a retailer observing the informative signal, the cartel must distort the price of p_ϕ and occasionally initiate price wars. Because of these challenges in adopting the responsive pricing strategy, the cartel can be better off choosing the rigid pricing scheme over the responsive pricing scheme.

When the retailers choose the rigid pricing scheme, the manufacturer understands that by refusing to accept the shared information, she earns a profit of $A_\mu^2/8$ during all periods. By choosing a different equilibrium than setting S2, the manufacturer cannot prevent the retailers from colluding, but she, rather, induces the retailers to adopt setting S3. Thus, the proposition suggests that when the number of retailers is high, colluding by using vertical information exchange Pareto dominates any other equilibrium studied in this paper.

From the retailers' perspective, choosing to share information with the manufacturer enables them to benefit from the information about the market demand. Choosing a rigid pricing scheme in scenario S3 solves the retailers' coordination problem, but this coordination is achieved at the cost of ignoring the demand information available to the retailers. When the retailers share information with the manufacturer, they are able to collude and enjoy the value of better forecast.

Proposition 3 highlights another factor that results in vertical information-exchange equilibrium. If the precision level is relatively low, the coordination problem the cartel faces is severe. A retailer observing the informative signal has a small probability of gaining some market share by setting the high price, since there is a high probability of another retailer observing the noisy signal when the precision is low. Because of this coordination problem, the retailers are better off sharing their private information with the manufacturer than adopting any alternative collusion mechanism.

The next proposition sheds additional light on the incentives of the manufacturer to choose the equilibrium with vertical information exchange.

Proposition 4. *If* $\mathrm{Var}(A_i) - E[\mathrm{Var}(A_i \mid \mathbf{Y}_n)] \geq A_\mu^2$ *and* $\psi \geq 3$, *the manufacturer is better off in setting S2 compared even with the competitive outcome.*

Proposition 4 demonstrates that there are cases in which the manufacturer is better off playing the equilibrium studied in setting S2, even compared with the competitive outcome. Although in a Bertrand competition among the retailers, the latter set the retail price to equate to the wholesale price, which results in the manufacturer capturing all of the profit in the market, the proposition suggests that under certain conditions, the manufacturer can do better by receiving the information from the retailers and allowing the retailers to collude. This can occur when the market volatility (captured by $\mathrm{Var}(A_i)$) is relatively high and the amount of information contained in the vector of messages $\mathbf{Y}_n$ about the future market demand is also high, such that the expected variability given $\mathbf{Y}_n$ is small. In the absence of information sharing, the market is characterized by high volatility, which compromises the ability of the manufacturer to set the appropriate wholesale price. Observing the vector of signals $\mathbf{Y}_n$ allows the manufacturer to reduce this level of uncertainty. To reduce her risk level, the manufacturer is willing to assist the retailers to form the cartel in exchange for the retailers' information.

Proposition 4 captures the trade-off from the manufacturer's perspective: by receiving the information, she assists in forming the cartel, but also obtains valuable information about the market demand. The value of the information about the market demand is high when the market is characterized by high volatility, and the vector of signals provides an accurate estimation of the future market demand. In a similar manner, from the retailers' perspective, the trade-off is that when the information is divulged to the manufacturer, the latter sets the wholesale price in a manner consistent with 1the future market demand, which in turn harms the retailers. However, the same wholesale price allows the retailers to make inference about the signals the other retailers received, thus solving the cartel's coordination problem.

7.2. Vertical Information Exchange and Consumer Welfare

The next issue worth exploring is the effect of banning horizontal communication on the consumer surplus. I compare setting S1, in which the retailers are allowed to exchange information horizontally, and setting S2, in which the retailers collude by sharing information with the manufacturer because of their inability to share information horizontally. The main reason behind the policy that monitors horizontal information exchange among competing firms is the risk that such practice would lead to collusion, which, in turn, would result in reduced consumer surplus. This paper suggests that as a result of the unavailability of the option to share information directly, the retailers would not forego the option of colluding, but rather they could choose to obtain this desired result by sharing information with the manufacturer. The next proposition compares the effects of direct information sharing between the retailers and vertical information sharing with the manufacturer on the consumer surplus.

Proposition 5. *Let* CS^{S1} *be the consumer surplus in setting* S1, *and let* CS^{S2} *be the consumer surplus in setting* S2. *When* $\psi \geq 3$

$$CS^{\mathrm{S1}} > CS^{\mathrm{S2}}.$$

The proposition demonstrates that limiting the ability of the retailers to establish a cartel by sharing information horizontally can actually result in lower consumer surplus if the retailers choose to form a cartel by sharing information with the manufacturer. Therefore, the current antitrust policy may have the perverse effect of not limiting the scope of collusion and actually lowering the consumer surplus. In fact, the social planner would be better off allowing the competing retailers to meet face to face and exchange their private information directly than having them share this information with the manufacturer due to the fear of being exposed to antitrust sanctions. The intuition behind this result is that in both settings the retailers are able to collude, but in setting S2 they share information about the market demand with their manufacturer. As a result, the manufacturer is able to set the wholesale price to better match demand, thus exacerbating the problem of double marginalization (Spengler 1950, Pasternack 1985) and hurting the consumers.

7.3. The Critical Discount Factor

In the analysis so far, I have adopted the assumption that the retailers' discount factor is high enough that full collusion is possible. I now revisit this assumption and explore the minimum discount factor that supports cartel formation in each setting.[9]

Proposition 6. *Let* $\delta^{\mathrm{S}i}$ *be the minimum discount factor that supports full collusion in setting* Si, $i \in \{2, 3\}$. *Under the conditions of Proposition 3,* $\delta^{\mathrm{S2}} < \delta^{\mathrm{S3}}$.

Proposition 6 compares the critical discount factor in the different settings discussed in this paper. It shows that full collusion can be supported in setting S2 for a lower discount factor than the critical discount factor that supports cartel formation in setting S3. Thus, vertical information exchange not only presents a new way for the retailers to collude, it can also support cartel formation for a wider parameter region of the discount factor.

The critical discount factor that supports cartel formation is determined by the incentive constraint of a retailer observing or inferring that the demand is high. When demand is high, the incentive to deviate from the cartel's scheme is higher than when the demand is low, since in the former case the profit from deviation is higher than in the latter case. In setting S2, when demand is high, the manufacturer sets the high wholesale price of w_H^{S2}, whereas in setting S3 the wholesale price is lower (since the manufacturer has no information about the demand); consequently, under the high demand state, a deviating retailer earns a higher profit in setting S3 compared with a deviating retailer in setting S2. In addition, under the conditions of Proposition 3, cooperation results in a higher profit for the retailers in setting S2. Therefore, in setting S2, a deviation is less attractive than in setting S3, and, in addition, by deviating, a retailer forgoes higher future profits in setting S2 compared with the stream of future profits that stems from cooperation in setting S3. As a result, it is easier to support collusion in setting S2, and the critical discount factor that is required to support collusion in this case is lower than the discount factor that supports collusion in setting S3.

8. Conclusions

In this paper, I examine retailers' ability to share information with a mutual manufacturer to achieve collusion without resorting to direct communications. In particular, the retailers share their private information on the expected market demand with the manufacturer and use the wholesale price to determine the collusive supracompetitive price. I show that, in certain instances, establishing a cartel based on the information flow from the retailers to their mutual manufacturer results in a higher profit to the retailers than any other traditional cartel formation mechanism. Moreover, even a manufacturer cognizant of the fact that the retailers exploit her wholesale price to coordinate on a monopoly-pricing scheme would find it beneficial to accept the retailers' private information.

Information sharing has received considerable attention in the literature on channel coordination. Recent research has demonstrated that in a complex environment—such as one with multiple competing retailers and a mutual manufacturer—firms might be reluctant to share information, because their private information can be leaked to unintended recipients. Contrary to this stream of literature, I demonstrate that retailers may use the fact that their shared information can reach a third party to establish a cartel. Each retailer shares his private information with the manufacturer, anticipating that this information will be leaked to the competing retailers via the wholesale price. In this paper, I highlight the positive effect of "information leakage" from the retailers' perspective.

In addition to the theoretical interest, the results presented in this research have important implications from the perspective of antitrust policy. In the absence of antitrust regulations, rational, patient, profit-maximizing firms would freely act in consort to reach a consensus and to maintain prices above the competitive level. However, in light of potential exposure to antitrust investigations with the potential risk of bearing heavy sanctions, competing firms devise schemes to achieve supracompetitive prices while escaping authorities' scrutiny. Vertical information sharing in a distribution channel, such as the one analyzed in this work, is one strategy by which a group of retailers, endowed with private information about market demand, can achieve collusive outcomes. In fact, in this case, the retailers exchange information and achieve collusion while eluding the scrutiny of the antitrust enforcement, which currently fails to appreciate the fact that the vertical sharing of information in a distribution channel may generate collusive outcomes.

More specifically, this work suggests that antitrust enforcement policy, which currently focuses on horizontal information sharing as a practice that can lead to an inference of illegal collusion, should be updated to include some forms of vertical information sharing in a distribution channel as supplementary evidence for possible collusive behavior. The paper also highlights a few factors that may indicate when vertical information sharing can be viewed as a signal of collusion. In particular, as the market volatility increases, it is more likely that the colluding retailers and the manufacturer will engage in vertical information sharing.

This paper examines the scope of collusion between a group of retailers in a market with a strategic manufacturer. In such a setting, many issues arise, such as the relative bargaining power of each firm in the distribution channel, the contractual form between the manufacturer and the retailers, etc. This paper adopts the assumption that the manufacturer is a Stackelberg leader in determining the wholesale price, and that the contract form is the simple uniform wholesale price. These assumptions were made because of their popularity in both practice and research (Arrow 1985, Desai and Srinivasan 1995, Bajari and Tadelis 2001, Iyer and Villas-Boas 2003, Gal-Or et al. 2008, He et al. 2008, Chu et al. 2016). It is left for future research to explore the way additional contracting schemes (e.g., two-part tariff) and different bargaining procedures influence the ability of the retailers to form a cartel by sharing information in a vertical manner.

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Appendix A. Proofs of Main Results

Proof of Lemma 1. (a) I first show that when $\delta \geq \bar{\delta}$, the retailers are able to set the monopoly price for any given wholesale price w. In the extreme case, the manufacturer sets the wholesale price w so high that the retailers do not sell during periods in which they observe noisy signals. In this case, the IC constraint during periods of high demand is

$$\pi_i(A_H, \mathbf{p}_H^m, w) + \frac{\delta}{1-\delta}\pi_i(A_H, \mathbf{p}_H^m, w)\mu(1-(1-\rho)^n) \geq \pi_i(A_H, p_H^m - \epsilon, \mathbf{p}_H^m, w), \tag{A1}$$

which can simplified to the expression

$$\delta \geq \frac{n-1}{n-(1-\mu(1-(1-\rho)^n)} = \bar{\delta}.$$

Note that if the retailers are able to sell during periods in which they receive noisy signals, the LHS of (A1) increases, and therefore the retailers are able to set the monopoly price during periods of high demand even for a lower discount factor.

If δ is high enough, the retailers set the monopoly price during both information sets. In this case, $p_H = (A_H + w)/2$ and

$$p_\phi = \frac{\mu(1-\rho)^n A_H + (1-\mu)A_L}{2[\mu(1-\rho)^n + (1-\mu)]} + \frac{w}{2}.$$

The sold quantity, from the manufacturer's perspective, is given by $E[Q] = (\mu A_H + (1-\mu)A_L - w)/2$, and the optimal wholesale price is given by $w = (\mu A_H + (1-\mu)A_L)/2$.

(b) When all of the retailers share their private information with the manufacturer, she sets the wholesale price $w = A_H/2$ during the informative information set and the wholesale price $w = \tilde{A}_L/2$ in the information set I_ϕ. As a result, the ex ante profit of the retailers is given by

$$\begin{aligned}
\Pi_i &= \frac{\mu(1-(1-\rho)^n)}{16}A_H^2 + ((1-\mu)+\mu(1-\rho)^n)\frac{\tilde{A}_L^2}{16} \\
&= \frac{\mu(1-(1-\rho)^n)}{16}(A_\mu + A_H - A_\mu)^2 \\
&\quad + \frac{((1-\mu)+\mu(1-\rho)^n)}{16}(A_\mu + \tilde{A}_L - A_\mu)^2 \\
&= \frac{\mu(1-(1-\rho)^n)}{16}(A_\mu^2 + (A_H - A_\mu)^2 + 2(A_\mu(A_H - A_\mu)) \\
&\quad + \frac{(1-\mu)+\mu(1-\rho)^n}{16}(A_\mu^2 + (\tilde{A}_L - A_\mu)^2 + 2A_\mu(\tilde{A}_L - A_\mu)) \\
&= \frac{A_\mu^2}{16} + \frac{\mathrm{Var}(E[A_i \mid \mathbf{Y}_n])}{16} + \frac{1}{8}A_\mu \\
&\quad \cdot \Big[\underbrace{\mu(1-(1-\rho)^n)A_H + ((1-\mu)+\mu(1-\rho)^n)\tilde{A}_L}_{=A_\mu} - A_\mu\Big] \\
&= \frac{A_\mu^2}{16} + \frac{\mathrm{Var}(E[A_i \mid \mathbf{Y}_n])}{16}.
\end{aligned}$$

When the retailers do not share their information with the manufacturer, she sets the price during all periods to be $w = A_\mu/2$. In this case, the one-period ex ante profit of the cartel is given by

$$\begin{aligned}
\Pi_i &= [\mu(1-\rho)^n + 1 - \mu]\left(\frac{\tilde{A}_L}{2} - \frac{A_\mu}{4}\right)^2 \\
&\quad + \mu(1-(1-\rho)^n)\left(\frac{A_H}{2} - \frac{A_\mu}{4}\right)^2 \\
&= \frac{A_\mu^2}{16} + [\mu(1-\rho)^n + 1 - \mu]\frac{\tilde{A}_L^2}{4} + \mu(1-(1-\rho)^n)\frac{A_H^2}{4} - \frac{A_\mu^2}{4} \\
&= \frac{A_\mu^2}{16} + \frac{1}{4}E[\mathrm{Var}(A_i \mid Y_n)],
\end{aligned}$$

which is higher than the cartel's profit when information is shared with the manufacturer. $\square$

Proof of Proposition 1. The problem of the manufacturer is formulized as

$$\max_{w_H^{S2}, w_\phi^{S2}} V_M$$

s.t.

$$\pi_M(A_H, p_H^{S2}, w_H^{S2}) \geq \pi_M(A_H, p(\tilde{w}), \tilde{w}), \quad \text{for any } \tilde{w} \neq w_H^{S2}; \tag{M-IC-H}$$

$$E_\theta[\pi_M(A_\theta, p_\phi^{S2}, w_\phi^{S2}) \mid I_\phi] \geq E_\theta[\pi_M(A_\theta, p(\tilde{w}), \tilde{w}) \mid I_\phi], \quad \text{for any } \tilde{w} \notin w_\phi^{S2}. \tag{M-IC-ϕ}$$

The unconstrained solution to the complete information problem is given by $w_H = A_H/2$ and

$$w_\phi = \frac{\mu(1-\rho)^n A_H + A_L(1-\mu)}{2[\mu(1-\rho)^n + (1-\mu)]} = \tilde{A}_L \backslash 2.$$

For this set of wholesale prices, the constraint M-IC-ϕ is always satisfied. Furthermore, a manufacturer observing the informative signal will choose to set the price w_H if the following condition is satisfied:

$$\frac{A_H^2}{8} \geq \left(A_H - \frac{\mu(1-\rho)^n A_H + (1-\mu)A_L}{2[1-\mu+\mu(1-\rho)^n]} - \frac{w_\phi}{2}\right)w_\phi. \tag{A2}$$

The LHS denotes the profit of the manufacturer from setting the price w_H. The RHS denotes the manufacturer's profit from setting the price w_ϕ for the complete information scenario. It is possible to confirm that if

$$\psi = \frac{A_H}{(\mu(1-\rho)^n A_H + (1-\mu)A_L)/[1-\mu+\mu(1-\rho)^n]} \geq 3,$$

the set of wholesale prices $w_H = A_H \backslash 2$ and $w_\phi = \tilde{A}_L \backslash 2$ satisfies the condition given in Equation (A2). However, if $\psi < 3$, the condition given in Equation (A2) is not satisfied. In the latter case, solving Equation (A2) for equality provides the solution

$$w_\phi = \frac{2A_H - \tilde{A}_L - \sqrt{3A_H^2 - 4A_H\tilde{A}_L + (\tilde{A}_L)^2}}{2}. \tag{A3}$$

Note that I assume in this solution that an interior solution exists. The condition for the existence of such an interior solution is that $w_\phi \leq A_L$. This condition is given by

$$\frac{2A_H - \tilde{A}_L - \sqrt{3A_H^2 - 4A_H\tilde{A}_L + (\tilde{A}_L)^2}}{2} \leq A_L.$$

In equilibrium, the manufacturer will not choose any wholesale price that differs from w_H or w_ϕ. $\square$

Proof of Proposition 2. (a) When information is not shared, a set of prices is sustainable if the following conditions are satisfied:

$$E[\pi_i(A_H, p_H, w) \mid Y_i = H] + \delta V_i(h_{t+1}) \geq E[\pi_i(A_H, \tilde{p}, w) \mid Y_i = H], \quad \text{for every } \tilde{p} \notin \{p_H, p_\phi\}; \qquad \text{(IC-off-H)}$$

$$E[\pi_i(A_\theta, p_\phi, w) \mid Y_i = \phi] + \delta V_i(h_{t+1}) \geq E[\pi_i(A_\theta, p_H, w) \mid Y_i = \phi], \quad \text{for every } \tilde{p} \notin \{p_H, p_\phi\}; \qquad \text{(IC-off-}\phi\text{)}$$

$$E[\pi_i(A_H, p_H, w) \mid Y_i = H] + \delta V_i(h_{t+1}) \geq E[\pi_i(A_H, p_\phi, w) \mid Y_i = H] + \delta V_i(h_{t+1}); \qquad \text{(IC-on-H)}$$

$$E[\pi_i(A_\theta, p_\phi, w) \mid Y_i = \phi] + \delta V_i(h_{t+1}) \geq E[\pi_i(A_\theta, p_H, w) \mid Y_i = \phi] + \delta V_i(h_{t+1}). \qquad \text{(IC-on-}\phi\text{)}$$

The first two constraints are the off-schedule constraints (i.e., deviations that are detected immediately). It was shown by Lemma 1 that the off-schedule constraints are slack for a high discount factor.

Let us consider the incentive constraint of a retailer observing the noisy signal. When a retailer observes the noisy signal and sets the price p_H, there is a strictly positive probability that demand is actually low. In this case, all of the retailers can infer that a retailer setting the high price has deviated from the collusive plan, and they initiate a price war. As a result, for a high enough discount factor, such a deviation is not profitable, since there is a positive probability that this deviation will result in price wars.

I now analyze the incentive constraint of a retailer observing the informative signal. Let us first relax this incentive constraint and assume the cartel solves an unconstrained optimization problem. When observing the informative signal, the cartel member knows that the demand is high, and when observing the noisy signal, the cartel members update the belief about the status of demand. The price p_H is implicitly given by $\partial\pi^m(A_H, p_H, w)/\partial p_H = 0$. If the retailer observes the noisy signal, he sets the price p_ϕ, which is implicitly given by

$$\frac{\mu(1-\rho)}{1-\mu+\mu(1-\rho)} \frac{\partial\pi(A_H, p_\phi, w)}{\partial p_\phi} + \frac{1-\mu}{1-\mu+\mu(1-\rho)} \frac{\partial\pi(A_L, p_\phi, w)}{\partial p_\phi} = 0.$$

This price maximizes the retailer's profit given his belief system.

A retailer who observes an informative signal and sets the price p_H earns an expected profit of $\rho^{n-1}\pi_i(A_H, p_H, w)$. With probability ρ^{n-1}, all other retailers observe the informative signal as well, and with probability $(1-\rho^{n-1})$, at least one retailer observes the noisy signal. In the latter case, a retailer setting the price p_H earns zero profit. A retailer who decides to deviate and set the price p_ϕ earns the profit $\sum_{k=0}^{n-1} P_k^{n-1}\pi_i(A_H, p_\phi, w)$. Therefore, the cartel can implement this solution, if a retailer does not find it beneficial to deviate. This is given by the condition

$$\rho^{n-1}\pi_i(A_H, p_H^*, w) \geq \sum_{k=0}^{n-1} P_k^{n-1}\pi_i(A_H, p_\phi^*, w).$$

Therefore, the unconstrained optimization problem also satisfies the on-schedule constraint of a retailer observing the informative signal if the above condition is satisfied.

(b) When the condition given above is not satisfied, the cartel must use additional tools to align the incentives of a retailer observing the informative signal. The solution to this problem is out of the scope of this paper, but the interested reader may consult Athey and Bagwell (2001) and Athey et al. (2004) for a treatment of this case. These papers show how the use of price wars and price distortion may align the incentives of a retailer observing the informative signal to set the appropriate price. □

Proof of Lemma 2. When the retailers choose a rigid pricing scheme, they choose the price p by solving the problem

$$\max_p \mu(A_H - p)(p - w) + (1-\mu)(A_L - p)(p - w).$$

The solution to this problem is $p = (A_\mu + w)/2$, and the expected sold quantity is $Q = (A_\mu - w)/2$. In this case, the manufacturer sets the wholesale price $w = A_\mu/2$, and the profits of the manufacturer and the retailers are as suggested by the lemma. □

Proof of Proposition 3. The proof is constructed in three steps. In the first step, I show that the vertical information exchange is preferable to the retailers to the responsive pricing strategy for a high number of retailers. In the second step, I prove that for a high number of retailers, the vertical information exchange is preferable to the retailers to the rigid pricing scheme. Finally, in the third step I show that also the manufacturer prefers the vertical information exchange.

Step 1. The upper bound on the ex ante per-period profit of the cartel in scenario S3, under the responsive pricing, is given by

$$n\Pi_i^{S3} = (1-\mu)\pi^m(A_L, p_\phi^*, w_\mu) + \mu(1-\rho^n)\pi^m(A_H, p_\phi^*, w_\mu) + \mu\rho^n\pi^m(A_H, p_H^*, w_\mu).$$

The ex ante per-period profit of the cartel in scenario S2 is given by

$$n\Pi_i^{S2} = (1-\mu)\pi^m(A_L, p_\phi^{S2}, w_L^{S2}) + \mu(1-\rho)^n\pi^m(A_H, p_\phi^{S2}, w_L^{S2}) + \mu(1-(1-\rho)^n)\pi^m(A_H, p_H^{S2}, w_H^{S2}).$$

Note that $\pi^m(A_L, p_\phi^{S2}, w_L^{S2}) > \pi^m(A_L, p_\phi^*, w_\mu)$ and that $\pi^m(A_H, p_H^{S2}, w_H^{S2}) > \pi^m(A_H, p_\phi^*, w_\mu)$ for $\psi > 3$. Furthermore, Π_i^{S3} is decreasing in n, while Π_i^{S2} is increasing in n, and for n high enough, $\Pi_i^{S2} > \Pi_i^{S3}$.

Step 2. In scenario S3, the profit of the retailers, under the rigid pricing scheme, during each period is $\Pi_i^{S3} = A_\mu^2/16$. When they share information with the manufacturer and $\psi > 3$

$$\Pi_i^{S2} = \mu\left[(1-(1-\rho)^n)\frac{A_H^2}{16}\right] + \frac{(\mu(1-\rho)^n + (1-\mu))[\tilde{A}_L]^2}{16}.$$

Note that by Jensen's inequality, $\Pi_i^{S2} > \Pi_i^{S3}$.

Step 3. The manufacturer's profit in scenario S3 under the rigid pricing scheme is $\Pi_M^{S3} = A_\mu^2/8$, and in scenario S2 for $\psi > 3$ it is

$$\Pi_M^{S2} = \mu\left[(1-(1-\rho)^n)\frac{A_H^2}{8}\right] + \frac{(\mu(1-\rho)^n + (1-\mu))[\tilde{A}_L]^2}{8}.$$

Applying Jensen's inequality gives $\Pi_M^{S2} > \Pi_M^{S3}$.

Under the responsive pricing equilibrium, the profit of the manufacturer in each period is given by

$$\Pi_M^{S3} = (1-\mu)\pi_M(A_L, p_\phi^{S3}, w_\mu) + \mu(1-\rho^n)\pi_M(A_H, p_\phi^{S3}, w_\mu) + \mu\rho^n \pi_M(A_H, p_H^{S3}, w_\mu).$$

The profit of the manufacturer in the vertical information exchange for $\psi \geq 3$ is given by

$$\Pi_M^{S2} = (1-\mu)\pi_M(A_L, p_\phi^{S2}, w_\phi^{S2}) + \mu(1-\rho)^n \pi_M(A_H, p_\phi^{S2}, w_\phi^{S2}) + \mu(1-(1-\rho))^n \pi_M(A_H, p_H^{S2}, w_H^{S2}).$$

Note that $\pi_M(A_L, p_\phi^{S2}, w_\phi^{S2}) > \pi_M(A_L, p_\phi^{S3}, w_\mu)$ and that $\pi_M(A_H, p_H^{S2}, w_H^{S2}) > \pi_M(A_H, p_\phi^{S3}, w_\mu)$, and thus for a high value of n we have that $\Pi_M^{S2} > \Pi_M^{S3}$.

The analysis for ρ is very similar and hence was omitted. □

Proof of Proposition 4. Consider first the case, in which the cartel collapses and the retailers compete by pricing the product at their marginal cost of w. In this case, the expected profit of the manufacturer is $A_\mu^2/4$. In setting S2, when $\psi \geq 3$, the expected profit of the manufacturer is given by

$$\begin{aligned}\Pi_M^{S2} &= \mu(1-(1-\rho)^n)\frac{(A_H)^2}{8} + \left(\mu(1-\rho)^n + (1-\mu)\right)\frac{(\tilde{A}_L)^2}{8}\\ &= \mu(1-(1-\rho)^n)\frac{(A_H + A_\mu - A_\mu)^2}{8}\\ &\quad + \left(\mu(1-\rho)^n + (1-\mu)\right)\frac{(A_\mu - A_\mu + \tilde{A}_L)^2}{8}\\ &= \frac{(A_\mu)^2}{8} + \frac{\mu(1-(1-\rho)^n)}{8}\left[(A_H - A_\mu)^2 + 2(A_H - A_\mu)A_\mu\right]\\ &\quad + \frac{\mu(1-\rho)^n + (1-\mu)}{8}\left[(\tilde{A}_L - A_\mu)^2 + 2(\tilde{A}_L - A_\mu)A_\mu\right]\\ &= \frac{(A_\mu)^2}{8} + \frac{\operatorname{Var}(E[A_i \mid \mathbf{Y}_n])}{8},\end{aligned}$$

since

$$\frac{\mu(1-(1-\rho)^n)}{8}2(A_H - A_\mu)A_\mu + \frac{\mu(1-\rho)^n + (1-\mu)}{8}2(\tilde{A}_L - A_\mu)A_\mu = 0.$$

By the law of total variance, we have that $\operatorname{Var}(E[A_i \mid \mathbf{Y}_n]) = \operatorname{Var}(A_i) - E[\operatorname{Var}(A_i \mid \mathbf{Y}_n)]$. □

Proof of Proposition 5. Since the demand function is linear, the consumer surplus is $\frac{1}{2}Q^2$ and can be expressed as

$$CS^{S2} = \frac{1}{2}\left[\mu(1-(1-\rho)^n)\left(\frac{A_H}{4}\right)^2 + \mu(1-\rho)^n\left[A_H\left(\frac{3}{4}\tilde{A}_L\right)\right]^2 + (1-\mu)\left[A_L - \left(\frac{3}{4}\tilde{A}_L\right)\right]^2\right] \quad \text{(A4)}$$

and

$$\begin{aligned}CS^{S1} &= \frac{1}{2}\left[\mu(1-(1-\rho)^n)\left(\frac{A_H}{2} - \frac{A_\mu}{4}\right)^2 + \mu(1-\rho)^n\left[A_H - \left(\frac{1}{2}\tilde{A}_L\right) - \frac{A_\mu}{4}\right]^2\right.\\ &\quad \left. + (1-\mu)\left[A_L - \left(\frac{1}{2}\tilde{A}_L\right) - \frac{A_\mu}{4}\right]^2\right]\\ &= \frac{1}{2}\left[\mu(1-(1-\rho)^n)\left(\frac{A_H}{4} + \frac{A_H - A_\mu}{4}\right)^2 + \mu(1-\rho)^n\right.\\ &\quad \left. \cdot\left[A_H - \left(\frac{3}{4}\tilde{A}_L\right) + \frac{\tilde{A}_L - A_\mu}{4}\right]^2\right]\\ &\quad + \frac{1}{2}\left[(1-\mu)\left[A_L - \left(\frac{3}{4}\tilde{A}_L\right) + \frac{\tilde{A}_L - A_\mu}{4}\right]^2\right]. \end{aligned} \quad \text{(A5)}$$

Subtracting (A4) from (A5) gives

$$\begin{aligned}&CS^{S1} - CS^{S2}\\ &= \frac{1}{2}\left[\mu(1-(1-\rho)^n)\left(\underbrace{\left(\frac{A_H - A_\mu}{4}\right)^2}_{*} + \underbrace{A_H\left(\frac{A_H - A_\mu}{8}\right)}_{**}\right)\right]\\ &\quad + \frac{1}{2}\left[\left(\mu(1-\rho)^n + (1-\mu)\right)\underbrace{\left(\frac{\tilde{A}_L - A_\mu}{4}\right)^2}_{*}\right]\\ &\quad + \underbrace{\frac{1}{2}\left(\frac{\tilde{A}_L - A_\mu}{2}\right)\left(\frac{\tilde{A}_L}{4}\right)}_{**}\left(\mu(1-\rho)^n + (1-\mu)\right).\end{aligned}$$

Note that summation of the two terms marked by $*$ results in $\operatorname{Var}(E[A_i \mid \mathbf{Y}_n])$, and the summation of the two terms marked by $**$ results also in $\operatorname{Var}(E[A_i \mid \mathbf{Y}_n])$. Therefore, the expression $CS^{S1} - CS^{S2} > 0$. □

Proof of Proposition 6. Note that the incentive of a retailer to deviate is stronger when he observes the informative signal (i.e., the demand is high). In this case, under setting S2, the incentive compatibility constraint of the retailer is given by

$$\pi_i(A_H, \mathbf{p}_H^{S2}, w_H^{S2}) + \delta V_i(h_{t+1}) \geq \pi_i(A_H, \tilde{p}; \mathbf{p}_H^{S2}, w_H^{S2}), \quad \text{for any } \tilde{p} \neq p_H^{S2}.$$

The best deviation is to slightly cut the price. This constraint can therefore be rewritten as

$$\delta^{S2} \geq \frac{n-1}{n}\frac{\pi^m(A_H, p^m(A_H, w_H^{S2}), w_H^{S2})}{V_i^{S2}(h_{t+1})}. \quad \text{(A6)}$$

Under setting S3 in the rigid pricing scheme, the discount factor must satisfy the following constraint:

$$\delta^{S3} \geq \frac{n-1}{n}\frac{\pi^m(A_H, p^m(A_H, w^{S3}), w^{S3})}{V_i^{S3}(h_{t+1})}. \quad \text{(A7)}$$

The profit π^m is decreasing in w, and $w_H^{S2} > w^{S3}$, and therefore the numerator is higher in the RHS of Equation (A7) than in Equation (A6). Furthermore, under the parameter region of Proposition 3, $V_i^{S2} > V_i^{S3}$, and therefore

$$\frac{n-1}{n}\frac{\pi^m(A_H, p^m(A_H, w_H^{S2}), w_H^{S2})}{V_{i,t+1}^{S2}(h_{t+1})} < \frac{n-1}{n}\frac{\pi^m(A_H, p^m(A_H, w^{S3}), w^{S3})}{V_{i,t+1}^{S3}(h_{t+1})}.$$

Consequently, the minimum discount factor that supports setting S2 is lower than the minimum discount factor that supports setting S3 under the rigid pricing scheme.

The proof for the case of the responsive pricing scheme is similar. □

Appendix B. The Issue of Verifiable Information

In the main model, when information is shared in settings S1 and S2, I assume that information is shared truthfully. In this appendix, I relax this assumption, and show that the main results hold also when information can be manipulated.

Sharing Nonverifiable Information in Setting S1

The following result demonstrates that even when the retailers share nonverifiable information in setting S1, the equilibrium in Lemma 1 holds.

Proposition 7. *Truthful information sharing is supported in the equilibrium given in Lemma 1.*

The proposition suggests that even if the retailers share "soft forecast" information, i.e., information that cannot be verified, and hence can choose to share a different forecast than they have, in the equilibrium characterized by Lemma 1, the retailers will still share information truthfully. The intuition behind this result is that if a retailer does not plan to cut the price set by the other cartel members, then his incentive is to maximize the profit of the cartel. This can be achieved by sharing information truthfully. However, if a retailer plans to cut the price in the market, then by announcing a lower forecast than he observed (announcing $Y_i = \phi$ instead of $Y_i = H$), he can induce the other retailers to set a low price (p_ϕ instead of p_H), which results in profit reduction for this retailer compared with the case in which this retailer would cut slightly the price of p_H. If however, this retailer announces a higher forecast than he observed (announcing $Y_i = H$ instead of $Y_i = \phi$), he also does not gain anything, since even if the retailers set the price to be p_ϕ (the case when he truthfully reveals his forecast), the retailer can obtain the same profit by slightly cutting this price. As a result, even if we allow the retailers to manipulate the shared information, the set of prices characterized by Lemma 1 is supported in equilibrium.

Sharing Nonverifiable Information in Setting S2

The following result demonstrates that even when the retailers share nonverifiable information with the manufacturer in setting S2, under some conditions, the equilibrium in Proposition 1 holds.

Proposition 8. *Assume that $\psi \geq 3$. Then, truthful information sharing is supported in the equilibrium given in Proposition 1.*

The intuition behind this result is as follows. Manipulation of the shared information can assist a retailer in a few ways. The first case occurs when a retailer plans to deviate from the cartel's plan and cut the price of his competitors. However, for a high discount factor δ, this deviation is not profitable, since it is detected immediately and punished by playing repeatedly the competitive outcome of the Bertrand game—an outcome that leaves the retailer with zero profit during all subsequent periods. Another possible way information manipulation can assist the retailer is when this manipulation cannot be detected. For example, a retailer can try to induce the manufacturer to set a low wholesale price, by reporting the signal $Y_i = \phi$ when actually observing the signal $Y_i = H$. The problem from the retailer's perspective is that to not have this deviation detected, he must set the retail price in a manner consistent with the wholesale price. Therefore, although the deviating retailer knows that the demand is actually high, if the manufacturer sets the price to be w_ϕ, he must set the price to be p_ϕ. The condition of $\psi \geq 3$ ensures that when the demand is high, a retailer prefers the high wholesale price and high retail price over the low wholesale price and low retail price. Therefore, even when the retailers are free to share messages that differ from their observed signals, they will choose to share information truthfully with the manufacturer when the market volatility (captured by ψ) is also high.

Proof of Proposition 8. Note that in the vertical information-sharing case, if a retailer sets a price in a manner not consistent with the wholesale price, it triggers a price war between the retailers. Therefore, for a high discount factor, to avoid price wars, a retailer must set the price in a manner consistent with the wholesale price.

We need to analyze two cases: when the retailer observes $Y_i = \phi$ and when he observes $Y_i = H$.

Case 1. When the retailer observes the noisy signal, by manipulating the shared information and sending a message $Y_i = H$, he induces the manufacturer to set the high wholesale price w_H and for all other retailers to set the high price p_H as well. It is easy to see that when a retailer observes the noisy signal, the cartel is better off when the manufacturer sets the low wholesale price and the retailers set the low retail price over the case in which both the retailers and the manufacturer set the high prices. To see this, we can examine the following inequalities:

$$\begin{aligned} E[\pi_i(A_\theta, p_\phi, w_\phi) \mid I_\phi] &> E[\pi_i(A_\theta, p_H, w_\phi) \mid I_\phi], \\ &> E[\pi_i(A_\theta, p_H, w_H) \mid I_\phi]. \end{aligned}$$

The first inequality is since the retailers choose to set the price p_ϕ to maximize the cartel's profit, instead of choosing the price p_H when the manufacturer sets the wholesale price of w_ϕ and the information set is I_ϕ. The second inequality holds since the retailers prefer a low wholesale price over a high price. Combining the two inequalities we get that $E[\pi_i(A_\theta, p_\phi, w_\phi) \mid I_\phi] > E[\pi_i(A_\theta, p_H, w_H) \mid I_\phi]$, which implies that a retailer observing the signal $Y_i = \phi$ prefers to truthfully reveal this information to the manufacturer.

Case 2. When a retailer observes the informative signal and announces this information truthfully, the profit is given by $A_H^2/16$. When this retailer sends the message $Y_i = \phi$ (i.e., manipulating the information) and all other retailers observe the noisy signals as well (when $\psi \geq 3$), the retailers' profit is given by $(A_H - (3/4)\tilde{A}_L)\tilde{A}_L/4$. Therefore, a retailer is better off announcing the observed signal truthfully if

$$\left(A_H - \frac{3}{4}\tilde{A}_L\right)\frac{\tilde{A}_L}{4} \leq \frac{A_H^2}{16},$$

and this inequality holds if $\psi \geq 3$. □

Endnotes

[1] In this case (OFT decision of August 1, 2003; CA98/06/2003), the OFT discovered concerted practices to set a minimum price for certain football replica kits, achieved through indirect contacts between competing retailers via a mutual manufacturer.

[2] The OFT's finding (OFT decision of November 21, 2003; CA98/8/2003), subsequently upheld by the Competition Appeal Tribunal, included, among others, the finding that the pricing intentions of one retailer were disclosed by the manufacturer to the other retailer.

[3] A linear demand curve has been widely used in models with incomplete information (e.g., Anand and Goyal 2009, Shin and Tunca (2010). In addition to its tractability, the linear demand curve has an appealing interpretation as the utility-maximizing behavior of consumers with quadratic, additively separable utility functions (Vives 1984).

[4] For convenience, I will refer to the signal $Y_i = H$ as the informative signal, and to the signal $Y_i = \phi$ as the noisy signal since the latter provides less accurate information about the market condition.

[5] Gerlach (2009) also uses a similar information structure when studying the role of private information in cartel coordination.

[6] A similar assumption about the high discount factor of the colluding firms is also adopted in many other papers that study the scope of collusion without communication, such as Athey and Bagwell (2001), Athey et al. (2004), Gerlach (2009), and Rachmilevitch (2013, 2014).

[7] In the responsive pricing strategy, although the pricing decision of each retailer is based on a retailer's private information, this equilibrium satisfies the solution concept of PPE. In this case, the strategy of each player that determines whether the game is in its cooperative phase or retaliation phase is public knowledge, and only the actual pricing decision is influenced by the private information of each retailer.

[8] The reader can consult Athey and Bagwell (2001), Athey et al. (2004), Hanazono and Yang (2007), and Gerlach (2009) for some papers that provide a rigorous treatment of cartel formation without any communication.

[9] Note that throughout this analysis, no assumption regarding the discount factor of the manufacturer has been made. Therefore, this section explores the retailers' critical discount factor.

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