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# Pricing, Frills, and Customer Ratings

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This paper explores whether and how a firm should adapt its strategy in view of consumer use of prior customer ratings. Specifically, we consider optimal pricing and whether the firm should offer an unexpected frill to early customers to enhance their product experiences. We show that if price history is unobserved by consumers, a forward-looking firm should always modify its strategy from single-period optimal one, but it may be optimal to do so by lowering price, by lowering price and offering frills, or by raising price and offering frills, depending on the market growth rate. Specifically, the last strategy becomes optimal when market growth rate is high enough. The results are similar when the price history is observed by consumers, except that no deviation from single-period profit maximization choices is optimal when market growth is low enough. We also analyze whether the firm should prefer that the price information be stated in or left out of consumer reviews. In addition, in considering the effects of consumer heterogeneity, we conclude that the optimal firm's effort to affect ratings is higher when the idiosyncratic part of consumer uncertainty is larger.

*Key words:* game theory; product augmentation; customer satisfaction; uncertainty; forward-looking behavior; pricing; customer ratings

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## 1. Introduction

Consumer uncertainty about product value is a pervasive feature of many markets. Thankfully, the current technologies, especially the Internet, provide plenty of opportunities for information sharing among consumers. In particular, many firms selling through the Internet post customer ratings and reviews. For example, Hotels.com and Travelocity.com prominently display the average rating that customers have given to each hotel; Amazon.com, as well as other Internet merchants, display consumer ratings for the products they sell, and eBay provides seller ratings. These customer ratings are affected by the decisions of the firms: in the above examples, the ratings depend on actions of the hotels, the product manufacturers, and the sellers, respectively. Therefore, sellers or firms should take this effect into account and should consider how they can improve customer ratings through the marketing instruments at their disposal.

Broadly speaking, there are two potential marketing instruments a firm can use to affect ratings. First, the firm may increase the expected and actual value the consumer will obtain from the purchase, for example, by decreasing the price. This could increase customer satisfaction and therefore ratings because of the higher net-of-price value. In addition, such action would also affect the consumer's purchase decision and hence change the distribution of customers. Second, the firm may engage in postpurchase activities or product augmentation to raise an *existing* rather than a *potential* customer's evaluation.

An example of such postpurchase effort may be a firm's willingness to accept product returns beyond the return policy. For instance, in the 1990s, Ross, a growing discount apparel chain, had a policy to accept returns well beyond the 30-day money-back term printed on the receipt. In other examples, consider the American Airlines dining rewards program (aa.rewardsnetwork.com) that added 500 miles to an existing customer's account as a "thank you gift for being a valued network customer in the previous year," or a Radisson hotel in Richardson, TX that gave scratch off cards to customers *after* they checked in to "win" hotel miles; all cards added positive miles to the customers' *existing* frequent-traveler accounts. In these examples, the firms could have mentioned the respective offers at the time the purchase decisions were made to encourage the purchases or could have made the offers conditional on the next purchases. Instead, each firm chose not to announce the frill before the purchase was completed. We refer to this strategy as offering "frills"—extras that are valued by consumers but not considered by consumers or promised to them at the time of purchase. As opposed to the effect of a price reduction, the firms' decisions on how many frills to offer do not increase consumer demand in the current period.

The goal of this paper is to investigate how a firm should adapt its strategy in view of consumer uncertainty and consumer-to-consumer information dissemination through ratings. Specifically, we consider a firm's optimal use of these two instruments to

affect ratings and ask the following questions: When should a firm want to use either or both of the instruments? Note that one of the important trade-offs to consider is that both of these types of efforts are costly. Why not just give a lower price to future consumers instead of trying to affect their expected value indirectly through reports by prior customers? Moreover, if a firm chooses to affect ratings through one of the two instruments, which instrument should it use more and under what conditions? What are the determinants of these decisions?

To address the above questions, we set up an analytical model of a firm selling a product to consumers in two periods. Consumers have heterogeneous product valuation and face uncertainty about the product value. The first-period consumers make purchase decisions based on price and their expectations about the product. After the purchase, the consumers experience and learn their realized utilities and are therefore able to report whether they are satisfied. We assume that a certain percentage of first-period consumers report their satisfaction, and the second-period consumers observe the first-period customer ratings: a summary measure of the satisfaction of the first-period customers. The firm's decision variables are the prices and the amount of frills, the extra values the firm provides after the purchase to customers, in each of the two periods. By "extra," we mean the value beyond what is required from the firm in the purchase contract. Both this value and the price can be different across periods. Clearly, the optimal amount of frills is minimal in the second period, and thus the interesting variable is the amount of frills in the initial period.

Note that although customer satisfaction is often viewed as increasing customer loyalty, in this paper, we analyze the role customer satisfaction plays in informing future consumers. Specifically, this model captures the consumer inference problem discussed previously: the second-period consumers use ratings to update their beliefs about the product value, and the rating is a function of the uncertain part of the product value, the price, and the frills. Because the latter two variables are under the control of the firm, the firm decides whether, how much, and which way to distort them from the single-period optimal ones to influence the ratings and, through them, the second-period consumer beliefs in the desired direction. The second-period consumers realize the firm's incentives and form beliefs about product value based on their understanding of the optimal strategy of the firm. Because the importance of affecting the second-period consumers at the cost of distorting first-period variables depends on the market growth rate from the first to the second period, the market growth rate is one of the important model parameters. Naturally, when the market growth rate is higher, the firm has a

stronger incentive to distort the first-period variables. The interesting question is how this distortion is allocated between the price and the frills, and in which way the firm finds it optimal to distort the price.

The main results are the following. If the second-period consumers do not observe first-period price, a firm always wants to change its actions from the single-period optimal ones to increase customer ratings. In particular, if the market growth is low, the firm should decrease its first-period price to increase customer ratings. The firm will use frills if and only if the growth rate is high enough. This can be viewed as consistent with observed firm behavior. For example, Home Depot was a leader in customer satisfaction in the 1980s and early 1990s, in part driven by post-purchase service, when do-it-yourself home improvement was a growing trend. Then, when market growth of that segment slowed, Home Depot replaced many full-time employees with part-time ones and reduced employee incentives for good customer service, pushing customer satisfaction down the priority list (Grow and McMillan 2006). Consequently, Home Depot slipped to last place among major U.S. retailers in the University of Michigan's 2006 annual American Customer Satisfaction index (as cited in Grow and McMillan). Many Internet retailers in the late 1990s provide additional examples of the strategy consistent with these results: for example, going beyond a free-return policy, Amazon.com used to include a postage-paid return shipping label in the original package, and many Internet retailers would send a free gift (e.g., an extra piece of merchandise from ThingsYouNever-KnewExisted.com, a T-shirt from eCampus.com, or a teddy bear from Overstock.com). Note that for these results to hold, the important condition is not a growing market per se but the firm's *belief* that its sales potential will be growing. In other words, it is irrelevant whether the managerial expectations of the growth of many Internet start-ups were too high in the late 1990s; what is relevant is that those managers believed that the Internet market would continue growing. Also consistent with these predictions, established (i.e., not fast-growing) companies such as cosmetics manufacturers often inform potential consumers of any extras that come with purchase before the consumers purchase decisions are made.

Furthermore, when growth rate is sufficiently high, the firm should actually increase its first-period price relative to the single-period optimal and should instead put an extra effort into providing frills. In other words, as the growth rate increases, the firm should switch from price to frills as the instrument for increasing customer ratings.

If consumers observe past prices, i.e., if they know what the price was at the time of each review, the results about the use of frills are mostly unchanged.

However, in this case, when market growth is low, the firm should use the same price as it would if it were single-period profit optimizer.

Endogenizing the market growth rate by assuming that prior sales drive future market size, we find that the results about downward price distortion for low market growth and the optimality of offering frills are robust if and only if the market growth rate is high enough, but the upward distortion of price may never be optimal if the market growth depends on sales rather than on ratings. However, if the market growth depends on the ratings rather than sales, all results are the same except that the market growth rate parameter is replaced by the parameter governing the strength of the effect of ratings on the second-period market potential.

Extending the model to heterogeneous consumer experiences with a product, we find that higher heterogeneity in consumer uncertainty results in a higher incentive for a firm to increase customer ratings. Although such a result would be expected for the amount of uncertainty correlated across consumers, it is interesting that the firm would have a higher incentive to spend on improving customer ratings when the *idiosyncratic* part of the uncertainty is higher.

The rest of this paper is structured as follows. The following section discusses the related literature, and §3 formally defines the model. Section 4 solves this model and derives the optimal level of pricing and frills when price history is and is not observed by second-period consumers. Section 5 considers several model extensions. In particular, we consider endogenous market growth and the effects of consumer heterogeneity in the uncertain part of the utility, and we discuss the effects of competition. Section 6 further discusses the model implications and interpretation of the results and concludes.

## 2. Relation to Prior Literature

Existing literature on customer ratings has concentrated on considering optimal contracts to align incentives between channel members given a particular response function of the future demand to current customer satisfaction efforts (e.g., Hauser et al. 1994, Simester et al. 2000, Chu and Desai 1995), how a firm should respond to customer ratings (Chen and Xie 2005), or how word of mouth affects demand (Godes and Mayzlin 2004, Manchanda et al. 2008). In particular, Hauser et al. (1994) consider the optimal incentives a firm should provide its employees to exert optimal service effort, Simester et al. (2000) study the financial impact of customer satisfaction improvement programs in the United States and Spain, and Chu and Desai (1995) consider how to coordinate the manufacturer–retailer channel to provide optimal effort to increase future demand. Chen

and Xie (2005) examine how a firm should respond to product reviews after they are posted. This paper, on the other hand, considers how future demand should depend on customer ratings and which mechanisms a firm should employ to raise customer ratings under different market conditions.

In a broader context, this paper also relates to marketing literature on the effect of product uncertainty and firm effort to convey desirable value perception to consumers. Some examples in the marketing literature include Moorthy and Srinivasan (1995), who analyze how money-back guarantee communicate quality; Kalra et al. (1998), who consider how a firm may use delay in its response to competitive entry to convey that its product has higher quality; Anderson and Simester (1998), who study the informative role of sale signs about the price and future product availability; Iyer et al. (2005), who consider the optimal design of communication through advertising; and Kuksov and Lin (2010), who examine the optimality of information provision in a competitive environment. In contrast to the idea of a firm communicating to consumers, we consider the optimal firm investment in affecting consumer-to-consumer communication in the absence of signalling motives by explicitly concentrating on the case when both the firm and the consumers are unsure of the value they will obtain from the product or service. The firm's behavior in our context resembles “signal jamming” (Fudenberg and Tirole 1986)—interfering with rational consumer inference from customer ratings so as to bias it upward. In the equilibrium, rational consumers adjust for this behavior and are not biased.

## 3. The Model

We consider a two-period model where consumers are uncertain about product valuation and where second-period consumers can use first-period customer ratings to update their beliefs about the product's value. First, we discuss how ratings are formed. Then, we specify the game-theoretic model of consumer and firm decision making.

### 3.1. Consumer Information Exchange Through Ratings

To model information dissemination from earlier to later consumers, we must model what affects the earlier customers' evaluations and what is observed by the later consumers.

In relation to the former objective, note that even when customers report on the product quality rather than overall satisfaction with the purchase, they still often rate the quality relative to the price they paid. For example, a review of the home theater projector Optoma HD70 at Amazon.com states “so it's not the greatest projector ever made...but it is easily the

best for the price.”<sup>1</sup> The customer then rates it as five out of five stars. Given the reference to price-quality trade-off, a prospective consumer would like to know what price the reviewer paid for the projector. However, the entire several-paragraph text of the review never states the price paid. In fact, the price was not mentioned in any of the 30 reviews. This is not an uncommon situation with online reviews.<sup>2</sup> Conversely, it is also a frequent scenario that a high-end product is rated relatively low because it is too expensive. An implication is that even though an objective quality rating may not depend on the price, the actual product ratings are affected by the price. For the purpose of the model, we therefore assume that consumer evaluation of the product depends on *the total value net of price*.

In relation to the latter objective of what information the later consumers receive, we observe that the overall product rating prominently displayed is usually the average of individual product ratings or the percentage of positive ratings. For example, Amazon.com, Overstock.com, and SmartBargains.com ask for consumer ratings on a scale of one to five and summarize through the average rating, whereas eBay.com has merchant ratings as positive versus negative, with “x% positive” being the aggregate measure.<sup>3</sup> Overstock.com and SmartBargains.com, in addition to the product rating on a scale of one to five, ask for a “bottom-line” rating of “would you recommend this to a friend,” which is a binary rating. We adopt the latter (binary) rating example as a building block of our model, mostly because of its analytical tractability. Another reason to use the binary rating in the model is that it is easier to link it to the consumer utility—whether a customer would still buy if she knew then what she knows now—than to translate the qualitative evaluation in a consumer’s mind to the rating she posts on a five-point scale. Thus, we assume that a percentage of existing customers rate the product, i.e., they report whether they are satisfied, and the future consumers observe the percentage of ratings that are positive. Note that in the binary rating context, this summary measure is equal to the average of ratings and contains the same information as the whole distribution of ratings.

<sup>1</sup> See <http://www.amazon.com/Optoma-HD70-720p-Theater-Projector/product-reviews/B000IEN8QQ>.

<sup>2</sup> In fact, Amazon.com guidelines on how to write a review explicitly state that a review should *not* include “availability, price, or alternative ordering/shipping information” (see <http://www.amazon.com/gp/help/customer/display.html?nodeId=14279631>).

<sup>3</sup> We do not distinguish between uncertainty about the seller or about the product in our model because when a consumer is shopping for a product from a relatively unknown seller, she may be concerned not only about the product per se but also about product augmentation such as delivery, services, etc.

### 3.2. Payoffs, Actions, and Timing

Let us assume that a firm is selling one product to consumers in two time periods,  $t = 1, 2$ . The total mass of consumers in the first time period is normalized to 1, and the total mass in the second time period is denoted by  $M$ . Each consumer  $i$  of either time period  $t$  has a single-unit demand and has the utility of

$$U_i = h_i + \eta - p_t + s_t, \quad (1)$$

where  $h_i$  is the consumer expectation of the product value in the absence of information from prior customers,  $\eta$  is the uncertain (mean zero) part of the product value,  $p_t$  is the price, and  $s_t$  is the consumer value of the frills. We assume that  $h_i$  is independent and identically distributed (i.i.d.) across consumers. Furthermore, to be specific and for analytical tractability, assume  $h_i$  is distributed uniformly on  $[0, V]$  for  $V > 6$ , and, in keeping with prior belief,  $\eta$  is distributed uniformly on  $[-1, 1]$ .<sup>4</sup>

Consumers of the first period know their  $h_i$  and observe  $p_1$  before the purchase. They learn  $\eta$  and  $s_1$  only after the purchase (it is inconsequential whether they observe these separately or whether they only feel the “overall product experience”). Furthermore, we assume that the first-period consumers do not expect  $s$ . This assumption is to simplify the model analysis, and as we show in §5.4, it is not essential for the main results of the model to hold. At the same time, this assumption allows us to capture the intention of firms providing frills to “exceed expectations.” Empirically, exceeding expectations, i.e., providing unexpected surplus, has been shown to be beneficial for investor stock valuations (see Bartov et al. 2002) and has been suggested as a possible strategy by the marketing literature (e.g., Rust et al. 1999).

After the purchase, first-period customers rate the product by reporting whether they are satisfied with the purchase or not. As discussed in §2.1, we assume that a customer gives a rating of 1 (“satisfied”) if postpurchase, after experiencing the true value of the product, the consumer does not regret purchasing the product (i.e., does not wish she chose the no-purchase option).<sup>5</sup> All or a fraction of the first-period consumers who purchased post their ratings, which

<sup>4</sup> Setting the range of  $\eta$  to 2 is without loss of generality as it reflects a multiplicative scalar normalization of the utility function and the monetary units. Restricting  $V$  to be high enough simplifies calculations. We need  $h_i$  different across consumers to generate a downward-sloping consumer demand, and for simplicity, we assume that  $\eta$  is the same across consumers. In §5.5, we show the robustness of the results to this assumption by considering the uncertain part of the consumer utility that is heterogeneous across consumers.

<sup>5</sup> For a more general form of customer rating and to further capture the effect of expectation disconfirmation, one can introduce a different (higher) weight on expectation disconfirmation ( $U - EU$ ),

aggregate into the average customer rating, denoted by  $R$ , equal to the fraction of the first-period customers who experienced a positive utility from the purchase.

The second-period consumers also know their  $h_i$ , observe  $p_2$  prior to purchase, and do not observe  $s_2$  or  $\eta$ ; however, they observe the rating  $R$  of the first-period consumers and use it to update their expectation of  $\eta$ . We also assume that the second-period consumers do not observe  $s_1$  or  $p_1$ . In §4.4, we explicitly consider the implications of  $p_1$  being observed by the second-period consumers. For simplicity, we assume that in the first period, the firm does not know  $\eta$ . It is inconsequential whether the firm knows the true fit of the product to the market ( $\eta$ ) or only this customer rating in the second period, because without the possibility of product returns and no future periods, only the perceived value matters. We assume that in the second period, the firm knows at least the average first-period customer rating. We assume that the value of frills  $s_i$  cannot be negative, but in §5.1 we discuss what would change if we allow negative values of  $s$ . We normalize the marginal cost of the product to 0, assume that the cost of  $s_i$  is  $s_i$ , and assume that the firm can set time-dependent but not consumer-dependent  $p_i$  and  $s_i$ . Because the model is a multistage game with incomplete information, we use perfect Bayesian equilibrium as a solution concept.

Because the second-period consumers do not observe  $s_2$  prior to purchase, it is optimal for the firm to set  $s_2 = 0$ . We will therefore drop the time-period index on  $s_1$  and use  $s$  for  $s_1$ .

## 4. Model Analysis

To analyze the model, we first derive how the average consumer rating  $R$  depends on the model parameters and the firm's actions and how the second-period consumers use it to update their beliefs. We then use these results to derive optimal actions of the firm and compare them to the case of single-period profit maximization. This leads to conceptual implications of how the firm should adjust its actions from the single-period optimal ones to maximize the long-term profits. We then consider how the results would change if the price history, i.e., the first-period price, were observed by the second-period consumers, and we discuss the implications of price history observability on the firm's profits.

which would give an additional incentive for the firm to use post-purchase effort. This would not change main qualitative results of optimal pricing but could reduce the threshold of the market growth necessary to make postpurchase efforts optimal.

### 4.1. The Rating Function and Consumer Inference from Ratings

The aggregate first-period consumer rating  $R$  is the fraction of customers whose realized utility from purchase is positive, which is (see the appendix)

$$R(\eta, p_1) = \begin{cases} 1, & \text{if } \eta \geq -s; \\ 1 + \frac{\eta + s}{V - p_1}, & \text{if } \eta \leq -s. \end{cases} \quad (2)$$

Note that  $R$  is weakly increasing in  $\eta$  and  $s$ , which is intuitive because higher  $\eta$  or  $s$  means that every customer's utility is higher. Furthermore,  $R$  is weakly decreasing in  $p_1$  (see Figure 1). The intuition for this is that a customer can only be potentially disappointed when her expected (before purchase) net-of-price utility of purchase is between 0 and  $1 - \min(\eta)$ . When price decreases, the number of customers with the expected net-of-price utility above 1 increases, whereas the number of consumers with the net-of-price utility between 0 and 1 remains the same. Note also that  $R$  is invertible as a function of  $\eta$ , keeping  $p_1$  and  $s$  constant, if and only if  $\eta \leq -s$ .

Let us now turn to the second-period consumer inference of  $\eta$  from  $R$ . Bayesian updating implies that the second-period consumers know the  $R$  function in Equation (2) and that they try to invert it to deduce  $\eta$ . The second-period consumer expectation of  $\eta$  conditional on  $R$ , their expectation  $\hat{p}$  of  $p$ , and their expectation  $\hat{s}$  of  $s$  is

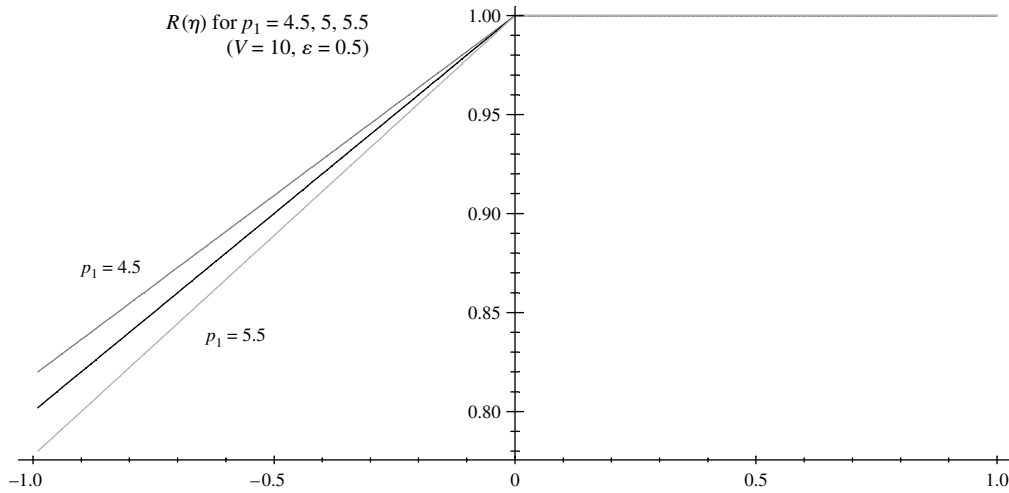
$$E(\eta | R, \hat{p}, \hat{s}) = \begin{cases} \frac{1 - \hat{s}}{2}, & \text{if } R = 1; \\ -\hat{s} - (1 - R)(V - \hat{p}), & \text{otherwise.} \end{cases} \quad (3)$$

This expression also allows us to consider what would change if consumer expectations are not fully rational (§4.5 further discusses this possibility).

For the firm's decision on pricing and frills, it is also important to consider how much the second-period consumer beliefs on  $\eta$  are affected by the price and frills of the first period. Substituting the value for  $R$  from Equation (2) into Equation (3), we obtain that the second-period consumer expectation of  $\eta$  given  $\eta$ , the firm's decisions, and the consumer beliefs about the firm's decision is

$$E(\eta | \eta, p_1, \hat{p}, s, \hat{s}) = \begin{cases} \frac{1 - \hat{s}}{2}, & \text{if } \eta \geq -s; \\ \max \left\{ -1, \frac{V - \hat{p}}{V - p_1}(\eta + s) - \hat{s} \right\}, & \text{otherwise.} \end{cases} \quad (4)$$

Note that if the first-period price is observed by the second-period consumers, Equation (4) implies that the second-period consumers are able to fully deduce the utility of the first-period consumers from the ratings as far as  $\eta < -s$ . If the ratings were on a finer

**Figure 1** Dependence of  $R(\eta)$  on Prices:  $V = 10$ ,  $s = 0$ , and  $p_1 = 4.5$  (Top, Dark Gray), 5 (Middle, Black), and 5.5 (Bottom, Light Gray)

Note. Increasing  $s$  increases  $R(\eta)$  by translating the graphs to the left.

scale, the inference would be conceptually similar: the second-period consumers would not be able to fully deduce the first-period consumer utility if and only if  $s + \eta$  is high enough. In other words, the binary nature of individual consumer feedback does not conceptually restrict the information flow in this case.

#### 4.2. Benchmark 1: Single-Period Profit Maximization

Because  $s$  is unobserved by the first-period consumers, it does not affect their demand. Therefore, it is single-period optimal not to provide frills ( $s^{sp} = 0$ ). Because the first-period consumer's expected utility is uniformly distributed on  $[0, V]$ , the optimal first-period price is  $p_1^{sp} = V/2$ .

#### 4.3. Optimal Price and Frills for Long-Term Profit Maximization

The equilibrium condition on second-period consumer beliefs is that  $\hat{p}$  and  $\hat{s}$  are equal to the equilibrium choices  $p_1$  and  $s$  of the firm, and that these values  $p_1$  and  $s$  are optimal to choose for the firm given that second-period consumer expectations of them are  $\hat{p}$  and  $\hat{s}$ .

Although the solution is conceptually simple (see the appendix), the analytical expressions for  $p_1$  and  $s$  are roots of fourth-degree polynomials. To gain insights into optimal firm behavior and trade-offs the firm is facing, the following partial equilibrium conditions are informative.

**LEMMA 1.** The optimal level of frills  $s^e$  as a function of the price  $p_1$  is given by

$$s^e(p_1) = \min \left\{ 1, \max \left\{ 0, 2V + 1 - 2\sqrt{(V-1)^2 + 8(V-p_1)/M} \right\} \right\}, \quad (5)$$

and the optimal price  $p_1^e$  as a function of  $s$  is

$$p_1^e(s) = \frac{3V + s}{4} - \frac{1}{12} \sqrt{9(V-s)^2 + 3(1-s)^2(3V-2-s)M}. \quad (6)$$

The simultaneous solution of the above two equations for  $p_1 = p_1^e$  and  $s = s^e$  determines the firm's optimal choice of  $p_1$  and  $s$ .

**PROOF.** See the appendix.  $\square$

Equation (5) implies that, conditional on any first-period price  $p_1$ , the market growth rate required to make the use of frills optimal ( $s > 0$ ) is

$$M > \frac{32(V-p_1)}{3(4V-1)}. \quad (7)$$

Combining this with Equation (6) on the optimal choice of price, we obtain that providing frills is optimal if and only if

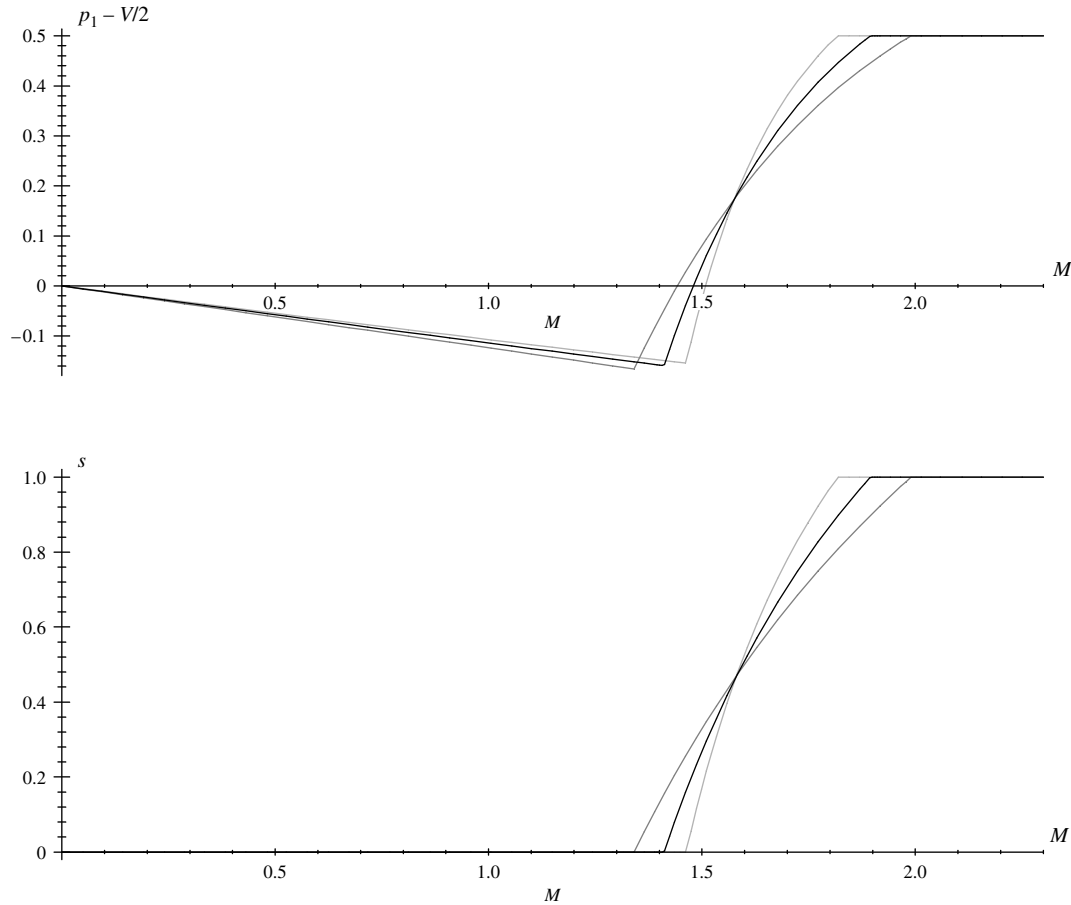
$$M > M_1 = \frac{16}{27} \cdot \frac{36V^2 + 3V - 8}{(4V-1)^2}. \quad (8)$$

Note that Equation (5) implies that the optimal frills increase in the price. The intuition for this result is that higher price leads to lower first-period demand, and therefore, offering frills is less costly to the firm on the total-cost basis. Figure 2 illustrates the optimal first-period price and frills as a function of the market growth rate  $M$ . Proposition 1 summarizes the main qualitative implications of Lemma 1.

**PROPOSITION 1.** There are critical values  $M_1 > 1$ , defined by Equation (8), and  $M_2 > M_1$  (see the appendix) of the market growth rate  $M$  such that

1. When  $M \leq M_1$ , the firm should set the first-period price below the single-period optimal one while not offering any frills (i.e.,  $p_1 < p_1^{sp}$  and  $s = 0$ ).

**Figure 2** Optimal  $p_1$  Relative to  $V/2$  (Top) and  $s$  (Bottom) for  $V = 6$  (Steepest, Light Gray), 10 (Black), and 100 (Dark Gray) When Price History Is Unobserved



*Notes.* The first (up) kink in every line defines  $M_1$  as the market growth rate after which  $s > 0$  is optimal; the second (flat) kink defines  $M_3$  as the market growth rate at which  $s$  reaches 1. The intersection of price function and single-period optimal  $V/2$  defines  $M_2$ .

2. When  $M_1 < M < M_2$ , the firm should set the first-period price below the single-period optimal one and offer some frills ( $p_1 < p_1^{sp}$  and  $s > 0$ ).

3. When  $M > M_2$ , the firm should optimally set the first-period price above the single-period optimal one and offer frills ( $p_1 > p_1^{sp}$  and  $s > 0$ ).

Furthermore, the optimal  $p_1$  decreases in  $M$  for  $M < M_1$ ; increases in  $M$  for  $M \in [M_1, M_3]$ , where  $M_3 = 4(V - 1)/(2V - 1)$ ; and is constant at  $(V + 1)/2$  thereafter. The optimal  $s$  increases in  $M$  from 0 to 1 for  $M \in [M_1, M_3]$  and is constant at 1 thereafter.

**PROOF.** See the appendix.  $\square$

An implication of Proposition 1 is that a forward-looking firm should always modify its strategy from a myopic one when later consumers look at the customer ratings. Because the optimal second-period price increases in  $s$ , offering frills to early customers to increase their ratings is dominated by giving a discount to future customers to increase their demand directly when the market growth rate is sufficiently low. However, in that case (low  $M$ ), reducing the

first-period's price is always optimal even if a market is rapidly declining ( $M \ll 1$ ). The intuition for the later result is that in the vicinity of the single-period optimal price, the effect of a price reduction on the first-period profits is a negative second-order effect, whereas the effect of a price reduction on customer ratings, and through them on the second-period demand, is a positive first-order effect. Therefore, the latter dominates the former at least for a small amount of downward price distortion. In other words, although giving discounts to second-period consumers is the most direct way to increase their demand, the firm should also decrease the first-period's price—in a sense, giving discounts to the early customers—to indirectly increase the second-period demand through favorable ratings.

Note that the model assumed that the consumer's expected gross-of-price value  $h_i$  of the product is uncorrelated with the variance of consumer uncertainty  $\eta$  (normalized to one) about this value. If these were correlated, this would be another factor affecting the firm's pricing decision. For example, if the correlation is positive, reducing price would even



further increase ratings because the variance of  $\eta$  would be lower for the marginal consumer when the price is lower. Alternatively, a negative correlation would be a factor in favor of a higher price.

Another implication of Proposition 1 is that the optimal adjustment of the price from the single-period optimal one is U-shaped as a function of the market growth rate. For low values of the market growth rate, the faster the growth rate, the more the firm should try to affect ratings by reducing price. However, when the growth rate is high enough, the firm should start to use frills to increase ratings. Once the firm uses frills, a lower price means a higher total cost of frills, because they have to be offered to more customers. Therefore, another negative effect of lower price on profits appears. For high enough market growth rate, this negative effect starts to dominate the positive effect through ratings, and it becomes optimal for the firm to increase the first-period price. Another intuition behind this result is that affecting ratings through frills is a more efficient, although more costly, method of increasing ratings. Thus, it should be used more when the importance of ratings is higher, i.e., when the market growth is higher.

An interesting consumer application of when  $s > 0$  is optimal is whether first-period consumers should expect a firm to do something extra after the purchase or expect it to shirk on its promises. If the market is not growing, consumers should be skeptical of a firm that does not put promises in writing, whereas if a firm is expecting the market to grow, consumers may believe that the firm will, on the contrary, try to please them even after the purchase and in ways not required by a sales contract. Note that recently, sales growth was increasingly cited as an important corporate objective. This paper suggests that it is not coincidental that recently, customer satisfaction has also become a corporate concern.

#### 4.4. Benchmark 2: Price and Frills with Observed Price History

Let us now compare the results of §4.3 with the case of second-period consumers being able to observe the first-period price. One of the purposes of this scenario is to consider how the results would change and how robust they would be if the consumer reviews were able to report price or if second-period consumers were able to observe the prices in the past.

The explicit solution in this case again involves roots of high-order polynomials. Therefore, in Proposition 2, we summarize only the conceptual implications of this case.

**PROPOSITION 2.** *When price history is observed, there are  $M_0$ ,  $M'_1$ , and  $M'_3$  (see the appendix) such that  $1 < M_0 < M'_1 < M'_3$  and*

1. *When  $M < M_0$ , the firm should optimally set  $p_1 = p_1^{sp}$  and  $s = 0$ .*

2. *When  $M_0 < M < M'_1$ , the firm should optimally set  $p_1 < p_1^{sp}$  and  $s = 0$ .*

3. *When  $M > M'_1$ , the firm should optimally set  $s > 0$ . Furthermore, the optimal  $s$  increases from 0 to 1 for  $M \in [M'_1, M'_3]$ . The optimal  $p_1$  decreases in  $M$  for  $M \in [M_0, M'_3]$  and is below the optimal first-period price in the case of nonobservable price history if and only if  $M \in (M'_1, M'_3)$ ; for  $M > M'_3$ , optimally,  $s = 1$  and  $p_1 = (V + 1)/2$ .*

**PROOF.** See the appendix.  $\square$

Figure 3 illustrates the optimal  $p_1$  and  $s$  in this case. The intuition for the optimal first-period price  $p_1$  being below the single-period optimal price  $V/2$  when  $M \in (M_0, M'_3)$  is that the firm would like to convince the second-period consumers that it sets a lower  $s$ . Because lowering  $p_1$  results in a lower expectation of  $s$  by second-period consumers, the firm prefers to set  $p_1$  lower than it would without this motivation. This reason is very different from the reason for the reduction of  $p_1$  in the case of unobserved price history: the reason for a reduction of  $p_1$  when it is observed by second-period consumers is not to increase the ratings but rather to convince second-period consumers that the firm did not engage too much in manipulation of ratings through  $s$ . Note that even when price history is not observable, second-period consumers could correctly anticipate it in the equilibrium. However, when price is observed, the actual price instead of the inferred price affects second-period consumer beliefs about  $\eta$ , and the firm takes this into account when choosing the first-period price. Specifically, when price history is observable, the firm chooses the first-period price not only to affect the ratings but also to affect the second-period consumer inference from ratings.<sup>6</sup>

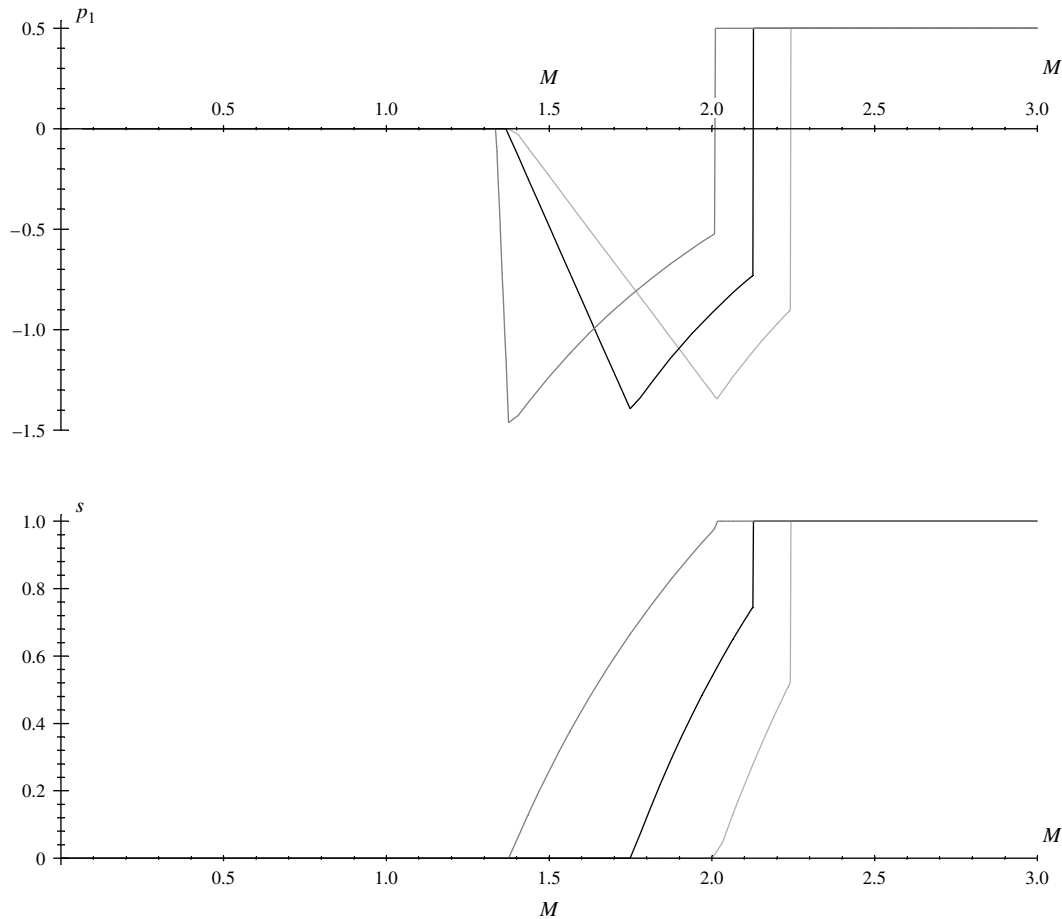
One of the implications of Proposition 2 is that the optimal price in the case of observed price history is higher than in the case of unobserved price history if the market growth rate is not very high, and the optimal price is lower if the market growth rate is such that  $s > 0$  (see Figure 4; Figure 5 clarifies the relation between the values of  $M$  where  $p_1$  changes directionally).

#### 4.5. Optimality of Price History Observability

When comparing the cases of observed and unobserved price history, a natural question to ask is when would the firm prefer to reveal or publicize its prior prices. It turns out that, in our model, it is always optimal for the firm to make prior prices observable. The intuition for this is the following: if the

<sup>6</sup> This rationale for price distortion is similar to the one for “inference effect” price distortion in Gal-Or et al. (2008).

**Figure 3** Optimal  $p_1$  Relative to  $V/2$  (Top) and  $s$  (Bottom) for  $V = 6$  (Rightmost, Light Gray), 10 (Black), and 100 (Leftmost, Dark Gray) When Price History Is Observed

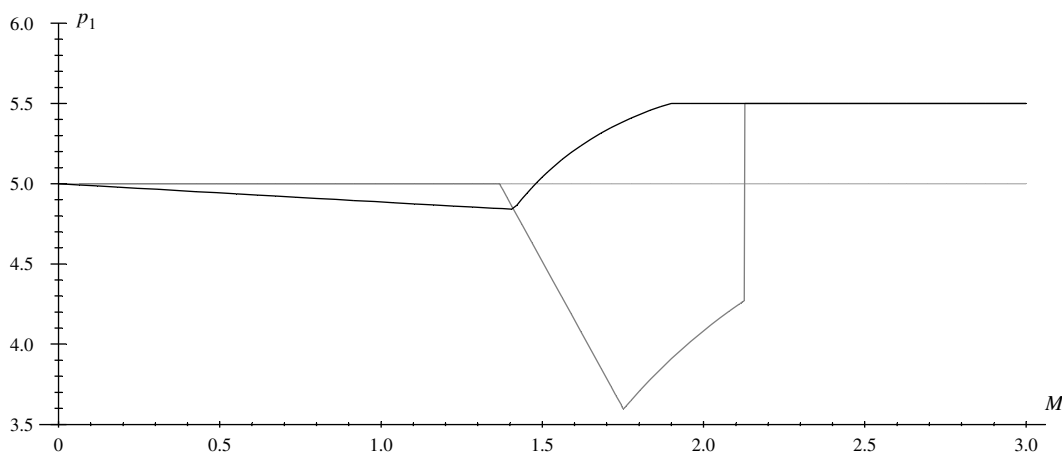


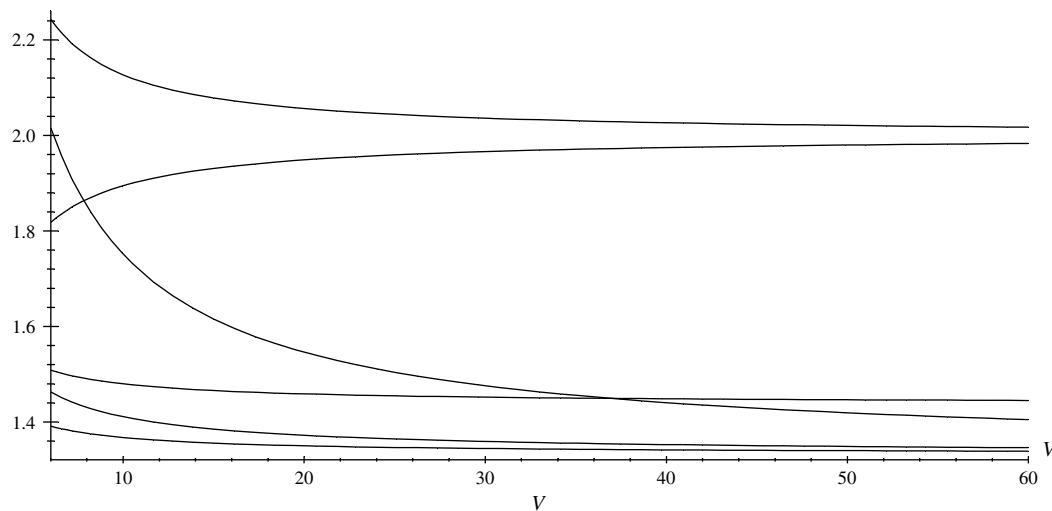
Notes. The first (down) kink in price function defines  $M_0$ , the first (up) kink in  $s$  function corresponds to the up-kink in price and defines  $M'_1$ , the second (up) kink in price and  $s$  show discontinuity and define  $M'_2$ .

price history is unobserved, then second-period consumers are able to correctly infer the optimal first-period price from the profit optimization problem of the firm. Therefore, if the firm reveals the price but

does not change any of the decisions on the price and frills, nothing changes and profits remain the same. Therefore, the firm cannot be worse off if it reveals the price history, but because the first-period price affects

**Figure 4** Optimal Price  $p_1$  When Price History Is Observed (Gray) and Unobserved (Black) as Functions of the Market Growth Rate for  $V = 10$



**Figure 5** Values of the Market Growth  $M$  at Which Price Changes Direction ( $M_0, M_1, M_2, M_3, M'_1$ , and  $M'_3$ ) for  $V$  from 6 to 60

Note. From top to bottom at  $V = 6$ :  $M'_3$ ,  $M'_1$ ,  $M_3$ ,  $M_2$ ,  $M_1$ , and  $M_0$ .

the decisions made by the second-period consumers, the firm can change it so as to induce the favorable change in consumer behavior. Therefore, when price history is observable and the firm chooses a different price from when the price history is not observable, the firm is strictly better off.

However, the above argument relies heavily on the following two assumptions present in the model: First, the second-period consumers are assumed to be fully rational and able to correctly deduce the firm's actions. If they are not, but rather follow some rule of thumb, e.g., use the single-period optimal or the current price instead of the deduced first-period's price, the firm may be better off if it does not reveal the price history. Essentially, the firm is better off only if it can successfully fool second-period consumers, but fully rational consumer assumption prevents this from happening. Second, even if fully rational, second-period consumers may not be able to deduce the first-period price because of some intrinsic uncertainty (e.g., about costs or the aggregate demand shape). In this case, the firm could benefit from unobservability of price history if the realization of the uncertainty is such that the firm sets a lower price than the mean of consumer expectations.

Note that although the profit implications depend on the second-period consumer being able to solve the firm's problem as discussed above, the conceptual implications about the optimal price and frills do not. If the second-period consumers use some rule of thumb for  $\hat{p}$  and  $\hat{s}$ , instead of the one derived, then, in their inference equation (Equation (3)), the shape of the effect of ratings on the second-period consumer demand is the same, and hence the incentives to change the price or frills are the same as well.

## 5. Extensions

In this section, we discuss several modifications of the model setup to examine the robustness of the results and how the predictions would change under different assumptions. We start with a discussion of the assumption about the proportion of customers providing reviews (see §5.1). In §5.2, we consider the implications of the ability of the firm to have hidden charges that would result in a negative  $s$ . In §5.3, we discuss the possibility that the market growth rate is endogenous and, in particular, depends on prior sales and on competitors' actions. In §5.4, we show that the model predictions are robust in the case when first-period consumers have rational expectations of  $s$ , and in §5.5, we consider the robustness to and additional implications of the possibility of idiosyncratic consumer uncertainty.

### 5.1. Which Customers Become Reviewers?

Conceptually, to analyze consumer use of prior customer ratings, one needs to consider how individual consumer opinion and rating are formed, which customers provide ratings, and how the later consumers use those ratings. In the discussion leading up to the model formulation, we concentrated mostly on the justification of what affects a customer rating, given that the customer will post a rating, and assumed that the probability of a customer posting a rating is independent of the customer's satisfaction. However, it could be that the probability that a customer reports her satisfaction with the product depends on whether or not she is satisfied. For example, it could be that dissatisfied customers provide ratings with a higher probability. If these probabilities are known to second-period consumers, second-period consumer belief updating as a function of the actual product

value and the firm's actions would be exactly the same, because there is a one-to-one mapping between the average rating in this case and what it would be if the probabilities were the same.

The situation is more interesting if second-period consumers are uncertain about the ratio of the probability of a satisfied customer posting a review from that of an unsatisfied customer. In this case, a second-period consumer should form beliefs not only about the product value but also about this ratio. Consequently, in this case, when a second-period consumer sees the rating, she needs to update both her belief about the product and her beliefs about the ratio of probabilities. This means that, for example, if a rating is lower than expected, a consumer now would only partially attribute this to the low value of the product and would partially attribute this to the higher-than-expected probability of dissatisfied consumers posting reviews. A similar attribution split would occur when the rating is higher than expected. As a result, customer ratings become less informative to consumers, which in turn means that it is less important for a firm to influence them. The effect on optimal decisions of the firm is then similar to that of a lower market growth rate—the other condition that makes a firm less worried about customer ratings. In other words, we should expect to see graphs of optimal price and frills functions (e.g., in Figures 2–4) to be stretched in the horizontal (market growth rate) dimension.

## 5.2. Optimality of Hidden Charges ( $s < 0$ )

In the main model, we have assumed that the firm is restricted to choose only positive values of  $s$ , i.e., to provide positive unexpected extra value to first-period consumers. In reality, it is possible that the firm could, on the contrary, try to shortchange consumers by including some hidden charges that consumers do not notice until after purchase. Alternatively, if first-period consumers expect positive frills, we can normalize their expectation of frills to zero so that  $h_i$  represents the expected product value net of the expected frills, and the above question becomes whether the firm would be willing to underperform expectations (see §5.4 for a formal consideration of first-period consumer expectations of  $s$ ).

If we completely relax the  $s \geq 0$  assumption by allowing arbitrarily large negative values, the optimal strategy of the firm would be to set  $s$  to negative infinity, thus generating infinite profits regardless of market growth rate  $M$  and other parameters. Clearly, some constraint on  $s$  from below is justified. Therefore, in this section, we consider relaxing the nonnegativity constraint to  $s \geq -\underline{s}$  for some  $\underline{s} > 0$ .

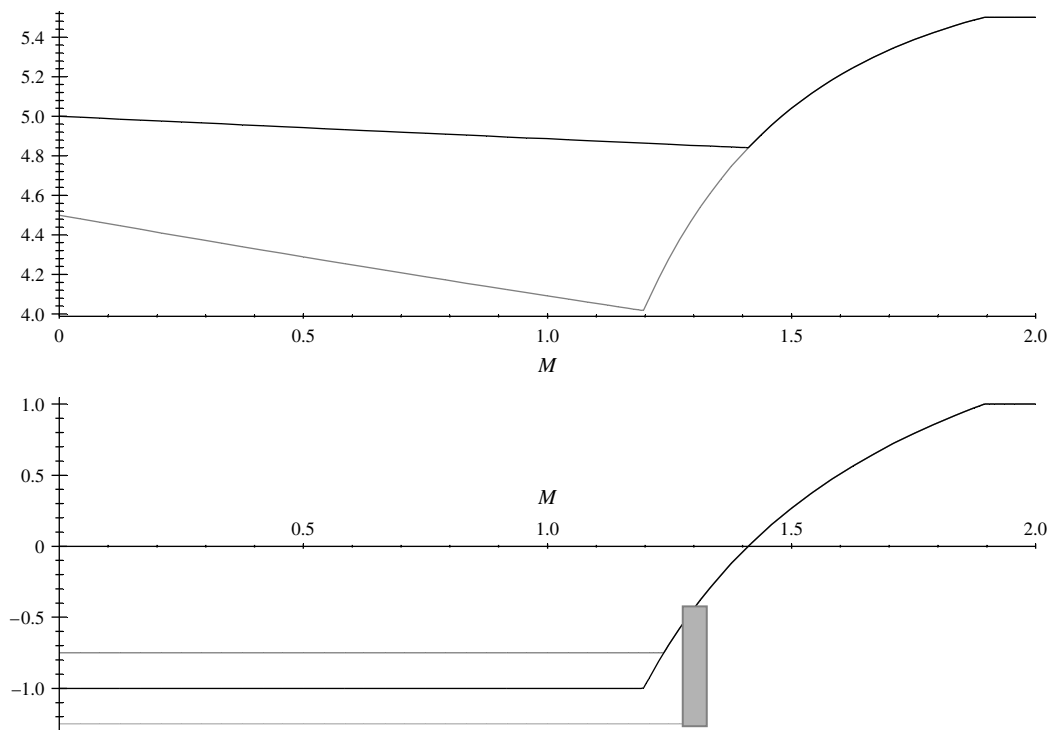
Following the same solution procedure as in the main model, one can show that when  $\underline{s} \in (0, 1)$ , the

optimal choice of  $p_1$  and  $s$  is the same as in the main model when  $M > M_1$ . This is because the  $s \geq 0$  constraint is not binding. When  $M < M_1$ , the optimal  $p_1$  is lower than the one predicted by the main model and the optimal  $s$  is negative. Specifically,  $p_1$  and  $s$  are given by Equations (5) and (6) with 0 replaced by  $-\underline{s}$  in Equation (5). If  $\underline{s} > 1$ , the profit function stops being concave in  $s$  for all  $M$  and  $V$ , and the solution is more complicated. Specifically, the first-order conditions are either never satisfied or result in Equations (5) and (6) with 0 replaced by  $-1$ , but the profit for those  $s$  and  $p_1$  should be compared to the profit when the firm deviates and sets  $s = -\underline{s}$ . This results in the optimal firm's strategy to set  $s = -\underline{s}$  for  $M < M_{\underline{s}}$  for some  $M_{\underline{s}}$ , which increases in  $\underline{s}$  as far as  $\underline{s} > 1$ . For higher  $M$ , it becomes optimal to set  $s > -1$ . This is because the firm strictly prefers  $s = -1$  to  $s = -\underline{s} < -1$  for any parameter values. Furthermore, it turns out that the incentive for the firm to charge higher  $s$  increases when second-period consumers have lower expectation of  $s$ . This leads to a mixed-strategy equilibrium when  $M$  is slightly above  $M_{\underline{s}}$ . For sufficiently high  $M$ , the solution follows the curve of the solution for  $\underline{s} = -1$ . Figure 6 illustrates the optimal choice of price and frills when negative  $s$  is allowed.

Besides showing the robustness of the model implications, the above consideration of negative  $s$  also shows how it is possible that the optimal strategy of the firm in providing frills could be not continuous as in the main model, but could be either “forget about the future” ( $s = -\underline{s}$ ) or “in, all the way” ( $s = 1$ ) for a sufficiently lax constraint on  $s$ . A consumer implication is that when market growth is high enough, it is not important to read through the small print because the firm would not be willing to take advantage of consumers even if it could.

## 5.3. Endogenous Market Growth

Until now, we have considered how optimal price ( $p_1$ ) and frills ( $s$ ) in the first period depend on the market growth rate ( $M$ ) from the first to the second period. However, the market growth rate itself can be dependent on the firm's actions. Specifically, in the spirit of the Bass (1969) model, one can speculate that future market potential may be driven by word of mouth, which could be proportional to prior sales or may depend on ratings (see Mahajan et al. 1990 for a discussion of interpretations and extensions of the Bass model). In the first case, lowering price in the first period would not only change the ratings but also increase market size or market attractiveness in the second period. To analyze this case, assume that the market size in the second period is  $(N - D_1)(a + bD_1)$ , where  $N$  is the total potential market size,  $a$  is related to the Bass model's innovation coefficient in the second period,  $D_1$  are the sales in the

**Figure 6** Price (Top) and Frills (Bottom) When Frills Can Be Negative

Notes. Price graph: the black line corresponds to the main model ( $s \geq 0$ ); the dark gray line is when  $s$  is restricted to be above  $-1$  instead. Frills graph: the black line corresponds to  $s \geq -1$ , the dark gray line (higher) is the change when  $s \geq -0.75$ , and the light gray line (lower) is the change when  $s \geq -1.25$  (filled rectangle illustrates mixed strategy).

first period,  $b$  is related to the Bass model's imitation coefficient, and the first-period market size is normalized to 1. Because the market potential in the first period is driven mostly by innovators, it may not be unreasonable to also use  $a = 1/N$ . If so, we have the following results.

When the market saturation effect is not too strong (i.e., when  $N$  is large), the optimal first-period price is always below  $V/2$  and is decreasing in  $b$ ; i.e., the optimal price should be lower when market grows faster. The optimal  $s$  is only positive when  $b$  is sufficiently high, and then  $s$  increases in  $b$  until it reaches  $s = 1$ . In other words, the results of the main model about the optimality of reducing price if the market does not grow very rapidly and the optimality of frills only when the market grows rapidly enough are robust. However, it is optimal to reduce price even if the market growth is very fast. This is because a lower price not only helps ratings but interacts positively with the market growth parameter  $b$ .

When the market saturation effect is strong enough (numerical calculations show that this condition is approximately  $N < 9.61$ ), it is optimal to set the first-period price  $p_1$  above  $V/2$  when  $b$  is small enough, but the optimal price is lower than if the ratings would not exist; i.e., we still have robustness of the result of the main model that the consideration of ratings leads to lower optimal price when market growth is

not too high. Also, the optimal  $p_1$  still decreases in  $b$ , and the result about the optimality of frills only for large enough  $b$  still holds. The intuition for optimality of higher  $p_1$  in this case is that now, contrary to the low-saturation case, lower sales in the first period imply a higher market potential in the second period.

There are several factors that can bring back the result that, at high market growth rates, the optimal price increases in the market growth rate and becomes higher than the optimal price when the effect of ratings is not considered. One such factor is if the potential market within a period is a function of previous period category sales, and the firm under consideration is one of two or more competitors. In this case, the cross-brand effect of prior period sales (e.g., Libai et al. 2009) could result in a large portion of the growth not being under control of this firm. Therefore, a large number of competitors could imply a large constant term  $a$  in the above formulation (which is now coming from both the innovators and the imitators of competitor customers), and thus the results would be the same as in the case of large  $M$  in the main model. Second, the growth of market potential can indeed be exogenous as a result of some macroeconomic or demographic trends (Mahajan et al. 1990). Third, the market potential can increase irrespectively of the first-period price because of the firm opening more retail locations or enlarging its distribution network.

It is also possible that market expansion depends not just on the prior sales but on how good initial customer ratings are (Mahajan et al. 1984). This would increase the importance of customer ratings as opposed to sales in generating future purchases. If the effect of customer ratings on future growth is strong enough relative to the effect of prior sales, then the major concern of a firm would be how and how much to influence customer ratings in a cost-effective way. This is exactly the same consideration we have in the main model. The results are then that if the effect of positive ratings on market growth is low, it is optimal for a firm to reduce first-period price and not offer frills; when the effect is strong enough, it is optimal for a firm to increase price and offer frills; and when in the intermediate range, it is optimal to reduce price and offer frills. In other words, conceptually, we have the results of Proposition 1 with the market growth rate parameter replaced by the strength of word of mouth as a function of ratings  $R$ .

#### 5.4. Robustness to First-Period Consumer Expectations of Frills

In the model setup, we have postulated that first-period consumers do not expect  $s$  and therefore act as if it were 0. A natural question arises of how the results would change if first-period consumers had rational expectations about  $s$ . A related question is whether first-period consumers or an outside observer should expect  $s$  to be positive.

To answer these questions, let us denote the first-period consumer expectation of  $s$  by  $\tilde{s}$  and denote the second-period consumer expectations of these first-period consumer expectations by  $\hat{\tilde{s}}$ . Again, we will look for a pure-strategy equilibrium, in which case both the firm actions and the consumer expectations of them are single-point values. Then, we have (see the appendix)

$$R(\eta, p_1, s, \tilde{s}) = \begin{cases} 1, & \text{if } \eta \geq -s + \hat{\tilde{s}}; \\ 1 + \frac{\eta + s - \tilde{s}}{V - p_1 + \tilde{s}}, & \text{if } \eta \leq -s + \tilde{s}. \end{cases} \quad (9)$$

The inversion of this function by the second-period consumers leads to the following function of expected  $\eta$  by the second-period consumers, conditional on actual  $\eta$ , the firm's actions, and consumer expectations:

$$E(\eta | \eta, \dots) = \begin{cases} \frac{1 - \hat{\tilde{s}} + \tilde{s}}{2}, & \text{if } \eta \geq -s + \tilde{s}, \\ \max \left\{ -1, \frac{V - \hat{p} + \hat{\tilde{s}}}{V - p_1 + \tilde{s}} \cdot (s - \tilde{s} + \eta) - \hat{\tilde{s}} + \tilde{s} \right\}, & \text{otherwise,} \end{cases} \quad (10)$$

where  $E(\eta | \eta, \dots) = E(\eta | \eta, p_1, \hat{p}, s, \hat{\tilde{s}}, \tilde{s})$ . Note that if the information available to the first- and second-period consumers is the same, then  $\hat{\tilde{s}} = \tilde{s}$  is an equilibrium condition. However, if  $p_1$  is observed by the first-period consumers but unobserved by the second-period consumers, as assumed in the main model, then  $\tilde{s}$  should generally be a function of  $p_1$ , whereas  $\hat{\tilde{s}}$  cannot be. The only consistency between the two is on the equilibrium path, i.e.,  $\tilde{s}(p_1^e) = \hat{\tilde{s}} = s^e$ , where  $p_1^e$  and  $s^e$  are the equilibrium price and postpurchase effort, respectively.

Note that when the original model predicted  $s = 0$ , the first-period consumers already had the correct expectations of  $s$ . Therefore, rational expectations of first-period consumers would not change any results. A consequence of this is that the statement of Propositions 1 and 2 about the minimal value of market growth ( $M_1$  and  $M_1'$ ) required for the firm to provide postpurchase effort remains valid when first-period consumers have fully rational expectations. In fact, we have the following proposition.

**PROPOSITION 3.** *When first-period consumers have rational expectations on  $s$ , we have*

1. *If  $M < M_1$ , there is an equilibrium with  $s = 0$  and prices as in the case of Proposition 1.*
2. *If  $M > M_1$ , then there is an equilibrium in which  $s > 0$  at least probabilistically.*

**PROOF.** See the appendix.  $\square$

Note that when  $s > 0$ , the actual value of  $s$  will be influenced by the difference in consumer behavior between these two models. Hence, only the condition under which  $s > 0$  is the same. To further explain the second point of the Proposition 3, please note the following. If we start with the beliefs by first-period consumers that  $\tilde{s} = 0$  when  $M > M_1$ , the firm wants to set  $s > 0$  according to Proposition 1; however, it is rational for first-period consumers to expect  $s > 0$ . This higher expectation leads to an even higher incentive for the firm to invest in  $s$ . However, at a high enough value of  $s$ , the firm will be willing to disappoint and set  $s < \tilde{s}$ , which leads to the possibility of a mixed-strategy equilibrium in  $s$ . In the case of a mixed-strategy equilibrium, the Bayesian belief updating by the second-period consumers is complicated because they need to attribute a given level of ratings partly to  $\eta$  and partly to the realization of the mixed strategy in  $s$ . Moreover, there is no mixed-strategy equilibrium where the firm mixes between only two values of  $s$ , and hence the probability distribution of  $s$  is complicated as well, even if the belief updating was not.

To derive more explicit results and show also the robustness of the result that the firm will find it optimal to raise the price in the first period when the growth rate is high enough, consider the following modification of the original model. Suppose the market growth rate is uncertain and could be either

low  $M_l$  or high  $M_h$  with equal probability. At the beginning of the first period, neither the consumers nor the firm know  $M$ , but everybody has rational expectations. Then the firm sets the price  $p_1$ , and the first-period consumers make their purchase decisions. Then the actual market growth is revealed to everybody, and the firm sets  $s_1(M)$ . The first-period consumers report their ratings, the firm then sets the second-period price, and the second-period consumers make their purchase decisions. In this case, we have the following.

**PROPOSITION 4.** *When the market growth rate is uncertain as specified above, and all market agents have rational expectations, we have*

1. If  $M_h$  is low enough, then  $p_1 < p_1^{sp}$  and  $s(M_l) = s(M_h) = 0$ .
2. There is a combination of  $M_l$  and  $M_h$  for which  $p_1 > p_1^{sp}$  and  $s(M_h) > 0$ .

**PROOF.** See the appendix.  $\square$

### 5.5. Consumer Heterogeneity in the Uncertain Part of Their Utility

In §3.2 we assumed that the expectation ( $h_i$ ) of the product value was heterogeneous across consumers and that the uncertain part  $\eta$  was the same across consumers. In this section, we relax this assumption by considering the following specification of consumer utility:

$$U_i = h_i + \eta + \varepsilon_i - p_t + s_t, \quad (11)$$

where in addition to the assumptions before, we assume that  $\varepsilon_i$  is distributed uniformly on  $[-\varepsilon, \varepsilon]$  for some  $\varepsilon$ , independent of  $h_i$  and across consumers, and unobserved to consumers prior to purchase. It is inconsequential whether consumers observe each of  $\eta$  and  $\varepsilon_i$  or only the sum, but a natural assumption is that they observe the sum only. In other words, the separation of the uncertain part of product value into  $\eta$  and  $\varepsilon_i$  is purely methodological in nature and is a way to model correlated but heterogeneous

uncertainty. We first consider  $\varepsilon < 1$ , but later we will note the differences of this case and that of  $\varepsilon > 1$ .

With the above specification of the consumer utility, we obtain (see the appendix)

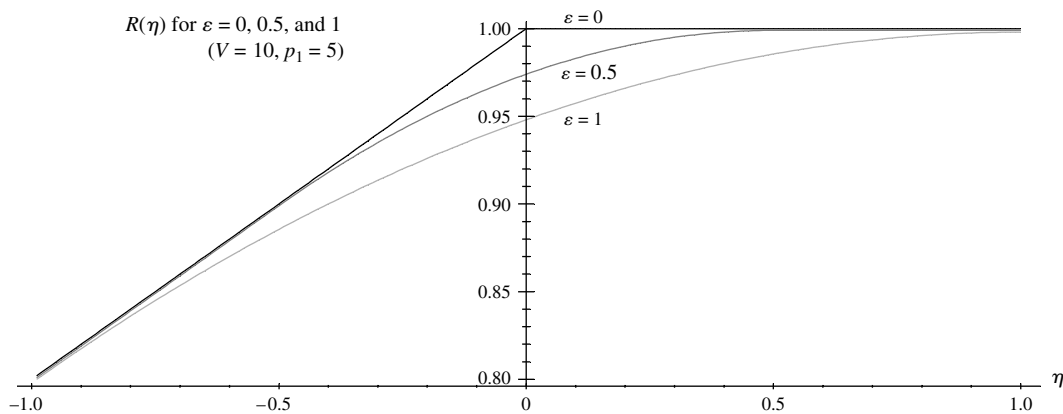
$$R(\eta, p_1, s) = \begin{cases} 1, & \text{if } \eta \geq \varepsilon - s; \\ 1 - \frac{(\varepsilon - \eta - s)^2}{4\varepsilon(V - p_1)}, & \text{if } \eta \in (-\varepsilon - s, \varepsilon - s); \\ 1 + \frac{\eta + s}{V - p_1}, & \text{if } \eta \leq -\varepsilon - s. \end{cases} \quad (12)$$

Figure 7 illustrates how ratings change as  $\varepsilon$  increases. Note that ratings weakly decrease in  $\varepsilon$ . This is because the higher the  $\varepsilon$ , the higher the uncertainty consumers are facing, and therefore, the higher the likelihood that they will be regretting the purchase decision made based on the basis of the expected value. Note also that  $R(\eta, p_1, s)$  is invertible as a function of  $\eta$ , keeping  $p_1$  and  $s$  constant when  $\eta \leq \varepsilon - s$ ; i.e.,  $R$  is more informative of  $\eta$  when  $\varepsilon$  is larger.

Although the second-period consumers are uncertain and would like to learn their  $\xi_i = \eta + \varepsilon_i$ , they realize that the prior customers' ratings are only informative about  $\eta$  and not about  $\varepsilon_i$ , because  $\varepsilon_i$  are i.i.d. across consumers. Therefore, the second-period consumers use  $R$  only to update their expectations of  $\eta$ . Similar to the main model, one then derives the following effect of firm's actions on the second-period consumer expectations:

$$E(\eta | \eta, p_1, \hat{p}, s, \hat{s}) = \begin{cases} \frac{1 + \varepsilon - \hat{s}}{2}, & \text{if } \eta \geq \varepsilon - s; \\ \varepsilon - \hat{s} - (\varepsilon - s - \eta) \sqrt{\frac{V - \hat{p}}{V - p}}, & \text{if } \eta \in (-\varepsilon - s, \varepsilon - s); \\ \max\left\{-1, \frac{V - \hat{p}}{V - p}(s + \eta) - \hat{s}\right\}, & \text{otherwise.} \end{cases} \quad (13)$$

**Figure 7** Dependence of  $R$  on  $\varepsilon$  (for  $V = 10$ ,  $p_1 = V/2$ ):  $\varepsilon = 0$  (Black), 0.5 (Dark Gray), and 1 (Light Gray)



The change in the above expression for consumer inference from Equation (4) leads to the following modification of the partial equilibrium results in Equations (5) and (6): the optimal first-period service as a function of the equilibrium first-period price is given by

$$s^e(p_1) = \min \left\{ 1 + \varepsilon, \max \left\{ 0, 2V + 1 + \varepsilon - 2\sqrt{(V-1)^2 + 8(V-p_1)/M} \right\} \right\}, \quad (14)$$

whereas the optimal first-period price as a function of the equilibrium first-period service is

$$p_1^e(s) = \begin{cases} \frac{3V+s}{4} - \frac{1}{12}\sqrt{9(V-s)^2 + 3(1-s)^2(3V-2-s)M + 9(V-s)\varepsilon^2 M}, & \text{if } s + \varepsilon \leq 1; \\ \frac{3V+s}{4} - \frac{1}{24}\sqrt{36(V-s)^2 + 6(1+\varepsilon-s)^2(3V+\varepsilon-2-s)M}, & \text{otherwise.} \end{cases} \quad (15)$$

The two cases arise because when  $s + \varepsilon > 1$ , the third case of Equation (12) disappears. Equation (14) now implies that the market growth rate required to make  $s > 0$  optimal is

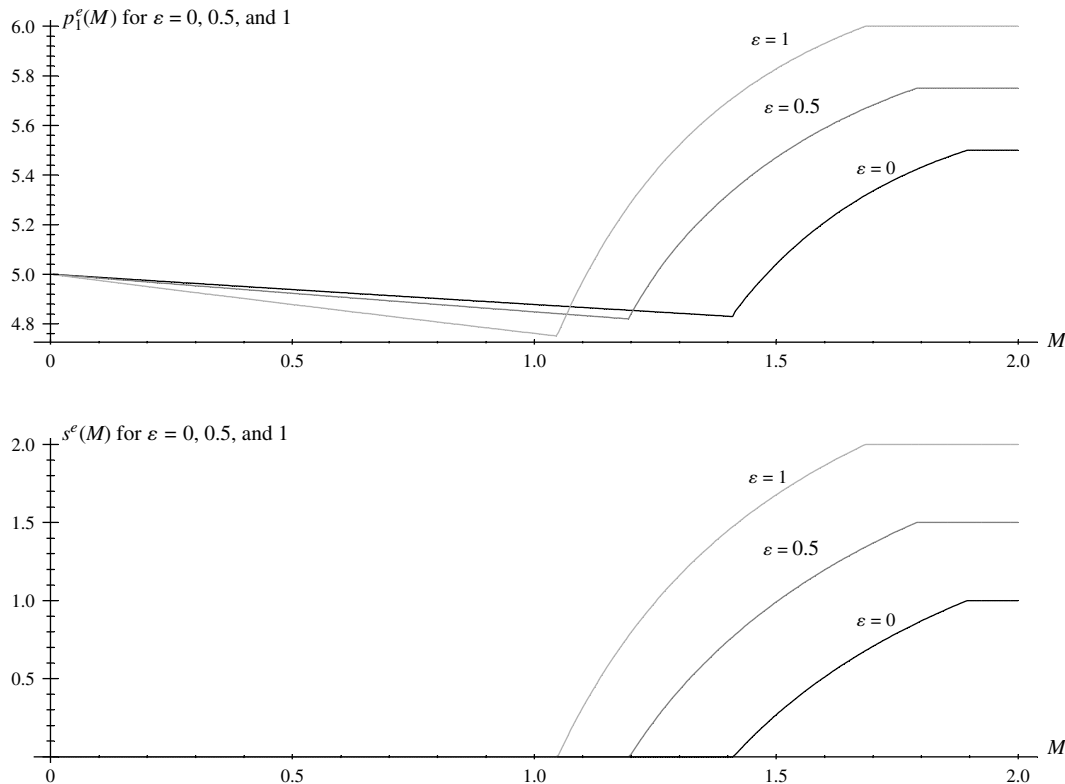
$$M > \frac{32(V-p_1)}{(3+\varepsilon)(4V-1+\varepsilon)} = \frac{16}{3} \cdot \frac{36V^2 + 12V^2\varepsilon + 3V + 15V\varepsilon^2 + 6V\varepsilon - 8}{(3+\varepsilon)^2(4V-1+\varepsilon)^2}, \quad (16)$$

and we have the following proposition (the proof is in the appendix).

**PROPOSITION 5.** *Propositions 1 and 2 hold under the alternative utility specification of this section, except with different critical values of market growth rate. Furthermore,  $M_1$ ,  $M_0$ , and  $M'_1$  decrease in  $\varepsilon$ .*

Figure 8 illustrates the effect of  $\varepsilon$  on optimal prices and frills. This effect is interesting as a higher  $\varepsilon$  implies that only the idiosyncratic part and not the common part of consumer uncertainty is greater. The intuition for the effect of  $\varepsilon$  on the optimal price and frills is that when  $\varepsilon$  is larger, the disappointed customers are more likely to be disappointed because of a low  $\varepsilon_i$  rather than a low  $\eta$ . Because the effect of  $\varepsilon_i$  on  $R$  averages out across consumers, the variability of  $R$  as a function of  $\eta$  is smaller when  $\varepsilon$  is larger.

**Figure 8** Optimal  $p_1$  (Top) and  $s$  (Bottom) for  $V = 10$ ;  $\varepsilon = 0$  (Black), 0.5 (Light Gray), and 1 (Light Gray)



**Notes.** The first (up) kink in every line corresponds to  $M_1$ , the market growth rate after which  $s > 0$  is optimal; the second (flat) kink corresponds to the market growth rate at which  $s$  reaches  $1 + \varepsilon$ .



This implies that with rational second-period consumer inference, a given change in  $R$  has a stronger effect on the second-period consumer perception of  $\eta$  when  $\varepsilon$  is larger. As a result, the benefit to the firm of increasing  $R$  by a given amount is higher when  $\varepsilon$  is larger. Therefore, the firm ends up investing more in affecting  $R$  either through the lower price or the frills when  $\varepsilon$  is higher. This may be consistent with market reality. For example, services such as hotel stays presumably have a high idiosyncratic component because consumer satisfaction may depend very much on the weather, a particular experience with the room service, the neighboring customer, or on the particular state of mind of the customer at the time he or she stayed at the hotel, etc. In line with the model predictions, hotels are very concerned about customer ratings.

In the case of  $\varepsilon > 1$ , the derivations and the qualitative results are the same, and Propositions 1 and 2 (with different cutoff values of  $M$ ) still hold. However, it turns out that the firm only wants to use  $s$  at a level that makes it possible for  $R$  to become 1. In other words, if  $s > 0$ , then  $s > \varepsilon - 1$ . This seems to be consistent with industry practice when a customer satisfaction goal is often stated as “strive for 100% customer satisfaction.”

To illustrate the results, note that if  $p_1 = V/2$  and  $V$  is large, the critical value of  $M$  for which  $s > 0$  becomes optimal decreases from  $4/3$  to 1 as  $\varepsilon$  increases from 0 to 1. Furthermore, for any  $V$  and  $\varepsilon$  in our model, it is only optimal to provide frills if the market is growing ( $M > 1$ ). Note, however, that this is based on the assumption that the cost of the frills is equal to the value they provide to the consumers. If the cost including the opportunity cost of selling the frill separately to the customers is below the consumer value, the cutoff value of  $M$  when it becomes optimal to provide frills would decrease. This trade-off is an interesting issue in itself (see, for example, Kim et al. 2001).

## 6. Discussion and Conclusion

Formally modelling the informative effect of ratings as the link through which future consumers obtain information from prior customers allows us to derive how a firm should optimally adjust its pricing and product augmentation (frills) strategy in view of this information dissemination. In particular, we derive conditions under which a decrease or increase in price is optimal and derive when it is optimal to provide frills. Furthermore, we identify the condition—namely, bounded rationality of consumers in how they process the information available in the consumer reviews—under which the firm benefits from concealing past price information. On the other hand, we show that if consumers are sophisticated enough,

it is best for the firm to encourage consumers to report price in their reviews.

Although we show that reducing price relative to the single-period optimal one increases ratings, we also identify the following two boundary conditions for this strategy to be optimal for the firm. First, if the price history is observed by future consumers, and future consumers are rational enough to see how the lower price affects the prior customer ratings, then changing the current price may not be effective in changing the beliefs of future consumers. Second, when the firm uses frills, then the beneficial effect of lowering price on customer ratings may be counteracted by the negative effect of making it more expensive for the firm to pay for customers to be satisfied, which can lead to the optimal price being *higher* than the single-period profit optimizing one. The latter strategy can be viewed as the firm shifting from maximizing current profits or revenue to maximizing customer satisfaction (surplus). Note that this strategy can only be optimal if the market is growing.

Because reducing price can be viewed as penetration or as a maximizing sales strategy, the above results suggest the following management rule of thumb when price history is not well observed: augment short-term profit maximization with market share maximization (i.e., reduce price) when the future value of the market is not very high and augment it with a strategy to increase the *current customer* surplus (i.e., “retention”) when the future value of the market is high enough. On the consumer side, these results provide insight on when to expect the firm to try to shirk on its promises and when this is not a concern.

Note that in the main model, we have defined the important market characteristic of the relative attractiveness of the second period’s profits to the firm as coming from growth in the market potential (parameter  $M$ ). Alternatively, the future value of the market can be positively affected by the increased availability of complementary products the firm may sell together with the main product in future periods, by the increased margins as a result of an increase in consumer valuation from a future product improvement, or by lower costs in the future periods. On the other hand, the future value can be negatively affected by an increase in competition, which one would expect to happen if the market is growing. These positive and negative factors would affect the results in much the same way as would a corresponding increase or decrease in  $M$ . The results will also be similar if we replace the exogenous parameter of market growth with an assumption that the second-period market potential is an increasing function of ratings with  $M$  being a parameter of the strength of this dependence. This is because, although we solved for the optimal second-period price in the main model, we

confirmed the main implications by modeling second-period profit through a reduced-form equation on margin, where the margin is affected by the above factors in the predicted direction. Specific parameters for the intercept and the effect of these factors turn out not to change the conceptual implications. We should note, however, that the positive effect of first-period sales on the future market value changes the implications in favor of using lower price instead of frills, although competition in the first period or the effect of the ratings on the market growth could restore the robustness of the implications of the main model. Another factor in favor of lower initial prices is a positive effect of larger sales on lower future costs through learning by doing. This factor is not likely to be affected by competition.

## Appendix

### Derivation of Equation (2)

A first-period consumer buys a product if and only if  $h_i \geq p_1$ . If  $\eta \geq -s$ , all who purchased have positive realized utility from the purchase; i.e.,  $R = 1$ . If  $\eta \leq -s$ , then  $(-s - \eta)/V$  customers with  $h_i - p + s + \eta < 0$  are dissatisfied and  $(V - p_1 + s + \eta)/V$  of the remaining customers are satisfied. Aggregating these, we obtain  $R = 1 + (s + \eta)/(V - p_1)$  if  $\eta \leq -s$ . Q.E.D.

### Derivation of Expressions (9) and (12)

A first-period consumer buys a product if and only if  $h_i + \tilde{s} \geq p_1$ .

If  $\eta \geq \varepsilon - s + \tilde{s}$ , we have

$$R(\eta, p_1, s, \tilde{s}) = \Pr(h_i + \eta + \varepsilon_i + s - p_1 \geq 0 \mid h_i + \tilde{s} - p_1 \geq 0) = 1.$$

If  $\eta \in (-\varepsilon - s + \tilde{s}, \varepsilon - s + \tilde{s})$ , we have (1)  $(V - p_1 + s + \eta - \varepsilon)/V$  customers with  $h_i \geq p_1 - s - \eta + \varepsilon$  are always satisfied, and (2) that there are  $(-s + \tilde{s} - \eta + \varepsilon)/V$  remaining first-period consumers who buy, and they are satisfied if  $h_i - p_1 + s + \eta + \varepsilon_i \geq 0$ ; i.e., the probability that a consumer of this type is satisfied is  $(\varepsilon - p_1 + h_i + s + \eta)/(2\varepsilon)$ . Averaging this probability over  $h_i \in (p - \tilde{s}, p - s - \eta + \varepsilon)$ , we obtain the average rating of these customers. Taking the weighted-by-demand average of the ratings of the two types of customers, we obtain  $R = 1 - (s - \tilde{s} + \eta - \varepsilon)^2/4\varepsilon(V - p_1 + \tilde{s})$ .

If  $\eta \leq -\varepsilon - s + \tilde{s}$ , consider the following three types of customers: (1) those with  $h_i - p + s + \eta + \varepsilon < 0$ , (2) those with  $h_i - p_1 + s + \eta \in (-\varepsilon, \varepsilon)$ , and (3) the rest. There are  $(-s - \eta - \varepsilon + \tilde{s})/V$  customers of the first type, and they are always dissatisfied. There are  $2\varepsilon/V$  customers of the second type, and the probability of their satisfaction linearly increases from  $-1$  to  $1$  as a function of  $h_i$ ; i.e., their average rating is  $1/2$ . There are  $(V - \varepsilon - p_1 + s + \eta)/V$  customers of the third type, and they are always satisfied. Taking a weighted-by-demand average of the above ratings, we obtain  $R = 1 + (s - \tilde{s} + \eta)/(V - p_1 + \tilde{s})$ .

Substituting  $\varepsilon = 0$  and  $\tilde{s} = 0$  in the above, we obtain Equations (9) and (12), respectively. Q.E.D.

**PROOF OF LEMMA 1.** We use here the more general specification of consumer utility in Equation (11). Equations in Lemma 1 follow by substituting  $\varepsilon = 0$ .

A firm's first-period profit is  $\pi_1(p_1, s) = (1 - (p_1/V)) \cdot (p_1 - s)$ , while the expected second-period profit given the optimal second-period price of  $p_2 = (V + E\hat{\eta}(\eta))/2$  is  $E\pi_2(p_1, \hat{p}, s, \hat{s}) = \int_{-1}^1 (M(V + E\hat{\eta})^2/4V) d\eta$ , where  $E\hat{\eta}$  is given by Equation (13). The optimal  $p_1$  and  $s$  conditional on consumer expectations satisfy the first-order conditions  $\partial(\pi_1 + E\pi_2)/\partial p_1 = 0$  and  $\partial(\pi_1 + E\pi_2)/\partial s = 0$ . Note that even though Equation (13) may not hold when  $\hat{s} \neq s$  or  $\hat{p} \neq p_1$ , using it is valid in the first-order conditions under the assumption of rational expectations. Substituting  $\hat{s} = s$  and  $\hat{p} = p_1$  after differentiation in the above leads to the equations on the equilibrium values of  $p_1$  and  $s$ . Specifically, the first-order condition on  $s$  simplifies to

$$M(3 + \varepsilon - s)(4V - 1 + \varepsilon - s) = 32(V - p_1). \quad (17)$$

This quadratic equation has two solutions, but only one satisfies the boundary condition  $s < 1 + \varepsilon$ , which leads to Equations (5) and (14). Note that by assumption,  $s$  must be nonnegative, and  $s > 1 + \varepsilon$  cannot be optimal because, in that case,  $E\pi_2 = M \int_{-1}^1 (V + (1 + \varepsilon - \hat{s})/2)^2/8V d\eta$  does not depend on  $s$ . Furthermore, the second-order condition is satisfied in the neighborhood of this solution. Therefore,  $s = s^*$  is the optimal  $s$  as far as it is within the range  $(0, 1 + \varepsilon)$ ; the optimal  $s$  is 0 if  $s^* \leq 0$  and the optimal  $s$  is  $1 + \varepsilon$  if  $s^* = 1 + \varepsilon$ . This completes the derivation of Equations (5) and (14).

To derive the equilibrium condition on  $p_1$ , consider first the case of  $0 \leq s < 1 - \varepsilon$ , which is the only possibility for optimal  $s$  if  $\varepsilon = 0$ . After substituting  $\hat{s} = s$  and  $\hat{p} = p_1$  in the first-order condition on  $p_1$ , it simplifies to

$$24(V - p_1)(V + s - 2p_1) = M(1 - s)(3V\varepsilon^2 + (1 - s)(3V - s - 2)),$$

which leads to Equation (6) and the top case of Equation (15). When  $1 - \varepsilon \leq s \leq 1 + \varepsilon$ , the third case of Equation (4) becomes empty and only two cases remain. In this case, the first-order condition on  $p_1$  simplifies to  $24(V - p_1)(V + s - 2p_1) = M(1 + \varepsilon - s)^2(3V + \varepsilon - s - 2)$ , which leads to the bottom case of Equation (15). Q.E.D.

**PROOF OF PROPOSITION 1.** To derive the exact condition under which equilibrium  $s$  equals zero, we substitute  $s = 0$  in Equation (15), substitute the resulting expression for price in Equation (14), and solve it for  $M$ . Note that Equation (15) implies that  $p_1^e < V/2$  and is decreasing in  $M$  when  $s = 0$ . This proves part 1. Also note that Equation (15) implies that when  $s = 1 + \varepsilon$ , the optimal  $p_1$  is  $(V + 1 + \varepsilon)/2$ . Equation (14) implies that optimal  $s$  increases whenever  $M$  or  $p_1$  increase. Thus, to prove parts 2 and 3 and the rest of Proposition 1, it suffices to prove that  $p_1$  increases in  $M$  when  $M$  is greater than  $M_1$  (i.e., when  $s \geq 0$  is not binding) and optimal  $s$  is below  $1 + \varepsilon$  ( $M_3$  is derived from simultaneous solution of Equations (14) and (15) for  $s = 1 + \varepsilon$ ). Computing  $dp/dM$  and  $ds/dM$  through implicit differentiation of the equilibrium equations, it is straightforward to check that  $dp/dM > 0$  given  $s^e \in (0, 1 + \varepsilon)$ ,  $0 \leq \varepsilon \leq 1$ , and  $V \geq 6$ . It also turns out that  $M_2$  can be expressed through a solution of a cubic equation involving  $\varepsilon$  and  $V$ . Q.E.D.

**PROOF OF PROPOSITION 2.** To solve the game with observed price history, we first consider the continuation game of setting the optimal  $s$  given  $p_1$  and then solve for the optimal  $p_1$ . The first-order condition on  $s$  (given  $p_1$ ) is

exactly the same as in the case of unobserved price history; i.e., Equations (5) and (14) still hold. However, the optimal first-period price is different because now the firm needs to take into account how the second-period consumers' expectations of  $s_1$  change as a function of  $p_1$ .

The first-order condition on  $p_1$  is obtained by substituting the optimal  $s$  given  $p_1$  for  $s$  and  $\hat{s}$  into the total profit of the firm and then differentiating it with respect to  $p_1$ . We will also use the following formulation of the first-order condition on  $p_1^e$  as a function of  $s$  when the first-order equation on  $s$  is binding. First, through implicit differentiation of Equation (17), we obtain  $\partial s/\partial p = 16/(M(2V + 1 + \varepsilon - s))$ . Using the above, the first-order condition on  $p_1^e$  can be written as

$$p_1^e = \frac{M(2sV + 4s\varepsilon - 3s^2 + 2V + 4V^2 + 2V\varepsilon - (1 - \varepsilon)^2) - 32V}{4(M(2V + 1 + \varepsilon - s) - 8)}, \quad (18)$$

which is valid as far as Equation (17) is binding, i.e., when it results in  $s \geq 0$ .

Note that if Equation (17) is not binding, then  $s = 0$  in the neighborhood of the optimal  $p_1$ , and hence  $ds/dp = 0$ . Using this, we obtain that the optimal price is  $V/2$ , i.e., the same as the single-period optimal one. When  $p = V/2$ , Equation (17) implies that  $s = 0$  if and only if  $M \leq M_0 \equiv 16V/((3 + \varepsilon)(4V - 1 + \varepsilon)) \geq 1$ , which is obtained by solving Equation (17) for  $M$  after substituting  $p = V/2$  and  $s = 0$ . On the other hand, when  $M > M_0$ , Equation (17) becomes binding. In this case, substituting  $p_1$  from Equation (18) with  $s = 0$  into Equation (17) and solving it for  $M$ , we obtain that the equilibrium  $s = 0$  if and only if

$$M \leq M'_1 \equiv \frac{16(2V^2 + (7 + 3\varepsilon)V - 1 + \varepsilon^2)}{(3 + \varepsilon)(2V + \varepsilon + 1)(4V - 1 + \varepsilon)}. \quad (19)$$

Similar to the proof of Proposition 1, implicit differentiation shows that  $dp/ds < 0$  when Equation (17) is binding. However, because optimal price  $p_1 = (V + 1 + \varepsilon)/2$ , to derive the optimal price the firm needs to compare the profit with  $p_1 = (V + 1 + \varepsilon)/2$  and  $s = 1 + \varepsilon$  to the profits when  $p_1$  satisfies Equation (18) and  $s$  satisfies Equation (17). This is because although the second-order conditions are satisfied when  $s < 1 + \varepsilon$ , the profit function has an upward kink in  $p_1$  when  $s$  reaches the boundary  $1 + \varepsilon$ . It then follows that  $M'_2$  is defined by the equality of the profit when  $p_1$  and  $s$  solve the first-order conditions to the profit when  $p_1 = (V + 1 + \varepsilon)/2$  and  $s = 1 + \varepsilon$ . Thus, unlike in the case of unobserved price history, in the case of observed price history, the optimal price as a function of  $M$  has a discontinuity at  $M = M'_3$ .

To prove the ordering  $M_0 < M_1 < M'_1$ , one can express

$$\begin{aligned} M'_1 - M_1 &= (16(-86V + 2\varepsilon + 17 + 3\varepsilon^4 + 6V\varepsilon^3 \\ &\quad - 6V\varepsilon + 192V^2 - 12\varepsilon^2 + 144\varepsilon V^2 \\ &\quad + 54V\varepsilon^2 + 6\varepsilon^3)) \\ &\quad \cdot (3(3 + \varepsilon)^2(2V + 1 + \varepsilon)(4V + \varepsilon - 1)^2)^{-1} \\ &> 0, \end{aligned} \quad (20)$$

and

$$M_1 - M_0 = \frac{64(3V\varepsilon^2 - 2 + 3V)}{3(3 + \varepsilon)^2(4V + \varepsilon - 1)^2} > 0. \quad \text{Q.E.D.} \quad (21)$$

**PROOF OF PROPOSITION 3.** As argued in the main text, whenever the main model results in  $s = 0$ , the first-period consumer expectations of  $s = 0$  were already rational. To prove part 2 of Proposition 3, we can derive the first-order conditions on  $p_1$  and  $s$  as in Proposition 1 with the difference that Equation (10) is used instead of Equation (13) in the expected profits of the second period. Note that the first-period consumer expectation  $\tilde{s}$  of  $s$  is a function of the price  $p_1$ , which in the equilibrium must be equal to the actual optimal  $s$  given any price  $p_1$ , whereas the second-period consumer expectations  $\hat{s}$  and  $\tilde{\hat{s}}$  of  $s$  and  $\tilde{s}$ , respectively, cannot depend on the actual first-period price. Therefore, they only equal  $s$  when  $p_1$  is the equilibrium one. The dependence of  $\tilde{s}$  on price  $p_1$  makes derivation of the first-order condition on  $p_1$  less straightforward. Therefore, we first derive the first-order condition on  $s$ .

When the boundary condition  $s \geq 0$  is not binding, using implicit function differentiation, we derive from the first-order condition on  $s$  that  $\partial s/\partial p = 1$ . Using this in the first-order conditions on price, we can derive the first-order condition on  $p_1$ , which, after using the rational expectations equilibrium conditions  $\hat{p} = p_1$ ,  $\tilde{s} = s$ ,  $\hat{s} = s$ , and  $\tilde{\hat{s}} = s$ , simplifies to

$$p_1 = s + V - \frac{3M(4V - 1)}{32}. \quad (22)$$

It turns out that the first-order condition on  $s$ , after using the rational expectation equilibrium conditions, simplifies to the same equation. In other words, any  $s$  satisfies the first-order conditions if  $p_1$  is set as above. Any combination of such  $p_1$  and  $s$  leaves the consumer utility, rating, and demand unchanged.

When  $s \geq 0$  is a binding constraint, we have  $\partial s/\partial p_1 = 0$ , and the first-order condition on  $p_1$  with rational expectations values is the same as the first-order condition on  $p_1$  when  $s = 0$  in the main model. It suggests a smaller  $p_1$  than in Equation (22) if and only if  $M < M_1$ , and it coincides with the price equation in the main model when  $s = 0$ .

Coming back to the case when  $s \geq 0$  is not binding, if a pure-strategy equilibrium exists, the firm can change  $p_1$  slightly and adjust  $s$  accordingly (the same happens in the case of observed price history). However, if  $p$  is set high enough, the firm may have the incentive to deviate strongly in  $s$  (for example, by setting it equal to zero), and thus an optimal mixed strategy in  $s$  may be preferred by the firm. Deriving the second-period consumer updating in the case of mixed strategy in  $s$  is very complicated. However, we can prove that when  $s \geq 0$  stops being a binding constraint, a pure-strategy equilibrium in  $s$  does not exist. This is because if it did, the firm could increase  $p_1$  and  $s$  until it is indifferent between deviating to a small  $s$  (i.e.,  $s = 0$ ) and not deviating. Because the firm has a higher incentive to deviate if more first-period consumers buy, the first-period consumer demand must start decreasing, i.e., the first-period consumer expectation of  $s$ ,  $E\tilde{s}$  becomes smaller than  $s$ . This lower expectation means that in the case where the firm does not shirk on frills (i.e., provides  $s$ ), this  $s$  results in the same ratings and consumer utilities as the optimal  $s$  when  $\tilde{s} = 0$  is exogenously fixed. Because the firm prefers this outcome to  $s = 0$  in the case of exogenously fixed  $\tilde{s} = 0$ , the firm will also decide to deviate to this now (instead of using a smaller  $p_1$ , which leads to pure strategy in  $s$  being optimal). Therefore, whenever  $s > 0$  in the main model,

a pure-strategy equilibrium in  $s$  does not exist. In other words, probabilistically,  $s > 0$ . Q.E.D.

**PROOF OF PROPOSITION 4.** The proof of part 1 of Proposition 4 is similar to the proof of part 1 of Proposition 1: when  $M$  is small enough, the cost of using  $s$  in the first period must outweigh the benefit in the second period. However, when  $s = 0$ , it is always optimal to set  $p_1$  strictly below  $V/2$ , because its cost is of the second order in  $p_1 - V/2$ , but the benefit (through higher ratings) is of the first order in  $p_1 - V/2$ . To prove part 2 of Proposition 4, it suffices to numerically solve the equilibrium conditions for particular parameter values that happen to yield  $p_1 > V/2$ . Specifically, we use  $M_l = 1/2$ ,  $M_h = 3/2$ ,  $V = 10$ , and  $\varepsilon = 0$ . In this case, when  $M = M_l$ , the optimal  $s$  is zero (the first-order condition leads to  $s < 0$ ); when  $M = M_h$ , the first-order condition on the optimal  $s$  (let us call it  $s_h$ ) becomes

$$3s_h^2/64 - 95s_h/16 + 4p_1 - 289/16 = 0; \quad \text{i.e.,} \quad (23)$$

$$s_h = \frac{190}{3} - \frac{4\sqrt{2473 - 48p_1}}{3},$$

which, as we will check, results in  $0 < s_h < 1$ . First-period consumers expect the above dependence of  $s$  on  $p_1$  in the event of  $M = M_h$ , and therefore, the firm has to take this into account when setting the optimal price. The resulting first-order condition on price can be written as (so far, the equations are exact and analytically derived)

$$\frac{37s^3 + 531s^2 + 16s^2p + 120678s - 15504sp - 188032p + 607808 + 12800p^2}{864(20 + s - 2p)} = 0. \quad (24)$$

Numerically solving the first-order conditions on  $s$  and  $p$  and choosing the solution with  $0 \leq p \leq V$ , we obtain 5.05173584173 and  $s_h = 0.36220513993$ . Note that in this case, the single-period optimal price is  $V/2 = 5 < p_1^e$ . Q.E.D.

**PROOF OF PROPOSITION 5.** The proofs of Propositions 1 and 2 are already generalized to be applicable for the utility function specified in Equation (15). The comparative statics with respect to  $\varepsilon$  immediately follow from differentiating the expressions for  $M_0$ ,  $M_1$ , and  $M_2$ . Q.E.D.

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