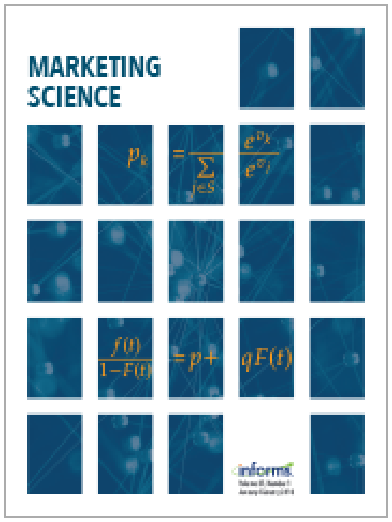

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Dynamic Incentives in Sales Force Compensation

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To inform the design of sales force compensation plans when carryover effects exist, we propose a dynamic model where these effects, together with present selling efforts, drive sales. Our results show that a salesperson with low risk aversion exerts effort to decrease attrition from existing business, whereas a salesperson with high risk aversion does not. Why? Because carryover increases not only expected sales but also sales uncertainty. Consequently, the manager should incentivize the high risk-aversion salesperson with a concave compensation plan to counterbalance suboptimal customer attrition, and the low risk-aversion salesperson with a convex compensation plan that limits coasting on past efforts. We generalize our results to when the firm employs multiple salespeople, and when advertising and personal selling are budgeted together.

Keywords: sales force; compensation; sales dynamics; agency theory; differential games; advertising
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1. Introduction

In many industries, such as pharmaceuticals, financial services, office equipment, or professional software, the effect of selling effort on sales persists over time (see, e.g., Tapiero and Farley 1975, Sinha and Zoltners 2001, Zoltners et al. 2006). From the perspective of a particular account, carryover sales may occur for several reasons, including buyer inertia, reputation and relationship of the salesperson with the account, or goodwill.

Firms internalize these carryover effects in compensation plans in various ways. Some compensate salespeople highly on new sales in anticipation of the continuing future revenue stream they represent. Other firms give a lower compensation on the initial sale, reasoning that the salesperson will benefit from recurring sales from the business without exerting much additional effort. Insurance agents, for instance, tend to be compensated highly on new sales, whereas firms that encourage long-term relationships, such as IBM, reward measures of continuing performance with the account (e.g., Hauser et al. 1994).

Although over 200 empirical studies document the long-term effects of personal selling activities on sales (Albers et al. 2010), the sales force compensation literature focuses mainly on moral hazard that results from the unobservability of selling effort. Thereby, managers cannot base compensation contracts directly on actions and instead rely on noisy performance outcomes to incentivize effort (e.g., Holmstrom 1979, Basu et al. 1985).

Carryover sales present a further layer of complexity. If carryover effects exist, but the compensation plan is designed without recognizing it, then the firm will lose money because it compensates sales generated through carryover as well as effort, but attributes sales only to effort. Furthermore, increasing the share of incentives in the salesperson's compensation package increases the temptation to coast on past actions. As a result, carryover effects expose firms to the peril of misallocating marketing resources. Managers might undersize the sales force if carryover effects are ignored (Sinha and Zoltners 2001), and carryover effects empower sales agents to receive compensation incommensurate with exerted efforts (Zoltners et al. 2006). Yet decreasing the share of incentives increases sales agents' temptation to shirk. The present study focuses on this hitherto unexplored problem.

Specifically, to inform the design of compensation plans when sales carryover effects exist, we propose a dynamic principal-agent model that sheds light on how to reconcile the conflicting incentives faced by the salesperson and the firm. The proposed model answers the following managerial questions: What is the optimal compensation plan when carryover effects exist, i.e., linear, convex, or concave in sales? How does the salesperson allocate her efforts in response to the compensation plan? How should the manager account for the agency relationship when budgeting advertising resources?

We characterize the equilibrium strategies and find that a salesperson *endogenously* allocates selling efforts between two goals; generating new business and managing attrition from existing business. A salesperson with a low risk aversion works toward decreasing customer's attrition, whereas a salesperson with a high risk aversion does not. Why? Because the carryover effect increases both the mean *and* the variance of future sales. This insight implies that a high risk-aversion salesperson gets more disutility from the additional uncertainty generated by carryover effects. As a result, whereas a salesperson with low risk aversion wants to increase sales by strategically influencing attrition, a salesperson with high risk aversion does not, to reduce future uncertainty. Thus, selling efforts make uncertainty endogenous when carryover effects exist. We find that when the salesperson has a high level of risk aversion compared to the noisiness of the sales response function, the manager should implement a concave compensation plan to countervail suboptimal customer attrition and incentivize new sales. Conversely, when the salesperson has a low risk aversion compared to the noisiness of the sales response function, the firm should implement a convex compensation plan with a threshold to disincentivize coasting on past effort. Finally, we examine how our results change when the manager employs multiple salespeople and when the firm shares the communications budget between selling and advertising.

2. Literature and Contributions

The principal-agent framework is widely used to understand the contracting relationship between the sales manager representing the firm, and the salesperson. These models have the following structure. Let x denote observable sales and v denote unobserved effort, and let their relationship follow the distribution $f(x, v)$ with support on $[\underline{x}, \bar{x}]$. Then the firm's goal is to provide a compensation $S(x)$ such that

$$\max_{S(x)} \int_{\underline{x}}^{\bar{x}} (x - S(x)) f(x, v)\, dx,$$

subject to the salesperson's individual rationality (IR) and incentive compatibility (IC) constraints, which are, respectively,

$$\int_{\underline{x}}^{\bar{x}} U(S(x)) f(x, v)\, dx - C(v) \geq R,$$

$$v = \arg\max_{u} \left[\int_{\underline{x}}^{\bar{x}} U(S(x)) f(x, u)\, dx - C(v) \right],$$

where $U(\cdot)$ is the utility from compensation, $C(\cdot)$ is the disutility from effort, and R is the outside reservation utility for the sales agent (see, e.g., Holmstrom 1979, Basu et al. 1985). For different specifications of the utility function and sales response function, different compensation plans will be obtained. Our model extends this static approach where compensation is based on total sales and not on sales dynamics, to a dynamic setting.

The sales force compensation literature offers little managerial guidance about how to incentivize strategic sales agents when carryover effects exist. On one hand, early literature informing allocation decisions of personal selling efforts over time omit agents' strategic behaviors (e.g., Beswick 1977, Darmon 1978, Tapiero and Farley 1975). On the other hand, although several mechanisms have been discovered to mitigate the risk of moral hazard, limited predictions exist to anticipate the outcomes of the dynamic agency relationship when sales are dynamic (e.g., Albers and Mantrala 2008, Coughlan and Joseph 2012). In particular, the dynamic sales force literature offers only partial predictive insights on how the interaction between sales carryover effects and risk aversion shapes the outcomes of the agency relationship (e.g., Caldieraro and Coughlan 2009, Lal and Srinivasan 1993, Krishnamoorthy et al. 2005, Mantrala et al. 1997). Framing precise guidelines requires understanding the full role of sales dynamics on the effort decisions by the salesperson and the effect of compensation in moderating effort.

Dynamics in principal-agent models have been studied in multiperiod models (e.g., Laffont and Martimort 2002, §8) where a good description of the repeated contracting between firms and salespeople is infinitely repeated moral hazard relationships introduced by Spear and Srivastava (1987). A simplified, two-period setting is considered by Laffont and Martimort (2002, §8.2.6). These papers omit within-period dynamics and show that constant high effort level is induced in each period. Holmstrom and Milgrom (1987) examine sales dynamics and show that a linear contract is optimal when sales follow a Brownian motion where the drift term depends solely on the salesperson's effort and not on current and past effort as in our case. Sannikov (2008) and Williams (2011, 2013) provide dynamic agency models where agent's observed performance relies also on *current* effort, similar to Holmstrom and Milgrom (1987), and solve the dynamic problem through a martingale approach.

Our model shares some common features with these dynamic agency models in that we consider a Brownian process where the sales agent's effort is a part of the drift term and that the salesperson has a utility function that exhibits constant risk aversion. However, we differ from them in important ways, viz., the introduction of sales carryover makes the problem fully dynamic, and the solution is parsimonious since it does not require the construction of the agent's continuation value with the martingale representation theorem (see, e.g., Sannikov 2008). Consequently, we show that the optimal compensation plan can be either convex or concave, and contrary to the recent dynamic agency literature (e.g., Sannikov 2008; Williams 2011, 2013), the dynamic optimal contract does not need to be based on the agent's *latent* continuation value as it suffices to offer a compensation plan that is a quadratic function of *observed* sales. Finally, our results add to the dynamic agency literature in that we allow the principal to have an impact on the performance measure through its own actions, i.e., advertising in our case.

3. Model

Consider an infinite horizon with time discounting where the salesperson is compensated continuously. In this setting, the infinite horizon setting resembles a long-term dynamic agency relationship in which the end time is not specified ex ante and time discounting means that neither the sales agent, nor the firm, is indifferent between present sales and carryover sales. The infinite horizon assumption is not only simpler as strategies depend only on the state but also practically relevant because often no certain terminal date exists in the firm-salesperson relationship.[1]

We assume the firm to be risk neutral, because of diversification, and the salesperson to be risk averse. We also adopt the common assumptions of absence of private information and that consumption is equal to wage, i.e., the salesperson does not access credit markets for borrowing or savings. The model is constructed as a stochastic Stackelberg differential game with a Markov perfect (closed-loop) solution concept. We summarize the notation in Table 1.

Table 1 **Notation**

Symbol	Description
$t \in [0, \infty)$	Timeline continuum.
$x(t), dx(t)/dt$	Sales and sales rate at time t, respectively.
δ, λ	Sales attrition parameter and sales carryover parameter, respectively. Note that $\delta + \lambda = 1$.
$S(x(t), t)$	Compensation contract at time t.
$v(t), C(v(t))$	Salesperson effort $v(t) \geq 0$ and disutility of effort, specified as $(v(t)) = v(t)^2/2$.
$J(S(\cdot), v(\cdot))$	The salesperson's concave utility function, a function of the compensation and effort.
$\sigma \geq 0,\ \theta \geq 0,\ r \geq 0$	Volatility parameter of the diffusion process, the agent's risk aversion, and the discount rate, respectively.
$V(x), \Pi(x)$	Value functions of the salesperson and the firm, respectively.

3.1. Sales Dynamics

The manager does not observe effort directly, but does observe sales, $x(t)$, at time t, with $t \in [0, \infty)$. As such, this is contracted upon as the basis for compensation. To incorporate carryover effects, we use a sales rate equation à la Nerlove-Arrow (1962), which has numerous empirical validations in advertising and sales research (see, e.g., Aravindakshan and Naik 2011). The canonical Nerlove-Arrow model is $dx(t)/dt = u(t) - \delta x(t)$, $x(0) = x_0$, where $u(t)$ is the advertising control, $\delta > 0$ is the decay or attrition parameter, and x_0 is the initial value of the state variable. It is a deterministic model, whereas we will develop a stochastic approach to ensure conformity with practice and the requirement for moral hazard that effort cannot be exactly inferred from sales.

Specifically, in our model, the sales rate, $dx(t)/dt$, is influenced by the salesperson's effort $v(t)$ at time t, carried-over sales, and demand shocks. The stochastic nature of the relationship between the sales rate and effort is captured through $\epsilon(t)$, where $\epsilon(t) = dB(t)/dt \sim N(0, 1)$, and $dB(t)$ is the increment of a standard Brownian motion.[2] We consider additive noise, as is commonly assumed in agency models in marketing. Thus the sales dynamics are given by the stochastic differential equation

$$dx(t)/dt = v(t) - (1-\lambda)x(t) + \sqrt{\sigma}\epsilon(t), \quad x(0) = x_0. \quad (1)$$

Here $\lambda \in [0, 1]$ is the parameter representing carryover, when λ increases or decreases, sales in the next period are higher or lower, respectively. Furthermore, $\sigma = \mathrm{Var}(dx(t)/dt)$ captures the noisiness of the sales response.

3.2. Compensation Contract

The manager offers the salesperson a contract to sell the firm's product, which the salesperson accepts or rejects based on its expected utility versus the utility, R, from their outside option. Owing to the dynamic nature of the managerial problem, the salesperson evaluates future earnings at any time and leaves the firm whenever the contract fails to exceed the value of the outside option. We examine the participation constraint at each instant of time and thus obtain time consistent compensation strategies (see, e.g., Jørgensen and Zaccour 2004).

To design the compensation plan, $S(x(t))$, the firm can restrict attention to specific compensation structures (e.g., Joseph and Kalwani 1998, Krishnamoorthy et al. 2005) or optimize over an unconstrained space

[1] Moreover, the firm, at least in theory, is infinitely lived, and salespeople spend time developing relationships with clients and the firm keeping the long term in mind. Finally, a finite horizon analysis would suppose a terminal stock target, but to compute the appropriate terminal stock requires taking the future after the horizon into account.

[2] The Brownian motion captures market uncertainties over the probability space $(\mathbb{P}, \Omega, \mathcal{F})$, where Ω is the countable set of all possible sales realizations, $\mathcal{F}$ is the filtration generated by B, and $\mathbb{P}$ is a probability measure such that for any time period t, $\mathbb{P}: \mathcal{F}t \to [0, 1]$, with $\mathbb{P}(\varnothing) = 0$ and $\mathbb{P}(\Omega) = 1$. In managerial terms, $\mathcal{F}t$ encodes the available information up to time t.

of functions. We focus on the latter. The nonlinear optimal solution can later be approximated in terms of commonly used compensation components, such as straight salary, commission, and quota (e.g., Basu and Kalyanaram 1990).

3.3. Salesperson Utility

The salesperson incurs the cost $C(v(t))$ from the selling effort $v(t)$. A convex cost function, i.e., $C' > 0$ and $C'' > 0$, is assumed and we use the common specification $C(v(t)) = v(t)^2/2$. The salesperson is compensated with the plan $S(x(t))$ that varies with observed sales $x(t)$. We note that the solution concept of Markov perfect (closed-loop) equilibrium is standard in the marketing and management science literature.[3] Consistent with the literature, the state variable is sales, $x(t)$, which encapsulates the profit relevant information for the firm. Hence, effort and compensation decisions are functions of sales, and the optimal decisions are truly optimal and not just in a class of prespecified functional forms. The time argument t will henceforth be suppressed when no confusion arises.

The salesperson is assumed to be risk averse with a concave utility function that exhibits constant risk aversion (e.g., Holmstrom and Milgrom 1987), such that the long-term utility of the salesperson is given by

$$J(x) = E\left[-\exp\left(-\theta \int_0^\infty e^{-rt}\{S(x) - v^2/2\}\,dt\right)\right]. \quad (2)$$

Here, r is the discount rate, E is the expectation operator, and $\theta > 0$ is the risk-aversion parameter.

The salesperson maximizes this long-term utility through the choice of an effort strategy that determines the agent's value function, i.e., $V(x) = J(x)$, subject to the sales dynamics in Equation (1). The Hamilton-Jacobi-Bellman (HJB) equation of the risk-sensitive control problem defined by (1)–(2) is

$$rV(x) = \max_v \{S(x) - v^2/2 + V_x(v - \delta x) + \sigma(V_{xx} - \theta V_x^2)/2\}, \quad (3)$$

where $V_x = \partial V(x)/\partial x$ and $V_{xx} = \partial V_x/\partial x$ (see, e.g., Jacobson 1973, Whittle 1986, Başar 1999, Bensoussan and Elliott 1995).

The right-hand side of (3) consists of three parts. The first term $S(x) - C(v)$, is the instantaneous difference between compensation and cost of effort. The second term $V_x(v - \delta x)$ consists of two components, namely, the expected evolution of the state $\mathbb{E}[dx/dt] = v - \delta x$, and the valuation V_x that measures the shadow price of an additional unit of sales. Thus, $V_x(v - \delta x)$ is the contribution of an incremental sale to the value function. Finally, the term $\sigma(V_{xx} - \theta V_x^2)/2$ captures the impacts of uncertainty (since $\mathrm{Var}[dx/dt] = \sigma$) on the value function, where $-\theta V_x$ is the additional cost of uncertainty borne because of risk aversion.

Differentiating (3) with respect to v and equating the resulting expression to zero, we obtain the first-order condition

$$v^* = V_x, \quad \text{(IC)}$$

which defines the salesperson's IC condition. Inserting it back into the HJB, we obtain the agent's IR condition

$$V(x) \geq R. \quad \text{(IR)}$$

3.4. Firm's Objective

The manager designs the optimal compensation plan to maximize the firm's long-term profit while ensuring that the salesperson's IR and IC constraints are respected. The firm's long-term profit is

$$\mathbb{E}\left[\int_0^\infty e^{-rt}\{x - S(x)\}\,dt\right]$$

and its value function is given by

$$\Pi(x) = \max_{S(x)} \mathbb{E}\left[\int_0^\infty e^{-rt}\{x - S(x)\}\,dt\right],$$

subject to (1), (IC), and (IR). As a result, the firm's HJB equation is

$$r\Pi(x) = \max_{S(x)}\{x - S(x) + \Pi_x(V_x - \delta x) + \sigma\Pi_{xx}/2\}, \quad (4)$$

where $\Pi_x = \partial\Pi/\partial x$, $\Pi_{xx} = \partial\Pi_x/\partial x$ and V_x as defined by the salesperson's IR condition.

Similar to the agent's HJB equation, Π_x measures the shadow price of an additional unit of sales to the firm and $\Pi_x(V_x - \delta x)$ is the contribution of an incremental sale to the value function of the firm, which will be, as shown later, an important component of the compensation strategies. We characterize the Markov perfect equilibrium strategies and obtain new results that inform sales management. We first present the results on the salesperson's dynamic allocation of selling efforts.

4. Optimal Selling Effort

Proposition 1. *The salesperson's optimal effort strategy is*

$$v^* = \begin{cases} \underbrace{\dfrac{\Pi_x}{1-\theta\sigma}}_{\substack{\text{Effort on}\\ \text{new business}}} + \underbrace{\dfrac{x\delta}{1-\theta\sigma}}_{\substack{\text{Effort on}\\ \text{existing business}}}, & \theta < \dfrac{1}{\sigma}, \\[2ex] \underbrace{\dfrac{\Pi_x}{\theta\sigma-1}}_{\substack{\text{Effort on}\\ \text{new business}}} - \underbrace{\dfrac{x\delta}{\theta\sigma-1}}_{\substack{\text{Effort on}\\ \text{existing business}}}, & \theta > \dfrac{1}{\sigma}. \end{cases}$$

[3] Furthermore, as noted by Maskin and Tirole (2001, p. 193), "Markov strategies prescribe the simplest form of behavior that is consistent with rationality." It posits that the players' decisions are based on the state of the game and not on the history taken to arrive at that state.

Table 2 **Uncertainty Levels Surrounding New and Existing Business**

	Low risk-aversion sales agent	High risk-aversion sales agent
Uncertainty of new business	$\frac{\sigma(\theta\sigma-1)}{2(A_2+\delta\theta\sigma)}\left(\frac{A_2}{1-\theta\sigma}\right)^2$	$\frac{\sigma(1-\theta\sigma)}{2(B_2-\delta\theta\sigma)}\left(\frac{B_2}{1-\theta\sigma}\right)^2$
Uncertainty of existing business	$\frac{\sigma(\theta\sigma-1)}{2(A_2+\delta\theta\sigma)}$	$\frac{\sigma(1-\theta\sigma)}{2(B_2-\delta\theta\sigma)}$

Where the value function of the firm satisfies

$$\Pi_x = \begin{cases} A_1 + A_2 x, & \theta < 1/\sigma, \\ B_1 + B_2 x, & \theta > 1/\sigma. \end{cases}$$

Expressions for constants $A_1 > 0$, $A_2 < 0$, $B_1 > 0$, *and* $B_2 < 0$ *are in the appendix.* □

Proposition 1 shows that the salesperson's effort is allocated between two goals—generating new business and managing attrition from existing business. To make this interpretation, replace the optimal effort expressions from Proposition 1 into the sales dynamics given in Equation (1), to obtain

$$\frac{dx}{dt} = \begin{cases} \underbrace{\frac{1}{1-\theta\sigma}(A_1 + A_2 x)}_{\text{New business}} - \underbrace{\delta\left(1 - \frac{1}{1-\theta\sigma}\right)}_{\substack{\text{Equilibrium} \\ \text{attrition rate}}} x \\ \quad + \sqrt{\sigma}\epsilon_t, \quad \theta < 1/\sigma, \\ \underbrace{\frac{1}{\theta\sigma - 1}(B_1 + B_2 x)}_{\text{New business}} - \underbrace{\delta\left(1 + \frac{1}{\theta\sigma-1}\right)}_{\substack{\text{Equilibrium} \\ \text{attrition rate}}} x \\ \quad + \sqrt{\sigma}\epsilon_t, \quad \theta > 1/\sigma. \end{cases} \tag{5}$$

Labeling the two cases $\theta < 1/\sigma$ and $\theta > 1/\sigma$ as low risk aversion and high risk aversion, respectively, we observe that the salesperson manages sales attrition differently depending on their level of risk aversion. A low risk-aversion agent works to reduce the rate of sales attrition, or said differently to increase carryover, since $1 - 1/(1-\theta\sigma) < 1$. Conversely, a high risk-aversion salesperson works to increase attrition or decrease carryover, because $1 + 1/(\theta\sigma - 1) > 1$, which can manifest in the field, for instance, by the salesperson not following up with existing clients and focusing mostly on generating new sales.

To understand why these different behaviors emerge especially when the salesperson has a high risk aversion, we first compute the mean and the variance of stationary distribution of $x(t)^*$, i.e., $\mathbb{E}[x(t)^*]$ and $\text{Var}[x(t)^*]$, respectively (the formula is reported in Corollary 1 in the appendix). Next, by differentiating $E[x(t)^*]$ and $\text{Var}[x(t)^*]$ with respect to the carryover effect, reveals that it increases not only expected future sales but also expected variance, which provides some insights about the salesperson's behavior. Specifically, a salesperson with a high risk aversion suffers more from the additional uncertainty generated by the carryover effect and thus favors decreasing uncertainty by reducing carryover sales by the factor $1 + 1/(\theta\sigma - 1)$. Conversely, a salesperson with low risk-aversion favors increasing future sales by curbing attrition and thus decreases δ by the factor $1 - 1/(1-\theta\sigma) < 1$.

Allocation of Selling Efforts. The impact of this asymmetric behavior also manifests in how the salesperson allocates selling efforts between old and new business. Why? Because under the optimal effort strategy of the salesperson, the levels of uncertainty for new and old business converge to different stationary mean values; see Table 2. Specifically, the level of uncertainty surrounding new business can be lower, higher, or equal to the level of uncertainty surrounding existing business.

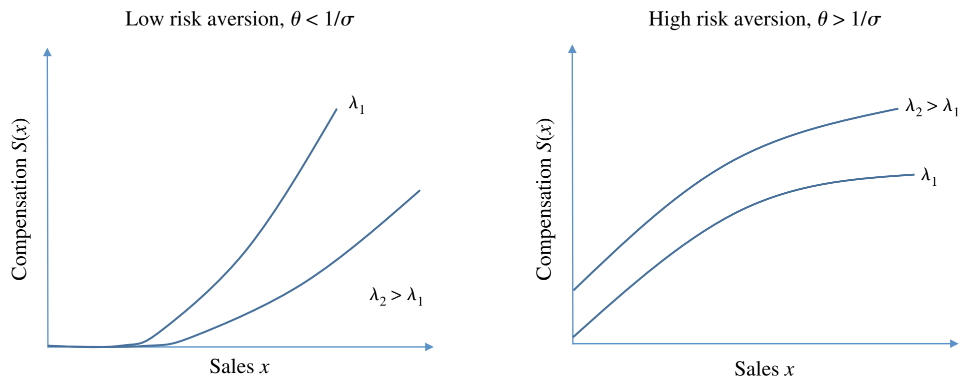
Moreover, how much new business is generated is proportional to how much the firm values a marginal sale, i.e., Π_x (see (19) in the appendix), which enters the firm's optimal compensation strategy. This shadow price decreases as x increases, due to concavity of the firm's value function with respect to sales, i.e., $\Pi_{xx} < 0$. Hence, the manager has fewer incentives to motivate the salesperson to work hard for a new sale. Moreover, as sales increase, so does the number of customers who can potentially leave the company, which increases the effort and hence the cost that the agent would have to put to retain existing business.[4] Therefore, how the agent allocates efforts between generating new and existing business is driven by sales uncertainty and how the manager compensates the salesperson through the incentive plan, which we detail next.

5. Optimal Compensation Plan

Proposition 2. *For a low risk-aversion salesperson* $(\theta < 1/\sigma)$ *the optimal compensation plan is convex in sales.*

[4] Thus, if the company has 100 customers and the carryover is 75% (as reported in Albers et al. 2010), the agent would have to focus on 25 existing customers who might churn, while when the company has 1,000 customers, the agent would have to focus on 250 customers who might churn. Hence the costs of managing new and existing business vary as sales vary, which ultimately impact how the agent works new and existing business.

Figure 1 (Color online) Optimal Compensation Plans



For a high risk-aversion salesperson ($\theta > 1/\sigma$) *it is concave in sales. Specifically,*

$$S(x)^* = \begin{cases} \max\{0; a_0 + a_1 x + a_2 x^2\}, & \theta < 1/\sigma, \\ b_0 + b_1 x + b_2 x^2, & \theta > 1/\sigma. \end{cases}$$

Where the constants are

$$a_0 = -\frac{\sigma(\delta + A_2) + A_1^2}{2(1-\theta\sigma)} < 0, \quad a_1 = -\frac{A_1 A_2}{1-\theta\sigma} > 0, \quad \textit{and}$$

$$a_2 = -\frac{(A_2-\delta)(A_2+\delta)}{2(1-\theta\sigma)} > 0; \quad b_0 = \frac{\sigma(\delta - B_2) + B_1^2}{2(\theta\sigma - 1)} > 0,$$

$$b_1 = \frac{B_1 B_2}{\theta\sigma - 1} > 0, \quad \textit{and} \quad b_2 = \frac{(B_2-\delta)(B_2+\delta)}{2(\theta\sigma-1)} < 0. \quad \square$$

In both cases the optimal compensation plan is quadratic in sales, which is a tractable form for theory and implementation. Specifically, for low risk-aversion salespeople, the optimal compensation plan is convex as $a_2 > 0$. Note also that there is a discontinuity due to the corner solution, and it specifies that when sales are below a threshold level such that the expression $a_0 + a_1 x + a_2 x^2$ is negative, the salesperson is paid a salary equal to the outside option (scaled to zero). Above this threshold, the compensation is convex in sales. Thus, when the agent has a low level of risk aversion, the firm implements a convex compensation plan that disincentivizes coasting on past efforts with a threshold, e.g., by implementing a quota.

Conversely, the optimal compensation plan for the high risk-aversion agent is concave in sales. Why? Because the carryover effect increases sales variance. Hence, the plan incentivizes the high risk-aversion sales agent to bring sales on target, despite the increased uncertainty resulting from the carryover effect. This result aligns with the insight of Zoltners et al. (2006) that concave compensation plans provide "protection" against demand uncertainty.

Figure 1 illustrates how the carryover parameter alters the optimal compensation plans depending on the agent's level of risk aversion. Based on numerical simulations,[5] we find that in the case of the low risk-aversion agent, an increase in the carryover effect increases the threshold and decreases the "convexity" of the plan. Such a result is intuitive as the manager internalizes that fact that more sales come from accumulated efforts with the account. Thus, it is optimal to increase the threshold after which the salesperson starts being compensated.

In the case of the high risk-aversion agent, the salary offered to the agent is larger than the value of the outside option, and it increases as the carryover effect increases. The total compensation also increases as the carryover effect increases. These two insights comport with our earlier finding that as the carryover effect increases, so does the level of risk to which the agent is exposed, which then requires a higher salary to ensure that the agent works for the firm.

To put these results in perspective, we note that several studies document that sales force compensation plans are often shown or assumed to be convex in sales to overcome the increasing marginal cost of effort of the agent to incentivize effort even at high levels of sales. In dynamic settings, however, Holmstrom and Milgrom (1987) show that the optimal contract is linear. Meanwhile, additional research shows that instead of a convex or linear compensation plan, managers often implement concave compensation contracts (e.g., Mantrala and Raman 1990, Hauser et al. 1996, Mantrala et al. 1997, Barnes 1986, Churchill et al. 1996). Proposition 2 adds to these insights by revealing that the optimal compensation is either convex or concave, depending on the sales agent's risk aversion, relative to the noisiness of the sales response function.

The results in Proposition 2 add to Mantrala and Raman (1990), Hauser et al. (1996), and Mantrala et al. (1997) not only in showing that a concave compensation

[5] Simulations were conducted with the function "Manipulate" implemented in Mathematica 10 with the parameter values $1 - \lambda = \delta \in [0, 0.5]$, $r \in [0, 1]$, and $\theta \in [0, 3]$.

plan can indeed be optimal but also in providing a rationale as for why managers should implement such plans, i.e., to countervail suboptimal customer attrition by the high risk-aversion salesperson.

Our results also add to the findings in Basu et al. (1985), where risk aversion also determines conditions for convex and concave shaped compensation plans. The results of the proposed model differ with respect to what determines the convexity of the plan. Specifically, in Basu et al. (1985) the optimal compensation plan of an agent with a constant absolute risk aversion (CARA) utility function is *always concave* in sales, whereas our results show that it can be *either concave or convex* depending on the agent's risk aversion and the noisiness of the sales response function, i.e., σ. Thus, a sales agent with a CARA utility function should be compensated with different shaped plans depending on the sales volatility of the products/territories to which they are assigned, which is not the case in Basu et al. (1985).

Equation (19) in the appendix details how the manager's dynamic shadow price for sales, i.e., Π_x, informs the compensation plan, which informs how marginal variations in the state variable, i.e., sales in our case, impacts the long-term profit. When sales are a function of effort and noise only, i.e., no carryover effect as in Holmstrom and Milgrom (1987) for instance, one can show that the principal's costate variable equals zero, whereas it evolves dynamically with sales when sales are dynamic. These different results occur because when carryover effects are absent, current sales *are not informative* about future profitability, i.e., how many products were sold today does not inform how many will be sold tomorrow. However, when carryover effects exists, current sales *are informative* about future profitability, i.e., how many products were sold today does inform how many will be sold tomorrow. Thus, the optimal compensation plans with and without carryover effects also differ because the principal's shadow prices for sales differ in the two cases.

To explore the consequences of omitting the carryover effect, we analyze the consequences if the firm does not take into account the long-term impact of selling effort in its value function. We show in the appendix that inefficiencies arise that yield a lower value function than if the manager had not ignored the long-term value of a marginal sale. Specifically, such a firm would try to hire only salespeople who it believes have low risk aversion, as it (incorrectly) finds it unprofitable to hire high risk-aversion salespeople. Consistent with this hiring strategy, it offers only convex compensation plans.

Finally, we report in the appendix how managers could use our findings to dynamically adjust compensation plans, depending on how sales vary. Specifically, we derive $\mathbb{E}[dS^*/dt]$ by applying Ito's lemma and find that

$$\mathbb{E}\left[\frac{dS^*}{dt}\right] = \frac{\partial S^*}{\partial x}\frac{dx}{dt} + \frac{\partial^2 S^*}{\partial x^2}\frac{\sigma}{2},$$

which reveals how the compensation plan evolves over *time* with respect to changes in sales. We then use discrete time approximations to obtain $S(t)^*$ as a function of $x(t)$ and $x(t-1)$.

Next, we extend our results to situations where the firm employs multiple salespeople and to situations where the firm invests in advertising.

6. Generalizations

We consider first an extension of the model where the firm hires multiple agents and observes only the aggregate sales that result from all of the salespeople.

6.1. Multiple Salespeople

Consider that the firm employs N salespeople, i.e., $i = \{1, 2, \ldots, N\}$, with utility functions

$$J_i(x) = \mathbb{E}\left[-\exp\left(-\theta \int_0^\infty e^{-rt}\{S_i(x) - v_i^2/2\}\,dt\right)\right], \quad (6)$$

such that the sales dynamics are now

$$dx(t)/dt = \sum_{i=1}^{N} v_i(t) - \delta x(t) + \sqrt{\sigma}\epsilon(t), \quad x(0) = x_0. \quad (7)$$

The HJB equation for agent i is

$$rV_i(x) = \max_{v_i}\left\{S_i(x) - v_i^2/2 + V_{ix}\left(v_i + \sum_{k\neq i} v_k - \delta x(t)\right) + \sigma(V_{ixx} - \theta_i V_{ix}^2)/2\right\}. \quad (8)$$

Differentiating (8) with respect to v_i and equating the resulting expression to zero, we obtain the IC condition for agent i

$$v_i^* = V_{ix}. \quad (\text{IC}_i)$$

Replacing (IC_i) in (8), we obtain the IR condition for agent i

$$V_i(x) \geq R. \quad (\text{IR}_i)$$

The firm's objective function becomes

$$\Pi_N(x) = \max_{S(x)} \mathbb{E}\left[\int_0^\infty e^{-rt}\left\{x - \sum_{i=1}^{N} S_i(x)\right\} dt\right], \quad (9)$$

where $S(x) = \{S_1(x), S_2(x), \ldots, S_N(x)\}$, subject to (7), the N IC and IR conditions. Solving for the optimal strategies we obtain the following proposition.

Proposition 3.
(a) *The optimal effort strategy for agent i is*

$$v_i^* = \begin{cases} \underbrace{\dfrac{\Pi_{Nx}}{2N-1-\theta\sigma}}_{\substack{\text{Effort on}\\ \text{new business}}} + \underbrace{\dfrac{x\delta}{2N-1-\theta\sigma}}_{\substack{\text{Effort on}\\ \text{existing business}}}, & \theta < \dfrac{2N-1}{\sigma}, \\ \underbrace{\dfrac{\Pi_{Nx}}{1-2N+\theta\sigma}}_{\substack{\text{Effort on}\\ \text{new business}}} - \underbrace{\dfrac{x\delta}{1-2N+\theta\sigma}}_{\substack{\text{Effort on}\\ \text{existing business}}}, & \theta > \dfrac{2N-1}{\sigma}. \end{cases}$$

Where the value function of the firm satisfies

$$\frac{\partial \Pi_N}{\partial x} = \Pi_{Nx} = \begin{cases} A_{N1} + A_{N2}x, & \theta < (2N-1)/\sigma, \\ B_{N1} + B_{N2}x, & \theta > (2N-1)/\sigma. \end{cases}$$

Expressions for constants $A_{N1} > 0$, $A_{N2} < 0$, $B_{N1} > 0$, *and* $B_{N2} < 0$ *are in the appendix.*
(b) *The optimal compensation plan is convex (or concave) in sales when* $\theta < (2N-1)/\sigma$ *(or* $\theta > (2N-1)/\sigma$*).*

Proposition 3 is qualitatively similar to Propositions 1 and 2 in that, first, sales agents' efforts are directed toward the two goals of generating new business and managing attrition of existing business, and, second, that the optimal compensation plan can be convex or concave depending on θ.

Proposition 3, however, reveals two new insights. First, it shows that the threshold level above (or below) which the firm should offer a concave (or convex) compensation plan to agents increases in the number of agents, N. Thus, a prediction is that firms managing a larger sales force are more likely to provide convex compensation plans than firms employing a smaller sales force.

Second, using the results from Proposition 3 allows us to investigate how the optimal strategies vary with the number of agents. Owing to the intricate mathematical expressions for the equilibrium strategies, we conduct such investigations numerically,[6] and obtain the following insights, (i) when $\theta < (2N-1)/\sigma$ agents exert less effort as N increases, as a result, the manager increases quotas and decreases total compensations. Conversely, (ii) when $\theta > (2N-1)/\sigma$, agents exert more effort as N increases since the manager increases total compensations. The numerical findings comport with our previous results in that when agents have a low risk aversion, the manager's optimal strategy is to implement a convex compensation structure with a threshold to prevent agents from coasting on free sales (Zoltners et al. 2006) that are generated by past efforts. Interestingly, when agents have a high risk aversion, then the manager implements a concave compensation plan to countervail suboptimal customer attrition and as a result motivates more efforts directed toward new business generation. Next, we investigate how the firm should account for the agency relationship when budgeting advertising resources.

6.2. Advertising and Sales Force

We begin by noting that despite the fact that firms usually use both advertising and personal selling to sell brands, few insights exist about how to optimally use advertising with sales force contract design. An exception is Murthy and Mantrala (2005) who provide guidelines on how managers should allocate marketing resources to advertising and *sales contests* taking into account the agency relationship and the related contractual problem.

Building on our earlier results, we augment the sales dynamics (1) to generalize it to situations where sales are not only driven by the agent's selling efforts but also by advertising, such that the sales dynamics becomes

$$dx/dt = v(t) + \gamma u(t) - \delta x + \sqrt{\sigma}\epsilon_t, \quad x(0) = x_0, \quad (10)$$

where $u(t)$ is the firm's advertising effort and γ its effectiveness on sales such that if $\gamma > 1$ advertising is more effective than personal selling, and vice versa if $\gamma < 1$. By setting $\gamma = 0$ we recover the main model. In such situations, the salesperson's HJB equation is now

$$rV(x) = \max_v \{S(x) - v^2/2 + V_x(v + \gamma u - \delta x) + \sigma(V_{xx} - \theta V_x^2)/2\}, \quad (11)$$

which yields the IC condition $v^* = V_x$ and the IR condition $(1/r)(S(x) + V_x^2(1-\theta\sigma)/2 + V_x(\gamma u - \delta x) + \sigma V_{xx}/2) \geq R$.

The manager determines both the optimal contract and advertising strategy so as to maximize the firm's long-term profit. Let $u^2/2$ represent the cost of advertising. Then the manager's problem is

$$\tilde{\Pi}(x) = \max_{\{S(x), u\}} \mathbb{E}\left[\int_0^\infty e^{-rt}\{x - S(x) - u^2/2\}\,dt\right],$$

subject to Equation (11) and the new IC and IR conditions.

We solve for the optimal strategies and value functions in the appendix and find that the optimal compensation plan comports with our earlier results in that for low risk-aversion salespeople the optimal compensation plan is convex in sales. For high risk-aversion salespeople it is concave in sales. Three additional insights are obtained from the analysis of the optimal strategies.

First, we learn that similar to the salesperson, the firm advertises to achieve two goals, namely, increase new

[6] Simulations were conducted with the function "Manipulate" implemented in Mathematica 10 with the parameter values $1-\lambda = \delta \in [0, 0.5]$, $r \in [0, 1]$, $\theta \in [0, 4]$, and $n = \{1, 2, \ldots, 10\}$.

business and manage attrition. Specifically, when $\theta < 1/\sigma$, the manager allocates $\max\{0, -\gamma\sigma\theta/(1+\gamma^2-\theta\sigma)\tilde{\Pi}_{x,1}\}$ to generate new sales and $\gamma x\delta/(1+\gamma^2-\theta\sigma)$ to control attrition; and when $\theta > 1/\sigma$, the manager allocates $\max\{0, -\gamma\sigma\theta/(1+\gamma^2-\theta\sigma)\tilde{\Pi}_{x,2}\}$ and $\gamma x\delta/(1+\gamma^2-\theta\sigma)$, to new and existing business, respectively. Second, despite the existence of no direct interaction between the promotional tools, variations in γ impact how the salesperson works. Without the agency relationship, and if the manager could directly control both u and v, this would not happen because the dynamics will resemble Naik and Raman (2003) without the interaction term. Third, we find that for $\sigma \in [1/\theta; (1+\gamma^2)/\theta]$, i.e., when a salesperson's level of risk aversion is intermediate, the optimal strategies equal zero, which means that the firm cannot optimally use both advertising and personal selling to sell the product and must therefore use only one instrument.

Furthermore, using these new insights, and the mathematical expressions for the optimal contracts (reported in the appendix), we obtain new results on the management of promotion budgets by firms when an agency relationship exists.

Specifically, when the manager uses both advertising and personal selling to sell the firm's brand, the total promotional budget, i.e., $B(x)$, equals the sum of the sales force compensation and advertising expenditures, such that $B(x) = S(x)^* + u^{*2}/2$. Consequently, the share of sales force compensation in this budget equals $\Lambda_1(x) = S(x)/B(x)$. Meanwhile, the actual total effort that is generated by both the firm and the agent is $M(x) = v^* + \gamma u^*$. Hence, the share of personal selling in the total promotional effort is different than the share of the sales force compensation in the firm's budget, i.e., $\Lambda_2(x) = v^*(x)/M(x)$. Said differently, owing to the nature of the promotional instruments, we find that $\Lambda_1(x) \neq \Lambda_2(x)$, which means that the impact ratio of the promotional instruments is different from the budget allocation. Using these results on the optimal promotion budgets and allocation rules, we obtain the following proposition.

Proposition 4. *When the agent has a low level of risk aversion, the manager adopts a proportional to sales budgeting strategy, i.e.,* $\partial B(x)/\partial x > 0$.

However, when the agent has a high level of risk aversion, the manager adopts an inverse to sales budgeting strategy, i.e., $\partial B(x)/\partial x < 0$.

Finally, in both cases, the share of personal selling in the total promotional budget increases as sales increases, i.e., $\partial \Lambda_1(x)/\partial x > 0$, *whereas its share in the total promotional impact decreases, i.e.,* $\partial \Lambda_2(x)/\partial x < 0$.

The first part of Proposition 4 comports with the managerial heuristics of budgeting marketing resources proportional to sales, as documented in the advertising literature (Farris et al. 1998). Conversely, the optimal promotion budget and total promotional impact decrease as sales increase when the agent has a low level of risk aversion, i.e., $\partial B(x)/\partial x < 0$ and $\partial M(x)/\partial x < 0$, thus obtaining an inverse allocation rule, which has been prescribed in some papers on advertising dynamics, e.g., Prasad and Sethi (2005). However, this and the preceding result have not been reconciled previously as both being optimal strategies, especially when one of the instruments is personal selling. We now conclude and explain how our results inform the sales force compensation practice.

7. Conclusions

Improvements in understanding compensation design in dynamic markets are worthwhile since companies invest about \$800 billion in personal selling, or about three times what they spend on advertising (Albers and Mantrala 2008). Similar to advertising, personal selling activities can have a long-term effect on sales (Albers et al. 2010). Yet, contrary to the well-developed dynamic policies of advertising, very few studies provide normative guidelines to help managers maximize the returns-on-investment of incentive compensation plans in dynamic markets. We provide an analytical treatment of the optimal design of sales force compensation contracts when sales carryover effects exist, that is when sales is understood as a dynamic variable.

By addressing this lacuna in the compensation design literature, we identify important factors that shape the outcomes of a sales force agency relationship in a dynamic market. The model shares assumptions with dynamic agency literature but has significant differences, i.e., carryover sales, infinite horizon, closed-loop concept, inclusion of advertising, and time discounting. Several analytical propositions are developed and the results are based on explicit analytical results rather than numerical simulations. Most important, the proposed approach yields answers to the managerial questions raised by dynamic sales.

We first discover that the degree of risk aversion of the salesperson, relative to the noisiness of the sales response function, plays an important role in determining the effort strategy of the salesperson and the optimal contract in the presence of carryover effects. This insight manifests because the carryover effect increases both the mean *and* the variance of future sales.

As a result, we find that the shape of the optimal compensation plan is convex in sales for a low risk-aversion salesperson and concave in sales for a high risk-aversion salesperson, which answers our first research question. This finding is significant because whereas there is evidence of the use of concave compensation plans (e.g., Mantrala et al. 1997), the literature has examined mainly convex and occasionally linear plans as optimal. We show that both forms are obtained

Table 3 Contributions to the Literature

Results	Comparison to literature
1. The carryover effect increases both the mean and the variance of sales.	Prior findings have not considered the role of carryover in the volatility of sales.
2. Low risk-aversion salespeople will prefer increased carryover and reduced sales attrition. High risk-aversion salespeople will prefer the opposite and focus on new sales generation while reducing uncertainty.	No study investigates the dynamic allocation of selling efforts by a salesperson when sales carryover effects exist.
3. The optimal compensation plan is convex for low risk-aversion salespeople and concave for high risk-aversion salespeople.	Theory mainly supports convex plans, but concave plans exist in practice (e.g., Mantrala et al. 1997). We obtain optimality of both forms from the same model.
4. Advertising does not qualitatively affect the form of the plan.	Previous models of multiple promotional activities do not consider the agency relationship and the issue of compensation design.
5. Optimal promotion budgets increase in sales for low risk-aversion sales forces and decrease in sales for high risk-aversion sales forces.	Heuristics such as the percentage of sales method lead to ad budgets being proportional to sales (Farris et al. 1998). Yet, other papers, e.g., Prasad and Sethi (2005), find that the inverse allocation rule is optimal. We reconcile these previous results.

from the same model thereby providing a resolution for the discrepancy between theory and practice.

Proposition 1 then reveals that the optimal effort strategy of the salesperson is driven by allocation considerations. According to this result, a low risk-aversion salesperson, who cares more about mean sales and (relatively) less about variance, will work to increase carryover and reduce sales attrition. On the contrary, a high risk-aversion salesperson will care more about variance and focus on new sales generation. Thus, Proposition 1 answers our second research question. Proposition 1 also shows the quadratic form of the firm's value function including the dependence on the carryover effect.

Moreover, we consider the inclusion of advertising budgeting considerations on the design of the compensation plan. Results show that whereas the coefficients of the optimal contract are different (see the appendix), the qualitative nature is unchanged, i.e., the optimal contract is convex for the low risk-aversion salesperson and concave otherwise. The analysis of the advertising and sales compensation budgets of the firm, and their ratio in the total promotional budget, yield some interesting results. In particular, we show that allocating more resources to compensating the sales force does not necessarily translate in increasing the share of personal selling in the total promotional effort of the firm. Thus, Proposition 4 addresses our third research question. Table 3 provides a summary of the main results and comparisons with the literature.

We close by explaining how our findings inform practice. The proposed model relies on a dynamic sales response model that is extensively used in empirical research in marketing. To implement our findings, managers should first jointly estimate the sales dynamics and the unobserved effort strategies provided in Proposition 1, via a Kalman filter, for example. On recovering the response parameters, a sales manager or consultancy like ZS Associates, could then use the estimated parameters to design the appropriate compensation plan based on the results provided by Proposition 2. Finally, a chief marketing officer can use our results in §6 to strategically allocate resources between advertising and sales taking into account the agency relationship.

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Appendix

Proof of Proposition 1. The dynamic optimal control problem faced by the salesperson is to determine the effort strategy, $v(t)$, that maximizes long-term utility

$$J(x) = \mathbb{E}\left[-\exp\left(-\theta\int_0^\infty e^{-rt}\{S(x)-C(v(t))\}\,dt\right)\right], \quad (12)$$

$$\text{s.t. } \frac{dx(t)}{dt} = v(t)-(1-\lambda)x(t)+\sqrt{\sigma}\epsilon(t), \quad x(0)=x_0. \quad (13)$$

The value function is $V(x)=J(x)$ s.t. (13)

The optimal control problem defined by (12)–(13) is of the risk-sensitive variety (see, e.g., Jacobson 1973, Whittle 1986) cast in continuous time. The salesperson's optimal selling effort is obtained through the HJB equation

$$rV(x) = \max_v\{S(x)-C(v)+V_x(v-\delta x) + \sigma(V_{xx}-\theta V_x^2)/2\}, \quad (14)$$

where $\delta = 1-\lambda$ (see, e.g., Bensoussan and Elliott 1995). After replacing $C(v)$ by $v^2/2$, we differentiate (14) with respect to v and equate the resulting expression to zero to obtain the necessary condition

$$v^* = V_x. \quad (15)$$

Equation (15) is the salesperson's IC condition. Next we replace (15) in (14) to find that the value function equals

$$V(x) = \frac{1}{r}\big(S(x)+V_x^2(1-\theta\sigma)/2 - V_x\delta x + \sigma V_{xx}/2\big). \quad (16)$$

From (16), the IR condition of the salesperson is

$$\big(S(x)+V_x^2(1-\theta\sigma)/2 - V_x\delta x + \sigma V_{xx}/2\big)/r \ge R, \quad (17)$$

where R is the value of the outside option.

The risk-neutral firm determines the compensation plan, $S(x)$, to maximize the long-term profit $\mathbb{E}[\int_0^\infty e^{-rt}\{x-S(x)\}\,dt]$, subject to sales dynamics, the IC, and the IR conditions, i.e., (12), (15), and (17), respectively.

Normalizing R to zero without loss of generality and assuming that (17) binds, we solve (17) as a quadratic in V_x (and hence in v^* according to (15)), or linear if $1-\theta\sigma=0$, to obtain

$$v^*=\begin{cases} v_1^*=\dfrac{x\delta+\sqrt{x^2\delta^2-(2S(x)+\sigma V_{xx})(1-\theta\sigma)}}{1-\theta\sigma}, & \theta<\dfrac{1}{\sigma},\\ \dfrac{1}{\delta x}\left(S(x)+\dfrac{\sigma V_{xx}}{2}\right), & \theta=\dfrac{1}{\sigma},\\ v_2^*=\dfrac{x\delta-\sqrt{x^2\delta^2-(2S(x)+\sigma V_{xx})(1-\theta\sigma)}}{1-\theta\sigma}, & \theta>\dfrac{1}{\sigma}.\end{cases}$$

The value function of the firm is characterized by the HJB equation

$$r\Pi(x)=\max_{S(x)}\{x-S(x)+\Pi_x(v^*-\delta x)+\sigma\Pi_{xx}/2\}. \quad (18)$$

Differentiating (18) with respect to $S(x)$ yields the necessary condition

$$S(x)^*=\frac{x^2\delta^2-\Pi_x^2}{2(1-\theta\sigma)}-\sigma\frac{V_{xx}}{2}. \quad (19)$$

Replacing $S(x)^*$ in v_1^* and v_2^* yields

$$v_1^*=\frac{\Pi_x}{1-\theta\sigma}+\frac{x\delta}{1-\theta\sigma} \quad\text{and}\quad v_2^*=-\frac{\Pi_x}{1-\theta\sigma}+\frac{x\delta}{1-\theta\sigma}. \quad (20)$$

From (20) we obtain $V_{xx}=(\Pi_{xx}+\delta)/(1-\theta\sigma)$ when $\theta<1/\sigma$ and $V_{xx}=(-\Pi_{xx}+\delta)/(1-\theta\sigma)$ when $\theta>1/\sigma$.

We verify the second-order condition that the HJB equation is concave with respect to $S(x)$ at the equilibrium. Specifically, we find that the second derivative of the HJB with respect to $S(x)$ equals $(1-\theta\sigma)/\Pi_x^2$ when $\theta>1/\sigma$ and $-(1-\theta\sigma)/\Pi_x^2$ when $\theta<1/\sigma$, which confirms that (19) is the optimal contract. When $\theta=1/\sigma$, there is no optimal contract.

Finally, we replace (19) and (20) in (18) and apply the method of undetermined coefficients to solve the resulting partial differential equation. Conjecturing that the value function of the firm equals $\Pi_1(x)=(A_2/2)x^2+A_1x+A_0$ when $\theta<1/\sigma$ and $\Pi_2(x)=(B_2/2)x^2+B_1x+B_0$ when $\theta>1/\sigma$, we find that the globally asymptotic equilibria are characterized by

$$A_2=\frac{(r-(r+2\delta)\sigma\theta-\sqrt{12\delta^2+(r-(r+2\delta)\theta\sigma)^2})}{6},$$

$$A_1=-\frac{1-\theta\sigma}{3A_2-r+\theta\sigma(r+\delta)}, \quad\text{and}$$

$$A_0=\frac{3A_1^2+\delta\sigma+A_2\sigma(2-\theta\sigma)}{2r(1-\theta\sigma)},$$

when $\theta<1/\sigma$ and

$$B_2=\frac{(-r+(r+2\delta)\sigma\theta-\sqrt{-4\delta^2+(r-(r+2\delta)\theta\sigma)^2})}{2},$$

$$B_1=\frac{1-\theta\sigma}{B_2+r-\theta\sigma(r+\delta)}, \quad\text{and}\quad B_0=-\frac{B_1^2-\delta\sigma+B_2\theta\sigma^2}{2r(1-\theta\sigma)}$$

when $\theta>1/\sigma$.

Corollary 1. *Equilibrium sales, $x(t)^*$, are normally distributed, with mean and variance as follows. For a low risk-aversion salesperson*

$$\mathbb{E}[x(t)^*]=-\frac{A_1}{A_2+\delta\theta\sigma}>0, \quad\textit{and}$$

$$\mathrm{Var}[x(t)^*]=\frac{\sigma(\theta\sigma-1)}{2(A_2+\delta\theta\sigma)}>0.$$

For a high risk-aversion salesperson

$$\mathbb{E}[x(t)^*]=\frac{B_1}{-B_2+\delta\theta\sigma}>0, \quad\textit{and}$$

$$\mathrm{Var}[x(t)^*]=\frac{\sigma(1-\theta\sigma)}{2(B_2-\delta\theta\sigma)}>0. \quad\square$$

Proof of Proposition 2. To obtain the mathematical expression of the optimal contract depending on the level of risk aversion, we replace $\Pi_x=A_2x+A_1$ and $V_{xx}=(A_2+\delta)/(1-\theta\sigma)$ in (19) when $\theta<1/\sigma$, and $\Pi_x=B_2x+B_1$ and $V_{xx}=(-B_2+\delta)/(1-\theta\sigma)$ in (19) when $\theta>1/\sigma$, respectively. We find that when $\theta<1/\sigma$, the optimal contract is $S(x)^*=\max\{0;\,a_0+a_1x+a_2x^2\}$, with

$$a_0=-\frac{\sigma(\delta+A_2)+A_1^2}{2(1-\theta\sigma)}<0, \quad a_1=-\frac{A_1A_2}{(1-\theta\sigma)}>0, \quad\text{and}$$

$$a_2=-\frac{(A_2-\delta)(A_2+\delta)}{2(1-\theta\sigma)}>0,$$

and $S(x)^*=b_0+b_1x+b_2x^2$, when $\theta>1/\sigma$ with

$$b_0=\frac{\sigma(\delta-B_2)+B_1^2}{2(\theta\sigma-1)}>0, \quad b_1=\frac{B_1B_2}{\theta\sigma-1}>0, \quad\text{and}$$

$$b_2=\frac{(B_2-\delta)(B_2+\delta)}{2(\theta\sigma-1)}<0.$$

Suboptimality of Omitting the Long-Term Value of a Marginal Sale. Ignoring the contribution of a marginal sale to the firm's value function yields with the following HJB equation:

$$r\Pi(x)=\max_{S(x)}\{x-S(x)+\sigma\Pi_{xx}/2\},$$

where the agent's IC and IR remain the same as in the proof for Proposition 2. Specifically, (16) can be rewritten as $S(x)=V_x\delta x-(\sigma V_{xx}+V_x^2(1-\theta\sigma))/2$. Thus the HJB equation can be rewritten as

$$r\Pi(x)=\max_{V_x}\{x-(V_x\delta x-(\sigma V_{xx}+V_x^2(1-\theta\sigma))/2)+\sigma\Pi_{xx}/2\}.$$

Differentiating this expression with respect to V_x and equating the resulting expression to zero yields the optimal effort strategy $v^*=\max\{0;\,x\delta/(1-\theta\sigma)\}$. The resulting optimal compensation plan is $S(x)^*=\max\{0;\,\delta(\delta x^2-\sigma)/2(1-\theta\sigma)\}$. Thus, we note that (i) the agent does not work and does not receive any compensation if her level of risk aversion is such that $\theta>1/\sigma$ and that (ii) the optimal contract remains convex with a quota. Thus, when $\theta>1\sigma$ the agent does not work and both sales and profit converge to zero, whereas this is not the case in Proposition 2 as sales and profit do not converge to zero. Conversely, when $\theta<1/\sigma$, the firm's value function in equilibrium equals $-\delta^2/((2r(1-\theta\sigma)))x^2 + x/r + \delta(r-\delta)\sigma/(2r^2(1-\theta\sigma))$, which has to be compared to $(A_2/2)x^2 + A_1 x + A_0$ where

$$A_2 = \frac{r-(r+2\delta)\sigma\theta - \sqrt{12\delta^2 + (2-(r+2\delta)\theta\sigma)^2}}{6},$$
$$A_1 = -\frac{1-\theta\sigma}{3A_2 - r + \theta\sigma(r+\delta)}, \quad \text{and}$$
$$A_0 = \frac{3A_1^2 + \delta\sigma + A_2\sigma(2-\theta\sigma)}{2r(1-\theta\sigma)}$$

(see proof of Proposition 2).

Dynamics of compensation plans. To inform how managers could use our findings to dynamically adjust quotas or sales targets, depending on the agent's level of risk aversion, we derive $\mathbb{E}[dS/dt]$ by applying Ito's lemma and use discrete time approximations to obtain $S(t)^*$ as a function of $x(t)$ and $x(t-1)$. We find that when the salesperson has low risk aversion, the optimal contract at time t is

$$S(t)^* = \begin{cases} 0, & x(t) < q[t \mid x(t-1)], \\ f[x(t) \mid x(t-1)], & x(t) \geq q[t \mid x(t-1)], \end{cases}$$

where $f[\cdot]$ is a convex reward function[7] paid to the salesperson after the threshold $q[t \mid x(t-1)]$ is reached.[8] Conversely, when the salesperson has high risk aversion, we find similarly that

$$S(t)^* = g[x(t) \mid x(t-1)],$$

where $g[\cdot]$ is a concave reward function[9] with an evolving target $(\tau[t \mid x(t-1)] = \frac{1}{2}x(t-1) - b_1/(4b_2))$ that is defined by setting $\partial S(t)/\partial x(t) = 0$.

Proof of Proposition 3.

Part (a). Given the firm's objective function (11), the sales dynamics (9), the N IC and IR conditions, we obtain that the firm's HJB equation is

$$r\Pi(x) = \max_{S(x)} \left\{ x - \sum_{i=1}^{N} S_i(x) + \Pi_x \left(\sum_{i=1}^{N} v_i^* - \delta x \right) + \sigma \Pi_{xx}/2 \right\}, \quad (21)$$

where

$$v^* = \begin{cases} v_1^* = \dfrac{x\delta + \sqrt{x^2\delta^2 + (2S(x) + \theta V_{xx})(1-2N+\theta\sigma)}}{2N-1-\theta\sigma}, & \theta < (2N-1)/\sigma, \\[2ex] \dfrac{1}{\delta x}\left(S(x) + \dfrac{\sigma V_{xx}}{2} \right), & \theta = (2N-1)/\sigma, \\[2ex] v_2^* = \dfrac{x\delta - \sqrt{x^2\delta^2 + (2S(x) + \theta V_{xx})(1-2N+\theta\sigma)}}{2N-1-\theta\sigma}, & \theta > (2N-1)/\sigma. \end{cases}$$

Differentiating (21) with respect to $S(x)$ yields the necessary condition

$$S(x)^* = \frac{x^2\delta^2 - \Pi_x^2}{2N-1-\theta\sigma} - \sigma \frac{V_{xx}}{2}. \quad (22)$$

We verify that the HJB equation is concave with respect to $S(x)$ at the equilibrium for $\theta < (2N-1)/\sigma$ and $\theta > (2N-1)/\sigma$ only. Replacing $S(x)^*$ in v_1^* and v_2^* yields,

$$v_1^* = \frac{\Pi_x}{2N-1-\theta\sigma} + \frac{x\delta}{2N-1-\theta\sigma} \quad \text{and}$$
$$V_{xx} = \frac{\Pi_{xx}}{2N-1-\theta\sigma} + \frac{\delta}{2N-1-\theta\sigma}$$

when

$$\theta < \frac{2N-1}{\sigma} \quad \text{and} \quad v_2^* = -\frac{\Pi_x}{2N-1-\theta\sigma} + \frac{x\delta}{2N-1-\theta\sigma}$$

and

$$V_{xx} = -\frac{\Pi_{xx}}{2N-1-\theta\sigma} + \frac{\delta}{2N-1-\theta\sigma}$$

when $\theta > (2N-1)/\sigma$, respectively. Finally, we replace these equations in (21) and apply the method of undetermined coefficients to solve the resulting partial differential equation. Conjecturing that the value function of the firm equals $\Pi_{N1}(x) = (A_{N2}/2)x^2 + A_{N1}x + A_{N0}$ when $\theta < (2N-1)/\sigma$ and $\Pi_{N2}(x) = (B_{N2}/2)x^2 + B_{N1}x + B_{N0}$ when $\theta > (2N-1)/\sigma$, we find that the globally asymptotic equilibria are characterized by

$$A_2 = \big[r(2N-1+\theta\sigma) + 2\delta(N-1-\theta\sigma) + \sqrt{12N^2\delta^2 + (r(1-2N) + 2\delta(1-2N) + (r+\delta)\theta\sigma)^2} \big] \cdot (6N)^{-1},$$
$$A_1 = \frac{1-2N+\theta\sigma}{3A_{N2}N + (1+\theta\sigma)(r+\delta) - N(2r+\delta)}, \quad \text{and}$$
$$A_0 = \frac{3A_{N1}^2 + \sigma(N\delta + A_{2N}((1+2\sigma)N - \sigma(1+\theta\sigma)))}{2r(2N-1-\theta\sigma)},$$

when $\theta < (2N-1)/\sigma$ and

$$B_{N2} = -\big[2N(r+\delta) - (r+2\delta)(1+\theta\sigma) + \sqrt{(r(1-2N) + 2\delta(1-N) + \theta\sigma(r+2\delta))^2 - 4N^2\delta^2} \big] \cdot (2N)^{-1},$$
$$B_1 = \frac{1-2N+\theta\sigma}{(r+\delta)(1+\theta\sigma) - N(2r+\delta+B_{2N})}, \quad \text{and}$$
$$B_0 = \frac{NB_{N1}^2 + \sigma(A_{2N}(N(1-2\sigma) + \sigma(1-\theta\sigma)) - n\delta)}{2r(1-2N+\theta\sigma)},$$

when $\theta > (2N-1)/\sigma$.

[7] $f[x(t) \mid x(t-1)] = a_0 + a_2(\sigma + x(t-1)^2) + (a_1 - 2a_2 x(t-1))x(t) + 2a_2 x(t)^2$.

[8] $q[t \mid x(t-1)] = (1/(4a_2))\{2a_2 x(t-1) - a_1 + [a_1^2 - 8a_2(a_0 + a_2\sigma) - 4a_2 \cdot x(t-1)(a_1 + a_2 x(t-1))]^{1/2}\}$

[9] $g[x(t) \mid x(t-1)] = b_0 + b_2(\sigma + x(t-1)^2) + (b_1 - 2b_2 x(t-1))x(t) + 2b_2 x(t)^2$.

Part (b). To obtain the mathematical expression of the optimal contract depending on the level of risk aversion, we replace $\Pi_{Nx} = A_{N2}x + A_{N1}$ and $V_{xx} = (A_{2N} + \delta)/(2N - 1 - \theta\sigma)$ in (22) when $\theta < (2N-1)/\sigma$, and $\Pi_{Nx} = B_{N2}x + B_{N1}$ and $V_{xx} = -(B_{N2-\delta})/(2N - 1 - \theta\sigma)$ in (22) when $\theta > (2N-1)/\sigma$, respectively. We find that when $\theta < (2N-1)/\sigma$, the optimal contract is $S(x)^* = \max\{0; a_{N0} + a_{N1}x + a_{N2}x^2\}$, with

$$a_{N0} = \frac{A_{N1}^2 + \sigma(A_{N2} + \delta)}{2(1 - 2N + \theta\sigma)} < 0, \quad a_{N1} = \frac{A_{N1}A_{N2}}{(1 - 2N + \theta\sigma)} > 0,$$

and

$$a_{N2} = -\frac{(A_{N2} - \delta)(A_{N2} + \delta)}{2(1 - 2N + \theta\sigma)} > 0,$$

and $S(x)^* = b_{N0} + b_{N1}x + b_{N2}x^2$, when $\theta > (2N-1)/\sigma$ with

$$b_{N0} = \frac{B_{N1}^2 + \sigma(\delta - A_{N2})}{2(1 - 2N + \theta\sigma)} > 0, \quad b_{N1} = \frac{B_{N1}B_{N2}}{(1 - 2N + \theta\sigma)} > 0,$$

and

$$b_{N2} = \frac{(B_{N2} - \delta)(B_{N2} + \delta)}{2(1 - 2N + \theta\sigma)} < 0.$$

Proof of Proposition 4. From the agent's IC and IR conditions, we obtain that

$$v^* = \begin{cases} v_1^* = \dfrac{x\delta - \gamma u + \sqrt{(\gamma u - \delta x)^2 - (2S(x) + \sigma V_{xx})(1 - \theta\sigma)}}{1 - \theta\sigma}, & \theta < 1/\sigma, \\ \dfrac{1}{\delta x - \gamma u}\left(S(x) + \dfrac{\sigma V_{xx}}{2}\right), & \theta = 1/\sigma, \\ v_2^* = \dfrac{x\delta - \gamma u - \sqrt{(\gamma u - \delta x)^2 - (2S(x) + \sigma V_{xx})(1 - \theta\sigma)}}{1 - \theta\sigma}, & \theta > 1/\sigma. \end{cases}$$

Furthermore, from the firm's objective function $\tilde{\Pi}(x) = \max_{\{S(x), u\}} \mathbb{E}[\int_0^\infty e^{-rt}\{x - S(x) - u^2/2\}\,dt]$, the sales dynamics (12), and the IC and IR condition we obtain that the firm's HJB equation is

$$r\Pi(x) = \max_{\{S(x), u\}} \{x - S(x) - u^2/2 + \Pi_x(v^* + \gamma u - \delta x) + \sigma\Pi_{xx}/2\}. \tag{23}$$

Differentiating (23) with respect to $S(x)$ we obtain that the first-order condition for the optimal compensation strategy is

$$S(x)^* = \frac{(\gamma u - \delta x)^2 - \Pi_x^2}{2(1 - \theta\sigma)} - \sigma\frac{V_{xx}}{2}, \tag{24}$$

which yields that $v_1^* = \Pi_x/(1 - \theta\sigma) + (x\delta - \gamma u)/(1 - \theta\sigma)$ when $\theta < 1/\sigma$ and $v_2^* = -\Pi_x/(1 - \theta\sigma) + (x\delta - \gamma u)/1 - \theta\sigma)$ and when $\theta > 1/\sigma$, respectively. As a result, the first-order condition for optimal advertising is

$$u^* = \gamma\frac{\delta x - \theta\sigma\Pi_x}{1 + \gamma^2 - \theta\sigma}. \tag{25}$$

Finally, we replace the optimal strategies in (23) and apply the method of undetermined coefficients to solve the resulting partial differential equation. Conjecturing that the value function of the firm equals $\tilde{\Pi}(x) = (K_2/2)x^2 + K_1x + K_0$ when $\theta < 1/\sigma$ and $\tilde{\Pi}(x) = (L_2/2)x^2 + L_1x + L_0$ when $\theta > 1/\sigma$, we find that the globally asymptotic equilibria are characterized by

$$K_2 = \frac{2\delta^2(\theta\sigma - 1)}{(\theta\sigma - 1)((r + 2\delta)\theta\sigma - r(1 + \gamma^2)) + \sqrt{(1 + \gamma^2 - \theta\sigma)(\theta\sigma - 1)\Delta}},$$

$$K_1 = \left[r - K_2\gamma^2 + \delta + \frac{(1 + \gamma^2)(K_2(1 + \gamma^2) - \delta)}{1 + \gamma^2 - \theta\sigma} + 4\frac{K_2}{\theta\sigma - 1}\right]^{-1},$$

and

$$K_0 = -\sigma\big[\delta(1 - \theta\sigma) + K_2\big(2(1 + \gamma^2) + \sigma\theta(\sigma\theta - 3)\big) + K_1^2\big(3(1 - \theta\sigma) + \gamma^2(3 + \theta^2\sigma^2)\big)\big] \cdot \big[(1 + \gamma^2 - \theta\sigma)(\theta\sigma - 1)\big]^{-1},$$

with

$$\Delta = \theta r\sigma(4\delta + (\gamma^2 + 2)r) - \theta^2\sigma^2(2\delta + r)^2 - 12\delta^2 - (\gamma^2 + 1)r^2,$$

when $\theta < (2N-1)/\sigma$ and

$$L_2 = -\big[r(1 + \gamma^2) - (r + 2\delta)\sigma\theta + \sqrt{(1 + \gamma^2 - \theta\sigma)(r^2(1 + \gamma^2) - 4\delta^2 - (r + 2\delta)^2\theta\sigma)}\big] \cdot \big[2(1 + (1 + \theta\sigma)\gamma^2)\big]^{-1},$$

$$L_1 = \frac{1 + \gamma^2 - \theta\sigma}{(L_2 + r)(1 + \gamma^2) - (r - L_2\gamma^2 + \delta)\theta\sigma}, \quad \text{and}$$

$$L_0 = -\frac{-\delta\sigma + L_2\theta\sigma^2 + L_1^2(1 + \gamma^2(1 + \theta\sigma))}{2r(1 + \gamma^2 - \theta\sigma)},$$

when $\theta > 1/\sigma$. As a result, we obtain that *For* $\theta < 1/\sigma$,

$$v^* = \left(\frac{2}{1 - \theta\sigma} - \frac{1 + \gamma^2}{1 + \gamma^2 - \theta\sigma}\right)\tilde{\Pi}_{x,1} + \frac{x\delta}{1 + \gamma^2 - \theta\sigma},$$

$$u^* = \max\left\{0; -\frac{\gamma\sigma\theta}{1 + \gamma^2 - \theta\sigma}\tilde{\Pi}_{x,1} + \frac{\gamma x\delta}{1 + \gamma^2 - \theta\sigma}\right\},$$

where $\tilde{\Pi}_{x,1} = K_1 + K_2x$, *and* $K_1 > 0$ *and* $K_2 < 0$ *are constants. Conversely for* $\theta > (1 + \gamma^2)/\sigma$, *we obtain that*

$$v^* = -\frac{1 + \gamma^2}{1 + \gamma^2 - \theta\sigma}\tilde{\Pi}_{x,2} + \frac{x\delta}{1 + \gamma^2 - \theta\sigma},$$

$$u^* = \max\left\{0; -\frac{\gamma\sigma\theta}{1 + \gamma^2 - \theta\sigma}\tilde{\Pi}_{x,2} + \frac{\gamma x\delta}{1 + \gamma^2 - \theta\sigma}\right\},$$

where $\tilde{\Pi}_{x,2} = L_1 + L_2x$, *and* $L_1 > 0$ *and* $L_2 < 0$, are constants. Finally, for $\sigma \in [1/\theta; (1 + \gamma^2)/\theta]$, the expressions for the optimal advertising and effort strategies imply that $u^* = v^* = 0$.

Optimal contract when the firm advertises. Using the results from Proposition 4 and in particular the optimal strategies and (24), we find that the optimal contract for the low risk-aversion agent is $S(x)^* = \max\{0; k_0 + k_1x + k_2x^2\}$ with

$$k_2 = -\frac{(K_2(1 + \gamma^2) - \delta)(K_2(1 + \gamma^2) + \delta - (K_2(1 - \gamma^2) + \delta)\theta\sigma)}{2(1 + \gamma^2 - \theta\sigma)^2} > 0,$$

$$k_1 = K_1\frac{-K_2(1 + \gamma^2)^2 + \theta\sigma(K_2(1 - \gamma^2) + \delta\gamma^2)}{(1 + \gamma^2 - \theta\sigma)^2},$$

$$k_0 = \left[\frac{(K_2(1 - \gamma^2) + \delta)\theta\sigma^2 - \sigma(K_2(1 + \gamma^2) + \delta)}{1 + \gamma^2 - \theta\sigma} + K_1^2\left(\frac{\gamma\theta\sigma}{1 + \gamma^2 - \theta\sigma}\right)^2 - 1\right] \cdot (2(1 - \theta\sigma))^{-1} < 0,$$

and $S(x)^* = l_0 + l_1 x + l_2 x^2$ for the high risk-aversion agent, with

$$l_2 = \frac{\delta^2(1-\theta\sigma) + L_2 2\gamma^2\delta\theta\sigma + L_2^2((1-\gamma^4)\theta\sigma - (1+\gamma^2)^2)}{2(1+\gamma^2-\theta\sigma)^2} < 0,$$

$$l_1 = L_1 \frac{\gamma^2\delta\theta\sigma + L_2((1-\gamma^4)\theta\sigma - (1+\gamma^2)^2)}{(1+\gamma^2-\theta\sigma)^2}, \quad \text{and}$$

$$l_0 = \frac{(1+\gamma^2-\theta\sigma)\sigma(L_2(1+\gamma^2)-\delta) + K_1^2((1-\gamma^4)\theta\sigma - (1+\gamma^2)^2)}{2(1+\gamma^2-\theta\sigma)^2} > 0,$$

i.e., the optimal contract for the low risk-aversion agent is convex in sales with a quota, whereas the optimal contract for the high risk-aversion agent is concave in sales.

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