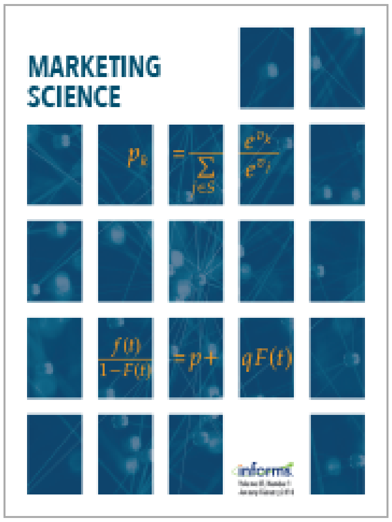

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Allowing Consumers to Bundle Themselves: The Profitability of Family Plans

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Abstract. Telecommunications service is an important and growing market, with worldwide revenue exceeding $2.2 trillion in 2016. In the U.S. market, the total number of mobile wireless connections has grown from 279.6 million in 2008 to 396 million in 2016. All the firms in this market offer consumers an option of purchasing either an individual plan or a family plan. Whereas a menu of individual plans can be thought of as a means to segment the market, the theoretical challenge is to understand how a firm stands to benefit from adding family plans to its product mix. In this paper, we use a game-theoretic framework to explore the role of family plans. Interestingly, we find that even when a family plan does not draw in any new consumers, a firm can still benefit from offering these plans. This occurs primarily because a family plan enables the firm to price discriminate more effectively. In particular, because some consumers can bundle themselves and join a family plan, the firm is able to charge a higher price to single high-valuation consumers who are unable to be part of a family. Furthermore, the presence of a family plan can have a negative impact on the plan offered to single low-valuation consumers who now have to pay a higher overage price. We also show that not all family plans are profitable and that the profitability depends on the sizes of different types of families.

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1. Introduction

Telecommunications service is an important and growing market with worldwide revenue projected to go from $2.2 trillion in 2015 to $2.4 trillion in 2019.[1] This growth is linked strongly to the adoption of smartphones—in the United States, smartphone penetration has grown from 2% in 2005 to 81% in 2016.[2] Over time, as the types and features of mobile devices and the market have grown, so too have the pricing plans offered to cellular subscribers. For example, as consumer adoption increased, firms moved away from pay-per-use plans to three-part plans that offered a bucket of minutes. In the typical three-part plan, a subscriber pays a fixed monthly fee that comes with a specific allowance level; if subscribers exceed the allowance, they pay an "overage" fee for this additional consumption. An important pricing innovation in this industry came in October 1999 when Bell Atlantic introduced its "Share-A-Minute" plan that allowed family members to share monthly airtime (Federal Communications Commission 2000). This sharing of minutes came to be known as a "family plan," and soon other firms began offering their own versions. Although three-part plans have been studied extensively in the literature, there has been significantly less attention paid to how a firm can benefit from offering family plans in addition to individual plans. In this paper, we offer a novel and heretofore unexplored rationale for family plans. We show that introducing an appropriately designed family plan changes the product line in a way that benefits the firm, because it is able to extract more surplus from consumers who are not part of a family. Importantly, a family plan allows some consumers to bundle themselves and buy a unique product that is not attractive to individual consumers.

The problem we pose specifically applies to situations in which consumers' needs for service vary over time, with the result that there always exists some uncertainty about the level of services that will be needed in any given period. For example, one may know that one needs a smartphone but still be uncertain about how many gigabytes of data will be needed over the course of a month. Moreover, the needs may vary from month to month, which makes estimating usage even more complicated. Knowing that consumers will face these and other types of uncertainties, firms need to devise contracts that anticipate consumers' expected usage and yet give some leeway for unanticipated consumption. In the market for wireless services, firms offer contracts in which they pay a single fee for a block of service and if consumers' usage exceeds the predetermined allotment, they pay an additional per unit fee—referred to broadly as an overage price—for the additional units of service they consume. For example, in 2017, Verizon offered a plan with 4 GB of data for $50 per month and an overage price of $15 per GB. A family plan follows essentially the same structure of the three-part price described above, but it allows multiple subscribers to share a single block of service. Although one may intuit that this approach would in some cases reduce consumers' level of uncertainty, this result is not enough to explain why a family plan makes sense from the perspective of the firm. In this paper, we show that, under certain situations, the introduction of family plans can reduce the cannibalization problem in the firm's product line design and the firm can maximize its profits by offering both individual and family plans.

We develop a model in which a firm offers a menu of three-part plans and consumers select one of these plans. Although consumers are uncertain about their consumption needs (e.g., they are uncertain about how many gigabytes of data they will need in the next period), they have to purchase the plan before they discover their needs for that period. In terms of consumers, we assume there are two segments that differ in their valuation of a unit of service—one has a high valuation and the other a low valuation. We capture uncertainty of consumption needs by assuming that all consumers can have either a "high" or a "low" consumption need. On the other hand, each segment has a specific probability of being either in the high or the low state. Within this framework, we determine the optimal plans that the firm designs for the high- and low-valuation segments.

We then extend our basic model by allowing the firm to introduce a family plan in which two consumers pay a single price and share a joint allowance. Defining a family as a couple, we consider three types of families that can coexist in the market: two high-valuation consumers, one high- and one low-valuation consumer, and two low-valuation consumers. Within this setup, we analyze how the presence of family plans affects equilibrium prices and profits. We find that compared with offering two individual plans, under certain conditions, the firm is able to increase its profits by offering a full product line consisting of two individual and three family plans. It is important to note that this result does not rely on any market expansion effects; that is, the family plans we study do not increase the number of consumers in the market. We find that the firm is better off because the family plans allow the firm to extract additional surplus from some family types. Furthermore, the family plans can reduce the firm's cannibalization problem among single consumers who are not part of a family.

This paper is at the intersection of the literatures on vertical differentiation and the growing literature on three-part pricing in telecommunications services. The vertical differentiation literature, beginning with Mussa and Rosen (1978) and Cooper (1984), has examined segmentation questions by explicitly modeling consumer self-selection. Much of this research extends the basic paradigm by embellishing the standard product design model, for example, by considering sequential introduction of new products (Moorthy and Png 1992, Dhebar 1994, Kornish 2001), bringing in competition (Desai 2001), implementing information provision policies (Mayzlin and Shin 2011), allowing for firm's uncertainty about consumer valuations (Biyalogorsky and Koenigsberg 2014), and examining socially responsible consumption (Jiang and Srinivasan 2014, Iyer and Soberman 2016). This literature, because of its focus on quality as the segmentation variable, commonly assumes that each consumer purchases a single unit of the product and does not examine the impact of consumers' uncertainty about their usage needs. However, in most telecommunications markets, consumers' use of the service is affected by a number of exogenous factors. As a result, consumers face uncertainty about their usage needs at the time of purchase. We specifically model consumers' uncertainty about their usage needs and examine how the firm can account for it in its segmentation scheme.

The empirical literature on three-part pricing for telecommunications services considers consumers' uncertainty about their usage. The general thrust of this research has been on understanding and documenting the *seemingly* irrational manner in which consumers react to nonlinear prices in the markets for telecommunications service. For example, by switching from a two-part to a three-part tariff, consumers use significantly less than the allowance they have purchased (Ascarza et al. 2012). This phenomenon, often referred to as a flat-rate bias, is interesting in that consumers would be better off in simple financial terms with a per-minute payment plan (i.e., the two-part tariff), but they pay more for the three-part plan and leave a significant number of minutes unused. The argument for why this occurs is that even if the payments are higher, consumers still prefer the certainty of payments under a three-part tariff; thus, a preference for three-part plans could also be explained by consumers' risk aversion (Herweg and Mierendorff 2013). Looking at this from the firm's perspective, if consumers are overconfident and have biased beliefs about their usage distributions (in particular, about the variance of their usage), the firm can exploit the situation by using a three-part plan (Grubb 2009, 2012). Similarly, the firm can exploit consumers' uncertainty about usage through a three-part tariff (Lambrecht et al. 2007, Jiang 2012, Fibich et al. 2018), but the profitability also depends on the cost of implementing three-part tariffs (Sundararajan 2004, Bagh and Bhargava 2013). On the other hand, plans such as these increase the likelihood of consumers switching to another service provider (Iyengar et al. 2011) and decrease consumers' overall satisfaction with the service (Gopalakrishnan et al. 2015). Our paper incorporates three-part prices and consumers' usage uncertainty validated by empirical work in this literature but differs from the prior work by focusing specifically on the firm's segmentation problem.

Motivated by family plans widely used in the telecommunications market, we allow some consumers to bundle themselves into a family. As a result, we have a novel result that, in equilibrium, consumers in the same segment can end up choosing different plans and consuming different quantities. Furthermore, by offering family plans, the firm is able to extract additional surplus from a group of consumers who are not members of a family. Importantly, we show how and when the firm can benefit from offering both individual and family plans. We know from the vertical differentiation literature that sometimes the optimal strategy involves the firm not serving some of the lower-valuation consumers or offering a common product to multiple consumer segments (e.g., Maskin and Riley 1984, Armstrong 1996). By contrast, with a family plan strategy, the firm can serve a given consumer segment with multiple different products. Thus, the addition of the family plan takes us in the opposite direction from traditional solutions.

Our paper is also related to the rich literature on bundling. The traditional argument for why bundling products can be optimal is that it makes price discrimination strategies more powerful by reducing the role of unpredictable and idiosyncratic components of valuations (see, e.g., Adams and Yellen 1976, Kolay and Shaffer 2003, Banciu et al. 2010, Bhargava 2013).[3] In our model, a family plan can be thought of as a bundle of consumers purchasing a unique product rather than the standard bundling scenario where a consumer purchases a bundle of different products. Importantly, the product that is purchased by the family is not a combination or weighted average of two individual products, and it cannot be purchased by consumers who are not part of a family. In this framework, we show that bundling consumers can be optimal because it allows the firm to offer them a unique product that is not available to individual buyers.

The rest of this paper is organized as follows. Section 2 presents the model, and Section 3 presents the optimal menu of individual plans. Section 4 analyzes the optimal menu of individual and family plans and compares the product line strategies available to the firm. Section 5 discusses several extensions of the main model, and Section 6 concludes.

2. Model

In this section, we lay out the assumptions related to the firm, consumers, and the structure of the game between them. Our overarching goal is to model the telecommunications industry, and this leads us to make specific assumptions about the firm and how consumers respond to the plans it offers. Other industries, such as gym memberships and residential broadband services, may not be characterized by exactly the same set of assumptions, but there are enough similarities that our general results can still provide useful insights. Note that the notation we use is summarized in Table 1. We begin with our assumptions about the firm.

2.1. Firm

A monopoly firm provides telecommunications service in the form of units of wireless data (the same applies to minutes of airtime) to consumers. The firm knows the distribution of different types of consumers (in terms of their valuations of data and their requirements for wireless data) but it cannot directly identify each individual's type or data requirement. We assume that the firm's marginal cost of providing a unit of data are zero.[4]

Table 1. Summary of Notation

Symbol	Definition
$i, -i$	Consumer type: $i, -i \in \{H, L\}$
H	High-valuation consumer
L	Low-valuation consumer
F	Family of two consumers, $F \in \{HH, HL, LL\}$
θ_i	Consumer i's valuation parameter
j	Plan targeted to segment $j \in \{H, L, F\}$
q_j	Allowance of plan j
p_j	Price of plan j
p_{Oj}	Overage price of plan j
λ	The size of the L-type consumer segment
f_F	The size of the family segment F, $F \in \{HH, HL, LL\}$
X	Consumer's need state, $X \in \{a, b\}$
X_F	A family's need state, $X_F \in \{aa, ab, ba, bb\}$
a	Low need state
b	High need state
α_H	Probability of an H-type consumer to be in the low state, $X = a$
α_L	Probability of an L-type consumer to be in the low state, $0 < \alpha_H < \alpha_L < 1$
α_F	Probability of an F-type family to have sufficient allowance
Q_{ijX}	Type i consumer's consumption level when choosing plan j with need state of X
$O_{ij} = Q_{ijX} - q_j$	Overage consumption when choosing plan j if $Q_{ijX} > q_j$
Q_{iF}	Type i family member's consumption level when choosing family plan F
*	Equilibrium outcomes when the firm offers only individual plans
**	Equilibrium outcomes when the firm offers individual and family plans

The firm offers consumers a menu of individual plans from which to choose. Each plan j is a three-part tariff, denoted by (q_j, p_j, p_{Oj}), where the three parts are as follows: (1) q_j is the data allowance of plan j, (2) p_j is the price consumers pay for q_j units of data regardless of their actual consumption, and (3) if consumers use additional data that exceeds the plan's allowance, q_j, they pay a per-unit price of p_{Oj}. We refer to any usage that exceeds the plan's allowance as "overage" for which consumers pay an overage fee of p_{Oj} per unit of overage consumption. As long as consumers use $q \le q_j$ units of data, they do not pay any overage fee.

In addition to the individual plans, the firm may decide to offer a full menu of family plans, where each family comprises two customers. The contract for the family (F) is given by (q_F, p_F, p_{OF}) and is similar to the three-part contract for individuals outlined above. The main difference is that in a family plan, as long as the *sum* of both members' consumption levels stays within the plan's allowance q_F, they do not incur any overage charges. On the other hand, if their total consumption exceeds the family plan's allowance, then they pay the per-unit fee associated with their overage consumption.

2.2. Consumers

We assume that there are two consumer segments, high (H) and low (L), which differ in their valuations for telecommunications service. In particular, each segment has a valuation parameter θ_i, where $i \in \{H, L\}$ and $\theta_H \ge \theta_L$, such that for a given level of consumption, an H-type consumer has a higher marginal utility than an L-type consumer. These segments capture consumers' heterogeneity in terms of their willingness to pay for data services; for example, busy executives have higher valuations for each unit than do college students. This assumption is consistent with the setup in Nevo et al. (2016), which analyzes the market for residential broadband services. The sizes of the L and H segments are given by λ and $(1-\lambda)$, respectively.

In addition to differences in per-unit valuations, consumers are uncertain about how much data they will need in any given period. This uncertainty arises from the stochastic nature of consumption in the telecommunications market; for example, one may not know how many important calls, e-mails, or videos one will consume in a particular period. To capture the uncertainty about how much data will be needed, we assume that a consumer's maximum usage of data is represented by X, where X is either at a low ($X = a$) or a high ($X = b$) level, such that $0 < a < b$.[5] Throughout this paper, we use the terms "usage need" and "consumption need" interchangeably. In both instances, they reflect a consumer's need state of X, which is the maximum amount of data that this consumer would like to use in the period. Even though all consumers are uncertain about their maximum usage in a particular period, on average, H-type consumers expect to require more data than L-type consumers (e.g., Narayanan et al. 2007, Gopalakrishnan et al. 2015). Thus, we assume that the L segment has an α_L probability of being in the low state (a), whereas the H segment has an α_H probability of being in the low state where $0 < \alpha_H < \alpha_L < 1$. Conversely, the probability of being in the high state (b) is greater for the H segment, $(1-\alpha_H) > (1-\alpha_L)$, and H-segment consumers expect to have a higher level of data needs than L-segment consumers (i.e., $\alpha_H a + (1-\alpha_H)b > \alpha_L a + (1-\alpha_L)b$). Table 2 summarizes the different need states and the associated probabilities for each segment.

Consumers derive utility from consumption of data (or equivalently, content), and the gross utility from this consumption is given by[6]

$$u(\theta_i; X, Q_i) = \theta_i\left(Q_i - \frac{Q_i^2}{2X}\right), \tag{1}$$

where Q_i is consumer i's ($i \in \{H, L\}$) actual consumption level. Recall that X captures the need state, which can either be low (a) or high (b). Note that the utility formulation in (1) has some important properties. First, it is clear that marginal utility decreases with consumption, $\frac{\partial u(.)}{\partial Q_i} \ge 0$ (for $Q_i \le X$), and $\frac{\partial^2 u(.)}{\partial Q_i^2} < 0$. Second, the satiation point for consumption is determined by the realized need state, $X \in \{a, b\}$. Thus, the formulation incorporates the consumers' need state as the satiation point in a logically consistent manner. If the realized need state is low, then consumers will satiate at a; otherwise, they will satiate at b. This assumption is consistent with the observation that consumers who have unlimited plans still end up consuming a finite amount of content (Nevo et al. 2016). This specification of gross utility also means that, given the same level of consumption, when the need state is high, the marginal utility is higher than when the need state is low. The gross utilities associated with each need state are graphically depicted in Figure 1. Finally, note that usage at the level of the need state either a or b (i.e., $Q_{iX} = X$) yields the maximum gross utility of $\frac{\theta_i a}{2}$ or $\frac{\theta_i b}{2}$, respectively.[7]

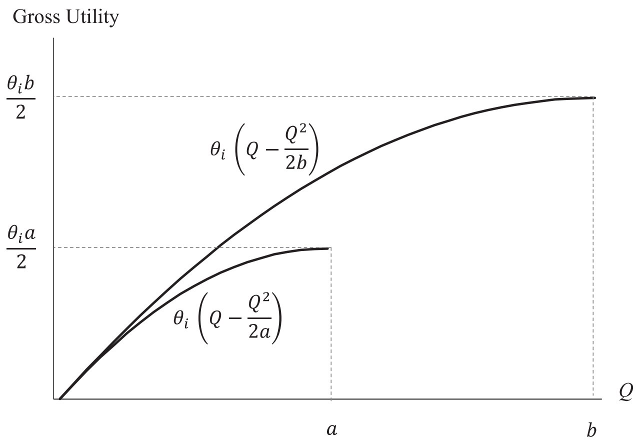
2.3. Consumer's Optimization Problem for an Individual Plan

Consistent with the pattern in the telecommunications market, we model the sequence of events as the following three stages: In the first stage, the firm announces

Table 2. Probabilities and Need States by Segment

	Usage or need state, X	
	a	***b***
L segment	α_L	$(1-\alpha_L)$
H segment	α_H	$(1-\alpha_H)$

Figure 1. Gross Utility of Consumption in Need States ***a*** and ***b***



the menu of plans. Next, on the basis of their expected utilities from each plan, consumers choose their plan and pay the price. In the third stage, consumers discover their usage needs and then decide how much to consume and, if needed, make the overage payment. It follows that a consumer's ultimate consumption choice will depend on the chosen plan as well as the revealed need state. We denote consumer i's consumption level under plan j when the need state is X by Q_{ijX}.

Below, we focus on the case where the firm offers only individual plans. Consumers choose a plan before they know their future need state, but they know the probability associated with each state. For a given plan (q_j, p_j, p_{Oj}), their expected utility will depend not only on the plan specifics but also on the amount of plan allowance, q_j, relative to the need states, a and b. Specifically, consumer i's expected utility of selecting plan (q_j, p_j, p_{Oj}) is as follows:

$$U(\theta_i; q_j, p_j, p_{Oj}) = \tag{2}$$

$$\begin{cases} \alpha_i\left(\theta_i \dfrac{a}{2} - p_j\right) + (1-\alpha_i)\left(\theta_i \dfrac{b}{2} - p_j\right), \quad \text{if } q_j \ge b; & (2.1) \\[2ex] \alpha_i\left(\theta_i \dfrac{a}{2} - p_j\right) + (1-\alpha_i)\left(\theta_i\left(Q_{ijb} - \dfrac{Q_{ijb}^2}{2b}\right) \right. \\ \qquad \left. - p_j - p_{Oj}(Q_{ijb} - q_j)\right), \quad \text{if } b > q_j \ge a; & (2.2) \\[2ex] \alpha_i\left(\theta_i\left(Q_{ija} - \dfrac{Q_{ija}^2}{2a}\right) - p_j - p_{Oj}(Q_{ija} - q_j)\right) \\ \qquad + (1-\alpha_i)\left(\theta_i\left(Q_{ijb} - \dfrac{Q_{ijb}^2}{2b}\right) - p_j \right. \\ \qquad \left. - p_{Oj}(Q_{ijb} - q_j)\right), \quad \text{if } q_j < a. & (2.3) \end{cases}$$

As can be seen above, the levels of expected utility depend on the plan allowance q_j relative to the need states, a and b. We discuss each of the levels below:

1. If $q_j \ge b$, then in both need states, consumers always have an adequate plan allowance, and their needs will be satisfied. Thus, $Q_{ija} = a$ and $Q_{ijb} = b$, and the gross utilities associated with these consumption levels are derived from Equation (1), yielding $\frac{\theta_i a}{2}$ and $\frac{\theta_i b}{2}$, respectively.
2. If $b > q_j \ge a$, then consumers who get $X = a$ will have a plan allowance that is sufficiently high to meet their needs. They simply consume $Q_{ija} = a$ and receive a gross utility of $\frac{\theta_i a}{2}$. This occurs with probability α_i and is given by the first part of Equation (2.2). However, consumers who get $X = b$ will have a plan allowance that is not adequate for their need state. In this case, they have to choose a consumption level $Q_{ijb} \ge q_j$ and, if necessary, pay an overage fee of p_{Oj} per unit of overage consumption. We elaborate on the consumption level Q_{ijb} in further details at the end of this section.
3. If $q_j < a$, then the plan allowance is insufficient for both need states a and b, and depending on the consumption levels chosen, the consumer may also have to pay overage fees. In this case, depending on the need state, the consumer chooses a consumption level of Q_{ija} or Q_{ijb}.

Having specified the net expected utility, we now determine the optimal consumption levels. In particular, once consumers choose plan j and then observe their need state of $X \in \{a, b\}$, they maximize utility by choosing their optimal consumption level. This optimization yields[8]

$$Q_{ijX} = \begin{cases} X, & X \le q_j; \\ \max\left\{q_j, X\left(1 - \dfrac{p_{Oj}}{\theta_i}\right)\right\}, & X > q_j. \end{cases} \tag{3}$$

As should be clear, if the consumers' need state is below the plan allowance, $X \le q_j$, they merely consume up to their need state. On the other hand, if $X > q_j$, then the optimal consumption is endogenously given by $\max\left\{q_j, X\left(1 - \frac{p_{Oj}}{\theta_i}\right)\right\}$. Note that the difference $O_{ij} = Q_{ij} - q_j \ge 0$ represents overage consumption, and it is easy to see that consumption Q_{ij} increases with valuation θ_i and decreases in the plan's overage price p_{Oj}: $\frac{\partial Q_{ij}}{\partial \theta_i} \ge 0$, $\frac{\partial Q_{ij}}{\partial p_{Oj}} \le 0$. If the overage price is sufficiently high, then consumers will not use any overage and simply consume their plan allowance of q_j. However, if overage consumption is positive, then the optimal level is scaled by the need state such that, holding all else equal, optimal consumption is higher when the need is higher. These comparative statics intuitively capture the impact of key factors that shape consumers' optimal overage consumption levels.

In summary, there are three key elements in our framework: (i) uncertainty about consumption needs, (ii) differences between consumer segments in terms of their valuations and their distribution of need states, and (iii) three-part tariffs. We also emphasize that these elements are consistent with the institutional intricacies of the telecommunications market and with previous research in this area. Next, we take into account how consumers choose an optimal level of overage consumption and analyze the firm's optimal strategy.

3. Individual Plans

In this section, we analyze the case in which the firm maximizes profits by offering two individual plans to consumers. In particular, the firm chooses two sets of allowances, prices, and overage fees in two individual plans: (q_H, p_H, p_{OH}) and (q_L, p_L, p_{OL}). The H plan is targeted to the high-valuation consumers and the L plan to the low-valuation consumers. After observing the two plans, consumers self-select the best plans for themselves. As is well known from the classic literature on self-selection, the firm must ensure that the H segment will not choose the L plan targeted at the L segment, and vice versa. In addition, the firm must ensure that both segments of consumers obtain nonnegative utilities from purchasing the plans targeted at them.

As can be surmised from Equations (1) and (3), the firm's expected profit is affected by the values of q_H and q_L, the need states a and b, and each segment's potential overage consumption. Consequently, we need to analyze several subcases, which are described in Online Appendix B.[9] In the main text, we focus on the case where the interior solution is such that $q_H > q_L \geq a$. This solution covers the lower need state and yet allows for overage consumption. Also, it represents the equilibrium outcome under a wide range of parameter values and is also the one that most closely resembles data plans offered by firms. All the other cases are detailed in Online Appendix B.

In the case under consideration, the firm's expected profit is given by

$$\Pi = (1-\lambda)[p_H + p_{OH}(1-\alpha_H)(Q_{HHb} - q_H)] + \lambda[p_L + p_{OL}(1-\alpha_L)(Q_{LLb} - q_L)]. \tag{4}$$

The first term in this profit function, $(1-\lambda)[p_H + p_{OH}(1-\alpha_H)(Q_{HHb} - q_H)]$, is the profit from the H segment where p_H is the price of the H plan, and the term $p_{OH}(1-\alpha_H)(Q_{HHb} - q_H)$ captures the H segment's expected overage payment after selecting the H plan. Similarly, the second term in the profit function, $\lambda[p_L + p_{OL}(1-\alpha_L)(Q_{LLb} - q_L)]$, is the profit from the L segment, where p_L is the plan price, and the term $p_{OL}(1-\alpha_L)\cdot(Q_{LLb} - q_L)$ captures the L segment's expected overage payment after selecting the L plan.

Formally, the firm sets its menu of plans to maximize its expected profit in Equation (4) subject to the following constraints:

$$U(\theta_H; q_H, p_H, p_{OH}) \geq U(\theta_H; q_L, p_L, p_{OL}), \qquad \text{(IC-H)} \quad (5)$$
$$U(\theta_L; q_L, p_L, p_{OL}) \geq U(\theta_L; q_H, p_H, p_{OH}), \qquad \text{(IC-L)} \quad (6)$$
$$U(\theta_H; q_H, p_H, p_{OH}) \geq 0, \qquad \text{(IR-H)} \quad (7)$$
$$U(\theta_L; q_L, p_L, p_{OL}) \geq 0. \qquad \text{(IR-L)} \quad (8)$$

The left-hand side of constraint (5), $U(\theta_H; q_H, p_H, p_{OH})$, is the H-type consumer's expected utility of purchasing the H plan. She pays the price p_H, gets an allowance of q_H, and faces the overage charge per unit, p_{OH}. The right-hand side of constraint (5), $U(\theta_H; q_L, p_L, p_{OL})$, is the H-type consumer's expected utility if she purchases the L plan. In this case, she pays the price p_L, gets an allowance of q_L, and faces the overage charge per unit p_{OL}. Similarly, the left and the right sides of constraint (6) capture an L-type consumer's expected utilities of buying the L and the H plans, respectively.

Constraints (5) and (6) ensure that both consumer segments voluntarily choose the plan directed to them, whereas constraints (7) and (8) ensure that each segment will buy the plan directed to it rather than not buy anything at all. As noted earlier, our focus is on the most plausible case where the binding constraints are the incentive compatibility (IC) constraint for the H segment, constraint (5), and the individual rationality (IR) constraint for the L segment, constraint (8). In this case, H-type consumers' best outside option is the L plan, (q_L, p_L, p_{OL}).

Solving the firm's constrained optimization problem yields its optimal menu of individual plans. We use a superscript asterisk (*) to denote the equilibrium prices, allowances, and overage fees when the firm only offers individual plans. All these expressions are detailed in the appendix. The equilibrium allowances of the two individual plans are given by the following proposition.

Proposition 1. *The individual H plan has an "unlimited" allowance, $q_H^* = b$, whereas the individual L plan has a limited allowance, $q_L^* = a < b$.*

The plan prices p_H^* and p_L^* are determined by the binding constraints and are detailed in the appendix. In terms of Proposition 1, note that the allowance for the H plan, $q_H^* = b$, is equal to the H segment's high need state. Given that no one would have a need greater than b, we can interpret $q_H^* = b$ as the firm offering H-type consumers an unlimited plan.[10] Therefore, because there is no need for consumption beyond b, the overage price for the H segment is moot. By contrast, L-type consumers never obtain their maximum requirement in their plan, $q_L^* = a < b$. This means that if their need state is low (a), then their plan allowance is adequate for their need. On the other hand, if their need state is high (b), then their plan allowance is below their maximum usage need, and in equilibrium, they have overage consumption. The optimal overage price and the expected overage consumption under the L plan are given by

$$p^*_{OL} = \frac{\theta_H\theta_L(b-a)(\alpha_L-\alpha_H)(1-\lambda)}{b(\theta_L(1-\alpha_H)(1-\lambda)-\theta_H(1-\alpha_L)(1-2\lambda))},$$

$$O^*_{OL} = (b-a) \cdot \left[1-\frac{\theta_H\theta_L(\alpha_L-\alpha_H)(1-\lambda)}{(\theta_L(1-\alpha_H)(1-\lambda)-\theta_H(1-\alpha_L)(1-2\lambda))}\right].$$

From the equations above, it is clear that overage consumption occurs only when the need state is b, and when it occurs, consumers do not consume their entire need: $O^*_{OL}<(b-a)$. Intuitively, overage consumption increases with b and decreases with a: $\frac{\partial O^*_{OL}}{\partial b}>0$ and $\frac{\partial O^*_{OL}}{\partial a}<0$.

It is useful to put these results within the context of the vertical differentiation literature in which the central result is that there is no distortion at the top; that is, the highest-valuation segment gets the highest quality level, whereas the quality levels for the lower-valuation segments are distorted downward. Within the context of our model, the H segment gets an unlimited plan, which can be interpreted as the highest quality (or data) level. By contrast, the firm pushes down the equilibrium allowance of the L plan to ensure that H-type consumers will not switch to this plan. This is also similar to the vertical differentiation result of quality being distorted down for the lower-valuation segment. In that sense, we get a similar result of no distortion at the top. However, that is only part of the story. In particular, note the top segment is purchasing more data ("quality") than it expects to consume. Given that any H segment consumer needs b only with a probability $(1-\alpha_H)<1$, they end up purchasing plan allowance that they may never use. This result can be interpreted as consumers overpurchasing data (quality). This additional quantity comes with a higher plan price and a higher profit for the firm. In addition, the third component of the three-part price, the overage price, plays an interesting role in segmenting the market. On the one hand, it allows the low-valuation segment to use additional data when it has a higher need state by paying the overage price. On the other hand, it creates disincentives for the high-valuation segment to buy the L plan and thus helps with self-selection. We also note an interesting feature of the solution: although the average price of data in the H plan may be higher, the L plan has a higher marginal cost of data when it comes to the overage price.

Thus far, we have shown that the optimal individual plans follow a familiar pattern: the H segment gets an unlimited plan, whereas the L segment gets a limited plan with an allowance that is sufficient only for the low (a) need state. Next, we examine the situation where the firm introduces a menu of family plans and individual plans.

4. Bundling Consumers with Family Plans

In this section, we consider the case where the firm offers individual plans as well as family plans that allow family members to share a joint allowance. In our analysis, the sizes and the types of families are exogenously specified. The individual plans are open to single subscribers, whereas family plans are available only to consumers who jointly subscribe (i.e., bundle themselves) to the family plan.[11] The optimization problem for consumers who purchase an individual plan is the same as the problem outlined in Section 2.3, so we do not repeat it here. However, the optimization problem for the family segment is different because it involves more than one consumer. We first detail our assumptions about the structure of the family and its consumption needs. Subsequently, we determine the specifics of the firm's offerings and discuss when it is optimal to offer individual and family plans.

4.1. Family Composition

All the major firms in the U.S. wireless market offer family plans in which a family shares a joint plan allowance. It is easy to see that if a family subscription plan expands the market—for example, by drawing in low-valuation consumers, such as children, who would otherwise not purchase the product—then a family plan is likely to be profitable. We acknowledge this possibility and present it in Section 5. However, when market expansion is not possible, then a priori it is unclear how or why adding a menu of family plans can increase profits for the firm.

Our main goal in this section is to understand how a family plan affects the self-selection problem and, in particular, how it changes the incentive compatibility constraints. Within the structure of our model, a family comprises any two consumers who share an account and its usage allowance. As such, there can possibly be three types of families—an LL family of two L consumers, an HH family of two H consumers, and an HL family that is a combination of one H and one L consumer. Denote the type of family by $F\in\{HH, HL, LL\}$ and the sizes of the three family segments by f_{HH}, f_{HL}, and f_{LL}. Because each family consists of two consumers, it follows that the size of the single H segment is $(1-\lambda-2f_{HH}-f_{HL})$ and that of the single L segment is $(\lambda-2f_{LL}-f_{HL})$, respectively. In our subsequent analysis, we focus on the cases where all three family types exist but the sizes of the individual segments (who cannot buy family plans) are large enough that it always warrants the firm to offer individual plans.

Each family member has a need state of a or b, and the distribution of consumption needs is independent of the other family member's need. Therefore, a family of two will have four need states denoted by $X_F \in \{aa, ab, ba, bb\}$. For example, the need state of an HL family (X_{HL}) is a combination of the need states of the H and L consumers in the family. These need states and the associated probabilities for each family type are highlighted in Table 3.

A family's utility is a simple sum of the two individual utilities as laid out in Equation (2). As in the previous case with individual plans, the optimal consumption level for the family will be influenced by the need states as well as the specifics of the plan. Specifically, having chosen a plan, each family would subsequently have to maximize its utility based on the realized need state of the family. Let $\theta_i, \theta_{-i} \in \{\theta_H, \theta_L\}$ denote the type of each member of family $F \in \{HH, HL, LL\}$. Denote the optimal consumption of the i and $-i$ members by Q_{iF} and Q_{-iF}, respectively. As before, note that the optimal consumption level will depend on whether the plan's allowance is above or below the realized need state. If the need state is such that a family's aggregate usage is within the family plan's allowance, $Q_{iF} + Q_{-iF} \le q_F$, then the family's utility is given by $\theta_i\left(Q_{iF} - \frac{Q_{iF}^2}{2X_{iF}}\right) + \theta_{-i}\left(Q_{-iF} - \frac{Q_{-iF}^2}{2X_{-iF}}\right) - p_F$. If the need state is such that a family's aggregate usage is beyond the family plan's allowance, $Q_{iF} + Q_{-iF} > q_F$, then the family's utility is given by $\theta_i\left(Q_{iF} - \frac{Q_{iF}^2}{2X_{iF}}\right) + \theta_{-i}\left(Q_{-iF} - \frac{Q_{-iF}^2}{2X_{-iF}}\right) - p_F - p_{OF}\,(Q_{iF} + Q_{-iF} - q_F)$.

In essence, both family members optimize their individual consumptions taking their family's total utility into account. In this case, the optimal joint family consumption under a family plan, F, is given by

$$Q_F = Q_{iF} + Q_{-iF} = \begin{cases} X_{iF} + X_{-iF}, & X_{iF} + X_{-iF} \le q_F; \\ \max\left\{q_F, X_{iF}\left(1 - \frac{p_{OF}}{\theta_i}\right) + X_{-iF}\left(1 - \frac{p_{OF}}{\theta_{-i}}\right)\right\}, & X_{iF} + X_{-iF} > q_F. \end{cases} \tag{F}$$

If the family consumes overage, then each member of the family will have an overage consumption level that is similar to the expressions for individual overage consumption in Equation (3). It is easy to see that the family's overage consumption decreases with the overage price, p_{OF}, and that conditional on the same need state, the H family member consumes more than the L family member because of the former's higher valuation.

Note that it is possible that different families could come up with different rules for optimizing consumption when the plan allowance is inadequate. The joint maximization rule we propose is reasonable and fits well within the context of family decision making where the focus is on maximizing the joint utility (e.g., Filiatrault and Ritchie 1980, Corfman and Lehmann 1987). We have also analyzed other rules (e.g., proportional allocation, preference given to the H member), and they all yield a similar set of results as those presented here.

Given this family structure, we now analyze the optimal menu of plans.

4.2. Analysis of the Full Product Line

In this subsection, we focus our attention on the case where the firm offers a full product line consisting of two individual plans and three family plans. The two individual plans, (q_H, p_H, p_{OH}) and (q_L, p_L, p_{OL}), are targeted to single H and L consumers, respectively, whereas the family plans, denoted by (q_F, p_F, p_{OF}), are targeted to an F-type family, where $F \in \{HH, HL, LL\}$. Although individual family members can always buy a separate individual plan, single buyers cannot purchase a family plan. The firm knows that a family comprises two consumers, but just as it cannot identify a single buyer's type, it also cannot identify a family's type.

As before, we focus on the case in which each individual plan's allowance is weakly greater than a and the family allowance is weakly greater than $2a$. The firm's expected profit is given by

$$\begin{aligned} \Pi = {} & (1 - \lambda - 2f_{HH} - f_{HL})[p_H + p_{OH}(1 - \alpha_H)(Q_{HHb} - q_H)] \\ & + (\lambda - 2f_{LL} - f_{HL})[p_L + p_{OL}(1 - \alpha_L)(Q_{LLb} - q_L)] \\ & + f_{HH}[p_{HH} + p_{OHH}(1 - \alpha_{HH})(Q_{HH} - q_{HH})] \\ & + f_{HL}[p_{HL} + p_{OHL}(1 - \alpha_{HL})(Q_{HL} - q_{HL})] \\ & + f_{LL}[p_{LL} + p_{OLL}(1 - \alpha_{LL})(Q_{LL} - q_{LL})], \end{aligned} \tag{9}$$

where $(1 - \alpha_F)$ is the probability that an F-type family has a realized need that exceeds the plan allowance, q_F. The first two lines in Equation (9) reflect the profit from individual plans and the next three lines capture the profit from family plans.

Given a plan allowance, the probability that a family F will need to use overage depends on the draws of both members of the family relative to the plan allowance. The probabilities of the family's need states are given in Table 3, and the probability that the family will need to use overage (i.e., the need state exceeds the allowance) is highlighted in Table 4.

The firm maximizes its profits above subject to the incentive compatibility and individual rationality constraints for the two individual segments and the three family segments. The constraints for the buyers of the two individual plans are the same as the case in Section 3 (Equations (5)–(8)) and are not repeated here. However, the three family segments require new incentive

Table 3. Probabilities and Need States for Families

	Family need states, $X_F = (X_{iF}, X_{-iF})$			
Family type	*a, a*	*a, b*	*b, a*	*b, b*
LL family	α_L^2	$\alpha_L(1-\alpha_L)$	$(1-\alpha_L)\alpha_L$	$(1-\alpha_L)^2$
HL family	$\alpha_H\alpha_L$	$\alpha_H(1-\alpha_L)$	$(1-\alpha_H)\alpha_L$	$(1-\alpha_H)(1-\alpha_L)$
HH family	α_H^2	$\alpha_H(1-\alpha_H)$	$(1-\alpha_H)\alpha_H$	$(1-\alpha_H)^2$

compatibility and individual rationality constraints, which we list below:

$$U(\theta_H, \theta_H; q_{HH}, p_{HH}, p_{OHH}) \geq \max\{U(\theta_H, \theta_H; q_{HL}, p_{HL}, p_{OHL}), U(\theta_H, \theta_H; q_{LL}, p_{LL}, p_{OLL}), 2U(\theta_H; q_H, p_H, p_{OH})\}, \tag{IC-HH}$$

$$U(\theta_H, \theta_L; q_{HL}, p_{HL}, p_{OHL}) \geq \max\{U(\theta_H, \theta_L; q_{LL}, p_{LL}, p_{OLL}), U(\theta_H, \theta_L; q_{HH}, p_{HH}, p_{OHH}), U(\theta_H; q_H, p_H, p_{OH}) + U(\theta_L; q_L, p_L, p_{OL})\}, \tag{IC-HL}$$

$$U(\theta_L, \theta_L; q_{LL}, p_{LL}, p_{OLL}) \geq \max\{U(\theta_L, \theta_L; q_{HH}, p_{HH}, p_{OHH}), U(\theta_L, \theta_L; q_{HL}, p_{HL}, p_{OHL}), 2U(\theta_L; q_L, p_L, p_{OL})\}, \tag{IC-LL}$$

$$U(\theta_H, \theta_H; q_{HH}, p_{HH}, p_{OHH}) \geq 0, \tag{IR-HH}$$

$$U(\theta_H, \theta_L; q_{HL}, p_{HL}, p_{OHL}) \geq 0, \tag{IR-HL}$$

$$U(\theta_L, \theta_L; q_{LL}, p_{LL}, p_{OLL}) \geq 0, \tag{IR-LL}$$

where $U(\theta_H, \theta_H; \cdot)$, $U(\theta_H, \theta_L; \cdot)$, and $U(\theta_L, \theta_L; \cdot)$ are the expected utilities of the HH, HL, and LL families, respectively. The incentive compatibility constraints for a family have to account for the multiple plan options available to the two family members. In particular, not only can families choose among alternative family plans, but each individual in a family can also choose an individual plan. Thus, for the HH-family to purchase the HH-family plan, the two members need to get a weakly greater utility from purchasing either the HL family plan or the LL family plan, or from purchasing two individual H plans (see Equation (IC-HH)). Similarly, for the HL family to purchase the HL family plan, the two members need to get a weakly greater utility from purchasing the LL family plan or the HH family plan, or from purchasing an individual H and an individual L plan (Equation (IC-HL)). Finally, the LL family needs to get a weakly greater utility from purchasing either of the other two family plans and from purchasing two individual L plans. Thus, both the HH and the HL families consider multiple family plans as well as individual plans, whereas the LL family only considers the LL family plan (it is easy to show that the HH or HL family plans are not viable options for the LL family) and individual plans. In equilibrium, the binding constraint for the HH family is to purchase an HL plan, and for the HL family, it is to purchase an LL plan. See Online Appendix C for more details.

By solving the firm's constrained optimization problem, we obtain its optimal menu of plans. We use double asterisk ($**$) to denote the equilibrium prices, allowances, and overage fees in the presence of family plans (refer to the appendix for details). In looking at the allowances of the plans, we have the following proposition.

Proposition 2. *In equilibrium, if the firm offers two individual plans and three family plans, the specifics of the plan allowances are as follows*:

1. *The individual H plan is "unlimited" and has an allowance of* $q_H^{**} = b$, *whereas the individual L plan is limited and has an allowance of* $q_L^{**} = a < b$.
2. *The HH family plan has an "unlimited" allowance of* $q_{HH}^{**} = 2b$ *and the allowances of the HL and the LL family plans are either* $a + b$ or $2a$, *such that* $q_{HH}^{**} > q_{HL}^{**} \geq q_{LL}^{**}$.

Proposition 2 is summarized in Table 5. Note that the optimal prices of the plans are determined by the binding constraints and are detailed in the appendix. This proposition highlights that whereas the HH family segment receives its maximum requirement and does not have to deal with overage charges, the HL and the LL family plans have allowances that are not sufficient for all need states, so they have a positive probability of incurring overage charges. Furthermore, the HL segment gets an allowance that is weakly greater than the allowance of the LL segment; more specifically, it gets a greater allowance when $q_{HL}^{**} = a + b$ and $q_{LL}^{**} = 2a$.

To develop our intuition about this proposition, consider the binding constraints associated with the HH, HL, and LL family plans. The HH plan should be such that HH families are indifferent between choosing the HH or the HL plan. By lowering the allowance of the HL plan below the unlimited quantity of $2b$, the

Table 4. Overage Probabilities for Family Segments by Need State and Plan Allowance

	Family need states relative to plan allowance, q_F		
$(1-\alpha_F)$	$2a \leq q_F < a + b$	$a + b \leq q_F < 2b$	$q_F = 2b$
HH family	$2\alpha_H(1-\alpha_H) + (1-\alpha_H)^2$	$(1-\alpha_H)^2$	0
HL family	$\alpha_H(1-\alpha_L) + \alpha_L(1-\alpha_H) + (1-\alpha_H)(1-\alpha_L)$	$(1-\alpha_H)(1-\alpha_L)$	0
LL family	$2\alpha_L(1-\alpha_L) + (1-\alpha_L)^2$	$(1-\alpha_L)^2$	0

Table 5. Equilibrium Family Plan Allowances

q^{**}_{HH}	q^{**}_{HL}	q^{**}_{LL}
$2b$	$a+b$	$a+b$
$2b$	$a+b$	$2a$
$2b$	$2a$	$2a$

firm is better able to keep HH families from deviating to the HL plan. Now consider the choice of HL families who can potentially deviate to the LL family plan. When the allowance of the LL plan is lower than that of the HL plan, which occurs when $q^{**}_{HL} = a+b$ and $q^{**}_{LL} = 2a$, it is easy to see that HL families will move to a lower allowance level if they switch to an LL plan and significantly increase their probability of having to consume overage. However, the more interesting case is where the allowances of the two plans are the same, $q^{**}_{HL} = q^{**}_{LL}$; in this case, it is less clear how the same allowance can separate the HL and the LL families. This leads to the following proposition.

Proposition 3. *In equilibrium, if the firm offers identical allowances for the HL and the LL family plans, the LL plan price is lower than the HL plan price,* $p^{**}_{LL} < p^{**}_{HL}$*, but the overage price of the LL plan is higher than the overage price of the HL plan,* $p^{**}_{OLL} > p^{**}_{OHL}$*.*

When the allowances are different across plans, it is easy to see how self-selection works across the high (HH), middle (HL), and low (LL) family plans. However, the problem becomes complicated when the low and the middle segments get the same allowances. In this case, as Proposition 3 shows, when the allowances are the same, the only way to prevent the middle segment (i.e., HL families) from buying the low (i.e., LL) plan is to increase the overage price of the LL family plan such that $p^{**}_{OLL} > p^{**}_{OHL}$. This higher overage price for the LL family plan separates the two segments because the probability that an HL family will need to consume overage is higher than the probability that an LL family will need to consume overage. As a result, an HL family has less of an incentive to move to an LL plan. Because the allowances of these two family plans are the same, this result is especially interesting in that it highlights the role of the overage price in market segmentation. In traditional vertical differentiation models, we see that the lower valuation segments get a lower quality and a lower price; in this instance, the lower-valuation segment gets the same quality (i.e., same allowance) level but gets a higher overage price, which is designed to prevent HL families from switching to the LL family plan.

Having characterized the three family plans, we now analyze how the introduction of these family plans affects the prices of the individual plans from Section 3. This comparison leads to the following.

Proposition 4. *If the firm adds to the product line of two individual plans by introducing three family plans, it changes the specifics of the individual plans as follows:*

1. *The H plan's allowance remains unchanged;* $q^{**}_H = q^*_H$*, but its price is higher,* $p^{**}_H > p^*_H$*.*
2. *The L plan's allowance remains unchanged;* $q^{**}_L = q^*_L$*, but its price is lower,* $p^{**}_L < p^*_L$*, and its overage price is higher,* $p^{**}_{OL} > p^*_{OL}$*.*

Recall that when the product line contains only two individual plans, the service provider accounts for the potential cannibalization problem by ensuring that the high-valuation H segment purchases the more profitable H plan by obtaining at least a utility of $U(\theta_H; q_L, p_L, p_{OL})$. At first blush, with $p^{**}_L < p^*_L$, it might appear that the L plan is now more attractive to H consumers. However, its overage price is now higher, and because H consumers are more likely to be in the high state, they are also more likely to be in the overage region. As a result, when the firm offers a family plan, the individual L plan ends up becoming a less attractive outside option for single H consumers. As the best outside option for single H consumers is the L plan, an increase in p^{**}_{OL} makes the outside option worse; this allows the firm to raise the H plan's price, $p^{**}_H > p^*_H$, and extract more surplus from the single H consumers. Another way to think about the impact of offering family plans is that H consumers who can join a family plan impose a negative externality on the single H consumers who cannot join the family plan. Of course, once the L plan's overage price is increased, the firm needs to reduce its price, $p^{**}_L < p^*_L$, to ensure that the single L segment still purchases this plan, $U(\theta_L; q_L, p_L, p_{OL}) \geq 0$. However, because the family plans draw in some L segment buyers, this loss in revenues from a lower price in the L plan is again attenuated by a smaller number of single consumers to purchase the L plan: $(\lambda - 2f_{LL} - f_{HL}) < \lambda$.

In the introduction, we suggested that a family plan could be thought of as an instrument that allows the firm to "bundle" some of its consumers. Note that this bundling is fundamentally different from the traditional product bundling where consumers who buy the bundle essentially purchase multiple products or, equivalently, the firm provides the bundle that is purchased by an individual consumer. In our context, no consumer has the need to buy two plans, but two consumers in a family can jointly purchase one family plan. This raises the question of whether a family plan gives consumers a benefit from pooling. Whereas pooling is certainly one of the benefits of a family plan, we find that family plans can be beneficial even when there are no benefits from pooling. In particular, when the optimal LL family plan offers an allowance of $2a$, it is exactly the same as the sum of the allowances of the two individual plans. In this situation, the benefit to the

LL family comes from a lower price and not from any changes in the allowance.

Having established how family plans affect the design of the full product line, we now analyze how the introduction of family plans affects the profits of the firm.

4.3. Profitability of the Full Product Line

Whereas we have characterized the nature of the family plans offered by the firm, it remains to be seen whether it is in the firm's interests to offer a full product line consisting of three family plans and two individual plans. In this subsection, we further develop our intuition about when and why this solution is preferred to our earlier benchmark in Section 3 of offering only individual plans.

The optimality of the full product line solution in Proposition 2 depends on the sizes of the three family segments. Recall that the sizes of these segments are exogenously specified and are such that the individual segments are always large enough (or family segments are small enough) that the firm chooses to serve the individual segments. In this framework, as we change the sizes of the family segments, we begin to move away from the full product line solution in a systematic manner. Beginning with the highest valuation or top family segment, we find that a decrease in the size of the HH family segment can eventually lead the firm to only offer one HL plan that appeals to both the HH and the HL families. In other words, it is not worth offering a separate plan for the HH family. Similarly, if we decrease the size of the HL segment, then the firm can drop an HL plan and let HL families purchase an LL family plan. Finally, as the size of LL families decreases, the firm can choose to drop the LL family plan altogether. This emphasizes that when families can optimally self-select their family plan, the number of family plans offered by the firm will depend critically on the sizes of the family segments.

From the discussion above, it is useful to consider how the sizes of family segments affects profitability. This leads to the following.

Proposition 5. *When the firm offers a menu of three family plans and two individual plans,*

(a) *profits increase as the size of the HH segment decreases,* $\frac{\partial \Pi^{**}_{HH}}{\partial f_{HH}} < 0$; *and*

(b) *when* $f_{HH} < \bar{f}_{HH}$, *it is more profitable to offer three family plans and two individual plans than to offer only two individual plans; otherwise, it is more profitable to offer only two individual plans.*

As we have shown earlier, family plans allow the firm to extract more surplus from single high-valuation consumers who cannot be part of a family. This suggests that the firm would prefer to keep the individual H segment as large as possible. This incentive comes from the fact that the HH family always gets a weakly greater surplus by purchasing a family plan as opposed to purchasing two individual H plans. In other words, the presence of an HH family simply lowers the surplus for the firm without any other compensating benefit. Therefore, if the firm were to offer only one family plan targeted at a family of two H-type consumers, it cannot strictly improve the profits of the firm. This comes from the idea that to induce two H-type consumers to purchase the HH family plan, the firm would need to offer them surplus beyond what they would have obtained from buying two single H plans. More generally, when the firm offers a full product line, as the size of the HH family segment grows, the individual L plan's price increases, $\frac{\partial p^{**}_L}{\partial f_{HH}} > 0$ and its overage price decreases, $\frac{\partial p^{**}_{OL}}{\partial f_{HH}} < 0$. This result means that the L plan has improved in the sense that the lower overage price will lead to a higher expected total usage by the L plan buyers. As a result, the price for the H plan decreases in the size of the HH family segment, $\frac{\partial p^{**}_H}{\partial f_{HH}} < 0$, hurting the firm's overall profits.

By contrast, if a family has at least one L consumer, the issues raised above do not apply. In particular, if some L consumers move into an LL or an HL family because they get additional surplus, the firm is able to compensate by charging higher prices to these consumers. Furthermore, the single L segment now has a higher overage price, which then allows the firm to extract more surplus from the single H consumers. In particular, note the following comparative statics with segment sizes: $\frac{\partial p^{**}_{OL}}{\partial f_{LL}} > 0$, $\frac{\partial p^{**}_{OL}}{\partial f_{HL}} > 0$, $\frac{\partial p^{**}_L}{\partial f_{LL}} < 0$, $\frac{\partial p^{**}_L}{\partial f_{HL}} < 0$, and $\frac{\partial p^{**}_H}{\partial f_{LL}} > 0$, $\frac{\partial p^{**}_H}{\partial f_{HL}} > 0$. In other words, when there are more L consumers who can join a family (regardless of the HL or the LL families), the individual L plan's overage price increases while its price decreases. By contrast, the individual H plan's price increases when there are fewer single L consumers, which then allows the firm to extract more surplus from single high types who purchase the H plan.

Note that Proposition 5 shows that a full product line solution is optimal when the size of the HH segment is below a critical level. In this case, for completeness, we note that there could be a four- or a three-plan solution (e.g., two individual plans and HL and LL family plans or two individual plans and an LL family plan) that could be more optimal than the five-plan (full product line) solution. In other words, offering multiple family plans can be the optimal strategy under many conditions. Although we have characterized all these solutions, the analytical complexity of the expressions prevents us from making a global comparison across these solutions.[12] However, Proposition 5 clearly establishes that there is a cutoff on f_{HH} that makes the full product line solution more profitable than a solution of only two individual plans. In a similar vein, it is easy to see that a large enough HH segment can also make any other family plan less profitable. Thus, we find that family plans can indeed be more profitable than individual plans alone.

Having established our central finding showing when and how family plans can be profitable for the firm, we now explore the robustness of our results to some of the main assumptions of our model.

5. Extensions

In this section, we briefly consider the implications of relaxing two key assumptions of our model. In particular, we analyze (1) the equivalence between having a middle segment and an HL family segment and (2) profitability of family plans with market expansion and contraction.

5.1. Do Family Plans Create a Middle Segment?

One may reasonably conjecture that by introducing an HL family plan, the firm is effectively adding an additional segment from which it can extract profits. Note that this is not the case for HH and LL families, which are simply a couple from the H and the L segments, respectively. On the other hand, an HL family plan effectively introduces a "middle" segment (M) between the H and the L segments, and it could be profitable to target this middle segment with its own product. In other words, an HL family plan can be thought of as a plan for two middle consumers, where this segment's valuation is a combination of the H and L valuations (as in the family case), and it is of the same size as the family segment. Because the LL and the HH families do not create a middle segment as defined here, we focus our attention solely on the case where the firm offers a single HL family plan. In this case, a natural question to ask is whether offering this HL family plan has the same effect as a third segment, as defined above.

At first blush, one might conjecture that the existence of the HL family segment affects the firm's product line design in the same way as introducing a middle valuation segment with the same effective size and valuation. Although there are similarities between the two cases, there are several important differences. First, because there is only one family plan, the outside options for the HL family are an individual H and an individual L plan. In other words, the HL family segment's surplus is equal to the single H segment's surplus plus the single L segment's surplus.

Now suppose there is a middle (M) segment that has an equivalent size and valuation to the HL family. In this case, if the market only consists of three single segments (H, M, and L segments), the outside option for the H segment is the M plan, and for the M segment, it is the L plan. On the other hand, when an HL family plan is offered, a single H-type consumer cannot choose the family plan, so this consumer's best outside option is the L plan. Moreover, adding a third segment exacerbates the firm's cannibalization problem, because it distorts downward the allowances of two segments, M and L. With a family plan, there is a single distortion associated with the L plan.

In summary, although a family can be thought of as a middle segment because of its average valuation lying between the H and the L segments' valuations, the incentive compatibility conditions imposed by second-degree price discrimination and the nature of the product created for the family lead to very different outcomes.

5.2. Market Expansion and Contraction Through Family Plans

In the main body of this paper, we focus on the situation in which it is more profitable for the firm to serve both the H and the L consumers. In this subsection, we present two cases where the firm finds it optimal to (1) expand the market by introducing a family plan and (2) contract the market by offering a family plan.

Market Expansion. Assume that consumers' valuations and the sizes of the two segments are such that it is optimal for the firm to serve only the H segment when it offers individual plans. In this case, it is easy to see that the equilibrium plan is unlimited (i.e., b), and the firm is able to extract the entire expected surplus from H consumers. Now suppose the firm extends its product line by adding an HL family plan as well as the individual H plan. In this case, the equilibrium allowance for the individual H plan remains unchanged at b. Similarly, the HL family also receives an unlimited allowance of $2b$, which ensures that both the H and the L consumers within a family can consume their entire consumption need. Thus, the firm increases its profits by introducing an HL family plan that expands the served market by drawing in low-valuation consumers who are part of an HL family.

Market Contraction. Recall that in Section 3, we focused our attention on the case where the firm serves all the consumers in the market by offering two individual plans. Under certain conditions, we see that if the firm introduces an HL family plan, it also chooses to drop the single L segment. In other words, it is optimal for the firm to serve the HL family and the single H consumers and exclude the single L consumers in the market. This finding highlights a pure bundling benefit of a family plan. By excluding the individual L segment, the firm loses the revenue from not serving these low-valuation consumers, but it is able to extract all the surplus from the single H segment whose best outside option now becomes zero; this also means that the firm can extract all the surplus from the H consumers in the family segment. Finally, as is usually the case, the firm extracts the entire surplus from the L consumers in the family.

6. Conclusion

In this paper, we study how family plans can help the firm price discriminate in a market characterized by heterogeneity and uncertainty in consumers' needs for the services provided by the firm. This framework captures two key features of telecommunications services: stochastic and differential requirements for services. Our model contains three stages of the game between the firm and the consumers. In the first stage, the firm announces a menu of three-part plans. Next, consumers who only know their valuation and their expected need states make their purchase decisions based on their expected utilities. Although the firm also knows consumers' expected need states, it cannot directly identify each consumer's type. In the third stage of the game, a random draw determines the individual consumer's need states. If the realization is below the chosen plan's allowance, consumers consume up to their need and ignore any remaining allowance. On the other hand, if the realization exceeds the chosen plan's allowance, consumers choose a level of consumption that factors in the overage price.

When the firm offers only individual plans, the firm offers the high-valuation segment an unlimited plan and offers the low-valuation segment a limited plan. When the firm extends its line by offering three family plans (one for each of the three family types) and two individual plans, we find that the optimal family plan for the family comprising two high-valuation consumers also includes an unlimited allowance, whereas all other family types are offered limited plans. Our central result is that offering family plans allows the firm to better price discriminate and capture additional surplus from some consumers. In this sense, if some consumers can bundle themselves into families, the firm is able to better price discriminate by offering them tailored family plans. The introduction of family plans to the product line has a specific effect on the individual plans offered by the firm. In particular, although the single H plan is still unlimited, it now has a higher price, whereas the single L plan has a lower price with a higher overage price. These findings suggest that an important reason a family plan is profitable is that it allows the firm an opportunity to extract additional surplus from high-valuation consumers who are not part of a family. Finally, we note that the firm would prefer the HH family segment to be as small as possible. Importantly, we find that when the size of this segment is below a threshold, it is more profitable for the firm to offer a full product line consisting of three family plans and two individual plans compared with the case of offering only two individual plans.

To our knowledge, our analysis offers the first systematic study of how the addition of a menu of family plans can help a firm increase its profits even when the strategy does not increase the market size. We also show that the addition of a family plan can be more profitable even when doing so reduces the number of consumers served by the firm. We also note another novel aspect of our work. By allowing some consumers to bundle themselves, the family plan strategy results in a given segment of consumers being served by two products. This is in contrast to traditional results that firms could benefit from dropping some lower-valuation consumers or serving multiple consumer segments with a common product.

In this paper, we have presented a first analysis of family plans. Given the importance and the popularity of these plans in a variety of contexts (healthcare insurance, fitness clubs, music streaming services, etc.), it is important to understand other ways in which they may work. For example, family plans can increase multiple users' switching costs and thus may soften the competition between competing firms. Furthermore, it would be interesting to study the optimal product line when the firm offers both nonlinear contract plans and pay-as-you-go plans. It would also be interesting to see how allowing allowance rollover would change the optimal product line design. Finally, it would be fascinating to empirically test the effects of family plans on the firm's strategy.

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Appendix. Proofs of Propositions

Proof of Proposition 1

As noted in the paper, we restrict our attention here to the case in which $q_H^* > q_L^* \geq a$. We first show that $q_H^* = b$, which helps us rule out a few subcases. We then analyze each of the remaining two subcases.

We prove the claim $q_H^* = b$ in equilibrium by contradiction. Suppose the optimal H plan's allowance is $q_H^* \in [a, b)$. The H-type consumers will then use a quantity $Q_{Hb} \in [\tilde{q}_H^*, b]$ if their need state is b. (Recall that consumers never use more than their need state). The expected utility for the H-type consumers with the H plan when their need state is b is $(1-\alpha_H)[\theta_H(Q_{HHb} - Q_{HHb}^2/(2b)) - p_{OH}^*(Q_{HHb} - q_H^*)]$. Now consider a deviation such that $q_H^* = b$. Then H-type consumers' expected utility when their need state is b becomes $(1-\alpha_H)\left[\theta_H\left(b - \dfrac{b^2}{2b}\right)\right]$, which is strictly greater than $(1-\alpha_H)\cdot\left[(\theta_H(Q_{HHb} - Q_{HHb}^2/(2b)) - p_{OH}^*(Q_{HHb} - q_H^*)\right]$ for any $Q_{HHb} < b$. Because $Q_{HHb} < b$ for any $p_{OH}^* > 0$, the H consumers will get a higher utility under the deviation, which will allow the firm to extract a greater revenue from the H consumers without affecting the IC-H constraint or the revenue from the L consumers. This contradicts the assumption that $q_H^* < b$.

Because $q_H^* = b$, the H consumers will not need to use overage in either state. However, with $a \leq q_L^* < b$, the L consumers may choose to use overage in the b state. It turns out that even when our analysis allows a nonnegative overage for these consumers, the subcase of zero overage is not always nested in this analysis. Therefore, we not only present the solution for the nonnegative overage but also separately discuss the subcase of zero overage.

We substitute $q_H^* = b$, p_H^* from the IC-H constraint and p_L^* from the IR-L constraint in the firm's optimization problem. The first-order condition for p_{OL} is given as follows:[13]

$$\frac{\begin{array}{c}-q_L(\alpha_L-\alpha_H)\theta_H\theta_L(1-\lambda)+b((\alpha_L-\alpha_H)\theta_H\theta_L(1-\lambda)\\ +p_{OL}((1-\alpha_L)(1-2\lambda)\theta_H-(1-\alpha_H)(1-\lambda)\theta_L))\end{array}}{\theta_H\theta_L}=0,$$

which gives $p_{OL}^* = \dfrac{\dfrac{(b-a)(\alpha_L-\alpha_H)}{\theta_H\theta_L(1-\lambda)}}{\begin{array}{c}b((1-\alpha_H)\theta_L(1-\lambda)\\ -(1-\alpha_L)\theta_H(1-2\lambda))\end{array}}$. The first-order condition for q_L is $(-p_{OL}(1-\lambda)(\alpha_L-\alpha_H))$, which is negative under our assumption of $\alpha_L > \alpha_H$. This gives $q_L^* = a$.

The plan prices are then as given below:

$$p_L^* = \frac{1}{2}\theta_L\left(a\alpha_L + b(1-\alpha_L)\cdot\left(1-\frac{(b-a)(\alpha_L-\alpha_H)\theta_H(1-\lambda)}{b((1-\alpha_H)\theta_L(1-\lambda)-(1-\alpha_L)\theta_H(1-2\lambda))}\right)^2 + \frac{2a(b-a)(\alpha_L-\alpha_H)(1-\alpha_L)\theta_H(1-\lambda)}{b((1-\alpha_H)\theta_L(1-\lambda)-(1-\alpha_L)\theta_H(1-2\lambda))}\right),$$

and

$$p_H^* = \frac{1}{2}\theta_L\left(\begin{array}{l}a\alpha_L+b(1-\alpha_L)\left(1-\dfrac{(b-a)(\alpha_L-\alpha_H)\theta_H(1-\lambda)}{\begin{array}{c}b((1-\alpha_H)\theta_L(1-\lambda)\\ -(1-\alpha_L)\theta_H(1-2\lambda))\end{array}}\right)^2\\ -\dfrac{\begin{array}{c}2a(b-a)(\alpha_L-\alpha_H)\\ \cdot(1-\alpha_H)\theta_H(1-\lambda)\end{array}}{\begin{array}{c}b((1-\alpha_H)\theta_L(1-\lambda)\\ -(1-\alpha_L)\theta_H(1-2\lambda))\end{array}}+\dfrac{\begin{array}{c}2a(b-a)(\alpha_L-\alpha_H)\\ \cdot(1-\alpha_L)\theta_H(1-\lambda)\end{array}}{\begin{array}{c}b((1-\alpha_H)\theta_L(1-\lambda)\\ -(1-\alpha_L)\theta_H(1-2\lambda))\end{array}}\\ +\dfrac{\begin{array}{c}(b-a)(\alpha_L-\alpha_H)(1-\alpha_H)\theta_H(b((2-\alpha_H+\alpha_L)\theta_L(1-\lambda)\\ -2(1-\alpha_L)\theta_H(1-2\lambda))-a(\alpha_H-\alpha_L)\theta_L(1-\lambda))(1-\lambda)\end{array}}{b((1-\alpha_H)\theta_L(1-\lambda)-(1-\alpha_L)\theta_H(1-2\lambda))^2}\end{array}\right).$$

The firm's profits are $\Pi^* = (1-\lambda)p_H^* + \lambda[p_L^* + (1-\alpha_L)\cdot p_{OL}^*(b - \frac{bp_{OL}^*}{\theta_L} - a)]$.

Finally, we discuss the subcase of zero overage for the L consumers within the current case of $q_L^* \geq a$. We show that this subcase can only arise if $q_H^* = q_L^* = b$; that is, the firm offers only one plan for both segments. Suppose that the L segment does not use overage in the b state, $Q_{LLa} = a$, $Q_{LLb} = q_L \geq a$. In this case, the firm's profit function is given by

$$\begin{aligned}\Pi &= (1-\lambda)(p_H + (1-\alpha_H)p_{OH}(Q_{HHb}-q_H))\\ &\quad + \lambda(p_L + (1-\alpha_L)p_{OL}(Q_{LLb}-q_L))\\ &= (1-\lambda)(p_H) + \lambda(p_L).\end{aligned}$$

Solving the constrained optimization problem, we obtain $q_L^* = q_H^* = b$, and $p_H^* = p_L^* = [\alpha_L a + (1-\alpha_L)b]\theta_L/2$. The overage price p_{OL}^* and p_{OH}^* can take any value because no consumer will use overage in this equilibrium. In other words, in this subcase the firm does not offer two different individual plans to target two different consumer segments. Therefore, we have shown that $q_H^* = b$ and $q_L^* = a$ in equilibrium.

Proof of Proposition 2

The derivation here is very similar to that used for Proposition 1. As noted in the paper, we continue to restrict our attention to the case in which $q_H^{**} > q_L^{**} \geq a$. We first show that $q_H^{**} = b$, which leaves us with two subcases similar to the two subcases in the proof of Proposition 1.

First, using the approach used for proposition, we can show that $q_H^{**} = b$ and $q_{HH}^{**} = 2b$. The key step is that by increasing the allowance of the H plan up to b and the HH family plan allowance up to $2b$, the firm can increase these consumers' utilities without affecting any constraints or its revenue from all other consumers. These higher utilities result in a higher profit for the firm.

After substituting p_{HH}^{**}, p_H^{**}, and p_L^{**} from IC-HH, IC-H, and IR-L constraints into the firm's profit function and noting $q_H^{**} = b$ and $q_{HH}^{**} = 2b$, we derive the first-order conditions for q_L and p_{OL}. The first-order condition for q_L is $-p_{OL}(1-\lambda-2f_{HH}-f_{HL})(\alpha_L-\alpha_H)$, which is negative because $\alpha_H < \alpha_L$, giving $q_L^{**} = a$. The first-order condition for p_{OL} is

$$\begin{aligned}&\frac{\begin{array}{c}-q_L(\alpha_L-\alpha_H)\theta_H\theta_L(1-\lambda-2f_{HH}-f_{HL})\\ +b((\alpha_L-\alpha_H)\theta_H\theta_L(1-\lambda-2f_{HH}-f_{HL}))\end{array}}{\theta_H\theta_L}\\ &+[p_{OL}((1-\alpha_L)(1-2f_{HH}+2f_{LL}-2\lambda)\theta_H\\ &-(1-\alpha_H)(1-\lambda-2f_{HH}-f_{HL})\theta_L)]/(\theta_H\theta)_L = 0,\end{aligned}$$

which gives $p_{OL}^{**} = \dfrac{\begin{array}{c}(b-a)(\alpha_L-\alpha_H)\theta_H\theta_L\\ \cdot(1-\lambda-2f_{HH}-f_{HL})\end{array}}{\begin{array}{c}b((1-\alpha_H)\theta_L(1-\lambda-2f_{HH}-f_{HL})\\ -(1-\alpha_L)\theta_H(1-2\lambda-2f_{HH}+2f_{LL}))\end{array}}$.

Depending on the HL and LL family plans' allowances, there are three distinct cases, labeled Cases 3.1, 3.2, and 3.3. To ensure the proper signs of the second-order conditions, we assume $(2\lambda f_{HH} + (1-2\lambda)f_{HL}) < 2(1-\lambda)f_{LL}$, $(1-\alpha_L)\theta_H(1-2f_{HH}+2f_{LL}-2\lambda) - (1-\alpha_H)\theta_L(1-2f_{HH}-f_{HL}-\lambda) < 0.$ Details are presented in Online Appendix C, where we show that there are three possible equilibria: $q_{HL}^{**} = a+b, q_{LL}^{**} = 2a$, $q_{HL}^{**} = q_{LL}^{**} = a+b$, and $q_{HL}^{**} = q_{LL}^{**} = 2a$.

In Case 3.1, where $q_{HL}^{**} = a+b, q_{LL}^{**} = 2a$, the equilibrium prices are given below:

$$p_L^{**} = \frac{1}{2}\theta_L\left(a\alpha_L + b(1-\alpha_L)\cdot\left(1-\frac{(b-a)(\alpha_L-\alpha_H)\theta_H(1-2f_{HH}-f_{HL}-\lambda)}{\begin{array}{c}b((1-\alpha_H)\theta_L(1-2f_{HH}-f_{HL}-\lambda)\\ -(1-\alpha_L)\theta_H(1-2f_{HH}-2f_{LL}-2\lambda))\end{array}}\right)^2 + \frac{2a(b-a)(\alpha_L-\alpha_H)(1-\alpha_L)\theta_H(1-2f_{HH}-f_{HL}-\lambda)}{\begin{array}{c}b((1-\alpha_H)\theta_L(1-2f_{HH}-f_{HL}-\lambda)\\ -(1-\alpha_L)\theta_H(1-2f_{HH}-2f_{LL}-2\lambda))\end{array}}\right),$$

$$P_h^{**} = \frac{1}{2}\theta \left(\begin{array}{l} \alpha\alpha_L + b(1-\alpha_L)\left(1 - \frac{(b-a)(\alpha_L-\alpha_H)\theta_H(1-2f_{HH}-f_{HL}-\lambda}{b(1-\alpha_H)\theta_L(1-2f_{HH}-f_{HL}-\lambda) - (1-\alpha_L)\theta_H(1-2f_{HH}=2f_{HH}-2\lambda)}\right) \\ - \frac{2a(b-a)(\alpha_L-\alpha_L)(1-\alpha_H)\theta_H(1-2f_{HH}f-_{HL}\lambda)}{b(1-\alpha_H)\theta_L(1-2f_{HH}-f_{HL}) - (1-\alpha_L)\theta_H(1-2f_{HH}-2f_{LL}-2\lambda)} \\ + \frac{2a(b-a)(\alpha_L-\alpha_H)(1-\alpha_L)\theta_H(1-2f_{HH}-f_{HL}-\lambda)}{b((1-\alpha_H)\theta_L)(1-2f_{HH}-f_{HL}-\lambda) - (1-\alpha_L)\theta_H(1-2f_{HH}-f_{HL}-\lambda)} \\ + \frac{\begin{array}{l} b(1-\alpha_H)(\alpha_L-\alpha_H)\theta_H(b(2-\alpha_H+\alpha_L)\theta_L(1-2f_{HH}-f_{HL}-\lambda) \\ -2(1-\alpha_L)\theta_H)(1-2f_{HH}2-f_{HL}2-\lambda) - \alpha(\alpha_H-\alpha_L) \\ \cdot\theta_L(1-2f_{HH}-f_{HL}-\lambda)(1-2f_{HH}2-f_{HL}2-\lambda)\end{array}}{b((1-\alpha_H)\theta_L(1-\lambda)-(1-\alpha_L))\theta_H(1-2f_{HH}-2f_{LL}-2\lambda)} \end{array}\right),$$

$$p_{OHL}^{**} = \frac{(b-a)(\alpha_L-\alpha_H)f_{HH}\theta_H\theta_L}{b\,(f_{HL}(1-\alpha_L)(\theta_H+\theta_L) + f_{HH}((-1+\alpha_L)\theta_H + (1-2\alpha_H+\alpha_L)\theta_L))},$$

$$p_{OLL}^{**} = (b-q)\,(f_{HH}+f_{Hl})(\alpha_H-\alpha_L)\theta_H\theta_L \Big/ \left(\begin{array}{l} b(2f_{LL}(-1\alpha_L)\theta_H + f_{HH}(\theta_H - \alpha_L\theta_H + (-1+\alpha_H)\theta_L \\ \quad + f_{HL}(\theta_H-\alpha_H)\theta_L)) \\ + a(2f_{LL}(-1+\alpha_L)\alpha_L\theta_H + f_{HH}(-2\alpha L2\theta_H - \alpha_H\theta_L \\ \quad + \alpha_L(\theta_H + \alpha_H\theta_H + \alpha_H\theta_L)) \\ + f_{HL}(-2\alpha_L 2\theta H - \alpha_L\theta H + \alpha_L(\theta_H + \alpha_H\theta_H + \alpha_H\theta_L)) \end{array}\right).$$

The three family plan prices are given by

$$p_{LL}^{**} = \frac{\begin{array}{c} b(1-\alpha_L)(p_{OLL}^{**}-\theta_L)^2 + 2ap_{OLL}^{**}(1-\alpha^2{}_{-\,\mathrm{L}})\theta_L \\ + a\alpha_L(p_{OLL}^{**2}(1-\alpha_L) - 2p_{OLL}^{**}(1-\alpha_L)\theta_L + \theta_L^2) \end{array}}{\theta L},$$

$$\begin{aligned} p_{HH}^{**} = \frac{1}{2\theta_H 2\theta_L}(2(p_{LL}^{**}(a+b)(1-\alpha_L) \\ & - 2ap_{OLL}^{**}(1-\alpha_H\alpha_L)\theta_H\theta_H - p_{OLL}^{**}\Big((1-\alpha_H)(-1\alpha_L)\theta_H\theta_L \\ & + p_{LL}^{**}(1-\alpha_H)(\theta_H+\theta_L) + p_{OLL}^{**}((1-\alpha_L)\theta_H \\ & + (1-\alpha_H)\theta_L)\Big) + ap_{OLL}^{**}(2(\alpha_H+\alpha_L-2\alpha_H\alpha_L)\theta_H\theta_L \\ & + p_{OLL}^{**}\Big(-\alpha_H\theta_H + \alpha_L((-1+\alpha_H)\theta_H + \alpha_H\theta_H)\Big), \end{aligned}$$

$$p_{HH}^{**} = \frac{\begin{array}{c} -bp_{OHL}^{**}(1-\alpha_H)^2(p_{OHL}^{**}-2\theta_H) \\ + \Big(p_{HL}^{**} - p_{OHL}^{**}(a+b)(1-\alpha_H)^2\Big)\theta_H \end{array}}{\theta_H}.$$

In Case 3.2, where $q_{HL}^{**} = a+b, q_{LL}^{**} = a+b$, the equilibrium prices p_L^{**} and p_H^{**} are identical to these in Case 3.1. Overage prices are given below:

$$p_{OHL}^{**} = \frac{(b-a)(\alpha_L-\alpha_H)f_{HH}\theta_H\theta_L}{b\,(f_{HL}(1-\alpha_L)(\theta_H+\theta_L) + f_{HH}((-1+\alpha_L)\theta_H + (1-2\alpha_H+\alpha_L)\theta_L))},$$

and

$$p_{OLL}^{**} = \frac{(b-a)(\alpha_L-\alpha_H)f_{HH}\theta_H\theta_L}{\begin{array}{l} b(2f_{LL}(1-\alpha_L)\theta_H - f_{HL}(1-\alpha_L)((1+\alpha_H-2\alpha_L)\theta_H \\ -(1-\alpha_H)\theta_L) - f_{HH}((1-2\alpha_L+\alpha_H)\theta_H(1-\alpha_H)\theta_L)) \end{array}}.$$

The three family plan prices are given by

$$p_{LL}^{**} = \frac{\begin{array}{l} \theta_L((a+b)p_{OLL}^{**}(1-\alpha_L)^2 + a\alpha_L\theta_L) \\ + b(1-\alpha_L)(p_{OLL}^{**2}(1-\alpha_L) + 2p_{OLL}^{**}(1-\alpha_L)\theta_L + \theta_L^2) \end{array}}{\theta L},$$

$$\begin{aligned} p_{HL}^{**} = \frac{1}{2\theta_H\theta_L}\Big(&2(p_{LL}^{**} + (p_{OHL}^{**}-p_{OLL}^{**})(a+b))(1-\alpha_H)(1-\alpha_L)) \\ & \cdot\theta_H\theta_L + b(p_{OHL}^{**}-p_{OLL}^{**})(1-\alpha_H)(1-\alpha_L) \\ & \cdot(-4\theta_H\theta_L + p_{OHL}^{**}(\theta_H+\theta_L) + p_{OLL}^{**}(\theta_H+\theta_L))\Big), \end{aligned}$$

and

$$p_{HH}^{**} = \frac{\begin{array}{c} -bp_{OHL}^{**}(1-\alpha_H)^2(p_{OHL}^{**}-2\theta_H) \\ + \Big(p_{HL}^{**} - p_{OHL}^{**}(a+b)(1-\alpha_H)^2\Big)\theta_H \end{array}}{\theta_H}.$$

In Case 3.3, where $q_{HL}^{**} = 2a, q_{LL}^{**} = 2a$, the equilibrium prices p_L^{**} and p_H^{**} are identical to these in Case 3.1. Overage prices are given below:

$$p_{OHL}^{**} = \frac{(b-a)(\alpha_L-\alpha_H)f_{HH}\theta_H\theta_L}{\begin{array}{l} b\Big(f_{HL}((1-\alpha_L)\theta_H - (1-\alpha_H)\theta_L) - f_{HH}((1-\alpha_L)\theta_H \\ -(1-\alpha_H)\theta_L)\Big) + a\,(f_{HL}(\alpha_H\theta_L + \alpha_L(\theta_H - \alpha_H\theta_H - \alpha_H\theta_L)) \\ + f_{HH}((1-2\alpha_H)\alpha_H\theta_L - ((1-\alpha_H)\theta_H + \alpha_H\theta_L)\alpha_L) \end{array}},$$

$$\begin{aligned} p_{OLL}^{**} = {} & (b-a)\,(f_{HH}+f_{HL})(\alpha_H-\alpha_H)\theta_H\theta_L / \\ & (b(2f_{LL}(-1+\alpha_L))\theta_H + f_{HH}(\theta_H - \alpha_L\theta_H + (-1+\alpha_H)\theta_L) \\ & + f_{HL}(\theta_H - \alpha_L\theta_H + (-1+\alpha_H)\theta_L) + a(2f_{LL}(-1+\alpha_L)\alpha_L\theta_H \\ & + f_{HH}(-2_L 2\theta_H - \alpha_H\theta_H + \alpha_L(\theta_H + \alpha_H\theta_H + \alpha_H\theta_L)) \\ & + f_{HL}(-2\alpha_L 2\theta_H - \alpha_L\theta_H + \alpha_L(\theta_H + \alpha_H\theta_H + \alpha_H\theta_H)). \end{aligned}$$

The three family plan prices are given by

$$p_{LL}^{**} = \frac{\begin{array}{c} b(1-\alpha_L)(p_{OLL}^{**}-\theta_L)^2 + (2a)p_{OLL}^{**}(1-\alpha_L^2)\theta_L \\ + a\alpha_L(p_{OLL}^{**2}(1-\alpha_L) - 2p_{OLL}^{**}(1-\alpha_L)\theta_L + \theta_L^2) \end{array}}{\theta L},$$

$$p_{HL}^{**} = \frac{1}{2\theta_H\theta_L}\left(\begin{array}{l} 2(p_{LL}^{**} + (2\alpha p_{OLL}^{**})(1-\alpha_H\alpha_L))\theta_H\theta_L \\ -b(p_{OHL}^{**}-p_{OLL}^{**})(2(2-\alpha_H-\alpha_L)\theta_H\theta_L \\ \quad -(p_{OLL}^{**}+p_{OHL}^{**})(1-\alpha_H\theta_H + (1-\alpha_H)\theta_L)) \\ -a(p_{OLL}^{**})(2(\alpha_H+\alpha_L-2\alpha_H\alpha_L)\theta_H\theta_L \\ \quad + p_{OHL}^{**}(-\alpha_H\theta_L + \alpha_L(-1+\alpha_H)\theta_H + \alpha_H\theta_H)) \\ + p_{OLL}^{**}(-\alpha_H\theta_L + \alpha_L((-1+\alpha_H)\theta_H + \alpha_H\theta_H)) \end{array}\right),$$

and

$$p_{HH}^{**} = \frac{\begin{array}{c} (b+a\alpha_H)p_{OHL}^{**}(1-\alpha_H)(2\theta_H - p_{OHL}^{**}) \\ + (p_{HL}^{**} - 2ap_{OHL}^{**}(1-\alpha_H^2))\theta_H \end{array}}{\theta_H}.$$

Please note that the overage prices for H and HH family plans can take any values because these consumers do not use overage in equilibrium. The firm's profits can be obtained by plugging in the equilibrium prices and overage price.

Finally, we describe a subcase that is not nested in the above analysis. In this subcase, the L consumers do not use any overage in either state. In this subcase, the H-type consumers' optimal consumption in the need state of b when selecting the L plan is $Q_{HLb} = b\left(1 - \frac{p_{OL}}{\theta_H}\right) \in (a, b]$. It is easy to see that $Q_{LLb} = q_L < Q_{HLb} \le b$.

Solving the firm's constrained optimization problem, we obtain $q_L^{**} = q_H^{**} = b$, $q_{HH}^{**} = 2b$, and $p_L^{**} = p_H^{**}$. In other words, there is only one individual plan, and there does not exist an equilibrium in which there are two different individual plans and L consumers never use any overage. This concludes the proof for Proposition 2.

Proof of Proposition 3

When the firm offers identical allowances for the HL and LL family plans, it is easy to see that $p_{LL}^{**} < p_{HL}^{**}, p_{OLL}^{**} < p_{OHL}^{**}$ cannot hold. Otherwise, the HL family would have switched to the LL family plan. Similarly, $p_{LL}^{**} > p_{HL}^{**}, p_{OLL}^{**} > p_{OHL}^{**}$ cannot hold. Otherwise, the LL family would have switched to the HL family plan. Now suppose $p_{LL}^{**} > p_{HL}^{**}, p_{OLL}^{**} < p_{OHL}^{**}$ holds. Given that the HL family segment is more likely to run into overages than the LL family segment regardless of the allowances, $(1 - \alpha_{HL}) > (1 - \alpha_{LL})$, it is easy to see that $U(\theta_H, \theta_L; q_F^{**}, p_{HL}^{**}, p_{OHL}^{**}) = U(\theta_H, \theta_L; q_F, p_{LL}^{**}, p_{OLL}^{**})$ and $U(\theta_L, \theta_L; q_F^{**}, p_{LL}^{**}, p_{OLL}^{**}) = 0$ cannot hold simultaneously. Because of its lower valuation, the LL family will not be willing to pay a higher price, $p_{LL}^{**} > p_{HL}^{**}$, for the same allowance, particularly because of its lower likelihood to use overages, and thus it does not benefit enough from a lower overage price $p_{OLL}^{**} < p_{OHL}^{**}$. Therefore, only $p_{LL}^{**} < p_{HL}^{**}, p_{OLL}^{**} > p_{OHL}^{**}$ holds in equilibrium.

Proof of Proposition 4

(i) It can be directly seen from Propositions 1 and 2 that $q_H^{**} = q_H^{*}$, and $p_H^{**} > p_H^{*}$ holds because of the second part of this proposition: $p_{OL}^{**} > p_{OL}^{*}$ and $p_L^{**} < p_L^{*}$. The increase in the overage price of the L plan has a more negative impact on the single H consumers (whose outside option is the L plan) compared with the single L consumers because $(1 - \alpha_H) > (1 - \alpha_L)$, and this allows the firm to charge a higher price for the H plan.

(ii) It can be directly seen from Propositions 1 and 2 that $q_L^{**} = q_L^{*}$;

$$p_{OL}^{**} - p_{OL}^{*} = \frac{\begin{array}{c}(b-a)(\alpha_L - \alpha_H)(1-\alpha_L)\theta_H^2\theta_L \\ \cdot\,(f_{HL} + 2f_{LL} - 2(f_{HH} + f_{HL} + f_{LL})\lambda)\end{array}}{\begin{array}{l}b((1-\alpha_H)\theta_L(1 - 2f_{HH} - f_{HL} + \lambda) \\ \quad -(1-\alpha_L)\theta_H(1 - 2f_{HH} + 2f_{LL} + 2\lambda)) \\ \quad \cdot\,(\theta_L(1 - \alpha_H - \lambda + \alpha_H\lambda) - (1-\alpha_L)\theta_H(1-2\lambda))\end{array}} > 0,$$

based on $p_{OL}^{**} > 0$ and $p_{OL}^{*} > 0$. Given $p_{OL}^{**} > p_{OL}^{*}$, the firm needs to decrease the L plan's price, $p_L^{**} < p_L^{*}$, to ensure that the L segment's IR-L constraint is binding. This concludes the proof of Proposition 4.

Proof of Proposition 5

From the envelope theorem, $\frac{\partial \Pi^{**}}{\partial f_{HH}} = p_{HH}^{**} - 2p_H^{*}$. Given the incentive compatibility constraint for the HH family segment, ICF-HH, it is easy to see that $p_{HH}^{**} - 2p_H^{*} < 0$. Otherwise, the two H members in the HH family would have bought the two single H plans and be better off. Therefore, $\frac{\partial \Pi^{**}}{\partial f_{HH}} < 0$; that is, profits with family plans decrease in the size of the HH family segment.

Define $\bar{f}_{HH}$ such that $\Pi^{*} = \Pi^{**}(\bar{f}_{HH})$. Recall that profits with two individual plans only, Π^{*}, do not depend on f_{HH}. Given $\frac{\partial \Pi^{**}}{\partial f_{HH}} < 0$, it is more profitable to offer multiple family plans when $f_{HH} < \bar{f}_{HH}$. When $f_{HH} > \bar{f}_{HH}$, it is more profitable to offer only two individual plans. To illustrate this result, consider the numerical example where $b = 3, a = 1, \lambda = 0.5$, $f_{LL} = 0.1$, $f_{HL} = 0.1$, $\alpha_L = 0.5$, $\alpha_H = 0.4$, $\theta_H = 2.5$, $\theta_L = 2.4$. In this case, the threshold $\bar{f}_{HH} = 0.0596$. When $f_{HH} = 0.02 < \bar{f}_{HH}$, $\Pi^{**} = 2.41816 > \Pi^{*} = 2.41389$. When $f_{HH} = 0.1 > \bar{f}_{HH}$, $\Pi^{**} = 2.41126 < \Pi^{*} = 2.41389$.

Endnotes

[1] See http://www.insight-corp.com/pr/2_12_15.asp, retrieved May 2018.

[2] See https://www.comscore.com/Insights/Blog/US-Smartphone-Penetration-Surpassed-80-Percent-in-2016, retrieved May 2018.

[3] Related to bundling is another recent literature on collaborative consumption, which allows buyers of a product to subsequently rent it out to nonbuyers when the product is not being used by the owner (e.g., Jiang and Tian 2018, Tian and Jiang 2018). This is similar to family plans in the sense that consumers are sharing their products, but it is different because of its explicit focus on durable goods and the fact that there are unserved customers in the market.

[4] We have also analyzed a model with positive, quadratic marginal costs and observe a similar set of results. We believe that the assumption of zero marginal cost is a better representation of the telecommunications industry.

[5] We have also analyzed a model in which the high need state is higher for the H segment than it is for the L segment. The results are essentially the same as those presented herein. Please refer to Online Appendix B2 for details.

[6] We have also analyzed a model where consumers' gross utility is $u(\theta_i; Q_i) = \theta_i\left(Q_i - \frac{Q_i^2}{2b}\right)$. The results remain essentially the same as those presented herein. Please refer to Online Appendix B3 for details.

[7] One implication of our utility framework is that a consumer's marginal utility decreases with consumption. In reality, when consumption occurs over time, then it is not clear that the highest valued shows, phone calls, etc., will necessarily be consumed before lower value phone calls. This can be seen as a limitation of our analysis but incorporating a random arrival rate makes our model intractable. We thank an anonymous reviewer for this observation.

[8] Throughout the main text, when referring to the consumption quantity, Q_{ijX}, we drop the X subscript when the need state is self-explanatory.

[9] See Online Appendices B and C.

[10] Note that even an "unlimited" plan in reality is not truly unlimited. Carriers typically throttle users' data speed once their data consumption exceeds a certain level. For instance, T-Mobile's unlimited plan starts to deprioritize users who exceed 30 GB of data in a month. See http://fortune.com/2017/03/09/how-t-mobile-unlimited-data-plan/, retrieved May 2018.

[11] Another situation in which consumers can bundle themselves is seen under "group buying," where a firm offers a quantity discount to consumers who are part of the group (e.g., Jing and Xie 2011, Hu et al. 2013, Chen and Zhang 2015). A central issue with group buying is to form a sufficiently large group. Our family plan has a three-part structure that

is limited to a family that faces uncertainty over consumption levels; it does not involve searching out individuals to form a family.

[12] Please refer to Online Appendix C for the complete analysis of 19 distinct cases, outlined there in Table C1, depending on the number of unique family plans being offered and their respective allowances.

[13] We have verified the second-order condition for every case presented throughout the appendix but have not provided them to save space.

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