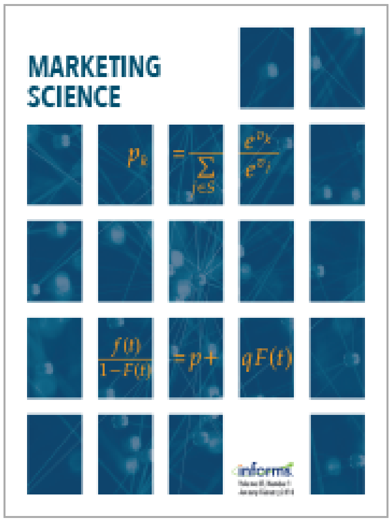

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Customized Advertising via a Common Media Distributor

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We show that when a single media content distributor (such as a television cable company or an Internet provider) delivers advertising messages on behalf of multiple competing brands, it can sometimes utilize customized advertising to implement monopoly pricing. Even though such monopolistic pricing can be implemented with varying degrees of customization of commercials, product revenues and consumer surplus are highest when the distributor chooses the highest level of customization feasible. Consumers would, obviously, prefer aggressive price competition in product markets. However, given that collusion on prices is facilitated anyway, when the distributor acts as a common agent, the welfare of consumers is enhanced when commercials are better aligned with their preferences.

Key words: personalized advertising; media content distributor; common agency; price competition
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1. Introduction

In this paper we investigate how media content distributors can utilize customized advertising to alleviate price competition in product markets. We focus, in particular, on distributors that act as common agents for competing advertisers by delivering commercials on their behalf. Sophisticated information technologies, combined with greater availability of information concerning consumption habits of consumers, offer tremendous opportunities to deliver targeted ads to different segments of the population. Television cable companies, radio broadcasters, and Internet providers can utilize the new technologies to deliver commercials to consumers that are better aligned with their preferences.

Consider, for instance, the emergence of the digital personal video recorder (PVR) technology in the television market. Because PVRs are programmable, they can record and transmit household viewing patterns.[1] Data mining techniques can be used to predict a household's characteristics from its viewing choices, and customized advertising that is relevant to the household's type can be sent to its PVR.[2] Such targeted advertisements are more likely to be viewed and influence purchasing behavior than nontargeted ones. They are also more efficient because fewer placements are necessary to achieve the same number of exposures to the desired demographic group. The emergence of the PVR has led to the creation of several start-up companies such as ACTV, Vision World, and MSA-Jovio to develop the necessary software to support customized advertising, with the anticipation that cable operators will find it profitable to subsidize the cost of the PVR to customers.[3]

The main result of our investigation is that under certain circumstances the distributor can implement monopoly pricing in product markets by controlling the extent of information that consumers have about competing products. Specifically, when consumers have only limited outside information about the specific product class and rely heavily on the commercials delivered by the distributor to gain such

[1] TiVo announced recently that it will sell to advertisers and broadcasters second-by-second information on the television commercials and shows its users are watching or skipping. (TiVo Plans to Sell Information on Customers' Viewing Habits, *New York Times*, June 3, 2003.)

[2] Some privacy concerns might arise with the use of the profiling system that monitors and analyzes viewing behavior (see Franzak et al. 2001, Phelps et al. 2000 for a summary of such concerns in the context of online shopping).

[3] Several patents that are related to individually addressable digital recording devices have been already granted (Patent No. 6,002,393, Hite et al. System and Method for Delivering Targeted Advertisements to Consumers Using Direct Commands, December 14, 1999).

information, monopoly pricing in the product market arises as an equilibrium. This result is related to the literature on a common agency in oligopoly (see Bernheim and Whinston 1985, 1986; Gal-Or 1991; Villas-Boas 1994). This previous literature has discussed the role of the common agent in implementing collusion among competitors and maximizing their joint profits. To obtain the maximization of industry profits, the common agent in this previous literature determines the terms of its agreement with the firms contingent upon product market variables such as sales volumes or prices. In contrast, in our setting the media distributor can condition its relationship with the advertisers only on the levels of advertising they choose to deliver to different groups of consumers. In spite of the more restricted class of variables that can be utilized in the contracts, the distributor can still sometimes implement monopoly pricing. However, given that such pricing is obtained by withholding information from some consumers, there is increased likelihood that consumers remain ignorant of the existence of the products and refrain from consumption altogether. Hence, in spite of monopoly pricing, industry profits are not necessarily maximized anymore.

We are familiar with only one other paper in the marketing literature that relates to targeted advertising. Similar to our investigation, Iyer et al. (2005) also consider the implications of targeted advertising on the extent of competition in product markets. However, whereas in our setting the media distributor acts as a common agent in delivering targeted ads to consumers, in Iyer et al. the advertisers themselves have the ability to target advertising messages to consumers. Their finding that advertisers choose to advertise more to consumers who have a strong preference for their products is similar to the one we obtain.

While the concept of targeted advertising received only scant attention in the marketing literature, the general idea of targeting individual consumers with tailored offers has already been widely discussed (see, for instance, Blattberg and Deighton 1991, Chen et al. 2001, Lederer and Hurter 1986, Montgomery 1997, Zhang and Krishnamurthi 2004). The main emphasis of this literature, however, has been on evaluating how targeted marketing can facilitate the practice of price discrimination by firms. In contrast, in our model, firms have to quote a single price to all consumers, even when customizing the type of commercials delivered to them (Iyer et al. consider targeted advertising with and without price discrimination).

The literature on targeted marketing uses purchase histories, panel data, or other types of consumer behavior as a basis for targeting (see, for instance, Blattberg and Deighton 1991, Bult and Wansbeek 1995, Rossi and Allenby 1993, Rossi et al. 1996). Recent research has also begun to focus on internet advertising, which presents other types of customer behavior that can be analyzed and modeled (Chickering and Heckerman 2003). Our theoretical model places no restrictions on the type of marketing related behavior that can be utilized as a basis for targeting ads. In particular, in the case of the PVR example mentioned above, it is television viewing data that are used in targeting commercials to different demographic groups.

In our model, advertising plays only an informative role of educating consumers of the attributes of the advertised products. Our assumptions are similar to those considered in Dukes and Gal-Or (2003) or Grossman and Shapiro (1984). Alternative models, where advertising plays a persuasive role, aimed at enhancing the willingness to pay of customers or the cross-price elasticity between products, have also been considered in the literature (see, for instance, Banerjee and Bandyopadhyay 2003, Fruchter 1999, Fruchter and Kalish 1997, Shaffer and Zettelmeyer 2004, von der Fehr and Stevik 1998). The idea of tailoring the type and volume of ads to the preferences of viewers was not considered in those earlier papers.

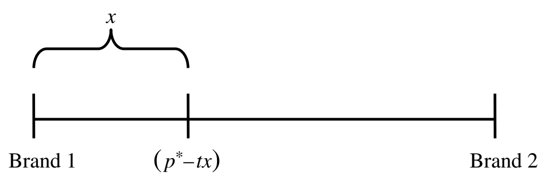
The rest of this paper is organized as follows. In the next section, we describe the main assumptions of the model. In §3 we derive the equilibria, and in §4 we conclude. The appendixes include proofs and several extensions of the basic model.

2. Model

Consider a media content distributor who has the capability to deliver targeted advertising to consumers. Two advertisers contemplate their advertising strategies with the distributor. The strategy selected by each determines both the overall level and the extent of targeting of its advertising messages to different groups of consumers.

The two advertisers offer competing brands of a given product class. We model the competition between them by using a Hotelling model. Specifically, the population of consumers is uniformly distributed on a line of unit length, with the two competing brands located at the edges of this line. Each consumer is willing to pay a maximum price of p^* for an "ideal" type of product, the characteristics of which exactly match her location on the line. This willingness to pay declines linearly, at the rate of $\$t$ per unit of distance, when the consumer buys a brand located at a point different than her location on the line. Figure 1 illustrates the willingness to pay of a representative consumer. In the sequel, we derive a condition to guarantee that at the symmetric equilibrium advertisers find it optimal to serve the entire population of

Figure 1 Competition in the Product Market



consumers. This condition is stated as

$$p^* > 2t. \tag{A1}$$

Our model focuses only on the informative role of advertising. Specifically, commercials inform consumers of the attributes and the price of the advertised product. Equipped with this information, consumers choose whether and which of the two brands to purchase. The larger the number of commercials sent to the consumer pertaining to this brand, the more likely the consumer is to be familiar with the brand. Consider, for instance, the PVR example discussed earlier. Because PVRs are programmable, they can record and transmit household viewing patterns. Data mining techniques can be used, in turn, to predict a household's characteristics from its viewing choices. Those techniques can enable the distributor to predict, in particular, the relative preference of the consumer between the two competing brands, as reflected by her location x on the line. For instance, if Figure 1 depicts competition between two brands of cereal, one having higher sugar content than the other, then the distributor can differentially send a larger number of commercials pertaining to the more sweetened brand to households whose viewing patterns indicate an interest in children's programming. Similarly, if the figure depicts competition between two types of automobiles, the distributor can differentially send a larger number of commercials pertaining to sports-like cars (e.g., convertibles) to young couples without children or to unmarried individuals. In contrast, mature adults without children are more likely to receive commercials pertaining to large sedans.

If a consumer is familiar with only one of the brands, she purchases it as long as its price p_i falls short of the consumer's willingness to pay for this brand, namely if $p^* - tx - p_i > 0$. If the consumer is familiar with both brands, she purchases brand i if it offers her a positive net payoff that exceeds the payoff she could derive from the competing brand j. Specifically, if $p^* - tx - p_i \geq p^* - t(1-x) - p_j$ and $p^* - tx - p_i > 0$. We also assume that brand producers cannot price discriminate, implying that they post a single price common to all consumers.[4]

[4] Iyer et al. (2005) discuss price discrimination in the context of targeted advertising; Chen et al. (2001) do it in the context of targeted marketing.

In the absence of any advertising with the distributor there is probability α that a consumer is familiar with a given brand. This probability is the same for both products. We refer to α as the base awareness level of consumers.[5] This base awareness level is implied by the existence of alternative sources of information that are available to consumers, including "word of mouth," and commercials on other media outlets such as newspapers, radio, and TV. Advertising with the distributor increases the probability that consumers are familiar with the advertised brand. The increased probability for a given consumer depends on the number of commercials that are transmitted to her by the distributor. Specifically, we assume that a larger number of transmitted commercials results in a more significant rise in the probability that the consumer becomes familiar with the advertised product.

We formulate the advertising strategy of i in terms of two parameters, a_i and b_i, which determine the extent to which different consumers are exposed to commercials of brand i. Specifically, the differential transmission of commercials to consumers results in a differential increase of the probability that a consumer becomes familiar with i. This increase for a consumer who is located at distance x from i amounts to $(a_i - b_i x)$. Hence, a higher value of a_i implies an overall increase in the number of commercials that are transmitted to all consumers. In contrast, a higher value of b_i indicates that the distributor differentiates more significantly between different types of consumers. Those who are judged by the profiling technology to have a stronger preference for i are exposed to many more commercials about this product than are the consumers who are identified as having a stronger preference for the competing brand j. Therefore, the parameter b_i measures the extent of targeting that is selected by advertiser i. The parameters a_i and b_i cannot violate the requirement that probabilities lie in the interval between zero and one. Moreover, the probability of reaching a consumer cannot fall short of her initial awareness level α. The restrictions imposed in A2 guarantee that to be the case:

$$\begin{gathered} 0 \leq a_i \leq (1-\alpha) \\ 0 \leq b_i \leq a_i. \end{gathered} \tag{A2}$$

The costs of the distributor might depend on the advertising strategies selected by the two advertisers. We designate those costs as $TC(a_i, b_i, a_j, b_j)$ and assume that this function is nondecreasing with a_k; $k = 1, 2$. Airing additional commercials on behalf of either one of the two brands could increase the costs

[5] It is possible to extend the analysis to allow for different base awareness levels on the part of consumers. This can be done by assuming that the initial probabilities of awareness are different across brands, namely, $\alpha_1 \neq \alpha_2$.

of the distributor for two reasons. First, if the distributor faces limited capacity of advertising slots, additional commercials allocated to one of the brands forces it to give up advertising revenues, accruing possibly from other product markets. Second, if the distributor's customers consider commercials a nuisance because they interrupt regular programming (such as commercial breaks on TV or radio, or pop-up ads on the Internet), they perceive the distributor to be less attractive if it raises its level of advertising. Hence, even in the absence of capacity constraints on advertising slots, the distributor could experience a decline of its customer base when increasing the number of commercials it airs. Such a decline, in turn, is likely to lower its rating[6] and, as a result, the price the distributor can command from advertisers. We summarize this opportunity cost facing the distributor by assuming the following:

$$\frac{\partial TC}{\partial a_k} \geq 0. \tag{A3}$$

An increase of the targeting parameters could have two counteracting effects on the cost facing the distributor. On one hand, an increase in b_k implies that advertiser k airs fewer commercials, thus potentially freeing up advertising space and reducing the opportunity cost of the distributor. On the other hand, higher values of b_k imply also that the distributor needs to invest additional resources to improve the profiling engine because a more precise identification of market segments is required when the type of advertising varies significantly across segments. We assume that the former favorable effect on the distributor's cost more than outweighs the latter unfavorable effect so that:[7]

$$\frac{\partial TC}{\partial b_k} \leq 0. \tag{A4}$$

We also assume that when a given advertiser increases both parameters by the same amount, the costs of the distributor can only rise; specifically:

$$\frac{\partial TC}{\partial a_k} + \frac{\partial TC}{\partial b_k} \geq 0. \tag{A5}$$

Consider, for instance, the case that the costs of the distributor consist of two components: a component that is independent of the value of the advertising parameters, measuring the fixed costs of developing the profiling engine, and a component measuring the opportunity costs of carrying the commercials of the two advertisers. The latter component is proportional to the average number of commercials that the two advertisers select. If each advertiser chooses to send some commercials to all viewers (i.e., if $a_i - b_i \geq 0$), the average number of advertising slots consumed by the two advertisers is proportional to $[(a_1 - b_1/2) + (a_2 - b_2/2)]$, and the costs incurred by the distributor can be expressed as follows:

$$TC(a_1, b_1, a_2, b_2) = F + \beta[(a_1 - b_1/2) + (a_2 - b_2/2)], \tag{1}$$

where the parameter β can be related to the price the distributor could have charged from advertisers in other industries or to the decline of its rating due to the loss of audience that additional commercials entail. The above linear specification satisfies all our earlier assumptions concerning the cost schedule $TC(\cdot)$.

For simplicity, we assume that, other than their payments to the distributor, advertisers incur no production costs.[8]

We model the game as consisting of three stages. In the first stage, the distributor makes a take-it-or-leave-it offer to the advertisers, specifying their payment schedule as a function of their selected advertising parameters a_i and b_i. In the second stage, the two brand producers choose their advertising strategies, and in the third and final stage, the two brands compete as Bertrand competitors, after observing the advertising strategies chosen in the second stage of the game. We assume that the distributor is prohibited from offering a different payment schedule to the two competing advertisers. This constraint can be implied by antitrust regulation or by "most favored nation" type of clauses between the distributor and the advertisers of a given industry.[9] We designate the payment schedule chosen by the distributor as $R(a_i, b_i)$. We do not impose any restrictions on the shape of this schedule.[10] In particular, consistent with empirical findings

[6] See Dukes and Gal-Or (2003) and Liu et al. (2004) for models that tie the revenues of broadcasters to their overall levels of advertising.

[7] In Iyer et al. (2005) advertising costs as incurred by the advertisers are proportional to the size of the market being targeted. Hence, intensified targeting reduces the costs incurred by the advertiser.

[8] This assumption can be easily relaxed without changing any of our results.

[9] This assumption guarantees that one advertiser can always observe the terms of the agreement offered to the competing advertiser, because those terms are identical to those he obtains for himself. In the technical appendix at http://mktsci.pubs.informs.org, we consider the possibility that one advertiser cannot observe the terms of the agreement offered to his competitor or the levels of the advertising parameters that the competitor selects. We show that even in this case, the distributor might be able to implement monopoly pricing in the product market.

[10] Notice that because the distributor is the entity who delivers the ads to the PVR according to the request of the advertiser, it obviously can verify the committed levels of a_i and b_i, and charge advertiser i accordingly.

of Koschat and Putsis (2002), we allow the schedule to be nonlinear.

Given that consumers are a priori equally familiar with the two products (i.e., ($\alpha_1 = \alpha_2 = \alpha$)) and given that the distributor cannot discriminate between the two advertisers, we restrict attention to the derivation of symmetric equilibria. At such equilibria, advertisers choose identical advertising parameters and prices for their products.

3. Solution

Because we focus only on the derivation of subgame perfect equilibria, we start by considering the third stage of the game. We characterize the competition in this stage as a function of the advertising strategies chosen by the two brand producers in the second stage of the game.

A consumer who is familiar with both brands will purchase brand 1 if her location on the line in Figure 1 satisfies the inequality

$$x \le \left(\frac{1}{2} + \frac{p_2 - p_1}{2t}\right).$$

Hence, given the advertising parameters a_i, b_i $i = 1, 2$ selected in the second stage, the proportion of the population that is fully informed and buys brand 1 is equal to

$$A_1 \equiv \int_0^{\frac{1}{2}+(p_2-p_1)/2t} (\alpha + a_1 - b_1 x)(\alpha + a_2 - b_2(1-x))\, dx.$$

The integrand of A_1 is the probability that a type x consumer is familiar with both products, and the upper limit of the integration is determined by the threshold consumer who is just indifferent between the two brands.

A consumer who is familiar with only brand 1 will buy it, assuming that equilibrium prices are sufficiently low so that $p_1 + tx \le p^*$. In the sequel, we show that Assumption A1 guarantees this inequality for all x at the symmetric equilibrium. Hence, the proportion of the consumers that buy brand 1 while being informed only of its existence and not of its competitor amounts to

$$\begin{aligned} B_1 &\equiv \int_0^1 [\alpha + a_1 - b_1 x][1 - (\alpha + a_2 - b_2(1-x))]\, dx \\ &= \left(\alpha + a_1 - \frac{b_1}{2}\right) - \left[\left(\alpha + a_1 - \frac{b_1}{2}\right) \right. \\ &\qquad \left. \cdot \left(\alpha + a_2 - \frac{b_2}{2}\right) - \frac{b_1 b_2}{12}\right]. \end{aligned}$$

The integrand of B_1 is the probability that consumer x is familiar with brand 1 but not with brand 2, and the integration is over the entire population of consumers.[11]

The above derivation implies that the payoff function of brand producer 1 gross of his payment to the distributor is equal to:[12]

$$\pi_1 = (A_1 + B_1) p_1. \tag{2}$$

Similarly, the payoff function of brand producer 2 gross of his payment to the distributor can be derived as

$$\pi_2 = (A_2 + B_2) p_2, \tag{3}$$

where

$$\begin{aligned} A_2 &\equiv \int_0^{\frac{1}{2}+(p_1-p_2)/2t} (\alpha + a_2 - b_2 x)(\alpha + a_1 - b_1(1-x))\, dx, \\ B_2 &\equiv \int_0^1 [\alpha + a_2 - b_2 x][1 - (\alpha + a_1 - b_1(1-x))]\, dx \\ &= \left(\alpha + a_2 - \frac{b_2}{2}\right) \\ &\quad - \left[\left(\alpha + a_1 - \frac{b_1}{2}\right)\left(\alpha + a_2 - \frac{b_2}{2}\right) - \frac{b_1 b_2}{12}\right]. \end{aligned} \tag{4}$$

Producer i chooses his price to maximize π_i, yielding the following two first-order conditions:

$$(A_i + B_i) - \frac{p_i}{2t}(\alpha + a_1 - b_1 \hat{x})(\alpha + a_2 - b_2(1-\hat{x})) = 0, \quad i = 1, 2, \tag{5}$$

where

$$\hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}.$$

Because the distributor is restricted from offering different payment schedules to the advertisers and because the advertisers face identical production costs (zero for both) and basic awareness levels on the part of consumers ($\alpha_1 = \alpha_2 = \alpha$), we focus primarily on the characterization of symmetric equilibria,[13] where

[11] In the above derivations we implicitly assume that advertisers choose to send messages to all consumers, including those who are located far from them (i.e., $\alpha + a_i - b_i x > \alpha$ for all $x_i < 1$). In the technical appendix at http://mktsci.pubs.informs.org, we derive a restriction to guarantee this outcome.

[12] Note that the payment to the distributor depends on the advertising parameters selected by the producer but not on his price. Hence, brand producers ignore their payments to the distributor when choosing prices in Stage 3. In essence, those payments are considered sunk at this stage.

[13] In the sequel, we show that even when restricting attention to symmetric equilibria, monopoly pricing in the product market can be implemented by the distributor. Our analysis can be extended to allow for asymmetry in the payment schedules offered by the distributor (see the technical appendix at http://mktsci.pubs.informs.org) or asymmetry in the inherent characteristics of the advertisers (by assuming that $\alpha_1 \neq \alpha_2$, for instance).

$a_1 = a_2 = a$, $b_1 = b_2 = b$, $p_1 = p_2 = p$, and $\hat{x} = 1/2$. Evaluating (5) at the symmetric equilibrium yields

$$p = \frac{2t\left[\left(\alpha + a - (b/2)\right) - \frac{1}{2}\left(\alpha + a - (b/2)\right)^2 + \frac{1}{24}b^2\right]}{\left(\alpha + a - (b/2)\right)^2}. \quad (6)$$

Note that the equilibrium price in (6) is higher than the transportation parameter t. This is in contrast to the outcome obtained in an environment with perfectly informed consumers, when equilibrium prices are equal to the transportation parameter t. Because in our model, consumers might sometimes be informed of only one brand, producers take advantage of this possible ignorance to raise prices above t. Also, to guarantee that every consumer participates in the market even when she is informed of only one product, we require that $p \leq p^* - t$. Because from (6), $p \geq t$, the region $t \leq p \leq p^* - t$ is nonempty only if $p^* \geq 2t$ (Assumption A1).

In Lemma 1 we summarize the restrictions on the advertising parameters that guarantee that (i) every consumer who is informed of at least one product participates in the market and (ii) producers choose to send at least some messages to every consumer.

Lemma 1. *To guarantee complete coverage of the market, both in terms of full participation of consumers and delivery of commercials by producers, the following restrictions on the advertising parameters have to be imposed at the symmetric equilibrium*:

$$\mathrm{Max}\{b + \alpha, LB\} \leq a + \alpha \leq 1, \quad (7)$$

where

$$LB \equiv \frac{b}{2} + \frac{t}{p^*} + \sqrt{\left(\frac{t}{p^*}\right)^2 + \frac{b^2 t}{12p^*}}. \quad (8)$$

In Figure 2, we describe the feasible region of the advertising parameters. In view of Assumption A1, either panel B or C of the figure can arise, implying that the feasible set is nonempty. Note that the lower boundary of this set corresponds to either monopoly pricing by producers (when $a + \alpha = LB$, $p = p^* - t$) or maximum level of targeting selected by them (when $a = b$, advertisers choose the highest level of targeting consistent with Assumption A2). In panel B, the lower boundary consists only of parameter values that implement monopoly pricing. In panel C, the lower boundary consists also of values that yield maximum targeting (when $b > b^2_{\max}$). The threshold level of the ratio (t/p^*) that distinguishes between panels B and C is designated in the figure by $\hat{v}(\alpha)$. This threshold level is a positive fraction less than 1/2, whose value depends upon α. In particular, when $\alpha = 0$ $\hat{v}(\alpha) = 3/13$.

It might be interesting to further investigate how the equilibrium prices chosen in the third stage of the game vary when the advertising parameters change. Total differentiation of system (5) around the symmetric equilibrium allows us to conduct a comparative statics analysis to evaluate the relationship between equilibrium price and the advertising parameters. We report the results of this analysis in Proposition 1.

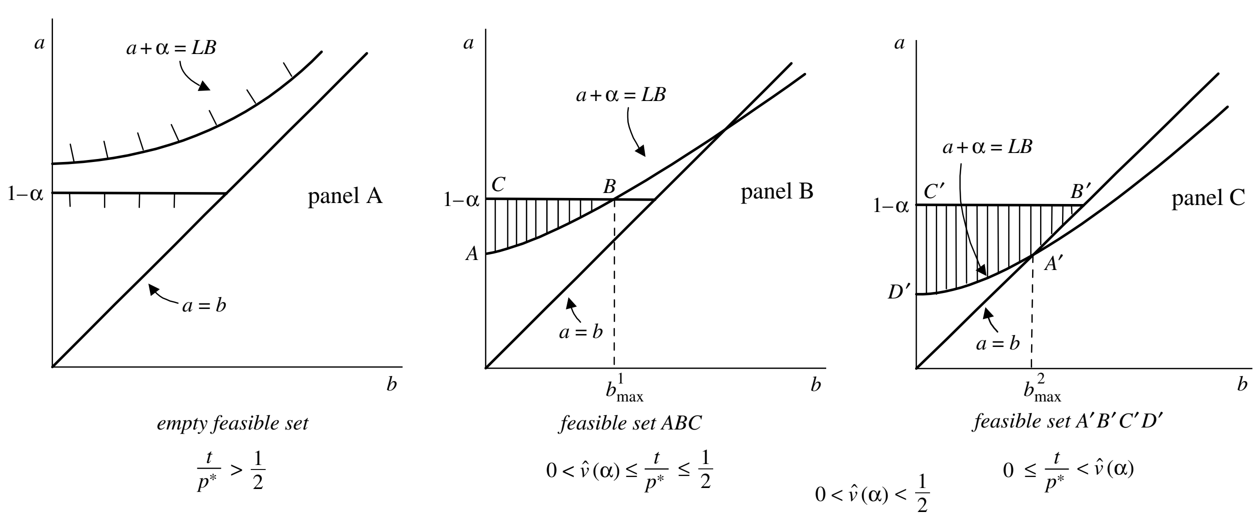
Proposition 1. *Changes in the advertising parameters a_i, b_i have the following effect on prices around the symmetric equilibrium*:

(i) $\dfrac{\partial p_j}{\partial a_i} \leq \dfrac{\partial p_i}{\partial a_i} \leq 0.$

(ii) $\dfrac{\partial p_j}{\partial b_i} \geq \dfrac{\partial p_i}{\partial b_i} \geq 0.$

According to Proposition 1, when advertisers reduce their overall levels of advertising and intensify the extent of targeting of their ads, they compete less aggressively in prices (a similar result is also derived in Iyer et al. 2005). When consumers receive fewer commercials, they are less informed about the options

Figure 2 Feasible Range of the Advertising Parameters



available to them in the marketplace. As a result, brand producers can raise their prices without having to worry about losing customers. Intensified targeting has the effect of reducing the number of commercials received by consumers who don't have a strong preference for either one of the two brands. Because loyal customers are inclined to buy their favorite products irrespective of their prices, it is usually the consumers with moderate tastes who determine how aggressively brand producers have to compete in prices. It is interesting to note that a change in the advertising strategy of one producer affects the competitor's price to a greater extent than it affects the producer's own price.

Having solved the third stage of the game, we can now consider the second stage, when each brand producer chooses his advertising strategy. This choice depends on the anticipated effect of the strategy on price competition in the third stage as well as on the payment schedule offered by the distributor in the first stage of the game.

Let $\pi_i(a_i, b_i, a_j, b_j)$ designate the revenues that accrue to each brand producer at the third stage equilibrium, then using system (5),

$$\pi_i(a_i, b_i, a_j, b_j) \equiv (\alpha + a_1 - b_1\hat{x})(\alpha + a_2 - b_2(1-\hat{x}))\frac{p_i^2}{2t}, \quad (9)$$

where

$$\hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t},$$

and p_1 and p_2 solve (5). Given the payment schedule $R(a_i, b_i)$ offered by the distributor, the second-stage payoff function of advertiser i becomes

$$u_i(a_i, b_i, a_j, b_j) = \pi_i(a_i, b_i, a_j, b_j) - R(a_i, b_i) \qquad i = 1, 2. \quad (10)$$

The advertiser chooses his strategy as expressed by (a_i, b_i) to maximize (10), yielding the following first-order conditions at an interior equilibrium:

$$\frac{\partial u_i}{\partial a_i} = \frac{\partial \pi_i}{\partial a_i} - \frac{\partial R}{\partial a_i} = 0; \qquad \frac{\partial u_i}{\partial b_i} = \frac{\partial \pi_i}{\partial b_i} - \frac{\partial R}{\partial b_i} = 0. \quad (11)$$

To guarantee that the equilibrium is interior, the objective defined in (10) should be a concave function of the decision variables a_i and b_i.

It is noteworthy that brand producer i chooses positive levels of advertising only if doing so enhances his profits. In the absence of advertising, the expected payoff of i amounts to $\pi_i(0, 0, a_j, b_j)$. Hence, producer i will choose to transact with the distributor only if

$$u_i \geq \pi_i(0, 0, a_j, b_j). \quad (12)$$

Conditions (11) and (12) constrain the maximization problem of the distributor in the first stage of the game. Those conditions have been referred to in the literature as incentive compatibility (IC) and individual rationality (IR) constraints.

In the first stage, the distributor chooses the payment schedules to maximize its proceeds from the advertisers net of the costs it incurs in carrying their ads. It chooses $R(a_i, b_i)$, therefore, to maximize

$$\text{Max} \quad W = R(a_1, b_1) + R(a_2, b_2) - TC(a_1, b_1, a_2, b_2),$$
$$\text{s.t. IC} + \text{IR constraints.}$$

It is clear that the solution to the above maximization entails extracting the most surplus possible from each brand producer, implying that the individual rationality constraint (12) is binding. We can express, therefore, the above objective of the distributor as follows:

$$\text{Max} \quad W = \sum_{\substack{1=1,2\\ j\neq i}} [\pi_i(a_i, b_i, a_j, b_j) - \pi_i(0, 0, a_j, b_j)] - TC(a_1, b_1, a_2, b_2), \quad (13)$$

s.t. a_1, b_1, a_2, b_2 solve the following conditions:

$$\frac{\partial \pi_i}{\partial a_i} = \frac{\partial R}{\partial a_i}; \qquad \frac{\partial \pi_i}{\partial b_i} = \frac{\partial R}{\partial b_i} \quad i = 1, 2. \quad \text{(IC)}$$

Note that the slope of the payment schedule $R(\cdot)$ determines the levels of the advertising parameters (a_i, b_i), as selected by advertiser i. Hence, the distributor can implement any desired pair of advertising parameters by selecting appropriate steepness of the schedule $R(\cdot)$. Its maximization problem can be interpreted, therefore, as a direct choice of the advertising parameters instead of the indirect choice of those parameters as implied by the schedule[14] of $R(\cdot)$.

It is noteworthy that the distributor's objective function in (13) coincides with total industry profits when $\pi_i(0, 0, a_j, b_j) = 0$. When producers do not have any viable outside alternative to reaching consumers, other than through the distributor, $\alpha = 0$ and the distributor can extract the entire profits generated in the industry.

While it is difficult, in general, to derive the expression for the "outside option" of the non advertising producer (the expression $\pi_i(0, 0, a, b)$), there are circumstances in which such a characterization is relatively simple. In Lemma 2 we state a condition that guarantees that when only one producer advertises, the equilibrium is characterized by significant

[14] An example of a payment schedule that will implement the pair (a_i, b_i) is a linear schedule specified as $R(a, b) = K + \alpha a + \beta b$, where the slopes α and β are selected to guarantee that desired pair (a_i, b_i) solves (9), and the fixed payment K guarantees that the (IR) constraint is binding. More sophisticated payment schedules can also implement the same outcome.

undercutting of prices on the part of the nonadvertising producer—so much so that this producer dominates the entire market of consumers who are fully informed of both products (if producer 1 is the only one to advertise, $\hat{x} = 0$ in (9), in this case). As a result, the advertising producer chooses to obtain revenues only from those consumers who are uninformed of the competing, nonadvertised brand (by charging them the highest price the market can bear, which is equal to $p^* - t$). The nonadvertising producer obtains revenues from all the consumers who are aware of his product, including those who are also familiar with the competing, advertised product. By significantly undercutting the price of the advertising producer and charging $p^* - 2t$, the producer dominates the entire market of fully informed consumers in this case.

Lemma 2. *If* $(t/p^*) > 2\alpha/(2 - 3\alpha)$, *then* $\pi_i(0, 0, a, b) = \alpha(p^* - 2t)$.

Lemma 2 states that for sufficiently small values of α and/or large values of the ratio t/p^*, the "outside option" $\pi_i(0, 0, a, b)$ of the nonadvertising producer is determined independently of the values of the advertising parameters selected by the advertising producer. As explained earlier, under the condition of the lemma, the nonadvertising producer charges the price $(p^* - 2t)$ from a proportion α of the population. The condition of the lemma holds definitely when $\alpha = 0$, namely when consumers do not have any other sources of information about the products. However, it may be valid even for positive values of α. For instance, when $t/p^* = 1/6$ the condition holds as long as $\alpha < 2/15$, and when t/p^* assumes its highest possible value of 1/2, this condition holds as long as $\alpha < 2/7$.

Given our focus on the derivation of symmetric equilibria, we substitute symmetry into (13), while utilizing the expression derived for equilibrium product prices in (6), to obtain the following objective of the distributor:

$$\underset{a,b}{\mathrm{Max}}\, w(a, b) = 4t\left[1 - \frac{1}{2}\left(\alpha + a - \frac{b}{2}\right) + \frac{b^2}{24(\alpha + a - b/2)}\right]^2 - 2\pi_i(0, 0, a, b) - G(a, b); \tag{14}$$

where $G(a, b) \equiv TC(a, b, a, b)$.

The condition derived in Lemma 2 guarantees that the objective function of the distributor is a strictly decreasing function of a and a strictly increasing function of b.

Lemma 3. *If*

$$\left(\frac{t}{p^*}\right) > \frac{2\alpha}{2 - 3\alpha},$$

the function $w(a, b)$ is a strictly decreasing function of a and a strictly increasing function of b.

It is noteworthy that the condition of the Lemma 3 is a sufficient condition that is much stronger than is necessary to guarantee the monotonicity[15] of the function $w(\cdot)$. As long as the derivatives of the outside option function $\pi_i(0, 0, a, b)$ are sufficiently flat, i.e.,

$$\frac{\partial \pi_i(0, 0, a, b)}{\partial a} \geq -k_\alpha \quad \text{and} \quad \frac{\partial \pi_i(0, 0, a, b)}{\partial b} \leq k_\beta$$

for some positive k_α and k_β, the monotonicity of $w(\cdot)$ holds.

Given the monotonicity of the objective function $w(\cdot)$, the solution to the distributor's maximization problem is a "corner solution." We use the feasible region described in Lemma 1, to characterize this solution in Proposition 2.

Proposition 2. (i) *If*

$$\frac{t}{p^*} > \frac{2\alpha}{2 - 3\alpha},$$

the advertising strategies that are selected at the equilibrium yield the highest possible product price that is consistent with full coverage of the population of consumers, namely $p = p^* - t$. The advertising parameters that implement this outcome lie on the lower boundary of the feasible region described in Figure 2. Because this lower boundary is upward sloping (i.e., $\partial a/\partial b > 0$), intensified targeting at the equilibrium results in more commercials being delivered to loyal customers.

(ii) *The expected revenues that accrue to each product at the equilibrium can be expressed as a function of the targeting parameter b and the ratio (t/p^*) as follows:*

$$\pi(b) = \frac{p^*}{2}\left(1 - \frac{t}{p^*}\right)^2 \left[1 + \sqrt{1 + \frac{b^2}{12(t/p^*)}}\right]^2 \left(\frac{t}{p^*}\right). \tag{15}$$

Hence, equilibrium product revenues increase with the value of the targeting parameter b.

According to Proposition 2, when acting as a common agent for the two producers, the distributor may be able to implement the collusive monopoly price in the product market when the base awareness level of consumers is sufficiently low. This result is related to the one obtained already in the literature on a common agency in oligopoly. The distributor in our setting acts as a common agent to alleviate the extent of price competition in the product market. However, in contrast to the common agency literature, monopoly pricing in our setting cannot guarantee the highest

[15] Similarly, the assumptions made concerning the function $TC(\cdot)$ (A3, A4, A5) are also stronger than necessary to guarantee the monotonicity of $w(\cdot)$. In particular, if in violation of A4 $\partial TC/\partial b_k > 0$, $\partial w/\partial b$ may still be positive if the first expression of $w(\cdot)$ in (14) is sufficiently steep.

possible revenues that would accrue if brand producers did not compete on prices. In the absence of price competition, each producer would set the price $(p^* - t/2)$, yielding the revenues $(p^*/2)(1 - t/(2p^*))$ to each.

This level of revenues is higher than that reported in (15), even for the highest possible value of b. A common agent would be able to implement this outcome only if the terms of its relationship with the producers could be determined contingent upon market variables such as sales volumes or prices (this is the standard assumption made in earlier common agency models such as Bernheim and Whinston 1985 and 1986, for instance). In our setting, the payment transferred between the producers and the distributor is determined as a function of advertising levels but not as a function of sales volumes or prices. As a result, while the agent can alleviate price competition, it cannot quite implement maximization of industry profits.

The vehicle used by the agent to alleviate price competition is the extent of information consumers have about competing products. By reducing the information available via advertising, consumers are more likely to be uninformed about competing products, thus encouraging producers to raise prices. Part (ii) of the proposition asserts that intensified targeting implies that additional commercials can be transmitted to viewers about their preferred brands without necessarily leading to more aggressive pricing by brand producers, because consumers having moderate tastes are still poorly informed, given that they are exposed to relatively few commercials. With intensified targeting, the distributor finds it optimal, therefore, to increase the exposure of loyal consumers to commercials about their preferred brands ($\partial a/\partial b > 0$ along the lower boundary of the feasible region). Because commercials are transmitted more efficiently to match the preferences of viewers, higher expected revenues can be achieved at the equilibrium.

Proposition 2 describes different combinations of the advertising parameters that can arise at the symmetric equilibrium. All of these combinations yield monopoly pricing in product markets. In general, it is unclear which pair of advertising parameters from the different combinations maximizes the distributor's objective in (14). Because this objective declines with a and increases with b, whereas the advertising parameters a and b move in the same direction when implementing monopoly pricing, the pair that maximizes the distributor's objective depends on the exact shape of the cost function $TC(\cdot)$ facing the distributor. For the linear cost function specified in (1), it is easy to show that the distributor will implement the maximum amount of targeting[16] consistent with the monopoly price $p^* - t$. We summarize this result in Corollary 1.

Corollary 1. *When the cost function facing the distributor is expressed by* (1), *the equilibrium is characterized by maximum targeting. Specifically, there exists a threshold level* $\hat{v}(\alpha) \in (0, 1)$ *so that*[17]

$$b = b^1_{\max} \equiv \frac{2\left[\left(1-(t/p^*)\right)-\sqrt{t/(3p^*)+\frac{1}{3}(t/p^*)^2}\,\right]}{\left(1-t/(3p^*)\right)} \qquad \hat{v}(\alpha) < \frac{t}{p^*} < \frac{1}{2},$$

$$b = b^2_{\max} \equiv \frac{2\left[(t/p^*-\alpha)+\sqrt{(t/p^*-\alpha)^2+(1-t/(3p^*))(2t/p^*-\alpha)\alpha}\,\right]}{\left(1-t/(3p^*)\right)} \qquad 0 < \frac{t}{p^*} \le \hat{v}(\alpha). \tag{16}$$

Note that the extent of targeting in (16) is not a monotone function of the ratio t/p^*: It is strictly increasing for small values of this ratio (when $t/p^* < \hat{v}(\alpha)$ and $a = b$) and strictly decreasing for larger values (when $t/p^* > \hat{v}(\alpha)$ and $a > b$). Also, the extent of targeting is nonincreasing with α (strictly decreasing for $t/p^* \le \hat{v}(\alpha)$ and independent of α otherwise).

It is interesting to note that consumers might also benefit from higher levels of targeting implemented by the distributor. In fact, if the distributor implements monopoly pricing, as reported in Proposition 2, consumer welfare is highest when the maximum level of targeting arises at the equilibrium. Specifically, the expression for the consumer surplus when $p = p^* - t$ can be derived as follows:

$$CS = t\left[(a+\alpha)\left(1-\frac{a+\alpha-b}{4}\right)-\frac{b}{3}-\frac{5}{96}b^2\right], \tag{17}$$

where $a+\alpha = b/2 + t/p^* + \sqrt{(t/p^*)^2 + b^2 t/(12p^*)}$.

The above expression is a strictly increasing function of b. Because the distributor can use its role as a common agent to implement monopoly pricing for any level of b, increased targeting has only a beneficial effect on the welfare of consumers. Specifically, it increases the likelihood that they are informed of their preferred brand (because $\partial a/\partial b > 0$ along the lower

[16] In the technical appendix at http://mktsci.pubs.informs.org, we consider a targeting technology less precise than the one considered in the main text. Specifically, the distributor can distinguish between only two segments of the population. As well, it cannot make "take it or leave it" offers to the advertisers. Instead, the distributor auctions off the rights to reach each of the two identifiable segments. We demonstrate that utilizing the targeting technology is disadvantageous to the distributor in this case.

[17] The notation is borrowed from Figure 2.

boundary of the feasible region of Figure 2). Such improved information reduces the expected transportation costs that consumers incur, thus enhancing their welfare. If the condition of Proposition 2 does not hold (when α is relatively large, for instance), then the distributor may not be able to implement monopoly pricing. In this case, higher values of b result in higher product prices, to the disadvantage of consumers. As a result, when α is sufficiently large the welfare of consumers may decline with intensified targeting.

4. Concluding Remarks and Possible Extensions

We have demonstrated that when a single media content distributor (such as a television cable company or an Internet provider) delivers advertising messages on behalf of multiple competing brands, it can sometimes utilize customized advertising to implement monopoly pricing. When consumers have only limited information about the given product class and rely heavily on the commercials delivered by the distributor, implementing monopoly pricing becomes feasible. Even though such monopolistic pricing can be implemented with varying degrees of customization of commercials, product revenues and consumer surplus are highest when the distributor chooses the highest level of customization feasible. Consumers would obviously prefer aggressive price competition in product markets. However, given that collusion on prices is facilitated anyhow, when the distributor acts as a common agent, the welfare of consumers is enhanced when commercials are better aligned with their preferences.

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Appendix

Proof of Lemma 1

To guarantee that the market is always served $p \le p^* - t$, so that even when the consumer is informed of only one product she chooses to buy it, irrespective of her location on the line. Setting the price in (6) equal to $p^* - t$ yields $a+\alpha = LB$, where LB is given in (8). Because $\partial p/\partial a \le 0$ in (6), it follows that $p \le p^* - t$ if and only if $a+\alpha \ge LB$.

To guarantee that some commercials pertaining to a given product are delivered even to the consumer who is located the farthest away from the product, $\alpha + a - b \ge \alpha$, thus implying that $a \ge b$. The set of inequalities in (7) guarantees, therefore, that the entire population of consumers participate both in the product market and in the coverage span of commercials.

The threshold level $\hat{v}(\alpha)$ in Figure 2 is obtained by finding the point of intersection of the equations $a = LB - \alpha$ and $a = b$. The intersection point yields the expression for $b^2_{\max}$ as follows:

$$b^2_{\max} = \frac{2[(t/p^*-\alpha)+\sqrt{(t/p^*-\alpha)^2+(1-t/(3p^*))(2t/p^*-\alpha)\alpha}]}{(1-t/3p^*)}. \tag{A.1}$$

Setting the requirement $b^2_{\max} \le (1-\alpha)$ yields the threshold level $\hat{v}(\alpha)$ for the ratio t/p^* that distinguishes between panels B and C. When $\alpha = 0$, $\hat{v}(\alpha) = 3/13$. In panel B, $LB - \alpha > b$ everywhere, implying that the binding constraint on a is $a \ge LB - \alpha$. In panel C, there is a region (for $b \ge b^2_{\max}$) over which $b > LB - \alpha$, implying that over this region the binding constraint on a is $a \ge b$. $\square$

Proof of Proposition 1

Total differentiation of system (5) yields the following system of equations:

$$\begin{aligned}
&\Big\{-\frac{1}{t}[\alpha+a_1-b_1\hat{x}][\alpha+a_2-b_2(1-\hat{x})] \\
&\quad -\frac{p_1}{2t}\Big[\frac{b_1}{2t}(\alpha+a_2-b_2(1-\hat{x}))-\frac{b_2}{2t}(\alpha+a_1-b_1\hat{x})\Big]\Big\}\partial p_1 \\
&\quad +\Big\{\frac{1}{2t}[\alpha+a_1-b_1\hat{x}][\alpha+a_2-b_2(1-\hat{x})] \\
&\qquad -\frac{p_1}{2t}\Big[-\frac{b_1}{2t}(\alpha+a_2-b_2(1-\hat{x}))+\frac{b_2}{2t}(\alpha+a_1-b_1\hat{x})\Big]\Big\}\partial p_2 \\
&\quad =\Big\{-(\alpha+a_2-b_2)\hat{x}-\frac{b_2\hat{x}^2}{2}-1+\Big(\alpha+a_2-\frac{b_2}{2}\Big) \\
&\qquad +\frac{p_1}{2t}(\alpha+a_2-b_2(1-\hat{x}))\Big\}\partial a_1 \\
&\quad +\Big\{\frac{(\alpha+a_2-b_2)\hat{x}^2}{2}+\frac{b_2\hat{x}^3}{3}+\frac{1}{2}-\frac{1}{2}(\alpha+a_2) \\
&\qquad +\frac{b_2}{6}-\frac{p_1}{2t}\hat{x}(\alpha+a_2-b_2(1-\hat{x}))\Big\}\partial b_1.
\end{aligned}$$

$$\begin{aligned}
&\Big\{\frac{1}{2t}[\alpha+a_2-b_2(1-\hat{x})][\alpha+a_1-b_1\hat{x}] \\
&\quad -\frac{p_2}{2t}\Big[-\frac{b_2}{2t}(\alpha+a_1-b_1\hat{x})+\frac{b_1}{2t}(\alpha+a_2-b_2(1-\hat{x}))\Big]\Big\}\partial p_1 \\
&\quad +\Big\{-\frac{1}{t}[\alpha+a_2-b_2(1-\hat{x})][\alpha+a_1-b_1\hat{x}] \\
&\qquad -\frac{p_2}{2t}\Big[\frac{b_2}{t}(\alpha+a_1-b_1\hat{x})-\frac{b_1}{2t}(\alpha+a_2-b_2(1-\hat{x}))\Big]\Big\}\partial p_2 \\
&\quad =\Big\{-(\alpha+a_2)(1-\hat{x})+\frac{b_2}{2}(1-\hat{x})^2+(\alpha+a_2) \\
&\qquad -\frac{b_2}{2}+\frac{p_2}{2t}(\alpha+a_2-b_2(1-\hat{x}))\Big\}\partial a_1 \\
&\quad +\Big\{\frac{(\alpha+a_2)}{2}(1-\hat{x})(1+\hat{x})-\frac{b_2}{2}(1-\hat{x})^2\Big(1-\frac{2}{3}(1-\hat{x})\Big) \\
&\qquad -\frac{1}{2}(\alpha+a_2)+\frac{b_2}{6}-\frac{p_2}{2t}\hat{x}(\alpha+a_2-b_2(1-\hat{x}))\Big\}\partial b_1.
\end{aligned} \tag{A.2}$$

Evaluating (A.1) at the symmetric equilibrium yields

$$\frac{(\alpha+a-b/2)^2}{t}\begin{pmatrix}-1 & \frac{1}{2}\\ \frac{1}{2} & -1\end{pmatrix}\begin{pmatrix}\partial p_1\\ \partial p_2\end{pmatrix}$$
$$=\begin{pmatrix}\big[(\alpha+a)/2-b/8-1+(p/(2t))(\alpha+a-b/2)\big], & \big[-\tfrac{3}{8}(\alpha+a)+b/12+\tfrac{1}{2}-(p/(4t))(\alpha+a-b/2)\big]\\ \big[(\alpha+a)/2-3/8b+(p/(2t))(\alpha+a-b/2)\big], & \big[-(\alpha+a)/8+b/12-(p/(4t))(\alpha+a-b/2)\big]\end{pmatrix}$$
$$\times\begin{pmatrix}\partial a_1\\ \partial b_1\end{pmatrix}.$$

Substituting for p from (6) we can solve for $\partial p_i/\partial a_i$ and $\partial p_i/\partial b_i$ $i=1,2$ as follows:

$$\begin{pmatrix}\dfrac{\partial p_1}{\partial a_1}\\[2mm] \dfrac{\partial p_2}{\partial a_1}\end{pmatrix}=\frac{4t}{3(\alpha+a-b/2)^2}\begin{pmatrix}-\dfrac{1}{2}-\dfrac{b}{16}-\dfrac{b^2}{(\alpha+a-b/2)16}\\[2mm] -1+\dfrac{b}{16}-\dfrac{b^2}{(\alpha+a-b/2)16}\end{pmatrix}, \tag{A.3}$$

$$\begin{pmatrix}\dfrac{\partial p_1}{\partial b_1}\\[2mm] \dfrac{\partial p_2}{\partial b_1}\end{pmatrix}=\frac{4t}{3(\alpha+a-b/2)^2}\times\begin{pmatrix}\dfrac{(\alpha+a)}{16}+\dfrac{1}{4}+b/16+\dfrac{b^2}{32(\alpha+a-b/2)}\\[2mm] \dfrac{-(\alpha+a)}{16}+\dfrac{1}{2}+b/16+\dfrac{b^2}{32(\alpha+a-b/2)}\end{pmatrix}. \tag{A.4}$$

Recall that $(\alpha+a-bx)$ is a probability measure. Evaluating it at $x=0$ implies that $\alpha+a\le 1$, and evaluating it at $x=1$ yields that $\alpha+a-b\ge\alpha$, implying that $b\le a$. Substituting those inequalities in (A.3) and (A.4) yields the results reported in the proposition. □

Proof of Lemma 2

Suppose that producer 1 is the only one to advertise. Hence, $a_1=a$, $b_1=b$, $a_2=0$, and $b_2=0$. Substituting into system (5) yields

$$p_1=\frac{(A_1+B_1)2t}{(\alpha+a-b\hat{x})\alpha},\qquad p_2=\frac{(A_2+B_2)2t}{(\alpha+a-b\hat{x})\alpha}, \tag{A.5}$$

where

$$\hat{x}=\frac{1}{2}+\frac{p_2-p_1}{2t},$$
$$A_1=\alpha\left[(\alpha+a)\hat{x}-\frac{b\hat{x}^2}{2}\right],\qquad B_1=\left(\alpha+a-\frac{b}{2}\right)(1-\alpha),$$
$$A_2=\alpha\left[(\alpha+a-b)(1-\hat{x})+\frac{b(1-\hat{x})^2}{2}\right], \tag{A.6}$$
$$B_2=\left(1-\left(\alpha+a-\frac{b}{2}\right)\right)\alpha.$$

Substituting (A.5) into (A.6) yields the following quadratic equation in $\hat{x}$ as unknown:

$$4b\hat{x}^2-\hat{x}[6(\alpha+a)-b]+\left[\alpha+a+b+2\left(\alpha-\frac{a(1-\alpha)}{\alpha}+\frac{b(1-2\alpha)}{2\alpha}\right)\right]=0.$$

Solving for $\hat{x}$ yields

$$\hat{x}=\Big[[6(\alpha+a)-b]-\sqrt{(6(\alpha+a)-b)^2-16b\big(\alpha+a+b+2(\alpha-\tfrac{a(1-\alpha)}{\alpha}+\tfrac{b(1-2\alpha)}{2\alpha})\big)}\Big]/8b. \tag{A.7}$$

The solution for $\hat{x}$ in (A.7) is negative if the following inequality holds:

$$\alpha+a+b+2\left(\alpha-\frac{a(1-\alpha)}{\alpha}+\frac{b(1-2\alpha)}{2\alpha}\right)<0,$$

which is equivalent to

$$a>\frac{3\alpha^2}{2-3\alpha}+\frac{b(1-\alpha)}{2-3\alpha}\equiv\hat{a}. \tag{A.8}$$

Hence, if (A.8) holds, there is no interior equilibrium with the advertising producer obtaining a positive market share of those consumers who are informed about both products. Recall also from (7) and (8) that to guarantee full participation of consumers in the market,

$$a>\left[\frac{b}{2}+\frac{t}{p^*}+\sqrt{\left(\frac{t}{p^*}\right)^2+\frac{b^2t}{12p^*}}-\alpha\right]\equiv LB-\alpha. \tag{A.9}$$

If the RHS of inequality (A.9) is bigger than the RHS of inequality (A.8), we are guaranteed that (A.8) holds, thus implying the nonexistence of an equilibrium with $\hat{x}>0$. Imposing the condition $LB-\alpha>\hat{a}$ yields the following inequality:

$$\frac{b^2}{4}\left[\frac{t}{3p^*}-\frac{\alpha^2}{(2-3\alpha)^2}\right]+\frac{b\alpha\big[\frac{t}{p^*}(2-3\alpha)-2\alpha\big]}{(2-3\alpha)^2}+\frac{4\alpha\big[\frac{t}{p^*}(2-3\alpha)-\alpha\big]}{(2-3\alpha)^2}>0. \tag{A.10}$$

When the condition of Lemma 2 holds, we are guaranteed that (A.10) is indeed valid. Hence $\hat{x}=0$, and the advertising producer obtains revenues only from those consumers who are uniformed about the competing nonadvertised product. Because he does not face any competition in serving those consumers, he charges them the monopoly price of (p^*-t). To dominate the entire market of the fully informed consumers, as is implied by $\hat{x}=0$, the nonadvertising producer undercuts the price of the advertising producer by an amount t so that $p_2=p^*-2t$. When charging this price, even the consumer who is located the farthest away from producer 2 chooses to buy his product. Given that producer 2 charges the price (p^*-2t) from a proportion α of the population (who are informed of his product), $\pi_2(0,\,0,\,a,\,b)=\alpha(p^*-2t)$.

Finally, we will show that under the condition of the lemma, no firm has an incentive to deviate from the above equilibrium (i.e., $p_1=p^*-t$, $p_2=p^*-2t$). Expressing the expected profits of firm 1 when it charges the price $p_1\le p^*-t$ and firm 2 charges the price p^*-2t yields

$$\pi_1=(A_1+B_1)p_1,$$

where A_1 and B_1 are given in (A.6) and

$$\hat{x} = \frac{p^* - p_1}{2t} - \frac{1}{2}$$

because

$$p_2 = p^* - 2t.$$

The function π_1 is a strictly concave function of p_1, implying a unique maximizing solution for p_1. Differentiating π_1 with respect to p_1 and evaluating the derivative at $p_1 = p^* - t$ yields

$$\frac{\partial \pi_1}{\partial p_1|_{p_1=p^*-t}} = (1-\alpha)\left(\alpha + a - \frac{b}{2}\right) - \frac{\alpha(\alpha+a)}{2t}(p^* - t).$$

The above derivative is strictly positive under the condition of the lemma. Because $\partial^2\pi_1/\partial p_1^2 < 0$, it follows that $\partial\pi_1/\partial p_1 > 0$ for all prices $p_1 \le p^* - t$, and $p_1 = p^* - t$ is the unique best response to $p_2 = p^* - 2t$. Similarly, expressing the expected profits of firm 2 when it charges the price $p_2 \ge p^* - 2t$ and firm 1 charges $p_1 = p^* - t$, yields that $\pi_2 = (A_2 + B_2)p_2$, where A_2 and B_2 are as defined in (A.6) and

$$\hat{x} = 1 - \frac{p^* - p_2}{2t} \quad \text{because } p_1 = p^* - t.$$

Once again, π_2 is a strictly concave function of p_2, and under the condition of the lemma $\partial\pi_2/\partial p_2$ evaluated at $(p^* - 2t)$ is negative. Hence, $\partial\pi_2/\partial p_2 < 0$ for all values of $p_2 \ge p^* - 2t$, and $p_2 = p^* - 2t$ is the unique best response to $p_1 = p^* - t$. □

Proof of Lemma 3
Given the condition of the lemma, $\pi_i(0, 0, a, b)$ is a constant function, and the derivatives of $w(\cdot)$ are determined by the slopes of the first and last terms of (14). Differentiating those terms yields the result, given Assumptions A3 and A4. □

Proof of Proposition 2
(i) Because $w(\cdot)$ is strictly monotone, the optimal solution lies on the lower boundary of the feasible region. For sufficiently high values of the ratio t/p^*, this boundary consists of (a, b) combinations that satisfy (6) when $p = p^* - t$ (panel B). For low values of t/p^* (panel C), the boundary includes also a segment of the 45° line when $b \ge b^2_{\max}$. We will show, however, that the profit of the distributor are strictly lower on this segment than they are at $b^2_{\max}$. Substituting $a = b$ into (14) yields

$$w(b,b) = 4t\left[1 - \frac{1}{2}\left(\alpha + \frac{b}{2}\right) + \frac{b^2}{24(\alpha + b/2)}\right]^2 - G(b,b) - 2\pi_i(0,0,b,b). \qquad \text{(A.11)}$$

By Assumption A5, we are guaranteed that the first two terms of (A.11) are strictly decreasing with b. Also, the condition of the proposition implies that $\pi_i(0,0,b,b) = \alpha(p^* - 2t)$ for $a = b \ge b^2_{\max}$. To see this, note that to guarantee (A.8) with $a = b$ implies that $a > 3\alpha^2/(1-2\alpha)$. It is easy to show that $b^2_{\max}$ in (A.1) is larger than $3\alpha^2/(1-2\alpha)$ under the condition of the proposition. As a result, for advertising parameters $a = b > b^2_{\max}$ in panel C we are guaranteed that the outside option of the nonadvertising producer is a flat function that is determined independently of the parameter b. Because $w(b, b)$ in (11) is a strictly decreasing function of b, the optimal solution will never entail a targeting parameter larger than $b^2_{\max}$. For $b \le b^2_{\max}$, the lower boundary of the feasible region implements, once again, the monopoly price $p^* - t$. This lower boundary is determined by the equation $a = LB - \alpha$, where LB is given by (8). Differentiating LB with respect to b yields the conclusion that $\partial a/\partial b > 0$.

(ii) When both producers set the price $p^* - t$ the expected revenues that accrue to each product are $(A_i + B_i)(p^* - t)$, where A_i and B_i satisfy Equation (4). Substituting into A_i and B_i, $\hat{x} = 1/2$ and the expression for $(\alpha + a)$ from (8) yields the expected revenues that accrue to each product as a function of the targeting parameter. Differentiating the RHS of (15) with respect to b yields the result. □

Proof of Corollary 1
Evaluating (14) for the value of a that satisfies (8) and the cost function given in (1) yields

$$w = 4p^*\left(1 - \frac{t}{p^*}\right)^2 \frac{t}{p^*}\left[1 + \frac{b^2}{24X}\right]^2 - F - 2\beta[X - \alpha] - 2\alpha(p^* - 2t), \qquad \text{(A.12)}$$

where

$$X \equiv \frac{t}{p^*} + \sqrt{\left(\frac{t}{p^*}\right)^2 + \frac{b^2 t}{12p^*}}.$$

Differentiating (A.12) with respect to b yields

$$\frac{\partial w}{\partial b} = \frac{bt}{6p^*}\left(\left(\frac{t}{p^*}\right)^2 + \frac{b^2 t}{12p^*}\right)^{-\frac{1}{2}} \cdot \left[\frac{4p^*(1 - t/p^*)^2(1 + b^2/(24X))^2(t/p^*)}{X} - \beta\right] > 0.$$

The latter inequality follows from (A.12), to guarantee that the distributor earns nonnegative profits. Hence, the distributor wishes to implement the highest possible level of targeting to maximize profits. Note that in (A.12) $\pi_i(0, 0, a, b) = \alpha(p^* - 2t)$, given our assumptions that

$$\left(\frac{t}{p^*}\right) > \frac{2\alpha}{3 - 2\alpha}. \quad \square$$

Derivation of the Expression for Consumer Surplus
Given that the distributor implements monopoly pricing, the expected consumer surplus can be derived as follows:

$$\int_0^{1/2}(a + \alpha - bx)(a + \alpha - b(1-x))[t - tx]\,dx + \int_{1/2}^1 (a + \alpha - bx)(a + \alpha - b(1-x))[t - t(1-x)]\,dx + \int_0^1 (a + \alpha - bx)(1 - (a + \alpha - b(1-x)))[t - tx]\,dx + \int_0^1 (1 - (a + \alpha - bx))(a + \alpha - b(1-x)) \cdot (t - t(1-x))\,dx. \qquad \text{(A.13)}$$

The first two terms of (A.13) correspond to the case in which the consumer is informed about both products, and the last two terms correspond to the case in which the consumer is informed about only one product. Conducting the integration yields (17).

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