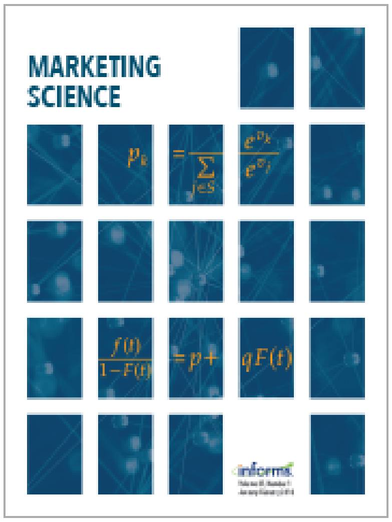

Marketing Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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To cite this article:
Dominique Olié Lauga, Elie Ofek, (2011) Product Positioning in a Two-Dimensional Vertical Differentiation Model: The Role of Quality Costs. Marketing Science 30(5):903-923. https://doi.org/10.1287/mksc.1110.0652

MARKETING SCIENCE
CELEBRATING 30 YEARS
Vol. 30, No. 5, September–October 2011, pp. 903–923
ISSN 0732-2399 | EISSN 1526-548X | 11 | 3005 | 0903

informs®
http://dx.doi.org/10.1287/mksc.1110.0652

Product Positioning in a Two-Dimensional Vertical Differentiation Model: The Role of Quality Costs

Dominique Olié Lauga
Rady School of Management, University of California, San Diego, La Jolla, California 92093, dlauga@ucsd.edu

Elie Ofek
Harvard Business School, Boston, Massachusetts 02163, eofek@hbs.edu

We study a duopoly model where consumers are heterogeneous with respect to their willingness to pay for two product characteristics and marginal costs are increasing with the quality level chosen on each attribute. We show that although firms seek to manage competition through product positioning, their differentiation strategies critically depend on how costly it is to provide higher quality. When the cost of providing quality is not too high, firms use only one attribute to differentiate their products: they maximally differentiate on one dimension and minimally differentiate on the other (a Max-Min equilibrium). Furthermore, they always differentiate along the dimension with the greater attribute range. As for the dimension with the smaller range and along which they agglomerate, firms either choose the highest quality level or the lowest quality level possible, depending on whether the marginal costs of quality provision are low or intermediate, respectively. However, for larger quality provision costs, firms exploit both dimensions to differentiate their products. In particular, we characterize a maximal differentiation equilibrium in which one firm chooses the highest quality level on both attributes while its rival offers the lowest quality level on both attributes (a Max-Max equilibrium). We discuss the managerial implications of our findings and explain how they enrich and qualify previous results reported in the literature on two-dimensional differentiation models.

Key words: product positioning; multiattribute products; differentiation; competitive strategy; game theory
History: Received: July 22, 2008; accepted: March 14, 2011; Eric Bradlow and then Preyas Desai served as the editor-in-chief and Sridhar Moorthy served as associate editor for this article. Published online in *Articles in Advance* July 15, 2011.

1. Introduction

Product positioning is perhaps one of the most important decisions that marketing managers need to make (Kotler and Keller 2008). It is a decision that affects the selection of key marketing mix variables, such as price, and that has considerable implications for a firm's profits. In many instances, the set of product dimensions relevant for consumer choice in the category are known, and the positioning decision boils down to determining how much of each attribute or product characteristic to provide. In making this decision, the firm has to balance a desire to offer a product that is more appealing to customers with the need to be mindful about how costly such a product is to produce. When facing competition, however, the positioning decision needs to take into account not only consumer preferences and production costs but also the positioning of rivals. Specifically, how differentiated should firms' products be from each other?

When there is only a single product dimension relevant for consumer choice, we would naturally expect firms to manage competition through differentiation along that dimension. For example, if consumers care mainly about the speed of a microprocessor, then competition would likely prompt one firm to offer a high-speed processor while its rival offers a low-speed one. Moreover, delivering faster processors that pack more transistors onto a single chip costs more to produce and is typically priced higher.[1]

However, in many markets customers base their purchase decisions on multiple product attributes. For example, in purchasing a hybrid electric vehicle, consumers might care about the effective miles per gallon the vehicle is estimated to achieve and the car's safety ratings or maximum cargo capacity. In a business-to-business context, digital storage solutions are typically evaluated on their capacity to store data as well as how quickly data can be accessed for processing.[2]

[1] In the case of PC microprocessors, for example, throughout the 1990s, Intel chips were on average 50% faster than AMD processors and were priced 50%–100% higher (Ofek 2007).

[2] For example, throughout the 2000s, the two prominent hybrid models sold, the Toyota Prius and Honda Insight, were mainly

In these multiattribute contexts, firms have much more flexibility in terms of how they position their products relative to their competitors, and the desire to differentiate can therefore take on a number of forms. For example, competing firms could locate their products to achieve differentiation on all attributes, or they could differentiate only on one attribute and remain undifferentiated on others. Moreover, in the case of differentiation on only one attribute, several configurations are possible, raising such questions as, which attribute should firms use to differentiate their products, and what level should firms offer on the nondifferentiated attribute?

In this paper, we study a duopolistic market in which consumer preferences are heterogeneous and are affected by the levels chosen on two vertical product attributes. Firms decide on their product positioning in the attribute space in the first stage and on pricing in the second stage. Our model incorporates the fact that the greater the performance or quality the firm wishes to provide on a given attribute, the more costly production becomes. In all the examples given above, this is clearly the case (more efficient hybrid fuel engines, more crash-mitigating safety features, greater storage capacity, and faster data retrieval all cost more to produce). In this context, where profitability depends on the confluence of demand generation, cost containment, and competitive intensity, we seek to understand how firms select their product positions in the multiattribute space.

Our analysis reveals several important findings. First, we show that when the cost of quality provision is not too high, then any equilibrium of the game must exhibit the Max-Min property: firms will choose to be maximally differentiated on one attribute, i.e., one firm selects the highest possible level while its rival selects the lowest possible level on that attribute, and to be minimally differentiated on the other attribute, i.e., both firms select the same level on the other attribute. Second, we shed light on the nature of the Max-Min equilibria to expect. Specifically, we find that the dimension that firms maximally differentiate on has the greatest attribute range or span. As for the attribute firms minimally differentiate on, the outcome depends on how costly it is to provide quality: if this cost is low, both firms will offer the highest possible level on that attribute. Conversely, if this cost is intermediate, they will both offer the lowest possible level. Finally, in cases where the cost of providing quality is high, we find that firms will differentiate on both attributes. We further characterize the conditions for such a Max-Max equilibrium in which one firm selects the highest possible level on both attributes while its rival selects the lowest possible levels. Taken together, our findings offer clarity on the topic of product positioning when there is more than one attribute to consider and uncover an interesting interplay between firms' strategic considerations and the need to manage the costs of quality provision.

The rest of this paper is organized as follows: in the next section we relate our work to the extant literature on product positioning when there is more than one product dimension. The model setup is described in §3. Section 4 analyzes the model starting with the pricing equilibrium in §4.1, followed by the equilibrium in choice of attribute levels in §4.2. In §5 we discuss in greater detail why certain equilibria presented in prior literature do not exist and also relax several model assumptions. Section 6 concludes and provides managerial implications. In Appendices A and B we provide the proofs of the claims, lemmas, and propositions presented in this paper.

2. Relevant Literature

The debate about the degree of competitive differentiation started with Hotelling (1929), who introduced the "principle of minimum differentiation"—suggesting that firms should sell the exact same good when consumers have different tastes along a horizontal line.[3] This principle was shown not to hold when prices are endogenous (d'Aspremont et al. 1979). Specifically, for a duopoly in which firms choose horizontal product location in the first stage and then compete in prices, the equilibrium is for firms to maximally differentiate (i.e., locate at opposite ends of the line).[4] In a vertical differentiation context with one dimension representing quality, Shaked and Sutton (1982) find that firms partially differentiate.

The above literature has provided an excellent understanding of competitive positioning when firms

evaluated on gas savings (based on the effective miles per gallon) and safety ratings. In the storage area network market in the 1990s and early 2000s, Brocade and McData were the major competitors, and storage capacity and data access were important purchase criteria (Gourville et al. 2002, Ofek and Mamoon 2005).

[3] Products are horizontally differentiated when consumers disagree about which type of product provides them with greater utility, holding price constant (for example, two equally priced 16 oz jars of peanut butter, one crunchy and the other creamy, may appeal to different consumers). By contrast, products are vertically differentiated when all consumers agree about the quality ordering of products but differ in how much they are willing to pay for higher quality. For example, all consumers would pick an 80 GB MP3 player over a 20 GB MP3 player—assuming all other characteristics including price are the same—but if the 80 GB version costs more, some consumers would prefer the 20 GB version if they do not place much value on memory capacity.

[4] This is true when consumers are uniformly distributed along the line and the transportation cost is quadratic.

decide on the level or location of only one product characteristic. In reality, however, products typically possess several characteristics (i.e., they are multiattribute). When firms have discretion over the levels of more than one dimension, several types of configurations are possible, ranging from firms choosing the same level on all dimensions (no differentiation, i.e., "Min-Min"), to firms choosing as much separation as possible on all dimensions (maximal differentiation, i.e., "Max-Max"), to firms choosing an "in-between" degree of differentiation (e.g., maximal differentiation on one dimension and minimal differentiation on the other, i.e., "Max-Min"; or maximal differentiation on one dimension and partial differentiation on the other, i.e., "Partial-Max"). Recent literature has begun looking at this issue. Much of the research has examined the horizontal case, with a general conclusion that firms never fully differentiate on all dimensions (see Irmen and Thisse 1998 for an analysis of n horizontal characteristics). In a model with one horizontal characteristic and one vertical characteristic, Neven and Thisse (1990) prove the possible coexistence of multiple Max-Min equilibria. Although they do not establish that the Max-Min property must always hold in equilibrium, the authors do show that firms never choose to fully differentiate on both dimensions. In the case of two vertically differentiated characteristics, Vandenbosch and Weinberg (1995) claim to have identified conditions for several types of equilibria to exist (Max-Min, Max-Max, and Partial-Max) with coexistence of multiple Max-Min equilibria over a wide range of the parameter space.

This stream of research on multiattribute product positioning has assumed constant marginal costs of production (typically taken to be zero). Thus, it has ignored the possibility that selecting a product position not only has demand and competitive implications but also affects costs. In particular, in many product categories it is typically the case that higher-quality provision (higher level on a desired attribute) comes with higher production costs as well.[5] In the single-attribute case, Moorthy (1988) studied the impact of increasing marginal costs and showed that such costs affect equilibrium outcomes.

Relative to the extant literature, our contribution is twofold. First, we revisit the two-dimensional vertical differentiation model and qualify the findings reported in prior work. Specifically, in the special case of marginal costs that are independent of attribute levels chosen (which is the setup in Vandenbosch and Weinberg 1995), we show that the product configurations in equilibrium must display the Max-Min property: one firm sells the best product (highest levels on all attributes) while its rival maximally differentiates along the attribute with the larger range and agglomerates along the smaller range attribute. Our results further show that for any given set of attribute ranges and cost of quality provision, the equilibrium product configuration is always unique. Second, we characterize the equilibria that arise in the two-dimensional vertical differentiation model when marginal costs do depend on the attribute levels chosen. This analysis reveals that having to consider production costs greatly affects firm positioning decisions and that different types equilibrium strategies can emerge as costs increase. In particular, we identify existence conditions for two distinct types of Max-Min equilibria, for a Max-Max equilibrium, and for a Partial-Max equilibrium. We believe the insights emanating from our analysis greatly enrich the understanding of firm differentiation strategies when there is more than one relevant dimension that affects consumer utility.

3. Model

Let product i in a given category comprise two relevant characteristics, x_i and y_i. Each of these characteristics is vertically differentiated in the sense that, keeping everything else constant, all consumers prefer higher levels on both characteristics. The market is heterogeneous in that consumers differ in terms of how much they care about quality levels. To be more specific, the utility a consumer derives from buying product i at price p_i is $V + \theta_x x_i + \theta_y y_i - p_i$, where θ_x represents willingness to pay for quality on characteristic x and θ_y represents willingness to pay for quality on characteristic y; θ_x and θ_y are independent and uniformly distributed over $[0, 1]$. Thus, consumer utility is increasing with respect to x_i and y_i, but the rates of increase reflect consumers' heterogeneity in taste.[6]

Two identical firms, indexed $i \in (1, 2)$, compete by choosing their product attributes[7] and their price. This duopoly setting is modeled in a two-stage game. In the first stage, firms simultaneously choose both product characteristics. Firms can choose any combination of (x, y) in the feasible set of characteristics $[\underline{x}, \bar{x}] \times [\underline{y}, \bar{y}]$. The marginal cost of production is linearly increasing with the quality level chosen for each characteristic and is given by $cx + cy$. The special case of constant marginal costs, which are independent of the quality level chosen, is captured simply by setting $c = 0$. In the second stage, both firms observe the product characteristics of their competitor and simultaneously choose the price at which

[5] See, for example, Srinivasan et al. (1997) for applications in product design where attribute-level decisions affect manufacturing costs.

[6] Section 5.5 explains how this setup encompasses more general cases in which the roles played by product characteristics in the utility function are less symmetric.

[7] We use the terms characteristic and attribute interchangeably.

to sell their product.[8] Firms compete à la Bertrand in the pricing stage and maximize their profits $\pi_i = D_i(p_i - cx_i - cy_i)$, where p_i and D_i are the price and demand of firm i, respectively. Finally, consumers decide to buy one unit from firm 1 or firm 2 because the reservation utility V is assumed to be large enough so that all consumers buy in equilibrium (in §5.5 we discuss the implications of relaxing this assumption to allow for the market to not be fully covered).

Denoting the span (or range) of the x characteristic as $\Delta x \equiv \bar{x} - \underline{x}$ and the span of the y characteristic as $\Delta y \equiv \bar{y} - \underline{y}$, for purposes of the analysis we can assume, without loss of generality, that characteristic x has a larger span than characteristic y; i.e., $\Delta x \geq \Delta y$. Firms are assumed to be risk neutral and maximize expected payoffs. We focus on subgame-perfect equilibria in pure strategies.

The game is solved through backward induction starting with the pricing subgame derived at the beginning of the next section.

4. Equilibrium Analysis

4.1. Pricing Equilibrium

In the pricing subgame, the product characteristics have already been chosen, and we are interested in the price competition that follows. If products are identical, prices are equal to marginal cost, and profits are zero. Therefore, going forward we study the price equilibrium when products are not identical. In solving for the equilibria, what matters is the degree of difference on each of the attributes. A firm's product is said to have a comparative advantage on a given characteristic if the level it offers on that characteristic is higher than what its competitor offers. It is convenient to label the attributes A and a such that A is the attribute with the larger comparative difference and a is the one with the smaller comparative difference. Let the firm whose product has the larger comparative advantage (A) be labeled firm F, and its rival is firm f.[9] The attributes of the products are thus (A_F, a_F) for firm F and (A_f, a_f) for firm f. Because by definition firm F has a comparative advantage in attribute A, we have $\Delta A \equiv A_F - A_f > 0$. Analogously, we can write $\Delta a \equiv a_F - a_f$ as the difference between the firms' products on attribute a. If firm F also has a comparative advantage in characteristic a, i.e., $\Delta a \geq 0$, firm F sells a *dominant product* that is better on both dimensions.[10] Otherwise, if $\Delta a < 0$ the products are said to have *asymmetric characteristics*; in this case firm f has a comparative advantage in a, and firm F has a comparative advantage in A. Finally, by definition the comparative advantage in A is larger than in a, which implies that $\Delta A \geq |\Delta a| \geq 0$.

Table 1 Existence of the Four Pricing Equilibria

Types	Existence conditions
Eq^{**}	$0 \leq c \leq (2\Delta a - \Delta A)/(\Delta a + \Delta A)$
Eq^{*}	$(2\Delta a - \Delta A)/(\Delta a + \Delta A) \leq c \leq (2\Delta A - \Delta a)/(\Delta a + \Delta A)$ and $0 \leq c \leq 2$
$Eq^{\dagger}$	$(2\Delta A - \Delta a)/(\Delta a + \Delta A) \leq c$ and $\Delta a \geq 0$
$Eq^{\dagger\dagger}$	$2 \leq c$ and $\Delta a \leq 0$

In the next subsection, we show that in each region of the parameter space (c, $\Delta a/\Delta A$), the subgame pricing equilibrium is unique, and we discuss the various demand structures that emerge. In §4.1.2, we provide the prices and profits in each of these pricing equilibria and characterize the comparative statics that will be relevant at the product positioning stage.

4.1.1. Characterization of the Pricing Equilibrium. Solving the last stage of the game between the firms, we obtain the following lemma.

Lemma 1. *For any cost of quality provision $c \geq 0$ and ratio of quality differences $\Delta a/\Delta A$, the pricing equilibrium is unique. The four types of pricing equilibria that may arise are denoted Eq^*, Eq^{**}, $Eq^{\dagger}$, and $Eq^{\dagger\dagger}$. Their existence conditions are given in Table* 1.

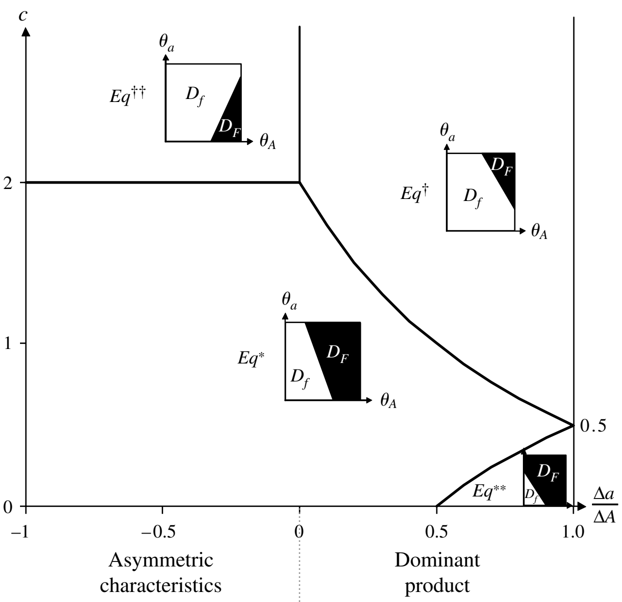
Figure 1 illustrates the existence regions for each of the four types of pricing equilibria as a function of the cost of quality provision c and the ratio of quality difference $\Delta a/\Delta A$ (per Table 1). The subgame equilibria Eq^{**} and $Eq^{\dagger}$ exist only when firm F has a comparative advantage on both product dimensions (dominant product case; $\Delta a/\Delta A \geq 0$), $Eq^{\dagger\dagger}$ exists only when firm f has a comparative advantage on attribute a (asymmetric characteristics case; $\Delta a/\Delta A \leq 0$), and Eq^* exists in both cases (provided the cost of quality provision is not too high).

[8] The multistage setup of our model is consistent with much of the industrial organization literature, whereby product decisions (quality level in our case) are made prior to pricing decisions, with the product decisions known to both firms at the pricing stage (see, for example, Moorthy 1988, Shaked and Sutton 1982, Tirole 1988). This structure is realistic because (a) product design typically needs to be determined before deciding what price is appropriate, (b) product decisions are much more difficult to change in the short run relative to changing prices, and (c) firms often need to make known their product designs in advance of the launch (to prepare the market, get suppliers of complementary products and distributors on board, etc.). As an example, consider videogame console makers that preannounce the exact specs of their next-generation devices years in advance but only set prices shortly before launch (Ofek 2008).

[9] Firm 1 can be either firm F or firm f, depending on whether its product possesses characteristic A. Similarly, characteristic x can either be A or a.

[10] We borrow this nomenclature from Vandenbosch and Weinberg (1995).

Figure 1 **Pricing Equilibria Depending on the Cost of Quality Provision c and the Relative Qualities $\Delta a/\Delta A$**



Note. In Eq^*, the line representing consumers indifferent between f and F is upward sloping in the asymmetric characteristics case and downward sloping in the dominant product case.

Figure 1 also depicts the demand structure for each of the four equilibria. Because every consumer is characterized by his or her tastes, θ_A for attribute A and θ_a for attribute a, the market demand for each firm can be represented in the unit square of the (θ_A, θ_a) plane. In each case, the demand for firm F is depicted in black and the demand for firm f in white. When in equilibrium a firm only serves consumers with very similar tastes, it is said to follow a *niche strategy*, and Figure 1 shows a triangular-shaped area in one of the corners of the appropriate market-demand graph. Specifically, it can be gleaned that firm F follows a high-end niche strategy in equilibria $Eq^{\dagger}$ and $Eq^{\dagger\dagger}$: consumers who buy firm F's product value high quality on both characteristics in $Eq^{\dagger}$, whereas they mainly care about the A characteristic in $Eq^{\dagger\dagger}$. Similarly, firm f follows a low-end niche strategy in Eq^{**} by only serving consumers who do not value quality much. By contrast, in Eq^* neither firm follows a niche strategy, with the shape of demand for both firms forming a trapezoid in the relevant market-demand graph in Figure 1.

Finally, the relative size of the demand for each firm also depends on the cost of quality provision c, because marginal costs affect the prices firms charge in equilibrium. Specifically, when c increases, firm F is relatively more affected than firm f because its product has the larger comparative advantage in quality and is more costly to produce. Firm F thus has a greater incentive than its rival to increase price as marginal costs increase, and consequently, consumers derive less net utility from its product. Figure 1 illustrates how firm F's demand shrinks while firm f's demand expands as c increases: the black area representing demand for F becomes smaller when moving up in Figure 1, and the white area becomes larger.

In the next subsection, we discuss the prices and profits in the different equilibria of the pricing subgame and provide comparative statics that will be relevant for solving the first stage of the game.

4.1.2. Prices and Profits. Table 2 provides the prices and profits in each of the four possible pricing equilibria. It can easily be gleaned that profits and contribution margins[11] depend only on characteristic-level differences, ΔA and Δa, and the cost of quality provision c. Firm F, which sells the product with the greater comparative advantage, always chooses a higher price than its rival: $p_F \geq p_f$. However, because quality is costly to produce, firm F earns more than its rival only in the low quality-cost region ($c < \frac{1}{2}$), where both its margin and demand are larger. Conversely, when $c > \frac{1}{2}$, firm f always earns greater profits than its rival, as both its margin and demand are larger; firm f thus benefits from selling a product that is relatively cheaper to produce. Only in the limit cases, when the comparative advantages are identical ($\Delta A = -\Delta a$) or when the cost of quality provision is exactly equal to $\frac{1}{2}$, do firms earn the same margins, split the market in half, and are equally profitable.

We note that for very large cost of quality provision ($c \geq 2$), firm F earns zero profits in any product configuration with minimal differentiation along the a characteristic ($\Delta a = 0$). In that case, firm f prices to capture the entire market, whereas firm F chooses a price exactly equal to its marginal cost, $p_F = ca_F + cA_F$, yet it gets no demand. Given these consequences when the cost of quality provision is extreme, we focus on the region $c < 2$ for the rest of our analysis. As such, the relevant pricing equilibria are Eq^*, Eq^{**}, and $Eq^{\dagger}$.

Comparative Statics. We now analyze how equilibrium prices vary with incremental changes in product characteristics. These comparative statics results will help in understanding firm strategies in the product positioning stage. Of particular interest is how a change in firm i's product attributes affects rival j's equilibrium price: if firm j reacts by increasing (decreasing) p_j, firm i faces a less (more) aggressive rival, and price competition is relaxed (intensified) as a result.

Firm j's decision to modify its price when the quality of firm i's product increases (either A_i or a_i

[11] Contribution margins are the difference between the price and marginal cost: $p_i - ca_i - cA_i$.

Table 2 Prices and Profits in the Four Pricing Equilibria: Eq^*, Eq^{**}, $Eq^{\dagger}$, **and** $Eq^{\dagger\dagger}$

Prices	Profits
Eq^*	
$p_F^* = ca_F + cA_F + \dfrac{(2-c)\Delta A + (\frac{1}{2}-c)\Delta a}{3}$	$\pi_F^* = \dfrac{((2-c)\Delta A + (\frac{1}{2}-c)\Delta a)^2}{9\Delta A}$
$p_f^* = ca_f + cA_f + \dfrac{(1+c)\Delta A + (c-\frac{1}{2})\Delta a}{3}$	$\pi_f^* = \dfrac{((1+c)\Delta A + (c-\frac{1}{2})\Delta a)^2}{9\Delta A}$
Eq^{**}	
$p_F^{**} = ca_F + cA_F + 3X^{**} - c\Delta a - c\Delta A$	$\pi_F^{**} = \left(1 - \dfrac{2X^{**2}}{\Delta a\Delta A}\right)(3X^{**} - c\Delta a - c\Delta A)$
$p_f^{**} = ca_f + cA_f + X^{**}$	$\pi_f^{**} = \dfrac{2X^{**3}}{\Delta a\Delta A}$
$Eq^{\dagger}$	
$p_F^{\dagger} = ca_F + cA_F + X^{\dagger}$	$\pi_F^{\dagger} = \dfrac{2X^{\dagger 3}}{\Delta a\Delta A}$
$p_f^{\dagger} = ca_f + cA_f + 3X^{\dagger} + (c-1)(\Delta a + \Delta A)$	$\pi_f^{\dagger} = \left(1 - \dfrac{2X^{\dagger 2}}{\Delta a\Delta A}\right)(3X^{\dagger} + (c-1)(\Delta a + \Delta A))$
$Eq^{\dagger\dagger}$	
$p_F^{\dagger\dagger} = ca_F + cA_F + X^{\dagger\dagger}$	$\pi_F^{\dagger\dagger} = \dfrac{2X^{\dagger\dagger 3}}{-\Delta a\Delta A}$
$p_f^{\dagger\dagger} = ca_f + cA_f + 3X^{\dagger\dagger} + c\Delta a + (c-1)\Delta A$	$\pi_f^{\dagger\dagger} = \left(1 - \dfrac{2X^{\dagger\dagger 2}}{-\Delta a\Delta A}\right)(3X^{\dagger\dagger} + c\Delta a + (c-1)\Delta A)$

Note. Where

$$X^{**} = \frac{c\Delta a + c\Delta A + \sqrt{c^2(\Delta a + \Delta A)^2 + 8\Delta a\Delta A}}{8}, \quad X^{\dagger} = \frac{(1-c)(\Delta a + \Delta A) + \sqrt{(1-c)^2(\Delta a + \Delta A)^2 + 8\Delta a\Delta A}}{8}, \quad \text{and}$$

$$X^{\dagger\dagger} = \frac{-c\Delta a - (1-c)\Delta A + \sqrt{(c\Delta a - (1-c)\Delta A)^2 - 8\Delta a\Delta A}}{8}.$$

increase) is governed by two effects. First, there is a direct effect that reflects how firm j would like to adjust its price p_j, assuming firm i keeps p_i constant. Specifically, when firm i increases an attribute level while firm j's attributes remain constant, firm j has an incentive to compensate for having to compete with a better product by lowering price to keep its offering attractive to consumers. The direct effect is thus always nonpositive. Second, there is a strategic effect that reflects how firm j would like to adjust price given that firm i is likely to change its price when the level of one of its attributes improves. Specifically, firm i has an incentive to raise price because consumers are willing to pay more for the higher-quality product and because of the greater marginal cost associated with its production. As prices are strategic complements, firm j has an incentive to react to a competitor price increase by raising price as well. Hence, the strategic effect is always positive. The overall impact of an attribute change by firm i on firm j's price thus depends on whether the direct or strategic effect dominates. As the next lemma shows, the overall impact differs by firm (F or f) and by attribute (A or a).

Lemma 2. *The comparative statics for how a change in a firm's attribute level affects its rival's equilibrium price is given in the following table.*

	Impact on p_f	Impact on p_F
Change in characteristic A	$\partial p_f/\partial A_F$	$\partial p_F/\partial A_f$
Eq^*, Eq^{**}, and $Eq^{\dagger}$	$+$	$-$
Change in characteristic a	$\partial p_f/\partial a_F$	$\partial p_F/\partial a_f$
Eq^{**} and $Eq^{\dagger}$	$+$	$-$
Eq^* $c < 1/2$	$-$	$+$
Eq^* $c > 1/2$	$-$	$+$

Recall that firm F has a comparative advantage along attribute A ($A_F > A_f$). Lemma 2 states that firm f increases its price p_f when firm F raises A_F (the strategic effect dominates in determining the sign of $\partial p_f/\partial A_F$), whereas firm F increases its price p_F when firm f lowers A_f (the direct effect dominates in determining the sign of $\partial p_F/\partial A_f$). Thus, both firms benefit from their products being located as far apart as possible along the A characteristic because this relaxes price competition and allows them to charge higher prices; this is true for all three subgame pricing equilibria and any cost of quality provision. As for changes in characteristic a, the comparative statics depend on which equilibrium is being considered. Eq^{**} and $Eq^{\dagger}$ occur only in the dominant product case when firm F has a comparative advantage on both characteristics. In these equilibria, both firms charge higher prices the further away their products are located along the a characteristic because, similar to the analysis of changes in characteristic A, this relaxes price competition.

Finally, Lemma 2 reveals that the comparative statics with respect to characteristic a in Eq^* depend on the cost of quality provision but not on whether F sells a dominant product ($a_F \lesseqgtr a_f$). When a firm's a characteristic increases, it raises its marginal costs, which contributes to the strategic effect by weakening the firm's competitive standing, and it also raises consumers' willingness to pay for its product, which contributes to both the direct effect and the strategic effect by strengthening the firm's competitive standing. In the low quality-cost region ($c < \frac{1}{2}$), the impact of marginal costs on the strategic effect is small, and hence the direct effect dominates: both firms would respond to an increase in their rival's a characteristic by lowering price (hence both $\partial p_F/\partial a_f$ and $\partial p_f/\partial a_F$ are negative). This suggests that both firms have an incentive to decrease their a characteristic level to soften price competition. Conversely, when $c > \frac{1}{2}$, the impact of marginal costs is large enough to make the strategic effect dominate: both firms react to an increase in a rival's a-characteristic level by increasing their own price (hence both $\partial p_F/\partial a_f$ and $\partial p_f/\partial a_F$ are positive). Finally, when $c = \frac{1}{2}$, the direct and strategic effects exactly offset each other and changing the level of the a characteristic has no effect on the rival's price. The difference in the role played by characteristics A and a in Eq^* is a major factor in understanding why a Max-Min product configuration arises in equilibrium. Furthermore, the dependence on the cost of quality provision c will be pivotal in determining the nature of the Max-Min equilibrium.

In the next section, we will use the results and intuitions obtained in solving the pricing subgame to examine how firms endogenously choose their product characteristic levels in the first stage.

4.2. Equilibrium Choice of Product Characteristics

To establish how firms position their products, we first identify all the product configurations that are locally stable, i.e., configurations in which neither firm wants to change its product positioning slightly given its competitor's product positioning. We then characterize the equilibrium among all locally stable product configurations by checking that neither firm can benefit from switching to a completely different product positioning.

The effects of a change in product characteristic q_i ($q_i \in \{a_i, A_i\}$) on firm i's profits can be decomposed using the envelope theorem as follows:

$$\frac{\partial \pi_i^{eq}}{\partial q_i} = \underbrace{\frac{\partial D_i}{\partial q_i}(p_i - ca_i - cA_i) - cD_i}_{\text{Demand-cost effect}} + \underbrace{\frac{\partial \pi_i}{\partial p_j}\frac{\partial p_j}{\partial q_i}}_{\text{Cross-price effect}}. \quad (1)$$

From (1) we see that there are several forces at play. First, holding rival actions constant, improvement along either characteristic increases overall product quality, which enhances the demand for such a product ($\partial D_i/\partial q_i > 0$) but at the same time makes the product more costly to produce ($-cD_i \leq 0$). As one effect is positive and the other is negative, what matters is the overall combined effect, which we denote the *demand-cost* effect. We use the first-order condition (FOC) for firm i from the pricing stage to rewrite the overall demand-cost effect as

$$\left(\frac{\partial D_i/\partial q_i}{-\partial D_i/\partial p_i} - c\right)D_i. \quad (2)$$

From (2), the sign of the demand-cost effect depends on the ratio of how sensitive demand is with respect to quality and with respect to price changes as compared to the cost of quality provision. We can expect the demand-cost effect to be positive when quality costs are relatively low and negative otherwise. This effect would also be relevant for a monopolist setting optimal quality levels, as we explain in §5.1.

From Lemma 2, however, we know that a modified product also has a competitive effect because a different configuration in the characteristics space affects downstream pricing decisions. This cross-price effect on profits is captured by the last term in (1). Because profits increase with the rival's price ($\partial \pi_i/\partial p_j > 0$), the sign of the cross-price effect boils down to figuring out whether firm j increases or decreases its price when firm i improves its characteristic q_i (i.e., the sign of $\partial p_j/\partial q_i$). Lemma 2 revealed that the cross-price effect can be positive or negative depending on the firm (F or f) and on the characteristic (A or a).

The next two propositions establish that when quality costs are low or intermediate, any equilibrium of the game must exhibit the *Max-Min* property: firms always choose to maximally differentiate on one dimension and minimally differentiate on the other dimension. Furthermore, maximal differentiation occurs on the characteristic with the greater span, with one firm selecting $\bar{x}$ and the other $\underline{x}$. Minimal differentiation occurs on the attribute with the smaller span y, and there are two possibilities that are characterized in Propositions 1 and 2.

Proposition 1. *When $0 \le c < \frac{1}{2}$, the Max-Min configuration $((\bar{x}, \bar{y}), (\underline{x}, \bar{y}))$ is the* unique *product positioning in equilibrium.*

Proposition 1 shows that when the marginal cost of providing quality is low, any equilibrium of the two-characteristic game must exhibit the Max-Min property. One firm sells the best possible product $(\bar{x}, \bar{y})$. The rival firm fully differentiates along the characteristic with the larger span and fully agglomerates along the characteristic with the smaller span. Specifically, the product configuration of the rival firm is $(\underline{x}, \bar{y})$.[12]

To understand the intuition, note that because the marginal cost of providing greater quality is low $(c < \frac{1}{2})$, the demand-cost effect is positive. Thus, holding rival actions constant, firms would like to increase product quality by offering greater attribute levels. However, a firm may not benefit from selling a higher-quality product if subsequent price competition becomes too intense, which is the case if the cross-price effect is negative and large enough to dominate the demand-cost effect. The sign of the cross-price effect depends on whether the subgame price equilibrium is Eq^* or Eq^{**} (see the existence conditions in Table 1 when $c < \frac{1}{2}$). We start by characterizing locally stable product configurations that lead to Eq^* in the pricing stage. From Lemma 2 we know that in the region of Eq^*, where the cost of quality provision is low, the cross-price effect is positive when A_F is increased and negative when A_f, a_F, or a_f are increased. Clearly then, both the demand-cost effect and the cross-price effect push firm F to increase A_F. For attribute A_f, however, the cross-price effect dominates; hence firm f prefers to decrease A_f to relax price competition. As for the a characteristic, since $c < \frac{1}{2}$, the demand-cost effect is positive and dominates the cross-price effect for both a_F and a_f. Hence both firms want to increase their a characteristic to raise demand for their product. Taken together, firms want to maximally differentiate along the A characteristic and choose the highest level on the a characteristic. The locally stable product configurations that satisfy these properties and lead to subgame equilibrium Eq^* are such that firm F sells the highest-quality product possible $(\bar{x}, \bar{y})$, and firm f max-min differentiates by selling either $(\underline{x}, \bar{y})$ or $(\bar{x}, \underline{y})$. In effect, either attribute x or y can serve as attribute A to form a locally stable configuration.

Another locally stable configuration may exist, one that leads to subgame equilibrium Eq^{**}. In Eq^{**}, firm F sells a dominant product with significant comparative advantages on both characteristics (see Figure 1). When a firm increases differentiation by locating further away from its rival on either attribute, price competition is relaxed per Lemma 2. The negative cross-price effect dominates the demand-cost effect for firm f on both attributes, and the two effects are positive for firm F and push it to improve quality on both attributes. Therefore, if the Max-Max product configuration is the strategy the firms play (where firms F and f choose positions $(\bar{x}, \bar{y})$ and $(\underline{x}, \underline{y})$, respectively) and the subgame pricing equilibrium is Eq^{**}, then this product configuration is also locally stable.

As is evident, in all the locally stable configurations, firm F selects the best product $(\bar{x}, \bar{y})$. To characterize the global equilibrium of the game, therefore, we must determine which corner of the characteristics space firm f would choose in order to maximize its profits $((\underline{x}, \bar{y}), (\bar{x}, \underline{y})$, or $(\underline{x}, \underline{y}))$. We first compare the two Max-Min configurations. When firm f chooses the location $(\underline{x}, \bar{y})$, its profits are $(1+c)^2 \Delta x/9$, and when it chooses product location $(\bar{x}, \underline{y})$, its profits are $(1+c)^2 \Delta y/9$. Clearly, because characteristic x has a larger range than characteristic y $(\Delta x \ge \Delta y)$, firm f prefers maximal differentiation along characteristic x and minimal differentiation on characteristic y. In other words, firm f benefits from as much separation as possible on the attribute used for differentiation purposes, and the attribute with the greater range affords this.

Given this conclusion, what remains is to compare firm f's profits in the Max-Min configuration $(\underline{x}, \bar{y})$ with those in the Max-Max configuration $(\underline{x}, \underline{y})$. Firm f essentially has to choose between serving a low-end niche market in the latter case and competing more directly with firm F by differentiating only along the x attribute in the former case. In the low-end niche strategy, firm f's profits are low because it sells a low-quality product at a low price. Firm f prefers to go after more lucrative high-willingness-to-pay consumers with a high-quality offer on the a characteristic. Head-to-head competition on the a characteristic is less of a concern given the maximum differentiation on the A characteristic. In sum, the Max-Min equilibrium with one firm offering the best possible product is unique up to a relabeling of the firms.[13] Moreover, differentiation occurs along

[12] Recall that, without loss of generality, we assumed that $\Delta x \ge \Delta y$. In the knife-edge case when $\Delta x = \Delta y$, the role of the characteristics are interchangeable, and $((\bar{x}, \bar{y}), (\bar{x}, \underline{y}))$ is sustainable in equilibrium.

[13] We thank the associate editor for helping us clarify this intuition.

the attribute with the larger span, and firm F always earns a larger profit than firm f.

We now turn to characterizing the equilibrium when the cost of quality provision is larger ($c > \frac{1}{2}$), whereby the demand-cost effect becomes negative. By decreasing attribute levels, a firm may benefit more from cost savings than it is penalized by the loss of demand as a result of selling a lower-quality product. The next proposition shows that, as a result, in an intermediate quality-cost region, one firm opts to sell the lowest quality product in equilibrium, yet the Max-Min property continues to hold.

Proposition 2. *For any $\Delta y/\Delta x$, there exists a cutoff for the quality provision cost $\hat{c}$ in $[1, 2]$ s.t. when $\frac{1}{2} < c < \hat{c}$, the Max-Min configuration $((\underline{x}, \underline{y}), (\bar{x}, \underline{y}))$ is the unique product positioning in equilibrium.*

Proposition 2 states that even in an intermediate quality-cost range, any equilibrium of the two-characteristic game must still exhibit the Max-Min property. However, in this case, one firm sells the worst possible product $(\underline{x}, \underline{y})$. The rival firm fully differentiates along the characteristic with the larger span and fully agglomerates along the characteristic with the smaller span. Specifically, the product configuration of the rival firm is $(\bar{x}, \underline{y})$.

The intuition behind Proposition 2, and the dependence of the cutoff value $\hat{c}$ on the attribute ranges, is as follows. As the demand-cost effect is now negative, firms would rather decrease their characteristic levels. This implies that one firm always selects the lowest-quality product in equilibrium. If the x characteristic has a span much larger than the y characteristic, the rival firm benefits from differentiating as much as possible on the x characteristic to relax price competition, but it would like to offer the lowest level on characteristic y to contain costs (thereby selecting the Max-Min positioning $(\bar{x}, \underline{y})$).[14] However, if the x characteristic has a span not too much larger than the y characteristic, there exists another configuration that is locally stable, namely, the Max-Max configuration $((\underline{x}, \underline{y}), (\bar{x}, \bar{y}))$. In choosing among all locally stable configurations the positioning that generates the highest profit, firm F compares selling to consumers who highly value characteristic A (Eq^* at $(\bar{x}, \underline{y})$) and selling only to the high-end niche, i.e., consumers who highly value both attributes ($Eq^\dagger$ at $(\bar{x}, \bar{y})$). When c is intermediate ($\frac{1}{2} < c < \hat{c}$), the cost disadvantage of firm F in the Max-Min configuration (Eq^*) compared to its competitor is not too large. Firm F can price its product at a level that is still attractive to a large portion of the market, and it achieves the greatest profits under the Max-Min configuration. However, as the cost of quality provision (c) increases, firm F finds it harder and harder to stay competitive under the Max-Min configuration because its costs rise faster than the rival's costs. The need to raise prices to cover these greater costs results in a big drop in demand. If c becomes too large, firm F finds it beneficial to switch strategies, and the Max-Min equilibrium ceases to exist. In sum, the Max-Min equilibrium with one firm offering the worst product exists only when the cost of quality provision is smaller than a cutoff $\hat{c}$. This cutoff depends on $\Delta y/\Delta x$, because the larger this ratio is, the more difficult it becomes for the firm at $(\bar{x}, \underline{y})$ to sustain this positioning—it has to incur the high costs associated with offering $\bar{x}$, and at the same time, minimal differentiation along the y characteristic intensifies price competition (see Figure 2 for a graphical representation of $\hat{c}$).

The next proposition shows that in the region with high costs of quality provision, the Max-Min property ceases to hold as firm F achieves its greatest profits by selling to high-end consumers only.

Proposition 3. *When $\hat{c} < c < 2$, one firm sells the* lowest-quality *product possible $(\underline{x}, \underline{y})$. The other firm positions at*

- *$(\bar{x}, \bar{y})$ if $\Delta x \le \Delta x^\dagger$ (Max-Max equilibrium) or*
- *$(x^\dagger, \bar{y})$ if $\Delta x \ge \Delta x^\dagger$ (Partial-Max equilibrium)*

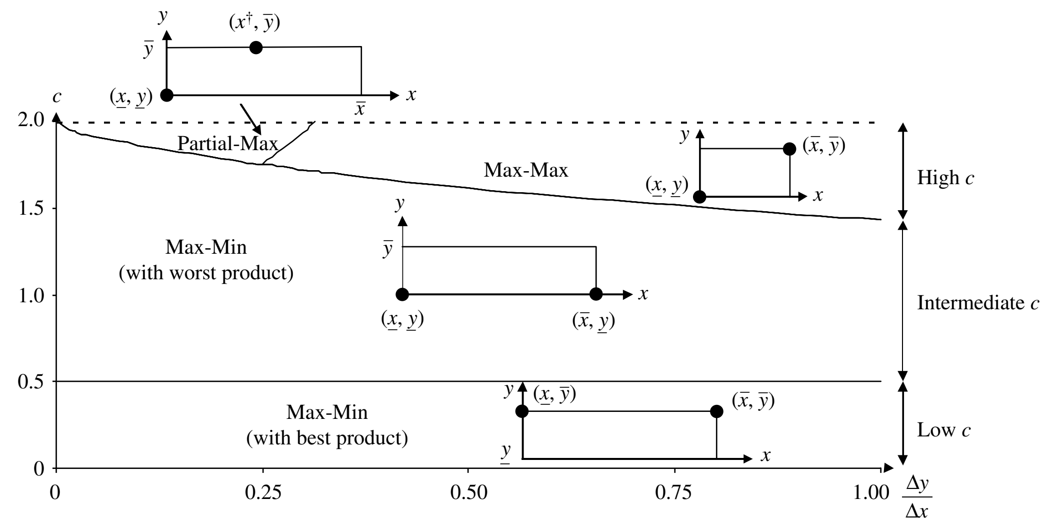
with

$$\Delta x^\dagger \equiv \left(5 + \frac{2}{(c-1)^2} + \sqrt{\left(5 + \frac{2}{(c-1)^2}\right)^2 - 16}\right)\frac{\Delta y}{4}.$$

Not surprisingly, in the high quality-cost region, one firm selects the lowest quality product possible $(\underline{x}, \underline{y})$. The other firm always fully differentiates along the characteristic with the smaller span by selecting $\bar{y}$, and it either fully or partially differentiates along the characteristic with the larger span. It might seem counterintuitive that when quality becomes much more expensive to provide, a firm chooses to increase its product quality by offering higher levels on all attributes. It happens because the firm with the larger comparative advantage in a Max-Min configuration finds it more and more difficult to sustain the $(\bar{x}, \underline{y})$ position when costs go up. The firm cannot raise prices to fully cover the costs of offering $\bar{x}$ because of the negative impact on demand, and competition is quite intense because it offers the same low level as the rival on attribute y (the rival is able to price very aggressively given that its product is much cheaper to produce). Hence, the firm's margin and demand decrease, and at some point (when $\hat{c} < c$), it finds switching to a new strategy more beneficial: the firm prefers to differentiate even more from its rival and focus mainly on the high-end niche because this alleviates price competition. To do so, the firm needs to

[14] We verify in the proof that the Max-Min configuration $(\underline{x}, \bar{y})$ is dominated by the Max-Min configuration $(\bar{x}, \underline{y})$.

Figure 2 Product Positioning Equilibrium as a Function of the Cost of Quality Provision c and the Relative Characteristic Span $\Delta y/\Delta x$



Notes. The boundary between the low and intermediate cost of quality regions is $c = 1/2$ per Propositions 1 and 2. The boundary between the intermediate and high cost of quality regions is $\hat{c}$ per Propositions 2 and 3. The boundary between the Partial-Max and the Max-Max equilibria is such that $\bar{x} = x^\dagger$ per Proposition 3.

exploit both product dimensions and select the positioning $(\bar{x}, \bar{y})$.

Figure 2 illustrates the equilibria in Propositions 1–3. The product configuration in equilibrium in each region of the parameter space is generically unique, except when the two characteristics have the same range ($\Delta x = \Delta y$) or at the limit between the different quality cost regions ($c = \frac{1}{2}$ or $\hat{c}$). We note that when the cost of quality is in the intermediate or high regions (equilibria characterized in Propositions 2 and 3), firm f always earns a greater profit than its rival because it serves more than half of the market and its margin is also larger per unit sold.

As indicated in Proposition 3, however, the firm may stop shy of providing the highest quality $\bar{x}$ when the range of the x characteristic is too large. This is because the trade-off in terms of fully differentiating on attribute x and the cost of selecting higher quality levels begins favoring the latter consideration. Note then that in this case, and in contrast to the Max-Min equilibria, the firms are maximally differentiated on the dimension with the *smaller* span and partially differentiated on the dimension with the larger span.

5. Discussion of the Model and Its Relationship to Existing Work

In this section, we come back to the assumptions of our model. We discuss in turn the benchmark case of a monopolist, the special case of marginal costs that are independent of quality, the special case of a single dimension, the difference between vertical and horizontal differentiation, how to deal with characteristics that are not symmetric, and finally, what happens when the market is not fully covered.

5.1. Benchmark Case: Monopolist

When duopolists agglomerate on the y attribute (as in Propositions 1 and 2), an interesting benchmark to compare against is the level that a monopolist would select for that attribute.[15] The question we ask in this section is whether a monopolist follows the same kind of strategy as the duopolists, i.e., select the highest level $\bar{y}$ for low costs of quality provision (c) and then jump to the lowest level $\underline{y}$ when c is above a certain threshold.[16] We show that a monopolist does not always follow this strategy: it might gradually decrease the level of y as c goes up. In effect, the monopolist's optimal strategy on the y attribute depends on how overlapping the ranges of the two attributes are. This contrasts the behavior of duopolists, whose equilibrium decisions do not depend on the degree of overlap in the ranges of the two attributes. Moreover, recall that when c is very high ($\hat{c} < c$), the duopolists cease to agglomerate on the attribute with the smaller range (per Proposition 3).

Two forces are at play when a monopolist selects its optimal product quality: improving quality boosts demand for the product while it makes the production costs go up. This is what we called the demand-cost effect in this paper (see (1)). When the cost of

[15] We thank the associate editor for suggesting that we compare the monopolist's optimal attribute selection to that of the duopolists.

[16] Because a monopolist never serves the full market, an assumption equivalent to full market coverage in the duopoly case is that V is large enough such that the monopolist serves all the consumers except the lower-end niche of customers, who do not care about quality: its market share is the full square except the bottom left corner in the (θ_x, θ_y) plane (similar to F's demand in Eq^{**} in Figure 1).

quality provision is low (c close to 0), the demand effect dominates, and the monopolist selects the best product $(\bar{x}, \bar{y})$. On the other hand, when the cost of quality provision is high (c close to 2), the cost effect dominates, and the monopolist selects the worst product position $(\underline{x}, \underline{y})$. However, whether the monopolist jumps abruptly from $\bar{y}$ to $\underline{y}$ at some threshold value or changes its y characteristic level gradually as a function of c depends on the absolute levels of x and y—specifically, on how much the range of the x characteristic overlaps with the range of the y characteristic. If characteristic ranges overlap substantially—for example, have the same lower bound $\underline{x} = \underline{y}$ and $\bar{x} \geq \bar{y}$—the monopolist uses a single cutoff c^m in determining whether to set the highest or lowest level for the y characteristic. In a sense, this result is similar to the $c = \frac{1}{2}$ cutoff in the duopolist case when the equilibrium is of a Max-Min type, except that the monopolist's cutoff c^m is smaller than the duopolist cutoff of $\frac{1}{2}$. Intuitively, cost containment $(-cD^m)$ kicks in at lower quality cost levels for the monopolist because it is the only firm serving the market; hence D^m is closer to 1 than in the case of duopolists who split the market.[17]

However, if characteristic ranges differ substantially—for example, when $\underline{x} > \bar{y}$ (i.e., all x levels are higher than any y level)—the monopolist changes its y characteristic continuously as a function of c. When c is low, the monopolist chooses the best product because quality is not costly to produce. As c increases, the monopolist needs to contain costs and starts doing so by decreasing the level of its most expensive characteristic, the x characteristic. However, once the monopolist reaches product position $(\underline{x}, \bar{y})$, the only way to contain costs is to start decreasing the y characteristic as well. The monopolist thus decreases its level on y continuously until it reaches $(\underline{x}, \underline{y})$. To understand why the monopolist acts this way, note that in the substantial overlapping range case, both high-end and low-end strategies are locally stable, and the monopolist always selects the one generating the greater profit. A small change in c is enough to favor one strategy at the expense of the other, thus explaining the discontinuous jump. In the latter case (nonoverlapping ranges), the monopolist has only one locally stable product location. This product location moves continuously with the value of c because the optimal level of y is used to balance demand enhancement and cost containment. These results are presented formally in the following proposition.

[17] The cutoff c^m depends on the absolute levels $\underline{y}$ and $\bar{y}$ and is always less than $\frac{1}{2}$, whereas both the demand-cost and the cross-price effects in a duopoly switch signs at $\frac{1}{2}$.

Proposition 4 (Monopolist Case).
- *When $\underline{x} \approx \underline{y}$ and $\bar{x} \geq \bar{y}$, there exists a cutoff value $c^m < \frac{1}{2}$ such that the monopolist positions at $\bar{y}$ if $c < c^m$ and at $\underline{y}$ if $c^m < c$.*
- *When $\underline{x} > \bar{y}$, there exist cutoff values $c_o^m < c_1^m < \frac{1}{2}$ and $y^m(c)$ such that the monopolist positions at $\bar{y}$ if $c \leq c_0^m$, at $y^m(c)$ if $c_0^m \leq c \leq c_1^m$, and at $\underline{y}$ if $c_1^m \leq c$, where $y^m(c)$ decreases continuously from $\bar{y}$ to $\underline{y}$ as c goes from c_o^m to c_1^m.*

5.2. Special Case: Single Dimension

In the special case when firms cannot differentiate along the y attribute $(\Delta y = 0)$, our model is about one-dimensional vertical differentiation. Now, firms have only the x attribute to manage both the demand-cost effect and the cross-price effect. Both of these effects are also behind the results in Moorthy (1988) and Shaked and Sutton (1982). Overall, Propositions 1 and 2 show that the cross-price effect is prevalent and that firms prefer to maximally differentiate: one firm selects the best product while its rival opts for the worst one.

Corollary 1. *In the one-dimension case, $\Delta y = 0$, firms only select the level of the x attribute; when $0 \leq c < 2$, the Max differentiation configuration $(\bar{x}, \underline{x})$ is the* unique *product positioning in equilibrium.*

When comparing the two-dimension case with the single-dimension case, one can think of the attribute with the larger range as playing the same role as the single dimension. Firms differentiate along x in order to relax price competition. They actually maximally differentiate along that attribute as long as it is not too costly to do so. Interestingly, the role of the second attribute is different. The prime concern of firms when choosing their y characteristic level is not price competition but demand stimulation or cost containment. In the low quality-cost region, firms use the smaller range attribute y to boost demand by choosing the highest quality level possible. In the intermediate quality-cost region, firms choose the lowest quality level possible to contain cost. Finally, in the high quality-cost region, one firm selects the lowest quality level to contain cost while its rival selects the highest level to target the niche of high-end consumers. In the two-dimensional case, by having two characteristics to work with, firms select one dimension to differentiate on and mitigate price competition and the other dimension to manage demand and cost considerations.

5.3. Special Case: Marginal Costs Independent of Attribute Levels Chosen

We now discuss the special case when marginal costs are independent of attribute levels ($c = 0$), which is covered by our results in Proposition 1. In that case, our model boils down to the setup in Vandenbosch and Weinberg (1995) (VW hereafter) but our conclusions regarding the equilibrium product configuration are not the same. We find the exact same prices and profits in the pricing stage as in VW, although we organize these results in a slightly different way. However, our Proposition 1 unequivocally states that when $c = 0$, the only equilibrium possible entails the Max-Min configuration. By contrast, VW propose several equilibria that do not satisfy the Max-Min property. To understand why the Max-Max and Partial-Max configurations proposed in VW cannot form an equilibrium when $c = 0$, start with either of these product configurations and note that both entail one firm selling the best product $(\bar{x}, \bar{y})$. The rival can then deviate to $(\underline{x}, \bar{y})$ and boost its demand by choosing the highest level of the smaller range attribute while keeping competition relaxed by maximally differentiating along the attribute with the larger span. Interested readers can refer to Appendix A for details of the profitable deviation for firm f. In the end, only the Max-Min configuration of Proposition 1 is sustainable in equilibrium. VW's conclusion that other product configurations could form an equilibrium stems from an error in the sign of certain profit derivatives with respect to product characteristics (specifically, in VW's Table 1, region ${}_dR_x^2$ contains sign errors that can lead to erroneous equilibrium conclusions).

We wish to note that the Max-Max and Partial-Max equilibria we characterize in Proposition 3 are the result of firms also taking into consideration greater costs in providing higher levels of product attributes—an issue that is not part of the VW model.

5.4. Vertical vs. Horizontal Differentiation

Our paper analyzes the case of two vertically differentiated characteristics: all consumers prefer higher characteristic levels but differ in their willingness to pay for higher quality. How do our conclusions relate to those of Neven and Thisse (1990), who study the case of one vertically and one horizontally differentiated characteristic? The horizontally differentiated characteristic is interpreted as variety instead of quality because consumers disagree on their preferred characteristic level keeping prices fixed. Because it is not clear why one variety would be more expensive to produce than another one, and consistent with the model assumptions of Neven and Thisse (1990), we assume in this discussion that marginal costs are independent of product positioning ($c = 0$). In Neven and Thisse (1990), a monopolist would choose the highest quality level $(\bar{q})$ and the central variety $(\frac{1}{2})$, whereas in our model, a monopolist chooses the top qualities ($\bar{x}$ and $\bar{y}$). Hence, the central variety location plays a similar role as the highest quality level on an attribute in our model.

Under duopolistic competition, Neven and Thisse (1990) identify two equilibria that satisfy the Max-Min property.[18] In one equilibrium, firms maximally differentiate along the quality characteristic (one firm chooses $\bar{q}$, and the other chooses $\underline{q}$) and agglomerate on variety (both locating at $\frac{1}{2}$). In the other equilibrium, firms agglomerate on quality (both locating at $\bar{q}$) and maximally differentiate along the variety dimension (one firm chooses to locate at 0, and the other chooses to locate at 1). Note that in both our paper and theirs, firms always agglomerate on the attribute level that would be chosen by a monopolist. However, when in Neven and Thisse (1990) differentiation occurs on the horizontal dimension, neither firm locates in the center, and the "best" product $(\bar{q}, \frac{1}{2})$, which would be chosen by a monopolist, is not chosen in equilibrium. By contrast, in our paper the best product $(\bar{x}, \bar{y})$ is always selected by one of the firms when $c = 0$. Another difference relates to uniqueness. Neven and Thisse (1990) show that both equilibria can coexist, whereas we find that the equilibrium is unique as firms always maximally differentiate along the larger range characteristic. Finally, both papers show that the Max-Max configuration cannot form an equilibrium when $c = 0$.

5.5. Other Model Assumptions Revisited

5.5.1. Attribute Roles and Preference Parameters. In our model, the two characteristics play a symmetric role in the utility of consumers $V + \theta_x x + \theta_y y - p$, with θ_x and θ_y uniformly distributed over $[0, 1]$. This setup is not as restrictive as it seems and, in fact, encompasses more-general cases. Let us examine two examples with characteristics that are not symmetric in the case when $c = 0$. First, one characteristic could play a more important role than the other, which can be modeled by having different weights in the utility function such as $V + \theta_x x + \omega\theta_y y - p$. If the weight ω is larger than 1, characteristic y is more important than characteristic x, and if ω is smaller than 1, the reverse is true. We can transform the above setup to conform to the model studied in this paper by rescaling the y characteristic. Specifically, by defining $y' = \omega y$ in $[\omega\underline{y}, \omega\bar{y}]$ and letting firms select x in $[\underline{x}, \bar{x}]$ and y' in $[\omega\underline{y}, \omega\bar{y}]$, we can compare the ranges of the characteristics (Δx and $\omega\Delta y$) to identify the product positioning equilibrium in analogy to the analysis presented in this paper.

The second issue around symmetry deals with taste parameters θ_x and θ_y that do not have the same range. As long as these taste parameters are uniformly distributed over $[0, \Theta_x]$ and $[0, \Theta_y]$, respectively, we can rescale both the characteristic levels and

[18] Neven and Thisse (1990) do not establish that these Max-Min equilibria are the only possible equilibria.

the taste parameters so that Propositions 1–3 apply. Specifically, we define $\theta'_x = \theta_x/\Theta_x$, $\theta'_y = \theta_y/\Theta_y$, $x' = \Theta_x x$, and $y' = \Theta_y y$ so that consumers' utility becomes $V + \theta'_x x' + \theta'_y y' - p$, with θ'_x and θ'_y uniformly distributed over $[0, 1]$. In the end, rescaling the characteristics so that they play a symmetric role in the utility function allows us to compare their ranges to be able to talk about the larger range and smaller range characteristics.

Rescaling the characteristics also changes the marginal cost, which becomes $c_x x + c_y y$ with $c_x \neq c_y$. In general, when the cost parameters differ for each of the attributes, our results qualitatively hold with a few caveats. When both costs are small, similar to Proposition 1, the product location equilibrium will exhibit the Max-Min property with one firm selling product $(\bar{x}, \bar{y})$ and the rival selling product $(\underline{x}, \bar{y})$. Yet if, for example, the cost parameter c_y increases substantially while c_x remains constant, firms will switch to the Max-Min equilibrium described in Proposition 2 to contain costs (although the threshold could be less than $1/2$). As c_y keeps increasing, both firms are affected in the same way (as both select the same minimal level of y), so firm F may not switch to the niche Max-Max configuration described in Proposition 3. Similarly, in the case of increasing c_x while keeping c_y constant, firms might not switch from the Max-Min configuration with both selecting $\bar{y}$ to the Max-Min configuration with both selecting $\underline{y}$ because now reducing the level of the x characteristic helps more in terms of cost containment.

5.5.2. Full Market Coverage. Throughout this paper we assume that the market is fully served. The value V of the product is large enough so that all consumers buy in equilibrium. We now discuss what happens if we relax the full market coverage assumption when marginal costs are independent of quality ($c = 0$). When $V \geq \max\{\Delta x/3, 3\Delta y/8\}$, for any product configuration the market is fully covered. In that case, Proposition 1 shows that the Max-Min property holds in equilibrium, that firms fully differentiate along the larger range attribute, and that both firms choose the highest quality level for the smaller range attribute. When $V < \max\{\Delta x/3, 3\Delta y/8\}$, the market will not be fully covered in equilibrium. In this case, firm f cares not only about losing consumers to its rival, in which case only the relative quality levels ΔA and Δa matter, but also about having consumers opting out of the market because they do not derive positive utility. Hence the absolute levels of a_f and A_f now also play an important role, and firm f may not want to offer the lowest possible level on attribute x so that it can create sufficient demand. We show in the electronic companion (available as part of the online version that can be found at http://mktsci.pubs.informs.org/) that as long as $\underline{x}$ and $\underline{y}$ are not too small, the unique equilibrium will have the market not fully served, and as in Proposition 1, one firm selects the best product possible $(\bar{x}, \bar{y})$ and firms agglomerate along the smaller range attribute by choosing the highest level $\bar{y}$. The only difference is that differentiation along the larger range attribute will not necessarily be maximal. In particular, when $\underline{x}$ is not large enough, firms might only partially differentiate on the x characteristic. But note that even though the market is not fully covered, the nature of the equilibrium—using only the larger span attribute to differentiate on while electing to be undifferentiated on the smaller span attribute—still holds.

6. Managerial Implications and Concluding Remarks

When products comprise multiple characteristics, firms have considerable flexibility with respect to how they position their offerings in the marketplace. In a duopoly model in which consumers are heterogeneous with respect to their willingness to pay for each of two product characteristics, we uncover an interesting interplay between the desire to differentiate in order to manage competition on the one hand and the need to contain the costs of delivering greater quality on the other hand.

A central result we obtain is that when the marginal cost of quality provision is not too high, firms will always choose to maximally differentiate on one dimension and minimally differentiate on the other, thereby ruling out equilibria that do not satisfy the Max-Min property. Furthermore, we show that differentiation should always occur along the attribute with the greater range.

Our results also provide clarity on what level the firms should provide on the nondifferentiated attribute (y). Specifically, when the marginal cost of quality provision is low, demand generation is a key consideration, and both firms should offer the highest possible attribute level. Conversely, when the cost of quality provision is intermediate, containing costs becomes a nontrivial concern, and both firms should offer the lowest possible attribute level.

We then show that the Max-Min property breaks down as the cost of quality provision increases further. Interestingly though, the higher costs prompt firms to become even more differentiated: one firm chooses a product with minimal levels on both product dimensions while its rival chooses maximal levels (a Max-Max equilibrium). The firm offering the highest qualities serves a small niche of consumers that value high-end products and charges them a high price. Given this high-end strategy, the rival firm offering the low-quality product does not have to price too aggressively and can have a sizable margin.

Our findings have important managerial implications for firms considering how to position their product in light of competition. One option firms often contemplate is to try to outdo rivals by offering a product that excels on all dimensions. Our findings suggest that a firm should consider such an approach only if (a) the marginal costs associated with increasing quality are low enough, and the rival is expected to offer a low level on the attribute with the larger span; or (b) the marginal costs associated with increasing quality are actually high, and the rival is expected to position at the low end on both attributes. Examined from the opposite perspective, when a firm faces a rival that is expected to choose high quality levels on both product attributes, then it makes sense to fully differentiate on all dimensions when the costs of quality provision are high but only differentiate on one dimension when those costs are low. Firms should also be aware of the fact that in case (a), the highest-quality product is also the most profitable, whereas in case (b), the lowest-quality product is the most profitable.[19]

Second, our findings indicate that as long as marginal quality costs are not too high, firms need to first understand which product attribute has the greatest range (in terms of the relevant levels consumers care about or the feasible range of development and/or production). After converting both attributes to a common utility scale, it is the dimension with the greater range that should serve for differentiation purposes.

Several of our findings may be relevant for empirical work that examines competitive differentiation. In particular, given that for a wide range of quality provision costs we should expect Max-Min equilibria, future work could test whether the degree of product differentiation among firms in a given market tends to be selectively greater around one attribute with less variance around the choice of other attributes (a percent difference from a baseline or some other mean-centering technique can be used to compare differentiation across each of the attributes). In addition, and consistent with our conditions for the Max-Max equilibrium, one can test whether industries that exhibit relatively high quality costs tend to exhibit more overall differentiation. To do so, one could create an index that accounts for differentiation on more than one dimension (say, an average differentiation measure across all attributes) and an estimate of the variable costs associated with quality provision (see, for example, Srinivasan et al. 1997, Horsky and Nelson 1992, for proposed techniques to do so); one further needs to devise a way to compare cost levels across industries. Alternatively, in industries where there was a significant change in the cost of quality provision, for example, because of an increase in the cost of inputs, one can look at how firms within the industry adjusted their product locations subsequent to the cost shock (looking for a positive correlation between variable cost changes and degree of overall differentiation). We note that because our model captures vertical differentiation, it would be relevant to look at industries where product attributes, and consumer heterogeneity, conform to this setup. Many high-tech markets—for example, microprocessors or graphics cards—could be appropriate for such an examination (noting also that in these particular cases we have duopolistic competition).

As with much of the literature in this stream, our analysis and findings come with a few caveats. First, we have assumed that variable costs linearly rise with attribute levels. In reality, higher-quality products may be increasingly more costly to produce, i.e., exhibit a convex quality-to-cost relationship (see Moorthy 1988 for an analysis of a one-dimensional case). Costs may also include some interaction terms between the two attribute levels; e.g., raising one attribute would be increasingly costly as the level of the other attribute goes up. In either case, we believe our results would hold for at least some cost regions, although we would expect the Max-Min equilibrium to cease existing sooner because costs increase faster than in the linear case. Second, choosing attribute levels often requires up-front fixed investments in research and development, which we have ignored. However, because up-front costs are incurred once and do not depend on volume sold, we expect our results to be less affected by them qualitatively. We leave the examination of these alternative cost structures for future research. Third, our analysis assumed uniformly distributed preferences for both attributes. In reality, consumer preferences may exhibit different distributions, which would likely affect the most lucrative product locations (see, e.g., Ansari et al. 1994). Fourth, we have assumed that the contribution of the two attributes to consumer utility is additive, in line with much of the work on multiattribute preferences (Green and Srinivasan 1990). Yet it is conceivable that in some settings, the contribution is multiplicative, particularly when the two attributes are complementary. This could lead to different conclusions than the ones reached here and is an area for future exploration. Fifth, it would be useful to examine whether the findings reported here extend to the case of more than two characteristics—in particular, whether we should expect the equilibrium when the cost of quality provision is not too high to

[19] These implications could also be relevant for an entrant given the product positioning chosen by an incumbent already in the market. Of course, such conclusions would require solving a sequential move game to verify the nature of the equilibria.

exhibit maximal differentiation on only one attribute and minimal differentiation on all others. Finally, in some contexts firms may seek to design more than one offering, which introduces product line considerations. Although we expect the forces identified here to still be relevant, each firm would also need to manage cannibalization concerns. Future research could examine whether the configurations identified in this paper still emerge in equilibrium, with firms adding products to occupy additional positions in the space, or whether entirely new positions emerge that are not predominantly at the corners of the $x-y$ space.

7. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mktsci.pubs.informs.org/.

Appendix A. Profitable Deviations from Max-Max and Partial-Max Configurations When $c=0$

When marginal cost is equal to 0 ($c=0$), our model setup boils down to the setup in VW. We provide below profitable deviations to show that certain product configurations suggested in VW cannot be part of an equilibrium.

First, VW claim on p. 240 that, if $\frac{128}{81}\Delta y \le \Delta x \le 2\Delta y$, there exists a Max-Max equilibrium in which firm 2 is positioned at $(\bar{x}, \bar{y})$, and firm 1 positions at $(\underline{x}, \underline{y})$. If firm 1 chooses a Max-Max configuration by locating at $(\underline{x}, \underline{y})$, its profits are $\frac{1}{4}\sqrt{\Delta x \Delta y/8}$ (see Eq^{**} in our Table 2 or VW's Table 1). However, if firm 1 chooses instead a Max-Min configuration by positioning at $(\underline{x}, \bar{y})$, its profits are $\Delta x/9$ (see Eq^{*} in our Table 2 or VW's Table 1). Firm 1 would never choose the product positioning with the lower profit. As $\frac{1}{4}\sqrt{\Delta x \Delta y/8} < \Delta x/9$ when $\frac{128}{81}\Delta y \le \Delta x$, firm 1 never chooses $(\underline{x}, \underline{y})$, implying that $((\bar{x}, \bar{y}), (\underline{x}, \underline{y}))$ cannot be an equilibrium.

Second, VW also claim that if $2\Delta y \le \Delta x$, there exists a Partial-Max equilibrium in which firm 2 is positioned at $(\bar{x}, \bar{y})$, and firm 1 positions at $(\bar{x}-2\Delta y, \underline{y})$. If firm 1 chooses $(\bar{x}-2\Delta y, \underline{y})$, its profits are $\Delta y/8$ (see Eq^{*} in our Table 2 or VW's Table 1). However, if firm 1 chooses $(\underline{x}, \bar{y})$ instead, its profits are $\Delta x/9$ (see Eq^{*} in our Table 2 or VW's Table 1), which are greater than $\Delta y/8$ as $2\Delta y \le \Delta x$. Hence, firm 1 has a profitable deviation, and $((\bar{x}, \bar{y}), (\bar{x}-2\Delta y, \underline{y}))$ is not an equilibrium.

We show in the proof of Proposition 1 that $((\bar{x}, \bar{y}), (\underline{x}, \bar{y}))$ is the only product configuration in equilibrium.

Appendix B. Proofs

Proof of Lemma 1. The profits are simply $\pi_F = D_F(p_F - c(a_F+A_F))$ and $\pi_f = D_f(p_f - c(a_f+A_f))$, where D_f and D_F are the demand of firms f and F, respectively. A consumer buys from firm F iff $\theta_A \ge (p_F-p_f)/\Delta A - \theta_a \Delta a/\Delta A$. Thus in the (θ_A, θ_a) plane, the indifferent consumers are represented by an indifference line that is increasing in the asymmetric characteristics case and decreasing in the dominant product case.

The assumptions of Propositions 15 and 6 in Caplin and Nalebuff (1991) are satisfied. Hence, the equilibrium is unique, and if a price vector (p_F, p_f) satisfies all the FOCs, it is a Nash equilibrium. In the rest of this proof, we look for prices solving the FOCs.

We first analyze the case when all consumers with $\theta_A=0$ buy product f and all consumers with $\theta_A=1$ buy product F; i.e., $\max\{0,\Delta a\} \le p_F - p_f \le \Delta A + \min\{0, \Delta a\}$. This case corresponds to the top left graph in Figure B.1. The demand for firm f is $D_f = (p_F-p_f)/\Delta A - \Delta a/(2\Delta A)$. The demand for firm F is $D_F = 1-D_f$. The FOCs ($\partial\pi_F/\partial p_F = 0, \partial\pi_f/\partial p_f=0$) are equivalent to

$$2p_F = p_f + \Delta A + \frac{\Delta a}{2} + c(a_F+A_F) \quad \text{and}$$
$$2p_f = p_F - \frac{\Delta a}{2} + c(a_f + cA_f).$$

Solving for prices leads to

$$p_F^* - c(a_F+A_F) = \frac{(2-c)\Delta A + (\frac{1}{2}-c)\Delta a}{3} \quad \text{and}$$
$$p_f^* - c(a_f+A_f) = \frac{(1+c)\Delta A + (c-\frac{1}{2})\Delta a}{3}.$$

Finally, we check that $\max\{0, \Delta a\} \le p_F^* - p_f^* \le \Delta A + \min\{0, \Delta a\}$ is satisfied iff $\Delta A + \Delta a = 0$ or ($c \le 2$ and $2\Delta a - \Delta A \le c(\Delta A + \Delta a) \le 2\Delta A - \Delta a$).

Second, we analyze the case when consumers buy product f only if they have a low willingness to pay for quality in both dimensions; i.e., $0 \le p_F - p_f \le \Delta a$. This case corresponds to the top right graph in Figure B.1. The demand for firm f is $D_f = (p_F - p_f^2)/(2\Delta A \Delta a)$. The demand for firm F is $D_F = 1-D_f$. Let us define $X = p_f - c(a_f+A_f)$. The FOCs ($\partial\pi_F/\partial p_F = 0, \partial\pi_f/\partial p_f=0$) are equivalent to $8X^2 - 2Xc(\Delta A + \Delta a) - \Delta a \Delta A = 0$ and $p_F - p_f = 2X$. The FOCs are satisfied iff

$$p_F^{**} - c(a_F+A_F) = 3X - c(\Delta A + \Delta a) \quad \text{and}$$
$$p_f^{**} - c(a_f+A_f) = \frac{c(\Delta A+\Delta a) + \sqrt{c^2(\Delta A + \Delta a)^2 + 8\Delta a \Delta A}}{8}.$$

Furthermore, $0 \le p_F^{**} - p_f^{**} \le \Delta a$ is true iff $c(\Delta A + \Delta a) \le 2\Delta a - \Delta A$.

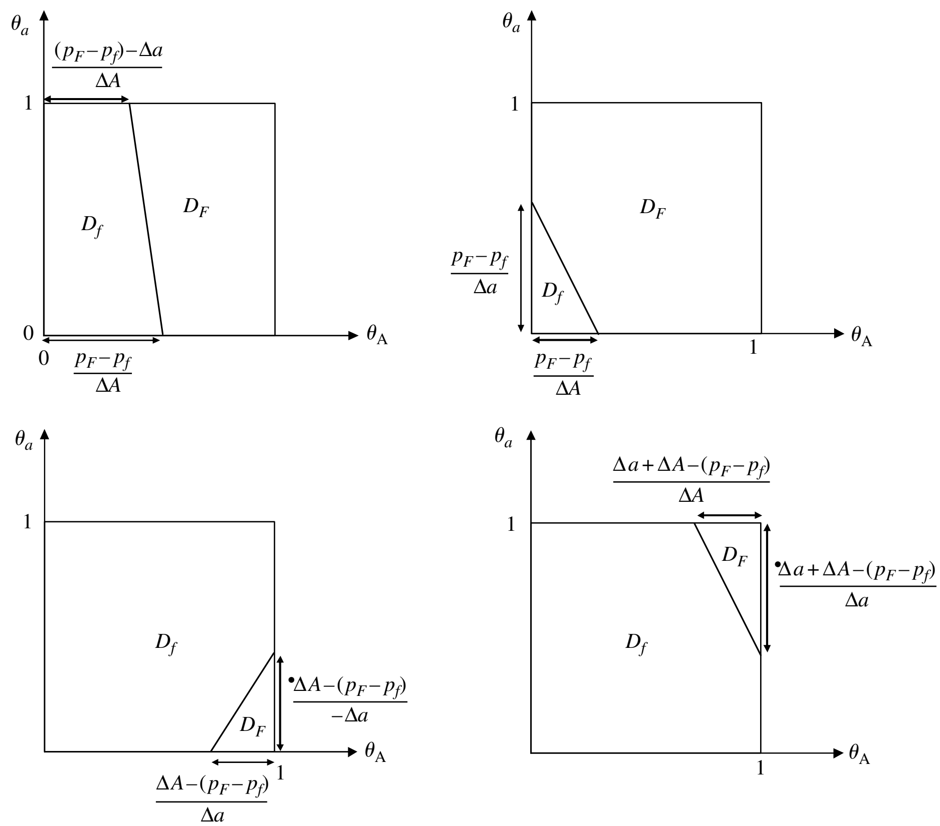
Third, we analyze the case when consumers buy product F only if they have a high willingness to pay for quality in both dimensions; i.e., $\Delta A \le p_F - p_f \le \Delta a + \Delta A$ ($\Delta a > 0$ dominant product case). This case corresponds to the bottom right graph in Figure B.1. The demand for firm f is $D_f = 1 - ((\Delta a + \Delta A - (p_F - p_f))^2)/(2\Delta a \Delta A)$. The demand for firm F is $D_F = ((\Delta a + \Delta A - (p_F-p_f))^2)/(2\Delta a\Delta A)$. Let us define $X = p_F - c(a_F + A_F)$. The FOCs ($\partial\pi_F/\partial p_F = 0, \partial\pi_f/\partial p_f=0$) are equivalent to $p_F - p_f = \Delta a + \Delta A - 2X$ and $8X^2 + 2X(c-1)(\Delta a + \Delta A) - \Delta a \Delta A = 0$. The FOCs are satisfied iff

$$p_F^{\dagger} - c(a_F+A_F) = \frac{(1-c)(\Delta A + \Delta a) + \sqrt{(1-c)^2(\Delta A+\Delta a)^2 + 8\Delta a \Delta A}}{8} \quad \text{and}$$
$$p_f^{\dagger} - c(a_f+A_f) = 3X + (c-1)(\Delta A + \Delta a).$$

Furthermore, $\Delta A \le p_F^{\dagger} - p_f^{\dagger} \le \Delta a + \Delta A$ is true iff $\Delta a > 0$ and $c(\Delta A + \Delta a) \ge 2\Delta A - \Delta a$.

Fourth, we analyze the case when consumers buy product F only if they have a high willingness to pay for the

Figure B.1 Market Shares in the Consumer Tastes (θ_A, θ_a) Plane



Notes. Top left, $\max\{0, \Delta a\} \le p_F - p_f \le \Delta A + \min\{0, \Delta a\}$; the indifference line can be increasing (asymmetric characteristics case) or decreasing (dominant product case). Top right, $0 \le p_F - p_f \le \Delta a$ ($\Delta a > 0$ dominant product case). Bottom left, $\Delta a + \Delta A \le p_F - p_f \le \Delta A$ ($\Delta a < 0$ asymmetric characteristic case). Bottom right, $\Delta A \le p_F - p_f \le \Delta a + \Delta A$ ($\Delta a > 0$ dominant product case).

A characteristic and a low willingness to pay for the a characteristic; i.e., $\Delta a + \Delta A \le p_F - p_f \le \Delta A$. This case corresponds to the bottom left graph in Figure B.1. The demand for firm f is $D_f = 1 - (\Delta A - (p_F - p_f))^2/(-2\Delta a\Delta A)$. The demand for firm F is $D_F = (\Delta A - (p_F - p_f))^2/(-2\Delta a\Delta A)$. Let us define $X = p_F - c(a_F + A_F)$. The FOCs ($\partial\pi_F/\partial p_F = 0$, $\partial\pi_f/\partial p_f = 0$) are equivalent to $p_F - p_f = \Delta A - 2X$ and $8X^2 + 2X((c-1)\Delta A + c\Delta a) + \Delta a\Delta A = 0$. The FOCs are satisfied iff

$$p_F^{\dagger\dagger} - c(a_F + A_F) = \frac{-(c-1)\Delta A - c\Delta a + \sqrt{((c-1)\Delta A + c\Delta a)^2 - 8\Delta a\Delta A}}{8} \quad \text{and}$$
$$p_f^{\dagger\dagger} - c(a_f + A_f) = 3X + (c-1)\Delta A + c\Delta a.$$

Furthermore, $\Delta a + \Delta A \le p_F^{\dagger\dagger} - p_f^{\dagger\dagger} \le \Delta A$ is true iff $\Delta a < 0$ and $c \ge 2$.

Finally, we double-checked the last case ($\Delta A + \min\{0, \Delta a\} \le p_F - p_f$), and as expected, these prices cannot form an equilibrium.

Therefore, we have characterized the unique Nash equilibrium in prices.[20] □

[20] Note that at the limit between several regions, all the equilibria lead to the same prices and demands.

Proof of Lemma 2. (i) In Eq^*,

$$\frac{\partial p_F^*}{\partial A_f} = -\frac{2-c}{3} < 0, \quad \frac{\partial p_f^*}{\partial A_F} = \frac{1+c}{3} > 0,$$
$$\frac{\partial p_F^*}{\partial a_f} = \frac{c-1/2}{3}, \quad \text{and} \quad \frac{\partial p_f^*}{\partial a_F} = \frac{c-1/2}{3}.$$

(ii) In Eq^{**},

$$\frac{\partial p_F^{**}}{\partial A_f} = -3\frac{\partial X^{**}}{\partial \Delta A} + c, \quad \frac{\partial p_f^{**}}{\partial A_F} = \frac{\partial X^{**}}{\partial \Delta A} > 0,$$
$$\frac{\partial p_F^{**}}{\partial a_f} = -3\frac{\partial X^{**}}{\partial \Delta a} + c, \quad \text{and} \quad \frac{\partial p_f^{**}}{\partial a_F} = \frac{\partial X^{**}}{\partial \Delta a} > 0.$$

Note that

$$\frac{\partial X^{**}}{\partial \Delta a} > \frac{\partial X^{**}}{\partial \Delta A} = \frac{1}{8}\left(c + \frac{c^2(\Delta a + \Delta A) + 4\Delta a}{\sqrt{c^2(\Delta A + \Delta a)^2 + 8\Delta a\Delta A}}\right) > 0.$$

We know that

$$\frac{\partial X^{**}}{\partial \Delta A} > \frac{c}{3} \iff 2c^4(\Delta a + \Delta A)^2 + c^2\Delta a(16\Delta A - 9\Delta a) - 18\Delta a^2 < 0,$$

where the left-hand side (LHS) of the second inequality is increasing wrt c; hence $\text{LHS} \le \text{LHS}|_{c=1/2} = \frac{1}{8}(\Delta a + \Delta A)^2 + \frac{1}{4}\Delta a(16\Delta A - 9\Delta a) - 18\Delta a^2$, which is increasing wrt ΔA and thus smaller than $\text{LHS}|_{c=1/2;\,\Delta A=2\Delta a} < 0$.

(iii) In $Eq^\dagger$,

$$\frac{\partial p_F^\dagger}{\partial A_f} = -\frac{\partial X^\dagger}{\partial \Delta A} < 0, \quad \frac{\partial p_f^\dagger}{\partial A_F} = 3\frac{\partial X^\dagger}{\partial \Delta A} + c - 1,$$

$$\frac{\partial p_F^\dagger}{\partial a_f} = -\frac{\partial X^\dagger}{\partial \Delta a} < 0, \quad \text{and} \quad \frac{\partial p_f^\dagger}{\partial a_F} = 3\frac{\partial X^\dagger}{\partial \Delta a} + c - 1.$$

Note that

$$\frac{\partial X^\dagger}{\partial \Delta a} > \frac{\partial X^\dagger}{\partial \Delta A} = \frac{1}{8}\left(1 - c + \frac{(1-c)^2(\Delta a + \Delta A) + 4\Delta a}{\sqrt{(1-c)^2(\Delta A + \Delta a)^2 + 8\Delta a \Delta A}}\right) > 0.$$

We know that when $c \geq 1$, $\partial p_f^\dagger / \partial A_F$, and $\partial p_f^\dagger / \partial a_F$ are > 0. When $\frac{1}{2} < c < 1$,

$$\frac{\partial X^\dagger}{\partial \Delta A} > \frac{1-c}{3} \iff 2(1-c)^4(\Delta a + \Delta A)^2 + (1-c)^2 \Delta a(16\Delta A - 9\Delta a) - 18\Delta a^2 < 0,$$

where the LHS of the second inequality is decreasing wrt c; hence $\text{LHS} \leq \text{LHS}|_{c=1/2} = \frac{1}{8}(\Delta a + \Delta A)^2 + \frac{1}{4}\Delta a(16\Delta A - 9\Delta a) - 18\Delta a^2$, which we just proved to be < 0. □

Lemma 3. *When $c < \frac{1}{2}$,*

$$\frac{\partial \pi_F}{\partial a_F} > 0, \quad \frac{\partial \pi_F}{\partial A_F} > 0, \quad \frac{\partial \pi_f}{\partial A_f} < 0,$$

$$\frac{\partial \pi_f}{\partial a_f} > 0 \quad \text{if } Eq^* \quad \text{and} \quad \frac{\partial \pi_f}{\partial a_f} < 0 \quad \text{if } Eq^{**}.$$

Proof. (i) If $c(\Delta a + \Delta A) > 2\Delta a - \Delta A$, the pricing equilibrium is Eq^* with

$$\pi_F^* = \frac{((2-c)\Delta A + (\frac{1}{2} - c)\Delta a)^2}{9\Delta A},$$

$$\pi_f^* = \frac{((1+c)\Delta A + (c - \frac{1}{2})\Delta a)^2}{9\Delta A}.$$

Thus $c < \frac{1}{2}$ implies $\partial \pi_F^* / \partial a_F > 0$ and $\partial \pi_f^* / \partial a_f > 0$.

$$\frac{\partial \pi_f^*}{\partial \Delta A} = \frac{(1+c)\Delta A + (c - \frac{1}{2})\Delta a}{9\Delta A^2}\left((1+c)\Delta A + (\tfrac{1}{2} - c)\Delta a\right) \geq 0,$$

$$\frac{\partial \pi_F^*}{\partial \Delta A} = \frac{(2-c)\Delta A + (\frac{1}{2} - c)\Delta a}{9\Delta A^2}\left((2-c)\Delta A - (\tfrac{1}{2} - c)\Delta a\right) \geq 0,$$

as $(2-c)\Delta A - (\frac{1}{2} - c)\Delta a$ is decreasing wrt c and positive when $c = 1/2$.

(ii) If $c(\Delta a + \Delta A) < 2\Delta a - \Delta A$, the pricing equilibrium is Eq^{**} with

$$\pi_F^{**} = \left(1 - \frac{2X^2}{\Delta a \Delta A}\right)(3X - c(\Delta a + \Delta A)), \quad \pi_f^{**} = \frac{2X^3}{\Delta a \Delta A},$$

where $X = c(\Delta a + \Delta A) + \sqrt{c^2(\Delta a + \Delta A)^2 + 8\Delta a \Delta A}/8$.

$$\frac{\partial \pi_f^{**}}{\partial \Delta A} = \frac{2X^2}{\Delta a \Delta A^2}\left(3\Delta A \frac{c + (c^2(\Delta A + \Delta a) + 4\Delta a)/\sqrt{c^2(\Delta A + \Delta a)^2 + 8\Delta a \Delta A}}{8} - X\right)$$

$$= \frac{X^2}{4\Delta a \Delta A^2 \sqrt{c^2(\Delta A + \Delta a)^2 + 8\Delta a \Delta A}}((2\Delta A - \Delta a)c8X + 4\Delta A \Delta a) \geq 0$$

as $2\Delta A - \Delta a \geq 0$.

Note that ΔA and Δa play a symmetric role in the above expressions. Hence

$$\frac{\partial \pi_f^{**}}{\partial \Delta a} = \frac{X^2}{4\Delta a^2 \Delta A \sqrt{c^2(\Delta A + \Delta a)^2 + 8\Delta a \Delta A}} \cdot ((2\Delta a - \Delta A)c8X + 4\Delta A \Delta a) \geq 0$$

as $2\Delta a - \Delta A > c(\Delta a + \Delta A) \geq 0$.

$$\frac{\partial \pi_F^{**}}{\partial A_F} = \frac{\partial \pi_F}{\partial p_f}\frac{\partial p_f^{**}}{\partial A_F} + \frac{(p_F^{**} - p_f^{**})^2}{2\Delta a \Delta A^2}(p_F^{**} - c(a_F + A_F)) - \left(1 - \frac{(p_F^{**} - p_f^{**})^2}{2\Delta a \Delta A}\right)c$$

(per the envelope theorem). Note that $\partial \pi_F / \partial p_f \geq 0$ and $\partial p_f^{**} / \partial A_F \geq 0$. Using the FOC of F, the last two terms are equal to

$$\frac{(p_F^{**} - p_f^{**})}{\Delta a \Delta A}(p_F^{**} - c(a_F + A_F))\left(\frac{p_F^{**} - p_f^{**}}{2\Delta A} - c\right),$$

which has the same sign as

$$X^{**} - c\Delta A = \frac{c(\Delta A + \Delta a) + \sqrt{c^2(\Delta A + \Delta a)^2 + 8\Delta a \Delta A} - 8c\Delta A}{8} \geq 0$$

$$\Leftrightarrow \sqrt{c^2(\Delta A + \Delta a)^2 + 8\Delta a \Delta A} \geq 8c\Delta A - c(\Delta A + \Delta a)$$

(the right-hand side of the second inequality is ≥ 0) $\Leftrightarrow \Delta a \geq c^2(6\Delta A - 2\Delta a)$ true as $c \leq (2\Delta a - \Delta A)/(\Delta A + \Delta a)$ implies that

$$c^2 \frac{6\Delta A - 2\Delta a}{\Delta a} \leq \left(\frac{2\Delta a - \Delta A}{\Delta A + \Delta a}\right)^2 \frac{6\Delta A - 2\Delta a}{\Delta a}$$

(which is decreasing wrt ΔA) $\leq ((2\Delta a - \Delta A)/(\Delta A + \Delta a))^2 (6\Delta A - 2\Delta a)/\Delta a|_{\Delta A = \Delta a} = 1$.

Note that a_F and A_F play a symmetric role in π_F^{**}, so $X^{**} > c\Delta a$ implies that $\partial \pi_F^{**} / \partial a_F$ is positive. We just proved that $X^{**} > c\Delta A \geq c\Delta a$. □

Proof of Proposition 1 (Max-Min Equilibrium with Best Product). Per the previous lemma, firm F has to sell the best product in equilibrium; otherwise, it could increase its profit by increasing the level of any of its characteristics. We now look at the best response of a firm when its rival sells the best product. This firm is necessarily firm f in equilibrium, and according to the previous lemma, firm f's best response is also to locate at a corner of the product space. Hence, $(\bar{x}, \underline{y})$ and $(\underline{x}, \bar{y})$ are always local maxima of firm f's profit function. Using a symmetry argument, firm f earns the same profit by locating at $(\bar{x}, \underline{y})$ or at $(\bar{x} - \Delta y, \bar{y})$, as in both cases $\Delta A = \Delta y$ and $\Delta a = 0$. We know that firm f strictly prefers $(\underline{x}, \bar{y})$ over $(\bar{x} - \Delta y, \bar{y})$, and therefore firm f prefers to position at $(\bar{x}, \underline{y})$ to $(\underline{x}, \bar{y})$.[21] If $c(\Delta x + \Delta y) < 2\Delta y - \Delta x$, $(\underline{x}, \underline{y})$ is also a local maximum of firm f's profit

[21] To be fully precise, if $\Delta x = \Delta y$, firm f is indifferent between $(\bar{x}, \underline{y})$ and $(\underline{x}, \bar{y})$.

function. Firm f chooses the product positioning with the higher profit:

$$\pi_f(\underline{x}, \bar{y}) = \frac{(1+c)^2\Delta x}{9} \quad \text{and}$$
$$\pi_f(\underline{x}, \underline{y}) = \frac{2}{\Delta y \Delta x}\left(\frac{c(\Delta y+\Delta x)+\sqrt{c^2(\Delta y+\Delta x)^2+8\Delta y\Delta x}}{8}\right)^3.$$

$$\frac{\pi_f(\underline{x}, \bar{y}) - \pi_f(\underline{x}, \underline{y})}{\Delta x} = \frac{(1+c)^2}{9} - \frac{2}{\Delta y/\Delta x}\left(\frac{c(\Delta y/\Delta x+1)+\sqrt{c^2(\Delta y/\Delta x+1)^2+8\Delta y/\Delta x}}{8}\right)^3$$

is a function of $(c, \Delta y/\Delta x)$, which is

$$\geq \left.\frac{\pi_f(\underline{x}, \bar{y}) - \pi_f(\underline{x}, \underline{y})}{\Delta x}\right|_{c=1/2;\, \Delta y/\Delta x=1} = 0$$

when

$$0 \leq \frac{\Delta y}{\Delta x} \leq 1 \quad \text{and} \quad 0 \leq c < \frac{2\Delta y/\Delta x - 1}{1+\Delta y/\Delta x}.$$

In the end, $(\underline{x}, \bar{y})$ is firm f's best response because it is the global maximum of firm f's profit function.

We need to check now that if the rival firm selects $(\underline{x}, \bar{y})$, a firm's best response is to choose the best product. Depending on its product positioning, this firm can either be firm f or firm F. In either case, however, $\Delta a < 0$, implying that $\partial\pi/\partial a > 0$. Hence the profit function has only one local maximum $(\bar{x}, \bar{y})$, which is the best response. Finally, the unique equilibrium—up to relabeling the firms—is $(\bar{x}, \bar{y})$ and $(\underline{x}, \bar{y})$.[22] □

Remark. In equilibrium, firms split the market based on how much consumers value characteristic A. The demand of firm f is $D_f = (c+1)/3$, which is smaller than $\frac{1}{2}$ as long as c is below $\frac{1}{2}$. As

$$\frac{\partial D_i}{\partial a_i}\bigg/\left(-\frac{\partial D_i}{\partial p_i}\right) = \frac{1}{2} \quad \text{and} \quad \frac{\partial D_i}{\partial A_i}\bigg/\left(-\frac{\partial D_i}{\partial p_i}\right) = D_f,$$

per Equation (2), the demand-cost effects for characteristics a_i and A_i are $(\frac{1}{2}-c)D_i$ and $(D_f - c)D_i$, respectively. Thus, as long as c stays below $\frac{1}{2}$, the demand-cost effect is positive for both firms and both characteristics, explaining the $\frac{1}{2}$ cutoff.

Lemma 4. *When $\frac{1}{2} < c$ and the equilibrium is Eq^*,*

$$\frac{\partial \pi_f}{\partial A_f} < 0, \quad \frac{\partial \pi_f}{\partial a_f} < 0;$$
$$\frac{\partial \pi_F}{\partial A_F} > 0 \quad \text{iff } c(\Delta A - \Delta a) < 2\Delta A - \frac{1}{2}\Delta a$$
$$\text{(which is true when } \Delta a \geq 0);$$
$$\frac{\partial \pi_F}{\partial a_F} < 0.$$

[22] Once again, if $\Delta x = \Delta y$, another equilibrium exists, which is $(\bar{x}, \bar{y})$ and $(\bar{x}, \underline{y})$, becuase no attribute has a strictly larger span.

Proof. The pricing equilibrium Eq^* generates

$$\pi_F^* = \frac{((2-c)\Delta A + (\frac{1}{2}-c)\Delta a)^2}{9\Delta A},$$
$$\pi_f^* = \frac{((1+c)\Delta A + (c-\frac{1}{2})\Delta a)^2}{9\Delta A}.$$

$c > \frac{1}{2}$ implies $\partial\pi_F^*/\partial a_F < 0$ and $\partial\pi_f^*/\partial a_f < 0$.

$$\frac{\partial \pi_f^*}{\partial \Delta A} = \frac{(1+c)\Delta A + (c-\frac{1}{2})\Delta a}{9\Delta A^2} \cdot (\Delta A + \tfrac{1}{2}\Delta a + c(\Delta A - \Delta a)) \geq 0$$
$$\text{as } \Delta A + \tfrac{1}{2}\Delta a \geq 0 \text{ and } \Delta A - \Delta a \geq 0.$$

$$\frac{\partial \pi_F^*}{\partial \Delta A} = \frac{(2-c)\Delta A + (\frac{1}{2}-c)\Delta a}{9\Delta A^2}\big(2\Delta A - \tfrac{1}{2}\Delta a - c(\Delta A - \Delta a)\big),$$

so $\partial\pi_F^*/\partial A_F > 0 \Longleftrightarrow c(\Delta A - \Delta a) < 2\Delta A - \frac{1}{2}\Delta a$. □

Lemma 5. *When $\frac{1}{2} < c$ and the equilibrium is $Eq^\dagger$,*

$$\frac{\partial \pi_f}{\partial A_f} < 0, \quad \frac{\partial \pi_f}{\partial a_f} < 0;$$
$$\frac{\partial \pi_F}{\partial A_F} > 0 \quad \text{if } \Delta A \leq 2\Delta a, \quad \text{and}$$
$$\frac{\partial \pi_F}{\partial A_F} \lesseqgtr 0 \quad \text{iff} \quad c \gtreqless \sqrt{\frac{2\Delta A \Delta a}{(2\Delta A - \Delta a)(\Delta A - 2\Delta a)}} + 1$$
$$\text{if } \Delta A > 2\Delta a.$$

Proof.

$$\frac{\partial \pi_f^\dagger}{\partial A_f} = \frac{\partial \pi_f}{\partial p_F}\frac{\partial p_F^\dagger}{\partial A_f} + \left(\frac{(\Delta a + \Delta A - (p_F - p_f))}{\Delta a \Delta A} - \frac{(\Delta a + \Delta A - (p_F - p_f))^2}{2\Delta a \Delta A^2}\right) \cdot (p_f - c(a_f + A_f)) - \left(1 - \frac{(\Delta a + \Delta A - (p_F - p_f))^2}{2\Delta a \Delta A}\right)c$$

(per the envelope theorem). Note that $\partial\pi_f/\partial p_F \geq 0$ and $\partial p_F^\dagger/\partial A_f < 0$ per Lemma 2. Let us now check the sign of the sum of the last two terms in $\partial\pi_f^\dagger/\partial A_f$. Using the FOC for f, we know that

$$1 - \frac{(\Delta a + \Delta A - (p_F - p_f))^2}{2\Delta a \Delta A} = \frac{\Delta a + \Delta A - (p_F - p_f)}{\Delta a \Delta A}(p_f - c(a_f + A_f)).$$

Hence these two terms become

$$\left(1 - \frac{\Delta a + \Delta A - (p_F - p_f)}{2\Delta A} - c\right) \cdot \frac{\Delta a + \Delta A - (p_F - p_f)}{\Delta a \Delta A}(p_f - c a_f - c A_f),$$

which is ≤ 0 when $c \geq 1$. When $c < 1$ (i.e., $\Delta A/2 < \Delta a$), $(1-c)2\Delta A \leq \Delta a + \Delta A - (p_F - p_f) = 2X \Leftrightarrow (1-c)^2 (6\Delta A - 2\Delta a) \leq \Delta a$, which is true as

$$1 - c \leq \frac{2\Delta a - \Delta A}{\Delta A + \Delta a} \quad \text{and} \quad \left(\frac{2\Delta a - \Delta A}{\Delta A + \Delta a}\right)^2 (6\Delta A - 2\Delta a) \leq \Delta a$$

when $\Delta A/2 < \Delta a < \Delta A$.

Note that A_f and a_f play a symmetric role in the above expressions. Hence

$$\frac{\partial \pi_f^\dagger}{\partial a_f} = \frac{\partial \pi_f}{\partial p_F}\frac{\partial p_F^\dagger}{\partial a_f} + \left(\frac{\Delta a + \Delta A - (p_F - p_f)}{\Delta a \Delta A} - \frac{(\Delta a + \Delta A - (p_F - p_f))^2}{2\Delta a \Delta A^2}\right) \cdot (p_f - ca_f - cA_f) - \left(1 - \frac{(\Delta a + \Delta A - (p_F - p_f))^2}{2\Delta a \Delta A}\right)c$$

(per the envelope theorem). Note that $\partial \pi_f/\partial p_F \geq 0$ and $\partial p_F^\dagger/\partial a_f < 0$ per Lemma 2. Moreover, we already know that the sum of the last two terms in $\partial \pi_f^\dagger/\partial a_f$ are ≤ 0.

$$\frac{\partial \pi_F^\dagger}{\partial \Delta A} = \frac{2X^2}{\Delta a \Delta A^2}\left(\frac{3\Delta A}{8}(1 - c + \frac{(1-c)^2(\Delta A + \Delta a) + 4\Delta a}{\sqrt{(1-c)^2(\Delta A + \Delta a)^2 + 8\Delta a \Delta A}}) - X\right)$$
$$= \frac{2X^2}{8\Delta a \Delta A^2 \sqrt{(1-c)^2(\Delta A + \Delta a)^2 + 8\Delta a \Delta A}} \cdot ((2\Delta A - \Delta a)(1-c)8X + 4\Delta A \Delta a) \geq 0$$

when $c \leq 1$ as $2\Delta A - \Delta a > 0$. When $c > 1$,

$$(2\Delta A - \Delta a)(1-c)8X + 4\Delta A \Delta a \geq 0 \Leftrightarrow 2\Delta A \Delta a/(2\Delta A - \Delta a) \geq (\Delta A - 2\Delta a)(c-1)^2,$$

which is true when $\Delta A/2 \leq \Delta a$. But when $\Delta A/2 > \Delta a$,

$$\frac{2\Delta A \Delta a}{(2\Delta A - \Delta a)(c-1)} \geq (\Delta A - 2\Delta a)(c-1) \Leftrightarrow \sqrt{\frac{2\Delta A \Delta a}{(2\Delta A - \Delta a)(\Delta A - 2\Delta a)}} + 1 \geq c. \quad \square$$

Lemma 6. *When $c(\Delta x + \Delta y) > 2\Delta x - \Delta y$, $c < 2$, and the rival firm chooses the worst product, there exists $\tilde{c}$ in $[(2\Delta x - \Delta y)/(\Delta x + \Delta y), 2]$ such that*

$$\pi_F(\bar{x}, \underline{y}) \lesseqqgtr \pi_F(\bar{x}, \bar{y}) \Leftrightarrow \tilde{c} \lesseqqgtr c.$$

Proof. If the rival firm positions on the worst product, the firm is necessarily firm F. If $c(\Delta x + \Delta y) > 2\Delta x - \Delta y$, positioning at $(\bar{x}, \bar{y})$ implies that $Eq^\dagger$ occurs. However, if firm F chooses $(\bar{x}, \underline{y})$, then Eq^* happens. Therefore

$$\pi_F(\bar{x}, \underline{y}) = \frac{(2-c)^2 \Delta x}{9} \quad \text{and}$$

$$\pi_F(\bar{x}, \bar{y}) = \frac{2}{\Delta x \Delta y}\left(\frac{(1-c)(\Delta x + \Delta y) + \sqrt{(1-c)^2(\Delta x + \Delta y)^2 + 8\Delta x \Delta y}}{8}\right)^3.$$

We checked that

$$\frac{(2-c)^2}{9} - \frac{2}{\Delta y/\Delta x} \cdot \left(\frac{(1-c)(\Delta y/\Delta x + 1) + \sqrt{(1-c)^2(\Delta y/\Delta x + 1)^2 + 8\Delta y/\Delta x}}{8}\right)^3$$

has a unique root between $(2\Delta x - \Delta y)/(\Delta x + \Delta y)$ and 2. Let us call it $\tilde{c}$. It then follows that $(\pi_F(\bar{x}, \underline{y}) - \pi_F(\bar{x}, \bar{y}))/\Delta x$ is positive when $(2\Delta x - \Delta y)/(\Delta x + \Delta y) < c < \tilde{c}$ and negative when $\tilde{c} < c < 2$. $\square$

Lemma 7. *When*

$$\Delta y < \frac{\Delta x}{2}, \quad c(\Delta x + \Delta y) > 2\Delta x - \Delta y,$$
$$\sqrt{\frac{2\Delta x \Delta y}{(2\Delta x - \Delta y)(\Delta x - 2\Delta y)}} + 1 < c < 2,$$

and the rival firm chooses the worst product, there exists $x^\dagger \in [\underline{x}, \bar{x})$ such that $\Delta x^\dagger = x^\dagger - \underline{x}$ solves

$$\sqrt{2\Delta x^\dagger \Delta y/(2\Delta x^\dagger - \Delta y)(\Delta x^\dagger - 2\Delta y)} + 1 = c,$$

and there exists $\tilde{\tilde{c}}$ such that $\pi_F(\bar{x}, \underline{y}) < \pi_F(x^\dagger, \bar{y}) \Leftrightarrow \tilde{\tilde{c}} < c$.

Proof. Let us define $\varphi(a) = \sqrt{2a/((2-a)(1-2a))} + 1$, which is an increasing function of a when $a \in [0, \frac{1}{2})$ and $\lim_{\frac{1}{2}} \varphi = +\infty$. Thus $\varphi(\Delta y/\Delta x) < c$ implies that there exists $2\Delta y < \Delta x^\dagger < \Delta x$ such that $\varphi(\Delta y/\Delta x^\dagger) = c$.

$$\frac{\Delta y}{\Delta x^\dagger} = \frac{1}{4}\left(5 + \frac{2}{(c-1)^2} - \sqrt{\left(5 + \frac{2}{(c-1)^2}\right)^2 - 16}\right).$$

Then define $x^\dagger \equiv \Delta x^\dagger + \underline{x}$.

If the rival firm positions on the worst product, the firm is necessarily firm F. If firm F chooses $(\bar{x}, \underline{y})$, then Eq^* occurs. Positioning at $(x^\dagger, \bar{y})$ implies that $Eq^\dagger$ occurs. Therefore

$$\pi_F(\bar{x}, \underline{y}) = \frac{(2-c)^2 \Delta x}{9} \quad \text{and}$$

$$\pi_F(x^\dagger, \bar{y}) = \frac{2}{\Delta x^\dagger \Delta y}\left(\frac{(1-c)(\Delta x^\dagger + \Delta y) + \sqrt{(1-c)^2(\Delta x^\dagger + \Delta y)^2 + 8\Delta x^\dagger \Delta y}}{8}\right)^3.$$

Let $\tilde{\tilde{c}}$ be the maximum of $(2\Delta x - \Delta y)/(\Delta x + \Delta y)$ and the root of

$$\frac{(2-c)^2}{9} - \frac{2\Delta y/\Delta x}{(\Delta y/\Delta x^\dagger)^2} \cdot \left(\frac{(1-c)(1 + \Delta y/\Delta x^\dagger) + \sqrt{(1-c)^2(1 + \Delta y/\Delta x^\dagger)^2 + 8\Delta y/\Delta x^\dagger}}{8}\right)^3,$$

which is between 1.5 and 2. It then follows that $(\pi_F(\bar{x}, \underline{y}) - \pi_F(x^\dagger, \bar{y}))/\Delta x$ is positive when

$$\sqrt{2\Delta x \Delta y/((2\Delta x - \Delta y)(\Delta x - 2\Delta y))} + 1 < c < \tilde{\tilde{c}}$$

and negative when $\tilde{\tilde{c}} < c < 2$. $\square$

Definition of $\hat{c}$. Define $\hat{c}$ as $\tilde{\tilde{c}}$ if

$$\tilde{c} > \sqrt{\frac{2\Delta x \Delta y}{(2\Delta x - \Delta y)(\Delta x - 2\Delta y)}} + 1$$

and as $\tilde{c}$ otherwise.

Note that $\tilde{c} \geq \hat{c}$ because per Lemma 5, if $\bar{x} > x^\dagger$, then $\pi_F(x^\dagger, \bar{y}) > \pi_F(\bar{x}, \bar{y})$.

Proof of Proposition 2 (Max-Min Equilibrium with Worst Product). When $\frac{1}{2} < c < 2$, Eq^* or $Eq^\dagger$ occurs. In equilibrium, firm f has to sell the worst product; otherwise, per Lemmas 4 and 5, it could increase its profit by decreasing the level of any of its characteristics. We now look at the best response of a firm when its rival sells the worst product. This firm is necessarily firm F in equilibrium, and according to Lemmas 4 and 5, $(\bar{x}, \underline{y})$ and $(\underline{x}, \bar{y})$ are always local maxima of firm F's profit function. Firm F earns the same profit by locating at $(\underline{x}, \bar{y})$ or $(\underline{x} + \Delta y, \underline{y})$, as in both cases $\Delta A = \Delta y$ and $\Delta a = 0$. We know that firm F strictly prefers $(\bar{x}, \underline{y})$ over $(\underline{x} + \Delta y, \underline{y})$, and therefore firm F prefers to position at $(\bar{x}, \underline{y})$ to $(\underline{x}, \bar{y})$.[23]

(i) If $c(\Delta x + \Delta y) > 2\Delta x - \Delta y$ and $\{\Delta x/2 \le \Delta y$ or $(\Delta x/2 > \Delta y$ and $c \le 1 + \sqrt{2\Delta x \Delta y/((2\Delta x - \Delta y)(\Delta x - 2\Delta y))})\}$, $(\bar{x}, \bar{y})$ is the only other local maximum of firm F's profit function. Per Lemma 6, $(\bar{x}, \underline{y})$ is firm F's best response (global maximum of firm F's profit function) when $\frac{1}{2} < c < \hat{c}$.

(ii) If $c(\Delta x + \Delta y) > 2\Delta x - \Delta y$, $\Delta x/2 > \Delta y$ and $c > 1 + \sqrt{2\Delta x \Delta y/(2\Delta x - \Delta y)(\Delta x - 2\Delta y)}$, $(x^\dagger, \bar{y})$ is the only other local maximum of firm F's profit function. Per Lemma 7, $(\bar{x}, \underline{y})$ is firm F's best response (global maximum of firm F's profit function) when $\frac{1}{2} < c < \hat{c}$.

We need to check now that if the rival firm selects $(\bar{x}, \underline{y})$, a firm's best response is to choose the worst product. Depending on its product positioning, this firm can be either firm f or firm F. In either case, however, $\Delta a < 0$, implying that $\partial\pi/\partial a < 0$. Hence the profit function has only one local maximum $(\underline{x}, \underline{y})$, which is the best response. Finally, the unique equilibrium up to relabeling the firms is $(\underline{x}, \underline{y})$ and $(\bar{x}, \underline{y})$.[24] □

Proof of Proposition 3. The first paragraph of the proof of Proposition 2 is still valid.

(i) If $c(\Delta x + \Delta y) > 2\Delta x - \Delta y$ and $\{\Delta x/2 \le \Delta y$ or $(\Delta x/2 > \Delta y$ and $c \le 1 + \sqrt{2\Delta x \Delta y/((2\Delta x - \Delta y)(\Delta x - 2\Delta y))})\}$, $(\bar{x}, \bar{y})$ is the only other local maximum of firm F's profit function. Per Lemma 6, $(\bar{x}, \bar{y})$ is firm F's best response (global maximum of firm F's profit function) when $\hat{c} < c < \min(2, \sqrt{2\Delta x \Delta y/((2\Delta x - \Delta y)(\Delta x - 2\Delta y))} + 1)$. We need to check now that if the rival firm selects $(\bar{x}, \bar{y})$, a firm's best response is to choose the worst product. Whatever its product positioning, this firm is firm f, implying that $\partial\pi_f/\partial q_f < 0$. Hence the profit function has only one local maximum $(\underline{x}, \underline{y})$, which is the best response. Finally, the unique equilibrium up to relabeling the firms is $(\underline{x}, \underline{y})$ and $(\bar{x}, \bar{y})$.

(ii) If $c(\Delta x + \Delta y) > 2\Delta x - \Delta y$, $\Delta x/2 > \Delta y$ and $c > 1 + \sqrt{2\Delta x \Delta y/((2\Delta x - \Delta y)(\Delta x - 2\Delta y))}$, $(x^\dagger, \bar{y})$ is the only other local maximum of firm F's profit function. Per Lemma 7, $(x^\dagger, \bar{y})$ is firm F's best response (global maximum of firm F's profit function) when $\max(\hat{c}, \sqrt{2\Delta x \Delta y/((2\Delta x - \Delta y)(\Delta x - 2\Delta y))} + 1) < c < 2$. We need to check now that if the rival firm selects $(x^\dagger, \bar{y})$, the firm's best response is to choose the worst product. Depending on its product positioning, this firm can be either firm f or firm F. Per Lemma 4, $(\underline{x}, \underline{y})$ is always a local maximum of the profit function. And if $\Delta x \ge \Delta x^\dagger + ((c - \frac{1}{2})/(2 - c))\Delta y$, $(\bar{x}, \underline{y})$ is also a local maximum of the profit function.[25] There exists no other local maximum. The firm chooses the product positioning for which profit is higher.

$$\pi_F^* = \frac{((2-c)(\Delta x - \Delta x^\dagger) + (\frac{1}{2} - c)\Delta y)^2}{9(\Delta x - \Delta x^\dagger)} \quad \text{and}$$

$$\pi_f^\dagger = \left(1 - \frac{2X^2}{\Delta y \Delta x^\dagger}\right)(3X + (c-1)(\Delta y + \Delta x^\dagger)),$$

where

$$X = \frac{(1-c)(\Delta y + \Delta x^\dagger) + \sqrt{(1-c)^2(\Delta y + \Delta x^\dagger)^2 + 8\Delta y \Delta x^\dagger}}{8}.$$

Note $\pi_f^\dagger$ is independent of Δx. Moreover, $\pi_F^* < \pi_f^\dagger$ iff $root_- < \Delta x < root_+$, where

$$root_\pm = \Delta x^\dagger + \frac{9\pi_f^\dagger - 2(2-c)(\frac{1}{2} - c) \pm \sqrt{\Delta\Delta}}{2(2-c)^2}\Delta y \quad \text{and}$$

$$\Delta\Delta = (9\pi_f^\dagger - 2(2-c)(\tfrac{1}{2} - c))^2 - 4(2-c)^2(\tfrac{1}{2} - c)^2.$$

We checked that

$$\Delta x^\dagger + \frac{c - \frac{1}{2}}{2 - c}\Delta y > root_- \quad \text{and} \quad \frac{1+c}{2-c}\Delta y < root_+.$$[26]

Therefore even when $(\bar{x}, \underline{y})$ is a local maximum of the profit function, the best response when the rival selects $(x^\dagger, \bar{y})$ is always $(\underline{x}, \underline{y})$. Finally, the unique equilibrium up to relabeling the firms is $(\underline{x}, \underline{y})$ and $(x^\dagger, \bar{y})$. □

Proof of Proposition 4. When $V \ge 3\max(\bar{x}, \bar{y}) + \min(\bar{x}, \bar{y})/2$ and $c \le 2$ the monopolist chooses

$$p^m = V + \frac{-(V - cx - cy) + \sqrt{(V - cx - cy)^2 + 6xy}}{3},$$

and its demand is $D^m = 1 - (p^m - V)^2/(2xy)$. Using the envelope theorem and plugging in the FOC, we get

$$\begin{aligned} \frac{\partial \pi^m}{\partial x} &= (p^m - cx - cy)\left(c\frac{\partial D^m}{\partial p} + \frac{\partial D}{\partial x}\right) \\ &= (p^m - cx - cy)\frac{(p^m - V)}{x^2 y}\left(-cx + \frac{p^m - V}{2}\right). \end{aligned}$$

Thus, $\partial\pi^m/\partial x > 0 \Leftrightarrow p^m - V > 2cx \Leftrightarrow y > 6c^2x + 2c(V - cx - cy) \Leftrightarrow y > (4c^2x + 2cV)/(1 + 2c^2)$ (1). Similarly, we get $\partial\pi^m/\partial y = (p^m - cx - cy)((p^m - V)/(xy^2))(-cy + (p^m - V)/2)$ and $\partial\pi^m/\partial y > 0 \Leftrightarrow x > 6c^2y + 2c(V - cx - cy) \Leftrightarrow y > ((1 + 2c^2)x - 2cV)/(4c^2)$ (2) When $c \ge 1/2$, the condition on V ensures that $\partial\pi^m/\partial x < 0$ and $\partial\pi^m/\partial y < 0$. When $c < 1/2$, conditions (1) and (2) split the (x, y) plane into four regions: $\partial\pi^m/\partial x$ and $\partial\pi^m/\partial y$ both positive, both negative, or one negative and one positive. Let $\hat{c} < 1/2$ be such that (1) and (2) hold with equality when $y = \bar{y}$, $c_0 < 1/2$ be such that (2) holds with equality when $(x, y) = (\underline{x}, \bar{y})$, and $c_1 < 1/2$ be such that (2) holds with equality when $(x, y) = (\underline{x}, \underline{y})$.

[23] To be fully precise, if $\Delta x = \Delta y$, firm F is indifferent between $(\bar{x}, \underline{y})$ and $(\underline{x}, \bar{y})$.

[24] Once again, if $\Delta x = \Delta y$, another equilibrium exists, which is $(\underline{x}, \underline{y})$ and $(\underline{x}, \bar{y})$, because no attribute has a strictly larger span.

[25] $x^\dagger$ is defined in the proof of Lemma 7.

[26] $c \ge \hat{c}$ and $\hat{c} \ge (2\Delta x - \Delta y)/(\Delta x + \Delta y) \Longrightarrow \Delta x \le (1 + c)/(2 - c)\Delta y$.

Case $\underline{x} = \underline{y}$ and $\bar{x} \geq \bar{y}$: A carefully analysis of all the signs of $\partial\pi^m/\partial x$ and $\partial\pi^m/\partial y$ shows that there exists $c^m \in (c_1, \hat{c})$ such that the monopolist chooses $\bar{y}$ if $c < c^m$ and $\underline{y}$ if $c^m \leq c \leq 2$. Using a continuity argument, the threshold strategy must also hold when $\underline{x} \approx \underline{y}$ and $\bar{x} \geq \bar{y}$.

Case $\underline{x} > \bar{y}$: Note that when $c \leq \hat{c}$, $\partial\pi^m/\partial y > 0$; and when $c \geq \hat{c}$, $\partial\pi^m/\partial x < 0$. Thus when $c < c_0$, the monopolist chooses $\bar{y}$; when $c_0 < c < c_1$, the monopolist chooses y_c^m such that condition (2) holds at $(\underline{x}, y_c^m)$ (i.e., $y_c^m > ((1+2c^2)\underline{x} - 2cV)/(4c^2)$); and when $c_1 < c$, the monopolist chooses $\underline{y}$. $\square$

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