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Dmitry Shapiro,

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Profitability of the Name-Your-Own-Price Channel in the Case of Risk-Averse Buyers

Dmitry Shapiro

Belk College of Business, University of North Carolina Charlotte, Charlotte, North Carolina 28223, dashapir@uncc.edu

In this paper, I study profitability of the name-your-own-price channel (NYOP) in the presence of risk-averse buyers. First, I provide conditions that guarantee that for the monopolistic seller the NYOP is more profitable than the posted price. Second, I consider a more competitive framework where buyers with rejected bids have access to an alternative option. I show that if under the posted-price scenario there are unserved customers with low valuations, then NYOP is more profitable than the posted price. Finally, I study whether adding the posted-price option to the NYOP will further increase the seller's profit and show that for the decreasing absolute risk-aversion utility and a monopolistic seller it does not. In the presence of an alternative option, the answer depends on whether buyers consider the posted-price option and the alternative option to be close substitutes or not. Adding the posted-price option will increase the profit in the former case and will not in the latter.

Key words: pricing; bidding; name-your-own-price; reverse auctions

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1. Introduction

A name-your-own-price (NYOP) mechanism is one in which a buyer of a good submits a bid (price) to an agency to procure a good. If that bid is greater than some unknown threshold provided to the agency by the firms it represents, then the consumer receives the good and pays the submitted price. If not, the consumer does not receive the good. In the late 1990s, priceline.com (Priceline) successfully pioneered this business model on the Internet, and their business has been growing rapidly ever since.¹

The increasing popularity of NYOP suggests that sellers should find this pricing format at least as profitable as a standard posted-price mechanism. In the literature, however, the results are somewhat mixed. Terwiesch et al. (2005), for example, develop a model with haggling cost that they estimate using the data from a German NYOP retailer. They conclude that the retailer could increase profit by using the posted price. Fay (2004) develops a theoretical framework of the NYOP mechanism and shows that in his framework, the NYOP format is weakly dominated by the posted price. In addition, it is argued that the NYOP format itself has several disadvantages that make it even less attractive. For example, uncertainty about the actual threshold makes consumers shade their bids (Spann et al. 2005), and the existence of haggling costs reduces consumers' willingness to pay (Hann and Terwiesch

2003). One argument in favor of NYOP was suggested by Fay (2008), who shows that NYOP allows a seller to profitably price discriminate consumers based on their haggling costs. Shapiro and Zillante (2009) show that in an experimental setting, the NYOP format is just as profitable as the posted price and, in fact, can increase the profit by as much as 15%.

In this paper I contribute to the discussion by studying profitability of NYOP when buyers are risk-averse. The necessity of introducing risk aversion into the model comes from Riley and Zeckhauser (1983, p. 289), who showed that with risk-neutral buyers, "[the] fixed price strategy . . . is optimal [for the seller] in comparison to any other, including all forms of buyer involvement such as quoting offers." In other words, when buyers are risk-neutral, the posted price is always optimal for the seller, and it will outperform the NYOP format. A natural question then is what happens when buyers are risk-averse.

In this paper, I compare the NYOP and posted-price formats in the presence of risk-averse buyers under two different scenarios. First, I study the profitability of the NYOP in the case of the monopolistic seller. In this setting all buyers submit their bids to the seller. If the bid is above an unknown threshold, it is accepted and the buyer receives the object and pays his bid. If the bid is rejected, the buyer does not acquire the product and is not allowed to bid again.²

¹ For example, see *Business Wire* (2006).

² NYOP websites differ in how many times they allow customers to bid for an object. Some websites, such as Priceline and eBay

The threshold is determined by a seller who draws it from a publicly known distribution. Buyers' optimal bids then are determined by their valuation of the object and the threshold distribution. I show that under assumptions of my model, the NYOP format is more profitable than the posted price. The intuition is similar to the auction theory result where the seller's expected profit from the first-price auction is higher when bidders are risk-averse. The reason is that risk-averse bidders are willing to submit higher bids because it decreases the riskiness of the lottery for them, which in turn profits the seller.

The framework above is somewhat restrictive in that, normally, rejected bidders can acquire the product from some other source. For example, rejected Priceline bidders can purchase airline tickets at the posted price on Orbitz. To capture this possibility, I relax the monopoly assumption by assuming that bidders have access to an alternative option of acquiring the product. Furthermore, because of many reasons such as search costs, transaction costs, or brand loyalty, the value of an alternative option can differ from the buyer's valuation of the seller's product. In this setting my main result is as follows. Assume that under the posted-price scenario, the seller sets price p and there are buyers with valuations just below p who are unserved by the market. Then, switching to the NYOP format will increase the seller's profit.³ This result is consistent with intuition that NYOP increases profit by enabling the seller to reach previously unserved customers with lower valuations.

The second question that I study in this paper is whether adding the posted-price option to the NYOP will further increase the seller's profit. One can immediately see that introducing the posted-price option cannot hurt the profit because it is always possible to set the price so high that it becomes redundant, and buyers would never use it. Furthermore, the posted-price option has a potentially positive effect as it enables the seller to capitalize on rejected bidders.

First, I consider the monopolistic setting and show that when buyers' utility is decreasing absolute risk aversion (DARA), then adding the posted price will not increase the seller's profit. Intuitively, if the seller increases the price, it will have two effects on the profit. On one hand, because the rejection leads to a lower payoff, the buyers will bid higher, and overall this will have a positive effect on the profit. On the other hand, the buyers with valuations between the

old and new prices will stay away if their bid gets rejected, and this effect is negative. As it turns out, in the optimal NYOP + PP setting (NYOP + PP is the setting where the seller uses both NYOP and the posted-price option) the last effect is zero. In the optimum, the seller should accept the bid of buyers with valuations close to the posted price with probability 1. Therefore increasing the price will only have a positive effect on the seller's profit, and so it is optimal to keep raising the price until it becomes redundant.

In the presence of an alternative option, the price change has three effects. In addition to the two described above, there is the third effect that is negative. As the seller increases the price, some buyers whose bids were rejected will now switch to the alternative option instead of the posted price. The strength of this effect depends on the substitutability between the alternative and posted-price options. When two options are close substitutes, this third effect will dominate and can make the price increase unprofitable for the seller. This result can explain why Priceline offers the posted-price option on its website. Given that horizontal differentiation among travel services is relatively low, Priceline's posted-price option and alternative options, say, Orbitz, are indeed close substitutes, and so it is better to profit from rejected buyers than let them use another option.

Overall, the contribution of this paper is as follows. First, to the best of my knowledge, this is the first paper that studies the profitability of the NYOP format in the presence of risk-averse buyers. Second, I study whether and when it is profitable for the seller to add a posted-price option to its NYOP channel. Third, the results provide guidance to practitioners with regard to when they should use the NYOP format and whether they should use the combination of the NYOP and posted price or just NYOP.

The rest of this paper is organized as follows. Section 2 provides a literature review. Section 3 shows the profitability of the NYOP channel when the seller is a monopolist. Section 4 introduces an alternative option and, finally, §5 studies whether adding the posted price can increase the seller's profit compared with the NYOP case. All proofs are given in the appendix.

2. Literature Review

One of the most common questions studied in the NYOP literature is the profitability of the NYOP format, and the results are rather mixed. On one hand, there are many papers that conclude that the NYOP mechanism is detrimental for profit. For example, Terwiesch et al. (2005) use the data from a German NYOP seller to show that with the exception of one product, the posted price would lead to a higher profit. Fay (2004) provides a framework where NYOP

Travel, allow consumers to bid only once for a given item; other websites such as All Cruise Actions do not put such restrictions on customers (Park and Wang 2009).

³ Intuitively, it is important that valuations of unserved buyers have to be close to p . If they are not, then serving these buyers will cannibalize the profit from existing customers.

profit is weakly dominated by the posted price. In addition, it has been argued that uncertainty will make consumers shade their bids (Spann et al. 2005), and frictional and haggling costs will decrease consumers' willingness to pay compared with the posted-price case (Hann and Terwiesch 2003).

At the same time, there are papers that show that NYOP can increase sellers' profit. Terwiesch et al. (2005) provide theoretical conditions on distributions of haggling costs and customer valuations for which the NYOP format will increase the profit. The source of the profit increase comes from price discrimination between consumers with different haggling costs. Fay (2008) develops a framework where similar price discrimination is profitable even if the conditions in Terwiesch et al. (2005) are not satisfied. Shapiro and Zillante (2009) study the performance of the NYOP pricing mechanism in the experimental setting. They find that it performs no worse than the posted price and can actually increase the seller's profit by as much as 15%. The results described in this paper are consistent with the second group of papers.

Another question commonly studied in the NYOP literature is whether a single or multiple bidding is better for sellers. In general, in an NYOP channel there is no consensus with regard to the optimal number of bids and, in particular, whether single or multiple bidding should be used. Both formats are used by NYOP sellers, and in the literature there are arguments for and against the multiple bidding. Hann and Terwiesch (2003), for example, show that allowing multiple bidding is advantageous for the NYOP seller as it increases total sales and enables the seller to price-discriminate. On the other hand, Fay (2004) shows that multiple bidding can be undesirable because consumers will start their bid sequences at a lower level and raise their bids in small increments. Fay (2009) also provides an argument that single bidding can be better because it softens the price competition. To complicate matters even further, it was argued that while some NYOP retailers (for example, Priceline) do use single bidding, it is possible to, at least partially, circumvent this constraint (Fay 2004). In this paper, I operate under the single-bidding assumption. First, given the variety of theoretical conclusions as well as actually used formats, the single-bid format is relevant and interesting in itself. Second, it keeps the theoretical model simple and intuitive.

Priceline, which is the most commonly cited example of an NYOP seller, has an additional so-called "opaque" feature, which is that bidders do not observe all the characteristics of the product. Although this opaque feature at first seems to be purely profit-destructive, there are several explanations with regard to how it can increase profit. For example, the opaque

feature can enable sellers to profitably price discriminate (Fay 2008, Shapiro and Shi 2008) or to respond to demand variations without jeopardizing existing branding and pricing policies (Wang et al. 2009).

The NYOP pricing mechanism has also been compared with select-your-price (SYP) mechanisms. With an SYP mechanism, a list of possible bids is provided by the seller, where the probability of that bid being accepted decreases as the bid decreases. Chernev (2003) conducts experiments using generation mechanisms (such as NYOP) and selection mechanisms (such as SYP) to determine how confident bidders feel in their likelihood of success. He finds that participants tend to feel more confident in selection mechanisms than in generation mechanisms. Spann et al. (2005) use field and lab experiments to determine whether NYOP or SYP mechanisms generate higher revenue. In particular, they use SYP mechanisms with a low range of values as well as with a high range of values. They find that the SYP mechanisms generate more revenue for the seller, particularly the SYP mechanism that has a high range of values.

The name-your-own-price mechanisms are also related to the auction literature in which the setting with risk-averse bidders has been studied intensively. In Maskin and Riley (1984), for example, it was shown that under nonincreasing absolute risk aversion, an optimal auction would be to ask the buyer to offer a bid for the object with the understanding that the seller will accept the bid on a probabilistic basis. Depending on particularities of the NYOP setup, it can be related to different types of auctions such as a multiple-unit auction as in Amaldoss and Jain (2008), the first-price auction as in the NYOP setting in this paper, or the first-price auction with the buy-it-now price as in the NYOP + PP setting in this paper. Despite these similarities, NYOP differs from the standard auction setting in that in auctions, bidders compete with each other for an object. This means that, first of all, a bidder should take into account the behavior of other bidders to make an optimal bid. Second, there is always a positive probability that the bidder will not receive an object. In the NYOP case, on the other hand, bidders do not directly compete with each other. Furthermore, if the seller uses the combination of NYOP and posted price, then all bidders with valuation above the posted price will receive the object with probability 1. It is the amount they pay (the submitted bid or the posted price) that is random.

3. Monopolist: NYOP vs. Posted Price

The goal of this section is to show that a monopolist serving risk-averse buyers can increase its expected profit by using the NYOP channel instead of the posted price. I assume that buyers have unit demand

for the monopolist's product. Their valuation, v , is distributed on $[0, 1]$ with cumulative distribution function (cdf) $F(v)$, which is differentiable and has a positive density. The cost of production is assumed to be equal to zero and there are no capacity constraints.⁴ The posted-price profit is then $p(1 - F(p))$. The optimal price p^m is determined by the first-order condition $1 - F(p^m) = p^m f(p^m)$, and the posted-price monopolist's profit is $\pi^m = p^m(1 - F(p^m))$. Buyers are risk-averse with utility function $u(\cdot)$. I assume that u is a strictly increasing and concave function with a positive relative risk-aversion coefficient. This assumption is stronger than concavity of u because it requires the relative risk-aversion coefficient to be positive at zero. For example, it is satisfied by constant relative risk-aversion (CRRA) utility functions but is not satisfied by constant absolute risk-aversion (CARA).

The alternative pricing mechanism that I consider is name-your-own price, or NYOP. Under NYOP a buyer makes an offer and the seller compares it with a randomly determined threshold. The threshold is unknown to the buyer, but the distribution used to generate the threshold is common knowledge. If the offer is greater than the threshold, then the seller accepts it and the buyer receives the object and pays his price. In this section I will assume that when the offer is below the threshold, then no transaction is made and the buyer's utility is zero. Notice that this setting is different from a standard auction setting in that buyers do not directly compete with each other. In particular, in the absence of capacity constraints, the probability of getting the product does not depend on the behavior of other bidders.

Clearly, the above-described mechanism will function only if the seller can credibly commit to such a format. Without commitment, the seller will accept any offer above the marginal cost, which will be anticipated by buyers and will drive the profit to zero. One way to make the seller's commitment credible is via publicly observed repeated interactions. In the case of Priceline, there are many Web forums such as BetterBidding.com, BiddingForTravel.com, and FlyerTalk.com that provide details about accepted and rejected bids. In the presence of these kinds of websites, should an NYOP seller decide to accept anything above and reject everything below

the marginal cost, such information would quickly become known to potential bidders, thereby giving the seller incentives to remain committed to the original mechanism. As discussed in Fay (2009), there is further evidence that NYOP sellers indeed use random threshold mechanisms. Priceline's acceptance decision, for example, is determined by a formula that includes a random element (Segan 2005). Furthermore, when accepting a bid for a hotel room, it uses a "randomizer" program that, instead of setting the threshold price equal to the lowest rate available, compares the bid to the rates set by two randomly selected hotels (Malhotra and Desira 2002, Haussman 2001).

Let $G(b)$ be a strictly increasing, twice-differentiable and log-concave function such that $G(0) = 0$ and $G'(b) = g(b) > 0$ when $b > 0$. I will also assume that $G(0)/g(0) = 0$, which is fairly unrestrictive as it is satisfied for any function G such that $G^{(n)}(0) \neq 0$ for some n . I will assume that the random threshold is distributed on interval $[l, h]$ with cdf $G(x - l)/G(h - l)$ so that the buyers' maximization problem is

$$\max_{b \in [l, h]} \frac{G(b - l)}{G(h - l)} u(v - b). \quad (1)$$

The first-order condition (FOC)

$$g(b - l)u(v - b) - G(b - l)u'(v - b) = 0 \quad (2)$$

implicitly defines function $b(v)$, and the fact that the second-order condition (SOC) is satisfied follows from log-concavity of G . When $b(v) \in [l, h]$, it will determine the bid submitted by the customer with value v . If $b(v) > h$, then we have a corner solution, and the submitted bid will be h .

In what follows, it will be convenient to denote $G(b)/g(b)$ as $\phi(b)$. From our assumptions on $G(\cdot)$, it follows that $\phi(\cdot)$ is differentiable, $\phi' > 0$, and $\phi(0) = 0$. Using this new notation, we can rewrite the FOC as $u(v - b) - \phi(b - l)u'(v - b) = 0$, from which we have that

$$b'(v) = \frac{u'(v - b) - \phi(b - l)u''(v - b)}{u'(v - b) + \phi'(b - l)u'(v - b) - \phi(b - l)u''(v - b)} > 0. \quad (3)$$

The positivity follows from the fact that the numerator is positive and the denominator is equal to the SOC multiplied by -1 and, therefore, is positive as well.

Let v_h be a solution to $b(v) = h$, if it exists. Since $b(\cdot)$ is an increasing function, it means that $b(v) > h$ when $v > v_h$ and $b(v) < h$ when $v < v_h$. Thus the bidding behavior can be summarized as follows. Buyers with $v < l$ will not bid at all. Buyers with $v \in [l, v_h]$ will bid according to function $b(v)$, and buyers with

⁴ In general, the capacity constraints can be relevant for NYOP sellers as, for example, discussed in Fay (2004). They are not, introduced here for the following reasons. First, the NYOP setting is most commonly used when service providers are undersold, in which case capacity constraints are not binding. Second, the main insights of this paper are unlikely to be affected by the presence of the capacity constraints. This is because in auctions, (for example, a single-unit auction where the capacity constraints are extreme), introducing risk aversion still leads to an increase in the seller's profit.

$v > v_h$ will bid h . If such v_h does not exist, then all buyers with $v > l$ will bid according to $b(v)$. Notice that because Equation (2) does not depend on h , the bids of buyers with $v \in [l, v_h]$ will not depend on h either. The only effect that h has on bids is that it determines the valuation v_h above which all buyers will bid h .

When $v_h < 1$, the monopolist's profit is

$$\pi(l, h) = \frac{1}{G(h-l)} \int_l^h \int_{b^{-1}(t)}^{v_h} b(v) f(v) g(t-l) dv dt + \frac{1}{G(h-l)} \int_l^h \int_{v_h}^1 h f(v) g(t-l) dv dt. \quad (4)$$

The second term is the profit that comes from the bidders with $v > v_h$ who submit bids h and whose bids are accepted with probability 1. It is simply equal to $h(1 - F(v_h))$. The first term is the profit from bidders whose valuations are between l and v_h . We can simplify it as follows. Denote the inside integral as $\Psi(t)$. Then the first term becomes

$$\begin{aligned} & \int_l^h \Psi(t) dG(t-l) \\ &= \Psi(t) G(t-l) \Big|_l^h - \int_l^h G(t-l) d\Psi(t) \\ &= \int_l^h \frac{\partial}{\partial t} b^{-1}(t) \cdot b(b^{-1}(t)) f(b^{-1}(t)) G(t-l) dt \\ &= \int_l^{v_h} b(v) f(v) G(b(v)-l) dv, \end{aligned}$$

where the first equality is an integration by parts, the second one follows from the fact that $G(0) = 0$ and $\Psi(h) = 0$, and the last one uses change of variables $v = b(t)$. Therefore,

$$\pi(l, h) = \frac{1}{G(h-l)} \int_l^{v_h} b(v) f(v) G(b(v)-l) dv + h(1 - F(v_h)). \quad (5)$$

PROPOSITION 1. *There exist l and h such that $\pi(l, h) > \pi^m$.*

PROOF. The proof is given in the appendix. \square

The intuition for this result is similar to the intuition from auction theory with risk-averse buyers. It is well known that risk-averse bidders tend to bid higher than risk-neutral bidders. The reason is that higher bids make the uncertain outcome of the auction less risky. As an extreme example, bidding your own value in a first-price auction would guarantee a certain payoff of zero. As a result, risk-averse bidders are willing to pay a higher price to achieve some risk reduction. The same effect is at play in the NYOP setting.

To visualize the effect of risk aversion on seller's profit, consider the following example. Assume that $u(v) = v^a$. Consumers valuations are distributed uniformly on $[0, 1]$ and $G(b) = b$ so that the threshold

distribution is uniform. In this case, the monopolist's price is $1/2$ and the monopolist's posted-price profit is $1/4$. When the threshold is distributed uniformly on $[l, h]$, the buyers' bidding function is linear and is equal to

$$b(v) = \frac{v + al}{1 + a},$$

when $v \in [l, \min\{h(1+a) - al, 1\}]$ and $b(v) = h$ when $v \geq \min\{h(1+a) - al, 1\}$. As a decreases from 1 to 0—so that the bidders become more risk-averse—the slope of the bidding function and bids themselves increase. In particular, as bidders become extremely risk-averse so that $a \rightarrow 0$, they bid closer to their valuation; that is, $b(v) \rightarrow v$. Naturally, this benefits the seller. Substituting the bidding function into (5), we get that the monopolist's profit becomes

$$\begin{aligned} \pi(l, h) &= -\frac{2}{3}(a+1)h^2 + \left(\frac{5}{6}al - \frac{1}{6}l + 1\right)h \\ &\quad - \frac{1}{6}l^2(a+1). \end{aligned} \quad (6)$$

By taking the first-order conditions, we can solve for optimal l and h , which are equal to

$$l = \frac{2(5a-1)}{5+14a-3a^2}, \quad h = \frac{4(1+a)}{5+14a-3a^2}.$$

Figures 1 and 2 show how the boundaries of optimal support change depending on a as well as the profit levels obtained when the optimal support is used. As we can see, when $a = 1$ so that agents are risk-neutral, the optimal support shrinks to one point, which is 0.5, or the monopoly price. In other words, for risk-neutral buyers the monopolist uses the degenerate NYOP, which coincides with the posted price and earns the posted-price profit of 0.25. When $a < 1$, however, using the NYOP mechanism becomes

Figure 1 Values of l and h That Maximize the Profit Function Given a

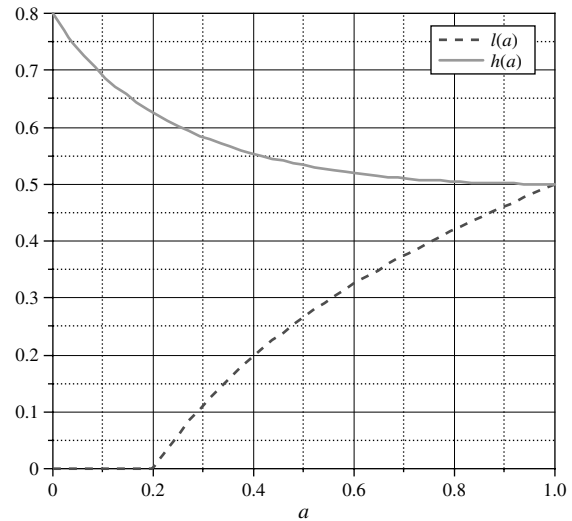
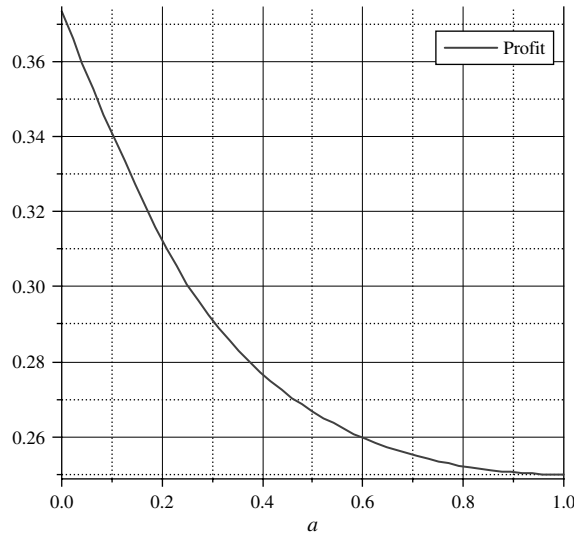


Figure 2 The Profit Under NYOP as a Function of a When the Optimal Support Is Used



strictly more profitable, and the profit increases with buyer's risk aversion. In particular, when buyers are extremely risk-averse (i.e., a is small) the monopolist's profit increases all the way up to 0.37, or 50% higher than the posted-price profit.

4. NYOP in the Presence of an Alternative Option

In the previous section I considered the case where the buyers have only one way to acquire the good—the NYOP channel. In particular, I assumed that when the bid is rejected, the buyers do not get the product at all. This is somewhat unrealistic because typically, rejected buyers can acquire the product at some other source. Naturally, having an alternative option is going to affect the bidding behavior, and in this section I will analyze the impact of this option.

Assume that the monetary value of the alternative option depends on the value of the product and is given by a continuously differentiable function $c(v)$. I assume that $c(v)$ is a monetary value so that the actual utility of the outside option is $u(c(v))$.

EXAMPLE 2. The same product is available at the same website at price p . Absent transaction and search costs, the value of this option is $c(v) = v - p$.

EXAMPLE 3. Consider the Hotelling linear city model with transportation cost d and customers' valuation \tilde{v} . Assume that firm 1 uses the NYOP channel and firm 2 charges a price equal to p_2 . The net value of the firm 1 product for the customer located at x is $v = \tilde{v} - dx$. Then, the value of an alternative option then is $c(v) = \tilde{v} - d(1-x) - p_2 = 2\tilde{v} - d - p_2 - v$. Here, in contrast to the previous example, $c(v)$ is a decreasing function.

As before, I model the NYOP channel using the function $G(\cdot)$ that generates a threshold distribution on the interval $[l, h]$. Bidders for whom $c(v) \leq 0$ will not use the alternative option, so their optimization problem is similar to that in the previous section. It is given by the equation

$$\max_b \frac{G(b-l)}{G(h-l)} u(v-b), \quad (7)$$

which leads to the first-order condition

$$u(v-b) - \phi(b-l)u'(v-b) = 0. \quad (8)$$

Bidders with $c(v) > 0$ will use the outside option in the case of rejection, and their optimization problem is therefore

$$\max_b \frac{G(b-l)}{G(h-l)} \cdot u(v-b) + \left(1 - \frac{G(b-l)}{G(h-l)}\right) \cdot u(c(v)). \quad (9)$$

The FOC is

$$u(v-b) - u(c(v)) - \phi(b-l)u'(v-b) = 0. \quad (10)$$

It follows from the FOC that buyers will never submit a bid such that $v-b < c(v)$; that is, a buyer's bid if accepted will always generate a surplus that is higher than the utility from the alternative option.

I will denote the part of the bidding function that is a solution to (8) as b_1 and the part of the bidding function that is a solution to (10) as b_2 . As before, the value of h does not enter the first-order conditions, so the *actual* submitted bid will be equal to $\max\{h, b_1(v)\}$ for those with $c(v) \leq 0$ and to $\max\{h, b_2(v)\}$ for those with $c(v) \geq 0$.

In this section I assume that the alternative option is provided by another supplier, and therefore the seller's profit is zero when the bid is rejected. First, consider the posted-price outcome. When the seller charges price p , the valuation of buyers who will purchase the product should satisfy two conditions: $v \geq p$ and $c(v) \leq v - p$. In what follows, I will assume that $c(\cdot)$ is such that the set $\{v \geq p, c(v) \leq v - p\}$ is either empty or an interval $[p, v^p]$ and that buyers with $v \in (p, v^p)$ strictly prefer paying price p to the alternative option. The upper bound of the interval $[p, v^p]$ is either a solution to the $c(v) = v - p$ equation, or 1 if such solution does not exist. The assumption is satisfied if, for example, $c'(v) \neq 1$, and its purpose is to rule out the case when valuations of buyers preferring the seller's posted price p is a group of disjointed intervals. For instance, the set of sellers' customers can be only low- or only high-value agents. However, it cannot be that low- and high-value buyers purchase the product from the seller, whereas buyers with intermediate values use the alternative option.

Let p^m be the price that maximizes the seller's posted-price profit. An important assumption that

I will use is that $c(p^m) < 0$. This assumption means that customers with valuations just below p^m would not use an alternative option and therefore would remain unserved under the posted-price scenario. I will show that when this assumption holds, the introduction of the NYOP channel will increase the seller's profit. This is consistent with the idea expressed in the literature that the seller can use the NYOP channel to increase profit by reaching price-sensitive customers who previously stayed out of the market.

PROPOSITION 4. *If the valuation of the alternative option $c(v)$ satisfies the assumptions above, then there exists l and h such that the NYOP profit is higher than the posted-price profit.*

The importance of the $c(p^m) < 0$ assumption is demonstrated in Figure 3. The graphs are built for the following parameter values: $v \in U[0, 1]$ and $u(x) = \sqrt{x}$. For the left graph I assumed $c(v) < 0$ so that there is effectively no alternative option, and for the right graph I assumed that $c(v) = 1 - v$. For both cases the monopoly price p^m is equal to $1/2$; however, in the former case, $c(p^m) < 0$, and in the latter case, $c(p^m) = 1/2 > 0$. On both charts I plot the seller's profit from using the NYOP channel with uniform distribution when h is set to be equal to $p^m = 1/2$ and l varies from 0 to $1/2$. We see that in the left graph, the one with $c(p^m) < 0$, as I decrease l from $l = 1/2$, the NYOP improves the seller's profit, which reaches its maximum at $l = 0.25$ (the fact that the profit at $l = 0$ is equal to the profit at $l = 0.5$ is purely coincidental for these parameter values). In the right graph, where $c(p^m) > 0$, the NYOP reduces the seller's profit so that the posted-price profit—when $l = h = p^m$ —is the highest. Furthermore, numerical simulations show that if

the threshold is distributed uniformly, then in the right graph there is no (l, h) that would increase the seller's profit above the posted-price level.

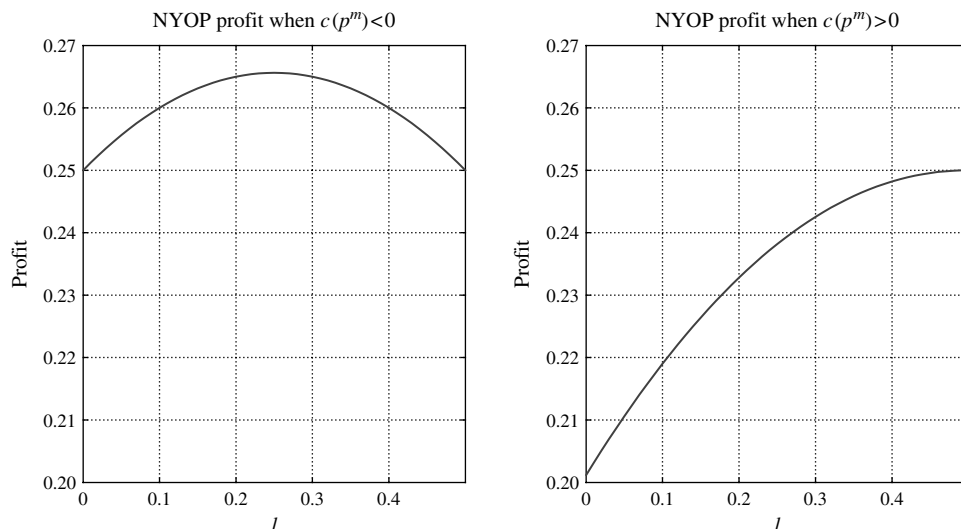
Figure 3 as well as numerical simulations do not, of course, indicate that when $c(p^m) > 0$ there is no NYOP that would increase the seller's profit. It still might be possible that for a different threshold distribution and some pair of $[l, h]$ there exists an NYOP that will increase the seller's profit. The main message of this example is that the NYOP's ability to increase the seller's profit is no longer robust when $c(p^m) > 0$.

5. Combination of NYOP and Posted Price

Until now, I have been considering the case when the seller completely abandons the posted price and uses the NYOP channel only. In this section, I study whether adding the posted-price option to the NYOP format can increase the seller's profit even further. Clearly, the optimal NYOP + PP format has to be at least as profitable as the NYOP format because the seller has an option of setting the price so high that it will become redundant and will not be used by buyers. Furthermore, adding the posted-price option has a clear potential to increase the profit as the seller can capitalize on rejected bidders.

It is worth emphasizing that the posted-price option in the NYOP setting differs from buy-it-now prices (BNPs) in online auctions. From the buyer's perspective, having a BNP is valuable because it helps to avoid time and participation costs (Wang et al. 2008, Mathews 2004). Furthermore, risk-averse bidders can execute BNP to reduce some risks, most notably, a risk of losing the auction to a bidder with

Figure 3 The Seller's NYOP Profit for Different Values of l Depending on Whether $c(p^m)$ Is Greater or Smaller Than 0



Notes. Buyers' valuations are uniformly distributed on $[0, 1]$ and $u(x) = \sqrt{x}$. The threshold is distributed uniformly on $[l, h]$ where l varies and h is set to be equal to $p^m = 1/2$.

a higher value (Budish and Takeyaman 2001, Hidvégi et al. 2006). In the NYOP setting, on the other hand, these concerns are absent or of less importance. Time costs are very low because the bidder learns the outcome almost immediately. Participation constraints are lower, for example, because the bidder does not have to keep track of other bidders or snipe. Finally, as long as $v > p$, bidders in NYOP will receive the object with probability 1. In particular, in my model because I set time and participation costs equal to zero, the bidder will never use the posted-price option without bidding first. From the seller's perspective, there is also a difference between BNP and the posted-price option. In the case of online auctions, the BNP can be set somewhat above the expected selling price, and because of aforementioned costs and risk aversion, some bidders will be willing to execute it (see Haruvy and Popkowski Leszczyc 2009). In the case of NYOP, the posted price benefits the seller by capitalizing on bidders whose bid got rejected.⁵

Assume that, as before, the seller uses function $G(\cdot)$ to generate a threshold distribution with support $[l, h]$, and assume that he also offers a posted-price option at price p . When the seller is a monopolist, the buyers thus can be divided into two groups: those buyers with $v < p$ who will not use the posted-price option and will bid according to (8), and those buyers with $v \geq p$ who have access to an alternative option and will bid according to (10) with $c(v) = v - p$.

When the seller sets the support for the threshold distribution as $[l, h]$ and charges the posted price p , it is easy to verify⁶ that his expected profit is equal to

$$\begin{aligned} \pi(l, h, p) = & \frac{1}{G(h-l)} \left[\int_l^p b_1(v) f(v) G(b_1(v) - l) dv \right. \\ & \left. + \int_p^1 (b_2(v) - p) f(v) G(b(v) - l) dv \right] \\ & + p(1 - F(p)). \end{aligned} \quad (11)$$

Intuitively, if the seller were to use only the posted price, his profit would be $p(1 - F(p))$ —the last term.

⁵ Priceline states that “In November, 2002, we began offering retail travel... that allows us to capitalize on the retail travel market as well as offer a retail alternative to those of our customers who fail to bind on our NAME YOUR OWN PRICE® path” (Priceline 2002 10-K report, p. 5).

⁶ Using the same logic as in §3, we get that the expected profit from bidders with $v < p$ is $(1/G(h-l)) \int_l^p b_1(v) f(v) G(b_1(v) - l) dv$. The profit from bidders with $v \geq p$ is equal to

$$\begin{aligned} & \frac{1}{G(h-l)} \left[\int_p^1 \int_l^{b(v)} b(v) f(v) g(t-l) dt dv + \int_p^1 \int_{b(v)}^h p f(v) g(t-l) dt dv \right] \\ & = \frac{1}{G(h-l)} \int_p^1 b(v) f(v) G(b(v) - l) dv + \int_p^1 p f(v) \left[1 - \frac{G(b(v) - l)}{G(h-l)} \right] dv \\ & = \frac{1}{G(h-l)} \int_p^1 (b(v) - p) f(v) G(b(v) - l) dv + p(1 - F(p)). \end{aligned}$$

Adding the NYOP affects the profit as follows. The seller gains by receiving b_1 from customers with $v < p$ instead of receiving zero. This is captured by the first term. For customers with $v \geq p$, if their bid is accepted, the seller gets b_2 instead of p and this is captured by the second term. If the bid is rejected, the seller receives p , which is the same as in the posted-price setting.

Given the function $G(\cdot)$, let (l, h, p) be the parameters of the NYOP + PP setting that will maximize the seller's profit. I say that the posted price is *redundant* if at (l, h, p) buyers never use the posted-price option. The next proposition shows that when the buyers' utility is DARA, and there is no alternative option available, which is the setting of §3, the posted price is redundant.

PROPOSITION 5. *If the buyers' utility exhibit DARA and there is no alternative option available, then the posted price is redundant.*

The proof, while being somewhat technical, is based on the following logic. Assume that (l, h, p) is an optimal triple and that the posted price is not redundant. That is, there are buyers who would use the posted price if their bid gets rejected. Fix l and h and see what happens as the seller increases p . There are two effects on the profit. On one hand, those buyers who would potentially use the posted price will bid more than before *and* if their bid is rejected, then the posted price they pay is higher than before. As it turns out, this effect is positive. On the other hand, the buyers with valuations just above $v = p$ will no longer use the posted-price option if their bid gets rejected. This effect is negative. At the optimum, the second effect is zero. The reason is that it is optimal for the seller to set h at such a level that $b(v) = h$ in the neighborhood of $v = p$. Then in this neighborhood no bid gets rejected and there is no profit loss for the seller. Thus, it is optimal to keep increasing p until it becomes redundant.

In the presence of an alternative option, there is a third effect that will impact the profit. It comes from a competition between the seller's posted-price option and an alternative option. Now, as the seller increases p , some buyers with rejected bids will use an alternative option instead of the posted-price option, which will have a negative impact on the profit. Let v_p be the valuation of a buyer indifferent between the two options; that is, $c(v_p) = v_p - p$. Then the rate with which buyers will switch from the posted price to an alternative option is given by

$$\frac{\partial v_p}{\partial p} = -\frac{1}{c'(v_p) - 1}.$$

The closer $c(v_p)$ is to 1, the larger the profit loss, and so, in particular, when $c(v_p)$ is sufficiently close to 1,

it will not be optimal to increase p until it becomes redundant.

PROPOSITION 6. *Let (l, h, p) be the optimal triple for the NYOP + PP mechanism. Whether the posted price is redundant or not depends on how close $c'(v_p)$ is to 1. It is not redundant when $c'(v_p)$ is close to 1 and is redundant if $c'(v_p)$ is sufficiently far from 1.*

From an economic perspective, the condition on $c'(v_p)$ has a very simple interpretation related to substitutability of the posted-price and alternative options. When $c'(v_p)$ is high it means that buyers just under v_p strongly prefer the posted-price option, whereas buyers just above v_p will strongly prefer the alternative option. On the other hand, when $c'(v_p)$ is close to 1, then it means that there are many buyers who are almost indifferent between the two options. That is, for these buyers they are almost perfect substitutes. In the latter case their behavior will be very responsive to the changes in posted price charged by the seller.

Proposition 6 can be used to explain why Priceline uses the posted price on its website. Whereas innovations such as frequent flier miles introduce some differentiation between airline companies, air travel is still a relatively homogeneous product, and demand is fairly responsive to the price, especially among price-sensitive bidders who are likely to use Priceline in the first place. Therefore, as Proposition 6 suggests, the seller should use the posted price to profit from rejected bidders, which is what we observe.

6. Concluding Remarks

In this paper I study the profitability of the NYOP channel when buyers are risk-averse and compare profitability with the posted-price profit. I also study whether and when it is profitable to add the posted-price option to the NYOP channel. First, I show that if the seller is a monopolist, then NYOP is more profitable than the posted price. Furthermore, if the buyers' utility is DARA, then adding the posted-price option to the NYOP will not produce any additional increase to the seller's profit. Next, I consider a more realistic case where buyers have an alternative option to the seller's product that they value as $c(v)$. I show that if under the posted-price scenario there are unserved customers, then the NYOP will increase the seller's profit. Effectively, the NYOP will enable the seller to reach those low-valuation customers and profit from that. The question as to what happens when this is not the case remains open; however, I provide some evidence in the paper to show that NYOP can fail to increase the seller's profit. From a practitioner's point of view this result suggests that the most promising way to increase the profit using the NYOP is by trying to expand the current customer

base with price-sensitive customers who would otherwise stay out of the market.

Finally, in the presence of an alternative option I study whether an NYOP + PP combination is more profitable than the NYOP or not. As it turns out, the result depends on the substitutability of the posted price and alternative options. When the two options are almost perfect substitutes, then adding the posted price to NYOP will benefit the seller. This is consistent with the fact that Priceline uses both pricing mechanisms simultaneously. Indeed, an alternative option of, say, purchasing the airline ticket at Orbitz is a close substitute and therefore using the NYOP + PP combination is more profitable.

Although this paper does address some of the issues related to profitability of the NYOP format in the presence of risk-averse buyers, there are many interesting questions that are left for future research. One such question is to solve for the optimal threshold distribution function $G(\cdot)$. The second question is whether the NYOP can increase the seller's profit when buyers with lower valuations are already served by another seller, i.e., when $c(p^m) > 0$. In this paper, results go only one way, which is that if there are unserved buyers with valuations close to the current seller's price, then NYOP will increase the profit. Studying what happens without the $c(p^m) < 0$ requirement will greatly enhance our understanding of the scope and limitations of the NYOP framework. Finally, some structural assumptions used in this paper can be relaxed to address a variability of the NYOP formats that exists in the real world. Particularly interesting topics would be introducing the multiple bidding and capacity constraints.

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Appendix

PROPOSITION 1. *There exist l and h such that $\pi(l, h) > \pi^m$.*

PROOF. First, I show that when l and h are close to p^m , then $v_h < 1$ so that firm's profit is given by (5). \square

LEMMA 7. *If l and h are sufficiently close to p^m , then $v_h < 1$.*

PROOF. First, from (2), it follows that $b(l) = l$ and $b(v) > l$ for any $v > l$. Thus, when $v < v_h$ or if v_h does not exist, then $b \in [l, h]$. Next, taking the expression for the derivative of the bidding function (3) and dividing both the numerator and denominator by $u'(v - b)$, we get that

$$b'(v) = \frac{1 + \phi(b - l)RA(v - b)}{1 + \phi'(b - l) + \phi(b - l)RA(v - b)} \geq \frac{1}{1 + \phi'(b - l)},$$

where $RA(v - b)$ is the coefficient of risk aversion at point $v - b$. Since $\phi'(0) > 0$, we have that $\phi'(b - l)$ is bounded

away from -1 when l and h are sufficiently close to each other, and therefore, $b'(v)$ is bounded away from 0 by a positive constant $c > 0$ on $[l, h]$. But then, since $b(l) = l$, we have that $b(l + (h - l)/c)$ should be greater than h when determined by the FOC. Therefore, $v_h < l + (h - l)/c < 1$, where the last inequality holds when l and h are sufficiently close to p^m . \square

When $v_h < 1$, the firm's profit is

$$\pi(l, h) = \frac{1}{G(h-l)} \int_l^{v_h} b(v)f(v)G(b(v)-l)dv + h(1-F(v_h)).$$

The idea of the proof is to take the derivative of the profit function with respect to h and calculate its limit as $h \rightarrow l = p^m$. As I will show, the limit is positive. Given that $\pi(p^m, p^m) = \pi^m$, it will imply that for h sufficiently close to p^m the NYOP's profit is higher than the posted-price monopoly profit.

First, I take the derivative of $h(1-F(v_h))$. It is equal to

$$1 - F(v_h) - hf(v_h) \frac{\partial v_h}{\partial h} \\ = 1 - F(v_h) - hf(v_h) \frac{1}{b'(v_h)} \rightarrow 1 - F(l) - lf(l) \frac{1}{b'(l)},$$

where I use the fact that $b(v_h) = h$ and therefore, $\partial v_h / \partial h = 1/b'(v_h)$. The derivative of the first term is

$$\left(\frac{1}{G(h-l)} \right)' \int_l^{v_h} b(v)f(v)G(b(v)-l)dv \\ + \frac{1}{G(h-l)} \left(\int_l^{v_h} b(v)f(v)G(b(v)-l)dv \right)'$$

Treating each term separately, we get that the second term is

$$\frac{1}{G(h-l)} b(v_h)f(v_h)G(b(v_h)-l) \frac{1}{b'(v_h)} \\ = hf(v_h) \frac{1}{b'(v_h)} \rightarrow lf(l) \frac{1}{b'(l)}.$$

The first term is

$$-\frac{\int_l^{v_h} b(v)f(v)G(b(v)-l)dv}{G(h-l)\phi(h-l)}; \quad (12)$$

recall that $\phi(b-l) = G(b-l)/g(b-l)$. To calculate the limit of (12), we use L'Hôpital's rule to show that it is equal to the limit of

$$-\frac{b(v_h)f(v_h)G(b(v_h)-l)1/b'(v_h)}{G(h-l)\phi'(h-l) + G(h-l)} \\ = -\frac{hf(v_h)}{1+\phi'(h-l)} \cdot \frac{1}{b'(v_h)} \rightarrow -\frac{lf(l)}{1+\phi'(0)} \cdot \frac{1}{b'(l)}.$$

Combining all the terms, we get that the limit of the profit derivative is equal to

$$1 - F(l) - \frac{lf(l)}{1+\phi'(0)} \cdot \frac{1}{b'(l)}. \quad (13)$$

To calculate $b'(l)$, I rewrite $b'(v)$ as

$$b'(v) = \frac{1 + \phi(b-l)RA(v-b)}{1 + \phi'(b-l) + \phi(b-l)RA(v-b)} \\ = \frac{1 + (u(v-b)/(u'(v-b)(v-b)))RR(v-b)}{1 + \phi'(b-l) + (u(v-b)/(u'(v-b)(v-b)))RR(v-b)},$$

where I used the fact that $\phi(b-l) = u(v-b)/u'(v-b)$ as follows from (2). When $h \rightarrow l$, term $v-b$ converges to 0, and therefore by L'Hôpital's rule,

$$\lim \frac{u(v-b)}{u'(v-b)(v-b)} = \lim \frac{u'(v-b)}{u'(v-b) + u''(v-b)(v-b)} \\ = \frac{1}{1-RR(0)}.$$

Plugging it back into the expression for the derivative, I get that

$$b'(l) = \frac{1}{1 + \phi'(0) - \phi'(0)RR(0)}.$$

Plugging this into (13), we have that as $h \rightarrow l = p^m$, the profit derivative is

$$1 - F(p^m) - p^m f(p^m) \frac{1 + \phi'(0) - \phi'(0)RR(0)}{1 + \phi'(0)}.$$

The expression above is positive. The monopoly price p^m satisfies the FOC, $1 - F(p^m) - p^m f(p^m) = 0$; since $\phi'(0) > 0$, the fraction that multiplies $p^m f(p^m)$ is strictly less than 1. \square

PROPOSITION 4. *If the valuation of the alternative option $c(v)$ satisfies the assumptions above, then there exist l and h such that the NYOP profit is higher than the posted-price profit.*

PROOF. The seller's expected profit is determined by how many customers will submit bids and by how much they will bid. The set of customers who will submit the bid is determined by two conditions: $v \geq l$ and $c(v) \leq v - l$. Because the lowest bid that can be accepted is l , the first constraint says that buyers with $v < l$ will never bid. When the second constraint is violated it means that buyers would rather use an alternative option than submit bid l . Let v_l denote the solution to $c(v) = v - l$, and let v_p denote the solution to $c(v) = v - p^m$. Because of the continuity of $c(\cdot)$, only two cases are possible: either for each l sufficiently close to p^m , there exists $v_l \in [p^m, 1]$; or as l gets close to p^m , there is no v_l in $[p^m, 1]$ and v_p , if it exists, is greater than or equal to 1.

Case 1. Assume that for each l sufficiently close to p^m there exists $v_l \in [p^m, 1]$. By the assumption on c , such v_l is unique, and from the continuity of c it follows that $v_l \rightarrow v_p$ as $l \rightarrow p^m$. I will consider an NYOP where the upper limit h is set equal to p^m and the lower limit l is arbitrarily close to h . Note that although throughout the entire proof the value of h is fixed at p^m , I will keep using the variable h to denote the upper limit of the NYOP distribution. This is done solely for expositional reasons and will help me to keep separated the usage of p^m as the optimal posted price from the usage of p^m as the upper bound of the threshold distribution.

Given l , the set of potential bidders is $[l, v_l]$. Let $V_+ = \{v \in [l, v_l]: c(v) \geq 0\}$ and $V_- = \{v \in [l, v_l]: c(v) \leq 0\}$. All bidders can be divided into two groups: those with $v \in V_-$ who bid according to (8) and those with $v \in V_+$ who bid according to (10). Let v_0 be the smallest root of $c(v)$ on interval $[l, v_l]$. Since $c(p^m) < 0$ and l is close to p^m , we can conclude that $v_0 > p^m > l$. One can use the same logic as in Lemma 7 to show that there exists $v_{h,1} \in V_-$ such that $l < v_{h,1} < v_0$ and such that $b(v_{h,1}) = h$. By monotonicity of $b_1(\cdot)$ it follows that $b_1(v) = h$ for any $v \in V_-$ that is greater than $v_{h,1}$. The next lemma establishes a similar result for the set V_+ .

LEMMA 8. When l is sufficiently close to h and $h = p^m$, there exists $v_{h,2}$ such that customers with $v \in V_+ \cap \{v \leq v_{h,2}\}$ will submit bid h and customers with $v \in [v_{h,2}, v_l]$ will bid according to (10).

The proof is somewhat technical but the idea is rather simple. I look at the function b_2 as defined by (10) with $l = p^m$ and ignore the constraint that bids should be below h . I will show that this function is strictly decreasing when it is close to v_p and that at v_p it reaches its unique minimum. Then I will show that it is possible to cap it with h so that the optimal bid is h everywhere except for a neighborhood of v_p . Since $c(v_p) > 0$, it follows that there is a neighborhood of v_p that belongs to V_+ , and therefore bidders from that neighborhood will bid according to (10). The final step is to notice that because everything is continuous I can slightly decrease l to make it strictly below p^m and yet everything established above will be satisfied.

PROOF. For a moment, I ignore the constraint that the submitted bid is always smaller than or equal to h , and I will only look at function $b_2(\cdot)$ as given by (10). The solution to (10) depends on v and l , and for the purposes of this lemma I will write it explicitly by using two arguments for function b_2 . Since $b_1(v_0) = b_2(v_0, l) > h$ and $b_2(v_l, l) = l$, it follows that there exists $v_{h,2}$ such that $b_2(v_{h,2}, l) = h$. The main difficulty is to show that $v_{h,2}$ is unique and sufficiently close to v_p so that $[v_{h,2}, v_p] \subset V_+$. Although this is generally not the case, this holds when l is sufficiently close to p^m .

Consider Equation (10) when $l = p^m$.⁷ First, from (10) it follows that $b_2(v_p, p^m) = p^m$. Second, since

$$\frac{\partial b_2}{\partial v} = \frac{u'(v-b) - u'(c(v))c'(v) - \phi(b-l)u''(v-l)}{u'(v-b) - \phi(b-l)u''(v-b)}$$

and the denominator is positive, the monotonicity of the bidding function with respect to v is determined by the sign of the numerator. When $v = v_p$ and $l = p^m$, we have that $c(v_p) = v_p - p^m$, $b = l = p^m$, and so the sign of the derivative is equal to the sign of $u'(v - p^m)(1 - c'(v_p))$. As the next lemma will show, this sign is negative, and therefore $b_2(\cdot, p^m)$ is a strictly decreasing function of v in the neighborhood of v_p . \square

LEMMA 9. If p^m is the profit-maximizing posted price and $c(p^m) < 0$, then $c'(v_p) > 1$.

PROOF. First I show that $c'(v_p) \geq 1$. Assume this is not the case. Then there is a neighborhood of v_p where $c'(v) < 1$, which would imply that $c(v) > v - v_p$ for $v < v_p$ that are sufficiently close to v_p . Together with the assumption that $c(p^m) < 0$, this is a contradiction because this would mean that $c(v) = v - p^m$ has at least two solutions (one being v_p and another belonging to (p^m, v_p)).

Next I show that $c'(v_p) \neq 1$. When the seller uses the posted price, the set of buyers is $[p^m, v_p]$. The profit is equal

⁷ It might be worthwhile to reiterate that if the optimal bid is interior, then it is determined by the FOC, which is (10). However, if FOC leads to a bid greater than h , then the optimal bid will be a corner solution $b = h$. Setting $l = p^m$ would mean that the optimal bid is always a corner solution and is equal to p^m . Equation (10) nonetheless will have a well-defined solution that I denote as function b_2 , and this is what I am interested in here.

to $p(F(v_p) - F(p))$ and the profit-maximizing posted price should satisfy the FOC

$$F(v_p) - F(p) + pf(v_p) \frac{\partial v_p}{\partial p} - pf(p) = 0.$$

Because v_p is determined by $c(v_p) = v_p - p$, we have that

$$\frac{\partial v_p}{\partial p} = \frac{1}{1 - c'(v_p)}.$$

If $c'(v_p)$ is equal to 1, then the derivative $\partial v_p / \partial p$ is infinite, and so the FOC above is not satisfied. \square

So far, I have established that $b_2(\cdot, p^m)$ is strictly decreasing in the neighborhood of v_p . The next step is to notice that there exists $\varepsilon > 0$ such that $\min_{v \in V_+ \cap \{v < v_p - \varepsilon\}} b_2(v, p^m) > b_2(v_p - \varepsilon, p^m)$ and $b_2(\cdot, p^m)$ is a strictly decreasing function on $[v_p - \varepsilon, v_p]$. Indeed, take any $\nu > 0$ such that $b_2(\cdot, p^m)$ is a decreasing function on $[v_p - \nu, v_p]$. Let b_ν be the minimum of $b_2(v, p^m)$ on the (compact) set $v \in V_+ \cap \{v \leq v_p - \nu\}$. By definition, $b_\nu > p^m$. If $b_\nu \geq b(v_p - \nu)$, we are done. If not, there exists $\nu_1 < \nu$ such that $b_{\nu_1} = b(v_p - \nu_1)$. Any $\varepsilon < \nu_1$ will suffice.

Next, notice that when l is sufficiently close to p^m there exists $\varepsilon > 0$ such that $\partial b_2(v, l) / \partial v < 0$ for $v \in [v_l - \varepsilon, v_l]$ and $b_2(v, l) > p^m$ otherwise. This follows from the result established in the previous paragraph and the fact that $c(\cdot)$ is continuous and $\partial b_2(v, l) / \partial l > 0$. In particular, since $h = p^m$ it means that bidders with $v \in V_+ \cap \{v \leq v_l - \varepsilon\}$ will bid h . Finally, let $v_{h,2} \in [v_l - \varepsilon, v_l]$ be such that $b_2(v_{h,2}, l) = p^m$. Such $v_{h,2}$ exists and is unique. Indeed, $b_2(v_l)$ is decreasing on $[v_l - \varepsilon, v_l]$, and its value at $v_l - \varepsilon$ is greater than p^m whereas its value at v_l is less than p^m . This proves the statement of the lemma as buyers with $v < v_{h,2}$ will bid h and buyers with $v > v_{h,2}$ will bid according to (10). \square

I have now established the following. Only buyers with $v \in [l, v_l]$ will submit their bids. Buyers with $v \in [v_{h,1}, v_{h,2}]$ will submit a bid equal to $h (= p^m)$, which will be accepted with probability 1. Bids of buyers with $v \in [l, v_{h,1}]$ will be determined by (8), and bids of buyers with $v \in [v_{h,2}, v_l]$ will be determined by (10).

Thus, the expected profit function is given by

$$\begin{aligned} & \frac{1}{G(h-l)} \int_l^{v_{h,1}} b_1(v) G(b_1(v) - l) f(v) dv \\ & + \frac{1}{G(h-l)} \int_{v_{h,2}}^{v_l} b_2(v) G(b_2(v) - l) f(v) dv + h(F(v_{h,2}) - F(v_{h,1})). \end{aligned}$$

I will take the derivative of the profit function with respect to l and will show that it is negative as $l \rightarrow p^m$, which means that instead of charging a price equal to p^m , the seller can do better by using the NYOP with limit $[l, p^m]$.

First, I will deal with the term

$$\frac{1}{G(h-l)} \int_l^{v_{h,1}} b_1(v) G(b_1(v) - l) f(v) dv - hF(v_{h,1}). \quad (14)$$

The value of $v_{h,1}$ is determined from the FOC $u(v_h - h) - \varphi(h-l)u'(v_h - h) = 0$. By an implicit function theorem,

$$\begin{aligned} \partial v_{h,1} / \partial l &= - \frac{\varphi'(h-l)u'(v_{h,1} - h)}{u'(v_{h,1} - h) - \varphi(h-l)u''(v_{h,1} - h)} \\ &= - \frac{\varphi'(h-l)}{1 + \varphi(h-l)RA(v_{h,1} - h)}. \end{aligned}$$

To find the limit of $\varphi(h-l)RA(v_{h,1}-h)$, I use the FOC to get that $\varphi(h-l) = u(v_{h,1}-h)/u'(v_{h,1}-h)$, and then

$$\begin{aligned} & \varphi(h-l)RA(v_{h,1}-h) \\ &= -\frac{u(v_{h,1}-h)}{u'(v_{h,1}-h)} \frac{u''(v_{h,1}-h)}{u'(v_{h,1}-h)} \\ &= RR(v_{h,1}-h) \frac{u(v_{h,1}-h)}{u'(v_{h,1}-h)(v_{h,1}-h)} \rightarrow \frac{RR(0)}{1-RR(0)}, \end{aligned}$$

where I used the fact that $v_{h,1}-h \rightarrow 0$ and L'Hôpital's rule. Therefore $\partial v_{h,1}/\partial l = -(1-RR(0))\varphi'(0)$.

The next step is to take the derivative of the integral that is equal to

$$\begin{aligned} & \left(\frac{1}{G(h-l)} \right)' \int_l^{v_{h,1}} b_1(v)f(v)G(b_1(v)-l)dv \\ &+ \frac{1}{G(h-l)} \left(\int_l^{v_{h,1}} b_1(v)f(v)G(b_1(v)-l)dv \right)'. \end{aligned}$$

By L'Hôpital's rule, the limit of the first part is equal to the limit of

$$-\frac{\left(\int_l^{v_{h,1}} b_1(v)f(v)G(b_1(v)-l)dv \right)'}{G(h-l)} \frac{1}{1+\varphi'(0)},$$

which is the limit of the second part divided by $-(1+\varphi'(0))$.⁸

As for the second part, after taking the derivative it becomes

$$\begin{aligned} & \frac{h \cdot f(v_{h,1})G(h-l)(\partial v_{h,1}/\partial l)}{G(h-l)} + \int_l^{v_{h,1}} \frac{\partial b_1}{\partial l} f(v) \frac{G(b_1(v)-l)}{G(h-l)} dv \\ &+ \frac{\int_l^{v_h} b_1(v)f(v)g(b_1(v)-l)(\partial b_1/\partial l - 1)dv}{G(h-l)}. \end{aligned}$$

The first term converges to $-p^m \cdot f(v_p)(1-RR(0))\varphi'(0)$, and the second converges to 0 because the expression inside the integral is bounded. Finally, since $(\partial b_1/\partial l - 1) = -\partial b_1/\partial v$ the limit of the last term is equal to $-p^m \cdot f(v_p)(\int_l^h g(b-l)db)/G(h-l) = -p^m \cdot f(v_p)$.

Combining terms, we get that at $l = p^m$,

$$\begin{aligned} & \frac{\left(\int_l^{v_{h,1}} b_1(v)f(v)G(b_1(v)-l)dv \right)'}{G(h-l)} \\ &= -p^m \cdot f(v_p)[1+\phi'(0)-RR(0)\phi'(0)]. \end{aligned}$$

Thus the derivative of the integral in (14) at $l = p^m$ is equal to

$$\begin{aligned} & -p^m \cdot f(v_p)[1+\phi'(0)-RR(0)\phi'(0)] \\ &+ p^m \cdot f(v_p) \frac{1+\phi'(0)-RR(0)\phi'(0)}{1+\phi'(0)}. \end{aligned} \quad (15)$$

⁸ One can write the derivative of $1/G(h-l)$ as $g(h-l)/G(h-l)^2 = 1/G(h-l)\phi(h-l)$. Applying L'Hôpital's rule would imply differentiating the numerator and denominator, which is $G(h-l)\phi(h-l)$. By definition of ϕ we will have that $[G(h-l)\phi(h-l)]' = -G(h-l) \cdot (1+\phi'(h-l))$.

The entire derivative of (14) is equal to (15) plus the derivative of $-h \cdot F(v_{h,1})$ and thus is equal to

$$-p^m \cdot f(v_p) + p^m \cdot f(v_p) \frac{1+\phi'(0)-RR(0)\phi'(0)}{1+\phi'(0)},$$

which is negative.

Next, I deal with the term

$$\frac{1}{G(h-l)} \int_{v_{h,2}}^{v_l} b_2(v)G(b_2(v)-l)f(v)dv + hF(v_{h,2}). \quad (16)$$

One thing that changes from the previous analysis is the limit of $\partial v_{h,2}/\partial l$. Applying the implicit function theorem to the FOC, I get that

$$\begin{aligned} & \frac{\partial v_{h,2}}{\partial l} \\ &= -\frac{\phi'(h-l)u'(v_{h,2}-h)}{u'(v_{h,2}-h) - u'(c(v_{h,2}))c'(v_{h,2}) - \phi(h-l)u''(v_{h,2}-h)}. \end{aligned}$$

When $l \rightarrow p^m$, then $\phi(p^m-l) \rightarrow 0$ and $v_{h,2} \rightarrow v_p$ and therefore $c(v_{h,2}) - (v_{h,2} - p^m) \rightarrow 0$. Thus,

$$\frac{\partial v_{h,2}}{\partial l} \Big|_{l=p^m} = \frac{\phi'(0)}{c'(v_p) - 1}.$$

As before,

$$\frac{1}{G(h-l)} \left(\int_{v_{h,2}}^{v_l} b_2(v)f(v)G(b_2(v)-l)dv \right)'$$

is equal to

$$\begin{aligned} & -\frac{h \cdot f(v_{h,2})G(h-l)\partial v_{h,2}/\partial l}{G(h-l)} + \int_{v_{h,2}}^{v_l} \frac{\partial b_2}{\partial l} f(v) \frac{G(b_2(v)-l)}{G(h-l)} dv \\ &+ \frac{\int_{v_{h,2}}^{v_l} b_2(v)f(v)g(b_2(v)-l)(\partial b_2/\partial l - 1)dv}{G(h-l)}. \end{aligned}$$

Again, the middle term converges to zero because the expression inside the integral is bounded. As for the last term, I will deal with it as follows. I will multiply and divide the expression inside the integral on the derivative of the bidding function with respect to v .

Then I use the fact that $b_2(v) \rightarrow p^m$, $f(v) \rightarrow f(v_p)$, and

$$\begin{aligned} & \frac{\partial b_2/\partial l - 1}{\partial b_2/\partial v} \\ &= -\frac{u'(v-b) - \phi(b-l)u''(v-b)}{u'(v-b) - u'(c(v))c'(v) - \phi(b-l)u''(v-b)} \rightarrow \frac{1}{c'(v_{h,2}) - 1} \end{aligned}$$

to get that

$$\begin{aligned} & \lim_{l \rightarrow p^m} \frac{\int_{v_{h,2}}^{v_l} b_2(v)f(v)g(b_2(v)-l)(\partial b_2/\partial l - 1)dv}{G(h-l)} \\ &= \lim_{l \rightarrow p^m} p^m f(v_p) \frac{1}{c'(v_p) - 1} \int_{v_{h,2}}^{v_l} \frac{g(b_2(v)-l)}{G(h-l)} \frac{\partial b_2}{\partial v} dv \\ &= -p^m f(v_p) \frac{1}{c'(v_p) - 1}. \end{aligned}$$

Therefore, we have that

$$\frac{1}{G(h-l)} \left(\int_{v_{h,2}}^{v_l} b_2(v) f(v) G(b_2(v)-l) dv \right)' \rightarrow -h f(v_h) \frac{1+\phi'(0)}{c'(v_{h,2})-1}.$$

Similar to the previous analysis, we get that

$$\begin{aligned} \lim_{l \rightarrow p^m} \left(\frac{1}{G(h-l)} \right)' \int_{v_{h,2}}^{v_l} b_2(v) f(v) G(b_2(v)-l) dv \\ = -\frac{1}{1+\phi'(0)} \lim_{l \rightarrow p^m} \frac{1}{G(h-l)} \left(\int_{v_{h,2}}^{v_l} b_2(v) f(v) G(b_2(v)-l) dv \right)' \end{aligned}$$

and is therefore equal to $(p^m \cdot f(v_p))/(c'(v_p)-1)$. Finally,

$$\frac{\partial h F(v_{h,2})}{\partial l} \rightarrow p^m f(v_p) \frac{\phi'(0)}{c'(v_p)-1},$$

and thus the limit of the derivative of (16) is equal to 0, and so the derivative of the entire profit is negative. Thus under conditions of Case 1, Proposition 4 holds.

Case 2. Now assume that when l is sufficiently close to p^m , v_l as well as v_p do not exist on the interval $[p^m, 1]$. This means that the set of bidders is given by $[l, 1]$ because even high-value bidders would submit the bid before trying the alternative option. If there is no v_0 , such that $c(v_0)=0$ then for all bidders $c(v)<0$, and this case is identical to the one considered in §3. Let v_0 be the lowest root of $c(\cdot)$ on the interval $(p^m, 1]$. Then, similar to Case 1, we can find $v_{h,1} < v_0$ so that bidders with $v \geq v_{h,1}$ will bid the highest bid h regardless of whether they belong to V_+ or V_- and bidders with $v < v_{h,1}$ will bid according to (8).⁹ The expected profit in this case is equal to

$$\frac{1}{G(h-l)} \int_l^{v_{h,1}} b_1(v) G(b_1(v)-l) f(v) dv + h(1-F(v_{h,1})), \quad (17)$$

and similar to Case 1, we can show that its derivative is negative when l gets sufficiently close to h . Thus Proposition 4 holds under conditions of Case 2 as well, which completes the proof. \square

PROPOSITION 5. *If the buyers' utility exhibits DARA and there is no alternative option available, then the posted price is redundant.*

PROOF. Consider an optimal NYOP+PP mechanism with support $[l, h]$ and posted price p . From the buyers' perspective, having the posted price is equivalent to having the outside option that has the value $c(v)=v-p$, and so their bidding behavior is still determined by (10). From the seller's perspective, however, the posted price is different from the outside option in that the posted-price option brings the profit to the seller.

The expected profit for the NYOP+PP case is given by

$$\begin{aligned} \pi(l, h, p) = \frac{1}{G(h-l)} \left[\int_l^p b_1(v) f(v) G(b_1(v)-l) dv \right. \\ \left. + \int_p^1 (b_2(v)-p) f(v) G(b_2(v)-l) dv \right] \\ + p(1-F(p)). \end{aligned}$$

⁹ Because there is no v_l , all bidders with $v > l$ will bid strictly higher than l , and therefore it is possible to cap the bidding function so that everyone except for $v \in [l, v_{h,1}]$ submits the highest possible bid.

First, optimal h cannot be a corner solution; that is, $h \neq l$. The reason is that when $h=l$, the seller's profit is equal to the posted-price profit and we already know that the seller can do better. Next, notice that it cannot be the case that $b(v) < h$ for any v . Indeed, the optimal level of h should satisfy the FOC

$$\frac{\partial \pi(l, h, p)}{\partial h} = 0.$$

If $b(v) < h$, then h affects the profit only via the term $1/G(h-l)$. Therefore, for $\partial \pi / \partial h$ to be equal to 0, the expression inside the square brackets should be equal to 0. But this is a contradiction because then the optimal profit would coincide with the posted-price profit.

To complete the proof of propositions, I will need the following lemma. The statement of the lemma is slightly more general than is needed for the proof because only the DARA case is relevant to Proposition 5.

LEMMA 10. When $c(v)=v-p$, the bidding function $b_2(v)$ —as determined by (10)—is decreasing when $u(\cdot)$ is DARA, increasing when $u(\cdot)$ is increasing absolute risk aversion (IARA), and constant when $u(\cdot)$ is CARA.

PROOF. From applying the implicit function theorem to the FOC, it follows that the sign of $\partial b / \partial v$ is equal to the sign of

$$u'(v-b) - u'(v-p) - \phi(b-l)u''(v-b). \quad (18)$$

From the FOC I can solve for $\phi(b-l)$ in terms of utilities. Substituting it into (18), I get

$$\begin{aligned} \text{sign} \frac{\partial b}{\partial v} &= \text{sign} \left\{ u'(v-b) - \frac{u(v-b)u''(v-b)}{u'(v-b)} \right. \\ &\quad \left. + \frac{u(v-p)u''(v-b)}{u'(v-b)} - u'(v-p) \right\} \\ &= \text{sign} \left\{ \left[u'(v-b) - \frac{u(v-b)u''(v-b)}{u'(v-b)} \right] \right. \\ &\quad \left. - \left[u'(v-p) - \frac{u(v-p)u''(v-b)}{u'(v-b)} \right] \right\} \\ &= \text{sign} \{ [u'(v-b) + u(v-b) \cdot RA(v-b)] \\ &\quad - [u'(v-p) + u(v-p) \cdot RA(v-b)] \}, \end{aligned}$$

where $RA(v-b)$ is the coefficient of absolute risk aversion at point $v-b$. Let $\psi(x)$ denote $u'(x) + u(x)RA(v-b)$ with v and b being fixed. Then

$$\text{sign} \frac{\partial b}{\partial v} = \text{sign} \{ \psi(v-b) - \psi(v-p) \}, \quad (19)$$

and therefore the sign of the derivative is positive if $\psi(x)$ is increasing and negative if $\psi(x)$ is decreasing on $[v-p, v-b]$.

The derivative of $\psi(x)$ is equal to $\psi'(x) = u''(x) + u'(x) \cdot RA(v-b)$ and so

$$\begin{aligned} \psi'(x) \geq 0 &\iff u''(x) \geq -u'(x) \left(-\frac{u''(v-b)}{u'(v-b)} \right) \\ &\iff RA(x) \leq RA(v-b). \end{aligned} \quad (20)$$

Therefore, when absolute risk-aversion is decreasing (increasing), so is $\psi(\cdot)$ and so is the bidding function on $[p, 1]$. \square

From Lemma 10, it follows that the maximum of the bidding function is reached at point $v=p$. In particular, it implies that $b(p)=h$. It will be convenient to rewrite the expected profit as

$$\begin{aligned}\pi(l, h, p) &= \int_l^p b_1(v) \frac{G(b_1(v)-l)}{G(h-l)} f(v) dv \\ &\quad + \int_p^1 b_2(v) \frac{G(b_2(v)-l)}{G(h-l)} f(v) dv \\ &\quad + \int_p^1 p \left(1 - \frac{G(b_2(v)-l)}{G(h-l)} \right) f(v) dv.\end{aligned}$$

The derivative with respect to p is then equal to

$$\begin{aligned}\frac{\partial \pi(l, h, p)}{\partial l} &= b_1(p) \frac{G(b_1(p)-l)}{G(h-l)} f(p) - b_2(p) \frac{G(b_2(p)-l)}{G(h-l)} f(p) \\ &\quad + \int_p^1 \left[\frac{\partial b_2}{\partial p} \frac{G(b_2(v)-l)}{G(h-l)} + b_2(v) \frac{g(b_2(v)-l)}{G(h-l)} \frac{\partial b_2}{\partial p} \right] f(v) dv \\ &\quad - p \left(1 - \frac{G(b_2(p)-l)}{G(h-l)} \right) f(p) \\ &\quad + \int_p^1 \left[1 - \frac{G(b_2(v)-l)}{G(h-l)} - p \frac{g(b_2(v)-l)}{G(h-l)} \frac{\partial b_2}{\partial p} \right] f(v) dv.\end{aligned}$$

The two first terms cancel, and because $b(p)=h$, what is left is

$$\begin{aligned}&\int_p^1 \left[\frac{\partial b_2}{\partial p} \frac{G(b_2(v)-l)}{G(h-l)} + b_2(v) \frac{g(b_2(v)-l)}{G(h-l)} \frac{\partial b_2}{\partial p} \right] dv \\ &\quad + \int_p^1 \left[1 - \frac{G(b_2(v)-l)}{G(h-l)} - p \frac{g(b_2(v)-l)}{G(h-l)} \frac{\partial b_2}{\partial p} \right] dv \\ &= \int_p^1 \frac{\partial b_2}{\partial p} \left[\frac{G(b_2(v)-l)}{G(h-l)} + b_2(v) \frac{g(b_2(v)-l)}{G(h-l)} - p \frac{g(b_2(v)-l)}{G(h-l)} \right] dv \\ &\quad + \int_p^1 \left(1 - \frac{G(b_2(v)-l)}{G(h-l)} \right) dv.\end{aligned}$$

Charging the posted price is redundant if, for any $v \in [p, 1]$, the buyer will submit the bid equal to h . In what follows, I will show that if this is not the case, then $\partial \pi / \partial p > 0$, and the seller will benefit from increasing the price until it becomes redundant.

Indeed, assume that there exists v such that $b_2(v) < h$. In this case the last integral is strictly positive and the derivative $\partial b_2 / \partial p$ is nonnegative. Thus, if

$$G(b(v)-l) + b(v)g(b(v)-l) - pg(b(v)-l) > 0,$$

or equivalently,

$$b(v) - p + \phi(b(v)-l) > 0,$$

then $\partial \pi / \partial p > 0$.

The FOC for the bidding function on interval $[p, 1]$ is

$$u(v-b) - u(v-p) - \phi(b-l)u'(v-b) = 0,$$

which can be rewritten as

$$u'(\xi)(p-b) - \phi(b-l)u'(v-b) = 0,$$

where $\xi \in (v-b, v-p)$. From the fact that $u(\cdot)$ is a concave function and $\xi < v-b$, it follows that $u'(\xi) > u'(v-b)$. Because, $b < p$, we have that

$$b(v) - p + \phi(b-l) > 0.$$

This completes the proof. \square

PROPOSITION 6. Let (l, h, p) be the optimal triple for the NYOP+PP mechanism. Whether the posted price is redundant or not depends on how close $c'(v_p)$ is to 1. It is not redundant when $c'(v_p)$ is close to 1, and it is redundant if $c'(v_p)$ is sufficiently far from 1.

PROOF. When both posted-price and alternative options are available, there are several cases regarding buyers' behavior, particularly with regard to the set of bidders and which option rejected bidders prefer. In what follows I will consider the case when $c(v) < 0$ if $v \in [l, p]$. The remaining cases are similar.¹⁰ When $v \in [l, p]$ rejected bidders use neither the posted-price nor the alternative option, the rejected buyers with $v \in [p, v_p]$ use the posted price, and when $v \in [v_p, v_i]$, the rejected buyers will prefer the alternative option. In this case, the expected profit is equal to

$$\begin{aligned}\pi(l, h, p) &= \int_l^p b_1 \frac{G(b_1-l)}{G(h-l)} dv + \int_p^{v_p} b_3 \frac{G(b_3-l)}{G(h-l)} dv \\ &\quad + \int_p^{v_p} p \left(1 - \frac{G(b_3-l)}{G(h-l)} \right) dv + \int_{v_p}^1 b_2 \frac{G(b_2-l)}{G(h-l)} dv,\end{aligned}$$

where b_3 denotes bids of those buyers who prefer the posted price to the alternative option. The first term is the profit received from bidders with $v \in [l, p]$ who do not use either the posted-price or the alternative option; the second term is from bidders with $v \in [p, v_p]$ if their bid is accepted, and the third term is the profit from the same bidders if their bid gets rejected and they use the posted price. Finally, the last term comes from the bidders with $v > v_p$ who use the alternative option if their bid is rejected.

The profit derivative with respect to p is equal to

$$\begin{aligned}\frac{\partial \pi}{\partial p} &= b_1(p) \frac{G(b_1(p)-l)}{G(h-l)} - b_3(p) \frac{G(b_3(p)-l)}{G(h-l)} \\ &\quad + \frac{\partial v_p}{\partial p} b_3(v_p) \frac{G(b_3(v_p)-l)}{G(h-l)} - p \left(1 - \frac{G(b_3(p)-l)}{G(h-l)} \right) \\ &\quad + \frac{\partial v_p}{\partial p} p \left(1 - \frac{G(b_3(v_p)-l)}{G(h-l)} \right) - \frac{\partial v_p}{\partial p} b_2(v_p) \frac{G(b_2(v_p)-l)}{G(h-l)} \\ &\quad + \int_p^{v_p} \frac{\partial b_3}{\partial p} \left[\frac{G(b_3-l)}{G(h-l)} + b_3 \frac{g(b_3-l)}{G(h-l)} - p \frac{g(b_3-l)}{G(h-l)} \right] dv \\ &\quad + \int_p^{v_p} \left(1 - \frac{G(b_3-l)}{G(h-l)} \right) dv.\end{aligned}$$

When the utility function is DARA, then the maximum is reached at point $v=p$. We know this because at interval $[p, v_p]$ the function is decreasing. Without the alternative

¹⁰ Similarity will come from the fact that the term $\partial v_p / \partial p$ is the only potentially unbounded term and will always enter $\partial \pi / \partial p$ negatively. The only exception is the case when v_p does not exist at all. In this case, the two options do not compete at all, and the analysis coincides with that of Proposition 5.

option the bidding function would continue to decrease, that is, without the alternative option $b(v) < b(v_p)$. The alternative option is *even more* attractive than the posted price; thus, those buyers will bid even less. Therefore, it is still the case that $b(v) < b(v_p)$ when $v > v_p$. Because there are bidders for whom h is binding, it has to be the case that $b(p) = h$. Therefore the expression for the profit simplifies to

$$\begin{aligned} \frac{\partial \pi}{\partial p} &= \frac{\partial v_p}{\partial p} p \left(1 - \frac{G(b_3(v_p) - l)}{G(h - l)} \right) \\ &+ \int_p^{v_p} \frac{\partial b_3}{\partial p} \left[\frac{G(b_3 - l)}{G(h - l)} + b_3 \frac{g(b_3 - l)}{G(h - l)} - p \frac{g(b_3 - l)}{G(h - l)} \right] dv \\ &+ \int_p^{v_p} \left(1 - \frac{G(b_3 - l)}{G(h - l)} \right). \end{aligned}$$

The last two terms are positive, and everything depends on the derivative of $\partial v_p / \partial p$, which is equal to

$$\frac{\partial v_p}{\partial p} = - \frac{1}{c'(v_p) - 1}.$$

Given our assumptions, $c(v)$ intersects $v - p$ from below. This means that $c'(v_p) > 1$, which means that the sign is negative. Thus, the derivative of the profit with respect to the price can be negative when $c'(v_p)$ is very close to 1. Intuitively, that means that the alternative option is almost a perfect substitute for the posted price. Indeed, in this case, by slightly increasing the price the seller would lose a large group of buyers who would go for the alternative option if their bids are rejected. At the same time decreasing p would lure a large group of customers whose bids got rejected into using the posted price instead of the alternative option. \square

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