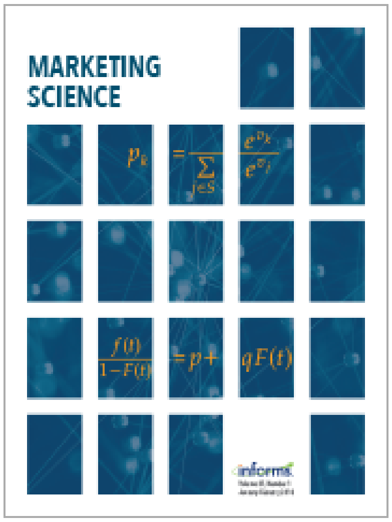


Marketing Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Research Note—Quantity Discounts in Differentiated Consumer Product Markets

Ramanathan Subramaniam, Esther Gal-Or,

MARKETING SCIENCE
Vol. 28, No. 1, January–February 2009, pp. 180–192
ISSN 0732-2399 | EISSN 1526-548X | 09 | 2801 | 0180

DOI 10.1287/mksc.1080.0381

Research Note

Quantity Discounts in Differentiated Consumer Product Markets

Ramanathan Subramaniam
School of Business, University of Kansas, Lawrence, Kansas 66045, srama@ku.edu

Esther Gal-Or
Katz Graduate School of Business, University of Pittsburgh, Pittsburgh, Pennsylvania 15260, esther@katz.pitt.edu

In this paper, we extend the standard Hotelling model of product differentiation to incorporate a second dimension of consumer heterogeneity that relates to the quantity of the product consumers wish to buy. This extension allows us to derive optimal nonlinear pricing rules chosen by competing sellers when offering differentiated products in the marketplace. It also permits us to assess whether sellers find it optimal to offer quantity discounts in such a setting, and the implications of such discounts on their profitability. We find that offering quantity discounts corresponds, indeed, to equilibrium behavior. The extent of discounting declines the less differentiated the products. Surprisingly, when sellers offer to consumers a choice between two different-sized packages, their profits are, at most, as high as when such a choice is unavailable. Moreover, when utilizing nonlinear pricing rules is not feasible, the profits of the sellers actually decline when they offer consumers a choice between different-sized packages. A limited empirical investigation supports the comparative statics we derive in our theoretical model.

Key words: price discrimination; game theory; nonlinear pricing; pricing research; competition
History: Received: April 3, 2007; accepted: December 13, 2007. Published online in *Articles in Advance* July 3, 2008.

1. Introduction

Offering quantity discounts for high-volume purchases is a marketing strategy widely used by both retailers and manufacturers. It can effectively segment consumers according to their intensity of consumption to facilitate more successful price discrimination among heterogeneous consumers. Our objective in the present paper is to investigate the optimality of this strategy in a market that consists of competing firms offering differentiated products. With this objective in mind, we extend the standard Hotelling (1929) model of product differentiation to incorporate a second dimension of consumer heterogeneity that relates to the quantity of the product consumers wish to buy. This extension allows us to derive optimal nonlinear pricing rules chosen by competing sellers when offering differentiated products in the marketplace. It also permits us to assess whether sellers find it optimal to offer quantity discounts in such a setting, and the implications of such discounts on their profitability.

In the model we develop, the population of consumers is heterogeneous along two dimensions. The horizontal dimension measures the relative preference of the consumers between the two sellers and the vertical dimension measures the marginal utilities of the consumers from increasing their levels of consumption. The strategy of each seller consists of his location choice along the horizontal axis and the prices he charges for two different-sized packages that he offers for sale. We restrict the characterization of the equilibrium to mature markets, where the entire population of consumers is active. Consistent with the result derived in the standard Hotelling model (d'Aspremont et al. 1979), sellers choose to maximally differentiate their products. They also offer quantity discounts on the larger-sized package at the equilibrium. This discount declines the less differentiated the products and the stickier the preferences of the consumers are. The measure of preference stickiness that we introduce in the paper relates to the extent of brand loyalty of heavy versus light users of the product. When the preferences of the consumers are relatively sticky, we find that the extent of quantity discount declines when the reservation price of the consumers and/or the unit cost of production are higher.

Surprisingly, when sellers offer a choice to consumers between two different-sized packages, their profits are, at most, as high as in an environment where such a choice is unavailable. Moreover, if utilizing nonlinear pricing is not feasible, due to the existence of secondary markets for the products, for instance, sellers are actually worse off when the added flexibility between different-sized packages is provided to consumers. The practice of nonlinear pricing is needed in order to just maintain the level of profits that accrue to the sellers in the absence of choice. It appears that in a mature market, where the entire population of consumers is served, the competition between the sellers for consumers having moderate tastes is so fierce that consumers are able to extract the entire added surplus that becomes available as a result of extended choice. Sellers are unable to retain any portion of this added surplus in the form of higher profits.

The comparative statics we derive in our theoretical model are supported by a regression analysis we conducted using a sample of 92 different food items sold in stores in the Kansas City area. We found, for instance, that the average discount offered on large packages of processed food items such as cakes, pizzas, and cereal is higher than on packages of unprocessed food items such as fruits, vegetables, eggs, and milk. Since the extent of differentiation in processed food markets is likely to be higher than in unprocessed food markets, this observation is consistent with our theoretical prediction.

There are many studies in the marketing literature that address issues related to quantity discounts. Most of those studies address, however, the role quantity discounts play in improving channel coordination in the relationship between producers and their retailers (see, for instance, Jeuland and Shugan 1983, 1988; McGuire and Staelin 1983; Coughlan 1985; Moorthy 1987; Coughlan and Mantrala 1994; Padmanabhan and Png 1997; Chintagunta et al. 2000; Raju and Zhang 2005; Liu and Zhang 2006; and Rust and Chung 2006). In contrast, our emphasis in the present paper is on direct sales from sellers to consumers, and the role quantity discounts play in improved market segmentation of the population of consumers. Allenby et al. (2004) address a similar issue empirically, by considering the optimal choice of consumers among a discrete number of pack sizes in a market of strong substitutes. Similarly, Lewis et al. (2006) conduct an empirical study to investigate the impact discounts in shipping charges for high volume purchases have on the order incidence and order size of consumers. In the above two papers, though, the prices chosen by the sellers are exogenously determined. In contrast, in our theoretical model, the nonlinear pricing strategies are determined endogenously as part of equilibrium behavior.[1]

In the economics literature there are many studies that pertain to nonlinear pricing. Most of these studies relate, however, to monopolistic markets (see, for instance, Oi 1971, Leland and Meyer 1976, Murphy 1977, Spence 1977, and Oren et al. 1982). All these studies demonstrate the optimality of nonlinear pricing as a vehicle to support price discrimination among heterogeneous consumers. In an extension to their earlier work, Oren et al. (1983, 1984) introduce competition among firms in addressing the incentives to utilize nonlinear pricing. However, their focus is primarily on competitors who offer homogenous products. In contrast, in our extended Hotelling model, sellers offer differentiated products. Their result that nonlinear pricing collapses and converges to marginal cost pricing as the number of firms is infinitely large is consistent with the result we obtain when the degree of differentiation between the sellers disappears. Three other studies in the economics literature that consider nonlinear pricing in a competitive setting are Spulber (1989), Stole (1995), and Amstrong and Vickers (2001). In those studies, however, consumers can choose from a continuum of feasible quantity levels, and sellers can smoothly adjust their pricing over such a continuum. We consider a more realistic environment, where consumers have a choice between a discrete number of package sizes. This formulation allows us to conduct a comparative static analysis that the earlier formulation cannot support. We investigate, in particular, how changes in the consumers' preferences, the degree of differentiation between products, and unit production cost affect the extent of quantity discounts and profitability of the sellers.

Another study in the economics literature that addresses, like us, price discrimination in the Hotelling model is by Thisse and Vives (1988, henceforth referred to as T&V). However, in their model the heterogeneity of consumers is unidimensional and each consumer buys a single unit at a price that depends upon her location on the line. In contrast, in our setting consumers' heterogeneity is bidimensional and each can choose between two different package sizes. Since the larger-sized package can be considered a bundle of multiple units, our paper is also indirectly related to the literature on bundling in a competitive setting (see, for instance, Matutes and Regibeau 1992). However, in contrast to the papers on bundling, where the components that comprise the bundle are each horizontally differentiated, in our model all consumers value the larger package more than the smaller one. They differ, however, in their willingness to pay for the additional unit of consumption. Hence, the bidimensional heterogeneity of consumers in our model has an element similar to models of vertical differentiation, where consumers

[1] Another paper that is related to our analysis is by Kumar and Rao (2006). In contrast to our focus on segmentation that is facilitated by offering multiple package sizes of a given product, Kumar and Rao focus on segmentation supported by baskets comprising of multiple products.

have different valuations for higher quantity, but they all agree that higher quantity is better.

The rest of this paper is organized as follows. In §2 we describe the main assumptions of the model. In §3, we derive the equilibrium when the extent of disutility experienced when buying a product with characteristics different from those the consumer prefers best (a measure of brand loyalty of the consumers that is referred to in the Hotelling model as transportation costs) is independent of the volume of consumption. We refer to this environment as nonsticky preferences of the consumers. In §4, we derive the equilibrium with sticky preferences, which in the context of our model imply that the extent of disutility (transportation costs) increases with volume. Section 5 reports the results of a regression analysis we conducted, and §6 concludes the paper. The proofs of the propositions and some extensions are included in two appendices. Numerical calculations for the case that preferences are sticky and a detailed description of our regression analysis are included in two Technical Appendices, available at http://mktsci.pubs.informs.org.

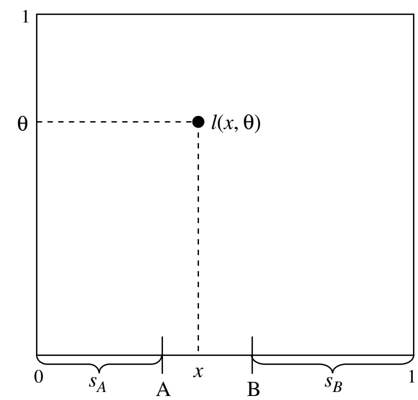
2. Model

Consider a market that consists of two competing sellers offering differentiated products. Consumers differ in their relative preference for the products of the two sellers as well as in the marginal benefit they derive from increased consumption of the product. We assume that the level of consumption is not perfectly divisible. Specifically, there are only two feasible levels that are available for sale from each firm. We refer to those two levels as packages containing one unit or two units of the product.[2]

We describe the preferences of a representative consumer in terms of her location inside a square having an area of one unit. The horizontal axis captures the location of the consumer relative to that of the two sellers (denoted as x), as in the regular Hotelling model. The vertical axis extends the Hotelling model by introducing a second dimension of heterogeneity. This second dimension measures the extent of diminishing marginal utility experienced by the consumer when increasing her consumption (denoted as θ). Each axis is normalized to have a maximum value of one unit.

In Figure 1 the locations of the two sellers are denoted as points A and B on the horizontal axis. Firm A is located at distance s_A from the left-end corner and Firm B is located at distance s_B from the right-end corner of the horizontal axis. Each consumer is designated by her location $l(x, \theta)$, where x is the horizontal displacement and θ is the vertical displacement of the consumer's location. The coordinate x determines, therefore, the relative preference of the consumer between the products of the two sellers. The coordinate θ measures the extent of diminishing marginal utility experienced by the consumer when she increases her consumption from one to two units. Specifically, we assume that the reservation price the consumer is willing to pay for one unit of her "ideal" product is r and her reservation price for two units of her "ideal" product is $r(2-\theta)$. Hence, a bigger value of θ implies a higher degree of diminishing marginal utility[3] derived from the second unit consumed. When $\theta = 0$, the marginal utility of the second unit is equal to that of the first unit (and equal to r), implying that the consumer does not experience any diminishing marginal utility in this case. When $\theta = 1$, the consumer experiences the highest possible level of diminishing marginal utility, since the second unit does not add any benefit to the consumer in this case.[4] We assume that consumers are uniformly distributed according to their location inside the square.[5]

Figure 1 **Distribution of Preferences of the Consumers**



[2] In this paper we do not address the issue of allowing firms to strategically choose the sizes of the packages offered. Instead, the two sizes are assumed to be exogenously determined.

[3] A bigger value of θ may also capture the additional inventory costs of buying in large quantities.

[4] The assumption that all consumers have a common reservation price r for the first unit of consumption is consistent with the standard assumption of the uni-dimensional Hotelling model. Relaxing this assumption would significantly complicate the analysis, since the heterogeneity of consumers would be three-dimensional as a result. Since our objective in the paper is to investigate the optimality of quantity discounts, we choose to introduce the second dimension of heterogeneity with regard to the degree of diminishing marginal utility of consumers. This type of heterogeneity yields different levels of consumption by consumers at the equilibrium.

[5] The assumption that the length of both the horizontal and vertical axes of the box in Figure 1 is the same and equal to one is made without any loss of generality. It is possible to scale both the extent of product differentiation and the degree of diminishing marginal utility, so that each is restricted to the unit interval. Since the horizontal coordinate of the consumer's location is assumed to be independent of her vertical coordinate, the qualitative results of the paper remain the same irrespective of the particular scale chosen.

The decision of the consumer consists of two components: which product to consume and the quantity of consumption. We designate the four options open to the consumer as $\{1_A, 2_A, 1_B, 2_B\}$. Thus choosing 1_A (1_B) implies that the consumer buys a package of one unit from seller A (B), and choosing 2_A (2_B) implies that she buys a package of two units of A's (B's) product, respectively.[6]

The utility of the consumer is obtained by subtracting from her reservation price (r or $r(2-\theta)$) the disutility she experiences by buying a product whose characteristics differ from those of her "ideal" product. In the context of our location model, this disutility is captured by the transportation costs incurred by the consumer. We assume these costs to be a quadratic function[7] of the horizontal distance between the consumer's and the firm's locations. Moreover, we initially assume that the transportation costs incurred by the consumer are independent of the number of units she consumes[8] (a similar assumption is made in Armstrong and Vickers 2001). In §4 we modify this assumption and allow the transportation costs to increase with the quantity consumed.

With transportation costs that are independent of the number of units, the disutility experienced by a consumer located at horizontal distance x is equal to $t_A(x) = (x - s_A)^2$ when buying from A, and $t_B(x) = (1 - s_B - x)^2$ when buying from B. If $m(x, \theta)$ denotes the consumption choice of the consumer, then her utility is given as follows:

$$U(m, x, \theta) = \begin{cases} r - (x - s_A)^2 & \text{when } m = 1_A, \\ r(2-\theta) - (x - s_A)^2 & \text{when } m = 2_A, \\ r - (1 - s_B - x)^2 & \text{when } m = 1_B, \\ r(2-\theta) - (1 - s_B - x)^2 & \text{when } m = 2_B. \end{cases} \tag{1}$$

[6] We do not consider the possibility that the consumer diversifies her consumption by buying one unit of product A and one unit of product B, since such a choice can never arise, given that sellers find it optimal to offer quantity discounts at the equilibrium.

[7] Quadratic transportation costs guarantee the existence of an equilibrium in pure strategies when sellers can choose their locations strategically (see d'Aspremont et al. 1979).

[8] If the distance between the consumer's and seller's locations corresponds to a geographic distance, then the assumption that the transportation costs are independent of quantity implies that most of the transport cost is fixed in nature. If the distance captures the dissatisfaction of the consumer from buying a product whose characteristics differ from her "first choice," this assumption implies the same level of dissatisfaction irrespective of whether the consumer buys one or two units of the product. Essentially, heavy users of the product have the same degree of brand loyalty as light users of the product.

Each seller chooses the prices of the two different packages he offers, designated by $p(m)$. It is clear that $p(2_i) \le 2p(1_i)$ for $i = A, B$, since otherwise no consumer will ever find it optimal to purchase the two-unit package. If $p(2_i) > 2p(1_i)$, the consumer who wishes to buy two units of product i will choose to buy two packages, each containing one unit, instead of buying the two units in a single package. Hence, without loss of generality we can restrict attention to the case that the sellers offer discounts on the larger volume of consumption. We assume that the sellers incur the cost c per unit produced.

Each consumer chooses her consumption bundle m to maximize her net utility. The implication of this maximization on the choice of the consumer is reported in Lemma 1 and depicted in Figure 2. Without loss of generality we describe the choice of the consumer only for the case where[9] $[p(1_B) - p(1_A)] \ge [p(2_B) - p(2_A)]$. This restriction guarantees that the boundaries of the different regions in Figure 2 satisfy the inequalities $x^* \ge x^{**}$ and $\theta_B^* \ge \theta_A^*$ (for the exact expressions of the threshold levels described in Figure 2, see Appendix A).

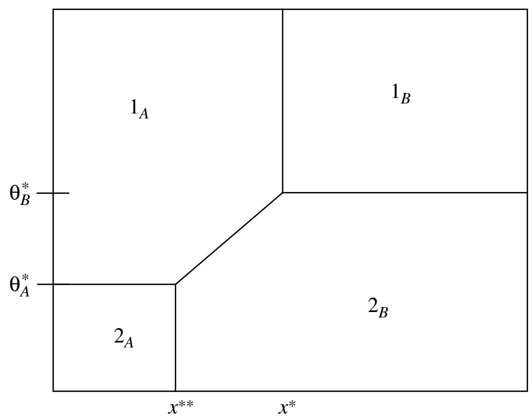
Lemma 1.
If $\theta \le \theta_A^*$, $m = 2_A$ if $x \le x^{**}$ and $m = 2_B$ if $x > x^{**}$.
If $\theta > \theta_B^*$, $m = 1_A$ if $x \le x^*$ and $m = 1_B$ if $x > x^*$.
If $\theta_A^* < \theta \le \theta_B^*$, $m = 1_A$ if $x \le \hat{x}(\theta)$ and $m = 2_B$ if $x > \hat{x}(\theta)$, where $\hat{x}(\theta)$ is the boundary separating regions 1_A and 2_B.

Before we interpret the results reported in Lemma 1 it is important to clarify that even when focusing on the derivation of symmetric equilibria, it is essential to characterize the choice of the consumers when sellers behave in an asymmetric manner. For the symmetric equilibrium to prevail, each seller should have no incentive to deviate. A unilateral deviation from symmetric behavior yields asymmetry between the sellers, and it is essential to understand, therefore, how consumers react to such asymmetry. This is the reason Lemma 1 and its depiction in Figure 2 characterizes the possible asymmetric outcome when $p(1_B) - p(1_A) \ge p(2_B) - p(2_A)$, even though at the equilibrium this inequality holds as an equality.

According to Lemma 1, if the consumer experiences very limited diminishing marginal utility (i.e., $\theta \le \theta_A^*$), she definitely buys the two-unit package. Her choice between the two sellers depends on her relative proximity to the locations of the two sellers and their relative prices. On the other hand, if she experiences significant diminishing marginal utility (i.e., $\theta > \theta_B^*$), she buys the single-unit package. Her choice

[9] In the sequel we show that at the equilibrium $p(1_B) - p(1_A) = p(2_B) - p(2_A)$. Hence, the derivation of the equilibrium would remain the same if we started with the reverse inequality.

Figure 2 **Optimal Choice of the Consumer** ($p(1_B) - p(1_A) \geq p(2_B) - p(2_A)$)



of the seller depends, once again, upon the relative locations of the sellers and their prices. For intermediate values of diminishing returns (i.e., $\theta_A^* < \theta \leq \theta_B^*$), the consumer chooses between the single-unit package of seller A and the two-unit package of seller B depending[10] upon the values of both the horizontal and vertical coordinates of her location. In particular, the boundary that separates the region of consumers who choose 1_A from those who choose 2_B is upward sloping ($\partial x/\partial\theta = r/2[1-(s_1+s_2)]$ along the boundary).

3. Derivation of the Equilibrium—Transportation Costs Independent of Volume

We formulate the game as consisting of two stages. In the first stage, each seller chooses the characteristics of his product, which determine his location on the horizontal axis (s_A for A and s_B for B). In the second stage, each seller chooses the prices of the packages he offers for sale $\{p(1_i), p(2_i)\}$ for $i = A, B$. Subsequently, consumers decide which product and what level of consumption to purchase.[11]

Before characterizing the equilibrium of the second stage subgame, in Proposition 1 we first derive the equilibrium of the standard, unidimensional Hotelling model, where consumers are not given the flexibility to choose between two different-sized packages. Hence, irrespective of their vertical coordinate θ all consumers are forced to purchase a common-sized package.

[10] If we assumed that $p(1_A) - p(1_B) \geq p(2_A) - p(2_B)$, $\theta_B^* \leq \theta_A^*$ and the choice of the consumer for $\theta_B^* \leq \theta < \theta_A^*$ would be between 1_B and 2_A.

[11] We implicitly assume that both sellers offer both types of packages for sale. In particular, no seller finds it optimal to restrict his offering to a single-sized package. In Appendix B we discuss the necessary approach to support such an assumption as an equilibrium outcome.

Proposition 1. (i) *For fixed locations s_A and s_B and transportation costs that are independent of volume, if the sellers offer only single-unit packages for sale, equilibrium prices are*

$$p_i = c + (1 - s_i - s_j)\left(1 + \frac{s_i - s_j}{3}\right), \quad i, j = A, B;\ i \neq j. \quad (2)$$

If the sellers offer only two-unit packages for sale, equilibrium prices are

$$p_i = 2c + (1 - s_i - s_j)\left(1 + \frac{s_i - s_j}{3}\right), \quad i, j = A, B;\ i \neq j. \quad (3)$$

(ii) *The expected profits of seller i as a function of the location choices s_A and s_B are given as*

$$\pi_i(s_i, s_j) = \frac{1}{2}(1 - s_i - s_j)\left(1 + \frac{s_i - s_j}{3}\right)^2. \quad (4)$$

According to part (i) of Proposition 1, the profit margin of each seller remains the same irrespective of the size of the package he sells. When the market is fully covered, sellers end up competing over the threshold consumer who is just indifferent between their products, irrespective of whether this consumer buys one or two units of her preferred product. The margin the sellers are able to retain is the same either way. It depends solely on the extent of differentiation between the two products (the locations s_A and s_B). Expressions (2) and (3) also imply that the seller offers the two-unit package at a discount relative to the one-unit package.

Next, we return to our environment, where each seller offers two different-sized packages. Given the optimal choice of the consumers characterized in Lemma 1, we report in Proposition 2, that the equilibrium prices of the two different-sized packages remain the same as in the standard Hotelling model, where choice is unavailable.

Proposition 2. (i) *For fixed locations s_A and s_B, when sellers offer a choice to consumers between two different-sized packages, equilibrium prices remain the same as when choice is unavailable. Specifically, the expressions for $p(1_i)$ and $p(2_i)$ are given by Equations* (2) *and* (3), *respectively.*

(ii) *The extent of discount on the two-unit package can be expressed, therefore, as*

$$2p(1_i) - p(2_i) = (1 - (s_i + s_j))\left[1 + \frac{s_i - s_j}{3}\right]. \quad (5)$$

Proposition 2 reports that offering a choice between two different-sized packages to the consumers does not change the pricing behavior of the seller in comparison to the standard Hotelling model where choice is unavailable. In particular, the profit margin of the seller on either type of package is identical. It should be pointed out that this result is directly implied by the assumption that transportation costs are independent of volume. Because of this assumption when the threshold consumer chooses to increase her consumption from one to two units, she does not incur any additional transportation costs. This forces each seller to offer the second unit of the larger-sized package at marginal cost (i.e., $p(2_i) = p(1_i) + c$). In §4 we show that this result is not valid when transportation costs increase with volume. The expression for the discount on the two-unit package reported in part (ii) of the proposition indicates that the size of the discount declines when the extent of differentiation between the sellers diminishes (i.e., when they move closer to each other).

Propositions 1 and 2 suggest that the sellers do not benefit from the added flexibility they offer to consumers. It appears that the entire added benefit generated from allowing consumers to choose the size of the package they wish to purchase is transferred to the consumers, and the sellers are unable to retain any portion of this added benefit. To illustrate this point, in Proposition 3 we compare the consumer surplus when consumers have the flexibility to choose the quantity of consumption to the case when consumers do not have this flexibility (i.e., the regular Hotelling model).

Proposition 3. *Consumers benefit when they are given the flexibility to choose between two different-sized packages. This added benefit amounts to* $((r-c)^2/2r)[c^2/2r]$ *when compared to an environment with only single-unit* [*two-unit*] *packages available, respectively.*

Since the second stage expected profits of each seller in our environment, when choice is offered to the consumers is identical to his expected profits in the regular Hotelling model, it follows that "the principle of maximum differentiation" derived in the standard model (d'Aspremont et al. 1979) applies in our setting as well. We report this result in Proposition 4.

Proposition 4. *At the Nash equilibrium of the two-stage game, the sellers choose to maximally differentiate their products, namely* $s_A = s_B = 0$. *As a result, the expected profits of each seller and the expected consumer surplus are equal to* $E\pi = 1/2$ *and* $ECS = (r-c)\cdot(3/2 - c/2r) - 13/12$.

The result reported in Proposition 4 is not that obvious since the additional dimension of heterogeneity introduced in our model is used by the sellers to practice nonlinear pricing. Normally, such pricing allows sellers to more successfully engage in price discrimination and the extraction of consumer surplus. As a result, one could have predicted less concern on the part of each seller to move closer to the location of his competitor in order to increase the size of his "captive market." Our analysis demonstrates, however, that the sellers are still very much concerned about intensified competition for consumers who have moderate preferences between the products of the two sellers.

Next we demonstrate that the practice of nonlinear pricing is essential for the seller to be able to maintain the level of profits of the standard Hotelling model. Specifically, if sellers are unable to practice nonlinear pricing, because they cannot prevent the emergence of secondary markets for their products, for instance, offering a choice between two different-sized packages reduces the profits of each seller. Restricting attention to the symmetric case, in Proposition 5 we characterize the equilibrium with linear pricing (i.e., $p(2_i) = 2p(1_i)$).

Proposition 5. *When sellers cannot practice nonlinear pricing and are symmetrically located, so that* $s_A = s_B = s$, *if each offers a choice to consumers between two different-sized packages, the equilibrium price per unit can be expressed as follows*:

$$\begin{aligned}(p_L - c) &= \frac{[4r-3c+2(1-2s)] - \sqrt{(4r-3c)^2 + 4(1-2s)^2 - 8r(1-2s)}}{6} \\ &< (1-2s). \end{aligned} \tag{6}$$

The profits of each seller are $E\pi_L = (p_L - c)(1 - p_L/2r)$ *where* p_L *is given in* (6). *These profits fall short of the seller's profits with nonlinear pricing, which amount to* $(1-2s)/2$.

Note that in contrast to the solution derived with nonlinear pricing, the profit margin $(p_L - c)$ derived in (6) depends upon the reservation price of the consumer and the unit cost of production. With symmetric locations of the sellers, nonlinear pricing leads to a profit margin that is independent of r and c and is equal to $(1-2s)$ for either type of package. It is easy to show that the profit margin expressed in (6) is a decreasing function of s and an increasing function of c. Hence, as the degree of differentiation between the sellers declines, their profit margins decline as well. In contrast, as the unit cost of production increases, sellers benefit from larger profit margins. This last result is implied by the fact that a higher unit cost reduces the incentive of the sellers to serve consumers who are interested in buying two-unit packages (consumers having low θ values). They focus, instead, on the segment of consumers who purchase the single-unit package. This transforms the environment to more closely resemble the regular Hotelling model, where consumers are offered only single-unit packages. The markup on such packages in the regular Hotelling model is $(1-2s)$. Since from (6) the markup always falls short of $(1-2s)$, moving closer to the single-unit Hotelling model implies that the per unit markup has to rise. Note also that the markup in (6) is a nonmonotonic function of the reservation price r.

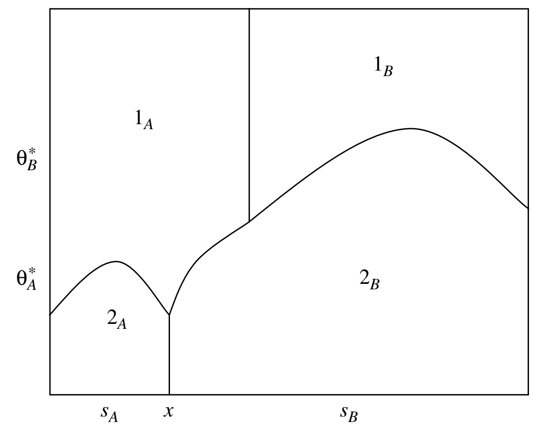
As was pointed out earlier, T&V consider the implication of using price discrimination in the standard, unidimensional Hotelling model. They demonstrate that when sellers can use personalized pricing, contingent upon the location of the consumer on the line, their equilibrium profits actually decline in comparison to uniform pricing. In contrast, in our setting, where the heterogeneity of consumers is bidimensional and price discrimination is implemented via the use of nonlinear pricing and not the location of the consumers on the horizontal axis, sellers can actually maintain (but not enhance) the level of profitability of the standard Hotelling model. However, according to Proposition 5, the practice of nonlinear pricing is essential in order to maintain this level of profitability. It is also noteworthy to mention that whereas T&V assume that sellers can actually observe the location of each consumer and base the discrimination in prices on this observable variable, we assume that the sellers cannot observe the diminishing utility parameter θ of different consumers. Instead, their pricing of the two different-sized packages induces self-selection by consumers between the two sizes available. The literature has referred to these two different types of discrimination as first-degree (T&V) versus second-degree (our model) price discrimination. We believe that our nonobservability assumption is more realistic than that in T&V.

4. Numerical Characterization of the Equilibrium—Transportation Costs Depend on Volume

In this section we offer a limited equilibrium analysis under the assumption that the transportation costs incurred by the consumer when buying the two-unit package are double the costs of buying the single unit package. Specifically, the disutility experienced by the consumer when buying a product whose characteristics are different from her "ideal" choice double as the consumer doubles her level of consumption. This assumption translates to the following utility function of the consumer:

$$U(m;x,\theta) = \begin{cases} r-(x-s_A)^2 & \text{when } m=1_A, \\ r(2-\theta)-2(x-s_A)^2 & \text{when } m=2_A, \\ r-(1-s_B-x)^2 & \text{when } m=1_B, \\ r(2-\theta)-2(1-s_B-x)^2 & \text{when } m=2_B. \end{cases} \quad (7)$$

Figure 3 **Optimal Choice of the Consumer with Volume Related Transportation Cost**



$[p(1_B)-p(1_A)] \geq [p(2_B)-p(2_A)]$

We start, once again, by characterizing the standard Hotelling model where consumers do not have a choice between two different-sized packages.

Proposition 6. *For fixed and symmetric locations of the sellers and transportation costs that are increasing with volume (sticky preferences): When sellers restrict the choice of the consumers by offering only single-unit packages,* $p_1^s = c+1-2s$, $E\pi^s = (1-2s)/2$. *When sellers restrict the choice of the consumers by offering only two-unit packages,*[12] $p_2^s = 2c+2(1-2s)$, $E\pi^s = 1-2s$.

Since the disutility from buying a product different from her "ideal" location increases with volume, the loyalty of the consumer to her preferred brand is higher when only two-unit packages are available. As a result, Proposition 6 reports that price competition is alleviated when only two-unit packages are available for sale, and the expected profits of each seller are higher.

Next we consider the case that sellers offer a choice between two different-sized packages. We assume, once again, that $[p(1_B)-p(1_A)] \geq [p(2_B)-p(2_A)]$, and describe the optimal choice of the consumer[13] in Figure 3 (for the exact derivation of the boundaries, see Technical Appendix 1, available at http://mktsci.pubs.informs.org).

[12] This is true only if the market remains completely covered; namely, the reservation price r is sufficiently big so that even a consumer with the highest value of diminishing marginal utility and transportation costs derives a positive surplus (i.e., $\theta=1$ and an x value equal to either 0 or 1/2). The constraint on r is, therefore, $r > 2(c+1-2s) + \text{Max}\{2s^2, 2(1/2-s)^2\}$.

[13] As pointed out earlier, we have to consider the choice of the consumer under asymmetric pricing by the sellers, even when our focus is on the derivation of symmetric equilibria. For a symmetric equilibrium to exist, each seller should not benefit from unilateral deviation. Such a deviation results in asymmetry.

Table 1 **Comparison of Sticky and Nonsticky Preferences—Fixed $r = 6$**

c	p_1^s	p_2^s	$D^s = p_1^s - p_2^s/2$	p_1^{NS}	p_2^{NS}	$D^{NS} = p_1^{NS} - p_L^{NS}/2$	$E\pi^s$	$E\pi_1^{NS}$
0	1.4190	1.8862	0.476	1	1	0.5	0.9217	0.5
0.5	1.7927	2.83871	0.3733	1.5	2	0.5	0.86797	0.5
0.75	2.0047	3.32016	0.3446	1.75	2.5	0.5	0.8442	0.5
1	2.225	3.8029	0.3235	2	3	0.5	0.8214	0.5
1.25	2.451	4.2861	0.3077	2.25	3.5	0.5	0.7994	0.5
1.5	2.6805	4.7694	0.2958	2.5	4	0.5	0.7781	0.5

Since the boundaries separating the different regions are not linear anymore (in contrast to Figure 2), the derivation of the equilibrium is far more cumbersome. As a result, we are unable to fully characterize the equilibrium analytically. In Technical Appendix 1, available at http://mktsci.pubs.informs.org, we derive the first-order conditions that characterize the second-stage pricing game. We solve those conditions numerically only for the case $s_A = s_B = 0$; namely, when sellers are maximally differentiated. Maximum differentiation was established as the Nash equilibrium of the two-stage game when transportation costs were assumed to be independent of volume. For the sake of comparison with this case, we restrict the numerical calculation to maximum differentiation here as well.[14] Tables 1 and 2 include the results of the numerical calculations. Table 1 provides the characterization of the equilibria for different values of the unit cost c, and Table 2 provides the characterization for different values of the reservation price r. For comparison, we also include in the tables the results derived in the previous section, when transportation costs were independent of volume. We use the superscript S and NS to designate sticky and nonsticky preferences of the consumers, and the variable D designates the size of the discount on the two-unit package. To guarantee that the market is fully covered, we assume that the reservation price of the consumers is sufficiently large (specifically, $r > p_1^s + 0.5$).

According to Tables 1 and 2, when transportation costs increase with volume, the profit margin on the two-unit package is bigger than the margin on the one-unit package ($p_2^s - 2c > p_1^s - c$). This result is similar to that reported in Proposition 6 for the standard Hotelling model without choice. Note also that when choice between two different-sized packages is offered to the consumers, the expected profits of each seller are lower than his profits when only two-unit packages are available, but higher than his profits when only one-unit packages are available ($0.5 < E\pi^s < 1$). Moreover, as the unit cost of production declines and/or the reservation price of the consumers increases, expected profits increase and get closer to the level of profits derived by each seller in the regular Hotelling model where choice is unavailable and only the bigger-sized packages are offered for sale ($E\pi^s \to 1$ as r increases and c decreases). When c declines or r increases a bigger segment of consumers chooses to buy the two-unit packages. Since such consumers experience higher disutility by defecting to the competing product, sellers can raise prices and increase profits. The situation is reversed if c increases or r decreases, in which case a bigger segment of consumers chooses the single-unit packages. Such consumers are less loyal to their preferred brand, thus leading to intensified price competition and reduced profits (as c increases and r declines, $E\pi^s \to 0.5$, which is the profit derived when only single unit packages are available for sale). Tables 1 and 2 also illustrate that the extent of quantity discount is smaller when the preferences are sticky. Moreover, with sticky preferences the size of the quantity discount declines as both c and r increase. Discounts are independent of c and r with nonsticky preferences.

The above discussion suggests also that sellers may actually be worse off by extending the choice they offer to consumers. Even when practicing nonlinear pricing, the expected profits of the sellers may decline when multiple-sized packages are available for sale instead of only the bigger-sized packages. This result is different from the one derived in the previous section. When transportation costs are determined independently of volume, the profits of the seller remain the same irrespective of whether or not he offers a choice between two different-sized packages, as long as he can practice nonlinear pricing. The results we derive with sticky preferences are similar to that derived in T&V, where the practice of price discrimination leads to lower profits of competing firms. However, T&V derive their result in a model where discrimination is based upon the observable location of the consumer (first-degree price discrimination), whereas we establish it when discrimination is based upon the choice of the consumer (second-degree price discrimination, based upon quantity of consumption).

It is important to note that in spite of the fact that producers are worse off when both extend their offerings and consumer preferences are sticky, they may nevertheless be forced to do it at the equilibrium.

[14] We cannot prove analytically that maximum differentiation remains an equilibrium with sticky preferences.

Table 2 Comparison of Sticky and Nonsticky Preferences—Fixed $c = 5$

r	p_1^s	p_2^s	$D^s = p_1^s - p_2^s/2$	p_1^{NS}	p_2^{NS}	$D^{NS} = p_1^{NS} - p_L^{NS}/2$	$E\pi^s$	$E\pi_1^{NS}$
10	6.0849	11.7538	0.2079	6	11	0.5	0.6845	0.5
20	6.1052	11.9087	0.15086	6	11	0.5	0.8360	0.5
30	6.1097	11.9442	0.1377	6	11	0.5	0.8898	0.5
40	6.1117	11.9598	0.1318	6	11	0.5	0.9170	0.5
50	6.1128	11.9686	0.1285	6	11	0.5	0.9335	0.5
200	6.1159	11.9927	0.1196	6	11	0.5	0.9833	0.5

Extending the mix offered to consumers may arise as equilibrium behavior if such behavior constitutes a "best response" strategy given that the competitor chooses to do the same. While we are unable to analytically prove whether or not producers would choose multiple-sized packages at the equilibrium, in Appendix B we address this question via numerical calculations. Implicit in the derivations included in Tables 1 and 2, is the assumption that both producers choose, indeed, to offer two different-sized packages to consumers. The calculations illustrate that a "Prisoner's Dilemma"-type of environment might arise, as a result, since the producers could potentially increase their profits if they agreed each to only offer bigger-sized packages.

5. Empirical Study

To investigate whether our theoretical predictions are supported by evidence, we collected data on prices of different-sized packages of 92 food items (from stores in the Kansas City area). The sample included 50 nonorganic and 42 organic food items. We classified each item as being either processed or unprocessed. Examples of items included in the processed category are cake, pizza, cereal, dressing, ice cream, and so on. Examples of items included in the unprocessed category are vegetables, eggs, milk, cottage cheese, vinegar, and so on. The full list of the items in each category is included in Technical Appendix 2, available at http://mktsci.pubs.informs.org. We ran three different regression models. In all three, the dependent variable was the percentage of discount on the larger-sized package of the item. In calculating the discount, we accounted for the relative sizes of the smaller versus the larger packages (measured in the relevant units for the item under consideration). The set of independent variables with which we experimented included two dummy and one continuous variable. The first dummy distinguished between processed and unprocessed items (processed = 1), and the second dummy distinguished between organic and nonorganic items[15] (organic = 1). The continuous variable was the price of the smaller-sized package per unit of the item. In the first regression we included the entire data set with all three independent variables. In the second regression we included the entire data set with only the processed/unprocessed dummy and the price as independent variables, and in the last regression we only considered the nonorganic items with the same two independent variables as in the second regression.

In all three regressions we found that only the coefficient of the processed/unprocessed dummy variable is positive and significant (at the 1% level in the first two regressions and 5% in the last regression). Specifically, the coefficient was on average 0.08, implying that processed items benefit from an increased discount of 8% in comparison to unprocessed. Since processed food items tend to be more highly differentiated than unprocessed items, this result is consistent with our prediction that greater differentiation among competing products leads to higher quantity discounts. The fact that the overall price level of the item is statistically insignificant in explaining the variations in the quantity discounts provides support to the assumption that preferences are nonsticky rather than sticky for the items we consider in our data set. Our theoretical model predicts that nonsticky preferences should lead to discounts that are independent of the unit cost or the reservation price of the consumer, and sticky preferences should lead to discounts that are diminishing with unit cost or the reservation price. Since prices tend to be higher if either unit cost and/or the reservation price is higher, the existence of sticky preferences would require the price coefficient to be negative and significant. This is not the case in our data set. To test for the possibility that consumers of organic food items respond differently to discounts than consumers of nonorganic items, we included the organic/nonorganic dummy variable in the first regression. This variable turned out to be insignificant.

6. Concluding Remarks

We have developed a theoretical model to evaluate the profitability of offering quantity discounts in a market with competing but differentiated products. We found that the extent of discounting declines when products are less differentiated. The effect of unit cost or the reservation price of the consumer

[15] Initially, we also included an interaction term for the two dummy variables. Since this interaction term proved to be statistically insignificant, we decided to drop it from the three models.

on the extent of discounting depends on the relative brand loyalty of heavy versus light users of the product. Discounts are independent of those variables if brand loyalty remains the same irrespective of volume. Discounts decline if loyalty is higher for heavy users. An empirical investigation lends support to the former assumption concerning the preferences of the consumers. It also supports our theoretical prediction that reduced differentiation results in lower quantity discounts.

Appendix A. Transportation Costs Independent of Volume Derivation of the Regions of Figure 2

Each consumer chooses her consumption bundle m to maximize her net utility as follows:

$$\operatorname*{Max}_{m} W(m; x, \theta) = [U(m, x, \theta) - p(m)], \quad \text{where } m \in \{1_A, 2_A, 1_B, 2_B\}.$$

This maximization yields the following comparisons for the consumer:

$$W(1_A) \geq W(1_B) \quad \text{if } x \leq \frac{1+s_A-s_B}{2} + \frac{p(1_B)-p(1_A)}{2[1-(s_A+s_B)]} \equiv x^*,$$

$$W(2_A) \geq W(2_B) \quad \text{if } x \leq \frac{1+s_A-s_B}{2} + \frac{p(2_B)-p(2_A)}{2[1-(s_A+s_B)]} \equiv x^{**},$$

$$W(1_i) \geq W(2_i) \quad \text{if } \theta \geq 1 - \frac{p(2_i)-p(1_i)}{r} \equiv \theta_i^*,$$

$$W(1_A) \geq W(2_B) \quad \text{if } x \leq \frac{1+s_A-s_B}{2} + \frac{[p(2_B)-p(1_A)-r(1-\theta)]}{2[1-(s_A+s_B)]} \equiv \hat{x}(\theta),$$

$$W(2_A) \geq W(1_B) \quad \text{if } x \leq \frac{1+s_A-s_B}{2} - \frac{[p(2_A)-p(1_B)-r(1-\theta)]}{2[1-(s_A+s_B)]} \equiv \check{x}(\theta).$$

From the above definitions, $x^* - x^{**} = \Delta/2[1-(S_A+S_B)]$ and $\theta_B^* - \theta_A^* = \Delta/r$, where $\Delta \equiv [p(1_B)-p(1_A)] - [p(2_B)-p(2_A)]$. Since $\Delta \geq 0$ it follows that $x^* \geq x^{**}$ and $\theta_B^* \geq \theta_A^*$.

Proof of Lemma 1. When $\theta \leq \theta_A \leq \theta_B$, $W(2_A) \geq W(1_A)$ and $W(2_B) \geq W(1_B)$. Hence, the choice of the consumer is between 2_A and 2_B. This choice is determined from Figure 2 by comparing the coordinate x with x^{**}. When $\theta > \theta_B \geq \theta_A$, $W(1_A) > W(2_A)$ and $W(1_B) \geq W(2_B)$. Hence, the choice of the consumer is between 1_A and 1_B. This choice in determined from Figure 2 by comparing x with x^*.

When $\theta_A < \theta \leq \theta_B$, $W(1_A) > W(2_A)$ but $W(2_B) \geq W(1_B)$. Hence, the choice of the consumer is between 1_A and 2_B. From Figure 2, this choice is determined by comparing x with $\hat{x}(\theta)$, where $\partial\hat{x}/\partial\theta = r/2[1-(s_1+s_2)]$. Q.E.D.

Proof of Proposition 1. In the regular Hotelling model with quadratic transportation costs that are independent of volume, the threshold consumer who is indifferent between the two sellers satisfies the equation:

$$x^* = \frac{1+s_A-s_B}{2} + \frac{p_B-p_A}{2(1-s_A-s_B)}.$$

Hence, the market shares of the two sellers are x^* for A and $(1-x^*)$ for B. Seller i maximizes his profits given as $\text{MS}_i(p_i-k)$, where MS_i is his market share and k is the unit cost per package he incurs. When only single-unit packages are sold, $k=c$ and when only two-unit packages are sold, $k=2c$. Solving for the Nash equilibrium prices yields (2) and (3). Substituting back into the objective functions of the sellers, yields the expressions for equilibrium profits. Q.E.D.

Proof of Proposition 2. (i) Given the choice of the consumer described in Lemma 1, we can formulate the objective functions of the firms in the second-stage pricing game. Let K_i^j $j=1,2;$ $i=A,B$ designate the areas of the regions depicted in Figure 2. Then the profits of the sellers can be expressed as follows:

$$\begin{aligned}\pi_i(p(1_i), p(2_i), p(1_j)p(2_j), s_A, s_B) \\ = K_i^1(p(1_i)-c) + K_i^2(p(2_i)-2c) \quad i=A,B;\ i\neq j,\end{aligned}$$

where

$$K_A^1 \equiv (1-\theta_B^*)x^* + \left(\frac{x^*+x^{**}}{2}\right)(\theta_B^*-\theta_A^*), \qquad K_A^2 \equiv \theta_A^* x^{**},$$

$$K_B^1 \equiv (1-\theta_B^*)(1-x^*),$$

$$K_B^2 \equiv \theta_A^*(1-x^{**}) + \left(1-\frac{x^*+x^{**}}{2}\right)(\theta_B^*-\theta_A^*).$$

θ_A^*, θ_B^*, x^*, x^{**} are defined above in the derivation of the regions of Figure 2. In the second stage, seller i chooses $p(1_i)$ and $p(2_i)$ to maximize his objective. For seller A this objective is given as follows:

$$\begin{aligned}\pi_A = {} & (p(1_A)-c)\Bigg[\frac{p(2_B)-p(1_B)}{r}\left(\frac{p(1_B)-p(1_A)}{4(1-(s_A+s_B))} - \frac{p(2_B)-p(2_A)}{4(1-(s_A+s_B))}\right) + \frac{(p(2_A)-p(1_A))}{r} \\ & \cdot\left[\frac{1+s_A-s_B}{2} + \frac{p(1_B)-p(1_A)}{4(1-(s_A+s_B))} + \frac{p(2_B)-p(2_A)}{4(1-(s_A+s_B))}\right]\Bigg] \\ & + (p(2_A)-2c)\left(1-\frac{(p(2_A)-p(1_A))}{r}\right) \cdot \left(\frac{1+s_A-s_B}{2} + \frac{p(2_B)-p(2_A)}{2(1-(s_A+s_B))}\right).\end{aligned}$$

Differentiating with respect to $p(1_A)$ and $p(2_A)$ yields the following two first-order conditions:

$$\begin{aligned}\frac{\partial\pi_A}{\partial p(1_A)} = {} & \frac{p(2_B)-p(1_B)}{r}\left[\frac{p(1_B)-p(1_A)}{4(1-(s_A+s_B))} - \frac{p(2_B)-p(2_A)}{4(1-(s_A+s_B))}\right] \\ & + \left[\frac{p(2_A)-p(1_A)}{r}\right] \cdot \left[\frac{1+s_A-s_B}{2} + \frac{p(1_B)-p(1_A)}{4(1-(s_A+s_B))} + \frac{p(2_B)-p(2_A)}{4(1-(s_A+s_B))}\right] - \left[\frac{(p(1_A)-c)}{r}\right] \\ & \cdot\left[\frac{p(2_B)-p(1_B)}{4(1-(s_A+s_B))} + \frac{p(2_A)-p(1_A)}{4(1-(s_A+s_B))} + \frac{1+s_A-s_B}{2} + \frac{p(1_B)-p(1_A)}{4(1-(s_A+s_B))} + \frac{p(2_B)-p(2_A)}{4(1-(s_A+s_B))}\right] \\ & + \left[\frac{(p(2_A)-2c)}{r}\right] \cdot \left[\frac{1+s_A-s_B}{2} + \frac{p(2_B)-p(2_A)}{2(1-(s_A+s_B))}\right] = 0, \qquad \text{(A1)}\end{aligned}$$

$$\begin{aligned}
&\frac{\partial \pi_A}{\partial p(2_A)}\\
&= \frac{(p(1_A)-c)}{r}\left[\frac{p(2_B)-p(1_B)}{4(1-(s_A+s_B))}-\frac{p(2_A)-p(1_A)}{4(1-(s_A+s_B))}\right.\\
&\qquad\left.+\frac{1+s_A-s_B}{2}+\frac{p(1_B)-p(1_A)}{4(1-(s_A+s_B))}+\frac{p(2_B)-p(2_A)}{4(1-(s_A+s_B))}\right]\\
&\quad+(p(2_A)-c)\left[\frac{p(2_A)-p(1_A)}{2r(1-(s_A+s_B))}-\frac{1+s_A-s_B}{2r}\right.\\
&\qquad\left.-\frac{p(2_B)-p(2_A)}{2r(1-(s_A+s_B))}-\frac{1}{2(1-(s_A+s_B))}\right]\\
&\quad+\left(1-\frac{p(2_A)-p(1_A)}{r}\right)\\
&\quad\cdot\left(\frac{1+s_A-s_B}{2}+\frac{p(2_B)-p(2_A)}{2(1-(s_A+s_B))}\right)=0. \qquad \text{(A2)}
\end{aligned}$$

It is easy to show that the solution in (2) and (3) satisfies first-order conditions (A1) and (A2). The first-order conditions for seller B can be similarly derived. Once again, the solution in (2) and (3) solves those conditions as well. It is possible to show that second-order conditions are also satisfied.

(ii) Follows by using the expressions derived in part (i). Q.E.D.

Proof of Proposition 3. The consumer surplus with flexible choice is calculated as follows:

$$\begin{aligned}
\mathrm{CS}_F=&\int_0^{x^*}\left[\int_0^{1-c/r}(r(2-\theta)-p(2_A))\,d\theta\right.\\
&\left.+\int_{1-c/r}^{r}(r-p(1_A))\,d\theta-(x-s_A)^2\right]dx\\
&+\int_{x^*}^{1}\left[\int_0^{1-c/r}(r(2-\theta)-p(2_B))\,d\theta\right.\\
&\left.+\int_{1-c/r}^{r}(r-p(1_B))\,d\theta-(1-x-s_B)^2\right]dx, \qquad \text{(A3)}
\end{aligned}$$

where $x^*=1/2+(s_A-s_B)/6$ and $p(1_j)$ and $p(2_j)$ are given in (2) and (3). Integrating (A3) and collecting terms yields

$$\mathrm{CS}_F=(r-c)\left[\frac{3}{2}-\frac{c}{2r}\right]-(1-s_A-s_B)\left[1+\frac{(s_A-s_B)^2}{9}\right]-k(s_A,s_B),$$

where $k(s_A, s_B)$ measures the average transportation costs incurred by the consumers due to their consumption of products different from their "ideal" points. For fixed s_A and s_B, those average transportation costs can be expressed as follows:

$$\begin{aligned}
k(s_A,s_B)\equiv&\ \frac{1}{12}+\frac{(s_A-s_B)^2}{36}\\
&-\frac{1}{4}\left[(s_A+s_B)+\frac{(s_A+s_B)(s_A-s_B)^2}{9}+\frac{2}{3}(s_A-s_B)^2\right]\\
&+\frac{1}{2}\left[s_A^2+s_B^2+\frac{(s_A-s_B)^2(s_A+s_B)}{3}\right].
\end{aligned}$$

In the absence of choice, when only single-unit packages are available, the consumer surplus can be calculated as follows:

$$\begin{aligned}
\mathrm{CS}_{NF}^1=&\int_0^{x^*}(r-p(1_A)-(x-s_A)^2)\,dx\\
&+\int_{x^*}^{1}(r-p(1_B)-(1-x-s_B)^2)\,dx, \qquad \text{(A4)}
\end{aligned}$$

where $p(1_A)$ and $p(1_B)$ are given by (2). Conducting the integration yields

$$\mathrm{CS}_{NF}^1=(r-c)-(1-s_A-s_B)\left[1+\frac{(s_A-s_B)^2}{9}\right]-k(s_A,s_B).$$

When only two-unit packages are available:

$$\begin{aligned}
\mathrm{CS}_{NF}^2=&\int_0^1\left[\int_0^{x^*}((2-\theta)r-p(2_A)-(x-s_A)^2)\,dx\right.\\
&\left.+\int_{x^*}^{1}((2-\theta)r-p(2_B)-(1-x-s_B)^2)\,dx\right]d\theta, \qquad \text{(A5)}
\end{aligned}$$

where $p(2_A)$ and $p(2_B)$ are given by (3). Conducting the integration yields

$$\mathrm{CS}_{NF}^2=\left(\frac{3}{2}r-2c\right)-(1-s_A-s_B)\left[1+\frac{(s_A-s_B)^2}{9}\right]-k(s_A,s_B).$$

The result reported in the proposition follows by comparing the expressions obtained for CS_F, CS_{NF}^1, and CS_{NF}^2. Q.E.D.

Proof of Proposition 4. Differentiating the expected profits of seller i from (4) with respect to s_i yields

$$\frac{\partial \pi_i(s_i,s_j)}{\partial s_i}=-\left(1+\frac{s_i-s_j}{3}\right)\left(\frac{1}{6}+\frac{s_i}{2}+\frac{s_j}{6}\right)<0.$$

Hence, each seller chooses the smallest possible value of s_i; namely, $s_i=s_j=0$. Substituting into (4) and the expression for CS_F in the proof of Proposition 3, yields the expected profits and consumer surplus, respectively. Q.E.D.

Proof of Proposition 5. Designate by p_L^i the price of the single-unit package charged by seller i with linear pricing. Then $p(1_i)=p_L^i$ and $p(2_i)=2p_L^i$. If $p(1_B)\le p(1_A)$, $(p(1_B)-p(1_A))\ge p(2_B)-p(2_A)=2(p(1_B)-p(1_A))$; and Figure 2 still characterizes the choice of the consumers. Hence, the objective function of each seller is still as derived in the proof of Proposition 2. Differentiating this objective with respect to p_L^i and substituting symmetry into the first-order condition yields the following equation for the equilibrium price in the second-stage game:

$$3(p_L-c)^2-(p_L-c)(4r+2(1-2s)-3c)+(1-2s)(2r-c)=0.$$

The solution in (6) is obtained by solving the quadratic equation for (p_L-c) and choosing the root that also satisfies the second-order condition. It is easy to show that this solution is less than $(1-2s)$. At the symmetric equilibrium $x^*=x^{**}=0.5$ and $\theta_A^*=\theta_B^*=(1-p_L/r)$. Substituting into the seller's objective:

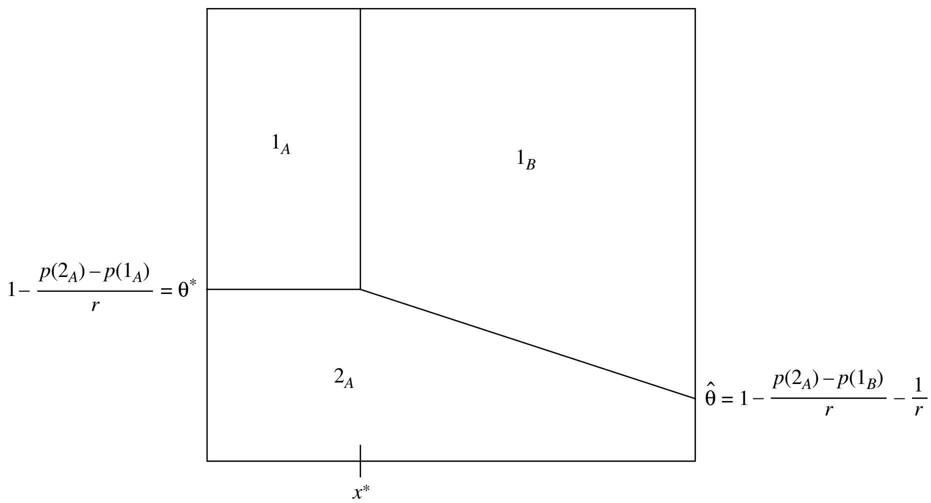
$$\begin{aligned}
E\pi_L&=\frac{1}{2}\left(\frac{p_L}{r}\right)(p_L-c)+\frac{1}{2}\left(1-\frac{p_L}{r}\right)(2p_L-2c)\\
&=\left(1-\frac{p_L}{2r}\right)(p_L-c). \qquad \text{(A6)}
\end{aligned}$$

The above is definitely less than $(1-2s)/2$ for $c<(1-2s)/2$ since in this case $(p_L-c)<(1-2s)/2$. Numerical calculations illustrate that $E\pi_L<(1-2s)/2$ for $c>(1-2s)/2$ as well. Recall that only values of c that are consistent with full coverage of the population have to be considered in those numerical calculations. Q.E.D.

Appendix B. Is Offering Choice to Consumers an Equilibrium?

In this appendix we illustrate the approach necessary in order to verify whether offering choice to consumers

Figure B.1 **Distribution of Consumers When Seller A Offers Choice and Seller B Does Not**



between two different-sized packages corresponds to equilibrium behavior. For simplicity, we restrict attention to the case that transportation costs are independent of volume, sellers are maximally differentiated so that $s_A=s_B=0$, and nonlinear pricing is feasible.

Suppose that Seller A offers choice and Seller B offers only single-unit packages. Seller A chooses the prices $p(1_A)$ and $p(2_A)$ for the two different sized packages he offers and Seller B chooses the price $p(1_B)$. The consumers' choice is determined as follows:

$$W(1_A)\geq W(1_B)\quad \text{if } x\leq \frac{1}{2}+\frac{p(1_B)-p(1_A)}{2}\equiv x^*,$$
$$W(1_A)\geq W(2_A)\quad \text{if } \theta\geq 1-\frac{p(2_A)-p(1_A)}{r}\equiv \theta^*,$$
$$W(2_A)\geq W(1_B)\quad \text{if } x\leq \frac{1}{2}+\frac{r(1-\theta)}{2}+\frac{p(1_B)-p(2_A)}{2}.$$

Figure B.1 illustrates the consumers' choice.

In Figure B.1 we assume that the parameters of the problem (r and c) support an interior equilibrium so that $0<\hat{\theta}<\theta^*<1$ and $0<x^*<1$. Given the choice of the consumers, the objective functions of the sellers can be formulated as follows:

$$\begin{aligned}\pi_A&=x^*(1-\theta^*)(p(1_A)-c)\\&\quad+\left[x^*\theta^*+\frac{(\theta^*+\hat{\theta})}{2}(1-x^*)\right](p(2_A)-2c),\\ \pi_B&=(1-x^*)\left[1-\frac{\theta^*+\hat{\theta}}{2}\right](p(1_B)-c).\end{aligned}\tag{B1}$$

Differentiating the objective function we can obtain the following three first-order conditions for prices:

$$\begin{aligned}\frac{\partial\pi_A}{\partial p(1_A)}&=x^*(1-\theta^*)-\left(\frac{1}{2}(1-\theta^*)+\frac{x^*}{r}\right)(p(1_A)-c)\\&\quad+\left(\frac{1+x^*}{2r}-\frac{(\theta^*-\hat{\theta})}{4}\right)(p(2_A)-2c)-0,\\ \frac{\partial\pi_A}{\partial p(2_A)}&=\frac{x^*}{r}(p(1_A)-c)-\frac{(p(2_A)-2c)}{r}\\&\quad+\left(x^*\theta^*+\frac{(\hat{\theta}+\theta^*)}{2}(1-x^*)\right)=0,\\ \frac{\partial\pi_B}{\partial p(1_B)}&=(1-x^*)\left(1-\frac{(\theta^*+\hat{\theta})}{2}\right)-\frac{(p(1_B)-c)}{2}\\&\quad\cdot\left[\left(1-\frac{(\theta^*+\hat{\theta})}{2}\right)+\frac{(1-x^*)}{r}\right]=0.\end{aligned}\tag{B2}$$

We numerically solve system (B2) for values of r and c that support the regions depicted in Figure B.1.

From Table B.1 the profits of the seller who does not offer choice are always less than 0.5. If this seller were to deviate and offer a choice between the two different-sized packages, his profits would rise to 0.5, as derived in the main text. Hence, restricting the choice to only single-unit packages cannot be a best response for Seller B. To complete the proof

Table B.1 **Solution of Pricing Subgame When Seller A Offers Choice and Seller B Does Not**

	r	$p(2_A)$	$p(1_A)$	$p(1_B)$	π_B
$c=0$	2	1.46231	1.20495	0.807428	0.26972
	2.5	1.68517	1.23917	0.858	0.26938
	3	1.89975	1.2665	0.8997	0.26984
	3.5	2.10827	1.28878	0.933173	0.270585
	4	2.31223	1.30727	0.96092	0.271382
	4.5	2.51268	1.32283	0.984252	0.272158
$c=0.5$	2	2.11156	1.48252	1.2978	0.28861
	2.5	2.34699	1.56466	1.34699	0.286957
	3	2.56906	1.6231	1.38636	0.285802
	3.5	2.78251	1.6668	1.41863	0.284977
	4	2.99002	1.7007	1.44559	0.284384
	4.5	3.1932	1.72809	1.46847	0.283956
$c=1$	2.5	3.02268	1.94691	1.83151	0.302998
	3	3.24767	2.01537	1.86969	0.299769
	3.5	3.46376	2.06862	1.90138	0.297416
	4	3.67356	2.1111	1.92812	0.295617
	4.5	3.87871	2.14578	1.95099	0.294199

that offering choice by both firms is an equilibrium, it is necessary to also consider the possibility that B restricts the choice to packages that include two units while his competitor offers both sized packages. If Seller B's profits in this scenario are also less than 0.5, then the "best response" of each seller is to offer choice if his competitor offers choice as well.

A similar approach can be utilized with linear pricing or when transportation costs increase with volume. If in such environments offering choice to consumers corresponds to equilibrium behavior, sellers may actually face a Prisoner's Dilemma, since the restriction of choice by *both* sellers can increase their profits in this case.

References

Allenby, G. M., T. S. Shivley, S. Yang, M. J. Garratt. 2004. A choice model for packaged goods: Dealing with discrete quantities and quantity discounts. *Marketing Sci.* **23**(1) 95–109.

Armstrong, M., J. Vickers. 2001. Competitive price discrimination. *RAND J. Econom.* **32**(4) 579–605.

Chintagunta, P., V. Kadiyali, N. Vilcassim. 2000. Manufacturer-retailer channel interactions and implications for channel power: An empirical investigation of pricing in a local market. *Marketing Sci.* **19**(2) 127–149.

Coughlan, A. 1985. Competition and cooperation in marketing channel choice: Theory and application. *Marketing Sci.* **4**(2) 110–129.

Coughlan, A., M. K. Mantrala. 1994. Dynamic competitive retail pricing behavior with uncertainty and learning. *Managerial Decision Econom.* **15**(1) 3–21.

d'Aspremont, C., J. J. Gabszewicz, J. F. Thisse. 1979. On Hotelling's "stability in competition." *Econometrica* **47**(5) 1145–1151.

Hotelling, H. 1929. Stability in competition. *Econom. J.* **39**(153) 41–57.

Jeuland, A., S. M. Shugan. 1983. Managing channel profits. *Marketing Sci.* **2**(3) 239–272.

Jeuland, A., S. M. Shugan. 1988. Reply to: Managing channel profits: Comment. *Marketing Sci.* **7**(1) 103–106.

Kumar, N., R. C. Rao. 2006. Using basket composition data for intelligent supermarket pricing. *Marketing Sci.* **25**(2) 188–199.

Leland, H., R. Meyer. 1976. Monopoly pricing structures with imperfect discrimination. *Bell J. Econom.* **7**(2) 449–462.

Lewis, M., V. Singh, S. Fay. 2006. An empirical study of the impact of nonlinear shipping and handling fees on purchase incidence and expenditure decisions. *Marketing Sci.* **25**(1) 51–64.

Liu, Y., Z. J. Zhang. 2006. The benefits of personalized pricing in a channel. *Marketing Sci.* **25**(1) 97–105.

Matutes, C., P. Regibeau. 1992. Compatibility and bundling of complementary goods in a duopoly. *J. Indust. Econom.* **40**(1) 37–54.

McGuire, T., R. Staelin. 1983. An industry equilibrium analysis of downstream vertical integration. *Marketing Sci.* **2**(2) 161–192.

Moorthy, S. K. 1987. Managing channel profits: Comment. *Marketing Sci.* **6**(4) 375–379.

Murphy, M. M. 1977. Price discrimination, market separation, and multi-part tariff. *Econom. Inquiry* **15**(4) 587–599.

Oi, W. Y. 1971. A disneyland dilemma: Two-part tariffs for a Mickey Mouse monopoly. *Quart. J. Econom.* **85**(1) 77–96.

Oren, S. S., S. A. Smith, R. B. Wilson. 1982. Nonlinear pricing in markets with interdependent demand. *Marketing Sci.* **1**(3) 287–313.

Oren, S. S., S. A. Smith, R. B. Wilson. 1983. Competitive nonlinear tariffs. *J. Econom. Theory* **29**(1) 49–71.

Oren, S. S., S. Smith, R. B. Wilson. 1984. Pricing a product line. *J. Bus.* **57**(1) S73–S99.

Padmanabhan, V., I. P. L. Png. 1997. Manufacturer's returns policies and retail competition. *Marketing Sci.* **16**(1) 81–96.

Raju, J., Z. J. Zhang. 2005. Channel coordination in the presence of a dominant retailer. *Marketing Sci.* **24**(2) 254–263.

Rust, R. T., T. S. Chung. 2006. Marketing models of service and relationships. *Marketing Sci.* **25**(6) 560–580.

Spence, M. 1977. Nonlinear prices and welfare. *J. Public Econom.* **8**(1) 1–18.

Spulber, D. 1989. Product variety and competitive discounts. *J. Econom. Theory* **48**(2) 510–525.

Stole, L. 1995. Nonlinear pricing and oligopoly. *J. Econom. Management Strategy* **4**(4) 529–562.

Thisse, J.-F., X. Vives. 1988. On the strategic choice of spatial price policy. *Amer. Econom. Rev.* **78**(1) 122–137.

Wilson, R. B. 1993. *Nonlinear Pricing*. Oxford Press, New York.