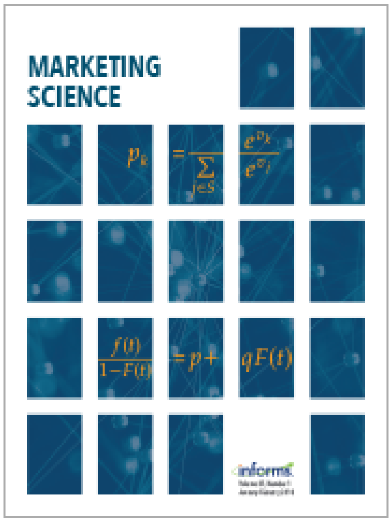

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Incorporating Direct Marketing Activity into Latent Attrition Models

David A. Schweidel
Goizueta Business School, Emory University, Atlanta, Georgia 30322, dschweidel@emory.edu

George Knox
LeBow College of Business, Drexel University, Philadelphia, Pennsylvania 19104, george.knox@drexel.edu

When defection is unobserved, latent attrition models provide useful insights about customer behavior and accurate forecasts of customer value. Yet extant models ignore direct marketing efforts. Response models incorporate the effects of direct marketing, but because they ignore latent attrition, they may lead firms to waste resources on inactive customers.

We propose a parsimonious model that allows direct marketing to impact three relevant behaviors in latent attrition models—the frequency with which customers conduct transactions, the size of the transactions, and the duration for which customers remain active. Our model also accounts for how the organization targets its direct marketing across individuals and over time.

Using donation data from a nonprofit organization, we find that direct marketing increases donation incidence for active donors. However, our analysis also shows that direct marketing has the potential to shorten the length of a donor's relationship. We find that our proposed model offers superior predictive performance compared with models that ignore the impact of direct marketing activity or latent attrition. We demonstrate the managerial applicability of our modeling approach by estimating the impact of direct marketing on donation behavior and identifying those donors most likely to conduct transactions in the future.

Key words: latent attrition; customer relationship management; simultaneity; direct marketing
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1. Introduction

Latent attrition models lie at the heart of transactional (i.e., noncontractual) customer base analysis, because customer attrition is not observed and hence must be inferred (Kumar and Reinartz 2006, p. 103). These models can be used to forecast future purchasing for both new and existing customers (e.g., Schmittlein et al. 1987, Fader et al. 2010) and aid managers in areas such as customer valuation, customer targeting, and resource allocation (e.g., Reinartz and Kumar 2000, 2003).

Early models ignored covariates entirely (e.g., Schmittlein et al. 1987, Fader et al. 2005a), yet more recent research has incorporated time-*invariant* predictors like demographics that covary with purchasing and attrition (e.g., Abe 2009, Neslin et al. 2009, Singh et al. 2009). Though these models represent an important advance, the next step is to consider the effects of time-varying marketing efforts, a necessary tool for managing customer relationships and evaluating the financial impact of marketing. The key challenge in estimating the impact of such marketing activity is carefully adjusting for how it is targeted.

In addition to omitting marketing, latent attrition models often assume that the time in between transactions and the transaction sizes are independent (e.g., Schmittlein and Peterson 1994, Fader et al. 2005b) or are conditionally independent within customers over time (e.g., Borle et al. 2008). We relax these assumptions by linking the intertransaction times and transaction amounts in two distinct ways. Like Borle et al. (2008), we account for the cross-sectional relationship between intertransaction times and transaction amounts, e.g., customers who make frequent purchases may spend more. We also link the intertransaction time process and the transaction amount process at the individual level (e.g., Jen et al. 2009) so that, for example, customers may compensate for a longer-than-expected intertransaction time by spending more than the average. In sum, we account for correlated transaction timing and spending within and across customers.

After incorporating marketing and relaxing the independence assumption, we demonstrate that our modeling framework provides superior guidance to managers allocating marketing resources across customers. The key feature of the model is that it probabilistically classifies donors as active or inactive based on the sequence of past transactional activity and responsiveness to marketing. Distinguishing between inactive customers who will not respond to a firm's marketing and active customers who are responsive to marketing is critical for ensuring that the firm is not wasting resources by sending marketing to inactive customers. We next review two streams of literature relevant to our research and to which we contribute.

1.1. Marketing Response Models

Marketing response models relate customer purchase incidence and/or quantity decisions to marketing or detailing efforts, and they capture dynamics by allowing past events (marketing and response variables) to affect current response (e.g., Manchanda et al. 2004, Donkers et al. 2006, Van Diepen et al. 2009, Li et al. 2011). Though these models provide useful estimates of the incremental value of direct marketing, they ignore the phenomenon of latent attrition, which is "of prime importance" in noncontractual businesses (Blattberg et al. 2008, p. 121). Prior research has considered the role of marketing efforts in contexts where the end of the relationship is observed (e.g., Reinartz et al. 2005, Schweidel et al. 2008b, Sun and Li 2011), but our focus is on marketing's impact when attrition is unobserved and must be inferred, which, to the best of our knowledge, has not been considered previously.

In this research, we contribute to both market response models and latent attrition models by allowing direct marketing activity to affect (1) an individual's transaction rate while he or she is active, (2) the size of the transaction, and (3) the length of the individual's latent relationship with the firm. Direct marketing may benefit the firm if it reduces attrition and consequently extends a customer's latent lifetime. However, it may adversely affect the firm if such efforts irritate customers (e.g., Fournier et al. 1997, Venkatesan and Kumar 2004, Van Diepen et al. 2009) and lead them to become inactive more quickly. We offer a comprehensive view of direct marketing by distinguishing the "short-term" effects on an individual's transaction rate and size from the "long-term" impact on an individual's latent lifetime (e.g., Montoya et al. 2010), and we demonstrate how the overall impact of direct marketing hinges on the balance of these effects. If direct marketing increases the transaction rate and the latent lifetime, it will drive up the total number of transactions unequivocally. If direct marketing diminishes the latent lifetime but increases the transaction rate, the change in the total number of transactions depends on the relative magnitude of each effect. Whether such a trade-off exists is critical for organizations, as their direct marketing may be front-loading transactions into the early part of the relationship and exacerbating attrition rather than generating incremental transactions.

1.2. Nonrandom Marketing Activity

The very nature of *direct* marketing is one of the reasons that it has been a challenge to incorporate its effect into latent attrition models. By design, direct marketing activity is nonrandom and targeted at specific individuals at specific times. Ignoring this will lead to biased estimates of the effectiveness of direct marketing. Hence, in the model, we account for two ways in which direct marketing efforts may be nonrandom. First, marketers may target select customers with direct marketing activity (e.g., Manchanda et al. 2004). Though these targeting decisions may be partially driven by observable summaries of customer behavior such as the recency and frequency of transactions, other factors such as expert judgment that are unobservable to the researcher may also drive this decision (e.g., Donkers et al. 2006). Managers may also time their direct marketing opportunistically to coincide with local events, greater-than-average website visits, or other unobserved time-varying factors. These unobserved shocks can affect both the firm's likelihood of sending direct marketing and the customer's likelihood of responding (Gupta and Park 2012). To the best of our knowledge, our research is the first to account for these two possibilities in a latent attrition framework.

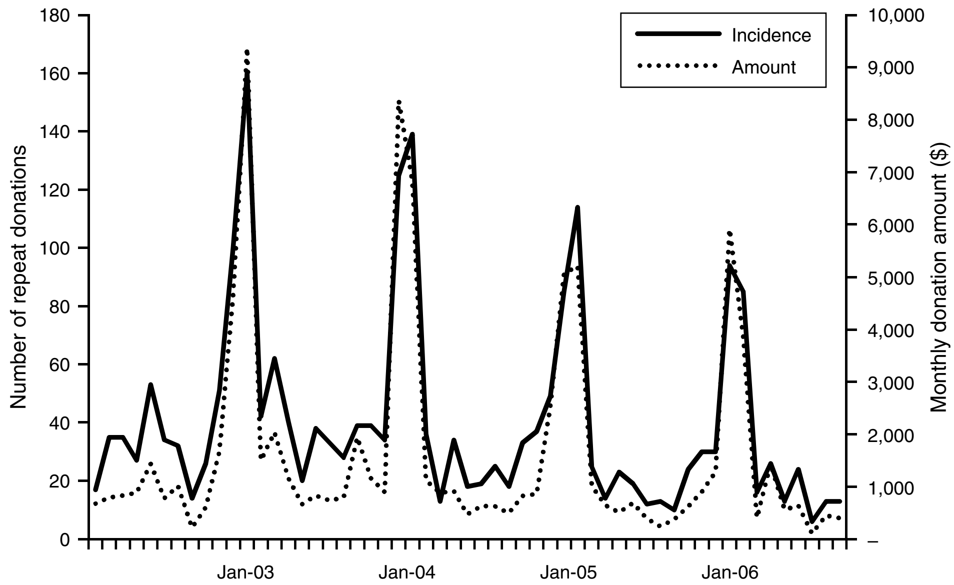
In sum, we propose a model that allows direct marketing to influence customer transactional behavior through three distinct mechanisms, we account for the nonrandom nature of direct marketing activity, and we allow for associations between transaction timing and spending. Using scenario analysis, we demonstrate how our model can quantify the bottom-line impact of direct marketing activity over time. We also demonstrate how it can identify those individuals who are most likely to conduct transactions in the future. In doing so, we also illustrate an important caveat about the usefulness of $P(Alive)$, a popular measure derived from latent attrition models that has been incorporated into several decision-making frameworks (e.g., Reinartz and Kumar 2000, Venkatesan and Kumar 2004).

The remainder of this paper proceeds as follows. We first describe the data used in our analysis, and next develop our model. We then present our empirical findings and demonstrate how the model can quantify the impact of direct marketing and identify individuals likely to donate in the future. We conclude with a discussion of the limitations of the current research and avenues for future work.

2. Data

Our donation and mailing data come from a nonprofit organization in the United States and were made

Figure 1 Observed Donation Behavior for January 2002 Cohort



publicly available by the Direct Marketing Educational Foundation for its Lifetime Value and Customer Equity Modeling Competition in 2008.[1] The data set contains all donation and mailing history for 21,166 donors who made their first donation in the first half of 2002 through the end of August 2006. Our analysis is based on a 20% random sample (4,234 donors). We use the first 40 months (January 2002–April 2005) for model calibration and the remaining 16 months (May 2005–August 2006) for model validation.

Subsequent to each donor's first contribution, we model repeat donation behavior on a monthly basis, because the organization sends its direct marketing at the start of each month.[2] Donors made, on average, 1.54 repeat donations and spent $38.69 on each donation. Similar to many applications, the distribution of donations across donors is skewed: 50% of donors make no repeat donations, 17% make a single repeat donation, and 10% make two repeat donations. Among donors making at least one repeat donation, donors made, on average, 3.07 repeat donations. The average total donation level per month is $6,094, and the average amount per donor making a contribution is $37.97.

We use the data on donations and direct marketing to motivate several aspects of our model and provide model-free evidence that is consistent with latent attrition. Figure 1 illustrates the repeat donation activity of a cohort of donors who made their first contributions in January 2002.

Consistent with latent attrition, we see that both the number of donations and total donation amounts decline over time. A similar pattern emerges among donors in other cohorts. Examining the time since donors in our sample made their last contribution to the organization, we observe an increasing pattern over time: in January 2003, the average number of months elapsed since a donor's last donation was 8.68 months; in January 2004, it was 15.98 months; and in January 2005, it was 23.77 months. Figure 1 also shows seasonal peaks in donations activity in November, December, and January.

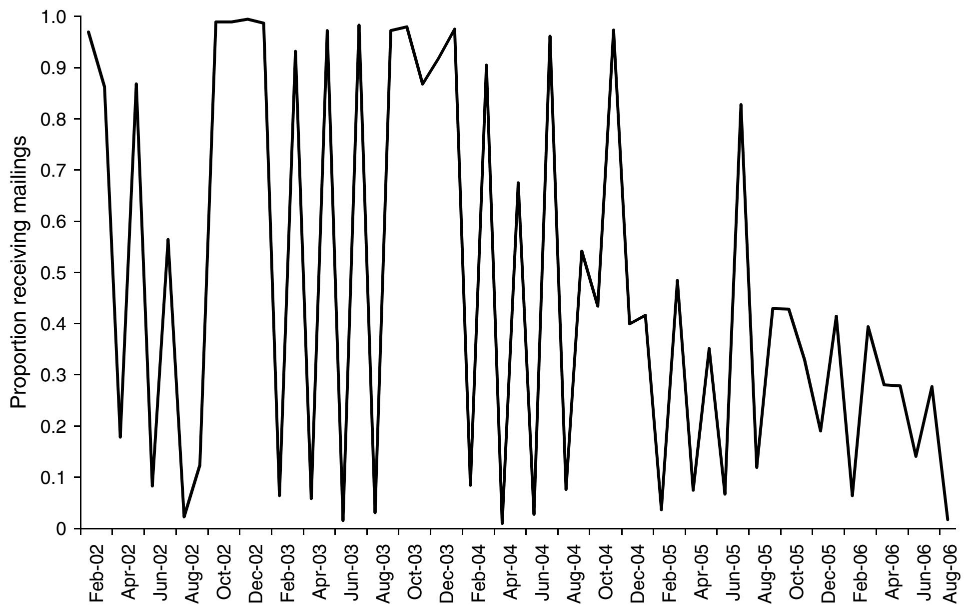
Figure 2 shows the sample proportion receiving a mailing each month subsequent to their first donation to the organization. Early in our observation period, most donors appear to receive a mailing every other month, with the exception of the winter, in which the organization appears to send mailings more frequently. Moreover, Figure 2 shows that there is variation over time with direct marketing activity declining in later months, consistent with an organization curbing its efforts in response to donors who have likely become inactive.

The most noticeable shift in direct marketing activity occurs at the beginning of 2005. With the exception of a single peak in mailing activity that occurs in July 2005, less than half of the donors in our sample receive mailings each month during 2005 and 2006. Ninety-eight percent of donors received mailings in January 2004, yet only 42% received them in January 2005. Among those who donated within one

[1] As our data are from a nonprofit organization, we use the terms "donor" and "organization" in place of "customer" and "firm," respectively. In discussing the organization's direct marketing activity, we use the terms "mailing" and "direct marketing" interchangeably.

[2] Although we analyze the data at the monthly level, corresponding with the frequency of the organization's direct marketing efforts, our analysis could be applied at a more granular (e.g., daily or weekly) or coarse (e.g., quarterly or annually) level.

Figure 2 Organization's Direct Marketing Activity



year of January 2005 (21% of our sample), 93% of those donors received a mailing, whereas only 28% of donors who did not make a contribution within the last year received a mailing. This evidence suggests that mailing decisions are responsive to donors' past activity; hence we simultaneously model an individual's donation activity conditional on the organization's direct marketing efforts and the organization's direct marketing conditional on the individual's past actions.

3. Model Development

We begin by developing a joint model of the time between donations and donation amount for an active donor. Next, we incorporate latent attrition by allowing for the possibility that donors can become inactive permanently. Finally, we model the organization and use observed donor history, unobserved donor characteristics (e.g., Donkers et al. 2006), and temporal shocks (e.g., Gupta and Park 2012) to drive its mailing decisions. Figure 3 provides a general schematic of the model and the order of decisions made by the organization and donor.

3.1. Interdonation Times and Donation Amount

While active, we assume that a given donor i's decision to make a donation at time t, measured in months, follows a discrete-time Bernoulli process with probability p_{it}. Let z_{it} be the time of donor i's most recent donation prior to month t. The cumulative distribution for the time until the next donation, a spell of length $t-z_{it}$, is given by

$$F(t \mid z_{it}) = 1 - \prod_{k=z_{it}+1}^{z_{it}+t} (1-p_{ik}). \tag{1}$$

We can then derive the probability of a donation occurring at time t by conditioning on the donor not having made a contribution by $t-1$:

$$f(t \mid z_{it}) = \frac{F(t \mid z_{it}) - F(t-1 \mid z_{it})}{1 - F(t-1 \mid z_{it})}. \tag{2}$$

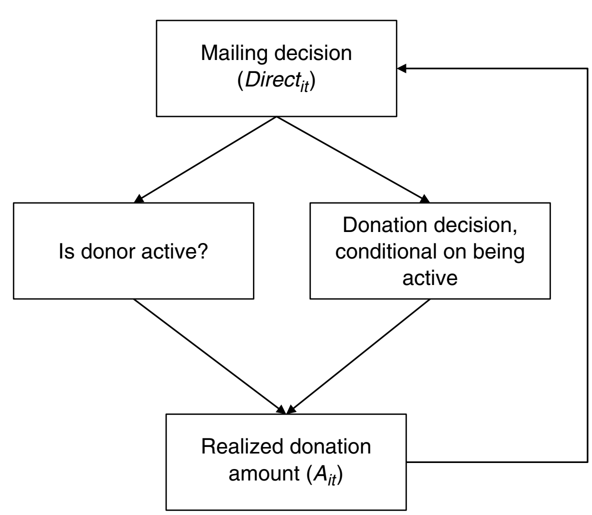
If $p_{it} = p_i$ for all t, the interdonation time process follows a geometric distribution, the individual-level interdonation time model assumed by Fader et al. (2010).

We allow for p_{it} to vary over time based on seasonality (see Figure 1) and the organization's direct marketing efforts:

$$p_{it} = \Phi(\gamma_0 + \gamma_1 \cdot Winter_t + \gamma_2 \cdot DM_{it} + \gamma_3 \cdot DM_{it}^2), \tag{3}$$

where $\Phi(x)$ is the cumulative distribution function (c.d.f.) of the standard normal distribution

Figure 3 Model Schematic



at x; $Winter_t$ is an indicator variable equal to 1 in November, December, and January and is equal to 0 otherwise; and DM_{it} is a stock variable for the amount of direct marketing activity that donor i has received by month t (e.g., Van Diepen et al. 2009), where[3]

$$DM_{it} = Direct_{it} + \lambda \cdot DM_{i,\,t-1}, \tag{4}$$

which allows the impact of direct marketing at time t ($Direct_{it}$) to carry over from one period to the next. We include both a linear and quadratic effect of direct marketing activity to allow for the possibility of wearout at high levels of direct marketing. Conditional on donor i making a donation in month t, we assume that the donation amount A_{it} follows a log-normal distribution with parameters μ_{it} and σ:

$$\begin{gathered}\mu_{it} = \beta_0 + \beta_1 \cdot Winter_t + \beta_2 \cdot DM_{it} + \beta_3 \cdot DM_{it}^2,\\ \log(A_{it}) \sim \mathrm{N}(\mu_{it}, \sigma^2).\end{gathered} \tag{5}$$

We denote the corresponding c.d.f. of the amount model as $G(a)$ and the probability density function (p.d.f.) as $g(a)$.

We use a bivariate Gaussian copula (Danaher and Smith 2011) to correlate the marginal distribution of interdonation times and donation amounts to account for compensating behavior. For example, if the correlation is positive, an individual will donate a larger-than-expected amount after a larger-than-expected interdonation time (e.g., Jen et al. 2009). Let $t^* = \Phi^{-1}(F(t \mid z_{it}))$ and $a^* = \Phi^{-1}(G(A_{it}))$, where $\Phi^{-1}(x)$ is the probit function. We assume that (t^*, a^*) follow a standard bivariate normal distribution with correlation ρ. If (X, Y) are distributed according to a standard bivariate normal distribution with correlation ρ, the conditional distribution for X given $Y = y$ is a normal distribution with mean $\rho \cdot y$ and variance $1-\rho^2$. Thus, conditional on the value $A_{it} = a$, t^* follows a normal distribution with mean $\rho \cdot \Phi^{-1}(G(a))$ and variance $1-\rho^2$. Substituting the c.d.f. of the log-normal distribution for $G(a)$ and simplifying, the conditional distribution of t^* can be written as

$$t^* \mid a \sim \mathrm{N}\left(\rho \cdot \left(\frac{\log(a) - \mu_{it}}{\sigma}\right), 1-\rho^2\right). \tag{6}$$

The cumulative distribution associated with the conditional distribution of $F(t \mid z_{it}, A_{it})$ can be expressed as

$$F(t \mid z_{it}, A_{it}) = \Phi\left(\frac{\Phi^{-1}(F(t \mid z_{it})) - \rho \cdot ((\log(A_{it}) - \mu_{it})/\sigma)}{\sqrt{1-\rho^2}}\right). \tag{7}$$

When $\rho = 0$ and the interdonation timing and donation amount processes are independent, $F(t \mid z_{it}, A_{it})$ in Equation (8) is given by $F(t \mid z_{it})$.

Let $h(t, a \mid z_{it})$ denote the joint likelihood of a donation in month t given the most recent donation occurred in month z_{it} and the donation being for an amount a, conditional on not having observed a donation in month $t-1$:

$$h(t, A_{it} \mid z_{it}) = \begin{cases} g(A_{it}) \dfrac{F(t \mid z_{it}, A_{it}) - F(t-1 \mid z_{it}, A_{it})}{1 - F(t-1 \mid z_{it})} & \text{if } A_{it} > 0, \\[2ex] \dfrac{1 - F(t \mid z_{it})}{1 - F(t-1 \mid z_{it})} & \text{if } A_{it} = 0. \end{cases} \tag{8}$$

If the donation timing and amount processes are independent ($\rho = 0$), when a donation occurs, the joint probability $h(t, A_{it} \mid z_{it})$ is simply the product of the probability associated with the interdonation time t (Equation (2)) and the probability associated with the amount donated a, given by $g(a)$.[4]

There are two advantages of our donation timing approach over the popular Type II Tobit, which models donation incidence. First, because we model the time since the last donation, we can account for compensating donor behavior mentioned earlier. Jen et al. (2009) demonstrate that ignoring this behavioral phenomenon leads to poorer predictive performance and customer overvaluation in two different empirical settings. Second, our timing model accounts for the path over time (within the interdonation time interval) of direct marketing; the timing of the donation may depend on the stock of direct marketing at the time of donation as well as the stock of direct marketing faced in the months since the last donation (Franses and Paap 2001, p. 158). Thus the expected donation amounts conditional on a donation occurring are a function of both the amount of time that has elapsed since the last donation and past direct marketing activity, as these two factors impact $F(t \mid z_{it})$.

3.2. Incorporating Latent Attrition in Donation Behavior

The model outlined in Equations (1)–(8) accounts for *active* donor behavior. At some point in time that is unobserved by the firm, donors may become inactive. Hence the attrition process is latent and must

[3] In Appendix A, we consider an alternative specification for the direct marketing stock variable *DM* that allows for a postdonation dip in the likelihood of a donation occurring. This model specification yielded poorer predictive performance using both aggregate and individual-level error measures.

[4] Though we condition on the donation amount A_{it} in Equations (6)–(8), we do not assume that A_{it} is decided prior to the decision to make a donation. We construct the model in this manner to derive the joint distribution of donation incidence and amount by employing the known conditional distribution of a bivariate normal distribution.

be inferred from prior activity. To allow for the possibility that donor i becomes inactive after the tth month, we assume that the time-varying probability that donor i remains active is given by q_{it}:

$$q_{it} = \Phi(\alpha_0 + \alpha_1 \cdot Winter + \alpha_2 \cdot DM_{it} + \alpha_3 \cdot DM_{it}^2). \quad (9)$$

If $q_{it} = q_i$, resulting from $\alpha_1 = \alpha_2 = \alpha_3 = 0$, then donor i's latent lifetime follows a geometric distribution, as in Fader et al. (2010).

Let $A_{i\cdot}$ be a vector of length T that contains the amount donated in each month of the observation period by donor i. As a latent attrition model is a hidden Markov model (HMM) with two states (active and inactive), the likelihood associated with donor i's sequence of activities can be written as follows (MacDonald and Zucchini 1997):

$$L(A_{i\cdot}) = \boldsymbol{\pi}\tilde{\mathbf{A}}_{\mathbf{i}1}\mathbf{Q}_{\mathbf{i}1}\tilde{\mathbf{A}}_{\mathbf{i}2}\mathbf{Q}_{\mathbf{i}2}\cdots\mathbf{Q}_{\mathbf{i},T-1}\tilde{\mathbf{A}}_{\mathbf{i}T}\mathbf{1}', \quad (10)$$

where $\boldsymbol{\pi} = [1 \ \ 0]$ denotes the initial distribution in which all donors are assumed to begin in the active state, $\mathbf{1}$ is a 2×1 vector of ones, the matrix $\mathbf{Q_{it}}$ is the transition matrix given by

$$Q_{it} = \begin{bmatrix} q_{it} & 1 - q_{it} \\ 0 & 1 \end{bmatrix} \quad (11)$$

such that the inactive state is assumed to be an absorbing state, and the matrix $\tilde{\mathbf{A}}_{\mathbf{it}}$ is a diagonal matrix of the likelihoods conditional on being in each state of the hidden Markov model at time t:

$$\tilde{\mathbf{A}}_{\mathbf{it}} = \begin{bmatrix} h(t, A_{it} \mid z_{it}) & 0 \\ 0 & I(A_{it} = 0) \end{bmatrix}. \quad (12)$$

Donor i can be in the inactive state at time t only if we observe no donations. Hence the likelihood conditional on donor i being in the inactive state at time t is captured by the indicator function $I(A_{it} = 0)$, which has a value equal to 1 if $A_{it} = 0$ and is equal to 0 otherwise.

3.3. Identification

As with extant latent attrition models (e.g., Schmittlein et al. 1987, Fader et al. 2005a), our model probabilistically classifies donors as active or inactive based on the sequence of past transactional activity. To provide some intuition for how our model is identified, let us consider two scenarios. For a frequent donor with short interdonation times, when many months have elapsed since his last contribution, it is unlikely that such an interval would occur if the donor were still active. In contrast, an infrequent donor will have long observed interdonation times. In this situation, a longer interval would need to pass without a donation occurring for it to be inconsistent with his previously observed interdonation intervals. For both of these individuals, as more time elapses without a transaction, it becomes increasingly likely that they are inactive. The estimate of whether a donor is active or inactive, however, is probabilistic and represents the uncertainty of the inference. The likelihood that an infrequent donor is inactive may be close to 0.5, representing maximum uncertainty, whereas for the frequent donor, it may be closer to 0.9 or 0.1. Latent attrition models assign a probability to whether an unobservable change point (active versus inactive) has occurred based on prior observed transactional activity and the length of the censored intertransaction time. Such changes in behavior are identified similarly in more general HMMs (e.g., Netzer et al. 2008, Schweidel et al. 2011), of which latent attrition models can be considered a subset.[5]

In addition to the standard latent attrition model arguments, responsiveness to direct marketing is another source of information that helps us identify whether a donor is currently active. If direct marketing increases the likelihood that a donor conducts a transaction and a donor has received a high volume of direct marketing, he is more likely to conduct a transaction, assuming he is still active. As such, if he has not conducted a transaction after receiving a high volume of direct marketing, it is more likely that he is inactive.

3.4. Likelihood

From Equations (10)–(12), we can calculate the probability with which a donor is in the active state after each period t and hence prior to each period $t+1$. The posterior probabilities of being active and inactive at time t are

$$\begin{aligned}&[P_{it}(Alive)(1 - P_{it}(Alive))] \\ &\quad = \left(\frac{\boldsymbol{\pi}\tilde{\mathbf{A}}_{\mathbf{i}1}\mathbf{Q}_{\mathbf{i}1}\tilde{\mathbf{A}}_{\mathbf{i}2}\mathbf{Q}_{\mathbf{i}2}\cdots\mathbf{Q}_{\mathbf{i},t-2}\tilde{\mathbf{A}}_{\mathbf{i},t-1}}{\boldsymbol{\pi}\tilde{\mathbf{A}}_{\mathbf{i}1}\mathbf{Q}_{\mathbf{i}1}\tilde{\mathbf{A}}_{\mathbf{i}2}\mathbf{Q}_{\mathbf{i}2}\cdots\mathbf{Q}_{\mathbf{i},t-2}\tilde{\mathbf{A}}_{\mathbf{i},t-1}\mathbf{1}'}\right)\mathbf{Q}_{\mathbf{i},t-1}, \quad (13)\end{aligned}$$

where the term in parenthesis is the posterior belief of state membership after the donation decision at time $t-1$ from applying the Bayes rule (Netzer et al. 2008). Multiplying this by the transition matrix $\mathbf{Q_{i,t-1}}$ yields the posterior state membership prior to the donation decision at time t. Using these probabilities, the likelihood of donor i's donation decision at time t can be written as

$$L(A_{it}) = \begin{cases} \left(P_{it}(Alive)\dfrac{1 - F(t \mid z_{it})}{1 - F(t-1 \mid z_{it})}\right) & \\ \quad + (1 - P_{it}(Alive)) & \text{if } A_{it} = 0, \\ P_{it}(Alive)h(t, A_{it} \mid z_{it}) & \text{if } A_{it} > 0. \end{cases} \quad (14)$$

[5] HMMs such as those developed by Netzer et al. (2008) and Schweidel et al. (2011) incorporate marketing activity, but these models do not accommodate the nonrandom nature of marketing actions.

If $A_{it} = 0$, the likelihood is the sum of two probabilities: (1) that donor i is still active (from Equation (13)) yet does not donate at time t (from Equation (8)), and (2) that donor i is no longer active. If $A_{it} > 0$, donor i must be active, and the joint likelihood for the donation incidence and amount is given by $h(t, A_{it} \mid z_{it})$.

In sum, we allow direct marketing to affect donation timing (Equation (3)) and size (Equation (5)) when the donor is active, as well as how long donors remain active (Equation (9)). Hence the total impact will depend on whether the short- and long-term effects complement each other (e.g., increase both the likelihood of donation incidence and the time horizon) or oppose each other (e.g., increase the incidence of donations while shortening the time horizon). In the latter case, an organization must balance the short-term benefits of direct marketing against the long-term consequences (e.g., Montoya et al. 2010).

3.5. Accounting for the Organization's Mailing Process and Heterogeneity

In addition to modeling the individual's donation behavior, we also must account for the organization's nonrandom targeting of direct marketing activity. Consistent with the industry and extant literature (e.g., Gönül and Shi 1998), we model the organization's decision to send marketing to a donor in a given month as a function of donor history, as captured by recency, frequency, monetary (RFM) variables and a seasonality variable, because the organization's mailing efforts are higher during the winter months (see Figure 2). We assume that the decision to send donor i a mailing in month t ($Direct_{it}$) occurs if $u_{it} + \varepsilon_{it} > 0$, where

$$u_{it} = \kappa_0 + \kappa_1 \cdot Recency_{it} + \kappa_2 \cdot Frequency_{it} + \kappa_3 \cdot Monetary_{it} + \kappa_4 \cdot Winter_t. \qquad (15)$$

If we assume that ε_{it} follows a standard normal distribution, this results in a binary probit model. The values for *Recency*, *Frequency*, and *Monetary* at each point in time are calculated based on donor i's donation history prior to month t. We operationalize *Recency* as the number of months that have elapsed since the last donation, *Frequency* as the number of donations made to date, and *Monetary* as the logarithm of the average amount that has been donated.

In addition to donation history, we expect the organization may base its decision on unobserved donor characteristics such as giving capacity or wealth. Such unobserved heterogeneity may also be related to differences in donation behavior and marketing responsiveness. We establish an association between donation and mailing processes by using a latent-class framework in which the parameters governing the donation process and the organization's direct marketing decisions are class specific (e.g., Donkers et al. 2006, Schweidel et al. 2008a). For example, one class of donors may have a higher frequency of giving and be less likely to become inactive compared with others, and the organization may be more likely to send direct marketing to this group. The latent-class model induces an association among the parameters governing the frequency of transactions, amount of transactions, and latent attrition, akin to Abe's (2009) assumption that the model parameters are correlated. Moreover, this approach is similar to models that link marketing decisions to individual-level response parameters through specific parametric models (Manchanda et al. 2004, Van Diepen et al. 2009, Li et al. 2011). As in such models, we jointly model a given individual's decision conditional on marketing activity *and* the organization's marketing activity toward the given individual.

In addition to the cross-sectional donor targeting as described above, the organization may take advantage of special events, such as a spike in website visits, by timing its marketing to coincide with them. These unobserved, time-varying events represent a common shock that affects both the donation decision and the direct marketing decision (e.g., Gupta and Park 2012). To account for this possibility, we link the organization's direct marketing decision to an individual's donation behavior. Rather than linking the organization's direct marketing activity to the individual model components (the time until the next donation, the amount donated conditional on a donation occurring, and the latent lifetime), we establish a relationship between the mailing decision and the distribution of donation amounts at time t, $D_{it}(a \mid z_{it})$, which synthesizes the individual model components. Using the posterior probability of a donor being active (Equation (13)), $D_{it}(a \mid z_{it})$ is given by

$$\begin{aligned} &D_{it}(a \mid z_{it}) \\ &\quad = \left(P_{it}(Alive) \frac{1 - F(t \mid z_{it})}{1 - F(t-1 \mid z_{it})} \right) + (1 - P_{it}(Alive)) \\ &\qquad + I(a > 0) P_{it}(Alive) \\ &\qquad \cdot \left\{ \left(\Phi_2\left(\left[\frac{\log(a) - \mu_{it}}{\sigma} \quad \Phi^{-1}(F(t \mid z_{it})) \right], \rho \right) \right. \right. \\ &\qquad\qquad \left. - \Phi_2\left(\left[\frac{\log(a) - \mu_{it}}{\sigma} \quad \Phi^{-1}(F(t-1 \mid z_{it})) \right], \rho \right) \right) \\ &\qquad\qquad \left. \cdot (1 - F(t-1 \mid z_{it}))^{-1} \right\}, \end{aligned} \qquad (16)$$

where $\Phi_2([x \;\; y], \rho)$ is the standard bivariate normal c.d.f. evaluated at (x, y) with correlation ρ. The first two terms of Equation (16), as in Equation (14), account for the probability that a donation does not occur and hence yield a probability mass at $a = 0$.

The remaining terms "turn on" for donation amounts $a > 0$, which may only occur if donor i is still active at time t, and the donation occurs at time t given that it has not occurred by $t-1$. When $\rho = 0$, the term in curly braces simplifies to the probability of a donation at time t conditional on it not having occurred by $t-1$, multiplied by the normal c.d.f.

Having derived the cumulative distribution of donation amount, we use a Gaussian copula to link it to direct marketing. Letting $d^* = \Phi^{-1}(D_{it}(a \mid z_{it}))$, we assume that (d^*, ε) follow a standard bivariate normal distribution with correlation τ. If $\tau > 0$, individuals who are prone to make a larger-than-expected contribution at time t are also more likely to receive a mailing at time t. Having derived the model for donors' behavior at time t with the likelihood $L(A_{it})$, we now construct the joint likelihood of the decision to send direct marketing to donor i at time t ($Direct_{it}$) and i's donation decision at time $t(A_{it})$, denoted $J\ (A_{it}, Direct_{it})$. Recall that the organization sends a mailing to donor i in month t if $m_{it} + \varepsilon > 0$. If a donation occurs ($A_{it} > 0$), then ε is normally distributed with mean $\tau \cdot d^*_{it}$ and variance $1 - \tau^2$. The marginal probability with which a donation does not occur ($A_{it} = 0$) is given by $D_{it}(0 \mid z_{it})$ and $d^*_{it} = \Phi^{-1}(D_{it}(0 \mid z_{it}))$. The joint probability of donor i receiving direct marketing and not making a donation, conditional on a donation not occurring, can be expressed as the quotient of $\Phi_2([d^*_{it} \quad m_{it}], \tau)$ to $D_{it}(0 \mid z_{it})$. The joint likelihood of donor i's donation decision at time t and the organization's decision to mail to that donor can then be written as

$$J(A_{it}, Direct_{it}) = \begin{cases} L(A_{it}) \times \left[\left(\dfrac{\Phi_2([d^*_{it} \quad m_{it}], \tau)}{D_{it}(0 \mid z_{it})} \right)^{Direct_{it}} \cdot \left(1 - \dfrac{\Phi_2([d^*_{it} \quad m_{it}], \tau)}{D_{it}(0 \mid z_{it})} \right)^{1-Direct_{it}} \right] & \text{if } A_{it} = 0, \\ L(A_{it}) \times \left[\left(\Phi\left(\dfrac{m_{it} + \tau \cdot d^*_{it}}{\sqrt{1-\tau^2}} \right) \right)^{Direct_{it}} \cdot \left(1 - \Phi\left(\dfrac{m_{it} + \tau \cdot d^*_{it}}{\sqrt{1-\tau^2}} \right) \right)^{1-Direct_{it}} \right] & \text{if } A_{it} > 0. \end{cases} \tag{17}$$

If the vector π denotes the size of each latent class, the log likelihood for the set of donation decisions made by donor i and the organization's direct marketing activity is

$$LL(A_{i\cdot}, Direct_{i\cdot}) = \log\left(\sum_{s=1}^{S} \pi_s \prod_{t=1}^{T} J_s(A_{it}, Direct_{it}) \right). \tag{18}$$

It is worth pointing out now how our context and model differ from several recent direct marketing response models. In Donkers et al. (2006) and Van Diepen et al. (2009), response observability is endogenous because individuals must respond to a particular mailing in their empirical setting. In contrast, we observe individuals making donations months after they have received direct marketing. Hence we develop a timing model of monthly donation decision, taking into account the time that has elapsed since the last donation in a manner consistent with prior literature (e.g., Fader et al. 2010). Because our model accounts for the impact of current and prior direct marketing efforts on donation occurrence and amount, it is generalizable to other direct marketing contexts.

In addition to differences in the empirical context, our methodological approaches also differ. In contrast to Donkers et al. (2006) and Van Diepen et al. (2009), we incorporate latent attrition to allow donors to become permanently inactive.[6] By allowing direct marketing to impact the donation decision while a donor is active and the likelihood of that donor becoming inactive, our model can quantify the increase in revenue due to direct marketing activity. Our methodological approach also offers a means of accounting for common shocks that may simultaneously affect direct marketing and donation decisions, as well as allowing the direct marketing process to vary across donors.

4. Empirical Analysis

We calibrate a series of nested models using maximum likelihood estimation with up to three latent classes. We find that the predictive performance does not improve after two classes and that our substantive findings remain unchanged. We therefore conduct our model comparisons using the model with two latent classes. First, we estimate a model in which we do not take into account the role of time-varying covariates (No covariates). Second, we estimate a model in which we assume that the direct marketing decisions are independent of the donation decisions (Independent direct marketing). Under this specification, we assume that the vector κ is the same across the two latent classes and that $\tau = 0$. We then allow direct marketing decisions and the donation decisions to be linked by assuming that κ is specific to the latent class, though we still assume that $\tau = 0$ (Linked direct marketing). Finally, we estimate our full model in which we relax the assumption that $\tau = 0$ (Common shock). In addition to these nested models based on the latent attrition framework, we estimate a Type II Tobit model similar to Donkers et al. (2006) with two

[6] To ensure that our modeling approach is consistent with the data, in the next section, we test the predictive ability of our latent attrition model against a model similar to Donkers et al. (2006) that does not incorporate latent attrition.

Table 1 Model Performance

Model	Calibration LL	Calibration BIC	Validation: Aggregate monthly donations RMSE ($)	Validation: Individual monthly donations RMSE ($)
No covariates	−118,260	236,713	2,565	22.55
Independent direct marketing	−117,056	234,405	2,184	22.46
Linked direct marketing	−116,255	232,844	2,168	22.45
Common shock	−116,067	232,407	2,986	22.61
RFM model	−114,098	228,564	2,500	22.46

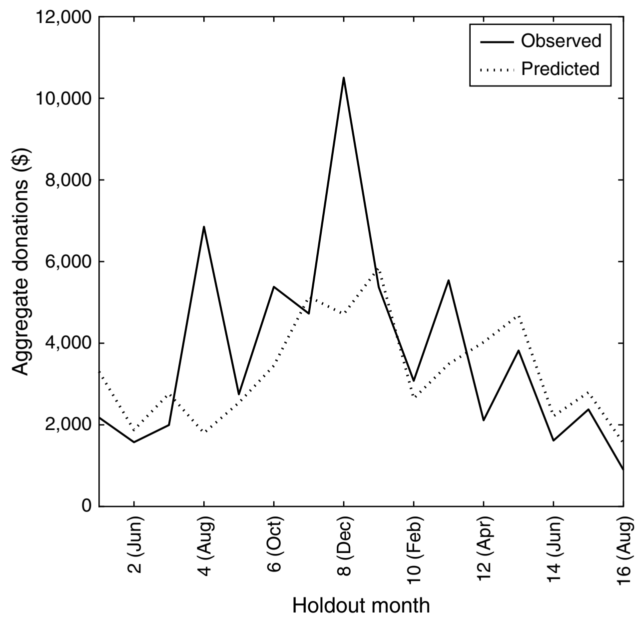
latent classes and RFM variables as predictors both in the organization's mailing decision and in the donor's behavior (RFM model).

4.1. Model Results

We compare model performance during the calibration period using the Bayesian information criteria (BIC) for each of the model specifications. To assess the predictive ability of the modeling framework, we conduct 10,000 simulations of expected donor activity during the validation period and average across the simulations. We calculate the root mean square error (RMSE) for the aggregate monthly donations during the validation period, averaged across months, and the RMSE on the amount contributed by each individual in each month of the validation period to assess individual-level forecasting performance. We present these model fit statistics in Table 1.

There are larger differences in model performance based on aggregate RMSE than individual RMSE, because most observations have no donations at the individual monthly level. The model that performs best out of sample at the individual and aggregate levels during the holdout period is the Linked direct marketing model, a nested version of the Common shock model that is based on the latent attrition framework.[7] The latter model performs better in the calibration sample but worse in the validation sample. The RFM model has the lowest BIC of all models during the calibration period; but good in-sample fit, often at the expense of holdout fit (as seen in the RMSE of aggregate monthly donations), is a well-known concern with RFM models (e.g., Malthouse 1999). Hence we focus on the Linked direct marketing model, because it provides the best out-of-sample fit at both the individual and monthly levels.[8]

Figure 4 Monthly Donation Amount During the Holdout Period



To better gauge model fit, we plot the total amount size donated each month (see Figure 4) and the number of monthly donations made during the holdout period (see Figure 5). Figure 4 shows that our model forecasts reflect the general pattern of donations, though it underpredicts donations in months 4 and 8.

Figure 5 shows that we capture the general pattern in donation incidence, though we underpredict in month 8. Including additional predictors, such as monthly differences rather than a single *Winter* variable, may provide the model with additional flexibility to capture this spike in behavior.

Table 1 suggests that it is not necessary to account for the way in which direct marketing is targeted if the only objective is to forecast future donation activity. However, our key focus and an issue of managerial importance is evaluating the impact of direct marketing. This requires not only incorporating direct marketing activity in the model of donation behavior but also accounting for the way in which the decision to send direct marketing may be linked to donation behavior. Beyond yielding marginally improved forecasts, as we will show, our analysis offers a new

[7] We do not observe substantive differences in the effects of direct marketing activity between these two model specifications. The correlation coefficient $\tau = 0.04$ (S.E. $= 0.01$) suggests a small association between the mailing process and expected donation amount. As this model has poorer predictive ability than the nested Linked direct marketing model, we focus the remainder of our discussion on results under the more parsimonious specification.

[8] For the purposes of comparison, we estimated the linked direct marketing model with three latent classes. Whereas in-sample fit improves with the addition of model parameters (log likelihood = −115,572; BIC = 231,647), predictive performance does not improve (aggregate monthly donation RMSE = $2,173; individual monthly donation RMSE = $22.46). Given no improvement in forecasting ability, we present the results of the more parsimonious model.

Figure 5 Monthly Donation Incidence During the Holdout Period

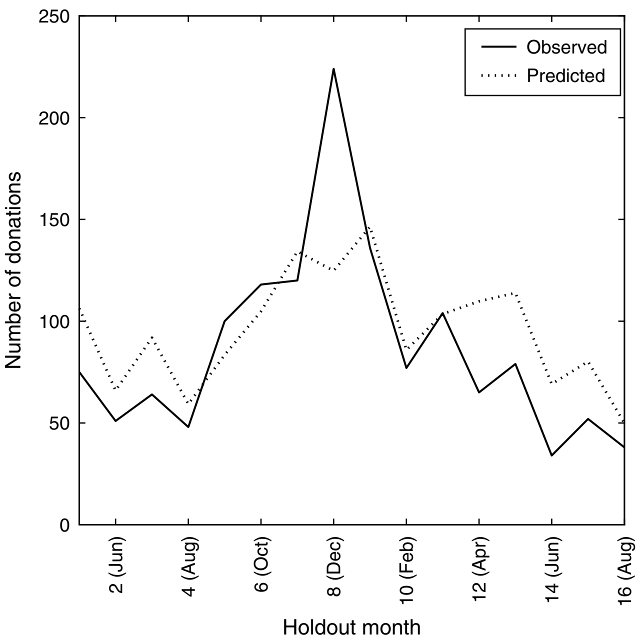


Table 2 Donation Behavior Parameter Estimates

Parameter	Description	Class 1	Class 2
γ_0	Incidence: Intercept	−2.27 (0.01)	−2.00 (0.01)
γ_1	Incidence: *Winter*	0.22 (0.02)	0.01 (0.01)
γ_2	Incidence: *DM*—linear	0.18 (0.01)	0.07 (0.00)
γ_3	Incidence: *DM*—quadratic	0.04 (0.00)	0.10 (0.00)
β_0	Amount: Intercept	3.37 (0.03)	2.91 (0.01)
β_1	Amount: *Winter*	−0.04 (0.04)	0.05 (0.01)
β_2	Amount: *DM*—linear	−0.20 (0.01)	−0.05 (0.00)
β_3	Amount: *DM*—quadratic	0.14 (0.01)	0.01 (0.00)
σ	Amount: Shape parameter	1.22 (0.02)	0.46 (0.02)
ρ	Incidence and Amount correlation	0.22 (0.02)	0.07 (0.02)
α_0	Remain active: Intercept	1.43 (0.02)	4.87 (0.05)
α_1	Remain active: *Winter*	4.74 (332.07)	2.78 (14.42)
α_2	Remain active: *DM*—linear	−1.16 (0.02)	−1.45 (0.02)
α_3	Remain active: *DM*—quadratic	0.81 (0.02)	0.13 (0.01)
κ_0	Mailing decision: Intercept	0.37 (0.00)	−0.10 (0.01)
κ_1	Mailing decision: *Winter*	1.19 (0.01)	2.69 (0.10)
κ_2	Mailing decision: *Recency*	−0.04 (0.00)	−0.01 (0.00)
κ_3	Mailing decision: *Frequency*	0.00 (0.00)	0.11 (0.00)
κ_4	Mailing decision: *Monetary*	0.00 (0.00)	0.00 (0.00)
π	Class size	0.73 (0.04)	0.27 (—)

perspective on the impact of direct marketing by considering its effect on latent attrition.

To ensure the estimation procedure can recover the model parameters accurately, including those related to the latent attrition process, we conduct a simulation study. We simulated 100 data sets using parameters estimated from our empirical application. Each data set contained 4,000 donors observed over 40 months, similar to our calibration sample.[9] The results show that the true parameters (on which the simulated data are based) are within the 95% interval of the parameter estimates from the simulated data sets, indicating that we are able to recover the model parameters. Although the maximum likelihood procedure generates bias, our simulation results suggest this is low.[10] To test the stability of our latent classes, we also conducted cross-sample validation by estimating the model on a second sample of the same size from the donor base. The key substantive findings from this analysis were similar to those resulting from our analysis of the calibration sample, providing further support for our results.

Next, we examine the parameters governing donation behavior. The decay rate for direct marketing is $\lambda = 0.68$ (S.E. $= 0.01$). We present the parameters (and standard errors) specific to each latent class in Table 2.[11]

The parameters governing the incidence of donation when the donor is active show that class 2 has a higher baseline probability of making a donation (γ_0) compared with class 1. For both classes, there is approximately the same propensity to donate in winter months ($\gamma_0 + \gamma_1$), and direct marketing activity increases the probability with which donations occur (γ_2), which increases at higher levels of *DM* (γ_3).

Whereas donors in class 1 have a lower baseline tendency to make donations compared with those in class 2, they tend to donate larger amounts (β_0). Additionally, we find a positive correlation between the interdonation time process and the donation amount process (ρ). This is consistent with compensating behavior, with donors giving larger amounts when more time has elapsed (e.g., Jen et al. 2009).

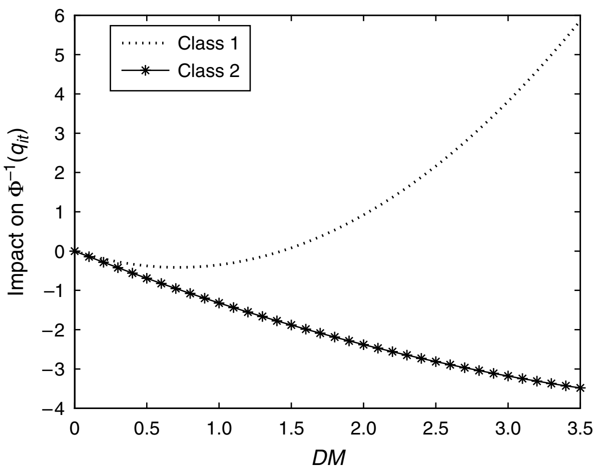
The key differences between the two latent classes are in the attrition process. The donors in class 2 have a higher baseline tendency to remain active (α_0) compared with those in class 1. The segments differ in how they respond to direct marketing. In both latent classes, we find $\alpha_2 < 0$ and $\alpha_3 > 0$, suggesting a U-shaped relationship between *DM* and the likelihood with which an individual remains active. As such, depending on the value of *DM*, direct marketing may adversely affect the likelihood of donors

[9] The simulated data sets match key features of our data, including the distribution of donations across donors, the proportion of periods with no observed donations.

[10] We calculate the bias and RMSE for each parameter in the model. The bias of our estimates ranged from −0.062 to 2.52, with an average of 0.084; the RMSE ranged from 2.34×10^{-4} to 2.88, with an average of 0.331.

[11] Small perturbations to the model parameters can yield larger changes in the log likelihood, given the number of observations in our data. This results in small standard errors for the parameter estimates.

Figure 6 **Effect of Direct Marketing on Latent Attrition**



remaining active and making contributions in the future. Examining the range of *DM* observed in our data (min(*DM*) = 0; max(*DM*) = 3.16), we find that for class 1, low levels of *DM* have a slight negative impact on the latent lifetime, whereas higher levels of *DM* increase the latent lifetime. Thus, direct marketing increases the latent time horizon over which donations occur for those donors in class 1 at sufficiently high levels. Alternatively, for those donors in class 2, the observed values of *DM* are contained in the downward portion of the U. Consequently, the horizon over which a customer in class 2 makes donations diminishes with *DM*. We illustrate the effect of *DM* on $\Phi^{-1}(q_{it})$ in Figure 6.

The impact of direct marketing on donation amounts (β_2 and β_3) follows a U shape for individuals in class 1; although there is a slight negative effect for low levels of *DM*, the effect is positive and increases for larger values of *DM*. In class 2, direct marketing has a slight negative effect on donation amount.

In sum, donors in class 1 positively react to higher levels of direct marketing with long latent lifetimes, increased donation incidence, and donation amounts; for donors in class 2, direct marketing increases donation incidence (for active donors), yet it shortens the latent lifetime and diminishes donation amounts. One explanation for these findings is that direct marketing temporally compresses donation activity. That is, rather than a given number of donations occuring over a given latent lifetime, direct marketing activity may increase the frequency of donations but shorten the time period over which they occur. The net effect of these opposing forces would determine whether the total amount an individual donates remains the same, increases, or decreases. Our results for donors in class 2 regarding the effect of marketing on donation incidence and latent attrition are consistent with this, but the negative effect on donation amounts is not necessarily so. Alternatively, it may be that direct marketing contributes to irritation, consistent with the finding of Van Diepen et al. (2009) that donation amounts diminish with increased direct marketing. Although the authors also find that donation incidence diminishes with increased direct marketing, their analysis did not incorporate latent attrition. We believe that our results are consistent with the view that high levels of direct marketing contribute to irritation, which affects donation behavior (both incidence and amount) while a donor is active and reduces the likelihood of remaining active, which can reduce donation incidence in the long run.

The negative impact of direct marketing on the tendency to remain active has important consequences for the organization. For donors in the larger latent class (class 1), direct marketing has a positive impact on the tendency to make donations while active. When there is a sufficient stock of *DM*, direct marketing has a positive impact on the amount donated and the latent length of the relationship. Thus, on all three of these dimensions, direct marketing has a positive impact on donation behavior. In the smaller latent class (class 2), direct marketing has a positive impact on the tendency to donate while active but a negative impact on the length of the relationship and donation amount. Whether the organization should continue to send direct marketing to this class depends on whether the upside potential from increasing the likelihood of a donation among active donors outweighs the risks associated with increasing the chances that a donor becomes inactive. We investigate this issue shortly.

Turning to the mailing process, again we observe important differences across the two latent classes. The intercepts for the latent classes (κ_0) reveal a higher baseline rate of sending direct marketing to class 1 compared with class 2. This is consistent with donors in class 1 reacting positively to direct marketing; donors in class 2 exhibit mixed reactions to direct marketing. As expected, direct marketing activity declines as more time elapses since the last donation (κ_2). This decline occurs more rapidly in class 1 than in class 2, which is consistent with the organization implementing different direct marketing plans for its donors. As donors in class 2 are less likely to become inactive, the organization may continue to market to them for a longer period of time following their last donation. We also find that the organization is more likely to send direct marketing to donors in class 2 who have a higher frequency of past repeat donations (κ_3).

4.2. Managerial Application: Valuing Direct Marketing Efforts

By quantifying the value of direct marketing efforts, we can illustrate the managerial applicability of our model and, more specifically, investigate whether the increase in donations for active donors in class 2 outweighs the risks of driving active donors to defect. Based on an individual's posterior probability of class membership as of the end of the calibration period, we simulate latent-class membership and whether the donor is still active at the beginning of the forecasting period. For those donors who are still active, we simulate their donation behavior during the forecasting period. We repeat this simulation procedure 1,000 times and average the donation behavior across the simulations.

As a baseline scenario, we first conduct our simulation procedure assuming that no direct marketing is sent during the forecasting period. We then simulate donation behavior by incorporating the organization's observed direct marketing activity through the first t months of the forecasting period. Taking the difference between the scenario that employs the direct marketing activity through month t of the forecasting period and the scenario that omits direct marketing provides a measure of the cumulative increase in donation activity (e.g., Schweidel et al. 2011).[12]

In Figure 7, we present the cumulative effects of direct marketing during the first t months of the forecasting period on the amount donated during the forecasting period, averaged across all donors and simulations. Figure 7 reveals a general increase in the overall cumulative impact of direct marketing (solid black line), suggesting that marketing activity is increasing donation activity, with an average increase of \$7.26 across all donors compared with the scenario in which they receive no direct marketing.

In interpreting Figure 7, we note two important caveats. First, the rate at which the cumulative impact of direct marketing increases is related to the organization's observed marketing activity. This is, in part, to ensure that we are not simulating too far out of the range of normal policy. Here, the organization sends more direct marketing during the winter months, which may contribute to a faster increase. Second, though we observe what appear to be diminishing returns to direct marketing toward the end of the holdout period, we note that it may be attributable to there being few remaining months in the 16-month time horizon in which direct marketing may impact donation behavior. A mailing sent in the first month of the holdout period may impact behavior throughout the entire 16-month period, whereas a mailing sent in the final month may only impact donation activity in that month.[13]

Figure 7 **Cumulative Impact of Direct Marketing During Forecasting Period**

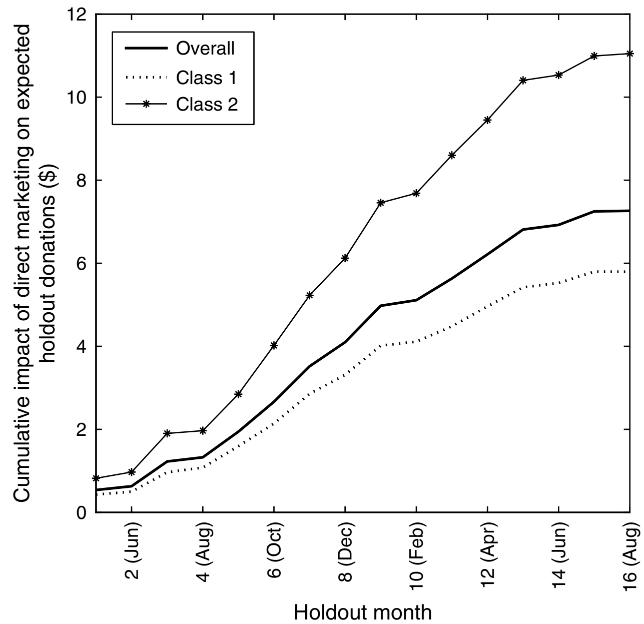


Figure 7 also presents the cumulative impact of direct marketing for the donors most likely to be in each of the two latent classes.[14] The increasing cumulative effect of direct marketing pattern holds in both latent classes; the expected increase is \$11.05 for class 2 donors (28%) and \$5.79 for class 1 donors (72%). At first glance, this may seem counterintuitive, given that direct marketing has a negative impact on the latent lifetime of class 2. However, because the baseline tendency to remain active (α_0) is high for class 2, direct marketing's impact on the donation behavior of active donors outweighs the risks of driving active customers to defect.

The above analysis, though, ignores differences in the organization's direct marketing efforts across individuals. Consistent with the difference in recency of donations across classes at the start of the validation period (31.0 months for class 1 donors versus 12.4 months for class 2 donors) and the difference in $P(Alive)$ at the end of the calibration period (0.20 for class 1 donors versus 0.80 for class 2 donors),

[12] One could also construct measures of the incremental effect of direct marketing by comparing the scenario with direct marketing through month t with the scenario with direct marketing through month $t-1$ (e.g., Braun and Schweidel 2011).

[13] Should the organization have a direct marketing schedule it intends to employ over a longer time horizon, the same approach could be employed to gauge marketing's impact over that time frame.

[14] We assign customers to the latent class for which they have the highest posterior class membership in Figure 5, but our simulation procedure does not "hard code" these assignments. For each of the 1,000 simulations conducted, donors are assigned to the latent class probabilistically based on the posterior probability of class membership.

class 1 donors receive fewer mailings than donors from class 2 (2.84 mailings versus 9.18 mailings). Comparing the expected increase in holdout donations per mailing, mailings to class 1 donors generate an average increase of \$2.04 per mailing (5.79/2.84), whereas mailings to class 2 donors generate an average increase of \$1.20 per mailing (11.05/9.18). Contributing to this difference is the higher baseline donation amount for donors in class 1 compared with class 2. Thus, despite the longer average recency and lower average $P(Alive)$ for donors believed to be in class 1, it would be premature for the organization to write off these donors.

To further investigate the overall impact of marketing in the presence of latent attrition for the two latent classes, we simulated 100 data sets, each one comprising donation activity and direct marketing received for 4,000 donors observed over 40 periods, using the estimated model parameters as before. We compared average total repeat donations observed in this scenario to one in which no direct marketing is sent. Repeat donations increased by 0.55 in class 1 and 1.97 in class 2 when direct marketing was included. We can compare these results to those when the latent attrition component of the model is "turned off" to quantify its impact. When latent attrition is omitted, direct marketing increases repeat donations by 1.23 in class 1 and 2.47 in class 2, revealing the extent to which latent attrition diminishes the impact of direct marketing in both latent classes. Note that whereas the impact of direct marketing on latent lifetime is negative in class 2, the overall impact on donation behavior is positive.

4.3. Managerial Application: Identifying Likely Donors

In addition to valuing the impact of direct marketing activity, identifying the prospects most likely to conduct future transactions is a common problem faced by managers. To demonstrate how our modeling framework can provide guidance in this task, we derive two different metrics that could be used to rank individuals in terms of how likely they are to make a donation during the forecasting period. First, we calculated the probability that a donor is still active (i.e., $P(Alive)$) at the end of the calibration period, a metric that has been proposed in much extant customer relationship management research as a means of scoring customers (e.g., Reinartz and Kumar 2003). However, although $P(Alive)$ captures the likelihood that an individual is active at a particular point in time and hence has the potential to donate in the future, it does not take into account the forecasting timeframe. A more appropriate measure we derive is the probability that an individual donates at least once during the forecasting period ("incidence probability"). This measure considers the direct marketing activity in which the organization engages during the forecasting period, as well as the tendency to conduct transactions while active.

We first construct deciles of customers based on their score on each of these metrics and compute the proportion of customers in each decile who make a donation during the forecasting period. This is displayed in Figure 8.

As expected, we find that a smaller proportion of donors in "lower" deciles make contributions during the forecasting period. Moreover, we find a stark difference between the two metrics in their respective top deciles. Whereas approximately 60% of the donors with the highest $P(Alive)$ scores make a contribution during the forecasting period, approximately 90% of donors with the highest incidence probabilities make a contribution. Donors with high values of $P(Alive)$ are expected to still be active and hence able to make a contribution at some point in the future, but high values of $P(Alive)$ do not ensure that this will happen during a specific time horizon. In contrast, the incidence probability takes this into account.

We can also use the deciles analysis above to calculate the economic value of our model. A full-blown individual-level optimization of direct marketing based on the model is beyond the scope of the paper. Instead, we provide an illustration of how our modeling approach could be employed, following the exercise in Blattberg et al. (2008, p. 279). We sort donors into deciles from most likely to least likely to donate in the forecasting period according to the incidence probability. We assume that the organization

Figure 8 **Donation Frequency by Decile**

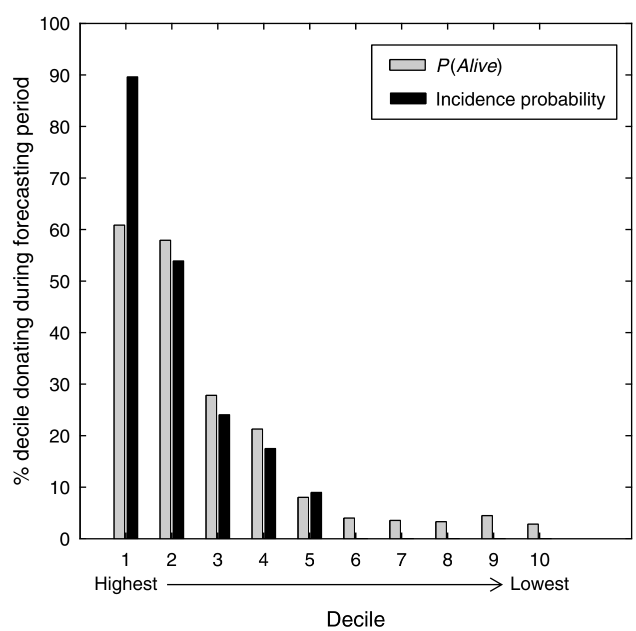
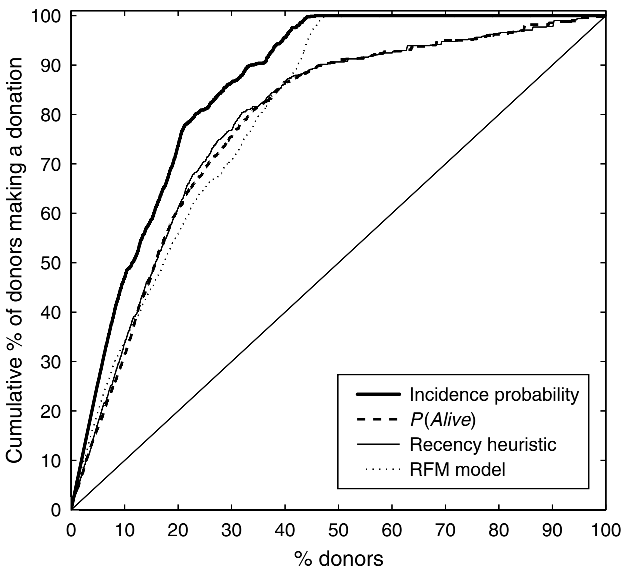


Figure 9 Comparison of Alternative Donor Rankings



decides whether to send direct marketing to donors in a given decile based on decile-specific profitability; if they send marketing, they do so according to the estimated parameters of the mailing model. For each decile, we calculate a predicted profit based on the decile-specific predicted average response rate, decile-specific predicted donation amount, and decile-specific predicted average number of direct mailings received.[15] The predicted profit per donor targeted is \$13.50; with no model-based targeting, the predicted profit per average donor is \$6.44. In the absence of such marketing activity, the predicted profit per average donor is \$2.41. Hence the economic value of direct marketing and using the model to target direct marketing efforts is considerable.

To further explore our modeling framework's ability to identify likely donors, we consider two additional measures that are not based on the latent attrition framework: the predicted probability with which donors make donations during the holdout period under the RFM model and a heuristic that ranks customers first in terms of their recency and then in terms of their frequency.[16]

In Figure 9, we present the proportion of donors making a donation, as ranked by each of the four methods described above. As a point of reference, we include a 45° line. If the proposed metric classifies donors well, we should find that the curve lies above the 45° line, suggesting that the measure more accurately identifies those individuals who make donations during the forecasting period than one would select by chance. We see that all four of the proposed methods of ranking donors are better able to distinguish likely donors better than chance.

Figure 9 shows that the incidence probability measure dominates the other three measures until the proportion of donors reaches 0.5, at which point it is tied with the RFM model. Overall, this indicates that the incidence model scores likely donors best compared to $P(Alive)$, the recency heuristic or the RFM model. To the best of our knowledge, the comparison between using the incidence probability and $P(Alive)$ to score customers has not previously been conducted in the literature employing latent attrition models and it is of managerial relevance. Although $P(Alive)$ is a popular metric, it does not take into account the likelihood that a transaction will occur within a specific time frame. In our empirical application, it performs no better than the recency heuristic. If such forecasts are desired, calculating the probability of the event occuring (while accounting for the likelihood that the individual is still active) may offer more accurate forecasts.

5. Conclusion

Models that incorporate latent attrition ignore the role of marketing, whereas marketing response models have omitted the phenomenon of latent attrition. Our model aims to fill this gap by incorporating time-varying covariates into a latent attrition framework; it allows marketing to affect the rate at which individuals make contributions, the amount of the contributions, and the length of the latent lifetime over which they make contributions. It also accounts for within-donor correlation between interdonation times and donation amounts, as well as the nonrandom nature—both at a cross-sectional level and at a time-varying level—of the organization's mailings. Though we find that direct marketing increases the donation rate, our analysis reveals that high levels of direct marketing can adversely affect the latent lifetime for some.

By accounting for direct marketing's impact on three dimensions that comprise transactional activity (latent lifetime, transaction frequency, and transaction amount), we demonstrate how our model can quantify the financial impact of direct marketing over a specified time horizon; this can provide guidance as far as mail scheduling. Organizations may then weigh the cost of their marketing efforts against the expected gains, as well as assess on which individuals to focus their efforts. As our empirical application shows, such analysis may reveal that it is still worthwhile to send

[15] We assume a contribution margin of 90%, a fulfillment cost of \$1, and a mailing (contact) cost of \$1 per direct mailing sent. The details are in Appendix B.

[16] We also considered a heuristic in which donors were ranked first in terms of frequency of donation and then by recency. This did not differ substantially from the heuristic we present in Figure 9.

direct marketing to individuals who have not donated in quite some time, as the amount they donate may be larger if they are spurred to action. In addition, we demonstrate how our model can be employed to identify the most likely donors, yielding more accurate forecasts compared to alternative metrics.

There are a number of directions in which the current research can be extended. First, although we establish a link between donation behavior and direct marketing to account for both cross-sectional targeting and common shocks, alternative methods of linking customers' and organizational behavior could be examined. For example, a structural model that incorporates latent attrition may allow for the evaluation of alternative direct marketing schedules. Second, though our data do not include information on the message content, our framework could be used to examine which message is most effective for which individuals, enabling the organization to tailor its activities based on an individual's response to past marketing efforts. For example, the degree of overlap in content among messages may moderate the effects on incidence, quantity, and lifetime (Blattberg et al. 2008).

Future research may consider the short-term and long-term effects of marketing activity in contexts where customers conduct repeat purchases. In such settings, customers' utilities from later consumption experiences may systematically differ from their early consumption experiences and the effects of marketing may differ across these stages (e.g., Danaher et al. 2001, Schweidel and Fader 2009). Generalizing our latent attrition framework by incorporating additional active states into the HMM may offer a path toward investigating such phenomenon. Research should also examine the role of other forms of marketing, such as coupling direct marketing efforts with electronic communications, personal contact with development officers, mass marketing, and social media strategies (Moe and Trusov 2011, Moe and Schweidel 2012). Though some tactics may reduce the time between transactions, other forms may influence the duration of an individual's latent relationship. Such a nuanced understanding is key not only in estimating customer value but also in estimating the effectiveness of marketing actions so that resources may be allocated appropriately based on both the impact and cost of different marketing decisions.

Appendix A. Alternative Specification for Direct Marketing Stock Variable

In a field experiment, Anderson and Simester (2004) found evidence of a postpromotion dip for customers who received a special promotional catalog, suggesting purchase acceleration (Neslin et al. 1985). To allow for such a pattern, we consider a model specification in which our stock variable *DM* reflects the amount of direct marketing activity received since donor *i*'s most recent donation; that is, the stock variable "resets" following a contribution, which would capture the empirical pattern of a postpromotion dip. This is similar to the implementation by Moe and Fader (2004) in which the effect of prior website visits accumulates until a customer makes a purchase, at which point it resets to zero. Under this model specification, we assume that

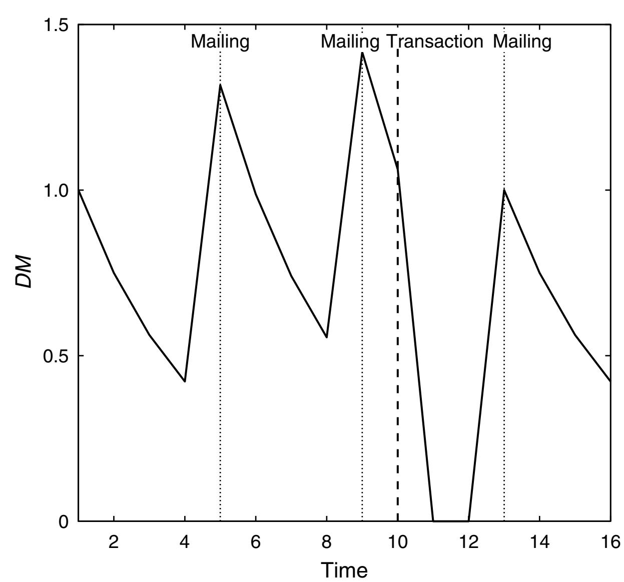
$$DM_{it} = Direct_{it} + \lambda \cdot DM_{i,\,t-1} \cdot I(A_{i,\,t-1} = 0), \tag{A1}$$

where A_{it} is the amount donated by individual i at time t. If a donation were to occur at $t-1$, the stock variable DM_{it} would be equal to 1 if direct marketing is received at time t ($Direct_{it} = 1$) and 0 otherwise. In contrast, if $A_{i,\,t-1} = 0$, then DM_{it} comprises the sum of direct marketing activity at time t ($Direct_{it}$) and the decayed prior stock ($\lambda \cdot DM_{i,\,t-1}$).

We provide a stylized illustration of how the *DM* stock variable evolves under Equation (A1) in Figure A.1 with $\lambda = 0.75$.

Following receipt of the first mailing, $DM = 1$. In subsequent months, it decays at a rate of λ until the donor receives the next mailing (indicated by the dotted vertical line in Figure A.1). The *DM* stock variable again diminishes until the donor receives the next mailing. When a transaction (i.e., donation) is conducted, as indicated by the dashed vertical line in Figure A.1, the stock variable resets in the next month to $DM = 0$. It remains at this level until the next mailing is received, at which time $DM = 1$ and subsequently decays. If *DM* positively affects the likelihood of making a donation, the specification presented in Equation (A1) would result in a postdonation dip in the probability of making a donation as a result of *DM* decreasing from its previous stock to $DM = 0$ following a donation, consistent with the empirical pattern of a postpromotion dip when acceleration is present. This specification for *DM* contrasts from that presented in Equation (4) by resetting following each transaction. The direct marketing stock described in Equation (4) continues to accumulate,

Figure A.1 Illustration of Direct Marketing Stock Variable *DM*



whereas that described in Equation (A1) assumes that the stock is "spent" on a transaction and then begins again at 0, which gives rise to the postpromotion dip and accelerated transactional behavior.

Recall that direct marketing may also affect the latent attrition process. If *DM* positively affects donation incidence when a donor is active and also increases the likelihood of latent attrition, we may observe that donation activity is temporally compressed into a shorter latent lifetime. This is distinct from the mechanism that gives rise to the postpromotion dip that is commonly associated with accelerated transactional behavior. We may find that data are or are not consistent with the postpromotion dip while a donor remains active; we may also find data that do or do not suggest that *DM* increases the likelihood of latent attrition. We therefore consider it an empirical question as to whether the direct marketing stock variable *DM* resets following a transaction.

To test this, we estimated the model that assumes *DM* resets, and it was found to yield poorer performance during both the calibration and forecasting periods. This suggests that the observed data are more consistent with a model in which the *DM* stock variable does not reset following a transaction. As such, we do not find evidence to suggest a postdonation dip characteristic of accelerated activity while donors are active. We therefore present result under the model specification in which *DM* accumulates across donations, as presented in Equation (4).

Appendix B. Economic Value of the Model

To evaluate the economic value, we use the decile cutoff approach of Blattberg et al. (2008, p. 279) to select customers to target with direct marketing. We assume that the organization mails (using the estimated mailing model) to all customers in a given decile if the predicted profit is positive. We calculate predicted profit per customer for a given decile d as

$$\Pi_d = a_d m - r_d f - c n_d. \tag{B1}$$

On the revenue side, a is the expected donation amount (taking into account both the likelihood of donation incidence and the expected donation amount when a donation incidence occurs), and m is the contribution margin (which we assume is 0.90). In terms of the costs incurred by the organization, r is the predicted response rate to direct marketing activity, f is the fulfillment cost (which we assume is \$1.00), c is the contact cost (which we assume is \$1.00), and n is the number of mailings sent. All predictions are based on model forecasts for the validation sample period, months 41–56; for the purposes of this exercise, we assume that this is one period by aggregating across months.

Table B.1 shows that only the top five deciles have positive predicted profits. The average predicted profit per donor targeted is \$13.50. If the organization randomly targeted its donors instead of using the model, the average predicted profit per donor is \$6.44. If no mailings are sent, as shown in Table B.1, this yields an expected profit of \$2.41 per donor. Hence the economic value of using the model to target donors raises predicted profits considerably.

Table B.1 Economic Value of the Model

Decile (based on the proposed model)	Predicted response rate (r)	Predicted donation amount (a)	Number of mailings sent (n)	Predicted profit per customer (Π)
1	0.90	35.87	11.9	19.44
2	0.54	42.37	10.8	26.78
3	0.24	26.84	8.6	15.28
4	0.18	13.90	6.7	5.67
5	0.00	4.43	3.5	0.37
6	0.00	0.43	1.0	−0.62
7	0.00	0.33	0.9	−0.64
8	0.00	0.31	0.9	−0.62
9	0.00	0.26	0.8	−0.59
10	0.00	0.24	0.9	−0.63
Average	0.19	12.50	4.6	6.44
Average with no mailings	0.09	2.77	0	2.41

References

Abe M (2009) "Counting your customers" one by one: A hierarchical Bayes extension to the Pareto/NBD model. *Marketing Sci.* 28(3):541–553.

Anderson ET, Simester DI (2004) Long-run effects of promotion depth on new versus established customers: Three field studies. *Marketing Sci.* 23(1):4–20.

Blattberg RC, Kim B-D, Neslin SA (2008) *Database Marketing: Analyzing and Managing Customers* (International Series in Quantitative Marketing) (Springer, New York).

Borle S, Singh SS, Jain DC (2008) Customer lifetime value measurement. *Management Sci.* 54(1):100–112.

Braun M, Schweidel DA (2011) Modeling customer lifetimes with multiple causes of churn. *Marketing Sci.* 30(5):881–902.

Danaher PJ, Smith MS (2011) Modeling multivariate distributions using copulas: Applications in marketing. *Marketing Sci.* 30(1):4–21.

Danaher PJ, Hardie BGS, Putsis WP Jr (2001) Marketing-mix variables and the diffusion of successive generations of a technological innovation. *J. Marketing Res.* 38(4):501–514.

Donkers B, Paap R, Jonker J-J, Franses PH (2006) Deriving target selection rules from endogenously selected samples. *J. Appl. Econometrics* 21(5):549–562.

Fader PS, Hardie BGS, Lee KL (2005a) "Counting your customers" the easy way: An alternative to the Pareto/NBD model. *Marketing Sci.* 24(2):275–284.

Fader PS, Hardie BGS, Lee KL (2005b) RFM and CLV: Using iso-value curves for customer base analysis. *J. Marketing Res.* 42(4):415–430.

Fader PS, Hardie BGS, Shang J (2010) Customer-base analysis in a discrete-time noncontractual setting. *Marketing Sci.* 29(6):1086–1108.

Franses PH, Paap R (2001) *Quantitative Models in Marketing Research* (Cambridge University Press, Cambridge, UK).

Fournier S, Dobscha S, Mick DG (1997) Preventing the premature death of relationship marketing. *Harvard Bus. Rev.* 75(1):42–49.

Gönül F, Shi MZ (1998) Optimal mailings of catalogs: A new methodology using estimable structural dynamic programming models. *Management Sci.* 44(9):1249–1262.

Gupta S, Park S (2012) Handling endogenous regressors by joint estimation using copulas. *Marketing Sci.* 31(4):567–586.

Jen L, Chou C-H, Allenby GM (2009) The importance of modeling temporal dependence of timing and quantity in direct marketing. *J. Marketing Res.* 46(4):482–493.

Kumar V, Reinartz WJ (2006) *Customer Relationship Management: A Database Approach* (John Wiley & Sons, New York).

Li S, Sun B, Montgomery A (2011) Cross-selling the right product to the right customer at the right time. *J. Marketing Res.* 48(4):683–700.

MacDonald IL, Zucchini W (1997) *Hidden Markov and Other Models for Discrete-Valued Time Series* (Chapman & Hall, London).

Malthouse EC (1999) Ridge regression and direct marketing scoring models. *J. Interactive Marketing* 13(4):10–23.

Manchanda P, Rossi PE, Chintagunta PK (2004) Response modeling with nonrandom marketing-mix variables. *J. Marketing Res.* 41(4):467–478.

Moe WW, Fader PS (2004) Dynamic conversion behavior at e-commerce sites. *Management Sci.* 50(3):326–335.

Moe WW, Schweidel DA (2012) Online product opinions: Incidence, evaluation, and evolution. *Marketing Sci.* 31(3):372–386.

Moe WW, Trusov M (2011) Measuring the value of social dynamics in online product forums. *J. Marketing Res.* 48(3):444–456.

Montoya R, Netzer O, Jedidi K (2010) Dynamic marketing resource allocation for long-term profitability: A pharmaceutical application. *Marketing Sci.* 29(5):909–924.

Neslin SA, Henderson C, Quelch J (1985) Consumer promotions and the acceleration of product purchases. *Marketing Sci.* 4(2):147–165.

Neslin SA, Rhoads EE, Wolfson P (2009) A model and empirical analysis of patient compliance and persistence in pharmaceuticals. Working paper, Dartmouth College, Hanover, NH.

Netzer O, Lattin JM, Srinivasan V (2008) A hidden Markov model of customer relationship dynamics. *Marketing Sci.* 27(2): 185–204.

Reinartz WJ, Kumar V (2000) On the profitability of long-life customers in a noncontractual setting: An empirical investigation and implications for marketing. *J. Marketing* 64(4):17–35.

Reinartz WJ, Kumar V (2003) The impact of customer relationship characteristics on profitable lifetime duration. *J. Marketing* 67(1):77–99.

Reinartz WJ, Thomas JS, Kumar V (2005) Balancing acquisition and retention resources to maximize customer profitability. *J. Marketing* 69(1):63–79.

Schmittlein DC, Peterson RA (1994) Customer base analysis: An industrial purchase process application. *Marketing Sci.* 13(1):41–67.

Schmittlein DC, Morrison DG, Colombo R (1987) Counting your customers: Who they are and what will they do next? *Management Sci.* 33(1):1–24.

Schweidel DA, Fader PS (2009) Revisiting dynamic changepoints: An evolving process model of new product sales. *Internat. J. Res. Marketing* 26(2):119–124.

Schweidel DA, Fader PS, Bradlow ET (2008a) A bivariate timing model of customer acquisition and retention. *Marketing Sci.* 27(5):829–843.

Schweidel DA, Fader PS, Bradlow ET (2008b) Understanding subscriber retention behavior across and within cohorts using limited information. *J. Marketing* 72(1):82–94.

Schweidel DA, Bradlow ET, Fader PS (2011) Portfolio dynamics for customers of a multiservice provider. *Management Sci.* 57(3):471–486.

Singh S, Borle S, Jain D (2009) A generalized framework for estimating customer lifetime value when customer lifetimes are not observed. *Quant. Marketing Econom.* 7(2):181–205.

Sun B, Li S (2011) Learning and acting on customer information: A simulation-based demonstration on service allocations with offshore centers. *J. Marketing Res.* 48(1):72–86.

Van Diepen M, Donkers B, Franses PH (2009) Dynamic and competitive effects of direct mailings: A charitable giving application. *J. Marketing Res.* 46(1):120–133.

Venkatesan R, Kumar V (2004) A customer lifetime value framework for customer selection and resource allocation strategy. *J. Marketing* 68(4):106–125.