



## Marketing Science

Publication details, including instructions for authors and subscription information:  
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To cite this article:

Jeffrey D. Shulman (2014) Product Diversion to a Direct Competitor. Marketing Science 33(3):422-436. <https://doi.org/10.1287/mksc.2013.0816>

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# Product Diversion to a Direct Competitor

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A manufacturer will often limit competition among downstream partners by authorizing only a select group of retailers to carry its product. However, it is not uncommon for authorized retailers to create an additional competitor by diverting units to an unauthorized seller. This paper presents an analytical model that demonstrates how diversion from authorized retailers to an unauthorized direct competitor can occur under circumstances not considered by the prior literature. In fact, diversion can represent a prisoner's dilemma whereby retailers diminish their own profit by selling to the unauthorized direct seller. The authorized retailer's profit loss actually increases as the per-unit diversion costs incurred by the authorized retailer decrease. The model also shows that the unauthorized direct seller earns greater profit by strategically procuring a unilaterally constraining quantity, even though this procurement strategy results in an equivalent increase in the quantity sold by the retailers. Combined, the results identify a new reason for diversion and its consequences for retailers and the unauthorized direct seller.

**Keywords:** pricing; gray markets; game theory

**History:** Received: June 22, 2011; accepted: September 10, 2013; Preyas Desai served as the editor-in-chief and John Zhang served as associate editor for this article. Published online in *Articles in Advance* October 24, 2013.

## 1. Introduction

Manufacturers often sell only to select authorized retailers. Among many reasons for this practice, selective distribution can reduce channel conflict and reward valuable retailers with higher profit (Antia et al. 2006). However, retailers themselves often undermine this process by selling the products to unauthorized direct sellers. For instance, salon product manufacturers often authorize only salons to serve as resellers of the product, yet salons divert to online competitors such as *drugstore.com* (Palmer 2004). Authorized resellers of Pioneer and Harman Kardon electronics also divert to unauthorized online sellers within the same market (*Consumer Electronics* 1999). In the flea control industry, it is estimated that diversion to unauthorized direct sellers totals as much as \$300 million per year (Luechtefeld 2008). This practice stretches into many industries including sporting goods, scuba gear, and electronics. Whereas diversion to different geographic markets takes advantage of the arbitrage opportunities created by disparity in wholesale prices, the above examples represent diversion within the same market. This is a practice known as *channel flow diversion* (Lowe and Rubin 1986). It is noteworthy that channel flow diversion can occur even if the unauthorized direct sellers ultimately steal customers from the retailers who divert the product. For instance, diversion costs salons an estimated \$100 million a year in lost sales (Gordon 1997), with one salon estimating a 10% loss in Aveda sales to unauthorized

sellers (Palmer 2004). Also, National Bicycle Dealers Association members Charlie McCorkell and Chris Holmes purportedly face increased price competition from Internet sellers of products diverted from authorized channels (see Clements 2013).

Prior research offers insight into why diversion occurs from one market to another (e.g., Duhan and Sheffet 1988, Ahmadi and Yang 2000, Xiao et al. 2011) but is relatively quiet on why an authorized retailer may divert to a direct competitor. The purpose of this paper is to offer a new explanation for channel flow diversion. A common rationale given for diversion within a market is a quantity discount schedule by the manufacturer that incents some retailers to buy in bulk to achieve the discount and to divert the additional units to another retailer (see Duhan and Sheffet 1988). However, profitable diversion of discounted units is at odds with profit loss estimates by retailers of salon products. Although there are many examples of diversion consistent with prior explanations (Antia et al. 2006), this research aims to demonstrate that diversion to a direct competitor can occur in situations not predicted by prior research.

The purpose of this paper is to answer the following research questions:

1. Why might diversion to a direct competitor occur in the absence of a quantity discount schedule?
2. What are the profit implications of the retailers' cost to divert when diversion to a direct competitor occurs?

3. What is the optimal number of units that the unauthorized direct seller will want to divert from authorized retailers?

Concerning the first research question, the model establishes an additional reason for channel flow diversion. I find that diversion can occur even if the unauthorized reseller directly competes with the authorized retailers and the retailers buy at a constant marginal cost per unit (i.e., no quantity discount schedule). In fact, retailers may divert to an unauthorized direct seller even if diversion represents a prisoner's dilemma.

The second research question is motivated by the fact that some products are more costly to divert than others. For instance, it costs more to ship heavier bottles of hair products than trading cards. Given the conditions under which diversion occurs, common intuition would suggest that retailers earn greater profit when they have lower costs. Also, from an operations perspective, logic would suggest that if diversion is going to take place, it would be better for the process to be more efficient and less costly. However, this paper identifies a strategic effect of diversion costs. In fact, the model finds that retailers' profits can increase with increases in the cost to divert.

In contrast to the two retailer-focused research questions, the final research question examines the unauthorized direct seller's decision. Specifically, because the procurement of diverted units occurs before the retail prices are set, an important question becomes whether the unauthorized direct seller will procure as many units as can profitably be sold. Research on quantity choice suggests that a Stackelberg leader will commit to a greater quantity than in a simultaneous-move game as a means to encourage the followers to sell a lesser quantity. However, strategic quantity commitments by all sellers have been shown to shift competition from Bertrand to Cournot and thus boost profit. Thus it is unclear whether an unauthorized direct seller would choose to (1) procure as many units as can be profitably sold, (2) strategically procure excess units, or (3) strategically procure fewer units. I show that in this situation, the unauthorized direct seller maximizes profit by strategically committing to buy a quantity of diverted units that will *unilaterally* constrain the unauthorized direct seller in subsequent pricing to consumers.

In sum, the findings of this paper help to explain the intriguing phenomenon of within-market diversion and illustrate the consequences for pricing and profits. They demonstrate that manufacturers may not be able to avoid diversion simply by ensuring that there are no price differentials between authorized markets. The findings show that technologies that reduce retailers' diversion costs do not necessarily increase their profit even if diversion occurs, and

they demonstrate that the procurement of diverted units by unauthorized direct sellers has strategic consequences that should be considered.

## 2. Literature Review

The literature offers several reasons for why a retailer may divert units. Diversion may arise because a gray marketer can procure units in one market at a lower cost and divert them to another market where the selling price is higher (Duhan and Sheffet 1988). Ahmadi and Yang (2000) and Xiao et al. (2011) demonstrate that diversion of this type can increase manufacturer profit. Transshipment literature finds that a retailer who experiences demand at the location exceeding the supply on hand will be able to fulfill the order by buying from another retailer. For instance, a Macy's in Chicago that has a customer order for an item not currently in its inventory will be able to supply this order by getting a Macy's in a nearby suburb to transship to the store in Chicago. A body of transshipment research examines ordering and inventory decisions when transshipments between resellers are coordinated centrally (e.g., Lee 1987, Axsäter 1990, Robinson 1990). Companies using centralized transshipments benefit from risk pooling and hence offset the costs associated with transshipping. Transshipments between retailers can also occur when inventory orders and transshipment purchase decisions are made locally by the independent resellers (e.g., Rudi et al. 2001, Hu et al. 2007, Huang and Sošić 2010). In contrast, this paper examines whether diversion to a direct competitor can occur when the direct competitor would not exist without diversion from the authorized retailers.

Another line of prior research examines the impact of *authorized* direct sellers on brick-and-mortar retailers. In a seminal paper, Balasubramanian (1998) identifies equilibrium pricing when local retailers face competition from a direct seller and shows that the direct seller may earn greater profit by strategically limiting the number of customers who are aware of its offering. A body of research examines the causes and implications of a manufacturer selling directly to consumers through a vertically integrated channel that competes with its independent resellers (e.g., Chiang et al. 2003, Vinas and Anderson 2005, Kumar and Ruan 2006, Yoo and Lee 2011). Another body of research looks at multichannel "bricks-and-clicks" retailers who sell both online and in stores (Viswanathan 2005, Zhang 2009, Ofek et al. 2011). Several papers empirically examine pricing competition between online sellers and traditional retailers (Brynjolfsson and Smith 2000, Brynjolfsson et al. 2009, Forman et al. 2009, Overby and Jap 2009). In contrast, this paper focuses on situations in which the manufacturer does not authorize a direct channel, yet the

authorized retailers endogenously choose to divert to an unauthorized direct seller.

Although Chen and Riordan (2007) and Ishibashi and Matsushima (2009) do not directly consider diversion, they do find that profit can increase with the number of competitors, which would conceivably make sellers willing to create an additional competitor. The Chen and Riordan result requires that the additional competitors decrease the demand elasticity. The Ishibashi and Matsushima result is driven by the fact that the consumers served by the additional competitors have a different sensitivity to inter-firm price differences than the high-end market competitors. The situation studied in the current paper is materially different in that the addition of a direct seller does not decrease demand elasticity, and the consumers who ultimately purchase from the direct seller need not be unique in their sensitivity to inter-firm price differences. Although a similar conclusion could be drawn from the literature that a firm will be willing to create a competitor, the mechanism behind the finding in this paper differs substantially. This leads to divergent predictions. Whereas the profit improvement effect of competition in each of these papers corresponds with an increase in prices and retail markups, the current paper finds that diversion to a direct competitor who intensifies price competition diminishes retail markups.

The finding that retailers may divert to a direct competitor even if the end result is a reduction in profit contributes to the growing body of research identifying the prisoners' dilemmas firms face in a broad range of marketing contexts. Prior research finds that prisoner's dilemmas arise in targeted coupons (e.g., Shaffer and Zhang 1995, Chen et al. 2001), channel partner choice (Coughlan 1985), loss leader pricing with rain checks (Hess and Gerstner 1987), advertising (e.g., Chen et al. 2009), and exchange programs (Desai et al. 2013). Prior research in the durable goods literature also finds that unauthorized sales can occur, though for different reasons, in the form of secondary markets for used durables (e.g., Desai et al. 2004, Shulman and Coughlan 2007, Gümüş et al. 2013). This research demonstrates when self-defeating diversion by retailers to an unauthorized direct seller occurs in equilibrium.

Finally, I identify the optimal number of diverted units ordered by the unauthorized direct seller. Kreps and Scheinkman (1983) find that capacity commitment can shift undifferentiated competitors from Bertrand (and hence, zero profit) to Cournot (and hence, positive profit) competition. Similarly, Maggi (1996) finds that differentiated competitors can mitigate competition via quantity commitments. Thus competition intensity can be reduced when *both firms* commit to the quantity before choosing pricing. When

only one firm commits to the quantity, Daughety (1990) finds a Stackelberg leader will commit to a *greater* quantity than it would in a simultaneous-move game. In contrast, I find a scenario in which one seller (the unauthorized direct seller) commits to a *lesser* quantity, even though its competitors are not only unconstrained but also sell greater quantities than if no quantity commitment were made.

In summary, I make several main contributions to the existing literature. First, I demonstrate how diversion can occur in situations not predicted by the prior literature. This occurs even if diversion results in less profit and lower retail price markups for the authorized retailers who divert. Second, this is the first research to my knowledge to show that the profit of a retailer who diverts increases with the per-unit cost of diversion. Finally, the paper is the first to my knowledge to show how a unilateral commitment to a lower quantity by an unauthorized direct seller can improve its profit even when such a commitment results in an equivalent increase in quantity sold by the competing retailers.

### 3. Model

In this section, I describe the players of the game and their payoff functions. All notations are summarized in Table 1. There are four types of players in the game. There is a manufacturer who produces a single product and relies on intermediaries to reach consumers. There are four horizontally differentiated authorized retailers who procure units from the manufacturer. There is a direct seller who, when left unauthorized, only has access to units if it can buy them from the authorized retailers. There are consumers who are heterogeneous in their preference for sellers. I describe each player in more detail below.

**Table 1** Parameters and Decision Variables

Variable	Definition
$u$	Consumer reservation utility of owning the product
$t$	Transportation cost for consumers
$\mu$	Consumer lack of fit with the direct seller
$\theta_i$	Consumer $i$ 's location
$c$	Marginal cost to authorized retailer per unit diverted
$x_j$	Location of retailer $j$
$p_j$	Retail price for retailer $j$
$p_d$	Selling price for unauthorized direct seller
$w_d$	Per-unit diversion price paid by unauthorized direct seller to retailers
$w$	Manufacturer per-unit wholesale price charged to authorized retailers
$q_d$	Total quantity sold to consumers by unauthorized direct seller
$q_{jt}$	Quantity of units sold to unauthorized direct seller by retailer $j$
$q_j$	Quantity sold to consumers by retailer $j$
$\hat{q}_{kt}^j$	Retailer $j$ 's belief about the quantity of units diverted by retailer $k$
$e_j$	Effort by retailer $j$
$\xi$	Sum of effort over all retailers (i.e., $\sum_{j=1}^4 e_j$ )

### 3.1. Consumers

Consumer preferences are uniformly distributed along a circular location model (see also Salop 1979, Balasubramanian 1998, Shulman et al. 2011). The utility of consumer  $i$  located at  $\theta_i$  buying from retailer  $j$  located at  $x_j$  is equal to  $u - t|x_j - \theta_i| - p_j$ , where  $u$  is the reservation utility from owning the product,  $t$  is the transportation cost, and  $p_j$  is the retail price from retailer  $j$ . All consumers along the circle buy one product (i.e., it is assumed that the reservation utility  $u$  is sufficiently high relative to the transportation cost; any  $u > t/3$  will suffice). The size of the market depends on retailer effort, which is described in more detail in §3.3. The discussion in §5 remarks on the robustness of the findings to this assumption. Consumers who buy from the direct seller will derive utility equal to  $u - \mu - p_d$ , where  $\mu$  is a measure of the lack of fit with the direct seller and  $p_d$  is the price from the direct seller. The lack of fit from the direct seller  $\mu$  captures the inconvenience of waiting for delivery, the inability to talk to a live salesperson, the cost of shipping, and/or the perceived reliability of the direct seller. Thus, consumer  $i$  will buy from the direct seller if  $u - t|x_j - \theta_i| - p_j < (u - \mu - p_d)$  for all  $j$  and will otherwise purchase from the retailer  $j$ , whose price and location result in the greatest value of  $u - t|x_j - \theta_i| - p_j$ . I focus on situations in which a direct seller can earn a positive contribution on a positive market share:  $\mu < t/4$ . Notice that although  $\mu$  is constant across consumers (as in Balasubramanian 1998, Liu and Zhang 2006, and Jeffers and Nault 2011), the formulation of the model captures the empirically validated phenomenon that the consumer's preference for buying online versus buying in the store depends on her location (Forman et al. 2009).

### 3.2. Direct Seller

There is a direct seller located in the center of the circle. It is assumed that the direct seller's fixed cost of entry,  $K_d$ , is sufficiently high such that only one direct seller enters the market, though the qualitative findings are robust to this assumption. The direct seller sets its selling price,  $p_d$ , simultaneously with the authorized retailers. If the direct seller is authorized by the manufacturer, it procures units from the manufacturer at a constant per-unit wholesale price.

If the direct seller is unauthorized, it can only access units if it can buy them from the authorized retailers at a diversion price,  $w_d$ . It should be noted that gray-market activity is a legal practice. Moreover, it is well documented that manufacturer efforts to deter diversion are often unsuccessful (Antia et al. 2006). The current model applies to situations in which the manufacturer either passively allows diversion or is unsuccessful in any enforcement attempts occurring outside of this model. In the interest of parsimony,

I assume that the direct seller makes simultaneous take-it-or-leave-it offers to the authorized retailers regarding  $w_d$  and the quantity to buy from retailer  $j$ ,  $q_{jt}$ . Note that the qualitative result that diversion can occur is robust to the take-it-or-leave-it offer assumption and is preserved with different bargaining processes between the unauthorized direct seller and the retailers.

### 3.3. Authorized Retailers

The four authorized retailers are distributed evenly along the circle, with the location of retailer  $j \in \{1, 2, 3, 4\}$  denoted by  $x_{j,}$ , and they procure products from the manufacturer at a per-unit rate of  $w$ . Note that the assumption of four authorized retailers allows for parsimony in the analysis while capturing certain complexities of a general model of  $N$  retailers.<sup>1</sup> Retailer  $j$  chooses a retail price  $p_j$  to charge to consumers. The quantity sold to consumers by retailer  $j$  is  $q_j$ . An authorized retailer may accept the opportunity to sell  $q_{jt} \geq 0$  units to the unauthorized direct seller at a per-unit diversion price  $w_d$ . Authorized retailers do not observe the diversion decisions by the competing retailers, but each retailer  $j$  forms beliefs regarding the diversion quantity by each retailer  $k \neq j$  denoted by  $\hat{q}_{kt}^j$ . Authorized retailers incur an additional cost,  $c$ , for each unit diverted to the unauthorized direct seller. Examples of this cost include the opportunity cost of time spent in preparing each unit for diversion and any costs paid to third-party shippers.

I consider markets for which the number of authorized retailers is established before the possibility of authorizing a direct seller emerges. This applies to mature companies for which the retailer relationships have long been established. Alternatively, there may be an exogenous shock to direct seller entry costs occurring after the authorization decision that makes direct seller entry possible when it was previously cost prohibitive. For example, there may be an unanticipated drop in technology and logistics infrastructure costs. It also implies that exit and relocation costs are nontrivial so that the number of authorized retailers persists if a direct seller is added. In the interest of parsimony, I consider a market with four established authorized retailers. In the appendix, I describe conditions that will endogenously result in four authorized retailers and prove that the conditions are consistent with those that generate the key insights of this paper.

<sup>1</sup> In particular, a model of  $N > 4$  shares with the current model the possibility that a retailer competes for customers with its adjacent retailers while experiencing a strategic effect of the prices of the competitors located farther away. A model of  $N > 4$  also shares with the current model an allowance for the unauthorized direct seller to procure units from multiple retailers while not engaging in trade with multiple other retailers (a strategy I will show to be dominated).

The primary interest of the paper is retailer diversion to an unauthorized direct seller. The aforementioned assumptions will generate our key results regarding retailers' decisions to divert to the unauthorized direct seller. However, for diversion to occur in equilibrium, it must also be that the direct seller is not endogenously authorized by the manufacturer. Carlton and Chevalier (2001) hypothesize that manufacturers who rely heavily on physical sales effort may limit the availability of their products on the Internet to control the free-rider problem. For example, manufacturers of salon products cite the importance of salons in helping consumers understand the value of the products (Palmer 2004). The assumptions on retailer effort are sufficient, though not necessary, for the diversion equilibrium.

I allow a retailer to exert effort that expands the entire market. The total market size is then equal to  $\sum_{j=1}^4 e_j$ , with the uniform distribution of consumer preferences for firms. Retailer  $j$  effort costs are convex:  $e_j^2/2$ . It is important to note that the results are robust to several modifications to how effort is modeled. The qualitative results are preserved if effort expands the market only locally, where "locally" can be defined to be the arc of length  $1/2$  surrounding the retailer's location (covering a distance  $1/4$  in either direction from the retail location) or a distance  $1/8$  in either direction of the retailer's location.

As an example of retailer effort, bicycle retailers can attract more consumers to the cycling market by hosting events and advertising so as to build greater awareness of and induce new customers to try cycling. Consider also the value of salons in terms of helping a customer use products best suited to his or her hair type, thereby increasing the demand of the hair product. Although the retailer benefits from the increased brand value and customer base, some consumers may choose to buy from the online seller (if there is one).

### 3.4. Manufacturer

The manufacturer first decides whether to authorize a direct seller to enter the market. Following the authorization decision, the manufacturer sets the per-unit wholesale price  $w$ , charged to intermediaries in

anticipation of the market equilibrium. A simple contracting structure is consistent with the literature in channels (e.g., Liu and Zhang 2006, Cui et al. 2007). Without loss of generality, unit production costs are normalized to 0. In a fully covered market, the manufacturer's payoff is  $w \sum_{j=1}^4 e_j$ .

### 3.5. Timing of the Model

In the initial stage, the manufacturer decides whether to authorize the direct seller. The manufacturer sets the wholesale price before authorized retailers choose effort levels. Following the effort decision, if the direct seller is left unauthorized, it makes take-it-or-leave-it offers of diversion terms to the authorized retailers, who simultaneously accept or reject based on their beliefs about the direct seller's agreements with the remaining retailers. If the direct seller is authorized, this stage (Stage 3 of Figure 1) does not occur. All sellers then set prices simultaneously. In the final stage, consumers decide from which seller to buy. Figure 1 summarizes the timing of the model (though Stage 3 does not occur when the direct seller is authorized), and Table 1 summarizes the parameters and decision variables.

## 4. Results

The model is solved using backwards induction. First, consider the consumers' Stage 5 purchase decision. Consider consumer choice in the presence of the direct seller. A consumer located a distance  $y = |\theta_i - x_j|$  from retailer  $j$  is indifferent between buying from retailer  $j$  and buying from the direct seller if  $p_j + ty = p_d + \mu$ . Notice that if the direct seller makes positive sales, it must be that the consumer who is indifferent between buying from retailer  $j$  and buying from the adjacent retailer gets a greater surplus buying from the direct seller than from any retailer. Although in this model the presence of the direct seller implies that consumers choose between their closest retailer and the direct seller (rather than a nearby retailer), the model results hold when a proportion of consumers chooses between retailers.

Consider the interval  $\theta_i \in [x_j - 1/8, x_j + 1/8]$  for each  $j$ . Comparing utilities, consumers in the interval  $\theta_i \in [x_j - y, x_j + y]$  will buy from retailer  $j$ , where  $y = (p_d - p_j + \mu)/t$ . Consumers in the intervals

Figure 1 Timing of the Model

Stage 0	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
Manufacturer decides whether to authorize direct seller	Manufacturer sets the wholesale price	Authorized retailers choose effort levels	Unauthorized direct seller offers diversion terms to retailers who simultaneously accept or reject	Retail prices and direct selling price are set simultaneously	Consumers decide from whom to buy

$\theta_i \in [x_j - 1/8, x_j - y]$  and  $\theta_i \in [x_j + y, x_j + 1/8]$  will prefer to buy from the direct seller than retailer  $j$ . Let  $e_j$  denote the effort level of retailer  $j$ . The thickness of the circle is  $\sum_{j=1}^4 e_j$ . Denoting  $\sum_{j=1}^4 e_j$  by  $\xi$ , demand for any retailer  $j$ ,  $q_j$ , and demand for the direct seller,  $q_d$ , can thus be written as

$$q_j = 2\xi(p_d - p_j + \mu)/t, \quad (1)$$

$$q_d = \xi \left( 1 - \sum_{j=1}^4 \frac{2(p_d - p_j + \mu)}{t} \right). \quad (2)$$

Before examining the manufacturer's Stage 0 decision of whether to authorize a direct seller, I first examine the equilibrium outcome if the direct seller is left unauthorized. Upon examining whether diversion will occur if the direct seller is not authorized, I subsequently check whether the manufacturer will leave the direct seller unauthorized in Stage 0.

I first solve for the retailers' and unauthorized direct seller's simultaneous pricing decisions for any positive procurement by the unauthorized direct seller (i.e.,  $q_{jt} \geq 0$ ,  $\sum_{j=1}^4 q_{jt} > 0$ ). The direct seller has the following constrained optimization problem, where  $\lambda$  is the Lagrange multiplier:

$$\begin{aligned} \max_{p_d, \lambda} \quad & p_d q_d - \lambda \left( q_d - \sum_{j=1}^4 q_{jt} \right) - w_d \sum_{j=1}^4 q_{jt} \\ \text{s.t.} \quad & \lambda \geq 0, \quad q_d - \sum_{j=1}^4 q_{jt} \leq 0, \quad \lambda \left( q_d - \sum_{j=1}^4 q_{jt} \right) = 0, \end{aligned}$$

where  $q_d$  is a function of prices defined in Equation (2). Note that in the pricing game, the per-unit procurement cost is sunk and does depend on the number of units sold by the direct seller.

Also in Stage 4, each authorized retailer chooses the price to maximize her own profit  $\pi_j = (p_j - w)q_j + X_j$ , where  $q_j$  is defined in Equation (1) and  $X_j = (w_d - w - c)q_{jt}$  if the retailer diverts and  $X_j = 0$  otherwise. Since  $\{w_d, q_{jt}\}$  are set in the prior stage and  $w$  is the same for all retailers, the pricing game is symmetric with respect to retailers.

Since the retailer pricing game is symmetric, I simultaneously solve for the retail prices for each retailer  $j$ ,  $p_j^*$ , and unauthorized direct seller price,  $p_d^*$ , such that  $\partial \pi_j / \partial p_j = 0$ ,  $\partial \pi_d / \partial p_d = 0$ ,  $\lambda(q_d - \sum_{j=1}^4 q_{jt}) = 0$ , and  $\lambda \geq 0$ . The subgame prices are presented in Table A.1 of the appendix. The equilibrium prices depend on whether the unauthorized direct seller is constrained by the diversion quantity chosen in the prior stage.

In Stage 3, the unauthorized direct seller chooses  $q_{jt}$  for  $j \in \{1, 2, 3, 4\}$  and  $w_d$  to maximize profit subject to the constraint that retailer  $j$  finds it optimal to accept the offer. There are two possibilities

to consider with respect to the total number of procured units. If  $\sum_{j=1}^4 q_{jt} > (2\xi(t + 4(w - \mu)))/(3t)$ , then the direct seller will be unconstrained by quantity in the subsequent pricing game. If  $\sum_{j=1}^4 q_{jt} < (2\xi(t + 4(w - \mu)))/(3t)$ , the direct seller will be constrained by quantity in the subsequent pricing game. There are four possibilities regarding the source of the diverted quantity to consider: (a) all retailers divert, resulting in  $q_{jt} > 0$  for all  $j$ ; (b) a single retailer does not divert (without loss of generality (w.l.o.g.),  $q_{1t} > 0$  for  $j \in \{2, 3, 4\}$  and  $q_{1t} = 0$ ); (c) only two retailers divert (w.l.o.g., only  $q_{3t} > 0$ ,  $q_{4t} > 0$ ); and (d) only one retailer diverts (w.l.o.g., only  $q_{4t} > 0$ ). It is also possible that all retailers reject the offer, resulting in  $q_{jt} = 0$  for  $j \in \{1, 2, 3, 4\}$ . I first determine the optimal source(s) of diverted quantity for the unconstrained and constrained quantity cases and then compare profits to determine the direct seller's optimal diversion contracts to offer the retailers.

I show in the appendix that in the absence of diversion, the Stage 4 prices are  $p_j^{nd} = w + t/4$  and authorized retailer profits are  $\pi_j^{nd} = t\xi/16$ . The unauthorized direct seller payoff in the absence of diversion is 0. Now consider each retailer's decision of whether to accept a diversion contract. Let retailer  $j$ 's beliefs about the quantity diverted by retailer  $k \neq j$  be denoted by  $\hat{q}_{kt}^j \geq 0$ . If retailer  $j$  has beliefs such that  $\sum_{k \neq j} \hat{q}_{kt}^j > 0$ , then retailer  $j$  will accept the offer to divert  $q_{jt}$  units if and only if  $\pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j) > \pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j | q_{jt} = 0)$ , where the function  $\pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j)$  is determined by the Stage 4 pricing game and is derived by replacing  $\sum_{j=1}^4 q_{jt}$  with  $q_{jt} + \sum_{k \neq j} \hat{q}_{kt}^j$  in the equations presented in Table A.1. If retailer  $j$  has beliefs such that  $\sum_{k \neq j} \hat{q}_{kt}^j = 0$ , then retailer  $j$  will accept the offer to divert  $q_{jt}$  if and only if  $\pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j) > \pi_j^{nd}$ .

To constitute an equilibrium, the retailers' beliefs about diversion quantities must be mutually consistent. Thus, I solve for the direct seller's optimal diversion quantities given any set of retailer beliefs about the diversion quantities of the remaining retailers  $\hat{q}_{kt}^j \geq 0$  and then solve for the diversion quantities and retailer beliefs such that each retailer's beliefs about each  $q_{jt}$  are correct. I first establish the equilibrium given retailers hold consistent beliefs such that  $\sum_{k \neq j} \hat{q}_{kt}^j > 0$  for each  $j$ . In the appendix, I prove that  $\sum_{k \neq j} \hat{q}_{kt}^j = 0$  is not a rational belief for sufficiently low  $\mu$ .

If the unauthorized direct seller chooses to buy diverted units from each retailer, then the unauthorized direct seller chooses  $q_{jt}$  for  $j \in \{1, 2, 3, 4\}$  and  $w_d$  to maximize profit subject to the constraint that all retailers accept the offers. The unauthorized

direct seller has the following constrained optimization problem:

$$\begin{aligned} \max_{\lambda_{dj}, q_{jt}, w_d} & \left\{ \pi_d^*(q_{jt}, w_d) + \lambda_{dj} [\pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j) \right. \\ & \quad \left. - \pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j | q_{jt}=0)] \right\} \\ \text{s.t. } & \lambda_{dj} [\pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j) \\ & \quad - \pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j | q_{jt}=0)] = 0, \quad \lambda_{dj} \geq 0 \\ & \text{for } j = \{1, 2, 3, 4\}. \end{aligned} \quad (3)$$

If the unauthorized direct seller chooses to procure diverted units from all but one retailer (w.l.o.g., retailer 1 is excluded), the unauthorized direct seller has the following constrained optimization problem:

$$\begin{aligned} \max_{\lambda_{dj}, q_{jt}, w_d} & \left\{ \pi_d^*(q_{jt}, w_d | q_{1t}=0) + \lambda_{dj} [\pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j) \right. \\ & \quad \left. - \pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j | q_{jt}=0)] \right\} \\ \text{s.t. } & \lambda_{dj} [\pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j) \\ & \quad - \pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j | q_{jt}=0)] = 0, \quad \lambda_{dj} \geq 0 \\ & \text{for } j = \{2, 3, 4\}. \end{aligned} \quad (4)$$

If the unauthorized direct seller chooses to procure diverted units from only two retailers (w.l.o.g., retailers 3 and 4), the unauthorized direct seller has the following constrained optimization problem:

$$\begin{aligned} \max_{\lambda_{dj}, q_{jt}, w_d} & \left\{ \pi_d^*(q_{jt}, w_d | q_{1t}=q_{2t}=0) + \lambda_{dj} [\pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j) \right. \\ & \quad \left. - \pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j | q_{jt}=0)] \right\} \\ \text{s.t. } & \lambda_{dj} [\pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j) \\ & \quad - \pi_j^*(w_d, q_{jt}, \hat{q}_{kt}^j | q_{jt}=0)] = 0, \quad \lambda_{dj} \geq 0 \\ & \text{for } j = \{3, 4\}. \end{aligned} \quad (5)$$

If the unauthorized direct seller chooses to procure diverted units from retailer 4 only, then the only set of beliefs that can be confirmed as rational for retailer 4 is  $\sum_{k \neq 4} \hat{q}_{kt}^k = 0$ . Thus, I can examine the unauthorized direct seller optimization problem as choosing  $q_{4t}$ ,  $w_d$  to maximize profit subject to retailer 4 accepting the offer to be the only retailer that diverts. The unauthorized direct seller has the following constrained optimization problem:

$$\begin{aligned} \max_{\lambda_{d4}, q_{4t}, w_d} & \left\{ \pi_d^*(q_{4t}, w_d | q_{1t}=q_{2t}=q_{3t}=0) \right. \\ & \quad \left. + \lambda_{d4} [\pi_4^*(w_d, q_{4t}, \hat{q}_{kt}^4) - \pi_4^{nd}] \right\} \\ \text{s.t. } & \lambda_{d4} [\pi_4^*(w_d, q_{4t}, \hat{q}_{kt}^4) - \pi_4^{nd}] = 0 \quad \lambda_{d4} \geq 0. \end{aligned} \quad (6)$$

From each of the optimization problems of Equations (3)–(6), I find unique solutions such that each

retailer's beliefs are consistent with the solution. I compare direct seller payoffs across each optimization problem for both the constrained pricing game and the unconstrained pricing game. The result is summarized in the following proposition.

**PROPOSITION 1.** *Diversion from both retailers to the unauthorized direct seller occurs in equilibrium if the consumer lack of fit with the direct seller is not too great. Specifically, if  $\mu < \hat{\mu} \equiv 3t/16 - c$ , a diversion equilibrium exists in which each authorized retailer diverts  $q_{jt}^* = \xi(3t - 16c - 16\mu)/(28t)$  units. The unauthorized direct seller procures units at a diversion price of  $w_d^* = w + (144c + 5t + 16\mu)/128$ .*

I first remark on the significance of Proposition 1. I then describe the intuition for the result. I also further clarify the generality of the finding. Proofs of all statements are in the appendix.

Proposition 1 demonstrates that it can be an equilibrium strategy for retailers to engage in channel flow diversion to a direct competitor who serves the same market of consumers. This result is new to the literature. It is noteworthy that diversion occurs in equilibrium even though the number of diverted units by each retailer in the diversion equilibrium is equal to the number of customers the direct seller steals from each authorized retailer (i.e.,  $\forall j, q_{jt}^* = \xi(\frac{1}{4} - q_{jt}^*)$ ). Moreover, the diversion price markup,  $w_d - w$ , is less than the retail price markup had the direct seller not existed (i.e.,  $p - w = t/4$ ).

Although it is counterintuitive for a retailer to sell its stock to a direct competitor at a lower markup than could be sold to consumers, the finding makes sense when one considers the strategic interaction between players. The profit loss associated with an authorized retailer stocking the competition with diverted units is distributed across all competing authorized sellers. Therefore, the marginal loss to each firm from the additional competitor is less than the marginal gain from the trade. Moreover, each firm correctly believes that if it does not cooperate with the unauthorized direct seller, the competing authorized retailers will. Thus diversion to a direct competitor can still occur even if the diversion price is less than the retail price of the sale that is lost to the direct seller.

Notice that Proposition 1 is developed in advance of solving for the equilibrium wholesale price and effort levels. This is because the diversion quantity holds for any wholesale price and is expressed in terms of general effort levels (for which equilibrium solutions are subsequently presented). Note the diversion result persists if the unauthorized direct seller does not make take-it-or-leave-it offers, though the specific prices and quantities depend on the bargaining process between the unauthorized direct seller and the retailers. I next look at the unauthorized direct seller's ordering decision.



**PROPOSITION 2.** *The unauthorized direct seller strategically buys a quantity of diverted units that leaves it unilaterally constrained by quantity in setting the selling price. In other words, the equilibrium quantity sold by the unauthorized direct seller is less than the quantity that would be sold in an unconstrained pricing game.*

See the appendix for the proof.

In the pricing subgame, the unauthorized direct seller can sell no more than the number of units diverted by the authorized retailers in the previous stage. The equilibrium prices described in the appendix (and the resulting quantity sold) depend on whether the unauthorized direct seller procures a sufficient supply to satisfy the resulting demand. If the unauthorized direct seller has a limitless supply, the equilibrium prices will lead to direct sales equal to  $q_d^{**} = 2\xi(t + 4w - 4\mu)/(3t)$ . However, the unauthorized direct seller will limit itself to procuring  $q_d^* < q_d^{**}$  units. A typical game in which a firm can Stackelberg lead in its choice of quantity will lead to a higher choice of quantity than that of a simultaneous-move game (Daughety 1990); here, the unauthorized direct seller boosts profit by strategically buying fewer units than would subsequently be optimal to sell. In the current model, the inability to procure units directly from the manufacturer presents the unauthorized direct seller with an opportunity to credibly constrain capacity in its offer to buy from authorized retailers. By buying a lesser quantity from the authorized retailers, the unauthorized direct seller mitigates price competition and raises equilibrium prices by all sellers. This is counterintuitive given that the *unilateral* commitment to a lower quantity results in a greater quantity sold by the authorized resellers, who could still choose to drive down prices.

This capacity constraint, however, limits the unauthorized direct seller's incentive to cut price because it cannot serve demand beyond its supply of units. Also, the diversion price  $w_d$  paid to the authorized retailers is higher with the constrained capacity, further diminishing the unauthorized direct seller's incentive to cut price. As a consequence, the competing retailers can be less aggressive on price and still achieve sales of a given quantity. Thus, the unauthorized direct seller's unilateral commitment to divert fewer units than would subsequently be optimal to sell softens price competition and benefits all sellers.

It should be noted that this result occurs when the diversion agreements are established *before* the selling prices. However, logic suggests it is robust to the assumption that the unauthorized direct seller makes take-it-or-leave-it offers. This is because, in a bargaining game, the equilibrium quantity and wholesale price will lie somewhere between the retailers' preferred values and the unauthorized direct seller's

preferred values. If the unauthorized direct seller has sufficient bargaining power, the resulting quantity will be close to the take-it-or-leave-it offer equilibrium and will still constrain the pricing game.

I now turn attention to retailer profitability when diversion occurs. Equilibrium retailer effort levels must first be established. In Stage 2, each retailer  $j$  chooses effort to maximize  $\max_{e_j} \{(p_j^* - w)q_j^* + (w_d^* - w - c)q_{jt}^* - e_j^2/2\}$ . Notice that at Stage 4 prices and the Stage 3 diversion price (each presented in Table 2), the objective function is independent of the manufacturer's wholesale price. The equilibrium prices, quantities, contribution, effort levels, and wholesale price are presented in Table 2. The following proposition demonstrates that the presence of the unauthorized direct seller can reduce retailer profit even though the unauthorized direct seller relies on retailers as the source of supply.

**PROPOSITION 3.** *The opportunity to divert to the unauthorized direct seller can represent a prisoner's dilemma whereby each authorized seller earns less profit than if diversion were not feasible.*

The key insight from Proposition 3 is that improved profitability is not a necessary condition for diversion. In fact, retailers may choose to help a direct competitor come into existence even if the diversion results in diminished profit. This is because the retailers correctly believe that the unauthorized direct seller will be able to come to mutual agreements with competing authorized retailers who recognize that the  $q_{jt}$  units diverted will steal shares equally from each of the retailers. Each retailer would prefer to sell to the direct competitor than have the direct competitor's request fulfilled by the remaining authorized retailers. Therefore, even if the direct competitor reduces retailer profit, diversion can occur in equilibrium.

It should be noted that Proposition 3 is not meant to suggest that diversion always represents a prisoner's dilemma. In this model, market size does not grow in the presence of a direct seller. This assumption helps isolate a novel strategic reason for diversion, but it is possible that the unauthorized direct seller instead increases the potential market. The profitability of diversion for the authorized retailers relative to when diversion is infeasible logically increases with the degree to which the unauthorized direct seller attracts new consumers to the market. Proposition 3 is robust to small levels of market growth by the unauthorized direct seller since the profit difference between when diversion is feasible and when it is infeasible is strictly negative without the direct seller increasing the size of the total market.

Next I consider the impact of retailer diversion costs on profitability. Recall that retailers face a per-unit cost of transshipping units to the unauthorized

**Table 2** Equilibrium Prices, Quantities, and Profits

Equilibrium outcome	$\mu < 3t/16 - c$ (diversion occurs in equilibrium)	$\mu \geq 3t/16 - c$ (no diversion occurs in equilibrium)
Retail price for each authorized retailer	$p_r^* = w^* + (4c + t + 4\mu)/14$	$p_r^* = w^* + t/4$
Unauthorized direct seller price	$p_d^* = w^* + (4c + t - 3\mu)/7$	N/A
Diversion price	$w_d^* = w^* + (144c + 5t + 16\mu)/128$	N/A
Quantity sold to consumers by each authorized retailer	$q_r^* = 4e^*(4c + t + 4\mu)/(7t)$	$q_r^* = e^*$
Unauthorized direct seller quantity	$q_d^* = 4e^*(t - 4c - 4\mu)/(5t)$	0
Contribution for each authorized retailer	$\pi_r^* = 4e^*(48c + 19t + 48\mu)^2/(25088t) - (e^*)^2/2$	$\pi_r^* = e^*(t/4) - (e^*)^2/2$
Unauthorized direct seller contribution	$\pi_d^* = 31e^*(16c - 3t + 16\mu)^2/(1568t)$	0
Wholesale price charged by manufacturer	$w^* = u - (4c + t + 4\mu)/7$	$w^* = u - 3t/8$
Retailer effort level	$e^* = (48c + 19t + 48\mu)^2/(25088t)$	$e^* = t/16$

direct seller. These costs include administrative hours in processing, packing, and reshipping, as well as shipping charges paid to a third party. In the following proposition, I describe how these diversion costs affect profitability.

**PROPOSITION 4.** *The profit retailers earn when diverting to the unauthorized direct seller increases with the per-unit cost to the retailers of diverting (i.e.,  $\partial\pi_r^*/\partial c > 0$ ).*

Proposition 4 demonstrates the counterintuitive result that a reduction in costs would actually reduce profit. This result is driven by the strategic implications of diversion costs. When a retailer incurs a greater cost per unit to divert, it will demand a greater diversion price from the unauthorized direct seller. The greater diversion price has two implications for the unauthorized direct seller. First, the unauthorized direct seller passes these costs on to consumers in the form of higher prices. Second, the unauthorized direct seller will prefer to buy fewer units from the retailers than with a lower diversion price. These effects result in two benefits to the retailers: a greater number of units sold to consumers and a higher retail price. So although the direct effect of the diversion cost is a lower profit from diversion itself, the strategic effects boost profit via higher prices and retail quantities.

I next turn my attention to the Stage 1 wholesale price decision by the manufacturer. As in Liu and Zhang (2006) and proven in the appendix, the manufacturer will set its wholesale price such that the market is just covered. Thus, in the presence of an unauthorized direct seller, the manufacturer chooses wholesale price  $w^* = u - (4c + t + 4\mu)/7$  and earns a profit of  $4e^*w^*$ .

Finally, I turn my attention to the Stage 0 manufacturer decision of whether to authorize the direct seller. If the direct seller is authorized, Stage 5 demand will be as in Equations (1) and (2). In Stage 4, each retailer  $j$  chooses  $p_j$  to maximize  $\pi_j = (p_j - w)q_j$ , and the authorized direct seller chooses  $p_d$  to maximize  $\pi_d = (p_d - w)q_d$ . Simultaneously solving the first-order

conditions, the Stage 4 prices are  $p_j = t/24 + w + \mu/3$  and  $p_d = t/12 + w - \mu/3$ , resulting in a retailer  $j$  payoff equal to  $\pi_j = \xi(t/24 + w + \mu/3 - w)(t + 8\mu)/(12t) - e_j^2/2$ . In Stage 2, each authorized retailer simultaneously chooses effort levels, which straightforwardly results in  $e^{**} = (t + 8\mu)^2/(288t)$ . In Stage 1, the high reservation value consumers have implies that the manufacturer optimally chooses wholesale price such that the market is just covered;  $w^{**} = u - t/12 - 2\mu/3$ . Comparing manufacturer profitability with an authorized direct seller to manufacturer profitability with an unauthorized direct seller, it can be seen that the manufacturer will authorize the direct seller if and only if  $(t + 8\mu)^2(12u - t - 8\mu)/(1728t) - (7u - 4c - t - 4\mu)(48c + 19t + 48\mu)^2/(87808t) > 0$ .

There are two opposing forces in the manufacturer's Stage 0 decision of whether to authorize the direct seller. On the one hand, authorizing the direct seller increases the wholesale price that the manufacturer can charge (i.e.,  $w^{**} \equiv u - t/12 - 2\mu/3 > w^* = u - (4c + t + 4\mu)/7$ ). On the other hand, the authorized direct seller decreases the authorized retailer's incentive to provide market-expanding effort. This is because the retailers' market-expanding effort is better rewarded when the retailers earn a share of the free-riding direct seller's sales (via diversion) than when the direct seller operates completely independently of authorized retailers. Each sale that the unauthorized direct seller steals from the authorized retailer results in a modest profit margin for the retailer. Although the retailers would earn greater profit and exert greater effort if there were no direct seller, the effort levels will be higher when the retailers supply their direct competitor than when the manufacturer does.

Although this same logic would seem to suggest that the manufacturer should authorize the existing brick-and-mortar retailers to sell directly as well (and then get the entire direct sales margin instead of a split with the unauthorized seller), such a scenario represents Bertrand competition (Jeffers and Nault 2011). Allowing the four authorized retailers to

also sell directly results in marginal cost price and zero effort. Therefore, authorizing the current retailers to also sell directly is a dominated strategy in this context.

Therefore, in Stage 0, the manufacturer will authorize the direct seller only if the positive effect of the increased wholesale price outweighs the negative effect of diminished retailer effort, implying a diminished quantity sold by the manufacturer. As shown in the appendix, the direct seller will be authorized for a sufficiently low reservation value,  $u < \bar{u}$ . The reason authorizing the direct seller is unappealing to the manufacturer for high values of  $u$  is because the wholesale price with or without a direct seller increases with  $u$ . Thus the negative effect of losing a sale as a result of lower retailer effort is amplified because of the higher wholesale price associated with a higher  $u$ .

To summarize, if  $u > \bar{u}$ , the manufacturer will choose not to add an authorized direct seller to its market of four authorized retailers. Any gain in wholesale price from adding the direct seller will be negated by the diminished effort exerted by the retailers. If  $\mu < 3t/16 - c$ , authorized retailers divert to an unauthorized direct seller in equilibrium. Thus, diversion to an unauthorized direct competitor by the four authorized retailers occurs in equilibrium if  $u > \bar{u}$  and  $\mu < 3t/16 - c$ . I confirm in the appendix that values of  $u$  such that the model's assumption of four retailers arises endogenously can satisfy these conditions.

#### 4.1. Can Authorized and Unauthorized Direct Sellers Coexist?

In this section, I consider the coexistence of an authorized direct seller and an unauthorized direct seller. The purpose of this section is to demonstrate that the results of the main analysis are preserved under this modification to the manufacturer's choice set. The game is equivalent to the main model with the direct sellers occupying the same location.

Denote the unauthorized direct seller's price as  $p_{ud}$  and the authorized direct seller's price as  $p_{ad}$ . In Stage 5, consumers who prefer the direct selling option over either retail option will choose the direct seller with the lowest price. I will show that the existence of the authorized direct seller makes it such that an unauthorized direct seller cannot profitably exist at the same location. To this end, I will prove that any prices that can be sustained in the Stage 4 pricing game preclude the unauthorized direct seller from earning a positive margin on units acquired from the authorized retailers.

Consider the authorized direct seller's objective in the Stage 4 pricing game. Following the logic of Bertrand competition, the unauthorized direct seller will not charge  $p_{ud} > w$ . To see this, consider other possibilities. If  $p_{ad} > p_{ud}$ , then the authorized direct

seller earns zero profit and will choose to charge a lower price. If  $p_{ad} < w$ , then the authorized direct seller earns negative profit and will choose to charge a higher price. If  $p_{ad} = p_{ud} > w$ , the authorized direct seller splits the consumers who prefer the direct selling option over the retail option and can profitably deviate by marginally lowering price and capturing the entire market of consumers who prefer the direct selling option over the retail option. For the unauthorized direct seller, whose procurement cost was sunk in Stage 3, any price  $p_{ud} > p_{ad}$  is similarly dominated by  $0 < p_{ud} \leq p_{ad}$ . Any  $p_{ud} > w$  in the pricing game can therefore be ruled out. However, the unauthorized direct seller must procure units at  $w_d < w$  to earn a profit. The authorized retailers will not divert if  $w_d < w$  because each unit directly results in profit loss. Thus there is no mutually agreeable arrangement in Stage 3 between the unauthorized direct seller and the authorized retailers such that  $p_{ud} > w_d > w$ .

In conclusion, an authorized direct seller precludes the possibility of an unauthorized direct seller occupying the same location. As shown in the main analysis, the manufacturer prefers an unauthorized direct seller to an authorized direct seller provided  $u > \bar{u}$ . As such, the manufacturer will not authorize a direct seller to compete with the unauthorized direct seller from the same location.

## 5. Discussion

This paper provides a new explanation for product diversion. There is evidence that suggests that retailers will sell to unauthorized dealers, even though the action intensifies competition to reduce unit price markups. This outcome has not been predicted by prior literature. The model provides a reason for why retailers might take this detrimental action and identifies the consequences of such an action.

Whereas diversion may be an equilibrium strategy for authorized retailers, it is shown that this equilibrium strategy may result in lower profit than if diversion were not possible. Thus channel flow diversion can represent a prisoner's dilemma. The finding adds to the growing body of marketing research identifying marketing variables implemented by firms that reduce equilibrium profit.

When channel flow diversion occurs, the model finds retailers actually earn greater profit when incurring a greater cost of diverting. For example, if the product being sold is bulky and expensive to ship to the unauthorized direct seller, then profit will be greater than if the product is easy and inexpensive to ship. Whereas higher costs have a negative direct effect on retailer profit, the strategic implications are higher retail prices *and* higher quantities sold to consumers. In this context, the strategic effects outweigh the direct effect and result in greater profitability.

In applying the results, it is beneficial to consider the elements of the demand model that drive them. Note that the direct seller competes with multiple retailers. An authorized retailer is willing to divert in part because it recognizes that the diverted units will compete with several authorized retailers. If the direct seller instead were seen as a strong substitute to only a single retailer, that retailer would not be willing to divert; however, the remaining retailers would be more likely to divert. The authorized retailer is also willing to divert to the direct competitor, in part because it correctly anticipates that other retailers will be willing to divert. The availability of a mechanism to credibly commit to not diverting will undermine the latter incentive.

It should be noted that the model abstracts from the fact that, in some industries, the unauthorized direct seller has the ability to expand the total market of consumers. This is a positive direct effect, and logically, the prisoner's dilemma could be transformed to a conditional effect depending on the level of market expansion by the unauthorized direct seller. The model also examines a linear wholesale contract. As described in previous literature, the combination of a quantity discount schedule and a market-expanding unauthorized channel creates conditions ripe for diversion and would serve as another direct positive effect of diversion for retailers. These conditions would also amplify the negative direct effect of diversion cost on retailer profits. As such, both results are conditional on the extent to which the unauthorized seller increases the size of the total market and the discount schedule offered by the manufacturer.

There are several managerial implications of this research. Managers of upstream firms should recognize that retailers in closed markets have incentive to engage in channel flow diversion to unauthorized direct sellers even without quantity discounts. Thus, actions to carefully write retailer agreements and monitor and enforce those agreements may be necessary steps in reducing diversion. Retailers should welcome very strict policy interventions that can crack down on diversion, especially in industries in which the diversion can be done with little cost to the retailers. Unauthorized direct sellers should show restraint and buy fewer diverted units than could ultimately be sold in order to soften competition. Combined, the results help manufacturers diagnose why diversion might occur, help retailers understand the implications of diversion and diversion costs, and help unauthorized direct sellers understand the strategic implications of their procurement decisions.

### Acknowledgments

The author thanks the editor-in-chief, the associate editor, and the anonymous reviewers for offering extraordinarily constructive feedback and thoughtful insights in the

review process. The author also thanks Fabio Caldieraro, Ted Klastorin, and Eyal Bialogorsky for valuable comments on an earlier draft and seminar participants at the University of California, Berkeley. The author gratefully acknowledges the financial support of the Michael G. Foster Faculty Fellowship.

### Appendix

#### Proof of Propositions 1 and 2

I first determine the retailer's payoff if there is no diversion. In Stage 5, a consumer in the interval  $\theta \in [0, 1/4]$  will buy from retailer 1 located at  $x_1 = 0$  if  $u - t\theta_i - p_1 > u - t(1/4 - \theta_i) - p_2$  and will otherwise buy from retailer 2 located at  $x_2 = 1/4$ . A consumer in the interval  $\theta \in [1/4, 1/2]$  will buy from retailer 2 if  $u - t(\theta_i - 1/4) - p_2$  and will otherwise buy from retailer 3 located at  $x_3 = 1/2$ . Similar analysis over the remaining interval of the circle yields the following demand for each retailer:  $q_1 = \xi(1/4 - (2p_1 - p_2 - p_4)/(2t))$ ,  $q_2 = \xi(1/4 - (2p_2 - p_1 - p_3)/(2t))$ ,  $q_3 = \xi(1/4 - (2p_3 - p_2 - p_4)/(2t))$ , and  $q_4 = \xi(1/4 - (2p_4 - p_1 - p_3)/(2t))$ . In Stage 4, retailers simultaneously set prices to maximize  $(p_j - w)q_j$ , which results in prices  $p_j^{nd} = w + t/4$  and authorized retailer profits  $\pi_j^{nd} = t\xi/16$ .

I solve the equilibrium Stage 3 actions and beliefs as follows. First, I solve for the optimal diversion quantities purchased from each retailer as functions of the retailers' beliefs about the competitors' diversion quantities. I then solve for the diversion quantities and beliefs for each retailer such that the beliefs are confirmed to be mutually consistent.

There are  $2 \times 4$  possibilities to consider in Stage 3. The direct seller may offer diversion terms that will leave it constrained or unconstrained in the next stage. For each of these possibilities, the direct seller may buy from all retailers, three retailers, two retailers, or a single retailer. I first examine the procurement decisions in anticipation of being constrained by quantity in the pricing game.

*Direct Seller Quantity Is Constrained in the Pricing Game* (See the Second Column of Table A.1). Consider first the subcase in which the unauthorized direct seller procures units from all retailers. There is a unique solution to the Kuhn-Tucker conditions for the constrained optimization problem of Equation (3). The diversion quantities are each functions of the retailers' beliefs about the competitors' diversion quantities. Setting  $\hat{q}_{1t}^j = q_{1t}$ ,  $\hat{q}_{2t}^j = q_{2t}$ ,  $\hat{q}_{3t}^j = q_{3t}$ , and  $\hat{q}_{4t}^j = q_{4t}$  for each  $j$  at this solution, I solve for the diversion quantities such that each retailer's beliefs about each  $q_{jt}$  are confirmed in equilibrium. The unique solution is given by

$$q_{jt} = \frac{\xi(3t - 16c - 16\mu)}{28t}, \quad w_d = w + (144c + 5t + 128w + 16\mu)/128, \quad \lambda_{dj} = 1 \quad (7)$$

for all  $j$ .

The diversion quantity from each retailer is positive if and only if  $\mu < \hat{\mu} \equiv -c + 3t/16$ . To verify that this is indeed a profit-maximizing contract (rather than a profit-minimizing one), consider the effect of changing each of the choice variables. Define  $Y_j = \pi_j^*(w_d, q_{jt}, \hat{q}_{jt}^j) - \pi_j^*(w_d, q_{jt}, \hat{q}_{jt}^j | q_{jt} = 0)$  as the participation constraint for retailer  $j$ . A decrease in  $w_d$  or  $q_{jt}$  will violate the participation constraint (i.e.,  $Y_j = 0$  at

**Table A.1 Subgame Equilibrium Prices with a Single Unauthorized Direct Seller**

Equilibrium outcome	Condition	
	Unconstrained pricing $\sum_{j=1}^4 q_{jt} > 2\xi(t + 4(w - \mu))/(3t)$	Constrained pricing $\sum_{j=1}^4 q_{jt} < 2\xi(t + 4(w - \mu))/(3t)$
$p_r$	$\frac{t/8 + 2w + \mu}{3}$	$w + \frac{t}{8} \left(1 - \frac{t \sum_{j=1}^4 q_{jt}}{\xi}\right)$
$p_d$	$\frac{t/4 + w - \mu}{3}$	$w + \frac{t}{4} - \mu - \frac{t \sum_{j=1}^4 q_{jt}}{4\xi}$
$\pi_d(w_d, q_{jt})$	$\frac{\xi(t + 4(w - \mu))^2}{18t} - w_d \left(\sum_{j=1}^4 q_{jt}\right)$	$\frac{\sum_{j=1}^4 q_{jt}}{4\xi} \left[ \xi(t + 4w - 4\mu - 4w_d) - t \sum_{j=1}^4 q_{jt} \right]$
Contribution for retailer $j$ who diverts	$\frac{\xi(t + 8(\mu - w))^2}{288t} + q_{jt}(w_d - w - c)$	$\frac{t(\xi - \sum_{j=1}^4 q_{jt})^2}{32\xi} + q_{jt}(w_d - c - w)$
Contribution for retailer who does not divert	$\frac{\xi(t + 8(\mu - w))^2}{288t}$	$\frac{t(\xi - \sum_{j=1}^4 q_{jt})^2}{32\xi}$

solution from (7),  $\partial Y_j / \partial w_d = q_{jt} > 0$ , and  $\partial Y_j / \partial q_{jt}$  evaluated at the solution from (7) equals  $(1/896)(3t - 16c - 16\mu) > 0$  for  $\mu < \hat{\mu}$ , whereas an increase will reduce profit (i.e.,  $\partial \pi_d(q_{jt}, w_d) / \partial w_d = -\sum_{j=1}^N q_{jt} < 0$  and  $\partial \pi_d(q_{jt}, w_d) / \partial q_{jt}$  evaluated at the solution from (7) equals  $(1/896)(16c - 3t + 16\mu) < 0$  for  $\mu < \hat{\mu}$ ).

Procuring units from all four retailers results in a payoff for the unauthorized direct seller equal to

$$\pi_d^* = \frac{31\xi(16c - 3t + 16\mu)^2}{6272t}. \quad (8)$$

I will show that the unauthorized direct seller's profit from Equation (8) is greater than the profit from buying from a subset of retailers.

There is a unique solution to the Kuhn–Tucker conditions for the constrained optimization problem of Equation (4) such that the unauthorized direct seller procures units from all but retailer 1. Setting  $\hat{q}_{1t}^j = 0$ ,  $\hat{q}_{2t}^j = q_{2t}$ ,  $\hat{q}_{3t}^j = q_{3t}$ , and  $\hat{q}_{4t}^j = q_{4t}$  for each  $j$  at this solution, the only candidate equilibrium is

$$\begin{aligned} q_{jt} &= \frac{\xi(3t - 16c - 16\mu)}{21t}, \quad q_{1t} = 0, \\ w_d &= w + \frac{47c}{42} + \frac{9t}{224} + \frac{5\mu}{42}, \quad \lambda_{dj} = 1 \quad \text{for } j \in \{2, 3, 4\}. \end{aligned} \quad (9)$$

In the same manner as above, it is straightforward to verify that this solution is profit-maximizing rather than profit-minimizing. The resulting payoff is equal to

$$\pi_d = \frac{23\xi(16c - 3t + 16\mu)^2}{4704t}. \quad (10)$$

There is a unique solution to the Kuhn–Tucker conditions for the constrained optimization problem of Equation (5) such that the unauthorized direct seller procures units from retailers 3 and 4 only. Setting  $\hat{q}_{1t}^j = 0$ ,  $\hat{q}_{2t}^j = 0$ ,  $\hat{q}_{3t}^j = q_{3t}$ , and  $\hat{q}_{4t}^j = q_{4t}$  for each  $j$  at the solution, the only candidate equilibrium is

$$\begin{aligned} q_{jt} &= \frac{\xi(3t - 16c - 16\mu)}{14t}, \quad q_{1t} = q_{2t} = 0, \\ w_d &= w + \frac{31c}{28} + \frac{19t}{448} + \frac{3\mu}{28}, \quad \lambda_{dj} = 1 \quad \text{for } j \in \{3, 4\}. \end{aligned} \quad (11)$$

In the same manner as above, it is straightforward to verify that this solution is profit-maximizing rather than profit-minimizing. The resulting payoff is equal to

$$\pi_d = \frac{30\xi(16c - 3t + 16\mu)^2}{6272t}. \quad (12)$$

There is a unique solution to the Kuhn–Tucker conditions for the constrained optimization problem of Equation (6) in the text such that the unauthorized direct seller procures units from retailer 4 only:

$$\begin{aligned} q_{4t} &= \frac{\xi(3t - 16c - 16\mu)}{7t}, \quad q_{1t} = q_{2t} = q_{3t} = 0, \\ w_d &= \frac{15c}{14} + \frac{\mu}{14} + \frac{1}{224}t \left(11 + \frac{49t}{3t - 16c - 16\mu}\right), \quad \lambda_{d4} = 1. \end{aligned} \quad (13)$$

In this case, the participation constraint  $Y_4 = \pi_4^*(w_d, q_{jt}, \hat{q}_{kt}^j) - \pi_4^{nd}$  is different from the previous cases. However, it is still the case that a decrease in  $w_d$  or  $q_{jt}$  will violate the participation constraint (i.e.,  $Y_4 = 0$  at the solution from (13),  $\partial Y_4 / \partial w_d = q_{4t} > 0$  and  $\partial Y_4 / \partial q_{4t}$  evaluated at the solution from (13) equals  $(1/224)(3t - 16c - 16\mu + 49t^2/(3t - 16c - 16\mu)) > 0$  for  $\mu < \hat{\mu}$ , whereas an increase will reduce profit (i.e.,  $\partial \pi_d(q_{jt}, w_d) / \partial w_d = -\sum_{j=1}^N q_{jt} < 0$  and  $\partial \pi_d(q_{jt}, w_d) / \partial q_{jt}$  evaluated at the solution from (13) equals  $-(1/224)(3t - 16c - 16\mu + 49t^2/(3t - 16c - 16\mu)) < 0$  for  $\mu < \hat{\mu}$ ). Thus the solution to the Kuhn–Tucker conditions is profit-maximizing rather than profit-minimizing. The resulting payoff for the direct seller is equal to

$$\pi_d = \frac{\xi(128c^2 - 48ct + t^2 + 256c\mu - 48t\mu + 128\mu^2)}{112t}. \quad (14)$$

Comparing payoffs from (8), (10), (12), and (14), it can be seen that for  $\mu < \hat{\mu}$ , the direct seller will prefer to buy from all four retailers than any subset of the retailers. Thus, when each retailer  $j$  believes  $\sum_{k \neq j} \hat{q}_{kt}^j > 0$ , the unique equilibrium procurement in anticipation of a constrained pricing game is given by Equation (7).

Note that retailers 1, 2, and 3 cannot rationally believe  $\sum_{k \neq j} \hat{q}_{kt}^j = 0$  if the unauthorized direct seller can come to a mutual agreement with retailer 4 even when retailer 4

believes  $\sum_{k \neq j} \hat{q}_{kt}^4 = 0$ . Thus,  $\sum_{k \neq j} \hat{q}_{kt}^j > 0$  is the only rational belief if (14) is positive. Therefore, if  $\mu < (1/16)(3 - \sqrt{7})t - c$ , then only beliefs such that  $\sum_{k \neq j} \hat{q}_{kt}^j > 0$  are rational.

*Direct Seller Quantity Is Unconstrained in the Pricing Game* (See the First Column of Table A.1). In anticipation of an unconstrained pricing game, the unique solution to the optimization problem of Equation (3) with positive diversion quantities from all retailers is

$$q_{jt} = \frac{\xi(t+4w-4\mu)}{6t}, \quad w_d = c + \frac{1}{192}(5t+164w+28\mu), \quad (15)$$

$$\lambda_{dj} = 1 \quad \text{for all } j \in \{1, 2, 3, 4\}.$$

This leads to a direct seller payoff of

$$\pi_d^{**} = \frac{\xi(-192c + 11t - 100w - 92\mu)(t + 4w - 4\mu)}{288t}. \quad (16)$$

The unauthorized direct seller's payoffs from the optimization problems of Equations (4), (5), and (6) are  $\pi_d = \xi(-36c + 2t - 19w - 17\mu)(t + 4w - 4\mu)/(54t)$ ,  $\pi_d = \xi(-96c + 5t - 52w - 44\mu)(t + 4w - 4\mu)/(144t)$ , and  $\pi_d = -\xi(t^2 + 192c(t + 4w - 4\mu) + 64(w - \mu)(7w + 5\mu) + 16t(5w + 7\mu))/(288t)$ , respectively. Each of these is less than (16) for any  $w$  such that  $q_{jt} > 0$  in any of the optimization problems.

*Comparing Quantity-Constrained Profit to Unconstrained Profit.* Comparing payoffs from (8) and (16) above, the difference between constrained profit and unconstrained profit is equal to  $\pi_d^* - \pi_d^{**} = \xi(48c + 5t + 56w - 8\mu)(1488c + 71t + 1400w + 88\mu)/(56448t) > 0$  for any  $\mu$  such that (15) results in positive diversion quantity.

To verify the claim that the diversion price in the unconstrained case is less than the diversion price in the constrained case, comparison of diversion prices across cases yields  $w_d^* - w_d^{**} = (1/384)(48c + 5t + 56w - 8\mu) > 0$  for  $\mu < \hat{\mu} \equiv -c + 3t/16$ .

*Conclusion.* When  $\mu < \hat{\mu} \equiv -c + 3t/16$ , the unauthorized direct seller will earn greater profit procuring units from each retailer than from a subset of retailers. The unauthorized direct seller will also earn greater profit by choosing a quantity that will leave it constrained, rather than unconstrained, in the pricing game. Given each retailer believes at least one other retailer will accept the unauthorized direct seller's offer (i.e.,  $\sum_{k \neq j} \hat{q}_{kt}^j > 0$ ), a belief structure that is mutually consistent, all retailers will accept the unauthorized direct seller's offer. Thus,  $q_{jt} = \xi(3t - 16c - 16\mu)/(28t)$ ,  $w_d = w + (144c + 5t + 128w + 16\mu)/128$ , and  $\lambda_{dj} = 1$  for all  $j$  constitutes an equilibrium. If  $\mu < (1/16)(3 - \sqrt{7})t - c$ , this equilibrium is unique because retailer beliefs that  $\sum_{k \neq j} \hat{q}_{kt}^j = 0$  are not mutually consistent. Q.E.D.

#### Proof of Propositions 3 and 4

Define  $\psi(\cdot) = (48c + 19t + 48\mu)^2/(25088t)$ , which is the retailer profit multiplier on effort. In Stage 2, retailer  $j$  chooses  $e_j$  to maximize profit:  $\pi_j(\cdot) = \max_{e_j} \{\xi\psi(\cdot) - e_j^2/2\}$ . This results in  $e_j^* = (48c + 19t + 48\mu)^2/(25088t)$  for all  $j$  and profit for retailer  $j$  equal to  $\pi_j^* = (48c + 19t + 48\mu)^4/(179830784t^2)$ , which is increasing in  $c$ .

In the absence of diversion, retailers choose effort to maximize  $\pi_j(\cdot) = \max_{e_j} \{\xi(t/16) - e_j^2/2\}$ . This results in  $e_j = t/16$  and a profit of  $\pi_j = 7t^2/512$ . The retailer's payoff with diversion minus the payoff without diversion increases with  $\mu$

and is equal to  $-21t^2/2048$  at  $\mu = \hat{\mu}$ . The difference is thus negative for any  $\mu$  such that the diversion quantity is positive, and thus the retailers earn less profit with diversion than when diversion does not occur. Q.E.D.

#### Proof That Optimal Wholesale Price Is Set to Just Cover the Market

Let  $w^*$  denote the wholesale price such that  $\max\{u - p_r^* - t/8, u - p_d^* - \mu\} = 0$ . I rule out other wholesale prices. If  $w < w^*$ , then the market is fully covered, demand is inelastic with respect to  $w$ , and thus manufacturer profit  $4e^*w < 4e^*w^*$  because  $e^*$  does not depend on  $w$  when the market is covered. Suppose  $w > w^*$ , and thus there exist consumers who do not buy. In this case, retailers are local monopolies. A consumer will buy from a typical retailer  $j$  if  $u - p_j - t|x_j - \theta_j| \geq 0$ —in other words, if  $\theta \in [x_j - (u - p_j)/t, x_j + (u - p_j)/t]$ . Therefore, retailer  $j$  chooses  $p_j^* = (u + w)/2$  to maximize  $2\xi(u - p_j)(p_j - w)/t$ , which results in  $q_j = (u - w)/t$ . In the prior stage, the retailer chooses effort  $e_j^* = (u - w)^2/(2t)$  to maximize  $\xi(u - w)^2/(2t) - e_j^2/2$ . Consider first when there is not a direct seller. The manufacturer's profit is then  $16e_j^*w(u - w)/t$ , which is concave in  $w$  if  $w < u/2$ . If  $w > u/2$  then profit is strictly decreasing in  $w$ . The first-order condition is satisfied at  $w = u/4$  and  $w = u$ , the latter of which results in zero profit. Thus, the manufacturer will maximize profit by choosing  $w = u/4$  if the market is not fully covered. However, there is a contradiction if  $u > t/3$  because the consumer located farthest from all retailers gets positive utility from buying from one of the retailers:  $u - p - t/8 = (3u - t)/8 > 0$ . Now suppose that  $w > w^*$ , the market is not fully covered, and there is a direct seller who is a local monopolist. Consumers will buy if  $u - p_d - \mu \geq 0$ . Direct seller profit is strictly increasing in  $p_d$ ; thus  $p_d^* = u - \mu$ , and the utility from buying from the direct seller is 0 for all consumers. Thus, a contradiction is created, because the active presence of the direct seller means all consumers earn nonnegative utility from making a purchase. Q.E.D.

#### Conditions Endogenously Resulting in Four Authorized Retailers

The authorized retailers are assumed to be established prior to the emergence of the direct selling option. I allow for an integer  $N$  retailers to be spaced at equal distances along the unit circle and demonstrate when  $N$  will equal 4 in equilibrium. Let  $K_r$  denote the retailers' fixed cost of entry. Assume that when unconstrained by the manufacturer's authorization decision, entry will be governed by the zero profit entry condition. Thus, the number of authorized retailers will be the minimum of the manufacturer's profit-maximizing number of authorized retailers and the free-entry equilibrium, with no other barriers to entry other than the entry cost. I will show that either the combination of  $7t^2/512 \geq K_r > 9t^2/1250$  and  $u \leq 3t/4$  or the combination of  $u = 3t/4$  and  $K_r \leq 7t^2/512$  will result in  $N = 4$ .

The assumption on effort in the general  $N$  authorized retailers model is that the total market size is then equal to  $\sum_{j=1}^N e_j$ , with the uniform distribution of consumer preferences for firms. In the absence of a direct seller, Stage 5 demand and Stage 4 prices are straightforward, as derived in Tirole (2001, p. 283):  $p_j = w + t/N$ . Each retailer  $j$  chooses effort levels to maximize  $\max_{e_j} \{\xi(w + t/N - w)(1/N) - e_j^2/2\}$ .

Taking first-order conditions straightforwardly results in  $e^* = t/N^2$ . Manufacturer profit,  $Ne^*w$ , is maximized at  $w^* = u - 3t/(2N)$ . The manufacturer profit in the absence of a direct seller is thus  $Ne^*w^* = (t/N)(u - 3t/(2N))$ . The manufacturer will choose  $N$  to maximize  $(t/N)(u - 3t/(2N))$ . This profit represents a trade-off that occurs when increasing the number of retailers. A higher  $N$  will increase the wholesale price but will decrease the retailer's sales effort and thus manufacturer quantity. The profit function is concave in  $N$  over the interval  $(0, 9t/(2u))$  and decreasing in  $N$  for greater values. As such, manufacturer profit is maximized at the value of  $N$  such that the derivative with respect to  $N$  is 0:  $N^m = 3t/u$ .

Now consider whether the manufacturer's authorization decision is constrained by the entry decisions of the retailers. Retailer profit is  $-K_r + N(t/N^2)(t/N^2) - (t/N^2)^2/2 = -K_r + (2N - 1)t^2/(2N^4)$ . Given consideration of integer values of  $N$ ,  $7t^2/512 \geq K_r > 9t^2/1250$  results in  $N^{fe} = 4$ , with free entry where entry costs above the upper bound will result in  $N^{fe} \leq 3$  and entry costs below the lower bound will result in  $N^{fe} \geq 5$ .

Thus, prior to the emergence of the direct selling option, the equilibrium number of authorized retailers will be  $N = 4$  if either of the following two conditions hold: (1) the manufacturer's profit-maximizing number of retailers equals 4 (i.e.,  $N^m = 3t/u = 4$ ), and the number of retailers who would be willing to enter is greater than or equal to 4 (i.e.,  $K_r \leq 7t^2/512$ ); or (2) the manufacturer's profit-maximizing number of retailers is greater than 4 (i.e.,  $N^m = 3t/u > 4$ , occurring because  $u < 3t/4$ ), but the number of retailers who would be willing to enter is equal to 4 (i.e.,  $7t^2/512 \geq K_r > 9t^2/1250$ ).

It should be noted that  $u = 3t/4$  is greater than the minimum value on  $u$  such that there is full market coverage (i.e.,  $u = t/3$ ). Furthermore,  $u = 3t/4$  is sufficient such that the manufacturer will choose not to authorize a direct seller even if this implies an authorized direct seller will arise. Recall from the text that the manufacturer will not authorize the direct seller if

$$\frac{(t + 8\mu)^2(12u - t - 8\mu)}{1728t} - \frac{(7u - 4c - t - 4\mu)(48c + 19t + 48\mu)^2}{87808t} \leq 0. \quad (17)$$

Simplified at  $u = 3t/4$ , (17) becomes  $(10976(t - \mu)(t + 8\mu)^2 + 27(4c - 17t/4 + 4\mu)(48c + 19t + 48\mu)^2)/(2370816t)$ . This condition is convex in  $\mu$  over  $0 < \mu < \hat{\mu}$ , negative at  $\mu = \hat{\mu}$  for any  $t$  such that  $\hat{\mu} > 0$ , and negative at  $\mu = 0$  for any  $t$  such that  $\hat{\mu} > 0$ . Therefore, it is negative for all  $0 < \mu < \hat{\mu}$ .

The general condition on  $u$  such that the manufacturer leaves the direct seller unauthorized is derived by setting (17) equal to 0 and noting the derivative of (17) with respect to  $u$  is negative for all  $0 < \mu < \hat{\mu}$ . The condition on  $u$  is as follows:

$$u > \bar{u} \equiv (8(-48276c + 28699t) + (233289(16c + t)(48c + 13t)) / (144c + 29t - 80\mu) + 1630240\mu) + (11025(48c + 13t)(112c + 15t)) / (144c + 85t + 368\mu)) \cdot (2221800)^{-1}.$$

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