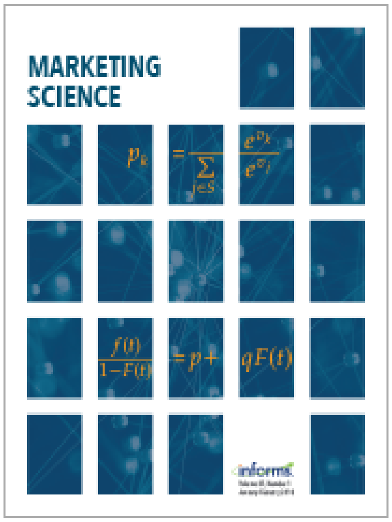

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Optimal Bundling Strategies in Multiobject Auctions of Complements or Substitutes

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We consider a problem at the interface of auctions and bundling. Our revenue-maximizing seller seeking to auction one unit each of two complements or substitutes in the best-of-three formats: the auction of the bundle, separate auctions of the individual items, and a combinatorial auction. We draw on an analytical model to address the following questions: (i) Which of the auctioning strategies is optimal under the second-price, sealed-bid format? (ii) What is the optimal strategy for the bidders? (iii) When the objects are asymmetrically valued (e.g., Super Bowl ticket versus souvenir), what is the optimal auctioning sequence under the pure components strategy? Our results suggest that separate auctions of the two objects are superior to the auction of the bundle for most substitutes and even moderate complements when there are at least four bidders. The auction of the pure bundle is better suited for strong complements or with too few bidders. When the combinatorial auction is an available option, it weakly dominates the auction of the pure bundle but has domains of inferiority relative to the separate auctions. When the objects are asymmetric in value, it is optimal to auction the higher-valued object first.

Key words: auctions; bidding; bundling; game theory; price discrimination; pricing
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1. Introduction

Auctions and bundling have entered the mainstream in both business-to-business and business-to-consumer markets. Net transaction revenues at eBay alone jumped from \$86.1 million in 1998 to \$5.97 billion in 2006.[1] The growing prominence of bundling is evident in its application to emerging topics such as affinity programs (Gans and King 2006), customization (Hitt and Chen 2005), and versioning (Shapiro and Varian 1999). Our objective is to address a problem at the interface of auctions and bundling.

Product portfolios such as a pair of Super Bowl tickets and telecast rights of the summer and winter Olympics share some defining characteristics. The products are complements or substitutes and are in short supply. The seller must decide how best to offer the products to maximize payoffs. We develop a stylized analytical model using a *second-price, sealed-bid format*—to address the following questions for a revenue-maximizing seller:

• Under what conditions (e.g., the degree of interrelatedness and the number of prospective bidders) should a portfolio of products be auctioned jointly—as a bundle—or sequentially—as individual objects? How does the availability of the combinatorial auction influence the seller's decision?

• If the objects are valued asymmetrically (e.g., a more valuable Super Bowl ticket plus a less valuable souvenir), which object should be auctioned first?

Our bidders are strategic too, so we examine when they should bid more or less than their reservation prices for the items in question (see Shugan 2005).

Our study considers the three well-known bundling strategies—pure bundling (i.e., offering the products only as a package), pure components (i.e., offering the products separately), and mixed bundling (i.e., offering both the bundle and the individual objects) (Schmalensee 1984). Because articles on bundling (e.g., Venkatesh and Kamakura 2003) point out that treating complementarity and substitutability as a simple dichotomy is prone to misleading conclusions, we delineate the optimal strategy on the basis of the *degree* of interrelatedness. A limiting aspect of the bundling literature is its assumption of (fairly) unrestricted supply. We add to this stream by examining the role of *limited* supply and the opportunity for first-degree price discrimination.

[1] eBay profit and loss statement (1998) and 2006 fourth-quarter earnings presentation at http://investor.ebay.com/.

The literature on auctions is our other key pillar (see Klemperer 1999). A small but growing number of these studies are in marketing (e.g., Bradlow and Park 2007, Fay 2004, Greenleaf et al. 1993). Whereas most articles focus on auctions of *individual* objects, our motivation comes from the limited research on multiobject auctions (e.g., Chakraborty 1999, Feng and Chatterjee 2005, Levin 1997). The second-price, sealed-bid auction is the format of choice (e.g., Krishna and Rosenthal 1996). The scope and findings of our study are distinct from—and complement—the literature on multiobject auctions in the following ways:

• Unlike extant studies, we consider a *broader* strategy space consisting of pure components (sequential) auctions, auction of the pure bundle, *and* combinatorial auction (equivalent of mixed bundling).[2] We show that no one strategy is dominant.

• Whereas earlier auction studies have emphasized the extreme scenarios involving independently valued products (e.g., Chakraborty 2002, Palfrey 1983) *or* perfect substitutes (e.g., Feng and Chatterjee 2005) *or* strong complements (e.g., Benoit and Krishna 2001), our focus is on the intermediate scenarios of weak/moderate/strong substitutes or complements.[3] For the intermediate scenarios, we show, for example, that the optimal auctioning and bidding strategies depend on the specific *combination* of degree of interrelatedness and number of bidders. Depending on the number of bidders, the pure components (sequential) auctions can be optimal even for moderate complements, and the auction of the bundle can be better for certain substitutes.

• Whereas asymmetry in product value has been examined for complements in English auctions (e.g., Branco 1997), we show that, for both complements and substitutes, the higher-valued object should be offered first in pure components Vickrey auctions.

We present in §2 our main model comparing the auction of the pure bundle against those of the pure components. We then present the related normative guidelines as three propositions (§3). In §4 we consider the mixed bundled, combinatorial auction.[4] We conclude with a discussion (§5) of the study's key findings and limitations and future research directions. The detailed proofs are in the Technical Appendix, which can be found at http://mktsci.pubs.informs.org.

2. The Main Model

2.1. The Product Market

Our seller is also the auctioneer. He or she has one unit each of objects A and B and seeks to maximize his/her expected revenues. Maximizing expected revenue and maximizing expected profit are equivalent in a typical auction setting because the seller already owns the two objects and so the procurement costs are sunk. The objects are hard for interested consumers to find outside of the auction. The two-object formulation is consistent with the setting in several extant articles on auctions (e.g., Avery and Hendershott 2000, Hausch 1986) and bundling (e.g., Adams and Yellen 1976, Schmalensee 1984). The seller has to choose between the auction of the pure bundle and the pure components auctions. This decision of the seller will be common knowledge before the auction begins. Our seller has chosen the second-price, sealed-bid Vickrey auction format following earlier precedents (e.g., Krishna and Rosenthal 1996, Chakraborty 2002). Also, the format closely resembles those on eBay and Yahoo!.[5]

There are $N+1$ potential bidders. This number is known exogenously as in Krishna and Rosenthal (1996) and Chakraborty (2002). The minimum number of bidders is 2. The bidders have private values for the objects and maximize their expected surplus. The bidders see the two auctioned objects as complements or substitutes of one another. Drawing on precedents in bundling (e.g., Venkatesh and Kamakura 2003), we parameterize the *degree* of complementarity or substitutability as θ such that if any bidder j's reservation prices for objects A and B taken separately are R_{jA} and R_{jB}, and that for the bundle is R_{jAB}, then

$$\theta = \left(\frac{R_{jAB} - (R_{jA} + R_{jB})}{R_{jA} + R_{jB}}\right). \tag{1}$$

Parameter $\theta > 0$ ($\theta < 0$) for complements (substitutes). For independently valued objects, θ is zero. For strong complements (or strong substitutes), $\theta \gg 0$ (or $\theta \ll 0$). For object pair AB, the bidders in our model perceive the same degree of interrelatedness θ—an assumption shared by Bakos and Brynjolfsson (1999) and Venkatesh and Kamakura (2003). Consistent with McAfee and Vincent (1997) and Riley and

[2] We thank Professor Vijay Krishna for suggesting to us the combinatorial auction—through the VCG (Vickrey-Clarke-Groves) mechanism—as the equivalent of mixed bundling.

[3] The optimal strategies in the extreme scenarios are "easier" to predict. For example, sequential auctions would be meaningless for perfect complements because the first-round winner would be the obvious second-round winner. With two perfect substitutes, the bundle reservation price collapses to the reservation price for the higher-valued object; two separate auctions will weakly dominate the auction of the pure bundle.

[4] Comparing pure components and pure bundling first is consistent with the template in bundling (e.g., Schmalensee 1984); doing so clarifies the mechanisms favoring either pure strategy. Additionally, in our auctions context, these are the only two feasible strategies on popular business-to-consumer auction sites (e.g., eBay and Yahoo!).

[5] Both sites permit proxy bidding in which it is optimal to bid at the last moment (cf. Wilcox 2000). Thus, if all bidders bid at the end, the auctions reduce to the second-price, sealed-bid format.

Samuelson (1981) in auctions and Carbajo et al. (1990) and Seidmann (1991) in bundling, we assume that the bidders' reservation prices for each object taken alone are independent draws from a uniform distribution $[0, V]$. This means the products are symmetric in value at the market level. Furthermore, as we consider distinct objects, each bidder's reservation prices R_{jA} and R_{jB} for the objects taken alone are independent of each other (as in Bakos and Brynjolfsson 1999). Per Equation (1), the bidders' reservation prices for the bundle follow a triangular distribution, with support $[0, 2V(1+\theta)]$ and mean $V(1+\theta)$. All of the above information is common knowledge *except* the individual-level realizations of reservation prices R_{jA} and R_{jB}, which are private values as in Palfrey (1983).

2.2. The Bidding Process

The seller chooses first between the pure components and pure bundling auctions. The buyers then place their bids. We derive equilibrium strategies for the bidders in the subgames where the bidders make their choices and, in turn, infer the seller's optimal strategy. Thus, we follow backward induction, and our strategies are subgame perfect.

When the seller auctions the bundle, each bidder's best bid, denoted b_{jAB}, can be no better than her reservation price for the bundle. Bidding one's reservation price is weakly dominant.

Result 1. The pure strategy Nash equilibrium bid of bidder j for bundle AB is $\text{Max}\{R_{jA}, R_{jB}, R_{jAB}\}\ \forall j$.

Result 1 holds because reservation prices are uncorrelated (i.e., one bidder's reservation price does not affect another's) and private, and in a second-price sealed-bid auction it is then optimal to bid the valuation (Milgrom and Weber 1982, Vickrey 1961). The revenue is the amount of the second-highest bid (i.e., the second-highest bidder's reservation price).

If the seller chooses the pure components auctions, each bidder considers the following:

- The bidder could (i) win the first object and go on to win the second object, (ii) win the first object but lose the second, (iii) lose the first object and yet win the second, or (iv) lose both objects. The bidder tries to maximize one's expected surplus.
- For complements, the winner of the first object could gain an upper hand in the eventual bid for the second object. This is because none of the first-round losers can realize the benefit of complementarity. Hence, there is some incentive in the first auction to bid aggressively (i.e., bid more than one's reservation price for the first object). This must be tempered by the realization that the winning bidder could lose the second object and incur a negative surplus overall. This could happen if, for example, one of the first-round losers has a high reservation price for the second taken alone and makes a strong bid.
- For substitutes, we assume away the possibility of arbitrage but invoke the property of free disposal. A bidder might choose to do so if he or she has a higher reservation price for either product but the overdose of consuming both triggers disutility from the bundle. That is, a single item may be preferable to a bundle.

With the first bid over and the results announced, some of the above uncertainties are resolved. Each bidder knows whether she won or lost the object and the price paid by the winner. The winner is aware of her surplus from the first object. She would bid on the second object so as not to bring down—and possibly to increase—her surplus from the first. All of the first-round losers bid their reservation prices for the second object. Doing so is a weakly dominant strategy for reasons discussed under the earlier case of auctioning the bundle. The winner of the second auction and the price paid by her are subsequently revealed.

The bidders' equilibrium bids determine whether the seller should choose pure components or pure bundling. We derive our Nash equlibria by backward induction.

Result 2. For the pure components—sequential—auctions, the pure strategy Nash equilibrium bids of j for objects A and B are as in Table 1.

The outline of the proof is in the appendix, and the complete proof is in the Technical Appendix, which can be found at http://mktsci.pubs.informs.org.

We will draw on Results 1 and 2 to address the bidders' problem. To address the seller's problem, we must compare the revenues from these two results. Because Result 2 is rather messy, we infer the seller's decision by simulating a large number of auctions. We will now present normative guidelines in the form of propositions.

3. Propositions from the Main Model

In our framework, the number of bidders $(N+1)$ and the degree of complementarity or substitutability (θ) are common knowledge at or before the start of an auction. For *each* combination $(N+1, \theta)$, we analyzed 30,000 sets of auctions, where each set proceeds as follows. First, from a uniform distribution [0, 100] where 100 (an arbitrary upper limit) represents V, we randomly draw reservation prices R_{jA} and R_{jB} for objects A and B of each of the $N+1$ bidders. The bundle reservation price R_{jAB} is then $(1+\theta)(R_{jA}+R_{jB})$ for each bidder j ($j \in \{1, \ldots, j, \ldots, N+1\}$). The bids for A and B, and bundle AB, correspond to the equilibrium bids in Results 1 and 2. The revenue to the seller from pure components is the sum of the second-highest bids from the two auctions. From across the 30,000 auctions for each $(N+1, \theta)$, we determine the seller's *expected* (i.e., mean) revenues from the auction of the

Table 1 Optimal Bids Under the Pure Components Auctions

Condition 1	Condition 2	Optimal bid for the first object A
(A) Complements		
$(R_{jAB} - R_{jA}) \le V$	$0 \le R_{jB} \le \theta V$	$b^*_{jA} = R_{jA} + \frac{(R_{jAB} - R_{jA})^{N+1}}{(N+1)V^N} - \frac{R_{jB}^{N+2}}{2(N+2)V^{N+1}\theta(1+\theta)}$
	$\theta V < R_{jB} \le V$	$b^*_{jA} = R_{jA} + \frac{(R_{jAB} - R_{jA})^{N+1}}{(N+1)V^N} - \frac{2N(N+2)R_{jB}^{N+1} - (N+1)(N+2)\theta V R_{jB}^N + 2(\theta V)^{N+1}}{2N(N+1)(N+2)V^N(1+\theta)}$
$(R_{jAB} - R_{jA}) > V$	$R_{jB} < \theta V$	$b^*_{jA} = R_{jAB} - \left(\frac{N}{N+1}\right)V - \frac{R_{jB}^{N+2}}{2(N+2)V^{N+1}\theta(1+\theta)}$
	$\theta V \le R_{jB} \le V$	$b^*_{jA} = R_{jAB} - \left(\frac{N}{N+1}\right)V - \frac{2N(N+2)R_{jB}^{N+1} - (N+1)(N+2)\theta V R_{jB}^N + 2(\theta V)^{N+1}}{2N(N+1)(N+2)V^N(1+\theta)}$
Condition		Optimal bid for the first object A
(B) Substitutes		
$0 \le R_{jB} \le (1+2\theta)V$		$b^*_{jA} = R_{jA} + \frac{(R^{\oplus}_{jAB} - R_{jA})^{N+1}}{(N+1)V^N} + \frac{\theta R_{jB}^N}{2N(1+\theta)V^{N-1}} - \frac{R_{jB}^{N+1}}{(N+1)(1+\theta)V^N} + \frac{\theta^2 R_{jB}^{N+2}}{2(1+\theta)(1+2\theta)(N+2)V^{N+1}}$
$(1+2\theta)V \le R_{jB} \le (1+2\theta)V/(1+\theta)$		$R_{jA} + \frac{(R^{\oplus}_{jAB} - R_{jA})^{N+1}}{(N+1)\,V^N} - \left(\frac{R_{jB}^{N+2}}{2\theta V^{N+1}(N+2)}\right) + \left(\frac{R_{jB}^{N+1}}{\theta\,(N+1)\,V^N}\right) - \left(\frac{(1+2\theta-2\theta^2-2\theta^3)\,R_{jB}^N}{\theta\,(1+2\theta)\,V^{N-1}}\right) + \frac{(1+2\theta)^{N+1}}{2\theta\,(1+\theta)}\left(\frac{V\,(1+2\theta)}{N+2} + \frac{(1+\theta)\,(1-\theta)\,(N+1)\,V - 2N\,(1+V\,(1+\theta))}{N\,(N+1)}\right)$
$(1+2\theta)V/(1+\theta) \le R_{jB} \le V$		$b_{jA} = R_{jA} + \frac{(R^{\oplus}_{jAB} - R_{jA})^{N+1}}{(N+1)\,V^N} - \frac{R_{jB}^N}{NV^{N-1}}\left[\frac{1+3\theta+3\theta^2}{2(1+\theta)^2}\right] - \frac{R_{jB}^{N+1}}{(N+1)\,V^N} + \frac{R_{jB}^{N+2}}{2V^{N+1}\,(N+2)} + \frac{(1+2\theta)^{N+1}\,V\,(\theta N^2 + 2N\theta - 1)}{2\,(1+\theta)\,N\,(N+1)\,(N+2)}$

Notes. Object A is the first of the two objects auctioned. The optimal bids for object B are $R_{jAB} - R_{jA}$ for the first-round winner and R_{jB} for the first-round losers.
Notation:
$N+1$, number of bidders;
θ, degree of complementarity or substitutability between objects A and B;
V, upper bound of the uniform distribution from which bidders' reservation prices for each standalone object are "drawn";
R_{jA} and R_{jB}, bidder j's reservation prices for objects A and B taken separately;
R_{jAB}, bidder j's reservation price for bundle AB;
$R^{\oplus}_{jAB} = \text{Max}\{R_{jA}, R_{jB}, R_{jAB}\}$.

bundle and from the pure components auctions. We repeat this process for different $(N+1, \theta)$, varying $N+1$ in steps of 1 from 2 to 11[6] and θ from -0.5 to $+0.8$. This analysis is presented as a phase diagram in Figure 1.

In the propositions, we treat complementarity or substitutability as "weak" when $0 < |\theta| \le 0.2$, "moderate" when $0.2 < |\theta| \le 0.4$, and "strong" when $|\theta| > 0.4$.

[6] Our preliminary review of eBay suggests that many auctions have bidders numbering in the single digits (see Hossain and Morgan 2003). Furthermore, in business-to-business auctions, the sample size could be small even for scarce products such as the radio spectrum. Allowing the bidder pool to be small to moderate to large also ensures completeness.

3.1. Guidelines on the Optimal Auction Format and Bidding Strategies

Our first proposition compares the domain of superiority of pure components auctions against that of the auction of the pure bundle.

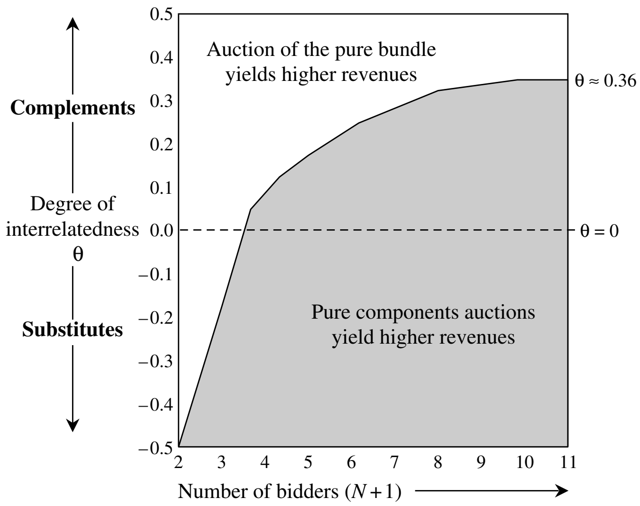
Proposition 1 (P1). (A). *Across complements and substitutes, the greater the number of bidders, the wider the domain of optimality of pure components auctions.*

(B) *For complements*:

(i) *Pure components auctions are optimal for the seller for weak to moderate complements if there are four or more bidders.*

(ii) *Auction of the pure bundle is optimal for strong complements, and for weak to moderate complements when there are two or three bidders.*

Figure 1 The Relative Attractiveness of Pure Components and Pure Bundling Auctions: Symmetric Case



Note. In the symmetric case, bidders' reservation prices for two individual objects are distributed i.i.d. $U[0, V]$.

(C) *For substitutes*:

(i) *With just two bidders, auction of the pure bundle is always optimal.*

(ii) *With three bidders, auction of the pure bundle is optimal for weak substitutes; pure components auctions are optimal for strong substitutes.*

(iii) *With four or more bidders, pure components auctions are always optimal.*

Comments. The phase diagram in Figure 1 shows that the combination of $N+1$ and θ matters.[7]

On P1(A). A larger bidder pool makes the first-round bidding more aggressive because the danger of losing the first object (and the resulting benefit of complementarity) is higher. Second, because more bidders are likely to value object B highly, the first-round winner has to maintain an aggressive stance on the second-round bid. This chain of competition boosts sequential auctions.

On P1(B). "Conventional wisdom" would suggest that a bundled auction would be the best for complements. The proposition clarifies that this intuition applies for strong complements only. In an auction setting, a counterbalancing advantage for pure components auctions arises from the size of the bidder pool. With weak to moderate complements, for which the complementarity boost is limited, the seller benefits from playing the bidders against each other in two competitive events. The seller gains from the first auction by tapping the bidders' desire to bid aggressively on object A in anticipation of also clinching object B later; the seller then gains from the second auction by playing off the winning bidder from the first auction against those who have a high value for object B alone.

On P1(C). Substitutability weakens the first-round winner's interest in the second object. Interestingly, the anticipated mildness in the second-round competition brings down the first bid—each bidder sees some incentive in losing the first object to walk away with the second. Of course, the bidders act strategically, and the combined revenue to the seller from sequential auctions is weakly lower than the auction of the bundle when there are only two bidders (P1(C-i)). However, as P1(C-iii) underscores, a large enough bidder pool triggers a fight on both rounds, and the seller gains more from pure components. P1(C-ii) is a "compromise" scenario, so the auction of the bundle is appealing for weak substitutes only.

We next consider the bidders' point of view in the context of pure components auctions.

Proposition 2 (P2). *With pure components auctions*:

(A) *It is optimal to overbid on the first object A if it is a complement of the second object B and to underbid if it is a substitute of B.*

(B) *For the second object B,* (i) *it is optimal for the first-round winner to bid more (or less) than her reservation price for B if it is a complement (or substitute) of A;* (ii) *The first-round losers' optimal bids are equal to their reservation prices for B.*

Comments. The basis for P2(A) is that winning the first of two complements opens up the prospect of tapping the value boost of complementarity later, and so the bidder should try harder on the first object. By contrast, aggressive bidding on the first auction is not optimal with two substitutes because there is *dis*-synergy from owning both.[8]

The significant point from P2(B-i) is that the aggressive bidding (i.e., bidding more than one's reservation price) on the first of two complements persists on the second round for the first-round winner alone. This is driven by the danger of losing the second object, which could then negate the advantage of capturing complementarity. The converse is true for the second of two substitutes. P2(B-ii) highlights that first-round losers are the ones who bid in a neutral fashion for the second object. They have lost their chance of benefiting from complementarity and seek only to earn

[7] For the "nested" case of $\theta = 0$, we replicate Chakraborty's (1999) result that unbundled sales will lead to higher revenues than bundled sales when there are four or more bidders (see Figure 1).

[8] P2(A) goes beyond a seemingly similar result in Menezes and Monteiro (2003, MM). MM's focus is on *identical* objects, so the standalone reservation prices are perfectly correlated. So, in their setup, a first-round winner is the de facto second-round winner (for complements and weak substitutes) and second-round loser (for strong substitutes). Our setup is more general and does not force such outcomes.

a nonnegative surplus from the second auction. That said, first-round losers with a high reservation price for object B could still threaten the first-round winner. Such a prospect is diminished in a bundled auction in which bundling would work to decrease bidder heterogeneity.

3.2. Impact of Value Asymmetry on Auction Sequence

As noted earlier, an asymmetric pair would consist of, say, a Super Bowl ticket that is valued highly and a related, less valuable souvenir. For the seller, asymmetry raises the important question of *sequencing*, i.e., which of the two objects should be auctioned first.[9]

Bidders' reservation prices for object A taken alone are distributed, $U[0, V]$, whereas those for object B are $U[0, 0.5V]$, the difference between V and $0.5V$ capturing the market-level asymmetry. The case of $0.5V$ chosen here is representative. The analysis and the resulting equilibrium are exactly the same for any kV where k is in the interval (0, 1]. The formulation allows stray bidders to have a higher reservation price for B than for A. Other aspects of the main model remain unchanged. The seller's strategic options are (i) auction the bundle; (ii) auction object A first, then B; (iii) auction B first, then A. We propose

Proposition 3 (P3). *Given significant asymmetry in market-level valuations for objects A and B*:

(A) *Pure components auctions are still optimal for the seller of weak to moderate complements if there are at least four bidders*; *it is also optimal for most substitutes*. *The auction of the bundle is optimal for strong complements only*.

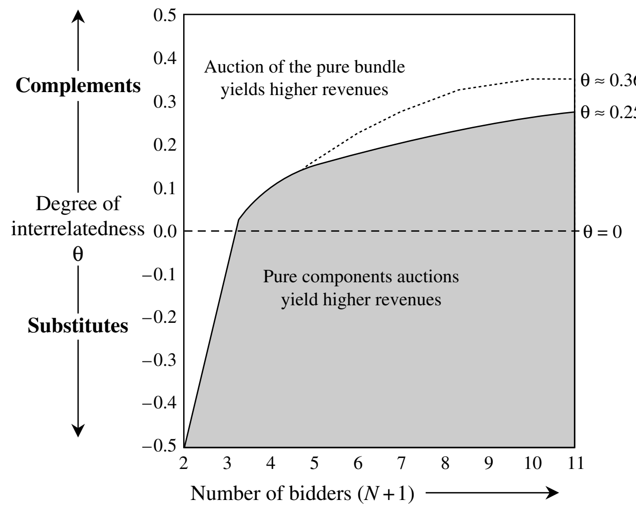
(B) *When pure components auctions are optimal*, *the higher-valued object should be auctioned first*.

Comment. Figure 2 contains the phase diagram delineating the domains of the seller's strategies.

P3(A) underscores the robustness of Proposition 1. Symmetry or lack thereof is not quite why pure components auctions are appealing. The multiple waves of competition and the resulting desire to bid aggressively, as well as the threat of losing out on the complementarity boost if the first object is not won, are what favor the pure components auctions. P3(B) makes the significant point that, under the domain of optimality of pure components, *the specific sequence matters to the seller*. Because the first auction is decidedly the more competitive of the two auctions, putting up the higher-valued object first helps in better extraction of the surplus. The proposition holds for substitutes also even though there is some underbidding on the first round.

[9] Our motivation comes from prior studies on sequencing (in very different settings). In English auctions of complements and with three bidders, Branco (1997) finds that expected prices are higher in the first auction. In reverse auctions involving sellers with varying capacities and costs, Elmaghraby (2003) shows that specific sequences lower procurement costs. We consider all complements and substitutes, focus on Vickrey auctions, and allow the bidder size to vary.

Figure 2 The Relative Attractiveness of Pure Components and Pure Bundling Auctions: Asymmetric Case



Notes. In the asymmetric case, reservation prices for the more valuable object are distributed, $U[0, V]$, whereas those for the other object are $U[0, 0.5V]$. The higher-valued object is auctioned first. The dotted line in the figure represents the indifference frontier for the symmetric case from Figure 1.

3.3. Possible Implications of Relaxing Two Assumptions

Our main model assumes that (a) the bidders' valuations are drawn from a *uniform* distribution, and (b) valuations for the two objects are *uncorrelated* across bidders. We speculate here on the implications of relaxing these assumptions for independently valued objects ($\theta = 0$).

What if the underlying distribution is unimodal (e.g., normal distribution) or bimodal, with the extreme supports representing the two modes? *Ceteris paribus*, given two supports equidistant from the mean, reservation prices for an object are less (or more) heterogeneous under the unimodal (or bimodal) distribution relative to the uniform. Compared to the uniform, the reduction in buyer heterogeneity through bundling would be higher (or lower) for the bimodal (or unimodal) case. We would speculate that competition across two rounds of auctions would be more (or less) intense when the distributions are unimodal (or bimodal). Thus, domain of attractiveness of pure components auctions vis-à-vis the auction of the pure bundle should be larger (or smaller) for the unimodal (or bimodal) case relative to the uniform case.

What if the bidders' valuations of the two objects are correlated? We consider a perfect positive or negative correlation coefficient (i.e., $\rho = +1$ or -1). The case of $\rho = +1$ implies that the same bidder will have the highest reservation prices for both objects. Therefore, the first-round winner will be the obvious winner of the second object. Conversely, $\rho = -1$ would mean that the winner of the first auction will lose the second. The presence of correlation acts as a signal to the bidders and makes the outcomes easier to predict.

The added presence of complementarity or substitutability would lead to higher-order effects that should be examined in future studies on the topic (discussed later).

4. Implications When the Combinatorial Auction Is Available

The combinatorial auction, the equivalent of mixed bundling, is a *simultaneous* auction in which bidders may submit separate but concurrent bids for the bundle and individual objects. Despite its generality, the format has been used only in selective business-to-business settings such as the auction of bus routes in London (Cantillon and Pesendorfer 2004) and industrial procurement (Ausubel and Milgrom 2006). Although it is efficient (in the sense of Pareto optimal allocation of goods), it is not used by popular online business-to-consumer auction sites, arguably because ordinary consumers might find the rules harder to grasp and see the auction as less transparent. That said, it is an attractive option to study mixed bundling. This section applies only when combinatorial auction can be used.[10]

We consider the VCG, or Vickrey-Clarke-Groves, combinatorial auction format. Here, it is optimal for bidders to report their true valuations (cf. Levin 1997, Myerson 1981).

The allocation in our VCG mechanism is as follows (from Ausubel and Milgrom 2001):

Step 1. Bids are placed for the three items: object A, object B, and bundle AB.

Step 2. The highest bid placed is recorded for each. Call these X, Y, and Z for A, B, and AB, respectively.

Step 3. If $X + Y > Z$, the items are allocated separately; otherwise the bundle is awarded at the second-highest bid.

Step 4. When the items are allocated separately, they are done so that the payment by the winning bidder equals the highest bid placed less the incremental revenue stemming from that particular bidder's participation. This increment would be the difference between the current auction and the revenue generated if the highest bidder had not participated.

As with the sequential and bundled auctions, we simulate 30,000 auctions for each $(N+1, \theta)$ combination discussed earlier. For the sequential and the bundled cases we use the equilibrium strategy derived earlier, and for the combinatorial case we use the mechanism described in the four steps above. Calculating the average revenue generated by each mechanism in each case (represented by a unique $(N+1, \theta)$) we ascertain regions where each format yields the highest revenues. Based on this we propose:

Proposition 4 (P4). *Among the auctions of the bundle*:

(A) *The VCG mechanism weakly dominates the auction of the pure bundle*.

(B) *For complements*:

(i) *Pure components auctions are optimal for the seller for weak to moderate complements when there are five or more bidders*.

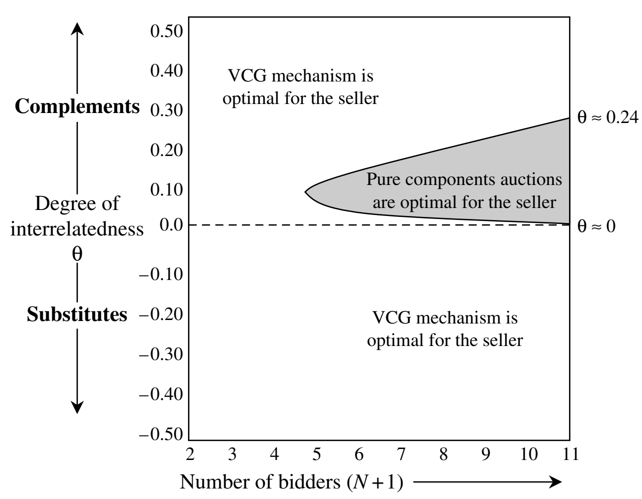
(ii) *The VCG mechanism is optimal for weak to moderate complements when there are four or fewer bidders and for strong complements*.

(C) *For substitutes*, *the VCG mechanism is always optimal*.

Comment. Figure 3 delineates the regions in which the different auction formats are optimal.

On P4(*A*). The VCG mechanism has all the strengths of the auction of the pure bundle and more. The bids for the bundle under both auction formats represent the bundle reservation prices. Therefore, the VCG mechanism will do at least just as well. When auctioning the two objects separately is advantageous,

Figure 3 **Optimality of Combinatorial (VCG) and Pure Components Auctions: Symmetric Case**



Notes. The combinatorial auction dominates the auction of the pure bundle, and so the latter is never optimal.

[10] We consider a generalized version of Krishna and Rosenthal (1996), who focus on identical objects. Although we do not obtain a closed-form solution, we find the optimal bids to be $b_i = R_i + \theta(R_i + R_j)G(b_j)$, where $G(\cdot)$ is the probability of winning the auction. In the derivation, we use the same assumptions as in the sequential case.

the VCG mechanism has the flexibility to allocate the objects separately (unlike the auction of the pure bundle). This explains P4(A).

On P4(B). For complements, the choice between VCG and pure components auctions depends on which effect is stronger—the benefit of complementarity that favors VCG or cascading competition that favors the pure components auctions. A smaller θ suggests that the complementarity effect is tepid. Therefore, when the number of bidders is moderate or high, the overbidding for the first of two auctions under pure components makes this auction format better. The converse is true for higher levels of θ or with fewer bidders, favoring VCG.

On P4(C). With substitutes, pure components auctions involve underbidding. This reduces the revenue generated from the sequential format compared with the VCG format, where no such underbidding takes place. Moreover, under pure components, winning the first object automatically reduces the winner's second-round bid, which is again absent in VCG.

5. Discussion

Despite the boom in auctions, the topic of multiobject auctions involving complements or substitutes has received limited attention. We have examined this problem by integrating the auctions and bundling perspectives. We address whether and when a revenue-maximizing seller should pursue pure components, pure bundling, or mixed bundling (combinatorial) auctions. We also offer guidelines on auction sequence and optimal bidding strategies.

5.1. Managerial Contributions

We would alert sellers and auctioneers that auctioning the objects as a bundle is not a given even for complements. Although bundling is well known to reduce heterogeneity in buyers' valuations (cf. Schmalensee 1984), sequential auctions have the advantage (from the seller's viewpoint) of creating a cascading (loosely multiplicative) effect of competition. This effect is increasing in the size of the bidder pool. As a result, pure components auctions are the best for the seller even when the objects are moderate complements, so long as there are at least a few bidders (four or more). For substitutes, pure components auctions are widely superior to the auction of the bundle, although the latter does prevail in a small band.

Sellers are likely to be better off recognizing the importance of auction sequence. In general, the seller benefits from auctioning the more valuable object first.

If the seller can pursue the mixed bundling (combinatorial) auctions, pure bundling becomes redundant. Although the combinatorial auction dominates pure components for substitutes, it is inferior to the latter for weak or moderate complements (with five or more bidders).

5.2. Theoretical Contributions

By considering a problem at the intersection of the topic areas of auctions and bundling, we see ourselves making a contribution to both areas. Extant work on bundling draws strongly on second-degree price discrimination, with the related assumption of fairly abundant supply. By considering two objects in limited supply, we bring an added perspective. Also, there is no competition among consumers in a typical bundling problem, whereas the auctions case makes the bidders competitive and strategic.

Adding to the work on multiobject auctions, we have examined the attractiveness of all three bundling strategies, examined the interactive role of the number of bidders and degree of interrelatedness, and assessed the import of auction sequence. These have yielded new insights.

5.3. Research Limitations and Future Research Directions

We have developed a stylized model that is based on some restrictive assumptions. We discuss a few specific limitations of the study and propose directions for future research.

In §3.3 we speculated on the possible implications of nonuniformity in the underlying reservation price distributions as well as correlated reservation prices, but a thorough investigation of these issues is likely to yield significant additional insights.

Whereas ours is a normative study based on a stylized model, we urge studies of an empirical nature. A cursory look at eBay auctions of strong complements suggests that (consistent with our proposition) the auction of the bundle yields a higher transaction price than the sum of those from separate auctions, for example, \$550 for a bundle of *Guitar Hero* game and Xbox $>$ \$89 $+$ \$400 (separate auctions), and \$660 for a set of six *Harry Potter* books (signed by the author) versus \$95 for a signed copy of *Harry Potter and The Chamber of Secrets* (specifically, \$660 $>$ 6 $\times$ \$95). The proliferation of auctions should aid empirical work.

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Appendix

We provide an outline of the proof for the equilibrium bids under the pure components (sequential) auctions. The actual proof is in the Technical Appendix, which can be found at http://mktsci.pubs.informs.org.

We denote bidder j's bid for object A as b_{jA}. If j is the winner of the first auction, then her bid for object B is b_{jB}; otherwise, it is b'_{jB}. The bidders are strategic while bidding on the first object. Whereas for complements, $\text{Max}\{R_{jA}, R_{jB}, R_{jAB}\} = R_{jAB}$, the same need not be true for substitutes for reasons in §2.2. Therefore, we define a new valuation of the bundle $R^{\oplus}_{jAB} = \text{Max}\{R_{jA}, R_{jB}, R_{jAB}\}$.

Because the winning bidder pays the equivalent of the second-highest bid, each bidder j maximizes the expected surplus from the auction. The bidder's expected surplus is given by

$$\text{Expected surplus of } j = \text{I} + \text{II} + \text{III}, \quad \text{where} \tag{1}$$

$$\begin{aligned} \text{I} &= (\text{probability of winning both objects}) \\ &\quad \times (\text{surplus from the bundle}), \end{aligned} \tag{2}$$

$$\begin{aligned} \text{II} &= (\text{probability of winning } A \text{ and losing } B) \\ &\quad \times (\text{surplus from the object } A), \end{aligned} \tag{3}$$

$$\begin{aligned} \text{III} &= (\text{probability of losing } A \text{ and winning } B) \\ &\quad \times (\text{surplus from the object } B). \end{aligned} \tag{4}$$

The formulation in Equation (1) is equivalent to the backward induction rule of a bidder maximizing P(winning A) $\times$ (surplus from A) + P(winning B | won A) $\times$ (incremental surplus) + P(winning B | lost A) $\times$ (surplus from B).

The probability of bidder j winning (or losing) an object refers to the probability that her bid is greater (or less) than the highest of the remaining bids, and the surplus from the object contingent on winning it represents the difference between the bidder's reservation price for the object less the second-highest bid. Losing both objects leads to a surplus of zero. Based on the notation developed thus far, and noting that $\text{Max}\{R_{jA}, R_{jB}, R_{jAB}\} = R^{\oplus}_{jAB}$ and k represents the first-round winner, Equation (1) can be represented mathematically as

Surplus

$$\begin{aligned}
&= \Bigg[\left\{ \begin{matrix} \text{P}(\text{Max}\{b_{iA}\} < b_{jA}) \\ \text{P}(\text{Max}\{b'_{iB}\}_{i\neq j} < b_{jB}) \end{matrix} \right\} \\
&\quad \times \left\{ R^{\oplus}_{jAB} - E\left(\text{Max}\{b_{iA}\} + \text{Max}\{b'_{iB}\} \,\middle|\, \begin{matrix} \text{Max}\{b_{iA}\} < b_{jA} \\ \text{Max}\{b'_{iB}\}_{i\neq j} < b_{jB} \end{matrix} \right) \right\} \\
&\quad + \left\{ \begin{matrix} \text{P}(\text{Max}\{b_{iA}\} < b_{jA}) \\ \text{P}(\text{Max}\{b'_{iB}\}_{i\neq j} > b_{jB}) \end{matrix} \right\} \\
&\quad \times \{ R_{jA} - E(\text{Max}\{b_{iA}\} \mid \text{Max}\{b_{iA}\} < b_{jA}) \} \\
&\quad + \left\{ \begin{matrix} \text{P}(\text{Max}\{b_{iA}\} > b_{jA}) \\ \text{P}(\text{Max}\{\text{Max}\{b'_{iB}\}_{i\neq j,k}, b_{kB}\} < b_{jB}) \end{matrix} \right\} \\
&\quad \times \left\{ R_{jB} - E\left(\begin{matrix} (\text{Max}\{\text{Max}\{b'_{iB}\}_{i\neq j,k}, b_{kB}\} < b_{jB}) \\ \big|(\text{Max}\{\text{Max}\{b'_{iB}\}_{i\neq j,k}, b_{kB}\} < b_{jB}) \end{matrix} \right) \right\} \Bigg].
\end{aligned} \tag{5}$$

The three multiplicative pairs on the right-hand side of Equation (5) correspond to those of Equations (2), (3), and (4).

To determine the equilibrium bid for A (i.e., the first of the auctioned objects), each bidder j must assess how her reservation prices for the objects (taken separately) and for the bundle measure up against specific thresholds. Let us note parenthetically that this is an analytical requirement intended to assess the cumulative probabilities correctly. The bidders' equilibrium bids are then as shown in Table 1.

References

Adams, W. J., J. L. Yellen. 1976. Commodity bundling and the burden of monopoly. *Quart. J. Econom.* **90**(3) 475–498.

Ausubel, L. M., P. Milgrom. 2006. The lovely but lonely Vickrey auction. P. Cramton, Y. Shoham, R. Steinberg, eds. *Combinatorial Auctions*. MIT Press, Cambridge, MA.

Avery, C., T. Hendershott. 2000. Bundling and optimal auctions of multiple products. *Rev. Econom. Stud.* **67**(3) 483–497.

Bakos, Y., E. Brynjolfsson. 1999. Bundling information goods: Pricing profits and efficiency. *Management Sci.* **45**(12) 1613–1630.

Benoit, J.-P., V. Krishna. 2001. Multipleobject auctions with budget constrained bidders. *Rev. Econom. Stud.* **68** 155–179.

Bradlow, E. T., Y.-H. Park. 2007. Bayesian estimation of bid sequences in Internet auctions using a generalized record-breaking model. *Marketing Sci.* **26**(2) 218–229.

Branco, F. 1997. Sequential auctions with synergies. *Econom. Lett.* 159–163.

Cantillon, E., M. Pesendorfer. 2004. Auctioning bus routes: The London experience. L. M. Ausubel, P. Cramton, R. Steinberg, eds. *Combinatorial Auctions*. MIT Press, Cambridge, MA.

Carbajo, J., D. de Meza, D. J. Seidmann. 1990. A strategic motivation for commodity bundling. *J. Indust. Econom.* **38**(March) 283–298.

Chakraborty, I. 1999. Bundling decisions for selling multiple products. *Econom. Theory* **13**(3) 723–734.

Chakraborty, I. 2002. Bundling and the reduction of the winner's curse. *J. Econom. Management Strategy* **11**(4) 663–684.

Elmaghraby, W. 2003. The importance of ordering in sequential auctions. *Management Sci.* **49**(5) 673–582.

Fay, S. 2004. Partial repeat bidding in the name-your-own-price channel. *Marketing Sci.* **23**(3) 407–418.

Feng, J., K. Chatterjee. 2005. Simultaneous vs. sequential auctions, intensity of competition and uncertainty. Working paper, Smeal College of Business, Pennsylvania State University, University Park.

Gans, J. S., S. P. King. 2006. Paying for loyalty: Product bundling in oligopoly. *J. Indust. Econom.* **54**(1) 43–62.

Greenleaf, E., A. Rao, A. Sinha. 1993. Guarantees in auctions: The auction house as negotiator and mangerial decision maker. *Management Sci.* **39**(9) 1130–1145.

Hausch, D. B. 1986. Multi object auctions: Sequential vs. simultaneous sales. *Management Sci.* **32**(12) 1599–1610.

Hitt, L. M., P.-Y. Chen. 2005. Bundling with customer self-selection: A simple approach to bundling low marginal cost goods. *Management Sci.* **51**(10) 1481–1493.

Hossain, T., J. Morgan. 2003. A test of revenue equivalence theorem using field experiments on eBay. Working paper, University of California, Berkeley, Berkeley.

Klemperer, P. 1999. Auction theory: A guide to the literature. *J. Econom. Surveys* **13**(3) 227–286.

Krishna, V., R. Rosenthal. 1996. Simultaneous auctions with synergies. *Games Econom. Behav.* **17** 1–31.

Levin, J. 1997. An optimal auction for complements. *Games Econom. Behav.* **18** 176–192.

McAfee, R. P., D. Vincent. 1997. Sequentially optimal auctions. *Games Econom. Behav.* **18** 246–276.

Menezes, F., P. Monteiro. 2003. Synergies and price trends in sequential auctions. *Rev. Econom. Design* **8** 85–98.

Milgrom, P., R. J. Weber. 1982. A theory of auctions and competitive bidding. *Econometrica* **50**(5) 1089–1122.

Myerson, R. B. 1981. Optimal auction design. *Math. Oper. Res.* **6**(1) 58–73.

Palfrey, T. R. 1983. Bundling decisions by a multi-product monopolist with incomplete information. *Econometrica* **51**(2) 463–483.

Riley, J., W. Samuelson. 1981. Optimal auctions. *Amer. Econom. Rev.* **71** 381–392.

Schmalensee, R. 1984. Gaussian demand and commodity bundling (in pricing of product bundles). *J. Bus.* **57**(1, Part 2) S211–S230.

Seidmann, D. J. 1991. Bundling as a facilitating device: A reinterpretation of leverage theory. *Economica* **58** 491–499.

Shapiro, C., H. R. Varian. 1999. *Information Rules: A Strategic Guide to the Network Economy*. Harvard Business School Press, Boston.

Shugan, S. M. 2005. Marketing and designing transaction games. *Marketing Sci.* **24**(4) 525–530.

Venkatesh, R., W. Kamakura. 2003. Optimal bundling and pricing under a monopoly: Contrasting complements and substitutes from independently valued products. *J. Bus.* **76**(2) 211–231.

Vickrey, W. 1961. Counterspeculation, auctions and competitive sealed tenders. *J. Finance* **16**(1) 8–37.

Wilcox, R. 2000. Experts and amateurs: The role of experience in Internet auctions. *Marketing Lett.* **11**(4) 363–374.