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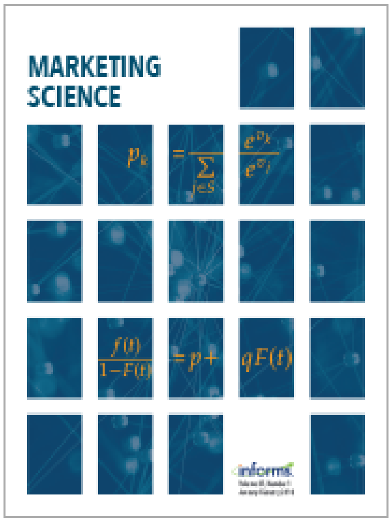

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Cross-Market Discounts

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Firms in several markets attract consumers by offering discounts in other unrelated markets. This promotion strategy, which we call "cross-market discounts," has been successfully adopted in the last few years by many grocery retailers in partnership with gasoline retailers across North America, Europe, and Australia. In this paper, we use an analytical model to investigate the major forces driving the profitability of this novel promotion strategy. We consider a generalized scenario in which purchases in a source market lead to price discounts redeemable in a target market. Our analysis shows that this strategy can be a revenue driver by simultaneously increasing prices as well as sales in the source market, even though we assume the demand curve to be downward sloping in price. Moreover, it distributes additional consumption (motivated by the discount) in two markets, and under diminishing marginal returns from consumption, this can simultaneously increase firm profits and consumer welfare more effectively than traditional nonlinear pricing strategies. Our study provides many other interesting insights as well, and our key results are in accordance with anecdotal evidence obtained from managers and industry publications.

Key words: fuelperks!; retail promotions; nonlinear pricing; competition; game theory
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1. Introduction

Giant Eagle is a dominant grocery chain in the Pittsburgh, PA metropolitan area. In contrast, the gasoline market in Pittsburgh has many players and is highly competitive.[1] However, one of the gas station chains, GetGo, is owned by Giant Eagle. A few years ago, Giant Eagle started the "fuelperks!" program under which a consumer, upon purchasing $50 of groceries from Giant Eagle, earns 10¢ off per gallon of gasoline purchased from GetGo at the next purchase occasion. This per-gallon discount increases in a (stepwise) linear fashion: $50 gets 10¢ off, $100 gets 20¢ off, and so on; if consumers spend enough money on groceries, they can even purchase fuel from GetGo for free. fuelperks! has been a tremendously successful promotional program for the Giant Eagle–GetGo combine, leading to a significant increase in sales and profits (*Progressive Grocer* 2006). The other gas station operators in Pittsburgh, on the other hand, have been hit hard, to the extent that they jointly filed (and lost) a lawsuit against Giant Eagle, in which they accused it of employing unfair sales practices (*Progressive Grocer* 2005).

Attracted by the success of fuelperks! in Pittsburgh, other grocery retailers and gas station operators across the United States are quickly adopting this promotion strategy. In fact, Giant Eagle has started a new company, Excentus Corporation, through which it is implementing the fuelperks! program for retailers across the United States: Giant Eagle in Pennsylvania, Ohio, and Maryland; BI-LO in Georgia, North Carolina, South Carolina, and Tennessee; Roundy's in Minnesota and Wisconsin; Winn-Dixie in Florida, Alabama, Louisiana, Georgia, and Mississippi; and Ukrop in Virginia. Although most of these grocery retailers do not own their own fuel pumps, they have set up their programs jointly with local fuel pumps, e.g., BI-LO with Sunoco, Roundy's and Ukrop with British Petroleum, etc. Likewise, Safeway, which has stores located throughout the western and central United States and western Canada; Genuardi's, which is run by Safeway in Pennsylvania, New Jersey, and Delaware; and Tom Thumb, which is run by Safeway in Texas, have recently started their own independent "Power Pump" rewards programs, which are very similar to the fuelperks! program, in

[1] Giant Eagle has more than 50% share of the grocery market, with the rest distributed between Walmart, Shop 'n Save, and some other smaller retailers (Lindeman 2007). In contrast, in the gasoline market, many firms such as Exxon, Shell, Sunoco, British Petroleum, Gulf, GetGo, CitGo, and CoGo have a sizable presence, and no firm is dominant.

partnership with British Petroleum.[2] Shaw's has introduced a fuelperks!-type scheme in its stores throughout Maine, Massachusetts, New Hampshire, Rhode Island, Connecticut, and Vermont; and Kroger has also introduced a similar program throughout its stores in the midwestern and southern United States.[3] Clearly, the idea of grocery retailers offering discounts on gasoline has quickly caught on all over North America. Moreover, supermarket chains in the United Kingdom, France, and Australia have also recently started similar promotional programs (Gans and King 2004).

At its core, fuelperks! is a "cross-market discount" program—purchases in one market lead to a discount in an otherwise unrelated market. This is a new and intriguing addition to the vast array of promotional tools that marketing managers have at their disposal, and its widespread success in the grocery–fuel combine raises a number of interesting questions regarding such cross-market discounts in general. Why are retailers adopting them en masse at such a rapid rate? How do they influence prices and consumption in both markets, and where do the profits come from? What is the impact of competition on the rate of the cross-market discount offered, and what are the incentives of competitors to introduce similar programs? Which market pairs are ideal for implementing such a scheme? Are such discounts good or bad for consumers? Finally, how does this compare with other similar-looking promotion strategies such as bundling, loss leadership, and quantity discounts?

Suppose one "parent firm" owns a grocery store and a fuel pump,[4] and consumers derive higher utility from higher consumption in both markets but have diminishing marginal returns from consumption. We start with a simple scenario in which both the grocery store and the fuel pump are monopolies in their respective markets. Upon purchasing groceries, consumers get lower per-unit prices for fuel, and the size of this discount depends on the quantity purchased of groceries. In this scenario, we find that price decreases and sales increase in the fuel market, as expected. However, the price in the grocery market rises above even the monopoly price that was already being charged before the discount was introduced. Moreover, grocery sales also increase. In other words, because of a "cross-market leverage" effect, both price and sales for groceries increase *together* (despite the fact that, in the demand function, demand is assumed to be reducing in price). Overall, the total profits of the parent firm increase as a larger cross-market discount is offered, and under certain conditions, the parent firm might even sell fuel at a loss to achieve this. On the other hand, we find that consumers are worse off—because there is no competition, the firm can set the cross-market discount to extract the full consumer surplus across the two markets.

In effect, a cross-market discount implements a nonlinear pricing schedule across market boundaries by allowing consumers to purchase in one market and avail a discount in a different market. This motivates a comparison to a nonlinear pricing schedule within a market, i.e., a quantity discount-type scheme in which the unit price of a product decreases as more of it is purchased. Our analysis shows that under diminishing marginal returns from consumption, firms will typically find a cross-market discount more profitable than a similar quantity discount in the same market. A cross-market discount distributes the additional consumption (motivated by the price discount) across both the markets rather than motivating more consumption in the same market. This delays the point at which reduced marginal utility from additional consumption sets in for both markets, which therefore leads to increased total consumption. In other words, besides the posted prices in the two markets, this pricing strategy introduces a "third lever to pull" in the form of the cross-market discount rate, which allows the firm to exploit the less price-sensitive portions of the consumers' utility functions.

Two features of our model are important for our results: first, price discounts motivate greater purchasing and consumption; second, marginal utility from consumption decreases as consumption increases. Both assumptions are common in the economics and marketing literature, and we believe that they are also appropriate in our setting. First, it is well established that price discounts lead to greater purchasing, and moreover, greater purchasing also endogenously leads to greater consumption (Ailawadi and Neslin 1998, Chandon and Wansink 2002, Sun 2005, Wansink 1996). Moreover, to further induce greater purchasing on the grocery side, grocery retailers have come up with the novel idea of heavily promoting gift cards of *other* nongrocery retailers. For instance, the advertisements and the websites for the fuelperks! and Power Pump Rewards programs prominently display gift cards for other retailers such as Best Buy, Macy's, Toys "R" Us, The Home Depot, the iTunes store, etc., on them. Second, it is well accepted that marginal

[2] Interestingly, Excentus, which has patented the fuelperks! idea, filed and won a patent-infringement lawsuit against Safeway in January 2010 (Excentus 2010). At the time of writing this manuscript, Safeway had temporarily halted its Power Pump Rewards program while it worked out a solution with Excentus.

[3] Some details regarding these programs are available at the following websites: http://www.fuelperks.com (fuelperks!), http://www.shaws.com/pages/promotionsIrving.php (Shaw's promotion), and http://www.kroger.com/in_store/fuel/pages/default.aspx (Kroger promotion).

[4] For ease of exposition, we use grocery and fuel as the two markets to discuss the insights in §1, but our modeling studies the general case. We also provide some other examples in the §6.

utility from consumption decreases as consumption increases, and this assumption is often invoked in analytical models (Mas-Colell et al. 1995, Singh and Vives 1984, Varian 1992). A primary reason is that consumers typically get satiated from increased consumption of a good (Brickman and Campbell 1971, Coombs and Avrunin 1977). They also face increasing holding and storage costs upon purchasing larger quantities of any good (Bell et al. 2002, Sun 2005, Wansink 1996). For instance, it can be very effort-intensive and costly for consumers to store excess gasoline after they fill up the fuel tank of their car.

Next, we bring competition into the picture by analyzing a scenario in which the grocery market is a monopoly but the fuel market is a competitive duopoly, and the parent firm owns the grocery store and one fuel pump. (We call the parent firm's fuel pump "fuel pump 1" and the competitor's fuel pump "fuel pump 2.") This is a closer representation of our motivating example from the Pittsburgh market. Suppose a fuelperks! scheme is introduced under which consumers can avail a discount at fuel pump 1 upon purchasing groceries. In this case, if the competitive intensity in the fuel market is low, the insights derived without competition largely carry over, whereas if this intensity is high, there are significant new dynamics at play. Most notably, because of a strong pricing reaction from fuel pump 2, the total profit of the parent firm has an inverted U shape in the rate of cross-market discount, unlike in the previous scenario. And what is the impact of the intensity of competition in the fuel market on the optimal rate of the cross-market discount? Will this rate be higher (to attract more consumers to the joint offering) or lower (to avoid a strong reaction from fuel pump 2)? From our model, we find that the second effect dominates—to avoid dissipating its profits by inviting a stronger pricing reaction from the competitor, the parent firm reduces the rate of the cross-market discount as the degree of competition in the fuel market increases.

We further extend our analysis to a scenario in which both the grocery and fuel markets are competitive. This raises an interesting possibility—the second grocery store can form a strategic alliance with the second fuel pump and also offer a competing cross-market discount. We find that these two players will indeed always make this joint offering, but the implications for profits and consumer surplus vary significantly with the intensity of competition. If the intensity of competition in both markets is low, then both partnerships see increased profits from cross-market discounts while consumers are worse off. If the intensity of competition is high, both partnerships are in a "prisoners' dilemma" situation; i.e., both lose profit from cross-market discounts but both offer them, and consumer surplus increases. However, when the intensity of competition is in a medium range, then both partnerships see increased profits from this scheme *and* consumers are also better off. Therefore, cross-market discounts can increase social welfare.

This result is in sharp contrast with the conclusions in Gans and King (2006), the only other study on cross-market discounts that we know of. They find that competing partnerships will see no increase in profit from cross-market discounts and consumer surplus will always decrease. However, unlike us, they assume demand to be price inelastic, which is the main difference in assumptions (among others) driving this difference in conclusions. Anecdotal evidence indicates that cross-market discounts have visibly increased household-level consumption of both grocery and fuel. This, coupled with the fact that fuelperks!-type programs are being adopted increasingly, makes a strong case supporting the results we obtain.

A natural question that arises is whether there is something special about the grocery–fuel combine that makes this strategy successful. Could it work for any two unrelated markets (or even two categories in a market, or two products in a category)? From our analysis, we find that the total profit increase from a cross-market discount program will be small if the "importance of consumption" (defined more precisely in §3) in either of the two markets is small, and this increase in profit is larger as the importance of consumption increases. Therefore, we can expect to see cross-market discounts in pairs of markets in which consumption is high enough (as measured by, say, frequency of purchasing and average dollar amount spent per purchase). Both grocery and fuel constitute a large fraction of a typical household's budget and therefore will typically satisfy this criterion, whereas any two arbitrary market combinations might not. Because programs such as fuelperks! and Power Pump Rewards have provided a "proof of concept" in the grocery–fuel combine, we might soon observe cross-market discounts in other settings that satisfy this criterion. These possibilities further motivate development of a deeper understanding of this promotion strategy.

The rest of this paper is organized as follows. In the next section, we compare cross-market discounts with related promotion strategies. In §3, we describe our model, and in §4, we present the results from our analysis. In §5, we consider extensions to the basic model. In §6, we summarize our results and conclude with a discussion.

2. Comparison with Related Promotion Strategies

Pricing and promotion strategies have been studied in great depth in economics and marketing literature (Neslin 2002, Winer 2006), and here, we discuss existing research that shares some common features with cross-market discounts. A first related stream of research is on *product bundling*—the strategy of selling two or more products in a package at a discount (Stremersch and Tellis 2002). Bundling is similar to cross-market discounting in that both strategies provide a lower total price when two goods are jointly purchased. However, they differ in several regards. The primary arguments for bundling rest on negative correlation among valuations of the products in the bundle (Adams and Yellen 1976, Venkatesh and Kamakura 2003) and complementarity among utilities from bundled products (Matutes and Regibeau 1992). We do not make any of these assumptions and show that cross-market discounts can be profitable even when implemented in completely independent markets. Furthermore, following Adams and Yellen (1976), much of the academic research on bundling rests on a price discrimination argument and uses a reservation-price paradigm with inelastic demand. In our model, elasticity of demand is an important driver of the results.

Complementary pricing strategies also share similarities with cross-market discounts. From Tellis's taxonomy of pricing strategies (Tellis 1986), two variants are closely related: captive pricing and loss leadership. In a typical *captive pricing* program, one product is sold at a low margin to penetrate the market and lock customers into purchasing a more profitable "tied" product (e.g., razors are sold cheap while the profit is made on blades that need to be used with that razor).[5] Several theories have been proposed to describe captive pricing (Slade 1998, Mathewson and Winter 1997), price discrimination once again being the prominent one. In our case, however, tying requirements are not necessary, and modulo the cross-market discount itself, demand in the two markets is completely independent.

Complementary pricing can also take the form of *loss leadership*, a slightly different arrangement in which a retailer offers certain products at low prices to lure customers into the store with the hope that they will purchase other more profitable products (Hess and Gerstner 1987). A successful loss-leader program typically requires economies of scale in shopping for consumers (Lal and Matutes 1994), a feature that our results do not depend on.

[5] In the economics literature, this strategy is also referred as *requirements tying* to emphasize that consumers have to purchase both the *tying* good and *tied* good from the same seller.

Using *cross-ruff coupons* (discount coupons that are obtained upon purchasing one brand and are redeemable on another brand at a later time) is another complementary pricing strategy that works in a similar fashion to cross-market discounts and can be modeled similarly. Dhar and Raju (1998) study this but focus on a very different question—namely, the time-lagged impact on sales of various types of offerings of these coupons. Moreover, they only consider a monopolist making these decisions, and prices and the amount of discount are treated as exogenous, whereas these are all critical elements of our study.

An important distinction between our study and the above complementary pricing strategies is that there is no explicit relationship between the prices paid for the various goods in any of these complementary pricing strategies. In the fuelperks!-type of cross-market discounts that grocery retailers are implementing nowadays, increased purchasing in the source market leads to a larger per-unit discount in the target market, which leads to very different incentives from the above.

Previous work has also shown how *nonlinear pricing* strategies can be used to increase firm profitability in a monopoly (Spence 1977) and in competitive environments (Oren et al. 1983). However, unlike our research, this literature focuses on a single market. Although there are various kinds of nonlinear pricing schedules, quantity discounts are the most relevant to our paper. This is similar to our analysis comparing cross-market discounts to similarly specified quantity discounts redeemable in the source market itself, and we characterize conditions under which the firm prefers the former over the latter.

3. Model

We assume that a firm, called firm 1, operates in two distinct and independent markets, s and t, and sells a product in each market at prices p_{s1} and p_{t1}, respectively.[6] Another firm, called firm 2, operates in market t and sells one product in this market at price p_{t2}. Therefore, market s is a monopoly and market t is a duopoly. We assume that the consumption utility function of a representative consumer is given by

$$\mathcal{U}(q_{s1}, q_{t1}, q_{t2}) = \alpha_s\left(q_{s1} - \frac{q_{s1}^2}{2}\right) + \alpha_t\left(q_{t1} - \frac{q_{t1}^2}{2} + q_{t2} - \frac{q_{t2}^2}{2} - \theta_t q_{t1} q_{t2}\right),$$

[6] We have assumed that firm 1 operates in both markets and, therefore, optimizes jointly in the two markets. If the entities in the two markets are owned separately, our formulation is equivalent to assuming that the two entities cooperatively bargain, which is equivalent to joint optimization in the two markets. Sharing of total profits can be modeled through a lump sum transfer payment based on the relative bargaining powers of the two entities, but this will not impact the variables that we focus on here.

where q_{s1} and q_{t1} denote the quantities consumed of firm 1's products in markets s and t, respectively, and q_{t2} denotes the quantity consumed of firm 2's product in market t. The terms $(q_{s1} - q_{s1}^2/2)$ and $(q_{t1} - q_{t1}^2/2 + q_{t2} - q_{t2}^2/2 - \theta_t q_{t1} q_{t2})$ denote the consumption utilities in markets s and t, respectively, from consuming the amounts q_{s1}, q_{t1}, and q_{t2}, where θ_t denotes the degree of substitutability between the products of the two firms in market t $(0 \le \theta_t < 1)$. Therefore, α_s and α_t denote the "importance" of consumption utility from the markets s and t, respectively, in the aggregated consumption utility function. Note that higher importance of consumption in a market implies lower price sensitivity in that market. Because θ_t denotes the degree of substitutability between the products of the two firms in market t, it serves as a measure of the intensity of competition in market t, with a larger value of θ_t denoting a greater intensity of competition. Note that we have assumed the consumption utility function to be concave in all quantities consumed. This captures diminishing marginal returns from consumption; i.e., as a consumer consumes more of a product, the marginal utility from consuming an extra unit of this product decreases. Using such a quadratic consumption utility function is a standard practice in economics and marketing literature (e.g., Arya et al. 2008, Jerath and Zhang 2010, Singh and Vives 1984).

Suppose that firm 1 offers a cross-market discount, $\delta \ge 0$, from s, the source market, to t, the target market. Specifically, if the consumer purchases quantity q_{s1} in the source market, she pays a unit price of $p_{t1} - \delta q_{s1}$ in the target market.[7] In this case, the expenditure function associated with consuming quantities q_{s1}, q_{t1}, and q_{t2} is given by

$$\begin{aligned}&\mathscr{E}(q_{s1}, q_{t1}, q_{t2} \mid p_{s1}, p_{t1}, p_{t2}, \delta)\\&\quad = p_{s1}q_{s1} + (p_{t1} - \delta q_{s1})q_{t1} + p_{t2}q_{t2}.\end{aligned}$$

An important point to note here is that in the above formulation the price discount in the target market depends on the quantity purchased in the source market, whereas in our motivating fuelperks! example, the price discount depends on the total expenditure in the source market. We start with this simpler formulation to obtain basic insights into the working of cross-market discounts while keeping the model analytically tractable. In §5.2, we analyze the case of expenditure-based cross-market discounts. There we have to resort to a partial numerical analysis and find that, although there are some new insights, the simpler model captures the key insights related to cross-market discounts very well.

The net consumer surplus is obtained by subtracting the expenditure from the consumption utility and is given by

$$\begin{aligned}&\mathscr{CS}(q_{s1}, q_{t1}, q_{t2} \mid p_{s1}, p_{t1}, p_{t2}, \delta)\\&\quad = \mathscr{U}(q_{s1}, q_{t1}, q_{t2}) - \mathscr{E}(q_{s1}, q_{t1}, q_{t2} \mid p_{s1}, p_{t1}, p_{t2}, \delta).\end{aligned}$$

The profit of firm 1 is given by $\Pi_{st,1}(p_{s1}, p_{t1}, \delta) = \Pi_{s1} + \Pi_{t1} = p_{s1}q_{s1} + (p_{t1} - \delta q_{s1})q_{t1}$, where Π_{s1} and Π_{t1} denote the profits of firm 1 in markets s and t, respectively, and $\Pi_{st,1}$ denotes the joint profit of firm 1 from the two markets. The profit of firm 2 is given by $\Pi_{t2}(p_{t2}) = p_{t2}q_{t2}$.[8]

We model the game in three stages. In stage 1, firm 1 decides the cross-market discount δ. In stage 2, the two firms simultaneously decide the posted prices p_{s1}, p_{t1}, and p_{t2} in the two markets, given δ. In stage 3, the consumer decides how much to purchase of each product in each market, given the posted prices and the cross-market discount. Our assumption that δ is decided before the prices are set is meant to reflect the fact that the cross-market discount is a long-term decision, whereas prices can be changed more often. For instance, in the Pittsburgh market, Giant Eagle has not changed the rate of fuelperks! in the last several years. This might also be important to clearly convey the characteristics of the fuelperks! scheme to consumers. Prices, on the other hand, change almost on a daily basis in both the grocery and fuel markets.

We solve for the subgame-perfect equilibrium of the game using backward induction. In stage 3, given the posted prices and the cross-market discount, the consumer solves $\max_{\{q_{s1}, q_{t1}, q_{t2}\}} \mathscr{CS}(q_{s1}, q_{t1}, q_{t2} \mid p_{s1}, p_{t1}, p_{t2}, \delta)$. In stage 2, given δ, the two firms simultaneously solve $\max_{\{p_{s1}, p_{t1}\}} \Pi_{st,1}(p_{s1}, p_{t1} \mid \delta)$ and $\max_{p_{t2}} \Pi_{t2} \cdot (p_{t2} \mid \delta)$. In stage 1, firm 1 solves $\max_{\delta} \Pi_{st,1}(\delta)$. In each stage we also need to impose the following conditions to guarantee that the model is well defined: we require nonnegative prices and quantities, and in the consumer utility maximization problem we require a nonnegative net consumer surplus (implicitly assuming that the consumer's outside option, corresponding to the case in which she does not purchase anything, is zero).

[7] We assume that the discount on the per-unit price of firm 1's product in market t grows linearly with the amount of firm 1's product purchased in market s. This is an approximation of the actual fuelperks! discount that grows in a stepwise-linear manner on the per-gallon price of fuel, as discussed in §1.

[8] Note that although we are designating one market as the source market and the other as the target market, this distinction is primarily for ease of exposition and ease of accounting of profits—the expenditure-reduction for the consumer and the profit reduction for the firm have the symmetric multiplicative form of $\delta q_{s1} q_{t1}$, whatever the "direction" of the discount. In §5.2 we consider expenditure-based cross-market discounts and find that they are not symmetric.

4. Analysis

We first analyze a monopoly–monopoly scenario. Understanding this simplified scenario helps us to obtain some basic insights into the dynamics of cross-market discounts before we proceed to more complicated scenarios.

4.1. A Simple Case: The Monopoly–Monopoly Scenario

In this scenario we assume that both market s and market t are monopolies (i.e., $\theta_t = 0$). Because our focus is on firm 1, and firm 2's actions have no effect on firm 1's actions when $\theta_t = 0$, we can simply ignore firm 2.

In stage 3, given the posted prices and the cross-market discount, the consumer decides the optimal quantities to purchase by maximizing her surplus. This gives the following functions for quantities consumed in each market:

$$q_{s1} = \frac{\alpha_t(\alpha_s + \delta)}{\alpha_s\alpha_t - \delta^2} - \frac{\alpha_t}{\alpha_s\alpha_t - \delta^2} p_{s1} - \frac{\delta}{\alpha_s\alpha_t - \delta^2} p_{t1}, \quad \text{and}$$

$$q_{t1} = \frac{\alpha_s(\alpha_t + \delta)}{\alpha_s\alpha_t - \delta^2} - \frac{\alpha_s}{\alpha_s\alpha_t - \delta^2} p_{t1} - \frac{\delta}{\alpha_s\alpha_t - \delta^2} p_{s1}.$$

It is insightful to discuss some salient properties of these demand functions. First, if $\delta = 0$, i.e., no cross-market discount is offered, the demand functions are given by $q_{s1} = 1 - (1/\alpha_s)p_{s1}$ and $q_{t1} = 1 - (1/\alpha_t)p_{t1}$, and the quantities demanded in the two markets are completely independent. In this case, the price sensitivity in market s is given by $1/\alpha_s$, which implies that, as the importance of the source-market product increases, consumers become less price sensitive for this product. The parameter α_t plays the same role in market t. Second, the quantities consumed are linear in prices but nonlinear in the cross-market discount. Third, the cross-market discount leads to a "coupling" between the two markets, and a price increase in *either* market leads to consumers purchasing lesser in *both* markets even though they were originally independent. Fourth, as the cross-market discount (δ) increases, the base demand, the own-price sensitivity, and the cross-price sensitivity of the consumers increase in both markets, all else equal.

In stage 2, the firm foresees the above response by the consumers and sets the optimal posted prices, p_{s1} and p_{t1}, by solving its profit maximization problem. The prices and the corresponding quantities consumed are given in Table 1. These expressions reveal an interesting "cross-market leverage" effect that the firm uses to its advantage. For a fixed δ, the effective per-unit price in the target market decreases with quantity consumed in the source market. Therefore, consumers are incented by a cross-market discount to increase consumption in the source market, even at a higher price in this market. Foreseeing this, the firm *raises* the price in the source market. In other words, despite the fact that consumers have a negative price elasticity of demand, they increase the quantity they purchase in the source market at a higher price when a cross-market discount is in place. (Specifically, in our model, both price and quantity simultaneously increase because the base demand, i.e., the intercept of demand, increases with δ.) Therefore, there is increased consumption in both markets, at a higher price in the source market and a lower effective price in the target market. We state the above result in the following proposition.

Table 1 Prices and Quantities in Stage 2 in the Monopoly–Monopoly Scenario

Prices	
p_{s1}	$\frac{\alpha_s\alpha_t(2\alpha_s + \delta)}{4\alpha_s\alpha_t - \delta^2}$
p_{t1}	$\frac{\alpha_s\alpha_t(2\alpha_t + \delta)}{4\alpha_s\alpha_t - \delta^2}$
$p_{t1} - \delta q_{s1}$	$\frac{\alpha_t(2\alpha_s\alpha_t - \alpha_s\delta - \delta^2)}{4\alpha_s\alpha_t - \delta^2}$
Quantities	
q_{s1}	$\frac{\alpha_t(2\alpha_s + \delta)}{4\alpha_s\alpha_t - \delta^2}$
q_{t1}	$\frac{\alpha_s(2\alpha_t + \delta)}{4\alpha_s\alpha_t - \delta^2}$

Proposition 1. *After the introduction of a cross-market discount, both price and quantity sold in the source market increase simultaneously.*

The above proposition holds whenever the model is well defined, the condition for which is $\delta < \sqrt{\alpha_s\alpha_t}$. The firm makes a larger profit in the source market (because both price and consumption increase in this market) but a lower profit in the target market. Overall, as δ increases, the combined profit from the two markets increases because of a larger cross-market leverage effect. From the consumers' point of view, their surplus *decreases* as a result of the cross-market discount, because they are purchasing more in the target market at a lower price and more in the source market but at a higher price. The decrease in surplus as a result of the price increase in the source market dominates any increases in surplus as a result of higher consumption in both markets or lower price in the target market. Therefore, when a cross-market discount is being offered, the prices charged are such that it is optimal for consumers to avail it, but they would have been better off if it were not offered in the first place.

Given the results of stage 2 above, in stage 1 the firm decides the optimal level of the cross-market discount rate (δ) to maximize its joint profit from the two markets. The expression for the joint profit is given by $\Pi_{st,1}(p_{s1}, p_{t1}, \delta) = \alpha_s \alpha_t(\alpha_s + \alpha_t + \delta)/(4\alpha_s\alpha_t - \delta^2)$. From the expressions and the discussion above, we can see that this profit monotonically increases in δ, whereas the consumer surplus and the effective price in the target market monotonically decrease in δ. Therefore, the firm will keep increasing δ up to the point at which either the consumer surplus is zero or the effective price in the target market is zero, and this gives the optimal level of δ. At this point, the total profit of the firm from the two markets is higher than the sum of the monopoly profits that it was making in the two markets without using a cross-market discount.

If consumption in the target market is not important (i.e., α_t is considerably smaller than α_s), prices in the target market will be low because consumers are not willing to spend a lot in this market. In this case, the firm increases δ up to the point at which the effective price in the target market is zero. On the other hand, if consumption in the target market is important enough, the firm can charge a higher price in this market, and in this case, the constraint that consumer surplus should be nonnegative is binding; i.e., the firm can extract the full consumer surplus by employing a cross-market discount and increasing price in the source market. Note that the firm can achieve this even when the price elasticity of demand is negative in both markets, and this was not possible without the cross-market discount. We state the above result as a proposition.

Proposition 2. *If neither market is competitive, the firm can always use a cross-market discount to extract greater consumer surplus across the two markets. Furthermore, if the target market is less price sensitive than the source market (i.e., $\alpha_t > \alpha_s$), then the firm can set the cross-market discount rate (δ) to extract the full consumer surplus across the two markets.*

Furthermore, the optimal cross-market discount rate increases with both α_s and α_t. This is because a larger value for either of these parameters increases the importance of these categories in the consumption utility function because of which consumers purchase more of both. Therefore, they attain more consumption utility, all of which the firm extracts by setting a higher value of δ, and leaves them with no surplus. We now provide some generalizations to the above model and compare them with quantity discounts restricted to the source market.

4.1.1. Different Rates of Diminishing Marginal Utilities. In the above analysis, for analytical simplicity, we assume that the rates of diminishing marginal utility from consumption are equal in the source and the target markets. We now consider a more general case in which the consumption utility function is given by $\mathcal{U}(q_{s1}, q_{t1}) = \alpha_s(q_{s1} - \psi_s q_{s1}^2/2) + \alpha_t(q_{t1} - \psi_t q_{t1}^2/2)$, where $\psi_s > 0$ and $\psi_t > 0$. By increasing or decreasing the values of ψ_s and ψ_t, we can increase or decrease the rates of decrease of marginal utility from consumption (or, the rates of "satiation" from consumption) in the source and target markets, respectively. (In our basic model, $\psi_s = \psi_t = 1$.) When no cross-market discount is applied ($\delta = 0$), the demand functions are given by $q_{s1} = 1/\psi_s - 1/(\alpha_s\psi_s)p_{s1}$ and $q_{t1} = 1/\psi_t - 1/(\alpha_t\psi_t)p_{t1}$. Therefore, if ψ_s and ψ_t increase, the base demand in the market in question decreases, and the demand in this market becomes less sensitive to price.

As before, the equilibrium in this scenario is derived using backward induction (details are provided in §TA1.1 in the electronic companion, available as part of the online version that can be found at http://mktsci.pubs.informs.org). The basic insights of Propositions 1 and 2 remain unchanged because both prices and quantities are increasing in δ. However, larger values of ψ_s and ψ_t imply that demand is less price sensitive (as mentioned above), which reduces the rates at which prices and quantities, and therefore profit, increase with an increasing cross-market discount rate. The other key impact of the rates of diminishing marginal utilities is on the binding constraints in stage 1. It turns out that both consumer surplus and effective price in the target market decrease at a slower pace for larger values of rates of diminishing marginal utilities, enlarging the region of feasible values of δ.

Therefore, when consumer satiate faster, we have two forces moving in opposite directions. Although profit increases at a slower rate with δ, the firm can impose a larger rate of the discount. In this scenario, the former effect dominates, providing the firm smaller gains in profits when consumers have larger rates of satiation. The ability of the firm to extract consumer surplus also depends on the rates of diminishing marginal utilities, and the effect of the rate in each market is different. We find that the set of values of α_s and α_t where the firm can fully extract consumer surplus is enlarged by larger values of ψ_s and smaller values of ψ_t.

Finally, it is worthwhile to note that if marginal utility from consumption is constant rather than decreasing (i.e., $\psi_s = \psi_t = 0$), then cross-market discounts will be futile. This is because this is a degenerate case in which if the price in a market is lower than marginal utility, then the consumer will consume the maximum amount available. In other words, with constant marginal utility, if the consumer has the incentive to consume one unit, then she has the incentive to consume all the units available. Therefore, there is no reason to offer any discount to induce more consumption.

4.1.2. Quantity Discounts Limited to the Source Market. How do cross-market discounts compare to similar discounts in the source market itself? Such a "self-market discount" scheme could be considered similar to the more standard quantity discounts or loyalty programs in which consumers are offered more grocery rebates as they purchase more.[9] In this section, we show that as a result of the diminishing marginal returns from consumption, a firm may prefer cross-market discounts to self-market discounts. We consider only the monopoly–monopoly scenario; the same basic insights hold for the other scenarios (that we subsequently discuss) as well.

To model the impact of diminishing marginal returns, we assume that the consumption utility function is given, as above, by $\mathcal{U}(q_{s1}, q_{t1}) = \alpha_s(q_{s1} - \psi_s q_{s1}^2/2) + \alpha_t(q_{t1} - \psi_t q_{t1}^2/2)$, where $\psi_s > 0$, $\psi_t > 0$. In the case of a cross-market discount, denoted by δ, the expenditure function is given as before. In the case of a self-market discount, denoted by δ_s, the effective per-unit price of the product in market s is given by $p_{s1} - \delta_s q_{s1}$, and the expenditure function is given by $\mathcal{E}(q_{s1}, q_{t1} \mid p_{s1}, p_{t1}, \delta_s) = (p_{s1} - \delta_s q_{s1})q_{s1} + p_{t1}q_{t1}$. We conduct the analysis in the same way as in §4.1; the details are in §TA1.2 in the electronic companion.

Consider the simplified case in which $\psi_s = \psi_t = \psi$; i.e., the rate of diminishing marginal returns is the same in both the source and the target markets. A self-market discount induces more consumption only in the source market, whereas a cross-market discount induces more consumption partly in the source market and partly in the target market, as shown earlier. This implies that a self-market discount leads to consumers reaching the point of reduced consumption utility earlier than a cross-market discount, because the latter slows down "satiation" in both markets by distributing additional consumption (motivated by the discount) across the two markets. Consequently, the optimal cross-market discount is larger than, and induces more total consumption than, the optimal self-market discount. Therefore, as ψ increases, the firm finds it optimal to employ a cross-market discount scheme rather than an analogous self-market discount scheme, because cross-market discounts allow the firm to exploit the less price-sensitive parts of the consumers' utility functions in the two markets.

The above insights carry over to the cases in which $\psi_s \neq \psi_t$. Notably, if $\psi_s > \psi_t$, then the rate at which marginal utility decreases with additional consumption is faster in the source market than in the target market, and therefore, cross-market discounts are even more likely to be offered compared with self-market discounts. On the other hand, if ψ_s is significantly smaller than ψ_t, then self-market discounts are preferred. The exact condition under which the firm will prefer a cross-market discount strategy over a self-market discount strategy is $2\alpha_t\psi_s - 3\alpha_s\psi_t + \sqrt{8\psi_s\psi_t\alpha_s\alpha_t + \alpha_s\psi_t} > 0$. We summarize the discussion above in the following proposition.

Proposition 3. *The firm prefers a cross-market discount over a self-market discount, except when the marginal utility of consumption in the target market diminishes significantly faster than it does in the source market.*

4.1.3. Nonzero Marginal Cost. In our analysis above, we assume that the marginal cost for the firm is zero in both markets and impose the condition that the effective price in the target market should be nonnegative. In §TA1.3 in the electronic companion, we allow a positive marginal cost in the target market and find that the firm may offer a cross-market discount large enough to price below marginal cost in this market. In our motivating example in §1 (in which a discount of 10¢ per gallon is offered for every \$50 spent on groceries), this indeed seems to be the case because fuel pumps across the United States are known to make a profit of only 3¢ to 15¢ per gallon (Robbins 2008). Besides this, there is no qualitative difference in the results. (Note that a zero marginal cost in the source market is not a restrictive assumption because prices in the source market increase after the introduction of a cross-market discount.)

4.2. The Monopoly–Duopoly Scenario

In the monopoly–duopoly scenario, we allow for competition in the target market by assuming $0 \le \theta_t < 1$. This scenario is a closer representation of our motivating example. We build on the insights obtained from the analysis of the monopoly–monopoly scenario, some of which carry over qualitatively. The new effects that arise can directly be attributed to the strategic interaction between competitors in the target market.

As before, we start with stage 3, in which the consumer decides the optimal quantities to purchase by maximizing her surplus, given the posted prices and the cross-market discount. This gives the following demand functions:

$$q_{s1} = \frac{1}{\alpha_s\alpha_t(1-\theta_t^2) - \delta^2}(\alpha_t(1-\theta_t)(\alpha_s(1+\theta_t)+\delta) - \alpha_t(1-\theta_t^2)p_{s1} - \delta p_{t1} + \delta\theta_t p_{t2}),$$

[9] In a typical quantity discount schedule, the unit price of a good decreases at a decreasing rate as more quantity is purchased. However, to study the self-market discount analog of the fuelperks! scheme, we model a "supercharged" quantity discount in which the unit price of the good decreases linearly as more quantity is purchased.

$$q_{t1} = \frac{1}{\alpha_s\alpha_t(1-\theta_t^2)-\delta^2}((\alpha_s\alpha_t(1-\theta_t)+\alpha_s\delta)-\delta p_{s1} - \alpha_s p_{t1} + \alpha_s\theta_t p_{t2}), \quad \text{and}$$

$$q_{t2} = \frac{1}{\alpha_s\alpha_t(1-\theta_t^2)-\delta^2}((\alpha_s\alpha_t(1-\theta_t)-\alpha_s\theta_t\delta-\delta^2) + \delta\theta_t p_{s1} + \alpha_s\theta_t p_{t1} - (\alpha_s\alpha_t-\delta^2)p_{t2}).$$

First, note that if $\theta_t = 0$, we obtain exactly the demand functions for firm 1 in the monopoly–monopoly scenario. When $\theta_t > 0$, i.e., there is competition in market t, the quantities consumed are linear in prices but nonlinear in the cross-market discount. As before, if $\delta = 0$, the two markets are completely independent, and if $\delta > 0$, the two markets are "coupled." If firm 1 increases price in either market, all else equal, the quantities demanded for its products in both markets reduce and the quantity demanded of firm 2's product in market t increases. If firm 2 increases its price in market t, its own demand decreases, while the quantities demanded of firm 1's products in both markets increase. If firm 1 increases δ, the base demands for both of firm 1's products increase and the base demand for firm 2's product in market t decreases, and the above discussed reactions of consumers to prices become stronger.

The firms foresee the above response by the consumers and, in stage 2, they simultaneously solve $\max_{\{p_{s1}, p_{t1}\}} \Pi_{st,1}(p_{s1}, p_{t1} \mid \delta)$ and $\max_{p_{t2}} \Pi_{t2}(p_{t2} \mid \delta)$. The prices and the corresponding quantities are given in Table 2. Figure 1 shows equilibrium prices, quantities demanded, and profits as functions of the discount rate δ for a representative case. As in the monopoly–monopoly scenario, this analysis also reveals the "cross-market leverage" effect when $\delta > 0$—for firm 1, posted prices are higher in both markets (whereas price after discount is lower in market t), quantities consumed are higher in both markets, and total profit is higher (decomposing this, profit is higher in market s but lower in market t).

But the story is richer in this scenario because of the strategic reaction of the competitor in market t. If $\delta > 0$, firm 1's effective price in market t decreases, in response to which firm 2 lowers its price in market t as well, and as δ increases, this reaction from firm 2 becomes stronger. For δ large enough, firm 2 charges a very small price to increase its own sales, with the result that the cross-market leverage effect for firm 1 starts to decrease because fewer units are purchased from it in market t. Because of reduced cross-market leverage, any further increase in δ leads to a decrease in posted prices, sales, and profits for firm 1 in both markets. In summary, for firm 1, the joint profit from the two markets has an inverted U shape in δ, unlike in the monopoly–monopoly scenario in which it always monotonically increases with δ. This is due to the strategic reaction by the competitor in market t.[10]

Given the results of stage 2, in stage 1, firm 1 solves $\max_\delta \Pi_{st,1}(\delta)$ to decide the optimal level of the cross-market discount rate δ. From the analysis above, we can see that the optimum solution for δ may be interior. At this optimum, firm 2 makes lower profits than when no cross-market discount was being offered, whereas firm 1 makes higher profits because of the cross-market leverage.[11] Note that the consumer surplus at the optimum level of the cross-market discount may be positive and larger than without a cross-market discount, unlike in the monopoly–monopoly case. In the monopoly–duopoly case, after cross-market discounts have been introduced, consumers obtain smaller surplus in the grocery market (because of higher prices in this market through the cross-market leverage effect) but a significantly larger surplus in the target market because of competition in this market. Overall, if the competitive intensity in the target market is high, total consumer surplus increases after the introduction of a cross-market discount, whereas if it is low, the total consumer surplus decreases (as in the monopoly–monopoly case). Therefore, both firm profit and consumer surplus can simultaneously increase in this case after cross-market discounts are introduced.

We now study the characteristics of the optimal cross-market discount rate. First, the optimal value of δ increases with both α_s and α_t, keeping other parameters fixed. This is because a larger value for either of these parameters increases the importance of these categories in the consumption utility function because of which consumers purchase more of both. Therefore, they attain more consumption utility, but the firm also sets δ at a larger value and can extract more of the consumer surplus (even if it is unable to extract all of it).

The impact of the degree of competition in the target market, θ_t, on the optimal cross-market discount rate, δ, is more interesting. Arguably, an increase in θ_t can lead to an increase or a decrease δ. On the one hand, one can argue that if the target market is

[10] At extreme values of θ_t, however, the trends are still monotonic. At one extreme, if θ_t is close to zero, the model resembles the monopoly–monopoly scenario and therefore prices and quantities demanded are monotonically increasing in δ. At the other extreme, if θ_t is close to one, prices are very low because of intense competition. In this case, the constraint that the effective price in the target market should be nonnegative is binding before the decreasing part of firm 1's profit curve is reached.

[11] The profit of firm 2 might also go to zero at the boundaries of the parameter space when the target market has no attractiveness to the consumer ($\alpha_t \to 0$), when asymptotically the importance of the source market dominates consumer's decision ($\alpha_s \to \infty$), or when the degree of competition approaches its maximum ($\theta_t \to 1$).

Table 2 Prices and Quantities in Stage 2 in the Monopoly–Duopoly Scenario

Prices	
p_{s1}	$\frac{\alpha_s\alpha_t(\alpha_s^2\alpha_t(\theta_t^4-5\theta_t^2+4)+\alpha_s\delta(\alpha_t(2-\theta_t^2-\theta_t)-\delta(4-3\theta_t^2))+\delta^3(2-\theta_t))}{2\alpha_s^2\alpha_t^2(\theta_t^4-5\theta_t^2+4)+\alpha_s\alpha_t\delta^2(7\theta_t^2-10)+2\delta^4}$
p_{t1}	$\frac{\alpha_s\alpha_t(1-\theta_t^2)(\alpha_s\alpha_t(2\alpha_t(2-\theta_t^2-\theta_t)+\delta(2-\theta_t^2))-2\delta^2(\delta+\alpha_t(2-\theta_t)))}{2\alpha_s^2\alpha_t^2(\theta_t^4-5\theta_t^2+4)+\alpha_s\alpha_t\delta^2(7\theta_t^2-10)+2\delta^4}$
p_{t2}	$\frac{\alpha_t(\delta^2-\alpha_s\alpha_t(1-\theta_t^2))(\alpha_s(\delta\theta_t-2\alpha_t(2-\theta_t^2-\theta_t))+\delta^2)}{2\alpha_s^2\alpha_t^2(\theta_t^4-5\theta_t^2+4)+\alpha_s\alpha_t\delta^2(7\theta_t^2-10)+2\delta^4}$
$p_{t1}-\delta q_{s1}$	$\frac{\alpha_t(2\alpha_s^2\alpha_t(1-\theta_t^2)(\alpha_t(2-\theta_t^2-\theta_t)-\delta)+\alpha_s\delta^2(\alpha_t(-2\theta_t^3+5\theta_t^2+3\theta_t-6)+\delta(2-\theta_t^2))+\delta^4((2-\theta_t)))}{2\alpha_s^2\alpha_t^2(\theta_t^4-5\theta_t^2+4)+\alpha_s\alpha_t\delta^2(7\theta_t^2-10)+2\delta^4}$
Quantities	
q_{s1}	$\frac{\alpha_t(\alpha_s^2\alpha_t(\theta_t^4-5\theta_t^2+4)+\alpha_s\delta(\alpha_t(2-\theta_t^2-\theta_t)-\delta(4-3\theta_t^2))+\delta^3(2-\theta_t))}{2\alpha_s^2\alpha_t^2(\theta_t^4-5\theta_t^2+4)+\alpha_s\alpha_t\delta^2(7\theta_t^2-10)+2\delta^4}$
q_{t1}	$\frac{\alpha_s(\alpha_s\alpha_t(2\alpha_t(2-\theta_t-\theta_t^2)+\delta(2-\theta_t^2))-2\delta^2(\delta-\alpha_t(\theta_t-2)))}{2\alpha_s^2\alpha_t^2(\theta_t^4-5\theta_t^2+4)+\alpha_s\alpha_t\delta^2(7\theta_t^2-10)+2\delta^4}$
q_{t2}	$\frac{\alpha_s^2\alpha_t(2\alpha_t(2-\theta_t^2-\theta_t)-\delta\theta_t)+\alpha_s\delta^2(\alpha_t(2\theta_t^2+2\theta_t-5)+\delta\theta_t)+\delta^4}{2\alpha_s^2\alpha_t^2(\theta_t^4-5\theta_t^2+4)+\alpha_s\alpha_t\delta^2(7\theta_t^2-10)+2\delta^4}$

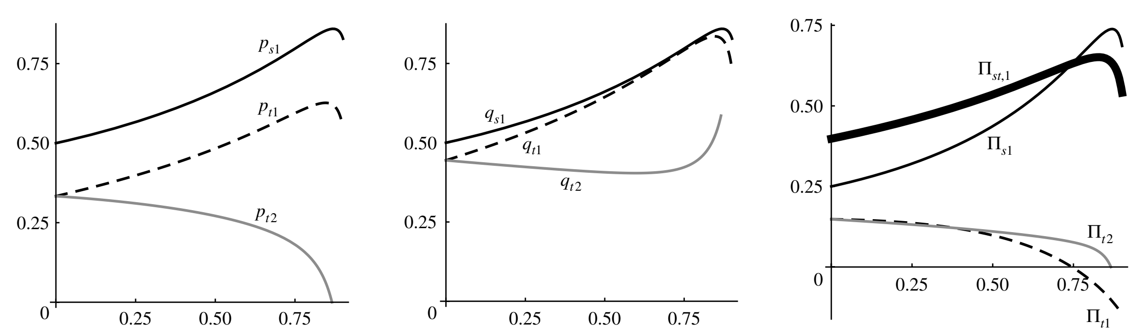
more competitive, the discount rate should be higher to induce more consumers to purchase from firm 1 in market t. On the other hand, a larger cross-market discount in a more competitive market could lead to a stronger reaction by firm 2, which can lead to overall lowering of prices and reduce everybody's profits.

We find that, in our model, the second effect dominates the first; i.e., the cross-market discount rate is smaller in a more competitive market. As θ_t increases from zero, i.e., the intensity of competition in the target market increases, the profit that can be made in this market and the extra profit from cross-market leverage both decrease. Because there is less profit to extract, the optimal value of the cross-market discount rate δ decreases. This is shown in Figure 2 for a representative case. Furthermore, as long as θ_t is small, firm 2 does not react very strongly to a cross-market discount. Thus, although the cross-market discount rate decreases with increasing θ_t, it decreases slowly. However, as θ_t becomes larger, the reaction by the competitor becomes progressively stronger and optimal δ starts decreasing in a concave fashion, i.e., decreasing at an increasing rate. As θ_t gets close to one, price sensitivities in the target market become very large and prices in the target market approach zero (equal to marginal cost), and there is no profit in the target market to extract. Therefore, offering a cross-market discount is of no use to firm 1, so that the optimal value of δ is zero. We confirm all the results above through a comprehensive analysis allowing for different values of the exogenous parameters of the model (θ_t, α_s, and α_t; details are provided in §TA2 in the electronic companion) and summarize them in the following proposition.

Proposition 4. *If the target market is competitive, the optimal value of the cross-market discount rate* (δ):

- *Decreases in the competitive intensity in the target market* (θ_t).
- *Increases in the importance of the source-market consumption utility* (α_s) *and in the importance of the target-market consumption utility* (α_t).

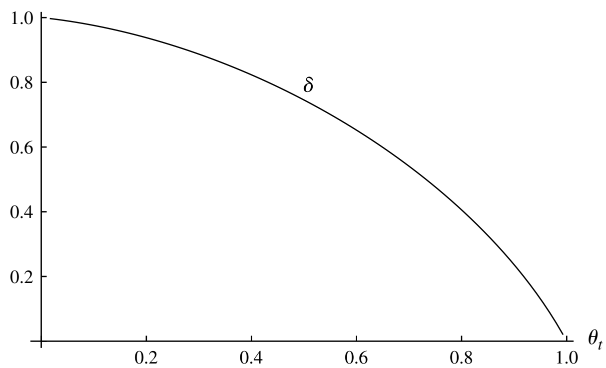
Figure 1 Prices, Quantities, and Profits as Functions of the Cross-Market Discount Rate (δ) in Stage 2 in the Monopoly–Duopoly Scenario



Posted prices as a function of δ | Quantities demanded as a function of δ | Profits as a function of δ

Note. For these plots, we use $\alpha_s=\alpha_t=1$ and $\theta_t=1/2$.

Figure 2 Plot of the Optimal Rate of the Cross-Market Discount (δ) with Competitive Intensity in the Target Market (θ_t), When $\alpha_s = \alpha_t = 1$



5. Extensions

5.1. The Duopoly–Duopoly Scenario

Although it is reasonable to assume that the source market is a monopoly for our motivating example of the Pittsburgh market, grocery retailers that are adopting the fuelperks! and Power Pump Rewards schemes in other areas might be facing significant competition in the grocery market as well. Therefore, we extend our analysis to a duopoly–duopoly scenario by assuming that the focal firm faces competition in the source market as well.

In this scenario, when only the focal firm offers a cross-market discount and the competitors in the two markets act independently, the focal firm is better off and the competitors in both markets are worse off. Furthermore, competition in the source market has similar effects as competition in the target market, and the basic insights from the model remain largely unchanged. More details are available in §TA3.1 in the electronic companion.

However, if competing firms are present in both markets, then these firms (i.e., the focal firm's competitor in the source market and its competitor in the target market) might have the incentive to join hands and introduce their own cross-market discount program in competition with the focal firm's cross-market discount program. We assume that two firms (1 and 2) compete simultaneously in both source (s) and target (t) markets. (Note that we have clubbed the competing entities in the two markets into "firm 2," and firms 1 and 2 are assumed to be completely symmetric.) We keep all notation used in previous scenarios except for the introduction of an index for the rate of cross-market discount offered by each firm (δ_1 and δ_2). Then, in the symmetric duopoly–duopoly scenario, the consumer utility and expenditure are given by

$$\mathcal{U}(q_{s1}, q_{s2}, q_{t1}, q_{t2}) = \alpha_s\left(q_{s1} - \frac{q_{s1}^2}{2} + q_{s2} - \frac{q_{s2}^2}{2} - \theta_s q_{s1} q_{s2}\right) + \alpha_t\left(q_{t1} - \frac{q_{t1}^2}{2} + q_{t2} - \frac{q_{t2}^2}{2} - \theta_t q_{t1} q_{t2}\right),$$

$$\mathcal{E}(q_{s1}, q_{s2}, q_{t1}, q_{t2} \mid p_{s1}, p_{s2}, p_{t1}, p_{t2}, \delta_1, \delta_2) = p_{s1}q_{s1} + p_{s2}q_{s2} + (p_{t1} - \delta_1 q_{s1})q_{t1} + (p_{t2} - \delta_2 q_{s2})q_{t2}.$$

We again model the game in three stages. In stage 3, the consumer decides how much she will consume of each product in each market. In stage 2, the two firms simultaneously decide all retail prices in both markets. In stage 1, both firms simultaneously decide their optimal cross-market discount rates.[12] Below, we discuss the insights obtained from our analysis; more details are available in §TA3.2 in the electronic companion.

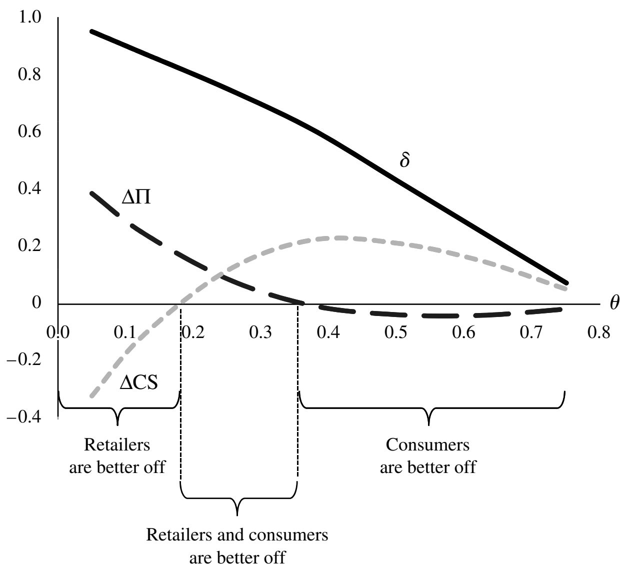
We are interested in analyzing how firm profits and consumer surplus change in comparison to the case in which cross-market discounts are not implemented. To simplify the analysis and focus on the key insights, we assume that both markets are equally important for the consumers and equally competitive for the firms; i.e., $\alpha_s = \alpha_t = \alpha$ and $\theta_s = \theta_t = \theta$. Our analysis shows that both firms always offer cross-market discounts. In Figure 3, we plot for a representative case the optimal rate of the cross-market discount (which, in equilibrium, is equal for both firms because of symmetry; i.e., $\delta_1 = \delta_2 = \delta$), the change in profit for the firms, and the change in surplus for the consumers as a result of the cross-market discounts.

As expected, when the competitive intensity (θ) is small, this scenario resembles the monopoly–monopoly scenario, and the retailers are better off and consumers are worse off with cross-market discounts. On the other extreme, when the competitive intensity is high, cross-market discounts actually hurt firms' profits because they invite stronger reactions from the competing firm. However, each firm still offers them because of a classic prisoners' dilemma situation. In this case, consumer surplus is higher after cross-market discounts are introduced.

The more interesting result, however, occurs when the intensity of competition is medium. In this case, *both* the consumers and the firms are better off through cross-market discounts. This is because consumption is higher, and because competitive intensity is medium, prices neither rise too much (which would have hurt the consumers) nor decrease too much (which would have hurt the firms). Therefore, the benefit from a cross-market discount strategy is positive even when a competitor decides to imitate the strategy, and moreover, this benefit is not at the

[12] As we described in Footnote 6 in §3, if the competing entities in the two markets are owned separately, our formulation is equivalent to assuming that the two entities cooperatively bargain with lump sum transfer payments (i.e., $s1$ and $t1$ cooperatively bargain, and $s2$ and $t2$ cooperatively bargain), which is equivalent to joint optimization by these entities in the two markets (i.e., joint optimization by $s1$ and $t1$, and joint optimization by $s2$ and $t2$).

Figure 3 **Impact of the Competitive Intensity (θ) on the Optimal Rate of the Cross-Market Discount (δ) Offered by Both Firms, the Change in Profit for Each Firm ($\Delta\Pi$), and the Change in Consumer Surplus (ΔCS) as a Result of the Cross-Market Discounts**



Note. For these plots, we use $\alpha = 1$.

expense of the consumers. We confirm that these results are replicated for different values of the exogenous parameters (θ and α) by conducting a comprehensive numerical analysis (details are available in §TA3.2 in the electronic companion) and summarize them in the proposition below.

Proposition 5. *If both the source market and the target market are competitive, both firms always offer cross-market discounts. Under moderately intense competition, profit of each firm increases, and consumer surplus also increases.*

The above result is in sharp contrast with the results in Gans and King (2006), the only other study on cross-market discounts that we know of. They model cross-market discounts as a bundling problem and obtain the conclusion that nobody benefits from cross-market discounts under any conditions—competing partnerships see no increase in profit and consumer surplus also decreases. However, they consider cross-market discounts primarily as a way to poach a competitor's consumers and do not allow consumer demand to be price elastic. This is a main difference between the two models (among others), which leads to these very different conclusions. In actual fuelperks! implementations, there is evidence that consumption of both grocery and fuel has increased. Indeed, when explaining why fuelperks! works, a Giant Eagle executive stated that "essentially fuelperks! is funded by the amount of additional store sales that we get inside our multiple store formats—both GetGo and Giant Eagle" (McTaggart 2006, p. 26). We believe that this evidence offers strong support for our formulation and insights.

5.2. Discount Based on Expenditure in the Source Market

In the basic model, we assume that a cross-market discount is tied to quantity purchased in the source market. In this extension, we consider a cross-market discount tied to the total amount spent by a consumer in the source market, as we see in several implementations of this strategy. We find that most of our key results from the previous sections are reproduced, and we discuss some new insights as well.

Suppose that a cross-market discount, δ_r, is offered from market s to market t such that if a consumer purchases an amount q_{s1} at the price p_{s1} in market s, she obtains a unit price of $p_{t1} - \delta_r p_{s1} q_{s1}$ in market t. The only change from the original monopoly–duopoly model is in the expenditure function, which is now given by $\mathscr{E}(q_{s1}, q_{t1}, q_{t2} \mid p_{s1}, p_{t1}, p_{t2}, \delta_r) = p_{s1}q_{s1} + (p_{t1} - \delta_r p_{s1} q_{s1})q_{t1} + p_{t2}q_{t2}$. The analysis in this case is algebraically intractable in stage 1, so we resort to a numerical study. The details are provided in §TA4 in the electronic companion.

First, we find that the qualitative impact of the discount rate δ_r on prices and quantities is the same as in our basic model. Profits also follow exactly the same patterns as before: total profit for firm 1 increases in δ_r if competitive intensity in the target market is low, and it has an inverted U shape in δ_r if this competitive intensity is high. Consumer surplus also follows the same trends as before: it decreases in δ_r if competitive intensity is low and increases in δ_r if competitive intensity is high enough. Consequently, the qualitative effect of each parameter of the model on the optimal level of the cross-market discount also remains unchanged. We conclude that, compared with this formulation, the basic model is significantly more parsimonious and captures the relevant insights of cross-market discounts.

However, a new and interesting feature of this extension is that a discount from market s to market t may not be symmetric to a discount from market t to market s, because the prices in the two markets may be different. Therefore, we can explore which direction of the discount is more profitable for the focal firm. Our numerical analysis shows two interesting results. First, when market s is relatively more important to the consumer than market t, the focal firm makes larger profits by allowing consumers to accumulate discounts in market s to redeem them in market t. When market t is relatively more important, the relation reverses. Second, competitive structure of a market also plays a role. We find that when both markets have the same importance, the focal firm will make larger profits by offering a discount from the monopoly to the duopoly market. In other words, it is more profitable if cross-market discounts are redeemable in the market that faces competition.

Both of the above results are in line with what we observe in the actual implementations of cross-market discounts from grocery to gasoline. First, according to the Consumer Expenditure Survey (CES) conducted by the Bureau of Labor Statistics, consumers spend on average more than double on groceries than on gasoline (CES 2009).[13] Second, gasoline is a highly commoditized market, making it more competitive than groceries. These observations indicate that, in actual implementations, the source market (grocery) is indeed the more important market and the less competitive market than the target market (gasoline).

5.3. Other Assumptions

We now briefly discuss some simplifying assumptions that we have made in our model. In this paper, for both cross-market discounts and self-market discounts, we have only considered price discounts that are linear in quantity consumed. We find that such a discount motivates higher consumption in both cases and typically motivates higher consumption in the cross-market discount case. Other "more-accelerated" forms of discount, such as one in which the price discount increases faster than linear with quantity consumed, can motivate even higher total consumption. However, even for such an accelerated discount schedule, we can expect that cross-market discounts may be more beneficial because they distribute additional consumption across two markets, thus delaying satiation in both. Future research can explore this question in more detail.

Another simplifying assumption we make is that consumption utility functions are concave. However, these functions might have different forms, such as an S-shaped form. In this case, a cross-market discount will not have a significant impact at low levels of consumption (i.e., the initial flat part of the consumption utility function). However, we expect our results to be applicable at higher levels of consumption. Therefore, we expect our results and insights to hold, although their impact will be "delayed." Furthermore, this might, based on the specific characteristics of the S-shaped utility functions in the two markets, also have an impact on the direction of the cross-market discount.

[13] We compare the total expenditure in the categories "food at home" and "gasoline and motor oil" as defined in the Consumer Expenditure Survey. The average ratio from 1997 to 2007 is 2.22. This is a conservative estimate because "grocery" typically contains many more categories than those included under "food at home" in the CES.

On a similar note, studies have shown that there can be significant uncertainty about the functional form of demand outside observed consumption/purchasing ranges (Montgomery and Bradlow 1999). Our analysis, which assumes demand to be linear in posted prices, cannot directly speak to these situations. Although we believe that even in these cases managers will still have the incentive to increase prices using cross-market discounts, it is important to bring other factors into the mix while making this decision, such as stronger consumer reactions to higher prices in the source market.

6. Conclusions and Discussion

Promotional programs such as fuelperks! and Power Pump Rewards have become very popular with grocery retailers in the last few years. In these cross-market discount programs, consumers can accumulate discounts by purchasing groceries at a particular grocery store (grocery being the "source market") and redeem these discounts when purchasing fuel at partnering gas stations (fuel being the "target market"). In this paper, we use an analytical model to obtain a deeper understanding of the working of such cross-market discounts.

We find that, across different competitive structures, profits in this scheme accrue from a simultaneous increase in both prices and sales in the source market, whereas the effective price that consumers pay in the target market reduces. In fact, firms can have an incentive to sell the product in the target market below marginal cost to increase total profits across the two markets. We also provide a characterization of the attributes of market pairs that favor a profitable cross-market discount program. Our model predicts that cross-market discounts will be preferred from a higher-importance-of-consumption market to a lower-importance-of-consumption market, and from a less competitive to a more competitive market. Both predictions are consistent with the fuelperks!-type promotions observed in reality. We also find that cross-market discounts can be more profitable than comparable nonlinear pricing strategies that focus on only one market, because they exploit less price-sensitive portions of consumers' utility functions (when there is diminishing marginal utility).

Another trend is that, in most markets, new grocery–gasoline combines are introducing fuelperks!-type schemes in response to such programs started by competitors. In accordance with this, our model predicts that when both source and target markets are competitive, competing partnerships will simultaneously offer cross-market discounts. Furthermore, when the competitive intensity is low, these partnerships will benefit at the cost of consumers. When the competitive intensity is high, both will see reduced profits from cross-market discounts but still offer them because they are stuck in a prisoners' dilemma situation, and consumers will see increased surplus. Interestingly, when competitive intensity is medium, both the competing partnerships and the consumers will benefit simultaneously.

Although we focus on the important grocery–gasoline combine throughout this paper, our results might also be applicable in certain other similar settings. For instance, airlines typically offer deals under which miles accumulated on one flight route can be redeemed for a discount on a different flight route or for a hotel stay or car rental. Another such interesting program is the Aeroplan rewards program from Air Canada in which, upon purchasing air tickets, consumers obtain points that can be used for subsidized fuel at Esso gas stations in Canada.

We now discuss some avenues for future work on cross-market discounts. We assume for simplicity that the focal firm owns stores in both the source market and the target market. If this is not the case, the incentives of firms in different markets to participate in such a promotional program need to be understood. For example, although a cooperative bargaining solution will still lead to the same pricing and discounting decisions that we find, the issue of dividing profits in these strategic alliances will arise. Furthermore, we assume that partnering firms in different markets sign exclusive partnership contracts. Future work can allow nonexclusive partnerships, which can lead to interesting incentives, and resulting configurations, for cross-market alliances.

We have compared cross-market discounts with self-market discounts only in the source market. However, self-market discounts could be used in both the source and target markets. We have conducted some basic exploratory analysis that shows that, under certain conditions, a cross-market discount in addition to two self-market discounts can further increase firm profits by motivating more consumption. A full analysis of this strategy is outside the scope of this paper, and future work can study it in more detail.

We also assume independence between demands in the two markets. This assumption helps to show that cross-market discounts are distinct from bundling, as they can be useful even when the usual incentives for bundling are absent. Analyzing cases in which market demands and valuations are correlated can further enhance our understanding of cross-market discounts. It is also possible that the importance of consumption parameters for the two markets could be impacted by a cross-market discount between the markets. This could lead to interesting implications for these discounts.

We also limit our analysis to a static setting that allows us to obtain some key insights using a simplified model. However, in a dynamic model, in which consumers can purchase in the source market and redeem discounts in the target market at different times, interesting dynamics such as discount accumulation and dynamically optimizing consumption jointly in the two markets will arise. A dynamic model can also explicitly build a "loyalty program" or a "reward program" component into the analysis of cross-market discounts. This can capture dynamics through which current purchasing from a firm promotes future purchasing from this firm or its partner in a different market, which can help to examine the time-lagged impact of cross-market discounts on prices, demand, and intensity of competition (Kim et al. 2001).

Finally, in 2009, Giant Eagle and GetGo started a "foodperks!" program simultaneously with the fuelperks! program. Consumers can now not only purchase groceries to earn discounts redeemable on fuel, but the money spent on fuel further earns discounts redeemable on groceries. This, of course, gives consumers an added incentive to increase consumption in both markets and is likely to further increase total profits. However, under diminishing marginal returns from consumption, if the foodperks! program operates in addition to the fuelperks! program, its marginal impact on profits might not be as great as the impact of the original fuelperks! program. Although in this paper we limit ourselves to fuelperks!, a joint analysis of fuelperks! and foodperks! is a fruitful opportunity for future work.

7. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mktsci.pubs.informs.org/.

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