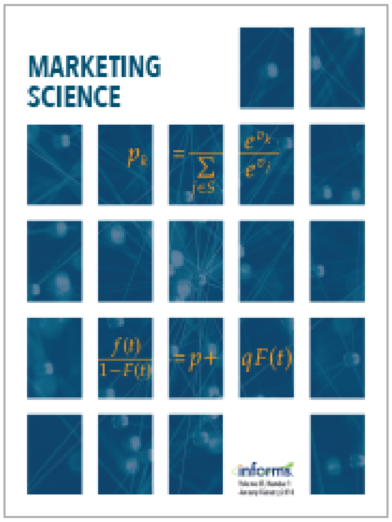

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Can Retail Sales Volatility be Curbed Through Marketing Actions?

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Abstract. For many years, marketing managers have used dynamic sales response models to compute expected sales conditional on the available information. These models fail to recognize that the volatility (conditional variance) of sales can vary over time. Moreover, the covolatilities (conditional covariances) between sales and marketing-mix variables can be time varying. Both concepts introduce a new range of strategic and tactical considerations for product and brand managers. Using a multivariate volatility model, we investigate the covolatility of sales and the marketing mix of a focal brand and competing brands in the market. We also examine carryover effects from a volatility perspective. The methodology is applied to six product categories sold by Dominick's Finer Foods. The results reveal valuable implications for marketing managers.

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Keywords: sales • volatility • marketing mix • time-series econometrics

1. Introduction

Creating noticeable and long-lasting gains in business performance while further improving the firm's competitive position is a key objective for product and brand managers in almost all industries. To achieve these objectives, managers widely use tactical decisions on marketing-mix planning such as on-again, off-again promotional campaigns and price discounts. As a result, marketing scholars have developed sales response models that help gauge the effectiveness of marketing-mix actions over top- and bottom-line performance metrics (Hanssens et al. 2001). Most of these models have focused on conditional mean of sales, while assuming that the conditional variance of sales—namely, *volatility*—is constant over time.[1] However, sales are more liable to change unexpectedly in some periods than in others. When such a pattern occurs, sales volatility varies over time. Consider the following example from the fast-moving consumer goods industry. The marketing manager of a particular grocery brand runs a price promotion campaign with unexpected strong price reductions to stimulate stagnant demand. In the presence of price reductions, sales may jump quickly and sharply. However, when the discount is withdrawn, sales often drop significantly, as the price promotion may have encouraged customers to buy unevenly, which in turn induces frequent swings in demand. This example illustrates that rapid changes can occur around average sales over time, culminating in time-varying volatility. Similar examples are available in the marketing literature for other products, such as computers (Hanssens 1998) and diapers Lee et al. (1997).

Time-varying sales volatility occurs for several reasons. First, heterogeneity in customers' purchase behavior may explain a portion of sales volatility, and such heterogeneity can change over time, partly because of differences in learning across customers (Ching 2010). Second, with homogeneous customers, purchase behavior can be more rational at certain periods. As such, customers may buy what is rationally expected (conditional mean), which results in small sales volatility. By contrast, in other periods, customers may engage in more impulse buying. In such a case, larger sales deviations occur in relation to the conditional mean, and thus sales volatility is also larger. In addition, demand volatility is often magnified over time when products are brought to customers through long distribution channels, known as the "bullwhip effect" (see Lee et al. 1997, 2004).

Not only can sales conditional variances vary over time, but the conditional covariance between sales and the marketing mix, or *covolatility*, can also evolve over time. Covolatility measures the strength of the contemporaneous association between sales and the marketing mix of a focal brand, or between sales and the marketing mix of competing brands during a certain period.[2] Indeed, marketing decisions may have an impact on sales in terms of brand switching and stockpiling that is reflected in increased volatility and covolatility. For example, increasing prices often reduces mean sales, while the impact on volatility depends on the heterogeneity of price sensitivity—i.e., with heterogeneous customers, volatility increases. By contrast, a sharp increment in promotional activity can imply (1) an increase in the sales mean and (2) an increase in volatility if some customers are prone to promotions and others are not. The following examples show how marketing may induce volatility and covolatility: promotions may make consumers less loyal, which in turn leads to irregular purchase patterns and less stable performance of a brand in the market; price discounts may provoke consumer stockpiling behavior, which prompts volatile sales performance; and setting a higher price level for a brand may signal high quality in the eyes of consumers (Milgrom and Roberts 1986), which in turn may induce less brand switching because of the differentiated positioning of the product and therefore result in lower volatility (Vakratsas 2008). These scenarios also happen in a competitive context. Therefore, it is not trivial to determine sales response a priori in both the conditional mean and conditional variance.

Figures 1 and 2 provide model-free evidence of volatility and covolatility relationships in weekly retail data. Figure 1 shows historical volatilities[3] in the logarithmic growth rates of the detergent brand Wisk's sales, prices, and promotions in the grocery chain Dominick's Finer Foods in Chicago. It provides visual evidence of time-varying volatility as historical volatilities change across different time windows. The highest sales volatility is at time window 6. Price and promotion historical volatilities also change over time. Specifically, we find high price volatility at time windows 8, 10, and 13, while promotion volatility reaches the maximum level at time window 6, which is the same period as the maximum sales volatility. This suggests that there is a strong covolatility between sales and promotions during this time window. Figure 2 shows historical covolatilities[4] for the detergent brand Wisk's sales and marketing mix. We observe time-varying covolatility between Wisk's sales and price (Figure 2(a)) and between Wisk's sales and promotion (Figure 2(b)). In particular, there are certain periods in which sales and prices and sales and promotions are more strongly associated than in other periods. The covolatility between the logarithmic growth rates of sales and price is generally negative and has a small magnitude, but these signs change over time. The covolatility between sales and promotion is positive and higher in magnitude, with the maximum level during time window 6, indicating that the contemporaneous promotional effectiveness is highest during that period. If managers can forecast these situations in advance, they may be able to intensify promotions during these weeks and thus improve the effectiveness of promotions.

Why are sales volatility and covolatility managerially important? On average, volatility is lower than unconditional variance. Erroneously assuming constant sales

Figure 1. Wisk's Unconditional Variance and Historical Volatilities for Sales, Prices, and Promotion

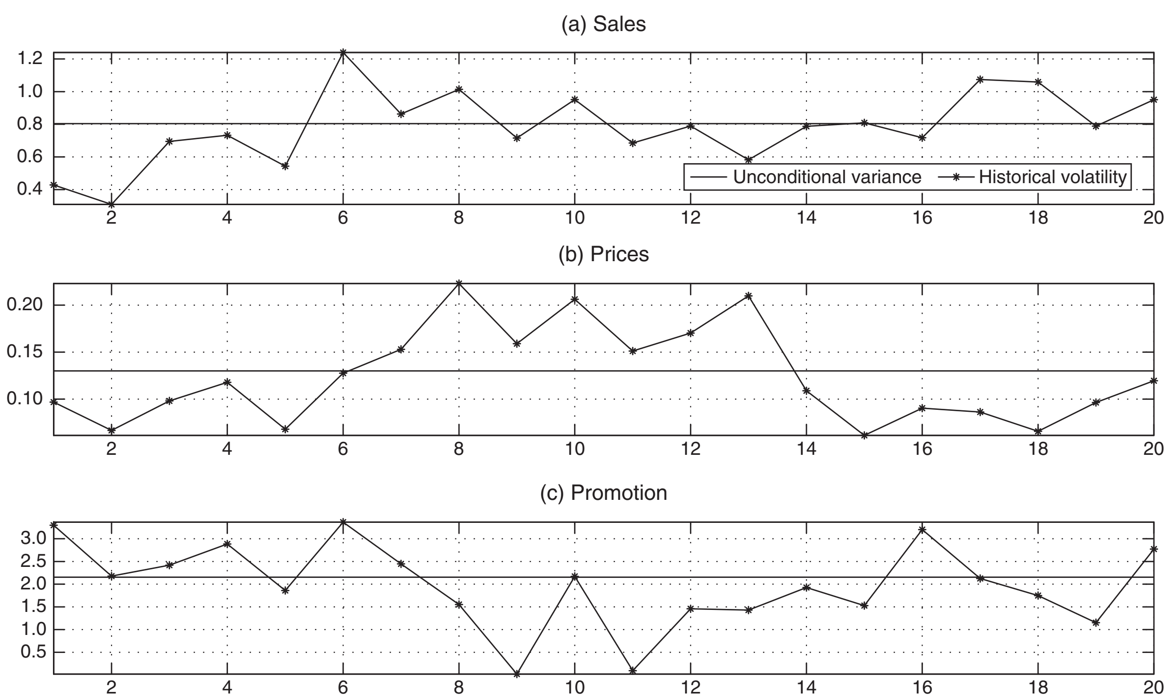
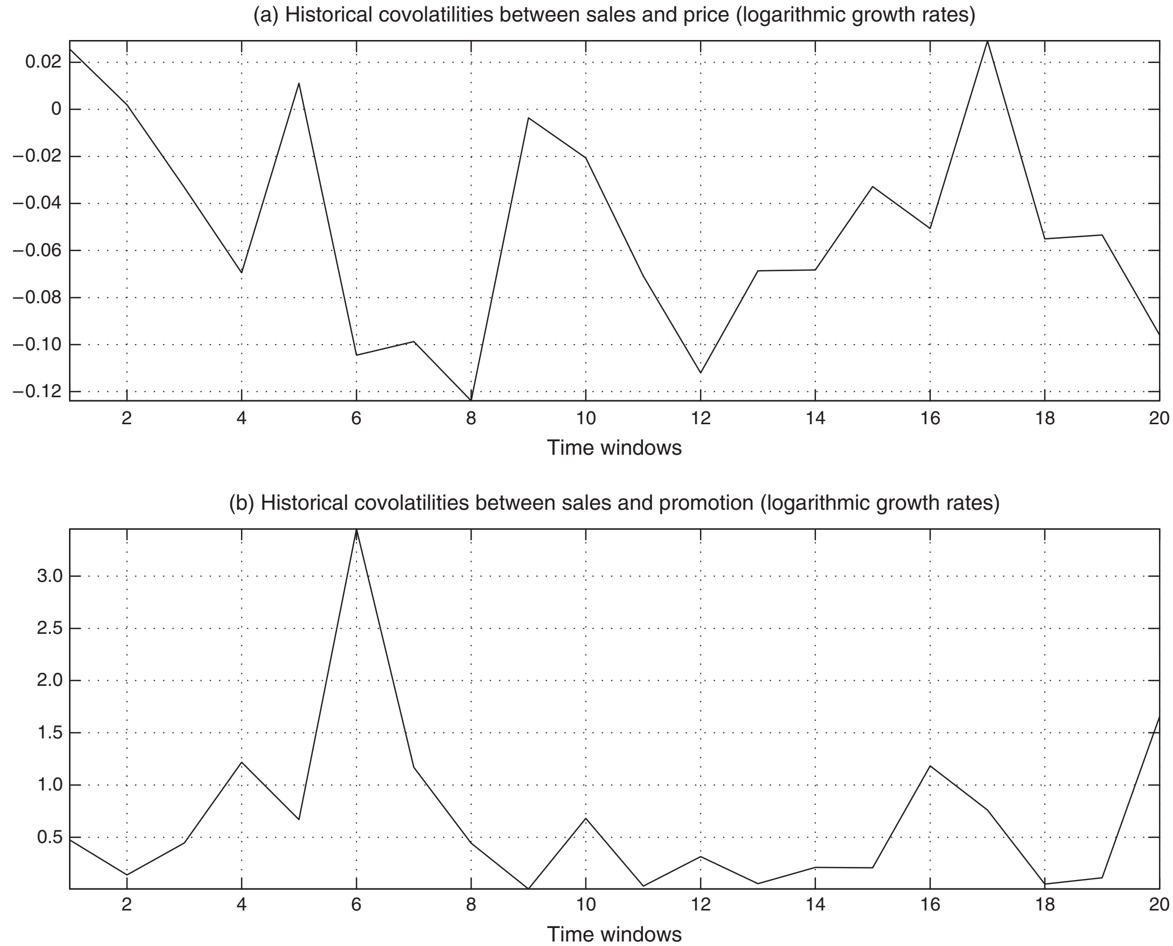


Figure 2. Historical Covolatilities Between Wisk's Sales and the Marketing Mix



volatility typically leads to oversized inventories to hedge against stockouts. Such inventory may increase firm costs unnecessarily, thus eroding a firm's competitive advantage. Nevertheless, on occasion, volatility is higher than unconditional variance. During these periods, managers basing their plans on sales unconditional variance face the considerable risk of stockouts. Indeed, these stockout situations may result in negative consequences for product and brand managers. For example, Kumar et al. (1998) find that stockouts affect sensitivity to external price references. About half the stockouts result in the purchase of a substitute product. Moreover, customers may move to another retail store if the trip cost is not too high (Gruen et al. 2002). Marketing mix actions may play a key role in reducing the risk of such shortfalls (Liu and van Ryzin 2008, Matsa 2011). Covolatility is another important metric that managers can track. Using this metric, managers can detect periods when the contemporaneous associations between sales and the marketing mix are strong (and weak) and use this knowledge to formulate their tactical marketing plans. More specifically, managers can intensify promotional actions during the periods with strong associations because the contemporaneous impact of promotions on sales will be higher.

The potential effects of a marketing-mix action generally extend well beyond the period when this action is implemented. The literature has traditionally measured these effects using impulse response functions (IRFs; see Dekimpe and Hanssens 1999). This tool is based on the assumption that all effects are channeled through the mean and that volatility is constant. This is why marketing managers tend to believe that they can influence the future level of expected sales through marketing-mix actions and that there is little they can do to influence the variation around the conditional mean. However, marketers can actually influence future sales volatility and future covolatilities by adopting appropriate marketing actions. The first step in this direction is to compute the IRFs of current marketing actions on future volatilities and covolatilities to understand more thoroughly the influence of present actions on future sales.

In this research, we employ a multivariate approach to assess the dynamic effects between sales and marketing-mix actions of one or several brands, accounting for both mean and volatility-type effects. Multivariate volatility modeling offers substantial benefits over univariate volatility models (for further details, see the online appendix. First, multivariate modeling allows for the covolatility between sales and the marketing mix to change over time, not only for the focal brand but also for the competitor brands. Second, a multivariate volatility model enables us to derive the over-time impact of a shock to marketing-mix action on sales volatility from the volatility impulse-response functions, whereas univariate modeling can accommodate a shock only in past sales. Third, unlike univariate modeling, multivariate volatility modeling can incorporate the two-way relationships in volatility—that is, past marketing-mix actions affect current sales volatility and past sales influence current marketing-mix volatilities. These methodological techniques from econometrics help us uncover novel and insightful results regarding the effectiveness of the marketing mix.

Relying on the multivariate modeling approach, we contribute to the marketing science literature in two distinctive ways. First, using the dynamic multivariate volatility Vector Autoregressive–Baba, Engle, Kraft, and Kroner (VAR-BEKK) model, we link the sales conditional mean and volatility directly with marketing-mix actions. With this model, we compute the covolatility between sales and the marketing mix. In tracking the conditional correlation (normalized covolatilities), managers may decide to intensify marketing-mix efforts in the appropriate periods. Covolatilities are particularly relevant for examining the contemporaneous effects. Second, we also consider the carryover effects for sales and marketing-mix actions. To this end, we introduce two types of IRFs: the mean IRF (MIRF) and volatility IRF (VIRF). These functions are useful to determine the effectiveness of marketing actions on future mean sales and on future volatilities and covolatilities over time. They are also relevant for strategic competitive analysis because today's marketing actions can influence the future sales volatility of a competitor. MIRFs and VIRFs differ from classic IRFs in that the latter measure only linear effects, and thus are not compatible with the nonlinearity in time-varying volatility. In addition, the standard Granger causality test and conditional independence tests applied in the sales response literature account for the effects only in the conditional mean. Building on this, we propose a test that accounts for both mean and volatility effects simultaneously. Specifically, we consider a Granger causality test between sales and the marketing mix. (The marketing-mix actions cause sales, in the Granger sense, if the current marketing-mix affects the probability distribution of future sales, but current sales do not affect the probability distribution of the future marketing mix.) We also test the dynamic independence between sales and the marketing mix (i.e., sales and marketing mix do not influence each other over time). We apply the methodology to sales and marketing-mix actions (price and promotion) for selected leader brands in six product categories sold by Dominick's Finer Foods.

2. Marketing Literature Review

The marketing literature on volatility mostly has a financial focus, linking marketing variables (marketing mix, customer satisfaction, channel and partner relationships) to financial performance (Srivastava et al. 1998, Anderson et al. 2004, Tuli and Bharadwaj 2009, Fornell et al. 2010, Luo et al. 2010). Few marketers have paid attention to sales volatility, and even fewer have investigated how sales volatility responds to marketing actions.

Raju (1992) examines the effect of brand market share and price on sales variability and finds that the magnitude of discounts partially explains sales variability. He computes sales variability as 25-week average deviations from *baseline sales* (minimum sales during the 25 weeks). Although this may be a useful metric, it is not a rigorous measure of volatility. Tuli et al. (2010) use a different proxy for volatility (the logarithm of a coefficient of variation (CV) for sales over three consecutive periods) as a dependent variable in a dynamic-panel log-linear regression model and find that an increase in the number of different types of ties with a customer results in a decrease in sales volatility for that customer. Vakratsas (2008) examines the volatility of the natural logarithms of a firm's market share in a univariate context, introducing mean effects as a dynamic regression model and volatility as an Exponential Generalized Autoregressive Conditional Heteroskedasticity (EGARCH) model, which emphasizes asymmetric dynamic effects on volatility. However, this approach has several limitations. First, market shares are by construction confined to the interval $[0,1]$, so that the logarithm of this variable falls in $(-\infty,0)$ and has an asymmetric distribution, which is probably the reason to consider an EGARCH model. Engle and Ng (1993) find that the EGARCH model usually overweighs the effects of larger shocks on volatility, providing a poorer fit than standard Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models. Second, Vakratsas's (2008) focus on market shares restricts the utility of the results from a managerial perspective. As it is infeasible to distinguish which part of the market share volatility is caused by own sales and which part is caused by those of competitors, it is difficult to interpret market share volatility; that is, if own sales and competitor sales fluctuate in a nearly proportional way, sales volatility could have a strong time variation for all brands, while the volatility of market share ratios could have no variation over time. Fischer et al. (2015) use a model with a microeconomic foundation to assess the

Table 1. Literature Review

Article	Research objective	Main findings	Modeling approach
Raju (1992)	How magnitude and frequency of discounts are associated with category sales variability	The magnitude of discounts is positively associated with sales variability, whereas frequency is negatively correlated.	Variability is the dependent variable in a log-linear model
Vakratsas (2008)	How marketing mix actions affect market share volatility	Market share volatility is significantly influenced by advertising, pricing, and distribution. Price has a significantly negative effect on volatility in both markets. Advertising and distribution both have a significant effect only in the minivan market, both in the expected direction; i.e., they reduce volatility.	Multiplicative mean model and EGARCH model
Tuli et al. (2010)	How multiple types of ties with a customer affect a supplier's performance ("sales growth" and "sales volatility")	An increase in the number of different types of ties with a customer results in an increase in supplier sales to the customer and a decrease in sales volatility to that customer.	Dynamic panel log-linear regression model
Fischer et al. (2015)	How revenue and cash-flow volatility are influenced by own and competitive marketing spending volatility, by the level of marketing spending, by the responsiveness of own marketing spending, and by competitive reactivity	Volatile marketing spending may incur negative financial side effects such as greater financing costs or higher opportunity costs of cash holdings. Common volatility-increasing marketing practices such as advertising pulsing may be effective at the top line, but could turn out to be ineffective after all costs are taken into account.	Structural model
This study	Sales volatility is analyzed to improve product management; the relationship between marketing mix actions and sales considering both contemporaneous and dynamic volatility effects for brand management	Sales volatility is estimated as conditional variance, finding evidence of conditional heteroskedasticity. The unexpected components in marketing actions affect future sales volatility over time. The conditional contemporaneous correlation between marketing effort and sales vary noticeably over time, with periods where marketing effectiveness is higher. There are cross-effects between brands competing in the same market.	VAR for sales mean and BEKK for sales volatility

impact of marketing actions on the volatility of revenue cash flows. They compute volatility using *historical volatilities* and estimate the coefficients of the model using linear regression methods. Table 1 provides an overview of the studies focusing on volatility.

By contrast, we use multivariate time series analysis and focus on sales and the marketing mix for all key competitors. In a multivariate context, we link sales mean and volatility to marketing-mix actions for all brands in the same market, but we do not impose a priori any causal direction between response (sales) and decision variables (price and promotion). We assess the contemporaneous effects between these variables using covolatilities and carryover effects using new IRFs (MIRFs and VIRFs). We discuss the managerial use of these tools, which can be applied to a focal brand or several competitors, to gain deeper insights into the effectiveness of marketing-mix actions. To our knowledge, no research to date has addressed all of the dynamic relationships between sales and marketing-mix actions in a market by accounting for mean and volatility effects simultaneously.

3. Conceptual Framework

In this article, we consider the standard definition of *volatility* in a time series: the conditional variance with respect to past information. In a multivariate setting, volatility is given by the conditional covariance matrix. More specifically, consider a vector with d random variables $X_t = (X_{t1}, \ldots, X_{td})'$, including sales and the marketing mix for one or several brands (or a suitable transformation of these variables, such as logarithm or logarithmic growth rates[5]). The multivariate time series $\{X_t\}$ follows a stationary time series process with finite unconditional moments (μ, Σ), where $\mu = E[X_t]$ and $H = E[(X_t - \mu)(X_t - \mu)']$. The conditional mean is denoted by $\mu_t = E[X_t \mid X_{t-1}, X_{t-2}, \ldots]$, and the conditional covariance matrix (volatility matrix) is $H_t = E[(X_t - \mu_t)(X_t - \mu_t)' \mid X_{t-1}, X_{t-2}, \ldots]$. The main diagonal contains the volatilities of the considered series $H_{ii,t} = \mathrm{Var}[X_{it} \mid X_{t-1}, X_{t-2}, \ldots]$ and off-diagonal entries contains the covolatilities $H_{ij,t} = E[(X_{it} - \mu_{it})(X_{jt} - \mu_{jt}) \mid X_{t-1}, X_{t-2}, \ldots]$. The conditional moments μ_{it}, H_{ijt} can vary noticeably over time around the analogous unconditional moments μ_i, H_{ij} in (μ, Σ). Standard sales response models assume that $H_t = H$ with probability 1 for all t (also known as *conditional homoskedasticity*), which can be restrictive. Time-varying volatility H_t is known as *conditional heteroskedasticity*.

Without loss of generality, we can write $X_t = \mu_t + u_t$, where u_t is the innovation or shock to the series and can be interpreted as the surprise or unexpected part of X_t, so that $E[u_t \mid X_{t-1}, X_{t-2}, \ldots] = 0$ and $H_t = E[u_t u_t' \mid X_{t-1}, X_{t-2}, \ldots]$. Researchers have developed a variety of models for the conditional mean μ_t. Here, we consider that X_t follows a VAR(r) model, such that $X_t = c + \Pi_1 X_{t-1} + \cdots + \Pi_r X_{t-r} + u_t$, and define the conditional mean μ_t as

$$\mu_t = c + \Pi_1 X_{t-1} + \cdots + \Pi_r X_{t-r}. \tag{1}$$

Note that the conditional mean μ_t is a function of its past and past error terms u_t. To assess time-varying volatility, we need to model the dynamics of H_t. Let us define $h_t = vech(H_t)$, where $vech(M)$ indicates the half-vectorization transformation of a symmetric matrix M stacking the lower triangle columns (on and below the main diagonal) in a vector. The half-vectorization transformation is used to eliminate duplicated upper elements in the symmetric covariance matrix. *Multivariate volatility models* determine H_t as

$$h_t = w + \sum_{j=1}^{q} A_j vech(u_{t-j} u_{t-j}') + \sum_{j=1}^{p} B_j \; h_{t-j}. \tag{2}$$

The coefficients w, A_j, and B_j must be constrained to ensure that H_t is positive definite at any time and the process is stationary, and to do so, researchers have proposed several multivariate volatility models. The most successful so far is the BEKK model introduced by Baba et al. (1991) and Engle and Kroner (1995), where the acronym BEKK stands for Baba, Engle, Kraft, and Kroner (for additional details, see the online appendix).

We can use the VAR-BEKK model to forecast future values of μ_t and h_t. To simplify the exposition, we consider the case of a VAR(1)-BEKK(1,1,1) model (Equations (1) and (2)), where $r = p = q = 1$. We can write this model as

$$\begin{aligned} \mu_t &= c + \Pi \mu_{t-1} + \Pi u_{t-1}, \\ h_t &= w + B \; h_{t-1} + A \; vech(u_{t-1} u_{t-1}'), \end{aligned} \tag{3}$$

since $\mu_t = c + \Pi X_{t-1} = c + \Pi(\mu_{t-1} + u_{t-1})$ using $X_{t-1} = \mu_{t-1} + u_{t-1}$. After estimating the parameters, we can use Equation (3) to understand the effect of shocks u_t over future expectations. If T denotes the last observed period, we forecast μ_{T+l} and h_{T+l} recursively for $l = 1, \ldots, L$ starting from $(\mu_T, h_T, \hat{u}_T)$. Specifically, we forecast μ_{T+l} substituting future shocks u_{T+l} by the expected value 0, and we forecast h_{T+l} substituting the unknown $vech(u_{T+l-1} u_{T+l-1}')$ by the previously computed forecast h_{T+l-1}. We can also use Equation (3) to simulate paths of expected values and volatility, drawing future shocks u_{T+j} from the normal distribution $N(0, H_{T+l})$, and then $X_{T+l} = \mu_{T+l} + u_{T+l}$.

Why is it relevant to consider the conditional variance H_t instead of the unconditional variance H? There are two reasons, one technical and one managerial. From a technical standpoint, erroneously assuming conditional homoskedasticity (constant volatility) typically leads to wrong inferences of the parameters in the conditional mean μ_t because the variances of the typical estimators (ordinary least squares (OLS) or pseudo-maximum likelihood) are very different when there is conditional heteroskedasticity. Thus, the standard hypothesis tests on the parameters of μ_t will have an erroneous significance level. From a managerial standpoint, the erroneous assumption of constant volatility renders erroneous inventory decisions. The variance equation analysis states that $H = E[H_t] + \mathrm{Var}[\mu_t]$, implying that $(H - E[H_t]) \geq 0$ (the inequality of this matrix difference indicates that it is positive definite). On one hand, this means that prediction confidence intervals based on H will be too large on average; that is, the risk is generally overrated using the unconditional variance H, and inventories will be too large in general. On the other hand, the inequality $(H - E[H_t]) \geq 0$ is compatible with occasional reverse scenarios where $(H - H_t) < 0$; this implies that the safety stocks determined from H can occasionally be too low for insurance against the ordinary stockout risk.

Taking volatility into account, managers can adapt their product stock to hedge against unexpected demand fluctuations. For example, if X_{it} are sales of a particular brand at time t, with conditional distribution $N(\mu_{it}, H_{ii,t})$, managers can hedge the risk of a stockout with probability $\alpha \in (0,1)$ (e.g., $\alpha = 0.05$) by setting inventories equal to the upper quantiles $\mu_{it} + \Phi^{-1}(1-\alpha)\sqrt{H_{ii,t}}$, where $\Phi^{-1}(p)$ denotes the p-quantile of a standard normal distribution; this decision can be quite different when we replace $\sqrt{H_{ii,t}}$ by $\sqrt{H_{ii}}$. This rule can be modified to accommodate other features. Cost considerations or deviations from conditional normality (e.g., when sales follow a log-normal conditional distribution $\ln X_{it} \sim N(\mu_{it}, H_{ii,t})$) may lead to different inventory and production decisions, but even in this case, volatility is a relevant metric for inventory management (for details, see the online appendix).

Demand volatility can be considered relative to sales volume. In this sense, we can consider the sales conditional CV of X_{it}, defined as $CV_{it} = \sqrt{H_{ii,t}}/\mu_{it}$, which typically fluctuates around the marginal $CV_i = \sqrt{H_{ii}}/\mu_i$. Both measures can be multiplied by 100 to convert them to percentages. The conditional CV measures the dispersion per average unit sold. An advantage of this measure is that it is scale free and can be used to compare volatility risks for different products selling different amounts. A drawback is that when μ_{it} falls toward zero, CV_{it} diverges to infinity.

Why have marketers overlooked sales volatility? Temporal aggregation of time series observed at higher frequencies tends to disguise their volatility (Diebold 1988). For example, sales aggregated over high-frequency periods (hourly, daily, or weekly sales) tend to present more volatility than those aggregated with lower frequencies (monthly, quarterly, or annual sales); however, the latter usually have marginal distributions with heavier tails (high kurtosis). Heavy tails are a well-known problem in retailing, occurring, for example, with aggregated sales from a wide range of unique products in which few units are sold for most items (Anderson 2004). Managers focusing on aggregated sales over long periods may be able to smooth out volatility, but at the cost of higher kurtosis and, therefore, more extreme events in sales.[6] Product and brand managers curbing demand volatility in daily series are typically less likely to face extreme events than those managing at a monthly level. By controlling volatility in time series with low levels of temporal aggregation, managers might be able to reduce extreme events occurring in aggregated series.

3.1. Marketing Mix and Volatility

This section focuses on the interrelationship between sales and the marketing mix from a multivariate perspective, accounting for both mean and volatility dynamic effects. First, we evaluate the current effect of marketing-mix actions (price or promotion) on sales. After estimating model (3) and computing the volatility matrices H_t, we determine the contemporaneous correlation between any pair of variables X_{it} and X_{jt} conditional on previous history using the metric

$$\rho_{ij,t} = \mathrm{Corr}[X_{it}, X_{jt} \mid X_{t-1}, \ldots] = H_{ij,t} \Big/ \sqrt{H_{ii,t} H_{jj,t}}. \quad (4)$$

The metric $\rho_{ij,t}$ measures the strength of the association between X_{it} and X_{jt}. Consider that X_{it} denotes the sales of a brand, and X_{jt} an element of its marketing mix. When $\rho_{ij,t}$ is close to 1, marketing-mix actions have a strong and positive contemporaneous effect on sales in the same period. Conversely, when $\rho_{ij,t}$ is close to 0, marketing-mix actions have little effect on sales. This relationship can change frequently over time t. These fluctuations explain why marketing-mix actions can have a strong contemporaneous impact on sales in some periods and little impact in others. For example, consider the case in which X_{it} is sales and X_{jt} is promotional effort. If the contemporaneous correlation ρ_{ij,t_0} is positive and high at time t_0, a large increment of promotion with respect to its baseline μ_{jt_0} (conditional mean of promotion)—that is, a large $u_{jt_0} = (X_{jt_0} - \mu_{jt_0})$—will be statistically associated with a contemporaneous large increment of sales with respect to its baseline $(X_{it_0} - \mu_{it_0}) = u_{it_0}$. By contrast, when a positive $\rho_{ij,t}$ is close to 0, a large deviation in promotion will be associated with a negligible deviation in sales with respect to its baseline. Therefore, it seems sensible to promote more when ρ_{ij,t_0} is higher (and positive) and less when it is lower. Note that a strong correlation, however, is not proof of promotional causality, as promotion and sales could covary because of the influence of other variables. In any case, if ρ_{ij,t_0} is close to 0, the promotional effort will hardly be effective. Similarly, when X_{jt} is the brand price, managers considering price reductions to increase sales should search for periods when the conditional correlation between sales and prices $\rho_{ij,t}$ is negative and large. In addition, they should charge higher prices in periods when the conditional correlation is closer to 0. Covolatilities and conditional correlations can also be used to measure the contemporaneous association between the marketing mix of a company and the sales of a competitor.

Second, we focus on quantifying the dynamic interrelationships between sales and the marketing mix beyond contemporaneous effects. To measure these carryover effects, we consider appropriate IRFs representing the impact of a shock in one variable $u_{j,T}$ at an arbitrary time T on another variable $X_{i,T+l}$ over time $l > 0$. Marketers have intensively used the classic IRF curves for multivariate time-series models (see Lüetkepohl 2005) in the context of dynamic sales response models (see Dekimpe and Hanssens 1999). However, under time-varying volatility, the carryover effects are nonlinear; therefore, they cannot be represented by classic IRFs (see the online appendix). To overcome these problems, we introduce two new tools; that is, we consider the MIRF, defined as the curve depicting the effects of a unit shock $u_{j,T}$ on the mean of $\mu_{i,T+l}$ over time $l \geq 0$, and the VIRF, defined as the curve reporting the effects of a unit shock $u_{j,T}$ on the volatility $H_{ii,T+l}$ over time $l \geq 0$. With X_{it} denoting the sales of a brand and X_{jt} an element of its marketing mix, we can use these tools to track the impact of marketing-mix decisions on future sales expectations and sales volatility. MIRF and VIRF are recursively defined by the paths (μ_{T+l}, h_{T+l}) for $l \geq 0$ given in Equation (3), in which we set $u_{j,T}$ equal to a unit impulse at time T (i.e., $u_{j,T} = 1$ and $u_{j,T+l} = 0$ for $l \neq 0$) and remove the intercepts. Alternatively, we can compute the curves analytically (see the online appendix).

We focus on the impact of marketing mix on one brand's sales, but the reciprocal MIRF and VIRF curves (impact of past sales on future marketing mix in both mean and volatility) are also useful to determine the reactiveness of the marketing mix to a previous sales impulse. In addition, IRFs are relevant for competitive analysis because they allow marketers to examine the carryover effects of one brand's marketing-mix surprises over the competitors brands' sales conditional mean and sales volatility. Note that if the analysis is based on a univariate sales model (an Autoregressive Moving Average (ARMA)-GARCH combination, discussed in the online appendix), we can only compute IRFs with respect to an impulse in past sales. Because the effect of changes in the marketing mix is ignored, a univariate model can lead to misleading conclusions about the evolution of sales.

Carryover effects of the marketing-mix actions can also affect future covolatilities between sales and marketing mix, $H_{ij,T+l}$. *Covolatility IRFs* can be computed similarly to VIRFs. In covolatility IRFs, the focus is on off-diagonal elements of H_t. Using this tool, firms can understand how current promotions affect future covolatility between price and sales. Moreover, a manager can plan a marketing action at time T to enhance a stronger contemporaneous correlation $\rho_{ij,T+l}$ between sales and the marketing mix at time $T+l$. For example, two weeks before Black Friday, a manager could act to increase the future contemporaneous conditional correlation between sales and promotion in the following weeks.

In addition, we can measure the carryover effects of a simultaneous change in two variables (e.g., price and promotion) considering a combination of impulses (u_{j_1t}, u_{j_2t}) (instead of a single impulse $u_{j,T}$). For the MIRF, this impact is the sum of both shock effects due to the linearity of the VAR model. The VIRF has more complex cross-effects because of the products in the term $vech(u_{T+l-1}u'_{T+l-1})$ in Equation (3) (we just need to aggregate the components associated with $u^2_{j_1t}$ to $u^2_{j_2t}$ and $u_{j_1t}u_{j_2t}$). Sometimes, it might be useful to compute IRFs of sales with respect to an impulse for a meaningful linear combination of shocks γu_T instead of one specific shock $u_{j,T}$. This procedure is related to the structural modeling approach. It usually involves the Cholesky factorization of H to derive the coefficients γ, so that the process $\{\gamma u_t\}$ is standardized to have a diagonal unconditional covariance matrix. In the online appendix, we discuss how this type of IRF can be computed.

Note that MIRFs and VIRFs are symmetric with respect to an upward or downward shock. If a positive impulse in price decreases sales, a negative identical impulse will increase sales in the same amount. This is because the VAR (conditional mean) and BEKK (volatility) models are essentially linear, and the effect of any lagged variable is also linear. If we model the logarithm of sales (or log-differentiated sales), the effects on actual sales can be nonlinear, with asymmetries.

MIRFs and VIRFs can have many visual types of patterns, depending on the parameters of the data generation time-series process. Typical IRF dynamic patterns show initial spikes followed by an exponential decay toward zero (where the effects can be all positive, all negative, or alternating in sign, or follow a sinusoidal wave). Furthermore, MIRFs and VIRFs can be quite different—that is, a specific marketing action at time T can have different effects on future sales mean $\mu_{i,T+l}$ and volatility $h_{i,T+l}$ for some $l \geq 1$. Sometimes a marketing-mix impulse $(u_{j,T})$ in a marketing action can lead to a positive (respectively, negative) change in sales mean and volatility, but sometimes both IRFs can show opposite effects (increasing sales mean while decreasing volatility, decreasing mean while increasing volatility). From a marketing behavioral perspective, this variety is not surprising. Several effects can concur in the market. For example, surprises about promotions or prices can generate brand switching, which tends to increase volatility. In a market with *loyal customers* and *price-sensitive random switchers*, a sharp increase in prices might deter switchers and therefore reduce mean sales and volatility. Alternatively, if we consider an impulse surprise in promotional activity, mean sales might increase. At the same time, we might obtain higher volatility in markets in which customers show heterogeneity in deal proneness. Thus, each instrument of the marketing mix might lead to different patterns. Moreover, these effects can change over time. For example, successful promotions often have positive effects on mean sales in the short run while volatility is reduced, provided that most consumers buy. In the following weeks, these effects can be compensated by consumers purchasing less on average but with higher variability, depending on the quantity of home-stock consumed (see Lee et al. 1997).

From a strategic perspective, the interpretation of MIRFs and VIRFs depends on the goal of the brand manager and the time-planning horizon weeks. From a dynamic perspective, the typical goal is to increase the sales mean and decrease sales volatility after implementing the marketing action at time T. Then, we can distinguish four generic scenarios, which are summarized in Table 2. Consider Scenario 1, in which the action increases both $\mu_{i,T+l}$ and $h_{i,T+l}$. Naturally, this may introduce a conflict between both goals, though not necessarily a strong one. If $CV_{i,T+l} = \sqrt{h_{i,T+l}}/\mu_{i,T+l}$ increases, there is a conflict, but if it decreases, the standard deviation per unit of expected sales is reduced, such that the conflict is milder. In Scenario 2, the action increases $\mu_{i,T+l}$ and decreases $h_{i,T+l}$. This is an optimal situation for the brand, and in this scenario $CV_{i,T+l}$ will decrease. In Scenario 3, the action decreases $\mu_{i,T+l}$ and increases $h_{i,T+l}$, which is the worst scenario because the manager fails to achieve both goals. In Scenario 4, the action decreases both $\mu_{i,T+l}$ and $h_{i,T+l}$, which is an ambiguous scenario. Consider, for example, that if $CV_{i,T+l}$ decreases, the standard deviation per average unit sold will be reduced, such that the scenario

Table 2. Possible Effect of Marketing-Mix Impulse Shocks

Scenarios	Expected sales	Sales volatility	Coefficient of variation
Scenario 1	$\uparrow \mu_{1,T+j}$	$\uparrow h_{1,T+j}$	$\uparrow CV_{1,T+j}$ or $\downarrow CV_{1,T+j}$
Scenario 2	$\uparrow \mu_{1,T+j}$	$\downarrow h_{1,T+j}$	$\downarrow CV_{1,T+j}$
Scenario 3	$\downarrow \mu_{1,T+j}$	$\uparrow h_{1,T+j}$	$\uparrow CV_{1,T+j}$
Scenario 4	$\downarrow \mu_{1,T+j}$	$\downarrow h_{1,T+j}$	$\uparrow CV_{1,T+j}$ or $\downarrow CV_{1,T+j}$

will not be as negative for the brand. It is important to highlight that these scenarios can change over the planning horizon, because the impact of a marketing action on $\mu_{i,T+l}$ and $h_{i,T+l}$ can be quite different in the short or long run, and the changes can be stronger for products whose marketing–sales IRFs present cycles or sawtooth shapes involving sign changes. Therefore, analysts must be cautious when analyzing the strategic consequences of the actions implemented. We have discussed these four scenarios in the context of a typical goal; however, there could be other targets. For example, some managers could seek high volatility as an entry barrier to deter potential competitors, while others might be interested in reducing $\mu_{i,T+l}$ (e.g., when they have a demarketing goal).

In general, both MIRFs and VIRFs are computed with estimated coefficients. Confidence intervals for IRFs can be computed, but they are often misleading (see the online appendix). Therefore, we consider more formal evidence of the overall significance of causal effects between the marketing mix (price and promotion) and sales, either within the brand or across competitors. We show this with *Granger causality* and *conditional independence* tests. *Granger causality* means that the matrices in Π, A, and B are block triangular (when the matrices are partitioned into four submatrix blocks based on causal and noncausal variables). For example, the marketing mix causes sales in the Granger sense if the previous marketing mix influences the sales mean and volatility but the past sales do not influence the marketing mix (i.e., the marketing mix is exogenous with respect to sales). Sales with no impact on the marketing mix would indicate passive marketing management, with no reaction to changes in past sales. *Conditional independence* means that the matrices are block diagonal (the previous marketing mix does not influence sales and the past sales do not influence the marketing mix). Note that the standard *Granger causality tests* assume conditional homoskedasticity; therefore, they only take into account in-mean effects. In the online appendix, we present a Wald test to analyze Granger causality under conditional heteroskedasticity. We also use this approach to test *conditional independence*.

Granger causality is relevant for making decisions about the use of simpler models. For example, if we consider the partition of vector X_t in sales of one brand X_{1t} and all of the other variables X_{2t}, when X_{2t} is strongly exogenous (this concept requires Granger causality), instead of the full VAR-BEKK model, we could estimate a *transfer function model* with volatility and use it to predict X_{1t}. When X_{1t} and X_{2t} are block independent, we can estimate and predict each variable X_{1t} and X_{2t} separately. However, in many contexts, strong exogeneity or independence assumptions do not hold, and a full multivariate VAR-BEKK model should be considered (for a more detailed explanation, see the online appendix).

4. Empirical Application

4.1. Data

We use a store-level scanner database, made available by the James M. Kilts Center at the University of Chicago, from Dominick's Finer Foods, the largest grocery retailer in the Chicago market at the time of data collection. This database includes all weekly sales, shelf price, possible presence of sales promotions (coupons, bonus buys, and price reductions), retail margin, and daily store traffic by individual item (referenced by Universal Product Code), for 29 categories of consumer packaged goods. The original data were collected for 96 stores operating in the Chicago area during an almost eight-year period, from September 1989 to April 1997. This resulted in 398 weekly observations.

Our empirical analysis includes six different fast-moving consumer product categories (i.e., products sold quickly and at relatively low cost): cheese, refrigerated juice, laundry detergent, toilet tissue, paper towels, and toothpaste. For each category, we pooled data across stores, computing (1) weekly brand sales aggregated across stores; (2) brand average price as a weighted average price across stores, in which the weight is the relative share of a store's overall total sales each week; and (3) brand "promotion" as the weekly percentage of Dominick's stores implementing a sales promotion for the brand. In all categories, except refrigerated juice and laundry detergent, observation 211 is missing (corresponding to the week of September 23 to September 29, 1993), so we base it on a figure equal to the average of the nearest weeks.

Our analysis centers on the top national brands for the selected categories. For the cheese and refrigerated juice categories, we consider two brands with the highest market share, forming 80% and 82% of the total category volume, respectively, whereas for the laundry detergent, toilet tissue, paper towel, and toothpaste categories, we focus on the top three selling brands, constituting 70%, 66%, 60%, and 73% of the market, respectively. We selected these product categories because in all of them, only a few leader brands account for most of the market. Therefore, the dimension of the model is not too large when we include all of the key players in the model. We do not include product categories with a large number of small competitors, as doing so would require us to have a very large sample size to estimate the model because we deal with sales and marketing-mix variables for each competitor. For such categories, we would need to marginalize some firms to reduce the size of the model, or include some simplifying assumptions that limit brands' heterogeneity (e.g., a factor model that we discuss in the online appendix).

4.2. Model Specification and Estimation

Step 1: Preliminary Analysis

We performed the standard exploratory data analysis to detect outliers, nonstationarities, and autocorrelation patterns and to determine the convenience of transforming the variables (e.g., taking logarithms and/or differences). We analyzed data graphs, autocorrelation functions (ACFs), partial autocorrelation functions (PACFs), and cross-autocorrelations. First, we decided to take the natural logarithm for all variables (for sales and promotions we took $\ln(1+x)$ to prevent $\ln(0)$ cases). Second, we examined the ACFs for the logarithmic series, detecting a slow decay, which is typical of a nonstationary time series. This conclusion is also consistent with the results of the augmented Dickey–Fuller, Phillips–Perron, and Kwiatkowski–Phillips–Schmidt–Shin unit root tests. We considered the possibility of deterministic trends but ruled them out. Given the evidence to obtain stationary time series, we took first-differences for all logarithmic variables. The output is a stationary time series that can be interpreted as the growth rates of the original series in levels.

In addition, we accounted for seasonality. For weekly data, this requires careful examination, because data are not exactly periodic, but we might find some type of seasonal pattern, typically annually (roughly every 52 weeks) or monthly (every 4 to 5 weeks). Seasonality may generate two types of problems: nonstationarity in first or second moments and periodic effects in the statistical dependence of the observations. Our preliminary analysis ruled out seasonal nonstationarity and found neither the need to take seasonal differentiation (a procedure applied when there are seasonal unit roots) nor the need to regress the variables with respect to seasonal dummies (a procedure used for deterministic seasonality, which is less realistic, but can still be a convenient assumption for short time series). Practitioners should be aware that when working with different retailers and/or product categories, some type of seasonal transformation might be required to obtain stationary data.

We also considered model-free evidence of volatility for the (stationary) logarithmic growth rates of sales and marketing-mix variables of the selected brands in each category, analyzing historical volatilities (as in Figures 1 and 2).[7] In addition, Figure 3 shows the log-differenced sales of one brand in each category. The plots show that the variance is higher in some periods than others, which is a typical pattern indicating the presence of time-varying volatility. Furthermore, during the exploratory steps of the analysis, we examined the correlograms and cross-correlograms for the squared log-differenced sales. These curves also show structure suggesting the presence of statistical dependence in second-order moments.

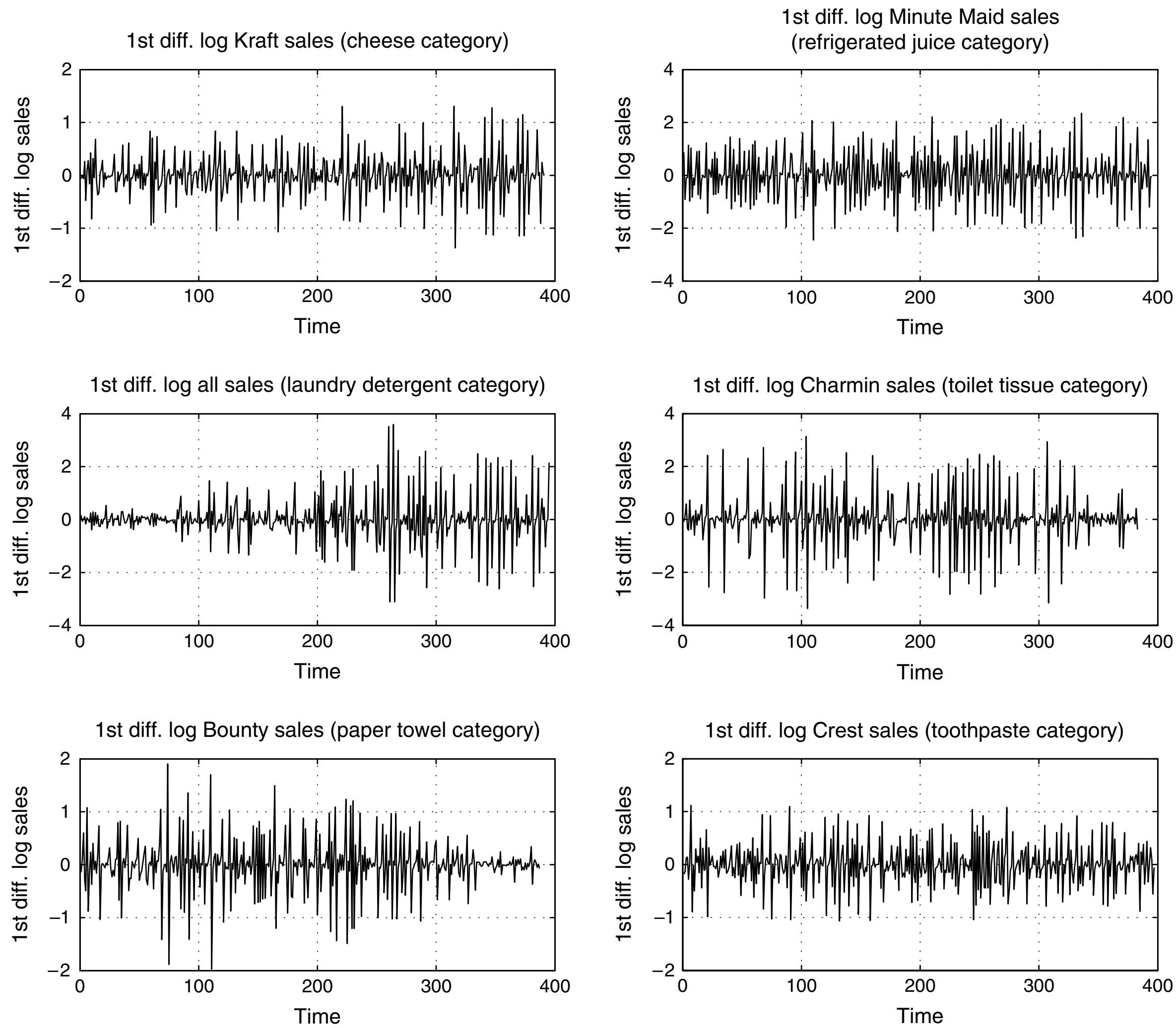
Step 2: Conditional Mean Modeling

Next, we conducted a Johansen's cointegration test to determine whether the integrated variables are cointegrated, that is, if they have a long-run equilibrium in levels. Cointegration would imply the specification of a vector error correction model instead of a VAR model for variables in differences. For all categories, we accept the null hypothesis that the variables are not cointegrated. Note that, in general, standard unit-root and cointegration tests do not account for conditional heteroskedasticity. If such a phenomenon exists, the output of the tests is somewhat exploratory. In our analysis, the output of these tests confirms the findings of the graphical analysis. Therefore, we proceed to estimate a VAR(r) model, including all variables in logarithmic first differences without intercept (as there are no deterministic trends), where $\mu_t = \Pi_1 X_{t-1} + \cdots + \Pi_r X_{t-r}$, and the determinant of the matrix $(I - \sum_{l=1}^{r} \Pi_l L^l)$ is a polynomial with all its roots outside the unit circle (for methodological details, see Lüetkepohl 2005). In particular, we chose the lag length of the VAR model to be $r = 1$ based on a visual inspection of the sample ACFs, PACFs, and cross-correlation functions of the series (in levels and log-differenced).

Note that the standard confidence intervals for an ACF usually assume that either X_t are uncorrelated data (if so, we obtain flat interval bands) or data are correlated but conditionally homoskedastic; however, both approaches are potentially misleading under volatility, so these confidence intervals should be handled with appropriate caution. Even in a homoskedastic context, these confidence intervals refer to individual coefficients, such that a simultaneous confidence interval for the whole curve will have a different significance level (the curve might have some spurious spikes out of interval bands). We also checked for seasonal dependence, analyzing these series' graphs (in levels and log-differenced) and their correlograms. We did not include seasonal lags in the VAR because the evidence of seasonal mean dependence in our data was not particularly strong, and in these circumstances, we prefer to use a parsimonious model (otherwise, we might consider a large r, for which most elements Π_j are set equal to a zero matrix, except seasonal lags and the initial regular lags). We also computed the information criteria commonly used in the marketing literature (see Dekimpe and Hanssens 1999, Pauwels et al. 2004). The Schwarz information criterion suggests one lag for all categories as a sensible specification.

As a result, we specify the VAR(1) model $\mu_t = \Pi X_{t-1}$, estimating Π by OLS. We assessed the significance of the estimated coefficients (using a heteroskedasticity and autocorrelation-consistent variance estimator because of the potential presence of time-varying volatility). Next, we obtained the

Figure 3. Examples of Sales Log Growth Rates



residuals $\hat{u}_t = X_t - \hat{\Pi}X_{t-1}$ and assessed the correlation structure. We analyzed the ACFs and PACFs of the residuals, finding weak evidence of correlation. (We also compared these results with those of VAR(4), VAR(8), and VAR(10), but the residuals of these models had more structure.) Regardless, we estimated a parsimonious model compatible with data-based evidence; time-series modelers should always acknowledge the possibility of misspecification regarding r (if so, some lagged variables would be included additively in u_t).

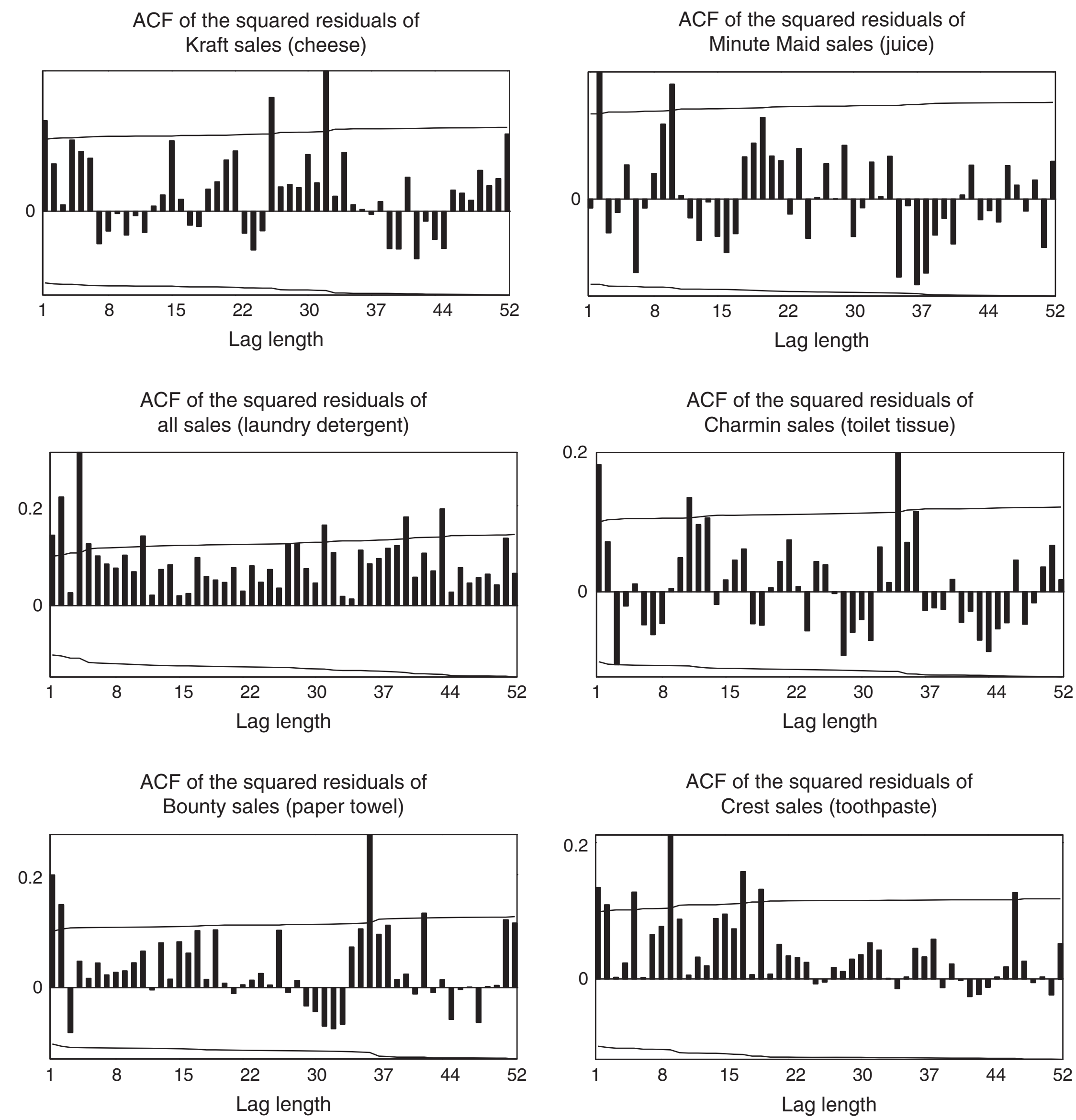
Step 3: Analysis of Volatility in Residuals

We explored the presence of volatility by analyzing the residuals $\hat{u}_t$. We examined the volatility of $\hat{u}_t$ using a univariate analysis for each coordinate of vector $\hat{u}_t$. The analysis was based on the following tools:

(i) *ACFs and PACFs of the squared OLS residuals* $\hat{u}_t$ *from the* VAR(1) *model*. We find substantial evidence of autoregressive conditional heteroskedasticity (ARCH) effects as judged by the autocorrelations of the squared residuals. (The results for sales in all brands and categories are available on request.) As Figure 4 shows, although the magnitude of the autocorrelations is sometimes small after lag 1 or lag 2, the ACF plots of the squared residuals of sales variables show the presence of autocorrelation patterns. (Here, the confidence intervals must be interpreted with some flexibility, focusing on the whole structure of the first periods.) This is, again, a typical pattern associated with dynamic volatility, suggesting the existence of conditional heteroskedasticity in the form of ARCH/GARCH models. We found no evidence of seasonal dependence on the volatility structure.

(ii) *ARCH test*. We formally tested the hypothesis of conditional heteroskedasticity applying Engle's (1982) ARCH test. The null hypothesis is that there is no autocorrelation in the squared residuals (and therefore no ARCH effect). For all brands (except Wisk in the laundry detergent category[8] and Colgate in the toothpaste

Figure 4. ACFs of Squared VAR Residuals of Sales



category), we rejected the no-ARCH hypotheses, thus confirming our findings in (i) and (ii).

We conclude that there are volatility and covolatility relationships. We specified a BEKK model for the conditional variance of the residuals vector. To determine the number of lags to include in the volatility model, we used the ACFs and PACFs of the squared residuals (for details, see the online appendix). We also estimated several BEKK models and compared them to select the final one. In our case, the identification procedure suggested a BEKK(1, 1, 1) model for all product categories.

Step 4: Estimation of the Volatility BEKK Model

We computed a preliminary estimation of the volatility model h_t using the residuals $\hat{u}_t$. Let θ denote the vector of parameters of the BEKK(1, 1, 1) model and $H_{t,\theta}$ the volatility matrix as a function of θ. Then, we can estimate θ by maximizing the conditional pseudolikelihood (Sheppard 2009)—minimizing the minus log-likelihood function $Q(\theta) = \sum_{t=1}^{T}(\ln|H_{t,\theta}| + \hat{u}_t' H_{t,\theta}^{-1}\hat{u}_t)$, using the residuals $\hat{u}_t = X_t - \hat{\Pi} X_{t-1}$ previously computed in Step 2. This estimation is consistent, but inefficient, as it is based on inefficient OLS estimations of the VAR model.

Step 5: Improving Efficiency

We gained efficiency by reestimating the parameters of the VAR(1) and BEKK(1, 1, 1) models simultaneously. Now, let θ denote all parameters in the VAR(1) and BEKK(1, 1, 1) models. Then, we estimate θ by minimizing $Q^{full}(\theta) = \sum_{t=1}^{T}(\ln|H_{t,\theta}| + (X_t - \mu_{t,\theta})' H_{t,\theta}^{-1} (X_t - \mu_{t,\theta}))$. The problem is solved using the Newton–Raphson algorithm, initialized at the estimators computed in Steps 2 and 4 (which are consistent but inefficient estimators of VAR and BEKK models, respectively).[9]

We applied the five-step process to each of the six product categories, performing diagnosis analysis. In all cases, most parameters were individually significant, and the model was globally significant. We also recursively computed the residuals

$$\hat{\eta}_t = vech(\hat{u}_t \hat{u}_t') - w - (A+B)vech(\hat{u}_{t-1}\hat{u}_{t-1}') - B\hat{\eta}_{t-1}. \quad (5)$$

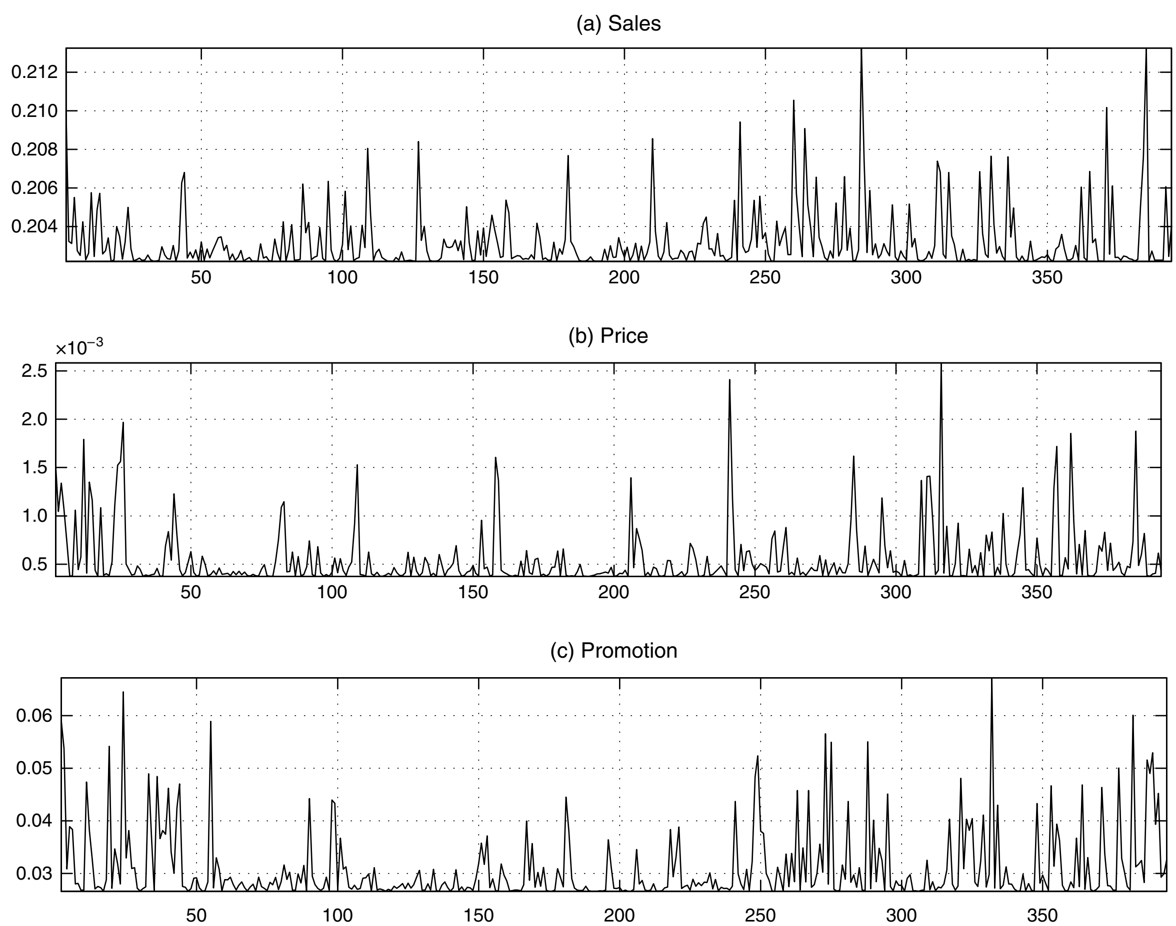
If the BEKK specification is correct, these residuals should be uncorrelated (see the online appendix). We assessed this by computing their ACFs and PACFs (results available on request), and we found no evidence of misspecification. We offer a warning regarding the overall diagnostic analysis using ACFs and PACFs. First, the usual confidence intervals do not account for time-varying volatility. Second, given the large dimensionality of $(\hat{u}_t, vech(\hat{u}_t\hat{u}_t')')'$, by individually analyzing autocorrelations for each variable and category with standard confidence intervals (specific for a single coefficient on a single curve), we are implicitly changing the overall size of the global significance test. Therefore, ACFs and PACFs should be handled with appropriate caution.

With the estimated parameters and the residuals of the model, we computed the conditional means and volatilities (μ_t, h_t) and the IRFs for sales and the marketing mix of all considered brands in each category. (These results are also available on request from the authors.)

4.3. Key Results

The results provide clear evidence of time-varying volatility in sales and the marketing mix of all brands in the six product categories. To preserve space, we discuss only the refrigerated juice category, focusing on Minute Maid and its relationship to its competitor Tropicana (the market leader). Figure 5 shows sales and marketing-mix volatility of Minute Maid refrigerated juice. In some weeks, volatility reaches relatively high levels, but in general, it is relatively low. Similar patterns hold in all of the considered categories, with one exception, the toothpaste category (as promotions for the three brands in this category show nearly constant volatility over time, though prices and sales have time-varying volatility). Because we are modeling

Figure 5. Sales, Price, and Promotion Volatilities of Minute Maid's Refrigerated Juice



time series of logarithmic growth rates, the changes in the conditional mean and volatility of X_t are relatively small in absolute terms, but this implies relatively large effects for the time series in levels (actual sales and marketing mix).

We also computed the conditional contemporaneous covariances h_{ijt} between sales and the marketing mix and their conditional correlations ρ_{ijt} for each of the considered brands and categories. For all of the brands and categories, the covolatilities show the strength of contemporaneous association between sales and the marketing mix over time (results available on request). Figure 6 presents the conditional correlation between the sales of Minute Maid and its own marketing mix. For this particular brand, the conditional correlation with respect to price is negative and positive with respect to promotion, though the correlation varies significantly in intensity over time. We also observe that prices and promotions are negatively correlated, suggesting that they are used as complementary managerial tools. As we mentioned previously, conditional correlation can be used to select appropriate periods to intensify promotional activity. For example, Figure 6(b) shows that promotion is positively contemporaneously correlated with sales conditioning on the past, but a promotion is likely to be (contemporaneously) less effective if it is implemented at time $t = 100$ (with a correlation of nearly 0.35) than at time $t = 75$ (when the correlation is nearly 0.5).

The conditional contemporaneous correlation of a brand's sales with competitors' marketing mix also provides valuable information about the situations in which a brand is more sensitive to the contemporaneous actions of a competitor. Figure 7 shows the conditional correlation between Minute Maid sales and the marketing mix by its rival Tropicana. Note that at the market equilibrium, the conditional correlation between Minute Maid sales and Tropicana's prices is negative in general, while for Minute Maid sales and Tropicana's promotions, it is positive. These signs seem counterintuitive, but they can be easily explained when

Figure 6. Sales, Price, and Promotion Conditional Correlations of Minute Maid's Refrigerated Juice

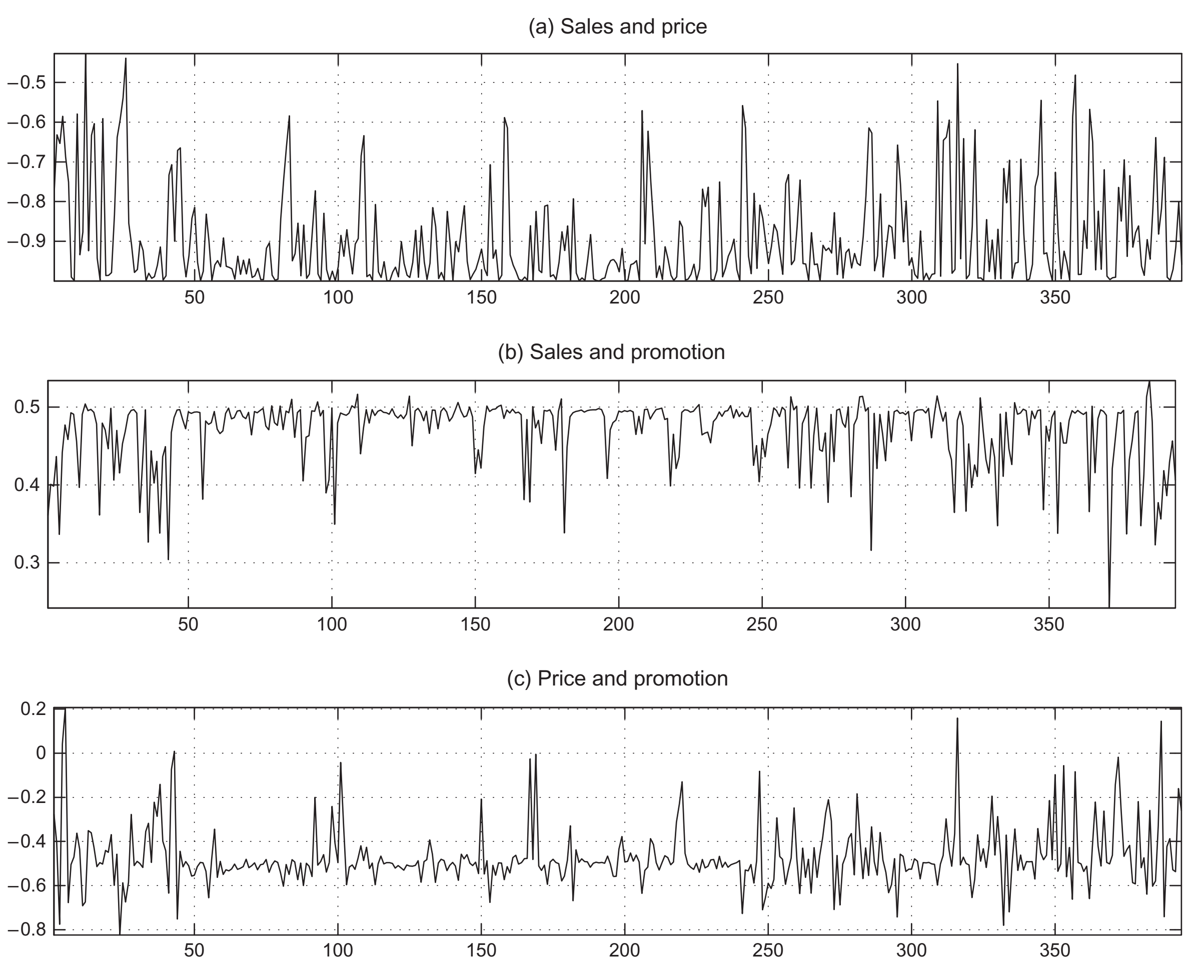
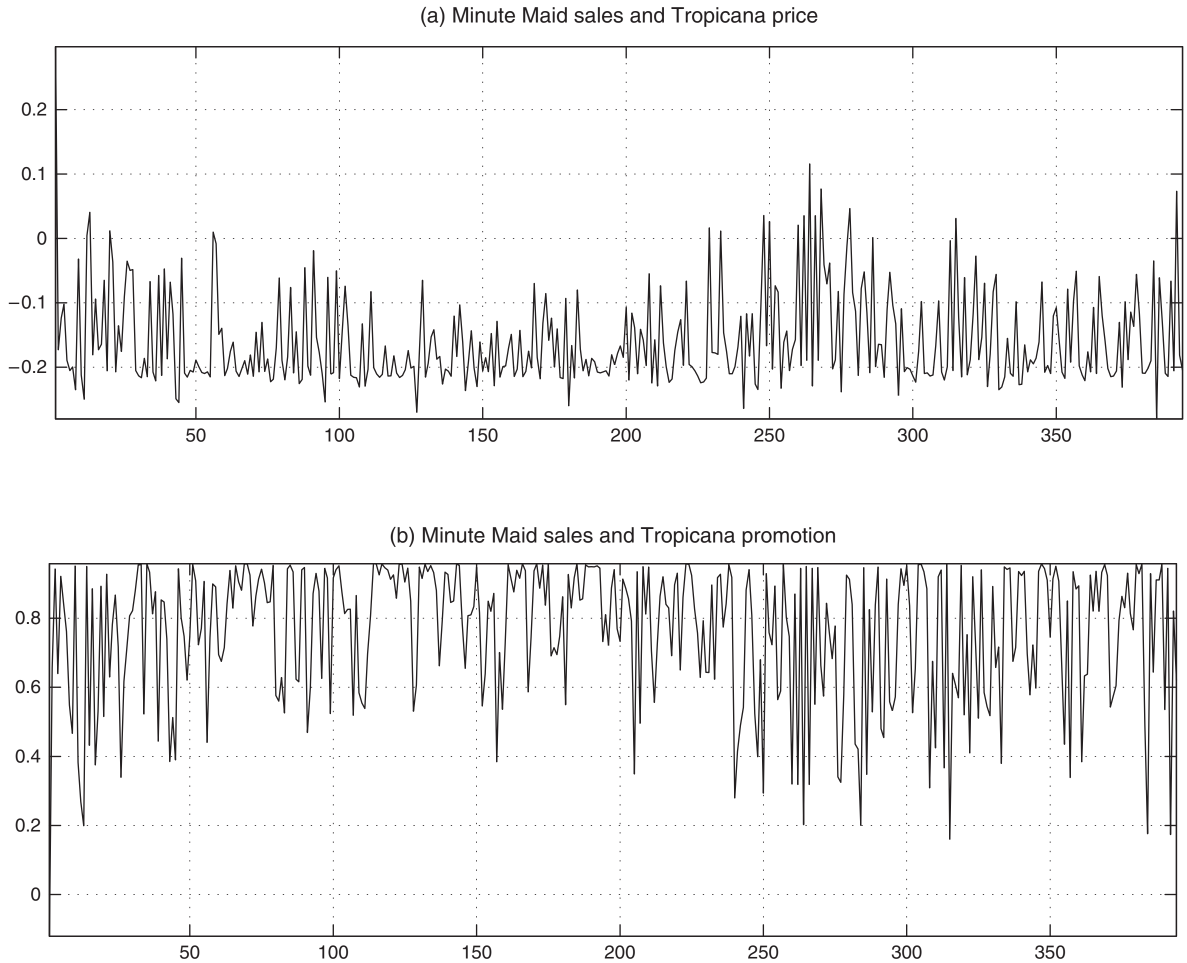


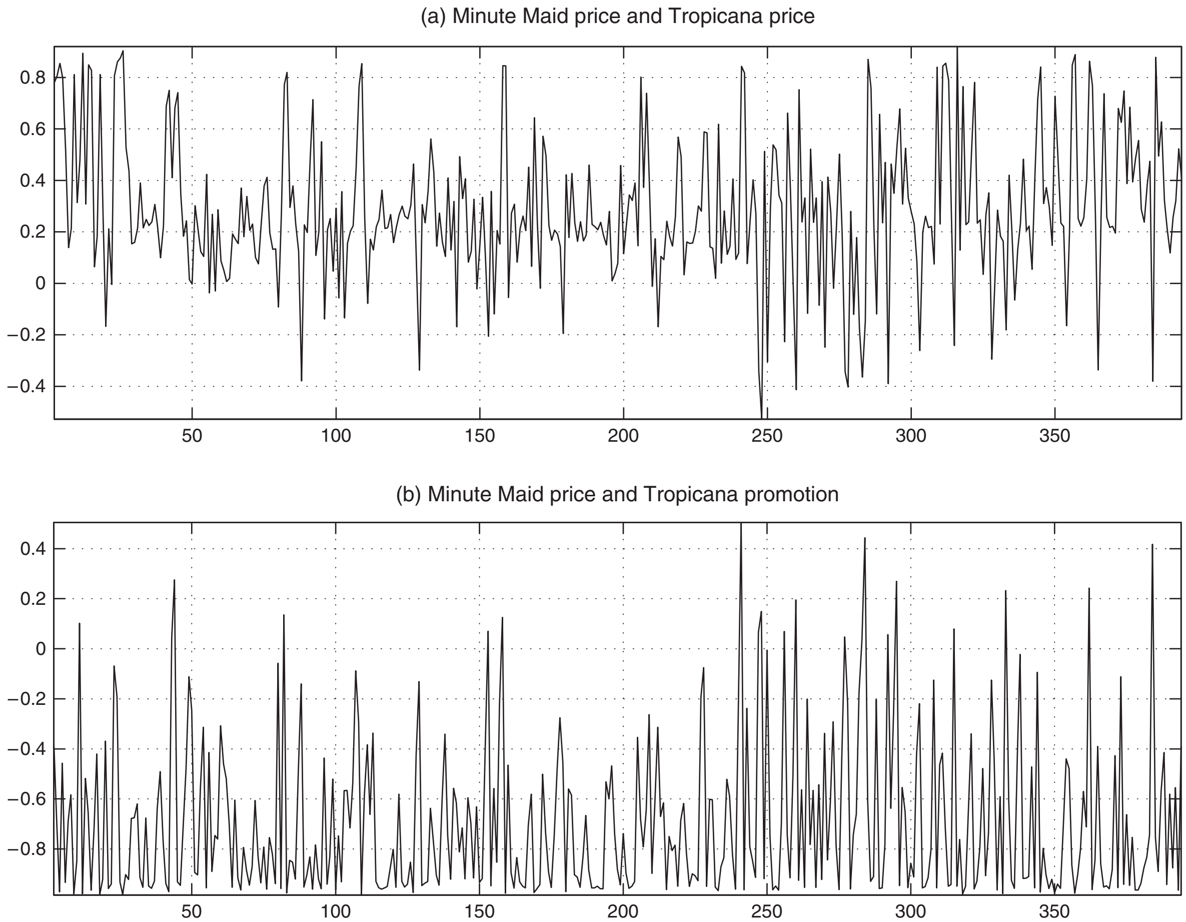
Figure 7. Conditional Contemporaneous Correlation Between Minute Maid Sales and Tropicana Marketing Mix



we consider the influence of another variable, namely, the contemporaneous response of Minute Maid to Tropicana's actions. Figures 8 and 9 show that when Tropicana increases prices, Minute Maid also increases prices (this is why its sales are reduced when Tropicana raises prices), and when Tropicana increases promotional activity, Minute Maid also intensifies promotions. In addition, time-varying covolatility occurs between prices and promotions in the category.

To analyze the carryover effects between sales and marketing-mix log growth rates, we examined MIRFs and VIRFs in the refrigerated juice category. We computed the MIRFs and VIRFs with respect to a (direct) impulse in the shocks u_t (for technical details, see the online appendix). Figure 10 represents the MIRFs with respect to shock impulses for the two refrigerated juice brands. For both brands, a price impulse has a negative impact on expected sales one week ahead, followed by an oscillating pattern. Minute Maid promotions have a positive impact over its expected sales in one week and a negative impact in the following week. By contrast, Tropicana sales react negatively to its promotions in one week and positively in the following week. Figure 11 shows the VIRFs for sales of Minute Maid and Tropicana with respect to an impulse in own marketing mix. For both brands, an impulse in price or promotion increases volatility essentially for one week. Thus, if the goal of Minute Maid is to increase mean sales one week ahead, a price cut will increase mean sales and reduce volatility simultaneously (the MIRF is negative and the VIRF is positive for one lag), whereas by increasing promotion it will achieve a mean sales improvement at the expense of higher volatility. We can analyze the different strategies for Minute Maid on the basis of Table 2. If the company is concerned with the effects one week after the promotion, a price reduction strategy falls in Scenario 2 (reciprocally, a price increment falls in Scenario 3), and a promotion increment falls in Scenario 1 (promotional reduction falls in Scenario 4). Note that impacts shown in Figures 10 and 11 vanish in a few weeks, but because they reflect log growth rates, they will have a persistent effect on levels. We present additional results and discuss the analysis of IRFs with

Figure 8. Conditional Contemporaneous Correlation Between Minute Maid Price and Tropicana Marketing Mix

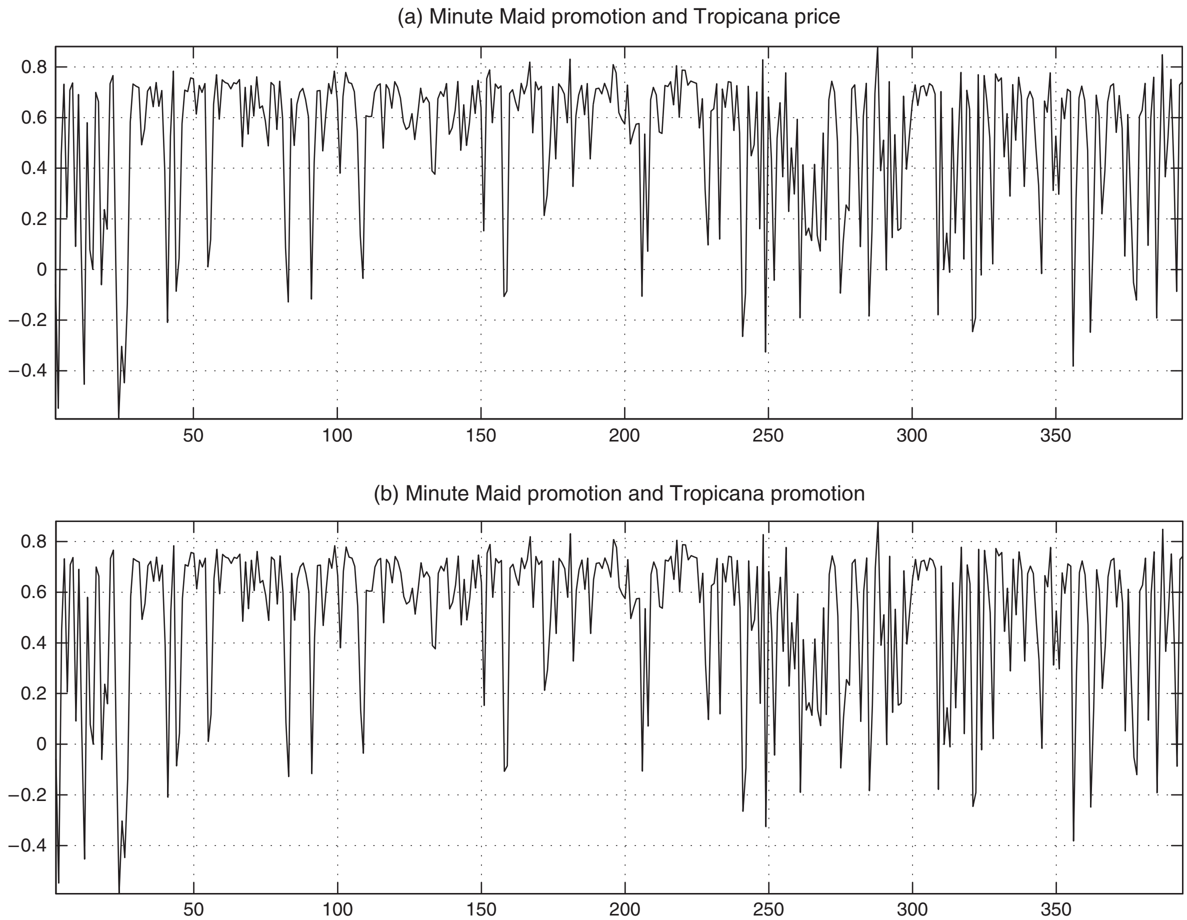


respect to some types of standardized shocks in the online appendix.

In general, any marketing surprise encountered in the presence can change the contemporaneous effectiveness (covolatility) of future marketing actions in the same or other instruments. In this sense, we computed the VIRF to show the response of future covolatility $H_{ij,T}$ between sales and the marketing mix (sales–price and sales–promotion) with respect to an impulse in the (original shock) of current marketing-mix action (price or promotion) at time T. Figure 12 shows the Minute Maid covolatility IRFs with respect to its marketing mix. We observe that an impulse in any marketing instrument (prices and promotions) will have a negative impact on future sales–price covolatilities and a positive impact on future sales–promotion covolatilities. All of these effects, however, are relatively small and essentially wear off in two weeks (in log growth rates).

For all six categories considered, we tested the Granger causality (in conditional mean and variance) from the marketing mix (price and promotions) to sales and the conditional block independence between sales and the marketing mix. We implemented the study from two perspectives. First, *within brands* we tested these hypotheses within the context of one brand (exogeneity of the marketing mix of the brand from its sales and the independence between sales and the marketing mix of the same brand). The results show that for most brands in all categories (laundry detergent, toilet tissue, toothpaste, paper towels, cheese, and refrigerated juice), we reject the exogeneity of the marketing-mix hypotheses as well as the strongest conditional independence hypotheses with 95% significance. This means that for each brand, the empirical evidence confirms sales means and variances depending on past sales and previous marketing-mix actions, and vice versa (marketing-mix actions are set on the basis of past sales and previous marketing actions). Second, *between brands* we tested the exogeneity of the brands' marketing mix (price and promotion) with respect to the competitors' sales and the independence between these variables, the exogeneity of the brands' marketing mix with respect to the competitors' marketing mix and the independence between these variables, and the

Figure 9. Conditional Contemporaneous Correlation Between Minute Maid Sales and Tropicana Marketing Mix



exogeneity of the brands' sales with respect to the competitors' sales and the independence between these variables. In all cases, we reject conditional block independence between the sales series of all competitors for all brands in all categories (a full report is available on request). These tests involve different data categories. We do not include Bonferroni corrections.

4.4. Planning Marketing Interventions

When determining the marketing mix, the model considered here enables managers to monitor sales response in mean and volatility. As we have previously discussed, marketers should intensify their actions when the conditional correlation between sales and marketing mix is strong. In this section, we further explain this issue.

To simplify the exposition, consider the bivariate case with just one brand, where X_{it} are brand sales and X_{jt} promotional actions. Then, a manager following the "business-as-usual" inertia implicit in the historical data would typically apply a marketing action $X_{jt} = \mu_{jt} + u_{jt}$, where μ_{jt} is the predictable component of the marketing mix, and $u_{jt} \approx N(0, H_{jj,t})$ is the innovative or unexpected component of the marketing mix. This information is observed by the market, rendering a sales surprise $u_{it} \mid u_{jt}$ conditionally Normal,[10] and sales are given by $X_{it} = \mu_{it} + u_{it}$. This specification is equivalent to stating that $X_t = \mu_t + u_t$ and $u_t \approx N(0, H_t)$ jointly. In the alternative "what-if" exceptional scenario, at each period t, the manager computes the cocorrelation $\rho_{ij,t}$ and considers an increase of promotion activity x, when $\rho_{ij,t} > \epsilon$ for some threshold $\epsilon > 0$. The threshold ϵ should be feasible, in that $\Pr(\rho_{ij,t} > \epsilon) > 0$. Then, instead of applying the marketing shock $u_{jt} \approx N(0, H_{jj,t})$, the manager should actually implement $\tilde{u}_{jt} = u_{jt} + x$ for a deterministic value $x > 0$. The market accounts for the intervention, and within the same week, sales response is updated by $X_{it} = \mu_{it} + u_{it}$ according to the conditional sales surprise $u_{it} \mid \tilde{u}_{jt}$. This promotional intervention effectively modifies the contemporaneous sales X_{it} (at the period t when the increase of promotion activity x occurs) and the sales of the following periods (because of the carryover effects). This explanation is for a bivariate model, though the same argument can be applied when there are more variables (including in

Figure 10. MIRFs for Sales of Minute Maid (Top Row) and Tropicana (Bottom Row) with Respect to Impulses in Their Own Prices and Promotions

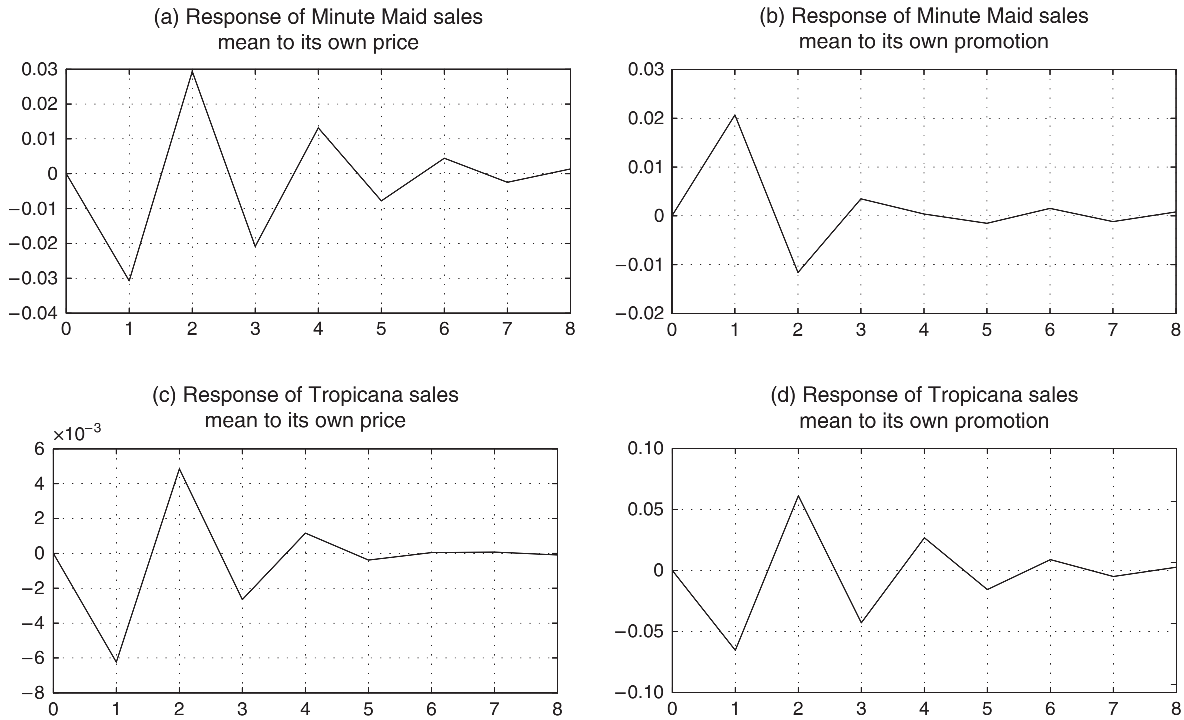


Figure 11. VIRFs for Sales of Minute Maid (Top Row) and Tropicana (Bottom Row) with Respect to Impulses in Their Own Marketing Mix

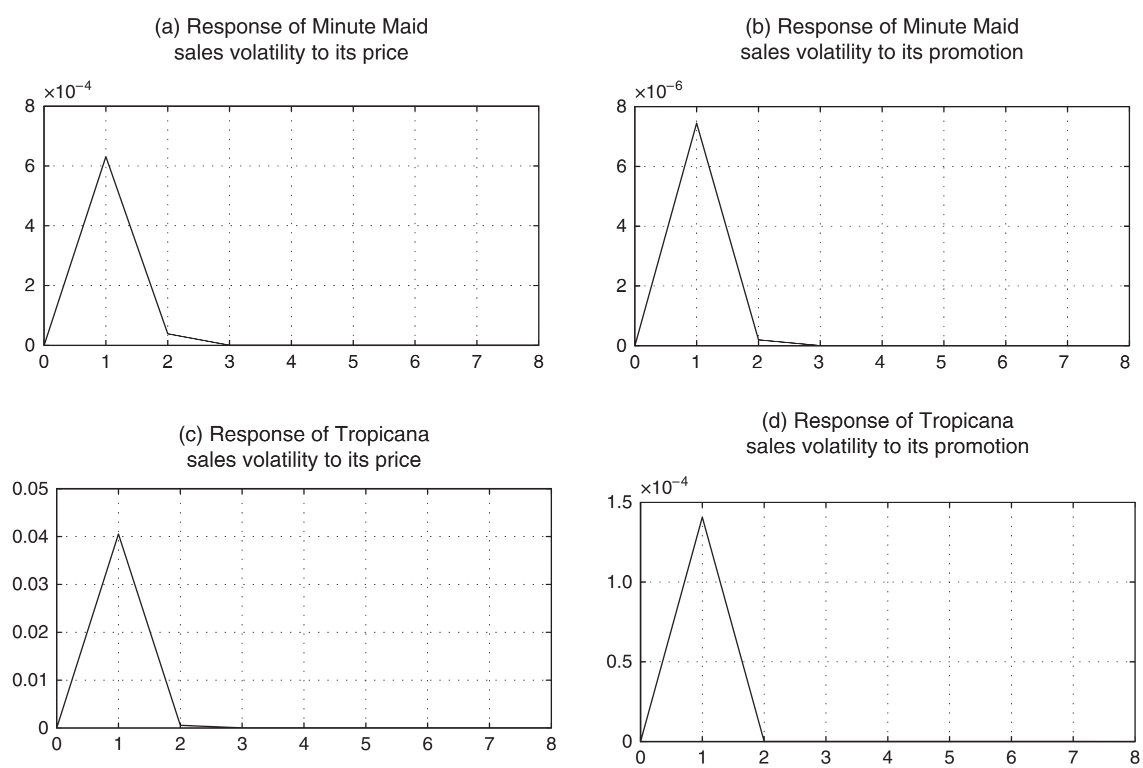
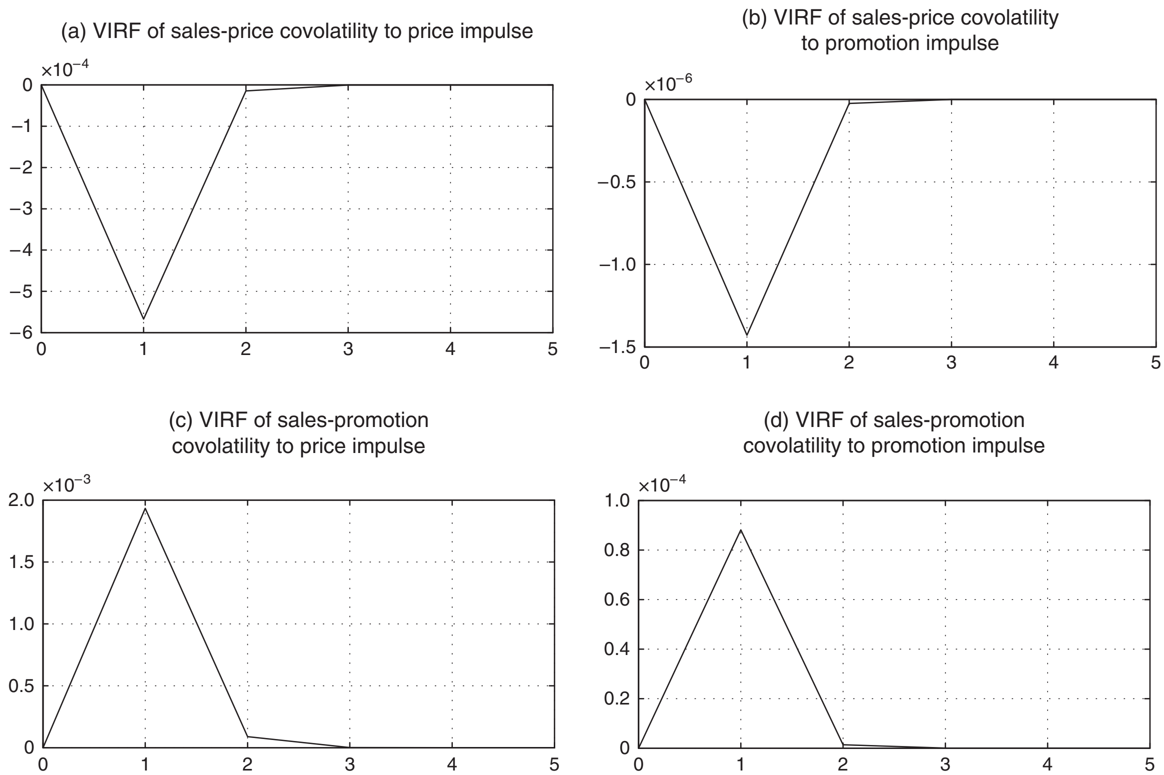


Figure 12. VIRF of Minute Maid Future Covolatilities with Respect to an Impulse in Its Own Marketing Mix

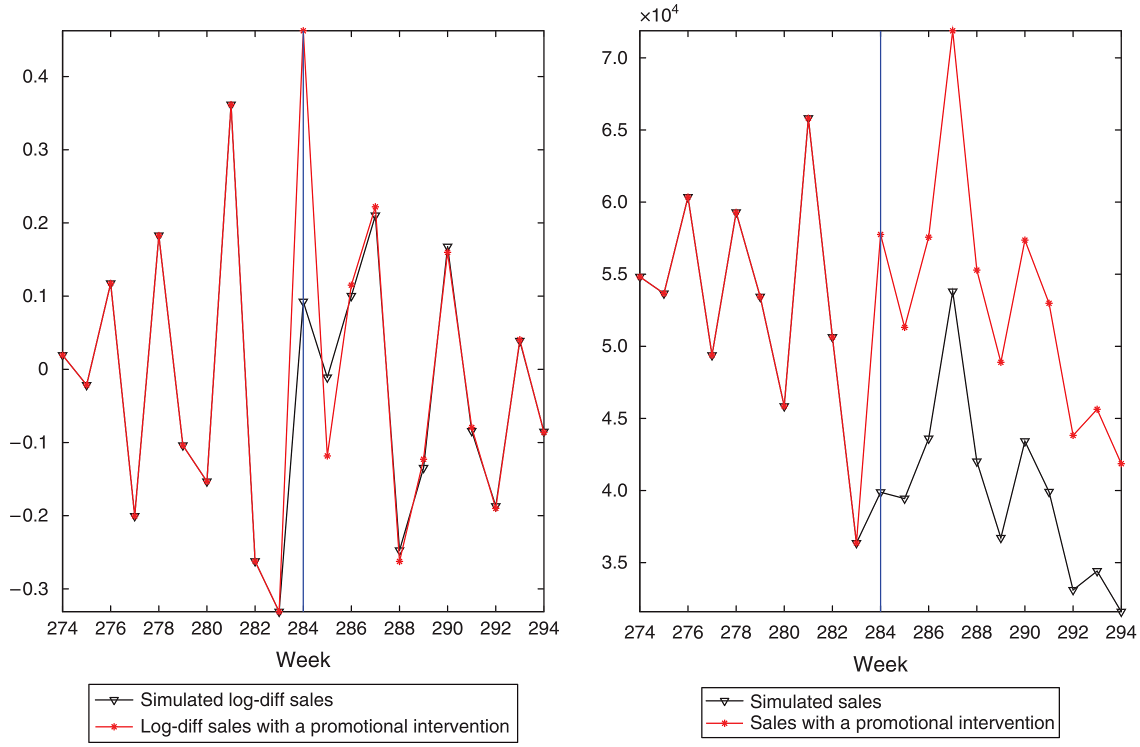


$(X_{it}, \mu_{it}, u_{it})$ not only sales and promotion, but marketing variables other than promotion as well).

To assess the effectiveness of the method, we applied this type of intervention to Tropicana refrigerated juice promotions. First, we simulated a path of the estimated VAR-BEKK model for this product category in the regular business-as-usual scenario. Second, we compared the simulated path of Tropicana's sales with the path that the firm would have obtained implementing an intervention based on covolatilities (what-if scenario). More specifically, in the business-as-usual scenario we simulated the dynamics of the VAR-BEKK model, generating $u_t \approx N(0, H_t)$, and then computed sales and marketing mix $X_t = \mu_t + u_t$ and used this information to compute future (μ_{t+1}, H_{t+1}). More specifically, we considered initial moments (μ_0, H_0) and generated a sequence $\{\varepsilon_t\}$ of white noise $N(0, I)$. Starting from the initial period, we iterated the following steps: at time t, given (μ_t, H_t) compute $u_t = H_t^{1/2}\varepsilon_t \approx N(0, H_t)$, then compute $X_t = \mu_t + u_t$, and finally update the moments computing (μ_{t+1}, H_{t+1}) using the model. In the simulation, the first observations are discarded to remove the effect of the initial values (μ_0, H_0). This simulation provides the baseline scenario. In addition, we computed the 0.99 quantile for the conditional correlations $\{\rho_{ij,t}\}$. (To compute the quantile, we generated an independent baseline scenario path, evaluating the conditional correlations $\{\rho_{ij,t}\}$ in this path, and then computed the smallest value ϵ, such that $\Pr(\rho_{ij,t} > \epsilon) = 0.01$ in the data distribution.) This sample quantile ϵ is around 0.5; we take this value as the threshold. In the alternative what-if scenario, at each period t the manager computes the cocorrelation $\rho_{ij,t}$, modifying the promotional plan when $\rho_{ij,t} > \epsilon$ for some threshold $\epsilon > 0$. Then, instead of applying the marketing shock $u_{jt} \approx N(x, H_{jj,t})$ previously computed, the manager actually implements an exceptional $\tilde{u}_{jt} = u_{jt} + x$ for a deterministic value $x > 0$. At these periods, sales and all other competitor shocks are determined by the conditional distribution $u_{it} \mid \tilde{u}_{jt}$. Then, we update (μ_{t+1}, H_{t+1}) and continue with the simulation using the same shocks $\{\varepsilon_t\}$ in both scenarios to make both paths comparable (the baseline using "promotions as usual" and the path with enhanced promotion). We set the threshold ϵ equal to the 0.99 sample quantile of $\{\rho_{ij,t}\}$. In other words, we record the periods for which there is a high and positive cocorrelation between sales and promotion; note that managers should increase promotional activities in these particular periods to attain higher sales responses than usual.

Figure 13 shows a realization of both processes: sales with promotions as usual and sales with a what-if increase in promotion activity. Figure 13(a) shows the actual logarithmic growth rates of Tropicana sales as well as the growth rates the company would have obtained applying an increase of promotion activity of $x = 0.5$. The impact of this intervention in sales level

Figure 13. (Color online) Promotional Intervention Impact on Tropicana Sales (0.99 Quantile Cutoff)



is considerably large, as Figure 13(b) shows. Note that the increase of promotion activity is applied just once, at week 284 (Figure 13 displays a vertical line at period 284, indicating the intervention in promotion), as we considered a relatively stringent requirement (a 0.99 quantile) to intensify promotion. We did this to obtain a single promotion when running the simulation and, therefore, to show an informative figure. If the threshold is reduced, the policy is applied more often (and the impact of these enhanced promotions overlaps).

Analogously, managers can consider an exceptional price reduction when the contemporaneous cocorrelation between sales and prices is high and negative. Moreover, they can simultaneously apply a price–sales conditional correlation rule to modify prices, and a promotions–sales conditional correlation rule to modify promotions.

There is an assumption involved in these tactics: we require that consumers do not anticipate these interventions x; otherwise, they might vary their behavior, and then the whole VAR-BEKK rule would change (this problem is known as the "Lucas' critique"). In practice, though, this is an unlikely problem in our context, because the aggregated data involve multiple retail shops, and for each individual consumer it is nearly impossible to track the general strategy of the whole retail chain. In any case, the intervention frequency $\Pr(\rho_{ij,t} > \epsilon)$ should not be too high. Note that the retailer could even vary ϵ and x over time to make it even more difficult to anticipate or even detect these interventions.

5. Conclusion

Product management requires planning inventory buffer stocks not only to cover the forecast but also to prevent stockouts due to unexpected demand deviations from the expected sales. Volatility should be a central metric for this task because sales can have time-varying volatility, and the use of unconditional variances is a poor substitute. Overlooking volatility brings inefficiencies into production and inventory management; most of the time, it leads to larger inventories than required, but during specific periods, it leads to stock shortfalls. Not only is volatility a relevant metric to be monitored, but volatility itself can also be managed. Brand managers can use the marketing mix to modify future volatility when they expect it to deviate from their objectives. To address this issue, managers can build multivariate volatility models and examine the MIRFs and VIRFs and Granger causality tests to understand cross-effects between sales and the marketing mix. Furthermore, in this research we used covolatility to track periods with a higher contemporaneous correlation between sales and the marketing mix conditional on history. Marketers can use this approach to select promotional timing. For measuring either contemporaneous or carryover effects, marketers can analyze the link between sales and the marketing-mix volatility from two perspectives: (1) how managers influence sales volatility through their own marketing-mix decisions and (2) how managers influence their competitors' volatility.

The objective of this research is not to establish an empirical generalization. We do not claim that sales of all products show volatility, though many do, as we show in the empirical application. As mentioned previously, sales time series aggregated over high-frequency periods (hourly, daily, or weekly sales) tend to present more volatility than those with lower sampling frequencies (monthly, quarterly, or annual sales), as changes in conditional variances tend to be smoothed out. Time aggregation is not the solution, as it often causes analysts to face more unexpected extreme events, which are observed in conditional distributions with heavier tails.

Working with high sampling frequencies, product and brand managers often encounter volatility, which should not be ignored; that is, managers who do not pay attention to this phenomenon and instead focus just on conditional expected sales may confront negative consequences due to undetected volatility. In this case, marketing analysts face two problems caused by the misspecified model. First, they will typically make erroneous inferences on the conditional mean parameters (maximum likelihood estimators for the conditional mean model have a different asymptotic distribution with conditional heteroskedasticity). If there is evidence of volatility, by estimating a full VAR-BEKK model, analysts would make correct inferences. Second, if focusing only on expected sales, they will make *risk neutral* management decisions because the risk dimension is ignored. As such, *risk averse* managers should take into account the volatility. What is right for sales performance in mean might also generate higher volatility. This article provides managers with powerful metrics to plan their marketing mix; it assesses conditional contemporaneous correlations between sales and the marketing mix for a focal brand and several competing brands, and delves into the carryover effects of marketing actions on sales. A managerial implication of this research is that the marketing mix (at least, price and promotional actions) can be a useful tool for product and brand managers to curb volatility.

The goal of this research is to contribute to the marketing literature on *sales response models*, paying attention to the link between sales volatility and optimal marketing-mix planning by volatility-aware managers. However, this study is far from being the last word on this topic, and our goal is to stimulate additional research in an area that needs further development. Our work sets up an agenda for further research on how own and competitive marketing actions drive sales and marketing effectiveness, accounting for conditional mean and volatility sales responses. We discuss some limitations (e.g., dimensionality) and potential extensions (e.g., structural models) in the online appendix. Herein, we measure promotions using a continuous variable. When using store-level models, analysts often need to consider dummy variables such as feature and display. In the online appendix, we briefly explain how to handle volatility in this context.

The methodology considered herein can aid in eventually smoothing out the bullwhip effect at the retail level (the starting point). Lower price and promotional growth rates lead to less volatility in sales growth. Managers should balance the positive effects on expected sales and the negative effects on volatility. This article presents a perspective complementary to the work of Hanssens (1998), which proposes an improvement of expected sales forecasts as an instrument to handle the bullwhip effect. Our contribution in this direction is limited, however, as we did not have manufacturer data. Further research could model both retail- and manufacturer-level data to provide clearer insights into the transmission mechanisms involved in the bullwhip effect. For example, what is the role of manufacturer versus retail promotions in causing volatility? This exercise could be carried out by building a VAR-BEKK model with the sales and marketing mix of a manufacturer and its distributors. Nevertheless, volatility deserves attention even at the retail level, and marketing strategic and operational decisions can be richer if this key business dimension is given the attention it deserves.

Finally, we mention a more general caveat. Volatility is a powerful metric for any manager. However, managers must bear in mind that risk and volatility are not interchangeable. Volatility is an important type of risk and accounts for changes in regular oscillation patterns, including rare events, because of its connection with kurtosis. Nevertheless, other risks are not included in volatility, such as unanticipated structural breaks (distribution shifts caused by changes in exogenous omitted or unpredictable influential variables) and unexpected instances of anomalous outliers (incidental exogenous changes in the probability distribution). Volatility has other limitations as well; for example, it does not take into account asymmetries. The harm caused by volatility on the left tail of the sales distribution is not the same as the harm on the right tail, and managers can make different tactical decisions depending on the type of volatility effects they face. For example, if the conditional distribution of sales is lognormal, the upper tail typically needs to be examined.[11] Decision making accounting for volatility is a relevant part of the wider area known as *risk management*. However, this study is not about risk in the broad sense; rather, it is about measuring and controlling the changes of magnitude in the regular intrinsic variability of the phenomenon. Even if managers have sound control of sales volatility, there could be unexpected shocks, as doing business, after all, involves risks.

In summary, this work challenges the widely accepted assumption that sales have time-constant conditional variances (conditional homoskedasticity). It introduces a new domain of inquiry in marketing, though additional research may try to find this phenomenon in other retail databases. If so, and time-varying sales volatility becomes an empirical generalization, product and brand managers will need to make substantial changes in their beliefs and behavior about retail inventory and marketing-mix management. This research presents the tools to help them in this process.

Acknowledgments

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Endnotes

[1] Constant volatility is identical to the unconditional variance.

[2] When modeling sales and the marketing mix together, the conditional covariance matrix (*volatility matrix*) contains volatilities as the main diagonal elements and covolatilites as the off-diagonal elements.

[3] Historical volatility is a descriptive approach to estimate volatilities. It considers a partition of the observation horizon in equal-length subintervals (time windows) and computes the sample variances of the observations in the same subinterval. In this case, we consider 20 subintervals, each having 39 weekly observations.

[4] Historical covolatility between two variables is a descriptive approach to estimate covolatilities. It considers covariances for pairs of observations in the same time windows.

[5] By log-differentiating a positive series Z_t, we obtain a proxy for its growth rate $r_t = (Z_t - Z_{t-1})/Z_{t-1}$ because $\ln Z_t - \ln Z_{t-1} = \ln(1 + r_t) \approx r_t$, when r_t is small, as $\ln(1 + x) = x + o(1)$ when $\rightarrow 0$.

[6] We explain the relationship between volatility and kurtosis in the online appendix.

[7] Similar results for the other categories are available on request from the authors.

[8] Yet graphical evidence indicates the presence of volatility (see, e.g., Figures 1 and 2).

[9] The estimated parameters and their t-values for each product category are available on request from the authors.

[10] In other words, $u_{it} \mid u_{jt} \approx N(H_{ij,t}H_{jj,t}^{-1}u_{jt}, [H_{ii,t} - H_{ij,t}H_{jj,t}^{-1}H_{ii,t}])$.

[11] To address this issue, we propose some useful metrics in the last section of the online appendix.

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