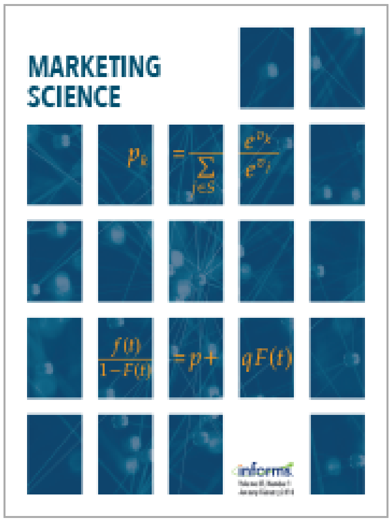

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Examining Demand Elasticities in Hanemann's Framework: A Theoretical and Empirical Analysis

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This paper examines demand elasticities using an integrated framework proposed by Hanemann [Hanemann, M. W. 1984. Discrete/continuous models of consumer demand. *Econometrica* **52**(3) 541–561], which models the incidence, brand choice, and quantity decisions of a consumer as an outcome of her utility maximization subject to budget constraints. Although the Hanemann framework has been the mainstay of earlier efforts to examine these decisions jointly, empirical researchers who have used the it to study purchase behavior have often found that the quantity elasticities are around -1, regardless of the brand or category. We attempt to uncover the underlying reasons for this finding and propose approaches to get as close to the "true" quantity elasticities as possible. We do this by (i) analytically demonstrating how assumptions on the distribution of the brand-specific econometrician's errors imply certain restrictions that in turn force quantity elasticities to -1, (ii) discussing how these restrictions can be alleviated by considering a suitable specification of unobserved parameter heterogeneity, and (iii) using scanner data to empirically illustrate the impact of the restrictions on quantity elasticities and the relative efficacy of multiple specifications of unobserved heterogeneity in easing those restrictions. We find that the specification of unobserved heterogeneity *crucially* influences estimates of quantity elasticities and that the mixture normal specification outperforms the alternatives.

Key words: microeconomic theory of demand; quantity elasticities; Hanemann's framework; stochastic properties of distribution functions of econometrician's errors; stochastic properties of finite mixture/normal random/mixture normal specifications of heterogeneity; structural approach
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1. Introduction

Ever since the availability of scanner panel data, researchers in marketing have been concerned with jointly modeling the three important decisions facing a consumer of a frequently purchased packaged good: whether to buy in the category (incidence), which brand to buy (choice), and how much of the chosen brand to buy (quantity). Almost all the prior literature that has modeled these three decisions simultaneously from an underlying utility maximizing framework has used Hanemann's framework (Hanemann 1984); relevant marketing papers are Chiang (1991), Chintagunta (1993), Arora et al. (1998), Nair et al. (2005), and Song and Chintagunta (2007). Table 1 provides further details on each of these papers.

Unfortunately, the Hanemann framework suffers from a widely acknowledged and important limitation that restricts its applicability to a number of problems. For many commonly used error distributions, the price elasticities related to the quantity decision are restricted to be equal to (or very close to) "one" in magnitude, regardless of brand, consumer, or category. In the context of those categories in which consumers make all three purchase decisions, restricted quantity elasticity estimates of this kind clearly hinder any empirical examination or policy analysis of a firm's decisions related to pricing or price promotions. For instance, consider a much-studied research question—namely, deciding which categories or brands a retailer should promote. An accurate estimate of quantity elasticities is clearly central to answering this question because a common response to promotions is purchase acceleration and stockpiling, the magnitudes of which are impossible to assess if one restricts quantity elasticities. Similarly, suppose one wanted to perform counterfactuals such as examining the change in profits or the change in consumer welfare if a product were deleted from a product line or if the retailer were to change his pricing policies. One needs an appropriate structural

Table 1 Related Empirical Literature in Marketing Based on Hanemann's Framework

Paper	Product category	Treatment of unobserved heterogeneity	Distribution of the econometrician's errors	Estimated population-level quantity elasticities
Chiang (1991)	Ground caffeinated coffee	Did not consider unobserved heterogeneity	Logistic	Constrained to −1 across all brands
Chintagunta (1993)	Yogurt	Considered unobserved heterogeneity in brand choice and incidence decisions, but not in the quantity decision	IID EV	Constrained to −1 across all brands
Arora et al. (1998)	Canned vegetable soup	Considered normal random unobserved heterogeneity in all three decisions	IID EV	Not reported
Nair et al. (2005)	Refrigerated orange juice	Considered normal random unobserved heterogeneity in all three decisions	Logistic	Constrained to −1 across all brands
Song and Chintagunta (2007)	Paper towels, toilet tissues, laundry detergents, and fabric softeners	Considered finite mixture heterogeneity in all three decisions	Logistic	Constrained to −1 across all brands in paper towels and toilet tissues; between −0.8 to −0.9 for brands in detergents; and between −0.74 to −0.89 for brands in softeners

Note. The three decisions referred to above are purchase incidence, brand choice, and quantity choice.

model that models all three purchase decisions, and one needs the model to provide correct estimates of quantity elasticity.

This brings us directly to the focus of this paper: examining why Hanemann's framework suffers from this restriction and suggesting ways to overcome it. Before we proceed, it is important to clarify that obtaining elasticities different from −1 is *not* an end in itself and is certainly not the aim of this paper. The intention is to get as close to the "true" quantity elasticities as possible. To do this, it is important to identify restrictions that might lead to biased estimates. We therefore attack the problem in the following three logical steps. In the *first* step, we point out three restrictions imposed by assumptions on the joint distribution of the brand-specific econometrician's errors[1] in forcing quantity elasticity to be close to one in magnitude. Specifically, we discuss the *nature of these restrictions* in the absence of unobserved heterogeneity and show that the three restrictions hold true when the econometrician's errors are independent and identically distributed (IID) extreme valued, logistic or generalized extreme value (GEV) distributed (an assumption used in all five prior papers based on Hanemann's framework). In the *second* step, we show how the three restrictions on the quantity elasticities can be alleviated at the population level by considering unobserved heterogeneity in the parameters. Specifically, having understood the nature of the restrictions imposed by the econometrician's errors in the first step, we use this knowledge to understand the *properties* required of the specification of unobserved heterogeneity that would help relax these restrictions. This, in turn, allows us to hypothesize on the extent to which different specifications of unobserved heterogeneity popularly used in prior literature (namely, normal random, finite mixture, and mixture normal heterogeneity) would serve as suitable specifications that would satisfy the properties and thereby relax the three restrictions.[2]

Finally, in the *third* step, we empirically demonstrate the nature of the restrictions imposed by the distributional assumptions of the econometrician's errors on the quantity elasticities, and we test our hypotheses on the extent to which each restriction gets relaxed at the population level over different specifications of unobserved heterogeneity. We do so using scanner panel data in the yogurt category.

Our findings can be simply stated. First, the specification of unobserved heterogeneity crucially influences estimates of quantity elasticities. Thus, it is not enough to merely account for unobserved heterogeneity because different specifications relax the restrictions to differing extents. For example, two popular

[1] Intuitively, "brand-specific econometrician's errors" are similar to the usual econometrician's errors in brand utilities in standard discrete-choice models. For brevity of expression, we use the term "econometrician's errors" and clarify the terminology when necessary.

[2] Since our analysis rests on the restrictions caused by the distribution of econometrician's errors, and the extent to which different specifications of unobserved heterogeneity satisfy properties needed to relax these restrictions, it is useful to provide some key details on the distributional specifications of the econometrician's errors and the unobserved heterogeneity used in prior papers that have used Hanemann's framework. Table 1 provides these details.

specifications, the finite mixture and the normal, do not do a good job of relaxing restrictions. This brings us to the second finding: the *mixture normal* specification of heterogeneity does the best job of relaxing the restrictions and, hence, of obtaining the most accurate estimates of quantity elasticities.

The rest of the paper is organized as follows. In §2, we formally elucidate three restrictions imposed by the choice of the distribution of the errors on the estimates of the quantity elasticities. In §3, we show how the restrictions can be relaxed at the population level by considering unobserved heterogeneity in the parameters. This is followed by §4, where we empirically demonstrate the restrictions on the quantity elasticities and the relative efficacy of different specifications of unobserved heterogeneity in relaxing the restrictions. In §5, we discuss the substantive implications of our empirical findings. Section 6 concludes with limitations and suggestions for future research.

2. Model

In §2.1, we provide the specifications of the incidence, quantity, and brand choice decisions in Hanemann's framework. Using these specifications, in §2.2 we derive the expressions for the price elasticities related to the quantity decision. With this in place, in §2.3 we detail the restrictions on quantity elasticities implied by the choice of the distribution of the econometrician's errors. Note that the discussion on the restrictions is based on the assumption of *no unobserved heterogeneity* in the parameters. It is important to emphasize, however, that the results we derive are *not restricted* to the no-heterogeneity case. Considering the no-heterogeneity case, however, helps us gain a clearer intuition into the nature of the restrictions and provides a natural segue to §3, where we discuss ways to relax the restrictions by considering unobserved heterogeneity.

2.1. Specifications of the Incidence, Choice, and Quantity Decisions

We start by outlining the theoretical specification of the incidence, brand choice, and purchase quantity model based on the joint utility maximization framework laid out by Hanemann (1984). Because much of this treatment is standard and has been used in the prior marketing papers listed in Table 1, we provide only key details here. Consider the case where there are $m = 1..M$ brands in the product category of interest. Let p_m be the unit price and ψ_m be the consumer's quality index of brand m. Furthermore, let ψ_z be the quality index of the composite commodity and p_z be its unit price, which is taken as a numeraire. Finally, let y be the total basket expenditure of the consumer on the given trip. Given these covariates, the consumer's utility maximization problem can be characterized as

$$\max_{\{q_m\}_{m=1}^{M}} \; u\left(\sum_{m=1}^{M} \psi_m q_m, \psi_z z\right)$$
$$\text{subject to } \sum_{m=1}^{M} p_m q_m + p_z z = y, \tag{1a}$$
$$q_z > 0, \quad q_m \geq 0 \quad \text{for } m = 1..M,$$

where $u(.)$ is the direct utility, and q_m and q_z are the quantities of brand m and of the composite commodity, respectively. The linearity of the first argument in the utility in (1a) implies that at most one brand can be purchased in the category, namely, the one with the lowest quality-adjusted price, p_m/ψ_m, amongst all $m = 1..M$ brands in the category. To get the functional form specifications of the purchase decisions, we choose the homothetic translog (HTL) specification of the indirect utility (this is the most widely used specification in the five prior papers) that corresponds to the direct utility in (1a) between the composite commodity z and the chosen brand k. This is given as

$$\nu^{HTL} \equiv \ln y - a \ln \frac{p_k}{\psi_k} - (1-a) \ln \frac{p_z}{\psi_z} + \frac{1}{2} b_{11} \left(\ln \frac{p_k}{\psi_k}\right)^2 + \frac{1}{2} b_{22} \left(\ln \frac{p_z}{\psi_z}\right)^2 + b_{12} \left(\ln \frac{p_k}{\psi_k}\right)\left(\ln \frac{p_z}{\psi_z}\right), \tag{1b}$$

where a, b_{11}, b_{12}, and b_{22} are the parameters of the HTL indirect utility, with $b_{11} = -b_{12}$ and $b_{11} = b_{22}$. Furthermore, we specify the quality index of any brand m, ψ_m, and that of the composite commodity ψ_z as follows:

$$\ln(\psi_m) \equiv (\beta_m H_m + \eta_m)/\mu, \tag{2a}$$

$$\ln(\psi_z) \equiv \mu_z \eta_z. \tag{2b}$$

In (2a), the parameter μ is the inverse of the consumer's quality sensitivity in the category and is restricted to be positive, and H_m is a vector of explanatory variables that include the brand dummies, presence of promotions, state dependence, and inventory for the category. Finally, β_m is a vector of parameters that represent the consumer's sensitivity toward the explanatory variables, and η_m is a random variable that is IID across all households and purchase occasions. In (2b), η_z is a random aggregate shock that is independent of the errors $\{\eta_m\}$ in (2a), and μ_z is the parameter by which the aggregate shock is scaled.

Based on the HTL specification and the functional forms of the quality indices, we make the transformations $V_m \equiv (\beta_m H_m - \mu \ln p_m)/\mu$, $\varepsilon_m \equiv (\eta_m - \mu_z \eta_z \mu)/\mu$, and $\tau \equiv a/b_{11}$ to get the stochastic specifications of the incidence, brand choice, and quantity decisions as follows:

Purchase Incidence Decision. A purchase will be made in the category (that is, $I = 1$) if

$$\max_{m=1..M} (V_m + \varepsilon_m) > \tau. \tag{3}$$

Brand Choice Decision. Conditional on purchase in the category (that is, $I = 1$), brand k will be chosen (that is, $d_k = 1$) if

$$V_k + \varepsilon_k \geq V_m + \varepsilon_m \quad \forall\, m = 1..M. \tag{4}$$

Budget Share/Purchase Quantity Decision. Conditional on purchase in the category and choice of brand k (that is, $I = 1$ and $d_k = 1$), the budget share of brand k is given as

$$s = b_{11}(V_k + \varepsilon_k - \tau), \tag{5}$$

where the purchase quantity of brand k is related to its budget share as $q_k \equiv ys/p_k$ (for tractability reasons, we generally work with the budget share of a brand instead of its purchase quantity; because one is easily obtained from the other, this choice has no material consequences). Notice that the stochastic specifications of all three decisions in (3), (4), and (5) are summarized in terms of two important elements: "$V_m + \varepsilon_m$" and "τ." These are explained as follows.

The first element $V_m + \varepsilon_m$ can be interpreted as the *consumer's subutility* of brand m, with V_m as the deterministic part and ε_m as the stochastic part. The deterministic part V_m is a function of the brand's price p_m and the explanatory variables in its quality index H_m. The stochastic part ε_m, which we henceforth refer to as the *econometrician's error*, is, in turn, a function of the error η_m and the aggregate shock η_z. The interpretation of $V_m + \varepsilon_m$ as brand m's subutility follows from the fact that in all three decisions, the impact of the market mix variables of any brand m only comes through its subutility. Finally, the second element τ can be interpreted as the *consumer's threshold utility* for purchase in the category. This interpretation follows from the purchase incidence decision in (3)—for a purchase to be made in the category, the subutility for at least one brand has to be greater than τ.

Given the stochastic specifications of the three decisions, we turn next to the derivation of the price elasticities for the purchase quantity/budget share decision of a brand conditional on its purchase. Note that the only stochastic terms in the three decisions in (3), (4), and (5) are the econometrician's errors $\{\varepsilon_m\}_{m=1}^M$. Thus, the computation of the elasticities requires the specification of the joint distribution of these errors. Since $\varepsilon_m \equiv (\eta_m - \mu_z \eta_z \mu)/\mu$, one obtains various specifications of the distribution for the econometrician's errors, depending on one's assumptions about the distribution of the errors $\{\eta_m\}_{m=1}^M$ and the composite good error η_z. The resulting specifications of the distributions of $\{\varepsilon_m\}_{m=1}^M$ in each of the five papers based on the Hanemann's framework are given in Table 1. Essentially, Chiang (1991), Nair et al. (2005), and Song and Chintagunta (2007) assumed $\mu_z = 1/\mu$ and the errors $\{\eta_m, \eta_z\}$ to be IID extreme valued (EV) distributed, which resulted in a *logistic distribution* for $\{\varepsilon_m\}_{m=1}^M$. Chintagunta (1993) and Arora et al. (1998) assumed $\mu_z = 0$ and the errors $\{\eta_m\}$ to be IID EV distributed, which resulted in an *IID EV distribution* for $\{\varepsilon_m\}_{m=1}^M$.

Because our main objective is to show how the choice of the joint distribution of the econometrician's errors can restrict the price elasticities related to the purchase quantity decision (or simply the quantity elasticities), we depart from prior literature and do not derive the elasticities using a *specific functional* form of the joint distribution of these errors. Instead, we start in §2.2 by deriving the specifications of the quantity elasticities related to the purchase quantity decision using a *general form* for the joint distribution function of the errors $F(\{\varepsilon_m\}_{m=1}^M)$. Based on that, in §2.3, we discuss three restrictive properties of the joint distribution function that would restrict the estimates of the quantity elasticities.

2.2. Specifications of Elasticities Related to the Purchase Quantity/Budget Share Decision

Using the specifications of the three decisions in (3), (4), and (5), we get the distribution of the budget share of brand k conditional on its choice and purchase in the category as

$$\begin{aligned} G_k&(s \mid d_k = 1, I = 1) \\ &= \Pr\Big(V_k + \varepsilon_k \leq \tau + s/b_{11} \mid V_k + \varepsilon_k \geq V_m + \varepsilon_m \\ &\qquad \forall\, m = 1..M,\ \max_{m=1..M}(V_m + \varepsilon_m) > \tau\Big). \end{aligned} \tag{6}$$

Making the transformation $\zeta = V_k + \varepsilon_k$ in (6), the distribution of the conditional budget share can be written in terms of the general form of the joint distribution of the errors, $F(\{\varepsilon_m\}_{m=1}^M)$ as

$$G_k(s \mid d_k = 1, I = 1) = 1 - \frac{\int_{\tau+s/b_{11}}^{\infty} F_k(\{\zeta - V_m\}_{m=1}^M)\, d\zeta}{\int_{\tau}^{\infty} F_k(\{\zeta - V_m\}_{m=1}^M)\, d\zeta}, \quad \text{where } F_k = \frac{\partial F}{\partial \varepsilon_k}. \tag{7}$$

From (7), we get the expected value of the conditional budget share of brand k as

$$\begin{aligned} E&(s \mid d_k = 1, I = 1) \\ &= \int_s s \frac{\partial G_k(s \mid d_k = 1, I = 1)}{\partial s}\, ds \\ &= \frac{1}{b_{11}} \frac{\int_s s F_k(\{\tau + s/b_{11} - V_m\}_{m=1}^M)\, ds}{\int_{\tau}^{\infty} F_k(\{\zeta - V_m\}_{m=1}^M)\, d\zeta}, \end{aligned} \tag{8}$$

which yields the price elasticity of the expected conditional budget share (or simply the conditional budget share elasticity) of brand k as

$$\mathscr{E}_k^{bs} = \frac{\partial \ln E(s \mid d_k = 1, I = 1)}{\partial \ln p_k}. \tag{9}$$

Because the purchase quantity is related to the budget share as $q_k \equiv ys/p_k$, we can relate the quantity elasticity of brand k to its conditional budget share elasticity in (9) as

$$\mathscr{E}_k^{pq} = \mathscr{E}_k^{bs} - 1. \quad (10)$$

This completes the specifications of the conditional budget share/quantity elasticities. We now turn to the central focus of this section, namely, a discussion of how the choice of the functional form of the distribution of the errors can restrict the estimates of the quantity elasticities.

Before we do so, there are two important points we would like to make. First, although we discuss the impact of the distributional assumptions of the errors on quantity elasticities for a particular demand system, one based on HTL indirect utility, our results can be generalized to all demand systems (the relevant proofs for a general demand system are given in §4 of the electronic companion to this paper, available as part of the online version that can be found at http://mktsci.pubs.informs.org). Second, we assume that the purchase quantity is continuous. This is in keeping with most of the prior literature based on Hanemann's framework (Chiang 1991, Chintagunta 1993, Nair et al. 2005). More importantly, Nair et al. (2005) empirically compared the predictions of the continuous quantity assumption with those of the discrete quantity approximation proposed by Arora et al. (1998). They found the results to be very similar across the two, implying that the continuous quantity assumption is fairly robust.

2.3. Restrictions Imposed by the Specification of $F(\{\varepsilon_m\}_{m=1}^M)$ on Quantity Elasticities

We discuss three restrictions that can be imposed by the specification of the distribution function of the econometrician's errors on the quantity elasticities given in (10). We do so in the following two lemmas, where Lemma 1 deals with the first restriction and Lemma 2 deals with the other two restrictions.

2.3.1. The First Restriction.

Lemma 1. *If the joint distribution function of the errors $F(\{\varepsilon_m\}_{m=1}^M)$ is such that for any brand k, the function $L_k(x)$ given as*

$$L_k(x) \equiv \Pr\Big(V_k + \varepsilon_k \leq x \mid V_k + \varepsilon_k \geq V_m + \varepsilon_m \quad \forall\, m = 1..M,\ \max_{m=1..M}(V_m + \varepsilon_m) > \tau\Big) \quad (11)$$

is identical across all brands, that is, $L_k(x) \equiv L_j(x)$ $\forall\, j, k \in \{1..M\}$, then the specifications of the expected values of the conditional budget shares (Equation (11)) will also be identical across all brands; i.e., $E(s \mid d_k = 1, I = 1) \equiv E(s \mid d_j = 1, I = 1)$ $\forall\, j, k \in \{1..M\}$.

Proof. See §2 of the electronic companion.

An Outline of a Proof of Lemma 1. Notice that the distribution function of the conditional budget share of brand k ($G_k(s \mid d_k = 1, I = 1)$ in Equation (6)) is a simple convolution of $L_k(x)$, given in Equation (11). In other words, if we replace "x" in the expression for $L_k(x)$ in Equation (11) with "$\tau + s/b_{11}$," we get the distribution function of the conditional budget share of brand k as given in Equation (6). Now if $L_k(x)$ is identical across all brands $k = 1..M$, it implies that the distribution of the conditional budget share in (6) will also be identical across all brands. This would, in turn, imply that the expected values of the conditional budget shares will be identical across all brands; that is, $E(s \mid d_k = 1, I = 1) \equiv E(s \mid d_j = 1, I = 1)$ $\forall\, j, k \in \{1..M\}$.

Corollary. *The condition stated in Lemma* 1 *holds true when the errors $\{\varepsilon_m\}_{m=1}^M$ follow a GEV, an IID EV, or a logistic distribution.*

Proof. See §2 of the electronic companion.

Implication of Lemma 1. If the specifications of the expected values of the conditional budget shares of all brands are identical, that is, $E(s \mid d_k = 1, I = 1) \equiv E(s \mid d_j = 1, I = 1)$ $\forall\, j, k \in \{1..M\}$, the conditional budget share elasticities of all brands $j = 1..M$ with respect to the price of brand k will also be identical; that is,

$$\frac{\partial \ln E(s \mid d_k = 1, I = 1)}{\partial \ln p_k} \equiv \frac{\partial \ln E(s \mid d_j = 1, I = 1)}{\partial \ln p_k} \quad \forall\, j, k \in \{1..M\}. \quad (12)$$

The above result follows from the fact that because $E(s \mid d_k = 1, I = 1) \equiv E(s \mid d_j = 1, I = 1)$ is an identity, differentiating both sides with respect to $\ln(p_k)$ will preserve the identity. Before discussing the implication of (12), it is useful to clarify what the right-hand side (RHS) in (12), which is the conditional budget share elasticity of brand j with respect to the price of another brand $k \neq j$, really captures. This elasticity captures the impact of brand k's price on a consumer's budget share decision (i.e., her allocation of the total shopping expenditure between the chosen brand j and the composite commodity, *conditional* on the fact that she has chosen to purchase brand j over brand k). Thus this elasticity refers *only* to consumers who have decided to purchase brand j and not brand k in the category. Intuitively, we would expect the budget share decision of buyers of brand j to not be affected by changes in the price of brand k because they do not buy brand k. Thus we would expect the RHS in Equation (12) to be close to zero. However, note that as per the identity in (12), this implies that the left-hand side (LHS), which represents the conditional budget share elasticity of brand k with respect to its own price, *would be biased toward zero*. Because the quantity elasticity of the chosen brand k with respect to its own price is nothing but the conditional budget share elasticity of the chosen brand k with respect to its own price minus one (Equation (10)), this implies that *the quantity elasticity of brand k would be biased toward* -1.[3]

Implication of the Corollary. The corollary implies that if there is no unobserved heterogeneity and if we use a GEV, IID EV, or logistic distribution for the errors, then the quantity elasticities of all brands will be biased toward -1. Linking this to prior literature, Chiang (1991) employed a logistic distribution and Chintagunta (1993) employed an IID EV distribution for the econometrician's errors (neither paper considered unobserved parameter heterogeneity in the quantity choice decision); both reported quantity elasticities close to -1. However, note that if the errors are IID normal, then the condition in Lemma 1 does not hold true. Thus, the quantity elasticities will not suffer from the first restriction when the errors are normally distributed.[4]

2.3.2. The Second and Third Restrictions.

Lemma 2. *The magnitude and sign of the deviation of the quantity elasticity of any brand k from "-1" depend, respectively, on the magnitude and sign of the slope of the function $M_k(x)$, given as*

$$M_k(x) \equiv -\frac{\int_x^\infty F_{kk}(\{\zeta - V_m\}_{m=1}^M)\, d\zeta}{\int_x^\infty F_k(\{\zeta - V_m\}_{m=1}^M)\, d\zeta} \tag{13}$$

for values of $x > \tau$ (recall that $F_k = \partial F/\partial \varepsilon_k$, $F_{kk} = \partial^2 F/\partial \varepsilon_k^2$, and τ is the threshold utility of the consumer in the category). In particular,

(a) *If the joint distribution of the errors $F(\{\varepsilon_m\}_{m=1}^M)$ is such that $\partial M_k(x)/\partial x > 0$ for all $x > \tau$, then the quantity elasticity of brand k will always be greater than one in magnitude; i.e., $|\mathscr{E}_k^{pq}| > 1$. Similarly, if $\partial M_k(x)/\partial x < 0$ for all $x > \tau$, then the quantity elasticity of brand k will always be less than one in magnitude; i.e., $|\mathscr{E}_k^{pq}| < 1$.*

(b) *If the joint distribution of the errors, $F(\{\varepsilon_m\}_{m=1}^M)$, is such that $\partial M_k(x)/\partial x \approx 0$ for all for $x > \tau$, then the quantity elasticity of brand k will be close to one in magnitude; i.e., $|\mathscr{E}_k^{pq}| \approx 1$.*

Proof. See §2 of the electronic companion.

Corollary 1. *If $F(\{\varepsilon_m\}_{m=1}^M)$ is distributed either GEV, IID EV, logistic, or independent normal, then $\partial M_k(x)/\partial x > 0\ \forall\, x > \tau$ for all brands $k = 1..M$. Thus, as per part* (a) *of Lemma* 2, *we will always get the quantity elasticities of all brands to be* greater than one in magnitude; *i.e.*, $|\mathscr{E}_k^{pq}| > 1$.

Proof. See §2 of the electronic companion.

Corollary 2. *If $F(\{\varepsilon_m\}_{m=1}^M)$ is distributed either GEV, IID EV, or logistic, and if the purchase incidence probability in the category is small, then for all brands $k = 1..M$, the function $\partial M_k(x)/\partial x \approx 0\ \forall\, x > \tau$. Thus, as per part* (b) *of Lemma* 2, *we will get the quantity elasticities for all brands to be close to one in magnitude; i.e.*, $|\mathscr{E}_k^{pq}| \approx 1$.

Proof. See §2 of the electronic companion.

An Outline of a Proof of Lemma 2*:* Consider the survivor function of the distribution of the conditional budget share of brand k, Ψ_k, which is related to the cumulative distribution function (cdf) of the conditional budget share of brand k as follows:

$$\Psi_k \equiv 1 - G_k(s \mid d_k = 1, I = 1). \tag{14}$$

Next, consider the case where $\partial \ln \Psi_k/\partial \ln p_k < 0$ for all values of the price of brand k, p_k, and the share of brand k. This inequality implies that $\partial \ln G_k/\partial \ln p_k > 0$. As a result, the cdf of the share of brand k at a higher value of p_k will be first-order stochastically dominated by the cdf of the share of brand k at a lower value of p_k. In other words, as the price of brand k increases, the probability mass in the cdf of the budget share of brand k moves toward zero, which in turn implies that the expected value of s at a higher value of p_k will be smaller than that at a lower value of p_k. As a result, we will get the conditional budget share elasticity of brand k to be negative and, consequently, the quantity elasticity of brand k to be greater than one in magnitude (since $\mathscr{E}_k^{pq} = \mathscr{E}_k^{bs} - 1$). In a similar vein, we can see that if $\partial \ln \Psi_k/\partial \ln p_k > 0$, we would get the quantity elasticity of brand k to be less than one in magnitude; and if $\partial \ln \Psi_k/\partial \ln p_k \approx 0$, we would get the quantity elasticity of brand k to be close to one in magnitude.

Next, using the specification of $G_k(s \mid d_k = 1, I = 1)$ given in Equation (7), we get the specification of $\partial \ln \Psi_k/\partial \ln p_k$ in terms of the function M_k as given in Equation (13) as

$$\partial \ln \Psi_k/\partial \ln p_k = M_k(\tau) - M_k(\tau + s/b_{11}). \tag{15}$$

Consider the RHS of Equation (15), which is $M_k(\tau) - M_k(\tau + s/b_{11})$. Since $b_{11} > 0$, it implies that $\tau + s/b_{11} > \tau$, which in turn implies that

(i) If $\partial M_k(x)/\partial x > 0$ for values of $x > \tau$, then $M_k(\tau) - M_k(\tau + s/b_{11}) < 0$. As a result, we will get

[3] To be clear, the bias results when the true distribution departs from what is assumed. We thank an anonymous reviewer for helping us clarify this point.

[4] We also used a simulation exercise to examine the biases caused by the first restriction, when the econometrician's errors are assumed to be either IID EV or logistic distributed. Details are in §1 of the electronic companion.

$\partial \ln \Psi_k / \partial \ln p_k < 0$. Thus, following the line of argument given before, we will get the quantity elasticity to be greater than one in magnitude. Similarly, if $\partial M_k / \partial x < 0$ for values of $x > \tau$, we will get the quantity elasticity to be less than one in magnitude.

(ii) If $\partial M_k(x)/\partial x \approx 0$ for values of $x > \tau$, then $M_k(\tau) - M_k(\tau + s/b_{11}) \approx 0$, and as a result, $\partial \ln \Psi_k / \partial \ln p_k \approx 0$. Thus, following the line of argument given before, we will get the quantity elasticity of brand k to be close to one in magnitude.

This completes the sketch of a proof for Lemma 2. Note that Lemma 2 thus implies two restrictions on the estimates of the conditional budget share/quantity elasticities. The first restriction, implied by part (a) of Lemma 2 (which we henceforth refer to as R2a), relates the *sign* of $\partial M_k(x)/\partial x$ for $x > \tau$ to whether the quantity elasticity of brand k will be less than or greater than one in magnitude. The second restriction, implied by part (b) of Lemma 2 (which we henceforth refer to as R2b), relates the *magnitude* of $\partial M_k/\partial x$ for $x > \tau$ to whether the quantity elasticity of brand k will be close to one in magnitude. Furthermore, the corollaries relate the two restrictions to the specific distributions of the econometrician's errors. Specifically, if these errors are IID EV/GEV/logistic distributed, then the quantity elasticities will suffer from both R2a and R2b. If the errors are independent normal, then the quantity elasticities will only suffer from R2a.

Given the above intuitive discussion, it is useful to delve deeper into the statistical properties of the distribution of the errors that result in the restrictions implied by Lemma 2. To do this, it helps to examine the term $\partial M_k/\partial x$ in the region $x > \tau$. Consider this for the case when there is only one brand k in the category. (Note that this is a salient difference between the restrictions implied by Lemma 2 and the restriction implied by Lemma 1. The restriction implied by Lemma 1 only holds when the total number of brands in the category is greater than one. However, the restrictions implied by Lemma 2 hold even if there is just one brand in the category.) In such a case, after making the substitution $\varepsilon_k = x - V_k$, the slope of the function $M_k(x)$ given in (13) simplifies to

$$\frac{\partial M_k(x)}{\partial x} \equiv \frac{\partial}{\partial \varepsilon_k}\left[\frac{F_k(\varepsilon_k)}{1 - F(\varepsilon_k)}\right], \tag{16a}$$

and the space spanned by the condition $x > \tau$ simplifies to

$$\varepsilon_k > \tau - V_k, \tag{16b}$$

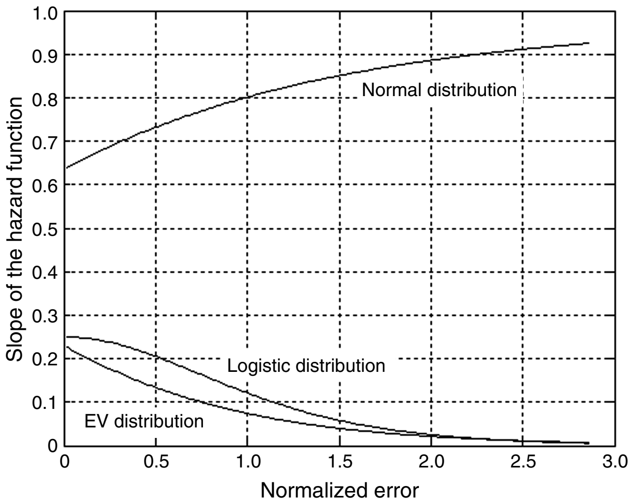
where F is the distribution function of the econometrician's error in brand k's subutility, ε_k, and F_k is its density. Now, notice the RHS of (16a) is simply the slope of the *hazard function* of ε_k. Moreover, the space spanned by the condition in (16b) is nothing but the region in which a purchase is made in the category, which can be further couched in terms of the purchase incidence probability in the category as[5] $\varepsilon_k > F^{-1}(1 - \Pr(I = 1))$. Thus, the slope of $M_k(x)$ in the region $x > \tau$ can be intuitively interpreted as *the slope of the hazard function of the error* ε_k *in the region in which a purchase is made in the category; i.e.,* $\varepsilon_k > F^{-1}(1 - \Pr(I = 1))$.

Based on this interpretation, the restriction R2a implied by part (a) of Lemma 2 can be interpreted as follows: *If the hazard function of the econometrician's error in brand k's subutility is increasing in the region in which the category is purchased, then the quantity elasticity of brand k will be greater than one in magnitude*. Given this interpretation, an intuitive explanation of Corollary 1 follows immediately. Note that if ε_k is either EV, logistic, or normally distributed, then its hazard function is always increasing. Thus, if we assume the errors $\{\varepsilon_m\}_{m=1}^{M}$ to be EV, logistic, or normally distributed, we will always restrict the quantity elasticities to be greater than one in magnitude.

The restriction R2b implied by part (b) of Lemma 2 can similarly be interpreted as follows: *For a given purchase incidence probability in the category* $\Pr(I = 1)$, *if the magnitude of the hazard function of the econometrician's error* ε_k *for values of* $\varepsilon_k > F^{-1}(1 - \Pr(I = 1))$ *is small, then the quantity elasticity of brand k will be close to one in magnitude*. Next, we use this interpretation to understand why Corollary 2 holds true for EV and logistic errors but not for normal errors. In Figure 1, we plot the slope of the hazard function for EV, normal, and logistic distributions (after normalizing the scale and location parameters across the three distributions) for the single brand case. Observe in Figure 1 that for high values of ε_k (or low values of the purchase incidence probability), the value of the slope of the hazard function becomes very small for EV and logistic distributions. For instance, if we consider $\Pr(I = 1) = 0.15$ as in the earlier simulation exercise, we can see that for values of $\varepsilon_k > F^{-1}(1 - 0.15)$ (which is approximately "1.0" for all three distributions), the slopes of the hazard functions for EV and logistic distributions become small. However, that is not the case for the normal random errors. Thus as per the interpretation of part (b) of Lemma 2, it follows that if the purchase incidence probability in the category is low (which it typically is for most data sets), then assuming EV or logistic distributions for

[5] For a single brand case, using the stochastic specification of purchase incidence decision in (3), we get $\varepsilon_k > \tau - V_k$ as the region in which a purchase is made in the category. Because F is the distribution function of the error term ε_k, it follows that the purchase incidence probability will be $\Pr(I = 1) = 1 - F(\tau - V_k)$. Inverting this purchase incidence probability, we get $\tau - V_k = F^{-1}(1 - \Pr(I = 1))$. Substituting $\tau - V_k$ into (16b), we get the condition $\varepsilon_k > \tau - V_k$ as $\varepsilon_k > F^{-1}(1 - \Pr(I = 1))$.

Figure 1 Slope of the Hazard Function for Normal, Logistic, and EV Distributions



the econometrician's errors will yield values of quantity elasticities close to one in magnitude.[6]

Table 2 summarizes the discussion above, and §3 discusses how to relax these restrictions.

Table 2 Summary of Restrictions on the Quantity Elasticities, Assuming No Unobserved Parameter Heterogeneity

Restriction	Implication of the restriction	Distribution of the econometrician's errors for which the restriction holds true
R1: Restriction implied by Lemma 1	The specification of the expected conditional budget share is identical across all brands, which biases the quantity elasticities of all brands toward −1	IID EV, GEV, logistic
R2a: Restriction implied by part (a) of Lemma 2	Because the slope of the hazard function is positive, the quantity elasticities of all brands are restricted to be greater than one in magnitude	IID EV, GEV, logistic, independent normal
R2b: Restriction implied by part (b) of Lemma 2	If the purchase incidence probability is low, the slope of the hazard function for these distributions is small, which implies that the quantity elasticities of all brands will be close to one in magnitude	IID EV, GEV, logistic

3. Relaxing the Three Restrictions on Quantity Elasticities

One can relax the three restrictions R1, R2a, and R2b in two possible ways. The first alternative is to use a distribution for the errors that does not suffer from the three restrictions, i.e., a distribution other than IID EV, GEV, independent normal, or logistic. The only distribution for the errors (which has a support from $-\infty$ to $+\infty$) that comes to mind is the multivariate normal distribution. Although an empirical question, it is not clear that a multivariate normal distribution for the errors will relax restriction R2a.[7] The second alternative, which we focus on, is to relax the three restrictions on the quantity elasticities at the population level by considering unobserved heterogeneity in the parameters in the brand subutilities.

We start in §3.1 by explaining how the three restrictions discussed so far in the absence of unobserved heterogeneity can be extended to the case when we have unobserved heterogeneity in the parameters in the subutilities of brands. Given this, in §§3.2 and 3.3, we discuss the properties that are required of a specification of unobserved heterogeneity that will help relax each of the three restrictions. Finally, in §3.4, we hypothesize on the extent to which different specifications of unobserved heterogeneity used in prior marketing literature would satisfy the properties needed to relax the restrictions.

3.1. Understanding the Restrictions in the Presence of Unobserved Heterogeneity

To see how the three restrictions can be extended to the unobserved heterogeneity case, it first helps to note the differences in the constituents of the stochastic terms in the subutilities of brands between the "no unobserved heterogeneity case" and the "unobserved heterogeneity case." If there is no heterogeneity, recall that the subutility of any brand m is given as $V_m+\varepsilon_m$, where the deterministic part of brand m's subutility is V_m, and the error in any brand m's subutility consists of *only* the econometrician's error, ε_m. However, once we consider unobserved heterogeneity in the parameters $\{\beta_m, \mu\}$ in the subutilities of brands, the subutility of any brand m will be given as $\bar{V}_m+\omega_m$, in which the deterministic part of brand m's subutility is $\bar{V}_m$ (which is the expected value of the deterministic part of the subutility of brand m, V_m, after taking out the errors due to unobserved heterogeneity in the parameters),

[6] This argument mirrors that used by Nair et al. (2005), who also find that when the purchase incidence probability is low, using a logistic distribution restricts the purchase quantity elasticities to be close to one in magnitude.

[7] It is difficult to formally prove the existence of restriction R2a for the general case of $M+1$ alternatives when the econometrician's errors are multivariate normally distributed. However, in the electronic companion, we show a formal proof for a simple case of three alternatives (two brands + the no purchase option) when the errors are bivariate normally distributed. We show that in such a case, the purchase quantity elasticity of at least one brand will always be greater than one in magnitude, implying that R2a will not be relaxed.

and the error in brand m's subutility ω_m will consist of *not only* the econometrician's error ε_m *but also* the error resulting from the unobserved heterogeneity (we refer to the composite of these two errors, in the unobserved heterogeneity case as the *overall stochastic term* ω_m).

Note that the conditions stated in Lemmas 1 and 2 only deal with the statistical properties of the distribution of the *errors in the subutilities of the brands*. Now, in the "no-heterogeneity case," to see if the three restrictions hold true, we only need to check whether the statistical properties of the *distribution of the econometrician's errors* $\{\varepsilon_m\}_{m=1}^M$ satisfy the conditions stated in Lemmas 1 and 2 (which is what we did in §2). On the other hand, in the presence of heterogeneity, the extent to which the three restrictions hold true at the population level depends on the extent to which the statistical properties of the *distribution of the overall stochastic terms* $\{\omega_m\}_{m=1}^M$ satisfy the conditions in Lemmas 1 and 2. Thus, in summary, in the presence of heterogeneity the restrictions discussed in §2 *remain as they are*, except that the conditions are not on the distribution of the econometrician's errors but on the overall stochastic term.[8]

3.2. Relaxing Restriction R1

Based on the above discussion, the restriction R1 implied by Lemma 1 can be extended to the case of unobserved heterogeneity as follows. If the joint distribution function of the overall stochastic terms $\{\omega_m\}_{m=1}^M$ is such that for any brand k, the function $L_k(x)$ given as

$$L_k(x) \equiv \Pr\Big(\bar{V}_k + \omega_k \le x \mid \bar{V}_k + \omega_k \ge \bar{V}_m + \omega_m \quad \forall\, m = 1..M,\ \max_{m=1..M}(\bar{V}_m + \omega_m) > \tau\Big) \quad (17)$$

is identical across all brands, that is, $L_k(x) \equiv L_j(x)$ $\forall\, j, k \in \{1..M\}$, then the specifications of the expected values of the conditional budget shares will also be identical across all brands.

To reiterate, the restriction in (17) differs from its no-heterogeneity counterpart in (11) in that the subutility of brand m was represented as $V_m + \varepsilon_m$ in the no-heterogeneity case in (11) versus $\bar{V}_m + \omega_m$ in the heterogeneity case in (17). Given the discussion above, it is easy to see that even if the joint distribution of the econometrician's errors $\{\varepsilon_m\}_{m=1}^M$ satisfies the condition in Lemma 1, it does not imply that the joint distribution of the overall stochastic terms $\{\omega_m\}_{m=1}^M$ will also satisfy the condition in Lemma 1.

Having said that, it is important to note that the extent to which R1 gets relaxed at the population level depends on one crucial property, which is the extent to which the specification of the unobserved heterogeneity imposed by the analyst can adequately recover the true heterogeneity in the data. Thus, if the specification of the unobserved heterogeneity is not able to capture the true heterogeneity in the data (and, as a result, underestimates it), then the statistical properties of the overall stochastic term will be dictated more by the properties of the econometrician's error ε_k than by the error due to heterogeneity; in that case, adding heterogeneity will not help to fully relax R1.

3.3. Relaxing Restrictions R2a and R2b

Similar to Lemma 1, the restrictions R2a and R2b as implied by Lemma 2 can be extended to the heterogeneity case as follows. If the slope of the hazard function of the overall stochastic term ω_m in the subutility of brand m is restricted to be positive (negative), then the quantity elasticity of brand m will be greater than one (less than one) in magnitude; and if the slope of the hazard function of the overall stochastic term ω_m is close to zero in magnitude in the region in which a purchase is made in the category, then the quantity elasticity of brand m will be close to one in magnitude. Given this extension, it is easy to see that the extent to which the two restrictions implied by Lemma 2 get relaxed depends on the extent to which the distribution of the overall stochastic term in the brand subutilities is flexible enough to allow for the slope of its hazard function to take both positive and negative values (and thereby relax R2a) over a wide range of magnitudes (and thereby relax R2b). Clearly, even if the distribution of the econometrician's errors ε_m satisfies the condition in Lemma 2, it does not imply that the joint distribution of the overall stochastic term ω_m will also satisfy the condition in Lemma 2.

To sum, the extent to which R2a and R2b get relaxed depends on *two* factors. First, as in the case of R1, it is important that the specification of unobserved heterogeneity be able to recover the true heterogeneity in the data adequately. Second, the extent to which R2a and R2b get relaxed depends on how flexible the distribution of the unobserved heterogeneity is in terms of the slope of its hazard function; the greater the flexibility, the more likely that the slope of the hazard function of the overall stochastic term can take a wide range of positive or negative values (and, thereby, relax R2a and R2b).

[8] Note that the proofs of Lemmas 1 and 2 for the heterogeneity case follow exactly the same line of reasoning as the proofs of Lemmas 1 and 2 given in §2.3 for the no-heterogeneity case. As pointed out, the no-heterogeneity case and the heterogeneity case only differ vis-à-vis the specifications of the subutilities of brands. Thus, the proofs of lemmas in the heterogeneity case remain as they are in the no-heterogeneity case, expect that in the proofs, (i) the deterministic part of the subutility of any brand m, V_m, is replaced by $\bar{V}_m$, and (ii) the joint distribution of the econometrician's errors $\{\varepsilon_m\}$ is replaced by the joint distribution of the overall stochastic terms $\{\omega_m\}$.

3.4. Relative Efficacy of Different Specifications of Unobserved Parameter Heterogeneity in Relaxing the Restrictions

Based on prior literature, there are three choices of unobserved heterogeneity distribution that come to mind readily as candidates for relaxing the three restrictions on the quantity elasticities at the population level. These are (a) normal random heterogeneity, (b) finite mixture heterogeneity (latent class segmentation), and (c) mixture normal random heterogeneity.

3.4.1. Normal Random Specification. In this specification, we consider the parameters β_m in the deterministic part of the subutility of brand m, V_m, to be normally distributed, and the parameter μ (which is restricted to be positive as per the model specification) to be log normally distributed across the consumer population. Note that introducing the log normal distribution over μ is a departure from previously used normal random specifications of heterogeneity. This is because the log normal distribution has a flexible hazard function whose slope is neither strictly positive nor strictly negative (Lancaster 1992). This gives more flexibility to the slope of the hazard function of the overall stochastic term, letting it take positive or negative values and, hence, potentially overcoming R2a.[9]

Having said that, it remains to be empirically seen whether using such a specification of unobserved heterogeneity would satisfy the two properties needed to relax the restrictions. Pertaining to the first property, if the true heterogeneity in the data is multimodal, this specification may do a poor job of recovering it. Pertaining to the second property, although this is an empirical question, it is not clear that using a simple fix of assuming log normal heterogeneity over μ will make the distribution of the overall stochastic term in a brand's subutility flexible enough to enable the slope of its hazard function to take a wide range of positive or negative values and, hence, overcome R2a and R2b.

3.4.2. Finite Mixture Specification (Latent Class Segmentation). Because this specification is semiparametric, we would it to be flexible enough to enable the slope of the hazard function of the overall stochastic term to take a wide range of positive or negative values. However, as shown by Allenby et al. (1998), the finite mixture specification does not capture heterogeneity as well as the normal random specification because it does a poor job of describing tail behavior. Thus, we would expect this specification to satisfy one of the properties needed to relax the restrictions (namely, flexibility), but it remains to be seen empirically whether it does a good job of satisfying the other property (namely, capturing the true heterogeneity in the data adequately).

3.4.3. Mixture Normal Random Specification. In this specification, for a given segment, the parameters β_m are assumed to be normally distributed while the parameter μ is assumed to be log normally distributed across the consumer population. Prior research (Allenby et al. 1998, Rossi and Allenby 2003) has shown that the mixture normal specification does an excellent job of capturing the underlying heterogeneity because, with enough components, it can approximate virtually any multivariate density. Furthermore, because the mixture normal specification has a semiparametric component, it would lend flexibility to the slope of its hazard function, which would in turn enable the slope of the hazard function of the overall stochastic term to take a wide range of positive or negative values and, hence, overcome R2a and R2b. Thus we would expect this specification to satisfy both the properties needed for alleviating the three restrictions.

4. Empirical Analysis

4.1. Design of the Empirical Analysis

Our objective in this section is to empirically test the extent of relaxation of the restrictions for a given distribution of the econometrician's errors as we move across different specifications of heterogeneity, focusing on an IID EV distribution for the econometrician's errors.[10] The four IID EV models that we estimate are (i) no heterogeneity (model 1), (ii) normal random heterogeneity (model 2), (iii) finite mixture heterogeneity (model 3), and (iv) mixture normal heterogeneity (model 4). Details of the models estimated are provided in Table 3.

4.2. Data and Variables

4.2.1. Data Description. We use scanner panel data on the yogurt category.[11] The data are from

[9] Note that if we do not consider the log normal heterogeneity over μ, then the random error in a brand's subutility that results from unobserved heterogeneity on β_m only will be normally distributed. Because a normal distribution has a strictly increasing hazard function, it will not lend flexibility to the slope of the hazard function of the overall stochastic term. Recall from Table 1 that this was the distribution of unobserved heterogeneity used by Nair et al. (2005), who considered the normal heterogeneity on β_m but no unobserved heterogeneity on μ. As a result, Nair et al. (2005) found quantity elasticities to be close to and greater than one in magnitude across all brands. We have modified specification of Nair et al. (2005) by taking log normal heterogeneity over μ.

[10] We have also estimated the models for each of the four heterogeneity cases when the econometrician's errors are IID normally distributed. We have provided the results and discussion for the four IID normal models in §7 of the electronic companion.

[11] Bucklin et al. (1998), who used this data set, found the quantity elasticities of brands to be significantly different from "one"

Table 3 Details of Models Estimated on Yogurt Data

Model number	Model description: econometrician's errors	Model description: unobserved heterogeneity	Number of parameters
1	IID EV	None	11
2	IID EV	Normal random with log normal heterogeneity on μ	18
3	IID EV	4-segment finite mixture	35
4	IID EV	2-segment mixture normal	33

Table 4 Descriptive Statistics of Yogurt Data

Brand	Mean price in cents/unit (std. dev.)	Promotion frequency (%)	Share in the category (%)
Nordica (6 oz.)	41.87 (6.10)	14.22	19.3
Yoplait (6 oz.)	61.89 (6.66)	2.92	34.0
Private label (8 oz.)	38.43 (10.65)	16.31	25.3
Dannon (8 oz.)	65.22 (5.26)	4.99	13.6
WBB (8 oz.)	43.71 (7.44)	4.45	7.8

ACNielsen and cover the Sioux Falls, South Dakota market from 1986 to 1988. To keep the model tractable, we limit ourselves to the top five yogurt brands in the market—Nordica, 6 oz.; Yoplait, 6 oz.; private label, 8 oz.; Dannon, 8 oz.; and WBB, 8 oz. Together, these brands account for around 70% of the category sales in dollars. Descriptive statistics for all brands are reported in Table 4.

The total sample consists of the purchase activities of 150 households over 2 years; the households are chosen such that all have purchased at least once in the yogurt category. The sample consists of 19,516 purchase observations, of which yogurt was purchased on 4,769 occasions. We randomly split the total sample into estimation and hold out samples. The estimation sample consists of 100 households with 13,392 purchase observations and the hold out sample consists of 50 households with 6,124 observations.

4.2.2. Variables. For each purchase observation, we have three sets of dependent variables: (i) an indicator variable representing the purchase incidence in the category, (ii) an $M \times 1$ vector of indictor variables (where M is the total number of brands) representing the brand choice decision conditional on purchase in the category, and (iii) the budget share/purchase quantity of the brand conditional on purchase in the category and brand choice.

The explanatory variables for each purchase observation are variables that enter the subutilities of the brands. Recall that in the deterministic part of the subutility of a brand m, the explanatory variables in the vector H_m impact the preference for brand m and p_m is the per unit price of brand m. The variables that we include in H_m are (i) brand dummies, (ii) presence of promotions on the brand (that includes features and displays), (iii) brand loyalty, and (iv) inventory of the category. The choice of these variables is driven by prior literature in marketing (e.g., Chiang 1991, Chintagunta 1993). Note that the inventory variable, unlike the first three variables, is the same across all brands in the category. Thus, it will only impact the purchase incidence and budget share decisions and not the brand choice decision. In the yogurt category, consumers' consumption rates depend on the inventory at hand (Ailawadi and Neslin 1998, Sun 2005); thus, we update the inventories using a flexible consumption rate as proposed by Ailawadi and Neslin (1998).

The parameters for each of the four IID EV models are discussed in §5 of the electronic companion. The number of segments for the finite mixture heterogeneity models and the mixture normal heterogeneity models are chosen such that the Bayesian information criterion (BIC) is minimized. Based on that, we end up with four segments for the finite mixture heterogeneity models and two segments for the mixture normal heterogeneity models. We use the method of simulated maximum likelihood to estimate the following parameters in each model: (i) the parameters in the subutilities of brands $\{\beta_m, \mu\}$, which are homogenous in model 1 but heterogeneous in models 2–4; and (ii) the parameter b_{11}. Note that we cannot separately identify the threshold utility τ from the brand dummy parameters (the brand intercepts) that enter the subutilities of brands in any of the four models. Thus, we set $\tau = 0$ and, instead, estimate the brand dummy parameters of all the brands. We will discuss our estimation methodology briefly; further details are provided in §5 of the electronic companion.

We simultaneously estimate all the parameters by maximizing the joint likelihood of all three purchase decisions. Because our interest lies in accounting for unobserved heterogeneity in all three purchase decisions (especially the quantity decision), we need to integrate out the unobserved heterogeneity over the joint likelihood for all three decisions while estimating the parameters. This lets us account for the impact of unobserved heterogeneity in brand subutilities in the quantity decision, which in turn allows us to use unobserved heterogeneity as a fix to the quantity elasticity problem.[12]

in magnitude. The data thus provide a good test of our method's ability to relax the restrictions.

[12] This is in contrast to prior work (e.g., Chintagunta 1993) that has used Heckman's two-stage estimator to estimate the likelihood for the three decisions. In that approach, one estimates the incidence and brand choice decisions along with the unobserved heterogeneity in the parameters in the subutilities of brands in the first stage. In the second stage, however, one estimates the quantity decision conditional on the mean values (over the population) of the het-

Formally, the dependent variables for each consumer i at occasion t are the incidence decision I_t^i, the brand choice decision if a purchase has been made in the category, $d_t^i \equiv \{d_{mt}^i\}_{m=1}^M$, and the budget share decision $s_t^i \equiv \{s_{mt}^i\}_{j=1}^M$. We represent the consumer-specific parameters by θ_{2i} and the population-level parameters that capture these consumer-specific parameters by θ_2. The likelihood estimates of the parameters $\hat{\theta}_2$ are computed by maximizing the joint log likelihood (for the entire sample of N consumers with T_i observations for each consumer i) as follows:

$$\hat{\theta}_2 = \arg\max_{\theta_2} \sum_{i=1}^N \ln\left(\int_{\theta_{2i}} \left(\prod_{t=1}^{T_i} L(d_t^i, I_t^i = 1, s_t^i \mid \theta_{2i})^{I_t^i} \cdot L(I_t^i = 0 \mid \theta_{2i})^{1-I_t^i}\right) f(\theta_{2i}/\theta_2)\, d\theta_{2i}\right), \quad (18)$$

where $f(\theta_{2i}/\theta_2)$ is the joint pdf of θ_{2i}. We integrate out the consumer-specific parameters using Monte Carlo simulation with $R = 200$ Halton draws. Note that the inner expression in the likelihood in (18) consists of two terms for each consumer i for a specific occasion t, namely, the observational likelihood when a purchase is not made in the category $L(I_t^i = 0 \mid \theta_{2i})$ and the observational likelihood when a purchase is made in the category $L(I_t^i = 1, d_t^i, s_t^i \mid \theta_{2i})$. When the econometrician's errors are IID EV distributed, these two terms are given as

$$L(I_t^i = 0 \mid \theta_{2i}) = \exp(-\exp(W_t^i)),$$

$$L(I_t^i = 1, d_t^i, s_t^i \mid \theta_{2i}) = \prod_{k=1}^M \left[\left(\frac{\mu^i}{b_{11}}\right)\frac{\exp(\mu^i V_{kt}^i)}{\exp(W_t^i)} \exp\left(W_t^i - \frac{\mu^i}{b_{11}} s_{kt}^i\right) \cdot \exp\left(-\exp\left(W_t^i - \frac{\mu^i}{b_{11}} s_{kt}^i\right)\right)\right]^{d_{kt}^i}, \quad (19)$$

where $W_t^i \equiv \ln(\sum_{m=1}^M \exp \mu^i V_{m,t}^i)$ and $V_{m,t}^i$ is the subutility of brand m at occasion t for consumer i.

4.3. Results

The log likelihood and goodness of fit of both the estimation and holdout samples for each of the four IID EV models are given in Table 5. We focus our discussion on the quantity elasticities for each of these models, reported in Table 6.[13] To aid the reader, we have based the following discussion closely on the information given in Table 2 (which presents a summary of the three restrictions discussed in §2.3).

Table 5 Goodness of Fit for the Four IID EV Models

Model number	Model description: unobserved heterogeneity	−Log likelihood: estimation sample	BIC estimation sample	−Log likelihood: Holdout sample	BIC holdout sample
1	None	2,734.7	2,786.9	1,260.8	1,308.8
2	Normal/log normal	1,721.7	1,807.2	884.4	962.9
3	4-segment finite mixture	1,986.0	2,152.3	1,014.6	1,167.2
4	2-segment mixture normal	1,635.3	1,792.1	817.0	960.9

Notes. The BIC is given as $-L + k\ln(n)/2$, where L is the log likelihood value, k is the number of parameters, and n is the number of observations. A lower BIC is preferred.

4.3.1. Model 1—No Unobserved Heterogeneity. As pointed out in Table 2, all the restrictions (R1, R2a, and R2b) on the quantity elasticities would hold in this model. As per R1, we should see expected conditional budget shares to be identical across all brands, which would force the own conditional budget share elasticities to be the same as the cross-conditional budget share elasticities. To verify this, we first computed the expected conditional budget shares of all brands at the mean values of the explanatory variables. As expected, they are identical across brands (0.0479 for Nordica, 0.0479 for Yoplait, 0.0479 for private label, 0.0479 for Dannon, and 0.0479 for WBB). The impact of this can be seen from the fact that the cross-conditional budget share elasticities are identical to the own conditional budget share elasticities.[14]

Next, as per R2a, we should see quantity elasticities greater than one in magnitude across all brands. Furthermore, because the purchase incidence probability is low (around 24%), we should also see all quantity elasticities being close to one in magnitude, as per R2b. Turning to the results (Table 6), we find these conjectures confirmed—the quantity elasticities of all brands are close to and greater than one in magnitude.

4.3.2. Model 2—Normal Random Coefficients Unobserved Heterogeneity. This model uses a normal random specification for unobserved heterogeneity (with log normal specification of unobserved heterogeneity over the parameter μ). Population-level elasticities for model 2 are reported in Table 6. Comparing these to the quantity elasticities predicted by model 1 (no unobserved heterogeneity), three points are noteworthy.

erogeneous parameters in the subutilities of brands (which were estimated in the first stage). Thus, for the quantity choice decision, the estimation is done assuming no unobserved heterogeneity in the parameters in the subutilities. Clearly then, one cannot take unobserved heterogeneity into account while calculating quantity elasticities, which suggests that one cannot relax the restrictions in the manner we suggest.

[13] Parameter estimates for all models, both the IID EV and IID Normal, are reported in §6 of the electronic companion. Also, §8 of the electronic companion provides the choice and incidence elasticities for all models.

[14] The own and cross-conditional budget share elasticities for model 1 are available from the authors upon request.

Table 6 Population-Level Quantity Elasticities for the Models 1–4: IID EV Distribution for the Econometrician's Errors

	Model 1: no unobserved heterogeneity (std. dev.)	Model 2: normal random unobserved heterogeneity (std. dev.)	Model 3: 4-segment finite mixture unobserved heterogeneity (std. dev.)	Model 4: 2-segment mixture normal unobserved heterogeneity (std. dev.)
Nordica (6 oz.)	−1.0206 (0.0019)	−0.8373 (0.0166)	−0.8696 (0.0403)	−0.5715 (0.0332)
Yoplait (6 oz.)	−1.0362 (0.0028)	−0.8643 (0.0140)	−0.8804 (0.0289)	−0.6668 (0.0185)
Private label (8 oz.)	−1.0270 (0.0015)	−1.0627 (0.0462)	−0.8849 (0.0452)	−0.8530 (0.0403)
Dannon (8 oz.)	−1.0145 (0.0010)	−0.9217 (0.0314)	−0.8486 (0.0415)	−0.6008 (0.0263)
WBB (8 oz.)	−1.0084 (0.0003)	−0.7920 (0.0193)	−0.8093 (0.0378)	−0.5107 (0.0226)

First, unlike model 1, the quantity elasticities predicted by model 2 are not as close to −1 (they range from −0.7920 to −1.0627), thus implying that R2b is more relaxed in model 2 than in model 1. Second, unlike model 1, the quantity elasticities predicted by model 2 are less than one in magnitude for four out of the five brands, which implies that restriction R2a is also more relaxed in model 2 than in model 1. Third, unlike model 1, the expected conditional budget shares calculated at the mean values of the explanatory variables in model 2 are different across all brands (they are 0.0397 for Nordica, 0.0474 for Yoplait, 0.0427 for private label, 0.0495 for Dannon, and 0.0406 for WBB), thus implying that R1 is relaxed compared to model 1.

Thus, we see that by comparison with the no-heterogeneity case (model 1), all three restrictions seem to get relaxed for the normal random heterogeneity model (model 2). The reasons for this follow from our hypotheses in §3.3 regarding the extent to which the normal random heterogeneity satisfies the two properties needed to relax the restrictions. In terms of the first property, the normal random heterogeneity does a good job of capturing the actual heterogeneity in the data, which can be inferred by observing a drastic difference in the BIC of model 2 (BIC = 1807.2) compared to model 1 (BIC = 2786.9). In terms of the second property, assuming log normal heterogeneity over the parameter μ gives model 2 some degree of flexibility to allow the slope of the hazard function of the overall stochastic term to take negative values.

4.3.3. Model 3—Finite Mixture for Unobserved Heterogeneity (Latent Class Segmentation). Population-level elasticities for model 3 are reported in Table 6. Comparing these to the quantity elasticities predicted by model 1 (no unobserved heterogeneity), we note three points.

First, unlike model 1, the quantity elasticities of *all* brands are less than one in magnitude in model 3. This suggests a relaxation of restriction R2a to the extent that the slope of the hazard function of the overall stochastic term can take negative values. This also matches with the findings of Bucklin et al. (1998), who report all quantity elasticities less than one in magnitude, using the same data set. Second, unlike model 1, the quantity elasticities in model 3 are not as close to −1 (they range from −0.8093 to −0.8849). This implies that R2b gets relaxed for model 3. Third, unlike model 1, the expected conditional budget shares calculated at the mean values of the explanatory variables in model 3 are different across all brands (they are 0.0421 for Nordica, 0.0475 for Yoplait, 0.0423 for private label, 0.0479 for Dannon, and 0.0428 for WBB), implying that R1 is relaxed. Thus, in summary, R1, R2a, and R2b seem to get relaxed for the finite mixture heterogeneity model as compared to the no-heterogeneity case (model 1).

Comparing model 3 with model 2 (normal random heterogeneity), we see, first, that while model 3 yields quantity elasticities less than one in magnitude across *all* brands, model 2 does not. This implies that model 3 does better than model 2 in relaxing restriction R2a. Second, we see a smaller dispersion of the quantity elasticities from −1 in model 3 as compared to model 2. This implies that model 3 performs worse than model 2 in its ability to relax restriction R2b. Third, model 3 yields a smaller dispersion in the values of the expected conditional budget shares across the five brands as compared to model 2. This implies that model 3 performs worse than model 2 in its ability to relax restriction R1.

The results above relate closely to our hypotheses in §3.3 on the extent to which the two specifications satisfy the properties needed to relax the restrictions. Because the finite mixture specification (model 3) is semiparametric, it is more flexible than normal random heterogeneity (model 2), which enables model 3 to yield values of the quantity elasticities of all brands less than one in magnitude (and thereby relax R2a more). However, the finite mixture specification does worse than the normal random specification in capturing the underlying heterogeneity (the BIC for model 2 is 1,807.2 versus 2,152.3 for model 3). As a result, the finite mixture heterogeneity model performs worse than the normal random heterogeneity model in its ability to relax restrictions R2b and R1.[15]

4.3.4. Model 4—Mixture Normal for Unobserved Heterogeneity. As pointed out in §3.3, because the mixture normal is a very flexible specification, we would expect it to do a good job of capturing the actual heterogeneity in the data. This is borne out—the BIC for model 4 is 1,792.1 versus 1,807.2 for model 2 and 2,152.3 for model 3. An added advantage of the flexibility of the mixture normal distribution is that the overall stochastic term will be flexible enough to enable the slope of its hazard function to take a wide range of positive or negative values. Comparing population-level elasticities for model 4 given in Table 6 with those of model 2 (normal random heterogeneity) and model 3 (finite mixture heterogeneity), we see three points of note.

First, similar to model 3 and unlike model 2, the quantity elasticities of all brands are less than one in magnitude in model 4. This implies that R2a is more relaxed for model 4 as compared to model 2. Second, similar to model 2, and unlike model 3, the quantity elasticities in model 4 are not close to one in magnitude (they range from −0.5107 to −0.8530). This implies that R2b is more relaxed for model 4 as compared to model 3. Third, similar to model 2, the expected conditional budget shares calculated at the mean values of the explanatory variables show a wide dispersion across brands (0.0387 for Nordica, 0.0484 for Yoplait, 0.0386 for private label, 0.0526 for Dannon, and 0.0424 for WBB at the mean values of the explanatory variables). This implies that R1 is relaxed for model 4.

In summary, model 4 does a better job in relaxing the three restrictions as compared to both model 2 (normal random heterogeneity) and model 3 (finite mixture heterogeneity). This stems from the fact that, unlike models 2 and 3, model 4 satisfies *both* the properties needed to relax the restriction: similar to the finite mixture specification (and unlike the normal random specification), it provides flexibility. Similar to the normal random specification (and unlike the finite mixture specification), it captures the underlying heterogeneity well.

4.3.5. Summarizing. For the case where the econometrician's errors are IID EV distributed, as we go from the no-heterogeneity case to the most flexible specification of heterogeneity, restrictions R1, R2a, and R2b each get relaxed. In particular, we find that for the no-heterogeneity case (model 1), all three restrictions hold. For the mixture normal specification of heterogeneity (model 4), all three restrictions get relaxed the most. For the other two cases—namely, normal random (model 2) and finite mixture heterogeneity (model 3)—the restrictions only get relaxed partially.

5. Implications

We compare model 4 with models 1–3 along two dimensions. First, for each model, we discuss whether the predicted hierarchy across the five brands in terms of their quantity elasticities is consistent on prior research. Following that, we compare the models in terms of their predictions on primary versus secondary demand elasticities.

5.1. Hierarchy in Quantity Elasticities Across Brands

We rank order the brands in terms of their quantity elasticities for model 4 (mixture normal heterogeneity), which relaxed the restrictions the most. The ranking is (i) −0.8530 (private label), (ii) −0.6668 (Yoplait), (iii) −0.6008 (Dannon), (iv) −0.5715 (Nordica),[16] and (v) −0.5107 (WBB).

It is natural to ask if this hierarchy is consistent with prior research. Bell et al. (1999) found that the two factors that differentiate brands on their quantity elasticities are (i) the extent of brand loyalty (the greater the loyalty, the greater the magnitude of the quantity elasticity) and (ii) price variability (the greater the price variability of a brand, the greater the quantity elasticity).[17] In Table 7, we report the brand loyalty and price variability (as percentages) across all five brands.[18] Note that the private label ranks the highest along both factors, which implies that its quantity elasticity should be the highest, which is what we get. Although Yoplait's brand loyalty is almost the same as that of private label, its price variability is low. Thus, its quantity elasticity would be expected to be lower than the private label, as our results show. Dannon and Nordica rank lower than

[15] As mentioned earlier, Song and Chintagunta (2007), who use the finite mixture specification of heterogeneity, find quantity elasticities to be close to −1 across all brands in two categories and between −0.74 to −0.9 across brands in the other two categories. This is similar to our finding that the finite mixture heterogeneity succeeds only partially in relaxing the restrictions on quantity elasticities.

[16] Note that the elasticities for Dannon and Nordica are not statistically different from each other.

[17] Note that, unlike our model, Bell et al. (1999) use a statistical approach for estimating the quantity decisions, which entails linearly regressing the quantities with respect to the marketing mix variables after correcting for the selectivity bias that results from the correlation between the errors in the quantity decision and those in the incidence and brand choice decisions.

[18] Following Bell et al. (1999), we construct the brand loyalty variable as the average number of purchases of a brand by all consumers who purchase the brand, normalized by the total number of purchases by the consumers who have purchased that brand. Price variability is the coefficient of variation of the brands price.

Table 7 Brand Loyalty and Price Variability of Brands in Yogurt Data

Brand	Brand loyalty measure (%)	Price variability construct (%)
Nordica (6 oz.)	23.33	14.56
Yoplait (6 oz.)	42.15	10.76
Private label (8 oz.)	42.88	27.70
Dannon (8 oz.)	24.40	8.07
WBB (8 oz.)	14.24	17.05

Yoplait on both factors and thus have lower quantity elasticities than Yoplait. Finally, WBB ranks the lowest on both factors and thus has the lowest quantity elasticity.

Next, we compare the hierarchy across brands in terms of their quantity elasticities as predicted by models 1–3. In model 1 (no unobserved heterogeneity), because the quantity elasticities of all brands are restricted to be very close to one in magnitude, we do not observe any hierarchy across brands. Similarly, for model 3 (finite mixture heterogeneity), we do not observe a clear hierarchy across brands because the quantity elasticities are not statistically different from each other. Finally, for model 2 (normal random heterogeneity), we see that the private label has the highest quantity elasticity (-1.0627) and WBB has the lowest (-0.7920), which is reasonable based on the two factors given in Table 8. However, the quantity elasticities for the other three brands do not follow the hierarchy based on the two factors.

5.2. Primary vs. Secondary Demand Effects

We follow Bell et al. (1999) and decompose the total elasticity into primary and secondary demand components. The primary demand elasticity component is computed as incidence elasticity plus quantity elasticity, as a percentage of the total elasticity. We report the primary demand elasticity components across all five brands for models 1–4 in Table 8. Observe that, across all brands, model 4 predicts the lowest primary demand elasticity percentage. Furthermore, averaged across all brands, we see that the primary demand component is 52.5% for model 1, 52.1% for model 2, 50.4% for model 3, and 43.6% for model 4. There are two points to note here. First, the average primary demand elasticity component for model 4 is very similar to that obtained by Bucklin et al. (1998), who used the same data set. Second, this decomposition tells us that one would overestimate the primary demand component if one did not correct for the restrictions properly.

Table 8 Elasticity Based Decomposition Across Brands as Predicted by Models 1–4

Brand	Primary demand elasticity (% of the total demand elasticity) as predicted by			
	Model 1	Model 2	Model 3	Model 4
Nordica (6 oz.)	49.69	47.40	47.59	38.34
Yoplait (6 oz.)	57.83	56.17	56.88	48.82
Private label (8 oz.)	53.25	57.54	51.15	49.07
Dannon (8 oz.)	46.12	50.95	43.30	36.41
WBB (8 oz.)	42.65	38.46	39.98	28.67

6. Conclusions

The principal objective of this paper was to formally examine the reasons behind the restricted quantity elasticity estimates obtained in the Hanemann's framework and suggest solutions for these restrictions. Our analytical results and empirical examination suggest that the specification of unobserved heterogeneity should be a crucial consideration for future researchers attempting to use Hanemann's framework to examine consumer behavior. More specifically, it is important to use a specification of heterogeneity that (i) adequately captures the true underlying heterogeneity in the data and (ii) is flexible in terms of the slope of its hazard function taking a wide range of positive and negative values. In particular, we show that the mixture normal unobserved heterogeneity does a better job in relaxing restrictions on quantity elasticities than the normal or finite mixture specifications of heterogeneity. This suggests that empirical modelers building integrated models of the incidence, choice, and quantity decisions are best served by using a mixture normal specification of unobserved heterogeneity.

We view our work as providing a foundation for a number of empirical industrial organization questions pertaining to those categories where consumers buy multiple units of an alternative on a purchase occasion. For instance, suppose one wishes to examine the change in consumer welfare and firm profits over different pricing policies, e.g., quantity discounts or bundling. To address this appropriately, one would have to model demand carefully, which would entail modeling a consumer's incidence, choice, and quantity decisions as arising from a joint utility maximization framework. Because the policies under consideration would be expected to affect the consumer's quantity choice decision, it would be crucial to assess quantity response accurately. Hanemann's framework with our proposed solution to ease the restriction on quantity elasticities is precisely suited to such an exercise. Coupled with a suitable supply-side specification, researchers can obtain a number of useful recommendations on pricing and product line policies. For instance, is a "buy one, get one" deal profitable in the short run? Would it be profitable to prune the assortment of products offered in a particular category?

Our work suffers from some limitations that could provide opportunities for further research. Perhaps most importantly, one needs to conduct an analysis similar to ours on a number of additional data sets across diverse product categories. The restrictions we have shown formally in the lemmas are independent of data, but the relative efficacies of the remedies we suggest rely on the extent of the true underlying heterogeneity in the data, suggesting the need for analysis across a variety of data sets.

7. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mktsci.pubs.informs.org/.

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