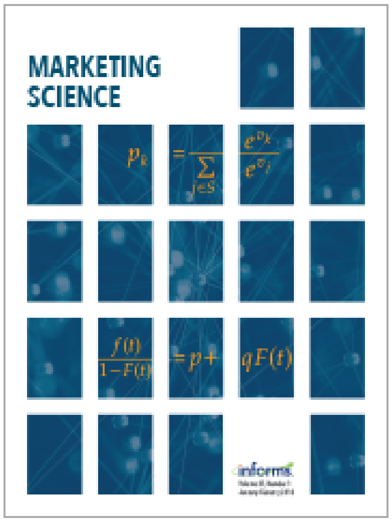

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Optimal Design of Return Policies

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Abstract. Quota-based and partial-refund return policies abound in practice between manufacturers and their resellers. While the literature has provided insights into the design of the partial-refund policy, little attention has been directed at the design of the quota-based return policy. Accordingly, this paper explores the relative preference of a quota-based policy vis-à-vis a partial-refund policy. We do this, first, in the context of risk-neutral channel partners to identify the strategic decisions of each party and the effect of demand uncertainty on the variation of their respective profits. Our results reveal that the manufacturer faces higher profit variation (between the different demand realizations) under the quota policy. The variance in profits for the reseller is, however, higher under the partial-refund policy. We explain the source of profit variations by comparing it across different channel structures (centralized and decentralized). Next, we formally extend the model to include a disutility associated with profit variation and show that when the manufacturer has a variation-induced disutility, the partial-refund contract should be used, as it is the dominating contract. Similarly, when the retailer has a variation-induced disutility, the quota contract should be used. This is consistent with the pattern of profit variations in the risk-neutral case where the manufacturer has lower variation with the partial-refund contract while the reseller has lower variation with the quota contract. Finally, our analysis also shows how the manufacturer may employ a combination policy to better manage its own profit variation while providing adequate overstocking protection for the reseller.

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1. Introduction

It is common for retailers to accept consumer returns for a refund, with or without a restocking fee (Davis et al. 1995, 1998; Hart 1988; Moorthy and Srinivasan 1995). A number of factors contribute to the existence of such returns, including money-back guarantees, consumers returning defective items under warranty for replacement or repair, etc. (for details on other factors, see, e.g., Shulman et al. 2009, 2010, 2011; Yin et al. 2010). The focus of our paper, however, is on studying channel returns accepted by a supplier to induce resellers facing uncertain demand to stock appropriately. In 2010, retail returns in the United States amounted to $194 billion, almost 8% of the estimated $2.45 trillion in sales (The Retail Equation 2010), and were largely responsible for the increased logistics costs to channel partners (Nawotka 2008, Neary 2008, and Gumus et al. 2013). To counter misuse and to control costs, suppliers are placing different types of restrictions on the return of unsold inventory by their channel partners (Longo 1995, Netgear 2006, Chen and Krakovsky 2010, Eaton 2014). As stringent return policies may adversely affect resellers' stocking quantities and, consequently, consumer demand, the manufacturer (or supplier) needs to carefully select its policies to maximize profit.

The goal of this paper is to develop a stylized model to characterize the relative preferences of manufacturers and resellers for three commonly observed return policies. We do this, first, in the context of risk-neutral channel partners to identify the strategic decisions of each party along with the effect of demand uncertainty on the variation of their respective profits. Later, we extend the model to study the setting where the manufacturer and reseller experience a variation-induced disutility; here, their optimal decisions are made not only to maximize profits, but to simultaneously minimize the disutility caused by the variation in profit induced by market uncertainty. Our analytical findings help explain the observed (anecdotal) preferences of manufacturers and resellers for the different types of return policies. In this paper, our focus is on the following return policies: The first, referred to as the *partial-refund* policy, allows for an unlimited quantity of returns by the reseller at a price less than the wholesale price charged by the manufacturer.[1] The second, referred to as the *quota* or *stock-rotation* policy, allows for a limited quantity of returns for a full refund of the wholesale price. Finally, in an extension, we study the *combination* policy, which includes a limited quantity of returns for full refund followed by unlimited returns at a reduced refund price.

The academic literature has examined the use of the partial-refund policy in various industries such as new books, software, fashion wear, and winter clothing—see, for example, Pasternack (1985), Padmanabhan and Png (1997), Emmons and Gilbert (1998), and Gurnani et al. (2010) for additional details. The quota (or stock-rotation) policy is often seen in practice, and there are a number of examples in the trade press. For instance, Cisco allows its distributors to claim stock rotation once every quarter, which is limited to a certain percentage of the total dollar value of purchase (Goh 2006). Until a few years ago, Hewlett-Packard offered a stock-rotation option to its distributors, but it recently discontinued the practice (Chen and Krakovsky 2010). In another example, communications product manufacturer Netgear offers a 10% stock-rotation allowance to its distributors (Netgear 2006). The combination policy of using a partial refund along with an initial quota is also observed in practice. Eaton, a diversified power management company, uses the combination stock-rotation policy as part of its inventory planning process. Eaton's Distributor Commitments Program rewards its distributors with varying levels of quota (with full refund) followed by a restocking fee for any excess returns (Eaton 2014).

While there is widespread use of the quota and combination policies, the academic literature has explored neither the optimal design of such policies nor the differential profit implications for the manufacturer and its reseller.[2] Our paper aims to address this gap in the literature by examining the following research questions:

- How does the optimal structure of the quota policy compare with the partial-refund policy in terms of the strategic decisions made by each party as well as their profits and the variation in these profits under market demand uncertainty?
- How are the profit variations affected by different channel structures (centralized and decentralized systems)?
- What is the dominating policy (quota or partial refund) when each party incurs a disutility induced by variation in profits?
- What role does the combination policy play in influencing the preferences of the manufacturer and the reseller?

We first derive the optimal structure of the quota policy for risk-neutral players and compare it with that of the partial-refund policy. We show that under the quota policy, while the reseller is protected from incurring the cost of carrying excess inventory (up to the quota), the manufacturer charges a higher wholesale price—relative to that under the partial-refund policy—to offset its potential cost of accepting returns. This results in two opposing effects on the order quantity: (1) the cost protection under the quota induces the reseller to order more stock, but (2) the higher wholesale price exerts a downward pressure on the order quantity. The manufacturer aims to balance these effects when designing the optimal quota policy. By contrast, under the partial-refund policy, the manufacturer and the reseller share the cost of unsold inventory. In this case, the manufacturer's wholesale price is lower than that under the quota contract. A key observation emerges upon comparing the two return policies: the variation in profit for the reseller is *diametrically opposite* to that of the manufacturer; this pattern may ultimately influence the preference for a certain policy when each player faces a disutility due to profit variation.

Intuitively, the use of a higher wholesale price under the quota policy—in comparison to the partial-refund policy—allows the manufacturer to earn a larger profit if the market happens to be in the higher demand state, but results in a smaller profit in the lower state because of paying out the full refund for the returned inventory. This leads to a *higher variation* in profit for the manufacturer. The reverse pattern is noted for the reseller; that is, its variation in profit is lower under the quota policy. The partial-refund policy has the opposite ordering in profit variation compared to the quota policy. Here, the manufacturer's (reseller's) profit variation is lower (higher) compared to the quota contract.

We also examine the source of profit variations as a function of the channel structure.[3] The effects identified above arise in a decentralized channel, wherein the two parties are independent and make decisions based on their self-interest. Accordingly, prices and quantity are set subject to double marginalization (Armstrong 2006). A comparison between this decentralized channel and a centralized one, wherein the manufacturer and the reseller are integrated, reveals how double marginalization affects the profit variation. Specifically, in the centralized channel, double marginalization is absent, resulting in higher quantity and lower prices; however, the reduction in prices—in comparison with the decentralized system—is not as steep as the increase in quantity. As such, the variation in channel profits between the demand states is further magnified in the centralized system.

Next, we formally include a variation-induced disutility for the players into our model. When players are averse to fluctuation in their profit under different demand states, a higher profit variation becomes less desirable. In an effort to control the variation in its profits, such a manufacturer can reduce the buyback price and quota when using the partial-refund and quota policies, respectively. Both decisions are eventually costly to the manufacturer, as the reseller will reduce the order quantity in response. However, the magnitude of reduction in the buyback price under a partial-refund policy is relatively less than that of the reduction in the quota because the variation in the manufacturer's profits is lower under the former policy. In other words, the partial-refund policy is more efficient than the quota policy in managing profit variation and can arise as the dominating contract when the manufacturer faces a variation-induced disutility. Conversely, when the reseller incurs a variation-induced disutility, the quota policy—which exhibits a lower variation in the reseller's profit—is more efficient. These preferences are shown to be *consistent* with the anecdotal evidence in practice.

Finally, in an extension, we illustrate how the optimal structure for the combination policy enables suitable sharing of costs associated with unsold inventory between the two players (for expositional ease, we do so using the setting with no disutility for profit variation). The manufacturer can increase (decrease) its share by raising (lowering) the quota, and more importantly, by doing so, can effectively control the variation in profit between the different demand realizations for both players. Overall, our analytical findings are consistent with the anecdotal evidence that manufacturers offer a higher quota to their most preferred resellers, indicating the resellers' preference for the use of quotas. By contrast, the nonpreferred resellers get a lower quota, indicating the manufacturer's preference to limit the use of quota, when possible. Furthermore, our analysis shows that the optimal design of the combination policy allows the manufacturer to manage its own profit variation by adjusting the quota allowance while providing adequate overstocking protection to the resellers.

The rest of this paper is organized as follows. Section 2 discusses the relevant literature, and Section 3 outlines our basic model. The results of the risk-neutral case and the analysis of profit variation under alternative channel structures are discussed in Sections 4 and 5, respectively. Section 6 investigates the optimality of the quota and partial-refund policies when each channel member experiences a disutility from market uncertainty induced profit variation. The penultimate section focuses on the model extension with a combination contract, and Section 8 wraps up this paper. All relevant proofs are in the appendix as well as in the online appendix.

2. Related Literature

A vast literature on consumer returns in the business-to-consumer context has focused on the design of return policies by retailers (both with and without retail competition) to better educate consumers about the fit and valuation for their products. We refer to papers by Shulman et al. (2009, 2010, 2011) and references therein for examples. This stream of research also studies how accepting returns from consumers may help retailers influence demand for their product by signaling product quality (Moorthy and Srinivasan 1995), screening for consumers with higher valuations and willingness to pay (Che 1996), or increasing long-term value through increased future purchases (Petersen and Kumar 2009). At the same time, accepting returns is costly to the firms, and this cost could be magnified by consumers' opportunistic behavior (e.g., abusive returns). This moral hazard problem has been studied by Hess et al. (1996), Chu et al. (1998), and Davis et al. (1998), among others.

Our paper, by contrast, studies the use of return policies in the business-to-business context; returns in this context are referred to as channel returns. The primary motivation for manufacturers to use return policies is to share the cost of demand–inventory mismatch with their resellers through a guaranteed buyback price if the resellers have unsold inventory.[4] Researchers have examined the use of return policies by manufacturers to elicit market information from channel partners (Arya and Mittendorf 2004). Pasternack (1985), for example, shows that a partial-refund policy (i.e., one with no restrictions on quantity of returns but with a restocking fee) can coordinate the channel when the resale prices are exogenous.[5] Returns contracts have also been studied in the presence of retail competition (Padmanabhan and Png 1997, Wang 2004) and manufacturer competition (Lan et al. 2013). As noted in Section 1, different types of returns policies are commonly used in practice, including the partial-refund policy and the stock-rotation policy. Gurnani et al. (2010) show that the partial-refund policy is optimal for a manufacturer when demand for the product is variable, and strictly dominates the no returns and full returns policies (examined in Padmanabhan and Png 1997) for the manufacturer. Both of the above papers include flexible pricing by the retailer in response to uncertain demand, whereas Emmons and Gilbert (1998) demonstrate the optimality of the partial-refund policy when fixed pricing is adopted by retailers, as is the case with catalog style goods.

While the academic literature has studied the use of the partial-refund policy in some detail, the design of optimal stock-rotation contract has received limited attention. Moreover, while these policies can be used individually, there is also evidence that a combination approach has been adopted by certain firms (see Table 1).

Accordingly, our paper makes the following contributions: (1) It derives the optimal policy structures and identifies the players' strategic decisions, along

Table 1. Examples of Return Policies

Return policy	Description	Related papers
Partial	All unsold stock can be returned with a restocking fee	Pasternack (1985), Padmanabhan and Png (1997), Emmons and Gilbert (1998), Gurnani et al. (2010)
Quota (also referred to as stock rotation)	A limited amount of excess stock can be returned for full refund	Trade articles: Cisco (Goh 2006), Netgear (Netgear 2006) This paper: Design of optimal quota policy
Combination	A limited amount of excess stock receives full refund and the remaining can be returned with a restocking fee	Trade article on Eaton Corporation (Eaton 2014) This paper: Design of optimal combination returns policy

with the effects of each policy on profits of channel members and their variations across demand states. (2) Traditionally, the literature has focused on comparing expected profits for the players to establish the preference for a certain return policy. We build on this by formally extending the model to account for the disutility incurred by channel members when facing a variation in their profits. This enables us to explain the observed preferences of the channel partners for different returns policies.

3. The Model

A manufacturer, M, distributes its products via a reseller, R, in a given market where consumer demand is uncertain. The terms of a general contractual agreement between M and R can include three components: (i) a wholesale price w, at which R buys stock from M, (ii) a fraction γ of the amount of stock bought by R that can be returned to M at the end of the selling cycle for a full refund of w per unit, and (iii) an amount b ($b < w$) that M refunds for each unit of unsold inventory over the γ fraction of the stock ordered from M. For example, suppose R bought q units of stock but is left with an unsold inventory of q_o, which is returned to M at the end of the selling cycle. If $q_o \le \gamma q$, then R will receive wq_o in refund from M; if $q_o > \gamma q$, then R receives a full refund on the γq units but only a partial refund on the remaining $(q_o - \gamma q)$ units—so the total refund here is $w\gamma q + b(q_o - \gamma q)$. For brevity, we will employ this (w, γ, b) contract—referred to as the combination (C) contract—to specify our model. Initially, however, our focus in this paper is on two specific cases: $(w, \gamma, b = 0)$ and $(w, \gamma = 0, b)$, which are referred to as the quota (Q) and partial-refund (P) contracts, respectively. Later, in Section 7, we analyze the combination contract.

The potential market size for M's product is captured by the random parameter $\alpha \in \{\alpha^h, \alpha^l\}$, $0 < \alpha^l < \alpha^h$, with α^h and α^l arising with probabilities λ and $1 - \lambda$, respectively. We refer to the realization of α^h (α^l) as the high (low) state of the market. Consistent with the model setting used extensively in the marketing literature (see Padmanabhan and Png 1997, Wang 2004, Gurnani et al. 2010), for a realization of α, the market demand for M's product is

$$D(p;\alpha) = \alpha - \beta p, \tag{1}$$

where β reflects consumers' sensitivity to the retail price p.[6] In the remainder of this section, we complete the description of the model with risk-neutral players, who make decisions to maximize their respective expected profits. Later, in Section 6, we analyze the setting wherein M and R experience a disutility from market uncertainty induced variation in profits. Finally, M is assumed to incur a constant marginal cost of production c.

The sequence of events is as follows: In Stage 1, M specifies the terms of the contract, that is, $\{w, \gamma, b\}$. (The Q and P contracts specify $\{w, \gamma\}$ and $\{w, b\}$, respectively.) Next, in Stage 2, R orders stock, q, from M. Finally, in Stage 3, demand uncertainty is resolved with α's value becoming known; subsequently, R sets p and consumer demand is realized according to (1). The selling season then ends with any unsold inventory returned to M, and profits are realized.

In Stage 3, for a given realization of α and contract terms $\{w, \gamma, b\}$, along with the stock level q already ordered, R sets the retail price to maximize its total revenue $V(p;\alpha)$, which includes both the cash flow from selling the product and any refund from M on the unsold inventory as follows:

$$V(p;\alpha) = \begin{cases} pD(p;\alpha) + w[q - D(p;\alpha)], & \text{if } (1-\gamma)q \le D(p;\alpha) \le q, \\ pD(p;\alpha) + w\gamma q \\ \quad + b[(1-\gamma)q - D(p;\alpha)], & \text{if } 0 < D(p;\alpha) \le (1-\gamma)q. \end{cases} \tag{2}$$

Let the optimal retail price be $p^*(q;\alpha) \stackrel{\text{def}}{=} \arg\max_p V(p;\alpha)$; the corresponding optimal revenue is given by $V^*(q;\alpha) \stackrel{\text{def}}{=} V(p^*(q;\alpha);\alpha)$. Next, in Stage 2, when uncertainty regarding α is yet to be resolved, R's expected profit is $\mathrm{E}[\Pi_R(q;w,\gamma,b)] \stackrel{\text{def}}{=} \lambda V^*(q;\alpha^h) + (1-\lambda)V^*(q;\alpha^l) - qw$. To maximize this expected profit, R sets the order quantity at the optimal level, denoted by $q^*(w,\gamma,b) \stackrel{\text{def}}{=} \arg\max_q \mathrm{E}[\Pi_R(q;w,\gamma,b)]$. Note that both $p^*(\cdot)$ and $V^*(\cdot)$ are functions of the ordered quantity q with $\alpha \in \{\alpha^h, \alpha^l\}$ being a parameter, and the optimal order quantity depends on the terms of the contract (i.e., w, γ, and b), which are set by M in Stage 1.

Let $\Pi_M^i(w,\gamma,b)$ be M's profit when the market is in state i ($i = h, l$). We show in the appendix that when $\alpha = \alpha^h$, R will optimally clear $q^*(\cdot)$, and accordingly,

M's profit is given by $\Pi_M^h(\cdot) = (w - c)q^*(w, \gamma, b)$. By contrast, if the low state is realized (i.e., $\alpha = \alpha^l$), then R will end up with unsold inventory. If this unsold inventory is less than or equal to the quota, it can be returned to M for a full refund; otherwise, if it is more than the quota, M refunds the wholesale price w for the amount up to the quota and pays the buyback price b for the remainder. Denote the sales that R optimally generates in the low state by $D^*(w, \gamma, b; \alpha^l) \stackrel{\text{def}}{=} D(p^*(q^*(w, \gamma, b); \alpha^l); \alpha^l)$. The profit that M can earn in the low state is equal to

$$\Pi_M^l(\cdot) = \begin{cases} (w-c)q^*(\cdot) - w[q^*(\cdot) - D^*(\cdot;\alpha^l)], \\ \quad \text{if } (1-\gamma)q^*(\cdot) \le D^*(\cdot;\alpha^l) \le q^*(\cdot), \\ (w-c)q^*(\cdot) - w\gamma q^*(\cdot) - b[(1-\gamma)q^*(\cdot) - D^*(\cdot;\alpha^l)], \\ \quad \text{if } 0 < D^*(\cdot;\alpha^l) \le (1-\gamma)q^*(\cdot). \end{cases} \tag{3}$$

Here M's expected profit is given by $\mathrm{E}[\Pi_M(w, \gamma, b)] \stackrel{\text{def}}{=} \lambda\Pi_M^h(\cdot) + (1-\lambda)\Pi_M^l(\cdot)$. The terms of the contract are chosen to maximize this profit. In the following analysis, we define $\bar{\alpha} \stackrel{\text{def}}{=} \lambda\alpha^h + (1-\lambda)\alpha^l$ and $\bar{\alpha}^2 \stackrel{\text{def}}{=} \lambda{\alpha^h}^2 + (1-\lambda){\alpha^l}^2$, and assume that (1) $\lambda(\alpha^h - \alpha^l) \ge \beta c$ and (2) $\lambda\bar{\alpha}^2 \ge \beta^2 c^2$. The former condition implies sufficient variation in market demand between the high and low states to warrant the existence of returns (in the low state; see the appendix), while the latter ensures regularity in players' decision variables (i.e., nonnegative wholesale price; see Table 2).

4. Optimal Contracts: Partial-Refund vs. Quota

In this section, we derive the terms of the partial-refund and quota contracts when both M and R maximize their respective expected profits accordingly. The following lemma describes the optimal partial-refund contract.

Lemma 1. *Under a partial-refund contract, M sets the wholesale price at $w_P^* = (\bar{\alpha} + \beta c)/(2\beta)$ and refunds any (and all) unsold inventory at a buyback price $b_P^* = \alpha^l/(2\beta)$ per unit.*

This result is consistent with Gurnani et al. (2010), whose model is developed specifically for the partial-refund policy only. Prices, order quantity, and amount of returns are summarized in Table 2. A partial-refund policy allows the reseller to return all unsold inventory but does not provide a full refund of the wholesale price. This implies that—in contrast to the quota policy that will be discussed subsequently—both the manufacturer and reseller *share* the cost of unsold inventory, which arises in the low state. (All of the inventory will be cleared in the high state.) This shared responsibility—and the lack thereof under the quota contract—is the driving factor in R's choice of how much quantity of stock to order and the subsequent retail prices. When M accepts returns with a quota, the terms of the contract are as summarized in Lemma 2.

Table 2. The Optimal Contracts and Reseller's Pricing/Stocking Decisions

	Quota ($j = Q$)	Partial-refund ($j = P$)	Comparison
w_j^*	$\dfrac{\lambda\bar{\alpha}^2 - \beta^2c^2}{2\lambda\beta(\bar{\alpha} - \beta c)}$	$\dfrac{\bar{\alpha} + \beta c}{2\beta}$	$Q > P$
γ_j^*	$1 - \dfrac{\lambda\alpha^l}{\lambda\alpha^h - \beta c}$	—	—
b_j^*	—	$\dfrac{\alpha^l}{2\beta}$	—
q_j^*	$\dfrac{\lambda\alpha^h - \beta c}{4\lambda}$	$\dfrac{\lambda\alpha^h - \beta c}{4\lambda}$	$Q = P$
High state			
p_j^h	$\dfrac{1}{4}\left(\dfrac{3\alpha^h}{\beta} + \dfrac{c}{\lambda}\right)$	$\dfrac{1}{4}\left(\dfrac{3\alpha^h}{\beta} + \dfrac{c}{\lambda}\right)$	$Q = P$
Returns	No returns	No returns	
Low state			
p_j^l	$\dfrac{3\alpha^l}{4\beta}$	$\dfrac{3\alpha^l}{4\beta}$	$Q = P$
Return quantity			
Full refund	$\dfrac{\lambda(\alpha^h - \alpha^l) - \beta c}{4\lambda}$	—	—
Partial refund	—	$\dfrac{\lambda(\alpha^h - \alpha^l) - \beta c}{4\lambda}$	—

Lemma 2. *Under the quota contract, M sets the wholesale price at $w_Q^* = (\lambda\bar{\alpha}^2 - \beta^2c^2)/(2\lambda\beta(\bar{\alpha} - \beta c))$ and allows R to return excess stock, up to a fraction $\gamma_Q^* = 1 - \lambda\alpha^l/(\lambda\alpha^h - \beta c)$ of the ordered quantity for a full refund of w_Q^*.*

See Table 2 for the prices, order quantity, and returned stock under the quota contract. This contract provides an insurance with *full coverage* up to the set quota, and the reseller takes full advantage of the policy in the sense that the excess stock under a low demand condition exactly matches the quota. For such a protection, however, the retailer pays a relatively high wholesale price (compared to that under the partial-refund contract, that is, $w_Q^* > w_P^*$; see Table 2).

Note that while the cost protection provides an incentive for the reseller to order more stock, the higher wholesale price puts an upward pressure on the retail price, potentially reducing sales and eventually limiting the quantity of ordered stock. By contrast, under the partial-refund policy, the reseller has to share the cost of excess inventory—and, consequently, has less incentive to order more stock because of the potential cost of unsold inventory—but benefits from a lower wholesale price, which allows for a higher profit margin. When both players are unconcerned about the variation in their profit, these two forces balance each other out in our model setting, and in equilibrium, the quantity of ordered stock is identical under both contracts.

In addition, the retail prices are also identical under both contracts. This result is straightforward in the high state, when the entire stock is cleared. However, in the low state, it is not optimal to clear the entire stock, and the reseller's pricing decision is affected by the excess inventory that will remain. Under the Q contract, this amount is optimally equal to the quota. Under partial refund, the excess inventory will be costly to the reseller, and the buyback price determines the magnitude of that cost. However, the wholesale price is relatively low (compared to that of the quota contract), allowing the reseller to earn a higher profit margin, which helps cover part of the cost of returns. These two opposing effects counterbalance each other, and the reseller sets identical retail prices under both contracts—resulting in an identical amount of returns—in the low state.[7] Table 3 provides the detailed comparison of the two contracts.

Now, we can compare the spread in profit of each player—that is, the variations in the profits earned in the high and low states—under the two contracts. Denote the variations in the profits of R and M by $\Delta_{Rj} \stackrel{\text{def}}{=} \Pi_{Rj}^{h*} - \Pi_{Rj}^{l*}$ and $\Delta_{Mj} \stackrel{\text{def}}{=} \Pi_{Mj}^{h*} - \Pi_{Mj}^{l*}$, respectively, where $j \in \{P, Q\}$ is the type of returns contract. We have $\Delta_{RQ} - \Delta_{RP} = -((\lambda\alpha^h - \beta c)[\lambda^2(\alpha^h - \alpha^l)^2 - \beta^2 c^2])/(8\lambda^2\beta \cdot (\bar{\alpha} - \beta c)) < 0$ and $\Delta_{MQ} - \Delta_{MP} = ((\lambda\alpha^h - \beta c)[\lambda^2(\alpha^h - \alpha^l)^2 - \beta^2 c^2])/(8\lambda^2\beta(\bar{\alpha} - \beta c)) > 0$. This result is formalized in Proposition 1.

Proposition 1. *Variation in profits of the manufacturer (between the high and low states) is larger under the quota contract than under the partial-refund contract. By contrast, profits of the reseller vary less between the high and low states under the quota contract than under the partial-refund contract.*

To understand the intuition underlying this result, consider the benefits and costs of returns to each party under different realizations of the market. It is clear that carrying excess inventory is costly and the responsibility for this cost depends on how returns are accepted by the manufacturer. More importantly, the manner in which the two parties share this responsibility affects the optimal wholesale price and the resulting profit. Under the Q contract, M fully covers the cost of excess inventory, up to the allowed amount, and eventually incurs a marginal loss of c per each returned unit. To offset this cost, it raises the wholesale price. This allows M to earn higher profit in the high state—when the entire stock is cleared and no return occurs—but lower profit in the low state because of the responsibility to pay the full refund for returns (up to the quota). The opposite effect arises regarding the reseller's profits. It earns lower profit in the high state under the quota policy (compared to the partial-refund policy) because of paying the higher wholesale price, and higher profit in the low state because of not incurring the cost of excess inventory. Under this contract, the manufacturer's full responsibility for returns and incentive to raise the wholesale price are the underlying forces that drive up (down) the manufacturer's (reseller's) profit variation between the high and low states.

By contrast, under the P contract, R is responsible for the cost of carrying excess inventory and incurs a marginal loss of $w - b$ per each unsold item. This requires the manufacturer—who still earns a positive marginal profit of $w - b - c$ per each returned unit—to reduce the wholesale price and induce the reseller to order the optimal quantity of stock. As such, the manufacturer earns a lower profit in the high state (because of the lower wholesale price) but a higher profit in the low state (because of the additional profit earned from the returned inventory). Here, too, the reverse is true for the reseller's profits. The lower wholesale price allows it to earn higher profit in the high state but the cost of carrying excess inventory reduces its profit in the low state. In other words, the partial-refund policy is accommodated by a lower wholesale price but puts the responsibility for carrying excess inventory on the reseller, resulting in lower (higher) variation in profits for the manufacturer (reseller). In summary, the spread in profits of the manufacturer is higher under the quota policy, and vice versa, for the reseller (see Table 3).

The forces identified above arise in a decentralized channel, wherein the two parties are independent and make their respective decisions based on their self-interest. To better understand the nature of profit variation—specifically, how the interaction between channel members may affect the magnitude of profit variation—in Section 5, we extend the analysis to compare this result to that of a centralized channel, in which the manufacturer owns the reseller and sells directly to consumers.

5. Profit Variation Under Different Channel Structures

In the following, we derive the optimal prices and quantity of the centralized channel, referred to as CC, and compare the profits of such a channel in the high and low states as well as the associated profit variation—denoted by Π_{CC}^h, Π_{CC}^l, and $\Delta_{CC} \stackrel{\text{def}}{=} \Pi_{CC}^h - \Pi_{CC}^l$, respectively—to those of the decentralized channel using the quota and partial-refund contracts, which are referred to as DQ (decentralized quota) and DP (decentralized partial), respectively. The decentralized channel's profits in the high and low states are derived from the previous analysis as $\Pi_j^i = \Pi_{Mj}^i + \Pi_{Rj}^i$ (where $i \in \{h, l\}$ and $j \in \{DQ, DP\}$), and the variations in profits are given by $\Delta_j \stackrel{\text{def}}{=} \Pi_j^h - \Pi_j^l$.

In the centralized channel, the manufacturer is the sole decision maker who decides on the quantity to

Table 3. Reseller's and Manufacturer's Profits

	Quota ($j = Q$)	Partial refund ($j = P$)	Comparison
High state			
Π^{h*}_{Rj}	$\frac{(\lambda\alpha^h - \beta c)\{\lambda[\bar{\alpha}^2 + 3(1-\lambda)(\alpha^h - \alpha^l)\alpha^l] + (\bar{\alpha} - 3\lambda\alpha^h)\beta c + \beta^2 c^2\}}{16\lambda^2\beta(\bar{\alpha} - \beta c)}$	$\frac{(\lambda\alpha^h - \beta c)\{\lambda[\bar{\alpha} + 3(1-\lambda)(\alpha^h - \alpha^l)] - (2\lambda - 1)\beta c\}}{16\lambda^2\beta}$	$P > Q$
Π^{h*}_{Mj}	$\frac{(\lambda\alpha^h - \beta c)[\lambda\bar{\alpha}^2 - 2\lambda\bar{\alpha}\beta c + (2\lambda - 1)\beta^2 c^2]}{8\lambda^2\beta(\bar{\alpha} - \beta c)}$	$\frac{(\lambda\alpha^h - \beta c)(\bar{\alpha} - \beta c)}{8\lambda\beta}$	$Q > P$
Low state			
Π^{l*}_{Rj}	$\frac{\alpha^{l2}}{16\beta} - \frac{\alpha^l(\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) + \beta c]}{8\lambda\beta(\bar{\alpha} - \beta c)}$	$\frac{\alpha^{l2}}{16\beta} - \frac{(\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) + \beta c]}{8\lambda\beta}$	$Q > P$
Π^{l*}_{Mj}	$\frac{\alpha^{l2}}{8\beta} + \frac{(\alpha^l - 2\beta c)(\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) - \beta c]}{8\lambda\beta(\bar{\alpha} - \beta c)}$	$\frac{\alpha^{l2}}{8\beta} + \frac{(\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) - \beta c]}{8\lambda\beta}$	$P > Q$
Expected profits			
$\mathrm{E}[\Pi^*_{Rj}]$	$\frac{(\lambda\alpha^h - \beta c)^2}{16\lambda\beta} + \frac{(1-\lambda)\alpha^{l2}}{16\beta}$	$\frac{(\lambda\alpha^h - \beta c)^2}{16\lambda\beta} + \frac{(1-\lambda)\alpha^{l2}}{16\beta}$	$P = Q$
$\mathrm{E}[\Pi^*_{Mj}]$	$\frac{(\lambda\alpha^h - \beta c)^2}{8\lambda\beta} + \frac{(1-\lambda)\alpha^{l2}}{8\beta}$	$\frac{(\lambda\alpha^h - \beta c)^2}{8\lambda\beta} + \frac{(1-\lambda)\alpha^{l2}}{8\beta}$	$P = Q$
Variation in profits			
$\Pi^{h*}_{Rj} - \Pi^{l*}_{Rj}$	From above	From above	$P > Q$
$\Pi^{h*}_{Mj} - \Pi^{l*}_{Mj}$	From above	From above	$Q > P$

produce in Stage 1 under market uncertainty and sets the respective retail prices in Stage 2 after observing the realization of α. It earns a profit based on the revenue generated by the consumer demand and incurs the cost of unsold inventory, if any. All parameters, including those characterizing market demand under uncertainty and manufacturing cost, remain the same as in the previous analysis. Using backward induction, we first solve the optimization problem in Stage 2 to determine the optimal retail prices, and subsequently derive the optimal quantity chosen in Stage 1. See Table 4 for the optimal retail prices, quantity, profits, and variation in the centralized channel. All derivations are provided in the appendix.

Now, we can compare the profits and their variation between the high and low states of the centralized channel with those of the decentralized channel. This comparison (summarized in Table 4) is formalized in the following lemma.

Lemma 3. *Variation in the channel profit between the high and low states of the centralized channel is higher than that of the decentralized channel under both quota and partial-refund policies.*

Note that in the decentralized system, the retailer's decisions regarding quantity and retail prices are made subject to double marginalization, which exerts a downward pressure on the quantity and an upward pressure on the retail prices. However, the magnitude of these effects differs depending on the state of the market. Specifically, in the high state, the reduction in quantity is larger (compared to the centralized system) than the increase in the retail price. (From Table 4, quantity is reduced by double, $(q_{DQ} - q_{CC})/q_{CC} = \frac{1}{2}$, while the retail price is increased by less than double, $(p^h_{DQ} - p^h_{CC})/p^h_{CC} = \frac{1}{2} - \beta c/(\lambda\alpha^h + \beta c)$.) Meanwhile, in the low state, the retail price is increased at the same rate as the quantity (i.e., by double, as seen by $(p^l_{DQ} - p^l_{CC})/p^l_{CC} = \frac{1}{2}$). As a result, channel profit is reduced because of double marginalization at a higher rate in the high state than in the low state. These forces effectively reduce the variation in channel profits of the decentralized system between the high and low demand states in comparison with the centralized system. In other words, double marginalization—and the lack thereof in the centralized system—exerts a moderating effect on the relationship between market uncertainty and the variation of the channel's profit.[8]

In summary, market uncertainty exerts a direct effect on channel members' decisions regarding quantity and prices, and results in profit variation between the high and low demand states. Furthermore, this effect is moderated by double marginalization, which arises when channel members are independent and make decisions based on their self-interest.

6. Variation-Induced Disutility and Contract Preferences

In an uncertain market, a risk-neutral player—which can be either the manufacturer or the reseller in our model—focuses solely on the expected profit and is unconcerned about the variation associated with such profit. However, players may be concerned with both

Table 4. Profits and Variations Under Various Channel Structures

	Centralized channel	Decentralized channel	
	CC	*DQ* (Quota)	*DP* (Partial)
w	n/a	$\frac{\lambda\bar{\alpha}^2-\beta^2c^2}{2\lambda\beta(\bar{\alpha}-\beta c)}$	$\frac{\bar{\alpha}+\beta c}{2\beta}$
γ, b	n/a	$\gamma_{DQ}=\gamma_{CQ}$	$b_{DP}=\frac{\alpha^l}{2\beta}$
q	$\frac{\lambda\alpha^h-\beta c}{2\lambda}$	$q_{DQ}=q_{DP}=\frac{1}{2}q_{CC}$	
p^h	$\frac{\lambda\alpha^h+\beta c}{2\lambda\beta}$	$p^h_{DQ}=p^h_{DP}=\frac{1}{4}\left(\frac{3\alpha^h}{\beta}+\frac{c}{\lambda}\right)$	
p^l	$\frac{\alpha^l}{2\beta}$	$p^l_{DQ}=p^l_{DP}=\frac{3\alpha^l}{4\beta}$	
Π^h	$\Pi^h_{CC}=\frac{(\lambda\alpha^h-\beta c)[\lambda(\alpha^h-\beta c)+(1-\lambda)\beta c]}{4\lambda^2\beta}$	$\Pi^h_{DQ}=\Pi^h_{DP}=\frac{(\lambda\alpha^h-\beta c)[3\lambda(\alpha^h-\beta c)+(1-\lambda)\beta c]}{16\lambda^2\beta}$	
Π^l	$\Pi^l_{CC}=\frac{1}{4}\left(-2\alpha^h c+\frac{\alpha^{l2}}{\beta}+\frac{2\beta c^2}{\lambda}\right)$	$\Pi^l_{DQ}=\Pi^l_{DP}=\frac{1}{16}\left(-4\alpha^h c+\frac{\alpha^{l2}}{\beta}+\frac{4\beta c^2}{\lambda}\right)$	
Δ	$\Delta_{CC}\overset{\text{def}}{=}\Pi^h_{CC}-\Pi^l_{CC}$	$\Delta_{DQ}\overset{\text{def}}{=}\Pi^h_{DQ}-\Pi^l_{DQ}$, $\Delta_{DP}\overset{\text{def}}{=}\Pi^h_{DP}-\Pi^l_{DP}$	
Comparison	$\Pi^h_{CC}>\Pi^h_{DQ}=\Pi^h_{DP}$ $\Pi^l_{CC}\lesseqgtr\Pi^l_{DQ}=\Pi^l_{DP}$ $\Delta_{CC}>\Delta_{DQ}=\Delta_{DP}$	(Centralized > decentralized variation)	

the expected value and the variation in profit, with uncertainty imposing a loss on their utility. Essentially, such a player's utility is less than the expected value of its profit.

In this context, the literature on decision making (e.g., Cyert and DeGroot 1987; see also Mantrala et al. 1997 for a marketing application) highlights how a "loss function" may be employed by firms to account for the uncertainty associated with possible outcomes. Specifically, for an uncertain outcome x and an associated target $\hat{x}$, the firm's objective function can include an absolute loss term $\alpha|x-\hat{x}|$ to account for the disutility due to uncertainty (see Granger 2002, Equation (5), wherein $\theta=1$, and Bollerslev and Mikkelsen 1996, Equation (10)). Accordingly, in the following analysis, we assume that the manufacturer makes its decisions based on an adjusted expected profit that accounts for the disutility/loss induced by the variation in profit between the high and low states of market demand. We refer to this as the variation-adjusted expected profit, denote it by $E^v[\Pi_M(\cdot)]$, and calculate it as follows.

Let r^h_M and r^l_M be the coefficients of the manufacturer's loss function when the realized profits in the high and low states—that is, $\Pi^h_M(\cdot)$ and $\Pi^l_M(\cdot)$—deviate from their expected value: $E[\Pi_M(\cdot)]=\lambda\Pi^h_M(\cdot)+(1-\lambda)\Pi^l_M(\cdot)$, respectively. Specifically, the loss terms in the high and low states are $r^h_M|\Pi^h_M(\cdot)-E[\Pi_M(\cdot)]|$ and $r^l_M|\Pi^l_M(\cdot)-E[\Pi_M(\cdot)]|$, respectively. Then the variation-adjusted expected profit is

$$E^v[\Pi_M(\cdot)]\overset{\text{def}}{=}E[\Pi_M(\cdot)]-\lambda r^h_M|\Pi^h_M(\cdot)-E[\Pi_M(\cdot)]| - (1-\lambda)r^l_M|\Pi^l_M(\cdot)-E[\Pi_M(\cdot)]|, \quad (4)$$

which can be further simplified to $E^v[\Pi_M(\cdot)]=\lambda\Pi^h_M(\cdot)+(1-\lambda)\Pi^l_M(\cdot)-\lambda(1-\lambda)(r^h_M+r^l_M)[\Pi^h_M(\cdot)-\Pi^l_M(\cdot)]$, because $\Pi^h_M(\cdot)\geq\Pi^l_M(\cdot)$.[9] Defining $r_M\overset{\text{def}}{=}r^h_M+r^l_M$, we obtain $E^v[\Pi_M(\cdot)]=\lambda\Pi^h_M(\cdot)+(1-\lambda)\Pi^l_M(\cdot)-\lambda(1-\lambda)r_M[\Pi^h_M(\cdot)-\Pi^l_M(\cdot)]$. Analogously, the variation-adjusted expected profit of the reseller, who incurs a variation-induced disutility with an overall coefficient of r_R $(=r^h_R+r^l_R)$, is defined by $E^v[\Pi_R(\cdot)]=\lambda\Pi^h_R(\cdot)+(1-\lambda)\Pi^l_R(\cdot)-\lambda(1-\lambda)\cdot r_R[\Pi^h_R(\cdot)-\Pi^l_R(\cdot)]$.

In the following analysis, we investigate the optimality of the partial-refund versus quota contracts in two settings: (1) manufacturer experiences a variation-induced disutility, that is, $r_M>0$ while $r_R=0$, and (2) reseller experiences a variation-induced disutility, that is, $r_R>0$ while $r_M=0$. In both settings, the sequence of events remains the same as in the base model, that is, the manufacturer sets the terms of the respective contracts in Stage 1, and the reseller decides on quantity and retail prices in Stages 2 and 3, respectively.

Case 1: Variation-Induced Disutility for the Manufacturer

Here, the manufacturer sets the terms of the respective contracts to maximize its variation-adjusted expected profit. By contrast, the reseller is risk neutral, and therefore, its decisions—regarding the retail prices, made in Stage 3, and the order quantity, made in Stage 2—remain identical to those under the base model setting. We derive the optimal contracts in this setting as follows. We first summarize the reseller's optimal decisions, from the analysis of the base model, and then specify the manufacturer's decision problem under the partial-refund and quota policies. The optimal terms of the two contracts are provided and a comparison of their profitability, which determines the dominating contract, then follows. All derivations are provided in the appendix.

First, consider the partial-refund contract. In Stage 3, the reseller observes the realization of α and sets the retail price accordingly. If the market is in the high state, it sets the retail price at $p^*(q;\alpha^h)=(\alpha^h-q)/\beta$ to clear all inventory and earns a revenue equal to $V(q;\alpha^h)=q(\alpha^h-q)/\beta$. By contrast, in the low state, it sets the price at $p^*(\cdot;\alpha^l)=(\alpha^l+\beta b)/(2\beta)$ and sells a quantity equal to $D(\cdot;\alpha^l)=(\alpha^l-\beta b)/2$; the amount of unsold inventory, which is $q-D(\cdot;\alpha^l)$, is returned to the manufacturer for a partial refund. As a result, it earns a revenue of $V(\cdot;\alpha^l)=D(\cdot;\alpha^l)p^*(\cdot;\alpha^l)+[q-D(\cdot;\alpha^l)]b=(\alpha^l-\beta b)^2/(4\beta)+qb$. Next, in Stage 2, it orders stock, whose optimal quantity is given by $q^*(w,b)=\arg\max_q \mathrm{E}[\Pi_R(q;w,b)]=\arg\max_q\{\lambda V(q;\alpha^h)+(1-\lambda)V(\cdot;\alpha^l)-qw\}=(\lambda\alpha^h+(1-\lambda)\beta b-\beta w)/(2\lambda)$.

Given the reseller's optimal order quantity and the amount of returns in the low state, the manufacturer's variation-adjusted expected profit is equal to $\mathrm{E}^v[\Pi_M(w,b)]=q^*(w,b)(w-c)-(1-\lambda)[q^*(w,b)-D(\cdot;\alpha^l)]b-\lambda(1-\lambda)r_M[q^*(w,b)-D(\cdot;\alpha^l)]b$. To maximize its variation-adjusted expected profit (i.e., $\max_{w,b}\mathrm{E}^v[\Pi_M(w,b)]$), the manufacturer sets its optimal wholesale and buyback prices at $w^*=((1+\lambda r_M)\cdot\{\lambda[2-(1-\lambda)r_M]\alpha^h+(1-\lambda)(2+\lambda r_M)\alpha^l\}+(2+r_M+\lambda r_M)\beta c)/(\beta[(2+\lambda r_M)^2-\lambda r_M^2])$ and $b^*=(2(1+\lambda r_M)\alpha^l-\lambda r_M\alpha^h+r_M\beta c)/(\beta[(2+\lambda r_M)^2-\lambda r_M^2])$, respectively. The expressions of the optimal profits and order quantity are provided in the appendix.

Under the quota contract, the reseller's pricing and quantity decisions are as follows. In Stage 3, if the market is in the high state, it sets the retail price at $p^*(q;\alpha^h)=(\alpha^h-q)/\beta$, clears the entire stock, and earns a revenue equal to $V(q;\alpha^h)=q(\alpha^h-q)/\beta$. In the low state, it sells at $p^*(q;\alpha^l)=(\alpha^l-(1-\gamma)q)/\beta$ and ends up with an amount of unsold inventory equal to the quota, which is then returned for a full refund of the wholesale price; its revenue is equal to $V(q;\alpha^l)=((1-\gamma)\cdot q[\alpha^l-(1-\gamma)q])/\beta+\gamma qw$. In Stage 2, the reseller maximizes its expected profit by ordering the optimal quantity, which is $q^*(w,\gamma)=\arg\max_q\mathrm{E}[\Pi_R(q;w,\gamma)]=\arg\max_q\{\lambda V(q;\alpha^h)+(1-\lambda)V(\cdot;\alpha^l)-qw\}=(\lambda\alpha^h+(1-\lambda)(1-\gamma)\alpha^l-[1-(1-\lambda)\gamma]\beta w)/(2[\lambda+(1-\lambda)\cdot(1-\gamma)^2])$. Accordingly, in Stage 1, the manufacturer's variation-adjusted expected profit is given by $\mathrm{E}^v[\Pi(w,\gamma)]=q^*(w-c)-(1-\lambda)\gamma q^*w-\lambda(1-\lambda)\gamma q^*w$, which is maximized by setting the wholesale price at $w^*=(\lambda\alpha^h+(1-\lambda)(1-\gamma)\alpha^l)/(2\beta[1-(1-\lambda)\gamma])+c/(2[1-(1-\lambda)(1+\lambda r_M)\gamma])$ for a given $\gamma\in[0,1]$, which can be determined by a simple line search. See the appendix for the expressions of the optimal profits and quantity.

Now, we can compare the two contracts to establish the dominating one—which gives the manufacturer higher profit, and therefore will be offered to the reseller in equilibrium—when the manufacturer experiences a variation-induced disutility. Table 5 summarizes the result of this comparison for a wide range of parameter values and highlights the dominating contracts accordingly. Observation 1 describes this result.

Observation 1. When the manufacturer experiences a variation-induced disutility, the partial-refund contract can arise in equilibrium as the dominating contract.

This result is consistent with the pattern of profit variation identified in Proposition 1 for the risk-neutral case. Specifically, compared to the quota contract, the partial-refund contract results in a smaller variation in profits for the manufacturer and, therefore, a smaller variation-induced disutility.

In this setting, the manufacturer's objective is not only to maximize the expected profit but to simultaneously minimize this disutility by reducing the magnitude of profit variation. It can do so by lowering the wholesale price and the buyback price in the partial-refund contract or the quota in the quota contract; see Table 5, wherein w^*, b^*, and γ^* decrease as r_M increases across different combinations of parameter values. This approach will effectively reduce the manufacturer's profit in the high state (because of the sufficiently lower wholesale price) while increasing its profit in the low state (because of lower cost of returns in both contracts). Recall from the discussion of Proposition 1 that the partial-refund contract features a wholesale price lower than that of the quota contract, and therefore requires a smaller reduction in the wholesale price. Because cutting the wholesale price reduces the manufacturer's profit, the partial-refund contract becomes more profitable to the manufacturer than the quota contract.

As an aside, the partial-refund contract also becomes more profitable to the reseller in this setting. Recall that the manufacturer reduces the buyback price and quota while lowering the wholesale price. Both actions negatively affect the reseller's profit: the lower buyback price raises the reseller's cost of carrying excess inventory, while the smaller quota reduces the allowance that it can benefit from if the low state arises. Because the pressure to reduce the wholesale price is smaller under the partial-refund contract (as the manufacturer has lower profit variation in the partial-refund contract), the magnitude of reduction in the buyback price is less than that of the quota contract. This serves to benefit the reseller in equilibrium. In summary, the partial-refund contract features lower variation in profits of the manufacturer and is more efficient/optimal

Table 5. Quota vs. Partial-Refund Contracts in Case 1

$r_R = 0$		Quota					Partial				
$\{\alpha^h, \alpha^l, \beta, \lambda, c\}$	r_M	w^*	γ^*	q^*	$\mathrm{E}^v[\Pi_M^*]$	$E[\Pi_R^*]$	w^*	b^*	q^*	$\mathrm{E}^v[\Pi_M^*]$	$E[\Pi_R^*]$
$\{0.6, 0.4, 0.1, 0.5, 0.1\}$	0.0	2.6510	0.3103	0.1450	0.3103	0.15513	2.5500	2.0000	0.1450	0.3103	0.15513
	0.1	2.6361	0.2696	0.1422	0.3075	0.15496	2.5402	1.9321	0.1426	**0.3082**	0.15511
	0.3	2.6067	0.1843	0.1360	0.3033	0.15402	2.5281	1.8198	0.1382	**0.3050**	0.15488
$\{0.6, 0.4, 0.1, 0.5, 0.5\}$	0.0	2.8333	0.2000	0.1250	0.2563	0.12813	2.7500	2.0000	0.1250	0.2563	0.12813
	0.1	2.8104	0.1467	0.1217	0.2547	0.12788	2.7451	1.9416	0.1226	**0.2552**	0.12808
	0.3	2.7645	0.0361	0.1147	0.2532	0.12696	2.7422	1.8460	0.1181	**0.2537**	0.12771
$\{0.6, 0.4, 0.3, 0.5, 0.1\}$	0.0	0.9156	0.2593	0.1350	0.0941	0.04704	0.8833	0.6667	0.1350	0.0941	0.04704
	0.1	0.9094	0.2127	0.1319	0.0934	0.04696	0.8809	0.6456	0.1326	**0.0936**	0.04703
	0.3	0.8971	0.1156	0.1253	0.0924	0.04663	0.8784	0.6110	0.1281	**0.0928**	0.04693
$\{0.7, 0.3, 0.1, 0.5, 0.1\}$	0.0	2.9571	0.5588	0.1700	0.3453	0.17263	2.5500	1.5000	0.1700	0.3453	0.17263
	0.1	2.9223	0.5222	0.1675	0.3385	0.17232	2.5277	1.4199	0.1682	**0.3419**	0.17271
	0.3	2.8555	0.4474	0.1618	0.3270	0.17089	2.4904	1.2845	0.1652	**0.3362**	0.17321
$\{0.7, 0.3, 0.1, 0.5, 0.5\}$	0.0	3.1667	0.5000	0.1500	0.2813	0.14063	2.7500	1.5000	0.1500	0.2813	0.14063
	0.1	3.1222	0.4543	0.1468	0.2757	0.13990	2.7326	1.4294	0.1482	**0.2786**	0.14067
	0.3	3.0362	0.3615	0.1400	0.2667	0.13789	2.7045	1.3108	0.1451	**0.2742**	0.14092
$\{0.7, 0.3, 0.1, 0.5, 1.5\}$	0.0	3.5000	0.2500	0.1000	0.1563	0.07813	3.2500	1.5000	0.1000	0.1563	0.07813
	0.1	3.4142	0.1679	0.0958	0.1545	0.07759	3.2448	1.4533	0.0982	**0.1554**	0.07811
	0.3	3.2557	0.0061	0.0878	0.1531	0.07660	3.2398	1.3763	0.0948	**0.1542**	0.07793
$\{0.7, 0.3, 0.3, 0.5, 0.1\}$	0.0	1.0220	0.5313	0.1600	0.1041	0.05204	0.8833	0.5000	0.1600	0.1041	0.05204
	0.1	1.0089	0.4903	0.1572	0.1020	0.05186	0.8767	0.4749	0.1582	**0.1031**	0.05206
	0.3	0.9838	0.4070	0.1508	0.0986	0.05125	0.8658	0.4326	0.1551	**0.1014**	0.05219
$\{0.7, 0.3, 0.3, 0.5, 0.5\}$	0.0	1.1667	0.2500	0.1000	0.0521	0.02604	1.0833	0.5000	0.1000	0.0521	0.02604
	0.1	1.1381	0.1679	0.0958	0.0515	0.02587	1.0816	0.4844	0.0982	**0.0518**	0.02604
	0.3	1.0852	0.0061	0.0878	0.0510	0.02553	1.0799	0.4588	0.0948	**0.0514**	0.02598
$\{0.8, 0.2, 0.1, 0.5, 0.1\}$	0.0	3.4674	0.7436	0.1950	0.4053	0.20263	2.5500	1.0000	0.1950	0.4053	0.20263
	0.1	3.3974	0.7058	0.1929	0.3932	0.20211	2.5152	0.9077	0.1939	**0.4019**	0.20283
	0.3	3.2695	0.6308	0.1875	0.3720	0.19980	2.4528	0.7493	0.1922	**0.3963**	0.20424
$\{0.8, 0.2, 0.1, 0.5, 0.5\}$	0.0	3.7222	0.7143	0.1750	0.3313	0.16563	2.7500	1.0000	0.1750	0.3313	0.16563
	0.1	3.6385	0.6686	0.1720	0.3202	0.16423	2.7201	0.9172	0.1739	**0.3283**	0.16578
	0.3	3.4852	0.5786	0.1651	0.3015	0.16054	2.6669	0.7755	0.1721	**0.3235**	0.16682

Note. The shaded columns highlight the manufacturer's variation-adjusted expected profits under the two contracts and the dominating one is highlighted in bold.

for it to use to manage the variation-induced disutility. Note that this observation is *consistent* with the anecdotal evidence where manufacturers such as HP have preferred to use partial-refund contracts and discontinued the use of quota contracts.

Case 2: Variation-Induced Disutility for the Reseller

Next, we consider the setting in which the reseller experiences variation-induced disutility, that is, $r_R > 0$ while $r_M = 0$. Note that in Stage 3, market uncertainty is resolved and the reseller sets the retail price after observing the realization of the market demand. Therefore, its pricing decision (under both the partial-refund and quota policies) remains the same as described in Case 1. By contrast, the decision on quantity is made in Stage 2 under market uncertainty.

Under the partial-refund policy, given the pricing decision and revenue earned in Stage 3, the reseller's variation-adjusted expected profit is equal to $\mathrm{E}^v[\Pi_R(q; w, b)] = \lambda V(q; \alpha^h) + (1-\lambda)V(\cdot; \alpha^l) - qw - \lambda \cdot (1-\lambda) r_R [V(q; \alpha^h) - V(\cdot; \alpha^l)] = -(\lambda[1-(1-\lambda)r_R]/\beta) q^2 + \{(\lambda[1-(1-\lambda)r_R]\alpha^h)/\beta - w + (1-\lambda)(1+\lambda r_R) b\} q + ((1-\lambda)(1+\lambda r_R)(\alpha^l - \beta b)^2)/(4\beta)$. In Stage 2, the reseller maximizes its variation-adjusted expected profit by ordering the optimal quantity, which is $q^*(w, b) = \arg\max_q \mathrm{E}^v[\Pi_R(q; w, b)] = (\lambda[1-(1-\lambda)r_R]\alpha^h - \beta w + (1-\lambda)(1+\lambda r_R)\beta b)/(2\lambda[1-(1-\lambda)r_R])$. Recall that the reseller will clear the entire stock in the high state and remains with an excess inventory equal to $q^*(w, b) - (\alpha^l - \beta b)/2$ in the low state. Accordingly, the manufacturer's expected profit is $\mathrm{E}[\Pi_M(w, b)] = q^*(w, b)(w - c) - (1-\lambda)[q^*(w, b) - (\alpha^l - \beta b)/2] b$. Solving the manufacturer's optimization problem in Stage 1, that is, $\max_{w, b} E[\Pi(w, b)]$, we obtain the optimal wholesale and buyback prices as follows: $w^* = ([1 - (1-\lambda) r_R]\{\lambda[2 - (1-\lambda) r_R]\alpha^h + (1-\lambda)(2 + \lambda r_R)\alpha^l\} + [2 - 3(1-\lambda) r_R - \lambda(1-\lambda) r_R^2]\beta c)/(\{[2 - (1-\lambda) r_R]^2 - (1-\lambda) r_R^2\}\beta)$ and $b^* = ([1 - (1-\lambda) r_R](\lambda r_R \alpha^h + 2\alpha^l) - r_R \beta c)/(\{[2 - (1-\lambda) r_R]^2 - (1-\lambda) r_R^2\}\beta)$.

Under the quota contract, the risk-averse reseller sets the order quantity in Stage 2 to maximize its variation-adjusted expected profit, that is, the value

$\max_q \mathrm{E}^v[\Pi_R(q;w,\gamma)] = \lambda V(q;\alpha^h) + (1-\lambda)V(q;\alpha^l) - qw - \lambda(1-\lambda)r_R[V(q;\alpha^h) - V(q;\alpha^l)] = -(((1-\lambda)(1+\lambda r_R)\cdot(1-\gamma)^2 + \lambda[1-(1-\lambda)r_R])/\beta)q^2 + ((\lambda[1-(1-\lambda)r_R]\alpha^h + (1-\lambda)\ldots(1+\lambda r_R)(1-\gamma)\alpha^l - [1-\gamma(1-\lambda)(1+\lambda r_R)]\beta w)/\beta)q$. Solving this optimization problem, we obtain the optimal quantity, which is $q^*(w,\gamma) = (\lambda[1-(1-\lambda)r_R]\alpha^h + (1-\lambda)(1-\gamma)(1+\lambda r_R)\alpha^l - [1-(1-\lambda)\gamma](1+\lambda r_R)\beta w)/(2\{(1-\lambda)(1-\gamma)^2(1+\lambda r_R) + \lambda[1-(1-\lambda)r_R]\})$. The reseller then clears this stock in the high state and returns the amount of unsold inventory equal to the quota in the low state. Accordingly, the risk-neutral manufacturer earns an expected profit equal to $\mathrm{E}[\Pi_M(w,\gamma)] = q^*(w,\gamma)(w-c) - (1-\lambda)\gamma q^*(w,\gamma)w$. In Stage 1, to maximize its expected profit, the manufacturer sets the wholesale price at the optimal level, which is $w^* = (\lambda[1-(1-\lambda)r_R]\alpha^h + (1-\lambda)(1-\gamma)(1+\lambda r_R)\alpha^l)/(2\beta[1-\gamma(1-\lambda)(1+\lambda r_R)]) + c/(2[1-(1-\lambda)\gamma])$ for a given $\gamma \in [0,1]$, which can be determined by a simple line search.

Table 6 presents the result of a comparison between the two contracts across the same range of parameter values as in Case 1. The following observation characterizes this result.

Observation 2. When the reseller incurs a variation-induced disutility, the quota contract can arise in equilibrium as the dominating contract.

Here, the reseller's optimal decisions serve to minimize its variation-induced disutility. Recall from Proposition 1 that the quota contract features a lower variation in the reseller's profit for the risk-neutral case. Accordingly, it incurs a smaller disutility, and therefore requires a lesser effort/cost—compared to that under the partial-refund contract—to achieve this goal. More importantly, it is in the interest of the manufacturer to accommodate the reseller by raising the quota and the buyback price (as demonstrated in Table 6).

These actions allow the reseller to earn more profit in the low state, and thus reduce the variation in its profits. However, the magnitude of these adjustments and the associated cost/benefit to the manufacturer differ between the two contracts. It is less costly for the manufacturer to increase the quota given the lower variation

Table 6. Quota vs. Partial-Refund Contracts in Case 2

$r_M = 0$		Quota					Partial				
$\{\alpha^h, \alpha^l, \beta, \lambda, c\}$	r_R	w^*	γ^*	q^*	$E[\Pi_M^*]$	$\mathrm{E}^v[\Pi_R^*]$	w^*	b^*	q^*	$E[\Pi_M^*]$	$\mathrm{E}^v[\Pi_R^*]$
$\{0.6, 0.4, 0.1, 0.5, 0.1\}$	0.0	2.6510	0.3103	0.1450	0.3103	0.15513	2.5500	2.0000	0.1450	0.3103	0.15513
	0.1	2.6417	0.3480	0.1475	**0.3072**	0.15197	2.5378	2.0737	0.1475	0.3065	0.15199
	0.3	2.6220	0.4165	0.1529	**0.3021**	0.14508	2.5241	2.2309	0.1540	0.3001	0.14573
$\{0.6, 0.4, 0.1, 0.5, 0.5\}$	0.0	2.8333	0.2000	0.1250	0.2563	0.12813	2.7500	2.0000	0.1250	0.2563	0.12813
	0.1	2.8292	0.2432	0.1271	**0.2523**	0.12525	2.7324	2.0632	0.1264	0.2517	0.12512
	0.3	2.8200	0.3229	0.1315	**0.2452**	0.11906	2.7050	2.1954	0.1303	0.2433	0.11894
$\{0.6, 0.4, 0.3, 0.5, 0.1\}$	0.0	0.9156	0.2593	0.1350	0.0941	0.04704	0.8833	0.6667	0.1350	0.0941	0.04704
	0.1	0.9134	0.2997	0.1373	**0.0929**	0.04604	0.8784	0.6895	0.1369	0.0927	0.04601
	0.3	0.9086	0.3737	0.1422	**0.0909**	0.04386	0.8715	0.7377	0.1421	0.0902	0.04391
$\{0.7, 0.3, 0.1, 0.5, 0.1\}$	0.0	2.9571	0.5588	0.1700	0.3453	0.17263	2.5500	1.5000	0.1700	0.3453	0.17263
	0.1	2.9508	0.5932	0.1721	**0.3400**	0.16641	2.5253	1.5859	0.1718	0.3365	0.16648
	0.3	2.9382	0.6552	0.1762	**0.3305**	0.15316	2.4864	1.7654	0.1769	0.3205	0.15492
$\{0.7, 0.3, 0.1, 0.5, 0.5\}$	0.0	3.1667	0.5000	0.1500	0.2813	0.14063	2.7500	1.5000	0.1500	0.2813	0.14063
	0.1	3.1683	0.5400	0.1518	**0.2752**	0.13506	2.7199	1.5754	0.1508	0.2720	0.13459
	0.3	3.1736	0.6133	0.1553	**0.2642**	0.12332	2.6673	1.7298	0.1532	0.2545	0.12297
$\{0.7, 0.3, 0.1, 0.5, 1.5\}$	0.0	3.5000	0.2500	0.1000	0.1563	0.07813	3.2500	1.5000	0.1000	0.1563	0.07813
	0.1	3.5138	0.3032	0.1006	**0.1489**	0.07381	3.2064	1.5491	0.0981	0.1476	0.07334
	0.3	3.5499	0.4060	0.1018	**0.1353**	0.06508	3.1195	1.6410	0.0940	0.1309	0.06411
$\{0.7, 0.3, 0.3, 0.5, 0.1\}$	0.0	1.0220	0.5313	0.1600	0.1041	0.05204	0.8833	0.5000	0.1600	0.1041	0.05204
	0.1	1.0212	0.5684	0.1620	**0.1022**	0.05008	0.8742	0.5269	0.1613	0.1011	0.05000
	0.3	1.0199	0.6359	0.1658	**0.0988**	0.04593	0.8589	0.5825	0.1651	0.0954	0.04611
$\{0.7, 0.3, 0.3, 0.5, 0.5\}$	0.0	1.1667	0.2500	0.1000	0.0521	0.02604	1.0833	0.5000	0.1000	0.0521	0.02604
	0.1	1.1713	0.3032	0.1006	**0.0496**	0.02460	1.0688	0.5163	0.0981	0.0492	0.02445
	0.3	1.1833	0.4060	0.1018	**0.0451**	0.02169	1.0399	0.5470	0.0940	0.0436	0.02137
$\{0.8, 0.2, 0.1, 0.5, 0.1\}$	0.0	3.4674	0.7436	0.1950	0.4053	0.20263	2.5500	1.0000	0.1950	0.4053	0.20263
	0.1	3.4960	0.7803	0.1966	**0.3994**	0.19331	2.5128	1.0981	0.1962	0.3904	0.19349
	0.3	3.5833	0.8508	0.1989	**0.3896**	0.17315	2.4486	1.2998	0.1999	0.3623	0.17680
$\{0.8, 0.2, 0.1, 0.5, 0.5\}$	0.0	3.7222	0.7143	0.1750	0.3313	0.16563	2.7500	1.0000	0.1750	0.3313	0.16563
	0.1	3.7683	0.7575	0.1764	**0.3248**	0.15745	2.7074	1.0876	0.1751	0.3161	0.15659
	0.3	3.9075	0.8433	0.1785	**0.3141**	0.13988	2.6295	1.2643	0.1762	0.2870	0.13968

Note. The shaded columns highlight the manufacturer's variation-adjusted expected profits under the two contracts and the dominating one is highlighted in bold.

in the reseller's profits under such a policy. On the other hand, because the reseller is entirely insured from incurring the cost of unsold inventory with the quota contract—by contrast, it shares part of the cost under the partial-refund contract—the increase in order quantity in response to an increase in the quota is much more significant than that when the manufacturer raises the buyback price under the partial-refund policy. Lower cost and higher benefit when using the quota make this contract more profitable to the manufacturer as well. We note from the Eaton example that the preferred reseller gets a higher quota compared to the nonpreferred ones. This is consistent with our result highlighted in Table 6.

7. Model Extension: Combination Contract

In addition to the partial-refund and quota policies, the combination policy is also observed in practice (e.g., see Table 1) but has not been addressed in the extant literature. We first derive the optimal structure of the combination contract and discuss how it offers the manufacturer the flexibility to effectively share the costs of returns with the reseller. Later, we illustrate how the ability to share the cost of returns helps the manufacturer structure the contract for its preferred reseller in a cost-effective manner.[10]

The following lemma formalizes the optimal terms of the combination contract.

Lemma 4. *Under a combination contract with risk-neutral players, M accepts returns for full refund, up to a quota $\gamma_C^* = \gamma \in (0, 1 - \lambda\alpha^l/(\lambda\alpha^h - \beta c))$; any returns above this quota receive a partial refund of $b_C^* = \alpha^l/(2\beta)$. The wholesale price is a function of γ and is set accordingly at $w_C^* = (\lambda\alpha^h + (1-\lambda)(1-\gamma)\alpha^l + \beta c)/(2\beta[1-\gamma(1-\lambda)])$.*

First, note that the combination contract features a quota, whose value can be chosen from a specific range. Essentially, there exist a multitude of (w, γ) combinations that yield the same expected profit even though the profit variation may depend on the specific value of γ. As we discuss next, this allows the manufacturer to better control the profit variation for both players in the channel.

Note that γ sets the cap for the manufacturer's full coverage for unsold inventory, above which the reseller starts sharing the cost of returns. In other words, the chosen quota determines how the cost of carrying excess inventory will be apportioned between the manufacturer and the reseller. A higher quota implies a higher cost of returns to the manufacturer, and this effect is eventually accounted for when setting the wholesale price. Specifically, if offering a higher quota, the manufacturer will raise the wholesale price to cover the increase in the associated cost: $\partial w_C^*/\partial\gamma = ((1-\lambda)\cdot[\lambda(\alpha^h - \alpha^l) + \beta c])/(2\beta[1-\gamma(1-\lambda)]^2) > 0$. In contrast to the wholesale price, the partial-refund price, b_C^*, is independent of γ, and because there is no limit on the quantity of stock qualified for partial refund, this price is optimally set based on the price that a reseller would charge in the low state.

While a higher quota provides incentive for the reseller to order more stock, the correspondingly higher wholesale price exerts an upward pressure on the retail price and, consequently, a downward pressure on sales quantity as well as the amount of stock to be ordered. These opposing effects counter each other, leaving the quantity of ordered stock unaffected by γ. Keeping this in mind, we examine the profits of M and R under the combination contract.

Proposition 2. (a) *Under the combination contract, both the reseller's and manufacturer's expected profits are unaffected by the choice of γ and are identical to those under the quota and partial-refund contracts.*

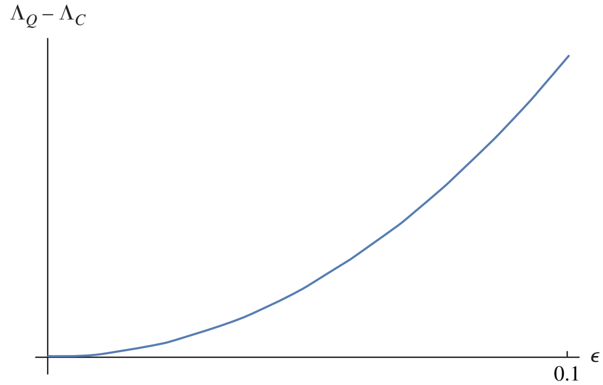
(b) *Variation in the profit of the manufacturer (between the high and low states) under the combination contract is monotonically increasing in γ. By contrast, the variation in the reseller's profit is monotonically decreasing in γ.*

The extension of the forces discussed under the Q and P contracts contributes to the intuition underlying this result. Recall that if the manufacturer accepts a higher quota in a combination contract, it will raise the wholesale price accordingly. This results in a higher initial profit margin and eventually higher manufacturer's profit in the high state when no returns occur. Yet, if the market ends up in the low state, a high quota implies a larger cost of returns to the manufacturer and, consequently, lower profit. These forces lead to an opposite impact on the reseller's profits, which explains the lower variation in profits of the reseller when the quota is raised. Moreover, as shown in the appendix (in the proof of Proposition 2), the variations in profits for the manufacturer are ordered such that $Q > C > P$, whereas the variation in profits for the reseller are ordered such that $P > C > Q$.

Given the opposite effects of γ on the variations of profits of the two parties, how should the manufacturer use the flexible quota under a combination contract—in contrast to using the quota contract—to control the variation of profits? Furthermore, will it be more (or less) costly for M to use the combination versus quota contract to better engage with its preferred resellers?

Consider the setting in which M is serving a reseller using either the optimal quota or combination contract, whose terms are $\{w_Q^*, \gamma_Q^*\}$ and $\{w_C^*, \gamma_C^*, b_C^*\}$, respectively; see Lemmas 2 and 3. In this analysis, we let $\gamma_C^* = \gamma_Q^* = 1 - \lambda\alpha^l/(\lambda\alpha^h - \beta c)$ for comparison; this results in a wholesale price of $w_C^* = (\lambda\bar{\alpha}^2 - \beta^2c^2)/(2\lambda\beta(\bar{\alpha} - \beta c))$. Now, suppose that a preferred reseller, concerned about market uncertainty, requests a higher quota. The manufacturer can accommodate such a request by offering either a combination or quota contract with

Figure 1. (Color online) Loss in M's Expected Profit When Raising Quota



the following terms:[11] $\{w_C^*, \gamma_C^* + \epsilon, b_C^*\}$ and $\{w_Q^*, \gamma_Q^* + \epsilon\}$, respectively (where $\epsilon > 0$).

We follow the analysis of the base model setting to derive the expected profits of M under these two contracts, denoted by $\mathrm{E}[\Pi_C^\epsilon]$ and $\mathrm{E}[\Pi_Q^\epsilon]$. Clearly, raising the quota above the optimal levels (i.e., γ_C^* and γ_Q^*) reduces M's profit. Define the loss in M's expected profit when using the adjusted combination and quota contracts by $\Lambda_C \stackrel{\text{def}}{=} \mathrm{E}[\Pi_{MC}^*] - \mathrm{E}[\Pi_C^\epsilon]$ and $\Lambda_Q \stackrel{\text{def}}{=} \mathrm{E}[\Pi_Q^*] - \mathrm{E}[\Pi_Q^\epsilon]$.

Figure 1 depicts the difference between Λ_C and Λ_Q as a function of ϵ, in a setting where $\alpha^h = 6$, $\alpha^l = 4$, $\lambda = \frac{1}{2}$, $\beta = 1$, and $c = \frac{1}{2}$. (See the appendix for the derivation.) It shows that the cost of appeasing the preferred reseller is lower by using the combination contract. In the quota contract, the manufacturer absorbs the entire cost of returns, which will be higher when the preferred reseller expects a higher quota. However, by using the combination contract, with an additional contract mechanism (using buyback price along with quota and the wholesale price), the manufacturer is able to exert more control and provide the extra incentive to the preferred reseller cost-effectively.

8. Conclusion

We began by noting that quota-based returns (also referred to as stock-rotation policies) as well as combination policies that employ partial refund along with an initial quota are commonly observed in practice. Extant academic research, however, has not devoted much attention to the properties of these policies. In this paper, we employed a stylized model to examine three returns policies from the perspective of the manufacturer while accounting for the reseller's behavior: a partial-refund contract, a quota contract, and a combination contract.

Comparatively speaking, under the quota contract, the manufacturer fully covers the reseller's cost of unsold inventory (up to the allowed amount). To offset this cost, however, the manufacturer raises its wholesale price. Conversely, because of the quota, the reseller enjoys protection under adverse demand conditions but pays a premium via higher wholesale prices. Under the partial-refund contract, the manufacturer does not bear the entire cost of unsold inventory; this leads to a lower (compared to a quota contract) wholesale price. The reseller enjoys this lower price, but pays its dues under adverse demand conditions.

Our analysis highlights that the variation in the manufacturer's profit (across favorable and adverse demand conditions) is lower under the partial-refund contract than under the quota contract, while the variation in the reseller's profit is precisely the opposite. We explained the underlying rationale for these diametrically opposing variations. In addition, we noted that this pattern of profit variations remains unchanged when the manufacturer coordinates the pricing and quantity decisions of the reseller. Interestingly, when the structure of the channel changes and the manufacturer owns the reseller and sells directly to consumers, the level of profit variation is magnified. Finally, we extended the base model to accommodate either player experiencing a disutility associated with any variation in profit. This extended analysis shows that the ordering in profit variations identified in the base model serves to explain the preference for specific returns policies and is consistent with the anecdotal evidence.

After examining the interactions between the manufacturer and the reseller under quota and partial-refund contracts, we then focused on the potential value offered by a combination contract. Note that the flexibility offered by quotas is often hailed in the sales force literature (e.g., Kishore et al. 2013)—it is an easily implementable approach to manage a diverse set of agents. In an analogous manner, our analysis shows that the optimal combination contract provides a framework for the manufacturer to adjust the quota level (and the corresponding wholesale price level) depending on the reseller's type (preferred versus non-preferred types). If a given reseller is of the preferred type, consistent with the observed practice, our model shows that the manufacturer may increase its quota offering (thereby granting more full refunds for a given order quantity), which also benefits the reseller by reducing the profit variation between the high and low demand realizations. Furthermore, in practice, even if optimality cannot be maintained, our analysis allows the cost of such deviations to be controlled. As an example, say the wholesale and buyback prices have to remain fixed (because of legal or industry norms); then the optimal quota for that wholesale price can be offered to some resellers while a higher quota is offered to the preferred ones—it is straightforward to compute the magnitude of loss for deviating from optimality.

Overall, to highlight the main issues, we focused on a stylized model. Several opportunities exist for further work; for instance, future research can explore competition at the reseller and manufacturer levels. Furthermore, information asymmetry regarding demand is another avenue worthy of exploration.

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Appendix

Based on the observation that the Q and P contracts are two special cases of the C contract—which arise when $b = 0$ and $\gamma = 0$, respectively—we first derive the optimal C contract in the proof of Lemma 4, followed by the proof of Proposition 2, which discusses the expectation and variation in profits under the C contract. Based on these analyses, the proofs of Lemmas 1 and 2 and Proposition 1 are provided.

Proof of Lemma 4 (The Optimal C Contract Under Risk Neutrality). In Stage 3, for a given realization of $\alpha \in \{\alpha^h, \alpha^l\}$, R sets the retail price to maximize its revenue, $\max_p V(p;\alpha)$, wherein $V(p;\alpha)$ is a piecewise function defined by (2). We solve this program by considering two constrained optimization problems: (1) $\max_p V(p;\alpha) = pD(p;\alpha) + w[q - D(p;\alpha)]$, s.t. $(1-\gamma)q \le D(p;\alpha) \le q$ and (2) $\max_p V(p;\alpha) = pD(p;\alpha) + w\gamma q + b[(1-\gamma)q - D(p;\alpha)]$, s.t. $0 < D(p;\alpha) \le (1-\gamma)q$, where $D(p;\alpha) = \alpha - \beta p$.

The Lagrangian function associated with the first problem is $\mathcal{L} = pD(p;\alpha) + w[q - D(p;\alpha)] + \mu_1(q - D(p;\alpha)) + \mu_2[D(p;\alpha) - (1-\gamma)q]$. Solving the Karush–Kuhn–Tucker (KKT) first-order condition $\partial\mathcal{L}/\partial p|_{\mu_1=\mu_2=0} = 0$, we obtain the interior solution: $p = (\alpha + \beta w)/(2\beta)$. This solution requires $(1-\gamma)q < D(p;\alpha)|_{p=(\alpha+\beta w)/(2\beta)} < q \Leftrightarrow (\alpha - \beta w)/2 < q < (\alpha - \beta w)/(2(1-\gamma))$. In addition, there are two corner solutions: (1) $q = D(p;\alpha) \Leftrightarrow p = (\alpha - q)/\beta$ and (2) $(1-\gamma)q = D(p;\alpha) \Leftrightarrow p = (\alpha - (1-\gamma)q)/\beta$, which arise under the conditions (1) $q \le (\alpha - \beta w)/2$ and (2) $q \ge (\alpha - \beta w)/(2(1-\gamma))$, respectively.

Analogously, we solve the second constrained optimization problem and obtain the following optimal prices: (1) $p = (\alpha + \beta b)/(2\beta)$ and (2) $p = (\alpha - (1-\gamma)q)/\beta$, which arise if (1) $q > (\alpha - \beta b)/(2(1-\gamma))$ and (2) $q \le (\alpha - \beta b)/(2(1-\gamma))$, respectively. Together with the solution of the first problem, the optimal price, $p^*(q;\alpha)$, and the corresponding revenue, $V^*(q;\alpha)$, are given by

$$p^*(q;\alpha) = \begin{cases} \dfrac{\alpha - q}{\beta} & \text{if } q \le \dfrac{\alpha - \beta w}{2}, \\ \dfrac{\alpha + \beta w}{2\beta} & \text{if } \dfrac{\alpha - \beta w}{2} < q < \dfrac{\alpha - \beta w}{2(1-\gamma)}, \\ \dfrac{\alpha - (1-\gamma)q}{\beta} & \text{if } \dfrac{\alpha - \beta w}{2(1-\gamma)} \le q \le \dfrac{\alpha - \beta b}{2(1-\gamma)}, \\ \dfrac{\alpha + \beta b}{2\beta} & \text{if } q > \dfrac{\alpha - \beta b}{2(1-\gamma)} \end{cases} \quad \text{(A.1)}$$

and

$$V^*(q;\alpha) = \begin{cases} V_1(q;\alpha) \stackrel{\text{def}}{=} \dfrac{q(\alpha - q)}{\beta} & \text{if } q \le \dfrac{\alpha - \beta w}{2}, \\ V_2(q;\alpha) \stackrel{\text{def}}{=} \dfrac{(\alpha - \beta w)^2}{4\beta} + qw & \text{if } \dfrac{\alpha - \beta w}{2} < q < \dfrac{\alpha - \beta w}{2(1-\gamma)}, \\ V_3(q;\alpha) \stackrel{\text{def}}{=} \dfrac{(1-\gamma)q[\alpha - (1-\gamma)q]}{\beta} + \gamma q w & \text{if } \dfrac{\alpha - \beta w}{2(1-\gamma)} \le q \le \dfrac{\alpha - \beta b}{2(1-\gamma)}, \\ V_4(q;\alpha) \stackrel{\text{def}}{=} \dfrac{(\alpha - \beta b)^2}{4\beta} + [(1-\gamma)b + \gamma w]q & \text{if } q > \dfrac{\alpha - \beta b}{2(1-\gamma)}, \text{ respectively.} \end{cases} \quad \text{(A.2)}$$

Next, in Stage 2, R chooses an order quantity that maximizes its expected profit: $\max_q \mathrm{E}[\Pi_R(q;w,\gamma,b)] = \lambda V^*(q;\alpha^h) + (1-\lambda)V^*(q;\alpha^l) - qw$. Here, $\mathrm{E}[\Pi_R(q;\cdot)]$ is a piecewise function with six cutoffs—three in each state, as implied by Equation (A.2)—which include (1) $(\alpha^l - \beta w)/2 \stackrel{\text{def}}{=} q_1$, (2) $(\alpha^l - \beta w)/(2(1-\gamma)) \stackrel{\text{def}}{=} q_2$, (3) $(\alpha^l - \beta b)/(2(1-\gamma)) \stackrel{\text{def}}{=} q_3$, (4) $(\alpha^h - \beta w)/2 \stackrel{\text{def}}{=} q_4$, (5) $(\alpha^h - \beta w)/(2(1-\gamma)) \stackrel{\text{def}}{=} q_5$, and (6) $(\alpha^h - \beta b)/(2(1-\gamma)) \stackrel{\text{def}}{=} q_6$. Since (1) the ordering of these cutoffs depends on w as shown in Table A.1 and (2) the specification of $\mathrm{E}[\Pi_R(q;\cdot)]$ depends on this ordering, we solve the above optimization problem (i.e., $\max_q \mathrm{E}[\Pi_R(q;\cdot)]$) by considering five specific problems, which are denoted by [P-a]–[P-e] in correspondence with the five cases listed in Table A.1, respectively. For instance, if $w \le w_1$, then $q_1 < q_2 < q_3 < q_4 < q_5 < q_6$, and consequently, R solves [P-a]: $\max_q \mathrm{E}[\Pi_R(q;w,\gamma,b)]$, wherein

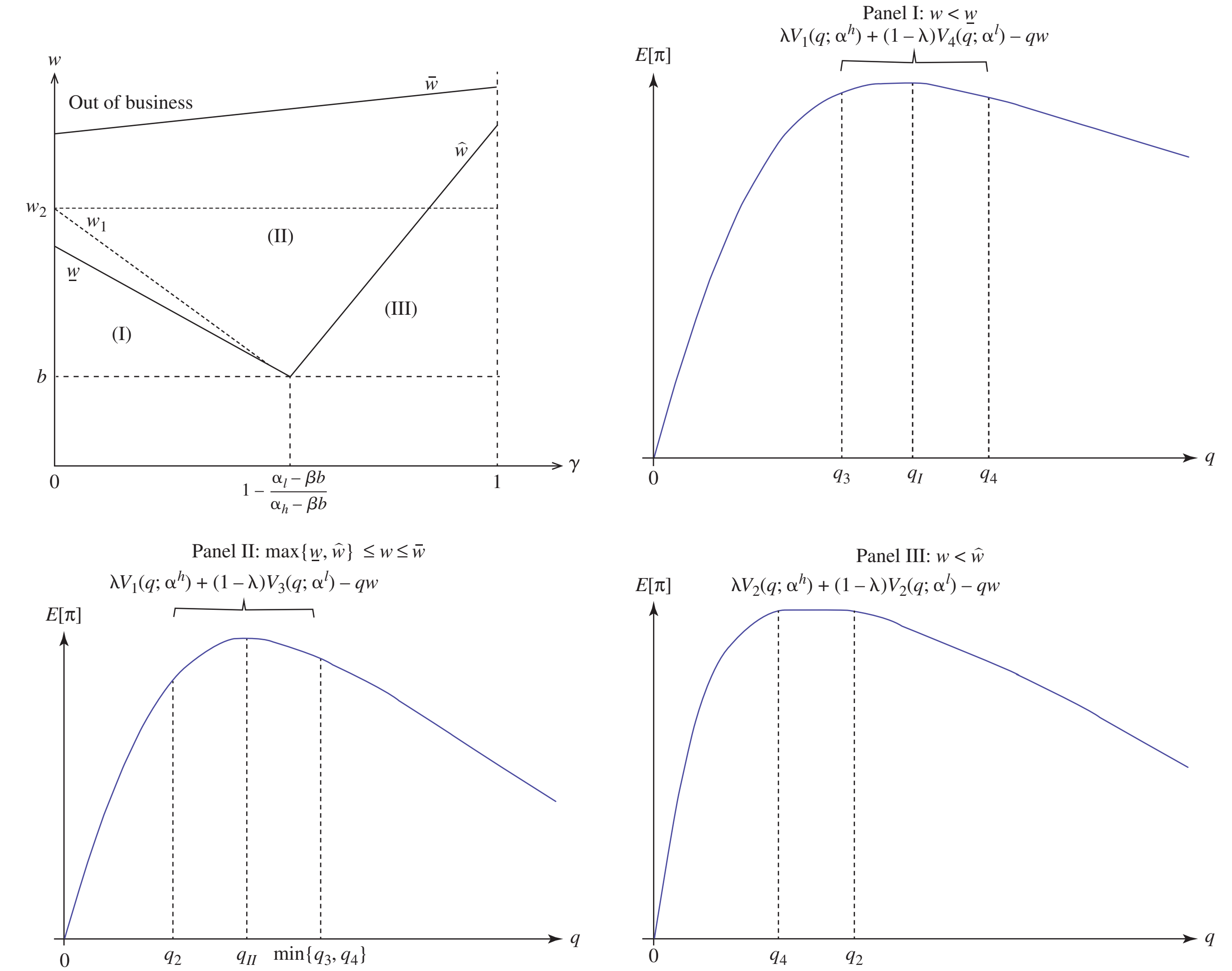
$$\mathrm{E}[\Pi_R(q;w,\gamma,b)] = \begin{cases} \lambda V_1(q;\alpha^h) + (1-\lambda)V_1(q;\alpha^l) - qw & \text{if } 0 \le q < q_1, \\ \lambda V_1(q;\alpha^h) + (1-\lambda)V_2(q;\alpha^l) - qw & \text{if } q_1 \le q < q_2, \\ \lambda V_1(q;\alpha^h) + (1-\lambda)V_3(q;\alpha^l) - qw & \text{if } q_2 \le q < q_3, \\ \lambda V_1(q;\alpha^h) + (1-\lambda)V_4(q;\alpha^l) - qw & \text{if } q_3 \le q < q_4, \\ \lambda V_2(q;\alpha^h) + (1-\lambda)V_4(q;\alpha^l) - qw & \text{if } q_4 \le q < q_5, \\ \lambda V_3(q;\alpha^h) + (1-\lambda)V_4(q;\alpha^l) - qw & \text{if } q_5 \le q < q_6, \\ \lambda V_4(q;\alpha^h) + (1-\lambda)V_4(q;\alpha^l) - qw & \text{if } q \ge q_6. \end{cases} \quad \text{(A.3)}$$

Table A.2 provides the specifications of $\mathrm{E}[\Pi_R(q;w,\gamma,b)]$ in the four remaining problems, that is, [P-b]–[P-e]. To solve these problems, we first determine the shape of the respective

Table A.1. Ordering of Quantity Cutoffs

Case	Ordering	Condition
(a)	$q_1 < q_2 < q_3 < q_4 < q_5 < q_6$	$w \le w_1 \stackrel{\text{def}}{=} \dfrac{1}{\beta}\left(\alpha^h - \dfrac{\alpha^l - \beta b}{1-\gamma}\right)$
(b)	$q_1 < q_2 < q_4 < q_3 < q_5 < q_6$	$\max\{w_1, w_3\} < w \le w_2$, where $w_2 \stackrel{\text{def}}{=} \dfrac{\alpha^h - \alpha^l}{\beta} + b$ and $w_3 \stackrel{\text{def}}{=} \dfrac{\alpha^l - (1-\gamma)\alpha^h}{\beta\gamma}$
(c)	$q_1 < q_4 < q_2 < q_3 < q_5 < q_6$	$w \le \min\{w_2, w_3\}$
(d)	$q_1 < q_2 < q_4 < q_5 < q_3 < q_6$	$w > \max\{w_2, w_3\}$
(e)	$q_1 < q_4 < q_2 < q_5 < q_3 < q_6$	$w_2 < w \le w_3$; this case does not arise if $\alpha^h \ge 2\alpha^l - \beta b$

Figure A.1. (Color online) Reseller's Optimal Profit and Order Quantity



piecewise objective functions and then use this information to identify the optimum. For instance, consider [P-a]; in the first interval, that is, when $q\in[0,q_1)$, we have (1) $\mathrm{E}[\Pi_R(q;\cdot)]=\lambda V_1(q;\alpha^h)+(1-\lambda)V_1(q;\alpha^l)-qw=-(1/\beta)q^2+[(\lambda\alpha^h+(1-\lambda)\cdot\alpha^l-\beta w)/\beta]q$, which is a quadratic and concave function in q; (2) $\mathrm{E}[\Pi_R(q;\cdot)]|_{q=0}=0$; and (3) $\lim_{q\to q_1^-}\partial\mathrm{E}[\Pi_R(q;\cdot)]/\partial q=\lambda(\alpha^h-\alpha^l)/\beta>0$. Accordingly, $\mathrm{E}[\Pi_R(q;\cdot)]$ is an increasing, concave quadratic function in $q\in[0,q_1)$. We use this procedure for the remaining intervals of [P-a] to determine the shape of $\mathrm{E}[\Pi_R(q;\cdot)]\ \forall q\in[0,+\infty)$. This result is depicted in Figure A.1 (panels I and II). (The technical derivation is available from the authors on request.) Specifically, if $w\le(\lambda[(1-\gamma)\alpha^h-\alpha^l]+[\lambda+(1-\lambda)(1-\gamma)^2]\beta b)/(\beta(1-\gamma)[1-\gamma(1-\lambda)])\stackrel{\text{def}}{=}\underline{w}$, then the problem in panel I arises, and $\mathrm{E}[\Pi_R(q;\cdot)]$ is maximized at $q^*(\cdot)=\arg\max_q\lambda V_1(q;\alpha^h)+(1-\lambda)V_4(q;\alpha^l)-qw=(\lambda\alpha^h-[1-\gamma(1-\lambda)]\beta w+(1-\lambda)(1-\gamma)\beta b)/(2\lambda)\stackrel{\text{def}}{=}q_I\in(q_3,q_4)$. Otherwise, if $\underline{w}<w\le w_1$, then the problem in panel II arises, and $\mathrm{E}[\Pi_R(q;\cdot)]$ is maximized at $q^*(\cdot)=\arg\max_q\lambda V_1(q;\alpha^h)+(1-\lambda)V_3(q;\alpha^l)-qw=(\lambda\alpha^h+(1-\lambda)(1-\gamma)\alpha^l-[1-\gamma(1-\lambda)]\beta w)/(2[\lambda+(1-\lambda)(1-\gamma)^2])\stackrel{\text{def}}{=}q_{II}\in(q_2,q_3)$. Here, $q_{II}\ge0$ requires $w\le\bar{w}\stackrel{\text{def}}{=}(\lambda\alpha^h+(1-\lambda)\cdot(1-\gamma)\alpha^l)/([1-\gamma(1-\lambda)]\beta)$.

Analogously, we solve the remaining problems: [P-b]–[P-e]; the results are provided in Table A.2 and depicted in Figure A.1, wherein $\hat{w}\stackrel{\text{def}}{=}w_3=(\alpha^l-(1-\gamma)\alpha^h)/(\beta\gamma)$. Put together, we arrive at the optimal quantity $q^*(w,\gamma,b)$ that R will order in response to (w,γ,b) as follows:

$$q^*(w,\gamma,b)=\begin{cases}\dfrac{\lambda\alpha^h-[1-\gamma(1-\lambda)]\beta w+(1-\lambda)(1-\gamma)\beta b}{2\lambda}\\ \quad=q_I\quad\text{if } b\le w<\underline{w},\\ \dfrac{\lambda\alpha^h+(1-\lambda)(1-\gamma)\alpha^l-[1-\gamma(1-\lambda)]\beta w}{2[\lambda+(1-\lambda)(1-\gamma)^2]}\\ \quad=q_{II}\quad\text{if }\max\{\underline{w},\hat{w}\}\le w\le\bar{w},\\ \text{any } q\in\left[\dfrac{\alpha^h-\beta w}{2},\dfrac{\alpha^l-\beta w}{2(1-\gamma)}\right]\\ \quad=[q_4,q_2]\quad\text{if } b\le w<\hat{w}.\end{cases}\tag{A.4}$$

Next, to derive M's expected profit function, which is maximized in Stage 1, we observe from (A.4)—along with (A.1) and (A.2)—that (1) if $b\le w<\underline{w}$, then R will order q_I and clear this quantity under α^h but end up with excess inventory exceeding the quota under α^l; (2) if $\max\{\underline{w},\hat{w}\}\le w\le\bar{w}$, then R will order q_{II} and clear it under α^h, but end up with an excess of inventory equal to the quota under α^l; and (3) if

Table A.2. R's Expected Profit Functions and Optimal Order Quantities

[P-b] (Figure A.1, panel II)

$$\mathrm{E}[\Pi_R]=\begin{cases}\lambda V_1(q;\alpha^h)+(1-\lambda)V_1(q;\alpha^l)-qw, & \text{if } 0\le q<q_1\\ \lambda V_1(q;\alpha^h)+(1-\lambda)V_2(q;\alpha^l)-qw, & \text{if } q_1\le q<q_2\\ \lambda V_1(q;\alpha^h)+(1-\lambda)V_3(q;\alpha^l)-qw, & \text{if } q_2\le q<q_4\\ \lambda V_2(q;\alpha^h)+(1-\lambda)V_3(q;\alpha^l)-qw, & \text{if } q_4\le q<q_3\\ \lambda V_2(q;\alpha^h)+(1-\lambda)V_4(q;\alpha^l)-qw, & \text{if } q_3\le q<q_5\\ \lambda V_3(q;\alpha^h)+(1-\lambda)V_4(q;\alpha^l)-qw, & \text{if } q_5\le q<q_6\\ \lambda V_4(q;\alpha^h)+(1-\lambda)V_4(q;\alpha^l)-qw, & \text{if } q\ge q_6\end{cases}$$

$$q^*(\cdot)=\arg\max_q \lambda V_1(q;\alpha^h)+(1-\lambda)V_3(q;\alpha^l)-qw$$
$$=\frac{\lambda\alpha^h+(1-\lambda)(1-\gamma)\alpha^l-[1-\gamma(1-\lambda)]\beta w}{2[\lambda+(1-\lambda)(1-\gamma)^2]}=q_{II}\in(q_2,q_4)$$

[P-d] (Figure A.1, panel II)

$$\mathrm{E}[\Pi_R]=\begin{cases}\lambda V_1(q;\alpha^h)+(1-\lambda)V_1(q;\alpha^l)-qw, & \text{if } 0\le q<q_1\\ \lambda V_1(q;\alpha^h)+(1-\lambda)V_2(q;\alpha^l)-qw, & \text{if } q_1\le q<q_2\\ \lambda V_1(q;\alpha^h)+(1-\lambda)V_3(q;\alpha^l)-qw, & \text{if } q_2\le q<q_4\\ \lambda V_2(q;\alpha^h)+(1-\lambda)V_3(q;\alpha^l)-qw, & \text{if } q_4\le q<q_5\\ \lambda V_3(q;\alpha^h)+(1-\lambda)V_3(q;\alpha^l)-qw, & \text{if } q_5\le q<q_3\\ \lambda V_3(q;\alpha^h)+(1-\lambda)V_4(q;\alpha^l)-qw, & \text{if } q_3\le q<q_6\\ \lambda V_4(q;\alpha^h)+(1-\lambda)V_4(q;\alpha^l)-qw, & \text{if } q\ge q_6\end{cases}$$

$$q^*(\cdot)=\arg\max_q \lambda V_1(q;\alpha^h)+(1-\lambda)V_3(q;\alpha^l)-qw$$
$$=q_{II}\in(q_2,q_4)$$

[P-c] (Figure A.1, panel III)

$$\mathrm{E}[\Pi_R]=\begin{cases}\lambda V_1(q;\alpha^h)+(1-\lambda)V_1(q;\alpha^l)-qw, & \text{if } 0\le q<q_1\\ \lambda V_1(q;\alpha^h)+(1-\lambda)V_2(q;\alpha^l)-qw, & \text{if } q_1\le q<q_4\\ \lambda V_2(q;\alpha^h)+(1-\lambda)V_2(q;\alpha^l)-qw, & \text{if } q_4\le q<q_2\\ \lambda V_2(q;\alpha^h)+(1-\lambda)V_3(q;\alpha^l)-qw, & \text{if } q_2\le q<q_3\\ \lambda V_2(q;\alpha^h)+(1-\lambda)V_4(q;\alpha^l)-qw, & \text{if } q_3\le q<q_5\\ \lambda V_3(q;\alpha^h)+(1-\lambda)V_4(q;\alpha^l)-qw, & \text{if } q_5\le q<q_6\\ \lambda V_4(q;\alpha^h)+(1-\lambda)V_4(q;\alpha^l)-qw, & \text{if } q\ge q_6\end{cases}$$

$$q^*(\cdot)=\arg\max_q \lambda V_2(q;\alpha^h)+(1-\lambda)V_2(q;\alpha^l)-qw$$
$$=\text{any } q\in[q_4,q_2]$$

[P-e] (Figure A.1, panel III)

$$\mathrm{E}[\Pi_R]=\begin{cases}\lambda V_1(q;\alpha^h)+(1-\lambda)V_1(q;\alpha^l)-qw, & \text{if } 0\le q<q_1\\ \lambda V_1(q;\alpha^h)+(1-\lambda)V_2(q;\alpha^l)-qw, & \text{if } q_1\le q<q_4\\ \lambda V_2(q;\alpha^h)+(1-\lambda)V_2(q;\alpha^l)-qw, & \text{if } q_4\le q<q_2\\ \lambda V_2(q;\alpha^h)+(1-\lambda)V_3(q;\alpha^l)-qw, & \text{if } q_2\le q<q_5\\ \lambda V_3(q;\alpha^h)+(1-\lambda)V_3(q;\alpha^l)-qw, & \text{if } q_5\le q<q_3\\ \lambda V_3(q;\alpha^h)+(1-\lambda)V_4(q;\alpha^l)-qw, & \text{if } q_3\le q<q_6\\ \lambda V_4(q;\alpha^h)+(1-\lambda)V_4(q;\alpha^l)-qw, & \text{if } q\ge q_6\end{cases}$$

$$q^*(\cdot)=\arg\max_q \lambda V_2(q;\alpha^h)+(1-\lambda)V_2(q;\alpha^l)-qw$$
$$=\text{any } q\in[q_4,q_2]$$

$b\le w<\hat{w}$, then R is indifferent between any $q\in[q_4,q_2]$, and accordingly, it simply orders $q_4=(\alpha^h-\beta w)/2$, clears this quantity under α^h, and remains with an excess inventory less than the quota under α^l. Accordingly, M earns $\Pi_M^h(\cdot)=q^*(w,\gamma,b)(w-c)$ in the high state. In the low state, its profit is given by

$$\Pi_M^l(\cdot)=\begin{cases}q_I(w-c)-\gamma q_I w-\left[(1-\gamma)q_I-D\left(\dfrac{\alpha^l+\beta b}{2\beta};\alpha^l\right)\right]b\\ \qquad\qquad\text{if } b\le w<\underline{w},\\ q_{II}(w-c)-\gamma q_I w\quad \text{if } \max\{\underline{w},\hat{w}\}\le w\le\bar{w},\\ q_4(w-c)-\left[q_4-D\left(\dfrac{\alpha^l-\beta w}{2\beta};\alpha^l\right)\right]w\quad \text{if } b\le w<\hat{w}.\end{cases}\tag{A.5}$$

The optimization problem in Stage 1 is

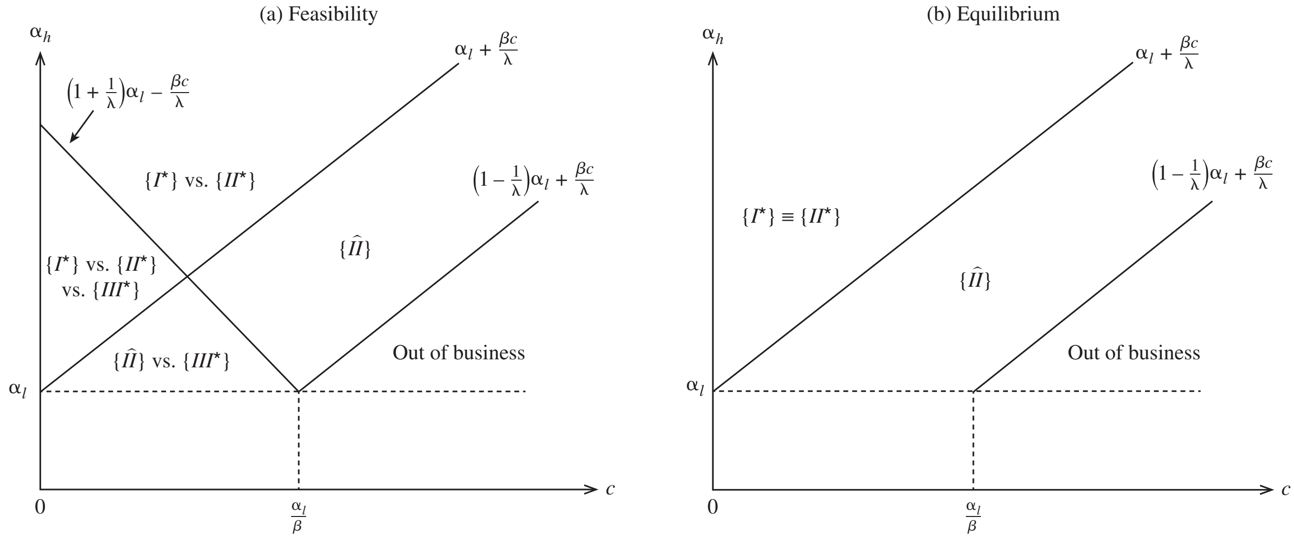
$$\max_{w,\gamma,b}\mathrm{E}[\Pi_M(w,\gamma,b)]=\lambda\Pi_M^h(\cdot)+(1-\lambda)\Pi_M^l(\cdot)$$
$$=\begin{cases}q_I(w-c)+(1-\lambda)\left[\gamma q_I w+\left((1-\gamma)q_I-\dfrac{\alpha^l-\beta b}{2}\right)b\right]\\ \qquad\qquad\text{if } b\le w<\underline{w},\\ q_{II}(w-c)+(1-\lambda)\gamma q_{II}w\quad \text{if } \max\{\underline{w},\hat{w}\}\le w\le\bar{w},\\ \dfrac{\alpha^h-\beta w}{2}(w-c)-(1-\lambda)\dfrac{\alpha^h-\alpha^l}{2}w\quad \text{if } b\le w<\hat{w}.\end{cases}\tag{A.6}$$

To solve this problem, we consider three constrained optimization problems, namely, (1) [P-I], $\max_{w,\gamma,b} q_I(w-c)+(1-\lambda)[\gamma q_I w+((1-\gamma)q_I-(\alpha^l-\beta b)/2)b]$, s.t. $b\le w<\underline{w}$ and $0\le\gamma\le 1$; (2) [P-II], $\max_{w,\gamma,b} q_{II}(w-c)+(1-\lambda)\gamma q_{II}w$, s.t. $\max\{\underline{w},\hat{w}\}\le w\le\bar{w}$ and $0\le\gamma\le 1$; and (3) [P-III], $\max_{w,\gamma,b}((\alpha^h-\beta w)/2)(w-c)-(1-\lambda)((\alpha^h-\alpha^l)/2)w$, s.t. $b\le w<\hat{w}$ and $0\le\gamma\le 1$.

By solving the KKT first-order conditions of [P-I], we obtain (1) an interior solution, denoted by $\{I^*\}$, which consists of $w_I^*=(\lambda\alpha^h+(1-\lambda)(1-\gamma)\alpha^l+\beta c)/(2\beta[1-\gamma(1-\lambda)])$, $b_I^*=\alpha^l/(2\beta)$, and γ taking on any value in $[0,1-\lambda\alpha^l/(\lambda\alpha^h-\beta c))$, and (2) two corner solutions, denoted by $\{\hat{I}\}$ and $\{\tilde{I}\}$, which are given by (a) $\hat{w}_I=(\lambda[\lambda(\alpha^h)^2+(1-\lambda)(\alpha^l)^2]-\beta^2c^2)/(2\lambda\beta[\lambda\alpha^h+(1-\lambda)\alpha^l-\beta c])$, $\hat{b}_I=\alpha^l/(2\beta)$, and $\hat{\gamma}_I=1-\lambda\alpha^l/(\lambda\alpha^h-\beta c)$, and (b) $\tilde{w}_I=(\lambda\alpha^h+(1-\lambda)\alpha^l+\beta c)/(2\beta)$, $\tilde{b}_I=\alpha^l/(2\beta)$, and $\tilde{\gamma}_I=0$, respectively. The interior solution arises when $\alpha^h>\alpha^l+\beta c/\lambda$, while the corner solutions require $\alpha^h\ge\alpha^l+\beta c/\lambda$. Here, note that (1) $\hat{w}_I=w_I^*|_{\gamma=1-\lambda\alpha^l/(\lambda\alpha^h-\beta c)}$, (2) $\tilde{w}_I=w_I^*|_{\gamma=0}$, and most importantly, the optimal expected profit of M is identical at these solutions: $\Pi_I^*=\hat{\Pi}_I=\tilde{\Pi}_I=(\lambda\alpha^h-\beta c)^2/(8\lambda\beta)+(1-\lambda)(\alpha^l)^2/(8\beta)$. This implies that the two corner solutions arise as special cases of and can be represented by $\{I^*\}$. (The technical derivation of this and the subsequent problems is available from the authors on request.)

Next, we solve [P-II] and obtain (1) an interior solution, denoted by $\{II^*\}$, at which $w_{II}^*=(\lambda[\lambda(\alpha^h)^2+(1-\lambda)(\alpha^l)^2]-\beta^2c^2)/(2\lambda\beta[\lambda\alpha^h+(1-\lambda)\alpha^l-\beta c])$, $\gamma_{II}^*=1-\lambda\alpha^l/(\lambda\alpha^h-\beta c)$, and b takes on any value in $[0,\alpha^l/(2\beta))$, and (2) a corner solution, denoted by $\{\hat{II}\}$, featuring $\hat{w}_{II}=(\lambda\alpha^h+(1-\lambda)\alpha^l+\beta c)/(2\beta)$, $\hat{\gamma}_{II}=0$, and b taking on any value in $[0,((1+\lambda)\alpha^l-\lambda\alpha^h+\beta c)/(2\beta))$. If offering a contract based on the interior solution $\{II^*\}$, which is feasible when $\alpha^h\ge\alpha^l+\beta c/\lambda$, the manufacturer earns an optimal expected profit $\Pi_{II}^*=(\lambda\alpha^h-\beta c)^2/(8\lambda\beta)+(1-\lambda)(\alpha^l)^2/(8\beta)$; by contrast, if choosing the corner solution, which is feasible when $-((1-\lambda)/\lambda)\alpha+\beta c/\lambda\le\alpha^h<\alpha^l+\beta c/\lambda$, it earns $\hat{\Pi}_{II}=[\lambda\alpha^h+(1-\lambda)\alpha^l-\beta c]^2/(8\beta)$. Finally, problem [P-III] has one solution, denoted by $\{III^*\}$, which consists of $w_{III}^*=(\lambda\alpha^h+(1-\lambda)\alpha^l+\beta c)/(2\beta)$, γ taking any value in $[2(\alpha^h-\alpha^l)/((1-\lambda)(\alpha^h-\alpha^l)+\alpha^h-\beta c),1]$, and b taking any value less than w_{III}^*. This solution arises when

Figure A.2. The Optimal Contract in Equilibrium



$\alpha^h \le (1+1/\lambda)\alpha^l - \beta c/\lambda$ and allows M to earn $\Pi^*_{III} = ([\lambda\alpha^h + (1-\lambda)\alpha^l - \beta c]^2 - 4(1-\lambda)(\alpha^h - \alpha^l)\beta c)/(8\beta)$.

In summary, we have four solution candidates, that is, $\{I^*\}$, $\{II^*\}$, $\{\widehat{II}\}$, and $\{III^*\}$, which are feasible under different conditions as shown in Figure A.2(a). Accordingly, if $\alpha^h \ge \max\{\alpha^l + \beta c/\lambda, (1+1/\lambda)\alpha^l - \beta c/\lambda\}$, then both $\{I^*\}$ and $\{II^*\}$ are feasible and provide M with identical profits: $\Pi^*_I = \Pi^*_{II} = (\lambda\alpha^h - \beta c)^2/(8\lambda\beta) + (1-\lambda)(\alpha^l)^2/(8\beta)$. Next, if $\alpha^l + \beta c/\lambda \le \alpha^h < (1+1/\lambda)\alpha^l - \beta c/\lambda$, among the three feasible candidates, that is, $\{I^*\}$, $\{II^*\}$, and $\{III^*\}$, M will choose either $\{I^*\}$ or $\{II^*\}$—and is indifferent between them—because $\{III^*\}$ is dominated by these two: $\Pi^*_I - \Pi^*_{III} = (1-\lambda)[\lambda(\alpha^h - \alpha^l) + \beta c]^2/(8\lambda\beta) \ge 0 \Rightarrow \Pi^*_I = \Pi^*_{II} \ge \Pi^*_{III}$. If $\alpha^h < \min\{\alpha^l + \beta c/\lambda, (1+1/\lambda)\alpha^l - \beta c/\lambda\}$, then M will choose $\{\widehat{II}\}$ since $\hat{\Pi}_{II} - \Pi^*_{III} = ((1-\lambda)(\alpha^h - \alpha^l)c)/2 \ge 0 \Rightarrow \hat{\Pi}_{II} \ge \Pi^*_{III}$. Finally, if $\max\{(1+1/\lambda)\alpha^l - \beta c/\lambda, (1-1/\lambda)\alpha^l + \beta c/\lambda\} \le \alpha^h < \alpha^l + \beta c/\lambda$, then only $\{\widehat{II}\}$ is feasible. Put this together, we arrive at the optimal solution that arises in equilibrium; that is, (1) if $\alpha^h \ge \alpha^l + \beta c/\lambda$, then M will choose either $\{I^*\}$ or $\{II^*\}$, and (2) if $(1-1/\lambda)\alpha^l + \beta c/\lambda \le \alpha^h < \alpha^l + \beta c/\lambda$, then $\{\widehat{II}\}$ is chosen in equilibrium. This result is graphically presented in Figure A.2.

Focus on $\{I^*\}$ with $\gamma \in (0, 1 - \lambda\alpha^l/(\lambda\alpha^h - \beta c))$, which effectively describes the combination contract, and define $w^*_C \overset{\text{def}}{=} w^*_I$ and $b^*_C \overset{\text{def}}{=} b^*_I$. The statement of Lemma 4 then follows. □

Proof of Proposition 2 (The Expectation and Variation in Profits Under the C Contract). Under the C contract, R will order a quantity of $q^*_C = q_I|_{w=w^*_C, b=b^*_C} = (\lambda\alpha^h - \beta c)/(4\lambda)$. If the high state arises, then it clears this stock by setting the price at $p^h_C = p^*(q^*_C; \alpha^h)|_{w=w^*_C, b=b^*_C} = \frac{1}{4}(3\alpha^h/\beta + c/\lambda)$. The profits earned by R and M are $\Pi^{h*}_{RC} = q^*_C(p^h_C - w^*_C) = (\lambda\alpha^h - \beta c)\{\lambda[\lambda\alpha^h + (1-\lambda)\cdot(1-\gamma)(3\alpha^h - 2\alpha^l)] - [\lambda - (1-\lambda)(1-\gamma)]\beta c\}/(16\lambda^2\beta[1-\gamma\cdot(1-\lambda)])$, where $\gamma \in (0, 1 - \lambda\alpha^l/(\lambda\alpha^h - \beta c))$ and $\Pi^{h*}_{MC} = q^*_C(w^*_C - c) = (\alpha^h - \beta c)\{\lambda\alpha^h + (1-\lambda)(1-\gamma)\alpha^l - [1 - 2\gamma(1-\lambda)]\beta c\}/(8\lambda\beta\cdot[1-\gamma(1-\lambda)])$, respectively.

If the low state arises, then R sets the price at $p^l_C = p^*(q^*_C; \alpha^l)|_{w=w^*_C, b=b^*_C} = 3\alpha^l/(4\beta)$ and generates sales $D(p^l_C; \alpha^l) = \alpha^l/4$. All excess inventory is returned; however, only a fraction of it, that is, γq^*_C, is fully refunded, and the remaining, that is, $(1-\gamma)q^*_C - D(p^l_C; \alpha^l)$, receives a partial refund. Accordingly, R earns a profit equal to $\Pi^{l*}_{RC} = D(p^l_C; \alpha^l)p^l_C + \gamma q^*_C w^*_C + [(1-\gamma)q^*_C - D(p^l_C; \alpha^l)]b^*_C - q^*_C w^*_C = \alpha^{l^2}/(16\beta) - ((1-\gamma)(\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) + \beta c])/(8\lambda\beta[1-\gamma(1-\lambda)])$, and M earns $\Pi^{l*}_{MC} = q^*_C(w^*_C - c) - \gamma q^*_C w^*_C - [(1-\gamma)q^*_C - D(p^l_C; \alpha^l)]b^*_C = \alpha^{l^2}/(8\beta) + (\lambda\alpha^h - \beta c)\{\lambda(1-\gamma)(\alpha^h - \alpha^l) - [1-\gamma(1-2\lambda)]\beta c\}/(8\lambda\beta[1-\gamma(1-\lambda)])$.

The expected profits of R and M are then given by $\mathrm{E}[\Pi^*_{RC}] = \lambda\Pi^{h*}_{RC} + (1-\lambda)\Pi^{l*}_{RC} = (\lambda\alpha^h - \beta c)^2/(16\lambda\beta) + (1-\lambda)\cdot\alpha^{l^2}/(16\beta)$ and $\mathrm{E}[\Pi^*_{MC}] = \lambda\Pi^{h*}_{MC} + (1-\lambda)\Pi^{l*}_{MC} = (\lambda\alpha^h - \beta c)^2/(8\lambda\beta) + (1-\lambda)\alpha^{l^2}/(8\beta)$, respectively. Denote the variation in profits of R and M by $\Delta_{RC} \overset{\text{def}}{=} \Pi^{h*}_{RC} - \Pi^{l*}_{RC}$ and $\Delta_{MC} \overset{\text{def}}{=} \Pi^{h*}_{MC} - \Pi^{l*}_{MC}$. We have $\partial\Delta_{RC}/\partial\gamma = -(\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) + \beta c]/(8\lambda\beta[1-\gamma(1-\lambda)]^2) < 0$ and $\partial\Delta_{MC}/\partial\gamma = (\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) + \beta c]/(8\lambda\beta[1-\gamma(1-\lambda)]^2) > 0$. The statement of Proposition 2 then follows. □

Proof of Lemma 1 (The Optimal P Contract). It follows from the derivation of the C contract that M can choose $\{I^*\}$ with γ set equal to 0. Such a contract features a partial-refund policy. Accordingly, we define $w^*_P \overset{\text{def}}{=} w^*_I|_{\gamma=0}$ and $b^*_P \overset{\text{def}}{=} b^*_I$ and obtain the statement of Lemma 1. The order quantity and retail prices under this contract are $q^*_P = q_I|_{w=w^*_P, b=b^*_P, \gamma=0} = (\lambda\alpha^h - \beta c)/(4\lambda)$, $p^h_P = p^*(q^*_P; \alpha^h)|_{w=w^*_P, b=b^*_P} = \frac{1}{4}(3\alpha^h/\beta + c/\lambda)$, and $p^l_P = p^*(q^*_P; \alpha^l)|_{w=w^*_P, b=b^*_P} = 3\alpha^l/(4\beta)$, respectively. Eventually, R clears all q^*_P in the high state, and returns the unsold inventory, which is equal to $q^*_P - D(p^l_P; \alpha^l) = (\lambda(\alpha^h - \alpha^l) - \beta c)/(4\lambda)$, for a partial refund in the low state. See Table 2. □

Proof of Lemma 2 (The Optimal Q Contract). Follow the derivation of the C contract, and let M choose $\{II^*\}$. Define $w^*_Q \overset{\text{def}}{=} w^*_{II}$ and $\gamma^*_Q \overset{\text{def}}{=} \gamma^*_{II}$. This solution describes the quota contract, which is formalized in Lemma 2. Under this contract, R will order $q^*_Q = q_{II}|_{w=w^*_Q, \gamma=\gamma^*_Q} = (\lambda\alpha^h - \beta c)/(4\lambda)$, which is cleared in the high state at a retail price equal to $p^h_Q = p^*(q^*_Q; \alpha^h)|_{w=w^*_Q, \gamma=\gamma^*_Q} = \frac{1}{4}(3\alpha^h/\beta + c/\lambda)$. In the low state, the retail price is $p^l_Q = p^*(q^*_Q; \alpha^l)|_{w=w^*_Q, \gamma=\gamma^*_Q} = 3\alpha^l/(4\beta)$, and the quantity of unsold inventory is $q^*_Q - D(p^l_Q; \alpha^l) = (\lambda(\alpha^h - \alpha^l) - \beta c)/(4\lambda)$, which is equal to the quota (i.e., $\gamma^*_Q q^*_Q$ and is returned for a refund of the wholesale price; see Table 2). (Note that though $\{II^*\}$ mathematically includes $b \in [0, \alpha^l/(2\beta))$, this term is immaterial in equilibrium.) □

Proof of Proposition 1 (Profit Variations Under the Q and P Contracts). In the high state, R clears the entire stock (which is $q_Q^* = q_P^* = (\lambda\alpha^h - \beta c)/(4\lambda)$) by setting the retail prices at $p_Q^h = p_P^h = \frac{1}{4}(3\alpha^h/\beta + c/\lambda)$ (see the proofs of Lemmas 1 and 2), and earns profits equal to $\Pi_{RQ}^{h*} \overset{\text{def}}{=} q_Q^*(p_Q^h - w_Q^*) = (\lambda\alpha^h - \beta c)\{\lambda[\bar{\alpha}^2 + 3(1-\lambda)(\alpha^h - \alpha^l)\alpha^l] + (\bar{\alpha} - 3\lambda\alpha^h)\beta c + \beta^2 c^2\}/(16\lambda^2\beta(\bar{\alpha} - \beta c))$ and $\Pi_{RP}^{h*} \overset{\text{def}}{=} q_P^*(p_P^h - w_P^*) = (\lambda\alpha^h - \beta c)\{\lambda[\bar{\alpha} + 3(1-\lambda)(\alpha^h - \alpha^l)] - (2\lambda - 1)\beta c\}/(16\lambda^2\beta)$, wherein $\bar{\alpha} \overset{\text{def}}{=} \lambda\alpha^h + (1-\lambda)\alpha^l$ and $\bar{\alpha}^2 \overset{\text{def}}{=} \lambda{\alpha^h}^2 + (1-\lambda){\alpha^l}^2$, under the quota and partial-refund contracts, respectively. With no returns, M earns profits $\Pi_{MQ}^{h*} \overset{\text{def}}{=} q_Q^*(w_Q^* - c) = (\lambda\alpha^h - \beta c)[\lambda\bar{\alpha}^2 - 2\lambda\bar{\alpha}\beta c + (2\lambda - 1)\beta^2 c^2]/(8\lambda^2\beta \cdot (\bar{\alpha} - \beta c))$ and $\Pi_{MP}^{h*} \overset{\text{def}}{=} q_P^*(w_P^* - c) = (\lambda\alpha^h - \beta c)(\bar{\alpha} - \beta c)/(8\lambda\beta)$.

Now, if the low state arises, then under the Q contract, R returns the unsold inventory, which is equal to the quota, for a refund of the wholesale price, and earns $\Pi_{RQ}^{l*} \overset{\text{def}}{=} V^*(q_Q^*;\alpha^l)|_{w=w_Q^*,\gamma=\gamma_Q^*,b=0} - q_Q^* w_Q^* = {\alpha^l}^2/(16\beta) - (\alpha^l(\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) + \beta c])/(8\lambda\beta(\bar{\alpha} - \beta c))$. After paying the refund, M earns $\Pi_{MQ}^{l*} \overset{\text{def}}{=} \Pi^l(w_Q^*, \gamma_Q^*, 0; \alpha^l) = q_Q^*(w_Q^* - c) - \gamma q_Q^* w_Q^* = {\alpha^l}^2/(8\beta) + ((\alpha^l - 2\beta c)(\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) - \beta c])/(8\lambda\beta(\bar{\alpha} - \beta c))$. By contrast, under the partial-refund contract, R only receives a refund of b_P^* per each unit of the unsold stock; its profit is given by $\Pi_{RP}^{l*} \overset{\text{def}}{=} V^*(q_P^*;\alpha^l)|_{w=w_P^*,\gamma=0,b=b_P^*} - q_P^* w_P^* = {\alpha^l}^2/(16\beta) - (\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) + \beta c]/(8\lambda\beta)$. The manufacturer's profit is then $\Pi_{MP}^{l*} \overset{\text{def}}{=} \Pi^l(w_P^*, 0, b_P^*; \alpha^l) = q_P^*(w_P^* - c) - [q_P^* - D(p_P^l;\alpha^l)]b_P^* = {\alpha^l}^2/(8\beta) + (\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) - \beta c]/(8\lambda\beta)$.

Denote the variations in profits of R and M by $\Delta_{Rj} \overset{\text{def}}{=} \Pi_{Rj}^{h*} - \Pi_{Rj}^{l*}$ and $\Delta_{Mj} \overset{\text{def}}{=} \Pi_{Mj}^{h*} - \Pi_{Mj}^{l*}$, respectively, where $j = P, Q, C$ is the type of contract. We have (1) $\Delta_{RP} - \Delta_{RC} = (\gamma(\lambda\alpha^h - \beta c)[\lambda \cdot (\alpha^h - \alpha^l) + \beta c])/(8\beta\lambda[1 - \gamma(1-\lambda)]) > 0$ and $\Delta_{RC} - \Delta_{RQ} = ((\lambda\alpha^h - \beta c)(\lambda(\alpha^h - \alpha^l) + \beta c)[(1-\gamma)(\lambda\alpha^h - \beta c) - \lambda\alpha^l])/(8\beta\lambda^2(\gamma(\lambda - 1) + 1)[\lambda\alpha^h + (1-\lambda)\alpha^l - \beta c]) > 0$, and (2) $\Delta_{MC} - \Delta_{MP} = (\gamma(\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) + \beta c])/(8\beta\lambda[1 - \gamma(1-\lambda)]) > 0$ and $\Delta_{MQ} - \Delta_{MC} = ((\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \alpha^l) + \beta c][(1-\gamma)(\lambda\alpha^h - \beta c) - \lambda\alpha^l])/(8\beta\lambda^2[1 - \gamma(1-\lambda)][\lambda\alpha^h + (1-\lambda)\alpha^l - \beta c]) > 0$. This implies that $\Delta_{RP} > \Delta_{RC} > \Delta_{RQ}$ and $\Delta_{MQ} > \Delta_{MC} > \Delta_{MP}$. The statement of Proposition 1 then follows. □

Proof of Lemma 3 (Centralized Channel (CC) vs. Decentralized Channel (DQ/DP)). Recall that in the centralized channel, M sells directly to consumers. In Stage 2, after observing α, it sets the retail price to maximize revenue: $\max_p V_M(p;\alpha) = (\alpha - \beta p)p$, subject to $q \geq \alpha - \beta p$ (wherein $\alpha \in \{\alpha^h, \alpha^l\}$). Solving the respective first-order conditions of the associated Lagrangian function—that is, $\mathcal{L}(p;\alpha) = V_M(p;\alpha) + \mu(q - \alpha + \beta p)$—we obtain an interior solution equal to $\alpha/(2\beta)$ when $q > \alpha/2$ and a corner solution equal to $(\alpha - q)/\beta$ when $q \leq \alpha/2$. Put together, M sets the retail price as follows:

$$p^*(q;\alpha) = \begin{cases} \dfrac{\alpha}{2\beta} & \text{if } q > \dfrac{\alpha}{2}, \\ \dfrac{\alpha - q}{\beta} & \text{if } q \leq \dfrac{\alpha}{2}. \end{cases} \tag{A.7}$$

Here, M remains with an amount of unsold inventory if $q > \alpha/2$, but clears all inventory if $q \leq \alpha/2$. Its revenue is given by

$$V_M^*(q;\alpha) = \begin{cases} \dfrac{\alpha^2}{4\beta} & \text{if } q > \dfrac{\alpha}{2}, \\ \dfrac{q(\alpha - q)}{\beta} & \text{if } q \leq \dfrac{\alpha}{2}. \end{cases} \tag{A.8}$$

In the first stage, M makes the decision on the quantity under market uncertainty. Given $\alpha \in \{\alpha^h, \alpha^l\}$ and how the retail price is set in Stage 1 (see Equation (A.7)), three possibilities may arise: (1) if $q > \alpha^h/2 > \alpha^l/2$, M will carry unsold inventory in both states; (2) if $\alpha^h/2 \geq q > \alpha^l/2$, M will clear all inventory in the high state and be left with some unsold stock in the low state; and (3) if $q \leq \alpha^l/2 < \alpha^h/2$, M will clear all stock in both states. Accordingly, its expected profit function is given by

$$E[\Pi_M(q)] \overset{\text{def}}{=} \lambda V_M(q;\alpha^h) + (1-\lambda)V_M(q;\alpha^l) - qc$$
$$= \begin{cases} \lambda\dfrac{{\alpha^h}^2}{4\beta} + (1-\lambda)\dfrac{{\alpha^l}^2}{4\beta} - qc \overset{\text{def}}{=} \Pi_1 & \text{if } q > \dfrac{\alpha^h}{2}, \\ \lambda\dfrac{q(\alpha^h - q)}{\beta} + (1-\lambda)\dfrac{{\alpha^l}^2}{4\beta} - qc \overset{\text{def}}{=} \Pi_2 & \text{if } \dfrac{\alpha^l}{2} < q \leq \dfrac{\alpha^h}{2}, \\ \lambda\dfrac{q(\alpha^h - q)}{\beta} + (1-\lambda)\dfrac{q(\alpha^l - q)}{\beta} - qc \overset{\text{def}}{=} \Pi_3 & \text{if } q \leq \dfrac{\alpha^l}{2}. \end{cases} \tag{A.9}$$

Now, we can solve M's optimization problem, that is, $\max_q E[\Pi_M(q)]$. To do so, we solve three constrained optimization problems as defined by the three respective piecewise functions and compare their solutions to determine the optimal one. First, consider $\max_q \Pi_1$ s.t. $q > \alpha^h/2$. Because Π_1 is a linear and decreasing function in $q \in (\alpha^h/2, +\infty)$, its maximum is attained at $q = \alpha^h/2$. Next, consider $\max_q \Pi_2$ s.t. $\alpha^l/2 < q \leq \alpha^h/2$. Solving the first-order conditions of the associated Lagrangian function, we obtain an interior solution equal to $\alpha^h/2 - \beta c/2\lambda$ and a corner solution equal to $\alpha^l/2$, which arise under the conditions $\alpha^h \geq \alpha^l + \beta c/\lambda$ and $\alpha^h < \alpha^l + \beta c/\lambda$, respectively. Finally, the third constrained optimization problem, that is, $\max_q \Pi_3$ s.t. $q \leq \alpha^l/2$, has an interior solution equal to $(\lambda\alpha^h + (1-\lambda)\alpha^l - \beta c)/2$ and a corner solution equal to $\alpha^l/2$, which arise under the conditions $\alpha^h \leq \alpha^l + \beta c/\lambda$ and $\alpha^h > \alpha^l + \beta c/\lambda$, respectively.

Recall that our analysis focuses on the setting under which $\alpha^h \geq \alpha^l + \beta c/\lambda$; see Figure A.2 and the proof of Lemma 4. Accordingly, we compare $\Pi_1|_{q=\alpha^h/2}$ to $\Pi_2|_{q=\alpha^h/2-\beta c/2\lambda}$ and $\Pi_3|_{q=\alpha^l/2}$ to obtain the optimal quantity, which is $q_{CC} = \alpha^h/2 - \beta c/(2\lambda)$. This quantity is cleared in the high state at a retail price equal to $p_{CC}^h = \alpha^h/(2\beta) + c/(2\lambda)$; by contrast, in the low state, the retail price is $p_{CC}^l = \alpha^l/(2\beta)$, and the amount of unsold inventory is equal to $(\lambda(\alpha^h - \alpha^l) - \beta c)/(2\lambda)$. The manufacturer then earns $\Pi_{MCC}^h = (\lambda\alpha^h - \beta c)[\lambda(\alpha^h - \beta c) + (1-\lambda) \cdot \beta c]/(4\lambda^2\beta)$ and $\Pi_{MCC}^l = \frac{1}{4}(-2\alpha^h c + {\alpha^l}^2/\beta + (2\beta c^2)/\lambda)$ in the high and low states, respectively. Define $\Delta_{CC} \overset{\text{def}}{=} \Pi_{MCC}^h - \Pi_{MCC}^l$. By comparing Δ_{CC} to $\Delta_{DQ} \overset{\text{def}}{=} (\Pi_{MQ}^{h*} + \Pi_{RQ}^{h*}) - (\Pi_{MQ}^{l*} + \Pi_{RQ}^{l*})$ and $\Delta_{DP} \overset{\text{def}}{=} (\Pi_{MP}^{h*} + \Pi_{RP}^{h*}) - (\Pi_{MP}^{l*} + \Pi_{RP}^{l*})$, we obtain the result, which is described in Lemma 3. See also Table 4 for a summary of this result. □

Endnotes

[1] Here, the difference between the wholesale price and the return price is equivalent to a restocking fee charged by the manufacturer.

[2] Pasternack (1985) considers the use of a limited quantity of returns, but the focus of the paper is on channel coordination, and it is noted that allowing unlimited returns for full credit is never optimal. By contrast, the focus of our paper is to derive the optimal contract structure for the different types of return policies and to discuss the relative preferences of the manufacturer and the retailer.

[3] We thank the reviewers and the associate editor for the suggestion and have added a new section in this paper (Section 5).

[4] Manufacturers may also use various vertical restraints, such as resale price maintenance, quantity forcing, franchise fees, etc., to influence retailers' incentives and to align them with those of the manufacturers (e.g., Mathewson and Winter 1984).

[5] However, when resale prices are endogenously determined (as in our model), returns policies without additional contractual instruments (e.g., a two part tariff) do not achieve channel coordination (Song et al. 2008).

[6] Note that both the variable(s) and focal parameter(s)—which are p and α, respectively—are explicitly specified and separated by a semicolon in the notation of the demand function $D(p;\alpha)$. However, where the focus is not on the variable(s) and/or parameter(s), the notation is simplified to $D(\cdot)$. This convention is used consistently to define other functions in this paper.

[7] The opposing effects counterbalance perfectly in our model, which is based on the assumption of binary demand outcomes and risk-neutral players.

[8] We can also show that the pattern of each party's profit variation will remain unchanged when an independent manufacturer attempts to coordinate the reseller's pricing and quantity decisions by designing the contracts (with partial-refund and quota policies along with, say, a two-part tariff) to induce the prices and quantity that the centralized manufacturer would set optimally. Details are available from the authors on request.

[9] The proof is available from the authors on request.

[10] For simplicity, we omit the variation-induced disutility in this section, and as in the analysis in Sections 3 and 4, we extend the model to study the combination contract with risk-neutral players.

[11] Raising the quota can also be used to attract new resellers/distributors and/or reward the loyalty of the current ones, as evidenced in the Eaton Corporation's Distributor Commitments Program (Eaton 2014). Eaton offers a 3% quota to its most preferred distributors and 2% to the rest, indicating that it is more willing to absorb the cost of full refunds (up to the quota) for its preferred customers. In other cases, as noted in Section 1, HP used to offer a quota to its preferred distributors in the past, but no longer does so, indicating its preference to use the partial-refund (restocking fee) contract only.

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