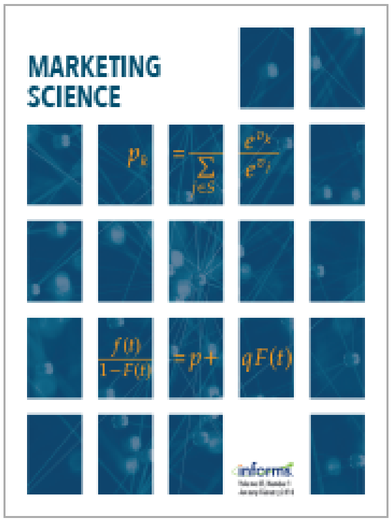

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A Dynamic Model of Repositioning

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Abstract. Consumer preferences change through time and firms must adjust their product positioning for their products to continue to be appealing to consumers. These changes in product positioning require fixed investments such that firms reposition only occasionally. I construct a model that can include predictable and unpredictable consumer preference changes, and where a firm optimally repositions its product given the current market conditions, and expected future repositionings. When unpredictable consumer preferences evolve away from a current firm's positioning, the decision to reposition is like exercising an option to be closer to current consumer preferences, or waiting to reposition later or for those preferences to return so as to be closer to the firm's current position. We characterize this optimal repositioning strategy, how it depends on the discount factor, variance of preferences, and repositioning costs. I compare the optimal policy of the firm with what could be optimal from a social welfare point of view, and find that the firm repositions more frequently than is efficient when there is full market coverage. With predictable changes in consumer preferences, the optimal repositioning strategy involves overshooting and asymmetric repositioning thresholds.

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1. Introduction

Consumer preferences change over time and firms must adjust product positioning to continue to be closer to those preferences. At the same time changes in product positioning require fixed investments and cannot be optimally done for the product to continuously match consumer preferences. That is, firms reposition a product only when those preferences are sufficiently far away from the current product position. In fact, firms only occasionally change their products, packages, logos or positioning communication. Examples include Morton Salt which changes its packaging every few years, and car manufacturers who release a major redesign every three to five years (e.g., BMW, Honda, Mercedes, Toyota).[1] Most companies also adjust their basic logo every few years (e.g., Apple, IBM). Some of these changes are to ensure that products or communications adhere to changing styles. For example, a necktie manufacturer may adjust the length and width of that accessory to match style preferences, which change over time. Similarly, fashion designers will adjust the length of skirts to match periodic changes in customer preferences.

Consumer preference variations involve predictable and unpredictable changes. There are trends in how preferences change, but the way in which they change at any given time may be unknown. When repositioning, firms must be aware of the potential need to reposition in the future, and that consumer preferences may come back to where the product is positioned. This provides an incentive for firms to reposition only when those preferences are sufficiently far away.[2]

I construct a continuous-time model that fully takes into account the option of when to reposition, while also considering possible future repositionings. When a firm repositions a product, there is a trade-off between the benefits of being at the center of the market and the fixed costs of that action. Consider first the case in which there are no trends in consumer preferences. The optimal strategy involves a threshold such that if consumer preferences are sufficiently far away from the firm's current positioning, the firm repositions to the center of those preferences. This threshold is greater, the greater the discount rate, as in that case the present value of the benefits of repositioning are lower. The firm adjusts by repositioning less often.

Consider the effect of the variance of the process by which consumer preferences change. If that variance is greater, the threshold of repositioning becomes larger: The firm becomes more hopeful that consumer preferences will return to the firm's current positioning. This effect of the repositioning threshold being greater with a greater uncertainty of consumer preferences is smaller when the variance is greater: Being too far away from consumer preferences becomes too costly, and the firm has greater incentive to reposition. Interestingly, we can obtain that the expected time between repositionings is lower the greater the variance of preferences. That is, with a greater variance of consumer preferences, the adjustment of the greater threshold is not enough to overcome the effect of more quickly reaching a threshold. Thus, the firm must reposition more frequently.

As expected, the effect of repositioning costs on the threshold to reposition is also monotonically increasing. More interestingly, this effect occurs at a decreasing rate: When consumer preferences are too far away from the firm's current position, it becomes too costly not to reposition. Obviously, if repositioning costs are too high, then the firm chooses never to reposition. Repositioning costs may also be increasing in the distance by which the product is repositioned. For example, repositioning a product a short distance can involve lower costs in product redesign and communication than repositioning a product over a greater distance. This paper also explores this effect, showing that with repositioning costs increasing in the distance traveled, and with total repositioning costs fixed, the firm may choose to reposition more frequently, and not all the way to the center of the market.

When the market is partially covered, the optimal repositioning strategy involves less frequent repositionings when consumer heterogeneity is greater. This is consistent with the possibility of frequent repositionings in the early days of a new product category, when consumer preferences may be less heterogeneous, with less frequent repositionings later on, when the product category is more established, and those preferences possibly more heterogeneous.[3]

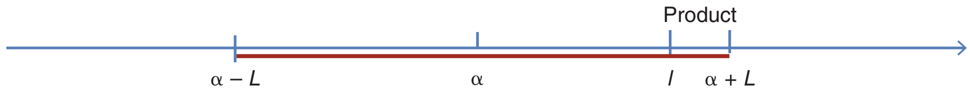
We can also compare the firm's optimal behavior with what would be optimal from a social welfare point of view. We find that the firm ends up repositioning too frequently compared to what would be optimal from a social welfare point of view in a situation where the market is always fully covered. The intuition is that the firm's profits fall more steeply than social welfare when consumer preferences move away from where the product is positioned. Thus, the firm is incentivized to reposition sooner than would be optimal in terms of social welfare.

When there are trends in consumer preferences the firm has two different thresholds, depending on the direction of those trends. When consumer preferences are trending away from the firm's current position, the firm is less tolerant and repositions sooner. When consumer preferences are trending towards the firm's current position, the firm only repositions if it is too far away from those preferences. In this case, with trends in consumer preferences, the optimal repositioning is to overshoot current preferences. This way the preferences will trend to the firm's new position, and the firm will save on repositioning costs. In markets where technology is a major component of the repositioning decision, anecdotal evidence suggests that newer products come with more features than most consumers may demand in the short run; this may be seen as overshooting the product's repositioning. In this case, with trends in consumer preferences, we fully and analytically solve the case with no uncertainty. We also show that the degree of overshooting and the two thresholds increase at a decreasing rate on the intensity of the trend, and on repositioning costs. In addition, we present simulations for the case wherein there are trends and unpredictable changes in the consumer preferences.

There has been substantial research on static positioning in markets (e.g., Hauser and Shugan 1983, Moorthy 1988, Hauser 1988, Sayman et al. 2002, Kuksov 2004, Lauga and Ofek 2011, Hauser et al. 2016), with a particular focus on competitive interaction. There is also work on the effects of the firm's resources on their strategic positioning (e.g., Wernerfelt 1989). As to dynamics, there is work on investments in research and development (R&D) (e.g., Harris and Vickers 1987, Ofek and Sarvary 2003) that generates results with some probability of success in product repositioning. By contrast, this paper allows the repositioning decision to have immediate effects; thus, the timing of the product reposition decision becomes crucial. A similar decision to that considered here is adoption of new technologies, and when to adopt, which is considered in a two-state version in Villas-Boas (1992). Here we consider a richer, uncertain environment, where the decision on when to reposition is investigated in greater depth for the monopoly setting. Another related stream of work considers richer environments of dynamic R&D competition among firms: This research stream is primarily empirical and has models that can be solved numerically (e.g., Erickson and Pakes 1995, and, especially with dynamic repositioning, Sweeting 2013 and Jeziorski 2014). As it relates to that work, our paper presents, in a monopoly setting, a sharper analysis of when to reposition, and how that decision depends on the degree of market uncertainty, the discount rate, and any market trends.[4]

The remainder of the paper is organized as follows. Section 2 presents the general set-up of consumer preferences and how those changes affect profits. Section 3 presents the case when there are no trends in consumer preferences, and all of the changes in those preferences are unpredictable. Section 4 introduces the possibility of trends in consumer preferences and examines what happens when all changes in those preferences are deterministic. Section 5 presents simulations on the optimal policy when consumer preferences exhibit predictable and unpredictable changes. Concluding remarks are provided in Section 6.

Figure 1. (Color online) Illustration of Consumer Preferences on the Real Line Centered at a, with Length $2L$, and with the Product Positioning at l



2. Market Set-Up

Consider a market where consumer preferences are described by the location of consumers on the real line, where a consumer at a point on the line values a product at a distance x from the consumer as $v - x$ where v is assumed to be large. Consumer preferences are uniformly distributed on a segment of distance $2L$ with a midpoint a. The mass of consumers is one. The product cannot be stored and can be consumed in every period.

There is one firm offering one product in this market. Let l be the product's location, and let $z \equiv l - a$, such that the distance between the product location and the midpoint of the consumer preferences is $|z| = |l - a|$. Figure 1 illustrates the positioning of the firm, and the location of consumer preferences in the real line. If the firm charges a price $P = v - L - |z|$ it attracts all consumers. If v is large enough ($v > 3L + |z|$), charging this price is optimal, and the profit is $\pi(z) = v - L - |z|$. The firm cannot price discriminate between consumers.

If the product is positioned at the center of consumer preferences, $l = a$, then the profit is the largest possible and equal to $\pi = v - L$.

Now consider that the center of consumer preferences, a, changes over time, to the right or left. At some point, if a gets too far away from the firm's positioning l, the distance z becomes too high, and the firm must lower the price to attract all consumers, with resulting profits that are quite low. That is, if a moves sufficiently far away from l, the firm decides to reposition the product, paying a fixed cost K, and moving to a new positioning l', which is relatively close to a. The firm must make this decision, taking into account that consumer preferences will continue to evolve over time, and that future repositionings will be needed.[5] Thus, our focus questions are: When is it optimal for the firm to reposition? How does the firm decide how far to reposition?

Consider now that the center of consumer preferences, a, continuously evolves over time as a Brownian motion

$$da = b\,dt + \sigma\,dw, \tag{1}$$

where dw is the standardized Brownian motion, $b\,dt$ represents the deterministic component of how a evolves, and $\sigma\,dw$ represents the random component of how a evolves. The parameter b represents the speed and direction at which the firm expects consumer preferences to evolve over time. The parameter σ represents the randomness of how a evolves. In Section 3, we focus on the case when $b = 0$, such that a only evolves at random. In Section 4 we consider the case where $b \neq 0$ but $\sigma = 0$, such that a only evolves deterministically. Section 5 considers numerically the case of when b and σ are different from zero.

Per the definition of z, we then have that while the product is not repositioned the evolution of z is

$$dz = -b\,dt + \sigma\,dw. \tag{2}$$

When the firm chooses to reposition its product, z moves instantly to where the firm wants it to be. The firm has the option to reposition or wait for preferences to return to the firm's current position. If preferences move too far from where the firm is positioned, it chooses to reposition, knowing that further repositionings will be necessary in the future.

3. Purely Random Evolution of Consumer Preferences

Consider first the case in which all changes in consumer preferences are unpredictable, $b = 0$. In this case, the firm repositions to $l = a$, which means $z = 0$. That is, when the firm repositions, it moves to the center of the market, a, as the market is equally likely to evolve in either direction.

Because the market is equally likely to evolve in either direction, we also have that the threshold a, where the firm chooses to reposition, is equally distant from l in both directions. That is, there will be a Δ, such that when the distance between a and l is Δ, the firm chooses to reposition: The firm chooses to reposition when $|z| = \Delta$, where Δ must be optimal for the firm. Now consider the case of $v > 3L$, such that the market is fully covered for $|z|$ small, and v large, compared with K ($K < \tilde{K}$ where $\tilde{K}$ is defined in (A.1)), such that it is optimal for the firm to keep the market fully covered on the equilibrium path.[6]

Let $V(z)$ be the expected net present value of profits when the firm is at a point z with respect to the center of the market (as noted above $z \equiv l - a$). Then when the firm is not repositioning, $V(z)$ can be written as

$$V(z) = \pi(z)\,dt + e^{-r\,dt} E V(z + dz), \tag{3}$$

where r is the instantaneous discount rate. Using a Taylor approximation of $V(z + dz)$ and applying Itô's Lemma we get

$$V(z) = \pi(z)\,dt + e^{-r\,dt}\left[V(z) + V'(z)E(dz) + V''(z)\frac{E(dz^2)}{2}\right].$$

Using the fact that $E(dz) = 0$, and $E(dz^2) = \sigma^2\,dt$, we then divide by dt and make $dt \to 0$, to obtain

$$rV(z) = \pi(z) + \frac{\sigma^2}{2}V''(z). \tag{4}$$

With $V(z)$ being the value of the firm, Equation (4) states that the return on the asset (the left-hand side (LHS) of (4)) is equal to the flow payoff, $\pi(z)$, plus the expected value of the capital gain, $(\sigma^2/2)V''(z)$, which is positive because the function $V(z)$ is convex (as the firm has an option to reposition if consumer preferences move too far away from the current product position).

As $\pi(z) = v - L - |z|$ and $V(z) = V(-z)$ by symmetry, we obtain the solution to the differential Equation (4) as

$$V(z) = C_1 e^{\sqrt{2r/\sigma^2}|z|} + C_2 e^{-\sqrt{2r/\sigma^2}|z|} - \frac{|z|}{r} + \frac{v-L}{r}, \tag{5}$$

where C_1 and C_2 are two constants to be determined.

When the firm chooses to reposition at distance Δ from the center of the market we must have

$$V(\Delta) = V(0) - K, \tag{6}$$

and the "smooth-pasting condition" at distance Δ (see, e.g., Dixit 1993)

$$V'(\Delta) = 0. \tag{7}$$

Condition (6) says that when the firm decides to reposition, it is indifferent between repositioning to the center of the market, and getting the present value of profits $V(0)$, while paying the repositioning costs K, and continuing at distance Δ from the center of the market without repositioning. If the LHS of (6) is greater than the right-hand side (RHS), the firm would be better off not repositioning and Δ would not be the repositioning threshold. If the LHS of (6) is smaller than the RHS, the firm should have repositioned before reaching the distance Δ from the center of the market, and then Δ would not be the repositioning threshold. Condition (7) states that Δ is the optimal repositioning threshold. Furthermore, the "smooth-pasting condition" must hold when the firm is at the center of the market, which means

$$V'(0^+) = V'(0^-). \tag{8}$$

Combining (6)–(8), we obtain C_1, C_2, and Δ to solve

$$C_1 = \sqrt{\frac{\sigma^2}{2r}}\frac{1}{r(1+e^{\sqrt{2r/\sigma^2}\Delta})}, \tag{9}$$

$$C_2 = C_1 - \frac{1}{r}\sqrt{\frac{\sigma^2}{2r}}, \tag{10}$$

$$rK = \Delta - 2\sqrt{\frac{\sigma^2}{2r}}\frac{e^{\sqrt{2r/\sigma^2}\Delta} - 1}{e^{\sqrt{2r/\sigma^2}\Delta} + 1}. \tag{11}$$

This completes the characterization of the value function and the optimal policy of the firm. Figure 2 illustrates the value function for $z \in [0, \Delta)$ for some parameter setting showing $V'(0) = V'(\Delta) = 0$. Figure 3 illustrates the evolution of the preferences over time and the optimal repositioning for a sample path.

Figure 2. (Color online) Value Function for Only Random Evolution of Preferences Case for $r = 0.1$, $\sigma^2 = 2$, $K = 3$, $v - L = 10$, and $\Delta = 3.4$

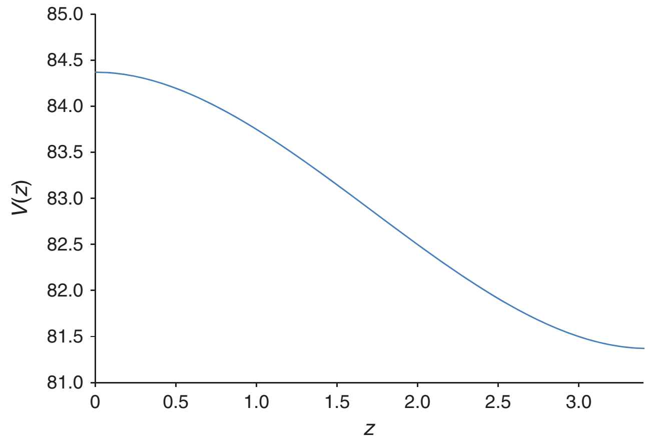
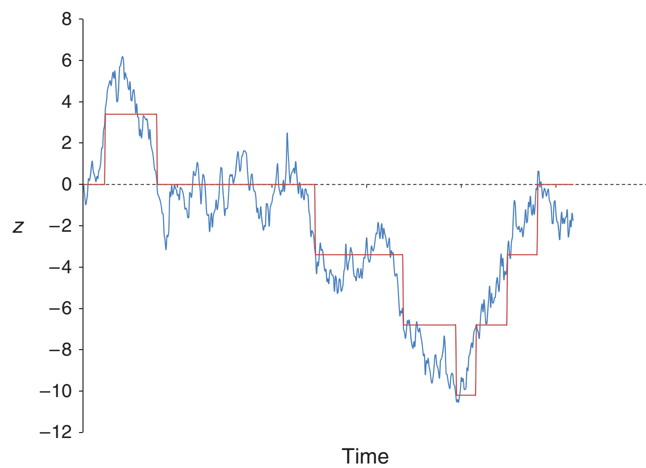


Figure 3. (Color online) Optimal Dynamic Repositionings: Example of a Sample Path and Optimal Repositionings for $r = 0.1$, $\sigma^2 = 2$, $K = 3$, $v - L = 10$, and $\Delta = 3.4$

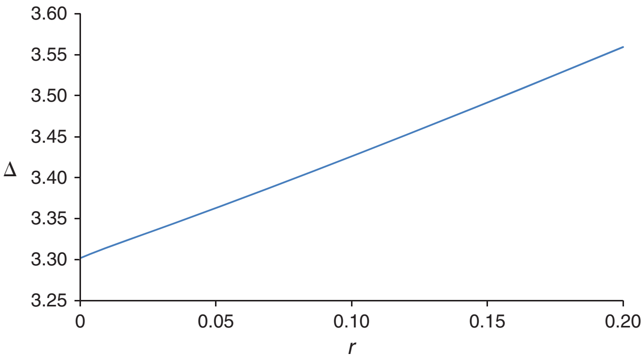


From (11) we can compute the comparative statics of the repositioning threshold Δ with respect to the discount rate r, variance of preferences σ^2, and repositioning cost. The following proposition states the results.

Proposition 1. *Consider the purely random evolution of preferences case. Then the repositioning threshold Δ is increasing in the discount rate r. Furthermore, the repositioning threshold Δ is increasing at a decreasing rate in the variance of the evolution of preferences σ^2, and in the repositioning cost K.*

As the discount rate increases, the present value of the benefits of repositioning decreases with respect to not repositioning. Therefore, as the discount rate increases the firm wants to reposition less often and

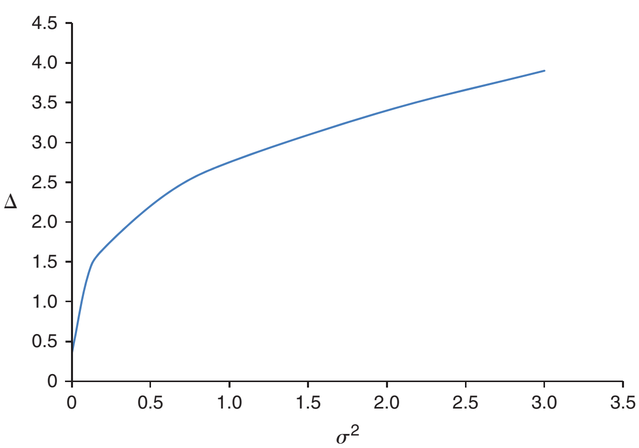
Figure 4. (Color online) Effect of the Discount Rate: Evolution of the Threshold Δ as a Function of the Discount Rate r for $\sigma^2 = 2$ and $K = 3$



waits for the preferences to be farther away. As expected, as the cost of repositioning increases, the firm prefers to reposition less often.

More interestingly, when the variance of the evolution of preferences increases, the firm realizes that even if consumer preferences are far away from the firm's current position, there is a greater likelihood of those preferences returning to the current position. Moreover, the value of being in the center of the market (having just repositioned) is lower when the variance of the evolution of preferences increases, as the preferences are more likely to move away from the firm's current position. Thus the firm decides to wait a little longer before repositioning when preferences move away. Furthermore, when the variance of preferences increases, this effect is reduced. When the preferences get too far away from the firm's current position, the firm's profits fall too much. That is, as the variance of the evolution of preferences increases, the threshold to reposition increases but at a decreasing rate. Figure 4 illustrates how the threshold Δ varies with the discount rate. Figure 5 illustrates how the threshold Δ varies with the variance of the preferences' evolution.

Figure 5. (Color online) Effect of the Variance: Evolution of the Threshold Δ as a Function of the Variance σ^2 for $K = 3$ and $r = 0.1$



When the discount rate converges to zero, we obtain an explicit expression for the threshold Δ. In fact, we obtain from (11) that $\lim_{r\to 0} \Delta = \sqrt[3]{6K\sigma^2}$.

Similarly, one obtains an explicit expression for the threshold Δ when the variance of the evolution of preferences converges to zero. We obtain from (11) that $\lim_{\sigma^2\to 0} \Delta = rK$, which is intuitive. If there is no variance in the evolution of preferences the firm wants to reposition if the present value of the benefits of repositioning, Δ/r, is greater than the repositioning costs K.

We then compare the optimal threshold for repositioning with positive variance of the evolution of preferences, with what the repositioning strategy would be if consumer preferences were fixed. Note also that the case in which those preferences are fixed can also be seen as the case wherein the firm does not believe that there will be changes in consumer preferences in the future. Looking at (11) we can see that the optimal repositioning strategy is to wait longer than a firm would wait if it did not believe there would be future changes in consumer preferences (see also Figure 5). That is, the firm waits to see whether preferences will return to the firm's current position before deciding to reposition, and only repositions when consumer preferences move farther away. Note that this effect can be arbitrarily high in relative terms. For example, for $r \to 0$, a firm that does not believe in future consumer preference changes, would reposition at any small change in those preferences, while a firm that is aware of possible future preference changes would wait until the center of consumer preferences moves to a distance $\sqrt[3]{6K\sigma^2}$ from the current product's position.

Note also that with the possibility of future repositionings the firm is more willing to reposition: If consumer preferences return to the current product's position, the firm can always reposition back to the current position. If the firm believes it can only reposition one more time, it may wait longer (so as not to waste the option) when there is still a possibility of consumer preferences returning to the firm's current position.

We can also calculate the expected time between repositionings. Denoting z_t as the difference from the center of the consumer preferences of the product positioning at time t, and letting time zero be the time of the last repositioning, we have $z_0 = 0$, and $E(z_t^2 - t\sigma^2) = 0$, given that z_t is a Brownian motion with variance σ^2. Letting t_Δ be the first time that z_t reaches Δ or $-\Delta$, we then have $\Delta^2 - \sigma^2 E(t_\Delta)$, which gives the expected time between repositionings $E(t_\Delta) = \Delta^2/\sigma^2$. See, for example, Dixit (1993). As Δ is increasing in the discount rate, from Proposition 1, we immediately get that the expected time between repositionings is increasing in the discount rate. More interestingly, we also show how the expected time between repositionings is affected by the variance of the consumer preferences.

Proposition 2. *For $r \to 0$, the expected time between repositionings is decreasing in the variance of the evolution of preferences at a decreasing rate.*

As the variance of the evolution of preferences increases, the repositioning threshold also increases, which would be a force towards less frequent repositionings. At the same time, the preferences can also evolve faster along the preference space, which would be a force towards more frequent repositionings. We find that the latter effect dominates, i.e., that a greater variance of the evolution of preferences leads to more frequent repositionings. In fact, when the variance of the evolution of preferences goes to zero, the time between repositionings goes to infinity, as the need to reposition decreases because of slow changing consumer preferences.

Finally, note that the degree of consumer heterogeneity in the market, L, does not affect the optimal repositioning strategy in this case of full market coverage. This can be seen intuitively in the value function (5), as L enters in an additively separable way. That is, in this case of full market coverage consumer heterogeneity L does not change, at the margin, the effect of the firm's positioning relative to the center of consumer preferences. Yet L affects the present value of profits: A lower L is preferred (which makes the value function move vertically, while keeping the same shape). This also means that if L varies over time, but stays in this region of full market coverage ($v > 3L$ and K are relatively small), the optimal repositioning strategy remains unchanged. In the case below, when the market is not always fully covered (for example, K large) we show that L affects the repositioning strategy.

Social Welfare

Next, we compare the optimal repositioning of the firm with what could be socially optimal. To understand the social optimum, note that given v is sufficiently large, as assumed above, the quantity supplied by the firm, given its positioning, is optimal. Therefore, comparison with the social welfare repositioning optimal policy depends on how the total utility generated by a product is affected when the product is not exactly at the center of the consumer preferences.

When the distance between the product's positioning and the center of consumer preferences, $|z|$, is less than L (that is, there are still some consumers for whom the product offered is ideal), we have that the gross surplus offered is $S(z) = \int_0^{L-|z|}(v-x)/(2L)\,dx + \int_0^{L+|z|}(v-x)/(2L)\,dx = v - L/2 - z^2/(2L)$. If $|z| > L$, then similarly we get $S(z) = v - |z|$.

Comparing to the flow payoff for the firm, $\pi(z) = v - L - |z|$, we note that for $|z| < L$ the gross surplus is less affected by z than the firm's profit, $|S'(z)| < |\pi'(z)|$. This will have implications for the optimal repositioning for social welfare, in addition to $S(z) > \pi(z)$. Note also that for $|z| > L$, the effect of z on $S(z)$ is exactly the same as on $\pi(z)$.

The optimal social welfare policy will be a threshold distance, Δ_w, such that when the distance between the product's positioning and the center of the consumer preferences reaches Δ_w, it would be optimal to reposition. The question, then, is what is the relationship between Δ_w and Δ obtained above. For example, if $\Delta_w > \Delta$ then the firm repositions more frequently than would be desirable from a social welfare point of view.

To determine Δ_w, first consider the case in which the repositioning cost, K, is small (or L large) such that $\Delta_w < L$. Similar to the analysis above, we obtain the value function when no repositioning is occurring, $V_w(z)$, as in (4), as

$$rV_w(z) = S(z) + \frac{\sigma^2}{2}V_w''(z), \tag{12}$$

from which we can obtain for $z > 0$

$$V_w(z) = C_w\left[e^{\sqrt{2r/\sigma^2}z} + e^{-\sqrt{2r/\sigma^2}z}\right] - \frac{z^2}{2Lr} + \frac{2v-L}{2r}, \tag{13}$$

given that $V_w'(0) = 0$ because of the smoothness at $z = 0$. As above, because $V_w(\Delta_w) = V_w(0) - K$ and $V_w'(\Delta_w) = 0$, we then obtain

$$C_w = \sqrt{\frac{\sigma^2}{2r}}\frac{\Delta_w}{Lr}\frac{e^{\sqrt{2r/\sigma^2}\Delta_w}}{e^{2\sqrt{2r/\sigma^2}\Delta_w} - 1}, \tag{14}$$

$$\Delta_w^2 - 2LrK = 2\sqrt{\frac{2r}{\sigma^2}}\Delta_w\frac{e^{\sqrt{2r/\sigma^2}\Delta_w} - 1}{e^{\sqrt{2r/\sigma^2}\Delta_w} + 1}, \tag{15}$$

where Δ_w can be implicitly obtained from (15).

Comparing (15) with (11) we can see how the frequency of repositionings compares with what would be optimal from a social welfare point of view. In the appendix we show that $\Delta < \Delta_w$. For example, in the case of $r \to 0$, we obtain that $\Delta_w \to \sqrt[4]{12LK\sigma^2}$, which is greater than the corresponding value for the firm's optimal decision mentioned above, $\sqrt[3]{6K\sigma^2}$, for the condition considered of K small, $\Delta_w < L$ (see the appendix). The same result, $\Delta < \Delta_w$, can be obtained for K large, with a more complicated analysis: This is also shown in the appendix, as the function $V_w()$ now has two regions with different functional forms. The result is presented in the following proposition.

Proposition 3. *The firm repositions more frequently than is optimal from a social welfare point of view, $\Delta < \Delta_w$.*

Comparing the optimal repositioning of the firm with what would be optimal from a social welfare point of view, one might think that the social welfare optimal policy would be to reposition more frequently than a firm would like, as the social welfare is greater than just the firm's profit. It turns out that this does not hold as social welfare is not overly affected by small deviations in consumer preferences while profits are affected at a greater rate. In fact, social welfare is affected at a rate of z/L for $z < L$, $S'(z) = -z/L$, and at a rate of 1 for $z > L$, while the profit is affected at a rate of one, $\pi'(z) = -1$, throughout.

The result obtained here on the comparison between a firm's repositioning decision and what is optimal from a social welfare point of view depends on the stylized model considered. In fact, this comparison is similar to the question of whether a monopolist provides the efficient quality level (e.g., Spence 1975), in which case one could take into account the extent of market coverage provided by the firm and, for a given market coverage, comparison between the benefit of quality to the marginal consumer and the average consumer. In this case, the market is fully covered by the firm; therefore, the question is only one of the effect of repositioning on the marginal consumer versus the average consumer. The effect on the marginal consumer is that the consumer is now closer to the product's positioning. The effect on the average consumer is less clear: While some consumers are now closer to the product's positioning, others are now farther away.[7] This shows that the effect on the marginal consumer is greater than the effect on the average consumer, and the firm repositions more often than it is optimal.

Note that this result need not necessarily hold in other model formulations. For example, if all consumers are at the same location (potentially with different valuations) the benefit of repositioning to the average and marginal consumer could be the same, and the firm would reposition as often as would be efficient. Another interesting example is the case in which the market is not fully covered, $v < 2L$, but the price is chosen by the firm, given the product's positioning. In that case, for small deviations in consumer preferences from the product's positioning, the firm's profit would remain unchanged (and equal to $v^2/4L$), but welfare would be negatively affected. This would be a force towards the firm repositioning less frequently than what would be efficient. In sum, the result of the firm repositioning more frequently than would be socially optimal must be interpreted with care, and can then be seen as a possibility even though social welfare is greater than profit.

Repositioning Costs Depending on the Extent of Repositioning

In some cases, repositioning costs could be an increasing function of the extent of the repositioning. That is, if a firm wants to reposition to a greater distance, repositioning costs will be higher. In terms of the analysis above, this means that the repositioning costs would not just be fixed costs K, but would include additional costs that would depend on the extent of repositioning.

One simple way to consider this possibility is to have the repositioning costs equal $K + \alpha\Delta$ where Δ is the extent of the repositioning and α is some parameter with $\alpha > 0$. Suppose that α is small.

In terms of the analysis above, one must now account for the possibility that the firm chooses not to reposition to the center of the market because, at the margin, not being positioned at the center involves losses of the second order, while the cost of repositioning to the center is of the first order. Let d be the distance to where the firm repositions, which happens when the center of the market is at a distance Δ from where the firm is currently positioned. That is, when the firm repositions it will be at a distance $\Delta - d$ from the center of the market. In the analysis above, when the cost of repositioning had a fixed component, we had $d = \Delta$. Now the firm may choose to save on repositioning costs, and not move all the way to the center of the market, hoping that consumer preferences will return to the firm's current position.

In terms of the analysis above, we then replace (6) and (7) with

$$V(\Delta) = V(\Delta - d) - K - \alpha d \quad \text{and} \tag{16}$$

$$V'(\Delta) = V'(\Delta - d), \tag{17}$$

respectively. Furthermore, the place, d, to which the firm repositions, must be optimal, which requires that

$$V'(\Delta - d) + \alpha = 0. \tag{18}$$

With an analysis similar to that shown above (details are presented in the appendix), we obtain that the threshold to reposition Δ, and the place to reposition to d are determined by

$$rK = d - \alpha r d - 2\sqrt{\frac{\sigma^2}{2r}}\,\frac{e^{\sqrt{2r/\sigma^2}\Delta} - e^{\sqrt{2r/\sigma^2}(\Delta-d)}}{e^{\sqrt{2r/\sigma^2}(\Delta-d)} + 1} + 2\alpha r\sqrt{\frac{\sigma^2}{2r}}\,\frac{e^{\sqrt{2r/\sigma^2}(2\Delta-d)} - e^{2\sqrt{2r/\sigma^2}(\Delta-d)}}{e^{2\sqrt{2r/\sigma^2}(\Delta-d)} - 1}, \tag{19}$$

$$d = \Delta + \sqrt{\frac{\sigma^2}{2r}}\ln\left[\frac{e^{\sqrt{2r/\sigma^2}\Delta}(1-\alpha r) - 1}{e^{\sqrt{2r/\sigma^2}\Delta} - 1 + \alpha r}\right]. \tag{20}$$

From (20) one obtains that $d < \Delta$, as expected. That is, when repositioning, the firm approaches the center of the market but does not move all the way to the center.

To investigate the effect of the variable cost α on the repositioning strategy, consider the case of $r \to 0$. In this case, we show that the expression of the repositioning distance is the same as when repositioning costs are not increasing in the repositioning distance

$$d = \sqrt[3]{6K\sigma^2}, \tag{21}$$

and that

$$\Delta(\Delta - d) = \alpha\sigma^2. \tag{22}$$

This last expression shows how the threshold of consumer preferences to decide to reposition depends positively on the marginal cost of repositioning α, and the variability of the preferences σ^2. Greater marginal repositioning costs makes the firm choose to reposition only when consumer preferences are farther away from the firm's current position. Greater variability of consumer preferences makes the firm more hopeful that those preferences will return to the firm's current position. When repositioning, the firm optimally ends up farther away from the center of the market. Note that having repositioning costs increasing in the distance repositioned does not affect the distance actually repositioned for $r \to 0$, but affects when to reposition, with the threshold to reposition increasing in α.

More interestingly, we can check the effect of the degree to which repositioning costs increase in the distance repositioned when the total repositioning costs remain constant. That is, when α increases, we reduce the fixed repositioning costs K, such that $K + \alpha d$ remains constant. To see this, note that for the repositioning costs to remain constant, we have $\partial K/\partial \alpha = -d^3/(d^2 + 2\alpha\sigma^2) < 0$, which leads to $\partial d/\partial|\alpha_{K+\alpha d=\text{Const.}} < 0$. That is, as expected, if the share of the overall repositioning costs is more related to the distance repositioned, the firm chooses to reposition with shorter distances.

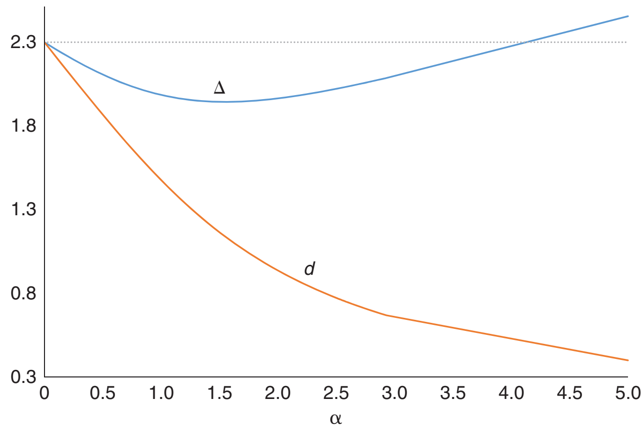
Note also that $\partial\Delta/\partial\alpha_{|K+\alpha d=\text{Const.}} = (\sigma^2/(2\Delta - d)) \times (1 - 2(\Delta d/(d^2 + 2\alpha\sigma^2)))$, which is negative for α small. This means that when the repositioning costs remain constant with a greater share of these costs depending on the distance repositioned, when α is small, the firm repositions more frequently. The intuition is that with the increasing repositioning costs per unit of distance repositioned, the firm chooses an increasingly lower repositioning distance; this makes the firm choose a lower threshold of consumer preferences moving away from the firm's current positioning to decide to reposition. However, note that if the degree to which the repositioning costs increase in the distance repositioned is sufficiently large, we can be in a situation where the threshold to reposition is greater than when there are no repositioning costs increasing in the distance repositioned. To see this, note that when $\alpha \to \infty$, we have d, $K \to 0$, which leads, by (22), to $\Delta \to \infty$. That is, when overall repositioning costs remain constant, increasing the degree to which the repositioning costs increase in the distance repositioned has a non-monotone effect on the threshold of repositioning. When overall repositioning costs remain constant, Figure 6 presents an example of how Δ and d evolve as a function of α for the case of $r \to 0, K + \alpha d = 2$, and $\sigma^2 = 1$.

Large Costs of Repositioning

Consider now the case in which K is large such that the market is not always fully covered on the equilibrium path.[8]

To consider this case, suppose as above that v is large compared to L such that if the product is close to the center of the market, the firm chooses to fully cover the market. In particular, this occurs if $v > 3L$.[9] Depending on how far the center of consumer preferences is from the current product positioning, the firm's price and profit can be in different cases. If the center of consumer preferences is close to the product's current position, in particular, if $|z| < v - 3L$, the optimal price is as noted above, $P = v - L - |z|$, which yields an optimal profit of $\pi(z) = v - L - |z|$.

Figure 6. (Color online) Threshold for Repositioning When Repositioning Costs Are a Function of the Extent of Repositioning with r Converging to Zero, $\sigma^2 = 1$, and K Such That the Total Costs of Repositioning $K + \alpha d$ Stay Constant at 2, for Several Values of α



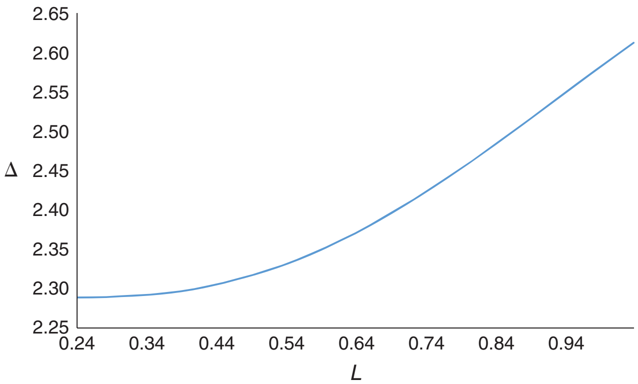
When $|z|$ is greater than $v - 3L$ but less than $v + L$, the optimal price maximizes $((v - P)/(2L))P + ((L - |z|)/(2L))P$, which gives an optimal price equal to $P = (v + L - |z|)/2$ and an optimal profit $\pi(z) = (v + L - |z|)^2/(8L)$. Finally, for $|z| > L + v$, the firm cannot generate any profit and $\pi(z) = 0$.

With this profit function $\pi(z)$ defined for the different regions of z, we use an analysis similar to that presented above to obtain the value function $V(z)$ and the optimal threshold Δ where the firm decides to reposition, while keeping continuity and smoothness of $V(z)$ throughout. This analysis is fully presented in the appendix.

When $r \to 0$, one can obtain the optimal threshold for repositioning to satisfy (A.31). From this, one can obtain that, as expected, greater K or greater σ^2 leads to a greater Δ. If the repositioning costs are greater, the firm prefers to wait longer for consumer preferences to move away from where the product is currently positioned. Similarly, if the variance of the evolution of consumer preferences is greater, the firm again prefers to wait longer to reposition itself, as the likelihood of the consumer preferences returning to where the product is currently positioned is higher.

In this case of partial market coverage, we can see the effect of consumer heterogeneity L on the optimal repositioning strategy. From (A.31) we obtain that a greater L leads to a greater threshold Δ. As there is

Figure 7. (Color online) Evolution of Δ as a Function of L for the Case of Large Repositioning Costs (Partial Market Coverage): $v = 3$, $K = 2$, and $\sigma^2 = 1$



greater consumer heterogeneity, the firm does not see the need to reposition as often, as it is covering the market to some degree. Figure 7 illustrates how Δ changes with L, for an example with $v = 3, K = 2$, and $\sigma^2 = 1$. The figure illustrates how L varies from around 0.24 (at which case $\tilde{K} = 2$, and the equilibrium would be full market coverage) to 1 (at which case $v = 3L$, and the equilibrium would never involve full market coverage). For a numerical example of the case analyzed here, consider $r \to 0$, $v = 3$, $L = 0.7$, and $\sigma^2 = 1$. Then, we get $\tilde{K} = 0.12$ and $\hat{K} = 5.44$ (see Endnote 9). If $K = 0.1$, the market is always fully covered and $\Delta = 0.84$. For $K = 2$, the market is sometimes partially covered and we have $\Delta = 2.41$.

One may also consider the case in which L evolves stochastically over time. Consider such a case under the assumption that for all possible L, we have $v > 3L$ and $K \in (\tilde{K}, \hat{K})$. In such a setting we would expect the repositioning strategy to be based on the current L and firm expectations about the future L. If L is positively serially correlated, we would then expect to have the same comparative statics of Δ increasing in L as presented above, but with a softened effect because of future L uncertainty. For example, consider a model where L starts at a low level, and then with a constant hazard rate moves to a high level where it remains. Then, when L is at a high level, we are back in the situation above because L does not change more going forward. When L is at a low level, the firm knows that it will remain there for some time, and then move to the high level. Then, the repositioning threshold when the firm is at a low level would be expected to be somewhere between the repositioning threshold for the low and high level of the model above with fixed L.

4. Purely Deterministic Evolution of Consumer Preferences

Consider now the case in which the evolution of consumer preferences is completely deterministic. This means that in (2) we have $\sigma = 0$ and $b > 0$. Studying this case is especially important for markets in which there are some trends on the evolution of preferences over time. For example, in markets where technology is important, one could expect that consumer preferences in terms of technical features become more demanding over time. In the car market, if positioning is considered on the technical features, one would expect that consumers would be more demanding on this type of dimension over time.

In this case, with deterministic evolution of preferences, the optimal policy will involve two thresholds on the distance of the product's positioning to the center of consumer preferences, and a target repositioning placement. First, there is a low threshold $\underline{\Delta}$, which is negative; when z is sufficiently low, the firm repositions towards the center of the market. Because the deterministic trend is for z to move downwards, this threshold will be reached often. Second, there is a high threshold $\bar{\Delta}$, which is positive; when z is sufficiently high, the firm repositions towards the center of the market. Because the deterministic trend is for z to move downwards, this threshold will not be reached often. In fact, it will never be reached when the evolution of preferences is purely deterministic as assumed in this section. We also expect $\bar{\Delta}$ to be farther away from zero than $\underline{\Delta}$ as the deterministic trend is downwards; when z is very high we know that after some time the center of consumer preferences will likely be close to the current product's position. Third, the firm must decide where it will reposition to with regard to the center of consumer preferences, d. We expect d to be positive as z trends downwards; in that way, after some period, the center of the consumer preferences will be close to the product's current position. Overall, we then have $\underline{\Delta} < 0 < d < \bar{\Delta}$.

Note that in Section 3 the optimal repositioning policy was completely determined by Δ. That is, we had $\bar{\Delta} = -\underline{\Delta} = \Delta$ and $d = 0$. As there was no deterministic trend, the thresholds for the center of consumer preferences were at the same distance of the product's positioning: When repositioning, the firms always wanted to go to the center of consumer preferences.

In this case of purely deterministic evolution of consumer preferences after repositioning, it always takes time $T = (d - \underline{\Delta})/b$ for the firm to want to reposition again. As the payoffs repeat every period T, we can write the present value of profits after repositioning as

$$V = \frac{e^{rT}}{e^{rT} - 1}\left[\int_0^T (v - L - |d - bt|)e^{-rt}\,dt - K\right]. \qquad (23)$$

Maximizing V with respect to T and d yields the optimal policy for the firm.

For sharper results, consider the case in which $r \to 0$. To do that, consider the average continuous profit rV when $r \to 0$

$$\lim_{r \to 0} rV = \frac{1}{T}\left[\int_0^T (v - L - |d - bt|)\, dt - K\right]. \tag{24}$$

Maximizing this expression with respect to T and d yields

$$T = 2\sqrt{\frac{K}{b}}, \tag{25}$$

$$d = \sqrt{bK}. \tag{26}$$

From this we can obtain $\underline{\Delta} = -\sqrt{bK}$.

To obtain the upper threshold $\bar{\Delta}$, we are looking for a high z, such that the firm is indifferent between repositioning to d with cost K and not repositioning. If the firm repositions, the average payoff is $v - L - d/2 - K/T = v - L - \sqrt{bK}$. If the firm does not reposition, it will be for the period $(\bar{\Delta} - \underline{\Delta})/b$ until the next repositioning. For a fraction of time $\bar{\Delta}/(\bar{\Delta} - \underline{\Delta})$, the firm will have $z > 0$ with an average profit of $v - L - \bar{\Delta}/2$. For a fraction of time $-\underline{\Delta}/(\bar{\Delta} - \underline{\Delta})$, the firm will have $z < 0$ with an average profit of $v - L - (-\underline{\Delta}/2)$. The average profit if the firm does not immediately reposition is then $v - L - (\bar{\Delta}/(\bar{\Delta} - \underline{\Delta}))(\bar{\Delta}/2) - (-\underline{\Delta}/(\bar{\Delta} - \underline{\Delta}))(-\underline{\Delta}/2)$. Making this equal to the average payoff if the firm immediately repositions leads to $\bar{\Delta}^2 + d^2 = 2d(\bar{\Delta} + d)$, where we use the result $d = -\underline{\Delta}$. Using the optimal value for d we then obtain that $\bar{\Delta} = \sqrt{bK}(1 + \sqrt{2})$.

These results lead to some interesting observations. First, note that with a deterministic trend in the evolution of consumer preferences, the firm chooses to optimally overshoot its repositioning from $\underline{\Delta}$ to $d > 0$, while the center of consumer preferences would be zero. This is because with the fixed repositioning costs, overshooting allows the firm to be, on average, closer to the center of consumer preferences. The extent of overshooting is increasing in the deterministic trend b. As this trend increases, the firm increases the extent of overshooting for its product positioning to be about the same time above as below the center of the consumer preferences. The extent of overshooting is also increasing in the repositioning costs K, as higher repositioning costs give the firm incentives to reposition less often: The firm must increase the extent of its repositioning overshooting, so that it is about the same above and below the center of consumer preferences.

Note also that given $r \to 0$, the extent of the overshooting is exactly equal to the distance of the center of consumer preferences to the lower threshold, $d = -\underline{\Delta}$. As the firm is infinitely patient, it wants to be as much above consumer preferences as below them. Thus, as the discount rate r increases, the extent of the overshooting decreases in relation to the lower threshold, so that $d < -\underline{\Delta}$. With $r > 0$, the firm cares more about profits now than profits in the future, and does not overshoot as much to be closer to the center of the market and earn greater profits sooner.

When the deterministic trend b increases, the firm adjusts the time between repositionings and the threshold to reposition. The firm not only shortens the time between repositionings T as now the preferences are evolving faster but it also increases the threshold to reposition, $|\underline{\Delta}|$, not to reposition often. As expected, the thresholds to reposition increase in the repositioning costs, but they do so at a decreasing rate to soften the profit lost from not having the product that is a perfect fit to consumer preferences.

Note also that the upper threshold $\bar{\Delta}$ is farther away from the chosen point of repositioning, d, than the lower threshold $\underline{\Delta}$ is from the center of consumer preferences. This reflects the fact that if z is high, the firm prefers to save on repositioning costs and let the evolution of consumer preferences bring the center of those preferences to where the product is currently positioned.

5. Both Random and Deterministic Evolution of Consumer Preferences

Consider now the case when the deterministic trend, b, and the variance of the evolutions of consumers preferences, σ^2, are different from zero. We present the conditions for the optimal policy, followed by numerical analysis of that policy.

In this case, the value function must satisfy the differential equation

$$rV(z) = \pi(z) - bV'(z) + \frac{\sigma^2}{2}V''(z), \tag{27}$$

where the difference from (4) comes from $E(dz) = -b\, dt$. The solution to this differential equation is

$$V(z) = C_1 e^{x_1 z} + C_2 e^{x_2 z} - \frac{|z|}{r} + \frac{v - L}{r} + (-1)^{1 + 1[z > 0]}\frac{b}{r^2}, \tag{28}$$

where $x_1 \equiv (b + \sqrt{b^2 + 2r\sigma^2})/\sigma^2$ and $x_2 \equiv (b - \sqrt{b^2 + 2r\sigma^2})/\sigma^2$ are the solutions to the characteristic equation $(\sigma^2/2)x^2 - bx - r = 0$, $1[z > 0]$ is the indicator function that takes the value of one if $z > 0$ and zero otherwise, and C_1 and C_2 are parameters to be obtained below. These parameters C_1 and C_2 will be different for z negative (C_1^- and C_2^-) and z positive (C_1^+ and C_2^+).

To find the optimal values of the thresholds to reposition, $\bar{\Delta}$ and $\underline{\Delta}$, and the optimal value to which the firm repositions, d, we consider continuity and smoothness at $\underline{\Delta}$, $\bar{\Delta}$, and $z = 0$, and optimality of d.

Continuity and smoothness at $z = 0$, $V(0^+) = V(0^-)$, and $V'(0^+) = V'(0^-)$, yields

$$C_1^- + C_2^- = C_1^+ + C_2^+ + \frac{2b}{r^2}, \tag{29}$$

$$x_1 C_1^- + x_2 C_2^- + \frac{2}{r} = x_1 C_1^+ + x_2 C_2^+. \tag{30}$$

Continuity and smoothness at $\underline{\Delta}$, $V(\underline{\Delta}) = V(d) - K$ and $V'(\underline{\Delta}) = 0$, yields

$$C_1^- e^{x_1 \underline{\Delta}} + C_2^- e^{x_2 \underline{\Delta}} + \frac{\underline{\Delta} + d}{r} = C_1^+ e^{x_1 d} + C_2^+ e^{x_2 d} + \frac{2b}{r^2} - K, \tag{31}$$

$$x_1 C_1^- e^{x_1 \underline{\Delta}} + x_2 C_2^- e^{x_2 \underline{\Delta}} + \frac{1}{r} = 0. \tag{32}$$

Similarly, continuity and smoothness at $\bar{\Delta}$, $V(\bar{\Delta}) = V(d) - K$, and $V'(\bar{\Delta}) = 0$, yields

$$C_1^+ e^{x_1 \bar{\Delta}} + C_2^+ e^{x_2 \bar{\Delta}} - \frac{\bar{\Delta} - d}{r} = C_1^+ e^{x_1 d} + C_2^+ e^{x_2 d} - K, \tag{33}$$

$$x_1 C_1^+ e^{x_1 \bar{\Delta}} + x_2 C_2^+ e^{x_2 \bar{\Delta}} - \frac{1}{r} = 0. \tag{34}$$

Finally, optimality of d, $V'(d) = 0$, yields

$$x_1 C_1^+ e^{x_1 d} + x_2 C_2^+ e^{x_2 d} - \frac{1}{r} = 0. \tag{35}$$

Using Equations (29)–(35), we can then obtain the seven unknowns, $\underline{\Delta}$, $\bar{\Delta}$, d, C_1^+, C_1^-, C_2^+, and C_2^-. Although it is not possible to analytically solve for these, we can numerically obtain the optimal policy.

Figure 8 illustrates how the optimal policy changes as a function of the discount rate r, for $b = 0.5$, $\sigma^2 = 1$, and $K = 4$. As in the case of $b = 0$, the repositioning thresholds, $\bar{\Delta}$ and $\underline{\Delta}$ move away from zero as the discount rate increases. As argued above, as the discount rate increases, the present value of future profits is lower, and the firm optimally chooses to reposition only when consumer preferences are farther away from the product's current position. The optimal point to

Figure 8. (Color online) Evolution of d and Repositioning Thresholds as a Function of r for $b = 0.5$, $\sigma^2 = 1$, and $K = 4$

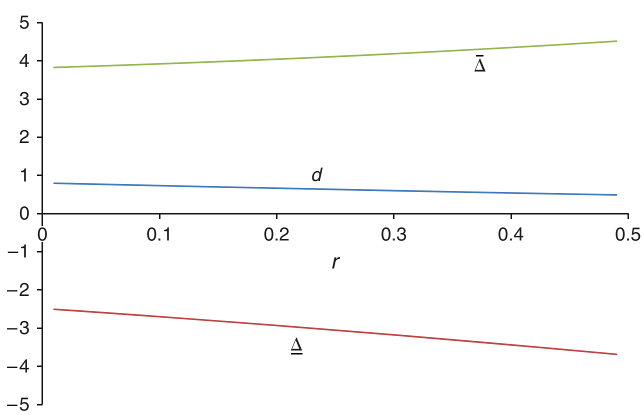
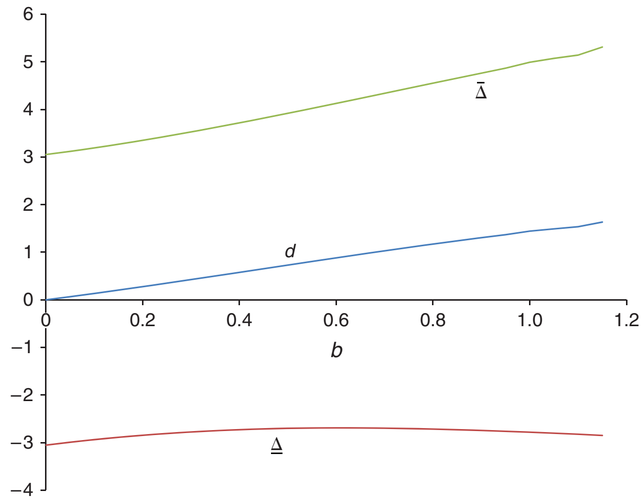


Figure 9. (Color online) Evolution of d and the Repositioning Thresholds as a Function of b for $r = 0.1$, $\sigma^2 = 1$, and $K = 4$

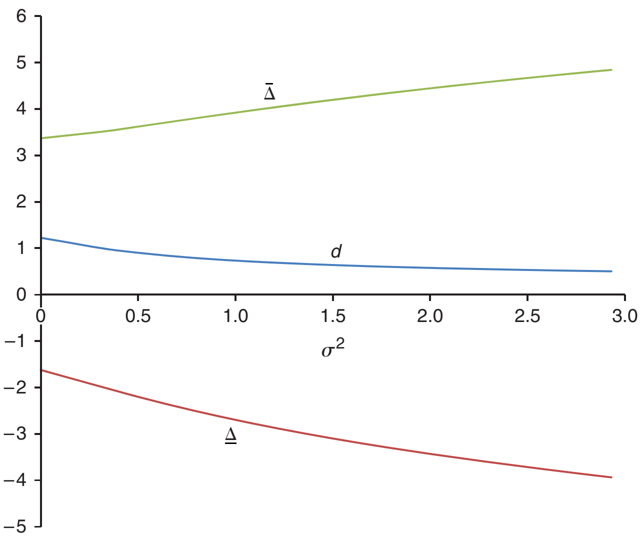


reposition to, d, falls as the discount rate r increases: As the discount rate increases, the firm discounts more the future payoffs; when it repositions it wants to be closer to the center of consumer preferences.

The way in which the optimal policy evolves as a function of the extent of the deterministic changes in consumer preferences, b, is illustrated in Figure 9 for $r = 0.1$, $\sigma^2 = 1$, and $K = 4$. For $b = 0$, we are in the case of Section 3, and we have $\bar{\Delta} = -\underline{\Delta}$. As the deterministic component of the evolution of consumer preferences increases, the firm adjusts by increasing the point at which to reposition, d, to be for a longer period closer to the center of the market. This idea is as presented in Section 4. As the deterministic component increases, the firm also increases the upper threshold of repositioning $\bar{\Delta}$, as in Section 4: A greater b brings the firm's positioning closer to the center of the market if the current center is above the firm's position. Also, as presented in Section 4, the upper threshold gets to be farther away from zero than the lower threshold: Above zero, the firm knows that the deterministic component will bring consumer preferences closer to the product's current position. The lower threshold of repositioning, $\underline{\Delta}$, first increases and then decreases with the deterministic component b. This is different than the case in Section 4 with $\sigma^2 = 0$, where $\underline{\Delta}$ was decreasing in b for all b. For $\sigma^2 > 0$ and b small, the optimal policy can have the lower threshold $\underline{\Delta}$ increasing in b, as the firm adjusts d upwards to be, on average, closer to the center of the market. At some point, $\underline{\Delta}$ starts decreasing in b to save on repositioning costs.

Figure 10 illustrates how the optimal policy evolves as a function of the importance of the random component in the evolution of consumer preferences, σ^2. As in Section 3, the greater the variance in the evolution of consumer preferences, the farther away from

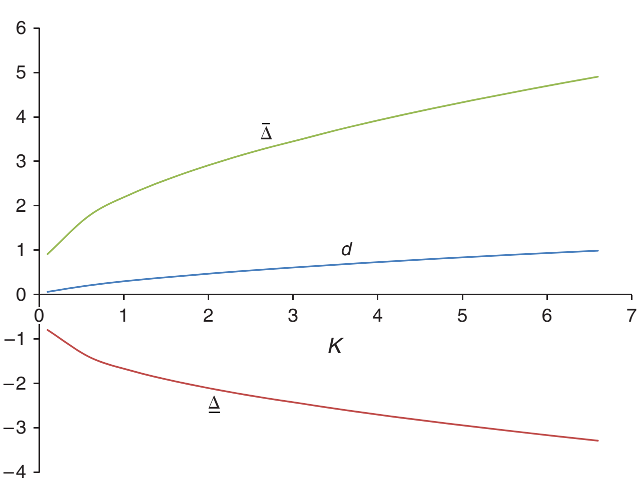
Figure 10. (Color online) Evolution of d and the Repositioning Thresholds as a Function of σ^2 for $r = 0.1$, $b = 0.5$, and $K = 4$



zero are the repositioning thresholds, $\bar{\Delta}$ and $\underline{\Delta}$. Furthermore, the greater σ^2, the lower the optimal d. This is because, the greater σ^2, the lower the relative importance of the deterministic component b. The firm adjusts its optimal policy by being closer to the center of the market when repositioning, d closer to zero.

Finally, the effect of the repositioning cost K on the optimal policy is illustrated in Figure 11. As in Section 3, as the repositioning cost K increases, the firm chooses to reposition less often, which leads to repositioning thresholds farther away from zero. When K increases, as more time passes without repositioning, the firm also increases the point to which it repositions to, d.

Figure 11. (Color online) Evolution of d and the Repositioning Thresholds as a Function of K, for $r = 0.1$, $b = 0.5$, and $\sigma^2 = 1$



6. Concluding Remarks

This paper investigates the way in which a firm optimally repositions when facing random and deterministic variations in consumer preferences. For random variations in consumer preferences, the firm must optimally trade off the repositioning cost and being close to the center of the market, with the possibility of consumer preferences returning to the firm's current position. When the market is always fully covered, a firm repositions more often than would be optimal from a social welfare point of view: Profits decline faster than welfare when consumer preferences move away from the product's current position.

In the case of deterministic variations in consumer preferences, we find that the firm chooses to overshoot in its repositioning to save on repositioning costs and be closer to the center of the market for a longer period.

The continuous-time framework considered here is a tractable way to consider dynamic repositioning decisions. Other marketing situations where dynamics are involved can also benefit from this modeling framework. Examples can include advertising decisions, capacity decisions (e.g., Gardete 2016), consumer search for information (e.g., Ke et al. 2016), sales force management, and branding evolution.

This paper investigates successive product repositionings by a firm to follow the evolution of consumer preferences. In future research, it would be interesting to explore how competition could affect these successive repositioning decisions.

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Appendix

Optimal Price for Different v and L

Suppose that the distance from the center of the market is z, and assume z small. There are several candidate optimal prices. These options include: (1) fully covering the market, which would lead to an optimal price of $P = v - L - z$ and a profit of $\pi = v - L - z$; (2) fully covering the market on one side of the firm, but not fully covering it on the other side, and having marginal consumers on both sides with zero surplus, which would lead to an optimal price of $P = v - L + z$ and a profit of $\pi = (v - L + z)(L - z)/L$; (3) fully covering the market on one side of the firm, but not fully covering it on the other side, and having the marginal consumer on only one side with zero surplus, which would lead to an optimal price of $P = (v + L - z)/2$ and a profit of $\pi = (v + L - z)^2/(8L)$; (4) not fully covering the market on both sides of the firm, which would lead to an optimal price of $P = \arg\max_p P(v - P) = v/2$ and a profit of $\pi = v^2/(4L)$.

In cases (3) and (4) we must ensure that the demand generated by the corresponding prices satisfies the assumptions for those cases. For case (3), this means that all consumers not served have a negative surplus, which requires $z > v - 3L$, and that on one extreme of consumer preferences the consumer has a positive surplus, which requires $v - 3L + 3z > 0$. For case (4), this means that all consumers not served have a negative surplus, which requires $v - 2L < -2z$. Then if $v < 2L$, the optimal price will be case (4) for z small, and the market is never fully covered.

Comparing the profits of cases (1) and (2) we show that case (1) is more profitable if $v - 3L > -z$. Then, if $v > 3L$, for z small, the optimal price is that for case (1), and the market is always fully covered.

Next we see what is optimal for $v \in (2L, 3L)$ for z small. We know that neither case (1) nor case (4) is optimal in this parameter range for v. We also know that for case (3) to be optimal, we would need $v - 3L > 0$ for z small. So, this yields that the optimal is case (2), where the market is fully covered for $z = 0$, and not fully covered otherwise.

Derivation of Threshold $\tilde{K}$

To obtain the threshold $\tilde{K}$ such that for $K < \tilde{K}$ the optimum is always full market coverage; for any possible z on the equilibrium path, we must have $|z| < v - 3L$. This occurs if $\Delta < v - 3L$. So, from (11) we have

$$r\tilde{K} = v - 3L - 2\sqrt{\frac{\sigma^2}{2r}}\,\frac{e^{\sqrt{2r/\sigma^2}(v-3L)} - 1}{e^{\sqrt{2r/\sigma^2}(v-3L)} + 1}. \tag{A.1}$$

Derivation of Δ When $r \to 0$ When v Is Large

Defining $X \equiv e^{\sqrt{2r/\sigma^2}\Delta}$ and $y \equiv \sqrt{r}$, we can rewrite (11) as

$$K(X+1) = \frac{y\Delta(X+1) - \sqrt{2\sigma^2}(X-1)}{y^3}. \tag{A.2}$$

When $r \to 0$ the LHS converges to $2K$. When $r \to 0$ the RHS is indeterminate. Taking L'Hôpital's rule three times, we show that it converges to $\Delta^3/(3\sigma^2)$. We then obtain that when $r \to 0$ we get $\Delta \to \sqrt[3]{6K\sigma^2}$.

Proof of Proposition 3

For the case of K small, following the analysis in the text, note that (15) can be written as

$$rK = \frac{\Delta_w}{2L}\left(\Delta_w - 2\sqrt{\frac{\sigma^2}{2r}}\,\frac{X_w - 1}{X_w + 1}\right), \tag{A.3}$$

where $X_w \equiv e^{\sqrt{2r/\sigma^2}\Delta_w}$, and where the RHS is increasing in Δ_w as the derivative of $\Delta_w - 2\sqrt{\sigma^2/(2r)}((X-1)/(X+1))$ is proportional to $(X-1)^2 > 0$. It follows that Δ_w is increasing in K. Similarly, from (11), we have

$$rK = \Delta - 2\sqrt{\frac{\sigma^2}{2r}}\,\frac{X-1}{X+1}, \tag{A.4}$$

where $X \equiv e^{\sqrt{2r/\sigma^2}\Delta}$, which yields Δ increasing in K. For $\Delta = \Delta_w$ the RHS of (A.3) equals the RHS of (A.4) multiplied by $\Delta_w/(2L)$. As Δ and Δ_w are increasing in K, it follows that for $\Delta_w < L$, we have $\Delta_w > \Delta$.

Next consider the case of K large such that the optimal Δ_w could be greater than L. For this case the value function has a different shape for $z > L$, as the social welfare function is $S(z) = v - z$ for $z > L$. We then show in this region for z that $V_w(z) = C_{w1}e^{\sqrt{2r/\sigma^2}z} + C_{w2}e^{-\sqrt{2r/\sigma^2}z} - z/r + v/r$. With continuity and smoothness at $z = L$, we have $V_w(L^+) = V_w(L^-)$ and $V_w'(L^+) = V_w'(L^-)$, which leads to $C_{w1} = C_w - \sigma^2/(4r^2LW)$ and $C_{w2} = C_w - \sigma^2 W/(4r^2L)$, where W is defined as $W \equiv e^{\sqrt{2r/\sigma^2}L}$.

Given continuity and smoothness at $z = \Delta_w$, $V_w(\Delta_w) = V_w(0) - K$ and $V_w'(\Delta_w) = 0$, which then leads to

$$C_w = \frac{X_w}{X_w^2 - 1}\left[\frac{1}{r}\sqrt{\frac{\sigma^2}{2r}} + \frac{\sigma^2}{4r^2L}\left(\frac{X_w}{W} - \frac{W}{X_w}\right)\right], \tag{A.5}$$

$$rK = \Delta_w - \frac{L}{2} - \sqrt{\frac{\sigma^2}{2r}}\,\frac{X_w - 1}{X_w + 1} + \frac{\sigma^2(X_w)(1-W)}{2rLW(X_w+1)}, \tag{A.6}$$

where $X_w \equiv e^{\sqrt{2r/\sigma^2}\Delta_w}$. Subtracting $\Delta - 2(\sqrt{\sigma^2/(2r)})((X-1)/(X+1))$ from the RHS of (A.6) when $\Delta = \Delta_w$, one obtains

$$g(\Delta) = -\frac{L}{2} + \sqrt{\frac{\sigma^2}{2r}}\,\frac{X-1}{X+1} - \frac{\sigma^2}{2r}\,\frac{(X-W)(W-1)}{LW(X+1)}. \tag{A.7}$$

If we can show that (A.7) is negative, then $\Delta < \Delta_w$ as Δ and Δ_w are increasing in K.

To see this note that

$$g(L) = -\frac{L}{2} + \sqrt{\frac{\sigma^2}{2r}}\,\frac{W-1}{W+1}, \tag{A.8}$$

which has the same sign as $\tilde{g}(x) = -x(e^x + 1) + 2(e^x - 1)$, by making $x = \sqrt{2r/\sigma^2}L$. Noting that $\tilde{g}(0) = 0$, $\tilde{g}'(0) = 0$, and $\tilde{g}''(x) < 0$ for $x > 0$, we have that $g(L) < 0$.

Differentiating now $g(\Delta)$ one obtains

$$g'(\Delta)\frac{(X+1)^2}{X'}\frac{2r}{\sigma^2}W = h(x) = 2xe^x - e^{2x} + 1, \tag{A.9}$$

where $x = \sqrt{2r/\sigma^2}L$. One can obtain $h(0) = 0, h'(0) = 0$, and $h''(x) < 0$ for $x > 0$. So, $h(x) < 0$ for $x > 0$, which means that $g'(\Delta) < 0$ for $\Delta > L$, which yields $g(\Delta) < 0$. That means $\Delta < \Delta_w$.

Derivation of Δ_w for $r \to 0$ and K Small

Defining $X_w \equiv e^{\sqrt{2r/\sigma^2}\Delta_w}$ and $y \equiv \sqrt{r}$, Equation (15) can be rewritten as

$$2(X_w+1)\frac{LK}{\Delta_w} = \frac{y(X_w+1)\Delta - \sqrt{2\sigma^2}(X_w - 1)}{y^3}. \tag{A.10}$$

When $r \to 0$ the LHS converges to $4(LK/\Delta_w)$. To obtain the value to which the RHS converges when $r \to 0$, we must apply L'Hôpital's rule three times, which yields $4\Delta_w^3/3$. We then show that when $r \to 0$, then $\Delta_w \to \sqrt[4]{12LK\sigma^2}$.

Derivation of the Case When Repositioning Costs Depend on the Extent of Repositioning

Using (5) we can write (16) and (17) as

$$C_1X + C_2\frac{1}{X} = C_1\frac{X}{Z} + C_2\frac{Z}{X} + \frac{d}{r} - K - \alpha d, \tag{A.11}$$

and

$$C_1X - C_2\frac{1}{X} = C_1\frac{X}{Z} - C_2\frac{Z}{X}, \tag{A.12}$$

respectively, where $X \equiv e^{\sqrt{2r/\sigma^2}\Delta}$ and $Z \equiv e^{\sqrt{2r/\sigma^2}d}$. Recall also that $V'(0^+) = V'(0^-)$ leads to

$$C_1 = C_2 + \frac{1}{r}\sqrt{\frac{\sigma^2}{2r}}. \tag{A.13}$$

Finally, note that (18) can be written as

$$C_1\frac{X}{Z} - C_2\frac{Z}{X} = \left(\frac{1}{r} - \alpha\right)\sqrt{\frac{\sigma^2}{2r}}. \tag{A.14}$$

Using (A.11)–(A.14) we then obtain, C_1, C_2, Δ, and d. In particular, using (A.13) and (A.14) we obtain

$$C_1 = \sqrt{\frac{\sigma^2}{2r}}\frac{1}{(X/Z)^2-1}\left[\frac{X/Z-1}{r} - \alpha\frac{X}{Z}\right], \tag{A.15}$$

$$C_2 = \sqrt{\frac{\sigma^2}{2r}}\frac{X/Z}{(X/Z)^2-1}\left[\frac{1-X/Z}{r} - \alpha\right]. \tag{A.16}$$

Using (A.11) and (A.12), and taking out $d/r - K - \alpha d$ (by addition of those two equations and subtraction), we obtain $C_1X + C_2(Z/X) = 0$. Substituting for C_1 and C_2 from (A.15) and (A.16), one then obtains

$$Z = X\frac{X-1-\alpha rX}{X-1+\alpha r}. \tag{A.17}$$

Taking logs on both sides and dividing by $\sqrt{2r/\sigma^2}$, one obtains (20). Finally, adding (A.11) and (A.12), and substituting for (A.15)–(A.17) one obtains (19).

Consider now what happens to (A.17) when $r \to 0$. Subtracting 1 from both sides of (A.17), dividing by $\sqrt{2r/\sigma^2}$, and then making $r \to 0$, and applying L'Hôpital's rule on both sides of the equation, one obtains (22).

Rearranging (19) one can obtain

$$\begin{aligned}\frac{\sigma^2}{2}(K+\alpha d) = \big[&2(X - X/Z) - d\beta - (2-\alpha\sigma^2\beta^2)X^2/Z \\ &+ (2-\alpha\sigma^2\beta^2 + d\beta)(X/Z)^2\big] \\ &\cdot \big[\beta^3((X/Z)^2-1)\big]^{-1},\end{aligned} \tag{A.18}$$

where $\beta \equiv \sqrt{2r/\sigma^2}$. Making $\beta \to 0$, and applying L'Hôpital's rule four times on the RHS, one obtains using (22)

$$d = \sqrt[3]{6K\sigma^2}. \tag{A.19}$$

Derivation of Optimal Δ for Large K and Market Sometimes Not Fully Covered

To obtain the optimal $V(z)$, we must consider the differential equation $rV(z) = (\sigma^2/2)V''(z) + \pi(z)$ and the two regions for $\pi(z)$ as described in the text, i.e., $z \in [0, v-3L]$ and $z \in (v-3L, \Delta)$.

For $z \in [0, v-3L]$, we have $\pi(z) = v - L - z$, and given smoothness at $V(0)$, $V'(0^+) = V'(0^-)$, we have

$$V(z) = C_1 e^{\sqrt{2r/\sigma^2}z} + C_2 e^{-\sqrt{2r/\sigma^2}z} - \frac{z}{r} + \frac{v-L}{r}, \tag{A.20}$$

and

$$C_2 = C_1 - \frac{1}{r}\sqrt{\frac{\sigma^2}{2r}}, \tag{A.21}$$

where C_1 and C_2 are constants to be determined later.

For $z \in (v-3L, \Delta)$, we have $\pi(z) = (v+L-z)^2/(8L)$, which yields

$$V(z) = C_3 e^{\sqrt{2r/\sigma^2}z} + C_4 e^{-\sqrt{2r/\sigma^2}z} + \frac{(v+L-z)^2}{8Lr} - \frac{\sigma^2}{8Lr^2}. \tag{A.22}$$

Continuity and smoothness at $V(v-3L)$, $V(v-3L^+) = V(v-3L^-)$, and $V'(v-3L^+) = V'(v-3L^-)$ yields

$$C_1X + C_2\frac{1}{X} = C_3X + C_4\frac{1}{X} + \frac{\sigma^2}{8Lr^2}, \tag{A.23}$$

$$C_1X - C_2\frac{1}{X} = C_3X - C_4\frac{1}{X}, \tag{A.24}$$

where $X \equiv e^{\sqrt{2r/\sigma^2}(v-3L)}$.

Finally, to determine the threshold Δ we must have $V(\Delta) = V(0) - K$ and $V'(\Delta) = 0$. These two conditions mean that

$$\begin{aligned}C_3Y + C_4\frac{1}{Y} + \frac{(v+L-\Delta)^2}{8Lr} + \frac{\sigma^2}{8Lr^2}& \\ = C_1 + C_2 + \frac{v-L}{r} - K&,\end{aligned} \tag{A.25}$$

$$C_3Y - C_4\frac{1}{Y} = \sqrt{\frac{\sigma^2}{2r}}\frac{v+L-\Delta}{4Lr}, \tag{A.26}$$

where $Y \equiv e^{\sqrt{2r/\sigma^2}\Delta}$. Using (A.21)–(A.26) we can obtain

$$\begin{aligned}C_3Y + C_4\frac{1}{Y} - C_1 - C_2 = &-\frac{\sigma^2}{8Lr^2}\frac{X^2+Y}{X(1+Y)} \\ &+ \frac{1}{r}\frac{Y-1}{Y+1}\left(\frac{v+L-\Delta}{4L}+1\right).\end{aligned} \tag{A.27}$$

Using (A.27) in (A.25) we obtain an equation to determine Δ as

$$\begin{aligned}rK = \frac{\sigma^2}{8Lr}\frac{X^2+Y}{X(1+Y)} - \sqrt{\frac{\sigma^2}{2r}}\frac{Y-1}{Y+1}\left(\frac{v+L-\Delta}{4L}+1\right) \\ + v - L - \frac{\sigma^2}{8Lr} - \frac{(v+L-\Delta)^2}{8L}.\end{aligned} \tag{A.28}$$

Evaluating (A.28) at $\Delta = v + L$ we obtain the upper threshold on K, $\hat{K}$, such that the market is always covered, even if partially, as

$$\begin{aligned}r\hat{K} = \frac{\sigma^2}{8Lr}\frac{X^2+\hat{Y}}{X(1+\hat{Y})} - \sqrt{\frac{\sigma^2}{2r}}\frac{\hat{Y}-1}{\hat{Y}+1}\left(\frac{v+L-\Delta}{4L}+1\right) + v \\ - L - \frac{\sigma^2}{8Lr} - \frac{(v+L-\Delta)^2}{8L},\end{aligned} \tag{A.29}$$

where $\hat{Y} \equiv e^{\sqrt{2r/\sigma^2}(v+L)}$.

To obtain the Δ when $r \to 0$, rearrange (A.28) to obtain

$$\begin{aligned}&4LKX(Y+1)\sigma^2 \\ &= \big(\beta^2X(1+Y)[8L(v-L) - (v+L-\Delta)^2] \\ &\quad -2\beta(Y-1)(v+5L-\Delta)X + 2(Y-X)(1-X)\big)(\beta^4)^{-1},\end{aligned} \tag{A.30}$$

where $\beta \equiv \sqrt{2r/\sigma^2}$. Applying L'Hôpital's rule four times on the RHS, we show that when $r \to 0$, we have

$$\begin{aligned}f(\Delta, L, \sigma^2, K, v) \equiv 144\sigma^2LK + 3\Delta^4 - 8v\Delta^3 + 2(v-3L)\Delta^3 \\ + 6(v-3L)^3\Delta - 3(v-3L)^4 = 0.\end{aligned} \tag{A.31}$$

Note that $f()$ is increasing in σ^2 and K. Note also that $\partial f/\partial\Delta = 12\Delta^3 - 18(v+L)\Delta^2 + 6(v-3L)^3$, which is negative for $\Delta \in (v-3L, v+L)$ which was assumed in this case (for $\Delta < v - 3L$ we are in the case of full market coverage; for $\Delta > v + L$

we are in the case where sometimes no consumer is served). Finally, note that

$$\begin{aligned} L\frac{\partial f}{\partial L} &= -3\Delta^4 + 6v\Delta^3 + 3(v-3L)^4 - 6(v-3L)^3\Delta \\ &\quad + 36L(v-3L)^3 - 54L\Delta(v-3L)^2 \\ &= 3(\Delta - v + 3L)^2[(\Delta + v - 3L)(3L + v - \Delta) + 6L(v-3L)], \quad \text{(A.32)} \end{aligned}$$

which is positive given that $v > 3L$, and $\Delta \in (v-3L, v+L)$. Then, we have $\partial\Delta/\partial\sigma^2$, $\partial\Delta/\partial K$, $\partial\Delta/\partial L > 0$.

Endnotes

[1] In the car manufacturing industry, model updates can include the latest technological developments (which could be interpreted as a preference for more technology), but may also include important redesign features.

[2] We consider a one-dimension model of consumer preferences and repositionings. Our model is simplified for the potential multidimensional "style" changes in the examples mentioned above. It is also possible to project several potential dimensions into one. In car manufacturing, for example, the dimension could be sporty versus functional.

[3] Obviously, there are many other factors involved when a product category evolves over time, including potentially decreasing variance in the evolution of consumer preferences (which is included in the model, and would be consistent with a similar pattern), and competition (which is not included as it is beyond the scope of this paper).

[4] See also, for example, Shen (2014) for an empirical analysis of dynamic entry and exit in a growing industry. Also related to this paper is the literature on portfolio choice with transaction costs, where an investor only adjusts the portfolio occasionally because of transaction costs and the portfolio evolves stochastically (Magill and Constantinidis 1976), and the literature on (S, s) economies from inventory problems (e.g., Scarf 1959, Sheshinski and Weiss 1983).

[5] We assume that the firm can choose to reposition at any time. Alternatively, one could have a model where firms can only reposition on random occasions (see, e.g., Calcagno et al. 2014).

[6] Later in this section, we also consider the case of K large, such that the market is not always fully covered. If $v \in (2L, 3L)$, the market is fully covered if $z = 0$, and otherwise is partially covered (see the appendix). For $v < 2L$ the market is always partially covered. This latter case is available in the online appendix.

[7] For example, suppose a consumer is at 1 to the right of the center of the market and that $z = 1$. Then, if the firm repositions to the center of the market, that consumer becomes worse off while a consumer at the center of the market is better off.

[8] We consider the case in which K is large ($K > \tilde{K}$), such that the firm sometimes chooses to serve the market partially, but not too large ($K < \hat{K}$ where $\hat{K} > \tilde{K}$ is defined in (A.29)), such that the firm may never choose to have zero profit waiting for the possibility that consumer preferences return to where the firm is positioned. The case of very large K, such that sometimes the market is not served at all, is shown in the online appendix.

[9] If $v < 3L$, the seller chooses not to fully cover the market if the product is not exactly at the center.

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