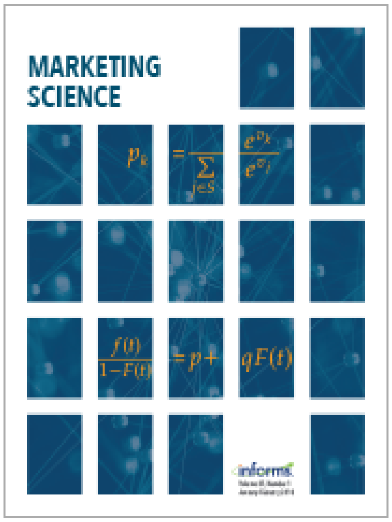

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Increasing Retailer Loyalty Through the Use of Cash Back Rebate Sites

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Abstract. Cash back sites are referral intermediaries that help retailers attract consumers and serve consumers through rebate offers. What exactly is the strategic impact of cash back sites on retailer pricing? We examine this by analyzing two competing retailers that use a cash back site to serve consumers, some of whom are loyal, some of whom are switchers, and some of whom are searchers. We formulate a multistage game by innovating on extant models of consumer price search and solve for the subgame perfect Nash equilibrium prices. We find that the cash back site can allow retailers to profitably eliminate consumer search and that makes retailer sites more sticky. Thus, cash back sites can act as strategic partners of retailers. Surprisingly, even consumers that use the cash back site can be worse off in the presence of cash back sites under some conditions. In particular, if search prevention is profitable even without a cash back site, then cash back sites result in higher prices to consumers. We also offer practical guidance through our finding that the optimal discount offer should be proportional to price rather than a flat sum.

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1. Introduction

A common practice in e-commerce is for sellers to gain customers through affiliate sites. An affiliate site has links to the seller and when a customer is directed to the seller, the affiliate is compensated for "generating" the lead. The contract between the seller and the affiliate site could be pay per click or pay per sale, consisting of a commission proportional to the sale. The affiliate site serves as a way to acquire customers similar to search engine advertising. What is different is the way a potential customer is identified: the cue is not a search keyword but a visit to the site. Thus, how the site attracts visitors and whether visitors are sorted are key to the effectiveness of the site. In this paper, we study the strategic role of affiliate sites that attract customers with offers of rebates.

Online cash back is a new type of reward for consumers who make a purchase via affiliated links provided by cash back websites. Examples of such websites include Rakuten.com (original Ebates.com), BeFrugal.com, Coupon Cactus, Mr. Rebates, and LafaLafa.com. In practice, sites differ on the emphasis and implementations of cash back. Rakuten.com and Mr. Rebates can be thought of as pure cash back sites. Coupon Cactus, on the other hand, bundles both coupons and cash back. BeFrugal.com positions itself as a more general portal. The cash back offers may also carry restrictions. In Appendix A, we display relevant pages from these sites that illustrate the specifics (Figures A.1–A.6). An interesting aspect of these sites is that a retailer may be found on all of them. For example, we see that Walmart appears on all sites, though the cash back rebate rate is not identical across sites. These sites often list more than 2,000 retailers, including large merchants like Walmart, Amazon, eBay, and Macy's, offering cash back. In 2014, Rakuten Inc., Japan's biggest online shopping mall, paid $952 million to acquire Rakuten.com. It was reported that in fiscal year 2013, Rakuten.com posted an operating income of $13.7 million on a net revenue of $167.4 million from its 2.5 million active members. Thus, cash back websites are an integral part of e-commerce.

How do online rebate websites differ from their offline counterparts? First, users of cash back websites can know ahead of time how much they can get back for each purchase at a specific retailer. In order to be listed on cash back websites, retailers have to negotiate a contract with the cash back site. After settling on all the terms, it may not be that easy to change the amount of cash back frequently. Thus, frequent users of cash back websites could have a general idea about how much they can get by using online cash back. Second, because consumers make their purchase on the retailers' websites, unlike Groupon, for example, cash back websites invest little to provide customer support for each purchase. They are more like portals through which consumers can access additional benefits but at the same time have to expend some effort to do so. For instance, users have to log in as a member in order to use the affiliated links. Then they have to locate the link and go through it in order to be eligible for cash back. This effort can serve to profitably sort consumers. Third, the amount of cash back will be automatically calculated and recorded for each purchase. Technically, to get cash back, users do not have to do anything after shopping. Technology, such as browser extensions, allows the site to monitor each individual's purchase activity once he or she goes through the affiliated links. Extra effort is not needed in the postpurchase stage for consumers, so online cash back reduces uncertainty about whether consumers would really redeem their eligible rebates. That is a common concern that must be accounted for when modeling offline rebates. Finally, we should note that rebates on purchases at retailers are also available when consumers use certain credit cards for their purchases. Such rebates are determined by and paid for by the credit card issuing banks, not the retailer. Thus, these rebates appear to be part of the strategy of the credit card issuers. We study the situation in which the cash back sites are compensated by the retailer, so our emphasis is on the strategies of the retailer.

Another question that comes to mind is: what is the difference between cash back websites and shopbots studied by Iyer and Pazgal (2003)? We know that consumers can find suitably aggregated and sorted information, such as prices, by visiting online infomediaries or a shopbot and in this way reduce search costs. Viswanathan et al. (2007) study the potential segmentation effect of online buying services, that is, online infomediaries. What they found is that consumers pay different amounts for the same product depending on what kind of information they obtain. In particular, those getting price information obtain a lower price while those getting product information pay a higher price. Note that when consumers visit a cash back site, they cannot obtain price information but learn only the level of cash back for each retailer. In other words, consumers must still visit retailers' websites to obtain price information, and so comparing retailer prices entails costly search. Cash back websites do not directly affect consumers' search, so their strategic role differs from that of shopbots. Cash back sites can, however, help to segment the market just as shopbots do in Iyer and Pazgal (2003). Do they have an additional strategic role?

Our goal in this research is to explore the strategic role of cash back websites. Our main finding in this paper is that cash back sites can make it profitable for retailers to limit consumer search and thereby increase the size of the effective loyal segment of consumers. Thus, the existence of cash back sites can serve to increase retailer stickiness. We also find that consumers using the cash back site can benefit from the discounts, but contrary to intuition, under some conditions they pay higher prices than what they would in the absence of a cash back site. Several questions arise naturally: How does online cash back affect price competition among retailers? What would be the equilibrium pricing strategy in this situation? Who benefits from having a third-party website that offers online cash back? Is it better to have a contract that specifies cash back as a percentage discount or flat discount? Is there an optimal cash back level? Are equilibrium transaction prices lower for users of the cash back site compared with a situation without these sites? In this paper, we develop a model of cash back sites, consumer search, and retailer pricing and, by analyzing it, provide answers to these questions. We also contribute from a modeling perspective by extending the models of Narasimhan (1988) and Stahl (1989) on competitive strategies by incorporating a segment of searching consumers that is absent in Narasimhan's model and a segment of loyal consumers that is absent in Stahl's model.

2. Literature Review

Although online cash back has grown into a large industry, there are limited past research findings. Ho et al. (2017) build an analytical model to shed light on the potential benefit of online cash back. They argue that cash back websites help to price discriminate consumers based on transaction costs that consumers must incur to become eligible for the cash back. They find that use of cash back sites is profitable for a monopolist only when consumer product valuation is sufficiently diverse and the fraction of low-valuation consumers is relatively small. In a duopoly, they show that when the merchants are asymmetric in valuations of the products, the low-valuation merchant has more incentive to use online cash back. This implies that it is more likely to see low-valuation merchants listed on sites, but casual observation fails to confirm that. Our work is different, because we do not model the merchants' decision to use a cash back site. We take as given that a cash back site can help segment the market, and then we ask whether retailers can enjoy additional benefits. It is tempting to think that the extra discount that cash back sites offer would make the competition among retailers more intense. We find, counter to intuition, that if the segmentation is successful, cash back changes the nature of price competition and indeed retailers' profits are higher. Thus, the whole industry would benefit from having cash back websites in the marketing channel.

Vana et al. (2018) conduct an empirical study on the influence of cash back payments. They find that receiving cash back payments increases the purchase likelihood through the cash back sites in subsequent weeks. Besides, receiving cash back payments increases the size of the new purchase. They also provide an interesting discussion of possible explanations behind their findings. Ballestar et al. (2016) study consumer behavior when using cash back sites. They found that when the number of consumers increases, the total amount of transaction increases and consumers respond differently in different categories.

Online cash back is sometimes referred to as online rebates. There is prior work on offline rebates in the literature. Chen et al. (2005) argue that rebates are able to discriminate between consumers based on their postpurchase states. However, consumers will increase up-front willingness to pay when offered rebates. Lu and Moorthy (2007) compare coupons and rebates, pointing out that the uncertainty in terms of redemption cost for consumers is a key difference between these two promotion tools. Although both of them can act as price discrimination methods, rebates are more efficient in surplus extraction, whereas coupons offer more control over whom to serve. Moreover, consumers may underestimate the cost of redeeming the rebates at the time of purchase (Akerlof 1991, Kahneman and Lovallo 1993, Loewenstein 1996, Soman 1998). Empirical work includes Bruce et al. (2006), who show that in durable markets, such as cars, manufacturers can use cash back to reduce negative equity problem. Dogan et al. (2010) examine rebates both theoretically and empirically. Their empirical findings support the theoretical result that lower-end products benefit from rebates used to price discriminate in a competitive setting.

3. Model

We are interested in exploring whether a cash back site can help retailers to profitably limit consumer search by innovative pricing strategies. To this end we consider a market with two stores indexed by i, $i = 1, 2$. They sell to consumers who are in the market for a basket of goods. In our model all consumers are willing to pay V for the basket but differ in their sensitivity to price. To simplify the calculation, we normalize V to 1. It is useful to think about the reasons for the heterogeneity in retail price sensitivity from a practical point of view. First, on a given purchase occasion a consumer may differ in the size and composition of her planned purchase basket causing the returns to price search to vary. Second, across consumers, and possibly even across purchase occasions for a given consumer, there would be differences in the cost of time. Finally, although stores are identical in our model, in practice, stores' assortments would overlap, but not be identical, so depending on the composition of the basket, the returns to price search may also vary. Keeping these considerations in mind we have chosen to model heterogeneity in price sensitivity by supposing that there are three types of consumers.[1] We modify Narasimhan's (1988) model to capture consumer heterogeneity. We denote a proportion α, $\alpha > 0$, of consumers as being loyal to one or the other store, and further assume that each store serves $\frac{\alpha}{2}$ proportion of loyal consumers. The loyal consumers buy from their "preferred" store, or go-to store, if the price at the retailer $p_i \leq V$. It is useful to understand what store loyalty means. On a given occasion, because of the composition of the basket and store assortment, some consumers may wish to visit only one store. These then are "loyal" to that store. Note that they may still be price sensitive for the basket, as will become clear when we discuss cash back sites. Consumers who are not loyal in this sense are assumed to be one of two types, depending on their cost to search across stores. A proportion γ, $\gamma > 0$, consists of consumers with zero search costs, corresponding to "switchers" in Narasimhan's model, and buys from the lower priced store. We label this the switching segment. This segment can be thought of as extremely price sensitive. Finally, a fraction β, $\beta > 0$, of consumers is assumed to be moderately price sensitive. Consumers in this segment go to the store offering higher expected utility. Having arrived there, they stop search if they see a sufficiently low price, reflecting their moderate levels of price sensitivity. We denote S as search cost and label the β proportion as the searching segment. It is useful to see how our model is related to prior work. Stahl (1989) studied a model with only switching and searching segments as defined in our model, whereas Narasimhan's model consists of loyal and switching consumers. Our model consists of all three segments and so can be thought of as a combination of the models of Narasimhan (1988) and Stahl (1989).

An important innovation in our model is the presence of an online cash back website. Consumers who visit the cash back site first and then subsequently visit store i, using a link, become eligible for a discount D_i at store i usually on their entire basket. To see why our model of consumer heterogeneity in price sensitivity that combines the Narasimhan and Stahl models is appropriate for studying the cash back site, note that the essential feature of a cash back site is to offer price incentives. Moreover, the cash back site represents competing stores on its site, making it more attractive to consumers who are not store loyal. Finally, the price incentives require a consumer to invest in time, provide private information, and receive rewards after the purchase. Thus, it is reasonable to suppose that store loyalty and price sensitivity both matter in who would use the cash back site, and to what extent. We know from practitioners that not all consumers use cash back sites. We do not know the exact fraction of consumers that uses these sites, but it is thought to be well below 50%. The use of cash back sites involves a time cost due to the need to login to the site and possibly a cost due to loss of privacy. Thus, given our reasoning behind the three segments, it makes sense to assume that some loyal consumers would use the cash back site. Of course, the fraction of loyal consumers using the cash back site is likely to be higher if the discount were higher. Casual evidence suggests that, even for holiday shopping, nearly 33% of consumers patronize their "go to store" rather than shop around for the best deal on the Internet, based on Accenture's (2016) finding in a survey that only ". . . 67% of shoppers will purchase items from different stores or websites (than their usual go to store) to get the lowest price . . . ." These could be thought of as loyal consumers in our model. Price sensitive consumers are the most likely to use the cash back site given their extreme price sensitivity. What can we say about searching consumers? Because searching consumers are not store loyal, they can potentially avail themselves of discounts at either store and so are more likely to search than the loyal consumers, but perhaps less than the switching consumers. To keep the analysis tractable, we assume that switching consumers and searching consumers all use the cash back site. We capture the behavior of different segments depending on the level of the discount. Thus, in the presence of a cash back site, each store's loyal segment consists of two subsegments: those that use the cash back site and those that do not. In Section 5, we will specify the fraction of the loyal segment that uses the cash back site as a function of D. Our approach is that of a reduced form formulation of the loyal consumers' decision to use the cash back site. A fully endogenous representation would consist of a utility framework and a characterization of loyal consumers' expectations of prices of their store. Of course, these expectations would depend on the competing brand's strategy, in addition to own brand's strategy. We do not pursue this approach, which would complicate the analysis, and we think would not provide additional insights.

It is useful to think about the role of the cash back site. Note that the site does not display the prices at the stores. Thus, it is not a price comparison website. In our case, the cash back site informs consumers of the discount and provides them with a link that makes them eligible for a discount. It will turn out that the equilibrium pricing strategy in our model is in mixed strategies. Therefore, knowing only the discount does not tell a searching consumer whether or not she should engage in search. That will depend on the price realization at the retailer.

We model the strategic interaction between retailers and consumers as a three-stage game of perfect information. In the first stage, retailers choose the discount. In the second stage, some consumers in the loyal segment and all consumers in the switching and searching segments visit the cash back site and become eligible for discounts. Furthermore, conditioned on the discounts, searching consumers choose the probability of visiting first each store, and retailers choose prices. Finally, in the third stage, searching consumers decide whether to stop search and buy from the first store or, as switching consumers do, continue to search and buy from the store with the lower discounted price. The retailers' decisions in two stages captures the reality that discounts are fixed over a longer horizon and prices are changed more frequently. This is analogous to airlines choosing baggage fees (negative discounts) that are fixed over time while prices vary. Next, we derive, in Section 4, the retailers' equilibrium pricing strategies when there is no intermediary in the form of a cash back site and in Section 5 in the presence of the cash back site.

4. Equilibrium Without an Intermediary

In the absence of a cash back site, equilibrium outcomes can be one of two types. In one outcome, the retailers' pricing is such that searchers do not find search profitable and so in equilibrium they do not search. Essentially, the pricing strategies are such that they eliminate search by searching consumers. This is basically Stahl's result. In the other outcome, the retailers do not find it profitable to eliminate search. Rather, they charge prices such that for a range of prices consumers search in equilibrium, but for a (lower) range of prices they do not search. So search is partially prevented. We consider each of these outcomes in turn to identify the equilibrium.

4.1. Outcome 1: Eliminate Search

This outcome, were it to occur in equilibrium, can be characterized by extending Stahl's (1989) analysis with the additional condition $\alpha > 0$. Of course, switchers buy from the lower-priced store. Searchers, on the other hand, first decide to visit one store. Given that the retailers are symmetric, searchers have no reason to believe that one store has a lower price and so we assume that half of them go to one store and the other half to the other store.[2] A consumer that visits store 1 will encounter a price p_1, and must decide whether to buy or to visit the other store. Denote $f_i^1(x)$, $x \in (l_i, \overline{p_i})$, $\overline{p_i} \le V = 1$ to be the p.d.f. of price p_i at store i as expected by the searching consumers, with the superscript denoting outcome 1. Note that consumers expect prices to be not greater than 1. For a consumer to want to search after vising store i, we need for $j = 3 - i$,

$$\int_{l_j}^{p_i} (p_i - x) f_j^1(x) dx > S. \tag{1}$$

Inequality (1) says that the expected gain from search given by the left-hand side should be greater than the cost of search, S. In that case, searchers will continue search and purchase at the lower price. If, however, inequality (1) does not hold and so $\int_{l_j}^{p_i} (p_i - x) \times f_j^1(x) dx \le S$, then searchers will stop search and purchase at the first store without comparing prices. Denote Π_i^1 to be the profit of retailer i. Then each retailer must choose l_i, $\overline{p_i}$, and f_i^1 to maximize Π_i^1.[3]

Stahl (1989) has characterized the mixed strategy Nash equilibrium that eliminates search. Stahl's analysis is based on $\alpha = 0$, so we must extend his analysis to the case $\alpha > 0$. To characterize the equilibrium that eliminates search, we must solve for the store's profit Π^1, l, $\overline{p}$, and f^1, noting that in light of symmetry we can drop the subscript i. Moreover, from Stahl (1989), we know that equilibrium consists of a continuous function f^1 over $[l_i, \overline{p_i}]$. For the mixed strategy Nash equilibrium, we require that a retailer's profit be equal for all prices in $[l_i, \overline{p_i}]$. Mathematically, this can be written as

$$\Pi^1 = \pi(\overline{p}) = \left(\frac{\alpha}{2} + \frac{\beta}{2}\right)\overline{p}, \tag{2}$$

$$\Pi^1 = \pi(l) = \left(\frac{\alpha}{2} + \frac{\beta}{2} + \gamma\right)l, \tag{3}$$

$$\Pi^1 = \pi(p) = \left(\frac{\alpha}{2} + \frac{\beta}{2} + \gamma\left(1 - F^1(p)\right)\right)p, p \in \left(l, \overline{p}\right). \tag{4}$$

Consider equality (2). The profit when a retailer chooses the highest price $\overline{p}$ is given by multiplying the price by the sales to its loyal consumers and the half of searchers who come to it first. This must be equal to Π^1. Similarly equality (3) says that the profit when charging l must also equal Π^1. Note, however, that at this low price the store sells not only to its loyal consumers and the half of searchers who come to it first but also to all the switching consumers. Finally, equality (4) says that when a store charges any intermediate prices the profit should again be equal to Π^1. In this situation, the probability of selling to switchers is the probability that the rival's price is higher, given by $1 - F^1(p)$. We also require that in equilibrium search is completely eliminated. For this to happen even when a searcher encounters the highest price $\overline{p}$, it must be unprofitable for her to search. Mathematically, following Weitzman (1979), after simplification we can get

$$S \ge \int_l^{\overline{p}} F^1(x) dx. \tag{5}$$

The four conditions (2)–(5) can be used to solve for Π^1, l, $\overline{p}$, and f^1. We refer to this outcome as the outcome of preventing search.

Lemma 1. *If and only if* $S \ge (1 - \frac{\ln(1+z)}{z}) \frac{\alpha}{\alpha+\beta}$, *where* $z = \frac{2\gamma}{\alpha+\beta}$, *there exists an equilibrium that prevents search, and moreover* $F^1(p) = 1 - \frac{1}{z}(\frac{\overline{p}}{p} - 1)$, $\overline{p} = \min\{\frac{S}{1-\frac{\ln(1+z)}{z}}, 1\}$, $l = \frac{1}{1+z}\overline{p}$ *and equilibrium profit* $\Pi^1 = (\frac{\alpha}{2} + \frac{\beta}{2})\overline{p}$.

Proof of Lemma 1. First, by equating (2) and (3), we can get $l = \frac{1}{1+z}\overline{p}$. Then, by equating (2) and (4), we can get $F^1(p) = 1 - \frac{1}{z}(\frac{\overline{p}}{p} - 1)$. Plug l and $F^1(p)$ back into Equation (5) to get $S \ge \int_{\frac{\overline{p}}{1+z}}^{\overline{p}} F^1(x) dx = \overline{p}(1 - \frac{\ln(1+z)}{z})$. Note that multiple $\overline{p}$ can satisfy the condition $S \ge \overline{p}(1 - \frac{\ln(1+z)}{z})$. However, in equilibrium, the stores would choose the one with the highest profit. If $S \ge \int_{\frac{1}{1+z}}^{1} F^1(x) dx$, corresponding to the highest profit, $\overline{p} = 1$ would be 1. The profit in this case is higher than the security level, $\frac{\alpha}{2}$. On the other hand, if $S < \int_{\frac{1}{1+z}}^{1} F^1(x) dx$, the optimal $\overline{p}$, given that the search is prevented in equilibrium, would be the one that satisfies $S = \overline{p}(1 - \frac{\ln(1+z)}{z})$, equivalently $\overline{p} = \frac{S}{1-\frac{\ln(1+z)}{z}}$. Because $\overline{p} < 1$, for this to be an equilibrium, we must verify that a deviation to $p = 1$, yielding the security level of profits, $\frac{\alpha}{2}$, generates a lower profit than the one in equilibrium. This implies $\overline{p} \ge \frac{\alpha}{\alpha+\beta}$. The search benefit is increasing in $\overline{p}$, so the equivalent condition on S is $S \ge (1 - \frac{\ln(1+z)}{z}) \frac{\alpha}{\alpha+\beta}$. □

Lemma 1 is easy to understand. Preventing search profitably requires a sufficiently high search cost. For example, as the cost approaches zero, searchers' behavior resembles that of switchers, and preventing search is possible only if $\overline{p}$ is sufficiently small. But then this results in decreased profits from the loyal consumers, and so preventing search is not profitable compared with the security level of charging a price of $V = 1$ to the loyal consumers. Indeed, if S is very high, search can be prevented even at $\overline{p} = 1$, the maximum willingness to pay.

4.2. Outcome 2: Searchers Search Partially

To solve for the equilibrium pricing strategies in this case, we must extend Narasimhan's (1988) analysis, because, unlike the switchers in Narasimhan's model, searchers continue to search at high prices and do not search at low prices. Thus, in contrast to Narasimhan's model, in which the relative size of loyal consumers and switchers is exogenous, in our model it can be thought of as being endogenous. As opposed to outcome 1, search by searchers is not completely prevented in this equilibrium. There exists a price point in the support, a reservation price, such that if the searchers see a price that is higher than this price after arriving at the first store, they continue searching. If on the other hand, the searchers see a price that is less than or equal to this price, they would remain and purchase at the first store they visit.

We can use an approach similar to that of Gangwar et al. (2014) and additionally incorporate the Weitzman's search condition to solve for the equilibrium of this outcome. Doing so leads to a mixed strategy of the type in Gangwar et al. (2014). It consists of two continuous functions $f_l^2(p)$, $p \in [l, \underline{h}]$ and $f_u^2(p)$, $p \in [\overline{h}, \overline{p}]$, $0 < l; \underline{h} \le \overline{h}; \overline{p} \le V = 1$ and a hole with $f^2(x) = 0, x \in (\underline{h}, \overline{h})$. As before, we can drop the subscript i in light of symmetry. Intuitively, the hole exists because the expected sales are different at the two prices $p = \underline{h}$ and $p = \overline{h}$, which we will discuss later. The superscript 2 on Π^2 and f^2 correspond to outcome 2.

$$\Pi^2 = \pi(\overline{p}) = \frac{\alpha}{2}\overline{p} \tag{6}$$

$$\Pi^2 = \pi(l) = \left(\frac{\alpha}{2} + \frac{\beta}{2} + \gamma + \frac{\beta}{2}\left(1 - F_l^2(\underline{h})\right)\right) l \tag{7}$$

$$\begin{aligned}\Pi^2 &= \pi\left(p|p \ge \overline{h}\right)\\ &= \left(\frac{\alpha}{2} + \frac{\beta}{2}\left(1 - F_u^2(p)\right) + \gamma\left(1 - F_u^2(p)\right)\right.\\ &\quad \left. + \frac{\beta}{2}\left(1 - F_u^2(p)\right)\right) p, p \in \left(\overline{h}, \overline{p}\right)\end{aligned} \tag{8}$$

$$\begin{aligned}\Pi^2 = \pi(p|p \le \underline{h}) &= \left(\frac{\alpha}{2} + \frac{\beta}{2} + \gamma\left(1 - F_l^2(p)\right)\right.\\ &\quad \left. + \frac{\beta}{2}\left(1 - F_l^2(\underline{h})\right)\right) p, p \in (l, \underline{h})\end{aligned} \tag{9}$$

Consider equality (6). At $\overline{p}$, the profit to each retailer is $\frac{\alpha}{2}\overline{p}$, because only loyal consumers purchase at their preferred store, while all the other consumers search.[4] Thus, the security level of profits is $\frac{\alpha}{2}\overline{p}$. And, as in Narasimhan (1988), it is easy to see that the security level is maximized at $\overline{p} = 1$, and equals Π^2. Equality (7) says that the profit when charging l must also equal Π^2. Note that equality (7) is different from equality (4). This is because the store now has an additional source of profit from those searchers who go to the competitor store first but finding a high price, decide to search and finally purchase at the focal store. This additional profit is $\frac{\beta}{2}(1 - F_l^2(\underline{h}))l$. To complete the equi-profit argument, we require that the profit at any price $p \in [l, \underline{h}] \cup [\overline{h}, 1]$ also must be equal to Π^2. This condition leads to equalities (8) and (9). Equality (8) says that when the store prices above $\overline{h}$, the searchers who first come to it actually search. The profit from these consumers is $\frac{\beta}{2}(1 - F_u^2(p))p$, because they buy only if the competitor's price is higher than the focal store's price, p. Also, note that searchers who first go to the competitor's store may stop search or continue to search depending on the competitor's price. So the profit from these consumers is also $\frac{\beta}{2}(1 - F_u^2(p))p$, because they buy only if the competitor's price is higher than p. Similarly, equality (9) shows the profit when the retailer's price is lower than $\underline{h}$. Now, the half of searchers who first come to it stop further search. The other half of searchers who first go to the other store and decide to continue search also purchases at the focal store, because they only search when they find the competitor's price to be higher than $\overline{h}$ and $p \le \underline{h} \le \overline{h}$. In sum, equalities (6)–(9) represent the equi-profit conditions for a mixed strategy Nash equilibrium.

Two other conditions must hold in equilibrium. The first arises because the cumulative discount factor of the mixing distribution must be monotonically nondecreasing. In other words, $f^2(p) = 0, p \in (\underline{h}, \overline{h})$, and so we have

$$F_l^2(\underline{h}) = F_u^2\left(\overline{h}\right). \tag{10}$$

We also require that in equilibrium $\underline{h}$ be such that when consumers encounter a price below $\underline{h}$, they do not search. For this to hold, it must be unprofitable for her to search when seeing a price $p \in [l, \underline{h}]$. Mathematically, following Weitzman, after simplification we can get

$$S = \int_l^{\underline{h}} F_l^2(x)dx. \tag{11}$$

We can now solve for $\Pi^2, l, \overline{p}, \underline{h}, \overline{h}$, and f^2 using the six conditions (6)–(11).

Lemma 2. *When $S < (1 - \frac{\ln(1+z)}{z})\frac{\alpha}{\alpha+\beta}$, there exists an equilibrium in which both stores allow search. Moreover, the pricing strategy is given by the mixing distribution:*

$F_l^2(p) = \frac{(p-l)\alpha}{2pl\gamma}$, *if* $l \le p \le \underline{h}$

$F_l^2(\underline{h}) = F_u^2(\overline{h})$

$F_u^2(p) = 1 - \frac{(1-p)\alpha}{2(\beta+\gamma)p}$, *if* $\overline{h} \le p \le \overline{p}$

$\underline{h} = \frac{l\alpha\beta}{\alpha(\beta+2\gamma) - 2l\gamma(\alpha+2(\beta+\gamma))} < \overline{h} = \frac{l\alpha\beta}{2\alpha(\beta+\gamma) - l(\beta+2\gamma)(\alpha+2(\beta+\gamma))}$

l solves $S = \int_l^{\underline{h}} F_l^2(x)dx$

$\overline{p} = 1$

The equilibrium profit is $\Pi^2 = \frac{\alpha}{2}$.

Proof of Lemma 2. We first equate (6) and (7) to solve for $F_l^2(\underline{h})$. Note that, according to our previous discussion, $\overline{p}$ is equal to $V = 1$, because both stores maximize the expected profit in equilibrium. Here, we get $F_l^2(\underline{h}) = \frac{(\alpha+2(\beta+\gamma))l-\alpha}{\beta l}$. Next, we equate equality (6) and (8) to get $F_u^2(p) = 1 - \frac{(1-p)\alpha}{2(\beta+\gamma)p}$. Similarly, if we equate equality (6) and (9), and at same time, plug in the value of $F_l^2(\underline{h})$, we can get $F_l^2(p) = \frac{(p-l)\alpha}{2pl\gamma}$.

Now we are able to solve for $\underline{h}$ and $\overline{h}$ from $F_l^2(p=\underline{h}) = F_l^2(\underline{h})$ and $F_u^2(\overline{h}) = F_l^2(\underline{h})$.

$$\underline{h} = \frac{l\alpha\beta}{\alpha(\beta+2\gamma) - 2l\gamma(\alpha+2(\beta+\gamma))},$$
$$\overline{h} = \frac{l\alpha\beta}{2\alpha(\beta+\gamma) - l(\beta+2\gamma)(\alpha+2(\beta+\gamma))} \quad (12)$$

Finally, we can solve for l from Equation (11) after plugging in $\underline{h}$ and $F_l^2(p)$.

$$S = \frac{\alpha}{2\gamma}\left(\frac{\alpha\beta}{\alpha(\beta+2\gamma) - 2\gamma l(\alpha+2(\beta+\gamma))} - \ln\left(\frac{\alpha\beta}{\alpha(\beta+2\gamma) - 2\gamma l(\alpha+2(\beta+\gamma))}\right) - 1\right) \quad (13)$$

Two conditions are necessary for this equilibrium to exist. First, it must be that $\underline{h} < 1$, equivalently $l < \frac{\alpha(\beta+2\gamma)}{2\alpha\gamma+4\beta\gamma+4\gamma^2+\alpha\beta}$. If $\underline{h} = 1$, such an equilibrium cannot exist, because the searching consumers will never search in equilibrium. Second, it must be that $F_l^2(\underline{h}) < 1$ or, equivalently, $l < \frac{\alpha}{\alpha+\beta+2\gamma}$ for outcome 2 to exist. Moreover, $\frac{\alpha}{\alpha+\beta+2\gamma} < \frac{\alpha(\beta+2\gamma)}{2\alpha\gamma+4\beta\gamma+4\gamma^2+\alpha\beta}$. From (13), we can get $\frac{dS}{dl} > 0$. Hence, outcome 2 will exist if $l < \frac{\alpha}{\alpha+\beta+2\gamma}$, equivalently $S < (1 - \frac{\ln(1+z)}{z})\frac{\alpha}{\alpha+\beta}$, where $z = \frac{2\gamma}{\alpha+\beta}$.

Can we rule out the possibility that $\underline{h} \geq \overline{h}$? We have calculated $\underline{h}$ and $\overline{h}$ in (12). Note that the numerators are identical. We can therefore compare the denominator to get the relationship between $\underline{h}$ and $\overline{h}$. From calculation, we have $\alpha(\beta+2\gamma) - 2l\gamma(\alpha+2(\beta+\gamma)) - (2\alpha(\beta+\gamma) - l(\beta+2\gamma)\ (\alpha+2(\beta+\gamma))) = \beta(l(\alpha+2(\beta+\gamma)) - \alpha)$. We know, by using the equi-profit conditions, that this value will always be larger than zero, unless $F_l^2(\underline{h}) = 0$. In the equilibrium, $F_l^2(\underline{h})$ is always larger than zero. □

The mixed strategy in Lemma 2 has an intuitive interpretation. First, it says that if search cost is sufficiently small, then complete search prevention is not a profitable strategy, because it would yield lower profits than the security level. So consumers search at high prices and do not search at low prices. Thus, the relative size of the segment of consumers who compare prices is a function of price and so the mixed strategy must account for this. This leads to a hole because the demand at the store has a jump decrease if the price p is higher than $\underline{h}$. When a searcher sees a price like this, he or she would continue to search, and purchase at the other store with a probability $F_u^2(p)$. Hence, the difference between the demands at the store when pricing at $\underline{h}$ and when pricing at $\overline{h}$ is larger than zero. This loss of the demand from searching consumers forces the store to increase price by a positive amount to maintain the expected profit at the same level. That is why we observe a hole in the equilibrium price distribution in outcome 2.

As we can see from Lemma 2, there is no analytical solution for l. So it is useful to briefly illustrate the equilibrium strategies numerically. Figure 1 does this. We present a numerical example with $V = 1$, $S = 0.1$, $\alpha = 0.25$, $\beta = 0.25$, and $\gamma = 0.5$. The dashed line corresponds to the hole where $f^2(p) = 0$.

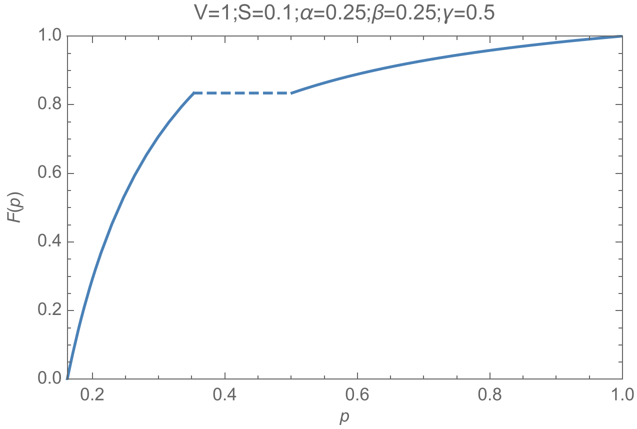
4.3. Equilibrium When There Is No Intermediary

We can combine our results in Sections 4.1 and 4.2 on the expected profits in the two outcomes, and determine the equilibrium depending on the parameters. We state this formally in Proposition 1. The proof is omitted, because the parameter regions are nonoverlapping, and so there is a unique equilibrium in each region.

Proposition 1. *Suppose there is no cash back site. If $S < (1 - \frac{\ln(1+z)}{z})\frac{\alpha}{\alpha+\beta}$, where $z = \frac{2\gamma}{\alpha+\beta}$, then both stores adopt strategies such that the searchers search partially in equilibrium; if $S \geq (1 - \frac{\ln(1+z)}{z})\frac{\alpha}{\alpha+\beta}$, then both stores adopt strategies such that search is prevented in equilibrium.*

The intuition behind Proposition 1 is as follows. Lemma 1 shows that when the search cost is extremely large ($S \geq (1 - \frac{\ln(1+z)}{z})$), the stores can prevent search without decreasing prices. In this case, their profits are higher than what they can obtain from the loyal consumers. As the search cost decreases, the stores still find it profitable to prevent search but in order to do that, they have to lower prices. Thus, there is a trade-off between preventing search to increase profit, on the one hand, and losing profit from loyal consumers by lowering prices, on the other. This is outcome 1. However, if the search cost is too small, the stores no longer prefer to prevent search, because the prices will be too low. In this case, they choose to allow search and the equilibrium will result in outcome 2. Moreover, their profits are simply the security level they can obtain from the loyal consumers.

Figure 1. (Color online) Cumulative Probability Distribution of Prices in Outcome 2



Outcome 2 profits are lower than outcome 1, because search is not prevented in outcome 2.

Next, we derive the retailers' equilibrium pricing strategies when there is an intermediary who is a cash back site.

5. Equilibrium with an Intermediary Present

With online cash back, a store can offer a "discount" only to those people who use the cash back site. In other words, the store is able to engage in price discrimination, permitting the store to prevent search by searching consumers without lowering the price to consumers that do not use the cash back site. The effect of this is different in the two outcomes identified in the absence of a cash back site. In outcome 1, search prevention is profitable, and so with the ability to price discriminate, stores would continue to prevent search but could also obtain higher profits with a discount strategy. On the other hand, if the equilibrium is outcome 2 in the absence of a cash back site, stores must ask whether they can prevent search profitably and earn higher profits. More important, would that be part of a Nash equilibrium with stores competing not only on prices but also on discounts at the cash back site? That is the question we answer in this section.

As noted earlier, in our model, all consumers in the searching and switching segments use the cash back site. Turning to the consumers in the loyal segments, we model the fraction that does not use the cash back site for store i as $\frac{1}{2}\alpha(1-D_i)^2$, with the remaining loyal consumers using the site.[5] Thus, if $D_i = 0$, no loyal consumers of store i use the site, while if $D_i = 1$, so that there is a 100% discount, leakage is at a maximum and all loyal consumers use the site. Our model of consumers' use of the cash back site implies that as the discount approaches 1, only a vanishingly small segment of consumers does not use the cash back site. In reality, like loyal consumers, some searching consumers also can be expected to not use the cash back site, and stores serve these consumers too, and so to capture that within our model we assume that both stores serve all consumers.[6]

As in the case of no intermediary, the equilibrium turns out to be in mixed strategies. However, deriving the equilibrium strategies is more complicated, because the game is now multistage and involves the choice of both discount and prices. We wish to solve for a subgame perfect pure strategy Nash equilibrium in discounts under complete information. Our analysis proceeds backward by first examining the second-stage equilibrium in Section 5.1 and then the first stage in Section 5.2. We focus on second-stage equilibria in subgames induced by first-stage discounts: $D_1 = D_2 = D > 0$ that are part of a symmetric first-stage candidate equilibrium. Given our interest in search prevention, we derive in Lemma 3, the necessary conditions for second-stage search prevention to be a Nash equilibrium for the subgame.

Turning to the first stage, in Section 5.2, in Lemma 5, we show that if the search cost is not too high, then, in any symmetric Nash equilibrium, the first-stage discount must be higher than a minimum level, D^*. In Proposition 2, we establish our main result and show that if the search cost is also not too low, the profits from a discount strategy $D_1 = D_2 = D^*$ leads to higher profits by preventing search in the second stage. Having established that the cash back site can increase profits, we elaborate in Section 5.3 on the implications of the presence of the cash back site. In particular, we discuss how it affects price paid by consumers, and its role if either searching or switching consumers were not present. Then, in Section 5.4, we establish subgame perfectness of the D^* in Lemmas 6, 7, 8, and 9 contained in the online appendix. This is an analytical result for a complicated game. To provide shaper insights, we illustrate the equilibrium in Proposition 2 through a numerical example and discuss the intuition behind our findings. Following that, we consider the case if S is large and provide a discussion of the equilibrium that obtains in this case with $D < D^*$, omitting a formal proof of subgame perfectness. We demonstrate subgame perfectness for this case through a numerical illustration.

5.1. Second-Stage Equilibrium

It will be convenient to work with the discounted price to characterize the mixed strategies in the second stage, so let us denote pp_i to be store i's price after discount so that $pp_i = (1-D_i)p_i$ with p_i denoting the undiscounted price. We only will be interested in search preventing pricing strategies in the second stage. This is because if the second-stage equilibrium does not prevent search, then such a discount strategy can be ruled out as a Nash equilibrium candidate, because it would result in lower profits than the security level of $\frac{\alpha}{2}$.[7] We define the effective loyal segment, denoted by ϕ, as consisting of consumers who buy at the store they visit without comparing prices. This includes two groups of consumers. One group consists of the searching consumers who do not search in equilibrium as well as loyal consumers who use the cash back site. This group, in equilibrium, pays pp_i. The other group consists of loyal consumers who do not use the cash back site. This group, in equilibrium, pays $p_i = \frac{pp_i}{1-D_i}$. To simplify the analysis of the mixed strategies in terms of pp_i, we treat $\frac{\alpha(1-D_i)^2}{2(1-D_i)}$ as the effective size of the second group and the price paid as pp_i. Let searching consumers visit store i with probability $\theta_i(D_1, D_2)$, $0 < \theta_i < 1 = \theta_1 + \theta_2$. Then

$$\phi_i = \frac{\alpha(1-D_i)^2}{2(1-D_i)} + \frac{1}{2}\alpha\left(1-(1-D_i)^2\right) + \theta_i\beta. \tag{14}$$

Focusing on symmetric first-stage discounts, $D_1 = D_2 = D$, we will seek stores' mixed strategies of prices $pp \in [l, \overline{pp}]$, $\overline{pp} \le 1 - D$ such that searching consumers do not want to search at any price in the support. Faced with equal discounts, we assume consumers choose $\theta_1 = \theta_2 = \frac{1}{2}$. So $\phi_1 = \phi_2 = \phi = \frac{1}{2}\alpha(1 + D - D^2) + \frac{\beta}{2}$, and the resultant symmetric mixed strategies in the second stage render the expected prices at the two stores to be equal. Moreover, the continuation profits in second stage from a search prevention strategy, given discount D in the first stage, $\widehat{\Pi}^3(D)$, is

$$\widehat{\Pi}^3(D) = \overline{pp}(D)\phi(D). \tag{15}$$

Then Lemma 3 states the necessary conditions for search prevention to be the outcome in a symmetric Nash equilibrium mixed strategy.

Lemma 3 (Necessary Conditions for Second-Stage Search Prevention Equilibrium). *Suppose that in the first stage, both stores choose an identical discount, D. Then the necessary conditions for search prevention to be the outcome in a symmetric Nash equilibrium mixed strategy are*

i. *searching consumers visit each store with probability $\frac{1}{2}$,*

ii. *stores' mixed strategy prices, $F^3(pp) \equiv F_i^3(pp)$, $i = 1, 2$, satisfy the following conditions:*

a. *profits at the supports must be equal: $\widehat{\Pi}_i^3(D) = \overline{pp}\phi(D) = l(\phi(D) + \gamma)$,*

b. *the mixing distribution renders profits equal over the interval: $\widehat{\Pi}^3(D) = pp(\phi(D) + \gamma(1 - F^3(pp)))$, implying the mixing distribution $F^3(pp) = \frac{\phi+\gamma}{\gamma}(1 - \frac{l}{pp})$, $pp \in (l, \min(1 - D, \widehat{pp}))$, where $\widehat{pp} \triangleq \frac{\gamma S}{\gamma + \phi \ln(\frac{\phi}{\phi+\gamma})}$,*

c. *searching consumers do not search if $pp \le \overline{pp}$ so that $S \ge \int_l^{\overline{pp}} F^3(x)dx$, and*

iii. *search prevention profits exceed security level: $\overline{pp} \ge (1 - D)(1 - \frac{\beta}{2\phi(D)})$.*

Proof of Lemma 3. (See Appendix B.)

Corollary 1. *For any D, $\exists$ a search cost $S(D)$ such that the equilibrium profits are $\widehat{\Pi}_i^3(D) = \widehat{pp}\phi(D)$ where $S(D)$ satisfies $(1 - \frac{\beta}{2\phi(D)})(1 - D)\frac{\gamma + \phi\ln(\frac{\phi+\gamma}{\gamma})}{\gamma} < S(D) < (1 - D)\frac{\gamma + \phi\ln(\frac{\phi+\gamma}{\gamma})}{\gamma}$.*

Proof of Corollary 1. Given the condition on S, we see that $(1 - D)(1 - \frac{\beta}{2\phi(D)}) < \widehat{pp} < 1 - D$, where $\widehat{pp} = \frac{\gamma S}{\gamma + \phi\ln(\frac{\phi}{\phi+\gamma})}$. It is easy to verify that the necessary conditions of Lemma 3 are satisfied in this case. □

It is useful to investigate the equilibrium when $\overline{pp} = \widehat{pp}$. In particular, we can explore the effect of $\phi(D)$ and γ on Π^3 if S. That would offer an insight into the effect of segment sizes on profits.

Corollary 2. *If $(1 - \frac{\beta}{2\phi(D)})(1 - D)\frac{\gamma + \phi\ln(\frac{\phi+\gamma}{\gamma})}{\gamma} < S(D) < (1 - D)\frac{\gamma + \phi\ln(\frac{\phi+\gamma}{\gamma})}{\gamma}$, then $\frac{d\hat{\Pi}^3}{d\gamma} < 0$ and $\frac{d\hat{\Pi}^3}{d\phi(D)} > 0$.*

Proof of Corollary 2. From Equation (B.4), we know that $\overline{pp} = \widehat{pp} = \frac{\gamma S}{\gamma + \phi(D)\ln(\frac{\phi(D)}{\phi(D)+\gamma})}$. After taking the derivatives with respect to γ, we have $\frac{d\overline{pp}}{d\gamma} = \frac{S\phi(D)(\gamma + (\gamma + \phi(D))\ln(\frac{\phi(D)}{\gamma + \phi(D)}))}{(\gamma + \phi(D))(\gamma + \phi(D)\ln(\frac{\phi(D)}{\gamma + \phi(D)}))^2} < 0$. Next, we take the derivative with respect to $\phi(D)$ and get $\frac{d\overline{pp}}{d\phi(D)} = -\frac{\gamma S(\gamma + (\gamma + \phi(D))\ln(\frac{\phi(D)}{\gamma + \phi(D)}))}{(\gamma + \phi(D))(\gamma + \phi(D)\ln(\frac{\phi(D)}{\gamma + \phi(D)}))^2} > 0$.

Therefore, because $\hat{\Pi}^3 = \overline{pp}\phi(D)$, $\frac{d\hat{\Pi}^3}{d\gamma} < 0$ and $\frac{d\hat{\Pi}^3}{d\phi(D)} = \overline{pp} + \phi(D)\frac{d\overline{pp}}{d\phi(D)} > 0$. □

Lemma 3 and its corollaries tell us how second-stage equilibrium outcome depends on the first-stage decision D when $D_1 = D_2 = D$, which in turn implies that the effective loyal segments of the two stores are equal: $\phi_1 = \phi_2 = \phi$.[8] Note that if the conditions in Corollary 2 are satisfied, then $\overline{pp} = \widehat{pp}$, and so in that case the profit in the second stage is $\widehat{pp}\phi$. Moreover, $\widehat{pp}$ depends on ϕ and the parameters. So the continuation value in the subgame induced by D can be solely expressed in terms of the corresponding ϕ. This key point allows us to characterize the second-stage equilibrium profits in terms of ϕ. Now, we can see from Corollary 2 that $\overline{pp}$ is increasing in ϕ. And ϕ is increasing in D if $D < \frac{1}{2}$ and decreasing in D if $D > \frac{1}{2}$. Therefore, if $D < \frac{1}{2}$, a higher D results in higher $\overline{pp}$ and higher ϕ and thus higher continuation profits.

It is useful to elaborate on the intuition behind why $\overline{pp}$ increases with D, up to $\frac{1}{2}$.[9] In our model, there are two groups of consumers who pay different prices, with or without discount for the same basket. For example, it is better to charge 0.8 as part of a strategy with $D = 10\%$ to people vising the site rather than charge 0.8 as part of a strategy with no discount. The reason is that one of the groups (loyal consumers not visiting the site) is paying a higher price than 0.8, namely, $\frac{0.8}{1-0.1}$. Moreover, as D increases, the price this group pays is even higher. If charging any price p is more beneficial for a larger D, the price competition is less severe. Hence, the price distribution shifts upward, allowing for a larger value of $\overline{pp}$. Note that if all loyal consumers were to visit the cash back site, the cost of preventing search cannot be reduced by making consumers pay different prices and so the equilibrium outcome would be the same as what obtains when there is no cash back site.

Turning to the case when $D > \frac{1}{2}$, a higher D results in lower ϕ because of "leakage" of loyal consumers shrinking the size of the group paying the higher price even though the price they pay relative to those who use the site increases. Thus, the benefit of larger D is actually diminishing and even could be negative if the size of this group shrinks faster. This explains why $\frac{d\overline{pp}}{dD} > 0$ when D is small, while $\frac{d\overline{pp}}{dD} < 0$ when D is large.

5.2. First-Stage Equilibrium

Discounts chosen in the first stage must consider the consequences of attracting more searching consumers or deriving greater benefit from price discrimination or both. The discounts then determine how searching consumers decide to visit each store and also the stores' pricing strategies. If stores offer symmetric discounts in the first stage, then consumers visit each store with probability $\frac{1}{2}$.[10] Conceptually, the discounts can range from 0 to 1. We proceed by showing first that if the search cost is not too high, a symmetric Nash equilibrium has a minimum discount level, D^*. Lemma 5 proves this result by showing that in any candidate symmetric equilibrium with discount $D < D^*$ one store can deviate profitably to D^*. This allows us to restrict our attention to possible candidates for Nash equilibrium. The proof of Lemma 5 invokes Lemma 4 in which we characterize searching consumers' choice of θ if one store chooses D^* and the other chooses $D < D^*$. Our central result then is in Proposition 2 in which we identify the conditions on search cost S for a profitable discount strategy to be a Nash equilibrium outcome: search cost must be bounded from below in addition to satisfying the condition in Corollary 2. The lower bound on S ensures that search prevention results in profits higher than the security level. What is also interesting is that the D^* equilibrium has higher profits than any other candidate Nash equilibrium with symmetric discount $D > D^*$.

Lemma 4 (Choosing θ to make the expected prices equal). *Suppose $\alpha < 8\beta$ and the first-stage discounting strategy is $\{0 < D_1 = D \le \frac{1}{2}, D_2 = \widetilde{D} \in [0,1]\}$. Then if consumers choose θ, $0 < \theta = \frac{\alpha(\tilde{D}-D)(1-D-\tilde{D})+2\beta}{4\beta} < 1$, the size of the effective loyal segments of store 1 and store 2 are equal, and, therefore, the expected prices are equal.*

Proof of Lemma 4. (See Appendix B.)

An immediate consequence of stores' effective loyal segments being symmetric is that their search prevention mixed strategies are also symmetric, the expected prices are equal, and so it makes sense for consumers to randomize across stores.

Corollary 3. *Suppose $\alpha < 8\beta$. If $S < \min(1 - D_1, 1 - D_2)\frac{\gamma + \phi\ln(\frac{\phi+\gamma}{\gamma})}{\gamma}$, then $\frac{d\widehat{\Pi}^3}{d\gamma} < 0$ and $\frac{d\widehat{\Pi}^3}{d\phi} > 0$.*

Proof of Corollary 3. (See Appendix B.)

Lemma 5 ($D = D^* \ge \frac{1}{2}$ in a symmetric Nash equilibrium). *Suppose $\alpha < 8\beta$ and $S < \frac{(\frac{5\alpha}{4}+\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+2\gamma}{4\gamma}$. Then any symmetric first-stage discounting strategy $D_1 = D_2 = D < \frac{1}{2}$ cannot be a Nash equilibrium in the first stage.*

Proof of Lemma 5. (See Appendix B.)

The implication of Lemma 5 is that if a symmetric Nash equilibrium exists, then the equilibrium discount must be greater than or equal to $\frac{1}{2}$ in the first stage. We turn to candidate Nash equilibria with $D > D^* = \frac{1}{2}$. We cannot rule out such equilibria, but the profits from any of these equilibria are lower than the D^* equilibrium. To see this, recall that the constraint $\overline{pp} \le 1 - D$ is either binding or not. If it is not binding, we know from Corollary 3 that profits are increasing in ϕ. Moreover, if $D > \frac{1}{2}$, ϕ is decreasing in D. Therefore, in this case, profits are lower than in the D^* equilibrium. If the constraint is binding, then note that again ϕ is decreasing in D and so is $1 - D$. Thus, profits are higher in the D^* equilibrium than in any candidate symmetric Nash equilibrium with $D > D^*$.

Next, Proposition 2 identifies the upper bound on search cost S so that with a symmetric discount strategy $D_1 = D_2 = D < \frac{1}{2}$ the profits from search prevention are higher than what the profits would be were there no cash back site.

Proposition 2 (Profitability of Cash Back Site).

i. *Suppose outcome 1 is the equilibrium without cash back site. Then there exists a candidate symmetric equilibrium when using a cash back site such that the expected store profit is always higher than without the cash back site.*

ii. *Suppose $\alpha \le \frac{4}{3}\beta$ and $\frac{1}{2}\alpha\left(\frac{\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})}{\gamma} + \frac{8}{5\alpha+4\beta}\right) \le S < \frac{(\frac{5\alpha}{4}+\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+2\gamma}{4\gamma}$. If the equilibrium without a cash back site is outcome 2, then $D = D^* = \frac{1}{2}$ is a candidate symmetric equilibrium that yields a higher expected retailer profit than without a cash back site.*

Proof of Proposition 2. Because $S < \frac{(\frac{5\alpha}{4}+\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+2\gamma}{4\gamma}$, we know from Lemmas 3 and 5 that $\widehat{pp} = \overline{pp} = \frac{\gamma S}{\gamma + \phi(D)\ln(\frac{\phi(D)}{\phi(D)+\gamma})}$. Recalling that the continuation value is $\widehat{pp}\phi$ and substituting $D = \frac{1}{2}$, $\widehat{\Pi}^3 = \widehat{pp}\phi(\frac{1}{2},\frac{1}{2}) = \frac{\gamma S(5\alpha+4\beta)}{(5\alpha+4\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+8\gamma}$.

For search prevention to be profitable, we need $\widehat{\Pi}^3 \ge \frac{\alpha}{2}$ or, equivalently,

$$\frac{1}{2}\alpha\left(\frac{\ln\left(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma}\right)}{\gamma} + \frac{8}{5\alpha+4\beta}\right) \le S. \tag{16}$$

Proposition 2 establishes our central result that a cash back site can lead to higher profits by preventing search on the part of searching consumers even if such search prevention strategies are unprofitable in the absence of a cash back site. In other words, if equilibrium is outcome 2 with no cash back site, then there exist conditions under which a symmetric discounting strategy with a cash back site can increase profits by preventing search. Moreover, if profitable search prevention occurs even in the absence of a cash back site, corresponding to outcome 1, a cash back site can increase profits by preventing search with less leakage to loyal consumers. What remains to investigate is if such strategies are also subgame perfect. In other words, would one store want to deviate from the symmetric strategy identified in Proposition 2? Subgame perfectness imposes further restrictions, in addition to those identified in Proposition 2. The mathematical challenge is to characterize, for every possible deviation, the second-stage Nash equilibrium in the three-player game consisting of consumers' choice of θ and mixed strategy pricing by the two stores that also satisfy search conditions. We discuss subgame perfectness in Section 5.4 along with a numerical illustration of subgame perfect equilibria corresponding to both outcomes 1 and 2. In Section 5.4, we also explore the equilibrium if search cost S is large. It is not possible to analytically establish sufficient conditions for subgame perfectness in this case. Nevertheless, subgame perfect equilibria exist, as a numerical illustration demonstrates. We then discuss the nature of this equilibrium and provide useful insights.

5.3. Discussion of the Results

Proposition 2 is our central result. An obvious consequence of the cash back site is the ability to sort consumers, in particular at least a fraction of the loyal consumers from the rest. What is interesting, however, is that this sorting can allow firms to prevent search by searching consumers and do so profitably. This is because the discount consumers can avail of at the cash back site accrues to only a fraction of the loyal consumers. In the absence of a cash back site all consumers face the same price, and if the search cost is not too high, $S < (1 - \frac{\ln(1+z)}{z})\frac{\alpha}{\alpha+\beta}$, search prevention would not be profitable as established in Proposition 1. In such a situation, the cash back site that sorts consumers offers a way to achieve higher profits by increasing retailer loyalty or stickiness and preventing search by a segment of a store's consumers, because discounts are targeted at searching consumers. Of course, if search cost is high, $S \geq (1 - \frac{\ln(1+z)}{z})\frac{\alpha}{\alpha+\beta}$, stores can profitably prevent search even in the absence of a cash back site. In this situation too, the cash back site can lead to higher profits by selectively lowering prices to searching consumers. We emphasize that the cash back site need not be perfect in its sorting function. Indeed, our result obtains in a model in which the leakage of loyal consumers is related to the (lower) differential price targeted at the searching consumers.

We can now ask who benefits from the presence of a cash back site. Clearly retailers can benefit under the conditions identified by us. Because retailer profits are higher, the wedge between the profits with and without a cash back site makes it possible to make the cash back site also profitable. What is the effect on consumers? Consider first the case in which stores find it profitable to prevent search even in the absence of a cash back site. So we have outcome 1. In this case, all loyal consumers pay the same price as searching consumers. On the other hand, in the presence of a cash back site, only a subsegment of loyal consumers that visits the site pays the same price as searching consumers. So the effect of this increased loyalty makes prices higher, and all consumers end up paying higher expected price, even after the discount. Thus, we can see that cash back sites may prove to be harmful to consumers. This effect on consumers is counterintuitive, because we may expect cash back sites to always make users of the site better off than if there were no cash back site.

We can also contrast outcome 1 finding with the case of outcome 2 in the absence of a cash back site. Outcome 2 prevails if without the cash back site, retailers do not prevent search, and so prices do not have to satisfy a restrictive search constraint. They remain high. However, once a cash back site appears, retailers find it profitable to prevent search. This causes prices to be lower for consumers that use the cash back site but higher for those that do not. In other words, the sorting of consumers leads to a differentiated pricing of the kind we would expect. The different consequences of the cash back site in outcomes 1 and 2 also can be verified in the numerical illustration in the online appendix and discussed in Section 5.4.

It is interesting to explore the role of a cash back site if the size of the switching segment $\gamma = 0$, and so the market has only loyal and searching consumers. In the absence of a cash back site, a Nash equilibrium consists of searching consumers visiting each store with probability $\frac{1}{2}$ based on their expectation of each store's price to be 1; and stores' choosing prices to be 1. This is a rational expectations Nash equilibrium exhibiting a Diamond (1971) paradox and occurs also in Lal and Matutes (1994) and Lal and Rao (1997). Note that each store's profit is $\frac{\alpha+\beta}{2}$. What would happen if there were a cash back site? In this case, we can construct the following subgame perfect Nash equilibrium. Store i chooses a discount D_i in the first stage and searching consumers visit the store who offers the higher discount, and visit each store with probability $\frac{1}{2}$ if the discounts are equal. Moreover, their expectation is that each store's undiscounted price is 1. In the second stage, stores choose their prices to be 1. This is clearly a subgame perfect Nash equilibrium if the first-stage discounts constitute a Nash equilibrium. The problem resembles Narasimhan's model, because searching consumers essentially choose the store with the lower price. It is easy to see that Nash equilibrium discounts are in mixed strategies and each store's profit is $\frac{\alpha}{2}$. This is lower than the profit with no cash back site. Thus, in the absence of switching consumers in our model, cash back sites lead to lower profits. In this case, if all loyal consumers visit the cash back site, the equilibrium will reduce to Narasimhan's model although the mixed strategies will be different from the model in which the leakage is partial.

Now we ask how a cash back site would affect pricing if the size of the searching segment $\beta = 0$, and so the market consisted only of loyal and switching consumers. With no cash back site, this case again resembles Narasimhan's model and so equilibrium prices are in mixed strategies and each store's profit is $\frac{\alpha}{2}$. In the presence of a cash back site, the security level of profits remains $\frac{\alpha}{2}$, corresponding to a discount $D_1 = D_2 = 0$ in the first stage. Suppose store 1 chooses $D_1 = 0$. What is store 2's best response? If it also chose $D_2 = 0$, the two stores would compete away the profits from switching consumers, and in equilibrium each store's profit would be the security level. But, in fact, store 2 can do better by choosing a discount $D_2 > 0$ and compete for the switching consumers with low discounted prices that are not available to the loyal consumers who do not visit the cash back site. When $D_1 = 0$ and $D_2 > 0$, store 2 is more willing to lower its price, because a proportion of loyal consumers still pay the high price. In this case, store 1 cannot do better than the security level. The question is: how high should D_2 be? Note that store 1's price in the second stage is bounded from below by $\frac{\alpha}{\alpha+2\gamma}$, and so store 2 can assure itself of the entire segment at a discounted price less than this bound. If no loyal consumers used the cash back site, the best response would be to choose a discount that corresponds to this lower bound. But leakage of loyal consumers increases with discount, so the optimal discount must take that into account. Store 2 has two strategies: $D_2 = 1 - \frac{\alpha}{\alpha+2\gamma}$, and, thus, a pure strategy with $pp_1 = 1$ and $pp_2 = 1 - D_2$ and $D_2 = \frac{1}{2}$, and, thus, a mixed strategy with $\overline{pp}_1 = 1$, $\overline{pp}_2 = \frac{1}{2}$, and $l = \frac{\alpha}{\alpha+2\gamma}$. The reason is as follows. When the equilibrium is in pure strategy, $\pi_2(0, D_2) = (\frac{\alpha}{2}(1 + D_2 - D_2^2) + \gamma)(1 - D_2)$, $D_2 > 1 - \frac{\alpha}{\alpha+2\gamma}$. It is easy to show that $\pi_2(0, D_2)$ is decreasing in D_2, and, thus, $\pi_2(0, D_2)$ is maximized at $D_2 = 1 - \frac{\alpha}{\alpha+2\gamma}$. When the equilibrium is in mixed strategies, $\pi_2(0, D_2) = (\frac{\alpha}{2}(1 + D_2 - D_2^2) + \gamma)l$. It is easy to show that $\pi_2(0, D_2)$ is maximized at $D_2 = \frac{1}{2}$. Hence, the best response of store 2 given $D_1 = 0$ would be either $D_2 = 1 - \frac{\alpha}{\alpha+2\gamma}$ or $D_2 = \frac{1}{2}$, whichever gives the higher profits. We denote such D_2 as $D^*(\beta = 0)$. So far, we have proved that $D_2 = D^*(\beta = 0)$ is the best response to $D_1 = 0$. When $D_2 = D^*(\beta = 0)$, will store 1 choose a $D_1 \neq 0$? If $D^*(\beta = 0)$ is such that the equilibrium results in mixed strategies, or equivalently $D^*(\beta = 0) = \frac{1}{2}$. It is easy to show that $D_1 = 0$ is a best response to $D_2 = \frac{1}{2}$. If $D^*(\beta = 0)$ is such that the equilibrium results in pure strategies, or equivalently $D^*(\beta = 0) = 1 - \frac{\alpha}{\alpha+2\gamma}$. Increasing D_1 will decrease the profits from the loyal consumers. Thus, unless store 1 can sell to more switching consumers, equivalently $D_1 > D_2 = 1 - \frac{\alpha}{\alpha+2\gamma}$, the deviation is not profitable. If $D_1 > 1 - \frac{\alpha}{\alpha+2\gamma}$, we can use the same arguments as above and show that store 1 will not want to deviate when α is not too small. In this case, there are two equilibria (D_1, D_2) corresponding to $(0, D^*(\beta = 0))$ and $(D^*(\beta = 0), 0)$. Note that these are asymmetric equilibria, and in either equilibrium the store with zero discount has profits equal to the security level, and so could choose to not use the cash back site. The other store has profits that exceed the security level. Thus, in the absence of searching consumers the cash back site could perform a profit-increasing role but only if only one store uses the cash back site.[11] Note that this occurs because some loyal consumers do not visit the cash back site. If all loyal consumers were to visit the cash back site, then we get Narasimhan's model.

Do our results have any implications for the cash back site? In our model only loyal consumers may or may not use the cash back site. So a natural question is: would it help for the cash back site to attract loyal consumers by making it easier to use the site? If retailers can increase profits and share it with the cash back site, that would be beneficial. So we can now ask: is leakage beneficial to stores? When leakage is less, high discounts have less adverse effect on profits from loyal consumers. In a competitive situation, this may cause the competitor to increase discount and competition shifts to discounts. On the other hand, if leakage is higher, there is less loyalty, so second-stage prices are lower. So stores would like some leakage to keep the competition on discounts manageable. Thus, the cash back site should not adopt policies to completely discourage loyal consumers. At the same time, if leakage is too high, search prevention is not profitable because the profitability of loyal consumers decreases. Therefore, neither should the cash back site make the site too accessible to loyal consumers.

Our next task is to find out whether the candidate equilibrium identified in Proposition 2 can be sustained as a subgame perfect equilibrium. Section 5.4 provides a road map for the lemmas that characterize the conditions for subgame perfectness and also discusses the practical implications of the same. The proofs of the lemmas used to characterize subgame perfectness are contained in the appendix.

5.4. Subgame Perfectness

Proposition 3 (Subgame perfectness of D^* equilibrium). *Suppose* $S < \frac{(\frac{5\alpha}{4}+\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+2\gamma}{4\gamma}$ *and conditions of Proposition 2 are satisfied. Then the discounting strategy of* $D_1 = D_2 = \frac{1}{2}$ *constitutes a subgame perfect Nash equilibrium in the presence of a cash back site if*

i. $\alpha < 8\beta$ *and* $S < \frac{(\frac{5\alpha}{4}+\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+2\gamma}{4\gamma}$ (Lemma 5)

ii. $\alpha \le \frac{4}{3}\beta$ *and* $S \ge \frac{1}{2}\alpha\left(\frac{\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})}{\gamma} + \frac{8}{5\alpha+4\beta}\right)$ (Proposition 2)

iii. $\gamma > \frac{1}{8}(5\alpha+4\beta)$ (Lemma 8)

iv. $(\frac{5}{8}\alpha+\beta+\gamma)\frac{5\alpha}{10\alpha+16\gamma} < \frac{\gamma S(5\alpha+4\beta)}{(5\alpha+4\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+8\gamma}$ (Lemma 9)

Proof of Proposition 3. (Follows from Lemmas 6–9.)

The subgame perfectness property is an important one albeit mainly technical. It is useful here to elaborate on the considerations that we must have in mind in order to establish this property.

Consider the candidate symmetric equilibrium $D_1 = D_2 = \frac{1}{2}$. For this to be a subgame perfect equilibrium, we must rule out deviations by one store. Let us fix store 1's choice and ask if $D_2 \neq \frac{1}{2}$. For example, if $D_2 > \frac{1}{2}$, more loyal consumers of store 2 would use the cash back site. Given store 2's choice, suppose that more searching consumers visit it first, and so $\theta < \frac{1}{2}$, say $\tilde{\theta}$. Then

$$\phi_1 = \frac{1}{2}\alpha\left(1 + D_1 - D_1^2\right) + \tilde{\theta}\beta \quad and$$
$$\phi_2 = \frac{1}{2}\alpha\left(1 + D_2 - D_2^2\right) + \left(1 - \tilde{\theta}\right)\beta. \tag{17}$$

A second-stage Nash equilibrium then must satisfy the necessary conditions: (i) if $0 < \tilde{\theta} < 1$, then consumers must be indifferent between choosing between the two stores;[12] (ii) stores' mixed strategies must satisfy equi-profit conditions at prices with measure greater than zero; (iii) search cost is greater than or equal to search benefit; and (iv) the maximum discounted price of store i must satisfy $\overline{pp}_i \le 1 - D_i$ and in particular $\overline{pp}_2 \le 1 - D_2$.

One way of satisfying (i) is to choose $\tilde{\theta}$ such that $\phi_1 = \phi_2$. Lemma 4 addresses this. Lemma 6 takes the explicitly solved $\tilde{\theta}$ from Lemma 4 and invoking Corollary 3 shows that the induced second-stage Nash equilibrium makes the deviation unprofitable, assuming (iv) holds with the strict inequality. This results in symmetric mixed strategies with $\overline{pp}_1 = \overline{pp}_2$. Lemmas 7 and 8 consider the case in which (iv) holds with equality $\overline{pp}_2 = 1 - D_2$, and establishes conditions that such deviations are unprofitable. The proof of Lemma 8 exploits the fact that $\tilde{\theta}$ is smaller than the one that makes $\phi_1 = \phi_2$, and the deviation can then be shown to be unprofitable without explicitly solving for the $\tilde{\theta}$ that satisfies (i). Lemma 9 takes up the case in which D_2 is so large that $\tilde{\theta} = 0$, and so the mixed strategies are asymmetric with $\frac{1}{2} = 1 - D_1 = \overline{pp}_1 > \overline{pp}_2 = 1 - D_2$. Using an upper bound for profits from such a deviation we establish the conditions that make the deviation unprofitable. Lemmas 6–9 thus establish conditions that make the candidate symmetric equilibrium $D_1 = D_2 = \frac{1}{2}$ identified in Proposition 2 also subgame perfect. In other words, we analytically established an existence result for a cash back site to be profitable. The numerical example in Appendix C provides an illustration of this. We can see that with the cash back site the D^* equilibrium can occur corresponding to either outcome 1 or 2 in the absence of a cash back site.

What is the practical significance of subgame perfectness? Recall that in the original Stahl model, consumers split equally across stores, equivalent to $\theta = \frac{1}{2}$ in our model. In the Stahl model, if consumers deviated from $\theta = \frac{1}{2}$, they would end up facing higher reservation prices. So apart from the symmetry argument, we can also see the consumers' choice as utility maximizing. In our model too when stores choose equal discounts, the situation is like the Stahl model. But if $D_1 \neq D_2$, consumers' choice of θ is more complicated. Nevertheless, from a game-theoretic viewpoint it is necessary to see whether the proposed equilibrium is robust to subgame perfectness.

Note that the D^* equilibrium requires $S < \frac{(\frac{5\alpha}{4}+\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+2\gamma}{4\gamma}$. A natural question is: if $S > \frac{(\frac{5\alpha}{4}+\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+2\gamma}{4\gamma}$, does a subgame prefect equilibrium exist? The answer is in the affirmative though it is not possible to analytically establish the counterpart of Proposition 2 in that case. However, the existence of a subgame perfect equilibrium can be established by construction through a numerical illustration, which we do next. We are also able to gain further insights into the equilibrium strategies from that.

Specifically, we refer to the numerical illustration in the appendix.[13] We provide two examples, one that corresponds to outcome 1 without the cash back site, and the other to outcome 2. Let us first consider the example of outcome 1. In this case the solution is not subgame perfect because a profitable deviation to a higher D exists.[14] What we can see is that such a deviation is accompanied by $\theta > 0.5$. This is possible in the solution because $\frac{\partial\phi}{\partial D}|_{D=\frac{1}{2}} > 0$, and so it is no longer necessary for $\theta < 0.5$ to hold in order to satisfy the condition that the expected prices of the two stores be equal. Now, if $\theta > 0.5$, obviously a profitable deviation exists, because the lower limit of the support of the mixed strategy in the subgame Nash equilibrium increases. Indeed, it is possible that both stores' profits can increase as a result of the deviation. That is exactly what we find. So the question is what would the equilibrium be in this case? When we look for a fixed point in discounts we find that the resultant subgame perfect equilibrium in pure strategies is asymmetric with $D_1 \neq D_2$, $\max\{D_1, D_2\} > D^*$, where D^* is the solution.[15] Moreover, in this case $\min\{D_1, D_2\} = D^*$.

Next, if we consider the example of outcome 2, we again find that the solution is not subgame perfect and a deviation to higher D is profitable. θ is still slightly higher than 0.5. Again, the subgame perfect equilibrium in pure strategies is of the type $D_1 \neq D_2$, $\max\{D_1, D_2\} > D^*$.

Although we are unable to provide a general result on the conditions for these types of equilibria, importantly what we have been able to show, analytically and by construction, is that for all S there can be a profitable subgame perfect equilibrium. Moreover, cash back sites can prove profitable to retailers for both outcomes 1 and 2 without the cash back site.

6. Implications for the Discount

In this section, we address a practical issue that could be useful to managers. Would it make more sense to offer cash back to consumers, as a percentage or a flat discount? Rather than derive the subgame perfect equilibrium with a flat fee, we proceed by examining a related problem and draw implications for the relative merit of percentage discount and flat sum. We know the equilibrium discount in our model, and, corresponding to this discount, each store has a segment of consumers that is loyal and does not use the cash back site. The combined (over the two stores) size of this segment is $\alpha' = \alpha(1-D)^2$. Moreover, in equilibrium, the combined size of the discount availing nonsearching segment is $\beta' = \alpha(2D - D^2) + \beta$. Finally, the size of the searching segment is still γ. We then ask, given this configuration of consumers what is the "cost" of preventing search using the two methods: percentage discount and flat fee?

There are two possible ways to offer cash back to consumers. If a cash back site offers cash back as a percentage, then consumers get a percentage of the price paid. Alternatively, if the offer is a flat sum, then consumers get a fixed amount regardless of price paid. Both implementations can be observed in practice. Here, we use our model to analyze the cost of each offer, ensuring that the offer prevents search.

Proposition 4. *An offer of percentage cash back can prevent search at a lower cost than an offer of a flat sum.*

Proof of Proposition 4. We first examine the case in which the amount of the leakage is fixed, and then take into account the leakage also. So, α' and β' are fixed. Denote the flat sum offered by φ. In this case, the necessary conditions for a symmetric equilibrium strategy that prevents search when cash back is offered as a flat sum, φ, are

$$\begin{aligned}
\Pi^4(\overline{p}) &= \frac{\alpha'}{2}\overline{p} + \frac{\beta'}{2}(\overline{p} - \varphi)\\
\Pi^4(l) &= \frac{\alpha'}{2}l + \left(\frac{\beta'}{2} + \gamma\right)(l - \varphi)\\
\Pi^4(p) &= \frac{\alpha'}{2}p + \left(\frac{\beta'}{2} + \gamma\left(1 - F^4(p)\right)\right)(p - \varphi)\\
S &= \int_l^{\overline{p}} \left((p - \varphi) - (x - \varphi)\right) f^4(x)dx = \int_l^{\overline{p}} F^4(x)dx.
\end{aligned} \tag{18}$$

From equality (18), we can get $\overline{p} = \varphi + \frac{2\gamma S}{(\alpha'+\beta')\ln(\frac{\alpha'+\beta'}{\alpha'+\beta'+2\gamma}) + 2\gamma}$. Given $\overline{p}$, the store should set $\varphi^* = \overline{p} - \frac{2\gamma S}{(\alpha'+\beta')\ln(\frac{\alpha'+\beta'}{\alpha'+\beta'+2\gamma}) + 2\gamma}$.

The strategy for our proof is to compare the cost of preventing search in the two cases. We will show that the cost is always lower in the case of the percentage discount assuming that the stores' profit is identical in both cases.

Assume that the higher bounds of the equilibrium pricing distributions are the same in both the case of the percentage discount and the case of the flat discount. Denote it as $\overline{p}'$. Under the percentage discount, the expected profit can be written as $\frac{\alpha'}{2}\overline{p}' + \frac{\beta'}{2}(1 - D^*)\overline{p}'$. Let $H = D^*\overline{p}'$. From Equation (18), we know that the profit under the flat discount is $\frac{\alpha'}{2}\overline{p}' + \frac{\beta'}{2}(\overline{p}' - \varphi^*)$. From Lemma 3, we know that the search benefit under the percentage discount is given by $\overline{p}'((1-D) + \frac{(\alpha'+\beta'-\beta'D)\ln(\frac{\alpha'+\beta'-\beta'D}{\alpha'+(1-D)(\beta'+2\gamma)})}{2\gamma}) = S$. Next, if $\varphi = H$, then the search benefit under the flat discount is $\frac{(\overline{p}'-\varphi)((\alpha'+\beta')(\ln(\frac{\alpha'+\beta'}{\alpha'+\beta'+2\gamma})) + 2\gamma)}{2\gamma}$. Moreover, because $\varphi = H$, the expected profits with the two discounts are equal. We will show $S - \frac{(\overline{p}'-\varphi)((\alpha'+\beta')(\ln(\frac{\alpha'+\beta'}{\alpha'+\beta'+2\gamma})) + 2\gamma)}{2\gamma} < 0$. We take the difference between the search benefits in the two cases when $H = \varphi$ and define it as $g(H)$.

$$g(H) = \frac{\begin{aligned}&(\alpha' + \beta')(H - \overline{p}')\ln\left(\tfrac{\alpha'+\beta'}{\alpha'+\beta'+2\gamma}\right)\\ &+ \left(\overline{p}'(\alpha'+\beta') - \beta' H\right)\ln\left(\tfrac{\overline{p}'(\alpha'+\beta') - \beta' H}{\overline{p}'(\alpha'+\beta'+2\gamma) - H(\beta'+2\gamma)}\right)\end{aligned}}{2\gamma} \tag{19}$$

Note that when $H = \varphi = 0$ and $H = \varphi = \overline{p}'$, $g(H) = 0$. We take the first and the second derivatives of $g(H)$.

$$\begin{aligned}
g'(H) = \frac{1}{2\gamma}\Bigg((\alpha' + \beta')\ln\left(\frac{\alpha' + \beta'}{\alpha' + \beta' + 2\gamma}\right)&\\
+ \frac{2\alpha'\gamma\overline{p}'}{\overline{p}'(\alpha' + \beta' + 2\gamma) - H(\beta' + 2\gamma)}&\\
- \beta'\ln\left(\frac{\overline{p}'(\alpha' + \beta') - \beta' H}{\overline{p}'(\alpha' + \beta' + 2\gamma) - H(\beta' + 2\gamma)}\right)\Bigg)&
\end{aligned} \tag{20}$$

$$g''(H) = \frac{2\alpha'^2\gamma'\overline{p}'^2}{\left(\overline{p}'(\alpha'+\beta') - \beta' H\right)\left(H(\beta'+2\gamma) - \overline{p}'(\alpha'+\beta'+2\gamma)\right)^2} > 0 \qquad (21)$$

From Equation (21), we know that $g'(H)$ is increasing in H. From Equation (20), when $H = \varphi = 0$, $g'(H) = \frac{\alpha'}{\alpha'+\beta'+2\gamma} + \frac{\alpha' \ln(\frac{\alpha'+\beta'}{\alpha'+\beta'+2\gamma})}{2\gamma}$. It can be shown that $g'(H=0) < 0$.[16] Thus, it must be that as H increases from 0 to $\overline{p}'$, $g(H)$ first decrease from zero and then increase to zero. Thus $g(H)$ is always less than zero. This implies that $S - \frac{(\overline{p}'-\varphi)((\alpha'+\beta')(\ln(\frac{\alpha'+\beta'}{\alpha'+\beta'+2\gamma}))+2\gamma)}{2\gamma} < 0$. In other words, the flat fee must be higher than $D^*\overline{p}'$ to prevent search assuming that the higher bounds of the equilibrium price distributions are the same in both cases. Of course, the store can also lower the highest price to prevent search instead of offering a higher flat discount. But in either case, the profit will be lower than the one when using the percentage discount. Therefore, the flat discount that prevents search is more costly and less profitable than the percentage discount. □

We have proved that to prevent search, the stores will incur a higher cost when using the flat discount if the amount of the leakage is fixed. We know that the stores have to give a larger discount or lower prices when using the flat discount to prevent search, so the amount of the leakage in this case must be larger than if the offer were a percentage discount. Taking this into consideration, the flat discount will further reduce profit.

Proposition 4 provides practical guidance on how this insight can be used. In particular, the discount offered to the consumer to prevent search is best made proportional to the price rather than a fixed amount. Note that if search can be prevented, say by a \$5 discount at a higher price, then it is also prevented at all lower prices by that discount. Indeed, at a lower price a discount less than \$5 should suffice. In other words, when a store charges a lower price, it can prevent search with a smaller dollar discount.[17] And therefore a percentage discount results in higher profits than fixed fee when ensuring that search is prevented.

In practice, we can observe flat cash back also, although over time they have become less common. Keep in mind that the percentage discount is best if the desired effect is mainly to limit search. As we have noted, cash back sites can act as a sorting mechanism and so if the desired effect is to convey a promotional reward it may make sense to offer flat cash back especially if the dollar value is substantial. A recent casual examination of the sites: Rakuten (original Ebates), BeFrugal, Coupon Cactus, and Mr. Rebates provides some confirmation of this. The flat cash back is offered for Dish (TV), FIOS (WiFi), Cheap-o-air (air travel), AT&T (U-verse), and Super Chewer (pet supply with a subscription option). The amounts tend to be substantial also.

Finally, it is useful to recognize that a lot of the cash back is associated with multiproduct retail stores. Thus, consumers' baskets could be split across stores if consumers decide to search for a subset of their basket. This search can be reduced or eliminated by percentage discounts. Such discounts have an effect similar to accumulating points through frequent reward programs (Lal and Bell 2003). Variation across consumers of basket size (Kumar and Rao 2006) also makes percentage discount attractive to prevent search by heterogeneous consumers.

7. Conclusions

As e-commerce continues to grow, channel arrangements also evolve to leverage possibilities that online transactions offer. One such emerging innovation is the cash back site. In this paper, we have provided an insight into how the cash back site helps the retailer to prevent search by consumers that visit the retailer. In other words, the cash back site can make the retailer site more sticky. The reason this is possible is because preventing search is a profitable strategy if the retailer can sort the loyal and the searching consumers. A cash back site serves the sorting function. The retailers' pricing strategies are such that they prevent search in equilibrium. We also find that, contrary to intuition, in the presence of cash back sites, users of the site in some situations may end up facing higher prices than they would have if there were no cash back site. Our results are based on the analysis of a multistage game involving profit maximizing retailers and cash back site.

We are also able to provide a practical guideline on how best search can be prevented. It is optimal for the retailers to offer a discount proportional to the price through the cash back site. Moreover, this discount prevents search by searching consumers at the highest possible price.

Given our interest in retailer pricing strategies, in our model we treated the site as a nonstrategic agent. In practice, the site could decide whether to allow some or all retailers to be on the site. To address this issue, one must explicitly model the sharing rule between the platform and the cash back site as a function of the number of retailers on the site. This could be a useful direction for future research.

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Appendix A. Demonstrations of Online Cash Back Sites

A.1. Rakuten.com

Figure A.1. (Color online) Main Page of Rakuten.com

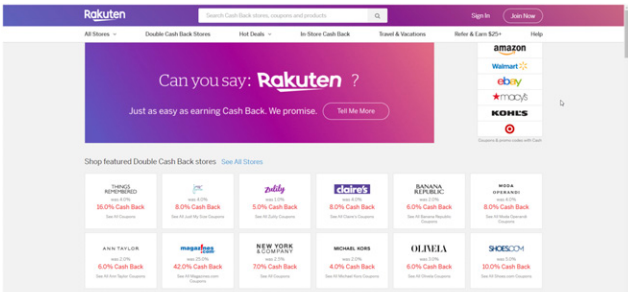
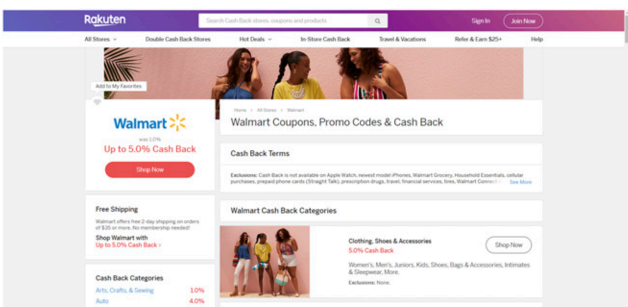


Figure A.2. (Color online) Cash Back for Walmart on Rakuten.com



A.2. Coupon Cactus

Figure A.3. (Color online) Main Page of Coupon Cactus

Figure A.4. (Color online) Cash Back for Walmart on Coupon Cactus

A.3. BeFrugal.com

Figure A.5. (Color online) Main Page of BeFrugal.com

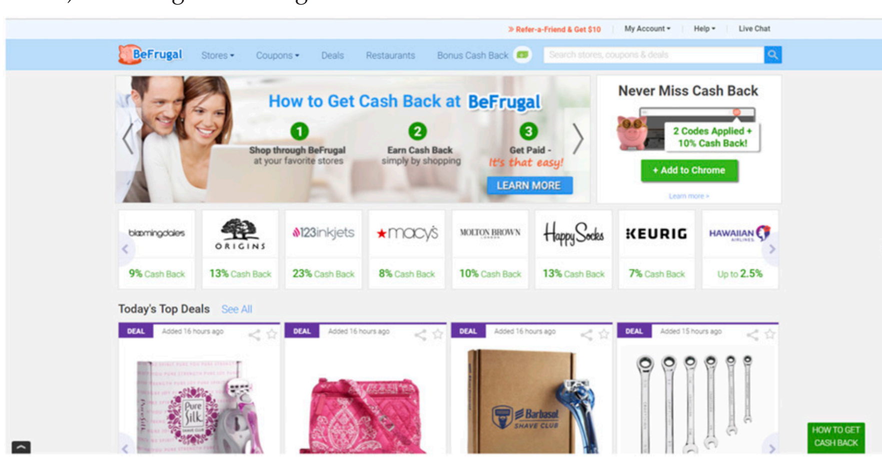
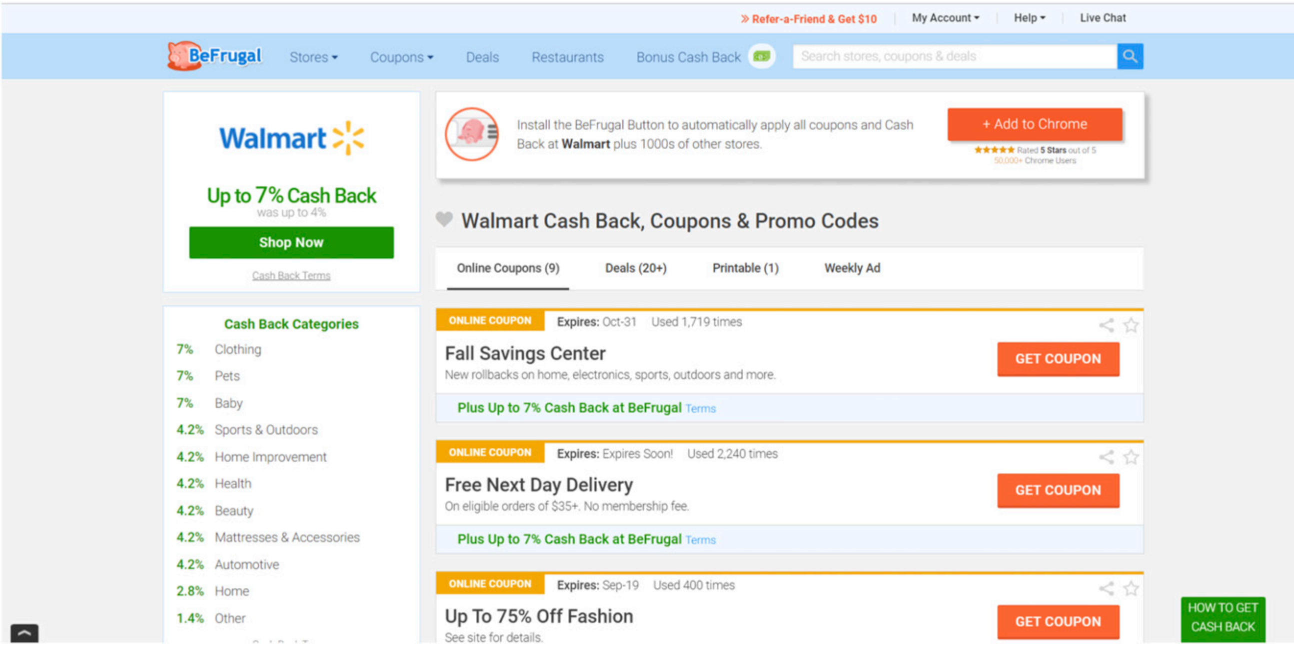


Figure A.6. (Color online) Cash Back for Walmart on BeFrugal.com



Appendix B. Proofs of Lemmas 3, 4, and 5 and Corollary 3

Proof of Lemma 3. We derive the equilibrium $\overline{pp}$ ignoring the constraint $\overline{pp} \le 1 - D$ and then return to it. Denote also $\pi_i(pp_i, pp_{3-i}; D)$ as store i's profits in the second stage given prices. Then if $F_i^3(p) = \Pr(pp_i \le p)$ denotes store i's mixed strategy, a necessary condition for Nash equilibrium prices is

$$\begin{aligned}\pi_i(pp_i, pp_{3-i}; D) &= \left(\phi(D) + \gamma\left(1 - F_{3-i}^3(pp_i)\right)\right)pp_i \\ &= (\phi(D) + \gamma)\, l, \qquad i = 1, 2.\end{aligned} \tag{B.1}$$

The foregoing necessary condition leads to the following strategy:

$$F_1^3(pp) = F_2^3(pp) = F^3(pp) = \frac{\phi(D)+\gamma}{\gamma}\left(1 - \frac{l}{pp}\right), \; pp \in [l, \overline{pp}]. \tag{B.2}$$

Because $F^3(\overline{pp}) = 1$, $l = \overline{pp}\,\frac{\phi(D)}{\phi(D)+\gamma}$. This can be used to evaluate the expected price, $\mathrm{E}(p)$, to be

$$\mathrm{E}(p) = l\frac{\phi(D) + \gamma}{\gamma}\ln\left(\frac{\overline{pp}}{l}\right) = -\overline{pp}\frac{\phi(D)}{\gamma}\ln\left(\frac{\phi(D)}{\phi(D) + \gamma}\right). \tag{B.3}$$

For search to be prevented, we require that the search cost be equal to or exceed search benefit. The highest search benefit occurs at $\overline{pp}$. So this implies $S \ge \overline{pp} - \mathrm{E}(p)$ or, equivalently, $\overline{pp} \le \mathrm{E}(p) + S$. Recall that we also require $\overline{pp} \le 1 - D$. If the second constraint is not binding, we can solve for $\overline{pp}$ as follows:

$$\overline{pp} = \frac{\gamma S}{\gamma + \phi(D)\ln\left(\frac{\phi(D)}{\phi(D)+\gamma}\right)}. \tag{B.4}$$

Denote the right-hand side of Equation (B.4) by $\widehat{pp}$. Note that $\widehat{pp}$ is increasing in S. Moreover, it is increasing in D if $D < \frac{1}{2}$ and decreasing in D if $D > \frac{1}{2}$.[18]

If the constraint $\overline{pp} \le 1 - D$ is binding, then $\overline{pp} = 1 - D$. In this case, the search cost is greater than the search benefit at $1 - D$.

Finally, if $\overline{pp} < 1 - D$, for the second-stage pricing strategy to be a Nash equilibrium, the equilibrium profits must exceed the security-level $\sigma(D)$ that a store can obtain by deviating to the maximum price $1 - D$ and possibly not preventing search. This yields the security level of profits, $\sigma(D) = \pi_i(pp_i = 1 - D, pp_{3-i}; D)$, $pp_{3-i} \sim F(pp)$, or, equivalently, $\sigma(D) = (1 - D)(\phi(D) - \frac{1}{2}\beta)$.

For search prevention mixed strategies to constitute a Nash equilibrium, the third necessary condition is

$$\overline{pp} \ge (1 - D)\left(1 - \frac{\beta}{2\phi(D)}\right). \tag{B.5}$$

Second-stage Nash equilibrium strategies for symmetric D that also prevent search are fully characterized by (B.3), (B.4), and (B.5) and by $l = \overline{pp}\,\frac{\phi(D)}{\phi(D)+\gamma}$. □

Proof of Lemma 4. Given the searching consumers' strategy θ, we know that the size of the effective loyal segments of store 1 and store 2 are given by $\phi_1(D_1, \theta) \triangleq \frac{\alpha(D_1)}{2(1-D_1)} + \frac{\alpha - \alpha(D_1)}{2} + \theta\beta$ and $\phi_2(D_2, \theta) \triangleq \frac{\alpha(D_2)}{2(1-D_2)} + \frac{\alpha - \alpha(D_2)}{2} + (1 - \theta)\beta$. By equating these two, we can solve for θ, which makes the effective loyal segments $\phi_1(D_1, \theta)$ and $\phi_2(D_2, \theta)$ equal. This implies

$$\theta = \frac{\alpha\left(\widetilde{D} - D\right)\left(1 - D - \widetilde{D}\right) + 2\beta}{4\beta} \tag{B.6}$$

Of course, for this to make sense, we require that $0 < \theta < 1$. We will show that a sufficient condition for this is $\alpha < 8\beta$.

Consider $0 < D_1 = D_2 = D \le \frac{1}{2}$ in the first stage.

First, suppose $\widetilde{D} < D$. Then $(\widetilde{D} - D)(1 - D - \widetilde{D}) < 0$, because $1 - D - \widetilde{D} > 0$. So, $\theta < 1$. Also, $\theta > 0$ if $\alpha(D - \widetilde{D})(1 - D - \widetilde{D}) < 2\beta$ or, equivalently, if $\alpha < \frac{2\beta}{(D-\widetilde{D})(1-D-\widetilde{D})} = \frac{2\beta}{D(1-D)-\widetilde{D}(1-\widetilde{D})}$, or

$$\alpha < \min\left\{\frac{2\beta}{\left(D - \widetilde{D}\right)\left(1 - D - \widetilde{D}\right)}\right\} = \frac{2\beta}{\max\left\{D(1 - D) - \widetilde{D}\left(1 - \widetilde{D}\right)\right\}} = \frac{2\beta}{\frac{1}{4}} = 8\beta. \quad \text{(B.7)}$$

Next, suppose $\widetilde{D} > D$. If $\widetilde{D}$ is sufficiently large so that $(\widetilde{D} - D)(1 - D - \widetilde{D}) < 0$, then the previous argument implies that $0 < \theta < 1$. Suppose $(\widetilde{D} - D)(1 - D - \widetilde{D}) > 0$. Now denote $\mu = \frac{1}{2} - D$. Clearly $0 < \mu < \frac{1}{2}$. Moreover, $(\widetilde{D} - D)(1 - D - \widetilde{D}) = (\mu + (\widetilde{D} - \frac{1}{2}))(\mu - (\widetilde{D} - \frac{1}{2})) < \mu^2$. So $\theta < 1$ if

$$\alpha < \min\left\{\frac{2\beta}{\left(\widetilde{D} - D\right)\left(1 - D - \widetilde{D}\right)}\right\} = \frac{2\beta}{\max\{\mu^2\}} = 8\beta. \quad \text{(B.8)}$$

Thus, if $\alpha < 8\beta$, there exists θ that consumers can choose rendering stores symmetric in the effective loyal segment. □

Proof of Corollary 3. From Lemma 4, we know that if $\alpha < 8\beta$, there exists a θ, $0 < \theta < 1$, such that $\phi_1(D_1; \theta) = \phi_2(D_2; \theta) = \phi$. This allows us to exploit the results of Corollary 2. From Lemma 3 and Corollary 1, we know that if $S < \min(1 - D_1, 1 - D_2)\frac{\gamma + \phi\ln(\frac{\phi+\gamma}{\gamma})}{\gamma}$, $\overline{pp} = \widehat{pp} = \frac{\gamma S}{\gamma + \phi\ln(\frac{\phi}{\phi+\gamma})} \le \min(1 - D_1, 1 - D_2)$.

Therefore, recalling that $\hat{\Pi}^3 = \overline{pp}\phi$, we get $\frac{d\hat{\Pi}^3}{d\gamma} < 0$ and $\frac{d\hat{\Pi}^3}{d\phi} = \overline{pp} + \phi\frac{d\overline{pp}}{d\phi} > 0$. □

Proof of Lemma 5. Note that if both stores choose discount D, then consumers choose $\theta = \frac{1}{2}$ and so

$$\phi_1\left(D, \frac{1}{2}\right) = \frac{\alpha}{2}(1 + D - D^2) + \frac{\beta}{2}. \quad \text{(B.9)}$$

However, if store 1 chooses discount D, then we will show that there exists a profitable deviation for store 2, $D_2 = \widetilde{D} = \frac{1}{2}$. First, because $\alpha < 8\beta$, invoking Lemma 4 we know that under deviation by store 2, consumers can choose θ as

$$\theta = \frac{\alpha\left(\widetilde{D} - D\right)\left(1 - D - \widetilde{D}\right) + 2\beta}{4\beta} = \frac{\alpha(\frac{1}{2} - D)(1 - D - \frac{1}{2}) + 2\beta}{4\beta}. \quad \text{(B.10)}$$

This yields, for deviation,

$$\widetilde{\phi} = \phi_1(D, \theta) = \phi_2\left(\frac{1}{2}, \theta\right) = \frac{\alpha}{2}(1 + D - D^2) + \frac{\beta}{2} + \frac{\alpha(\frac{1}{2} - D)(\frac{1}{2} - D)}{4\beta} > \phi\left(D, \frac{1}{2}\right). \quad \text{(B.11)}$$

In other words, the effective loyal segment under deviation, $\widetilde{\phi}$, is higher. Next, from Corollary 2,[19] we know that $\frac{d\overline{pp}}{d\phi} > 0$. Finally, we can verify that $\overline{pp} < \min\{1 - D, 1 - \widetilde{D}\} = \min\{1 - D, \frac{1}{2}\} = \frac{1}{2}$ if

$$\frac{\gamma S}{\gamma + \widetilde{\phi}\ln\left(\frac{\widetilde{\phi}}{\widetilde{\phi}+\gamma}\right)} < \frac{1}{2}. \quad \text{(B.12)}$$

To see this, consider $\widetilde{\phi} < \phi(\frac{1}{2}, \frac{1}{2}) \triangleq \overline{\phi}$. Because $\frac{d\overline{pp}}{d\phi} > 0$, a sufficient condition for $\overline{pp} < \frac{1}{2}$ is $\frac{\gamma S}{\gamma + \overline{\phi}\ln(\frac{\overline{\phi}}{\overline{\phi}+\gamma})} < \frac{1}{2}$, or $S < \frac{1}{2\gamma}(\gamma + \overline{\phi}\ln(\frac{\overline{\phi}}{\overline{\phi}+\gamma}))$. Substitute $\overline{\phi} = \frac{1}{2}(\frac{5\alpha}{4} + \beta)$ to get the sufficient condition as

$$S < \frac{\left(\frac{5\alpha}{4} + \beta\right)\ln\left(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma}\right) + 2\gamma}{4\gamma}. \quad \text{(B.13)}$$

Collecting the results, we have, under deviation by store 2 to discount $\widetilde{\phi} = \frac{1}{2}$, $\widetilde{\phi}$ is higher, $\overline{pp}$ is higher, implying that the profits $\widetilde{\overline{pp}\phi}$ are also higher; moreover, $\overline{pp} < \min\{1 - D, 1 - \widetilde{D}\}$. Thus, there exists a profitable deviation if stores choose $D_1 = D_2 = D < \frac{1}{2}$ in the first stage. □

Appendix C. Numerical Examples: $S < \frac{(\frac{5\alpha}{4}+\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+2\gamma}{4\gamma}$

In this section, we present a numerical example to illustrate the symmetric equilibrium described in Lemmas 6–9. Suppose $V = 1$, $\alpha = 0.25$, $\beta = 0.3$ and $\gamma = 0.45$. Then, from Proposition 1, we know that if $S < 0.185$, the equilibrium is outcome 2 with no cash back site, while if $S \ge 0.185$, the equilibrium is outcome 1 with no cash back site. From Lemma 4, we know that if $S < 0.192$, then in the proposed symmetric equilibrium, $D^* = \frac{1}{2}$. In other words, D^* is a best response to D^*. In Proposition 2, we establish that if $D = D^* = \frac{1}{2}$, in order for a cash back site to lead to higher profits, it must be that $\alpha \le \frac{4}{3}\beta$ and $S \ge \frac{1}{2}\alpha(\frac{\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})}{\gamma} + \frac{8}{5\alpha+4\beta})$. The first condition is satisfied, because $0.25 < (\frac{4}{3})0.3 = 0.4$. The second condition is equivalent to $S \ge 0.157$. Hence, for this example, either outcome 1 or outcome 2 can exist when there is no cash back site in market. Specifically, if $S \in [0.157, 0.185)$, then outcome 2 is the equilibrium without cash back site. On the other hand, if $S \in [0.185, 0.1922)$, then outcome 1 is the equilibrium without cash back site. Moreover, in both cases, a cash back site leads to higher profits for the stores.

Next, we show that for this example, all the sufficient conditions for the subgame perfectness are satisfied. In Lemma 5, we require that $\alpha \le 8\beta$. This condition is automatically satisfied, because Proposition 2 requires that $\alpha \le \frac{4}{3}\beta$. In Lemma 8, we have a sufficient condition that $\gamma > \frac{1}{8}(5\alpha + 4\beta)$. This condition is satisfied, because $0.45 > \frac{1}{8}(1.25 + 1.2) = 0.306$. Finally, Lemma 9 requires that when $D_2 = \frac{5\alpha+16\gamma}{10\alpha+16\gamma} = 0.871$, the pure strategy's profit must be less than Π^3. In this case, the store 2's profit if it deviates to such D_2 is less than 0.117. According to Corollary 2, the stores' equilibrium profit is increasing in S. Moreover, because $S \ge 0.157$, $\Pi^3 > 0.125$. Hence, the sufficient condition in Lemma 9 is also satisfied. Finally, we use the same example to illustrate the differences between the equilibrium prices with and without a cash back site.

C.1. Prices

When $S = 0.19$, as shown in Proposition 2, outcome 1 would be the equilibrium without a cash back site. Again, all the conditions for a subgame perfect equilibrium are satisfied. Using Lemma 1, we get that $\Pi^1 = 0.128$ and the expected price is equal to 0.276. If there is a cash back site in market, we use Lemma 3 to get that $\Pi^3 = 0.151$ and the expected price is equal to 0.303. We can see that the expected price in the symmetric equilibrium is higher than the one in outcome 1. Similarly, when $S = 0.16$, outcome 2 would be the equilibrium without a cash back site. Using Lemma 2, we get that $\Pi^2 = 0.125$ and the expected price is equal to 0.277. If there is a cash back site in market, we use Lemma 3 to get that $\Pi^3 = 0.135$ and the expected price is equal to 0.271. We can see that the expected price in the symmetric equilibrium is lower than the one in outcome 2.

Endnotes

[1] Of course, a single consumer can be any of the three types on a purchase occasion, and the size of segments will reflect the probability of a consumer being each type.

[2] Because the retailers in our model do not advertise prices, it is reasonable to suppose that a searching consumer visits either store with probability $\frac{1}{2}$.

[3] It is easy to show that there is no pure strategy Nash equilibrium for this problem.

[4] This is also because in equilibrium there can be no mass point at $\overline{p}$.

[5] We have chosen this functional form, so we can obtain analytical results. Other functional forms of nonusers of cash back site could be $\frac{1}{2}\alpha H(D)$, where $\frac{H(D)}{1-D} + 1 - H(D)$ is quasiconcave in D. For example, $H(D) = (1 - D)^n$, $n > 1$.

[6] Thus, stores in our model cannot price any consumer out of the market. This, in turn, implies that the undiscounted price must satisfy the constraint $p_i \le V = 1$. So the discounted price of store i cannot exceed $1 - D_i$. This restriction helps us to capture reality in our model without having to account for some searchers searching at high prices, but not at low prices, as that would make the analysis intractable.

[7] Of course, only if $D_i = 0$, then store i could have a second-stage pricing strategy that does not prevent search as part of a Nash equilibrium.

[8] In Corollary 3 following Lemma 4 in the Appendix B, we analyze the case $D_1 \ne D_2$ by exploiting the idea that the searching consumers' equilibrium behavior can render $\phi_1 = \phi_2 = \phi$.

[9] We thank a reviewer for suggesting this discussion.

[10] The reason for proceeding in this fashion is that the second-stage equilibrium for deviations is complicated to establish for all possible first-stage equilibrium pairs (D_1, D_2).

[11] There is a symmetric mixed strategy in discounts that has both stores using the cash back site but that yields profits equal to the security level.

[12] If one store offers a higher utility, then $\tilde{\theta} = 0$ or 1.

[13] If $S > \frac{(\frac{5\alpha}{4}+\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+2\gamma}{4\gamma}$, the obvious discount strategy in a candidate symmetric equilibrium maximizes $(1 - D)(\phi + \frac{\beta}{2})$.

[14] A deviation to a lower D will cause consumers to search and so is not profitable.

[15] Obviously, there are two pure strategy equilibria in the asymmetric case and one mixed strategy equilibrium also.

[16] $g'(H = 0) = \frac{\alpha'}{2\gamma}(\frac{2\gamma}{\alpha'+\beta'+2\gamma} + \ln(1 - \frac{2\gamma}{\alpha'+\beta'+2\gamma})) < 0$.

[17] Indeed, the theoretically optimal strategy is a price-discount pair over the support of prices. The percentage captures the essential property of this pair that the two are positively correlated.

[18] $\frac{d\widetilde{pp}}{dD} = -C(1 - 2D)(\gamma + (\gamma + \phi(D))\ln(\frac{\phi(D)}{\phi(D)+\gamma}))$, where C is a function of α, β, γ, D, and S. C is always positive. Because $\ln(\frac{\phi(D)}{\phi(D)+\gamma}) < -\frac{\gamma}{\gamma+\phi(D)}$, we know that $\gamma + (\gamma + \phi(D))\ln(\frac{\phi(D)}{\phi(D)+\gamma}) < 0$.

[19] Note that $S < \frac{(\frac{5\alpha}{4}+\beta)\ln(\frac{5\alpha+4\beta}{5\alpha+4\beta+8\gamma})+2\gamma}{4\gamma}$ is the same as the upper bound condition in Corollary 2 with $D = \frac{1}{2}$.

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