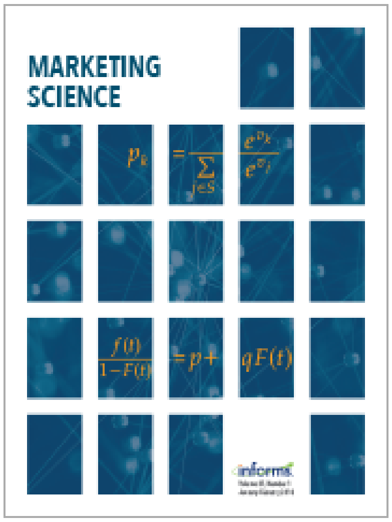

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Regulating Deceptive Advertising: False Claims and Skeptical Consumers

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Abstract. Nowadays firms often claim that their products are superior, but product statements may not be truthful. Knowing firms' potential dishonesty, consumers are skeptical about these possibly false statements and may investigate. To protect consumers, regulators can penalize firms who deceive consumers. In response to consumers and regulators, firms can make their false claims deceptive to impede investigation. We develop a game theoretical model to study interactions between dishonest firms, skeptical consumers, and regulations. We show that increasing the penalty for false statements can surprisingly reduce consumer surplus, firm profits, and social welfare. The welfare reduction is due to higher spending on deceptiveness, which hinders consumers from investigating potentially false claims. The lack of information discourages consumers from identifying product quality, thus decreasing welfare. Furthermore, when it is costless to adjust the penalty, the optimal penalty that maximizes both consumer surplus and welfare is the minimum penalty that ensures truthful claims, and it increases with firms' quality difference and the probability of encountering a high-quality firm. In an extension, we allow regulators to detect false claims through consumer complaints. We find that higher penalty leads to lower consumer surplus if and only if the average product value is sufficiently high.

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Keywords: false advertising • consumer skepticism • regulation • signaling • game theory

1. Introduction

It is not uncommon to see firms engage in "deceptive" advertising in order for consumers to believe that their product quality is superior. A firm may use false or misleading claims in its advertisements to exaggerate the benefits of its product. Oftentimes, the misleading statements are deceptive, and firms can even use truth to tell lies. For example, a mobile phone manufacturer can present sample photos shot by professional photographers to showcase the superior quality of the camera(s) on its mobile phone. Yet the manufacturer is usually reluctant to reveal the photographers and the professional equipment (e.g., lighting equipment) used when those sample photos are shot.[1]

To protect consumers from deceptive ads, regulators such as the Federal Trade Commission (FTC) implement regulations and guidelines, which penalize firms who lie to or mislead consumers. For instance, according to FTC (2013), "[w]hen consumers see or hear an advertisement, whether it's on the Internet, radio or television, or anywhere else, federal law says that ad must be truthful, not misleading, and, when appropriate, backed by scientific evidence." Despite truth-in-advertising laws, many firms still manipulate their ad content to deceive consumers. For example, in 2015 a toothpaste brand Crest hired a famous TV host to illustrate to Chinese consumers its toothpaste's whitening effect, but people found out that in the advertisement the brand used digital software to make teeth whiter. The regulator in China eventually fined Crest almost one million US dollars for false advertising.[2]

Although deceptive ads are used to raise consumer demand, consumers are not equally vulnerable. Rao and Wang (2017) find that false claims primarily affect newcomers. Many consumers are skeptical about product claims and may actively investigate and verify those claims (e.g., see Ford et al. 1990), in order to find out the truth behind them. For example, consumers can verify product claims by conducting their own research on the internet or consulting with their friends who have the expertise.

In this paper, we propose a game theoretical model of false claims. In our model, a firm is offering a product to consumers, the quality of which is a priori unknown to consumers. A high-quality firm can issue a truthful product claim to reveal its high quality, whereas a low-quality firm can either claim to be a high-quality firm, or be completely honest and issue a less positive statement. False statements are subject to a penalty imposed by a regulator. Our model allows the low-quality firm to determine the deceptiveness of its false statement, which affects how likely the deceptive advertising is caught by consumers and the regulator. Based on the posted price and the firm's statement, each consumer can determine whether to verify the statement with a cost. The verification cost can differ for different consumers. Given the price, the statement, and the verification result (if any), consumers will then update their belief about the product quality and decide whether to make a purchase.

Our theoretical model allows us to investigate the impact of regulation on firm decisions, consumer surplus, firm profits, and social welfare, in the presence of skeptical consumers. Surprisingly, we find that stricter regulation (i.e., a higher penalty for fabrication) can lead to lower consumer surplus. When regulation is loose (i.e., low penalty), a low-quality firm makes a false claim with little justification. Expecting the false claim, a significant proportion of consumers verifies the statement, and most of them will be able to identify the low-quality firm from a high-quality firm. The identification enables some consumers to avoid purchasing a low-quality product at an inflated price. This is consistent with the empirical evidence that consumers in developing countries (where regulation is underdeveloped) are very skeptical about marketing practices (e.g., Varadarajan and Thirunarayana 1990, Chan and Cui 2002). By contrast, when regulation becomes relatively strict (i.e., penalty becomes relatively high), in order to reduce the chance of being caught by regulators, the low-quality firm is incentivized to spend more money to justify its product claim. However, the increased deceptiveness level hinders consumer verification, thus making it hard for consumers to spot a low-quality firm. Because only a few consumers can avoid buying the low-quality product at an inflated price, consumer surplus is lower under relatively strict regulation than under loose regulation.

Furthermore, the low-quality firm's profit can decrease with stricter regulation because of the increased penalty, even though the firm responds by increasing the level of deceptiveness to reduce the probability of being caught. Although both consumer surplus and firm profit may suffer from stricter regulation, in general social welfare does not exhibit a monotonic relationship with regulation strictness because the higher penalty can bring more monetary income for regulators (and hence for the society). These results suggest that regulators should be cautious in tightening the regulation on false claims, as it can lower consumer surplus, firm profit, and social welfare. However, if regulators are able to significantly raise the penalty on false claims, welfare will increase because firms will be honest and consumers do not need to spend effort/time on verifying the claims.

In addition, we identify the optimal penalty that maximizes both consumer surplus and social welfare, when the regulator incurs no cost to adjust the level of penalty. This optimal penalty is the minimum penalty that secures truthful statements. We demonstrate that the optimal penalty is increasing in quality difference between firms and prior quality belief. The result is driven by the low-quality firm's incentive to lie. When there is bigger quality difference between firms or when consumers are more likely to encounter the high-quality firm, the low-quality firm can benefit more from being dishonest, thereby increasing the minimum penalty that ensures truthful product claims.

In an extension, we allow regulators to catch false claims through consumer complaints. We find that the negative effect of stricter regulation on consumer surplus holds if and only if the average product value is sufficiently large. The reason is that, when average product value is small, if penalty increases, it can be shown that the low-quality firm has incentive to reduce the level of deceptiveness, in order to limit consumer demand and the associated complaints. In this case, the lower deceptiveness can lead to higher consumer surplus.

Our paper is most related to the theoretical literature on false advertising and its regulation. Barigozzi et al. (2009) discover that a new entrant in an industry can use comparative advertising, which may be false advertising, to signal its quality. This is because consumers know that an incumbent has the option to appeal to court to validate ad content and penalize false claims. Piccolo et al. (2015) underline a benefit of deceptive advertising for consumers: When a low-quality seller advertises deceptively, competing sellers' products look like closer substitutes to consumers, thereby reducing all sellers' pricing power and increasing consumer surplus. In contrast to these two studies, our paper suggests that even in the absence of competitive pressure a firm may find it optimal to deceive consumers.

Corts (2013, 2014) investigates a monopoly firm's false advertising decisions. The firm does not have perfect information about its product quality and can either make false claims or unsubstantiated claims (speculative claims). Corts (2013, 2014) highlights that strict regulation on unsubstantiated claims encourages a firm to find out its own product quality through costly information acquisition, but too strict regulation can induce socially wasteful costly information acquisition. By contrast, in our paper, the firm knows its quality, and it is the consumers who may acquire quality information with a cost.

Rhodes and Wilson (2018) reveal that a low-quality firm may choose to use false advertising with a positive probability, and a lower penalty for false claims can lead to higher consumer surplus. The reason is that, a lower penalty increases the probability of issuing a false claim, which decreases consumers' willingness-to-pay upon seeing a positive product claim. This essentially reduces the firm's pricing power and may leave consumers higher surplus. The change of the pricing power is the key of Rhodes and Wilson's result. In our paper, we find that a lower penalty for false claims can raise consumer surplus without any change of a firm's equilibrium price.[3] Our result is, therefore, not driven by a shift of pricing power, but is driven by consumers' endogenous information acquisition. Furthermore, in Rhodes and Wilson (2018), although the low-quality firm can use a mixed strategy of false claims and truthful claims, it is always indifferent between advertising deceptively and advertising truthfully. By contrast, in our paper, the low-quality firm can prefer advertising deceptively to advertising truthfully, and vice versa. When the firm chooses to lie, we investigate the firm's decision on the deceptiveness of its false claim, which is not considered in Rhodes and Wilson (2018) but plays a key role in determining the impact of regulation in our paper.

In short, our paper contributes to the theoretical literature on false advertising by proposing a new theory that shows why regulation may have a negative impact on consumer welfare. Our theory hinges on consumers' endogenous information acquisition, which is absent from previous theoretical studies.

In addition to the theoretical research, researchers have investigated false advertising empirically and experimentally (e.g., Darke and Ritchie 2007, Zinman and Zitzewitz 2016). Our model setup and theoretical predictions are consistent with some of the empirical findings. Jin and Kato (2006) find that in an online market that sells baseball cards, some buyers are misled by bold claims and they pay inflated prices for inferior quality. Rao and Wang (2017) reveal that deceptive advertisement primarily affects consumers who are newcomers. Consistent with these two findings, in our paper, because consumers differ in their ability to judge the firms' claims (as reflected in different verification costs), some consumers are more likely to be deceived by false statements. This phenomenon is not captured in previous theoretical studies but is critical in our paper.

Our research is also related to the literature on advertising and quality signaling (e.g., Kihlstrom and Riordan 1984, Milgrom and Roberts 1986, Joshi and Musalem 2018). In a recent study, Chakraborty and Harbaugh (2014) reveal that unsubstantiated claims can serve as a credible quality signal because, given limited bandwidth, emphasizing a strength (without providing evidence) is at the expense of not talking about another strength. Our research differs from this literature in that our focus is not to provide a new signaling mechanism through false advertising. Instead, in our paper a low-quality firm advertises deceptively in order to be undistinguishable from a high-quality firm.

Because consumers' endogenous information acquisition is a key model component in our paper, our research is also broadly related to the literature on search models (e.g., Diamond 1982, Branco et al. 2016). It is worth highlighting a recent paper by Mayzlin and Shin (2011), which is related to both advertising and consumer search. Mayzlin and Shin (2011) propose a new signaling mechanism that enables firms to use uninformative advertising as a credible quality signal. A high-quality firm can choose uninformative advertising as opposed to informative advertising to separate from a medium-quality firm by incentivizing consumers to search for quality information. The key difference in our paper is that Mayzlin and Shin (2011) consider honest firms only, whereas we focus on false claims and the low-quality firm's dishonest behavior. Furthermore, although Mayzlin and Shin (2011) emphasize the high-quality (and medium-quality) firms' strategies to signal their superior qualities, we highlight the low-quality firm's strategy to disguise its inferior quality.

The rest of our paper proceeds as follows. Section 2 introduces the model setup. Section 3 offers the equilibrium analysis. Section 4 discusses an alternative formulation of penalty based on consumer complaints. Section 5 provides concluding remarks. Technical details and proofs are relegated to the appendix and the online appendix.

2. Model Setup

Our model is composed of a representative firm, a unit mass of consumers, and a regulator. The firm sells a product to each consumer at a price p. There are two possible types of firms, the high type (H) and the low type (L). The value of firm j's product is V_j for consumers ($j = L, H$, and $V_L < V_H$), which is a priori unknown to them and the regulator. The firm is the high type with prior probability ϕ. To signal its quality to consumers, the firm issues a product statement.[4] A false statement may be penalized by the regulator.

2.1. Product Claims and Verification

Each type of firm decides on the statement $w \in \{0, 1\}$ about its product: $w = 1$ indicates a positive statement (labeled as h-statement) and $w = 0$ represents a less positive statement (labeled as l-statement). The l-statement should not be interpreted as a negative statement. It just does not claim the product to be a high-quality one. Because we consider only two types, the binary message space is without loss of generality.[5] Formally, the h-statement represents any statement that, if issued by firm L, is subjected to a penalty by the regulator; the l-statement represents any statement, including no statement or a neutral statement, that is not subjected to a penalty. An h-statement issued by firm L is labeled as a *false claim* or a *false statement*. If the firm chooses $w = 1$, it can decide how deceptive the product claim is. Let $x \in [0, 1]$ represent the deceptiveness of the false claim: a higher x indicates a more deceptive false claim. The firm decision x is not observed by consumers, but in equilibrium consumers have rational expectation of x, and consumers' decisions are based on their expectation of x.

Upon receiving a positive statement $w = 1$, we allow consumers to verify the statement with a cost $s > 0$. The cost s incorporates all time and effort a consumer spends on verification. The verification decision is denoted by $y \in \{0, 1\}$, where $y = 1$ means that a consumer verifies the statement. The verification cost s is distributed among consumers according to a twice-differentiable CDF $F(\cdot)$; the corresponding PDF is denoted by $f(\cdot)$.[6] Throughout the paper, we will use $\overline{F}(\cdot) \equiv 1 - F(\cdot)$ to denote the complimentary CDF. Our results hold for a general distribution, and we only assume that $\overline{F}((1-\phi)(V_H - V_L))$ is sufficiently large (compared with $f(\cdot)$)—that is, there are sufficiently many consumers with high verification cost.[7] Verification of an l-statement does not provide any value for consumers, whereas verification of an h-statement can help consumers distinguish firm L from firm H. Formally, after verification of an h-statement, the consumer receives a noisy signal $t \in \{0, 1\}$ of the product statement, where $t = 1$ indicates a positive signal and can be interpreted as confirmation of the h-statement and $t = 0$ indicates a negative signal and can be interpreted as disconfirmation. The high-quality firm's h-statement always produces a positive signal, that is, $\Pr(t = 1 | j = H) = 1$. By contrast, the low-quality firm's h-statement yields a positive signal with probability x, which denotes the deceptiveness of the false claim. That is, $\Pr(t = 1 | j = L) = x$ and $\Pr(t = 0 | j = L) = 1 - x$. Put differently, the more deceptive a false claim is, the harder it is for consumers to identify the false claim. To exclude the unrealistic case where the low-quality firm can be perfectly deceptive, we assume that the deceptiveness x is bounded by a threshold $\overline{x} < 1$ (i.e., $x \in [0, \overline{x}]$). It is worth noting that, although $t = 0$ implies that the firm is the low type, $t = 1$ does not completely resolve the uncertainty of firm type.

Our model best applies to situations where most consumers do not have complete knowledge about product quality even after purchase or the quality information cannot be easily and reliably spread out. Otherwise, within a short period of time after product launch, the quality becomes common knowledge, and false claims become inessential. For example, a firm may claim that its food product provides certain health benefits, but even after consumers consume the product, this quality information can be hard to evaluate.[8]

To make the false claim deceptive in order to avoid the penalty and consumers' identification of the false claim, firm L needs to incur a cost $c(x)$ (with $c(0) = c'(0) = 0$), which is twice-differentiable and strictly increasing and convex in x. Put differently, the more deceptive the low-quality firm wants to be to reduce the chance of being caught, the more money it needs to spend. To see how a false claim costs the low-quality firm, let's take the sample photo example from Section 1: When issuing false claims (high-quality sample photos), a mobile phone manufacturer can be deceptive by spending money on very expensive professional photographers and extra professional equipment, which are usually not revealed to consumers. Another example is the *angel dusting* strategy (see Gabriel 2008)—a firm can add a minute amount of an ingredient to a cosmetic or food product, and advertise the benefit of such ingredient, although the amount of the ingredient added to the product is too small to offer any real benefit to consumers. In this example, adding such an ingredient is costly to the firm. Note that, because its statement will not be penalized, firm H always chooses $x = 0$.

Our model is different from a conventional signaling model, where the focus is typically the separating equilibrium, and it is usually the firm H who invests resources to separate from firm L. In our model, the pooling equilibrium is crucial, and it is the firm L who spends on deceptive advertising to pool with firm H. If firm H has other instruments (e.g., advertisement and product warranty) to credibly signal its quality, firm L may find it harder to deceive consumers. We leave this for future study and assume that other than product statement and price, the firm has no readily available instrument to signal its quality.

To clearly illustrate our results, our model assumes that product quality is exogenously given and not observable to consumers. If quality improvement is allowed for both types of firms, then a firm will consider quality investment only if it is observable to consumers. Observable quality investment offers firm L an opportunity to reveal its improved quality, and therefore can eliminate firm L's incentive to spend on deceptive advertising if deception is too costly compared with quality investment.

2.2. Regulation

Product claims are regulated. A false statement results in an (expected) fine $b \cdot (1 - x)$, where b measures the penalty for false statements and indicates the strictness of regulation. The more deceptive the low-quality firm's false claim is, the less likely the firm will be caught. The punishment can be interpreted as the result of a regulator's detection process. For example, of such detection process, FTC actively monitors advertising practices regarding certain industries and pays close attention to "advertising claims that can affect consumers' health or their pocketbooks" (FTC 2013). Formally, the detection process is modeled as a noisy signal received by the regulator, $t' \in \{0, 1\}$. A truthful claim always yields $t' = 1$. A false claim produces a negative signal $t' = 0$ with probability $\Pr(t' = 0|j = L) = \beta(1 - x)$, where β measures the effectiveness of the regulator's detection process. The regulator can impose a fine b_o on the firm upon receiving $t' = 0$—leading to an expected fine $b_o \cdot \Pr(t' = 0|j = L) = b \cdot (1 - x)$, where $b \equiv b_o\beta$.[9]

The regulator in our paper is not necessarily a public authority or government agency, but it can be interpreted as media or the society. Given the alternative interpretations, the penalty can be seen as the loss of goodwill or the loss of future revenue (Suleman 2016). For simplicity, in this paper we use the term "regulator," but our insights can be generalized to a broader context.

2.3. Consumer Utility

Given the prior probability ϕ and the firm's observed decisions p and w, consumers update their belief about product quality. The posterior probability (after observing p and w) that the firm is type H is denoted by $\psi(p, w)$. Each consumer decides whether to verify the firm's statement (i.e., determining y) based on $\psi(p, w)$. If a consumer chooses $y = 1$, she spends s and acquires a noisy signal t, which allows her to further update the belief into $\psi(p, w; t)$.[10] Consumers can verify the product claim by, for example, doing their own research on the internet or consulting their friends who have the expertise.

The a priori expected utility of a consumer with verification cost s is

$$U(s) = \max_{y \in \{0,1\}} y \cdot U_I(s) + \left(1 - y\right) \cdot U_N, \tag{1}$$

where

$$U_I(s) = \mathbb{E}_t\left[\psi\left(p, w; t\right)V_H + \left(1 - \psi\left(p, w; t\right)\right)V_L - p\right]^+ - s; \tag{2}$$

$$U_N = \left[\psi\left(p, w\right)V_H + \left(1 - \psi\left(p, w\right)\right)V_L - p\right]^+. \tag{3}$$

In Equations (1), (2), and (3), $U_I(s)$ refers to the expected utility when the consumer verifies, and U_N denotes the expected utility when the consumer does not verify. The consumer chooses to verify if and only if $U_I(s) \geq U_N$. In the equations, $(\cdot)^+$ denotes $\max\{\cdot, 0\}$, which implies that we normalize consumers' outside option to 0.

2.4. Firm Profit

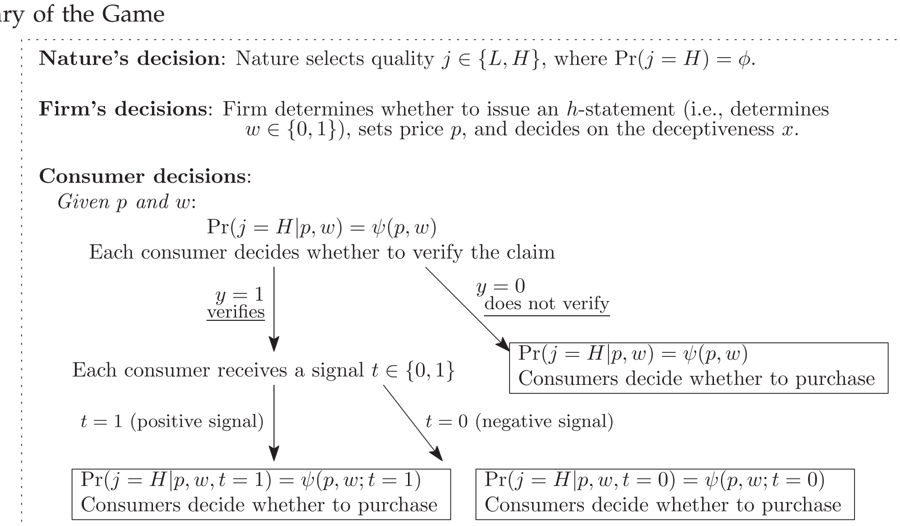
Given consumers' responses, each type of firm maximizes its profit:

$$\pi_H = p_H \cdot D_H; \tag{4}$$

$$\pi_L = p_L \cdot D_L - w_L \cdot [c(x) + b(1 - x)], \tag{5}$$

where D_j is the consumer demand for firm j and the demand depends on the observed decisions (p_j, w_j) of firm j, the low-quality firm's actual deceptiveness x, and its deceptiveness expected by consumers x^e, which in equilibrium coincides with the low-quality firm's actual deceptiveness (i.e., $x = x^e$ in equilibrium).[11] It is worth noting that we assume that it is costless for the firm to make a truthful statement.

Figure 1. Summary of the Game



The cost $c(x)$ refers to the cost of making a false statement deceptive.[12]

The game is summarized in Figure 1.[13] Firms determine the statement to issue and the product price. Consumers observe the statement and the price, but they have incomplete information about firm type j (i.e., product quality) and imperfect information about the low-quality firm's deceptiveness decision x. Each consumer needs to determine whether to verify the product claim, based on their prior belief ϕ and the firm's observed actions p and w. After that, depending on the verification result (if there is any), each consumer infers the firm's product quality and decides whether to make a purchase. The notation is summarized in Table 1.[14]

2.5. Solution Concept

In this paper, we adopt pure-strategy Perfect Bayesian Equilibrium (PBE) (Fudenberg and Tirole 1991). The firm's statement w and price p serve as signals for its product quality. Based on the prior belief ϕ about the firm type, consumers update their belief in light of the Bayes' rule, whenever applicable. The firm decides w, p, and x, in order to maximize its profits according to consumers' posterior belief and best responses.

The arbitrariness of the out-of-equilibrium beliefs in PBE leads to multiple equilibria. We adopt the *intuitive criterion* (Cho and Kreps 1987) to rule out equilibria that are driven by "unreasonable" out-of-equilibrium beliefs.[15] Under intuitive criterion, multiple equilibria can still arise. To guarantee the equilibrium uniqueness, whenever there exist multiple PBEs that survive the intuitive criterion, we use the *D1 criterion* (Fudenberg and Tirole 1991) to further refine the equilibria.[16]

There are two possible types of equilibria. In a *separating equilibrium*, the low-quality firm's observed decisions $(p_L, w_L) = (p_L^{sep}, w_L^{sep})$ differ from the high-quality firm's decisions $(p_H, w_H) = (p_H^{sep}, w_H^{sep})$, in which the superscript "sep" refers to the separating equilibrium. In the separating equilibrium, after observing p and w, consumers can distinguish the two types of firms and do not need to verify the claim—that is, equilibrium beliefs are $\psi(p_H^{sep}, w_H^{sep}) = 1$ and $\psi(p_L^{sep}, w_L^{sep}) = 0$. In a *pooling equilibrium*, the two types of firms implement the same strategy $(p_H, w_H) = (p_L, w_L) = (p^{pool}, w^{pool})$, where the superscript "pool" signifies the pooling equilibrium. The belief before verification $\psi(p^{pool}, w^{pool}) = \phi$ coincides with the prior belief.

3. Equilibrium Analysis

3.1. Consumer Decisions

After observing an l-statement ($w = 0$), because verification offers no value to consumers, they do not verify and they purchase if and only if $p \le \psi(p, 0)V_H + (1 - \psi(p, 0))V_L$. After observing an h-statement ($w = 1$),

Table 1. Summary of Notation

V_j	Value of firm j's product for consumers, $j = L, H$
ϕ	Prior probability that the firm is the high-quality firm
$\overline{V}$	$\equiv \phi V_H + (1 - \phi)V_L$, the average product value, a priori
ΔV	$\equiv V_H - V_L$, quality difference
w_j	Firm j's decision on product statement: 1—h-statement; 0—l-statement, $j = L, H$
p_j	Firm j's price, $j = L, H$
x	Deceptiveness of firm L's false claim
$\overline{x}$	Upper bound of x
χ	Unbounded solution of the equilibrium deceptiveness x in the pooling equilibrium
x^e	Expected deceptiveness of firm L's false claim, from consumers' perspective
$c(x)$	Cost of deceptiveness
D_j	Firm j's demand, $j = L, H$
π_j	Firm j's profit, $j = L, H$
s	Consumers' cost of verification
$\widetilde{s}$	Value of information verification can provide
$F(\cdot), f(\cdot)$	CDF, PDF of s
y	A consumer's decision on verification: 1—verifies; 0—does not verify
t	The signal a consumer receives after verification
$\psi(p, w)$	Consumers' belief after observing price p and product statement w
$\psi(p, w; t)$	Consumers' belief after verification, given p, w, and signal t
$U(s)$	Expected utility of a consumer with verification cost s
$U_I(s)$	Expected utility when a consumer (with verification cost s) verifies the claim
U_N	Expected utility when a consumer does not verify the claim
$\square^{sep}$	Superscript that denotes a variable in the separating equilibrium
$\square^{pool}$	Superscript that denotes a variable in the pooling equilibrium
b	Penalty for false statements
CS	Consumer surplus
W	Social welfare

verification can make consumers more certain about the product quality, and thus they need to decide whether to verify the claim before making any purchase. In order to determine the verification decision, we need to evaluate how the verification results affect consumers' willingness-to-pay. If a consumer verifies the product claim, a negative signal $t = 0$ indicates that the firm is a low-quality firm (i.e., $\Pr(j = H|p, w = 1, t = 0) = \psi(p, 1; 0) = 0$), and a positive signal $t = 1$ improves the firm's quality perception because $\Pr(j = H|p, w = 1, t = 1) = \psi(p, 1; 1) = \Pr(j = H, t = 1|p, w = 1)/\Pr(t = 1|p, w = 1) = \Pr(j = H|p, w = 1)\Pr(t = 1|j = H, p, w = 1)/(\Pr(j = H|p, w = 1)\Pr(t = 1|j = H, p, w =, 1) + \Pr(j = L|p, w = 1)\Pr(t = 1|j = L, p, w = 1)) = \psi(p, 1) \cdot 1/[\psi(p, 1) \cdot 1 + (1 - \psi(p, 1)) \cdot x^e] \geq \psi(p, 1)$ (the equality holds only for $\psi(p, 1) = 0$ or 1), where x^e is the deceptiveness of the low-quality firm expected by consumers.

Intuitively, a consumer chooses to verify only when the verification can help her make a more informed decision—that is, she would purchase if and only if $t = 1$. Thus, for any $p \in (V_L, \psi(p,1;1)V_H + (1-\psi(p,1;1))V_L]$,[17] verification provides the consumer a utility $U_I(s) = \Pr(t = 1|p,\ w = 1)[\psi(p, 1; 1)V_H + (1 - \psi(p, 1; 1))V_L - p]^+ - s = \psi(p, 1)V_H + (1 - \psi(p, 1))x^e V_L - [\psi(p, 1) + (1 - \psi(p, 1))x^e]p - s$. The consumer verifies if and only if $U_I(s) \geq U_N$, which translates into $s \leq \tilde{s}$, where

$$\begin{aligned}\tilde{s} = \psi(p, 1)V_H + (1 - \psi(p, 1))x^e V_L \\ - [\psi(p, 1) + (1 - \psi(p, 1))x^e]p \\ - [\psi(p, 1)V_H + (1 - \psi(p, 1))V_L - p]^+. \qquad (6)\end{aligned}$$

Note that $\tilde{s}$ is the value of information that the verification process can provide. As a result, there are $F(\tilde{s})$ consumers who verify and then decide whether to purchase. If the firm is a high-quality firm, all of these consumers receive $t = 1$ and buy the product after verification; if the firm is a low-quality firm, only $xF(\tilde{s})$ of these consumers receive $t = 1$ and make a purchase. Meanwhile, there are $\overline{F}(\tilde{s})$ consumers with high verification cost. They do not verify the product claim and they purchase when $p \leq \psi(p, 1)V_H + (1 - \psi(p, 1))V_L$.

3.2. Equilibrium Outcome

3.2.1. Complete-Information Benchmark. Under complete information about firm type, the low-quality firm has no incentive to issue a false statement. Instead, it is completely honest and makes an *l*-statement. Because consumers can tell the firm type, the choice of product statements is irrelevant to the high-quality firm's profit, and no consumer needs to verify the statement. As a result, the two types of firms can appropriate all consumer surplus by charging their product values V_H and V_L, respectively.

3.2.2. Incomplete Information. Before characterizing the equilibrium outcome, we first establish the importance of product statements when signaling quality:

Lemma 1. *Price alone cannot signal product quality. Specifically, in any separating equilibrium firm H issues an h-statement ($w_H^{sep} = 1$) and firm L issues an l-statement ($w_L^{sep} = 0$).*

In the absence of complete information about firm type, the low-quality firm has incentives to issue a false statement, in order to deceive consumers. The purpose of regulation on false statements is to raise the low-quality firm's lying cost. As formally shown by Lemma 2 and summarized by Proposition 1, the low-quality firm makes a false statement if and only if the penalty for false statements is not too large.

Lemma 2. *The equilibrium is characterized as follows in Table 2.*

Proposition 1. *When the penalty b is small, the unique equilibrium is a pooling equilibrium, in which the low-quality firm issues a false statement about its product. When b is large, the unique equilibrium is a separating equilibrium, where the low-quality firm is honest and does not claim to be a high-quality firm.*

When the penalty b is relatively small (i.e., $b \leq \kappa$), the low-quality firm chooses to be dishonest and it charges the same price as the high-quality firm. No consumer can distinguish the two types before the verification process. However, consumers with low verification cost (i.e., $s \leq \tilde{s}$) are willing to investigate the statement and then make the purchase if and only if the investigation yields a positive outcome (i.e., $t = 1$). Because the signal t is informative, the consumers who investigate and receive a positive signal $t = 1$ have higher willingness-to-pay for the product than those who do not verify the statement. Consumers with high verification cost (i.e., $s > \tilde{s}$) purchase without any verification. Because there are sufficiently many consumers who have high verification cost and do not investigate, both types of firms set a price $p^{\text{pool}} \equiv \phi V_H + (1 - \phi)V_L$ that appropriates all surplus from those consumers.

The threshold $\tilde{s} = \phi(1 - \phi)(1 - x^{\text{pool}})(V_H - V_L)$ represents the value of information from the verification process in equilibrium. Intuitively, the value of information is proportional to the variance of product quality $\phi(1 - \phi)$ and the quality difference $(V_H - V_L)$. Furthermore, the value of information is negatively affected by the deceptiveness of the low-quality firm's claim x^{pool}. The reason is the following. When the false statement is coated with more deceptive elements, it is more likely to produce a positive signal $t = 1$. Put differently, the low-quality firm can jam the signal t by being more deceptive. As a result, the signal t consumers receive becomes noisier, thereby reducing the value of information.

In the pooling equilibrium, the low-quality firm's deceptiveness decision is governed by three effects on

Table 2. Equilibrium Characterization

	Pooling equilibrium	Separating equilibrium
Existence condition	$b \leq \kappa$	$b > \kappa$
κ is uniquely determined by the value of b s.t. $[1-(1-x^{\text{pool}})F(\widetilde{s})]\overline{V} - c(x^{\text{pool}}) - b(1-x^{\text{pool}}) = V_L$.		
Firm decisions		
Price		
p_H	$p^{\text{pool}} \equiv \phi V_H + (1-\phi)V_L$	$p_H^{\text{sep}} \equiv \min\{V_H, V_L + \min_x[c(x) + b(1-x)]\}$
p_L		$p_L^{\text{sep}} \equiv V_L$
False claim?	Yes: $w_H = w_L = w^{\text{pool}} \equiv 1$	No: $w_H = w_H^{\text{sep}} \equiv 1$ and $w_L = w_L^{\text{sep}} \equiv 0$
Deceptiveness	$x^{\text{pool}} \equiv \min\{\overline{x}, \chi\}$	
$\chi \in (0, +\infty)$ is uniquely determined by $F(\phi(1-\phi)(1-\chi)(V_H - V_L))[\phi V_H + (1-\phi)V_L] - c'(\chi) + b = 0$. The out-of-equilibrium belief can be specified as $\psi(p,w) = 0$ for any off-equilibrium strategy (p,w).		
Consumer decisions		
Who verify?	Consumers with $s \leq \widetilde{s}$, where $\widetilde{s} = \phi(1-\phi)(1-x^{\text{pool}})(V_H - V_L)$	No consumer needs to verify.
Who purchase?	Consumers with $s > \widetilde{s}$ and consumers with $s \leq \widetilde{s}$ who receive a positive signal $t = 1$	All consumers purchase.

the firm profit, which can be seen from the first derivative of π_L with respect to x. Firm L's profit is $\pi_L = [1-(1-x)F(\widetilde{s})]p^{\text{pool}} - c(x) - b(1-x)$, where $p^{\text{pool}} = \phi V_H + (1-\phi)V_L$. The first derivative is thus $d\pi_L/dx = F(\widetilde{s})p^{\text{pool}} - c'(x) + b$. The first term, $F(\widetilde{s})p^{\text{pool}}$, represents a *demand effect*: A higher x increases the likelihood of a positive signal after verification and it thus leads to larger demand and higher revenue. The second term, $-c'(x)$, indicates a *cost effect*, which is the marginal cost firm L needs to incur, in order to be more deceptive. The third term, b, refers to a *penalty avoidance effect*, which is the marginal saving on penalty when firm L becomes more deceptive. As x increases, the demand effect becomes smaller and the cost effect becomes larger (in magnitude), whereas the penalty avoidance effect remains the same. Therefore, an interior solution ($x^{\text{pool}} = \chi < \overline{x}$) is achieved when the sum of the two positive effects equal to the cost effect, that is, when the first-order condition $d\pi_L/dx = 0$ is satisfied (see the equation that determines χ in Table 2).

When the penalty exceeds κ, it becomes very costly for the low-quality firm to issue an h-statement (and set the same price) to pool with the high-quality firm. Instead, it would rather be completely honest and charge consumers their willingness-to-pay V_L. Meanwhile, the high-quality firm makes a truthful positive claim and charges a higher price. However, the high-quality firm still needs to set a price below consumers' willingness-to-pay V_H in order to prevent the low-quality firm from mimicking the h-statement and the pricing strategy. This is because, as the low-quality firm is unwilling to be indistinguishable from the high-quality firm, consumers stop verifying the claim. In the absence of consumer verification, the low-quality firm has strong incentive to lie and pretend to have high product quality. As a consequence, the high-quality firm needs to reduce its price to separate from the low-quality firm. Specifically, the high-quality firm would charge a price $p_H^{\text{sep}} \equiv \min\{V_H, V_L + \min_x[c(x) + b(1-x)]\}$. The inequality $p_H^{\text{sep}} \leq V_L + \min_x[c(x) + b(1-x)]$ ensures that the low-quality firm has no incentive to mimic the high-quality firm.

3.3. Effects of Regulation

Given the equilibrium outcome, in this subsection, we examine regulation's welfare impact—specifically, how regulation (i.e., penalty) affects consumer surplus, firm profits, and social welfare. We first discuss the regulation effect on consumer surplus (denoted by CS). In the separating equilibrium, consumers know the firm type and do not need to verify the statement. Hence, consumer surplus is equal to consumers' willingness-to-pay minus price. In the pooling equilibrium, both types of firms charge a price to appropriate all surplus from consumers who have high verification cost and do not investigate the product claim. Meanwhile, because the firm cannot price-discriminate consumers, the consumers with low verification cost will verify the statement with a cost s and obtain the value of information $\widetilde{s}$. Therefore, the aggregate consumer surplus in the pooling equilibrium is $CS = \int_0^{\widetilde{s}}(\widetilde{s} - s)f(s)ds$, which hinges on $\widetilde{s}$.

Recall that the value of information is $\widetilde{s} = \phi(1-\phi) \times (1-x^{\text{pool}})(V_H - V_L)$. It is proportional to $(1-x^{\text{pool}})$, which refers to the chance that the verification process can identify the low-quality firm via observing a bad signal $t = 0$, given that the firm is a low-quality firm. Because both the variance of product quality $\phi(1-\phi)$ and the quality difference $(V_H - V_L)$ are invariant in the penalty b, how regulation affects consumer surplus is determined by its impact on x^{pool}.

As discussed before, the decision x^{pool} is governed by three effects on firm profit—the demand effect, the cost effect, and the penalty avoidance effect. As the penalty b increases, the demand effect and the cost effect remain the same, but the penalty avoidance effect is magnified. Therefore, in order to alleviate the raised penalty, the low-quality firm would like to be more deceptive (i.e., spending more money to justify its false claim). This result is formally presented in Lemma 3.

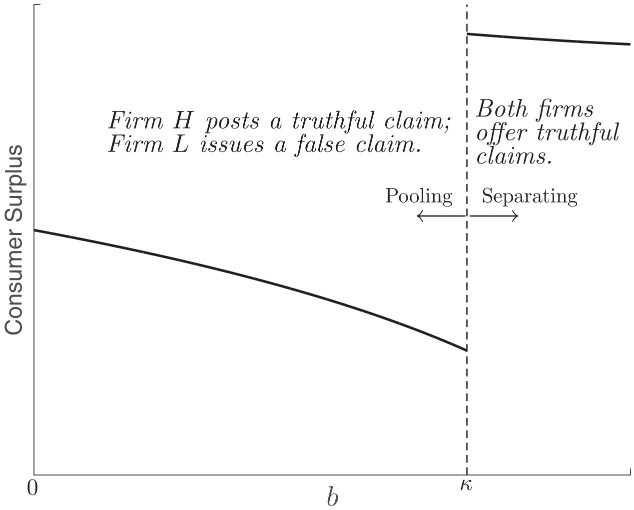
Lemma 3. *When the low-quality firm provides a false statement ($b \le \kappa$), the deceptiveness x^{pool} is increasing in penalty b.*

As the deceptiveness increases, consumers find it harder to identify the low-quality firm via verification. Thus, the value of information $\tilde{s}$ is decreasing in the penalty b. This implies that, as the regulation becomes stricter (i.e., b becomes larger), consumer surplus falls. This result is formally presented in the first part of Proposition 2 and illustrated by Figure 2.

Proposition 2. *As the penalty b increases, the consumer surplus CS decreases for $b \le \kappa$, increases discontinuously at $b = \kappa$, and then decreases again for $b > \kappa$.*

Proposition 2 confirms that, in the presence of a false statement ($b \le \kappa$), consumer surplus drops with penalty because the value of information diminishes. The result suggests that regulators and policymakers should be cautious in raising the penalty for false statements, which can lead to lower consumer surplus.

Figure 2. Effect of Regulation on Consumer Surplus



Once b goes beyond κ, the pooling equilibrium cannot sustain because it is not profitable for the low-quality firm to issue a false statement (and charge the same price) to become indistinguishable from the high-quality firm. As such, the equilibrium is a separating equilibrium, where only the high-quality firm claims to have superior product quality. When the penalty b surpasses the threshold κ, consumer surplus increases discontinuously. The intuition is two-fold. First, in the separating equilibrium consumers can infer the product quality perfectly and thus do not need to verify the claim, saving the cost of information acquisition. Second, the high-quality firm needs to reduce its price for separation because the low-quality firm has strong incentive to pretend to be a high-quality firm in the absence of consumer verification. Because of the above two reasons, consumers have a larger surplus $CS = \phi(V_H - p_H^{\text{sep}})$ in the separating equilibrium for b just above κ than in the pooling equilibrium for b just below κ. However, as b increases further from κ, consumer surplus decreases again. This is because, as the mimicking cost of the low-quality firm increases, the high-quality firm can charge a higher price p_H^{sep} that can maintain separation in equilibrium, and this lowers consumer surplus.

The discussion is focused on consumer surplus. We next examine the regulation effects on firm profits and total social welfare. The total social welfare is defined as $W = CS + \phi\pi_H + (1-\phi)\pi_L + (1-\phi)w_L b(1-x)$, which is the sum of consumer surplus, firm profit, and total fine for false claims (if there is any). Note that, according to the definition, we implicitly assume that the penalty fees collected by the regulator are equally valuable for the society. Proposition 3 formally presents the results, and Figure 3 offers graphical illustrations.

Proposition 3. *As the penalty b increases,*

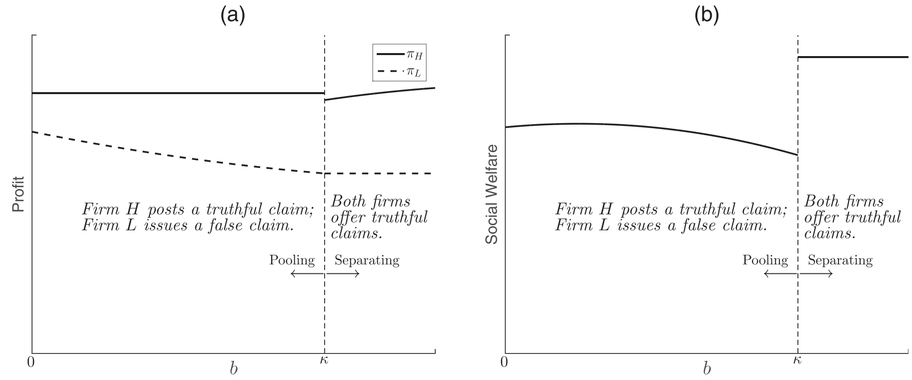
i. *the low-quality firm's profit decreases for $b \le \kappa$ and then stays at V_L for $b > \kappa$;*

ii. *the high-quality firm's profit stays at $\phi V_H + (1-\phi)V_L$ for $b \le \kappa$, decreases discontinuously at $b = \kappa$, and then increases for $b > \kappa$;*

iii. *social welfare is in general not monotonic for $b \le \kappa$;*[18] *social welfare increases to $\phi V_H + (1-\phi)V_L$ once $b > \kappa$.*

Proposition 3(i) and (ii) demonstrate how firm profits vary with the penalty b. When $b \le \kappa$, the low-quality firm's profit decreases with the penalty because of the increased fine. The high-quality firm's profit stays the same because it is not penalized by the regulator. Once b surpasses κ, the low-quality firm behaves truthfully and its profit is not affected by any further increase in b. Meanwhile, the high-quality firm's profit drops discontinuously at $b = \kappa$, because the firm needs to cut its price in order to

Figure 3. Effect of Regulation on Firm Profits and Social Welfare



separate from the low-quality firm. The lower price hurts the high-quality firm's profitability. When b further increases ($b > \kappa$), the high-quality firm's profit rises because the low-quality firm's mimicking incentive diminishes.

Proposition 3(iii) reveals that the effect of regulation on total social welfare is not monotonic in general. When $b \leq \kappa$, consumer surplus and firm profit (low type) decrease, whereas the total fine $(1-\phi)b(1-x)$ can increase. As a result, social welfare does not have a monotonic relationship with b in general. When b surpasses κ such that both types of firms are truthful, social welfare rises to the expected product value $\phi V_H + (1-\phi)V_L$. The welfare increase is driven by the absence of verification process and deceptive ads, which saves the society both the verification costs and the spending on deceptiveness.

In short, Propositions 2 and 3 imply that, in the presence of deceptive ads, consumer surplus, firm profit, and social welfare can all decrease with the penalty for false statements. Regulators and policymakers should carefully evaluate the impact of regulation—when the penalty is not very high ($b \leq \kappa$), raising the penalty can have negative impact on not only consumers and firms but also the society as a whole.

It is worth noting that in our model, the penalty is from the low-quality firm to the regulator. If part of the penalty is given back to consumers who have bought the product from firm L in the presence of false claims, then from consumers' perspective, they do not suffer too much from being deceived by firm L. Knowing this, both types of firms are able to raise their price level and appropriate consumers' gain from the compensation in the pooling equilibrium. This redistribution of penalty essentially increases the "product value" of firm L in the pooling equilibrium. Although most of our results remain intact qualitatively, given this redistribution of penalty, firm H's profit can rise with penalty in the pooling equilibrium (instead of staying the same as shown by Proposition 3). This is because a higher penalty can increase consumers' potential compensation and lead to higher equilibrium price, which benefits firm H. Furthermore, compared with the case in the absence of penalty redistribution, firm L's higher product value reduces the "quality difference" between firms, resulting in lower consumer surplus and higher social welfare (as will be shown in Proposition 5).

3.4. Optimal Regulation

In the previous discussions, we treat the penalty for false statements as an exogenous parameter and analyze its impact. If the regulator can modify the regulation, she will choose an optimal penalty to maximize her objective. We consider two possible objectives: consumer surplus and social welfare. For simplicity, we assume that the regulator can adjust the penalty at no cost. Figures 2 and 3(b) suggest that the penalty just above κ maximizes both objectives. The result is formally presented in Lemma 4.

Lemma 4. *Consumer surplus and social welfare are maximized at $b = \kappa + \varepsilon$, where $\varepsilon \to 0^+$ and both firms make truthful claims.*[19]

According to Lemma 4, to maximize consumer surplus or social welfare, the regulator would ensure that the penalty is sufficiently high that the low-quality firm makes a truthful claim (l-statement), so that consumers are not deceived by the low-quality firm and can save the unnecessary cost of verification. At the same time, the regulator prefers the penalty just above κ to keep the price level of the high-quality firm low, thus maximizing the surplus left for consumers. Therefore, the optimal penalty (that maximizes both consumer surplus and social welfare) is the minimum penalty that can ensure separation (i.e., $\kappa + \varepsilon$). Mathematically, there does not exist a maximal consumer surplus because the existence condition for the separating equilibrium is $b > \kappa$ but not $b \geq \kappa$. As a result, the consumer surplus can only infinitely approach but never reach the supremum $\lim_{b \to \kappa^+} CS$. We follow Chen et al. (2010) to use $\kappa + \varepsilon$ to denote the optimal level, even though the optimal point does not exist mathematically. Note that in practice it can be costly for the regulator to adjust the penalty. A significant increase in penalty for false statements may raise the expense of regulation and litigation. If so, the regulator needs to conduct a cost-benefit analysis, in order to determine whether to set a high penalty to induce truthful claims.

The optimal penalty depends on two important determinants, quality difference and consumers' prior belief about product quality. Quality difference $\Delta V \equiv V_H - V_L$ is what motivates the low-quality firm to make a false statement and why consumers may engage in information acquisition before purchase. Consumers' prior quality belief ϕ influences consumers' expectation of the product quality and the uncertainty of product quality (i.e., the variance of product quality, $\phi(1-\phi)$), both of which affect the firm's decisions. In the comparative statics with regard to ΔV, we keep the average product value $\overline{V} \equiv \phi V_H + (1-\phi)V_L$ constant to ensure that the results are driven by the change of quality difference but not the change of quality level. Proposition 4 shows that both ΔV and ϕ have a positive effect on the optimal penalty.

Proposition 4. *The optimal penalty (i.e., $\kappa + \varepsilon$) increases in quality difference ΔV and consumers' prior belief ϕ about product quality.*

The optimal penalty is determined by the threshold κ, above which the low-quality firm cannot profit from pooling with the high-quality firm. A larger ΔV implies a greater gain from pooling for the low-quality firm, compared with its profit in the separating equilibrium. Similarly, a higher ϕ raises the average product value $\overline{V}$, which increases the low-quality firm's profit in the pooling equilibrium. Therefore, the minimum penalty $(\kappa + \varepsilon)$ to achieve separation, which is the optimal penalty that maximizes consumer surplus and social welfare, increases with both ΔV and ϕ.

The regulator may desire a smaller quality difference and a less positive prior quality belief so that the optimal penalty is not too high to impose. However, Proposition 5 implies that, given a specific penalty level, consumer surplus and social welfare can increase in ΔV and ϕ.

Proposition 5.

i. *As the quality difference ΔV increases,*

a. *in the pooling equilibrium, consumer surplus increases and social welfare decreases;*

b. *in the separating equilibrium, consumer surplus increases and social welfare stays the same;*

ii. *As the prior belief ϕ becomes more positive,*[20]

a. *in the pooling equilibrium, consumer surplus and social welfare are in general not monotonic;*

b. *in the separating equilibrium, both consumer surplus and social welfare increase.*

In the pooling equilibrium, the low-quality firm makes a false claim, and consumers with low verification cost investigate the false claim. Recall that consumer surplus is determined by the value of information $\widetilde{s} = \phi(1-\phi)(1-x^{\text{pool}})\Delta V$, which is proportional to ΔV and $\phi(1-\phi)$. As a result, keeping the deceptiveness x^{pool} the same, consumer surplus is increasing in ΔV and nonmonotonic in ϕ. Proposition 5 implies that, even though the low-quality firm may adjust x^{pool} in response to a change in ΔV or ϕ, the impact of x^{pool} does not negate the direct effects of ΔV and ϕ. Although a larger quality difference leads to higher consumer surplus, it results in lower social welfare. This is because, when ΔV increases, more consumers verify the product claim, and thus more consumers would end up not purchasing the product (from the low-quality firm). The reduced transaction volume lowers social welfare. Furthermore, as more consumers investigate, the low-quality firm has incentive to increase the deceptiveness of its false claim, thereby increasing the socially wasteful lying cost and decreasing welfare.

Because of the nonmonotonic effect of ϕ on the value of information $\widetilde{s}$, the transaction volume does not exhibit a monotonic relationship with the prior belief. As a result, social welfare is nonmonotonic in ϕ. The nonmonotonic effect of ϕ on consumer surplus and social welfare implies a surprising result: When the firm is more likely to have high quality, consumer surplus and social welfare can decrease.

In the separating equilibrium, both firms make truthful claims, and verification is not needed. When the quality difference increases, the low-quality firm has greater incentive to mimic the *h*-statement and the pricing strategy, and thus the high-quality firm needs to set a lower price to prevent the low type's mimicking attempt. Therefore, the lower pricing power implies higher consumer surplus. However, social welfare remains the same because the higher consumer surplus is at the expense of the high-quality firm's profit.

When the prior belief ϕ becomes more positive, consumers are more likely to encounter the high-quality firm. Recall that the consumer surplus in the separating equilibrium is $CS = \phi(V_H - p_H^{\text{sep}})$. As a result, a higher ϕ leads to larger consumer surplus. Social welfare also increases with ϕ because the welfare is equal to the average product value $\overline{V}$, which is increasing in ϕ.

4. Extension: Alternative Formulation of Penalty Based on Consumer Complaints

In our basic model, the regulator detects false claims via a process similar to consumers' investigation process. Although our formulation of penalty is consistent with FTC (2013), one may argue that besides active monitoring, the regulator may detect false claims with the help of consumer complaints. To explore the role played by consumer complaints in detecting deceptive advertising, in this section we discuss an alternative formulation of the penalty, based on consumer complaints after purchase. Because the number of consumer complaints should be positively affected by the number of consumers who have made a purchase, it is reasonable to assume that the expected penalty is not only inversely related to the deceptiveness x but also proportional to the consumer demand. Formally, we assume that a false statement will result in an expected fine $b \cdot (1-x) \cdot D_L$, where D_L is consumer demand endogenously determined in equilibrium and $[(1-x) \cdot D_L]$ measures the volume of consumer complaints.

Given the formulation of penalty, it is worth noting the difference between the deceptiveness level x and an actual quality investment (which is not considered in the model). Increasing deceptiveness and improving quality can both reduce consumer complaints after purchase. However, increasing deceptiveness does not provide real benefit to consumers but only deceives them (see the angel dusting example we have introduced in Section 2), whereas improving quality can offer real benefit to consumers.

Similarly to the basic model, under complete information about firm type, the market is free of false claims. In this extension, therefore, we focus on the analysis under incomplete information. Because a higher penalty weakens the low-quality firm's incentive to issue a false statement, Proposition 1 continues to hold. Specifically, there exists a threshold $\widehat{\kappa}$, such that the unique equilibrium is a pooling equilibrium if and only if $b \le \widehat{\kappa}$, in which the low-quality firm uses a false claim. This result is formalized in Lemma 5. Throughout this extension, to ensure concavity of firm L's profit in x, we assume that $c'' > 2b$.

Lemma 5. *The equilibrium is characterized as follows in Table 3.*[21]

The equilibrium outcome shows that the main difference from the basic model is the equilibrium deceptiveness decision (i.e., interior solution is $\widehat{\chi}$ as opposed to χ) in the pooling equilibrium. To illustrate the intuition, we decompose the first derivative (w.r.t. x) of firm L's profit $\pi_L = D_L p^{\text{pool}} - c(x) - b(1-x)D_L$ (where $D_L = 1-(1-x)F(\widetilde{s})$): $d\pi_L/dx = F(\widetilde{s})p^{\text{pool}} - c'(x) - b \cdot d\,[(1-x)D_L]/dx$. The first two terms respectively correspond to the demand effect and the cost effect, which have been discussed in the basic model. The third term refers the effect of deceptiveness on expected penalty. It can be further decomposed into two terms: $-b \cdot d[(1-x)D_L]/dx = bD_L - b(1-x)F(\widetilde{s})$. The first part, bD_L, is a penalty avoidance effect, which is similar to the one in the basic model—given any fixed consumer demand, higher deceptiveness reduces consumer complaints and expected penalty. The second part, $-b(1-x)F(\widetilde{s})$, is a *negative demand effect*—given any fixed level of deceptiveness, higher deceptiveness leads to higher demand, which raises the number of total consumer complaints and expected penalty. The negative demand effect is absent from the basic model but plays a key role in this extension.

Recall that in the basic model, as the penalty b rises, in the pooling equilibrium the deceptiveness of false claims increases because of the elevated penalty avoidance effect. In this extension, a higher b not only raises the penalty avoidance effect but also aggravates the negative demand effect. As a result, whether the deceptiveness increases in b depends on

Table 3. Equilibrium Characterization

	Pooling equilibrium	Separating equilibrium
Existence condition	$b \le \widehat{\kappa}$	$b > \widehat{\kappa}$
$\widehat{\kappa}$ is uniquely determined by the value of b s.t. $[1-(1-x^{\text{pool}})F(\widetilde{s})][\overline{V} - b(1-x^{\text{pool}})] - c(x^{\text{pool}}) = V_L$.		
Firm decisions		
Price	The same as those in Table 2	
False claim?	Yes: $w_H = w_L = w^{\text{pool}} \equiv 1$	No: $w_H = w_H^{\text{sep}} \equiv 1$ and $w_L = w_L^{\text{sep}} \equiv 0$
Deceptiveness	$x^{\text{pool}} \equiv \min\{\overline{x}, \widehat{\chi}\}$	
$\widehat{\chi} \in (0, +\infty)$ is uniquely determined by $F(\phi(1-\phi)(1-\widehat{\chi})(V_H - V_L))[\overline{V} - 2b(1-\widehat{\chi})] - c'(\widehat{\chi}) + b = 0$. The out-of-equilibrium belief can be specified as $\psi(p, w) = 0$ for any off-equilibrium strategy (p, w).		
Consumer decisions: The same as those in Table 2		

the relative magnitude of the coefficients of b in the two effects, D_L and $(1-x)F(\widetilde{s})$. Lemma 6 provides the condition under which x^{pool} is increasing in b.

Lemma 6. *When the low-quality firm provides a false statement ($b \le \widehat{\kappa}$), there exists a threshold $\widetilde{V}$ such that, if $\overline{V} > \widetilde{V}$,*[22] *the deceptiveness x^{pool} is increasing in penalty b; otherwise, x^{pool} is decreasing in b.*

The intuition of Lemma 6 can be illustrated by examining how average product value $\overline{V}$ affects D_L and $(1-x)F(\widetilde{s})$. A larger $\overline{V}$ leads to a higher gain from deception and thus incentivizes firm L to be more deceptive (i.e., setting a higher x). The higher x implies a larger D_L and smaller $(1-x)F(\widetilde{s})$ because $D_L = 1 - (1-x)F(\widetilde{s})$ increases with x and $(1-x)F(\widetilde{s})$ decreases with x. Therefore, for sufficiently large $\overline{V}$, the penalty avoidance effect is more crucial than the negative demand effect. As a result, the same intuition from the basic model applies, and thereby x^{pool} is increasing in b. Given a small $\overline{V}$, the negative demand effect is more important than the penalty avoidance effect, and thus x^{pool} is decreasing in b. In other words, for a small $\overline{V}$, firm L is worried about the negative impact of a large customer base on the expected penalty.

As in the basic model, in the pooling equilibrium the consumer surplus is positively related to the value of information, which is proportional to $(1-x^{\text{pool}})$. Therefore, Lemma 6 implies that Proposition 2 is preserved only for sufficiently large $\overline{V}$ (i.e., $\overline{V} > \widetilde{V}$). For small $\overline{V}$, the consumer surplus is increasing in b in the pooling equilibrium. Proposition 6 formally states this result.

Proposition 6. *As the penalty b increases,* (i) *if $\overline{V} > \widetilde{V}$, the consumer surplus CS decreases for $b \le \widehat{\kappa}$; otherwise, CS increases for $b \le \widehat{\kappa}$;* (ii) *CS increases discontinuously at $b = \widehat{\kappa}$;* (iii) *CS decreases for $b > \widehat{\kappa}$.*

Proposition 6 suggests that in the pooling equilibrium the effect of penalty on consumer surplus hinges on how penalty affects the deceptiveness of false claims in the market. When the deceptiveness level increases with the penalty for false statements, the regulator needs to act carefully, in order to avoid the negative impact on consumer surplus. Regarding the effects of the penalty b on profit and welfare, the results in Proposition 3 are preserved qualitatively.

5. Discussions and Concluding Remarks

In this paper, we have proposed a game theoretical framework to study deceptive ads and the impact of their regulation. When a low-quality firm issues a false claim and misrepresents its product quality, consumers can actively engage in information acquisition and verify the claim. Surprisingly, we find that, when the penalty for false statements increases, consumer surplus can decrease. The reason is that, as penalty rises, the low-quality firm becomes more deceptive about its product quality, and the increased deceptiveness reduces consumers' incentives to verify the product claim. The lack of verification results in more purchases of the low-quality product at an inflated price and leads to lower consumer surplus. Furthermore, we demonstrate that firm profit and social welfare can both decrease with the penalty for false statements.

Our results suggest that, in a market with loose regulation (e.g., in a developing country), low-quality firms do not spend much on making their product statements deceptive. As a result, consumers (and the society) can benefit from investigating firms' product claims and distinguishing a high-quality firm from a low-quality firm. By contrast, in a market with relatively strict regulation (e.g., in a developed country), firms have to be very deceptive about their false claims in order to reduce the chance of being caught by regulators. Consequently, consumers cannot easily identify a low-quality product, thereby having little incentive to investigate product claims and suffering from the lack of information.

For instance, a global conglomerate claimed that one of its skin care products can heal burn scars. The company made such a bold statement only in a large developing country.[23] Consumers who know other languages could easily find out the truth by going to the company's official websites in other countries. In developed countries, the same company acknowledged that they had engaged in false advertising by intentionally altering some of their testing,[24] which is much more deceptive and harder for consumers and regulators to detect. Another example is related to the toothpaste company introduced in Section 1, who claimed in a large developing country that consumers can whiten their teeth in one day. To support the claim, the company used digital software to make a celebrity's teeth whiter in the advertisement.[25] The deceptive practice was not very hard to identify because the celebrity appeared in multiple popular TV shows at the time. In a large developed country, the same company, however, was much more deceptive by conducting well-designed studies, in order to support a false claim that its toothpaste for sensitive teeth can relieve tooth pain within minutes.[26]

Our analysis shows that, in order to increase consumer surplus and total social welfare, regulators should significantly raise the penalty for false statements to completely eliminate false claims. When this is done, consumers do not need to collect information about product claims and can simply tell the product quality from the truthful reports. However, in the real world, it is not straightforward for regulators to substantially increase the penalty for deceptive ads because stronger punishment is often associated with higher litigation cost. As a consequence, the regulators should carefully evaluate the impact of regulation beyond the model setup in this paper in order to ensure that a large increase in penalty is economically beneficial for the society.

When it is costless to adjust the penalty, the optimal penalty that maximizes both consumer surplus and social welfare is the minimum penalty that induces truthful statements. However, it is worth noting that the maximal welfare is at the expense of firm profits. Our analysis reveals that the optimal level of penalty is increasing in quality difference and prior quality belief. The result is driven by the low-quality firm's incentive to be dishonest. With a larger quality difference or better prior quality belief, the low-quality firm has stronger incentive to lie, thus raising the minimum penalty that leads to truthful claims.

In our model, given the optimal penalty, the market is free of false claims. However, it is not our intent to claim that in the real world the optimal penalty is achieved when false statements are completely eliminated. There are factors, not considered in our model, that can affect the incidence of false claims. For example, a firm may not have complete information about its product quality, and thus the firm may be uncertain about a product statement it makes. What we would like to emphasize in the paper is that, given a low penalty level, raising penalty can reduce consumers' incentives to verify product statements. Thus, regulators need to take into account these negative effects of penalty on consumer surplus when drafting and enforcing the regulation.

In an extension, we explore an alternative formulation of the penalty for false statements. Specifically, we assume that the regulator can detect false claims with the help of consumers who have purchased the product and complained. We highlight an important factor the low-quality firm needs to consider—as higher deceptiveness allows the firm to raise its demand, the number of consumer complaints increases too. We show that, if the average product value is high, all of our results preserve qualitatively. Otherwise, due to the concern about consumer complaints, the equilibrium deceptiveness is decreasing in penalty, and the consumer surplus is increasing in penalty.

There are a few research directions that are worth exploring. First, our paper abstracts from how regulators utilize the penalty they receive. We also assume away the cost of penalty adjustment (e.g., a change in litigation costs). It could be interesting to examine how those factors affect the presence and the deceptiveness of false statements and welfare. Second, on e-commerce platforms, the platforms themselves also serve as regulators. Depending on the revenue model, different platforms may regulate deceptive advertising differently. It could be interesting to study how online platforms should regulate deceptive advertising and the associated welfare implications.

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Appendix
Proof of Lemma 1
First, in any separating equilibrium firm L adopts its optimal strategy under complete information and thus sets $w_L^{sep} = 0$ and $p_L^{sep} = V_L$. Next, we prove by contradiction that $w_H^{sep} = 1$. Suppose $w_H^{sep} = w_L^{sep} = 0$ in equilibrium. We have $p_H^{sep} \neq p_L^{sep}$ and $p_H^{sep} \le V_H$. The firm with lower price can make a profitable deviation by mimicking the equilibrium strategy of the firm with higher price, a contradiction. Hence, it is not possible that $w_H^{sep} = w_L^{sep}$ and $p_H^{sep} \neq p_L^{sep}$.

Proof of Lemma 2
We prove Lemmas 7, 8, and 9, which will imply Lemma 2.

Lemma 7. Under the intuitive criterion, there exists a unique separating equilibrium.[27] In equilibrium, $p_H = p_H^{sep} \equiv \min\{V_H, V_L + \min_x[c(x) + b(1-x)]\}$, $p_L = p_L^{sep} \equiv V_L$, $w_H = w_H^{sep} \equiv 1$, and $w_L = w_L^{sep} \equiv 0$. The out-of-equilibrium belief can be specified as $\psi(p, w) = 0$ for any off-equilibrium strategy (p, w).

Proof. We first prove that $(p_H^{sep}, p_L^{sep}, w_H^{sep}, w_L^{sep})$ constitute a PBE, which survives the intuitive criterion. By mimicking firm H, firm L can earn a profit $p_H^{sep} - \min_x[c(x) + b(1-x)] \le V_L = p_L^{sep}$, and so firm L has no incentive to deviate. To show that the PBE exists and survives the intuitive criterion, we just need to prove that for any off-equilibrium action (p, w), (A) neither type can make a profitable deviation to (p, w) under the specified out-of-equilibrium belief $\psi(p, w) = 0$ and (B) the specified out-of-equilibrium belief *survives* the intuitive criterion. Note that, in order to *fail* the intuitive criterion, if firm L is not willing to deviate to (p, w) under any possible belief, then firm H can make a profitable deviation to (p, w) when consumers perceive the firm as the high type (i.e., when $\psi(p, w) = 1$). Therefore, part (B) is equivalent to the following: firm L is willing to deviate under the most optimistic belief $\psi(p, w) = 1$, or firm H cannot make a profitable deviation even when it is perceived as the high type (i.e., even when $\psi(p, w) = 1$).

For any off-equilibrium action (p, w), if $p < p_H^{sep}$, firm H cannot make a profitable deviation under any belief, and firm L would not deviate under $\psi(p, w) = 0$. If $p_H^{sep} < p \le V_H$, then both types would deviate under $\psi(p, w) = 1$, and neither type can make a profitable deviation under $\psi(p, w) = 0$. For $p > V_H$, the consumer demand is zero, and thus no firm is willing to deviate.

We next prove the uniqueness by contradiction. Suppose there exists another separating equilibrium that survives the intuitive criterion. Lemma 1 implies that $w_L^{sep} = 0$, $w_H^{sep} = 1$, and $p_L^{sep} = V_L$. Firm H's equilibrium price must satisfy $p_H^{sep} < \min\{V_H, V_L + \min_x[c(x) + b(1-x)]\}$ because otherwise demand will be zero, or firm L can make a profitable deviation by mimicking the equilibrium pricing strategy p_H^{sep}. Now consider an off-equilibrium action $(p, w = 1)$, where $p \in (p_H^{sep}, \min\{V_H, V_L + \min_x[c(x) + b(1-x)]\})$. Firm L has no incentive to deviate to (p, w) under any belief. However, under the only reasonable belief $\psi(p, w) = 1$ in light of the intuitive criterion, firm H can make a profitable deviation to (p, w), a contradiction. □

Lemma 8. In any pooling equilibrium that survives the intuitive criterion, $w^{pool} = 1$.

Proof. We prove this lemma by contradiction. Suppose that $w^{pool} = 0$. Recall that in Section 3.4, we have defined $\overline{V} \equiv \phi V_H + (1-\phi)V_L$. We have $V_L \le p^{pool} \le \overline{V} < V_H$. Consider an off-equilibrium action $(p, w = 1)$, where $p = p^{pool} + \varepsilon$ and $\varepsilon > 0$ is sufficiently small. Firm L is not willing to deviate to (p, w) under any belief because of the penalty. However, under the only reasonable belief $\psi(p, w) = 1$ in light of the intuitive criterion, firm H can make a profitable deviation to (p, w), a contradiction. □

Under the intuitive criterion, there can still be multiple pooling equilibria. To guarantee a unique equilibrium, we adopt a stronger refinement, the D1 criterion, in the presence of multiple equilibria. In our model, the D1 criterion requires that, by deviating to an off-equilibrium strategy (p, w), if firm H can benefit from a larger set of out-of-equilibrium beliefs than firm L,[28] then the out-of-equilibrium belief should be $\psi(p, w) = 1$.

Lemma 9. There exists a threshold κ, such that there exists a pooling equilibrium that survives the intuitive criterion iff $b \le \kappa$. For $b \le \kappa$, the unique equilibrium that survives the D1 criterion is the following pooling equilibrium: $p_H = p_L = p^{pool} \equiv \phi V_H + (1-\phi)V_L$, $w_H = w_L = w^{pool} \equiv 1$, and $x^* = x^{pool} \equiv \min\{\overline{x}, \chi\}$, where χ is uniquely determined by

$$G(\chi) \equiv F\big(\phi(1-\phi)(1-\chi)(V_H - V_L)\big) \cdot \big[\phi V_H + \big(1-\phi\big)V_L\big] - c'(\chi) + b = 0. \tag{A.1}$$

The out-of-equilibrium belief can be specified as $\psi(p, w) = 0$ for any off-equilibrium strategy (p, w).

Proof. The proof proceeds in four steps: (i) The stated pooling equilibrium constitutes a PBE iff b is sufficiently small (i.e., $b \le \kappa$, where κ is determined in the proof). (ii) It survives the D1 criterion. (iii) For any $b > \kappa$, there does not exist any pooling equilibrium that survives the intuitive criterion (or the D1 criterion). (iv) For any $b \le \kappa$, the stated pooling equilibrium is the unique PBE that survives the D1 criterion.

(i) Given the equilibrium price, $\psi(p^{pool}, w^{pool}) = \phi$. Therefore, from Equation (6), $\widetilde{s} = (1-\phi)(1-x^e)(\overline{V} - V_L) = \phi(1-\phi) \times (1-x^e)(V_H - V_L)$. Thus, given any x, we have $D_H = 1$ and $D_L = \overline{F}(\widetilde{s}) + F(\widetilde{s})x = 1 - (1-x)F(\widetilde{s})$. Therefore, $\pi_H = \overline{V}$ and $\pi_L = [1 - (1-x)F(\widetilde{s})]\overline{V} - c(x) - b(1-x)$. We have

$$\frac{d\pi_L}{dx} = F(\widetilde{s})\overline{V} - c'(x) + b, \tag{A.2}$$

which is decreasing in x. In equilibrium, the expected deceptiveness x^e coincides with the actual deceptiveness chosen by firm L. Therefore, the equilibrium deceptiveness is $x^* = x^{pool} \equiv \min\{\overline{x}, \chi\}$, where $\chi \in (0, +\infty)$ is determined by the first-order condition shown in Equation (A.1). Note that $\chi \in (0, +\infty)$ exists because $G(0) > 0$ and $G(+\infty) < 0$ ($\because c''(\cdot) > 0$ and $F(\widetilde{s})|_{x \ge 1} = 0$). Because $G(x)$ is decreasing in x, χ is unique. The value of χ can exceed 1, but $x^{pool} \equiv \min\{\overline{x}, \chi\}$ is always between 0 and 1.

Firm H has no incentive to deviate to any off-equilibrium strategy under $\psi(p, w) = 0$ because $\pi_H = \overline{V} > V_L$, which is the highest profit it can earn under $\psi(p, w) = 0$. Firm L's highest profit it can earn under $\psi(p, w) = 0$ is V_L. Therefore, firm L has no incentive to deviate to any off-equilibrium strategy iff $\pi_L = [1 - (1-x^{pool})F(\widetilde{s})]\overline{V} - c(x^{pool}) - b(1-x^{pool}) \ge V_L$. Therefore, to show that the equilibrium sustains for a sufficiently small b, it is remaining to show that π_L is decreasing in b. If $x^{pool} = \overline{x}$, then π_L is strictly decreasing in b. Otherwise ($x^{pool} = \chi$), according to the envelope theorem, we have

$$\frac{d\pi_L}{db} = \frac{\partial \pi_L}{\partial b} + \frac{\partial \pi_L}{\partial x^e}\frac{d\chi}{db}, \tag{A.3}$$

where

$$\frac{\partial \pi_L}{\partial b} = -(1-\chi); \tag{A.4}$$

$$\frac{\partial \pi_L}{\partial x^e} = (1-\chi)\phi(1-\phi)(V_H - V_L)f(\widetilde{s})\overline{V}; \tag{A.5}$$

$$\frac{d\chi}{db} = -\frac{\frac{\partial G(\chi)}{\partial b}}{\frac{\partial G(\chi)}{\partial \chi}} = \frac{1}{\phi(1-\phi)(V_H - V_L)f(\widetilde{s})\overline{V} + c''(\chi)}. \tag{A.6}$$

Therefore,

$$\frac{d\pi_L}{db} = -\frac{(1-\chi)c''(\chi)}{\phi(1-\phi)(V_H - V_L)f(\widetilde{s})\overline{V} + c''(\chi)} < 0. \tag{A.7}$$

Let κ be the value of b such that $\pi_L = [1 - (1-x^{pool})F(\widetilde{s})]\overline{V} - c(x^{pool}) - b(1-x^{pool}) = V_L$. Then, the stated pooling equilibrium exists iff $b \le \kappa$.

(ii) Next, we show that the pooling equilibrium survives the D1 criterion. We just need to show that for any off-equilibrium action, firm L can benefit from a weakly larger set of out-of-equilibrium beliefs than firm H. For any off-equilibrium action $(p, w = 0)$, both types of firms can earn the same profit under any belief because consumers never investigate given $w = 0$. Because firm L's equilibrium profit is lower than firm H's equilibrium profit, whenever firm H can make a profitable deviation under a belief, firm L can do so too.

For any off-equilibrium action $(p, w = 1)$ and out-of-equilibrium belief $\psi \le \phi$, firm H cannot make any profitable deviation. For any off-equilibrium action $(p, w = 1)$ and out-of-equilibrium belief $\psi > \phi$, firm H is willing to deviate iff $\overline{V} < p \le \psi V_H + (1-\psi)V_L$. To prove that firm L can also make a profitable deviation iff $\overline{V} < p \le \psi V_H + (1-\psi)V_L$, we just need to show that firm L's profit is increasing in $p \in (V_L, \psi V_H + (1-\psi)V_L]$ given any fixed belief ψ.

We have $\widetilde{s} = (1-\psi)(1-x^e)(p - V_L)$, $D_L = 1 - (1-x)F(\widetilde{s})$, and $\pi_L = [1 - (1-x)F(\widetilde{s})]p - c(x) - b(1-x)$. Given consumers' rational expectation, the deceptiveness $x = \min\{\overline{x}, \chi'\}$, where χ' is uniquely determined by $g(\chi') \equiv F((1-\psi)(1-\chi') \times (p - V_L))p - c'(\chi') + b = 0$.

For $x = \overline{x}$, we have

$$\frac{d\pi_L}{dp} = \frac{\partial \pi_L}{\partial p} = 1 - (1-\overline{x})F(\widetilde{s}) - \big(1-\psi\big)(1-\overline{x})^2 f(\widetilde{s})p. \tag{A.8}$$

For $x = \chi'$, we have

$$\frac{d\pi_L}{dp} = \frac{\partial \pi_L}{\partial p} + \frac{\partial \pi_L}{\partial x^e}\frac{d\chi'}{dp}, \tag{A.9}$$

where

$$\frac{\partial \pi_L}{\partial p} = 1 - (1-\chi')F(\widetilde{s}) - (1-\psi)(1-\chi')^2 f(\widetilde{s})p; \tag{A.10}$$

$$\frac{\partial \pi_L}{\partial x^e} = (1-\chi')(1-\psi)(p - V_L)f(\widetilde{s})p; \tag{A.11}$$

$$\frac{d\chi'}{dp} = -\frac{\frac{\partial g(\chi')}{\partial p}}{\frac{\partial g(\chi')}{\partial \chi'}} = \frac{(1-\psi)(1-\chi')f(\widetilde{s})p + F(\widetilde{s})}{(1-\psi)(p-V_L)f(\widetilde{s})p + c''(\chi')}. \tag{A.12}$$

Therefore,

$$\begin{aligned}\frac{d\pi_L}{dp} &= 1 - \frac{(1-\chi')[(1-\psi)(1-\chi')f(\widetilde{s})p + F(\widetilde{s})]c''(\chi')}{(1-\psi)(p-V_L)f(\widetilde{s})p + c''(\chi')} \\ &\geq \frac{(1-\chi')[\overline{F}(\widetilde{s}) - (1-\psi)(1-\chi')f(\widetilde{s})p]c''(\chi')}{(1-\psi)(p-V_L)f(\widetilde{s})p + c''(\chi')}.\end{aligned} \tag{A.13}$$

Because $\overline{F}((1-\phi)(V_H - V_L))$ is assumed to be sufficiently large compared with $f(\cdot)$, $d\pi_L/dp > 0$ for any x. This completes the proof that both types of firms are willing to deviate iff $\overline{V} < p \leq \psi V_H + (1-\psi)V_L$, and thus the aforementioned pooling equilibrium survives the D1 criterion.

(iii) We prove by contradiction that for any $b > \kappa$, there does not exist any pooling equilibrium that can survive the intuitive criterion. Lemma 8 shows that $w^{\text{pool}} = 1$. Among the pooling equilibrium with $w^{\text{pool}} = 1$, part (ii) implies that the pooling equilibrium with $p_L = p_H = p^{\text{pool}} = \overline{V}$ can generate the highest profit for both firms. Even in this pooling equilibrium the profit π_L is lower than V_L for $b > \kappa$ (see Equation (A.7) and the definition of κ after the equation). Therefore, in any pooling equilibrium, the low-quality firm's profit is lower than V_L for $b > \kappa$. Thus, firm L can always make a profitable deviation to $(p_L, w_L) = (V_L, 0)$, a contradiction.

(iv) We prove by contradiction that for $b \leq \kappa$ the stated pooling equilibrium is the unique PBE that survives the D1 criterion. First, suppose there exists another pooling equilibrium with equilibrium action $(p^{\text{pool}\prime}, w^{\text{pool}\prime})$. We have $p^{\text{pool}\prime} < \overline{V}$ and $w^{\text{pool}\prime} = 1$. Consider an off-equilibrium price $p = p^{\text{pool}\prime} + \varepsilon$, where $\varepsilon > 0$ is sufficiently small. Firm H can make a profitable deviation to $(p, w = 1)$ for any belief $\psi \geq \psi_H \equiv (p^{\text{pool}\prime} + \varepsilon - V_L)/(V_H - V_L)$ ($\Longleftrightarrow p \leq \psi_H V_H + (1-\psi_H)V_L$). This is because all consumers who investigate will make a purchase. Firm L is willing to deviate to $(p, w = 1)$ for any belief $\psi \geq \psi_L \equiv \phi - \varepsilon'$, where $\varepsilon' > 0$ and $\lim_{\varepsilon \to 0^+} \varepsilon' = 0$. This is because a lower ψ increases the number of consumers who investigate and thus reduces consumer demand. Because $\lim_{\varepsilon \to 0^+} \psi_H = (p^{\text{pool}\prime} - V_L)/(V_H - V_L) < \phi = \lim_{\varepsilon \to 0^+} \psi_L$, for sufficiently small ε we have $\psi_H < \psi_L$. As a result, firm H can benefit from a strictly larger set of beliefs by deviating to p than firm L, a contradiction to the D1 criterion.

Second, suppose the separating equilibrium stated in Lemma 7 survives the D1 criterion. We first show that the separating equilibrium price $p_H^{\text{sep}} < \overline{V}$ for $b \leq \kappa$. Recall that $b = \kappa$ satisfies $[1 - (1 - x^{\text{pool}})F(\widetilde{s})]\overline{V} - c(x^{\text{pool}}) - b(1 - x^{\text{pool}}) = V_L$, and the left-hand side is decreasing in b. Thus, for any $b \leq \kappa$, we have

$$\begin{aligned}&\left[1 - \left(1 - x^{\text{pool}}\right)F(\widetilde{s})\right]\overline{V} \geq V_L + c\left(x^{\text{pool}}\right) + b\left(1 - x^{\text{pool}}\right) \\ &\quad \geq V_L + \min_x[c(x) + b(1-x)] \geq p_H^{\text{sep}} \quad \Longrightarrow \quad p_H^{\text{sep}} < \overline{V}.\end{aligned} \tag{A.14}$$

As a result, $p_H^{\text{sep}} = V_L + \min_x[c(x) + b(1-x)] < \overline{V}$. Similar to the proof above regarding the pooling equilibrium, we can show by contradiction that the separating equilibrium fails the D1 criterion: Consider an off-equilibrium price $p = p_H^{\text{sep}} + \varepsilon$, where $\varepsilon > 0$ is sufficiently small. Firm H is willing to deviate to $(p, w = 1)$ for any belief $\psi \geq \psi_H \equiv (p_H^{\text{sep}} + \varepsilon - V_L)/(V_H - V_L)$ ($\Longleftrightarrow p \leq \psi_H V_H + (1-\psi_H)V_L$). Firm L can make a profitable deviation to $(p, w = 1)$ for any belief $\psi \geq \psi_L \equiv 1 - \varepsilon'$, where $\varepsilon' > 0$ and $\lim_{\varepsilon \to 0^+} \varepsilon' = 0$. Because $\lim_{\varepsilon \to 0^+} \psi_H = (p^{\text{sep}} - V_L) \times (V_H - V_L) < 1 = \lim_{\varepsilon \to 0^+} \psi_L$ for sufficiently small ε we have $\psi_H < \psi_L$. Thus, firm H can benefit from a strictly larger set of beliefs by deviating to p than firm L, a contradiction to the D1 criterion. □

Lemmas 7, 8, and 9 imply Lemma 2.

Proof of Lemma 3

This lemma is implied by the proof of Lemma 9 (see part (i)): The boundary solution $x^{\text{pool}} = \overline{x}$ is invariant in b. The interior solution $x^{\text{pool}} = \chi$ is increasing in b because Equation (A.6) shows that $d\chi/db > 0$.

Proof of Proposition 2

The proof proceeds in three steps as follows.

i. We first show that $dCS/db \leq 0$ for the pooling equilibrium ($b \leq \kappa$). For $x^{\text{pool}} = \overline{x}$, $dCS/db = 0$. For $x^{\text{pool}} = \chi$, we have

$$\frac{dCS}{db} = \frac{dCS}{d\widetilde{s}} \cdot \frac{d\widetilde{s}}{d\chi} \cdot \frac{d\chi}{db}, \tag{A.15}$$

where

$$\frac{dCS}{d\widetilde{s}} = F(\widetilde{s}); \tag{A.16}$$

$$\frac{d\widetilde{s}}{d\chi} = -\phi(1-\phi)(V_H - V_L); \tag{A.17}$$

$$\frac{d\chi}{db} = -\frac{\frac{\partial G(\chi)}{\partial b}}{\frac{\partial G(\chi)}{\partial \chi}} = \frac{1}{\phi(1-\phi)(V_H - V_L)f(\widetilde{s})\overline{V} + c''(\chi)}. \tag{A.18}$$

Therefore, $dCS/db < 0$.

ii. Next we show that when the equilibrium moves from the pooling equilibrium to the separating equilibrium at $b = \kappa$, consumer surplus increases. Recall that in the proof of Lemma 9, we have shown that the separating equilibrium price $p_H^{\text{sep}} < \overline{V}$ (see Inequalities (A.14)). Let CS^{sep} and CS^{pool} denote the consumer surplus for the separating equilibrium and the pooling equilibrium respectively at $b = \kappa$. We have

$$\begin{aligned}CS^{\text{pool}} &= \int_0^{\widetilde{s}} (\widetilde{s} - s)f(s)ds < \int_0^{\widetilde{s}} \widetilde{s}f(s)ds < \widetilde{s} \\ &= \phi(1-\phi)(1 - x^{\text{pool}})(V_H - V_L) < \phi(1-\phi)(V_H - V_L) \\ &= \phi(V_H - \overline{V}) < \phi(V_H - p_H^{\text{sep}}) \\ &= CS^{\text{sep}}.\end{aligned} \tag{A.19}$$

iii. To complete the proof of the proposition, we just need to show that $dCS/db \le 0$ for the separating equilibrium $(b > \kappa)$. We have $CS = \phi(V_H - p_H^{\text{sep}})$. Because $p_H^{\text{sep}} = \min\{V_H, V_L + \min_x[c(x) + b(1-x)]\}$ is (weakly) increasing in b, consumer surplus is (weakly) decreasing in b.

Proof of Proposition 3

i and ii. The proof of Lemma 9 shows that for $b \le \kappa$ the equilibrium is a pooling equilibrium and π_L is decreasing in b (see Inequality (A.7)), and that at $b = \kappa$ firm L's profit $\pi_L = V_L$. For $b > \kappa$, the equilibrium is a separating equilibrium, and $\pi_L = V_L$. Firm H's profit π_H is $\overline{V}$ in the pooling equilibrium ($b \le \kappa$) and p_H^{sep} in the separating equilibrium ($b > \kappa$), which is (weakly) increasing in b and $p_H^{\text{sep}}|_{b \to \kappa^+} < \overline{V}$ (see Inequalities (A.14)).

iii. In the separating equilibrium, the expected social welfare is always $W = \overline{V}$ because consumers do not need to verify the statement and the firm chooses $w = 0$. In the rest of the proof, we focus on the pooling equilibrium. Because some consumers will verify the statement and firm L will choose $w = 1$ and incur a cost $c(x^{\text{pool}})$, the expected social welfare is always below $\overline{V}$.

To show the unimodality of W, we take $c(x) = kx^2/2$ and suppose s follows a uniform distribution on $[0, T]$, where k is a cost parameter and T is the upper bound of the verification cost. According to the proof of Lemma 3, the interior solution $x^{\text{pool}} = \chi$ is increasing in b. Note that the welfare can be written as the total purchased product value minus the cost of verification and the cost of deceptiveness: $W = \phi V_H + (1-\phi)V_L[1 - (1 - x^{\text{pool}})F(\tilde{s})] - \int_0^{\tilde{s}} sf(s)ds - (1-\phi) \times c(x^{\text{pool}})$. Thus, welfare is affected by b through x^{pool} only, we just need to focus on the interior solution $x^{\text{pool}} = \chi$ and prove that W is unimodal in χ (recall $\tilde{s} = \phi(1-\phi)(1-\chi) \times (V_H - V_L)$). According to Equation (A.1), we have $\phi(1-\phi) \times (1-\chi)(V_H - V_L)\overline{V}/T - k\chi + b = 0 \Longrightarrow \chi = [\phi(1-\phi)(V_H - V_L) \times \overline{V} + bT]/[\phi(1-\phi)(V_H - V_L)\overline{V} + kT]$. Therefore,

$$
\begin{aligned}
\frac{dW}{d\chi} &= (1-\phi)V_L F(\tilde{s}) - (1-\phi)V_L(1-\chi)f(\tilde{s})\frac{d\tilde{s}}{d\chi} \\
&\quad - (1-\phi)(1-\chi)(\overline{V} - V_L)f(\tilde{s})\frac{d\tilde{s}}{d\chi} - (1-\phi)c'(\chi) \\
&= (1-\phi)[V_L F(\tilde{s}) + \phi(1-\phi)(1-\chi)(V_H - V_L)\overline{V} \\
&\quad \times f(\tilde{s}) - c'(\chi)] \\
&= (1-\phi)\left[\frac{\phi(1-\phi)(1-\chi)(V_H - V_L)(\overline{V} + V_L)}{T} - k\chi\right] \\
&= \frac{\begin{array}{c}(1-\phi)[\phi(1-\phi)(V_H - V_L)(\overline{V} + V_L)(k-b) \\ -k\phi(1-\phi)(V_H - V_L)\overline{V} - kbT]\end{array}}{\phi(1-\phi)(V_H - V_L)\overline{V} + kT} \\
&= \frac{\begin{array}{c}(1-\phi)\{k\phi(1-\phi)(V_H - V_L)V_L \\ -[\phi(1-\phi)(V_H - V_L)(\overline{V} + V_L) + kT]b\}\end{array}}{\phi(1-\phi)(V_H - V_L)\overline{V} + kT},
\end{aligned}
\tag{A.20}
$$

where the denominator is positive and the numerator is decreasing in b. Therefore, W is unimodal in b.

Proof of Lemma 4

According to the proof of Proposition 3 (see part (iii)), the social welfare is at its maximal level in the separating equilibrium, and so it is maximized at any $b > \kappa$. From the proof of Proposition 2 (see part (ii)), the consumer surplus in the pooling equilibrium (for any $b \le \kappa$) is lower than that in the separating equilibrium for b just above κ. Because in the separating equilibrium consumer surplus is decreasing in b, the optimal b is achieved at $\kappa + \varepsilon$, where $\varepsilon \to 0^+$.

Proof of Proposition 4

Quality difference ΔV.

Let π_L^{sep} and π_L^{pool} denote firm L's profit in the separating equilibrium and in the pooling equilibrium respectively at $b = \kappa$. Recall that $b = \kappa$ is determined by $\pi_L^{\text{pool}} = V_L = \pi_L^{\text{sep}}$. We have

$$
\frac{d\kappa}{d\Delta V} = -\frac{\frac{d\left(\pi_L^{\text{pool}} - \pi_L^{\text{sep}}\right)}{d\Delta V}}{\frac{d\left(\pi_L^{\text{pool}} - \pi_L^{\text{sep}}\right)}{db}},
\tag{A.21}
$$

where $d(\pi_L^{\text{pool}} - \pi_L^{\text{sep}})/db = d\pi_L^{\text{pool}}/db < 0$ according to the proof of Lemma 9 (see Inequality (A.7)), and $d\pi_L^{\text{sep}}/d\Delta V = -\phi$. Now we derive $d\pi_L^{\text{pool}}/d\Delta V$: For $x^{\text{pool}} = \bar{x}$, we have

$$
\frac{d\pi_L^{\text{pool}}}{d\Delta V} = \frac{\partial \pi_L^{\text{pool}}}{\partial \Delta V} = -(1-\bar{x})^2 f(\tilde{s})\overline{V}\phi(1-\phi).
\tag{A.22}
$$

For $x^{\text{pool}} = \chi$, we have

$$
\frac{d\pi_L^{\text{pool}}}{d\Delta V} = \frac{\partial \pi_L^{\text{pool}}}{\partial \Delta V} + \frac{\partial \pi_L^{\text{pool}}}{\partial x^e}\frac{d\chi}{d\Delta V},
\tag{A.23}
$$

where

$$
\frac{\partial \pi_L^{\text{pool}}}{\partial \Delta V} = -(1-\chi)^2 f(\tilde{s})\overline{V}\phi(1-\phi);
\tag{A.24}
$$

$$
\frac{\partial \pi_L^{\text{pool}}}{\partial x^e} = (1-\chi)\phi(1-\phi)\Delta V f(\tilde{s})\overline{V};
\tag{A.25}
$$

$$
\frac{d\chi}{d\Delta V} = -\frac{\frac{\partial G(\chi)}{\partial \Delta V}}{\frac{\partial G(\chi)}{\partial \chi}} = \frac{f(\tilde{s})\overline{V}\phi(1-\phi)(1-\chi)}{\phi(1-\phi)\Delta V f(\tilde{s})\overline{V} + c''(\chi)}.
\tag{A.26}
$$

Therefore,

$$
\frac{d\pi_L^{\text{pool}}}{d\Delta V} = -\frac{(1-\chi)^2 f(\tilde{s})\overline{V}\phi(1-\phi)c''(\chi)}{\phi(1-\phi)(V_H - V_L)f(\tilde{s})\overline{V} + c''(\chi)}.
\tag{A.27}
$$

Similar to the proof of Lemma 9 (see part (ii)), because $f(\tilde{s})$ is sufficiently small compared with $\overline{F}(\tilde{s})$, it can be shown that $d\pi_L^{\text{pool}}/d\Delta V > -\phi = d\pi_L^{\text{sep}}/d\Delta V$ for any x^{pool}. Therefore, $d(\pi_L^{\text{pool}} - \pi_L^{\text{sep}})/d\Delta V > 0$, and thus $d\kappa/d\Delta V > 0$.

Prior belief ϕ.

We have

$$
\frac{d\kappa}{d\phi} = -\frac{\frac{d\left(\pi_L^{\text{pool}} - \pi_L^{\text{sep}}\right)}{d\phi}}{\frac{d\left(\pi_L^{\text{pool}} - \pi_L^{\text{sep}}\right)}{db}},
\tag{A.28}
$$

where $d(\pi_L^{\text{pool}} - \pi_L^{\text{sep}})/db = d\pi_L^{\text{pool}}/db < 0$, and $d(\pi_L^{\text{pool}} - \pi_L^{\text{sep}})/d\phi = d\pi_L^{\text{pool}}/d\phi$. For $x^{\text{pool}} = \bar{x}$,

$$
\begin{aligned}
\frac{d\pi_L^{\text{pool}}}{d\phi} = \frac{\partial \pi_L^{\text{pool}}}{\partial \phi} &= [1 - (1-\bar{x})F(\tilde{s})]\Delta V \\
&\quad - (1-\bar{x})^2 f(\tilde{s})\overline{V}(1-2\phi)\Delta V.
\end{aligned}
\tag{A.29}
$$

For $x^{\text{pool}} = \chi$, we have

$$\frac{d\pi_L^{\text{pool}}}{d\phi} = \frac{\partial\pi_L^{\text{pool}}}{\partial\phi} + \frac{\partial\pi_L^{\text{pool}}}{\partial x^e}\frac{d\chi}{d\phi}, \tag{A.30}$$

where

$$\frac{\partial\pi_L^{\text{pool}}}{\partial\phi} = [1-(1-\chi)F(\tilde{s})]\Delta V - (1-\chi)^2 f(\tilde{s})\overline{V}(1-2\phi)\Delta V; \tag{A.31}$$

$$\frac{\partial\pi_L^{\text{pool}}}{\partial x^e} = (1-\chi)\phi(1-\phi)\Delta V f(\tilde{s})\overline{V}; \tag{A.32}$$

$$\frac{d\chi}{d\phi} = -\frac{\frac{\partial G(\chi)}{\partial \Delta V}}{\frac{\partial G(\chi)}{\partial\chi}} = \frac{f(\tilde{s})\overline{V}(1-2\phi)(1-\chi)\Delta V + F(\tilde{s})\Delta V}{\phi(1-\phi)\Delta V f(\tilde{s})\overline{V} + c''(\chi)}. \tag{A.33}$$

Therefore,

$$\frac{d\pi_L^{\text{pool}}}{d\phi} = \Delta V - \frac{(1-\chi)\Delta V\big[F(\tilde{s}) + f(\tilde{s})\overline{V}(1-2\phi)(1-\chi)\big]c''(\chi)}{\phi(1-\phi)(V_H - V_L)f(\tilde{s})\overline{V} + c''(\chi)}. \tag{A.34}$$

Similar to the proof with regard to ΔV, it can be shown that $d\pi_L^{\text{pool}}/d\phi > 0$ for any x^{pool}. Therefore, $d\kappa/d\phi > 0$.

Proof of Proposition 5

i.a. In the pooling equilibrium, $CS = \int_0^{\tilde{s}}(\tilde{s} - s)f(s)ds$, which is increasing in $\tilde{s} = \phi(1-\phi)(1-x^{\text{pool}})\Delta V$ because $dCS/d\tilde{s} > 0$ (see Equation (A.16)). If the deceptiveness is at the boundary $x^{\text{pool}} = \bar{x}$, then $\tilde{s}$ is increasing in ΔV, and thus CS is increasing in ΔV. Otherwise, the deceptiveness is interior and $x^{\text{pool}} = \chi$. We have

$$\frac{d\chi}{d\Delta V} = -\frac{\frac{\partial G(\chi)}{\partial \Delta V}}{\frac{\partial G(\chi)}{\partial\chi}} = \frac{(1-\chi)\phi(1-\phi)f(\tilde{s})\overline{V}}{\phi(1-\phi)\Delta V f(\tilde{s})\overline{V} + c''(\chi)} > 0. \tag{A.35}$$

Therefore, a larger ΔV leads to a higher χ, which implies a higher $\tilde{s}$ because $G(\chi) = 0$ and because G is increasing in $\tilde{s}$ and decreasing in χ (see Equation (A.1)). Hence, CS is increasing in ΔV.

According to the proof of Proposition 3 (see part (iii)), social welfare can be written as $W = \overline{V} - (1-\phi)V_L(1 - x^{\text{pool}})F(\tilde{s}) - \int_0^{\tilde{s}} sf(s)ds - (1-\phi)c(x^{\text{pool}})$. We have

$$\begin{aligned}\frac{dW}{d\Delta V} &= (1-\phi)\big(F(\tilde{s})V_L - c'(x^{\text{pool}})\big)\frac{dx^{\text{pool}}}{d\Delta V} \\ &\quad - \big[(1-\phi)(1-x^{\text{pool}})V_L + \tilde{s}\big]f(\tilde{s})\frac{d\tilde{s}}{d\Delta V} \\ &\quad - \phi(1-\phi)(1-x^{\text{pool}})F(\tilde{s}),\end{aligned} \tag{A.36}$$

where $d\tilde{s}/d\Delta V > 0$ as shown. If $x^{\text{pool}} = \bar{x}$, then $dx^{\text{pool}}/d\Delta V = 0$, and thus $dW/d\Delta V < 0$. Otherwise, $x^{\text{pool}} = \chi$. This implies that $dx^{\text{pool}}/d\Delta V > 0$ (see Inequality (A.35)) and $c'(x^{\text{pool}}) = F(\tilde{s})\overline{V} + b > F(\tilde{s})V_L$ (the equality comes from Equation (A.1)). Therefore, $dW/d\Delta V < 0$. Therefore, social welfare is decreasing in ΔV.

i.b. In the separating equilibrium, $CS = \phi(V_H - p_H^{\text{sep}})$, where $p_H^{\text{sep}} = \min\{V_H,\ V_L + \min_x[c(x) + b(1-x)]\}$. Because $V_H = \overline{V} + (1-\phi)\Delta V$ and $V_L = \overline{V} - \phi\Delta V$, CS is (weakly) increasing in ΔV. Social welfare $W = \overline{V}$ stays the same.

ii.a. For $x^{\text{pool}} = \bar{x}$, $\tilde{s}$ is unimodal in ϕ, and so CS is unimodal in ϕ. We have $dW/d\phi = -[(1-\phi)(1-\bar{x})V_L + \tilde{s}]f(\tilde{s})\frac{d\tilde{s}}{d\phi} + \Delta V + V_L(1-\bar{x})F(\tilde{s}) + c(\bar{x})$, which is positive when ϕ is very large (i.e., close to 1) but can be negative when ϕ, ΔV, and $c(\bar{x})$ are small. For $x^{\text{pool}} = \chi$, both $\tilde{s}$ and CS is positive when ϕ is close to 0 and negative when ϕ is close to 1. We have $dW/d\phi = -[(1-\phi)(1-\chi)V_L + \tilde{s}]f(\tilde{s})\frac{d\tilde{s}}{d\phi} + \Delta V + V_L(1-\chi)F(\tilde{s}) + c(\chi) + (1-\phi)(V_L F(\tilde{s}) - c'(\chi))\frac{d\chi}{d\phi}$, which is positive when ϕ is very large (i.e., close to 1) but can be negative when ϕ, ΔV, $c(\chi)$, and $c'(\chi)$ are small. Therefore, CS and W are in general not monotonic in ϕ.

ii.b. In the separating equilibrium, $CS = \phi(V_H - p_H^{\text{sep}})$, which is increasing in ϕ because p_H^{sep} is invariant in ϕ. Social welfare $W = V_L + \phi\Delta V$, which is also increasing in ϕ.

Endnotes

[1] See an example on https://tinyurl.com/yys6enl8 (Fortune.com, retrieved from waybackmachine, August 21, 2018).

[2] See https://tinyurl.com/y2qy4q64 (chinadaily.com.cn, retrieved from waybackmachine, November 8, 2017).

[3] See Table 2 for the detail about the pooling equilibrium price, which is independent of the penalty for false claims.

[4] Both the product statement and the price serve as signals for product quality.

[5] See Rhodes and Wilson (2018) for the same assumption.

[6] CDF refers to the cumulative distribution function, and PDF refers to the probability density function.

[7] Otherwise, in equilibrium firms will not target at those consumers and the firms will charge a price higher than those consumers' willingness-to-pay. In this case, consumers with high verification cost would never consider a purchase. As will be shown later, the assumption ($\overline{F}(\cdot)$ is sufficiently large) also ensures that firms' pricing power remains unchanged in the presence of false statements, and we can exclude the change of pricing power as a reason for regulation's negative impact on consumer surplus, proposed by Rhodes and Wilson (2018). See Table 2 for the detail about the pooling equilibrium price, which is independent of the penalty for false claims.

[8] See an example on https://tinyurl.com/y2qwhkdn (BusinessInsider.com, accessed April 26, 2018).

[9] In Section 4, we will discuss an alternative formulation of penalty, where the regulator can detect false claims based on consumercomplaints.

[10] A *belief* refers to the probability that the firm is a high-quality firm.

[11] The equilibrium concept corresponds to the fulfilled expectation equilibrium (e.g., Katz and Shapiro 1985). Specifically, in our model consumers' decisions are based on the expected deceptiveness x^e, but the verification signal they receive hinges on the actual x chosen by the low-quality firm. We need to distinguish between x^e and x because, when the low-quality firm considers a possible deviation of x (unobserved by consumers), the expected deceptiveness x^e will remain unchanged.

[12] Note that our model does not consider certification agencies who can verify product statements for consumers (see Dranove and Jin 2010 for a review of the economic literature on certification).

[13] In any equilibrium, consumers do not verify an *l*-statement. Thus, in the figure the verification process is for the *h*-statement.

[14] Some of the variables in Table 1 have not been introduced yet and will be used later in the paper.

[15] According to the intuitive criterion, consumers engage in certain type of forward induction. Specifically, in our model if firm L is worse off under any possible belief by deviating to an off-equilibrium strategy

(p, w), then the out-of-equilibrium belief should be $\psi(p, w) = 1$. If (p, w) is dominated for any firm j under any belief, then neither firm would like to deviate to (p, w). In such case, it is not necessary to require $\psi(p, w) = 1$.

[16] The D1 criterion is also based on the idea of forward induction and is stronger than the intuitive criterion. The details about the D1 criterion are relegated to the appendix. Note that our results hold under the equilibrium selection rule, *Undefeated Equilibrium* (UE), proposed by Mailath et al. (1993) (see also Miklós-Thal and Zhang 2013, Jiang et al. 2014, and Subramanian and Rao 2015).

[17] If $p \le V_L$, all consumers purchase without verification. If $p > \psi(p, 1; 1)V_H + (1 - \psi(p, 1; 1))V_L$, no consumer is willing to purchase the product or verify the product claim.

[18] For example, it is unimodal in b for a quadratic cost $c(\cdot)$ if the verification cost follows a uniform distribution. See the proof of Proposition 3 in the appendix.

[19] The social welfare is at the maximal level for all $b > \kappa$.

[20] In the paper, a more positive prior belief means a larger ϕ.

[21] The expression of the threshold $\widehat{\kappa}$ can be found in the proof of Lemma 5.

[22] The (implicit) expression of $\widetilde{V}$ is given in the online appendix. When discussing the magnitude of $\overline{V}$ in the extension, we hold the quality difference ΔV constant. If $F(\phi(1-\phi)(V_H - V_L)) < 1/2$, then $\widetilde{V} = 0$ and the condition $\overline{V} > \widetilde{V}$ is satisfied automatically.

[23] See https://tinyurl.com/yyzb9eks (BusinessInsider.sg, retrieved from waybackmachine, July 10, 2019).

[24] See https://tinyurl.com/y4jqwjhg (Bloomberg.com, retrieved from waybackmachine, December 18, 2018).

[25] See https://tinyurl.com/y2qy4q64 (chinadaily.com.cn, retrieved from waybackmachine, November 8, 2017).

[26] See https://tinyurl.com/y3lfbabu (ASRCReviews.org, retrieved from waybackmachine, July 27, 2017).

[27] In this paper, uniqueness refers to the equilibrium strategy, not the out-of-equilibrium belief.

[28] According to the definition of the D1 criterion in Fudenberg and Tirole (1991), the condition is that firm H can benefit from a larger set of consumers' best responses associated with all out-of-equilibrium beliefs. In our paper, benefiting from a larger set of out-of-equilibrium beliefs is equivalent to benefiting from a larger set of consumers' best responses.

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