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A Theory of Combative Advertising

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In mature markets with competing firms, a common role for advertising is to shift consumer preferences towards the advertiser in a tug-of-war, with no effect on category demand. In this paper, we analyze the effect of such “combative” advertising on market power. We show that, depending on the nature of consumer response, combative advertising can reduce price competition to benefit competing firms. However, it can also lead to a procompetitive outcome where individual firms advertise to increase their own profitability, but collectively become worse off. This is because combative advertising can intensify price competition such that an “advertising war” leads to a “price war.” Similar to price competition, advertising competition can result in a prisoner’s dilemma where all competing firms make less profit even when the effect of each firm’s advertising is to enhance consumer preferences in its favor. Given such procompetitive effects, we further show that cost of combative advertising could be a blessing in disguise—higher unit cost of advertising resulting in lower equilibrium levels of advertising, leading to higher prices and profits. We conduct a laboratory experiment to investigate how combative advertising by competing brands influences consumer preferences. Our experimental analysis offers strong support for our conclusions.

Key words: advertising; persuasion; game theory; competitive strategy; prisoner’s dilemma; preference shifts

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1. Introduction

In some of the earliest writings on the role of advertising in influencing firm prices and profitability, the renowned economist Alfred Marshall (1919, p. 304) noted the following:

...the nation [America], which excels all others in the energy and inventive ability devoted to developing the efficiency of retail trade, is also the nation that pays the most dearly for the services of that trade.

He believed that part of the reason why one observed high prices in many markets was the advertising efforts of firms; and the fact that much of this advertising spending was not *constructive*, but *combative*. He defined *constructive* advertising as all advertising designed to draw the attention of people to the opportunities of buying or selling of which they may be willing to avail themselves; and *combative* advertising as the iterative claims made by a firm in an effort to identify itself with consumers without expanding the market. His contention was that combative advertising helped a firm increase its market power and support a high price.

Over the past century, researchers have proposed many theories on how advertising may affect a firm’s market power. Three main views have emerged: the informative, complementary, and persuasive views of advertising (Bagwell 2005). According to informative view, advertising works by increasing consumer awareness and reducing search costs. This results in an increase in price sensitivity and reduction in market power (the Stigler-Telser-Nelson school of thought: Stigler 1961, Telser 1964, Nelson 1975). Of course, informative advertising can also enhance a firm’s market power if it merely informs consumers of product differences and hence increases product differentiation (Meurer and Stahl 1994). The complementary view recognizes the importance of advertising in the consumption process by providing additional utility to consumers—such as creating a feeling of greater social *prestige* when the product is appropriately advertised, or by signalling high quality. Therefore, advertising is *good* or *bad* complementary to consumption. According to this view, advertising can increase a firm’s market power if it

enhances consumption utility, but need not always do so (Becker and Murphy 1993).

The view that is closest to capturing combative advertising described by Marshall is the persuasive view of advertising (the Kaldor-Bain-Comanor-Wilson school of thought: Kaldor 1950, Bain 1956, Comanor and Wilson 1967). This view supports Marshall's thesis that advertising can alter consumer tastes to create mostly spurious product differentiation. Ultimately, such advertising decreases consumer price sensitivity making demand less elastic, thus supporting higher prices and profits (Gasmi et al. 1992, Tremblay and Polasky 2002, Tremblay and Martins-Filho 2001, Bagwell 2005).

As combative advertising is often observed in mature markets for consumer goods such as detergents and soft drinks, according to this view, one would expect that as a firm increases its advertising expenditure, demand will shift in its favor. However, research in marketing does not always corroborate this view. Past research has shown that advertising can indeed have an impact on consumer's price sensitivity in the long term (Mela et al. 1997). While there is quite a bit of agreement that advertising in its various forms (rational, emotional, creative) helps shift consumer preferences towards the advertising firm (Batra et al. 1996), there is skepticism about advertising being of much help to firms in increasing short-term sales or profitability. Erickson (1985, 2003) suggests that when competing firms are equally matched (symmetric), combative advertising is harmful (albeit necessary) for firms as it does not provide any additional increase in market share or profits.¹ Studying disaggregate scanner data on detergents, Tellis and Weiss (1995) further the skepticism of advertising's positive sales impact by showing empirically that the effect of advertising on current sales is either weak or nonsignificant and appears to be present only when data are aggregated across consumers or over time. This implies that any sales effect of advertising or any gain in market power may be spurious, most likely due to data aggregation. Other researchers have presented a more mixed view. Kaul and Wittink (1995) reviewed 14 empirical studies on the effects of nonprice advertising² and concluded,

based on the majority results (9 out of 14 studies), that nonprice advertising reduces consumer price sensitivity (mostly for frequently purchased consumer goods). They cite studies that suggest nonprice advertising could increase price sensitivity particularly in the airline industry (Gatignon 1984) as well as for some categories of consumer goods such as aluminum foil and dog food (Kanetkar et al. 1992). Albion and Farris (1981) and Lodish et al. (1995) also have reported mixed evidence about the effect of advertising on sales, thus calling into question the prevalent belief that nonprice (combative) advertising is primarily anticompetitive in nature. Recent research by Pauwels and Hanssens (2007) reports that advertising can lead to an increase or decrease in market share in mature markets depending on the brand's current performance regime. We believe this raises two questions currently unaddressed in the literature:

- Is it possible that combative advertising, while altering consumer preferences to favor competing firms, could also lead to procompetitive effects?
- If the answer to the previous question is yes, then through what mechanism does combative advertising lead to both pro- and anticompetitive effects?

The answers to these two related questions have important implications for our understanding of the role that advertising plays in competition, and therefore for advertising research and practice. If combative advertising can only lead to higher prices, then firms can legitimately consider advertising as an investment and a firm's advertising decision is a matter of weighing the costs and benefits of additional advertising expenditures. However, if combative advertising can also intensify price competition, the decision becomes more complex as now a firm must decide whether or how much to advertise in a product category in order to benefit from advertising and advertising professionals must decide how to implement an advertising campaign to benefit their clients. Therefore, marketing researchers will face a new line of inquiry: how different consumers may respond to different advertising messages such that either procompetitive or anticompetitive effects may result in the marketplace for different types of products.

In this paper, we seek to provide some answers to these two questions by developing a game theoretical model. Past empirical research on reaction functions has looked at how a competing firm would react to advertising and pricing decisions of the other firms in the same market, and has uncovered a high degree of competitive interaction depending on the product category (Lambin et al. 1975, Steenkamp et al. 2005, Horvath et al. 2005, Bass et al. 2007). We examine the interactions of competing firms in advertising and pricing decisions by building a model that

¹ The key focus of the literature on dynamic effects of advertising is to understand how advertising affects market share by bringing over competitors' consumers. The more a firm spends on advertising, the more sales it stands to gain, but if both competing firms spend on advertising, then the efforts are negated. There is no proposed change to the preference structure of consumers, and firms do not make their pricing decisions. Dynamic models are primarily awareness driven, whereas in this paper we shall focus on a preference-driven model.

² In their nomenclature, they classify persuasive advertising as non-price advertising.

incorporates the preference-changing effects of combative advertising. In our model, advertising does not directly enhance a consumer's willingness to pay for the advertised product, and it does not alter the advertising firm's positioning in the product space. Rather, the advertising we model is purely persuasive in nature: "each firm tries to convince consumers that what they *really* want is its particular variety" (von der Fehr and Stevik 1998, p. 115). In other words, advertising changes the distribution of consumer ideal points.

Two previous studies, von der Fehr and Stevik (1998) and Bloch and Manceau (1999), have investigated this preference-transforming effect of advertising in a competitive context. von der Fehr and Stevik (1998) anchor their study in a Hotelling model where advertising by a firm moves consumers located along the Hotelling line uniformly toward the advertising firm. They conclude that in any competitive equilibrium, such advertising will not change the distribution of consumer ideal points and hence should not have any price or sales effect. They came to this conclusion by assuming that all consumers along the Hotelling line are unaffected by advertising whenever competing firms advertise at the same level. Bloch and Manceau (1999), however, suggest that persuasive advertisements may alter consumer preferences in a way that reduces or intensifies price competition. Their study is also anchored in a Hotelling model. According to this study, a firm's advertising can trigger more aggressive pricing by the competing (non-advertising) firm to the detriment of the advertising firm, if consumers already prefer the product of the advertising firm. In other words, persuasive advertising intensifies price competition only under two conditions. First, when consumer preferences are biased in favor of one of the two competing firms' products, the favored firm is the one that advertises. Second, the firm in consumers' disfavor cannot advertise and can only respond to the rival's advertising with price.³

Similar to von der Fehr and Stevik (1998) and Bloch and Manceau (1999), our analysis is anchored in the Hotelling model of a horizontally differentiated duopoly. Compared to von der Fehr and Stevik (1998), we offer a more general specification for the preference distribution and model combative advertising as causing a shift in consumer preference towards the advertising firm in a more heterogeneous way, consistent with empirical and experimental findings

from the existing literature (Zajonc 1968, Hoch and Ha 1986, Carpenter and Nakamoto 1989, D'Souza and Rao 1995, Tellis 2004). We also incorporate the commonly observed diminishing returns to advertising in the sales response function. Compared to Bloch and Manceau (1999), we conduct an equilibrium analysis with competing firms setting both prices and advertising levels simultaneously. This analysis allows us to identify the mechanism through which combative advertising may reduce or intensify price competition when all competing firms can advertise. Such a mechanism is absent in both studies.

In our model, advertising always exerts a force pulling consumers closer to that firm. It turns out that the effect of such a force can be anticompetitive (all competing firms charge higher prices) or procompetitive (all competing firms charge lower prices). We discuss how these pro- and anticompetitive effects of combative advertising come about and what mediates these different outcomes first in a parsimonious discrete model in §2. In §§3 and 4 we extend this to a more general model to study how firms' advertising intensities and consumer responsiveness to advertising in a market jointly determine the effect of combative advertising. Because of the procompetitive effect we have identified, we can shed some new light on the effects of ever escalating advertising costs in §5: the rising costs of advertising might be a blessing in disguise. We demonstrate the possibility that with higher unit costs of advertising, the equilibrium levels of advertising could be lower, resulting in higher profits for competing firms. As all our conclusions hinge on the procompetitive effect, we experimentally test the external validity of this procompetitive effect in §6. Before we present the general model, in the next section, we start with a simple discrete model to illustrate the forces at work when firms compete in advertising.⁴

2. Basic Mechanics of Combative Advertising

In the absence of advertising, assume that consumers are uniformly distributed along a Hotelling line of unit length $[0, 1]$, with Firm 1 located at 0 and Firm 2 at 1. This means that the density of consumer distribution is 1 everywhere along the line in the absence of any advertising in this market.⁵ Each consumer buys at most one unit of the product, and incurs a linear

³ Strickly speaking, Bloch and Manceau (1999) do not set up an equilibrium model for advertising decisions: their main focus is on investigating a firm's incentives to advertise in as general a way as possible. As they point out in the paper, "at this level of generality, we are unable to analyze the firms' choice of the level of advertising expenses and cannot capture the effects of advertising spending by both firms on the distribution of consumers" (p. 565).

⁴ We thank the anonymous area editor for suggesting the addition of this discrete analysis.

⁵ In some cases, firms may have loyal consumers. In our model, these consumers would translate to mass points at the two ends of the line. Having additional mass points will not affect the qualitative nature of our results. To keep our analysis simple, we focus our attention on the case of a uniform distribution.

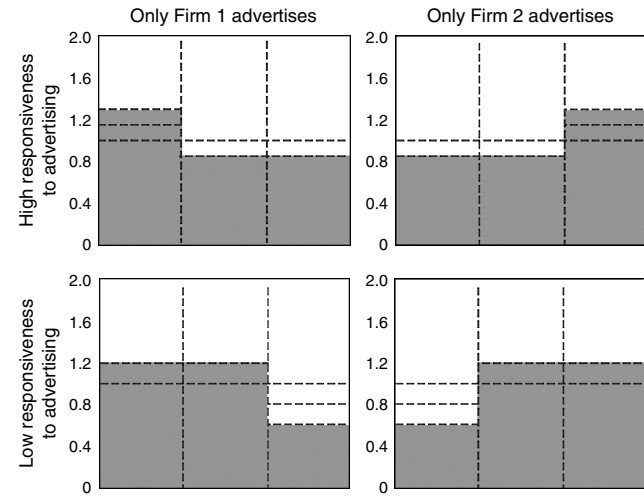
transportation cost. This transportation cost measures the disutility that a consumer suffers by consuming a nonideal product. We denote the unit transportation cost by t . The reservation price of each consumer, V , is assumed to be sufficiently large compared to t so that the market is always covered in equilibrium, with or without advertising. This allows us to assume away any possible market expansion effect due to advertising in this market. Let the price for firm i 's product be p_i . Thus, the utility for Firm 1's product for a consumer located at x will be $V - p_1 - tx$. Similarly, the utility for Firm 2's product for the same consumer would be $V - p_2 - t(1 - x)$.

In the past, the effect of persuasive advertising in a Hotelling model has been modeled as increasing either the reservation price V or the differentiation parameter t , or as shifting the distribution of consumer ideal points (von der Fehr and Stevik 1998, Tremblay and Polasky 2002). We take the latter modeling approach to better capture combative advertising "as a tug-of-war in which each firm attempts to attract consumers by molding their preferences to fit the characteristics of its product" (von der Fehr and Stevik 1998, p. 115). This means that the distribution of consumers along the Hotelling line depends on both firms' choices of advertising levels. For now, we assume that a firm can choose to advertise in this market with a fixed intensity k to shift consumers's ideal points closer to its location.

To incorporate heterogeneity in consumer responsiveness to advertising, we divide consumers in this market into three segments based on their preadvertising ideal points: consumers located in $[0, \frac{1}{3}]$ who prefer Firm 1, those in $[\frac{2}{3}, 1]$ who favor Firm 2, and those in $[\frac{1}{3}, \frac{2}{3}]$ who are ambivalent in terms of which firm they favor.

We distinguish two possible mechanisms through which advertising affects the distribution of consumer ideal points. In the first case, consumers are very responsive to advertising so that all consumers in the market except those who already prefer a firm's product are susceptible to the influence of advertising and may change their ideal points. Past research suggests that advertising is not particularly effective in changing the preference for consumers who have a strong preference to start with, as they are already convinced of buying from the preferred firm and it is harder to reinforce their preferences further in favor of the preferred firm (Winter 1973). Specifically, when only Firm 1 advertises, any consumer in $[\frac{1}{3}, \frac{2}{3}]$ and $[\frac{2}{3}, 1]$ relocates their ideal points uniformly to $[0, \frac{1}{3}]$ with probability vk such that the density of consumer distribution in $[\frac{1}{3}, \frac{2}{3}]$ and $[\frac{2}{3}, 1]$ each reduces by vk and that in $[0, \frac{1}{3}]$ increases correspondingly by $2vk$, where v is a constant and k is the level of advertising. Symmetrically, when only Firm 2 advertises, any

Figure 1 Consumer Response to Unilateral Advertising



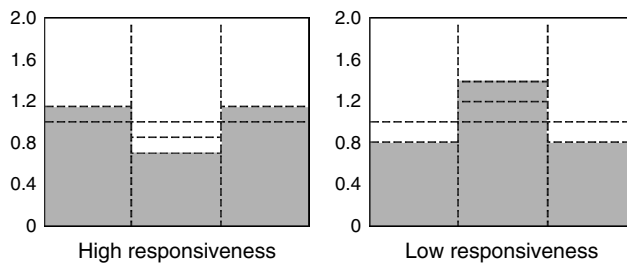
consumer in $[0, \frac{1}{3}]$ and $[\frac{1}{3}, \frac{2}{3}]$ relocates uniformly to $[\frac{2}{3}, 1]$ with probability vk and corresponding density changes. Thus, whenever a firm advertises alone, it succeeds in persuading more consumers to like its products.

In the second case, consumers are *not* very responsive to advertising and only those who like a firm's product the least are affected by the firm's advertising. Thus, when only Firm 1 advertises, any consumer in $[\frac{2}{3}, 1]$ relocates their ideal points uniformly to $[0, \frac{1}{3}]$ and $[\frac{1}{3}, \frac{2}{3}]$ with probability uk such that the density in $[\frac{2}{3}, 1]$ reduces by uk and that in $[0, \frac{1}{3}]$ and $[\frac{1}{3}, \frac{2}{3}]$ each increases by $\frac{1}{2}uk$ (u is also a constant). Symmetrically, when only Firm 2 advertises, any consumer in $[0, \frac{1}{3}]$ relocates uniformly to $[\frac{1}{3}, \frac{2}{3}]$ and $[\frac{2}{3}, 1]$ with probability uk and corresponding density changes. The effects of unilateral advertising are illustrated in Figure 1 for both cases.⁶

What happens if both firms choose to advertise? At a given level of consumer responsiveness to advertising, the effect of combative advertising on each of the three segments of the market will, of course, depend on how many consumers are attracted into the segment and how many are pulled out of it. For instance, when consumers are very responsive to advertising, at the same level of advertising intensity k for both firms, Firm 1's advertising will increase consumer distribution density in $[0, \frac{1}{3}]$ by $2vk$, but Firm 2's advertising will reduce the density in that segment by vk so that the net density increase in that segment is vk . This is also the case in $[\frac{2}{3}, 1]$. In the middle region $[\frac{1}{3}, \frac{2}{3}]$, each firm's advertising will reduce the density by vk so that combative advertising will reduce the density in $[\frac{1}{3}, \frac{2}{3}]$ by $2vk$. Thus, combative advertising will reshape the distribution of consumer ideal points

⁶ ($k = 1$, $u = 0.15$, $v = 0.2$).

Figure 2 Consumer Response When Both Firms Advertise



in this market from a uniform distribution to a nonuniform distribution illustrated on the left in Figure 2. Similarly, for the case where consumers are not very responsive to advertising, we have a different density function as illustrated on the right in Figure 2.

As readers might have already realized, depending on how responsive consumers are to advertising, combative advertising can have quite different competitive implications depending on its impact on the distribution of consumer ideal points. In the first case, consumers are very responsive and combative advertising generates more *partisan customers*, the customers who strongly prefer a firm's product, and reduces the number of *indifferent customers*, the customers who do not have a strong preference for either firm's product. In that case, when firms compete on price, they will each set a higher price, relative to the case of a uniform consumer distribution, to capitalize on stronger consumer preferences or less elastic demand. In this case, combative advertising supports a higher price, consistent with what Marshall and other economists have pointed out.

In the second case, consumers are not very responsive to advertising and combative advertising actually pulls more consumers to the middle—a region also referred to as the combat zone (Syam et al. 2005). This reduces the number of partisan customers and increases the number of indifferent customers. In this case, combative advertising results in higher price elasticity. Therefore, firms will price lower than what they would in the absence of advertising as they compete for a more sizable body of indifferent customers. It is this case that previous studies (Gasmi et al. 1992, Tremblay and Polasky 2002, Tremblay and Martins-Filho 2001, Bagwell 2005) have overlooked.

3. A Model for Combative Advertising

What the previous simple model establishes is the fact that depending on consumer responsiveness to advertising, it is possible for combative advertising to cause more intense price competition in a market by increasing the number of indifferent customers and reducing the number of partisan customers. It thus raises two

critical questions. First, could combative advertising intensify price competition when firms choose their advertising intensity simultaneously? Second, is the phenomenon an artifact of the discrete segments of consumers that we have specified as a way to model heterogeneity in consumer responsiveness to advertising? We now address these questions by developing a more general model of combative advertising. This more general model allows us to develop a better understanding of combative advertising and provides more insights about its competitive implications.

We make two modifications to the illustrative model in the previous section. First, we abandon the discrete segmentation of consumers and specify a continuous density function f for this market. This general specification allows all consumers along the Hotelling line to respond to combative advertising. Second, we formally specify the game that two competing firms play and allow firms to set their respective advertising intensity simultaneously. Firms in our model play a two-stage game. In the first stage, firms simultaneously decide advertising levels, k_i . We normalize k_i to be within $[0, 1]$, where $k_i = 0$ denotes no advertising and $k_i = 1$ denotes maximum advertising. We assume that advertising costs are quadratic, $\frac{1}{2}ck_i^2$ (Tirole 1988, Bagwell 2005), where c is a constant. In the second stage, after observing the other firm's advertising level, each firm simultaneously decides its price p_i and then consumers optimally make their purchases. For simplicity, we assume that both firms' marginal costs of production are zero.

Various factors, such as brand loyalty, product attributes, brand perceptions, etc., can potentially influence consumer preferences for products in a given market. In this paper, we focus the effects of combative advertising on shaping consumer preferences, given a baseline distribution of consumer preferences. We illustrate these effects for the case when baseline preferences are distributed uniformly.

We formulate the density function $f(k_1, k_2, x)$ starting from a general specification. With the density function pinned down, we derive the advertising response functions. With these response functions, we show that combative advertising can generate anti-competitive effects in equilibrium. However, the same forces that give rise to the anticompetitive effects of advertising can, under other conditions, generate pro-competitive effects.

In Appendix 1, we show that we can parameterize the density function $f(k_1, k_2, x)$ relying only on the regularity conditions and the known effects of advertising to generate a tractable but still fairly general model that lends itself to game-theoretic analysis. This modeling approach differs from von der Fehr and Stevik (1998) and Bloch and Manceau (1999) in that we do not parameterize the density function in

an ad hoc fashion. In von der Fehr and Stevik (1998), the distribution function of consumer ideal points is uniform and if competing firms advertise at the same level, none of the consumers along the Hotelling line are affected so that the distribution function does not change. Bloch and Manceau (1999) allow for more general density functions, however, allow only one firm to advertise, and their model is not sufficiently parameterized. Our parameterized density function is given by⁷

$$f(k_1, k_2, x) = v_1(k_1, x) + v_2(k_2, x), \quad (1)$$

such that we have

$$v_1(k_1, x) = \frac{1}{2} + k_1[(12a - 1) - 6(8a - 1)x + 6(6a - 1)x^2] - 12bk_1^2[1 - 4x + 3x^2], \quad (2)$$

$$v_2(k_2, x) = \frac{1}{2} + k_2[(12a - 1) - 6(8a - 1)(1 - x) + 6(6a - 1)(1 - x)^2] - 12bk_2^2[1 - 4(1 - x) + 3(1 - x)^2], \quad (3)$$

where $\frac{1}{8} < a < \frac{5}{12}$ and $\max\{0, a - \frac{1}{4}\} < b < \min\{a - \frac{1}{8}, a/2 - \frac{1}{24}\}$. Intuitively, v_i captures the effects of firm i 's advertising. We thus view the density at any given x , i.e., $f(k_1, k_2, x)$, as resulting from two opposing advertising forces: consumers at x may be pulled away from x and at the same time consumers at other locations may be pulled to x when subject to combative advertising. This derived density function has only two parameters, a and b . To understand what they represent, we first derive the mean location of consumers:

$$m(k_1, k_2) = \int_0^1 xf(k_1, k_2, x) dx = \frac{1}{2} - a(k_1 - k_2) + b(k_1 - k_2)(k_1 + k_2). \quad (4)$$

It is straightforward to show that the density function satisfies the following conditions:

$$\begin{aligned} \frac{\partial m(k_1, k_2)}{\partial k_1} < 0, \quad \frac{\partial^2 m(k_1, k_2)}{\partial k_1^2} > 0; \quad \text{and} \\ \frac{\partial m(k_1, k_2)}{\partial k_2} > 0, \quad \frac{\partial^2 m(k_1, k_2)}{\partial k_2^2} < 0. \end{aligned} \quad (5)$$

This implies that, given the rival's advertising level, the average consumer location moves toward a firm as the firm increases its advertising intensity, but the rate of such movements diminishes as the firm's advertising level increases. In other words, a firm's increased advertising should, on average, make the firm's product closer to being the ideal product for the consumers in the market (the first-order effect),

given any level of the rival's advertising, but such preference-shifting effect diminishes with higher levels of advertising (the second-order effect). Therefore, holding the prices to be the same in the market, the sales response function for a firm's advertising is increasing and concave—an attractive feature of our parameterized density function. Many studies have shown that advertising exposure has a favorable effect on consumer preferences toward the advertised product, and that the consumer response to advertising is concave (Winter 1973, Simon and Arndt 1980, Vakratsas and Ambler 1999). This concave nature of advertising response functions has been reported by Little (1979), Albion and Farris (1981), and also endorsed by Lilien et al. (1992, p. 267), who note that “while a good deal of discussion and modeling concerns S-shaped response, most of the empirical evidence supports concavity.” Even when advertising response is S-shaped, typically, the equilibrium advertising levels belong to the upper concave part of the S-curve, making the assumption of concavity more reasonable (also see Chen and Xie 2007).

In Equation (4), we see how these two effects come about. The term $(k_1 - k_2)$ captures the relative advertising intensity, while $(k_1 + k_2)$ captures the aggregate advertising intensity in the market. As a firm (say Firm 1) increases its advertising intensity, consumers prefer Firm 1's product so that the mean consumer location shifts toward Firm 1 by the distance of a per unit of advertising intensity. This is the first-order effect of advertising in shifting consumer preference. However, this mean shift is tempered by the second-order effect of diminishing returns to advertising, captured in the term $b(k_1 + k_2)$. This second-order effect depends on the aggregate advertising intensity in the marketplace, as expected. Thus, larger a means that the consumers are, on average, more responsive to advertising and larger b means that the consumers are more easily satiated with advertising appeals. If one firm has an advantage in terms of relative advertising intensity, it stands to gain more when the aggregate advertising intensity is low rather than high. If two competing firms have the same advertising level, then the mean location of consumers, or the mean consumer preference, does not change and is fixed at $\frac{1}{2}$. This is rather expected in the context of combative advertising when two competing firms are equally matched.

To understand how the density function may shift with advertising, we compute its variance:

$$\begin{aligned} \sigma^2(k_1, k_2) = & \frac{1}{12} + (k_1 + k_2) \left(\frac{6a - 1}{30} - \frac{b}{10}(k_1 + k_2) \right) \\ & - (k_1 - k_2)^2 \left(\frac{b}{10} + (a - b(k_1 + k_2))^2 \right). \end{aligned}$$

It is important to point out that compared to the variance of a uniform distribution of consumers

⁷ We expect our qualitative conclusions to remain unchanged when f is nonseparable in advertising. For more details, please see Appendix 1.

($\sigma_{\text{uniform}}^2 = \frac{1}{12}$), the variance of this density function f can increase as well as decrease, depending on the relative levels of a , b , and k_i . We shall return to the particular conditions under which this may happen later in the analysis.

In this market, the two competing firms engage in a tug-of-war in advertising. If only one firm has the ability to advertise, that firm will pull consumers over, gaining market share (Bloch and Manceau 1999). If both firms advertise to the same extent and yet the distribution of consumer preferences is not altered as a result, the tug-of-war is at a dead draw everywhere along the Hotelling line. In that case, as pointed out by von der Fehr and Stevik (1998), combative advertising does not generate any price effect. Of course, when consumers in the market are all subject to two opposing forces, it is perhaps unrealistic to expect that the distribution of consumer preferences does not change at all. With this more general and empirically grounded consumer preference distribution function, we can conduct an explicit equilibrium analysis next to investigate how the consumer distribution and hence price competition may change due to combative advertising.

Let d be the location of the marginal consumer who is indifferent between the two firms: $d = (t - p_1 + p_2)/2t$. The demand functions for the two firms are $D_1 = \int_0^d f dx = F(d)$; and $D_2 = \int_d^1 f dx = 1 - F(d)$. Given demand,⁸ the profit functions for the two firms are $\pi_1 = p_1 D_1 - \frac{1}{2} c k_1^2$ and $\pi_2 = p_2 D_2 - \frac{1}{2} c k_2^2$. We now proceed to derive the equilibrium using backwards induction.

In the second stage, both firms simultaneously set their prices for any given pair of advertising levels (k_1, k_2) chosen in the previous stage. In equilibrium, both firms' prices must satisfy the following two first-order conditions:

$$\frac{\partial \pi_1(p_1, p_2, k_1, k_2)}{\partial p_1} = 0; \quad \frac{\partial \pi_2(p_1, p_2, k_1, k_2)}{\partial p_2} = 0. \quad (6)$$

As we show in Appendix 2, although the equilibrium prices $p_1^*(k_1, k_2)$ and $p_2^*(k_1, k_2)$ cannot be solved explicitly without any further simplifying assumptions, they exist and are unique. In other words, the second-stage equilibrium always exists and is unique for any pair of (k_1, k_2) . When $k_1 = k_2$, we can write out the second-stage price equilibrium explicitly below:⁹

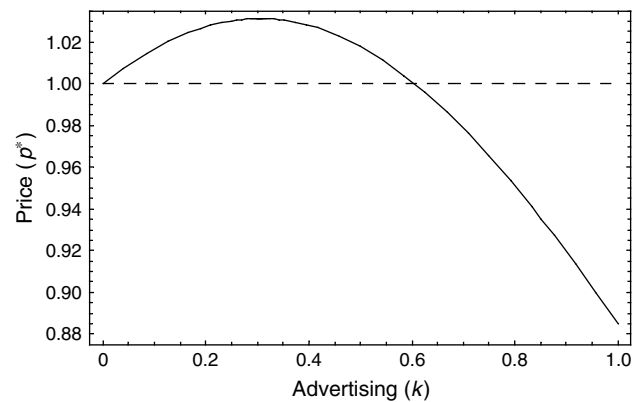
$$p^*|_{k_1=k_2=k} = \frac{t}{1+k-6ak+6bk^2}, \quad (7)$$

$$\pi^*|_{k_1=k_2=k} = \frac{t}{2(1+k-6ak+6bk^2)} - \frac{1}{2}ck^2.$$

⁸ One can easily verify that the sales response to advertising in this market is concave for both firms.

⁹ We assume symmetry to allow for the derivation of closed form solutions for the pricing subgame and, subsequently, the advertising game. Future research could investigate alternative models that would permit a relaxation of this assumption.

Figure 3 Effect of Combative Advertising on the Equilibrium Price



The above expressions reveal the impact of two equally matched firms on price competition. When $k = 0$, the above equations reduce to the familiar Hotelling (1929) results. Surprisingly, however, the relationship between advertising and price is not monotonic. Even more surprisingly, the market price may be lower because of combative advertising. In Figure 3, we illustrate the optimal price in the second stage as a function of advertising.¹⁰

PROPOSITION 1. *When competing firms are equally matched in advertising, equilibrium prices can be increasing or decreasing with the level of advertising in the market, dependent on the level of advertising ($\partial p^*/\partial k > 0$ when $k < (6a - 1)/12b$). More importantly, combative advertising can intensify or moderate price competition in a market, relative to the case of no advertising.*

Given that empirical evidence in the literature largely suggests that manufacturer prices are increasing in advertising (Steiner 1973, Farris and Reibstein 1979, Tellis 2004), the anticompetitive effect of advertising in Proposition 1 is perhaps not so surprising. However, the procompetitive effect in Proposition 1 is. At high levels of advertising ($k > (6a - 1)/12b$, which corresponds to $\partial \sigma^2/\partial k|_{k_1=k_2=k} < 0$), an increase in advertising can lead to a decrease, rather than an increase, in equilibrium prices. Indeed, if the advertising level is sufficiently high ($k > (6a - 1)/6b$, which also corresponds to $\sigma^2|_{k_1=k_2=k} < \frac{1}{12}$), the market price with advertising falls below the price without any advertising.

How does this procompetitive effect come about? Equation (7) offers some clues. We can see that the equilibrium price increases as the advertising response becomes steeper (a increases) and decreases as the rate of diminishing returns becomes stronger (b increases), in addition to responding to the level of advertising. Further clues are offered by changes

¹⁰ $t = 1$, $a = 0.2$, $b = 0.055$.

in the variance of the consumer density function. Observe that $\partial \sigma^2 / \partial k|_{k_1=k_2=k} > 0$ when $k < (6a - 1)/12b$. Thus, advertising affects prices via changes in the distribution of consumers in the market. When advertising levels are low (high), as advertising increases (decreases), the variance of the consumer density function increases (decreases). In other words, the shape of the advertising response function can have significant competitive implications. This does not come as a surprise, of course, given the illustrative model we have discussed previously.

Before we discuss how higher levels of advertising may lead to lower prices, we first show that competing firms may indeed optimally choose a level of advertising that would lead to a lower market price. We can do this by solving for the first-stage equilibrium.

4. Competitive Effects of Combative Advertising

In the first stage, both firms set their advertising level, anticipating how their choice may affect their subsequent pricing decisions. Then, in the first-stage equilibrium, the following two first-order conditions must be satisfied:

$$\begin{aligned} & \frac{\partial \pi_1(p_1^*(k_1, k_2), p_2^*(k_1, k_2), k_1, k_2)}{\partial k_1} \\ &= \frac{\partial \pi_1}{\partial k_1} + \frac{\partial \pi_1}{\partial p_1} \frac{\partial p_1^*}{\partial k_1} + \frac{\partial \pi_1}{\partial p_2} \frac{\partial p_2^*}{\partial k_1} = 0, \\ & \frac{\partial \pi_2(p_1^*(k_1, k_2), p_2^*(k_1, k_2), k_1, k_2)}{\partial k_2} \\ &= \frac{\partial \pi_2}{\partial k_2} + \frac{\partial \pi_2}{\partial p_1} \frac{\partial p_1^*}{\partial k_2} + \frac{\partial \pi_2}{\partial p_2} \frac{\partial p_2^*}{\partial k_2} = 0. \end{aligned} \quad (8)$$

Note that $(\partial \pi_i / \partial p_i)(\partial p_i^* / \partial k_i) = 0$, by the envelope theorem. It is fairly tricky to solve for the equilibrium advertising levels based on the implicit expressions of $p_1^*(k_1, k_2)$ and $p_2^*(k_1, k_2)$ as defined by Equation (6). We first note that in the second-stage equilibrium, we must have for any $k_i \in [0, 1]$, where $i = 1, 2$:

$$\frac{\partial \pi_1(p_1^*, p_2^*, k_1, k_2)}{\partial p_1} = 0, \quad \frac{\partial \pi_2(p_1^*, p_2^*, k_1, k_2)}{\partial p_2} = 0. \quad (9)$$

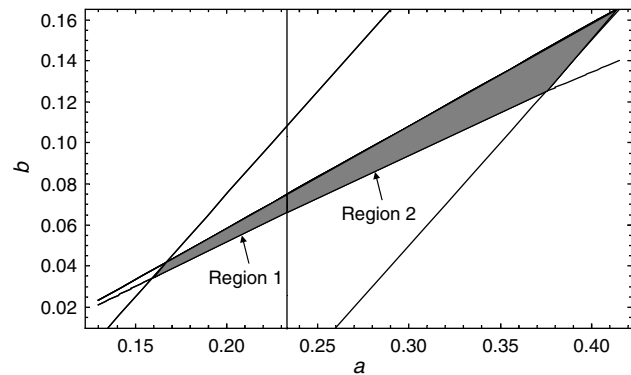
By differentiating these two identities, we have

$$\begin{aligned} & \frac{\partial^2 \pi_1}{\partial p_1 \partial k_1} + \frac{\partial \pi_1^2}{\partial p_1^2} \frac{\partial p_1^*}{\partial k_1} + \frac{\partial \pi_1^2}{\partial p_1 \partial p_2} \frac{\partial p_2^*}{\partial k_1} = 0, \\ & \frac{\partial^2 \pi_1}{\partial p_1 \partial k_2} + \frac{\partial \pi_1^2}{\partial p_1^2} \frac{\partial p_1^*}{\partial k_2} + \frac{\partial \pi_1^2}{\partial p_1 \partial p_2} \frac{\partial p_2^*}{\partial k_2} = 0, \\ & \frac{\partial^2 \pi_2}{\partial p_2 \partial k_1} + \frac{\partial \pi_2^2}{\partial p_1 \partial p_2} \frac{\partial p_1^*}{\partial k_1} + \frac{\partial \pi_2^2}{\partial p_2^2} \frac{\partial p_2^*}{\partial k_1} = 0, \\ & \frac{\partial^2 \pi_2}{\partial p_2 \partial k_2} + \frac{\partial \pi_2^2}{\partial p_1 \partial p_2} \frac{\partial p_1^*}{\partial k_2} + \frac{\partial \pi_2^2}{\partial p_2^2} \frac{\partial p_2^*}{\partial k_2} = 0. \end{aligned} \quad (10)$$

To obtain the symmetric equilibrium, we note that all second-order derivatives in the above system of equations, i.e., $\partial^2 \pi_i / \partial p_i \partial k_i$, $\partial^2 \pi_i / \partial p_i \partial k_j$, $\partial \pi_i^2 / \partial p_i^2$, and $\partial \pi_i^2 / \partial p_i \partial p_j$, as well as the first-order derivatives in Equation (8), i.e., $\partial \pi_i / \partial p_i$ and $\partial \pi_i / \partial p_j$, can be obtained directly from differentiating the profit expressions and then substituting in $p^*|_{k_1=k_2=k}$ from Equation (7). This means that we can solve $\partial p_i^* / \partial k_i$ and $\partial p_i^* / \partial k_j$ from Equation (10) and substitute them into Equation (8). By imposing the symmetry condition $k_1 = k_2 = k$ in the first stage, the two independent first-order conditions in Equation (8) will be reduced to one, leading to a unique symmetric solution for k^* , the candidate equilibrium we are looking for. In the case of $c = 0$, we obtain a closed-form solution, which is given in Appendix 2. In Figure 4, we illustrate the parameter space where we have $k^* \in [0, 1]$ for the case of $c = 0$. In the case of $c > 0$, we solve numerically for the equilibrium for all feasible parameters. In both cases, we numerically verify that the proposed equilibrium is indeed the equilibrium of the game, as neither firm has any incentive to deviate. For now, we will focus on the first case, as this is the simpler case that would give us all the qualitative conclusions. We will pick up the second case in §5.

Our analysis shows, as expected, that a firm's optimal advertising level k^* is increasing in a . This suggests that as consumers are more responsive to firms' advertising, firms will optimally choose to increase their advertising level. Also as expected, k^* is decreasing in b , suggesting that if the advertising response exhibits higher diminishing returns, firms will optimally choose to do less advertising. With this equilibrium value of k^* , we now assess the consequences of combative advertising on equilibrium pricing and profitability. The comparison of profits and prices with and without combative advertising is indicated in Figure 4. Firms' prices and profits fall in the vertically shaded region (Region 1), and they rise in the horizontally shaded region (Region 2).

Figure 4 Combative Advertising and Equilibrium Profitability



In equilibrium, the variance of the consumer density function depends on the responsiveness parameters a , b , and k^* is given by¹¹

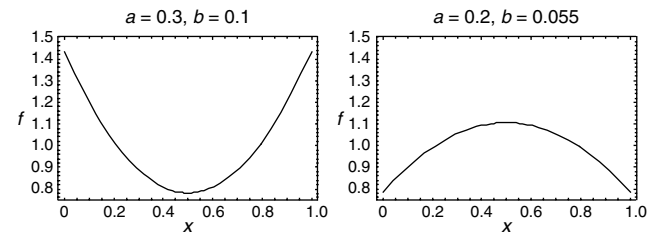
$$\sigma^{*2} = \frac{1}{12} + \frac{6a-1}{15}k^* - \frac{2}{5}bk^{*2}.$$

PROPOSITION 2. *Under combative advertising, competing firms' prices and profits are higher as compared to the case of no advertising if consumers in the market are sufficiently responsive to advertising ($\sigma^{*2} > \frac{1}{12}$, in Region 2). However, prices and profits are lower when consumers are not sufficiently responsive ($\sigma^{*2} < \frac{1}{12}$, in Region 1).*

Proposition 2, based on the first-order effect, suggests that whether or not a firm can benefit from combative advertising will depend on whether consumers in the market are sufficiently impressionable. The conclusions in the proposition are expected in light of the simple model we have discussed in §2. The driving force behind Proposition 2 is similar to that in the discrete model. Combative advertising determines the equilibrium distribution of consumers along the Hotelling line. As illustrated in Figure 5, when consumers are sufficiently responsive to advertising, the variance of the consumer density function increases.¹² Thus, combative advertising creates more partisan customers, with more consumers located toward the two ends as compared to a market without advertising, and thus increases product differentiation in the minds of consumers. Increased product differentiation, in turn, leads to higher prices and profits. When consumers are not sufficiently responsive, the variance of the consumer density function decreases.¹³ Thus, combative advertising creates more indifferent customers, manifested by a net accumulation of consumers in the middle and fewer consumers located closer to firms, reducing product differentiation. Reduced differentiation encourages more price competition and lowers the price and profit for competing firms. The main difference between the current model and the discrete model is that we do not need to restrict the pattern of consumer responses to advertising along the Hotelling line, except for imposing some mild conditions at the level of aggregate advertising response functions, which are supported by past empirical research. We now have in equilibrium a continuous, endogenous distribution density function f , which is convex in the former case and concave in the latter case.

While the profit implications of Proposition 2 follow directly from price implications, the price implications themselves are new and thought provoking.

Figure 5 Effect of Combative Advertising on the Distribution of Consumer Preferences



Most importantly, Proposition 2 seems consistent with what we frequently observe in the marketplace. As competing firms combat, engaging in extensive advertising campaigns touting the virtues of their own product, one of two things can happen. In the first instance, consumers are bombarded with so much advertising that they become partisan customers. In that case, it would take a lot of convincing for these consumers to consider purchasing a product from a rival firm.¹⁴ In the second instance, consumers receive a good dose of advertising from both firms, see merits in both firms' products, and therefore feel more indifferent about buying from either firm. It is this latter possibility—the creation of indifferent customers—that the previous literature on advertising overlooks. Our model shows that the critical moderating factor for these two outcomes is how responsive consumers are to advertising in a market.

It is important to note here that the concave distribution function of consumer preferences in Figure 5 is an endogenous outcome of the equilibrium strategies and is not assumed a priori. Given $a = 0.2$ and $b = 0.055$, the distribution function is concave only when the advertising intensity in the market is larger than 0.606061, as can be inferred from Equation (7). When the advertising intensity is less than that cutoff point, the distribution of consumer preferences in the market is convex, as shown in Figure 6. However, competing firms are incentivized in this market to escalate their advertising intensities above and beyond that cutoff point and the equilibrium level of advertising intensity for both firms is 0.946128, resulting in the concave distribution function in Figure 5. What this means is that *even* in advertising, competing firms can be caught in a prisoner's dilemma situation where firms escalate their own level of advertising intensity, motivated by self-interest, and end up becoming worse off collectively because advertising competition leads to price competition.

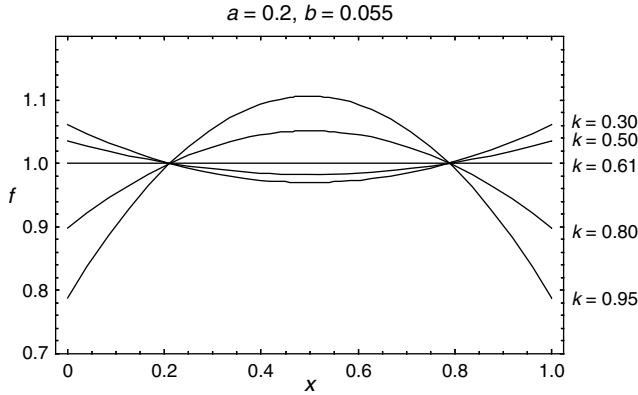
¹¹ k^* , in turn, depends on a and b , as shown in Appendix 2.

¹² For instance, $a = 0.3$, $b = 0.1 \Rightarrow \sigma^{*2} = 0.0978 > 1/12$.

¹³ For instance, $a = 0.2$, $b = 0.055 \Rightarrow \sigma^{*2} = 0.0763 < 1/12$.

¹⁴ The case that best illustrates this possibility is, perhaps, from the political marketplace where competing candidates use media-based political campaigns to garner more votes. By and large, such combative political persuasion "mainly reinforces voters' preexisting partisan loyalties" (Iyengar and Simon 2000, p. 150).

Figure 6 Variation in Preference Distribution with Advertising



5. The Role of Advertising Costs

In their study of informative advertising, Grossman and Shapiro (1984) found that an increase in advertising costs could have two effects: a direct increase in total advertising costs resulting in a reduction in profits; and also a reduction in the degree of competition as measured by demand elasticities, which results in higher profits. Their explanation was that advertising improves information available to consumers and reduces profits. With increased costs, the amount of advertising decreases, resulting in a reduction in the information available to consumers, as a consequence allowing firms to charge higher prices.

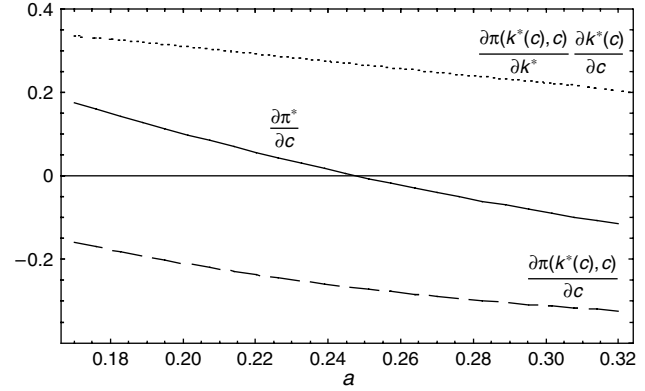
In our analysis of combative advertising, an increase or decrease in advertising does not have any effect on the amount of information available to consumers. Hence, the above reasoning is not applicable to markets where consumers are already informed about the available products. Do the same relationships between costs and firms' prices and profits hold in the context of combative advertising?

To answer this question, we note that for any $c > 0$, we can no longer write down an explicit expression for k^* as a function of c ; however, we can numerically analyze this case. We find, as expected, that the equilibrium level of advertising decreases as the unit cost of advertising increases ($\partial k^*/\partial c < 0$). But what does this mean in terms of equilibrium profit levels? Note that a firm's equilibrium profit can be written as $\pi^* = \pi(k^*(c), c)$. Therefore, we have

$$\frac{d\pi^*}{dc} = \frac{\partial \pi(k^*(c), c)}{\partial k^*} \frac{\partial k^*(c)}{\partial c} + \frac{\partial \pi(k^*(c), c)}{\partial c}. \quad (11)$$

The second term in Equation (11) is the direct effect of the advertising cost on a firm's profitability, which is always negative. The first term captures the indirect effect of advertising cost on a firm's profitability through advertising. As mentioned before, $\partial k^*(c)/\partial c$ in the first term is negative. Furthermore, it can be shown that $\partial \pi(k^*(c), c)/\partial k^*$ is also negative. This is

Figure 7 Effect of Advertising Costs



because firms always advertise too much to maximize their profits, a feature of combative advertising. Thus, the first term, or the indirect effect, is always positive. This implies that a firm's profitability must decrease (increase) with the costs of advertising c , i.e., $d\pi^*/dc < 0$ ($d\pi^*/dc > 0$), if the direct effect is larger (smaller) than the indirect effect. The relative sizes of these two effects depend, in turn, on consumer responsiveness to advertising. The direct effect is larger (smaller) than the indirect effect, as we show in Figure 7, when consumers in the market are sufficiently responsive (unresponsive) so that firms are already (not) engaging in a high level of advertising.

PROPOSITION 3. *With combative advertising, as the unit cost of advertising increases, equilibrium profit decreases when consumers are sufficiently responsive to advertising, and equilibrium profit increases when consumers are not sufficiently responsive to advertising.*

Proposition 3 thus suggests that the intuition we have gained about the effects of advertising costs in the context of informative advertising does not necessarily carry over to combative advertising. Higher advertising costs can indeed hurt the bottom line of competing firms, as much anecdotal evidence seems to suggest. In fact, as shown in Figure 7, firms that tend to get hurt by higher advertising cost are the firms in markets where the consumers are sufficiently responsive to advertising, and the firms that tend to benefit from higher advertising costs are the firms in markets where consumers are sufficiently unresponsive to advertising. This implication of our model is testable with suitable data.¹⁵

¹⁵ Understanding how the variation in profits with costs relates to variation in prices with costs could further facilitate conducting an empirical test. Since our modeling assumptions imply a fixed-size market and symmetric market share, expected variation in profits would be positively correlated with the expected variation in prices. Thus, we expect that as the unit cost of advertising increases, equilibrium prices would decrease if consumers are sufficiently responsive to advertising, and equilibrium prices would increase

6. Experimental Analysis

Past experimental research on consumer reactions to advertising messages suggests that advertising can have a dramatic effect on consumer perceptions of a product, especially when the product experience is of an ambiguous nature (Hoch and Ha 1986). In our modeling context, this change in perception could correspond to one of three things: a change in reservation price, or in travel costs, or in the distance of the consumer's ideal point from the product. These claims are further supported by the work of Carpenter and Nakamoto (1989), who show that a firm can gain pioneering advantage by shifting consumer preferences toward itself—either by shifting the taste distribution towards the firm's position, and/or by influencing attribute weights used by consumers to evaluate products. Additional support is found in D'Souza and Rao (1995) who show that in the context of mature product categories, advertising repetition can have a positive effect on relative brand preference. Tellis (2004) reports that loyals respond to advertising for their preferred brand more quickly at low levels of advertising than nonloyals, whereas high levels of advertising is likely to gain the attention of and positively influence nonloyals more so than loyal consumers of the brand. The shift in ideal points that we propose as a basic premise of our model is also consistent with the widely documented mere exposure effect (Zajonc 1968), which suggests that the mere act of repetitive exposure of a stimulus can lead to a preference for it, even if consumers do not remember the exposure.

Our analysis so far has demonstrated that if combative advertising results in more indifferent consumers, it can result in lower prices and profits. While it is well documented that combative advertising can influence consumer preferences in favor of the firm, to the best of our knowledge, there are no studies that examine whether combative advertising can indeed result in more indifferent consumers. Therefore, we conducted experiments to investigate the effects of combative advertising on consumer preferences. Our objective in this study is not to test the validity of our model's predictions but to demonstrate that combative advertising can result in more indifferent consumers, a key driver behind our pricing implications.

6.1. Description of the Study

We limited our attention to a few product categories with two equally matched competitors actively

engaged in combative advertising to sway consumer preferences and with a high likelihood for us to observe the phenomenon of advertising generating more indifferent customers. Although many product categories can fit this description, our choice was further limited by the availability of current TV commercials, and also whether these product categories were relevant to our experimental subjects—undergraduate students at a major East Coast business school. The product categories and the brands chosen were as follows: Credit cards (Visa and MasterCard), Courier services (FedEx and UPS), Batteries (Duracell and Energizer), Toothpastes (Colgate and Aquafresh), and Cars (BMW and Audi).¹⁶

The study was conducted in a state-of-the-art behavioral lab. Two hundred seventy-two subjects participated in the study. The subject entered the lab, was greeted by the lab administrator, and was asked to fill out a questionnaire to report preferences for each of the two brands in five product categories by allocating 100 points between each brand pairs. The responses were collected. Then, subjects (164) were exposed to advertising stimuli (nonprice commercials) of the two brands in each of the five categories. The order in which the advertisements were shown was rotated across categories and also within each category.¹⁷ After viewing the TV commercials for a product category, the subjects in the exposure group were asked to report their preferences for the two brands again by allocating 100 points to the two brands—more to the one they preferred. The process was repeated for all five product categories. The subjects in the control group (108) were asked to perform the same task, but we replaced the commercials shown to subjects in the treatment condition with the same number of commercials not related to any of the product categories under study.

6.2. Analysis and Results

The summary statistics are reported in Tables 1 and 2.

To analyze the effect of combative advertising on consumer preferences, we first placed all subjects on a Hotelling line, bounded by $[0, 100]$, on the basis of their reported preferences prior to being exposed to the stimuli by (arbitrarily) locating the two competing firms at the two ends of the line. For example, we located FedEx at 0 and UPS at 100. If a subject assigned 60 points to FedEx and 40 points to UPS prior to the advertising exposure, we located the subject on the Hotelling line at 40, indicating that the subject prefers FedEx to UPS. Once all subjects were thus located on the Hotelling line, we grouped both

if consumers were not sufficiently responsive to advertising. Since changes in prices and costs are relatively easier to monitor in a market than changes in profits and costs, these pricing implications of our cost analysis would be amenable to empirical testing in future research.

¹⁶ Students were familiar with the brands chosen.

¹⁷ The stimuli used in the study are available from the corresponding author.

Table 1 Summary Statistics for the Control Group ($n = 108$)

	Courier services [0 = Fdx, 100 = UPS]	Toothpastes [0 = Aqua, 100 = Colg]	Credit cards [0 = MC, 100 = Visa]	Cars [0 = Audi, 100 = BMW]	Batteries [0 = Dura, 100 = Energ]
Mean before	44.68	64.69	64.54	55.33	49.12
Mean after	47.07	64.31	62.3	54.82	50.38
Median before	50	65	60	50.5	50
Median after	50	60	55	50	50
St. dev. before	18.46	22.91	23.39	20.58	17.9
St. dev. after	19.07	22.79	22.71	19.37	17.92

Table 2 Summary Statistics for the Exposure Group ($n = 164$)

	Courier services [0 = Fdx, 100 = UPS]	Toothpastes [0 = Aqua, 100 = Colg]	Credit cards [0 = MC, 100 = Visa]	Cars [0 = Audi, 100 = BMW]	Batteries [0 = Dura, 100 = Energ]
Mean before	46.92	60.82	61.4	63.26	49.33
Mean after	51.26	61.18	54.4	58.23	39.27
Median before	50	60	50	70	50
Median after	50	60	50	60	40
St. dev. before	16.58	23.51	20.8	20.26	17.58
St. dev. after	17.05	21	17.34	16.75	18.9

Table 3 Analysis of the Shift in Conditional Means

<i>t</i> -test: Two-sample with unequal variances											
Location		Exposure group				Control group				<i>t</i> -stat	<i>p</i> -value
		Mean bef.	Mean after	Mean shift	No. of obs	Mean bef.	Mean after	Mean shift	No. of obs		
0	FedEx	37.84	48.68	10.85	97	36.27	38.81	2.54	67	4.07	<0.0001
100	UPS	60.07	55.00	−5.08	67	58.41	60.59	2.17	41	−2.65	0.0095
0	Aquafresh	34.43	48.21	13.77	53	37.83	41.03	3.21	29	2.75	0.0075
100	Colgate	73.42	67.37	−6.05	111	74.56	72.85	−1.71	79	−2.38	0.0184
0	MasterCard	43.06	47.50	4.44	54	41.03	40.17	−0.86	29	1.86	0.0660
100	Visa	70.42	57.80	−12.62	110	73.16	70.42	−2.75	79	−4.63	<0.0001
0	Audi	37.38	48.71	11.33	42	35.26	38.95	3.68	38	2.90	0.0050
100	BMW	72.17	61.50	−10.67	122	66.23	63.44	−2.79	70	−4.71	<0.0001
0	Duracell	38.81	34.82	−3.99	84	38.33	39.91	1.57	54	−2.54	0.0126
100	Energizer	60.38	43.94	−16.44	80	59.91	60.85	0.94	54	−7.53	<0.0001

experimental and control group subjects into two subgroups: those who preferred one firm and those the other. The subjects assigning exactly 50 points were randomly assigned to these two subgroups.¹⁸ We then computed the pre- and postadvertising preferences for each of these two subgroups.

If advertising generates more indifferent consumers, we would expect the conditional means of both these subgroups in the treatment condition to move closer to 50 and such movements should be statistically significant relative to the movements displayed by the corresponding subjects in the control group. If advertising generates more partisan consumers, we expect that the conditional means to move

away from 50 in a statistically significant way relative to the control group. The results are reported in Table 3.

Results in Table 3 suggest that for three of the five product categories (courier services, toothpaste, and cars), the conditional means move closer to 50, and this change is significant at the 5% level relative to the change in the control group. For credit cards, it is also in the same direction, but the results are statistically significant at the 10% level ($p = 0.066$). For the fifth product category (batteries), the conditional means move toward Duracell, implying a more effective Duracell advertising. These results do not depend on whether the two groups were formed based on mean or median.

We further analyze the data by focusing only on the within-subject information from the exposure group. In this case, we need to rule out the regression-to-the-

¹⁸ The reader may recognize that this is consistent with assigning the consumers to a Hotelling line.

mean bias (RTM). As we show in Appendix 3, if experimental subjects tend to give a small number after giving a large number or vice versa, the conditional means as we have calculated will move closer to the indifference point, regardless of whether advertising has any effect. To rule this out as a possible explanation, we conducted two additional statistical tests.

The first test we conduct is an F -test on the variance of pre- and postadvertising data. The basic idea is as follows: the variance in the data consists of two parts—the true variance in the data (σ_B or σ_A) and the error variance (σ_0) that causes the RTM drift (Morrison 1973). As the latter variance should be the same for both the pre- and postadvertising data, it is easy to see that any significant difference in the measured total variance between pre- ($\sigma_B + \sigma_0$) and postadvertising data series ($\sigma_A + \sigma_0$) must imply that σ_B and σ_A are significantly different. The difference must be due to advertising exposures, not due to any RTM effect. However, when the difference between ($\sigma_B + \sigma_0$) and ($\sigma_A + \sigma_0$) is not significant by the F -test, the difference between σ_B and σ_A can still be significant. In other words, the F -test here is an overly restrictive test for our purposes. Table 4 shows that in the categories of cars and credit cards, subjects become significantly more indifferent due to advertising exposures, as the postadvertising variance is significantly smaller than the preadvertising variance (at the 5% level). In the toothpaste category, subjects become more indifferent at the 10% significance level. In the battery category, the variance increases to indicate more partisan customers; however, the difference is not statistically significant.

Another test that incorporates a clearer indifference point is the distance dispersion test. With this test, we can also statistically eliminate the preference bias in favor of a brand and control for the possible differences in the commercials' potency. To conduct this test, we first compute the mean for both the pre- and postadvertising preferences. Then, we subtract the respective mean from each pre- and postadvertising observations to de-mean the two data sets. In other words, we statistically center both the pre- and postadvertising preference distributions at zero.

Table 4 Analysis of Variance in the Pre- and Postexposure Preferences, Using the F -Test

	Courier services	Toothpastes	Credit cards	Cars	Batteries
Preadvertising variance	275.0	552.8	432.5	410.6	309.1
Postadvertising variance	290.6	441.1	302.0	280.5	357.4
F -statistic	0.9463	1.2532	1.4321	1.4638	0.8649
p -value	0.372	0.076	0.011	0.008	0.188

Table 5 Analysis of Pre- and Postexposure Distances

	Courier services	Toothpastes	Credit cards	Cars	Batteries
Difference in pre- and postexposure distances	−0.528257	1.996356	3.805399	3.349048	−3.21163
t -statistic	−0.492479	1.728975	3.664976	3.302581	−2.867211
p -value (2-tailed)	0.623043	0.085707	0.000334	0.001177	0.004688

Then, for each subject i , we compute the distance of his or her preference from zero in absolute value, respectively, for both pre- and postadvertising cases. The difference in the pre- and the postadvertising distances then measures the change in a subject's preference: the subject's preference moves closer to the center when the difference is positive and further away from the center when the difference is negative. We compute such a difference measure for all experimental subjects and find the mean difference for the population. As we show in Appendix 4, this procedure helps mitigate the RTM effect. We expect that the mean difference for the population is not zero due to the effect of the advertisements. Table 5 reports the results of this analysis.

Consistent with the analysis contained in Tables 3 and 4, Table 5 shows that the dispersion in preferences for the two product categories of credit cards and cars shrinks even after controlling for any mean shift. The shrinkage is strongly significant statistically, implying that subjects become more indifferent between two brands in those two product categories after the ad exposure. For toothpaste, the change in subjects' preferences is in the same direction of indifference, but it is statistically significant only at the 10% level as in the F -test. However, with this test, the dispersion of subjects' preferences increases for the battery category with a strong statistical significance. Overall, this test suggests that combative advertisements can produce indifferent customers.

6.3. Pricing Implications

Although preferences may change due to advertisements in a statistically significant way, one may still question whether the change is economically significant so as to modify a brand's pricing behavior as predicted by our model. We address this question in the context of our experiment by fitting continuous cumulative distribution functions to the preference data within a product category. We estimate two such functions for each category, using the pre- and postadvertising data, respectively. Of course, we expect these functions to be highly nonlinear. To accommodate nonlinearity, we specify each function as a Taylor series to the fifth power to achieve the proper good-

ness of fit.¹⁹ We subsequently assume a common reservation price as in the Hotelling model and set the unit transportation cost t (a scaling factor) to one and marginal costs for both brands to zero. This allows us to numerically solve for the equilibrium in the pre- and postadvertising pricing games and see if the postadvertising price competition intensifies in a product category, as predicted by our model.

As illustrated in Figure 8, the preadvertising price equilibrium for the category of cars is 0.68 for BMW and 0.48 for Audi and the postadvertising equilibrium prices are 0.52 and 0.41, respectively. In other words, price competition intensifies in the postadvertising market, because combative advertisements have generated more indifferent customers.

Correspondingly, the equilibrium profit for BMW decreases from 0.3956 to 0.2924 and for Audi from 0.2007 to 0.1795. In Table 6, we report the equilibrium analysis for the rest of the product categories. Note that the postadvertising equilibrium prices go down for two additional categories—credit cards and toothpastes, and they increase for the batteries category, consistent with the preference shifts in Table 4. Interestingly, even if equilibrium prices decrease (increase) for both competing brands, not every brand is worse off (better off), as shown in the case of MasterCard (Energizer), because of preference-induced market share changes.²⁰

Overall, our experimental analysis demonstrates that combative advertisements can indeed generate indifferent customers and lead to intensified price competition.

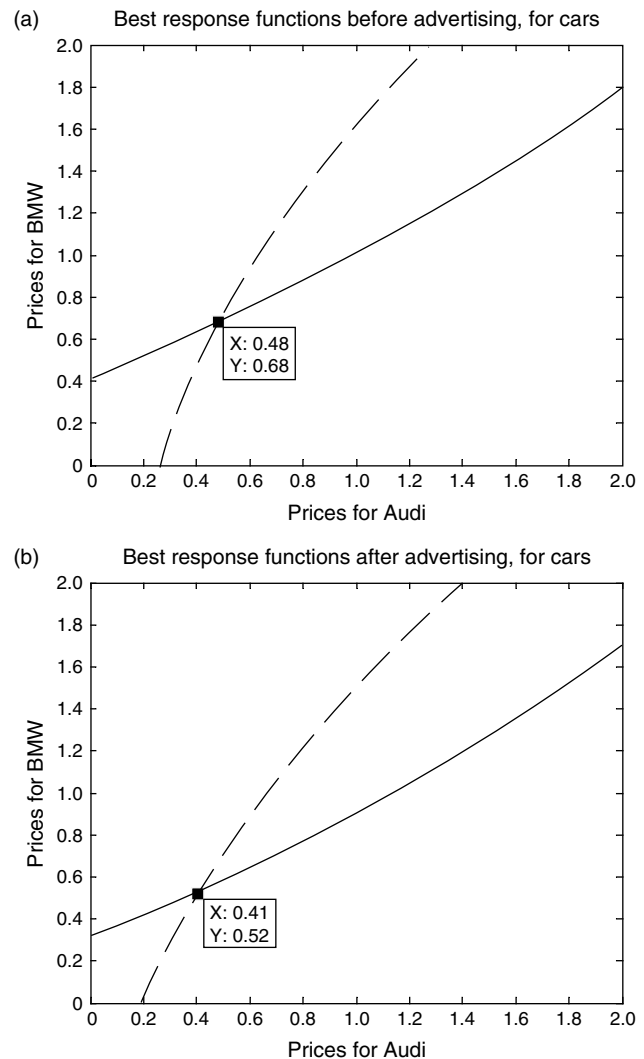
7. Conclusions and Future Research

Our analysis suggests that although from an individual firm's perspective combative advertising can change consumer preferences in its favor, through competitive interactions in the marketplace, such advertising may not favor the firm. We show that combative advertising can generate either more partisan customers or more indifferent customers. In the former case, more consumers will have a stronger preference for a particular firm's product and hence

¹⁹ We need to fit a continuous *cdf* to guarantee the existence of price equilibrium. Otherwise, a brand's pricing strategy is continuous, but its demand is not, such that a pricing equilibrium may not exist. In fitting the *cdf*, we ensure $F(0) = 0$ and $F(100) = 1$. For all 10 estimated functions, the minimum R^2 we get is 0.81 and the highest is 0.96, with the average being 0.92.

²⁰ While our analysis in this section confirms the possibility of intensified/reduced price competition as a consequence of combative advertising, it would be worthwhile to empirically observe this intensified/reduced price competition in test markets, given changes in advertising levels and differences in consumer responsiveness. Such an analysis is outside our current scope, but we believe it would be an interesting avenue for future research.

Figure 8 (a) Preadvertising Price Equilibrium; (b) Postadvertising Price Equilibrium



reduce price competition. In the latter case, more consumers feel indifferent about buying from either firm so as to intensify price competition through reduced product differentiation. Therefore, with advertising, as with pricing, firms can be caught in a prisoner's dilemma where all firms become worse off because of their independent decisions to advertise in order to gain advantages over competition. Our analysis shows that what ultimately mediates the two different outcomes is how responsive consumers are to advertising. In markets where consumers are sufficiently responsive, heavy combative advertising will create more partisan customers to the benefit of competing firms. In contrast, in markets where consumers are not sufficiently responsive, combative advertising might simply create more indifferent customers. As a result, price competition in the market could intensify.

Looking at it from a slightly different perspective, consumers in the market are subject to opposing

Table 6 Equilibrium Analysis

	Visa	MasterCard	Engergizer	Duracell	Colgate	Aquafresh	UPS	FedEx
Price before	0.52	0.42	0.36	0.38	0.74	0.6	0.4	0.44
Price after	0.42	0.39	0.42	0.58	0.68	0.53	0.42	0.41
Profit before	0.2889	0.1866	0.1796	0.1904	0.4083	0.2689	0.1852	0.2363
Profit after	0.2181	0.1875	0.1750	0.3384	0.3823	0.2321	0.2142	0.2009

forces when firms engage in combative advertising. Our analysis reveals that depending on the strengths of these opposing forces, consumers are either pulled away from the middle (more partisan customers) or pulled towards the middle (more indifferent customers). To the best of our knowledge, our model is the first in the literature that studies the mechanics of combative advertising, analyzing the full range of competitive implications of the preference-changing effects of advertising. The conclusion about indifferent customers is surprising but plausible. Indeed, our experimental analysis shows that the outcome we have uncovered is not just a theoretical curiosity—it can happen with real people watching real commercials and evaluating real products.

Our analysis, informed by empirical research on advertising, also provides a number of insights that may guide practice and future research on advertising. First, if combative advertising can have anti-competitive effects in one case, but procompetitive effects in another, it implies that advertising decisions are more complex. While making these decisions, it is important for us to ascertain the mediators for these two kinds of effects. Our model suggests that the mediators are most likely related to the consumer responsiveness to advertising, which in turn is related perhaps to consumer characteristics, product characteristics, and message effectiveness. Future research, most suitably experimental research, can further explore those mediators. Second, by recognizing the phenomenon of combative advertising generating partisan and indifferent customers, we find an intuitive, measurable way of gauging its competitive effects in practice. As the creation of partisan or indifferent consumers is a phenomenon that could be observed even in the presence of more than two firms in the market, we are inclined to believe that the findings from our model are easily generalizable to product categories where oligopolistic competition is more prevalent as compared to duopolistic competition. Third, the competitive implications of advertising may differ in different markets depending on how responsive consumers are to advertising. For instance, lumping different product categories together to measure competitive effects may not be advisable, as it may lead to cancelling out of opposing effects. Additional research investigating the movement of each individual utility maximizing consumer may be able

to shed more light on this issue. Fourth, in the context of combative advertising, lower advertising costs may or may not be a blessing for a firm. In markets where advertising creates indifferent customers, firms may want to embrace, rather than fight, the increase in advertising costs. Finally, advertising is not always wasteful at the expense of consumers, as it can encourage more intense price competition.

Our model can be extended in many ways. For instance, it may be worthwhile to allow for other marketing decision variables.²¹ Also, it might be desirable to have a more detailed model at the micro-level. Another possibility is to introduce dynamic advertising competition between firms over time. In addition, future empirical or experimental research can further look into the magnitude of effects of advertising in shifting consumer preferences. While in our experiment the effects are quite pronounced, in real-life settings, they may not be. We hope that the first step we have taken here shows the fruitfulness of research on combative advertising.

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Appendix 1. Derivation of the Continuous Distribution Function

To parameterize the density function $f(k_1, k_2, x)$, we shall rely on the regularity conditions and the known effects of advertising to generate a tractable but still fairly general model that lends itself to game-theoretic analysis. As a benchmark, we assume that in the absence of any advertising, consumers are uniformly distributed in the market, so that we have

$$f(0, 0, x) = 1. \quad (12)$$

²¹ One could argue that in place of our functional specification for f , a traditional beta distribution (as typically used in marketing to specify consumer preferences) could be used to generate the two distributions illustrated in Figure 5. Although that is a possibility, the beta distribution was not amenable to closed-form solutions for problem setup considered in this paper.

Consumers in this market are fully informed about the existence of the two firms, their locations, and prices; hence the effect of a firm's advertising is not to inform consumers about its existence, location, or price, but to engage in a combat with the competitor in shifting consumer preferences towards itself. Thus, under combative advertising, some consumers at location x may move towards firm i due to its advertising k_i . In other words, they consider the firm's product closer to their ideal product. Consequently, the distribution of consumers along the unit line will change from the uniform distribution to $f(k_1, k_2, x)$, with the mean location of consumers in this market being given by

$$m(k_1, k_2) = \int_0^1 x f(k_1, k_2, x) dx. \quad (13)$$

As a probability density function, $f(k_1, k_2, x)$ must satisfy the following regularity condition, for all k_1 and k_2 :

$$\int_0^1 f(k_1, k_2, x) dx = 1. \quad (14)$$

We further impose symmetry on the density function to assume away any firm-specific factors and to focus on the competitive effect of combative advertising. Algebraically, this means the following condition must hold:

$$f(k_1, k_2, x) = f(k_2, k_1, 1 - x). \quad (15)$$

This condition rules out the mechanism identified in Bloch and Manceau (1999) as the driving force for competitive effects. To introduce the preference-shifting effect of a firm's advertising, we assume

$$\frac{\partial f(k_1, k_2, 0)}{\partial k_1} > 0, \quad \frac{\partial f(k_1, k_2, 1)}{\partial k_2} > 0. \quad (16)$$

Equation (16) shows the boundary conditions that ensure that at any given level of advertising by its rival, a firm can, through more of its own advertising, increase the number of customers who would consider its product as the ideal. Hence the density of consumers at $x = 0$ ($x = 1$) increases as Firm 1 (Firm 2) unilaterally increases its advertising intensity. We further impose a boundary condition below to help us to parameterize the density function:

$$\begin{aligned} \frac{\partial f(k_1, 0, 1)}{\partial k_1} < 0, \quad \frac{\partial f(0, k_2, 0)}{\partial k_2} < 0, \\ \frac{\partial^2 f(k_1, 0, 1)}{\partial^2 k_1} = \frac{\partial^2 f(0, k_2, 0)}{\partial^2 k_2} = 0. \end{aligned} \quad (17)$$

The conditions imposed in Equation (17) are essentially technical in nature and they ensure that the density of consumers at the extreme location ($x = 0$ or $x = 1$) who consider the respective firm's product as ideal will decrease with the rival's advertising level, and do so at a constant rate if the rival firm does not do any advertising at all. To scale such preference-shifting effects, we assume

$$f(1, 0, 1) = 0, \quad f(0, 1, 0) = 0. \quad (18)$$

Equation (18) implies that the effect of a firm's maximum advertising is so strong that it can shift the preference of all

consumers farthest away from it toward itself, if the other firm does not advertise at all.²² Finally, we assume

$$\frac{\partial f(k_1, 0, x)}{\partial x} \leq 0, \quad \frac{\partial f(0, k_2, x)}{\partial x} \geq 0. \quad (19)$$

These two conditions simply state that if the rival firm does not advertise, a firm's advertising should skew the consumer distribution in its favor or should have favorable effects on consumer preferences. Together with (18), (19) ensures that the density of consumer distribution can never become negative.

To determine a specific density function that satisfies all conditions (12)–(19), let us assume that

$$f(k_1, k_2, x) = v_1(k_1, x) + v_2(k_2, x); \quad (20)$$

i.e., $f(k_1, k_2, x)$ is of an additive form in terms of the impact of k_1 and k_2 on the shifts in the distribution of preferences.²³ Intuitively, v_i captures the effects of firm i 's advertising. We thus view the density at any given x , i.e., $f(k_1, k_2, x)$, as resulting from two opposing advertising forces: consumers at x may be pulled away from x and at the same time consumers at other locations may be pulled to x when subject to combative advertising. We let v_1 take on the following, general quadratic form²⁴ with respect to both x and k_1 :

$$\begin{aligned} v_1(k_1, x) = & a_0 + a_1 k_1 + a_2 k_1^2 + a_3 x + a_4 k_1 x + a_5 k_1^2 x + a_6 x^2 \\ & + a_7 k_1 x^2 + a_8 k_1^2 x^2. \end{aligned} \quad (21)$$

We can analogously specify $v_2(k_2, x)$. Now it is straightforward to use conditions (12)–(19) to derive the following parameterized functions:²⁵

$$\begin{aligned} v_1(k_1, x) = & \frac{1}{2} + k_1[(12a - 1) - 6(8a - 1)x + 6(6a - 1)x^2] \\ & - 12bk_1^2[1 - 4x + 3x^2], \\ v_2(k_2, x) = & \frac{1}{2} + k_2[(12a - 1) - 6(8a - 1)(1 - x) \\ & + 6(6a - 1)(1 - x)^2] \\ & - 12bk_2^2[1 - 4(1 - x) + 3(1 - x)^2]. \end{aligned}$$

The mean and variance for f are

$$\begin{aligned} \text{Mean} = & \frac{1}{2} - a(k_1 - k_2) + b(k_1 - k_2)(k_1 + k_2); \\ \text{Variance} = & \frac{1}{12} + (k_1 + k_2) \left(\frac{6a - 1}{30} - \frac{b}{10}(k_1 + k_2) \right) \\ & - (k_1 - k_2)^2 \left(\frac{b}{10} + (a - b(k_1 + k_2))^2 \right). \end{aligned}$$

²² Note that as this assumption is only one of the many possible ways to scale the effect of advertising, we can easily relax this assumption by using any $0 \leq f(k_1, 0, 1) = f(0, k_2, 0) < 1$ without changing our main results.

²³ We expect our qualitative conclusions to remain unchanged when f is nonseparable in advertising. If $\partial^2 f / \partial k_1 \partial k_2 < 0$, the equilibrium advertising level would be lower than the case when $\partial^2 f / \partial k_1 \partial k_2 = 0$. This is because the marginal benefit from advertising becomes smaller in a symmetric advertising equilibrium. Thus, a convex density shape is more likely. In contrast, if $\partial^2 f / \partial k_1 \partial k_2 > 0$, the equilibrium advertising level would be higher than the case when $\partial^2 f / \partial k_1 \partial k_2 = 0$. Here, a concave density shape is more likely.

²⁴ v_2 also takes on a similar form, given Equation (15).

²⁵ Where $a = (1 + a_1)/12$ and $b = -(a_2/12)$. For additional details on the intermediate steps in this derivation, please see the Technical Appendix, found at <http://mktsci.pubs.informs.org>.

Appendix 2. Establishing Existence and Uniqueness

We first establish the existence and uniqueness of the second stage pricing equilibrium. From the first-order conditions (§3), we have

$$\begin{aligned} \frac{\partial[p_1 F(d)]}{\partial p_1} &= 0, & \frac{\partial[p_2(1-F(d))]}{\partial p_2} &= 0 \\ \Rightarrow p_1^* &= \frac{2tF(d^*)}{f(d^*)}, & p_2^* &= \frac{2t[1-F(d^*)]}{f(d^*)}, \end{aligned}$$

where

$$\begin{aligned} d &= \frac{t - (p_1 - p_2)}{2t} & d^* &= \frac{t - (p_1^* - p_2^*)}{2t}, \\ \Rightarrow p_1^* - p_2^* &= t - 2td^* = \frac{2tF(d^*)}{f(d^*)} - \frac{2t[1-F(d^*)]}{f(d^*)}, \\ \Rightarrow g(d^*) &= 4F(d^*) - (1 - 2d^*)f(d^*) - 2 = 0. \end{aligned}$$

We now show that $g(d)$ has a unique solution for $d \in [0, 1]$. First, note that $g(0) < 0$ and $g(1) > 0$; hence a solution exists for $d \in (0, 1)$. Given our specification of $f(x)$, $F(d)$ is of the form $F(d) = c_1 d + c_2 d^2 + c_3 d^3$. With this functional form for $F(d)$, solving for $g(d) = 0$ (in *Mathematica*) gives us three roots of which two are imaginary. Therefore, $g(d) = 0$ has a unique solution on $d \in (0, 1)$. We can show that p_i^* solved from the unique solution for d^* from $g(d^*) = 0$ constructs the unique equilibrium. p_i^* is unique because d^* is unique. We now argue that p_i^* is an equilibrium solution. Let us consider the case where it is not the equilibrium solution: then p_i^* is a local minimum. From the uniqueness of p_i^* , p_i^* must then be the unique local minimum, and this implies that the profit at $p_i = 0$ is higher than the profit at p_i^* given the same p_j^* . However, we know $p_i D$ is 0 at $p_i = 0$ but $p_i^* D$ is positive. Hence, p_i^* cannot be a local minimum; thus it has to be a local maximum. The uniqueness of p_i^* then implies that p_i^* is also the unique global maximum. This completes the proof of the existence and uniqueness of the second-stage pricing equilibrium.

We establish uniqueness of the first-stage advertising equilibrium numerically by checking for deviations in the entire advertising strategy space followed by competitive price setting by both firms. Such a deviation is never profitable, establishing that the first-stage equilibrium exists and is unique.²⁶

The expression for k^* is

$$k^* = \frac{1}{1,152b^2} \left(\frac{64(2^{2/3})(9(1-3a)a + 45b - 1)b^2}{\beta_1} + 64(9a - 1)b - 32(2^{1/3})\beta_1 \right), \quad \text{where}$$

$$\beta_1 = \sqrt[3]{-27b^4 + 2(9a - 2)(9a - 1)b^3 + \beta_2},$$

$$\beta_2 = \sqrt{b^6(16(9(1-3a)a + 45b - 1)^3 + (54a(3a - 1) - 27b + 4)^2)}.$$

²⁶ We numerically check for the stability of both the second-stage and first-stage equilibrium. Specifically, the following two conditions: $(\partial^2 \pi_1 / \partial p_1^2)(\partial^2 \pi_2 / \partial p_2^2) - (\partial^2 \pi_1 / \partial p_1 \partial p_2)(\partial^2 \pi_2 / \partial p_2 \partial p_1) > 0$, and $(\partial^2 \pi_1 / \partial k_1^2)(\partial^2 \pi_2 / \partial k_2^2) - (\partial^2 \pi_1 / \partial k_1 \partial k_2)(\partial^2 \pi_2 / \partial k_2 \partial k_1) > 0$, are always satisfied for $1/8 < a < 5/12$, $\max\{0, a - 1/4\} < b < \min\{a - 1/8, a/2 - 1/24\}$, and $k^* \in [0, 1]$, the relevant parameter space examined in the paper.

Appendix 3. The RTM Effect

Assume the individual true mean as x_i^* and x_{1i} and x_{2i} are the values for individual i pre- and postadvertising. If the only effect presented is a regression to mean, x_i^* will be unchanged pre- and postadvertising. We can define $x_{1i} = x_i^* - e_i$ and $x_{2i} = x_i^* + e_i$, where e_i can be negative or positive with 0 mean and follows a symmetric distribution (e.g., a normal distribution).

For any symmetric distribution of preferences, its mean m is equal to its median so that there are an equal number of people on both sides of the mean. Also, this mean should not change pre- and postadvertising. The conditional mean change for those with preadvertising preference less than m is

$$\begin{aligned} \Delta m_c(\text{post} - \text{pre}) &= \left[\sum_{x_i^* - e_i \leq m} (x_i^* + e_i) - \sum_{x_i^* - e_i \leq m} (x_i^* - e_i) \right] / (n/2) \\ &= \left[2 \sum_{x_i^* - e_i \leq m} e_i \right] / (n/2) = 4 \left(\sum_{x_i^* - e_i \leq m} e_i \right) / n \\ &= 4 \left(\sum_{e_i \geq x_i^* - m} e_i \right) / n > 0, \end{aligned}$$

where n is the number of people. In the above proof, we used the fact $\sum_{e_i \geq -\infty} e_i = 0$ so that $\sum_{e_i \geq x_i^* - m} e_i > 0$ given the symmetry in the distribution of e_i . Similarly, we can show that the conditional mean change for those with preadvertising preference larger than m is less than 0. Thus, together they imply that conditional means can move to the middle purely due to the RTM effect.

Appendix 4. Overcoming the RTM Effect

Assume that an individual's true mean is x_i^* and x_{1i} and x_{2i} are the values for individual i pre- and postadvertising. If only the RTM effect is present, x_i^* will be unchanged before and after advertising exposures. We can define $x_{2i} = x_i^* - e_i$ and $x_{1i} = x_i^* + e_i$, where e_i can be negative or positive with 0 mean and follows a symmetric distribution (e.g., normal distribution). For any symmetric distribution of preferences, its mean is equal to its median so that the number of people on both sides are equal. Also, this mean does not change pre- and postadvertising. The total preadvertising distance to this mean m across all people is

$$\begin{aligned} D_{\text{before}} &= \sum_i |x_i^* - e_i - m| \\ &= \sum_{x_i^* - e_i > m} (x_i^* - e_i - m) + \sum_{x_i^* - e_i < m} (m - x_i^* + e_i) \\ &= \sum_{x_i^* - e_i > m} -e_i + \sum_{x_i^* - e_i < m} e_i + \sum_{x_i^* - e_i > m} x_i^* + \sum_{x_i^* - e_i < m} -x_i^*. \end{aligned}$$

In the above proof, we used the fact that the preference distribution is symmetric so that the number of people with $x_i^* - e_i > m$ is equal to the number of people with $x_i^* - e_i < m$. Similarly, the total postadvertising distance to this mean m across all people is

$$\begin{aligned} D_{\text{after}} &= \sum_i |x_i^* + e_i - m| = \sum_{x_i^* + e_i > m} (x_i^* + e_i - m) \\ &\quad + \sum_{x_i^* + e_i < m} (m - x_i^* - e_i) \\ &= \sum_{x_i^* + e_i > m} e_i + \sum_{x_i^* + e_i < m} -e_i + \sum_{x_i^* + e_i > m} x_i^* + \sum_{x_i^* + e_i < m} -x_i^*. \end{aligned}$$

Therefore, $D_{\text{before}} - D_{\text{after}}$ is

$$\begin{aligned}
& \left(\sum_{x_i^* - e_i > m} -e_i + \sum_{x_i^* - e_i < m} e_i - \sum_{x_i^* + e_i > m} e_i - \sum_{x_i^* + e_i < m} -e_i \right) \\
& + \left(\sum_{x_i^* - e_i > m} x_i^* - \sum_{x_i^* - e_i < m} x_i^* - \sum_{x_i^* + e_i > m} x_i^* + \sum_{x_i^* + e_i < m} x_i^* \right) \\
& = \left(- \sum_{x_i^* - e_i > m, x_i^* + e_i > m} 2e_i + \sum_{x_i^* - e_i < m, x_i^* + e_i < m} 2e_i \right) \\
& + \left(\sum_{x_i^* - e_i > m, x_i^* + e_i < m} 2x_i^* - \sum_{x_i^* - e_i < m, x_i^* + e_i > m} 2x_i^* \right) \\
& = \left(- \sum_{x_i^* - m > e_i, x_i^* - m > -e_i} 2e_i + \sum_{x_i^* - m < e_i, x_i^* - m < -e_i} 2e_i \right) \\
& + \left(\sum_{x_i^* - m > e_i, x_i^* - m < -e_i} 2x_i^* - \sum_{x_i^* - m < e_i, x_i^* - m > -e_i} 2x_i^* \right) \\
& = \left[\sum_{e_i > 0} \left(- \sum_{x_i^* - m > e_i} 2e_i + \sum_{x_i^* - m < -e_i} 2e_i \right) \right. \\
& \quad \left. + \sum_{e_i < 0} \left(- \sum_{x_i^* - m > -e_i} 2e_i + \sum_{x_i^* - m < e_i} 2e_i \right) \right] \\
& + \left[\sum_{e_i > 0} \left(\sum_{|x_i^* - m| > |e_i|} 2x_i^* - \sum_{|x_i^* - m| < |e_i|} 2x_i^* \right) \right. \\
& \quad \left. + \sum_{e_i < 0} \left(\sum_{|x_i^* - m| < |e_i|} 2x_i^* - \sum_{|x_i^* - m| > |e_i|} 2x_i^* \right) \right] \\
& = \left[\sum_{e_i > 0} \left(- \sum_{x_i^* - m > e_i} 2e_i \right) + \sum_{e_i < 0} \left(- \sum_{x_i^* - m > -e_i} 2e_i \right) \right] \\
& \quad + \left[\sum_{e_i > 0} \sum_{x_i^* - m < -e_i} 2e_i + \sum_{e_i < 0} \sum_{x_i^* - m < e_i} 2e_i \right] \\
& \quad + \left[\sum_{e_i > 0} \sum_{|x_i^* - m| > |e_i|} 2x_i^* - \sum_{e_i < 0} \sum_{|x_i^* - m| > |e_i|} 2x_i^* \right] \\
& \quad + \left[\sum_{e_i < 0} \sum_{|x_i^* - m| < |e_i|} 2x_i^* - \sum_{e_i > 0} \sum_{|x_i^* - m| < |e_i|} 2x_i^* \right] \\
& = \left[\sum_{e_i < 0} \sum_{x_i^* - m > -e_i} 2e_i + \sum_{e_i < 0} \left(- \sum_{x_i^* - m > -e_i} 2e_i \right) \right] \\
& \quad + \left[\sum_{e_i < 0} \left(- \sum_{x_i^* - m < e_i} 2e_i \right) + \sum_{e_i < 0} \sum_{x_i^* - m < e_i} 2e_i \right] + 0 + 0 \\
& = 0 + 0 + 0 + 0 = 0.
\end{aligned}$$

The last two steps are due to the fact that e is symmetrically distributed with zero mean. What this means is that by looking at the difference in the distances across the subjects, we cancel out any RTM effect so that any detected difference in distances can only be due to advertising exposures.

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