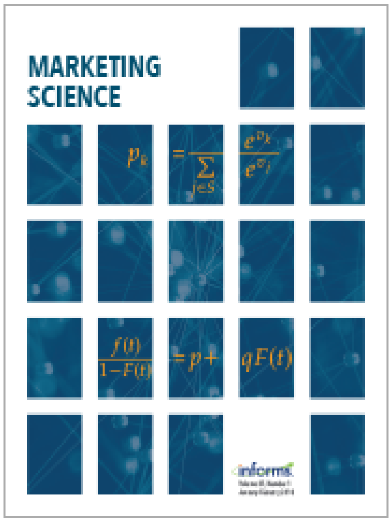

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Consumer Search Activities and the Value of Ad Positions in Sponsored Search Advertising

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Consumer search activities can be endogenously determined by the ad positions in sponsored search advertising. We model how advertisers compete for ad positions in sponsored listings and, conditional on the list of sponsored ads, how online consumers search for information and make purchase decisions. On the consumer side, assuming that users browse information from top to bottom and adopt a sequential search strategy, we develop a two-stage model of consumer search (whether to click and whether to stop the search) that extends the standard sequential search framework in economics literature. On the advertiser side, it is very difficult to fully specify the optimal strategies of advertisers because the equilibrium outcome depends on variables that researchers do not observe. As we have an incomplete model of advertiser competition, we propose using the necessary condition that, at equilibrium, no advertiser will find another available ad position more valuable than the one it has chosen. Using a data set obtained from a search engine, we find that consumers can be classified into two segments that exhibit distinct search behaviors. For advertisers, the value of search advertising comes primarily from terminal clicks, which represent the last link (including organic results) clicked by an online user. We also demonstrate that the value of ad positions depends not only on the identities and positions of the advertisers in sponsored listings but also the composition of online consumers who exhibit distinct search behaviors.

Keywords: search advertising; advertiser value; advertising competition; impressions; clicks; terminal clicks; moment inequality

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1. Introduction

Sponsored search advertising, like other advertising formats, can impact different stages of the consumer decision-making process. When running sponsored search advertising campaigns, some advertisers might seek to increase their brand awareness and preferences from online users and thus will value consumers' browsing and clicking their sponsored ads. Other advertisers may instead target optimizing the purchase conversions from consumers' browsing and clicking behavior. The likelihood that a consumer browses a sponsored ad, clicks the link, and purchases at the advertiser's website vary across different ads in sponsored listings. They could depend not only on the identities and positions of the advertisers in sponsored listings but also on the characteristics of the consumers who may have different information needs and thus exhibit different search behaviors. By investigating how advertisers value consumers' browsing, clicking, and purchasing behaviors, and what factors drive these types of consumer search activities in keyword search, we can advance our understanding of the underlying mechanism that drives the willingness-to-pay of advertisers for ad positions in search advertising markets.

This paper proposes a methodology to address the questions described above. We first model how consumers search on a search results page. Assuming that users browse information from top to bottom and adopt a sequential search strategy, we develop a two-stage model of consumer search (whether to click and whether to stop the search) that extends the standard sequential search framework in economics literature (McCall 1970). This structural model of consumer search allows us to estimate the likelihood of three levels of consumer search activities ((1) effective impression, (2) click, and (3) terminal click) from data for each advertiser at each ad position. An *effective impression* implies that a user indeed browses the ad. A *click* indicates that the user not only browses the ad but also clicks the sponsored link to search for information, which may enhance the consumer's awareness and good will for the advertiser. A *terminal click*, which is a subset of clicks, represents the last link (including organic results) clicked by the user. If the user takes an action (e.g., purchase) following the search query, a terminal click is a precondition that indicates the link the user has chosen for the action. We also adopt a latent class approach to capture the heterogeneity in consumer search behavior; consumers in different segments may exhibit different click-through rates (CTRs) and terminal click-through rates (TCTRs). Furthermore, we explicitly model how advertisers at different ad positions compete for consumer search activities. Our approach thus differs from standard position competition models in the literature (e.g., Athey and Ellison 2011), which assume that the value of an ad position is exogenous and independent from other positions.

Using estimation results from the consumer search model as an input, we use a revealed preference approach to infer advertisers' values for the three types of consumer search activities from their decisions on paying the prices for ad positions. To this end, we estimate a model of advertiser competition with a finite number of ad positions. The main methodological challenge for estimating such a model is that the optimal strategies of advertisers are very difficult to fully specify, mainly because the equilibrium outcome depends on variables that researchers do not observe. For example, the order of advertisers' bidding for a keyword or paying prices for ad positions may impact which advertisers take what ad positions, but such an order is typically not in data. We therefore have an "incomplete" model. To address this challenge, we use the necessary condition that, at equilibrium, no advertiser will find another available ad position more valuable than the one it has chosen. Derived from this condition, we construct lower bounds for the advertiser's payoff function for a search query, and use the method of moment (MM) inequalities (Pakes et al. 2014) in model estimation. This method allows us to estimate the advertiser competition model without imposing restrictive assumptions on how equilibrium outcomes are generated.

We apply the proposed methodology to estimate consumer search activities and advertiser values from a unique data set obtained from a leading search engine in Korea. The search engine considered in this research adopts the cost-per-impression (CPM) pricing mechanism, and all advertisers purchase ad positions by exercising the buy-it-now (BIN) option in position auctions. Although we only study one particular keyword in a specific context, the proposed methodology can be used for a wide range of empirical applications. The two-stage consumer search model, for example, can be applied to other online and offline contexts where consumers decide in a sequential manner where to search for product or price information and where to purchase. The model helps predict what attracts website traffic (or store visits) and what drives purchase conversions, as well as how consumers differ in their search processes. Moreover, the advertiser competition model and its estimation have general applications, allowing researchers to avoid restrictive assumptions on the data-generating process. In the market of online display advertising, for example, ad slots are limited and the price for each slot is predetermined by publishers. Which advertisers get what ad slots at equilibrium depends on the order that advertisers make purchase decisions, but such order is typically unobserved by researchers. In this research, we use necessary conditions for the equilibrium in model estimation and apply it under fixed prices in the context of sponsored search advertising. It can also be applied to other pricing mechanisms, including CPM, cost-per-click (CPC), and the generalized second-price (GSP) auction mechanism that is commonly adopted by search engines such as Google and Yahoo!. Thus it is simple to construct lower bounds for the advertiser's payoff function based on the bids submitted from the advertiser and its competitors.

Applying the proposed methodology to a keyword in search advertising, we show that consumers can be classified into two segments that exhibit distinct search behaviors. Users in the larger, low-involvement segment are less likely to click sponsored links, and once they do, they are more likely to stop the search. By contrast, users in the smaller, high-involvement segment are more likely to click multiple links and less likely to stop the search. We also find that the value of an ad comes from terminal clicks only, rather than from effective impressions or from nonterminal clicks. Although our results may not be generalized to all other advertisers, they illustrate that TCTR can be an important metric that search engines should provide to sponsored advertisers. Finally, we highlight how the value of a search query for advertisers depends not only on the identities and positions of the advertisers in sponsored listings but also on the composition of online consumers who exhibit different search behaviors.

1.1. Literature Review

The commercial success of sponsored search advertising has motivated substantial research into its various aspects. Theoretical research has offered important insights about optimal bidding strategies for sponsored links and appropriate designs for auction mechanisms (e.g., Edelman et al. 2007, Varian 2007, Katona and Sarvary 2010, Desai et al. 2014). Such literature typically treats each ad position as an auction item whose value is exogenously given. Recognizing this as a major limitation, a few studies (e.g., Athey and Ellison 2011, Chen and He 2011, Jerath et al. 2011) examine how the ad value comes from the consumer information search that is determined by ad positions. Other studies (e.g., Aggarwal et al. 2008, Das et al. 2008, Kempe and Mahdian 2008) use statistical approaches to capture the effect of competition from other ads on the probabilities of consumer browsing and clicking an ad position. Unlike these streams of research, this paper develops a structural model of consumer search that can be empirically applied. Our search model helps describe and predict how ad positions compete for effective impressions, clicks, and terminal clicks from consumers. Whereas the model derives probabilities of these search activities in a manner similar to the click model in Aggarwal et al. (2008), the structural approach is useful when we use counterfactuals to estimate the bounds for advertiser values. Furthermore, empirical research in marketing (e.g., Ghose and Yang 2009, Agarwal et al. 2011, Rutz and Bucklin 2011, Rutz et al. 2012) has documented the effects of ad positions on consumer clicks and conversions. Because of data limitations, these studies do not consider the competition effect. Our paper thus extends the literature by studying how advertisers compete for consumer search activities in search advertising markets.

Economics literature has investigated both nonsequential (e.g., Stigler 1961) and sequential (e.g., McCall 1970, Rothschild 1974) search, in which the latter is in general considered a better strategy since it does not require pre-commitment on the number of searches. Yet some researchers offer empirical evidence in support of both sequential (Zhang et al. 2012) and nonsequential (De los Santos et al. 2012, Honka and Chintagunta 2014) search, whereas others propose an optimal sequential process (Kim et al. 2010). In this tradition, our two-stage search model represents a modified version of the standard sequential search framework.

Empirical research has developed useful methodologies to study advertiser competition in keyword auctions. Yao and Mela (2011), for example, model competition for ad positions and investigate strategic behaviors by advertisers. Athey and Nekipelov (2012) propose a homotopy-based method to compute equilibrium outcomes when advertisers face uncertainty, and develop a means to estimate the advertiser value. Because of the complexity of modeling the equilibrium conditions in GSP auctions (see Edelman et al. 2007), several empirical studies have adopted an approach proposed by Haile and Tamer (2003) to estimate the bounds of advertiser values for multiple ad positions (e.g., Edelman and Ostrovsky 2007, Varian 2007). Our approach of estimating the bounds of advertiser values is similar to Haile and Tamer (2003). Whereas previous works treat the advertiser value for each ad position as exogenously given, we investigate how the value is determined by consumer search activities, which are endogenously determined by the competition across ad positions. This paper therefore represents a further development in the literature by considering both the consumers' and advertisers' decisions. Finally, we model how advertisers compete for ad positions, which is contiguous to the location competition wherein interfirm competition results from the substitutability of demand among neighboring firms (e.g., Thomadsen 2005, Seim 2006, Chan et al. 2007). Yet our empirical context is more complicated. With an incomplete competition model, the equilibrium conditions are difficult to fully specify, so we adopt a unique estimation strategy.

The rest of the paper is organized as follows. We develop the consumer search model in §2. In §3, we describe the advertiser competition model in sponsored search advertising, characterize the necessary condition for equilibrium outcomes, and develop the estimation methodology. Section 4 describes the data of our empirical application, and discusses the results and implications. We conclude with some directions for future work in §5.

2. Consumer Search

As an overview of our modeling approach, we model a two-stage game. In the first stage, advertisers decide what ad positions to purchase using an auction or fixed price format. Decisions are based on their valuations of three types of consumer search activities (effective impressions, clicks, and terminal clicks) that each ad position can attract. Consumers arrive in the second stage and decide how to optimally search for information on the listed sponsored links. After browsing a sponsored link, a consumer first decides whether to click the link to search for more information at the advertiser's website and, if she does, then decides whether to terminate the search. This section develops the structural model of consumer search that helps infer the effective impressions, clicks, and terminal clicks that each ad position will attract. In §3, we build the model of advertiser competition and discuss the estimation strategy.

To develop the consumer search model, we make a few assumptions about how a consumer behaves in a keyword search. We assume that she responds to search listings sequentially, starting with the sponsored ad listed at the topmost position, then the one at the second position, and so on. After searching for information in each sponsored link, the consumer begins to browse the organic results if she continues the search. Consistent with standard sequential search literature in economics (e.g., McCall 1970, Gastwirth 1976), we assume that the consumer does not revisit links that have been clicked.

2.1. A Model of Sequential Search

Conditional on consumer i browsing advertiser j's link listed at a certain position, we assume that she makes decisions in two steps, i.e., whether to click the link and whether to stop the search. The selling propositions of an ad, which typically consist of a few phrases or words describing the products and offers to consumers, provide the consumer partial information about what would be revealed if she clicks the link. If she does not click the link, the consumer browses the next sponsored or organic links; if she clicks, the consumer finds full information from the advertiser's website. If she does not stop the search, the consumer browses sponsored or organic links listed below on the search results page. In day t, the utility of advertiser j's offering for consumer i who searches is

$$u_{ijt} = X_{jt}\beta + w_{ijt}, \tag{1}$$

where X_{jt} is a vector of observed attributes at advertiser j's sponsored link, including the advertiser's identity and the selling propositions, that we assume to be exogenous. The stochastic component w_{ijt} captures the advertiser's offering to the consumer, unobserved to researchers, that may coincide in different ways with individual needs. For example, if advertiser j promotes running shoes that the consumer is searching for, w_{ijt} will be higher than average.

After browsing the link, the consumer observes the selling propositions. They may include other information (I_{jt}), unobserved to researchers, on the products as well as offers to consumers. For example, it may advertise that running shoes are on sale at the website, which will increase the consumer's expected utility. Still, she has to click on the website to obtain the exact prices for different models or styles of running shoes, and whether the advertiser carries the models or styles that she likes. We assume that the consumer uses the information to form her expectation $e_{ijt} \equiv E[w_{ijt} \mid X_{jt}, I_{jt}]$. Her expected utility after browsing the link will be

$$E[u_{ijt} \mid X_{jt}, I_{jt}] = X_{jt}\beta + e_{ijt}. \tag{2}$$

If the consumer clicks the link and searches for information, w_{ijt} will be revealed. Defining $\varepsilon_{ijt} \equiv w_{ijt} - e_{ijt}$, Equation (1) can be rewritten as

$$u_{ijt} = X_{jt}\beta + e_{ijt} + \varepsilon_{ijt}. \tag{3}$$

We assume that consumers are rational in forming expectations. The stochastic term ε_{ijt} represents a shock to the consumer which has zero mean (i.e., no systematic bias) and is uncorrelated with e_{ijt}; otherwise she has not fully used the information. Define F_X, F_e, and F_ε as the distributions of X_{jt}, e_{ijt}, and ε_{ijt}, respectively. We also assume that the consumer knows the distributions, but not the exact values, prior to the search.

We assume an individual-specific marginal cost c_{it}^1 to browse a link, equal to the time and cognitive effort the consumer exerts to process the ad, and that this cost remains constant throughout the search. We also assume a marginal cost c_{it}^2 to click a link, which remains constant during the search. The two search costs differ in time and cognitive costs required for consumer decision making. If a consumer clicks a link, she incurs both c_{it}^1 and c_{it}^2 because she would have browsed the link before clicking it.

Our model constitutes a variation of the dynamic sequential search model in economics (e.g., McCall 1970). We define $V(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt})$ as the consumer's value of the optimal search decision in the first stage, conditional on search costs (c_{it}^1, c_{it}^2) and attributes (X_{jt}, e_{ijt}) obtained from browsing advertiser j's link. We also define $W(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}, \varepsilon_{ijt})$ as the value of the optimal search decision in the second stage, conditional on clicking advertiser j's link and observing ε_{ijt}. After clicking the link, the consumer decides whether to stop the search by maximizing the value in the following Bellman's equation:

$$\tilde{W}(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}, \varepsilon_{ijt}) = \max\{X_{jt}\beta + e_{ijt} + \varepsilon_{ijt}, EV_{it}\}, \tag{4}$$

where $EV_{it} = \int_{X', e'} V(c_{it}^1, c_{it}^2; X', e')\, dF_X(X')\, dF_e(e')$ is the consumer's expectation of the value of continuing the search, measured by integrating out the attributes (X', e') of the next links yet to be observed by the consumer. The value of the optimal search in the second stage is $W(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}, \varepsilon_{ijt}) \equiv \tilde{W}(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}, \varepsilon_{ijt}) - c_{it}^2$. If we define $z_{it}^2 = EV_{it}$ as the reservation utility for stopping the search, we can apply a simple optimal stopping rule in the second stage: If $u_{ijt} \geq z_{it}^2$, the consumer stops; otherwise, she continues the search. We assume that F_X, F_e, and F_ε are continuous and independent across advertisers. Previous research on sequential search models has shown that a unique z_{it}^2 exists under these conditions (e.g., McCall 1970).

Unlike sequential search models in the literature, our approach also characterizes the optimal decision in the first stage. Conditional on browsing advertiser j's link, the consumer has yet to observe ε_{ijt} and must decide whether to click the link by maximizing the expected utility through the following Bellman's equation:

$$\tilde{V}(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}) = \max\left\{\int_\varepsilon W(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}, \varepsilon_{ijt})\, dF_\varepsilon(\varepsilon_{ijt}), EV_{it}\right\}. \tag{5}$$

That is, if she clicks, the expected utility is $\int_\varepsilon W(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}, \varepsilon_{ijt})\, dF_\varepsilon(\varepsilon_{ijt})$, the value of optimal search in the second stage as defined in Equation (4) after integrating out ε_{ijt}; otherwise, the user continues to search the next link with an expected value of EV_{it}. Thus, the value of the optimal search decision in the first stage is $V(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}) = \tilde{V}(c_{it}^1, c_{it}^2; X_{jt}, e_{ijt}) - c_{it}^1$.

Given z_{it}^2, let z_{it}^1 be the reservation utility of clicking the link such that

$$-c_{it}^2 + \int_{z_{it}^2 - z_{it}^1}^{\infty} (z_{it}^1 + \varepsilon_{ijt})\, dF_\varepsilon(\varepsilon_{ijt}) - z_{it}^2 \cdot [1 - F_\varepsilon(z_{it}^2)] = 0. \quad (6)$$

In Appendix A, we derive the optimal stopping rule in the first stage: If $\bar{u}_{ijt} \equiv X_{jt}\beta + e_{ijt} \geq z_{it}^1$, the consumer clicks the link. Given z_{it}^2, Equation (6) offers a unique solution, so a unique reservation utility z_{it}^1 exists.

2.2. Model Estimation

Under the sequential search assumption, the distribution functions F_X, F_e, and F_ε for the next link that the consumer has not browsed remain unchanged in the expectation function during the search process. We also assume that the distributions are the same for every consumer. The heterogeneity in reservation utilities z_{it}^1 and z_{it}^2 is driven by the difference in consumer search costs. We show in Appendix A that the higher the search cost c_{it}^2, the higher z_{it}^1 is in the first stage. It is also easy to see that the higher the search cost c_{it}^1, the lower z_{it}^2 is in the second stage. To estimate the sequential search model, we adopt an approach similar to Kiefer and Neumann's (1979), using a reduced-form approximation for z_{it}^1 and z_{it}^2, as follows:

$$z_{it}^1 = Z_{it}\gamma_1 + v_{1it} \quad \text{and} \quad z_{it}^2 = Z_{it}\gamma_2 + v_{2it}, \quad (7)$$

where Z_{it} is a vector of consumer characteristics and time-related variables of consumer searches, including month and weekday dummies that may influence consumer search costs and utility. The stochastic components v_{1it} and v_{2it} capture the unobserved individual-specific effects.[1]

For notational simplicity, we observe K sponsored links in day t, where $j = 1, \ldots, K$ represents the advertiser at the jth position ("1" for the topmost, "2" for the second position, and so on). Let y_{ijt} be an indicator that equals 1 if consumer i clicks advertiser j's link, and 0 otherwise. Let y_{i0t} be an indicator that equals 1 if consumer i clicks any of the organic links. For the data on consumer search activity in the full list, the assumption of top-down sequential search implies (1) an effective impression on link j if $y_{ikt} = 1$ for $k \geq j$, or $y_{i0t} = 1$; (2) a click on link j if $y_{ijt} = 1$; and (3) a terminal click on link j if $y_{ijt} = 1$ and $y_{ikt} = 0$ for all $k > j$, and $y_{i0t} = 0$. From the optimal decision rules in our model, two conditions must be satisfied for a terminal click: $e_{ijt} - v_{1it} \equiv \tilde{\omega}_{ijt}^1 \geq Z_{it}\gamma_1 - X_{jt}\beta$ and $e_{ijt} + \varepsilon_{ijt} - v_{2it} \equiv \tilde{\omega}_{ijt}^2 \geq Z_{it}\gamma_2 - X_{jt}\beta$. For a nonterminal click, $\tilde{\omega}_{ijt}^1 \geq Z_{it}\gamma_1 - X_{jt}\beta$ and $\tilde{\omega}_{ijt}^2 < Z_{it}\gamma_2 - X_{jt}\beta$. If the consumer browses but chooses not to click, $\tilde{\omega}_{ijt}^1 < Z_{it}\gamma_1 - X_{jt}\beta$.

We next build a general formulation of the likelihood function. Define consumer i's search as $y_{it} = \{y_{i1t}, y_{i2t}, \ldots, y_{iKt}; y_{i0t}\}$. From the definition, $\tilde{\omega}_{ijt}^1$ and $\tilde{\omega}_{ijt}^2$ are driven by the advertiser's offers (observed before and after clicking the link) and the consumer's individual-specific preferences and search costs that are not observable to researchers. Under general conditions these error terms may correlate with each other and across sponsored links. They may also correlate across consumers who browse link j in the same day. To account for these correlations, we specify $\tilde{\omega}_{ijt}^1$ and $\tilde{\omega}_{ijt}^2$ as follows:

$$\tilde{\omega}_{ijt}^1 = \omega_{jt}^1 + \omega_{it}^1 + \omega_{ijt}^1 \quad \text{and} \quad \tilde{\omega}_{ijt}^2 = \omega_{jt}^2 + \omega_{it}^2 + \omega_{ijt}^2, \quad (8)$$

where ω_{jt}^1 and ω_{jt}^2 represent advertiser-specific attributes observed by consumers at time t, but not by researchers; ω_{it}^1 and ω_{it}^2 refer to the consumer preferences and time-cost shocks common to all sponsored links; and ω_{ijt}^1 and ω_{ijt}^2 are the idiosyncratic shocks, identically and independently distributed (i.i.d.) across consumers and sponsored links.

Suppose we observe N consumers in day t. Let $\mathbf{y_t} = (y_{1t}, y_{2t}, \ldots, y_{Nt})$ be the collection of consumer searches. From the assumptions for stochastic terms in Equation (8), we can write the likelihood of $\mathbf{y_t}$ as

$$\Pr(\mathbf{y_t}) = \int_{\omega_{1t}^1, \omega_{1t}^2; \ldots; \omega_{Kt}^1, \omega_{Kt}^2} \left\{ \prod_{i=1}^{N} \int_{\omega_{it}^1, \omega_{it}^2} \Pr(y_{it} \mid \omega_{1t}^1, \omega_{1t}^2, \ldots, \omega_{Kt}^1, \omega_{Kt}^2; \omega_{it}^1, \omega_{it}^2)\, dF^1(\omega_{it}^1, \omega_{it}^2) \right\} \cdot \prod_{j=1}^{K} dF^2(\omega_{jt}^1, \omega_{jt}^2),^{2} \quad (9)$$

where F^1 is the joint distribution function of consumer-specific errors $(\omega_{it}^1, \omega_{it}^2)$, and F^2 is the joint distribution function of advertiser-specific errors $(\omega_{jt}^1, \omega_{jt}^2)$. In this expression, the errors are integrated out first in the evaluation of search likelihood.

We differentiate the search into two different types: one in which the search ends at the jth link such that for all $k > j$, $y_{ikt} = 0$ and $y_{i0t} = 0$, and the second in which the search ends at an organic link such that $y_{i0t} = 1$. The conditional likelihood

[1] Because z_{it}^1 and z_{it}^2 capture the consumer's expectation of the attributes from links that she has not browsed, as well as her search costs, z_{it}^1 and z_{it}^2 are not a function of X_{jt} of the sponsored link that she has browsed.

[2] For analytical simplicity, we assume that for advertiser j, ω_{jt}^1, and ω_{jt}^2 are independent over time. This assumption appears reasonable because we include indicators to capture advertiser-fixed effects as part of X_{jt}. We assume each search was conducted by a separate consumer and that ω_{it}^1 and ω_{it}^2 are i.i.d. across consumers.

$\Pr(y_{it} \mid \omega^1_{1t}, \omega^2_{1t}, \ldots, \omega^1_{Kt}, \omega^2_{Kt}; \omega^1_{it}, \omega^2_{it})$ in Equation (9) can be written as

$$\begin{aligned}
&\Pr(y_{it} \mid \omega^1_{1t}, \omega^2_{1t}, \ldots, \omega^1_{Kt}, \omega^2_{Kt}; \omega^1_{it}, \omega^2_{it})\\
&= 1\{y_{i1t}, \ldots, y_{ijt} = 1, y_{i,j+1,t} = 0, \ldots, y_{iKt} = 0, y_{i0t} = 0\}\\
&\quad \cdot \prod_{k=1}^{j-1} \big[\Pr(\omega^1_{ikt} \geq Z_{it}\gamma_1 - X_{kt}\beta - \omega^1_{kt} - \omega^1_{it} \text{ and}\\
&\qquad\quad \omega^2_{ikt} < Z_{it}\gamma_2 - X_{kt}\beta - \omega^2_{kt} - \omega^2_{it}) \cdot 1\{y_{ikt} = 1\}\\
&\qquad\quad + \Pr(\omega^1_{ikt} < Z_{it}\gamma_1 - X_{kt}\beta - \omega^1_{kt} - \omega^1_{it}) \cdot 1\{y_{ikt} = 0\}\big]\\
&\qquad\quad \cdot \Pr(\omega^1_{ijt} \geq Z_{it}\gamma_1 - X_{jt}\beta - \omega^1_{jt} - \omega^1_{it} \text{ and}\\
&\qquad\qquad \omega^2_{ijt} \geq Z_{it}\gamma_2 - X_{jt}\beta - \omega^2_{jt} - \omega^2_{it})\\
&\quad + 1\{y_{i1t}, \ldots, y_{iKt}, y_{i0t} = 1\}\\
&\quad \cdot \prod_{k=1}^{K} \big[\Pr(\omega^1_{ikt} \geq Z_{it}\gamma_1 - X_{kt}\beta - \omega^1_{kt} - \omega^1_{it} \text{ and}\\
&\qquad\quad \omega^2_{ikt} < Z_{it}\gamma_2 - X_{kt}\beta - \omega^2_{kt} - \omega^2_{it}) \cdot 1\{y_{ikt} = 1\}\\
&\qquad\quad + \Pr(\omega^1_{ikt} < Z_{it}\gamma_1 - X_{kt}\beta - \omega^1_{kt} - \omega^1_{it})\\
&\qquad\quad \cdot 1\{y_{ikt} = 0\}\big]. \qquad (10)
\end{aligned}$$

Lines 2 through 7 in Equation (10) represent the first type of search,[3] whereas the other lines represent the second type. Because ω^1_{ikt} and ω^2_{ikt} may correlate, the functions in this equation must be jointly estimated. For day t, we evaluate the likelihood $\Pr(\mathbf{y}_t)$ by simulating NS times of ω^1_{jt} and ω^2_{jt} from F^2 for link j, as well as simulating NS times of ω^1_{it} and ω^2_{it} from F^1 for consumer i, following the distribution assumptions for these stochastic terms.[4]

We posit that consumers who search the keyword come from several different latent segments (Kamakura and Russell 1989) that allow for the heterogeneity of search behavior among users. For a user who belongs to segment q, her search behavior is captured by the segment-specific parameters $(\beta^q, \gamma^q_1, \gamma^q_2)$.[5] Based on those parameters, we can evaluate the simulated segment-specific likelihood function $\Pr(y_{it} \mid q)$. Let Q be the number of consumer segments and π_q be the probability with which a consumer is in segment q. The model estimates $(\beta^1, \gamma^1_1, \gamma^1_2; \ldots; \beta^Q, \gamma^Q_1, \gamma^Q_2)$ maximize the full likelihood function $\prod_{i,t}[\sum_q (\pi_q \cdot \Pr(y_{it} \mid q))]$.

With the estimation results, we compute the probabilities of effective impression, click, and terminal click for a specific link, as shown in Appendix B. This allows us to infer the value per effective impression, click, and terminal click for each advertiser.

2.3. Discussion of Model Assumptions

We have made a few important assumptions in our consumer search model. These assumptions may be quite restrictive in describing consumer search behavior and, if they are invalid, the model is misspecified and estimation results could be biased. The top-down search assumption has been well supported in the literature. In particular, using eye-tracking data, Granka et al. (2004) show that online consumers generally investigate a list of ranked results from the top down. Hoque and Lohse (1999) and Ansari and Mela (2003) also find supportive evidence in other contexts. Under the assumption that the distributions F_X and F_w are i.i.d. across advertisers, no other browsing strategies (e.g., bottom to top, random) will dominate top-down search. We thus believe that this is a reasonable assumption that is consistent with optimal search strategy. This assumption has also been made in the economics literature (e.g., Aggarwal et al. 2008, Das et al. 2008, Kempe and Mahdian 2008).

Sequential search models have been widely studied in economics (e.g., McCall 1970, Rothschild 1974, Gastwirth 1976, Arbatskaya 2007). These models assume no updating for the distribution of advertiser offers (e.g., product, price) from previous searches, which can be justified if consumers have sufficient search experience and distribution knowledge. Hence, consumers would not revisit a link that has been previously clicked. Several behavioral studies indicate that consumers process alternatives sequentially (e.g., Saad and Russo 1996, Moorthy et al. 1997).[6] We use these assumptions to infer effective impressions and terminal clicks. Although competing models, such

[3] A consumer, after the last click, may continue browsing additional links below without making further clicks. As a result, the likelihood function may be incomplete because it does not incorporate this behavior. The omission of this component from the likelihood, however, is unlikely to affect the result under (i) the i.i.d. assumption of error terms across links, and (ii) the assumption that advertisers do not treat these additional browsing activities as effective impressions.

[4] We assume F^1 and F^2 are joint normal distributions and fix NS to be 1,000 in the estimations.

[5] We do not estimate c^1_{it} and c^2_{it} in the model. These parameters affect z^1_{it} and z^2_{it} in Equation (7). Estimated z^1_{it} and z^2_{it}, together with estimated β, give us the predicted probabilities of effective impression, click, and terminal click.

[6] Sequential search is a better strategy than nonsequential search, as long as users are not too impatient in getting results. Honka and Chintagunta (2014) test the two search models based on the prices of the searched options in a buyers' consideration set, and find evidence supporting nonsequential search. De los Santos et al. (2012) test the no-revisit assumption from the sequential search model using the comScore clickstream data. Although the test rejects the sequential search model, their data reveals that, among the 10% of transactions in which consumers visited more than one bookstore, 62% bought from the last one in the search sequence. This is consistent with the predictions from the proposed model. Only 38% revisited a previously searched bookstore. For 90% of transactions, consumers only visited one bookstore. This suggests that the no-revisit assumption may be a reasonable approximation of consumer search behavior in reality.

as nonsequential (finite sample) search and sequential search with updates are available, they cannot help identify consumer search activities if the entire sequence of consumer clicks on sponsored and organic links is not observed. If the actual sequence of clicks is observed, researchers could test the assumptions of no-revisit and top-down browsing with data. Still, a structural model of consumer search is needed to infer how advertisers value consumer search activities in the advertiser competition model. This is discussed in §3. As shown in §4, this model fits the data considered in this study very well. If advertisers use these assumptions to approximate consumer search behaviors and guide their decisions in search advertising, we can treat it as a good as-if model, since our main objective is to infer the advertiser values for consumer search activities.

Note also that the infinite horizon assumption in our sequential search model is an approximation of the fact that users typically find a large number of relevant results on the search results page. Because the probability of terminating the search at any link is positive, the probability that a user will continue the search quickly declines as more links are browsed. We expect that the predicted search behavior is not too different from a finite horizon model. Finally, consumers likely take actions following their search, although different consumers arrive at the search engine with distinct information objectives. As data from search engines do not include post-click conversion behavior of consumers, we cannot distinguish purchase from no-purchase behavior following a search query. The value per terminal click for advertisers therefore constitutes the expected value, accounting for the probability of no purchase.

3. Advertiser Competition

Given the consumer search model, this section proposes a strategy to estimate an incomplete model of advertiser competition with a finite number of ad positions in which the optimal strategies of players are not fully specified because of the complicated nature of the game. We focus on the necessary conditions that, at equilibrium, no advertisers can increase their payoffs by changing strategies. Based on the necessary conditions we develop an estimation strategy to infer how advertisers value the three types of consumer search activities. This strategy can be applied to various pricing mechanisms, including fixed prices and GSP auctions, and CPM and CPC price formats.

Suppose that J advertisers compete for K ad positions, $J > K$. We denote A_{jt} as the ad position that advertiser j acquires at day t, where "1" indicates the topmost position. If the advertiser does not acquire any ad position, $A_{jt} = 0$. Let $\bar{A}_t = (A_{1t}, A_{2t}, \ldots, A_{Jt})'$ be the collection of positions that advertisers purchased. Let $c(A_{jt})$ be the cost of advertiser j for a search at day t that depends on the ad position. Under CPM, $c(A_{jt})$ is the price the advertiser pays for a search query that may or may not bring an effective impression; under CPC, it is the price paid for a click, multiplied by the probability that the consumer will click the sponsored link. If the advertiser chooses not to advertise, $c(A_{jt}) = 0$.

Given $\bar{A}_t$, let π_j^1 and π_j^2 be advertiser j's expected value per effective impression and click, respectively. We denote the value per terminal click as $\tilde{\pi}_{jt}^3 = \pi_j^3 + \xi_{jt}$, where π_j^3 is the average value over the data period and ξ_{jt} captures the time-varying profit shock for the advertiser. If a manufacturer runs a trade promotion in day t, for instance, the margin of the advertiser selling the manufacturer's products is higher; ξ_{jt} is positive. This stochastic term may be serially correlated and correlated across advertisers (e.g., the manufacturer offers the same promotion to all advertisers), and by definition $E(\xi_{jt}) = 0$ for advertiser j.

Let the probability of browsing link j (i.e., effective impression), conditional on the nominal positions $\bar{A}_t$ and a vector of variables S_t, including observed attributes X_{jt} of all advertisers and the set of variables Z_{it} that affects reservation utilities, be $P_j^1(\bar{A}_t, S_t)$. Let the probability of clicking link j be $P_j^2(\bar{A}_t, S_t)$. Finally, let the probability of the click being the terminal click be $P_j^3(\bar{A}_t, S_t)$. The three probabilities for advertiser j at an ad position L can be derived from (B1)–(B3) in Appendix B. If $A_{jt} = 0$, $P_j^1(\bar{A}_t, S_t) = P_j^2(\bar{A}_t, S_t) = P_j^3(\bar{A}_t, S_t) = 0$. The payoff function of advertiser j for acquiring ad position A_{jt} from a search query is

$$\begin{aligned} V_j(\bar{A}_t, S_t) = P_j^1(\bar{A}_t, S_t) \cdot \pi_j^1 + P_j^2(\bar{A}_t, S_t) \cdot \pi_j^2 \\ + P_j^3(\bar{A}_t, S_t) \cdot (\pi_j^3 + \xi_{jt}) - c(A_{jt}). \quad (11) \end{aligned}$$

If $A_{jt} = 0$, $V_j(\bar{A}_t, S_t) = 0$.[7] We assume that given $\bar{A}_t$, the probabilities of search activities attracted by each advertiser as well as the advertiser's valuation of the activities are common knowledge. However, other advertisers only know the distribution, not the exact value of ξ_{jt}. Advertiser j has to maximize its expected value in Equation (11) when it makes the purchase decision.

It is difficult to characterize optimal strategies that advertisers use in such a game. Advertisers in reality make their purchase decisions of ad positions sequentially. We therefore think of the competition

[7] We assume the advertiser cannot generate effective impressions, clicks, or terminal clicks from organic results. From the data that we use to estimate the model, we find little overlap between the two sets of search listings, which also differ in nature (i.e., information-based links in organic results versus commercial links in sponsored results).

as a sequential game. The order of moves, however, is stochastic and unobserved by researchers. When ad positions are sold under the fixed-price format,[8] suppose advertiser j chooses A_{jt} which generates the highest profit to the advertiser. This assumption may be invalid because the advertiser may want to, but cannot, buy another ad position as it has been taken by a competitor who moved earlier. When ad positions are sold by GSP auctions, advertisers may frequently change bids so that the advertiser competition can be treated as an infinitely repeated game. The optimal strategies are very complex since it does not have an equilibrium in dominant strategies. Furthermore, depending on how advertisers strategically change bids, the game has a large set of equilibria (Edelman et al. 2007). Because of this problem, it is challenging to estimate the game from the data to infer advertiser values.

3.1. Equilibrium Conditions and Inequalities

Our estimation strategy follows the recent literature in economics (e.g., Andrews et al. 2007, Chernozhukov et al. 2007, Pakes et al. 2014) that infers model parameters from incomplete econometric models. Researchers search for a set of model parameters (which can be a singleton) that are consistent with the necessary conditions for equilibrium. Any value within the set is an acceptable candidate for the estimated parameters. Although less precise than traditional econometric methods that generate point estimates, this approach allows for estimation of a model without requiring that optimal strategies of players are fully specified.

We adopt the MM inequalities, as proposed by Pakes et al. (2014), in model estimation. This method applies a lower computational burden than other methods (e.g., Andrews et al. 2007, Chernozhukov et al. 2007), and it does not rely on the distributional assumption of the error term (i.e., the profit shock ξ_{jt}). Therefore, ξ_{jt} can be serially correlated and correlated across advertisers. The consistency of estimators requires only that the expectation of ξ_{jt}, conditional on instrumental variables, is zero. Estimation results are robust to various behavioral assumptions about the data-generation process in an empirical context.

To adopt such an estimation strategy, we assume that the ad positions acquired by advertisers observed in data are at equilibrium. We use this assumption and construct bounds for the payoff function in Equation (11) to estimate model parameters. We first derive the bounds under fixed prices and then GSP auctions. Under fixed prices, we assume that at equilibrium no advertiser can obtain a higher value by switching to an alternatively available position, including no advertising.[9]

Conditional on observed $\bar{A}_t$, let A^U_{jt} be an alternative higher position for advertiser j and A^L_{jt} be an alternative lower position for it, which also includes no advertising. Let $\bar{A}^{U,j}_t = (A_{1t}, \ldots, A^U_{jt}, \ldots, A_{Jt})$ be the vector of advertisers' positions in the higher position scenario, where advertiser j moves up to position A^U_{jt} and all others remain at their original positions. Similarly, let $\bar{A}^{L,j}_t = (A_{1t}, \ldots, A^L_{jt}, \ldots, A_{Jt})$ be the alternative scenario in the lower position scenario for advertiser j. Under this assumption, two conditions must be satisfied to ensure that the observed advertisers' final choices are at equilibrium. The first (higher position scenario) is as follows:

i. Suppose A^U_{jt} exists. The value of A_{jt} to advertiser j is greater than the value of A^U_{jt}. That is, $V_j(\bar{A}_t, S_t) \geq V_j(\bar{A}^{U,j}_t S_t)$.

Because $P^3_j(\bar{A}_t, S_t) \leq P^3_j(\bar{A}^{U,j}_t, S_t)$, we can derive the following inequality condition:

$$\frac{P^1_j(\bar{A}_t, S_t) - P^1_j(\bar{A}^{U,j}_t, S_t)}{P^3_j(\bar{A}^{U,j}_t, S_t) - P^3_j(\bar{A}_t, S_t)} \cdot \pi^1_j + \frac{P^2_j(\bar{A}_t, S_t) - P^2_j(\bar{A}^{U,j}_t, S_t)}{P^3_j(\bar{A}^{U,j}_t, S_t) - P^3_j(\bar{A}_t, S_t)} \cdot \pi^2_j - \pi^3_j - \frac{c(A_{jt}) - c(A^U_{jt})}{[P^3_j(\bar{A}^{U,j}_t, S_t) - P^3_j(\bar{A}_t, S_t)]} \geq \xi_{jt}. \tag{12}$$

If $A_{jt} = 0$, i.e., advertiser j is not currently advertising, the inequality can be simplified to

$$-\frac{P^1_j(\bar{A}^{U,j}_t, S_t)}{P^3_j(\bar{A}^{U,j}_t, S_t)} \cdot \pi^1_j - \frac{P^2_j(\bar{A}^{U,j}_t, S_t)}{P^3_j(\bar{A}^{U,j}_t, S_t)} \cdot \pi^2_j - \pi^3_j + \frac{c(A^U_{jt})}{P^3_j(\bar{A}^{U,j}_t, S_t)} \geq \xi_{jt}. \tag{13}$$

The second necessary equilibrium condition, similar to the first, is as follows:

ii. Suppose A^L_{jt} exists. The value of A_{jt} to advertiser j is greater than the value of A^L_{jt}. That is, $V_j(\bar{A}_t, S_t) \geq V_j(\bar{A}^{L,j}_t, S_t)$.

Because $P^3_j(\bar{A}_t, S_t) \geq P^3_j(\bar{A}^{L,j}_t, S_t)$, we can derive the following inequality condition:

$$\frac{c(A_{jt}) - c(A^L_{jt})}{[P^3_j(\bar{A}^{U,j}_t, S_t) - P^3_j(\bar{A}_t, S_t)]} - \frac{P^1_j(\bar{A}_t, S_t) - P^1_j(\bar{A}^{L,j}_t, S_t)}{P^3_j(\bar{A}^{L,j}_t, S_t) - P^3_j(\bar{A}_t, S_t)}$$

[8] This game can be applied to other contexts, e.g., online display advertising, and even traditional advertising media (e.g., TV, print) in which ad slots are sold at prices predetermined by publishers.

[9] This assumption ignores the possibility of a class of perfect Bayesian equilibria, in which an advertiser may choose not to pay for a more valuable position because it expects that competitors will respond aggressively such that the outcome for the advertiser will not be profitable. It also does not allow for collusive behavior among advertisers. Furthermore, it requires that the cost of switching does not dominate the increased profit of switching to a more valuable position.

$$\cdot\,\pi_j^1-\frac{P_j^2(\bar{A}_t,S_t)-P_j^2(\bar{A}_t^{L,j},S_t)}{P_j^3(\bar{A}_t^{L,j},S_t)-P_j^3(\bar{A}_t,S_t)}\cdot\pi_j^2+\pi_j^3\geq-\xi_{jt}. \quad (14)$$

If A_{jt}^L means no advertising, Equation (14) can be simplified to

$$\frac{P_j^1(\bar{A}_t,S_t)}{P_j^3(\bar{A}_t,S_t)}\cdot\pi_j^1+\frac{P_j^2(\bar{A}_t,S_t)}{P_j^3(\bar{A}_t,S_t)}\cdot\pi_j^2+\pi_j^3 -\frac{c(A_{jt})}{P_j^3(\bar{A}_t,S_t)}\geq-\xi_{jt}. \quad (15)$$

In GSP auctions, conditional on observed $\bar{A}_t$, let A_{jt}^U be one position higher than advertiser j's current position, and let the alternative scenario $\bar{A}_t^{U,j}$ denote that advertiser j exchanges the position with the advertiser at A_{jt}^U, while other ad positions remain the same. We make use of the following condition to ensure that $\bar{A}_t$ is at equilibrium:

i. An advertiser cannot improve its payoff by exchanging bids with the advertiser ranked one position above (i.e., A_{jt}^U).

This is the locally envy-free assumption in Edelman et al. (2007). This condition suggests that

$$V_j(\bar{A}_t,S_t)\geq V_j(\bar{A}_t^{U,j}S_t)=P_j^1(\bar{A}_t^{U,j}S_t)\cdot\pi_j^1+P_j^2(\bar{A}_t^{U,j}S_t) \cdot\pi_j^2+P_j^3(\bar{A}_t^{U,j}S_t)\cdot(\pi_j^3+\xi_{jt})-c(A_{jt}^U), \quad (16)$$

which will derive the same inequality as in Inequality (12).

Let b_{jt} be the observed final bid of the advertiser and $c(b_{jt})$ be the implied cost for each search query if the advertiser pays for the bid. Under CPM $c(b_{jt})$ is equal to b_{jt} and under CPC it is equal to $P_j^2(\bar{A}_t,S_t)\cdot b_{jt}$. In GSP auctions it is optimal for bidders to shade their bids. We therefore assume that at equilibrium the following condition will hold:

ii. Advertisers do not bid more than their valuation for ad positions.

This condition helps us construct an inequality that is analogous to Inequality (15)

$$\frac{P_j^1(\bar{A}_t,S_t)}{P_j^3(\bar{A}_t,S_t)}\cdot\pi_j^1+\frac{P_j^2(\bar{A}_t,S_t)}{P_j^3(\bar{A}_t,S_t)}\cdot\pi_j^2+\pi_j^3 -\frac{c(b_{jt})}{P_j^3(\bar{A}_t,S_t)}\geq-\xi_{jt}.\text{[10]} \quad (17)$$

[10] This assumption is motivated by the concept of using the "Generalized English Auction" to approximate how, in GSP auctions, advertisers will converge to a long-run steady state in Edelman et al. (2007). They show that in such an auction the bid an advertiser submits will be lower than its valuation. Haile and Tamer (2003) estimate the bounds of the distribution of the valuation of bidders in English auctions, using two assumptions: (a) Bidders do not bid more than their valuation, and (b) bidders do not let opponents win at a price they are willing to pay. These assumptions are very similar to the equilibrium conditions we use here for GSP auctions. Varian (2007) models the auction as a simultaneous move game with complete information and derive bounds for advertisers' value of each consumer click based on the Nash equilibrium assumption. He constructs a bound based on the payoff that an advertiser can obtain by moving to lower positions, giving an inequality that is different from Inequality (17).

3.2. Boundaries and Inequalities

Suppose $A_{jt}=0$. Advertiser j cannot move to a lower position, $A_{jt}^L=\phi$. Inequalities (14) and (15) cannot be constructed. Because the advertiser does not submit a bid, Inequality (17) also cannot be used. We are at a boundary condition. These observations cannot be ignored from model estimation; otherwise we have the selection issue and our estimates will be biased. When there is only one boundary, Pakes et al. (2014) propose using the symmetry assumption for the distribution of ξ_{jt}. One may infer the bounds for the ξ's when $A_j=0$ from the distribution of high-valued ξ's, under which $A_j\neq 0$ and one can derive upper bounds. This procedure is valid as long as the distribution of ξ is symmetric. In our case, however, we may have two-way boundaries: In some periods, the advertiser cannot move down, and in other periods it cannot move up. The latter occurs when the advertiser is at the topmost position or, in the context of fixed prices, when all higher positions are occupied by other advertisers. To address this problem, we assume that the value per terminal click is never negative for advertisers such that $\tilde{\pi}_j^3=\pi_j^3+\xi_{jt}\geq 0$ for all t. This means that to maximize profits, an advertiser never sells at a loss, and the value per terminal click is never negative. We then obtain the following inequality:

$$\pi_j^3\geq-\xi_{jt}, \quad (18)$$

providing us the upper bound for the (negative of) low-valued ξ's. When $A_{jt}=0$, we use this inequality instead of Inequalities (14), (15), or (17). We then follow Pakes et al. (2014), assuming that the distribution of ξ_{jt} is symmetric with mean 0 which implies that the maximum of ξ_{jt} is equal to the maximum of $-\xi_{jt}$. As $\pi_j^3\geq\max_t(-\xi_{jt})\geq\xi_{jt}$, this gives the following inequality:

$$\pi_j^3\geq\xi_{jt}, \quad (19)$$

providing us the upper bound for high-valued ξ's. When $A_{jt}^U=\phi$, we use this inequality instead of Inequalities (12) and (13).

Combining Inequalities (12), (13), and (19), and given $E(\xi_{jt})=0$, we recognize that for $\bar{A}_t$ to be an equilibrium, we have the following inequality condition:

$$\begin{aligned}\frac{1}{T}\Bigg\{&\sum_{\substack{A_{jt}^U\neq\phi,\\ A_{jt}\neq 0}}\Bigg[\frac{P_j^1(\bar{A}_t,S_t)-P_j^1(\bar{A}_t^{U,j},S_t)}{P_j^3(\bar{A}_t^{U,j},S_t)-P_j^3(\bar{A}_t,S_t)}\cdot\pi_j^1\\ &\quad+\frac{P_j^2(\bar{A}_t,S_t)-P_j^2(\bar{A}_t^{U,j},S_t)}{P_j^3(\bar{A}_t^{U,j},S_t)-P_j^3(\bar{A}_t,S_t)}\cdot\pi_j^2-\pi_j^3\\ &\quad-\frac{c(A_{jt})-c(A_{jt}^U)}{\kappa\cdot[P_j^3(\bar{A}_t^{U,j},S_t)-P_j^3(\bar{A}_t,S_t)]}\Bigg]\\ &+\sum_{\substack{A_{jt}^U\neq\phi,\\ A_{jt}=0}}\Bigg[-\frac{P_j^1(\bar{A}_t^{U,j},S_t)}{P_j^3(\bar{A}_t^{U,j},S_t)}\cdot\pi_j^1-\frac{P_j^2(\bar{A}_t^{U,j},S_t)}{P_j^3(\bar{A}_t^{U,j},S_t)}\cdot\pi_j^2\\ &\quad-\pi_j^3+\frac{c(A_{jt}^U)}{\kappa\cdot P_j^3(\bar{A}_t^{U,j},S_t)}\Bigg]+\sum_{A_{jt}^U=\phi}\pi_j^3\Bigg\}\\ \geq{}&\frac{1}{T}\sum_t\xi_{jt}\rightarrow E(\xi_{jt})=0\quad\text{as } T\rightarrow\infty.\end{aligned}\tag{20}$$

For GSP auctions, we combine Inequalities (12) and (19) to obtain a similar condition.

Combining Inequalities (14), (15), and (18), we obtain another inequality condition

$$\begin{aligned}\frac{1}{T}\Bigg\{&\sum_{\substack{A_{jt}^L\neq\phi,\\ A_{jt}^L\neq 0}}\Bigg[\frac{c(A_{jt})-c(A_{jt}^L)}{\kappa\cdot[P_j^3(\bar{A}_t^{U,j},S_t)-P_j^3(\bar{A}_t,S_t)]}\\ &\quad-\frac{P_j^1(\bar{A}_t,S_t)-P_j^1(\bar{A}_t^{L,j},S_t)}{P_j^3(\bar{A}_t^{L,j},S_t)-P_j^3(\bar{A}_t,S_t)}\cdot\pi_j^1\\ &\quad-\frac{P_j^2(\bar{A}_t,S_t)-P_j^2(\bar{A}_t^{L,j},S_t)}{P_j^3(\bar{A}_t^{L,j},S_t)-P_j^3(\bar{A}_t,S_t)}\cdot\pi_j^2+\pi_j^3\Bigg]\\ &+\sum_{A_{jt}^L=0}\Bigg[\frac{P_j^1(\bar{A}_t,S_t)}{P_j^3(\bar{A}_t,S_t)}\cdot\pi_j^1+\frac{P_j^2(\bar{A}_t,S_t)}{P_j^3(\bar{A}_t,S_t)}\cdot\pi_j^2+\pi_j^3\\ &\quad-\frac{c(A_{jt})}{\kappa\cdot P_j^3(\bar{A}_t,S_t)}\Bigg]+\sum_{A_{jt}=0}\pi_j^3\Bigg\}\\ \geq{}&\frac{1}{T}\sum_t-\xi_{jt}\rightarrow -E(\xi_{jt})=0\quad\text{as } T\rightarrow\infty.\end{aligned}\tag{21}$$

For GSP auctions, we combine Inequalities (17) and (18) to obtain a similar condition.

3.3. Method of Moment Inequalities

We use the MM inequalities proposed by Pakes et al. (2014) to estimate advertisers' valuations π_j^1, π_j^2, and π_j^3. Let $\bar{A}$ be the collection of $\bar{A}_t$ and S be the collection of S_t for all t. Also let $\theta_j=(\pi_j^1,\pi_j^2,\pi_j^3)'$ be the vector of value parameters to be estimated. Furthermore, let $\Delta^U V(\bar{A},\bar{A}^{U,j},S,\theta_j)$ denote the left-hand side of Inequality (20), and $\Delta^L V(\bar{A},\bar{A}^{L,j},S,\theta_j)$ denote the left-hand side of Inequality (21), where $\bar{A}^{U,j}$ and $\bar{A}^{L,j}$ are the collections of $\bar{A}_t^{U,j}$ and $\bar{A}_t^{L,j}$, respectively.

For the model estimation, we use instruments W_{jt}, which provide a vector of variables such that $E(\xi_{jt}\mid W_{jt})=0$.[11] Define $\mathbf{\Delta V}(\bar{A}_t,\bar{A}_t^{U,j},S_t,\theta_j)=[\Delta^U V(\bar{A}_t,\bar{A}_t^{U,j},S_t,\theta_j)\mid\Delta^L V(\bar{A}_t,\bar{A}_t^{L,j},S_t,\theta_j)]$, and

$$\mathbf{W}_{jt}=\begin{pmatrix}W_{jt} & 0\\ 0 & W_{jt}\end{pmatrix}.$$

Let $m(W_{jt},\theta_j)=\mathbf{W}_{jt}'\mathbf{\Delta V}(\bar{A}_t,\bar{A}_t^{U,j},S_t,\theta_j)$ and

$$\mathbf{P}_T m(\mathbf{W}_j,\theta_j)=\frac{1}{2T}\sum_{t=1}^{T}m(W_{jt},\theta_j).$$

Furthermore, we have $(\mathbf{D}_{jT}^{1/2}\mathbf{P}_T m(\mathbf{W}_j,\theta_j))_-=\min\{0,(\mathbf{W}_{jt}'\mathbf{W}_{jt}')^{-1/2}\mathbf{P}_T m(\mathbf{W}_j,\theta_j)\}$. The value thus is negative if the inequality conditions are violated, and zero otherwise. The estimator of the MM inequalities can be obtained from minimizing the violation of the inequality conditions, as follows:

$$\hat{\theta}_j=\underset{\theta\in\Theta}{\arg\min}(\mathbf{D}_{jT}^{1/2}\mathbf{P}_T m(\mathbf{W}_j,\theta_j))_-'(\mathbf{D}_{jT}^{1/2}\mathbf{P}_T m(\mathbf{W}_j,\theta_j))_-.\tag{22}$$

There is no guarantee that $\hat{\theta}_j$ is a singleton in the parameter space because there may be a set of estimates all of which can satisfy Inequalities (20) and (21). Therefore, we search for the set estimates in the parameter space.

Finally, to calculate the standard errors for estimators, Pakes et al. (2014) offer an analytical asymptotic distribution, though it cannot be applied directly because of the complexities in our study. First, ξ_{jt} may be serially correlated and correlated across advertisers within any period. Second, we must account for the errors of our estimates in the first-stage consumer search model, which affect the calculation of $P_j^1(\bar{A}_t,S_t)$, $P_j^2(\bar{A}_t,S_t)$, and $P_j^3(\bar{A}_t,S_t)$. To our knowledge, there is no existing analytical distribution that accounts for these issues. Instead, we propose using a semi-parametric overlapping-block bootstrapping procedure to calculate the standard errors for $\hat{\theta}_j$. The estimation and bootstrapping procedures are discussed in Appendix C.

4. An Empirical Application

4.1. Data

We apply the proposed methodology of estimating the models of consumer search and advertiser competition to a data set obtained from a leading search

[11] We can include in W_{jt} all variables in X_{jt} (advertiser identities and selling propositions) in Equation (3), and variables in Z_{it} (month and day of week indicators) in Equation (7).

engine firm in Korea. We observe which sponsored ads are displayed in response to the consumer's search query, and which sponsored ads are clicked. We also have data on whether the consumer clicks on any organic links. However, we observe neither the sequence of clicks nor the post-click conversion behavior.

The search engine uses CPM pricing, and offers potential advertisers up to five ad positions in the sponsored section of the search results page. Ad positions are sold through first-price position auctions with a BIN option for each position.[12] In our data, advertisers always exercised BIN options to acquire ad positions, implying that the market features are essentially fixed prices. Wang et al. (2008) show that, when the BIN option is adopted in auctions, the resulting price format at equilibrium could be fixed prices (i.e., buyers always exercise the BIN option) if participation costs for advertisers are sufficiently high. They also show that the BIN option can increase sellers' revenue, relative to the pure auction mechanism. Other studies that assume bidders to be risk averse (e.g., Budish and Takayama 2001, Reynolds and Wooders 2009) or impatient (e.g., Mathews 2004) also obtain similar results. We assume that fixed prices are the unique equilibrium in our empirical context because advertisers have high participation costs. Another important feature of the data is that the search engine adopts a "no regret" selling rule that allows advertisers, after purchasing ad positions, to change positions and pay different prices as long as the positions have not been sold. This practice represents an effort to maintain long-term relationships with advertisers, rather than confronting them with regret. It supports our equilibrium assumption since advertisers would have an incentive to switch to an available position if doing so increased their payoffs.

We apply the proposed methods to a single keyword, related to a brand of sporting goods (e.g., footwear, apparel, accessories). During the data period (1/1/2008 to 9/10/2008), six advertisers, all popular retailers in Korea, purchased ad positions in sponsored listings. For each sponsored link, we observe the selling propositions of the advertiser, which typically describes the products and offers to consumers. We control for the two variables, price discount and assortment information, that may influence how users respond to an ad.[13] On average, users conduct 211 search queries daily. We observed that 65.6% of searches led to clicks only on organic listings, compared with 3.7% of searches that produced only clicks on sponsored listings, and 5.5% with clicks for both. For searches that prompted clicks on sponsored links, approximately 30% involved clicking multiple links.

In Figure 1, we illustrate the BIN prices of the ad positions throughout the data period. The topmost position is the most expensive (mean = 2.8 cents) and the bottom position is the least expensive (mean = 1.5 cents).[14] Prices have changed considerably over the data period. For the topmost position, the price started at 3.2 cents, dropped to 2.7 cents, and then fell to 2.3 cents. Lower positions also experienced price declines. Our model assumes that BIN prices are exogenously set. Figure 2 shows the ad positions obtained by advertisers. These advertisers purchase nominal ad positions, but the actual positions will move up if a high position has not been sold. The directional characteristics of the sponsored listings imply that the position of an advertiser's ad on the search results page depends on the decisions of other advertisers, as well as its own. On average 2.98 ads are displayed each day, with a standard deviation of 1.45. Advertisers generally do not change their positions, indicating stickiness in the value of ad positions. However, we also observe some abrupt changes in positions (e.g., Advertiser 1).

4.2. Estimation Results

Actual ad positions, denoted by $\bar{a} = (a_{1t}, a_{2t}, \ldots, a_{Jt})'$, can differ from the nominal ad positions $\bar{A}_t$ that advertisers purchased. We infer $\bar{a}_t$ from $\bar{A}_t$ as $\bar{a}_t = \text{rankindex}(\bar{A}_t \mid \bar{A}_t \neq 0)$, where rankindex is an operator that returns the rank of $\bar{A}_t$ (conditional on $\bar{A}_t \neq 0$) from smallest to largest. The likelihood function in Equation (10) is estimated based on the actual ad positions.

We estimate the proposed search model with various numbers of consumer segments, each with a different set of model parameters. The two-segment model performs best, in terms of the Akaike information criterion (AIC) and the Bayesian information criterion (BIC). To determine the fit of this two-segment model, we computed the predicted CTR and TCTR

[12] Auctions open at 9:00 A.M. and close at 4:00 P.M. each day. All ad positions are available as long as they have not been sold or the day has not passed. The BIN option remains in effect throughout the auction as long as it has not been exercised. Advertisers can bid for any unsold ad position auction, and may buy a position for multiple days. We do not have data on the time at which advertisers make their decisions.

[13] All six retailers sell products of the brand. By clicking a sponsored link, consumers are directed to the retailer's website where branded products can be found. In addition, selling propositions are all related to the brand. This characterizes the distinctive feature of search advertising that gives advertisers the opportunity of reaching consumers at the time when they are ready to make purchases.

[14] All prices are in U.S. currency. During the data collection period, foreign exchange rates were highly volatile, but we simplify matters by equating 1,000 Korean won to approximately \$1 U.S.

Figure 1 BIN Prices

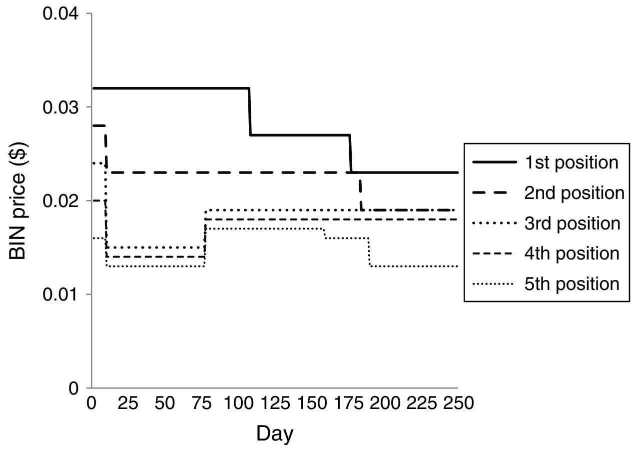


Figure 2 Advertiser Positions

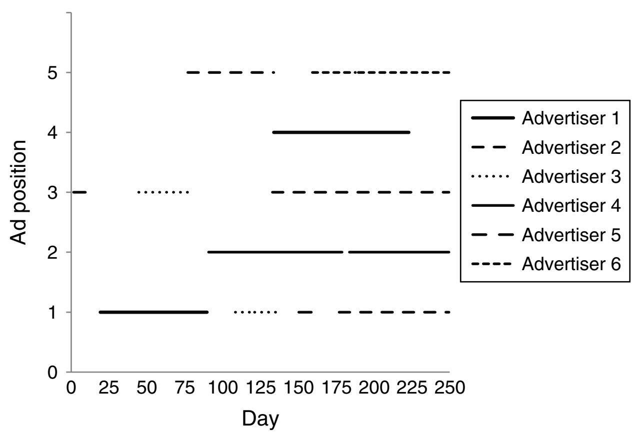
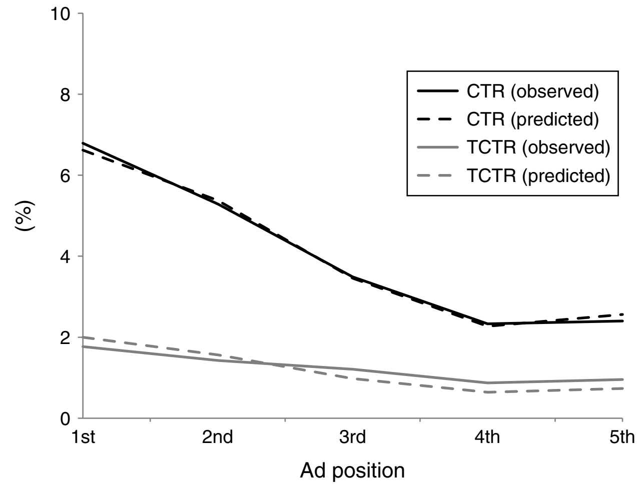


Figure 3 Model Fit



from the model, compared with the observed values at each ad position over the data period. Figure 3 presents the average observed and the predicted CTR and TCTR of the five ad positions, which are closely matched. Overall, the model predicts CTR at a 0.24% mean absolute error (MAE) and TCTR at a 0.21% MAE. This reinforces the validity of the proposed model.[15] Moreover, our model can explain the nonmonotonic relationship in CTR and TCTR between the fourth and fifth ad positions. Therefore, although the top-down sequential search assumptions may be restrictive, our model captures consumer search behavior well.

The key parameter estimates of the two-segment consumer search model are presented in Table 1. The upper panel contains the coefficients for variables that influence the reservation utility for clicking a sponsored link (z_1); the middle panel features the coefficients for variables that influence the reservation utility for stopping the search (z_2). A positive coefficient indicates that the higher the value of the corresponding variable, the lower the probability of clicking a sponsored link or stopping the search after having clicked. Finally, the lower panel of Table 1 contains the coefficients of ad attributes that affect consumers' expected utility of clicking the advertised website. A positive coefficient indicates that a higher value of the corresponding attribute increases the probability that a consumer will click and then terminate the search.[16]

We have one large segment (94.8%) and one small segment (5.2%) of consumers.[17] Those in the low-involvement segment 1 are less likely to click any of the links, judging from the large positive constant coefficient in z_1. Once they click a link, these users are more likely to stop the search, since the constant coefficient in z_2 is much smaller than that for segment 2. By contrast, those in the smaller, high-involvement

[15] We also compared the observed and predicted CTR and TCTR of different ad positions daily; despite large daily fluctuations, the variations are highly correlated (e.g., observed and predicted CTR [TCTR] for the top position correlate at 0.68 [0.50]).

[16] In the model estimation, we assume ω's in Equation (8) are normally distributed and normalize the variances of ω^1_{ijt} and ω^2_{ijt} to 1. We also assume that they are uncorrelated. Estimated variances for consumer-specific ω_{it} and advertiser-specific ω_{jt}, and their covariances, are small and insignificant, probably due to the sparseness of clicking on sponsored links. To identify variances and covariances of ω^1_{it} and ω^2_{it}, for example, we need sufficient variation in the number of clicks across consumers. As more than 90% of searches do not lead to any clicks on sponsored links, and only about 3% of searchers click multiple links, it is difficult to obtain precise estimates for the variances and covariances. Advertisers (especially those at lower ad positions) only attract a few daily clicks on average, so it is also difficult to obtain precise estimated variances and covariances of ω^1_{jt} and ω^2_{jt}.

[17] On average, users conduct 211 search queries daily, and the data period spans more than several months. Assuming they come from unique users, the sample size is large enough to obtain significant estimates even though segment 2 is small.

Table 1 Consumer Search Model Results

	Two-segment model			
	Segment 1		Segment 2	
Parameter	Estimate	Std. err.	Estimate	Std. err.
z_1				
Constant	**1.243**	0.017	**−0.165**	0.022
Tuesday	**−0.083**	0.042	**−0.102**	0.053
Wednesday	−0.074	0.044	0.036	0.066
Thursday	−0.072	0.043	0.005	0.060
Friday	0.079	0.050	0.045	0.056
Saturday	**−0.127**	0.043	**−0.118**	0.062
Sunday	−0.028	0.052	**−0.188**	0.062
z_2				
Constant	0.117	0.063	**1.549**	0.042
Tuesday	**0.313**	0.162	−0.082	0.100
Wednesday	−0.071	0.154	−0.063	0.109
Thursday	0.013	0.158	−0.176	0.106
Friday	−0.096	0.180	−0.011	0.114
Saturday	−0.027	0.156	**−0.374**	0.114
Sunday	−0.062	0.181	−0.051	0.126
X				
Advertiser 2	**−0.141**	0.037	**0.298**	0.058
Advertiser 3	−0.935	0.623	−0.054	0.071
Advertiser 4	**−0.437**	0.044	**0.649**	0.043
Advertiser 5	**−0.111**	0.029	−0.003	0.047
Advertiser 6	**−1.328**	0.135	0.014	0.066
ln(Discount + 1)	**0.037**	0.008	−0.005	0.011
ln(#Categories + 1)	**−0.381**	0.025	**−0.712**	0.032
Pr(Segment 1)	**0.948**	0.002		

Notes. Parameter estimates for month dummies are not included. Numbers in bold are significant at the 95% significance level.

segment 2 are more likely to click any of the links, then less likely to stop the search.

With respect to the advertiser-specific attributes, we first note that for segment 1, the fixed effects for most advertisers are significantly smaller than that of Advertiser 1 (which is normalized to 0). This indicates that the majority of consumers prefer Advertiser 1 over the others. Even though only about 5% of consumers fall into segment 2, they are more likely to click. Because they also have a high preference for Advertisers 2 and 4, this small group of consumers helps generate higher CTR and subsequently higher TCTR for the two advertisers than the others. Advertisers 3 and 6 show the lowest CTR and TCTR. These results illustrate that, depending on which segment consumers belong to, advertisers can be heterogeneous in their attractiveness for clicks and terminal clicks from consumers.

Consumers in segment 1 are attracted by discount-related information, according to the positive and significant estimate for ln(Discount + 1), but price discount has no effect on consumers in segment 2. Assortment-related information in the selling propositions reduces consumer clicks; the estimates for ln(#Categories + 1) are significantly negative for both segments. Perhaps consumers expect popular online retailers to carry a full line of products associated with the keyword, so promoting only a few selected product categories (e.g., men's shoes) fails to convey value to consumers.

To investigate the value of consumer search for advertisers, we estimate two model specifications, using the results from the two-segment consumer model. Model 1 (upper panel in Table 2) assumes that every advertiser has the same value per effective impression, click, and terminal click. We also investigate the possibility that the values differ across advertisers. Because Advertisers 1 and 2 adopt an open market format, their profits from sales may differ from other retailers. In Model 2, we estimate the value per effective impression, click, and terminal click of Advertisers 1 and 2 separately from the others (lower panel in Table 2).

To estimate the advertiser values using moments of inequality, we use instruments including the advertisers' identities and day of the week indicators. We also include the CPM for each ad position and ad attributes as instruments. The first two columns in Table 2 contain the lower and upper bounds of each estimated value in the two models, in which all instruments are used. The lower and upper bounds are the same, which implies that we have obtained point estimates. Surprisingly, the values per effective impression and click are all zero; advertisers do not appear to perceive any benefits from search activities that do not lead to immediate transactions. The value of ad positions comes from terminal clicks only. On average, one terminal click is valued at \$34.4. The next two columns in Table 2 report the bootstrapped 90% confidence intervals of the estimates.[18] We caution that the results may differ for keywords in which the market is highly fragmented, with a large number of small businesses, because visits to these websites and ad browsing can help increase consumer awareness of these websites, which would be valuable to such advertisers.

Results from Model 2 again confirm that the values per effective impression and click are all zero. Although the upper 90% confidence interval for Advertisers 1 and 2 under Model 2 are greater than zero, their monetary values are trivially small, compared with the estimated value per terminal click. The estimated value per terminal click is much higher for Advertisers 1 and 2 than for other advertisers (\$64.2 versus \$12), and the difference is statistically significant according to the bootstrapped 90% confidence intervals.

[18] We use the lowest 5th percentile of the bootstrapped lower-bound estimates to measure the lower intervals, and the highest 5th percentile of the bootstrapped upper-bound estimates to measure the upper intervals.

Table 2 Advertiser Valuation ($)

	All instruments				Without CPMs		Without CPMs and X_{jt}	
	Estimates		90% CI		Estimates		Estimates	
	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound	Lower bound	Upper bound
Model 1								
Per effective impression	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Per click	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Per terminal click	34.4	34.4	16.9	55.8	34.6	34.6	31.8	31.8
Model 2								
Advertisers 1 and 2								
Per effective impression	0.0	0.0	0.0	0.04	0.0	0.0	0.0	0.0
Per click	0.0	0.0	0.0	0.05	0.0	0.0	0.0	0.0
Per terminal click	64.2	64.2	32.1	98.0	65.6	65.6	66.5	55.5
Advertisers 3, 4, 5, and 6								
Per effective impression	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Per click	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Per terminal click	12.0	12.0	6.6	19.0	12.4	12.4	12.6	12.5

The validity of the instruments W_{jt} requires $E(\xi_{jt} \mid W_{jt}) = 0$. To test the exogeneity assumption of these variables, we estimate the models with different sets of instruments. We have more instruments than necessary to estimate advertisers' values, and our test is similar to the over-identification test for instrument validity (Hansen 1982). To check whether the CPMs of ad positions correlate with ξ_{jt}, such as when the search engine sets prices strategically, we exclude CPMs from W_{jt} (see the "Without CPMs" columns in Table 2). We further exclude X_{jt} from W_{jt} because they include a price discount that may correlate with ξ_{jt} (see the last two columns of Table 2). The estimation results are close to the estimates with the whole set of instruments. Such robustness implies that the exogeneity assumptions of CPMs and advertiser attributes are likely valid.[19]

These findings have important managerial and economic implications. Search engines typically report CTR and impressions of a keyword to advertisers as key performance metrics. Our results suggest that search engines may also report the TCTR for each ad position because it would help advertisers evaluate the returns of their search advertising investments. As typical clickstream data tracks the time of consumer clicks, search engines and advertisers do not have to rely on the sequential search model proposed in this study to infer terminal clicks. With such concrete performance information, search engines can attract advertisers by reducing their uncertainty about search advertising. We also emphasize that one cannot infer TCTR just from CTR. As shown in Figure 3, for example, TCTR is not proportional to CTR across ad positions: CTR for the topmost position is 6.80%, with about 26% of TCTR. By contrast, for the fourth (fifth) position, CTR is 2.33% (2.40%), with about 37% (40%) of TCTR.

4.3. Determinants of the Value of Ad Positions

The unique feature of search advertising is that it gives an advertiser the opportunity to reach consumers when they are ready to make purchases. The value of an ad position is not exogenously given; it is the consumer search activities that the ad position can attract, multiplied by the advertiser values. Consumer search activities depend not only on the advertisers in sponsored listings at different ad positions but also on the composition of users who exhibit different search behaviors.

To demonstrate how the value of ad positions is generated, we conduct simulations to compare each of the three metrics (i.e., rate of effective impression P_j^1, CTR P_j^2, and TCTR P_j^3) and the advertiser value under two scenarios, each with different advertisers in sponsored listings. In the first scenario, Advertiser 1 occupies the topmost position and, in the second scenario, Advertiser 2 is on top. Lower positions 2 to 5 are occupied by the same set of advertisers. The three metrics and advertiser values are calculated from the estimation results from Model 2 (see Table 2), where the values from a terminal click for Advertisers 1 and 2 are the same. The difference in value between the two scenarios, therefore, is not driven by the difference in the advertiser value for consumer search activities.

The top panel in Table 3 reports the results of the first scenario, and the bottom panel reports the results

[19] We believe that CPM can be exogenous to the profit shock because we have already incorporated month indicators (and weekday indicators) to capture macro demand shocks in the consumer model. Suppose the profit shocks are independent across advertisers. The search engine may not have information on how the profit of each advertiser, who carries tens of thousands of branded products, changes daily for the particular brand considered in this research. Thus, CPM for each sponsored position may not be affected by the shocks.

Table 3 Search Activities and the Value of Ad Positions

Scenario 1		Segment 1				Segment 2				Two segments combined			
Rank	Advertiser	P_j^1	P_j^2	P_j^3	Ad value ($)	P_j^1	P_j^2	P_j^3	Ad value ($)	P_j^1	P_j^2	P_j^3	Ad value ($)
1	1	1.000	0.121	0.058	3.748	1.000	0.551	0.038	2.429	1.000	0.144	0.057	3.679
2	5	0.942	0.094	0.041	0.496	0.962	0.529	0.036	0.433	0.943	0.117	0.041	0.493
3	4	0.900	0.049	0.015	0.184	0.926	0.724	0.146	1.749	0.902	0.084	0.022	0.265
4	3	0.885	0.016	0.003	0.031	0.780	0.413	0.026	0.307	0.880	0.036	0.004	0.045
5	6	0.882	0.006	0.000	0.006	0.755	0.420	0.030	0.356	0.876	0.027	0.002	0.024
Sum													
All ranks		4.609	0.286	0.118	4.464	4.423	2.638	0.275	5.274	4.600	0.408	0.126	4.506
Ranks 2–5		3.609	0.164	0.060	0.716	3.423	2.087	0.237	2.844	3.600	0.264	0.069	0.827

Scenario 2		Segment 1				Segment 2				Two segments combined			
Rank	Advertiser	P_j^1	P_j^2	P_j^3	Ad value ($)	P_j^1	P_j^2	P_j^3	Ad value ($)	P_j^1	P_j^2	P_j^3	Ad value ($)
1	2	1.000	0.095	0.040	2.600	1.000	0.665	0.078	5.015	1.000	0.125	0.042	2.725
2	5	0.960	0.096	0.042	0.505	0.922	0.507	0.035	0.415	0.958	0.118	0.042	0.501
3	4	0.917	0.050	0.016	0.187	0.887	0.694	0.140	1.675	0.916	0.083	0.022	0.265
4	3	0.902	0.016	0.003	0.031	0.748	0.396	0.020	0.294	0.894	0.036	0.004	0.045
5	6	0.899	0.006	0.000	0.006	0.723	0.403	0.028	0.341	0.890	0.026	0.002	0.023
Sum													
All ranks		4.678	0.263	0.101	3.330	4.280	2.665	0.305	7.740	4.657	0.387	0.112	3.559
Ranks 2–5		3.678	0.167	0.061	0.730	3.280	2.000	0.227	2.725	3.657	0.263	0.069	0.833

for the second. For each case, the first (second) four columns under Segment 1 (2) report the three metrics and the advertiser value from a search query in segment 1 (2). The last four columns report the aggregated results weighted by segment sizes. Advertiser 1, as compared to Advertiser 2, generates more value by 27% ($3.679 for Advertiser 1 in scenario 1 versus $2.725 for Advertiser 2 in scenario 2) by placing its ad at the top. This is because Advertiser 1 is more likely to generate terminal clicks than Advertiser 2. For the same reason, however, having Advertiser 1 on top implies that advertisers listed below can attract fewer terminal clicks. The total value for lower-ranked advertisers in the first scenario is smaller by 1% than that in the second scenario ($0.827 in scenario 1 versus $0.833 in scenario 2). By comparison, the increased value for these advertisers, when we move their ads up by one position, is about 5%. This suggests that changing the competing advertiser placed at the top has a non-negligible profit impact on the advertisers listed below.

Table 3 also highlights the impact of different types of consumers on the advertiser value from the search query. For each search query, a consumer in segment 2 on average clicks 2.6 sponsored links in the two scenarios, compared to 0.3 clicks from a consumer in segment 1. The probability that a consumer in segment 2 will terminate the search is approximately 30%, compared to approximately 10% for a consumer in segment 1. Their search behaviors are distinctly different. In line with these search behaviors, a search query from the high-involvement segment 2 generates much higher value than the low-involvement segment 1 ($5.274 versus $4.464 in scenario 1; $7.740 versus $3.330 in scenario 2). Yet Advertiser 1 generates more value from the latter segment ($3.748 from segment 1 versus $2.429 from segment 2). The consumer type is especially important for Advertiser 4, which is placed at the third position in both scenarios. Its value from a search query from segment 2 is about ten times higher than from segment 1. Our results demonstrate that for advertisers the value of a search query depends not only on the identities and positions of advertisers in the sponsored listings but also on what type of consumers conduct the keyword search. The search engine may use past search history to identify which segment a consumer belongs to and create targeted advertisements to maximize the value for advertisers.

5. Conclusions

This paper proposes a methodology to study how consumer search activities drive the value of ad positions in sponsored search advertising and to identify the determinants for the search activities at ad positions. We develop a structural model of consumer search that allows us to construct measures of effective impression, click, and terminal click. This model also describes how consumer search activities at each position depend on the positions occupied by competitors. To estimate the model of advertiser competition, we use the necessary equilibrium conditions and adopt the MM inequalities (Pakes et al. 2014) to tackle the associated estimation challenges. While we apply this estimation strategy under fixed prices, it can be applied to other pricing mechanisms, e.g., GSP auctions, CPM, and CPC pricing mechanisms. Using a data set obtained from a search engine, we demonstrate that the value of ad positions depends not only on the identities and positions of the advertisers in sponsored listings but also on the composition of online consumers who exhibit very different search behaviors.

Several limitations of this study might be addressed by further research. Our empirical application only studies a particular branded keyword, and all of the advertisers were popular online retailers. Future research can build on our modeling approach to explore how and why advertiser values for consumer search activities may vary with other types of keywords and advertisers. Moreover, we used terminal clicks to proxy for consumer actions. Richer data, including sales data from advertisers and users' search activities on the search results page, could further enhance understanding of consumer behavior in keyword search. Finally, online retailers usually purchase multiple, similar keywords to attract consumer demand. Future research should examine the competition among advertisers for multiple keywords. Our study provides a possible framework for such further empirical explorations.

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Appendix A. Consumer Search

1. If $\bar{u} \geq z_1$, the consumer clicks the link.

Proof. Substitute z_2 from the optimal decision rule in the second stage into Equation (3). Then,

$$V(u) = -c_1 + \max\left\{-c_2 + \int_{z_2}^{\infty} (u - z_2)\, dF(u \mid X, e) + z_2, z_2\right\}$$
$$= -c_1 + z_2 + \max\left\{-c_2 + \int_{z_2}^{\infty} (u - z_2)\, dF(u \mid X, e), 0\right\}.$$

This expression indicates that, given z_2 for the optimal rule of stopping the search, the consumer clicks the link if

$$-c_2 + \int_{z_2}^{\infty} (u - z_2)\, dF(u \mid X, e) \geq 0. \tag{A1}$$

Because $u = X\beta + e + \varepsilon$, the left-hand side in Equation (A1) can be rewritten as

$$-c_2 + \int_{z_2-(X\beta+e)}^{\infty} (X\beta + e + \varepsilon)\, dF(\varepsilon) - z_2[1 - F(z_2)],$$

using change of variable and $d\varepsilon/du = 1$. Let z_1 be the reservation utility of the clicking decision, such that

$$-c_2 + \int_{z_2-z_1}^{\infty} (z_1 + \varepsilon)\, dF(\varepsilon) - z_2[1 - F(z_2)] = 0. \tag{A2}$$

The optimal clicking rule in Equation (A1) can be rewritten as follows: If $\bar{u} \geq z_1$, the consumer clicks the link.

2. Equation (A2) has a unique solution.

Proof. Let $\kappa(z_1 \mid z_2) \equiv \int_{z_2-z_1}^{\infty} (z_1 + \varepsilon)\, dF(\varepsilon)$. Then we can derive

$$\frac{\partial \kappa(z_1 \mid z_2)}{\partial z_1} = [1 - F(z_2 - z_1)] + z_2 \cdot f(z_2 - z_1) > 0.$$

That is, $\kappa(z_1 \mid z_2)$ is an increasing function of z_1. Therefore, given z_2, Equation (A2) has a unique solution, and a unique z_1 exists for the optimal clicking rule in the first stage.

3. $\partial z_2/\partial c_1 > 0$.

Proof. Using Equation (3) for the second-stage decision, and $z_2 = EV(u)$, we know

$$z_2 = -c_1 + E_{X,\xi} \max\left\{-c_2 + \int_{z_2}^{\infty} (u - z_2)\, dF(u \mid X, \xi) + z_2, z_2\right\}$$
$$\Rightarrow \quad c_1 = E_{X,\xi} \max\left\{-c_2 + \int_{z_2}^{\infty} (u - z_2)\, dF(u \mid X, \xi), 0\right\}.$$

Furthermore, using the optimal clicking rule in the first stage, we have

$$c_1 = \int_{z_1}^{\infty} \left[-c_2 + \int_{z_2-\bar{u}}^{\infty} (\bar{u} + \varepsilon - z_2)\, dF(\varepsilon)\right] dF(\bar{u}(X, \xi)). \tag{A3}$$

Let $h(z_2) = \int_{z_2-\bar{u}}^{\infty} (\bar{u} + \varepsilon - z_2)\, dF(\varepsilon)$. Then,

$$\partial h(z_2)/\partial z_2 = -[1 - F(z_2 - \bar{u})] - 0 \cdot f(z_2 - \bar{u}) < 0.$$

To maintain equality in Equation (A3), a higher search cost in the first stage, conditional on c_2 and z_1, leads to a lower reservation utility in the second stage. That is, $\partial z_2/\partial c_1 > 0$.

Appendix B. Probabilities of Effective Impression, Click, and Terminal Click

1. Pr(Effective impression)

For advertiser j at an ad position L, the probability of effective impression is as follows:

$$\begin{aligned}
&\Pr(i \text{ browses } j)\\
&\quad = \prod_{k=1}^{L-1} \Pr(i \text{ does not terminate at } k)\\
&\quad = \int_{\omega_1^1, \omega_1^2} \cdots \int_{\omega_{L-1}^1, \omega_{L-1}^2} \cdot \int_{\omega_i^1, \omega_i^2} \prod_{k=1}^{L-1} [1 -\\
&\qquad \Pr(\omega_{ik}^1 \geq Z_i\gamma_1 - X_k\beta - \omega_k^1 - \omega_i^1 \text{ and}\\
&\qquad \omega_{ik}^2 \geq Z_i\gamma_2 - X_k\beta - \omega_k^2 - \omega_i^2)]\\
&\qquad \cdot dF(\omega_i^1, \omega_i^2)\, dF(\omega_1^1, \omega_1^2, \ldots, \omega_{L-1}^1, \omega_{L-1}^2)\\
&\quad \equiv P_j^1(X, Z; L).
\end{aligned} \tag{B1}$$

2. Pr(Click)

$$\begin{aligned}
&\Pr(i \text{ browses and searches } j)\\
&\quad = \prod_{k=1}^{L-1} \Pr(i \text{ does not stop at } k)\\
&\qquad \cdot \Pr(i \text{ clicks into } j \mid i \text{ does not stop at } k)
\end{aligned}$$

$$= \int_{\omega_1^1,\omega_1^2} \cdots \int_{\omega_j^1,\omega_j^2} \cdot \int_{\omega_i^1,\omega_i^2} \left\{ \prod_{k=1}^{L-1} [1 - \Pr(\omega_{ik}^1 \geq Z_i\gamma_1 - X_k\beta - \omega_k^1 - \omega_i^1 \text{ and } \omega_{ik}^2 \geq Z_i\gamma_2 - X_k\beta - \omega_k^2 - \omega_i^2)] \cdot \Pr(\omega_{ij}^1 \geq Z_i\gamma_1 - X_j\beta - \omega_j^1 - \omega_i^1) \right\} \cdot dF(\omega_i^1, \omega_i^2)\, dF(\omega_1^1, \omega_1^2, \ldots, \omega_j^1, \omega_j^2)$$
$$\equiv P_j^2(X, Z; L). \tag{B2}$$

3. Pr(Terminal click)

Pr(i browses, clicks, and terminates search at j)

$$= \prod_{k=1}^{L-1} [\Pr(i \text{ does not terminate at } k)] \cdot \Pr(i \text{ terminates at } j \mid i \text{ does not terminate at } k]$$
$$= \int_{\omega_1^1,\omega_1^2} \cdots \int_{\omega_j^1,\omega_j^2} \cdot \int_{\omega_i^1,\omega_i^2} \left\{ \prod_{k=1}^{L-1} [1 - \Pr(\omega_{ik}^1 \geq Z_i\gamma_1 - X_k\beta - \omega_k^1 - \omega_i^1 \text{ and } \omega_{ik}^2 \geq Z_i\gamma_2 - X_k\beta - \omega_k^2 - \omega_i^2)] \cdot \Pr(\omega_{ij}^1 \geq Z_i\gamma_1 - X_j\beta - \omega_j^1 - \omega_i^1 \text{ and } \omega_{ij}^2 \geq Z_i\gamma_2 - X_j\beta - \omega_j^2 - \omega_i^2) \right\} \cdot dF(\omega_i^1, \omega_i^2)\, dF(\omega_1^1, \omega_1^2, \ldots, \omega_j^1, \omega_j^2)$$
$$\equiv P_j^3(X, Z; L). \tag{B3}$$

Appendix C. Estimation Algorithm and Bootstrapping Standard Errors

Model estimates $\hat{\theta}_j$ minimize the value of the criterion function in Equation (22). The criterion function is not smooth and differentiable everywhere; therefore, one should use nonderivatives-based algorithms such as the simplex method to search for $\hat{\theta}_j$. When the dimensionality of model parameters is small (e.g., in our empirical application we only estimate the advertiser values π^1, π^2, and π^3 for each advertiser), one can also use the grid search method. After finding a point in the parameter space that minimizes the criterion function value, one should increase and decrease the value of each estimate to search for the upper bound and the lower bound of the estimate. This procedure will continue until the criterion function value grows when the estimate value further increases or decreases. Any value within the range of the upper bound and the lower bound minimizes the criterion function value thus is in the set estimates.

Pakes et al. (2014) derived a closed-form asymptotic distribution for the estimator $\hat{\theta}_j$. However, the derivation is based on the i.i.d. assumption of the stochastic term ξ across periods and advertisers, which may not be valid in our context. The MM inequality estimator is also based on the estimation of the first-stage consumer search model. Correct standard errors of $\hat{\theta}_j$ must account for the standard errors of the first-stage estimators. Because of these complications, we use the following semi-parametric bootstrapping procedure to calculate the standard errors for $\hat{\theta}_j$:

1. Let $\hat{\mathbf{B}} = (\hat{\gamma}_1, \hat{\gamma}_2, \hat{\beta})$ be the estimated parameters, and $\hat{\boldsymbol{\Sigma}}_{\mathbf{B}}$ be the estimated variance-covariance matrix for $\hat{\mathbf{B}}$, in the first-stage estimation. During each bootstrapping, we simulate $\hat{\mathbf{B}}^b = (\hat{\gamma}_1^b, \hat{\gamma}_2^b, \hat{\beta}^b)$ from the distribution $N(\hat{\mathbf{B}}, \hat{\boldsymbol{\Sigma}}_{\mathbf{B}})$. It is the parametric component in the bootstrapping procedure.

2. We non-parametrically bootstrap advertisers' purchasing decisions of ad positions from our data. We use the overlapping-blocked bootstrapping procedure. We randomly select (uniformly draw from the entire periods) a starting period and the following $l-1$ periods from our data (with l fixed to 10 periods). The choices of all advertisers A^b, c^b, and other variables that affect consumer click behavior (S^b) during these l periods, become data for bootstrapping. We repeat this procedure T/l times until we have simulated T periods of data. By bootstrapping blocks of periods and choices of all advertisers during these periods, we take the temporal and cross-sectional correlations of the stochastic term ξs into account.

3. With bootstrapped data $\hat{\mathbf{B}}^b$, A^b, c^b, and S^b, we reestimate the structural parameter θ using the MM inequalities and obtain $\hat{\theta}^b$.

4. We repeat the bootstrapping procedure B times (fixed to 1,000 times) and obtain $(\hat{\theta}^1, \hat{\theta}^2, \ldots, \hat{\theta}^B)$, then calculate the confidence interval of the estimate $\hat{\theta}$.

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