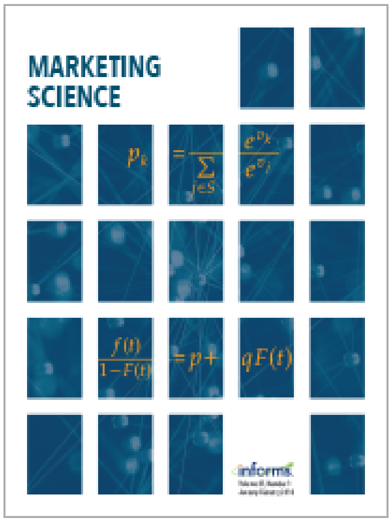

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Costs and Benefits of Inducing Intrabrand Competition: The Role of Limited Liability

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When is inducing intrabrand competition (via nonexclusive distribution) an optimal strategy? To address this issue, a static model is developed to examine two settings. The manufacturer uses exclusive distributors in the first setting and nonexclusive distributors in the second. The analysis indicates that the choice of distribution rests critically on whether the manufacturer can effectively extract surplus from the distributors. Due to a variety of institutional reasons, the distributors' liability is often limited in performing on behalf of the manufacturer; such limited liability restricts how much of the distributors' surplus can be extracted. When the distributors' surplus cannot be fully extracted, the manufacturer may prefer nonexclusive distribution even when distributors can free-ride on each other's efforts.

Key words: channels of distribution; agency theory; intrabrand competition; free-riding; limited liability; vertical contractual restrictions
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1. Introduction

Whether or not to assign exclusive territories has been a longstanding question in the literature. While extant research (see e.g., Rey and Stiglitz 1995, Mathewson and Winter 1984) provides useful insights on this choice, other important dimensions of the problem remain unexplored. This analysis examines a tradeoff associated with intrabrand competition and highlights the impact of distributors' limited liability on a manufacturer's decision to use exclusive territories.

More specifically, this paper develops a mathematical model to examine two scenarios. In the first, the manufacturer uses exclusive territories and protects its distributors from intrabrand competition. In the second, the distributors compete against each other to sell the manufacturer's product. The analysis compares these two scenarios and illustrates how the manufacturer's profit is affected by the limits on the distributors' liability.

Explicit limits on liability (or limited liability clauses) are a common feature in business contracts. A principal reason these clauses arise is due to equity considerations that mandate the guarantee of an appropriate level of well-being for all parties involved in the contract (Sappington 1983, Barton 1972). These limits on liability can take the form of bankruptcy clauses, statements outlining the conditions under which breach of contract is permissible, and other clauses involving product returns or cash compensation (Miller and Jentz 2000).[1]

In distribution channels, limited liability issues are likely to arise when demand is uncertain and distributors can experience significant losses under adverse demand conditions. To understand this, consider a typical characterization of the interaction between a manufacturer and its distributor: to eliminate all incentive problems, the manufacturer can sell to the distributor at cost and extract the surplus via a fixed fee. (In expectation, the profit balances out the loss. Another way to eliminate incentive problems would be to design a suitable quantity discount schedule to coordinate the distributor's actions (Jeuland and Shugan 1983).) The fixed fee will extract the expected surplus, and when demand is uncertain, the distributor will have a loss when demand is "low," but a strictly positive profit when demand is "high."

Limits on the distributor's liability, however, restrict how much loss the distributor can bear, and therefore all of the surplus cannot be extracted via the fixed

[1] For instance, business failure is one setting under which the distributor's liability can be limited. In the wholesale trade, which encompasses a variety of product categories, Dun and Bradstreet reports an overall failure rate of 71 per thousand firms in 1995 and 69 per thousand firms in 1996. These rates can be viewed as lower bounds on the prevalence rate of liability clauses in wholesale distribution channels.

fee. Consequently, to increase revenues, the manufacturer will tend to raise the wholesale price above cost. The higher wholesale price will distort the distributor's price from the requisite levels (due to the standard double-marginalization problem). Moreover, since effort is costly and demand is uncertain, the distributor has an opportunity to shirk its efforts to sell the manufacturer's product. These considerations are particularly relevant in the context of assigning exclusive territories.

In assigning exclusive territories, conventional wisdom suggests evaluating the costs of intrabrand competition. Practitioners indicate that exclusive territories often serve as an inducement to the selected distributors to put forth the requisite marketing effort,[2] including investments in demonstration equipment and hiring a technical expert. Admittedly, in the absence of exclusive territories, some of these efforts can help generate buyers who may subsequently purchase from a competing distributor selling the same manufacturer's products in that territory.

In other words, a distributor's investments can be jeopardized by the presence of intrabrand competition, especially in the early stages of market development, when returns are uncertain. Therefore, a positive linkage between the marketing investments of one distributor and the sales of a competing distributor can encourage free-riding[3] and decrease a distributor's incentive to invest in the manufacturer's products. In the presence of such positive linkages, even some academic research supports the imposition of exclusive territories (e.g., Mathewson and Winter 1984).

In contrast, other research suggests that intrabrand competition can motivate distributors to expend the requisite marketing efforts on the manufacturer's behalf. First, tournament theory (see e.g., Green and Stokey 1983, Kalra and Shi 2001) argues that, in a principal-agent context, competition induces agents to work harder. Similarly, Nalebuff and Stiglitz (1983) indicate that in oligopolistic industries, competition can make firms raise their efforts to sell the products. The main tenet in these streams of work is that, in the presence of a "common" uncertainty, linking rewards to *relative* performance reduces the agents' incentive to shirk. Extending these arguments to the present context suggests that, compared to an exclusive territories setting, when two distributors compete to sell the same product, each may have to work harder to obtain and retain customers.

Overall, in its choice of distribution method, the manufacturer balances the costs arising from free-riding with the benefits of intrabrand competition. This paper argues that as the potential loss borne by the distributor is reduced (i.e., limits on liability become tighter), there will be greater opportunity for distributors in exclusive territories to shirk (because intrabrand competition is not present to discipline their incentives). Consequently, the manufacturer may prefer to use nonexclusive territories; this preference will hold even when distributors can potentially free-ride on each other's efforts.

This paper refines the above intuition by analyzing a model for each type of territorial (exclusive or nonexclusive) arrangement. The analysis demonstrates that the moderating influence of limited liability becomes increasingly apparent as the magnitude of the distributor's liability is progressively reduced.

Relevant Literature

There is related literature in both marketing and economics that bears on this paper's model development and analysis. In marketing, the research on channel coordination, particularly Jeuland and Shugan (1983)[4] and Iyer (1998) are relevant. Jeuland and Shugan propose quantity discounts as a mechanism to coordinate the channel. In his analysis of channel coordination under different types of consumer heterogeneity, Iyer notes that nonlinear pricing may not be sufficient to coordinate the channel when distributors compete on both price and service. In contrast, this paper considers settings where distributors *do* and *do not* compete against each other, and where the distributor's surplus cannot be extracted completely. The manufacturer's inability to extract distributors' surplus is captured via the limited liability constraints. In this manner, the present analysis complements the contributions of the existing research.[5]

Next, in economics, there are two streams of research on exclusive distribution. First is the literature on vertical restraints that deals with settings in which all the relevant information is known to all the channel members (e.g., Blair and Kaserman 1985, Mathewson and Winter 1984). In contrast to that, there is the literature that explicitly acknowledges the difficulties of monitoring and observing distributors' actions (Rey and Stiglitz 1995, Rey and Tirole 1986).

Similar to the latter stream of research, here the analysis takes into account that distributors' actions cannot be observed. Further, in the nonexclusive distribution model, a distributor's demand is positively affected by the effort of a competing distributor.

[2] The cost of these efforts easily ranges from \$150,000 to \$225,000 (Corey et al. 1989, p. 79).

[3] That is, suppose firm A benefits from the efforts of firm B. Then A may reduce its efforts and free-ride on B's efforts.

[4] See also Moorthy (1987) and Jeuland and Shugan (1988).

[5] For an analysis of exclusivity outside the context of channel management, see Dukes and Gal-Or (2003).

Examining the impact of this linkage serves as the primary distinction between the analysis presented here and those of Rey and Stiglitz (1995) and Rey and Tirole (1986). More specifically, they examine other tradeoffs (e.g., Rey and Tirole 1986 consider costs and benefits of risk sharing versus the effective use of decentralized information) and do not model distributors' effort explicitly.

The rest of the paper is organized as follows. Section 2 develops the basic model and analyzes the exclusive and nonexclusive distribution versions of that model. The implications of the analysis are highlighted with the help of a numerical example in §3, and the paper concludes in §4. All proofs are available in the technical appendix posted on this journal's website.

2. The Model

A risk-neutral manufacturer, M, sells its product through two distributors, A and B. The sales ($Q^A, Q^B \in R^+$) of the manufacturer's product are random and the focus is on a simple case in which each of Q^A and Q^B takes one of two values: $0 \le q_L^i < q_H^i$, $i = A, B$. $g^i(\cdot)$, $i = A, B$, is used to represent the probability that q_H^i is realized, and $g^A(\cdot)$ and $g^B(\cdot)$ are parameterized by the prices ($p^A, p^B \in R^+$) charged to customers by the two distributors and the levels of marketing effort ($e^A, e^B \in R^+$) put forth by the distributors. The following notation is used to denote expected sales:

$$q^i(\cdot) = q_L^i + g^i(\cdot)(q_H^i - q_L^i), \quad \forall i = A, B. \tag{1}$$

Both the manufacturer and the distributors observe the realized sales of the distributors in the market, but the effort supplied by each distributor is observable only to that one distributor.[6] It is assumed that all the parties observe prices charged to final customers, and that the density functions, $g^A(\cdot)$ and $g^B(\cdot)$, are common knowledge.

The sequence of events in the model is as follows: (1) Manufacturer announces its wholesale price and fixed fees. (2) Based on the manufacturer's announcement, the distributors decide independently whether to work for the manufacturer. If a distributor does not agree to participate, the manufacturer makes zero profit through that distributor, and that distributor makes its reservation level of profit, π_0, from other sources. (3) If the distributors agree to participate, then each distributor chooses final price and effort levels simultaneously and noncollusively. Sales are then realized (as characterized by the density functions) and profits are made.

[6] "Observable" means costlessly observable. Technically, if a variable were observable, it may be contracted upon; here, the focus is on settings where efforts are not explicitly included in the manufacturer-distributor contract.

Table 1 List of Symbols

Symbol	Represents
e^i, p^i	Distributor i's effort and price, respectively
q^i	i's sales, takes one of two values, q_L^i and q_H^i, where $0 \le q_L^i < q_H^i$
$g^i(\cdot)$	i's probability of obtaining q_H^i
w	Manufacturer (M)'s wholesale price
F^i	Fixed fee or liability that can be imposed on i
Π^M, π^i	Manufacturer's and distributor i's profits, respectively
λ_i, γ_i	Lagrange multiplier on i's IR and LL constraints, respectively
μ_{ie}	Lagrange multiplier on i's effort selection constraints
μ_{ip}	Lagrange multiplier on i's price selection constraints
K	Demand function's intercept in the example in §3
α	Price sensitivity parameter in the example
β	Effort sensitivity parameter in the example
θ	Cross-price sensitivity parameter in the example
δ	Cross-effort sensitivity parameter in the example
Free	Magnitude of the free-rider effect
Slack	Magnitude of the slack effect
Priceff	Magnitude of the price effect
Loss	Difference between M's first- and second-best profit
% Loss	$\dfrac{\text{Loss}}{\text{First-best profit}} \times 100$

The manufacturer's expected profit, Π^M, is given by

$$\Pi^M = (w - c)(q^A + q^B) + F^A + F^B, \tag{2}$$

where c is the manufacturer's variable cost of production, w the manufacturer's common wholesale price to the distributors,[7] and F^i is a fixed fee paid to the manufacturer by distributor i.[8]

Both distributors are assumed to be risk neutral, and their individual expected profit, π^i, $i = A, B$, is given by

$$\pi^i = (p^i - w)q^i - F^i - C^i(e^i), \tag{3}$$

where $C^i(\cdot)$ is the cost of effort to the ith distributor, with $\partial C^i(\cdot)/\partial e^i > 0$ and $\partial^2 C^i(\cdot)/\partial {e^i}^2 > 0$, $\forall e^i > 0$ and p^i is distributor i's price in the final market. Table 1 summarizes all the notation employed in the paper.

Under either form of distribution, the manufacturer's objective is to maximize its profit with respect to the wholesale price and fixed fees, subject to the constraints that each of the distributors participates and selects prices and efforts to maximize individual profit and that there is an upper bound on the level

[7] There are at least two problems if the manufacturer charges different wholesale prices to different distributors. First, there is the possibility of arbitrage among distributors, particularly when transaction costs are not too high. Second, unless it offers a menu, the manufacturer can be prosecuted for price discrimination. For these reasons, and for ease of exposition, the analysis assumes a common price.

[8] While there has not been much legal precedent for this, different fixed fees for different distributors may also be argued as discrimination. The qualitative insights from the analysis continue to hold even when the same F is charged to both distributors.

of fixed fee (due to limited liability). These constraints are stated below:

For a given (w, F^i) that M sets, distributor i will participate if its expected profit exceeds the reservation profit level. Formally,

$$\pi^i \geq \pi_0 \quad \Leftrightarrow \quad (p^i - w)q^i - F^i - C^i(e^i) \geq \pi_0. \tag{4}$$

Next, using F_0 to denote the upper bound on the fixed fee, limited liability requires

$$F^i \leq F_0 \quad \equiv \quad -F^i \geq -F_0. \tag{5}$$

Finally, when participating in the (w, F^i) contract with the manufacturer, distributor i selects price, $p^i(w, F^i)$, and effort, $e^i(w, F^i)$, to maximize its profit:

$$e^i(w, F^i), p^i(w, F^i) \in \operatorname*{Arg\,max}_{e^{i\prime}, p^{i\prime}} \{\pi^i(e^{i\prime}, p^{i\prime})\}. \tag{6}$$

The analysis uses the following first-order conditions to capture distributor i's (above) choices of price and effort:[9]

$$\frac{\partial \pi^i}{\partial e^i} = 0 \quad \equiv \quad (p^i - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = 0, \tag{7}$$

and

$$\frac{\partial \pi^i}{\partial p^i} = 0 \quad \equiv \quad q^i + (p^i - w)\frac{\partial q^i}{\partial p^i} = 0. \tag{8}$$

To facilitate employing the above first-order conditions, π^i is assumed to be concave in p^i and e^i, $i = A, B$. Notice that concavity implies the following (e.g., see Takayama 1985):

$$\frac{\partial^2 \pi^i}{\partial p^{i2}} < 0, \qquad \frac{\partial^2 \pi^i}{\partial e^{i2}} < 0, \tag{9}$$

and,

$$\frac{\partial^2 \pi^i}{\partial p^{i2}}\frac{\partial^2 \pi^i}{\partial e^{i2}} - \left(\frac{\partial^2 \pi^i}{\partial e^i \partial p^i}\right)^2 > 0. \tag{10}$$

Using (3), we note the following:

$$\frac{\partial^2 \pi^i}{\partial p^{i2}} = 2\frac{\partial q^i}{\partial p^i} + (p^i - w)\frac{\partial^2 q^i}{\partial p^{i2}}, \tag{11}$$

$$\frac{\partial^2 \pi^i}{\partial e^{i2}} = (p^i - w)\frac{\partial^2 q^i}{\partial e^{i2}} - \frac{\partial^2 C^i}{\partial e^{i2}}, \tag{12}$$

and

$$\frac{\partial^2 \pi^i}{\partial e^i \partial p^i} = \frac{\partial q^i}{\partial e^i} + (p^i - w)\frac{\partial^2 q^i}{\partial e^i \partial p^i}. \tag{13}$$

The expressions on the right-hand side of (11), (12), and (13) are therefore assumed to satisfy the conditions in (9) and (10). To ensure this, under either form of distribution, it is assumed that the following relatively standard conditions hold: $\forall i = A, B$,

$$\frac{\partial g^i}{\partial e^i} > 0, \quad \frac{\partial^2 g^i}{\partial e^{i2}} \leq 0, \quad \frac{\partial g^i}{\partial p^i} < 0, \quad \text{and} \quad \frac{\partial^2 g^i}{\partial p^{i2}} \geq 0. \tag{14}$$

Further, for expositional convenience, the cross-partials of the density functions are all set equal to zero; that is,[10]

$$\frac{\partial^2 g^i}{\partial e^i \partial p^i} = \frac{\partial^2 g^i}{\partial e^i \partial p^j} = \frac{\partial^2 g^i}{\partial e^i \partial e^j} = \frac{\partial^2 g^i}{\partial p^i \partial p^j} = \frac{\partial^2 g^i}{\partial p^i \partial e^j} = 0.$$

Note that (13) now reduces to

$$\frac{\partial^2 \pi^i}{\partial e^i \partial p^i} = \frac{\partial q^i}{\partial e^i}. \tag{15}$$

The spotlight here is on either a new product entering a market or a relatively established product entering a new market. In either case, distributors' efforts will have a critical role in category expansion. In such instances, one distributor's efforts are likely to have a positive effect on the expected sales of the competing nonexclusive distributor. It is this feature of the setting that facilitates free-riding. Note that prices continue to play the same role as in standard analyses: If a distributor raises the price of an item, its expected demand goes down while the other nonexclusive distributor's expected demand goes up.

The above considerations are captured in the following additional assumptions on the density functions, $g^i(\cdot)$.[11] To capture free-riding, in the nonexclusive distribution setting it is assumed that $\partial g^i/\partial e^j > 0$; under exclusive distribution, it is assumed that $\partial g^i/\partial e^j = 0$, where $i \neq j$ and $i, j = A, B$. Similarly, for nonexclusive distribution it is assumed $\partial g^i/\partial p^j > 0$, and $\partial g^i/\partial p^j = 0$ for exclusive distribution.[12]

[9] In the nonexclusive setting, the distributor selects price and effort levels as best responses to the other distributor's choices.

[10] In a previous version of the manuscript, the analysis explicitly accounted for the role of the derivative $\partial^2 g^i/\partial e^j \partial e^i$; while the main insights in this paper are not hinged on it, including that interaction term adds some expositional complexity. Investigating and discussing the impact of the various cross-partials in a comprehensive manner is beyond the scope of the current paper.

[11] It is worth noting that the density functions satisfy both the monotone likelihood ratio condition (MLRC) and convexity of the distribution function condition (CDFC) specified in Rogerson (1985) (see the appendix for a formal proof). For additional sufficient conditions that ensure (9) and (10) hold, see the example in §3. There, $\partial^2 q^i/\partial p^{i2} = \partial^2 q^i/\partial e^{i2} = 0$, and (9) is automatically satisfied while (10) reduces to

$$-2\frac{\partial q^i}{\partial p^i}\frac{\partial^2 C^i}{\partial e^{i2}} - \left(\frac{\partial q^i}{\partial e^i}\right)^2 > 0. \tag{16}$$

For the selected parameter values in that example, (16) is indeed satisfied.

[12] The manufacturer is assumed to use two distributors in both forms of distribution. This helps to control, at least to some degree, any differences in demand that would otherwise have arisen. See more on this in the example in §3.

In the above model setup, there are several factors that can potentially impact any difference in the manufacturer's solutions to the two forms of distribution (e.g., is it because distributors are *choosing* both prices and efforts or is it due to the binding limited liability constraints? etc.). Therefore, to understand what is driving any difference in the results, the analysis considers various combinations of territorial exclusivity and the level of manufacturer's control over distributors' actions.

More specifically, six separate scenarios are examined for each form of distribution. The first three of these scenarios assume that the manufacturer controls the distributors' final prices and in the latter three scenarios this assumption is relaxed.[13] Under either assumption, the three scenarios correspond to (i) distributors' efforts are observable,[14] (ii) efforts are unobservable but limited liability constraints are not binding, and (iii) efforts are unobservable and limited liability constraints are binding.

Under each form of distribution, the solution to the scenario where M controls final prices and where efforts are observable is referred to as the first-best solution. This solution serves as a benchmark to understand how the manufacturer's profit is affected as the distributors obtain more control over final prices and efforts. The technical appendix systematically analyzes the above 12 scenarios; Table 2 highlights the specific constraints examined along with the primary finding (i.e., whether or not the first-best solution can be regained) from each analysis. In the main body of the paper, the focus is on the most complex scenario for each form of distribution.

Table 2 Scenarios Investigated

No.	Form of distribution	Constraints in the optimization	Main result (FB = first-best)
1	Exclusive	Participation ($IR-i$)	FB (exclusive territories) solution
2	Exclusive	Participation ($IR-i$) Effort selection ($E-i$)	FB solution regained
3	Exclusive	Participation ($IR-i$) Effort selection ($E-i$) Limited Liability ($LL-i$)	FB solution not regained Effort less than first-best level
4	Exclusive	Participation ($IR-i$) Price selection ($P-i$)	FB solution regained
5	Exclusive	Participation ($IR-i$) Price selection ($EP-i$) Effort selection ($EP-i$)	FB solution regained
6	Exclusive	Participation ($IR-i$) Price selection ($EP-i$) Effort selection ($EP-i$) Limited Liability ($LL-i$)	FB solution not regained (see Finding 1 in the text)
7	Nonexclusive	Participation ($IR-i$)	FB (nonexclusive) solution
8	Nonexclusive	Participation ($IR-i$) Effort selection ($E-i$)	FB solution regained
9	Nonexclusive	Participation ($IR-i$) Effort selection ($E-i$) Limited Liability ($LL-i$)	FB solution not regained Effort less than first-best level
10	Nonexclusive	Participation ($IR-i$) Price selection ($P-i$)	FB solution regained
11	Nonexclusive	Participation ($IR-i$) Price selection ($EP-i$) Effort selection ($EP-i$)	FB solution not regained
12	Nonexclusive	Participation ($IR-i$) Price selection ($EP-i$) Effort selection ($EP-i$) Limited Liability ($LL-i$)	FB solution not regained (see Finding 2 in the text)

2.1. Exclusive Distribution

A formal statement of the manufacturer's problem, denoted $[M-E]$, is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$

$$\text{subject to} \quad \pi^i\left(e^i(w,F^i),p^i(w,F^i)\right)\geq \pi_0, \quad \forall i=A,B, \quad (IR-i)$$

$$e^i(w,F^i),p^i(w,F^i)\in \underset{e^{i'},p^{i'}}{\operatorname{Arg\,max}}\{\pi^i(e^{i'},p^{i'})\}, \quad \forall w,F^i\in R^+,\ i=A,B, \quad (EP-i)$$

$$\text{and} \quad -F^i\geq -F_0, \quad i=A,B, \quad (LL-i)$$

where $(IR-i)$ refers to distributor i's participation constraint—see Equation (4), $(EP-i)$ to the manner in which each distributor selects effort and price levels—see Equations (7) and (8), and $(LL-i)$ to the limited liability constraints—see Equation (5). In solving this nonlinear programming problem, several Lagrange multipliers are employed: λ_i represents the multiplier for distributor i's participation constraint, μ_{ie} and μ_{ip} for i's effort and price selection constraints respectively, and γ_i for the limited liability constraint. See Table 3 for the Lagrangian.

Table 3 The Lagrangian

$$\begin{aligned} L = {} & (w-c)(q^A+q^B)+F^A+F^B+\lambda_A[(p^A-w)q^A-F^A-C^A(e^A)] \\ & +\lambda_B[(p^B-w)q^B-F^B-C^B(e^B)]+\mu_{Ae}\left[(p^A-w)\frac{\partial q^A}{\partial e^A}-\frac{\partial C^A}{\partial e^A}\right] \\ & +\mu_{Be}\left[(p^B-w)\frac{\partial q^B}{\partial e^B}-\frac{\partial C^B}{\partial e^B}\right]+\mu_{Ap}\left[q^A+(p^A-w)\frac{\partial q^A}{\partial p^A}\right] \\ & +\mu_{Bp}\left[q^B+(p^B-w)\frac{\partial q^B}{\partial p^B}\right]+\gamma_A[-F^A+F_0]+\gamma_B[-F^B+F_0]. \end{aligned}$$

Note. (i) The demand structure for exclusive distribution is different from that for nonexclusive distribution, so the above Lagrangian gets suitably modified for the two forms of distribution. (ii) The λs, μ_{ie}, μ_{ip}s, and γs represent the Lagrange multipliers for the IR constraints, the EP (effort and price selection) constraints, and the limited liability constraints, respectively.

[13] See Liu et al. (2003) for another context in which agents compete without price. Bhardwaj (2001) examines settings in which a principal does (not) delegate pricing authority to its agent.

[14] This implies that efforts are contractible—that is, effort level can be specified by the manufacturer in the contract.

Before considering the solution to $[M-E]$, it is worth noting the following benchmark solutions to M's problem:[15]

Benchmark 1. *For all $i=A,B$,*

(a) *When M controls final prices and distributors efforts are observable* (*i.e., constraints* $(EP-i)$ *and* $(LL-i)$ *do not hold*), *the optimal contract, referred to as the first-best solution, has the following properties*:

(i) $\pi^i=\pi_0$ (*i.e., distributor profit is restricted to the reservation profit level*)

(ii) $(p^i-c)\partial q^i/\partial e^i=\partial C^i/\partial e^i$ (*i.e., marginal benefit of distributor effort equals its marginal cost to the channel*)

(iii) $q^i=-(p^i-c)\partial q^i/\partial p^i$ (*i.e., marginal benefit of final price equals its marginal cost to the channel*)

(b) *When M does not control final prices and distributors' efforts are unobservable, but limited liability constraints are not binding* (*i.e., constraints* $(EP-i)$ *hold but not* $(LL-i)$), *the first-best solution is regained when M sets $w=c$ and extracts all of distributor i's surplus via the fixed fee, F^i.*

Now consider the properties of a solution to $[M-E]$ that are summarized below.

Finding 1. *For all $i=A,B$,*

(a) *In the presence of binding limited liability constraints, inefficiency exists in the distributors' choices of efforts and final prices. More formally, both μ_{ie} and μ_{ip} cannot equal zero, and the first-best solution* (*where both μ_{ie} and μ_{ip} equal zero*) *cannot be achieved.*

(b) *At optimality, distributor i's choices of effort and price are given by*

$$\text{(i)}\quad (p^i-c)\frac{\partial q^i}{\partial e^i}-\frac{\partial C^i}{\partial e^i} = -\mu_{ie}\left[(p^i-w)\frac{\partial^2 q^i}{\partial e^{i2}}-\frac{\partial^2 C^i}{\partial e^{i2}}\right]-\mu_{ip}\frac{\partial q^i}{\partial e^i}$$

and

$$\text{(ii)}\quad q^i+(p^i-c)\frac{\partial q^i}{\partial p^i} = -\mu_{ie}\frac{\partial q^i}{\partial e^i}-\mu_{ip}\left[2\frac{\partial q^i}{\partial p^i}+(p^i-w)\frac{\partial^2 q^i}{\partial p^{i2}}\right].$$

Proof. See the appendix.

Benchmark 1 reiterates the standard result that when limited liability is not an issue, the manufacturer can eliminate any incentive problems by suitably designing w and F (i.e., the double-marginalization problem is eliminated via the two-part tariff). However, when limited liability is relevant (for any of a variety of institutional reasons), M cannot extract as large an F as it would like. Then, Finding 1(a) indicates that there is inefficiency under exclusive territories (even with a two-part tariff). The exact nature of the inefficiency (in the choice of effort[16]) depends on the endogenously determined expressions,

$$-\mu_{ie}\left[(p^i-w)\frac{\partial^2 q^i}{\partial e^{i2}}-\frac{\partial^2 C^i}{\partial e^{i2}}\right]$$

(dubbed the *slack* effect for expositional convenience), and $-\mu_{ip}\partial q^i/\partial e^i$ (the *price* effect), given in Finding 1(b). To provide further insight, §3 discusses an example in which $\mu_{ie}>0$ and $\mu_{ip}<0$ (i.e., too little effort and too high prices), and the appendix shows an approach to develop sufficient conditions for unambiguously signing these multipliers.

In contrast to the exclusive distribution setup, under nonexclusive distribution, even in the absence of binding limited liability constraints, the first-best solution will not be obtained. For a formal discussion of this see Iyer (1998). The main implication is that, when limited liability is not an issue, the manufacturer strictly prefers exclusive distribution over nonexclusive distribution. The preference is driven by the fact that there is no loss in efficiency with exclusive distributors. With binding limited liability constraints however, as Finding 1(a) reveals, there is inefficiency under exclusive territories.

2.2. Nonexclusive Distribution

A formal statement of the manufacturer's problem, $[M-NE]$, is

$$\begin{aligned}
&\underset{w,F,e,p}{\text{Maximize}}\quad \Pi^M\\
&\text{subject to}\quad \pi^i(e^i(\cdot),p^i(\cdot),e^j(\cdot),p^j(\cdot))\geq\pi_0,\\
&\qquad\qquad\qquad \forall i\neq j,\ i,j=A,B,\quad (IR-i)\\
&\qquad e^i(w,F^i,F^j),p^i(w,F^i,F^j)\\
&\qquad\quad \in \underset{e^{i\prime},p^{i\prime}}{\text{Arg}\max}\{\pi^i(e^{i\prime},p^{i\prime},e^j(w,F^i,F^j),\\
&\qquad\qquad\qquad p^j(w,F^i,F^j))\},\\
&\qquad\quad \forall w,F^i,F^j\in R^+,\ i\neq j,\ i,j=A,B,\quad (EP-i)\\
&\text{and}\quad -F^i\geq -F_0,\quad i=A,B,\qquad (LL-i)
\end{aligned}$$

where the constraints (and the notation employed for the Lagrange multipliers) have the same meaning as in the exclusive distribution problem. The distributors here, though, simultaneously select prices and efforts as best responses to each other's actions.

[15] These are referred to as benchmark solutions also in the sense that some variation of the findings is currently available in the literature.

[16] The components of the expressions determining the distributors' choices of prices are not discussed here. These are available in the appendix.

As in the exclusive setting, it is worth noting the following benchmark solutions to M's problem:

BENCHMARK 2. *For all $i = A, B$,*

(a) *When M controls final prices and distributors' efforts are observable, the first-best (nonexclusive distribution) solution has the following properties: For $i \neq j$; $i = A, B$,*

(i) $\pi^i = \pi_0$ (*i.e., distributor profit is restricted to the reservation profit level*)

(ii) $(p^i - c)\partial q^i/\partial e^i + (p^j - c)\partial q^j/\partial e^i - \partial C^i/\partial e^i = 0$ (*i.e., the marginal benefit of distributor effort equals its marginal cost to the channel*)

(iii) $q^i + (p^i - c)\partial q^i/\partial p^i + (p^j - c)\partial q^j/\partial p^i = 0$ (*i.e., the marginal benefit of final price equals its marginal cost to the channel*)

(b) *When M does not control final prices and distributors' efforts are unobservable, but limited liability constraints are not binding, inefficiency exists in the distributors' choices of efforts and final prices. More formally, both μ_{ie} and μ_{ip} cannot equal zero and the first-best nonexclusive distribution solution (where both μ_{ie} and μ_{ip} equal zero) cannot be regained.*

Next, a solution to $[M - NE]$ has the following properties:

FINDING 2. *For all $i = A, B$,*

(a) *In the presence of binding limited liability constraints, inefficiency exists in the distributors' choices of efforts and prices.*

(b) *At optimality, distributor A's choice of effort and price are given by*[17]

$$\text{(i)}\quad (p^A - c)\frac{\partial q^A}{\partial e^A} + (p^B - c)\frac{\partial q^B}{\partial e^A} - \frac{\partial C^A}{\partial e^A} = \left[\gamma_B(p^B - w) - \mu_{Bp}\right]\frac{\partial q^B}{\partial e^A} - \mu_{Ae}\left[(p^A - w)\frac{\partial^2 q^A}{\partial e^{A2}} - \frac{\partial^2 C^A}{\partial e^{A2}}\right] - \mu_{Ap}\frac{\partial q^A}{\partial e^A},$$

and

$$\text{(ii)}\quad q^A + (p^A - c)\frac{\partial q^A}{\partial p^A} + (p^B - c)\frac{\partial q^B}{\partial p^A} = \left[\gamma_B(p^B - w) - \mu_{Bp}\right]\frac{\partial q^B}{\partial p^A} - \mu_{Ae}\frac{\partial q^A}{\partial e^A} - \mu_{Ap}\left[2\frac{\partial q^A}{\partial p^A} + (p^A - w)\frac{\partial^2 q^A}{\partial p^{A2}}\right].$$

PROOF. See the appendix.

Consistent with previous research, Benchmark 2(b) shows that under intrabrand competition, there is inefficiency in the channel. Finding 2(a) indicates that such inefficiency continues to exist even under binding limited liability constraints. Further, Finding 2(b) reveals that the distributor's choice of effort[18] is affected by (as in the solution to $[M - E]$) the slack effect, $(-\mu_{Ae}[(p^A - w)\partial^2 q^A/\partial e^{A2} - \partial^2 C^A/\partial e^{A2}])$, and the price effect, $(-\mu_{Ap}\partial q^A/\partial e^A)$; in addition, the free-rider effect, $([\gamma_B(p^B - w) - \mu_{Bp}]\partial q^B/\partial e^A)$, arises from the positive interlinkage between the distributors' demand functions. Since these effects are determined endogenously, an example in the next section provides a better sense of their relative magnitude.

The main purpose of that example, though, is as follows. The analysis so far has revealed that either form of distribution has inefficiencies (in terms of the distributors' effort/price choices). Conventional wisdom and previous research, however, suggest that when firms can free-ride on each other's efforts, exclusive territories are the preferred form of distribution. However, such suggestions presume that distributors under exclusive territories automatically have appropriate incentives to work on the manufacturer's behalf. Therefore, the next section develops a simple but plausible example in which the manufacturer prefers nonexclusive (over exclusive) territories even when free-riding among distributors is possible.

3. An Example

For the purposes of this example, the reservation level of profit and the manufacturer's marginal cost of production are each set equal to zero (i.e., $\pi_0 = 0$ and $c = 0$). Each distributor's cost of effort is given by $C^i(e) = e^2/2$, $i = A, B$. It is also assumed that $q_L^i = 0$, and $q_H^i = 100{,}000$. Therefore, expected sales are now a product of $g^i(\cdot)$ and q_H^i. Expected sales of each distributor are assumed to have the following structure:

$$q^A = K - \alpha p^A + \beta e^A + \theta p^B + \delta e^B, \tag{17}$$

$$q^B = K - \alpha p^B + \beta e^B + \theta p^A + \delta e^A. \tag{18}$$

When the distributors are exclusive, θ and δ are set equal to zero. Essentially, (17) and (18) represent the probabilities $g^A(\cdot)$ and $g^B(\cdot)$, scaled up by q_H^i.[19] The parameter values ensure that the linear structure in (17) and (18) does not result in probabilities outside the open interval $(0, 1)$.

This example assumes that both distributors are identical. The values for the parameters of the above functions have been chosen to imitate a real-world

[17] Analogous expressions arise for distributor B.

[18] The corresponding effects for the distributors' choices of prices are not explicitly identified here. These are available in the appendix.

[19] It may be worth repeating that while the two states, q_L^i and q_H^i, are fixed *a priori*, the probability with which either state arises is affected by the distributors' prices and efforts, and the expressions in (17) and (18) represent the expected demand levels.

setting, namely, the sales of different brands of beer in the United States in the 1980s.[20]

It is useful to recall that beer manufacturers frequently used exclusive wholesalers and were concerned about free-riding among wholesalers. Wholesale sales of beer in the year 1980 were of the order of hundreds of thousands of dollars, ranging from \$35,000 in Alaska to \$1,897,750 in California, with Florida at \$885,500, and Delaware at \$48,500. The wholesale price per case of beer was of the order of ones of dollars—in Indianapolis, the price per case of Pabst was about \$6 in November 1982—where a case is either a six pack of 12-oz. cans or a four pack of 24-oz. cans. Given these data,[21] the following parameter values were assigned to obtain reasonable optimal prices and quantities. Note that the parameter values are in the order of 10^3; that is, a value of 50 would imply 50,000, and 0.001 represents 1. For expositional convenience, the zeros are suppressed and the reader is cautioned to make a note of the "actual" magnitude. However, prices are to be read at face value.

(a) For exclusive distribution, the parameter values are $K = 50$, $\alpha = 7$, and $\beta = 2$.

(b) For nonexclusive distribution, the parameter values are: $\alpha = 7$, $\beta = 2$, $\theta = 1$, $\delta = 0.001$, and $K = 46.64$.

Notice that the value of K in the nonexclusive case is different from that in the exclusive territories setting. This is due to an attempt to control for any differences in the industry-level (expected) demand across the two forms of distribution. The following normalization procedure was followed. The procedure begins by giving a specific value to the intercept for the exclusive setting (in this example, $K = 50$) and treating the intercept for the nonexclusive setting as an unknown value. To make the two settings equivalent, the optimal first-best solution is derived for both the exclusive and the nonexclusive cases. This solution involves optimal prices and effort levels. At those respective levels, the demand is computed for the exclusive and nonexclusive settings, and the two demand levels are equated to one another to solve for the unknown intercept term for the nonexclusive setting. This procedure ensures that, in the first-best setting, the two types of distribution have identical expected demands when the first-best optimal prices and efforts are selected.[22] The goal of this normalization is to reduce any inherent bias in the model toward either form of distribution.

Table 4 **Exclusive Distribution**

γ_A	F^A	Π^M	Free	Slack	Priceff	Loss	% Loss
0.0	125.00	250.00	0.00	0.00	0.00	0.00	0.00
0.1	103.31	246.93	0.00	0.91	0.39	2.07	0.83
0.2	86.81	243.06	0.00	1.67	0.71	6.94	2.78
0.3	73.97	236.69	0.00	2.31	0.99	13.31	5.33
0.4	63.78	229.59	0.00	2.86	1.23	20.41	8.16
0.5	55.56	222.22	0.00	3.33	1.43	27.78	11.11
0.6	48.83	214.84	0.00	3.75	1.61	35.16	14.06
0.7	43.25	207.61	0.00	4.12	1.77	42.39	16.96

Note. Parameter values: $\alpha = 7$, $\beta = 2$, and $K = 50$.

With the first-best solutions in hand, the manufacturer's profits in the second-best scenarios (i.e., at the solution to $[M - E]$ and $[M - NE]$) are determined. Then the loss in efficiency for each form of distribution is compared with the other, rather than a summary comparison of manufacturer profit across the two forms of distribution.[23] *Mathematica* (Wolfram Research Inc.) helped generate the equilibrium values of the relevant variables in the first- and second-best scenarios. The notation used for the column headings in the simulation tables is given in Table 1. All the entries in Tables 4 and 5 are at optimality.

Observation 1. *First, as the maximum liability that the distributor can bear becomes smaller, the Lagrange multiplier γ increases in value. Next, as γ increases, the effects that distort distributors' effort and price choices (identified in Findings 1 and 2) increase; the manufacturer's profits decrease and the manufacturer's percentage losses (from the first-best solution) increase.*

This can be seen by going down the columns in Tables 4 and 5.

Observation 2. *For smaller values of γ ($= 0, 0.1, 0.2, 0.3$), the manufacturer incurs a smaller loss with exclusive territories. For larger values of γ ($= 0.4, 0.5, 0.6, 0.7$), the manufacturer incurs a smaller loss with nonexclusive territories.*

This can be seen from a comparison of the percentage losses in Table 4 with those in Table 5. The intuition behind this observation is that under more severe limits on liability, the relatively low fixed fees do not make it profitable for the manufacturer to induce the exclusive distributor to work diligently. Instead, under those conditions, it is more profitable to provide the proper incentives via intrabrand

[20] As an aside, note that the choice of exclusive distribution continues to be an issue in such product categories (e.g., see An Act to Amend the Wine Franchise Law to Provide For Exclusive Territories, a bill introduced in 1999 in North Carolina).

[21] See, for instance, *Malt Beverage Interbrand Competition Act: Hearings on S. 1215 Before the Committee on the Judiciary*, 97th Congress, 2nd Session (1982), p. 211.

[22] The analysis also determined the value of the nonexclusive intercept by equating the manufacturer's first-best profit from the two forms of distribution. There are no qualitative differences in the results of the analysis in either approach.

[23] Another approach would be to compare directly the manufacturer's profit under each form of distribution. This approach, however, is much more complex—since it must be ensured that the two regimes are truly comparable by controlling for differences in the marketing mix's effect on demand—and does not add to the qualitative insights of the paper.

Table 5 Nonexclusive Distribution

γ_A	F^A	Π^M	Free	Slack	Priceff	Loss	% Loss
0.00	122.47	269.32	−0.0005	1.66	−0.95	2.77	1.02
0.10	102.90	267.45	0.0002	2.42	−0.48	4.64	1.71
0.20	87.67	262.94	0.0008	3.08	−0.08	9.15	3.36
0.30	75.59	256.94	0.0013	3.63	0.26	15.14	5.57
0.40	65.85	250.15	0.0017	4.12	0.55	21.93	8.06
0.50	57.87	243.00	0.0021	4.54	0.81	29.08	10.69
0.60	51.26	235.75	0.0024	4.91	1.03	36.34	13.35
0.70	45.72	228.57	0.0027	5.25	1.24	43.52	15.99

Note. Parameter values: $\alpha = 7$, $\beta = 2$, $\theta = 1$, $\delta = 0.001$, and $K = 46.64$.

competition. Consequently, nonexclusive territories are the preferred form of distribution.

It is worth pointing out that in many institutional settings, the imposable fixed fees are minimal, if not zero (Iyer and Villas-Boas 2001). In such settings, this paper's analysis highlights why manufacturers may prefer nonexclusive distribution even when firms at the distribution level can free-ride on one another.

4. Conclusion

Suppliers in many product categories often enter major markets through wholesalers or distributors. For instance, a wine supplier wishing to enter the North Carolina market would consider one or more of several wine distributors, each typically with an ability to reach the entire retail market. Similar deliberations arise when suppliers or manufacturers enter foreign markets. In such contexts, the supplier often considers whether to assign exclusive territories to its distributors. Previous research and conventional wisdom suggest that if intrabrand competition encourages free-riding, then exclusive territories can be employed to fix that problem.

This paper argues that, even when distributors can free-ride on the efforts of one another, intrabrand competition can be beneficial, since distributors in exclusive territories may not always work diligently. In other words, tolerating some amount of free-riding among the distributors can be optimal for the manufacturer. This benefit becomes important particularly when the manufacturer cannot extract a significant portion of the distributor's surplus; the analysis captured this feature via the presence of binding limited liability constraints. When such constraints are not sufficiently binding, the analysis is consistent with prior research and conventional wisdom that exclusive territories offer a better solution for dealing with the relevant incentive problems.

One possible direction for further research is to explore the impact of the various demand elasticities (or demand sensitivity to price and effort) on the preference for intrabrand competition. As argued here, these sensitivities would not have any impact in the absence of binding limited liability constraints. A systematic analysis of these sensitivities, along with the impact of the various cross-partial derivatives on the value of the two forms of distribution, seems warranted.

This paper's primary contribution lies in identifying the sources of the inefficiency that drives the choice between exclusive and nonexclusive forms of distribution. Overall, the analysis underscores the importance of assessing the benefits of intrabrand competition.

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Appendix

This appendix is organized as follows. Section A.1 shows that the density functions, $g^i(\cdot)$, satisfy MLRC and CDFC (cf. Rogerson 1985). Section A.2 analyzes the exclusive distribution model while §A.3 focuses on the nonexclusive distribution model. Sections A.2 and A.3 each examine six separate scenarios: the first three scenarios assume that the manufacturer controls the distributors' final prices; in the latter three scenarios this assumption is relaxed.[24] Under either assumption, the three scenarios correspond to (i) distributors' efforts are observable, (ii) efforts are unobservable and limited liability constraints are not binding, and (iii) efforts are unobservable and limited liability constraints are binding.

Note that each of the above scenarios is considered in a separate subsection. Each subsection begins with the formal statement of the manufacturer's problem, denoted $[M.x]$, where x is that subsection's number. This formal statement is followed by a "result" that highlights the principal finding for that subsection. This result is then substantiated with a proof that employs the standard Kuhn-Tucker (KT) analysis. The analysis includes writing the entire Lagrangian along with the appropriate KT necessary conditions and using these conditions to derive the result. Throughout the ensuing analysis, μs represent the Lagrange multipliers on the distributors' incentive compatibility constraints (i.e., effort and price choice constraints), and λs and γs represent the multipliers on the individual rationality constraints and the limited liability constraints, respectively. Finally, wherever appropriate, the intuition and implication(s) of the result(s) are discussed briefly.

[24] The rationale for considering these cases is explained at the end of §A.3.3.

A.1. Showing that MLRC and CDFC Hold

When a density function satisfies MLRC and CDFC, the first-order approach is valid in the sense that a distributor's choice of action (e.g., effort) can be replaced by the corresponding first-order condition (see Rogerson 1985). The following parallels between the model setup here and §§2 and 4 in Rogerson (1985) are worth noting.

Here, the outcome for distributor i is either q_L^i or q_H^i (where $0 \le q_L^i < q_H^i$), and the probability that q_H^i is realized is denoted by g^i. These outcomes $\{q_L^i, q_H^i\}$ and the corresponding probabilities of these outcomes $\{(1-g^i), g^i\}$ correspond to Rogerson's outcomes $\{x_1, x_2\}$ and probabilities $\{p_1, p_2\}$, respectively. Finally, the cumulative probability distribution here is $\{(1-g^i), 1\}$ and corresponds to Rogerson's $\{F_1, F_2\}$. Now, the definition of MLRC (see Rogerson 1985, p. 1361) can be rewritten as follows:

Definition 1. The functions $\{(1-g^i), g^i\}$ are said to satisfy the MLRC if two effort levels, e_1 and e_2, are such that $e_1 \le e_2$ implies that $g^i(e_1)/g^i(e_2) \le (1-g^i(e_1))/(1-g^i(e_2))$.

The following verifies that the model setup here satisfies MLRC: Since $e_2 \ge e_1$, and g^i is increasing in effort by assumption, it follows that

$$g^i(e_2) \ge g^i(e_1) \tag{A.1.1.1a}$$

or

$$-g^i(e_2) \le -g^i(e_1). \tag{A.1.1.1b}$$

Adding one to both sides of the inequality in (A.1.1.1b) gives

$$1-g^i(e_2) \le 1-g^i(e_1) \tag{A.1.1.2}$$

or

$$1 \le \frac{1-g^i(e_1)}{1-g^i(e_2)}. \tag{A.1.1.3}$$

Further, from (A.1.1.1a), it follows that

$$\frac{g^i(e_1)}{g^i(e_2)} \le 1. \tag{A.1.1.4}$$

Combining the inequalities in (A.1.1.3) and (A.1.1.4) gives

$$\frac{g^i(e_1)}{g^i(e_2)} \le 1 \le \frac{1-g^i(e_1)}{1-g^i(e_2)}. \tag{A.1.1.5}$$

Consequently:

$$\frac{g^i(e_1)}{g^i(e_2)} \le \frac{1-g^i(e_1)}{1-g^i(e_2)}, \tag{A.1.1.6}$$

as required by MLRC.

Next, the definition of CDFC (see Rogerson 1985, p. 1362) can be rewritten as follows:

Definition 2. The functions $\{(1-g^i), g^i\}$ satisfy the CDFC if the second derivative of each of the cumulative probabilities is nonnegative.

Here, the cumulative probabilities are $\{(1-g^i), 1\}$, and the second derivatives are $\{-(g^i)'', 0\}$. Further, by assumption, $(g^i)' > 0$, and $(g^i)'' \le 0$. Consequently, $-(g^i)'' \ge 0$. Therefore, CDFC is also satisfied.

A.2. Analysis of the Exclusive Distribution Model

A.2.1. Manufacturer Controls Final Prices in a Full-Information Setting

In this setting, the manufacturer observes distributor effort costlessly. A solution to the manufacturer's problem in this setting will serve as a benchmark for the analysis of channel settings in which distributor effort is unobservable.

A formal statement of the manufacturer's problem, denoted [M.A.2.1], is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$
$$\text{subject to} \quad \pi^i \ge \pi_0, \quad \forall i = A, B. \tag{$IR-i$}$$

A solution to [M.A.2.1] has the following properties:

Result 1. For $i = A, B$,
(a) $\pi^i = \pi_0$
(b) $(p^i - c)\partial q^i/\partial e^i = \partial C^i/\partial e^i$
(c) $q^i = -(p^i - c)\partial q^i/\partial p^i$.

Proof. The Lagrangian, L, for this problem (also note that without loss of generality, $\pi_0 = 0$ here and throughout the analysis) is

$$L = (w-c)(q^A+q^B)+F^A+F^B+\lambda_A[(p^A-w)q^A-F^A-C^A(e^A)] + \lambda_B[(p^B-w)q^B-F^B-C^B(e^B)]. \tag{A.2.1.1}$$

Assuming an interior solution for F^i, $i = A, B$, gives the following necessary conditions for optimality:

$$\frac{\partial L}{\partial F^i} = 1-\lambda_i = 0 \tag{A.2.1.2a}$$

or

$$\lambda_i = 1. \tag{A.2.1.2b}$$

Using (A.2.1.2b) and assuming interior solutions for p^i and e^i, $i = A, B$, the necessary conditions for optimality are

$$\frac{\partial L}{\partial p^i} = q^i + (p^i - c)\frac{\partial q^i}{\partial p^i} = 0 \tag{A.2.1.3}$$

and

$$\frac{\partial L}{\partial e^i} = (p^i - c)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = 0. \tag{A.2.1.4}$$

The statement of the result follows. □

Result 1(a) indicates that in a full-information setting, both distributors are restricted to their reservation level of profit. Result 1(b) indicates that the marginal benefit (to the channel) of increasing the final price by \$1 equals the marginal cost incurred through a loss in demand due to that \$1 increase in price. Result 1(c) indicates that the marginal cost of the optimal effort from a distributor exactly equals the marginal benefit to the channel from that effort. In summary, Results 1(b) and 1(c) indicate that efforts and prices are chosen efficiently.[25] For expositional purposes, the paper refers to the above solution as either the first-best solution or as the full-information efficient solution.

[25] The notion of efficiency referred to here and throughout the ensuing analysis is that, from a channel perspective, marginal benefit equals marginal cost.

A.2.2. Manufacturer Controls Final Prices and Distributor Effort Is Unobservable

The formal statement of the manufacturer's problem, [M.A.2.2], is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$

$$\text{subject to} \quad \pi^i\big(e^i(w,p^i,F^i)\big) \geq \pi_0, \quad \forall i = A, B. \qquad (IR-i)$$

$$e^i(w,p^i,F^i) \in \operatorname{Arg}\max_{e^{i'}}\{\pi^i(e^{i'})\},$$

$$\forall w, p^i, F^i \in R^+, \; i = A, B. \quad (E-i)$$

A solution to [M.A.2.2] has the following properties:

Result 2. In this setting, the manufacturer recovers the first-best solution by setting $w = c$.

Proof. Here the analysis employs the first-order approach (cf. Rogerson 1985) to solve the manufacturer's problem. That is, the $(E-i)$ constraint is represented by the appropriate first-order condition from each distributor's profit maximization (i.e., $\partial\pi^i/\partial e^i = 0$). Since the density functions satisfy both MLRC and CDFC,[26] and the distributor is selecting only effort (because the manufacturer controls final prices), this setting is similar to the standard analyses of the principal-agent model.

The Lagrangian, L, for this problem is

$$\begin{aligned} L = {} & (w-c)(q^A+q^B)+F^A+F^B+\lambda_A[(p^A-w)q^A-F^A-C^A(e^A)] \\ & +\lambda_B[(p^B-w)q^B-F^B-C^B(e^B)]+\mu_{Ae}\left[(p^A-w)\frac{\partial q^A}{\partial e^A}-\frac{\partial C^A}{\partial e^A}\right] \\ & +\mu_{Be}\left[(p^B-w)\frac{\partial q^B}{\partial e^B}-\frac{\partial C^B}{\partial e^B}\right]. \end{aligned} \qquad (A.2.2.1)$$

Assuming an interior solution for F^i, $i = A, B$, gives

$$\frac{\partial L}{\partial F^i} = 1 - \lambda_i = 0 \qquad (A.2.2.2a)$$

or

$$\lambda_i = 1. \qquad (A.2.2.2b)$$

Next, the KT condition on w is

$$\frac{\partial L}{\partial w} = -\mu_{Ae}\frac{\partial q^A}{\partial e^A} - \mu_{Be}\frac{\partial q^B}{\partial e^B} = 0 \qquad (A.2.2.3a)$$

or

$$\mu_{Ae}\frac{\partial q^A}{\partial e^A} + \mu_{Be}\frac{\partial q^B}{\partial e^B} = 0. \qquad (A.2.2.3b)$$

(A.2.2.3b) indicates either that (a) μ_{Ae} and μ_{Be} are each equal to zero or (b) μ_{Ae} and μ_{Be} are of opposite signs. The analysis will now show that the two multipliers cannot be of opposite signs.

Recall the $(E-i)$ constraint:

$$\frac{\partial L}{\partial \mu_{ie}} = (p^i - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = 0. \qquad (A.2.2.4a)$$

(A.2.2.4a) can be rewritten as

$$(p^i - c)\frac{\partial q^i}{\partial e^i} + (c - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = 0 \qquad (A.2.2.4b)$$

or

$$(p^i - c)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = (w - c)\frac{\partial q^i}{\partial e^i}. \qquad (A.2.2.4c)$$

Notice that the left-hand side of (A.2.2.4c) represents the sum of the marginal benefit and the marginal cost to the channel. Next, since $\partial q^i/\partial e^i > 0$ by assumption, the sign of the right-hand side in (A.2.2.4c) is the same as the sign of $(w-c)$. It follows that for any given value of w, the left-hand side in (A.2.2.4c) has the same sign for either distributor. This implies that μ_{Ae} and μ_{Be} have the same sign. (For example, when $w > c$, $(p^i - c)\partial q^i/\partial e^i - \partial C^i/\partial e^i > 0$, and the marginal benefit to the channel is greater than the marginal cost, and $\mu_{ie} > 0$.) Consequently, μ_{Ae} and μ_{Be} cannot have opposite signs and the only solution that satisfies (A.2.2.3b) involves $\mu_{Ae} = \mu_{Be} = 0$. From (A.2.2.4c), it follows that $w = c$.

When $w = c$, the KT conditions on the prices and efforts satisfy

$$\frac{\partial L}{\partial p^i} = q^i + (p^i - c)\frac{\partial q^i}{\partial p^i} = 0 \qquad (A.2.2.5)$$

and

$$\frac{\partial L}{\partial e^i} = (p^i - c)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = 0. \qquad (A.2.2.6)$$

Comparing these with the first-best solution, notice that when $w = c$, the manufacturer regains its full-information efficient solution. The statement of the result follows. □

Setting the wholesale price equal to the marginal cost of production succeeds in aligning the exclusive distributors' objectives (at the margin) with those of the manufacturer. At the aggregate level, the manufacturer extracts distributors' rents through the fixed fees and restricts distributors' expected profits to their reservation level. However, this scheme forces the distributors to bear all the risk in the market. Therefore, the manufacturer capitalizes on the risk neutrality of the distributors to achieve the first-best profit.

A.2.3. Manufacturer Controls Final Prices, Distributor Effort Is Unobservable, and Limited Liability Constraints Are Binding

Here, the formal statement of the manufacturer's problem, [M.A.2.3], is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$

$$\text{subject to} \quad \pi^i\big(e^i(w,p^i,F^i)\big) \geq \pi_0, \quad \forall i = A, B. \qquad (IR-i)$$

$$e^i(w,p^i,F^i) \in \operatorname{Arg}\max_{e^{i'}}\{\pi^i(e^{i'})\},$$

$$\forall w, p^i, F^i \in R^+, \; i = A, B \quad (E-i)$$

$$\text{and} \quad -F^i \geq -F_0, \quad i = A, B. \qquad (LL-i)$$

A solution to [M.A.2.3] has the following properties:

Result 3. For all $i = A, B$,

(a) $F^i = F_0$

(b) $w > c$

(c) $(p^i - c)\partial q^i/\partial e^i > \partial C^i/\partial e^i$.

[26] See §A.1 for details.

Proof. Employing the first-order approach, the Lagrangian, L, is

$$L=(w-c)(q^A+q^B)+F^A+F^B+\lambda_A[(p^A-w)q^A-F^A-C^A(e^A)] +\lambda_B[(p^B-w)q^B-F^B-C^B(e^B)]+\mu_{Ae}\left[(p^A-w)\frac{\partial q^A}{\partial e^A}-\frac{\partial C^A}{\partial e^A}\right] +\mu_{Be}\left[(p^B-w)\frac{\partial q^B}{\partial e^B}-\frac{\partial C^B}{\partial e^B}\right]+\gamma_A[-F^A+F_0]+\gamma_B[-F^B+F_0]. \quad \text{(A.2.3.1)}$$

Assuming interior solutions, the necessary conditions for F^i, $i=A,B$ are

$$\frac{\partial L}{\partial F^i}=1-\lambda_i-\gamma_i=0 \quad \text{(A.2.3.2a)}$$

or

$$\lambda_i=1-\gamma_i. \quad \text{(A.2.3.2b)}$$

Notice from (A.2.3.2b) that if the limited liability constraints are not binding, the principal can implement the full-information efficient solution, as outlined in Result 2.[27] Consequently, here the focus is on the following case:

$$\gamma_i>0 \quad \text{for all } i=A,B. \quad \text{(A.2.3.3)}$$

Next, assuming an interior solution, the KT condition on w is

$$\frac{\partial L}{\partial w}=q^A+q^B-\lambda_A q^A-\lambda_B q^B-\mu_{Ae}\frac{\partial q^A}{\partial e^A}-\mu_{Be}\frac{\partial q^B}{\partial e^B}=0. \quad \text{(A.2.3.4)}$$

Substituting from (A.2.3.2b) in (A.2.3.4) gives

$$\frac{\partial L}{\partial w}=\gamma_A q^A+\gamma_B q^B-\mu_{Ae}\frac{\partial q^A}{\partial e^A}-\mu_{Be}\frac{\partial q^B}{\partial e^B}=0 \quad \text{(A.2.3.5a)}$$

or

$$\mu_{Ae}\frac{\partial q^A}{\partial e^A}+\mu_{Be}\frac{\partial q^B}{\partial e^B}=\gamma_A q^A+\gamma_B q^B>0. \quad \text{(A.2.3.5b)}$$

Next, note that

$$\frac{\partial L}{\partial \mu_{ie}}=(p^i-w)\frac{\partial q^i}{\partial e^i}-\frac{\partial C^i}{\partial e^i}=0. \quad \text{(A.2.3.6)}$$

The KT condition on i's effort is

$$\frac{\partial L}{\partial e^i}=(w-c)\frac{\partial q^i}{\partial e^i}+\lambda_i\left[(p^i-w)\frac{\partial q^i}{\partial e^i}-\frac{\partial C^i}{\partial e^i}\right] +\mu_{ie}\left[(p^i-w)\frac{\partial^2 q^i}{\partial e^{i2}}-\frac{\partial^2 C^i}{\partial e^{i2}}\right]=0. \quad \text{(A.2.3.7a)}$$

[27] Further, even if the limited liability constraint binds for only one of the two distributors, deviation from first-best will be induced in the behavior of both distributors. This is due to the *common* wholesale price; the reason the common price induces distortions from efficiency in this case is similar to the one provided below. Given that no new insights may be gained from further detailed analysis of asymmetric binding of limited liability constraints (i.e., one distributor's constraint binds and the other distributor's constraint is not binding), the analysis focuses on those settings where the liability constraints bind for both distributors.

Recalling (A.2.3.2b), (A.2.3.7) can be rewritten as

$$\frac{\partial L}{\partial e^i}=(w-c)\frac{\partial q^i}{\partial e^i}+\left[(p^i-w)\frac{\partial q^i}{\partial e^i}-\frac{\partial C^i}{\partial e^i}\right] -\gamma_i\left[(p^i-w)\frac{\partial q^i}{\partial e^i}-\frac{\partial C^i}{\partial e^i}\right]+\mu_{ie}\left[(p^i-w)\frac{\partial^2 q^i}{\partial e^{i2}}-\frac{\partial^2 C^i}{\partial e^{i2}}\right]=0. \quad \text{(A.2.3.7b)}$$

Noting that $(w-c)\partial q^i/\partial e^i+(p^i-w)\partial q^i/\partial e^i$ simplifies to $(p^i-c)\partial q^i/\partial e^i$, and (A.2.3.6) implies that the coefficient of γ_i in (A.2.3.7b) equals zero, (A.2.3.7b) is rewritten as

$$\frac{\partial L}{\partial e^i}=\left[(p^i-c)\frac{\partial q^i}{\partial e^i}-\frac{\partial C^i}{\partial e^i}\right]+\mu_{ie}\left[(p^i-w)\frac{\partial^2 q^i}{\partial e^{i2}}-\frac{\partial^2 C^i}{\partial e^{i2}}\right]=0 \quad \text{(A.2.3.7c)}$$

or

$$\left[(p^i-c)\frac{\partial q^i}{\partial e^i}-\frac{\partial C^i}{\partial e^i}\right]=-\mu_{ie}\left[(p^i-w)\frac{\partial^2 q^i}{\partial e^{i2}}-\frac{\partial^2 C^i}{\partial e^{i2}}\right]. \quad \text{(A.2.3.7d)}$$

The right-hand side on (A.2.3.7d) represents the distortion from the first-best effort choice. Notice that the concavity of the distributor's profit expression ensures that the term in the square brackets on the right-hand side of (A.2.3.7d) is strictly negative. Therefore, the sign of that right-hand side depends on the sign of μ_{ie}.

First, (A.2.3.5b) implies that at least one of the μs is strictly positive. Further, for $i=A,B$, (A.2.3.6) can be rewritten as

$$(p^i-c)\frac{\partial q^i}{\partial e^i}+(c-w)\frac{\partial q^i}{\partial e^i}-\frac{\partial C^i}{\partial e^i}=0 \quad \text{(A.2.3.8a)}$$

or

$$(p^i-c)\frac{\partial q^i}{\partial e^i}-\frac{\partial C^i}{\partial e^i}=(w-c)\frac{\partial q^i}{\partial e^i}. \quad \text{(A.2.3.8b)}$$

From (A.2.3.8b) it follows that for $i=A,B$, μ_{ie} is strictly positive only when $w>c$ (this is analogous to the argument in the proof of the solution to [M.A.2.2]). When $w>c$, however, both μ_{Ae} and μ_{Be} are strictly positive. Therefore, satisfying (A.2.3.5b) requires

$$w>c \quad \text{(A.2.3.9)}$$

and

$$\mu_{ie}>0 \quad \text{for all } i=A,B. \quad \text{(A.2.3.10)}$$

From (A.2.3.10) it follows that the sign of the right-hand side in (A.2.3.7d) is strictly positive, and

$$(p^i-c)\frac{\partial q^i}{\partial e^i}>\frac{\partial C^i}{\partial e^i}. \quad \text{(A.2.3.11)}$$

The statement of the result follows. □

Result 3(a) indicates that in the presence of binding limited liability constraints, the manufacturer imposes the maximum possible fixed fees on the distributors. From Result 3(b) it follows that it is optimal for the manufacturer to set the wholesale price above the marginal cost of production. The intuition is that, if the manufacturer sets a wholesale price equal to its marginal cost of production, then the distributors have the proper incentives at the margin; however, due to restrictions on the magnitudes of the fixed fees, the manufacturer cannot fully benefit from those proper incentives. Therefore, the manufacturer no longer wants to align the distributors' incentives with its own. Instead, it supplements the restricted fixed fees with some positive profit through a wholesale price that exceeds the marginal cost of production.

Finally, Result 3(c) indicates that each distributor selects an effort level such that the marginal benefit to the channel from that effort exceeds its marginal cost. This implies that, in the presence of binding limited liability constraints, exclusive distributors do not work hard enough on the manufacturer's behalf; they slack off because they receive too little incentive to exert the first-best level of effort.

It is worth noting that in all three scenarios analyzed above (i.e., [M.A.2.1]–[M.A.2.3]), it is legitimate to use the standard first-order approach (cf. Rogerson 1985). The extant literature, however, is silent on the validity of the first-order approach when the manufacturer does not control the final prices. Establishing (i.e., identifying exogenous conditions that ensure) the validity of the first-order approach in this new scenario is technical and is outside the scope of this paper.

Finally, the above analysis has been conducted in a setting where the manufacturer has complete control over the final prices charged to customers. The qualitative considerations identified in the above results do not vanish when the manufacturer ceases to control the distributors' final prices. The next three sections illustrate this.

A.2.4. Manufacturer Does Not Control Final Prices and Distributor Effort Is Observable

Here, distributors' effort is observable, but their prices are beyond the manufacturer's direct control. The formal statement of the manufacturer's problem, [M.A.2.4], is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$

$$\text{subject to} \quad \pi^i(e^i(w,p^i,F^i)) \geq \pi_0, \quad \forall i = A,B. \qquad (IR-i)$$

$$p^i(w,e^i,F^i) \in \underset{p^{i'}}{\text{Arg}\max}\{\pi^i(p^{i'})\},$$

$$\forall w, e^i, F^i \in R^+,\ i = A,B. \quad (P-i)$$

The $(P-i)$ constraints take into account the fact that each distributor controls its own price. A solution to [M.A.2.4] has the following property:

Result 4. In this setting, too, the manufacturer recovers the first-best solution by setting $w = c$.

Proof. The proof technique is identical to that for Result 5.

$$L = (w-c)(q^A+q^B)+F^A+F^B+\lambda_A[(p^A-w)q^A-F^A-C^A(e^A)] + \lambda_B[(p^B-w)q^B-F^B-C^B(e^B)]+\mu_{Ap}\left[q^A+(p^A-w)\frac{\partial q^A}{\partial p^A}\right] + \mu_{Bp}\left[q^B+(p^B-w)\frac{\partial q^B}{\partial p^B}\right]. \qquad (A.2.4.1)$$

Assuming an interior solution for F^i, $i = A,B$, gives

$$\frac{\partial L}{\partial F^i} = 1-\lambda_i = 0 \qquad (A.2.4.2a)$$

or

$$\lambda_i = 1. \qquad (A.2.4.2b)$$

The necessary condition for an interior solution for w is

$$\frac{\partial L}{\partial w} = q^A+q^B-\lambda_A q^A-\lambda_B q^B-\mu_{Ap}\frac{\partial q^A}{\partial p^A}-\mu_{Bp}\frac{\partial q^B}{\partial p^B} = 0. \qquad (A.2.4.3)$$

Substituting from (A.2.4.2b) into (A.2.4.3) gives

$$\mu_{Ap}\frac{\partial q^A}{\partial p^A}+\mu_{Bp}\frac{\partial q^B}{\partial p^B} = 0. \qquad (A.2.4.4)$$

Next:

$$\frac{\partial L}{\partial \mu_{ip}} = q^i+(p^i-w)\frac{\partial q^i}{\partial p^i} = 0 \qquad (A.2.4.5a)$$

or

$$q^i+(p^i-c)\frac{\partial q^i}{\partial p^i}+(c-w)\frac{\partial q^i}{\partial p^i} = 0 \qquad (A.2.4.5b)$$

or

$$q^i+(p^i-c)\frac{\partial q^i}{\partial p^i} = (w-c)\frac{\partial q^i}{\partial p^i}. \qquad (A.2.4.5c)$$

Note that (A.2.4.4) requires that either $\mu_{Ap} = \mu_{Bp} = 0$, or μ_{Ap} and μ_{Bp} have opposite signs. From (A.2.4.5c), however, it is clear that both μ_{Ap} and μ_{Bp} will have the same sign. Consequently, at the solution to [M.A.2.4], $w = c$ and the incentive problem is eliminated. The statement of the result follows. □

A.2.5. Manufacturer Does Not Control Final Prices and Distributor Effort Is Unobservable

A formal statement of the manufacturer's problem, [M.A.2.5], is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$

$$\text{subject to} \quad \pi^i(e^i(w,F^i),p^i(w,F^i)) \geq \pi_0,$$

$$\forall i = A,B. \quad (IR-i)$$

$$\text{where} \quad e^i(w,F^i),p^i(w,F^i) \in \underset{e^{i'},p^{i'}}{\text{Arg}\max}\{\pi^i(e^{i'},p^{i'})\},$$

$$\forall w, F^i \in R^+,\ i = A,B. \quad (EP-i)$$

A solution to [M.A.2.5] has the following property:

Result 5. By setting $w = c$, the manufacturer regains the full-information efficient solution.

Proof. The proof technique is identical to that in the previous result. The Lagrangian, L, for this problem is

$$L = (w-c)(q^A+q^B)+F^A+F^B+\lambda_A[(p^A-w)q^A-F^A-C^A(e^A)] + \lambda_B[(p^B-w)q^B-F^B-C^B(e^B)]+\mu_{Ae}\left[(p^A-w)\frac{\partial q^A}{\partial e^A}-\frac{\partial C^A}{\partial e^A}\right] + \mu_{Be}\left[(p^B-w)\frac{\partial q^B}{\partial e^B}-\frac{\partial C^B}{\partial e^B}\right]+\mu_{Ap}\left[q^A+(p^A-w)\frac{\partial q^A}{\partial p^A}\right] + \mu_{Bp}\left[q^B+(p^B-w)\frac{\partial q^B}{\partial p^B}\right]. \qquad (A.2.5.1)$$

Consider the distributor A's effort selection constraint (analogous results hold for distributor B):

$$\frac{\partial L}{\partial \mu_{Ae}} = (p^A - w)\frac{\partial q^A}{\partial e^A} - \frac{C^A}{e^A} = 0. \qquad \text{(A.2.5.2a)}$$

Rewriting (A.2.5.2a) gives

$$(p^A - c + c - w)\frac{\partial q^A}{\partial e^A} - \frac{C^A}{e^A} = 0. \qquad \text{(A.2.5.2b)}$$

Rearranging gives

$$(p^A - c)\frac{\partial q^A}{\partial e^A} - \frac{C^A}{e^A} = (w-c)\frac{\partial q^A}{\partial e^A}. \qquad \text{(A.2.5.2c)}$$

Next, distributor A's price selection constraint provides

$$\frac{\partial L}{\partial \mu_{Ap}} = q^A + (p^A - w)\frac{\partial q^A}{\partial p^A} = 0. \qquad \text{(A.2.5.3a)}$$

Rewriting and rearranging (A.2.5.3) gives

$$q^A + (p^A - c)\frac{\partial q^A}{\partial p^A} = (w-c)\frac{\partial q^A}{\partial p^A}. \qquad \text{(A.2.5.3b)}$$

From (A.2.5.2c) and (A.2.5.3b) it is clear that setting $w = c$ will eliminate any deviation from efficiency. That is, at $w = c$, the final prices and efforts satisfy conditions (b) and (c) in Result 1. Consequently, the statement of the result follows. □

It is worth noting that the full-information efficient solution is regained in this case even when distributor effort is unobservable to the manufacturer. By selling the product at its marginal cost of production, the manufacturer is making each exclusive distributor a residual claimant. Consequently, the distributor's objectives at the margin are completely in line with those of the manufacturer. Of course, the manufacturer extracts all rents through the fixed fees and restricts the expected profit of each exclusive distributor to its reservation level. This standard result is an essential benchmark in discussing the next subsection's results.

To this point, the analysis has not found any new distortions in the distributors' behavior due to incomplete manufacturer control over final prices. The next case provides the greatest opportunity for distributors to deviate from the full-information efficient levels.

A.2.6. Manufacturer Does Not Control Final Prices, Distributor Effort Is Unobservable, and the Limited Liability Constraints Are Binding

A formal statement of the manufacturer's problem, $[M-E]$, is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$

$$\text{subject to} \quad \pi^i\big(e^i(w,F^i), p^i(w,F^i)\big) \geq \pi_0, \quad \forall i = A, B. \qquad (IR-i)$$

$$e^i(w,F^i), p^i(w,F^i) \in \underset{e^{i'},\, p^{i'}}{\operatorname{Arg\,max}}\{\pi^i(e^{i'}, p^{i'})\},$$

$$\forall w, F^i \in R^+, \ i = A, B. \qquad (EP-i)$$

$$\text{and} \quad -F^i \geq -F_0, \quad i = A, B. \qquad (LL-i)$$

A solution to $[M - E]$ is summarized below (this corresponds to Finding 1 in the main paper):

Result 6. For all $i = A, B$,

(a) In the presence of binding limited liability constraints, inefficiency exists in the distributors' choices of efforts and final prices. More formally, both μ_{ie} and μ_{ip} cannot equal zero and the first-best solution (where both μ_{ie} and μ_{ip} equal zero) cannot be achieved.

(b) At optimality, the inefficiency arising from distributor i's choice of effort is comprised of the following two components:

(i) the slack effect, $-\mu_{ie}[(p^i - w)\partial^2 q^i/\partial e^{i^2} - \partial^2 C^i/\partial e^{i^2}]$
(ii) the price effect, $-\mu_{ip}\partial q^i/\partial e^i$

(c) Further, suppose $\partial^2 q^i/\partial p^i \partial e^i = \partial^2 q^i/\partial p^{i^2} = 0$ and $(\partial q^i/\partial p^i)(\partial^2 \pi^i/\partial e^{i^2}) - (\partial^2 \pi^i/\partial p^i \partial e^i)^2 > 0$. When these sufficient conditions hold,[28] then

(i) $w > c$
(ii) $(p^i - c)\partial q^i/\partial e^i > \partial C^i/\partial e^i$
(iii) $q^i < -(p^i - c)\partial q^i/\partial p^i$.

Proof. The Lagrangian is

$$\begin{aligned}L &= (w-c)(q^A+q^B)+F^A+F^B+\lambda_A[(p^A-w)q^A-F^A-C^A(e^A)]\\ &\quad+\lambda_B[(p^B-w)q^B-F^B-C^B(e^B)]+\mu_{Ae}\left[(p^A-w)\frac{\partial q^A}{\partial e^A}-\frac{\partial C^A}{\partial e^A}\right]\\ &\quad+\mu_{Be}\left[(p^B-w)\frac{\partial q^B}{\partial e^B}-\frac{\partial C^B}{\partial e^B}\right]+\mu_{Ap}\left[q^A+(p^A-w)\frac{\partial q^A}{\partial p^A}\right]\\ &\quad+\mu_{Bp}\left[q^B+(p^B-w)\frac{\partial q^B}{\partial p^B}\right]+\gamma_A[-F^A+F_0]+\gamma_B[-F^B+F_0].\end{aligned} \qquad \text{(A.2.6.1)}$$

The necessary (KT) conditions for F^i, $i = A, B$ are

$$\frac{\partial L}{\partial F^i} = 1 - \lambda_i - \gamma_i = 0 \qquad \text{(A.2.6.2a)}$$

or

$$\lambda_i = 1 - \gamma_i. \qquad \text{(A.2.6.2b)}$$

Notice from (A.2.6.2b) that if the limited liability constraints are not binding, the principal can implement the full-information efficient solution, as outlined in Result 5. Again, for the same reasons as those given for (A.2.3.3), the focus is on the following case:

$$\gamma_i > 0 \quad \text{for all } i = A, B. \qquad \text{(A.2.6.3)}$$

Next, assuming an interior solution, the KT condition on w is

$$\begin{aligned}\frac{\partial L}{\partial w} &= q^A + q^B - \lambda_A q^A - \lambda_B q^B - \mu_{Ae}\frac{\partial q^A}{\partial e^A} - \mu_{Ap}\frac{\partial q^A}{\partial p^A}\\ &\quad - \mu_{Be}\frac{\partial q^B}{\partial e^B} - \mu_{Bp}\frac{\partial q^B}{\partial p^B} = 0.\end{aligned} \qquad \text{(A.2.6.4)}$$

Substituting from (A.2.6.2b) in (A.2.6.4) gives

$$\mu_{Ae}\frac{\partial q^A}{\partial e^A} + \mu_{Ap}\frac{\partial q^A}{\partial p^A} + \mu_{Be}\frac{\partial q^B}{\partial e^B} + \mu_{Bp}\frac{\partial q^B}{\partial p^B} = \gamma_A q^A + \gamma_B q^B > 0. \qquad \text{(A.2.6.5)}$$

From (A.2.6.5) it follows that all the μs cannot equal zero, and the first-best solution cannot be regained. The statement of Result 6(a) follows.

[28] These hold in the example in the main text of the paper.

Further:

$$\frac{\partial L}{\partial \mu_{ie}} = (p^i - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = 0 \qquad \text{(A.2.6.6)}$$

and

$$\frac{\partial L}{\partial \mu_{ip}} = q^i + (p^i - w)\frac{\partial q^i}{\partial p^i} = 0. \qquad \text{(A.2.6.7)}$$

The necessary condition on the ith distributors' choice of effort and price are, for all $i = A, B$:

$$\begin{aligned}\frac{\partial L}{\partial e^i} &= (w-c)\frac{\partial q^i}{\partial e^i} + \lambda_i\left[(p^i - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i}\right]\\ &\quad + \mu_{ie}\left[(p^i - w)\frac{\partial^2 q^i}{\partial e^{i2}} - \frac{\partial^2 C^i}{\partial e^{i2}}\right]\\ &\quad + \mu_{ip}\left[\frac{\partial q^i}{\partial e^i} + (p^i - w)\frac{\partial^2 q^i}{\partial e^i \partial p^i}\right] = 0. \qquad \text{(A.2.6.8)}\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial p^i} &= (w-c)\frac{\partial q^i}{\partial p^i} + \lambda_i\left[q^i + (p^i - w)\frac{\partial q^i}{\partial p^i}\right]\\ &\quad + \mu_{ie}\left[\frac{\partial q^i}{\partial e^i} + (p^i - w)\frac{\partial^2 q^i}{\partial p^i \partial e^i}\right]\\ &\quad + \mu_{ip}\left[2\frac{\partial q^i}{\partial p^i} + (p^i - w)\frac{\partial^2 q^i}{\partial p^{i2}}\right] = 0. \qquad \text{(A.2.6.9)}\end{aligned}$$

Substituting λ_i with $(1 - \gamma_i)$, (A.2.6.8) and (A.2.6.9) are rewritten as

$$\begin{aligned}\frac{\partial L}{\partial e^i} &= (w-c)\frac{\partial q^i}{\partial e^i} + \left[(p^i - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i}\right]\\ &\quad - \gamma_i\left[(p^i - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i}\right]\\ &\quad + \mu_{ie}\left[(p^i - w)\frac{\partial^2 q^i}{\partial e^{i2}} - \frac{\partial^2 C^i}{\partial e^{i2}}\right]\\ &\quad + \mu_{ip}\left[\frac{\partial q^i}{\partial e^i} + (p^i - w)\frac{\partial^2 q^i}{\partial e^i \partial p^i}\right] = 0. \qquad \text{(A.2.6.10)}\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial p^i} &= (w-c)\frac{\partial q^i}{\partial p^i} + \left[q^i + (p^i - w)\frac{\partial q^i}{\partial p^i}\right] - \gamma_i\left[q^i + (p^i - w)\frac{\partial q^i}{\partial p^i}\right]\\ &\quad + \mu_{ie}\left[\frac{\partial q^i}{\partial e^i} + (p^i - w)\frac{\partial^2 q^i}{\partial p^i \partial e^i}\right]\\ &\quad + \mu_{ip}\left[2\frac{\partial q^i}{\partial p^i} + (p^i - w)\frac{\partial^2 q^i}{\partial p^{i2}}\right] = 0. \qquad \text{(A.2.6.11)}\end{aligned}$$

Note that (a) $(w - c)\partial q^i/\partial e^i + (p^i - w)\partial q^i/\partial e^i$ simplifies to $(p^i - c)\partial q^i/\partial e^i$; (b) $(w - c)\partial q^i/\partial p^i + (p^i - w)\partial q^i/\partial p^i$ simplifies to $(p^i - c)\partial q^i/\partial p^i$; (c) from (A.2.6.6) it follows that the coefficient of γ_i in (A.2.6.10) is zero; and (d) from (A.2.6.7) it follows that the coefficient of γ_i in (A.2.6.11) is zero. Given these, the KT conditions on prices and efforts can be rewritten as

$$\begin{aligned}\frac{\partial L}{\partial e^i} &= \left[(p^i - c)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i}\right] + \mu_{ie}\left[(p^i - w)\frac{\partial^2 q^i}{\partial e^{i2}} - \frac{\partial^2 C^i}{\partial e^{i2}}\right]\\ &\quad + \mu_{ip}\left[\frac{\partial q^i}{\partial e^i} + (p^i - w)\frac{\partial^2 q^i}{\partial e^i \partial p^i}\right] = 0, \qquad \text{(A.2.6.12)}\end{aligned}$$

$$\begin{aligned}\frac{\partial L}{\partial p^i} &= \left[q^i + (p^i - c)\frac{\partial q^i}{\partial p^i}\right] + \mu_{ie}\left[\frac{\partial q^i}{\partial e^i} + (p^i - w)\frac{\partial^2 q^i}{\partial p^i \partial e^i}\right]\\ &\quad + \mu_{ip}\left[2\frac{\partial q^i}{\partial p^i} + (p^i - w)\frac{\partial^2 q^i}{\partial p^{i2}}\right] = 0, \qquad \text{(A.2.6.13)}\end{aligned}$$

or

$$\begin{aligned}\left[(p^i - c)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i}\right] &= -\mu_{ie}\left[(p^i - w)\frac{\partial^2 q^i}{\partial e^{i2}} - \frac{\partial^2 C^i}{\partial e^{i2}}\right]\\ &\quad - \mu_{ip}\left[\frac{\partial q^i}{\partial e^i} + (p^i - w)\frac{\partial^2 q^i}{\partial e^i \partial p^i}\right], \qquad \text{(A.2.6.14)}\end{aligned}$$

$$\begin{aligned}\left[q^i + (p^i - c)\frac{\partial q^i}{\partial p^i}\right] &= -\mu_{ie}\left[\frac{\partial q^i}{\partial e^i} + (p^i - w)\frac{\partial^2 q^i}{\partial p^i \partial e^i}\right]\\ &\quad - \mu_{ip}\left[2\frac{\partial q^i}{\partial p^i} + (p^i - w)\frac{\partial^2 q^i}{\partial p^{i2}}\right]. \qquad \text{(A.2.6.15)}\end{aligned}$$

From the above expressions, the statement of Result 6(b) follows.

Next, rewriting (A.2.6.6) and (A.2.6.7) gives, for all $i = A, B$:

$$(p^i - c)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = (w - c)\frac{\partial q^i}{\partial e^i} \qquad \text{(A.2.6.16)}$$

and

$$q^i + (p^i - c)\frac{\partial q^i}{\partial p^i} = (w - c)\frac{\partial q^i}{\partial p^i}. \qquad \text{(A.2.6.17)}$$

Also, recall that

$$\frac{\partial^2 \pi^i}{\partial e^{i2}} = \left[(p^i - w)\frac{\partial^2 q^i}{\partial e^{i2}} - \frac{\partial^2 C^i}{\partial e^{i2}}\right],$$

$$\frac{\partial^2 \pi^i}{\partial p^i \partial e^i} = \left[\frac{\partial q^i}{\partial e^i} + (p^i - w)\frac{\partial^2 q^i}{\partial e^i \partial p^i}\right],$$

and

$$\frac{\partial^2 \pi^i}{\partial p^{i2}} = \left[2\frac{\partial q^i}{\partial p^i} + (p^i - w)\frac{\partial^2 q^i}{\partial p^{i2}}\right].$$

Now, using (A.2.6.16) and (A.2.6.17), Equations (A.2.6.14) and (A.2.6.15) can be rewritten as follows:

$$(w - c)\frac{\partial q^i}{\partial e^i} = -\mu_{ie}\frac{\partial^2 \pi^i}{\partial e^{i2}} - \mu_{ip}\frac{\partial^2 \pi^i}{\partial p^i \partial e^i} \qquad \text{(A.2.6.18)}$$

and

$$(w - c)\frac{\partial q^i}{\partial p^i} = -\mu_{ie}\frac{\partial^2 \pi^i}{\partial p^i \partial e^i} - \mu_{ip}\frac{\partial^2 \pi^i}{\partial p^{i2}}. \qquad \text{(A.2.6.19)}$$

(A.2.6.18) can be rewritten as

$$(w - c) = -\mu_{ie}\frac{\partial^2 \pi^i/\partial e^{i2}}{\partial q^i/\partial e^i} - \mu_{ip}\frac{\partial^2 \pi^i/\partial p^i \partial e^i}{\partial q^i/\partial e^i}. \qquad \text{(A.2.6.20)}$$

Similarly, (A.2.6.19) can be rewritten as

$$(w - c) = -\mu_{ie}\frac{\partial^2 \pi^i/\partial p^i \partial e^i}{\partial q^i/\partial p^i} - \mu_{ip}\frac{\partial^2 \pi^i/\partial p^{i2}}{\partial q^i/\partial p^i}. \qquad \text{(A.2.6.21)}$$

Equating the right-hand side of (A.2.6.20) with the right-hand side of (A.2.6.21) gives

$$-\mu_{ie}\frac{\partial^2\pi^i/\partial e^{i2}}{\partial q^i/\partial e^i}-\mu_{ip}\frac{\partial^2\pi^i/\partial p^i\partial e^i}{\partial q^i/\partial e^i}=-\mu_{ie}\frac{\partial^2\pi^i/\partial p^i\partial e^i}{\partial q^i/\partial p^i}-\mu_{ip}\frac{\partial^2\pi^i/\partial p^{i2}}{\partial q^i/\partial p^i}. \quad (A.2.6.22)$$

Rearranging (A.2.6.22) so that the μ_{ie} terms are one side and the μ_{ip} terms are on the other gives

$$\mu_{ie}\left[-\frac{\partial^2\pi^i/\partial e^{i2}}{\partial q^i/\partial e^i}+\frac{\partial^2\pi^i/\partial p^i\partial e^i}{\partial q^i/\partial p^i}\right]=\mu_{ip}\left[\frac{\partial^2\pi^i/\partial p^i\partial e^i}{\partial q^i/\partial e^i}-\frac{\partial^2\pi^i/\partial p^{i2}}{\partial q^i/\partial p^i}\right] \quad (A.2.6.23)$$

or

$$\frac{\mu_{ie}}{\mu_{ip}}=\frac{\dfrac{\partial^2\pi^i/\partial p^i\partial e^i}{\partial q^i/\partial e^i}-\dfrac{\partial^2\pi^i/\partial p^{i2}}{\partial q^i/\partial p^i}}{-\dfrac{\partial^2\pi^i/\partial e^{i2}}{\partial q^i/\partial e^i}+\dfrac{\partial^2\pi^i/\partial p^i\partial e^i}{\partial q^i/\partial p^i}}=\frac{\dfrac{\partial q^i}{\partial p^i}\dfrac{\partial^2\pi^i}{\partial p^i\partial e^i}-\dfrac{\partial q^i}{\partial e^i}\dfrac{\partial^2\pi^i}{\partial p^{i2}}}{\dfrac{\partial q^i}{\partial e^i}\dfrac{\partial^2\pi^i}{\partial p^i\partial e^i}-\dfrac{\partial q^i}{\partial p^i}\dfrac{\partial^2\pi^i}{\partial e^{i2}}}. \quad (A.2.6.24)$$

The signs on the Lagrange multipliers μ_{ie} and μ_{ip} will depend on the sign of the right-hand side in (A.2.6.24). For instance, if the numerator and the denominator on the right-hand side of (A.2.6.24) have different signs, then μ_{ie} and μ_{ip} cannot have the same sign. Here, the analysis will focus on the case of a simple demand structure that is employed in the example in §3 of the main paper. There, the following additional assumptions are met:

(1) First, $\partial^2 q^i/\partial p^i\partial e^i=\partial^2 q^i/\partial p^{i2}=0$. This implies:

$$\frac{\partial^2\pi^i}{\partial p^{i2}}=2\frac{\partial q^i}{\partial p^i} \quad (A.2.6.25)$$

and

$$\frac{\partial^2\pi^i}{\partial e^i\partial p^i}=\frac{\partial q^i}{\partial e^i}. \quad (A.2.6.26)$$

(Note: $\partial^2\pi^i/\partial e^{i2}$ is not affected. Recall $\partial^2\pi^i/\partial e^{i2}=(p^i-w)\cdot\partial^2 q^i/\partial e^{i2}-\partial^2 C^i/\partial e^{i2}$.)

(2) $(\partial q^i/\partial p^i)(\partial^2\pi^i/\partial e^{i2})-(\partial^2\pi^i/\partial p^i\partial e^i)^2>0$.

(Recall that concavity requires $(\partial^2\pi^i/\partial p^{i2})(\partial^2\pi^i/\partial e^{i2})-(\partial^2\pi^i/\partial p^i\partial e^i)^2>0$. Since the absolute value of $\partial^2\pi^i/\partial p^{i2}$ is larger than the absolute value of $\partial q^i/\partial p^i$, additional assumption (2) automatically ensures concavity.)

Under these assumptions, (A.2.6.24) reduces to

$$\frac{\mu_{ie}}{\mu_{ip}}=\frac{\dfrac{\partial q^i}{\partial p^i}\dfrac{\partial q^i}{\partial e^i}-2\dfrac{\partial q^i}{\partial e^i}\dfrac{\partial q^i}{\partial p^i}}{\dfrac{\partial q^i}{\partial e^i}\dfrac{\partial q^i}{\partial e^i}-\dfrac{\partial q^i}{\partial p^i}\dfrac{\partial^2\pi^i}{\partial e^{i2}}}=\frac{-\dfrac{\partial q^i}{\partial p^i}\dfrac{\partial q^i}{\partial e^i}}{\left(\dfrac{\partial q^i}{\partial e^i}\right)^2-\dfrac{\partial q^i}{\partial p^i}\dfrac{\partial^2\pi^i}{\partial e^{i2}}}<0. \quad (A.2.6.27)$$

The numerator in (A.2.6.27) is positive, while from additional assumption (2), the denominator is negative; consequently, μ_{ie} and μ_{ip} have to be of opposite signs.

Next, for ease of exposition, define $(MR-MC)_e\equiv(p^i-c)\cdot\partial q^i/\partial e^i-\partial C^i/\partial e^i$, and $(MR-MC)_p\equiv q^i+(p^i-c)\partial q^i/\partial p^i$. Now, (A.2.6.14) and (A.2.6.15) can be rewritten as follows:

$$(MR-MC)_e=-\mu_{ie}\frac{\partial^2\pi^i}{\partial e^{i2}}-\mu_{ip}\frac{\partial^2\pi^i}{\partial p^i\partial e^i}=\mu_{ie}\left[-\frac{\partial^2\pi^i}{\partial e^{i2}}-\frac{\mu_{ip}}{\mu_{ie}}\frac{\partial^2\pi^i}{\partial p^i\partial e^i}\right], \quad (A.2.6.28)$$

$$(MR-MC)_p=-\mu_{ie}\frac{\partial^2\pi^i}{\partial p^i\partial e^i}-\mu_{ip}\frac{\partial^2\pi^i}{\partial p^{i2}}=\mu_{ip}\left[-\frac{\mu_{ie}}{\mu_{ip}}\frac{\partial^2\pi^i}{\partial p^i\partial e^i}-\frac{\partial^2\pi^i}{\partial p^{i2}}\right]. \quad (A.2.6.29)$$

Note that from (A.2.6.16) and (A.2.6.17) it follows that for a given w ($\neq c$), $(MR-MC)_p$ and $(MR-MC)_e$ are of opposite signs. Next, (A.2.6.27) indicates that μ_{ie} and μ_{ip} are of opposite signs. The analysis now shows that μ_{ie} and μ_{ip} have the same sign as $(MR-MC)_e$ and $(MR-MC)_p$, respectively. To show that, first assume μ_{ie}'s sign is opposite that of $(MR-MC)_e$ and μ_{ip}'s sign is opposite that of $(MR-MC)_p$, and then arrive at a contradiction as shown below.

Since the Lagrange multipliers do not have the same sign as $(MR-MC)$, the square bracketed terms on the right-hand side of (A.2.6.28) and (A.2.6.29) must each be negative. That is

$$-\frac{\partial^2\pi^i}{\partial e^{i2}}-\frac{\mu_{ip}}{\mu_{ie}}\frac{\partial^2\pi^i}{\partial p^i\partial e^i}<0 \quad (A.2.6.30)$$

and

$$\frac{\mu_{ie}}{\mu_{ip}}\frac{\partial^2\pi^i}{\partial p^i\partial e^i}-\frac{\partial^2\pi^i}{\partial p^{i2}}<0. \quad (A.2.6.31)$$

(A.2.6.30) implies

$$\frac{\partial^2\pi^i}{\partial e^{i2}}>-\frac{\mu_{ip}}{\mu_{ie}}\frac{\partial^2\pi^i}{\partial p^i\partial e^i} \quad (A.2.6.32)$$

and (A.2.6.31) implies

$$\frac{\partial^2\pi^i}{\partial p^{i2}}>-\frac{\mu_{ie}}{\mu_{ip}}\frac{\partial^2\pi^i}{\partial p^i\partial e^i}. \quad (A.2.6.33)$$

However, in (A.2.6.32), the right-hand side is strictly positive (since the μs are of opposite sign and $\partial^2\pi^i/\partial p^i\partial e^i>0$), while the left-hand side is negative by assumption. (A similar argument holds for (A.2.6.33).) Consequently, the Lagrange multipliers μ_{ie} and μ_{ip} have the same sign as $(MR-MC)_e$ and $(MR-MC)_p$, respectively. (Also, from (A.2.6.16) it follows that μ_{Ae} and μ_{Be} must have the same sign, and from (A.2.6.17) that μ_{Ap} and μ_{Bp} must have the same sign.)

Given the above, two distinct possibilities need to be considered: either $\mu_{ie}>0$ and $\mu_{ip}<0$ or $\mu_{ie}<0$ and $\mu_{ip}>0$. Suppose $\mu_{ie}<0$ and $\mu_{ip}>0$; then the left-hand side of (A.2.6.5) is negative and that equation cannot be satisfied. Consequently, at the solution to $[M-E]$, $\mu_{ip}<0$ and $\mu_{ie}>0$, $\forall i=A,B$, and this requires $w>c$.

Finally, notice that in the KT condition for the ith distributor's effort (see (A.2.6.14)), the effort distortion is comprised of two components: the one involving μ_{ie} is analogous to the one in the solution to [M.A.2.3]. Here, however, since the distributor also selects its prices, the effort distortion has another component that involves μ_{ip}. The statement of the result follows. □

As the results indicate, the manufacturer imposes the maximum possible fixed fees on both its exclusive distributors and sets a wholesale price above its marginal cost of production. The intuition behind this result can be explained as follows: If the manufacturer sets $w = c$, the distributors have the proper incentives at the margin and will exert first-best efforts and choose first-best prices. However, due to limitations on the fixed fees, the manufacturer cannot fully benefit from such optimal behavior. Next, if manufacturer sets $w < c$, distributors will choose effort levels higher than the first-best levels and set final prices below the first-best levels. Both these actions serve to enhance demand; however, once again, the manufacturer cannot benefit from these activities because of limitations on the fixed fees. Further, selling more at a loss only serves to decrease the manufacturer's profit even further.

On the other hand, if the manufacturer sets $w > c$, then the distributors do not have the first-best incentives at the margin. But if the maximum possible fixed fees can be imposed while ensuring that the distributors participate, then $w > c$ gives the manufacturer some additional profit. Therefore, the manufacturer will set w above c.

At optimality, the exclusive distributors in turn set their prices too high and put forth too little effort on behalf of the manufacturer. Next, in contrast to Result 3, Result 6 reveals that the effort distortion has an additional component, $-\mu_{Ap}\partial q^A/\partial e^A$, due to distributors controlling their own prices.

A.3. Analysis of the Nonexclusive Distribution Model

A.3.1. Manufacturer Controls Final Prices and Distributor Effort Is Observable

As in the exclusive territories setting, the analysis begins by recording a benchmark solution where the manufacturer observes distributor effort costlessly. The formal statement of the manufacturer's problem, [M.A.3.1], is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$
$$\text{subject to} \quad \pi^i \ge \pi_0, \quad \forall i = A, B. \qquad (IR-i)$$

A solution to [M.A.3.1] has the following properties:

Result 7. For $i \neq j$; $i = A, B$,
(a) $\pi^i = \pi_0$
(b) $(p^i - c)\partial q^i/\partial e^i + (p^j - c)\partial q^j/\partial e^i = \partial C^i/\partial e^i$
(c) $q^i = -(p^i - c)\partial q^i/\partial p^i - (p^j - c)\partial q^j/\partial p^i$.

Proof. The Lagrangian, L, for this problem is

$$\begin{aligned} L = {} & (w-c)(q^A+q^B)+F^A+F^B \\ & + \lambda_A[(p^A-w)q^A - F^A - C^A(e^A)] \\ & + \lambda_B[(p^B-w)q^B - F^B - C^B(e^B)]. \end{aligned} \qquad \text{(A.3.1.1)}$$

Assuming an interior solution for F^i, $i = A, B$, the necessary KT conditions for optimality are

$$\frac{\partial L}{\partial F^i} = 1 - \lambda_i = 0 \qquad \text{(A.3.1.2a)}$$

or

$$\lambda_i = 1. \qquad \text{(A.3.1.2b)}$$

Similarly, assuming interior solutions for p^i and e^i, $\forall i \neq j$ and $i, j = A, B$, the necessary conditions for optimality are

$$\frac{\partial L}{\partial p^i} = q^i + (p^i - c)\frac{\partial q^i}{\partial p^i} + (p^j - c)\frac{\partial q^j}{\partial p^i} = 0, \qquad \text{(A.3.1.3)}$$

$$\frac{\partial L}{\partial e^i} = (p^i - c)\frac{\partial q^i}{\partial e^i} + (p^j - c)\frac{\partial q^j}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = 0. \qquad \text{(A.3.1.4)}$$

The statement of the result follows. □

Result 7(a) indicates that both distributors are held to their reservation level of profit. In this nonexclusive distribution model, note that when a distributor changes its price or effort, there are two effects: First, the distributor's own demand is affected, and second, the other distributor's demand is affected. Results 7(b) and 7(c) indicate that the optimal prices and efforts are selected efficiently, taking into account the manner in which the two distributors' demands are interlinked. For this reason, the optimal prices and efforts here are different from those in the exclusive distribution scenario (see Result 1); the exact difference in magnitude and direction between the two sets of prices and efforts will be contingent on the specified demand structure.

A.3.2. Manufacturer Controls Final Prices and Distributor Effort Is Unobservable

The formal statement of the manufacturer's problem, [M.A.3.2], is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$
$$\text{subject to} \quad \pi^i(e^i(\cdot), e^j(\cdot)) \ge \pi_0, \quad \forall i = A, B. \qquad (IR-i)$$
$$\text{where} \quad e^i(w, p^i, p^j, F^i, F^j) \in \operatorname*{Arg\,max}_{e^{i'}}\{\pi^i(e^{i'}, e^j(\cdot))\},$$
$$\forall w, p^i, p^j, F^i, F^j \in R^+,\ i \neq j,\ i, j = A, B. \quad (E-i)$$

In the effort selection constraints, $(E-i)$, above (and those that will be encountered later on), each distributor chooses his effort level to maximize his own profit as a best response to the other distributor doing the same. A solution to [M.A.3.2] has the following property:

Result 8. The manufacturer recovers the full-information efficient solution when it sells to its distributors at a wholesale price, w^*, which is below the marginal cost of production, c. At that wholesale price, the following condition is required to hold:

$$(p^B - c)\frac{\partial q^B/\partial e^A}{\partial q^A/\partial e^A} = (p^A - c)\frac{\partial q^A/\partial e^B}{\partial q^B/\partial e^B}.$$

Proof. The Lagrangian, L, for this problem is

$$\begin{aligned} L = {} & (w-c)(q^A+q^B)+F^A+F^B+\lambda_A[(p^A-w)q^A-F^A-C^A(e^A)] \\ & +\lambda_B[(p^B-w)q^B-F^B-C^B(e^B)]+\mu_{Ae}\left[(p^A-w)\frac{\partial q^A}{\partial e^A}-\frac{\partial C^A}{\partial e^A}\right] \\ & +\mu_{Be}\left[(p^B-w)\frac{\partial q^B}{\partial e^B}-\frac{\partial C^B}{\partial e^B}\right]. \end{aligned} \qquad \text{(A.3.2.1)}$$

Assuming an interior solution for F^i, $i = A, B$, gives the following necessary conditions for optimality:

$$\frac{\partial L}{\partial F^i} = 1 - \lambda_i = 0 \tag{A.3.2.2a}$$

or

$$\lambda_i = 1. \tag{A.3.2.2b}$$

Next, assuming an interior solution, the KT condition on w is

$$\frac{\partial L}{\partial w} = -\mu_{Ae}\frac{\partial q^A}{\partial e^A} - \mu_{Be}\frac{\partial q^B}{\partial e^B} = 0 \tag{A.3.2.3}$$

or

$$\mu_{Ae}\frac{\partial q^A}{\partial e^A} + \mu_{Be}\frac{\partial q^B}{\partial e^B} = 0. \tag{A.3.2.4}$$

(A.3.2.4) is satisfied either when μ_{Ae} and μ_{Be} each equal zero or they have opposite signs. Further:

$$\frac{\partial L}{\partial \mu_{ie}} = (p^i - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = 0 \tag{A.3.2.5a}$$

or

$$(p^i - c)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = (w - c)\frac{\partial q^i}{\partial e^i}. \tag{A.3.2.5b}$$

From (A.3.2.5b) it is clear that there will not be any incentive problems if the wholesale price satisfies the following condition (if it is satisfied, marginal revenue to the channel equals the marginal cost as in Result 7):

$$-(w - c)\frac{\partial q^i}{\partial e^i} = (p^j - c)\frac{\partial q^j}{\partial e^i} \tag{A.3.2.6}$$

or

$$-(w - c) = \frac{(p^B - c)\partial q^B/\partial e^A}{\partial q^A/\partial e^A} = \frac{(p^A - c)\partial q^A/\partial e^B}{\partial q^B/\partial e^B}. \tag{A.3.2.7}$$

When (A.3.2.7) is satisfied, both μs equal zero[29] and the first-best solution is regained. The statement of the result follows. □

The condition in Result 8 reveals the flexibility enjoyed by a manufacturer when it has direct control over final prices. Further, the condition indicates the problems that can arise when the manufacturer has only indirect control over final prices.

Notice that to regain the first-best solution, the manufacturer chooses a wholesale price lower than its marginal production cost. When one distributor chooses its effort level, it ignores the positive effect of that effort on the other distributor. Consequently, left to itself, each distributor chooses effort levels that are lower than the full-information efficient levels. To motivate the distributors to put forth the full-information efficient levels, the manufacturer sets the wholesale price lower than cost. As usual, however, the manufacturer recovers any lost revenues through suitably designed fixed fees, which ensure binding $(IR - i)$ constraints for the distributors.

[29] We can verify this by solving the first-order conditions with respect to e^i simultaneously for the μs.

A.3.3. Manufacturer Controls Final Prices, Distributor Effort Is Unobservable, and the Limited Liability Constraints Are Binding

Here, the formal statement of the manufacturer's problem, $[M.\text{A}.3.3]$, is

$$\underset{w, F, e, p}{\text{Maximize}} \;\; \Pi^M$$

$$\text{subject to} \;\; \pi^i\big(e^i(\cdot), e^j(\cdot)\big) \geq \pi_0, \quad \forall i = A, B. \tag{$IR - i$}$$

$$e^i(w, F^i, F^j, p^i, p^j) \in \operatorname*{Arg\,max}_{e^{i'}}\big\{\pi^i(e^{i'}, e^i(w, F^i, F^j, p^i, p^j))\big\},$$
$$\forall w, F^i, F^j, p^i, p^j \in R^+, \; i \neq j, \; i, j = A, B. \tag{$E - i$}$$

$$\text{and} \;\; -F^i \geq -F_0, \quad i = A, B. \tag{$LL - i$}$$

A solution to $[M.\text{A}.3.3]$ has the following properties:

Result 9. For all $i \neq j$ and $i, j = A, B$,

(a) $F^i = F_0$

(b) $w > w^*$, where w^* is the first-best wholesale price in Result 8

(c) $(p^A - c)\partial q^A/\partial e^A > \partial C^A/\partial e^A$, and the distortion from efficiency in a distributor's choice of effort is comprised of two components:

(i) the free-rider effect $\{\gamma_B(p^B - w)\partial q^B/\partial e^A\}$

(i) the slack effect
$-\mu_{Ae}[(p^A - w)\partial^2 q^A/\partial e^{A^2} - \partial^2 C^A/\partial e^{A^2}]$.

Proof. Employing the first-order approach, the Lagrangian, L, is

$$\begin{aligned} L = {} & (w-c)(q^A+q^B)+F^A+F^B+\lambda_A[(p^A-w)q^A-F^A-C^A(e^A)] \\ & + \lambda_B[(p^B - w)q^B - F^B - C^B(e^B)] \\ & + \mu_{Ae}\left[(p^A - w)\frac{\partial q^A}{\partial e^A} - \frac{\partial C^A}{\partial e^A}\right] + \mu_{Be}\left[(p^B - w)\frac{\partial q^B}{\partial e^B} - \frac{\partial C^B}{\partial e^B}\right] \\ & + \gamma_A[-F^A + F_0] + \gamma_B[-F^B + F_0]. \end{aligned} \tag{A.3.3.1}$$

Assuming interior solutions, the necessary conditions for F^i, $i = A, B$ are

$$\frac{\partial L}{\partial F^i} = 1 - \lambda_i - \gamma_i = 0 \tag{A.3.3.2a}$$

or

$$\lambda_i = 1 - \gamma_i. \tag{A.3.3.2b}$$

Notice from (A.3.3.2b) that if the limited liability constraints are not binding, the principal can implement the full-information efficient solution, as outlined in Result 8.[30] Consequently, the focus is on the following case:

$$\gamma_i > 0 \quad \text{for all } i = A, B. \tag{A.3.3.3}$$

Next, assuming an interior solution, the KT condition on w is

$$\frac{\partial L}{\partial w} = q^A + q^B - \lambda_A q^A - \lambda_B q^B - \mu_{Ae}\frac{\partial q^A}{\partial e^A} - \mu_{Be}\frac{\partial q^B}{\partial e^B} = 0. \tag{A.3.3.4}$$

[30] See also the rationale offered in §A.2.3.

Substituting from (A.3.3.2b) in (A.3.3.4) gives

$$\frac{\partial L}{\partial w} = \gamma_A q^A + \gamma_B q^B - \mu_{Ae}\frac{\partial q^A}{\partial e^A} - \mu_{Be}\frac{\partial q^B}{\partial e^B} = 0 \quad \text{(A.3.3.5)}$$

or

$$\mu_{Ae}\frac{\partial q^A}{\partial e^A} + \mu_{Be}\frac{\partial q^B}{\partial e^B} = \gamma_A q^A + \gamma_B q^B > 0. \quad \text{(A.3.3.6)}$$

From (A.3.3.6) it is clear that at least one of μ_{Ae} and μ_{Be} is strictly positive. Next, note that $\forall i = A, B$,

$$\frac{\partial L}{\partial \mu_{ie}} = (p^i - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = 0. \quad \text{(A.3.3.7)}$$

Rewriting (A.3.3.7) gives

$$(p^i - c)\frac{\partial q^i}{\partial e^i} + (p^j - c)\frac{\partial q^j}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = (w - c)\frac{\partial q^i}{\partial e^i} + (p^j - c)\frac{\partial q^j}{\partial e^i}. \quad \text{(A.3.3.8)}$$

Suppose $w \le w^*$, where w^* is the wholesale price that induces the first-best solution to $[M.\text{A}.3.2]$, the manufacturer's problem considered in the previous subsection. For such a w, the right-hand side in (A.3.3.8) is negative and the distributor's effort is (weakly) greater than the first-best level.[31] Moreover, the corresponding μ_{ie} is (weakly) negative for $i = A, B$; these values of the multipliers do not satisfy (A.3.3.6). Consequently, the solution must have

$$w > w^*. \quad \text{(A.3.3.9)}$$

At such a wholesale price

$$\mu_{ie} > 0, \quad \forall i = A, B. \quad \text{(A.3.3.10)}$$

Clearly, the presence of binding limited liability constraints induces inefficiency in the channel and the manufacturer cannot achieve the first-best solution.

It is useful to consider the necessary condition on i's effort, $i \ne j$ and $i, j = A, B$:

$$\begin{aligned}\frac{\partial L}{\partial e^i} &= (w - c)\left[\frac{\partial q^i}{\partial e^i} + \frac{\partial q^j}{\partial e^i}\right] + \lambda_i\left[(p^i - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i}\right] \\ &\quad + \lambda_j(p^j - w)\frac{\partial q^j}{\partial e^i} + \mu_{ie}\left[(p^i - w)\frac{\partial^2 q^i}{\partial e^{i2}} - \frac{\partial^2 C^i}{\partial e^{i2}}\right] \\ &\quad + \mu_{je}(p^j - w)\frac{\partial^2 q^j}{\partial e^i \partial e^j} = 0. \end{aligned} \quad \text{(A.3.3.11)}$$

Equation (A.3.3.11) can be rewritten to characterize the inefficiency that arises due to the binding limited liability constraints. Substituting from (A.3.3.2b), (A.3.3.11) can be rewritten as (for expositional ease, focus on the expression for distributor A)

$$\begin{aligned}\frac{\partial L}{\partial e^A} &= (w - c)\frac{\partial q^A}{\partial e^A} + (w - c)\frac{\partial q^B}{\partial e^A} \\ &\quad + (1 - \gamma_A)\left[(p^A - w)\frac{\partial q^A}{\partial e^A} - \frac{\partial C^A}{\partial e^A}\right] + (1 - \gamma_B)(p^B - w)\frac{\partial q^B}{\partial e^A} \\ &\quad + \mu_{Ae}\left[(p^A - w)\frac{\partial^2 q^A}{\partial e^{A2}} - \frac{\partial^2 C^A}{\partial e^{A2}}\right] + \mu_{Be}(p^B - w)\frac{\partial^2 q^B}{\partial e^A \partial e^B} = 0. \end{aligned} \quad \text{(A.3.3.12a)}$$

[31] Because when $w = w^*$, distributors' effort is at the first-best level; when $w < w^*$, a distributor puts forth more effort than the first-best level.

Notice that $(w - c)\partial q^A/\partial e^A + (p^A - w)\partial q^A/\partial e^A$ simplifies to $(p^A - c)\partial q^A/\partial e^A$, and $(w - c)\partial q^B/\partial e^A + (1 - \gamma_B)(p^B - w)\partial q^B/\partial e^A$ simplifies to $(p^B - c)\partial q^B/\partial e^A - \gamma_B(p^B - w)\partial q^B/\partial e^A$. Further, the term multiplying γ_A is zero from (A.3.3.7). Consequently, (A.3.3.12a) can be rewritten as

$$\begin{aligned}(p^A - c)\frac{\partial q^A}{\partial e^A} &+ (p^B - c)\frac{\partial q^B}{\partial e^A} - \frac{\partial C^A}{\partial e^A} \\ &= \gamma_B(p^B - w)\frac{\partial q^B}{\partial e^A} - \mu_{Ae}\left[(p^A - w)\frac{\partial^2 q^A}{\partial e^{A2}} - \frac{\partial^2 C^A}{\partial e^{A2}}\right] \\ &\quad - \mu_{Be}(p^B - w)\frac{\partial^2 q^B}{\partial e^A \partial e^B}. \end{aligned} \quad \text{(A.3.3.12b)}$$

Since $\partial^2 q^B/\partial e^A \partial e^B = 0$ by assumption in this model, the statement of the result follows. □

The results indicate that when the manufacturer imposes the maximum possible fixed fees on both his distributors, distortions from first-best are induced in the distributors' behavior. This distortion from efficiency can be better understood when its magnitude is split into two components, as in the statement of Result 9.

The nature of each component becomes clear from its sign. The first component, $\gamma_B(p^B - w)\partial q^B/\partial e^A$, is positive since $\gamma_B > 0$, $p^B - w > 0$, and $\partial q^B/\partial e^A > 0$. This component is referred to as the free-rider effect since its magnitude is related to the positive interlinkage between the sales of one distributor and the effort of another.

The second component, $-\mu_{Ae}[(p^A - w)\partial^2 q^A/\partial e^{A2} - \partial^2 C^A/\partial e^{A2}]$, is referred to as the slack effect, and it characterizes how diligently a distributor is working on behalf of the manufacturer. Notice that the concavity of the distributor's profit function ensures $[(p^A - w)\partial^2 q^A/\partial e^{A2} - \partial^2 C^A/\partial e^{A2}] < 0$; since $\mu_{Ae} > 0$, the slack effect serves to distort the effort choice from efficiency. This effect is analogous to the corresponding effect under exclusive distribution with binding limited liability constraints.

In equilibrium, the net distortion from efficiency is endogenously determined by the sum of the above-described effects. The sign of each effect, however, provides a clue to the dynamics of the inefficiency. Any "positive" effect tends to push the marginal benefit over the marginal cost of that distributor's effort, implying that too little effort is being exerted. In contrast, a "negative" effect drives the marginal cost above the marginal benefit, suggesting that too much effort is being exerted. Given this, notice that the free-rider effect is always positive and consequently induces distributors to put forth too little effort. Next, the slack effect is positive and indicates that distributors tend to work too little in the presence of binding liability constraints.

The above three components support the intuition that there are inefficiencies associated with each form of distribution. Under exclusive distribution, a manufacturer has to contend with the slack effect; under nonexclusive distribution, the manufacturer deals with the costs of the free-rider effect and the slack effect (this slack effect may be smaller than the one under exclusive territories). Clearly, the relative magnitude of these effects plays an important role in the manufacturer's choice of distribution method.

A.3.4. Manufacturer Does Not Control Final Prices and Distributor Effort Is Observable

The formal statement of the manufacturer's problem, [M.A.3.4], is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$

$$\text{subject to} \quad \pi^i(p^i(\cdot), p^j(\cdot)) \geq \pi_0, \quad \forall i = A, B. \qquad (IR-i)$$

$$\text{where} \quad p^i(w, e^i, e^j, F^i, F^j) \in \underset{p^{i'}}{\text{Arg}\max}\{\pi^i(p^{i'}, p^j(w, e^i, e^j, F^i, F^j))\},$$

$$\forall w, e^i, e^j, F^i, F^j \in R^+, \; i \neq j, \; i, j = A, B. \qquad (P-i)$$

Notice that in the $(P-i)$ constraints above, each distributor selects its final price to maximize its own profits as a best response to the other distributor doing the same. A solution to [M.A.3.4] has the following properties:

Result 10. The manufacturer achieves the full-information efficient solution that was feasible when it had control over the final prices. To do this, the manufacturer sets $w > c$, and at that wholesale price, the following condition will hold:

$$(p^B - c)\frac{\partial q^B/\partial p^A}{\partial q^A/\partial p^A} = (p^A - c)\frac{\partial q^A/\partial p^B}{\partial q^B/\partial p^B}.$$

Proof. The Lagrangian, L, for this problem is

$$L = (w-c)(q^A+q^B)+F^A+F^B+\lambda_A[(p^A-w)q^A-F^A-C^A(e^A)] + \lambda_B[(p^B-w)q^B-F^B-C^B(e^B)]+\mu_{Ap}\left[q^A+(p^A-w)\frac{\partial q^A}{\partial p^A}\right] + \mu_{Bp}\left[q^B+(p^B-w)\frac{\partial q^B}{\partial p^B}\right]. \qquad \text{(A.3.4.1)}$$

Assuming an interior solution for F^i, $i = A, B$, gives

$$\frac{\partial L}{\partial F^i} = 1 - \lambda_i = 0 \qquad \text{(A.3.4.2a)}$$

or

$$\lambda_i = 1. \qquad \text{(A.3.4.2b)}$$

The necessary condition for an interior solution for w is

$$\frac{\partial L}{\partial w} = q^A + q^B - \lambda_A q^A - \lambda_B q^B - \mu_{Ap}\frac{\partial q^A}{\partial p^A} - \mu_{Bp}\frac{\partial q^B}{\partial p^B} = 0. \qquad \text{(A.3.4.3)}$$

Substituting from (A.3.4.2b) into (A.3.4.3) gives

$$\mu_{Ap}\frac{\partial q^A}{\partial p^A} + \mu_{Bp}\frac{\partial q^B}{\partial p^B} = 0. \qquad \text{(A.3.4.4)}$$

Further

$$\frac{\partial L}{\partial \mu_{ip}} = q^i + (p^i - w)\frac{\partial q^i}{\partial p^i} = 0. \qquad \text{(A.3.4.5)}$$

Rewrite (A.3.4.5) as follows:

$$q^i + (p^i - c)\frac{\partial q^i}{\partial p^i} + (p^j - c)\frac{\partial q^j}{\partial p^i} = (w - c)\frac{\partial q^i}{\partial p^i} + (p^j - c)\frac{\partial q^j}{\partial p^i}. \qquad \text{(A.3.4.6)}$$

The first-best contract requires—

$$-(w - c)\frac{\partial q^i}{\partial p^i} = (p^j - c)\frac{\partial q^j}{\partial p^i}. \qquad \text{(A.3.4.7)}$$

If (A.3.4.7) is satisfied, μ_{ip} will equal zero and the first-best prices will be selected. Since $\partial q^j/\partial p^i > 0$ and $\partial q^i/\partial p^i < 0$ by assumption, (A.3.4.7) will hold only when $w > c$. Further, (A.3.4.7) implies

$$-(w - c) = \frac{(p^B - c)\partial q^B/\partial p^A}{\partial q^A/\partial p^A} = \frac{(p^A - c)\partial q^A/\partial p^B}{\partial q^B/\partial p^B}. \qquad \text{(A.3.4.8)}$$

The statement of the result follows. □

A.3.5. Manufacturer Does Not Control Final Prices and Distributor Effort Is Unobservable

A formal statement of the manufacturer's problem, [M.A.3.5], is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$

$$\text{subject to} \quad \pi^i(e^i(\cdot), p^i(\cdot), e^j(\cdot), p^j(\cdot)) \geq \pi_0, \quad \forall i \neq j, \; i, j = A, B, \qquad (IR-i)$$

$$e^i(w, F^i, F^j), p^i(w, F^i, F^j) \in \underset{e^{i'}, p^{i'}}{\text{Arg}\max}\{\pi^i(e^{i'}, p^{i'}, e^j(w, F^i, F^j), p^j(w, F^i, F^j))\},$$

$$\forall w, F^i, F^j \in R^+, \; i \neq j, \; i, j = A, B. \qquad (EP-i)$$

The solution to [M.A.3.5] has the following property:

Result 11. In this setting, the manufacturer cannot achieve the full-information efficient solution.

Proof. Suppose that the manufacturer naïvely attempted to implement the first-best solution of Result 10 (i.e., w is set above the marginal cost, and fixed fees are chosen to ensure that the distributors' individual rationality constraints bind). Consider the rate of change of a distributor's expected profit with respect to its effort level at the first-best solution:

$$\frac{\partial \pi^i}{\partial e^i} = (p^i - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = (p^i - c + c - w)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i}. \qquad \text{(A.3.5.1)}$$

(A.3.5.1) can be rewritten as

$$\frac{\partial \pi^i}{\partial e^i} = (p^i - c)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} - (w - c)\frac{\partial q^i}{\partial e^i}. \qquad \text{(A.3.5.2)}$$

Next, recall (from (A.3.1.4)) the first-best solution:

$$(p^i - c)\frac{\partial q^i}{\partial e^i} + (p^j - c)\frac{\partial q^j}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = 0. \qquad \text{(A.3.5.3)}$$

(A.3.5.3) can be rewritten as

$$(p^i - c)\frac{\partial q^i}{\partial e^i} - \frac{\partial C^i}{\partial e^i} = -(p^j - c)\frac{\partial q^j}{\partial e^i} < 0. \qquad \text{(A.3.5.4)}$$

Substituting from (A.3.5.4) into (A.3.5.2) gives

$$\frac{\partial \pi^i}{\partial e^i} = -(p^j - c)\frac{\partial q^j}{\partial e^i} - (w - c)\frac{\partial q^i}{\partial e^i} < 0. \qquad \text{(A.3.5.5)}$$

When distributors' efforts are unobservable and the manufacturer attempts to implement the first-best effort levels, (A.3.5.5) reveals that each distributor can increase profit by decreasing the level of effort. Similarly, if the manufacturer sets $w < c$, to induce efficient effort levels (as in Result 8), analogous analysis shows that the distributors will have an incentive to set final prices inefficiently.

Also notice that to induce efficient effort levels, w must be set below c; however, to induce efficient price levels, w must be set above c. Therefore, it is not feasible to induce both efficient effort levels as well as price levels via one wholesale price. It follows that the manufacturer cannot obtain the first-best solution in this setting. □

A.3.6. Manufacturer Does Not Control Final Prices, Distributor Effort Is Unobservable, and the Limited Liability Constraints Are Binding

The formal statement of the manufacturer's problem, $[M-NE]$, is

$$\underset{w,F,e,p}{\text{Maximize}} \quad \Pi^M$$

$$\text{subject to} \quad \pi^i(e^i(\cdot), p^i(\cdot), e^j(\cdot), p^j(\cdot)) \geq \pi_0, \quad \forall i \neq j,\ i,j = A,B, \quad (IR-i)$$

$$e^i(w, F^i, F^j), p^i(w, F^i, F^j) \in \underset{e^{i'}, p^{i'}}{\text{Argmax}} \{\pi^i(e^{i'}, p^{i'}, e^j(w, F^i, F^j), p^j(w, F^i, F^j))\},$$

$$\forall w, F^i, F^j \in R^+,\ i \neq j,\ i,j = A,B, \quad (EP-i)$$

$$\text{and} \quad -F^i \geq -F_0, \quad i = A,B. \quad (LL-i)$$

The properties of a solution to $[M-NE]$ are summarized below (this corresponds to Finding 2 in the main text):

RESULT 12. For all $i = A,B$,

(a) In the presence of binding limited liability constraints, inefficiency exists in the selection of efforts and final prices by distributors.

(b) At optimality, the inefficiency arising from a distributor's choice of effort is comprised of the following three components:

(i) the free-rider effect, $[\gamma_B(p^B - w) - \mu_{Bp}]\partial q^B/\partial e^A$

(ii) the slack effect, $-\mu_{Ae}[(p^A - w)\partial^2 q^A/\partial e^{A2} - \partial^2 C^A/\partial e^{A2}]$

(iii) the price effect, $-\mu_{Ap}\partial q^A/\partial e^A$.

(c) At optimality, the inefficiency arising from a distributor's choice of price is comprised of the following three components:

(i) $[\gamma_B(p^B - w) - \mu_{Bp}]\partial q^B/\partial p^A$

(ii) $-\mu_{Ae}\partial q^A/\partial e^A$

(iii) $-\mu_{Ap}[2\partial q^A/\partial p^A + (p^A - w)\partial^2 q^A/\partial p^{A2}]$.

PROOF.

$$\begin{aligned} L = {} & (w-c)(q^A+q^B)+F^A+F^B+\lambda_A[(p^A-w)q^A-F^A-C^A(e^A)] \\ & +\lambda_B[(p^B-w)q^B-F^B-C^B(e^B)]+\mu_{Ae}\left[(p^A-w)\frac{\partial q^A}{\partial e^A}-\frac{\partial C^A}{\partial e^A}\right] \\ & +\mu_{Be}\left[(p^B-w)\frac{\partial q^B}{\partial e^B}-\frac{\partial C^B}{\partial e^B}\right]+\mu_{Ap}\left[q^A+(p^A-w)\frac{\partial q^A}{\partial p^A}\right] \\ & +\mu_{Bp}\left[q^B+(p^B-w)\frac{\partial q^B}{\partial p^B}\right]+\gamma_A[-F^A+F_0]+\gamma_B[-F^B+F_0]. \end{aligned} \quad (A.3.6.1)$$

According to KT analysis, the necessary conditions for F^i, $i = A,B$ are

$$\frac{\partial L}{\partial F^i} = 1 - \lambda_i - \gamma_i = 0 \quad (A.3.6.2a)$$

or

$$\lambda_i = 1 - \gamma_i. \quad (A.3.6.2b)$$

Since the aim is to study settings where the limited liability constraints are binding, the analysis focuses on the following case (see also the earlier discussion in §§A.2.3, A.2.6, and A.3.3):

$$\gamma_i > 0 \quad \text{for all } i = A,B. \quad (A.3.6.3)$$

Next, assuming an interior solution, the KT condition on w is

$$\begin{aligned} \frac{\partial L}{\partial w} = {} & q^A + q^B - \lambda_A q^A - \lambda_B q^B - \mu_{Ae}\frac{\partial q^A}{\partial e^A} - \mu_{Ap}\frac{\partial q^A}{\partial p^A} \\ & - \mu_{Be}\frac{\partial q^B}{\partial e^B} - \mu_{Bp}\frac{\partial q^B}{\partial p^B} = 0. \end{aligned} \quad (A.3.6.4)$$

Substituting from (A.3.6.2b) in (A.3.6.4) gives

$$\mu_{Ae}\frac{\partial q^A}{\partial e^A} + \mu_{Ap}\frac{\partial q^A}{\partial p^A} + \mu_{Be}\frac{\partial q^B}{\partial e^B} + \mu_{Bp}\frac{\partial q^B}{\partial p^B} = \gamma_A q^A + \gamma_B q^B > 0. \quad (A.3.6.5)$$

It follows from (A.3.6.5) that not all the μs equal zero; consequently, there are inefficiencies in this setting, as indicated in the statement of the result.

The necessary condition on A's effort is (since the term multiplying λ_A is zero, that term is not shown here)

$$\begin{aligned} \frac{\partial L}{\partial e^A} = {} & (w-c)\left[\frac{\partial q^A}{\partial e^A} + \frac{\partial q^B}{\partial e^A}\right] + \lambda_B(p^B - w)\frac{\partial q^B}{\partial e^A} \\ & + \mu_{Ae}\left[(p^A - w)\frac{\partial^2 q^A}{\partial e^{A2}} - \frac{\partial^2 C^A}{\partial e^{A2}}\right] + \mu_{Be}(p^B - w)\frac{\partial^2 q^B}{\partial e^A \partial e^B} \\ & + \mu_{Ap}\left[\frac{\partial q^A}{\partial e^A} + (p^A - w)\frac{\partial^2 q^A}{\partial e^A \partial p^A}\right] \\ & + \mu_{Bp}\left[\frac{\partial q^B}{\partial e^A} + (p^B - w)\frac{\partial^2 q^B}{\partial e^A \partial p^B}\right] = 0. \end{aligned} \quad (A.3.6.6)$$

Rewriting (A.3.6.6) gives

$$\begin{aligned} & (p^A - c)\frac{\partial q^A}{\partial e^A} + (p^B - c)\frac{\partial q^B}{\partial e^A} - \frac{\partial C^A}{\partial e^A} \\ & = [\gamma_B(p^B - w) - \mu_{Bp}]\frac{\partial q^B}{\partial e^A} - \mu_{Ae}\left[(p^A - w)\frac{\partial^2 q^A}{\partial e^{A2}} - \frac{\partial^2 C^A}{\partial e^{A2}}\right] \\ & \quad - \mu_{Ap}\left[\frac{\partial q^A}{\partial e^A} + (p^A - w)\frac{\partial^2 q^A}{\partial e^A \partial p^A}\right] \\ & \quad - (p^B - w)\left[\mu_{Be}\frac{\partial^2 q^B}{\partial e^A \partial e^B} + \mu_{Bp}\frac{\partial^2 q^B}{\partial e^A \partial p^B}\right]. \end{aligned} \quad (A.3.6.7)$$

The right-hand side in (A.3.6.7) contains the components listed in Finding 2(b).

Next, the KT condition on p^A is

$$\begin{aligned} \frac{\partial L}{\partial p^A} = {} & (w-c)\left[\frac{\partial q^A}{\partial p^A} + \frac{\partial q^B}{\partial p^A}\right] + \lambda_B(p^B - w)\frac{\partial q^B}{\partial p^A} \\ & + \mu_{Ae}\left[\frac{\partial q^A}{\partial e^A} + (p^A - w)\frac{\partial^2 q^A}{\partial p^A \partial e^A}\right] + \mu_{Be}(p^B - w)\frac{\partial^2 q^B}{\partial p^A \partial e^B} \\ & + \mu_{Ap}\left[2\frac{\partial q^A}{\partial p^A} + (p^A - w)\frac{\partial^2 q^A}{\partial p^{A2}}\right] \\ & + \mu_{Bp}\left[\frac{\partial q^B}{\partial p^A} + (p^B - w)\frac{\partial^2 q^B}{\partial p^A \partial p^B}\right] = 0. \end{aligned} \quad (A.3.6.8)$$

Rewriting (A.3.6.8) gives

$$\begin{aligned} & q^A + (p^A - c)\frac{\partial q^A}{\partial p^A} + (p^B - c)\frac{\partial q^B}{\partial p^A} \\ & = [\gamma_B(p^B - w) - \mu_{Bp}]\frac{\partial q^B}{\partial p^A} \end{aligned}$$

$$
\begin{aligned}
&-(p^B - w)\left[\mu_{Be}\frac{\partial^2 q^B}{\partial p^A \partial e^B} + \mu_{Bp}\frac{\partial^2 q^B}{\partial p^A \partial p^B}\right]\\
&-\mu_{Ae}\left[\frac{\partial q^A}{\partial e^A} + (p^A - w)\frac{\partial^2 q^A}{\partial p^A \partial e^A}\right]\\
&-\mu_{Ap}\left[2\frac{\partial q^A}{\partial p^A} + (p^A - w)\frac{\partial^2 q^A}{\partial {p^A}^2}\right]. \qquad \text{(A.3.6.9)}
\end{aligned}
$$

Note that the second-cross-partial derivatives of q^i with respect to price and effort are zero in this model. The terms on the right-hand side of (A.3.6.9) are the components listed in the statement of the result. □

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