Marketing Science

Practice Prize Winner—Dynamic Marketing Budget Allocation Across Countries, Products, and Marketing Activities

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Practice Prize Winner

Dynamic Marketing Budget Allocation Across Countries, Products, and Marketing Activities

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Previous research on marketing budget decisions has shown that profit improvement from better allocation across products or regions is much higher than from improving the overall budget. However, despite its high managerial relevance, contributions by marketing scholars are rare.

In this paper, we introduce an innovative and feasible solution to the dynamic marketing budget allocation problem for multiproduct, multicountry firms. Specifically, our decision support model allows determining near-optimal marketing budgets at the country–product–marketing–activity level in an Excel-supported environment each year. The model accounts for marketing dynamics and a product's growth potential as well as for trade-offs with respect to marketing effectiveness and profit contribution. The model has been successfully implemented at Bayer, one the world's largest pharmaceutical and chemical firms. The profit improvement potential is more than 50% and worth nearly €500 million in incremental discounted cash flows.

1. Introduction

Determining the marketing budget has been of paramount importance to marketers for many decades. Global players such as Procter & Gamble (P&G) spend approximately US$8.5 billion on advertising per year (P&G 2008). Because marketing expenditures are immediately recognized as costs on the income statement, but their total impact on sales often fully unfolds only in future periods, they need to be evaluated in terms of an investment decision. In view of limited financial resources, the global annual marketing budget of a company is usually set in the previous year; i.e., it is fixed. If companies offer a broad product portfolio to customers from various countries and use a variety of communication channels, they need to break down the fixed annual budget into expenditures across countries, products, and communication activities. For many firms, this task requires determining individual budgets for hundreds of allocation units. As a result, firms face a complex decision problem: they need to allocate a fixed budget across a multitude of allocation units by evaluating the impact of these investment decisions on future cash flows. Technically, management needs to solve a dynamic optimization problem for an investment portfolio under a budget constraint. Because marketing budgets are set on an annual basis, this management challenge recurs regularly.

1.1. State of the Art of Marketing Budget Allocation

Marketing practitioners frequently use heuristic methods when it comes to determining the marketing budget. Bigné (1995) reviews 16 studies published between 1975 and 1991 on actual budgeting behavior of North American and European firms from diverse industries. He finds that by far the most often used budget rules are the "percentage-of-sales," "objective-and-task," and "affordability" methods. These rules usually yield results that are rather far from the optimal profit-maximizing budget. Analytic methods that

are based on the principle of marginal returns analysis produce optimal budgets but are only considered by a minority of firms.

The academic literature has been dealing with budget questions for a long time. A large body of work focuses on optimizing the budget for a single product in a static environment (for an overview, see Hanssens et al. 2001). Among the earliest and most influential contributions is the work by Dorfman and Steiner (1954). They derive necessary conditions that must hold for static profit maximization when optimal levels for several marketing-mix variables are set simultaneously. The solution offers important general insights into the budgeting problem but does not offer guidance for implementation into marketing practice. In addition, it does not consider dynamics and the perspective of a multicountry, multiproduct firm.

A large stream of papers takes a dynamic perspective (for an overview, see Erickson 2003). The recent paper by Naik et al. (2005), for example, considers interaction effects between advertising and promotion under dynamic oligopolistic competition. The focus of these studies, however, remains on single products. They do not inform on how budgets are simultaneously set for several products in view of limited financial resources.

This question can only be answered by an integrated *allocation* approach. Previous research (e.g., Tull et al. 1986) has shown that profit improvement from better allocation across products or regions is much higher than from improving the overall budget. However, despite its high managerial relevance and potential for profit improvement, contributions by marketing scholars are rare (Reibstein et al. 2009).[1] An important emerging literature stream (e.g., Kumar et al. 2008, Reinartz et al. 2005) deals with the problem of resource allocation across customers. Typically, these approaches require data on individual customer behavior and focus on service industries. Other articles focus on problems of sales territory design and sales force size (e.g., Skiera and Albers 1998, Zoltners and Sinha 2005) but do not address allocation decisions for products in multiproduct, multicountry businesses. Only a few approaches are based on aggregate market response models that can be calibrated with sales and marketing data at the product level, which is the primary data source in many industries. Lodish et al. (1988) propose an allocation algorithm for a specific type of market response that has been adopted by a pharmaceutical company. Doyle and Saunders (1990) derive a closed-form allocation solution under a budget constraint

for the semi-log response model and apply it to a British retailer. Albers (1998) generalizes the solution to the case of an arbitrary response function and allocation unit. Because a closed-form solution in terms of response parameters no longer exists, he proposes a heuristic rule and shows via simulation that it quickly converges to the optimal numerical solution. Although these approaches consider trade-offs among products of a portfolio for budget decisions, they are focused on short-term profit maximization. Marketing decisions, however, need to account for dynamics as well. On the one side, dynamic considerations result from lagged effects that can be represented by a marketing stock variable. On the other side, dynamic considerations arise from the fact that a portfolio mixes products with different ages and growth opportunities. Requirements for marketing support change as the product evolves along its life cycle. To the best of our knowledge, a dynamic marketing budget allocation approach for a product portfolio has not been suggested so far.

1.2. Contribution to Allocation Theory and Practice

In this paper, we propose an allocation method for breaking down a global marketing budget into individual budgets at the country–product–marketing–activity level. We take the position of an international firm that offers a broad portfolio of products to customers from different countries. Products are promoted by various activities, including classical advertising, below-the-line activities, personal selling, etc. Each year the firm sets a global marketing budget that is to be spent by the various allocation units in the year ahead. The portfolio is composed of products that differ in their life cycle stage. The firm wishes to maximize the discounted total profits of its portfolio. Although we propose a method that recommends how to allocate the annual global budget across countries, products, and marketing activities, we do not address the tactical problem of intertemporal allocation of an individual budget within the year (for a summary of this literature, see Doganoglu and Klapper 2006).

We contribute to allocation *theory* by offering a solution to the dynamic portfolio-profit maximization problem. The theoretical solution provides important insights into how individual budgets should be set so that they account for differences in profit contribution, marketing effectiveness, and growth potential. The optimal budget describes an endogenous relationship where various variables need to be in their global optimum. This relationship also holds under Nash competition. Under both monopoly and Nash competition, however, it can only be solved with numerical methods. Numerical optimization often

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[1] We acknowledge other research traditions that deal with allocation problems. For example, international trade theory discusses issues of dynamic resource allocation across countries at a macro level (e.g., Wong 1995).

faces significant acceptance barriers in practice, which may be one reason for the frequent use of suboptimal budgeting heuristics (Bigné 1995). Although the numerical method produces the optimal budget, the product manager cannot reproduce the result on its own. Therefore, he or she does not understand why the recommended budget level should be optimal for his or her product.

Hence, our second contribution is to allocation *practice*. We develop a near-optimal allocation rule that addresses the demand for simple allocation rules by practitioners. The rule is directly derived from the theoretical solution. It provides insights into the solution structure and can be used with a spreadsheet. In a simulation study, we demonstrate that the allocation heuristic quickly converges to the optimal solution under varying conditions. Although easy to understand and to implement, the heuristic goes beyond widespread budgeting rules such as the percentage-of-sales method (size of the business). Specifically, it integrates and trades off information about the following:

—the size of the business,
—the profit contribution margin,
—the (short-term) effectiveness of marketing investments,
—the carryover effect of marketing investments,
—the growth potential, and
—the time value of money.

Together with the management of Bayer, we developed and implemented the heuristic for the product portfolio of Bayer's primary care business unit. This portfolio includes 36 products from four strategic therapeutic areas that are marketed worldwide. Product managers can choose among six different types of marketing activities such as detailing or print advertising. The project had significant impact on the marketing budgeting practice at Bayer. It initiated an important change in the understanding of the allocation task by providing structure and solution to a complex problem. The empirical application revealed a profit improvement potential of more than 50% or nearly €500 million of incremental discounted cash flows over the next five years. Finally, the project significantly contributed to an organizational change that resulted in the creation of a new marketing intelligence unit. One of the main tasks of this unit is to support top management in evaluating the financial impact of marketing decisions.

The rest of this paper is organized as follows. In the next section, we describe our analytic approach to derive the proposed heuristic allocation rule and the associated simulation study. Section 3 provides information about Bayer and the market background. The fourth section focuses on the empirical application. We discuss the data, the estimation of the market response model, and validation issues. Section 5 presents the implementation of the allocation heuristic in Excel. We further evaluate the empirical findings and the impact of the project on Bayer. We close with limitations and suggestions for future research.

2. A Heuristic Rule for Dynamic Marketing Budget Allocation

2.1. Theory

Assume an international firm that operates across the world and sells several products, which may belong to the same or different categories. The number of products offered may differ across countries. Product management can choose among various marketing activities, such as print advertising, personal selling, direct mailing, etc., to enhance current and future sales. At the end of each year, marketing investment plans for the next year are developed. We assume that the firm wishes to maximize the net present value Π of its product portfolio over a planning period T, e.g., five years. We further assume that a total marketing budget R has already been set at the firm level. We do not model this process; i.e., R is exogenous. In addition, the total budget is assumed to be constant over the planning horizon. Top management, however, may decide to adjust the level during next year's planning cycle.

2.1.1. Allocation Solution for an Arbitrary Growth Function. Denote $q(t, S, \mathbf{Z})$ as the sales of a product in period t that depends on S, the marketing stock, and other variables (e.g., competitive marketing stock) that are summarized in the row vector $\mathbf{Z}$. Without loss of generality, we focus on only one own stock variable. Let us decompose sales into two components,

$$q(t, S, \mathbf{Z}) = g[t, S(t)]f[S(t), \mathbf{Z}(t)], \qquad (1)$$

where $g[\cdot]$ is a growth function that represents a basic pattern of growth dynamics as known from diffusion and product life cycle research, and $f[\cdot]$ is a separate response function that measures the direct impact of S and $\mathbf{Z}$ on sales. Note that this decomposition is helpful for interpretation but does not limit the generality of our model development. The growth function describes the evolution of new product sales over the life cycle and is assumed to be influenced by investments in the marketing stock. Research on diffusion processes and product life cycles provides broad evidence for the dependence of growth dynamics on marketing-mix variables (e.g., Bass et al. 2000, Fischer et al. 2010). The marketing stock S follows a dynamic process that satisfies the differential equation

(Nerlove and Arrow 1962):

$$\frac{dS}{dt} = -\delta S + x, \quad x \geq 0, \text{ and } S(0) \text{ known}, \quad (2)$$

where x denotes marketing expenditures and δ is the depreciation rate of the marketing stock.

Let k denote the country with the index set K, and let i denote the product, whereas the set of products offered in country k may vary and is given by I_k. Let n denote the type of marketing activity or spending category, respectively, and let N_i be the associated index set that may vary across products. We omit the time argument unless it is needed for an unambiguous understanding. $\mathbf{S}_{ki}$ is an N_i-dimensional row vector summarizing the activity-specific marketing stocks for product i. Let ET measure the elapsed time since the launch of a product in $t = 0$; let r be a discount rate, $0 < r < \infty$; let p denote price; let c be marginal cost; and let x_n be activity-specific marketing expenditures. The constrained dynamic profit maximization problem of the firm is

$$\max_{\mathbf{S}_{ki}} \Pi = \int_{t=0}^{T} \underbrace{e^{-rt}}_{\text{Discounting}} \left\{ \left[\underbrace{\sum_{k \in K} \sum_{i \in I_k} (p_{ki} - c_{ki})}_{\text{Profit contribution}} \right. \right.$$

$$\left. \underbrace{\cdot q_{ki}(ET_{ki} + t, \mathbf{S}_{ki}, \mathbf{Z}_{ki})}_{\text{Unit sales}} \right]$$

$$\left. - \underbrace{\sum_{k \in K} \sum_{i \in I_k} \sum_{n \in N_i} x_{kin}}_{\text{Marketing expenditures}} \right\} dt$$

$$\underbrace{}_{\text{Discounted net value of product portfolio}} \quad (3)$$

subject to

$$R = \sum_{k \in K} \sum_{i \in I_k} \sum_{n \in N_i} x_{kin},$$

$$\text{with } \frac{dR}{dt} = 0 \text{ (budget constraint)}, \quad (3.1)$$

$$\frac{dS_{kin}}{dt} = -\delta_{kin} S_{kin} + x_{kin},$$

$$\text{with } x_{kin} \geq 0 \text{ (state variable equation)}, \quad (3.2)$$

$$S_{kin} \geq 0, \quad S_{kin}(0) = S_{kin0}, \quad \text{and} \quad S_{kin}(T) = S_{kinT}$$

$$\text{(boundary conditions).} \quad (3.3)$$

The final stock S_{kinT} is free but must be nonnegative and satisfy the budget constraint. In the appendix, we show how this problem can be solved by employing the calculus of variations together with the Lagrange approach (Kamien and Schwartz 1991). Specifically, the solution to the $1 + \sum_{k \in K} \sum_{i \in I_k} |N_i|$ Euler–Lagrange equations is

$$\frac{x_{kin}^*}{R} = ((p_{ki} - c_{ki})q_{ki}^*(\varepsilon_{f(x_{kin}^*)} + \varepsilon_{g(x_{kin}^*)}))/(r + 1 - \gamma_{kin}))$$

$$\cdot \left(\sum_{l \in K} \sum_{j \in I_l} \sum_{m \in N_j} ((p_{lj} - c_{lj})q_{lj}^*(\varepsilon_{f(x_{ljm}^*)} + \varepsilon_{g(x_{ljm}^*)}) \right.$$

$$\left. \Big/ (r + 1 - \gamma_{ljm})) + \lambda \sum_{l \in K} \sum_{j \in I_l} \sum_{m \in N_j} (dS_{ljm}^*/dt) \right)^{-1}$$

$$+ \frac{1}{R} \frac{dS_{kin}^*}{dt}, \quad \forall k \in K, i \in I_k, n \in N_i, t \in [0, T], \quad (4)$$

where $\varepsilon_{f(x)}$ denotes the *current-period* elasticity of sales with respect to marketing expenditures, $\varepsilon_{g(x)}$ measures the sales growth elasticity, γ measures the marketing carryover (with $\gamma = 1 - \delta$), λ is the dynamic Lagrange multiplier, and all other terms are as defined earlier. The asterisk indicates that variable values correspond to the optimal solution for the marketing budget, which is measured as the optimal share in the fixed total budget R in Equation (4). In a common product portfolio, the number of allocation units tends to be quite large. Because the total budget R to be allocated is fixed, some stocks increase and others decrease in the dynamic optimum. As a result, gains and losses tend to cancel each other out, and the second summand in the denominator of Equation (4) is close to zero. Considering the restriction that optimal budget shares must sum to one (see the appendix), we obtain the following general solution for the optimal budget that is close to solution (4):

$$x_{kin}^*(t) \cong \frac{\max\{w_{kin}^*(t), 0\}}{\sum_{l \in K} \sum_{j \in I_l} \sum_{m \in N_j} \max\{w_{ljm}^*(t), 0\}} R,$$

$$\forall k \in K, i \in I_k, n \in N_i, t \in [0, T], \quad (5)$$

with

$$w_{kin}^*(t) = \underbrace{(p_{ki} - c_{ki})}_{\substack{\text{Profit} \\ \text{contribution}}} \underbrace{q_{ki}^*(t)}_{\substack{\text{Unit} \\ \text{sales}}} (\underbrace{\varepsilon_{f[x_{kin}^*(t)]}}_{\substack{\text{Sales elasticity} \\ \text{w.r.t. marketing}}} + \underbrace{\varepsilon_{g[x_{kin}^*(t)]}}_{\substack{\text{Growth elasticity} \\ \text{w.r.t. marketing}}})$$

$$\Big/ \underbrace{(r + 1 - \gamma_{kin})}_{\substack{\text{Discounted marketing} \\ \text{multiplier}}}, \quad (6)$$

where w is an allocation weight, and all other terms are as defined earlier.

The optimal solution considers dynamics in two different ways. First, it incorporates the dynamic effects of building and leveraging the marketing stock, which is reflected in the marketing carryover coefficient γ. Second, it accounts for the growth potential of a product that is related to marketing investments as reflected in the growth elasticity $\varepsilon_{g(x)}$. Note that our sales response in Equation (1) includes a growth function, $g[\cdot]$, that describes the evolution of new product sales along its life cycle. The growth elasticity measures the power of marketing to shape the life cycle. Hence, we assume that the

growth process is endogenous with respect to marketing expenditures. A recent empirical study on drugs by Fischer et al. (2010) supports this premise. The authors find that the shape of the life cycle is indeed influenced by investments in the marketing stock. More importantly, their results suggest that marketing investments in the growth potential of a new product have a strong impact on future cumulative sales and discounted cash flows. On the basis of a parametric growth model, we show subsequently how the optimal solution favors shifting marketing resources to newer products so that they can leverage their endogenous growth potential.

Equations (5) and (6) represent the first-order conditions of the constrained dynamic maximization problem. These conditions also need to be fulfilled by each firm under Nash competition. Here, each firm sets marketing budgets independently of its competitors by taking the competitor budgets as given. Equilibrium values may be obtained by numerically and simultaneously solving the budget equations for the portfolio of each competitor. Note that our general sales model in (1) assumes product sales being influenced by competitor variables such as competitive price and competitive marketing expenditures. Competitor actions thus have an impact on the optimal solution because they change q^*, $\varepsilon_{f(x*)}$, and $\varepsilon_{g(x*)}$ in Equation (6).

2.1.2. Allocation Solution for a Specific Growth Function. We now introduce a parametric growth function and derive the growth elasticity for this specific case. This enables us to demonstrate the effects of the growth potential on the allocation solution in more detail. Following the study on drug life cycles by Fischer et al. (2010) and consistent with our empirical application at Bayer, we specify the growth function as follows:

$$g_{ki}(t, \mathbf{S}_{ki}) = \alpha_{ki} t^{a_{ki}(\mathbf{S}_{ki})} e^{-b_{ki}(\mathbf{S}_{ki})t},$$

$$\text{with } \alpha_{ki}, a_{ki}, b_{ki} > 0 \text{ and } t \in [0, \infty), \quad (7)$$

where α is a scaling constant, and a and b are growth parameters that depend on the marketing stock. The model describes an asymmetric growth path that leads to a single peak in the life cycle, which occurs at $t^{\text{Peak}} = a/b$. Hence, the growth parameters determine the time-to-peak sales and, as Fischer et al. (2010) also show, the height-of-peak sales. In addition, they define the shape of the life cycle. Equation (7) is equivalent to the gamma distribution, which has been frequently used by researchers because of its flexibility to capture many shapes (see the appendix). For example, if $a = 0$, Equation (7) reduces to the exponential distribution that is characteristic for many media products such as movies. Most importantly, we

Figure 1 **Illustrative Product Life Cycles for Different Marketing Investment Levels**
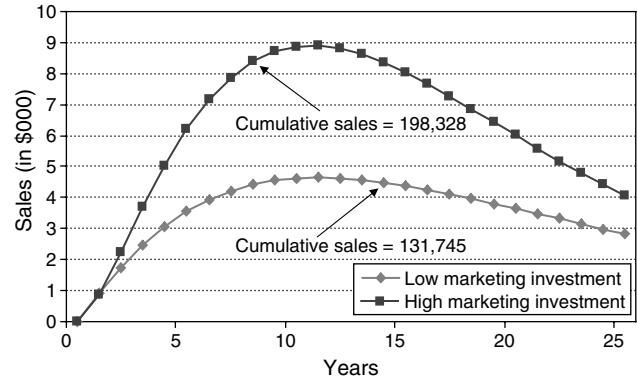
Notes. See Equation (7). Growth parameters in low investment case: $a = 1.1$, $b = 0.10$ (scale parameter $\alpha = 1$). Growth parameters in high investment case: $a = 1.6$, $b = 0.15$ (scale parameter $\alpha = 1$).

assume that marketing investments have a long-term impact on cumulative sales that is mediated by the growth parameters. Figure 1 compares two life cycles that peak around the same time. Cumulative sales are, however, quite different because of the differences in growth parameters that are assumed to arise from either low or high marketing investments. In the appendix, we show that cumulative sales are always higher for the life cycle whose difference between growth parameters a and b is larger.

From Equation (7), we obtain for the growth elasticity

$$\varepsilon_{g(x_{kin})} = \varepsilon_{a,kin} \ln(t) a_{ki} - \varepsilon_{b,kin} t b_{ki},$$

which can be inserted into (6) to yield the optimal allocation weight in planning period t

$$w_{kin}^*(t) = (p_{ki} - c_{ki}) q_{ki}^* [\varepsilon_{f(x_{kin}^*)} + \varepsilon_{a,kin}^* \ln(ET_{ki} + t) a_{ki}^*$$
$$- \varepsilon_{b,kin}^* (ET_{ki} + t) b_{ki}^*] / (r + 1 - \gamma_{kin}), \quad (8)$$

where $\varepsilon_{a,kin}$ and $\varepsilon_{b,kin}$ measure the elasticity of the growth parameters with respect to expenditures on marketing activity n, and all other terms are as defined earlier. Note that ET (elapsed time since the launch in $t = 0$) accounts for differences in launch times among products in the portfolio context.

2.1.3. Implications for Budget Allocation. The optimal solution provides a number of intuitive insights into the allocation problem. Equations (5) and (6) show that a fixed budget should be allocated according to a simple proportional rule. The optimal budget for a product relative to other products increases with its contribution margin $p - c$ and its sales base q. Similarly, the larger that product i's long-term marketing effectiveness for activity n is, the higher its budget. The long-term marketing effectiveness is composed of the short-term sales elasticity,

the discount rate, and the marketing carryover: $\varepsilon_{f(x*)}/(r+1-\gamma)$. Consequently, if long-term marketing effectiveness is larger across all of product i's activities compared with other products, the total budget for product i increases. Finally, Equations (5) and (6) reveal the importance of a product's growth potential for budget setting as reflected by the sales growth elasticity. This term varies over the life cycle. It is largest at the beginning when most of the sales are yet to come. Hence, the potential impact of marketing expenditures on future cash flows is greatest at this stage, which is why newer products get a higher allocation weight and thus a larger share in total budget. Because of the growth potential, the optimal marketing budget might even be higher than are the revenues of a new product at the beginning of its life; i.e., the solution may suggest spending money on products that involve a temporary loss.

The role of the growth potential term becomes clearer when we consider a specific growth function such as in Equation (7). Now, the optimal allocation weight is expressed in terms of growth parameters. From (8), it follows that the larger the difference $\varepsilon_a^* \ln(ET+t)a^* - \varepsilon_b^*(ET+t)b^*$ is, the higher the budget for a product. For products that have been launched in the same year, we know that cumulative sales are higher for those products for which the distance between a and b is larger (see the appendix). The distance may be enlarged by marketing investments to a certain extent as reflected in the elasticity parameters ε_a and ε_b. The growth expectations of a product also change over time. Because the growth potential term varies with t, this term accounts for this. To facilitate interpretation, assume that $\varepsilon_a^* = \varepsilon_b^* = 1$. Then, our measure simplifies to $\ln(ET+t)a^* - (ET+t)b^*$. For mature products, it gets smaller and may turn negative at some point in time. In the decline stage, the budget is likely to be zero because the sum of the short-term marketing elasticity and the growth potential measure in Equation (8) eventually becomes smaller than zero.

2.2. Proposed Near-Optimal Allocation Rule

The optimal budget for spending category n of product i in country k describes an endogenous relationship where various variables need to be in their optimum. To obtain the optimal values, we need to solve the profit maximization problem (3)–(3.3) numerically. The use of numerical methods, however, has two disadvantages. First, it requires that we explicitly specify the sales response function, which limits the generalizability of the solution approach. Second, marketing managers are reluctant to accept results from numerical optimization because they do not understand how the budget recommendation is derived. Although the optimization algorithm implicitly evaluates and trades off factors such as marketing

effectiveness, growth potential, or the size of the business, this process is not transparent for the manager.

Consistent with Little (1970), we believe that the simplicity of the allocation rule is important because it enables the manager to understand the allocation solution. The wide distribution of heuristic budget rules among companies (see Bigné 1995) despite the advances in the analytical marketing literature seems to support the need for simplicity in allocation methods in practice. We derive an allocation heuristic directly from the theoretical solution that produces near-optimal budgets and is easy to understand for managers:

$$\tilde{x}_{kint} = \frac{\tilde{w}_{kint}}{\sum_{l\in K}\sum_{j\in I_l}\sum_{m\in N_j}\tilde{w}_{ljmt}}R_t,$$

$$\forall k \in K, i \in I_k, n \in N_i, t \in [0,T], \quad (9)$$

$$\text{with } \tilde{w}_{kint} = \underbrace{\frac{\varepsilon_{kin,t-1}}{(r+1-\gamma_{kin})}}_{\text{Long-term marketing effectiveness}}$$

$$\cdot \underbrace{d_{ki}\cdot RV_{ki,t-1}}_{\text{Profit contribution}} \cdot \underbrace{\rho_{kit}}_{\text{Growth potential}}, \quad (10)$$

where

$\tilde{x}_{kint}$: Near-optimal budget for marketing activity n and product i in country k and period t;

$\tilde{w}_{kint}$: Heuristic allocation weight for marketing activity n and product i in country k and period t;

R_t: Total budget to be allocated in period t;

r: Discount rate (capital cost of firm, strategic business unit, etc.);

γ_{kin}: Carryover coefficient of marketing activity n for product i in country k;

$\varepsilon_{kin,t-1}$: Short-term sales elasticity with respect to product i's marketing expenditures on activity n in country k available from period $t-1$;

d_{ki}: (Percentage) contribution margin for product i in country k [$=(p_{ki}-c_{ki})/p_{ki}$];

$RV_{ki,t-1}$: Revenue level of product i in country k available from period $t-1$ ($=p_{ki,t-1}\cdot q_{ki,t-1}$); and

ρ_{kit}: Multiplier to measure the growth potential of product i in country k and period t.

The basic idea of the heuristic is to explicitly map Equations (5) and (6), the true optimum, to Equations (9) and (10), the heuristic approach. We do so by substituting currently available values for revenues and sales elasticity for their optimal values that are only endogenously determined by solving the equation system iteratively. We approximate the growth potential ρ by a multiplier that divides expected revenues in five years (planning horizon) by the current revenue level. By this heuristic approach, we ensure that products get a greater share of the total budget as long as they are expected to grow. In contrast, when they are expected to turn into their decline

stage, their budget is reduced. Current values of revenues are available from last year, and the contribution margin is a target figure decided by management. Data for the carryover coefficient, sales elasticity, and the growth multiplier are not readily available but must be estimated. In our empirical application, we specify a parametric response model to estimate these quantities econometrically. However, we note that this is not a prerequisite of the allocation heuristic. The user may adopt other, nonparametric approaches to estimate the required data.

Basically, the proposed heuristic is a simple proportional rule that integrates relevant information from three areas:

—the long-term effectiveness of marketing investments in the focal product,

—the profit contribution of the focal product, and

—the focal product's growth expectations.

The logic behind the selection and integration of information into a proportionality rule is well founded in theory but at the same time easy to understand for practitioners.

2.3. Testing the Near Optimality of the Allocation Rule via Simulation

By definition, the heuristic solution is likely to differ from the optimal solution, but it should not deviate too much to be useful. Because the heuristic rule is a contraction mapping on the theoretical optimum, it exhibits a fixed-point property. According to the Banach fixed-point theorem, an iterative sequence such as (9) and (10), where values are subsequently replaced by values closer to the fixed point, will converge to the fixed point, which is in our case the true optimum (Granas and Dugundji 2003). Note that this holds also under Nash competition because the Nash equilibrium establishes the fixed point. The interesting question is how fast the convergence process is.

To analyze the performance of the heuristic, we therefore conducted an experimental simulation study (for full details, see the electronic companion to this paper, which is available as part of the online version that can be found at http://mktsci.pubs.informs .org/). In this study, we analyze a firm with a product portfolio of four products.[2] We consider two scenarios, a single-firm scenario and a competitive scenario including a second firm with a portfolio of four products. Sales are generated by a multiplicative market response function, the most frequent type of response in empirical studies (Hanssens et al. 2001). The response function includes an asymmetric growth function, consistent with Equation (7), and two expenditure categories whose stocks evolve according to

Equation (2). Six factors that characterize the products in the portfolio were experimentally manipulated: current-period elasticities, carryover coefficients, size of the revenue bases, profit contribution margins, growth parameters, and launch dates as reflected in the elapsed times since launch. Each factor has two levels. The initial condition assumes equally distributed budgets across the two marketing activities and four products. We use a five-year planning horizon, and the objective criterion is the discounted profit over the five years.

Optimal budgets are obtained by numerically solving the dynamic optimization problem as described by Equations (3)–(3.3). To reduce overall computation time, which is especially high in the competitive scenario, we construct an efficient Latin-square design that contains eight portfolio profiles. Profiles are randomly assigned to the two competitors. Consistent with practice, we simulate an annually recurring budget planning process and investigate 12 planning cycles. We compare the performance of the heuristic with the optimal solution in terms of (profit) suboptimality and match with the optimally allocated budget (for details, see the electronic companion). Figure 2 shows how these two performance criteria develop over time (the number of planning cycles).

Values in Figure 2 represent mean values across the 16 experimental conditions. If we do not apply the heuristic rule to improve the initial naïve budget allocation, the deviation from discounted profits of the optimal solution amounts to 19.2% on average. This suboptimality increases to 28.6% after 12 planning cycles (not shown in Figure 2). The match with the optimal budget allocation is 47.1% and remains around this level (50% after 12 planning cycles). As Figure 2 shows, we already achieve a dramatic improvement with our heuristic rule in the first planning cycle (4.3% profit suboptimality and 74.6% match with optimal budget allocation). Moreover, the rule quickly converges to the optimal solution when it is repeatedly used in the following planning cycles (0.95% profit suboptimality and 90.7% match with optimal budget allocation). This result holds under both the single-firm and the competitive scenario. Hence, the proposed rule appears to be a useful allocation heuristic.

3. Background for Implementation in Practice

3.1. Company Background

Together with the management of Bayer, we implemented and adapted the proposed heuristic to the specifics of Bayer's primary care business unit in the period 2005–2006 and derived budget recommendations for 2007. Bayer is one of the leading companies in the pharmaceuticals and chemicals business

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[2] We also tried larger product portfolios, e.g., with eight products. Results do not change, but computation time increases exponentially.
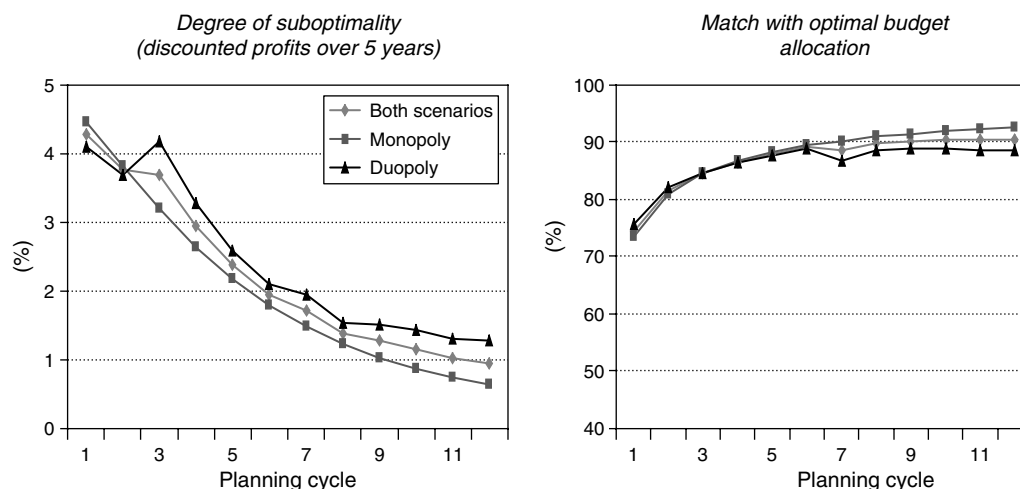
sector of the world. As of 2008, the company had €32.9 billion sales and approximately 108,600 employees (Bayer 2009). Bayer consists of three major business areas: Bayer HealthCare, Bayer CropScience, and Bayer MaterialScience. Bayer HealthCare is the largest area in terms of sales, contributing almost 50% to total sales. In 2008, the business area reported €15.4 billion in sales, positioning Bayer among the top 10 pharmaceutical firms worldwide. Bayer HealthCare is divided into a prescription drug business (pharmaceuticals, €10.7 billion) and an over-the-counter drug business (consumer health, €4.7 billion). The prescription drug business is composed of several business units. Primary care is the largest unit (€3.1 billion) and our focus for implementation of the allocation heuristic. Three business units, women's health, diagnostic imaging, and specialized therapeutics, are rather new to the company because they mainly belong to Schering, a pharmaceutical competitor Bayer acquired in 2006.

3.2.   Market Background

The primary care business unit of Bayer comprises prescription drugs that operate in four separate competitive market environments or therapeutic areas, respectively. These drugs treat diabetes, hypertension, erectile dysfunction, and infectious diseases. The hypertension segment is the largest one and includes several subcategories, such as beta blockers, calcium channel blockers, angiotensin-converting enzyme inhibitors, and AII-antagonists. Bayer has several offerings in this segment. With €626 million, the calcium channel blocker Adalat was its best-selling primary care drug (Bayer 2009) in 2008 and has been in the market since the mid-1970s. Although the drug has lost patent protection more than 20 years ago and is facing increasing generic competition, it contributes substantially to sales and profits of the primary care business unit. Avelox and

Ciprobay are Bayer's drugs in the anti-infectives business (€462 and €338 million, respectively). Whereas Avelox is an innovative, young drug under patent protection, Ciprobay recently lost patent protection. In the antidiabetes segment, Glucobay is also off-patent and generated €304 million in sales in 2008. All three above-mentioned therapeutic areas represent established areas that are in their saturation stage. Because of the aging of population in industrialized societies and innovative new product introductions, they are, however, expected to continue to grow at moderate rates in the future. The biggest challenge for Bayer in these areas is to keep its market position. Innovative drugs by other global players are the main competitors for the Bayer drugs. In contrast, the market for the treatment of erectile dysfunction is a new category that was pioneered by Pfizer with its Viagra brand in 1998. Bayer and Eli Lilly followed in 2003 with the introduction of their brands Levitra and Cialis, respectively. Levitra generated €341 million in 2008. The market is still growing and does not yet face generic competitors.

To summarize, the primary care business unit of Bayer holds a broad portfolio of drugs that are at different stages in their life cycle, face varying conditions of competition, and differ in their contributions to sales and profits. Hence, the challenge for the management was to find a balance in the allocation of marketing resources that trades off the size of the business, the growth expectations, and, eventually, the effectiveness of marketing expenditures. The main objective was to improve the process and results of annual budget allocation in order to maximize discounted profits from the product portfolio over a planning horizon of five years.

Bayer invests substantial resources in marketing and sales activities. Total marketing and selling expenditures were €7.6 billion (~23.1% of total sales)

in 2008. For confidentiality reasons, we cannot report the exact figures for the primary care product portfolio. The lion's share is spent on detailing targeted at general practitioners and specialists. Competitors spent a significant share of their budget on pharmacists' detailing. In addition, Bayer invests in print advertisements, direct mailing activities, invitations of physicians to symposia, and other marketing activities. The implementation of the allocation tool is targeted at the five main European countries that contribute the largest share to total sales. The U.S. market also provides a substantial portion of sales. However, the Bayer products are marketed here by licensee firms. Hence, budget decisions are not under the control of the Bayer management.

4. Data and Model Estimation

4.1. Data

To calibrate the heuristic allocation tool for Bayer, we need to estimate a number of input variables. Specifically, we require product-specific data on the short-term sales elasticity of different types of marketing investments, carryover coefficients, and information to compute the growth multiplier. For this purpose, we use 10 years (1996–2006) of quarterly marketing and sales data at the product level to estimate a market response model for each product market. IMS Health, Inc., provided data on unit sales counted in standard units, revenues (all in euros), and the dates

of product launches, which we use to obtain order-of-entry and life cycle information. We computed prices from revenues and unit sales. Via their CAM database, CEGEDIM, S.A., provided information on detailing expenditures targeted at general practitioners, specialists, and pharmacists. In addition, we have information available on professional journal advertising expenditures (including direct mailing); expenditures on physicians for invitations to symposia, meetings, etc. (hereafter denoted as meeting invitations); and other expenditures (hereafter denoted as OME).

The database covers the four strategic Bayer primary care prescription drug businesses, antidiabetes, hypertension, erectile dysfunction, and anti-infectives, in five countries, Germany, France, the United Kingdom, Italy, and Spain. Bayer management helped us to identify the relevant subcategories and competitors within each therapeutic area by country. Subcategories vary from 12 for anti-infectives to 1 for erectile dysfunction. Products vary from 15 for the erectile dysfunction area to 306 for the Hypertension area (see Table 1).

Table 1 also shows mean values and standard deviations for the variables used in estimation. The detailing stocks for general practitioners are highest, followed by the stocks for specialists. Stocks are computed consistent with Equation (2) (see also Berndt et al. 1995). Details on estimation are given in Table 1. The carryover is highest for hypertension, which is a chronic disease, and lowest for anti-infectives, which

Table 1   Descriptive Statistics

	Antidiabetes		Hypertension		Erectile dysfunction		Anti-infectives	
	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.	Mean	Std. dev.
Unit sales (in thousand standard units)	16,319	20,674	11,891	16,649	1,008	649	5,291	8,004
Elapsed time since launch (in years)	14.50	12.69	10.00	7.42	2.75	1.91	12.25	10.45
Order of entry (median)	3		4		2		3	
Price in EUR per standard unit	0.16	0.26	0.50	2.96	7.00	0.48	2.01	1.97
Marketing stock variables								
Detailing at general practitioners (in thousand EUR)	22,519	36,566	64,595	87,134	55,026	30,326	44,259	34,930
Detailing at specialists (in thousand EUR)	2,081	4,068	8,803	13,701	14,498	12,771	10,380	11,353
Detailing at pharmacies (in thousand EUR)	588	1,453	1,930	3,039		[a]	1,766	2,598
Professional journal advertising (in thousand EUR)	149	341		[a]	458	502	165	295
Meeting invitations (in thousand EUR)	730	2,030	1,361	3,062	3,884	2,481	471	837
Other marketing expenditures (in thousand EUR)		[a]	2,558	9,278	3,912	4,404		[a]
No. of countries	5		5		5		5	
No. of subcategories	6		10		1		12	
No. of products	104		306		15		100	
No. of observations	2,398		7,908		233		2,916	

Notes. All units and EUR figures are on a quarterly basis. The marketing stock S_{kint} for activity n of drug i in country k and period t is defined as $S_{kint} = \sum_{\tau=0}^{t}(1-\delta_{TA})^{\tau}x_{kint}$, where δ_{TA} is the quarterly decay rate, specific for each therapeutic area TA, and x measures the marketing expenditures. We used a numerical search algorithm to estimate the decay coefficient in a first-stage nonlinear regression of Equation (11) that minimizes the residual sum of squares. Because of the complexity of our model, we could only identify decay parameters at the level of the therapeutic area. With better data, a brand- and marketing activity-specific parameter may be obtained. If we did not observe initial stocks, we imputed the first quarter by dividing the average quarterly expenditures of the first observed year by the decay coefficient.

[a]Spending category was only rarely used by firms.

Table 2   Estimation Results for Market Response Models (Equation (11)): Antidiabetes and Hypertension Categories

	Antidiabetes				Hypertension			
	Est. parameter	SE	Est. parameter SD	SE	Est. parameter	SE	Est. parameter SD	SE
Constant	5.32	(0.202)	0.904	(0.019)	9.06	(0.154)	1.98	(0.021)
Ln(elapsed time since launch) × Total marketing stock	0.225×10^{-5}	(0.155×10^{-12})			0.897×10^{-8}	(0.470×10^{-9})		
Elapsed time since launch × Total marketing stock	-0.531×10^{-9}	(0.598×10^{-14})			-0.503×10^{-9}	(0.383×10^{-10})		
Ln(own price)	−0.597	(0.026)			−0.911	(0.013)		
Ln(average competitor price)	−0.449	(0.024)			−0.049	(0.018)		
Ln(order of entry)	−0.256	(0.016)			−0.225	(0.011)		
Marketing stock variables								
Category-specific carryover coefficient (annual level)	0.57				0.78			
Ln(detailing at general practitioners)	0.103	(0.005)	0.046	(0.004)	0.193	(0.004)	0.100	(0.003)
Ln(detailing at specialists)	0.016	(0.007)	0.089	(0.005)	0.047	(0.004)	0.085	(0.003)
Ln(detailing at pharmacies)	0.035	(0.005)	0.034	(0.003)	0.035	(0.003)	0.070	(0.003)
Ln(professional journal advertising)	0.060	(0.010)	0.032	(0.006)	[a]			
Ln(meeting invitations)	0.023	(0.006)	0.030	(0.005)	0.019	(0.003)	0.016	(0.003)
Ln(other marketing expenditures)	[a]				0.001	(0.003)[NS]	0.033	(0.002)
Ln(cumulative competitive marketing expenditures)	−0.008	(0.015)[NS]			−0.224	(0.011)		
Log likelihood	−11,859.02				−52,608.69			
Pseudo R^2	0.949				0.933			
No. of observations	2,398				7,908			
No. of products	104				306			

Notes. NS, not significant ($p > 0.05$). Product-specific parameter estimates for Bayer brands cannot be shown for confidentiality reasons. Effects for country dummies and seasonal dummies are not shown but can be obtained from the authors upon request.

[a]Spending category was only rarely used by firms.

are usually used for a one-time therapy (see also Tables 2 and 3). Note that not all marketing spending categories are equally utilized across the different markets. For example, OME for antidiabetes and anti-infective drugs are rarely used, so the data are not rich enough for estimating reliable marketing effects. Prices are highest in the youngest category, the erectile dysfunction category. Finally, we note that sample sizes differ to a great extent because of the number of brands. The erectile dysfunction category was launched only in 1998, so we have the smallest sample size here, which limits model estimation to some extent. Finally, note that the samples are unbalanced; i.e., several drugs were launched after the start of the observation period and a few drugs left the market during that period. Thus, we observe 25.6 quarters per drug on average.

4.2. Specification of Market Response Model

Following Fischer and Albers (2010), we specify a double-log sales response function for each therapeutic area. Let sales of drug i in country k and period t be defined as follows:

$$\ln q_{kit} = \alpha_{0ki} + \alpha_{1ki} \ln gp\_sdet_{kit} + \alpha_{2ki} \ln sp\_sdet_{kit}$$

$$+ \alpha_{3ki} \ln ph\_sdet_{kit} + \alpha_{4ki} \ln sadv_{kit} + \alpha_{5ki} \ln smeet_{kit}$$

$$+ \alpha_{6ki} \ln sOME_{kit} + \beta_1 \ln scomp_{kit} + \beta_2 \ln prc_{kit}$$

$$+ \beta_3 \ln comprc_{kit} + \beta_4 \ln OE_{ki} + \beta_5 stot_{kit}$$

$$\times \ln ET_{kit} + \beta_6 stot_{kit} \times ET_{kit} + \sum_{l=1}^{M-1} \gamma_l CTY_{lk}$$

$$+ \sum_{l=1}^{M} \sum_{h=1}^{H-1} \gamma'_{lh} SD_{ht} \times CTY_{lk} + v_{kit},$$

$$\text{with } v_{kit} \sim N(0, \sigma_v^2), \quad (11)$$

where
q_{kit}: Unit sales of drug i in country k and period t;
gp_sdet_{kit}: Stock of detailing expenditures at general practitioners of drug i in country k and period t;
sp_sdet_{kit}: Stock of detailing expenditures at specialists of drug i in country k and period t;
ph_sdet_{kit}: Stock of detailing expenditures at pharmacists of drug i in country k and period t;
$sadv_{kit}$: Stock of professional journal advertising expenditures of drug i in country k and period t;
$smeet_{kit}$: Stock of expenditures on meeting invitations of drug i in country k and period t;

Table 3   Estimation Results for Market Response Models (Equation (11)): Erectile Dysfunction and Anti-infectives Categories

	Anti-infectives				Erectile dysfunction			
	Est. parameter	SE	Est. parameter SD	SE	Est. parameter	SE	Est. parameter SD	SE
Constant	8.95	(0.216)	1.50	(0.054)	0.138	$(0.626)^{NS}$	2.84	(0.315)
Ln(elapsed time since launch) × Total marketing stock	0.133×10^{-7}	(0.885×10^{-13})			0.477	$(0.130)^a$		
Elapsed time since launch × Total marketing stock	-0.299×10^{-9}	(0.516×10^{-14})			-0.036	$(0.017)^a$		
Ln(own price)	-0.803	(0.070)			-0.848	(0.255)		
Ln(average competitor price)	-0.023	$(0.068)^{NS}$			^{a}			
Ln(order of entry)	-0.267	(0.011)						
Pioneer dummy					0.540	(0.109)		
Marketing stock variables								
Category-specific carryover coefficient (annual level)	0.33				0.52			
Ln(detailing at general practitioners)	0.254	(0.009)	0.107	(0.007)	0.464	(0.042)	0.201	(0.049)
Ln(detailing at specialists)	0.032	(0.005)	0.029	(0.004)	0.080	(0.031)	0.032	$(0.026)^{NS}$
Ln(detailing at pharmacies)	0.035	(0.004)	0.021	(0.003)	^{b}			
Ln(professional journal advertising)	0.026	(0.004)	0.037	(0.003)	0.079	(0.034)	0.075	(0.023)
Ln(meeting invitations)	0.004	$(0.003)^{NS}$	0.011	(0.003)	0.059	$(0.047)^{NS}$	0.080	$(0.042)^{NS}$
Ln(other marketing expenditures)	^{b}				0.034	(0.014)	0.032	(0.012)
Ln(cumulative competitive marketing expenditures)	-0.273	(0.014)			-0.007	$(0.008)^{NS}$		
Log likelihood	$-3{,}851.37$				-50.63			
Pseudo R^2	0.972				0.973			
No. of observations	2,916				233			
No. of products	100				15			

Notes. NS, not significant ($p > 0.05$). Product-specific parameter estimates for Bayer brands cannot be shown for confidentiality reasons. Effects for country dummies and seasonal dummies are not shown but can be obtained from the authors upon request.

[a]Because of the small number of observations and associated collinearity issues, we were unable to fit a model that includes competitor price and interactions of the elapsed-time-since-launch variables with total marketing stock. Therefore, results do not reflect interactions but main effects of elapsed time since launch.

[b]Spending category was only rarely used by firms.

$sOME_{kit}$: Stock of other marketing expenditures of drug i in country k and period t;

$scomp_{kit}$: Stock of cumulative marketing expenditures by drug i's competitors in country k and period t;

prc_{kit}: Price of drug i in country k and period t;

$comprc_{kit}$: Average price by drug i's competitors in country k and period t;

OE_{ki}: Order of entry by subcategory of drug i in country k;

$stot_{kit}$: Stock of drug i's total marketing expenditures in country k and period t;

ET_{kit}: Elapsed time since launch of drug i in country k and period t;

CTY_k: Country dummy variable for country k (1 for $k = l$, 0 otherwise);

SD_{ht}: Seasonal dummy variable for quarter h and period t (1 if period t corresponds to quarter h, 0 otherwise);

$\alpha, \beta, \gamma, \gamma'$: (Unobserved) parameter vectors;

v, σ^2: Error terms and error variances;

i: Index for drug that belongs to country-specific set I_k;

$k = 1, 2, \ldots, l, \ldots, M$ (number of countries);
$t = 1, 2, \ldots, T_i$ (number of periods per drug); and
$h = 1, 2, \ldots, H$ (quarters of the year).

The α_{1-6} parameters measure the effects of the own marketing expenditure stock variables; β_1 captures the effect of competitive marketing expenditures that are observable to competitors. We combine all expenditure types in a cumulative stock variable. We could have specified a greater number of more differentiated competitor variables. Because our interest does not rest on competitive effects, we save degrees of freedom by using a composite variable. The same argument applies for the average competitor price, which we include in addition to the own price. The sales model does not incorporate a distribution variable. Because pharmacies in Europe are required to list every prescription drug, there is no variation in this variable.

We include interactions of the stock of total marketing expenditures with elapsed time and the log of elapsed time to measure an asymmetric growth function that is consistent with Equation (7). By this specification, we assume that the growth parameters

a and *b* are scaled by the stock, whereas β_5 and β_6 measure the two scaling factors and are to be estimated. Note that the resulting growth parameters *a* and *b* are drug-specific because they are determined by a drug's total marketing stock.

Finally, our model incorporates a number of control variables that have been shown to impact sales of pharmaceuticals. With order of entry, we control for the disadvantage of a late market entry (e.g., Berndt et al. 1995). Because order of entry is defined at the subcategory level, we may have more than one pioneer drug in a therapeutic area. We account for product quality, brand equity, and other unobserved time-invariant variables by specifying a *random* drug-specific constant (α_{0ki}). Because we include the randomness into the conditional mean function but not the error term, we avoid potential *endogeneity* issues that arise from the correlation of unobserved product quality, brand equity, etc., with marketing-mix variables (Fischer and Albers 2010). Even though we do not model endogeneity in budget setting, e.g., allocating resources to more effective activities as represented by elasticities α_{1-6}, we effectively control for it and obtain consistent parameter estimates. We account for market size differences by including country dummies. Seasonal dummy variables by country control for seasonal variation in demand.

4.3. Estimation and Results

4.3.1. Estimation. We estimate four models, one for each therapeutic area. The specification of the sales model accounts for heterogeneity in the constant term and marketing effectiveness. We impose the following heterogeneity structure on these parameters:

$$\alpha_{kiv} = \bar{\alpha}_v + \lambda_{v1}\eta_{1ki} + \lambda_{v2}\eta_{2ki},$$

$$\text{with } \eta_{1ki}, \eta_{2ki} \sim N(0,1) \text{ and } \text{Cov}(\eta_{1ki}, \eta_{2ki}) = 0, \quad (12)$$

where α_{kiv} represents an unknown drug-specific parameter associated with predictor $v \in [0,6]$; $\bar{\alpha}_v$, λ_{v1}, and λ_{v2} are heterogeneity parameters to be estimated; and η_{1ki} and η_{2ki} denote variance components that vary by drug and country. The implied variance of α_{kiv} is $(\lambda_{v1}^2 + \lambda_{v2}^2)$. The variance–covariance matrix for α_{ki} is given by $\Sigma = \Lambda\Lambda'$.

We adopt the estimation approach used by Fischer and Albers (2010). Estimation also produces a set of posterior means of the drug-specific elasticity parameters (for details, see Fischer and Albers 2010).

4.3.2. Results. Tables 2 and 3 show the results of model estimations. Because of confidentiality reasons, we cannot show individual estimates for Bayer products. Reported estimates therefore reflect market averages. In-sample model fit is very good across all four therapeutic areas. Pseudo R^2, which is based on the

squared correlation between predicted and observed values of the criterion variable, ranges from 0.933 (hypertension) to 0.973 (erectile dysfunction). Because we account for drug heterogeneity, it is quite high. In a few cases, a marketing spending category was used by only a very small number of firms, leading to an inflation of zero-stock values (e.g., OME for antidiabetes and anti-infectives). Estimation of marketing effects was unreliable in such cases, so we excluded this variable from the model. The relatively low number of 233 observations in the young erectile dysfunction category created collinearity issues for the interactions of total marketing stock with the elapsed-time variables and for the price variables. Because we could not separate the associated effects, we estimated only main effects with respect to elapsed time since launch and the own-price effect. In addition, we include a dummy variable for the pioneer Viagra, because only two competitors followed in the same quarter, and the common order-of-entry variable lacks variation.

In a double-log model, parameter estimates for marketing-mix variables correspond to elasticities. These elasticities refer to marketing stock variables and reflect long-term elasticities with respect to current-period expenditures. To obtain short-term elasticities, the stock elasticity needs to be multiplied with the decay coefficient. Elasticities for detailing and other marketing activities vary substantially across the different therapeutic areas. In general, they are highest in the erectile dysfunction category, which is not surprising because this category is the youngest category and still in its growth phase. Among the detailing elasticities, GP detailing appears to be more effective than is detailing at specialists and pharmacists. However, considering that specialists account only for a share of approximately 20% in antidiabetes and approximately 27% in hypertension, segment-specific specialist detailing elasticities are four to five times higher. Note that for the application of our allocation heuristic, the sales elasticities with respect to total brand sales as reported in Tables 2 and 3 are relevant. Elasticities for professional journal advertising, meeting invitations, and OME are usually considerably smaller than are elasticities for detailing at physicians. Finally, we note that the estimated effects are within the range of results of recent studies on pharmaceuticals (e.g., Albers et al. 2010, Fischer and Albers 2010).

In terms of control variables, we find significant but inelastic own-price effects. For competitive prices, we find negative cross-effects. This finding is consistent with Fischer and Albers (2010), who provide an explanation for negative cross-effects. The impact of competitive marketing expenditures is negative across all therapeutic areas, although it is not always

statistically significant. We find a negative elasticity for order of entry, as expected. Although not reported in Tables 2 and 3, seasonal effects are only relevant for anti-infectives, which experience a high season in autumn and winter.

4.3.3. Model Validation. We checked whether our model specification and estimation are appropriate for the data in several ways. First, we split the data sets into an estimation and a holdout sample. For the holdout, we used the four quarters of the last year of our observation period. Pseudo R^2 in the holdout samples ranged from 0.922 (hypertension) to 0.972 (erectile dysfunction) and were only slightly lower than were those of the estimation samples. The same picture emerges with respect to the mean absolute percentage error (MAPE) that ranges from 1.14% (erectile dysfunction) to 4.24% (hypertension) and strongly supports the predictive validity of our response model. Second, we compared the suggested log-log brand sales model with a linear model, a semilog model, and an S-shaped model. The Davidson and MacKinnon (1981) test for unnested models suggests that the proposed specification is superior to the alternative specifications. By adding predicted values from an alternative response model to the predictor set of the focal model, the test checks for the additional explanatory power of the alternative specification. Finally, we checked whether the residuals follow an autoregressive process by using the test for common factors (Greene 2006). We did not find evidence for it. Note that our sales model already incorporates dynamics in terms of marketing stock variables and the life cycle function.

5. Model Implementation and Impact
In this section, we describe how we implemented the allocation heuristic into a decision support tool in a spreadsheet environment. Furthermore, we discuss the various impacts the new tool and the project had on the Bayer organization.

5.1. Excel-Based Decision Support Tool
We developed a decision support tool that integrates the proposed allocation heuristic into an Excel-based software program. Excel is particularly suitable for applications in practice because it is widely used and easy to understand (Albers 2000). The tool is to assist the management with providing budget scenarios and their implications for the development of market shares and profits over the next five years. Specifically, the tool produces a recommendation for the allocation of the total marketing budget that is based on data on the effectiveness of marketing expenditures, including carryover and discounting effects, the size of the product's business, product profitability, and growth expectations (see Equations (9) and (10)).

The tool applies to Bayer's primary care product portfolio and covers expenditures in six spending categories for 36 products in four therapeutic areas and five countries as described previously. Hence, at the product–country–activity level, 36 (products) × 6 (spending categories) = 216 allocation decisions are made. It may easily be applied to other product portfolios that may be smaller or larger in size. Consistent with the periodicity of the response model estimation, metrics such as carryover coefficients, growth multipliers, etc., are defined at the quarterly level. The same applies to market share and profit simulations. Based on the response model (11), the tool demonstrates the impact of budget decisions on sales and profits by extrapolating the evolution of sales and profits over the next five years.

The heuristic rule requires us to compute an allocation weight for each marketing spending category and each drug (see Equation (10)). Input data have been obtained either from econometric analysis or internal records. The plausibility of input data, especially the estimated sales elasticities, has been extensively discussed with different groups of managers in several workshops (global marketing, market research, product management, sales management, controlling, etc.). Internal records provided data on the discount rate, the profit contribution margin, and last year's product revenues. Estimation of the sales response model (11) produced data on the carryover coefficient, short-term sales elasticities, and the growth potential multiplier. Computation of the growth potential multiplier ρ is based on the life cycle function (7) that is incorporated into (11). Specifically,

Growth potential multiplier (ρ_{kit})

$$= \left(\frac{\text{Elapsed Time since Launch}_{kit} + T}{\text{Elapsed Time since Launch}_{kit}} \right)^{\hat{a}_{kit-1}}$$
$$\cdot \exp(-\hat{b}_{kit-1} \cdot T), \tag{13}$$

where T is the forecast horizon (20 quarters, or 5 years), and $\hat{a}$ and $\hat{b}$ are estimated growth parameters. Because they depend on the marketing stock, we obtain estimated values from the last period.

Following the needs of management, we extended the tool in two ways. First, we included a threshold for product budgets. Although our demand analysis did not find evidence for an S-shaped response that justifies a threshold, management required a threshold because of internal setup costs that are fixed at the product- and marketing-activity level. Second, we allowed for manual adjustments to budgets recommended by the heuristic. By this feature, management can account for exogenous restrictions to budget setting, e.g., to counter competitive attacks in a predetermined way. In addition, it enables management to

investigate the effects of budget scenarios on market share and profit as well as on the recommended budgets for other products and marketing activities. Technically, the budget for an allocation unit is exogenously set and subtracted from the total budget. The remaining budget is allocated according to the heuristic.

The Excel-based decision support system offers a powerful tool to generate budget allocation options and analyze these options with respect to their economic consequences. The tool is easy to use and flexible enough to adapt to varying conditions of decision making.

5.2. Impact on Managerial Decision Making

The effort to develop and implement the budget allocation tool had significant impact on managerial decision making, which is reflected in several aspects.

5.2.1. Providing Structure to the Problem. The suggested allocation heuristic provides structure to a complex decision problem: 216 budget decisions arise from allocating a total budget across six spending categories for 36 drugs that are marketed in different countries and therapeutic areas. The market positions of these products are quite diverse and are determined by product age and competition. Depending on age and expected changes in the competitive and market environment, products offer different growth potentials. As a first benefit, the allocation rule provides the required information to solve the problem. This information falls into three groups. The first group refers to the effectiveness of marketing expenditures to build goodwill and affect sales in the long run (short-run elasticity and discounted carryover). The second group includes information on a product's contribution to profit. This depends on the contribution margin (price minus marginal cost) as well as the size of the revenue base. The third group emphasizes the growth expectations of the product. It uses information on where the product stands in its life cycle.

5.2.2. Providing Solution to the Problem. Although management had a good understanding of the type of information required for budget decisions, it benefited much from the new insights offered by the heuristic. Specifically, the allocation rule suggests that information on (1) long-term marketing effectiveness, (2) profit contribution, and (3) growth potential is to be combined in a multiplicative fashion. Implications from this rule are straightforward. (1) Products that generate more incremental sales with the same budget should get a larger slice of the total budget. Of course, relative incremental sales tend to decline as sales and budgets increase because of saturation effects. The budget ratio of two products reflects their ratio in terms of sales elasticity. (2) The same principle of proportionality applies to the size of sales or profit contribution, respectively. Products with a higher level of profit contribution generate more financial resources to cover their own marketing expenditures and contribute more to overall profits. (3) Marketing should support growing and not declining products and shift resources over the life cycle.

The rule also teaches that the drivers of a product's near-optimal budget share interact with each other; i.e., there exist synergies between them. Finally, it makes the trade-offs in budget allocation transparent. For example, a product with high marketing effectiveness but a low profit contribution level could get a lower budget than a product with a high level of profit contribution but lower marketing effectiveness. Even though that product's spending is less effective, it may still contribute more to overall profit because of its larger sales base.

5.2.3. Understanding the Limitations of Separate Return on Investment Analysis. Management was initially very focused on comparing incremental returns on investments (ROIs) that result from raising or lowering marketing expenditures for individual products and marketing activities (hereafter denoted as separate ROI analysis). Profit calculations with the allocation tool quickly revealed the limitations of such an analysis. First, separate ROI analyses for individual marketing activities do not consider synergies between marketing activities that interact with each other. Profit simulations for several brands, for example, showed that the ROI of a 10% budget increase in a specific spending category is negative but turns positive if the budget increase is accompanied by a reallocation across the different spending categories. Hence, the synergy between marketing activities is exploited only by the allocation heuristic but not by separate ROI considerations. Second, separate ROI analyses do not consider the trade-offs that exist with respect to potential profit improvements by other products and activities. For example, even though simulated ROIs for a few products were positive, the allocation heuristic suggested reducing the current budget on these products. The reason is that free budget resources were transferred to other products where the incremental return was even higher. Third, separate ROI analyses do not inform us about the magnitude of budget changes for products and activities, given a fixed total budget. Marginal returns analysis teaches us that the budget should be increased until the ROI reaches zero. However, if other products' budgets are also raised, the budget increase may exceed the total budget constraint. The allocation heuristic produces exact results for the recommended allocation of a fixed budget within one step.

5.3. Organizational Impact

The introduction of the allocation tool had a considerable impact on the organization. The project was part of a larger effort aimed at revising the organization's tools and processes to evaluate marketing initiatives in terms of their financial implications. This effort had the full attention of the managing board of Bayer. Budget decisions are often associated with several rounds of intensive discussions that follow a bottom-up process; i.e., product and country managers communicate their budget needs for the next year upwards. Budget discussions in companies are probably never fully free of politics and individual agendas. The allocation tool adds an independent, top-down perspective. Because it is strictly based on a range of verifiable input information, its recommendations are fully fact based. Assumptions about marketing effectiveness and other drivers may be discussed. Their implications for budget allocation are immediately transparent through application of the tool. Because of its transparency and top-down perspective, the allocation tool ameliorates the decision process that often appears emotional and inefficient.

Although the allocation tool is not the only source used by Bayer to generate budget options, it had a significant impact on the efficiency and quality of the decision process. The project contributed substantially to an organizational transformation that eventually resulted in the creation of a completely new marketing intelligence unit called global business support. This unit supports global marketing management and sales, including the global management board with tools, results, and recommendations for a more efficient and effective use of marketing resources.

5.4. Strategic Impact

Application of the tool initiated an important strategic discussion within the firm. The results suggested that some older products that still hold a strong position in sizable markets did not get sufficient marketing resources anymore. The allocation tool showed a substantial profit improvement potential from shifting more resources to these older products.

The results also initiated a discussion about the targets of sales calls and the relevance of accompanying marketing activities. In terms of targets, the results suggest reconsidering the strong focus on specialists. It seems that because of a higher frequency of sales calls at specialists by competitors, effectiveness is lower relative to sales calls at general practitioners. Consequently, the tool proposed reallocating resources between those two target groups. In addition, the results suggest that the potential of accompanying activities such as meeting invitations and OME were not yet fully exploited.

5.5. Financial Impact

The tool enables the user to simulate the financial impact of different budget allocation options. By analyzing the simulation results, it provides transparency about the impact of different assumptions on financial results. Based on data from the year 2007, the simulation suggested an increase in discounted profits of 55% over the next five years because of an optimized allocation. This is worth €493 million. In contrast, changing the overall budget by 20% promised a profit impact of less than 5%. Even if only a small portion of this increase can be realized, the additional profit for a business unit such as primary care with €3 billion worldwide sales is substantial.

Actual profit improvements are hard to evaluate. First, management did not completely follow the suggested reallocation by the tool for several reasons (e.g., varying personal experiences, concerns about errors in IMS data). Second, activities by competitors and exogenous influences on market dynamics affect profit results. Nevertheless, the business area Bayer HealthCare reports an increase in EBIT of 12% (€273 million) compared to a 4% revenue increase for the year 2008 (Bayer 2009). Although we have no validation from a field test, these results are consistent with prior observations that reallocation really focuses on the bottom line.

5.6. Generalizability

Although the tool was applied to prescription drugs, we emphasize that it is suitable for many other industries, such as consumer durables, consumer packaged goods, etc. In all these markets, rich information is available at the aggregate product level that allows the calibration of market response models; however, even if data on marketing effectiveness, carryover, etc., cannot be estimated with aggregate market response models, other data and methods including choice models and managerial experiments are available to generate the required input data for allocation. In this respect, we are not aware of a limitation to apply the allocation heuristic in other industries.

6. Conclusions, Limitations, and Future Research

In this paper, we suggest an innovative approach to allocate a global marketing budget across countries, products, and marketing activities. Based on the theoretical solution to the dynamic optimization problem, we derive a simple but comprehensive heuristic that accounts for dynamics in marketing effects and product growth. It suggests allocating a budget proportionally to the size of the business (sales and profit contribution margin), the effectiveness of the marketing activities (short-term elasticity and carryover coefficient), and the growth potential of the

product (growth multiplier accounting for time discounting). A simulation study demonstrates that the heuristic quickly converges to the optimal solution under both monopoly and competitive conditions. The implementation of the heuristic at Bayer had various significant effects on the organization. It revealed substantial profit improvement potentials by reallocating marketing resources for the primary care business unit. It had a significant effect on the quality and efficiency of the budget allocation process and contributed to organizational change.

Our research has limitations that may stimulate future research. First, we have analyzed budget allocation issues under the assumption of a specific response function that has been found to best represent the data in this study. It would be interesting to extend the application to other response functions, including different growth functions. Second, our simulation study covers only a limited range of conditions. Additional conditions such as more competitors and errors in input data may be analyzed and the number of scenarios extended. It would also be useful to understand which conditions have a critical influence on the performance of the heuristic. Third, the tool may be extended to compute uncertainty bounds for recommended budget and market share and profit simulations. This would add a risk-analysis perspective to the application. Finally, we note that our research lacks an experimental field test that is hard to implement in a global portfolio worth €3 billion. Future applications to smaller portfolios might, however, overcome this limitation and test the superiority of the suggested heuristic. Finally, we assume that the overall marketing budget is set exogenously. Unless the budget level is optimal, there is still profit improvement potential. The flat maximum principle, however, suggests that this potential is very small, provided the budget is set within a reasonably wide range around the true optimal level (Tull et al. 1986).

7. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mktsci.pubs.informs.org/.

Appendix

Derivation of Theoretical Allocation Solution for Arbitrary Growth and Response Functions

We consider the constrained dynamic profit maximization problem as stated in Equations (3)–(3.3). We assume that the sales function in Equation (1) is twice differentiable in t and S. Note that it is sufficient to maximize profit contribution

before marketing cost because these cost are fixed by the total budget and thus not relevant to the optimization. Using the state variable Equation (3.2) to substitute x_{kin} in the objective function (3), we can write the following Lagrange objective functional:

$$L = \int_{t=0}^{T} e^{-rt}\left\{\left[\sum_{k\in K}\sum_{i\in I_k}(p_{ki}-c_{ki})q_{ki}\right] + \lambda\left(R - \sum_{k\in K}\sum_{i\in I_k}\sum_{n\in N_i}\frac{dS_{kin}}{dt} + \delta_{kin}S_{kin}\right)\right\}dt. \quad (14)$$

Note that the budget constraint (3.1) has to be fulfilled in each period, which may entail a time-varying Lagrange multiplier. A solution requires solving the $1 + \sum_{k\in K}\sum_{i\in I_k}|N_i|$ Euler–Lagrange equations, which constitute the first-order conditions

$$\frac{\partial F}{\partial S_{kin}}\left[t, S_{kin}^*, \frac{dS_{kin}^*}{dt}, \lambda^*\right] - \frac{d}{dt}\left[\frac{\partial}{\partial(dS_{kin}/dt)}F\left(t, S_{kin}^*, \frac{dS_{kin}^*}{dt}, \lambda^*\right)\right]$$
$$= 0, \quad \forall k\in K, i\in I_k, n\in N_i, t\in[0,T] \quad (15a)$$

and

$$\frac{\partial F}{\partial \lambda} = 0, \quad \forall t\in[0,T], \quad (15b)$$

where $F(t, S_{kin}^*, (dS_{kin}^*/dt), \lambda^*)$ is the Lagrangean integrand, and the asterisk indicates that variable values correspond to the optimal solution for the marketing budget. Note that each competitor has to satisfy these conditions under Nash competition. The required derivatives to solve (15a) are

$$\frac{\partial F}{\partial S_{kin}} = e^{-rt}[(p_{ki}-c_{ki})\frac{\partial q_{ki}}{\partial S_{kin}} - \lambda\delta_{kin}], \quad (16a)$$

$$\frac{\partial F}{\partial(dS_{kin}/dt)} = -\lambda e^{-rt}, \quad (16b)$$

$$\frac{d}{dt}\frac{\partial F}{\partial(dS_{kin}/dt)} = r\lambda e^{-rt}. \quad (16c)$$

Setting (16c) equal to (16a) yields

$$(p_{ki}-c_{ki})\frac{\partial q_{ki}}{\partial S_{kin}} = \lambda(r+\delta_{kin}). \quad (17)$$

From Equation (1), we obtain

$$\frac{\partial q_{ki}}{\partial S_{kin}} = \frac{\partial f_{ki}}{\partial S_{kin}}g_{ki} + \frac{\partial g_{ki}}{\partial S_{kin}}f_{ki},$$

which may be expanded into

$$\frac{\partial q_{ki}}{\partial S_{kin}} = \frac{\partial f_{ki}}{\partial S_{kin}}\frac{S_{kin}}{f_{ki}}g_{ki}f_{ki}\frac{1}{S_{kin}} + \frac{\partial g_{ki}}{\partial S_{kin}}\frac{S_{kin}}{g_{ki}}f_{ki}g_{ki}\frac{1}{S_{kin}}$$
$$= (\varepsilon_{f(S_{kin})} + \varepsilon_{g(S_{kin})})\frac{q_{ki}}{S_{kin}}. \quad (18)$$

Inserting (18) into (17) and solving for S_{kin}^* yields

$$S_{kin}^* = \frac{(p_{ki}-c_{ki})q_{ki}^*(\varepsilon_{f(S_{kin}^*)} + \varepsilon_{g(S_{kin}^*)})}{\lambda(r+\delta_{kin})}. \quad (19)$$

We multiply both sides of Equation (19) with δ_{kin} and use the identities $\delta_{kin}S_{kin}^* = x_{kin}^* - dS_{kin}^*/dt$, $\gamma_{kin} = 1 - \delta_{kin}$, $\varepsilon_{f(x_{kin}^*)} = \delta\varepsilon_{f(S_{kin}^*)}$, and $\varepsilon_{g(x_{kin}^*)} = \delta\varepsilon_{g(S_{kin}^*)}$ to obtain

$$x_{kin}^* = \frac{(p_{ki}-c_{ki})q_{ki}^*(\varepsilon_{f(x_{kin}^*)} + \varepsilon_{g(x_{kin}^*)})}{\lambda(r+1-\gamma_{kin})} + \frac{dS_{kin}^*}{dt}, \quad (20)$$

where γ measures the carryover coefficient, and $\varepsilon_{f(x_{kin}^*)}$ and $\varepsilon_{g(x_{kin}^*)}$ are (short-term) sales elasticities with respect to marketing expenditures.

Recall that the budget constraint is binding and has to be satisfied in the optimum; i.e., $R = \sum_{k \in K} \sum_{i \in I_k} \sum_{n \in N_i} x_{kin}^*$, $\forall t \in [0, T]$. Because this constraint also applies to the end period, S_{kinT} is free but only within the constraint. This turns the problem into a fixed-endpoint problem, and we do not need a general transversality condition. From (20), we obtain the optimal share of the budget that is allocated to marketing activity n of product i in country k by

$$
\frac{x_{kin}^*}{R} = ((p_{ki} - c_{ki}) q_{ki}^* (\varepsilon_{f(x_{kin}^*)} + \varepsilon_{g(x_{kin}^*)})/(r + 1 - \gamma_{kin}))
$$
$$
\cdot \left(\sum_{l \in K} \sum_{j \in I_l} \sum_{m \in N_j} ((p_{lj} - c_{lj}) q_{lj}^* (\varepsilon_{f(x_{ljm}^*)} + \varepsilon_{g(x_{ljm}^*)})/(r + 1 - \gamma_{ljm})) \right.
$$
$$
\left. + \lambda \sum_{l \in K} \sum_{j \in I_l} \sum_{m \in N_j} (dS_{ljm}^*/dt) \right)^{-1}
$$
$$
+ \frac{1}{R} \frac{dS_{kin}^*}{dt}, \quad \forall k \in K, i \in I_k, n \in N_i, t \in [0, T], \quad (21)
$$

which is equivalent to Equation (4).

From the budget constraint, we know that the following linear restriction must hold:

$$
\sum_{k \in K} \sum_{i \in I_k} \sum_{n \in N_i} \frac{x_{kin}^*}{R}
$$
$$
= \sum_k \sum_i \sum_n \left[((p_{ki} - c_{ki}) q_{ki}^* (\varepsilon_{f(x_{kin}^*)} + \varepsilon_{g(x_{kin}^*)})/(r + 1 - \gamma_{kin})) \right.
$$
$$
\cdot \left(\sum_{l \in K} \sum_{j \in I_l} \sum_{m \in N_j} ((p_{lj} - c_{lj}) q_{lj}^* (\varepsilon_{f(x_{ljm}^*)} + \varepsilon_{g(x_{ljm}^*)}) \right.
$$
$$
\left. \left. /(r + 1 - \gamma_{ljm})) + \lambda \sum_{l \in K} \sum_{j \in I_l} \sum_{m \in N_j} (dS_{ljm}^*/dt) \right)^{-1} \right]
$$
$$
+ \frac{1}{R} \sum_{k \in K} \sum_{i \in I_k} \sum_{n \in N_i} \frac{dS_{kin}^*}{dt} = 1.
$$

In a typical product portfolio that includes several products of different ages and therefore different levels of marketing activity stocks, some stocks will increase and others will decrease from one period to the next because the total budget to be allocated is limited. For a fairly large number of allocations units, which are defined at the country–product–marketing–activity level, gains and losses in stocks tend to cancel each other out, so $\sum_{k \in K} \sum_{i \in I_k} \sum_{n \in N_i} dS_{kin}^*/dt \cong 0$, with $k = 1, \ldots, l, \ldots K$, $i = 1, \ldots, j, \ldots, I_k$, and $n = 1, \ldots, m, \ldots, N_i$. As a result, we have

$$
\sum_{k \in K} \sum_{i \in I_k} \sum_{n \in N_i} \frac{x_{kin}^*}{R}
$$
$$
= \sum_k \sum_i \sum_n \left[((p_{ki} - c_{ki}) q_{ki}^* (\varepsilon_{f(x_{kin}^*)} + \varepsilon_{g(x_{kin}^*)})/(r + 1 - \gamma_{kin})) \right.
$$
$$
\cdot \sum_{l \in K} \sum_{j \in I_l} \sum_{m \in N_j} ((p_{lj} - c_{lj}) q_{lj}^* (\varepsilon_{f(x_{ljm}^*)} + \varepsilon_{g(x_{ljm}^*)})
$$
$$
\left. /(r + 1 - \gamma_{ljm}))^{-1} \right] \cong 1
$$

and obtain a solution for the optimal budget share that is very close to (21):

$$
\frac{x_{kin}^*}{R} \cong \frac{(p_{ki} - c_{ki}) q_{ki}^* (\varepsilon_{f(x_{kin}^*)} + \varepsilon_{g(x_{kin}^*)})/(r + 1 - \gamma_{kin})}{\sum_{l \in K} \sum_{j \in I_l} \sum_{m \in N_j} (p_{lj} - c_{lj}) q_{lj}^* (\varepsilon_{f(x_{ljm}^*)} + \varepsilon_{g(x_{ljm}^*)})/(r + 1 - \gamma_{ljm})}.
$$
$$
(22)
$$

Because we also need to satisfy the condition $x_{kin}^* \geq 0$ that is violated if $\varepsilon_{f(x_{kin}^*)} + \varepsilon_{g(x_{kin}^*)} < 0$ and $\varepsilon_{g(x_{kin}^*)} < 0$, the optimal marketing budget for marketing activity n of product i in country k is given by

$$
x_{kin}^* \cong \frac{\max\{w_{kin}^*, 0\}}{\sum_{l \in K} \sum_{j \in I_l} \sum_{m \in N_j} \max\{w_{ljm}^*, 0\}} R,
$$
$$
\forall k \in K, i \in I_k, n \in N_i, t \in [0, T], \quad (23)
$$
$$
\text{with } w_{kin}^* = \frac{(p_{ki} - c_{ki}) q_{ki}^* (\varepsilon_{f(x_{kin}^*)} + \varepsilon_{g(x_{kin}^*)})}{r + 1 - \gamma_{kin}},
$$

which is equivalent to Equation (5).

The solution establishes a global maximum because the integrand $F(t, S_{kin}^*, (dS_{kin}^*/dt), \lambda^*)$ is concave in S_{kin}^* and dS_{kin}^*/dt. For a fixed-endpoint problem, the Euler–Lagrange Equations (15a) and (15b) are sufficient for an absolute maximum (Kamien and Schwartz 1991).

Parametric Growth Model
Consistent with (6), we consider the following parametric growth model

$$
g_i(t) = \alpha_i t^{a_i} e^{-b_i t}, \quad \text{with } \alpha_i, a_i, b_i > 0 \text{ and } t \in [0, \infty). \quad (24)
$$

Equivalence with the Gamma Distribution. The probability density function of a gamma distributed random variable t is defined as follows

$$
\text{gamma}(t) = \frac{\phi^\theta t^{\theta - 1} e^{-\phi t}}{\Gamma(\theta)}, \quad (25)
$$

where ϕ and θ are characteristic parameters that define the shape of the distribution. Let the parameters α, a, and b of (24) be defined as $\alpha = b^{a+1}/\Gamma(a+1)$, $a = \theta - 1$, and $b = \phi$. Then it can be shown that (24) results in (25).

Properties of Cumulative Sales. Let (24) measure unit sales. We obtain cumulative sales over the total lifetime of product i by solving the integral

$$
\text{CumSales}_i = \alpha_i \cdot \int_0^\infty t^{a_i} \cdot e^{-(b_i)t} \, dt = \alpha_i \cdot \frac{\Gamma(a_i + 1)}{b_i^{a_i + 1}},
$$
$$
\text{with } \alpha_i, a_i, b_i > 0. \quad (26)
$$

Let $a_i - b_i = \omega_i$ measure the distance in growth parameters for i. Substituting b_i for $a_i - \omega_i$ in (26) and differentiating this expression with respect to the distance ω_i yields

$$
\frac{d\text{CumSales}_i}{d\omega_i} = (a_i + 1) \alpha_i \Gamma(a_i + 1)(a_i - \omega_i)^{-a_i - 2}. \quad (27)
$$

Expression (27) is always greater than zero because all terms are greater than zero. Note that $a_i - \omega_i > 0$ because $a_i, b_i > 0$. From $a_i - b_i = \omega_i$, it follows that $a_i > \omega_i$.

Hence, cumulative sales increase with the difference in the growth parameters a and b. This result also holds for

discounted cumulative sales. Discounting (24) at rate r and differentiating with respect to ω_i leads to

$$\frac{d\text{CumSales}_i}{d\omega_i} = (a_i+1)\alpha_i\Gamma(a_i+1)(a_i-\omega_i+r)^{-a_i-2}. \quad (28)$$

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