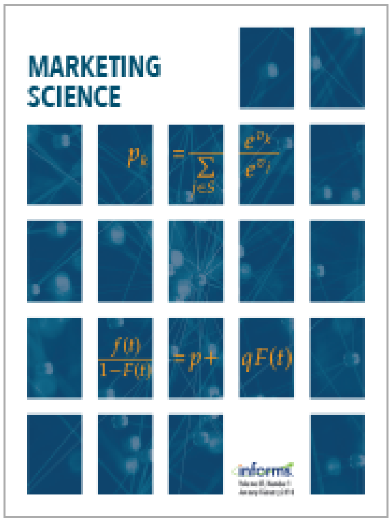


Marketing Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Communication Strategies and Product Line Design

J. Miguel Villas-Boas,

MARKETING SCIENCE
Vol. 23, No. 3, Summer 2004, pp. 304–316
ISSN 0732-2399 | EISSN 1526-548X | 04 | 2303 | 0304

DOI 10.1287/mksc.1030.0048

Communication Strategies and Product Line Design

J. Miguel Villas-Boas
Haas School of Business, University of California at Berkeley, Berkeley, California 94720-1900,
villas@haas.berkeley.edu

When selling a product line, a firm has to consider the costs of communicating about the different products to the consumers. This may affect the product line design in general, and which products or services are offered in particular. The problem is that firms have to communicate to consumers, possibly through advertising, to make them consider buying the products that firms are selling. This results in the firm offering a smaller number of products than is optimal when advertising has no costs. This effect is greater the extent of consumer confusion about the advertising messages, and is reduced by a greater ability to target advertising. When offering vertically differentiated products (second-degree price discrimination), under general conditions it is optimal to advertise so that one has a greater proportion of sales of a lower-quality product than if advertising had no cost. This situation also allows the firm to charge a lower price for the high-quality product and offer a higher quality of the low-quality product than it would if advertising were without cost.

Key words: communication strategies; advertising; product policy; product line; segmentation; price discrimination
History: This paper was received January 2, 2003, and was with the authors 14 months for 2 revisions; processed by Duncan Simester.

1. Introduction

When selling a product line, a firm has to consider the costs of communicating about its different products to the consumers. This may affect the product line design in general, and which products or services are to be offered, in particular. In this sense, firms sometimes prefer not offer many products so they do not confuse consumers. For example, automobile manufacturers offer a limited number of models or brands,[1] manufacturers of consumer products offer a limited number of brands or packages, and banks offer only a few types of checking accounts rather than a continuum of them.[2]

Firms have to communicate to consumers, possibly through advertising, to make them consider buying the products that the firms are selling.[3] If firms invest in advertising more products, they offer a better fit of each product to the consumer preferences and can charge higher prices. However, advertising more products is costly and may result in consumers receiving advertising messages on more that one product, which could generate confusion. As noted by Wells et al. (1992, p. 247): "An important requirement of informational advertising is that the explanation be clear and relevant to the prospect. Consumers have little patience with ads that are confusing, vague, or unfocused."

When deciding to have a marketing program for a particular segment, a company has to make sure that the "segment should be the largest possible homogeneous group worth going after with a tailored marketing program" (Kotler 1997, p. 269). The firm can decide between undifferentiated marketing, where there is a single marketing program for all the market segments, and differentiated marketing, where there are different marketing programs. The extent to which a segment is "accessible" (i.e., the ability to target it with a particular marketing program) is an important factor in deciding whether to use differentiated marketing. Similarly, a company's decision of how many brands to have "involves gauging the trade-off between the value that the brand might create and the cost that it might incur" (Aaker 1996, p. 264). Firms also make different choices. For example, General Motors has more than 30 brands or subbrands (e.g., under Buick, Roadmaster, Park Avenue, and Riviera) while BMW or Mercedes basically each have one brand, with the models indicated by numbers (e.g., the BMW 700 series). At 3M the criterion

[1] Sometimes, when including the different potential options, one can have a large number of combinations. However, the combinations are all along particular quality dimensions.

[2] One academic researcher in the beginning of her career did not want to circulate a paper outside her main area of research, to avoid confusing the market about her positioning. Communicating about this new area with only one paper would take effort without helping much in what the market considered her main area of research.

[3] By "consumers considering buying a product" we mean that the consumers have that product in their consideration set. For evidence that consumers do not consider buying all products available, and that there may be substantial heterogeneity in the products being considered, see, for example, Newman and Staelin (1972) and Mehta et al. (2003).

for a new brand is that the segment of that brand be large enough to support the necessary brand-building investment (Aaker 1996). Similarly, one has to worry about how new brands may affect the sales of existing brands and the interaction between the brands or products (Urban 1969, Dobson and Kalish 1988, Wilson and Norton 1989, Shugan and Desiraju 2001). One way in which a new advertised brand may affect the other brands is by creating confusion for consumers about which brand best fits the consumer preferences. As noted by Keller (1998, p. 464), "Different varieties of line extensions may confuse and perhaps even frustrate consumers as to which version of the product is the 'right one' for them. As a result, they may reject new extensions for 'tried-and-true' favorites or all-purpose versions that claim to supersede more specialized product versions."[4] This paper addresses these issues by looking carefully at consumer preferences, pricing, product design, and the costs of communicating information on the product attributes to the potential consumers.[5]

This tradeoff between advertising expenditures and the products-consumers fit yields a lower optimal number of products being advertised (and, therefore, being offered for sale) than when advertising has no cost. The lack of ability to perfectly target the advertising results in too much overlap in advertising expenditures. Firms may then find it in their best interest to advertise a smaller number of products. The cost is that there will then be a worse fit between products and consumers, which translates into the firms being able to charge less for the products. Under these conditions (less-than-perfect targeting) firms advertise a smaller number of products than they would if advertising had no cost. This effect is greater if there is greater consumer confusion about multiple advertising messages, and is smaller if the firm has a greater ability to target its advertising. Furthermore, one finds that there are circumstances where the firm chooses to offer both a specialized product targeted at only one consumer segment (with a high price) and a generic product targeted at all segments (with a low price). This is because the waste of advertising from a consumer receiving multiple messages is reduced by a lower advertising level of a specialized product.

In the general problem of product line design, the different products being advertised may be vertically differentiated (second-degree price discrimination). In such a case, one could think that the firm should advertise the products for which it has a higher margin more, that is, the higher-quality products. However, another important factor is that consumers who like quality more are still going to buy the low-quality products when they are aware of them and not the high-quality products, while consumers who have a lower preference for quality will not buy the high-quality products even when they are only aware of them. That is, more consumers end up buying the low-quality product than in a costless advertising situation for the same product line configuration. This gives a greater incentive to advertise the lower-quality products more.

In fact, if the firm wants to price discriminate between the consumers who consider both products, it is optimal to advertise so that one has a greater proportion of sales of the lower-quality product than when advertising is costless. In comparison to the case in which advertising is costless, the high-quality product sells for a lower price while the lower-quality product is offered at a higher price and higher quality.

The results of this paper are more applicable in industries in which firms communicating to consumers about product attributes is an important issue and perfect targeting of advertising is not possible. One idea that has been previously mentioned to explain the limited number of products offered by firms is the existence of fixed costs per product sold (see, e.g., Dixit and Stiglitz 1977, Iyer and Seetharaman 2001).[6] Typically these fixed costs have been interpreted as production or setup costs. The point of this paper is that because of imperfect targeting, communication about the product's existence through advertising can itself generate specific costs per product through the relative convexity of advertising costs, which yields an optimal number of products to be advertised.

Product line design leading to consumer confusion can also have important implications on distribution channels, as discussed in Bergen et al. (1996). That paper shows empirically that consumer confusion about product assortments can lead to more retail differentiation (because the retailers adopt different subsets of the product line), less retail price competition, more retail service, and enhanced distribution. This argument may lead the manufacturer to create more confusion in the product line, in order to give greater incentives for retailers to advertise.

Also related to this work is the idea of economies of scope in building and using reputations (Wernerfelt 1988). The idea is that larger umbrella brands have more to lose (loss of reputation in more products) from having one poor-quality product. This could

[4] This can also have particular effects in vertical product lines (Randall et al. 1998).

[5] This confusion about the benefits could possibly lead to consumers buying multiple products (e.g., Guo 2003, Dubé 2004).

[6] See also Karmarkar and Pitbladdo (1994) for the role of common fixed costs for several products.

potentially be a force toward having a smaller number of brands. This paper does not look at asymmetric information with respect to product quality; it instead focuses on the communication to create consideration of the products by consumers.

The rest of the paper is organized as follows. The next section presents a parsimonious model that illustrates the interaction between product line decisions and advertising. Section 3 considers the problem of the optimal number of products. Section 4 addresses the issue of price discrimination and advertising. Section 5 concludes and discusses how firms may find ways to overcome these communication constraints.

2. Product Line Decisions and Advertising

2.1. Preliminaries

As an illustration of the issue of product line decisions with advertising, consider a market where consumers can be of two types. One type of consumer values product design A at v and values product design B at $v-t$. The other type of consumer values product design B at v and values product design A at $v-t$. Suppose that in the population of consumers, these two types are equally represented, each with mass 1. Furthermore, assume $2(v-t) > v$ so that if a firm were selling only one product and advertising were costless, the firm would prefer to sell to both types of consumers.

The firm can choose to sell either product design A or B or both. The marginal cost of production for each product is zero. The firm cannot sell any other product designs.

For the consumers to consider purchasing one particular product design, they have to receive advertising about it. In what follows, we describe the effect of advertising in terms of consumer consideration about the products, which should be interpreted as a state such that the consumer will buy the product if the price is sufficiently low. This requires not only awareness about the product but also being convinced that the product will meet the consumer preferences.

The cost of making ϕ_A percent consumers consider product A and ϕ_B percent consumers consider product B is $C(\phi_A, \phi_B)$, which is assumed symmetric, that is, $C(\phi_A, \phi_B) = C(\phi_B, \phi_A)$, $\forall \phi_A, \phi_B$. Note that we have $0 \le \phi_A, \phi_B \le 1$. Also assume that the probability of considering buying product A and that of considering buying product B are independent.[7] Consider an advertising cost function such that there is never full consideration at the optimal communication strategy, $\phi_i < 1$, for all i.[8]

If the firm chooses $\phi_i = 0$, we interpret it as the firm not selling product i. The question to be answered is then whether the firm chooses to offer the two products or just one of them.

2.2. Main Analysis

If the firm chooses $p = v$ for both products, its profit is $\pi(v) = (\phi_A + \phi_B)v - C(\phi_A, \phi_B)$. The first-order condition is then $\phi_i v = C_i$ for $i = A, B$.[9] The second-order conditions are just that $C_{ii} \ge 0$ and $C_{ii}^2 \ge C_{AB}^2$. For the remainder of the paper assume that $C(\cdot)$ satisfies these conditions. They just mean that there are decreasing returns to advertising expenditures, and that the cross effects are not larger than the second-order own effects.

Consider now a firm charging $p = v - t$ for both products. The profit is then $\pi(v-t) = 2(v-t) \cdot (\phi_A + \phi_B - \phi_A\phi_B) - C(\phi_A, \phi_B)$. The first-order condition for product A in an interior maximum is $2(v-t)(1-\phi_B) = C_A$. The first-order condition for product B is similar. The second-order conditions for an interior maximum are $C_{ii} \ge 0$ for $i = A, B$, and $C_{AA}C_{BB} - [2(v-t) + C_{AB}]^2 \ge 0$. In particular, note that this last condition can be violated while the second-order conditions for the price $p = v$ are still satisfied. This is because for $p = v - t$, when advertising more than one brand the firm does not gain all the additional consideration of that product, because some of the new consumers considering any product were already considering buying the other product. This overlap of the advertising expenditures may then cause the firm to choose not to advertise one of the products. This result is stated in the following proposition.

Proposition 1. *Suppose that $C_A(1, \phi_B) > 2(v-t)$, $\forall \phi_B$ and that $p = v - t$ for both products. Then if*

$$C_{AA}C_{BB} - [2(v-t) + C_{AB}]^2 < 0, \quad \forall \phi_A, \phi_B, \qquad (1)$$

the firm chooses to advertise only one of the two products.

This result only states that if $p = v - t$ for both products, then one can have the firm advertising only one product. However, it is important to know under which conditions advertising one product is

[7] The case of nonindependence is briefly discussed below in §5. As argued there, the likely case of positive correlation reinforces the conclusion of a limited number of products.

[8] The size of the market could potentially affect the advertising cost function, and therefore have an effect on the optimal advertising strategy. This issue is not explored in this paper. Note also that, as an alternative to receiving information through advertising, consumers could potentially learn about the available products through active search (Kuksov 2004). This possibility is not considered in this paper.

[9] The term C_i represents the partial derivative of C with respect to ϕ_i. Similarly, C_{ij} represents the second derivative of C with respect to ϕ_i and ϕ_j.

better than advertising two products and charging any price.

This is presented in the following proposition.

Proposition 2. *Suppose that $C_A(1, \phi_B) > 2(v - t)$, $\forall \phi_B$ and that condition (1) is satisfied. Then, if consumer heterogeneity is sufficiently small (small t), the firm chooses to advertise only one of the products.*

This result shows that if there is a sufficiently small consumer heterogeneity, we will have just one product being advertised. The intuition is that if consumer heterogeneity is small (t is small), then, by charging a price of $v-t$ for both products instead of a price of v when advertising both products, the firm gets greater demand with little sacrifice on the revenue per unit sold. Then by Proposition 1 we know that advertising only one product is better than advertising both products.

Note also that the greater the economies of scope in advertising, $C_{AB} < 0$, the less likely we have condition (1) being satisfied. This is intuitive in the sense that greater economies of scope make advertising both products less costly. In the limit, when advertising two products is as costly as advertising just one product, we have that the optimum is obviously advertising two products.

Note that this result can be seen as showing that consumer confusion is endogenous to the product line design choice of the firm. If the firm chooses to advertise two products, it may create more consumer confusion, which then requires more advertising expenditures to achieve the same level of consideration for each product. Because of these additional communication costs, the firm may end up offering only one product and therefore reducing consumer confusion.

This result also illustrates that the location of the product line can be affected by the communications about the products. In some conditions the firm decides to offer only one product, and locates the product for the preference of only one consumer segment. If we also allowed the firm to position a product between the two products targeted for each segment, we could also have that the firm, when only advertising one product, would position its product between the two products targeted to each segment and would be able to charge a price greater than $v-t$.

If consumer segments are not symmetric, it could be that the firm chooses to advertise only the product for the largest segment at price v. Note also that with a slight ability to target advertising, a firm may choose to advertise only the product that is most appreciated by the largest consumer segment.

For the general case of additively symmetric separable advertising cost function, $C(\phi_A, \phi_B) = c(\phi_A) + c(\phi_B)$, note that condition (1) reduces to $c'(\phi) > (1 - \phi)c''(\phi)$. That is, the advertising cost function for each product cannot be too convex. Note that the case where the advertising cost function is constructed by using independent equal messages and only one exposure is enough for consideration (e.g., as in Butters 1977), we have $c(\phi) = a\log(1/(1-\phi))$, which has the property that $c'(\phi) = (1-\phi)c''(\phi)$; that is, the firm is indifferent about advertising one or two products. In this sense, in order to be optimal for the firm to advertise only one product, one needs the advertising cost function to be less convex than the cost function for the case of independent but equal messages.

One interesting possibility is that the cost of making a certain percentage ϕ of consumers consider the product is the same whether the firm advertises two products equally or advertises only one product; that is, $C(\phi, 0) = C(1-\sqrt{1-\phi}, 1-\sqrt{1-\phi})$, because if both products are advertised at $\phi_A = \phi_B$, the number of consumers aware of any product is $1-(1-\phi_A)^2$. Note, however, that this does not necessarily need to hold because of potential economies of scope in advertising both products, or because consumers can become more likely to consider a product after a later advertising exposure than after an earlier one, given that they are not considering the product until then. For example, denote by $\beta(n)$ the probability to begin considering a product after the nth advertising exposure given that a consumer was not considering the product at the $(n-1)$th exposure. Then, the probability of one consumer considering the product after n exposures is $1 - \prod_{j=1}^{n}(1 - \beta(j))$. Note that if $\beta(n)$ is independent of n, then it is similar to the case of independent equal messages, with one exposure being enough for consideration (Butters 1977). If $\beta(n)$ is increasing in n, then we get an advertising cost function that is less convex than the case of independent equal messages, and the firm may gain more from advertising only one product. Note that having $\beta(n)$ increasing in n could be derived from a threshold advertising effect, where consumers need more than a certain number of advertising exposures to start considering one product.

2.3. Confusion in Advertising

One important issue in advertising a product line is the possible consumer confusion about the attributes of the different products in the line. One way this effect could be modeled is through the idea that it is more difficult to communicate the attributes of one product if the firm is also communicating the attributes of other products. Denote by A_i the amount spent advertising product i. Then we could define $\phi_i(A_i, \gamma A_j)$, with $i \neq j$, as the fraction of consumers who understand the attributes of product i given that the firm spent A_i advertising product i and A_j advertising product j. The parameter γ, greater or equal to zero, represents the degree of confusion as seen below. Furthermore, if the firm did not advertise one product, then no consumer would understand the attributes of that product, $\phi_i(0, \gamma A_j) = 0 \, \forall A_j$, and more advertising for one product would result in more consumers understanding the attributes of that product $\partial\phi_i/\partial A_i > 0$.

Consumer confusion about the advertising messages would then be defined by $\partial\phi_i/\partial A_j = \gamma(\partial\phi_i/\partial(\gamma A_j)) < 0$, with $\gamma \geq 0$ and $\partial\phi_i/\partial(\gamma A_j)$ bounded away from zero.[10] Note then that a greater γ is associated with greater consumer confusion, because for the same advertising expenditures, A_i and A_j, the fraction of consumers who become informed about the attributes of product i is smaller. Note also that greater advertising of A_j, for $\gamma > 0$, leads to a smaller fraction of consumers being informed about product i, greater consumer confusion, and this effect increases as the index of consumer confusion γ increases.

In terms of the notation above, note that if we invert the system of equations $\phi_A = \phi_A(A_A, \gamma A_B)$, $\phi_B = \phi_B(A_B, \gamma A_A)$, we get the advertising spending for each product that yields considerations ϕ_A and ϕ_B, $A_A(\phi_A, \phi_B)$ and $A_B(\phi_A, \phi_B)$, and then we have $C(\phi_A, \phi_B) = A_A(\phi_A, \phi_B) + A_B(\phi_A, \phi_B)$.

One can then obtain the following proposition:

Proposition 3. *If consumer confusion is sufficiently large* (*large* γ) *and if consumer heterogeneity is sufficiently small* (*small* t), *then the firm chooses to advertise only one of the products.*

The intuition is that for a large index of consumer confusion, a firm, when advertising both products, has to advertise each product very intensely to compensate for the confusion created by the advertising of the other product. Then the firm may be better off advertising only one product and not creating consumer confusion. Even with an additively separable advertising cost function, one could interpret the nonexistence of economies of scope on the advertising cost function as the extra cost of consumer confusion about the advertising messages.

2.4. Targeting of Advertising

If the firm has the ability to target its advertising to different consumer segments, then it may reduce some of the overlap of the considerations for both products and may possibly charge higher prices. To see this, consider an index of targeting ability τ such that if the firm gets $2\phi_i$ consumers considering product i, a fraction τ is fully targeted and is composed of the consumers in segment i, and a fraction $(1-\tau)$ is composed of consumers of segments i and j in equal proportions.[11] That is, of the $2\phi_i$ consumers considering product i, $2\phi_i\tau + (1-\tau)\phi_i$ consumers are from segment i and $(1-\tau)\phi_i$ consumers are from segment j. When there is no target ability, $\tau = 0$, we get back to the case considered above. When there is maximum targeting ability, $\tau = 1$, all adverting of product i goes to segment i.

If the firm charges the price $v - t$ for either of the two products, the profit function reduces to $2(v-t)\cdot[\phi_A + \phi_B - (1-\tau)\phi_A\phi_B] - C(\phi_A, \phi_B)$. The equivalent to condition (1) is then $C_{AA}C_{BB} - [2(v-t)(1-\tau) + C_{AB}]^2 < 0$. That is, if the targeting ability is sufficiently large, condition (1) is not satisfied and the firm is better off advertising both products (instead of only one product).

Consider now the effect of targeting ability on prices. If the firm charges price v for both products, it gets a profit of $v[2\tau\phi_A + 2\tau\phi_B + (1-\tau)(\phi_A + \phi_B)] - C(\phi_A, \phi_B)$. Note that when $\tau = 1$, the demand is the same as when the firm charges any other prices, and therefore this profit is greater than when the firm charges any other prices. By continuity we then have that if the targeting ability is sufficiently large, the firm decides to charge higher prices, while advertising both products. That is, greater targeting ability allows a firm to charge higher prices when both products are advertised.

2.5. An Example

As an illustration of the results above, consider the example of the quadratic and additively separable advertising costs functions, $C(\phi_A, \phi_B) = a(\phi_A + \phi_B) + (b/2)(\phi_A^2 + \phi_B^2)$ with $v > a$ and $b > 2(v-t) - a$.[12] For $p = v$ for both products we would then have $\phi_A = \phi_B = (v-a)/b$ and a profit of $\pi(v) = (v-a)^2/b$. Given the assumptions on $C(\phi_A, \phi_B)$, note that we have $0 < \phi_A = \phi_B < 1$.

Given this cost function, condition (1) reduces to $b < 2(v-t)$. Under this condition, if the firm charges $p = v - t$ for both products, it will only advertise one product, say product A, and advertise it at $\phi_A = (2(v-t) - a)/b$, and have a profit $\pi(v-t) = [2(v-t) - a]^2/2b$. If condition (1) is not satisfied, we have $\phi_A = \phi_B = (2(v-t) - a)/(2(v-t) - b)$ and a profit of $\pi(v-t) = [2(v-t) - a]^2/(2(v-t) + b)$.

Under condition (1) one obtains that advertising only one product at $p = v - t$ is better than advertising

[10] Assume that there are decreasing effects of advertising, $\partial^2\phi_i/\partial A_i^2 < 0$ and $\partial^2\phi_i/\partial(\gamma A_j)^2 < 0$. Alternatively, one could have a demand side characterization of confusion having to do with an imperfection in the fit signals received by the consumers. This alternative is left for future research.

[11] One could think of alternative formulations of targeting ability, for which advertising costs could be functions of the size of the consumer segments being targeted.

[12] The condition $b > 2(v-t) - a$ is not crucial and only guarantees that when charging $p = v - t$ and advertising only one product, the consideration of that product is less than 100%.

both products at $p = v$ or $p = v - t$ if and only if $4(v-t)^2 - 2v^2 + 4ta - a^2 > 0$. The worst case for this last condition not to be satisfied is $a = v$, which reduces the condition to $(v-2t)^2 > 0$, which is always satisfied. That is, under condition (1), $b < 2(v-t)$, it is always better to advertise only one product than to advertise both products at either $p = v$ or $p = v - t$. If condition (1) is violated, then comparing $\pi(v-t)$ with $\pi(v)$ one finds that $\pi(v-t)$ is greater if t is sufficiently small. For $t = 0$ we have $\pi(v-t) = (2v-a)^2/(2v+b) > \pi(v) = (v-a)^2/b$, while for $t = v/2$ we have $\pi(v-t) = (v-a)^2/(v+b) < \pi(v) = (v-a)^2/b$, and $\pi(v-t)$ is decreasing in t.

The intuition for the effect of condition (1) on the number of products to advertise is that if the marginal costs of advertising increase very quickly (high b), then the firm prefers to advertise different products in a small amount. On the other hand, if the marginal costs of advertising do not increase very quickly (small b), then the firm prefers to advertise only one product and does not "waste" advertising expenditures by making consumers consider both products.

2.6. Generic and Specialized Products

Under some conditions it may be endogenously optimal for a firm to advertise a generic product priced for all the segments and a specialized product priced only for the segment that most appreciates that product. Note that this can happen even if the two products and consumer segments are ex ante symmetric. To see this, consider, for example, the case in which the firm prices product A, the specialized product, at $p_A = v$ and product B, the generic product, at $p_B = v - t$. The specialized product is only bought by the consumer segment that most likes it, while the generic product is bought by either of the consumer segments. The profit for these prices is $\pi(v, v-t) = v\phi_A + (v-t)[\phi_B + \phi_B(1-\phi_A)] - a(\phi_A + \phi_B) - (b/2)\cdot(\phi_A^2 + \phi_B^2)$. From the first-order conditions for an interior optimum, one obtains

$$\hat{\phi}_A = \frac{b(v-a) - (v-t)[2(v-t)-a]}{b^2 - (v-t)^2}, \tag{2}$$

$$\hat{\phi}_B = \frac{b[2(v-t)-a] - (v-t)(v-a)}{b^2 - (v-t)^2}, \tag{3}$$

and the second-order conditions reduce to $b > v - t$. Note that under the second-order conditions we have that the generic product is advertised more than the specialized product, $\hat{\phi}_A < \hat{\phi}_B$, because it can attract greater demand. To have an interior optimum we need the second-order conditions to be satisfied and $b(v-a) - (v-t)[2(v-t)-a] > 0$. Note that there are parameter values under which an interior optimum exists in this case (advertising both products) while the interior optimum does not exist (advertising only one product) under $p_A = p_B = v - t$, which occurs under condition (1).

The optimal profit under ($p_A = v$, $p_B = v - t$) can then be obtained:

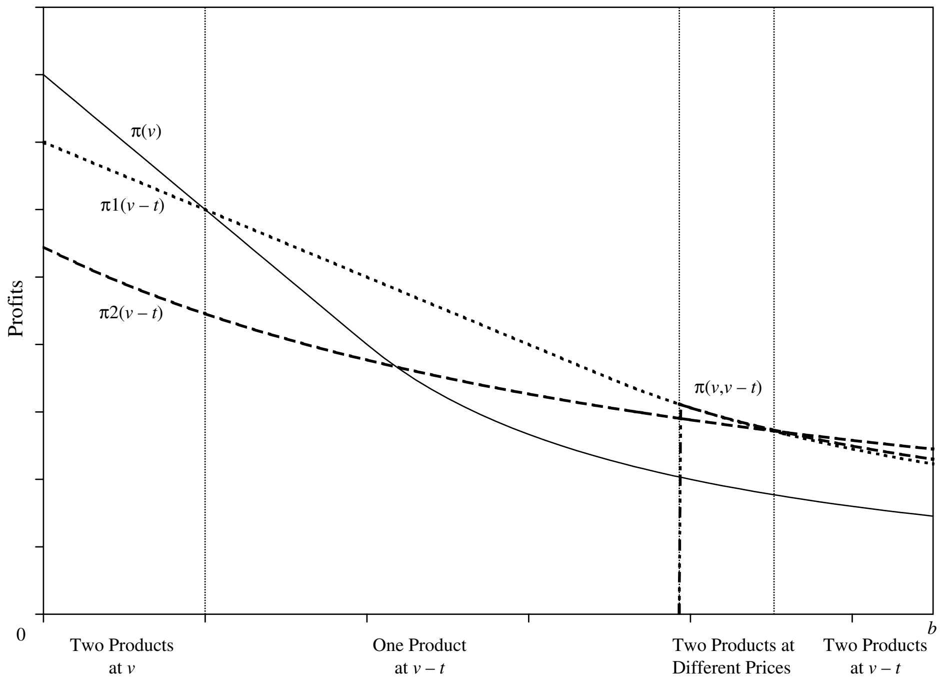
$$\pi(v, v-t) = \frac{b\{(v-a)^2 + [2(v-t)-a]^2\} - 2(v-a)(v-t)[2(v-t)-a]}{2[b^2 - (v-t)^2]}.$$

We can check that for t close to zero this profit is lower than $\pi(v-t)$, as stated in Proposition 2. Consider now that condition (1) is satisfied and that $(v-a)$ is close to $\sqrt{2}[2(v-t)-a]$ such that the profit $\pi(v) = (v-a)^2/b$ is close to the profit $\pi(v-t) = [2(v-t)-a]^2/2b$. Then if b is close to $v - t$, we have that the profit $\pi(v, v-t)$ is greater than both $\pi(v)$ and $\pi(v-t)$; that is, advertising a generic and a specialized product is the optimal strategy. The intuition is that if the firm advertised both products for both consumer segments (at price $v - t$), there would be too much overlap of consideration of both products. The firm can reduce the advertising of one of the products, therefore reducing the overlap of considerations, but charging a higher price for that product because there is less demand to loose from charging a higher price. From an ex ante symmetric situation one can then obtain that the firm advertises both a generic (for all consumer segments) and a specialized product (for only one of the segments).

Figure 1 presents the profits of the different possible strategies as a function of b (the second derivative of the quadratic advertising cost functions) for the case $v = 3$, $a = 1$, $t = 0.75$. Let us consider the optimal strategies when b increases. For b small the optimal strategy is to advertise both products at $p = v$ because advertising is not very costly (and advertising both products at 100% consideration). When b increases, it is better to avoid the loss in the overlap of advertising both products, and only one product is advertised at $p = v - t$. When b increases further, optimal advertising decreases, the overlap problem becomes less serious, and the firm gains from advertising at a low level an additional product at $p = v$ while continuing to advertise the other product at $p = v - t$, the generic and specialized product strategy. Finally, for very high b, advertising decreases further, the overlap problem becomes even less serious, and the firm decides to advertise both products at $p = v - t$.

In Figure 2 profits for the different strategies are presented as a function of consumer heterogeneity, t, for $v = 3$, $a = 1$, $b = 4.5$. Consider the optimal strategies when t increases. For small consumer heterogeneity (small t), the lower price $v - t$ is close to v and the firm advertises only one product at $p = v - t$. When consumer heterogeneity increases, the firm advertises less because the revenue from selling

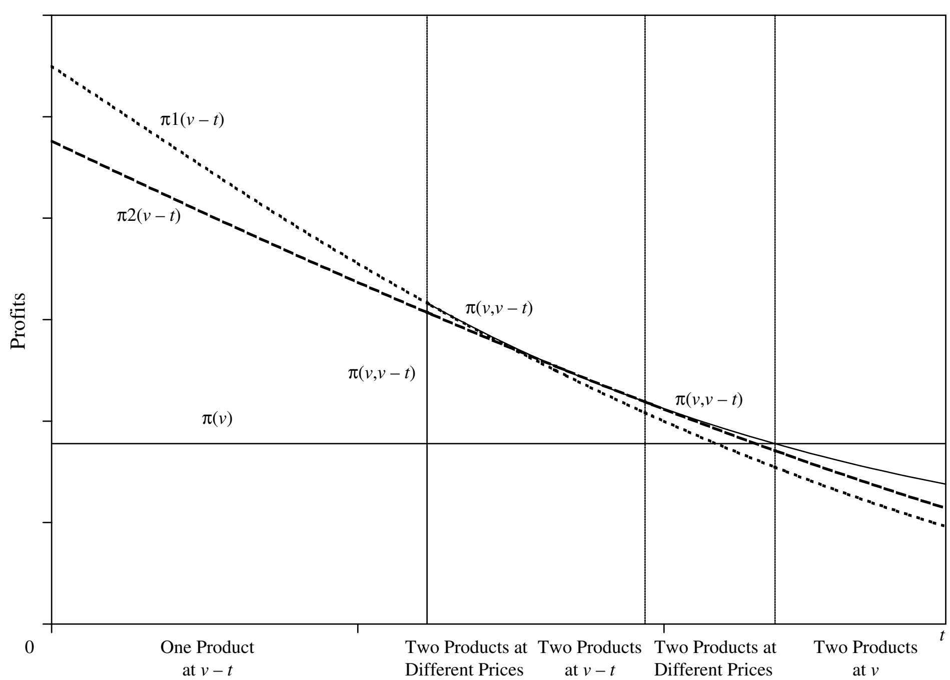
Figure 1 **Profits for Different Alternatives and Optimal Strategy for Each b (Second Derivative of Cost Function)**



Note. $\pi j(v-t)$ represents the profit with j products with $p = v - t$.

a product is lower, and at some point the firm decides to add another product advertised at $p = v$, the generic and specialized product strategy. When consumer heterogeneity increases further, advertising is reduced so much that the firm decides to sell to more consumers by advertising both products at $p = v - t$. Finally, when consumer heterogeneity increases even further, the effect of the reduced price of higher t is so large that the firm decides to advertise both products at $p = v$.

Figure 2 **Profits for Different Alternatives and Optimal Strategy for Each t (Consumer Heterogeneity)**



Note. $\pi j(v-t)$ represents the profit with j products with $p = v - t$.

3. The Optimal Number of Products

Consider now the question of the optimal number of products to advertise. Suppose that the consumer preferences are uniformly distributed in a circle of length one.[13] If a consumer buys a product located at a distance x from where the consumer preferences are located, the consumer gets a utility of $v - tx - p$, where p is price and t is a measure of consumer heterogeneity. Assume the mass of consumers to be one.

If advertising were costless, the firm would choose to offer (advertise) a continuum of products, one matching each consumer preference.[14] Because each consumer would have a perfect fit, the firm could charge a price $p = v$. The profit being generated would be $\pi = v$.

With costly advertising (and imperfect targeting) we may expect firms to decide to advertise a finite number of products. Suppose that the firm advertises n products, $i = 1, \ldots, n$, a product i advertised to obtain consideration ϕ_i. Assuming additive separability of advertising costs the cost of advertising the n products is then $C(\phi_1, \ldots, \phi_n) = \sum_{i=1}^{n} c(\phi_i)$. Assume $c''() > 0$, and $c'(1)$ high enough such that no product is advertised with consideration one.

Consider the pricing decision for sufficiently low consumer heterogeneity.

Proposition 4. *Given that n products are being advertised, if the consumer heterogeneity t is sufficiently small, the optimal price of each of the n products being advertised is equal to $v - t/2$.*

If the consumer heterogeneity is sufficiently small, the firm prefers to set prices such that all of its products are attractive to any consumer in the market, that is, price at $v - t/2$. A higher price would lead to some loss in demand, but without much increase in revenue per unit because t is small. As shown in the proof of the proposition, a sufficient condition for t small can be written as $t \leq (v - t/2)(1 - \bar{\phi})^{n-1}$, where $\bar{\phi}$ is defined by $v = c'(\bar{\phi})$. When consumer heterogeneity is not too small (larger t), the firm may set prices such that not all consumers receiving an advertising message are willing to buy that product. That is, and as shown in the previous section, the firm may choose to offer more specialized products. Furthermore, because the potential waste of the overlap of advertising messages is reduced by advertising some products less, it may be that, as illustrated in the previous section, the firm chooses to advertise and price a priori equal potential products differently.

To focus on the question of the number of products, from now on let us assume sufficiently low consumer heterogeneity.

Consider now the total demand for the n products being advertised. The mass of consumers not considering any product is $\prod_{i=1}^{n}(1 - \phi_i)$. The total demand for the n products is $1 - \prod_{i=1}^{n}(1 - \phi_i)$. The total profit from advertising n products is then $\pi(\phi_1, \ldots, \phi_n) = (v - t/2)[1 - \prod_{i=1}^{n}(1 - \phi_i)] - \sum_{i=1}^{n} c(\phi_i)$. Maximizing $\pi(\phi_1, \ldots, \phi_n)$ with respect to ϕ_i one obtains $(v - t/2) \cdot \prod_{j \neq i}(1 - \phi_j) = c'(\phi_i)$. If $c''()$ is larger than $v - t/2$ the second-order conditions are satisfied and the optimum is symmetric with $\phi_1 = \phi_2 = \cdots = \phi_n = \phi$. The objective function becomes then $\pi(n, \phi) = (v - t/2) \cdot [1 - (1 - \phi)^n] - nc(\phi)$. From the first-order condition with respect to n one obtains

$$(1 - \phi)^n = \frac{c(\phi)}{(v - t/2)\log(1/(1 - \phi))} \quad \text{and}$$

$$n = \log\left(\frac{c(\phi)}{(v - t/2)\log(1/(1 - \phi))}\right)\frac{1}{\log(1 - \phi)}.$$

Substituting into $\pi(n, \phi)$ one obtains that the problem becomes $\max_z (v - t/2)(1 - z + z\log z)$, where $z \equiv c(\phi)/[(v - t/2)\log(1/(1 - \phi))]$. Because $0 \leq z \leq 1$ we know that the objective function above is decreasing in z. Then the optimum ϕ can be obtained by $\min_\phi c(\phi)/\log(1/(1 - \phi))$. This can be seen as finding the efficient advertising amount, given that there is overlap of the advertising messages. From these conditions one can obtain that the amount of advertising for each product is independent of the parameters of consumer heterogeneity, t, and the maximum valuation for the product, v.

Proposition 5. *In an interior optimum the amount of advertising for each product is independent of t and v and is given by*

$$(1 - \phi)c'(\phi)\log(1 - \phi) + c(\phi) = 0. \qquad (4)$$

This means that when we have an interior optimum all the adjustment to changes in the consumer heterogeneity or in the product valuation is done through the number of products being advertised. Note now that the case where the advertising cost function is constructed by using independent equal messages with one exposure being enough for consideration (e.g., as in Butters 1977) we have $c(\phi) = a\log(1/(1 - \phi))$ for which condition (4) is satisfied for any ϕ. For each ϕ there is an optimal number of products, n. Note that for this advertising cost function when we make ϕ tend to zero, the optimal number of products goes to infinity while the total expenditures in advertising remains constant.[15]

[13] For the case of competition with each firm carrying one product, see, for example, Grossman and Shapiro (1984) and Soberman (2001).

[14] Note that a continuum of products is not possible if there is a fixed production cost per product.

[15] Proposition 4 continues to hold when the number of products goes to infinity. This is immediate for $t = 0$, and the result can also be obtained for $t > 0$ but small.

Note that by perturbing the advertising cost function away from this one, one gets that (4) is not satisfied for all ϕ. There are some $c(\phi)$ for which condition (4) is only satisfied at $\phi = 0$, which means that at the optimum there is an infinite number of products being advertised. A sufficient condition for ϕ to be strictly positive and the optimal number of products to be finite is $c'(0) > c''(0)$ because the limit when ϕ goes to zero of the first-order condition is negative under this condition,

$$\lim_{\phi\to 0} \frac{c'(\phi)\log(1/(1-\phi)) - c(\phi)/(1-\phi)}{\log(1/(1-\phi))^2} = \frac{c''(0) - c'(0)}{2}.$$

For example, this condition is satisfied for $c(\phi) = a\phi + b/(1-\phi)$ and $a > b$. For this advertising cost function one can also obtain that the optimal advertising per product is positive and with consideration less than 100%. Finally, note that the optimal number of products is increasing in the price $v - t/2$ and decreasing in the optimal ϕ. When the revenue per unit increases, the firm wants to advertise a greater number of products. When the optimal amount of advertising per product increases, the firm can reach a similar number of consumers by advertising a smaller number of products. Because the optimal amount of advertising per product is decreasing in the convexity of the advertising cost function, the optimal number of products decreases with lower convexity of the advertising cost function.

4. Price Discrimination and Advertising

Another important issue to consider is that when firms sell vertically differentiated products they may want to change the product design because of advertising considerations. Moreover, in such a setting it may be important to understand which factors determine which products are advertised more intensely. Suppose that consumers have net utility, $U = V(\theta, q) - p$, where θ determines the relative preference for quality, q represents quality (or quantity), and p is price. Assume $V_q > 0$, consumers like products with greater quality,[16] $V(\theta, 0) = 0$, the gross utility of a product of zero quality is zero, $V_{\theta q} > 0$, consumers with greater θ have a greater marginal utility for quality, and $V_{qq} < 0$, consumers have a decreasing marginal utility for quality. Suppose that there are two types of consumers, θ_1 and θ_2, with $\theta_2 > \theta_1$, and with proportions N_1 and N_2, respectively ($N_1 + N_2 = 1$). Each consumer has private information on θ but the firm can offer different products for consumers to self-select (Mussa and Rosen 1978, Moorthy 1984).[17] Denote product i as the product targeted at the θ_i consumers, with $i = 1, 2$. Producing one unit of product of quality q has a cost of gq.[18]

Suppose also that consumers are initially not considering buying either of the products but can be made to consider either through advertising. Denote the chosen consideration for product i as ϕ_i, which can be achieved at cost $c(\phi_i)$. Suppose for now that the θ_1 consumers never buy product 2, even when only considering that product, and that the firm wants the θ_2 consumers to buy product 2 when they are aware of both products.[19]

If consumers have both products in their consideration set, they will buy the product targeted to their type. This yields the standard incentive compatibility (IC) and individual rationality (IR) constraints (presented in the appendix). The proportion of consumers that has both products in their consideration set is $\phi_1\phi_2$. Note that any type of consumer considering only product 1 will buy that product because for type 1 this is guaranteed by its IR constraint, and for type 2 this is implied by the IR constraint for type 1 and $V_\theta > 0$. The proportion of consumers considering only product 1 is $\phi_1(1-\phi_2)$.

By assumption for this case, only the θ_2 consumers buy if consumers consider only product 2. This means that $V(\theta_1, q_2) - p_2 < 0$. The proportion of consumers who consider only product 2 is $\phi_2(1-\phi_1)$. Because of the IC for type 2, the IR for type 1, and $V_\theta > 0$, we know that the IR for type 2 is not binding. Furthermore, because we assumed $V(\theta_1, q_2) - p_2 < 0$, we have that the IC for type 1 is not binding. We will have the IR for type 1 binding, because otherwise the firm would gain from increasing p_1 and we will have the IC for type 2 binding because otherwise the firm would gain from increasing p_2. These binding constraints are exactly as in the traditional second-degree price discrimination problem with costless advertising (Mussa and Rosen 1978). Substituting for the binding constraints we have $p_1 = V(\theta_1, q_1)$ and $p_2 = V(\theta_2, q_2) - V(\theta_2, q_1) + V(\theta_1, q_1)$. One can then use these expressions to rewrite the problem of the firm (presented in appendix). Note that because the firm is choosing both advertising and product qualities, it

[16] The term F_x means partial derivative of F with respect to x.

[17] See also Villas-Boas and Schmidt-Mohr (1999) and Desai (2001) for the case of selling a product line under competition, or Villas-Boas (1998) for the case of a distribution channel. In a different setting, Shugan (1989) also considers the question of the optimal product line in an oligopoly where firms are vertically differentiated.

[18] Similar results can be obtained by assuming that the unit cost of a product is convex in quality.

[19] The cases where θ_1 consumers may buy product 2 and where θ_2 consumers only buy product 2 when only aware of that product are discussed below.

does not matter whether these are chosen sequentially or simultaneously. The first-order conditions with respect to q_1 and q_2 then yield

$$V_q(\theta_2, q_2) = g, \tag{5}$$

$$\frac{\phi_1}{\phi_2}[N_1 + (1-\alpha_2)N_2][V_q(\theta_1, q_1) - g] - N_2[V_q(\theta_2, q_1) - V_q(\theta_1, q_1)] = 0. \tag{6}$$

Note that the optimal q_2 is the efficient one, such that the marginal utility is equal to the marginal cost of quality. Furthermore, by (6), the optimal quality for the lower-quality product, q_1, is distorted downward and is less than the efficient one. These results are as in the traditional case with costless advertising (Mussa and Rosen 1978). More interestingly, from (6), the distortion on q_1 is greater the more the firm advertises the high-quality product and the less it advertises the lower-quality product. Therefore, note that when one increases the advertising expenditures on the lower-quality product or decreases the advertising expenditures on the higher-quality product, the margin of the lower-quality product, $V(\theta_1, q_1) - gq_1$, is increased, and the margin of the higher-quality product, $V(\theta_2, q_2) - V(\theta_2, q_1) + V(\theta_1, q_1) - gq_2$, is decreased.

Consider now the first-order conditions with respect to the advertising intensities of both products, ϕ_1 and ϕ_2. To simplify notation denote the margins of both products as, respectively, $m_1 \equiv V(\theta_1, q_1) - gq_1$ and $m_2 \equiv V(\theta_2, q_2) - V(\theta_2, q_1) + V(\theta_1, q_1) - gq_2$. These first-order conditions are

$$[N_1 + (1-\phi_2)N_2]m_1 = c'(\phi_1), \tag{7}$$

$$-N_2\phi_1 m_1 + N_2 m_2 = c'(\phi_2). \tag{8}$$

The second-order conditions of this problem given m_1 and m_2 are satisfied if $c''(\phi) > m_1 N_2$, $\forall \phi$, which is assumed. If the function $c()$ is not too convex, low $c''()$, then the second-order conditions will not be satisfied and we end up in the same case as in Proposition 2 above, with the firm choosing to advertise only one product. If there is little consumer heterogeneity, θ_1 close to θ_2, that product is priced to be attractive to both consumer segments.

From (7) one can obtain $\phi_2 = (m_1 - c'(\phi_1))/m_1 N_2$, which we can substitute in (8) to obtain $N_2 m_2 - N_2 m_1 \phi_1 = c'((m_1 - c'(\phi_1))/m_1 N_2)$. Totally differentiating this equation with respect to ϕ_1 and each m_i one obtains that advertising the lower-quality product, ϕ_1, is increasing in the margin of that product and decreasing in the margin of the higher-quality product. Similarly, one can obtain that advertising the higher-quality product is increasing in the margin of the higher-quality product and decreasing in the margin of the lower-quality product.

Subtracting (7) from (8) one obtains $c'(\phi_1) - c'(\phi_2) - m_1 N_2(\phi_1 - \phi_2) = m_1 - N_2 m_2$, which can be written as $(\phi_1 - \phi_2)[c''(\phi_{12}) - m_1 N_2] = m_1 - N_2 m_2$, where ϕ_{12} is a number between ϕ_1 and ϕ_2. Noting that by the assumption on the second-order conditions, $c''(\phi_{12}) > m_1 N_2$, we then have the following results on which product should be advertised more intensely.

Proposition 6. *The lower-quality product should be advertised more intensely if and only if $m_1 > N_2 m_2$.*

This result shows that which product is advertised more depends on the size of its margin and on the size of the high type segment. Interestingly, the product with the lower margin, the lower-quality product, can be advertised more intensely. For example, if the margins are not too different, the firm chooses to advertise the lower-quality product more. The intuition for this result is that the consumers who appreciate more quality will also buy the lower-quality product when only considering that product, while the consumers who appreciate quality less do not buy the higher-quality product if they are only considering that product. That is, the low-quality product is also sold to the consumers who appreciate quality more, while the high-quality product is never sold to the consumers who appreciate quality less. This gives a greater incentive to advertise the lower-quality product. Note that the condition in the proposition will occur when $\theta_2 - \theta_1$ is small enough. Comparing the incentives to distort the lower-quality product with the case in which advertising is costless, one finds that if the preferences across consumers are not too different, then costly advertising causes the quality of the lower-quality product to be less distorted.

Proposition 7. *If $m_1 > N_2 m_2$, then costly advertising causes the quality of the lower-quality product to be less distorted from the efficient one in comparison to the case in which advertising is costless.*

This proposition then implies that, with costly advertising, if the consumer preferences are relatively similar, then the firm gets a greater margin from the lower-quality product and lowers the price of the higher-quality product in comparison to the case of costless advertising. Note that even if $m_1 < N_2 m_2$, one may still get less quality distortion under costly advertising than under costless advertising. To solve for the optimum one has to consider (5), (6), (7), and (8) together. One can then obtain comparative statics on the proportion of consumers who have a higher preference for quality and on the similarity of consumer preferences.

Proposition 8. *Consider $c''(\cdot)$ large. Then the margin and the advertising intensity on the lower-quality product, m_1 and ϕ_1 respectively, are decreasing in the proportion of consumers who have a higher preference for* *quality, N_2. Similarly, the margin and the advertising intensity on the higher-quality product, m_2 and ϕ_2 respectively, are increasing in N_2.*

This proposition gets the expected result: When the proportion of consumers who most appreciate quality increases, the firm distorts the lower-quality product more, generating a lower margin for the lower-quality product and a higher margin for the higher-quality product. At the same time the firm decreases advertising for the lower-quality product and increases advertising for the higher-quality product.

Suppose now that the firm decides to offer a product such that the θ_1 consumers may also buy product 2. This means that $V(\theta_1, q_2) - p_2 \geq 0$. It can then be seen that at the optimum $p_2 = V(\theta_1, q_2)$, which yields that the optimal q_1 and q_2 are determined by $V_q(\theta_1, q) = g$, that is, $q_1 = q_2$ and $p_1 = p_2$. This means that the firm decides to offer only one product. This will happen when the proportion of θ_2 consumers, N_2, is high and θ_1 is close to θ_2. This case is a particular case of the firm's problem above.

Finally, another possibility is that the firm chooses to offer product 2 at a price such that the θ_2 consumers only buy that product when they are aware of product 2 and unaware of product 1. In this case the firm would not need to worry about the IC constraint for type 2 and would charge $p_2 = V(\theta_2, q_2)$. The quality levels being offered would then be the efficient ones, $V_q(\theta_i, q_i) = g$, and the margins m_1 and m_2 would be at the maximum possible. Note that this can be potentially optimal if the increase in the margins has a bigger effect than the transfer of demand from the high-quality product to the low-quality product, which is $\phi_1\phi_2 N_2$. This can be the case if ϕ_1, ϕ_2, or N_2 are small, or if θ_2 is much higher than θ_1.

5. Discussion and Concluding Remarks

This paper models the firms' costs of communicating to consumers the existence of their products and the influence of these costs on the optimal design of the product line. In particular, under some general advertising cost functions a firm may choose to offer a smaller number of products than the number of consumer preferences that exist in the market. The intuition for this result is that by advertising several products a firm is going to have some consumers consider more than one of these products, while buying only one of them. This overlap of considerations creates a greater cost in advertising expenditures, which is reduced when the firm advertises a smaller number of products. The firm has to tradeoff this effect against offering less fit between different consumer preferences and the products being offered, which translates to lower prices. This tradeoff then determines the optimal number of products to be advertised or sold. Note also that if advertising messages for different products are positively correlated across consumers, then the results of a limited number of products being advertised would be strengthened, because there would be greater overlap of the consideration of different products.

There are, however, some techniques that firms can use to solve this problem of overlap of advertising effects. As noted above, better targeting may allow a firm to target each advertising message to the intended audience, therefore reducing the overlap and consumer confusion. Improvements on the profiling of the audience for different types of media allow firms to better manage the overlap and confusion of advertising, and therefore to offer an expanded product line that better fits the different consumer preferences.

Another technique that firms can use to deal with overlap and consumer confusion is to offer relatively few base models together with a range of options, thus allowing customers to tailor products to their needs. The description of the options can be communicated through a sales force after the customers understand their preferences with respect to the base models. While understanding their preferences with respect to the base models, customers are aware that they will be able to choose among more detailed options within each base model. The automobile industry is a good example of this technique at work, with dealers offering detailed information on different options.

Providing comparisons in a single advertisement may also allow firms to make customers aware of the products and their different features. Finally, the price level and the amount of advertising could also serve a signaling role to reveal the horizontal and/or vertical differentiation between products.

Note also that the preference for a product could also be a function of the products being offered in the market (e.g., Wernerfelt 1995). This is a potentially interesting issue that should be looked at in future research, as well as its potential interactions with communication strategies. In this paper this issue is only being included through consumers having or not having a preference for a certain product (whether that product is being considered by the consumers) and through the communication costs of having consumers consider a product or products.

In a setting where vertically differentiated products are offered in order to self-select different types of consumers, one obtains that because the binding incentive compatibility constraint is for consumers who value quality more not to buy the lower-quality products, then advertising costs generate less distortion of the lower-quality product. This is because consumers who value quality less buy only the lower-quality product, while consumers who value quality more have a positive surplus from buying any product. This brings the actual proportion of consumers buying the lower-quality product higher than when advertising is costless. This also yields a force for a firm to advertise its lower-quality product more.

Acknowledgments
Comments from the editor, the area editor, three anonymous reviewers, Jean-Pierre Dubé, Ganesh Iyer, Dmitri Kuksov, and Surendra Rajiv are gratefully appreciated.

Appendix

Proof of Proposition 1. Because $C_A(1, \phi_B) > 2(v-t)$, $\forall \phi_B$, we know that at the optimum $\phi_A < 1$. Similarly, because of symmetry of $C(\cdot)$ we have $\phi_B < 1$. Then, because of condition (1), we know that an interior optimum does not exist, which means that $\phi_i = 0$ for one i. Q.E.D.

Proof of Proposition 2. The case of $p = v-t$ and advertising two products is already ruled out by Proposition 1. Therefore, we now only need to worry about the cases of advertising two products with at least one of them being charged the price $p = v$. Consider first the case where both products are priced at $p = v$. Define ϕ_A^* and ϕ_B^* as the advertising levels satisfying the first-order conditions for the case where $p = v$. Then the profit for $p = v$ is $\pi(v) = v(\phi_A^* + \phi_B^*) - C(\phi_A^*, \phi_B^*)$, which for t close to zero is strictly smaller than the profit under $p = v-t$ for (ϕ_A^*, ϕ_B^*), which is $\pi(v-t) = 2(v-t)(\phi_A^* + \phi_B^* - \phi_A^*\phi_B^*) - C(\phi_A^*, \phi_B^*)$, because $2(\phi_A^* + \phi_B^* - \phi_A^*\phi_B^*) > \phi_A^* + \phi_B^*$. Now, because by Proposition 1 we know that the profit for $p = v-t$ under (ϕ_A^*, ϕ_B^*) is smaller than the optimum profit for $p = v-t$ under condition (1) (which means only advertising one product), we have that advertising only one product is better than the highest profit under $p = v$.

Consider now the case where one product is charged at $p = v$ (say product A) and the other product is charged at $p = v-t$ (say product B). Define $\hat{\phi}_A$ and $\hat{\phi}_B$ as the advertising levels satisfying the first-order conditions for this case. Then the profit for these prices is $\pi(v, v-t) = v\hat{\phi}_A + (v-t)\cdot(2\hat{\phi}_B - \hat{\phi}_A\hat{\phi}_B) - C(\hat{\phi}_A, \hat{\phi}_B)$, which for t close to zero is strictly smaller than the profit under $p_A = p_B = v-t$ for $(\hat{\phi}_A, \hat{\phi}_B)$ because $2(\hat{\phi}_A + \hat{\phi}_B - \hat{\phi}_A\hat{\phi}_B) > \hat{\phi}_A + 2\hat{\phi}_B - \hat{\phi}_A\hat{\phi}_B$. Now, using Proposition 1, we have that advertising both products and charging $p_A = v$ and $p_B = v-t$ is dominated by advertising only one product. Q.E.D.

Proof of Proposition 3. Because t is small the firm prefers to charge a price $v-t$ for the products it advertises because it gets a greater demand. If the firm only advertises one product, the profit can be written as $\max_{A_i}[2(v-t)\phi_i(A_i, 0) - A_i]$, which is independent of γ. If the firm advertises both products, the profit is $\max_{A_A, A_B}[2(v-t)[\phi_A(A_A, \gamma A_B) + \phi_B(A_B, \gamma A_A) - \phi_A(A_A, \gamma A_B)\phi_B(A_B, \gamma A_A)] - A_A - A_B]$. Using the envelope theorem, we get that the derivative of this profit with respect to γ has the same sign as $(1-\phi_B)(\partial\phi_A/\partial(\gamma A_B))A_B + (1-\phi_A)\cdot(\partial\phi_B/\partial(\gamma A_A))A_A < 0$. Note that because $\partial\phi_i/\partial(\gamma A_j)$ is assumed bounded away from zero, as long as either A_A or A_B is not close to zero, the derivative of the profit is also bounded away from zero. Then, if either A_A or A_B is not close to zero, if γ is sufficiently large, the profit of advertising only one product is greater than the profit of advertising both products. Finally, if both A_A and A_B go to zero when γ increases, then the profit of advertising both products goes to zero, which is less than the profit of advertising only one product. Q.E.D.

Proof of Proposition 4. Consider the price for one product i, p_i. If $p_i > v - t/2$ then $2(v-p_i)/(2t)\phi_i \prod_{j\neq i}(1-\phi_j)$ is the demand from the consumers who consider firm i and do not consider any other firm. By cutting the price to $p_i - dp$, the firm loses something less than $\phi_i dp$ because of lower margins (because the maximum potential demand for product i is ϕ_i) and gains at least $p_i(1/t)\phi_i \prod_{j\neq i}(1-\phi_j)\, dp$ because of greater demand. Because the maximum a firm would advertise for a product would be smaller than $\bar{\phi}$ defined by $v = c'(\bar{\phi})$ (given that this is the case with the maximum possible price and the maximum marginal payoff for more advertising), the gain from cutting prices is greater than the loss if $t \leq (v - t/2)(1-\bar{\phi})^{n-1}$, given that the firm never gains from having a product with a price below $v - t/2$. That is, if t is sufficiently small, then the gain is greater than the loss and the firm is better off undercutting p_i. Finally, as argued above, if $p_i < v - t/2$, the firm can increase this price slightly without losing any demand and therefore raising revenue. Q.E.D.

Incentive Compatibility and Individual Rational Constraints in Price Discrimination Case

$$V(\theta_1, q_1) - p_1 \geq V(\theta_1, q_2) - p_2, \tag{9}$$

$$V(\theta_2, q_2) - p_2 \geq V(\theta_2, q_1) - p_1, \tag{10}$$

$$V(\theta_1, q_1) - p_1 \geq 0, \tag{11}$$

$$V(\theta_2, q_2) - p_2 \geq 0. \tag{12}$$

Problem for the Firm in Price Discrimination Case

$$\begin{aligned} \max_{\phi_1, \phi_2, q_1, q_2} \; & \phi_1[N_1 + (1-\phi_2)N_2][V(\theta_1, q_1) - gq_1] \\ & + \phi_2 N_2[V(\theta_2, q_2) - V(\theta_2, q_1) + V(\theta_1, q_1) - gq_2] \\ & - c(\phi_1) - c(\phi_2). \end{aligned} \tag{13}$$

Proof of Proposition 7. From (6) the incentive to distort q_1 under costly advertising is determined by the ratio of N_2 to $(\phi_1/\phi_2)[N_1 + (1-\phi_2)N_2]$. Under costless advertising the incentive to distort q_1 is determined by the ratio of N_2 to N_1. Subtracting N_1 from $(\phi_1/\phi_2)[N_1 + (1-\phi_2)N_2]$ one obtains $(1/\phi_2)[(\phi_1-\phi_2)N_1 + (1-\phi_2)N_2]$, which is strictly positive given Proposition 6. Q.E.D.

Proof of Proposition 8. Consider a quadratic approximation of $c(\cdot)$, $c(\phi) = a\phi + (b/2)\phi_2$, where b is large by assumption. From (7) and (8) for b large one obtains approximately, $\phi_1 = (m_1 - a)/b$, $\phi_2 = (m_2N_2 - a)/b$. Substituting into the objective function (with b large) one obtains

$$\max_{q_1} \frac{m_1(m_1-a)}{b} + N_2^2 \frac{m_2(m_2-a)}{b}.$$

The maximization over q_1 then yields that m_1 decreases in N_2 while m_2 increases in N_2. Substituting into ϕ_1 and ϕ_2 one gets that ϕ_1 is decreasing and ϕ_2 is increasing in N_2. Q.E.D.

References

Aaker, D. 1996. *Building Strong Brands*. The Free Press, New York.

Bergen, M., S. Dutta, S. M. Shugan. 1996. Branded variants: A retail perspective. *J. Marketing Res.* **33** 9–19.

Butters, G. 1977. Equilibrium distributions of sales and advertising prices. *Rev. Econom. Stud.* **44** 465–491.

Desai, P. 2001. Quality segmentation in spatial markets: When does cannibalization affect product line design? *Marketing Sci.* **20** 265–283.

Dixit, A., J. Stiglitz. 1977. Monopolistic competition and optimum product diversity. *Amer. Econom. Rev.* **67** 297–308.

Dobson, G., S. Kalish. 1988. Positioning and pricing a product line. *Marketing Sci.* **7** 107–125.

Dubé, J.-P. 2004. Multiple discreteness and product differentiation: Demand for carbonated soft drinks. *Marketing Sci.* **23** 66–81.

Grossman, G. M., C. Shapiro. 1984. Informative advertising with differentiated products. *Rev. Econom. Stud.* **51** 63–81.

Guo, L. 2003. Consumption flexibility, product configuration, and market competition. Working paper, University of California, Berkeley, CA.

Iyer, G., P. B. Seetharaman. 2001. To price discriminate or not: The endogenous choice of product-type in retail gasoline markets. Working paper, University of California, Berkeley, CA.

Karmarkar, U., T. Pitbladdo. 1994. Product-line selection, production decisions and allocation of common fixed costs. *Internat. J. Production Econom.* **34** 17–33.

Keller, K. L. 1998. *Strategic Brand Management: Building, Measuring, and Managing Brand Equity*. Prentice Hall, Upper Saddle River, NJ.

Kotler, P. 1997. *Marketing Management*. Prentice Hall, Upper Saddle River, NJ.

Kuksov, D. 2004. Buyer search costs and endogenous product design. *Marketing Sci.* Forthcoming.

Mehta, N., S. Rajiv, K. Srinivasan. 2003. Price uncertainty and consumer search: A structural model of consideration set formation. *Marketing Sci.* **22** 58–84.

Moorthy, K. S. 1984. Market segmentation, self-selection, and product line design. *Marketing Sci.* **3** 288–307.

Mussa, M., S. Rosen. 1978. Monopoly and product quality. *J. Econom. Theory* **18** 141–168.

Newman, J., R. Staelin. 1972. Prepurchase information seeking for new cars and major household appliances. *J. Marketing Res.* **9** 249–257.

Randall, T., K. Ulrich, D. Reibstein. 1998. Brand equity and vertical product line extent. *Marketing Sci.* **17** 356–379.

Shugan, S. M. 1989. Product assortment in a tripoly. *Management Sci.* **35** 104–320.

Shugan, S. M., R. Desiraju. 2001. Retail product-line pricing strategy when costs and products change. *J. Retailing* **77** 17–38.

Soberman, D. 2001. Informative advertising: An alternate viewpoint and implications. Working paper, INSEAD.

Urban, G. L. 1969. Mathematical modeling approach to product line decisions. *J. Marketing Res.* **6** 40–47.

Villas-Boas, J. M. 1998. Product line design for a distribution channel. *Marketing Sci.* **17** 156–169.

Villas-Boas, J. M., U. Schmidt-Mohr. 1999. Oligopoly with asymmetric information: Differentiation in credit markets. *RAND J. Econom.* **30** 375–396.

Wells, W., J. Burnett, S. Moriarty. 1992. *Advertising Principles and Practice*. Prentice Hall, Englewood Cliffs, NJ.

Wernerfelt, B. 1988. Umbrella branding as a signal of new product quality: An example of signalling by posting a bond. *RAND J. Econom.* **19** 458–466.

Wernerfelt, B. 1995. A rational reconstruction of the compromise effect: Using market data to infer utilities. *J. Consumer Res.* **21** 627–633.

Wilson, L. O., J. A. Norton. 1989. Optimal entry timing for a product line extension. *Marketing Sci.* **8** 1–17.