



Marketing Science

Publication details, including instructions for authors and subscription information:
<http://pubsonline.informs.org>

Procuring Commodities: First-Price Sealed-Bid or English Auctions?

Jason Shachat, Lijia Wei,

To cite this article:

Jason Shachat, Lijia Wei, (2012) Procuring Commodities: First-Price Sealed-Bid or English Auctions?. Marketing Science 31(2):317-333. <https://doi.org/10.1287/mksc.1120.0704>

Full terms and conditions of use: <https://pubsonline.informs.org/Publications/Librarians-Portal/PubsOnLine-Terms-and-Conditions>

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2012, INFORMS

Please scroll down for article—it is on subsequent pages



With 12,500 members from nearly 90 countries, INFORMS is the largest international association of operations research (O.R.) and analytics professionals and students. INFORMS provides unique networking and learning opportunities for individual professionals, and organizations of all types and sizes, to better understand and use O.R. and analytics tools and methods to transform strategic visions and achieve better outcomes.

For more information on INFORMS, its publications, membership, or meetings visit <http://www.informs.org>

Procuring Commodities: First-Price Sealed-Bid or English Auctions?

Jason Shachat, Lijia Wei

Wang Yanan Institute for Studies in Economics (WISE) and the MOE Key Laboratory of Econometrics, Xiamen University,
Xiamen, 361005 China {jason.shachat@gmail.com, ljwei.wise@gmail.com}

We use laboratory experiments to examine the relative performance of the English auction (EA) and the first-price sealed-bid auction (FPA) when procuring a commodity. The mean and variance of prices are lower in the FPA than in the EA. Bids and prices in the EA agree with game-theoretic predictions, but they do not agree in the FPA. To resolve these deviations found in the FPA, we introduce a mixture model with three bidding rules: constant absolute markup, constant percentage markup, and strategic best response. A dynamic specification in which bidders can switch strategies as they gain experience is estimated as a hidden Markov model. Initially, about three quarters of the subjects are strategic bidders, but over time, the number of strategic bidders falls to below 65%. There is a corresponding growth in those who use the constant absolute markup rule.

Key words: procurement auction; experiment; hidden Markov model; decision rules of thumb

History: Received: February 14, 2009; accepted: November 21, 2011; Eric Bradlow and then Preyas Desai served as the editor-in-chief and Teck Ho served as associate editor for this article. Published online in *Articles in Advance* February 16, 2012.

1. Introduction

Large organizations, which procure a wide range of goods and services, are increasingly finding themselves contemplating changes from traditional request for quote mechanisms to dynamic reverse auctions. Practitioners (*Purchasing* 2001, Chafkin 2007) and academics (Jap 2002, Elmaghraby 2007) commonly suggest that when organizations consider this change they should start with their commodity purchases. For current purposes, commodities are goods for which the organization considers price the sole differentiating attribute among suppliers. When price is the sole determinant of supplier choice, a request for quote mechanism reduces to a first-price sealed-bid auction (FPA), and a dynamic reverse auction reduces to an English auction (EA). This raises the need for some basic comparative empirical knowledge of the two auction formats for procurement officials in such organizations. Toward filling this need, this study provides insights into the relative empirical price performance between the FPA and the EA in the context of procuring commodities, the nature of how suppliers bid when having little to no experience in such auctions, and how their bidding behaviors adjust as they gain experience.

We conduct a controlled laboratory experiment designed so that inferences on performance, behavior, and dynamics are made without concern for estimating suppliers' costs or the procuring organization's

willingness to pay. We report statistical tests comparing properties of the empirical price distributions generated within the FPA and EA versus each other and against theoretical benchmarks. The results of these tests show that the mean and variance of prices are lower in the FPA, prices in the EA first-order stochastically dominate those in the FPA, and the Nash equilibrium predictions regarding prices and individual bids are accurate in the EA but not in the FPA.

Our investigation of the procurement context is an important extension in the broader experimental study of FPA and EA auctions with independent and private information.¹ Previous experimental and behavioral studies have almost exclusively considered the forward context; i.e., the auctioneer is selling an object, and a bidder's private information is his valuation of that object. Our results demonstrate that the previously observed robustness of game-theoretic predicted performance in the EA extends to the procurement context. Likewise, we replicate previous deviations from Nash equilibrium predictions in the FPA; however, the procurement setting permits us to identify a new explanation for these deviations. Namely, there is a discrete heterogeneity in the bidding rules subjects follow. Two of these rules are non-strategic and have more sensible interpretations in the

¹ "Independent and private" here refers to the fact suppliers' costs are independent random variables for which the realizations are private information.

procurement setting than in the forward one. One rule is to use an absolute markup in which a constant amount is added to a bidder's realized cost. We believe that demanding a constant profit margin is a more natural decision heuristic in a selling context relative to its forward auction counterpart, in which a bidder demands a constant amount of consumer surplus independent of the valuation. The other nonstrategic rule is to use a fixed proportional markup of realized cost. Requiring a certain rate of return is a common rule of thumb used by businesses when setting prices, and it generates a positive relationship between costs and the absolute amount of profit demanded. In a procurement FPA, this rule is distinct from Nash equilibrium bidding, which generates a negative relationship between cost and profit demanded. In contrast, both these rules generate an increasing relationship between valuation and demanded consumer surplus in the forward FPA. This makes econometric identification of the Nash equilibrium and percentage markup rules difficult, if not impossible, in the forward FPA.

To explain and estimate this discrete bidding heterogeneity, we present a quasi-equilibrium model of bidding in the FPA based on a mixture of strategic and rule-of-thumb bidders. Then a dynamic version of this model is estimated as a hidden Markov model; the latent variable is the rule a subject follows in each auction period. In the initial stage of the experiment, we estimate about three-quarters of the subjects follow a strategic bidding rule, and the remaining subjects follow either an absolute or a percentage markup rule. The estimated dynamics are surprising because bidders become less strategic as they gain experience. During the sequence of 30 auctions, the proportion of strategic bidders falls below two-thirds, and there is a corresponding rise in the proportion of bidders who simply bid a small constant absolute amount above cost.

The rest of this paper proceeds as follows. In §2, we review the relevant literatures. This is followed by the development of hypotheses regarding the performance of the two auction types in §3. Next, we present the details of the experimental design and procedures in §4. Then, in §5, there is an empirical analysis of the price performance and bidder behavior in the experiment. In §6, we discuss the theoretical mixture bidding model for the FPA, and the estimation results of a hidden Markov dynamic version of the model are presented in §7. In §8, we demonstrate the robustness of this model through specification tests against multiple alternative models found in the literature. We conclude the paper in §9 by commenting on implications of the results and possible extensions.

2. Related Auction Literature

Our evaluation of the relative price performances of the procurement FPA and EA contributes to the extensive experimental literature testing the revenue equivalence theorem in basic auction formats. The original version of the theorem (Vickrey 1961) states that expected prices are the same for the FPA, the Dutch auction (a dynamic auction with the same normal game form as the FPA), the second-price sealed-bid auction (which has the same normal game form as the EA), and the EA when bidder types are independent and identically distributed private information. As far as we know, all other experimental tests of the revenue equivalence theorem use forward auctions. Most of these early studies focus on making comparisons within pairs of normal game form-equivalent auctions. For example, Coppinger et al. (1980) and Cox et al. (1982) find that the EA and the second-price sealed-bid auction generate the same expected prices, whereas the FPA generates higher prices than the Dutch auction. In a well-known field experimental study, Lucking-Reiley (1999) finds the same revenue equivalence between the EA and the second-price sealed-bid auction but observes that the average Dutch auction price exceeds that of the FPA—contradicting the results of earlier laboratory studies. These and other studies speak little to the comparison of the EA and the FPA, which vary both in the normal game form and Nash equilibrium solution. However, the nature of our applied procurement problem begs this comparison.

Considering the procurement rather than the forward context is a key aspect of our study, despite this difference in context having no effect on Nash equilibrium and the prediction of revenue equivalence. The impact in auctions of nonstrategic factors, such as context and framing, has many precedents. Many auction formats have the same normal game form representations as the second-price auction, and researchers often exploit this relationship by analyzing such auctions as second-price ones. However, Zeithammer and Adams (2010) demonstrate that in EAs with bidder proxies on eBay, this second-price auction abstraction fails. Hossain and Morgan (2006) and Brown et al. (2010) find a failure of revenue equivalence in a series of eBay auctions that are identical except for the framing of fixed shipping and handling fees. An additional example, Dholakia and Simonson (2005) demonstrate in a field experiment that including explicit reference points to competing prices—which does not impact the equilibrium predictions—raises auction prices. In a related work, Fay (2004) shows that multiple-bid versus single-bid formats of name-your-own-price auctions are not empirically revenue equivalent—the auctioneer's

profit increases in the multiple-bid format as the number of sophisticated bidders grows. In response to the revenue equivalence failure, each of these studies formulates an appropriate behavioral model to capture the effect of the framing variation. We also argue from a behavioral perspective that the procurement context facilitates the adoption of simple pricing heuristics, which have less intuitive counterparts in the forward context.

The performance of an auction is a product of the individual bidding behavior, as these previous studies found that identifying better models of individual behavior leads to better explanations of auction performance. Thus identification of the bidding behavior is crucial to understanding why auction formats perform differently. A large segment of the structural econometric modeling of auctions literature proceeds by assuming that bidders follow Nash equilibrium strategies and then use properties of the Nash equilibrium to establish identification and consistent estimation of parameters such as the bidder's underlying values and beliefs. Hendricks and Porter (1988), Laffont et al. (1995), Guerre et al. (2009), and Krasnokutskaya (2011) represent a very incomplete list of examples from this literature that analyze the private value, single object case. Reliance on the strong rationality assumptions of Nash equilibrium is a natural criticism of this approach. In response, recent alternatives have built structural empirical bidding models without relying on the Nash equilibrium assumption for identification (Goeree et al. 2002, Park and Bradlow 2005, Chan et al. 2007, Yao and Mela 2008, Abbas and Hann 2010) or have taken a reduced-form approach (Jap and Naik 2008, Bradlow and Park 2007) to build forecasting models. In many ways, our work is in the spirit of structural models that relax the assumption of Nash equilibrium, although, given the nature of our experimental data, we mostly focus on the estimation of bidding rule parameters and the distribution of rules in the population.

The performance differences we observe between the EA and the FPA are driven by individual deviations from Nash equilibrium bidding in the FPA. Thus our modeling approach to individual bidding brings new insights into the extensive literature examining non-Nash bidding in FPA forward auctions. In one of the earliest experimental studies of a forward FPA with independent private values, Cox et al. (1982) find that the vast majority of bids exceed the Nash equilibrium-predicted ones, thus leading to higher-than-predicted prices. They further propose a new Nash equilibrium model in which the bidders are characterized by heterogeneous risk attitudes. Their simple elegant solution is highly scrutinized by Kagel and Levin (1993) and others (see Kagel 1995; Kagel

and Levin 2008, 2012 for thorough reviews), who generate data that cannot be explained by models of risk aversion. Proposed alternatives to explain the data from these subsequent experiments use principles from behavioral economics such as anticipated regret (Engelbrecht-Wiggans and Katok 2007, 2008; Filiz-Ozbay and Ozbay 2007) and nonlinear probability weighting (Armantier and Treich 2009a, b), or they adopt alternative game-theoretic solutions to the Nash equilibrium (Goeree et al. 2002, Morgan et al. 2003, Ockenfels and Selten 2005).

Our alternative behavioral model for bidding in the FPA, in its static form, follows a recent approach of allowing discrete strategic heterogeneity. In an influential study, Crawford and Iriberri (2007) replace the stringent belief assumptions of Nash equilibrium with a level- k strategic model to analyze FPAs. In the level- k model, each bidder is characterized by a non-negative integer k , indicating the number of steps of iterated best response he performs when selecting a strategy. The iterated best-response process starts with some boundedly rational rule for step $k = 0$. For the forward FPA, Crawford and Iriberri consider two possible $k = 0$ strategies: bid the true value or bid randomly according to a uniform distribution over the interval from the minimum allowable bid to value. A $k = 1$ type believes all other bidders follow a particular $k = 0$ strategy and best responds; correspondingly, a $k = 2$ type believes all other bidders are $k = 1$ and best responds, and so on. Crawford and Iriberri apply this model to an FPA experiment with a discrete valuation space reported in Goeree et al. (2002) and find that it performs quite favorably to other models. Moreover, they estimate a full mixture model and find that approximately 4%, 76%, and 20% of the subjects follow the level $k = 0$, 1, and 2 rules, respectively.

In a second paper using mixtures of bidding rules, Kirchkamp and Reiss (2008) consider a model of two bidders in which there is some probability a bidder bids a fixed markdown of his valuation; otherwise, the bidder is rational and best responds while taking full account of the mixture distribution of rational and fixed markdown bidders. This model differs from Crawford and Iriberri (2007) because it specifies a different $k = 0$, and the $k = 1$ type best responds according to the true distribution of the level types rather than the belief that all bidders are $k = 0$. Kirchkamp and Reiss (2008) test their model in a series of forward FPAs that allow bids less than the lowest possible valuation. By estimating individual bid functions, they find that roughly 30% of the subjects follow the markdown, whereas the remaining subjects follow the more rational rule. It should be noted that these models are static; Crawford and Iriberri (2007) state that

their level- k formulation is a model of how inexperienced individuals bid (accordingly, they only consider the first five rounds of each experimental session in their analysis), and the Kirchkamp and Reiss (2008) model is quasi-equilibrium. Our model first extends the Kirchkamp and Reiss model by allowing multiple participants and additional bidding heuristic rules. Moreover, we take a step forward in this modeling approach by allowing bidders to switch rules over time and model this dynamic to examine how individuals adapt with experience.

Our dynamic extension is a significant advance in identifying how bidders in the FPA adapt with experience. Despite the large number of FPA studies, surprisingly few of them address how individuals learn to bid over time. The most developed empirical literature on learning in FPAs focuses on the impact of the informational feedback provided to bidders. If a forward FPA provides the winning bid, or the whole vector of bids, then bid functions are estimated to increase to levels above the Nash equilibrium (Selten and Buchta 1998, Guth et al. 2003, Neugebauer and Selten 2006). However, in the absence of this feedback, behavior converges toward the risk-neutral Nash equilibrium (Neugebauer and Perote 2008). The workhouse dynamic principle of this literature is the directional learning model introduced by Selten and Buchta (1998). Directional learning is more of a behavioral principle than a complete model, as it only specifies that individuals adjust their strategy toward those who offer ex post higher payoffs conditional on available information.

The set of investigations regarding structural learning models is less developed. Bajari and Hortacsu (2005) find that econometric models that assume subjects follow an adaptive learning process do not outperform equilibrium models with risk aversion in recovering structural parameters. Recent efforts to study learning in FPA auctions using Camerer and Ho's (1999) experienced weighted attraction (EWA) model also have had limited success. James and Reagle (2009) find that EWA models can generate aggregate data similar to aggregate human subject data, but the EWA parameters are extremely difficult to recover econometrically. Furthermore, Dittrich et al. (2011) find great between-subject heterogeneity in their EWA model estimates as well as significant estimated loss aversion. The dynamic rule-switching model we study advances these efforts by demonstrating that individual heterogeneity is well explained by a small set of discrete rules and that observed increasing (or decreasing, in our procurement setting) deviations from Nash equilibrium in FPA arises from individuals increasingly adopting absolute markup strategies that demand small profit margins.

3. Environment Formulation and the Development of Auction Performance Hypotheses

Let us begin by considering the simple situation with a procurement official whose task is to purchase an indivisible unit of a commodity as cheaply as possible. There are n potential suppliers indexed by i . Each supplier can provide a unit of the commodity for the cost of c_i , which is incurred only if they supply the unit. The cost c_i is only known by supplier i —i.e., it is private information—and will typically vary across suppliers. Suppliers are symmetric in that none has an ex ante cost advantage. Specifically, each supplier's costs is an independent realization from a random variable whose distribution is uniform on the interval $[c_L, c_H]$. Whereas each supplier knows his own realized costs, the other suppliers and the procurement official only know its distribution.

We consider two sourcing methods for this scenario. First is the EA, which starts at an initial high price, and all n suppliers are still participants. Then the price sequentially decrements. At each decrement, a participant can take the action to irreversibly exit the auction, after which he is no longer considered a participant. At each decrement, the number of remaining participants and current price are publicly posted. The auction closes when the second-to-last participant exits. The last remaining participant wins the auction and receives the closing auction price, p . This winning supplier i receives a profit of $p - c_i$, and all other suppliers receive zero profit.² The EA has a weakly dominant strategy: a supplier chooses to exit the auction when the price equals his unit cost.³ When everyone follows the weakly dominant strategy, the losing suppliers reveal their true costs, and the supplier with the lowest realized cost wins the auction and subsequently receives a price equal to the second-lowest realized cost. Consequently, the price paid by the procurement official is a random variable whose distribution is the second-order statistic of the n suppliers' cost realizations.

In an FPA, each supplier privately submits a price. Then the procurement official purchases from the lowest-priced supplier, and that supplier receives a price equal to his bid. Under the crucial assumption

² There are other formats of the EA. For example, in an open outcry format, individual suppliers can announce successively lower bids until there is no supplier willing to improve on the current existing price. The supplier submitting the last price wins the auction at that price. In the strategic analysis, optimal bidder behavior is equivalent in the open outcry format and the version we describe.

³ One can find standard arguments for this weakly dominant strategy in texts such as Krishna (2002). A weakly dominant strategy in this context means that regardless of the other suppliers' strategies, there is never an instance in which the supplier can strictly increase his expected payoff by deviating from this strategy.

bids depend on the associated costs that the suppliers are all risk neutral, the pure-strategy symmetric Nash equilibrium calls for a supplier to submit a price according to the following function of his realized cost and the number of potential suppliers (see Vickrey 1961):

$$b_i(c_i) = \frac{c_H + (n-1)c_i}{n}.$$

This bidding strategy has an interesting behavioral interpretation; a supplier's bid is equal to the expected second-lowest realized cost conditional on his cost being the lowest. Thus, the unconditional expected winning price is the amount that the unconditional lowest expected cost type bids. One can show that this is just the unconditional expected value of the second-lowest realized cost, same as the expected auction price in the EA. This is an example of the revenue equivalence theorem⁴ and forms our first hypothesis.

HYPOTHESIS 1. *The expected prices in the FPA and the EA are the same.*

Let us consider an example with three suppliers whose costs are independent and uniformly distributed on the interval $[0, 20]$. In this case the expected value of the lowest, second-lowest, and highest costs are 5, 10, and 15, respectively. Figure 1 depicts the expected outcomes in both auction formats. In the EA, the winner's expected cost is five. Furthermore, the expected second-lowest cost, and the corresponding auction price, is 10. In the FPA, we expect the winning supplier's cost is also 5 and for him to bid 10, the expected second-lowest cost conditional upon 5 being the lowest.

Of course, although the expected—or average—prices are the same, the distributions of prices are not. In the EA it is easy to see that the actual winning price can occur anywhere on the interval $[0, 20]$. At the same time, Figure 1 shows that the support of possible prices in the FPA is smaller. The lower end of this support occurs when a supplier's cost is 0 and he bids $6\frac{2}{3}$. The upper end of this support occurs when the lowest supplier's cost is 20 and he bids 20. Thus the distribution of prices in the EA is a mean preserving spread of the prices in the FPA. In fact, Vickrey (1961) shows that with independent and uniformly distributed costs and n bidders, the variance of the price in the EA is $(2(n-1)(c_H - c_L)^2)/((n+2)(n+1)^2)$

and in the FPA is $((n-1)(c_H - c_L)^2)/(n(n+2)(n+1)^2)$. So the variance in the EA price is greater than that of the FPA by a factor of $(2n)/(n-1)$; this is our second hypothesis.

HYPOTHESIS 2. *The variance of price in the EA is greater than in the FPA.*

Next we formulate an *alternative* hypothesis by setting aside the assumption that suppliers are risk neutral, and instead, we assume that they are risk averse. Because risk neutrality is a necessary condition for revenue equivalence, we should now be able to order the expected price in the two auctions. First, in the EA, because it has a weakly dominant strategy, risk aversion does not affect suppliers' predicted behavior or the expected price. This is not true for the FPA. Holt (1980) shows that if all suppliers have the same risk-averse, von Neumann–Morgenstern expected utility function, then there exists a symmetric Nash equilibrium in which the expected price is lower than that of the EA. Consequently, we have the following alternative to Hypothesis 1.

HYPOTHESIS 3. *The expected or average price in the FPA will be lower than in the EA.*

Note that replacing the assumption of risk neutrality does not change the hypothesis regarding ranking of price variances.

So far, we have only considered the procuring organization's welfare—price in this case—but what about the suppliers? First, under the assumption of risk neutrality, the expected profit is the same in both the EA and the FPA. This is the expected lowest cost less the expected second-lowest cost multiplied by the probability of being the lowest-cost supplier. In our example, this is \$1.67. However, the difference is the variance of a supplier's payoff (profit) is much higher than for the price. The variances for a supplier's profit in the EA is $(n(c_H - c_L)^2)/((n-2)(n+1)^2)$ and in the FPA is $(c_H - c_L)^2/(n(n+2)(n+1)^2)$; the difference is a factor of n^2 . We summarize these observations with the following pair of hypotheses.

HYPOTHESIS 4. *The expected profit of a supplier in the FPA and the EA is the same.*

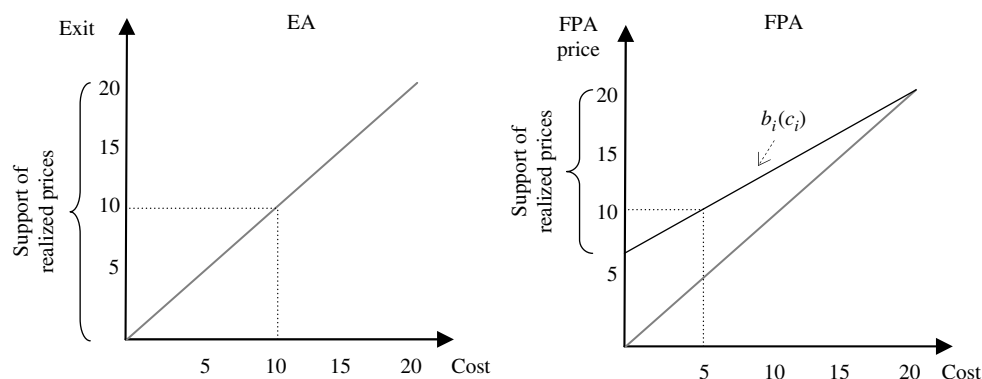
HYPOTHESIS 5. *The variance of a supplier's profit in the EA is greater than in the FPA.*

Once again, replacing the assumption of supplier risk neutrality with risk aversion gives us an alternative hypothesis to equal expected supplier profit.

HYPOTHESIS 6. *The expected profit of a supplier is lower in the FPA than in the EA.*

⁴ The revenue equivalence theorem was first proven by Vickrey (1961) and is applicable to our scenario. It was proven for a wider class of scenarios by Myerson (1981). The version used here states that if sellers are risk neutral, if they have independent and symmetric costs, and if payment is a function of the bid only, then the FPA and the EA will have the same expected price.

Figure 1 Nash Equilibrium Bidding Behavior



Note. This figure illustrates that although the expected prices are the same, the distribution of prices differs for the EA and FPA.

4. Experimental Design and Procedures

All experimental sessions were conducted at the Economics Laboratory at the University of California, San Diego (UCSD) and at the National University of Singapore (NUS) School of Business. All participants were either undergraduates or master's students at one of the two universities. FPA and EA sessions were conducted at both facilities. The number of participants in an experimental session consisted of some multiple of 3 between 9 and 18. Experimental sessions lasted no more than one hour, and subjects earned between \$8 (S\$12) and \$60 (S\$89) inclusive of a show-up fee. Table 1 provides these and further details.

We adopted the simple previous three-supplier example as the basis for our experiment. In a session, the subjects participated in series of 32 rounds of either FPAs or EAs. In each round, the subjects were randomly repartitioned into a set of triads. The first two rounds were for practice; the subjects earned no money and the data are not reported. For the remaining rounds, the participants' earnings were given in an experimental currency. The exchange rates were

one experimental dollar to \$0.33 (or S\$0.50). In total, we conducted 720 FPAs and 360 EAs.⁵

Each subject sat in a partition designed to ensure privacy and made decisions using a personal computer running a custom-designed software program. In the FPA, at the start of each period each subject is shown his realized private cost (a new cost was drawn each period), the period number, and the number of participants in the auction. Then he was prompted to submit a price (restricted to be between 0 and 30), but he could take as much time desired to do so. After all prices were submitted, the auction results were revealed. These results consisted of his bid, the amount of the winning bid, and his period profit. This information, along with the cumulative profit, was then entered into a display at the bottom of the computer screen for future reference. At the conclusion of the experiment, subjects were privately paid their total earnings.

The EA sessions followed the same procedures, except for the execution of the auction. The computer screen contained a display of the current auction price and a button that a subject clicked on to irreversibly exit the auction. The auction started with an initial price of \$21. Then the price was decremented at the rate of 10 cents every half a second. As auction participants exited the auction, subjects could observe the decrement of the displayed number of participants remaining in the auction. At the close of the auction, the subject's cost, exit price, auction price, period profit, and cumulative profit were entered in his or her history viewing area.

5. Data Analysis and Performance Results

We first address the relative performance of the two alternative procurement procedures and observe that

Table 1 Summary of Experimental Sessions

Session	Treatment	Location	No. of subjects	Rounds	Minimum earnings	Maximum earnings
1	FPA	UCSD	15	32	16.72	52.62
2	FPA	UCSD	15	32	11.00	58.98
3	FPA	UCSD	15	32	13.23	42.45
4	FPA	NUS	9	32	23.24	48.94
5	FPA	NUS	18	32	21.60	55.72
6	EA	UCSD	9	32	7.05	62.70
7	EA	NUS	9	32	30.78	87.84
8	EA	NUS	18	32	32.84	88.74

Notes. Earnings are US\$ for UCSD sessions and S\$ for NUS sessions; the exchange rate at the time of the sessions was approximately US\$1 = S\$1.5. The show-up fee was US\$5 at UCSD and S\$10 at NUS.

⁵ We collected an unbalanced sample because the data in the EA strongly agreed with the weakly dominant bidding strategy.

procurement costs are lower and less variable in the FPA. Furthermore, the empirical distribution of EA prices first-order stochastically dominates that of the FPA. For suppliers, average profits are higher but, at the same time, more variable in the EA.

We start by considering the distributions of realized FPA and EA prices. Figure 2 presents the empirical cumulative distribution function (CDF) of prices for each auction types. For now, let us assume that each of the $m = 720$ FPA prices is an independent realization from the continuous distribution W_{FPA} generating the empirical CDF, \bar{W}_{FPA} , and that each of the $n = 360$ EA prices is an independent realization from the continuous distribution W_{EA} generating the empirical CDF, \bar{W}_{EA} . We test the hypothesis that these two distributions are equal, versus the alternative that are not, with the Kolmogorov–Smirnov two-sample test. The test statistic is $\bar{S} \max_p = |\bar{W}_{FPA}(p) - \bar{W}_{EA}(p)| = 0.372$ and has a p -value of essentially 0. The hypothesis test clearly rejects the underlying distributions of auction prices are the same. Casual inspection of Figure 2 also suggests that W_{EA} first-order stochastically dominates W_{FPA} ; in other words, for every p , $W_{EA}(p) \leq W_{FPA}(p)$. We use the Barrett and Donald (2003) test to evaluate this hypothesis versus the alternative that for some p , $W_{EA}(p) > W_{FPA}(p)$. The test statistic is

$$\bar{S} = \left(\frac{mn}{(m+n)} \right)^{1/2} |\bar{W}_{FPA}(p) - \bar{W}_{EA}(p)| = 0.002.$$

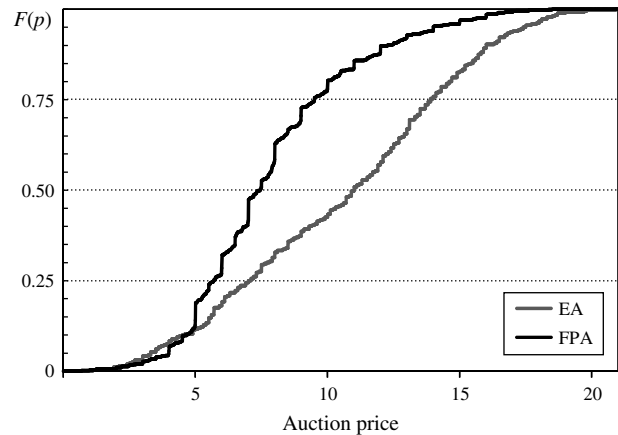
The p -value of this test statistic is greater than 0.999, which leads us to not reject the hypothesis of first-order stochastic dominance.⁶ Under the first-order stochastic dominance criteria in decision making (Seshadri et al. 1991, Levy 1992), the EA more strongly favors the suppliers rather than the procurement official.

RESULT 1. Procurement prices in the EA first-order stochastically dominate those in the FPA.

The FPA delivers lower average price and lower price variability to the procurement official. Table 2 presents the means and variances of the two price distributions and the differences in these values. The first column shows the mean, the standard error of the mean, and the percentage difference in means. Two-tailed t -tests reject that the EA mean price (5% level of significance) and the FPA mean price (1% level of significance) are equal to the risk-neutral Nash equilibrium prediction of 10. Moreover, we reject that the mean prices are the same in favor of the hypothesis that the FPA mean price is lower by conducting a t -test for unequal variances (1% level of significance).

⁶ The p -value of this test statistic is calculated as $\exp(-2\bar{S}^2)$.

Figure 2 Empirical Cumulative Distribution Functions of FPA and EA Prices



RESULT 2. Procurement prices are lower—25.5%, in fact—for the FPA. We reject Hypothesis 1 in favor of its alternative, Hypothesis 3.

In the second column, we report information for the variance of prices. Under the risk-neutral Nash equilibrium bidding models, the variance of the prices in the EA and FPA are 20 and $6\frac{2}{3}$, respectively. We give the sample variances, and beneath those, we give bootstraps of the 95% confidence intervals of the variance under Nash equilibrium. We cannot reject that the variance is equal to the predicted value in the EA, but we do reject this for the FPA, as the estimated variance is well above the confidence interval. When comparing the variances, we reject the hypothesis that they are the same in favor of the hypothesis that the EA price has greater variance (Levin F -tests for heterogeneous variances at the 1% level of significance). However, the price variance in the EA is only twice as large (i.e., the –50.4% change noted in the table), not triple as predicted by Nash equilibrium.

Table 2 Summary Statistics on Prices and Profits

	Mean price	Price variance	Mean supplier profit	Variance supplier profit
EA				
Observed	10.60	19.11	1.68	10.67
Theoretical	10	20	1.67	15.03
Std. error or CI	(0.230)	[18.47, 21.61]	(0.099)	[12.79, 17.14]
FPA				
Observed	7.90	9.63	0.90	3.04
Theoretical	10	10.67	1.67	1.67
Std. error or CI	(0.116)	[5.63, 7.69]	(0.037)	[1.50, 1.84]
% Difference	–25.5	–50.4	–46.3	–71.5

Notes. Because the distribution of prices under Nash equilibrium is not normal, simple t -tests of the estimated variances are not valid. Accordingly, we construct 95% confidence intervals (CIs) by bootstrap with 10,000 draws from the known distribution.

RESULT 3. Price variance is greater in the EA than the FPA. Our experimental results are consistent with Hypothesis 2.

Thus, unless a procurement organization is very risk loving, the lower average price and lower price variability make the FPA the better method in the commodity situation.

What about the welfare of the suppliers? As we see in the third column, we cannot reject (t -test at 1% level of significance) that the mean supplier profit in the EA is equal to the theoretical value of 1.67. The average supplier profit in the FPA is 46.3% lower, and we reject these profits are the same as the risk-neutral Nash equilibrium prediction and the empirical mean of the average profit in the EA experiments (both at the 1% level of significance). Although suppliers actually do better, on average, in the EA, the variability of the payoffs are about two-and-a-half times more volatile.

RESULT 4. Supplier's profits are higher in the EA rather than the FPA, and we reject Hypothesis 4 in favor of its alternative, Hypothesis 6. And we also find that these profits have greater variance in the EA as well, confirming Hypothesis 5.

The relative performances of the two procurements methods are most consistent with the risk-averse Nash equilibrium model. However, it is natural to ask how robust these results are to other types of procurement environments. Recall that the Nash equilibrium model with risk-averse bidders is formulated as a description of individual-level behavior. So a likely necessary condition for the generalization of our results on performance is that the model explains what happens at the individual level as well.

As noted earlier, risk aversion does not affect the weakly dominant strategy in the EA. Accordingly, we should observe those who exit an auction to do so at their true costs. Figure 3 plots sellers' exit prices

Figure 3 Exit Price vs. Cost in the EA

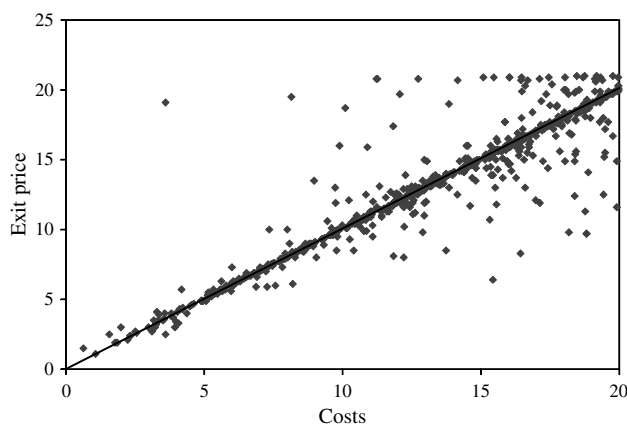
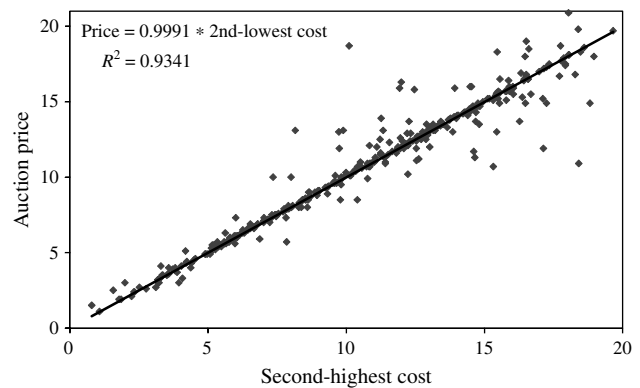


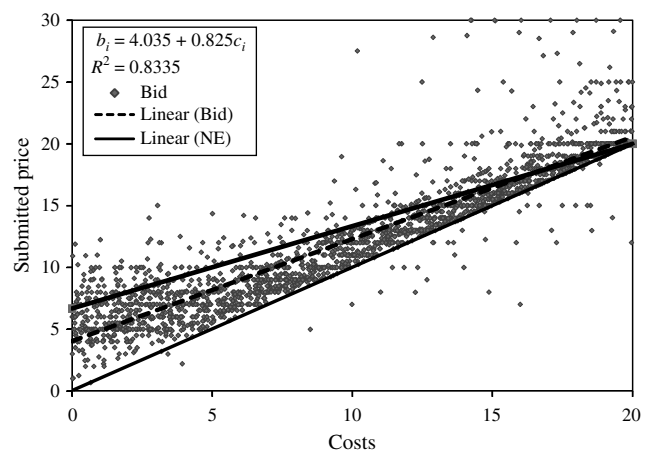
Figure 4 EA Price vs. Second to Exit Participant's Cost



versus their realized costs. Whereas much of the data adhere closely to the 45° line as expected, there is a surprising amount that does not. Much of these occurrences are at high cost levels, and the seller opts out as soon as the auction opens at the price of \$21. Perhaps such a bidder is frustrated by receiving a high cost? It is a different story for the observed exit prices below cost. In most of these cases, the errant subject remains in the auction while there are still two other remaining suppliers. Then, as soon as one of the other suppliers exits, this individual exits as well. We filter these likely nonsalient observations to see how price-determining bids depend on the associated costs. In Figure 4, we plot the realized auction price versus the second-to-exit participant's realized cost. Here, we see much crisper conformity with the 45° line. To quantify this we present an ordinary least squares (OLS) fitted trend line through the origin. The slope coefficient is essentially 1, and this regression explains more than 93% of the variation as indicated by the R^2 statistic.

Now let us turn our attention to the individual bids in the FPA sessions. Figure 5 plots the 2,160 submitted

Figure 5 FPA Submitted Bid vs. Cost



Note. NE, Nash equilibrium.

FPA bids versus realized costs, the risk-neutral Nash equilibrium bid function, the bid equal to cost line (the 45° line), and an OLS-fitted line for the data. Clearly, the majority of bids are above cost but below the risk-neutral Nash equilibrium bid line—just as a risk-averse Nash model predicts. However, there is also a surprising amount of bids outside (particularly above) this “cone of risk-averse bids.” We proceed by conjecturing that there is significant heterogeneity in how subjects behave in the FPA. Then we formulate a model of general discrete heterogeneity in bidding strategies that accord rationality only to some bidders.

6. A Model of Sophisticated and Rule-of-Thumb Bidders

The structure of the first-price auction and the uniform distribution of costs allow us to consider the special case of linear functions for both the rational and boundedly rational bidders. Consider a setting in which a supplier follows a rule r that belongs to a set of rules $R = \{AM, PM, BR\}$. Let Π denote a probability distribution on this set for which π_r is the probability a supplier is a bidder of type r . An *AM*, or absolute markup, bidder always demands a fixed profit amount independent of his costs. In other words, *AM* bidders price according to

$$b_{AM}(c_i) = c_i + \kappa,$$

where κ is the fixed absolute markup. A *PM*, or proportional markup, bidder always demands a fixed percentage margin based on realized costs. His bidding rule is

$$b_{PM}(c_i) = (1 + \rho)c_i,$$

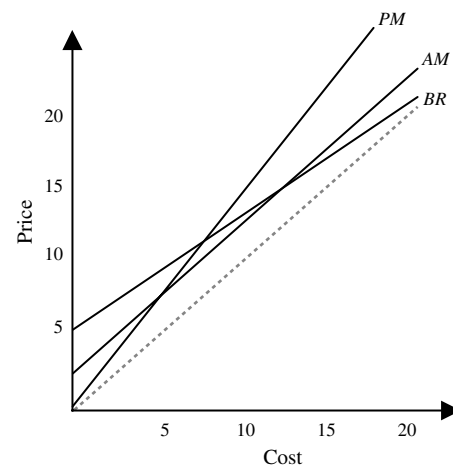
where ρ is the proportional markup. A *BR*, or strategic best-response, bidder selects a price that maximizes his expected utility conditional on the parameters Π , κ , and ρ and his realized cost. When a *BR* bidder formulates his bidding strategy, it is assumed that he knows the parameters Π , κ , and ρ . Also, we assume he has the von Neumann–Morgenstern expected utility function $U(y) = \eta y^{1/\eta}$, where y is a nonnegative change in wealth. In the appendix we show there is a symmetric *BR* price rule of the form

$$b_{BR}(c_i) = \frac{(c_H + \pi_{AM}\kappa)(1 + \rho)}{(1 + \rho(1 - \pi_{PM}))M} + \frac{M - 1}{M} c_i,$$

where $M = \eta(N - 1) + 1$.

There are several interesting things to note about the *BR* bid function. It is linear in cost, with the slope depending only on the parameter of the expected utility function and the number of other bidders. Most surprising is that the probabilities of the absolute and percentage markup bidders, and the size of these

Figure 6 Crossing Pattern of Bidding Functions



markups, only affect the intercept term; changes in these values only vertically shift the *BR* bid function.

From this general static model, we can generate several alternative models with appropriate parameter restrictions and formulations of beliefs. The Bayes Nash equilibrium model occurs when $\pi_{AM} = \pi_{PM} = 0$ and $\eta = 1$. We can replace the equality constraint on η with the appropriate inequality constraint to obtain the risk-averse Nash equilibrium model. The model of Kirchkamp and Reiss (2008) is recovered by setting $\pi_{PM} = 0$ and setting $N = 2$. Finally, we can obtain a level- k model by setting either $\pi_{AM} = 1$ or $\pi_{PM} = 1$.⁷

With respect to predicted change in performance of the FPA under this heterogeneous bidding mode, the mean and variance of the winning FPA price depends on the distribution of each supplier i 's bid. This is given by

$$Q(z) = \Pr(p_i \leq z) = \sum_{r \in R} \pi_r F(b_r^{-1}(z)),$$

and the distribution of the winning, or minimum, bid is

$$Q_1(z) = [1 - Q(z)]^{N-1}.$$

When calculating the expectation and variance of the winning FPA price, it will involve piecewise integration of this function. The boundary points of these integrals depend on the intersection points of the three bidding rules. Figure 6 depicts the scenario found in the subsequent data analysis.

7. Rule-Switching Dynamics: Specification and Estimation of a Hidden Markov Model

Because the subjects participate in repeated auctions with changing sets of bidders, as suppliers likely do

⁷ In this example, $k \geq 1$ results in the same bid function, b_{BR} .

in practice, we incorporate adaptation into our model through the switching of bidding rules. We start by assuming that there is an initial portion of subjects adopting each of the three bidding rules. Then rule switching is described by a stationary Markov chain. Finally, we assume a bidder follows his adopted rule with some random noise, thus making his bidding rule a latent variable. Such a state-space model with latent states is called a hidden Markov model (HMM). HMMs are a natural way to model dynamic decision making with hidden information.⁸

Consider the following dynamic process. Let z_{it} be the rule used by subject i in period t , and let z_i be the 30 element sequence of rules seller i adopts over the course of the experiment. Define π_{r1} as the probability a seller follows rule r in period 1, let Π_1 be the multinomial probability distribution these initial probabilities constitute, and let A be the matrix of transition probabilities with element a_{sr} . In other words, this is the probability that a subject who uses rule s in period $t-1$ transitions to rule r in the subsequent period, $a_{sr} = \Pr(z_{it} = r | z_{it-1} = s)$. Also, denote μ_{rt} as the proportion of the subjects who are using rule r in period t . And finally, let π_{rt} be the period t probability a randomly selected subject is using rule r , which is

$$\pi_{rt} = \sum_{s \in R} \mu_{st-1} a_{sr}.$$

A key element of our HMM is that subjects follow rules imprecisely, resulting in an unbiased random perturbation to each submitted bid. In particular, we assume that when a subject follows rule r , the price he submits is an independent random variable that follows a normal distribution G with mean $b_r(c_{it})$ and rule-specific variance σ_r^2 . The probability densities of the three bidding rules are

$$\begin{aligned} h_{AM}(p_{it} | c_{it}, \kappa) &= G(c_{it} + \kappa, \sigma_{AM}^2), \\ h_{PM}(p_{it} | c_{it}, \rho) &= G((1 + \rho)c_{it}, \sigma_{PM}^2), \quad \text{and} \\ h_{BR}(p_{it} | c_{it}, \Pi_t, \kappa, \rho) &= G\left(\frac{(c_H + \pi_{AMt}\kappa)(1 + \rho)}{(1 + \rho(1 - \pi_{PMt}))M} + \frac{M-1}{M}c_{it}, \sigma_{BR}^2\right). \end{aligned}$$

The set of parameters of this model is $\Theta = \{\Theta_M, \Theta_B\}$, where Θ_M is the set of variables governing the stochastic process of rule adoption, $\{\Pi_1, A\}$, and Θ_B is the set of parameters that determine the bidding rules, $\{\kappa, \rho, \eta, \sigma_r\}$. The likelihood of subject i 's

sequence of prices conditional on the parameters and the realization of z_i is

$$L_i(p_{i1}, \dots, p_{i30} | z_i, \Theta) = \prod_{t=1}^{30} h_{z_{it}}(p_{it} | c_{it}, \Theta),$$

and the likelihood of z_i conditional on Θ_B is

$$L_i(z_i | \Theta_M) = \pi_{z_{i1}} \prod_{t=2}^{30} a_{z_{it-1}, z_{it}}.$$

We can express the likelihood of the joint sequences of bids and bidding rule adoption as

$$L_i(p_{i1}, \dots, p_{i30}, z_i | \Theta) = L_i(p_{i1}, \dots, p_{i30} | z_i, \Theta) L_i(z_i | \Theta_M).$$

However, because the realized sequence of states is not observable, we need to integrate out the marginal likelihood of z_i by summing over all possible sequences of rule adoption:

$$L_i(p_{i1}, \dots, p_{i30} | \Theta) = \sum_{z_i \in Z} L_i(p_{i1}, \dots, p_{i30} | z_i, \Theta) L_i(z_i | \Theta_M).$$

Finally, the likelihood for the whole sample is

$$L(X | \Theta) = \prod_{i=1}^{72} \sum_{z_i \in Z} L_i(p_{i1}, \dots, p_{i30} | z_i, \Theta) L_i(z_i | \Theta_M). \quad (1)$$

A common way to find the parameter values that maximize Equation (1) is to use the Baum–Welch algorithm, which is a special case of the EM algorithm.⁹ The output of this algorithm provides posterior modes of Θ_M , an estimated most likely path \hat{z}_i for each subject, and maximum likelihood estimates of Θ_B .

One issue remains that prevents us from estimating the model as specified; the intercept term of strategic bidding rule depends on the distribution over types and thus is a function of the parameters in Θ_M . To address this issue, we change the specification of b_{BR} to a fixed-effect model:

$$b_{BR}(c_{it}) = \sum_{d=1}^{30} I_{(d=t)} \alpha_t + \frac{M-1}{M} c_{it},$$

where $M = \eta(N-1) + 1$, (2)

and I is an indicator function. We call this specification HMM with fixed effects specifications (HMM FE hereafter).

Table 3 presents the maximum likelihood estimates of the parameters of each of the three bidding rules. Note that all parameter estimates are significant. For the AM rule, we see the estimated markup is a mere

⁸ For example, Netzer et al. (2008) study the relationships between charitable donors and institutions, and Hui et al. (2009) present an HMM framework for general forms of path data over time. However, ours is the first example we know of that defines the state space as the strategies within a multiperson interaction and the Harrobian adjustments as an adaptive learning process.

⁹ For details on these two algorithms, we refer the reader to the classic presentation in Rabiner (1989).

Table 3 Estimates of Bidding Rule Parameters in the HMM FE

Model	Intercept	<i>t</i> -Statistic	Slope	<i>t</i> -Statistic	σ_r^2
AM	0.80	37.36	1.00	—	0.50
PM	0.00	—	1.30	10.93	5.27
BR	4.43	25.59	0.79	35.54	1.30

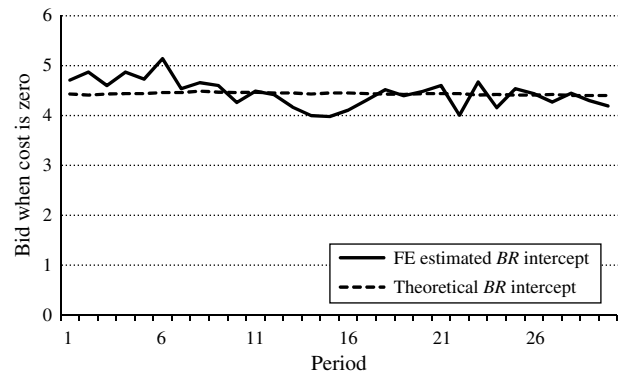
Note. The dash for *t*-statistic indicates that this parameter is constrained and is the given value by the model.

80 cents, which suggests quite aggressive bidding to win the FPA. On the other hand, for the *PM* rule the estimated markup is 30% over costs, which is consistent with bidding aggressively for a large reward from winning the auction. The standard error for this rule is much higher than the standard errors of the other two rules, symptomatic of possible heterogeneity in the percentage markup across subjects. For the *BR* bidding rule, the inferred value of η is 1.84, which yields a constant coefficient of relative risk aversion of 0.56—consistent with the estimates from other studies of private value first-price sealed-bid auctions.¹⁰

After estimating the HMM FE, we calculate the theoretical values of the intercept of b_{BR} for each period t based on the maximum likelihood estimates of the markups and the proportions of subjects estimated to be using each rule in period t according to the \hat{z}_i . Then we compare these theoretical calculated intercepts to the estimated dummy variables. Figure 7 presents a comparison of these estimated fixed effects versus the theoretical values of the *BR* rule intercepts. Visual inspection reveals that the theoretical value of the intercept varies little over time and that the dummy variable model also does not vary a lot. In fact, only three of the fixed effects are significantly different than the mean effect, suggesting that there is little dramatic movement in the use of different rules over time. We now examine this more explicitly.

The posterior mode estimates of the transition probabilities reveal three surprising results: (1) there is inertia in rule adoption, (2) subjects only switch between the sophisticated rule and a rule of thumb—or vice versa—but never switch between the two simple rules of thumb, and (3) overall subject sophistication *decreases* over time. The estimated initial distribution over the rules is $\hat{\Pi}_1 = (\hat{\pi}_{AM1}, \hat{\pi}_{PM1}, \hat{\pi}_{BR1}) = (0.12, 0.14, 0.74)$, and the estimated matrix of transition probabilities \hat{A} is given in Table 4. The strong inertia in the rules subjects follows from the high estimated continuation probabilities that range from 78% to 88%. The estimated transition matrix also indicates that there is no switching between the two simple

Figure 7 Theoretical Values of the *BR* Rule Intercept Over Time and Estimated Fixed Effects of the *BR* Rule



rules of thumb. The only rule transitions are between *AM* and *BR* and *PM* and *BR*. While it is not surprising that subjects switch from simple markup price rules to the more strategically sophisticated strategy, it is surprising that there are subjects who switch from the *BR* rule to simple markup strategies. In these cases, it is mostly the absolute markup rule that is switched to. Why would someone make such a switch? A natural conjecture is that estimated *AM* rule has a small markup and that following that rule will lead to more frequent wins in the auction.¹¹

The dynamics of the estimated model suggest that the rule adjustments mostly occur early in the experiment and involve a decaying proportion of sophisticated sellers. In Figure 8, we present the sequence of estimated proportions of subjects for each period in time, μ_{rt} . Notable here is that initially 76% of the subjects are estimated to be using a strategic best-response strategy, but that proportion over the course of 30 auctions *falls* to 64%. Meanwhile, we see an accordingly strong increase in the proportion of *AM* bidders from 11% to 29%, and the percentage of *PM* bidders falls from 13% to 9%. In this estimated rule-switching model, would the subjects learn to be more sophisticated in a much longer sequence of auctions? Actually, the prospect of further learning is not strong. The limiting distribution of the matrix of transition probabilities is $\Pi_\infty = (0.28, 0.10, 0.62)$. Inspection of Figure 8 shows that the experiment has already converged close to these values.

Does this estimated model explain the deviations of price performance from the predictions of the risk-neutral Nash equilibrium model? The sample mean has a small but statistically significant negative time trend, and the sample variance appears to have a

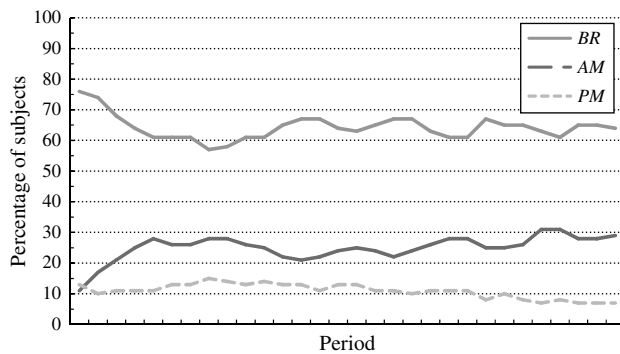
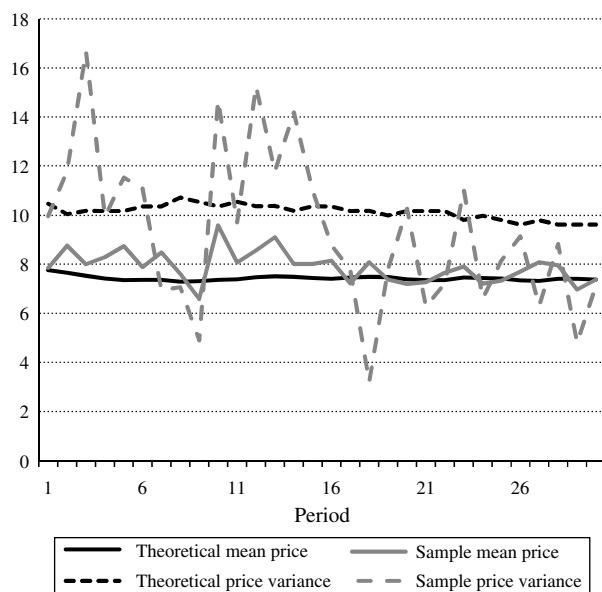
¹⁰ For example, consider some other estimates of constant coefficient of relative risk aversion of 0.67, 0.52, 0.48, and 0.57, respectively, from Cox and Oaxaca (1996), Goeree et al. (2002), Chen and Plott (1998), and Berg et al. (2005).

¹¹ The natural way to explore hypotheses of this nature would be to model the transition probabilities as functions of the history of actions and payoffs. Unfortunately, there simply is not enough variation in the rule switching to generate estimated transitional probabilities that have covariates.

Table 4 Posterior Mode Estimates of Rule-Switching Probabilities

	$Z_{it} = AM$	$Z_{it} = PM$	$Z_{it} = BR$
$Z_{it-1} = AM$	0.78	0.00	0.22
$Z_{it-1} = PM$	0.00	0.87	0.13
$Z_{it-1} = BR$	0.10	0.02	0.88

negative time trend but is not significant. Figure 9 depicts the theoretical mean and variance calculated using the estimated rule-switching model parameters as well as the empirical measurements of price mean and variance. Visual inspection suggests that the rule-switching model qualitatively captures the values and trend, and we find this encouraging. However, regressions of the observed averages on the theoretical predictions are not significant—although the coefficients are close to one in both cases. For this experiment the time variance of price average and variance dynamics

Figure 8 The Estimated Sequence of Proportions of Subjects Using the Three Rules**Figure 9** Empirical and Theoretical Means and Variances of Price

Note. Theoretical values are calculated using HMM parameter estimates.

adjust too little over time to provide a dynamic model a strong opportunity to explain, or fail to explain, these dynamics. There is a plan to run future experiments that we hope will create such an opportunity.

8. Specification Test of the Dynamic Rule-Switching HMM

Our HMM framework provides a behaviorally intuitive formulation and a tractable extension to both strategic heterogeneity and learning in FPAs, and it can be estimated with well-developed econometric tools. Furthermore, the insights it provides into bidding behavior, the mixture of strategic/heuristic bidders, and the learning to be unsophisticated are thought provoking and could reorient the direction of behavioral game-theoretic models of auctions. These successes, however, are predicated on the HMM FE being a better specification than other existing alternative models. In this penultimate section, we report on a series of specification hypothesis tests in which we evaluate the HMM FE against an assortment of alternative models.

The first set of alternative specifications we consider is a variety of static models that are nested within the HMM FE; appropriate restrictions to parameters in the HMM FE yield each of the alternative models. Each of these static models requires that the Markov transition matrix be restricted to the identity matrix, whose size equals the number of bidding rules in the alternative. We evaluate the following homogeneous static models: linear bidding, a Nash equilibrium bidding rule (linear bidding with the restriction that the highest-cost-type demands zero profit), *PM*, and *AM*. Each of these models is estimated by maximum likelihood and the results reported in columns 2–5 of Table 5. For each of the rules, we report estimates of the appropriate intercepts, coefficients, and standard deviations of errors in terms of our HMM parameter labels. We also report the number of free parameters of the model in the row labeled “Parameter count.” For evaluating specifications, in the last three rows we report the log-likelihood value of the estimated model, the Akaike information criteria (AIC), and the likelihood ratio test statistic along with the result of the hypothesis test that the restricted model is true versus the unrestricted HMM FE. For reference, we provide the relevant results of the estimated HMM FE in the last column. Clearly, the likelihood ratio test soundly rejects each of the homogeneous static models we consider, which is confirmed by the relative ranking of the AIC values.

Next we consider a subset of nondegenerate static mixture models that are a special case of the HMM FE. In particular, we consider the mixture of *PM* & *BR*,

Table 5 Specification Tests of the HMM FE with Respect to Alternative Static Pure and Mixture Models

	Linear model	Restricted linear model $b_{BR}(20) = 20$	Pure PM	Pure AM	Mixture $PM\&BR$	Mixture $AM\&BR$	Full mixture	HMM FE
<i>Intercept</i>	4.04	4.30	—	—	3.78	3.79	4.40	4.71
<i>Cost</i>	0.83	0.79	—	—	0.82	0.82	0.79	0.79
σ_{BR}	2.14	2.15	—	—	1.44	1.40	1.49	1.30
$(1 + \rho)$	—	—	1.13	—	1.23	—	1.23	1.30
σ_{PM}	—	—	2.96	—	4.38	—	4.44	5.20
κ	—	—	—	2.31	—	3.52	1.16	0.82
σ_{AM}	—	—	—	2.37	—	3.49	0.98	0.50
π_{BR1}	1.00 ^a	1.00 ^a	—	—	0.81	0.78	0.61	0.74
π_{PM1}	—	—	1.00 ^a	—	0.19	—	0.18	0.14
π_{AM1}	—	—	—	1.00 ^a	—	0.22	0.21	0.12
Parameter count	3	2	2	2	6	6	9	44
Log-likelihood	−4,704	−4,720	−5,404	−4,923	−4,359	−4,275	−4,231	−3,943
AIC	9,414	9,445	10,811	9,850	8,731	8,561	8,480	7,974
LRT: model vs. HMM FE	1,522***	1,556***	2,921***	1,960***	832***	663***	576***	—

Note. LRT, likelihood ratio test.

^aIndicates a value that is set by model constraint and not estimated.

***Statistically significant at the 1% level; **statistically significant at the 5% level; *statistically significant at the 10% level.

the mixture of $AM\&BR$ (this is the Kirchkamp and Reiss 2008 model for multiple bidders), and the full mixture of all three models. In addition to the previous pointed-out parameter estimates, we also report the estimated mixture probabilities that in the case of the HMM FE are the initial probabilities, Π_1 . Each of the static mixture models is estimated by the EM algorithm. Here, we once again soundly reject each static mixture model in favor of the dynamic HMM FE in both likelihood ratio tests and in AIC comparison.

The level- k model of Crawford and Iriberri (2007) is an obvious alternative model for comparison. In the static formulation, the HMM FE and level- k model follow a similar approach; however, several key differences should be raised before making this comparison. First, if one uses Crawford and Iriberri's specified $k = 0$ rules in our setting, then the corresponding $k = 1$ strategies are simply the Nash equilibrium ones.¹² To avoid making this comparison trivial, we adopt the AM and PM as the $k = 0$ rules. Second, there are two $k = 1$ types: BR_{PM} , who believes with probability 1 the other bidders follow the PM rule; and BR_{AM} , who believes with probability 1 the other bidders follow the AM rule. This is in contrast to the HMM FE in which the BR bidders form accurate beliefs regarding the distribution over the bidding rules at each point in time. A consequence of having two $k = 1$ types is that neither the level- k model nor the HMM FE nests the other. Third, the

level- k model is a formulation of how individuals initially bid in an FPA. Correspondingly, we introduce an additional model that expands the scope of the level- k model by incorporating an exogenous first-order Markov rule-switching process; we call this the level- k Markov model. This will allow us to separate differences in model performance that relate to the bidding rule structure and subjects' beliefs from those that relate to adaptation through rule switching.

The Baum–Welch algorithm estimates of the relevant level- k and level- k Markov model parameters and other statistical measures are presented in the first two columns of Table 6. Because we cannot apply a likelihood ratio for model specification, we report the values of the Bayesian information criteria (BIC). The estimation results for our version of the static level- k are strikingly similar to those of Crawford and Iriberri (2007). We find our estimated PM rule has a markup of $\rho = 0$, which collapses to their bid-your-cost type $k = 0$. Also, we find that no subjects follow the PM rule nor the best response to our AM rule BR_{AM} . The subjects are roughly divided between the AM rule and the best response to PM (which is the Nash bidding strategy). However, the model performs poorly against the HMM FE when comparing the respective AIC and BIC values. When we allow rule switching in the level- k Markov model, we observe a sharp increase in the log-likelihood value, and the parameter estimates move toward those in our HMM models. But we can see this model specification still fares poorly with respect to our HMM FE in both the AIC and BIC.

Finally, we consider dynamic mixture specifications that are nested within the HMM FE. We consider three such models: an HMM with $BR\&PM$, an HMM

¹² When types are independent draws from identical uniform distributions, the Nash equilibrium strategy is also a best response against any linear strategy for which the bidder bids his own value for the least competitive type.

Table 6 Specification Tests of the HMM FE with Respect to Level- k and Alternative Dynamic Mixture Models

	Level- k		Level- k Markov		HMM	HMM	HMM	HMM FE
	BR_{PM}	BR_{AM}	BR_{PM}	BR_{AM}	$BR\&PM$	$BR\&AM$		
<i>Intercept</i>	6.67 ^a	7.19	8.61	7.08	3.93	3.61	4.44	4.71
<i>Cost</i>	0.67 ^a	0.67 ^a	0.67 ^a	0.67 ^a	0.83	0.84	0.79	0.79
σ_{BR}	2.66	0.76	5.21	1.74	1.19	1.04	1.30	1.30
$(1 + \rho)$		1.00		1.29	1.28	—	1.29	1.30
σ_{PM}		0.82		1.57	4.99	—	5.13	5.20
κ		1.57		1.24	—	4.28	0.81	0.82
σ_{AM}		1.33		0.94	—	3.44	0.50	0.50
π_{BR1}	0.54	0.00	0.09	0.35	0.79	0.71	0.73	0.74
π_{PM1}		0.00		0.09	0.21	—	0.17	0.14
π_{AM1}		0.46		0.47	—	0.29	0.10	0.12
Parameter count		11		23	8	8	15	44
Log-likelihood		−4,538		−4,224	−4,182	−4,061	−3,964	−3,943
AIC		9,094		8,491	8,439	8,197	7,958	7,974
BIC		8,610		8,610	8,649	8,407	8,043	8,224
LRT: model vs. HMM FE		—		—	478***	236***	42*	—

Note. LRT, likelihood ratio test.

^aIndicates a value that is set by model constraint and not estimated.

***Statistically significant at the 1% level; **statistically significant at the 5% level; *statistically significant at the 10% level.

with $BR\&AM$, and an HMM with all three bidding rules. Both HMM models with only two bidding rules are strongly rejected by the likelihood ratio test. On the other hand, the pure HMM is only rejected at the 10% level of significance by the likelihood ratio test. This is not surprising, as we reported earlier that only 3 of the 29 effects were statistically significant. However, we still believe the HMM FE retains value as it allows the BR rule to track changes in the relative proportions at which bidders adopt the differing bidding rules. In conclusion, the battery of specification test suggests that the HMM and HMM FE approaches are a strong improvement in the explanations of heterogeneity and learning in FPAs.

9. Discussion

The insights of this study should prove valuable to both the procurement practitioner and the researcher interested in behavioral models of bidding in auctions. Our experiments provide a clear demonstration of the relative performance of the FPA versus the EA in a commodity procurement setting. In light of these results, any procurement organization should proceed cautiously when initiating a new auction strategy for the purchase of commodities. From the procurement organization's perspective, EAs not only deliver higher expected prices but also greater price variability. It is worth noting that increased price variability may have a negative impact beyond a procurement organization's distaste of increased price uncertainty. Recent studies, such as Jap (2007) and Jap and Haruvy (2008), document that the relationship between the supplier and buyer is negatively impacted by the adoption of the EA. The documented increase in price

variance could be a partial source of this increased animosity.

Despite this negative result, we can offer some thoughts regarding when changing from an FPA to an EA might be successful. In this study we make several strong assumptions, and relaxing any of these could lead to different outcomes. For example, we assume suppliers independently draw costs from the same distribution. However, a likely scenario is that one supplier has a clear advantage in location and a likely lower cost; in this case it is not clear which auction format is better. Theoretical studies, such as Maskin and Riley (2000) and Cantillon (2008), suggest that whether an EA will lead to a lower expected price depends crucially on the distributions of costs. One area where EA-like auctions show promise is in the procurement of goods for which price is not the sole differentiating attribute between suppliers. Researchers have studied two cases: when nonprice attributes are exogenous and when these attributes are endogenously determined within the auction. In the first case, Engelbrecht-Wiggans and Katok (2007) find significant gains to the procurement official when suppliers bid on price, and then the buyer chooses the winner (as opposed to awarding the contract to the lowest bidder). The performance of the buyer determined winner EA versus the FPA in these settings is studied in Haruvy and Katok (2008), and they find the FPA is better if suppliers have accurate and precise information regarding the quality of other sellers. On the other hand, Shachat and Swarthout (2010) find that an EA with buyer-assigned bidding credits can provide better outcomes than the FPA for both the buyer and the suppliers. For the latter case, there is also a large and promising literature on successful dynamic reverse auction examples where the

quality is determined within multiattribute versions of the EA—for example, Chen-Ritzo et al. (2005) and Parkes and Kalagnanam (2005).

Our HMM of sophisticated and rule-of-thumb bidders both answers some questions and raises others. Our results provide strong evidence for discrete heterogeneous bidding behavior, and the procurement setting has allowed for easy identification of some bidding rules of thumb. Furthermore, we make a significant advancement in the learning of bidding behavior through the introduction of a rule-switching dynamic. Somewhat disconcerting is the conclusion that individuals unlearn strategic sophistication as they gain experience. Clearly, a next step is to generalize the transition probabilities from being purely exogenous to being functions of the subjects' experiences. There must be something in the way participants cognitively process feedback that leads them from the expected utility-maximizing strategy. One conjecture we find attractive is that subjects more strongly react to whether they win or lose when following a particular strategy than they do to the full opportunity costs of following alternative strategies. To test such a conjecture, as well as others, one would need experimental sessions with many more repetitions to obtain significant estimates on transition probability covariates. Another direction of inquiry is to ask whether the simple bidding heuristics we observe also hold in the forward auction case. The experiments and subsequent analysis in Kirchkamp and Reiss (2008) suggest that they do, particularly the absolute markup strategy. But to identify the percentage markup strategy, experiments need to be run for which the number of auction participants is a within-subject treatment variable. Finally, an important question is whether one can use a model like ours in a structural econometric estimation to recover individual parameters and improve upon approaches that assume pure Nash equilibrium play.

Acknowledgments

Ernan Haruvy, J. Todd Swarthout, John Wooders, James C. Cox, Elana Katok, Mark Walker, Peter Reiss, Eric T. Bradlow, Teck Ho, and especially two anonymous referees provided helpful comments on earlier drafts of the paper. The paper has benefitted from audience members' comments in departmental seminars at Georgia State University, the University of Arizona, Xiamen University, Shanghai University of Finance and Economics, Shanghai Jiao Tong University, and others. The authors thank the University of California, San Diego, and the National University of Singapore for allowing them to conduct experiments in their facilities. For financial support, they also thank Xiamen University and the Fujian Province High Talent Program. The authors also acknowledge the IBM T. J. Watson Research Lab for financial support and use of their software.

Appendix. Derivation of the Best-Response Bidding Rule

Here, we derive the best response (BR) bidding rule assuming the multinomial distribution function Π over the set of bidding rules $R = \{AM, PM, BR\}$. For now, let us assume that a BR bidder prices according to a strictly increasing bounded function, thus having an inverse, denoted as $b_{BR}(c_i)$. When a BR bidder formulates his bidding strategy, it is assumed that he knows the parameters Π , κ , and ρ . Also assume that he also has the von Neumann–Morgenstern expected utility function $U(y) = \eta y^{\frac{1}{\eta}}$, where y is a nonnegative change in wealth. If the BR wins the forward auction, his award is his value minus his bid; otherwise, there is no award. The BR bidder's expected utility is the utility of his profit times the probability of winning the FPA, i.e., the probability his bid is lower than all other bids. This probability is

$$\Pr(p_j < p_i) = (1 - \pi_{AM} - \pi_{PM}) \cdot \Pr(p_j < b_{BR}(c_i)) + \pi_{AM} \cdot \Pr(p_j < c_i + \kappa) + \pi_{PM} \cdot \Pr(p_j < (1 + \rho)c_i).$$

As each of the three bidding rules has an inverse, we can restate this probability as

$$\Pr(p_j < p_i) = (1 - \pi_{AM} - \pi_{PM}) \cdot F(b_{BR}^{-1}(p_j)) + \pi_{AM} \cdot F(p_j - \kappa) + \pi_{PM} \cdot F\left(\frac{p_j}{(1 + \rho)}\right),$$

where F denotes the uniform distribution. In an FPA with $N - 1$ other suppliers, the probability of winning with a submitted price p_j is

$$\Pr(p_j < p_i; i = 1, \dots, N - 1) = [\Pr(p_j < p_i)]^{N-1}.$$

Thus the expected utility of a bid p_j conditional on realized value c_j is

$$E[U(p_j | c_j)] = \eta(p_j - c_j)^{1/\eta} \cdot \left[(1 - \pi_{AM} - \pi_{PM}) \cdot \frac{c_H - b_{BR}^{-1}(p_j)}{c_H - c_L} + \pi_{AM} \cdot \frac{c_H - (p_j - \kappa)}{c_H - c_L} + \pi_{PM} \cdot \frac{c_H - (p_j/(1 + \rho))}{c_H - c_L} \right].$$

The first-order condition of maximizing expected utility with respect to the bid price is

$$\begin{aligned} & \frac{1}{\eta} \left[(1 - \pi_{AM} - \pi_{PM}) \cdot (c_H - b_{BR}^{-1}(p_j)) + \pi_{AM} \cdot (c_H - (p_j - \kappa)) + \pi_{PM} \cdot \left(c_H - \frac{p_j}{(1 + \rho)} \right) \right] \\ & = (N - 1)(p_j - c_j) \left[\frac{1 - \pi_{AM} - \pi_{PM}}{b'_{BR}(c_j)} + \pi_{AM} + \frac{\pi_{PM}}{1 + \rho} \right]. \end{aligned}$$

Now if one assumes that all the BR bidders are using the same bid function, the above expression reduces to the following differential equation:

$$\begin{aligned} & \frac{c_H - c_j + \pi_{AM}\kappa + (\rho/(1 + \rho))\pi_{PM}}{p_j - c_j} \\ & = \frac{\eta(N - 1)(1 - \pi_{AM} - \pi_{PM})}{b'_{BR}(c_j)} + (\eta(N - 1) + 1) \left(\pi_{AM} + \frac{\pi_{PM}}{1 + \rho} \right). \end{aligned}$$

The solution and BR bidding function is

$$b_{BR}(c_j) = \frac{(c_H + \pi_{AM} \cdot \kappa) \cdot (1 + \rho)}{(1 + \rho \cdot (1 - \pi_{PM})) \cdot M} + \frac{M}{(M - 1)} \cdot c_j,$$

where $M = \eta \cdot (N - 1)$.

References

- Abbas, A. E., I.-H. Hann. 2010. Measuring risk aversion in a name-your-own-price channel. *Decision Anal.* 7(1) 123–136.
- Armantier, O., N. Treich. 2009a. Star-shaped probability weighting functions and overbidding in first-price auctions. *Econom. Lett.* 104(2) 83–85.
- Armantier, O., N. Treich. 2009b. Subjective probabilities in games: An application to the overbidding puzzle. *Internat. Econom. Rev.* 50(4) 1079–1102.
- Bajari, P., A. Hortacsu. 2005. Are structural estimates of auction models reasonable? Evidence from experimental data. *J. Political Econom.* 113(4) 703–741.
- Barrett, G. F., S. G. Donald. 2003. Consistent tests for stochastic dominance. *Econometrica* 71(1) 71–104.
- Berg, J., J. Dickhaut, K. McCabe. 2005. Risk preference instability across institutions: A dilemma. *Proc. Natl. Acad. Sci. USA* 102(11) 4209–4214.
- Bradlow, E. T., Y.-H. Park. 2007. Bayesian estimation of bid sequences in Internet auctions using a generalized record-breaking model. *Marketing Sci.* 26(2) 218–229.
- Brown, J., T. Hossain, J. Morgan. 2010. Shrouded attributes and information suppression: Evidence from the field. *Quart. J. Econom.* 125(2) 859–876.
- Camerer, C. F., T.-H. Ho. 1999. Experience-weighted attraction learning in normal form games. *Econometrica* 67(4) 827–874.
- Cantillon, E. 2008. The effect of bidders' asymmetries on expected revenue in auctions. *Games Econom. Behav.* 61(1) 1–25.
- Chafkin, M. 2007. Reverse auctions: A supplier's survival guide. *Inc.* (April 25) <http://inc.com/incr/2007/04/reverse-auctions-a-suppliers-survival-guide.html>.
- Chan, T. Y., V. Kadiyali, Y.-H. Park. 2007. Willingness to pay and competition in online auctions. *J. Marketing Res.* 44(2) 324–333.
- Chen, K.-Y., C. R. Plott. 1998. Nonlinear behavior in sealed bid first price auctions. *Games Econom. Behav.* 25(1) 34–78.
- Chen-Ritzo, C.-H., T. P. Harrison, A. M. Kwasnica, D. J. Thomas. 2005. Better, faster, cheaper: An experimental analysis of a multiattribute reverse auction mechanism with restricted information feedback. *Management Sci.* 51(12) 1753–1762.
- Coppinger, V. M., V. L. Smith, J. A. Titus. 1980. Incentives and behavior in English, Dutch, and sealed-bid auctions. *Econom. Inquiry* 18(1) 1–22.
- Cox, J., R. L. Oaxaca. 1996. Is bidding behavior consistent with bidding theory for private value auctions? R. M. Issac, ed. *Research in Experimental Economics*, Vol. 6. JAI Press, Greenwich, CT, 131–148.
- Cox, J. C., B. Roberson, V. L. Smith. 1982. Theory and behavior of single object auctions. V. L. Smith, ed. *Research in Experimental Economics*, Vol. 2. JAI Press, Greenwich, CT, 537–579.
- Crawford, V. P., N. Iriberri. 2007. Level- k auctions: Can a nonequilibrium model of strategic thinking explain the winner's curse and overbidding in private-value auctions? *Econometrica* 75(6) 1721–1770.
- Dholakia, U. M., I. Simonson. 2005. The effect of explicit reference points on consumer choice and online bidding behavior. *Marketing Sci.* 24(2) 206–217.
- Dittrich, D. A. V., W. Guth, M. Kocher, P. Pezanis-Christou. 2011. Loss aversion and learning to bid. *Economica*, ePub ahead of print August 18.
- Elmaghraby, W. 2007. Auctions within e-sourcing events. *Production Oper. Management* 16(4) 409–422.
- Engelbrecht-Wiggans, R., E. Katok. 2007. Regret in auctions: Theory and evidence. *Econom. Theory* 33(1) 81–101.
- Engelbrecht-Wiggans, R., E. Katok. 2008. Regret and feedback information in first-price sealed-bid auctions. *Management Sci.* 54(4) 808–819.
- Fay, S. 2004. Partial-repeat-bidding in the name-your-own-price channel. *Marketing Sci.* 23(3) 407–418.
- Filiz-Ozbay, E., E. Y. Ozbay. 2007. Auctions with anticipated regret: Theory and experiment. *Amer. Econom. Rev.* 97(4) 1407–1418.
- Goeree, J., C. A. Holt, T. R. Palfrey. 2002. Quantal response equilibrium and overbidding in private-value auctions. *J. Econom. Theory* 104(1) 247–272.
- Guerre, E., I. Perrigne, Q. Vuong. 2009. Nonparametric identification of risk aversion in first-price auctions under exclusion restrictions. *Econometrica* 77(4) 1193–1227.
- Guth, W., R. Ivanova-Stenzel, M. Königstein, M. Strobel. 2003. Learning to bid—An experimental study of bid function adjustments in auctions and fair division games. *Econom. J.* 113(487) 477–494.
- Haruvy, E., E. Katok. 2008. An experimental investigation of buyer determined procurement auctions. Working paper, Pennsylvania State University, University Park.
- Hendricks, K., R. H. Porter. 1988. An empirical study of an auction with asymmetric information. *Amer. Econom. Rev.* 78(5) 865–883.
- Holt, C. A., Jr. 1980. Competitive bidding for contracts under alternative auction procedures. *J. Political Econom.* 88(3) 433–445.
- Hossain, T., J. Morgan. 2006. ...Plus shipping and handling: Revenue (non) equivalence in field experiments on eBay. *B. E. J. Econom. Anal. Policy* 6(2). http://works.bepress.com/john_morgan/1.
- Hui, S. K., P. S. Fader, E. T. Bradlow. 2009. Path data in marketing: An integrative framework and prospectus for model building. *Marketing Sci.* 28(2) 320–335.
- James, D., D. Reagle. 2009. Experience weighted attraction in the first price auction and Becker Degroot Marschak. R. S. Anderssen, R. D. Braddock, L. T. H. Newham, eds. *18th World IMACS Congress MODSIM09 Internat. Congress Modelling Simulation*, Modelling and Simulation Society of Australia and New Zealand and International Association for Mathematics and Computers in Simulation, 1437–1441.
- Jap, S. D. 2002. Online reverse auctions: Issues, themes, and prospects for the future. *J. Acad. Marketing Sci.* 30(4) 506–525.
- Jap, S. D. 2007. The impact of online reverse auction design on buyer-supplier relationships. *J. Marketing* 71(1) 146–159.
- Jap, S. D., E. Haruvy. 2008. Interorganizational relationships and bidding behavior in industrial online reverse auctions. *J. Marketing Res.* 45(5) 550–561.
- Jap, S. D., P. A. Naik. 2008. BidAnalyzer: A method for estimation and selection of dynamic bidding models. *Marketing Sci.* 27(6) 949–960.
- Kagel, J. H. 1995. Auctions: A survey of experimental research. J. H. Kagel, A. E. Roth, eds. *The Handbook of Experimental Economics*. Princeton University Press, Princeton, NJ, 501–586.
- Kagel, J. H., D. Levin. 1993. Independent private value auctions: Bidder behavior in first-, second-, and third-price auctions with varying numbers of bidders. *Econom. J.* 103(419) 868–879.
- Kagel, J. H., D. Levin. 2008. Auctions: Experiments. S. N. Durlauf, L. E. Blume, eds. *The New Palgrave Dictionary of Economics*, 2nd ed. Palgrave Macmillan, Basingstoke, Hampshire, UK. DOI:10.1087/9780230226203.0072.
- Kagel, J. H., D. Levin. 2012. Auctions: A survey of experimental research 1995–2008. J. H. Kagel, A. E. Roth, eds. *The Handbook of Experimental Economics*, Vol. 2. Princeton University Press, Princeton, NJ. Forthcoming.
- Kirchkamp, O., J. P. Reiss. 2008. Heterogeneous bids in auctions with rational and markdown bidders—Theory and experiment. Jena Economic Research Paper 2008-066, Friedrich-Schiller-University Jena, Max-Planck-Institute of Economics, Jena, Germany.
- Krasnokutskaya, E. 2011. Identification and estimation of auction models with unobserved heterogeneity. *Rev. Econom. Stud.* 78(1) 293–327.

- Krishna, V. 2002. *Auction Theory*. Academic Press, London.
- Laffont, J.-J., H. Ossard, Q. Vuong. 1995. Econometrics of first-price auctions. *Econometrica* **63**(4) 953–980.
- Levy, H. 1992. Stochastic dominance and expected utility: Survey and analysis. *Management Sci.* **38**(4) 555–593.
- Lucking-Reiley, D. 1999. Using field experiments to test equivalence between auction formats: Magic on the Internet. *Amer. Econom. Rev.* **89**(5) 1063–1080.
- Maskin, E. S., J. G. Riley. 2000. Asymmetric auctions. *Rev. Econom. Stud.* **67**(3) 413–438.
- Morgan, J., K. Steiglitz, G. Reis. 2003. The spite motive and equilibrium behavior in auctions. *B. E. J. Econom. Anal. Policy* **2**(1) Article 5.
- Myerson, R. B. 1981. Optimal auction design. *Math. Oper. Res.* **6**(1) 58–73.
- Netzer, O., J. M. Lattin, V. Srinivasan. 2008. A hidden Markov model of customer relationship dynamics. *Marketing Sci.* **27**(2) 185–204.
- Neugebauer, T., J. Perote. 2008. Bidding “as if” risk neutral in experimental first price auctions without information feedback. *Experiment. Econom.* **11**(2) 190–202.
- Neugebauer, T., R. Selten. 2006. Individual behavior of first-price auctions: The importance of information feedback in computerized experimental markets. *Games Econom. Behav.* **54**(1) 183–204.
- Ockenfels, A., R. Selten. 2005. Impulse balance equilibrium and feedback in first price auctions. *Games Econom. Behav.* **51**(1) 155–170.
- Park, Y.-H., E. T. Bradlow. 2005. An integrated model for bidding behavior in Internet auctions: Whether, who, when, and how much. *J. Marketing Res.* **42**(4) 470–482.
- Parkes, D. C., J. Kalagnanam. 2005. Models for iterative multiattribute procurement auctions. *Management Sci.* **51**(3) 435–451.
- Purchasing. 2001. E-auction playbook. (June 21) S1–S3.
- Rabiner, L. R. 1989. A tutorial on hidden Markov models and selected applications in speech recognition. *Proc. IEEE* **77**(2) 257–286.
- Selten, R., J. Buchta. 1998. Experimental sealed bid first price auctions with directly observed bid functions. D. V. Budescu, I. Erev, R. Zwick, eds. *Games and Human Behavior: Essays in Honor of Amos Rapoport*. Lawrence Erlbaum Associates, Mahwah, NJ, 79–102.
- Seshadri, S., K. Chatterjee, G. L. Lilien. 1991. Multiple source procurement competitions. *Marketing Sci.* **10**(3) 246–263.
- Shachat, J., J. T. Swarthout. 2010. Procurement auctions for differentiated goods. *Decision Anal.* **7**(1) 6–22.
- Vickrey, W. 1961. Counterspeculation, auctions and competitive sealed tenders. *J. Finance* **16**(1) 8–37.
- Yao, S., C. F. Mela. 2008. Online auction demand. *Marketing Sci.* **27**(5) 861–885.
- Zeithammer, R., C. Adams. 2010. The sealed-bid abstraction in online auctions. *Marketing Sci.* **29**(6) 964–987.