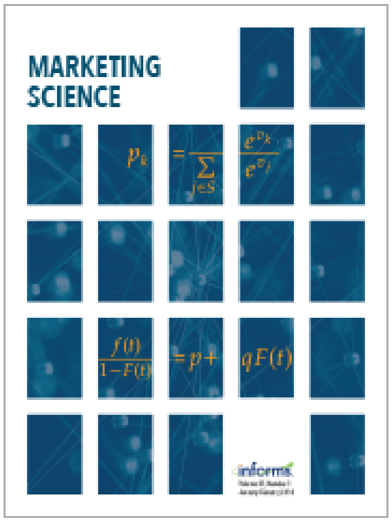

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Optimizing E-tailer Profits and Customer Savings: Pricing Multistage Customized Online Bundles

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Online retailing provides an opportunity for new pricing options that are not feasible in traditional retail settings. This paper proposes an interactive, dynamic pricing strategy from the perspective of customized bundling to derive savings for customers while maximizing profits for electronic retailers ("e-tailers"). Given product costs, posted prices, shipping fees, and customers' reservation prices, we propose a nonlinear mixed-integer programming model to increase e-tailers' profits by sequentially pricing customized bundles. The model is flexible in terms of the number and variety of products customers may choose to incorporate during the various stages of their online shopping. Our computational study suggests that the proposed model not only attracts more customers to purchase the discounted bundle but also noticeably increases profits for e-tailers. This online dynamic bundle pricing model is robust under various bundle sizes and scenarios. It improves e-tailer profit and customer savings the most when facing divergent views about product values, lower budgets, and higher cost ratios.

Key words: e-tailing; online retailing; bundling; customized bundle; multistage dynamic pricing; nonlinear mixed-integer programming; customer budget; reservation price
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1. Introduction

The past decade has witnessed an astonishing growth of Internet retailing. Online shopping has become a daily phenomenon for some consumers, with estimated U.S. Internet sales growing to $172.9 billion in 2010 and expected to reach $250 billion in 2014 (Schonfeld 2010). This trend is projected to continue, and undoubtedly this rapid growth will attract further competition. For instance, when interested in a laptop computer, a customer can find one at all top Internet retailers (hereafter referred to as "e-tailers"). To remain competitive, e-tailers need to continue looking for ways to incentivize existing customers and to attract new ones.

Customer preferences and product prices have been documented as the two main factors influencing purchasing decisions (Keeney 1999). In practice, e-tailers often motivate customers by recommending products of interest and presenting savings opportunities. For example, online recommendation systems (ORS) regularly use a customer's purchase history and online behaviors, expert evaluations, product characteristics, and customer characteristics to recommend products; retailers often promote merchandise through bundling at lower price (Ansari et al. 2000, Bakos and Brynjolfsson 2000). However, the current ORS tend to focus on customers' prior online behavior, with little emphasis on e-tailer profitability or customer savings. Also, the bundles promoted are usually established off-line in advance by e-tailers. Customers do not have the freedom of selecting the content or the total number of items in a bundle. The potential for interactively pricing products in real time is not made available to customers who may be looking for better deals.

Dana (2008) and Elmaghraby and Keskinocak (2003) point out that the extant literature has not fully incorporated pricing strategies into the Internet environment. The detachment of customers' online behavior from retailers' pricing strategies can lead to limitations. Without incorporating customer preferences and savings concerns, systems designed to maximize e-tailers' profits fail to capitalize on customers' interest and to convert online browsers into buyers, whereas focusing on customer savings without linking them to the e-tailer's profit may improve sales volume but not necessarily a firm's profit. In the traditional cross-selling (e.g., an appliance and its extended warranty), customers achieve savings only if they buy the specific package. Such fixed offerings can now be seen as overly rigid because customers receive no savings when a greater number of unbundled products are purchased. To date, e-tailers have implemented a few relatively unsophisticated pricing strategies to encourage customer purchases. For example, Amazon.com's "everyday low price" regularly offers customers to "buy A and get B at an additional 5% off," where the discounted price only links with certain products. Customers gain no savings when other products are chosen.

In this paper, we propose an online dynamic bundle pricing (ODBP) model for e-tailers to exploit the real-time information available from tracking customers' decision-making processes online. We offer e-tailers an interactive pricing scheme and provide a customized price to enhance consumers' savings and maximize e-tailers' profits in a manner not available to bricks-and-mortar retailers. Customers' online shopping behavior is a multistage process where they sequentially add products to a shopping cart and often buy multiple products in one transaction (Häubl and Trifts 2000). After placing a few products in the shopping cart, a customer may choose to remove certain items. Each "add" or "delete" event would update the shopping cart and advance the customer's shopping process to the next "stage." Based on customers' online behavior (e.g., clicking on Web pages, rating products, and/or adding friends in their online social networks), the ORS can capture these real-time events, estimate customers' preferences, update customer models, and generate new recommendation lists. For each recommended item, the ODBP model determines the bundle price by combining the new product with the products already in the cart. Because the bundle price will inherently be cheaper than the sum of the individually posted prices (the standalone selling price) shown on the e-tailer's website, customers are enticed to buy more products in one transaction.

Based on the data collected from Amazon.com and through a numerical study, we found the proposed approach is a "win–win" strategy because the ODBP model provides more profits for e-tailers and extra savings for customers. The contributions of the proposed model are threefold.

1. The model considers customer preferences, customer savings, and e-tailer profits. Compared with existing methods that explore the three aspects independently (e.g., Bodapati 2008, Geng et al. 2005, Ghosh and Balachander 2007), the ODBP model should attract more customers because of its emphasis on customers' savings. Also, because the optimization model focuses on profit maximization and balances between discounts and profits, the price cut will not come at the expense of e-tailer's earnings because of higher sales volume.

2. Traditional pricing models regard customers' purchasing behavior as a buy-or-not-buy one-stage decision process. Our model more realistically allows customers to explore as many times as they please, in terms of the number and variety of products. The ODBP model allows e-tailers to instantaneously provide an attractive discount for any selected product mix. Incorporating real-time pricing capability significantly enhances the information available and provides better customer service.

3. The optimization model and heuristics developed in this paper advance the viability of online pricing. It achieves near-optimal solutions in a negligible time that satisfy the real-time interactive environment while helping customers to make better decisions.

The rest of this paper is organized as follows. In §2 we review the literature and identify the unique characteristics of our model; §3 proposes a nonlinear mixed-integer programming model and solution methodology to interactively solve the online adaptive bundle pricing problem. A computational study is conducted in §4 to understand the benefits of the proposed ODBP model. In §5 we conduct sensitivity analyses and examine the robustness of the model. A summary, conclusions, and future research are given in §6.

2. Literature Review

Customers are key to a firm's survival; as a result, several methods have been proposed to attract new patrons and to maintain old ones. Lu and Moorthy (2007) study the application conditions of coupons and rebates, whereas Subramaniam and Venkatesh (2009) employ auction-based models. Among the available tactics, ORS and pricing strategies are two effective approaches that are gaining popularity. In this section we review the theoretical and practical models of both.

2.1. Online Recommendation Systems

An ORS is a decision aid that analyzes customers' prior online behavior and suggests products to meet the needs of a particular customer (Ansari et al. 2000). Most ORS gather data to extract information and to understand customer preferences, and then they recommend the products most likely to be purchased by the customer based on her preferences as expressed through her online behavior (Huang et al. 2007). Ansari et al. (2000) point out that an ORS should integrate five types of information: preferences of target customers (by detecting customer's purchase history and online behavior), preferences of other customers, expert evaluations, product characteristics, and customer characteristics.

To improve customer acceptance of an ORS, some researchers have focused on ORS design issues to enhance customers' shopping experiences (Cooke et al. 2002), whereas others have examined the impact of recommendation systems on a customer's buying decisions (Fitzsimons and Lehmann 2004, Fleder and Hosanagar 2009).

A practical online shopping aid should consider both customer need and e-tailer want (Bohte et al. 2004). To date, most systems focus on predicting customers' preferences rather than customers' savings and e-tailers' profits (Garfinkel et al. 2008, Chen et al. 2008). Currently, none of the ORS or shopbots (price comparison services) has been developed with such integration. Thus, the existing systems are less successful in translating recommended products into sales than the market potential (Wang et al. 2007, Wu et al. 2008). These findings motivate us to integrate all these aspects into one model so as to address the concerns of both e-tailers and customers.

2.2. Price Differentiation and Bundle Pricing

Price differentiation has been adopted in a number of industries because it is an important strategy that both aids customer retention and creates a competitive advantage (Sahay 2008). All else being equal, economists favor price discrimination (differential pricing) because it is generally welfare-enhancing (Varian 1985). There are three degrees of price discrimination. The first degree relies on consumer identification. Because it appears discordant with current views on privacy, it is the least likely to be adopted. Second-degree discrimination is based on different product versions or quantities and is a more easily justifiable and publicly acceptable method. Using observable group characteristics, third-degree discrimination separates the market into segments, e.g., business versus leisure classes, to maximize a seller's profit. It is also a relatively common market practice.

E-tailers may change prices, either across customers or across products, by dynamically updating the posted prices or by offering auctions or quantity discounts (Kannan and Praveen 2001). For example, Jain and Kannan (2002) study the pricing strategies for information goods online. Khouja and Park (2007) propose that customers with different attitudes toward piracy be charged differently for digital goods such as music and video. In our model, the price difference is due to product variety and quantity chosen, an example of second-degree price discrimination.

With an aim of selling two or more products jointly, bundling is another attractive marketing practice (Ghosh and Balachander 2007, Venkatesh and Mahajan 1993). Bundle researchers determine whether products should be sold as pure components (only individual products), pure bundling (only bundle), or mixed bundling (individual and bundle), as well as the corresponding pricing. Others subject bundling to the constraints of customer demand, arrival process, supply information, and fixed-price bundles (Bakos and Brynjolfsson 2000, Basu and Vitharana 2009, Hitt and Chen 2005). Our multistage online pricing system allows customers to interactively select the products of their choice and provides shoppers with a dynamic price menu in real time.

2.3. Distinctive Features of Our ODBP Model

The ODBP model proposed in this research contains a number of distinctive characteristics compared with prior work. First, the model emphasizes motivating customers, because emphasizing only profits without inspiring customers to participate is less likely to improve sales, ceteris paribus. Given the posted prices of the recommended products, the ODBP determines the bundle prices most likely to entice customers and to generate profits. Second, traditional pricing models regard customers' purchasing behavior as a one-stage process, where they decide whether or not to buy a product or a bundle as a single decision. However, given that online shopping is a multistage process (Häubl and Trifts 2000), a method to attract customers at each interactive stage becomes much more relevant.

Our model allows shoppers to realistically have flexibility and choice in terms of when and what to put in their shopping cart. The model incorporates customers' views and seeks savings for customers, thereby enhancing customer satisfaction with the likely concomitant increase in e-tailers' sales. Compared with cross-selling (Netessine et al. 2006), our bundles are formed freely by online shoppers and are guaranteed to derive savings every time a new item is added to the cart.

3. The Online Dynamic Bundle Pricing Model

3.1. Problem Description and Model Assumptions

In this section we formulate the ODBP problem as a nonlinear mixed-integer programming model for e-tailers. Before detailing the model, we first discuss four model assumptions. (i) Following Wu et al. (2008), we assume the default reservation price for a bundle is the sum of the reservation prices of individual products in the bundle. For products with dependent reservation prices, e-tailers can employ the superadditive or subadditititive methods to increase or decrease the reservation prices for the bundle (Venkatesh and Kamakura 2003, Jedidi et al. 2003). (ii) Customers' purchasing decisions are governed by their consumer surplus, defined as the reservation price minus the price paid. Customers would prefer a product (bundle) that has the highest consumer surplus. (iii) Because the individual posted price is a function of market competition and demand popularity, we deem it the retailer's optimal price that generates the highest possible profit when sold alone. Methods for establishing optimal posted prices can be found in McCardle et al. (2007).[1] (iv) Customers have budget limits that restrict their ability to pay. See Ulkumen et al. (2008) for methods to estimate customer budgets.

Two types of data are needed for our model. One is product information, e.g., product cost, posted price, and shipping rate. The other is customer information, e.g., purchasing history and reservation price. On the individual level, the shopper's online interaction with the system, such as shopping cart contents, needs to be tracked. At the group level, e-tailers have to survey or analyze past data to estimate reservation prices, budgets, and customers' shipping preferences. Of course, sufficient transaction records are necessary to generate online recommendations to customers. However, these data are increasingly available in online environments.

To estimate customers' reservation prices, Wertenbroch and Skiera (2002), Jedidi et al. (2003), Wang et al. (2007), and Bitran and Ferrer (2007) have proposed several practical methods. For mature products that have known prices and demand records, e-tailers can use the posted price to approximate the reservation price (see Footnote 1). For new products, e-tailers could first estimate reservation prices through a market survey and later adjust them according to the customer's response to the price changes and market condition (Jedidi and Zhang 2002).

3.2. The Proposed ODBP Model

Suppose an e-tailer has N products and M potential customers who might patronize the business (Wu et al. 2008, Venkatesh and Kamakura 2003). Each product has a posted price, and different customers have different reservation prices for each product. Suppose I products are already in the shopping cart, $G_S = \{g_1^S, \dots, g_i^S, \dots, g_I^S\}$, and the corresponding posted prices, costs, reservation prices, and shipping fees are $\{p_1^S, \dots, p_i^S, \dots, p_I^S\}$, $\{c_1^S, \dots, c_i^S, \dots, c_I^S\}$, $\{r_{m,1}^S, \dots, r_{m,I}^S, \dots, r_{m,I}^S\}$, and $\{f_{m,1}^S, \dots, f_{m,I}^S, \dots, f_{m,I}^S\}$, respectively, where $m = 1, 2, \dots, M$, and $i = 1, 2, \dots, I$. The shipping fee $f_{m,I}^S$ is established based on the products to ship, the customer's shipping option (e.g., express or ground shipping), and the e-tailer's shipping rate. Once the customer places products into the shopping cart and selects a shipping option, the shipping fee of the order can be determined. The bundle price of the shopping cart is p^S, and $p^S = p_1^S$ if there is only one product in the shopping cart. Based on customer's online behavior and the products in G_S, ORS recommends an additional J products, $G_R = \{g_1^R, \dots, g_j^R, \dots, g_J^R\}$.

The corresponding posted prices, costs, reservation prices, and shipping fees are $\{p_1^R, \dots, p_j^R, \dots, p_J^R\}$, $\{c_1^R, \dots, c_j^R, \dots, c_J^R\}$, $\{r_{m,1}^R, \dots, r_{m,j}^R, \dots, r_{m,J}^R\}$, and $\{f_{m,1}^R, \dots, f_{m,j}^R, \dots, f_{m,J}^R\}$, respectively, where $j = 1, 2, \dots, J$. The budgets of the M potential customers are denoted as $\{b_1, \dots, b_m, \dots, b_M\}$. For each recommended product g_j^R in G_R, we combine it with the products already in the cart G_S to form a new bundle, $\{g_1^S, \dots, g_i^S, \dots, g_I^S, g_j^R\}$, and we determine the best bundle price that would win over a customer purchase while generating profit for the e-tailer. In all, J different new bundles are formed and J bundle prices are calculated by the ODBP in each shopping stage. Customers will presumably pick one out of the J candidate new bundles and continue to the next shopping stage. We use X_m, a binary decision variable, to denote whether customer m will purchase the specific bundle under consideration. Another decision variable, p, is the bundle price for the bundle $\{g_1^S, \dots, g_i^S, \dots, g_I^S, g_j^R\}$. X_m is dependent on the price p. The percentage of customers who actually made the purchase among all M shoppers is $(\sum_{m=1}^{M} X_m)/M$. In addition, p_{-i} is the bundle price before product g_i^S is added to the bundle; i.e., p_{-i} is the price of bundle $\{g_1^S, \dots, g_{i-1}^S, g_{i+1}^S, \dots, g_I^S, g_j^R\}$. The notations necessary to formulate the ODBP model are listed in the appendix.

The objective of the ODBP model is to maximize profit for the e-tailer when recommending product g_j^R to potential customers who have already picked G_S. Thus, the e-tailer's profits are the sum of profits obtained from all customers who would choose to buy g_j^R and Gs:

$$\max \sum_{m=1}^{M} \left(p - \left(\sum_{u=1}^{I} c_u^S + c_j^R \right) \right) X_m. \tag{1}$$

The profit obtained from each customer is the difference between the bundle price (p) and the total bundle cost, which includes the cost of all products already in the cart, $\sum_{u=1}^{I} c_u^S$, and the cost of the newly recommended product, c_j^R. The binary variable X_m in the objective function (1) equals 1 if customer m chooses to buy the bundle $\{g_1^S, \dots, g_i^S, \dots, g_I^S, g_j^R\}$ at price p, and 0 otherwise. Therefore, among the M

[1] When customers' reservation prices for product g_n follow the distribution $f(x)$ between $[r_l, r_u]$, e-tailers' profits can be determined by $Profit_n = (M \times \int_{p_n}^{r_u} f(x)\,dx) \times (p_n - c_n)$. Function $f(x)$ can be $\mathrm{U}(u-b, u+b)$, $\mathrm{N}(u, \sigma^2)$, or other shapes. The value of $M \times \int_{p_n}^{r_u} f(x)\,dx$ corresponds to the actual demand of product g_n when market size is M and g_n is sold at price p_n. The optimal selling price p_n^* can be found at the point that maximizes profit: $Profit^* = \mathrm{Max}_{p_n}[(M \times \int_{p_n}^{r_u} f(x)\,dx)] \times (p_n - c_n)$.

shoppers in the market, only those with decision variables X_m equal to 1 are the actual buyers.

A serious difficulty faced by firms is that deep discounts offered to boost sales do not drive enough traffic volume to generate a profit. To avoid such a predicament, we carefully incorporate both product costs and customers' reservation prices into the proposed ODBP model by employing nine constraints. Constraints (2)–(5) determine whether customers would be interested in buying the bundle at price p. The reservation prices for all products customer m placed in the shopping cart is $\sum_{u=1}^{I} r_{m,u}^{S}$, and $\sum_{u=1}^{I} r_{m,u}^{S} + r_{m,j}^{R} - p - f_m$ is customer m's consumer surplus when buying the recommended bundle $\{g_1^S, \ldots, g_i^S, \ldots, g_I^S, g_j^R\}$ at price p and paying shipping fee f_m. To attract customer m to buy the recommended g_j^R, the customer's surplus derived from $G_S \cup \{g_j^R\}$ has to be greater than that generated by buying g_j^R alone at the posted price. In other words, the consumer surplus of the new bundle should be no less than that of g_j^R:

$$\left[\left(\sum_{u=1}^{I} r_{m,u}^{S} + r_{m,j}^{R} - p - f_m\right) - (r_{m,j}^{R} - p_j^R - f_{m,j}^{R})\right] X_m \geq 0, \quad m = 1, \ldots, M. \tag{2}$$

Similarly, customer m's surplus derived from the bundle has to be at least equal to that of buying g_i^S individually. Only when the consumer surplus from the bundle is no less than that of purchasing the individual item at price p_i^S would the customer keep g_i^S in the bundle:

$$\left[\left(\sum_{u=1}^{I} r_{m,u}^{S} + r_{m,j}^{R} - p - f_m\right) - (r_{m,I}^{S} - p_i^S - f_{m,I}^{S})\right] X_m \geq 0, \quad i = 1, \ldots, I, \quad m = 1, \ldots, M. \tag{3}$$

In constraints (4), customer m would buy the bundle only if her reservation price for the bundle is no less than her expenses:

$$\left(\sum_{u=1}^{I} r_{m,u}^{S} + r_{m,j}^{R} - p - f_m\right) X_m \geq 0, \quad m = 1, 2, \ldots, M. \tag{4}$$

Constraints (5) ensure that customer m's actual expenditure is no more than her budget b_m; otherwise, customers cannot afford the products:

$$(p + f_m - b_m) X_m \leq 0, \quad m = 1, 2, \ldots, M. \tag{5}$$

When adding product g_j^R to the shopping cart, constraint (6) ensures that the marginal bundle price $(p - p^s)$ for the newly recommended product, g_j^R, is no more than its posted price. It would be irrational to pay a higher marginal price for g_j^R in the bundle than to buy the item separately:

$$p - p^S - p_j^R \leq 0. \tag{6}$$

Besides adding product g_j^R to the shopping cart, there are an additional I ways to form bundle $\{g_1^S, \ldots, g_i^S, \ldots, g_I^S, g_j^R\}$. That is, instead of having g_j^R as the last item to be added into the cart, customers may actually add g_i^S to the cart that already contains $\{g_1^S, \ldots, g_{i-1}^S, g_{i+1}^S, \ldots, g_I^S, g_j^R\}$. Recall that we let p_{-i} be the bundle price of $\{g_1^S, \ldots, g_{i-1}^S, g_{i+1}^S, \ldots, g_I^S, g_j^R\}$. Constraints (7) ensure that the marginal bundle price $(p - p_{-i})$ is no more than its posted price p_i^S. The bundle prices for all I cases in Equation (7) are the same, because the same contents $\{g_1^S, \ldots, g_{i-1}^S, g_i^S, g_{i+1}^S, \ldots, g_I^S, g_j^R\}$ are in the bundle. The cart-entering sequence of products i does not affect the bundle price:

$$p - p_{-i} - p_i^S \leq 0, \quad i = 1, \ldots, I. \tag{7}$$

Given constraints (7) and possible economies of scale in shipping, i.e., $f_{m,i}^S + f_{m,-i}^S \geq f_m$, we found customer m's surplus derived from the bundle is at least equal to the sum of the surplus of buying g_i^S individually and that of buying other products in the bundle $\{g_1^S, \ldots, g_{i-1}^S, g_{i+1}^S, \ldots, g_I^S, g_j^R\}$; i.e., $\sum_{u=1}^{I} r_{m,u}^S + r_{m,j}^R - p - f_m \geq (r_{m,i}^S - p_i^S - f_{m,i}^S) + (\sum_{u=1}^{i-1} r_{m,u}^S + \sum_{u=i+1}^{I} r_{m,u}^S + r_{m,j}^R - p_{-i} - f_{m,-i}^S)$, where $f_{m,-i}^S$ is m's shipping charge for products $\{g_1^S, \ldots, g_{i-1}^S, g_{i+1}^S, \ldots, g_I^S, g_j^R\}$. Constraint (8) guarantees that the bundle price is no less than the total cost of the products in the bundle. Otherwise, e-tailers will incur a loss when selling the bundle:

$$p \geq \sum_{u=1}^{I} c_u^S + c_j^R. \tag{8}$$

Constraint (9) ascertains that the bundle price of the shopping cart is no more than the sum of the individual posted prices of all products in the bundle:

$$p \leq \sum_{u=1}^{I} p_u^S + p_j^R. \tag{9}$$

However, given constraints (6) and (7), constraint (9) becomes redundant, so we will remove it when solving the model. Constraints (10) define X_m as a binary variable, and it equals 1 if customer m purchases the bundle at price p, and 0 otherwise:

$$X_m = 0 \text{ or } 1, \quad m = 1, \ldots, M. \tag{10}$$

Given constraints (2)–(10), the nonlinear mixed-integer ODBP model aims to maximize (1). The optimal bundle price p derived by the ODBP is based on the reservation prices of all M potential customers rather than by a single customer's valuation of the product. This is because in each shopping stage, the J bundles and their ODBP-derived prices serve as an online price menu; all shoppers are quoted the same price as long as the same bundle contents are selected.

However, if an e-tailer chooses to differentiate prices across customers based on their profile, he can estimate the reservation price distribution for the customer segment and personalize the prices by entering the respective reservation price distributions directly into the ODBP model. According to the customer's online browsing and purchasing behavior, the e-tailer can also rematch the customer to the appropriate customer segment in real time. This way, when the online information tracked suggests that the customer may be willing to pay more (or less) than others for that item, the customer's reservation price distributions can be revised accordingly, and the ODBP can thus set the bundle price in accordance with customer's online behavior and willingness to pay. In short, when adopting first-degree price discrimination, or when engaging in target recommendations, the ODBP implementation is unchanged except for the segment match and real-time update of the reservation price distributions. Market survey data and tracked online information are key inputs to dynamically determine the reservation price distribution and thus the bundle price for potential customers.

3.3. Illustration of the Multistage Online Shopping Process

Figure 1 illustrates shopper's decision-making process and its relationship with the proposed ODBP. Suppose the e-tailer sells eight products (A–H) with posted prices (p_j^R) of \$9.00, \$11.99, \$16.47, \$13.72, \$7.53, \$6.59, \$14.75, and \$15.63, respectively. The shopper's reservation price for each product is shown in the second row of each subtable. For ease of illustration, we assume the shopper chooses ground shipping and the shipping fee is \$3.00 + \$0.99 × number of products. The Marginal_price_UBL is the stand-alone posted price plus the marginal shipping fee ($= p_j^R +$ \$0.99), and the Marginal_price_BL is the change in bundle price from last stage plus the marginal shipping fee ($= (p - p^s) +$ \$0.99).

After the shopper logs on, the ORS recommends products {A, B, C} based on the customer profile and shopping history. To maximize consumer surplus (\$16 − \$11.99 − \$3.99 shipping fees), the shopper selects B. At this point, she may either check out or continue. If she chooses to continue, the ORS recommends {A, C, D}, and the ODBP instantly determines the optimal bundle price for {B, A}, {B, C}, and {B, D}, whose corresponding marginal bundle prices are \$8.69, \$17.18, and \$14.51, respectively. Note that without the bundle discount, the shopper will not buy product A because of the associated negative consumer surplus (\$9.50 − \$9.99). However, because the marginal price of the bundle (\$8.69) is smaller than her reservation price (\$9.50) for product A, she is better motivated to buy the bundle. By applying the maximum consumer surplus rule, she will choose A among the recommended {A, C, D}, and again in stage 3, she will choose F owing to its maximum surplus.

After adding products B, A, and F to the shopping cart, the customer, now in shopping stage 4, may decide to remove A. Once again, the ODBP is applied to determine the bundle price for {B, F}. Thereafter, the ORS may recommend products {C, E, H}[2] in stage 5.

3.4. Solution Methodology

Three relationships are defined between products: substitutes, independence, and complements (e.g., Venkatesh and Kamakura 2003). For example, to a Hewlett-Packard laptop, an Acer Laptop is a substitute (relation 1), an MP3 player is an independent product (relation 2), and a notebook case is a complement (relation 3). Customers generally buy only one of the substitutes at a time, not both. Thus, the following two cases should be solved differently when implementing the ODBP.

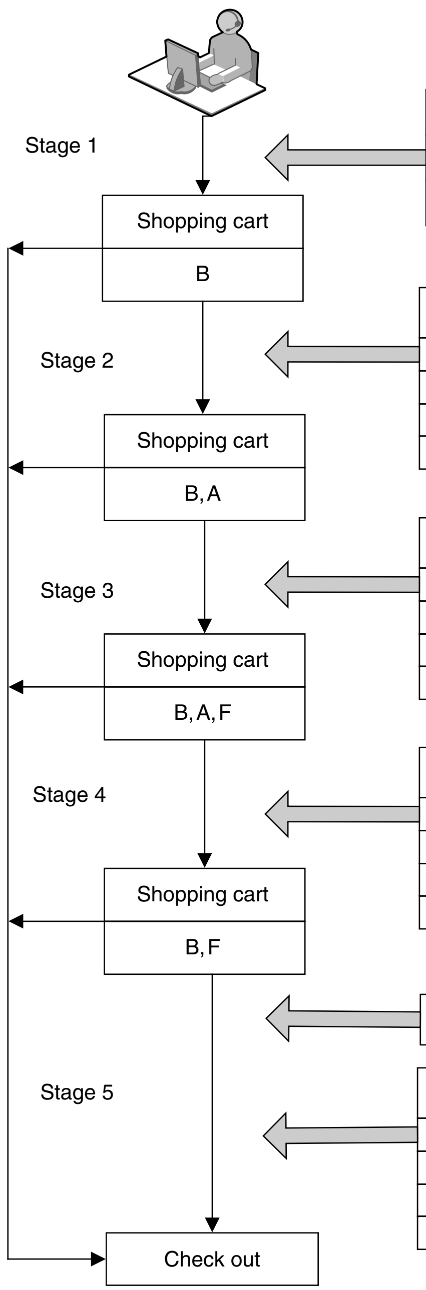
For relation 1, given that product g_j^R is a substitute for g_I^S, the ODBP should calculate the bundle price of $\{g_j^R\} \cup \{g_1^S, \ldots, g_i^S, \ldots, g_{I-1}^S\}$. For relations 2 and 3, the recommended product is independent or complementary to the products in the cart. The proposed model can be employed directly to determine the price of the bundle, $\{g_j^R\} \cup \{g_1^S, \ldots, g_i^S, \ldots, g_{I-1}^S, g_I^S\}$. In addition, if customers choose a product not in the recommendation list, the ODBP can also compute the bundle price by combining the selected product with those in the shopping cart.

Note that constraints (7) inherently demand solving the model repeatedly I times. If a customer's transaction contains many products, the recurring application of the nonlinear mixed-integer programming algorithm would require excessive computation time to reach optimality, which is computationally infeasible for a commercial online environment. Because conventional optimization techniques such as relaxation and decomposition methods (Nowak 2005) are relatively slow and cannot solve our problem in real time, we propose a quasi-optimal method to achieve a near-optimal solution, as outlined in Figure 2 and described next.

3.4.1. The Quasi-Optimal Method. Although solving small-sized nonlinear mixed-integer programming problems with optimization software is possible, a drawback is that they are generic tools and do not consider the special structure of the problem in question. Consequently, they require a relatively long solution time. In the online shopping environment, a

[2] Different products in the cart portray different customer characteristics and thus a unique recommendation list. After removing product A from the cart, bundle {B, F} may communicate new customer attributes and extract a different recommendation list.

Figure 1 Illustration of the Role of the ODBP in the Shopper's Decision-Making Process



Recommendation	A	B	C
Reservation price	9.50	16.00	16.50
Posted price	9.00	11.99	16.47

Recommendation (When B in the cart)	A	C	D
Reservation price	9.50	16.50	15.00
Marginal_price_UBL	9.99	17.46	14.71
Marginal_price_BL	8.69	17.18	14.51
Consumer surplus	0.81	−0.68	0.49

Recommendation (When B and A in the cart)	C	F	G
Reservation price	16.50	7.00	15.50
Marginal_price_UBL	17.46	7.58	15.74
Marginal_price_BL	16.81	6.55	15.49
Consumer surplus	−0.31	0.45	0.01

Recommendation (When B, A, F in the cart)	D	E	H
Reservation price	15.00	8.00	16.00
Marginal_price_UBL	14.71	8.52	16.62
Marginal_price_BL	14.34	7.59	16.27
Consumer surplus	0.66	0.41	−0.27

Recommendation (When B and F in the cart)	C	E	H
Reservation price	16.50	8.00	16.00
Marginal_price_UBL	17.46	8.52	16.62
Marginal_price_BL	16.65	7.81	16.59
Consumer surplus	−0.15	0.19	−0.59

Notes. Note that Marginal_price_UBL = stand-alone posted price + marginal shipping fee = $p_j^R + \$0.99$. Marginal_price_BL = change in bundle price from the last stage + marginal shipping fee = $(p - p^s) + \$0.99$. Consumer surplus = reservation price of individual product − Marginal_price_BL.

waiting time of more than a few seconds is unacceptable and may cause customers to renege. Therefore, rapid response time is essential.

To promptly determine the optimal bundle price p for $\{g_1^S, \ldots, g_i^S, \ldots, g_I^S, g_j^R\}$, we need to establish the prices of p_{-i} for $i = 1, 2, \ldots, I$. This requires repetitive execution of the procedures in Figure 2, with $\sum_{v=2}^{I-1} {}_I C_v + 1$ nested loops to arrive at the solution. Clearly, this would be a time-consuming procedure. To make it practical for an online real-time application, we propose a computationally efficient heuristic method in Figure 3 to establish the price for p_{-i}, which is then used to replace p_{-i} in Step 2 of Figure 2.

3.4.2. A Heuristic Algorithm to Expedite the Quasi-Optimal Method. The rationale for the need of a heuristic in Figure 3 is best illustrated with an example. Suppose that there are three products, g_1, g_2, and g_3, with prices of \$10, \$100, and \$200, respectively. The bundle $\{g_1, g_2, g_3\}$ may be formed in three ways: (i) after the cart has already contained g_2 and g_3, g_1 is added, i.e., $\{g_2, g_3\} \cup \{g_1\}$; (ii) $\{g_1, g_3\} \cup$

Figure 2 The Quasi-Optimal Method for the Proposed ODBP Model

1 Calculate the lower bound *lowBound* of p according to constraint (8).

$$lowBound = \sum_{i=1}^{I} c_i^S + c_j^R.$$

2 Calculate the upper bound *upBound* of p according to constraints (6) and (7).
For $i = 1$ to I
Calculate the bundle price p_{-i} of the products $\{g_1^S, \ldots, g_{i-1}^S, g_{i+1}^S, \ldots, g_I^S, g_j^R\}$.
End For

$$upBound = \min\{\{p_{-i} + p_i^S,\ i = 1, \ldots, I\} \cup \{p^S + p_j^R\}\}.$$

3 Search the optimal price with the following fixed step-length method
Set the step length *stepLen* in the search of optimal price to a constant integer.
Initialize the alternative price point: $altPrice = upBound$.
Initialize the maximum profit and intermediate variable of profit: $maxProfit = altProfit = 0$.
While $altPrice >= lowBound$, Do
Count the number NC of customers whose budget and reservation prices for the bundle are both larger than the sum of *altPrice* and shipping fee f_m.
Calculate the intermediate variable of maximum profit: $altProfit = NC \times (altPrice - cost)$.
Real number *cost* is the cost of the bundle.
4 If *altProfit* is larger than *maxProfit*
$p = altPrice$; $maxProfit = altProfit$.
End If
$altPrice = altPrice - (upBound - lowBound)/stepLen$
End While

$\{g_2\}$; and finally, (iii) $\{g_1, g_2\} \cup \{g_3\}$. Suppose that the existing bundle price for $\{g_2, g_3\}$ in scenario (i) is \$285. To attract customers, the ODBP model has to increase customer savings when g_1 is added. Therefore $\{g_2, g_3\} \cup \{g_1\}$ should be less than \$295 (=\$285 + \$10). Similarly, if $\{g_1, g_3\}$ is \$200 and $\{g_1, g_2\}$ \$104, the corresponding upper bounds for scenarios (ii) and (iii) would be \$300 (=\$200 + \$100) and \$304 (=\$104 + \$200), respectively. In the end, the value \$295 (=Min{\$295, \$300, \$304}) would be the effective upper bound for the bundle price of $\{g_1, g_2, g_3\}$, regardless of which scenario has formed the bundle.

In Figure 3, we define the upper bound for p_{-i} as $p_{-v} + p_v$, and its lower bound is the sum of the costs of all products in the cart, $\sum_{u=1}^{i-1} c_u^S + \sum_{u=i+1}^{I} c_u^S + c_j^R$. In the heuristic, the nested loop will only be executed $I - 1$ times to determine p_{-i} and $I \times (I-1) + 1$ times to determine p if I is larger than 2. By taking advantage of the problem structure, we develop the heuristic in Figure 3 to provide the p_{-i} for Step 2 of Figure 2. The heuristic-based solution approach is both effective (accurate) and efficient (with small execution time) when solving the ODBP (see the numerical study in §4.2).

Figure 3 The Heuristic Method to Calculate p_{-i} in Step 2 of Figure 2

The heuristic method to calculate the bundle price p_{-i}:
Calculate the lower bound of p_{-i}:

$$lowBoundP_{-i} = \sum_{u=1}^{i-1} c_u^S + \sum_{u=i+1}^{I} c_u^S + c_j^R.$$

Calculate the upper bound of p_{-i} as follows:
Find the product g_v^S from $\{g_1^S, \ldots, g_{i-1}^S, g_{i+1}^S, \ldots, g_I^S, g_j^R\}$ that has the lowest price.
Calculate the upper bound of p_{-i} as follows:

$$upBoundP_{-i} = p_{-v} + p_v,$$

where p_{-v} is the optimal price for the product bundle: $\{g_1^S, \ldots, g_{i-1}^S, g_{i+1}^S, \ldots, g_I^S, g_j^R\} - \{g_v^S\}$, which is also calculated by the heuristic method.
Search the optimal price of p_{-i} in $[lowBoundP_{-i}, upBoundP_{-i}]$ with the fixed step-length method.

4. Numerical Study of the ODBP Model

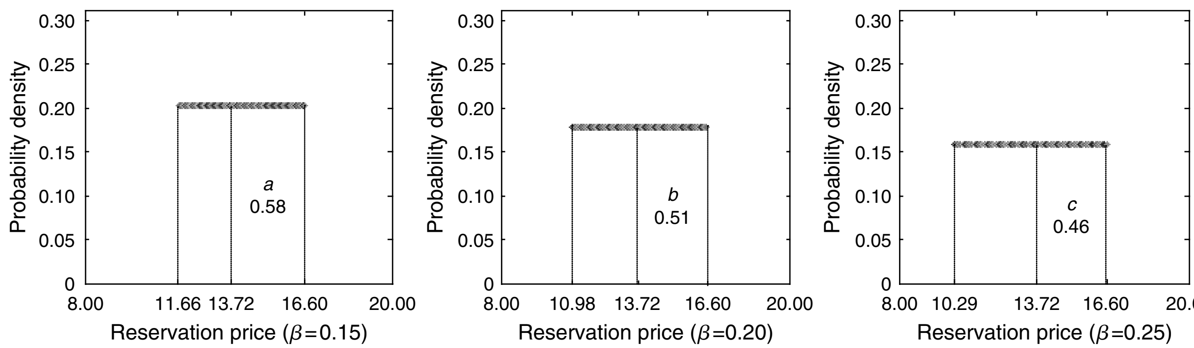
In this section we study the effectiveness of the proposed ODBP model from the perspectives of both e-tailer profit improvement and customer savings, and we compare the ODBP with the existing pricing method. We also investigate the computational efficiency of the proposed heuristics so as to understand its suitability for online real-time implementation.

4.1. The Data Sets and Experiment Procedure

4.1.1. Posted Price and Product Costs. Data of the top-100-ranked books from Amazon.com, including book titles and posted prices, were collected in April 2010. Although Amazon.com knows the exact costs of their products, because of commercial confidentiality, we are not able to obtain this information. We therefore follow the literature (Sampson 2007) and assume book costs are uniformly distributed at U(0.60, 0.80) of the posted price. Impacts of potential cost variation on e-tailer profits are examined in §5.5.

4.1.2. Reservation Prices. Reservation prices have been widely used in the literature to develop customized pricing models (Chen and Iyer 2002) and auction strategies (Yao and Mela 2008). Following Wu et al. (2008), we assume customers' reservation prices are uniformly distributed, $U(r_l, r_u)$. The prices

Figure 4 **The Reservation Price U(r_u, r_l) Under Uniform Probability Distribution**



posted by Amazon.com are assumed to give optimal profit because they are the results of market competition. For uniformly distributed reservation prices, McCardle et al. (2007) show that the optimal selling price p_n equals the average of the upper bound of reservation price and the product's cost; i.e., $p_n^* = (r_u + c_n)/2$. The upper bound of the reservation price thus can be derived by $r_u = 2 \times p_n - c_n$. As for the lower bound of reservation prices, we let $r_l = (1-\beta) \times p_n$, with β being the range index that signifies the degree of heterogeneity in customers' valuation of the product. Profits are generated when a customer's reservation price is greater than p_n; any reservation price lower than p_n will not generate revenue for the e-tailer. A larger β gives a smaller r_l, which corresponds to a wider dispersion of reservation prices.

Using Malcolm Gladwell's book *Outliers: The Story of Success*[3] as an example, Figure 4 shows three different uniform reservation price distributions, with $\beta = (0.15, 0.20, 0.25)$ and p_n being \$13.72. By assuming p_n is the optimal price, we found $r_u = 2 \times p_n - c_n = \$16.60$, and the lower bounds computed by $r_l = (1-\beta) \times p_n$ are (\$11.66, \$10.98, \$10.29), respectively. The probabilities corresponding to areas (a, b, c) are $(0.58, 0.51, 0.46)$, indicating the chance of selling at $p_n = \$13.72$ under different customer valuations. The small probability in c indicates that at a high β, fewer customers will buy the product at p_n.

4.1.3. Shipping Charge. Consistent with Amazon .com, we assume three shipping options: next-day, two-day, and ground shipping. Shipping charges are determined by

$$\textit{Shipping fee per order} = \textit{“Per shipment” charge} \\ + \textit{No. of items per order} \\ \times \textit{“Per item” fee.}$$

The "Per shipment" charges by Amazon.com for the ground, two-day, and next-day shipping are (\$3.00, \$9.99, \$12.99), and the "Per item" fees are (\$0.99, \$1.99, \$4.99), respectively. To be consistent with our survey results[4] that buyers often choose ground shipping when shopping online, and that only a few opt for express shipping, the percentages of customers who require ground, two-day, and next-day shipping are estimated at 70%, 20%, and 10%, respectively.

4.2. Improvement of E-tailer Profit and Customer Savings

Chen et al. (2008) suggest that customers, in general, order no more than eight items in one transaction. We thus assume the bundle size follows U(1, 8) and use the item-to-item collaborative filtering technology of Amazon.com (Linden et al. 2003) to make recommendations. As in Figure 1, each customer starts with a list of recommendations and selects the product that gives her the highest consumer surplus. From that, the ORS generate another recommendation list, the ODBP determines the corresponding prices, and the consumer picks a product to add to the shopping cart. The process continues until she checks out. Customers may pick items not recommended and can leave the process at any time.

To examine the benefit of adopting ODBP, we let M, the market population, equal 500. Note that M could assume any number. However, a larger M takes a longer time to replicate. In Table 1, column (1) displays the e-tailer's total profits when buyers pay at Amazon's prices, and column (2) gives the e-tailer's total profits when buyers pay the ODBP bundle prices. The positive values of percentage of profit improvement in column (3) indicate that the ODBP outperforms its counterpart in profit generation, whereas column (6) represents the average customer savings in dollars. In addition, we found that profits and savings increase with the bundle size. For example, the percentage of profit improvement increases from 121% (=(860.69 − 389.41)/389.41) to

[3] Published in 2008 by Little, Brown, New York.

[4] Administered to students at the University of Pittsburgh. Details are available from the authors upon request.

Table 1 E-tailer's Profits and Customers' Savings by the Proposed Strategy

	E-tailer's profits			Average customer savings (in $)			
No. of products	(1) Unbundling profit	(2) Bundling profit	(3) = ((1) − (2))/(1) Profit improvement (%)	(4) Unbundling price	(5) Bundling price	(6) = (4) − (5) ODBP savings	(7) % of new buyers
2	389.41	860.69	121.0	30.29	27.16	3.13	358.3
3	678.43	1,698.98	150.4	46.53	41.72	4.81	406.4
4	1,033.27	2,702.22	161.5	61.44	55.18	6.26	452.6
5	1,177.21	3,737.69	217.5	76.33	68.32	8.01	515.1
6	1,396.55	4,729.40	238.6	90.60	81.09	9.51	554.2
7	1,256.96	5,624.85	347.5	105.85	94.54	11.31	770.3
8	1,345.65	6,545.49	386.4	120.33	107.54	12.78	844.3

386.4% (=(6,545.49 − 1,345.65)/1,345.65) when bundle size rises from two to eight. Similarly, the average savings per customer increases from \$3.13 to \$12.78 (see column (6)), whereas the number of new buyers increases from approximately 358.3% to 844.3% (see column (7)).

Because customers' purchasing decisions are influenced by price and shipping cost, offering savings opportunity for larger bundles is effective. Rational customers are better motivated to buy larger orders under the ODBP discounts. Thus, although unit profit decreases, overall profits increase as a result of larger volume. The sizeable increase in profit provides a convincing argument as to the attractiveness of the ODBP strategy and gives e-tailers a clear incentive to adopt the model.

To ensure prompt response for the online environment, we replace Figure 3 for p_{-i} in Figure 2. Table 2 shows that for small bundle, both the quasi-optimal method (Figure 2) and the heuristics-based method (Figure 3) require comparable computation time. However, when the bundle size increases, the computational efficiency of the heuristic-based method is much more pronounced. The heuristic-based method improves computational efficiency by up to 99.8% while maintaining the same solution quality (see the rightmost two columns). We thus adopt the more efficient heuristic-based method to solve the ODBP model and for our sensitivity analysis.

5. Sensitivity Analysis

We investigate the robustness of the ODBP by examining how uncertainties in input parameters affect the model performance. We first study the impacts of changes in the range and shape of customer reservation price distribution on e-tailer and customers. We subsequently examine the sensitivity when customers employ different purchase decision rules and when their budgets vary. Next, we analyze how changes in product cost ratios affect the e-tailer's profitability. Finally, a multiple-factor sensitivity analysis is conducted to study the overall model robustness.

5.1. Changes in the Range of Reservation Prices

The range index, β, is used to measure the heterogeneity of customers; a larger β indicates wider-ranging views of merchandise value. Table 3 shows the simulated e-tailer profits and customer savings under different βs. A larger β corresponds to higher percentage of profit improvement because more customers will regard the product as low value (see Figure 4). Correspondingly, fewer customers are willing to pay for the product at the posted price. This implies that the price discount from a large bundle has a better chance to attract previously uninterested customers in the market. Therefore, the percentage of profit improvement under a higher β and a larger bundle size is more significant. Similarly, customer savings show the same trend; e.g., for the

Table 2 Comparison of the Heuristic Method and Quasi-Optimal Method

	Execution time (seconds)			Average optimal price (in $)	
No. of products	Quasi-optimal method	Heuristic-based method	Efficiency improvement (%)	Quasi-optimal method	Heuristic-based method
2	0.000	0.000	0.0	27.16	27.16
3	0.000	0.000	0	41.72	41.72
4	0.004	0.001	75	55.19	55.18
5	0.017	0.003	82.3	68.32	68.32
6	0.105	0.005	95.2	81.09	81.09
7	0.729	0.008	98.9	94.55	94.54
8	6.215	0.013	99.8	107.54	107.54

Notes. The quasi-optimal method is given in Figure 2. The heuristic-based method uses Figure 3 to replace p_{-i} in Figure 2.

Table 3 **Impact of Changes in the Range of Reservation Prices**

	β							
	E-tailer's profit improvement (%)				Customers' savings			
No. of products	0.1	0.15	0.2	0.25	0.1	0.15	0.2	0.25
2	97.4	118.1	121.0	156.3	2.83	3.01	3.13	3.13
3	94.0	127.2	150.4	175.0	3.84	4.32	4.81	5.01
4	94.7	123.3	161.5	215.6	4.62	5.44	6.26	6.84
5	110.8	154.2	217.5	308.6	5.58	6.85	8.01	9.03
6	105.4	166.5	238.6	357.3	6.16	8.00	9.51	10.63
7	130.1	215.1	347.5	512.6	7.28	9.44	11.31	12.99
8	128.1	239.4	386.4	623.8	7.96	10.50	12.78	14.72

eight-product bundle, customer savings is \$14.72 at $\beta = 0.25$.

5.2. Changes in the Distributions of Reservation Price

In §4.1.2 we assume that the posted price is optimal when reservation prices are uniformly distributed, and from this we determine r_u. To make a fair comparison when reservation prices follow different distributions, we derive the optimal posted prices based on McCardle et al. (2007); i.e., $\mathit{Profit}^* = \mathrm{Max}_{p_n}[(M \times \int_{p_n}^{r_u} f(x)\,dx)] \times (p_n - c_n)$. Four cases are examined in Table 4. The reservation prices in Case 1 follows $\mathrm{N}(u, \sigma^2)$, with $u = (r_u + r_l)/2$ and $\sigma = (r_u - r_l)/4$. Case 2 is a uniform distribution. Case 3 allocates 30% of its products' reservation price to the normal distribution and the other 70% of its products to the uniform distribution. Case 4 is the inverse of Case 3.

As shown in Table 4, regardless of distribution type, ODBP universally outperforms the unbundling strategy, as evidenced by the positive improvements. Under the same bundle size, the values are comparable among all cases. However, the performance improves with the bundle sizes; e.g., for the eight-product bundle in Case 4, the percentage of profit improvement using ODBP is 312.1% and the customer savings is \$10, much higher than those of the two-product bundle.

5.3. Changes in Purchase Decision Rules

A customer's purchase decision is a complex process and may be influenced by the seller's marketing strategy (Bodapati 2008, Sun 2005) and the consumer's ability to process the product information (Chen et al. 2010, Sriram et al. 2010). When multiple alternatives from which to choose are available, a customer's purchase decision may be affected by factors such as the e-tailer's recommendation strategy (Cooke et al. 2002, Fitzsimons and Lehmann 2004), coupons or rebates (Lu and Moorthy 2007, Gönül and Srinivasan 1996), and website style (Hauser et al. 2009).

To examine whether the ODBP remains valid under different purchase decision rules, we vary the rule from maximizing consumer surplus to maximizing price savings, defined as the sum of posted price minus bundle price. We found in Table 5 that, although customers receive slightly better savings when applying the "max price savings" rule because of its direct focus on price, e-tailers can nonetheless obtain comparable percentage of profit improvements when customers employ the surplus rule. This is because the customer's price savings do not automatically counteract the e-tailer's profit margin (bundle price − bundle cost), and therefore applying the max price savings rule does not necessarily cause more profit decline for e-tailers than applying the surplus rule. Overall, purchase decision rules do not significantly affect the e-tailer's profit or customer savings.

Table 4 **Impacts of the Distribution Type of Reservation Prices**

	Distribution							
	E-tailer's profit improvement (%)				Customers' savings			
No. of products	Case 1: Normal	Case 2: Uniform	Case 3: 0.3N&0.7U	Case 4: 0.7N&0.3U	Case 1: Normal	Case 1: Uniform	Case 1: 0.3N&0.7U	Case 1: 0.7N&0.3U
2	191.8	121.0	152.1	159.10	2.73	3.13	2.83	2.59
3	190.3	150.4	177.2	177.70	4.07	4.81	4.45	4.15
4	201.0	161.5	200.5	206.70	5.19	6.26	5.85	5.43
5	224.9	217.5	242.0	228.70	6.29	8.01	7.23	6.64
6	258.5	238.6	284.4	255.00	7.25	9.51	8.51	7.81
7	298.0	347.5	331.6	283.40	8.19	11.31	9.83	8.92
8	325.6	386.4	350.9	312.10	9.18	12.78	10.98	10.00

Table 5 Impacts of Purchase Decision Rule

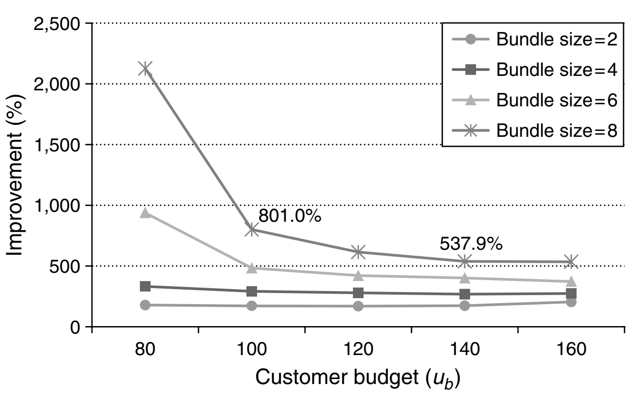
	Decision rule			
	Applying maximum consumer surplus rule		Applying maximum price savings rule	
No. of products	E-tailer's profit improvement (%)	Customers' savings ($)	E-tailer's profit improvement (%)	Customers' savings ($)
2	121.0	3.13	111.2	3.48
3	150.4	4.81	140.6	5.20
4	161.5	6.26	155.4	6.68
5	217.5	8.01	212.3	8.41
6	238.6	9.51	247.1	9.91
7	347.5	11.31	341.1	11.82
8	386.4	12.78	395.5	13.20

5.4. Changes in Budget Level

Suppose that customers' budgets follow a normal distribution $N(u_b, \sigma^2)$, with $\sigma^2 = 0.2 \times u_b$. We vary u_b from \$80 to \$160, and Figure 5 shows such effects. Limited by budgets, customers cannot afford certain products even though their reservation prices are high. Therefore, relative to no budget limitation, the e-tailer's overall profits will fall. For small bundles the percentage of profit improvements at low budget are comparable with that of unlimited budgets. This is because even with a low budget, customers who are interested in only few products can still afford to buy. Therefore, unlimited budgets do not attract many more buyers for small bundles, and thus no extra profits.

However, Figure 5 shows that when the bundle size is large, the percentage of profit improvement is more significant under the lower budget situation; e.g., the profit improvement for the eight-product bundle is 801.0% when u_b is 100, but it is only 537.9% when u_b is 140. For the larger bundle size the ODBP has more room to offer savings opportunities, which, in turn, bring more sales and make more profits.

Figure 5 The Impact of Customers' Budgets on the E-tailer's Profits



The simulation results indicate that our model performs significantly better when the budget is tight. This implies that the ODBP is more valuable in improving e-tailers' profit when customers' purchasing power decreases, as in an economic downturn. E-tailers should provide greater discounts during recessions when consumer budgets are low, as might be expected.

5.5. Changes in the Cost Ratio of Products

The e-tailer's cost ratio (unit cost/posted price) is varied from 60% to 80% in increments of 5%. Because the e-tailer has limited room to earn profit when the cost ratio is high, the dollar profit decreases at the rise of the ratio (see Figure 6(a)). However, the percentage improvement by ODBP grows as the ratio increases (see Figure 6(b)). At a high cost ratio, e-tailers often charge a higher price, which, in turn, deters potential buyers. Using the ODBP bundle discount, the e-tailer could attract more customers, boost sales volume, offset the high costs, and continue to make profits.

The above experiments show that the ODBP is most effective when customers have a diverse view about

Figure 6 The Impact of E-Tailer's Cost Ratio

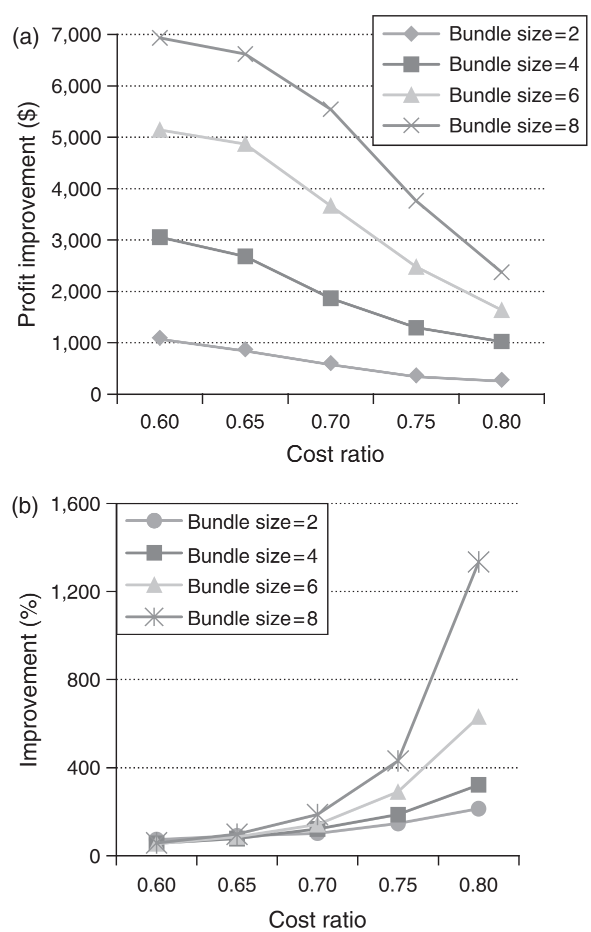


Table 6 A $3^4 \times 2 = 162$ Sensitivity Analysis and Results

	Level for sensitivity factors					Profit improvement		
ID	Reservation price distributions	Reservation price range (β)	Cost ratio	Customer budget	Purchase decision rule	Bundle size = 2	Bundle size = 3	Bundle size = 4
1	Uniform	0.23	0.805	76.5	Price saving	346.25	680.44	1,179.73
2	Uniform	0.17	0.805	76.5	Price saving	396.03	832.05	1,387.76
—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—
38	Mixed	0.20	0.595	90	Surplus	914.17	1,454.04	1,980.61
—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—
91	Uniform	0.23	0.595	76.5	Surplus	905.29	1,716.28	2,882.23
—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—
162	Normal	0.20	0.70	103.5	Surplus	638.26	1,188.97	1,872.89

the value of the product (high β), when customers' budgets are low, and when the cost ratio is high. It is relatively indifferent to the distribution of reservation prices and to the customer's purchase decision rules. To understand the ODBP's overall quality in withstanding uncertainties in different environments, we next conduct a multifactor sensitivity analysis.

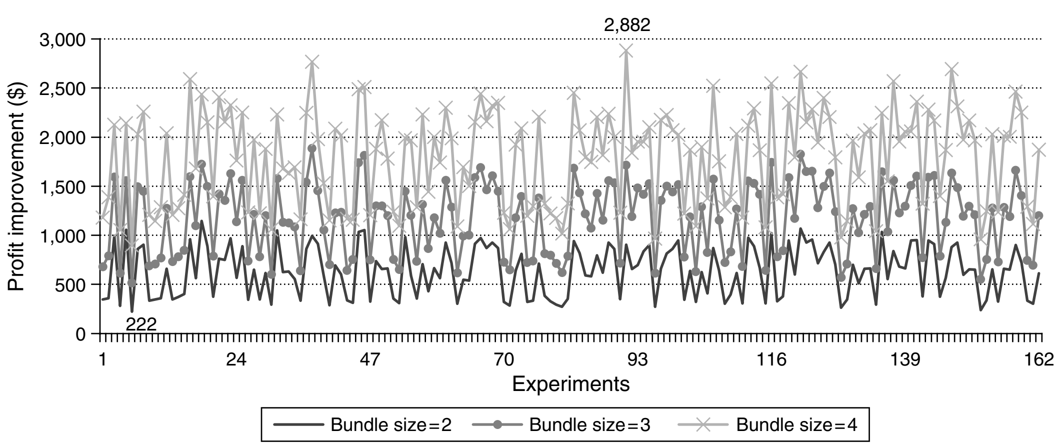
5.6. Multifactor Robustness Study

The ODBP is deemed robust if it can cope with significant uncertainty in its operating environment. Using a $3^4 \times 2$ sensitivity analysis, we conduct 162 experiments for each bundle size to concurrently test five factors: (i) reservation price distribution, (ii) reservation price range, (iii) product cost ratio, (iv) customer budget, and (v) purchase decision rule (see Table 6). The quantitative factor levels assumed in §4 are varied by $\pm 15\%$ of their average values. The corresponding factor levels for the experiment are as follows: distribution = [Uniform, Normal, Mixed (=0.5U&0.5N)], $\beta = [0.17, 0.20, 0.23]$, cost ratio = $[0.595, 0.70, 0.805]$, and budget = $[76.5, 90, 103.5]$, as well as the purchase decision rule at two levels: [max consumer surplus, max price savings]. Each experiment (scenario) corresponds to a unique combination of these factors. For example, in Table 6 the first experiment uses a uniformly distributed reservation price, with $\beta = 0.23$, cost ratio = 0.805, customer budget = \$76.5, and the max price savings decision rule. The profit improvements are shown in the three rightmost columns, whose complete values are graphically displayed in Figure 7.

Figure 7 shows that the highest profit improvement is \$2,882 under the four-product bundle case, whereas the smallest is \$222 under the two-product bundle case. For ease of illustration and clarity, we only show the results of bundle sizes of two to four products. Similar patterns are found for the bundle sizes of five to eight products, i.e., the profit improvements are all positive, and again increase with bundle size.

This sensitivity analysis suggests that the ODBP model is robust under various bundle sizes and scenarios. As far as the percentage of profit improvement is concerned, we found that when facing

Figure 7 Profit Improvements Under Multiple-Factor Sensitivity Analysis



divergent views about product values, lower budgets, and higher cost ratios, the ODBP performs significantly better, whereas the customers' purchase decision rule is inconsequential, as is the reservation price distribution.

6. Summary and Conclusions

Attractive and profitable pricing is essential for business survival and success. This paper provides a new approach to promote online customer spending by offering interactive bundling and pricing, contingent on the products chosen by shoppers at various browsing stages. Product bundling is a widely used tactic for differential pricing. But because the number of bundles increases exponentially with the number of products, pricing all possible bundle combinations and displaying them off-line is a practical impossibility. As a result, traditional pricing strategies prespecify discount rates, bundle sizes, and bundle contents in promotions. Such approaches severely limit the choices to customers, and their application online would squander the valuable information available from the interactive shopping environment.

The method we propose starts with an insight into consumer motivation and ends in a stream of profit enhancement. It includes product selection flexibility in terms of bundle size and product variety, coupled with a dynamic pricing model that integrates customers' preferences, customers' savings, and e-tailers' profits. Furthermore, the incorporation of customers' multistage purchasing behavior in the decision process and the development of the heuristics afford the ODBP model the capability to provide real-time online pricing information that appeals to customers regardless of the mixture of the products they choose.

The proposed model ensures that the price presented online is independent of the sequence of products entering into the shopping cart—customers see the same price for the same bundle. Any product combination is allowed in the shopping cart, and an extra discount is guaranteed when additional products are selected. The price of the customized bundle can be prompted instantaneously online. To sensibly implement the proposed model, we design a heuristic-based solution procedure that is capable of arriving at a near-optimal bundle price with negligible computation time. The numerical studies show that the proposed model is a win–win strategy. It offers monetary savings for customers, enhances product differentiation with numerous discount scenarios, helps firms gain competitive advantage, and ultimately enhances e-tailers' profits.

In terms of future research, one possibility is that in actual applications, e-tailers may adjust the selling price according to their inventory level. They could provide a bigger discount when the inventory level of a product is high and a smaller discount when the inventory level is low. Incorporating a product's inventory level to the dynamic bundle pricing strategy could be a future extension to our model. Of course, such an extension would only be of value for sellers of tangible goods. Sellers of digital goods, e.g., information goods such as software, videos, news reports, stock prices, etc., would not have inventories subject to this type of constraint.

Another possible extension is to link the ODBP model with other marketing strategies, such as coupon offering. For sellers to make the optimal coupon-offering decision endogenously, it is necessary to have information about the market response of the deal-prone segment, the number of customers who are loyal to the brand, the profit margin, and the cost of managing coupons. Integrating a coupon-offering decision with the ODBP sequential bundle pricing model through understanding the market structure would be an important and interesting research topic.

Because reservation price is a cornerstone to pricing decision, a third research extension could be improving the estimation of the consumer reservation price. The current literature offers useful guidelines (e.g., Venkatesh and Kamakura 2003, Jedidi et al. 2003, Wang et al. 2007). But, for the online environment, e-tailers should also exploit the readily available information such as consumer ratings, feedback, and competitors' pricing when estimating reservation prices for bundling policy and marketing strategy.

In addition to these specific research extensions to the model, greater effort could be focused on more tightly integrating the ODBP model with continuing research in ORS. For example, ORS can update recommendation lists according to customers' online behavior, such as clicking on Web pages, rating products, and adding friends in their online social networks.

In recent years, there has been a burst of online retailing activities. As is typical in the adoption of information technology, the initial implementation tends to model the virtual world rather strictly after its real-world analogue. As e-commerce progresses, we expect e-tailers to actively take advantage of the unique characteristics of the online environment and, in particular, the opportunity to dynamically customize and price goods in ways that benefit both sellers and buyers.

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Appendix. Definition of Parameters and Variables

Given parameters

M	The total number of potential customers of the e-tailer.
N	The total number of products for sale online.
I	The number of products in the shopping cart.
J	The number of products in the recommendation list.
g_i^S	The products in the shopping cart, $i=1,2,\ldots,I$.
p^S	The bundle price of all products in the shopping cart.
p_i^S	The posted price of product g_i^S.
c_i^S	The cost of product g_i^S.
$r_{m,i}^S$	Customer m's reservation price for g_i^S.
$f_{m,i}^S$	Customer m's shipping charge of product g_i^S.
$f_{m,-i}^S$	Customer m's shipping charge for products $\{g_1^S,\ldots,g_{i-1}^S,g_{i+1}^S,\ldots,g_I^S,g_j^R\}$.
g_j^R	The products in the recommendation list, $j=1,2,\ldots,J$.
p_j^R	The price of product g_j^R.
c_j^R	The cost of product g_j^R.
$r_{m,j}^R$	Customer m's reservation price for g_j^R.
$f_{m,j}^R$	Customer m's shipping charge of product g_j^R.
b_m	The budget of customer m, $m=1,\ldots,M$.
f_m	Customer m's shipping charge of product bundle $\{g_1^S,\ldots,g_i^S,\ldots,g_I^S,g_j^R\}$.

Decision variables or intermediate variables

p_{-i}	Bundle price of products $\{g_1^S,\ldots,g_{i-1}^S,g_{i+1}^S,\ldots,g_I^S,g_j^R\}$, $i=1,2,\ldots,I$.
p	The decision variable that is the bundle price for the bundle $\{g_1^S,\ldots,g_i^S,\ldots,g_I^S,g_j^R\}$.
X_m	The decision variable that is 1 if customer m buys the bundle $\{g_1^S,\ldots,g_i^S,\ldots,g_I^S,g_j^R\}$, and 0 otherwise.

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