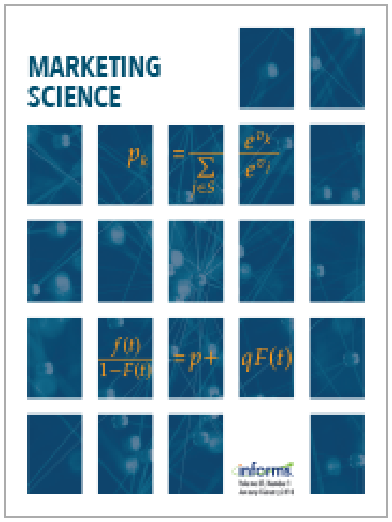

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Research Note

Prerelease Demand Forecasting for Motion Pictures Using Functional Shape Analysis of Virtual Stock Markets

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Prerelease demand forecasting is one of the most crucial yet difficult tasks facing marketers in the $60 billion motion picture industry. We propose functional shape analysis (FSA) of virtual stock markets (VSMs) to address this long-standing challenge. In VSMs, prices of a movie's stock reflect the dynamic demand expectations prior to the movie's release. Using FSA, we identify a small number of distinguishing shapes, e.g., the last-moment velocity spurt, that carry information about a movie's future demand and produce early and accurate prerelease forecasts.

We find that although forecasting errors from the existing methods, e.g., those that rely on movie features, can be as high as 90.87%, our approach results in an error of only 4.73%. Because demand forecasting is especially useful for managerial decision making when provided *long before* a movie's release, we further demonstrate how our method can be used for early forecasting and compare its power against alternative approaches. We also discuss the theoretical implications of the discovered shapes that may help managers identify indicators of a potentially successful movie early and dynamically.

Key words: prerelease demand forecasting; motion pictures; virtual stock market; functional shape analysis
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1. Introduction

Introductions of new motion pictures are confronted with enormous financial stakes, alarming failure rates, and highly uncertain demand. For instance, major Hollywood studios spend on average $106.7 million to develop and market a movie, yet each year only a handful of movies become blockbusters or at least break even (Motion Picture Association of America (MPAA) 2007). It is also commonplace that large budget films such as *Alexander* produce abysmal box office returns, whereas lesser-known films such as *Slumdog Millionaire* surprisingly become the audience favorites. To increase returns on investment, marketers are enormously interested in deriving early demand forecasts long before a movie's release to guide their decision making on, e.g., release timing, advertising, and distribution (see Appendix 1; an electronic companion to this paper is available as part of the online version that can be found at http://mktsci.pubs.informs.org). Such forecasts are particularly crucial for short life-cycle products such as motion pictures because their initial demand comprises a large portion of the overall revenues.

Prerelease demand forecasting is not only crucial but also well-recognized as a challenging task facing marketers in many industries. In particular, for the leading U.S. export motion picture industry with highly experiential products, developing accurate early forecasting models remains a top challenge and priority despite recent progress in this area (see Eliashberg et al. 2006 for an excellent review). We contribute to the growing stream of research that has used such approaches as product features and consumer surveys to derive demand forecast by analyzing prerelease trading prices of movie virtual stocks to provide greatly improved forecast accuracy.

Online virtual stock markets (VSMs) are online marketplaces where participants trade virtual stocks whose values are tied to future outcomes of events.

For example, prices of a movie stock on the Hollywood Stock Exchange (HSX), one of the best-known VSMs, are determined by prerelease expectations of the movie's box office revenues. Thus, one intriguing question is this: *How can we utilize the dynamic prerelease price changes to improve demand forecasting?* We propose the use of functional shape analysis (FSA) to automatically identify a small number of distinguishing shapes, e.g., trending up or down, that characterize the heterogeneous VSM price histories. These shapes carry information about demand and produce accurate prerelease forecasts of demand for upcoming films.

The results show that our approach greatly improves upon existing methods, particularly when valuable early forecasts are needed long before a movie's release. We find that four shapes, a high average trading price, a strong upward trend, a velocity spurt toward the time of product release, and an early velocity spurt, are associated with strong demand for a new movie. We further interpret these shapes, respectively, as indicators of overall product appeal, intertemporal sentiment, last-moment hype, and early preannouncement hype.

This research introduces a novel and powerful prerelease forecasting method to address a challenging task facing the motion picture industry. The method could be adapted to address similar needs in other industries, as we will discuss later. Our study also reflects an attempt toward demonstrating the potential of VSMs as an important new information resource for forecasting and FSA as a powerful tool to extract new knowledge from such resources. The rest of this paper unfolds as follows. We first introduce VSMs and motivate why we analyze trading price histories for demand forecasting. We then describe three major steps of FSA in §3. Although FSA can be applied to the entire price histories, it is practically most useful when applied to only the early partial price histories for early forecasting. Thus we demonstrate how to conduct such early forecasting and report the results in §4. We conclude with open questions and future research avenues in §5.

2. Virtual Stock Markets

Online VSMs have been used to forecast a wide range of events—for example, presidential elections, sports wins and losses and, more recently, demand for new products. For example, a stock is listed for initial public offering (IPO) on the HSX when a movie's development is publicized. The two million HSX participants worldwide are each endowed with $2 million virtual currency initially and can increase their net worth by strategically selecting and trading movie stocks. The current trading price of a movie stock (e.g., $70 per share) reflects the traders' collective expectation of the movie's first four weekend revenues (e.g., $70 million), which typically constitute the majority of the movies's overall theatrical demand. Similar to real-life stock markets, new information about a movie emerges over time and influences traders' beliefs. Traders who believe in a higher demand buy (or "long") the stock, whereas those who believe otherwise sell (or "short") it. Traders are motivated by the opportunities to exchange the accrued wealth for merchandise and to appear on the daily Leaderboard that features the most successful traders. On the Friday of a movie's nationwide release, trading of its stock is halted. On the following Sunday at midnight, the price is adjusted to its actual opening weekend revenues times a multiplier (e.g., 2.7), and trading is resumed. Four weeks later, the stock is liquidated at a price equal to the movie's actual first four weekend revenues.

VSMs such as the HSX have several desirable features for forecasting purposes. As opposed to the traditionally used product features that are mostly static, VSMs are *dynamic* and continuously reflect the most up-to-date demand expectations. VSMs furthermore provide *incentives* such as entertainment and competition with other traders to reward participants for active information discovery and revelation, hence engendering reliable forecasts (Dahan et al. 2007). They are also *economic* to implement, using readily available software like Inkling. A rapidly growing number of major corporations, including Hewlett-Packard, Google, Eli Lilly, and Best Buy, are also establishing employee-only VSMs to forecast new-product release timing, sales, or competitive reactions, and VSMs also provide guidance to executive decision making.

Several studies report that the *latest* VSM trading prices (i.e., immediately before a movie's release) provide reliable forecasts (Spann and Skiera 2003). Although these latest prices are straightforward to analyze, they are unavailable when crucial *early* marketing decisions need to be made weeks or months prior to releases. Also, VSM prices may not follow a random walk. Thus price *histories* may provide additional information and improve forecast accuracy above and beyond the latest prices. For instance, the augmented Dickey-Fuller test (1979) shows that the HSX prices in our sample do not follow a random walk ($F = 32.4$, significant at 0.01 level for testing if $\beta = \gamma = 0$ in $\Delta y_t = \mu + \beta t + \gamma y_{t-1} + \sum_j \phi_j \Delta y_{t-j} + \varepsilon_t$). The Lo and MacKinlay (1988) variance ratio test gives similar results.

This notion is further supported by a large volume of behavioral finance and marketing research that documents behavioral anomalies such as herding and overreaction in both real and virtual stock trading (e.g., Thaler 2005). In VSMs, the current or latest prices may not instantaneously incorporate all available information. Instead, information cascades from a handful of insiders to a larger number of less knowledgeable traders, which results in herding. Even if information diffuses immediately, traders may not interpret the information properly: they may over- or underreact to the information or delay acting upon the information. Many VSMs also limit the maximum number of shares that each participant can trade, which leads to reduced market efficiency. For VSMs such as the HSX, which trade fashionable products of enormous public interest (i.e., movies), a product whose trading prices increase very sharply toward the time of release may experience much stronger last-moment hype that stimulates higher demand as compared to one whose prices increase at a lower rate. In §3, we will introduce FSA as a method to uncover the underlying shapes of the observed trading price histories and produce reliable prerelease forecasts.

3. Functional Shape Analysis

FSA is housed within a more general framework of functional data analysis (FDA). In contrast to classical statistics that focus on a sample of data points or vectors, FDA analyzes functional observations such as curves and images (Ramsay and Silverman 2005). Although research on functional forecasting is sparse, FDA has recently been used to address a variety of marketing topics such as auctions (Wang et al. 2008) and diffusion (Sood et al. 2009). We propose FSA of VSM prices to address another critical marketing topic: prerelease demand forecasting for motion pictures.

Note that our method bears similarities to prior studies that use principal component analysis (PCA) to extract key features from repeated measurement data (Bradlow 2002, Tucker 1958, Jones and Rice 1992). We use a functional version of PCA to analyze the trading dynamics that play an important role in our forecasting model, and we also draw explicit value from the heterogeneous VSM price curves as opposed to the use of similar curves as in, e.g., Jones and Rice (1992).

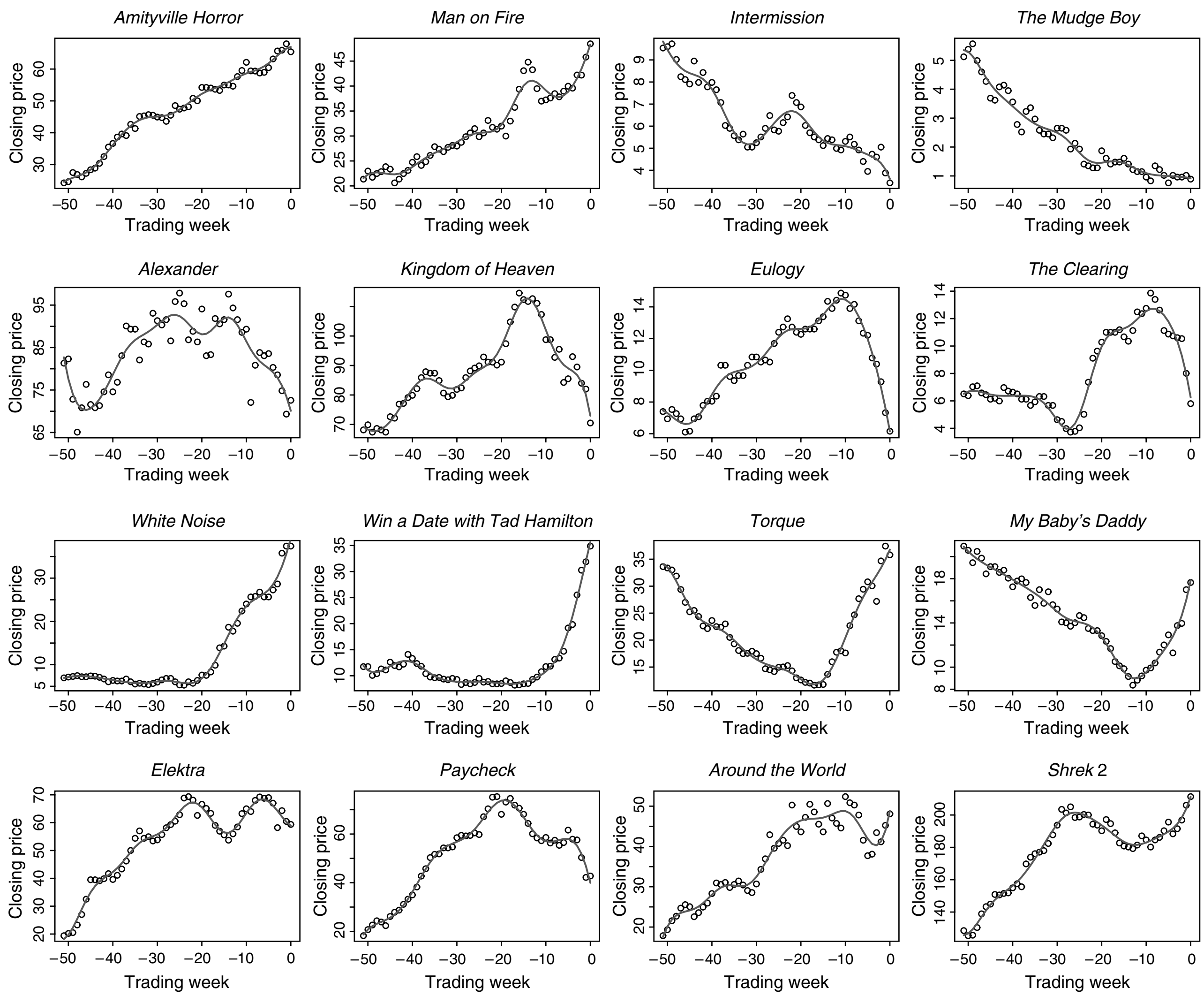
We first describe the three major steps of our approach in the context of the *entire price histories* (i.e., using all the information until a movie's release). However, FSA becomes more powerful and practically relevant when used for early forecasting (i.e., by applying it to the *early partial price histories* ending weeks or months before a movie's release). We will discuss such early forecasting in §4.

Step 1. Smooth price histories using penalized smoothing splines (see Appendix A). Smoothing eliminates noise from the raw data that might result from recording errors, system errors, or random fluctuations over time. A smooth price path also admits the *trading dynamics* in the form of first derivatives (i.e., *velocities*) and second derivatives (i.e., *accelerations*). As we will show later, these trading dynamics constitute critical elements of our forecasting model.

Our sample consists of the last 52 prerelease weekly trading prices, operationalized as the averages of each Friday's daily high and daily low of 262 movies released between December 2003 and July 2005. Although other lengths of the prerelease trading period are also admissible, we choose 52 weeks to allow for early forecasting up to one year prior to release. Figure 1 shows that the shapes of these *price paths* are considerably heterogeneous across movies. For example, although prices trend up toward the release week 0 for movies like *Amittyville Horror*, they decrease for others like *Intermission*. Movies like *Alexander* display an overall concave shape, whereas movies like *Torque* display an overall convex shape. Also, the speed at which prices trend up or down differs greatly, especially early on (*Win a Date with Tad Hamilton* versus *Paycheck*) and close to release (*Win a Date with Tad Hamilton* versus *Elektra*). Moreover, although not shown here, the shapes of the *velocity* and *acceleration* curves, obtained respectively as the first and second derivatives of these smooth price paths, also vary greatly across movies. Overall, these different shapes may contain important information about demand (e.g., last-moment hype surrounding a movie), and our goal is to identify the most indicative shapes and then use them for forecasting.

Step 2. Apply functional principal component analysis (FPCA) to extract key shapes from the trading price paths, velocities, and accelerations. FPCA is a functional generalization of ordinary PCA. Similar to PCA, FPCA projects the original data to a new space of reduced, orthogonal dimensions to capture only the most important features of the original data (see Appendix B). To better explain the fundamentals of FPCA, assume for now that each price path is measured at p discrete time points. Let $Y^s = [y_1^s, \ldots, y_n^s]$ denote the $[n \times p]$ matrix of the n smooth price paths. Its $[p \times p]$ correlation matrix, $R := \text{Corr}(Y^s)$, can be decomposed into $P^T \Lambda P$, where Λ is the diagonal matrix of eigenvalues $[\lambda_1, \lambda_2, \ldots, \lambda_p]$ and $P = [e_1, e_2, \ldots, e_p]$ is the corresponding matrix of eigenvectors. In this simplified example, each e_i is a $[1 \times p]$ vector, but in reality e_i is a continuous function in time; i.e., $e_i = e_i(t)$. Each e_i captures a unique characteristic of the price history such as trending up and down or a last-moment velocity spurt. Because each e_i captures $(\lambda_i / \sum_j \lambda_j) * 100\%$ of the variability in Y^s, the e_i corresponding to the largest (or the second-largest, etc.) λ_i is denoted as the first (or the second, etc.) principal component (PC). Only the first few PCs, or key

Figure 1 **Examples of HSX Trading Price Histories and Smooth Price Paths**



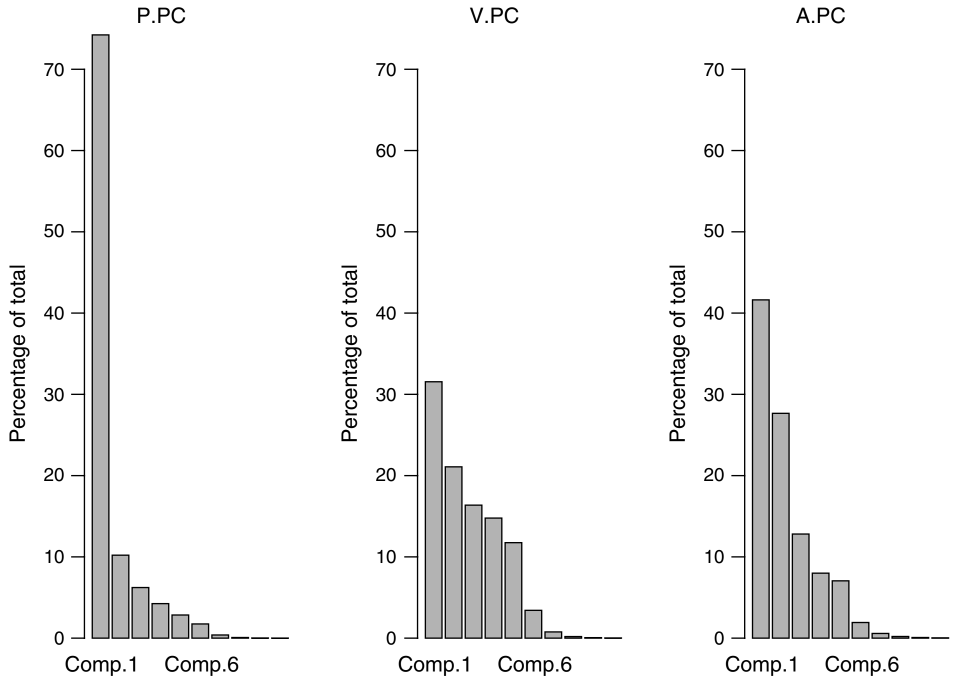
shapes, are selected to capture the most crucial features of the price histories. We similarly obtain the PCs of the trading velocities and accelerations by conducting FPCA on the velocity and acceleration curves.

Figure 2 shows that the first PC of the price paths (denoted as P.PC1; we use similar notations for other PCs of the price paths, velocities, and accelerations) explains nearly 75% of the variation. Although P.PC2 and P.PC3 appear to be carrying much less information, we incorporate them into our variable selection procedure in Step 3. We also initially retain the first five PCs of the velocities (V.PC1–V.PC5) and accelerations (A.PC1–A.PC5).

Figure 3 displays the first three PCs of the price paths and velocities. To interpret these PCs, we introduce the principal component scores (PCSs) computed from their corresponding PCs. Recall that P.PC1–P.PC3 are orthogonal to one another (similarly, V.PC1–V.PC5 are orthogonal, etc.) Thus each PC can be viewed as a weighting factor of the original data. For example, movie i's first PCS of the price path, P.PCS1_i, is the inner product of its price path, $y_i^s = [y_{i1}^s, \ldots, y_{ip}^s]$, and the first PC of the price path, $\text{P.PC1} = [e_{11}, \ldots, e_{1p}]$; i.e., $\text{P.PCS1}_i = y_{i1}^s e_{11} + \cdots + y_{ip}^s e_{1p}$.

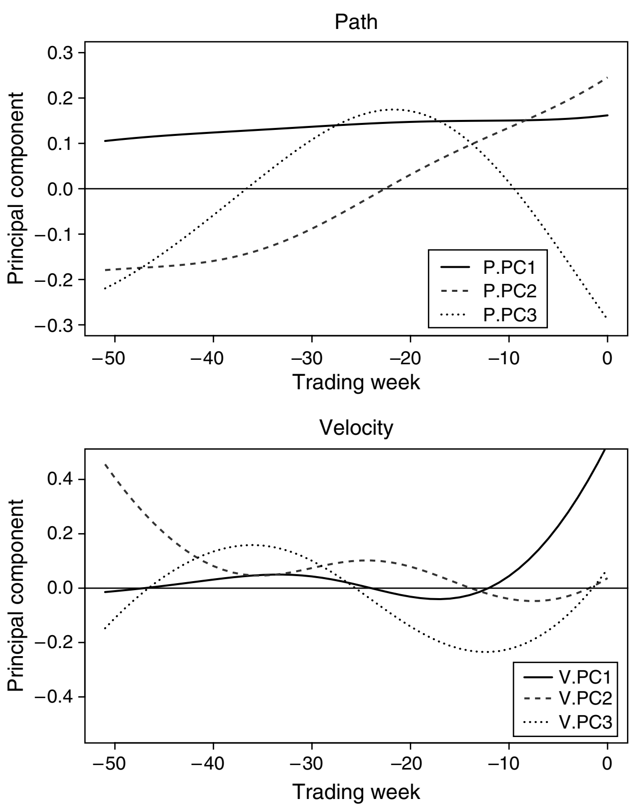
Notice that P.PC1 in Figure 3 remains almost constant and positive over the entire trading period. Thus it places almost equal and positive weight on each price along the price path when used to calculate P.PCS1_i. That is, P.PC1 captures the difference across movies in their 52-week averages: a movie with a relatively high (low) 52-week average has a positive (negative) P.PCS1. To see this more clearly, the 52-week average is \$27 for *13 Going on 30*, \$84 for *Alexander*, \$47 for *Amityville Horror* (the top panel of Figure 4), and \$32 across all movies in the sample. Accordingly, P.PCS1 is negative for *13 Going on 30* but positive for *Amityville Horror*, and even larger (and positive) for *Alexander* (the black bars in the bottom panel of Figure 4).

Figure 2 Percent Variance Explained by First Nine PCs of Price Paths, Velocities, and Accelerations



Analogously, we can interpret P.PC2 as capturing the difference between early and late prices: a movie with an upward (downward) trend has a positive (negative) P.PCS2. P.PC3 reflects the difference between midterm and early–late prices. V.PC1 emphasizes the last-moment velocity spurt: a movie with a positive (negative) last-moment velocity spurt has a positive (negative) V.PC1. Similarly, V.PC2 points to the early velocity spurt. Note that although these PCs are common across all movies, the PCSs are movie specific and are used later in our forecasting model.

Figure 3 P.PC1–P.PC3 (Top) and V.PC1–V.PC3 (Bottom)

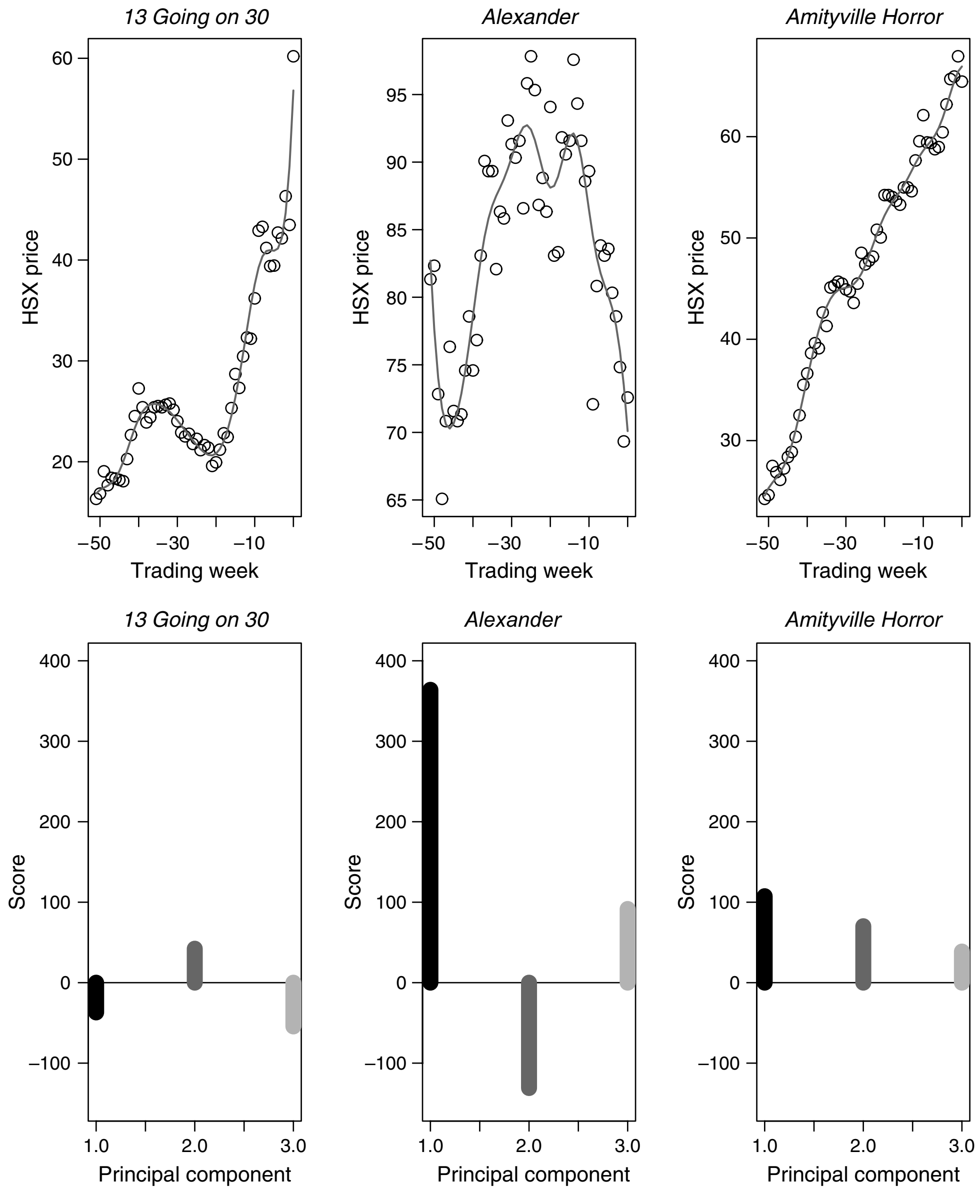


Step 3. Forecast demand using the most important shapes. Using a linear[1] regression model, we initially link the opening weekend revenues to all 13 key shapes identified in Step 2: P.PCS1–P.PCS3, V.PCS1–V.PCS5, and A.PCS1–A.PCS5. We focus on forecasting the opening weekend revenues because they often account for a third or even half of a movie's overall theatrical revenues and also provide a strong signal of the movie's revenue potential in subsequent channels, e.g., videos (see Appendices 4–7 in the electronic companion for our results from forecasting the first four weekend revenues).

We then use stepwise regression to successively eliminate the less important predictors based on the Bayesian information criteria (BIC) (Table 1). This variable selection procedure results in the following four most important shapes used in our final forecasting model: P.PCS1 (average), P.PCS2 (early versus late), V.PCS1 (last-moment velocity spurt), and V.PCS2 (early velocity spurt). Note that although the scree plot (Figure 2) suggests that P.PCS1 is the most important price path variable, our variable selection procedure suggests that we should also include P.PCS2.

[1] We also estimated a log-linear model whose fit and prediction are worse than the linear model.

Figure 4 Price Paths (Top) and P.PCS1 (Black), P.PCS2 (Dark Grey), and P.PCS3 (Light Grey) (Bottom)



Model Comparison. We evaluate the predictive performance of the proposed model (C1) versus alternative models, holding out one movie at a time. That is, we hold out the ith movie, estimate the model based on the remaining $n-1$ $(=262-1)$ movies and then compute the predictive performance for movie i using the estimated parameters and movie i's four PCSs. We repeat the procedure for all movies in the data. The predictive performance is evaluated by the mean absolute percentage errors (MAPEs) and mean squared errors (MSEs), both leading to similar conclusions (Table 2).

Model A1 is based on movie features commonly used in the literature: genre, sequel, production budgets, MPAA ratings, run time, and studios.[2] Model A2

Table 1 Variable Selection via Stepwise Regression

Model	BIC	Model	BIC	Model	BIC
3 paths, 5 vel., 5 acc.	9,026.77	**2 paths, 2 vel.**	**9,012.12**	2 vel.	9,358.20
3 paths, 5 vel.	9,029.58	2 paths	9,136.52	1 path, 2 vel.	9,048.56
3 paths, 2 vel.	9,015.79	1 path	9,194.80	1 path, 1 vel.	9,064.34

Note. Bold indicates the best-fitting model.

[2] Other movie features such as star salary are highly correlated with the included covariates, e.g., production budgets, and are thus excluded. Augmenting the movie features with screenings, critics' ratings, Oscars, and audience ratings reduces the forecast errors but does not outperform the proposed model (see Appendix 3 in the electronic companion). We also applied Bayesian estimation

Table 2 Model Comparison

Model	Description	MAPE (%)	MSE (10^{12})
A1	Movie features	90.87	221.29
A2	Movie features + Ad	69.37	225.80
B1	Latest price	10.35	49.02
B2	Latest price + Movie features	11.64	50.04
B3	Latest price + Movie features + Ad	12.54	57.17
C1	**4 shapes (proposed model)**	**4.73**	**42.03**
C2	2 path shapes, no dynamics	11.82	72.34
C3	All 13 shapes	8.87	51.13
C4	4 shapes + Movie features	12.79	59.30
C5	4 shapes + Ad	5.79	59.22
C6	4 shapes + Movie features + Ad	13.69	67.70
C7	4 shapes + Latest price	9.43	45.46
D1	Avg.	18.60	98.11
D2	Volume-weighted avg.	7.39	72.07
D3	Median	25.25	103.54
D4	Avg. + Linear	10.72	73.02
D5	Avg. + Linear + Nonlinear	18.56	69.27
E1	Avg. + End.early + Late spurt + Early spurt	11.96	42.46
E2	CART on all 13 shapes	21.42	106.98
E3	GAM on all 13 shapes	9.43	67.48
E4	CART on 4 shapes	33.10	99.76
E5	GAM on 4 shapes	4.08	72.23

Note. Bold indicates the best-fitting model.

includes both movie features and weekly advertising dollars in the last 10 prerelease weeks when the majority of prerelease advertising takes place. Both yield the highest forecast errors among all models. Augmenting the latest prices (B1) with movie features and advertising (B2 and B3) does not reduce errors, which suggests that information embedded in movie features and advertising may have been incorporated in the latest prices. To validate this, we reestimate Models B1–B3 on all 262 movies. The coefficients for movie features and advertising are all insignificant, and the F-tests cannot reject the null hypotheses that the coefficients for movie features in B2 ($F = 1.45$, p-value $= 0.07$) and for both movie features and advertising in B3 ($F = 1.19$, p-value $= 0.22$) are jointly zero.

The proposed model (C1) has the lowest errors (except when compared to E5, as discussed later) and reduces the errors of B1 based on the latest prices by up to 54%. In light of the costly investment and low return on investment of each movie, such improvement is substantial and managerially important for studios in prerelease marketing planning (e.g., media purchase, theater contracts, and merchandise deals). To better understand why using the shapes (C1) above and beyond the latest prices (B1) improves forecast accuracy, we further regress the gap between B1 and C1's forecast errors on movie features and trading covariates. We find that the gap is linked to higher last-moment trading volumes for any given price and higher last-moment advertising effort (see Appendix 8 in the electronic companion). This suggests that the last-moment spurt in hype about a movie is likely captured by the shapes but not entirely by the latest prices alone.

Consistent with this finding, we also observe that the forecast errors considerably increase when the two velocity shapes are removed (C2) from the four-shape model (C1), confirming the importance of the trading dynamics. The parsimonious four-shape model (C1) also outperforms C3 using all 13 key shapes. When enriching the four shapes with movie features (C4), prerelease advertising (C5), movie features and prerelease advertising (C6), or the latest prices (C7), the forecast errors actually increase due to overfitting.

To determine if summary statistics or simple measures of trends may capture the price histories equally well, we examine the averages (D1), volume-weighted averages (D2), medians (D3), averages plus linear trends (from fitting a zero-intercept linear model on the 52 de-meaned weekly prices) (D4), and averages plus linear trends plus nonlinear trends (from fitting a quadratic model on the 52 de-meaned and detrended weekly prices) (D5). They yield higher forecast errors than the four-shape model, indicating that the summary statistics and simple measures of trends not only require user input but also have limited capability of characterizing the price histories. In contrast, FSA automates the discovery of the key shapes in the data and thus alleviates possible omissions of important patterns and facilitates ready applications to different forecasting tasks.

We further validate the four-shape model using four simple measures (E1): the 52-week averages (Avg.), latest-minus-earliest prices (End.early), Late spurt (linear slope of the de-meaned, week -9 to week 0 prices), and Early spurt (linear slope of the de-meaned, week -51 to week -42 prices). Although these measures do not perfectly replicate the proposed model, the model's forecast performance confirms the value of the four shapes even if constructed with simpler measures. We also estimate two classification and regression tree (CART) models (E2 and E4) that automatically select the most important predictors through recursive, tree-type partitions of the data. They are outperformed by the proposed model (C1), which supports our variable selection procedure that renders a parsimonious yet powerful forecasting model. To test the linearity assumption of our model, we further estimate two nonparametric generalized additive models (GAMs) that do not assume any strict functional relationships (E3 and E5). E5 fares slightly

to those models in Table 2 that do not take advantage of the cross-sectional information as in our model (see Appendix 7 in the electronic companion).

Table 3 Forecasting Opening Weekend Revenues Using Four Shapes

	Estimate	Std. err.	p-value
Intercept	14,605,722	417,257	0.00
P.PCS1: Average	53,463	1,956	0.00
P.PCS2: Early versus late	80,789	12,084	0.00
V.PCS1: Last-moment velocity spurt	1,254,831	95,339	0.00
V.PCS2: Early velocity spurt	327,533	151,810	0.00

better than the proposed model in the MAPE, suggesting that the relationship between the revenues and four shapes may be nearly, but not perfectly, linear.

To summarize, the proposed four-shape model accommodates the dynamics in the price histories and greatly improves the forecast accuracy over the alternative methods, e.g., those based on product features, latest prices, summary statistics, or simple measures of trends.

Parameter Estimates. Table 3 shows that the coefficients of all four shapes are significant: a higher 52-week average, a steeper upward trend, and more rapid and more positive last-moment and early velocity spurts are all associated with higher opening weekend revenues. These indicators may be collectively used as rules of thumb for managerial decision making.

To better understand why these shapes are indicative of demand for a movie, we propose plausible theoretical explanations. P.PCS1 is associated with a stock's average price and represents the *overall appeal* of a movie relative to other movies. It reflects beliefs of potential buyers' preferences for product features, marketing effort, and potential competition. For example, the difference in the average prices between *Kingdom of Heaven* ($87.56) and *Eulogy* ($10.60) may arise from Orlando Bloom's star appeal, a much stronger prerelease ad support for the former ($31 million versus $0.2 million), or both. Overall, the same set of information that impacts the average trading prices, or demand expectations, also drives the actual demand (Henard and Szymanski 2001). Also interesting and consistent with the well-known *recency effect*, P.PC1 slightly increases over time and thus weighs the recent information more heavily than earlier information.

P.PCS2 pertains to early versus late prices and reflects *intertemporal sentiment* toward a movie: a positive (negative) trend in the sentiment is a strong indicator of potential success (failure). Consistent with the *prospect theory* (Kahneman and Tversky 1979), decreasing expectations over time, driven by negative information such as release delays, can lead to loss aversion among potential adopters and thus reduced demand at the time of release. This is also consistent with why firms are vastly interested in strategic management of prerelease expectations (Kopalle and Lehmann 2004).

V.PCS1 reflects *last-moment hype* about a movie. Hype is a set of prerelease activities that contribute to the acceptance of a new product, including distribution, media, and opinion leaders. It is a key element that should be incorporated in forecasting models. A rapid increase in the last-moment trading prices may suggest a swift diffusion of the last-moment information to potential adopters and a dramatic rise in preferences and desire for a movie. The impact of last-moment hype on demand also rationalizes why studios ramp up their advertising spending drastically toward release.

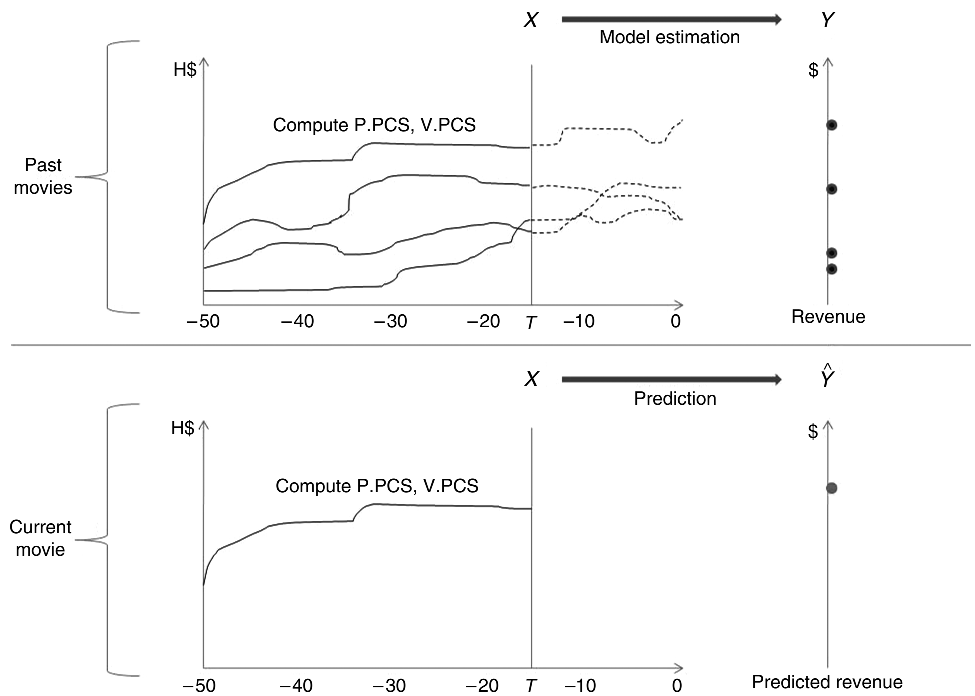
V.PCS2 points to *early preannouncement hype* about a movie. Prior research shows that preannouncements may create pent-up demand and network externalities, generate word of mouth (WOM), promote brand preference and perceptions, and deter releases of rival products (e.g., Bayus et al. 2001). A rapid increase in early trading prices suggests a rapid diffusion of awareness among potential adopters and strong interest in a movie. Accordingly, we conjecture that V.PCS2 is associated with sequels (i.e., extensions of a prior successful product) and production budgets that are often associated with probabilities of new-product development success. Indeed, the average V.PCS2 in our data is 2.89 for sequels versus 0.37 for nonsequels, and the correlation between V.PCS2 and budget is 0.43.

4. Early and Dynamic Prerelease Forecasting

The results so far suggest that our approach is capable of producing accurate prerelease forecasts based on the *entire price histories* up to a movie's release. In reality, early and dynamic forecasts, long before a movie's release, are most valuable to managers. In §3 we outlined the general ideas of FSA; we now illustrate how our method can be readily applied to *early partial price histories* for early forecasting.

Consider Figure 5 as an illustration. Our goal is to forecast the ith movie's opening weekend (week 0) revenue long before its release, given that we only observe its early partial price history up to $T \ll 0$ (the solid line in the bottom panel of Figure 5). With the observed opening weekend revenues of the remaining $n-1$ (=261) movies in our training sample (or, in practice, a sample of past movies), we first apply the three-step FSA procedure to these movies' early partial price histories from week -52 to week T (the solid lines in the top panel of Figure 5). This procedure outputs parameter estimates similar to those in Table 3. We then smooth the early partial price history of movie i from week -52 to week T (Step 1), extract its four PCSs (Step 2), and forecast its opening weekend

Figure 5 Using Early Partial Price Histories to Forecast



revenue by plugging in the derived parameter estimates from the training sample (Step 3).

Applying this procedure to early partial price histories of varying lengths (i.e., $T = -52, -51, -50, \ldots, 0$), we derive early and dynamic forecasts for all models from Table 2. To avoid visual clutter, we only show in Figure 6 the resulting MAPEs and their 95% confidence bands for the proposed model (C1), those using movie features and prerelease advertising (A1, A2, and C6 in the left panel of Figure 6), and those based on the latest prices from time T (B1 and B3 in the right panel of Figure 6).

Not surprisingly, managers can derive increasingly accurate forecasts as more information becomes

Figure 6 Early and Dynamic Prerelease Forecasting via FSA of Early Partial Price Histories

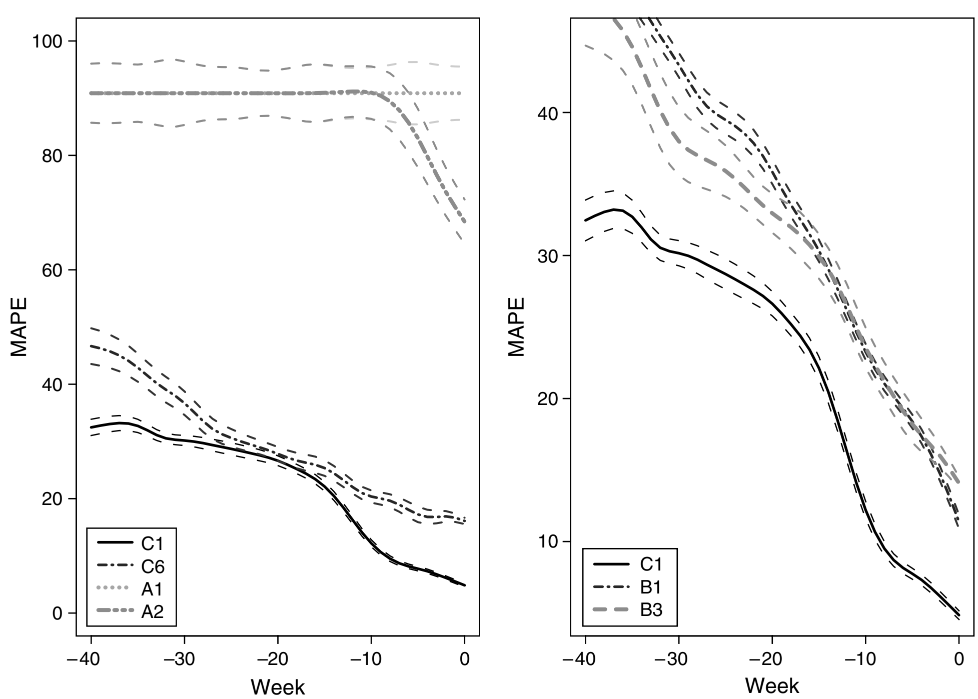


Table 4 Comparison Between Proposed Model C1 and Alternative Models in Figure 6

		Alternative models' MAPE (%) − C1's MAPE (%)				
Model	Description	Wk −40	Wk −30	Wk −20	Wk −10	Wk 0
A1	Movie features	58.75	60.03	63.51	79.49	86.13
A2	Movie features + Ad	58.75	60.03	63.51	79.49	64.33
C6	4 shapes + Movie features + Ad	13.65	6.43	0.34	7.93	8.96
B1	Latest price	17.50	13.19	9.24	11.19	5.61
B3	Latest price + Movie features + Ad	15.43	6.24	5.14	11.24	8.58

available over time, especially close to release week 0. The forecast errors of the proposed model (C1) decrease only gradually early on but rather rapidly after week −15. This suggests that the amount of information about a movie increases at a nonconstant speed, especially after prerelease advertising commences (typically around week −15). The proposed four-shape model (C1) significantly outperforms all other models, and Table 4 displays a more detailed numerical comparison.

Interestingly, Table 4 shows that augmenting the four shapes (C1) with product features and prerelease advertising (C6) does not lead to lower errors. This suggests that much of the information about movies has been captured by their trading shapes: thus, incorporating additional information may lead to overfitting. In contrast, enriching the latest prices (B1) with the same information (B3) initially reduces forecast errors, indicating that these latest prices are less capable of capturing all relevant information compared with shapes, especially early on.

Overall, our results show that the proposed four-shape model produces accurate, early, and dynamic forecasts. Similar forecasts cannot be accomplished, at least not easily, by using product features that are mostly time invariant, sales in other channels or markets that are not available early on or are not suitable for forecasting sales in the first channel or market (e.g., theaters for movies), or consumer surveys that are expensive to administer repeatedly, to name a few examples.

5. Discussion

We introduce functional shape analysis of virtual stock markets to address a crucial yet challenging task facing the $60 billion motion picture industry: prerelease demand forecasting. The results show that our approach leads to greatly improved forecast accuracy compared with existing methods. More importantly, our method produces early and dynamic prerelease forecasts that are most valuable to managerial decision making. We also discover that four shapes—a high trading average, a strong upward trend, and early and late velocity spurts—are linked to higher demand for a new movie. We further propose that these shapes are indicators of overall product appeal, intertemporal sentiment, last-moment hype, and early preannouncement hype.

Our research capitalizes on VSMs' capability to aggregate diversified sources of information and FSA's capability to capture the trading price dynamics that reflect the speed of information diffusion, learning, and potential demand changes. It introduces VSMs to the literature as an interesting new source of data and a potentially powerful tool to address important marketing topics such as forecasting. It also introduces a new functional forecasting methodology that can be readily adapted to other forecasting tasks. Notwithstanding these contributions, our research has a number of limitations.

We acknowledge that our empirical findings apply primarily to the motion picture industry and that further research is needed to establish external validity. It would be intriguing to examine if similar conclusions can be drawn from examining other product categories that share similar characteristics (e.g., frequent introductions of new, unique, and experiential products; pop cultural appeal; and strong influence of hype on demand) to movies. Such research would be made possible by the increasing availability of data from VSMs for books (MediaPredict.com), music (HSX.com), TV shows (Inklingmarkets.com), and video games (simExchange.com).

We also recognize that our research is highly exploratory and primarily aims at developing more accurate *predictive models* for better managerial decision making. It is not directly geared toward the development or confirmation of theories. Nonetheless, predictive models such as ours can be very valuable for theory building in emerging and fast-changing environments such as VSMs, where data are plentiful but theories are scarce (Shmueli and Koppius 2008). We hope that our work is one step toward that direction. In that context, several recent studies have already identified similar shapes in different domains. For example, Sood et al. (2009) identified a similar set of shapes in market penetration curves of 21 categories of new products across 70 countries. Further research will help establish if similar shapes can be identified in other product categories and will thus engender theoretical generalization that identifies and, more importantly, accelerates the creation of prerelease success factors.

We also want to point out several caveats about our FSA approach. In this research, we apply FSA to a balanced data sample by using only the most recent 52 trading weeks for each movie. Using a balanced sample avoids methodological complications that have been recognized by prior studies (Bradlow 2002). In fact, left censoring of the price histories renders our results conservative in demonstrating the true predictive power of FSA because potentially useful information has been eliminated by the censoring process. Although FSA can also be applied to unbalanced samples, it may require differential weighing of individual price histories. As there has been no research on this topic yet, we consider it a fruitful area for future endeavors.

In contrast to technical analysis (TA) of financial data, FSA can be used in a wide array of domains such as auctions (Wang et al. 2008) and diffusion (Sood et al. 2009). In fact, it may prove valuable for many longitudinal marketing data such as pricing, advertising, click streams, and online WOM. Furthermore, FSA discovers key shapes in an automated data-driven fashion, requires minimal user input, and can thus be readily and repeatedly applied to many different forecasting tasks. In contrast, TA uses manually created summary statistics to characterize price histories and thus relies heavily on user input (Dahan et al. 2007).

We also want to point out that the stock prices in our data do not follow a random walk, and as a result, our approach can take advantage of the price histories to further improve forecast accuracy. As VSMs become more mature and efficient over time, such advantages may disappear. Thus, researchers who wish to apply our methods to data from other VSMs may want to first perform such tests as the augmented Dickey-Fuller test and Lo and MacKinlay variance ratio test to establish that their data histories contain additional information above and beyond the last data point.

Finally, VSMs also present great potential for marketing research and practice. For example, it would be of great managerial interest to examine how to dynamically optimize such decisions as advertising spending or release date selections, given the demand forecasts derived from the VSM prices. In fact, the nonlinear increase of information in Figure 6 suggests that some time points are more amenable to decision making than others. An increasing number of marketing studies are also exploiting the ability of these markets to aggregate information and facilitate learning and are using their incentive-aligned market-based mechanisms to measure preferences, generate new-product ideas, and assess the effectiveness of marketing mix (e.g., Dahan et al. 2007, Ding et al. 2009).

6. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mktsci.pubs.informs.org/.

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Appendix A. Deriving Smooth Trading Price Paths

FDA operates on a set of continuous functional objects, e.g., a set of continuous curves describing the daily temperature changes (Ramsay and Silverman 2005), the prices in an online auction (Wang et al. 2008), or the trading prices of an online VSM. Despite their continuous nature, limitations in human perception and measurement capability allow us to record only discrete (for example, weekly observations of these curves). Thus, the first step is to recover the underlying continuous functional objects from the observed data using smoothing techniques. There are a variety of data smoothers. We use a flexible and computationally efficient technique called the penalized smoothing spline.

Specifically, let $\tau_1, \ldots, \tau_L$ be a set of knots. Then, a polynomial spline of order p is given by $f(t) = \beta_0 + \beta_1 t + \beta_2 t^2 + \cdots + \beta_p t^p + \sum_{l=1}^{L} \beta_{pl}(t-\tau_l)_+^p$, where $u_+ = uI_{[u\geq 0]}$ denotes the positive part of the function u. Define the roughness penalty $PEN_m(t) = \int \{D^m f(t)\}^2\,dt$, where $D^m f$, $m = 1, 2, 3, \ldots,$ denotes the mth derivative of the function f. The penalized smoothing spline f minimizes the penalized squared error $PENSS_{\lambda,m} = \int \{y(t) - f(t)\}^2\,dt + \lambda PEN_m(t)$, where $y(t)$ denotes the observed data at time t, and the smoothing parameter λ controls the trade-off between data fit and smoothness of the function f. Using $m = 2$ leads to the commonly encountered cubic smoothing spline. Other possible smoothers include the use of B-splines or radial basis functions.

Estimation of the smoothing splines is conducted similarly to ordinary least squares. Define the $(L+p+1)$ vector of spline basis function $x(t) = (1, t, t^2, \ldots, t^p, [(t-\tau_1)_+]^p, \ldots, [(t-\tau_L)_+]^p)$. Then, we can write $f(t) = x(t)\beta$, where $\beta = (\beta_0, \beta_1, \ldots, \beta_p, \beta_{p1}, \ldots, \beta_{pL})'$ is the $(L+p+1)$ parameter vector. The roughness penalty can now be written as $P_m = \beta' D \beta$, where the symmetric positive semidefinite penalty matrix D is defined as $D = \int \{D^m x(t)\}'\{D^m x(t)\}\,dt$.

We then rewrite the penalized residual sum of squares as $Q_{\lambda,m} = \lambda\beta' D\beta + \sum_{i=1}^{n}\{y_i - x(t_i)\beta\}^2$. Let $y = (y_1, \ldots, y_n)'$ denote the vector of the observed trading prices, and define the matrix of spline basis functions $X = (x(t_1), x(t_2), \ldots, x(t_n))'$. We can now write $Q_{\lambda,m} = \lambda\beta' D\beta + (y - X\beta)'(y - X\beta)$. Setting the gradient equal to zero and rearranging the terms yields the estimating equations $(X'X + \lambda D)\beta = X'y$. Solving for β gives the penalized spline estimator $\hat{\beta}_{ps} = (X'X + \lambda D)^{-1}X'y$. Note that the Hessian matrix equals $2(X'X + \lambda D)$. Because the matrix $X'X$ is positive definite and λD is positive semidefinite, the Hessian matrix is positive definite and, thus, $\hat{\beta}_{ps}$ indeed minimizes the penalized residual sum of squares.

In this study, we use smoothing splines of order $p = 4$ and a smoothing parameter of $\lambda = 50$. We place a knot at every week of the 52-week trading period, which results in 52 knots per price path. Although the choice of the smoothing parameter value can appear arbitrary, our specific selection is guided by the goal of obtaining smooth functional objects that visually represent the original data well. Moreover, we also conduct a robustness study and find that the results do not vary much, given different choices of the smoothing parameter values (see Appendix 2 in the electronic companion).

Appendix B. Functional Principal Component Analysis (FPCA)

FPCA is a functional generalization of ordinary PCA, which operates on a set of data vectors $x_1, \ldots, x_n$, where each observation is a p-dimensional data vector $x_i = [x_{i1}, \ldots, x_{ip}]$. The goal of PCA is to find a projection of $x_1, \ldots, x_n$ onto a new space of orthogonal dimensions while maximizing the variance along each dimension. Specifically, a PC vector $e_1 = [e_{11}, \ldots, e_{1p}]$ needs to be determined such that its PCS, $S_{i1} = \sum_j e_{1j}x_{ij} = e_1^T x_i$, maximizes $\sum_i S_{i1}^2$ subject to $\sum_j e_{1j}^2 = \|e_1\|^2 = 1$. This yields the first PC, e_1. Similarly, in the next step, the second PC, $e_2 = [e_{21}, \ldots, e_{2p}]$, is computed for which the PCS, $S_{i2} = e_2^T x_i$, maximizes $\sum_i S_{i2}^2$ subject to $\|e_2\|^2 = 1$ and the additional constraint $\sum_j e_{2j}e_{1j} = e_2^T e_1 = 0$, which ensures that the resulting PCs are orthogonal. The above steps are repeated for the remaining PCs, $e_3, \ldots, e_p$.

FPCA is similar in nature except that it operates on a set of continuous curves instead of discrete vectors. Thus, summations are replaced by integrations. Specifically, assume that each of the curves $x_1(s), \ldots, x_n(s)$ is measured on a continuous scale indexed by s. The goal is to find a corresponding set of PC curves $e_i(s)$ that maximize the variance along each orthogonal dimension. First, a PC function $e_1(s)$ is determined such that its PCS, $S_{i1} = \int e_1(s)x_i(s)\,ds$, maximizes $\sum_i S_{i1}^2$ subject to $\int e_1^2\,ds = \|e_1\|^2 = 1$. Then, the next step involves finding e_2, whose PCS, $S_{i2} = \int e_2(s)x_i(s)\,ds$, maximizes $\sum_i S_{i2}^2$ subject to $\|e_2\|^2 = 1$ and the additional constraint $\int e_2(s)e_1(s)\,ds = 0$.

In practice, the above integrals are circumvented by either operating on a fine grid or finding a lower-dimensional expression for the PC functions $e_i(s)$ through basis expansions. For example, let $\phi(s) = [\phi_1(s), \ldots, \phi_K(s)]$ be a suitable basis expansion. Then, we can write $e_i(s) = \sum_{k=1}^K b_{ik}\phi_k(s) = \phi(s)^T b_i$ for a set of basis coefficients $b = [b_{i1}, \ldots, b_{iK}]$. The integral in $\int e_2(s)e_1(s)\,ds$ becomes $\int e_2(s)e_1(s)\,ds = b_1^T W b_2$, where $W = \int \phi(s)\phi(s)^T\,ds$ (Ramsay and Silverman 2005).

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