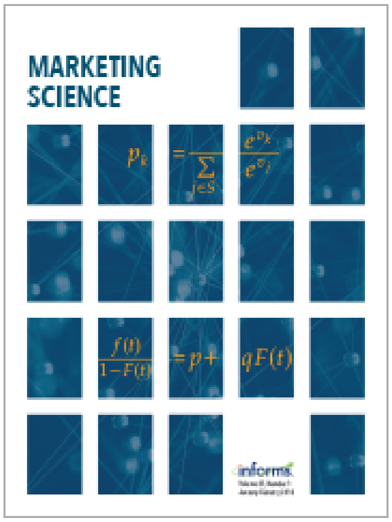

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Click Fraud

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Click fraud is the practice of deceptively clicking on search ads with the intention of either increasing third-party website revenues or exhausting an advertiser's budget. Search advertisers are forced to trust that search engines detect and prevent click fraud even though the engines get paid for every undetected fraudulent click. We find conditions under which it is in a search engine's interest to allow some click fraud.

Under full information in a second-price auction, if x% of clicks are fraudulent, advertisers will lower their bids by x%, leaving the auction outcome and search engine revenues unchanged. However, if we allow for uncertainty in the amount of click fraud or change the auction type to include a click-through component, search engine revenues may rise or fall with click fraud. A decrease occurs when the keyword auction is relatively competitive because advertisers lower their budgets to hedge against downside risk. If the keyword auction is less competitive, click fraud may transfer surplus from the winning advertiser to the search engine. Our results suggest that the search advertising industry would benefit from using a neutral third party to audit search engines' click fraud detection algorithms.

Key words: advertising; auctions; click fraud; game theory; Internet marketing; search advertising
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1. Introduction

Search advertising revenues grew from virtually nothing in 1996 to more than \$7 billion in 2006, constituting 43% of online advertising revenues (*Advertising Age* 2006). The primary benefits of search advertising for advertisers are its relevance and accountability. It tends to reach consumers as they enter the market for the advertised product, and advertisers' ability to track consumers' actions online allows for accurate measurements of advertising profitability.

The downside of this accountability is a practice known as "click fraud." Website publishers or rival advertisers may impersonate consumers and click search ads, driving up advertising costs without increasing sales, effectively stealing a firm's paid advertising inventory. The Click Fraud Network, which defines itself as "a community of online advertisers, agencies and search providers," estimated that 16.6% of all paid clicks on search engine home pages and 28.3% of all paid clicks on search engines' content networks in the fourth quarter of 2007 may have been fraudulent. Discussions with executives in the search advertising industry indicate that the amount of click fraud varies widely across industries and keywords. The perceived threats of click fraud may outweigh the benefits of using search advertising for some firms in high-risk categories.

Seventy-three percent of search advertisers say that click fraud is a concern (*Advertising Age* 2006). The question of click fraud is vexing because search engines cannot give advertisers full information about how they detect and prevent click fraud. Doing so would be tantamount to providing unscrupulous advertisers with directions on how to commit click fraud. Advertisers are therefore forced to trust that search engines do their utmost to prevent click fraud, even though the search engines get paid every time they fail to detect a fraudulent click. This trust was called into question in 2006 when Google CEO Eric Schmidt was quoted as saying

> Eventually the price that the advertiser is willing to pay for the conversion will decline because the advertiser will realize that these are bad clicks. In other words, the value of the ad declines. So, over some amount of time, the system is, in fact, self-correcting. In fact, there is a perfect economic solution, which is to let it happen (Ghosemajumder 2006).

His remarks were interpreted as suggesting that market forces would eliminate any negative effects of click fraud in the long run, possibly undermining the need for click fraud detection.[1]

The primary objective of this paper is to understand how click fraud affects search engines' advertising

[1] However, Google CFO George Reyes had previously said "I think something has to be done about [click fraud] really, really quickly, because I think, potentially, it threatens our business model" (Stone 2005, p. 52).

revenues. We also hope to gain insights into what actions search engines may be able to take to mitigate click fraud. We present an analytical model of the auction market for search advertising keywords and then introduce the possibility that third-party websites or rival bidders may engage in click fraud. The strengths of our model are its parsimony and generality because firms' search advertising objectives and the degree of competition in keyword auctions vary widely across keywords.

We model the elemental unit of competition in the industry: Two advertisers bid in an auction for a single advertising slot sold by a monopoly search engine. We find that, in a second-price auction, when firms know that x% of all clicks will be fraudulent, they lower their bids by x%. In equilibrium, this adjustment leaves advertising expenditures and the auction result unchanged. However, when the amount of click fraud is uncertain or when the auction contains a click-through rate component, search engine revenues may increase or decrease with click fraud. A decrease may occur in relatively competitive keyword auctions as high bidders hedge their advertising budgets to protect against the threat of a high realization of click fraud. On the other hand, advertising revenues may increase in relatively uncompetitive auctions if the foregone profits of exiting the ad auction outweigh the effects of click fraud, resulting in a transfer from a very profitable advertiser to the search engine.

The surge in Internet usage and advertising revenues has attracted substantial academic interest (see, e.g., He and Chen 2006, Iyer and Pazgal 2003, Manchanda et al. 2006, Prasad 2007). For example, Danaher (2007) estimates a comprehensive model of website-visit behavior based on a large-scale panel of Internet users, showing how to predict reach and frequency for various schedules of online advertisements. Theoretical research on search advertising has focused mainly on competition in advertising auctions and consumer search. Baye and Morgan (2001) analyze a homogeneous-products market organized by a search engine ("gatekeeper") and show that the gatekeeper's incentive is to maximize consumer adoption but limit the number of advertisers using the platform because it can extract more revenues when competition among advertisers is lessened. Chen and He (2006) analyze optimal consumer search and advertiser bid strategies and show that advertisers' bid order mirrors their products' relevance order. Consumers then optimally engage in sequential search. Edelman et al. (2007) solve for equilibrium in what they call the "generalized second-price" auction, a mechanism similar to what Yahoo! used to allocate search ads until 2007. Varian (2007) independently discovers some similar properties in a matching problem between advertisers and ad slots. Borgers et al. (2007) generalize advertiser preference functions beyond the previous two papers, finding that this implies multiple equilibria, many of which are inefficient. Katona and Sarvary (2007) extend these analyses by introducing click-through rates into the auction mechanism and allowing for interplay between paid and organic search results.

Empirical work on search advertising has focused mainly on the link between keyword prices and advertiser profitability. Goldfarb and Tucker (2007) show that keyword prices increase in advertisers' profitability of advertising and decrease with the availability of substitute advertising media. Rutz and Bucklin (2007a) develop a model to enable advertisers to decide which keywords to keep in a campaign and showed that keyword characteristics and ad position influence conversion rates. Rutz and Bucklin (2007b) show that there are spillovers between search advertising on branded and generic keywords, as some customers may start with a generic search to gather information, but later use a branded search to complete their transaction. Ghose and Yang (2007) estimate a model of consumer search and advertiser behavior, linking keyword characteristics to purchase rates and evaluating the optimality of advertiser bids.

We are not aware of any previous analyses of the economic effects of click fraud.

In the next section we discuss the institutional details of the industry that guide our analysis. Section 3 presents a baseline model of search advertising sales in a second-price auction, to which we compare click fraud equilibria under various assumptions in §§4 and 5. In §6, we derive click fraud's effects in what we call the "click through" auction. Section 7 provides a nontechnical discussion of the managerial implications of our analysis. Section 8 discusses limitations and future research opportunities. Proofs of all lemmas and propositions are contained in the appendix.

2. Industry Background

In this section, we describe the market for search advertising, types of click fraud, advertiser perceptions of click fraud, and issues in click fraud detection and measurement.

2.1. The Search Advertising Marketplace

Search advertising, also known as "cost-per-click" (CPC) or "pay-per-click" advertising, is sold on a per-click basis. Advertisers bid on a word or phrase related to their business and enter a maximum advertising budget per time period. When consumers enter that "keyword" into a search engine or read a third-party Web page relevant to the keyword, the advertiser's ad may then be displayed along with the consumer's search results or Web page content. If the consumer clicks on the advertiser's ad, she is redirected to a Web address chosen by the advertiser, and the advertiser is charged a fee. Advertising costs and quantity of searches available vary widely across keywords.

The cost-per-click business model was pioneered by GoTo.com, which was later renamed Overture and acquired by Yahoo!. Overture sold keywords in a public-information, first-price auction. It later changed its auction mechanism to a private-value variation on Vickrey's (1961) second-price auction, the generalized second-price auction described by Edelman et al. (2007). Google adopted the CPC auction in 2002, and introduced the use of consumers' "click-through rate" (CTR), the number of consumers who clicked on the ad divided by all consumers who saw the ad, as an element in its auction mechanism (Battelle 2005). Google's early ad-ranking algorithm was the product of advertisers' per-click bid and click-through rate (Goodman 2006). In 2005, Google changed its auction mechanism to include a "quality score" that included CTR and additional predictors of consumer response to ads (such as the match between ad text and search term). Yahoo! introduced elements of consumer acceptance into its auction in 2007. Neither Google's or Yahoo!'s current auction mechanism is publicly known (Gupta 2007).

The market leaders are Google, Yahoo!, and Microsoft, with 66%, 21%, and 5% of clicks, respectively.[2] Google receives about 75% of paid search advertising revenues.[3]

Keyword prices vary according to advertiser profitability, media competition, and keyword characteristics. Although not representative, the keyword "mesothelioma attorney" cost an average of $35 per click, but region-specific keyword costs reached as high as $80 per click (Goldfarb and Tucker 2007). Rutz and Bucklin (2007b) illustrate the dramatic differences between keywords containing branded and generic terms. In a search advertising campaign for a hotel chain, branded keywords on Google created 3.5 million impressions, with a click-through rate of 13.3% and a cost per reservation of $2.76. Generic keywords generated 19.9 million impressions, with a click-through rate of 0.3% and a cost per reservation of $61.71.

There is some evidence that higher ad positions are more desirable because not all consumers read through all of the ads. For example, Wilk (2007) reported that 62% of all searchers do not read past the first page of ads, and 23% do not read past the first few ads. He also noted that consumers often refine their search if they do not find a good ad among the first few slots. Chen and He (2006) find that a higher ad listing sends a quality signal to uninformed consumers. Rutz and Bucklin (2007a) and Ghose and Yang (2007) demonstrate empirically that higher ad positions result in higher conversion rates.

Other forms of online advertising include cost per thousand (CPM), in which websites are compensated on an impression basis, and cost per action (CPA), in which advertisers pay per sale or lead. Prasad (2007) discussed "impression fraud," a problem in CPM advertising that is conceptually similar to click fraud but operationally different. CPA advertising has the potential to resolve click fraud concerns, but has a principal/agent problem in which advertisers are incented to conceal customer leads and conversions from the search engine. Google piloted a CPA beta test in 2007, but participating advertisers were required to use Google software to track their conversions. It may be that if advertisers reveal enough revenue information to the search engine to resolve the principal/agent problem, the search engine would be able to design its auction mechanism to extract maximal advertising revenues. We speculate that CPA will cannibalize some CPC revenues, but we do not expect it to completely replace the CPC business model.

2.2. Types of Click Fraud

Search advertisers are charged when their ads are clicked, regardless of who does the clicking. Clicks may come from potential customers, employees of rival firms, or computer programs. We refer to all clicks that do not come from potential customers as click fraud.

Click fraud is sometimes called "invalid clicks" or "unwanted clicks." This is partly because the word "fraud" has legal implications that may be difficult to prove or contrary to the interests of some of the parties involved. Google calls click fraud invalid clicks and says it is "any clicks or impressions that may artificially inflate an advertiser's costs or a publisher's earnings . . . [including] a publisher clicking on his own ads, a publisher encouraging clicks on his ads, automated clicking tools or traffic sources, robots, or other deceptive software."[4]

There are two main types of click fraud:

• **Inflationary click fraud:** Search advertisements often appear on third-party websites and compensate those website owners on a per-click basis or with a share of advertising revenues. These third parties may click the ads to inflate their revenues.

[2] *Source.* http://hitwise.com/datacenter/searchengineanalysis.php. Accessed February 2008.

[3] *Source.* http://www.microsoft.com/presspass/press/2008/feb08/02-03statement.mspx. Accessed February 2008.

[4] *Source.* https://www.google.com/adsense/support/bin/answer.py?answer = 16737. Accessed February 2008.

• **Competitive click fraud:** Advertisers may click rivals' ads with the purpose of driving up their costs or exhausting their ad budgets. When an advertiser's budget is exhausted, it exits the ad auction. A common explanation for competitive click fraud is that firms have the goal of driving up rivals' advertising costs, but such an explanation may not be subgame perfect. If committing competitive click fraud is costly, then driving up competitors' costs comes at the expense of driving down one's own profits. A more convincing explanation may be found in the structure of the ad auction. When a higher-bidding advertiser exits the ad auction, its rival may claim a better ad position without paying a higher price per click.

There are myriad other types of click fraud, such as fraud designed to boost click-through rates, to invite retaliation by search engines against rival websites, or to do malicious harm based on philosophical or economic grounds. These other types are thought to be relatively infrequent, so we do not consider them in this paper.

2.3. Advertiser Perceptions of Click Fraud

Search advertisers say click fraud is troubling. *Advertising Age* (2006) reported the following results of a survey of search advertising agencies:

"In your experience, how much of a problem is click fraud with regard to paid placement?"

16% "a significant problem we have tracked"
23% "a moderate problem we have tracked"
35% "we have not tracked, but are worried"
25% "not a significant concern"
2% "never heard of it"

"Have you been a victim of click fraud?"

42% Yes
21% No
38% Don't know

"What type of click fraud did you experience?"

78% Inflationary click fraud
53% Competitive click fraud

Google implicitly acknowledged the problem when it paid \$90 million to settle a click fraud lawsuit, *Lane's Gifts v. Google* (CV-2005-052-1 (Ark. Cir. Ct., Miller County 2006)), in July 2006. Google acknowledges and describes the risks posed by click fraud in its annual reports.

2.4. Click Fraud Detection and Prevention

Search engines do not claim that they can fully detect click fraud. Google states: "we use both automated systems and human reviews, analyzing all ad clicks and impressions for any invalid click activity that may artificially drive up an advertiser's costs or a publisher's earnings.... Our system enables us to filter out most invalid clicks and impressions, and our advertisers are not charged for this activity."[5] We surmise it is especially difficult to detect invalid clicks if they come from IP addresses that are used by many people or if the invalid clicks are designed to resemble clicks generated by normal human use.

Most search engines claim to offer advertisers some protections against click fraud, although they do not explain specifically how they identify fraudulent clicks. Tuzhilin (2006) defined the "fundamental problem of click fraud prevention": A search engine can not explain specifically how it detects click fraud to its advertisers without providing explicit instructions to unscrupulous advertisers on how to avoid detection. Advertisers are forced to either blindly trust that search engines seek to prevent click fraud or they may hire third-party firms to detect click fraud and pursue refunds for any such fraud detected.

Empirical research on click fraud's effects will have two challenges. The first challenge is that probabilistic judgments are required to detect click fraud, because a smart click fraudster would design its fraudulent clicks to complicate detection. For example, fraudulent clicks may be generated by a widely distributed "botnet" (Daswani and Stoppelman 2007) and timed to mimic normal click activity. The second challenge is that if click fraud can be detected by the researcher, it also could have been detected by the advertiser. Search engines' standard business practice is to refund advertising expenditures when advertisers present evidence of undetected click fraud, so empirical evidence of click fraud's effects could be impacted by advertiser detection of click fraud. We speculate that these concerns could be resolved by analyzing data from a company that previously did not try to detect click fraud, or perhaps by using an experimental approach.

3. A Baseline Model of Search Advertising

We begin with a simple setting to establish how the market operates in the absence of click fraud. This aids interpretation of equilibrium results when we introduce inflationary and competitive click fraud in later sections.

3.1. Clicks

We assume there is a fixed period of length one, n customers click, and clicks arrive at a constant rate $1/n$. Firm $j \in \{1, 2\}$ receives π_{jW} per customer click when its ad is in the top spot, and π_{jL} otherwise. We define $\Delta_j = n(\pi_{jW} - \pi_{jL})$ as the total value to advertiser j of remaining in the top spot for the entire period of time. It must be that $\min(\Delta_1, \Delta_2) > 0$, else firms will never enter positive bids.

[5] *Source.* https://www.google.com/adsense/support/bin/answer.py?answer=9718&ctx=sibling. Accessed February 2008.

3.2. Search Advertising Technology

Each firm enters a bid per click, b_j, for a single advertising slot sold by a monopoly gatekeeper. The high bidder claims the slot and pays the low bidder's bid per click.[6] We assume the two firms' ads have identical click-through rates, so the high bidder wins the top ad position. The high bidder then enters a capacity, $K_j \geq 1$, the maximum number of clicks for which it is willing to pay. If the total number of clicks exceeds K_j, the high bidder exits the advertising market when its capacity has been exceeded. The low bidder then claims the top spot at the next-highest advertiser's per-click bid, which we normalize to zero. We assume that each bidder knows its rival's value per click.

Given that the advertiser's maximum expenditure is the product of K and b, the advertiser's capacity choice is equivalent to choosing an advertising budget, as is required by all major search engines (Google, Yahoo!, MSN). In the absence of an ad budget, we presume that an advertiser could choose to stop remitting payments to the search engine at some point, which would give the same effect. The capacity-setting assumption simplifies the analysis, but the results are unchanged under a budget-setting assumption.

Edelman et al. (2007) showed that with two bidders and one slot, the generalized second-price auction reduces to a Vickrey second-price auction. We will appeal several times to the standard result that in a second-price auction, a weakly dominant strategy is for firms to bid their reservation price.

We have made two restrictive assumptions here: each firm knows its rival's profits from advertising, and the firms have identical click-through rates. We relax both of these assumptions in §6, when we consider an auction mechanism that takes click-through rates into account.

3.3. Structure of the Game

The game is played in two stages. First, each firm enters a bid per click and observes its position, with the high bidder in the top spot. The high bidder observes the low bidder's bid, the amount it must pay per click. In the second stage, the high bidder chooses its capacity. The reason for this structure is that, in reality, the high bidder may observe its payment per click (the low bidder's bid) immediately, but the low bidder may only discover its rival's capacity if it has been exhausted. We seek a subgame-perfect equilibrium in pure strategies under full information: each firm anticipates its rival's action.

[6] Results in §§3–5 continue to hold if the auction winner is determined by the advertising budget rather than by a comparison of per-click bids alone.

3.4. Equilibrium

We analyze second-stage profits, then first-stage actions. When firm j wins the auction (W), its profit is

$$\Pi_{jW}(b_k) = \begin{cases} n(\pi_{jW} - b_k) & \text{when } K_j \geq n \\ n\pi_{jW}\dfrac{K_j}{n} + \left(1 - \dfrac{K_j}{n}\right) n\pi_{jL} - b_k K_j & \text{when } K_j < n, \end{cases} \tag{1}$$

where K_j/n is the fraction of customer clicks firm j receives while on top, in the event it does not remain on top for the entire time period. If firm k wins the auction and sets a capacity $K_k \geq n$, firm j receives $\Pi_{jL} = n\pi_{jL}$.

In the first stage, each firm will anticipate the second-stage outcome and choose the bid b that makes it indifferent between winning and losing the auction. Thus, b_j is chosen to equate $\Pi_{jW}(b_j) = \Pi_{jL}$.

We summarize equilibrium behavior in Proposition 1.

Proposition 1. *In the absence of click fraud, firm j bids Δ_j/n per click and wins the advertising auction if $\Delta_j > \Delta_k$. The auction winner remains on top for the entire time period and earns a profit of $n\pi_{jW} - \Delta_k$. The gatekeeper earns* $\min\{\Delta_1, \Delta_2\}$.

Proofs of all lemmas and propositions are in the appendix. Proposition 1 serves as a useful benchmark to which we compare equilibria under inflationary and competitive click fraud.

4. Search Advertising with Inflationary Click Fraud

We now introduce inflationary click fraud into the baseline model. Major search engines pay third-party websites to display search ads relevant to their site content. Inflationary click fraud results when those website owners click the ads to inflate their advertising revenues.

We first analyze the individual website's problem of choosing a click fraud level. Next, we solve for equilibrium bids and capacities, given a known amount of inflationary click fraud. Finally, we consider the case of stochastic inflationary click fraud.

4.1. Websites' Choice of Inflationary Click Fraud

Ads associated with a particular keyword are placed on I third-party websites, indexed by w. We consider two different compensation schemes. If website w is paid γ_w per click generated, its revenues are $\gamma_w(n_w + r_w)$, where n_w is the number of customer clicks generated through the website, and r_w is its inflationary click fraud level. If website w is paid a fraction $\delta_w \in (0, 1)$ of the ad revenues it generates, its revenues are $\delta_w b(n_w + r_w)$, where b is the advertiser's payment per click. b does not vary across websites and is a function of inflationary click fraud, with $\partial b/\partial r_w < 0$. (We show in §4.2 that $\partial b/\partial(\sum_w r_w) < 0$ in equilibrium under certain conditions.)

We assume that the cost of r_w fraudulent clicks is an increasing and convex function $c_w(r_w)$, because a greater number of fraudulent clicks increases the risk that the search engine will detect the fraudulent activity. The search engine could then retaliate by excluding the website from its content network or initiating a costly legal action against the website if click fraud constitutes a breach of contract.

Under a per-click compensation scheme, website w's profits are

$$\pi_w = \max_{r_w} \gamma_w(n_w + r_w) - c_w(r_w), \tag{2}$$

yielding a first-order condition $\gamma_w = c'_w(r^*_w)$ and a choice of $r^*_w = c'^{-1}_w(\gamma_w)$ in equilibrium.

Under a revenue-sharing compensation scheme, website w's profits are

$$\pi_w = \max_{r_w} \delta_w b(n_w + r_w) - c_w(r_w) \tag{3}$$

and site w's first-order condition, $\delta_w b + \delta_w(n_w + r_w) \cdot (\partial b/\partial r_w) = c'_w(r_w)$, yields a unique r^*_w.

Proposition 2. *Holding click fraud constant, for $\gamma_w = \delta_w b$, per-click and revenue-sharing compensation schemes yield identical payouts to websites. Allowing for endogenous click fraud, the revenue-sharing compensation scheme will reduce inflationary click fraud, because it incents content network partners to partially internalize the effect of inflationary click fraud on advertisers' bids.*

The larger the derivative $\partial b/\partial r_w$ is in absolute value, the less inflationary click fraud the third-party websites will produce. Search ads are typically displayed on a large number of websites in search engines' content networks, suggesting that $|\partial b/\partial r_w|$ is small. The implications of this result for search engines are clear: click fraud is reduced when search ads are not rotated across a large number of websites, and each website is compensated with a percentage of the advertising revenues it generates.

If $\partial^2 b/\partial r^2_w > 0$ (we later show $\partial^2 b/\partial(\sum_w r_w)^2 > 0$ under certain assumptions), then $\partial b/\partial r_w$ is greatest in absolute value when $I = 1$. Thus, websites' incentives to internalize the effects of their inflationary click fraud on search engine revenues are maximized when other sites' click fraud levels do not affect their share of advertising revenues. This suggests that search engines should allow advertisers to enter site-specific keyword bids b_w to maximally reduce sites' incentives to engage in click fraud. Although it may be difficult for a human to manage site-specific bids when I is large, software could be designed to accomplish this task.

Although a revenue-sharing compensation scheme suggests an equilibrium relationship between bids and inflationary click fraud levels, we are not going to model this relationship explicitly. Current search engine policies make information transmission between websites and advertisers prohibitively difficult. Search engines do not reveal the distribution of γ_w or δ_w to either side. In addition, websites do not know b, and advertisers do not know $c_w(r_w)$, nor how they vary across websites or advertisers. (However, note that websites need not know $\partial b/\partial r_w$ exactly for Proposition 2 to hold; they need only know $\partial b/\partial r_w < 0$.) Finally, advertisers have only limited information about where their ads will appear, and website owners do not know in advance what ads will appear on their sites.

We proceed under each of two assumptions: Either both advertisers can anticipate the inflationary click fraud level $r = \sum_w r_w$, or they share a common belief $f(r)$ about its distribution. Advertisers may share a common belief about $f(r)$ because they both observe the low bidder's bid, which is the only bid that is paid, so it is the only bid that could influence r.

Next, we analyze the equilibrium effects of inflationary click fraud on advertisers' bids.

4.2. Deterministic Inflationary Click Fraud

We assume that customers generate $n = n_0 + \sum_w n_w$ clicks, where n_0 is the number of customer clicks that come directly from the search engine. Website owners generate r fraudulent clicks. Both advertisers can anticipate r. We relax this assumption in §4.3.

4.2.1. Equilibrium. Equation (4) gives firm j's profit when it wins the auction.

$$\Pi_{jW}(b_k) = \begin{cases} n\pi_{jW} - b_k(n+r) & \text{when } K_j \ge n+r \\ n\pi_{jW}\dfrac{K_j}{n+r} + \left(1 - \dfrac{K_j}{n+r}\right) n\pi_{jL} - b_k K_j & \\ & \text{when } K_j < n+r. \end{cases} \tag{4}$$

The first segment of the profit function represents the outcome in which firm j stays on top for the entire time period. The second segment occurs if firm j's capacity will be exhausted at some point, in which case it is on top for $n(K_j/(n+r))$ customer clicks, and has exited the auction for the remaining $(1 - K_j/(n+r))n$ customer clicks.

Proposition 3 summarizes equilibrium behavior.

Proposition 3. *When inflationary click fraud is deterministic and known to both bidders, and there is no competitive click fraud, firm j bids $\Delta_j/(n+r)$ and wins the advertising auction if $\Delta_j > \Delta_k$. Advertisers reduce their bids by a proportion of $r/(n+r)$, pricing out the effect of* *click fraud. Firm j remains on top for the entire time period and earns a profit of* $n\pi_{jW} - \Delta_k$. *The gatekeeper's revenues are* $\min\{\Delta_1, \Delta_2\}$, *as in the baseline model.*

The auction mechanism completely internalizes the effect of inflationary click fraud when the number of fraudulent clicks is known to both bidders. Advertiser profits are unaffected; bids adjust endogenously to counter the detrimental effects of the fraudulent clicks. The gatekeeper's revenues are unchanged, although its profits must fall if it makes larger transfers to third-party websites.

4.3. Stochastic Inflationary Click Fraud

It is perhaps more intuitive to assume that advertisers do not know how many fraudulent clicks will occur because they may not know where their ads will appear, or the distribution of δ_w, γ_w, or $c_w(r_w)$ across websites. We assume here that advertisers maximize expected profits under a common belief about the probability density $f(r)$ of the inflationary click fraud level r.

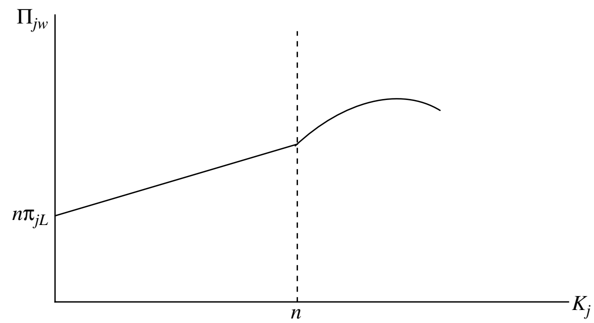
4.3.1. Capacity Choice. We now add uncertainty about r into firm j's profit function. If $K_j < n$, firm j's capacity will be exhausted for any realization of r. When $K_j \geq n$, firm j's capacity is only exhausted for some realizations of r. Equation (5) displays firm j's profit function when it wins the auction.

$$\Pi_{jW}(b_k) = \begin{cases} \displaystyle\int_0^\infty \left[n\pi_{jW}\frac{K_j}{n+r} - b_k K_j + \left(1 - \frac{K_j}{n+r}\right) n\pi_{jL} \right] f(r)\,dr & \text{when } K_j < n \\ \displaystyle\int_0^{K_j - n} [n\pi_{jW} - b_k(n+r)] f(r)\,dr + \int_{K_j-n}^\infty \left[n\pi_{jW}\frac{K_j}{n+r} - b_k K_j + \left(1 - \frac{K_j}{n+r}\right) n\pi_{jL} \right] f(r)\,dr & \text{when } K_j \geq n. \end{cases} \tag{5}$$

The uncertainty in the first segment of the profit function concerns the number of customer clicks for which the firm will remain on top. On the second segment of the profit function, the first term is the firm's expected profits when it remains on top, weighted by the probability that r is small enough that firm j is never knocked off. The second term is the firm's expected profits in the event that its capacity is exhausted, weighted by the probability that r is large enough to exhaust the firm's capacity.

Figure 1 depicts Π_{jW}. For $K_j < n$, Π_{jW} changes linearly with K_j at a constant rate $\int_0^\infty (\Delta_j/(n+r)) f(r)\,dr - b_k$.

Figure 1 **Firm *j*'s Profit Function**



Note. It also might be that Π_{1W} is increasing everywhere above n.

For $K_j \geq n$,

$$\frac{\partial \Pi_{jW}}{\partial K_j} \equiv MR(K_j) - MC(K_j) = \int_{K_j-n}^\infty \frac{\Delta_j}{n+r} f(r)\,dr - b_k[1 - F(K_j - n)]. \tag{6}$$

Both $MR(K_j)$ and $MC(K_j)$ are decreasing in K_j.

Firm j will choose a K_j larger than n if $\int_0^\infty (\Delta_j/(n+r)) f(r)\,dr > b_k$. This holds in equilibrium when $b_j > b_k$. K_j is therefore determined by the first-order condition on the second segment of the profit function.

$$\int_{K_j-n}^\infty \frac{\Delta_j}{n+r} f(r)\,dr - b_k[1 - F(K_j - n)] \geq 0. \tag{7}$$

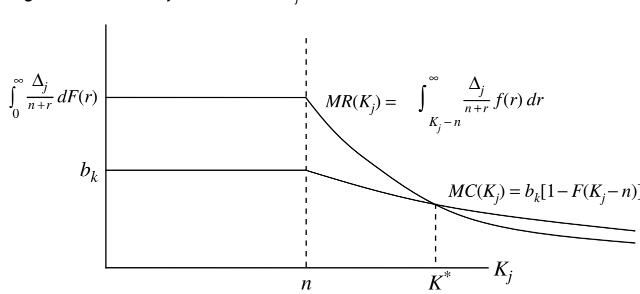
K_j will be finite if $MR(K_j)$ crosses $MC(K_j)$. At $K_j = n$, $MR(K_j)$ is above $MC(K_j)$ and steeper than $MC(K_j)$. As K_j increases, $dMR(K_j)/dK_j = -(\Delta/K_j) f(K_j - n)$ and $dMC(K_j)/dK_j = -b_k f(K_j - n)$, so $MR(K_j)$ later becomes flatter than $MC(K_j)$. If $MR(K_j)$ does not cross $MC(K_j)$, $K_j = \infty$.

Lemma 1. *When* (7) *holds with equality,* K_j *is unique.*

Figure 2 shows the case when K_j is finite. If $K_j < \infty$, K_j is increasing in Δ_j.

4.3.2. Bids. As before, we calculate b_j as the per-click payment that makes firm j indifferent between acquiring the advertising right and not acquiring it.

Figure 2 **Firm *j*'s Choice of K_j**



Thus, b_j is found by setting $\Pi_{jW}(b_j) = \Pi_{jL}$. We consider two cases: $n < K_j < \infty$ and $K_j = \infty$.

In the first case, K_j will be finite when the firms are sufficiently similar that $MC(K)$ does not lie everywhere below $MR(K)$. To aid interpretation of the results, we assume symmetry between the two firms, $\Delta_j = \Delta_k = \Delta$, implying $K_j = K_k = K$ and $b_j = b_k = b$. We find b by equating firm j's expected winning profits to its expected losing profits, but we now must consider that when firm j loses, it will claim the top spot when $n + r > K_k$. Thus,

$$\Pi_{jW} = \int_0^{K-n} [n\pi_{jW} - b(n+r)] f(r)\,dr + \int_{K-n}^{\infty} \left(\Delta \frac{K}{n+r} + n\pi_{jL} - bK\right) f(r)\,dr, \tag{8}$$

$$\Pi_{jL} = \int_0^{K-n} (n\pi_{jL}) f(r)\,dr + \int_{K-n}^{\infty} \left(n\pi_{jW} - \Delta \frac{K}{n+r}\right) f(r)\,dr, \tag{9}$$

and b is determined by the equality of Π_{jW} and Π_{jL}.

Proposition 4. *When inflationary click fraud is stochastic, there is no competitive click fraud, and firms are identical and set a finite K, expected gatekeeper revenues are strictly lower than the baseline model.*

In the second case, firm j wins and sets a capacity $K_j = \infty$. This occurs when $\Delta_j - \Delta_k$ is sufficiently large that $MR(K_j)$ lies everywhere above $MC(K_j)$.

Proposition 5. *When inflationary click fraud is stochastic, there is no competitive click fraud, and the losing bid is sufficiently low that the high bidder sets $K_j = \infty$, expected gatekeeper revenues are strictly higher than the case when inflationary click fraud is deterministic.*

Uncertainty about the amount of inflationary click fraud may either increase or decrease gatekeeper revenues. It is likely to lower gatekeeper revenues when firms' incremental profits of winning the auction are similar. In such situations, for example in auctions for generic keywords, bidding is more intense and the auction winner realizes a smaller profit from winning the auction. Low profits induce the auction winner to strategically limit its capacity to avoid paying for a large number of fraudulent clicks.

Gatekeeper revenues may rise with inflationary click fraud when one firm's profits of winning are much larger than its rival's (for example, in auctions for branded keywords). In this case, the high bidder gains very large rents in the baseline model, and its rents are so large that it never chooses to strategically limit its capacity. Click fraud may then have the effect of transferring some of the winner's profits to the gatekeeper.

What we learn in this section is that inflationary click fraud does not harm advertisers in a second-price auction when they know exactly how much to expect; this seemingly verifies the executive's comment that perhaps no solution to click fraud is necessary. However, under the more realistic assumption that firms face uncertainty in the level of inflationary click fraud, we see two things. First, search engines certainly have a strong incentive to detect and limit click fraud in very competitive keyword auctions. Second, when keyword auctions are less competitive, it may be in the gatekeeper's interest to allow some click fraud.

5. Search Advertising with Inflationary and Competitive Click Fraud

We have previously considered the effects of third-party invalid clicks on market equilibria. Now we extend the analysis to consider what happens when the low bidder may click the high bidder's ad to hasten the high bidder's exit from the advertising auction.

We start by proving our earlier assertion that competitive click fraud may not be subgame perfect. In a model where the number of inflationary fraudulent clicks is known and the number of competitive fraudulent clicks is rationally anticipated, the high bidder will shade its capacity upward in equilibrium. Assuming that click fraud is costly, the low bidder then will not commit any competitive click fraud.

In §5.2, we show that uncertainty in the total number of clicks may lead to competitive click fraud in equilibrium. Competitive click fraud unambiguously decreases advertisers' bids, but it also may increase the high bidder's budget.

Assumptions About Competitive Click Fraud. We assume that the low bidder chooses a level of competitive click fraud, z, at cost $c(z)$. We assume that $c(z)$ is increasing and convex because a larger number of clicks will increase the probability that the high bidder or the gatekeeper can verify the identity of the firm committing click fraud and retaliate (e.g., through civil lawsuits or business channels).[7]

We assume the low bidder chooses z simultaneously with the high bidder's choice of K. The total

[7] One might also posit a competitive click fraud cost function $c(z, r)$, where $dc/dr < 0$, to allow for the probability of competitive click fraud detection to fall with inflationary click fraud. We expect that the two types of click fraud can be independently detected, given that website owners' fraudulent clicks will come exclusively from their own sites, whereas competitive click fraud is more likely to occur on search engines' main pages. The results presented below are qualitatively unchanged under the assumption that $c(z) = c(z, r)$.

number of clicks is now $z+n+r$. We seek a rational expectations equilibrium in pure strategies under full information: each firm anticipates its rival's action.

5.1. Deterministic Inflationary and Competitive Click Fraud

In the case that r is deterministic and known to both firms, firm j's profit when it wins the initial auction is

$$\Pi_{jW}(b_k) = \begin{cases} n\pi_{jW} - b_k(n+z_k+r) & \text{when } K_j \geq n+z_k+r \\ n\pi_{jW}\dfrac{K_j}{n+z_k+r} + \left(1-\dfrac{K_j}{n+z_k+r}\right)n\pi_{jL} - b_k K_j & \\ \qquad\qquad \text{when } K_j < n+z_k+r. \end{cases} \tag{10}$$

Firm k's profit when it loses the initial auction is

$$\Pi_{kL} = \begin{cases} n\pi_{kL} - c(z_k) & \text{when } K_j \geq n+z_k+r \\ n\pi_{kL}\dfrac{K_j}{n+z_k+r} + \left(1-\dfrac{K_j}{n+z_k+r}\right)n\pi_{kW} - c(z_k) & \\ \qquad\qquad \text{when } K_j < n+z_k+r. \end{cases} \tag{11}$$

These profit functions are similar to those analyzed in §4.2. Proposition 6 describes equilibrium behavior.

Proposition 6. *When both firms know the inflationary click fraud level r, if firm j wins the auction, then in equilibrium $K_j \geq n+z_k+r$ and $z_k = 0$. Firm j never loses the top spot, and firm k therefore does not engage in competitive click fraud.*

5.2. Stochastic Inflationary and Competitive Click Fraud

Here, we set up the problem under the general distribution $f(r)$ and discuss results and intuition from the full model. We describe the set of equilibria in pure strategies in the Technical Appendix, which can be found at http://mktsci.pubs.informs.org.

5.2.1. Profits. As before, there are two parts to firm j's profit function. When $K_j < n+z_k$, firm j is always knocked off the top spot. When $K_j \geq n+z_k$, firm j is only knocked off for some realizations of r.

$$\Pi_{jW}(b_k) = \begin{cases} \displaystyle\int_0^\infty \left(\Delta_j \frac{K_j}{n+z_k+r} + n\pi_{jL} - b_k K_j\right) f(r)\,dr \\ \quad \text{when } K_j < n+z_k \\ \displaystyle\int_0^{K_j-n-z_k} [n\pi_{jW} - b_k(n+z_k+r)] f(r)\,dr \\ \quad + \displaystyle\int_{K_j-n-z_k}^\infty \left[\frac{\Delta_j K_j}{n+z_k+r} + n\pi_{jL} - b_k K_j\right] f(r)\,dr \\ \quad \text{when } K_j \geq n+z_k. \end{cases} \tag{12}$$

Π_{jW} is continuous at $K_j = n+z_k$, although its slope falls at this point.

The problem facing firm k in the case that it loses is choosing z_k to maximize

$$\Pi_{kL} = \begin{cases} \displaystyle\int_0^\infty \left[n\pi_{kL}\frac{K_j}{n+z_k+r} - c(z_k) \right. \\ \qquad \left. + \left(1-\frac{K_j}{n+z_k+r}\right)n\pi_{kW}\right] f(r)\,dr \\ \quad \text{when } K_j < n+z_k \\ \displaystyle\int_0^{K_j-n-z_k} [n\pi_{kL} - c(z_k)] f(r)\,dr \\ \quad + \displaystyle\int_{K_j-n-z_k}^\infty \left[n\pi_{kW} - \frac{\Delta_k K_j}{n+z_k+r} - c(z_k)\right] f(r)\,dr \\ \quad \text{when } K_j \geq n+z_k. \end{cases} \tag{13}$$

Π_{kL} is continuous at $K_j = n+z_k$ although its slope falls at this point.

Proposition 7. *Under stochastic inflationary click fraud, gatekeeper revenues may be increasing or decreasing in the level of competitive click fraud z.*

We prove Proposition 7 for two special cases of the model in the appendix.

The presence of both uncertain inflationary and competitive click fraud limits the high bidder's ability to react to either one. The high bidder mitigates inflationary click fraud by limiting its capacity to protect against paying for a large realization of r. The high bidder mitigates competitive click fraud by increasing its capacity, to prevent the low bidder from knocking it off with a large z. Thus, when we add both types of click fraud into the model, the auction winner cannot respond optimally to either one without being hurt by the other. What we learn in this section is that search engine revenues may rise with competitive click fraud.

6. Asymmetric Click-Through Rates

Search engines commonly use advertisers' CTRs in conjunction with per-click bids to determine ad position listings (Rutz and Bucklin 2007a). It is therefore interesting to consider whether our main results would change under a more realistic keyword auction. We consider here the auction mechanism used by Google from 2002 to 2005, the last publicly known auction mechanism used by that company.

A related analysis is Katona and Sarvary (2007), which provides necessary conditions for equilibrium strategies in a multiunit click-through auction. However, we are not aware of any previous papers to solve for dominant bidding strategies in an auction with a CTR component. We show that moving from a second-price auction to what we term a "click-through auction" (bid-per-click $*$ total-clicks) has three effects. First, advertisers no longer bid their reservation price, and the less-profitable firm may win the auction, even when it has a lower click-through rate. Second, search engine revenues may be lower in a click-through auction than in a second-price auction. Third, inflationary click fraud may increase search engine revenues in a click-through auction, even when the amount of fraud is deterministic and known to both bidders.

6.1. Baseline Model with a Click-Through Auction

We now assume that firm j will get n_j clicks during the unit time period. What follows is identical if n_j is a fraction $\eta_j \in (0, 1)$ of total potential clicks N, where $n_j = \eta_j N$ (as click-through rates are commonly defined). We assume each bidder knows its rival's click level.

It is useful to denote firm j's per-click value of winning as $v_j \equiv \pi_{jW} - \pi_{jL}$. We assume that firms are numbered such that $v_j \geq v_k$ and reservation prices are private information.

Firm asymmetry in clicks is different from firm asymmetry in advertising profits. It may be that firms' clicks are identical ($n_j = n_k$), but their variable profits are different because of price or cost factors. It may also be that firms' reservation prices are identical ($v_j = v_k$), but their clicks are different because of factors such as brand recognition or ad quality. It seems likely that n_j and v_j would be positively related, but we do not require it.

We distinguish between two types of auction:

(1) The *second-price* (*SP*) *auction*, which we used in previous sections. The highest bid per click determines the auction winner, and the winner pays the loser's bid on each click it receives.

(2) The *click-through* (*CT*) *auction*, in which the gatekeeper allocates the advertising slot to firm j if and only if $n_j b_j > n_k b_k$. The winning bidder then pays the loser's bid per click. We assume that n_j and n_k are known to both advertisers and the gatekeeper.

As before, we solve the second stage first. When firm j wins the CT auction, its profits are

$$\Pi_{jW} = \begin{cases} n_j(\pi_{jW} - b_k) & \text{for } K_j \geq n_j \\ \pi_{jW} K_j + \left(1 - \dfrac{K_j}{n_j}\right) n_j \pi_{jL} - b_k K_j & \\ & \text{for } 1 \leq K_j < n_j. \end{cases} \tag{14}$$

First-order conditions indicate the firm will set $K_j \geq n_j$ if $v_j > b_k$. If firm j loses the auction and firm k sets a $K_k \geq n_k$, its profits will be $\Pi_{jL} = n_j \pi_{jL}$.

In the SP auction, firm j chose its bid b_j by setting $\Pi_{jW}(b_j) = \Pi_{jL}$. However, it is not optimal for both firms to bid their reservation price in the CT auction. To see this, assume that $v_j = v_k + \varepsilon$, $n_j < n_k$, and $b_k = v_k$, for an arbitrarily small $\varepsilon > 0$. If $b_j = v_j$, firm j will lose the auction. However, for $b_j = (n_k/n_j)v_j > v_j$, firm j can win the auction and earn a positive profit ε on each click.

Lemma 2. *A weakly dominant strategy is to bid $b_j^* = (n_k/n_j)v_j$. Firm j will win the CT auction if and only if $n_k v_j > n_j v_k$.*

Lemma 2 shows that the low-click firm bids more aggressively in the CT auction than in the SP auction, whereas the high-click firm bids more passively. This happens because the auction mechanism handicaps the high-click firm. It does so by making it susceptible to the threat of negative variable profits produced by an aggressive bid by the low-click firm.

Substituting firm j's equilibrium bid into the gatekeeper's auction mechanism $n_j b_j > n_k b_k$ implies that $v_j > b_k$. In equilibrium neither firm wins the auction at an unprofitable per-click payment. Thus, when firm j wins the auction, it optimally sets a capacity $K_j \geq n_j$.

When both firms play optimal strategies, firm j will win if and only if $v_j/v_k > n_j/n_k$. Thus, even if firm j has a higher per-click profit and a higher click-through rate, it may lose the keyword auction because of its rival's ability to bid aggressively. The high-value firm is only assured of winning the auction if its relative profit advantage is larger than its relative click-through advantage.

We now compare gatekeeper revenues in the CT and SP auctions.

Proposition 8. *A switch from a second-price auction to a click-through auction increases gatekeeper revenues if and only if $v_j/v_k > n_j/n_k > 1$. Otherwise, the click-through auction produces lower revenues than a second-price auction.*

The click-through auction only produces higher revenues than the second-price auction if the high-value firm's relative click advantage is not too large compared to its relative profit advantage. The possibility that the CT auction can lower search engine revenues is counterintuitive and contrary to the conventional wisdom. It may suggest that Google's early adoption of the CT auction lowered advertisers' costs, encouraging them to buy more keywords on Google's platform than on rival platforms. It also may help explain Yahoo!'s late adoption of a relevance-related auction, or Google's switch to its unspecified use of "quality scores" in 2005.

We are motivated by practice to consider these two particular auctions; the SP auction is a simplified version of the generalized second-price auction used by Yahoo! until 2007, and the CT auction was known to be Google's auction mechanism from 2002 to 2005.

We have left unanswered the question of whether there exists another auction mechanism that dominates both the SP and CT auctions. This question is interesting and compelling, but sufficiently difficult in that it lies beyond the scope of this paper. We refer the interested reader to Borgers et al. (2007), Edelman and Schwarz (2006), and Feng (2008).

6.2. Deterministic Inflationary Click Fraud in a Click-Through Auction

Our motivation to consider the CT auction is to determine whether it reverses our result that gatekeeper revenues may increase with click fraud. In the SP auction, both bidders can price out the effects of inflationary click fraud when its quantity is known, yielding no effect on search engine revenues. In the CT auction, however, inflationary click fraud alters the ratio of firms' click-through rates. Click fraud may increase search engine revenues when it reduces the high-value advertiser's relative advantage in clicks.

We again assume that content network websites generate r fraudulent clicks, and both bidders know r. The gatekeeper now awards the advertising slot to firm j if and only if $(n_j + r)b_j > (n_k + r)b_k$. The previous analysis indicates that firm j will win if and only if $v_j/v_k > (n_j + r)/(n_k + r)$. (To see this, relabel each firm's click level with $n'_j = n_j + r$.) The gatekeeper's revenues are $((n_j + r)^2/(n_k + r))v_k$ if $v_j/v_k > (n_j + r)/(n_k + r)$, or $((n_k + r)^2/(n_j + r))v_j$ otherwise.

Proposition 9. *When firm j wins the CT auction, if deterministic inflationary click fraud does not reverse the auction result, gatekeeper revenues will increase if $r > n_j - 2n_k$.*

Note that we have ignored the possibility that the click fraud level r responds to advertisers' bids. What would happen if we allowed r to depend on the auction winner? Let us assume that $n_j > n_k$, $r_j > r_k$ (where r_j is the number of inflationary fraudulent clicks accruing to firm j), and $v_j/v_k > (n_j + r_j)/(n_k + r_k)$, so the high-profit firm is also the high-traffic firm and the auction winner. Gatekeeper revenues are then $((n_j + r_j)^2/(n_k + r_k))v_k$, greater than gatekeeper revenues for any common click fraud level $r \in (r_k, r_j)$.

Inflationary click fraud has two effects in the click-through auction. First, it lowers the threshold at which a high-value, high-click firm wins the auction, expanding the parameter space in which the high-value firm wins. Second, it alters the low-value firm's ability to threaten the high-value firm. The smaller the high-value firm's relative click advantage is, the more likely a given level of click fraud will be beneficial to the search engine.

We have shown that our main result, that search engine revenues may increase with click fraud, is not an artifact of our assumed auction mechanism. We proceed with the normative implications of our analysis.

7. Managerial Implications

Our analysis has produced several results that could influence search engines' and advertisers' business practices. We note that our implications are subject to the limitations discussed in the final section.

7.1. Tuzhilin's "Fundamental Problem of Click Fraud Prevention"

Tuzhilin (2006) defined the "fundamental problem of click fraud prevention." Search engines may try vigorously to detect and prevent click fraud, but they cannot tell advertisers specifically how they do so, because this would constitute explicit instructions on how to avoid click fraud detection. It also may be that in identifying fraudulent clicks, search engines must make probabilistic judgments balancing "false positives" against "false negatives." Presumably, search engines would prefer to minimize false negatives, whereas advertisers would prefer to maximize false positives.

To resolve this problem, we suggest that the search advertising industry form a neutral third party to authenticate search engines' click fraud detection efforts. Such a party could maintain the confidentiality needed by search engines while allaying advertisers' concerns. It could even introduce a new dimension of competition between search engines if, for example, it "graded" each engine's click fraud detection algorithms.

Similar third parties are used in other media industries. For example, Nielsen Media Research's audience measurements underpin transactions between television networks and advertisers, the Audit Bureau of Circulations authenticates newspapers' and magazines' subscription figures, and comScore and other companies measure website audiences for display (CPM) advertising transactions. In the absence of such a neutral third party, it may be possible to design some creative incentive-compatible contracts to provide verifiable evidence of click quality. For example, if human searchers are each assigned individual-specific accounts, advertisers could enter different bids for clicks made from individuals' accounts, and "anonymous" clicks.

Our result that search engines are sometimes helped, and sometimes hurt, by click fraud reinforces the need for such a neutral third party. Advertisers may perceive the risk that search engines do not apply the same click fraud detection algorithms to all keyword auctions. Our results suggest that a profit-maximizing search engine might exert maximal efforts to prevent click fraud in competitive keyword auctions but do less to prevent click fraud in relatively uncompetitive auctions such as those for branded keywords. Or it may be that search engines try vigorously to prevent click fraud but are unable to credibly convey the depths of their efforts to concerned advertisers.

Click fraud can theoretically be detected by two means. The first is to use sophisticated statistical models to try to detect patterns of fraudulent clicks. The second means is to use humans; pose as potential customers contacting botnet owners, or offer rewards for information leading to evidence of click manipulation. In theory, statistical algorithms cannot detect sophisticated click fraud that is generated by widely distributed botnets and designed to mimic human use. However, we are not aware of any search engines proactively using human means to catch click fraudsters, although some search engines currently employ people to investigate specific cases in response to advertisers' complaints about click fraud.

We have been asked why we assume that search engines do not commit click fraud on their own networks. First, we think it is unlikely that major search engines would commit such unethical acts. Second, we think that a search engine would put its market value at great risk if it were found to be directly responsible for click fraud on its own network. However, the search engine could indirectly benefit with less risk if it failed to detect fraudulent clicks generated by users of its network.

We have only modeled one gatekeeper; would competition between gatekeepers resolve the click fraud problem? We think not, for two reasons. First, the "fundamental problem of click fraud detection" would still prevent search engines from sending credible signals to advertisers about their click fraud detection efforts. Second, so long as advertisers realize profits per click, and consumers are distributed across search engines, the profit-maximizing advertiser is likely to buy keywords from all search engines (although its expenditures may vary across search engines).

7.2. Advertiser Information

We showed that click fraud does advertisers no harm when advertisers have full information in a second-price auction. This suggests that search engines should take actions to increase the amount of information at advertisers' disposal. Specifically, they can issue keyword-specific reports on how and when they punish advertisers and websites suspected of engaging in click fraud, issue keyword-specific reports on when and how much click fraud they detect, and give advertisers information about the identity and frequency of the content network sites on which their ads will appear.

7.3. Content Network Management

We showed that third-party websites' incentives to engage in click fraud are reduced when a revenue-sharing compensation scheme is used in place of a per-click compensation scheme, and when search ads are not rotated across a large number of websites. Content networks should not only adopt these strategies, they should make them public to increase transparency and build advertiser confidence.

We found that content network partners' incentives to engage in click fraud are minimized when advertisers may enter site-specific bids. Any site that generates a large amount of inflationary click fraud would then be penalized through a lower site-specific bid. We are not aware of any content networks that currently allow advertisers to enter site-specific bids in CPC auctions, but it seems within the realm of technical possibility.

Note that we have not modeled websites' choice to enter or remain in a content network. It may be that decreasing websites' incentives to commit click fraud could also reduce search engines' inventory of customer clicks by encouraging websites to enroll in competing content networks or reducing their incentives to invest in content.

7.4. Will Click Fraud Destroy the Market?

Our results suggest it is unlikely that click fraud will ever completely destroy the search advertising industry. First, we found that when advertisers have full information in an SP auction, they can strategically adjust their bids and advertising budgets to mitigate the effects of click fraud. Second, so long as search engines are able to maintain a positive probability of detecting some click fraud and punishing those responsible, we will see limited click fraud in equilibrium. It seems the CPC business model will likely remain viable in the long run.

8. Discussion

We have presented the first analysis of the effects of inflationary and competitive click fraud on search advertising markets. We found that when advertisers know the level of inflationary click fraud in a second-price auction, they lower their bids to the point that click fraud has no impact on total advertising expenditures. However, when the level of inflationary click fraud is uncertain, total advertising expenditures may rise or fall. They rise when the keyword auction is relatively less competitive because advertising is so profitable for the high bidder that it is willing to pay to remain on top for any realization of click fraud. Advertising expenditures may fall when the keyword auction is more competitive because the high bidder faces higher advertising costs and therefore shades its capacity downward to protect against paying for large levels of inflationary click fraud. Even when inflationary click fraud is known to both bidders in a click-through auction, it can enhance the low-value firm's ability to bid aggressively, thereby increasing gatekeeper revenues.

We also analyzed the effects of competitive click fraud in the second-price auction. We found that when inflationary click fraud is deterministic, a high-bidding firm may effectively deter its rival from committing click fraud by choosing a large capacity. However, when the number of clicks is stochastic, the high bidder may shade its bid downward and the low bidder may then profitably engage in competitive click fraud. We showed that gatekeeper revenues may be increasing or decreasing in the level of competitive click fraud.

As in all models, we have made several simplifying assumptions. Three assumptions in particular suggest directions for future research. The first is the assumption that the gatekeeper offers only one advertising slot. Edelman et al. (2007) show that when only one search ad is available, the generalized second-price (GSP) auction mechanism reduces to a standard second-price auction. However, when more than one ad is offered, the GSP does not have an equilibrium in dominant strategies, and firms do not engage in truth-telling. This technical concern limits our ability to make predictions about firms' equilibrium click fraud strategies in a multiple-slot auction. We suspect that adding more slots and advertisers to the model would increase competitive click fraud, because more advertisers would stand to gain from knocking off the highest bidder.

The second assumption that could be relaxed in future work is the assumption of a single gatekeeper. Expanding the analysis to multiple gatekeepers could introduce elements of two-sided market competition. Search engines might choose their policies based on the possibility of advertiser, searcher, or content-network website defection to competitors. In this paper we have not considered that advertiser adoption of a search engine's platforms could be a function of its business model.

A third assumption that could be modified deals with advertisers' information about each other. In §§3–5, we assumed that advertisers had no private information, and in §6 we assumed they knew each other's click-through rates. Although these assumptions are common in the literature, they may not always hold in practice. The main effect of allowing for private information on the analysis would be to remove our ability to use backward induction to solve the model. This may increase equilibrium predictions about click fraud (by, for example, invalidating Proposition 6), but the specific effects would depend on assumptions about advertisers' beliefs.

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Appendix

Proposition 1. Π_{jW} does not change with K_j when $K_j \geq n$. When $K_j < n$, Π_{jW} changes linearly with K_j at rate $(\pi_{jW} - \pi_{jL} - b_k)$. If firm j wins, it cannot be that $b_k > \pi_{jW} - \pi_{jL}$, because firm j would be better off losing in this scenario. Thus, when firm j wins the auction, $\partial\Pi_{jW}/\partial K_j > 0$ for $K_j \leq n$, so firm j sets $K_j \geq n$ and earns $\Pi_{jW} = n(\pi_{jW} - b_k)$. $\Pi_{jW} = \Pi_{jL}$, then gives $b_j = \Delta_j/n$.

Proposition 2. The right-hand sides of site w's first-order conditions are identical under the two compensation schemes, but the left-hand side is strictly lower under the revenue-sharing compensation scheme, because $\partial b/\partial r_w < 0$.

Proposition 3. Π_{jW} does not change with K_j when $K_j \geq n + r$. When $K_j < n + r$, Π_{jW} changes linearly with K_j at rate $((\pi_{jW} - \pi_{jL})/(n + r) - b_k)$. If firm j wins, it cannot be that $b_k > (\pi_{jW} - \pi_{jL})/(n + r)$, because firm j would be better off losing in this scenario. Thus, when firm j wins the auction, $\partial\Pi_{jW}/\partial K_j > 0$ for $K_j \leq n + r$, so firm j sets $K_j \geq n + r$ and earns $\Pi_{jW} = n\pi_{jW} - b_k(n + r)$. $\Pi_{jW}(b_j) = \Pi_{jL}$, then gives $b_j = \Delta_j/(n + r)$.

Lemma 1. Here, we prove that, if $K_j < \infty$, it is unique. For $K_j > n$, firm j's second-order condition is

$$\frac{\partial^2\Pi_{jW}}{\partial K^2} = \left(\frac{-\Delta_j}{K_j} + b_k\right) f(K_j - n). \tag{15}$$

We can show that if the first-order condition is satisfied, the second-order condition is strictly negative, implying Π_{jW} is strictly concave. $\partial\Pi_{jW}/\partial K = 0$ implies

$$\frac{\Delta_j}{b_k} = \frac{[1 - F(K_j - n)]}{\int_{K_j-n}^{\infty} \frac{1}{n+r} f(r)\, dr}. \tag{16}$$

Substituting this into the second-order condition gives

$$\frac{\partial^2\Pi_{jW}}{\partial K^2} = \left[\frac{-[1 - F(K_j - n)]}{\int_{K_j-n}^{\infty} \frac{K_j}{n+r} f(r)\, dr} + 1\right] b_k f(K_j - n) \tag{17}$$

when the first-order condition holds with equality. Under the bounds of the integral, we have $K_j/(n + r) \leq 1$ for every term $r \geq K_j - n$. Thus,

$$\left[\int_{K_j-n}^{\infty} \frac{K_j}{n+r} f(r)\, dr - [1 - F(K_j - n)]\right] < \left[\int_{K_j-n}^{\infty} f(r)\, dr - [1 - F(K_j - n)]\right] = 0. \tag{18}$$

Therefore, the second-order condition is strictly satisfied whenever the first-order condition holds with equality.

Proposition 4. Gatekeeper revenues are equal to firm j's expenditure $\int_0^{K-n} [b(n + r)] f(r)\, dr + \int_{K-n}^{\infty} bK f(r)\, dr$

$$\begin{aligned} &= \int_0^{K-n} (n\pi_{jW}) f(r)\, dr + \int_{K-n}^{\infty} \left[\Delta \frac{K}{n+r} + n\pi_{jL}\right] f(r)\, dr \\ &\quad - \int_0^{K-n} n\pi_{jL} f(r)\, dr - \int_{K-n}^{\infty} \left(n\pi_{jW} - \Delta \frac{K}{n+r}\right) f(r)\, dr \\ &= \Delta\left\{2\left[\int_0^{K-n} f(r)\, dr + \int_{K-n}^{\infty} \left(\frac{K}{n+r}\right) f(r)\, dr\right] - 1\right\}. \end{aligned} \tag{19}$$

Note that $\int_{K-n}^{\infty}(K/(n+r))f(r)\,dr < \int_{K-n}^{\infty} f(r)\,dr$, because $K/(n+r)<1$ for every $r\in(K-n,\infty)$, so

$$\Delta\left\{2\left[\int_0^{K-n} f(r)\,dr+\int_{K-n}^{\infty}\left(\frac{K}{n+r}\right)f(r)\,dr\right]-1\right\}<\Delta. \qquad (20)$$

The right-hand side is gatekeeper revenues when inflationary click fraud is deterministic.

Proposition 5. Here, we prove that under stochastic inflationary click fraud and no competitive click fraud, when $K_j=\infty$, expected gatekeeper revenues are larger than in the baseline model. We have

$$\Pi_{jW}=\int_0^{\infty}[n\pi_{jW}-b_k(n+r)]f(r)\,dr \qquad (21)$$

and

$$\Pi_{jL}=\int_0^{K_k-n}(n\pi_{jL})f(r)\,dr+\int_{K_k-n}^{\infty}\left(n\pi_{jW}-\Delta_j\frac{K_j}{n+r}\right)f(r)\,dr \qquad (22)$$

when $n\le K_k<\infty$. Gatekeeper revenue when firm j wins is $\int_0^{\infty} b_k(n+r)f(r)\,dr$, so we need to find b_k.

Firm k chooses b_k to set $\Pi_{kW}=\Pi_{kL}$. From above, we have

$$\Pi_{kW}=\int_0^{K_k-n}[n\pi_{kW}-b_k(n+r)]f(r)\,dr +\int_{K_k-n}^{\infty}\left(\Delta_k\frac{K_k}{n+r}+n\pi_{kL}-b_kK_k\right)f(r)\,dr \qquad (23)$$

and $\Pi_{kL}=n\pi_{kL}$. $\Pi_{kW}(b_k)=\Pi_{kL}$ gives

$$\int_0^{K_k-n} b_k(n+r)f(r)\,dr+\int_{K_k-n}^{\infty} b_kK_kf(r)\,dr =\Delta_k\left(\int_0^{K_k-n} f(r)\,dr+\int_{K_k-n}^{\infty}\frac{K_k}{n+r}f(r)\,dr\right). \qquad (24)$$

From firm k's FOC in K_k, we have

$$\int_{K_k-n}^{\infty}\left[\frac{\Delta_k}{n+r}-b_j\right]f(r)\,dr=0, \qquad (25)$$

and $b_j>b_k$ so

$$\int_{K_k-n}^{\infty}\left[\frac{\Delta_k}{n+r}-b_k\right]f(r)\,dr>0. \qquad (26)$$

Therefore, we know that

$$\begin{aligned}\int_0^{K_k-n}&[b_k(n+r)]f(r)\,dr\\ &=\Delta_k\int_0^{K_k-n} f(r)\,dr+K_k\int_{K_k-n}^{\infty}\left(\frac{\Delta_k}{n+r}-b_k\right)f(r)\,dr\\ &>\Delta_k\int_0^{K_k-n} f(r)\,dr.\end{aligned} \qquad (27)$$

We can now look at expected gatekeeper revenues,

$$\begin{aligned}\int_0^{\infty}& b_k(n+r)f(r)\,dr\\ &>\int_0^{\infty}\frac{\Delta_k\int_0^{K_k-n} f(r)\,dr}{\int_0^{K_k-n}(n+r)f(r)\,dr}(n+r)f(r)\,dr\\ &=\Delta_k\frac{E(n+r)}{E(n+r\mid n+r<K_k)}>\Delta_k.\end{aligned} \qquad (28)$$

Δ_k is gatekeeper revenues in the baseline model, so we have shown that gatekeeper revenues are strictly larger when the two firms are sufficiently different that the high bidder sets an infinite capacity in spite of uncertain inflationary click fraud.

Proposition 6. Suppose not. If $z_k>0$ and $K_j<n+z_k+r$, firm j's profit is $n\pi_{jW}(K_j/(n+z_j+r))+(1-K_j/(n+z_k+r))n\pi_{jL}-b_kK_j$. This is strictly less than the case in which $K_j\ge n+z_k+r$. Therefore, firm j will always increase K_j until $K_j\ge n+z_k+r$. Firm k's best response to this strategy is $z_k=0$.

Proposition 7. We prove this proposition with two special cases of the model. In the extreme case that $c(z)=0$, the low bidder's best strategy is to set $z=\infty$, erasing the high bidder's profit and driving bids to zero. Thus, competitive click fraud may cause gatekeeper revenues to fall.

To prove that gatekeeper revenues may rise with competitive click fraud, we solve for equilibrium for a special case of $f(r)$ and $c(z)$. We assume that $r=\underline{r}$ with probability $1-\theta$ and $r=\bar r$ with probability θ, where $\underline{r}<\bar r$. This discrete distribution $f(r)$ is the only distribution that yields analytical solutions in this model. We also assume $\Delta_j=\Delta_k=\Delta$ and $c(z)=cz$ for simplicity.

If firm j wins the auction, its profit is

$$\Pi_{jW}=\begin{cases} n\pi_{jW}-b_k(1-\theta)(n+\underline{r}+z_k)\\ \quad -b_k\theta(n+z_k+\bar r);\quad \text{when } K_j\ge n+\bar r+z_k\\ (1-\theta)[n\pi_{jW}-b_k(n+\underline{r}+z_k)]\\ \quad +\theta\left(\dfrac{K_j\Delta}{n+\bar r+z_k}+n\pi_{jL}-b_kK_j\right);\\ \quad \text{when } n+\underline{r}+z_k\le K_j<n+\bar r+z_k\\ (1-\theta)\left[\dfrac{K_j\Delta}{n+\underline{r}+z_k}\right]+\theta\left(\dfrac{K_j\Delta}{n+\bar r+z_k}\right)\\ \quad +n\pi_{jL}-b_kK_j;\qquad \text{when } K_j<n+\underline{r}+z_k.\end{cases} \qquad (29)$$

Π_{jW} is continuous and piecewise linear. It is flat for $K_j\ge n+\bar r+z_k$.

$$\frac{\partial\Pi_j}{\partial K_j}=\begin{cases}\theta\left(\dfrac{\Delta}{n+\bar r+z_k}-b_k\right)\\ \quad \text{for } n+\underline{r}+z_k\le K_j<n+\bar r+z_k\\ \Delta\left(\dfrac{1-\theta}{n+\underline{r}+z_k}+\dfrac{\theta}{n+\bar r+z_k}\right)-b_k\\ \quad \text{for } K_j<n+\underline{r}+z_k.\end{cases} \qquad (30)$$

We first show it cannot decrease on the first segment and then increase on the second. From the slope expressions, if it did, then

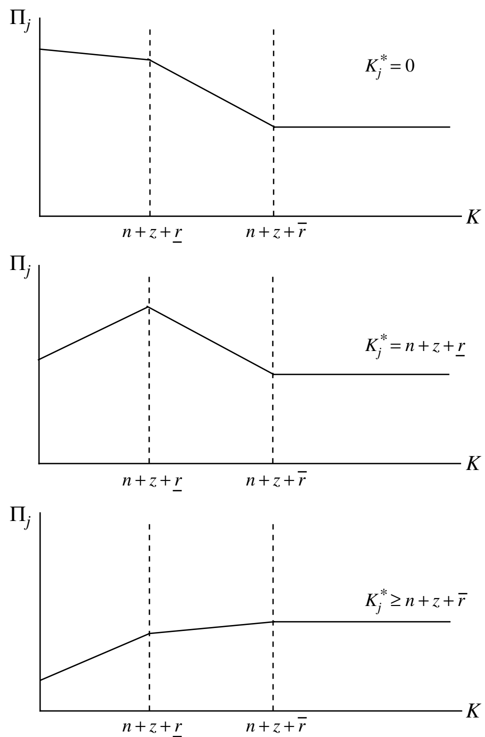
$$\Delta\left(\frac{1-\theta}{n+\underline{r}+z_k}+\frac{\theta}{n+\bar r+z_k}\right)<b_k<\frac{\Delta}{n+\bar r+z_k}, \qquad (31)$$

which cannot happen because $\underline{r}<\bar r$.

Figure A.1 shows the three possible shapes Π_j can take in K_j.

Equilibrium capacity may be given by $K_j^*=0$, $K_j^*=n+z_2+\underline{r}$, or $K_j^*\ge n+z_2+\bar r$, depending on the shape of Π_j. The

Figure A.1 **Possible Shapes of Π_{jW}**



middle case occurs when (rewriting the slope conditions and evaluating at $K_j = n + \underline{r} + z_k$)

$$\Delta\left(\frac{1-\theta}{K_j} + \frac{\theta}{K_j + \bar{r} - \underline{r}}\right) > b_k > \frac{\Delta}{K_j + \bar{r} - \underline{r}}. \tag{32}$$

If the first inequality is violated, $K_j^* = 0$. If the second inequality is violated, $K_j^* \geq n + z_k + \bar{r}$.

Now we consider the auction loser. Firm k's profit function is

$$\Pi_{kL} = \begin{cases} n\pi_{kL} - cz_k, & \text{when } K_j \geq n + \bar{r} + z_k \\ (1-\theta)n\pi_{kL} + \theta\left(\dfrac{-K_j\Delta}{n + \bar{r} + z_k} + n\pi_{kW}\right) - cz_k, & \\ \quad \text{when } n + \underline{r} + z_k \leq K_j < n + \bar{r} + z_k & \\ (1-\theta)\left(\dfrac{-K_j\Delta}{n + \underline{r} + z_k}\right) + \theta\left(\dfrac{-K_j\Delta}{n + \bar{r} + z_k}\right) & \\ \quad + n\pi_{kW} - cz_k \quad \text{when } K_j < n + \underline{r} + z_k. & \end{cases} \tag{33}$$

For $K_j \geq n + \bar{r} + z_k$, $\partial\Pi_{kL}/\partial z_k = -c$, and $z_k = 0$. For $n + \underline{r} + z_k \leq K_j < n + \bar{r} + z_k$, $\partial\Pi_{kL}/\partial z_k = \theta K_j \Delta/(n + \bar{r} + z_k)^2 - c = 0$ and $z_k = \sqrt{\theta K_j \Delta/c} - n - \bar{r}$. For $K_j < n + \underline{r} + z_k$, $\partial\Pi_k/\partial z_k = K_j\Delta[(1-\theta)/(n + \underline{r} + z_k)^2 + \theta/(n + \bar{r} + z_k)^2] - c = 0$ defines z_k. Second-order conditions are satisfied for $K_j < n + \bar{r} + z_k$.

We find two equilibria in pure strategies. In the first, Π_{jW} is rising in its third segment, $K_j \geq n + \bar{r} + z_k$, and $z_k = 0$.

The other possibility is that Π_{jW} peaks at $K_j = n + \underline{r} + z_k$, in which case firm k responds according to its first-order condition. We then have

$$z_k^* = \sqrt{\frac{\theta^2\Delta^2}{4c^2} + \theta\Delta(\underline{r} - \bar{r})} + \frac{\theta\Delta}{2c} - n - \bar{r} \tag{34}$$

$$K_j^* = \sqrt{\frac{\theta^2\Delta^2}{4c^2} + \theta\Delta(\underline{r} - \bar{r})} + \frac{\theta\Delta}{2c} - \bar{r} + \underline{r}. \tag{35}$$

There are three necessary conditions for this equilibrium. First, it must be that K_j^* is in the prescribed range, which implies $z_k^* < \bar{r} - \underline{r}$. Second, it must be that $z_k^* > 0$. Third, it must be that firm k prefers $\Pi_{kL}(K_j^*, z_k^*)$ to $\Pi_{kL}(K_j^*, 0)$; this implies $\theta\Delta(1 - K_j^*/(K_j^* + \bar{r} - \underline{r})) > cz_k^*$. The equilibrium level of competitive click fraud is increasing with $\bar{r}$ and decreasing with c.

Gatekeeper Revenues. We now evaluate gatekeeper revenues. Total payments made to the gatekeeper in equilibrium are b^*K^* as determined by firm j's indifference to winning and losing: $\Pi_{jW}^*(K^*, z^*, b^*) = \Pi_{jL}^*(K^*, z^*)$ implies

$$\begin{aligned} n\pi_{jW} - \theta\Delta\left(1 - \frac{K^*}{K^* + \bar{r} - \underline{r}}\right) - b^*K^* \\ = n\pi_{jL} + \theta\Delta\left(1 - \frac{K^*}{K^* + \bar{r} - \underline{r}}\right) - cz^* \end{aligned} \tag{36}$$

or

$$b^*K^* = \Delta\left[1 - 2\theta\Delta\left(1 - \frac{K^*}{K^* + \bar{r} - \underline{r}}\right)\right] + cz^*. \tag{37}$$

It can be seen that gatekeeper revenues, b^*K^*, rise with z^*.

Lemma 2. Consider two mutually exclusive cases. For $b_k < v_j$, winning the auction produces a positive profit for firm j. No $b_j > b_j^*$ will increase firm j's profits, whereas a $b_j < b_j^*$ can only decrease profits by reversing the profitable auction outcome. For $b_k > v_j$, firm j cannot profitably win the auction. $b_j = b_j^*$ ensures that it loses. No $b_j < b_j^*$ can improve profits, whereas a $b_j > b_j^*$ can only change profits by unprofitably reversing the auction outcome. The proof for firm k is symmetric.

Proposition 8. From §3, firm j will always win the SP auction and pay the gatekeeper $n_j v_k$. Consider two mutually exclusive cases. If $n_j/n_k < v_j/v_k$, firm j also wins the CT auction and pays the gatekeeper $n_j b_k^* = (n_j^2/n_k)v_k$. This is more than in the SP auction if and only if $n_j > n_k$. In the second case, if $n_j/n_k > v_j/v_k$, firm k wins the CT auction and pays the gatekeeper $n_k b_j^* = ((n_k)^2/n_j)v_j$. This is less than gatekeeper revenues in the SP auction: $n_j/n_k > v_j/v_k \iff (n_k^2/n_j)v_j < n_j v_k$.

Proposition 9. When firm j wins the CT auction it pays the gatekeeper $(n_j + r)b_k^* = ((n_j + r)^2/(n_k + r))v_k$. Taking the derivative shows that gatekeeper revenues are increasing in r if and only if $r > n_j - 2n_k$.

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