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### Research Note

### A Cross-Category Model of Households' Incidence and Quantity Decisions

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This paper advances the literature on multicategory demand models by simultaneously handling more than one purchase decision of the household. We propose a two-stage bivariate logit model of incidence and quantity outcomes in multiple categories. Our results show that cross-category promotional spillovers are asymmetric between the two product categories of bacon and eggs. The total retail profit responds more to bacon price than to egg price. Promoting bacon is found to have a bigger impact on egg profit than the impact of egg promotion on bacon profit. We decompose (1) the total retail profits, as well as (2) the cross-category profit impact of a price promotion, into its two components, and find that (1) 23% (67%) of the total retail profit impact of a promotion on bacon (eggs) arises on account of quantity effects, and (2) 40% (33%) of the increase in egg (bacon) profit from promoting bacon (eggs) is on account of quantity effects.

*Key words*: multicategory; multivariate choices; bivariate logit; incidence; quantity; basket data *History*: This paper was received June 17, 2002, and was with the authors 37 months for 4 revisions; processed by Scott Neslin.

### 1. Introduction

There is widespread recognition of the fact that consumers make multicategory purchase decisions across a variety of contexts, for example, in grocery products (pasta and pasta sauce), durable goods (washer and dryer), financial products (stocks, bonds, mutual funds), etc. It is clear that manufacturers and retailers recognize the dependencies in consumers' behaviors across related categories (Terbeek 1993). Manufacturers' utilization of cross-category promotions and retailers' shelf-space allocation decisions across product categories within a store evidence this reality.

Only recently have academic researchers begun to understand cross-category relationships in consumers' decision making using *multicategory models* (see Seetharaman et al. 2005 for a review of this emerging literature). The recognition of cross-category dependencies implies that a consumer's purchase decisions across categories are not independent. In other words, the consumer's (a) decision of when (or whether) to buy in one category depends on the consumer's corresponding decision in the related category, and (b) decision of how much to buy in one category

depends on the consumer's corresponding decision in the related category. The existing research on multicategory models focuses only on (a), but ignores (b) (see Chintagunta and Haldar 1998, Manchanda et al. 1999, Russell and Peterson 2000, Chib et al. 2002, Chung and Rao 2003, Jedidi et al. 2003). The focus of this paper is to estimate a multicategory model that simultaneously allows for both (a) and (b). Given that single-category analyses have been extensively used over the years to quantify promotional lifts that accrue to retailers from offering price deals (see, for example, Blattberg et al. 1981, Neslin et al. 1985, Bucklin and Gupta 1992, Mela et al. 1998, Bell et al. 1999, Seetharaman 2004), it is natural to ask whether such increased quantity buying would lead to increased consumption in complementary product categories. By focusing on quantity outcomes across related product categories, we are able to address this issue. In doing this, ours becomes the first effort in the literature on multicategory choice models that simultaneously accommodates cross-category dependencies in more than one consumer purchase decision at the same time (for cross-category models of brand choice decisions only, see Erdem 1998, Ainslie and Rossi 1998, Erdem and Winer 1999, Seetharaman et al. 1999, Erdem and Sun 2002, Iyengar et al. 2003, Singh et al. 2005, Hansen et al. 2006). Using our proposed model, we are able to investigate two research questions:

- 1. We quantify the profit impact of *promotional spillovers* (i.e., cross-category effects of retail promotions) to the retailer, and study whether it is asymmetric across product categories. Such an analysis would be managerially instructive to the retailer from the standpoint of figuring out which of two product categories to promote in a given week.
- 2. We decompose the estimated profit impact of promotional spillovers into incidence and quantity components. Such a decomposition would allow the retailer to understand to what extent a promotion in one product category would simply stimulate simultaneous purchase of the related category, rather than also leading to increased quantity buying of the related category.

In our analyses using bacon and egg data, we find that cross-category promotional spillovers are asymmetric between the two product categories of bacon and eggs. We find that the retailer benefits much more (from the standpoint of increasing total retail profit) from promoting bacon rather than eggs, i.e., the total retail profit responds more to bacon price than to egg price. Further, promoting bacon is found to have a bigger impact on egg profit than the impact of egg promotion on bacon profit. We decompose (1) the total retail profits, as well as (2) the cross-category profit impact of a price promotion, into its two components, and find that (1) 23% (67%) of the total retail profit impact of a promotion on bacon (eggs) arises on account of quantity effects, and (2) 40% (33%) of the increase in egg (bacon) profit from promoting bacon (eggs) is on account of quantity effects. These differences are substantively meaningful for fast-moving product categories such as bacon and eggs, where product volume movements, and therefore product category profits, in a given week at stores of a retail chain are substantial. Explicit consideration of cross-category effects in quantities, in addition to such effects in incidence, will lead to better managerial decision making on retail promotional planning across categories.

To reiterate, the objective of this paper is to investigate the usefulness of jointly estimating incidence and quantity decisions while estimating correlations in household purchase behavior across product categories.<sup>1</sup> Neoclassical economic theory suggests two

opposing ways in which a price cut (or, more generally, a retail promotion) in one product category could influence quantity buying in a complementary product category: First, by making the focal (i.e., promoted) product more attractive for current consumption, the price cut induces the consumer to purchase more of the focal product, which in turn creates the need for the consumer to purchase more of the complementary product (because the consumer gains additional satisfaction from consuming the two products jointly rather than separately), e.g., two boxes of pasta require more pasta sauce for joint consumption than does one box of pasta. We call this the complementarity effect. Second, increased quantity buying of the focal product may lead to decreased money available to spend on other product categories (including the complementary product) that belong to the same "mental account" of the consumer (Thaler 1985), which in turn leads to decreased quantity buying of the complementary product. We call this the expenditure effect. The relative strengths of these two effects and, therefore, the net effect of a price cut, depends on (1) how versatile the product is, in terms of whether it can or needs to be consumed with other products (see Wansink 1993); and (2) how expensive the product is and, therefore, what fraction of the consumer's shopping budget is allocated to the product. In our empirical application, we use the product pair of bacon and eggs, where bacon is the more expensive product and eggs is the more versatile product. Other retail promotional instruments, such as store displays and newspaper features, are also likely to show the above-mentioned two effects in terms of stimulating consumer demand for related products in the store. However, there is an interesting difference between the two instruments. To the extent that newspaper features influence consumer purchasing decisions at the home, i.e., prior to visiting the store, they are more likely to show complementarity effects rather than expenditure effects, because the consumer can more consciously alter his shopping budget before leaving for the store. Store displays, on the other hand, stimulate unplanned consumer purchasing behavior at the store. Therefore, display-based purchases are more likely to lead to budget constraint effects that affect other product purchases at the store (perhaps adversely, if the expenditure on the promoted product ends up being large on account of increased quantity buying).

We propose a *two-stage bivariate logit* model that explicitly allows for cross-category correlations in incidence and quantity decisions of households across two product categories. Using the proposed model, we investigate the two research questions discussed earlier. The rest of the paper is organized as follows. In the next section, we derive the proposed model.

<sup>&</sup>lt;sup>1</sup> We ignore households' brand-choice decisions. The retailer is interested in first figuring out which categories to copromote before figuring out which specific brands to copromote in the chosen categories. We leave the brand-choice component as an important extension of our proposed model.

In the third section, we discuss the data and estimation results. The fourth section concludes with a summary of our main findings.

### 2. Model Development

We develop the model in four steps: first, we present the *bivariate logit* (BVL) model and show how it can be used to model a household's incidence decisions across two product categories; second, we develop a BVL for the household's conditional quantity decisions across the two categories; third, we show how to extend the proposed model of incidence and quantity outcomes (from steps 1 and 2) to handle multiple package sizes; fourth, we derive the sample likelihood function that is required for estimating the model parameters using the maximum-likelihood technique.

## 2.1. Bivariate Logit (BVL) Model of Joint Incidence

According to the BVL model, first proposed by Cox (1972), the household's utility for a bundle of two product categories at a shopping occasion t can be written as follows.<sup>2</sup>

$$U_{10t} = X_{1t}\beta_{11} + X_{2t}\beta_{21} + \varepsilon_{10t} \tag{1}$$

$$U_{01t} = X_{1t}\beta_{12} + X_{2t}\beta_{22} + \varepsilon_{01t}$$
 (2)

$$U_{11t} = X_{1t}\beta_{11} + X_{2t}\beta_{21} + X_{1t}\beta_{12} + X_{2t}\beta_{22}$$

$$+\gamma_{11} + IV_t \eta + \varepsilon_{11t} \tag{3}$$

$$U_{00t} = \varepsilon_{00t}, \tag{4}$$

where  $U_{10t}$ ,  $U_{01t}$ ,  $U_{11t}$ , and  $U_{00t}$  stand for the household's joint utilities, during shopping trip t, from buying (i) category 1 but not category 2 (i.e., 10), (ii) category 2 but not category 1 (i.e., 01), (iii) both categories (i.e., 11), and (iv) neither category (i.e., 00), respectively. Further,  $X_{ct}$  refers to a vector of relevant covariates for category c at shopping trip t,  $\beta_{11}$  and  $\beta_{21}$  are vectors of parameters that, respectively, capture the effects of category 1 covariates and category 2 covariates on the household's utility for category 1;  $\beta_{12}$  and  $\beta_{22}$  are vectors of parameters that, respectively, capture the effects of category 1 covariates and category 2 covariates on the household's utility for category 2. The parameter  $\gamma_{11}$  captures the additional utility obtained by the household from jointly purchasing both categories at shopping trip t. The variable  $IV_t$ , called the *inclusive value variable* (which will be explained later in §2.2), together with the parameter  $\eta$ , called the *inclusive value parameter*, jointly capture the dependence of the household's incidence outcomes on its quantity decisions (see, for example, Mela et al. 1998). Lastly,  $\varepsilon_{10t}$ ,  $\varepsilon_{01t}$ ,  $\varepsilon_{11t}$ , and  $\varepsilon_{00t}$  are iid Gumbel variates, capturing the effects of unobserved variables on the household's utilities for the bundles, with location and scale fixed at zero and one, respectively. This yields the following expressions for the household's purchase probabilities during shopping trip t for all possible bundles of the two categories.

$$P_{I_{ct},I_{c't},t} = \left(e^{(X_{1t}\beta_{11}+X_{2t}\beta_{21})I_{ct}+(X_{1t}\beta_{12}+X_{2t}\beta_{22})I_{c't}+(\gamma_{11}+IV_{t}\eta)I_{ct}I_{c't}}\right) \cdot \left(e^{X_{1t}\beta_{11}+X_{2t}\beta_{21}}+e^{X_{1t}\beta_{12}+X_{2t}\beta_{22}}\right. \\ \left. + e^{X_{1t}\beta_{11}+X_{2t}\beta_{21}+X_{1t}\beta_{12}+X_{2t}\beta_{22}+\gamma_{11}+IV_{t}\eta}+1\right)^{-1}, (5)$$

where  $I_{ct}$  ( $I_{c't}$ ) is an indicator variable that takes the value one if category c (c') is bought at t and zero otherwise. When  $\gamma_{11}=0$ , this reduces to two independent binary logit (BNL) incidence models (i.e., the joint probability under the BVL equals the product of the two BNL marginal probabilities). It is useful to note that the BVL admits both positive and negative covariance between the household's incidence outcomes in the two product categories (because the covariance  $\gamma_{11}$  is a free parameter that is not restricted to be only positive or only negative). Further, crosscategory complementarity effects are captured in our BVL both through (1) the cross effects of marketing variables, i.e.,  $\beta_{12}$  and  $\beta_{21}$ , and (2) the complementarity fixed effect  $\gamma_{11}$ .

We would expect  $\gamma_{11} > 0$  if the two product categories were *complements*, and  $\gamma_{11}$  < 0 if they were substitutes. Just like the bivariate probit (BVP) model of Manchanda et al. (1999)—wherein crosscategory complementarity is captured by the crosscategory effects of marketing variables—our BVL model also accommodates cross-category complementarity on account of cross-category effects of marketing variables, i.e., through  $\beta_{12}$  and  $\beta_{21}$ . In addition to this effect, our model also captures the effects of intrinsic consumption complementarity between product categories, as reflected in  $\gamma_{11}$ , a fixed effect in the household's utility for a product bundle containing both products. This is an attractive property of the BVL model compared to the BVP model. However, the BVP model captures cross-category correlations in the error terms of the households' random utilities for the categories (which will be reflected in the error terms of the households' random utilities for bundles in our model), which the BVL model ignores. Such correlated random effects represented in a BVP could capture such phenomena such as the unannounced arrival of guests (that are unobserved

<sup>&</sup>lt;sup>2</sup> For another paper that explains a household's joint purchase of two product categories using the household's utilities for bundles, see Jedidi et al. (2003).

<sup>&</sup>lt;sup>3</sup> For a recent application of the BVL on incidence data from four product categories, see Russell and Peterson (2000).

in scanner panel data) leading to a simultaneous purchase of both product categories. <sup>4</sup> That said, it is important to note that the computational burden associated with estimating the multivariate logit (MVL), of which the BVL is a special case, increases only linearly with the number of product categories in the data, because it entails the estimation of additional  $\gamma$  parameters within a closed-form likelihood function. On the other hand, the computational burden associated with estimating the multivariate probit (MVP), of which the BVP is a special case, would increase in a convex manner with the number of product categories because the likelihood function involves high-dimensional integration, where the order of integration equals the number of product categories.

## 2.2. Bivariate Logit Model of Joint Conditional Quantities

The household's conditional utility for a quantity bundle of two product categories (given that both categories are jointly purchased) at a shopping occasion t can be written as follows.

$$u_{21t} = Z_{1t}\delta_{11} + Z_{2t}\delta_{21} + \zeta_{21t} \tag{6}$$

$$u_{12t} = Z_{1t}\delta_{12} + Z_{2t}\delta_{22} + \zeta_{12t} \tag{7}$$

$$u_{22t} = Z_{1t}\delta_{11} + Z_{2t}\delta_{21} + Z_{1t}\delta_{12} + Z_{2t}\delta_{22} + \tau_{22} + \zeta_{22t}$$
 (8)

$$u_{11t} = \zeta_{11t}, (9)$$

where  $u_{21t}$ ,  $u_{12t}$ ,  $u_{22t}$ , and  $u_{11t}$  stand for the household's joint utilities (conditional on purchasing both categories), during shopping trip t, from buying (i) two (or more) units of category 1, but one unit of category 2 (i.e., 21); (ii) two (or more) units of category 2, but one unit of category 1 (i.e., 12); (iii) two (or more) units of both categories (i.e., 22); and (iv) one unit of both categories (i.e., 11), respectively. Further,  $Z_{ct}$ refers to a vector of relevant covariates<sup>5</sup> for category *c* at t;  $\delta_{11}$  and  $\delta_{21}$  are vectors of parameters that, respectively, capture the effects of category 1 covariates and category 2 covariates on the household's utility from buying two (or more) units in category 1;  $\delta_{12}$  and  $\delta_{22}$ are vectors of parameters that, respectively, capture the effects of category 1 covariates and category 2 covariates on the household's utility from buying two (or more) units of category 2. The parameter  $\tau_{22}$  captures the additional utility obtained by the household from jointly purchasing two (or more) units of both categories at shopping trip t. Lastly,  $\zeta_{21t}$ ,  $\zeta_{12t}$ ,  $\zeta_{22t}$ , and  $\zeta_{11t}$  are iid Gumbel variates, capturing the effects of unobserved variables on the household's utilities for

the quantity bundles, with location and scale fixed at zero and one, respectively. This yields the following expressions for the household's conditional probabilities during shopping trip *t* for all possible conditional quantity bundles of the two categories.

$$p_{Q_{ct},Q_{c't},t} = \left(e^{(Z_{1t}\delta_{11} + Z_{2t}\delta_{21})Q_{ct} + (Z_{1t}\delta_{12} + Z_{2t}\delta_{22})Q_{c't} + \tau_{22}Q_{ct}Q_{c't}}\right) \cdot \left(e^{Z_{1t}\delta_{11} + Z_{2t}\delta_{21}} + e^{Z_{1t}\delta_{12} + Z_{2t}\delta_{22}} + e^{Z_{1t}\delta_{11} + Z_{2t}\delta_{21} + Z_{1t}\delta_{12} + Z_{2t}\delta_{22} + \tau_{22}}\right)^{-1}, \quad (10)$$

where  $Q_{ct}$  ( $Q_{c't}$ ) is an indicator variable that takes the value one if two (or more) units of category c (c') are bought at t, and zero otherwise.

We define the *inclusive value variable*,  $IV_t$  (used in §2.1 earlier; see Equations (3) and (5)), as  $Z_{1t}\delta_{11} + Z_{2t}\delta_{21} + Z_{1t}\delta_{12} + Z_{2t}\delta_{22} + \tau_{22}$ . This variable captures the attractiveness of jointly buying both products in large quantities (i.e., buying two or more units in either category) to the household. By including this variable in the incidence model in §2.1, we capture the effects of the attractiveness of jointly buying both products in large quantities (i.e., buying two or more units in both categories) on the household's likelihood to jointly buy both product categories. This renders the incidence and quantity outcomes of a household to be correlated with each other.

It is important to note that  $\tau_{22}$  under the quantity model flexibly captures the cross-category correlation in a household's quantity decisions, as distinct from the cross-category correlation in the household's incidence decisions (as reflected in  $\gamma_{11}$ ). Further, crosscategory complementarity effects are captured in the BVL both through (1) the cross effects of marketing variables, i.e.,  $\delta_{12}$  and  $\delta_{21}$ ; and (2) the complementarity fixed effect  $\tau_{22}$ . Comparing the estimated magnitudes of  $\gamma_{11}$  ( $\beta_{12}$  and  $\beta_{21}$ ) versus  $\tau_{22}$  ( $\delta_{12}$  and  $\delta_{21}$ ) allows us to understand the relative strengths of complementarity between two product categories for incidence versus quantity. Further,  $\beta_{cc'}$ ,  $(c \neq c')$  and  $\delta_{cc'}$ ,  $(c \neq c')$ can be used to understand possible asymmetries in promotional spillovers across categories, and then to decompose these cross-category spillovers into incidence and quantity components. We treat the quantity decision as a binary decision because in our data set, more than 95% of the mass of the conditional quantity distribution is accounted for by the first two supports, i.e., quantity = 1 unit, and quantity = 2 units, of the distribution (with a 60-40 mass split between them).<sup>6</sup>

To handle the *conditional* quantity outcomes when the household purchases either (i) two (or more) units of category 1 and none of category 2 (i.e., 20);

<sup>&</sup>lt;sup>4</sup> We thank the area editor for alerting us to this issue.

<sup>&</sup>lt;sup>5</sup> In the empirical application we will restrict  $Z_{ct} = X_{ct}$ , i.e., the same set of covariates will be allowed to influence both incidence and quantity.

 $<sup>^6</sup>$  In fact, we estimated our model on two additional data sets, which also satisfied this property. In §2.3, we show how to relax this assumption.

or (ii) two (or more) units of category 2 and none of category 1 (i.e., 02), we use the following BNLs.

$$p_{20t} = \frac{e^{Z_{1t}\theta_{11} + Z_{2t}\theta_{21}}}{1 + e^{Z_{1t}\theta_{11} + Z_{2t}\theta_{21}}}, \qquad p_{10t} = \frac{1}{1 + e^{Z_{1t}\theta_{11} + Z_{2t}\theta_{21}}}, \qquad (11)$$

$$p_{02t} = \frac{e^{Z_{1t}\theta_{12} + Z_{2t}\theta_{22}}}{1 + e^{Z_{1t}\theta_{12} + Z_{2t}\theta_{22}}}, \qquad p_{01t} = \frac{1}{1 + e^{Z_{1t}\theta_{12} + Z_{2t}\theta_{22}}}, \qquad (12)$$

$$p_{02t} = \frac{e^{Z_{1t}\theta_{12} + Z_{2t}\theta_{22}}}{1 + e^{Z_{1t}\theta_{12} + Z_{2t}\theta_{22}}}, \qquad p_{01t} = \frac{1}{1 + e^{Z_{1t}\theta_{12} + Z_{2t}\theta_{22}}}, \quad (12)$$

where  $\theta_{11}$  and  $\theta_{21}$  are vectors of parameters that, respectively, capture the effects of category 1 covariates and category 2 covariates on the probability  $p_{20t}$ ;  $\theta_{12}$  and  $\theta_{22}$  are vectors of parameters that, respectively, capture the effects of category 1 covariates and category 2 covariates on the probability  $p_{02t}$ . By using a separate set of parameters to handle quantity outcomes when both categories are not jointly purchased, we are able to model quantity decisions of households in a highly flexible manner.

To reiterate, we have developed a marginal BVL model of joint incidence outcomes (given by Equation (5)), and a conditional BVL model of joint quantity outcomes (given by Equations (10), (11), and (12)). The following table (Table 1) enumerates the purchase likelihoods for the nine possible scenarios that can arise in the data, and these likelihoods sum to one.

It is useful to note here that our proposed two-stage bivariate model of incidence and quantity outcomes allows for two types of complementarity effects across product categories:

- 1. The cross-category effects of marketing variables captured through  $\beta_{12}$  and  $\beta_{21}$  (in the incidence model) and  $\delta_{12}$  and  $\delta_{21}$  (in the quantity model). We call this marketing-mix complementarity.
- 2. The cross-category effects captured through  $\gamma_{11}$ and  $\tau_{22}$  (in the incidence and quantity models respectively). We call this intrinsic complementarity.

### 2.3. Handling Multiple Package Sizes

Our model has thus far been developed under the assumption that both product categories contain only one package size, and that consumers choose to buy one unit or two units of the respective package size in either category. Two generalizations to this model are in order. First, one may encounter product categories (unlike the one used in our application) where consumers buy not just one or two units, but also three units on a significant number of purchase occasions.

**Purchase Likelihoods for Nine Purchase Scenarios** Table 1

Purchase outcome	No buy in category 2	Buy 1 unit of category 2	Buy 2 (or more) units of category 2
No buy in category 1 Buy 1 unit of category 1 Buy 2 (or more) units of category 1	$P_{00} \\ P_{10} * p_{10} \\ P_{10} * p_{20}$	$P_{01} * p_{01}$ $P_{11} * p_{11}$ $P_{11} * p_{21}$	$P_{01} * p_{02} P_{11} * p_{12} P_{11} * p_{22}$

In such cases, one can extend our quantity model by defining the household's utilities for bundles of two product quantities (each of which can be one, two, or three units) as follows.

$$u_{11t} = Z_{1t}\delta_{111} + Z_{2t}\delta_{211} + \zeta_{11t} \tag{13}$$

$$u_{12t} = Z_{1t}\delta_{112} + Z_{2t}\delta_{212} + \zeta_{12t} \tag{14}$$

$$u_{13t} = Z_{1t}\delta_{113} + Z_{2t}\delta_{213} + \zeta_{13t} \tag{15}$$

$$u_{21t} = Z_{1t}\delta_{121} + Z_{2t}\delta_{221} + \zeta_{21t} \tag{16}$$

$$u_{22t} = Z_{1t}\delta_{122} + Z_{2t}\delta_{222} + \tau_{22} + \zeta_{22t}$$
 (17)

$$u_{23t} = Z_{1t}\delta_{123} + Z_{2t}\delta_{223} + \tau_{22} + \zeta_{23t}$$
 (18)

$$u_{31t} = Z_{1t}\delta_{131} + Z_{2t}\delta_{231} + \zeta_{31t}$$
 (19)

$$u_{32t} = Z_{1t}\delta_{132} + Z_{2t}\delta_{232} + \tau_{22} + \zeta_{32t} \tag{20}$$

$$u_{33t} = Z_{1t}\delta_{133} + Z_{2t}\delta_{233} + \tau_{22} + \tau_{33} + \zeta_{33t}, \tag{21}$$

where  $\tau_{22}$  and  $\tau_{33}$  capture two distinct levels of correlations among the two quantity outcomes (with  $\tau_{22}$  being a fixed effect shared among the household's utilities for bundles that include at least two units of either product, whereas  $\tau_{33}$  is a fixed effect shared among the household's utilities for bundles that include at least three units of either product). Assuming that  $\zeta_{11t}$ ,  $\zeta_{12t}$ ,  $\zeta_{13t}$ ,  $\zeta_{21t}$ ,  $\zeta_{22t}$ ,  $\zeta_{23t}$ ,  $\zeta_{31t}$ ,  $\zeta_{32t}$ , and  $\zeta_{33t}$  are iid Gumbel variates, with location zero and scale one, yields a likelihood function for the quantity outcomes that is the multiunit generalization of Equation (10).<sup>7</sup> Correspondingly, one can extend Equations (11) and (12) to represent trinomial (as opposed to binomial) quantity choice probabilities. It is straightforward to see that one can extend this model more generally to handle four, five, etc. quantity outcomes.

Second, one may encounter product categories that contain multiple package sizes. For example, ketchup is typically available in three sizes: 8 oz., 16 oz., and 24 oz. In such cases, one can employ our extendedquantity model (discussed above) after going through the following three steps: (1) The least common multiple of the available package sizes is first calculated; in the ketchup example, this multiple is 8; (2) successive multiples of this number are treated as defining the space of discrete-quantity outcomes, e.g., 8 oz. = 1 unit, 16 oz. = 2 units, 24 oz. = 3 units, 32 oz. = 4 units etc.; (3) for each discrete-quantity outcome (say, two units), among all possible combinations of package sizes that yield that particular quantity outcome (say, two packages of 8 oz., versus one package of 16 oz.), one can choose the combination that has the lowest effective price (on a perounce basis) as representing that quantity outcome,

 $<sup>^{7}</sup>$  For identification purposes, one would have to set  $\delta_{111}=0$  and  $\delta_{211} = 0.$ 

and plug in the explanatory variables associated with that combination for  $Z_{1t}$  or  $Z_{2t}$  on the right-hand side of Equations (13)-(21). In product categories where each package size is not a multiple of the other, for example, 8 oz., 15 oz., and 21 oz., our proposed solution is to treat the smallest size (i.e., 8 oz.) as 1 unit, 15 oz. as 1.875 units, 16 oz. as 2 units, 21 oz. as 2.625 units, 24 oz. as 3 units, and then allow for all possible permutations of the three possible basic sizes to create higher-ordered units, and then apply the same quantity model that is represented in Equations (13)–(21). This will work because our proposed quantity model relies only on the ordering of, and not the actual magnitudes of, allowable quantity outcomes. Said differently, our proposed distribution of quantity outcomes is semiparametric in the sense of imposing probability mass only over allowable quantities (that may not be equally spaced or even integers), as constructed above.8

#### 2.4. Sample Likelihood Function

We maximize the following sample likelihood function using incidence and quantity data from a panel of households on two product categories.

$$L = \prod_{h=1}^{H} \sum_{s=1}^{S} \pi_{s} \prod_{t=1}^{n_{h}} \left\{ \prod_{a=0}^{1} \prod_{b=0}^{1} P_{sabt}^{I_{abt}} \prod_{g=0}^{2} \prod_{h=0}^{2} (g_{+}h \neq 0) p_{sght}^{i_{ght}} \right\}, \quad (22)$$

where H stands for the total number of households in the sample,  $n_h$  stands for the total number of shopping trips undertaken by household h, S stands for the total number of supports of the unobserved heterogeneity distribution (which is assumed to be semiparametric as in Kamakura and Russell 1989),  $\pi_s$  stands for the probability mass associated with support s,  $P_{sabt}$  is the incidence probability (given in Equation (5)) associated with the incidence indicator  $I_{abt}$ , and  $p_{sght}$  is the quantity probability (given in Equations (10)–(12)) associated with the quantity indicator  $i_{ght}$ . This completes our formulation of the proposed model.

### 3. Data and Estimation Results

To estimate the proposed model, we use IRI's Market Basket Data (MBD), which tracks grocery purchases of 1,042 panelists over a two-year period from May 1991 to May 1993 across 24 product categories and 10 stores. We pick bacon and eggs as the two product categories for our estimation.<sup>9</sup> We retain only those

Table 2 Descriptive Statistics

Outcome	%Observations (%)
Incidence outcomes (42,274)	
Egg only (10)	5.9
Bacon only (01)	13.5
Both (11)	2.9
Neither (00)	77.7
Quantity outcomes (9,427)	
One unit each of eggs and bacon (11)	7.4
Two (or more) units of bacon, one unit of eggs (21)	2.2
Two (or more) units of eggs, one unit of bacon (12)	2.4
Two (or more) units each of eggs and bacon (22)	1.1
One unit of bacon, none of eggs (10)	19.3
Two (or more) units of bacon, none of eggs (20)	7.0
One unit of eggs, none of bacon (01)	46.3
Two (or more) units of eggs, none of bacon (02)	14.3

households that make at least five bacon purchases and four egg purchases over the two-year period (883 and 467 households, respectively). From among these households, we retain only households that purchase the most popular package sizes, i.e., 16 oz. bacon and 12-pack eggs, more than 80% of the time. This leaves us with 293 households, making 42,274 shopping trips over a two-year period, for estimation. Table 2 reports descriptive statistics on outcomes.

We use the following covariates in the vectors  $X_{ct}$ and  $Z_{ct}$ : (i) Price (share-weighted price across all SKUs in the household's consideration set, where the share and the consideration set are both specific to the household, and computed on the basis of the household's purchases over the study period); (ii) display (indicator variable that takes the value one if any of the SKUs in the household's consideration set is on display, and zero otherwise); (iii) feature (indicator variable that takes the value one if any of the SKUs in the household's consideration set is featured in a newspaper advertisement, and zero otherwise); (iv) inventory. As in previous literature, starting with an initial inventory equal to the household's average consumption rate, household inventory is imputed using the following inventory flow equation: Current period inventory = Max(previous-period inventory + previous-period quantity purchased - average consumption rate \* days elapsed since previous shopping trip, 0). This variable is mean centered for each household to eliminate the effects of heterogeneity in consumption rates across households. As far as descriptive statistics on marketing variables are concerned, the average prices of bacon and eggs over the study period are 13.42 cents per oz. and 6.7 cents per egg, respectively. Bacon and eggs were displayed during 24% and 11% of the weeks, respectively, whereas they were featured during 9% and 3% of the weeks, respectively. The standard size of bacon and the eggs is 16 oz. and 12 pack, respectively.

<sup>&</sup>lt;sup>8</sup> Given the meager prevalence of alternative package sizes in our data set, we have chosen not to implement these extended-quantity models on our data.

<sup>&</sup>lt;sup>9</sup> We also estimated the proposed model on two additional pairs: detergents and softeners, detergents and analgesics. These results have been suppressed in the paper, and are available from the authors.

The likelihood function, given in Equation (22), is maximized using the DFP algorithm in the Gauss programming environment. To ensure that the identified maximum is indeed the global maximum, we start from different starting points for the parameter values to ensure that convergence is achieved at the same set of parameter values. For benchmarking purposes, we estimate a simultaneous model of incidence and quantity independently for each category, which is obtained by setting  $\gamma_{11} = 0$ ,  $\tau_{22} = 0$ ,  $\beta_{12} = 0$ ,  $\beta_{21} = 0$ ,  $\delta_{12} = 0$ , and  $\delta_{21} = 0$  in our proposed model (i.e., this model ignores cross-category complementarity effects). We compare the predictive ability of the estimates of our proposed model to those of this benchmark model using a holdout sample. Specifically, we estimate the two models using 80% of the observations in the sample and predict the outcomes in the remaining 20% of the observations. The validation log-likelihood of the proposed model is -7,091, whereas that of the benchmark model is -7,118.

The estimation results obtained for the proposed model using the full sample of data are presented in Tables 3 and 4. In each table, we present the estimates—along with their standard errors within parentheses—both at the support level and at the aggregate market level (by averaging over the supports of the heterogeneity distribution). We found that a two-support specification of the unobserved heterogeneity distribution was appropriate for the data. Going to three supports produced a third segment that was not large enough to be of substantive interest. Therefore, we present the two-support heterogeneity solution in all tables.

Table 3 reports the estimated parameters for the incidence component of our proposed model. We uncover significant cross-category covariances between the household's incidence outcomes in the two categories ( $\gamma_{11} = 0.92$ ). This means that households' purchases of bacon and eggs are not independent of each other. In fact, the two categories, if purchased jointly, increase the household's utility for the product bundle beyond the simple sum of the household's utilities for each category (because  $\gamma_{11} > 0$ ). The own-effects of marketing variables and product inventory are all significant and have the expected signs (i.e., price and inventory have negative coefficients, whereas display and feature have positive coefficients) in both categories. This gives strong face validity to our incidence model specification. In terms of cross-effects, egg price has a negative effect ( $\beta_{21\nu} = -0.05$ ) on the household's bacon incidence decision, and bacon price has a negative effect ( $\beta_{22n} = -0.03$ ) on the household's egg incidence decision, both of which are consistent with the household treating the two products as consumption complements (as also reflected in the estimated  $\gamma_{11}$  being

Table 3 Estimation Results for Incidence Component of Proposed Model

Parameter	Support 1	Support 2	Market-level
Joint incidence $(\gamma_{11})$ Inclusive value $(\eta)$	<b>0.99</b> (0.32) <b>0.03</b> (0.01)	<b>0.82</b> (0.30) <b>0.02</b> (0.00)	<b>0.92</b> (0.31) <b>0.03</b> (0.01)
Bacon incidence			
Intercept $(\beta_{0B})$	<b>-1.60</b> (0.48)	<b>-1.74</b> (0.20)	<b>-1.66</b> (0.38)
Bacon price $(\beta_{11p})$	-0.09 (0.02)	-0.02 (0.01)	-0.06 (0.02)
Bacon display $(\beta_{11d})$	<b>0.80</b> (0.09)	<b>0.71</b> (0.11)	
Bacon feature $(\beta_{11f})$	<b>0.46</b> (0.08)	<b>0.54</b> (0.08)	, ,
Bacon inventory $(\beta_{11i})$	-0.01 (0.00)	-0.01 (0.00)	' '
Egg price $(\beta_{21p})$	-0.06 (0.01)	- <b>0.04</b> (0.01)	-0.05 (0.01)
Egg display $(\beta_{21d})$	-0.50 (0.09)	-0.16 (0.09)	-0.35 (0.09)
Egg feature ( $\beta_{21f}$ )	<b>0.10</b> (0.04)	<b>−0.16</b> (0.07)	-0.01 (0.06)
Egg inventory $(\beta_{21i})$	0.00 (0.00)	0.00 (0.01)	0.00 (0.01)
Egg incidence			
Intercept $(\beta_{0E})$	<b>-2.05</b> (0.48)	<b>−0.18</b> (0.09)	- <b>1.25</b> (0.37)
Bacon price $(\beta_{12p})$	-0.01 (0.00)	-0.06 (0.01)	-0.03 (0.01)
Bacon display $(\hat{\beta}_{12d})$	- <b>0.15</b> (0.07)	- <b>0.31</b> (0.07)	-0.22 (0.07)
Bacon feature $(\beta_{12f})$	<b>0.05</b> (0.02)	<b>0.11</b> (0.02)	0.08 (0.02)
Bacon inventory $(\beta_{12i})$	0.00 (0.00)	0.00 (0.01)	0.00 (0.01)
Egg price $(\beta_{22p})$	-0.04(0.01)	-0.01(0.00)	-0.03(0.01)
Egg display $(\beta_{22d})$	<b>1.15</b> (0.09)	<b>0.65</b> (0.08)	<b>0.94</b> (0.09)
Egg feature $(\beta_{22f})$	<b>0.24</b> (0.06)	<b>0.11</b> (0.03)	<b>0.18</b> (0.05)
Egg inventory $(\beta_{22i})$	-0.01(0.00)	-0.01(0.00)	-0.01(0.00)
Support probabilities $(\pi_s)$	0.57	0.43	( /

Note. Significant estimates in bold.

positive). Further, bacon feature has a positive effect on the household's egg incidence decision ( $\beta_{12f}$  = 0.08), which is also consistent with the complementarity explanation. However, display activity in either category is found to have a negative effect on the household's incidence decision in the other category  $(\beta_{21d} = -0.35, \beta_{12d} = -0.22)$ . One rationale for this (as explained in the introduction section) could be that to the extent that display activities trigger impulse (i.e., unplanned) purchasing in a category, budget constraint effects—the household, having bought an unplanned product, then conserves on the shopping budget by cutting back on the other category—make households less likely to buy the complementary category. Last, but not least, we find that the estimated inclusive value parameter is positive ( $\eta = 0.03$ ), which implies that as the household's conditional propensity to jointly purchase two or more units of either product category increases, the household's joint incidence probability for the two categories increases. This makes intuitive sense, and also shows that the household's incidence and quantity outcomes are correlated with each other (which means that they cannot be treated as independent). The two estimated segments are of roughly equal size, with 57% of consumers being in segment 1, and 43% in segment 2. Between the two segments, consumers in segment 1 show higher sensitivity to own marketing variables for both products, higher propensity to jointly purchase the two complementary products, as well as

Table 4 Estimation Results for Quantity Component of Proposed Model

Parameter	Support 1	Support 2	Market-level
Joint two units quantity $( au_{22})$	<b>1.18</b> (0.04)	<b>0.21</b> (0.04)	0.76 (0.04)
Bacon quantity			
Intercept $(\delta_{0B})$	- <b>2.63</b> (0.71)	<b>-2.12</b> (0.45)	- <b>2.41</b> (0.61)
Bacon price $(\delta_{11p})$	-0.02(0.00)	-0.04(0.01)	-0.03(0.01)
Bacon display $(\delta_{11d})$	<b>0.72</b> (0.29)	0.96 (0.28)	0.82 (0.29)
Bacon feature $(\delta_{11f})$	-0.06(0.04)	<b>0.34</b> (0.11)	0.11 (0.08)
Bacon inventory $(\delta_{11i})$	-0.02 (0.01)	-0.01 (0.00)	
Egg price $(\delta_{21p})$	0.14 (0.12)		
Egg display $(\delta_{21d})$	<b>0.48</b> (0.12)	<b>0.10</b> (0.04)	<b>0.32</b> (0.09)
Egg feature $(\delta_{21f})$	-0.10 (0.07)	-0.24 (0.13)	-0.16 (0.10)
Egg inventory $(\delta_{21i})$	0.01 (0.01)	-0.01 (0.00)	0.00 (0.01)
Egg quantity			
Intercept $(\delta_{0F})$	-1.03(0.60)	-0.13(0.12)	-0.64(0.46)
Bacon price $(\delta_{12p})$	-0.13(0.03)	-0.01(0.00)	-0.08(0.02)
Bacon display $(\delta_{12d})$	<b>0.65</b> (0.11)	-0.14(0.17)	<b>0.31</b> (0.14)
Bacon feature $(\delta_{12f})$	-0.22(0.12)	-0.44(0.27)	-0.32(0.20)
Bacon inventory $(\delta_{12i})$	-0.01 (0.00)	0.00 (0.00)	-0.01 (0.00)
Egg price $(\delta_{22p})$	0.04 (0.03)	-0.12 (0.05)	-0.03(0.04)
Egg display $(\delta_{22d})$	<b>1.87</b> (0.42)	<b>1.02</b> (0.37)	<b>1.50</b> (0.40)
Egg feature $(\delta_{22f})$	-0.45 (0.20)	0.04 (0.02)	-0.24 (0.15)
Egg inventory $(\delta_{22i})$	-0.01 (0.00)		-0.01 (0.00)
Support probabilities $(\pi_s)$	0.57	0.43	

Note. Significant estimates in bold.

higher correlation between incidence and quantity outcomes.

Table 4 reports the estimated parameters for the quantity component of our proposed model for the case when both categories are jointly purchased. 10 Interestingly, we uncover significant cross-category covariances in the quantity decisions ( $\tau_{22} = 0.76$ ). In other words, bacon and eggs are not only systematically copurchased by households during the same shopping trip (as observed in Table 3), but also are systematically bought in larger quantities at the same time. This quantity effect is ignored in the multivariate incidence models of Chintagunta and Haldar (1998), Manchanda et al. (1999), Russell and Peterson (2000), and Chib et al. (2002). For bacon quantities, all marketing variables and inventory have significant effects (with the expected signs). For egg quantities, own price and display, as well as inventory, have significant effects (with the expected signs). In terms of cross effects, egg display increases bacon quantities ( $\delta_{21d} = 0.32$ ). Coupled with our earlier finding that egg display has a negative effect on bacon incidence, this indicates an interesting second-order effect, i.e., conditional on purchasing both categories, a household's propensity to buy greater quantities of

bacon increases with the display activities of eggs. Said differently, although the household's unconditional probability of buying bacon is directly decreasing in egg's display activities (on account of  $\beta_{21d}$  < 0 in Table 3), there is an increasing indirect effect through the inclusive value (on account of  $\eta > 0$  in Table 3). Similarly, bacon display increases egg quantities ( $\delta_{12d} = 0.31$ ), which is the opposite of our earlier finding that bacon display decreases egg incidence (note that  $\beta_{12d}$  < 0 in Table 3). The price of bacon decreases egg quantities ( $\delta_{12p} = -0.08$ ), which is consistent with the negative (i.e., complementary) effect of bacon price on egg incidence discussed earlier (see Table 3). The price of eggs, however, has no significant effect on bacon quantities. Between the two segments, consumers in segment 1 show higher propensity to jointly purchase two or more units of the two complementary products, but lower baseline purchase quantities in both products, as well as lowering sensitivity to own marketing variables for bacon.

### 3.1. Managerial Implications

The "Marketing Profit" concept (Chen et al. 1999), is about the own and cross-category profit implications of retail promotional activity. Our methodology allows quantification of the profit implications of the estimated cross-category effects to the retailer. Consequently, we can provide retail managers with answers to what-if questions about the optimality of various promotional offerings. Consider the situation where a retail manager needs to decide promotional spending across bacon and eggs. For example, assume that the promotional option being considered is a price cut of, say, 10%. Is it better for the manager to promote bacon or eggs? We answer this question based on the estimated model parameters. The simulated demand and profits—as well as their elasticity counterparts—for the base case (i.e., both products at their average prices), as well as the two promotional scenarios (i.e., (1) bacon at 10% price promotion, and (2) eggs at 10% price promotion), are presented in Table 5. For these computations, we assume a market size of 10,000 consumers and that retail pass-through

Table 5 Price Promotion Simulation Results

Simulated quantity	Base case	10% price cut bacon only	10% price cut egg only
Bacon demand (units)	1,263	1,325	1,265
Egg demand (units)	2,360	2,368	2,402
Bacon profits (\$)	1,840	1,930	1,843
Egg profits (\$)	1,372	1,377	1,397
Bacon profit elasticity	0	-0.4887	-0.0133
Egg profit elasticity	0	-0.0325	-0.1785
Total profits (\$) Total profit elasticity	3,213 0	3,307 -0.2938	3,240 -0.0839

<sup>&</sup>lt;sup>10</sup> To conserve space, we have suppressed the estimates of the quantity component of our proposed model for the case when only one category is bought (see Equations (11) and (12)). All the coefficients for this case are consistent, in sign, with those reported in Table 4 and are available from the authors.

is 100%, i.e., a retail price cut is in response to a retail cost (i.e., wholesale price) decrease of the same magnitude.<sup>11</sup>

It is observed in Table 5 that the price elasticity of either product's profit is higher in response to a price cut on the same product rather than on the other product. For example, bacon (eggs) has an own-price profit elasticity of 0.49 (0.18) versus a crossprice profit elasticity of 0.01 (0.03). This finding makes intuitive sense because one would expect a product's profit to respond more to its own price than to the price of a complementary product. In terms of retail profits, we uncover an interesting asymmetry: The total retail profit responds more to bacon price than to egg *price.* In other words, the total retail profit increases by 0.29% for a 1% price cut on bacon, whereas it increases only by 0.08% for a 1% price cut on eggs. This suggests that the retailer is better off promoting bacon than egg. A 1% price cut on bacon translates to a 0.49% profit gain for bacon and a 0.03% profit gain for eggs (on account of complementarity effects), whereas a 1% price cut on eggs translates to a 0.18% profit gain for eggs and a 0.01% profit gain for bacon. Because bacon brands are procured by retailers from national manufacturers (who typically have trade deal schedules for their brands), in contrast to egg brands that are typically purchased from local suppliers on account of perishability and breakage concerns, our finding about the relative efficacy of bacon as a product category suitable for retail price promotion will be practically useful to the retailer from the standpoint of planning their retail promotion calendar in advance for the coming year.

Next, we quantify the incremental value of the quantity component of our proposed model. We do this by assuming that only incidence outcomes matter (and, therefore, using Equation (5) for simulation purposes, and ignoring Equations (10)–(12)), and assuming that the conditional purchase quantity at a purchase occasion is equal to the modal value of the observed purchase quantities in the data, i.e., one unit. This generates the effects summarized in Table 6. The quantity effects can then be deduced by subtracting the effects in Table 6 from those in Table 5.

By using Table 6 as a comparison, one can decompose the estimated demand and profits given in Table 5 (under each scenario) into incidence and

Table 6 Price Promotion Simulation Results—Incidence Effects

Simulated quantity	Base case	10% price cut bacon only	10% price cut egg only
Bacon demand (units)	961	1,008	962
Egg demand (units)	1,872	1,878	1,886
Bacon profits (\$)	1,400	1,469	1,402
Egg profits (\$)	1,089	1,092	1,097
Bacon profit elasticity	0	-0.4917	-0.0100
Egg profit elasticity	0	-0.0340	-0.0744
Total profits (\$) Total profit elasticity	2,489 0	2,561 0.2915	2,498 -0.0381

quantity components. As a consequence, one can also decompose the *incremental* profits obtained going from the base case to either promotional scenario in Table 5 into incidence and quantity components. The results of this decomposition are given in Table 7. We can see that 23% of the increase in total retail profits from promoting bacon is on account of quantity effects, whereas 67% of the increase in total retail profits from promoting eggs is on account of quantity effects. This underscores the importance of accounting for quantity effects in multicategory models of consumer choice behavior. More interestingly, 40% (33%) of the increase in egg (bacon) profit from promoting bacon (eggs) is because of quantity effects. This underscores the importance of accounting for cross-category complementarity effects in quantity outcomes.

We decompose the estimated cross-category complementarity effects into (a) marketing-mix complementarity effects (i.e., the effects of  $\beta_{12}$ ,  $\beta_{21}$ ,  $\delta_{12}$ , and  $\delta_{21}$ ), versus (b) intrinsic complementarity effects (i.e., the effects of  $\gamma_{11}$  and  $\tau_{22}$ ). We find that 85% of the estimated cross-category complementarity effects are due to the effects of marketing-mix complementarity, whereas only 15% are due to the effects of intrinsic complementarity between products. In other words, consumers appear to be much more likely to buy the two products together when at least one is on retail promotion than when neither is on promotion.

### 4. Conclusions

It is widely accepted that consumers make multicategory purchase decisions across a variety of shopping contexts. The academic literature in marketing has recently begun to pay attention to this aspect

Table 7 Price Promotion Simulation Results—Percentage of Profit Increase Due to Quantity Effects

Simulated quantity	10% price cut bacon only (%)	10% price cut egg only (%)
Bacon profits	23	33
Egg profits	40	68
Total profits	23	67

<sup>&</sup>lt;sup>11</sup> We also undertook a dynamic profit analysis over multiple periods, taking into account the estimated inventory effects and, therefore, their impact on product demand in future periods. Because the estimated inventory effects were of little to no substantive significance, such a dynamic analysis yielded results that were identical to the results reported here. We thank the area editor for alerting us to check this issue.

<sup>&</sup>lt;sup>12</sup> Even if we use the mean value of the observed purchase quantities in the data, which is 1.2 units, the results remain similar.

of consumer behavior by developing and estimating multicategory incidence models. We contribute to this literature by extending multicategory incidence models to handle quantity decisions. Using scanner panel data on bacon and eggs, we estimate strong crosscategory associations in both incidence and quantity outcomes. We assess the impact of promotional spillovers across categories, and find that promoting bacon is more profitable to the retailer than promoting eggs. We find that 23% and 67%, respectively, of the multicategory profit impact of a price promotion on bacon and eggs arises because of quantity effects. We also find that 40% and 33%, respectively, of crosscategory profit spillovers of price promotions arise on account of quantity effects. This underscores the importance of specifying and estimating not only incidence outcomes—as in Manchanda et al. (1999)—but also quantity outcomes in multicategory models of consumer choice behavior.

By employing a flexible statistical model of a household's conditional quantity outcomes across product categories (i.e., BVL when both categories are purchased, and two separate BNLs when just one category is purchased), our approach is able to estimate a household's quantity decisions without imposing (untested) restrictions on the data. However, a few caveats are in order:13 (1) By ignoring brand choice outcomes, we have to impute the marketing variables of brands using a household-specific share-weighted average across all available brands. Including brand choice outcomes would avoid our having to undertake this imputation procedure (because the consumer's utility for each brand will be a function of that brand's marketing variables, and the consumer's composite utility for the product category would then be a function of these brand-specific utilities). (2) By ignoring the effects of observed characteristics of households—such as family size, average consumption rate, etc.—on purchasing behavior, our model may be underrepresenting the effects of heterogeneity across households. Therefore, it is possible that we may be underestimating the effects of marketing-mix variables on households' incidence and quantity decisions. (3) Although our two-category analysis indicates that promoting bacon is more profitable to the retailer than promoting eggs, it is possible that including other categories (that complement eggs, but not bacon) in our analysis may reveal that promoting eggs is very profitable to the retailer. (4) Ignoring possibly correlated error terms across categories that capture the effects of, say, unexpected arrival of guests on households' propensities to buy both product categories may be understating the extent of cross-category effects estimated using our model.

(5) Our model, by including an inclusive value variable in the incidence specification to provide a link between incidence and quantity outcomes, implies that if the covariates at a purchase occasion make increased quantity buying more likely, these covariates also make incidence more likely. It may be worthwhile to investigate the sensitivity of our results to the inclusion of an inclusive value variable in the quantity specification instead.

In light of the above discussion, we believe that there are some interesting directions for future research: First, it will be useful to extend the proposed model to handle brand choice outcomes and/or store choice outcomes (Bodapati and Srinivasan 2006); second, it will be instructive to develop, and then compare to our proposed model, a multicategory model of incidence and quantity outcomes that is based on product attributes (see Chung and Rao 2003 for an example of an attribute-based multicategory incidence model); third, it is necessary to extend our empirical application to more than two categories—in addition to bacon and eggs—to see if our substantive findings about the relative efficacy of promoting bacon and eggs remain the same; fourth, it will be useful to investigate whether a theory-driven modeling approach (such as Song and Chintagunta 2007) that imposes parametric restrictions based on economic theory can be used to obtain a more parsimonious model of such multicategory decision making. Such a theory-driven approach can also be used to incorporate the effects of flexible consumption and forward buying in households' quantity decisions (see Seetharaman et al. 2005 for a discussion about the benefits of statistical over theory-driven models in the multicategory context); fifth, it will be interesting to understand the cross-category effects of product assortment decisions of the retailer (see Borle et al. 2005 for a study of within-category effects of assortment cuts).

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