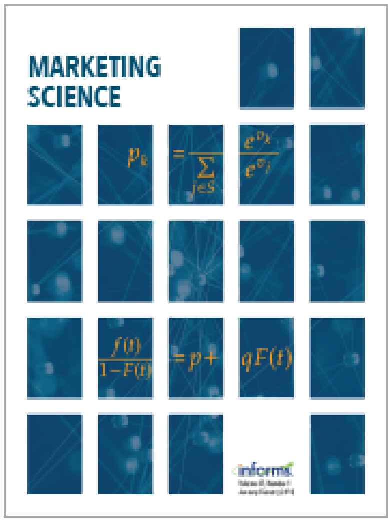

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Position Auctions with Budget Constraints: Implications for Advertisers and Publishers

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This paper examines position auctions with budget-constrained advertisers, a dominant bidding environment used by publishers to allocate positions in online advertising. Budget constraints play a crucial role in equilibrium bidding by inducing advertisers to strategically deplete a higher-ranked advertiser's budget to gain in rank. This strategic consideration has consequences for the advertisers' profits and the publisher's revenue. An advertiser's profit can strictly decrease with her budget when competition for an advertising space (e.g., a keyword) is intense. The publisher's revenue can also strictly decrease when an increase in the higher-ranked advertiser's budget induces the lower-ranked rival to reduce her bid, due to her inability to deplete the higher-ranked advertiser's budget. Several managerial implications for advertisers and publishers are discussed.

Keywords: position auctions; generalized second-price auctions; budget constraints; Internet marketing; online advertising; game theory

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1. Introduction

The position auction, typically a *generalized second-price* auction, is the dominant format used by online *publishers,* such as search engines (e.g., Google, Yahoo, and Bing), social media (e.g., Facebook and LinkedIn), and media websites (e.g., CNN.com and Fox.com), to allocate advertising space in ranked listings. Advertisers are ranked in ad listings according to their bids; the highest bids receive the best ranks. Advertisers also pay a cost-per-click (CPC), which is determined by the bid of the subsequently ranked advertiser. An advertiser's total online advertising cost, however, depends not just on the CPC, but also on the volume of clicks made throughout the duration of the listing.

In these online bidding environments, which can generate thousands of clicks on high-traffic publishers, marketers can inadvertently spend more than they budgeted for online advertising. To help marketers control their ad spending, publishers require every advertiser to indicate the daily budget for each auction. This budget is the maximum amount of money the advertiser can spend in one day on a given position auction. If an advertiser's daily budget is depleted before the end of the day, she is removed from the listing and all lower-ranked advertisers move up one position.

Any advertiser who ascends in position correspondingly sees an improvement in her click-through rate (CTR), but still pays her original CPC. Theoretically, this feature may induce an advertiser to strategically bid in a way that raises the cost of the advertiser ranked just above her, deplete the above-ranked advertiser's budget, and subsequently move into a higher slot. Indeed, according to industry (Stokes 2010) and academic (e.g., Lahaie 2006, Ganchev et al. 2007, Zhou and Lukose 2007, Iyengar et al. 2007, Bu et al. 2010, and Zhang and Feng 2011) accounts, some advertisers bid aggressively with the explicit aim of depleting the budget of higher-ranked advertisers. Thus, in a position auction with budget-constrained advertisers, there is a fundamental connection between an advertiser's choice of bid and the budget constraints of rival advertisers. This connection, between budgets and equilibrium bids, raises two questions about the impact of advertisers' budgets on their profits and on the publisher's revenue.

First, how does the size of an advertiser's budget affect its profit? Intuitively, a larger budget should bring more clicks and therefore higher profits. Yet this intuition can break down due to the connection between budgets and equilibrium bids mentioned above. In fact, an advertiser's bid can, in equilibrium, be a function of her rival's budget. To illustrate, suppose advertiser i is ranked just above advertiser j. If i's budget is insufficient to remain in a higher position, then j can target her bid to the budget of i to ensure that she eventually drives i out of the ad listing. In this case, advertiser i's CPC, which equals advertiser j's bid, increases in i's budget. Advertiser i's profit can, therefore, decrease as her budget increases. We call this situation a *budget trap* and identify the conditions under which it occurs.

The second question we address is: How do advertisers' budgets affect the publisher's revenue? We find that, in some situations, an advertiser with a larger budget will spend (weakly) more in the auction, which leads to (weakly) more revenue for the publisher. Yet in other situations, the opposite can occur. As noted above, an advertiser may bid aggressively to deplete a small daily budget of a higher-ranked advertiser. However, if the higher-ranked advertiser's budget increases beyond a certain level, the lower-ranked advertiser will relax her bid because she has no chance of ascending in position. This suggests that an increase in an advertiser's budget can actually lower her CPC in equilibrium, thereby reducing the publisher's revenue. This result is important in light of Google's practice of offering credit services to help small- and medium-sized advertisers expand their budgets.[1]

This article adds to the growing literature on position auctions. Previous analytical studies have examined various topics such as firms' bidding strategies (Edelman et al. 2007, Varian 2007, Börgers et al. 2013), the interplay between organic and sponsored links (Katona and Sarvary 2010), linking consumers' search patterns to advertisers' bidding strategies (Chen and He 2011, Athey and Ellison 2011), the impact of uncertainty on bids (Athey and Nekipelov 2012), new pricing metrics (Zhu and Wilbur 2011, Dellarocas 2012, Liu and Viswanathan 2014), fraudulent behavior (Wilbur and Zhu 2009), the effect of quality on advertising performance (Even-Dar et al. 2007, Jerath et al. 2011, Feng and Xie 2012), the integration of advertising auctions and price competition (Xu et al. 2011), buying a competitor's keyword (Desai et al. 2014), and cyclical bidding patterns (Zhang and Feng 2011). There is also a burgeoning empirical literature on paid-search advertising, which is often sold through position auctions (e.g., Ghose and Yang 2009, Rutz and Bucklin 2011, Yao and Mela 2011, Joo et al. 2014, Yang et al. 2014, Abhishek et al. 2015, Jeziorski and Segal 2015). A common feature of these studies is that they did not account for firms' budget constraints in position auctions. By introducing budget constraints into a comprehensive equilibrium analysis, we find significant strategic considerations for advertisers' bidding decisions. To our knowledge, these considerations have not been identified in earlier work.

Recent research has recognized the implications of budget constraints for strategies in position auctions.[2,3] One of the first articles to introduce budget constraints for advertisers in position auctions is Abhishek and Hosanagar (2013). This article is aimed at helping advertisers determine an optimal bid strategy across multiple keywords for a given budget. Sayedi et al. (2014) studied advertisers' poaching behavior between online and offline channels under a fixed advertising budget. Amaldoss et al. (2015) found that search engines can benefit from the first-page bid estimate when a high-value advertiser has a limited budget. In contrast to the above-mentioned research, we examine how the depletion of daily budgets affects an advertiser's exit from the listing as well as the corresponding aspects of strategic bidding.

Our article is closest to Shin (2015), who independently examines the strategic aspects of budget constraints in position auctions. However, our study differs from Shin (2015) in terms of research focus and modeling approach. Shin's (2015) objective is to understand how the presence of budget constraints implies differences in positions and bidding behavior found in the previous literature. He examines a model with multiple advertisers and shows that advertisers can actually bid above their value-per-click in a variety of situations. By contrast, our research focuses on how advertisers' budget constraints affect their profits and the publisher's revenue. Therefore, we study a model with two strategic advertisers who are budget-constrained, and characterize the equilibrium across a full range of budgets.

[1] Exclusive: Google offers credit card to advertisers. *Reuters*. http://www.reuters.com/article/2011/07/20/us-google-creditcard-idUSTRE76J6XU20110720, accessed March 2015.

[2] The auction theory literature in economics has considered the strategic implications of budget constraints for classic single-unit auctions (Che and Gale 1998) and sequential, multi-unit auctions (Benoît and Krishna 2001). An important difference from these two articles is that our work focuses on the position auction, which has been shown to have distinctive strategic properties relative to these traditional auction settings (e.g., Varian 2007).

[3] The mechanism design literature has assessed the efficiency of various auction mechanisms used for online advertising (e.g., Borgs et al. 2005, Chen et al. 2009, Fiat et al. 2011, Dobzinski et al. 2012). Our research differs from this literature by taking the allocation mechanism as fixed and studying the position auctions as currently practiced.

The rest of this article is organized as follows: §2 describes the model. Section 3 presents the bidding-equilibrium analysis, the impact of budgets on advertiser profits and publisher revenue, and the discussion of model extensions. Section 4 concludes our paper. Proofs of all lemmas, propositions, and supplementary analyses are provided in the online appendix (available as supplemental material at http://dx.doi.org/10.1287/mksc.2015.0931).

2. The Model

Our model is a simultaneous-move static game of complete information played by two budget-constrained bidders in a position auction. We model two focal advertisers competing for two ad positions (i.e., "slots") sold by a publisher. We allow advertisers to be different in two dimensions: (π_k, B_k), where π_k is the value-per-click and B_k is the daily budget constraint defined as the daily spending limit in the position auction for advertiser $k = i, j$. Budgets and values-per-click are exogenous and common knowledge to reflect the high level of competitive information available to advertisers. In §3.4 we relax the exogenous budget assumption by discussing a case wherein advertisers choose budgets before bidding in the auction.

The flow rate of clicks from consumers is assumed to be constant. Without loss of generality, we normalize the total daily click volume for the second position to be one and denote the click ratio between the first and second position as σ. We assume $\sigma > 1$ to reflect the positive effect of a higher rank on CTR (Ghose and Yang 2009, Agarwal et al. 2011).

Each advertiser chooses her bid $b_k \in (0, +\infty)$, $k = i, j$. Henceforth, we assume that $\pi_i > \pi_j$ to reflect potential differences in the advertisers' values-per-click. The advertiser with the highest bid initially wins the first position and pays the second-highest bid, whereas the second advertiser pays a reservation value r per-click.[4] The reservation value can be interpreted as the highest bid among a set of nonstrategic, lower-ranked advertisers. In this way, the level of r captures the intensity of external competition among other advertisers for this particular advertising space (e.g., a keyword). We assume that both advertisers' values-per-click are sufficiently large that they have incentives to bid in the auction: $\pi_i > \pi_j > r$.

Depending on the size of an advertiser's budget, the volume of clicks can deplete the budget before the end of the day. An advertiser whose budget has been depleted is removed from the listing. If the advertiser who was initially awarded the first position has her budget depleted, the second advertiser moves up to the first position, receives the improved CTR of σ, and pays her original CPC, which is r. Meanwhile, we suppose there is an unranked advertiser, z, filling the second position and paying the remaining click volume at a CPC of ρr with $\rho \in [0, 1)$.[5] For simplicity, we assume this unranked advertiser z is nonstrategic and always bids r. See §3.4 for a discussion of the case with three strategic bidders.

Define t_i, $t_j \le 1$ as the portion of the day the budget of advertiser i or j, respectively, remains unexhausted. Whether an advertiser's budget is exhausted (either or both t_i, $t_j < 1$) and the order of their departure (i.e., $t_i \gtrless t_j$) are determined by both advertisers' budgets and bids. Table 1 presents the process of ranked listing when advertiser i wins the first position under two scenarios: (i) Neither i nor j has her budget depleted ($t_i = t_j = 1$); and (ii) Only i has her budget depleted ($t_i = B_i/(\sigma b_j) < t_j = 1$).

We characterize advertisers' payoffs in this one-shot game as functions of their bids (b_i, b_j) given the exogenous parameters $\{\sigma, r, (\pi_i, B_i), (\pi_j, B_j)\}$.

When $b_i \ge b_j$,

$$\Pi_i(b_i, b_j) = \begin{cases} \sigma(\pi_i - b_j)t_j + \sigma(\pi_i - r)(t_i - t_j) & \text{if } t_i \ge t_j, \\ \sigma(\pi_i - b_j)t_i & \text{if } t_i < t_j \end{cases} \quad (1)$$

When $b_i < b_j$,

$$\Pi_i(b_i, b_j) = \begin{cases} (\pi_i - r)t_j + \sigma(\pi_i - r)(t_i - t_j) & \text{if } t_i \ge t_j, \\ (\pi_i - r)t_i & \text{if } t_i < t_j \end{cases} \quad (2)$$

where Π_i stands for daily advertising return (Table A2 in the online appendix provides a detailed characterization of each advertiser's profit). Suppose advertiser i is initially awarded the first position ($b_i > b_j$). She receives σt_i clicks during her time in the listing. For each click, she receives her value-per-click π_i less her CPC, which is initially the second advertiser's bid, b_j. If the second advertiser's budget is depleted before the first advertiser's ($t_j < t_i \le 1$), the first advertiser's CPC reduces to r for the remainder of her time in the first position ($t_i - t_j$). Otherwise, her CPC is b_j for the duration, t_i.

[4] Some publishers rank advertisers by the product of advertisers' bids and quality scores. In position auctions with quality scores, we can reinterpret the reservation price r as the highest weighted bid of an unranked advertiser. We show in Claim 2 in the online appendix that our results are robust to the consideration of quality scores under the complete information assumption.

[5] The assumption that $0 \le \rho r < r$ captures the fact that another unmodeled advertiser rising to the second position has a CPC less than her bid of r. Because the parameter ρ has no effect on focal advertisers' payoffs, it does not affect the equilibrium bids of advertiser i and j. It also does not affect our results.

Table 1 Example of Positions and Payments When $b_i > b_j$ and $t_j = 1$

Non-depletion ($t_i = 1$)			Depletion of bidder i's budget ($t_i < 1$)			
Bidder	Position	Payment	Bidder	Position before depletion	Position after depletion	Payment
i	1	σb_j	i	1	Unranked	$t_i \sigma b_j$
j	2	r	j	2	1	$t_i r + (1 - t_i)\sigma r$
z	Unranked	0	z	Unranked	2	$(1 - t_i)\rho r$

Alternatively, suppose advertiser i is initially in the second position ($b_i < b_j$). For each click she receives π_i less her CPC, which is r for her entire time in the listing. If the first advertiser's budget is not exhausted before the end of the day ($t_j = \min\{B_j/(\sigma b_i), 1\} = 1 \geq t_i$), advertiser i receives a total of t_i clicks. Otherwise, if $t_j < t_i$, advertiser i occupies the second position, receives t_j clicks, and ascends to the first position at time t_j, where she receives $\sigma(t_i - t_j)$ clicks.

Like many models of position auctions, our game has multiple Nash equilibria. To select the most intuitive equilibrium, we use the notion of an *Undominated Nash Equilibrium* (UNE).[6] A pair of bids $(\bar{b}_i, \bar{b}_j)$ is a UNE if it is a Nash equilibrium and neither $\bar{b}_i$ nor $\bar{b}_j$ is a weakly dominated strategy.

3. Equilibrium Analysis

We derive our main results by studying the equilibrium outcomes of the game described above. Our derivation relies on an important implication of our equilibrium concept: For each $(B_i, B_j) \in \Theta \equiv \{(B_i, B_j) \in \mathbb{R}^2 \mid B_i, B_j > 0\}$, there exists a *unique* UNE outcome defined by (i) the ranking, and (ii) the CPC of both advertisers throughout the duration of the listing.[7]

Sketch of the equilibrium derivation: We use a series of indifference conditions that define advertisers' preferences over the two positions to solve for the UNE. For each $(B_i, B_j) \in \Theta$, the indifference condition is characterized by an advertiser's *willingness-to-pay* (per click), which is defined as a CPC such that the advertiser is indifferent between initially staying at the first and second positions. We then partition the whole budget space Θ into smaller regions based on the corresponding *budget thresholds*, defined as the smallest budget that is inexhaustible when paying the willingness-to-pay (per click) at the first position. Under such a partition, an advertiser's profit takes the same functional form in each region. This allows us to characterize the UNE in three steps: (1) Characterize the best-response correspondences of advertisers; (2) Eliminate weakly dominated bids; (3) Find the UNE as the intersection of the best-response correspondences. Because the detailed derivation of the UNE in the whole Θ is quite involved, it is provided in the online appendix. Nevertheless, as Lemmas 1 and 2 will show, the set of all equilibrium outcomes can be categorized into four types. This allows us to partition Θ into four regions, each of which has the same type of equilibrium outcome.

The first equilibrium outcome is the *budget-independent equilibrium* (*BI*). The set of budget-pairs (B_i, B_j) leading to the *BI* equilibrium is defined as $\Theta_{BI} = \Theta_{BI}^L \cup \Theta_{BI}^S \subset \Theta$. Budgets in Θ_{BI} are either sufficiently large (Θ_{BI}^L) or at least one advertiser's budget is so small (Θ_{BI}^S) that budgets play no strategic role in advertisers' bidding decisions.[8] Henceforth, we ignore discussion of this region due to the lack of interesting interactions among advertisers. Please refer to the proof of Lemmas 1 and 2 in Section A of the online appendix for a formal definition of Θ_{BI} and all other regions.

In regions $\Theta\backslash\Theta_{BI}$, there are three other types of equilibrium outcomes, which we call Type-I, Type-II, and Type-III. We superscript $\Theta\backslash\Theta_{BI}$ by the advertiser who is initially assigned the first position. We define and characterize these equilibrium outcomes in §§3.1 and 3.2. These outcomes depend on the level of the reservation value r. The full characterization of the UNE in Θ is reported in Table A4 of the online appendix. In §3.3, we present the main results implied by the equilibrium analysis. Finally, we discuss the generality of our main results in §3.4 by considering two model extensions.

3.1. Bidding Equilibrium with a Low Reservation Value

When the reservation value is relatively small, $r < \min\{B_i/\sigma, B_j/\sigma\}$, advertisers initially in the second position can afford the highest possible daily expenses σr. This implies that only the first advertiser may have her budget depleted before the end of the day. In this case, the second advertiser may bid one penny

[6] The UNE is an equilibrium concept established in Palfrey and Srivastava (1991) and has been extensively used in previous auction literature (e.g., Benoît and Krishna 2001, Börgers et al. 2013).

[7] The UNE bids are unique up to the first advertiser's bid. In certain budget regions, there is a range of the first advertiser's bids that satisfy the UNE conditions.

[8] When budgets are in Θ_{BI}^L, the equilibrium outcome is the same as if advertisers have unlimited budgets. When budgets are in Θ_{BI}^S, the second-ranked advertiser's budget is so small that she bids the reservation value r.

below the first advertiser's bid to accelerate the budget exhaustion. When this occurs, we call it a *Type*-I equilibrium. It is also possible that the first advertiser's budget is not depleted in equilibrium yet both advertisers' bids are a function of budgets. We call this a *Type*-II equilibrium. Lemma 1 provides a characterization of these two equilibrium outcomes.

Lemma 1. *If $r < \min\{B_i/\sigma, B_j/\sigma\}$, there are two classes of equilibrium outcomes in addition to the budget-independent equilibrium. Let index k denote the advertiser initially in the first position, and $\epsilon > 0$ be a "penny", i.e., the smallest unit of payment.*

(i) *Type*-I: *Let $\Theta_{\mathrm{I}} = \Theta_{\mathrm{I}}^i \cup \Theta_{\mathrm{I}}^j$ be the set of budget pairs (B_i, B_j) leading to a Type-I equilibrium. If $(B_i, B_j) \in \Theta_{\mathrm{I}}^k$, then $\bar{b}_k^{\mathrm{I}} > \bar{b}_{-k}^{\mathrm{I}} = \bar{b}_k^{\mathrm{I}} - \epsilon$. Advertiser k's budget is depleted.*

(ii) *Type*-II: *Let $\Theta_{\mathrm{II}} = \Theta_{\mathrm{II}}^i \cup \Theta_{\mathrm{II}}^j$ be the set of budget pairs (B_i, B_j) leading to a Type-II equilibrium. If $(B_i, B_j) \in \Theta_{\mathrm{II}}^k$, then $\bar{b}_k^{\mathrm{II}} > \bar{b}_{-k}^{\mathrm{II}}$ and $\bar{b}_{-k}^{\mathrm{II}} = \pi_{-k}B_{-k}/(\pi_{-k} + B_{-k} - r) < w_{-k}^*$, where w_{-k}^* is the equilibrium bid without budget constraints.*[9] *Advertiser k's budget is not depleted.*

Figure 1(a) graphically depicts the equilibrium regions of Lemma 1. In region Θ_{I}, the high-value or low-value advertiser might take the second position and bid one penny less than the first advertiser, depending on their relative budget sizes. The Type-I equilibrium occurs under one of the following two conditions: (i) The discrepancy in budgets is small so that the competition for the first position is intense (the lower portion of Θ_{I}^i and all of Θ_{I}^j); (ii) The low-value advertiser j has a larger budget and the high-value advertiser i's budget is moderate (the upper portion of Θ_{I}^i). In the latter case, advertiser i has a higher willingness-to-pay (per click) than advertiser j. This causes advertiser i to outbid j and initially stay at the first position. Furthermore, because i's budget is not too large, advertiser j can bid one penny less than i to quickly deplete i's budget and then ascend to the first position.

The Type-II equilibrium occurs for $(B_i, B_j) \in \Theta_{\mathrm{II}}$. As shown in Figure 1(a), one advertiser's budget is sufficiently larger than the other's in this region. Consequently, the first-ranked advertiser's budget is inexhaustible in equilibrium because of her significant budget advantage. On the other hand, because of the second advertiser's relatively small budget, she shades her bid below w^*, i.e., her willingness-to-pay (per click) without a budget constraint. Specifically, if $(B_i, B_j) \in \Theta_{\mathrm{II}}^i$, then advertiser j takes the second position with the bid $\bar{b}_j = \pi_j B_j/(\pi_j + B_j - r) < \bar{b}_i - \epsilon$ to eliminate the incentive of advertiser i to bid one penny less than $\bar{b}_j$ and deplete j's budget. Therefore, in equilibrium, advertiser j's bid must equate her profit at the second position, $\pi_j - r$, with that of initially obtaining the first position and having her budget depleted at time t_j: $\sigma\pi_j t_j - B_j$, where $t_j = B_j/(\sigma\bar{b}_j)$. Note that the second advertiser's bid is always below w^* in Θ_{II} because the potential threat of budget exhaustion drives her to bid cautiously. By bidding conservatively, the second advertiser provides the incentive for the large-budget advertiser to stay at the first position due to the lower CPC. In other words, the large-budget advertiser's willingness-to-pay (per click) is greater than that of the other advertiser; she therefore outbids the small-budget advertiser. This can happen even if the large-budget advertiser has a lower value-per-click. Jerath et al. (2011) call this a "position paradox."

3.2. Bidding Equilibrium with a High Reservation Value

We now investigate the instance wherein the reservation value is large, $r \geq \min\{B_i/\sigma, B_j/\sigma\}$. In this case, the external competition in the position auction is so intense that at least one advertiser will drop from the listing before the end of the day. Furthermore, due to the larger reservation value, or equivalently, the higher CPC paid by the second advertiser, the second advertiser might not have enough remaining budget to digest the additional click volume should she ascend to the first position.

The potential budget exhaustion by the reservation value gives rise to a new Type-III equilibrium. Under this new equilibrium the initially second-ranked advertiser bids proportionally to the first-ranked advertiser's budget to deplete it at a slower rate than in the Type-I equilibrium. We denote the budget region associated with the Type-III equilibrium as Θ_{III}. Lemma 2 describes the Type-III equilibrium outcomes and Figure 1(b) depicts all equilibrium regimes for the case of a high reservation value.

Lemma 2. *Let $r \geq \min\{B_i/\sigma, B_j/\sigma\}$ and $(B_i, B_j) \notin \Theta_{BI}^S$. In addition to Type-I and Type-II, there is a Type-III equilibrium outcome. Let index k denote the advertiser initially in the first position.*

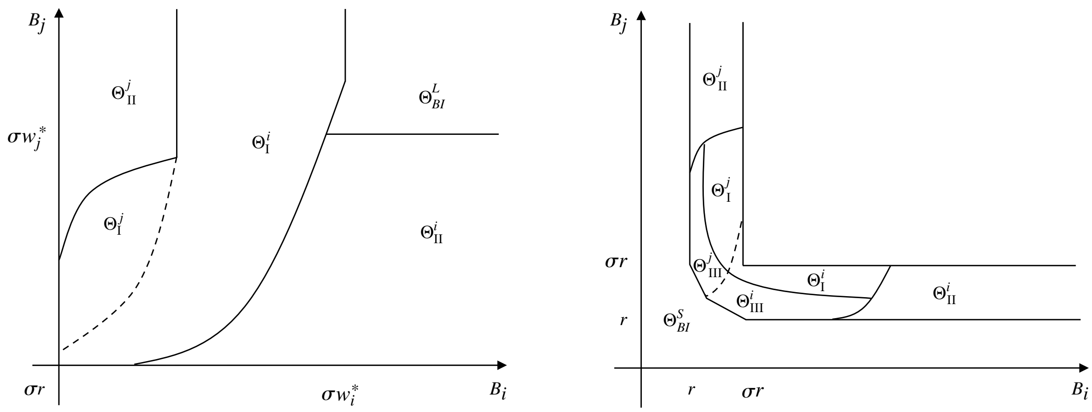
Type-III: *Let $\Theta_{\mathrm{III}} = \Theta_{\mathrm{III}}^i \cup \Theta_{\mathrm{III}}^j$ be the set of budget pairs (B_i, B_j) leading to a Type-III equilibrium. If $(B_i, B_j) \in \Theta_{\mathrm{III}}^k$ then $\bar{b}_k^{\mathrm{III}} > \bar{b}_{-k}^{\mathrm{III}} = (\sigma - 1)rB_k/(\sigma(\sigma r - B_{-k}))$. Advertiser k's budget is depleted before the end of the day and advertiser $-k$'s budget is depleted exactly at the end of the day.*

The Type-III equilibrium requires two conditions, which can be seen in Figure 1(b): (i) The first-ranked advertiser has a relatively small, but inexhaustible, budget; (ii) The second-ranked advertiser's budget is sufficiently small that she can only afford a fraction

[9] When both advertisers have unlimited budgets, our model reduces to a standard second-price auction with each advertiser k bidding her willingness-to-pay (per click), which equals $w_k^* = (1 - 1/\sigma)\pi_k + r/\sigma$. In Claim 1 in the online appendix, we prove that w_k^* is the unique equilibrium bid.

Figure 1 **Budget Regions with Different Bidding Equilibrium**

(a) Low reservation value (b) High reservation value



of the total click volume in the first position by paying r per click on ascension. By contrast to the Type-I equilibrium, the second advertiser in a Type-III equilibrium does not quickly drive out the first advertiser by bidding higher. This is because doing so brings no extra benefit to the second advertiser and provides the rival with the incentive to undercut her bid by one penny and deplete her budget. To reduce such an incentive, the second advertiser will choose the lowest bid among a set of bids that can achieve maximum profit. This suggests that the second advertiser bids proportionally to the first-ranked advertiser's budget so that the second advertiser's budget is depleted exactly at the end of the period (see a more detailed explanation along with a numerical example in Section A in the online appendix).

3.3. Impact of Budgets on Advertiser Profits and Publisher Revenue

Based on the results from Lemmas 1 and 2, we now examine the impact of budgets on advertisers' profits and publisher's revenue. Formal expressions of each party's payoff are provided in Table A5 in the online appendix.

Proposition 1. *If* $r \geq \min\{B_i/\sigma, B_j/\sigma\}$ *and* $(B_i, B_j) \in \Theta_{\mathrm{III}}$, *the first advertiser's profit strictly decreases with her budget. Otherwise, each advertiser's profit weakly increases with her budget.*

We find that an advertiser's profit always weakly increases with her budget except in Θ_{III}, where the external competition is intense (i.e., $r \geq \min\{B_i/\sigma, B_j/\sigma\}$). To illustrate this intuition, suppose $(B_i, B_j) \in \Theta^i_{\mathrm{III}}$. According to Lemma 2, advertiser j bids proportionally to advertiser i's budget and initially occupies the second position. By bidding proportionally to B_i, advertiser j depletes advertiser i's budget at a specific time point so that her remaining budget can afford the increased CTR from the time of ascension to the end of the day. This equilibrium bid choice of advertiser j links advertiser i's budget directly to advertiser i's CPC in equilibrium. In other words, advertiser i's CPC increases with her own budget, but her total click volume, $B_i/\bar{b}_j^{\mathrm{III}} = \sigma(\sigma r - B_j)/(\sigma - 1)r$, does not. In this scenario, what we call a budget trap, an increase in the first advertiser's budget results only in a higher CPC and, therefore, a lower profit.

Outside of Θ_{III}, a larger daily budget brings an advertiser advantages in the position auction in two situations. First, for the second-ranked advertiser, a larger budget affords her the ability to bid more aggressively to exhaust the above-ranked competitor's budget and ascend in position sooner. Second, when the first-ranked advertiser's budget exceeds a threshold, the below-ranked competitor will shade her bid, and consequently reduce the first advertiser's CPC.

Next we summarize our finding of the impact of budgets on the publisher's revenue.

Proposition 2. *The publisher's revenue weakly increases with an advertiser's budget except in the following two scenarios*:

(i) *Change in equilibrium type*: *An increase in the first-ranked advertiser's budget causes the equilibrium to change from Type-*I *equilibrium to Type-*II *or BI equilibrium.*

(ii) *Change in ad position*: *An increase in the high-value advertiser's budget,* B_i, *causes her initial equilibrium position to change from the second to the first.*

Intuitively, an increase in an advertiser's budget should always benefit the publisher: For a fixed CPC, a larger budget means a larger amount of clicks an advertiser may receive, thereby transferring more surplus from the advertiser to the publisher. However, after accounting for the strategic impact of budgets on advertisers' bid choices, a counterintuitive result arises: The publisher's revenue can strictly *decrease* with an advertiser's budget. As indicated in Proposition 2, this can happen in two scenarios. The first is when the first advertiser has a more-than-necessary daily budget. In this case, the first advertiser's budget is no longer exhaustible even if the second advertiser bids one penny less than her bid. As a response, the second advertiser shades her bid in equilibrium. This reduces the first advertiser's payment and therefore hurts the publisher's revenue. Figure 1 illustrates this situation in which an increase in either advertiser's budget causes a switch from Θ_{I} into Θ_{II} or Θ_{BI}^{L}.

The second scenario is when an increase in the high-value advertiser's budget induces her to bid for the first position despite a disadvantaged budget. This corresponds to the case wherein an increase in B_i causes a switch from Θ_{I}^{j} to Θ_{I}^{i} (for any r), from Θ_{II}^{j} to Θ_{I}^{i} (only for $r < \min\{B_i/\sigma, B_j/\sigma\}$) or from $\Theta_{\mathrm{III}}^{j}$ to $\Theta_{\mathrm{III}}^{i}$ (only for $r \geq \min\{B_i/\sigma, B_j/\sigma\}$). In Figure 1, these are represented by an increase in B_i, which crosses the boundary indicated by the dotted line or the vertical line that separates Θ_{II}^{j} and Θ_{I}^{i}. In all of these situations, the publisher benefits from the larger-budget advertiser staying in the first position because of the greater surplus transferred from advertisers' budgets to the publisher. Hence, when the high-value advertiser's budget increases to the level that forces the low-value, but larger-budget, competitor to step down from the top, the publisher suffers revenue losses.

3.4. Extensions to the Basic Model

In our main model, we limited ourselves to only two strategic advertisers for simplicity. To determine whether our main results still hold when the number of bidders increases, we consider a model extension with three strategic bidders competing for two ad slots. This extension is analytically intractable and we resort to a numerical analysis in Section C in the online appendix. Consistent with our main findings, our simulation shows that there can be a negative impact of budgets on advertisers' profits and on the publisher's revenue.

Our main analysis focuses on advertisers with exogenous budgets to capture firms' liquidity constraints. Yet advertisers may have the flexibility to soften their budget constraints through the financial services, which are sometimes directly offered by the publisher (e.g., Google's credit card). To provide a deeper understanding of the strategic impact of budgets, we examine a model extension in Section D in the online appendix in which advertisers endogenously set budgets before bidding. Our finding from this extension echoes the result from Proposition 2: The publisher can be worse off by incentivizing advertisers to expand their budgets.

4. Discussion and Implications

This paper studies the impact of liquidity constraints on advertisers' bidding behaviors in position auctions. We conduct a comprehensive equilibrium analysis and our results provide several managerial insights for advertisers and publishers in the online advertising industry.

Our analysis suggests that advertisers in position auctions with budget constraints face complex considerations when determining their bids. For instance, an advertiser with a sufficient budget may consider bidding aggressively to deplete the budget of a higher-ranked advertiser. Indeed, as reported in §1, there is evidence of advertisers bidding one penny below the bid of the above-ranked advertiser, just as the bidder initially staying at the second position in a Type-I equilibrium (e.g., Ganchev et al. 2007). Alternatively, an advertiser may play conservatively to avoid being aggressively driven out of the listing. Google's own AdWords advises budget-constrained advertisers to consider reducing bids,[10] which is in line with the smaller-budget advertiser's bid strategy in a Type-II equilibrium. To address such a complexity, advertisers should actively collect budget-related information and adjust their bids accordingly.

Advertisers should also be aware of a peculiar budget trap situation, in which an advertiser's CPC increases proportionally (and profit decreases) in her own budget. Our analysis indicates that a budget trap will arise when both advertisers' budgets are relatively small and the external competition is fierce. Recently some advertisers have stated that their CPC actually increases with their own budgets in a linear rate. This pattern is consistent with our prediction in a Type-III equilibrium.[11]

One major finding in this paper is that the publisher's revenue can be negatively affected by advertisers' budgets in position auctions. Thus, publishers should be careful when encouraging advertisers to expand their budgets by offering financial services. For example, Google recently offered small-sized advertisers a new credit card with a competitive interest rate and an ample credit line to promote larger budgets in position auctions. This credit card can only be used to pay for online advertising expenses and is exclusively offered to small advertisers selected by Google. This practice can be regarded as an effort to encourage advertisers to allocate more money to online advertising. However, as indicated in Proposition 2, a firm with a high value-per-click but a small budget may induce a higher bid from the lower-ranked advertiser. Easing the budget of the higher-ranked advertiser may provoke conservative

[10] Google AdWords, https://support.google.com/adwords/answer/2375418?hl=en, accessed March 2015.

[11] "Why did my CPC increase when I increased my budget?" *Mediative Blog*. http://blog.mediative.com/en/2012/09/05/cpc-increase-increased-budget, accessed March 2015.

bidding by the lower-ranked advertiser and correspondingly decrease publisher revenue. Therefore, publishers may want to refrain from providing budget assistance if such a practice induces advertisers to bid less aggressively.

In this paper, we made several simplifying assumptions, which open directions for future research. First, we assumed that the advertisers' submitted budgets are real financial constraints. This simplification allows us to abstract away any strategic gaming by budget misreporting to focus on advertisers' interactions at the bidding stage. Future research can extend our study by considering whether/how this strategic decision may affect advertisers' incentive to bid. Second, we focused on understanding the role of budget constraints in advertisers' bidding strategies in a stylized setting. We did not consider the strategic role of a publisher in the model. A full analysis of the publisher's incentive in a position auction is beyond the objective of this paper, but it may point to a fruitful direction for future research. Finally, by relaxing our complete information assumption on values-per-click and budgets, an extension with a two-dimensional, incomplete information game for position auctions could be an interesting but challenging opportunity for future work.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mksc.2015.0931.

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References

Abhishek V, Hosanagar K (2013) Optimal bidding in multi-item multi-slot sponsored search auctions. *Oper. Res.* 61(4):855–873.

Abhishek V, Hosanagar K, Fader P (2015) Aggregation bias in sponsored search data: The curse and the cure. *Marketing Sci.* 34(1):59–77.

Agarwal A, Hosanagar K, Smith MD (2011) Location, location, location: An analysis of profitability of position in online advertising markets. *J. Marketing Res.* 48(6):1057–1073.

Amaldoss W, Desai PS, Shin W (2015) Keyword search advertising and first-page bid estimates: A strategic analysis. *Management Sci.* 61(3):507–519.

Athey S, Ellison G (2011) Position auctions with consumer search. *Quart. J. Econom.* 126(3):1213–1270.

Athey S, Nekipelov D (2012) A structural model of sponsored search advertising. Working paper, Harvard University, Cambridge, MA.

Benoît J, Krishna V (2001) Multiple-object auctions with budget constrained bidders. *Rev. Econom. Stud.* 68(1):155–179.

Börgers TM, Cox I, Pesendorfer M, Petricek V (2013) Equilibrium bids in sponsored search auctions: Theory and evidence. *Amer. Econom. J., Microeconomics* 5(4):163–187.

Borgs C, Chayes J, Immerlica N, Mahdian M, Saberi A (2005) Multi-unit auctions with budget-constrained bidders. *Proc. 6th ACM Conf. Electronic Commerce (EC)* (ACM, New York), 44–51.

Bu T, Liang L, Qi Q (2010) On robustness of forward-looking in sponsored search auction. *Algorithmica* 58(4):970–989.

Che Y, Gale I (1998) Standard auctions with financially-constrained buyers. *Rev. Econom. Stud.* 65(1):1–21.

Chen J, Liu D, Whinston AB (2009) Auctioning keywords in online search. *J. Marketing* 73(4):125–141.

Chen Y, He C (2011) Paid placement: Advertising and search on the Internet. *Econom. J.* 121(556):F309–F328.

Dellarocas C (2012) Double marginalization in performance-based advertising: Implications and solutions. *Management Sci.* 58(6):1178–1195.

Desai PS, Shin W, Staelin R (2014) The company that you keep: When to buy a competitor's keyword. *Marketing Sci.* 33(4): 485–508.

Dobzinski S, Lavi R, Nisan N (2011) Multi-unit auctions with budget limits. *Games Econom. Behav.* 74(2):486–503.

Edelman B, Ostrovsky M, Schwarz M (2007) Internet advertising and the generalized second price auction: Selling billions of dollars worth of keywords. *Amer. Econom. Rev.* 97(1):242–259.

Even-Dar E, Kearns M, Wortman J (2007) Sponsored search with contexts. Deng X, Graham FC, eds. *Proc. 3rd Internat. Conf. Internet and Network Econom.* (Springer-Verlag, Berlin Heidelberg), 312–317.

Feng J, Xie J (2012) Performance-based advertising: Price and advertising as signals of product quality. *Inform. Systems Res.* 23(3–2): 1030–1041.

Fiat A, Lenoardi S, Saia J, Sankowski P (2011) Single valued combinatorial auctions with budgets. *ACM Conf. Electronic Commerce (EC)* (ACM, New York), 223–232.

Ganchev K, Kulesza A, Tan J, Gabbard R, Liu Q, Kearns M (2007) Empirical price modeling for sponsored search. Deng X, Graham FC, eds. *Proc. 3rd Internat. Conf. Internet and Network Econom.* (Springer-Verlag, Berlin Heidelberg), 541–548.

Ghose A, Yang S (2009) An empirical analysis of search engine advertising: Sponsored search in electronic markets. *Management Sci.* 55(10):1605–1622.

Iyengar G, Phillips D, Stein C (2007) Models of malicious behavior in sponsored search. *Proc. 2007 Spring Simulation Multiconference*, Vol. 3, Society for Computer Simulation International, 143–151.

Jerath K, Ma L, Park YH, Srinivasan K (2011) A "position paradox" in sponsored search auctions. *Marketing Sci.* 30(4):612–627.

Jeziorski P, Segal IR (2015) What makes them click: Empirical analysis of consumer demand for search advertising. *Amer. Econom. J.: Microeconomics* 7(3):24–53.

Joo M, Wilbur KC, Cowgill B, Zhu Y (2014) Television advertising and online search. *Management Sci.* 60(1):56–73.

Katona Z, Sarvary M (2010) The race for sponsored links: Bidding patterns for search advertising. *Marketing Sci.* 29(2):199–215.

Lahaie S (2006) An analysis of alternative slot auction designs for sponsored search. *Proc. 7th ACM Conf. Electronic Commerce (EC)* (ACM, New York), 218–227.

Liu D, Viswanathan S (2014) Information asymmetry and hybrid advertising. *J. Marketing Res.* 51(5):609–624.

Palfrey TR, Srivastava S (1991) Nash implementation using undominated strategies. *Econometrica* 59(2):479–501.

Rutz OJ, Bucklin RE (2011) From generic to branded: A model of spillover in paid search advertising. *J. Marketing Res.* 48(1): 87–102.

Sayedi A, Jerath K, Srinivasan K (2014) Competitive poaching in sponsored search advertising and strategic impact on traditional advertising. *Marketing Sci.* 33(4):586–608.

Shin W (2015) Keyword search advertising and limited budgets. *Marketing Sci.* 34(6):882–896.

Stokes R (2010) *Ultimate Guide to Pay-per-Click Advertising* (Entrepreneur Press, Irvine, CA).

Varian HR (2007) Position auctions. *Internat. J. Indust. Organ.* 25(6):1163–1178.

Wilbur KC, Zhu Y (2009) Click fraud. *Marketing Sci.* 28(2):293–308.

Xu L, Chen J, Whinston A (2011) Price competition and endogenous valuation in search advertising. *J. Marketing Res.* 48(3): 566–586.

Yang S, Lu S, Lu X (2014) Modeling competition and its impact on paid-search advertising. *Marketing Sci.* 33(1):134–153.

Yao S, Mela CF (2011) A dynamic model of sponsored search advertising. *Marketing Sci.* 30(3):447–468.

Zhang XM, Feng J (2011) Cyclical bid adjustments in search-engine advertising. *Management Sci.* 57(9):1703–1719.

Zhou Y, Lukose R (2007) Vindictive bidding in keyword auctions. *Proc. Ninth Internat. Conf. Electronic Commerce* (ACM, New York), 141–146.

Zhu Y, Wilbur KC (2011) Hybrid advertising auctions. *Marketing Sci.* 30(2):249–273.

CORRECTION

In this article, "Position Auctions with Budget Constraints: Implications for Advertisers and Publishers" by Shijie Lu, Yi Zhu, and Anthony Dukes (first published in *Articles in Advance*, July 23, 2015, *Marketing Science*, DOI:10.1287/mksc.2015.0931), Figure 1 on page 902 has been corrected.