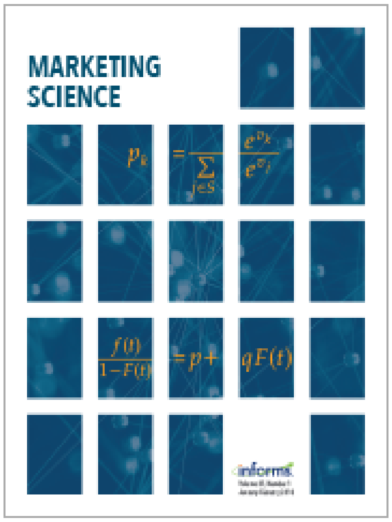

Marketing Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

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To cite this article:
Niladri B. Syam, Amit Pazgal (2013) Co-Creation with Production Externalities. Marketing Science 32(5):805-820. https://doi.org/10.1287/mksc.2013.0791

MARKETING SCIENCE
Vol. 32, No. 5, September–October 2013, pp. 805–820
ISSN 0732-2399 (print) | ISSN 1526-548X (online)
http://dx.doi.org/10.1287/mksc.2013.0791

Co-Creation with Production Externalities

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Co-creation, the participation of customers in the design and production of goods and services, has been gaining popularity in recent years. In this research we incorporate firm pricing into the joint production process allowing us to study (1) production externalities between firm and customers, (2) production externalities among customers, and (3) optimal pricing by firms.

We show that given a choice, a monopoly firm will opt for co-creation with customers rather than deal with passive price-taking consumers. Furthermore, the firm will increase the effort it devotes to co-creation as the number of potential co-creating customers increases.

We show that the profit of a firm facing a centralized pattern of externalities among customers (with an expert, or lead user, in the center) can be higher than its profit when facing a decentralized pattern of externalities among customers and clearly dominates its profit when customers do not have any cross externalities. Thus, we provide a different justification for the use of lead users, one that depends on their network centrality and not on having lower cost, more information, or greater ability than the firm. Because the decentralized pattern has more links than the centralized pattern, our results demonstrate the importance of the pattern of links between customers, and not just their number, in determining the profitability of co-creation.

Furthermore, we find that the lead user's externality spillover to other connected users, her neighbors, acts as a force multiplier on the efforts exerted by all participants in equilibrium. Specifically, a higher spillover from the lead user increases the efforts of the firm, the neighbors, and the lead user herself, and this may lead to beneficial outcomes for all.

Finally, we show that in co-creation environments, a monopolist firm may benefit by committing to a single price rather than exercising price discrimination. This is because the pricing structure affects customers' incentive to invest effort in the innovation-production stage.

Key words: co-creation; customization; networks; externalities; game theory
History: Received: May 3, 2012; accepted: March 16, 2013; Preyas Desai served as the editor-in-chief and Yuxin Chen served as associate editor for this article. Published online in *Articles in Advance* July 26, 2013.

1. Introduction

Instances of co-creation, the participation of potential customers in the production of goods and services, continue to increase in number and importance (Pine and Gilmore 1999). Collaborating with customers to co-create products has always been commonplace in business-to-business (B2B) settings, well before the current proliferation of business-to-consumer (B2C) instances. Specifically, user participation in the design of products, especially in business-to-business contexts, is the earliest and most prominent manifestation of product co-creation (Griffin and Hauser 1993, Thomke and von Hippel 2002, von Hippel and Katz 2002). Recent advances in information technology and flexible manufacturing have greatly enhanced the ability of firms and customers to co-create products such as cameras, copier finishers, laptop bags, laundry products, etc. (Redstrom 2006, Sawhney et al. 2005). Paris Miki, a Japanese eyewear retailer, has developed a system where designs are created in close collaboration with its customers; customers complete the design task by selecting options for the nose bridge, hinges, and arms (Gilmore and Pine 1997). eMachineshop.com provides individuals the ability to create their own tools by allowing access to an infrastructure that was hitherto available only to "real" manufacturers.

Academic researchers have also turned their attention to co-creation, with an emphasis on viewing the customer as a coproducer of goods and services (Jaworski and Kohli 2006; Kalaignanam and Varadarajan 2006; Prahalad and Ramaswamy 2000, 2004; Vargo and Lusch 2004). Our goal in this paper is to rigorously model co-creation and to determine the optimal cooperative productive efforts of the firm and the consumer as well as the pricing by the firm.

There are two types of co-creation: proprietary and nonproprietary (Bessen 2006, Rossi and Bonaccorsi 2006). The types differ in the ownership structure and distribution of the final product. Nonproprietary co-creation, which has the flavor of public goods, describes environments wherein the firm and its customers invest in creating a better product that is then distributed freely to all users (open source software, for example). Bramoulle and Kranton (2007), Ballester et al. (2006), and Calvó-Armengol and Zenou (2004) interpreted the efforts of the agents as product innovations and treated such innovations as strict public goods. Proprietary co-creation, on the other hand, refers to situations where customers participate in the development of the product but the firm retains ownership and can sell the final product. Proprietary co-creation has flavors of both public and private goods.

In this paper, we study co-creation environments where the price mechanism is crucial in mediating the value exchanged between the firm and the customer, i.e., the proprietary setting. Customers can benefit not only from the effort of the firm but also from the efforts and resources of their fellow customers. Specifically, some users may be "experts"[1] in that their needs are similar to others and their efforts can substantially benefit other customers. Consider the phenomenon of *lead users* in business markets.[2] Lead users are at the forefront of emerging trends and can help other users with their product solutions. In this paper, we broaden the definitions and denote as experts/lead users those customers who are able to substantially benefit other users through their innovation efforts. Consider, for example, the oil and gas industry. Original equipment manufacturers of generators and turbines (e.g., GE, Simmons, Mitsubishi) develop their products jointly with big advanced multinational oil companies (e.g., ExxonMobil, Royal Dutch Shell, Chevron). After the development stage, the manufacturer aims to sell the equipment to all potential end users (including many less sophisticated users, such as Petronas or Sonangol) with the appropriate fine-tuning and customizations. This forms an oil and gas network of co-creators where the firm is connected to its lead users and regular customers. The former help in the development of the new product, and their expertise benefits all potential users. The efforts of the latter are concentrated on working with the manufacturers to fine-tune and customize the products for their own use.

This paper captures three different aspects of co-creation: (1) Customers and firms both contribute to the design of products. Thus there are production externalities between customers and the firm. (2) Some customers have capabilities or expertise that are of value to other customers in the market. Thus there are production externalities among customers. (3) Firms charge for products co-created with the help of customers.

We compare and contrast optimal effort levels as well as firm prices and profits in several representative environments. We compare the basic no co-creation case to the intermediate case, where there are no lead users but every customer can improve his or her product directly with the firm (no links between users), to the advanced cases that include lead users whose efforts improve the products of all users or more diffused connections between similar users. We further investigate the optimality of price discrimination by the firm when there are lead users (i.e., when they charge different prices to lead and regular users).

The rest of the paper is organized as follows. Section 2 describes the relevant literature; §3 presents our general findings and the intuition behind them. The general model is developed in §4. Section 5 provides robustness checks for the main results of §4. Section 6 is devoted to the analysis of the benefits of a single price over price discrimination, and §7 concludes.

2. Literature Review

Much of the research that deals with the production externalities of the type that we study has appeared in the networks literature. The existing literature that examines networks and externalities can be broadly divided into three streams. The first and earliest research stream covers only network externalities. Crucially, this literature (implicitly) assumes that every agent (be it a firm or a customer) is connected to every other agent. In the terminology of graph theory, these papers assume a "complete" graph and ignore the specific pattern of connections between agents (e.g., Katz and Shapiro 1985, Farrell and Saloner 1985) The second stream of work examines network topology, but it tackles only the problem of agent contributions (or efforts) and how agents optimally determine their contributions given the network of connections. Thus, it has the flavor of contributions to a "public good," where the pattern of externalities is given by the topology of local network effects. Prominent papers in this group include Bramoulle and Kranton (2007), Ballester et al. (2006), and Calvó-Armengol and Zenou (2004). We differ in two respects. First, and most importantly, we investigate proprietary situations where firm pricing plays an important role. Second, we focus on specific patterns of externalities (i.e., network structures) that incorporate the fact that certain agents, the lead users, are experts who benefit their fellow users. Both aspects are critical

[1] We use the terms "expert" and "lead user" interchangeably to denote those customers whose innovative efforts are universally beneficial.

[2] For extensive discussions, see von Hippel (1986, 1998) and Urban and von Hippel (1988).

to co-creation and have not been examined by the above public goods literature. A recent paper, Chennamaneni and Desiraju (2011), investigates these public good inefficiencies in the context of comarketing alliances and offers a contractual solution to these problems, though the issues of network topology and ownership and the selling of a product are not investigated.

The third and nascent research stream examines pricing in networks and accounts for local network effects, but the firms and customers play completely distinct and different roles. Firms set prices, and the customers determine their contributions. There is no effort contribution on the part of firms, and the only decision for consumers to make is whether to buy or not buy the offered products. Thus, these papers can be thought of as investigating consumption externality rather than production externality (Banerji and Dutta 2009, Bloch and Quérou 2013).

Co-creation involves both the joint contribution of the firm and customers in designing a product and the firm's pricing. This paper takes a first step towards understanding the importance of both network connections and pricing in the co-creation process.

3. Results and Intuition

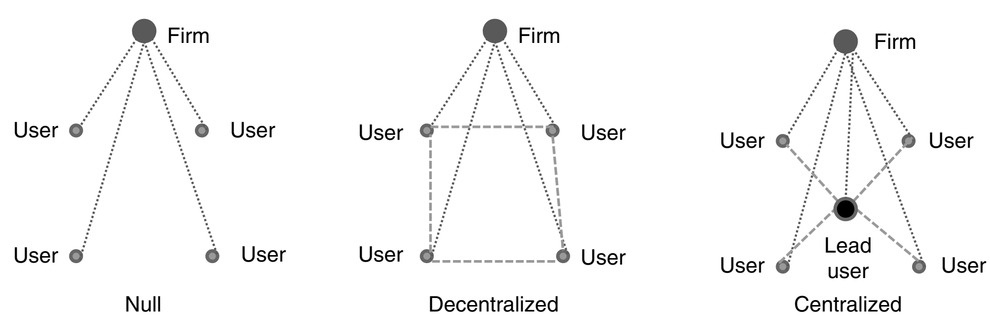
In this paper, we analyze co-creation between a firm and a set of users, where the users can further be connected among themselves according to specific patterns of externalities. We analyze three canonical externality patterns. In the network literature, these patterns correspond to different networks: the first has no connections among users (null pattern), the second has an expert/lead user at the center who is connected to the other users called neighbors (centralized pattern), and the third has no recognized expert among the users, all of whom have symmetric connections (decentralized pattern). See Figure 1 for a depiction of these externality patterns.

We have three main results. First, a centralized pattern of externalities, with a lead user in the center, can be more profitable than both the null and decentralized patterns of externalities, even though for a given number of customers, the decentralized pattern has more links than the centralized pattern. The networks literature has shown that the pattern of links as well as the number of links matters, but more importantly, we demonstrate in our specific context exactly how the pattern of links affects co-creation efforts and profits. Specifically, a firm facing a centralized pattern of links can be more profitable than one facing a decentralized pattern, even when all spillovers among customers are the same in both cases. Thus, we provide a novel justification for the use of a lead user by a profit-seeking firm. The benefit of the lead user does not depend on her having a lower cost or greater ability compared with the firm (the usual reasons advanced in the trade press to justify a lead user) but on the fact that she is central in the production network. When the spillover from the lead user to other customers increases, the firm increases its effort and therefore can increase its price. This force is absent in a decentralized network. Therefore for high spillover, a centralized network is always more profitable than a decentralized network.

Our second result states that if users are connected by a centralized externality pattern, then when the lead user increases her spillover to the other users, *all* the involved parties benefit: the firm's profit, the lead user's expected utility, and the other users' expected utilities all increase. The spillover of the lead user captures the extent to which her effort can benefit the other users connected to her. Thus we show that when the lead user's effort is more beneficial to the other users, both she and the firm benefit. As mentioned above, a higher spillover from the lead user increases the effort the firm exerts, and this benefits the lead user. Even though the price increases, the positive effect dominates, and the lead user's expected utility increases in her spillover.

An interesting implication of the above is that the lead user and the firm are not mere substitute experts in the creation of value. In fact, they complement each other. Having a better (more representative or better connected) lead user should not cause the firm to reduce its creative effort; instead, the firm should increase it.

Figure 1 **Externality Patterns**



Our third finding shows that when customers are connected by a centralized pattern of externalities (with a clear distinction between lead users and regular users), it is better for the firm to charge a single price than to price discriminate, even if it is a monopolist. It is well documented that through price discrimination, the firm can better extract the surplus of heterogeneous customers. However, in co-creation environments, this turns out to be problematic for the firm. Price discrimination leads to reduced consumer surplus, which in turn causes consumers to rationally reduce their co-creation efforts. Because the firm's profit increases in the total efforts exerted, price discriminating reduces profit compared with charging a single price. In a sense, by agreeing to not price discriminate, the firm signals to the customers that they will retain more of their co-creation efforts, leading them to increase their contribution enough to offset the firm's losses from not customizing the prices. The extant literature has shown the suboptimality of price discrimination when firms compete for customers (especially in behavior-based schemes). Our result is similar to the potential extra profitability of a monopolist's ability to commit to a single price for a good over several time periods (see Fudenberg and Villas-Boas 2006). The point of difference lies in the mechanism by which this occurs in the two situations: in co-creation, the commitment to a single price changes consumers' incentives to expend effort on product development; in the behavior-based pricing literature, it changes the consumers' timing of purchase.

4. A Model of Co-Creation with Production Externalities

Consider a situation where several agents engage in joint production (of a product or service) with a monopolist firm. In co-creation each agent is a user-creator who will participate in joint production with the firm and perhaps with other fellow user-creators, and this agent may purchase the final product for its own use. We conceptualize cooperative production as a network of contributors, with each user-creator represented by a node. The links in the network of externalities represent the potential effect of one user's effort on the other's overall utility. In the remainder of the paper, we will refer to the user-creators simply as *users.* We also assume that the firm is (by definition) connected to all users. Thus, the specific patterns of externalities that we will investigate (null, centralized, and decentralized) refer only to the pattern of links between the users.

4.1. No Externalities Among Users

In this section we analyze the case where there are no links between the users themselves, so that each user's effort does not benefit other users. We can think of each user as exerting effort to better fit the product for his or her own specific circumstances. We refer to this base case as the *null network*.

As a base case, we investigate an environment with a single firm and U symmetric users. Let the production "effort" of the firm be e_f and that of a user be e_u. It is important that our use of the term "effort" does not confuse the reader; we use the term here to capture the amount of innovation or product design that an agent contributes. The literature on contributions to open source software uses the term in the same spirit (Hindricks and Pancs 2002, Johnson 2002). The newer literature on public goods in networks, e.g., Bramoulle and Kranton (2007) and Ballester et al. (2006), also interprets an agent's efforts as innovation attempts. Agents could innovate by experimenting with new technology or by generating new information that is shared among their network of partners in a collaborative venture. In this sense the innovation efforts of various agents are observable.[3]

Each user benefits from the combined total effort of all agents connected to him. Specifically, we define the total effort $\tilde{e}_u$ at a user's node as

$$\tilde{e}_u = e_u + \delta_f e_f, \tag{1}$$

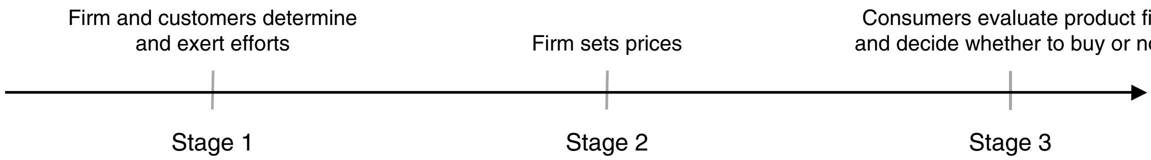
where $\delta_f \leq 1$ represents the degree of effort substitutability between the firm and all the users. The user's benefit is thus a function $b(\cdot)$ of the above total effort. We also assume a convex cost of effort $c(\cdot)$ with $c(0) = 0$. The total consumer surplus is given by $CS_u = b(\tilde{e}_u) - ce_u^2 - p_u + \varepsilon$, where p_u is the price charged to a user for the co-created product, and ε is a zero mean random variable representing uncertainty the user has about the fit of the products to her needs. This fit parameter is unknown to the consumer during the development stage and is revealed to her once the product is available in the market.[4] The firm only knows the distribution of the fit parameters in the population, never its actual realizations. Throughout the paper we assume that ε is distributed uniformly on $[-\theta, \theta]$. To concentrate on the network effect, we assume that the cost coefficient c is also the same across all users.

The competitive environment unfolds in three stages. In the first stage, both the customers and the firm determine their innovation effort levels and then exert that effort. In the second stage, the prototype

[3] We do not use the term "effort" in the sense of the principal–agent models where the agent's efforts are unobservable (e.g., Holmstrom 1979).

[4] In many new product development cases, consumers can articulate their needs but cannot fully evaluate the benefit of the product or their willingness to pay for it until a prototype is developed and is shown to them.

Figure 2 **Timeline of the Competitive Environment**



is available, and by observing effort levels, the firm determines the prices. In the third stage customers evaluate the fit of the product and make their purchase decisions. Figure 2 depicts this timeline.

Note the importance of the first stage in the co-creation game. When users exert their innovation effort, they do not know the fit of the final product to their needs. Thus, depending on the magnitude of the fit parameter, there is a positive probability that the customers will not purchase the co-created product (despite exerting a costly effort). Rational consumers must take into account the probability of non-purchase when making their effort decisions in the first stage. Moreover, at the last stage, when agents make their purchase decisions, their cost of effort has already been incurred in the first stage and is considered to be sunk for the buying decision.

To solve for the optimal prices and effort level, we use the concept of a subgame-perfect equilibrium and backwards induction. Starting at the last stage, given a price p_u, a user will purchase the product with probability given by $\Pr[\varepsilon > p_u - b(\tilde{e}_u)]$. The probability that a user purchases is thus

$$\Pr[\text{User } u \text{ buys}] = \frac{\theta + b(\tilde{e}_u) - p_u}{2\theta}. \tag{2}$$

Notice that the cost, $c(e_u)$, does not figure in the probability calculation because when the user makes the purchase decision, the cost of co-creation is already sunk (determined in the first stage).

The expected profit of the firm is

$$\Pi_f = U \cdot \left(\frac{\theta + b(\tilde{e}_u) - p_u}{2\theta}\right) p_u - c(e_f). \tag{3}$$

In the second stage, the firm chooses p_u optimally; the first-order condition (FOC) gives the price in terms of the efforts:

$$p_u = \frac{\theta + b(\tilde{e}_u)}{2}. \tag{4}$$

Note that the price charged to a user is increasing in the benefit a user gets from joint production with the firm, $b(\tilde{e}_u)$, and so it is increasing in the user's own effort, e_u. Intuitively, the firm can extract the benefit that the user enjoys by consuming the product, and the product's benefit increases in e_u.

We now turn to the first stage and determine the optimal efforts of the firm and the users. The expected profit of the firm after price has been incorporated from (4) is

$$\Pi = U \frac{(\theta + b(\tilde{e}_u))^2}{8\theta} - c(e_f). \tag{5}$$

Next, we need to calculate the expected consumer surplus of the users. A given user, u, will realize the exact value of the product fit only after the effort choice. Thus if the product is ultimately a poor fit, the user may not buy in the future despite the efforts exerted in the development stage. This fact needs to be incorporated in the calculation of the expected consumer surplus:

$$\begin{aligned} E[CS_u] &= E[CS_u \text{ if } u \text{ buys} \mid u \text{ buys}] \Pr[u \text{ buys}] \\ &\quad + E[CS_u \text{ if } u \text{ doesn't buy} \mid u \text{ doesn't buy}] \\ &\quad \cdot \Pr[u \text{ doesn't buy}] - c(e_u). \end{aligned}$$

Notice that the user incurs the cost of effort, regardless of whether she buys the product in the future or not. Also, as already noted, the probability of purchase in the future is independent of the cost of effort. Thus,

$$\begin{aligned} E[CS_u] &= E[b(\tilde{e}_u) - p_u + \varepsilon \mid \varepsilon \geq p_u - b(\tilde{e}_u)] \\ &\quad \cdot \Pr[\varepsilon \geq p_u - b(\tilde{e}_u)] - c(e_u) \\ &= (b(\tilde{e}_u) - p_u + E[\varepsilon \mid \varepsilon \geq p_u - b(\tilde{e}_u)]) \\ &\quad \cdot \left(\frac{\theta + b(\tilde{e}_u) - p_u}{2\theta}\right) - c(e_u). \end{aligned}$$

This gives us $E[CS_u] = (1/4\theta)(\theta + b(\tilde{e}_u) - p_u)^2 - c(e_u)$. After substituting the price from (4), we get

$$E[CS_u] = \frac{1}{16\theta}(\theta + b(\tilde{e}_u))^2 - c(e_u). \tag{6}$$

The optimal effort profile $\mathbf{e} = (e_f, e_u)$ is solved by maximizing (5) and (6) simultaneously.

The user's FOC $\partial E[CS_u]/\partial e_u = 0$ gives $(1/(8\theta))(b(\tilde{e}_u) + \theta)b'(\tilde{e}_u) = c'(e_u)$, and the firm's FOC $\partial E[\Pi]/\partial e_f = 0$ gives $U((\theta + b(\tilde{e}_u))/(4\theta))\delta_f b'(\tilde{e}_u) - c'(e_f) = 0$. For the sake of tractability, we assume that the benefit function is linear and the cost is quadratic. Yet, as §5 shows, the results hold for more general functions. Specifically, we assume that $b(\tilde{e}_u) = \tilde{e}_u = e_u + \delta_f e_f$ and $c(e_u) = ce_u^2$. The solutions for the users' and the firm's efforts are shown in (7) and (8), respectively;[5] all optimal quantities are denoted with a superscript that corresponds to the network type:

$$e_u^{\mathrm{Null}} = \frac{\theta}{16c\theta - 2\delta_f^2 U - 1}, \tag{7}$$

$$e_f^{\mathrm{Null}} = \frac{2\theta\delta_f U}{16c\theta - 2\delta_f^2 U - 1}. \tag{8}$$

For participants to exert an effort (an interior solution), we need $e_f^{\mathrm{Null}} > 0$ and $e_f^{\mathrm{Null}} > 0$. For these to be satisfied, we need

$$16c\theta - 2\delta_f^2 U - 1 > 0. \tag{9}$$

In the rest of the paper, we maintain this assumption.[6] Note that the parameter restriction is satisfied as long as the number of users is not too large; that is, $U < (16c\theta - 1)/\delta_f^2$.

Clearly, the optimal effort of the firm increases in U, the number of users. This is due to the pricing effect. Without pricing, the firm would be just another agent creating a product with other economic agents; this is akin to a public goods situation. The public goods literature has found free riding to be prominent, and as the number of agents increases, the problem of free riding is exacerbated. We show that allowing the firm to own the product and set its prices reverses the free-riding effect caused by the public goods nature of co-creation. Finally, the optimal price and profit of the firm can be obtained by suitable substitutions. We state these criteria as Proposition 1.

Proposition 1. *When a monopolist firm co-creates a product with U homogeneous users with no connections between them (forming a null pattern of externalities), then the optimal effort of each user is $e_u^{\mathrm{Null}} = \theta/(16c\theta - 2\delta_f^2 U - 1)$, and the optimal firm effort is $e_f^{\mathrm{Null}} = (2\theta\delta_f U)/(16c\theta - 2\delta_f^2 U - 1)$. The firm's profit is given by $\Pi^{\mathrm{Null}} = 4c\theta^2 U(8c\theta - \delta_f^2 U)/(16c\theta - 2\delta_f^2 U - 1)^2$.*

It is straightforward to check that if $8c\theta - 1 > 0$, the optimal profit of the firm increases in the number of users U that it is connected to.[7] This is an immediate consequence of the price effect. Even though the firm's own effort increases in the number of users, leading to a higher cost of co-creation, the increase in price with an increase in U more than makes up for that increased cost.

[5] The second-order condition (SOC) for the user's effort requires $16c\theta - 1 > 0$, and the SOC for the firm's effort requires $8c\theta - \delta_f^2 U > 0$. We assume they are satisfied.

[6] We focus on interior solutions; otherwise, the parties would put in zero effort, and this would not be a true co-creation environment.

[7] That is,

$$\frac{\partial \Pi^{\mathrm{Null}}}{\partial U} = \frac{8c\theta^2 \lfloor 4c\theta(16c\theta - 1) + \delta_f^2 U(8c\theta - 1) \rfloor}{(16c\theta - 2\delta_f^2 U - 1)^3}.$$

As a benchmark we analyze the case without co-creation, where the users are merely price takers as in traditional markets.

4.1.1. Benchmark Without Co-Creation. A monopolist firm creates and sells a product to U homogeneous users who do not exert effort. Suppose the firm first chooses its design effort and then its price. The optimal effort and profit of the firm are

$$e_f^{\text{No–Co-creation}} = \frac{\delta_f \theta U}{8c\theta - \delta_f^2 U}, \quad \Pi^{\text{No–Co-creation}} = \frac{c\theta^2 U}{8c\theta - \delta_f^2 U}.$$

Proof. See the appendix.

A direct comparison of the monopolist's profits with and without co-creation yields the following result.

Corollary. *Suppose that a monopolist has the option of co-creating products with its customers. The monopolist's profit from choosing co-creation, even when there are no links among users (Π^{Null}), is higher than that with no co-creation ($\Pi^{\text{No–Co-creation}}$).*

This result follows from the convexity of the cost function, but it is still useful to point out. In subsequent sections we analyze the more challenging cases where there are different networks of externalities between the users, and we perform profit comparisons between networks under different functional forms of the cost and benefit functions. Because users do not innovate or create in a vacuum, an analysis of the patterns of externalities between users is a salient aspect of co-creation.

4.2. Centralized Pattern of Externalities Among Users

Although theoretically there could be many patterns of links between users, the most common in practice is that with an expert user whose productive input benefits all other users. A prominent example is the lead user phenomenon, which has been the most well established and widely studied method for user design, especially in B2B markets (von Hippel 1986, 1998; Urban and von Hippel 1988).

Though very common in business markets, the expert user phenomenon finds resonance in consumer markets as well. For example, at Threadless.com, customers, usually amateur designers at the forefront of fashion trends, can send in their own T-shirt designs. Popular designs are then printed and sold as a newly created product. In software design, beta testers are often expert users.

In this subsection we analyze the case where there are links among the users themselves. These links form a centralized network with the expert/lead user at the center. In terms of the network literature, the centralized network is an important canonical one, which captures situations where one expert agent has more connections, and more impact, than all other agents.

The analysis of this case is more complex than that presented in §4.1 because the agents are not symmetric with respect to their numbers of links, and so we have to allow for an asymmetric equilibrium in effort and choice. We denote the users linked to the lead user as her *neighbors*, and the number of neighbors is exogenously fixed at N.

The total efforts at a neighbor's node $\tilde{e}_n$ and at the lead user's node $\tilde{e}_l$ are, respectively,

$$\begin{aligned}\tilde{e}_n &= e_n + \delta_f e_f + \delta_l e_l \quad \text{and}\\ \tilde{e}_l &= e_l + \delta_f e_f + \delta_n N e_n,\end{aligned} \tag{10}$$

where $\delta_l \leq 1$ is the benefit to a neighbor from a unit effort exerted by the lead user, and $\delta_n \leq 1$ denotes the benefit to the lead user from a unit effort exerted by his neighbor. We assume that the lead user has higher spillover than her neighbors: $\delta_l \geq \delta_n$.

For purposes of comparison with §§4.1 and 4.3, we will set $N+1 = U$. We assume that the firm can charge different prices to the neighbors and the lead user. It is straightforward to assume different benefit functions $b_l(\cdot)$ and $b_n(\cdot)$ for the lead user and neighbors, respectively, but we lay these aside to focus on the network structure.

Proceeding similarly to §4.1, the probability that a customer of type i purchases is $\Pr[\text{User } i \text{ buys}] = (\theta + b(\tilde{e}_i) - p_i)/(2\theta)$, where $i = 1, n$. The expected profit of the firm is

$$\begin{aligned}\Pi &= \left(\frac{\theta + b(\tilde{e}_l) - p_l}{2\theta}\right) p_l \\ &\quad + N\left(\frac{\theta + b(\tilde{e}_n) - p_n}{2\theta}\right) p_n - c(e_f).\end{aligned} \tag{11}$$

The firm will choose p_1 and p_n optimally, and the FOCs give the two prices in terms of the efforts:

$$p_i = \frac{\theta + b(\tilde{e}_i)}{2}, \quad i = 1, n. \tag{12}$$

As in §4.1 (see Equation (6)), each neighbor's and lead user's surpluses, after prices are incorporated into Equation (11), are $CS_i = (1/(16\theta))(\theta + b(\tilde{e}_i))^2 - c(e_i)$, $i = 1, n$. Finally, the expected profit of the firm after prices have been incorporated is

$$\Pi = \frac{(\theta + b(\tilde{e}_l))^2}{8\theta} + N\frac{(\theta + b(\tilde{e}_n))^2}{8\theta} - c(e_f). \tag{13}$$

To characterize the effort profile $\mathbf{e} = (e_f, e_l, e_n)$ in equilibrium, we first focus on the problem of customers 1 and n. The two FOCs for the lead user and neighbor, $\partial CS_i/\partial e_i = 0$ for $i = 1, n$, give us, respectively,

$$(\theta + b(\tilde{e}_l)) = 8\theta c'(e_l) \quad \text{and} \quad (\theta + b(\tilde{e}_n)) = 8\theta c'(e_n). \tag{14}$$

The firm's FOC,

$$\begin{aligned}\frac{\partial E[\Pi]}{\partial e_f} &= \frac{\theta + b(\tilde{e}_l)}{4\theta}\,\frac{\partial b(\tilde{e}_l)}{\partial e_f} + N\,\frac{\theta + b(\tilde{e}_n)}{4\theta}\,\frac{\partial b(\tilde{e}_n)}{\partial e_f}\\ &= c'(e_f),\end{aligned}$$

becomes

$$c'(e_l)b'(\tilde{e}_l) + Nc'(e_n)b'(\tilde{e}_n) = \frac{1}{2\delta_f}c'(e_f). \tag{15}$$

For the linear benefit and quadratic cost case, the three FOCs solved simultaneously give the optimal efforts of the lead user, neighbors, and the firm, respectively, as

$$\begin{aligned}e_l^{\text{Centr}} &= \big[\theta(16c\theta - 1 + \delta_n N)\big]\\ &\quad \cdot \Big((16c\theta - 1)(16c\theta - 1 - 2\delta_f^2(N+1))\\ &\quad - 2\delta_f^2 N(\delta_l + \delta_n) - \delta_l\delta_n N\Big)^{-1},\end{aligned} \tag{16}$$

$$\begin{aligned}e_n^{\text{Centr}} &= \big[\theta(16c\theta - 1 + \delta_l)\big]\\ &\quad \cdot \Big((16c\theta - 1)(16c\theta - 1 - 2\delta_f^2(N+1))\\ &\quad - 2\delta_f^2 N(\delta_l + \delta_n) - \delta_l\delta_n N\Big)^{-1},\end{aligned} \tag{17}$$

$$\begin{aligned}e_f^{\text{Centr}} &= \big[2\delta_f\theta[(16c\theta - 1)(N+1) + N(\delta_l + \delta_n)]\big]\\ &\quad \cdot \Big((16c\theta - 1)(16c\theta - 1 - 2\delta_f^2(N+1))\\ &\quad - 2\delta_f^2 N(\delta_l + \delta_n) - \delta_l\delta_n N\Big)^{-1}.\end{aligned} \tag{18}$$

Substituting the optimal efforts in (16)–(18) in the prices given in (12) gives the optimal prices:

$$p_l^{\text{Centr}} = 8c\theta e_l^{\text{Centr}}, \quad p_n^{\text{Centr}} = 8c\theta e_n^{\text{Centr}}. \tag{19}$$

From (16), (17), and (19), we get the following proposition regarding optimal efforts exerted and prices paid by the lead user and by each neighbor.

Proposition 2. *When the spillover from the lead user to the neighbors is not too large ($\delta_l < N\delta_n$), the lead user exerts a greater effort and also faces a higher price than the neighbors.*

From (12) we see that the price paid by a user increases in the total benefit she receives, and therefore in the total effort accruing at the user's node. The total benefit received by the lead user is higher than a neighbor if the spillover from the lead user to the neighbors (δ_1) is smaller than the total spillover from all the neighbors to the lead user ($N\delta_n$). By contrast, the neighbor pays a higher price and the lead user a lower price when the spillover from the lead user is higher than the spillover from the neighbor.

In equilibrium, after substituting the optimal efforts, the probabilities of purchase by the lead user and neighbor are given by

$$\Pr[\text{Lead user buys}] = 4ce_l^{\text{Centr}},$$
$$\Pr[\text{Neighbor buys}] = 4ce_n^{\text{Centr}}.$$

The optimal profit of the firm in the centralized network can be expressed conveniently in terms of efforts as

$$\Pi^{\text{Centr}} = 32c^2\theta(e_l^{\text{Centr}})^2 + 32c^2\theta N(e_n^{\text{Centr}})^2 - c(e_f^{\text{Centr}})^2. \quad (20)$$

The following result states the main finding of this section.

Theorem 1. (a) *When the users have a centralized pattern of externalities among them, the lead user's and the neighbors' expected surplus as well as the firm's profit all increase in the lead user's spillover,* δ_l.

(b) *The firm's equilibrium profit and effort are higher if it co-creates with users who are connected to each other by a centralized pattern of externalities than if it co-creates with users who do not have any connections with each other* ($\Pi^{\text{Centr}} > \Pi^{\text{Null}}$ *and* $e_f^{\text{Centr}} > e_f^{\text{Null}}$).

Part (a) of Theorem 1 provides one of the main insights of the paper: when the lead user's spillover increases, i.e., when her effort is more beneficial to her neighbors, *all parties—the lead user, the neighbor, and the firm—benefit*. Interestingly, the symbiotic relationship between the lead user and firm occurs independently of their costs and abilities and also independently of the spillover δ_f between them. It can be argued that a lead user is more effective when her efforts are more beneficial to the other users. Our analysis shows that everyone benefits when the lead user becomes more effective, even though "effective" is defined here only as being advantageous for other users (the neighbors) and not for the lead user or the firm directly.

A further implication of Theorem 1, part (a) is that the efforts of the firm and the lead user are not substitutes: a better (more representative or better connected) lead user is not a reason for the firm to lower its creative effort. Thus, the lead user and the firm are not mere substitute experts in the creation of value. In fact, they complement each other.

The intuition is as follows. In a centralized network, as the spillover of the lead user (δ_l) increases from a given initial level, the benefit to the neighbors goes up as a result of two different effects: the direct benefit increases and the probability of buying increases. The price charged by the firm will increase as well but only by half the benefit (see Equation (12)); overall, the neighbors will benefit. In fact, the neighbors will now choose to exert a greater effort themselves.[8] As a result of increased spillovers by the lead users, the neighbors are not content to just receive the extra benefit; they actually increase their own effort as well to gain even more utility. In response to this increase, the firm will not only increase its price to the neighbors but will also increase its effort to make even higher profits (using a similar argument to the neighbors' increase). Now, the lead user is faced with greater firm and neighbor efforts and thus finds it beneficial to increase her own effort as well. Greater efforts by all parties create a larger benefit for the lead user, and though the price increases as well, it increases by only half of the extra benefit. This leads to a higher surplus for the lead user.

The endogenous efforts also explain why the profit from a centralized network of externalities, with a lead user in the center, is higher than co-creation with a null network of externalities. Because the efforts of the firm, lead user, and neighbors in the centralized network are greater than the corresponding efforts in the null network, the total efforts are greater for the centralized network. This results in higher prices and thus higher profits for the centralized network compared to the null network. Thus the firm benefits from the lead user.

The rationale for using the lead user described in the previous paragraph is novel. We have *not* assumed any differential cost or information between the firm and the lead user, the usual argument presented in the trade press for why lead users may be valuable.[9] The business press holds that lead users are privy to information that firms do not have, especially product use information since they are users themselves. Lacking the knowledge required for product-use-related innovations, firms find it beneficial to co-opt the lead user's innovations on that dimension. This information asymmetry is not needed to obtain our result, which is driven purely by the network centrality of the lead user.

Note that there is an increase in the number of links when we go from a null network to a centralized network; part of the reason why the centralized network is more profitable could be attributed to the number of links. We demonstrate below the importance of the *pattern* of links as opposed to just the *number* of links. We prove the stronger result that the centralized externality network can be more profitable than the decentralized externality network even though the decentralized network has more links between users.

[8] Recall that $CS_n = (1/(16\theta))(\theta + b(\tilde{e}_n))^2 - c(e_n)$. So the marginal benefit equals marginal cost equation is $(1/(8\theta))(\theta + b(\tilde{e}_n))b'(\tilde{e}_n) = c'(e_n)$. Since increasing δ_1 unequivocally increases the left-hand side, the optimal choice of e_n increases.

[9] By assumption, the marginal cost of effort, c, is the same for both the firm and the lead user.

4.3. Decentralized Pattern of Externalities Among Users

We now analyze the case where links between the users form a decentralized network. We focus only on links among users. For U users, a decentralized (circle) network has U links among users, whereas a centralized (star) network has $U-1$ links among users (if we include links to the firm as well, then the number of links is $2U$ and $2U-1$, respectively). The main reason for analyzing a decentralized network is to establish that a centralized network of externalities with a recognized expert (lead user) can yield higher profits than the decentralized network, despite having fewer links than its counterpart.

In the networks literature, the decentralized network is one of the important canonical networks. It captures situations where the agents are symmetric and connected to each other but there is no recognized expert in the network. A decentralized network can emerge when there is social learning among agents who are "close" to each other in terms of their tastes and/or preferences, and there is only local observation among agents (e.g., Goyal 2007, p. 97; Banerji and Dutta 2009).

We number the U nodes from 1 through U. The total effort $\tilde{e}_u$ at a user u's node $(1 < u < U)$ is

$$\tilde{e}_u = e_u + \delta_u e_{u-1} = \delta_u e_{u+1} + \delta_f e_f.$$

Each node u benefits from the efforts expended by her two neighbors, $(u-1)$ and $(u+1)$. Here, δ_u is the effort spillover between users. As in the two previous sections, the firm's optimal price is $p_u = (\theta + b(\tilde{e}_u))/2$, and a user's consumer surplus after incorporating the firm's optimal price is $CS_u = (1/(16\theta))(\theta + b(\tilde{e}_u))^2 - c(e_u)$ with $b(\tilde{e}_u) = b(e_u + \delta_u e_{u-1} + \delta_u e_{u+1} + \delta_f e_f)$.

The FOC for user u is $(1/8\theta)(\theta + b(\tilde{e}_u)) = c'(e_u)$. Using symmetry, linear benefits and quadratic costs yield the optimal efforts of the firm and the users with a decentralized network:

$$e_u^{\text{Decentr}} = \frac{\theta}{16c\theta - 2\delta_f^2 U - 1 - 2\delta_u},$$
$$e_f^{\text{Decentr}} = \frac{2\theta\delta_f U}{16c\theta - 2\delta_f^2 U - 1 - 2\delta_u}. \tag{21}$$

The firm's profit is given by

$$\Pi^{\text{Decentr}} = \frac{4c\theta^2 U(8c\theta - \delta_f^2 U)}{(16c\theta - 2\delta_f^2 U - 1 - 2\delta_u)^2}. \tag{22}$$

Comparing the profit with the null network given in Corollary 1, we immediately see that the firm's profit with a decentralized network is higher.

The main result of this section compares the firm's profit from a centralized network with that from a decentralized network. To make the comparison, we assume that all users in the decentralized network have the same spillover as the neighbors in the centralized network; that is, $\delta_u = \delta_n$. In this way we can focus on the lead user's spillover δ_l, since it is the presence of the expert/lead user that is the major difference between the two networks.

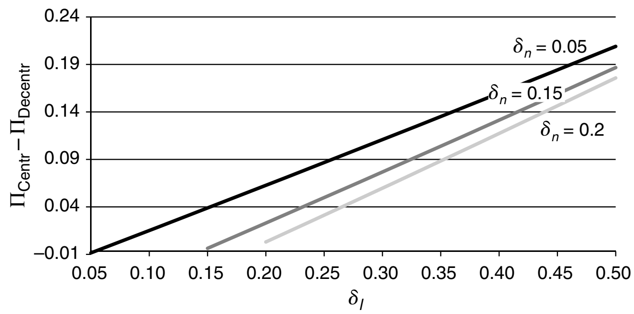
Theorem 2. *There exist critical values $\hat{\delta}_1$ and $\hat{\delta}_n$ of the lead user's and neighbor's spillovers such that, for $\delta_n < \hat{\delta}_n$ and $\delta_1 > \hat{\delta}_1$, environments with centralized externality patterns among users are more profitable for the firm than environments with decentralized externality patterns.*

Proof. See the appendix.

To understand the above, first note that if $\delta_n = 0$, the decentralized network reduces to the null network. We have already shown in part (b) of Theorem 1 that the firm is better off under a centralized network than with the null network. Using continuity arguments, the same holds for small δ_n. Now, the decentralized network does not have a lead user, and the firm's profit clearly does not depend on δ_l. On the other hand, the firm's profit with the centralized network increases in δ_l, as seen in Theorem 1, part (a). So if we let $\delta_u = \delta_n$ and allow δ_l to increase, the profits of the centralized network will eventually exceed that of the decentralized one. The only question is, would we get the same result for the allowed values of the spillover parameter, $\delta_l \leq 1$? If we set $\delta_l = 1$, one can see that for values of δ_n that are not too large, the firm's profit under the decentralized network is indeed lower than under a centralized one. The decentralized network has more links than the centralized network, which shows that the centralized network can be more profitable because of the pattern of links and not the number of links. Our finding is consistent with the networks literature, which has shown that the *pattern* of links matters. More importantly, we demonstrate exactly how it matters in our specific context. In co-creation it can be more profitable for the firm when the customers form a centralized pattern of links among themselves than when they form a decentralized pattern. As shown below, this is true even when the spillovers among users are the same in both cases.

Note that it is possible for the centralized network of externalities to dominate the decentralized one even when the lead user and the neighbors have the same spillovers on each other, $\delta_l = \delta_n$. This occurs when the minimum of the profit difference $(\Pi^{\text{Centr}} - \Pi^{\text{Decentr}})$ is positive, where the minimum difference corresponds to $\delta_l = \delta_n$ (recall that, by assumption, $\delta_l \geq \delta_n$). Thus the centralized network can dominate the decentralized network even when all the spillovers in both networks are exactly the same; $\delta_u = \delta_n = \delta_l$. For example, with $c = 1$, $\theta = 1$,

Figure 3 **Difference in Profit Between Centralized and Decentralized Networks vs. δ_l**



$\delta_u = \delta_l = \delta_n = 0.2$, and U ranging from 10 to 25, the profits of the centralized structure exceed those of the decentralized one, with improvement values ranging from 0.4% to 0.8%.

Figure 3 depicts the profit difference as a function of the lead user's spillover δ_l for several values of the neighbor spillover. Since $\delta_l \geq \delta_n$, each graph starts at the point $\delta_l = \delta_n$. The graph shows that the profit difference is indeed increasing in δ_l, but it is also higher for lower values of δ_n.

5. Different Cost and Benefit Functions

To check the robustness of our previous results, we solved the equilibria of our model allowing for different specifications of the cost and benefit functions. We began by assuming a linear benefit function and generalizing the cost to the family of convex functions $c(x) = cx^p$ with $p > 1$. The appropriate FOCs do not allow for a closed-form analytical solution, but they provide for a unique numerical solution. The graph in Figure 4 shows the difference between the firm's profits in centralized and decentralized externality networks as a function of δ_l for the following parameter values: $N = 10$, $\delta_f = 0.25$, and $\delta_n = \delta_u = 0.1$. This clearly demonstrates that Theorem 2 still holds.

Figure 5 demonstrates that, for the same parameter values, Theorem 2 holds even when we assume quadratic cost and change the benefit function.

Figure 4 **Difference in Profits for Various Cost Functions vs. δ_l**

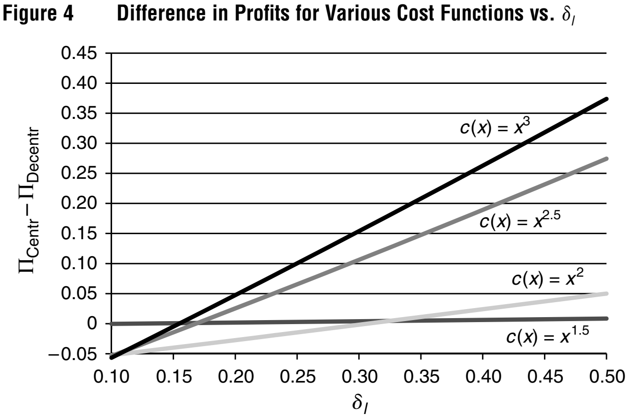
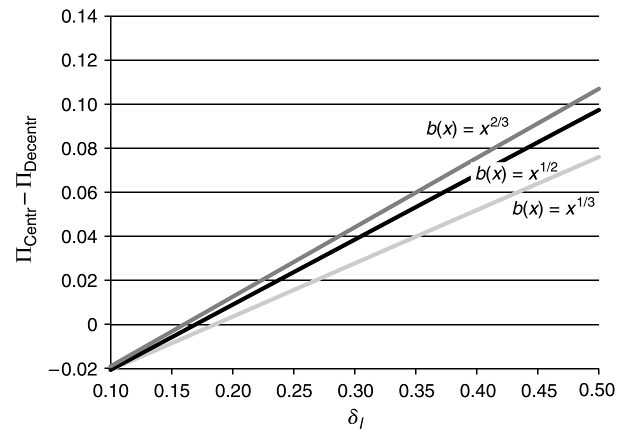


Figure 5 **Difference in Profits for Various Benefit Functions vs. δ_l**



6. Centralized Pattern of Externalities with a Single Price

In §4.2 we analyzed the centralized externality pattern where the firm price discriminates between the lead user and neighbors based on its knowledge of the degrees of the nodes. In this section we examine the centralized network where the monopolist firm sets a single price for both types of users. The main purpose here is to compare the potential advantage of price discrimination in a co-creation setting.

Though the main result of this section (see Theorem 3) holds for the general centralized network where both the lead user and neighbors exert effort as in §4.2, for analytical simplicity and to clearly bring out the forces involved, we will present the case where only the firm and lead user exert effort and the neighbors do not.

The total efforts at a neighbor's node and at the lead user's node $\tilde{e}_n$ and $\tilde{e}_l$ are, respectively,

$$\begin{aligned} \tilde{e}_n &= \delta_f e_f + \delta_l e_l \quad \text{and} \\ \tilde{e}_l &= e_l + \delta_f e_f. \end{aligned} \tag{23}$$

The consumer surpluses are $CS_i = b_i(\tilde{e}_i) - ce_i^2 - p + \varepsilon$, $i = l, n$. The *single* price charged to all the agents is p. Proceeding similarly to §4.2, the probability that a customer of type i purchases the product is $(\theta + b(\tilde{e}_i) - p)/(2\theta)$, where $i = 1, n$.

The expected profit of the firm is[10] $\Pi^{SP} = ((\theta + b(\tilde{e}_l) - p)/(2\theta))p + N((\theta + b(\tilde{e}_n) - p)/(2\theta))p - ce_f^2$. The optimal price in terms of the efforts is

$$p = \frac{\theta(N+1) + b(\tilde{e}_l) + Nb(\tilde{e}_n)}{2(N+1)}. \tag{24}$$

[10] We denote the various quantities in the single-price case with the superscript SP.

The lead user's surplus in terms of price is the same as in §4.2, and after incorporating the optimal price from (24), the expected surplus becomes

$$E[CS_1] = \frac{1}{4\theta}\left(\frac{2N+1}{2(N+1)}b(\tilde{e}_l) - \frac{N}{2(N+1)}b(\tilde{e}_n) + \frac{\theta}{2}\right)^2 - ce_l^2.$$

The expected profit of the firm after incorporating price is $\Pi^{SP} = [\theta(N+1) + b(\tilde{e}_l) + Nb(\tilde{e}_n)]^2/(8(N+1)\theta) - ce_f^2$. The FOC for the lead user is

$$\frac{1}{4(N+1)\theta}\left(\frac{2N+1}{2(N+1)}(e_l+\delta_f e_f) - \frac{N}{2(N+1)}(\delta_f e_f + \delta_l e_l) + \frac{\theta}{2}\right) \cdot (2N+1-\delta_l N) - 2ce_l = 0.$$

The firm's FOC is

$$\frac{1}{4(N+1)\theta}\left[(N+1)\theta + (1+\delta_l N)e_l + (N+1)\delta_f e_f\right] \cdot (N+1)\delta_f - 2ce_f = 0. \tag{25}$$

Solving the FOCs simultaneously gives the optimal efforts e_l^{SP} and e_f^{SP}. These expressions are complex and are recorded in the appendix in (35) and (36), respectively. Substituting back gives the firm's optimal profit in the centralized network when it does not price discriminate and charges only a single price, Π^{SP}. This is given in the appendix in Equation (37).

To compare the monopolist's profit with and without price discrimination, we need to provide the solution for an environment where the firm price discriminates between the lead user and the neighbors in a centralized network of externalities, but where only the lead user and the firm itself exert effort (note that the analysis in §4.2 had the neighbors also exerting effort). The details are straightforward and are relegated to the appendix. The optimal efforts of the lead user and firm when the firm price discriminates between lead users and neighbors are, respectively,[11]

$$e_l^{PD} = \frac{8}{120 - \delta_f^2[16 + (15+\delta_l)N]} \quad \text{and} \quad e_f^{PD} = \frac{\delta_f[16 + (15+\delta_l)N]}{120 - \delta_f^2[16+(15+\delta_l)N]}. \tag{26}$$

The optimal profit is

$$\Pi^{PD} = \frac{2{,}048 + 8(15+\delta_l)^2 N - \delta_f^2[16+(15+\delta_l)N]^2}{(120-\delta_f^2[16+(15+\delta_l)N])^2}. \tag{27}$$

The following theorem states the main result of this section.

[11] To simplify expressions, we set $c=1$ and $\theta=1$, and we denote the various quantities in the price discrimination case with the superscript *PD*.

Theorem 3. *When a monopolist co-creates a product with customers who form a centralized pattern of externalities, its optimal profit is higher if it charges a single price to all agents than if it price discriminates between the lead user and neighbors. Both the lead user and firm's optimal efforts are also higher with a single price.*

Proof. See the appendix.

We find that even in the more general centralized network, with both the lead user and neighbors exerting co-creation effort (as in §4.2), the monopolist is better off without price discrimination compared with a situation where it price discriminates. This result stands in contrast to the situation in traditional markets without co-creation, where customers do not exert product development efforts—in those cases, price discrimination is always better for the monopolist.[12] The reason for this result is that price discrimination by the monopolist has two opposite effects: First, it can better extract the surplus of heterogeneous customers; this is the standard positive effect of price discrimination on profits, as is well documented in the literature. Second, because of the reduced surplus left with them, consumers rationally reduce their co-creation efforts; this reduces the "benefit" of the product as captured by the total efforts exerted by all network partners. Because the firm's price is increasing in the efforts exerted, this second force has a negative effect on firm profits, and Theorem 3 shows that the negative effect dominates. The second effect is clearly seen from the fact that for the lead user, $e_l^{SP} > e_l^{PD}$. In fact, in so far as the efforts are concerned, there is a double effect when the firm price discriminates—not only does the lead user exert less effort but so does the firm. That is, $e_f^{SP} > e_f^{PD}$ (see the proof of Theorem 3 in the appendix).

Our finding above has practical ramifications. Despite the opportunity to price discriminate, in many B2B settings, firms often have a strict "no-discount" book price policy. Our analysis shows that for co-creation, this self-imposed lack of pricing flexibility may actually be profit enhancing. This result is reminiscent of similar game-theoretic findings where a commitment by a monopolist to a certain action (thus limiting its flexibility) causes a change in customer behavior that is profitable to the monopolist. For example, a monopolistic firm that can profitably employ behavior-based price discrimination may wish instead to commit to a single price over time. This commitment will change the purchase timing of the customers and lead to higher

[12] In the appendix we analyze the case of a centralized network where *only* the firm exerts effort, so there is no co-creation with the customers. We analyze both the cases with and without price discrimination and show that if there is no co-creation, the monopolist is better off with price discrimination, as is to be expected.

profits (see Fudenberg and Villas-Boas 2006 for a survey and Fudenberg and Tirole 2000 for a similar result in a competitive setting). However, the mechanism through which this occurs in co-creation (changing consumers' incentives to expend effort on product development) is different than the mechanism through which it occurs in the literature on behavior-based pricing, changing consumers' timing of purchase.

7. Conclusion

The idea that producers and consumers collaborate to co-create goods and services has a long history in business and marketing thought. In B2B contexts, co-creation has always been a common phenomenon. Recently, however, with increases in information technology support, flexible manufacturing, and advanced user interfaces, it has become increasingly feasible for customers to participate in the production process even in the B2C world. Despite the great deal of interest in co-creation, there has been little academic research in this field, especially with a view to developing economic models of this phenomenon. This paper takes a first step toward addressing this gap.

We model co-creation and the interaction between firm and users/consumers as a network of externalities. Each user is located on a node of the network. Any two users whose innovation efforts affect each other are depicted as connected nodes. This particular network topology can be used to capture the specific pattern of production externalities between different users; of course, the firm is assumed to be connected to all of them. This setting allows us to model the important role played by lead users in the innovation process. Lead users can be modeled in the network simply as central nodes with many connections to other nodes, each with a user on it.

Variants of our model can be used to analyze many other phenomena that involve team production or joint production. In team selling, especially for complex products, some team members are more likely to benefit other members. These asymmetric effects in sales teams can be captured by the pattern of links in a network and by different spillover coefficients. As another example, it is well known that people often construct their preferences for novel products rather than having preformed preferences. Such construction occurs over social networks wherein people discuss novel product features with each other. Clearly, these social connections can be modeled using our network approach, and the disproportionate influence of some members can be captured by the number of connections and the size of the spillover effects.

In our specific model, we find that the firm's profit from co-creation dominates its profit in a no co-creation environment. Also, pricing by the firm reverses the usual free-riding effect found in public goods, so that in co-creation, the firm's effort increases in the number of users it is connected to. If there are connections among users such that they form a centralized pattern of externalities with a lead user in the center, the optimal profit is higher than that without user connections. Furthermore, the centralized externality network can generate higher profit than the decentralized one even though the latter has more links, even in cases where the spillovers among users are identical in both cases. This highlights the importance of the pattern of links in co-creation production networks as well as the number of links. Another insight our model provides is that as the spillovers from the lead user to the neighbors increase, all parties can benefit, including the neighbors, the firm, and the lead user herself. Finally, we find that it is better for the monopolist firm to charge a single price in a centralized network than to price discriminate between the lead user and her connected neighbors.

Our analysis has clear managerial implications. First, firms should try to identify and develop alliances with users that are central to the network, i.e., users who have production externalities with many other users, *even when* such central users do not have lower marginal costs or more product knowledge compared to the firm. The user innovation literature exhorts firms to identify and build alliances with lead users who can foresee emerging trends in the marketplace (von Hippel 1986). Another economic reason for a firm to ally with customers is that customers may have lower costs or higher innovation ability than the firm, especially in B2B contexts. Clearly, it is difficult to identify customers who can foresee future trends and who have lower costs or higher abilities. Thus identifying lead users according to the usual prescriptions may be difficult. The lead user literature views the identification of lead users as a major obstacle. We show that it may be beneficial to just identify users who are linked to many other customers informally or through strategic alliances and other types of research and development networks. In some circumstances a user's network of connections may be more publicly observable than its costs and abilities, which are internal to its functioning.

Second, in terms of pricing regimes, firms should consider their effect on the effort exerted in the innovation and production stages. It is sometimes better for the firm to commit to charging a single price than to price discriminate in the presence of lead users in the network. In B2B settings, despite the opportunity to price discriminate, many firms often have a strict no-discount book price policy. Our analysis shows that for co-creation, this self-imposed lack of pricing flexibility may actually be profit enhancing.

Appendix

Proof of a Benchmark Without Co-Creation

Because there is no co-creation by the users, the effort at a given user's node only comes from the firm. Thus, $\tilde{e}_u = \delta_f e_f$, and the surplus a user receives is $b(\tilde{e}_u) - p_u + \varepsilon$. Given a price p_u, a user will purchase the product with probability given by $\Pr\{\varepsilon \geq p_u - b(\tilde{e}_u)\}$. The probability that a user purchases is $(\theta + b(\tilde{e}_u) - p_u)/(2\theta)$. The expected profit of the firm is

$$\Pi_f = U\left(\frac{\theta + b(\tilde{e}_u) - p_u}{2\theta}\right)p_u - ce_f^2. \quad (28)$$

The firm will choose p_u optimally, and the FOC gives the price in terms of the efforts:

$$p_u = \frac{\theta + b(\tilde{e}_u)}{2}. \quad (29)$$

As in §4.1, the profit after incorporating price is

$$\Pi = U\frac{(\theta + b(\tilde{e}_u))^2}{8\theta} - ce_f^2. \quad (30)$$

Specializing the benefit function to $b(e) = e$, the firm's FOC gives $\partial E[\Pi]/\partial e_f = U((\theta + \delta_f e_f)/(4\theta))\delta_f - 2ce_f = 0$. This gives the optimal effort of the firm as

$$e_f^{\text{No-Co-creation}} = \frac{U}{8}\frac{\delta_f}{c - \delta_f^2 U/(8\theta)}. \quad (31)$$

After simplifications, the optimal profit of the firm without co-creation is

$$\Pi^{\text{No-Co-creation}} = \frac{U\theta}{8}\left\{1 + \frac{U}{8\theta}\;\frac{\delta_f^2}{c - \delta_f^2 U/(8\theta)}\right\}. \quad (32)$$

Clearly, for the effort and the profit to be positive, we need $c > \delta_f^2 U/(8\theta)$, and we make this assumption. We can see that the optimal effort increases in U. $\square$

Proof of Theorem 1. (a) The lead user's expected surplus is $E[CS_1] = (1/(16\theta))(\theta + b(\tilde{e}_l))^2 - ce_l^2$. After substituting the optimal efforts, the derivative of the expected surplus with respect to (w.r.t.) δ_1 is

$$\begin{aligned}\frac{\partial E[CS_l]}{\partial \delta_l} &= \frac{1}{8\theta}(\theta + e_l^{\text{Centr}} + \delta_f e_f^{\text{Centr}} + N\delta_n e_n^{\text{Centr}})\\ &\quad\cdot\left(\frac{\partial e_l^{\text{Centr}}}{\partial \delta_l} + \delta_f\frac{\partial e_f^{\text{Centr}}}{\partial \delta_l} + N\delta_n\frac{\partial e_n^{\text{Centr}}}{\partial \delta_l}\right)\\ &\quad - 2ce_l^{\text{Centr}}\frac{\partial e_l^{\text{Centr}}}{\partial \delta_l}.\end{aligned}$$

Collecting terms involving $\partial e_l^{\text{Centr}}/\partial \delta_l$, we can rewrite the derivative as

$$\begin{aligned}\frac{\partial E[CS_l]}{\partial \delta_l} &= \left\{\frac{1}{8\theta}(\theta + e_l^{\text{Centr}} + \delta_f e_f^{\text{Centr}} + N\delta_{nl} e_n^{\text{Centr}}) - 2ce_l^{\text{Centr}}\right\}\\ &\quad\cdot\frac{\partial e_l^{\text{Centr}}}{\partial \delta_l} + \frac{1}{8\theta}(\theta + e_l^{\text{Centr}} + \delta_f e_f^{\text{Centr}} + N\delta_l e_n^{\text{Centr}})\\ &\quad\cdot\left(\delta_f\frac{\partial e_f^{\text{Centr}}}{\partial \delta_l} + N\delta_n\frac{\partial e_n^{\text{Centr}}}{\partial \delta_l}\right).\end{aligned} \quad (33)$$

From the lead user's FOC given in (15), the term in curly braces on the right-hand side (RHS) of (33) is zero. Also, from (18) and (19), it is clear that $\partial e_n^{\text{Centr}}/\partial \delta_l > 0$ and $\partial e_f^{\text{Centr}}/\partial \delta_l > 0$. Since the efforts are positive by assumption, we can therefore conclude that $\partial E[CS_l]/\partial \delta_l > 0$. Using a similar approach, it can be shown that $\partial E[CS_n]/\partial \delta_l > 0$; i.e., the neighbor's surplus increases in δ_l.

Now consider the derivative of the firm's profit given in (14), after substituting the optimal efforts w.r.t. δ_l:

$$\begin{aligned}\frac{\partial \Pi^{\text{Centr}}}{\partial \delta_l} &= \frac{1}{4\theta}(\theta + e_l^{\text{Centr}} + \delta_f e_f^{\text{Centr}} + N\delta_l e_n^{\text{Centr}})\\ &\quad\cdot\left(\frac{\partial e_l^{\text{Centr}}}{\partial \delta_l} + \delta_f\frac{\partial e_f^{\text{Centr}}}{\partial \delta_l} + N\delta_n\frac{\partial e_n^{\text{Centr}}}{\partial \delta_l}\right)\\ &\quad + \frac{N}{4\theta}(\theta + e_n^{\text{Centr}} + \delta_f e_f^{\text{Centr}} + \delta_l e_l^{\text{Centr}})\\ &\quad\cdot\left(\frac{\partial e_n^{\text{Centr}}}{\partial \delta_l} + \delta_f\frac{\partial e_f^{\text{Centr}}}{\partial \delta_l} + e_l^{\text{Centr}} + \delta_l\frac{\partial e_l^{\text{Centr}}}{\partial \delta_l}\right)\\ &\quad - 2ce_f^{\text{Centr}}\frac{\partial e_f^{\text{Centr}}}{\partial \delta_l}.\end{aligned}$$

Rearranging and combining all terms involving $\partial e_f^{\text{Centr}}/\partial \delta_l$, we get

$$\begin{aligned}\frac{\partial \Pi^{\text{Centr}}}{\partial \delta_l} &= \left\{\frac{1}{4\theta}(\theta + e_l^{\text{Centr}} + \delta_f e_f^{\text{Centr}} + N\delta_l e_n^{\text{Centr}})\delta_f\right.\\ &\quad\left. + \frac{N}{4\theta}(\theta + e_n^{\text{Centr}} + \delta_f e_f^{\text{Centr}} + \delta_{1n} e_l^{\text{Centr}})\delta_f - 2ce_f^{\text{Centr}}\right\}\\ &\quad\cdot\frac{\partial e_f^{\text{Centr}}}{\partial \delta_l} + \frac{1}{4\theta}(\theta + e_l^{\text{Centr}} + \delta_f e_f^{\text{Centr}} + N\delta_{nl} e_n^{\text{Centr}})\\ &\quad\cdot\left(\frac{\partial e_l^{\text{Centr}}}{\partial \delta_{1l}} + N\delta_n\frac{\partial e_n^{\text{Centr}}}{\partial \delta_l}\right) + \frac{N}{4\theta}(\theta + e_n^{\text{Centr}} + \delta_f e_f^{\text{Centr}}\\ &\quad + \delta_l e_l^{\text{Centr}})\left(\frac{\partial e_n^{\text{Centr}}}{\partial \delta_l} + e_l^{\text{Centr}} + \delta_l\frac{\partial e_l^{\text{Centr}}}{\partial \delta_l}\right).\end{aligned} \quad (34)$$

The entire term in curly braces on the RHS of (34) is zero by the firm's FOC given in (16). Again, from (17) and (18), it is clear that $\partial e_l^{\text{Centr}}/\partial \delta_l > 0$ and $\partial e_n^{\text{Centr}}/\partial \delta_l > 0$. Since all the efforts are positive by assumption, we therefore proved that $\partial \Pi^{\text{Centr}}/\partial \delta_l > 0$.

(b) In part (a) we have shown that $\partial \Pi^{\text{Centr}}/\partial \delta_l > 0$. Using the same technique, it can also be shown that $\partial \Pi^{\text{Centr}}/\partial \delta_n > 0$. Thus the minimum profit with the centralized network (minimum w.r.t. δ_l and δ_n) corresponds to $\delta_l \to 0$ and $\delta_n \to 0$. This minimum profit is $\Pi^{\text{Centr}}|_{\delta_l \to 0,\, \delta_n \to 0} = 4c\theta^2 U(8c\theta - \delta_f^2 U)/(16c\theta - 2\delta_f^2 U - 1)^2$. From Lemma 1, this is the same as the profit with a null network, Π^{Null}. Thus, for all positive δ_l and δ_n, we have $\Pi^{\text{Centr}} > \Pi^{\text{Null}}$.

Also, $\partial e_f^{\text{Centr}}/\partial \delta_l > 0$ and $\partial e_f^{\text{Centr}}/\partial \delta_n > 0$ imply that the minimum effort of the firm with the centralized network corresponds to $\delta_l \to 0$ and $\delta_n \to 0$. With $U = N + 1$, the minimum effort is $e_f^{\text{Centr}}|_{\delta_l \to 0,\, \delta_n \to 0} = 2\theta\delta_f U/(16c\theta - 2\delta_f^2 U - 1)$. From Lemma 1, we can see that this is the same as the firm's effort with a null network, e_f^{Null}. Thus, for all positive δ_{1n} and δ_{nl}, we have $e_f^{\text{Centr}} > e_f^{\text{Null}}$. In fact, it can be similarly shown that both the lead user and the neighbors in the centralized network exert more effort than a typical user in the null network; i.e., $e_l^{\text{Centr}} > e_u^{\text{Null}}$ and $e_n^{\text{Centr}} > e_u^{\text{Null}}$. $\square$

Proof of Theorem 2. In the proof of Theorem 1, we already established that $\partial \Pi^{\text{Centr}}/\partial \delta_l > 0$; that is, the profit of the firm with the centralized network increases in the spillover from the lead user to her neighbors. Also, it is clear that the firm's profit with the decentralized network is independent of δ_{1n}. If $\Pi^{\text{Centr}} > \Pi^{\text{Decentr}}$ even at the minimum value of δ_l, which is $\delta_l = \delta_n$, then we are done. If $\Pi^{\text{Centr}} < \Pi^{\text{Decentr}}$ at $\delta_l = \delta_n$, then the difference $(\Pi^{\text{Centr}} - \Pi^{\text{Decentr}})$ increases as δ_l increases from its minimum value δ_n. We will show that $\Pi^{\text{Centr}} > \Pi^{\text{Decentr}}$ when Π^{Centr} is evaluated at $\delta_l = 1$. This will establish that there exists a value $\hat{\delta}_l$ of the lead user's spillover such that the centralized network is more profitable than the decentralized network for every $\delta_l > \hat{\delta}_l$.

Since the expressions are cumbersome, we will simplify them by setting $c = 1$ and $\theta = 1$. These parameters are not of interest to us and do not affect any results in the paper.

Consider $\Pi^{\text{Centr}}|_{\delta_1 \to 1}$, the firm's profit evaluated at $\delta_l = 1$. We need to show that $\Pi^{\text{Centr}}|_{\delta_l \to 1} > \Pi^{\text{Decentr}}$. Now, using the same techniques as in Theorem 1, we can show that $\partial \Pi^{\text{Centr}} / \partial \delta_{nl} > 0$. Since this holds for any δ_l, including $\delta_l = 1$, therefore $\Pi^{\text{Centr}}|_{\delta_l \to 1, \delta_n \to 0}$ is clearly a lower bound of $\Pi^{\text{Centr}}|_{\delta_l \to 1}$. In other words, $\Pi^{\text{Centr}}|_{\delta_l \to 1, \delta_n \to 0} < \Pi^{\text{Centr}}|_{\delta_l \to 1}$. Hence, we are done if we can show that $\Pi^{\text{Centr}}|_{\delta_l \to 1, \delta_n \to 0} > \Pi^{\text{Decentr}}$. Substituting $\delta_l = 1$ and $\delta_n = 0$, the profit from the centralized network reduces to

$$\Pi^{\text{Centr}}|_{\delta_l \to 1, \delta_n \to 0} = \frac{4[2{,}048U - 248 - \delta_f^2 (16U - 1)^2]}{225 - 2\delta_f^2 (16U - 1)]^2}.$$

The firm's profit with the decentralized network is $\Pi^{\text{Decentr}} = (4U(8 - \delta_f^2 U))/(15 - 2\delta_f^2 U - 2\delta_n)^2$. Observe that $\Pi^{\text{Centr}}|_{\substack{\delta_l \to 1 \\ \delta_n \to 0}}$ does not depend on δ_n, which appears only in the denominator of Π^{Decentr}. Therefore, $\Pi^{\text{Centr}}|_{\substack{\delta_l \to 1 \\ \delta_n \to 0}} > \Pi^{\text{Decentr}}$ if δ_n is not too large. Specifically, a sufficient, but by no means necessary, condition is

$$\delta_n < \hat{\delta}_n \equiv \frac{1}{2} \left\{ 15 - 2\delta_f^2 U - [225 - 2\delta_f^2 U (16U - 1)] \cdot \sqrt{\frac{U(8 - \delta_f^2 U)}{2{,}048U - 248 - \delta_f^2 U (16U - 1)^2}} \right\}. \quad \square$$

Proof of Theorem 3. We will prove this result in three steps.

Step 1. We will show that $e_l^{SP} > e_l^{PD}$ and $e_f^{SP} > e_f^{PD}$. That is, the efforts of the lead user and firm are higher when charging a single price than they are when using price discrimination. We will also show that $\partial e_l^{SP} / \partial \delta_f > \partial e_l^{PD} / \partial \delta_f$.

Step 2. Using the inequalities established in part (a) above, we will show that $\partial (\Pi^{SP} - \Pi^{PD}) / \partial \delta_f > 0$. That is, the difference $(\Pi^{SP} - \Pi^{PD})$ increases in δ_f, and so the minimum difference corresponds to $\delta_f = 0$.

Step 3. Finally, we will show that the difference of profits evaluated at $\delta_f = 0$ is positive. That is, the minimum difference $(\Pi^{SP} - \Pi^{PD})|_{\delta_f = 0} > 0$. Then, by (b) above, the difference is always positive.

Proof of Step 1. We first record the optimal efforts and firm profit with the centralized network when the firm charges a single price:

$$e_l^{SP} = \big[4(N+1)[1 + (2 - \delta_l)N]\big] \cdot \Big[4\big\{16(N+1)^2 + [1 + (2 - \delta_l)N]\big\} - \delta_f^2 (N+1) \cdot \{8(N+1)^2 - (1 - \delta_l)N[1 + (2 - \delta_l)N]\}\Big]^{-1}, \quad (35)$$

$$e_f^{SP} = \big[\delta_f (N+1)\{8(N+1)^2 - (1 - \delta_l)N[1 + (2 - \delta_l)N]\}\big] \cdot \Big[4\big\{16(N+1)^2 + [1 + (2 - \delta_l)N]\big\} - \delta_f^2 (N+1) \cdot \{8(N+1)^2 - (1 - \delta_l)N[1 + (2 - \delta_l)N]\}\Big]^{-1}. \quad (36)$$

Substituting back gives the firm's optimal profit as

$$\Pi_f^{SP} = \Big[(N+1)[8 - \delta_f^2 (N+1)]\big\{8(N+1)^2 - (1 - \delta_l) \cdot N[1 + (2 - \delta_l)N]\big\}^2\Big] \cdot \Big[\big(4\{16(N+1)^2 + [1 + (2 - \delta_l)N]\} - \delta_f^2 (N+1) \cdot \{8(N+1)^2 - (1 - \delta_l)N[1 + (2 - \delta_l)N]\}\big)^2\Big]^{-1}. \quad (37)$$

The optimal efforts and firm profit when the firm price discriminates, but only the lead user and firm exert effort, are

$$e_l^{PD} = \frac{8}{120 - \delta_f^2 [16 + (15 + \delta_l)N]}, \qquad e_f^{PD} = \frac{\delta_f [16 + (15 + \delta_l)N]}{120 - \delta_f^2 [16 + (15 + \delta_l)N]}. \quad (38)$$

The optimal profit is

$$\Pi^{PD} = \frac{2{,}048 + 8(15 + \delta_l)^2 N - \delta_f^2 [16 + (15 + \delta_l)N]^2}{\{120 - \delta_f^2 [16 + (15 + \delta_l)N]\}^2}. \quad (39)$$

For notational simplicity, we write the efforts as $e_l^{SP} = 4(N + 1)[1 + (2 - \delta_l)N]/D_1$ and $e_1^{PD} = 8/D_2$, where $D_1 = 4\{16(N + 1)^2 + [1 + (2 - \delta_l)N]\} - \delta_f^2 (N + 1)\{8(N + 1)^2 - (1 - \delta_l)N[1 + (2 - \delta_l)N]\}$ and $D_2 = 120 - \delta_f^2 [16 + (15 + \delta_l)N]$. Since the numerators of e_l^{SP} and e_l^{PD} are positive, the denominators are $D_1 > 0$ and $D_2 > 0$. Simple algebra shows that

$$e_l^{SP} - e_l^{PD} = \frac{4(1 - \delta_l)N[17 + (18 - \delta_l)][8 - \delta_f^2 (N+1)]}{D_1 D_2} > 0. \quad (40)$$

Similarly,

$$e_f^{SP} - e_f^{PD} = \frac{4\delta_f (1 - \delta_l)N[17 + (18 - \delta_l)](1 + \delta_l N)}{D_1 D_2} > 0. \quad (41)$$

The inequality in (40) follows because, by assumption from §4.1, $8c\theta - \delta_f^2 U > 0$. With $c = 1$, $\theta = 1$, and $U = N + 1$, the condition becomes $8 - \delta_f^2 (N+1) > 0$.

The derivative of e_l^{SP} w.r.t. δ_f is

$$\frac{\partial e_l^{SP}}{\partial \delta_f} = \frac{8\delta_f (N+1)^2 [1 + (2 - \delta_l)N]\{8 + (15 + \delta_l)N + [6 + (3 - \delta_l)\delta_l]N^2\}}{D_1^2}. \quad (42)$$

The derivative of e_l^{PD} w.r.t. δ_f is

$$\frac{\partial e_l^{PD}}{\partial \delta_f} = \frac{16\delta_f [16 + (15 + \delta_l)N]}{D_2^2}. \quad (43)$$

We can show that $(\partial / \partial \delta_{1n})(\partial e_1^{SP} / \partial \delta_f) < 0$. Thus, the derivative $\partial e_1^{SP} / \partial \delta_f$ decreases in δ_l and attains its minimum value at $\delta_l = 1$. The minimum value of the derivative is $(\partial e_l^{SP} / \partial \delta_f)|_{\delta_l = 1} = (4\delta_f (N+1))/[15 - 2\delta_f^2 (N+1)]^2$. Taking the difference between this minimum value of $\partial e_l^{SP}/\partial\delta_f$ and the RHS of (43) gives us

$$\left.\frac{\partial e_l^{SP}}{\partial\delta_f}\right|_{\delta_l=1}-\frac{\partial e_l^{PD}}{\partial\delta_f}=\frac{4\delta_f(1-\delta_l)N\{900-\delta_f^4(N+1)[16+(15+\delta_l)N]\}}{120-\delta_f^2[16+(15+\delta_l)N]^2[15-2\delta_f^2(N+1)]^2}. \quad (44)$$

Clearly, the RHS of (44) will be positive, and we will have shown that $\partial e_l^{SP}/\partial\delta_f > \partial e_l^{PD}/\partial\delta_f$ if the expression in curly braces in the numerator is positive. We know from §4.1 that $16c\theta-1-2\delta_f^2U>0$. Since $c=1$, $\theta=1$, and $U=N+1$, this condition therefore reduces to $\delta_f^2(N+1)<7.5$. The expression in curly braces will be positive if $\delta_f^4(N+1)[16+(15+\delta_l)N]$ is less than 900. Setting $\delta_{1n}=1$ increases the value of the expression, which now becomes $16\delta_f^4(N+1)^2$. If this is less than 900, we are done. Since, by assumption, $\delta_f^2(N+1)<7.5$, therefore we indeed have $16\delta_f^4(N+1)^2<900$. This completes Step 1.

Proof of Step 2. We will take the derivatives of the firm's profits w.r.t. δ_f. With $c=1$ and $\theta=1$, the profits with a single price and under price discrimination are

$$\Pi^{SP}=\frac{1}{8(N+1)}\big[(N+1)+(1+\delta_lN)e_l^{SP}+(N+1)\delta_fe_f^{SP}\big]^2-(e_f^{SP})^2, \quad (45)$$

$$\Pi^{PD}=\frac{1}{8}[1+e_l^{PD}+\delta_fe_f^{PD}]^2+\frac{N}{8}[1+\delta_fe_f^{PD}+\delta_le_l^{PD}]^2-(e_f^{PD})^2. \quad (46)$$

The derivative of Π^{SP} w.r.t. δ_f is

$$\frac{\partial\Pi^{SP}}{\partial\delta_f}=\frac{1}{4(N+1)}\big((N+1)+(1+\delta_lN)e_l^{SP}+(N+1)\delta_fe_f^{SP}\big)\cdot\left((1+\delta_lN)\frac{\partial e_l^{SP}}{\partial\delta_f}+(N+1)\delta_f\frac{\partial e_f^{SP}}{\partial\delta_f}+(N+1)e_f^{SP}\right)-2e_f^{SP}\frac{\partial e_f^{SP}}{\partial\delta_f}. \quad (47)$$

Rearranging and combining all terms involving $\partial e_f^{SP}/\partial\delta_f$, we get

$$\frac{\partial\Pi^{SP}}{\partial\delta_f}=\left\{\frac{1}{4(N+1)}\big[(N+1)+(1+\delta_lN)e_l^{SP}+(N+1)\delta_fe_f^{SP}\big](N+1)\delta_f-2e_f^{SP}\right\}\frac{\partial e_f^{SP}}{\partial\delta_f}+\frac{1}{4(N+1)}\big[(N+1)+(1+\delta_lN)e_l^{SP}+(N+1)\delta_fe_f^{SP}\big]\cdot\left((1+\delta_lN)\frac{\partial e_l^{SP}}{\partial\delta_f}+(N+1)e_f^{SP}\right).$$

The term in curly braces is zero by the firm's FOC (Equation (25) in text), and so the derivative reduces to

$$\frac{\partial\Pi^{SP}}{\partial\delta_f}=\frac{1}{4(N+1)}\big[(N+1)+(1+\delta_lN)e_l^{SP}+(N+1)\delta_fe_f^{SP}\big]\cdot\left((1+\delta_lN)\frac{\partial e_l^{SP}}{\partial\delta_f}+(N+1)e_f^{SP}\right). \quad (48)$$

Using a similar approach for the price discrimination case, and using the firm's FOC (see (56)), we get

$$\frac{\partial\Pi^{PD}}{\partial\delta_f}=\frac{1}{4}\big[(1+\delta_lN)+(1+\delta_l^2N)e_l^{PD}+(1+\delta_lN)\delta_fe_f^{PD}\big]\frac{\partial e_l^{PD}}{\partial\delta_f}+\frac{1}{4}\big[(N+1)+(1+\delta_lN)e_l^{PD}+(N+1)\delta_fe_f^{PD}\big]e_f^{PD}. \quad (49)$$

Taking the difference between (48) and (49) and rearranging the terms gives us

$$\frac{\partial\Pi^{SP}}{\partial\delta_f}-\frac{\partial\Pi^{PD}}{\partial\delta_f}=\frac{1}{4}\left\{\left((1+\delta_lN)+\frac{(1+\delta_lN)^2}{N+1}e_l^{SP}+(1+\delta_lN)\delta_fe_f^{SP}\right)\frac{\partial e_l^{SP}}{\partial\delta_f}-\big[(1+\delta_lN)+(1+\delta_l^2N)e_l^{PD}+(1+\delta_lN)\delta_fe_f^{PD}\big]\frac{\partial e_l^{PD}}{\partial\delta_f}\right\}+\frac{1}{4}\Big\{((N+1)+(1+\delta_lN)e_l^{SP}+(N+1)\delta_fe_f^{SP})e_f^{SP}-\big[(N+1)+(1+\delta_lN)e_l^{PD}+(N+1)\delta_fe_f^{PD}\big]e_f^{PD}\Big\}. \quad (50)$$

Since we have established that $e_l^{SP}>e_l^{PD}$, $e_f^{SP}>e_f^{PD}$, and $\partial e_l^{SP}/\partial\delta_f>\partial e_l^{PD}/\partial\delta_f$, by inspecting the first set of curly braces, we can see that it is positive if $((1+\delta_lN)^2/(N+1))e_l^{SP}>(1+\delta_l^2N)e_l^{PD}$. We can show that this is indeed the case. The term in the second set of curly braces is clearly positive. Thus, we have established that $\partial(\Pi^{SP}-\Pi^{PD})/\partial\delta_f>0$.

Proof of Step 3. We will now evaluate the profits at $\delta_f=0$. The firm's profit with a single price is

$$\Pi^{SP}|_{\delta_f=0}=\frac{(N+1)(8+N[16+\delta_l+6N+(3-\delta_l)\delta_lN])^2}{2[5+(6-\delta_l)N]^2[3+(2+\delta_l)N]^2}. \quad (51)$$

With price discrimination, it is

$$\Pi^{PD}|_{\delta_f=0}=\frac{256+(15+\delta_l)^2N}{1{,}800}. \quad (52)$$

Simple algebra and some rearrangement shows that

$$\Pi^{SP}|_{\delta_f=0}-\Pi^{PD}|_{\delta_f=0}=\frac{A+BN+CN^2+DN^3+EN^4}{3{,}600[5+(6-\delta_l)N]^2[3+(2+\delta_l)N]^2}, \quad (53)$$

where $A=7{,}935+222\delta_l$, $B=4[5{,}759+\delta_l(2{,}386+15\delta_l)]$, $C=2\{10{,}734+\delta_l[13{,}658+(101-13\delta_l)\delta_l]\}$, $D=4\{1{,}584+\delta_l[6{,}150+\delta_l(629-\delta_l(209+\delta_l))]\}$, and $E=(18-\delta_l)\delta_l\{360+\delta_l[162-\delta_l(41+\delta_l)]\}$. All these are clearly positive, and we have thus established the minimum value of $(\Pi^{SP}-\Pi^{PD})>0$. Theorem 3 is proved. □

Price Discrimination in a Centralized Network Where Only the Lead User and Firm Exert Effort

The consumer surpluses are $CS_i=b_i(\tilde{e})_i^2-ce_i^2-p_i+\varepsilon$, $i=1,n$. The prices charged to the neighbor and lead user are p_n and p_l, respectively. Furthermore, let $b(\tilde{e}_n)=\tilde{e}_n=\delta_le_l+\delta_fe_f$ and $b(\tilde{e}_l)=\tilde{e}_l=e_l+\delta_fe_f$.

The expected profit of the firm is

$$\Pi = \left(\frac{\theta + b(\tilde{e}_l) - p_l}{2\theta}\right)p_l + N\left(\frac{\theta + b(\tilde{e}_n) - p_n}{2\theta}\right)p_n - ce_f^2. \quad (54)$$

The two prices in terms of the efforts are

$$p_i = \frac{\theta + b(\tilde{e}_i)}{2}, \quad i = 1, n. \quad (55)$$

The lead user's surplus after price is $CS_1 = (1/(16\theta))(b(\tilde{e}_l) + \theta)^2 - ce_l^2$. The neighbor does not exert effort; the expected profit of the firm after prices have been incorporated is $\Pi = (\theta + b(\tilde{e}_l))^2/(8\theta) + N((\theta + b(\tilde{e}_n))^2/(8\theta)) - ce_f^2$. The firm's FOC is

$$\frac{1}{4\theta}(e_l + \delta_f e_f + \theta)\delta_f + \frac{N}{4\theta}(\delta_f e_f + \delta_l e_l + \theta)\delta_f - 2ce_f = 0. \quad (56)$$

The lead user's FOC is similarly obtained. Solving these simultaneously, we get the optimal efforts as in (26), and substituting back gives us the firm's profit as in (27). □

Centralized Network with Only Firm's Effort: With and Without Price Discrimination

Price Discrimination: The consumer surpluses are $CS_i = b_i(\tilde{e}_i) - ce_i^2 - p_i + \varepsilon$, $i = 1, n$. The prices charged to the neighbor and lead user are p_n and p_l, respectively. Furthermore, let $b(\tilde{e}_n) = \tilde{e}_n = \delta_f e_f$ and $b(\tilde{e}_l) = l + \tilde{e}_l = l + \delta_f e_f$, $l > 0$. We introduce $l > 0$ merely to create heterogeneity between the lead user and neighbor so that there is a role for price discrimination. Because this is just an additive constant, none of the results in the paper would change if we had assumed $b(\tilde{e}_l) = l + \tilde{e}_l$ throughout. The expected profit of the firm and the prices in terms of effort remain the same as (54) and (55). Following the usual method, the optimal effort of the firm and the optimal profit are

$$e_f^{\text{TwoPrices}} = \frac{\delta_f(l + \theta U)}{8c\theta - \delta_f^2 U},$$
$$\Pi_f^{\text{TwoPrices}} = \frac{8c\theta(l^2 + 2l\theta + \theta^2 U) - \delta_f^2 l^2 (U-1)}{8\theta(8c\theta - \delta_f^2 U)}. \quad (57)$$

Single Price: The consumer surpluses and benefit functions remain as above. We record the optimal effort and profit of the firm:

$$e_f^{\text{OnePrice}} = \frac{\delta_f(l + \theta U)}{8c\theta - \delta_f^2 U}, \quad \Pi_f^{\text{OnePrice}} = \frac{c(l + \theta U)^2}{U(8c\theta - \delta_f^2 U)}. \quad (58)$$

Simple algebra shows that $\Pi_f^{\text{TwoPrices}} - \Pi_f^{\text{OnePrice}} = l^2(U-1)/(8\theta U) > 0$. Thus, when there is no co-creation, the monopolist firm is better off exercising price discrimination than committing to a single price. □

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