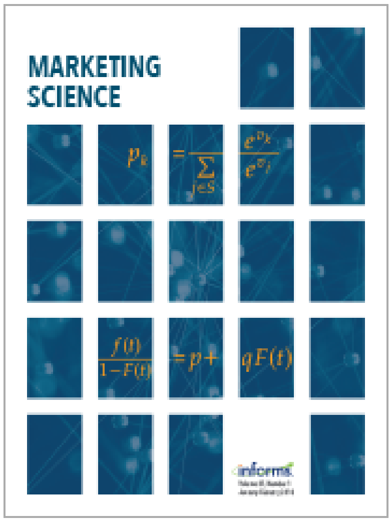

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Product Differentiation and Collusion Sustainability When Collusion Is Costly

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A widely debated question in recent years by both strategy theorists and antitrust practitioners is what role product differentiation between firms plays in their ability to sustain a collusive agreement in order to reduce the strength of competition and gain higher profits. This paper addresses the following question: What happens to the "product differentiation–collusion sustainability" relationship when setting up and maintaining an agreement is costly? We show that introducing collusion costs into the discussion has relevant implications. Indeed, sufficiently high collusion costs modify the underlying market structure, thus altering the product differentiation–collusion sustainability relationship with respect to the case where collusion costs are absent or low. In particular, if the gains from collusion are increasing (decreasing) with the degree of product differentiation, the relationship between product differentiation and collusion sustainability is always positive (negative), whereas if the gains from collusion are inverted U-shaped, the relationship is inverted U-shaped too. These results stress the importance of considering those markets where the coordination between firms is sufficiently costly as structurally different from those markets where coordination has no costs for firms.

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1. Introduction

Product differentiation is a widely explored issue by both marketing and industrial organization theorists. It has been discussed with regard to the positioning choices of firms in the product space and with regard to the effects that various positioning configurations have on, for example, the firms' equilibrium profits. Particularly relevant is the role that different market structures play in firms' ability to reduce the strength of competition. Reducing fierceness of competition is a key issue for competitors in the real world. Therefore, identifying those market configurations where a tacit collusive agreement is easier to enforce (i.e., where competition is less fierce, such as in a cartel) is a relevant theoretical and practical concern for strategy researchers as well as for antitrust authorities. Along these lines, the Horizontal Merger Guidelines (U.S. Department of Justice/Federal Trade Commission 1992) claim that a high level of product differentiation is anticollusive, as homogeneity increases "both the ability to reach terms of coordination and to detect or punish deviations from those terms" (§2.1). However, and in contrast to the clear-cut conclusion of those guidelines, the discussion among scholars about the impact of product differentiation on collusion sustainability is still open, and such a clear-cut assessment has not yet been reached. The reason for such theoretical ambiguity is that when firms producing differentiated goods compete, a high degree of substitutability determines a low equilibrium price and low equilibrium profits. In response, the firms may be willing to jointly raise the price above the competitive level. However, not only does the opportunity for collusion increase when product differentiation is low but the incentive to deviate from a cartel increases as well; as with scarcely differentiated products, any cheating firm captures a large share of the market when it lowers the price unilaterally (i.e., when it deviates). Therefore, because the degree of sustainability in equilibrium of a collusive agreement is determined by the level of collusive, deviation, and punishment profits, it is not surprising that, depending on the specific model adopted, the impact of product differentiation over collusion sustainability may have a different sign. The findings in the theoretical literature seem to confirm such ambiguity. For example, Chang (1991) and Häckner (1995), in a model with uniform pricing firms, find that the more the firms' products are similar, the more collusion is difficult to sustain in equilibrium, whereas the opposite occurs in the case of price-discriminating firms (Gupta and Venkatu 2002) or partial collusion (Friedman and Thisse 1993).[1]

[1] See also Tyagi (1999), who analyzes the implications of demand specifications for the relationship between product differentiation and collusion sustainability. See Colombo (2010) for more references on this issue.

All the above-mentioned articles are based on the assumption that collusion is costless—that is, there are no monitoring or implementation costs linked to the creation and the maintenance of a cartel. However, these costs are likely to exist, and they are often relevant. Indeed, setting up a collusive agreement entails the costs of negotiation, maintaining collusion requires the continuous monitoring of the other members of the cartel to avoid secret defections, and—given the illegal nature of cartels—avoiding prosecution from antitrust authorities generates additional costs. Several examples exist that demonstrate how setting up and maintaining a collusive agreement may be costly for the participating firms. For example, firms participating in a food flavor enhancers' cartel met several times a year to coordinate their actions.[2] These meetings required time, effort, and money. Sometimes, direct contacts may not be sufficient to maintain a collusive agreement, and a formal infrastructure has to be created to support collusion.[3] In other cases, colluding firms have used joint sales agency to enhance their collusive agreement.[4]

In this article, we introduce collusion costs in a highly general model of collusion encompassing previous specific models. The aim of this paper is to provide a general theory about the "product differentiation–collusion sustainability" relationship in the presence of (sufficiently high) collusion costs. We shall show that the impact of product differentiation on the degree of collusion sustainability can be easily characterized and that the introduction of sufficiently high collusion costs might dramatically modify the impact of product differentiation on collusion sustainability with respect to the case where collusion costs are absent or low. That is, the way the market structure (product differentiation) influences the ability of firms to reduce competition depends in a nonobvious way on the costs for firms to set up and maintain an agreement.

To the best of our knowledge, the only article considering this issue is Thomadsen and Rhee (2007). The authors show that when collusion costs are high enough, the relationship between product differentiation and collusion sustainability is always negative. However, this result is based on some specific assumptions on the sign of the derivative of the collusive, the deviation, and the punishment profits functions with respect to the level of product differentiation.[5] Our contribution is broader and provides a general theory about the impact of product differentiation on collusion sustainability without recourse to specific modelling assumptions: we shall show that the impact of product differentiation over the sustainability of collusion may also be positive and nonmonotonic with sufficiently high collusion costs, and we shall provide simple necessary and sufficient conditions for this to occur.

The rest of this paper proceeds as follows. In §2, we characterize the impact of collusion costs on the relationship between product differentiation and collusion sustainability in a general model of costly collusion. In §3, we illustrate our results by analyzing a Hotelling-style model with quantity-setting firms and a price discrimination model. Section 4 concludes with some final remarks.

2. A General Model of Costly Collusion

We consider a general model of collusion sustainability. The analysis is conducted here in the case of Nash reversion when a deviation from a cartel occurs (Friedman 1971).[6] We define $d \in [\underline{d}, \bar{d}]$ as the degree of differentiation between firms, where $\underline{d} \in R^+ \cup \{0\}$ and $\bar{d} \in R^+$ indicate the minimum and the maximum levels of product differentiation, respectively. We denote by $\pi^N(d)$, $\pi^C(d)$, and $\pi^D(d)$ the static Nash equilibrium profits, the per-period collusive profits, and the per-period deviation profits, respectively. The profit functions are continuous and once differentiable. To avoid trivial cases where collusion is never sustainable in equilibrium or always sustainable in equilibrium,

[2] *Food Flavour Enhancers* (case 37671, D.Comm. December 17, 2002).

[3] In the *Cast Iron and Steel Rolls* case (D.Comm. October 17, 1983) the firms involved in the cartel created a central office in charge of collecting and sharing relevant information (such as prices and outputs) across the cartel's members.

[4] Levenstein and Suslow (2006) contain a comprehensive discussion of the different types of costs emerging when a collusive agreement is constituted and participants act to preserve it from defection of members. Evidence of these costs is provided in numerous antitrust cases. Apart from the two previously cited cases, existence of collusion costs can also be found, for example, in *Imperial Chemical Industries Ltd. v. Commission of the European Communities* (ECJ July 14, 1972), *PVC II* (D.Comm. July 27, 1994), *British Sugar plc et al.* (D.Comm. October 14, 1998), *Industrial and Medical Gases* (case 36700, D.Comm. July 24, 2002), *Methylglucamine* (case 37978, D.Comm. November 27, 2002), and *Organic Peroxide* (case 37857, D.Comm. December 10, 2003).

[5] Namely, Thomadsen and Rhee (2007) assume that the gains from colluding with and from deviating from the agreement strictly decrease with product differentiation. Although these assumptions hold in some models discussed in literature, they do not hold in other common models, such as those discussed in §3.

[6] It can be easily shown that our main result applies identically also to the case of optimal punishment (Abreu 1988) once the Nash profits in the presented formulae are substituted by the average profits during the punishment stage, provided that the collusive profits (including the collusion costs) are higher than the average punishment profits. Details are available from the author upon request. The second example in §3 refers to a case of optimal punishment in the case of a deviation from the cartel.

it must be $\pi^D(d) \geq \pi^C(d) \geq \pi^N(d)$.[7] To save on notation, the argument of the profits functions shall be omitted in the rest of the article when not necessary.

Let us denote by δ the market discount factor, which indicates the weight attached by firms to the future. Collusion is sustainable as a subgame-perfect Nash equilibrium if and only if the discounted value of the profits that each firm obtains under collusion exceeds the discounted value of the profits that each firm obtains deviating from the cartel agreement. Formally stated, the following incentive-compatibility constraint must be satisfied in equilibrium for each firm participating in the cartel: $\sum_{t=0}^{\infty} \delta^t \pi^C(d) \geq \pi^D(d) + \sum_{t=1}^{\infty} \delta^t \pi^N(d)$. After rearranging, the constraint can be rewritten as $\delta \geq \delta^* \equiv (\pi^D(d) - \pi^C(d))/(\pi^D(d) - \pi^N(d))$. We shall refer to δ^* as the critical discount factor. The incentive-compatibility condition says that when the market discount factor is greater than the critical discount factor, collusion is sustainable in equilibrium; otherwise, it is not sustainable. Therefore, the critical discount factor represents a natural measure of the sustainability of the collusive agreement: the greater the δ^*, the smaller the set of market discount factors that support collusion (i.e., collusion is *more difficult* to sustain); conversely, the lower the δ^*, the larger the set of market discount factors that support collusion (i.e., collusion is *less difficult* to sustain).

We introduce a per-period cost of sustaining collusion, which is denoted by C. Therefore, the collusion profits in each period become $\pi^{C^*} = \pi^C - C$. In this general context, we ask what the impact of a sufficiently high cost of sustaining collusion is on the relationship between product differentiation and collusion sustainability—that is, we investigate the sign of $\partial\delta^*/\partial d$. Our goal is to derive a simple and general characterization of the sign of $\partial\delta^*/\partial d$.

First, note that for collusion to be sustainable in equilibrium—at least for a nonempty subset of parameters—the costs of sustaining collusion cannot be too high. In particular, the following constraint must be satisfied at least for a subset of parameters: $C \leq \bar{C}(d) \equiv \pi^C - \pi^N$. If the constraint is not satisfied, collusion (once the collusion costs are taken into account) is less profitable than competition; therefore, it cannot be sustainable. We assume that $\bar{C}(d)$ is monotonically increasing, monotonically decreasing, or inverted U-shaped.[8] Let us define $\hat{d} \equiv \arg\max_d \bar{C}(d)$, which is unique by assumption. Because we are looking for the impact of high collusion costs, we focus on the neighborhood of $\hat{d}$: by the definition of $\hat{d}$, if collusion costs satisfy the constraint $C \leq \bar{C}(d)$ outside the neighborhood of $\hat{d}$, they must also satisfy the constraint near $\hat{d}$, although the reverse is not necessarily true. That is, if the constraint $C \leq \bar{C}(d)$ is satisfied for some d,[9] it is always possible to find a sufficiently high C such that the constraint $C \leq \bar{C}(d)$ holds in the neighborhood of $\hat{d}$. Consider the derivative of the critical discount factor with respect to d. We have

$$\frac{\partial\delta^*}{\partial d} = \left(\frac{\partial(\pi^D - \pi^C)}{\partial d}(\pi^D - \pi^N) - \frac{\partial(\pi^D - \pi^N)}{\partial d}(\pi^D - \pi^C + C)\right) \cdot ((\pi^D - \pi^N)^2)^{-1}. \quad (1)$$

If $\partial(\pi^D - \pi^N)/\partial d \leq 0$, by (1) we have that $\partial\delta^*/\partial d \geq (\leq)\, 0 \Leftrightarrow C \geq (\leq)\, C^*$, where $C^* \equiv (\pi^D - \pi^N) \cdot (\partial(\pi^D - \pi^C)/\partial d)/(\partial(\pi^D - \pi^N)/\partial d) - (\pi^D - \pi^C)$. On the other hand, if $\partial(\pi^D - \pi^N)/\partial d \geq 0$, we have that $\partial\delta^*/\partial d \geq (\leq)\, 0 \Leftrightarrow C \leq (\geq)\, C^*$. Proposition 1 provides the central result of this article.

Proposition 1. (a) *If $\hat{d} = \underline{d}$, there always exists a level of C such that the critical discount factor increases with the level of differentiation.*

(b) *If $\hat{d} \in (\underline{d}, \bar{d})$, there always exists a level of C such that the critical discount factor is a U-shaped function of the level of differentiation.*

(c) *If $\hat{d} = \bar{d}$, there always exists a level of C such that the critical discount factor decreases with the level of differentiation.*

Proof. See the appendix. □

Proposition 1 has relevant implications. Even when the gain from colluding is not maximal at the minimum degree of differentiation, the impact of high costs of collusion over the relationship between product differentiation and collusion sustainability can still be easily characterized. Indeed, Proposition 1 shows that, if the gain from colluding has an interior maximum, there always exists a sufficiently high collusion cost such that the relationship between product differentiation and the critical discount factor is U-shaped, whereas if the gain from colluding is maximum when the degree of differentiation is maximum, there always exists a sufficiently high collusion cost such that the relationship between product differentiation and the critical discount factor is negative. In other

[7] Clearly, if $\pi^N(d) \geq \pi^C(d)$, collusion is never profitable. If $\pi^C(d) \geq \pi^D(d)$, collusion is always sustainable (provided that it is also profitable) because no firm has an incentive to deviate from the collusive agreement.

[8] This guarantees the existence of no more than one stationary point for $\bar{C}(d)$. It should be noted that Proposition 1 holds also for more general shapes of function $\bar{C}(d)$, provided that $\hat{d}$ is unique. I thank a reviewer for this clarification.

[9] Clearly, if $C \geq \bar{C}(d)$ for every d, the analysis is meaningless because collusion is never sustainable, as it is never profitable for the firms to participate in the cartel.

words, the shape of the gain from collusion function determines the impact of collusion costs on the relationship between product differentiation and collusion sustainability.

The intuition behind Proposition 1 is as follows (in the following discussion, we consider only the case where $\partial(\pi^D - \pi^N)/\partial d \le 0$; the other case can be treated by symmetry). With positive collusion costs, the critical discount factor becomes $\delta^* = (\pi^D - \pi^C)/(\pi^D - \pi^N) + C/(\pi^D - \pi^N)$. It follows that $\partial\delta^*/\partial d = \partial[(\pi^D - \pi^C)/(\pi^D - \pi^N)]/\partial d + C\partial[1/(\pi^D - \pi^N)]/\partial d$. The latter equation shows that if C is sufficiently high, the impact of the differentiation degree over the critical discount factor is positive. We already indicated the critical threshold for the sign of the critical discount factor derivative by $C^*(d)$. Let us denote the condition $C \ge C^*(d)$ as the *positivity constraint*, because when it is satisfied, the relation between the differentiation degree and the critical discount factor is positive; otherwise, it is negative. However, recall that for collusion to be possible, the collusion costs must also satisfy the constraint regarding participation in the cartel—that is, $C \le \bar{C}(d)$. Therefore, for the impact of product differentiation over the critical discount factor to be positive, there must exist a level of C that simultaneously satisfies the participation constraint $C \le \bar{C}(d)$ and the positivity constraint $C \ge C^*(d)$. The existence of such a level of collusion costs depends on the relative value of $C^*(d)$ and $\bar{C}(d)$. In particular, for the relation between product differentiation and the critical discount factor to be positive for sufficiently high collusion costs, the necessary condition is that $C^*(d)$ must always be lower than $\bar{C}(d)$. In this case, it is always possible to find a level of costs that satisfies both constraints. On the other hand, under general shapes of the profits functions, it may be that $C^*(d) \ge \bar{C}(d)$: in this case, the participation constraint becomes binding at levels of collusion costs that do not satisfy the positivity constraint. Therefore, it is not possible to find collusion costs satisfying the positivity constraint while simultaneously maintaining the profitability of collusion, and the relation between product differentiation and the critical discount factor is negative or nonmonotonic. In sum, the impact of product differentiation on collusion sustainability under sufficiently high collusion costs is driven by both the participation and the positivity constraints. This opens the way for nonmonotonic relations between product differentiation and collusion sustainability.

3. Two Examples

3.1. First Example

In the first example, we consider the differentiated Hotelling model with quantity-setting competitors. This class of models was introduced by Hamilton et al. (1989) and has received consistent attention by scholars.[10] The following analysis is based on Hamilton et al. (1989), Arevalo Tomé and Chamorro-Rivas (2007), and Colombo (2012).[11] Different from these articles, we introduce the positive costs of collusion. The profits functions are

$$\pi^N(d) = \frac{12 - 6t(1+d^2) + t^2(1+27d^2-16d^3)}{108},$$

$$\pi^C(d) = \frac{12 - 6t(1-2d+2d^2) + t^2(1-3d+3d^2)}{96}, \quad \text{and}$$

$$\pi^D(d) = \frac{60 - 6t(5-2d+6d^2) + t^2(5-3d+39d^2-16d^3)}{384},$$

where $d \in [0,1]$ measures the product differentiation degree, and $t \in [0, 1/2]$ indicates the disutility costs. Note that the function $\pi^C - \pi^N$ is globally maximized at $\hat{d} = (20 + 63t - \sqrt{400 - 2{,}088t + 5{,}121t^2})/(128t)$. Figure 1 illustrates what happens to the critical discount factor when positive costs of sustaining collusion are introduced.[12]

Figure 1 **The Impact of Collusion Costs (First Example)**

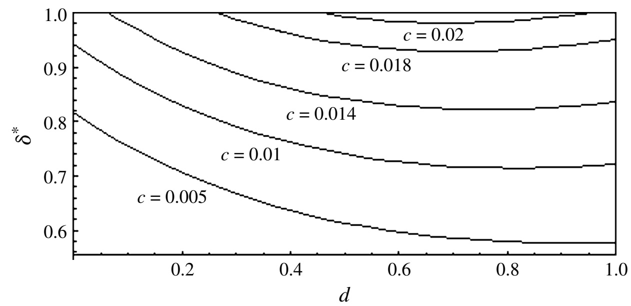

Figure 1 should be compared with Figures 1–3 in Thomadsen and Rhee (2007). In Thomadsen and Rhee (2007), one observes that when the collusion costs become sufficiently high, the critical discount factor for collusion sustainability increases in the level of product differentiation. In contrast, Figure 1 shows that, for sufficiently high collusion costs, the critical discount factor becomes U-shaped for all levels of differentiation such that the critical discount factor is

[10] See Colombo (2012) for an extensive review of this class of models.

[11] Examples of cartels where firms collude by sharing the market and fixing the quantity to deliver are *Amino Acids cartel—Archer Daniels Midland Co. and others* (case 36545, D.Comm. June 7, 2000), *Food Flavour Enhancers* (case 37671, D.Comm. December 17, 2002), and *Methylglucamine* (case 37978, D.Comm. November 27, 2002). The last antitrust case states, for example, "According to this agreement, [customer 2]'s yearly needs of meglumine would be supplied by RPB up to a minimum of 25 tonnes. If requested by [customer 2], Merck could supply [customer 2] with its remaining needs" (see paragraph 91, http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:32004D0104:EN:HTML, accessed April 2, 2012).

[12] Figures 1 and 2 have been drawn for $t = 1/4$.

lower than 1 at least for some values of d. Interestingly, no level of the collusion costs satisfying the participation constraint can be found such that the critical discount factor is a monotonically increasing function of the degree of product differentiation between the firms. This occurs because, in contrast to the examples in Thomadsen and Rhee (2007), the gain of collusion does not monotonically decrease for the whole range of product differentiation degrees. In particular, the gain of collusion has an interior maximum: we are in case (b) of Proposition 1, where we show that for sufficiently high collusion costs the relation between product differentiation and the critical discount factor must be U-shaped.

3.2. Second Example

In the second example, we consider the following collusive agreement between price-discriminating firms: firms coordinate on a uniform price but deviate and punish by adopting discriminatory prices. This kind of collusive agreement has been studied by Liu and Serfes (2007) and Colombo (2011). Price discrimination implies that the firms are able to set price schedules instead of a uniform price: *price schedules* mean that the price is a function of the location of consumers, where the *location* of consumers can be intended both in a geographical sense (Thisse and Vives 1988) and in a product-characteristic-space sense (Shaffer and Zhang 1995). Sufficient conditions for price discrimination to be possible are that the firms are able to recognize the consumers' location and that the consumers cannot arbitrate. The type of collusive agreement considered here assumes that the firms collude on a uniform price, even if they are individually able to set discriminatory prices. This type of collusive scheme can be rationalized as follows. Colluding on discriminatory prices is difficult to implement because it requires negotiating on a huge number of prices (one for each consumer/market). A less extreme collusive agreement is when firms try to coordinate on a uniform price. This scheme is less profitable, but it is easier to implement because it requires the firms to agree only on one price. On the other hand, during the deviation stage and during the punishment stage, there is no agreement between firms; thus each firm can use discriminatory prices.[13] From Liu and Serfes (2007) and Colombo (2011), we derive the relevant profits functions: $\pi^C(d) = [4 - t(1-2d+2d^2)]/8$ and $\pi^D(d) = [4 - t(1+2d^2)]/4$, with $d \in [0, 1]$ and $t \in [0, 1/2]$. We suppose here that firms adopt optimal punishment in the case of a deviation from the agreement. In the case of price discrimination, Miklós-Thal (2008) shows that optimal punishment implies zero profits.[14] It is immediate to verify that π^C has a global maximum at $\hat{d} = 1/2$.[15] Figure 2 illustrates what happens to the critical discount factor when positive costs of sustaining collusion are introduced.

Figure 2 **The Impact of Collusion Costs (Second Example)**

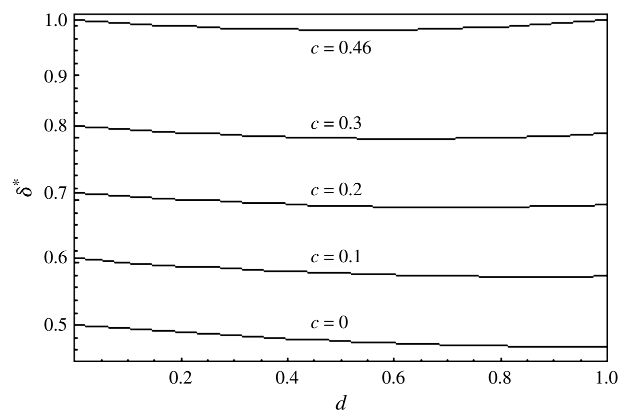

Figure 2 is similar to Figure 1 and contrasts with Figures 1–3 in Thomadsen and Rhee (2007). It shows that when the costs of collusion are low, the relationship between product differentiation and the critical discount factor is negative. On the other hand, when the collusion costs increase, the critical discount factor becomes U-shaped with the differentiation level. No admissible collusion costs level exists such that the critical discount factor strictly increases with the degree of differentiation between firms. As indicated in Proposition 2, the U-shaped relation between the degree of product differentiation and the critical discount factor for high levels of collusion costs depends on whether the gain of collusion has an interior maximum.[16]

4. Conclusions

Setting up and maintaining a collusive agreement is often costly for firms. Collusion costs include, for

[13] Examples of this collusive scheme can be found in *Austrian Banks* (D.Comm. June 12, 2002) and *Specialty Graphite* (D.Comm. December 17, 2002).

[14] It follows that the collusive profits (including also the collusion costs) must be higher than the average profits in the case of punishment, which are zero in this case.

[15] Proposition 1 is still valid once the Nash profits are substituted by the average profits in case of punishment.

[16] The same result emerges when firms engage in imperfect price discrimination; that is, they are able to price discriminate across different groups of consumers but not across consumers belonging to the same group, and they adopt Nash reversion in the case of a deviation (see Liu and Serfes 2004, Colombo 2010). In this case, when the collusion costs are low, the critical discount factor increases with the level of product differentiation, but when the collusion costs become sufficiently high, the relationship between the critical discount factor and product differentiation becomes U-shaped. Details are available from the author upon request.

example, the costs of negotiation and monitoring. This article considers collusion costs as regards a heavily debated question: What is the impact of product differentiation on the sustainability of a collusive agreement? It is shown that for sufficiently high collusion costs, the impact of product differentiation on the sustainability of a collusive agreement can be easily characterized through simple necessary and sufficient conditions. In particular, if the gain of collusion is maximized at a nonextreme degree of differentiation, for sufficiently high collusion costs, the relationship between product differentiation and the critical discount factor is U-shaped. For example, a U-shaped relationship between product differentiation and the critical discount factor for collusion sustainability is shown to exist for the case of a differentiated demand à la Hotelling, with firms competing with quantities, and for the case of firms being able to price discriminate across consumers having different tastes. Therefore, we have shown that the introduction of sufficiently high collusion costs might dramatically modify the impact of product differentiation on collusion sustainability with respect to the case where collusion costs are absent. For this reason, the existence of costs of collusion—which there surely are—is a key factor when assessing the impact of product differentiation over the likelihood that a collusive agreement arises in equilibrium.

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Appendix

Proof of Proposition 1. If $\bar{C}(d) \geq C^*(d)$, there must exist a level of C such that $C \in [C^*(d), \bar{C}(d)]$. In this case, if $\partial(\pi^D - \pi^N)/\partial d \leq 0$, we have $\partial\delta^*/\partial d \geq 0\ \forall C \in [C^*(d), \bar{C}(d)]$, whereas if $\partial(\pi^D - \pi^N)/\partial d \geq 0$, we have $\partial\delta^*/\partial d \leq 0\ \forall C \in [C^*(d), \bar{C}(d)]$. Instead, if $\bar{C}(d) \leq C^*(d)$, for all the admissible values of C we have $C(d) \leq C^*(d)$. In this case, if $\partial(\pi^D - \pi^N)/\partial d \leq 0$, we have $\partial\delta^*/\partial d \leq 0$, whereas if $\partial(\pi^D - \pi^N)/\partial d \geq 0$, we have $\partial\delta^*/\partial d \geq 0$. After straightforward simplifications, condition $\bar{C}(d) \geq (\leq)\, C^*(d)$ can be rewritten as $(\partial\bar{C}/\partial d)/(\partial(\pi^D - \pi^N)/\partial d) \geq (\leq)\, 0$. Suppose that $\partial(\pi^D - \pi^N)/\partial d \leq 0$ in the neighborhood of $\hat{d}$. In this case, condition $\bar{C}(d) \geq (\leq)\, C^*(d)$ requires that $\partial\bar{C}/\partial d \leq (\geq)\, 0$. Suppose that $\partial(\pi^D - \pi^N)/\partial d \geq 0$ in the neighborhood of $\hat{d}$. In this case, condition $\bar{C}(d) \geq (\leq)\, C^*(d)$ requires that $\partial\bar{C}/\partial d \geq (\leq)\, 0$. By the definition of $\hat{d}$, $\bar{C}(d)$ must be decreasing in the right around of $\hat{d}$, and it must be increasing in the left around of $\hat{d}$. Therefore, depending on $\hat{d}$, we have three possible cases. We analyze each in turn.

(a) $\hat{d} = \underline{d}$: Only the right around of $\hat{d}$ exists. Therefore, only condition $\partial\bar{C}/\partial d \leq 0$ is verified in the neighborhood of $\hat{d}$. It follows that a sufficiently high level of C must exist such that $\partial\delta^*/\partial d \geq 0$ for any d satisfying $C \leq \bar{C}(d)$.

(b) $\hat{d} \in (\underline{d}, \bar{d})$: Both the left and the right around of $\hat{d}$ exist. Therefore, condition $\partial\bar{C}/\partial d \geq 0$ is verified in the left around of $\hat{d}$, and condition $\partial\bar{C}/\partial d \leq 0$ is verified in the right around of $\hat{d}$. It follows that a sufficiently high level of C must exist such that $\partial\delta^*/\partial d \leq 0$ at the left of $\hat{d}$ and $\partial\delta^*/\partial d \geq 0$ at the right of $\hat{d}$ for any d satisfying $C \leq \bar{C}(d)$.[17]

(c) $\hat{d} = \bar{d}$: Only the left around of $\hat{d}$ exists. Therefore, only condition $\partial\bar{C}/\partial d \geq 0$ is verified in the neighborhood of $\hat{d}$. It follows that a sufficiently high level of C must exist such that $\partial\delta^*/\partial d \leq 0$ for any d satisfying $C \leq \bar{C}(d)$.

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[17] A straightforward implication is that C^* must intersect $\bar{C}$ in all the stationary points (if they exist) of $\bar{C}$.