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An Empirically Tractable Dynamic Oligopoly Model: Application to Store Entry and Exit in Dutch Grocery Retail

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Abstract. I develop a simple dynamic oligopoly model for empirical work. A unique feature of the model is that any Markov-perfect equilibrium that survives some intuitive refinements can be quickly computed from low-dimensional contraction mappings in seconds. After computation, it is easy to check the uniqueness of the refined equilibrium. These results facilitate fast estimation using a full-solution approach and produce reliable counterfactual analysis. Model estimation at its minimum only requires panel data on firm presence, yet it quantifies important market primitives, such as toughness of competition and entry costs. I provide a step-by-step illustration of the estimation approach by applying it to the Dutch retail grocery industry, in which chain stores slowly replace local stores. A counterfactual simulation computing equilibrium under a large number of different primitive values shows that relaxing restrictions on chain-store entry will not only quicken the destruction of local stores, but also hurt chain stores' profits.

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1. Introduction

Since the seminal work of Ericson and Pakes (1995), models of Markov-perfect industry dynamics have received increasing attention from economists and marketing researchers. Recently developed two-step estimation methods (e.g., Aguirregabiria and Mira 2007, Bajari et al. 2007, and Pakes et al. 2007) significantly reduce the difficulty of estimating these models by sidestepping equilibrium calculation and, hence, have spurred the rise in successful empirical applications (e.g., Yao and Mela 2011, Ellickson et al. 2012, and Ryan 2012). Nonetheless, two hurdles remain before the model can be disseminated to a broader audience, including industry practitioners. First, structural parameters are rarely of interest per se to marketing researchers but rather serve as intermediate inputs to counterfactual analysis; as such, equilibrium calculation is often inevitable even if it can be circumvented in estimation. The lack of a quick and reliable computation algorithm renders the task difficult. Compared with single-agent models, Markov-perfect dynamic games require, in many cases, that one firm's equilibrium payoff and other firms' strategies be interdependent. Existing equilibrium computation algorithms (e.g., Pakes and McGuire 1994, 2001) typically address this simultaneity by iterating between updating payoffs and updating strategy, which

is time consuming and does not guarantee convergence to the correct result. Second, little is known about the number of Markov-perfect equilibria in these models (Besanko et al. 2010), which renders the computation of all equilibria nearly impossible. As a consequence, the results of counterfactual simulations are questionable because researchers may fail to predict outcomes generated by alternative equilibria.

Although complex models are more broadly appealing to empirical demand, they often lead to weaker theoretical results. In a series of papers (Abbring et al. 2015, 2017, 2018), the authors take an alternative route in developing simpler Markov-perfect dynamic models with improved theoretical understanding and practical calculation of the equilibrium set. This class of models (ACTY models hereafter) restricts attention to characterizing oligopoly firms' forward-looking *entry and exit* decisions in markets with uncertainties. Their unique feature is that an intuitively refined symmetric Markov-perfect equilibrium (MPE) always exists, possibly in a mixed strategy, and all such equilibria can be computed by an algorithm that guarantees quick convergence.

In this paper, I present an econometric extension to the ACTY models and, in particular, the one developed by Abbring et al. (2015). Unlike Abbring et al. (2017, 2018), the model allows firms to have persistent heterogeneity

in profitability type. I intend to achieve two objectives. First, by providing a detailed description of the equilibrium computation algorithm, the paper serves as a practitioner's guide for solving the proposed model and ACTY models (Section 3). Even without data, these models may be used as building blocks in the analytical modeling of market dynamics. Second, and more importantly, I develop a nested fixed-point (NFXP) procedure for the proposed model's estimation. In an empirical example of retail grocery competition in the Netherlands, I demonstrate step by step the estimation procedure using panel data on store presence and demand in local markets (Section 4). The estimation takes only hours on an off-the-shelf personal computer, and it quantifies the determinants of grocery stores' entry and exit decisions and reveals how entry influences incumbent stores' profitability and survival. Estimation results contribute to our understanding of the retail grocery industry's evolution: although chain stores have a significant profitability advantage over local store rivals, a high entry cost delays the former's replacement of the latter. The counterfactual analysis is made easy and reliable by the algorithm: I examine the change in industry landscapes under a wide range of entry barriers at low computational cost while watching for the possibility of equilibrium multiplicity.

Similar to Ericson and Pakes (1995), the model developed in this paper features market uncertainties, fixed costs of operation, sunk costs of entry, and firms that are heterogeneous in profitability. As in Ericson and Pakes's (1995) model, firms sequentially decide to enter in each period,¹ followed by simultaneous decisions on exit. Unlike Ericson and Pakes's (1995) model, however, the proposed model leaves out firms' investment decisions and assumes that their profitability types have two levels and they remain unchanged over time. This assumption is the key to simplifying equilibrium computation and estimation. It also allows the model to encompass persistent and unobserved profitability types in estimation. Under this restriction, researchers can adopt the model to analyze markets in which dynamics can be well characterized by entries and exits and not by R&D. The retail grocery industry in the illustrative example and many other service industries are among the candidates.

Following Abbring et al. (2015), the proposed equilibrium computation algorithm breaks the interdependence between equilibrium payoff and strategy by using one key insight on mixed-strategy symmetric MPE: in states in which multiple firms with equal profitability type mix between continuation and exit, the expected payoff from continuation always equals the exogenous outside option of exit. This allows calculation of firms' equilibrium payoff in these states without knowing competitors' strategy. Using payoff in these states as a starting point, a sequence of contraction mapping-based calculations traverses through all states, uses

results obtained from past steps as input for future steps, and computes the equilibria quickly.

Taking advantage of the quick computation, I propose a full-solution estimation approach—the NFXP procedure—that requires repeatedly solving MPE for each trial value of model parameters. In contrast, the aforementioned two-step estimation methods recover primitive parameters without computing the equilibrium. This distinction warrants discussion. First, the two-step methods rely on an accurate “first-stage” reduced-form estimation of policy functions at *every state* of the game. This exercise requires that data densely cover the state space and offer a large amount of variation. In empirical applications in which entry and exit are rare events, such requirements may turn out to be too demanding. In these cases, the full-solution approach is the more appropriate empirical tool. Second, without nontrivial extensions, the two-step methods cannot handle persistent unobserved heterogeneity because policy functions estimated in the first stage cannot depend on a hidden persistent state.² On the other hand, this paper shows that incorporating persistent unobserved heterogeneity is straightforward in the full-solution approach. Third, by using a full-information maximum likelihood estimation, the NFXP procedure proposed here achieves statistical efficiency. Finally, although the two-step methods allow researchers to estimate flexible models while being agnostic about equilibrium solutions, the full-solution approach's assurance that MPE can be computed multiple times with ease expands the scope of counterfactual analysis.

2. The Model

Consider a longitudinal data set of firms' presence in a cross-section of markets over a number of periods. The proposed model characterizes the evolution of firms' presence by explaining entries and exits as outcomes of a dynamic game. The model's setup is rooted in Abbring et al. (2015). Because their model's predicted market outcome is sometimes deterministically related to the observed market conditions for any given set of parameter values, it cannot be directly applied to analysis of real market data. Following Rust (1987), I introduce various sources of unobserved transitory shocks into firms' decision problems to rationalize the real data.

In the remainder of the paper, I use a capital letter to denote a random variable or vector; the corresponding lower-case letter is reserved for its realization. The expectation taken over the random variable X is denoted with \mathbb{E}_X . The conditional density function for Y is written as $f(y|X = x)$ when random variable X takes the value x .

2.1. Primitives

The model is a complete information game with an infinite horizon in discrete time $t \in \{1, 2, \dots\}$. In a local market, firms of either high \mathcal{H} or low \mathcal{L} profitability types

make forward-looking entry and exit decisions that change the market over time. A countable number of firms of each type potentially serve a local market. A firm is born *inactive*. It then becomes *active* by entering the market and inactive again and forever upon exiting. A 2×1 vector \mathbf{N}_t , the variable for *market structure*, records the number of active type- \mathcal{H} (\mathcal{L}) firms in its first (second) element. I use ι_k , a 2×1 vector with its first (second) element being one and the other element being zero, to denote a monopoly market structure with one type- \mathcal{H} (\mathcal{L}) firm. Thus, for instance, $2\iota_{\mathcal{H}}$ ($2\iota_{\mathcal{L}}$) indicates a duopoly market with two type- \mathcal{H} (\mathcal{L}) firms, and $\iota_{\mathcal{H}} + \iota_{\mathcal{L}}$ indicates a duopoly market with one type each.

Figure 1 illustrates the sequencing of firms' actions within period t . It starts with the market structure (\mathbf{N}_t), a persistent shock C_t , and a transitory shock $W_{M,t}$. These values are inherited from period $t-1$. The persistent shock is stochastic and has support \mathcal{C} . The transitory shock is at market level and has the full support \mathbb{R} . This model does not require firm-specific shocks (e.g., the one used by Doraszelski and Satterthwaite 2010) to ensure that no observed entry or exit in the data can result in a zero-likelihood contribution. All active firms begin the period by serving the market. The profits from this stage are $\pi_k(\mathbf{N}_t, C_t, W_{M,t})$ for a type- k firm. I make the following regularity assumptions on the expectation of π .

Assumption 1 (Per-Period Profit). For any $k \in \{\mathcal{L}, \mathcal{H}\}$, $c \in \mathcal{C}$, $w_M \in \mathbb{R}$, and market structure \mathbf{n} , which includes at least one type- k firm,

(1) the expected profit is bounded from above: there exists a $\check{\pi} \in \mathbb{R}$ such that

$$\mathbb{E}_{C_{t+1}, W_{M,t+1}}[\pi_k(\mathbf{n}, C_{t+1}, W_{M,t+1}) \mid C_t = c, W_{M,t} = w_M] \leq \check{\pi} < \infty;$$

(2) facing a type- \mathcal{L} rival yields a weakly higher expected profit than facing a type- \mathcal{H} rival:

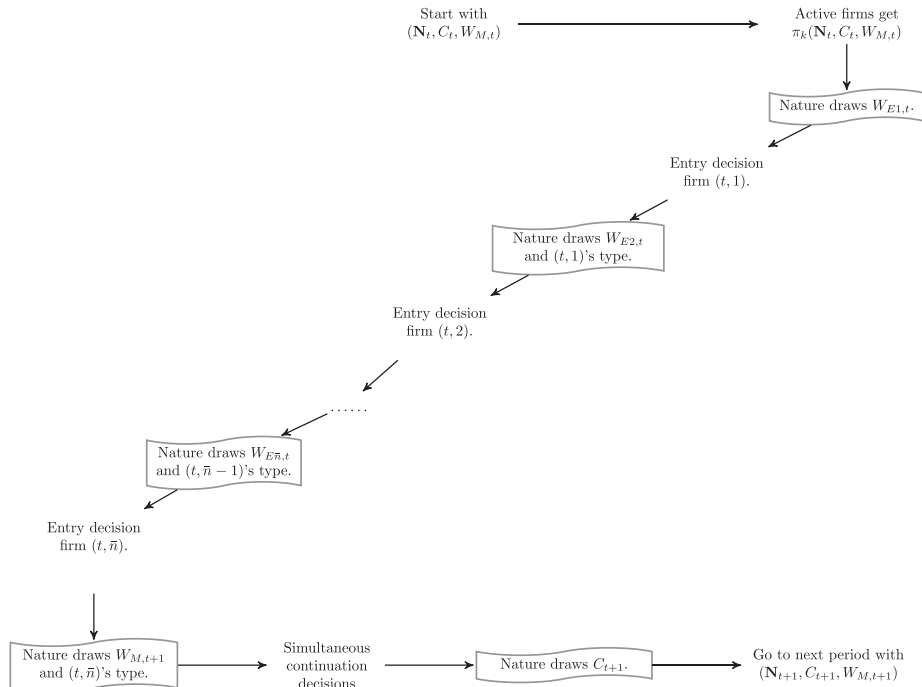
$$\mathbb{E}_{C_{t+1}, W_{M,t+1}}[\pi_k(\mathbf{n} + \iota_{\mathcal{L}}, C_{t+1}, W_{M,t+1}) \mid C_t = c, W_{M,t} = w_M] \geq \mathbb{E}_{C_{t+1}, W_{M,t+1}}[\pi_k(\mathbf{n} + \iota_{\mathcal{H}}, C_{t+1}, W_{M,t+1}) \mid C_t = c, W_{M,t} = w_M];$$

(3) only a finite number of firms can earn positive expected profit in a market: there exists an $\check{n} \in \mathbb{N}$ such that $\mathbb{E}_{C_{t+1}, W_{M,t+1}}[\pi_k(\mathbf{n}, C_{t+1}, W_{M,t+1}) \mid C_t = c, W_{M,t} = w_M] < 0$ if the number of firms in \mathbf{n} is larger than \check{n} ; and

(4) a type- \mathcal{H} firm earns a higher expected profit than a type- \mathcal{L} rival does in the same market: $\mathbb{E}_{C_{t+1}, W_{M,t+1}}[\pi_{\mathcal{L}}(\mathbf{n}, C_{t+1}, W_{M,t+1}) \mid C_t = c, W_{M,t} = w_M] \leq \mathbb{E}_{C_{t+1}, W_{M,t+1}}[\pi_{\mathcal{H}}(\mathbf{n}, C_{t+1}, W_{M,t+1}) \mid C_t = c, W_{M,t} = w_M]$ for any \mathbf{n} that includes at least one type- \mathcal{H} and one type- \mathcal{L} firm.

After market service, a fixed \bar{n} number of firms that have never attempted to enter the market sequentially make an entry decision. In period t , all potential entrants have names (t, f) with $f \in \{1, \dots, \bar{n}\}$. They are identical before actual entry except that having their names dictates that they make entry decisions in the ascending order of f , the second component in their

Figure 1. Sequence of Events and Actions Within a Period



names. Right after the $(t, f - 1)$ firm's entry decision and right before the (t, f) 's, nature draws $W_{Ef,t}$, a shock on entry cost that is specific to the (t, f) -named entrant and has an infinite support. If this (t, f) -named entrant firm decides to enter, it pays a sunk entry cost of $\varphi_{Ef}(W_{Ef,t}) \geq 0$. If it decides not to enter, it will not get another entry opportunity and will remain inactive forever with an outside option normalized to zero. After entry, it receives a profitability type k drawn from a Bernoulli distribution specific to the f -ranked entrant with the probability of being type \mathcal{H} equaling $\omega_{\mathcal{H},f}$. Item 3 in Assumption 1 and the nonnegative sunk costs of entry imply that an upper bound for the number of simultaneously active firms following an initially empty market is \bar{n} .

The entry phase is modeled delicately and requires more discussion of assumptions. As stated in the introduction, the choice of sequential entry is motivated by the original framework of Ericson and Pakes (1995). Abbring et al. (2018) show that alternative specifications of simultaneous entry complicate the equilibrium analysis and often lead to equilibrium multiplicity. On the other hand, other assumptions in this setup are disposable: the entrant knows the entry cost before its entry decision but not the profitability type, which reflects greater uncertainty about profitability than about cost. To better suit empirical realism, one may wish to alter the assumptions based on (1) whether cost information arrives before the profitability type's realization or after and (2) whether the realization of cost and/or profitability type is available before entry. These alterations maintain the sequential nature of the entry phase and only require algebraic modifications on equilibrium analysis and estimation.

After the entry phase, the transitory shock for period $t + 1$, $W_{M,t+1}$ is revealed to all active firms. Then all of them—including those that just entered the market—decide simultaneously between survival and exit. Exit is irreversible but otherwise costless. It allows firms to avoid future periods' negative profits but leads to the zero-valued outside option. All firms' entry and exit decisions maximize their expected discounted profits. They all discount the future with a common factor $\beta < 1$. In the period's final stage, C_t evolves exogenously following a first-order Markov process. This concludes the updating of all the exogenous random components in this market. With the updated values of N_{t+1} , C_{t+1} and $W_{M,t+1}$, the market moves on to the next period.

Throughout the paper and fitting my empirical exercise, I refer to C_t as "demand," $W_{M,t}$ as profit shock, and $W_{Ef,t}$ as entry cost shocks. In another empirical application, C_t can capture any observed, persistent, and exogenous uncertainties that affect firms' decisions, and $W_{M,t}$ and $W_{Ef,t}$ can be any unobserved (to an econometrician), transitory, and exogenous uncertainties that affect profits and entry, respectively. I follow Rust (1987)

and assume that the unobserved shocks $W_{M,t}$ and $W_{Ef,t}$ are conditionally independent in the following manner.

Assumption 2 (Markov and Conditional Independence). *For any $t \in \{1, 2, \dots\}$, random variables $\{C_t, W_{M,t}, W_{E1,t}, \dots, W_{E\bar{n},t}\}$ follows a first-order Markov process. The transition density of the process factors as*

$$g(C_{t+1}, W_{M,t+1}, W_{E1,t+1}, \dots, W_{E\bar{n},t+1} | C_t, W_{M,t}, W_{E1,t}, \dots, W_{E\bar{n},t}) = g_C(C_{t+1} | C_t) g_{W_M}(W_{M,t+1}) g_{W_{E1}}(W_{E1,t+1}) \dots g_{W_{E\bar{n}}}(W_{E\bar{n},t+1}),$$

in which g_C is the conditional density for the Markov variable C_t , g_{W_M} is the density for the profit shock $W_{M,t}$, and $g_{W_{Ef}}$ are the densities of the shocks on entry costs.

2.2. Markov-Perfect Equilibrium

I focus on the *Markov-perfect equilibria* of the model. This is a subgame-perfect equilibrium in strategies that are only contingent on payoff-relevant variables. For potential entrant (t, f) , the payoff-relevant variables are C_t , $W_{Ef,t}$ and the market structure $\mathbf{M}_{Ef,t}$ just before the firm's possible entry. Next, denote the market structure after the period's entry phase with $\mathbf{M}_{E,t}$. For an active type- k firm contemplating survival, the payoff-relevant variables are the pre-exit market structure, demand C_t , profit shock $W_{M,t+1}$, and type k . A *Markov strategy* is a set of functions (a_f^E, a^S)

$$\begin{aligned} a_f^E : \mathbb{Z}^2 \times \mathbb{C} \times \mathbb{R} &\rightarrow \{0, 1\} \\ a^S : \mathbb{Z}^2 \times \mathbb{C} \times \mathbb{R} \times \{\mathcal{L}, \mathcal{H}\} &\rightarrow [0, 1]. \end{aligned}$$

This allows for mixed strategies when firms exit. For each potential entrant (t, f) , this strategy's *entry rule* a_f^E assigns a probability of becoming active to any $(\mathbf{M}_{Ef,t}, C_t, W_{Ef,t})$. Similarly, its *survival rule* a^S assigns a probability of remaining active in the next period to each possible value of the payoff-relevant state $(\mathbf{M}_{E,t}, C_t, W_{M,t+1}, k)$ for all active firms. Because calendar time is not payoff relevant in a Markov-perfect equilibrium, I hereafter drop the subscript t from all variables and denote random variable Y 's next-period value by Y' . Throughout the paper, I focus on symmetric equilibria: firms facing the same values for state variables have the same equilibrium probability of survival or entry.

To characterize equilibria, it is useful to define two payoff functions, each corresponding to a particular node of the game tree within each period. The *postentry payoff* $v^E(\mathbf{m}_E, c, k)$ equals the expected discounted profits for a firm that has profitability type k and faces market structure \mathbf{m}_E and demand c just after all entry decisions have been realized and just before the profit shock W_M is revealed. For a potential entrant, this payoff function gives the expected discounted profits from entry, and therefore, it determines optimal entry

choices. The realized shocks on entry costs w_{Ef} do not enter this value because they are sunk upon entry and do not help in predicting future shocks (Assumption 2). The *postsurvival payoff* $v^S(\mathbf{m}_S, c, W'_M, k)$ equals the expected discounted profits of a type- k firm when it is facing market structure \mathbf{m}_S , demand c , and next-period profitability shock W'_M just after all survival decisions have been realized. This value equals the payoff to a surviving firm following continuation decisions, so it is central to the analysis of exit.

The payoff functions v^E and v^S satisfy

$$v^E(\mathbf{m}_E, c, k) = \mathbb{E}_{\mathbf{M}_S, W_M} [a^S(\mathbf{m}_E, c, W'_M, k) v^S(\mathbf{M}_S, c, W'_M, k) | \mathbf{M}_E = \mathbf{m}_E, C = c], \text{ and} \quad (1)$$

$$v^S(\mathbf{m}_S, c, W'_M, k) = \beta \mathbb{E}_{\mathbf{M}_E, C} [\pi_k(\mathbf{m}_S, C', W'_M) + v^E(\mathbf{M}'_E, C', k) | \mathbf{M}_S = \mathbf{m}_S, C = c]. \quad (2)$$

Equations (1) and (2) state the relationship between the two payoff functions: A firm's postentry payoff equals its expected postsurvival payoff weighted by the survival probability with the expectation taken over the postsurvival market structure \mathbf{M}_S and profit shock W_M . Its postsurvival payoff equals the expected flow profit plus postentry payoff in the next period with the expectation taken over the postentry market structure \mathbf{M}_E and demand C .

For (a_f^E, a^S) to form a symmetric Markov-perfect equilibrium, it is necessary and sufficient that no firm can gain from a one-shot deviation from (a_f^E, a^S) (e.g., Fudenberg and Tirole 1991, theorem 4.2):

$$a_f^E(\mathbf{m}_{Ef}, c, w_{Ef}) \in \arg \max_{a \in [0,1]} a(\mathbb{E}_{\mathbf{M}_E} [v^E(\mathbf{M}_E, c, K) | \mathbf{M}_{Ef} = \mathbf{m}_{Ef}] - \varphi_{Ef}(w_{Ef})) \quad (3)$$

$$a^S(\mathbf{m}_E, c, W'_M, k) \in \arg \max_{a \in [0,1]} a(\mathbb{E}_{\mathbf{M}_S} [v^S(\mathbf{M}_S, c, W'_M, k) | \mathbf{M}_E = \mathbf{m}_E]). \quad (4)$$

Conditions in Equations (3) and (4) state that (a_f^E, a^S) maximize the postentry and postsurvival payoffs, respectively, given that all other firms use (a_f^E, a^S) . The conditional expectations over market structures \mathbf{M}_S and \mathbf{M}_E in Equations (1)–(4) are computed given that other firms follow (a_f^E, a^S) and the firm of interest enters or remains active. Together, conditions (1)–(4) are necessary and sufficient for a strategy (a_f^E, a^S) to form a symmetric Markov-perfect equilibrium with payoffs v^E and v^S .

A symmetric MPE always exists in the Abbring et al. (2015) model without unobserved shocks. They prove this result by introducing an algorithm that always quickly computes such an equilibrium. In the next section, I present a modified version of the algorithm

that computes the MPE for the econometric model. In addition to establishing equilibrium existence, the algorithm is the cornerstone for estimation. From the researcher's viewpoint, because the shocks $W_{Ef,t}$ and $W_{M,t}$ are unobserved, a computed equilibrium strategy implies the probabilities of one market structure's changing to another under any given values of observed state variables. These probabilities are inputs when constructing the likelihood function for estimation.

3. Equilibrium Computation

The major hurdle in computing a Markov-perfect equilibrium is the interdependence between one firm's equilibrium payoff and other firms' strategies, which is reflected in Equations (1)–(4) as the conditional expectations' dependence on competitors' strategies. For the ACTY models, the way to circumvent this problem and make the equilibrium computation simple and quick is to explore a symmetric Markov-perfect equilibria's payoff structure: when firms base their continuation choices on a mixed strategy, their expected equilibrium payoffs from the continuation phase equal the payoff from exit, which is zero. This allows calculation of firms' expected continuation payoffs at some nodes of the game tree without knowing competitors' strategies; these nodes serve as the starting points of their equilibrium computation algorithm. The algorithm then traverses through partitions of the state space and computes an equilibrium in steps.

Before formally presenting the algorithm adapted for this model, I illustrate the intuition behind it by going through an example, which concerns a market that can simultaneously accommodate at most two active firms ($\bar{n} = 2$). I further restrict the number of potential entrants per period \bar{n} to one. This is the simplest case of the model that retains strategic interactions between firms. This illustration makes two important points. First, in each step, the entry and survival rules computed in previous steps are sufficient to compute the conditional expectations in Equations (1)–(4) that are relevant to the current step. Second, equilibrium payoffs are always computed as a unique fixed point of a contraction mapping in each step.

3.1. An Example

Throughout the example, I focus on calculating the postentry payoff $v^E(\mathbf{m}_E, c, k)$ and show how the rest of the equilibrium calculation follows from it. Substituting Equation (2) (the definition of v^S) into Equation (1) (the definition of v^E) gives a recursive definition for v^E

$$v^E(\mathbf{m}_E, c, k) = \mathbb{E}_{\mathbf{M}_S, W_M} [a^S(\mathbf{m}_E, c, W'_M, k) \times \beta \mathbb{E}_{\mathbf{M}_E, C} [\pi_k(\mathbf{M}_S, C', W'_M) + v^E(\mathbf{M}'_E, C', k) | C = c] | \mathbf{M}_E = \mathbf{m}_E]. \quad (5)$$

Recall that v^E is the payoff for an incumbent firm at the node in which all entry decisions are realized. Moving from this point forward, the keys to calculating the two conditional expectations in Equation (5) are (1) firms' survival strategy in the current period that transforms \mathbf{m}_E to \mathbf{M}_S and (2) firms' entry strategy in the next period that transforms \mathbf{M}_S to \mathbf{M}'_E . Although, for any *given* strategy, the right-hand side of Equation (5) defines a contraction mapping, the equilibrium strategy, in turn, depends on equilibrium payoffs. An existing method such as Pakes and McGuire (1994) would prescribe that one first fix a hypothetical strategy to compute payoffs, then update the strategy using computed payoffs, then update payoffs using the updated strategy, and so on. This iterative procedure is slow and does not guarantee convergence to an equilibrium. Instead, I demonstrate how to calculate the payoffs in steps with sure convergence.

Step 1. Duopoly Market with Two Type- \mathcal{H} Firms After Entry Phase.

Figure 2 depicts the possible market structure transition involved in the conditional expectations for computing $v^E(2l_{\mathcal{H}}, c, \mathcal{H})$. The parentheses “()” enclose values that are unknown before this step, and the brackets “[]” enclose known values. Note that these expectations are always conditional on the firm of interest's continuing in the current period. In this figure, the survival rule $a^S(2l_{\mathcal{H}}, c, W'_M, \mathcal{H})$ belongs to the firm of interest's rival. After the profit shock W'_M is drawn, the survival phase has three scenarios. (1) Upper branch: the rival exits for sure ($a^S(2l_{\mathcal{H}}, c, W'_M, \mathcal{H}) = 0$). In a symmetric MPE, the firm of interest would also face negative continuation

payoff from continuation. (2) Middle branch: the rival uses a mixed strategy ($0 < a^S(2l_{\mathcal{H}}, c, W'_M, \mathcal{H}) < 1$). By definition of mixed strategy in a symmetric MPE, the firm of interest would be indifferent between continuation and exit, and its continuation payoff would equal the outside option, zero. (3) Lower branch: the rival survives for sure ($a^S(2l_{\mathcal{H}}, c, W'_M, \mathcal{H}) = 1$), which occurs if both firms expect to receive positive value from continuation.

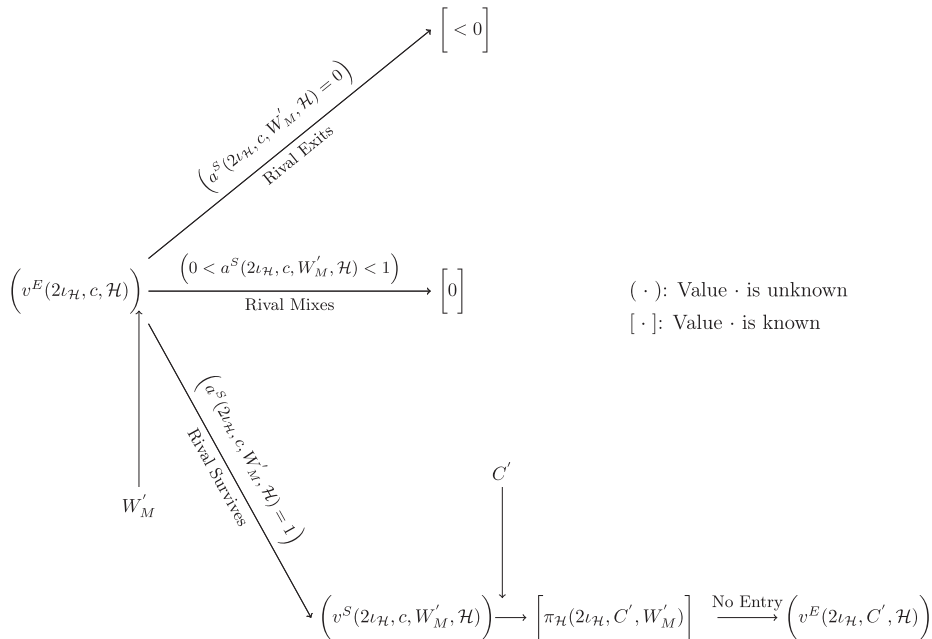
Moving beyond the survival phase and after C' is updated, the jointly survived incumbents earn $\pi_{\mathcal{H}}(2l_{\mathcal{H}}, C', W'_M)$. Because the market can accommodate at most two firms at the same time, there would be no more entry in the next period's entry phase, leading the market structure to remain at $2l_{\mathcal{H}}$ immediately after the entry phase in the next period.

Without knowing the value of $a^S(2l_{\mathcal{H}}, c, W'_M, \mathcal{H})$, one can still assert from this analysis that the firm of interest would only have a positive continuation payoff $v^E(2l_{\mathcal{H}}, c, \mathcal{H})$ if both firms continue for sure. In addition, the firm of interest would choose to exit for sure to avoid the negative continuation payoff in any MPE. These assertions lead to a necessary condition for the postsurvival payoff:

$$\begin{aligned} v^E(2l_{\mathcal{H}}, c, \mathcal{H}) &= \mathbb{E}_{W_M}[\max\{0, v^S(2l_{\mathcal{H}}, c, W'_M, \mathcal{H})\}] \\ &= \mathbb{E}_{W_M}[\max\{0, \beta \mathbb{E}_C[\pi_{\mathcal{H}}(2l_{\mathcal{H}}, C', W'_M) \\ &\quad + v^E(2l_{\mathcal{H}}, C', \mathcal{H}) | C = c]\}]. \end{aligned} \quad (6)$$

The expectations in Equation (6) are taken over the exogenous variables C' and W'_M only and do not depend on any survival or entry rule. This ensures that the

Figure 2. Market Transition Starting from Two Type- \mathcal{H} Firms (Step 1)



equation's right-hand side defines a contraction mapping with its unique fixed point pinning down $v^E(2\iota_{\mathcal{H}}, \cdot, \mathcal{H})$. For some functional form of $\pi_{\mathcal{H}}$ and distributions of W_M , the expectation over W_M in Equation (6) has a closed-form expression. By using the closed-form expression to compute $v^E(2\iota_{\mathcal{H}}, \cdot, \mathcal{H})$, one can avoid numerical integration over W_M . A standard successive approximation algorithm can quickly find this unique fixed point by numerically integrating over a single-dimensional C .

Step 2. A Type- \mathcal{L} Duopolist Facing a Type- \mathcal{H} Rival.

Next, consider a type- \mathcal{L} firm that faces a type- \mathcal{H} competitor. Because a type- \mathcal{H} firm earns higher flow profit than a type- \mathcal{L} firm in each period, I only focus on equilibria that are *natural* in their survival rules: a type- \mathcal{H} firm never exits when a type- \mathcal{L} competitor survives. In such a refined equilibrium, the type- \mathcal{L} firm's survival implies the survival of the type- \mathcal{H} rival.

Figure 3 depicts the possible market-structure transition involved in the conditional expectations for computing $v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{L})$. Given that the type- \mathcal{L} firm continues to the next period, the type- \mathcal{H} firm will also survive in a natural equilibrium. Hence, the type- \mathcal{H} firm's survival rule $a^S(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, W'_M, \mathcal{H})$ is taken as known. After joint continuation, no more entry would occur in the next period's entry phase. Therefore, a necessary condition for the type- \mathcal{L} firm's postentry payoff in a natural equilibrium is

$$\begin{aligned} v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{L}) &= \mathbb{E}_{W_M}[\max\{0, v^S(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, W'_M, \mathcal{L})\}] \\ &= \mathbb{E}_{W_M}[\max\{0, \beta \mathbb{E}_C[\pi_{\mathcal{L}}(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, C', W'_M) \\ &\quad + v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, C', \mathcal{L}) \mid C = c]\}]. \end{aligned} \quad (7)$$

As in Equation (6), the expectations in Equation (7) are taken over the exogenous variables C' and W'_M only and do not depend on any unknown survival or entry rule. A contraction mapping defined by the equation's right-hand side has a unique fixed point that determines $v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, \cdot, \mathcal{L})$. Equation (7) also guides how to obtain $v^S(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, W'_M, \mathcal{L})$ from $v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{L})$. This firm's survival rule follows as

$$a^S(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, W'_M, \mathcal{L}) = \mathbb{I}\{v^S(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, W'_M, \mathcal{L}) > 0\},$$

where $\mathbb{I}\{\cdot\}$ equals one if \cdot is true and zero otherwise.

At this point, one also has sufficient information to compute the entry rule for the single potential entrant (with name $f = 1$) facing a market monopolized by a type- \mathcal{H} incumbent. For this entrant, Figure 4 describes its postdecision scenarios. (1) Upper branch: enters the market, becomes a type- \mathcal{H} incumbent with probability $\omega_{\mathcal{H},1}$, and receives $v^E(2\iota_{\mathcal{H}}, c, \mathcal{H})$. (2) Middle branch: enters the market, becomes a type- \mathcal{L} incumbent with probability $1 - \omega_{\mathcal{H},1}$, and receives $v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{L})$. (3) Lower branch: does not enter and receives zero as an outside option. Thus, its entry rule is

$$\begin{aligned} a_1^E(\iota_{\mathcal{H}}, c, w_{E1}) &= \mathbb{I}\{\omega_{\mathcal{H},1} v^E(2\iota_{\mathcal{H}}, c, \mathcal{H}) \\ &\quad + (1 - \omega_{\mathcal{H},1}) v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{L}) > \varphi_{E1}(w_{E1})\}, \end{aligned}$$

which only depends on postentry payoffs that are known.

Step 3. A Type- \mathcal{H} Monopolist and a Type- \mathcal{H} Duopolist Facing One Type- \mathcal{L} Rival.

This step differs from the previous two by considering two market structures at the same time. This requires a more complicated figure to describe the possible market-structure transition. Nonetheless, the relevant survival and entry rules are known, and the relevant postentry payoffs are either known or being considered in the current step.

To see this point, first note that necessary conditions for the postentry payoffs under these two market structures are

$$\begin{aligned} v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{H}) &= \mathbb{E}_{W_M}[\max\{0, a^S(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, W'_M, \mathcal{L}) v^S(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, W'_M, \mathcal{H}) \\ &\quad + (1 - a^S(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, W'_M, \mathcal{L})) v^S(\iota_{\mathcal{H}}, c, W'_M, \mathcal{H})\}], \end{aligned} \quad (8)$$

which depends on whether the type- \mathcal{L} rival exits or not—a known survival rule $a^S(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, W'_M, \mathcal{L})$ calculated in Step 2. And

$$v^E(\iota_{\mathcal{H}}, c, \mathcal{H}) = \mathbb{E}_{W_M}[\max\{0, v^S(\iota_{\mathcal{H}}, c, W'_M, \mathcal{H})\}]. \quad (9)$$

Next, consider the postsurvival payoff v^S in Equations (8) and (9). As shown in Figure 5, if the type- \mathcal{H} firm continues to the next period with a type- \mathcal{L} rival, no further entry would occur in the same period.

Figure 3. (Natural Equilibrium) Market Transition for Type- \mathcal{L} Firm Facing Type- \mathcal{H} Rival (Step 2)

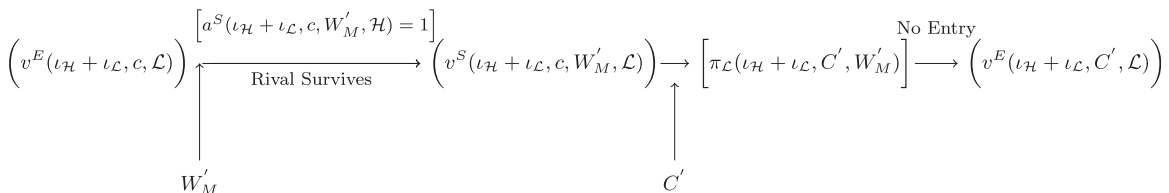
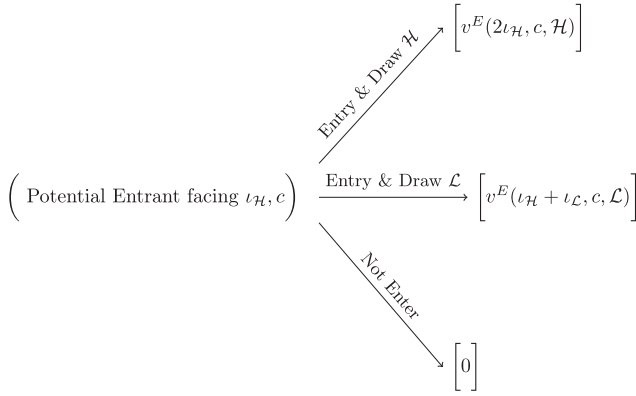


Figure 4. Entry Problem Facing a Type- \mathcal{H} Monopoly Incumbent (Step 2)

Therefore, the type- \mathcal{H} firm's postsurvival payoff satisfies

$$v^S(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, W'_M, \mathcal{H}) = \beta \mathbb{E}_C[\pi_{\mathcal{H}}(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, C', W'_M) + v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, C', \mathcal{H}) | C = c]. \quad (10)$$

If the type- \mathcal{H} firm continues to the next period alone, entry could occur, and the entering firm could become either a type- \mathcal{L} or a type- \mathcal{H} firm. These scenarios imply that the type- \mathcal{H} firm's postsurvival payoff satisfies

$$\begin{aligned} v^S(\iota_{\mathcal{H}}, c, W'_M, \mathcal{H}) &= \beta \mathbb{E}_C[\pi_{\mathcal{H}}(\iota_{\mathcal{H}}, C', W'_M) \\ &\quad + \mathbb{E}_{W_{E1}}[(1 - a_1^E(\iota_{\mathcal{H}}, c, W'_{E1}))v^E(\iota_{\mathcal{H}}, C', \mathcal{H}) \\ &\quad + a_1^E(\iota_{\mathcal{H}}, c, W'_{E1})((1 - \omega_{\mathcal{H}1})v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{H}) \\ &\quad + \omega_{\mathcal{H}1}v^E(2\iota_{\mathcal{H}}, c, \mathcal{H})) | C = c], \end{aligned} \quad (11)$$

where the second, third, and fourth lines correspond to the entrant's not entering, entering and becoming a type- \mathcal{L} firm, and entering and becoming a type- \mathcal{H} firm, respectively. In both Equations (10) and (11), the relevant entry rule and postentry payoffs either have been calculated in Step 1 ($v^E(2\iota_{\mathcal{H}}, c, \mathcal{H})$) and Step 2 ($a_1^E(\iota_{\mathcal{H}}, c, w_{E1})$) or are being considered in the current step ($v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, C', \mathcal{H})$ and $v^E(\iota_{\mathcal{H}}, C', \mathcal{H})$). Therefore, Equations (8)–(11) together define a contraction mapping with its unique fixed point simultaneously determining the postentry payoffs for the type- \mathcal{H} monopolist and the type- \mathcal{H} duopolist facing a type- \mathcal{L} rival. Associated postsurvival payoffs and the survival rule for a type- \mathcal{H} monopolist straightforwardly follow.

Step 4. Duopoly Market with Two Type- \mathcal{L} Firms.

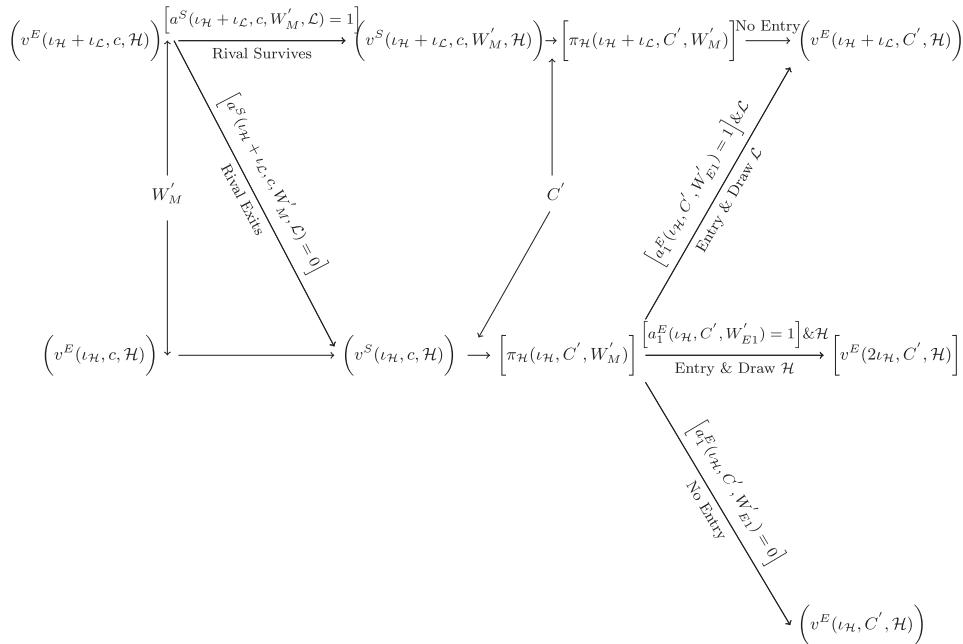
The analysis that leads to the calculation of $v^E(2\iota_{\mathcal{L}}, \cdot, \mathcal{L})$ is a carbon copy of Step 1. After obtaining $v^E(2\iota_{\mathcal{L}}, \cdot, \mathcal{L})$, one can also apply the entry rule to the single potential entrant facing a market monopolized by a type- \mathcal{L} incumbent. Again, the reasoning is identical to that in Step 2. This entry rule is

$$a_1^E(\iota_{\mathcal{L}}, c, w_{E1}) = \mathbb{I}\{\omega_{\mathcal{H}1}v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{H}) + (1 - \omega_{\mathcal{H}1})v^E(2\iota_{\mathcal{L}}, c, \mathcal{L}) > \varphi_{E1}(w_{E1})\},$$

which only depends on postentry payoffs that are known by now.

Step 5. The Rest.

The postentry payoff yet to be calculated is $v^E(\iota_{\mathcal{L}}, \cdot, \mathcal{L})$. The following figure depicts the postsurvival market evolution for this type- \mathcal{L} monopolist. This is analogous to the lower half of Figure 5, which describes

Figure 5. Market Transition for a Type- \mathcal{H} Firm Facing Type- \mathcal{L} Rival (Upper Half) and for a Type- \mathcal{H} Monopolist (Lower Half) (Step 3)

the market evolution faced by a type- \mathcal{H} monopolist. Figure 6 shows the market transition, and a necessary condition for $v^E(\iota_{\mathcal{L}}, \cdot, \mathcal{L})$ follows:

$$v^E(\iota_{\mathcal{L}}, c, \mathcal{L}) = \mathbb{E}_{W_M}[\max\{0, v^S(\iota_{\mathcal{L}}, c, W'_M, \mathcal{L})\}],$$

$$\begin{aligned} \text{where } v^S(\iota_{\mathcal{L}}, c, W'_M, \mathcal{L}) &= \beta \mathbb{E}_C[\pi_{\mathcal{L}}(\iota_{\mathcal{L}}, C', W'_M) \\ &+ \mathbb{E}_{W_{E1}}[(1 - a_1^E(\iota_{\mathcal{L}}, c, W'_{E1}))v^E(\iota_{\mathcal{L}}, C', \mathcal{L}) \\ &+ a_1^E(\iota_{\mathcal{L}}, c, W'_{E1})((1 - \omega_{\mathcal{H},1})v^E(2\iota_{\mathcal{L}}, c, \mathcal{L}) \\ &+ \omega_{\mathcal{H},1}v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{L})) \mid C = c], \end{aligned}$$

where the relevant entry rule $a_1^E(\iota_{\mathcal{L}}, c, W'_{E1})$ has been calculated in Step 4. Therefore, this necessary condition defines a contraction mapping that uniquely determines $v^E(\iota_{\mathcal{L}}, \cdot, \mathcal{L})$. The postsurvival payoff $v^S(\iota_{\mathcal{L}}, c, W'_M, \mathcal{L})$ and survival rule $a^S(\iota_{\mathcal{L}}, c, W'_M, \mathcal{L})$ straightforwardly follow.

Next, I compute the entry rule for the entrant to an empty market as

$$\begin{aligned} a_1^E(0, c, w_{E1}) &= \mathbb{I}\{\omega_{\mathcal{H},1}v^E(\iota_{\mathcal{H}}, c, \mathcal{H}) \\ &+ (1 - \omega_{\mathcal{H},1})v^E(\iota_{\mathcal{L}}, c, \mathcal{L}) > \varphi_{E1}(w_{E1})\}. \end{aligned}$$

Finally, all that remains undetermined is the survival rule for duopoly firms of identical type. Reconsider the analysis in Steps 1 and 4: in a symmetric equilibrium, both firms continue for sure when the duopoly postsurvival payoff is positive, and both exit for sure when the monopoly postsurvival payoff is negative. When the duopoly postsurvival payoff is negative but the monopoly payoff is positive, the game of attrition has no pure-strategy symmetric equilibrium. Instead, it admits a unique mixed-strategy equilibrium, in which each firm chooses a survival probability to render its rival indifferent between exiting and surviving. For

a type- k duopolist, $k \in \{\mathcal{L}, \mathcal{H}\}$, this survival probability is

$$a^S(2\iota_k, c, W'_M, k) = \frac{v^S(\iota_k, c, W'_M, k)}{v^S(\iota_k, c, W'_M, k) - v^S(2\iota_k, c, W'_M, k)}.$$

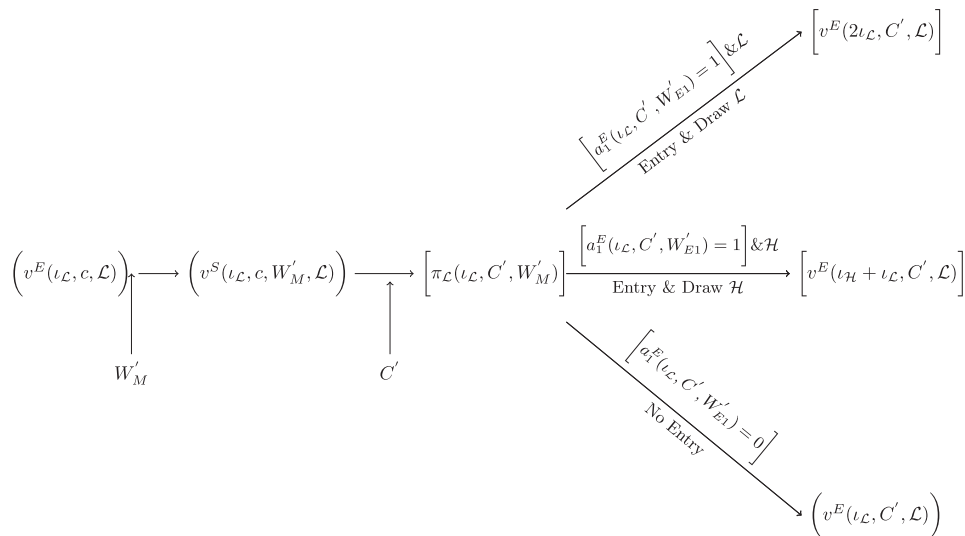
With this part of the survival rule determined, the equilibrium construction is concluded. This construction calculates a natural Markov-perfect equilibrium whose values are always uniquely determined by the fixed points of contraction mappings. In the next section, I generalize the result to arbitrarily finite \tilde{n} and \bar{n} .

3.2. Equilibrium Existence, Uniqueness, and Computation

The five-step example already uses equilibrium refinements. First, in Steps 1 and 4, two incumbents of the same type continue for sure if joint incumbency yields positive expected value. This rules out an odd equilibrium in which joint continuation is profitable but both firms fail to “renegotiate” and exit from the market altogether.³ Second, in Steps 2 and 3, a type- \mathcal{L} firm expects that any type- \mathcal{H} competitor will never exit when the type- \mathcal{L} firm survives. This is natural because a type- \mathcal{L} firm never earns a higher per-period expected profit than a type- \mathcal{H} firm. Cabral (1993) considers a similar refinement in selecting equilibria in dynamic games.

Following Abbring et al. (2015), I apply these two refinements and always restrict empirical analysis on *renegotiation-proof natural Markov-perfect equilibria* or RNMPE. The formal definitions of these refinements are as follows.

Figure 6. Market Transition for a Type- \mathcal{L} Monopolist (Step 5)



Definition 1. A renegotiation-proof natural Markov-perfect equilibrium is a symmetric Markov-perfect equilibrium in a strategy (a_f^E, a^S) such that

- (renegotiation-proof) for all c, w_M and all \mathbf{m} , $a^S(\mathbf{m}, c, w_M, k) = 1$ if

$$\arg \max_{a \in [0,1]} a(\mathbb{E}_{\mathbf{M}_S}[v^S(\mathbf{M}_S, c, w_M, k) | \mathbf{M}_E = \mathbf{m}]) = \{0, 1\}.$$

- (natural) for all c, w_M and all \mathbf{m} that include at least one type- \mathcal{H} and one type- \mathcal{L} firm, $a^S(\mathbf{m}, c, w_M, \mathcal{L}) > 0$ implies that $a^S(\mathbf{m}, c, w_M, \mathcal{H}) = 1$.

Straightforward extensions of the Abbring et al. (2015) theoretical results show that an RNMPE always exists and can be quickly computed using a sequence of contraction mappings. When $\check{n} = 2$, the five-step example illustrates the computation algorithm. When the upper bound for the number of simultaneously active firms is \check{n} , $(\check{n} + 2)(\check{n} + 1)/2 - 1$ different nonempty market structures can possibly arise when the market evolves. The algorithm to construct an RNMPE equilibrium begins by considering the market structure $\check{n}\mathcal{L}$ immediately after the entry phase and at the beginning of a survival game. Analogous to Step 1 in the duopoly example, no firm would enter this saturated market, and each incumbent earns a positive continuation payoff only if all competitors continue to the next period. Thus, $v^E(\check{n}\mathcal{L}, c, \mathcal{H})$ is uniquely determined by a contraction mapping. Afterward, this algorithm partitions the state space and traverses through the parts in steps. The order of the steps ensures that, in each step, after using values and entry/survival rules obtained in previous steps, the postentry payoff function restricted to the part of the state space is always determined by the fixed point of a contraction mapping. Algorithm 1 presents this as a flowchart.

In this algorithm, h indexes the number of type- \mathcal{H} firms, and l indexes the number of type- \mathcal{L} firms. In the course of the computation, h decreases from \check{n} to zero. For each level of h , l decreases from $\check{n} - h$ to zero. When $l = 0$, the algorithm enters its west branch. Otherwise, it enters the east branch. I discuss the branches in order.

For any (h, l) such that $l = 0$ (west branch), the procedure simultaneously computes the payoff for a type- \mathcal{H} firm in a market with $h - 1$ type- \mathcal{H} and $0, 1, \dots, \check{n} - h$ type- \mathcal{L} rivals. Steps 1 and 3 in the duopoly example are in this branch. The payoff is computed as the fixed point of a functional operator $T_{\mathcal{H}}$.

$$(T_{\mathcal{H}}f)(\mathbf{m}, c, k) = \mathbb{E}_{\mathbf{M}_S, W_M}[\max\{0, f^S(\mathbf{M}_S, c, W'_M, \mathcal{H})\} | \mathbf{M}_E = \mathbf{m}], \text{ with,} \quad (12)$$

$$f^S(\mathbf{m}, c, W'_M, k) = \beta \mathbb{E}_{\mathbf{M}'_E, C}[\pi_k(\mathbf{m}, C', W'_M) + g^E(\mathbf{M}'_E, C', k) | C = c], \text{ and}$$

$$g^E(\mathbf{M}'_E, C', k) = \begin{cases} f(\mathbf{M}'_E, C', k) & \text{if } (\mathbf{M}'_E, C', k) \in \mathbb{H}_S^{h,l} \\ v^E(\mathbf{M}'_E, C', k) & \text{if } (\mathbf{M}'_E, C', k) \in \mathbb{H}_S^{h+,l+}, \\ & \text{for } h+ \geq h, l+ > l \\ & \text{or } h+ > h, l+ \geq l. \end{cases}$$

Equation (6), in calculating the duopoly case, is an example of applying $T_{\mathcal{H}}$ as defined in Equation (12). Evaluating the operator $T_{\mathcal{H}}$ may require the survival rules for the type- \mathcal{L} competitors (encompassed by $\mathbb{E}_{\mathbf{M}_S}$), the entry rules for potential entrants (encompassed by $\mathbb{E}_{\mathbf{M}'_E}$), and the corresponding postentry payoffs. Equations (8)–(11) in the duopoly calculation are another example of applying $T_{\mathcal{H}}$, which uses both the survival and entry rules. The ordering of the procedure guarantees that the necessary values have been computed before this step. Therefore, $T_{\mathcal{H}}$ is a contraction mapping with a unique fixed point. After using $T_{\mathcal{H}}$ once, the procedure reduces h by one, resets $l = \check{n} - h$, and goes to the east branch.

For any (h, l) such that $l > 0$ (east branch), the post-entry payoff of type- \mathcal{L} firms facing h type- \mathcal{H} rivals, $v^E(l\mathcal{L} + h\mathcal{H}, \cdot, \mathcal{L})$, is computed as the fixed point of a functional operator $T_{\mathcal{L}}$,

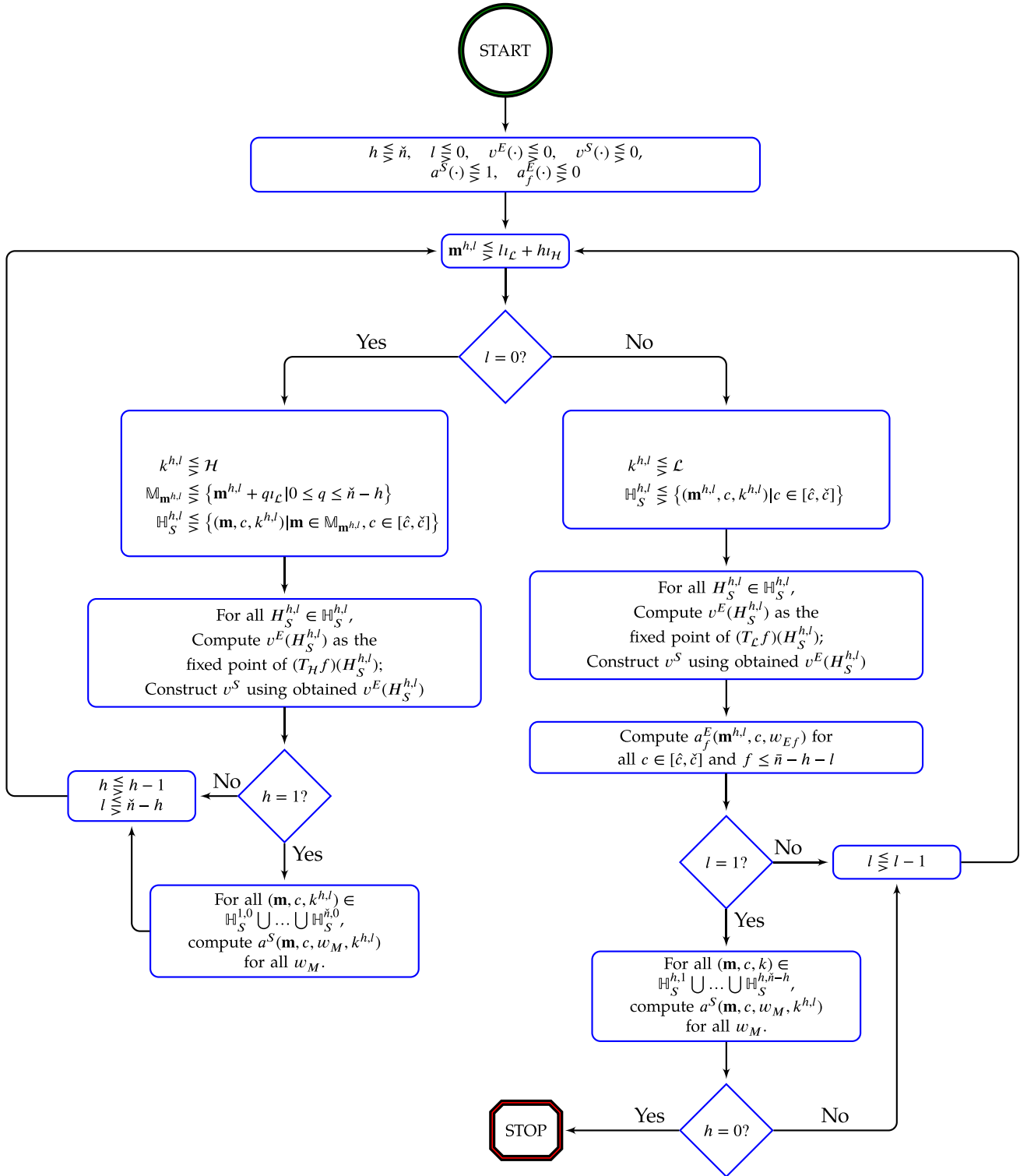
$$(T_{\mathcal{L}}f)(\mathbf{m}, c, k) = \mathbb{E}_{W_M}[\max\{0, f^S(\mathbf{m}, c, W'_M, k)\}], \quad (13)$$

where f^S is defined analogously as in Equation (12). Equation (7) in the duopoly calculation is an example of applying $T_{\mathcal{L}}$ as defined by Equation (13). In an RNMPE, a type- \mathcal{L} firm's value only depends on future states in which all currently active firms survive. To evaluate f^S , the procedure must consider all possible postentry market structures \mathbf{M}'_E . Because (1) any entry leads to a market structure with a higher h or a higher l , and (2) the procedure proceeds in descending order of (h, l) , the payoff-relevant entry rules and the postentry payoffs have been computed prior to this step. Therefore, $T_{\mathcal{L}}$ is a contraction mapping with a unique fixed point.

The procedure's east branch repeatedly uses $T_{\mathcal{L}}$ to compute values until l reduces to one, at which point the procedure also computes the probabilities of survival for $1, 2, \dots, \check{n} - h$ type- \mathcal{L} firms facing h type- \mathcal{L} firms. After that, l reaches zero and the computation again enters the west branch of the procedure. The procedure ends in the east branch when h reaches zero and l reaches one. Steps 2, 4, and 5 in the duopoly example belong to this branch.

When the algorithm finishes, the fixed points from both $T_{\mathcal{L}}$ and $T_{\mathcal{H}}$ pin down the equilibrium postentry payoff function v^E . The payoffs in any RNMPE necessarily satisfy the fixed-point conditions specified by operators $T_{\mathcal{H}}$ and $T_{\mathcal{L}}$. Because Algorithm 1 always computes the necessary survival and entry rules before they are used in $T_{\mathcal{H}}$ and $T_{\mathcal{L}}$, these two operators are both contraction mappings with unique fixed points. The only case in which Algorithm 1's output

Algorithm 1. (Color online) Calculation of a Candidate Equilibrium



could be multiple is when the equilibrium survival rule on (\mathbf{m}, c, w_M, k) involves mixed actions. The mixing probability p is determined by the following polynomial equation:

$$\sum_{i=0}^{m_k-1} (1-p)^{m_k-1-i} p^i \binom{m_k-1}{i} \cdot (v^S(\underline{\mathbf{m}} - (m_k - 1 - i)\iota_k, c, w_M, k)) = 0, \quad (14)$$

where $\underline{\mathbf{m}} = \mathbf{m}$ if $k = \mathcal{L}$, and $\underline{\mathbf{m}} = m_k \iota_k$ if $k = \mathcal{H}$.

The degree of the polynomial equals the number of mixing firms minus one. When the number of mixing firms exceeds two, multiple roots of the polynomial may exist between $[0, 1)$, resulting in multiple survival rules. Given each distinct set of survival rules, the contraction mappings $T_{\mathcal{H}}$ and $T_{\mathcal{L}}$ produce a unique equilibrium postentry payoff function. Thus, the uniqueness of RNMPE rests entirely on the uniqueness of the mixing probability.

If both C and W_M are discrete variables (or discretized for computational purposes), the state space for any firm facing a continuation decision has a finite number of points. This implies that one can only create a finite number of distinct sets of survival rules and, therefore, a finite number of RNMPE. By recording all the admissible mixing probabilities and repeating the algorithm for every possible set of these probabilities, I can compute the payoffs and strategies for all such RNMPE.

Nevertheless, equilibrium uniqueness is still an empirically desirable feature. It is also essential to ensure the reliability of policy experiments. The following proposition and its corollary further address the uniqueness issue.

Proposition 1 (Equilibrium Uniqueness). *The renegotiation-proof natural Markov-perfect equilibrium of the model is unique if for any (\mathbf{m}, c, w_M, k) such that $v^S(\underline{\mathbf{m}}, c, w_M, k) \leq 0$, the polynomial Equation (14) admits only one root in $[0, 1)$, where $\underline{\mathbf{m}} = \mathbf{m}$ if $k = \mathcal{L}$, and $\underline{\mathbf{m}} = m_k \iota_k$ if $k = \mathcal{H}$.*

A sufficient condition for the polynomial (14) to have a unique root is that $v^E(\mathbf{m}, c, k)$ is nonincreasing in the number of type- k stores in \mathbf{m} .

Corollary 1. *If the payoff functions of a natural Markov-perfect equilibrium satisfy $v^E(\mathbf{m}, c, k) \geq v^E(\mathbf{m} + \iota_k, c, k)$ for all (\mathbf{m}, c, k) , it is the unique renegotiation-proof natural Markov-perfect equilibrium.*

Three important remarks supplement this corollary. First, because Algorithm 1 determines all the postentry payoffs relevant to any particular mixing probability before computing the probability, payoff monotonicity is more easily testable than directly examining the number of admissible roots of the polynomial. Second, monotonicity should be checked in the same order as the equilibrium computation. If monotonicity is only violated in later steps, multiple equilibria, if any, still

agree on the payoffs and entry and survival rules computed in the earlier steps in which monotonicity remains good. Third, payoff monotonicity is only sufficient, but not necessary, for uniqueness. Even if monotonicity fails, one should solve for the roots of Equation (14) numerically. If Equation (14) always admits only one root between $[0, 1)$, Proposition 1 applies, and the RNMPE is still unique.

4. Estimation Playbook

In this section, I use an illustrative example to describe the model's empirical implementation in detail. In this example, the model is estimated to characterize the market dynamics for the Dutch retail grocery industry. I begin with a brief description of the data, followed by the likelihood construction. Presentation of the empirical results and a counterfactual analysis that showcases the model's computational ease conclude the demonstration. I close by comparing this model and its estimation procedure with their counterparts in the two-step method.

4.1. Data

The minimum data required for model estimation should have a longitudinal structure recording firms' presence in a cross-section of markets in at least two instances. "Presence" means that beyond firm count, one will need to observe *which* firms are active in the market. Knowing firms' identities is necessary to model firm heterogeneity. Business registries, tax records, and even Yellow Pages can be used to build such a data set, and the one I use in the following example comes from the first source. The Dutch Trade Register (Het Handelsregister) archive maintained by the Dutch Chamber of Commerce (Kamer van Koophandel) covers *all* business establishments' names, main activities, locations, dates of incorporation, ownership, and, if applicable, dates of ceasing operation. I focus on the retail grocery industry between January 1, 2002, and December 31, 2010. In the online appendix, I provide detailed descriptions of how the data set is constructed and a reduced-form overview of industry dynamics.

Market Definition. I define a local *market* as a populated postcode area; this is motivated by a unique Dutch market feature. As Van Lin and Gijbrecchts (2014) explain, the distance to a store is the first concern for Dutch grocery shoppers; this is arguably the most prominent characteristic that horizontally differentiates supermarkets. The average travel time to the supermarket is about nine minutes for a Dutch consumer (47% of shoppers either travel on foot or ride a bike; see Kolkman 2000). To stay close to their potential customers, grocery stores usually locate in residential areas. The principle of defining local markets is to avoid getting markets that are either too small or too large. If the

market is too small to support multiple firms, the characterization of oligopoly dynamics is unnecessary. Too-large markets are likely to contain several catchments in which firms mostly interact locally. Hence, I choose middle-sized postcode areas that had 4,000–12,000 inhabitants in 2009.

Demand. Because transitory shocks serve as “econometric errors” in the model, having a time-varying demand is not theoretically necessary for rationalizing data. Nonetheless, observed time-varying covariates may remarkably improve model fit in many empirical contexts. In this exercise, I use postcode-level population, obtained from publicly available Dutch census data StatLine, as proxies for local demand. Hereafter, I refer to these as “consumers.” From 2002 to 2010, the number of consumers grew in roughly half the markets and declined in the other half. The absolute population percentage change has an annual median of 0.7% and a median of 3.5% over the entire sample period. The online appendix provides more details on descriptive demographic statistics.

Players and Heterogeneity. In these markets, 2,588 grocery stores have been active for at least some period between 2002 and 2010; these are what the model refers to as the “firms.” For these stores, a natural identifier for heterogeneity is the store format: chain stores versus mom-and-pop local stores. Among all stores, 1,559 are chain store outlets and 1,029 are local stores.⁴ The literature has well documented the systematic differences in operation, strategy, and profitability between these two types of stores. However, if one assumes that all chain stores have profitability types superior to local stores, a natural equilibrium would require that *no* chain store ever exits, leaving behind a local-store rival. This theoretical prediction may contradict the data and result in zero likelihood contribution. To resolve this issue, one can introduce a “measurement error” when using store format to dictate profitability type. This approach assumes that the data set does not always precisely classify a store’s format although stores always know their own and competitors’ profitability types. Alternatively, and in a more “structural” manner, one can assume that stores face profitability uncertainty, and hence, a store’s format only stochastically determines its profitability type. In this application, I use the second approach and assume that, upon entry, a chain store has a probability $\omega_{\mathcal{H}}$ to become a type- \mathcal{H} firm and the complementary probability to become a type- \mathcal{L} firm.

4.2. Parameterization

To operationalize the estimation, I set the upper bound on the number of simultaneously active stores \tilde{n} to 11, the maximum number of any local market observed in the sample; feasible alternatives $\tilde{n} = 12$ and $\tilde{n} = 13$ change the estimation results negligibly. I restrict the

number of potential entrants per period, \bar{n} , to two. In the Dutch markets considered in the estimation, entry by more than two stores in the same year is very rare (less than 20 cases in the 877 markets over nine years). Hence, the model practically has a (almost) free-entry feature. Between the two potential entrants, a chain store moves first, and a local store follows. After its entry, the chain store becomes a type- \mathcal{H} firm with probability $\omega_{\mathcal{H}}$ and a type- \mathcal{L} one with the complementary probability. The type realizes before the entry decision of the local store, which has profitability type \mathcal{L} for sure. For lack of historical observations, I assume that the existing chain stores at the beginning of the sample period have an identical probability ω_0 to be a type- \mathcal{H} store. I estimate ω_0 simultaneously with other parameters. The assumption that the chain store decides on entry first is motivated by a longer preparation time for a chain establishment than a local one. Therefore, the local store entrant can condition its entry decision on the chain’s action. This assumption is empirically consequential only if the chain store’s entry is otherwise unexpected and it significantly deters the immediate entry of the local store in the same period. As discussed in Section 2, one can alter the order of entry and type realization with only straightforward modifications on equilibrium analysis and estimation. I parameterize the entry cost as $\varphi_{E1} \exp(W_{E1})$ for chain store entrants and $\varphi_{E2} \exp(W_{E2})$ for local store entrants. The entry cost shocks for chains W_{E1} and for local stores W_{E2} are normally distributed with mean zero and variances σ_{E1}^2 and σ_{E2}^2 , respectively.

Market demand C , the number of consumers in a postcode area, is between 4,000 and 12,000 in 2009. I discretize C on a 201-point equidistant grid with bounds $[3, 500, 12, 500]$. Each sampled value is located on its nearest grid point. The conditional distribution $g_C(C'|C)$ is assumed to be a mixture over 51 reflected random walks in C with uniformly distributed innovations. This mixture approximates a normally distributed innovation. The mean of the innovation is set to zero, and the standard deviation equals 161.38, which is the sample standard deviation of annual population.

With $\tilde{n} = 11$, a total of 77 possible market structures can arise in market dynamics. Because not every possible market structure is well represented in the sample, nonparametric estimation of the profit function $\pi_k(n, c, w_M)$ is impossible. Therefore, I adopt a parametric approach to specify the profit function. To this end, I use a conventional additive separable form $\pi_k(n, c, w_M) \equiv \bar{\pi}_k(n, c) - \varphi_M \exp(w_M)$, where $\bar{\pi}_k(n, c)$ is the revenue and $-\varphi_M \exp(w_M)$ with $\varphi_M \geq 0$ gives a nonpositive market-level profit shock, which can be seen as costs. To construct the revenue portion, I use a simple multinomial logit model to characterize consumers’ store choices: they choose a store from all the active ones in the same postcode area plus

an outside option (for instance, stores outside the postcode area). They receive utility $\theta_k + e$ if shopping in a type- k store ($k \in \{\mathcal{L}, \mathcal{H}\}$), in which e follows a multivariate extreme value distribution and is independent across stores and time. The mean utility from the outside option is normalized to zero.⁵ This demand model gives stores' market shares and motivates a specification for stores' profits $\bar{\pi}_k(n, c)$ as

$$\bar{\pi}_k(n, c) = \frac{\exp(\theta_k)(c/500)}{\exp(\theta_{\mathcal{H}})n_{\mathcal{H}} + \exp(\theta_{\mathcal{L}})n_{\mathcal{L}} + 1}, \quad k \in \{\mathcal{L}, \mathcal{H}\}. \quad (15)$$

In this expression, the parameter θ_k measures the profitability of a type- k store. A type- \mathcal{H} store earns a profit $\exp(\theta_{\mathcal{H}})/\exp(\theta_{\mathcal{L}})$ times that of a type- \mathcal{L} store in the same market. Therefore, this ratio captures the profitability advantage of a type- \mathcal{H} relative to a type- \mathcal{L} rival. Assumption 1 requires that this ratio is larger than one. However, this restriction is not imposed in the estimation. Because the identification is only up to scale, I use $c/500$ to rescale the profit such that the structural parameters' estimates are of the same order of magnitude. If store-level performance data were available, they could be readily incorporated to estimate the profit parameters θ_k and the profit shock parameters φ_M, σ_M . Note that with the additively separable form of $\pi_k(n, c, w_M)$, the profit shock w_M is also additively separable in the postsurvival value. Therefore, Equation (2) can be written as

$$v^S(\mathbf{m}_S, c, W'_M, k) = \bar{v}^S(\mathbf{m}_S, c, k) - \beta\varphi_M \exp(w_M),$$

where $\bar{v}^S(\mathbf{m}_S, c, k) \equiv \beta \mathbb{E}_{\mathbf{M}_E, C}[\bar{\pi}_k(\mathbf{m}_S, C') + v^E(\mathbf{M}_E, C', k) | \mathbf{M}_S = \mathbf{m}_S, C = c]$.

I assume that the profit shock W_M is normally distributed with mean zero and variance σ_M^2 . The normality assumption, together with π 's additive separable assumption and Assumption 2 that C and W_M are conditionally independent, leads to a closed-form solution when integrating out W_M . For instance, Equation (6) in Section 3 becomes

$$\begin{aligned} v^E(2l_{\mathcal{H}}, c, \mathcal{H}) &= \mathbb{E}_{W_M}[\max\{0, \bar{v}^S(2l_{\mathcal{H}}, c, \mathcal{H}) - \beta\varphi_M \exp(W'_M)\}] \\ &= \Pr(\bar{v}^S(2l_{\mathcal{H}}, c, \mathcal{H}) > \beta\varphi_M \exp(W'_M)) \left(\bar{v}^S(2l_{\mathcal{H}}, c, \mathcal{H}) \right. \\ &\quad \left. - \int_{-\infty}^{\log \bar{v}^S(2l_{\mathcal{H}}, c, \mathcal{H}) - \log(\beta\varphi_M)} \beta\varphi_M \exp(W'_M) \phi(W'_M) dW'_M \right) \\ &= \Phi\left(\frac{\log \bar{v}^S(2l_{\mathcal{H}}, c, \mathcal{H}) - \log(\beta\varphi_M)}{\sigma_M}\right) \bar{v}^S(2l_{\mathcal{H}}, c, \mathcal{H}) \\ &\quad - \exp(\sigma_M^2/2) \Phi\left(\frac{\log \bar{v}^S(2l_{\mathcal{H}}, c, \mathcal{H}) - \log(\beta\varphi_M) - \sigma_M^2}{\sigma_M}\right), \end{aligned}$$

where ϕ and Φ are the probability density function and cumulative distribution function for standard normal distribution, respectively. In this expression, $\log \bar{v}^S(2l_{\mathcal{H}}, c, \mathcal{H}) - \log(\beta\varphi_M)$ is the "ceiling" value of W_M to ensure profitable continuation for a type- \mathcal{H} incumbent. Finally, the annual discount rate $1 - \beta$ is set to 5%.

Before proceeding to the likelihood construction, I briefly discuss the intuitions for identification. Abbring et al. (2017) provide a formal discussion of the identification of a similar but simpler model without profitability heterogeneity and unobserved type uncertainty. To identify the initial type uncertainty parameters ω_0 in this model, consider a market with two incumbents: one chain and one local at the beginning of the data. Conditioning on that one store exits, the observed probability for the exited store to be the chain store equals $(1 - \omega_0)/2$: the chain must be a type- \mathcal{L} retailer and mixing between exit and survival with its local store rival. Therefore, this observed probability identifies ω_0 . Similarly, consider a market with one chain and one local in which the chain store entered after the beginning of the data. Conditioning on that one store exits, the observed probability for the exited store to be the chain equals $(1 - \omega_{\mathcal{H}})/2$ and identifies $\omega_{\mathcal{H}}$.⁶ With ω_0 and $\omega_{\mathcal{H}}$ identified, the rest of the identification arguments are similar to the discussion by Abbring et al. (2017): given the parametric assumption of W_M , the mixing probabilities give the differences in equilibrium payoffs to stores in various market structures that could be the outcomes of mixing. These differences in equilibrium payoffs, together with the parametric form of $\bar{\pi}_k$ in Equation (15), identify the profit parameters $\theta_{\mathcal{H}}, \theta_{\mathcal{L}}$ and the profit shock parameters φ_M, σ_M .⁷ Then, the probability for a chain/local store to enter an empty market identifies the entry cost parameters $\varphi_{E1}, \varphi_{E2}, \sigma_{E1}, \sigma_{E2}$.

4.3. Estimation Procedure

I construct the likelihood function by matching the observed market evolution with the equilibrium-implied probabilities under the conventional assumption that the data are generated by a single equilibrium. Throughout Section 3, the equilibrium strategy and payoff functions depend on stores' profitability types and market structure. The data, on the other hand, only contain information on store formats, which are noisy indicators of the active stores' profitability types. Therefore, to construct a likelihood function using choice and transition probabilities, I need to assess the underlying market structure through the joint distribution of active chain stores' types. However, for expositional ease, assume for the moment that the researcher can observe the profitability types of chain stores.

For the entry phase in market i in year t , suppose that the pre-entry market structure is $\mathbf{n}_{E1,it}$ and the demand is c_{it} . The log-likelihood contribution from a chain store's entry/no entry is

$$I_{it}^{Ch} \log \Pr(a_1^E(\mathbf{n}_{E1,it}, c_{it}, w_{E1,it}) = 1) \\ + (1 - I_{it}^{Ch}) \log \Pr(a_1^E(\mathbf{n}_{E1,it}, c_{it}, w_{E1,it}) = 0),$$

where $I_{it}^{Ch} = 1$ if a chain store enters market i in year t and $I_{it}^{Ch} = 0$ otherwise. Similarly, the log-likelihood contribution from a local store's entry/no entry when the pre-entry market structure is $\mathbf{n}_{E2,it}$ is

$$I_{it}^{Lo} \log \Pr(a_2^E(\mathbf{n}_{E2,it}, c_{it}, w_{E2,it}) = 1) \\ + (1 - I_{it}^{Lo}) \log \Pr(a_2^E(\mathbf{n}_{E2,it}, c_{it}, w_{E2,it}) = 0),$$

where $I_{it}^{Lo} = 1$ if a local store enters market i in year t and $I_{it}^{Lo} = 0$ otherwise. Rare as they are, in cases of multiple chain or multiple local entries in the same year and same market, I only consider the likelihood contribution from the first chain entry and the first local entry.

In the exit phase, the log-likelihood contribution is usually formed by multiple firms' survival rules and is more tedious to calculate. I discuss the general procedure here and use a flowchart in the online appendix to present the details. Specifically, five mutually exclusive cases are possible in a natural equilibrium.

(1) If all stores survive, it could be because either the realization of $w_{M,it}$ is sufficiently favorable for all of the lower-type stores (type \mathcal{L} or type \mathcal{H} when there are no type- \mathcal{L} stores) to continue for sure, or all of the lower-type stores are mixing between survival and exit and all happen to survive.

(2) If some but not all lower-type stores survive (and all higher-type stores, if any, must also survive in a natural equilibrium), it must be the case that the lower-type stores are mixing between survival and exit.

(3) If none of the lower-type stores survives and all higher-type stores survive, it could be because (i) $w_{M,it}$ is sufficiently unfavorable for even one lower-type store to continue but sufficiently favorable for all higher-type stores to survive for sure; (ii) $w_{M,it}$ is sufficiently unfavorable for even one lower-type store to continue, and all of the higher-type stores are mixing between survival and exit and all happen to survive; or (iii) all lower-type stores are mixing between survival and exit and all happen to exit, and all higher-type stores survive for sure.

(4) If none of the lower-type stores survives and some, but not all, higher-type stores survive, it must be the case that $w_{M,it}$ is sufficiently unfavorable for even one lower-type store to continue and all of the higher-type stores are mixing between survival and exit.

(5) If no store survives, it could be because either $w_{M,it}$ is sufficiently unfavorable for one of the higher-type

stores to continue for sure or all of the higher-type stores are mixing between survival and exit and all happen to exit.

Algorithm 1 always computes the necessary survival rules for assembling the probabilities of this market structure transition. Denote the probability for presurvival market structure $\mathbf{n}_{S,it}$ to become \mathbf{n}'_{it} when demand is c_{it} with $\mathbb{P}^S(\mathbf{n}'_{it}|\mathbf{n}_{S,it}, c_{it})$. The log-likelihood contribution in the survival phase is simple $\log \mathbb{P}^S(\mathbf{n}'_{it}|\mathbf{n}_{S,it}, c_{it})$.

A computational difficulty to get $\mathbb{P}^S(\mathbf{n}'_{it}|\mathbf{n}_{S,it}, c_{it})$ from survival rules is the integration over unobserved W_M . As an example, consider $\mathbb{P}^S(n'|\mathcal{L}|n_{\mathcal{L}}, c)$, the probability for a market with n type- \mathcal{L} firms to become $0 < n' < n$ in the survival phase. because some but not all stores survive, the market transition falls to case 2, and hence, the probability equals

$$\mathbb{P}^S(n'|\mathcal{L}|n_{\mathcal{L}}, c) \\ = \int_{\bar{w}_M(n_{\mathcal{L}}, c)}^{\bar{w}_M(n'_{\mathcal{L}}, c)} \binom{n}{n'} a^S(n_{\mathcal{L}}, c, w_M, \mathcal{L})^{n'} (1 - a^S(n_{\mathcal{L}}, c, w_M, \mathcal{L}))^{n-n'} \\ \cdot g_{W_M}(w_M) dw, \quad (16)$$

where $\bar{w}_M(n_{\mathcal{L}}, c)$ and $\bar{w}_M(n'_{\mathcal{L}}, c)$ give the lower and upper bounds on w_M , respectively, for the mixing to occur. When the realized w_M is lower (more favorable for incumbents) than $\bar{w}_M(n_{\mathcal{L}}, c)$, all n firms survive; when the realized w_M is higher (less favorable for incumbents) than $\bar{w}_M(n'_{\mathcal{L}}, c)$, all n firms exit. Note that the survival rule $a^S(n_{\mathcal{L}}, c, w_M, \mathcal{L})$, as a root of polynomial Equation (14), is a nonlinear function of w_M . A brute-force implementation of such integration is to first discretize or draw W_M , then solve Equation (14) and compute a^S under each grid point or realization of W_M . This approach is not only computationally expensive, owing to the necessity of finding high-order polynomials' roots, but also inaccurate because the infinite support of W_M challenges the precision of the approximation. As a solution, I use change of variables and substitute for w_M the value $x_M(a, n_{\mathcal{L}}, c, \mathcal{L})$ such that $a^S(n_{\mathcal{L}}, c, x_M, \mathcal{L}) = a$. The integration in Equation (16) then becomes

$$\mathbb{P}^S(n'|\mathcal{L}|n_{\mathcal{L}}, c) = \int_0^1 \binom{n}{n'} a^{n'} (1 - a)^{n-n'} \frac{dx_M(a, n_{\mathcal{L}}, c, \mathcal{L})}{da} \\ \cdot g_{W_M}(x_M(a, n_{\mathcal{L}}, c, \mathcal{L})) da. \quad (17)$$

To proceed, I further discretize a , a variable for probability, on a fine grid of S points over the interval $[0, 1]$. Denote these points as $p[1], \dots, p[S]$ with $p[1] = 0$ and $p[S] = 1$. Next, for each grid point $p[s]$, find $x_M(p[s], n_{\mathcal{L}}, c, \mathcal{L})$ satisfying

$$x_M(p[s], n_{\mathcal{L}}, c, \mathcal{L}) \\ = \log \sum_{n'=1}^n \binom{n-1}{n'-1} p[s]^{n'-1} (1 - p[s])^{n-n'} \bar{v}^S(n'|\mathcal{L}, c, \mathcal{L}) \\ - \log \beta \varphi_M.$$

This equation is a simple rearrangement of Equation (14) and makes use of the fact that $\varphi_M \exp(w_M)$ in all of the polynomial coefficients, except the scalar, cancels out. Note that computing $x_M(p[s], n_{\mathcal{L}}, c, \mathcal{L})$ only requires summation and multiplication—operations that computers can execute at much higher speed and with more precision than solving polynomials. Once all $x_M(p[s], n_{\mathcal{L}}, c, \mathcal{L})$ are nailed, its density function g_{w_M} defines the probability that each corresponding $p[s]$ has in calculating Equation (17) as the expected sum over $p[1], \dots, p[S]$.

This construction assumes that the researcher always observes the profitability types directly from data. Next, I use an example with two chain stores to illustrate how to deal with unobserved profitability types in the likelihood construction. Because all active stores' profitability types are public information for the players of the game, stores' equilibrium decisions are informative on the underlying market structure. Therefore, I infer the joint distribution for all active stores' types from their observed equilibrium actions using Bayes' rule. Consider market i with two active chain stores, A and B. Their joint type distribution has four points in the support. Denote the initial postentry probabilities in period t for these four points by $p_{it}^{E, \mathcal{H}\mathcal{H}}$, $p_{it}^{E, \mathcal{H}\mathcal{L}}$, $p_{it}^{E, \mathcal{L}\mathcal{H}}$, and $p_{it}^{E, \mathcal{L}\mathcal{L}}$, respectively (the types in the superscript are alphabetically ordered by the stores' names). These probabilities are either formed using the initial type probability ω_0 for $t = 1$ or inherited from the previous period's calculation for $t > 1$.

Now suppose that, for instance, both stores are observed to continue in period 1 under demand c_{it} . This observation is informative as to the underlying market structure. According to Bayes' rule, the probability that chain store A is type- k_1 and B is type- k_2 , conditional on joint continuation, is updated to

$$\tilde{p}_{it}^{E, k_1 k_2} = \frac{p_{it}^{E, k_1 k_2} \mathbb{P}^S(l_{k_1} + l_{k_2} | l_{k_1} + l_{k_2}, c_{it})}{\sum_{s \in \{\mathcal{L}, \mathcal{H}\}} \sum_{q \in \{\mathcal{L}, \mathcal{H}\}} p_{it}^{E, sq} \mathbb{P}^S(l_s + l_q | l_s + l_q, c_{it})},$$

in which $\mathbb{P}^S(l_s + l_q | l_s + l_q, c_{it})$ is the equilibrium-implied probability for market structure $l_s + l_q$ to remain unchanged under demand c_{it} . The numerator is the probability that A has type- k_1 , B has type- k_2 , and both survive under c_{it} . The denominator is the sum of probabilities that both A and B survive, taking all possible type combinations between them and under c_{it} .

With the updated probabilities, the observed joint continuations from A and B contribute to the log-likelihood by

$$\log(\mathbb{P}^S(2l_{\mathcal{L}} | 2l_{\mathcal{L}}, c_{it}) \tilde{p}_{it}^{E, \mathcal{L}\mathcal{L}} + \mathbb{P}^S(2l_{\mathcal{H}} | 2l_{\mathcal{H}}, c_{it}) \tilde{p}_{it}^{E, \mathcal{H}\mathcal{H}} + \mathbb{P}^S(l_{\mathcal{L}} + l_{\mathcal{H}} | l_{\mathcal{L}} + l_{\mathcal{H}}, c_{it}) (\tilde{p}_{it}^{E, \mathcal{H}\mathcal{L}} + \tilde{p}_{it}^{E, \mathcal{L}\mathcal{H}})),$$

which is the sum of the appropriate transition probabilities weighted by the probabilities of underlying market structures.

This completes the update for chain stores' unobserved types and the likelihood construction. Given the model's timing assumptions, the distribution of chain-store types is updated three times in each period: once after a chain store entrant's entry decision, once after a local store entrant's entry decision, and once after incumbent stores' continuation decisions. Likelihood contributions are also computed iteratively over time.

To search for the optimal value of the structural parameters, I use NFXP, which iterates between an outer loop and an inner loop. It starts with initial guesses of parameter values. In the inner loop, it uses procedure 1 to compute RNMPPE by finding the fixed points of a sequence of contraction mappings and returns the entry rules and market transition probabilities. Then it uses the likelihood construction procedure described in the online appendix to evaluate the partial likelihood for the structural parameters. In the outer loop, it searches for new parameter values to increase the likelihood value. When these values are updated, they are passed to the inner loop to solve the model again and regenerate the likelihood value. The algorithm stops when it cannot further improve the likelihood.

4.4. Results

The NFXP estimation is coded in MATLAB using the optimization library KNITRO. With 201 possible demand states, 77 possible market structures, and two profitability types, there are more than 30,000 points in the state space for v^E . Yet each likelihood evaluation typically takes less than 10 seconds with an off-the-shelf computer, and the entire estimation takes about two to four hours.

Estimates and inferences are reported in Table 1. I resample from all the markets with replacement 100 times to generate bootstrap standard errors and confidence intervals. Bootstrap inference suggests that all estimates are statistically significant.

In 2002, the beginning of the period examined, around 68% of the active chain stores are type- \mathcal{H} retailers. This is an outcome after market selection: 59% of the chain stores establish themselves as type- \mathcal{H} retailers in the market after entry. Estimates of $\theta_{\mathcal{H}}$ and $\theta_{\mathcal{L}}$ confirm the advantageous position of type- \mathcal{H} retailers relative to type- \mathcal{L} competitors. In the same market, a type- \mathcal{H} store enjoys a per-period revenue ($\bar{\pi}$) 6.6 times that of a type- \mathcal{L} . Also note that, because $\theta_{\mathcal{L}} < 0$, consumers enjoy a higher mean utility from the outside option than from a type- \mathcal{L} store. This is likely because many Dutch consumers prefer to shop in a chain store near their workplace or in neighboring markets instead of buying groceries in a nearby local store. If one has access to some revenue data, estimates of $\theta_{\mathcal{H}}$ and $\theta_{\mathcal{L}}$ can help to anchor normalized equilibrium values to euros.

In an average year and unconditionally on whether entry actually occurs, the estimated average sunk costs

Table 1. Estimates and Bootstrap Confidence Intervals

	Estimates	Standard error	90% Confidence intervals	95% Confidence intervals
Initial prob. type- \mathcal{H} (ω_0)	67.81%	9.47%	[65.45%, 70.22%]	[64.89%, 71.36%]
Prob. type- \mathcal{H} (ω)	58.69%	9.01%	[53.41%, 63.69%]	[52.41%, 64.01%]
Profit parameter \mathcal{H} ($\theta_{\mathcal{H}}$)	0.75	0.06	[0.66, 0.86]	[0.65, 0.87]
Profit parameter \mathcal{L} ($\theta_{\mathcal{L}}$)	-1.14	0.07	[-1.27, -1.03]	[-1.27, -1.01]
Sunk cost chain (φ_{E1})	157.81	17.05	[139.68, 190.43]	[134.24, 193.38]
Sunk cost local (φ_{E2})	14.10	1.71	[11.56, 17.12]	[11.42, 17.85]
Profit shock (φ_M)	1.34	0.09	[1.17, 1.46]	[1.17, 1.53]
Variance sunk cost chain (σ_{E1})	0.61	0.07	[0.54, 0.73]	[0.52, 0.80]
Variance profit shock (σ_M)	1.01	0.06	[0.94, 1.13]	[0.93, 1.13]

turn out to be large. Even for a potential monopolist chain-store entrant facing the largest market of 12,500 consumers, the average sunk cost is almost equal to the expected postentry value. For a local-store potential monopolist, the ratio is 1.4. A possible explanation for the barrier is the zoning regulation, which often greatly limits the availability of business space in residential areas. When such space is unavailable in a certain year, entry is virtually impossible.

Comparative Statics. Equilibrium payoff functions give the present value of stores' expected discounted profits. With the estimates in hand, I can evaluate a chain-store entry's impact on these values. When a chain store is certain to enter but has yet to realize its profitability type, Table 2 presents the expected percentage change in incumbent stores' values. Under the estimated parameter values of the model, loss of store value inflicted by the new chain store is expected to range from 19% to 24% for a type- \mathcal{H} incumbent retailer and from 28% to 58% for a type- \mathcal{L} retailer. Because entry only results in a decline of 11%–44% in flow profits for incumbents, a significant share of the damage to value is attributed to the reduced chance of survival: calculation shows that a type- \mathcal{L} retailer's survival probability can drop by 80%, to as much as eightfold, following a chain's entry.

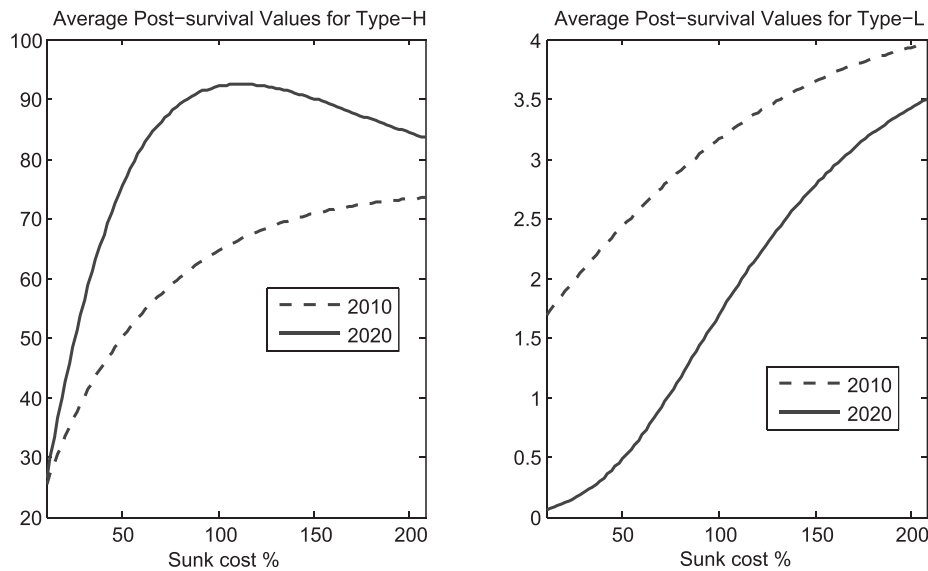
Policy Experiment. The estimation results suggest that a high average sunk cost often prevents such entry and slows down the chain store's expansion. Therefore, potential chain-store entrants are often tempted by possibilities to lower sunk cost—for example, petitioning for a relaxed zoning regulation.⁸ However, in a dynamic environment, reducing average sunk costs has two opposite effects on chain stores' values: (1) it lowers the barrier for chain stores to enter markets dominated by local stores, and (2) it intensifies the competition among chain stores. The net effect of changing sunk cost on chain stores' value must be empirically evaluated by a counterfactual simulation.

To this end, I simulate industry dynamics from 2010 to 2020, varying the average sunk cost for chain stores. The model's light computational burden allows me to quickly examine a wide range of possible values for average sunk cost. I first solve the RNMPE under 100 values of the parameter for chain stores' sunk cost φ_{E1} . These values are equidistant between 2.1 times the estimate (331.4) and its 90%-reduced value (15.8). Under each value, the obtained RNMPE is verified to be unique by verifying the monotonicity stated in Corollary 1 and by solving for polynomials' roots when monotonicity fails. I then simulate the market dynamics 1,000 times for 10 years and for all sampled markets, using market structure

Table 2. Expected Impact of One More Chain on Incumbents' Values and Flow Profits

	Zero type- \mathcal{L}	One type- \mathcal{L}	Two type- \mathcal{L}	Three type- \mathcal{L}	Four type- \mathcal{L}
Zero type- \mathcal{H}	—/— —	—/–58.4% (–44.2%)	—/–57.5% (–39.8%)	—/–56.7% (–36.3%)	—/–55.9% (–33.4%)
One type- \mathcal{H}	–23.8%/— (–27.6%)	–23.0%/–54.6% (–25.9%)	–22.5%/–53.1% (–24.4%)	–22.1%/–51.5% (–23.1%)	–21.8%/–49.7% (–21.9%)
Two type- \mathcal{H}	–21.7%/— (–19.3%)	–21.5%/–44.7% (–18.4%)	–21.4%/–43.0% (–17.7%)	–21.3%/–41.5% (–17.0%)	–21.2%/–40.1% (–16.3%)
Three type- \mathcal{H}	–20.4%/— (–14.8%)	–20.3%/–36.0% (–14.3%)	–20.3%/–34.9% (–13.9%)	–20.3%/–34.0% (–13.4%)	–20.3%/–33.1% (–13.0%)
Four type- \mathcal{H}	–19.4%/— (–12.1%)	–19.4%/–30.3% (–11.7%)	–19.4%/–29.7% (–11.4%)	–19.3%/–29.0% (–11.1%)	–19.3%/–28.4% (–10.9%)

Notes. The value in each cell before the “/” sign is the expected percentage change in a type- \mathcal{H} incumbent store's value. The value after the “/” sign is for a type- \mathcal{L} incumbent store. The percentage change in flow profit is included in parentheses. Change in value is computed using the estimated postentry values averaged over steady-state demand. Pre-entry markets in the same row (column) share the same number of type- \mathcal{H} (\mathcal{L}) incumbent stores.

Figure 7. Values for Stores Under Different φ_{E1} 

transition probabilities computed under these 100 values for φ_{E1} . Values for other primitives are held constant at their estimates in all simulations. Market structures at the end of 2010 are used to initialize the simulations.

Figure 7 illustrates the long-term impact of varying chain stores' sunk cost on the incumbent store. From left to right, panels show the average postsurvival values for type- \mathcal{H} and type- \mathcal{L} stores against different levels of φ_{E1} and averaged over all simulations and markets. Dashed lines represent the values in 2010 and solid lines in 2020. One interesting finding is that type- \mathcal{H} stores' average postsurvival value in 2020 is maximized at around 112% of the current level of φ_{E1} and not far away from the status quo. Different factors explain why this value is lower for both higher and lower values of φ_{E1} . On one hand, when the increase in φ_{E1} is more than moderate (20%), chain stores' entries are more rare, especially in markets with fewer consumers and initially occupied by some (possibly type- \mathcal{L}) incumbents. Further calculation shows that raising φ_{E1} to 200% almost halts chain stores' expansion, resulting in a smaller number of chain stores in 2020 than in 2010. Therefore, type- \mathcal{L} incumbents have higher postsurvival values, more of these type- \mathcal{L} incumbents will survive through 2020, and more will enter. This, in turn, dwarfs postsurvival values for type- \mathcal{H} incumbents in 2020. On the other hand, for lower values of φ_{E1} , chain stores enter more often and drive out many of the type- \mathcal{L} stores. In 2020, the lower φ_{E1} is, the fewer type- \mathcal{L} stores will still be active, and the competition will be more predominant among type- \mathcal{H} stores. Further calculation shows that when φ_{E1} is cut to 10%, the number of chain-store entrants in the 10 years will be seven times higher than in the baseline case, and the number of chain store incumbents in 2020

will be twice as many as the baseline value. With competition intensified, market selection on chain stores is more pronounced: The fraction of type- \mathcal{L} chain stores in 2020 is only 2.3%, and very few local stores will survive in 2020. This explains the low average post-survival values for chain stores in 2020.

The findings in this policy experiment suggest that the status quo promises close to the highest values for chain stores in a decade. This is because historical chain stores' entries primarily target markets by local stores, which limits the intensity of competition among chain stores. Therefore, investors in retail grocery should expect an improved return on chain stores in the next decade. Further restricting or relaxing chain stores' entries has an asymmetric effect: the former will benefit the incumbent local stores, and the latter will speed their destruction; in both cases, as long as the change in φ_{E1} is more than moderate, chain stores' profits will suffer.

4.5. Method Comparison

Before concluding, I wish to compare this model and its estimation procedure with their counterparts in the two-step method. This comparison focuses on three aspects: model specifications, computational burden, and the handling of multiple equilibria.

Model Specifications. The model developed here (henceforth "this model") is evidently less general than could be estimated using the two-step estimation methods (henceforth "two-step models"). In particular, firms' action space considered in this model only includes entry and exit, and two-step models typically accommodate continuous actions, such as technological investment, capacity building, and/or dynamic pricing.

Inclusion of these actions in this model will almost certainly result in two complications. First, continuous actions are an additional source of equilibrium multiplicity. This is because firms' best response functions in these actions usually have multiple crossings. Second, the algorithm proposed in Section 3 and other ACTY algorithms traverse through partitions of the state space in a particular order and use the strategy and payoffs calculated in previous steps as input for future steps. Some continuous actions may render such an ordering impossible. For instance, consider the duopoly example discussed in Section 3.1. If a type- \mathcal{L} incumbent can invest to become a type- \mathcal{H} firm and its type- \mathcal{H} rival may degenerate to type \mathcal{L} , then one would need to know $v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{H})$ when calculating $v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{L})$ in Step 2. Because $v^E(\iota_{\mathcal{H}} + \iota_{\mathcal{L}}, c, \mathcal{H})$ has not been calculated prior to Step 2, the algorithm's contraction mapping property breaks down.

Nonetheless, a few feasible extensions of this model enhance its generalizability. First, as discussed in Section 2, the timing of information arrival and firm actions in the entry phase can be altered to suit empirical needs. Second, Abbring et al. (2015) show that one can maintain most of the results in equilibrium existence, uniqueness, and computation while allowing for "learning by aging": Stores' profitability stochastically and exogenously improves as long as they survive market competition. However, estimating such a model with an unobserved and serially correlated state variable—the profitability type—considerably complicates identification and empirical implementation. Future work is required. Third, this model assumes that potential entrants cannot delay their entry opportunities. Relaxing this assumption to model adoption of the chain format upon entry over a retailer's lifespan can quantify the "option value" of delaying entry. This contributes to research on dynamic technology adoption (e.g., Ferrari et al 2010 and Yang and Ching 2013) by incorporating competition among adopters.

Computational Burden. The two-step methods gain computational speed by side-stepping equilibrium computation. Typically, a two-step estimation procedure spends most of its computation time on forward-simulating firms' actions to compute payoffs. In contrast, the most computationally expensive part of this model's estimation procedure is using value-function iteration to compute equilibrium payoffs as fixed points. Both methods' computational burden increases significantly when state space becomes larger and discount factor approaches one.

A major computational advantage that this model has over most of the two-step models is in allowing quick equilibrium calculation in counterfactual analysis. Because this model's computational algorithm comprises a sequence of contraction mappings, it is guaranteed to

converge to an RNMPE. As illustrated in the Dutch retail grocery example, this feature is particularly helpful when researchers need to examine a wide range of counterfactual parameters.

Handling Equilibrium Multiplicity. This model's RNMPE may still be multiple. Therefore, to handle possible equilibrium multiplicity in empirical work, its estimation procedure shares the same assumption with the two-step methods: the observed data are generated by a single equilibrium. This model differs from most of the two-step models in having a more "controllable" equilibrium multiplicity: for any set of parameter values, the number of this model's RNMPE is known and all RNMPE can be computed, which much improves on counterfactual analysis' reliability.

When equilibrium multiplicity occurs, ideally, the RNMPE that generates the highest likelihood value should be picked. However, the iterative nature of the NFXP algorithm does not allow a straightforward selection of the equilibria in its intermediate steps: under any set of trial parameter values, the RNMPE that maximizes the likelihood value is not necessarily the equilibrium under true parameter values. In the empirical exercise, I set the program to select the largest mixing probability when multiplicity occurs. Such a choice seems empirically plausible because a store survives much more often than exiting. It turns out that, in this case, no multiplicity is found on the path of parameter search and the selection rule is never used. A more systematic way of empirically treating the controllable equilibrium multiplicity warrants future research.

5. Conclusion

In this paper, I develop an econometric extension to ACTY models and provide a guide for its estimation. Two attractive features make this model a useful addition to the toolkit for market structure research. First, clear-cut results on equilibrium existence, uniqueness, and computation ensure the reliability of estimates and counterfactual analysis. In particular, any Markov-perfect equilibrium that survives some intuitive refinements can be quickly computed from low-dimensional contraction mappings. After computation, it is easy to check the uniqueness of the refined equilibrium. The contraction property ensures convergence of the estimation procedure. Second, the light computational burden allows for counterfactual analysis of many policy alternatives at a low cost. I demonstrate these features in an empirical application to the Dutch retail grocery industry.

The model empowers business analysts to gain useful insights with minimal data; market-level panel data on local demand and store presence are sufficient for

estimation. Such data are typically easy to procure from business registration records, industry surveys, or even the Yellow Pages. Although the minimal data requirement may be helpful in data-scarce applications, such as many business consultant and litigation cases, the model has the potential to accommodate richer data for better inference. For instance, if store-level sales data are available, they can easily be incorporated for a first-stage estimation of the per-period profit function. This should improve the efficiency of the estimation and relax parametric assumptions, such as the one posited in Equation (15), which would allow researchers to capture richer competition patterns, such as store complementarity (Vitorino 2012).

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Endnotes

¹ See their page 60: “We assume that, in each period, ex ante identical firms decide to enter sequentially until the expected value of entry falls sufficiently to render further entry unprofitable.”

² Arcidiacono and Miller (2011) propose to combine an expectation-maximization algorithm with a two-step method to estimate single-agent dynamic discrete choice models with persistent unobserved heterogeneity. To the best of my knowledge, empirical application of their method has not been extended to dynamic game models.

³ This could only arise for type- \mathcal{L} incumbents. If (1) all type- \mathcal{L} incumbents can deter a future entry by joint continuation and one type- \mathcal{L} firm’s continuation cannot, and (2) the gain from not having an entrant in the future dominates the harm from a decrease in present per-period profit, then solo continuation is less profitable than joint continuation.

⁴ I model entry and exit as stores’ individual decisions, which are independent across markets. That is, I abstract away from network considerations in chain-store headquarters’ expansion decisions. Both Jia (2008) and Holmes (2011) highlight the importance of networks in grocery chains’ entry decisions.

⁵ Although including active stores in neighboring areas is a more realistic way to model the outside option, it requires consideration of the network effect and is beyond the scope of the paper.

⁶ The algorithm developed by Abbring et al. (2015) allows more than two profitability types. In their model, these types can also stochastically improve over time. However, it is unclear how multiple unobserved and varying profitability types can be identified in empirical work. Thus, the econometric extension developed here restricts attention to two profitability types for chain stores.

⁷ Abbring et al. (2017) show that, in a model without profitability heterogeneity, the identification of profit functions can be nonparametric.

⁸ Several studies have documented zoning regulations’ entry-deterrence effect; for example, Bertrand and Kramarz (2002) and Datta and

Sudhir (2013). For a Dutch example of a chain store’s petition effort, see <https://www.raadvanstate.nl/uitspraken/zoeken-in-uitspraken/tekst-uitspraak.html?id=74608>.

References

- Abbring JH, Campbell JR, Yang N (2015) Simple Markov-perfect industry dynamics. Working paper, Federal Reserve Bank of Chicago, Chicago.
- Abbring JH, Campbell JR, Tilly J, Yang N (2017) Very simple Markov-perfect industry dynamics: Empirics. CentER Discussion Paper Series 2017-021, Tilburg University, Tilburg, Netherlands.
- Abbring JH, Campbell JR, Tilly J, Yang N (2018) Very simple Markov-perfect industry dynamics: Theory. *Econometrica* 86(2): 721–735.
- Aguirregabiria V, Mira P (2007) Sequential estimation of dynamic discrete games. *Econometrica* 75(1):1–53.
- Arcidiacono P, Miller R (2011) Conditional choice probability estimation of dynamic discrete choice models with unobserved heterogeneity. *Econometrica* 79(6):1823–1867.
- Bajari P, Benkard C, Levin J (2007) Estimating dynamic models of imperfect competition. *Econometrica* 75(5):1331–1370.
- Bertrand M, Kramarz F (2002) Does entry regulation hinder job creation? Evidence from the French retail industry. *Quart. J. Econom.* 117(4):1369–1413.
- Besanko D, Doraszelski U, Kryukov Y, Satterthwaite M (2010) Learning-by-doing, organizational forgetting, and industry dynamics. *Econometrica* 78(2):453–508.
- Cabral LM (1993) Experience advantages and entry dynamics. *J. Econom. Theory* 59(2):403–416.
- Datta S, Sudhir K (2013) Does reducing spatial differentiation increase product differentiation? Effects of zoning on retail entry and format variety. *Quant. Marketing Econom.* 11(1): 83–116.
- Doraszelski U, Satterthwaite M (2010) Computable Markov-perfect industry dynamics. *RAND J. Econom.* 41(2):215–243.
- Ellickson PB, Misra S, Nair HS (2012) Repositioning dynamics and pricing strategy. *J. Marketing Res.* 49(6):750–772.
- Ericson R, Pakes A (1995) Markov-perfect industry dynamics: A framework for empirical work. *Rev. Econom. Stud.* 62(1): 53–82.
- Ferrari S, Verboven F, Degryse H (2010) Investment and usage of new technologies: Evidence from a shared ATM network. *Amer. Econom. Rev.* 100(3):1046–1079.
- Fudenberg D, Tirole J (1991) *Game Theory* (MIT Press, Cambridge, MA).
- Holmes T (2011) The diffusion of Wal-Mart and economies of density. *Econometrica* 79(1):253–302.
- Jia P (2008) What happens when Wal-Mart comes to town: An empirical analysis of the discount retailing industry. *Econometrica* 76(6):1263–1316.
- Kolkman W (2000) The Netherlands retail food sector report 2000. Global Agriculture Information Network Report (NL0066), United States Department of Agriculture, Foreign Agricultural Service, Washington, DC.
- Pakes A, McGuire P (1994) Computing Markov-perfect Nash equilibria: Numerical implications of a dynamic differentiated product model. *RAND J. Econom.* 25(4):555–589.
- Pakes A, McGuire P (2001) Stochastic algorithms, symmetric Markov perfect equilibrium, and the “curse” of dimensionality. *Econometrica* 69(5):1261–1281.
- Pakes A, Ostrovsky M, Berry S (2007) Simple estimators for the parameters of discrete dynamic games, with entry/exit samples. *RAND J. Econom.* 28(3):373–399.

- Rust J (1987) Optimal replacement of GMC bus engines: An empirical model of Harold Zurcher. *Econometrica* 55(5): 999–1033.
- Ryan S (2012) The costs of environmental regulation in a concentrated industry. *Econometrica* 80(3):1019–1061.
- Van Lin A, Gijsbrechts E (2014) Shopper loyalty to whom? Chain versus outlet loyalty in the context of store acquisitions. *J. Marketing Res.* 51(3):352–370.
- Vitorino MA (2012) Empirical entry games with complementarities: An application to the shopping center industry. *J. Marketing Res.* 49(2):175–191.
- Yang B, Ching AT (2013) Dynamics of consumer adoption of financial innovation: The case of ATM cards. *Management Sci.* 60(4): 903–922.
- Yao S, Mela CF (2011) A dynamic model of sponsored search advertising. *Marketing Sci.* 30(3):447–468.

CORRECTION

In this article, “An Empirically Tractable Dynamic Oligopoly Model: Application to Store Entry and Exit in Dutch Grocery Retail” by Nan Yang (first published in *Articles in Advance*, November 16, 2018, *Marketing Science*, DOI:10.1287/mksc.2018.1117), a missing algorithm (Algorithm 1 on page 11) has been added.