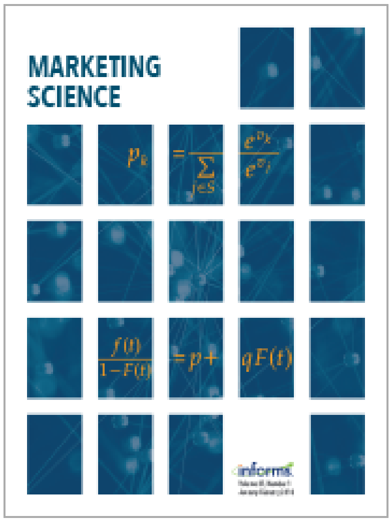

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Intertemporal Movie Distribution: Versioning When Customers Can Buy Both Versions

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We study a model of film distribution and consumption. The studio can release two goods, a theatrical version and a video version, and has to decide on its versioning and sequencing strategy. In contrast with the previous literature, we allow for the possibility that some consumers may watch both versions. This simple extension leads to novel results. It now becomes optimal to introduce versioning if the goods are not too substitute for one another, even when production costs are zero (pure information goods). We also demonstrate that the simultaneous release of the versions ("day-and-date" strategy) can be optimal when the studio is integrated with the exhibition and distribution channels. In contrast, a sequential release ("video window" strategy) is typically the outcome when the studio negotiates with independent distributors and exhibitors.

Key words: product segmentation; versioning; sequencing; distribution channels; movie industry; information goods
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1. Introduction

This paper examines the sale of movies through sequential distribution channels. Studios' revenues are determined by the number of versions being offered and by the timing of their releases. Movies are typically first shown in a theater, followed later by their video version. This corresponds to the principle of the "second-best alternative," according to which the producer should initially offer the movie in the channel that generates the highest revenue in the least amount of time. Then, the movie should cascade down to markets with lower returns per unit of time. Historically, this has resulted in theatrical screening, followed by pay-TV programming, home video, network television, and finally local television syndication (Owen and Wildman 1992, Lehmann and Weinberg 2000, Eliashberg et al. 2006, Vogel 2007).

An essential aspect in the sequencing strategy of a movie is the lapse of time between its initial release in theaters and its debut on video, which is called the "video window." In the last decade, the inter-release time has decreased gradually from the six months that was the norm for many years to approximately three or four months (Frank 1994, Nelson et al. 2007). Part of this change can be attributed to advances in the home video market, but there is also evidence of frictions between distribution channels, in particular between producers and exhibitors. For example, Disney decided in 2010 to accelerate the release of DVD and Blu-ray disc versions of *Alice in Wonderland*, a 3D motion picture directed by Tim Burton, shortly after its theatrical screening—both in the United States and in Europe.[1] In the United Kingdom, the big exhibitors first reacted by refusing to book any movie that would not have a guaranteed four-month run, but later they had to concede, and they allowed Disney to cut the video window to 12 weeks.[2] In this paper, we propose a model of movie marketing that can help the industry managers understand the consequences of the changes in the distribution agreements. First, we analyze versioning and sequencing in a general framework. Second, we examine the specific factors that influence the strategy of producers such as Disney when they negotiate the release sequence of their movies with independent exhibitors and DVD distributors.

The literature on versioning has shown that the release of a new version expands the market but also cannibalizes the existing versions, as some consumers switch to the new alternative. The seminal works of Mussa and Rosen (1978) and Moorthy (1984) show that versioning is optimal, whereas Stokey (1979)

[1] "Bob Iger wasn't bluffing. The Disney CEO has been telling Wall Street for months of his plans for studio executives to shorten traditional movie release schedules, and it appears the time has arrived for the first grand experiment" (DiOrio 2010).

[2] See *BBC News* (2010).

provides conditions under which it is not the best strategy. Salant (1989) reconciles these studies by showing that their differences stem from the marginal cost function. In fact, in the model specifications of Mussa and Rosen (1978), versioning is optimal if the marginal cost function of improving quality is sufficiently convex. By contrast, in the case of information goods, where the variable reproduction costs are constant or even zero, versioning is not profitable, and only the high-quality good should be supplied. Recent studies in the marketing and management literature have generalized this analysis (e.g., Bhargava and Choudhary 2008, Anderson and Dana 2009).

The literature has also tackled the fundamental question of whether to introduce the different versions simultaneously or sequentially. Moorthy and Png (1992) demonstrate that sequencing might be profitable when consumers are relatively more impatient than the seller. They employ a model with increasing marginal costs of production, which arguably does not fit information goods very well.[3]

Why, then, another paper on versioning? All the works mentioned above assume that consumers buy at most one version of a good, e.g., one cup of coffee or one model of car. However, this assumption does not really suit the movie industry. Although consumers might watch the same movie more than once in different channels, scant attention has been paid to developing theoretical models that explain how this affects the versioning and sequencing decisions of firms. Our model allows consumers to make multiple purchases over time, rather than restrict their choice set a priori.

Our first contribution to the marketing literature is to extend the models of Mussa and Rosen (1978) and Moorthy and Png (1992) to analyze versioning when consumers can buy the two variants of a product (the consumer's role) offered by an integrated monopolist (the manufacturer's role). This innovation has important implications in the commercialization of pure information goods. We show that if the single-unit purchase assumption is imposed, the firm never offers the two versions. In contrast, when consumers are allowed to buy both versions, versioning becomes optimal, with some consumers buying the high-quality good, some buying the low-quality good, and some consumers buying both. Versioning improves the firm's ability to price discriminate even among people who buy only one version. By decreasing the prices of *both* versions, the firm can considerably increase its overall profits. However, the profitability of this strategy depends on the degree of substitutability between versions, because if they are too substitute, one cannibalizes the other.

We also examine when the monopolist should use sequencing. We show that when the monopolist and the consumer have the same discount factor, the firm will *never* introduce the versions sequentially, because delaying the release of one version would only reduce future profits. However, when the consumers' time discount factor is lower than the seller's, sequencing emerges as long as the versions are imperfect substitutes and consumers can buy both versions. The firm now delays the release of the low-quality version as a mechanism to differentiate the products. A novelty of our model is that we endogenize the time when the firm releases the second version, whereas the previous literature typically considers a predetermined two-period game. We are able to determine the optimal release time and the factors that affect the length of the delay.

The second contribution of our paper is to show that versioning and sequencing strategies are more likely when there is vertical separation between producers and distribution channels (the channel's role). In the movie industry, theater exhibitors do not want too-short video windows, to avoid some consumers waiting for the video version. However, producers and video distributors might prefer a quicker video release, as this moves their video revenues ahead and increases the benefits of publicity. Although some papers such as Corts (2001), Mortimer (2007), and Gil (2009) take into account the vertical separation between the producer and the channels, to our knowledge, this is the first work that considers the full interaction among the roles of producers, consumers, and channels in the movie industry.

In our model, consumers can buy two versions, eventually, over two periods. Each version is not durable since consumption lasts only for one period. Still, because the utility of the version purchased in the second period depends on what was purchased in the first, there is an intertemporal link that allows us to relate our paper to the research on durable goods. As in this literature, consumers always form expectations about the prices of the versions that might be released in the future. Taking this into account, the seller tries to leverage multiple purchases by some consumers to improve its ability to price discriminate. The analogy with the durable goods literature is even clearer when we study the seller's price discrimination strategy in the presence of channel coordination problems. For instance, Desai and Purohit (1998) develop a model of a durable good that considers the effects of leasing and selling cars. Leases create a profitable secondary market that is in the manufacturer's

[3] Riggins (2004) extends Moorthy and Png (1992) to a setting where a seller markets its products in an online (Internet) channel and an off-line (bricks-and-mortar) store. Padmanabhan et al. (1997) analyze the distribution strategy when consumers are uncertain about the presence of demand externalities.

control. Purohit (1997) develops a two-period model where a manufacturer markets its cars through a rental agency and a dealer that sell the products to different sets of consumers. Channel conflict is also central in Shulman and Coughlan (2007), who analyze the negotiation between book manufacturers and retailers when consumers can resell books they no longer want.

Our model considers that the producer, as copyright holder, designs the whole commercialization strategy of movies. However, the producer must conduct separate wholesale bargains with the exhibitor and the distributor to determine the share of the revenues it keeps from each version. We show that with vertical separation, the producer supplies the two versions only if its bargaining power with the exhibitor is not too low. In addition, it might provide the two versions sequentially. In fact, independent DVD distributors set lower prices than an integrated monopolist. To moderate this coordination problem, the producer can delay the release of the DVD version.

Our findings are supported by recent empirical analysis that shows that the major studios ("majors") in the United States have longer video windows than independent integrated producers, who prefer a quicker introduction of videos (Waterman et al. 2010).[4] For many years the industry has discussed the possibility of using "day-and-date" strategies, meaning that a title is released across two or more channels on the same day.[5] A series of experiments have recently been carried out in this direction. For instance, the movie *Bubble*, directed by Steven Soderbergh, was released simultaneously across all channels back in 2006 by 2929 Entertainment, a company that is vertically integrated across production, distribution, and exhibition. In our model, we show that the shrinking of the video window observed in the last few years is related to both the increase of the producers' bargaining power vis-à-vis the various channels and the perceived convergence in quality offered by theatrical and video versions.

The rest of the paper is as follows. Section 2 analyzes the optimal versioning and sequencing strategies of an integrated monopolist. Section 3 reassesses the main results when the two versions are sold in a vertically separated chain. Section 4 concludes and offers directions for future research.

2. The Basic Model

We consider a single firm that offers two versions of a product, a high-quality version denoted as H and a low-quality version denoted as L. Our departure from the extant literature is that we allow some consumers to buy both versions. Thus consumers can buy one unit of H alone, one unit of L alone, or one unit of both versions (we refer to this case as B, a mnemonic for "both"), or they can decide to buy nothing (we denote this case by 0). In our application to the movie industry, the firm is an integrated producer, H represents watching a movie in a theater, and L represents renting a DVD for watching it at home.[6]

Let u_i denote the quality of product $i = \{H, L, B, 0\}$, where $u_H > u_L$ and $u_0 = 0$. At times it will be convenient to refer to $k = u_H/u_L > 1$, which denotes the "quality ratio" of the two versions. When both H and L are bought, the resulting quality of both versions consumed jointly is

$$u_B = u_H + u_L(1-s),$$

where s represents the level of substitutability of the products. The versions are partial substitutes for $0 \le s < 1$. Products H and B are perfect substitutes for $s = 1$, and versions H and L are independent for $s = 0$. Notice that $s = 1$ corresponds to the standard case in the literature, where consumers might only purchase a single unit of either version. In fact, in this case, if a consumer has already bought H, buying L confers no additional utility on top, and therefore she never buys the two versions. The more interesting case is when $0 < s < 1$, as it represents the situation where consumers are willing to watch a movie at home that they have already watched in a theater, though their liking is not as high as if they were watching the movie at home for the first time. Indeed, in an empirical model on versioning, Luan and Sudhir (2007) find that consumers obtain less utility from DVDs after having viewed the movie in a theater. Our model also accounts for the case of complements where theater viewing enhances the pleasure of watching the DVD, in which case $s < 0$.

There is a continuum of consumers who are heterogeneous in their preferences over quality. Following Mussa and Rosen (1978), we employ a standard model of vertical product differentiation, where each

[4] Several authors have analyzed the factors affecting the video window (Frank 1994, Lehmann and Weinberg 2000, Hennig-Thurau et al. 2007). August and Shin (2011) study sequencing release of movies when going early to the theater can generate negative congestion externalities.

[5] Some big Hollywood players have recently decided to distribute films via online service, video-on-demand channels, and movie theaters—sometimes simultaneously. See Kung (2011).

[6] In the movie industry, the utility derived from the theatrical version is allegedly higher than the one obtained from the video version. It is worth noting that a feature of videos, which is not considered here explicitly, is that they can be viewed several times and by several people (Varian 2000). To account for this, the model can be reformulated in terms of having comparable units so that the DVD streams are aggregated to generate a single overall stock, and the video price would realistically reflect the average price per viewer.

consumer is represented by her type θ, which is uniformly distributed over the segment $[0, 1]$. The net surplus of a consumer that buys a product of quality u_i at price p_i is given by $\theta u_i - p_i$.

We take the quality of the two versions as given; hence we do not study the problem of film production, which is beyond the scope of this paper. Our problem is therefore narrower in scope but also quite well defined: Imagine a movie has been shot, and its quality is already determined. How many versions of it should be distributed, and when?

We assume that the cost of supplying each unit of H is equal to or higher than the cost of providing L, $c_H \geq c_L \geq 0$. This specification is general, in that it can accommodate both "industrial goods," for which production and distribution costs are relevant and increase with the level of quality, and "information goods," for which reproduction costs are negligible once the product has been developed (Jones and Mendelson 2011). We show how this distinction in the cost structure is crucial to explain versioning.

Movies, as well as other forms of digital content such as TV programming and music, are a classic case of information goods. Taking this into account, in Proposition 1 we characterize the general solution of our model, and in the second part of the paper, we investigate in greater detail the case of "pure" information goods ($c_H = c_L = 0$). This assumption simplifies expressions and is useful for analyzing price discrimination and versioning, because it implies that differences in the prices of the versions are due only to differences in willingness to pay.

2.1. Simultaneous Release of the Two Versions

We start with a single-period analysis where we study the monopolist's optimal segmentation strategy when versions are released simultaneously. The second part of this section considers the possibility of releasing the two versions sequentially.

When the two versions are released simultaneously, the net payoff V of a type θ consumer can be summarized as follows, where $i = \{H, L, B, 0\}$ denotes the set of choices available to each consumer:

$$V(\theta, i) = \begin{cases} \theta u_H - p_H & \text{if } H, \\ \theta u_L - p_L & \text{if } L, \\ \theta[u_H + u_L(1-s)] - p_H - p_L & \text{if } B, \\ 0 & \text{if } 0. \end{cases} \tag{1}$$

We can now illustrate how the market is split, taking into account that consumers make incentive-compatible decisions. Define θ_{ij} as the consumer that is indifferent between buying good i and good j, where $j \neq i$, at a price $p_i = \{p_H, p_L, p_H + p_L, 0\}$, respectively. We thus obtain that $\theta_{LH} = (p_H - p_L)/(u_H - u_L)$ is the consumer that is indifferent between buying L and H separately; that is, $V(\theta_{LH}, L) \equiv V(\theta_{LH}, H)$. Similarly, $\theta_{HB} = p_L/(u_B - u_H) = p_L/[u_L(1-s)]$ is the consumer that is indifferent between buying the high-quality version and both versions, and $\theta_{i0} = p_i/u_i$ is the consumer that is indifferent between buying product $i = H, L, B$, and not buying anything. Different market shares for the two products can arise according to the relative magnitude of the various indifferent consumers θ_{ij}.

The timing of the game is the following: the firm decides how many versions to release and sets the prices p_L and p_H to maximize its profits, anticipating that consumers will purchase one particular version, both, or none. Proposition 1 describes the optimal strategy.

Proposition 1. *The firm's optimal segmentation strategy is the following.*

(1) *If* $c_H < c_L k$
- *when* $s^1 < s \leq 1$, *the firm only supplies* H (*no versioning*);
- *when* $s^2 < s \leq s^1$, *the firm segments consumers as* $L/H/B$ (*versioning*);
- *when* $s^3 < s \leq s^2$, *the firm segments consumers as* H/B (*versioning*); *and*
- *when* $s \leq s^3 < 0$, *the firm supplies* B (*strongly complementary products*).

(2) *If* $c_H = c_L k$
- *when* $s^1 < s \leq 1$, *the firm only supplies* H (*no versioning*);
- *when* $0 < s \leq s^1$, *the firm segments consumers as* $L/H/B$ (*versioning*); *and*
- *when* $s \leq 0$, *the firm supplies* B (*complementary products*).

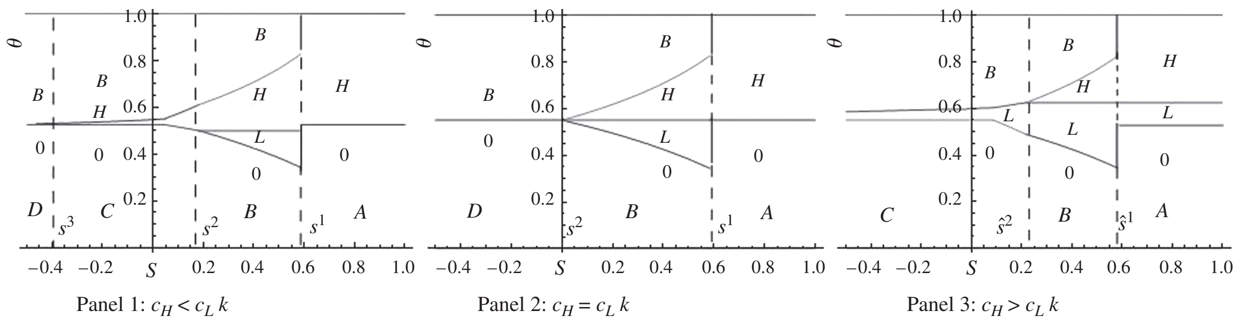
(3) *If* $c_H > c_L k$
- *when* $\hat{s}^1 < s \leq 1$, *the firm segments consumers as* L/H (*versioning*);
- *when* $\hat{s}^2 < s \leq \hat{s}^1$, *the firm segments consumers as* $L/H/B$ (*versioning*);
- *when* $\hat{s}^3 < s \leq \hat{s}^2$, *the firm segments consumers as* L/B (*versioning*); *and*
- *when* $s \leq \hat{s}^3 < 0$, *the firm supplies* B (*strongly complementary products*).

Proof. See the appendix.[7]

Proposition 1 shows that the firm's versioning strategy depends on the relative costs of the versions, on their degree of substitution, and on the utility they generate. We show that when the marginal costs of supplying the versions are sufficiently convex ($c_H > c_L k$), the firm always releases the two versions. This result has been previously identified in the literature in the case with a single-unit purchase, $s = 1$ (e.g., Bhargava and Choudhary 2001), and we have

[7] In the proof, we provide the expressions for prices, profits, indifferent types, and threshold values of s.

Figure 1 Market Segmentation



Notes. The figure shows how the firm optimally segments the market as a function of the degree of substitutability s, taking into account the incentive-compatible choices of consumers. Consumer types in region B purchase multiple products, those in region H buy the high-quality product, those in region L buy the low-quality product, and those in region 0 do not buy (parameter values $u_L = 1$, $k = 3$, $c_L = 0.1$; and $c_H = 0.1, 0.3, 0.6$ in panels 1, 2, and 3, respectively).

generalized it for any value of the degree of substitutability s. In Figure 1, this is the case of "industrial goods," depicted in region C of panel 3. Our newer and more interesting result is to show that, if costs are not too convex ($c_H \leq c_L k$)—and in particular, in the presence of "information goods" ($c_H = c_L = 0$)—the firm can segment consumers depending on the degree of substitutability of the versions: there is segmentation if some consumers are allowed to buy the two versions and the degree of substitutability is not too high.

The first part of Proposition 1 examines the case where $c_H < c_L k$. If the degree of substitutability is sufficiently high ($s > s^1$), the cannibalization effect of introducing the lower-quality variant always prevails over the market expansion effect, and the monopolist is better off by supplying only H. In Figure 1, this is represented as region A in panel 1. Despite this finding, for a lower level of substitutability, the firm finds it profitable to offer both variants, and consumers self-select the variant(s) that maximize their individual utility. For $s^2 < s \leq s^1$, the firm segments consumers as $L/H/B$, where this notation means that consumers with a very low θ buy nothing, those with a low θ buy only L, those with an intermediate θ buy H, and those with a high θ buy both versions (region B in panel 1). For $s^3 < s \leq s^2$ (region C), consumers are segmented as H/B, as no consumer buys L alone: intermediate types buy H, and higher types buy both versions. For $s \leq s^3$ (region D), the two versions are such strong complements that segmentation disappears altogether, and it is optimal to sell both versions to every buyer. In the proof (in the appendix), we show that the existence and size of these four regions depends on the costs of the versions and on their relative utility.

The second part of the proposition analyzes the case when cost differences match utility differences; $c_H = c_L k$. In this situation, $s^2 = s^3 = 0$, and region C disappears. For $s > s^1$, the monopolist only offers H (region A in panel 2), and for $0 \leq s \leq s^1$ (region B), consumers are segmented as $L/H/B$. In the particular case of information goods ($c_H = c_L = 0$), it is also found that $s^1 = 2/3$. The third part shows that for $c_H > c_L k$, the firm always releases the two versions. But also in this case, the firm's segmentation policy is determined by the degree of substitutability of the versions. For $\hat{s}^1 \leq s \leq 1$ (region A in panel 3), consumers are segmented as L/H (the "standard" case). For $s \leq \hat{s}^1$, the firm sets prices such that high-type consumers buy the two versions. In particular, for $\hat{s}^2 < s \leq \hat{s}^1$, consumers are segmented as $L/H/B$ (region B). For $\hat{s}^3 < s \leq \hat{s}^2$, the segmentation is L/B (region C), and for $s \leq \hat{s}^3$, only B is supplied (this region is not illustrated in panel 3).

It is important to take a closer look at the firm's segmentation appearing in region B of the three panels, whereby some consumers who view the movie in the theater might also want to buy the DVD version. As shown in the proof of Proposition 1, the optimal prices are as follows:

$$p_H = \frac{c_H + u_H}{2} - \frac{s u_L}{2(2-s)}, \quad p_L = \frac{c_L + u_L}{2} - \frac{s u_L}{2(2-s)}.$$

Imagine now that the manufacturer offered the two versions but *did not* account for the possibility that some consumers might buy both of them. In this case, prices would take the familiar expressions:

$$p_H^* = \frac{c_H + u_H}{2} > p_H, \quad p_L^* = \frac{c_L + u_L}{2} > p_L,$$

where s does not enter in the expressions because, by assumption, these are the prices when consumers purchase one unit at most. Comparisons are telling: when consumers can purchase the two units, the firm must decrease *both* prices by the same amount, according to the magnitude of s. Although this does not change the self-selection problem of the consumer indifferent between H and L, it does allow expanding the market share of those who buy both versions, which turns out to be profitable. To continue with this thought experiment, imagine that the firm charged the standard prices p_H^* and p_L^*, but consumers were left to buy as many units as they want. With pure information goods, it is immediate to compute the profit loss from setting suboptimal prices, which amounts to $-(s^2u_L)/(4(2-s)(1-s))$. This expression increases with s, as long as region B exists ($s < 2/3$). To put it in perspective, if the high-quality version is twice as good as the low-quality version, $k = 2$ and $s = 1/2$, the firm can increase its profits by 17% by charging p_H and p_L instead of p_H^* and p_L^*.

The main result in this section is that market segmentation is sustainable when some consumers are able to buy both versions and their degree of substitutability is not too high. Indeed, it is remarkable from the three panels in Figure 1 that versioning is a robust result for lower values of s, independent from the degree of convexity of costs. By contrast, it is only for high values of s that the nature of marginal costs determines the segmentation strategy.

2.2. Sequential Release

We now examine the case where the movie producer can introduce both versions sequentially. Imagine that H is offered in the first period, and L can be released simultaneously or at a later time. With this extension we are stipulating two dimensions of product substitutability: one is exogenous to the firm and is represented by s, and the other is endogenous and is created by delaying the introduction of L.

Imagine that the firm releases H at time t_0 and L at a possibly later time t_1. We denote by d the *compound* discount factor for t_1, so that choosing d implicitly determines the time t_1 when L is released. In particular, the period of time that elapses between t_0 and t_1 can be obtained from $d = \delta_p^t$, where δ_p is the producer's discount factor and $t = t_1 - t_0$ is the video window that separates the release of the two versions.[8] If $t_0 = t_1$, then $d = 1$, and H and L are supplied simultaneously. If $t_0 < t_1$, then $d < 1$, and L is introduced some time after H. The lower the value of d, the later the release of L. The case where H is released alone corresponds to the case of infinite delay of L; that is, $d = 0$.

We also account for possible differences in the discount factor of the producer and of the viewers. We define the consumer's discount factor as $\delta_c = x\delta_p$, where $0 \le x \le 1$ measures the consumer's relative impatience with respect to the firm. Hence, we can write the consumer's compound discount factor as $d_c = x^t d$. Consumer's utility function (1) is thus generalized to

$$V(\theta, i) = \begin{cases} \theta u_H - p_H & \text{if } H, \\ \theta d_c u_L - d_c p_L & \text{if } L, \\ \theta[u_H + d_c u_L(1-s)] - p_H - d_c p_L & \text{if } B, \\ 0 & \text{if } 0. \end{cases} \quad (2)$$

The expressions for the indifferent types are the same as before, with the exception of $\theta_{LH} = (p_H - d_c p_L)/(u_H - d_c u_L)$, which is the consumer that is indifferent between buying separately either H at t_0 or L at t_1.

We analyze the producer's sequential release decision when it can commit to its subsequent policy and viewers form correct expectations about it. Although the producer commits to a price path, the parameters of the contract clearly can and do change between the periods. The commitment assumption is particularly appropriate for the movie industry, because studios usually announce in advance the price and the release date of DVDs on their websites and in specialized magazines.[9] They also indicate recommended prices for DVD stores. These marketing campaigns are costly to modify and allow consumers to anticipate producers' future actions.

The timing of the game is now the following: at time t_0, the producer sets the retail prices p_H and p_L and chooses the time $t_1 \ge t_0$ at which L will be released. At time t_0, it then releases H, and at time t_1, it releases L. Taking into account the possibility of sequencing, we first establish a generalization of Proposition 1 that shows that our previous results are maintained when the firm is allowed to delay the introduction of L.

Corollary 1. *If consumers and the firm have the same discount factor* ($x = 1$), *sequencing never arises for any cost* $c_H \ge c_L \ge 0$ *and for any value of* s. *Products' segmentation is as in Proposition* 1.

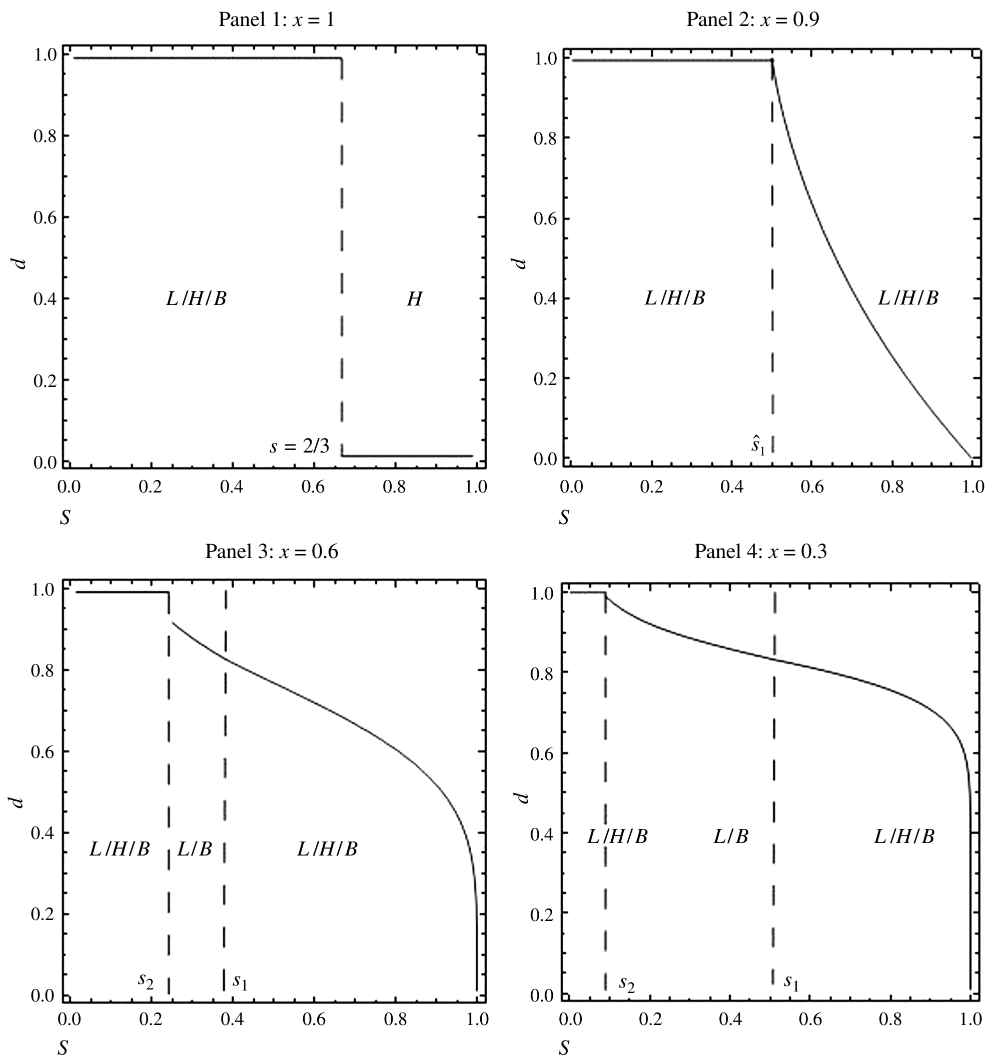
Proof. See the appendix.

When the producer's and the viewers' discount factors do not differ, the firm *never* introduces the products sequentially, because the loss generated by the postponement of profits of L does not compensate the reduction in the cannibalization over H (see panel 1 in Figure 2). More precisely, as the profit of the firm is convex in d, it is optimal either to introduce the two versions simultaneously ($d = 1$) or to never introduce L at all ($d = 0$). As described in Proposition 1, the latter outcome arises for large values of s; otherwise, there is versioning. It is interesting to note that

[8] From $d = \delta_p^t$, it follows that the endogenous release time is $t = \log(d)/\log(\delta_p)$. Most models on sequencing reviewed in §1 typically consider only two predetermined periods (an exception is Prasad et al. 2004) and analyze whether to introduce another version in the first or second period. In contrast, we endogenize optimal sequencing to examine *when* to introduce a new variant.

[9] For example, see the Disney release calendar (http://disneydvd.disney.go.com/release-calendar, accessed April 17, 2012) or Movie Insider DVD releases 2012 (http://www.movieinsider.com/dvds/2012/, accessed April 17, 2012).

Figure 2 Video Window



Notes. The figure plots the profit-maximizing value of d as a function of the degree of substitutability s and for different values of the relative degree of consumers' impatience x. When $d = 1$, there is immediate release; when $d = 0$, there is no versioning (parameters $u_L = 1$, $k = 3$, and $\delta_p = 0.9$).

this convexity of the profit function cannot be found in the extant literature that has considered only two exogenously predetermined periods.

This finding suggests that our model should be amended to understand why producers in the movie industry maintain a video window. We first do so by allowing discount factors to differ, making consumers relatively more impatient than the firm.

For simplicity, we concentrate on the limiting but empirically relevant case of pure information goods, for which marginal costs are assumed to be zero. This assumption fits well with the commercialization of movies, and it implies that, once the quality of the movie is determined, the firm has constant marginal costs of supplying each version, which we normalize to zero. For example, the costs of making an additional copy of a theatrical version or of a DVD are constant and quite small when compared to its (sunk) production costs. Similarly, the physical costs of broadcasting a movie through a pay-TV channel or a local free-to-air channel are close to zero. In general, we just require that the marginal cost of serving an extra consumer be negligible compared to the amortized development cost of the movie. The nature of our results is robust to introducing a constant and equal marginal cost for both versions. However, the zero marginal cost assumption yields manageable expressions that allow us to extend the model in several directions. We now study under which circumstances the firm will release the versions sequentially.

Proposition 2. *The firm's optimal versioning and sequencing strategy for information goods is as follows.*

(1) *If* $1 > \delta_c/\delta_p = x > x^*$, *the firm's optimal strategy is*

• *when* $s = 1$, *the firm supplies only H immediately* (*no versioning*);

• *when* $\hat{s}_1 < s < 1$, *the firm segments consumers as* $L/H/B$ *and* $0 < d < 1$ (*sequencing*); *and*

• *when* $0 < s \leq \hat{s}_1$, *the firm segments consumers as* $L/H/B$ *and* $d = 1$ (*versioning*);

(2) *If* $0 < \delta_c/\delta_p = x < x^*$, *the firm's optimal strategy is*

• *when* $s = 1$, *the firm supplies only H immediately* (*no versioning*);

• *when* $s_1 < s < 1$, *the firm segments consumers as* $L/H/B$ *and* $0 < d < 1$ (*sequencing*);

• *when* $s_2 < s \leq s_1$, *the firm segments consumers as* L/B *and* $0 < d < 1$ (*sequencing*); *and*

• *when* $0 < s \leq s_2$, *the firm segments consumers as* $L/H/B$ *and* $d = 1$ (*versioning*).

Proof. See the appendix.[10]

These results reveal that sequencing can emerge when consumers are relatively more impatient than the firm ($x < 1$). The firm supplies H alone in the first period *only* in the limiting case where $s = 1$ (H and B are perfect substitutes). That is, when L does not confer any additional utility to consumers, the firm does not release it, not even some time after the introduction of H.

For $s < 1$, the strategy of the firm depends on the ratio $x = \delta_c/\delta_p$. Panel 2 of Figure 2 shows that, for $x > x^*$,[11] when s is low enough ($0 < s \leq \hat{s}_1$), there is no sequencing (i.e., there is no video window). The two goods are rather independent, and the firm releases them immediately as there is no gain in introducing L later. For $\hat{s}_1 < s < 1$, however, there is now both versioning and sequencing. A low segment of consumers buys only L in the second period, an intermediate segment of consumers buys only H in the first period, and a high segment of consumers buys both goods sequentially. The screening problem with sequencing becomes more profitable with impatient customers; those especially with an intermediate willingness to pay prefer to buy H immediately instead of waiting for the delayed release of L. As we formally show in the proof (in the appendix), $\hat{s}_1$ becomes smaller (and therefore sequencing arises more often) as x gets lower (customers are relatively more impatient) or as δ_p gets higher (the firm does not discount much the future).

Panels 3 and 4 in Figure 2 are drawn for $x < x^*$. They present a story similar to that in panel 2 when s is high ($s_1 < s < 1$), in which case the firm segments the market as $L/H/B$ and the versions are introduced sequentially, and when s is low ($0 \leq s \leq s_2$), in which case it segments consumers as $L/H/B$ and versions are introduced simultaneously. For intermediate values of s ($s_2 < s < s_1$), there is an additional region where the firm segments consumers as L/B and the versions are released sequentially. In this case, the firm finds it optimal to set prices such that *all* consumers who bought H in the first period also buy L later.

Our results complement the findings of Moorthy and Png (1992). Recall that their model shows that when consumers only buy one version, there is versioning and sequencing if the monopolist has a convex cost function *and* it is less impatient than consumers. We show that when consumers can purchase the two versions, and these are not perfect substitutes, sequencing does not depend on the nature of the cost function, and it even arises with information goods. An additional contribution is that while these authors assume that the firm can only release the versions at two possible dates (in our notation, $t_1 - t_0 = t = 1$), our model allows the video length to be endogenous, taking any value.

To sum up, we have shown that under the multiple-purchase extension, a firm might market the versions sequentially, even in the case of an information good like a movie. This result, however, requires that the consumers' discount factor be lower than the producer's, which is empirically difficult to justify or validate. We next propose a new and simple reason for sequencing that relies on the vertical structure of the industry.

3. Sequential Distribution of Movies in a Vertically Separated Industry

This section extends our basic model and focuses on the case where the versions of a "pure" information good can be released through separated distribution "channels." It is customary to divide the movie industry into three vertically related sectors: production, distribution, and exhibition. In the United States, production and distribution are often performed by the same studios, which we call "producers" in our model. Hollywood's major studios (Universal Pictures, Paramount Pictures, Metro–Goldwyn–Mayer, 20th Century Fox, Columbia Pictures, Disney, and Warner Bros.) account for 80% to 90% of the total income from the sale of movies to theaters and other media in the United States (Vogel 2007). "Exhibitors," by contrast, run theaters and screen movies to attract audiences.

The vertical structure of the movie industry has undergone several changes over the last century. In the 1920s and 1930s, majors owned chains of theaters and formed cartels to assign the stage run and to set minimum prices and geographic and temporal clearances to theaters in urban areas (Orbach 2004). In 1948, the U.S. Supreme Court in *United States v. Paramount Pictures, Inc.* (334 U.S. 131) considered that

[10] In the proof, we provide the expressions for prices, video window, and threshold values of s.

[11] With the parameters of Figure 2, it is $x^* \simeq 0.62$.

these agreements were in violation of the Sherman Act and required the majors to divest themselves of their theater chains. The Court prevented majors from setting admission prices and exclusivity contracts with exhibitors. These restrictions were relaxed in the 1980s, and some majors acquired large theater chains.

Producers and exhibitors negotiate to share a percentage of the theater box office receipts (De Vany 2004), but their interests are not perfectly aligned. Exhibitors enjoy some market power (e.g., enjoy exclusivity deals, can determine the number of weeks the movie will be on screen) and use it to jointly maximize their admission prices and revenues from popcorn and other concessions (Vogel 2007, McKenzie 2008, Gil and Hartmann 2009) and determine the length of the movie run (Gil 2009). Producers cannot avoid this with resale price maintenance or with vertical integration, given the strict antitrust provisions.

Another important channel for producers is the home video; nowadays video rentals and DVD sales are the largest source of domestic revenue for studios in the United States (Waterman et al. 2010). Mortimer (2007) and Ho et al. (2012) analyze different pricing mechanisms used by producers in their contracts with video stores. They show that Blockbuster Video adopted revenue sharing agreements in 1998, and other retailers were quick to adopt the same instrument.

In this section, we examine how channel management modifies the versioning and sequencing release of movies, which we consider as pure information goods. Taking into account the literature on product lines, our assumption of zero marginal costs would stack the deck against the existence of market segmentation. In the presence of an integrated firm, several versions should not arise, let alone their sequencing. This makes the characterization of versioning in the presence of channel separation our most interesting and novel contribution.

To address this case, we imagine there is a producer (a studio), an exhibitor (a movie theater), and a distributor (a DVD store). The producer holds all the rights over the movie but needs to release it in theaters and/or through DVD stores. The studio bargains wholesale payments with each channel, and afterwards, the channels set the retail price of the versions they distribute. We first consider that the producer is vertically separated from every downstream channel. The producer cannot fully appropriate the revenues associated with each version, and its commercialization interests are not perfectly aligned with those of the exhibitor and of the independent video distributor (e.g., Blockbuster or Netflix). At the end of this section, we present the case where the producer sells the DVD versions via its own stores (e.g., Disney or Warner Bros. stores) or offers the movies directly via Internet streaming or other platforms (e.g., UltraViolet), as this is also a case of material relevance in the industry.

3.1. Negotiations Between the Producer and Two Independent Distribution Channels

Following the notation in §2, we call H the movie exhibited in theaters (high-quality version) and L the movie viewed on DVD (low-quality version). We stipulate that the prices of H and L are set independently by the exhibitor and distributor, respectively. Moreover, the producer bargains with the exhibitor over the share of the theater revenues it keeps (r_e) and the release time of the video version (t_1). Afterwards, it separately bargains with the distributor over a revenue sharing agreement of video sales (r_d).[12]

The fact that the video window is decided together by the producer and by the exhibitor follows directly from the example of *Alice in Wonderland* mentioned in §1, where exhibitors engaged in a struggle with Disney. This is just one instance of the tension existing between producers and exhibitors.[13]

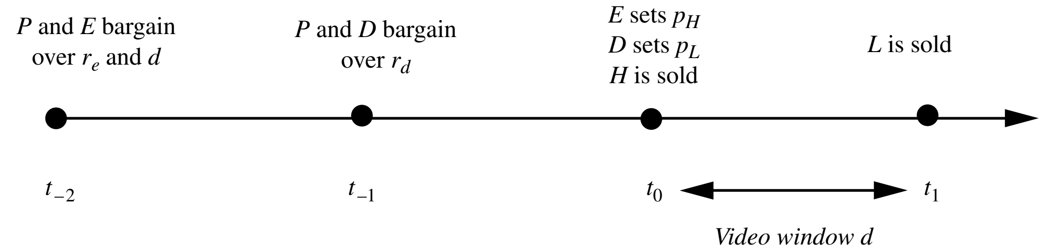
We next describe the game that is played between the producer and the channels. Figure 3 depicts the sequence of moves that are played along the equilibrium path as agreements between parties will be reached. However, we also need to specify what happens in disagreements. We denote by t_0 the earliest possible date to show any version of the movie. Prior to this date, the timing of the negotiations is as follows (see Figure 3):

- At time t_{-2} the producer and the exhibitor jointly decide whether or not to show the movie in the theater (version H). If negotiations break down, H is not shown, the exhibitor gets nothing, and the producer still can negotiate at time t_{-1} with the distributor the release of L. If, instead, the two parties agree to show H, they negotiate over the revenue share r_e that must accrue to the producer (the share $1 - r_e$ is kept by the exhibitor) and the release time $t_1 \geq t_0$ for the video version. We model bargaining using the generalized Nash axiomatic approach, whereas the two negotiating players maximize the weighted product

[12] Notice that revenue sharing contracts do not cause double markups with either the exhibitor or the distributor. Revenue sharing contracts are widely employed in the industry and have attracted the attention of the empirical literature on movie distribution.

[13] As another example of the views of exhibitors, John Fithian, President and CEO of the National Association of Theatre Owners, commented, "I believe that the two biggest threats to the movie business are shrinking theatrical release windows and movie theft (or 'piracy')" (National Association of Theatre Owners 2007). Instead, video stores do not directly participate in the determination of video windows.

Figure 3 Timeline in the Channels' Negotiation Game



of the payoffs received in excess of their disagreement payoffs. The producer's degree of bargaining power is denoted by α, and the exhibitor's by $1-\alpha$.

- At time t_{-1}, the producer and the video distributor jointly decide the revenue share r_d accruing to the producer from the DVD (the share $1-r_d$ is kept by the distributor). In this negotiation, β is the producer's degree of bargaining power, and $1-\beta$ is the distributor's.[14] If at time t_{-2} an agreement with the exhibitor has been reached, then DVD sales can occur at $t_1 \geq t_0$. If, instead, past negotiations with the exhibitor have failed, the distributor and the producer could commercialize the DVD at t_0. In either case, if the negotiation with the distributor breaks down, the distributor gets nothing while the producer might still get its share from each movie ticket that is sold in the theater, in case negotiations at t_{-2} have succeeded.[15]
- After these contractual terms over r_e, d, and r_d are set, at time t_0, the exhibitor and the distributor independently set the retail prices p_H and p_L, respectively. As is usually assumed in the literature on durable goods (Purohit 1997), the relationship between the producer and the channels involve long-term commitments that are difficult to change.
- The exhibitor then releases H at time t_0, and the distributor offers L at t_1. As in §2, we denote by d the compound discount factor for t_1, so that choosing d implicitly also determines t_1. We assume that d is common to all firms and consumers (i.e., $x=1$). Hence, according to the results of the previous section, in this setting we would never find sequencing under full integration of the producer and the channels.

Two remarks must be made about this timing. First, the producer bargains sequentially, first with the exhibitor and then with the distributor. We consider this timing because in our model the theatrical version is assumed to be the high-quality version, and according to the "second-best alternative," the producer should offer the movie first to the channel that generates the highest revenue in the least amount of time. Moreover, theater contracts are more rigid than retailing contracts. In the e-companion (at http://dx.doi.org/10.1287/mksc.1120.0716), we also solve the game where the two bargains are simultaneous, confirming the robustness of our results. Second, the fact that both prices are set simultaneously at time t_0 mirrors the same timing as in §2, and it makes the results directly comparable. In §4, we further discuss the role of this assumption.

We solve the game backwards; therefore we look for the subgame-perfect Nash equilibrium of the game. Proposition 3 presents the equilibrium strategies for firms when the degree of substitution between versions is sufficiently high ($s \geq 1/2$).

Proposition 3. *Imagine that the producer negotiates independently with the exhibitor and the distributor. The equilibrium segmentation strategy is the following.*

(1) *When $s > 7/11$, the producer offers H, and $d=0$ (no versioning). The negotiated wholesale contract specifies $r_e^0 = \alpha + (1-\alpha)\beta/k$.*

(2) *When $1/2 \leq s \leq 7/11$, the segmentation is*

- *if $\alpha \leq \alpha^0$, the producer offers H, and $d=0$ (no versioning). The negotiated wholesale contract specifies $r_e = r_e^0$;*
- *if $\max \alpha^0 < \alpha < \min[\alpha^1, 1]$, consumers are segmented as $L/H/B$, and $0<d<1$ (versioning and sequencing). The negotiated wholesale contracts are $r_e^* > \alpha$ and $r_d^* > \beta$; and*
- *if $\alpha \geq \min[\alpha^1, 1]$, consumers are segmented as $L/H/B$, and $d=1$ (versioning and immediate release). The negotiated wholesale contracts are $r_e^1 > \alpha$ and $r_d^1 > \beta$.*

Proof. See the appendix.[16]

Proposition 3 shows that when there is separation between the producer and the distribution channels, the results regarding versioning and sequencing

[14] In general, α and β will differ as they are affected by different factors such as the presence of other channels in the region, the affiliation to an association of exhibitors, and the impatience of negotiators. For instance, a "tough" negotiator such as Walmart, which is the number one DVD retailer in the United States, would be captured by a high value of $1-\beta$.

[15] For simplicity, we do not consider competition from the movies of other producers. If the exhibitor or the distributor could sell the movies of other producers, their outside options would not be zero.

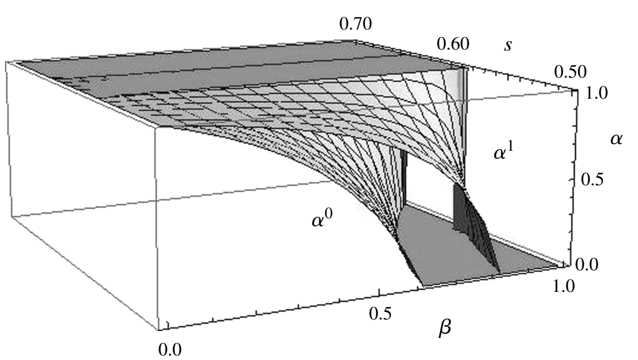
[16] In the proof, we provide the full expressions of the threshold values of α, as well as the revenue shares r and video window d. This remark also applies to Proposition 4.

change radically. Recall from Corollary 1 that when an integrated monopolist controls the commercialization of the two versions, it releases both versions immediately for $s \leq 2/3$ (for pure information goods), or it offers only H for higher values of s. This strategy is no longer sustainable with channel separation. The producer now supplies the two versions for $s \leq 7/11$ and if α is not too low. It may also provide the versions sequentially. Figure 4 presents the range of validity of the segmentation strategies, and Figure 5 plots the equilibrium value of the video window.

Two intuitions are useful before going into the details of our results. First, with vertical separation there is a coordination problem, as the prices of the two versions are set by competing channels instead of a single firm that internalizes cannibalization. As a consequence, equilibrium retail prices are typically set "too" low by the channels, compared to the integrated monopolist. The producer and the exhibitor can moderate this problem by delaying the introduction of L (sequencing). However, they have divergent interests over the length of the video window. The exhibitor always obtains more profits by delaying the introduction of L, as this reduces the cannibalization over H, but the producer maximizes its joint profits from the two channels, and its preferred video window depends on its bargaining power.

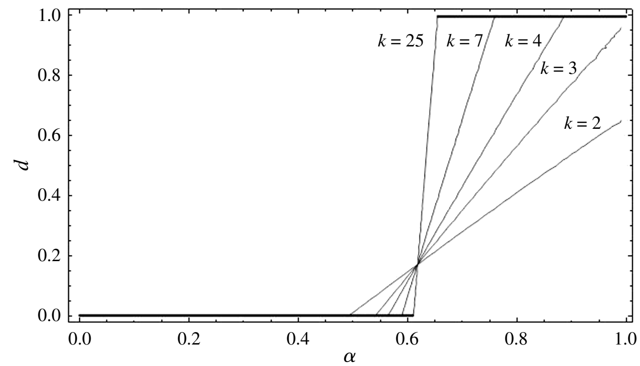
The second intuition is that the producer has a *strategic* reason for versioning. The producer's outside option from L becomes a valuable threat when negotiating with the exhibitor, allowing it to secure a better revenue share from H. Conversely, if the producer obtains a higher revenue share from H, this increases its outside option when negotiating with the distributor. In other words, there is a complementarity between the outside options. This strategic effect

Figure 4 Versioning and Sequencing



Notes. The figure plots the three regions characterized by Proposition 3 as a function of the producer's bargaining power. When $\alpha < \alpha^0$ falls in the lowest region, only H is released. When $\alpha^0 < \alpha < \alpha^1$ falls in the intermediate region, there is both versioning and sequencing. When $\alpha > \alpha^1$ falls in the highest region, there is versioning and simultaneous release of both versions (parameters values $u_L = 1$ and $k = 4$).

Figure 5 Negotiated Video Window



Notes. The figure plots the optimal length of the video window d negotiated between a producer and an exhibitor. As characterized by Proposition 3, the window can fall into three areas according to the value taken by k: $d = 1$ corresponds to the simultaneous release of both versions (day-and-date), $0 < d < 1$ is the sequential release of both versions (video window), and $d = 0$ implies the release of the theatrical version only (parameter values $u_L = 1$, $\beta = 0.5$, and $s = 0.55$).

explains why the revenue shares obtained by the producer are strictly higher than its bargaining ability, $r_e > \alpha$ and $r_d > \beta$.

The identification of these two forces allows us to interpret Proposition 3. If s is sufficiently high, the cannibalization problem is so strong that both the producer and the exhibitor agree to delay L indefinitely. This occurs even in some instances where an integrated producer would have instead opted for immediate versioning ($7/11 < s < 2/3$), because now the distributor would set prices too low. When the degree of substitution is not so strong ($s < 7/11$), the segmentation that emerges is determined by the parties' bargaining power as well as by the quality ratio $k = u_H/u_L$. First, when the producer's bargaining power is much smaller than the exhibitor's ($\alpha \leq \alpha^0$), only H is released (no versioning). This corresponds to the case where the exhibitor is a tough negotiator and avoids the introduction of L. Second, if the producer's bargaining power vis-à-vis the exhibitor ($\alpha^0 < \alpha < \alpha^1$) and/or vis-à-vis the distributor (β) increases, the two versions are released sequentially (video window). Third, when the producer's bargaining power is large ($\alpha > \alpha^1$), it can impose, in complete contrast with the exhibitor, the immediate release of the two versions (day-and-date strategy).

Note that the size of these three regions is affected by the quality ratio k. Since $\partial\alpha^0/\partial k > 0$, it is more likely that only H is introduced for high values of k (as long as $\alpha < \alpha^0$). Also, as $\partial\alpha^1/\partial k < 0$, the region with sequencing ($\alpha^0 < \alpha < \alpha^1$) becomes smaller the higher k is. Finally, it is more likely that both versions are introduced simultaneously for high values of k (as long as $\alpha > \alpha^1$). Taking this into account, for very high values of k, "bang-bang" solutions can arise almost abruptly: a small (but discrete) increase in α can bring the solution from $d = 0$ to $d = 1$ (consider the case where $k = 25$ in Figure 5).

For expositional reasons we have presented the full results for $s \geq 1/2$, as in this range equilibria always exist in pure strategies. For lower values of s, there would be no pure-strategy price equilibria because of the channel coordination problem. However, there would be equilibria in mixed strategies, which consist, for one channel, in playing a pure strategy and, for the other channel, in playing a low price with some probability and a high price with the remaining probability. Without entering into too much detail, the closer s is to zero, the higher the probability assigned to the high-price option, and the closer this option to the pure monopoly price.[17] Hence, the sequence of mixed-strategy equilibria converges to the monopoly prices for both versions as $s \to 0$ (independent versions). In this limiting case, both versions are released immediately, and the revenue-sharing contracts reflect the firm's bargaining power; i.e., $r_e = \alpha$ and $r_d = \beta$.

3.2. Discussion and Robustness of the Results

We have shown how the vertical separation between the producer and the channels affects the versioning and sequencing strategies. Next we argue that our results persist and are even reinforced under alternative market specifications. In particular, we investigate the role played by channel control, as well as the relevance of the contract design.

3.2.1. The Role of Channel Control: Integration Between the Producer and the Distributor. With the complete channel separation that we characterized in §3.1, even if β is very high and the producer appropriates most of the revenues generated by L, the independent distributor still sets p_L "too aggressively" because it does not internalize its impact on the sales of H. This raises the question of whether there would be more or less versioning if the producer were able to sell L *directly*, hence achieving a better channel control. In the movie industry, whereas the vertical separation between the studios and theater chains was dictated by an antitrust provision, producers have been allowed to sell DVDs directly via their websites or specialized stores. Recently, studios have suffered declining DVD and Blu-ray sales, as some services such as Netflix that allow viewers to watch movies streaming over the Internet have become very popular. This has incentivized studios to distribute movies digitally. For example, a multistudio venture called UltraViolet allows access to movies through connected TVs, PCs, and mobile devices.

Proposition 4 presents the case where the producer and the DVD distributor are integrated, but the producer still needs to strike an agreement with the exhibitor to show the movie in a theater. We consider that at t_{-1} there is one negotiation between the exhibitor and the producer, while at time t_0 the exhibitor and the producer set p_H and p_L, respectively.

Proposition 4. *Imagine that the producer sells L directly and negotiates with the exhibitor the commercialization of H. Versioning and sequencing can arise for a wider range of s than with full channel separation; that is, they arise for $s \leq \hat{s}$ with $\hat{s} > 7/11$. Similarly, the region where only H is sold (and $d = 0$) becomes smaller as it arises only when $\alpha < \hat{\alpha}^0 < \alpha^0$.*

Proof. See the appendix.

What is remarkable about this case is that the producer will be *more* likely (i.e., for a wider range of values of s) to introduce the DVD compared to the *full channel separation* of Proposition 3, and it will reduce the length of the video window.[18] The main reason is that with this partial integration, the producer has control over the price of L and can better internalize the cannibalization effect from DVD sales on the theater side. As a consequence, the range of s where only H is released is smaller. However, with *partial channel integration*, the producer is unable to appropriate all the profits generated by the movie. For this reason, it introduces the DVD for a smaller range of values of s than with *full channel integration*.

3.2.2. The Role of Contracts. Our results in this section have been derived by assuming that the firms negotiate revenue-sharing wholesale contracts. This type of arrangement is interesting for two reasons. First, from a theoretical point of view, they do not cause any particular inefficiency due to double markups as linear rental prices would entail. Second, revenue-sharing contracts are pervasive in the movie industry. Yet one might wonder if different contractual solutions would change our results. To answer this question, we have examined other possibilities, such as linear contracts, and a mix of linear and revenue sharing contracts, confirming the flavor of our main results. Both the price miscoordination problem when setting the price of the two versions independently as well as the misalignment of preferences over the duration of the video window survive. This causes sequencing to arise in contexts where an integrated monopolist would not introduce it.

Of course, the precise versioning and sequencing strategies do depend on the precise form of wholesale

[17] Because the revenue functions of both channels are continuous, mixed-strategy price equilibria must exist. As these functions are piecewise concave, mixed strategies must have only a finite support in prices.

[18] The prediction of the model matches the comments of Time Warner's CEO, Jeff Bewkes, who said in 2012 that an early UltraViolet release window could fit within the theatrical window to satiate consumer demand (see Gruenwedel 2012).

contracts. For instance, with linear contracts, double markup problems would be added on top of the forces that we have previously identified. As a consequence, versioning, and sequencing can arise even when $s = 1$.[19] This example illustrates that when firms use contracts that are more inefficient than revenue-sharing agreements, the producer has additional reasons to introduce versioning and sequencing.

4. Conclusions

We have presented a model of movie distribution and consumption across two channels that provides insights on how studios should time the window between theatrical and video releases. We focused the analysis on the movie industry, but our results can also inform all those products, such as books, computers, and video games, which are introduced sequentially through different channels.

Our main result is to characterize when a vertically integrated producer should release several versions of a movie in different channels, given that some consumers are willing to buy both versions of the product. When planning their distribution strategy, studio managers should take into account the degree of substitutability between theatrical and nontheatrical versions. If DVDs deter people from going to the theater, then versioning should be less likely. However, if consumers enjoy consuming movies through different channels, then day-and-date release should occur more often than previously thought by the literature. The studio managers should reduce the prices of the two versions, compared to a scenario where consumers would stick to a single-unit purchase, as this improves their ability to price discriminate.

The vertical structure of the supply chain is at the core of the second contribution of our paper. When studios are vertically separated from theater exhibitors and distributors, they should agree to supply the two versions sequentially if their bargaining power is sufficiently strong and if the versions are not too substitute for one another. In this case, sequencing moderates the pricing miscoordination problem that emerges when there are several competing distribution channels.

The possibility of delaying the introduction of a version still raises many questions. Waterman et al. (2010) found that the video window was quite stable (established at around six months) between 1988 and 1997; however, it has since fallen steadily to a current window of about four months. Our model predicts this trend as studios acquire stronger bargaining power than that of the exhibitors when deciding on the length of the video window, especially when they can control distributors. Luan and Sudhir (2007) calculate that the optimal window should be even shorter, at around 2.5 months. Their approach, though, considers the profits of an integrated producer/exhibitor, whereas we suggest that vertical separation might be one reason for longer video windows.

We hope that our work helps in better understanding why video windows have become shorter, as well as how short they might become with the digital distribution of movies. Features of the vertical chain should be accounted for in future empirical studies or in numerical computations aimed at calibrating the optimal time release of different versions.

Despite our contributions to the literature, several relevant aspects for the future movie industry have not been considered in full by this paper but could warrant interesting extensions. Most important, we have studied the optimal versioning strategy of a movie when the producer can form correct expectations about its future success. One fruitful extension would be to examine how demand uncertainty can affect the movie development costs. Also, we assumed that the producer announces the video release time early on and is able to commit to the announced date due to the costs associated to marketing campaigns. This commitment is further justified given the repeated nature of the interactions with the exhibitor and the reputational gains from sticking to the announced terms. However, with endogenous development costs and uncertain success, the model could be extended to permit delayed announcements after the movie is first shown in the theater.

In this paper, we also focused our attention on the optimal release for a single movie title supplied by a single studio, although versions may be distributed by different firms. This framework is useful to clarify some fundamental trade-offs that should also arise in the presence of competing titles. Despite this, we foresee the introduction of competition for scarce resources among content producers (in terms of both the talent that should determine the quality of a movie and the shelf space of retailers that distribute competing titles) as an essential issue for our future research.

Electronic Companion

An electronic companion to this paper is available as part of the online version at http://dx.doi.org/10.1287/mksc.1120.0716.

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[19] With a double markup, the full monopoly profits obtained from selling only H cannot be reaped. For more analytical details, see Calzada and Valletti (2011).

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Appendix

Proof of Proposition 1. When $s = 1$ and the firm only offers H, it sets the price p_H that maximizes $\pi_H = (p_H - c_H)(1 - \theta_{H0})$. The resulting price is $p_H = (u_H + c_H)/2$, and the indifferent type is $\theta_{H0} = 1/2 + c_H/u_H$. As a result, the firm obtains $\pi_H = (u_H - c_H)^2/4u_H$. Alternatively, the firm could offer L to the low segment of consumers and H to the high segment. In this case it sets p_H and p_L to maximize

$$\pi_{LH} = (p_H - c_H)(1 - \theta_{LH}) + (p_L - c_L)(\theta_{LH} - \theta_{L0}). \tag{3}$$

Solving this problem yields the following prices and profits, where we use $u_H = ku_L$:

$$p_L = \frac{c_L + u_L}{2}, \quad p_H = \frac{c_H + ku_L}{2};$$
$$\pi_{LH} = \frac{1}{4}\left[ku_L - 2c_H - \frac{c_H^2 - 2c_Hc_L + c_L^2k}{(1-k)u_L}\right]. \tag{4}$$

It is simple to verify that $\pi_H < \pi_{LH}$ for $c_H > c_Lk$. In this range, it is also straightforward to confirm that at the equilibrium prices, $\theta_{LH} = 1/2 + (c_H - c_L)/(2(k-1)u_L) > \theta_{L0} = 1/2 + c_L/(2u_L)$.[20] For $c_H < c_Lk$, by contrast, it is $\theta_{LH} < \theta_{L0}$, and the firm only releases H.

Next we analyze the firm's releasing strategy for general values of $s < 1$. Consider first the case $c_H < c_Lk$. If the firm offers L to the low segment of consumers, H to the intermediate segment, and B to the high segment, it sets p_H and p_L to maximize

$$\pi_{LHB} = (p_H + p_L - c_H - c_L)(1 - \theta_{HB}) + (p_H - c_H)(\theta_{HB} - \theta_{LH}) + (p_L - c_L)(\theta_{LH} - \theta_{L0}). \tag{5}$$

The prices that solve this problem and the corresponding profits are

$$p_H = \frac{c_H}{2} + \frac{[k(2-s) - s]u_L}{2(2-s)}, \quad p_L = \frac{c_L}{2} + \frac{(1-s)u_L}{2-s}; \tag{6}$$

$$\pi_{LHB} = \frac{1}{4}\left[ku_L - 2(c_H + c_L) + \frac{(c_H - c_L)^2}{(k-1)u_L} + \frac{c_L^2(2-s)}{(1-s)u_L} + \frac{(2-3s)u_L}{2-s}\right].$$

The value of s that equals π_H and π_{LHB} can be computed as

$$s^1 = \big[(3c_H(c_H - 2c_Lk) - c_L^2k(1-4k) + (6c_L - 5u_L)(1-k)ku_L + [(c_H^2 - 2c_Hc_Lk + k(c_L^2 + (u_L + 6c_L)(1-k)u_L)) \cdot (c_H^2 - 2c_Hc_Lk + k(c_L^2 + (u_L - 2c_L)(1-k)u_L))]^{1/2})\big] \cdot (2(c_H - c_Lk)^2 + (4c_L - 6u_L)(1-k)ku_L)^{-1}. \tag{7}$$

Thus, for $s^1 < s \le 1$, there is a region A where only H is provided (see panel 1 in Figure 1). For $s < s^1$, the prices in (6) are a candidate solution, as long as

$$1 \ge \theta_{HB} = \frac{1}{2-s} + \frac{c_L}{2u_L(1-s)} > \theta_{LH} = \frac{1}{2} - \frac{c_H - c_L}{2(1-k)u_L} > \theta_{L0} = \frac{1-s}{2-s} + \frac{c_L}{2u_L} \ge 0.$$

Notice that $\theta_{LH} = \theta_{L0}$ when $s = s^2 = 2(c_H - c_Lk)/(c_H - c_Lk + u_L(1-k))$. Hence, for $s^2 < s \le s^1$ there is a region B where the segmentation $L/H/B$ is offered.

For $s < s^2$, at the prices given by (6) the first marginal buyers will choose H instead of L, and no one buys L alone. The firm will then choose p_H and p_L to maximize

$$\pi_{HB} = (p_H + p_L - c_H - c_L)(1 - \theta_{HB}) + (p_H - c_H)(\theta_{HB} - \theta_{H0}). \tag{8}$$

The prices and the associated profit are

$$p_H = \frac{c_H + ku_L}{2}, \quad p_L = \frac{c_L + u_L(1-s)}{2};$$
$$\pi_{HB} = \frac{1}{4}\left[ku_L - 2(c_H + c_L) + \frac{c_H^2(1-s) + k(c_L^2 + (s-1)^2u_L^2)}{4(1-s)ku_L}\right]. \tag{9}$$

In this region, which we call C, the firm segments consumers as H/B, and the indifferent types are $\theta_{HB} = 1/2 + c_L/(2u_L(1-s))$ and $\theta_{H0} = 1/2 + c_H/(2ku_L)$. This solution holds as long as $\theta_{HB} \ge \theta_{H0}$, which results in $s \ge s^3 = (c_H - c_Lk)/c_H < 0$. Notice, however, that we also have to check that consumers may not want to buy L: $\theta_{H0}u_L - p_L \le 0$ at the prices given by (9). This is satisfied when $s \le s^* = (c_Lk - c_H)/(ku_L) < s^2$: the prices in (9) are the solutions for $s^3 \le s \le s^*$.

When $s^* \le s \le s^2$, we are still in region C as the segmentation is H/B, but the prices take a different expression. In particular, the firm sets $p_L = p_H(u_L/u_H)$ to make sure that $\theta_{L0} = \theta_{LH}$. The prices that satisfy this condition and maximize the profits are

$$p_H = \frac{k[c_L + c_H(1-s) + (1+k)(1-s)u_L]}{2[1 + k(1-s)]}, \quad p_L = p_H/k.$$

In this case the indifferent types are

$$\theta_{HB} = \frac{c_L + c_H(1-s) + (1+k)(1-s)u_L}{2[1 + k(1-s)]u_L(1-s)}$$
$$\text{and} \quad \theta_{H0} = \frac{c_L + c_H(1-s) + (1+k)(1-s)u_L}{2[1 + k(1-s)]u_L}.$$

Finally, when s is very negative (strong complementarity), all consumers who buy prefer to buy B. In particular, for $s < s^3$ the firm maximizes $\pi_B = (p_H + p_L - c_H - c_L)(1 - \theta_{B0})$. The optimal price is $p_H + p_L = (c_H + c_L + u_B)/2$, and the firm's profit is $\pi_B = (u_B - c_H - c_L)^2/4u_B$. The indifferent type is $\theta_{B0} = 1/2 + (c_H + c_L)/(2u_B)$. This situation corresponds to region D.

The various regions A, B, C, and D are plotted in panel 1 of Figure 1.[21] Their existence and size depend on the value of c_H and c_L and on the relative utility of the versions, k.

[20] The firm will segment consumers as L/H as long as H is not too costly, in which case no one will be interested in buying this version. To avoid this, we impose the natural restriction $c_H - c_L \le u_H - u_L$.

[21] The numerical set of parameters chosen for all figures in this paper is just for illustration.

We have already shown that $s^2 > s^* > s^3$. Therefore, we only need to discuss when $1 > s^1 > s^2$. The expression for s^1 given by (7) is cumbersome, but it can be shown that it decreases in c_H and that it takes a minimum when $c_H = c_L k$. Second, still at $c_H = c_L k$, s^2 simplifies to $s^2 = 0 < s^1$. By continuity, the four regions always exist for sufficiently high levels of c_H. Panel 2 of Figure 1 shows that when $c_H = c_L k$, $s^2 = s^* = s^3 = 0$. As a result, region C disappears, and the optimal segmentation is $L/H/B$ for all values $0 \le s < s^1$. Notice, moreover, that when $c_H = c_L = 0$ (information goods), the threshold further simplifies to $s^1 = 2/3$.

We analyze next the case where $c_H > c_L k$. From the profit functions in (4) and (6), it can be verified that the value of s that equals π_{LH} and π_{LHB} is

$$\hat{s}^1 = \frac{(u_L - c_L)[5u_L - c_L - (c_L^2 + 6c_L u_L + u_L^2)^{1/2}]}{2(3u_L - 2c_L)u_L}.$$

Taking this into account, for $\hat{s}^1 < s \le 1$, there is a region A where the firm segments consumers as L/H (see panel 3 in Figure 1). For $s < \hat{s}^1$, the prices in (6) are a candidate solution as long as $\theta_{HB} > \theta_{LH} > \theta_{L0}$. At these prices, it is found that $\theta_{HB} = \theta_{LH}$ for

$$\begin{aligned}\hat{s}^2 = \big[&3c_H - c_L(2+k) - u_L(1-k) - [(c_H - c_L(2-k))^2 \\ &+ 2u_L(1-k)(c_H + c_L(2-3k)) + (k-1)^2 u_L^2]^{1/2}\big] \\ &\cdot (2[c_L - c_H - (1-k)u_L])^{-1}.\end{aligned}$$

Therefore, for $\hat{s}^2 < s \le \hat{s}^1$ (region B) the segmentation is $L/H/B$. Notice that this region always exists for values of c_H sufficiently small and that $\hat{s}^2 = 0$ for $c_H = c_L k$. For $s < \hat{s}^2$, the second marginal buyers will consume both L and H, since now no one buys H alone. When this occurs, the firm will choose p_H and p_L to maximize

$$\pi_{LB} = (p_H + p_L - c_H - c_L)(1 - \theta_{LB}) + (p_L - c_L)(\theta_{LB} - \theta_{L0}). \quad (10)$$

The prices and the associated profit would be

$$\begin{aligned} p_H &= \frac{c_H + (k-s)u_L}{2}, \quad p_L = \frac{c_L + u_L}{2}; \\ \pi_{LB} &= \frac{1}{4}\left[(1+k-s)u_L + \frac{c_H^2 + c_L^2(k-s)}{(k-s)u_L} - 2(c_H + c_L)\right]. \end{aligned} \quad (11)$$

Moreover, the indifferent types are

$$\theta_{LB} = \frac{1}{2} + \frac{c_H}{2u_L(k-s)} \quad \text{and} \quad \theta_{L0} = \frac{1}{2} + \frac{c_L}{2u_L}.$$

The prices (11) are a candidate solution as long as $\theta_{LB} \ge \theta_{L0}$, which results in $s \ge \hat{s}^3 = k - c_H/c_L < 0$. This implies that for $\hat{s}^3 \le s \le \hat{s}^2$ (region C), consumers are segmented as L/B. We still need to verify that some consumers will not buy only version H: $\theta_{H0}u_H - p_H < 0$. This is satisfied for

$$s < \hat{s}^* = \frac{c_H - c_L + ku_L - [4(c_H - c_L k)u_L - (c_H - c_L + ku_L)^2]^{1/2}}{2u_L}.$$

Hence, the prices in (11) are valid in the range $\hat{s}^3 \le s \le \hat{s}^*$. For $\hat{s}^* \le s \le \hat{s}^2$, the segmentation is still L/B, but the firm sets $p_L = p_H(1-s)u_L/(u_H - su_L)$ to make sure that $\theta_{LH} = \theta_{LB}$. The price that satisfies this and maximizes the profit in (10) is

$$p_H = \frac{(k-s)[c_H + c_L(1-s) + (1+k-2s)u_L]}{2[1+k-(3-s)s]}.$$

The indifferent types are now

$$\theta_{LB} = \frac{(k-s)[(1+k-2s)u_L - c_L(1-s) - c_H]}{2(s-3)(1+k-(3-s)s)u_L}$$

$$\text{and} \quad \theta_{H0} = \frac{(k-s)(1-s)[(1+k-2s)u_L - c_L(1-s) - c_H]}{2(s-3)(1+k-(3-s)s)u_L}.$$

Finally, when $s < \hat{s}^3$, all consumers who buy prefer to buy B, and we get region D (not shown in the range plotted in panel 3 in Figure 1). Q.E.D.

Proof of Corollary 1. Imagine $x = 1$, which implies that $d_c = d$. Also imagine that the firm releases the two versions sequentially: it offers H at t_0 to the intermediate and high segments of consumers and L at t_1 to the low and high segments of consumers. When $c_H \ge c_L \ge 0$, the firm maximizes the following profit:

$$\begin{aligned}\pi_{LHB}^{\text{SEQ}} = {} & [p_H - c_H + d(p_L - c_L)](1 - \theta_{HB}) + (p_H - c_H) \\ & \cdot (\theta_{HB} - \theta_{LH}) + d(p_L - c_L)(\theta_{LH} - \theta_{L0}). \end{aligned} \quad (12)$$

The prices that solve this problem and the corresponding profits are

$$p_H = \frac{c_H}{2} + \frac{[k(2-s) - sd]u_L}{2(2-s)}, \quad p_L = \frac{c_L}{2} + \frac{(1-s)u_L}{2-s};$$

$$\begin{aligned}\pi_{LHB}^{\text{SEQ}} = \frac{1}{4}\bigg[& ku_L - 2(c_H + c_L) + \frac{d(2-3s)u_L}{(2-s)} \\ & - \frac{c_H^2(1-s) - 2c_H c_L d(1-s) - c_L^2 d(d - k(2-s))}{(1-s)(d-k)u_L}\bigg]. \end{aligned} \quad (13)$$

Next, to determine the firm's sequencing policy, we evaluate the first-order condition (FOC) with respect to d, and we obtain

$$\frac{\partial \pi_{LHB}^{\text{SEQ}}}{\partial d} = \frac{1}{4}\left[\frac{(2-3s)u_L}{(2-s)} + c_L\left(\frac{c_L}{(1-s)u_L} - 2\right) + \frac{(c_H - c_L k)^2}{(k-d)^2 u_L}\right].$$

It is also $\partial^2 \pi_{LHB}^{\text{SEQ}}/\partial d^2 = (c_H - c_L k)^2/(2(k-d)^3 u_L) > 0$. Hence π_{LHB}^{SEQ} is convex in d and takes its maximum either at $d = 0$ or at $d = 1$. In particular, the sign of $\partial \pi_{LHB}^{\text{SEQ}}/\partial d$ is positive for $s < s^1$, where s^1 is defined as in Equation (7) in the proof of Proposition 1 (recall that $s^1 = 2/3$ for information goods; $c_H = c_L = 0$). Thus for $s < s^1$, the firm optimally sets $d = 1$, and there is simultaneous release. For $s > s^1$, the firm sets $d = 0$ and offers H alone at t_0. Therefore, we obtain the same result as in Proposition 1. Q.E.D.

Proof of Proposition 2. When the firm releases the products sequentially, it maximizes the profit as in (12), where now $x < 1$ and $d_c = x^t d$. For simplicity, we study the case of information goods ($c_H = c_L = 0$). Taking this into account, profit maximization yields

$$\begin{aligned} p_L &= \frac{(1-s)(3+x^t)(k - dx^t)u_L}{4k(2-s) - d[1 + 6x^t + x^{2t} - s(1+x^t)^2]}, \\ p_H &= \frac{(k - dx^t)[2k(2-s) + d(1 - s - x^t(1+s))]u_L}{4k(2-s) - d[1 + 6x^t + x^{2t} - s(1+x^t)^2]}, \\ \pi_{LHB}^{\text{SEQ}} &= \frac{(k - dx^t)[k(2-s) + d(2 - s(2+x^t))]u_L}{4k(2-s) - d[1 + 6x^t + x^{2t} - s(1+x^t)^2]}. \end{aligned} \quad (14)$$

Next we analyze how the firm sets the video window. We first establish limiting cases. By evaluating the FOC with respect to d at $d=1$, we obtain

$$\left.\frac{\partial \pi_{LHB}^{\mathrm{SEQ}}}{\partial d}\right|_{d=1} = \frac{[(3s-2)\log(\delta_p)+s\log(x)]u_L}{4(s-2)\log(\delta_p)}.$$

The sign of this expression is positive for $s<\hat{s}_1 = 2\log(\delta_p)/(3\log(\delta_p)+\log(x))<1$; hence the corner solution $d=1$ exists in the range $0\le s\le \hat{s}_1$. For $\hat{s}_1<s<1$, there will be an interior solution with $0<d<1$ (sequential release). An example is plotted in panel 2 in Figure 2. Also note that the FOC calculated for $d=0$ is

$$\left.\frac{\partial \pi_{LHB}^{\mathrm{SEQ}}}{\partial d}\right|_{d=0} = \frac{9(1-s)u_L}{16(2-s)},$$

which is never negative. Thus, the firm sets $d=0$ and offers H alone *only* when $s=1$, while it always sets $d>0$ for $s<1$. Finally, it can be seen immediately that $\hat{s}_1$ gets smaller as x is low (customers are relatively more impatient) or δ_p is high (the firm does not discount the future much).

Therefore, for $\hat{s}_1<s<1$, the prices in (14) are a candidate solution, as long as the ranking of indifferent types follows $\theta_{HB}>\theta_{LH}>\theta_{L0}$. In this region the indifferent types are

$$\theta_{HB}=\frac{(3+x^t)(dx^t-k)}{4k(s-2)+d[1+6x^t+x^{2t}-s(1+x^t)^2]},$$
$$\theta_{LH}=\frac{2k(s-2)+d[(s-1)-2(s-2)x^t-(s-1)x^{2t}]}{4k(s-2)+d[1+6x^t+x^{2t}-s(1+x^t)^2]},$$
$$\theta_{L0}=\frac{(1-s)(3+x^t)(dx^t-k)}{4k(s-2)+d[1+6x^t+x^{2t}-s(1+x^t)^2]}.$$

It is always $\theta_{LH}>\theta_{L0}$. Also, if $d=1$, it is $\theta_{HB}>\theta_{LH}$. In the $\{s,d\}$ space, the condition $\theta_{HB}>\theta_{LH}$ is always satisfied by the candidate solution obtained from $\partial\pi_{LHB}^{\mathrm{SEQ}}/\partial d=0$ as long as x is higher than a limiting value, denoted as x^*. For lower values of x, however, the ranking is not preserved. Consider therefore the condition such that $\theta_{HB}=\theta_{LH}$, which gives

$$d[1-x^t+s(2x^t+(x^t)^2-1)]=k(x^t+2s-1). \qquad (15)$$

The limiting value x^* is obtained by looking at the first point of tangency between the curve that describes $\partial\pi_{LHB}^{\mathrm{SEQ}}/\partial d=0$ and the curve that describes (15). After computations, it is the highest value of x such that the equation

$$[k(3+x^{2t})+d(1-4x^t-x^{2t})]\log(\delta_p)-2(d+k)x^t(1-x^t)=0$$

has a root in the plausible domain $0<d<1$, $0<x<1$.

When $x<x^*$, it can be shown that the curve that describes $\partial\pi_{LHB}^{\mathrm{SEQ}}/\partial d=0$ intersects the curve (15) twice in the $\{s,d\}$ space: call these roots (s_1,d_1) and (s_2,d_2), where $s_1>s_2$. In particular, when $s<s_2$ or $s>s_1$, the ranking $\theta_{HB}>\theta_{LH}>\theta_{L0}$ is preserved, and therefore the firm will segment the market as $L/H/B$ with the video window previously described. For $s_2\le s\le s_1$, the second marginal buyer will buy B instead of H, and no consumer buys H alone. Thus, the firm segments the market as L/B and sets $p_L=(1-s)p_H/(k-dsx^t)$ to make sure that $\theta_{HB}=\theta_{LH}$. In particular, the firm now maximizes

$$\pi_{LB}^{\mathrm{SEQ}}=(p_H+dp_L)(1-\theta_{LB})+dp_L(\theta_{LB}-\theta_{L0}).$$

The prices and the associated profits are

$$p_L=\frac{(1-s)[k+d(1-s(1+x^t))]u_L}{2[k+d(1+s^2-s(2+x^t))]},$$
$$p_H=\frac{(k-dsx^t)[k+d(1-s(1+x^t))]u_L}{2[k+d(1+s^2-s(2+x^t))]}, \qquad (16)$$
$$\pi_{LB}^{\mathrm{SEQ}}=\frac{[k+d(1-s(1+x^t))]^2u_L}{4[k+d(1+s^2-s(2+x^t))]}.$$

With these prices, the indifferent types are

$$\theta_{LB}=\frac{k+d[1-s(1+x^t)]}{2[k+d(1+s^2-s(2+x^t))]}$$
$$\text{and}\quad \theta_{L0}=\frac{(1-s)[k+d(1-s(1+x^t))]}{2[k+d(1+s^2-s(2+x^t))]}.$$

Finally, in this intermediate range of s the video window chosen by the firm at t_0 is obtained from $\partial\pi_{LB}^{\mathrm{SEQ}}/\partial d=0$. Examples of strictly interior solutions with $0<d<1$ and the segmentation L/B are plotted in panels 3 and 4 in Figure 2. Q.E.D.

Proof of Proposition 3. Imagine that both versions are released sequentially, with consumers segmented as $L/H/B$. This approach covers as limit cases both the simultaneous release of the two versions ($d=1$) and the single release of H ($d=0$). We now study how the producer reaches wholesale agreements with the channels. Working backwards, at t_0 the exhibitor's retail problem is to set p_H to maximize $\pi_{LHB}^e=(1-r_e)p_H(1-\theta_{LH})$, and the distributor's is to set p_L to maximize $\pi_{LHB}^d=(1-r_d)dp_L(1-\theta_{HB}+\theta_{LH}-\theta_{L0})$, where the indifferent types are $\theta_{HB}=p_L/u_L(1-s)$, $\theta_{LH}=(p_H-dp_L)/(u_H-du_L)$ and $\theta_{L0}=p_L/u_L$. Computing the equilibrium in prices and substituting them in the profits yields

$$\pi_{LHB}^d=(1-r_d)(k-d)u_L\frac{9d(1-s)[k(2-s)-d]}{[4k(2-s)-d(5-s)]^2},$$
$$\pi_{LHB}^e=(1-r_e)(k-d)u_L\frac{[2k(2-s)-d(1+s)]^2}{[4k(2-s)-d(5-s)]^2}.$$

As a consequence, the producer obtains the following profit:

$$\begin{aligned}\pi_{LHB}^p&=r_ep_H(1-\theta_{LH})+dr_dp_L(1-\theta_{HB}+\theta_{LH}-\theta_{L0})\\&=r_d(k-d)u_L\frac{9d(1-s)[k(2-s)-d]}{[4k(2-s)-d(5-s)]^2}\\&\quad+r_e(k-d)u_L\frac{[2k(2-s)-d(1+s)]^2}{[4k(2-s)-d(5-s)]^2}.\end{aligned} \qquad (17)$$

At the equilibrium prices, it can be verified that $\theta_{HB}-\theta_{LH}=(k-d)(2s-1)/(4k(2-s)-d(5-s))$. Hence, this solution is defined only for $s\ge 1/2$.[22]

At time t_{-1}, the producer negotiates with the distributor over the revenue share r_d in a Nash bargain. Let $0<\beta<1$ be the producer's bargaining power vis-à-vis the distributor. In case the negotiation breaks down, the distributor's outside option is zero, while the producer can still agree with

[22] This is the only boundary of indifferent types that matters, as $\theta_{LH}-\theta_{L0}=(k-d)(1+s)/(4k(2-s)-d(5-s))>0$ is always satisfied. For lower values of $s<1/2$, there are equilibria in mixed strategies; see Gabszewicz and Wauthy (2003).

the exhibitor at t_{-2} to sell H. In this case, H would be sold at $p_H = u_H/2$, and the producer would get $\pi_H^p = r_e(u_H/4)$. Thus the firms solve the following problem:

$$\max_{r_d} \Omega_{LHB}^{p,d} = \left(\pi_{LHB}^p - r_e \frac{u_H}{4}\right)^\beta (\pi_{LHB}^d)^{1-\beta}.$$

This gives the equilibrium revenue share

$$r_d = \beta + \frac{r_e(1-\beta)[4d^2(1+s)^2 + 8k^2(2+s-s^2) + dk(13s^2-11-34s)]}{36(k-d)(1-s)[k(2-s)-d]}. \tag{18}$$

This revenue share is $r_d = 1$ when $\beta = 1$, but in all other cases, it is $r_d > \beta$ as long as $r_e > 0$. Hence the producer gets a revenue share in excess of its bargaining power as long as it obtains some income from theater sales. Its revenue share also increases in d.

Instead, if the negotiations between the producer and the exhibitor at t_{-2} had broken down, the producer's outside option would be zero when negotiating with the distributor. The revenue share, as well as the release date of L, would be determined from selling L alone:

$$\max_{r_d, d} \Omega_L^{p,d} = (\pi_L^p)^\beta (\pi_L^d)^{1-\beta} = d(1-r_d)^\beta r_d^{1-\beta} \frac{u_L}{4}.$$

This gives the result that $r_d = \beta$ and $d = 1$. That is, *if H is not commercialized, L is released immediately, and firms share the monopoly profits according to their bargaining power.* The producer's outside option when it negotiates with the exhibitor is then $\pi_L^p = \beta u_L/4$.

At time t_{-2}, the producer and the exhibitor bargain to determine the length of the video window and the revenue share r_e. In this negotiation, the exhibitor's outside option is zero, and the producer's amounts to π_L^p, as just described. Let $0 < \alpha < 1$ denote the producer's bargaining power vis-à-vis the exhibitor. The firms solve the following Nash bargain:

$$\max_{r_e, d} \Omega_{LHB}^{p,e} = \left(\pi_{LHB}^p - \frac{\beta u_L}{4}\right)^\alpha (\pi_{LHB}^e)^{1-\alpha},$$

where (18) is used in the expression (17) for π_{LHB}^p. The interior solution of this problem ($0 < d < 1$) is characterized by the following FOCs:

$$\frac{\alpha}{1-\alpha} \frac{\pi_{LHB}^e}{\pi_{LHB}^p - (\beta u_L)/4} = -\frac{\partial \pi_{LHB}^e / \partial r_e}{\partial(\pi_{LHB}^p - (\beta u_L)/4)/\partial r_e}, \tag{19}$$

$$\frac{\alpha}{1-\alpha} \frac{\pi_{LHB}^e}{\pi_{LHB}^p - (\beta u_L)/4} = -\frac{\partial \pi_{LHB}^e / \partial d}{\partial(\pi_{LHB}^p - (\beta u_L)/4)/\partial d}. \tag{20}$$

Solving (19) yields

$$\begin{aligned} r_e^* = \alpha + \big[&((1-\alpha)\beta\{16k^2(2-s)^2 + d^2[(5-s)^2 + 36k(3-4s+s^2)] \\ &- 4dk[2(5-s) + 9k(1-s)](2-s) - 36d^3(1-s)\}\big] \\ &\cdot \big[k[4k(2-s) - d(5-s)]^2 - \beta d[4d^2(1+s)^2 \\ &+ 8k^2(2+s-s^2) + dk(13s^2-11-34s)]\big]^{-1}. \end{aligned} \tag{21}$$

Notice that $r_e^* = 1$ only in the case $\alpha = 1$. Otherwise, it is always $r_e^* > \alpha$ to the extent that $\beta > 0$. Substituting r_e^* in (20) results in one last equation in d, which can be solved as a function of the parameters to get d^*. Substituting r_e^* and d^* in (18), we obtain r_d^*. This solves the problem completely. We do not report the explicit value for d^*, as this involves a long expression, but this solution takes values in the appropriate interval [0, 1] when parameters k, α, and β are in a relevant range (Figure 5 in the text plots d^* at equilibrium).

We now discuss what happens when an interior solution does not exist for some parameter configuration. Take first the corner solution $d = 0$ (i.e., version L is not released). In this case, from (19) we obtain $r_e^0 = \alpha + (1-\alpha)\beta/k$.

To define the range of validity of this solution, substitute r_e^0 and $d = 0$ in (20) to get

$$\text{sign}\left[\frac{\partial \Omega_{LH}^{p,e}}{\partial d}\right] < 0 \quad \text{iff} \quad \alpha < \alpha^0 = 1 - \frac{k\beta(7-11s)}{2(1-\beta)(k-\beta)(1+s)}.$$

Hence the corner solution $d = 0$ can occur as long as α^0 takes plausible values (between 0 and 1). First, notice that this condition is always satisfied for $s > 7/11$. Hence, in this range only H is sold. For $s \le 7/11$, α^0 is decreasing in β; hence it takes its maximum value $\alpha^0 = 1$ for $\beta = 0$. Also note that α^0 is increasing in k. Figure 4 reports a three-dimensional plot of α^0.

Consider now the corner solution $d = 1$. The rental share price in (21) simplifies to

$$\begin{aligned} r_e^1 = \alpha + \big[&(1-\alpha)\beta[11 - 26s - s^2 + 4k^2(2 - 11s + 5s^2) \\ &- 4k(7 - 22s + 7s^2)]\big] \\ &\cdot \big[k[4k(2-s) + s - 5]^2 - \beta[4(1+s)^2 \\ &- k(11 + 34s - 13s^2) + 8k^2(2 + s - s^2)]\big]^{-1}. \end{aligned}$$

By substituting this expression in (18) when $d = 1$, we also obtain the solution for r_d^1. To define the range of validity, we substitute r_e^1 in (20) when $d = 1$ to get $\text{sign}[\partial \Omega_{LB}^{p,e}/\partial d] > 0$ iff $\alpha > \alpha^1$, where the full expression is not reported here for the sake of brevity (see Calzada and Valletti 2011).

Hence the solution $d = 1$ can occur as long as α^1 takes admissible values (between 0 and 1). The expression for α^1 is complex, yet it can be shown that it is decreasing in β and reaches its maximum value $\alpha^1 = 1$ for $\beta = 0$. Also notice that α^1 is decreasing in k. In particular, it reaches $\alpha^1 = 1$ for $k = 1$ and the limit value $\lim_{k\to\infty} \alpha^1 = 1 - \beta(7 - 11s)/(2(1-\beta) \cdot (1+s)) = \lim_{k\to\infty} \alpha^0$. Since α^0 is increasing in k, this implies that $\alpha^1 > \alpha^0$ for *all* plausible parameter ranges. Figure 4 reports the three-dimensional plot of α^1. Q.E.D.

Proof of Proposition 4. The proof follows conceptually the same steps as the proof of Proposition 3. At time t_0, the exhibitor sets p_H to maximize $\pi_{LHB}^e = (1-r_e)p_H(1-\theta_{LH})$, and the producer sets p_L to maximize $\pi_{LHB}^p = r_e p_H(1-\theta_{LH}) + dp_L(1 - \theta_{HB} + \theta_{LH} - \theta_{L0})$. Solving these problems yields

$$\begin{aligned} \pi_{LHB}^p = \big[&(k-d)u_L\{r_e[2k(2-s) - d(1+s)^2] + d(3+r_e)(1-s) \\ &\cdot [(3-r_e)k(2-s) - d(3-sr_e)]\}\big] \\ &\cdot ([4k(2-s) - d(5 + r_e(1-s) - s)]^2)^{-1}, \end{aligned}$$

$$\pi_{LHB}^e = \frac{(1-r_e)(k-d)u_L[2k(2-s) - d(1+s)^2]}{[4k(2-s) - d(5 + r_e(1-s) - s)]^2}.$$

Contract terms are established at time t_{-1} by maximizing the following expression:

$$\max_{r_e, d} \Omega^{p,e}_{LHB} = \left(\pi^p_{LHB} - \frac{u_L}{4}\right)^\alpha (\pi^e_{LHB})^{1-\alpha}.$$

We are interested in characterizing the region of parameters when sequencing starts, as opposed to selling only H. By setting the corner solution $d = 0$ from (19), we get $r_e^0 = \alpha + (1-\alpha)/k$.

To define the range of validity of this solution, substitute r_e^0 and $d = 0$ into (20) to get

$$\frac{\partial \Omega^{p,e}_{LHB}}{\partial d} = \frac{(1-\alpha)u_L}{64k^2} \cdot \left\{ \frac{(2-3s)[(5-\alpha)k - 1 + \alpha][k(3+\alpha) + 1 - \alpha]}{2-s} - (1-\alpha)^2(1-k)^2 \right\}.$$

This expression is always negative for $s > 2/3$, in which case $d = 0$, and only H is sold. For values $s \le 2/3$, it is

$$\text{sign}\left[\frac{\partial \Omega^{p,e}_{LHB}}{\partial d}\right] < 0 \quad \text{iff} \quad \alpha < \hat{\alpha}^0 = 1 - \frac{2k\sqrt{(1-s)(2-3s)}}{(k-1)(1-s)}.$$

Thus versioning can arise also for $7/11 < s < 2/3$, when $\alpha \ge \hat{\alpha}^0$, in contrast with Proposition 3. Therefore there is sequencing for a wider range of s than with full channel separation. Additionally, notice that, for $s \le 7/11$, $\hat{\alpha}^0$ takes negative values,[23] while α^0 is positive for $k > 2(1-\beta) \cdot \beta(1+s)/(2(1+s) - 9\beta(1-s))$. Thus $\hat{\alpha}^0 < \alpha^0$, and for $s \le 7/11$, H alone is never introduced under partial integration, although it can arise under full channel separation. Q.E.D.

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[23] $\hat{\alpha}^0$ is increasing in s and simplifies to $\hat{\alpha}^0 = 1/(1-k) < 0$ for $s = 7/11$.

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