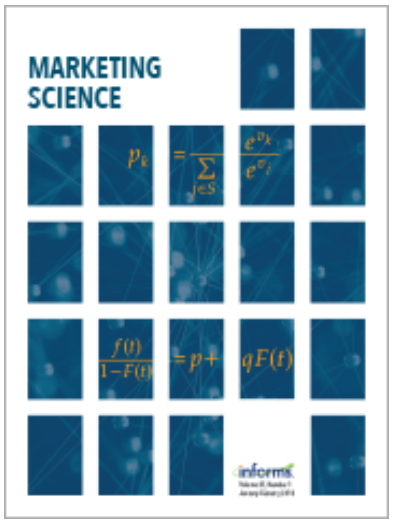

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The Equivalence of Bundling and Advance Sales

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We identify the conditions under which a problem of optimal advance selling strategy can be mathematically transformed into a problem of optimal bundle pricing. These conditions are as follows: (i) consumers and sellers have common priors on the probability of each state being realized in the future, (ii) consumers are risk-neutral, (iii) sellers can commit to spot prices, and (iv) consumers and sellers discount the future at the same rate. The result allows both researchers and practitioners to extend and/or apply the findings from the vast literature on bundling to advance selling problems, and vice versa. We highlight several insights that are particularly relevant, such as the importance of the dependence of consumer valuations across states on the profitability of advance selling in the base case of two states as well as in the cases of more than two states or with possible competition in some of the states.

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1. Introduction

The literature on advance selling postulates that it might be profitable to give discounts to consumers who purchase a product or service before the consumption date—and thus before some uncertainty is resolved.[1] We identify the conditions under which a problem of optimal advance selling strategy can be mathematically transformed into a problem of optimal bundle pricing. These conditions are that (i) consumers and sellers have common priors on the probability of each state being realized in the future, (ii) consumers are risk-neutral, (iii) sellers can commit to spot prices, and (iv) consumers and sellers discount the future at the same rate. This mathematical equivalence provides a new way of analyzing and comparing these problems. Using the findings of the well-developed bundling literature (starting with Adams and Yellen 1976),[2] this mathematical equivalence enables us to show the effects of the correlation of valuations across states on the profitability of advance selling by a monopoly firm: advance selling is more profitable than spot selling when consumer valuations across states are independent or negatively dependent or positively dependent up to a point.[3] Furthermore, we illustrate the effect of advance selling on spot prices. When the firm offers an advance purchasing option, it sets higher spot prices, so consumers who do not buy in advance are worse off as a result of the firm using advance selling.

We show that the equivalence result and the implied findings are robust to several extensions: when consumers and the firm have the same time preferences (i.e., when they discount future payments/consumption at the same rate), when there are more than two states of the world (see the online appendix, available as supplemental material at http://dx.doi.org/10.1287/mksc.2013.0833), and when the firm faces a competitor in only one of the states. We also illustrate when it is profitable (for an incumbent) to use advance selling to deter potential entry to the market.

On the methodological front, going beyond the existing literature, we allow for almost arbitrary ex ante (at the advance selling stage) consumer heterogeneity and ex post (at the spot selling stage) aggregate demands that vary across states. Finally, we take a first step toward joining the two literatures (bundling and advance selling), and we show both researchers

[1] See, e.g., Shugan and Xie (2000) and Xie and Shugan (2001).

[2] Chu et al. (2011) and Eckalbar (2010) both provide new methods and references to the previous primary literature in empirical estimation and computational methods. Stremersch and Tellis (2002), Kobayashi (2005), Armstrong (2006), and Stole (2007) provide surveys on bundling and on nonlinear pricing in general. These reviews have well over 200 citations to earlier literature and are a good place to start exploring the bundling literature after reading the newer research articles cited in this paper. See pp. 259–280 and pp. 417–420 of Belleflamme and Peitz (2010) for more influential papers in this literature.

[3] Dependence is essentially a nonlinear copula-based version of correlation. See Chen and Riordan (2013) and Clemen and Reilly (1999) for more.

and practitioners that one can use the well-developed bundling intuition for advance selling problems, and vice versa.

There is a simple insight behind the mathematical equivalence of an advance selling problem to a bundling problem. Suppose that the new basketball season is about to start, and a firm is selling tickets to one of the last games in the regular season. Everyone agrees on the probability with which that game will be important (the team will or will not be able to achieve a higher overall position in the league and/or a playoff position); however, this uncertainty is not going to be resolved until the end of the season. For ease of exposition, suppose there are just two future states of the world—in state 1 the game is important, and in state 2 it is not—and everyone agrees that the probability of state 1 occurring is a. The ticket to the last game can be viewed as a package (a bundle) of two single-state tickets: one of them is valid only in state 1 and the other is valid only in state 2. The consumer's valuation of the single-state ticket is the value from going to the game in that state multiplied by the probability of that state occurring. The value of an advance sale ticket is then the sum of the values of the two single-state tickets. Selling a ticket to the game before the uncertainty is resolved (advance selling) is therefore analogous to selling the two single-state tickets as a bundle. Another way to think about this is that the advance sale ticket is a bundle of several Debreu securities, with each security valid only in its state of the world. The states have to be mutually exclusive but do not have to be collectively exhaustive; more on that follows in the discussion.

The equivalence result is intuitive but is not straightforward: there are important implementation differences between an advance selling strategy and a bundling strategy. If a firm sells a bundle, it sells one unit of each good included in the bundle, and consumers decide whether to consume the bundle or the individual products separately, with the decisions being made at the same time. On the other hand, if the firm sells a good in advance, the good can actually only be consumed in one of the future states. Consumers make a sequential consumption decision: First, they decide whether to purchase the good in advance and then (after the uncertainty is resolved) the ones who did not buy in advance decide whether to buy the good at the spot price. Unlike a standard bundling problem, an advance selling problem has a time dimension. Some of the problems that we are interested in in the advance selling literature were of no particular interest in the contexts of bundling, and so the researchers in bundling did not analyze these situations. For instance, they did not analyze the impact of uncertainty (risk aversion), commitment to future prices, time preferences (of consumers and the firm), different arrival times of consumers, and the impact on production costs (i.e., advance selling enables longer lead times). Because of these differences, we have identified the previously stated four conditions to be satisfied for the mathematical equivalence of these problems to hold.

We discuss how stringent these conditions are and the implications of relaxing them in §5. In §5.1 we outline conditions under which consumers and firms will hold common priors on the state probabilities, and we also discuss what happens if these conditions do not hold. In §5.2 we discuss the potential effects of risk aversion on the demand for advance selling, explain why the case with risk-averse consumers would be particularly difficult to analyze within our advance selling model, and provide some empirical evidence to support our argument that the advance selling model with risk-neutral consumers is a reasonable approximation of many real-world advance selling situations. We also provide a Taylor expansion in the online appendix to show that if the consumer utility functions are close to being linear around the consumers' current wealth levels (have low absolute magnitude derivatives or, in other words, are close to being locally risk-neutral), then we also have an approximation.

The commitment assumption, which we discuss in §5.3, is in line with the most of the previous advance selling literature[4] and is technically hard to relax in our setup with ex ante heterogenous consumers. More important, in many real-world advance selling scenarios, we believe that the firm can commit to spot prices, in particular because of reputational concerns. Such examples include situations where firms can commit to one spot price (e.g., "advance price" and "gate price" for concert tickets, sport games, amusement parks, zoo tickets, fair tickets, exhibitions, early bird registration fees for conferences, recreation activities, professional training classes) and situations where firms can commit to different spot prices depending on the state (e.g., electricity prices with smart meters or energy supply prices based on an index).

When consumers and the firm discount future differently, we show (in the online appendix) that it is not possible to find the exact bundling equivalent to the advance selling model, but we can still specify a bundling model to understand the profitability of advance selling. In particular, we show that when consumers are more impatient than the firm, for all parameter spaces where bundling is not profitable in the bundling model, advance selling is not profitable

[4] According to Fay and Xie (2010, p. 1048), "The advance selling literature (e.g., Xie and Shugan 2001, Shugan and Xie 2005) suggests that a seller is capable of committing in advance to announced spot prices if the latter are observable at the time when customers are making advance purchases."

in the advance selling model, either. Also, in the extensions we derive a version of equivalence for consumers who value securing the product earlier (or later).[5]

We contribute to the advance selling literature that explains the profitability of advance selling solely by demand-related factors (see, e.g., Fay and Xie 2010, Shugan and Xie 2000, Xie and Shugan 2001). This literature usually assumes ex ante homogeneous and ex post heterogeneous consumers.[6] It is a good starting point for research, but it has some undesirable qualities: for example, without any exogenously imposed restrictions, either all consumers purchase in advance or none of them does. Another line of literature relates the profitability of advance selling to supply-side efficiency, such as using advance selling as a yield management tool to screen consumers by their uncertainty of demand while allocating capacity efficiently across periods when aggregate demand is uncertain (Dana 1998; Gale and Holmes 1992, 1993), improving capacity planning by using the information on advance sales (Boyaci and Özer 2010), and improving supply chain coordination by allocating inventory risk between the partners (Cachon 2004).

In our setup, however, consumer valuations for the good in a given state are drawn from a probability distribution function such that ex ante consumers know their valuations and the probability of each state. Thus, some consumers purchase the good in advance and some wait and purchase (or not) when the state of the world is realized. Allowing for ex ante, as well as ex post, consumer heterogeneity enables us to illustrate that advance selling might be profitable even when the seller cannot fully homogenize the consumer types. The ex ante heterogeneity allows us to illustrate when a monopolist wants to use advance selling as a price discrimination tool to exploit consumer heterogeneity in valuations across different states of world. The previous literature on yield management (Dana 1998; Gale and Holmes 1992, 1993) also showed advance selling as a profitable price discrimination tool. Different from that literature, we do not have costly capacity, so we do not link the profitability of advance selling to allocating capacity efficiently across the periods. We explain below how price discrimination works in our setup.

[5] The problem can be transformed into a bundling problem where products are complements or substitutes; see Venkatesh and Kamakura (2003). A less straightforward example is a reduced way of modeling a particular form of risk aversion. Suppose the consumer wants to secure the product in advance rather than risking anything unforeseen that might happen in the future, above and beyond the common-known state probabilities. In that case, the consumer values the advance purchase more than her expected valuation from purchasing the good at the spot, even if the expected spot price is the same as the advance sale price.

[6] An exception is an extension in the model of Shugan and Xie (2000), which we discuss below.

Fay and Xie (2010) draw a parallel between the profitability of advance selling and the profitability of pure bundling when consumer types are negatively correlated: both strategies work by aggregating consumers and thereby homogenizing consumer heterogeneity. We show that in our advance selling setup, any kind of negative dependence between consumer valuations that are distributed by any joint probability density function (p.d.f.) (and cumulative distribution function satisfying the standard second-order conditions) implies that advance selling homogenizes at least some of the consumer heterogeneity and thus improves profit compared with spot selling. Moreover, going beyond proving their conjecture, we show that a more profitable advance selling might arise even when consumer valuations are positively correlated.

Intuitively, advance purchasing discounts are profitable if there is a sufficiently high number of consumers who would purchase the good in only one state if there was no advance selling. This is the case if the correlation of the valuations across states is not very positive; then a consumer who has a high consumption value in state 1 is less likely to have a high value in state 2. In that case, advance selling discounts convince many consumers to buy the good in advance rather than buying in only one state.

Another big distinction from the previous advance selling literature is that our states are aggregate: all consumers are going to end up in the same state. The previous literature worked with individual states: each consumer independently drew the state that she was going to be in. The individual state assumption (coupled with a continuum of ex ante homogeneous consumers) results in the same ex post aggregate demand, regardless of the individual state realizations. Thus, once the uncertainty is resolved, the firm always faces the same demand curve, and so overall it does not face any uncertainty. The firm cannot charge different prices in different states because it cannot observe the individual state realizations of different consumers. The situation is different with our aggregate state assumption: the firm faces uncertainty; aggregate demand depends on the realized state, and thus the firm might want to charge different prices in different states. In our advance selling model, we would get the same aggregate spot demand regardless of the state if the marginal density function of the valuations is the same for the two states (for example, normal with a mean of 5 and a standard deviation of 1 in both states). In this case, even with ex ante heterogeneity, the firm charges the same spot price across two different states, and our model degenerates to the one where the firm chooses just two prices: the advance selling price and the spot price (which is the same, regardless of the state).

In general, the two models (individual states versus aggregate state) are not nested and correspond to different real-world phenomena. The aggregate state assumption (our model) captures the cases where all consumers experience the same state: a basketball game at the end of the season (since it would be important to all the fans) or a flight that turned out to be in demand (since it would be close to fully booked for those trying to get a seat). On the other hand, the individual state assumption is more reasonable when different consumers face different state realizations; for example, when deciding whether to purchase gas in advance when renting a car, consumers who do not buy in advance could end up in a good state where they find a gas station nearby, or they could end up in a bad state where they are late for their flight and do not have time to fill up the tank.

We are aware of only one paper analyzing competitive advance selling, Shugan and Xie (2005). The authors use different demand specifications to show that the impact of competition on the profitability of advance selling is unclear. The literature on competitive bundling is much more developed,[7] so our equivalence result in the extension to competition contributes to the advance selling literature by illustrating that a firm that faces a competitor in one state will find advance selling profitable under similar conditions to a monopoly firm and that an incumbent can use advance selling as a tool to deter potential entry to its market.

The equivalence that we prove is technical. It implies that the techniques for solving bundling problems can be used to solve advance selling problems, and vice versa. The intuition behind the two problems is very similar. This does not imply that in economic and marketing applications the two strategies are equivalent or that if a firm does one then there is no use in doing the other. Advance selling exploits consumer heterogeneity in the dimension of different valuations of the same product across different states of the world. Bundling exploits consumer heterogeneity in the dimension of different valuations for different products. The two might or might not be related, and the firm can use neither bundling nor advance selling, either one but not the other, or both at the same time.[8] Although the logic behind the two price discrimination instruments is the same, and the techniques used for one are applicable to the other, there might still be value in using both at the same time since they exploit different dimensions of consumer heterogeneity.

We only consider the revenue benefits of advance selling and do not deal, for example, with the issues of capacity constraints, risk sharing between the suppliers and the retailers, or short sales seasons and replenishment. Gale and Holmes (1993), Cachon (2004), and Tang et al. (2004) show how advance purchase discounts improve profits under these scenarios. We assume that the seller is not constrained on the production side and that production can occur instantaneously. We could build these and other supply features into our model; however, doing so would distract from the main purpose of this article.

Since the bundling literature is older and more developed than the advance selling literature, we mainly focus on which findings one can bring from the bundling literature to advance selling. This knowledge transfer can go the other way, too: research on advance selling can provide important insights about bundling since advance selling researchers have studied variations of the maximization problem that have not been considered previously in the bundling literature—for example, comparing advance selling to probabilistic selling, the effects of uncertainty and risk aversion, and supply-side savings. Exploring these issues is left for future research. Hence, the current paper encourages, rather than discourages, future research on advance selling. Overall, we hope that this paper joins two currently separate and developed literatures, allowing researchers, practitioners, and instructors to leverage tools from one stream to the other and to possibly combine these tools to create more.

2. The Framework

We first describe a model of advance selling and then show the mathematical equivalence of the optimal pricing decisions in this model to those in a bundling problem with appropriate parameter specifications.

2.1. A Model of Advance Selling

We consider one seller and a continuum (mass 1) of consumers. The seller offers one product and each consumer has a unit demand for the product. The value from consumption depends on the state of the world. For simplicity, we assume that there are two states of the world (see the online appendix for the extension of our results to more than two states of the world). State 1 occurs with probability $a \in [0, 1]$. A consumer receives utility v_1 if she consumes the product in state 1 and v_2 if she consumes it in state 2. For now, we assume that consumers are risk-neutral.[9] Consumers

[7] See, e.g., Whinston (1990), Carlton and Waldman (2002), Nalebuff (2004), and Chen and Riordan (2013).

[8] For example, some ski resorts start selling passes for the next season as early as April, taking advantage of the negative dependence of consumer valuations across several possible states of the world (winter with a lot of snow or winter with barely any snow). However, these ski resorts might also engage in the bundling of lodging and lift tickets, taking advantage of the negative dependence of consumer valuations between the values of lodging and lift tickets. We thank an anonymous referee for this example.

[9] We discuss how risk aversion of consumers would change the profitability of advance selling in §5.1.

differ in their valuations: (v_1, v_2) are drawn from a joint probability distribution function, $f(\cdot,\cdot)$ over region $[\underline{v}_1, \bar{v}_1] \times [\underline{v}_2, \bar{v}_2]$. We do not make any assumptions on the type of correlation between the valuations across the states.[10] Furthermore, we allow the marginal cost to the seller to be state dependent: in state j the marginal cost is c_j. We assume that at least some consumers have valuations above the respective marginal costs and that the monopolist's second-order condition (S.O.C.) is satisfied (there exists a global maximum).[11] We analyze the profitability of selling the product before the state is realized (advance selling) at price p^{AS} or after the state realization (spot selling) at price p_1^{SS} in state 1 and at price p_2^{SS} in state 2, or offering both an advance selling price and spot selling prices. The timing of the events is as follows.

Period 1:

- Consumers arrive. Each consumer knows her valuation from consumption in the two states and the probability of each state occurring (v_1, v_2, and a), but she does not know which state is going to occur.
- The firm knows the p.d.f. from which consumer valuations are drawn and the state probabilities ($f(\cdot,\cdot)$ and a). The firm announces the advance sale price and two spot prices conditional on the state of the world (p^{AS}, p_1^{SS}, and p_2^{SS}).
- Consumers decide whether to buy now (and pay p^{AS}) or to wait until the uncertainty about the state is resolved.

Period 2:

- The state is realized (say, state 2).
- Consumers decide whether to purchase at the preannounced spot price (p_2^{SS}).
- All buyers consume and receive their utility regardless of when they purchased.

Note that in this setup, the firm faces unobserved consumer heterogeneity both before the state is realized (ex ante) and after the state realization (ex post), since the firm only knows the p.d.f. from which consumer valuations were drawn and the state probabilities. Ex ante consumers know their value from consumption in a given state and the probability of the state occurring. Thus, in particular, consumers know their expected value from consumption ex ante. Ex post (in period 2), they know the exact value of their utility since they know which state occurred.

We solve the model by backward induction. A consumer who did not purchase in advance purchases the good in the spot period (after the state realization) if her valuation is higher than the spot price. In the first period, consumers learn their valuation in each state, the advance selling price and the spot prices, and then decide whether to purchase in advance. In the decision problem of the first period, consumers anticipate the second period choice problem and the corresponding utility of each outcome. The expected utility from buying in advance is

$$U^{AS} = \alpha v_1 + (1-\alpha)v_2 - p^{AS}.$$

If a consumer does not buy in advance and waits to buy at the spot price instead, then she buys at the spot only if her value in that state is higher than the spot price. So her expected utility from waiting until the spot period is the weighted average of the utility she gets in each state:

$$U^{SS} = \alpha(v_1 - p_1^{SS})\Gamma_1 + (1-\alpha)(v_2 - p_2^{SS})\Gamma_2,$$

where $\Gamma_i = 1$ if $v_i \geq p_i^{SS}$ and $\Gamma_i = 0$ otherwise. She prefers to purchase in advance if and only if $U^{AS} \geq U^{SS}$. Let D^{AS} denote the advance selling demand and D_j^{SS} denote the spot selling demand in state j.[12] The monopolist's profit is the sum of the *expected* profits from advance selling and from spot selling:

$$\begin{aligned}\Pi^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}) = {} & (p^{AS} - ac_1 - (1-a)c_2)D^{AS} \\ & + a(p_1^{SS} - c_1)D_1^{SS} \\ & + (1-a)(p_2^{SS} - c_2)D_2^{SS}. \qquad (1)\end{aligned}$$

We assume that there exists a unique price vector $(p_1^{SS}, p_2^{SS}, p^{AS})$ that maximizes the seller's profit given in (1). The following lemma shows that when $p^{AS} > \alpha p_1^{SS} + (1-\alpha)p_2^{SS}$, every consumer is better off waiting until the spot period instead of buying in advance, and so the firm would make the same profit if it lowered its advance selling price to $p^{AS} = \alpha p_1^{SS} + (1-\alpha)p_2^{SS}$.

Lemma 1. *There is no loss of generality from assuming that* $p^{AS} \leq \alpha p_1^{SS} + (1-\alpha)p_2^{SS}$.

Proof. Suppose not, and suppose that $p^{AS} > \alpha p_1^{SS} + (1-\alpha)p_2^{SS}$. The expected price of purchasing at the spot, regardless of what state it is, is lower than the advance

[10] More precisely, the valuations are independently distributed across the states if $\Pr(v_i \geq x \mid v_j \geq y) = \Pr(v_i \geq x)$ for $i \neq j$. Using the definition of Long (1984), the valuations are negatively (respectively, positively) correlated if $\Pr(v_i \geq x \mid v_j \geq y)$ decreases (respectively, increases) in y.

[11] Defining straightforward conditions for the S.O.C. is not a trivial task in this problem or in the bundling problem in the next subsection. First, the maximum might not be unique—for example, if advance selling is not profitable, then there are many potential advance selling prices such that no advance sales are made, all resulting in the optimal profit. In general, the models (of advance selling and of bundling) that we present in this paper are variations of a multidimensional screening problem. A common assumption, made to ensure the convexity of such problems, is some version of the monotone hazard rate assumption on the probability distribution function. See McAfee and McMillan (1988) for more.

[12] See Appendix A for the derivation of these demands.

purchase price. Thus, a consumer is better off waiting for the spot market instead of purchasing in advance. Therefore, the firm receives the same profit if it lowers the advance selling price to $p^{AS} = \alpha p_1^{SS} + (1-\alpha)p_2^{SS}$: still no consumer purchases in advance, assuming that indifferent consumers wait until the spot market.

Note that this lemma might not be true if consumers were risk averse or had some time preferences, or the firm cannot commit to future prices. We will discuss time preferences in §4.1, further in §5.4; and we discuss risk adverson in §5.1.

In particular, making comparisons with the earlier literature, this lemma is not true in the Xie and Shugan (2001) setup. In their case, this is due to the limited capacity assumption in that part of their model. This lemma is simply to cut down on the number of cases (rows in Table 1) discussed while obtaining the result in Proposition 1 and is not crucial to the logic of the proof.

Moreover, without loss of generality, the real value of the advance selling price is lower than the expected value of spot selling prices: if the consumer discount factor is δ, then $(1/\delta)p^{AS} \le ap_1^{SS} + (1-a)p_2^{SS}$. See §4 for more on discounting in our model.

We could therefore write the consumers' choice problem as if they face four options in period 1: (1) purchase in advance, (2) purchase on the spot only in state 1, (3) purchase on-the-spot only in state 2, and (4) do not purchase. Table 1 shows the utility a consumer gets from each of the four purchasing options.

2.2. The Equivalence of the Advance Selling Problem to a Bundling Problem

Now consider a static problem of monopoly bundling where the seller offers two products and consumers have a unit demand for each product. Suppose that a consumer receives utility av_1 from consuming product 1 and $(1-a)v_2$ from consuming product 2. As before, let $f(\cdot,\cdot)$ denote the joint distribution function of the valuations v_1, v_2 over region $[\underline{v}_1, \bar{v}_1] \times [\underline{v}_2, \bar{v}_2]$. There is unobserved consumer heterogeneity: consumers know their utilities (and a), but the seller only knows parameter $a \in [0,1]$ and the joint distribution function $f(\cdot,\cdot)$. Assume that the seller's marginal cost of product 1 is ac_1 and of product 2 is $(1-a)c_2$. Assume that the firm cannot prevent consumers from purchasing both products separately. The bundling problem involves analyzing the profitability of selling the products as a bundle at price p^B or selling only the individual products separately at prices p_1^I and p_2^I, or offering both a bundle price and individual product prices (mixed bundling).

Table 1 The Expected Utility of a Consumer from Different Purchasing Options in the Advance Selling Model

Purchasing options	Expected utility
Advance purchase	$av_1 + (1-a)v_2 - p^{AS}$
Spot purchase only in state 1	$a(v_1 - p_1^{SS})$
Spot purchase only in state 2	$(1-a)(v_2 - p_2^{SS})$
No purchase	0

First, we show that it is not feasible to sell the bundle at a price that is higher than the sum of the stand-alone prices—that is, implement a bundling premium—because the firm cannot prevent consumers from buying both of the items separately.[13]

Lemma 2. *There is no loss of generality from assuming that* $p^B \le \alpha p_1^I + (1-\alpha)p_2^I$.

Proof. Suppose that $p^B > \alpha p_1^I + (1-\alpha)p_2^I$, but then every consumer is better off buying both goods separately and paying the sum of the stand-alone prices rather than buying the bundle and paying a premium. So the firm would make the same profit if it lowered its bundle price to $p^B = \alpha p_1^I + (1-\alpha)p_2^I$.

Given the prices p^B for the bundle, p_1^I, p_2^I for the individual products, and Lemma 1, consumers have four options: (1) purchase both products as a bundle, (2) purchase only product 1, (3) purchase only product 2, and (4) do not purchase. Table 2 shows the utility a consumer gets from each of the four available options. Consumers choose the option that gives them the highest utility. The monopolist's profit in the bundling model is

$$\Pi^B(p_1^I, p_2^I, p^B) = (p^B - ac_1 - (1-a)c_2)D^B + (p_1^I - ac_1)D_1^I + (p_2^I - (1-a)c_2)D_2^I, \quad (2)$$

where D^B denotes the demand for the bundle and D_j^I refers to the demand for product j only.[14] Comparing consumer options in the advance selling model (Table 1) with those in the bundling model (Table 2), it is straightforward to see that if the price for the bundle is p^{AS} and the prices for product 1 and product 2 are ap_1^{SS} and $(1-a)p_2^{SS}$, respectively, the purchasing options of the two models would coincide; advance selling corresponds to selling the bundle, spot selling only in state j corresponds to selling only product j for $j = 1, 2$. Hence, at these prices, the demand for the bundle coincides with the demand for advance selling: $D^B \equiv D^{AS}$, and the demand for only product j coincides

[13] Indeed, the bundling literature shows that, if it is feasible to monitor purchases, the monopoly might want to charge a bundling premium rather than offering a bundling discount. Armstrong (2013) shows that this is the case if the demand for the bundle is less elastic than the demand for an individual item at the equilibrium prices without bundling. For example, when the valuations are independently distributed or negatively correlated, this cannot be the case, so the monopolist prefers a bundling discount.

[14] In Appendix B, we drive these demand functions explicitly.

Table 2 The Utility of a Consumer from Different Purchasing Options in the Bundling Model

Purchasing options	Utility
Bundle of products 1 and 2	$av_1 + (1-a)v_2 - p^B$
Only product 1	$av_1 - p_1^I$
Only product 2	$(1-a)v_2 - p_2^I$
No purchase	0

with the demand for spot purchase only in state j, $D_j^I \equiv D_j^{SS}$.[15] As a result, at these prices the profit in the bundling model would be equivalent to the profit in the advance selling model:

$$\Pi^B(ap_1^{SS}, (1-a)p_2^{SS}, p^{AS}) = \Pi^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}). \quad (3)$$

This proves our main result.

Proposition 1. *The problem of finding the optimal pricing strategy in the advance selling model described in §2.1 is mathematically equivalent to the problem of finding the optimal pricing strategy in the bundling problem described above. Moreover, if the optimal bundle price is p^*, and the optimal prices for the individual products are ap_1^* and $(1-a)p_2^*$, then the optimal advance selling price is p^*, and the optimal spot selling prices are p_1^* and p_2^*.*[16]

The main idea behind this result is that an advance sale ticket is a bundle of two tickets, each valid only in a particular state in the future. The weights in the proposition adjust the valuations and the marginal costs of stand-alone products to conform to the fact that neither state occurs with certainty, and to make sure that a consumer's expected value of each item (and the cost of serving the consumer) in the advance selling model is the same as her value for each item (and the cost of serving her) in the bundling model.

The equivalence result seems to be straightforward. However, there are important implementation differences between the two problems. If the monopolist sells a bundle, it sells one unit of product 1 and one unit of product 2 with certainty. Consumers decide whether to consume the bundle or the individual products separately, with the decisions being made at the same time. In the advance selling model, however, if the monopolist sells a ticket in advance, this ticket can actually be consumed in only one of the two states. Consumers make a sequential consumption decision: first, they decide whether to purchase the good in advance and then (after the uncertainty is resolved) the ones who did not buy in advance decide whether to buy the good at the spot price. A priori, it is unclear whether the monopolist's problem is the same in these models.

We prove the mathematical equivalence of these models by rewriting consumer purchasing options in the advance selling model to reflect the fact that when consumers decide whether to advance purchase, they compare their expected utility from advance purchasing with their expected utility from spot purchasing in a given state (or to no purchase). Thus, even if advance purchasing and spot purchasing occur at different points in time, consumers decide at the same time whether to advance purchase or whether to delay their purchase to the spot period. Since consumers know spot prices from the beginning (and we assume that the firm can commit to spot prices), from the beginning consumers know whether they will purchase the good at the spot market in any given state.

2.2.1. Example. Suppose that consumer valuations for the two states are independently and uniformly distributed, with the valuations for State 1 in the range of $[0, V_1]$, and the valuations for State 2 in the range of $[0, V_2]$. Furthermore, suppose that the probability of State 1 is a, the marginal costs are zero regardless of the state, and $(1-a)V_2 \geq aV_1/2$ (see Eckalbar 2010 for the case when this assumption is not satisfied).

Proposition 1 implies that this problem is equivalent to the following bundling problem: valuations for the two products are independently and uniformly distributed, with the valuations for product 1 in the range of $[0, aV_1]$, the valuations for product 2 in the range of $[0, (1-a)V_2]$, and the marginal cost being 0 for either product. From Eckalbar (2010), we know that the optimal mixed bundling solution for this problem is that optimal individual product prices are $p_1^I = \frac{2}{3}aV_1$ and $p_2^I = \frac{2}{3}(1-a)V_2$, that the optimal bundle price is $p^B = \frac{1}{3}(2aV_1 + (1-a)V_2 - \sqrt{2a(1-a)V_1V_2})$, and that mixed bundling is more profitable than selling two independent products and not selling a bundle.

Then, using Proposition 1, in the advance selling model the optimal spot selling prices are $p_1^{SS} = \frac{2}{3}V_1$ and $p_2^{SS} = \frac{2}{3}V_2$, and the optimal advance selling price is $p^{AS} = \frac{1}{3}(2aV_1 + (1-a)V_2 - \sqrt{2a(1-a)V_1V_2})$. Thus we find that advance selling along with spot selling is more profitable than spot selling and not offering advance selling, despite the correlation between the valuations in two states being zero.

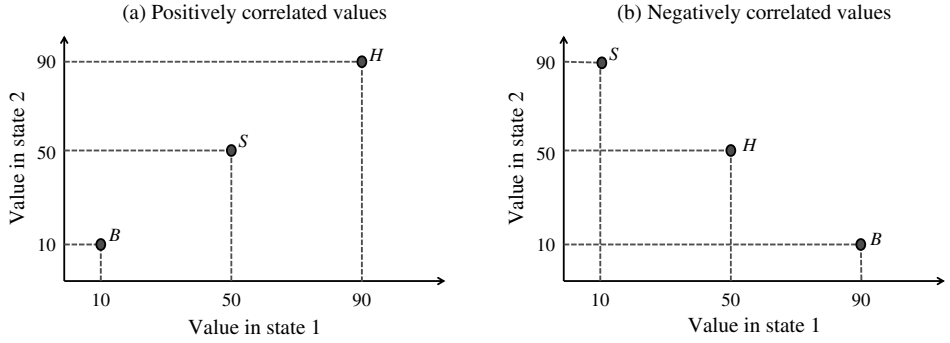
3. Implications of the Equivalence Result

In this section we outline which findings from the monopoly bundling literature we can bring over to the advance selling literature using our equivalence result (Proposition 1). First, we highlight an important insight from the bundling literature: the correlation of

[15] Comparing the demands derived in Appendix A to those derived in Appendix B shows the equivalence of those demands when $p^B = p^{AS}$, $p_1^I = ap_1^{SS}$, and $p_2^I = (1-a)p_2^{SS}$.

[16] We extend the case of two products/states to $N > 2$ products/states in §4.3 and discuss the implications of this result using the bundling literature on multiproduct bundling.

Figure 1 Correlation of Values Across States



valuations across states is an important determinant of the profitability of offering advance sales. In particular, we show that, despite the intuition from the advance selling literature, advance selling can be profitable with positive correlation. We show in general when advance selling increases spot market prices and when it lowers consumer welfare.

Since Stigler's (1968) examples of profitable bundling of movies, the bundling literature has developed significantly (see footnote 2 for references to review articles). A large proportion of the literature studied whether and when a multiproduct monopoly firm wants to sell its products as a bundle rather than separately or do mixed bundling (combining the two options): a price for the bundle and individual product prices. This literature emphasizes the role of bundling as a price discrimination tool. The main finding is that the profitability of bundling depends on the correlation between valuations for the two products.

Chen and Riordan (2013) differ from the previous work by allowing for (noncontinuous) general distribution functions of valuations (subject to the standard second-order condition) and deriving intuitive sufficient conditions using *copulas*[17] under which mixed bundling strictly dominates separate selling. They also examine the effect of bundling on stand-alone prices. We use their findings (unless otherwise specified) to derive the implications of our equivalence result. The following corollaries are direct equivalents of corresponding results from the bundling literature. See their work for the exact bounds for the results derived below.

Corollary 1. *If the valuations for consumption in the two possible states of nature are negatively dependent, independent, or not too positively dependent, then optimal advance selling discounts strictly increase profit.*

[17] To define the dependence between the distributions of valuations for the two products, they use a copula, which is a function that couples marginal distributions of random variables to form a joint distribution.

Offering an advance sale discount lowers profits from consumers who purchase the good in both states at spot prices, but it increases profits by convincing some of the consumers who would purchase only in one state of the world to buy in advance. The gains from advance sale discounts are likely to be higher than the losses when there are many consumers who switch from buying the good only in one state to buying it in advance. This is the case when consumer valuations across states are not too positively dependent: if a consumer's valuation from the good is high in one state, it is unlikely to also be high in the other state.

3.1. Example

We are used to interpreting correlation in bundling settings, but understanding correlation of consumer valuations for consumption across different states of the world is not as straightforward, at least not at first. This example aims to illustrate this issue. Let us return to the example of selling tickets to a future basketball game that might or might not be important. Say there are three types of fans: hard-core, bandwagon, and snobs.

If consumers receive the same value from the game regardless of the state, then consumer values are perfectly and positively correlated. For instance, if the hard-core fans value the game at \$90 both in the case when it is going to be important and when it is not, snobs value it at \$50 in either case, and bandwagon fans value it at \$10 in either case. Figure 1(a) plots the consumer types of this example, where the x axis refers to valuations for the game in the good state and the y axis refers to valuations in the bad state. Our equivalence result implies that, in this case, offering advance sale tickets is not profitable (correlation is 1, and so it is above any threshold where a positive correlation still results in increased profits).

When a consumer receiving a higher value from the game in one state gets a lower value in the other state, consumer valuations across states are negatively correlated. For instance, if the hardcore fans value the game at \$50 in either case, the bandwagon fans value it at \$90 when the game matters and at \$10 if it does not,

and the snobs value it at \$10 when it matters and at \$90 when it does not, then there is negative correlation, and advance selling improves profit. Figure 1(b) shows the negatively correlated consumer types of this example.

Corollary 2. *If the valuations for consumption in the two possible states of nature are negatively dependent, independent, or not too positively dependent, then advance selling discounts strictly increase spot prices.*

This corollary shows that consumers who purchase in one of the spot markets are worse off as a result of the availability of the advance selling discounts. The firm engages in price discrimination and effectively discriminates against consumers who purchase in one of the states. These consumers did not buy in advance because their expected valuation for the other state was not that high, but they are going to buy in the spot market, and thus their valuations for this state must be higher than those of the whole consumer group before the advance selling period.

4. Extensions

4.1. Time Preferences

4.1.1. Base Model. We extend our setup to account for some ways that the consumers' and firm's time preferences can display themselves. By purchasing early, consumers spend the money up front but get the actual product later, at the same time as those consumers who wait until the spot sale. If we normalize the value of unit utility/money to 1 at the consumption stage, the price paid up front should have a unit value higher than 1—to be more precise, it would be multiplied by the inverse of the discount factor, $1/\delta$ for $\delta \in (0, 1]$. This implies that the utility from buying in advance is now

$$av_1 + (1-a)v_2 - (1/\delta)p^{AS}.$$

Apart from this, the rest of Table 1 remains the same. If $\delta = 1$, there is no discounting, so we have the benchmark model. For $0 < \delta < 1$, consumers discount future utility/payments, so advance purchasing becomes less valuable to consumers. Furthermore, suppose that the firm values a unit profit at 1 at the spot sale stage and at $1/\delta$ at the advance selling stage. Also, assume that the firm's production takes place at the advance selling stage. So the firm's profit from advance selling and spot selling becomes

$$\begin{aligned}\Pi^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}) = {} & (1/\delta)(p^{AS} - ac_1 - (1-a)c_2)D^{AS} \\ & + a(p_1^{SS} - (1/\delta)c_1)D_1^{SS} \\ & + (1-a)(p_2^{SS} - (1/\delta)c_2)D_2^{SS}.\end{aligned}$$

So with $0 < \delta < 1$, advance selling becomes more profitable for the firm (keeping the demand and price constant).

The bundling model remains the same except that we scale the firm's marginal costs by $1/\delta$ to account for the firm's time preferences. To do so, assume that in the bundling model the marginal cost of product 1 is ac_1/δ and the marginal cost of product 2 is $(1-a)c_2/\delta$. With this parametrization, we obtain a result analogous to Proposition 1.

Proposition 2. *Suppose that in the advance selling model, consumers and the firm discount the future (consumption and payments at the spot selling period) by $0 < \delta \le 1$ (compared with the advance selling period). The problem of finding the optimal pricing strategy in the advance selling model with time preferences is mathematically equivalent to the problem of finding the optimal pricing strategy in the bundling problem where the marginal cost of product 1 is ac_1/δ and the marginal cost of product 2 is $(1-a)c_2/\delta$. Moreover, if the optimal bundle price is $(1/\delta)p^*$, and the optimal prices for the individual products are ap_1^* and $(1-a)p_2^*$, then the optimal advance selling price is p^*, and the optimal spot selling prices are p_1^* and p_2^*.*

Recall the example (from Eckalbar 2010) in §2 where consumer valuations are uniformly and independently distributed and the firm's marginal costs are zero. Using the closed-form solution to this bundling problem and Proposition 2 implies that in the advance selling problem where consumers and the firm discount future at the same rate, δ, the optimal spot selling prices are the same as the ones without discounting: $p_1^{SS} = \frac{2}{3}V_1$ and $p_2^{SS} = \frac{2}{3}V_2$, but the optimal advance selling price is lower reflecting the discount factor: $p^{AS} = (\delta/3)(2aV_1 + (1-a)V_2 - \sqrt{2a(1-a)V_1V_2})$. In this example, introducing time preferences does not affect spot selling prices but lowers the advance selling price by accounting for the discount factor.

4.1.2. Consumers Placing a Premium on Getting the Product Earlier. The value of a product (or a service) purchased in advance could be larger than the expected value of the same product purchased in all the possible future states. One example is tweaking the model to allow consumers who purchased the good in advance to enjoy it in period 1 as well. The utility in that period is certain because there is no uncertainty to resolve. Then, in period 2, we return to the model described earlier. In general, scenarios where a consumer gets an extra benefit from securing the transaction earlier fit this model.[18]

There are several papers in the bundling literature that allow nonadditive valuations: consumers' valuation from purchasing a bundle is not simply the sum of the valuations for the individual products included in

[18] In particular, this could be a reduced-form way of modeling risk aversion.

the bundle.[19] To model the situation where consumers get an extra utility from purchasing in advance, we use the advance selling equivalent of Venkatesh and Kamakura's (2003) bundling complements model.[20] Hence, we assume that the extra benefit from advance purchasing is likely dependent on the expected value from spot purchasing. Thus, the utility of the advance purchase in this scenario is $(1+\theta)(av_1+(1-a)v_2)-p^{AS}$, where $\theta>0$ implies that consumers prefer to secure their product early.

Proposition 3. *The problem of finding the optimal pricing strategy in the advance selling model with time preferences as described above is mathematically equivalent to the problem of finding the optimal pricing strategy in the bundling problem where goods are complements, as described above. Moreover, if the optimal bundle price is p^*, and the optimal prices for the individual products are ap_1^* and $(1-a)p_2^*$, then the optimal advance selling price is p^* and the optimal spot selling prices are p_1^* and p_2^*.*

Thus Propositions 2 and 3 both generalize Proposition 1 and show that all of our results and intuition still apply in these cases with time preferences.

Using Venkatesh and Kamakura's (2003) findings, this result implies the following.[21]

Corollary 3. *Assume that consumer valuations from consumption in different states are uniformly distributed over a compact interval $[0,a]$ and that the firm's marginal cost is the same in both states, $c_1=c_2=c$.*

- *When c/a is low, pure advance selling is more profitable than pure spot selling. Otherwise, pure spot selling is more profitable than pure advance selling except when the time preferences are very strong, in which case the two strategies are equally profitable.*
- *Simulations show that when c/a is low, advance selling together with spot selling is more profitable when time preferences are weak, whereas pure advance selling is more profitable when time preferences are very strong. When c/a is high, pure spot selling is more profitable when time preferences are weak and pure advance selling is more profitable otherwise. When c/a is moderate, pure spot selling is more profitable when time preferences are moderate, advance selling together with spot selling is more profitable when time preferences are weak, and pure advance selling is more profitable when time preferences are strong.*

4.2. Competition

Chen and Riordan (2013) also analyze the profitability of bundling when one firm offers both products and a competitor can offer only a differentiated version of one product and show that bundling is profitable under similar conditions to the case without competition. To analyze a similar competitive scenario in the advance selling model, we extend it to the case where the firm faces a competitor that offers a differentiated alternative to the firm's product only in one state—say, state 1.

For example, if the basketball game is not important, consumers might consider going to see a hockey game instead. Another example is the industrial transportation market between Detroit and Chicago. There are two ways of getting goods cheaply from Detroit to Chicago. If the weather is really cold, then there is only a train. If the weather is warm and there is no ice on the lakes, then there is also a boat. The question is, when does the train company have an incentive to sell in advance, knowing that consumers are going to get stuck with only one choice if the weather is cold? To analyze this question, we modify the benchmark setup. The full analysis, including the proposition describing the equivalence, is detailed in the online appendix. Below, we briefly describe the timing of the game and discuss the main conclusions.

The timing is as follows:

1. The firm sets an advance selling price (p^{AS}) and a spot selling price for each state (p_1^{SS}, p_2^{SS}). Simultaneously, the competitor sets its price, $\tilde{p}_1^{SS}$.
2. Consumers decide whether to purchase in advance or wait until the uncertainty about the state is resolved.
3. The state is realized. In state 1, the two firms compete for consumers who did not buy in advance. In state 2, the firm is the monopoly seller, and consumers who did not buy in advance decide whether to buy the product.

Corollary 4. *Suppose that the firm will face a competitor only in one state of the world. Then advance selling discounts are strictly more profitable if the valuations are negatively dependent, independent, or not too positively dependent.*[22]

The corollary shows that the firm facing a competitor only in one state has similar incentives to advance sell as the monopoly firm, even if the level of prices is going to change. As Chen and Riordan (2013) show, the bounds with competition require a stronger negative dependence property than the monopoly case. Intuitively, when the firm faces a competitor, consumers

[19] This assumption allows for cases where the products are complements so that consumers get an extra utility from buying two products together or where the products are substitutes so that consumers' marginal utility from consuming the second product is lower if they have already bought the first one.

[20] Venkatesh and Kamakura (2003) analyze optimal bundle pricing when the utility of a bundle of goods 1 and 2 is more than the sum of the values from independent consumption—more precisely, $(1+\theta)(u_1+u_2)$.

[21] Also see Armstrong (2013) for a more general model covering substitute products ($\theta<0$) or consumers preferring not to secure the transaction in advance. This could occur if consumers are cash constrained and do not want to commit any of their cash to this future purchase or simply do not have the money to pay now.

[22] See Chen and Riordan (2013) for the proof in the bundling case.

have one more option to choose from (spot purchase only from the competitor in state 1). Since competition lowers the spot selling price in state 1 and raises the expected utility from delaying to purchase, there will be less demand for advance purchasing, and therefore the optimal advance selling price would be lower than in the monopoly case. This implies that advance selling margins would be lower, and so the profitability of advance selling discounts would require more people to switch from purchasing the good only in one state to buying the good in advance, which would be the case if there was stronger negative dependence between the valuations across the states.

Most of the literature on competitive bundling has focused on anticompetitive bundling and analyzed when a multiproduct incumbent facing potential entry to a single market wants to bundle its monopoly good with the competitive good to deter entry (e.g., see Whinston 1990). To analyze whether advance selling can be used as a tool to deter potential entry in the spot period, we use the equivalent of Nalebuff's (2004) bundling model and modify our advance selling model as follows: consumer valuations across the states are uniformly and independently distributed over a unit square.[23] With some probability, the challenger who offers a perfect substitute to the incumbent's product enters the market either in state 1 or in state 2. The timing of the events is as follows: (1) The incumbent decides whether to sell its product in advance, and if so, it sets its advance selling price. Simultaneously, it sets its spot prices. (2) The challenger decides whether to enter in either of the two states, and if it does, then it sets its spot price. (3) Consumers decide which firm to patronize and whether to purchase in advance or wait for the spot markets. Regardless of whether the challenger enters, the incumbent cannot change its prices.[24]

Corollary 5. *In the advance selling model described above, advance selling makes entry less profitable without lowering the profits of the incumbent* (*if entry is deterred*).

Intuitively, if entry occurs, advance selling reduces the market share of the entrant. Without advance selling, if the entrant offers an ϵ lower spot price than the incumbent, it gains the entire market. With advance selling, if the entrant offers an ϵ lower spot price than the incumbent, it steals less than the entire market. Offering more advance purchasing discounts than an uncontested monopoly is profitable because it has only a second-order effect on the incumbent profit but a first-order effect on the entrant's profit.

[23] Nalebuff (2004) argues that uniform density is for the sake of simplicity, and qualitative arguments on how bundling deters entry would apply for the class of quasi-concave density functions, such as multivariate normal.

[24] Fixing prices of the incumbent pre-entry is favorable to the entrant and thus conservative for the entry deterrence role of advance selling. See Nalebuff (2004) for more on this.

5. Limitations

The key contribution of our paper is in identifying the conditions under which the findings from the bundling literature can be extended or applied to advance selling. They are the following:

- Consumers and sellers have common priors on the probability of each state being realized in the future.
- Consumers are risk neutral.
- Sellers can commit to spot prices.
- Consumers and sellers discount the future at the same rate.

In this section we discuss how stringent these assumptions are and to what extent they can be relaxed.

5.1. Common Priors

The assumption that consumers and sellers hold the same objective beliefs on the probability of future events is common in the literature. The economics literature has provided several justifications for this assumption (e.g., rationality, frequency of past events, symmetric information) and also discussed the reasons and situations where individuals and firms might hold subjective beliefs. (See Morris 1995 for an interesting discussion of the justifications for this assumption and reactions to those.) Assuming that both sellers and consumers have the same prior information, that they process this information rationally, and that neither side can change the probabilities of the future events is sufficient for consumers and sellers to have common priors over probabilities of a given state being realized. Thus, there are two particularly likely scenarios where this assumption is not satisfied. The first scenario is consumers being not fully rational and/or being misinformed. In that case, sellers are likely to take advantage of consumers' biases and offer a contract to exploit them. These issues have been analyzed extensively by the growing literature on behavioral industrial organization; see, for example, Gabaix and Laibson (2006), Heidhues and Kőszegi (2010), and the references therein.

The second scenario that typically violates this assumption is when sellers can affect the probabilities of the states of the world. To return to the basketball game example, the owner of the team might be able to trade its star player in the middle of the season, dramatically changing the probabilities of the states being realized. If consumers are aware that this can happen, then this issue is similar to the case of no commitment to spot prices discussed below, because then the seller can affect the value (or quality) of the ticket bought at the spot. Otherwise, we conjecture that it would be similar to the situations where sellers might exploit consumers' biases as discussed above.

5.2. Risk Neutrality of Consumers

Advance purchasing decisions involve uncertainty, and so risk aversion might be an important factor driving consumer decisions of whether to buy a product in advance. Risk aversion does not affect profitability and consumer decision making in a way that is convenient to analyze analytically. This was illustrated by Shugan and Xie (2001), who present some examples of risk-averse utility functions where consumers are assumed to be homogeneous. The analysis of risk aversion is even more complicated in our setup since we allow for ex ante consumer heterogeneity and ex post different aggregate demands depending on the state. Modeling this goes beyond the scope of our paper.

In the online appendix, we illustrate how one can incorporate risk aversion into our benchmark models and apply Taylor expansions to the versions of Tables 1 and 2 to show that the bundling problem provides an approximation to the advance selling problem even in the risk aversion case, with the market of risk-neutral consumers resulting in the already-stated equivalence. One can then use empirical estimates of the derivatives of consumers' utility functions to at least bound our risk-neutral (linear) approximation. We conjecture that for U.S. households of median wealth, for most everyday purchases (e.g., concert tickets) or even most airplane ticket purchases, assuming standard functional utility forms such as constant absolute risk aversion (CARA) or constant relative risk aversion (CRRA), the risk-neutral approximation is reasonably close.[25]

5.3. Commitment to Spot Prices

As noted in §1, the assumption of exogenous credibility is commonly held in the advance selling literature and is usually justified in real-world situations by reputation concerns (see, e.g., Fay and Xie 2010; Shugan and Xie 2000, 2005).[26]

Unfortunately, analyzing the no commitment case is particularly hard in our model since we allow consumers to be heterogeneous at the advance selling stage. And so depending on the price charged in advance, a distinct, nonrandom, and self-selected subset of consumers waits for the spot period. Consumers and the firm play a Bayesian Nash game, where consumers form expectations over who exactly purchases in the first period based on the advance selling price and thus what the monopolist will charge in the second. An equilibrium advance purchase price depends on the consumers' beliefs, and the spot price depends on who exactly purchases in advance and who does not. There are potentially many possible equilibria, including tipping and self-fulfilling expectations, when consumer beliefs about the identities of the advance purchasers might result in the spot markets shutting down. This topic is important, but it is arguably less relevant in many real-world examples as a result of reputation concerns—as discussed above and in §1—and is beyond the scope of this paper.

5.4. Same Discount Rate

This assumption is satisfied if both consumers and sellers are rational and have the same access to the credit markets. However, this might not be true. We have already shown in the previous section how to solve the model if consumers place a premium on getting a product earlier, a special case of different discount rates.

We cannot prove the equivalence for a more general formulation of different discount rates; however, we can place some bounds on the outcomes and the comparison between advance selling and bundling. For example, if the firm discounts the future less than consumers, then the region where the advance selling is profitable is smaller than the region where bundling is profitable, and we find that the latter includes the former. We direct the interested reader to the online appendix for the proof of the previous claim.

6. Conclusion

We have shown that the problem of optimal advance selling is technically equivalent to the problem of optimal bundle pricing after making the appropriate changes to the parameters of costs and valuations.

[25] We use the results of an empirical estimation of risk aversion; see Fullenkamp et al. (2003). They use the data from a game show to analyze the contestants' risk aversion patterns. The authors show that, for example, using the CARA and CRRA functional forms and estimating the models using several possible bounded rationality scenarios, the certainty equivalent of a 50/50 gamble between \$0 and \$1,000 ranges from \$491.26 to \$499.19, depending on the exact function and a possible bounded rationality scenario that they use. Compared with other studies, Fullenkamp et al. (2003, p. 225) point out that "for a gamble offering a 50–50 chance of winning nothing and \$1,000, the certainty equivalent ranges from \$461.40 (Gertner 1993) to \$496.73 (Hersch and McDougall 1997), with most estimates implying a certainty equivalent above \$490." Although the amount of risk aversion is not trivial, it shows that our approximation should not be too far off even for a gamble of this size, i.e., 50/50 between \$0 and \$1,000. However, we believe that most real-world advance sales products do not present as much of a gamble for a potential consumer, and thus our bundling approximation should be reasonably accurate.

[26] One of the few exceptions in the literature is Xie and Shugan (2001), who analyze an extension where the seller cannot precommit to spot prices and show that the seller's credibility is critical for the profitability of advance selling when consumers are ex ante homogeneous. With no exogenous credibility, they show that large marginal costs and/or capacity constraints might enable the seller to commit to high spot prices. They also find that limiting advance sales can be profitable when three conditions hold: (1) selling to all early arrivals would leave insufficient capacity in the spot period to sell to all second period arrivals with high valuations, (2) the optimal spot price is high, and (3) marginal costs are sufficiently small to make advance selling profitable.

We identify four conditions under which this equivalence holds. They are as follows: (i) consumers and the seller have common priors on the probability of each state occurring, (ii) consumers are risk neutral, (iii) the seller can commit to the preannounced spot prices, and (iv) consumers and the firm have the same time preferences.

This equivalence allows us to apply numerous insights from the bundling literature to the study of advance selling problems and provides researchers and practitioners with a new way of analyzing advance selling and bundling problems. In particular, it shows that the correlation of consumer valuations across states plays a crucial role in determining whether advance selling is profitable, including the cases of more than two states, the case where consumers and the firm have the same time preferences, and the case of potential competition in one of the states. We also illustrate a profitable use of advance selling as an entry deterrence tool.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/mksc.2013.0833.

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Appendix A. Spot Selling and Advance Selling Demands

Consumers who would purchase on the spot in either state (types $v_1 \geq p_1^{SS}$ and $v_2 \geq p_2^{SS}$) purchase in advance since the advance selling price is lower than the expected spot selling price (by Lemma 1). Consumers who would spot purchase only in state 1 (types $v_1 \geq p_1^{SS}$ and $v_2 < p_2^{SS}$) prefer to delay their purchase and purchase on the spot (only when state 1 occurs) if

$$a(v_1 - p_1^{SS}) \geq av_1 + (1-a)v_2 - p^{AS}$$

or, equivalently, if their state 2 valuation is very low ($v_2 < (p^{AS} - ap_1^{SS})/(1-a)$). Aggregation of these consumer types gives us the demand for spot purchase only in state 1:

$$D_1^{SS}(p_1^{SS}, p^{AS}) = \int_{p_1^{SS}}^{\bar{v}_1} \int_{\underline{v}_2}^{(p^{AS} - ap_1^{SS})/(1-a)} f(v_1, v_2)\, dv_2\, dv_1. \quad \text{(A1)}$$

Consumers who would spot purchase only in state 2 (types $v_1 < p_1^{SS}$ and $v_2 \geq p_2^{SS}$) prefer to delay their purchase and purchase on the spot (only when state 2 occurs) if

$$(1-a)(v_2 - p_2^{SS}) \geq av_1 + (1-a)v_2 - p^{AS}$$

or, equivalently, if their state 1 valuation is very low ($v_1 < (p^{AS} - (1-a)p_2^{SS})/a$). An aggregation of these consumer types gives us the demand for spot purchase only in state 2:

$$D_2^{SS}(p_2^{SS}, p^{AS}) = \int_{\underline{v}_1}^{(p^{AS} - (1-a)p_2^{SS})/a} \int_{p_2^{SS}}^{\bar{v}_2} f(v_1, v_2)\, dv_2\, dv_1. \quad \text{(A2)}$$

Consumers who would not purchase on the spot (types $v_1 < p_1^{SS}$ and $v_2 < p_2^{SS}$) purchase in advance if the advance selling price is sufficiently low ($av_1 + (1-a)v_2 \geq p^{AS}$) and do not purchase at all otherwise. Hence, the proportion of consumers who do not make any purchase is given by

$$D^{\text{NoPurchase}}(p_1^{SS}, p_2^{SS}, p^{AS}) = \int_{\underline{v}_2}^{p_2^{SS}} \int_{\underline{v}_1}^{(p^{AS} - (1-a)v_2)/a} f(v_1, v_2)\, dv_1\, dv_2, \quad \text{(A3)}$$

and the demand for advance selling is equal to

$$D^{AS}(p_1^{SS}, p_2^{SS}, p^{AS}) = 1 - D^{\text{NoPurchase}}(p_1^{SS}, p_2^{SS}, p^{AS}) - D_1^{SS}(p_1^{SS}, p^{AS}) - D_2^{SS}(p_2^{SS}, p^{AS}). \quad \text{(A4)}$$

Appendix B. Individual Product and Bundling Demands

Consumers who would purchase both products (types $v_1 \geq p_1^I/a$ and $v_2 \geq p_2^I/(1-a)$) buy the bundle since it costs less than purchasing both products separately (by Lemma 2). Consumers who would purchase only product 1 (types $v_1 \geq p_1^I/a$ and $v_2 < p_2^I/(1-a)$) prefer to buy only product 1 rather than the bundle if

$$av_1 - p_1^I \geq av_1 + (1-a)v_2 - p^B$$

or, equivalently, if their product 2 valuation is very low: $v_2 < (p^B - p_1^I)/(1-a)$. Aggregation of these consumer types gives us the demand for product 1 only:

$$D_1^I(p_1^I, p^B) = \int_{p_1^I/a}^{\bar{v}_1} \int_{\underline{v}_2}^{(p^B - p_1^I)/(1-a)} f(v_1, v_2)\, dv_2\, dv_1. \quad \text{(B1)}$$

Consumers who would purchase only product 2 (types $v_1 < p_1^I/a$ and $v_2 \geq p_2^I/(1-a)$) prefer to buy only product 2 rather than the bundle if

$$(1-a)v_2 - p_2^I \geq av_1 + (1-a)v_2 - p^B$$

or, equivalently, if their product 1 valuation is very low ($v_1 < (p^{AS} - p_2^I)/a$). Aggregation of these consumer types gives us the demand for product 2 only:

$$D_2^I(p_2^I, p^B) = \int_{\underline{v}_1}^{(p^B - p_2^I)/a} \int_{p_2^I/(1-a)}^{\bar{v}_2} f(v_1, v_2)\, dv_2\, dv_1. \quad \text{(B2)}$$

Consumers who would not purchase the products separately (types $v_1 < p_1^I/a$ and $v_2 < p_2^I/(1-a)$) purchase the bundle if the bundle price is sufficiently low ($av_1 + (1-a)v_2 \geq p^B$) and do not purchase at all otherwise. Hence, the proportion of consumers who do not make any purchase is given by

$$D^{\text{NoPurchase}}(p_1^I, p_2^I, p^B) = \int_{\underline{v}_2}^{p_2^I/(1-a)} \int_{\underline{v}_1}^{(p^B - (1-a)v_2)/a} f(v_1, v_2)\, dv_1\, dv_2, \quad \text{(B3)}$$

and the demand for the bundle is equal to

$$D^B(p_1^I, p_2^I, p^B) = 1 - D^{\text{NoPurchase}}(p_1^I, p_2^I, p^B) - D_1^I(p_1^I, p^B) - D_2^I(p_2^I, p^B). \quad \text{(B4)}$$

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