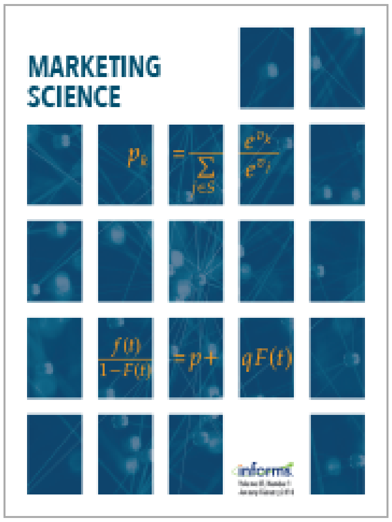

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Pricing Under Dynamic Competition When Loyal Consumers Stockpile

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Abstract. One goal of promotions for frequently purchased products is increasing short-term sales. Increases could be at competitors' expense, coming from consumers with relatively weak brand preferences. However, increased sales from brand-loyal consumers could well cannibalize future sales of the promoted brand. An unintended consequence of promotions is that loyal consumers otherwise willing to pay high prices may strategically stockpile at low prices. What is its impact on firms' profits? Who benefits from stockpiling? How should firms adapt their pricing to accommodate consumer stockpiling? For answers, we analyze an infinite horizon dynamic model of competition and derive the Markov perfect equilibrium pricing strategies that yield several managerial insights. We find strategic stockpiling does not reduce firms' long-run profits when managers adopt pricing strategies we identify. Turning to strategies, stockpiling causes firms to move away from frequently promoting below the stockpiling threshold, but it leads to mass points at reservation price and stockpiling threshold. Stockpiling is beneficial to consumers who stockpile but hurts those that do not stockpile, whereas switchers remain largely unaffected. State-dependent pricing as a result of stockpiling leads to positive intertemporal price correlation, implying, counterintuitively, that in equilibrium, deep promotions should be followed by deep promotions.

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1. Introduction

One goal of promotions by packaged goods manufacturers is to appeal to price-sensitive consumers who might buy a competing brand absent promotions. But promotions may have an unintended consequence that firms should consider. Neslin et al. (1985, p. 158) find that "loyal purchasers in the coffee market have learned to change their purchase patterns in order to take advantage of promotions for their preferred brand." Recent empirical evidence also suggests that brand-loyal consumers are more likely to engage in stockpiling in response to promotions than switching consumers (Krishnamurthi and Raj 1991, Sun et al. 2003, Chan et al. 2008). The intuition behind this finding is that loyal consumers derive relatively more benefit from stockpiling during a promotion because they do not intend to avail of future promotions by competing brands. By contrast, switchers are not committed to any particular brand and hence can take advantage of promotions offered by any brand, making stockpiling less attractive. For example, Coke lovers are likely to stockpile if Coke offers a lower price, but cola lovers can take advantage of promotions not only by Coke but also those offered by Pepsi and other competing brands of cola. If loyal consumers, who shun competing brands, stockpile their favorite brand opportunistically for later consumption, then future sales, potentially at higher prices, are cannibalized. Thus, when promoting to attract price-sensitive (switching) consumers, firms should internalize this possible downside. So the research question is, how are firms' equilibrium strategies affected by stockpiling? And how should managers address practical issues arising from stockpiling by loyal consumers? Thus, our work contributes to extant understanding of the effects of competitive promotions on firms' strategies and profits, and in doing so, it offers both managerial insights and possibly a more complete explanation for observed promotional price patterns constituting an equilibrium.

1.1. Literature Review

Price promotion is a well-studied topic in economics and marketing (Shilony 1977; Varian 1980; Narasimhan 1988; Raju et al. 1990; Rao 1991; Sinitsyn 2008a, 2008b). This body of theoretical work assumes that some consumers engage in switching brands as a response to promotions, and so the firm that offers a lower price sells to these switching consumers. The presence of such switching consumers along with other consumers who are not price sensitive—and so may be thought of as loyal—leads to equilibrium prices in mixed strategies with realizations of low prices interpreted as promotions. Early empirical research (Chiang 1991, Chintagunta 1993, Bucklin et al. 1998, Bell et al. 1999) has found substantial brand switching among consumers. Researchers have found that a large fraction (about 75%) of the demand expansion as a result of promotions can be attributed to brand switching and the rest to purchase acceleration and increased purchase quantity. More recent work (Pauwels et al. 2002, Heerde et al. 2003, Steenburgh 2007) that focuses on sales volume finds that in many instances only a third of the incremental unit sales in the promotional period can be attributed to brand switching. This suggests that there is significant purchase acceleration and increase in purchase quantity, which cannot be ignored in deriving equilibrium pricing strategies.

Chan et al. (2008) find that brand-loyal consumers respond to promotions by stockpiling, whereas switchers merely change brands and do not stockpile. Sun et al. (2003) have also found similar results. These studies, emphasizing modeling innovations, have findings that are consistent with the earlier evidence in Krishnamurthi and Raj (1991) that loyal consumers, though less price sensitive in the brand choice decision, are more sensitive to price in the quantity decision, further reinforcing the observations of Neslin et al. (1985) on the coffee market. In summary, empirical researchers have found loyal consumers to be more likely than switchers to stockpile because of promotions. This renders relevant our analysis in this paper of firms' equilibrium pricing strategies based on a model explicitly accounting for stockpiling by loyal consumers.

Our work should be seen in the context of prior exploration of consumer stockpiling. Salop and Stiglitz (1982) have shown that consumers' forward-looking behavior by itself may lead to promotions in the form of mixed strategy even when consumer preferences are homogeneous. Bell et al. (2002) innovate on Salop and Stiglitz's model of homogeneous consumers by making consumption state dependent, and so firms may induce stockpiling to increase consumption to get higher sales. Hendel et al. (2014) provide an alternative explanation for cyclical patterns in sales and prices and show that storability imposes a constraint on a monopolist's ability to extract surplus under nonlinear pricing that leads to cyclical patterns. Su (2010) develops a monopoly model with consumer stockpiling to show that consumer stockpiling itself leads to price promotions that happen at predictable time intervals. In contrast to these, we focus on profit and strategy implications of opportunistic stockpiling by brand-loyal consumers at low prices when faced with stochastic promotions that arise when firms compete for price-sensitive (brand-switching) consumers. The question then is, what should a firm's equilibrium pricing strategy be to balance the forces of competing for switching consumers and facing opportunistic buying by loyal consumers? By providing an answer to the question, we can offer managerial insights and guidelines.

Our work is closer to Hong et al. (2002), who study consumer stockpiling by assuming that all switching consumers stockpile at an exogenously specified threshold. Our work differs in two important ways. First, we explore stockpiling by loyal consumers. Second, we treat consumers' stockpiling rule as endogenous. This is important because consumers' stockpiling rule and firms' pricing strategies are mutually dependent. For example, if firms' equilibrium prices do not contain frequent deep promotions, consumers may find it profitable to stockpile at relatively higher prices. On the other hand, if deep promotions were frequent, the incidence of stockpiling at relatively higher prices would be lower. Firms' equilibrium pricing strategies will depend on consumers' stockpiling rule, as conceptualized in our work. Although there is extant work that makes consumer stockpiling rules endogenous, our work focuses on different issues and so adds to existing research. Guo and Villas-Boas (2007) consider a differentiated market in which consumer preferences for products are distributed over a Hotelling line. They, too, model consumers with strong brand preferences as more likely to stockpile. They find that when consumers have stable preferences, so that loyalty is not ephemeral, the firms' equilibrium pricing is such that stockpiling is never an equilibrium response for consumers. However, if consumer preferences randomly change from one period to another, making loyalty transient, then firms have an incentive to choose price that induces stockpiling by "locking" consumer loyalty in the first period. This pricing strategy reduces profits relative to the case of no stockpiling. We, on the other hand, model consumers with stable preferences and loyalty who are heterogeneous in price sensitivity. Some consumers are not price sensitive but are brand loyal, whereas others are price sensitive and switch brands.

Gangwar et al. (2014) analyze a model in which switching consumers stockpile. They show that firms' equilibrium profits are not reduced as a result of consumer stockpiling so long as firms recognize it in formulating their equilibrium promotional strategies. Intuitively, because all potential profits from switching consumers are competed away in equilibrium, stockpiling by them does not reduce firms' equilibrium profits.

1.2. Overview of Contributions and Results

We contribute to prior work by offering managerial insights and also modeling innovations. We obtain several substantive results. Prior work by Guo and Villas-Boas (2007) has pointed out that stockpiling by loyal consumers intensifies price competition in the subsequent period, and so firms' incentive to induce stockpiling disappears. We extend this insight by showing that even if stockpiling cannot be eliminated because of random prices, what managers should care about is not the extent of stockpiling but the price at which stockpiling occurs. Thus, instead of the facile reaction of eliminating promotions, shallow promotions should be favored over deep promotions. Other results are novel. We show that despite stockpiling, if firms design strategies taking it into account, then they can ensure that profits are not adversely affected. On the other hand, if they ignore stockpiling, the potential loss could be as high as 7%. We also show that although stockpiling enables loyal consumers to benefit from lower prices, this benefit decreases as more consumers stockpile. However, the price paid by switching consumers is insensitive to the extent of stockpiling. Finally, we show that state-dependent pricing strategies exhibit positive autocorrelation but are negatively correlated with competitor's price. On the technical dimension, we innovate by analyzing a dynamic model in which not only are firms strategic but also consumers. Moreover, firms have perfect information, whereas consumers are endowed with incomplete information. Interestingly, a novel characteristic of firms' equilibrium strategies is the presence of mass points at the stockpiling threshold in the interior of the price support. Thus, our work would be of interest to modelers.

The rest of the paper is organized as follows. In Section 2, we describe the consumer model and firms' technology and then define the equilibrium we seek. In Section 3, we state consumers' and firms' decision problems and analyze a two-period version of our model to obtain key insights and motivate the infinite horizon version. Section 4 analyzes the infinite horizon case and characterizes firms' value functions and their stationary equilibrium mixed strategies as well as consumers' stockpiling behavior by endogenizing their threshold rule for stockpiling. In Section 5 we discuss how our work relates to managerial concerns and strategies. Finally, Section 6 contains conclusions.

2. Model

We study a market with two firms serving consumers over an infinite horizon. We first describe our model of consumers, followed by our model of firms.

2.1. Consumers

In our model, there is a continuum of consumers. They each consume one unit of the product in every period. All consumers are assumed to have a reservation price of r. As in Narasimhan (1988), there are two types of consumers in our model, brand loyal and switchers: a proportion α of consumers in the market is loyal to each of the two brands. The remaining proportion of consumers of size $\beta = (1-2\alpha)$ consists of switching consumers. The switching consumers, at their turn, decide whether to buy and, if so, which brand to buy; their information set is $\{p_1, p_2\} \in \mathbb{R}_+^2$. Note that the switching consumers' decision depends on the prices of both brands.

Let us now turn to loyal consumers. Motivated by the empirical findings of Quelch (1985), Krishnamurthi and Raj (1991), Sun et al. (2003), and Chan et al. (2008) that consumers who are brand loyal are more likely to stockpile, we innovate with respect to Narasimhan (1988) by letting a fraction λ of loyal consumers stockpile for future consumption. Said differently, even though loyal consumers in our model need one unit of the product per period, they may buy more than one unit, or none, in some periods. The $(1-\lambda)$ fraction of loyal consumers never stockpiles. They decide whether to buy their favorite brand based on its price. Thus, for this fraction of loyal consumers of brand i who, at their turn, must choose whether to buy, the information set is $\{p_i\} \in \mathbb{R}_+$.

The fraction λ of the loyal consumers of each brand may stockpile for the next period if the price of their preferred brand is sufficiently low.[1] In our model consumers can stockpile for the next period.[2] Specifically, consumers can hold two units in pantry: one for current period and one for future consumption. We define a state variable $s = \{I_i, I_{3-i}\}$, where I_i for $i = 1, 2$, is the inventory of stockpiling consumers of brand i. This leads to four possible states: $s \in \{00, 01, 10, 11\}$.[3] The state s = 00 corresponds to the case in which stockpiling loyal consumers of neither firm have inventory. The state s = 01 (s = 10) corresponds to the case in which stockpiling loyal consumers of the focal (competing) firm do not have inventory but those of the competing (focal) firm have inventory. Finally, s = 11 corresponds to the case where stockpiling loyal consumers of both firms have inventory. We assume that stockpiling consumers of firm i in our model decide to stockpile whenever they encounter a price $p_i < t$.[4] So the relevant information set of stockpiling loyal consumers of brand i is $\{p_i\} \in \mathbb{R}_+$. These consumers decide by comparing the price of the brand they are loyal to with the stockpiling threshold. Note that we have assumed that stockpiling is independent of inventory. But, of course, the quantity bought depends on the inventory state. With no inventory, a consumer can contemplate buying one or two units, or none at all. With inventory, they would decide between buying one unit or no unit. In other words, their decision conditional on buying one or two units can be thought of as deciding whether to stockpile or not.

2.2. Firms

Firms in our model maximize expected profits discounted over an infinite horizon. Firms are assumed to discount the future using a discount factor δ_f, $0 < \delta_f < 1$. They choose prices simultaneously in each period. Without loss of generality, we normalize the marginal cost of both firms to 0, so per-period profits are simply the product of demand and price. The switching consumers and the nonstockpiling loyal consumers demand one unit each period. However, the stockpiling loyal consumers' demand can vary. In particular, if stockpiling loyal consumers have inventory, they demand zero units at high prices and one unit at low prices, and if they do not have inventory, they demand one unit at high prices and two units at low prices. Therefore, firms' pricing strategies would depend on the state of stockpiling consumers' inventory.

We should note that all payoff-relevant past information for firms in our model is captured by the inventory held by stockpiling consumers of each brand. Hence, the relevant information set for firms is s. Moreover, the information set is a singleton, so firms have complete information.

2.3. Equilibrium

We invoke a Markov perfect equilibrium (MPE) to characterize the firms' optimal pricing strategies that maximize expected profits of the firms given consumers' stockpiling decision. We focus on the expected profits in the stationary state. In other words, we derive firms' equilibrium pricing strategies conditional on state recognizing that continuation profits of firms depend on the equilibrium strategies in the next period. Our equilibrium is subgame perfect, ruling out non-Markovian deviations by firms (Maskin and Tirole 1988, 2001). Maskin and Tirole offered several reasons for invoking an MPE as a desirable equilibrium concept. Moreover, Markov strategies often have more predictive power compared with strategies that rely on punishment, because punishment strategies can have multiple equilibria. MPE has also been used for analyzing marketing strategies—for example, in Villas-Boas (1993) and Anderson and Kumar (2007). Finally, in our model, the payoff-relevant state variable is low dimensional and so the model remains tractable. Firms' MPE strategies are then defined as follows: firms take the current state as given and set prices simultaneously, $\sigma_f : s \in \{00, 01, 10, 11\} \to \mathbb{R}_+$, anticipating consumer strategies—switching consumers' strategies σ_1, nonstockpiling loyal consumers' strategies σ_2, and stockpiling loyal consumers' strategies σ_3.

Recall that switching consumers decide whether to buy and, if so, which brand to buy; nonstockpiling consumers decide whether to buy; and stockpiling consumers decide whether or not to stockpile. Thus, consumer strategies are $\sigma_1 : \mathbb{R}_+^2 \to \{0,1\} \times \{i, 3-i\},$, $\sigma_2 : \mathbb{R}_+ \to \{0,1\}$, and $\sigma_3 : \mathbb{R}_+ \to \{\text{Don't Stockpile, Stockpile}\}$.

We can view the multiperiod game as having two stages in each period with firms choosing prices in stage 1 and consumers buying decisions in stage 2. Consumers' decisions determine the state in stage 1 of the next period, and firms' strategies are state dependent. Thus, the game proceeds as follows: In some period n, in stage 2, loyal consumers of firm i observe price of firm i. Switching consumers observe prices of both firms. Consumers then act according to σ_1, σ_2, and σ_3. In period $n + 1$ firms, after observing consumers' action in period n, know the state s_{n+1} and so have complete information and choose their strategies.

Somewhat foreseeing the analysis to follow, we note that equilibrium pricing turns out to be in mixed strategies. The reason for this in our model, as in Narasimhan's model, is the discontinuity in demand that results from the behavior of switching consumers. Because switchers buy from the lowest-priced firm, a firm has an incentive to undercut its competitor to win all switchers and improve profit. No symmetric price pair is sustainable in the equilibrium, including the lowest price at which a firm would instead charge a reservation price and serve only loyal consumers to earn higher profits. However, if one firm charges a reservation price, another firm would also increase its price close to the reservation price to improve profits. Consequently, there is no equilibrium in pure strategies. What is novel in our work is these mixed strategies being state dependent. Next we consider a two-period version of our model that serves to illustrate the equilibrium and yields insights into firms' strategies. It will also help us to analyze the infinite period model.

3. Equilibrium in a Two-Period Model

We characterize the equilibrium working backward. Let us examine period 2, when consumers do not stockpile. Depending on the consumers' actions in the first period, based on the prices in the first period, the state s in period 2 could be one of four: $s_2 \in \{00, 01, 10, 11\}$. What distinguishes each of the states is the size of the loyal segment of one or both firms. Viewed in the framework of Narasimhan's model, the segment sizes and equilibrium profits in period 2 are displayed in Table 1. Subscript n in $\pi_{i,n}^{s_n}$ identifies period, and subscript i identifies firm and $M_{i,2}^{01} = M_{3-i,2}^{10} \triangleq \bar{M} = \lambda\alpha/(\alpha+\beta)$.

3.1. Demand and Profits with Loyal Consumer Stockpiling

First, consider, for example, the focal firm's demand, D_i^{00}, in state $s = 00$. The demand is given by

$$D_i^{00}(p_i, p_j, t) = \alpha + \lambda\alpha 1^+(t - p_i) + \beta 1^+(p_j - p_i),$$

$$\text{where (indicator function), } 1^+(z) = \begin{Bmatrix} 1 & z > 0 \\ 0 & z < 0 \end{Bmatrix}.$$

The three components of demand can be understood by noting that because the focal firm's stockpiling consumers have no inventory, all loyal consumers of proportion α will buy at least one unit, when the price is below their reservation price, r. Furthermore, if the price of the focal firm is less than t, all stockpiling loyal consumers (a fraction $\lambda\alpha$) will buy an additional unit. Finally, if the focal firm's price is less than the competitor's price, switchers of size β will also buy one unit from the focal firm (third term on right-hand side in the preceding equation).

There are two cases that require special consideration: (a) when both firms charge the same price and (b) when the focal firm charges a price equal to the stockpiling threshold. The first case, ($p_i = p_j$), arises only if both firms charge a certain price with positive probability. In that case, we assume that the switching segment buys from the focal firm with probability $\phi_i = 0.5$, and it buys from the competing firm with probability $(1 - \phi_i)$.[5] In the second case, ($p_i = t$), stockpiling consumers of brand i are indifferent between stockpiling and not stockpiling. In this case, we suppose that the stockpiling loyal segment of firm i decides to stockpile with probability $\eta_i = 0.5$. Equation (1) represents the demand taking into account all possibilities including $p_i = p_j$ and $p_i = t$:

$$D_i^s(p_i, p_j, t) = \begin{Bmatrix} \kappa + \lambda\alpha.1^+(t - p_i) & \\ \quad +\beta.1^+(p_j - p_i), & p_i \neq p_j, p_i \neq t \\ \kappa + \lambda\alpha.1^+(t - p_i) + \beta/2 & p_i = p_j, p_i \neq t \\ \kappa + \lambda\alpha/2 + \beta.1^+(p_j - p_i), & p_i \neq p_j, p_i = t \\ \kappa + \lambda\alpha/2 + \beta/2 & p_i = p_j = t \end{Bmatrix}, \tag{1}$$

$$\text{where } \kappa = \begin{Bmatrix} \alpha & \text{if state } s \in \{00,01\} \\ (1-\lambda)\alpha & \text{if state } s \in \{10,11\} \end{Bmatrix} \text{ and}$$

$$1^+(z) = \begin{Bmatrix} 1 & z > 0 \\ 0 & z < 0 \end{Bmatrix}.$$

Denote by $\pi_i^s(p_i) = p_i D_i^s(p_i, p_j, t)$ profits of firm i in state s when its price is p_i. We next derive the equilibrium mixed strategies in period 1, denoted by $F_i^s(p)$. Of course, in period 1 the initial state could also be one of four: $s \in \{00, 01, 10, 11\}$. Let us consider the state 00.

3.2. Period 1 Mixed Strategies in Symmetric State 00

First, we examine the symmetric mixed strategy equilibrium of period 1 in state 00. Denote the lower bound of the support of prices by l^{00}. Then, the profits of firm i evaluated at price $p_i \in [l^{00}, t)$ given state s, consist of the sum of current-period profits $\pi_{i,1}^s(p_i)$ and the discounted expected continuation profits in period 2 (see Table 1).[6] This is given by

$$p_i(\alpha(1+\lambda) + \beta(1 - F_{3-i}^{00}(p_i))) + \delta_f(\pi_{i,2}^{11} F_{3-i}^{00}(t) + \pi_{i,2}^{10}(1 - F_{3-i}^{00}(t))), \quad p_i \in [l^{00}, t).$$

Note that as firm i's price, $p_i \in [l^{00}, t)$ increases, current profits from the loyal consumers of the firm increase monotonically, and moreover, the expected continuation profits remain constant because the choice of p_i

Table 1. Period 2 (Last-Period) State-Dependent Demand and Profits

	Segment sizes			Period 2 profits	
	Loyal segments		Switching segment		
Period 2 state, s_2	Firm 1, α_1	Firm 2, α_2	β	Firm 1, $\pi_{1,2}^{s_2}$	Firm 2, $\pi_{2,2}^{s_2}$
00	α	α	$\beta = 1 - 2\alpha$	$\alpha\, r$	$\alpha\, r$
11	$\alpha(1-\lambda)$	$\alpha(1-\lambda)$	β	$\alpha(1-\lambda)r$	$\alpha(1-\lambda)r$
01	α	$\alpha(1-\lambda)$	β	αr	$\alpha(1-\lambda)r + \bar{M}\beta r$
10	$\alpha(1-\lambda)$	α	β	$\alpha(1-\lambda)r + \bar{M}\beta r$	$\alpha\, r$

has no effect on it. So for the equiprofit condition to hold, the profits from the switching consumers must decrease, caused by a decrease in $(1 - F^{00}_{3-i}(p_i))$, the probability of the competitor firm $(3 - i)$'s price being higher than the focal firm's price, p_i. We can make this precise by differentiating the above-mentioned equation and setting it to 0 to obtain the differential equation

$$p_i f^{00}_{3-i}(p) + F^{00}_{3-i}(p) = \frac{\alpha(1+\lambda)+\beta}{\beta}.$$

Solving for $F^{00}_{3-i}(p)$, and imposing the boundary condition $F^{00}_{3-i}(l^{00}) = 0$, we get

$$F^{00}_{3-i}(p) = \frac{\alpha(1+\lambda)+\beta}{\beta}\left(1 - \frac{l^{00}}{p}\right), \quad p \in [l^{00}, t). \tag{2}$$

Note that l^{00} can be determined in one of two ways. If the equilibrium strategies are such that r is dominated, then all prices $p, p \in (t, r]$, are also dominated. In this case, l^{00} solves $F^{00}_{3-i}(t) = 1$, and so the two-period cumulative discounted profits are $t\alpha(1+\lambda) + \delta_f \alpha r \times (1-\lambda)$.[7] If, on the other hand, r is not dominated, then l^{00} would be determined by the boundary condition that the cumulative distribution function of prices of firm $(3 - i)$ attains 1 at r. In that case the profits to firm i from choosing r in period 1 are given by $r\alpha + \delta_f \alpha r$. Therefore, a sufficient condition for r to be undominated in this case is

$$t < r\frac{1+\delta_f\lambda}{1+\lambda}.$$

3.2.1. Discontinuity in Mixed Strategies at Stockpiling Threshold *t*.

Equation (2) holds for $p < t$. But when $p = t$, current-period profits from loyal consumers are no longer increasing monotonically. In fact, when the price increases from t^- to t, two things happen, both arising from the behavior of the stockpiling consumers. Recall that at $p = t^-$, stockpiling consumers decide to stockpile with probability 1, whereas at $p = t$, they do so only with probability $\eta_i = 1/2$. The first consequence of this is that current profits decrease by $\frac{1}{2}\lambda\alpha t$. The second consequence is that the future profits remain unaffected only with probability 1/2, corresponding to stockpiling by consumers. When they do not stockpile, the continuation profits correspond to states 00 and 01: $\pi^{00}_{i,2}$ and $\pi^{01}_{i,2}$ are both are equal to αr (see Table 1).

Had consumers stockpiled, profits would have been $\pi^{10}_{i,2}$ and $\pi^{11}_{i,2}$. Also at t, we must entertain also the possibility that firm $(3 - i)$ has a mass point too. Denote this mass point by $m^{00}_{3-i} \geq 0$, and we assume that when the price of firm $(3 - i)$ is t, its consumers stockpile with probability $\eta_{3-i} = 1/2$. Then, the continuation profits $\pi^{00}_{i,2}$ and $\pi^{10}_{i,2}$ occur with probability $\frac{1}{2}\left(1 - F^{00}_{3-i}(t^-) - \frac{1}{2}m^{00}_{3-i}\right)$ and $\pi^{01}_{i,2}$ and $\pi^{11}_{i,2}$ with probability $\frac{1}{2}\left(F^{00}_{3-i}(t^-) + \frac{1}{2}m^{00}_{3-i}\right)$. We can now invoke the equiprofit condition at t^- and t. The profit at t^- is

$$\begin{aligned}&t^-(\alpha(1+\lambda) + \beta(1 - F^{00}_{3-i}(t^-)))\\&\quad + \delta_f r\left(\left(1 - F^{00}_{3-i}(t^-) - \frac{1}{2}m^{00}_{3-i}\right)(\alpha(1-\lambda) + \bar{M}\beta)\right.\\&\quad \left.+\left(F^{00}_{3-i}(t^-) + \frac{1}{2}m^{00}_{3-i}\right)\alpha(1-\lambda)\right).\end{aligned}$$

Similarly, the profit at t is[8]

$$\begin{aligned}&t\left(\alpha\left(1+\frac{1}{2}\lambda\right) + \beta\left(1 - F^{00}_{3-i}(t) - \frac{1}{2}m^{00}_{3-i}\right)\right)\\&+\delta_f r\left[\frac{1}{2}\left(\left(1 - F^{00}_{3-i}(t^-) - \frac{1}{2}m^{00}_{3-i}\right)(\alpha(1-\lambda) + \bar{M}\beta)\right.\right.\\&+\left.\left(F^{00}_{3-i}(t^-) + \frac{1}{2}m^{00}_{3-i}\right)\alpha(1-\lambda)\right)\\&+\left.\frac{1}{2}\left(\left(1 - F^{00}_{3-i}(t^-) - \frac{1}{2}m^{00}_{3-i}\right)\alpha + \left(F^{00}_{3-i}(t^-) + \frac{1}{2}m^{00}_{3-i}\right)\alpha\right)\right].\end{aligned}$$

Equating the profits at t^- and t and simplifying, we get

$$\begin{aligned}\frac{1}{2}t\beta m^{00}_{3-i} &= \frac{1}{2}\left(-t\alpha\lambda + \delta_f r\left(\alpha\lambda - \left(1 - F^{00}_{3-i}(t^-)\right.\right.\right.\\&\qquad \left.\left.\left.- \frac{1}{2}m^{00}_{3-i}\right)\bar{M}\beta\right)\right)\end{aligned}$$

or

$$\begin{aligned}m^{00}_{3-i}\beta t = \alpha\lambda(-t + \delta_f r) - \delta_f r(1 - F^{00}_{3-i}(t^-))\bar{M}\beta\\ + \frac{1}{2}\delta_f r\bar{M}m^{00}_{3-i}\beta.\end{aligned}$$

In the foregoing equation, as firm i's price goes from t^- to t, the left-hand side is the loss in profits from switching consumers, and the right-hand side is the net gain from continuation profits after accounting for loss in profits from stockpiling consumers. Define

$$\Delta^{00}_{3-i} = \alpha\lambda(-t + \delta_f r) - \delta_f r(1 - F^{00}_{3-i}(t^-))\bar{M}\beta.$$

We can see that the equiprofit condition is satisfied with mass point $m^{00}_{3-i} = 0$ if $\Delta^{00}_{3-i} = 0$. If $\Delta^{00}_{3-i} < 0$, and we set $m^{00}_{3-i} = 0$, then the profit at t is less than the profit at t^-, and so in this case, the probability density function $f^{00}_{3-i}(p) = 0$, $p \in [t, h^{00})$, with profit in this interval being lower than the profit at t^-. We call the interval $[t, h^{00})$ a hole. Note that the current-period profit from loyal consumers increases over the interval $[t, h^{00})$; profits from switching consumers stay constant because $f_{3-i}^{00}(p) = 0$ and so h^{00} solves

$$\alpha(h^{00} - t) + \Delta_{3-i}^{00} = 0, m_{3-i}^{00} = 0, \text{ if } \Delta_{3-i}^{00} < 0. \quad (3)$$

Turning to the case $\Delta_{3-i}^{00} > 0$, the equiprofit condition is satisfied if there is a mass point m_{3-i}^{00} at t:[9] Now if $\Delta_{3-i}^{00} > 0$, for the equiprofit condition to hold as the price goes from t^- to t, there must be a jump in $F_{3-i}^{00}(p)$ at $p = t$ so that expected profits from the switching segment decrease to compensate for any increased profits from loyal consumers and continuation profits. We can see that if $\Delta_{3-i}^{00} > 0$, then there is a mass point at t in firm $(3-i)$'s mixed strategy:

$$m_{3-i}^{00} = \frac{\Delta_{3-i}^{00}}{\beta\left(t - \frac{1}{2}\delta_f \bar{M} r\right)} > 0. \quad (4)$$

Thus, in general, the mixed strategy in state 00 contains either a mass point or a hole but not both. We can complete its characterization by solving for the strategy over $(h^{00}, r]$, $h^{00} \geq t$, to be

$$F_{3-i}^{00}(p) = \frac{\alpha + \beta}{\beta}\left(1 - \frac{h^{00}}{p}\right) + \frac{h^{00}}{p}\left(F_{3-i}^{00}(t^-) + m_{3-i}^{00}\right), \quad p \in \left[h^{00}, r\right). \quad (5)$$

The mixed strategy in state 00 is completely characterized by (2) and (5) if l^{00}, m_{3-i}^{00}, and h^{00} are known. In the symmetric state 00, there is no mass point at r in the mixed strategy, and so we have the boundary condition $F_{3-i}^{00}(r) = 1$. Along with this boundary condition and the second-period solution, we can use (3) and (4) to solve for l^{00}, m_{3-i}^{00}, and h^{00}.[10] Similarly, we can derive mixed strategies in states 11, 01, and 10 by following the procedure outlined for state 00 in Section 3.2. Please see the appendix for more details. Having derived the mixed strategies, we can now derive the value functions in period 1.

3.3. Period 1 Value Functions

Denote $V_{i,1}^{s_1}$ to be the value of firm i in the first period, given state s_1. Then the value function in period 1 can be written as the sum of period 1 profits, as a function of the current-period prices and state, and the continuation profit as a function of the state in period 2:

$$V_{i,1}^{s_1} = \pi_{i,1}^{s_1}(p_i; p_{3-i}|s_1) + \delta_f E_{s_2}(\pi_{i,2}^{s_2}).$$

To evaluate the state-dependent value functions, we set $p_i = r$. We know that in light of symmetry, $F_{3-i}^{00}(p)$ and $F_{3-i}^{11}(p)$ do not have mass points at $p = r$, and from Lemma A.1 (see the appendix), it is also true of $F_{3-i}^{01}(p)$ but not for $F_{3-i}^{10}(p)$.[11] So the current-period profits corresponding to the four possible states, are (see demand equation (1))

$$\pi_{i,1}^{s_1}(r) = \begin{cases} \alpha r & s_1 = 00, \\ \alpha(1-\lambda)r & s_1 = 11, \\ \alpha r & s_1 = 01, \\ \alpha(1-\lambda)r + M_{i,1}^{01}\beta r & s_1 = 10. \end{cases}$$

Moreover, if $p_i = r$, the future state s_2 is

$$(s_2|p_i = r; s_1 = 00) = \begin{cases} 00 & \text{with prob. } \left(1 - F_{3-i}^{00}(t) + \dfrac{m_{3-i}^{00}}{2}\right), \\ 01 & \text{with prob. } \left(F_{3-i}^{00}(t) - \dfrac{m_{3-i}^{00}}{2}\right); \end{cases}$$

$$(s_2|p_i = r; s_1 = 11) = \begin{cases} 00 & \text{with prob. } \left(1 - F_{3-i}^{11}(t) + \dfrac{m_{3-i}^{11}}{2}\right), \\ 01 & \text{with prob. } \left(F_{3-i}^{11}(t) - \dfrac{m_{3-i}^{11}}{2}\right); \end{cases}$$

$$(s_2|p_i = r; s_1 = 01) = \begin{cases} 00 & \text{with prob. } \left(1 - F_{3-i}^{01}(t) + \dfrac{m_{3-i}^{01}}{2}\right), \\ 01 & \text{with prob. } \left(F_{3-i}^{01}(t) - \dfrac{m_{3-i}^{01}}{2}\right); \end{cases}$$

$$(s_2|p_i = r; s_1 = 10) = \begin{cases} 00 & \text{with prob. } \left(1 - F_{3-i}^{10}(t) + \dfrac{m_{3-i}^{10}}{2}\right), \\ 01 & \text{with prob. } \left(F_{3-i}^{10}(t) - \dfrac{m_{3-i}^{10}}{2}\right). \end{cases}$$

From Table 1, we know that corresponding to the second-period states 00 and 01, continuation profits are $\pi_{i,2}^{00} = \pi_{i,2}^{01} = \alpha r$. Therefore, the values in period 1 are

$$V_{i,1}^{s_1} = \begin{cases} \alpha r + \delta_f \alpha r = (1+\delta_f)\alpha r & s_1 = 00, \\ \alpha(1-\lambda)r + \delta_f \alpha r = (1+\delta_f)\alpha r - \lambda \alpha r & s_1 = 11, \\ \alpha r + \delta_f \alpha r = (1+\delta_f)\alpha r & s_1 = 01, \\ \alpha(1-\lambda)r + M_{i,1}^{01}\beta r + \delta_f \alpha r & \\ \quad = (1+\delta_f)\alpha r - \lambda \alpha r + M_{i,1}^{01}\beta r & s_1 = 10. \end{cases}$$

We summarize our result on value functions in Lemma 1, stated without proof.

Lemma 1. The value in the first period, corresponding to the sum of discounted profits over the two periods, is given by $V_{i,1}^{00} = V_{i,1}^{01} = (1+\delta_f)\alpha r$, $V_{i,1}^{11} = V_{i,1}^{00} - \lambda \alpha r$, and $V_{i,1}^{10} = V_{i,1}^{11} + M_{i,1}^{01}\beta r$.

Proof. The proof has been omitted.

What is readily apparent is that firms' profits are lowest when stockpiling consumers of both firms have inventory, corresponding to state 11. This can be thought of as the adverse effect of past stockpiling. On the other hand, if the focal firm's consumers have no inventory, corresponding to states 00 and 01, the profits correspond to exactly what would be realized if consumers did not have the choice to stockpile. In other words, firms can devise suitable mixed strategies to nullify any potential adverse effect of consumer stockpiling on profits. It is important to recognize that this occurs despite consumers engaging in stockpiling in equilibrium. Finally, for the focal firm, firm 1, the adverse effects of stockpiling on profits are mitigated if the competitor's consumers do not have inventory, corresponding to state 10 as opposed to state 11. This can be understood by realizing that for a competitor with a loyal segment's demand larger than that of the focal firm, the trade-off between extracting surplus from switching consumers and loyal consumers favors the loyal consumers. That, in turn, allows the focal firm to extract greater surplus from the switching consumers.

We can also try to ask managerially relevant questions by computing the equilibrium strategies for various parameter values. For example, in the presence of stockpiling, should firms offer more or fewer promotions? Also, should promotions be deeper or shallower? Finally, how do consumer segment sizes affect the equilibrium frequency of consumer stockpiling? Unfortunately, these questions are difficult to answer in a two-period framework. In a two-period model, stockpiling is followed necessarily by no stockpiling. Thus, we see, for example, that in period 1, mixed strategies may contain a mass point at the threshold t, whereas in period 2, the strategy contains no such mass point. Said differently, to fully grasp the effect of stockpiling on strategies, we should examine a multiperiod model. The equilibrium strategies in a finite multiperiod model would depend not only on states but also on the number of periods. We therefore next consider an infinite period model to characterize the stationary MPE strategies that depend only on the state.

4. Equilibrium in an Infinite Period Model

In this section we address two important issues. First, we extend the two-period model of Section 3 to an infinite period case, to then solve for the stationary Markov perfect equilibrium strategies conditional on state. Second, we characterize the endogenous threshold for consumers to stockpile given firms' pricing strategies. The road map for this consists of Proposition 1 in Section 4.1, which characterizes the value functions using an induction argument to extend the two-period results. That enables us to then sketch the derivation of the stationary equilibrium mixed strategies in Section 4.2, relegating the mathematical details to the web appendix. Following that, we derive in Proposition 2 the condition for the consumers' optimal threshold for stockpiling. Finally, in Proposition 3 we establish a sufficient condition for stockpiling to occur in equilibrium.

4.1. Value Functions

For the infinite horizon case, we can write the firm's value function in period n as follows:

$$V_i^s = \pi_i^s(p_i; p_{3-i}|s) + \delta_f E_s(V_i^s).$$

Note that these are the stationary value functions that are independent of period but depend only on state. We solve for the firm's value function by deriving the expected profits in each state, in the multistate equilibrium in Proposition 1. Specifically, we use value function in steady state to derive closed-form expressions by evaluating the equiprofit conditions in each state at $p = r$.

Proposition 1. *The firm's value functions in each state in an infinite period model are*

$$V_i^{00} = V_i^{01} = \frac{r\alpha}{1-\delta_f}, V_i^{11} = \frac{r\alpha}{1-\delta_f} - r\alpha\lambda, \text{ and } \frac{r\alpha}{1-\delta_f} - r\alpha\lambda = V_i^{11} \le V_i^{10} = \frac{r\alpha}{1-\delta_f} - r\alpha\lambda + r\beta M_i^{01}.$$

Proof. See the appendix.

Proposition 1 can be understood intuitively in this way.[12] Think of a firm that always charges r. First, consider the future periods. Because at this price the stockpiling consumers never stockpile, in all periods after the current one, the firm would always sell to α loyal consumers one unit and remain in state "0."[13] When the focal firm is in state "0," the rival does not have a mass point at r, so the focal firm always loses the switchers. Thus, in each future period, the focal firm earns $r\alpha$. Now, consider the current period. If the current state of the focal firm is "0," it also earns $r\alpha$ in the current period, so the cumulative discounted values are $V^{00} = V^{01} = \frac{\alpha r}{1-\delta_f}$.[14] If the current state is 11, the focal firm sells to $(1-\lambda)\alpha$ instead of $r\alpha$ loyal consumers and loses the switchers; therefore $r\lambda\alpha$ is subtracted from the total payoff, and $V^{11} = V^{00} - r\lambda\alpha$. If the current state is 10, the focal firm sells to fewer loyal consumers but has a higher chance of selling to the switchers because with probability M^{01}, the rival charges r.

Note that the continuation profits are equal in statess $= \{00,01\}$. It is also interesting to note that in these states, V^{00} and V^{01} do not depend on λ. This is because loyal consumers of the focal firm do not have inventory in either state. Therefore, all loyal consumers, irrespective of λ, demand one unit at r.

By contrast, in states $s = \{10,11\}$, V^{10} and V^{11} do depend on λ. This occurs because only the loyal consumers who do not have inventory purchase at r. The size of this segment is $(1-\lambda)\alpha$, which is a function of λ. This, in turn, makes V^{11} less than V^{00} and V^{10}. This is in contrast to Hong et al. (2002), where the value function is independent of the consumers' inventory state. The intuition for lower profits in state 11 relative to 10 is that in state 11, switching consumers consist of a larger fraction of demand at r relative to the fraction of loyal consumers, for both firms, which in turn intensifies competition. By contrast, in state 10, the competing firm is not very aggressive because of a relatively higher proportion of loyal consumers at r.

4.2. Equilibrium Mixed Strategies

Proposition 1 allows us to obtain closed-form solutions for the symmetric mixed strategies in the infinite horizon case. Equilibrium in mixed strategies requires that the value function be equal across all prices in the support. For example, the value function of firm i in state 00 is as follows:

$$V_i^{00} = \begin{Bmatrix} p_i[\alpha + (1 - F_{3-i}^{00}(p_i))\beta] \ + \delta_f \bar{V}_i^0 & t < p_i \le r, \\ t\left[\alpha + \frac{1}{2}\alpha\lambda + \left(1 - F_{3-i}^{00}(t) + \frac{1}{2}m_{3-i}^{00}(t)\right)\beta\right] & \\ \quad + \delta_f\left(\frac{1}{2}\bar{V}_i^1 + \frac{1}{2}\bar{V}_i^0\right) & p_i = t, \\ p_i[(1+\lambda)\alpha + (1 - F_{3-i}^{00}(p_i))\beta] + \delta_f \bar{V}_i^1 & p_i < t, \end{Bmatrix}.$$

where $\bar{V}_i^1$ and $\bar{V}_i^0$, respectively, are[15]

$$\bar{V}_i^0 = \left[\left(1 - F_{3-i}^{00}(t) + \frac{1}{2}m_{3-i}^{00}(t)\right)V_i^{00} + \left(F_{3-i}^{00}(t) - \frac{1}{2}m_{3-i}^{00}(t)\right)V_i^{01}\right]$$

$$\bar{V}_i^1 = \left[\left(1 - F_{3-i}^{00}(t) + \frac{1}{2}m_{3-i}^{00}(t)\right)V_i^{10} + \left(F_{3-i}^{00}(t) - \frac{1}{2}m_{3-i}^{00}(t)\right)V_i^{11}\right]$$

Using Proposition 1, we obtain the analytical solutions for $F_{3-i}^{00}(p|p>t)$ and $F_{3-i}^{00}(p|p<t)$ as

$$F_{3-i}^{00}(p>t) = \frac{p(1-\alpha) - r\alpha}{p(1-2\alpha)},$$

$$F_{3-i}^{00}(p<t) = \frac{\begin{array}{c}(2\delta p[1-\alpha+\alpha\lambda] - \delta t[m_{3-i}^{00}(t)(1-2\alpha) \\ +2\alpha+2\alpha\lambda])rM_i^{01} + tp(1-\alpha+\alpha\lambda) \\ -2rt\alpha(1+\delta\lambda)\end{array}}{2p(1-2\alpha)(\delta r M_i^{01} + t)}.$$

Similarly, by invoking the equiprofit conditions in the other states 11, 01, and 10, we characterize the equilibrium mixing distributions in each state (see Web Appendix Section A.1).

In what ways do our theoretical results on mixed strategies differ from those in prior work and so are novel? This becomes clear if we compare the mixed strategies in our model with those in prior work. In the two symmetric states, the mixed strategies in our model resemble those in Varian's (1980) and Narasimhan's (1988) models with one important difference. The similarity is that both firms obtain identical profits and current-period profits are simply those that loyal consumers contribute. There is no mass point at r in light of symmetry. The difference is that in our model, there could be a mass point at the threshold t. This is because the continuation profits and current-period profits are discontinuous at t. Next, turning to the asymmetric states, we find that strategies in our model are qualitatively similar to those in Narasimhan. In particular, the mixed strategy of the firm with the larger segment of loyal consumers willing to buy at r has a mass point at r. Finally, when we contrast our model with that of Hong et al. (2002), we find that in both models, mixed strategies could have a hole over an interval above t. However, the difference is that in our model, there could also be a mass point at t. Moreover, in Hong et al. (2002), the states are always symmetric across firms because only switchers stockpile, but in our model, the states can be asymmetric.

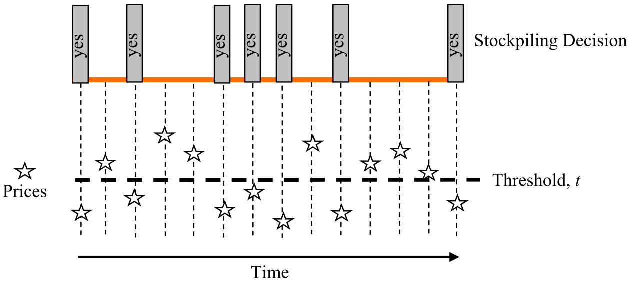
In our model, consumers are assumed to stockpile at or below the threshold t. An immediate question that arises is, how would strategic consumers decide what the threshold should be? We answer this question by explicitly deriving the optimal consumer threshold for stockpiling.

4.3. Consumer Optimal Stockpiling Threshold

Recall from Section 2.1 that stockpiling consumers decide whether to buy their favorite brand based on its price and their own stockpiling threshold t. So where does the threshold come from? We assume the consumers' stockpiling rule is a best response to the pricing strategy of their preferred brand.[16] In this way, our model is closed under rational expectations, thus providing a condition to solve for the threshold. We now turn to characterizing this threshold.

Imagine a brand-loyal stockpiling consumer who, keeping track of the prices of her favorite brand, encounters brand prices when she has inventory, $I = 1$ (or does not have inventory, $I = 0$). Figure 1 depicts this pictorially for either case. The prices allow the consumer to make inferences about two (equilibrium) price distributions. And so in our model we assume that loyal stockpiling consumers know the distributions of their brand's price conditional on their own inventory $I = \{0,1\}$, which is denoted by $G^0(p)$ and $G^1(p)$.[17] How can consumers use these distributions

Figure 1. (Color online) Prices Observed by Consumer and Their Stockpiling Decision



to characterize stockpiling threshold t? In Proposition 2, we derive the stockpiling rule that is the consumer's best response.

Because consumers, in general, may use different thresholds depending on their level of inventory, we do not restrict them to a single threshold across states.[18] However, it turns out that for consumers in our model, the optimal rule is such that the threshold is independent of the inventory state.

Proposition 2. *Stockpiling loyal consumers stockpile only if $p \le t$and do not stockpile if $p > t$, where the stockpiling threshold t is independent of the state and must satisfy*

$$t = \delta_c\left(\frac{\bar{\pi}^0(2\overline{pl}^0 - \overline{ph}^0) + \overline{ph}^0 - \bar{\pi}^1\overline{pl}^1}{1 + \delta_c(\bar{\pi}^0 - \bar{\pi}^1)}\right),$$

where $\bar{\pi}^0(\bar{\pi}^1)$ denotes the probability of stockpiling, $\overline{pl}^0$ $(\overline{pl}^1)$ denotes the expected price below the stockpiling threshold, and $\overline{ph}^0$ $(\overline{ph}^1)$ *denotes the expected price above the threshold, under inventory state $I = 0$ $(I = 1)$.*

Proof. See the appendix.

Note that the threshold t is independent of inventory state. So, regardless of the state, the stockpiling decision depends only on the current price of the brand she is loyal to and t. The equation in Proposition 2 contains an implicit solution for t. If consumers use threshold t, and they expect firms' strategies will be such as to render their choice to be optimal, then their expectations will indeed be fulfilled. Thus, our model is closed under rational expectations.

We wish to focus on an equilibrium in which consumers stockpile. Said differently, state 00 should not be an absorbing state. Thus, the threshold t should be in the support of mixed strategies. Proposition 3 delineates the sufficient conditions for this.

Proposition 3. *If $\delta_c > \delta^*$, stockpiling consumers stockpile with positive probability, where $\delta^* = \frac{1-2\alpha}{(1-\alpha)\ln\left(\frac{1-\alpha}{\alpha}\right)}$.*

Proof. See the appendix.

Proposition 3 gives the conditions to rule out an equilibrium in which the state is always 00. When the consumers' discount factor is sufficiently high, then in state 00, firms will charge prices below the stockpiling threshold with positive probability so that consumers will find it in their best interest to stockpile.

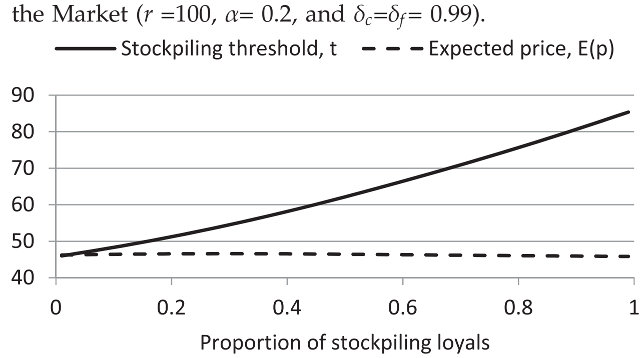
Recall that Proposition 2 provides the implicit solution for t, so to obtain sharper insights into the effect of stockpiling on the pricing strategies, we numerically solve for the endogenous threshold.[19] For example, equilibrium analysis shows that the (endogenous) stockpiling threshold increases as the proportion of stockpiling loyal consumers increases in the marketplace (see Figure 2). Although opportunistic loyal consumers start stockpiling at higher prices, the firm's equilibrium expected price does not change much with the increase in the proportion of stockpiling loyal consumers. However, firms do adjust their optimal pricing strategies, the frequency and depth of shallow and deep discounts, in such a way that the negative consequence of stockpiling is mitigated. We discuss this and our main results from infinite period analysis in detail in the next section.

5. Results and Managerial Implications

Our analysis enables us to address questions of interest on how stockpiling by loyal consumers affects firms' strategies and profits as well as consumer welfare. First, to what extent would firms' profits be reduced if their pricing and promotion strategies were agnostic to consumers' stockpiling behavior? Second, how does internalizing stockpiling behavior affect the frequency of shallow and deep promotions in equilibrium? Finally, if stockpiling benefits consumers who engage in it, how might switchers and nonstockpiling loyal consumers be affected?

Because we explicitly solve for the stationary strategies in a multiperiod setting, we can also identify intertemporal pricing patterns that stockpiling could lead to, which are distinct from those established in the extant literature. More specifically, static models of price promotions Varian (1980) and Narasimhan (1988) predict that prices charged by firms should not exhibit any

Figure 2. Expected Prices and Stockpiling Threshold for Varying Proportions of Stockpiling Loyal Consumers λ in the Market (r =100, α= 0.2, and δ_c=δ_f= 0.99).



temporal or contemporaneous correlation. In the model of Hong et al. (2002), stockpiling by price-sensitive consumers results in negative serial correlation in prices. By contrast, in our model, stockpiling by loyal consumers leads to positive serial correlation. The practical import of intertemporal pricing is captured in the state-dependent strategies that we will then elaborate on.

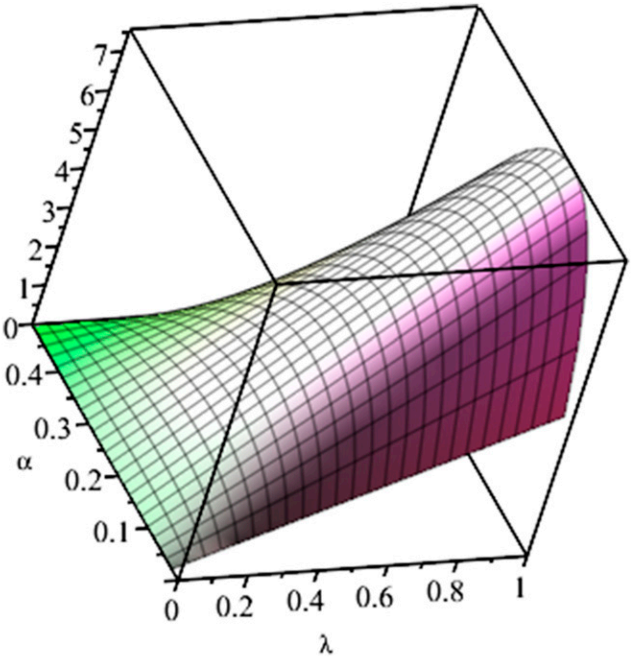
5.1. Reduction in Profits If Stockpiling Is Not Accounted For

It is obvious that not following the equilibrium strategy will lead to a loss, but how big is this loss of not accounting for loyal consumers' stockpiling behavior? We can compute the losses from not following the optimal strategies. Suppose firms do not internalize loyal consumers' stockpiling and follow the equilibrium strategy outlined in Narasimhan (1988). Some loyal consumers, however, will continue to take advantage of promotions and stockpile when prices are sufficiently low. The consumer stockpiling threshold in this case is simply the discounted expected price, but the equilibrium will still have four states depending on consumer inventories. Our analysis shows that the percentage loss from not accounting for consumer stockpiling can be significant (e.g., 7% in Figure 3), depending on proportion of loyal (α) and stockpiling (λ) consumers.

5.2. Pricing and Promotions Under Stockpiling

How different are the price distributions in a stockpiling equilibrium from those in a situation without

Figure 3. (Color online) Percentage Loss (Represented on the Vertical Axis) as a Result of Suboptimal Strategies, Not Taking into Account Loyal Consumers Stockpiling, at Various Levels of α and λ (δ_c=δ_f= 0.99)



stockpiling? To illustrate this, we compute the unconditional distribution of prices by integrating out the mixed strategies over the states. We display it in the right panel of Figure 4 and contrast it with the distribution of prices without stockpiling in the left panel.

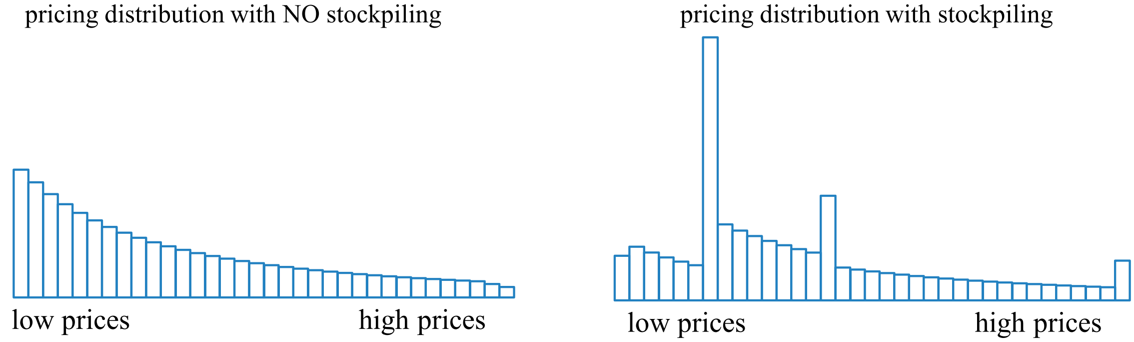
From Figure 4 we can see that in the presence of stockpiling by loyal consumers, interior modes appear, and managers move away from frequently promoting below the threshold. Moreover, there is an increase in the probability of charging the reservation price. Nevertheless, it is important to understand that to mitigate potential losses resulting from stockpiling requires changes in pricing strategies. The economic intuition for shifting the mass away from deep promotions to the interior of the support (right panel) is to price discriminate between stockpiling consumers and nonstockpiling consumers. On the one hand, offering deep promotions induces demand expansion as a result of stockpiling; on the other hand, this comes at the expense of nonstockpiling consumers also availing the reduced prices. Consequently, the frequency of deep promotions is lower.

5.3. Price Paid by Loyal and Switching Consumers

How much does stockpiling make consumers better off? Endogenizing the stockpiling threshold allows us to offer some interesting economic insights on this issue. So we begin by illustrating how the stockpiling threshold changes with respect to the size of the loyal segment (α) as well as the proportion of loyal consumers who stockpile (λ). It is useful to keep in mind that a higher α leads to higher prices, as well as the consumer threshold for stockpiling. Interestingly, the threshold is higher if λ is higher. The intuition for this finding is that if λ is higher, the expected prices in periods following stockpiling are decreasing in λ because the stock of loyal consumers purchasing at higher prices is reduced, and competition for switchers is more intense. This is a desirable state for the stockpiling loyal consumers; as a result, they are willing to stockpile at higher prices to transition into this desirable state. This illustrated in Figure 5.

We examined the difference in price that stockpiling consumers pay relative to nonstockpiling loyal consumers depending on two important parameters: size of the loyal segment α and the proportion of loyal consumers engaging in stockpiling λ. This price difference can be thought of as the benefit of stockpiling. We find that it decreases with λ, and also is lower for higher values of α, as displayed in Figure 6(a). In essence, the price paid by stockpiling consumers increases more rapidly than the price paid by loyal consumers who do not stockpile if α and λ are higher. Thus, as more consumers stockpile, the benefit decreases. This makes sense because as $\lambda \rightarrow 1$, there are no nonstockpiling loyal consumers, and the difference in price paid by stockpiling consumers and nonstockpiling

Figure 4. (Color online) Equilibrium Pricing Distribution Without (With) Consumer Stockpiling

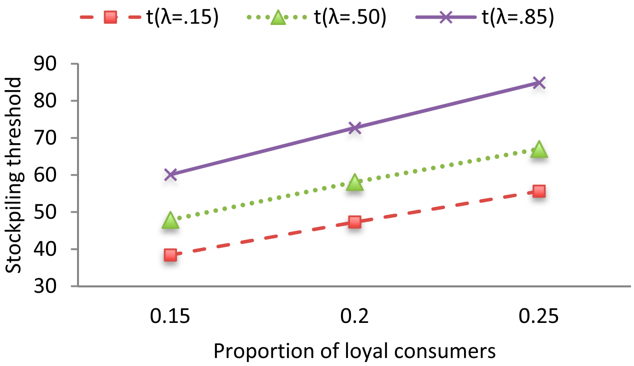


loyal consumers should converge to 0. How is the price paid by switchers affected by stockpiling? It turns out that only α affects that. That, of course, is because with high α, there are fewer switchers that firms compete for. What is more interesting is that the incidence of stockpiling λ does not affect the price paid, as seen in Figure 6(b).

5.4. Intertemporal Pricing Patterns

As noted earlier, an interesting result from our analysis is that a firm's prices exhibit positive serial correlation when loyal consumers stockpile (see examples in the web appendix). The intuition behind the positive correlation lies in the interplay of two forces. When a firm offers a low price, loyal consumers stockpile and so in the next period prices are low because the mix of loyal consumers and switchers is skewed toward switchers (higher proportion of switchers relative to loyal consumers at higher prices). Thus, when a firm does not induce stockpiling, the next-period prices are higher relative to prices after stockpiling. This result is distinct from that of Hong et al. (2002), who predicted negative temporal correlation in prices as a result of stockpiling. The reason for this is they assume that stockpiling is done by switching consumers. Our model, on the other hand, allows loyal consumers to benefit from stockpiling. What is interesting is that if we examine table A4 in Gangwar et al. (2014), showing intertemporal correlations of prices in IRI marketing data set, we find a positive correlation in such product categories as beer, diapers, soup, and sugar substitutes. These categories, on the face of it, can be expected to have more brand loyalty so that the effect of promotions is more likely to favor stockpiling by loyal consumers than by brand switching consumers. Thus, our model can possibly throw light on some of the unexplained empirical findings in Gangwar et al. (2014).

Figure 5. (Color online) Consumer Stockpiling Threshold, t

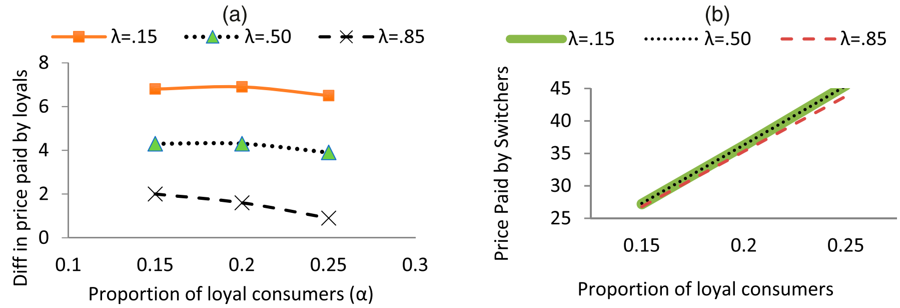


5.5. State-Dependent Pricing

We can use our model to derive managerial implications on "dynamic" pricing consisting of state-dependent pricing. Managers of consumer products with whom we have held discussions are indeed concerned with consumer stockpiling. Although they are aware of the increased demand as a result of stockpiling, they are also aware of, and unhappy with, current sales at low prices "borrowing" from future sales, possibly at higher prices. They are less clear about how to adapt their pricing strategy. For instance, if managers offer a discount that triggers stockpiling in one period, should they change their frequency and depth of discounts in the following period and, if so, how? We offer interesting and some counterintuitive prescriptions using the equilibrium strategies in our model by varying λ, the size of the consumer segment that stockpiles strategically.

The first question we pose is, if λ is higher, how should firms tailor their promotions? Said differently, what happens to prices above and below the threshold, and how often are prices below threshold? From Figure 7 we see that prices increase, but because the increase is greater for prices above the threshold relative to those below the threshold, stockpiling consumers are penalized less. Moreover, the frequency of "deep discounts" is higher, implying that the managers' goal should be not to get rid of stockpiling but to adapt prices in the face of consumers who choose the threshold strategically. Note from Figure 8 that if λ is higher, consumers find

Figure 6. (Color online) (a) Consumer Benefit from Stockpiling; and (b) Price Paid by Switchers



stockpiling more beneficial: compare the difference in expected prices following stockpiling (states 10 and 11) to not stockpiling (states 00 and 01).

Thus, although consumer stockpiling increases with λ, that is not a drawback in and of itself. What is key for managers is that the increased stockpiling occurs at higher prices. This is especially insightful, recalling a manager's reaction: "We try to cope by lopping off deeper promotions to prevent stockpiling", (personal interview). This uncertain response is resolved by the strategies that our model suggests.

The next question is this: How should a manager balance promotions, keeping in mind the twin goals of attracting switchers and checking stockpiling? In particular, should deep (shallow) promotions that (do not) induce stockpiling be followed by deep (or shallow) promotions? The answer depends not only on consumers' inventory but critically also on what the competitor did in the prior period. We can use our dynamic model to provide sharper guidelines. Conventional thinking may lead managers to raise price following deep promotions. Turning to Figure 8, consider $\lambda = 0.40$, and suppose in state 00 a discount induces stockpiling, leading to a next-period state of 10 or 11, depending on the competitor's price. If the state is 10, our model prescribes continuing to ignore stockpiling and following a strategy identical to 00. However, if the state is 11, our model recommends lower prices ($E(p|11) = 43$ versus $E(p|10) = 50$), inducing stockpiling with an even higher frequency ($Pr(p < t|11) = 0.85$ versus $Pr(p < t|10) = 0.76$). In this case, conventional wisdom fails, and the counterintuitive strategy is preferable. This result is general: we obtain similar results even when the proportion of stockpiling loyal consumers is relatively high ($\lambda = 0.80$).

Next, let us consider a shallow promotion in state 00 that induces no stockpiling, still with $\lambda = 0.40$. Should the manager offer a deep promotion in the next period? Once again, that depends on the competitor's action: a shallow or deep discount will render the next period to be 00 or 01, respectively. In state 00, our model recommends continuing the 00 strategy. However, if the state is 01, managers should avoid the knee-jerk reaction to offer a deep discount to counter the erosion in market share in the prior period. Indeed, the prudent thing is to do exactly the opposite. Said differently, it is to "lop off promotions"—something managers told us that they wish they could do but are afraid to. We would suggest that they go with their intuition on this. To understand this, note that following a deep discount in the previous period, the competitor's stock of loyal consumers it can serve at high prices decreases, and so it is in a better position to compete for switching consumers. Therefore, it should increase prices and not offer deep promotions. Because prices are strategic complements, it is prudent for the focal firm to increase the price rather than engage in a price competition with a firm that can compete

Figure 7. Left Panel: How Shallow (Deep) Discounts, $E(p|p > t)$ ($E(p|p < t)$), Change with the Proportion of Stockpiling Loyal Consumers; Right Panel: How the Frequency of Deep Discounts Changes with the Proportion of Stockpiling Loyal Consumers ($r = 100$, $\alpha = 0.2$, and $\delta_c = \delta_f = 0.99$)

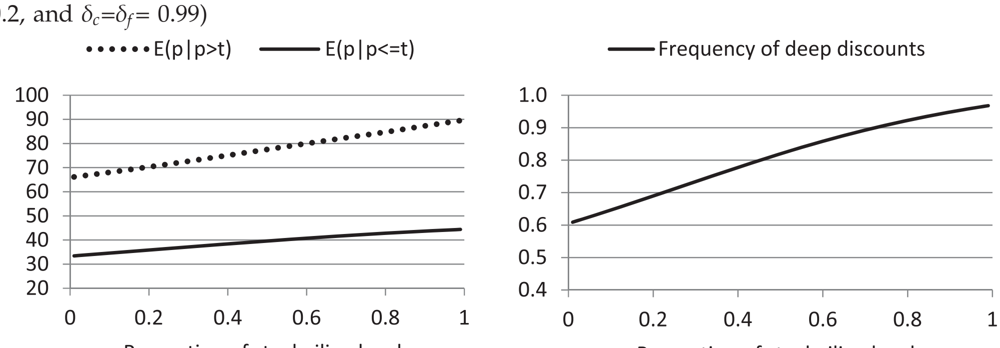
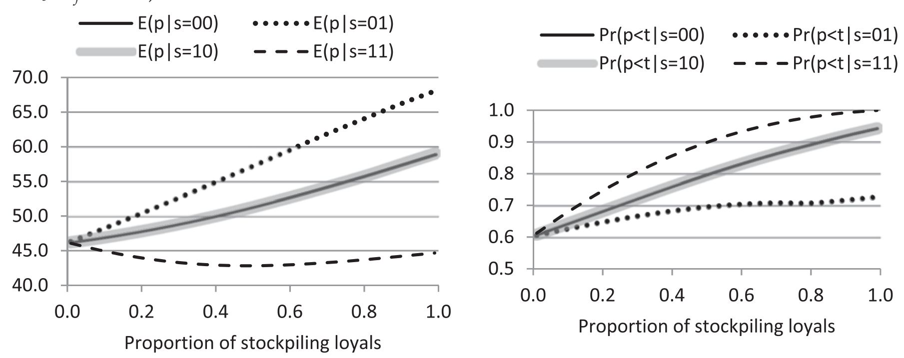


Figure 8. Left Panel: Changes in Expected Prices, E($p|s$), in Different States; Right Panel: Changes in Probability of Deep Discount in Different States, Pr(p < $t|s$), as the Proportion of Stockpiling Loyal Consumers Increases in the Marketplace (r = 100, α= 0.2, and δ_c=δ_f= 0.99)



more effectively. These results also hold for λ = 0.80 and, indeed, are general.

In nutshell, a firm's pricing strategy critically depends on the competitor's price in the prior period. If the competitor's price is high (low), it could be profitable (not) to offer deep promotions in turn, depending on consumers' inventory. Thus, our model captures the dynamics of competition in an essential way and provides sharper insights for managers as they develop their strategy.

6. Conclusions

In this paper, we have endogenized the stockpiling threshold and characterized the equilibrium promotional strategy for firms when they face stockpiling loyal consumers who are willing to stockpile if prices are sufficiently low. This provides us new economic insights into firms' promotional strategies and the effect of consumer stockpiling behavior on firms' profits.

Extant models find that stockpiling has no effect on equilibrium profits when only switching consumers stockpile. However, when brand-loyal consumers stockpile, this result may not hold. The loss could arise from the collateral effect as a result of firms competing for switching consumers. Firms induce switching by offering discounts to switching consumers, but the resulting promotion provides loyal consumers an opportunity to stockpile that leads to lower demand in future. One of our main results is that stockpiling by brand-loyal consumers results in no loss in profit, compared with a benchmark case of no stockpiling, if consumers' starting inventory is zero (state 00). By carefully modeling consumer heterogeneity in price sensitivity to purchase quantity separately from sensitivity to brand choice, we extend our understanding of the effect of consumers' opportunistic stockpiling on firms' profits. This has important implications for managers who deal with categories where consumption is relatively inelastic and brand preferences are strong. Chasing higher market share through promotions may not work in a firm's favor unless promotions are adjusted to account for stockpiling by loyal consumers.

The paper provides guidance to managers on how to adapt their promotional strategies that account for opportunistic stockpiling by loyal consumers. This is important because managers concerned with consumer stockpiling often attempt to impose restrictions to curb consumer stockpiling. These kinds of restrictions will leave some consumers unhappy. A better approach is to reoptimize promotions in such a way that the adverse effects of stockpiling are mitigated. The resulting optimal pricing strategies manifest positive intertemporal correlation in prices.

In this paper we have assumed that only loyal consumers can stockpile. In reality, switching consumers may also stockpile. So an interesting question is, how will that affect losses as a result of stockpiling? Fortunately, it turns out that we can address this question without explicitly analyzing a model in which both loyal consumers and switchers stockpile. We can invoke past research that has studied the case of stockpiling by switching consumers to offer some insights into what might happen if some fraction of all consumers stockpile. As far as profits are concerned, recall that stockpiling by switching consumers has no effect relative to the case when consumers do not stockpile as long as firms adapt their equilibrium strategies suitably (Hong et al. 2002, Gangwar et al. 2014). Recalling our discussion following Proposition 1, we can see that even when both types (loyal and switchers) of consumers stockpile, in three out of four states, equilibrium profits V^{00}, V^{01}, and V^{11} will be identical to what we find in our model; moreover, $V^{10} > V^{11}$. Thus, our main result will continue to hold.

However, the equilibrium pricing strategies will be different and will depend on whether switchers stockpile. One consequence is that stockpiling thresholds will differ across consumer types.[20] Given that switching consumers buy units at lower prices compared with loyal consumers, switching consumers can be expected to have a lower stockpiling threshold. This implies that in certain markets, depending on parameters, only loyal consumers may get the opportunity to stockpile and switching consumers may not. In that case, our results on what firms should do in equilibrium are unaffected. In markets in which switchers stockpile in equilibrium, we can point to likely outcomes. In our model, after stockpiling, deep discounts are more likely to be followed by another deep discount that induces stockpiling. By contrast, in the model where only switchers stockpile, we know that firms would raise prices after stockpiling. Given that very deep discounts may induce even switchers to stockpile, managers should modify the strategy prescribed in our model to accommodate switchers' stockpiling. Deep discounts that induce switchers also to stockpile need not be followed up by another deep discount in the next period. However, deep discounts that do not induce switchers to stockpile should still be followed by another deep discount.

Acknowledgments
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Appendix. Pricing Under Dynamic Competition When Loyal Consumers Stockpile

Period 1 Mixed Strategies in Symmetric State 11

We can now use arguments similar to the ones to characterize the mixed strategy in state 00 (see Section 3.2.1 in the main paper) to obtain the mixed strategy in state 11. Corresponding to (2), we get

$$F^{11}_{3-i}(p) = \frac{\alpha+\beta}{\beta}\left(1-\frac{l^{11}}{p}\right), \quad p\in[l^{11},t). \tag{A.1}$$

Define $\Delta^{11}_{3-i} \triangleq \alpha\lambda(-t+\delta_f r) - \delta_f r(1-F^{11}_{3-i}(t^-))\bar{M}\beta \geq 0$. Depending on whether $\Delta^{11}_{3-i}<0$ or $\Delta^{11}_{3-i}>0$, we have

$$\alpha(h^{11}-t)+\Delta^{11}_{3-i}=0 \text{ and } m^{11}_{3-i}=0, \ \text{ if } \Delta^{11}_{3-i}<0, \tag{A.2}$$

$$m^{11}_{3-i} = \frac{\Delta^{11}_{3-i}}{\beta\left(t-\frac{1}{2}\delta_f\bar{M}r\right)} > 0 \text{ and } h^{11}=t, \ \text{ if } \Delta^{11}_{3-i}\geq 0. \tag{A.3}$$

And, finally,

$$F^{11}_{3-i}(p) = \frac{(\alpha(1-\lambda)+\beta)}{\beta}\left(1-\frac{h^{11}}{p}\right) + \frac{h^{11}}{p}(F^{11}_{3-i}(t^-) + m^{11}_{3-i}) \ p\in[h^{11},r). \tag{A.4}$$

The mixed strategy in state 11 is completely characterized by (A.1) and (A.4) after solving for l^{11}, m^{11}_{3-i}, and h^{11} by using (A.2), (A.3), and the boundary condition that the mixing distribution $F^{11}_{3-i}(p)$ attain the value of 1 at $p=r$ along with period 2 results. As in the case of state 00, a sufficient condition for r to be undominated in state 11 is also derived by comparing profits and is $t<r(1-\lambda(1-\delta_f))$.

Of the two upper bounds for t, based on r being undominated in states 00 and 11, the second one is tighter, and so for r to be undominated in both states, a sufficient condition is $t<r(1-\lambda(1-\delta_f))$.

We now turn to the asymmetric states 01 and 10.

Period 1 Mixed Strategies in Asymmetric States 01 and 10

Using arguments similar to the ones used to derive $F^{00}_{3-i}(p)$ and $F^{11}_{3-i}(p)$, we can derive the following:

$$\begin{aligned} F^{01}_{3-i}(p) &= \frac{\alpha(1+\lambda)+\beta}{\beta}\left(1-\frac{l^{01}}{p}\right) > \frac{\alpha+\beta}{\beta}\left(1-\frac{l^{01}}{p}\right) \\ &= F^{10}_{3-i}(p), \quad p\in[l^{01},t) \\ \Delta^{01}_{3-i} &\triangleq \alpha\lambda(-t+\delta_f r) - \delta_f r(1-F^{01}_{3-i}(t^-))\bar{M}\beta \\ &> \alpha\lambda(-t+\delta_f r) - \delta_f r(1-F^{10}_{3-i}(t^-))\bar{M}\beta = \Delta^{10}_{3-i}. \end{aligned} \tag{A.5}$$

We can then see that the mass points at t fall into one of three cases. In the first case, if $\Delta^{10}_{3-i}>0$, then both $F^{01}_{3-i}(p)$ and $F^{10}_{3-i}(p)$ have mass points m^{01}_{3-i} and m^{10}_{3-i}, respectively, at t, and moreover, $m^{01}_{3-i} = \frac{\Delta^{01}_{3-i}}{\beta(t-\frac{1}{2}\delta_f\bar{M}r)} > \frac{\Delta^{10}_{3-i}}{\beta(t-\frac{1}{2}\delta_f\bar{M}r)} = m^{10}_{3-i} > 0$, implying $(F^{01}_{3-i}(t)=F^{01}_{3-i}(t^-)+m^{01}_{3-i}) > (F^{10}_{3-i}(t^-)+m^{10}_{3-i}=F^{10}_{3-i}(t))$. In the second case, if $\Delta^{01}_{3-i}>0>\Delta^{10}_{3-i}$, then there will be a hole in the strategy $F^{10}_{3-i}(p)$; therefore there will also be a hole over the same interval for $F^{01}_{3-i}(p)$, between $[t,h)$, and a mass point $m^{01}_{3-i} > \frac{\Delta^{01}_{3-i}}{\beta(t-\frac{1}{2}\delta_f\bar{M}r)} > 0$; so $(F^{01}_{3-i}(h)=F^{01}_{3-i}(t^-)+m^{01}_{3-i}) > (F^{10}_{3-i}(t^-)=F^{10}_{3-i}(h))$. Finally, in the third case, if $\Delta^{01}_{3-i}<0$, there will be a hole in both strategies, $F^{01}_{3-i}(p)$ and $F^{10}_{3-i}(p)$, and moreover, because the interval of the hole should be equal for both, there will be a mass point $m^{01}_{3-i}>0$; so, again, $(F^{01}_{3-i}(h)=F^{01}_{3-i}(t^-)+m^{01}_{3-i}) > (F^{10}_{3-i}(t^-)=F^{10}_{3-i}(h))$.

Therefore, in all three cases, we see that

$$m^{01}_{3-i}>0 \text{ and } F^{01}_{3-i}(h) > F^{10}_{3-i}(h), \quad h\geq t. \tag{A.6}$$

We are now in a position to completely specify the mixed strategies in states 01 and 10:

$$F^{10}_i(p) \equiv F^{01}_{3-i}(p) = \frac{\alpha(1+\lambda)+\beta}{\beta}\left(1-\frac{l}{p}\right), \ p\in[l,t), \tag{A.7}$$

$$F^{01}_i(p) \equiv F^{10}_{3-i}(p) = \frac{\alpha+\beta}{\beta}\left(1-\frac{l}{p}\right), \quad p\in[l,t), \tag{A.8}$$

$$\begin{aligned} &m^{01}_{3-i} = \frac{\Delta^{01}_{3-i}}{\beta\left(t-\frac{1}{2}\delta_f\bar{M}r\right)}, \ m^{10}_{3-i} = \frac{\Delta^{10}_{3-i}}{\beta\left(t-\frac{1}{2}\delta_f\bar{M}r\right)}, \text{ and} \\ &h=t \qquad \textit{if } 0\leq\Delta^{10}_{3-i}, \end{aligned} \tag{A.9}$$

$$m^{01}_{3-i} > m^{10}_{3-i} = 0, \text{ and } h = t-\frac{\Delta^{10}_{3-i}}{\alpha} \qquad \textit{if } \Delta^{10}_{3-i}<0, \tag{A.10}$$

$$F^{10}_i(p) \equiv F^{01}_{3-i}(p) = F^{01}_{3-i}(h) + \frac{\alpha+\beta}{\beta}\left(1-\frac{h}{p}\right), \qquad p\in[h,r), \tag{A.11}$$

$$F^{01}_i(p) \equiv F^{10}_{3-i}(p) = F^{10}_{3-i}(h) + \frac{\alpha(1-\lambda)+\beta}{\beta}\left(1-\frac{h}{p}\right), \ p\in[h,r). \tag{A.12}$$

We can solve for m^{01}_{3-i}, h and l by using (A.9), (A.10), and the boundary condition that the mixing distribution $F^{01}_{3-i}(p)$ attain the value of 1 at $p = r$ along with period 2 results. Once l is known, we can use (A.5) to obtain m^{10}_{3-i}, if needed, and use (A.12) to obtain $F^{10}_{3-i}(r^-)$ and infer from that $M^{01}_i \equiv M^{10}_{3-i} = (1 - F^{10}_{3-i}(r^-))$. To see whether r is undominated in states 01 and 10, first note from (A.7) and (A.8) that $F^{01}_{3-i}(p) > F^{10}_{3-i}(p), p \in [l, t)$. And so there are three possibilities: (i) r is dominated in both states 01 and 10; (ii) r is dominated in state 10, only for the focal firm, in which case competing firm's mixing distribution, $F^{10}_{3-i}(p)$, contains a mass point m_d at $p = r$; or (iii) r is undominated. It is easy to verify that if $t < r(1 - \lambda(1 - \delta_f))$, r is not dominated in both states 01 and 10. We can rule out the second case also, if $t(\alpha + \beta m_d) + \delta_f r(\alpha(1 - \lambda) + \beta\bar{M}) < \alpha r(1 - \lambda) + \delta_f \alpha r$, where $m_d = (1 - F^{10}_{3-i}(t)) = \frac{\alpha\lambda}{\alpha(1+\lambda)+\beta}$ because $F^{10}_i(t) = 1$. Thus, r is undominated in all states if

$$t < r\frac{1 - \lambda + \delta_f\lambda\eta_i - \delta_f\beta\overline{M}/\alpha}{1 - \lambda + \lambda\eta_i + \beta m_d/\alpha}.$$

We have derived the mixed strategies assuming that stockpiling consumers adopt a mixed strategy by choosing to stockpile with probability $\eta_i = 0.5$ when they encounter a price of t. This has force in our model because the price of t in equilibrium is chosen with probability greater than 0, as evidenced by the mass point at t in the mixed strategies. The mass points also raise the possibility that both firms can have equal prices, of t, with nonzero probability. That too has force in our model and we have assumed that switching consumers adopt a mixed strategy by choosing brands with equal probability of $\phi_i = 0.5$ when encountering equal prices.

Lemma A.1. *In the asymmetric state* 01, *in period* 1, *competing firm's pricing distribution,* $F^{01}_{3-i}(p)$ *does not a have mass point at* r. *Moreover, focal firm's pricing distribution,* $F^{01}_i(p)$ *has a mass point at* r, *denoted by* $M^{01}_i \ge 0$.

From (A.11) and (A.12), we know that

$$F^{10}_i(p) \equiv F^{01}_{3-i}(p) = F^{01}_{3-i}(h) + \frac{\alpha+\beta}{\beta}\left(1 - \frac{h}{p}\right), \qquad p \in [h, r),$$

$$F^{01}_i(p) \equiv F^{10}_{3-i}(p) = F^{10}_{3-i}(h) + \frac{\alpha(1-\lambda)+\beta}{\beta}\left(1 - \frac{h}{p}\right), \quad p \in [h, r).$$

Moreover, from (A.9) and (A.10) we know $F^{01}_{3-i}(h) > F^{10}_{3-i}(h), h \ge t$.

This implies that

$$F^{01}_{3-i}(p) > F^{10}_{3-i}(p), \quad p \in [h, r).$$

Note that $f^{01}_{3-i}(p) > f^{10}_{3-i}(p), \ p \in [h, r)$. And because at most only one firm can have a mass point at r, the desired result follows.

This completes the characterization of mixed strategies in the first period of the two-period model. The next section highlights the main results of an infinite period model; for a detailed characterization of the infinite period model, please see the web appendix.

Mixed Strategy Equilibrium in an Infinite Period Model

Proposition A.1. *The firm's value functions in each state in an infinite period model are*

$$V^{00}_i = V^{01}_i = \frac{r\alpha}{1-\delta_f}; V^{11}_i = \frac{r\alpha}{1-\delta_f} - r\alpha\lambda \textit{ and } \frac{r\alpha}{1-\delta_f} - r\alpha\lambda$$
$$= V^{11}_i \le V^{10}_i = \frac{r\alpha}{1-\delta_f} - r\alpha\lambda + r\beta M^{01}_i.$$

Proof. The proof is by induction. Consider a finite horizon problem with T periods. Denote the value functions in the (n)th period, $1 < n < T$, by $V^{00}_{i,n}$, $V^{01}_{i,n}$, $V^{10}_{i,n}$, and $V^{11}_{i,n}$. Suppose also $V^{00}_{i,n} = V^{01}_{i,n}$, $V^{11}_{i,n} < V^{10}_{i,n}$ and the mixing distribution $F^{01}_{i,n}(r^-) < 1$, so $M^{01}_{i,n} > 0$.

Now consider the situation in the n − 1th period. The value functions conditioned on state can now be evaluated:

$$\begin{aligned} V^{00}_{i,n-1} &= V^{00}_{i,n-1}(p_{i,n-1} = r) \\ &= \alpha r + \delta_f\left(\left(1 - F^{00}_{3-i,n-1}(t) + \frac{m^{00}_{3-i,n-1}}{2}\right)V^{00}_{i,n}\right. \\ &\quad \left. + \left(F^{00}_{3-i,n-1}(t) - \frac{m^{00}_{3-i,n-1}}{2}\right)V^{01}_{i,n}\right) \\ &= \alpha r + \delta_f V^{00}_{i,n} \qquad (\text{since } \mathrm{V^{00}_{i,n}} = \mathrm{V^{01}_{i,n}}). \end{aligned}$$

The current-period profit in period $n-1$ is $\pi^{01}_{i,n-1}(p_{i,n-1} = r) = \alpha r + M^{01}_{3-i,n}\beta r$. Note that because $M^{01}_{i,n} > 0$, it must be that $M^{10}_{i,n} = M^{01}_{3-i,n} = 0$. Therefore

$$\begin{aligned} V^{01}_{i,n-1} &= V^{01}_{i,n-1}(p_{i,n-1} = r) \\ &= \alpha r + \delta_f\left(\left(1 - F^{01}_{3-i,n-1}(t) + \frac{m^{01}_{3-i,n-1}}{2}\right)V^{00}_{i,n}\right. \\ &\quad \left. + \left(F^{01}_{3-i,n-1}(t) - \frac{m^{01}_{3-i,n-1}}{2}\right)V^{01}_{i,n}\right) \\ &= \alpha r + \delta_f V^{00}_{i,n} \qquad (\text{since } \mathrm{V^{00}_{i,n}} = \mathrm{V^{01}_{i,n}}). \end{aligned}$$

The foregoing implies $V^{00}_{i,n-1}(p_{i,n-1} = r) = V^{01}_{i,n-1}(p_{i,n-1} = r)$. Next, we see that

$$\begin{aligned} V^{11}_{i,n-1} &= V^{11}_{i,n-1}(p_{i,n-1} = r) \\ &= \alpha(1-\lambda)r + \delta_f\left(\left(1 - F^{11}_{3-i,n-1}(t) + \frac{m^{11}_{3-i,n-1}}{2}\right)V^{00}_{i,n}\right. \\ &\quad \left. + \left(F^{11}_{3-i,n-1}(t) - \frac{m^{11}_{3-i,n-1}}{2}\right)V^{01}_{i,n}\right) \\ &= \alpha(1-\lambda)r + \delta_f V^{00}_{i,n} \qquad (\text{since } \mathrm{V^{00}_{i,n}} = \mathrm{V^{01}_{i,n}}). \end{aligned}$$

Finally,

$$\begin{aligned} V^{10}_{i,n-1} &= V^{10}_{i,n-1}(p_{i,n-1} = r) \\ &= \alpha(1-\lambda)r + M^{10}_{3-i,n}\beta r + \delta_f\left(\left(1 - F^{10}_{3-i,n-1}(t) + \frac{m^{10}_{3-i,n-1}}{2}\right)V^{00}_{i,n}\right. \\ &\quad \left. + \left(F^{10}_{3-i,n-1}(t) - \frac{m^{10}_{3-i,n-1}}{2}\right)V^{01}_{i,n}\right) \\ &= \alpha r + \delta_f V^{00}_{i,n} \quad (\text{since } V^{00}_{i,n} = V^{01}_{i,n}) \\ &= V^{11}_{i,n-1} + M^{10}_{3-i,n}\beta r > V^{11}_{i,n-1}. \end{aligned}$$

Table A.1 Notation

Variable	Description
r	Reservation price
t	Stockpiling threshold
ϕ_i	Probability of switchers buying form focal firm when both firms offer same price
η_i	Loyal consumers stockpiling at threshold price t
N	Subscript for period n
α	Proportion of loyal consumers for each firm
$\beta = 1 - 2\alpha$	Proportion of switchers
λ	Stockpiling fraction of loyal consumer
s	Superscript for state $s = \{00, 01, 10, 11\}$
$F_i^s(p_i)$, $F^s(p)$	Cumulative pricing distribution functions
$F_i^s(t)$, $F^s(t)$	Probability that firm will charge a price below the stockpiling threshold
$V_{i,n}^s$, V_i^s, V^s	Firm's net payoff in state s
M_i^{01}, M^{01}	Mass point at r in state $s = 01$
$m_i^s(t)$, $m^s(t)$, m^s	Mass point at the stockpiling threshold t
$X_i^s = \left(F_i^s(t) - \frac{m_i^s(t)}{2}\right)$	Probability of stockpiling in state s
δ_f	Firms' discount factor
δ_c	Consumer discount factor
U_i^0,U_i^1	Stockpiling consumer's net utility from purchase when she has 0 inventory or inventory of 1, respectively
$\bar{\pi}^0$, $\bar{\pi}^1$	Probability of stockpiling by loyal consumers when she has 0 inventory or inventory of 1, respectively
$\overline{pl}^0$, $\overline{pl}^1$	Expected price of her favorite brand below the stockpiling threshold conditioned on her inventory state $I = 0$ or 1, respectively
$\overline{ph}^0$, $\overline{ph}^1$	Expected price of her favorite brand above the stockpile threshold conditioned on her inventory state $I = 0$ or 1, respectively
ψ^s	Probability of state $= s$ in equilibrium

To characterize $M_{i,n-1}^{01}$, we can proceed in a manner identical to the two-period case and show that $M_{i,n-1}^{01} > M_{i,n-1}^{10} = 0$. Thus, the value functions and the mass point in the $n-1$th period inherit the properties of the corresponding ones in the (n)th period. Indeed, by induction, these properties are inherited for all periods $m < n$.

Invoking Table 1, we can see that in the last period, in period T, $V_{i,T}^{00} = V_{i,T}^{01}$, $V_{i,T}^{11} < V_{i,T}^{10}$ and $M_{i,T}^{01} > 0$. The equilibrium for a finite horizon of arbitrary length, T, is then a sequence of strategies $\{\sigma_f(T), \sigma_1(T), \sigma_2(T), \sigma_3(T)\}$ that inherit the properties of the value functions. Moreover, because the equilibrium is Markov perfect and all strategies are therefore conditioned only on payoff-relevant states, the finite horizon equilibrium converges to a steady state as $T \to \infty$, and the infinite horizon value functions inherit the desired properties.[21]

We can solve for the values in the infinite period case by evaluating the profits at r, $V_i^{01} = \alpha r + \delta_f(X_{3-i}^{01} V_i^{01} + (1 - X_{3-i}^{01})V_i^{00})$, where X_{3-i}^{01} is the probability that the competing firm induces its loyal consumers to stockpile. Because $V_i^{00} = V_i^{01}$, we get

$$V_i^{01} = \frac{\alpha r}{1-\delta_f} = V_i^{00}.$$

Similarly, $V_i^{11} = \alpha(1-\lambda)r + \delta_f(X_{3-i}^{11} V_i^{01} + (1 - X_{3-i}^{11})V_i^{00})$, so $V_i^{11} = \frac{\alpha r}{1-\delta_f} - \lambda \alpha r$.
Finally, $V_i^{10} = \alpha(1-\lambda)r + M_i^{01}\beta r + \delta_f(X_{3-i}^{10} V_i^{01} + (1 - X_{3-i}^{10})V_i^{00})$, so $V_i^{10} = \frac{\alpha r}{1-\delta_f} - \lambda\alpha r + M_i^{01}\beta r$. Q.E.D.

Proposition A.2. *Stockpiling loyal consumers stockpile only if $p \le t$and do not stockpile if $p > t$, where the stockpiling threshold, t, is independent of the state and must satisfy*

$$t = \delta_c\left(\frac{\bar{\pi}^0(2\overline{pl}^0 - \overline{ph}^0) + \overline{ph}^0 - \bar{\pi}^1\overline{pl}^1}{1 + \delta_c(\bar{\pi}^0 - \bar{\pi}^1)}\right).$$

Proof. Define $\bar{U}^0$ and $\bar{U}^1$ to be the utilities of the loyal consumer conditioned on their corresponding inventory being $I \in \{0,1\}$. The cost of stockpiling is the payment for the extra unit, p, incurred in the current period. The benefit of stockpiling occurs in the next period. The benefit is given by the difference in continuation utilities that are discounted, $\delta_c(\bar{U}^1 - \bar{U}^0)$, where δ_c is the discount factor used by consumers. Therefore, for stockpiling to be optimal, we require $p < \delta_c(\bar{U}^1 - \bar{U}^0)$.[22] We denote $t = \delta_c(\bar{U}^1 - \bar{U}^0)$ as the stockpiling threshold. As we can see, the threshold is simply the difference in the appropriately discounted continuation utilities following a decision to either stockpile or not.

Recall that loyal consumers know the price distributions, $G^0(p)$ and $G^1(p)$, conditional on their own inventory state. Under steady-state conditions, suppose loyal consumers have no inventory, $I = 0$. Let $\bar{\pi}^0$ denote the probability that they stockpile, buy two units, and pay expected price $\overline{pl}^0$ when stockpiling. (Please refer to Web Appendix Section A.2 for how these quantities can be computed.) When they stockpile, they consume one unit in the current period and move to inventory state $I = 1$, in the next period. Conversely, with probability $(1 - \bar{\pi}^0)$, they buy only one unit at expected price of $\overline{ph}^0$ and remain in inventory state $I = 0$. Hence the steady-state utility under inventory state $I = 0$ will be

$$\bar{U}^0 = \bar{\pi}^0(r - 2\overline{pl}^0 + \delta_c \bar{U}^1) + (1 - \bar{\pi}^0)(r - \overline{ph}^0 + \delta_c \bar{U}^0).$$

Similarly, in inventory state $I = 1$with probability $\bar{\pi}^1$, they will buy one unit and remain in inventory state $I = 1$, and with probability $(1 - \bar{\pi}^1)$, they will not buy and move to inventory state$I = 0$, which yields

$$\bar{U}^1 = \bar{\pi}^1(r - \overline{pl}^1 + \delta_c \bar{U}^1) + (1 - \bar{\pi}^1)(r + \delta_c \bar{U}^0).$$

From these two equations, we solve for $t = \delta_c(\bar{U}^1 - \bar{U}^0)$to get the desired result.

Proposition A.3. *If $\delta^* < \delta_c$, stockpiling loyal consumers stockpile with positive probability, where* $\delta^* = \frac{1-2\alpha}{(1-\alpha)\ln\left(\frac{1-\alpha}{\alpha}\right)}$.

Proof. Recall that no stockpiling implies that $s = 00$ is an absorbing state. Suppose state $s = 00$ is an absorbing state. If there is no stockpiling, then every period is identical, and it is easy to characterize the mixed strategy pricing distribution following Narasimhan (1988). It is

$$F^{00}_{\text{absorb}}(p) = 1 - \frac{\alpha(r-p)}{(1-2\alpha)p}, \qquad \frac{r\alpha}{1-\alpha} \le p \le r,$$

and the expected price and the lower bound of the support of prices, respectively, are

$$E^{00}_{\text{absorb}} = \frac{r\alpha}{1-2\alpha}\ln\left(\frac{1-\alpha}{\alpha}\right) \text{ and } l^{00} = \frac{r\alpha}{1-\alpha}.$$

For $s = 00$ to be an absorbing state, stockpiling loyal consumers' thresholds satisfy $t_1^*, t_2^* < l^{00}$. We will prove that a unilateral deviation by consumers of one of the firms—say, firm 1—who use a different threshold to stockpile is unprofitable. Let such a threshold be $t_{\text{deviate}} > l^{00} > t^*$, which leads to their stockpiling when price $p_1 = l^{00} < t_{\text{deviate}}$ is unprofitable for the consumers. Consider a period in which $p_1 = l^{00}$. The next period state, given firm 1 consumers deviate to t_{deviate}, is (A.5). Firm 2's loyal consumers stay on the equilibrium path and do not stockpile. Firms would now employ the Markov perfect equilibrium mixed strategy conditional on state. It turns out that firm 1's strategy $F^{10}_{\text{deviate}}(p)$ in state (A.5) is identical to $F^{00}_{\text{absorb}}(p)$ because the equiprofit conditions for firm 2 after deviation are the same as on the equilibrium path. Therefore, in the periods following the deviation, firm 1's stockpiling loyal consumers face prices that are identical to what they would have faced had they stayed on the equilibrium path. This implies that a necessary condition for 00 to be an absorbing state is that a deviation to stockpiling in any period by firm 1 consumers should be unprofitable to them even when firm 1 charges a price of l^{00} in that period. This leads to

$$l^{00} = \frac{r\alpha}{1-\alpha} > \delta_c E^{00}_{\text{deviate}} = \delta_c E^{00}_{\text{absorb}} = \delta_c \frac{r\alpha}{1-2\alpha}\ln\left(\frac{1-\alpha}{\alpha}\right).$$

Therefore, a sufficient condition for 00 to not be an absorbing state is

$$\frac{r\alpha}{1-\alpha} < \delta_c \frac{r\alpha}{1-2\alpha}\ln\left(\frac{1-\alpha}{\alpha}\right),$$

or, equivalently,

$$\delta_c > \delta^* = \frac{1-2\alpha}{(1-\alpha)\ln\left(\frac{1-\alpha}{\alpha}\right)}.$$

Q.E.D.

Endnotes

[1] In our model there is no heterogeneity among consumers who have stockpiling capacity, and so either all of them stockpile or none does. We thank the area editor for alerting us on this.

[2] Niraj et al. (2008) and Ailawadi et. al. (2007) found that most of the purchases are concentrated at zero, one, or two units. Moreover, allowing stockpiling for only one period keeps our exposition and analysis simple and consistent with Hong et. al (2002) and Gangwar et. al. (2014).

[3] We use the state space notation using the perspective of the focal firm i. In other words, we follow the convention that the first digit of the state space corresponds to the current inventory position of stockpiling loyal consumers of the focal firm and the second digit corresponds to that of the competing firm's stockpiling loyal consumers. We use superscripts to denote the state and subscripts to denote the firm.

[4] We analyze the two-period model with an exogenously specified stockpiling rule to retain focus on characterizing the firms' strategies. The reader may naturally wonder if consumers would want to use such a stockpiling rule. In Section 4.3 we show that, indeed, such a rule is optimal, and we also derive the condition that the threshold t must satisfy. In Sections 2 and 3 we assume that t is sufficiently large so that it is in the support of the mixed strategies of firms' prices.

[5] In other words, we assume that when prices charged by the two firms are identical, switchers employ an exogenous cue to break ties and purchase from one of the two firms. This tiebreaking rule is assumed in the spirit that all switching consumers are homogeneous and make similar decisions. It is in the class of sunspot equilibria. For simplicity, we assume $\phi_i = 1/2$.

[6] For notational simplicity, we drop the first-period subscript in Sections 3.2 and 3.3 whenever they are not required (e.g., $F^{00}_{3-i,1} \triangleq F^{00}_{3-i}$). Similarly, whenever not necessary, we drop the subscript i for the firm (e.g., $F^{00}_i \triangleq F^{00}$); however, F^{00}_{3-i} remains F^{00}_{3-i}.

[7] This holds because (00) is a symmetric state and so there cannot be a mass point at t (if it is an upper bound) because of the presence of switching consumers. Although at price t consumers stockpile with probability $\eta_i = \frac{1}{2}$, we can ignore this because price t occurs with probability 0.

[8] If both firms charge t, then switching consumers buy from each firm with probability 0.5.

[9] This assumes that t is not too small—specifically, $t - \frac{1}{2}\delta_f \bar{M} r > 0$. If that were not the case, all prices $p_i < t$ would be dominated, and that is an uninteresting case. Moreover, we can verify whether this condition holds in all numerical experiments, and it does, indeed. For the two-period problem, the condition can be stated explicitly: $t > \frac{1}{2}\delta_f \frac{\alpha\lambda}{\alpha+\beta} r$.

[10] It is sufficient to ensure that the equiprofit condition holds on $\forall p \in [l^{00}, t) \cup [h^{00}, r)$ because all other prices are dominated.

[11] In the first period, competing firm's pricing distribution, $F_i^{01}(p)$ will have a mass point at r, denoted by $M_{i,1}^{01} \triangleq M_i^{01}$, that is different from second period's mass point, denoted by $M_{i,2}^{01} \triangleq \bar{M}$.

[12] We thank a reviewer for providing this sharp insight into the profits in our model.

[13] Abusing the notation a little bit, we refer to state 00 and 01 as state "0."

[14] Recall we often drop the subscript i for notational simplicity (e.g., $V_i^{00} \triangleq V^{00}$); however, V_{3-i}^{00} remains V_{3-i}^{00}.

[15] Note that $F_{3-i}^{00}(t) = F_{3-i}^{00}(t^-) + m_{3-i}^{00}(t)$.

[16] Note that for consumers who do not stockpile, the relevant information set in period n is the current price: for switchers, it is $\{p_{1,n}, p_{2,n}\}$; for loyal consumers of brand i, it is $\{p_{i,n}\}$. Stockpiling consumers loyal to brand i must anticipate future pricing by firms that depends on s_{n+1}, in turn depending on $\{p_{1,n}, p_{2,n}\}$. Thus armed only with $\{p_{i,n}\}$, they have incomplete information, and that has force in modeling stockpiling. Alternatively, one can assume that the loyal consumers' information set is $\{p_{1,n}, p_{2,n}\}$, endowing all agents with complete information. We analyzed this case also and found that the stockpiling threshold and our main results remain robust to this alternative specification; the analysis is available on request from the authors.

[17] How are $G^0(p)$ and $G^1(p)$ related to firms' pricing strategies? Because firms' strategies depend on the state characterized by the inventories of consumers of both firms, we integrate out the competing firms' pricing strategies suitably to obtain $G^0(p)$ and $G^1(p)$. Details are in Web Appendix Section A.2. The distributions $G(.)$ can be thought of as reflecting bounded rationality on the part of consumers: Simon (1972), Geigerenzer and Goldstein (1996), and Ellison (2006).

[18] This result is also because consumers in our model can only be at one of two levels of inventory. In general, if they could be at one of k levels of inventory, they could, in theory, use $k - 1$ thresholds.

[19] Please find numerical experiments in the web appendix.

[20] A formal analysis of a model with all consumers stockpiling gets complicated because of the multiple thresholds, and so we thank and agree with the senior editor that the additional costs significantly outweigh any additional insights we could obtain.

[21] It is well known in the case of repeated games that finite horizon results do not extend to the infinite horizon case because standard dynamic programming convergence results cannot be invoked, because strategies are conditioned on history that is not payoff relevant. We thank a reviewer for pointing this out.

[22] Intuitively, when current period price is lower than discounted expected price in the next period consumer should stockpile.

References

Ailawadi KL, Gedenk K, Lutzky C, Neslin SA (2007) Decomposition of the sales impact of promotion-induced stockpiling. *J. Marketing Res.* 44(3):450–467.

Anderson ET, Kumar N (2007) Price competition with repeat, loyal buyers (2007). *Quant. Marketing Econom.* 5(13):333–359.

Bell DR, Chiang J, Padmanabhan V (1999) The decomposition of promotional response: An empirical generalization. *Marketing Sci.* 18(4):504–526.

Bell DR, Iyer G, Padmanabhan V (2002) Price competition under stockpiling and flexible consumption. *J. Marketing Res.* 39(3): 292–303.

Bucklin RE, Gupta S, Siddarth S (1998) Determining segmentation in sales response across consumer purchase behaviors. *J. Marketing Res.* 35(2):189–197.

Chan T, Narasimhan C, Zhang Q (2008) Decomposing promotional effects with a dynamic structural model of flexible consumption. *J. Marketing Res.* 45(4):487–498.

Chiang J (1991) A simultaneous approach to the whether, what and how much to buy questions. *Marketing Sci.* 10(4):297–315.

Chintagunta PK (1993) Investigating purchase incidence, brand choice and purchase quantity decisions of households. *Marketing Sci.* 12(2):184–208.

Ellison G (2006) Bounded rationality in industrial organization. Blundell R, Newey WK, Persson T, eds. *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress* (Cambridge University Press, New York), 142–174.

Gangwar M, Kumar N, Rao R (2014) Consumer stockpiling and competitive promotional strategies. *Marketing Sci.* 33(1): 94–113.

Geigerenzer G, Goldstein DG (1996) Reasoning the fast and frugal way: Models of bounded rationality. *Psych. Rev.* 103(4):650–669.

Guo L, Villas-Boas JM (2007) Consumer stockpiling and price competition in differentiated markets. *J. Econom. Management Strategy* 16(4):827–858.

Heerde HJV, Gupta S, Wittink DR (2003) Is 75% of the sales promotion bump due to brand switching? No, only 33% is. *J. Marketing Res.* 40(4):481–491.

Hendel I, Lizzeri A, Roketskiy N (2014) Nonlinear pricing of storable goods. *Amer. Econom. J. Microeconom.* 6(3):1–34.

Hong P, McAfee RP, Nayyar A (2002) Equilibrium price dispersion with consumer inventories. *J. Econom. Theory* 105(2):503–517.

Krishnamurthi L, Raj SP (1991) An empirical analysis of the relationship between brand loyalty and consumer price elasticity. *Marketing Sci.* 10(2):172–183.

Maskin E, Tirole J (1988) A theory of dynamic oligopoly, II: Price competition, kinked demand curves, and Edgeworth cycles. *Econometrica* 56(3):571–599.

Maskin E, Tirole J (2001) Markov perfect equilibrium I. Observable actions. *J. Econom. Theory* 100(2):191–219.

Narasimhan C (1988) Competitive promotional strategies. *J. Bus.* 61(4):427–449.

Niraj R, Padmanabhan V, Seetharaman PB (2008) A cross-category model of households' incidence and quantity decision. *Marketing Sci.* 27(2):225–235.

Neslin AS, Henderson C, Quelch J (1985) Consumer promotions and the acceleration of product purchases. *Marketing Sci.* 4(2): 147–165.

Pauwels K, Hanssens DM, Siddarth S (2002) The long-term effects of price promotions on category incidence, brand choice, and purchase quantity. *J. Marketing Res.* 39(4):421–439.

Quelch JA (1985) How to build a product licensing program. *Harvard Bus. Rev.* 63(3):186–197.

Raju JS, Srinivasan V, Lal R (1990) The effects of brand loyalty on competitive price promotional strategies. *Management Sci.* 36(3): 276–304.

Rao R (1991) Pricing and promotions in asymmetric duopolies. *Marketing Sci.* 10(2):131–144.

Salop S, Stiglitz J (1982) The theory of sales: A simple model of equilibrium price distribution with identical agents. *Amer. Econom. Rev.* 72(5):1121–1130.

Shilony Y (1977) Mixed pricing in oligopoly. *J. Econom. Theory* 14(2):373–388.

Simon HA (1972) Theories of bounded rationality. McGuire CB, Radner R, eds. *Decision and Organization* (North-Holland, Amsterdam).

Sinitsyn M (2008a) Characterization of the support of the mixed strategy price equilibria in oligopolies with heterogeneous consumers. *Econom. Lett.* 99(2):242–245.

Sinitsyn M (2008b) Price promotions in asymmetric duopolies with heterogeneous consumers. *Management Sci.* 54(12):2081–2087.

Steenburgh TJ (2007) Measuring consumer and competitive impact with elasticity decompositions. *J. Marketing Res.* 44(4):636–646.

Su X (2010) Intertemporal pricing and consumer stockpiling. *Oper. Res.* 58(4, Part 2):1133–1147.

Sun B, Neslin SC, Srinivasan K (2003) Measuring the impact of promotions on brand switching when consumers are forward looking. *J. Marketing Res.* 40(4):389–405.

Varian HR (1980) A model of sales. *Amer. Econom. Rev.* 70(4):651–659.

Villas-Boas JM (1993) Predicting advertising pulsing policies in an oligopoly: A model and empirical test. *Marketing Sci.* 12(1):88–102.