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# Structural Workshop Paper

## Discrete-Choice Models of Consumer Demand in Marketing

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Marketing researchers have used models of consumer demand to forecast future sales, to describe and test theories of behavior, and to measure the response to marketing interventions. The basic framework typically starts from microfoundations of expected utility theory to obtain an econometric system that describes consumers' choices over available options, and to thus characterize product demand. The basic framework has been augmented significantly to account for quantity choices, to accommodate purchases of several products on a single purchase occasion (multiple discreteness and multicategory purchases), and to allow for asymmetric switching between brands across different price tiers. These extensions have enabled researchers to bring the analysis to bear on several related marketing phenomena of interest. This paper has three main objectives. The first objective is to articulate the main *goals of demand analysis*—forecasting, measurement, and testing—and to highlight several considerations associated with these goals. Our second objective is to describe the main *building blocks of individual-level demand models*. We discuss approaches built on direct and indirect utility specifications of demand systems, and we review extensions that have appeared in the marketing literature. The third objective is to explore a few *emerging directions in demand analysis*, including considering demand-side dynamics, combining purchase data with primary information, and using semiparametric and nonparametric approaches. We hope researchers new to this literature will take away a broader perspective on these models and see the potential for new directions in future research.

**Key words:** demand models; econometrics; utility functions

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You can never foretell what any one man will do, but you can say with precision what an average number will be up to. Individuals vary, but percentages remain constant. So says the statistician.

—Arthur Conan Doyle, *The Sign of Four*

### 1. Introduction

Demand analysis plays a more central role in marketing than perhaps any other field in the social sciences. Normatively, models of demand are used to forecast the effect of marketing interventions and to prescribe the implementation of better policies that increase the profits of firms or improve the welfare of consumers. Positively, models of demand are used to test theories of consumer response and to quantify the effects of marketing in competitive environments. The proliferation of data, contexts, and motivations has now resulted in large classes of demand models, differing both in their properties and in their intended use. This paper summarizes some of the recent developments

in demand analysis in marketing, focusing specifically on the goals underlying the use of these models and the microfoundations on which they are based. An important theme of the paper is to articulate the considerations that shape the nature of the model used, and we constrain the scope of the analysis using those models. Our objective is not to provide an exhaustive survey of demand models that have appeared in the marketing literature. Rather, our focus is on the building blocks of these demand models along with some ways in which the models have been augmented to study aspects of consumer purchase behavior that are of interest to marketers. We mostly focus attention on discrete-choice models of demand in posted-price environments. We also highlight the rich tradition in the field of sophisticated, individual-level models of consumer behavior, which form a foil to the recent interest in aggregate demand systems in the tradition of Berry et al. (1995). In keeping with the goals

of this issue, this paper is targeted primarily at doctoral students and at researchers new to this literature. Some understanding of utility theory and econometric issues is presumed.

We divide the rest of this paper into three broad sections, followed by a short conclusion. In the first section, our focus is on the main goals of demand analysis—forecasting, measurement, and testing. We emphasize how these goals drive the choice of a particular demand specification. In the second section, we focus on the microfoundations of discrete-choice demand and discuss the two broad approaches to building demand models—an indirect utility approach and a direct utility approach. We discuss extensions including discrete/continuous demand systems, models of “multiple discreteness,” multicategory models, and nonhomothetic demand systems that incorporate income effects. In the third section, we provide a brief look at emerging directions in demand analysis and discuss current topics of interest. These include the consideration of dynamics in demand, the recent trend toward enhancing demand models with primary data, and the interest in flexible, “nonparametric” approaches.

## 2. What Determines “Model-Form”?

One of the strengths of the marketing field is its interdisciplinary nature, incorporating ideas from microeconomics, psychology, statistics, and sociology. A consequence has been a rich proliferation of models, approaches, and philosophies for understanding consumer behavior, and in particular, for modeling consumer demand. One theme of this paper is to discuss the considerations that determine the choice of consumer demand models and to discuss which model works under what circumstance. We organize our discussion of model-form around three typical goals of demand analysis—namely, forecasting, measurement, and testing. We argue that model-form should be determined by the goal of the analysis and the feasibility of identification using available data.

Demand forecasts are important to firms for predicting future sales, inventory planning, and understanding the profit consequences of potential marketing strategies. Hence, demand systems are frequently embedded in firms’ decision-support systems, and forecasting is of interest to researchers per se. When the goal of the analysis is *forecasting demand in relatively stable environments*, the best option is a “descriptive” model. By a stable environment, we mean the counterfactual policies for which forecasts are sought have been observed in the data (or, at least, the counterfactual policies are in the neighborhood of the policies observed in the data). For example, this may include forecasting how sales will evolve in the future

if the firm continued to use the same pricing policy as it has in the data. Forecasts of sales under small changes around observed prices in the data also fit the “stable environment” description, but a radical departure in the pricing policy would not. By a descriptive model, we mean one that flexibly and parsimoniously captures the across-unit demand relationships in the data without being concerned about causality. For example, if forecasting across time is the main goal of model building, an approach in which sales are modeled as flexible functions of current sales drivers and past history—own and competing sales, prices, and other marketing-mix variables—would be termed “descriptive.” The model builder in this exercise focuses on using all variables available to the firm today to best forecast outcomes for the future. An emphasis on measuring the causal effect of history (or any other variable) on current or future outcomes is immaterial to such an exercise, and to insist on this may result in a worse-performing model.<sup>1</sup> Examples of such models for aggregate data include vector auto regression (VAR) systems (e.g., Dekimpe and Hanssens 1995), which model current sales and marketing mix as a function of past values, or Bass-type diffusion models (Bass 1969), which model current sales as a function of past cumulative sales. Examples for individual-level data include discrete-choice models incorporating “Guadagni and Little”-style functions of past purchase history (Guadagni and Little 1983). Such models perform impressively for forecasting aggregate or individual outcomes, respectively. The main concern for model building in this class of models is parsimony and an emphasis on avoiding overfitting in-sample. Overfitting has the potential to significantly impinge on the model’s out-of-sample forecasting ability. Overfitting considerations can be addressed by imposing informative structure (e.g., the notion of using theory as a Bayesian prior; see Montgomery and Rossi 1999) or via the use of model selection criteria that penalize parameter proliferation (e.g., use of marginal likelihoods or Bayes’ factors).

Descriptive models are indexed by policy-specific parameters and are unsuitable for forecasting the effects of *radically different counterfactuals* that have not been observed in the data. Intuitively, demand equations are a function of the interaction of underlying buyer behavior with a policy environment. In stable environments the demand parameters that occur from this interaction are fixed; however, in a radically different environment, the parameters cannot

<sup>1</sup> For example, a causal analysis may necessitate not using the full variation in past history on account of the fact that it is “endogenous” to current outcomes; finding exogenous sources of variation in history will help identify causality, but this will almost surely result in a poorer fit relative to a model that exploits all the variation in past history, thereby reducing forecasting ability.

be logically considered unchanged when the policy environment changes. For example, consumer beliefs about future prices change significantly when a firm moves its pricing policy from a Hi-Lo regime to an EDLP (everyday low price) regime. The *parameters* of a descriptive model of demand estimated on data from the Hi-Lo regime are functions of these beliefs; hence, these cannot logically be held fixed in forecasting the move to EDLP. The promise of “structural models,” derived from theoretical micro-foundations of consumer behavior, is built on the premise that these counterfactuals can be more credibly simulated by re-solving the model explicitly for agents’ policies given estimates of policy-invariant parameters that index primitive consumer preferences (see Chintagunta et al. 2006 and Reiss and Wolak 2007 for recent discussions). In essence, the approach involves estimating deep parameters indexing consumer behavior and then building up to a “new demand” structure under the counterfactual conditional on these primitives. In some sense, the models use theory to navigate the unknown, and in several contexts they have been shown to provide surprisingly good predictions of radically different counterfactuals and underlying primitives.<sup>2</sup> In addition, the recourse to a theory of underlying behavior implies that the model has a causal interpretation. The main concern for model building in this class of models is finding the right match among the theory, the data, and the econometric specification. This is a significantly challenging endeavor. A good structural model will need to demonstrate that the theory, combined with the chosen econometric specification, can explain key patterns in sample to convince the reader of the credibility of the reported out-of-sample predictions.

A second goal of demand analysis is *measurement*. Choosing the right demand model here depends on what is being measured. Sometimes, researchers are interested in recovering metrics from observed data that have meaning only in the context of a well-specified behavioral model. For instance, an analyst may be interested in measuring consumer welfare, or risk preferences, or compensating variation, which cannot be measured without taking a stance on the consumer’s utility. Or alternatively, the analyst may be interested in recovering primitives such as a consumer’s beliefs (in the context of a model of learning for experience goods, as in Erdem and Keane 1996) or consumer’s unobserved inventory-accumulating behavior (in the context of a stockpiling model for

storable goods, as in Erdem et al. 2003). If this is the goal, a structural model of demand that incorporates a theory of consumer choice and clarifies the consumer’s information sets, beliefs, and preferences may be required to demand data. If, on the other hand, the only goal is to measure causal effects as cleanly as possible from the data, the right model is one that imposes minimal structure. In essence, we want the data, and not the functional-form assumptions of the model, to drive the estimated effect. The ideal option then is to be able to run an experiment, where the treatment (e.g., marketing intervention) is randomly assigned to treated and control groups.

Randomization solves two problems endemic to empirical work: (a) it removes the criticism that results are driven by model structure, and (b) it provides exogenous variation to measure the causal effects of the treatment. The exogenous variation helps address the endogeneity induced by the code-termination of marketing variables with demand and by the potential targeting of marketing interventions by the firm. Although attractive, randomization is not necessarily a replacement for models. The experimental approach is often costly to run (or infeasible, in some cases) and provides only local average treatment effects. It also typically provides only summary measures (such as the mean) of the distribution of marketing treatments. For modeling distributional effects, a full model may be required, especially when treatment effects are heterogeneous. Even in laboratory settings, where randomization is less difficult to achieve, combining the experiment with a model of behavior is useful for inference of heterogeneous effects and to facilitate a less onerous and more efficient research design (e.g., Lenk et al. 1996). In the absence of an experiment, a researcher wishing to measure causal effects is forced to confront the twin challenges of identification: finding the right variation to measure the effect of interest and ensuring that the effect is driven by the data and not completely by functional-form assumptions.

Nonparametric models of demand, when combined with sources of exogenous variation, can address issues related to functional form and nonrandom variation. As examples, consider a common marketing problem of measuring the joint lift, or incremental effect, of prices and promotions. An example of an experimental approach to this problem is Fong et al. (2010), an example of a nonparametric approach to estimating these effects in the context of nontargeted promotions is Briesch et al. (2010), and an example of a nonparametric approach in the context of targeted promotions is Hartmann et al. (2011). This literature is often referred to as the “reduced-form, causal-effects” class because of its emphasis on measuring

<sup>2</sup> Some examples of studies that have assessed the external validity of predictions from structural models include Cho and Rust (2008), in the context of implementing new auto rental policies; Bajari and Hortacsu (2005), in the context of estimating bidder valuations in auctions; and Misra and Nair (2011), in the context of forecasting the effect of new incentive schemes.

causal effects with minimal assumptions. Reduced-form, causal-effects approaches are contrasted with descriptive models that measure correlations between variables and are therefore not concerned with causality. Hence, finding sources of exogenous variation is not a concern in using descriptive models for the goal of forecasting in stable environments. However, once the goals of the exercise are expanded beyond forecasting, the considerations outlined above will apply to descriptive approaches as well. Finally, both reduced-form, causal-effects and “structural” models are concerned with causality and require implicit or explicit modeling from economic or other behavioral fundamentals and an implicit or explicit theory of the data-generating process. However, under the former approach, fewer distributional and specification assumptions are required because simulating radically different counterfactuals is not a goal of the analysis.

A third goal of demand analysis is *testing*. For example, one may be interested in whether the Internet lowers prices to particular populations (e.g., Morton et al. 2003) or whether “\$9-endings” for product prices have causal demand effects (e.g., Anderson and Simester 2003). Testing is conceptually distinct from the measurement goal because it involves deciding between two or more alternative theories of the data-generating process, whereas measurement involves estimation conditional on a single theory. Models used for testing demand-side effects share features with models for measuring causal effects: incorporating as little structure as possible and incorporating data that can isolate the effects being tested. A randomized experiment is ideal. In the absence of access to an experiment, researchers typically use simple models that transparently exploit some source(s) of exogenous variation. One strategy is to provide several sensitivity analyses to convince the reader that the results are robust to the various choices made in the analysis, including the chosen functional form or the choice of exogenous instruments.

Our discussion above highlights the roles of different classes of demand models—descriptive models for forecasting in stable environments; structural models when the interest is in forecasting the effects of radically different counterfactuals; structural models and reduced-form, causal effects models for measurement; and reduced-form, causal effects models for testing. An important theme is the role of structure. Structure serves to generate parsimony in model specification and guide the specification of relationships between the various moving pieces of the model. However, as we tried to clarify above, the *extent* of the theory imposed depends on the goals of the analysis and the features of its intended use. Finally, in empirical models, theory typically specifies only a

set of relationships among variables. The quantitative strength of the relationship is indexed by parameters that will be estimated. What parameters can be estimated is constrained by the nature of the variation in the data. Thus, the available variation also constrains the scope of the structure that can be incorporated into the model. This is the *identification* problem (discussed in more detail in Misra 2011).

We close this section with a discussion on the interaction of supply-side issues in demand analysis. In equilibrium, demand, prices, advertising, and other marketing-mix allocations are codetermined, and hence, marketing factors are econometrically endogenous in empirical demand systems. Accounting for the endogeneity is critical to a credible estimation of the demand curve. The endogeneity can be addressed by finding some source of exogenous variation in the demand–supply system. In some situations, one option to address the codetermination is to impose restrictions from an assumed model of supply (e.g., weekly Bertrand pricing) into the demand estimation step. This improves efficiency and helps pin down structural parameters (e.g., Thomadsen 2005 shows how a Bertrand pricing model helps pin down substitution patterns between geographically differentiated firms, an intuition also utilized in Chan et al. 2007). The offsetting cost of the identification or efficiency gain is potential misspecification bias if the wrong supply model is assumed. Our view is that, when feasible, finding the right exogenous variation and estimating demand without imposing parametric supply-side assumptions is preferred. This does not imply that assumptions about the supply side are irrelevant: these implicitly drive the identification as they determine why a particular type of observed variation identifies a demand curve, not shifts in supply. If demand parameters thus estimated are available in a first step, the supply-side model can always be used in a second step to simulate counterfactuals and to address normative issues regarding the efficiency of alternative marketing strategies. Apart from reducing misspecification risk, this approach also has the advantage that one can test whether the assumed supply-side model is an adequate description of industry policies or whether observed behavior is suboptimal relative to the chosen supply model. A *very* strong test is to demonstrate that the assumed supply model, when simulated using the estimated demand system, can reproduce the patterns in, say, pricing or advertising actually observed in the data (e.g., a supply-side model of learning-by-doing explains the observed below-marginal-cost pricing in Benkard 2004; an advertising game with advertising threshold effects explains observed pulsing in Dubé et al. 2005; an intertemporal durable-good pricing model explains observed

declining life-cycle price paths in Nair 2007; a competitive entry game with network effects predicts observed nonrandom entry of ethanol gas retailers into regions with high ethanol automobile installed bases in Shriver 2011). A reader is more likely to be convinced by counterfactuals from a demand–supply system that has demonstrated this well. This kind of test has no power if restrictions from the supply model are used to estimate demand parameters. One exception is a situation when the exact marketing-mix allocation rule is known—then, there is no misspecification, and incorporating supply-side restrictions improves efficiency while adding little bias (e.g., Hartmann et al. 2011 exploit the exact rule firms use to target promotions to consumers).

Some may criticize this view as schizophrenic: we seem to be advocating using strong assumptions such as utility maximization for estimating consumer-level models, but not for firms; it would seem firms in competitive industries have more incentives to make economically rational decisions than consumers. Our response is that as an empirical matter, our models seem to do a better job of explaining demand data than they do supply data. The demand systems marketing empiricists routinely use fit the data well and also perform impressively out of sample. However, evidence of the reliable *out-of-sample* performance of supply-side models has been scarce in the literature (this is, indeed, an area where more empirical work would be welcome). It is also hard to reconcile static supply-side models with key features of firm-side variables: examples include the observed persistence of weekly prices (which requires solving an intertemporal price discrimination problem on the supply side with price cycles) or the nonrandom stocking of specific stock-keeping units (SKUs) across stores (which involves solving an oligopolistic, product-line choice problem with sunk costs and dynamics). Furthermore, firms often care about long-term outcomes, CEOs routinely value market share in addition to profits, and marketing managers respond to career concerns and agency issues. These suggest that marketing-mix allocation in competitive markets are complex, dynamic phenomena that may not be well approximated by simple, static models of supply. Unlike demand, which typically involves a single-agent model, credible supply-side models that can capture these phenomena are multiagent, dynamic systems encompassing multiple incentives, which are harder to test, estimate, and validate, especially given current data and computing power.

The rest of this paper is concerned largely with the specification of models of demand built on an underlying theory of consumer utility maximization. The derivation from an underlying model of utility helps clarify the role served by structure, the restrictions

imposed by the theory, and how economic models can be converted to econometric specifications by incorporating stochastic elements. In the next section, we provide a short review of the microfoundations of the workhorse discrete-choice demand systems popular in marketing. Such demand systems form the basic building blocks of understanding individual-level purchase behavior because much of micro data in marketing involve consumers choosing from a fixed set of alternatives within a category. We discuss two approaches to building up the model from first principles, one starting from the direct utility function and the other with the indirect utility function.

### 3. Marketing Models of Demand

The discussion here is motivated by five distinguishing aspects of the demand literature in marketing. First, the marketing literature emphasizes the disaggregate analysis of demand at the brand, product, or SKU levels, as this is the relevant unit of analysis for firms. At this level of disaggregation, demand at the individual consumer level is lumpy, featuring many zeros (corner solutions), and quantities purchased are discrete. Consequently, discrete-choice models of demand that accommodate the proliferation of zeros, augmented to allow for quantity choices, have flourished in marketing.<sup>3</sup> Second, since its earliest days, marketing models of demand typically accommodate differentiated products, treating both branding and attribute differences as sources of product differentiation. Third, empirical work on demand has a strong emphasis on heterogeneity, focused on uncovering differences across consumers that facilitate targeting and discrimination. As noted by Allenby and Rossi (1999), this emphasis differs from much of the econometric literature, which regards heterogeneity as nuisance parameters to be “integrated out” of the objective function. In contrast, the uncovering of heterogeneity is often the object of inference in several studies of demand. Furthermore, a robust finding across marketing data sets is the fact that observationally equivalent consumers tend to exhibit significantly differing patterns of behavior. This had led to a sustaining emphasis in marketing on

<sup>3</sup> Continuous demand systems such as the Rotterdam model (see Clements and Selvanathan 1988 and Vilcassim 1989 for marketing applications), the Stone–Geary demand system, and the AIDS model of Deaton and Muellbauer (see Israilevich 2004 for a marketing application) were developed early in economics for analyzing broad aggregates or classes of goods, such as food, clothing, and shelter. These methods have been adapted to analyze aggregate store- or market-level data in the marketing literature. A potential concern with using this approach is parameter proliferation when handling large numbers of products. The models of Pinkse et al. (2002) and Davis and Ribeiro (2010) provide more parsimonious approaches to these types of models.

allowing for unobserved heterogeneity. The demand literature in marketing thus leads in the development and adoption of methods for parsimoniously and flexibly accommodating heterogeneity. We will not separately address the issue of unobserved heterogeneity in this paper. Rather, the formulations of all the models we discuss will account for such heterogeneity. Fourth, individual consumer-level demand analysis in marketing has its roots in structural approaches, with a tight link to economic theory; structural work on demand is thus flourishing in the field. This emphasis on microfoundations contrasts with closely related fields of management science such as operations, which often models demand as the outcome of an exogenous arrival process (there is recent active work in operations research on consumer-driven demand systems; see, e.g., Netessine and Tang 2009). Fifth, because of strong linkages with firms, researchers have been fortunate to have access to rich demand data, containing individual-level purchase information that is linked with data on price and promotional variation.<sup>4</sup> Whereas the marketing literature has made significant contributions to the modeling of aggregate demand (see, e.g., the early work by Horsky and Nelson 1992), access to individual-level panel data sets implies an emphasis on rich consumer- or household-level demand specifications. In contrast, broadly speaking, related fields such as industrial organization have typically focused on demand systems for aggregate data.

To emphasize these five features, much of the work we review below has a strong link to an underlying theory of consumer optimization, involves discrete choice models of different products, accommodates the informational content of quantity choices, allows for rich observed and unobserved heterogeneity, and utilizes individual-level data. We point the reader to the reviews by Dubé et al. (2002), Reiss and Wolak (2007), and Akerberg et al. (2007) for more on models of aggregate demand.

### 3.1. Microfoundations: Two Approaches

Utility-theoretic approaches to demand analysis in marketing fall into two related but distinct categories. The first involves deriving demand from the specification of an indirect utility function, which by Roy's identity (Roy 1947) yields Marshallian demand functions. The other approach starts with the direct utility function and derives demand from the optimality or Karush–Kuhn–Tucker (KKT) conditions associated with the maximization problem. We discuss both approaches in sequence, focusing on the main

workhorse approaches in both literatures. Our goal is to also clarify the consequences of assumptions about preferences so that reasonable restrictions can be used to generate specifications for empirical work while unreasonable ones can be avoided.

**Basic Setup:** The goal of the theory is to describe demand for a basket of goods,  $\mathbf{x} = (x_1, \dots, x_J)$ , when facing prices,  $\mathbf{p} = (p_1, \dots, p_J)$ . It is typical to also include an outside good,  $z$ . The outside good represents that part of total income,  $y$ , spent on all goods other than the  $J$  inside goods. Econometrically, specification of the outside good in the demand function is important to allow for total category demand to respond to net changes in prices, and in discrete choice models, it is equivalent to including a “no-purchase” option.<sup>5</sup> The direct utility is specified over the demands as  $u(x_1, \dots, x_J, z)$ . With linear pricing, the budget constraint is  $\mathbf{x} \cdot \mathbf{p} + z = y$ .<sup>6</sup> As  $u(\cdot)$  is increasing, the consumer will spend all his income and buy at least one good. By construction, we choose  $z$  as the “essential” good: demand for the outside good is strictly positive. In this interpretation, the budget constraint is binding at the optimum. The consumer chooses demand by solving

$$\begin{aligned} \max_{x_1, \dots, x_J, z} \quad & u(x_1, \dots, x_J, z) \\ \text{s.t.} \quad & \mathbf{x} \cdot \mathbf{p} + z = y; (\mathbf{x}, z) \geq 0. \end{aligned} \quad (1)$$

Prices affect choices only through the budget constraint. This is the main aspect that imposes restrictions on the specification of utility-consistent demand functions.<sup>7</sup>

**3.1.1. Indirect Utility Approach.** The indirect utility approach is attractive because it avoids having to derive demand as the solution to the complicated nonlinear optimization problem in Equation (1). The indirect utility is a function of prices and expenditure and is obtained by substituting the optimal demands,  $(\mathbf{x}^*, z^*)$ , into  $u(\cdot)$ :

$$v(\mathbf{p}, y) = u(x_1^*(\mathbf{p}, y), \dots, x_J^*(\mathbf{p}, y), z^*(\mathbf{p}, y)).$$

The researcher starts by picking a specification of  $v(\mathbf{p}, y)$  and then obtains the implied Marshallian demand functions via Roy's identity:

$$\begin{aligned} x_j^*(\mathbf{p}, y) &= -\frac{\partial v(\mathbf{p}, y)/\partial p_j}{\partial v(\mathbf{p}, y)/\partial y}, \\ z^*(\mathbf{p}, y) &= y - \mathbf{x}^*(\mathbf{p}, y) \cdot \mathbf{p}. \end{aligned}$$

<sup>5</sup> Elasticities are biased in the absence of a no-purchase option, a point illustrated in Chintagunta (1993).

<sup>6</sup> With nonlinear prices, we can write the budget constraint as  $\mathbf{x} \cdot \mathbf{p}(\mathbf{x}) + \bar{z} = y$ . See Hausman (1985) for an early application.

<sup>7</sup> Exceptions include cases where prices signal quality (e.g., Anderson and Simester 2003, Shiv et al. 2005).

<sup>4</sup> Note the recent access to the IRI academic database (Bronnenberg et al. 2008) and the new Nielsen Data Center initiative at the University of Chicago's Booth School of Business.

The chosen indirect utility function corresponds to *some* direct utility function, which is typically not the object of interest. Access to the indirect utility function is sufficient for computing metrics of economic interest such as compensating variation.<sup>8</sup> An alternative equivalent approach is to start directly with the specification of the Marshallian demand functions,  $(\mathbf{x}^*(\mathbf{p}, y), \mathbf{z}^*(\mathbf{p}, y))$ , and to treat Roy's identity as a differential equation to solve for the implied  $v(\mathbf{p}, y)$  (see Dubin and McFadden 1984 for this approach).

The main details for taking this model to marketing data involve allowing for corner solutions to accommodate discrete choice, allowing for product differentiation, and specifying stochastic terms to produce an econometric specification. The typical approach follows the model of Hanemann (1984) and is extended by Lee and Pitt (1986) to allow for an outside good. It has been extensively applied in marketing, examples including Chiang (1991), Chintagunta (1993), Arora et al. (1998), Nair et al. (2005), Song and Chintagunta (2007), and Mehta et al. (2010). Our exposition follows Chiang and Lee (1992).

Most marketing applications of this model deal with consumers making a choice of a single brand or item from a category with a fixed set of alternatives (items, brands, etc.). In this case, the two goods of interest are the focal category and the outside good. To obtain a discrete/continuous demand system in which the purchase of potentially multiple units of one chosen brand emerges as the optimal choice, Hanemann (1984) suggested working with an indirect utility function corresponding to a bivariate direct utility function of the form

$$u = u\left(\sum_{j=1}^J \psi_j x_j, \psi_z z\right). \quad (2)$$

Here,  $\psi_j (>0)$  are weights reflecting the consumer's perceived quality of brand  $j$ . Maximization of Equation (2) subject to a linear budget constraint results in a corner solution in which at most one brand is purchased. This will be the case whenever indifference curves between any two pairs of products are linear or concave. Then, the indifference curve corresponding to maximal utility will intersect the budget constraint at one of the axes. Product differentiation is accommodated by allowing the weights  $\psi_j$  to be a function of brand attributes  $w_j$  and consumer  $i$ 's tastes for attributes  $\beta_i$ . Stochastic errors  $\epsilon = (\epsilon_1, \dots, \epsilon_J, \epsilon_z)$  enter the model via the weights, reflecting the econometrician's uncertainty regarding subjective brand evaluations. A popular specification is  $\psi_j = \exp(\psi_j(w_j, \epsilon_j; \beta_i))$ ,  $\psi_z = \exp(\psi_z(\epsilon_z; \beta_i))$ , which

guarantees the positivity of the weights. Consistent with a random utility formulation, the consumer is assumed to observe the realization of  $\epsilon$  prior to making purchase. From the perspective of the econometrician,  $\psi_j$  has a distribution induced by the distribution of  $\epsilon$ ,  $\mathcal{F}_\psi(\psi_1, \dots, \psi_J, \psi_z)$ , which generates a likelihood for the data. The Lagrangian for the problem is

$$\mathcal{L} = u\left(\sum_{j=1}^J \psi_j x_j, \psi_z z\right) + \lambda(y - \mathbf{x} \cdot \mathbf{p} - z),$$

with the nonnegativity constraints  $(\mathbf{x}, z) \geq 0$ . The key for corner solutions is to recognize that some nonnegativity constraints,  $\mathbf{x} \geq 0$ , will bind as equalities. The solution satisfies the KKT conditions:

$$\frac{\partial \mathcal{L}}{\partial x_j} \leq 0, \quad x_j \geq 0, \quad x_j \frac{\partial \mathcal{L}}{\partial x_j} = 0, \quad j = 1, \dots, J;$$

$$\frac{\partial \mathcal{L}}{\partial z} \leq 0, \quad z \geq 0, \quad z \frac{\partial \mathcal{L}}{\partial z} = 0.$$

There are two possible outcomes: no purchase, or the purchase of one of the inside goods.

**No Purchase.** If the consumer chooses not to purchase any of the inside goods, the bundle  $(0, \dots, 0, z^*)$  is optimal. The KKT conditions imply that for this bundle to be optimal, we need

$$u_1(0, z^*)\psi_j - \lambda p_j \leq 0 \quad j = 1, \dots, J, \quad (3)$$

$$u_2(0, z^*)\psi_z - \lambda = 0,$$

where  $u_1(\cdot)$  and  $u_2(\cdot)$  are the derivatives of  $u$  with respect to its first and second arguments, respectively. We can interpret Equation (3) as follows. When  $x_j^* = 0$  is optimal, complementary slackness implies that the first-order conditions for  $x_j$ ,  $j = 1, \dots, J$  are inequalities; when  $z^* > 0$  is also optimal, complementary slackness implies that the first-order conditions for  $z$  are binding as an equality (if  $z$  is positive, requiring  $z \times \partial \mathcal{L} / \partial z = 0$  implies  $\partial \mathcal{L} / \partial z = 0$ ). Equation (3) implies that no purchase will occur when all the quality-weighted prices  $p_j / \psi_j \geq (u_1 / u_2)(1 / \psi_z)$ ,  $j = 1, \dots, J$ . Intuitively, no brand has a quality-weighted price that is low enough to induce purchase. Calling the right-hand term  $R = (u_1 / u_2)(1 / \psi_z)$  a "reservation price" implies the following condition for no purchase:

$$\min_{j=1, \dots, J} \left( \frac{p_j}{\psi_j} \right) \geq R. \quad (4)$$

How do we obtain the reservation price while avoiding specifying  $u(\cdot)$ ? It is easy to see (e.g., Appendix 1 of Chiang 1991) that the conditional indirect utility corresponding to the direct utility function (2) has the form  $v_j = v(p_j / \psi_j, 1 / \psi_z, y)$ . The conditional indirect utility function is the indirect utility function conditional on  $j$  being the chosen brand. Applying Roy's

<sup>8</sup> See Anderson et al. (1992) for more on solving for the direct utility function corresponding to a specified indirect utility.



identity to  $v(\cdot)$  shows the implied demand function is of the form  $x_j = (1/\psi_j)x(p_j/\psi_j, 1/\psi_z, y)$ . The functional form of  $x(\cdot)$  is determined by the analyst's choice for  $v(\cdot)$ . Once the function  $x(\cdot)$  is known, we can find the implied  $R$  as the quality-weighted price that sets the conditional demand of the inside goods equal to zero:

$$0 = x\left(R, \frac{1}{\psi_z}, y\right). \quad (5)$$

**One Brand Is Chosen.** If the consumer chooses one of the inside goods, the bundle  $(0, \dots, x_j^*, \dots, 0, z^*)$  is optimal. The associated KKT conditions for this bundle to be optimal are

$$\begin{aligned} u_1(0, \dots, x_j^*, \dots, 0, z^*)\psi_j - \lambda p_j &= 0, \\ u_1(0, \dots, x_j^*, \dots, 0, z^*)\psi_k - \lambda p_k &\leq 0 \quad k=1, \dots, J, k \neq j, \\ u_2(0, \dots, x_j^*, \dots, 0, z^*)\psi_z - \lambda &= 0. \end{aligned}$$

Following the same logic as above, if  $j$  is chosen, it has to be that  $p_j/\psi_j = (u_1/u_2)(1/\psi_z)$  and  $p_k/\psi_k \leq (u_1/u_2)(1/\psi_z)$  for all  $k \neq j$ . Intuitively, conditional on purchase, the brand with the lowest quality-weighted price is chosen. Thus, we can write the implied condition for the  $j$ th brand to be chosen as

$$\frac{p_j}{\psi_j} = \min_{k=1, \dots, J} \left( \frac{p_k}{\psi_k} \right). \quad (6)$$

The associated quantity demanded of brand  $j$  is

$$x_j^* = \frac{1}{\psi_j} x\left(\frac{p_j}{\psi_j}, \frac{1}{\psi_z}, y\right). \quad (7)$$

For future reference, note that it is possible to invert Equation (7) for the quality index of the chosen good:

$$\psi_j = x^{-1}(x_j^*, p_j, \psi_z, y). \quad (8)$$

Under this setup, the consumer follows a simple decision rule. He decides to buy if the minimum quality-weighted price is less than his reservation price. Otherwise, he decides not to purchase in the category and spends the entire budget on the outside good. If he decides to buy, he chooses the brand with the lowest price per unit quality. From the econometrician's perspective, the brand and quantity decisions flow from one integrated utility function and are affected by the same stochastic element,  $\psi$ . This aspect will be important in forming the likelihood of the model.

**Likelihood.** The likelihood of the model is derived by the induced distribution on choices by the distribution of the quality weights,  $\mathcal{F}_\psi(\psi_1, \dots, \psi_J, \psi_z)$ . For the examples here, we will assume that the researcher has picked a functional form for  $v(\cdot)$  and has obtained the expressions for the conditional demand  $x_j$  from Equation (7) and for the reservation price  $R = R(\mathbf{w}, \mathbf{p}; \beta_i)$

from Equation (5). We now add the index  $i$  for consumer and  $t$  for time. We present the likelihood first for a no-purchase observation and then for an observation in which brand  $j$  is chosen.

The likelihood of a no-purchase bundle  $(0, \dots, 0, z_{it}^*)$  can be evaluated from Equation (4) as

$$\begin{aligned} \mathbb{L}(0, \dots, 0, z_{it}^* | \mathbf{w}, \mathbf{p}_t; \beta_i) \\ = \int \mathcal{F}_{\psi|\psi_z} \left( \frac{p_{1t}}{R_{it}}, \dots, \frac{p_{Jt}}{R_{it}} | \mathbf{w}, \mathbf{p}; \beta_i \right) f(\psi_{z,it}; \beta_i) d\psi_{z,it}, \end{aligned}$$

where  $f(\psi_z)$  is the marginal probability density function (pdf) of  $\psi_z$ . To obtain the likelihood of a bundle  $(0, \dots, x_{j,it}^*, \dots, 0, z_{it}^*)$  in which brand  $j$  is chosen, we combine the inversion in Equation (8) with the inequality conditions in Equation (6) to obtain

$$\begin{aligned} \mathbb{L}(0, \dots, x_{j,it}^*, \dots, 0, z_{it}^* | \mathbf{w}, \mathbf{p}_t; \beta_i) \\ = \int \mathcal{F}_{\psi|\psi_z}^j \left( \frac{p_{1t}}{p_{jt}} \psi_{j,it}, \dots, \psi_{j,it}, \dots, \frac{p_{Jt}}{p_{jt}} \psi_{j,it} | \mathbf{w}, \mathbf{p}_t; \beta_i \right) \\ \times |\mathcal{J}| f(\psi_{z,it}; \beta_i) d\psi_{z,it}, \end{aligned}$$

where  $\mathcal{F}_{\psi|\psi_z}^j$  is the partial of  $\mathcal{F}_{\psi|\psi_z}$  with respect to the  $j$ th quality index  $\psi_j$ , and  $\mathcal{J}$  is the Jacobian of the transformation from  $\psi_j$  to  $x_j^*$  in Equation (7). It is common in marketing to allow for heterogeneity in the underlying parameters of the demand system and to relate it to observed (e.g., sociodemographics) and unobserved components. Letting  $d_i$  denote observed sociodemographics for consumer  $i$ , heterogeneity is typically specified via random effects with hyperparameters  $\Theta$ :

$$\beta_i \sim \mathcal{F}_\beta(\beta_i | d_i; \Theta).$$

Combining all, we let  $y_{ijt}$  be an indicator of whether product  $j$  was purchased in period  $t$ , and let  $y_{i0t}$  be an indicator for no purchase; the overall likelihood for an individual with  $T_i$  observations is thus

$$\begin{aligned} \mathbb{L}_i(\Theta | \mathbf{w}, \bar{\mathbf{p}}, d_i) \\ = \int \prod_{t=1}^{T_i} \left\{ [\mathbb{L}(0, \dots, 0, z_{it}^* | \mathbf{w}, \mathbf{p}_t; \beta_i)]^{y_{i0t}} \right. \\ \left. \times \prod_{k=1}^J [\mathbb{L}(0, \dots, x_{k,it}^*, \dots, 0, z_{it}^* | \mathbf{w}, \mathbf{p}_t; \beta_i)]^{y_{ikt}} \right\} d\mathcal{F}_\beta(\beta_i | d_i; \Theta). \end{aligned}$$

**Choice of Utility and Homotheticity.** Empirical work with the above model requires specifying a functional form for the conditional utility function  $v_j = v(p_j/\psi_j, 1/\psi_z, y)$  and a distributional assumption for the stochastic terms in the model  $\epsilon$ . A popular choice in the marketing literature (e.g., Chiang 1991, Nair et al. 2005, Mehta et al. 2010) has been to use the flexible translog indirect utility function (Christiansen et al. 1975), combined with an extreme

value specification for  $\epsilon$ . This results in logit probabilities and a closed-form expression for conditional demands (see Chiang 1991, for example). Functions such as the translog have the advantage of allowing for flexible substitution patterns. However, they have the disadvantage of homotheticity, implying that as expenditure rises, relative preferences for chosen brands remain the same.<sup>9</sup> This is not unreasonable for categories in which included brands are close substitutes for each other, where shifts in income may not produce dramatic changes in preference ordering.

A nonhomothetic specification is required for categories in which there are wide differences in the qualities of the included goods or for specifying preferences across categories. Descriptive evidence has documented significant asymmetries in price response across brands of differing quality tiers (e.g., Pauwels et al. 2007). Allowing for nonhomothetic specifications allows for an income effect that can capture these asymmetries (Allenby and Rossi 1991). As income increases, consumers are likely to allocate a disproportionate share of expenditure to a higher-quality brand. Thus, for a high-quality brand, the substitution and income effects go in the same direction, but for a low-quality brand, the income effect goes in the opposite direction to the substitution effect, thus allowing for asymmetric switching. This can be accommodated in the above framework by choosing a nonhomothetic specification for  $v(\cdot)$ . A parsimonious way of capturing nonhomotheticity by parameterizing the quality weights as a function of total attainable utility is presented in Allenby and Rossi (1991) and Allenby et al. (2010).

### 3.1.2. Direct Utility Approach.

It is simpler to deal direct.

—Arthur Conan Doyle, *The Adventure of the Sussex Vampire*

The marketing literature has recognized that in some situations, it is simpler to derive the model starting with the direct utility formulation. The basic approach starts with Wales and Woodland (1983), who outline the derivation of demand from the KKT conditions associated with the maximization of a direct utility function, allowing for binding nonnegativity constraints and corner solutions. Variants and extensions of the model include Kim et al. (2002, 2007), Bhat (2008), Lee et al. (2010), and Satomura et al. (2011). The model has proven successful in

parsimoniously describing situations with “multiple discreteness,” where multiple units of potentially multiple brands are chosen. This approach is superior to multinomial (“pick any- $J$ ”) models because it enables exploiting quantity information for inference and facilitates policy analysis by retaining a link to a valid utility function. It is also superior to approaches that treat each brand–quantity combination as a choice alternative, because it prevents parameter proliferation and does not introduce new random utility error terms into consumer preferences for each potential quantity option for a given brand. We first present the framework introduced in Kim et al. (2002), and we discuss extensions presented by Bhat (2008).

Kim et al. (2002) suggest starting with a direct utility function of form

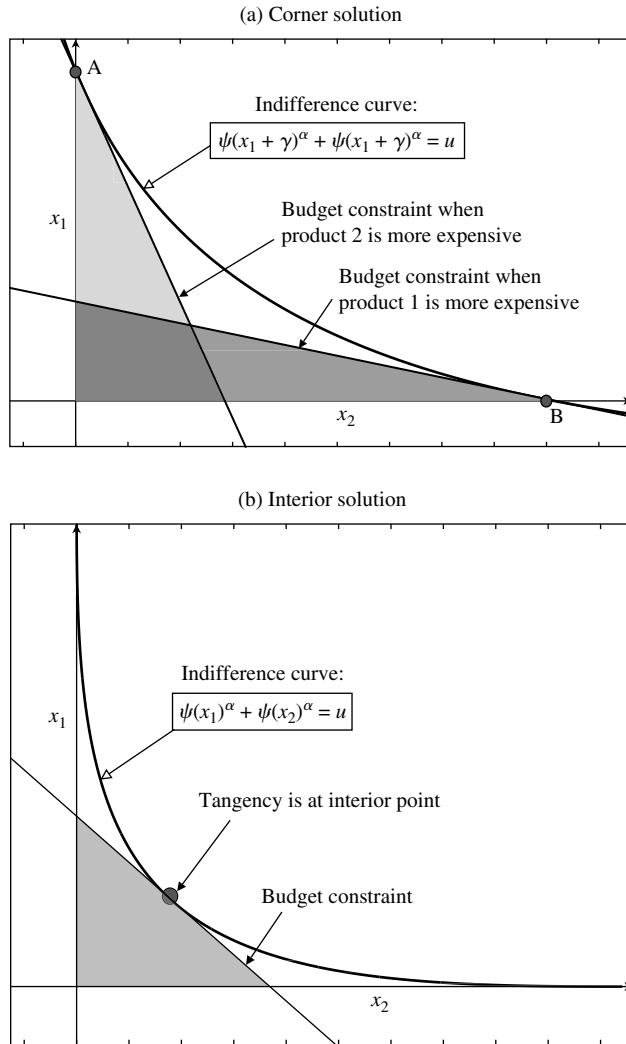
$$u(\mathbf{x}, \mathbf{z}) = \sum_{j=1}^J \psi_j(x_j + \gamma_j)^{\alpha_j} + \psi_z(z + \gamma_z)^{\alpha_z}, \quad (9)$$

where  $\psi$  are quality weights as before. The direct utility function in (9) augments a linear-in-consumption utility function with location ( $\gamma$ ) and scale ( $\alpha$ ) translations. The role of the location translation is to allow for the possibility of corner solutions. To see this, in Figure 1(a) we plot the indifference curve and the budget constraint for a two-goods case. For simplicity, we set  $\gamma_1 = \gamma_2 = \gamma$ , and  $\alpha_1 = \alpha_2 = \alpha$ . When  $\gamma > 0$ , the indifference curves strike the axis at an angle, allowing for the possibility that the tangency with the budget constraint is at one of the axis. Thus, corner solutions are accommodated. Which brand is selected depends on the relative prices (slope of the budget constraint) and the relative qualities of the products (level of indifference curve). An interior solution where both brands are purchased is also a possibility. Figure 1(b) depicts a situation where the location translation parameters  $\gamma_1 = \gamma_2 = 0$ . Now, the indifference curves are tangent to the axis, and only interior solutions are possible. Thus, allowing for  $\gamma$  allows for mixed discrete-continuous demand.

The role of the scale translation is to allow for satiation by building in curvature into the utility function. The marginal utility from consumption is  $\psi_j \alpha_j (x_j + \gamma_j)^{\alpha_j - 1}$ . When  $\alpha_j = 1$ , the marginal utility is constant, and there is no satiation. When all  $\alpha_j = 1$ ,  $j = 1, \dots, J$ , (9) collapses to a linear-in-consumption specification, and we go back to the case discussed in (2). Intuitively, when there is no satiation, the consumer spends all his expenditure on one inside good, which is the brand with the lowest price per unit quality,  $p_j/\psi_j$ . When  $\alpha_j < 1$ , the consumer’s marginal utility diminishes with increased consumption. This satiation is a force that pushes him toward multiple discreteness.

Bhat (2008) points out that (9) is a special case of a Box-Cox transformation of a constant elasticity of

<sup>9</sup> Homothetic preferences imply that utility can be produced from consumption according to a constant returns-to-scale technology; i.e., doubling consumption of all goods doubles utility. Thus,  $u(\tau \mathbf{x}) = \tau u(\mathbf{x})$ . An indirect utility function corresponding to homothetic preferences has the form  $v = b(\mathbf{p})y$ . Applying Roy’s identity,  $(x_j(\mathbf{p}, y))/(x_k(\mathbf{p}, y)) = (b_j(\mathbf{p}))/b_k(\mathbf{p})$ , which implies changes in  $y$  do not shift the relative demands of brands  $j, k$ .

**Figure 1** Translation Parameters Enable Corner vs. Interior Solutions

substitution utility function:

$$u(\mathbf{x}, z) = \sum_{j=1}^J \frac{\gamma_j}{\alpha_j} \psi_j \left( \left( \frac{x_j}{\gamma_j} + 1 \right)^{\alpha_j} - 1 \right) + \frac{\gamma_z}{\alpha_z} \psi_z \left( \left( \frac{z}{\gamma_z} + 1 \right)^{\alpha_z} - 1 \right). \quad (10)$$

The additional “−1” terms inside the subutility functions are cardinal normalizations to ensure “weak complementarity” (Mäler 1974), which is an intuitive requirement that the utility from a good  $j$  is zero if it is not consumed. The specification (10) ensures this as the subutility from brand  $j$  equals zero if  $x_j = 0$ . Bhat notes that when all the scaling parameters,  $\alpha_j \rightarrow 0$ ,  $j = 1, \dots, J$ , and  $\alpha_z \rightarrow 0$ , (10) collapses to the utility from a linear expenditure system, popular in the environment economics literature (see Phaneuf and Smith 2005, von Haefen and Phaneuf 2005):

$$u(\mathbf{x}, z) = \sum_{j=1}^J \gamma_j \psi_j \ln \left( \frac{x_j}{\gamma_j} + 1 \right) + \gamma_z \psi_z \ln \left( \frac{z}{\gamma_z} + 1 \right). \quad (11)$$

When all the  $\gamma$ s are normalized to 1, (10) collapses to specification similar to Kim et al. (2002):

$$u(\mathbf{x}, z) = \sum_{j=1}^J \frac{1}{\alpha_j} \psi_j ((x_j + 1)^{\alpha_j} - 1) + \frac{1}{\alpha_z} \psi_z ((z + 1)^{\alpha_z} - 1). \quad (12)$$

In practice, however, with existing data sets, researchers have found it difficult to estimate the general model (10) with fully specified scale and translation parameters. A choice between the restricted specifications above is thus necessary. Each of the specifications, (9), (11), or (12), are able to allow for both corner solutions and satiation. Unfortunately, they are not testable against one another with typical purchase data because each can fully rationalize the observed patterns of brand and quantity choices in a given data set; so a nonparametric test between the models is not possible. Hence, the choice of one over the other has to be based on the researcher’s preference and modeling goals, as well as the nature of the product category. In practice, we expect that the rate of satiation of the outside good is likely lower than the inside goods. Furthermore, if the goods are strong substitutes (e.g., flavors of the same product), we may expect that the rate of satiation across brands may not be too different. Then, a model with a common satiation parameter for all the inside goods, and a separate one for the outside good, may be a reasonable approximation.

We now discuss how the above model results in a demand system suitable for empirical work. We present the model treatment with Bhat’s (2008) specification. The derivation for the other utility functions is analogous. To reflect empirical work, stochastic elements and characteristics are introduced into the model in the same fashion as before, by parameterizing the baseline utility as  $\psi_j = \exp(w_j \beta_i + \epsilon_j)$  and  $\psi_z = \exp(\epsilon_z)$ . We can write the Lagrangian for the consumer’s problem as

$$\mathcal{L} = \sum_{j=1}^J \frac{\gamma_j}{\alpha_j} \exp(w_j \beta + \epsilon_j) \left( \left( \frac{x_j}{\gamma_j} + 1 \right)^{\alpha_j} - 1 \right) + \frac{\gamma_z}{\alpha_z} \exp(w_z \beta + \epsilon_z) \left( \left( \frac{z}{\gamma_z} + 1 \right)^{\alpha_z} - 1 \right) + \lambda(y - \mathbf{x} \cdot \mathbf{p} - z),$$

with the nonnegativity constraints  $(\mathbf{x}, z) \geq 0$ .

Following the same approach as outlined before, the KKT conditions corresponding to a bundle  $(x_1^*, \dots, x_K^*, 0, \dots, 0, z^*)$  in which  $K$  out of the  $J$  goods (along with the outside good, which is essential) are bought are

$$\eta_j = \mathcal{V}_z - \mathcal{V}_j \quad j = (1, \dots, K), x_j^* > 0, \\ \eta_j \leq \mathcal{V}_z - \mathcal{V}_j \quad j = (K+1, \dots, J), x_j^* = 0, \quad (13)$$

where  $\eta_j = \epsilon_j - \epsilon_z$ ; and

$$\mathcal{V}_j = w_j \beta_i + (\alpha_j - 1) \ln \left( \frac{x_j^*}{\gamma_j} + 1 \right) - \ln(p_j),$$

$$\mathcal{V}_j = (\alpha_z - 1) \ln \left( \frac{z^*}{\gamma_z} + 1 \right).$$

In writing (13), we have employed the usual procedure of differencing out the KKT conditions against the equality condition for the essential, outside good. The fact that the budget constraint is binding implies that the demand for one good (say,  $z$ ) is known once the demand for the other  $J$  goods are determined, as  $z^*(\mathbf{p}, y) = y - \mathbf{x}^*(p, y) \cdot \mathbf{p}$ . The differencing reflects this unitary reduction in the degrees of freedom for the problem.

**Likelihood.** The likelihood of the model is derived by the distribution induced on choices by the distribution of  $\epsilon$  on the quality weights. Given the assumed joint density on  $\epsilon$ , let  $f_\eta(\eta_1, \dots, \eta_J)$  denote the implied pdf of the error differences  $\eta$ . We now add the index  $i$  for consumers and  $t$  for time. Collect all parameters that are consumer-specific in a vector  $\theta_i \equiv (\beta_i, \psi_i, \gamma_i, \alpha_i)$ . The likelihood of a bundle  $(x_{1,it}^*, \dots, x_{K,it}^*, 0, \dots, 0, z_{it}^*)$  in which  $K$  out of the  $J$  goods are bought, and goods  $(K+1, \dots, J)$  are not bought, is

$$\mathbb{L}(x_{1,it}^*, \dots, x_{K,it}^*, 0, \dots, 0, z_{it}^* | \mathbf{w}, \mathbf{p}_t; \theta_i)$$

$$= \int_{-\infty}^{\mathcal{V}_{z,it} - \mathcal{V}_{K+1,it}} \dots \int_{-\infty}^{\mathcal{V}_{z,it} - \mathcal{V}_{J,it}} f_\eta(\mathcal{V}_{z,it} - \mathcal{V}_{1,it}, \dots, \mathcal{V}_{z,it} - \mathcal{V}_{K,it}, \eta_{K+1,it}, \dots, \eta_{J,it}) \|\mathcal{J}_{it}\| d\eta_{K+1,it} \dots d\eta_{J,it}, \quad (14)$$

where implicitly

$$(\mathcal{V}_{it}, \mathcal{V}_{z,it}) \equiv (\mathcal{V}(\mathbf{w}, \mathbf{p}_t; \theta_i), \mathcal{V}_z(\mathbf{w}, \mathbf{p}_t; \theta_i)).$$

$\mathcal{J}_{it}$  is the  $K \times K$  Jacobian matrix with cell  $(l, m)$  given by

$$\mathcal{J}_{lm,it} = \frac{\partial(\mathcal{V}_{z,it} - \mathcal{V}_{l,it})}{\partial x_{m,it}^*}, \quad l, m = (1, \dots, K).$$

The likelihood has two parts and can be understood as follows. First, for the chosen goods  $(1, \dots, K)$ , Equation (13) defines the inverse mapping from the unobservables to demand. Thus, the first part of the likelihood involves the density of  $(x_{1,it}^*, \dots, x_{K,it}^*)$  given by change-of-variable calculus. This generates the  $K \times K$  Jacobian  $\mathcal{J}$ . The second part involves the probability of not purchasing goods  $(K+1, \dots, J)$ . This is obtained by integrating  $(\eta_{K+1}, \dots, \eta_J)$  over the region consistent with no purchase as per the KKT inequalities in Equation (13).

The likelihood defined by Equation (14) is very complicated and involves integration over a truncated multivariate distribution. This is significantly

challenging for the case of probit specification with normally distributed  $\epsilon$ . Kim et al. (2002) propose a Markov chain Monte Carlo algorithm to solve the problem, employing a GHK (Keane 1994, Hajivassiliou et al. 1996) algorithm to efficiently simulate from a truncated multivariate normal. The Kim et al. approach also handles unobserved taste heterogeneity specified via random effects with hyperparameters  $\Theta$  and demographics for consumer  $d_i$ :

$$\theta_i \sim \mathcal{F}_\theta(\theta_i | d_i; \Theta).$$

For the case of Type 1 extreme value distributed  $\epsilon$ , Bhat (2008) shows that the likelihood (13) simplifies considerably as

$$\mathbb{L}(x_{1,it}^*, \dots, x_{K,it}^*, 0, \dots, 0, z_{it}^* | \mathbf{w}, \mathbf{p}_t; \theta_i)$$

$$= (K-1)! \left[ \prod_{j=1}^K \hat{f}_{j,it} \right] \times \left[ \sum_{j=1}^K \frac{p_{jt}}{\hat{f}_{j,it}} \right] \times \left[ \frac{\prod_{j=1}^K \exp(\mathcal{V}_{j,it})}{(\sum_{j=1}^J \exp(\mathcal{V}_{j,it}))^K} \right],$$

where  $\hat{f}_{j,it} = ((1 - \alpha_{ji})/(x_{j,it}^* + \gamma_{ji}))$ . This simplifying result facilitates the use of the model for the applied researcher.

**Other Approaches.** The above approach relies on satiation as a force to explain multiple discreteness. An alternative approach in Dubé (2004) explains multiple discreteness as a form of temporal variety seeking, wherein a consumer purchases multiple brands in response to uncertain future needs (e.g., Walsh 1995). Dubé's model follows Hendel's (1999) formulation and postulates that at the time of purchase, a consumer anticipates he may face  $\mathcal{N}$  future consumption occasions, and his preferences in consumption occasion  $n$  will be  $\theta_n$ . Both  $\mathcal{N}$  and  $\theta_n$  are deterministic from the perspective of the consumer (i.e., he has no uncertainty about his future needs or tastes), but are stochastic from the perspective of the researcher. This generates a likelihood for the data. Dubé assumes that consumer utility for the purchase of  $J$  inside goods and an outside good  $z$  is additively separable over the  $\mathcal{N}$  occasions, and it is given by

$$u(\mathbf{x}, z) = \sum_{n=1}^{\mathcal{N}} u_n(\mathbf{x}_n) + z, \quad (15)$$

subject to an overall budget constraint,  $\sum_n \mathbf{p} \cdot \mathbf{x}_n + z = y$ . The occasion-specific subutilities are defined over unobserved consumption bundles as  $u_n(\mathbf{x}_n) = (\sum_{j=1}^J \psi_{jn} x_{jn})^\alpha$ , where  $\psi_{jn} = \psi_j(\theta_n)$  are quality weights for brand  $j$  in occasion  $n$ . This is similar to the Hanemann formulation in Equation (2). Thus, multiple units of a single alternative will be chosen for each consumption occasion. The separability of the subutilities and the budget constraint ensures that the problem can be solved separately for each consumption occasion and aggregated to obtain the predicted

demand at the purchase stage. Finally, Chan (2005) presents an alternative approach in which utility is specified over characteristics rather than over consumption.

**Direct vs. Indirect Utility Approaches.** As the above discussion highlights, the direct and indirect utility approaches to studying individual-level demand share substantial commonality. The main difference is in how the purchase quantities are characterized. In the former approach, the researcher specifies a functional form for the direct utility function and obtains the likelihood for purchase quantities directly from the Karush–Kuhn–Tucker conditions. On the other hand, the indirect utility approaches typically specify a functional form for the indirect utility function and obtain the purchase quantities from Roy's identity. In our discussion of the indirect utility, discrete/continuous model, we mentioned the use of the translog indirect utility function to obtain purchase quantities. Such a utility function is consistent with the bivariate direct utility function in Equation (2). If one wanted to use the direct utility approach to the same problem, one example of a utility function corresponding to Equation (2) would be  $u(\mathbf{x}, z) = (\sum_{j=1}^J \psi_j x_j)^\alpha + z$ . This function is a simplification of the utility function in Dubé (2004) to a situation in which there is only one consumption occasion corresponding to each purchase occasion. Alternatively, one could use the specifications in Kim et al. (2002) or Bhat (2008). An additional consideration is that under the direct utility approach, the researcher has to compute the corresponding indirect utility in a subsequent maximization step to undertake welfare analysis or to measure compensating variation.

### 3.2. On Separability Assumptions

We conclude this section with a discussion of the implications of separability assumptions for the properties of demand derived from the above frameworks. We first discuss handling complementarity. Subsequently, we discuss how prices and marketing-mix effects in other categories may be handled when modeling demand for a focal category.

**3.2.1. Complementarities.** Many marketing situations involve complementarities. Models with additive utility implicitly assume that all products are substitutes and cannot allow for complementarities. To see this, suppose utility for goods  $(1, \dots, J)$  is given by the additive structure:

$$u(\mathbf{x}) = u(u_1(x_1) + \dots + u_J(x_J)).$$

The effect of price  $k$  on the compensated demand for good  $j$  then has the following structure (see Deaton and Muelbaeur 1980):

$$\frac{\partial x_j(\mathbf{p}, u)}{\partial p_k} = \mu \frac{\partial x_j(\mathbf{p}, y)}{\partial y} \frac{\partial x_k(\mathbf{p}, y)}{\partial y},$$

where  $\mu$  is a constant. Thus if both  $j, k$  are normal goods, it has to be they are substitutes. More restrictive specifications,  $u(\mathbf{x}) = u_1(x_1) + \dots + u_J(x_J)$ , will imply no possibility of interaction from joint consumption, as the marginal utility from consuming one product is unaffected by the consumption of others. Demand studies that accommodate complementarities essentially postulate utility specifications that relax additivity by allowing for interaction terms between the subutilities of products. For example, Bhat and Pinjari (2010) suggest adding cross-product interactions into the utility function presented in (10) to obtain

$$u(\mathbf{x}, z) = \sum_{j=1}^J \frac{\gamma_j}{\alpha_j} \left( \left( \frac{x_j}{\gamma_j} + 1 \right)^{\alpha_j} - 1 \right) \cdot \left( \psi_j + \frac{1}{2} \sum_{k \neq j} \theta_{kj} \frac{\gamma_j}{\alpha_j} \left( \left( \frac{x_j}{\gamma_j} + 1 \right)^{\alpha_j} - 1 \right) \right) + \frac{1}{\alpha_z} \psi_z (z + 1)^{\alpha_z}. \quad (16)$$

A feature is that when  $\alpha_j \rightarrow 0$ ,  $j = 1, \dots, J$ , (16) collapses to the popular translog utility function (Christiansen et al. 1975, see also Song and Chintagunta 2007), whereas when all  $\alpha_j = 1$ ,  $j = 1, \dots, J$ , it collapses to the quadratic utility used in Wales and Woodland (1983). Other examples include Lee et al. (2010), who propose interactions in  $\log(\text{quantities})$  in the direct utility function, and Gentzkow (2007), who suggests allowing for interactions in the conditional indirect utility for product bundles. Finally, the large literature on state-dependent demand using choice models with lagged dependent variables can be thought of as models with complementary goods where the complementarity is across time.

#### 3.2.2. The “Outside Good” and Multiple Categories.

The little things are infinitely the most important.

—Arthur Conan Doyle, *A Case of Identity*

One aspect of the discussion so far is that all previous models focused on the  $J$  inside goods while ignoring the characteristics and prices of all other goods. These were bundled into an “outside” option. We close this section with a discussion of the primitive assumptions that justify this focus. The justification for separating the demand of  $J$  inside goods from the overall problem of demand for all  $N$  possible goods relies on two different forms of separability.

The first relies on the notion of *Hicksian separability*, which requires the prices of all other goods,  $\mathbf{p}_{-j}$ , move in parallel; i.e.,  $\mathbf{p}_{-j} = c \bar{\mathbf{p}}_{-j}$ , where  $c$  is positive and  $\bar{\mathbf{p}}_{-j}$  is a vector of constant base-price levels for the other goods (so relative prices of all goods  $k \notin (1, \dots, J)$  always remain the same over time).

Define  $z = \mathbf{x}_{-J} \cdot \bar{\mathbf{p}}_{-J}$ , a base-price-weighted average of quantities;  $z$  is referred to as the Hicksian “composite good.” Let  $\tilde{u}(\cdot)$  be the utility function defined over all  $N$  possible goods that may be consumed. Then, the solution to the “full” problem,

$$\begin{aligned} \max_{x_1, \dots, x_J, x_{J+1}, \dots, x_N} \quad & \tilde{u}(x_1, \dots, x_J, x_{J+1}, \dots, x_N) \\ \text{s.t.} \quad & \mathbf{x} \cdot \mathbf{p} + \mathbf{x}_{-J} \cdot \bar{\mathbf{p}}_{-J} = y, \end{aligned}$$

is the same as the solution to the simpler problem,

$$\begin{aligned} \max_{x_1, \dots, x_J, z} \quad & u(x_1, \dots, x_J, z) \\ \text{s.t.} \quad & \mathbf{x} \cdot \mathbf{p} + cz = y, \end{aligned} \quad (17)$$

where  $u(\cdot)$  is interpreted as the solution to

$$\begin{aligned} u(x_1, \dots, x_J, z) = \max_{x_{J+1}, \dots, x_N} \quad & u(x_1, \dots, x_J, x_{J+1}, \dots, x_N) \\ \text{s.t.} \quad & \mathbf{x}_{-J} \cdot \bar{\mathbf{p}}_{-J} = z. \end{aligned}$$

Thus,  $c$  serves as a “price” for the composite good  $z$ . As the budget constraint does not change if all quantities are scaled (homogeneity of degree 1), we can normalize  $c$  to 1 in (17) to give us the standard form (1).

Hicksian separability is an unattractive justification for marketing studies, as all data sets contradict the fact that relative prices of goods in other categories stay constant over time, or stores. An alternative justification is “weak separability” of preferences. Assume the utility function is separable in the inside and outside goods as

$$\tilde{u}(x_1, \dots, x_J, x_{J+1}, \dots, x_N) = \tilde{u}(x_1, \dots, x_J, \vartheta_z(x_{J+1}, \dots, x_N)),$$

where  $\vartheta_z(\cdot)$  is an increasing subutility function. Then, we can think of the customer making a two-stage decision. In the first stage, he decides how much of total income  $y$  to allocate to the inside goods and the outside category. In the second stage, he decides to choose demand for each category conditioning on the expenditure allocation for that category (see Deaton and Muelbauer 1980 for more on such “multilevel” budgeting). Denote the expenditure allocated to the outside category as  $y^*$ . It is clear that the optimal demand for the outside goods is determined by the subproblem:

$$\begin{aligned} \max_{x_{J+1}, \dots, x_N} \quad & \vartheta_z(x_{J+1}, \dots, x_N) \\ \text{s.t.} \quad & \mathbf{x}_{-J} \cdot \mathbf{p}_{-J} = y^*. \end{aligned}$$

Let  $v^* = v(\mathbf{p}_{-J}, y^*)$  denote the corresponding indirect utility from spending expenditure  $y^*$  on the outside goods. Let  $y^* = y(\mathbf{p}_{-J}, v^*)$  be the corresponding cost function. Then, we can write the problem for choosing the inside goods as

$$\begin{aligned} \max_{x_1, \dots, x_J, v^*} \quad & u(x_1, \dots, x_J, v^*) \\ \text{s.t.} \quad & \mathbf{x} \cdot \mathbf{p} + y(\mathbf{p}_{-J}, v^*) = y. \end{aligned} \quad (18)$$

Following Gorman (1959), suppose we can write the expenditure function as  $y(\mathbf{p}_{-J}, v^*) = a(v^*)b(\mathbf{p}_{-J})$ , where  $a(\cdot)$  is an increasing function and  $b(\cdot)$  is degree-1 homogeneous in prices.<sup>10</sup> Then, we can write (18) as

$$\begin{aligned} \max_{x_1, \dots, x_J, v^*} \quad & u(x_1, \dots, x_J, v^*) \\ \text{s.t.} \quad & \mathbf{x} \cdot \mathbf{p} + a(v^*)b(\mathbf{p}_{-J}) = y, \end{aligned}$$

which is of the form (17). We interpret  $a(v^*)$  as a quantity index and  $b(\mathbf{p}_{-J})$  as a price index. Thus, under weak separability, we interpret the outside good  $z$  as  $v^*$ , the utility from the consumption of all other goods. Whether weak separability is justified for marketing demand data depends on how categories are defined. Following Deaton and Muellbauer (1980), weak separability implies that prices or characteristics of any product in the outside good will affect the demand for any inside good only via expenditures (i.e., there is only an income effect). Furthermore, all products in the outside group must be either substitutes or complements to each of the inside goods. Thus, forming an outside good by grouping together two products, one of which is a substitute to one of the inside goods and the other a complement, is inconsistent with weak separability.

The extant literature has been somewhat informal in its treatment of goods across categories and its analysis of market baskets. Models for multicategory demand have typically taken preference structures originally developed for modeling demand among substitutes within a category and allowed correlation across categories via error terms or correlated parameters. In our view, more work remains to be done in formally deriving multicategory demand systems from a transparent underlying model of expenditure allocation and well-articulated separability assumptions (see, e.g., Drèze et al. 2004).

## 4. New Directions

We discuss three new directions in recent work on demand: dynamics, use of data on unobservables, and nonparametric approaches. Again, we reiterate that our aim is not to provide an exhaustive survey of all possible directions but to provide a flavor for new demand-side work in marketing.

### 4.1. Dynamics in Demand

A sophisticated empirical literature in marketing now explicitly considers dynamic aspects of demand. The main demand-side factors are storability, durability, experience goods, and complementarities. We discuss these briefly in sequence.

<sup>10</sup> This implies homotheticity (see Deaton and Muellbauer 1980). The other option is to impose additive preferences, which is even more restrictive, or to use an approximate solution.

**4.1.1. Storability.** Storable goods are products that do not perish if not consumed within the same period as they are purchased. Clearly, the classification of a product as perishable or storable depends on the length of the time period considered. For a short-enough time period, all products are storable.<sup>11</sup> Demand under storability is a dynamic problem because current purchase increases inventory and, ceteris paribus, makes the consumer less likely to purchase tomorrow. Understanding the dynamic implications of storability is key to marketers because it affects the auditing of promotions. Storability implies that consumers can stockpile the product during periods of low prices and consume out of inventory at other times. If all promotions achieve to move sales from a high-price future to a low-price present (referred to as “purchase acceleration”), the sale is essentially losing money. However, if the promotion results in gainful brand switching or increases consumption, it may be beneficial. More generally, storability (or any negative state dependence in demand; see, e.g., Hartmann 2006) implies that demand is subject to intertemporal substitution. Hence, short-run price variation can overstate true price elasticities. In the short run, buying the product tomorrow is a substitute for buying the product today; hence there are many short-run substitutes to the product bought today. In the long run, one cannot substitute across time. Hence, long-run demand is less elastic, holding other factors fixed.<sup>12</sup>

Storability can be accommodated in a utility-theoretic model of demand by allowing inventory,  $i$ , to be a state that shifts utility. We denote  $x$  as per-period consumption,  $u(x)$  as the per-period utility from consumption,  $c(i)$  as the cost of carrying inventory  $i$ ,  $p_j$  as the current price for product  $j$ , and  $\epsilon_j$  as unobservable (to the econometrician) components of the utility from purchasing product  $j$ ; we can then write the value function from the purchase of product  $j$  with pack size  $q_j$  as

$$\begin{aligned} \mathcal{V}_j(i, p_j, \epsilon_j) &= \max_{x \geq 0} \left\{ u_j(x) - c(i) - \beta p_j + \epsilon_j \right. \\ &\quad \left. + \delta \mathbb{E}_{\mathbf{p}', \epsilon' | \mathbf{p}, j} \left[ \max_k \left( \mathcal{V}_k(i + q_j - x, p'_k, \epsilon'_k) \right) \right] \right\}, \end{aligned}$$

<sup>11</sup> Vegetables, meat, and poultry may be considered perishable if time is defined in weeks but storable if time is defined in hours. Typical marketing data are available in discrete time with calendar time coded in weeks.

<sup>12</sup> Firms such as IRI incorporate this insight in practice by measuring price elasticities from base-price changes only, dropping weeks with temporary price reductions or promotions from the estimation data set. Access to long time series (e.g., Mela et al. 1997) enables exploring the effect of changes in base prices on demand.

where the outer “max” over  $x$  indicates that consumption is endogenously chosen, and the future inventory conditional on choosing product  $j$  is modeled as  $i + q_j - x$  for any chosen  $x$ . The key empirical force determining stockpiling will be the specification of expectations over future prices,  $\mathbf{p}'$ . The empirical problem is complicated by the fact that inventory is a serially correlated unobserved state variable, which increases the computational complexity of the dynamic programming problem.

Erdem et al. (2005) were the first to estimate a dynamic demand system for storable goods. Their analysis was extended by Sun (2005) and Hendel and Nevo (2006) to allow for endogenous consumption; by Hartmann and Nair (2010) to allow for endogenous inventory accumulation across stores; and by Seiler (2010) to allow for search dynamics.

**4.1.2. Durability.** A durable good is a product that is “infinitely” storable and hence subject to one-time purchases. Durable good demand is a dynamic problem because purchase today implies that the consumer is out of the market tomorrow. Demand for a durable good subject to replacement is modeled similar to that of a storable good by replacing the inventory state by an indicator of the product adopted. Following Melnikov (2000), durability can be accommodated by allowing whether a product was purchased yesterday  $y$  to be a state. Letting  $p_j, \mathbf{w}_j$  be the current price and attribute vector for product  $j$ , and letting  $\epsilon_j, \epsilon_0$  be unobservable (to the econometrician) components of the utility from purchasing product  $j$  and delaying purchase, respectively, we can write the value function from the purchase of product  $j$  and for delaying purchase (option 0) as

$$\begin{aligned} \mathcal{V}_j(y, p_j, w_j, \epsilon_j) &= \frac{\alpha_j + \rho \mathbf{w}_j}{1 - \delta} - \beta p_j + \epsilon_j, \\ \mathcal{V}_0(y, \mathbf{p}, \mathbf{w}, \epsilon_0) &= \epsilon_0 + \delta \mathbb{E}_{\mathbf{p}', \mathbf{w}', \epsilon' | \mathbf{p}, \mathbf{w}, 0} \left[ \max \left( \mathcal{V}_0(y, \mathbf{p}', \mathbf{w}', \epsilon'_0), \right. \right. \\ &\quad \left. \left. \max_k \{ \mathcal{V}_k(y, p'_k, w'_k, \epsilon'_k) \} \right) \right], \end{aligned}$$

where  $\mathbf{w}'$  denotes expectations over future attributes. The value functions encapsulate two aspects. First, the purchase of  $j$  gives utility  $\alpha_j + \rho \mathbf{w}_j$  per period forever with present discounted value  $(\alpha_j + \rho \mathbf{w}_j)/(1 - \delta)$ . This is the implication of durability. Second, delaying purchase has an option value. By delaying, the consumer can make a potentially better decision tomorrow by choosing to adopt or further wait after observing tomorrow’s prices and product sets.

Recent empirical demand systems for durable goods trace their origins to Horsky (1990) and Chatterjee and Eliashberg (1990). More recently, Song and Chintagunta (2003) implemented the formal

framework above using data on the purchases of digital cameras. It is now a rich literature, including studies by Erdem et al. (2005; extension to search), Nair (2007; extension to dynamic pricing), Gordon (2009; extension to replacement), Ryan and Tucker (2011) and Dubé et al. (2010a) (both accommodating network effects), and Ishihara (2010; adding secondhand markets).

**4.1.3. Experience Goods.** Experience goods are characterized by ex ante uncertainty about quality, which is resolved by consumption. Demand for experience goods is a dynamic problem because purchase today provides a signal about quality, which updates the future information set. Experience goods can be accommodated by allowing beliefs about product quality to be a state. We denote  $x_j$  as the quality of brand  $j$ ,  $u_j(x_j)$  as the per-period utility from purchasing brand  $j$  under the belief that its quality is  $x_j$ ,  $b(x)$  as the density of the consumers beliefs about the vector of brand qualities,  $p_j$  as prices, and  $\epsilon_j$  as unobservable (to the econometrician) components of the utility from purchasing product  $j$ ; we can then write the value function from purchase of product  $j$  as

$$\mathcal{V}_j(b, p_j, \epsilon_j) = \int u_j(x_j) db_j(x_j) - \beta p_j + \epsilon_j \\ + \delta \mathbb{E}_{b', p', \epsilon'} [p, b, j \left[ \max_k (\mathcal{V}_k(b', p'_k, \epsilon'_k)) \right]].$$

The key force driving the dynamics is that buying product  $j$  provides a signal that updates current beliefs  $b$  to a posterior  $b | j$ . Thus, buying generates an option value—the updated posterior beliefs enable the consumer to make a potentially better decision tomorrow. The empirical problem is complicated by the fact that beliefs are a multivariate set of serially correlated unobserved state variables, which significantly increases the computational complexity of the dynamic programming problem. Following Miller (1984) and Eckstein et al. (1988), beliefs have been typically modeled by a normal distribution and consumers modeled as rational, Bayesian learners. Erdem and Keane (1996) estimated a dynamic demand system for experience goods, and that dynamic framework has become very popular in many studies, including Hitsch (2006; extension to learning about demand), Goettler and Clay (2011; application to demand under two-part tariffs), Zhang (2010; accommodating observational learning), and Osborne (2010; separating structural state dependence from learning). More generally, learning from consumption is one example of a broader set of consumption dynamics implied by a human capital model (Ratchford 2001).

**4.1.4. One-Way Complementarities.** One-way complements refer to systems of goods in which a secondary set of goods are purchased only after

adoption of a primary good (e.g., razors and blades, consoles, video games). When the purchase of the secondary good is temporally separated from adoption, this requires augmenting the model to accommodate dynamic considerations arising from the expectations of consumers about future secondary good availability and prices. Demand for such products is dynamic because the purchase of the primary good changes the choice set for the consumer tomorrow: by buying a Hewlett-Packard (HP) printer, the set of cartridge options compatible with HP is added to the choice set. Demand for one-way complements can be modeled by accommodating the current holdings of the primary good as a state. The primary good is typically treated as durable. We denote  $p_j^p$  as the price for the primary product  $j$ ,  $p_k^s$  as the price for the secondary product  $k$ ,  $\Omega$  as a  $J \times 1$  vector of indicators denoting the set of primary brands owned,  $\aleph_{\Omega_j}$  as the set of secondary products compatible with primary good  $j$ , and  $\epsilon_p, \epsilon_s$  as the vector of unobservables (to the econometrician) to the utility from purchasing the primary and secondary products, respectively; we can then write the value function from the purchase of primary good  $j$  as

$$\mathcal{V}_j(i, p_j, \epsilon_j) = \frac{\alpha_j}{1 - \delta} - \beta p_j^p + \epsilon_j \\ + \delta \mathbb{E}_{p^{s'}, \epsilon^{s'} | j} \left[ \max_{k \in \aleph_{\Omega_j}} (\mathcal{V}_k(p'_k, \epsilon'_k)) \right].$$

The key dynamic here is that the purchase of  $j$  allows the consumers to buy complementary secondary products from the set  $\aleph_{\Omega_j}$  in the future. The value function for secondary goods is similar to that for a storable goods problem. See Hartmann and Nair (2010) for a dynamic demand system for tied goods, Sriram et al. (2010) for dynamics with contingent adoption, and Dubé et al. (2010a) and Liu (2010) for dynamic demand for hardware/software systems with indirect network effects.

## 4.2. Enriching Demand Models with Primary Data

It is a capital mistake to theorize before one has data.

Insensibly one begins to twist facts to suit theories, instead of theories to suit facts.

—Arthur Conan Doyle, *A Scandal in Bohemia*

An important theme in this paper has been the role of structure and assumptions. Our view is that structure and assumptions are part and parcel of model building, and researchers have to be comfortable with the fact that some aspects of the model will remain untestable. On the one hand, the search for a “structureless” or “assumption-free” approach to scientific knowledge is likely to be elusive. At the other extreme, it is hard to accept a study where all the results are driven purely by the structure and not by



the variation in the data. Rather, as researchers, we would like to see that the key constructs of the model are identified by some source of variation in the data and not by unverifiable assumptions about unobservables or functional form (for more on nonparametric identification, see Misra 2011). Our view is that we should be more worried about unverifiable assumptions on *unobservables* than about testable functional-form specifications on observables. The treatment of unobservables drives empirical work and should not be treated merely as “error components” or “nuisance terms.” An important trend in the empirical marketing literature is a burgeoning set of applications that leverage better and more-detailed data on unobservables to improve the credibility of estimates and to relax several assumptions. The new direction is in the best tradition of marketing: obtaining direct data on aspects that underpin model structure.

We discuss several examples where marketing leads in this domain. One example is in the treatment of unobserved heterogeneity across consumers. A significant confound for empirical work is that the observed persistence in choices in typical panel data confounds state dependence with permanent unobserved heterogeneity. Horsky et al. (2006) augment a brand-choice model with survey data on self-reported consumer tastes for brands, and they find that controlling for such heterogeneity significantly attenuates evidence for structural state dependence. Shin et al. (2011) leverage the additional data to investigate demand for experience goods and find that evidence for consumer learning about packaged goods brands (a particular form of state dependence) goes away once unobserved heterogeneity is properly accounted for. Gauri et al. (2008) augment purchase data with primary survey data to control for unobserved heterogeneity in the search propensity of shoppers. Misra and Nair (2011) leverage detailed cross-sectional and panel data at the individual sales-agent level to circumvent pooling across agents altogether. Their data enable estimating a separate model for each agent, providing a semiparametric accommodation of heterogeneity. Bronnenberg et al. (2011) leverage new data on consumer’s migration patterns to better understand the sources of unobserved heterogeneity in consumer’s brand preferences. Ishihara (2010) augments data on sales and prices of new video games, with additional data on quantities sold to and by retailers of used goods, to identify unobserved heterogeneity in preferences between consumers participating in new and used durable goods markets. Albuquerque and Bronnenberg (2009) show how aggregate demand data can be combined with auxiliary data on summaries of consumer purchases to better estimate unobserved consumer heterogeneity. Significant progress has also been made in modeling heterogeneity at a very granular level. Teixeira

et al. (2010) use detailed data on eye movements of consumers watching ads to measure unobserved heterogeneity in advertising avoidance. They document that advertisements optimized on the basis of their model can reduce ad avoidance as much as 8% by incorporating brand pulsing within the commercial.

Another area is in the accommodation of unobserved beliefs. It is well known that outcomes in both single-agent problems (e.g., heterogeneous learning; see Narayanan and Manchanda 2009), and multi-agent problems (e.g., incomplete information entry games; see Zhu and Singh 2009, Orhun 2010, Goldfarb and Xiao 2011) are very sensitive to the specification of agent’s belief structures. A new literature obtains direct data on agents beliefs to relax strong assumptions such as rational expectations. Recent examples include Erdem et al. (2005) (primary data on beliefs about future computer prices incorporated into a dynamic adoption model for durables), Chintagunta et al. (2009) (primary data on patient satisfaction in a Bayesian learning model of prescription drug demand to inform belief updating), Nair and Rao (2010) (survey data on consumer beliefs about auto insurance price changes in the event of accidents, in a model of insurance demand with experience rating), and Dubé et al. (2010b) (conjoint-based beliefs data to estimate discount factors for durable goods adoption). More broadly, combining conjoint data on stated preferences with behavioral data on revealed preferences to improve identification of preferences is likely to be a fruitful and frontier area of research in the field. Louviere et al. (2000) provide a comprehensive discussion of the various issues, challenges, and progress in this area.

Advances have also been made in using better data to achieve cleaner identification of demand in the presence of social interactions. A primary confound in estimating causal social interactions from observed data on groups has been the lack of precise social network information, as well as spurious effects stemming from correlated unobservables that make group members behave similarly. Nair et al. (2010) and Iyengar et al. (2011) leverage primary data on individual-level social networks of physicians, and Nam et al. (2010) use direct data on correlated unobservables (location-specific signal quality that generates spatial correlation in the adoption of a movie-on-demand device) to improve the credibility of social effects measured in marketing settings.

With the increasing availability of better data, it is clear that the growth in this empirical literature will be exciting. Mitigating potential confounds can enable marketers to better pin down consumer preferences and sensitivities to marketing activities, which in turn allows for more efficient and effective use of marketing resources.

### 4.3. “Semi” and “Non”-Parametrics

“Data! Data! Data!” he cried impatiently, “I cannot make bricks without clay!”

—Arthur Conan Doyle, *The Adventure of the Copper Beeches*

The availability of large data sets is also spurring interest in the use of nonparametric approaches to demand analysis. In data-rich situations, these enable relaxing parametric assumptions to flexibly measure marketing-mix effects and to conduct inference. In models of discrete choice, one has to make a conceptual distinction between a nonparametric specification of the choice-specific indirect utilities and the random utility components, versus a nonparametric specification of the distribution of heterogeneity. It is rarely possible to achieve nonparametric identification of all three.<sup>13</sup> When one of these components is parametrically specified, the model is referred to as “semi-parametric.” In marketing, Briesch et al. (2002) has made some inroads into the first problem, specifying semiparametric brand-choice models with nonparametric specifications for the systematic and random components of indirect utility (but allowing no unobserved heterogeneity). There has been a significant amount of recent research on the second problem, with the development of several methods for flexibly accommodating unobserved heterogeneity while parametrically specifying the components of the indirect utility: examples include Fox et al. (2011) for finite-mixture distributions, Rossi et al. (2005) and Braun and Bonfrer (2011) for mixtures of normals specifications using Dirichlet processes, and Fong and Godsill (2002) for finite-mixture specifications of potentially time-varying heterogeneity using particle filters. Other applications include nonparametric controls for selection and endogeneity concerns. Two examples include Ellickson and Misra (2011), who outline methods for controlling for selectivity nonparametrically when observed data are outcomes of discrete games (e.g., demand in a social interactions setting), and Hartmann et al. (2011), who use kinks in firm’s targeting rules to nonparametrically control for the endogeneity of targeted marketing in demand under database marketing. This is a fast-growing, frontier area of research in the field.

## 5. Conclusions

This paper has discussed empirical models of consumer behavior in marketing. We hope our discussion

(1) has reiterated that the state of the demand analysis enterprise in marketing is strong and that it is an exciting time to be doing empirical work in the field; (2) has pointed out how empirical research continues to forge a closer connection with the theory, and the theory work in the field continues to be more closely motivated by and connected to the richness of empirical models; and (3) encourages researchers to do empirical work with (a) well-articulated goals, (b) clear identification, and (c) a tight and transparent link to a model of underlying behavior that realistically and flexibly describes the process generating the demand data.

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<sup>13</sup> The conditions for the joint nonparametric identification of the indirect utilities, the distribution of random-utility errors, and unobserved heterogeneity are stringent, and they are discussed in Briesch et al. (2010). Most importantly, we need a choice-specific “special regressor” to enter the indirect utilities additively with a known coefficient and the indirect utilities to be known a priori at some vector of observed choice characteristics.

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