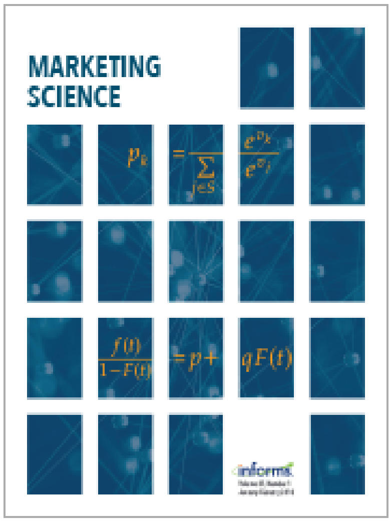


Marketing Science

Publication details, including instructions for authors and subscription information:
http://pubsonline.informs.org

Cooperative Search Advertising

Xinyu Cao, T. Tony Ke

To cite this article:
Xinyu Cao, T. Tony Ke (2019) Cooperative Search Advertising. Marketing Science 38(1):44-67. https://doi.org/10.1287/mksc.2018.1111

http://pubsonline.informs.org/journal/mksc/

MARKETING SCIENCE
Vol. 38, No. 1, January–February 2019, pp. 44–67
ISSN 0732-2399 (print), ISSN 1526-548X (online)

Cooperative Search Advertising

Xinyu Cao,[a] T. Tony Ke[b]

[a] Stern School of Business, New York University, New York, New York 10012; [b] Sloan School of Management, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139-4307

Contact: xcao@stern.nyu.edu, http://orcid.org/0000-0001-8309-9947 (XC); kete@mit.edu, http://orcid.org/0000-0002-8261-0640 (TTK)

Received: September 2, 2016
Revised: February 5, 2018; May 23, 2018
Accepted: May 27, 2018
Published Online in Articles in Advance: October 23, 2018

https://doi.org/10.1287/mksc.2018.1111

Abstract. Channel coordination in search advertising is an important but complicated managerial decision for both manufacturers and retailers. Because of the highly concentrated market of search advertising, a manufacturer's and its retailers' ads can compete instead of complementing each other. We consider a manufacturer, who coordinates with its retailers by sharing a fixed percentage of each retailer's advertising cost and, at the same time, competes with its retailers and outside advertisers in search ad position auctions. Our model prescribes the optimal cooperative advertising strategies from the manufacturer's perspective. We find that different from cooperative advertising in traditional media, it can be optimal for a manufacturer to cooperate with only a subset of its retailers even if they are ex ante the same. This reflects the manufacturer's trade-off between higher demand and higher bidding cost caused by more intense competition. We also find that with two asymmetric retailers, the manufacturer should support the retailer with higher channel profit per click to get a higher position than the other retailer, which demonstrates the effectiveness of the participation-rate mechanism. The manufacturer should take a higher position than a retailer when its profit per click via direct sales exceeds the channel profit per click of the retailer. The main results still hold when we endogenize retail price competition or wholesale contracts.

History: Ganesh Iyer served as the senior editor and Anthony Dukes served as associate editor for this article.
Supplemental Material: The online appendix is available at https://doi.org/10.1287/mksc.2018.1111.

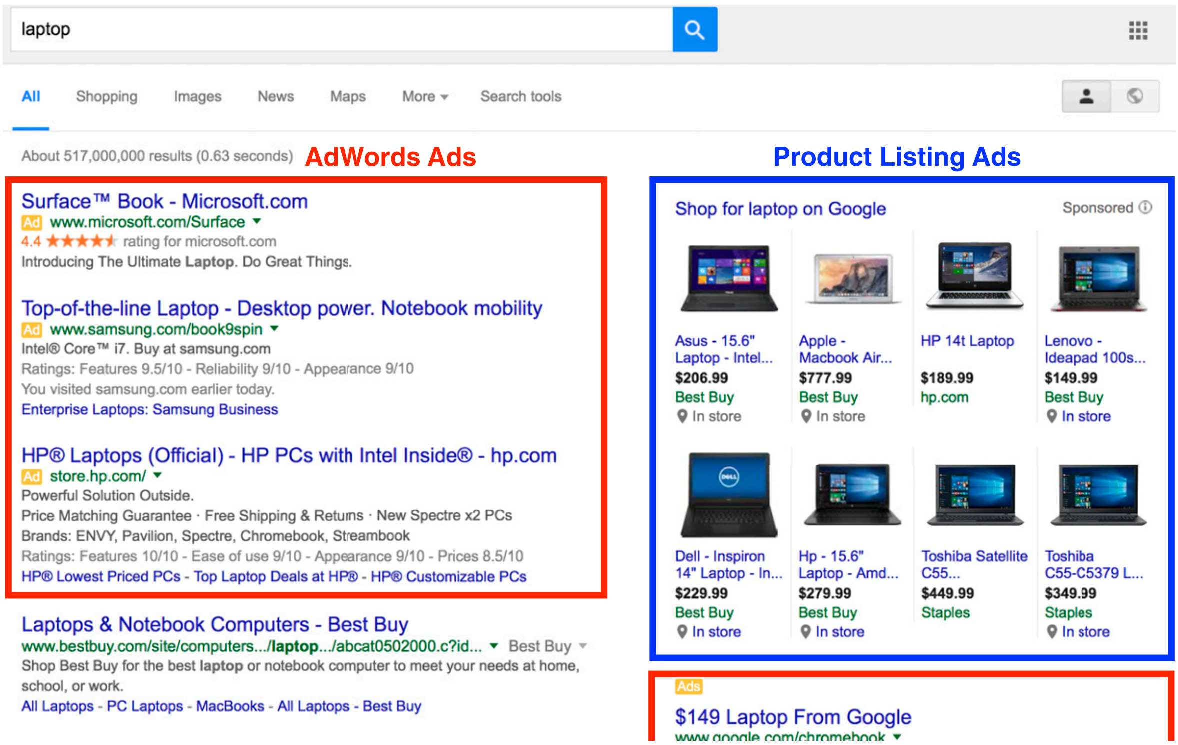
Keywords: search advertising • position auctions • cooperative advertising • channel coordination

1. Introduction

Search advertising is growing rapidly and has become a major advertising channel. In 2015, compared with a 6.0% growth of the entire advertising industry, search advertising grew fast at 16.2% and has reached a global expenditure of $80 billion dollars.[1] Retailing is a major contributor to search advertising. The number one spender on Google Adwords is Amazon, and five of the top 10 industries contributing most to Google Adwords are related to retailing, which together contribute more than one quarter of Google's revenue.[2] In all these industries, both brand owners (manufacturers) and retailers can advertise on the same keywords. For example, Figure 1 shows the advertisements on Google in one search query for "laptop."[3] In this example, some manufacturers advertise products via their own e-commerce sites (Microsoft Surface, Samsung, and Google Chromebook), some advertise products via retailers (Asus, Apple, Lenovo, Dell, and Toshiba), and one brand (HP) does both.

What lies behind the scene is the channel coordination between the manufacturers and retailers on search advertising. On one hand, manufacturers and their retailers compete directly with each other on search engine platforms; on the other hand, it is common for a manufacturer to coordinate with its retailers on search advertising spending. Specifically, a manufacturer can set up a "cooperative advertising" (co-op) fund and reimburse retailers when they advertise the manufacturer's products on search engines. According to one author's work experience at Walmart.com, many major brands provide co-op funds to Walmart.com for search ads. According to a survey from Borrell Associates in 2015, 70% of brand managers say that they offer digital marketing co-op programs. From 2012 to 2015, the percentage of local advertisers involved in digital marketing co-op programs increased from 25% to 61%. Among all digital advertising categories, search advertising ranked number one for the highest impact in the minds of both brand managers (68%) and local advertisers (62%), and 83% of brand managers and 85% of local advertisers agree that the use of search and display advertising is important in supporting the brand. Although 29% of brand managers said that they are now actually supporting paid search advertising in their co-op programs, the report by Borrell Associates points out that, "search advertising needs to be a far bigger part of the digital side of co-op."[4]

Figure 1. (Color online) An Example of Google Ads for "Laptop"



Cooperative advertising is not new, and it prevails in traditional media (Berger 1972, Bergen and John 1997). For example, manufacturers of fast-moving consumer goods can promote their products and get them better displayed on shelves by subsidizing supermarkets; phone manufacturers can get their products featured in the mobile carriers' TV commercials by cooperating with them on ad spending. The annual total spending on cooperative advertising is estimated to exceed \$50 billion in the United States alone.[5] However, distinct market and institutional features of the search advertising industry make cooperative search advertising very different from cooperative advertising in traditional media.

A prominent feature of search advertising is that the market is highly concentrated, and thus, advertisers face stronger competition. In the United States, the market is dominated by three players: Google, Bing, and Yahoo with Google taking more than 60% of the market share. For any business trying to reach customers via search ads, it is almost imperative to use Google's service. As a result, a manufacturer will inevitably compete with its own retailers directly for consumers' clicks on Google. In contrast, in traditional media, a manufacturer and its retailers face much weaker competition on advertisements with each other as their ads can appear in various media outlets. Furthermore, in traditional advertising, advertisements from manufacturers and retailers often serve different purposes and have positive spillovers (Bergen and John 1997). For example, a manufacturer can make national ads to increase brand awareness, and its retailers can make local ads about the product availability, price, promotion, etc. A consumer who has watched a TV ad from the manufacturer may choose to buy the product from a retailer, and a consumer may see ads from retailer A but make the purchase from retailer B. In contrast, in paid search advertising, both a manufacturer and its retailers' ads are mainly made to drive conversions. Because their differentiation becomes much smaller, the competition becomes stronger.

Another prominent feature is that search engine platforms sell keywords via generalized second-price position auctions, and this becomes crucial for cooperative advertising because of the high market concentration of the search advertising industry. It is important for advertisers to understand the auction mechanism when formulating their competitive strategies on search advertising. In the position auctions, a manufacturer and its retailers compete directly in bidding for a better position. One advertiser's higher bid will either increase other advertisers' costs or decrease their demands by moving them to less attractive positions. As a result, a higher advertising cost does not necessarily lead to larger demand but can be burning the manufacturer's own money. This is in stark contrast with the case in traditional advertising, in which ads from a manufacturer and its retailers usually complement each other.

These new features of cooperative search advertising have been noticed by brand managers and potentially prevent them from operating cooperative search advertising effectively. The survey by Borell indicated that brand managers and local advertisers are not participating more heavily in co-op search advertising because of the complexity of digital co-op programs and lack of understanding of the advertising landscape, leaving more advertising dollars—about $14 billion—unused. A report regarding digital cooperative advertising by the Interactive Advertising Bureau also pointed out that the key barriers to online co-op advertising are the complexity of digital channels, lack of knowledge required to advertise in digital channels, and lack of guidelines and requirements. They quoted from a senior manager of search engine and mobile marketing at HP who said, "We're driving each other's bidding up. HP's perspective is we don't think co-op is that positive if we're all going after the same term."[6]

In this paper, we build a game-theoretic model to investigate how a manufacturer and its retailers should coordinate in search advertising and prescribe the manufacturer's optimal cooperative search advertising strategy. Specifically, we aim to answer the following research questions: (1) Should a manufacturer cooperate with all its retailers? Is it indeed burning its own money by invoking competition on the search advertising platform? (2) If a manufacturer should not cooperate with all retailers, which one(s) should it cooperate with? (3) Given the profit margin via direct sales is often higher than that via retailers, should a manufacturer advertise directly to consumers instead of via retailers? (4) How does retailers' price competition influence the manufacturer's cooperative advertising strategy? (5) How to coordinate the channel by using both wholesale and cooperative advertising contracts?

Throughout the paper, we consider the setting in which one manufacturer sells via two retailers. This is the simplest setting under which we can study a manufacturer's choice of how many and which retailer(s) to cooperate with on search advertising. We build our basic model based on the assumption of exogenous wholesale contracts and retail prices. This is a reasonable assumption when search advertising is only part of the demand source, and thus, it is reasonable to assume that prices and wholesale contracts have been determined before the manufacturer and retailers make decisions on search advertising. We consider a simple coordination mechanism where a manufacturer covers a fixed percentage—the so-called *participation rate*—of a retailer's spending on search advertising. This coordination mechanism has been widely accepted in the industry as well as in previous literature (Dutta et al. 1995, Bergen and John 1997, Nagler 2006). The game proceeds as follows. First, a manufacturer determines the participation rate for each retailer. Then retailers and other advertisers submit their bids to a search engine platform. Finally, the auction outcome realizes. Following the literature of position auctions (Edelman et al. 2007, Varian 2007), we do not explicitly model consumers' searching and clicking behaviors but assume an exogenous click-through rate for each position, which is independent of advertisers' identities (an extension that allows the click-through rates to depend on advertiser identities is investigated in Section 4.1). We focus on the intrabrand coordination and competition among a manufacturer and its retailers and also account for interbrand competition by incorporating outside advertisers in the position auction.

The basic model (Section 2) considers the case in which the manufacturer cannot directly participate in search advertising.[7] This enables us to focus on the manufacturer's problem of how many retailers to sponsor, which retailer(s) to sponsor, and how much sponsorship to provide. In determining how many retailers with which to cooperate, a manufacturer makes a trade-off between larger demand brought by more retailers and higher bidding cost associated with intensified competition in bidding. We find that it may not be optimal for a manufacturer to cooperate with all the retailers even if they are ex ante the same. This finding is in contrast with that for traditional cooperative advertising with which a manufacturer should provide identical co-op plans to ex ante symmetric retailers (Bergen and John 1997).

With two asymmetric retailers, the manufacturer should support the retailer with higher channel profit per click to get a higher position than the other retailer. That is, when deciding the two retailers' relative positions, the manufacturer acts as if the channel is fully coordinated even though it is actually not. This demonstrates the effectiveness of this simple coordination mechanism and, thus, provides a rationale for its prevalence in industry. We discuss the optimality of the participation rate mechanism by considering the integrated channel as a benchmark and find that channel integration can lead to lower channel profit than the participation rate mechanism because the outside advertiser may strategically raise the bid knowing that the integrated channel has higher willingness to pay per click.

In Section 3, we study a more general setup in which the manufacturer can endogenously decide whether to participate in search advertising directly by bidding for its own website. The key finding in the basic model still holds; that is, the retailer with the higher channel profit per click still gets a higher position than the other retailer in equilibrium. We also find that when the manufacturer's profit per click via direct sales is higher than a retailer's channel profit per click, the manufacturer should get a higher position than the retailer. The manufacturer will definitely participate in the auction directly when the manufacturer's profit per click via direct sales is higher than both retailers' channel profit per click; otherwise, the manufacturer may or may not participate directly. Our result explains why we commonly observe retailers' search ads in practice given that manufacturers earn higher profits by selling directly to consumers. There are two reasons. First, a retailer may have a higher conversion rate, which means a higher return for each click. Second, selling through a retailer may result in a higher channel profit margin because retailers may operate more efficiently and have lower operational cost. When deciding whether to advertise directly to consumers or to advertise via retailers, the manufacturer should rely on the comparison of channel profits instead of its own profits. A retailer with high profit margin has a strong incentive to bid and, therefore, does not require much cooperative sponsorship from the manufacturer to win the search ad auction; on the other hand, if the retailer is not sponsored, it will become a strong competitor to the manufacturer, which makes the manufacturer's winning more costly.

We further consider three extensions to our basic model in Section 4. In Section 4.1, we extend our basic model by allowing the click-through rate to depend on the advertiser's identity. Our main results generalize nicely. With two retailers, the manufacturer will support the retailer with higher channel profit per *impression* to get a higher position than the other retailer.

In Section 4.2, we incorporate endogenous retail price with price competition. Consistent with the findings from the basic model, a manufacturer may optimally provide sponsorship to only one retailer given two ex ante symmetric retailers. Particularly, by sponsoring both retailers, the manufacturer encourages retail price competition, which lowers retail prices and increases demand. However, a lower retail price and, thus, a lower retail margin reduce retailers' incentives to bid high and win the position auctions. Consequently, the manufacturer has to provide more sponsorship to both retailers to get them displayed in the position auctions. Generally speaking, the manufacturer trades off demand expansion and sponsorship cost and chooses to sponsor only one retailer when the consumer heterogeneity between the two retailers is relatively small.

In Section 4.3, we endogenize both retail prices and wholesale contracts without explicitly considering retail price competition. We illustrate how a manufacturer can use the two devices—wholesale and cooperative advertising contracts—to coordinate the channel. We find that when the wholesale contracts are restricted to be linear, it can still be optimal for a manufacturer to cooperate with one retailer on advertising, but it is never optimal to support both retailers. When the wholesale contracts are two-part tariffs, we show that cooperative advertising is no longer needed. This is consistent with the general viewpoint that a sufficiently flexible wholesale contract can fully coordinate the channel, and there is no need for a manufacturer to cooperate with retailers separately on advertising. Nevertheless, we still observe that cooperative search advertising can play an important role in practice. The reason could be that search advertising is one of the demand sources, and changing wholesale contracts will affect the profits from other demand sources, or it could be that the market conditions of search advertising vary across time, whereas the wholesale contracts cannot be adjusted frequently.

To the best of our knowledge, this is the first paper that addresses cooperative search advertising. It is closely related to the literature of cooperative advertising in conventional channels (Berger 1972, Bergen and John 1997, Desai 1997, Kim and Staelin 1999, etc.). Berger (1972) is the earliest work, as far as we know, that analytically investigates traditional cooperative advertising. It solves for the best cooperative advertising plan for one manufacturer and one retailer, treating co-op funds as price subsidies. Desai (1997) studies a different mechanism for cooperative advertising: he shows that a franchisor can charge an advertising fee from each franchisee and decide where and how the advertising dollars are spent, thus overcoming free-riding problems among franchisees. Our paper is closest to Bergen and John (1997) in terms of two aspects: (1) we both consider the participation rate mechanism, and (2) we both assume that retailers are independent—manufacturers cannot directly control their pricing and advertising decisions, and they cannot be forced to participate in a co-op plan. However, Bergen and John (1997) show that a manufacturer will provide identical co-op plans to ex ante symmetric retailers, whereas we find that, in cooperative search advertising, it can be optimal for the manufacturer to sponsor only one of two retailers even when they are symmetric. The distinct results are driven by the distinct market and institutional features of the search advertising industry discussed earlier.

This paper also relates to a large literature of competitive strategies in search advertising. Existing theoretical works have studied the impact of click fraud on advertisers' bidding strategies and the search engine's revenue (Wilbur and Zhu 2009), the interaction between firms' advertising auction and price competition (Xu et al. 2011), the interplay between organic and sponsored links (Katona and Sarvary 2010), the bidding strategies of vertically differentiated firms (Jerath et al. 2011), the competitive poaching strategy (Desai et al. 2014, Sayedi et al. 2014), the impact of advertisers' budget constraints on their own profits and the platform's revenue (Lu et al. 2015), the effect of real-time bidding on advertisers' strategies and profits (Sayedi 2018), etc. We contribute to the literature by incorporating channel coordination in search advertising for the first time, as far as we know.

Finally, we contribute to the literature about position auctions. The auction mechanism design and equilibrium properties have been investigated extensively (Edelman et al. 2007, Varian 2007, Feng 2008, Athey and Ellison 2011, Chen and He 2011, Zhu and Wilbur 2011, Dellarocas 2012, etc.), but these studies all assume that bidders are independent. In our setting, a manufacturer's profit comes from not only its own website, but also its retailers' websites, so the bidders are not independent from each other anymore. Our analysis of the position auctions with nonindependent bidders makes a contribution to this stream of literature. There are also recent works investigating collusive bidding behaviors (Decarolis and Rovigatti 2017, Decarolis et al. 2017), in which competing bidders delegate their bidding decisions to a common marketing agency. Different from our paper, these papers have not considered vertical relationships in distribution channels.

The paper unfolds as follows. In Section 2, we lay out and analyze our basic model. We incorporate the manufacturer's direct participation in search advertising in Section 3, and consider three extensions from Section 4.1 to 4.3. Finally, Section 5 concludes the paper.

2. Basic Model

2.1. Position Auctions

We first introduce the assumptions on position auctions and also briefly review the equilibrium analysis of position auctions closely following Varian (2007).

We consider a generalized second-price (GSP) position auction with two positions. The highest bidder wins the first position and pays the second highest bid; the second highest bidder wins the second position and pays the third highest bid; all other bidders with lower bids will not get a position nor clicks and pay zero.[8] We consider a pay-per-click mechanism, which has been widely adopted in the industry. The click-through rate (CTR) of the ith position is denoted as d_i $(i = 1, 2)$, which is defined as the fraction of clicks on this position out of all impressions displayed to consumers. Suppose there are $n \geq 3$ bidders in the market. To simplify notations, we can view the current position auction equivalently as the one with n positions in which the positions three up to n have zero CTR; that is, $d_i = 0$ for $i = 3 \ldots n$. A higher position is assumed to have a higher CTR; that is, $d_1 \geq d_2 \geq 0$.[9] Furthermore, the CTR for each position is assumed to be independent of the identity of the advertiser who takes that position. We relax this assumption in Section 4.1. Following Varian (2007) and Edelman et al. (2007), we assume the auction is a complete-information simultaneous game, in which each bidder knows the others' valuations or payoffs per click. This assumption can be justified by considering that bidding takes place frequently, and as a result, after many rounds of bidding, the bidders will be able to infer each other's valuations.

It turns out that there are infinite Nash equilibria for the position auction. Varian (2007) and Edelman et al. (2007) have come up with some equilibrium refinement rules. We recap their results with two positions and n independent bidders $(n \geq 3)$ as follows. Suppose the n independent bidders have payoffs per click $v_1 > v_2 > \ldots > v_n$. In equilibrium, bidder i will get the ith position $(i = 1, \ldots, n)$. The equilibrium bids by bidder $i \geq 2$ are, respectively,

$$b_2^* = \frac{d_1 - d_2}{d_1} v_2 + \frac{d_2}{d_1} v_3, \tag{1}$$

$$b_i^* = v_i, \; i = 3, \ldots, n, \tag{2}$$

and bidder 1's equilibrium bid $b_1^* \geq b_2^*$.

Because we analyze equilibria for more complex bidding games with nonindependent bidders later, it is worthwhile to understand the aforementioned result. Let us first consider bidder 2's one potential deviation: by bidding higher, it may be able to get the first position. To guard against this deviation, we must have $d_2(v_2 - b_3) \geq d_1(v_2 - b_1)$. Varian (2007) proposed the concept of symmetric Nash equilibrium (SNE) by requiring that $d_2(v_2 - b_3) \geq d_1(v_2 - b_2)$. This is a stronger condition because $b_1 \geq b_2$; hence, the SNE is a subset of Nash equilibria (NE). A nice property of SNE is that bidders with higher payoff per click will always get a higher position. Varian (2007) further proposed the equilibrium selection criterion LB (short for *lower bound*), which selects the lowest possible bids from SNE.[10] The LB rule implies that $d_1(v_2 - b_2) \geq d_2(v_2 - b_3)$. The interpretation of this requirement is that, if it happens that bidder 1 has bid so low that bidder 2 slightly exceeded bidder 1's bid and moved up to the first position, bidder 2 would earn at least as much profit as it makes now at the second position. Combining the SNE and LB criteria, we have $d_2(v_2 - b_3^*) = d_1(v_2 - b_2^*)$, from which we can get the expression of b_2^* in Equation (1). One can verify that $b_i^* = v_i$ for $i \geq 3$ is also an SNE and satisfies the LB equilibrium selection rule. Therefore, under Varian's (2007) SNE and LB equilibrium selection criteria, the bidders that do not win a position will bid its true valuation, and the bidders who get a position will underbid.

2.2. Coordination Game

After introducing the position auctions previously, let us continue to specify the assumptions on the game of channel coordination. Consider a channel with one manufacturer, who produces and sells one product via two retailers. In the basic model, we assume that the manufacturer does not sell directly to consumers and, thus, does not participate in search ad auctions directly. This assumption not only simplifies the equilibrium analysis of the basic model, but it also helps isolate the manufacturer's key managerial question of how many and with which retailer(s) to cooperate in search advertising. In Section 3, we relax this assumption by allowing the manufacturer to sell directly to consumers and consider its endogenous participation in the position auction.

The manufacturer first signs a wholesale contract with each retailer. The wholesale contract between the manufacturer and each retailer i essentially determines the manufacturer's and retailer i's profit margins for each product sold via the retailer, which are denoted as m_i and r_i, respectively; m_i and r_i are assumed to be exogenously given when the manufacturer and retailers make their search advertising decisions. In reality, the manufacturer and retailers may sell products via both online and offline channels, and even for the online channel, search advertising is only one of the demand sources, so it is reasonable to assume that the wholesale and retail prices have been determined before they make search advertising decisions. We consider endogenous retail prices and wholesale contracts in Section 4.3.

The conversion rate at retailer i's site is denoted as θ_i, which means that θ_i fraction out of all clicks on the retailer's site will convert into purchases eventually. We assume that the conversion rate is independent of the position and, hence, the bid of the sponsored link. This assumption is consistent with some recent empirical findings (e.g., Narayanan and Kalyanam 2015). Given these assumptions, retailer i's profit per click is $\theta_i r_i$, the manufacturer's profit per click is $\theta_i m_i$, and the channel profit per click is $\theta_i(m_i + r_i)$.

We incorporate interbrand competition in the game by introducing one strategic outside advertiser representing other brands or other retailers. The outside advertiser is denoted as A, and its profit per click is assumed to be v_A. To summarize, in our basic model, there are in total $n = 3$ bidders: two retailers and one outside advertiser.

We consider the following game. First, the manufacturer decides the participation rate $\alpha_i \in [0, 1)$ for each retailer $i = 1, 2$, which means that the manufacturer will contribute α_i percentage of retailer i's spending on search advertising, and retailer i only needs to pay the remaining $1 - \alpha_i$ percentage. Second, each retailer i decides its bid b_i, and outside advertiser A decides its bid b_A. Finally, given everyone's bid, the auction outcome realizes, and the advertisers' demands and profits realize.

2.3. Equilibrium Analysis

We can solve the game by backward induction.

We first consider each retailer's bidding strategy. The following lemma characterizes how the participation rate α_i from the manufacturer changes retailer i's bidding strategy.

Lemma 1. *Given the manufacturer's participation rate α_i, retailer i's equivalent profit per click in the position auction will be $\theta_i r_i/(1 - \alpha_i)$. In other words, its bidding strategy will be the same as if its profit per click was $\theta_i r_i/(1 - \alpha_i)$ and there was no support from the manufacturer.*

Proof. Suppose a retailer will get position i and pay p_i in equilibrium. The Nash equilibrium condition that guards against the retailer deviating to position $j \neq i$ is that

$$d_i[\theta_i r_i - (1 - \alpha_i)p_i] \geq d_j[\theta_i r_i - (1 - \alpha_i)p_j], \quad (3)$$

which is equivalent to

$$d_i[\theta_i r_i/(1 - \alpha_i) - p_i] \geq d_j[\theta_i r_i/(1 - \alpha_i) - p_j]. \quad (4)$$

Similarly, we can write down and transform the SNE and LB conditions. Therefore, the retailer's equilibrium bid will be the same as if its profit per click was $\theta_i r_i/(1 - \alpha_i)$ and there was no support from the manufacturer. ∎

Lemma 1 shows that, by choosing participation rate α_i, the manufacturer essentially determines retailer i's equivalent profit per click on $[\theta_i r_i, +\infty)$. The manufacturer can incentivize retailer i to bid as high as possible by choosing α_i close to one; however, the manufacturer is not able to force the retailer to bid lower than $\theta_i r_i$ because $\alpha_i \geq 0$. We discuss the advantage and disadvantage of using negative α_i to coordinate channel in Section 2.4. Another implication of Lemma 1 is that, in the position auction, outside bidders do not need to observe the participation rate α_i or the profit per click $\theta_i r_i$; instead, they only need to know each retailer's equivalent profit per click $\theta_i r_i/(1 - \alpha_i)$, which has been assumed to be observable because of repeated bidding. Therefore, the results of our model do not rely on the observability of the channel coordination contract α_i, which has been shown to be a critical assumption that greatly influences the equilibrium channel structure (Coughlan and Wernerfelt 1989).

Before continuing to analyze the manufacturer's strategy in equilibrium, we set a restriction on the parameter space, which will facilitate the equilibrium analysis. We assume that $v_A > \theta_i r_i$ $(i = 1, 2)$. Under this condition, the two retailers are not able to win position 1 without the manufacturer's support, and different levels of participation rates can potentially move retailers to different positions. This is the most interesting case to study because it allows us to investigate whether it is optimal for the manufacturer to sponsor one or both retailers and how much support it should provide to each retailer. We analyze the case in which v_A is between $\theta_1 r_1$ and $\theta_2 r_2$ later as a robustness check. The case with v_A below both $\theta_1 r_1$ and $\theta_2 r_2$ is trivial because, in this case, the two retailers can get the first and second positions even without the manufacturer's sponsorship.

Given the manufacturer's participation rates α_1 and α_2, the position rank of the three bidders is determined by the order of $\theta_1 r_1/(1-\alpha_1)$, $\theta_2 r_2/(1-\alpha_2)$, and v_A. This results in six possible position configurations. For each position configuration, we can write down the manufacturer's profit function and maximize it with respect to α_1 and α_2. Then we compare the manufacturer's profits under all six position configurations to determine the manufacturer's optimal choice of participation rates α_1^* and α_2^*.

We use an array (X, Y) to denote a position configuration in which bidder X takes the first position and bidder Y takes the second position. Without loss of generality, we adopt the following tie-breaking rule: when R_1 bids the same as R_2 or A, R_1 wins; when R_2 bids the same as A, R_2 wins.

We first consider position configuration (R_1, R_2), that is, the configuration in which retailer 1 gets position 1 and retailer 2 gets position 2. According to the result in Section 2.1, we need to have $\theta_1 r_1/(1-\alpha_1) \geq \theta_2 r_2/(1-\alpha_2) \geq v_A$ for this position configuration to be the equilibrium. The outside advertiser's equilibrium bid is v_A, and retailer 2's equilibrium bid is $(d_1-d_2)/d_1 \cdot (\theta_2 r_2)/(1-\alpha_2) + d_2/d_1 v_A$. The manufacturer's profit will be

$$\pi_M(\alpha_1,\alpha_2) = d_1\left[\theta_1 m_1 - \alpha_1\left(\frac{d_1-d_2}{d_1}\frac{\theta_2 r_2}{1-\alpha_2} + \frac{d_2}{d_1}v_A\right)\right] + d_2(\theta_2 m_2 - \alpha_2 v_A), \tag{5}$$

which decreases with both α_1 and α_2. Therefore, the manufacturer will choose the smallest participation rates that ensure $\theta_1 r_1/(1-\alpha_1) \geq \theta_2 r_2/(1-\alpha_2) \geq v_A$, which are

$$\alpha_1^* = 1 - \frac{\theta_1 r_1}{v_A}, \tag{6}$$

$$\alpha_2^* = 1 - \frac{\theta_2 r_2}{v_A}. \tag{7}$$

Correspondingly, the manufacturer's profit under the optimal participation rates will be

$$\pi_M(\alpha_1^*,\alpha_2^*) = d_1[\theta_1(m_1+r_1) - v_A] + d_2[\theta_2(m_2+r_2) - v_A]. \tag{8}$$

Following the same procedure, we can work out the optimal participation rates and the manufacturer's profit under the other five position configurations. By comparing the manufacturer's profits among all six position configurations, we can determine the equilibrium position configuration. The following theorem summarizes the equilibrium outcome, with the proof in the appendix.

Theorem 1. *Consider a position auction, the participants of which consist of a manufacturer's two retailers and an outside advertiser. The manufacturer supports retailers in search advertising by sharing a fraction of each retailer's advertising expense. Assume that $v_A > \theta_i r_i$ for $i = 1, 2$.*

In equilibrium, the retailer i with a higher channel profit per click $\theta_i(m_i + r_i)$ will take a higher position than the other retailer. If it is optimal for the manufacturer to support only one retailer, it will choose to support this retailer.

Suppose $\theta_1(m_1+r_1) \geq \theta_2(m_2+r_2)$ without loss of generality; then the possible position configurations are (R_1, R_2), (R_1, A), and (A, R_1).

- *The equilibrium position configuration is (R_1, R_2) if and only if*

$$\theta_2 m_2 \geq v_A - \theta_2 r_2 + \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A}, \text{ and}$$

$$\frac{d_1-d_2}{d_2}\theta_1 m_1 + \theta_2 m_2 \geq \frac{d_1+d_2}{d_2}v_A - \frac{d_1-d_2}{d_2}\theta_1 r_1 - \theta_2 r_2 - \max\{\theta_1 r_1, \theta_2 r_2\}.$$

The corresponding equilibrium participation rates are $\alpha_i^* = 1 - \theta_i r_i/v_A$ for $i = 1, 2$.

- *The equilibrium position configuration is (R_1, A) if and only if*

$$\frac{d_1-d_2}{d_2}\theta_1 m_1 \geq \frac{d_1-d_2}{d_2}v_A + \min\{\theta_1 r_1, \theta_2 r_2\} - \frac{d_1-d_2}{d_2}\theta_i r_i - \frac{\theta_1 r_1 \cdot \theta_2 r_2}{v_A}, \text{ and}$$

$$\theta_2 m_2 < v_A - \theta_2 r_2 + \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A}.$$

The corresponding equilibrium participation rates are $\alpha_1^* = 1 - \theta_1 r_1/v_A$ and $\alpha_2^* = 0$ for $i \neq j = 1, 2$.

- *The equilibrium position configuration is (A, R_1) if and only if*

$$\frac{d_1-d_2}{d_2}\theta_1 m_1 < \frac{d_1-d_2}{d_2}v_A + \min\{\theta_1 r_1, \theta_2 r_2\} - \frac{d_1-d_2}{d_2}\theta_1 r_1 - \frac{\theta_1 r_1 \cdot \theta_2 r_2}{v_A}, \text{ and}$$

$$\frac{d_1-d_2}{d_2}\theta_1 m_1 + \theta_2 m_2 < \frac{d_1+d_2}{d_2}v_A - \frac{d_1-d_2}{d_2}\theta_1 r_1 - \theta_2 r_2 - \max\{\theta_1 r_1, \theta_2 r_2\}.$$

The corresponding equilibrium participation rates are $\alpha_1^* = \max\{1 - \theta_1 r_1/(\theta_2 r_2), 0\}$ and $\alpha_2^* = 0$ for $i \neq j = 1, 2$.

The theorem completely characterizes the manufacturer's optimal sponsorship strategy by essentially providing answers to three managerial questions: (1) which retailer to sponsor, (2) how much sponsorship to provide, and (3) how many retailers to sponsor. We discuss the implications of Theorem 1 on these three questions one by one.

Regarding the first question, the theorem shows that the two retailers' relative positions in equilibrium are entirely determined by their channel profits per click. When deciding which retailer to support, the manufacturer should rely its decision on the *channel profit per click* instead of its own profit per click. In this sense, the manufacturer acts as if the channel is fully coordinated when deciding the two retailers' *relative* positions. The intuition is that, in cooperative search advertising, the manufacturer needs to consider not only its own profit per click, but also how much it needs to pay for the retailer to get a good position. Hence, there are two reasons why not only the manufacturer's own profit per click, but also the retailers' profits per click, matter. First, when a retailer's own profit per click is relatively high, it already has relatively high willingness to pay for the position, and thus, the manufacturer can sponsor this retailer to get a higher position with a relatively low cost. Second, if the manufacturer chooses to sponsor a retailer with relatively low profit per click, the other retailer with relatively high profit per click will become a strong competitor in the position auction, which will raise the bidding cost and, thus, the manufacturer's sponsorship cost. Taking both rationales into consideration, the manufacturer should sponsor the retailer with a higher channel profit per click to get a higher position.[11]

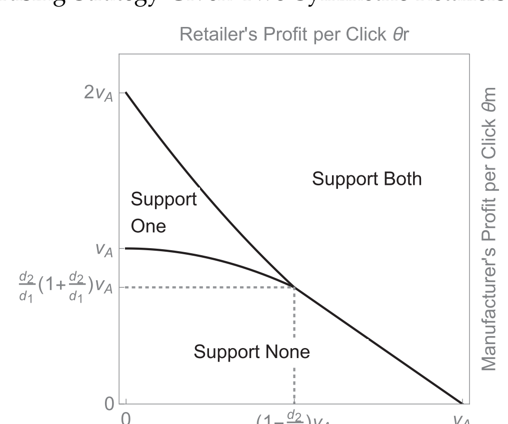
We should notice that we obtain these results based on the assumption that $v_A \geq \theta_i r_i$ $(i = 1, 2)$. We also consider the case in which v_A is between $\theta_1 r_1$ and $\theta_2 r_2$ with details of the analysis relegated to the online appendix. We find that given $\theta_i r_i \geq v_A \geq \theta_j r_j$, retailer i will get a higher position when $\theta_i(m_i + r_i) \geq \theta_j(m_j + r_j)$, but retailer j may not get a higher position when $\theta_j(m_j + r_j) > \theta_i(m_i + r_i)$. In other words, in this case, a retailer with higher channel profit per click may not necessarily get a higher position, yet a retailer with both higher total channel profit and higher own profit per click will always get a higher position than the other retailer. Therefore, the results in Theorem 1 rely on the assumption that $v_A \geq \theta_i r_i$ $(i = 1, 2)$, which we do think is a more reasonable case to study. This is because, given $\theta_i r_i \geq v_A \geq \theta_j r_j$, the manufacturer will never provide a positive participation rate to retailer i and, thus, will have no control of this retailer's bidding. It only needs to decide whether and how much to support retailer j. Thus, retailer i is like an outside advertiser that is not of the manufacturer's concern in terms of cooperative advertising. In contrast, when $v_A \geq \theta_i r_i$ $(i = 1, 2)$, the manufacturer will consider supporting both retailers under some circumstances.

Regarding the second question about optimal sponsorship rates, we interpret Theorem 1 under each position configuration, respectively. In configuration (R_1, R_2), the manufacturer provides positive participation rates to both retailers. Specifically, it carefully chooses the participation rates such that the equivalent profits per click for both retailers are equal to v_A, which is also the price per click for both positions. As a result, both retailers earn zero profit, and the manufacturer collects the entire channel profit. In configuration (R_1, A), the manufacturer only provides support to retailer 1. It will not provide a positive participation rate to retailer 2 because that will raise the equilibrium bid of advertiser A and, thus, increase the price per click for retailer 1. In configuration (A, R_1), the manufacturer still provides a zero participation rate to retailer 2, and it needs to provide a positive participation rate to retailer 1 only when this retailer's own profit is lower than that of retailer 2.

Finally, to answer the third question about how many retailers to sponsor, Theorem 1 describes the exact conditions for the manufacturer to sponsor two, one, or zero retailer(s) given two asymmetric retailers. The key trade-off here is higher demand versus higher bidding cost resulting from intensified competition. Specifically, by supporting more retailers, the manufacturer will get more demand, but at the same time, the bidding costs will go up as retailers now compete with each other by bidding higher. To understand the key trade-off more intuitively, we consider a special case in which the two retailers are ex ante symmetric, that is, $\theta_1 = \theta_2 = \theta$, $m_1 = m_2 = m$, and $r_1 = r_2 = r$, and plot Figure 2, which completely characterizes a manufacturer's optimal cooperative search advertising strategy for two symmetric retailers.

Figure 2 indicates that the manufacturer provides positive participation rates to both retailers when the channel profit per click is relatively high; it provides support to neither retailer if its own profit per click and each retailer's profit per click are both relatively low. More interestingly, we find that the manufacturer will provide a positive participation rate to only one retailer when its profit per click is relatively high but the retailers' profit per click is relatively low. In this case, retailers need high participation rates to move up to a higher position, but it is too expensive for the manufacturer to support both retailers. For

Figure 2. Manufacturer's Optimal Cooperative Search Advertising Strategy Given Two Symmetric Retailers



cooperative advertising, it has been noticed that the manufacturer should provide identical co-op programs to symmetric retailers (Bergen and John 1997), whereas our finding indicates that the manufacturer may optimally offer the co-op program only to a subset of retailers even if they are ex ante symmetric. The reason is that, in traditional advertising, ads from different retailers usually appear in different places and have positive spillovers; that is, a consumer may see ads from retailer A but make the purchase from retailer B. In search advertising, different retailers compete with each other directly in the position auction, and their competition for demand also becomes stronger because of smaller differentiation. Therefore, the manufacturer takes hold of the cooperative advertising money to reduce potential competition among retailers.

2.4. Integrated Channel

It is worthwhile to compare our basic model with the case of channel integration, in which the manufacturer has full control of both retailers' bids. First, we notice that the integration does not necessarily generate a higher channel profit (McGuire and Staelin 1983) because there is a strategic outsider advertiser who can choose its bid based on whether the channel is integrated. If the outside advertiser chooses to bid higher when the channel is integrated, the bidding cost for the integrated channel can get higher, and the channel profit can get lower. This is exactly what we find under certain circumstances.

When the channel is integrated, the two retailers are no longer independent bidders in the auction. The integrated channel as a whole gets profits from two retailers, and one retailer's bid will influence the channel profit by affecting the other retailer's cost per click even when the two retailers' positions are given, which differs from the case of independent bidders when each bidder's bid does not affect its own profit given its position. As a result, for any given position configuration, we cannot directly apply the result in Section 2.1 to determine each player's equilibrium bid at this position configuration. Instead, we use the following rule to determine each player's equilibrium bids at a given position configuration.

- For the outside advertiser, because it is still independent from other players, we still use Varian's (2007) SNE and LB equilibrium selection rules to determine its equilibrium bid.[12]
- For the two retailers, the SNE rule is no longer a sensible refinement rule to select equilibrium bids at a given position configuration. This is because SNE posits a stronger condition than NE, which raises the bid of the retailer at the lower position. As the lower-ranked retailer's bid gets higher, the higher-ranked retailer's cost per click increases and its profit decreases. Therefore, equilibrium bids selected by SNE do not maximize the channel profit, and the channel can deviate to other bids that lead to higher channel profit. Thus, the bids selected by SNE can fail to be an NE. Therefore, we use a new equilibrium selection rule—*profit maximization* (PM) criterion—to determine the two retailers' equilibrium bids at a given position configuration. Particularly, given a position configuration, among all bids b_1 and b_2 that satisfy the Nash equilibrium condition, we select the equilibrium that maximizes the integrated channel's profit. Given that our objective is to show that, under some circumstances, the integrated channel can result in a lower channel profit than a nonintegrated channel, the PM rule is conservative in that it selects the equilibrium with the maximum integrated channel profit.[13]

We determine the equilibrium position configuration in the following way. For a position auction with two retailers and one outside advertiser, there are six possible position configurations in total. For each position configuration, we determine the integrated channel and the outside advertiser's equilibrium bids, using the selection rules we described previously. Given that the integrated channel's objective is to maximize profit and it must have no incentive to deviate to any other position configuration, we can get the Nash equilibrium condition, that is, the parameter space that this position configuration can hold in equilibrium. It turns out that the Nash equilibrium conditions for the six position configurations are not mutually exclusive. For the parameter setting in which only one position configuration can exist, this position configuration will be the one in equilibrium. For the parameter setting in which multiple position configurations coexist, the integrated channel will select the equilibrium position configuration as the one with the highest channel profit. The idea is that the integrated channel can choose which position configuration to be in equilibrium by choosing its bids b_1 and b_2.

The detailed analysis is in the appendix. We summarize the main findings as follows.

Proposition 1. *Consider a channel with one manufacturer and two retailers. The two retailers and one outside advertiser A compete for two positions. Assume $v_A > \theta_i r_1$ for $i = 1, 2$.*

- *In both integrated and nonintegrated channels, the retailer with a higher channel profit per click $\theta_i(m_i + r_i)$ gets a higher position than the other retailer in equilibrium.*
- *The channel profit is the same for the integrated and nonintegrated channels when the two retailers win both positions.*
- *Under certain conditions, the channel profit could be lower in the integrated channel than in the nonintegrated channel.*[14]

The first two findings suggest the efficiency of the participation rate mechanism. We have discussed the intuition for the first finding. The intuition for the second finding is as follows. For the integrated channel, when the two retailers get the two positions, the channel maximizes channel profit by letting the retailer who takes the second position submit the lowest possible bid v_A. For the nonintegrated channel, the manufacturer can choose the participation rates so that the equivalent profit per click for both retailers are v_A and the retailer who takes the second position bids exactly v_A, and as a result, the manufacturer collects the total channel profit by itself, which is the same as that in the integrated case.

The third finding is surprising. It implies that it is not an easy task to come up with a "first best" benchmark that maximizes the channel profit. That is the reason why we instead focus on a simple and widely used coordination mechanism: participation rates. The intuition for this finding is as follows. Suppose $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$. When the channel is integrated, retailer 2's profit per click is $\theta_2(m_2 + r_2)$, whereas when the channel is nonintegrated, retailer 2's profit per click is $\theta_2 r_2$ if there is no sponsorship from the manufacturer. At the position configuration (R_1, A), retailer 2 in the integrated channel will have a higher incentive to move up to the second position compared with the nonintegrated channel because it has a higher profit per click. Knowing this, the outside advertiser A will bid higher in equilibrium to keep its position. As a result, the cost per click for retailer 1 at the first position will be higher when the channel is integrated, which leads to a lower total channel profit.

Given this finding, we can further consider whether there is a more efficient way to coordinate the channel than the channel integration. The answer is yes. In our participation-rate mechanism, we restrict that the participation rate α_i must be nonnegative. If we allow α_i to be negative, which means that the retailer needs to pay the manufacturer a certain proportion of its spending on search advertising, then under position configuration (R_1, A) and (A, R_1), the manufacturer can force R_2 to bid zero by choosing $\alpha_2 \to -\infty$. This minimizes the cost per click for R_1 and achieves a higher channel profit. However, it is worthwhile noticing that negative participation rates increase the channel profit at the price of a lower retailer profit. Therefore, retailers may have no incentive to participate in a cooperative advertising program with negative participation rates unless more complex contracts are involved, such as two-part tariffs. In contrast, by restricting the participation rates to be nonnegative in our basic model, we guarantee that both the manufacturer and retailers have the incentive to participate in the cooperative advertising program.

3. Manufacturer's Direct Participation in Search Advertising

In the basic model, we have assumed that the manufacturer does not participate in the position auction directly so that we can focus on the manufacturer's decision of which retailer(s) to support and how much support to provide. In this section, we relax this assumption by allowing the manufacturer to participate in the search advertising directly. We show that the results from the basic model are robust. Moreover, we have some new insights about when the manufacturer wants to advertise directly and when to advertise via the retailer(s).

We have four bidders in the auction: one manufacturer, its two retailers, and an outside advertiser. We denote m_0 as the manufacturer's profit margin when selling directly to consumers, which could be either greater or less than $m_i + r_i$, the channel profit margin when selling via retailer i. We further denote θ_0 as the conversion rate at the manufacturer's site. Therefore, the manufacturer's profit per click on its own site is $\theta_0 m_0$. The manufacturer's bid is denoted as b_0.

The game still proceeds in two stages. In the first stage, the manufacturer not only decides the participation rate $\alpha_i \in [0, 1)$ for each retailer $i = 1, 2$, but also decides whether to participate in the auction directly in the second stage. We assume that the manufacturer can credibly commit *not* to participate in the auction.[15] The reason why the manufacturer may want to commit not to participate in the auction is that the manufacturer's participation may increase the outside advertiser's bid, leading to higher bidding costs and lower profit for the manufacturer. In the second stage, each retailer i decides its bid b_i, outside advertiser A decides its bid b_A, and the manufacturer decides its bid b_0 if it has decided to participate. Finally, given everyone's bid, the auction outcome realizes, and the advertisers' demands and profits realize.

In the position auction, if the manufacturer participates in bidding directly, the manufacturer makes profits from its own site as well as the retailers' sites; therefore, the manufacturer and retailers are nonindependent bidders in the auction. Similar to the case of channel integration in Section 2.4, we cannot directly apply the result in Section 2.1 to determine each player's equilibrium bid at any given position configuration. For the retailers and the outside advertiser, we still use Varian's (2007) SNE and LB equilibrium selection rules to determine their equilibrium bids because they make profits only from their own sites, and their choice of bids does not affect their own profits directly at any given position. However, for the manufacturer, its choice of bid does affect its profit by affecting the price per click on the retailers' sites when it is at a lower position than one or both retailers. In such case, the SNE is no longer a sensible criterion to select the manufacturer's bid because it fails to be an NE. Therefore, for the position configurations in which the manufacturer is below one or both retailers, we use the manufacturer's PM criterion in determining the manufacturer's equilibrium bid (we have introduced the PM criterion in Section 4.3). For the position configurations in which the manufacturer is above the retailer, the manufacturer's bid does not affect its profit directly, so we still use the SNE and LB criterion to select the manufacturer's bid.

We solve the game by backward induction. In the second stage, given the manufacturer's participation decision and participation rates, each bidder's (equivalent) profit per click will be determined. Given four bidders competing for two positions, there are a total of $4 \times 3 = 12$ possible position configurations. For each position configuration, we determine the bids of the manufacturer, the retailers, and the outside advertiser and the manufacturer's optimal participation rates α_1, α_2. If the manufacturer's optimal bid b_M^* is zero, then the manufacturer should commit to quit the bidding in the first stage. Back to the first stage, the manufacturer compares its profits over the 12 position configurations to determine the equilibrium. The equilibrium condition for one position configuration is that the manufacturer's profit under this position configuration is greater than that under all other position configurations. We find that when this equilibrium condition holds, the Nash equilibrium condition for this position configuration always holds. Similarly, we restrict our attention to the case with $\theta_i r_i < v_A$ and $\theta_i r_i < \theta_0 m_0$ for $i = 1, 2$. That is, without the manufacturer's support, neither retailer can get displayed. Again, this is the most interesting case because it can be optimal for the manufacturer to sponsor zero, one, or two retailers, and different levels of participation rates will move the retailers to different positions. The following theorem characterizes the equilibrium with proof in the appendix.

Theorem 2. *Consider a position auction, the participants of which consist of a manufacturer, its two retailers, and an outside advertiser. The manufacturer supports the two retailers in search advertising by providing certain participation rates. Assume that neither retailer can get displayed without support ($\theta_i r_i < v_A$, and $\theta_i r_i < \theta_0 m_0$ for both $i = 1, 2$).*

- *The retailer who has a higher channel profit per click will get a higher position than the other retailer in equilibrium.*
- *The manufacturer will take a higher position than a retailer when its profit per click via direct sales exceeds the channel profit per click of the retailer.*
- *The manufacturer will definitely participate in bidding when its profit per click is higher than both retailers' channel profits per click and may participate in bidding when its profit per click is lower than one or both retailers' channel profit per click but not too low.*
- *When v_A is relatively big or small, retailer i will get a higher position than the manufacturer as long as $\theta_i(m_i + r_i) \geq \theta_0 m_0$. When v_A is in the middle range, retailer i will get a higher position than the manufacturer when $\theta_i(m_i + r_i) - \theta_0 m_0$ exceeds a positive threshold.*

Theorem 2 shows that one key finding in the basic model remains true when the manufacturer is allowed to participate in the search ad auction directly; that is, the retailer with a higher channel profit per click will get a higher position than the other retailer in equilibrium. Furthermore, the manufacturer will get a higher position than both retailers if its profit per click via direct sales is higher than both retailers' channel profit per click. On the other hand, the manufacturer may want to sponsor one or both of the retailers to get a higher position than itself when they get a higher channel profit per click even though the manufacturer may earn a higher profit margin from direct sales than via the retailers (i.e., $m_0 > m_1, m_2$). There are two reasons for this. First, the conversion rate at the manufacturer's site may be lower than that at the retailers' sites, that is, $\theta_0 < \theta_i, i = 1, 2$, which may be due to a better design of the retailers' websites or consumer loyalty, etc. Second, the retailers's total channel profit may be higher than the manufacturer's profit via direct sales, that is, $(m_i + r_i) > m_0, i = 1, 2$. This can be due to a higher efficiency of the retailers' operations and logistics, etc.

Similar to the basic model, we find that in deciding whether to participate in search advertising directly or indirectly via retailers, the manufacturer compares its profit per click with the retailers' channel profit per click. The intuition is that when a retailer's profit per click $\theta_i r_i$ is high, the retailer will bid high without the manufacturer's support, so sponsoring this retailer to get a higher position is less costly. On the other hand, if the manufacturer wants to get a high position for its own site, it needs to bid higher than the retailer, which can be very costly. As a result, it can be optimal for the manufacturer to support the retailer(s) rather than to get a higher position by itself.

The manufacturer's equilibrium bidding and sponsoring strategies can be very complicated under a general setting as shown in the online appendix. Therefore, it is informative to consider a special case in which the two retailers are symmetric and their channel profit is the same as the manufacturer's profit (i.e., $\theta_1 = \theta_2 = \theta_0 = \theta$, $m_1 = m_2 = m$, $r_1 = r_2 = r$, and $m + r = m_0$). In this case, $(M, R), (R, M), (R, R)$ will lead to exactly the same profit for the manufacturer as long as the channel takes both positions; $(A, M), (A, R)$ will also lead to the same profit for the manufacturer as long as the channel takes the second position, whereas (M, A) will always be more profitable for the manufacturer than (R, A). Figure 3 illustrates the manufacturer's optimal bidding and sponsoring strategies.

We find that the manufacturer will sponsor one or both retailers if and only if $\theta m + 2\theta r \geq 2v_A$; otherwise, the manufacturer will bid directly and takes the first position when the channel profit per click is higher than the outside advertiser's profit per click, that is, $\theta(m+r) = \theta m_0 \geq v_A$, and the manufacturer will neither bid nor sponsor when $\theta(m+r) = \theta m_0 < v_A$. The result indicates the key role of channel profit per click in determining the relative position rank and also implies that there exists the situation in which the manufacturer sponsors a retailer and bid by itself at the same time (under position configurations (R, M) or (M, R)).

4. Extensions

4.1. Identity-Dependent Click-Through Rate

In the basic model, we assume that the CTR at each position is independent of the identity of the advertiser. In reality, this assumption may not hold. For example, some consumers may be loyal to a retailer and more likely to click on its sponsored ads even if the retailer is not at the top position. In this subsection, we consider an extension of the basic model that allows the CTR of a sponsored ad to depend on both its position and the identity of its advertiser.

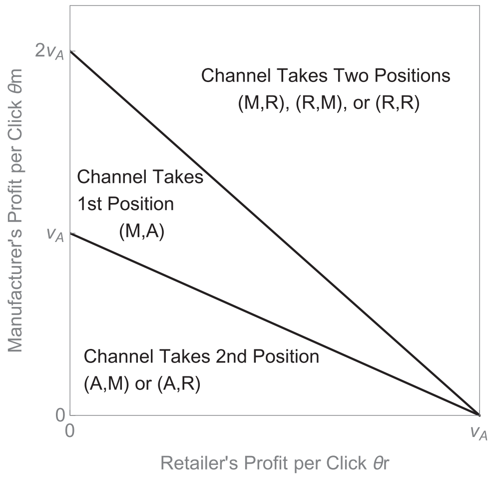
Specifically, we assume that the CTR of advertiser i at position j, d_{ij}, can be decomposed as $d_{ij} = e_i x_j$, where e_i is the "identity effect," which measures the attractiveness of the advertiser, and x_j is the "position effect," which measures the attractiveness of the position.[16] Similar to before, we assume that given an advertiser, the higher position it takes, the more clicks it will get; that is, x_j decreases in j. Moreover, it is common for search advertising platforms to adjust the position rank of advertisers according to their identity effects. For example, when deciding the positions ranks, Google augments each advertiser's bid with its quality score, which, roughly speaking, is a measure of the advertiser's predicted CTR, that is, the identity effect, besides other less important considerations, such as landing page quality. We follow Varian (2007) to assume that the positions of advertisers are ranked according to $e_i b_i$ in descending order, where b_i denotes the bid of the advertiser at position i. The advertiser at position i then pays $(e_{i+1}/e_i)b_{i+1}$ per click.

The equilibrium analysis of the model here parallels with the basic model (with details in the online appendix). We summarize the findings by the following theorem.

Theorem 3. *Consider a position auction with identity-dependent CTR participated in by an outside advertiser and two retailers who are supported by a manufacturer. Assume that $e_A v_A > e_i \theta_i r_i$ for $i = 1, 2$. In equilibrium, the retailer with higher total channel profit per impression will always take a higher position than the other retailer. If it is optimal for the manufacturer to support only one retailer, it will choose to support this retailer.*

Theorem 3 generalizes the results in Theorem 1 nicely. With identity-dependent CTR, the retailer with higher total channel profit per impression, that is, $e_i\theta_i(m_i + r_i)$, will always take a higher position than the other retailer in equilibrium.

Figure 3. Manufacturer's Optimal Cooperative Search Advertising Strategy Given Two Symmetric Retailers and Equal Channel Profits



4.2. Endogenous Retail Prices with Price Competition

As argued before, the wholesale prices and retail prices can be seen as exogenously given when search advertising is only part of the demand source for the manufacturer and retailers. However, when search advertising is the main source of demand for the channel, it is more reasonable to consider endogenous wholesale contracts and retail prices. That is, when manufacturers and retailers determine their wholesale and retail prices, they take the search advertising strategy into account.

In this section, we extend the basic model to consider the case in which search advertising is the main source of demand for the retailers. The retailers set retail prices endogenously, taking into account the potential price competition between them. We analyze the effect of price competition on manufacturer's sponsorship strategy and investigate whether the insights from the main model generalize here. That is, whether the manufacturer may still sponsor only one retailer even when the two retailers are symmetric. We take the manufacturer's wholesale contracts as exogenously given in this section, and we analyze endogenous wholesale contracts in next section.

The overall effect of retailers' price competition on the manufacturer's sponsorship strategy is ambiguous. On the one hand, price competition between retailers can lead to lower retail prices and, thus, higher total demand for the manufacturer. On the other hand, as price competition forces retailers to charge a lower retail price, they have less incentive to win the search ad auction, and consequently, the manufacturer may need to provide higher participation rates to the retailers to help them get displayed. In general, the manufacturer faces the tradeoff between demand expansion and sponsorship costs, both of which depend on the intensity of retail price competition.

The same as in the main model, we consider the setting in which one manufacturer sells via two retailers, and the manufacturer does not participate in the auction directly. Instead of having only one outside advertiser, we assume that there are two outside advertisers with the same profit per click, v_A.[17] The game proceeds in three stages. First, the manufacturer decides the participation rates, α_1 and α_2; second, the two retailers decide their prices p_1 and p_2 simultaneously; and finally, the two retailers and the outside advertisers bid in the position auction with two positions. To focus on the price competition between retailers, we assume exogenous and symmetric wholesale price for the two retailers, w. For tractability of the analysis, we do not explicitly consider the outside advertisers' pricing decisions. We restrict ourselves to text ads, for which consumers do not see the price information before clicking on an ad.

In practice, upon seeing two ads, some consumers may form rational expectations of their prices and only click on the ad that they prefer more, and others may be less sophisticated or more interested in evaluating both options and, thus, click on both ads before making a choice. To avoid the complexity of explicitly modeling consumers' clicking behaviors, we will focus on the extreme case in which each consumer who clicks on an ad will click on both ads before making a choice. That is, the click-through rates for the two positions are equal, $d_1 = d_2$, which are normalized as one. This is also the case in which the price competition is most severe.[18] The equal CTRs of the two positions also allow us to focus on the manufacturer's problem of how many retailers to sponsor instead of discerning between sponsoring a retailer to the first or second position.

We assume that the two retailers are ex ante symmetric. Out of all consumers who click on their ads, $\bar{\theta}$ of them will consider purchasing from one of the two retailers, and these consumers are uniformly distributed on a Hotelling line $[0, 1]$ with the two retailers located at the two end points. Given the two retailers' prices p_1 and p_2, a consumer at location x derives utility $v - tx - p_1$ if purchasing from retailer 1 and $v - t(1 - x) - p_2$ if purchasing from retailer 2. We assume that $v > w$, that is, the wholesale price is lower than the consumer's highest valuation of the product; t is the unit traveling cost that measures the degree of consumer heterogeneity.

We investigate whether the manufacturer will sponsor both, one, or neither of the retailers given symmetric retailers and symmetric wholesale prices. Similar to the main model, we assume that neither retailer can get displayed without support from the manufacturer (this amounts to some restrictions on t as we show later).

We first consider the case in which the manufacturer sponsors both retailers to get displayed. We focus on the case in which v is sufficiently high such that the Hotelling line is covered when both retailers get displayed (we explicitly provide the market coverage condition later).[19] The consumer at location $x^* = 1/2 + (p_2 - p_1)/(2t)$ will be indifferent between the two retailers. Given the retail prices, the conversion rate for retailer 1 is $\bar{\theta}x^*$, and the conversion rate for retailer 2 is $\bar{\theta}(1 - x^*)$. Because the two positions have the same CTR, the price per click for both positions is the profit per click of the first bidder who cannot get displayed, that is, the outside advertisers' profit per click, v_A. Retailer 1's profit will be $\pi_1 = \bar{\theta}x_1(p_1 - w) - (1 - \alpha_1)v_A$, and retailer 2's profit will be $\pi_2 = \bar{\theta}(1 - x_1)(p_2 - w) - (1 - \alpha_2)v_A$. By solving the best response function of the two retailers, we get the equilibrium retail prices $p_1^* = p_2^* = w + t$, and the equilibrium market share $x^* = 1 - x^* = 1/2$. Given the participation rates, the two retailers' profits in equilibrium are $\pi_1^* = \bar{\theta}t/2 - (1 - \alpha_1)v_A, \pi_2^* = \bar{\theta}t/2 - (1 - \alpha_2)v_A$, and the manufacturer's profit is $\pi_M = \bar{\theta}w - (\alpha_1 + \alpha_2)v_A$. The manufacturer's profit decreases in α_1, α_2, so the manufacturer will choose the lowest participation rates that can help the two retailers get displayed; that is, $\alpha_i^* = 1 - (\bar{\theta}t)/(2v_A)$, $i = 1, 2$. Notice that under the condition $t < 2v_A/\bar{\theta}$, $\alpha_i^* > 0$. The manufacturer's profit under the optimal participation rates is $\pi_M^* = \bar{\theta}(w + t) - 2v_A$.

In the online appendix, we analyze the other two cases in which the manufacturer sponsors one or neither retailer. Comparing the manufacturer's profits under these three cases (sponsoring two, one, or zero retailers), we get the following theorem.

Theorem 4. *Suppose that neither retailer can get displayed without support from the manufacturer, and the market is covered when the manufacturer sponsors both retailers to get displayed. These two conditions amount to* $t \in (\underline{t}, \bar{t}]$, *where* $\bar{t} = 2(v - w)/3$, *and*

$$\underline{t} \equiv \begin{cases} \frac{\bar{\theta}}{4v_A}(v - w)^2, & \text{if } \frac{3}{8}(v - w) < \frac{v_A}{\bar{\theta}} < \frac{v-w}{2} \\ v - w - \frac{v_A}{\bar{\theta}}, & \text{if } \frac{v_A}{\bar{\theta}} \geq \frac{v-w}{2}. \end{cases}$$

The manufacturer's optimal sponsorship strategy depends on the profit per click of the outside advertisers v_A and the degree of consumer heterogeneity t in the following way:

- *If $v_A \geq \bar{\theta}v$, the manufacturer sponsors neither retailer.*

- *If $\bar{\theta}(v+w)/2 \le v_A < \bar{\theta}v$, the manufacturer sponsors one retailer when $t \le t_1$, and sponsors neither retailer when $t > t_1$.*
- *If $3\bar{\theta}(v-w)/8 < v_A < \bar{\theta}(v+w)/2$, the manufacturer sponsors one retailer for $t < t_2$, sponsors neither retailer when $t_2 \le t < t_3$, and sponsors both retailers when $t \ge t_3$.*

The thresholds t_1, t_2, and t_3 are functions of v, w, v_A, and $\bar{\theta}$.

This theorem completely characterizes the manufacturer's optimal sponsorship strategy. The expressions of the thresholds t_1, t_2, and t_3 are complex and provided in the online appendix. Notice that we need to have $v_A > 3\bar{\theta}(v-w)/8$ for having $\underline{t} < \bar{t}$.

The results of Theorem 4 are intuitive. When v_A is so high that it exceeds the highest possible profit per click the channel can get, $\bar{\theta}v$, the manufacturer would sponsor neither retailer to get displayed and earn zero profit. As long as $v_A < \bar{\theta}v$, the manufacturer will consider sponsoring one or two retailers. When the degree of consumer heterogeneity t is relatively small, one retailer can already cover a large portion of consumers, so the manufacturer can already get a relatively high demand by sponsoring just one retailer. At the same time, if the manufacturer sponsors both retailers to get displayed, the spatial competition between retailers is intense when t is small, which makes the retail price and retailers' profits low. Consequently, the manufacturer needs to provide high participation rates to the two retailers when sponsoring both retailers. Therefore, it is optimal for the manufacturer to sponsor only one retailer when t is small. On the other hand, when the degree of consumer heterogeneity t is relatively large, the manufacturer will lose a significant portion of the market by sponsoring only one retailer. At the same time, with relatively large t, the price competition is weak, which provides retailers enough incentive to bid high. Thus, it is more profitable for the manufacturer to sponsor both retailers as it increases the total demand with relatively low sponsorship costs. Combining both sides, it is more profitable for the manufacturer to sponsor one retailer when t is small and to sponsor both retailers when t is large. For t in the middle range, the benefits from both sides are not large enough to overcome the cost of outbidding the outside advertisers, and thus, the manufacturer may optimally sponsor neither retailer, earning zero profit.

One thing to notice is that, when $\bar{\theta}(v+w)/2 \le v_A < \bar{\theta}v$, $\underline{t} < t_1$ always holds, and when $3\bar{\theta}(v-w)/8 < v_A < \bar{\theta}(v+w)/2$, $\underline{t} < t_2$ always holds. This implies that, as long as $v_A < \bar{\theta}v$, there always exists the case that it is optimal for the manufacturer to sponsor only one retailer given two symmetric retailers.

4.3. Endogenous Wholesale Contracts and Retail Prices

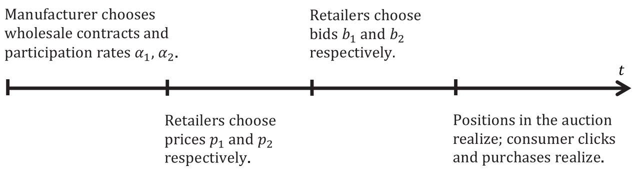
In this section, we further consider an extension when search advertising is the main source of demand for both manufacturer and retailers so that the manufacturer has to take search advertising into account when setting wholesale prices. We incorporate endogenous wholesale contracts and retail prices into the basic model without explicitly considering retail price competition.[20]

Let's consider a three-stage game with the timeline of events shown in Figure 4. First, the manufacturer chooses the wholesale contracts and participation rates α_1, α_2 for the two retailers. Second, given the manufacturer's choices, the two retailers decide retail prices p_1, p_2. Finally, after observing the retail prices, the two retailers submit bids b_1, b_2 in a position auction. By specifying this timeline, we have made several assumptions, and here are our justifications. First, we notice that both wholesale and co-op contracts require heavy administrative work and, thus, cannot be altered frequently. In contrast, online retailers can adjust their prices weekly or even daily, so they can treat the wholesale contracts and participation rates as given when determining the retail prices. Moreover, it is not uncommon for retailers to adjust their bids almost continuously with the help of automatic bidding support systems. Therefore, it is reasonable to assume that when choosing their bids, the retailers can treat the retail prices as given.

For the wholesale contracts, we consider both linear contracts and two-part tariffs. Generally speaking, two-part tariffs generate a higher channel profit, but linear contracts are simpler to implement and more robust to accommodate against changing environments or incomplete contracts (Villas-Boas 1998). We denote the wholesale prices as w_i for retailer i.

For sake of simplicity of the analysis, we assume that the click-through rate only depends on the rank of the position, independent of the retail price and the identity of the product. However, the price will affect how likely a consumer will be to make a purchase after the customer clicks the ad. That is, retail prices will influence the conversion rate. Consumers' valuation of the product is assumed to be uniformly distributed on $[0, 1]$. Therefore, the conversion rate of retailer i can be written as $\theta_i = \bar{\theta}_i(1 - p_i)$, where $\bar{\theta}_i$ is a constant.

Figure 4. Timeline of Events with Endogenous Wholesale Contracts and Retail Prices



Retailer i's profit margin is $r_i = p_i - w_i$, and the manufacturer's profit margin is $m_i = w_i - c$, where c is the marginal production cost. Retailers are ex ante symmetric; that is, $\bar{\theta}_1 = \bar{\theta}_2 = \bar{\theta}$.

We solve the equilibrium by backward induction. In fact, in the last stage when the wholesale contracts, participation rates, and retail prices have been determined, retailers face exactly the same problem as the basic model, and we have solved for the retailers' optimal bids given their profit per click. Now we consider the two retailers' decisions on retail prices. Notice that p_i enters into retailer i's profit function only via $\theta_i r_i$. Retailer i's equivalent profit per click is

$$v_i \equiv \frac{\theta_i r_i}{1-\alpha_i} = \frac{\bar{\theta}(1-p_i)(p_i-w_i)}{1-\alpha_i}, \tag{9}$$

which reaches the maximum value $v_i^* = \bar{\theta}(1-w_i)^2/[4(1-\alpha_i)]$ when $p_i = (1+w_i)/2$ and takes the minimum value of zero when p_i is equal to w_i or one. Therefore, when choosing the retail price $p_i \in [w_i, 1]$, retailer i is essentially choosing $v_i \in [0, v_i^*]$. Given the two retailers' choice of retail prices, their positions will be determined by the order of v_1, v_2, and v_A. The following lemma shows that, in fact, the bidders' positions in equilibrium are completely determined by the rank of v_1^*, v_2^*, and v_A (with proof in the online appendix).

Lemma 2. *In equilibrium, retail prices are set at $p_i^* = (1+w_i)/2$, and the positions of retailer 1, retailer 2, and the outside advertiser are given by the descending order of v_1^*, v_2^*, and v_A.*

4.3.1. Linear Wholesale Contracts. Given the retailers' retail prices and bids, now we consider the manufacturer's problem. Let's first consider linear wholesale contracts. By symmetry, without loss of generality, we can assume that retailer 1 takes a higher position than retailer 2. Similar to the basic model, there are three possible position configurations. Under each position configuration, we can write down the manufacturer's profit function and then maximize its profit with respect to $\alpha_1, \alpha_2, w_1, w_2$ subject to the conditions for the position configuration to hold in equilibrium. By comparing the manufacturer's profit among the three cases, we can get its wholesale prices and participation rates in equilibrium. We relegate the details of calculations to the online appendix. The following theorem completely characterizes the equilibrium.

Theorem 5. *Consider a position auction, the participants of which are two ex ante symmetric retailers supported by a manufacturer as well as an outside advertiser. The manufacturer provides linear wholesale contracts and participation rates to the two retailers. In equilibrium, the manufacturer's wholesale prices and participation rates are*

$$\begin{pmatrix} w_1^* \\ w_2^* \\ \alpha_1^* \\ \alpha_2^* \end{pmatrix} = \begin{cases} \left(\frac{1+c}{2}, \frac{1+c}{2}, 0, 0\right)^T, \quad v_A \le \frac{\bar{\theta}(1-c)^2}{16}, \ (R_1, R_2) \\ \left(1 - 2\sqrt{\frac{v_A}{\bar{\theta}}},\ 1 - 2\sqrt{\frac{v_A}{\bar{\theta}}}, 0, 0\right)^T, \ \frac{\bar{\theta}(1-c)^2}{16} \\ \quad \le v_A \le \left(\frac{d_1 + d_2 + d_2\sqrt{\frac{3d_1+d_2}{d_1+d_2}}}{d_1 + 3d_2}\right)^2 \frac{\bar{\theta}(1-c)^2}{4}, \ (R_1, R_2) \\ \left(\frac{d_1 c + d_2}{d_1 + d_2}, 1, 1 - \frac{\bar{\theta}(1-c)^2 d_1^2}{4v_A(d_1+d_2)^2}, 0\right)^T, \\ \quad \left(\frac{d_1 + d_2 + d_2\sqrt{\frac{3d_1+d_2}{d_1+d_2}}}{d_1 + 3d_2}\right)^2 \frac{\bar{\theta}(1-c)^2}{4} \\ \quad < v_A \le \frac{2d_1 + d_2}{d_1 + d_2}\frac{\bar{\theta}(1-c)^2}{8}, \ (R_1, A) \\ \left(\frac{1+c}{2}, 1, 0, 0\right)^T, v_A > \frac{2d_1 + d_2}{d_1 + d_2}\frac{\bar{\theta}(1-c)^2}{8}, \ (A, R_1) \end{cases} \tag{10}$$

where (X, Y) at the end of each row indicates the equilibrium position configuration under such condition. The retail prices in equilibrium are

$$p_i^* = \frac{1 + w_i^*}{2}, \ i = 1, 2. \tag{11}$$

Retailers 1 and 2 and the outsider advertiser's positions in equilibrium are determined by the order of $\bar{\theta}(1-w_1^*)^2/[4(1-\alpha_1^*)]$, $\bar{\theta}(1-w_2^*)^2/[4(1-\alpha_2^*)]$, and v_A.

Basically, the manufacturer has two devices at hand: wholesale prices w_1 and w_2 and participation rates α_1 and α_2. Depending on the outside advertiser's profit per click v_A, the manufacturer will optimally apply one or both devices to coordinate the channel so as to maximize its profit. We go through the four cases in Equation (10) together to understand the manufacturer's optimal wholesaling and advertising strategies. First, when v_A is very low, the two retailers will get the top two positions without the help of the manufacturer. In this case, the manufacturer sets the monopolistic wholesale prices $(1+c)/2$ and provides zero participation rates. As we increase v_A to the interval in the second case, the manufacturer still wants to keep the two retailers at the top two positions. It will achieve this goal by providing lower wholesale prices and, thus, higher profit margins for the retailers but still keeping the participation rates at zero. Intuitively, lowering the wholesale prices not only increases the retailers' profit margins and, thus, helps the retailers outbid the outside advertiser, but also increases the demand as the retail prices go down, whereas increasing participation rates only has the first effect. This is why the wholesale prices are the manufacturer's first choice when fighting against downstream competition in bidding. As we further increase v_A to the third case, wholesale prices alone do not suffice to grant the retailers the winners of the auction. The manufacturer will set a low wholesale price and at the same time provide a positive participation rate to retailer 1 so as to keep it at the first position. It will entirely drop retailer 2 by not selling to it, and as a result, retailer 2 will take the third position with zero demand. Finally, when v_A is very high, the manufacturer has to give up winning the auction. It will set the monopolistic wholesale price $(1+c)/2$ again for retailer 1 and provide zero participation rate to it. The manufacturer will not sell to retailer 2, who will take the third position.

Figure 5 clearly illustrates the manufacturer's optimal wholesaling and cooperative advertising strategies as described previously. By Lemma 2, the equilibrium retail price $p_i^* = (1+w_i^*)/2$. The average retail price for all consumers will be $\overline{p^*} = (d_{(1)}p_1^* + d_{(2)}p_2^*)/(d_{(1)} + d_{(2)})$, where $d_{(i)}$ denotes the CTR for retailer i given its position. According to Equation (10), it is straightforward to show that $\overline{p^*} = p_1^* = (1+w_1^*)/2$. Therefore, the relationship between $\overline{p^*}$ and v_A will be very similar to the relationship between w_1^* and v_A in Figure 5. As the outside advertiser's profit per click increases, the average retail price first decreases then increases.

4.3.2. Two-Part Tariffs. Now suppose the manufacturer uses two-part tariff wholesale contracts for channel coordination. Each retailer i pays the manufacturer w_i for each product as well as a fixed franchise fee. Under two-part tariffs, the retailers' problem is the same as before with their positions determined by the order of v_1^*, v_2^* and v_A; however, the manufacturer's objective now is to maximize the total channel profit by choosing w_1, w_2, α_1, and α_2. It uses the franchise fee to divide the channel profit with retailers. Similarly, we analyze the equilibrium given each of the three position configurations in the online appendix. By comparing the manufacturer's profits among the three position configurations, we get its equilibrium wholesale prices and participation rates. The following theorem characterizes the equilibrium.

Theorem 6. *Suppose a manufacturer's two ex ante symmetric retailers and one outside advertiser participate in the position auction. The manufacturer provides two-part tariff wholesale contracts and linear participation rates to the two retailers. In equilibrium, the manufacturer's wholesale prices and participation rates are*

$$\begin{pmatrix} w_1^* \\ w_2^* \\ \alpha_1^* \\ \alpha_2^* \end{pmatrix} = \begin{cases} \left(c, \left(1-\dfrac{d_2}{d_1}\right) + \dfrac{d_2}{d_1}c, 0, 0\right)^T, \\ \quad v_A \le \dfrac{d_2^2}{4d_1^2}\dfrac{\bar{\theta}(1-c)^2}{4}, (R_1, R_2) \\ (c, 1-2\sqrt{\frac{v_A}{\bar{\theta}}}, 0, 0)^T, \\ \quad \dfrac{d_2^2}{4d_1^2}\dfrac{\bar{\theta}(1-c)^2}{4} \le v_A \le \dfrac{\bar{\theta}(1-c)^2}{9}, \\ \quad \cdot (R_1, R_2),\ \mathit{if}\ \dfrac{d_1}{d_2} \ge \dfrac{3}{2}. \\ (c, 1, 0, 0)^T, \\ \quad \dfrac{\bar{\theta}(1-c)^2}{9} < v_A \le \dfrac{\bar{\theta}(1-c)^2}{4}, (R_1, A) \\ (c, 1, 0, 0)^T,\ v_A > \dfrac{\bar{\theta}(1-c)^2}{4}, (A, R_1) \end{cases} \tag{12}$$

$$\begin{pmatrix} w_1^* \\ w_2^* \\ \alpha_1^* \\ \alpha_2^* \end{pmatrix} = \begin{cases} \left(c, \left(1-\frac{d_2}{d_1}\right) + \frac{d_2}{d_1}c, 0, 0\right)^T, \\ \quad v_A \le \dfrac{d_2^2}{4d_1(3d_2-d_1)}\dfrac{\bar{\theta}(1-c)^2}{4}, (R_1, R_2) \\ (c, 1, 0, 0)^T, \\ \quad \dfrac{d_2^2}{4d_1(3d_2-d_1)}\dfrac{\bar{\theta}(1-c)^2}{4} < v_A \le \dfrac{\bar{\theta}(1-c)^2}{4}, \\ \quad (R_1, A)\ \mathit{if}\ \dfrac{d_1}{d_2} < \dfrac{3}{2}. \\ (c, 1, 0, 0)^T, \\ \quad v_A > \dfrac{\bar{\theta}(1-c)^2}{4}, (A, R_1) \end{cases} \tag{13}$$

The retail prices in equilibrium are

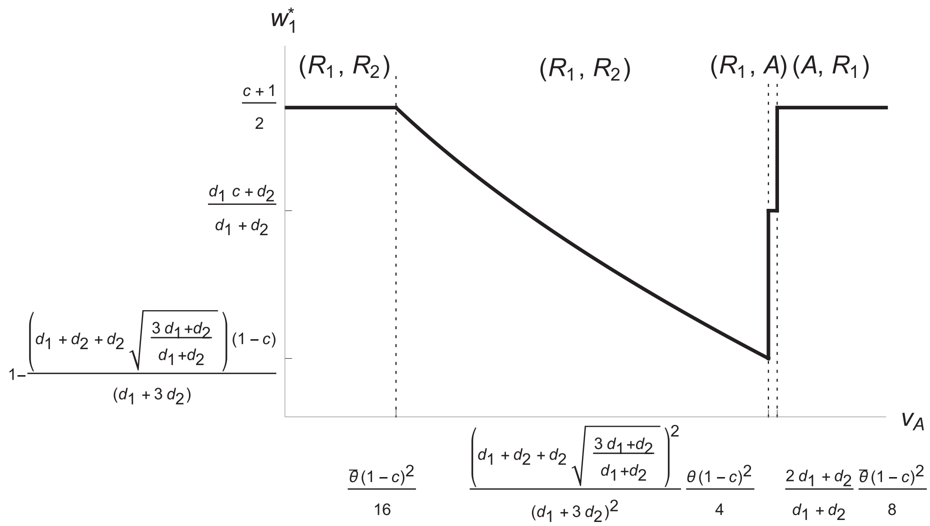
$$p_i^* = \frac{1+w_i^*}{2},\ i = 1, 2. \tag{14}$$

Retailers 1 and 2 and the outsider advertiser's positions in equilibrium are given by the descending order of $\bar{\theta}(1-w_1^*)^2/[4(1-\alpha_1^*)]$, $\bar{\theta}(1-w_2^*)^2/[4(1-\alpha_2^*)]$, *and* v_A.

Compared with linear contracts, we find that cooperative advertising in forms of participation rates is never used to coordinate the channel with two-part tariffs. This is consistent with the general observation that a sufficiently flexible wholesale contract will fully coordinate the channel and there will be no need for a manufacturer to cooperate with retailers separately on advertising. From a different angle, we can also view the lump-sum payment between the manufacturer and the retailers as a form of cooperative search advertising.

From Theorem 6, we find that the manufacturer always sets the wholesale price as the marginal production cost for retailer 1, who takes a higher position than the other retailer. This not only eliminates the double

Figure 5. Manufacturer's Optimal Wholesaling and Cooperative Advertising Strategy Under Linear Wholesale Contracts



marginalization conflicts in retail prices, but also maximizes the retailer's profit per click and, thus, chance of winning the auction. In contrast, it takes the manufacturer more deliberations when setting the wholesale price for retailer 2, who takes a lower position than retailer 1. When the outside advertiser's profit per click is relatively high, the manufacturer will not sell to retailer 2, who ends up in the third position; on the other hand, when the outside advertiser's profit per click is relatively low, the manufacturer will support retailer 2 to take the second position by providing a wholesale price that is higher than the marginal production cost in an effort to balance between supporting retailer 2 to outbid the outside advertiser and lowering the price per click for retailer 1.

5. Conclusion

To the best of our knowledge, this is the first paper that addresses the problem of channel coordination in the context of search advertising. Our paper studies cooperative search advertising by considering a simple form of coordination contract: a manufacturer shares a fixed percentage of a retailer's spending on search advertising. We consider intrabrand coordination and competition between one manufacturer and two retailers as well as interbrand competition with outside advertisers. We find that it may not be optimal for the manufacturer to sponsor both retailers even if they are ex ante the same. This reflects the manufacturer's tradeoff between higher demand and higher bidding cost resulting from intensified competition. We also find that the relative positions of retailers in equilibrium are determined by their channel profit per click. This illustrates the efficiency of this simple coordination mechanism despite that it does not fully coordinate the channel.

In general, our main results carry through the several extensions we consider. First, when the manufacturer can submit bids directly, the relative positions of retailers are still determined by the rank of their channel profit per click, and the manufacturer should get a higher position itself if its profit per click via direct sales is higher than both retailers' channel profit per click. Second, with endogenous retail price competition, the manufacturer may still optimally sponsor one retailer given two symmetric retailers in the market. Third, with endogenous linear wholesale contracts and retail prices, the manufacturer may still optimally sponsor one retailer; however, it is never optimal to sponsor both retailers, and it is no longer necessary to use cooperative advertising with two-part tariffs. Finally, when click-through rates depend on advertisers' identities, we demonstrate that our main results generalize nicely—now one's total channel profit per impression will determine its position rank.

This paper has several limitations. First, we do not make a distinction between branded and generic keywords, which could be a topic for future research. Second, we have not fully modeled consumers' search and click behaviors and, thus, cannot evaluate consumers' welfare in the context of cooperative search advertising. Last but not least, we discuss the problem of one focal manufacturer, incorporating the competition in a position auction from other brands by considering an outside advertiser, but we do not explicitly model the strategic interaction between manufacturers, especially when they sell to common retailers.

Acknowledgments

The authors thank Ron Borkovsky, Wes Hartmann, John Hauser, Sridhar Narayanan, Sherif Nasser, Navdeep Sahni, Stephan Seiler, Duncan Simester, Catherine Tucker, Birger

Wernerfelt, and Juanjuan Zhang for helpful discussions and suggestions. This paper has also benefited from the comments of seminar participants at Peking University, Stanford University, Tsinghua University, University of Toronto, Washington University in St. Louis, and attendees of the 2016 Marketing Science Conference, 2016 CEIBS Marketing Conference, 2016 Summer Institute in Competitive Strategy, 2016 MIT Micro@Sloan Conference, and 2016 Northeast Marketing Conference. The authors gratefully acknowledge the excellent suggestions of the senior editor, associate editor, and two anonymous reviewers. All remaining errors are the authors' own.

Appendix

A.1. Equilibrium Analysis of the Basic Model (Proof of Theorem 1)

We first consider the three cases in which retailer 1 gets a higher position than retailer 2.

- Consider the position configuration (R_1, R_2).

We lay out the equilibrium analysis for this case in the main text. The manufacturer's optimal participation rates are

$$\alpha_1^* = 1 - \frac{\theta_1 r_1}{v_A},$$
$$\alpha_2^* = 1 - \frac{\theta_2 r_2}{v_A}.$$

The manufacturer's optimal profit in this case is

$$\pi_M(\alpha_1^*, \alpha_2^*) = d_1\left[\theta_1(m_1 + r_1) - v_A\right] + d_2[\theta_2(m_2 + r_2) - v_A]. \tag{A.1}$$

Accordingly, profits of the two retailers and the channel under α_1^* and α_2^* are

$$\pi_{R_1}(\alpha_1^*, \alpha_2^*) = 0,$$
$$\pi_{R_2}(\alpha_1^*, \alpha_2^*) = 0,$$
$$\pi_C(\alpha_1^*, \alpha_2^*) = d_1[\theta_1(m_1 + r_1) - v_A] + d_2[\theta_2(m_2 + r_2) - v_A].$$

- Consider the position configuration (R_1, A).

According to the result in Section 2.1, we need to have $\theta_1 r_1/(1-\alpha_1) \geq v_A > \theta_2 r_2/(1-\alpha_2)$ for this position configuration to be the equilibrium. Retailer 2 will bid its equivalent value $\theta_2 r_2/(1-\alpha_2)$, and the outside advertiser's equilibrium bid is $\frac{d_1-d_2}{d_1}v_A + \frac{d_2}{d_1}\frac{\theta_2 r_2}{1-\alpha_2}$. The manufacturer's profit will be

$$\pi_M(\alpha_1, \alpha_2) = d_1\left[\theta_1 m_1 - \alpha_1\left(\frac{d_1 - d_2}{d_1}v_A + \frac{d_2}{d_1}\frac{\theta_2 r_2}{1-\alpha_2}\right)\right],$$

which decreases in both α_1 and α_2. Therefore, the manufacturer will choose the smallest α_1 and α_2 that ensure $\theta_1 r_1/(1-\alpha_1) > v_A > \theta_2 r_2/(1-\alpha_2)$, which are

$$\alpha_1^* = 1 - \frac{\theta_1 r_1}{v_A},$$
$$\alpha_2^* = 0.$$

The manufacturer's profit under the optimal participation rates is

$$\pi_M(\alpha_1^*, \alpha_2^*) = d_1\left[\theta_1(m_1 + r_1) - v_A\right] + d_2\frac{(v_A - \theta_1 r_1)(v_A - \theta_2 r_2)}{v_A}. \tag{A.2}$$

Accordingly, profits of the two retailers and the channel under α_1^* and α_2^* are

$$\pi_{R_1}(\alpha_1^*, \alpha_2^*) = d_1\left[\theta_1 r_1 - (1-\alpha_1^*)\left(\frac{d_1 - d_2}{d_1}v_A + \frac{d_2}{d_1}\frac{\theta_2 r_2}{1-\alpha_2^*}\right)\right]$$
$$= d_2\theta_1 r_1\left(1 - \frac{\theta_2 r_2}{v_A}\right),$$
$$\pi_{R_2}(\alpha_1^*, \alpha_2^*) = 0,$$
$$\pi_C(\alpha_1^*, \alpha_2^*) = d_1\left[\theta_1(m_1 + r_1) - v_A\right] + d_2\left[v_A - \theta_2 r_2\right].$$

- Consider the position configuration (A, R_1).

Similarly, we need to have $v_A > \theta_1 r_1/(1-\alpha_1) \geq \theta_2 r_2/(1-\alpha_2)$ for this position configuration to be the equilibrium. Retailer 2's bid at position 3 will be its equivalent profit per click, $\theta_2 r_2/(1-\alpha_2)$, and this is the cost per click for retailer 1. The manufacturer's profit will be

$$\pi_M(\alpha_1, \alpha_2) = d_2\left(\theta_1 m_1 - \alpha_1\frac{\theta_2 r_2}{1-\alpha_2}\right),$$

which decreases in α_1 and α_2. Therefore, the manufacturer will choose the smallest α_1 and α_2 that satisfy $v_A > \theta_1 r_1/(1-\alpha_1) > \theta_2 r_2/(1-\alpha_2)$. The optimal participation rates are

$$\alpha_1^* = \max\left\{1 - \frac{\theta_1 r_1}{\theta_2 r_2}, 0\right\},$$
$$\alpha_2^* = 0.$$

The manufacturer's profit under the optimal participation rates is

$$\pi_M(\alpha_1^*, \alpha_2^*) = d_2\left[\theta_1(m_1 + r_1) - \max\{\theta_1 r_1, \theta_2 r_2\}\right]. \tag{A.3}$$

Accordingly, profits of the two retailers and the channel under α_1^* and α_2^* are

$$\pi_{R_1}(\alpha_1^*, \alpha_2^*) = d_2\left[\theta_1 r_1 - (1-\alpha_1^*)\frac{\theta_2 r_2}{1-\alpha_2^*}\right]$$
$$= \max\{\theta_1 r_1 - \theta_2 r_2, 0\},$$
$$\pi_{R_2}(\alpha_1^*, \alpha_2^*) = 0,$$
$$\pi_C(\alpha_1^*, \alpha_2^*) = \pi_M(\alpha_1^*, \alpha_2^*) + \pi_{R_1}(\alpha_1^*, \alpha_2^*) + \pi_{R_2}(\alpha_1^*, \alpha_2^*)$$
$$= d_2\left[\theta_1(m_1 + r_1) - \theta_2 r_2\right].$$

So far, we have analyzed the three position configurations in which retailer 1 gets a higher position than retailer 2. We find that, given $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$, in each case, the manufacturer's profit will get lower if retailer 1 and retailer 2 are exchanged. That is, retailer 1 will always take a higher position than retailer 2 in equilibrium.

To know which position configuration will be chosen by the manufacturer in equilibrium, we only need to compare the manufacturer's profits under the three cases: Equations (A.1)–(A.3).

A.2. Integrated Channel (Proof of Proposition 1)

Step 1. We first show that, for the integrated channel, the retailer with a higher channel profit per click $\theta_i(m_i + r_i)$ gets a higher position than the other retailer.

To prove this, we write down the integrated channel's profit under each position configuration given the retailers' and advertiser's bids.

$$\begin{aligned}\pi_C^{(R_1,R_2)}(b_1,b_2,b_A) &= d_1\,[\theta_1(m_1+r_1)-b_2]\\&\quad + d_2\,[\theta_2(m_2+r_2)-b_A],\\ \pi_C^{(R_1,A)}(b_1,b_2,b_A) &= d_1\,[\theta_1(m_1+r_1)-b_A],\\ \pi_C^{(A,R_1)}(b_1,b_2,b_A) &= d_2\,[\theta_1(m_1+r_1)-b_2],\end{aligned}$$

where the superscripts denote the position configurations. We have only written down the profit functions for the three cases in which retailer 1 gets a higher position than retailer 2. The profit functions for the other three cases can be obtained by symmetry. It is straightforward to verify that when $\theta_1(m_1+r_1)\geq\theta_2(m_2+r_2)$, $\pi_C^{(R_1,R_2)}(b_1,b_2,b_A)\geq\pi_C^{(R_2,R_1)}(b_1,b_2,b_A)$, $\pi_C^{(R_1,A)}(b_1,b_2,b_A)\geq\pi_C^{(R_2,A)}(b_1,b_2,b_A)$, and $\pi_C^{(A,R_1)}(b_1,b_2,b_A)\geq\pi_C^{(A,R_2)}(b_1,b_2,b_A)$ for any b_1, b_2, and b_A. This implies that, given $\theta_1(m_1+r_1)\geq\theta_2(m_2+r_2)$, retailer 1 must get a higher position than retailer 2 in equilibrium, and the only possible equilibrium position configurations are (R_1,R_2), (R_1,A), and (A,R_1).

Step 2. Now, we assume $\theta_1(m_1+r_1)\geq\theta_2(m_2+r_2)$ without loss of generalizability. For each of the three position configurations in which retailer 1 gets a higher position than retailer 2, we work out the parameter space (i.e., requirement on the relationship between v_A, $\theta_1(m_1+r_1)$, and $\theta_2(m_2+r_2)$) in which the position configuration can be an equilibrium, the channel's optimal choice of bids in the parameter space, and the channel's optimal profit.

- Consider the position configuration (R_1,R_2).

For this position configuration to be an equilibrium, the bids should satisfy

$$b_1\geq b_2\geq b_A. \tag{A.4}$$

According to the SNE and LB that guard against advertiser A's deviation to position 2, the outside advertiser A's equilibrium bid should be $b_A=v_A$. Then Equation (A.4) implies $b_1\geq v_A$, and thus, it is not profitable for advertiser A to deviate to position 1.

To prevent the channel deviating from (R_1,R_2) to (R_1,A), the bids should satisfy $d_1[\theta_1(m_1+r_1)-b_2]+d_2[\theta_2(m_2+r_2)-b_A]\geq d_1[\theta_1(m_1+r_1)-b_A]$. Given $b_A=v_A$, this is equivalent to

$$\theta_2(m_2+r_2)\geq\frac{d_1}{d_2}(b_2-v_A)+v_A. \tag{A.5}$$

To prevent the channel deviating from (R_1,R_2) to (A,R_1), the bids should satisfy $d_1[\theta_1(m_1+r_1)-b_2]+d_2[\theta_2(m_2+r_2)-b_A]\geq d_2[\theta_1(m_1+r_1)-b_A]$, which is equivalent to

$$\frac{d_1-d_2}{d_1}\theta_1(m_1+r_1)+\frac{d_2}{d_1}\theta_2(m_2+r_2)\geq b_2. \tag{A.6}$$

The channel profit is

$$\pi_C(b_1,b_2)=d_1\,[\theta_1(m_1+r_1)-b_2]+d_2\,[\theta_2(m_2+r_2)-v_A]. \tag{A.7}$$

According to Equation (A.7), the channel profit decreases with b_2 at this position configuration, and the two NE conditions (A.5) and (A.6) cover a larger parameter space when b_2 is smaller, so the channel should choose the smallest possible b_2 that satisfies Equation (A.4); that is, $b_2^*=v_A$. Retailer 1's equilibrium bid b_1^* can be any value higher than or equal to v_A. The channel profit under configuration (R_1,R_2) is

$$\pi_C(b_1^*,b_2^*)=d_1\,[\theta_1(m_1+r_1)-v_A]+d_2\,[\theta_2(m_2+r_2)-v_A],$$

and the equilibrium can exist when $\theta_2(m_2+r_2)\geq v_A$. (Equation (A.6) is automatically satisfied given this and $\theta_1(m_1+r_1)\ \geq\ \theta_2(m_2+r_2)$.)

- Consider the position configuration (R_1,A).

For this position configuration to be an equilibrium, the bids should satisfy $b_1\geq b_A>b_2$.

Given any b_2, to prevent the outside advertiser A deviating from position 2 to position 1, advertiser A's bid in equilibrium is $b_A=\frac{d_1-d_2}{d_1}v_A+\frac{d_2}{d_1}b_2$ according to SNE and LB.

To prevent advertiser A deviating from position 2 to position 3, we need $d_2(v_A-b_2)>0$; that is, $b_2<v_A$.

To prevent the channel deviating from (R_1,A) to (R_1,R_2), we need $d_1[\theta_1(m_1+r_1)-b_A]>d_1[\theta_1(m_1+r_1)-b_A]+d_2[\theta_2(m_2+r_2)-b_A]$, which is equivalent to

$$b_A>\theta_2(m_2+r_2). \tag{A.8}$$

To prevent the channel deviating from (R_1,A) to (A,R_1), we need $d_1[\theta_1(m_1+r_1)-b_A]\geq d_2[\theta_1(m_1+r_1)-b_2]$. Given the expression of b_A, this is equivalent to $\theta_1(m_1+r_1)\geq v_A$.

The channel profit is

$$\begin{aligned}\pi_C(b_1,b_2)&=d_1\,[\theta_1(m_1+r_1)-b_A]\\&=d_1\left[\theta_1(m_1+r_1)-\frac{d_1-d_2}{d_1}v_A-\frac{d_2}{d_1}b_2\right]\end{aligned} \tag{A.9}$$

According to Equation (A.9), the channel profit decreases in b_2 given the position configuration. Thus, retailer 2 will choose the lowest possible bid that satisfies the equilibrium conditions. Notice the requirement of Equation (A.8) and $b_A=\frac{d_1-d_2}{d_1}v_A+\frac{d_2}{d_1}b_2$, when $\theta_2(m_2+r_2)<\frac{d_1-d_2}{d_1}v_A$, retailer 2 can choose $b_2^*=0$ in equilibrium; however, when $\theta_2(m_2+r_2)>\frac{d_1-d_2}{d_1}v_A$, to sustain the equilibrium, retailer 2 has to choose b_2^* such that $\theta_2(m_2+r_2)=\frac{d_1-d_2}{d_1}v_A+\frac{d_2}{d_1}b_2^*$.[21]

The channel profit under position configuration (R_1,A) is

$$\pi_C(b_1^*,b_2^*)=d_1\left[\theta_1(m_1+r_1)-\max\{\frac{d_1-d_2}{d_1}v_A,\theta_2(m_2+r_2)\}\right],$$

and the NE holds when $\theta_1(m_1+r_1)\geq v_A>\theta_2(m_2+r_2)$ (the latter inequality comes from $b_2<v_A$).

- Consider the position configuration (A,R_1).

For this position configuration to be an equilibrium, the bids should satisfy $b_A>b_1\geq b_2$.

To prevent advertiser A deviating from position 1 to position 2, we need $d_1(v_A-b_1)>d_2(v_A-b_2)$, which is equivalent to $b_1<\frac{d_1-d_2}{d_1}v_A+\frac{d_2}{d_1}b_2$. To prevent advertiser A deviating from position 1 to position 3, we need $d_1(v_A-b_1)>0$, which is equivalent to $b_1<v_A$.

To prevent the channel deviating from (A,R_1) to (R_1,R_2), we need $d_2[\theta_1(m_1+r_1)-b_2]>d_1[\theta_1(m_1+r_1)-b_A]+d_2[\theta_2(m_2+r_2)-b_A]$. To prevent the channel deviating from (A,R_1) to (R_1,A), we need $d_2[\theta_1(m_1+r_1)-b_2]>d_1[\theta_1(m_1+r_1)-b_A]$. Given any b_2, these two conditions can be satisfied when b_A is sufficiently large.

The channel profit is

$$\pi_C(b_1,b_2)=d_2\,[\theta_1(m_1+r_1)-b_2] \tag{A.10}$$

According to Equation (A.10), the channel profit decreases in b_2 at this position configuration. Thus, retailer 2 will choose the lowest possible bid in equilibrium; that is, $b_2^* = 0$. Retailer 1's equilibrium bid b_1^* can be any value between zero and $\frac{d_1-d_2}{d_1}v_A$. The channel profit under position configuration (R_1, A) is

$$\pi_C(b_1^*, b_2^*) = d_2\theta_1(m_1 + r_1),$$

and this equilibrium can exist on the entire parameter space given $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$.

Step 3. So far, for each of the three position configurations, we have worked out the largest possible parameter space on which the position configuration can be an equilibrium, the channel's optimal choice of bids in the parameter space, and the channel's optimal profit. Table A.1 summarizes the Nash equilibrium condition for each position configuration and the integrated channel's corresponding profit.

Then, we can divide the parameter space of $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$ into four mutually exclusive parts. In each part, if only one position configuration can exist according to this analysis, it will be the equilibrium position configuration, and if there are multiple possible position configurations, the channel will choose the one with the highest channel profit in equilibrium.

- Part 1. $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2) \geq v_A$.

The possible position configurations in equilibrium are (R_1, R_2) and (A, R_1). The channel will choose (A, R_1) when $\theta_1(m_1 + r_1) < \frac{d_1+d_2}{d_1-d_2}v_A - \frac{d_2}{d_1-d_2}\theta_2(m_2 + r_2)$ and will choose (R_1, R_2) otherwise.

- Part 2. $\theta_1(m_1 + r_1) \geq v_A, \theta_2(m_2 + r_2) < \frac{d_1-d_2}{d_1}v_A$.

The possible position configurations are $(R_1, A)^a$ and (A, R_1).[22] The channel will always choose $(R_1, A)^a$ in this area.

- Part 3. $\theta_1(m_1 + r_1) \geq v_A, \frac{d_1-d_2}{d_1}v_A \leq \theta_2(m_2 + r_2) < v_A$.

The possible configurations are $(R_1, A)^b$ and (A, R_1). The channel will choose (A, R_1) when $\theta_1(m_1 + r_1) < \frac{d_1}{d_1-d_2}\theta_2(m_2 + r_2)$ and will choose $(R_1, A)^b$ otherwise.

- Part 4. $\theta_2(m_2 + r_2) \leq \theta_1(m_1 + r_1) \leq v_A$.

The only possible position configuration is (A, R_1).

To summarize, the condition for each position configuration to be chosen is

- (R_1, R_2): $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2) \geq v_A$ and $\theta_1(m_1 + r_1) \geq \frac{d_1+d_2}{d_1-d_2}v_A - \frac{d_2}{d_1-d_2}\theta_2(m_2 + r_2)$
- (R_1, A): $\theta_1(m_1 + r_1) \geq v_A > \theta_2(m_2 + r_2)$ and $\theta_1(m_1 + r_1) \geq \frac{d_1}{d_1-d_2}\theta_2(m_2 + r_2)$
- (A, R_1): $\theta_2(m_2 + r_2) \leq \theta_1(m_1 + r_1) \leq v_A$ or $\theta_1(m_1 + r_1) < \min\{\frac{d_1}{d_1-d_2}\theta_2(m_2 + r_2), \frac{d_1+d_2}{d_1-d_2}v_A - \frac{d_2}{d_1-d_2}\theta_2(m_2 + r_2)\}$.

Step 4. Now, we compare the integrated channel's profit versus the nonintegrated channel's profit.

Table A.2 summarizes the integrated channel's profit under each position configuration and the manufacturer's profit and channel profit when the channel is nonintegrated.

One observation is that, under position configuration (R_1, R_2), the profit of the nonintegrated channel and that of the integrated channel are the same. When the channel profits per click for both retailers are relatively high, both the integrated channel and the nonintegrated channel will have position configuration (R_1, R_2) in equilibrium and will earn the same channel profit in equilibrium. This is evidence of the efficiency of the participation-rate mechanism.

An important and surprising finding is that the total channel profit could be lower in the integrated case than in the nonintegrated case. When $\frac{d_1-d_2}{d_1}v_A \leq \theta_2(m_2 + r_2) < v_A$ and $\theta_1(m_1 + r_1) \geq \frac{d_1}{d_1-d_2}\theta_2(m_2 + r_2)$, the equilibrium position configuration for the integrated channel is (R_1, A), and the equilibrium channel profit is $\pi^*_{inte-C} = d_1[\theta_1(m_1 + r_1) - \theta_2(m_2 + r_2)]$. Within this parameter space, when $\theta_2(m_2 + r_2) \in (v_A - \frac{d_2}{d_1}\frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A}, v_A)$,[23]

$$\begin{aligned}\pi^*_{inte-C} &= d_1[\theta_1(m_1 + r_1) - \theta_2(m_2 + r_2)]\\ &< d_1[\theta_1(m_1 + r_1) - v_A] + d_2\frac{(v_A - \theta_1 r_1)(v_A - \theta_2 r_2)}{v_A}\\ &= \pi^{R_1 A}_{noninte-M}\\ &\leq \pi^*_{noninte-M}.\end{aligned}$$

The last inequality comes from the fact that when the channel is nonintegrated, the manufacturer chooses the position configuration that maximizes its own profit in equilibrium. Further noticing that, at any position configuration, $\pi_{noninte-M} \leq \pi_{noninte-C}$, we know the equilibrium position configuration also satisfies $\pi^*_{noninte-M} \leq \pi^*_{noninte-C}$. Therefore, when $\theta_2(m_2 + r_2) \in (v_A - \frac{d_2}{d_1}\frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A}, v_A)$ and $\theta_1(m_1 + r_1) \geq \frac{d_1}{d_1-d_2}\theta_2(m_2 + r_2)$, $\pi^*_{inte-C} < \pi^*_{noninte-C}$, meaning that under such condition, the integrated channel's profit in equilibrium is lower than the nonintegrated channel's profit in equilibrium.

A.3. Manufacturer's Direct Participation (Proof of Theorem 2)

We solve the game by backward induction. In the second stage, given the manufacturer's participation decision and participation rates, each bidder's (equivalent) profit per click will be determined. Given four bidders competing for two positions, there are a total of $4 \times 3 = 12$ possible position configurations. In four of the 12 position configurations, the manufacturer is above both retailers, and in the other eight, the manufacturer is below one or both retailers. For each position configuration with the manufacturer above both retailers, we determine the bids of the manufacturer, the retailers, and the outside advertiser and then maximize the manufacturer's profit with respect to participation rates α_1, α_2, similar to the analysis in the main model; for each position configuration with the manufacturer below one or both retailers, we determine the retailer and outside advertiser's bids first and then maximize the manufacturer's profit with respect to α_1, α_2 and its bid b_M

Table A.1. Profits and NE Condition for Each Position Configuration: Integrated Channel

	π_C	NE condition
(R_1, R_2)	$d_1[\theta_1(m_1 + r_1) - v_A] + d_2[\theta_2(m_2 + r_2) - v_A]$	$\theta_2(m_2 + r_2) \geq v_A$
(R_1, A)	$d_1[\theta_1(m_1 + r_1) - \max\{\frac{d_1-d_2}{d_1}v_A, \theta_2(m_2 + r_2)\}]$	$\theta_1(m_1 + r_1) \geq v_A > \theta_2(m_2 + r_2)$
(A, R_1)	$d_2\theta_1(m_1 + r_1)$	any $\theta_1(m_1 + r_1) \geq \theta_2(m_2 + r_2)$

Table A.2. Profits Under Each Position Configuration for Integrated and Nonintegrated Channel

	Integrated	Nonintegrated	
	π_{inte-C}	$\pi_{noninte-M}$	$\pi_{noninte-C}$
(R_1, R_2)	$d_1[\theta_1(m_1+r_1)-v_A]+$ $d_2[\theta_2(m_2+r_2)-v_A]$	$d_1[\theta_1(m_1+r_1)-v_A]+$ $d_2[\theta_2(m_2+r_2)-v_A]$	$d_1[\theta_1(m_1+r_1)-v_A]+$ $d_2[\theta_2(m_2+r_2)-v_A]$
(R_1, A)	$d_1[\theta_1(m_1+r_1)$ $-\max\{\frac{d_1-d_2}{d_1}v_A, \theta_2(m_2+r_2)\}]$	$d_1[\theta_1(m_1+r_1)-v_A]+$ $d_2\frac{(v_A-\theta_1r_1)(v_A-\theta_2r_2)}{v_A}$	$d_1[\theta_1(m_1+r_1)-v_A]+$ $d_2[v_A-\theta_2r_2]$
(A, R_1)	$d_2\theta_1(m_1+r_1)$	$d_2[\theta_1(m_1+r_1)-\max\{\theta_1r_1, \theta_2r_2\}]$	$d_2[\theta_1(m_1+r_1)-\theta_2r_2]$

without imposing the Nash equilibrium condition that guards against the manufacturer's deviation to other positions. This is because the analysis gets very complicated by taking into account the Nash equilibrium condition directly. Instead, we show that this actually does not influence the equilibrium outcome; that is, the Nash equilibrium conditions for all position configurations are not binding. If the manufacturer's optimal bid b_M^* is zero, then the manufacturer should commit to quit the bidding in the first stage because, otherwise, the manufacturer may find it profitable to deviate in the second stage, and we cannot safely omit the Nash equilibrium condition. Back to the first stage, the manufacturer compares its profits over the 12 position configurations to determine the equilibrium. The equilibrium condition for one position configuration is that the manufacturer's profit under this position configuration is greater than that under all other position configurations. We find that when this equilibrium condition holds, the Nash equilibrium condition for this position configuration always holds. This means that the Nash equilibrium condition is not binding.

The detailed analysis for the 12 possible position configurations is rendered in the Online Appendix. From the analysis, it is easy to see that given the assumption $\theta_1(m_1+r_1) \geq \theta_2(m_2+r_2)$, retailer 2 cannot get a higher position than retailer 1 in equilibrium.

Given that retailer 1 always gets a higher position than retailer 2, there are seven possible position configurations in equilibrium. Table A.3 summarizes the manufacturer's profit and the optimal participation rates under each position configuration.

In the first stage, that is, when the manufacturer chooses the participation rates, it will compare its optimal profit under these possible position configurations and choose the position that leads to the highest profit $\pi_M(\alpha_1^*, \alpha_2^*, b_M^*)$. It chooses the optimal participation rates correspondingly. If the manufacturer finds that, to stay in the third position is optimal for itself, it will commit to quit the bidding.

To compare the optimal profits under the seven possible position configurations is too complicated, and the result will not be informative. So we focus on two managerially important questions: (1) when should the manufacturer participate in bidding, and (2) should the manufacturer sponsor the retailer(s) when it participates in bidding?

Before we answer these two questions, we first notice that it is relatively easier to classify the seven position configurations into three categories (the channel takes both positions, takes the first position, and takes the second position) and compare the manufacturer's profit within each category.

- Category 1. The channel takes both positions, that is, $(M, R_1), (R_1, M), (R_1, R_2)$. Within this category, the optimal position configuration is determined by the rank of channel profit per click. That is, if $\theta_0 m_0 \geq \theta_1(m_1+r_1) \geq \theta_2(m_2+r_2)$, the optimal position configuration out of the three is (M, R_1); if $\theta_1(m_1+r_1) > \theta_0 m_0 \geq \theta_2(m_2+r_2)$, the optimal one is (R_1, M); and if $\theta_1(m_1+r_1) \geq \theta_2(m_2+r_2) > \theta_0 m_0$, the optimal one is (R_1, R_2).
- Category 2. The channel takes only the first position, that is, $(M, A), (R_1, A)$. Within this category, (M, A) is the more profitable one if and only if

$$\theta_0 m_0 \geq \theta_1(m_1+r_1) - \frac{d_2}{d_1}\left(v_A - \max\{\theta_1 r_1, \theta_2 r_2\} - \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A}\right), \tag{A.11}$$

and (R_1, A) is the more profitable one if and only if

$$\theta_0 m_0 + \frac{d_2}{d_1}\left(v_A - \max\{\theta_1 r_1, \theta_2 r_2\} - \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A}\right) < \theta_1(m_1+r_1). \tag{A.12}$$

Notice that $v_A - \max\{\theta_1 r_1, \theta_2 r_2\} - \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A} > 0$, so Equation (A.11) is a weaker condition than $\theta_0 m_0 \geq \theta_1(m_1+r_1)$. Therefore, when $\theta_0 m_0 \geq \theta_1(m_1+r_1)$, (M, A) is definitely more profitable than (R_1, A), but $(R_1, A) \succ (M, A)$ needs a stronger condition than $\theta_1(m_1+r_1) > \theta_0 m_0$.

Table A.3. Optimal Participation Rates and Manufacturer's Profit

Position	$\pi_M(\alpha_1^*, \alpha_2^*, b_M^*)$	(α_1^*, α_2^*)	Requirement for NE
(M, R_1)	$d_1(\theta_0 m_0 - v_A) + d_2[\theta_1(m_1+r_1) - v_A]$	$(1-\frac{\theta_1 r_1}{v_A}, 0)$	$\theta_0 m_0 \geq \theta_1(m_1+r_1)$ and $\frac{d_1}{d_1+d_2}\theta_0 m_0 + \frac{d_2}{d_1+d_2}\theta_1(m_1+r_1) \geq v_A$
(R_1, M)	$d_1[\theta_1(m_1+r_1) - v_A] + d_2(\theta_0 m_0 - v_A)$	$(1-\frac{\theta_1 r_1}{v_A}, 0)$	$\theta_0 m_0 \geq v_A$
(R_1, R_2)	$d_1[\theta_1(m_1+r_1) - v_A] + d_2[\theta_2(m_2+r_2) - v_A]$	$(1-\frac{\theta_1 r_1}{v_A}, 1-\frac{\theta_2 r_2}{v_A})$	
(M, A)	$d_1(\theta_0 m_0 - v_A) + d_2(v_A - \max\{\theta_1 r_1, \theta_2 r_2\})$	$(0, 0)$	$\theta_0 m_0 \geq v_A$
(R_1, A)	$d_1[\theta_1(m_1+r_1) - v_A] + d_2\frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A}$	$(1-\frac{\theta_1 r_1}{v_A}, 0)$	
(A, M)	$d_2(\theta_0 m_0 - \max\{\theta_1 r_1, \theta_2 r_2\})$	$(0, 0)$	$v_A \geq \theta_0 m_0$
(A, R_1)	$d_2[\theta_1(m_1+r_1) - \max\{\theta_1 r_1, \theta_2 r_2\}]$	$(\max\{1-\frac{\theta_1 r_1}{\theta_2 r_2}, 0\}, 0)$	

- Category 3. The channel takes only the second position, that is, $(A,M),(A,R_1)$. (A,M) is the more profitable one if $\theta_0 m_0 \geq \theta_1(m_1+r_1)$, and (A,R_1) is the more profitable one if and only if $\theta_1(m_1+r_1) > \theta_0 m_0$.

The manufacturer can first select the winner from each category and then compare the candidates selected from the three categories.

We divide the parameter space into three cases according to the relationship between the manufacturer's channel profit per click and the two retailers' channel profit per click.

- Case 1. The manufacturer's channel profit per click is higher than both retailers; that is, $\theta_0 m_0 \geq \theta_1(m_1+r_1) \geq \theta_2(m_2+r_2)$.

The candidates from the three categories are (M,R_1), (M,A), and (A,M). No matter which one out of the three is the most profitable, the manufacturer participates in bidding in equilibrium. The manufacturer sponsors R_1 when (M,R_1) is the position configuration in equilibrium, that is, when $\theta_1(m_1+r_1) \geq 2v_A - \max\{\theta_1 r_1, \theta_2 r_2\}$.[24]

- Case 2. The manufacturer's channel profit per click is higher than R_2 but lower than R_1; that is, $\theta_1(m_1+r_1) > \theta_0 m_0 \geq \theta_2(m_2+r_2)$.

The candidates from the first and third categories are (R_1,M) and (A,R_1), respectively. In the second category, the winner is (R_1,A) if $\theta_1(m_1+r_1) > \theta_0 m_0 + \frac{d_2}{d_1}(v_A - \max\{\theta_1 r_1, \theta_2 r_2\} - \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A})$ and is (M,A) if $\theta_1(m_1+r_1) > \theta_0 m_0 \geq \theta_1(m_1+r_1) - \frac{d_2}{d_1}(v_A - \max\{\theta_1 r_1, \theta_2 r_2\} - \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A})$.

The manufacturer participates in bidding when (R_1,M) or (M,A) is the position configuration in equilibrium and sponsors R_1 in the former case.

(R_1,M) is more profitable than the other three position configurations if and only if

$$\theta_0 m_0 - v_A \geq \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A},$$
$$\theta_1(m_1+r_1) - v_A \geq \frac{d_1-d_2}{d_1}(\theta_0 m_0 - v_A) + \frac{d_2}{d_1}(v_A - \max\{\theta_1 r_1, \theta_2 r_2\}),$$
$$\text{and } \theta_1(m_1+r_1) - v_A \geq \frac{d_2}{d_1}\left(\theta_1(m_1+r_1) - \theta_0 m_0 + v_A - \max\{\theta_1 r_1, \theta_2 r_2\}\right).$$

Notice that the requirement for NE is automatically satisfied. Therefore, (R_1,M) is the position configuration in equilibrium when the aforementioned conditions are satisfied.

Similarly, (M,A) is the position configuration in equilibrium if and only if

$$\theta_1(m_1+r_1) - v_A < \frac{d_1-d_2}{d_1}(\theta_0 m_0 - v_A) + \frac{d_2}{d_1}(v_A - \max\{\theta_1 r_1, \theta_2 r_2\}),$$
$$\theta_1(m_1+r_1) - \theta_0 m_0 \leq \frac{d_2}{d_1}\left(v_A - \max\{\theta_1 r_1, \theta_2 r_2\} - \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A}\right),$$
$$\text{and } \theta_0 m_0 - v_A \geq \frac{d_2}{d_1}[\theta_1(m_1+r_1) - v_A].$$

- Case 3. The manufacturer's channel profit per click is lower than both retailers; that is, $\theta_1(m_1+r_1) \geq \theta_2(m_2+r_2) > \theta_0 m_0$.

The candidates from the first and the third categories are (R_1,R_2) and (A,R_1). In the second category, the winner is (R_1,A) if $\theta_1(m_1+r_1) > \theta_0 m_0 + \frac{d_2}{d_1}(v_A - \max\{\theta_1 r_1, \theta_2 r_2\} - \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A})$ and is (M,A) if $\theta_1(m_1+r_1) \geq \theta_2(m_2+r_2) > \theta_0 m_0 \geq \theta_1(m_1+r_1) - \frac{d_2}{d_1}(v_A - \max\{\theta_1 r_1, \theta_2 r_2\} - \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A})$.

The manufacturer participates in bidding only when (M,A) is the position configuration in equilibrium, that is, when

$$\theta_1(m_1+r_1) - v_A < \frac{d_2}{d_1}\left(2v_A - \theta_2(m_2+r_2) - \max\{\theta_1 r_1, \theta_2 r_2\}\right),$$
$$\theta_1(m_1+r_1) - \theta_0 m_0 \leq \frac{d_2}{d_1}\left(v_A - \max\{\theta_1 r_1, \theta_2 r_2\} - \frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A}\right),$$
$$\text{and } \theta_0 m_0 - v_A \geq \frac{d_2}{d_1}[\theta_1(m_1+r_1) - v_A].$$

In this position configuration, the manufacturer will not sponsor the retailers.

From these results, we can see that the manufacturer will definitely participate in bidding when its profit per click is higher than both retailers and may participate in bidding when its profit per click is lower than one or both retailers but not too low. When the manufacturer participates in bidding, it may still sponsor one retailer but will not sponsor both retailers. The manufacturer will support both retailers only when its profit per click is lower than both retailers and the manufacturer quits the bidding itself.

Endnotes

[1] Data source: eMarketer.com.

[2] These industries include retailing and general merchandise, home and garden, computer and consumer electronics, and vehicles as well as business and industrial. This is according to a breakdown of Google's 2011 revenue provided by WordStream.com.

[3] Google provides two kinds of search ads: AdWords ads in text and product listing ads in pictures. We do not explicitly distinguish them in our paper.

[4] https://www.netsertive.com/resources/borrell-research-report-changing-face-co-op-programs/, p. 15.

[5] Co-op Advertising Programs Sourcebook, by National Register Publishing Company.

[6] https://www.iab.com/wp-content/uploads/2015/07/CoopAdvertisingStudy.pdf, p. 10.

[7] This happens, for example, when the manufacturer does not have its own e-commerce site that sells to consumers directly.

[8] In practice, search engines usually use a modified GSP mechanism that adjusts the ranking according to advertisers' "quality score," which, roughly speaking, is the prediction of an advertiser's click-through rate. In the basic model, we consider the simple case that all advertisers have the same quality score. In Section 4.1, we incorporate heterogeneous advertiser quality scores and show that the findings of the basic model generalize nicely.

[9] Notice that a second-price auction with only one position is a special case of our model with $d_2 = 0$.

[10] Edelman et al. (2007) proposed the concept of "locally envy-free equilibria," which requires that each bidder cannot improve its payoff by exchanging bids with the bidder ranked one position above it, and it yields the same result as the LB and SNE in Varian (2007).

[11] This result is in contrast with that on cooperative advertising in classic channels with a dominant retailer. Geylani et al. (2007) shows that when a manufacturer sells via a dominant retailer and a weak retailer and the dominant retailer dictates the wholesale price while the manufacturer sets the wholesale price for the weak retailer, the manufacturer may use cooperative advertising to shift demand from the more profitable channel to the less profitable channel because its channel power is stronger there. Our result is based on an implicit assumption that the channel power of the manufacturer is the same over the two retailers.

[12] This means that the outside advertiser can anticipate the integrated channel's bids b_1 and b_2 and then decides its bid b_A.

[13] The PM criterion also facilitates the equilibrium analysis. Particularly, the SNE and LB rules for the outside advertiser will determine its bid as a function of the integrated channel's bids. Then when the integrated channel follows the PM criterion to pick the most profitable equilibrium, it essentially chooses its bids after taking into account the outsider advertiser's response to its bids. Mathematically, this is as if the integrated channel chooses its bids first while anticipating the outside advertiser's response and then the outside advertiser observes the integrated channel's choice and then decides its bid. Therefore, the PM criterion essentially transforms the current simultaneous bidding game equivalently into a dynamic game with the integrated channel bidding first. The equilibrium outcome is the same whether the outside advertiser anticipates or observes the integrated channel's bids. In fact, in Varian's (2007) setup with independent bidders, consider a dynamic game in which one bidder decides its bid first and other bidders observe the bid and decide their bids subsequently. This dynamic game will yield the same equilibrium outcome as a static game in which everyone bids simultaneously as long as the SNE and LB equilibrium selection rules are maintained.

[14] Suppose $\theta_1(m_1+r_1) \geq \theta_2(m_2+r_2)$; then the channel profit in the integrated case is lower compared with the nonintegrated case when $v_A - \frac{d_2}{d_1}\frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A} < \theta_2(m_2+r_2) < v_A$ and $\theta_1(m_1+r_1) \geq \frac{d_1}{d_1-d_2}\theta_2(m_2+r_2)$. This is a sufficient but not a necessary condition.

[15] The manufacturer can commit by, for example, not setting up a search engine marketing team.

[16] We have not fully modeled consumers' searching and clicking behaviors, which is beyond the scope of the paper. See Athey and Ellison (2011), Chen and He (2011), and Jerath et al. (2011), etc.

[17] If there is only one outside advertiser, when retailer 1 and the outside advertiser get displayed, their cost per click is the profit per click of retailer 2. However, in this case, retailer 2 is not displayed and, thus, does not compete with retailer 1 directly. Therefore, retailer 2's profit per click is ambiguous. To get over the complexity of defining equilibrium selection rules for retailer 2's profit per click when retailer 2 does not get a position, we assume that there are two outside advertisers in the market. With two outside advertisers, when retailer 1 and one outside advertiser get displayed, their cost per click is the profit per click of the other outside advertiser.

[18] We essentially consider the case in which consumers' cost of clicking is zero. If consumers face a positive cost of clicking and if prices are the only information consumers learn after clicking, the classic Diamond paradox (Diamond 1971) implies that each retailer will set a monopolistic price no matter how small the cost of clicking is, and there is no price competition.

[19] When the Hotelling line is not fully covered (i.e., t is large or v is small), there is no direct price competition between the two retailers. Each of the two retailers can be considered as a monopoly in its covered market. In such case, the manufacturer's profit of sponsoring two retailers equals the profit of sponsoring one retailer multiplied by two, and therefore, the manufacturer will sponsor both retailers or sponsor neither retailer. As Theorem 4 shows, when the Hotelling line is covered, the manufacturer tends to sponsor one retailer when t is relatively small and will sponsor zero or both retailers when t is relatively large. Here, when t is large enough, the Hotelling line is not covered, and the manufacturer will sponsor zero or two retailers, so it is consistent with the conclusion of Theorem 4.

[20] Considering endogenous wholesale contracts with retail price competition is challenging and beyond the scope of this paper. The focus of this section is to show that when wholesale contracts are fully flexible, the manufacturer should sponsor retailers in search advertising by first lowering wholesale prices rather than using participation rates.

[21] In other words, for (R_1, A) to be an equilibrium, advertiser A has to bid no less than $\theta_2(m_2+r_2)$.

[22] For the position configuration (R_1, A), we denote it as $(R_1, A)^a$ when $\theta_2(m_2+r_2) < \frac{d_1-d_2}{d_1}v_A$ and denote it as $(R_1, A)^b$ when $\frac{d_1-d_2}{d_1}v_A \leq \theta_2(m_2+r_2) < v_A$.

[23] Notice that $\frac{d_1-d_2}{d_1}v_A < v_A - \frac{d_2}{d_1}\frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A} < v_A$, so the set $(v_A - \frac{d_2}{d_1}\frac{(v_A-\theta_1 r_1)(v_A-\theta_2 r_2)}{v_A}, v_A)$ is a nonempty subset of $(\frac{d_1-d_2}{d_1}v_A, v_A)$.

[24] $(M, R_1) \succ (M, A)$is equivalent to $\theta_1(m_1+r_1) \geq 2v_A - \max\{\theta_1 r_1, \theta_2 r_2\}$, which implies $\theta_0 m_0 \geq \theta_1(m_1+r_1) > v_A$ and, thus, indicates $(M, R_1) \succ (M, A) \succ (A, M)$. The requirement for (M, R_1) to be an NE is also automatically satisfied.

References

Athey S, Ellison G (2011) Position auctions with consumer search. *Quart. J. Econom.* 126(3):1213–1270.

Bergen M, John G (1997) Understanding cooperative advertising participation rates in conventional channels. *J. Marketing Res.* 34(3): 357–369.

Berger PD (1972) Vertical cooperative advertising ventures. *J. Marketing Res.* 9(3):309–312.

Chen Y, He C (2011) Paid placement: Advertising and search on the internet. *Econom. J.* 121(556):F309–F328.

Coughlan AT, Wernerfelt B (1989) On credible delegation by oligopolists: A discussion of distribution channel management. *Management Sci.* 35(2):226–239.

Decarolis F, Rovigatti G (2017) Online auctions and digital marketing agencies. NET Institute Working Paper 17-18, Cambridge, MA.

Decarolis F, Goldmanis M, Penta A (2017) Marketing agencies and collusive bidding in online ad auctions. NBER Working Paper 23962, Cambridge, MA.

Dellarocas C (2012) Double marginalization in performance-based advertising: Implications and solutions. *Management Sci.* 58(6): 1178–1195.

Desai PS (1997) Advertising fee in business-format franchising. *Management Sci.* 43(10):1401–1419.

Desai PS, Shin W, Staelin R (2014) The company that you keep: When to buy a competitor's keyword. *Marketing Sci.* 33(4): 485–508.

Diamond PA (1971) A model of price adjustment. *J. Econom. Theory* 3(2):156–168.

Dutta S, Bergen M, John G, Rao A (1995) Variations in the contractual terms of cooperative advertising contracts: An empirical investigation. *Marketing Lett.* 6(1):15–22.

Edelman B, Ostrovsky M, Schwarz M (2007) Internet advertising and the generalized second-price auction: Selling billions of dollars worth of keywords. *Amer. Econom. Rev.* 97(1):242–259.

Feng J (2008) Optimal mechanism for selling a set of commonly ranked objects. *Marketing Sci.* 27(3):501–512.

Geylani T, Dukes AJ, Srinivasan K (2007) Strategic manufacturer response to a dominant retailer. *Marketing Sci.* 26(2): 164–178.

Jerath K, Ma L, Park Y-H, Srinivasan K (2011) A "position paradox" in sponsored search auctions. *Marketing Sci.* 30(4):612–627.

Katona Z, Sarvary M (2010) The race for sponsored links: Bidding patterns for search advertising. *Marketing Sci.* 29(2): 199–215.

Kim SY, Staelin R (1999) Manufacturer allowances and retailer pass-through rates in a competitive environment. *Marketing Sci.* 18(1): 59–76.

Lu S, Zhu Yi, Dukes A (2015) Position auctions with budget constraints: Implications for advertisers and publishers. *Marketing Sci.* 34(6):897–905.

McGuire TW, Staelin R (1983) An industry equilibrium analysis of downstream vertical integration. *Marketing Sci.* 2(2):161–191.

Nagler MG (2006) An exploratory analysis of the determinants of cooperative advertising participation rates. *Marketing Lett.* 17(2): 91–102.

Narayanan S, Kalyanam K (2015) Position effects in search advertising and their moderators: A regression discontinuity approach. *Marketing Sci.* 34(3):388–407.

Sayedi A (2018) Real-time bidding in online display advertising. *Marketing Sci.* Forthcoming.

Sayedi A, Jerath K, Srinivasan K (2014) Competitive poaching in sponsored search advertising and its strategic impact on traditional advertising. *Marketing Sci.* 33(4):586–608.

Varian HR (2007) Position auctions. *Internat. J. Indust. Organ.* 25(6): 1163–1178.

Villas-Boas JM (1998) Product line design for a distribution channel. *Marketing Sci.* 17(2):156–169.

Wilbur KC, Zhu Yi (2009) Click fraud. *Marketing Sci.* 28(2):293–308.

Xu L, Chen J, Whinston A (2011) Price competition and endogenous valuation in search advertising. *J. Marketing Res.* 48(3):566–586.

Zhu Y, Wilbur KC (2011) Hybrid advertising auctions. *Marketing Sci.* 30(2):249–273.