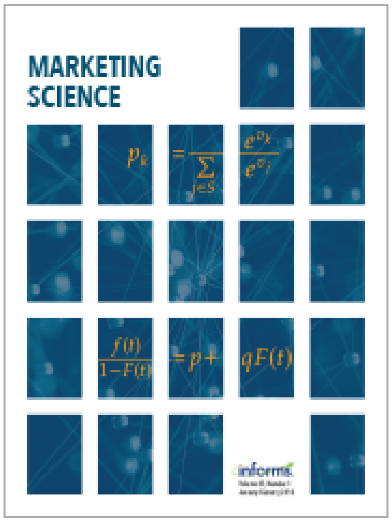

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Functional Regression: A New Model for Predicting Market Penetration of New Products

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The Bass model has been a standard for analyzing and predicting the market penetration of new products. We demonstrate the insights to be gained and predictive performance of functional data analysis (FDA), a new class of nonparametric techniques that has shown impressive results within the statistics community, on the market penetration of 760 categories drawn from 21 products and 70 countries. We propose a new model called Functional Regression and compare its performance to several models, including the Classic Bass model, Estimated Means, Last Observation Projection, a Meta-Bass model, and an Augmented Meta-Bass model for predicting eight aspects of market penetration. Results (a) validate the logic of FDA in integrating information across categories, (b) show that Augmented Functional Regression is superior to the above models, and (c) product-specific effects are more important than country-specific effects when predicting penetration of an evolving new product.

Key words: predicting market penetration; global diffusion; Bass model; functional data analysis; functional principal components; generalized additive models; functional clustering; spline regression; new products
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Introduction

Firms are introducing new products at an increasingly rapid rate. At the same time, the globalization of markets has increased the speed at which new products diffuse across countries, mature, and die off (Chandrasekaran and Tellis 2008). These two forces have increased the importance of the accurate prediction of the market penetration of an evolving new product. Although research on modeling sales of new products in marketing has been quite insightful (Chandrasekaran and Tellis 2007), it is limited in a few respects. First, most studies rely primarily, if not exclusively, on the Bass model. Second, prior research, especially those based on the Bass model, need data past the peak sales or penetration for stable estimates and meaningful predictions. Third, prior research has not indicated how the wealth of accumulated penetration histories across countries and categories can be best integrated for good prediction of penetration of an evolving new product. For example, a vital unanswered question is whether a new product's penetration can be best predicted from past penetration of (a) similar products in the same country, (b) the same product in similar countries, (c) the same product itself in the same country, or (d) some combination of these three histories.

The current study attempts to address these limitations. In particular, it makes four contributions to the literature. First, we illustrate the potential advantages of using functional data analysis (FDA) techniques for the analysis of penetration curves (Ramsay and Silverman 2005). Second, we demonstrate how information about the historical evolution of new products in other categories and countries can be integrated to predict the evolution of penetration of a new product. Third, we compare the predictive performance of the Bass model versus an FDA approach and some naïve models. Fourth, we indicate whether information about prior countries, other categories, the target product itself, or a combination of all three is most important in predicting the penetration of an evolving new product.

One important aspect of the current study is that it uses data about market penetration from most of 21 products across 70 countries, for a total of 760 categories (product × country combinations). The data include both developed and developing countries from Europe, Asia, Africa, Australasia, and North and South America. In scope, this study exceeds the sample used in prior studies (see Table 1), yet the approach achieves our goals in a computationally efficient and substantively instructive manner.

Table 1 Scope of Prior Studies

Authors	Categories	Countries
Gatignon et al. (1989)	6 consumer durables	14 European countries
Mahajan et al. (1990)	Numerous studies	
Sultan et al. (1990)	213 applications	United States, European countries
Helsen et al. (1993)	3 consumer durables	11 European countries and United States
Ganesh and Kumar (1996)	1 industrial product	10 European countries, United States, Japan
Ganesh et al. (1997)	4 consumer durables	16 European countries
Golder and Tellis (1997)	31 consumer durables	United States
Putsis et al. (1997)	4 consumer durables	10 European countries
Dekimpe et al. (1998)	1 service	74 countries
Kumar et al. (1998)	5 consumer durables	14 European countries
Golder and Tellis (1998)	10 consumer durables	United States
'Kohli et al. (1999)	32 appliances, housewares and electronics	United States
Dekimpe et al. (2000)	1 innovation	More than 160 countries
Mahajan et al. (2000)	Numerous studies	
Van den Bulte (2000)	31 consumer durables	United States
Talukdar et al. (2002)	6 consumer durables	31 countries
Agarwal and Bayus (2002)	30 innovations	United States
Goldenberg et al. (2002)	32 innovations	United States
Tellis et al. (2003)	10 consumer durables	16 European countries
Golder and Tellis (2004)	30 consumer durables	United States
Stremersch and Tellis (2004)	10 consumer durables	16 European countries
Van den Bulte and Stremersch (2004)	293 applications	28 countries
Chandrasekaran and Tellis (2007)	16 products and services	40 countries

Note. Adapted from Chandrasekaran and Tellis (2008).

Another important aspect of the study is that it uses FDA to analyze these data. Over the last decade, FDA has become a very important emerging field in statistics, although it is not well known in the marketing literature. FDA provides a set of techniques that can improve the prediction of future items of interest, especially in cases where prior longitudinal data are available for the same products, data are available from histories of similar products, or complete data are not available for some years. The central paradigm of FDA is to treat each function or curve as the unit of observation. We apply the FDA approach by treating the yearly cumulative penetration data of each category as 760 curves or functions. By taking this approach, we can extend several standard statistical methods for use on the curves themselves.

For instance, we use functional principal components analysis (PCA) to identify the patterns of shapes in the penetration curves. Doing so enables a meaningful understanding of the variations among the curves. An additional benefit of the principal component analysis is that it provides a parsimonious, finite-dimensional representation for each curve. In turn, this allows us to perform functional regression by treating the functional principal component scores as the independent variables and future characteristics of the curves, such as future penetration or time to takeoff, as the dependent variable. We show that this approach to prediction is more accurate than the traditional approach of using information from only one curve. It also provides a deeper understanding of the evolutions of the penetration curves.

Finally, we perform functional clustering by grouping the curves into clusters with similar patterns of evolution in penetration. The groups we form show strong clustering among certain products and provide further insights into the patterns of evolution in penetration. In particular, plotting the principal component scores allows us to visually assess the level of clustering among different products for all 760 curves simultaneously. Such a visual representation would be impossible using the original curves.

The rest of the paper is organized as follows. The next three sections present the method, data, and results. The last section discusses the limitations and implications of the research.

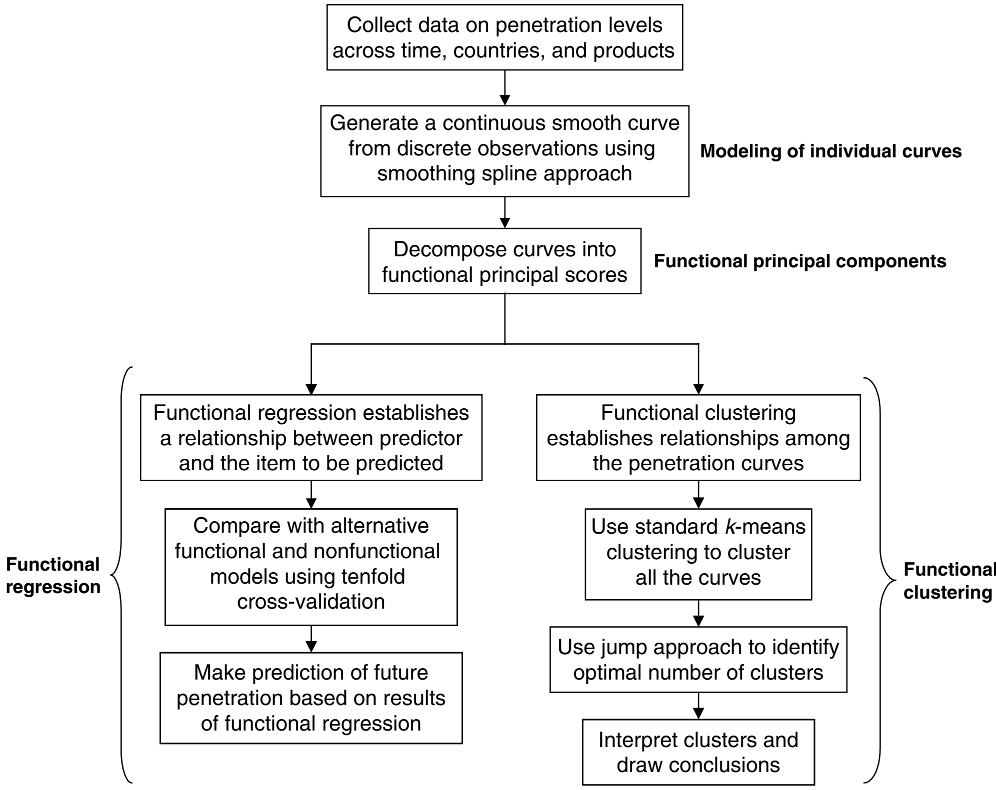
Method

We present the method in five sections. The first three sections outline various applications of FDA. Figure 1 provides a flowchart of the implementation of our three FDA techniques. The first section describes functional principal components. The second section shows how the functional principal component scores can be used to perform functional regression for predictions. The third section illustrates how the PCA scores can be used to perform functional cluster analysis and hence identify groupings among curves. The fourth section describes the alternate models against which we test the predictive performance of the FDA models. The last section details the method used for carrying out predictions.

Functional Principal Components

FDA is a collection of techniques in statistics for the analysis of curves or functions. Most FDA techniques assume that the curves have been observed at all time points, but in practice this is rarely the case. In some instances, curves might not be observed over all time periods. In other cases, the curves might be observed only over discrete intervals (e.g., annual estimates of adoption of new products). Because we have many observations for each curve, we first use a simple smoothing spline approach to generate a continuous smooth curve from our discrete observations. For example, a smoothing spline can be fit to a curve plotting the penetration of CD players, given 10 years

Figure 1 Flowchart of the Implementation of the Three FDA Techniques



of discrete data to obtain its penetration curve. The full details of our spline implementation are provided in Appendix A.

We denote by $X_1(t), X_2(t), \ldots, X_n(t)$ the n smooth curves that are our approximations to the penetration curves for each product-country combination and decompose these curves in the form

$$X_i(t) = \mu(t) + \sum_{j=1}^{\infty} e_{ij}\varphi_j(t) \quad i = 1, \ldots, n \tag{1}$$

subject to the following orthogonality constraints:

$$\int \varphi_j^2(s)\,ds = 1 \quad \text{and} \quad \int \varphi_j(s)\varphi_k(s)\,ds = 0 \quad \text{for } j \neq k.$$

The $\varphi_j(t)$s represent the principal component functions, the e_{ij}s, the principal component scores corresponding to the ith curve, and $\mu(t)$ the average curve over the entire population. As with standard principal components, $\varphi_1(t)$ represents the direction of greatest variability in the curves about their mean. $\varphi_2(t)$ represents the direction with next greatest variability subject to an orthogonality constraint with $\varphi_1(t)$, etc. The e_{ij}s represent the amount that $X_i(t)$ varies in the direction defined by $\varphi_j(t)$. Hence, a score of zero indicates that the shape of $X_i(t)$ is not similar to $\varphi_j(t)$, while a large score suggests that a high fraction of $X_i(t)$'s shape is generated from $\varphi_j(t)$.

To compute the functional principal components, we divide the time period $t = 1$ to $t = T$ into p equally spaced points and evaluate $X_i(t)$ at each of these time points. Note that the new time points are not restricted to be yearly observations, because the smoothing spline estimate can be evaluated at any point in time. Finally, we perform standard PCA on this p dimensional data. The resulting principal component vectors provide accurate approximations to the $\varphi_j(t)$s at each of the p grid points, and likewise the principal component scores represent the e_{ij}s. We opted to set $p = T$ and to evaluate the $\varphi_j(t)$s at the original yearly time points. Because our penetration curves were generally smooth, this approach generated smooth estimates for the $\varphi_j(t)$s.

In theory, n different principal component curves are needed to perfectly represent all n $X_i(t)$s. However, in practice, a small number (D) of components usually explain a substantial proportion of the variability (Ramsay and Silverman 2005), which indicates that

$$X_i(t) \approx \mu(t) + e_{i1}\varphi_1(t) + e_{i2}\varphi_2(t) + \cdots + e_{iD}\varphi_D(t) \quad i = 1, \ldots, n \tag{2}$$

for some positive $D \ll n$.

Note that the smooth functions, $X_i(t)$, are infinite dimensional in nature even though they are observed at only a finite number of time points. However, we use the e_{ij}s in Equation (2) to reduce the infinite dimensional functional data to a small set of dimensions. This reduction in dimensions is crucial because it allows us to perform functional clustering and functional regression, as described in the following two sections. In addition, it provides a parsimonious representation because it reduces the number of observations for each curve from T down to some small value D.

Note that even though the spline approach will not work in situations where only one or two time points are available for each curve, we can compute the functional principal components from sparsely observed data using other more sophisticated methods (see, e.g., James et al. 2000, Jank and Shmueli 2006, Rettinger et al. 2008). Hence the methods of functional clustering and functional regression that we describe in this paper can be applied even to products with only one or two years of penetration data.

Functional Regression

We use functional regression to predict several items of interest, such as future marginal penetration level in any given year or the year of takeoff. Let $X_i(t)$ be the smooth spline representation of the ith curve observed over time such as the first five years of cumulative penetration for a given category. Let Y_i represent a related item to be predicted, such as the marginal penetration in year six.

Functional regression establishes a relationship between predictor, $X_i(t)$, and the item to be predicted, Y_i, as follows:

$$Y_i = f(X_i(t)) + \varepsilon_i \quad i = 1, \ldots, n. \tag{3}$$

Equation (3) is difficult to work with directly because $X_i(t)$ is infinite dimensional. However, for any function f, there exists a corresponding function g such that $f(X(t)) = g(e_1, e_2, \ldots)$, where $e_1, e_2, \ldots$ are the principal component scores of $X(t)$. We use this equivalence to perform functional regression with the functional principal component scores as the independent variables. This approach is related to principal components regression, which is often used for nonfunctional, but high-dimensional, data. The simplest choice for g would be a linear function in which case Equation (3) becomes

$$Y_i = \beta_0 + \sum_{j=1}^{D} e_{ij}\beta_j + \varepsilon_j \tag{4}$$

for some $D \geq 1$. A somewhat more powerful model is produced by assuming that g is an additive, but nonlinear, function (Hastie and Tibshirani 1990). In this case, Equation (3) becomes

$$Y_i = \beta_0 + \sum_{j=1}^{D} g_j(e_{ij}) + \varepsilon_j, \tag{5}$$

where the g_js are nonlinear functions that are estimated as part of the fitting procedure. There are different ways to model the g_js but one common approach, which we use in this paper, is the smoothing spline discussed in Appendix A. One advantage of using Equation (4) or Equation (5) to implement a functional regression is that once the e_{ij}s have been computed via the functional PCA, we can then use standard linear or additive regression to relate Y_i to the principal component scores. We can also extend Equation (5) by adding covariates that contain information about the curves beyond the principal components, such as product or country characteristics or marketing variables.

Functional Clustering

We use functional clustering for the purpose of better understanding the penetration patterns in the data. In particular, we wish to identify groups of similar curves and relate them to observed characteristics of these curves such as the product and country. We use the principal components described in the previous section to reduce the potentially large number of dimensions of variability and cluster all the curves in the sample.

We apply the standard k-means clustering approach (MacQueen 1967) to the D-dimensional principal component scores, e_i, described in Equation (2) to cluster all the curves in the sample. Appendix B provides more details of k-means clustering.

We use the "jump" approach (Sugar and James 2003) to select the optimal number of clusters, k. We compute $\xi_k = \gamma_k^{-Z} - \gamma_{k-1}^{-Z}$ for a range of values of k, where γ_k is given by (B1) and Z is usually taken to be $D/2$. Sugar and James (2003) show through the use of information theory and simulations that setting the number of clusters equal to the value corresponding to the largest ξ_k provides an accurate estimate of the true number of clusters in the data.

Once we compute the cluster centers, we assign each curve to its closest cluster mean curve. We can then use Equation (2) to project the centers back into the original curve space and examine the shape of a typical curve from each cluster.

Comparing Alternative Models

To fully understand the advantages of FDA, we compare two implementations or models of FDA with five nonfunctional models. We name the two functional models—Functional Regression and Augmented Functional Regression—and the five nonfunctional models—Estimated Mean, Last Observation Projection, Classic Bass, Meta-Bass, and Augmented Meta-Bass. Table 2 classifies all the models based on their use of information across curves and nature of the model.

Table 2 Classification of Models

Uses information across curves	Analysis of curves	
	Nonfunctional analysis	Functional analysis
No	Classic Bass	
Yes	Estimation Mean Last Observation Projection Meta-Bass Augmented Meta-Bass	Functional Regression Augmented Functional Regression

The Functional Regression approach has three main strengths. First, it is able to incorporate information from other products to improve prediction accuracy. Second, it implements a nonparametric fitting procedure so it is not restricted by parametric assumptions. Third, it uses the functional nature of the penetration curves. We chose the five comparison models to gain an understanding of the gains from each of these strengths. For example, Classic Bass is parametric, does not use information from other products, and is nonfunctional so it provides a baseline case where none of the strengths are present. The Meta-Bass and Augmented Meta-Bass models extend Classic Bass to incorporate information from other products but are still parametric and nonfunctional so they illustrate the improvement from borrowing strength across curves. The Last Observation Projection model uses information from all products and is also nonparametric, so it illustrates the improvement from the first two strengths of FDA.

Estimated Mean. The Estimated Mean is a simple model, which fits the mean of the item to be predicted in the estimation sample as the predicted value of the item in the holdout sample. So, for example, to predict marginal penetration in year $T+1$, we use the mean marginal penetration in year $T+1$ among all curves in the estimation sample. Specifically, the prediction for the ith observation in the holdout sample, $\widehat{Y}_i$, is given by

$$\widehat{Y}_i = \overline{Y}, \tag{6}$$

where $\overline{Y}$ is the mean across all countries and products on the estimation sample. Note that this is a very simple model, which does not use any information from the first T periods of data.

Last Observation Projection. The Last Observation Projection is another simple model, which estimates the item to be predicted from only the last observation in each penetration curve. To do so, we first relate the item to be predicted, Y_i, to the final observed penetration level, $X_i(T)$, in the estimation sample. To estimate this relationship, we explore both a standard linear model (Equation (7)) as well as a more flexible nonlinear model (Equation (8)),

$$Y_i = \beta_0 + \beta_1 X_i(T) + \varepsilon_i, \tag{7}$$

$$Y_i = \beta_0 + g(X_i(T)) + \varepsilon_i. \tag{8}$$

We use the nonlinear model for our final results. For the prediction, we use the estimated g from Equation (8) and the final observed penetration level ($X_i(T)$) in the holdout sample to get the predicted item in the holdout sample.

Note that this is a slightly superior model to the Estimated Mean, because it uses at least the last observation from each curve to be predicted. However, it still does not use any other prior data from the curve. We also tested out a linear regression model incorporating all T time periods, $X_i(1), \ldots, X_i(T)$, as independent variables. We have not reported the results here because while this approach worked slightly better on some items and slightly worse on others, the overall results were not substantively different from the Last Observation predictions.

Classic Bass. The Classic Bass Model (Bass 1969) fits each curve in the sample separately by estimating the following model:

$$s(t) = m[F(t) - F(t-1)] + \varepsilon(t),$$
$$F(t) = \frac{1 - e^{-(p+q)t}}{1 + (q/p)e^{-(p+q)t}}, \tag{9}$$

where t = time period, $s(t)$ = marginal penetration at time t, p = coefficient of innovation, q = coefficient of imitation, and m = final cumulative penetration.

We estimate the model via the genetic algorithm because Venkatesan et al. (2004) provide convincing evidence that the genetic algorithm provides the best method for fitting the Bass model relative to all prior estimation methods. For each curve, we use the first T years of data to estimate the three Bass parameters, m, p, and q. We then predict the next five years of penetration levels by plugging the estimated parameters back into the Bass model and evaluating at times $T+1$ through $T+5$. We predict the time of peak marginal penetration by using $t = \log(q/p)/(p+q)$ and the peak marginal penetration using $s = m(p+q)2/4q$. We do not predict time to takeoff with the Classic Bass model. Note that the Classic Bass model does not distinguish between holdout and estimation samples because each curve is fit individually without using information from other curves.

Meta-Bass. In the Meta-Bass model, we extend the Classic Bass model to use information across curves. To do so, we first estimate m, p, and q for each curve using the genetic algorithm, as outlined above. Then, for each item to be predicted, we fit the nonlinear additive model

$$Y_i = \beta_0 + g_1(m_i) + g_2(p_i) + g_3(q_i) + \varepsilon_i \tag{10}$$

to the estimation sample, where g_1, g_2, and g_3 are smoothing splines as defined previously. We use the estimated parameters from this additive model and the estimates of m, p, and q for each curve in the holdout sample to compute the corresponding item to be predicted for each of the holdout curves. Note that the estimation of m, p, and q can also be done using a Bayesian formulation with a prior on $\{m, p, q\}$.

Augmented Meta-Bass. The Augmented Meta-Bass is the same nonlinear additive model used for the Meta-Bass, except that we add an indicator variable for each of the R products to which each curve belongs; thus

$$Y_i = \beta_0 + g_1(m_i) + g_2(p_i) + g_3(q_i) + \sum_{r=1}^{R-1} \delta_r I_{ir} + \varepsilon_i, \quad (11)$$

where $I_{ir} = 1$ if the ith curve belongs to product r, and 0 otherwise, and the δ_rs are regression coefficients that are estimated as part of the model-fitting procedure. Note that the Meta-Bass and Augmented Meta-Bass are extensions of the Classic Bass that make use of all of the information across curves, rather than just using each curve individually. Because using information across curves is an essential feature of functional regression, doing so puts the Meta-Bass and the Augmented Bass on the same platform as the FDA models (see Table 2).

Functional Regression. For the Functional Regression model, we compute four principal component scores, the first two each on the penetration curves, $X_i(t)$, and on the velocity curves, $X_i'(t)$. The principal component scores on the velocity curves are computed in an identical fashion to that for the penetration curves, except that we use the derivative of $X_i(t)$. We then use these four scores as the independent variables in an additive regression model, as shown in Equation (5), on the estimation sample. We then use the estimated parameters of this equation and the data from the curves in the holdout sample to compute the items to be predicted in the holdout sample.

Augmented Functional Regression. Our second functional approach enhances the power of Functional Regression by adding an indicator variable for each of the R products to which each curve belongs, as with the Augmented Meta-Bass model. Hence the Augmented Functional Regression model involves estimating a nonlinear additive model on the estimation sample as follows:

$$Y_i = \beta_0 + \sum_{j=1}^{4} g_j(e_{ij}) + \sum_{r=1}^{R-1} \delta_r I_{tr} + \varepsilon_i, \quad (12)$$

where the g_js are smoothing splines. We then compute the items to be predicted for each curve in the holdout sample from the estimated values of the above parameters and the data in each curve in the holdout sample. This model is directly comparable to Augmented Meta-Bass because both models use information across curves and from products.

Method for Prediction

We explain the specific procedure for carrying out the prediction in three parts: items being predicted, computation of errors, and partitioning of sample.

Items Being Predicted. We first truncate each curve at the Tth year. We use the penetration in years 1 to T to estimate the model and predict the marginal change in penetration for years $T+1$ to $T+5$. For each curve, we also predict the number of years to takeoff, the years to peak marginal penetration, and the level of peak marginal penetration. Takeoff is the first turning point in sales, marking the transition from the introductory to the growth stage of the product lifecycle. We identify the year of takeoff based on the definition proposed by Golder and Tellis (1997). Thus we predict a total of eight items for each of seven models, for a total of 56 model items. We do this whole process once each for $T = 5$ years and $T = 10$ years.

Computation of Errors. For each of these 56 model items to be predicted, we compute the mean absolute deviation (MAD) over all penetration curves; i.e.,

$$\text{MAD} = \frac{1}{n} \sum_{i=1}^{n} |Y_i - \widehat{Y}_i|, \quad (13)$$

where Y_i is a particular item for curve i and $\widehat{Y}_i$ is the corresponding estimate using a given model.

Partitioning of Sample. We use tenfold cross-validation by randomly partitioning the curves into 10 equal groups. We hold out one group, estimate each of the models on the remaining nine groups using data from years 1 to T, and then form predictions on the held out group for years $T+1$ to $T+5$. We repeat this process 10 times, for each of the 10 held out groups of data. T is the same for all countries and products. Figure 2 provides a graphical description of our process. For each of the 56 model items, we compute the MAD as an average of these 10 iterations. Note that k-fold cross-validation is superior to simple splitting of data into one holdout and training group, because all the data are used (randomly) as a holdout once.

Data

This section details our sample, sources, and procedure for data collection.

Figure 2 Analytic Framework Demonstrating Tenfold Cross-Validation

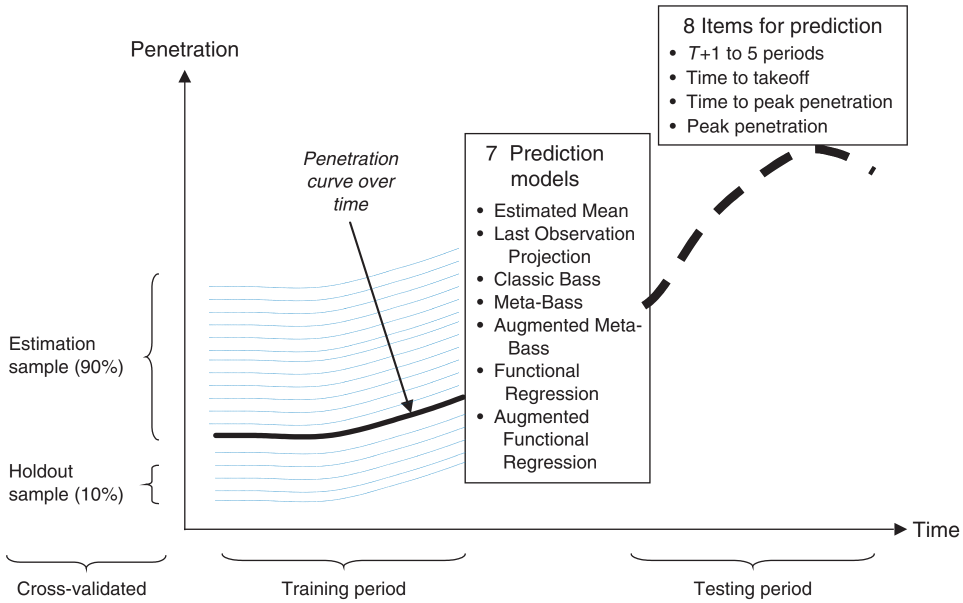
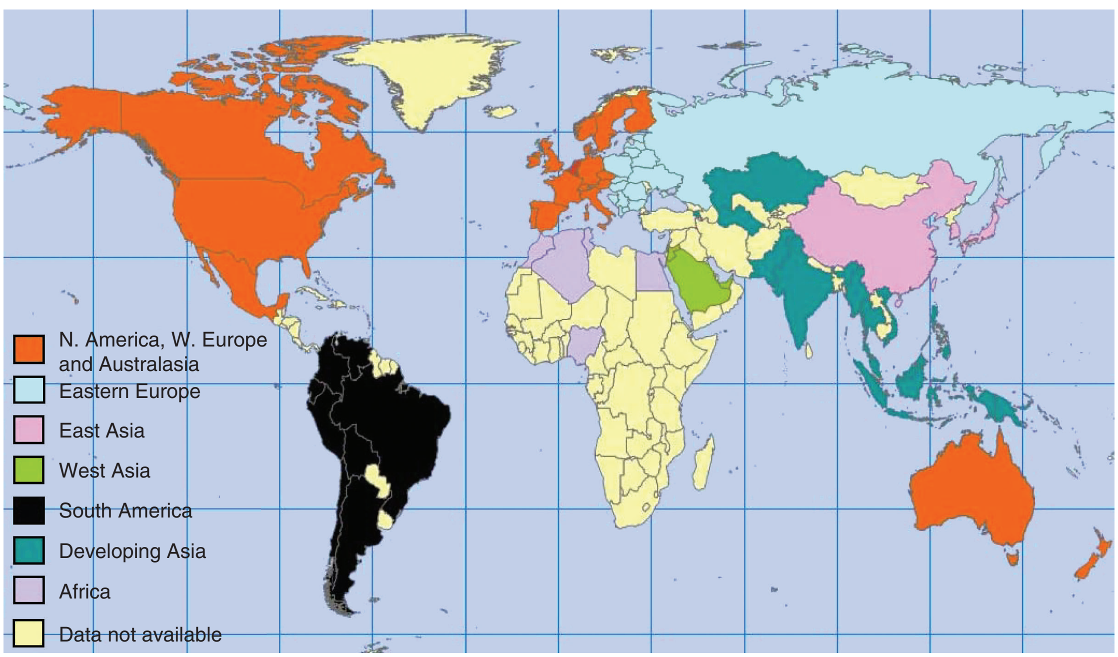


Figure 3 Distribution and Classification of Countries



Notes.

Cluster	Countries
North America, Western Europe, and Australasia	Canada, United States, Mexico, Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, UK, Australia, New Zealand
Eastern Europe	Belarus, Bulgaria, Croatia, Czech Rep., Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Russia, Slovakia, Slovenia, Ukraine
East Asia	China, Hong Kong, Japan, South Korea, Singapore, Taiwan
West Asia	Israel, Jordan, Kuwait, S. Arabia, U.A.E.
South America	Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Peru, Venezuela
Africa	Algeria, Egypt, Morocco, Nigeria, Tunisia
Developing Asia	Azerbaijan, India, Indonesia, Kazakhstan, Malaysia, Pakistan, Philippines, Thailand, Turkmenistan, Vietnam

Sample

Most of the prior studies are limited in scope in terms of both product type and geographical breadth (see Table 1). We collect data on 760 categories drawn from 21 products (see Table 3) and 70 countries (see Figure 3). The sample includes a broad sample from three categories: household white goods, computers and communication, and entertainment and lifestyle.

Sources

The information required for this study is penetration rates of different products introduced in different markets from the year of introduction to at least some time after the takeoff. The primary source of our data is the Global Market Information Database of Euromonitor International, which is an integrated online information system that provides business intelligence on countries, consumers, and lifestyles. We also use press releases, industry reports, and archived records to identify the year of introduction from databases like Factiva and Productscan.

Procedure

We follow the general rules for data collection for the historical method (Golder 2000). We explain specific problems we encounter and the rules we use to resolve them. We screen the categories to be used by three criteria. First, we suspect that all curves that have penetration rates above 1% in the first year might have missing early years of data. So for these categories, we check the year of introduction from historical reports or press releases. We exclude all categories where data are not available from the first year of introduction. Second, we exclude from our analysis any categories that do not contain at least $T+5$ years of observations or have not reached peak marginal penetration. Third, the data from this source are available only from 1977. Hence we exclude all categories where the product had been introduced or taken off earlier than 1977.

Table 3 Sample Categories

Entertainment and lifestyle	Household white goods	Computers and communication
Cable TV	Air conditioner	Internet personal computer (PC)
Camera	Dishwasher	PC
CD player	Freezer	Fax
Color TV	Microwave oven	Satellite TV
DVD player	Tumble drier	Telephone
Hi-Fi stereo	Vacuum cleaner	
Video camera	Washing machine	
Videotape recorder		
Video game console		

Results

We present the results on functional principal components, functional regression, and functional clustering.

Functional Principal Components

Figures 4(a) and 4(b) provide plots of $\varphi_1(t)$ and $\varphi_2(t)$ computed from the first 10 years of observations on the 760 penetration curves. The first principal component represents the amount by which a curve's penetration, at year 10, is above or below the global year 10 average of all 760 curves. Categories with a positive score on the first component end up with above average last-period penetration levels, while those with negative scores have below average last-period penetration. Alternatively, the second principal component represents the way the penetration levels evolve. Categories with a positive score on the second component grow most rapidly in the early years but slow down by year 10, while those with a negative score are associated with slow initial growth and a rapid increase toward year 10.

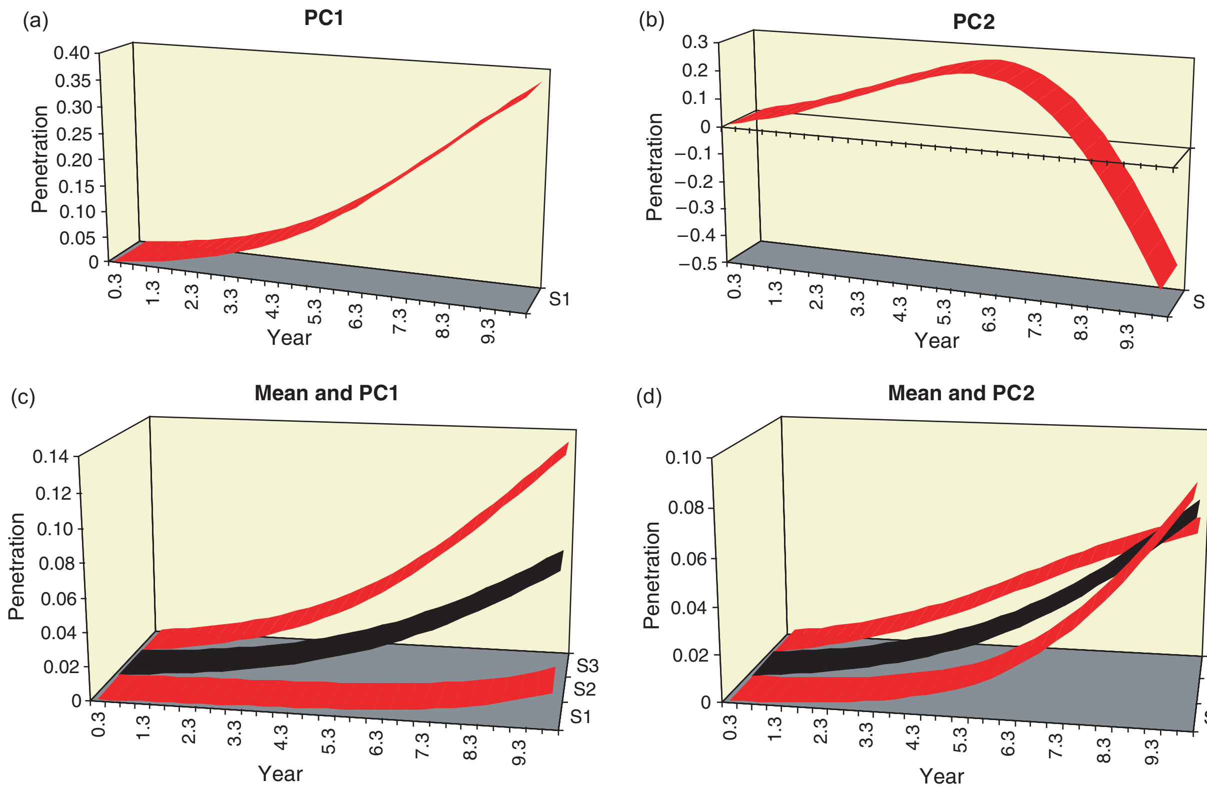
An alternative way of visualizing these curves is presented in Figures 4(c) and 4(d). Here, the black line corresponds to $\mu(t)$, the average penetration level over all 760 curves. The red lines represent $\mu(t) \pm \eta_j\varphi_j(t)$, where η_j is a constant proportional to the standard deviation of e_{ij}. Figure 4(c) shows that categories with a positive value for e_{i1} will have above average last period penetration levels at year 10, while ones with a negative e_{i1} will remain stagnant over time and will have last period penetration levels below the overall average. Alternatively, Figure 4(d) shows that categories with a positive value for e_{i2} will grow somewhat faster than average to begin with but then fall below average after 10 years, while curves with a negative e_{i2} will have the opposite pattern.

Remarkably, $\varphi_1(t)$ and $\varphi_2(t)$ together explain more than 99% of the variability in the smoothed penetration curves, which indicates that e_{i1} and e_{i2} provide a highly accurate two-dimensional representation of $X_i(t)$. However, it should be noted that the smoothing spline approach removes some of the variability in the data, so $\varphi_1(t)$ and $\varphi_2(t)$ explain somewhat less than 99% of the variation in the observed penetration data. As mentioned previously, one can also compute principal components for the velocity curves of the penetration levels. When we perform this decomposition on the penetration curves, the principal components of $X_i'(t)$ have a very similar structure to those for $X_i(t)$.

Functional Regression

This section presents the performance of the seven models on the eight items to be predicted. Tables 4(a) and 4(b) present the cross-validated MAD scores for each model using cutoffs of $T = 5$ and $T = 10$ years

Figure 4 Illustration of First Two Functional Principal Component Curves (Based on 10 Years of Training Data)



of training data, respectively. We also compute the fraction of curves for which Augmented Functional Regression outperforms each of the other methods (see Tables 5(a) and 5(b)).

To assess the ability of FDA to predict items of penetration curves, we compare the Functional Regression model to the five nonfunctional models. Functional Regression is superior to Estimation Mean, Last Observation Projection, and Classic Bass at predicting all eight items at both cutoff times (see Table 4). The reason is that the Estimation Mean and the Last Observation Projection use minimal information from prior time periods, while Classic Bass uses no information across curves. Functional Regression is also better than Meta-Bass on all items for both cutoff times, except for time to peak marginal penetration at cutoff time $T = 10$ years.

The performance of Functional Regression is mixed when compared with Augmented Meta-Bass. At the cutoff of $T = 5$ years, Functional Regression is superior to Augmented Meta-Bass for the $T+1$, $T+2$, and $T+3$ years, similar for $T+4$ years, but inferior for the other four items (see Tables 4(a) and 5(a)). At the cutoff of $T = 10$ years, Functional Regression outperforms Augmented Meta-Bass for $T+1$ through $T+5$ years as well as time to takeoff but not for time to peak marginal penetration and peak marginal penetration (see Tables 4(b) and 5(b)). The reason is that the Augmented Meta-Bass uses information about product while the Functional Regression does not.

On the other hand, with the sole exception of the Classical Bass predicting year $T+1$ with cutoff of $T = 10$, the Augmented Functional Regression model is superior to all nonfunctional models, including Augmented Meta-Bass, for every item to be predicted and for both cutoff times. The Augmented Functional Regression is also superior to Functional Regression, except in three instances where it is equal or slightly inferior (for $T+1$ years at cutoff of $T = 5$ years and $T+1$, $T+4$ years at cutoff of $T = 10$ years). The superiority over Functional Regression is most noticeable in the time to takeoff and time to peak marginal penetration.

When considering Table 5, Augmented Functional Regression is superior for at least 50% of the curves in 91 out of the 94 possible comparisons with other models. It appears that the Functional Regression model is slightly superior for predicting $T+1$, but the augmented version is preferable for any longer range predictions. Most of the differences in Tables 4 and 5 are highly statistically significant. We also tested out the Augmented Functional Regression model with the addition of a predictor for geographic region as defined in the clustering section but found that the performance deteriorated slightly. In summary, the Augmented Functional Regression model

Table 4 MAD by Model and Item

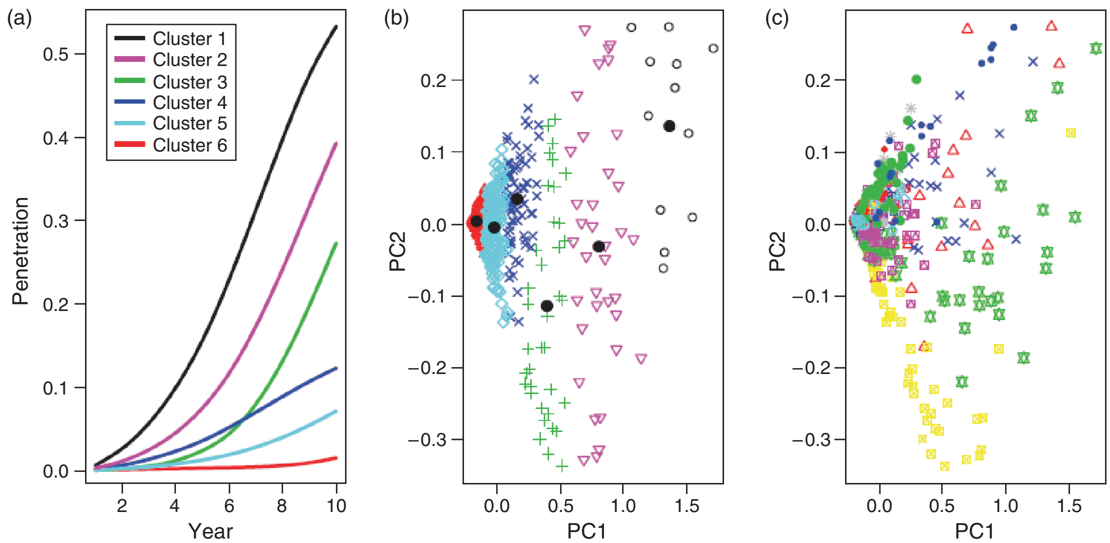
Item to be predicted	Method: Estimation Mean	Last Observation Projection	Classic Bass	Meta-Bass	Augmented Meta-Bass	Functional Regression	Augmented Functional Regression
			(a) Using 5 years ($T = 5$) of training data				
$T+1$	9.08	4.05	3.01	7.47	7.52	2.43	2.48
$T+2$	12.39	7.49	7.18	10.50	10.01	5.46	5.12
$T+3$	14.49	10.01	12.40	12.74	11.20	8.14	6.87
$T+4$	17.35	13.74	17.27	16.08	12.16	11.75	8.29
$T+5$	19.57	17.28	19.52	19.52	13.99	15.82	9.85
Takeoff	3.36	2.84	NA	2.69	2.41	2.66	2.35
Peak time	5.82	5.09	9.55	4.65	3.36	4.62	3.18
Peak marginal penetration	33.88	31.95	140.07	34.40	24.49	29.38	20.70
			(b) Using 10 years ($T = 10$) of training data				
$T+1$	10.83	5.37	4.02	8.17	8.17	4.17	4.70
$T+2$	11.19	6.59	6.38	8.69	8.73	5.48	5.69
$T+3$	11.56	7.25	8.43	10.90	10.83	6.31	6.11
$T+4$	11.58	8.44	11.02	9.40	9.26	8.08	7.81
$T+5$	11.65	9.72	11.96	11.48	11.11	9.07	8.40
Takeoff	3.61	2.93	NA	2.95	2.86	2.69	2.51
Peak time	4.69	3.95	7.54	3.50	2.94	3.66	2.92
Peak marginal penetration	26.66	23.65	42.71	25.99	22.32	23.18	18.18

Notes. All results, except those for Takeoff and Peak time, have been multiplied by 10^3. Using an alternative metric for error, the mean squared error, yields similar results.

Table 5 Superiority of Augmented Functional Regression over Other Models

Item to be predicted	Method: Estimation Mean	Last Observation Projection	Classic Bass	Meta-Bass	Augmented Meta-Bass	Functional Regression
		(a) Fraction of curves for which Augmented Functional Regression outperforms other models using 5 years of training data				
$T+1$	0.89	0.68	0.50	0.83	0.84	0.52
$T+2$	0.79	0.70	0.50	0.77	0.76	0.58
$T+3$	0.77	0.72	0.53	0.72	0.73	0.68
$T+4$	0.77	0.77	0.61	0.74	0.70	0.74
$T+5$	0.79	0.80	0.64	0.75	0.74	0.78
Takeoff	0.63	0.63	NA	0.58	0.53	0.60
Peak time	0.76	0.69	0.76	0.66	0.53	0.67
Peak marginal penetration	0.70	0.69	0.66	0.72	0.59	0.69
		(b) Fraction of curves for which Augmented Functional Regression outperforms other models using 10 years of training data				
$T+1$	0.87	0.51	0.33	0.72	0.74	0.36
$T+2$	0.81	0.62	0.50	0.72	0.75	0.50
$T+3$	0.79	0.63	0.56	0.70	0.70	0.57
$T+4$	0.75	0.61	0.64	0.62	0.62	0.59
$T+5$	0.67	0.61	0.62	0.62	0.62	0.60
Takeoff	0.67	0.60	NA	0.58	0.57	0.52
Peak time	0.69	0.61	0.77	0.53	0.49	0.56
Peak marginal penetration	0.72	0.67	0.63	0.66	0.60	0.68

Figure 5 Illustration of Functional Clustering



Notes. (a) The shapes of the average penetration curves within each of the six clusters. (b) The first two principal component scores for all 760 curves. A different color and plotting symbol has been used for each cluster with a black solid circle for the cluster centers. (c) Same as (b) but with different symbols for each product.

outperforms other models in more than 96% of the comparisons with six alternate models to predict seven items across two cutoff times.

Functional Clustering

Figure 5 provides several approaches to viewing the results from the functional clustering using k-means on e_{i1} and e_{i2}. The "jump" approach of Sugar and James (2003) suggests between six and nine clusters. We opt for six to provide the most parsimonious representation (see Table 6). Figure 5(a) plots the centers of the six clusters on the original time domain. The figure illustrates the pattern of growth of a typical curve in each cluster. Alternatively, Figure 5(b) plots all 760 curves in the reduced two-dimensional space, using the same colors as Figure 5(a) to represent each cluster. The six cluster centers are represented as solid black circles.

Each cluster differs from the other clusters in the pattern of penetration over time. Broadly speaking, Clusters 1–3 represent high-growth products, while the last three correspond to lower growth rates. Cluster 1 takes on large values in both the first and second principal component dimensions. Recall that a positive value in the first dimension corresponds to overall high last-period penetration, while a positive value in the second dimension represents a fast growth at the beginning but a slowdown by year 10. The black curve in Figure 5(a) shows this pattern with the fastest overall growth but a slight slowdown by year 10. Cluster 2 is close to zero for the second dimension, indicating no overall slowdown, as we can see from the pink curve. Clusters 3 and 4 provide an interesting contrast: Cluster 3 has a negative value in the second dimension, while Cluster 4 is positive. This suggests a slow start for Cluster 3 but with increasing momentum by year 10 and the opposite pattern for Cluster 4. Figure 5(a) shows precisely this pattern, with Cluster 4 starting ahead of Cluster 3 but then falling rapidly behind. Cluster 5 represents a moderate rate of growth, while Cluster 6, which contains the largest number of products, corresponds to a much slower improvement in penetration.

Table 6 Proportions of Each Type of Product Within Each Cluster

	Clusters (%)					
Product type	1	2	3	4	5	6
Cable TV	16.7	16.7	10.5	7.7	8.9	4.2
CD player	8.3	16.7	18.4	15.4	10.3	2.0
DVD player	8.3	16.7	44.7	5.1	20.5	2.7
Internet PC	58.3	36.1	10.5	7.7	8.2	5.6
Satellite TV	0.0	2.8	10.5	16.7	15.1	5.3
Videotape recorder	8.3	11.1	5.3	7.7	2.1	0.0
Camera	0.0	0.0	0.0	0.0	0.7	1.8
Color TV	0.0	0.0	0.0	2.6	0.7	2.4
Fax	0.0	0.0	0.0	3.8	2.1	0.0
Hi-Fi stereo	0.0	0.0	0.0	2.6	1.4	7.6
PC	0.0	0.0	0.0	2.6	4.8	8.9
Telephone	0.0	0.0	0.0	0.0	1.4	2.7
Video camera	0.0	0.0	0.0	2.6	4.8	1.8
Video game console	0.0	0.0	0.0	19.2	11.0	2.2
Air conditioner	0.0	0.0	0.0	0.0	0.7	12.9
Freezer	0.0	0.0	0.0	0.0	0.0	7.1
Microwave oven	0.0	0.0	0.0	3.8	4.1	11.1
Tumble drier	0.0	0.0	0.0	1.3	1.4	3.6
Vacuum cleaner	0.0	0.0	0.0	0.0	0.0	5.1
Washing machine	0.0	0.0	0.0	1.3	0.7	2.0

We also examine whether the penetration patterns differ across products. Figure 5(c) illustrates the growth patterns for the 21 different products in the sample. We plot all 760 curves in our two-dimensional space using a different plotting symbol for each product. There are very clear patterns within the same product. For example, the green stars correspond to Internet-compatible PCs and have almost uniformly large values on the first dimension, indicating rapid increases in penetration levels. Notice that one product might have both positive and negative values for the second dimension, suggesting more rapid takeoff in some markets over others. Alternatively, the yellow squares represent DVD players and have a very tight clustering with almost uniformly moderate scores on the first principal component and negative scores on the second principal component. These results suggest a slow initial growth with much more rapid expansion toward year 10. The tighter clustering suggests that the takeoff for these products is largely similar across different markets in the sample. Finally, the blue solid dots, representing videotape recorders, show the opposite pattern with large positive scores on the second dimension, suggesting fast initial growth but then a slowdown in later years.

Curves for each product are from a variety of countries. Table 6 provides the fraction of curves of each product that fall within each of the six clusters. The functional clustering suggests three groups: fast growth electronics, slower growth electronics, and household goods. The first three clusters capture a group of six fast-growth electronics products, with Cluster 1 primarily Internet PCs, Cluster 2 a mixture, and Cluster 3 mainly DVD players. The other three clusters capture a group of slow-growth products: video game consoles, satellite TV, and CD players make up the bulk of Cluster 4. Cluster 5 contains many products but seems to concentrate principally on countries with slower growth for CD and DVD players, satellite TV, and video game consoles. Finally, Cluster 6, the slowest growth cluster, contains the vast bulk of household appliances.

Similarly, we also examine whether the penetration patterns differ across countries. We categorize the data into seven economic groupings (see Table 7 and Figure 3). For each group, Table 7 shows the fraction of curves that fall in each of the six clusters. For example, for countries from Africa and developing Asia, 86% of curves fall into the slowest growth Cluster 6. In contrast, North American, Western Europe, and Australasia have curves that are more spread out over the six clusters, with only 31% in the slowest growth Cluster 6.

Table 7 Distribution of Each Economic Grouping over Clusters

	Clusters (%)					
Economic groupings	1	2	3	4	5	6
N. America, W. Europe, and Australasia	5.1	10.8	13.6	15.9	23.3	31.2
Eastern Europe	0.0	3.4	3.4	8.0	24.7	60.3
East Asia	5.1	8.5	3.4	20.3	13.6	49.2
West Asia	0.0	6.7	11.1	13.3	22.2	46.7
South America	0.0	0.0	0.9	10.0	23.6	65.5
Africa	0.0	0.0	0.0	3.1	10.9	85.9
Developing Asia	0.0	2.3	0.0	3.8	8.3	85.6

Discussion

Predicting the market penetration of new products is currently growing in importance because of increasing globalization, rapid introduction of new products, and rapid obsolescence of newly introduced products. Moreover, good record keeping has generated a wealth of new product penetration histories. The Bass model has been the standard model for analyzing such histories. However, the literature has not shown how exactly researchers should integrate the rich record of penetration histories across categories with the penetration of an evolving new product to predict future characteristics of its penetration. FDA, which has gained significant importance in statistics, is well suited for this task. Our goal is to demonstrate and assess the merit of FDA for predicting the market penetration of new products and compare it with the Bass model.

We compare the predictive performance of Functional Regression and Augmented Functional Regression with five other models—two simple or naïve models, the Classic Bass model, the Meta-Bass model, and the Augmented Meta-Bass model—on eight items to be predicted.

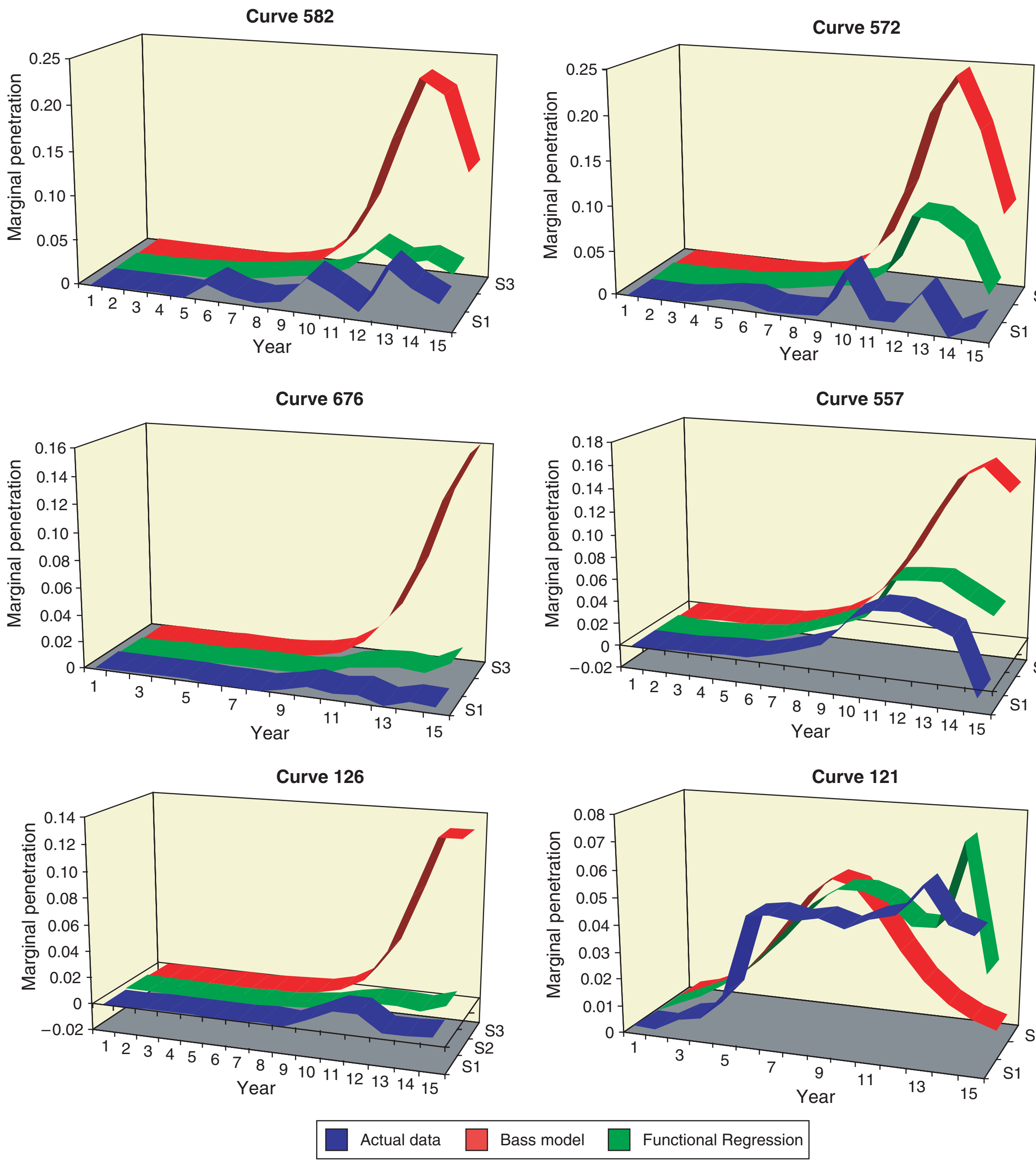
Our analysis leads to the following three important results:

(1) The essential logic of integrating information across categories, which is the foundation of FDA, provides superior prediction for an evolving new product.

(2) Specifically, an evolving category can be best predicted by integrating information from (a) past penetration of that category, (b) past penetration of other categories, and (c) knowledge of the product to which it belongs, via the framework of functional regression.

(3) For a vast variety of items that need to be predicted, the Augmented Functional Regression is distinctly superior to a variety of models, including simple or naïve models Classic and Enhanced Bass models and Functional Regression.

Figure 6 **Comparison of Predictive Accuracy of Classic Bass Model and Functional Regression Model**



Implications

Our functional regression has at least two clear managerial implications. First, our method can be used to make more accurate predictions of the future trajectory for both existing products and new products with only a few years of observations. One could also make predictions for the evolution of a new product without any data, based on the previously observed principal component scores of similar products. Second, although we have not done so here, it would be conceptually simple to add additional variables such as pricing and advertising information to the Functional Regression model. The addition of these variables would allow a manager not only to passively predict but to also control future penetration levels.

Our results raise the following questions with further managerial and research implications.

First, why don't simple extrapolative models work well for prediction, as some researchers assert they do (Fader and Hardie 2005; Armstrong 1984, 1978; Armstrong and Lusk 1983)? Our analysis makes it clear that there are two dimensions of information that are not captured by simple models. One, there is valuable information in the prior history of the new product, which as the Bass model suggests, probably arises from consumers' innovative and imitative tendencies. Two, there is intrinsic information across products and countries, which might be used effectively to predict the penetration of an evolving new product. Despite their intuitive appeal, simple models that do not capture these sources of information will fail to predict well.

Second, why does the Classic Bass model not work as well for prediction? We suspect that it does not fully capture the two dimensions of information. One, the Classic Bass model ignores other categories. This fact is borne out by the superiority of the Meta-Bass and the Augmented Meta-Bass in predicting items further into the future. Both of these latter models capture information from other categories. Thus, even in a parametric setting, increased predictive accuracy can be gained by incorporating information from multiple categories, especially when predicting further into the future. Two, the Classic Bass model is relatively flexible but nevertheless parametric, so it is limited in the range of shapes that it can take on. In particular, it is constrained to symmetric shapes for certain values of p and q. The relatively strong performance of the Last Observation Projection model shows that removing the parametric assumptions can cause additional improvements. In line with that result, FDA provides higher flexibility by using a nonparametric approach. So it can capture a variety of flexible patterns without overfitting, with the help of the principal components as explained earlier. The main disadvantage of a nonparametric method is that the increased flexibility can produce variability in the estimates. However, Functional Regression builds strength across the 760 curves to mitigate the problem of variability while generating more flexible estimates than those produced by the Classic Bass model.

Third, why does the Augmented Functional Regression outperform Functional Regression, especially for items further into the future? The probable reason is that a particular product has a distinct pattern of penetration over time. Adding knowledge of that product further stabilizes the variability of predictions around their true value. This pattern can be seen in both the improvement of Augmented Meta-Bass over Meta-Bass and Augmented Functional Regression over Functional Regression. Also note that the improvement is greatest in peak marginal penetration, an item that arguably is most closely associated with a product.

Fourth, why is product seemingly more relevant for predicting market penetration than is country? The probable reason is that the evolution of market penetration seems to follow more distinct patterns by the nature of the product than by the country. For example, electronic products with universal appeal diffuse rapidly across countries both large and small and developed and developing. On the other hand, culturally sensitive products such as food appliances diffuse slowly overall and very differently across countries. Moreover, our data are only after 1977. Because of increasing industrialization of developing countries and flattening of the world economy, intercountry differences are much smaller after 1977 than before it.

Fifth, is the exclusion of marketing variables a limitation of Functional Regression? We posit that it is not. Indeed, we show the superiority of Augmented Functional Regression, which includes a covariate for the product to which the curve belongs. In like manner, this model could also include covariates for marketing variables such as price, quality, or advertising.

To illustrate some of the above points, Figure 6 demonstrates six plots of the predictive performance of the Classic Bass model (red) and the Functional Regression model (green) relative to actual (blue). These six plots are drawn from among those where Functional Regression does the best. For each plot, the first 10 periods are fitted on the estimation sample, while the last five periods are predictions on the holdout sample. Both models do well in the estimation periods. However, performance varies dramatically in the holdout periods.

Note how for curves, 676, 557, and 126, a generally flat curve with a late takeoff in the last two periods, tricks the Classic Bass into overpredicting penetration for the holdout period. However, Functional Regression, which draws strength from other categories, is not so influenced by the last two periods. Also, note how for curves 582, 572, and 121, the parameterization of the Bass model leads it to predict symmetric curves, which are quite far from the actual.

This study has the following limitations. First, while the data are from a single source, the source itself does not record data before 1977. Indeed, we drop categories in some countries where we consider the year of introduction precedes 1977. We also drop categories in countries where penetration is not high enough until 2006, so the data are not balanced by country. Thus, substantive estimates about time to takeoff or about penetration by countries must be made with caution. However, that fact should not affect the comparison of the models, because all models have access to the same data. Second, depending on the release patterns of a particular product, the product predictor used in Augmented Meta-Bass and Augmented Functional Regression may or may not be available. Third, our data do not include any marketing variables. However, the strength of the Augmented Functional Regression is that it can include such marketing variables. Fourth, our approach applies to the prediction of market penetration using only aggregate historical data. Other approaches exist to predict based on survey and experimental data (Hauser et al. 2006) and disaggregate historical data (Tellis and Franses 2006). Future research could address how better improvements can be obtained by using such information when available. Fifth, future research could also address functional regression for predicting the evolution of underlying technologies (e.g., Sood and Tellis 2005).

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Appendix A. Modeling of Individual Curves
Suppose that a curve, $X(t)$, has been measured at times $t = 1, 2, \ldots, T$. Then, the smoothing spline estimate is defined as the function, $h(t)$, that minimizes

$$\sum_{t=1}^{T}(X(t) - h(t))^2 + \lambda \int \{h''(s)\}^2\, ds \tag{A1}$$

for a given value of $\lambda > 0$ (Hastie et al. 2001). The first squared error term in Equation (A1) forces $h(t)$ to provide a close fit to the observed data, while the second integrated second-derivative term penalizes curvature in $h(t)$. The tuning parameter λ determines the relative importance of the two components in the fitting procedure. Large values of λ force a $h(t)$ to be chosen such that the second derivative is close to zero. Hence, as λ gets larger, $h(t)$ becomes closer to a straight line, which minimizes the second derivative at zero. Smaller values of λ place more emphasis on $h(t)$s that minimize the squared error term and hence produce more flexible estimates. We follow the standard practice of choosing λ as the value that provides the smallest cross-validated residual sum of squared errors (Hastie et al. 2001). Remarkably, even though Equation (A1) is minimized over all smooth functions, it has been shown that its solution is uniquely given by a finite-dimensional, natural cubic spline (Green and Silverman 1994), which allows the smoothing spline to be easily computed. A cubic spline is formed by dividing the time period into L regions, where larger values of L generate a more flexible spline. Within the lth region, a cubic polynomial of the form

$$h(t) = a_l + b_l t + c_l t^2 + d_l t^3 \tag{A2}$$

is fit to the data. Different coefficients, a_l, b_l, c_l, and d_l are used for each region, subject to the constraints that $h(t)$ must be continuous at the boundary points of the regions and also have continuous first and second derivatives. In a natural cubic spline, the second derivative of each polynomial is also set to zero at the end points of the time period. In the more complicated situation where the curves are sparsely observed over time (e.g., because of a different data-generating process or data limitations), a number of alternatives have been proposed. For example, James et al. (2000) suggest a random effects approach when computing sparsely observed curves.

Appendix B. k-Means Clustering
k-means clustering works by locating D-dimensional cluster centers $c_1, \ldots, c_k$, which minimize the sum of squared distances between each observation and its closest cluster center, i.e., find $c_1, \ldots, c_k$ to minimize

$$\gamma_k = \sum_{i=1}^{n} \min_{c_1, c_2, \ldots, c_k} \|e_i - c_j\|^2. \tag{B1}$$

We use an iterative algorithm to minimize γ. First, we choose an initial set of candidate centers, $c_1, \ldots, c_k$, by randomly selecting k of the e_is and assign each curve to its closest center. Then, for each cluster, we define a new center by taking the average overall curves currently assigned to that cluster. We continue this algorithm until additional iterations do not yield significant changes in the cluster centers.

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