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Behavior-Based Discrimination: Is It a Winning Play, and If So, When?

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With advances in technology, the collection of information from consumers at the time of purchase is common in many categories. This information allows a firm to straightforwardly classify consumers as either “new” or “past” consumers. This opens the door for firms to implement marketing that (a) discriminates between new and past consumers and (b) entails making offers to them that are significantly different. Our objective is to examine the competitive effects of marketing that tailors offers to consumers based on their past buying behavior. In a two-period model with two competing firms, we assume that each firm is able to commit about whether or not to implement behavior-based discrimination (BBD), i.e., to add benefits to its offer for past consumers in the second period. When the firms are identical in their ability to add value to the second-period offer, BBD generally leads to lower profits for both firms. Past customers are so valuable in the second period that BBD leads to cutthroat competition in the first period. As a result, the payoffs associated with the implementation of BBD form a prisoner’s dilemma. Interestingly, when a firm has a significant advantage over its competitor (one firm has the capability to add more benefits for its past customers than the other), it can increase its profit versus the base case even when there is significant competition in the second period. Moreover, the firm at a disadvantage sometimes finds that the best response to BBD by a strong competitor is to respond with a uniform price and avoid the practice completely.

Key words: dynamic games; price discrimination; customer data; product design

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1. Introduction

1.1. Background

Consider the following situations:

1. Air Canada announces at the end of February that any Aeroplan member who joined in February will earn double frequent flyer miles in March.
2. In the last week of December 2004, the Société Nancéienne Varin-Bernier Bank, a major retail bank in France, offers the gold Visa card at 50% off the yearly fee to all clients who opened codevi savings accounts in the last quarter of 2004.
3. All Scandinavian Airlines (SAS) travellers who sign up for wireless access at the Copenhagen airport find that they are offered the same access plus automatic flight information the second time they log in to purchase the same service.

All three situations entail firms taking actions that are directly related to a consumer’s past behavior. The firms collect information at the time of the first purchase and know when interacting with the consumer

a second time that the consumer is a past consumer.¹ This information allows the firms to implement marketing that (a) discriminates between new and past consumers and (b) consists of offers to each segment that are significantly different.

The easiest action to discriminate between new and past customers is to offer them different prices (through couponing, introductory offers, or a repeat-buyer discounts). Accordingly, there are a number of studies that analyze the impact of prices that depend on the past behavior of customers (Fudenberg and Tirole 2000, Villas-Boas 1999). However, as the examples above demonstrate, a firm can go further than offering “tailored prices” when it bases marketing activity on past buying behavior. It can offer special services, options, or accessories that are of added value to the customer.

¹ Many firms collect information from consumers with whom they interact. Not only do consumers of durable goods complete warranty cards, but many frequently purchase items are sold over Internet sites or by retailers who have loyalty programs.

Our objective is to examine the interaction between two competitors who decide whether to implement behavior-based discrimination (BBD) in a two-period model that focuses on markets where firms are able to commit to not implementing BBD. In many markets, this may not be possible. For such a commitment to be possible, implementation of BBD must be observable. For example, firms need to collect information and identities of consumers in the first period (this information is used by firms to design benefits and to identify past consumers at the time of the second purchase). When the collection of identities and information is observable, firms can commit to not implementing BBD.

1.2. Literature Review

In the last decade, a wide array of firms have initiated the use of customer-level information to guide their marketing. For example, firms now offer customer-specific prices through a combination of special offers, coupons, and bundling. In fact, there is an important body of research that examines the impact of prices that are tailored to consumers based on their characteristics (Thisse and Vives 1988, Chen and Iyer 2002, Chen et al. 2002).

Our interest, however, is research that examines the impact of offers (modified products and corresponding prices) that are tailored to consumers based on past buying behavior.² In contrast to models where loyalty is exogenous or a function of network effects (Chen and Xie 2007), the incentive to buy again from the same firm is driven by benefits that are offered to past consumers in the second period. The information used to design these benefits is collected at the time of the first purchase through devices such as warranty cards; this allows a firm to make offers to consumers based on their past buying behavior.

The most important stream of research related to this topic examines the impact of offering tailored prices to consumers based on their past buying behavior (Fudenberg and Tirole 2000; Villas-Boas 1999, 2004). As noted in Fudenberg and Tirole (2000), research on behavior-based price discrimination is recent because it does not correspond to the traditional categories of price discrimination such as first, second, and third degree.

The models of Villas-Boas (1999, 2004) consider markets with infinitely lived firms and overlapping generations of consumers.³ These models are important because they identify the key forces that affect the

prices and behavior of firms when firms charge a different price to the consumers whom they have served previously. However, these studies do not address situations where past consumers are offered an additional benefit along with a unique price.

Fudenberg and Tirole (2000) examine behavior-based price discrimination when there is not significant turnover. They show that when consumer preferences are fixed over time and the market is characterized by short-term contracts, firms poach each others' previous customers, which leads to socially inefficient switching. In contrast, when consumer preferences over time are independent, short-term contracts are efficient.

Our analysis builds on Fudenberg and Tirole's analysis in two important ways. First, we consider a situation where firms need to take observable action to engage in BBD.⁴ Our objective is to examine the economic motivation for BBD, given that there are costs associated with its implementation (e.g., the cost of collecting and processing information from past consumers and the cost of identifying consumers at the time of purchase).

Second, our interest is in situations where firms can do more than offer different prices to consumers based on their past purchase. To be specific, we wish to examine how offering an *additional benefit* and a unique price to past consumers affects firm performance under competition. Recent work demonstrates that adding benefits (such as embedded premiums) can be more effective than offering price discounts of similar value (Arora and Henderson 2007). In contrast to situations where consumers themselves design the benefits (Randall et al. 2007), we focus on situations where firms collect information from groups of consumers to design benefits themselves. Beyond the three examples provided in the introduction, the benefits we consider could be free business services to repeat business customers at a downtown hotel (Internet access, printing, and computer services), expedited check-out to past customers (at Internet sites such as Amazon and Travelocity), and specialized promotions and services offered to past customers by casinos such as Harrah's (Loveman 2003).

To date there has been limited research on this topic. One paper that does examine the issue is Acquisti and Varian (2005). Although most of their analysis is focused on the impact of BBD in a monopoly context, they include a section on the effect of BBD under competition. In that section, Acquisti and Varian assume that all competitors are identical and sell a homogeneous good (after serving customers, firms are differentiated in the sense that previous customers obtain a benefit by buying again at the same firm). Because

² Syam et al. (2005) look at product customization in a context where firms position their offer along two dimensions. This is related to our work but does not address the impact of BBD because of its static nature.

³ Villas-Boas (2004) considers the case of an infinitely lived monopolist. Because our study is focused on price discrimination under competitive conditions, it relates primarily to Villas-Boas (1999).

⁴ This also implies that firms can commit not to implement BBD.

firms do not have market power, the solution is based on all firms earning zero profits. This analysis is incisive about what can happen in a market where BBD occurs, but it does not provide guidance about (a) whether a firm should implement BBD when it faces a *differentiated* competitor or (b) the expected outcome in a market where each firm makes a choice about implementing BBD.⁵

A second paper, by Zhang (2005), is related to this analysis, but there are two key differences. First, Zhang's paper is a dynamic analysis of product positioning and product line management. The logic of Zhang's model is that firms make positioning decisions in each period and offer different products to consumers based on their past decisions. Second, past consumers are offered a repositioned product that provides a different combination of benefits; it does not necessarily provide the consumer with more benefits. In contrast, our objective is to examine the effect of adding a vertical benefit (and not of offering a repositioned product) to past consumers. In cases where BBD means redesigning the physical product attributes and not simply adding benefits for past consumers, Zhang's model is more relevant.

Note that our analysis has links to the literature on switching costs (Fudenberg and Villas-Boas 2006). The similarity stems from the fact that the second-period decision for consumers is affected by a discrete change in the value of a previously consumed product. Chen (1997) finds that discriminatory pricing makes firms worse off in a two-period market where consumers incur a cost to switch firms in the second period. Taylor (2003) extends Chen's two-firm model to multiple firms and multiple periods. His analysis demonstrates how the addition of a third firm exacerbates price competition, driving economic profits to zero. These studies show that discriminatory pricing is not necessarily an advantage for firms in a competitive setting. In these models, switching costs occur endogenously as a function of the first purchase. In contrast, we analyze a context where the discrete change in the value of repurchasing from the same firm is a decision variable. In addition, the benefit a firm adds to its offer is not bounded in the way that switching costs are.

Our first objective is to examine the interaction between competing firms when they can add benefits for past consumers as well as charging them different prices. Second, we want to better understand the

economic incentives to implement BBD if firms can commit not to do so. This will provide guidance about whether firms have an incentive to implement BBD (assuming the costs of implementation are sufficiently low). It will also allow identification of conditions (if they exist) where BBD is mutually beneficial for competing firms.

1.3. Preview of Findings

A key finding of our analysis is that when firms are *identical* in their ability to add value to the second-period offer, BBD often leads to a game with the characteristics of a prisoner's dilemma: Both firms earn less profit than they would in the absence of BBD. This is explained by considering the mechanism that underlies BBD. The benefits that are added to offers made to past consumers tie them to the firm they bought from previously. Consequently, firms charge these past consumers high prices. The problem for firms is that this dynamic also increases the value of a past consumer. Firms recognize the added value of past consumers, and in the first period this leads to cutthroat competition. Also in the first period, firms offer prices lower than marginal cost to capture customers for the second period. (Interestingly, the findings are different when firms are restricted to charging prices greater than marginal cost in the first period; in this situation, BBD can be profitable even when both competitors implement it.)

When the benefits that firms can add for previous customers are below a threshold, the game has two equilibria, one where both firms implement BBD and one where neither of them does. Our analysis does not indicate which outcome is more likely, even though the equilibrium where firms implement BBD leads to strictly lower profits for both firms. However, when the firms make sequential decisions to implement BBD, the unique outcome is that no BBD occurs. Because the capability to implement BBD is recent, it is possible that the absence of BBD in some markets stems from the fact that the competitors have acquired their capabilities sequentially as opposed to simultaneously.

Our analysis paints a stark picture of how BBD should affect industry profitability. However, the model is based on firms adding the benefits (for past consumers) at zero cost. This naturally leads to the question of whether firms might be better off if they incur a cost to provide a benefit level to past consumers. In a modified version of the model, we show that the problem of the prisoner's dilemma is reduced but not eliminated by assuming that the level of benefit offered to past consumers is costly.

We also analyze the impact of BBD when firms have unequal capabilities for adding benefits for past consumers; i.e., one firm has a stronger capability to add

⁵ Acquisti and Varian (2005) also examine how the equilibrium changes when consumers have the ability to hide their identity. For example, in an Internet purchasing situation, a consumer can hide her identity by deleting cookies. Although we do not consider this possibility and assume that past customers are identified correctly, our analysis shows that consumers do not have an incentive to hide their identity.

these benefits. When a firm has a sufficient advantage over its competitor, it can increase its profit versus the base case even when there is significant competition (poaching) in the second period. In addition, the firm at a disadvantage sometimes finds that the best response to BBD by a strong competitor is a uniform price, avoiding the practice completely.

The model thus provides guidance to managers about the necessary conditions for BBD to be profitable. In addition, the model provides normative predictions about whether the lack of BBD in a market is caused by the *choice* or *inability* of firms to implement the practice. In the following section, we present the modelling framework. In §3, we present the analysis, and in §4, our conclusions.

2. The Model

The model is comprised of two firms and two stages. In the first stage, each firm decides whether to implement BBD. After the choice is made, the decisions become common knowledge. We also consider a modified game in which the firms make sequential decisions to implement BBD.

In the second stage, the firms engage in two periods of differentiated price competition. Thus, there are four potential second-stage games: Both firms choose to implement BBD, both choose not to implement BBD, and the two mixed cases where one firm chooses to implement BBD while the other does not. Our objective is to investigate a 2×2 metagame where firms decide whether to implement BBD.

To model the second stage of the game, we use a standard unitary hotelling market where each firm is at either end of the market. We denote the firm at the left end of the market as Firm 1 and the one at the right end as Firm 2. Each firm produces a single product at a constant marginal cost of production, c . The products differ with respect to an attribute, and each consumer is identified by an ideal point along the attribute that corresponds to preferred brand. Consumers are assumed to be uniformly distributed along the market with a density of one, and their location (their preferences) is fixed across both periods, similar to the model in Fudenberg and Tirole (2000).

We assume that values of surplus (realized by consumers) and profits (realized by firms) in both periods are of equivalent value.⁶ Each consumer buys no more than one unit of product per period and places a

value v on her ideal product in the first period.⁷ Consumers, however, cannot obtain their ideal product. In the first period, a consumer at a distance x from Firm i ($i = 1, 2$) obtains a surplus $v - tx - p_i$ by consuming Firm i 's product (t is the transportation cost, and it reflects the degree of differentiation in the market).

If a firm implements BBD, there are two observable actions.⁸ The first is that the identity of each consumer in the first period is recorded so that a consumer in the second period is correctly identified as either a new consumer or a past consumer. This relates to situations where consumers need to provide personal information to make a purchase. For example, to make a first purchase at the Price Club, a consumer needs to register and become a member. We assume that there are no legal restrictions against firms offering different deals to consumers based on their identity.

Second, in the first period, each firm collects information from its consumers related to needs that are not addressed by the first-period offer. This information allows the firm to add a benefit to its second-period offer that we denote by B_i ($i = 1, 2$). In the examples listed in the introduction, the benefits would be double frequent flyer miles, the reduced rate for the gold card, and the automatic flight information. The assumption is that the firms collect information at the time of the first purchase, suggesting that these benefits would be of significant value. Note that the benefits are specific to the group of consumers who purchase in the first period. We acknowledge that this representation of how information is collected and used to add benefits for past consumers is somewhat ad hoc. Nevertheless, it reflects three key aspects of BBD: (a) firms learn from consumers with whom they have had interactions (past consumers), (b) these consumers are similar in terms of preferences, and (c) there is no reason why the benefits in question would be of value to new consumers who by definition have different preferences.

In the second period, a consumer in group j has purchased from Firm j in the first period ($j = 1, 2$). When Firm i implements BBD, then a consumer in group j located a distance x from Firm i where $j = i$ will obtain a surplus of $B_i + v - tx - p_i^j$ by consuming Firm i 's product. Conversely, if $j \neq i$, then the same consumer obtains a surplus of $v - tx - p_i^j$ by consuming Firm i 's product. Note that a consumer always pays a price that is specific to the group to which she belongs. However, only a consumer who is a "past

⁶ Discounting the second period does not affect the findings. Reduced importance of the second period weakens the strategic importance of BBD (the directional impact on profits versus a base case of uniform pricing remains).

⁷ We assume that v is sufficiently high such that the participation constraint does not bind for any consumer in the market. Given the unitary length of the market, this means that $v > 3t/2$.

⁸ These actions are assumed to be observable by both consumers and the firms.

consumer” obtains the benefit B_i associated with Firm i ’s product. If the firm does not implement BBD, then all consumers in the second period obtain a surplus of $v - tx - p_i^b$ from Firm i ’s product. The superscript b indicates that all second-period consumers are offered an identical price (the critical difference between the model with BBD and the standard model is that in the second period, consumers are offered different things based on their behavior in the first period). When the benefit B_i associated with BBD is 0, we obtain a strict model of price discrimination, as in Fudenberg and Tirole (2000). We now specify demand in each period given these assumptions.

2.1. Consumer Demand

In the first period, we assume that consumers account for the expected surplus they will realize in the second period. This allows us to write the surplus for a consumer at x_0 who buys from Firm 1 in the first period as

$$CS_1^1(x) = v - tx_0 - p_1 + E(CS_2^1),$$

and from Firm 2 as

$$CS_1^2(x) = v - t(1 - x_0) - p_2 + E(CS_2^2). \quad (1)$$

Here, CS_1^i represents the surplus from buying at Firm i in the first period, and $E(CS_2^i)$ is the expected surplus in the second period, given that the consumer purchased from Firm i in the first period. The consumer who is indifferent between Firms 1 and 2 is found where these two expressions are equal. Let the indifferent consumer be at a location we denote as q ,

$$q = \frac{p_2 - p_1 + E(CS_2^1) - E(CS_2^2) + t}{2t}. \quad (2)$$

In the second period, there are two groups of consumers: those who purchased from Firms 1 and 2, respectively, in the first period. In period 2, let x_i be the location of the indifferent consumer in the group of consumers who purchased from Firm i in the first period.

We consider three cases. In the first, we assume that neither firm implements BBD. This implies that the two firms charge a price of p_i^b in the second period ($i = 1, 2$) and no additional benefits are added to the products. In this situation, the surplus for a consumer at x who buys from Firm 1 will be $CS_2^1(x) = v - tx - p_1^b$ and from Firm 2 will be $CS_2^2(x) = v - t(1 - x) - p_2^b$. As a result, the indifferent consumer is found at $x^* = (p_2^b - p_1^b + t)/2t$. Here, the location of the indifferent consumer is independent of decisions made in the first period.

In the second case, we assume that both firms implement BBD. In the group that purchased from Firm 1 in the first period, the surplus for a consumer at x

who buys from Firm 1 will be $CS_2^1(x) = B_1 + v - tx - p_1^1$ and from Firm 2 will be $CS_2^2(x) = v - (1 - x) - p_2^1$. Therefore, in period 2, the indifferent consumer in the group that purchased from Firm 1 in period 1 is given by $x_1 = (B_1 + p_2^1 - p_1^1 + t)/2t$. Similarly, $x_2 = (p_2^2 - p_1^2 + t - B_2)/2t$. Note that second-period demands for Firms 1 and 2, respectively, from the group that purchased from Firm 1 in period 1 are $d_1^1 = x_1$ and $d_2^1 = q - x_1$. Similarly, second-period demand for Firms 1 and 2, respectively, from the group that purchased from Firm 2 are $d_1^2 = x_2 - q$ and $d_2^2 = 1 - x_2$.⁹

In the third case, we assume that only one firm (say Firm 1) implements BBD. Using similar reasoning as in the first two cases, it is straightforward to show that the indifferent consumer in the group that purchased from Firm 1 in period 1 is given by $x_1 = (B_1 + p_2^b - p_1^1 + t)/2t$. Similarly, $x_2 = (p_2^b - p_1^2 + t)/2t$. Note that second-period demands for Firms 1 and 2—i.e., d_1^1, d_2^1, d_1^2 —and d_2^2 , are identical to the expressions presented for the second case.

To specify demand in the first period, we now show how consumers form expectations for consumer surplus in period 2. We consider each of the three cases in question. In the first (when BBD is not implemented by either firm), consumers who bought from Firms 1 and 2, respectively, in the first period know they are likely to buy from the same firm in the second period. These observations imply that

$$E(CS_2^1) = v - x_0 t - p_1^b \quad (3)$$

$$E(CS_2^2) = v - (1 - x_0)t - p_2^b. \quad (4)$$

In these expressions, $E(CS_2^1)$ and $E(CS_2^2)$ apply to the consumers to the left and right of the indifferent consumer in the first-period, respectively.

In the second case (when BBD is implemented by both firms), we assume that poaching occurs; i.e., consumers close to q who bought from Firm 1 in the first period will switch to Firm 2 in the second period and vice versa.¹⁰ These observations imply that

$$E(CS_2^1) = v - (1 - x)t - p_2^1 \quad (5)$$

$$E(CS_2^2) = v - xt - p_1^2. \quad (6)$$

In the third case, we assume that poaching occurs (similar to the second case) and use the same expressions for $E(CS_2^1)$ and $E(CS_2^2)$.

⁹ These expressions assume poaching of the competitor’s first-period customers by both firms. Fudenberg and Tirole (2000) demonstrate that this is the equilibrium outcome in the symmetric case when $B_1 = B_2 = 0$. When poaching does not occur, $d_1^1 = d_2^1 = 0$.

¹⁰ The expectations consumers form about second-period surplus do not depend on the magnitude of B . In other words, the model does not rely on consumers having a precise estimate of the benefit they will be offered in the second period. The model simply requires that consumers have an idea about whether the benefit will be big enough to eliminate switching.

Expected consumer surplus is a key input for the consumer's first-period decision. Because consumers are forward looking, the first-period decision is more complicated than the second-period decision. The solution entails first solving for second-period prices as a function of q (the split of the market in the first period). These expressions are then used to reduce the problem to a single-period optimization for each of the three cases.

2.2. Firms' Pricing Decision

When neither firm uses BBD, the pricing question in the first period is identical to the pricing question in the second period. To simplify the presentation of our analysis, we normalize t , the transportation cost, to 1.¹¹ Straightforward calculations show that the equilibrium prices when neither firm implements BBD are $p_i = p_i^b = 1$ (for $i = 1, 2$), and both firms earn profits of $\frac{1}{2}$ per period.

We now consider the second and third cases. When one or more of the firms uses BBD, the second-period decisions are a function of the fraction of the market that purchased from each firm in the first period. We write the second-period objective functions, π_1 and π_2 (for Firms 1 and 2, respectively), assuming that each firm poaches some of the competitor's previous consumers

$$\pi_1 = (p_1^1 - c)x_1 + (p_1^2 - c)(x_2 - q) \quad (7)$$

$$\pi_2 = (p_2^1 - c)(q - x_1) + (p_2^2 - c)(1 - x_2). \quad (8)$$

The values of x_1 and x_2 as presented in §2.1 are substituted into Equations (7) and (8) to obtain second-period demand as a function of second-period prices. Each firm maximizes its profits with respect to its second-period prices. This yields a series of first-order conditions (provided in the appendix), which are used to calculate the equilibrium prices in the second period as a function of q , the location of the indifferent consumer in the first period (in the first period, Firms 1 and 2 obtain demand of q and $1 - q$, respectively). The optimal prices in the second period as a function of q are shown in Equations (9) and (10).

$$\begin{aligned} p_1^1 &= c + \frac{1}{3}B_1 + \frac{1}{3} + \frac{2}{3}q, & p_1^2 &= c + 1 - \frac{4}{3}q - \frac{1}{3}B_2 \\ p_2^1 &= c + \frac{4}{3}q - \frac{1}{3} - \frac{1}{3}B_1, & p_2^2 &= c + \frac{1}{3}B_2 + 1 - \frac{2}{3}q \\ p_1^b &= c + \frac{1}{6}q + \frac{5}{12}B_1 + \frac{2}{3}, & p_1^b &= c + \frac{2}{3} - \frac{1}{12}B_1 - \frac{5}{6}q, \\ p_2^b &= c + \frac{1}{3}q - \frac{1}{6}B_1 + \frac{1}{3}. \end{aligned} \quad (10)$$

When a consumer makes her first-period purchase, naturally it depends on first-period prices. But it also

depends on the surplus that she expects to enjoy in the second period. As discussed earlier, consumers in the neighborhood of the indifferent consumer know that they will buy from different firms each period (they foresee poaching by both firms). We use the expressions for consumer surplus (Equations (3) and (4)) to write first-period demand (in the symmetric and asymmetric cases) as a function of first-period prices.

$$q_{\text{both}} = \frac{1}{8}B_1 - \frac{1}{8}B_2 - \frac{3}{8}p_1 + \frac{3}{8}p_2 + \frac{1}{2} \quad (11)$$

$$q_{\text{Firm 1}} = q = \frac{1}{14}B_1 - \frac{6}{7}p_1 + \frac{6}{7}p_2 + \frac{2}{7}. \quad (12)$$

The next step in the solution is to determine the optimal prices in the first period. The first-period objective functions for the symmetric and asymmetric cases can be written as functions of first-period prices by substituting for x_1 , x_2 , and q into Equations (7) and (8) (the expressions are provided in the appendix). For the two cases, these functions are optimized with regard to the first-period price chosen by each firm.

Having solved for the optimal prices in the first period, Equations (11) and (12) are used to identify first-period demand and hence the optimal second-period prices for each firm. To focus on the demand-side effects of BBD, we assume that the benefit and discriminatory price for past consumers are offered costlessly by a firm that implements BBD. We now consider the first stage of the game, where firms make a decision about whether to implement BBD.

2.3. The Decision by Firms to Implement BBD

The first stage of the game entails simultaneous decisions by the firms whether to implement BBD in the second period. The normal form of the game is represented by Figure 1. In Figure 1, Π_{BBD} and $\Pi_{\text{no BBD}}$ are the profits earned by each firm when both firms implement BBD and when neither firm implements BBD, respectively. When only one firm implements BBD, we define Π_a as the profit earned by the firm that implements BBD and Π_d as the profit earned by the firm that uses standard pricing in both periods. A Nash equilibrium involves identifying an outcome where neither firm has an incentive to deviate.

Figure 1 First Stage: BBD or Not?

		Firm 2	
		No BBD	BBD
Firm 1	No BBD	$\Pi_{\text{no BBD}}, \Pi_{\text{no BBD}}$	Π_d, Π_a
	BBD	Π_a, Π_d	$\Pi_{\text{BBD}}, \Pi_{\text{BBD}}$

¹¹ Normalizing t to 1 implies that the critical levels of B will be identified in absolute terms. These levels can be multiplied by t to obtain the critical levels without the normalization.

Note that once a firm decides to implement BBD in the first stage, implementation in the second stage is self-enforcing. That is, a firm that prices in the first period as if it were going to implement BBD is strictly worse off in the second period if it does not implement BBD.¹²

3. Analysis

3.1. The Case of Firms with Identical Capability to Add Benefits

We first analyze the two-period pricing subgame for firms with identical ability to add benefits ($B = B_1 = B_2$). Later in the paper, we consider a game between firms that are asymmetric; i.e., $B_1 > B_2$. The results from the two-period subgame are used to analyze the 2×2 metagame of Figure 1.

3.1.1. The Two-Period Pricing Subgame. As discussed earlier, the symmetric game where neither firm engages in BBD is two one-period subgames where each firm earns profits of $\frac{1}{2}$ per period. Accordingly, the role of this section is to present the outcomes where either one or both firms engage in BBD. We start by examining the outcomes when poaching occurs in the (BBD, BBD) subgame.¹³

3.1.2. The Subgame Outcome When Poaching Is Possible. When poaching occurs in the (BBD, BBD) subgame, the objective functions are based on the case where “poaching” occurs in the second period (see §2.2). Proposition 1 summarizes the optimal prices in each period and the total profits when poaching occurs. All proofs are provided in the appendix.

PROPOSITION 1. *When $B < 1$ and both firms engage in BBD, prices in the first period are $p_1 = p_2 = c + \frac{4}{3} - \frac{2}{3}B$, and the prices offered in the second period are $p_1^1 = p_2^2 = c + \frac{2}{3} + \frac{1}{3}B$ (to previous customers) and $p_1^2 = p_2^1 = c + \frac{1}{3} - \frac{1}{3}B$ (to previous customers of the competitor). The firms earn profits of $\frac{1}{9}B^2 - \frac{2}{9}B + \frac{17}{18}$.*

Proposition 1 shows that in the second period, each firm sets one price to retain its past consumers and another to capture demand from the competitor’s past consumers. The targeted price for past consumers of the competitor is low because (a) these consumers incur a high travel cost to change firms in the second period (consumers are located closer to the firm

they buy from in the first period) and (b) they lose the benefit B when they switch firms in the second period. This exacerbates price competition, and prices are lower in the second period than the base case (the optimal uniform prices are 1 in both periods).

Second, prices in the first period are higher than the base case when $B \in (0, \frac{1}{2})$ and less than the base case when $B \in (\frac{1}{2}, 1)$. This happens because there are two opposing forces. The first is that firms have little incentive to expand their first-period demand when they know that a significant fraction of their first-period demand will be poached by the competitor. This increases the relative incentive to charge high prices to nearby consumers. The second force is the attractiveness of having a significant number of past consumers. This force is directly related to size of B because the second-period price that can be charged to past customers is positively related to B . As B increases, the second force becomes stronger than the first, and this explains why prices fall below the base case in the upper half of the $(0, 1)$ interval.

When only one firm (Firm 1) implements BBD, the subgame equilibrium with poaching is based on the objective functions developed in §2.2. The conditions for poaching are satisfied when $B < \frac{23}{31}$. Proposition 2 summarizes the outcome in this range.

PROPOSITION 2. *When Firm 1 uses BBD and $B < \frac{23}{31}$, both firms poach the competitor’s previous customers in the second period. Prices in the first period are $p_1 = c + \frac{2}{3} - \frac{11}{92}B$ and $p_2 = c + \frac{11}{12} - \frac{7}{46}B$. In the second period, the prices are $p_1^1 = c + \frac{39}{92}B + \frac{3}{4}$, $p_1^2 = -\frac{11}{92}B + \frac{1}{4}$, and $p_2^1 = c - \frac{7}{46}B + \frac{1}{2}$. The profits of Firms 1 and 2 are $\Pi_1 = \frac{71}{276}B + \frac{777}{8,464}B^2 + \frac{31}{48}$ and $\Pi_2 = \frac{63}{2,116}B^2 - \frac{37}{138}B + \frac{17}{24}$.*

Proposition 2 shows that the main effects of one firm implementing BBD are to allow it to raise prices for its past consumers (because of the benefit that is added) and to increase price competition for the first-period consumers of Firm 2 in the second period. In particular, Firm 1 becomes aggressive in attacking its competitor when it implements BBD. Firm 2 prices aggressively precisely because it is vulnerable to the targeted marketing of Firm 1. Proposition 2 also underlines a second interesting characteristic of the outcome when only one firm implements BBD. When $B < (23\sqrt{3,553} - 1,334)/315 \approx 0.1173$, the firm implementing BBD earns lower profits than the competitor, which neither adds benefits nor uses discriminatory pricing in the second period. The reverse is true when $B \gtrapprox 0.1173$.

When $B \in (\frac{23}{31}, 1)$, poaching occurs when both firms implement BBD but not when only one firm does. The firm that does not implement BBD (Firm 2) is too weak in the second period to attract previous customers of Firm 1, so the outcome does not entail poaching. Moreover, Firm 1 obtains more than 50% of

¹² The analysis in §3.1.1 shows that BBD exacerbates price competition in period 1 but allows high prices to be charged to past consumers in period 2. This explains why the commitment to implement BBD is self-enforcing. A firm that abandons BBD in period 2 walks away from the high prices that can be charged to past consumers.

¹³ The (BBD, BBD) subgame refers to the case where both firms implement BBD.

the market because of the benefits it offers to its previous customers in the second period. The equilibrium is summarized in Proposition 3.

PROPOSITION 3. *When Firm 1 uses BBD and $B \in (\frac{23}{31}, 1)$, there are two possible equilibria:*

1. *Both firms serve exactly the same consumers in both periods. First-period prices are $p_1 = c + \frac{9}{10} - \frac{2}{5}B$ and $p_2 = c + \frac{7}{10} - \frac{1}{5}B$. In the second period, the prices are $p_1^1 = c + \frac{4}{5}B + \frac{7}{10}$, $p_1^2 = c$, and $p_2^1 = c + \frac{1}{2}$. The profits of Firms 1 and 2 are $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$, respectively.*

2. *Firm 1 poaches Firm 2's past consumers. First-period prices are $p_1 = c - \frac{4}{23}B - \frac{8}{69}$ and $p_2 = c - \frac{1}{23}B - \frac{2}{69}$. In the second period, the prices are $p_1^1 = \frac{11}{23}B + c + \frac{38}{23}$, $p_1^2 = c - \frac{6}{23}B + \frac{19}{23}$, and $p_2^1 = c - \frac{3}{23}B + \frac{21}{23}$. The profits of Firms 1 and 2 are $\Pi_1 = \frac{66}{529}B + \frac{99}{1,058}B^2 + \frac{573}{1,058}$ and $\Pi_2 = \frac{9}{529}B^2 - \frac{80}{529}B + \frac{1,243}{3,174}$, respectively.*

Simple comparisons using Proposition 3 show that Firm 2 suffers by not implementing BBD. However, independent of the equilibrium that results in the subgame, Firm 1's profits are lower than the base case. Note that the outcome with no poaching is strictly superior for both firms, yet either outcome is possible. In sum, Propositions 2 and 3 show that both firms are strictly worse off than the base case of uniform pricing when $B < 1$. It also appears that when only Firm 1 implements BBD, the weak position of Firm 2 causes it to compete aggressively, to the detriment of both firms. We now consider situations where B is sufficiently high to eliminate poaching in the second period.

3.1.3. The Subgame Outcome When Poaching Cannot Occur in the Second Period. When the benefits that firms can offer in the second period are sufficiently high, firms do not poach in the second period and each firm serves exactly the same customers in both periods. As shown in the appendix, when $B > B^*$ where $B^* = \frac{197}{84} - \frac{69}{308}\sqrt{22} \approx 1.3$, poaching does not occur in the second period.¹⁴ Proposition 4 describes the outcome in the subgame where both firms implement BBD.

PROPOSITION 4. *When $B > B^*$ and both firms implement BBD, first-period prices are $p_1 = p_2 = 1 + c - B$, second-period prices are $p_1^1 = p_2^2 = B + c$, and firms earn profits of $\Pi_1 = \Pi_2 = \frac{1}{2}$.*

Proposition 4 applies as long as marginal cost exceeds the threshold of $B - 1$ (otherwise, first-period

prices will be negative). When marginal costs are positive and firms can sell for less than marginal cost (but at positive prices), the profits in the (BBD, BBD) subgame are independent of the level of benefit and 50% lower than the base case of (no BBD, no BBD).

We now consider the case where only Firm 1 implements BBD and $B > B^*$. In the appendix, we show that Firm 2 is too weak to attract previous consumers of Firm 1, and an outcome where only Firm 1 poaches Firm 2's consumers is not stable. As a result, the equilibrium does not entail poaching, and it is summarized in Proposition 5.

PROPOSITION 5. *When Firm 1 uses BBD and*

1. *$B \in (B^*, 6)$, first-period prices are $p_1 = 2c + \frac{9}{10} - \frac{2}{5}B$ and $p_2 = 2c + \frac{7}{10} - \frac{1}{5}B$ and second-period prices are $p_1^1 = c + \frac{4}{5}B + \frac{7}{10}$, $p_1^2 = c$, and $p_2^1 = c + \frac{1}{2}$. Firms 1 and 2 earn profits of $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$, respectively.*

2. *$B > 6$, first-period prices are $p_1 = c - \frac{27}{20}$ and $p_2 = c - \frac{3}{10}$, leaving Firm 1 with the entire market. Second-period prices are $p_1^1 = B + c - 1$ and $p_2^1 = c$. Firms 1 and 2 earn profits of $\Pi_1 = B - \frac{47}{20}$ and $\Pi_2 = 0$, respectively.*

Proposition 5 shows that when only Firm 1 implements BBD, it earns higher profits than the base case of no BBD (because $\frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $B - \frac{47}{20}$ are greater than 1 in the relevant zone) at the expense of Firm 2. We now move the first stage of the game using the subgame outcomes as a basis for analyzing each cell of Figure 1.

3.1.4. The First Stage of the Game. First, we consider the equilibrium outcome when poaching is possible, i.e., when $B < 1$.

PROPOSITION 6. *When $B \in (0, 1)$, there are two pure strategy Nash equilibria: both firms implement BBD or neither implements BBD.*

Proposition 6 shows that when the benefits that firms can add are less than the differentiation between products, the game has two pure strategy Nash equilibria.¹⁵ This happens because the payoffs in the upper left and lower right cells of Figure 1 for both firms are strictly higher than the payoffs in the off-diagonal cells. Said differently, the best response to a competitor's strategy is to mimic it.

Proposition 6 leads to Corollary 1, which follows directly from the firm profit expressions. The corollaries highlight critical characteristics of the equilibrium

¹⁴ When $B \in (1, (669 - 39\sqrt{101})/230 \approx 1.205)$, the equilibrium involves only one of the two firms poaching the competitor's past consumers (an asymmetric outcome even though the competitors are symmetric). Details of this outcome are provided in the appendix.

¹⁵ As is always the case in a two-player/two-action game with two Nash equilibria, there is a mixed strategy equilibrium in which firms randomize between the two actions (implementing BBD and not implementing BBD). This equilibrium leads to strictly lower profits than either of the two pure strategy equilibria so we focus the discussion on the pure strategy outcomes. A complete analysis of the mixed-strategy equilibrium is available on request from the authors.

as a function of the magnitude of benefits offered to past consumers.

COROLLARY 1. *When $B \in (0, 1)$, the equilibrium where neither firm implements BBD results in higher profits than the equilibrium where both firms implement BBD.*

Corollary 1 indicates that when the game has two pure strategy equilibria ($B < 1$), the equilibrium where firms implement BBD leads to strictly lower profits. When the firms make simultaneous decisions about the implementation of BBD, the model makes no prediction about which outcome is more likely. However, when the firms make *sequential* decisions about the implementation of BBD, the unique outcome is that no BBD occurs. In other words, the plausibility of poaching by both firms (in equilibrium) depends on players making simultaneous and not sequential decisions to implement BBD. When the capability to implement BBD is acquired over time, the sequential game may be more relevant to understand whether BBD is likely to arise.

Note that in the case of $B = 0$ where firms implement behavior-based price discrimination alone, no benefit is added to a past customer's offer. This is the case of short-term contracts (where the consumer preferences are stable), which was examined by Fudenberg and Tirole (2000). Fudenberg and Tirole find that poaching and socially inefficient switching occur in equilibrium. When firms cannot commit to not implementing BBD, this is exactly what happens. In our model, however, it is assumed that firms can make a credible commitment to not implementing BBD. For that reason, the symmetric outcome of neither firm implementing behavior-based price discrimination is also possible.

We now consider the equilibrium when poaching cannot occur in the second period; i.e., $B > B^*$.

PROPOSITION 7. *When $B > B^*$, both firms implement BBD and earn profits of $\frac{1}{2}$.*

When $B > B^*$, the game has a unique Nash equilibrium where both firms implement BBD. Proposition 7 leads to Corollary 2.

COROLLARY 2. *When $B > B^*$, the firms are trapped in a prisoner's dilemma whereby both firms implement BBD yet earn lower profits than the base case.*

Corollary 2 shows that when the benefits that can be added are high, the unilateral incentive to implement BBD is positive, but when the firms implement it, they are both worse off. This suggests that firms will implement BBD across many markets. But we should also expect significant press about how the effort has yielded little in terms of profitability. In addition, Corollary 2 highlights the fierceness of first-period competition. Despite the fact that consumers

pay high prices for a benefit-enhanced offer in the second period, the impact of BBD on firm profits is negative because of unmitigated price competition in the first period.

3.2. Situations Where Price Competition Is Constrained in the First Period

The analysis in §3.1 is based on a market where the marginal cost, c , is sufficiently high to preclude negative prices being offered in the first period: When c is small the equilibrium first-period prices of $1 + c - B$ can be negative. This may lead to a moral hazard problem in that noncustomers will simply show up and ask for money. Moreover, in some situations, prices less than marginal cost may not be feasible.¹⁶ To examine such situations, we consider another version of the model where the marginal cost of the product is normalized to zero and prices are restricted to being positive.

When $B < 1$ and BBD leads to poaching, the findings are analogous to the case when marginal cost is positive. The prices given in Propositions 1, 2, and 3 are the solutions with marginal cost set to zero (profits are unaffected).

As shown in the appendix, when both firms implement BBD and $B > B^{**} = (669 - 39\sqrt{101})/230 \approx 1.2$, the outcome where one or both firms poach the other's past customers is not possible. Lemma 1 summarizes the outcome when both firms implement BBD in this range.

LEMMA 1. *When both firms engage in BBD and $B > B^{**}$, prices in the first period are $p_1 = p_2 = 0$ and the prices offered in the second period are $p_1^1 = p_2^2 = B$ (to previous customers) and $p_1^2 = p_2^1 = 0$ (to previous customers of the competitor). Both firms earn profits of $B/2$.*

When the firms are restricted to prices greater than or equal to zero (the marginal cost), firms give the product away in the first period and all profits obtain from second-period sales. Lemma 1 also identifies a change in the relationship of profits to B as the level of B increases. In particular, the relationship between B and firm profits is negative when $B < 1$ but positive when $B > B^{**}$, in other words, when competition in the first period is attenuated by the firms' inability to set prices below marginal cost: small benefits exacerbate competition but large benefits do not.

¹⁶ If firms can buy product from each other, an equilibrium with prices lower than marginal cost in the first period breaks down. Suppose Firms 1 and 2 sell identical products but are differentiated by location. If Firm 1 offers its product in the first period at a price less than marginal cost, Firm 2 can buy the product it sells from Firm 1 (versus paying the marginal cost c). The same is true for Firm 1.

We now consider the case where only Firm 1 implements BBD and $B > B^{**}$. The outcome does not entail poaching and is similar to the case of positive marginal costs; however, when $B > \frac{9}{4}$, the outcome is different.

LEMMA 2. *When Firm 1 uses BBD and*

1. $B \in (B^{**}, \frac{9}{4})$, first-period prices are $p_1 = \frac{9}{10} - \frac{2}{5}B$ and $p_2 = \frac{7}{10} - \frac{1}{5}B$. In the second period, the prices are $p_1^1 = \frac{4}{5}B + \frac{7}{10}$, $p_1^2 = 0$, and $p_2^1 = \frac{1}{2}$. Firms 1 and 2 earn profits of $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$, respectively.

2. $B > \frac{9}{4}$, first-period prices are $p_1 = 0$ and $p_2 = \frac{1}{4}$. In the second period, the prices are $p_1^1 = B + \frac{1}{4}$, $p_1^2 = 0$, and $p_2^1 = \frac{1}{2}$. Firms 1 and 2 earn profits of $\Pi_1 = \frac{5}{8}B + \frac{5}{32}$ and $\Pi_2 = \frac{9}{32}$, respectively.

Similar to the case of positive marginal costs, Lemma 2 demonstrates that when only Firm 1 implements BBD, it earns higher profits than the base case and Firm 2 is strictly worse off.

The results for the first stage of the game are unaffected when poaching is possible: There are two possible symmetric equilibria where both or neither of the firms implements BBD. When poaching is not possible, i.e., $B > B^{**}$, Proposition 8 describes the equilibrium in the first stage of the game.

PROPOSITION 8. *When $B > B^{**}$, there is a unique equilibrium where both firms implement BBD and the firms earn profits of $\frac{B}{2}$.*

When $B > B^{**}$, the best response to a competitor that does not implement BBD is to implement BBD. As a result, the upper-left square in the Figure 1 is not stable. Corollary 3 follows directly from the profit expressions.

COROLLARY 3. *When $B \in (B^{**}, 2)$, the firms are trapped in a prisoner's dilemma whereby both firms implement BBD yet generate strictly lower profit than the base case. When $B > 2$, the firms increase profit versus the base case.*

Corollary 3 shows that firms may find themselves in a prisoner's dilemma when the benefits are sufficient to eliminate poaching in the second period. However, if B is sufficiently large (>2), firms benefit from BBD even when both firms implement it. This does not happen when marginal costs are positive and firms set prices below marginal cost in the first period. In other words, when the ability of firms to compete on a price basis is limited because of (a) the inability to "pay" consumers to buy a product or (b) the infeasibility of charging a price below marginal cost, BBD is attractive (on an industry basis) at a much lower benefit level. This provides useful insight for managers. When competition for new customers is exogenously constrained, BBD can be profitable even when both firms implement the practice. In contrast,

when competition for new customers is unfettered, the implementation of BBD invariably leads to lower profits for both firms.

3.3. Endogenizing the Level of Benefit Offered to Past Consumers

Until now, we have assumed that the firms costlessly add benefits to the second-period offer for past consumers. This raises a concern. The model predicts that the ability to implement BBD is often a curse because it drives the profits of firms down; the firms find themselves in a prisoner's dilemma. This may be an overstatement of the potential negative effects of BBD; i.e., when the benefits offered to past consumers need to be financed, the likelihood of a prisoner's dilemma may be lower.

To address these concerns, we consider a version of the model where firms invest to add benefits to second-period offers for past customers. In particular, we assume that firms invest αB^2 prior to the start of the game to have the ability to offer a benefit B to past consumers in the second period. These investments become public knowledge. As before, in the second stage of the game, the firms engage in two periods of differentiated price competition. We also assume that firms that invest to offer a benefit B do, in fact, implement BBD. In contrast, firms that have not invested do not implement the BBD and price uniformly to the market in the second period. These investments can be thought of as financing for systems that (a) collect and process information from customers or (b) use first-period interactions with customers as a way of adding value in the second period.

The objective is to determine whether firms will invest a priori to create benefits and, if they do, what level of benefits they will offer. In other words, will the equilibrium involve investment by both firms (or only one), and what conditions on α lead to different market outcomes? The following proposition describes the equilibrium outcome, benefit level, and profits as a function of α .

PROPOSITION 9.

1. When BBD leads to poaching ($\alpha > \frac{13}{96}$), there are two Nash equilibria: both firms implement BBD or neither implements BBD. When both firms implement BBD, they choose a benefit level of $B = 23/(144(2\alpha - 1/9))$ and earn profits of $\Pi = 529/(2,304(18\alpha - 1)^2) - 23/(72(18\alpha - 1)) + \frac{17}{18}$.

2. When BBD does not lead to poaching $\alpha < (207\sqrt{22} + 5,863)/(5,175\sqrt{22} + 54,175)$, there are two regions.

a. When $\alpha \in (\frac{1}{9}, (207\sqrt{22} + 5,863)/(5,175\sqrt{22} + 54,175))$, firms do not implement BBD.

b. When $\alpha < \frac{1}{9}$, both firms implement BBD and earn profits of $\alpha(4,531/4,312\sqrt{22} - 512,597/77,616) + \frac{1}{2}$.

When BBD leads to poaching (i.e., $\alpha > \frac{13}{96}$), the asymmetric profits (where one firm implements BBD and the other does not) are strictly lower than the symmetric profits. For this reason, there are two equilibria. Thus, if the firms make sequential decisions about the implementation of BBD when $\alpha > \frac{13}{96}$, the unique outcome is for no BBD to occur. In any event, the key observation is that when the costs of creating benefits are high enough that firms poach each other's past consumers, endogenizing the benefit level does not affect the results substantially. There are two possible equilibria, and the creation of benefits simply becomes an added cost of doing business.

When BBD does not lead to poaching, point 2 in Proposition 9 shows that endogenizing the creation of the benefit reduces the prisoner's dilemma problem. For a significant fraction of the parameter zone, firms do not implement BBD. However, when the cost of creating benefits is sufficiently low ($\alpha < \frac{1}{9}$), the prisoner's dilemma problem reappears. This is explained by recalling that the unilateral incentive to implement BBD remains high even when the level of B is endogenized.

3.4. The Case of Firms with Unequal Capability to Add Benefits

To analyze the case of asymmetric firms ($B_1 \neq B_2$), we assume without loss of generality that $B_1 > B_2$. As in the symmetric case, we derive the results for the two-period subgame where both firms implement BBD. Note that the asymmetric subgame where one firm implements BBD and the other does not is identical to the analysis in §3.1. We then use these results to solve the 2×2 metagame of Figure 1.

The equilibrium outcomes in the subgame are derived in the appendix. The analysis shows that when the benefits of both firms are low and they implement BBD, the outcome entails poaching by both firms. The details of the poaching outcome provided in the appendix show that Firm 1 (the firm that offers a larger benefit) prices more aggressively in the first period. As a result, Firm 1 captures more than 50% of the market in the first period (in fact, $q = \frac{1}{32}\Delta B + \frac{1}{2}$ where $\Delta B = B_1 - B_2$). Not surprisingly, in the second period, Firm 1 is able to charge higher prices to its former customers than does Firm 2. This explains why Firm 1 is more aggressive in the first period. In the second period, Firm 2 is more aggressive with Firm 1's former customers than Firm 1 is with Firm 2's former customers. Firm 2 has no choice but to be more aggressive because of the higher benefit offered by Firm 1.

The analysis of the metagame when firms have unequal capability to add benefits entails comparing the profits for each of the four cells in Figure 1. Note

that the values of Π_a and Π_d are given straightforwardly by Propositions 1 to 5 but in contrast to the symmetric game, Π_a and Π_d are different depending on whether the strong firm (Firm 1) or the weak firm (Firm 2) implements BBD. A complete analysis of the metagame is provided in the appendix. We provide a visual summary of the results in Figure 2. The figure is a sketch of how each of the conditions defines different equilibrium zones as a function of the benefit levels that each firm can add to its second-period offer.

The figure shows that as we move from left to right, the equilibrium changes from poaching by both firms to partial poaching and ultimately to conditions where neither firm poaches. There are a number of insights generated in the analysis that lies behind Figure 2. The first is that when the benefit levels are relatively low (lower than the level of differentiation between firms), two equilibria are possible (one where both firms implement BBD and the other when they do not). Similar to the case of symmetric firms, the (BBD, BBD) outcome results in lower profits for both firms than the (no BBD, no BBD) outcome.

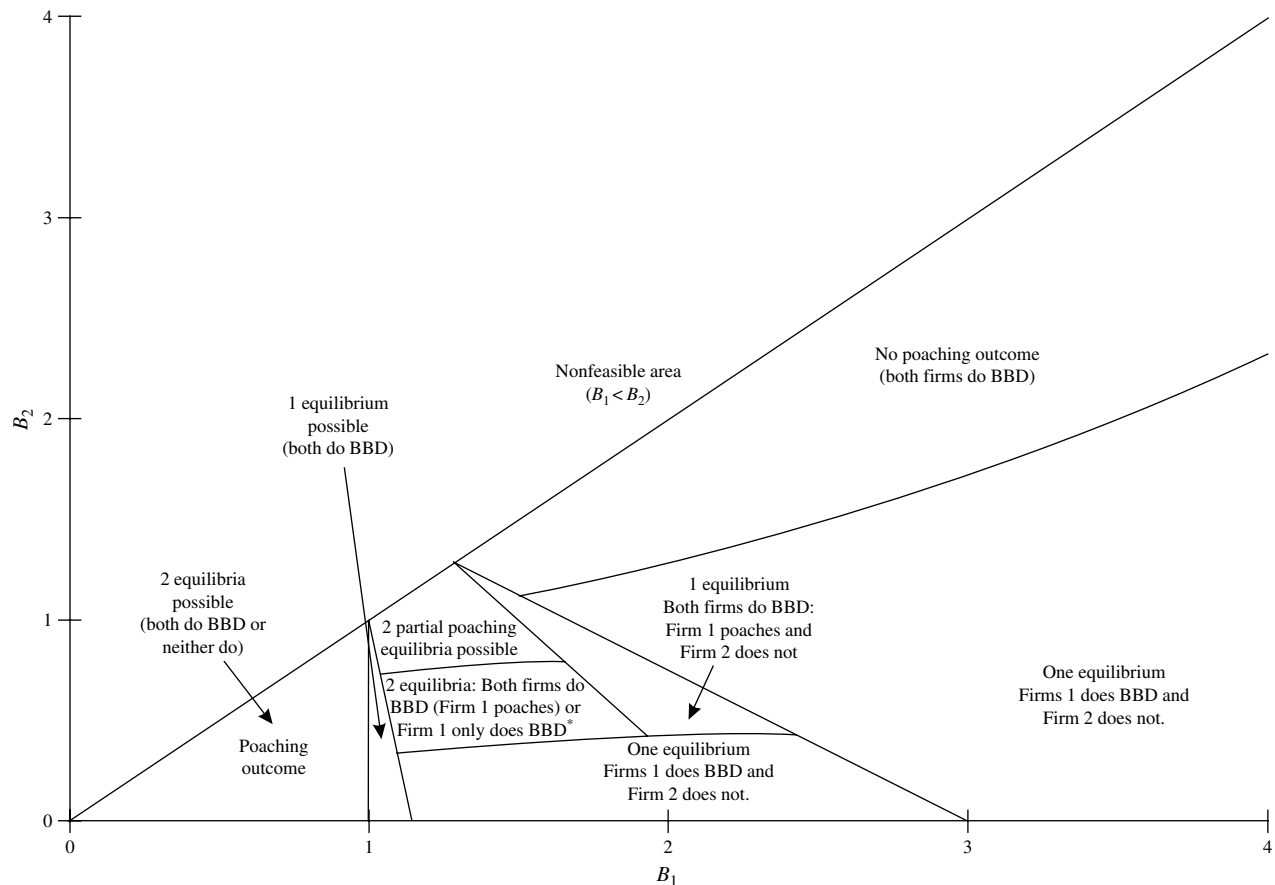
Second, there is a *unique* equilibrium where both firms poach (the second zone in Figure 2 when moving from left to right). Interestingly, when $B_1 = B_2$, poaching by both firms is *never* unique. This suggests that poaching by both firms is more likely to be observed when firms have different capabilities to add benefits for past consumers.

Third and most surprisingly, for a significant fraction of the parameter space, the best response of the firm at a disadvantage (Firm 2) is to respond to BBD by pricing uniformly in the second period and offering no benefits (even when it has the ability to do so). When a firm makes a decision about implementing BBD, its decision is based on two effects. The first is the competition-increasing effect of BBD (because tailored prices can be set for subsections of the market; there are less inframarginal consumers). The second effect is the ability to charge higher prices to past consumers as a result of the benefits that are added in the second period. Basically, when the benefits Firm 2 can offer are small, the first effect is larger than the second effect and Firm 2 benefits from lower competition associated with uniform pricing in the second period.

The profit expressions (provided in the appendix) lead to Proposition 10, which highlights a number of key differences between the cases of equal and unequal capability to add benefits.

PROPOSITION 10.

1. When $B_1 < 1$ and firms implement BBD, Firm 1 increases profits versus the base case when $\frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2,304}B_1B_2 + \frac{263}{4,608}B_1^2 + \frac{263}{4,608}B_2^2 - \frac{1}{18} > 0$.

Figure 2 Summary of Metagame Equilibrium When $B_1 > B_2$ 

*The outcome where both firms do BBD is an equilibrium. However, Firm 2 cannot enforce the “right” subgame after implementing BBD.

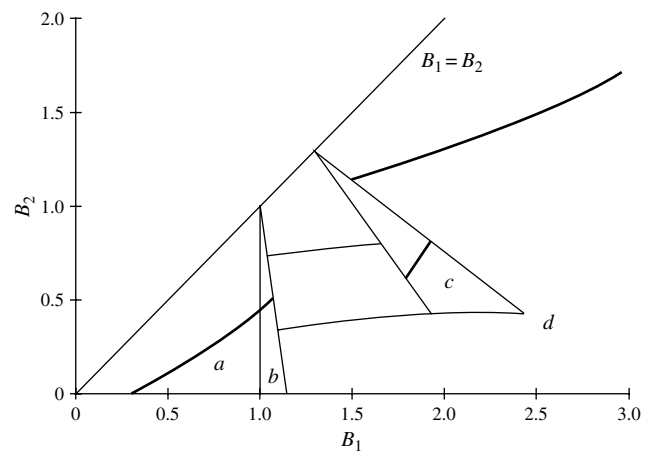
2. When $B_1 \in (1, \frac{8}{7} - \frac{1}{7}B_2)$ and $\frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2,304}B_1B_2 + \frac{263}{4,608}B_1^2 + \frac{263}{4,608}B_2^2 - \frac{1}{18} > 0$, Firm 1 increases profits versus the base case.

3. When both firms implement BBD and the equilibrium involves partial poaching by Firm 1, Firm 1 earns more than the base case when $\frac{33}{169}B_1 - \frac{60}{169}B_2 - \frac{11}{676}B_1B_2 + \frac{99}{1,352}B_1^2 + \frac{229}{4,056}B_2^2 - \frac{125}{338} > 0$.

4. When Firm 2 does not implement BBD, Firm 1 earns more than the base case.

Proposition 10 is illustrated in Figure 3. The areas containing the letters *a*, *b*, *c*, and *d* correspond to the regions specified by points 1–4 in the proposition. The most important insight highlighted by Proposition 10 is that BBD can be profit enhancing for a firm when it can offer benefits that provide higher value than those offered by the competitor. In particular, when Firm 1 has a sufficient advantage over Firm 2, BBD can lead to higher profits for Firm 1 (compared to no BBD) under conditions of poaching, partial poaching, and no poaching. Moreover, in region *b*, poaching by both firms is the unique outcome and Firm 1 earns strictly higher profit than the base case. The analysis thus demonstrates that BBD can be an effective

competitive tool for a firm when the benefits it adds are significantly higher than its competitor. The firm can increase profits versus the base case, and its gains come largely at the expense of a competitor who is not able to respond effectively.

Figure 3 Graph Showing Zones Where Firm 1 Gains from BBD vs. the Base Case

4. Conclusion

Our analysis shows that when firms have equal capability to add benefits for past consumers, BBD is generally a curse. For most parameter values, the ability to implement BBD creates a prisoner's dilemma. Only at low benefit levels (when there are two possible outcomes) can firms avoid the trap of reduced profits caused by implementing BBD.

The deleterious effects of BBD occur because the benefit that a firm adds to its offer in the second period (for past consumers) indirectly leads to fiercer competition. BBD makes the value of a first-period sale high because firms obtain a significant benefit from first-period customers in the second period. Firms account for this and price aggressively in the first period. We also consider a situation where firms are restricted to charging prices greater than marginal cost in the first period; in this situation, BBD can be profitable once the level of benefits is above a threshold.

To highlight the impact of adding benefits to past consumers' offers, we assume that benefits are added costlessly in the base model. This raises a question of whether the negative effects of BBD are due to this assumption. To investigate this possibility, we analyze a version of the model where the size of the benefits needs to be financed. We show that the prisoner's dilemma problem is reduced by assuming that the level of benefit offered to past consumers is costly but not eliminated.

We also analyze the impact of BBD when firms have unequal capabilities to add benefits for past customers, i.e., when one firm has a stronger capability to add benefits. Our analysis shows that when a firm has a sufficient advantage over its competitor, it can increase profit versus the base case. In addition, the firm at a disadvantage sometimes finds that the best response to BBD by a strong competitor is to respond with a uniform price and avoid implementing BBD.

The ultimate impact of BBD in a given market is a function of (a) how important the benefits are to past consumers, (b) how much the firm needs to invest to be able to offer these benefits, and (c) whether one firm has a significant advantage over its competitor in terms of its ability to add benefits.

It seems obvious that information collected from consumers is valuable for guiding firms about how to treat consumers in the future. Moreover, many firms have implemented sophisticated forms of BBD, notably retailers and airline companies. Nevertheless, for many firms, these programs have increased the cost of doing business without delivering significant benefits in terms of higher profits. This is precisely the prisoner's dilemma outcome highlighted by the model. It happens because significant gains are possible when BBD is implemented unilaterally.

As long as the competitors do not respond (or at least respond slowly), a significant advantage can be obtained. Perhaps that was the thinking behind American Airlines' launch of the AAdvantage frequent flyer program in 1982. Unfortunately for American Airlines, competitive airlines launched their own programs within weeks and the advantage evaporated (Kearney 1990).

Another insight of the model relates to the impact of having firms make sequential versus simultaneous decisions. The model underlines how decisions become strategic when competitors make sequential versus simultaneous decisions. The sequential game allows the first mover to transmit information to the follower about which outcome it prefers. When the game is sequential, both firms benefit by avoiding the dominated equilibrium of implementing BBD (whenever B or $B_1 < 1$). This raises a question related to the previous American Airlines example. By 1982, a dynamic of leader-follower competition in the U.S. airline industry was well established, with American Airlines as the leader. The model suggests that American Airlines might have been able to avoid the profit-reducing outcome of (BBD, BBD) by choosing not to implement BBD. How do we explain this? First, it seems that American Airlines may have misjudged the ability of its competitors to respond quickly to the new program. Second, American Airlines may have misjudged the magnitude of the benefits provided by its frequent flyer program. Customers do appreciate the miles and upgrades. Nonetheless, the most notorious aspect of these programs is perhaps the huge balance of unclaimed awards (*The Economist* 2002).

A final insight relates to the conditions where BBD is a winning play. When firms are equal in their capability to add benefits, BBD can be jointly beneficial when first-period competition is limited by an inability to charge negative prices or an inability to sell products for less than their marginal cost.¹⁷ Alternatively, a firm needs to have more ability to add benefits than its competitor for BBD to be a winning play. Regardless, the most likely situation for a firm to benefit from BBD is when the competitor is unable to respond and competition in the second period is eliminated. But the real winners with regard to the recent growth of BBD are consumers, who often enjoy prices at or below cost for their first purchase.

Acknowledgments

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¹⁷ For example, with software, marginal costs are negligible so prices lower than marginal cost may be infeasible.

Appendix

The First-Order Conditions Under Conditions of Poaching

When both firms employ BBD:

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_1^1} &= \frac{1}{2}(c + B_1 + 1 + p_2^1 - 2p_1^1) = 0 \\ \frac{\partial \pi_1}{\partial p_1^2} &= \frac{1}{2}(c + 1 - 2q + p_2^2 - 2p_1^2 - B_2) = 0 \\ \frac{\partial \pi_2}{\partial p_2^1} &= \frac{1}{2}(c + 2q - 1 - 2p_2^1 + p_1^1 - B_1) = 0 \\ \frac{\partial \pi_2}{\partial p_2^2} &= \frac{1}{2}(c + B_2 + 1 - 2p_2^2 + p_1^2) = 0.\end{aligned}\quad (13)$$

When only Firm 1 uses BBD:

$$\begin{aligned}\frac{\partial \pi_1}{\partial p_1^1} &= \frac{1}{2}(c + B_1 + 1 + p_2^b - 2p_1^1) = 0 \\ \frac{\partial \pi_1}{\partial p_1^2} &= \frac{1}{2}(c + 1 - 2q + p_2^2 - 2p_1^2) = 0 \\ \frac{\partial \pi_2}{\partial p_2^b} &= \frac{1}{2}(2c + 2q - B_1 + p_1^1 - 4p_2^b + p_1^2) = 0.\end{aligned}\quad (14)$$

PROOF OF PROPOSITION 1. When $B < 1$, the first-order conditions for second-period prices imply that

$$\begin{aligned}p_1^1 &= \frac{1}{3}B_1 + \frac{1}{3} + \frac{2}{3}q + c, & p_1^2 &= 1 - \frac{4}{3}q - \frac{1}{3}B_2 + c \\ p_2^1 &= \frac{4}{3}q - \frac{1}{3} - \frac{1}{3}B_1 + c, & p_2^2 &= \frac{1}{3}B_2 + 1 - \frac{2}{3}q + c.\end{aligned}$$

Using the reasoning of §2.2, we substitute to obtain $q = \frac{1}{8}B_1 - \frac{1}{8}B_2 - \frac{3}{8}p_1 + \frac{3}{8}p_2 + \frac{1}{2}$. We then substitute first, for x_1 and x_2 , second, for equilibrium second-period prices (shown above) and finally for q . This generates two objective functions in terms of first-period prices (too long for presentation purposes). Taking the first-order condition in terms of p_1 and p_2 and solving, we obtain equilibrium prices of

$$p_1 = \frac{4}{3} - \frac{11}{24}B_2 - \frac{5}{24}B_1 + c, \quad p_2 = \frac{4}{3} - \frac{5}{24}B_2 - \frac{11}{24}B_1 + c.$$

When $B = B_1 = B_2$, we obtain $p_1 = p_2 = \frac{4}{3} - \frac{2}{3}B + c$. Substitute this solution into Equations (11) and (12) to derive the remainder of the expressions. In the second period, $x_2 > x_1$ for poaching to occur. This implies that $(p_2^2 - p_1^2 + 1 - B_2)/2 > (B_1 + p_2^1 - p_1^1 + 1)/2$. When $B_1 = B_2$, this implies that $B < 1$. The equilibrium prices imply that $\Pi = \frac{1}{9}B^2 - \frac{2}{9}B + \frac{17}{18}$ in this region. Q.E.D.

PROOF OF PROPOSITION 2. Similar to the proof of Proposition 1, solve the first-order conditions to obtain $p_1 = c + \frac{2}{3} - \frac{11}{92}B$ and $p_2 = c + \frac{11}{12} - \frac{7}{46}B$. Substitute this solution into Equations (12) and then (10) to derive the remainder of the expressions. In the second period, $x_2 > q$ and $q > x_1$ for the poaching equilibrium to hold. Equation (12) implies that $q_{\text{Firm 1}} = \frac{1}{23}B + \frac{1}{2}$. Now $x_2 - q > 0$ implies that $\frac{1}{8} - \frac{11}{184}B > 0 \Rightarrow B < \frac{23}{11}$. Conversely, $q - x_1 > 0$ implies that $\frac{1}{8} - \frac{31}{184}B > 0 \Rightarrow B < \frac{23}{31}$. Because $\frac{23}{31} < \frac{23}{11}$, the limiting condition is $q > x_1$ and the solution applies when $B < \frac{23}{31}$. In this region, firms' profits are $\Pi_1 = \frac{71}{276}B + \frac{777}{8,464}B^2 + \frac{31}{48}$ and $\Pi_2 = \frac{63}{2,116}B^2 - \frac{37}{138}B + \frac{17}{24}$. Q.E.D.

PROOF OF PROPOSITION 3. When $B > \frac{23}{31}$, the outcome where both firms poach each other's previous customers is not feasible. Accordingly, there are three remaining outcomes that are possible: (a) Only Firm 2 poaches, (b) Only Firm 1 poaches, and (c) neither firm poaches. Only (c) survives as a reasonable equilibrium. When neither firm poaches, consumers who bought from each firm in the first period buy from them again. This implies two incentive compatibility conditions. In the first period, $B + 2v - p_1 - p_1^1 - 2q = 2v - p_2 - p_2^b - 2(1 - q)$, and in the second period, $B + v - q - p_1^1 = v - (1 - q) - p_2^b$. The first-period incentive compatibility (IC) constraint implies that $q = \frac{1}{4}B + \frac{1}{4}p_2^b - \frac{1}{4}p_1 + \frac{1}{4}p_2 - \frac{1}{4}p_1^1 + \frac{1}{2}$. In the second period, Firm 2 must find p_2^b to be optimal, and Firm 1 must have no demand (implying that $p_2^1 = c$). Therefore, $\pi_2^b = (p_2^b - c)(1 - (p_1^2 - p_2^b + 1)/2) \Rightarrow \partial \pi_2^b / \partial p_2^b = p_2^b - \frac{1}{2}p_1^2 + \frac{1}{2} - \frac{c}{2} = 0$. When $p_1^2 = c$, $p_2^b = c + \frac{1}{2}$, the two IC constraints then imply that $p_1^1 = B + c + p_1 - p_2 + \frac{1}{2}$ and $q = \frac{1}{2}p_2 - \frac{1}{2}p_1 + \frac{1}{2}$.

The first-period objective functions then become $\Pi_1 = (p_1 + p_1^1 - 2c)q$ and $\Pi_2 = (p_2 + p_2^b - 2c)(1 - q)$. Substituting and solving the first-order conditions, we obtain $p_1 = c + \frac{9}{10} - \frac{2}{5}B$ and $p_2 = c + \frac{7}{10} - \frac{1}{5}B$. Feasibility ($q < 1$) requires $B < 6$, which is satisfied.¹⁸ The profits that firms earn in this situation are $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$. Q.E.D.

PROOF OF PROPOSITION 4. As shown in Proposition 1, when $B > 1$, an equilibrium where both firms implement BBD and poach is not feasible. When only one firm implements BBD, poaching does not occur when $B > \frac{197}{84} - \frac{69}{308}\sqrt{22}$ (a full derivation of this limit is available in the Technical Appendix).¹⁹

When neither firm poaches, the second-period profits are $\pi_1 = (p_1^1 - c)q$ and $\pi_2 = (p_2^2 - c)(1 - q)$. For no poaching to occur, we know that $p_1^2 = p_2^1 = c$, or either firm could reduce the price offered to new customers and poach the competitor's previous customers.

This implies that $p_1^1 = 1 - 2q + B$ and $p_2^2 = 2q - 1 + B$. Moving to the first-period decision, the surplus from buying at Firm 1 is $CS_1 = B + 2v - 2tx - p_1 - p_1^1$ and from Firm 2 is $CS_2 = B + 2v - 2t(1 - x) - p_2 - p_2^2$. Substituting the values for p_1^1 and p_2^2 at the indifferent consumer in the first period, we obtain $CS_1 = 2v - p_1 - 1$ and $CS_2 = 2v - p_2 - 1$. This implies that when competition is eliminated in the second period, the decision in the first period is based entirely on price (no matter which decision the consumer makes, she is fully compensated for her location by the price that is charged in the second period). Strictly speaking, any division of the market in the first period is consistent with these incentive compatibility constraints. To focus on a "reasonable division," one can assume that payoffs in the first period have a higher weight than the payoff in the second period ($\delta < 1$). As $\delta \rightarrow 1$, the outcomes are exactly as presented in the paper, and $\delta < 1$ ensures that the split in the first period is symmetric.

The value of buying in period 1 for the indifferent consumer who buys from Firm 1 is $CS_1 = v - q - \delta - c\delta + v\delta + q\delta - p_1$ and from Firm 2 is $CS_2 = v + q - c\delta + v\delta - q\delta - p_2 - 1$.

¹⁸ When $B > 6$, Firm 1 obtains the entire market.

¹⁹ The Technical Appendix can be found at <http://mktsci.pubs.informs.org>.

Solving, we obtain $q = 1/(2\delta - 2)(\delta + p_1 - p_2 - 1)$. The first-period objective functions are $\Pi_1 = ((p_1 - c) + \delta(p_1^1 - c))q$ and $\Pi_2 = ((p_2 - c) + \delta(p_2^1 - c))(1 - q)$. Substituting and solving the first-order conditions, we obtain $p_1 = 1 + c - \delta B$ and $p_2 = 1 + c - \delta B$ in the first period. As $\delta \rightarrow 1$, the equilibrium prices in the first period approach $p_1 = 1 + c - B$ and $p_2 = 1 + c - B$. In this situation, $p_1^1 = p_2^1 = B + c$ and $p_1^2 = p_2^2 = c$ (in the second period), and firms earn profits of $\Pi_1 = \Pi_2 = \frac{1}{2}$. Note that the profits earned are independent of B . The feasibility of this outcome requires that $c > B - 1$. Q.E.D.

PROOF OF PROPOSITION 5. In the proof of Proposition 3, we show that the solution is $p_1 = c + \frac{9}{10} - \frac{2}{5}B$ and $p_2 = c + \frac{7}{10} - \frac{1}{5}B$ when only Firm 1 implements BBD (as long as $B > 6$). The profits that firms earn in this situation are $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$. When $B > 6$ and only Firm 1 implements BBD, the optimal first-period prices are $p_1 = c - \frac{27}{20}$ and $p_2 = c - \frac{3}{10}$, leaving Firm 1 with the entire market. Here, Firm 2 sets $p_2^1 = c$ (the lowest price that Firm 2 can set to capture past consumers of Firm 1). This implies that $p_1^1 = B + c - 1$. The resulting profits are $\Pi_1 = B - \frac{47}{20}$ and $\Pi_2 = 0$. Q.E.D.

PROOF OF PROPOSITION 6. When $B \in (0, \frac{23}{31})$, the profit expressions are as follows:

$$\begin{aligned}\Pi_{\text{BBD}} &= \frac{1}{9}B^2 - \frac{2}{9}B + \frac{17}{18}, & \Pi_{\text{no BBD}} &= 1 \\ \Pi_a &= \frac{71}{276}B + \frac{777}{8,464}B^2 + \frac{31}{48}, & \Pi_d &= \frac{63}{2,116}B^2 - \frac{37}{138}B + \frac{17}{24}.\end{aligned}$$

For the top-left and lower-right square of Figure 1 to be equilibria, we need $\Pi_{\text{no BBD}} > \Pi_a$ and $\Pi_{\text{BBD}} > \Pi_d$. It is straightforward to show that these inequalities are satisfied.

When $B \in (\frac{23}{31}, 1)$, the profit expressions are as follows:

$$\begin{aligned}\Pi_{\text{BBD}} &= \frac{1}{9}B^2 - \frac{2}{9}B + \frac{17}{18}, & \Pi_{\text{no BBD}} &= 1 \\ \Pi_a &= \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}, & \Pi_d &= \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25} \quad \text{or} \\ \Pi_a &= \frac{66}{529}B + \frac{99}{1,058}B^2 + \frac{573}{1,058}, & \Pi_d &= \frac{9}{529}B^2 - \frac{80}{529}B + \frac{1,243}{3,174}.\end{aligned}$$

Recall that Proposition 3 shows there are two stable outcomes in the subgame, where only one firm implements BBD. For the top-left and lower-right square to be equilibria, we need $\Pi_{\text{no BBD}} > \Pi_a$ and $\Pi_{\text{BBD}} > \Pi_d$ (for both potential asymmetric outcomes). As before, it is straightforward to show that these inequalities are satisfied (for both asymmetric outcomes). Q.E.D.

PROOF OF COROLLARY 1. When $B \in (0, 1)$, show that $\Pi_{\text{no BBD}} > \Pi_{\text{BBD}}$. If the reverse is true, then $\frac{1}{9}B^2 - \frac{2}{9}B + \frac{17}{18} - 1 > 0 \Rightarrow B > \frac{1}{2}\sqrt{6} + 1 \approx 2.2247$, or $B < 1 - \frac{1}{2}\sqrt{6} \approx -0.22474$, which implies a B outside the allowable range. Q.E.D.

PROOF OF PROPOSITION 7. When $B \in (\frac{197}{84} - \frac{69}{308}\sqrt{22} \approx 1.2945, 6)$, the profits of Firms 1 and 2 are

$$\begin{aligned}\Pi_{\text{BBD}} &= \frac{1}{2}, & \Pi_{\text{no BBD}} &= 1, & \Pi_a &= \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}, \\ \Pi_d &= \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}.\end{aligned}$$

For the lower-right square of Figure 1 to be an equilibrium, we need $\Pi_a > \Pi_{\text{no BBD}}$ and $\Pi_{\text{BBD}} > \Pi_d$. It is straightforward to show that these inequalities are satisfied.

Conversely, when $B > 6$, the profits of Firms 1 and 2 are

$$\Pi_{\text{BBD}} = \frac{1}{2}, \quad \Pi_{\text{no BBD}} = 1, \quad \Pi_a = B - \frac{19}{10}, \quad \Pi_d = 0.$$

As before, it is straightforward to show that the lower-right square of Figure 1 is the equilibrium because $\Pi_a > \Pi_{\text{no BBD}}$ and $\Pi_{\text{BBD}} > \Pi_d$. Q.E.D.

PROOF OF COROLLARY 2. When $B > \frac{197}{84} - \frac{69}{308}\sqrt{22}$, the equilibrium is the lower-right square, as shown in Proposition 7. This is a prisoner's dilemma because $\Pi_{\text{no BBD}} = 1 > \Pi_{\text{BBD}} = \frac{1}{2}$ throughout the range. Q.E.D.

PROOF OF LEMMA 1. An equilibrium where both firms poach is not feasible when $B > 1$. When $B > B^{**} = \frac{669}{230} - \frac{39}{230}\sqrt{101} \approx 1.2046$, an outcome where only one firm poaches is not feasible (this limit is derived in the detailed appendix). Thus, when $B > B^{**}$, neither firm poaches and the second-period profits are $\pi_1 = p_1^1 q$ and $\pi_2 = p_2^2 (1 - q)$. For no poaching to occur, p_1^1 and p_2^1 equal 0 or either firm could reduce the price offered to new customers and poach the competitor's previous customers.

Similar reasoning to the case of positive marginal cost implies that the indifferent consumer in the first period obtains $CS_1 = 2v - p_1 - 1$ by choosing Firm 1 and $CS_2 = 2v - p_2 - 1$ by choosing Firm 2. This implies that when competition is eliminated in the second period, the decision in the first period is based entirely on price (no matter which decision the consumer makes, she is compensated for her location by the price charged in the second period).²⁰ This implies that (a) prices in the first period are negative or (b) price in the first period is zero. Because we restrict our attention to positive prices, $p_1 = p_2 = 0$ in the first period and $q = \frac{1}{2}$. This outcome then results in profits of $\Pi = B/2$ for each firm. Q.E.D.

PROOF OF LEMMA 2. Using similar reasoning as for the case of positive marginal costs, the equilibrium is $p_1 = \frac{9}{10} - \frac{2}{5}B$ and $p_2 = \frac{7}{10} - \frac{1}{5}B$. However, this solution is valid if and only if $p_2 > 0 \Rightarrow B < \frac{9}{4}$. Therefore, $p_1 = \frac{9}{10} - \frac{2}{5}B$ and $p_2 = \frac{7}{10} - \frac{1}{5}B$ when $B < \frac{9}{4}$ and $p_1 = 0$ and $p_2 = \frac{1}{4}$ when $B > \frac{9}{4}$. The equilibrium values imply that $\Pi_1 = \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}$ and $\Pi_2 = \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}$ when $B \in (\frac{669}{230} - \frac{39}{230}\sqrt{101}, \frac{9}{4})$ and $\Pi_1 = \frac{5}{8}B + \frac{5}{32}$ and $\Pi_2 = \frac{9}{32}$ when $B > \frac{9}{4}$. Q.E.D.

PROOF OF PROPOSITION 8. When $B \in (\frac{669}{230} - \frac{39}{230}\sqrt{101}, \frac{9}{4})$, the profit expressions are as follows:

$$\begin{aligned}\Pi_{\text{BBD}} &= \frac{B}{2}, & \Pi_{\text{no BBD}} &= 1, & \Pi_a &= \frac{8}{25}B + \frac{1}{25}B^2 + \frac{16}{25}, \\ \Pi_d &= \frac{1}{50}B^2 - \frac{6}{25}B + \frac{18}{25}.\end{aligned}$$

Also note that in the zone $B \in (1, \frac{669}{230} - \frac{39}{230}\sqrt{101})$, $\Pi_{\text{BBD}1}$, and $\Pi_{\text{BBD}2} > B/2$. For the lower-right square of Figure 1 to be an equilibrium we need $\Pi_a > \Pi_{\text{no BBD}}$ and $\Pi_{\text{BBD}} > \Pi_d$. It is straightforward to show that these inequalities are satisfied.

When $B > \frac{9}{4}$, the profit expressions are as follows:

$$\Pi_{\text{BBD}} = B/2, \quad \Pi_{\text{no BBD}} = 1, \quad \Pi_a = \frac{5}{8}B + \frac{5}{32}, \quad \Pi_d = \frac{9}{32}.$$

²⁰ Any division of the market in the first period is consistent with these incentive compatibility constraints. Similar to the case of positive marginal costs, we examine a reasonable division by assuming that payoffs in the first period have a higher weight than the payoff in the second period ($\delta < 1$). As $\delta \rightarrow 1$, the outcomes are as presented, and $\delta < 1$ ensures that the split in the first period is symmetric.

For the lower-right square of Figure 1 to be an equilibrium we need $\Pi_a > \Pi_{\text{no BBD}}$ and $\Pi_{\text{BBD}} > \Pi_d$. It is straightforward to show that these inequalities are satisfied. Q.E.D.

PROOF OF COROLLARY 3. When $B \in (\frac{669}{230} - \frac{39}{230}\sqrt{101}, 2)$, the equilibrium is the lower-right square of Figure 1. If $\Pi_{\text{no BBD}} > \Pi_{\text{BBD}}$, then firm profits are reduced by BBD. Using the profit expressions, if the opposite is true, then $B/2 - 1 > 0 \Rightarrow B > 2$, which implies a B outside the allowable range. When $B > 2$, the equilibrium is the lower-right square, as shown in Proposition 8. To show that profits increase because of BBD, we need to show that $\Pi_{\text{BBD}} > \Pi_{\text{no BBD}}$. Assume the opposite; then $1 - B/2 > 0 \Rightarrow B < 2$, which implies a B outside the allowable range. Q.E.D.

PROOF OF PROPOSITION 9. When both firms implement BBD and the optimal level of benefit is less than 1 (poaching occurs), the profit functions are as follows:

$$\begin{aligned}\Pi_1 &= \frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2,304}B_1B_2 + \frac{263}{4,608}B_1^2 \\ &\quad + \frac{263}{4,608}B_2^2 + \frac{17}{18} - \alpha B_1^2 \\ \Pi_2 &= \frac{23}{144}B_2 - \frac{55}{144}B_1 - \frac{7}{2,304}B_1B_2 \\ &\quad + \frac{263}{4,608}B_1^2 + \frac{263}{4,608}B_2^2 + \frac{17}{18} - \alpha B_2^2.\end{aligned}$$

The first-order conditions imply that $B_i = -23/144(-2\alpha + 1/9)$. Substituting this solution back into the objective functions, we obtain $\Pi_i = 529/2,304(18\alpha - 1)^2 - 23/72(18\alpha - 1) + \frac{17}{18}$. This solution applies when $\alpha > \frac{13}{96}$ (i.e., $B_i < 1$). This is a Nash equilibrium as long as $\Pi_{\text{BBD}} > \Pi_d$. Following Propositions 2 and 3, $\Pi_d = \frac{63}{2,116}B_1^2 - \frac{37}{138}B_1 + \frac{17}{24}$ when $B < \frac{23}{31}$ and $\Pi_d = \frac{1}{50}B_1^2 - \frac{6}{25}B_1 + \frac{18}{25}$ when $B \in (\frac{23}{31}, 1)$. The proof to show that no BBD is an equilibrium is analogous to Proposition 6 (Π_a being strictly less compared to the case when benefits are costless).

Following Proposition 4, poaching does not occur if $B > \frac{197}{84} - \frac{69}{308}\sqrt{22}$. When only Firm 1 implements BBD, we use the profit expressions relative to one firm implementing BBD when $B > \frac{23}{31}$, from Proposition 3. Optimizing the expression for Firm 1 implies an optimal $B_1 = -8/25(-2\alpha + 2/25)$ when $\alpha > \frac{1}{15}$ (i.e., $B_1 < 6$) and $B = 1/2\alpha$ when $\alpha < \frac{1}{15}$ (i.e., $B_1 > 6$).

When both firms implement BBD and benefits are high enough to eliminate poaching $B > \frac{197}{84} - \frac{69}{308}\sqrt{22}$, the profit function for each firm is $\Pi_i = \frac{1}{2} - \alpha B_i^2$. This function is declining in B_i so firms will choose the level B_i that ensures no poaching but maximizes profit; i.e., $B_i = \frac{197}{84} - \frac{69}{308}\sqrt{22}$ (if a firm chooses $B < \frac{197}{84} - \frac{69}{308}\sqrt{22}$, the competitor may poach the firm's past consumers, leading to strictly lower profits).²¹ Therefore, the profits that firms earn when $\alpha < \frac{1}{9}$ are $\Pi_i = \alpha(\frac{4,531}{4,312}\sqrt{22} - \frac{512,597}{77,616}) + \frac{1}{2}$.

The following chart sketches how the equilibrium is determined for both regions, i.e., when $\alpha \in (\frac{1}{15}, \frac{1}{9})$ and when $\alpha < \frac{1}{15}$. First, we assume that a pure strategy equilibrium exists. Second, if a pure strategy equilibrium exists, then there are only three possible benefit levels that a firm will consider. The first is to choose $B = 0$ (i.e., do not implement BBD). The second is to set the benefit level as if the firm

faces a competitor that does not engage in BBD. The final level is to set the benefit level that would be chosen given that both firms decide to implement BBD. The metagame can then be thought of as a 3×3 , based on firms investing to achieve the three benefit levels described above. Straightforward mathematical comparison of the profit expressions in each case reveals that the unique equilibrium is for both firms to implement BBD and choose $B = \frac{197}{84} - \frac{69}{308}\sqrt{22}$ in both regions. Q.E.D.

PROOF OF PROPOSITION 10. The equilibrium prices derived in the Proof of Proposition 1 (when both firms poach) imply that

$$\begin{aligned}\Pi_1 &= \frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2,304}B_1B_2 + \frac{263}{4,608}B_1^2 + \frac{263}{4,608}B_2^2 + \frac{17}{18} \\ \Pi_2 &= \frac{23}{144}B_2 - \frac{55}{144}B_1 - \frac{7}{2,304}B_1B_2 + \frac{263}{4,608}B_1^2 + \frac{263}{4,608}B_2^2 + \frac{17}{18}.\end{aligned}$$

This occurs when $q > x_1$ (the expression for q is Equation (11) in the paper), $x_2 > q$, and $x_2 < 1$. The first condition to be violated is $q > x_1$, and this obtains when $-\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} > 0$.

When $\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} < 0$, partial poaching is possible when Firm 1 (the firm with stronger benefits) poaches Firm 2's past consumers. Solving the first-order conditions for second period and then the first period implies equilibrium prices in the first period of $p_1 = \frac{15}{13} - \frac{20}{39}B_2 - \frac{5}{13}B_1 + c$ and $p_2 = \frac{6}{13} - \frac{8}{39}B_2 - \frac{2}{13}B_1 + c$ and in the second period of $p_1^2 = \frac{17}{26}B_1 + \frac{1}{26}B_2 + \frac{7}{13} + c$, $p_1^2 = \frac{9}{13} - \frac{4}{13}B_2 - \frac{3}{13}B_1 + c$, $p_2^2 = c$, and $\frac{2}{26}B_2 - \frac{3}{26}B_1 + \frac{11}{13} + c$. The profits of Firms 1 and 2 under this regime are

$$\begin{aligned}\Pi_1 &= \frac{33}{169}B_1 - \frac{60}{169}B_2 - \frac{11}{676}B_1B_2 + \frac{99}{1,352}B_1^2 + \frac{229}{4,056}B_2^2 + \frac{213}{338} \\ \Pi_2 &= \frac{73}{507}B_2 - \frac{50}{169}B_1 - \frac{5}{676}B_1B_2 + \frac{45}{1,352}B_1^2 + \frac{227}{4,056}B_2^2 + \frac{241}{338}.\end{aligned}$$

The feasibility conditions imply that $x_2 < 1$ and $x_2 > q$. The limiting condition is $x_2 > q$, and this implies that $\frac{9}{26} - \frac{2}{13}B_2 - \frac{3}{26}B_1 > 0$.

When Firm 2 poaches Firm 1's past consumers, the expressions are identical, but with 1 and 2 reversed. Note that $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} < 0$ is more restrictive than the condition for the first partial poaching equilibrium.

This implies that when $-\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} < 0$ and $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} < 0$, the outcome when both firms implement BBD could be either of two partial poaching outcomes. In contrast, when $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} > 0$ and $\frac{9}{26} - \frac{2}{13}B_2 - \frac{3}{26}B_1 > 0$, the unique equilibrium is for Firm 1 to poach Firm 2's past consumers.

When $\frac{9}{26} - \frac{2}{13}B_2 - \frac{3}{26}B_1 < 0$, the only viable outcome is for neither firm to poach the other's past consumers in the second period. Similar to the no-poaching equilibrium when firms are symmetric, it is straightforward to derive the equilibrium prices of $p_1 = c - \frac{1}{2}B_1 - \frac{1}{2}B_2 + 1$ and $p_2 = c - \frac{1}{2}B_1 - \frac{1}{2}B_2 + 1$. In this situation, $p_1^2 = B_1 + c$ and $p_2^2 = B_2 + c$ and $p_1^2 = p_2^2 = c$ (in the second period). The firms earn profits of $\Pi_1 = \frac{1}{4}B_1 - \frac{1}{4}B_2 + \frac{1}{2}$ and $\Pi_2 = \frac{1}{4}B_2 - \frac{1}{4}B_1 + \frac{1}{2}$.

To summarize, the following are subgame outcomes as a function of B_1 and B_2 .

1. When $-\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} > 0$ and both firms implement BBD, the outcome involves poaching by both firms.

2. When $-\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} < 0$ and $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} < 0$, there are two partial poaching outcomes. Either Firm 1 poaches Firm 2's past consumers or Firm 2 poaches Firm 1's past consumers.

²¹ This observation is based on the existence of a partial poaching equilibrium when $B < \frac{197}{84} - \frac{69}{308}\sqrt{22}$. The reduction in profits occurs in this equilibrium when a firm's past customers are poached.

3. When $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} > 0$ and $\frac{9}{26} - \frac{2}{13}B_2 - \frac{3}{26}B_1 > 0$, the unique equilibrium is for Firm 1 (the strong firm) to poach Firm 2's past consumers in the second period 1.

4. When $\frac{9}{26} - \frac{2}{13}B_2 - \frac{3}{26}B_1 < 0$ and both firms implement BBD, the unique equilibrium does not involve poaching by either firm.

These outcomes provide the basis for solving the metagame.

Poaching Zone ($B_1 < \frac{23}{31}$).

1. We know from Proposition 6 that the no-BBD outcomes are higher than asymmetric payoffs for both firms. Hence, (no BBD, no BBD) is a Nash equilibrium.

2. For BBD by Firm 1 to be the best response to BBD by Firm 2, we need $\Pi_{BBD1} > \Pi_{d1}$. For Firm 1 $\Pi_{d1} = \frac{63}{2,116}B_2^2 - \frac{37}{138}B_2 + \frac{17}{24}$ (Firm 2 only implements BBD) and $\Pi_{BBD} = \frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2,304}B_1B_2 + \frac{263}{4,608}B_1^2 + \frac{263}{4,608}B_2^2 + \frac{17}{18}$. It is straightforward to show that this expressions is always negative. For BBD by Firm 2 to be the best response to BBD by Firm 1, we also need $\Pi_{BBD2} > \Pi_{d2}$. Similarly, it is straightforward to show that $\Pi_{BBD} - \Pi_d$ is strictly positive in this range.

Poaching Zone $B_1 \in (\frac{23}{31}, 1)$. Similar reasoning can be used to show that there two Nash equilibria in this zone as well, one where both firms implement BBD and one where neither do.

Poaching Zone $B_1 \in (1, \frac{8}{7} - \frac{1}{7}B_2)$. The second limit is simply the limit for poaching $-\frac{7}{48}B_1 - \frac{1}{48}B_2 + \frac{1}{6} > 0$ rewritten for B_1 in terms of B_2 . Note that $\Pi_a = \frac{8}{25}B_1 + \frac{1}{25}B_2^2 + \frac{16}{25}$ for Firm 1 is strictly greater than 1, so there is only one equilibrium (BBD, BBD) in this part of the parameter space.

Partial Poaching Zone (1) $B_1 \in (\frac{8}{7} - \frac{1}{7}B_2, \frac{9}{4} - \frac{3}{4}B_2)$. The second limit is simply the first limit for partial poaching $\frac{2}{13}B_1 + \frac{3}{26}B_2 - \frac{9}{26} = 0$, rewritten for B_1 in terms of B_2 .

1. Firm 1 will always implement BBD because $\Pi_a > 1$. The only question is whether Firm 2 responds by implementing BBD as well.

2. When $\frac{33}{169}B_2 - \frac{486}{4,225}B_1 - \frac{11}{676}B_1B_2 + \frac{3,697}{101,400}B_1^2 + \frac{99}{1,352}B_2^2 - \frac{759}{8,450} > 0$, there are two possible equilibria, one in which Firm 1 poaches (and Firm 2 does not) and vice versa. This is based on Π_2 (the profit of Firm 2 under partial poaching) being greater than Π_d for both cases. It is straightforward to show that the profits earned by Firm 2 when Firm 1 poaches, $\Pi_2 \text{ F1 Poaches} = \frac{73}{507}B_2 - \frac{50}{169}B_1 - \frac{5}{676}B_1B_2 + \frac{45}{1,352}B_1^2 + \frac{227}{4,056}B_2^2 + \frac{241}{338}$, are greater than the profits earned by Firm 2 when Firm 2 poaches, $\Pi_2 \text{ F2 Poaches} = \frac{33}{169}B_2 - \frac{60}{169}B_1 - \frac{11}{676}B_1B_2 + \frac{99}{1,352}B_2^2 + \frac{229}{4,056}B_1^2 + \frac{213}{338}$. This implies that if $\Pi_2 \text{ F2 Poaches}$ exceeds the no-BBD profits, then either partial poaching equilibrium is possible, i.e., $\Pi_2 \text{ F2 Poaches} - \Pi_d > 0 \Rightarrow \frac{33}{169}B_2 - \frac{486}{4,225}B_1 - \frac{11}{676}B_1B_2 + \frac{3,697}{101,400}B_1^2 + \frac{99}{1,352}B_2^2 - \frac{759}{8,450} > 0$.

3. When $\frac{33}{169}B_2 - \frac{486}{4,225}B_1 - \frac{11}{676}B_1B_2 + \frac{3,697}{101,400}B_1^2 + \frac{99}{1,352}B_2^2 - \frac{759}{8,450} < 0$, the partial poaching equilibrium where Firm 2 poaches generates less profit for Firm 2 than not implementing BBD. The area in partial poaching (1) where $\Pi_2 \text{ F1 Poaches} > \Pi_d > \Pi_2 \text{ F2 Poaches}$ has an upper bound (B_1, B_2) as given above and a lower bound given by $\Pi_2 \text{ F1 Poaches} - \Pi_d = 0$. This limit is $0 = \frac{73}{507}B_2 - \frac{236}{4,225}B_1 - \frac{5}{676}B_1B_2 + \frac{449}{33,800}B_1^2 + \frac{227}{4,056}B_2^2 - \frac{59}{8,450}$. In this area, there are two potential equilibria. In the first, Firm 2 does not implement BBD. In the second, Firm 2 implements BBD (as does Firm 1) and Firm 1

poaches but Firm 2 does not. This equilibrium is valid analytically but can be questioned because of Firm 2's inability to implement the "right" subgame after implementing BBD.

4. When $0 > \frac{73}{507}B_2 - \frac{236}{4,225}B_1 - \frac{5}{676}B_1B_2 + \frac{449}{33,800}B_1^2 + \frac{227}{4,056}B_2^2 - \frac{59}{8,450}$, the equilibrium is for Firm 1 to implement BBD and Firm 2 to employ uniform pricing in period 2 (without adding benefits for past consumers). Note that in this equilibrium, Firm 1's profits are strictly higher than the base case (because $\Pi_a > \Pi_{\text{no BBD}}$).

Partial Poaching Zone (2) $B_1 \in (\frac{9}{4} - \frac{3}{4}B_2, 3 - \frac{4}{3}B_2)$. As noted earlier, Firm 1 will implement BBD. If Firm 2 also implements BBD, the equilibrium entails Firm 1 poaching Firm 2's customers (and not vice versa). Thus, the question is whether Firm 2's preferred response is to implement BBD or not. This obtains by comparing $\Pi_2 \text{ F1 Poaches}$ to Π_d . As derived earlier, the boundary is given by $0 = \frac{73}{507}B_2 - \frac{236}{4,225}B_1 - \frac{5}{676}B_1B_2 + \frac{449}{33,800}B_1^2 + \frac{227}{4,056}B_2^2 - \frac{59}{8,450}$. Thus, when $\frac{73}{507}B_2 - \frac{236}{4,225}B_1 - \frac{5}{676}B_1B_2 + \frac{449}{33,800}B_1^2 + \frac{227}{4,056}B_2^2 - \frac{59}{8,450} > 0$, the equilibrium is for both firms to implement BBD. When the condition is not satisfied, only Firm 1 implements BBD.

No Poaching Zone. In this zone, Firm 1 will implement BBD. Firm 2 will implement BBD as long as its expected profits under BBD, i.e., $\Pi_2 = \frac{1}{4}B_2 - \frac{1}{4}B_1 + \frac{1}{2}$, are greater than profits earned by not implementing BBD, i.e., $\Pi_d = \frac{1}{50}B_1^2 - \frac{6}{25}B_1 + \frac{18}{25}$ as per Proposition 5. This implies that Firm 2 implements BBD when $B_1 \in (B_2, \frac{5}{4}\sqrt{8B_2 - 7} - \frac{1}{4})$. When $B_1 > \frac{5}{4}\sqrt{8B_2 - 7} - \frac{1}{4}$, Firm 2 does not implement BBD.

1. When the equilibrium entails poaching by both firms, Firm 1's profit is $\Pi_1 = \frac{23}{144}B_1 - \frac{55}{144}B_2 - \frac{7}{2,304}B_1B_2 + \frac{263}{4,608}B_1^2 + \frac{263}{4,608}B_2^2 + \frac{17}{18}$. Setting this equal to 1 generates the first boundary in the proposition.

2. When the equilibrium involves partial poaching by Firm 1, Firm 1's profit is given by $\Pi_1 = \frac{33}{169}B_1 - \frac{60}{169}B_2 - \frac{11}{676}B_1B_2 + \frac{99}{1,352}B_1^2 + \frac{229}{4,056}B_2^2 + \frac{213}{338}$. Setting this equal to 1 generates the second boundary in the proposition (under point 3).

3. When Firm 2 does not engage in BBD, Firm 1 earns Π_a . As noted earlier, $\Pi_a > 1$ for all $B_1 > 1$. Q.E.D.

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