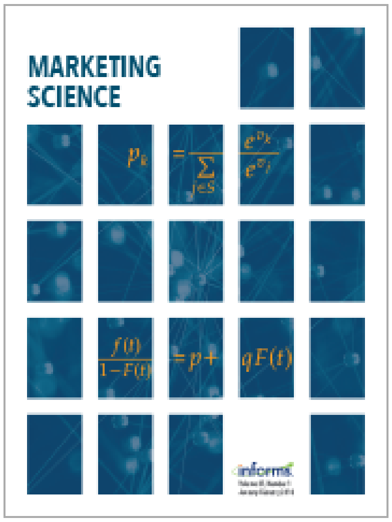

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Quality Segmentation in Spatial Markets: When Does Cannibalization Affect Product Line Design?

Preyas S. Desai,

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Preyas S. Desai
The Fuqua School of Business, Duke University, Durham, North Carolina 27708
preyas.desai@duke.edu

Abstract

Durable goods manufacturers often design product lines by segmenting their markets on quality attributes—attributes that exhibit a "more is better" property for all consumers. Since products within a product line are partial substitutes, and consumers can self-select the products they want to purchase, multiproduct firms have to carefully consider the cannibalization problem in designing their product lines. Existing research has analyzed the cannibalization problem for a monopolist who faces consumers who differ in their quality valuations. If lower-quality products are sufficiently attractive, higher-valuation consumers may find it beneficial to buy lower-quality products rather than the higher-quality products targeted to them. That is, lower-quality products can potentially cannibalize higher-quality products. The cannibalization problem forces the firm to provide only the highest-valuation segment with its preferred (efficient) quality. All other segments get qualities lower than their preferred (efficient) qualities. When the cannibalization problem is very severe, the firm may not serve some of the lowest-valuation segments.

However, not much is known about how and when the cannibalization problem affects product line design in an oligopoly. Also, consumers may differ not only in their quality valuations but also in their taste preferences. The objective of this paper is to fill these gaps by examining whether the cannibalization problem affects a firm's price and quality decisions in a model with consumer differences in quality valuations, as well as in their taste preferences, in both monopoly and duopoly settings. The paper addresses questions such as the following. With both types of consumer differences, should a firm, even a monopolist, provide efficient quality only to the top segment? Are there conditions under which other segments can also get their preferred quality levels? If so, how do consumer and firm characteristics affect the likelihood of different segments getting their preferred qualities? How does competition affect the firm's choice of qualities?

I develop a model in which the market is made up of two segments, with one segment valuing quality more than the other. Consumers within each segment are distributed over Hotelling's (1929) linear city. Consumers in the two segments can have different taste preferences (transportation costs). Firm locations in the two segments may also be different.

The paper begins with an analysis of the monopoly case. I find that when both segments are fully covered, the standard self-selection results of the high-valuation segment getting its preferred quality and the low-valuation segment getting less than its preferred quality do hold. Interestingly, when both segments are incompletely covered, under some conditions, the monopolist's price and quality choices are not determined by the cannibalization problem. In these cases, the monopolist finds it optimal to provide each segment with its preferred quality. Thus, the equilibrium quality levels in a second-degree price discrimination situation resemble the third-degree price discrimination solution. I characterize the relevant conditions in terms of consumer characteristics.

I then consider the case of two firms competing in the market, each offering two products—one for the high-valuation segment and the other for the low-valuation segment. Here also both types of outcomes are possible, depending on consumers and firm characteristics. Under some conditions, the cannibalization problem does not affect the firms' price and quality choices, and each firm provides each segment with that segment's preferred quality. Each firm finds it optimal to serve both segments. When these conditions do not hold, only the high-valuation segment gets its preferred quality. I interpret the conditions necessary for these results to exist in terms of characteristics of the consumers and the firms.

An interesting insight from the analysis is that as the taste preferences of the low-valuation segment become weaker (their "transportation cost" becomes lower), the more intense competition in the low-valuation segment makes it more attractive for the high-valuation consumers to buy the products meant for the low-valuation segment. This worsens the cannibalization problem, and the low-valuation segment may not get its preferred quality. On the other hand, when the taste preferences of the high-valuation segments are sufficiently weak, more intense competition in the high-valuation segment reduces that segment's incentives to buy the product meant for the low-valuation segment. This mitigates the cannibalization problem and makes it more likely for the low-valuation segment to get its preferred quality.

Similarly, when firms are less differentiated in the low-valuation segment, stronger competition between the firms makes the cannibalization problem worse, and the low-valuation segment may not get its preferred quality. When the differentiation between the firms is sufficiently weak in the high-valuation segment, the high-valuation segment is more likely to be better off buying the product meant for it. As the high-valuation segment's incentives to buy the lower-quality product are reduced, the low-valuation segment is more likely to get its preferred quality.

(*Cannibalization; Product Line Design; Price Discrimination; Vertical Differentiation; Horizontal Differentiation*)

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1. Introduction

1.1. Background

We see quality-based segmentation in almost all durable product categories.[1] For example, Sony's TV line includes 27-, 32-, and 35-inch models, and models within each screen size differ in terms of features such as picture-in-picture, freeze memo, wideband video amplifier, etc. Toyota makes luxury cars under the Lexus name and traditional cars under the Toyota name. Within its Toyota line are cars ranging from the small Tercel to the full-size Avalon. These are examples of firms creating product lines made up of different price-quality bundles in order to serve different consumer segments. Higher-price, higher-quality products are targeted to consumers who care more about quality and are willing to pay higher prices for higher-quality products. Lower-price, lower-quality products are meant for consumers who value quality less, and are not willing to pay higher prices for higher-quality levels. Since price and quality are the two dimensions that define different items in a given product line, it is critical to understand how price and quality levels are determined. I examine a firm's choice of price and quality levels for different items within its product line.

Several important papers such as Mussa and Rosen (1978), Katz (1984), and Moorthy (1984) have analyzed this problem in monopoly settings. They emphasize the fact that consumers *self-select* the product they purchase. If the lower-quality products are sufficiently attractive, higher-end consumers may find it beneficial to buy lower-quality products rather than higher-quality products targeted to them. That is, lower-quality products can potentially cannibalize sales of higher-quality products. This cannibalization problem becomes the primary determinant of the optimal price-quality profile in these papers. To alleviate this cannibalization problem, a firm makes lower-quality products relatively unattractive to higher-end consumers. Specifically, the firm provides the top-valuation segment with its preferred (efficient) quality.[2] However, it distorts the qualities of all other products, so that all other segments get quality levels lower than their preferred levels. This effect of the cannibalization problem in determining price and quality levels in a monopoly is well established, and several papers employ this basic setup to investigate a variety of interesting problems (see, for example, Desai et al. 2001). However, not much is known about how cannibalization can affect price and quality choice in an oligopoly.

In addition, consumer heterogeneity in monopoly models is typically restricted to quality valuations. That is, all consumers differ only in terms of how much they are willing to pay for a given level of quality. Clearly, consumers differ in other respects too, especially in terms of how they value nonquality—i.e., taste—attributes. The existing literature doesn't say much about quality segmentation in markets where consumers differ in terms of both their quality valuations and their taste preferences.

The objective of this paper is to fill these gaps by examining whether the cannibalization problem affects a firm's price and quality decisions in a model with consumer differences in quality valuations as well as taste preferences in both monopoly and duopoly settings. The paper addresses questions such as the following. With both types of consumer differences, should a firm, even a monopolist, provide efficient quality only to the top segment? Are there conditions under which other segments can also get their preferred quality levels? If so, how do consumer and firm characteristics affect the likelihood of different segments getting their preferred qualities? How does competition affect the firm's choice of qualities?

1.2. Overview of Model and Key Results

I develop a model in which the market is made up of two consumer segments that differ in their valuations for quality. The high-valuation segment is willing to pay a greater price for any given quality level than

[1]The term *quality* is used to mean a single attribute or a combination of attributes exhibiting the "more-is-better" property.

[2]Efficient quality of a segment is its most preferred quality when all products are available at their marginal costs. This is also the quality that a social planner would choose for that segment. The terms *preferred quality* and *efficient quality* are used interchangeably.

the low-valuation segment. The consumers within a segment are represented by a linear market as in Hotelling (1929). The taste preference of a specific consumer is represented by his/her location on the line. The "transportation cost" of a segment represents the strength of its taste preferences. The model also allows for the possibility that high-valuation consumers may also have stronger taste preferences (higher transportation cost) than low-valuation consumers.

Since most discrimination takes place in oligopolistic markets, I analyze both monopoly and duopoly settings. In duopoly settings, I allow competing firms to be more differentiated in the high-valuation market than in the low-valuation market (see Figure 1).

Depending on consumer and firm characteristics, both efficient and inefficient quality provisions are possible in both monopoly and duopoly. In other words, it is possible that even a monopolist provides efficient quality to both segments. On the other hand, it is also possible that the insights from the existing monopoly analysis are valid even in a duopoly setting. In both settings, there are conditions under which the self-selection constraints are not binding in equilibrium, so that the cannibalization problem does not determine the product line design and firms provide efficient qualities to both segments. Thus, the equilibrium quality levels in a second-degree price discrimination situation resemble the third-degree price discrimination solution. An interesting issue, then, is to understand the conditions under which the cannibalization problem does not affect the product line design. My analysis identifies and characterizes these conditions, which are discussed in detail later. Here I highlight two important factors. As the taste preferences of the low-valuation segment become weaker (their transportation cost becomes lower), the more intense competition in the low-valuation segment makes it more attractive for the high-valuation consumers to buy the products meant for the low-valuation segment. This worsens the cannibalization problem, and the low-valuation segment may not get its preferred quality. On the other hand, when the taste preferences of the high-valuation segments are sufficiently weak, more intense competition in the high-valuation segment reduces that segment's incentives to buy the product meant for the low-valuation segment. This mitigates the cannibalization problem and makes it more likely for the low-valuation segment to get its preferred quality.

Similarly, when firms are less differentiated in the low-valuation segment, stronger competition between the firms makes the cannibalization problem worse, and the low-valuation segment may not gets its preferred quality. When the differentiation between the firms is sufficiently weak in the high-valuation segment, the high-valuation segment is more likely to be better off buying the product meant for it. As the high-valuation segment's incentives to buy the lower-quality product are reduced, the low-valuation segment is more likely to get its preferred quality.

These and other results provide important insights on how firms should carry out quality-based segmentation. A key insight from the standard self-selection models is that the kind of segmentation that marketing textbooks typically recommend—providing each customer segment with its preferred quality—is infeasible because of the cannibalization problem. However, the cannibalization problem may not be the critical determinant of product quality even for a monopolist. In the presence of competition, the issue of cannibalization becomes even less serious because the within-segment competition between firms may be more important than between-segments competition between a single firm's products (cannibalization).

1.3. Related Literature

Product Line Quality in Monopoly. To provide more background on the monopoly self-selection models, the existing self-selection models of Mussa and Rosen (1978), Katz (1984), and Moorthy (1984) are reviewed with the help of the Moorthy (1984) model. In an influential paper, Moorthy (1984) considers a monopolist serving multiple consumer segments differing in their valuations for quality. Higher-valuation segments derive greater marginal utility from quality, and hence are willing to pay a higher price for a given quality product than lower-valuation segments. It is optimal for the monopolist to create multiple products (price-quality bundles) so that higher-valuation consumers buy higher-quality products at higher prices. However, he points out that since consumers self-select the products they purchase, higher-valuation consumers may find that a lower-quality product gives them higher utility than purchasing the product aimed at them and will purchase the lower-quality product. In marketing terms, lower-quality products can potentially cannibalize higher-quality products. Ensuring that this cannibalization problem does not undermine the segmentation scheme is the central task of the firm in Moorthy (1984). He finds that the optimal price-quality bundles are such that only the highest-valuation segment gets its preferred quality. The qualities of products aimed at all other segments are distorted downwards, and are at strictly less than efficient levels. In addition, under some conditions the cannibalization problem forces the firm not to serve some of the lowest-valuation segments. The price of the lowest-quality product is set in such a way that the lowest-valuation consumer gets no rents from purchasing the product. That is, the lowest-valuation segment is indifferent between buying the product targeted to it and not purchasing any product. Prices of all other products are set in such a way that each segment is indifferent between purchasing its product and the product targeted at the next lower-valuation segment. All segments other than the lowest-valuation segment get strictly positive rents from purchasing their products. Mussa and Rosen (1978) derive similar insights in a model with a continuous distribution of consumer valuations.[3]

Product Line Quality in Duopoly. Katz (1984) shows the interdependence between markets for high-quality and low-quality goods. His duopoly model is similar to mine; however, the firms' locations are fixed at the two extremes. Katz's (1984) monopoly results are similar to those of Mussa and Rosen (1978) and Moorthy (1984). In his duopoly analysis he assumes that both firms serve all consumers in both markets, and shows that standard self-selection results arise when consumers self-select. If the self-selection constraints are nonexistent, or if the firms are not differentiated, then the two firms provide efficient qualities. Although he raises the possibility that self-selection constraints may not be binding in a duopoly under some conditions, he does not show this result or identify such conditions. In contrast, this paper shows that even in the presence of self-selection constraints, both efficient and inefficient quality provisions are possible in both monopoly and duopoly. I also identify and characterize the conditions under which both segments get efficient qualities, and I consider incomplete market coverage, which is not analyzed in Katz (1984).

A recent working paper by Rochet and Stole (1998) has independently shown that when consumers' valuations for quality are sufficiently different and firms have symmetric costs, firms provide efficient quality to all consumers. They also more fully characterize the solution when consumers' downward self-selection constraints are binding. They consider a more general horizontal distribution of consumers but allow the segments to vary only in terms of quality valuations. Schmidt-Mohr and Villas-Boas (1997) analyze a similar model and show that the screening solution involves pooling of customer types at the top and at the bottom of the quality-valuation distribution. Villas-Boas and Schmidt-Mohr (1999) also analyze a similar model in the context of credit markets and show that more differentiated firms compete less intensively for the most profitable consumers, and may screen loan applications less intensively. Interestingly, they show that total profits may increase or decrease in the presence of asymmetric information. Armstrong and Vickers (1999) analyze competitive price discrimination and examine the effect of price discrimination on consumer welfare. They show, among other things, that in a model of consumers differing in their valuations for quantity, one of the equilibria involves firms pricing a given quantity at cost plus a constant. However, these papers do not allow for differences between segments in terms of taste preferences and different firm locations in the two markets. Also, my focus is on identifying the conditions under which the cannibalization problem

[3]It is also well known that the problem of a firm managing heterogeneous salespeople, as in Lal and Staelin (1985), also has a similar structure. However, competition is less of an issue in salesforce management.

does not determine the equilibrium price and quality levels.

Product Line Strategy with Exogenous Quality Levels. Gilbert and Matutes (1993) examine competition between the product lines of two spatially differentiated manufacturers. Assuming exogenous quality levels, they examine whether the two firms would specialize to serve one segment each. They show that firms produce full product lines when the differentiation between them is sufficiently large. However, if a firm can move first and commit not to serve a market, specialization may occur as the differentiation between the two firms approaches zero.

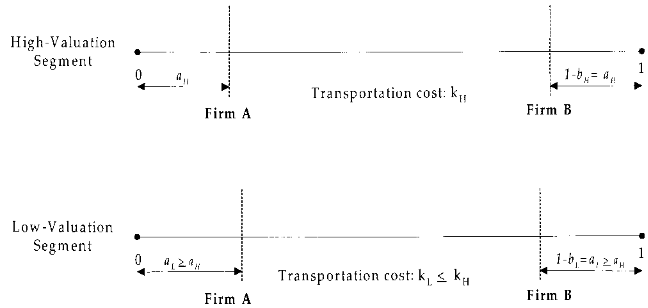
The rest of the paper is organized as follows. Section 2 describes the model. Section 3 presents the analysis for the monopoly case. The analysis of the duopoly case is given in §4. I conclude with a summary and discussion of key results in §5.

2. Model

I develop a model of a market characterized by both quality (vertical) and taste (horizontal) differentiation.

Consumers. There are two consumer segments, a high-valuation segment and a low-valuation segment. The high- and low-valuation segments are denoted by H and L, respectively. There are N_H consumers in the high-valuation segment, and N_L consumers in the low-valuation segment. A consumer in segment i (i = H, L) derives a utility of $\theta_H q$ from using a product of quality q. Here the quality variable, q, is a summary measure of all more-is-better attributes of the product. For example, for a personal computer, the quality variable q can be a composite of the CPU speed, bus size, number of drives, access speed, monitor size, etc. Consumers within each segment are uniformly distributed on a [0, 1] line segment representing a linear market (Hotelling 1929), and incur a transportation cost of k_i per unit length traveled. The horizontal distribution of consumers represents taste differences among consumers. Taste attributes refer to those product attributes for which all consumers are not in agreement. Each consumer may have his/her ideal product—his/her most preferred bundle of these attributes. A consumer's location on the [0, 1] line segment, x, describes the consumer's ideal point. The transportation cost is the disutility that a consumer suffers from using a product other than his/her ideal product and represents the strength of consumers' taste preferences. To model the possibility that higher valuation consumers also have stronger taste preferences, the two segments are allowed to have two different transportation costs. In particular, $k_H \geq k_L$.

Figure 1 **Firms and Consumers**



A consumer with valuation θ_i ($i = H, L$) and location x from the left extreme on the [0, 1] line segment derives a utility of $U(\theta_i, x) = \theta_i q - k_i t - p$, where q is the product quality, p is the retail price of the product, and t is a given product's distance from the consumer's ideal point—i.e., the distance that the consumer has to travel to purchase the product. Each consumer has the option of not purchasing any product on the market. The no-purchase option gives the consumer zero utility.

Firms. In the duopoly, there are two symmetric firms, labeled A and B, competing in the market. Each firm can produce up to one product for each segment. Firm j's ($j = A, B$) product for segment i ($i = H, L$) is referred to by its quality level, q_{ij}. Each firm has a fixed location on a given line segment, which represents consumers' *perceptions* of the taste attributes offered by the firm. The distance between the locations of the two firms represents taste-based differentiation between products of the two firms. As Figure 1 shows, on the line for segment i ($i = H, L$), Firm A is located at a distance a_i and Firm B is located at a distance b_i from the left extreme, with $a_i < ½$ and $b_i = 1 - a_i > ½$.

It is often more difficult to differentiate lower-quality products, which have fewer features and less functionality than higher-quality products. To model this, in the duopoly analysis a_L (and b_L) can be different from a_H (and b_H). In particular, $b_H - a_H \geq b_L - a_L$ in the duopoly analysis. Another way to think about this difference is to recognize that consumers' perceptions of the two firms may be different in the two segments. If the two firms are closely located, there will not be any pure strategy equilibrium in the price subgame (D'Aspremont et al. 1979). I assume that firms are sufficiently separated so that there is a pure strategy equilibrium in the price subgame.[4] In monopoly we consider only Firm A, with qualities q_i and prices p_i ($i = H, L$). Because product differentiation issues don't matter in monopoly, we set $a_H = a_L$ for simplicity.

While firms may target a given segment with a specific product (quality), they cannot identify the location or the valuation of a given consumer. The production cost functions are identical for both firms. The marginal cost of producing a product of quality q is $c(q) = \gamma q^2/2$. For ease of exposition, the cost of producing product q_{ij}, $c(q_{ij})$, is often denoted c_{ij}.

It is easy to see that when both products are priced at the marginal costs, Segment i's ($i = H, L$) optimal quality choice is $q_i^e = \theta_i/\gamma$. I refer to this quality level as Segment i's preferred quality level or efficient quality level.

There are three stages in the game. In Stage 1, each firm chooses its quality level for each segment. In Stage 2, each firm chooses the prices for its products. In Stage 3, consumers decide whether or not to purchase any product, and choose which product to purchase if they decide to purchase a product.

3. Monopoly Analysis

Consider a case in which Firm A is the only firm in the market. Recall that I set $a_L = a_H$ in this section. All other aspects of the model are the same as described in §2. Firm A offers two products, one directed to the high-valuation segment with quality q_H^M at price p_H^M, and the other directed to the low-valuation segment with quality q_L^M and price p_L^M.

Depending on the relative values of θ_i and k_i, the firm may or may not find it optimal to serve all the consumers in a given segment. Here I focus on two possibilities: the firm leaving out some consumers in both markets, and the firm covering both markets completely. I begin with the case of the firm serving both markets completely because it replicates the standard results and helps us understand results of the other case.

3.1. Full Coverage of Both Market Segments

When both market segments are fully served, the firm's profit function is given by $\Pi = N_H(p_H^M - c_H) + N_L(p_L^M - c_L)$. The firm chooses prices and qualities that maximize this profit function subject to self-selection and voluntary participation constraints for each segment. The solution to this problem is given by

$$p_L^{M*} = \theta_L q_L^{M*} - (1 - a_L)k_L,$$

$$p_H^{M*} = \theta_H(q_H^{M*} - q_L^{M*}) + \theta_L q_L^{M*} - (1 - a_L)k_L \quad \text{and} \tag{1}$$

$$q_L^{M*} = \frac{\theta_L}{\gamma} - \frac{N_H(\theta_H - \theta_L)}{N_L\gamma}, \qquad q_H^{M*} = \frac{\theta_H}{\gamma}. \tag{2}$$

It is easy to see that, like the standard self-selection solution, the above solution suggests that the high-valuation segment should be made just indifferent between buying q_H^{M*} and buying q_L^{M*}. As in the standard self-selection model, the high-valuation segment gets its preferred (efficient) quality, but the quality intended for the low-valuation segment is distorted downward. Finally, when the relative valuation and size of the low-valuation segment are small, ($N_L\theta_L < N_H(\theta_H - \theta_L)$), the firm is better off serving only the high-valuation segment.

I now consider the case in which the firm does not serve either market completely.

3.2. Incomplete Market Coverage

There are two possibilities—with the inactive consumers being on both sides of the firm or on only one (right) side of the firm. The results of the two incomplete market coverage cases turn out to be very sim-

[4]Specific conditions are derived in the Appendix C.

ilar. Therefore, I only discuss the case in which the inactive consumers are to the right of the firm.

The location of the high-valuation consumer indifferent between purchasing the product targeted to him/her and not purchasing any product, x_H^M is given by

$$x_H^M = \frac{a_H k_H - p_H^M + \theta_H q_H^M}{k_H}.$$

Similarly, the location of the low-valuation consumer indifferent between purchasing the low-quality product and not purchasing any product is given by

$$x_L^M = \frac{a_L k_L - p_L^M + \theta_L q_L^M}{k_L}.$$

Note that the difference in the high-valuation consumers' utility of purchasing the high-quality product and purchasing the low-quality product does not depend on x. That is, either the self-selection constraint is satisfied for all high-valuation consumers or it is not satisfied for any high-valuation consumers.

It is important to note an interesting difference between the current case of incomplete market on the one hand and the case of full market coverage, as well as existing self-selection models, on the other hand. In the latter case, the demand from a given segment does not change so long as that segment's self-selection and voluntary participation constraints are satisfied. In the present case, the demand from each segment changes with changes in prices. As p_L^M or p_H^M decreases, the firm can increase its demand from the low- or high-valuation segment. As a result, the firm has greater incentives to reduce prices here than in the previous case of full market coverage. Proposition 1 characterizes the solution for this case.

Proposition 1. *When $-2a_L\gamma(k_H - k_L) + (\theta_H - \theta_L)(\theta_H - 3\theta_L) \geq 0$, each segment gets its preferred product quality; otherwise, the low-valuation segment gets a quality lower than its preferred quality, and the high-valuation segment gets its preferred quality.*

Proofs. All proofs are in the appendix.

Proposition 1 shows that under certain conditions the monopolist's cannibalization problem does not affect its quality choices. The downward-sloping demand functions mentioned earlier play an important role here. Consider what would happen in the present case with the standard solution given in (1)–(2). Suppose the firm chooses prices such that the high-valuation consumers are just indifferent between buying high-quality and buying low-quality products. Now, by reducing the price of the high-quality product slightly, the firm can increase its demand from the high-valuation segment without violating any constraints. If the firm can increase its profits by increasing the demand of the high-quality product and giving more surplus to some of the high-valuation consumers in the process, the standard self-selection solution will not work. This is precisely what happens when the above condition is satisfied.

The above condition is more likely to be satisfied when segment heterogeneity in quality valuation ($\theta_H - \theta_L$) is much greater than that in taste preferences ($k_H - k_L$). Higher transportation costs are equivalent to lower price sensitivity and reduce a segment's response to price changes

$$\left(\frac{\partial x_i^M}{\partial p_i^M} = -\frac{1}{k_i} < 0\right).$$

With $k_H > k_L$, the high-valuation segment responds less to price changes than the low-valuation consumers. Therefore, higher transportation costs in the high-valuation segment reduce the firm's incentives to reduce prices in that segment compared with the other segment. On the other hand, when the $\theta_H - \theta_L$ difference is sufficiently high, the firm would like to get greater demand from the high-valuation segment than from the low-valuation segment, and its incentive to cover more of the high-valuation segment is increased. When the $\theta_H - \theta_L$ difference relative to the $k_H - k_L$ difference is sufficiently high, the self-selection constraint is not binding for any high-valuation consumer who purchases the product. At this stage, the firm does not have to use quality distortions to make purchasing the low-quality product unattractive to high-valuation consumers. In other words, even if we determine the optimal prices and qualities by ignoring the self-selection constraint, these prices and qualities satisfy the self-selection constraints for all active consumers.

Note that Proposition 1 is valid only for those parameter values for which the firm finds it optimal not to serve either segment fully. This is true if $\theta_i^2 < 2(2 - a_L)\gamma k_i$ for $i = H, L$. That is, for each segment taste preferences (k_i) should be relatively high in comparison to quality valuation (θ_i). If θ_H is sufficiently higher than k_H, specifically, if $\theta_H^2 \geq 2(2 - a_H)\gamma k_H$, the firm finds it optimal to serve the high-valuation segment fully. In that case, Proposition 1 results are not valid. In fact, it can be shown that if the high-valuation segment is fully served, the optimal strategy for the firm is to provide inefficient quality to the low-valuation segment. When $\theta_L^2 \geq 2(2 - a_L)\gamma k_L$ in addition to $\theta_H^2 \geq 2(2 - a_H)\gamma k_H$, both segments are fully covered and the standard self-selection results apply.

In summary, the monopoly analysis recovers the standard self-selection results and derives new insights into the relative effects of taste preferences and quality valuation in determining the price-quality profile for the firm. I now consider the duopoly case.

4. Duopoly Analysis

As in monopoly, I consider two possibilities—both segments are fully served by the two firms, and neither segment is fully served by the two firms. Recall that each firm can have different locations in the two segments, i.e., $a_H \leq a_L$. All other aspects of the model are identical to the monopoly model.

4.1. Both Market Segments Fully Served

In this case, each firm chooses its prices (p_{Hj}, p_{Lj}, ($j = A, B$)) and qualities (q_{Hj}, q_{Lj}, ($j = A, B$)) to maximize its own profits. For a given set of prices and qualities, let the high-valuation consumer indifferent between buying q_{HA} and q_{HB} be at the location x_H. Then x_H is given by

$$\theta_H q_{HA} - |x_H - a_H| k_H - p_{HA} = \theta_H q_{HB} - |b_H - x_H| k_H - p_{HB}.$$

Similarly, the location of the low-valuation-segment consumer indifferent between buying Firm A's and Firm B's products, x_L, is given by

$$\theta_L q_{LA} - |x_L - a_L| k_L - p_{LA} = \theta_L q_{LB} - |b_L - x_L| k_L - p_{LB}.$$

Note that so long as $b_H \geq x_H \geq a_H$, any high-valuation consumer at location $x < x_H$ prefers buying q_{HA} to buying q_{HB}, and any high-valuation consumer with location $x > x_H$ prefers buying q_{HB} to buying q_{HA}. A similar preference structure holds for the low-valuation consumers. If all the self-selection constraints are satisfied, the demand for Firm A's product q_{iA}, d_{iA}, is given by $d_{iA} = N_i x_i$, and the demand for Firm B's product q_{iB}, d_{iB}, is given by $d_{iB} = N_i(1 - x_i)$.

I first solve a more general problem, viz., unconstrained optimization of each firm's objective function. Simultaneously solving the two firms' unconstrained problems gives the following solution:

$$p_{iA}^{D*} = \frac{1}{6\gamma}[2(2 + a_i + b_i)\gamma k_i + 3\theta_i^2],$$

$$p_{iB}^{D*} = \frac{1}{6\gamma}[2(4 - a_i - b_i)\gamma k_i + 3\theta_i^2], \qquad q_{ij}^{D*} = \frac{\theta_i}{\gamma}, \quad (3)$$

where $i = H, L$ and $j = A, B$.

Equation (3) shows that when firms are not restricted by self-selection constraints, in equilibrium they provide each segment with that segment's preferred product quality. This is not surprising because it is the existence of self-selection constraints that induces firms in the standard self-selection models to distort the quality to the low-valuation segment. Katz (1984) also shows a similar result. However, what is surprising is that the above solution satisfies self-selection constraints under some conditions, and therefore becomes the equilibrium outcome even when each firm faces self-selection constraints.

PROPOSITION 2. *When $-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2 \geq 0$, each firm provides each segment with that segment's preferred quality in equilibrium; otherwise, the high-valuation segment gets its preferred quality, and the low-valuation segment gets lower than its preferred quality.*

As in the incomplete coverage monopoly case, each firm faces a downward-sloping demand curve rather than a step-function demand present in the standard self-selection model. The key difference is that in a duopoly, a firm's demand from a segment is affected not only by its own prices but also by the competitor's prices. Therefore, each firm has a greater incentive to reduce prices here than in the earlier case, and the cannibalization problem is less likely to determine the equilibrium quality choices.

Here also, if the two segments are more different in their quality valuations than in their taste preference, i.e., $(\theta_H - \theta_L)$ is significantly greater than $(k_H - k_L)$, the firms are more likely to provide each segment with its preferred quality levels. The intuition is similar to the previous case. With weaker taste preference, the low-valuation segment is more price sensitive. This increases the intensity of competition between the two firms and reduces prices more in the low-valuation segment than in the high-valuation segment. As prices in the low-valuation segment fall, the high-valuation consumers have greater incentives to purchase the lower-quality product, and self-selection issues become critical again. We might expect that more intense competition in the low-valuation segment would make it more likely that this segment gets its preferred quality. It turns out that the reverse is true.

The firms' quality choices are also affected by their locations and the extent of differentiation between them. If the two firms are more differentiated in the high-valuation segment than in the low-valuation segment, they are less likely to provide efficient quality to the low-valuation segment. In this case, the high-valuation consumer farther away from the firm incurs a greater transportation cost in buying the high-quality product than the low-quality product. This again increases the high-valuation consumers' incentives to purchase the low-quality product, making self-selection issues more serious. Here also, one might expect that reduced differentiation between the two firms in the low-valuation segment would give them greater incentives to provide efficient quality. However, reduced differentiation in the low-valuation market can make the cannibalization problem more serious, and therefore, makes it less likely that the low-valuation segment will get the efficient quality.

The traditional monopoly self-selection solution prescribes that the high-valuation consumers be given just enough surplus to make them indifferent between buying the high-quality product intended for them and buying the lower-quality product. Proposition 2 shows that when the above condition is satisfied, such a pricing policy is suboptimal in a duopoly. In particular, due to competitive pressures, a firm's ability to extract more surplus out of the high-valuation segment is limited. High-valuation consumers can not only buy the low-quality product of the same firm, but also the high-quality product of the competitor. Therefore, if either firm tries to make the high-valuation consumers just indifferent between purchasing its high-quality and low-quality products, its price for the high-quality product will end up being too high, and it will lose market share in the high-valuation segment to the competing firm. Similarly, neither firm can extract all the surplus from any low-valuation consumer, because doing so will result in a loss of market share to the competing firm.

Interestingly, the price that the high-valuation segment pays depends only on that segment's characteristics and not on the characteristics of the low-valuation segment, as in the standard self-selection models. Because the high-valuation segment has stronger taste preferences, the equilibrium price-cost margins are higher for the higher-quality product. This is in contrast to Armstrong and Vickers (1999) and Rochet and Stole (1999), who assume that both segments have the same strength of taste preferences and find that the price-cost margins are the same for all items in the product line in the efficient solution.

As in the Hotelling (1929) model, each firm may have incentives to cut prices and attract consumers from the other firm's ''territory'' (D'Aspremont et al. 1979). In Appendix C, I derive conditions under which such strategies are not profitable. Intuitively, these conditions require that the two firms be sufficiently separated so that neither firm can profitably raid the other firm's territory.

In the preceding analysis, each firm's strategy of serving both segments was taken as a given. I now consider whether a firm has the incentive not to serve either segment.

PROPOSITION 3. *When* $-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2 \geq 0$, *it is not optimal for either firm not to serve the low-valuation segment.*

Proposition 3 shows that in a competitive environment it may not be optimal to serve only the high-valuation segment due to cannibalization concerns. This is in contrast to the standard self-selection result of the firm not serving the low-valuation segment when the relative valuation and size of the low-valuation segment are small. The reason for Proposition 3 results is that the rents accruing to the high-valuation segment depend on the competitor's high-quality product's price and quality and the degree of competition between the two firms; they do not depend on the relative size and valuation of the low-valuation segment. In other words, not serving the low-valuation segment does not affect the profitability of serving the high-valuation segment. This result is related to Gilbert's and Matutes' (1993) result on differentiated firms choosing to offer full product lines. However, as noted before, the Proposition 3 result is based on endogenous quality choices by the two firms. Note that, in practice, firms may not have full product lines due to a variety of other reasons (see Kotler 2000).

Interestingly, in the standard self-selection analysis a firm is less likely to serve the lower-valuation segment as the difference between the two segments' quality valuations increases. Proposition 3 shows that a firm is more likely to serve both segments as the difference between quality valuations of the two segments increases.

4.2. Incomplete Coverage of Markets

As in the monopoly case, there are several possible configurations by which markets may not be fully served—inactive consumers being near the two extremes, or in the middle, or near the two extremes as well as in the middle of the line segments. If the inactive consumers are only in the middle of the line segments, then each firm enjoys a monopoly and the analysis is nearly identical to the analysis of the monopoly incomplete coverage case discussed in §3.1. Therefore, I do not repeat it here but note two important differences. With differences in a_H and a_L, the condition noted in Proposition 1 changes to $-2k_H\gamma(a_L - a_H) - 2a_L\gamma(k_H - k_L) + (\theta_H - \theta_L)(\theta_H - 3\theta_L) \geq 0$. The other difference is that full coverage of the market will be obtained when $\theta_i^2 > (2 - a_i)\gamma k_i$.

A more interesting case is the one in which the inactive consumers are only at the extremes of the line segments. Here the middle consumers are served by the firms, but some consumers at the two extremes may not be. This case is more likely to arise when the firms are closer to the midpoints than they are to the extremes.

The demand from Segment i can be characterized by three locations, x_{i1}, x_{i2}, and x_{i3} ($i = H, L$) defined as follows on each line segment

$$x_{1i} = \frac{a_i k_i + p_{iA} - \theta_i q_{iA}}{k_i},$$

$$x_{2i} = \frac{k_i - (p_{iA} - p_{iB}) + \theta_i(q_{iA} - q_{iB})}{k_i}, \quad \text{and}$$

$$x_{3i} = \frac{b_i k_i - p_{iB} + \theta_H q_{iB}}{k_i}.$$

It is easy to seee that x_{1i} defines the consumer indifferent between buying Firm A's product and not buying any product; x_{2i} defines the consumer indifferent between buying Firm A's product and buying Firm B's product; and x_{3i} defines the consumers indifferent between buying Firm B's product and not buying any product. Firm A's demand from Segment i is $(x_{2i} - x_{1i})$, and Firm B's demand from Segment i is $(x_{3i} - x_{2i})$.

Proposition 4. *When $\gamma k_i(2 + 6a_i) - 3\theta_i^2 > 0$, ($i = H$, L), both markets are served incompletely in the equilibrium. In the incomplete coverage equilibrium, each segment gets its preferred quality from each firm if $(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L] > 0$; otherwise, the high-valuation segment gets its preferred quality and the low-valuation segment gets lower than its preferred quality.*

The first part of Proposition 4 identifies conditions for the two segments to be incompletely served in equilibrium. For these conditions to be satisfied, a segment's valuation shouldn't be too high compared to the transportation costs incurred by consumers in the segment. The second part of Proposition 4 specifies when we can expect firms to provide the two segments with their preferred quality levels. In particular, if the two segments differ more in terms of their valuations and less in terms of their other characteristics—the strength of their taste preferences and their perceptions of the two firms—equilibrium outcome would involve both segments getting their preferred qualities from the two competitors in the equilibrium. If the segments differ less in terms of their valuations and more in terms of their taste preferences and perceptions of the two firms, then the standard self-selection results of firms distorting the quality for the low-valuations segment arise. The intuition for this result is similar to that for the earlier cases, and is not repeated here.

Similar results arise for the other incomplete market configuration in which the inactive consumers are in the middle as well as at the extremes.

As in the case of full market coverage, each firm may have incentives to cut prices to attract consumers from the other firm's "territory" (D'Aspremont et al. 1979). Appendix C gives strong sufficient conditions for such deviations to be unprofitable. Intuitively, these conditions require that the two firms be sufficiently separated.

4.3. Consumer Welfare Implications

It is well known that in the standard self-selection results, the low-valuation consumers get no rents and the high-valuation consumers get just enough rents to make them indifferent between purchasing the high-quality product targeted to them and purchasing low-quality products targeted to the other segment. Similar results arise in my model when the low-valuation segment gets lower than its preferred quality—all high-valuation consumers get just enough rents so that they purchase the product targeted to them. Among the low-valuation consumers, only the last consumer to purchase the low-quality product in the incomplete coverage cases gets no rents. All other low-valuation consumers get rents.

A more interesting case arises when both segments get their preferred qualities. Consider, for example, the duopoly equilibrium described in §4.1, in which both consumer segments are fully served and both segments get their preferred qualities.

The utility that a given Firm A consumer in the low-valuation segment derives, U_L^{DA}, is as follows:

$$U_i^{DA} = \begin{cases} \dfrac{3\theta_i^2 - 2\gamma k_i(2 - 2a_i + b_i + 3x)}{6\gamma} & \text{for } x \geq a_i \\ \dfrac{3\theta_i^2 - 2\gamma k_i(2 + 4a_i + b_i - 3x)}{6\gamma} & \text{for } x < a_i. \end{cases} \tag{4}$$

The utilities of Firm B consumers are symmetrically given.

It is easy to see that all consumers in both segments get strictly positive rents. Interestingly, both segments get symmetric treatment in this equilibrium. The high-valuation segment's equilibrium surplus does not depend on the low-valuation segment's valuation or the quality of the product targeted to the low-valuation segment, as is the case in the standard self-selection results. The difference in rents accruing to the two segments is simply a function of the difference in their characteristics θ, k, and firm locations.

Proposition 5 summarizes this discussion.

PROPOSITION 5. *In the duopoly equilibrium in which both segments are fully served and* $-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2 \geq 0$, *all the consumers in both segments get positive rents. Rents accruing to the two segments are symmetric.*

5. Summary and Discussion

Both efficient and inefficient quality provisions can arise in monopoly as well as in duopoly. I have identified and characterized conditions under which the cannibalization problem does not determine the equilibrium quality choices so that both segments get efficient quality. Under these conditions, the equilibrium quality levels in a second-degree price discrimination situation resemble the third-degree price discrimination solution. These conditions in general depend on three factors: (1) the trade-off between quality and taste preferences (θ_i and k_i) in the consumers' utility function, (2) the differences between the segments (θ_H vs. θ_L and k_H vs. k_L), and (3) the nature of competition between the two firms in a duopoly.

The trade-off between quality and taste preference determines whether the entire market is covered. If the taste preferences are relatively stronger, some consumers will not purchase any product. With incomplete market coverage, even a monopolist may find it optimal to provide efficient quality to each segment. On the other hand, if the market is fully covered, a monopolist will provide efficient quality only to the high-valuation segment. In a duopoly, the efficient quality provision can arise with either type of market coverage.

Firms are more likely to provide each segment with its preferred quality when the two segments differ more in terms of their quality valuations and less in terms of taste preferences. As the taste preference of the low-valuation segment becomes weaker, prices of the products targeted to that segment fall, and the high-valuation consumers have greater incentives to purchase the products targeted to the low-valuation segment. This worsens the cannibalization problem.

In a duopoly, if the firms are significantly more differentiated in the high-valuation segment than in the low-valuation segment, then lower prices for the products targeted to the low-valuation segment can make the cannibalization problem more serious. In such a case, the low-valuation segment is less likely to get its preferred quality. Thus, less differentiation in the low-valuation market makes it less likely that consumers in this market get their preferred quality.

A firm may choose not to serve all the consumer segments for a number of reasons. For example, Kotler (2000) lists excessive design and engineering costs, inventory carrying costs, etc., as possible reasons for not having a full product line. However, cannibalization should not be a reason for dropping a product in a competitive situation when the above conditions are met.

This analysis also has interesting implications for pricing a product line. When the above conditions are satisfied, making the high-valuation consumers indifferent between choosing the high-quality product and the low-quality product is no longer a critical issue. Instead, the pricing policy for a segment depends on the characteristics of the consumers in that segment and degree of differentiation between the firms in that segment. In duopoly with full market coverage, the per-unit margins depend on the strength of taste preferences. When both segments have identical taste preferences, the per-unit margins are the same for both products. When the high-valuation segment has stronger taste preferences, the per-unit margins are higher for higher-quality products. When the markets are incompletely covered, the per-unit margins in duopoly are higher for higher-quality products.

Limitations and Directions for Future Research. I have not fully characterized the inefficient solution. Interested readers are referred to Rochet and Stole (1998) and Schmidt-Mohr and Villas-Boas (1997) for details on this solution, including results on bunching. I have used Hotelling's (1929) linear market to model competition between the two firms. In practice, multiple firms compete in each segment. Therefore, it may be interesting to see if these results can be replicated in a model with a different type of competition and with multiple firms. We also observe firms occupying multiple locations in a given segment. For example, Ford Motor Company makes cars under Ford and Mercury brand names. Similarly, General Motors makes cars under several brand names. More research is needed to study how a firm's price and quality strategies change when they compete in a segment under multiple brand names, occupying multiple locations.[5]

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Appendix: A. Monopoly Analysis

Proof for the Full Coverage Case. When the firm serves the entire low-valuation segment, it does not gain anything by offering rents to the low-valuation consumer at $x = 1$. Similarly, the firm does not gain anything by giving more rents to the high-valuation

[5]Previous versions of the paper were entitled ''Quality-Segmentation in a Spatial Duopoly.''

consumers than the minimum necessary for the self-selection constraint to hold. Therefore,

$$p_L^{M*} = \theta_L q_L^M - (1 - a_L)k_L, \quad \text{and}$$

$$p_H^{M*} = \theta_H q_H^M + q_L^M(\theta_H - \theta_L) - (1 - a_L)k_L - (a_L - a_H)k_H.$$

The firm's profit function is

$$\Pi_A^M = N_L\left(p_L^{M*} - \frac{\gamma(q_L^M)^2}{2}\right) + N_H\left[p_H^{M*} - \frac{\gamma(q_H^M)^2}{2}\right].$$

We get the optimal qualities from solving the first-order conditions from the above profit function. The solution given in (1)–(2) is obtained by setting $a_H = a_L$. □

Proof for the Incomplete Coverage Case (Proposition 1). I find the optimal solution to the firm's problem by ignoring the self-selection constraints and then check whether self-selection constraints are satisfied by the solution to the unconstrained problem. If the unconstrained solution satisfies all the self-selection constraints, then it is also the solution to the constrained problem. If not, the constrained problem has a different solution. The firm's profit function in the unconstrained case is given by $\Pi^M = N_H(p_H^M - c_H)x_H^M + N_L(p_L^M - c_L)x_L^M$. Solving the first-order conditions for prices, I get

$$p_L^{M**} = \frac{a_L + \theta_L q_L^M + c_L}{2} \quad \text{and} \quad p_H^{M**} = \frac{a_H + \theta_H q_H^M + c_H}{2}.$$

After substituting these prices and solving the first-order conditions for q_H^M and q_L^M, I get $q_i^{M**} = \theta_i/\gamma$.

Now we need to ensure that this solution satisfies the self-selection constraints. When $x > a_L = a_H$, the self-selection constraint of the high-valuation consumer located at x is simplified as

$$\frac{-2\gamma a_L(k_H - k_L) + (\theta_H - 3\theta_L)(\theta_H - \theta_L)}{4\gamma} > 0. \tag{A1}$$

The self-selection constraint for the high-valuation consumer located at $x < a_L = a_H$ is

$$\frac{-2\gamma a_L(k_H - k_L) + (\theta_H - 3\theta_L)(\theta_H - \theta_L)}{4\gamma} > 0. \tag{A2}$$

It is easy to see that (A2) is identical to (A1). When (A1) is satisfied, the unconstrained solution satisfies the self-selection constraints for all high-valuation consumers and is the optimal solution for the constrained problem. □

For low-valuation consumers at x, both $x > a_L = a_H$ and $x \le a_L = a_H$, the self-selection constraint is simplified as

$$\frac{-2\gamma a_L(k_H - k_L) + (\theta_H - 3\theta_L)(\theta_H - \theta_L)}{4\gamma} + \frac{\theta_H^2 - \theta_L^2 + 2a_L\gamma(k_H - k_L)}{2\gamma}. \tag{A3}$$

It is easy to see that the second term in (A3) is always positive, therefore, the above constraint is satisfied when

$$\frac{-2\gamma a_L(k_H - k_L) + (\theta_H - 3\theta_L)(\theta_H - \theta_L)}{4\gamma} > 0$$

See Equation (A1). It can be shown that the upward self-selection constraints are also satisfied.

When the condition in (A1) is not satisfied, the self-selection constraints are binding in the equilibrium, and the low-valuation segment will get lower than its preferred quality (Moorthy 1984). □

Note that with the equilibrium solution under consideration,

$$1 - x_i^{M**} = \frac{2\gamma k_i[2 - a_i] - \theta_i^2}{4\gamma k_i}.$$

Therefore, when $2\gamma k_i[2 - a_i] \le \theta_i^2$, the firm covers the segment i completely. □

B. Duopoly Model

Solution for the Full-Coverage Case (Propositions 2 and 3)

As before, I solve the unconstrained maximization problems for the two firms and then check when the constraints are satisfied by the unconstrained solution.

Solving the first-order conditions for prices for the two firms, we get

$$p_{iA}^{D*} = \frac{2c_{iA} + c_{iB} + (2 + a_i + b_i)k_i + \theta_i(q_{iA} - q_{iB})}{3} \quad \text{and}$$

$$p_{iB}^{D*} = \frac{2c_{iB} + c_{iA} + (4 - a_i - b_i)k_i + \theta_i(q_{iB} - q_{iA})}{3}.$$

I solve the first-order conditions for the quality levels after substituting for the prices. Simultaneously solving the first-order conditions for qualities, we get $q_{ij}^{D*} = \theta_i/\gamma$ ($i = H, L$ and $j = A, B$). □

Now I show that the above solution satisfies the self-selection constraints for the high-valuation consumers. Depending on the location of a high-valuation consumer, his/her self-selection constraint may take one of three forms.

For a high-valuation consumer located at x, such that $x > a_L \ge a_H$, the self-selection constraint is simplified as

$$\frac{-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2}{2\gamma}. \tag{A4}$$

Therefore, when $-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2 \ge 0$, the self-selection constraints of the high-valuation consumers located to the right of a_H and a_L are satisfied by the solution to the unconstrained problem.

For a high-valuation consumer located at x, such that $x \le a_H \le a_L$, the self-selection constraint is simplified as

$$\frac{-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2}{2\gamma} + 2(a_L - a_H)k_H.$$

With $a_L \ge a_H$, the above expression is always weakly positive when $-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2 \ge 0$. Therefore, the self-selection constraint for the high-valuation consumers located to the left of a_H and a_L is satisfied by the solution to the unconstrained problem when $-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2 \geq 0$.

Finally, for a high-valuation consumer located at x such that $x > a_H$ and $x \leq a_L$, the self-selection constraint is simplified as

$$\frac{1}{6}[(-4 + 6a_L + 4a_H - 2b_H - 12x)k_H + (4 + 2a_L + 2b_L)k_L - 2\gamma(q_{HA}^2 - q_{LA}^2) - \gamma(q_{HB}^2 - q_{LB}^2) + 2\theta_H(2q_{HA} + q_{HB} - 3q_{LA}) + 2\theta_L(q_{LA} - q_{LB})].$$

The above expression is decreasing in x. Therefore, the constraint is strongest at the highest value of x in the relevant range, i.e., at $x = a_L$. At $x = a_L$, the constraint is identical to the expression in (A4) (the self-selection constraint for high-valuation consumers located to the right of a_H and a_L), and is weakly positive when $-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2 \geq 0$.

I now consider the low-valuation consumers' self-selection constraints. For a low-valuation consumer located to the right of a_H and a_L, the self-selection constraint is simplified as

$$\frac{-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2}{2\gamma} + (2 + a_L - a_H)k_H - (2 - a_L + a_H)k_L,$$

which is weakly positive when $-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2 \geq 0$.

For any low-valuation consumer located to the left of a_H and a_L, the self-selection constraint is simplified as

$$\frac{-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2}{2\gamma} + (2 + a_L - a_H)(k_H - k_L),$$

which is weakly positive when $-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2 \geq 0$.

Finally, for a low-valuation consumer located between a_H and a_L, the self-selection constraint is simplified as

$$\frac{2\gamma[k_H - (1 + a_L + a_H - 2x)k_L] + (\theta_H - \theta_L)^2}{2\gamma},$$

which is increasing in x. Therefore, the lowest value of the above expression occurs at the lowest value of x in this range, i.e., at $x = a_H$. At $x = a_H$, the expression simplifies to

$$\frac{2\gamma[k_H - (1 + a_L - a_H)k_L] + (\theta_H - \theta_L)^2}{2\gamma},$$

which is nonnegative when $-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2 \geq 0$.

Thus, when $-2\gamma[(1 + a_L - a_H)k_H - k_L] + (\theta_H - \theta_L)^2 \geq 0$, the low-valuation consumers' self-selection constraints are satisfied. It is easy to see that low-valuation consumers to the left of x_H and x_L are better off buying the high-quality product of Firm A than buying the high-quality product of Firm B. It can be shown that the upward self-selection constraints are also satisfied. When the self-selection constraints are not satisfied by the unconstrained solution, the low-valuation segment gets distorted quality (Moorthy 1984). □

To prove Proposition 3, I need to show that neither firm is better off by unilaterally deviating from offering two products to offering only one product. A deviation that targets only the low-valuation segment is clearly dominated. Therefore, I now consider a case in which Firm A serves only the high-valuation segment and Firm B serves both segments. Here there are two possibilites: (1) Firm B serves the entire low-valuation segment, or (2) Firm B serves only a part of the low-valuation segment.

In Case (1) it can be shown that if the condition in Proposition 2 is satisfied, then both offer efficient qualities, and Firm A's profit is $(1/18)(2 + a_H + b_H)^2k_HN_H$, which is strictly less than $(1/18)[(2 + a_H + b_H)^2k_HN_H + (2 + a_L + b_L)^2k_LN_L]$, Firm A's profit when it serves both segments in the equilibrium characterized by Proposition 2. Now consider Case 2. Suppose Firm B covers the $[\ddot{x}_L, 1]$ interval on the low-valuation segment and the $[\ddot{x}_H, 1]$ interval on the high-valuation segment. If $\ddot{x}_L > a_L$, Firm A can choose a price and a quality for the low-valuation segment such that the low-valuation consumer at $x = a_L$ gets a small positive utility ϵ. In the high-valuation segment, the consumers at $\ddot{x}_H$ get the lowest rents. Therefore, the high-valuation consumers at $x = a_L$ get at least $k_H(\ddot{x}_H - a_L) > \epsilon$ utility from buying Firm A's high-quality product, and will continue to buy the high-quality product from Firm A. Thus, Firm A can generate additional profits by introducing a distinct product for the low-valuation segment. Similar logic works if $\ddot{x}_L \leq a_L$. Here Firm A can make additional profits by serving the low-valuation consumers at a_L by choosing a price and a quality such that these consumers get a slightly higher utility from purchasing Firm A's product, than purchasing Firm B's product. Given that Firm A's high-valuation consumers are not purchasing Firm B's low-quality product, they will continue to purchase Firm A's high-quality product. This way, Firm A can make a higher profit by serving the low-valuation segment.

Using similar arguments, it can be shown that Firm A serving only one segment (e.g., high valuation) and Firm B serving the other segment (low valuation) is not an equilibrium. □

Incomplete Market Coverage (Proposition 4)

As before, I solve the unconstrained maximization problems for the two firms and then check that the unconstrained solution satisfies the self-selection constraints when

$$(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L] > 0. \tag{A5}$$

Simultaneously solving the first-order conditions for prices, we get

$$p_{iA}^{D**} = \frac{1}{3}[2c_{iA} + c_{iB} + 3k_i + \theta_i(q_{iA} - q_{iB})] \quad \text{and}$$

$$p_{iB}^{D**} = \frac{1}{3}[2c_{iB} + c_{iA} + 3k_i + \theta_i(q_{iB} - q_{iA})],$$

where $i = H, L$, and $j = A, B$; the superscript ** denotes the equilibrium values in the present case. After substituting these values in the firms' profit functions and solving for their first-order conditions for qualities, we get

$$q_{ij}^{D**} = \frac{\theta_i}{\gamma} \quad \text{for } i = H, L \quad \text{and} \quad j = A, B. \quad \square$$

I now show that the self-selection constraints are satisfied by the above unconstrained solution when $(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L] > 0$. As in the previous case, the self-selection constraint for the high-valuation consumers can take one of three forms depending on the location of a given consumer.

For high-valuation consumers located to the right of a_H and a_L, the self-selection constraint is simplified as

$$\frac{(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L]}{10\gamma},$$

which is nonnegative when $(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L] > 0$.

For a high-valuation consumer located to the left of both a_H and a_L, the self-selection constraint is simplified as

$$\frac{(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L]}{10\gamma} + 2(a_L - a_H)k_H,$$

which is nonnegative when $(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L] > 0$.

For a high-valuation consumer located between a_H and a_L, the self-selection constraint is simplified as

$$\frac{(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H - 10x)k_H - (1 - 2a_L)k_L]}{10\gamma},$$

which is decreasing in x.

Therefore, this constraint is strongest at $x = a_L$. At $x = a_L$, the constraint is

$$\frac{(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L]}{10\gamma},$$

which is nonnegative when $(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L] > 0$.

Similarly, the self-selection constraint for the low-valuation consumers can take one of the following three forms, depending on the location of a given consumer.

For a low-valuation consumer located to the right of a_H and a_L, the self-selection constraint is simplified as

$$\frac{(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L]}{10\gamma} + \frac{2(\theta_H^2 - \theta_L^2) + \gamma[(2 + 5a_L - 9a_H)k_H - (2 - 9a_L + 5a_H)k_L]}{5\gamma}.$$

Since the second term in the above expression is always positive for $a_L \geq a_H$ and $k_H \geq k_L$, the above expression is nonnegative when $(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L] > 0$.

For a low-valuation consumer located to the left of a_H and a_L, the self-selection constraint is simplified as

$$\frac{(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L]}{10\gamma} + \frac{2(\theta_H^2 - \theta_L^2) + \gamma[(2 + a_L - 5a_H)(k_H - k_L) + 4(a_L - a_H)k_L]}{5\gamma}.$$

The second term in the above expression is positive for $a_L \geq a_H$ and $k_H \geq k_L$, and the above expression is nonnegative when $(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L] > 0$.

Finally, for low-valuation consumers located between a_L and a_H,

$$\{(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 3a_L + 5a_H - 10x)k_L + (1 - 2a_H)k_H]\}/10\gamma,$$

which is increasing in x. Therefore, the constraint is strongest when $x = a_H$. At $x = a_H$, the constraint is

$$\frac{(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 3a_L - 5a_H)k_L + (1 - 2a_H)k_H]}{10\gamma}.$$

It can be shown that the above expression is nonnegative when $(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L] > 0$. $\square$

It can be shown that the upward self-selection constraints are also satisfied. For the market coverage to be incomplete, it has to be true that $x_{1i} > 0$ and $x_{3i} < 1$. With the solution under consideration,

$$x_{1i} = 1 - x_{3i} = \frac{1}{10}\left[2 + 6a_i - \frac{3\theta_i^2}{\gamma k_i}\right].$$

Therefore, the incomplete market coverage solution is valid only when

$$2 + 6a_i - \frac{3\theta_i^2}{\gamma k_i} > 0. \quad \square$$

When the unconstrained solution does not satisfy the self-selection constraints, the low-valuation segment gets distorted quality (Moorthy 1984).

C. Global Optimality and Equilibrium Existence in Price Subgame

D'Aspremont et al. (1979) show that if one of the firms can be better off by charging a price such that it captures the whole market, then there is no pure strategy equilibrium in the price subgame. Even if a pure strategy equilibrium exists in the price subgame, profitability of deviation would suggest our solution is a local optimal and not a global optimal. Here I show that if the two firms are sufficiently separated, then each of the following four deviations by Firm A is unprofitable:

1. Firm A gets all high-valuation consumrs in $[b_H, 1]$ interval by reducing the price of q_{HA}.
2. Firm A gets all low-valuation consumers in $[b_L, 1]$ interval by reducing the price of q_{LA}.
3. Firm A gets all high-valuation consumers in $[b_H, 1]$ interval by reducing the price of q_{LA}.
4. Firm A gets all low-valuation consumers in $[b_L, 1]$ interval by reducing the price of q_{HA}.

Full Coverage Model

1. Firm A gets all-high valuation consumers in $[b_H, 1]$ interval by reducing the price of q_{HA}.

(a) The price at which the high-valuation consumers at $x > b_H$ is indifferent between choosing q_{HA} and q_{HB} is given by $p^x_{HA} = p^{D*}_{HB} + \theta_H(q_{HA} - q_{HB}) + k_H(a_H - b_H)$. At this price, Firm A will get the entire high-valuation segment. If no low-valuation consumer finds it optimal to buy q_{HA} at this price, then Firm A's profit from the high-valuation segment with the deviation under consideration is $(1/3)N_H[6a_Hk_H - (q_{HA} - q_{HB})(\gamma(q_{HA} + q_{HB}) - 2\theta_H)]$. Firm A's profit from the high-valuation segment with the equilibrium strategy is

$$\frac{N_H[-6k_H + (q_{HA} - q_{HB})(\gamma(q_{HA} + q_{HB}) - 2\theta_H)]^2}{72k_H}.$$

The proposed deviation is unprofitable when $36k_H^2(-1 + 4a_H) - 12k_H(q_{HA} - q_{HB})(\gamma(q_{HA} + q_{HB}) - 2\theta_H) - (q_{HA} - q_{HB})^2(\gamma(q_{HA} + q_{HB}) - 2\theta_H)^2 < 0$. It is easy to see that with the equilibrium solution of $q^{D*}_{HA} = q^{D*}_{HB}$, the above condition reduces to the familiar $a_H \le 1/4$.

(b) It is also possible that some of the low-valuation consumers of Firm B may also find it optimal to buy q_{HA} at this price. Let the location of the q_{LB} consumer who is indifferent between buying q_{LB} at p^{D*}_{LB} and buying q_{HA} at p^x_{HA} be denoted by x^x_L. Note that if x^x_L is less than the equilibrium value of x_L, then no q_{LB} consumers will buy q_{HA}. It can be shown that if any q_{LB} consumer gains a greater utility from buying q_{HA} at p^x_{HA} than from buying q_{LB} at p^{D*}_{LB}, then all q_{LA} consumers also gain a greater utility from q_{HA} at p^x_{HA} than from buying q_{LA} at p^{D*}_{LA}. Therefore, in the current case, all the high-valuation consumers and all the low-valuation consumers in $[0, x^x_L]$ range will buy q_{HA}. No consumer buys q_{LA}. Firm A's profit from this deviation is

$$(p^x_{HA} - c_{HA})(N_H + N_Lx^x_L), \tag{A6}$$

where

$$x^x_L = \frac{1}{12k_L}[6(2 - a_L)k_L + 6a_H(k_L - 2k_H) - \gamma(q^2_{HA} + 2q^2_{HB} - q^2_{LA} - 2q^2_{LB}) - 4(q_{HA} - q_{HB})\theta_H + 2(3q_{HA} - q_{LA} + 2q_{LB})\theta_L].$$

Firm A's profit from both segments in the equilibrium is

$$\frac{N_H[-6k_H + (q_{HA} - q_{HB})(\gamma(q_{HA} + q_{HB}) - 2\theta_H)]^2}{72k_H} + \frac{N_L[-6k_L + (q_{LA} - q_{LB})(\gamma(q_{LA} + q_{LB}) - 2\theta_L)]^2}{72k_L}. \tag{A7}$$

The proposed deviation is unprofitable when the equilibrium profit given in (A7) is greater than its profit from the deviation given in (A6).

With the equilibrium quality levels $q^{D**}_{ij} = \theta_i/\gamma$ ($i = H, L$ and $j = A, B$), the condition necessary for the equilibrium to be unprofitable is

$$\{a_Hk_HN_L(\theta_H - \theta_L)^2 + \gamma[k_Hk_LN_H(1 - 4a_H) + N_L(2a_Hk_H - k_L)^2 + 2a_Hk_Hk_L(a_L - a_H)N_L]\} \div 2\gamma k_L > 0.$$

It is easy to see that with $a_L \ge a_H$, the above expression is positive for any $a_H \le 1/4$. With the equilibrium quality solutions, it can also be shown that

$$1 - x^x_L = \frac{2a_Lk_L + (k_H - k_L)(1 + a_H + a_L)}{2k_L} > 0.$$

It can be shown that it is not possible that no q_{LB} consumers buy q_{HA} at p^x_{HA}, but some q_{LA} consumers buy q_{HA} at p^x_{HA}.

2. Firm A gets all low-valuation consumers in $[b_L, 1]$ interval by reducing the price of q_{LA}.

(a) The value of p_{LA} at which q_{LB} consumers at $x \ge b_L$ become indifferent between buying q_{LA} and q_{LB} is given by $p^x_{LA} = p^{D*}_{LB} + \theta_L(q_{LA} - q_{LB}) - k_L(b_L - a_L)$. At this price, Firm A will get all of the low-valuation consumers. If no high-valuation consumer finds it optimal to buy q_{LA} at this price, then Firm A's profit from the low-valuation segment with the deviation under consideration is $(1/3)N_L[6a_Lk_L - (q_{LA} - q_{LB})(\gamma(q_{LA} + q_{LB}) - 2\theta_L)]$. Firm A's profit from the low-valuation segment with the equilibrium strategy is

$$\frac{N_L[-6k_L + (q_{LA} - q_{LB})(\gamma(q_{LA} + q_{LB}) - 2\theta_L)]^2}{72k_L}.$$

The proposed deviation is unprofitable when $36k_H^2(-1 + 4a_L) - 12k_L(q_{LA} - q_{LB})(\gamma(q_{LA} + q_{LB}) - 2\theta_L) - (q_{LA} - q_{LB})^2(\gamma(q_{LA} + q_{LB}) - 2\theta_L)^2 < 0$. It is easy to see that with the equilibrium solution of $q^{D*}_{LA} = q^{D*}_{LB}$, the above condition reduces to the familiar $a_L \le 1/4$.

(b) It is possible that some of the high-valuation consumers buying q_{HB} may also find it optimal to buy q_{LA} at p^x_{LA}. Let the location of the q_{HB} consumer who is indifferent between buying q_{HB} at p^{D*}_{HB} and q_{LA} at p^x_{LA} be x^x_H, where

$$x^x_H = [6(2 + a_L - a_H) - 12ak_L + \gamma(q^2_{HA} + 2q^2_{HB} - q^2_{LA} - 2q^2_{LB}) - \theta_H(2q_{HA} + 4q_{HB} - 6q_{LA}) - 4\theta_L(q_{LA} - q_{LB})]/12k_H.$$

It can be shown that when some q_{HB} consumers find it optimal to buy q_{LA}, there are two possibilities: If $b \ge x^{D*}_H$ all of q_{HA} consumers also buy q_{LA} at p^x_{LA} and if $b < x^{D**}_H$, q_{HA} consumers with $x \ge x^y_H$ buy q_{LA} at p^x_{LA}, where

$$x^y_H = [6(-1 + a_L - a_H) + 12ak_L + \gamma(-2q^2_{HA} - q^2_{HB} + q^2_{LA} + 2q^2_{LB}) + 2\theta_H(2q_{HA} + q_{HB} - 3q_{LA}) - 4\theta_L(q_{LA} - q_{LB})]/12k_H.$$

If all q_{HA} consumers buy q_{LA} at p^x_{LA}, then Firm A's profit under this deviation is given by

$$(p^x_{LA} - c_{LA})(N_L + N_Hx^x_H). \tag{A8}$$

The necessary condition for the deviation to be unprofitable is that Firm A's equilibrium profit given in (A7) be greater than its profit under deviation given in (A8). With the equilibrium quality levels $q^{D**}_{ij} = \theta_i/\gamma$ ($i = H, L$ and $j = A, B$), the above necessary condition is simplified as

$$[\gamma(k^2_H - 2a_L(2 + a_L - a_H)k_Hk_L + 4a^2_Lk^2_L)N_H + (1 - 4a_L)\gamma k_Hk_LN_L + a_LN_Hk_L(\theta_H - \theta_L)^2]/2\gamma k_H > 0. \tag{A9}$$

Substituting the minimum value of $(\theta_H - \theta_L)^2$ implied by the condition in Proposition 2, the above expression is simplified as

$$\frac{1}{2}(k_H - k_L)N_L + \frac{1}{2}(1 - 4a_L)k_L N_L + \frac{(-1 + 2a_L)k_L(-k_H + 2ak_L)N_H}{2k_H}$$

which is strictly positive for $a_L \leq ¼$ and $a_H \leq ¼$. If q_{HA} consumers with $x \geq x_H^y$ buy q_{LA} at p_{LA}^x, then Firm A's profit under this deviation is given by

$$(p_{LA}^x - c_{LA})(N_L + N_H(x_H^x - x_H^y)). \tag{A10}$$

The necessary condition for the deviation to be unprofitable is that Firm A's equilibrium profit given in (A7) be greater than its profit under deviation given in (A10).

With the equilibrium quality levels $q_{ij}^{D**} = \theta_i/\gamma$ ($i = H, L$ and $j = A, B$), $b > x_H^{D**}$ for any value of $a \leq ¼$, and the current case does not arise at all. It can be shown that it is not possible that no q_{HB} consumers buy q_{LA} at p_{LA}^x, but some q_{HA} consumers buy q_{LA} at p_{LA}^x.

3. Firm A gets all high-valuation consumers in $[b_H, 1]$ interval by reducing the price of q_{LA}. The price at which q_{HB} consumers become indifferent between buying q_{HB} at p_{HB}^{D**} and buying q_{LA}, p_{LA}^z, is given by $p_{LA}^z = p_{HB}^{D*} + k_H(a_L - b_H) + \theta_H(q_{LA} - q_{HB})$. It can be shown that at this price, there are two possibilities: If $b \geq x_H^{D*}$, then all q_{HA} consumers will also buy q_{LA} and if $b < x_H^{D*}$, then q_{HA} consumers with $x > x_H^w$ will buy q_{LA} where

$$x_H^w = \frac{6(-1 + 2a_L + 2a_H)k_H - (q_{HA} - q_{HB})(\gamma(q_{HA} + q_{HB}) - 2\theta_H)}{12k_H}. \tag{A11}$$

In addition, as the price of q_{LA} is reduced, some of the q_{LB} consumers will also buy q_{LA}. Let the location of the low-valuation consumer indifferent between buying q_{LA} and q_{LB} be x_L^z, where

$$x_L^z = \frac{1}{12k_L}[-6(a_L + a_H)k_H + 12k_L + \gamma(-q_{HA}^2 - 2q_{HB}^2 + q_{LA}^2 + 2q_{LB}^2) + 2(q_{HA} + 2q_{HB} - 3q_{LA})\theta_H + 4(q_{LA} - q_{LB})\theta_L].$$

If all q_{HA} consumers buy q_{LA}, Firm A's profit from this deviation is

$$(p_{LA}^z - c_{LA})(N_L x_L^z + N_H). \tag{A12}$$

The necessary condition for this deviation to be unprofitable is that Firm A's equilibrium profit given in (A7) be greater than the above profit. With the equilibrium quality levels $q_{ij}^{D**} = \theta_i/\gamma$ ($i = H, L$ and $j = A, B$), the above condition is simplified as

$$[4\gamma^2(1 - 2a_H - 2a_L)k_H k_L N_H + (a_L k_H + a_H k_H - k_L)^2 N_L + 4\gamma(-(a_L + a_H)k_H N_L + k_L(N_L + N_H))(\theta_H - \theta_L)^2 + N_L(\theta_H - \theta_L)^4] \div 8\gamma^2 k_L > 0.$$

Substituting the minimum value of $(\theta_H - \theta_L)^2$ from Proposition 1, I find that the above condition is satisfied when

$$\frac{(3 - 4a_H)k_H k_L N_H - 2k_L^2 N_H + (1 - 2a_H)^2 k_H^2 N_L}{2k_L}$$

is positive, which is true for any $a_H \leq ¼$. If q_{HA} consumers with $x \geq x_H^w$ buy q_{LA} at p_{LA}^x, then Firm A's profit under this deviation is given by

$$(p_{LA}^z - c_{LA})(N_L x_L^z + N_H(1 - x_H^w)). \tag{A13}$$

The necessary condition for the deviation to be unprofitable is that Firm A's equilibrium profit given in (A7) be greater than its profit under deviation given in (A13).

With the equilibrium quality levels $q_{ij}^{D**} = \theta_i/\gamma$ ($i = H, L$ and $j = A, B$), $b > x_H^{D**}$ for any value of $a \leq ¼$, and the current case does not arise at all.

4. Firm A gets all low-valuation consumers in $[b_L, 1]$ interval by reducing the price of q_{HA}. The price at which q_{LB} consumers become indifferent between buying q_{LB} at p_{LB}^{D*} and buying q_{HA}, p_{HA}^z, is given by $p_{HA}^z = p_{LB}^{D*} + k_L(a_H - b_L) + \theta_L(q_{HA} - q_{LB})$.

It can be shown that at this price, all q_{LA} consumers will also prefer to buy q_{HA} instead of q_{LA}. in addition, as the price of q_{HA} is reduced, some of the q_{HB} consumers will also buy q_{HA}. Let the location of the high-valuation consumer indifferent between buying q_{HA} and q_{HB} be x_H^z, where

$$x_H^z = \frac{1}{12k_H}[-6(a_L + a_H)k_L + 12k_H + \gamma(q_{HA}^2 + 2q_{HB}^2 - q_{LA}^2 - 2q_{LB}^2) + 2(q_{LA} + 2q_{LB} - 3q_{HA})\theta_H + 4(q_{HA} - q_{HB})\theta_H].$$

Firm A's profit from this deviation is

$$(p_{HA}^z - c_{HA})(N_H x_H^x + N_L). \tag{A14}$$

The necessary condition for this deviation to be unprofitable is Firm A's equilibrium profit given in (A7) be greater than the above profit. With the equilibrium quality levels $q_{ij}^{D**} = \theta_i/\gamma$ ($i = H, L$ and $j = A, B$), the above condition is simplified as

$$[4\gamma^2(1 - 2a_H - 2a_L)k_H k_L N_L + (a_L k_L + a_H k_L - k_H)^2 N_H + 4\gamma(-(a_L + a_H)k_L N_H + k_H(N_L + N_H))(\theta_H - \theta_L)^2 + N_H(\theta_H - \theta_L)^4] \div 8\gamma^2 k_H > 0.$$

Substituting the minimum value of $(\theta_H - \theta_L)^2$ from Proposition 1, the above expression is simplified as

$$\frac{N_H[(2 + a_L - a_H)k_H - (1 + a_L + a_H)k_L]^2}{2k_H} + \frac{N_L[k_L(1 - 4a_H) + 2(1 + a_L - a_H)(k_H - k_L)]}{2},$$

which is positive for any $a_H \leq ¼$.

Incomplete Coverage Model

The four deviations discussed for the complete coverage model are relevant here also. However, the necessary conditions are more complex. I derive simpler strong sufficient conditions for each of the four deviations to be unprofitable. Specifically, I derive conditions under which Firm A's per-unit margin is weakly negative, so that the firm's profits are weakly negative under each of the deviations.

1. Firm A gets all high-valuation consumers in $[b_H, x_{3H}]$ interval by reducing the price of q_{HA}. The lowest price at which the q_{HB}

consumer at b_H can be induced to buy q_{HA} is $\hat{p}_{HA} = p_{HB}^{D**} + \theta_H(q_{HA} - q_{HB}) + k_H(a_H - b_H)$. At this price, Firm A's per-unit margin is

$$\hat{p}_{HA} - \frac{\gamma}{2}q_{HA}^2.$$

A sufficient condition for ruling out the deviation is that

$$\hat{p}_{HA} - \frac{\gamma}{2}q_{HA}^2 \le 0.$$

With the equilibrium quality solution of $a_{ij}^{D**} = \theta_i/\gamma$ ($i = H, L$ and $j = A, B$), the condition is simplified as

$$\frac{\theta_H^2 - 4(1 - 2a_H)\gamma k_H}{5\gamma} \le 0.$$

I am only considering cases in which $\gamma k_H(2 + 6a_H) - 3\theta_H^2 > 0$ or

$$\frac{\gamma k_H(2 + 6a_H)}{3} > \theta_H^2$$

(see Proposition 4). For $a_H \le \frac{1}{3}$,

$$4(1 - 2a_H)\gamma k_H - \frac{\gamma k_H(2 + 6a_H)}{3} = \frac{10}{3}(1 - 3a_H) > 0.$$

Therefore, for $a_H \le 1/3$, $\theta_H^2 < 4(1 - 2a_H)\gamma k_H$ and

$$\frac{\theta_H^2 - 4(1 - 2a_H)\gamma k_H}{5\gamma} < 0.$$

2. Firm A gets all low-valuation consumers in $[b_L, x_{3L}]$ interval by reducing the price of q_{LA}. This deviation is symmetric to the deviation in (1) above and can be ruled out as in the previous case.

3. Firm A gets all high-valuation consumers in $[b_H, x_{3H}]$ interval by reducing the price of q_{LA}. The lowest price of q_{LA} at which the q_{HB} consumer at b_H can be induced to buy q_{LA}, $\hat{\hat{p}}$, is given by $\hat{\hat{p}}_{LA} = p_{HB}^{D**} - k_H(b_H - a_L) + \theta_H(q_{LA} - q_{HB})$. This price results in weakly negative margins when

$$\hat{\hat{p}}_{LA} - c_{LA} = (14(-4 + 5a_L + 3a_H)k_H + \gamma(3q_{HA}^2 + 18q_{HB}^2 - 35q_{LA}^2) - 2(3q_{HA} + 18q_{HB} - 35q_{LA})\theta_H)/70 \le 0.$$

With the equilibrium quality solution of $q_{ij}^{D**} = \theta_i/\gamma$ ($i = H, L$ and $j = A, B$), the per-unit margin is simplified as

$$\frac{2\gamma(-4 + 5a_L + 3a_H)k_H - 3\theta_H^2 + 10\theta_H\theta_L - 5\theta_L^2}{10\gamma}.$$

Using an approach similar to (1) above, it can be shown that

$$\frac{\theta_L^2 - 4(1 - 2a_L)\gamma k_L}{5\gamma} < 0 \quad \text{for } a_L < \frac{1}{3}.$$

$$\frac{\theta_L^2 - 4(1 - 2a_L)\gamma k_L}{5\gamma} - \frac{2\gamma(-4 + 5a_L + 3a_H)k_H - 3\theta_H^2 + 10\theta_H\theta_L - 5\theta_L^2}{10\gamma}$$
$$= \frac{-2\gamma(1 + 5a_L - 7a_H)k_H - k_L(1 - 2ak_L) + (3\theta_H - 7\theta_L)(\theta_H - \theta_L)}{10\gamma} + 2(a_L - a_H)k_L + (k_H - k_L)(1 - 2a_H).$$

We know that $-2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L] + (3\theta_H - 7\theta_L)(\theta_H - \theta_L) > 0$ from the condition in Proposition 4 and $2(a_L - a_H)k_L + (k_H - k_L)(1 - 2a_H) > 0$ for any $a_H \le a_{H,}$ $k_L \le k_H$, $a_H \le 1/2$. Therefore,

$$0 \ge \frac{\theta_L^2 - 4(1 - 2a_L)\gamma k_L}{5\gamma} > \frac{2\gamma(-4 + 5a_L + 3a_H)k_H - 3\theta_H^2 + 10\theta_H\theta_L - 5\theta_L^2}{10\gamma} \quad \text{for } a_L < \frac{1}{3}.$$

4. Firm A gets all low-valuation consumers in $[b_L, x_{3L}]$ interval by reducing the price of q_{HA}. The lowest price of q_{HA} at which the q_{HB} consumer at b_L can be induced to buy q_{HA}, $\hat{\hat{p}}_{HA}$, is given by $\hat{\hat{p}}_{HA} = p_{LB}^{D**} - k_L(b_L - a_H) + \theta_L(q_{HA} - q_{LB})$. This price results in weakly negative margins when

$$\hat{\hat{p}}_{HA} - c_{HA} = \frac{1}{70}(14(-4 + 3a_L + 5a_H)k_L - 35\gamma q_{HA}^2 + 3\gamma(q_{LA}^2 + 6q_{LB}^2) + 2(35q_{LA} - 3(q_{LA} + 6q_{LB}))\theta_L) \le 0.$$

With the equilibrium quality solution, the per-unit margin is simplified as

$$\frac{2(-4 + 3a_L + 5a_H)\gamma k_L - 5\theta_H^2 + 10\theta_H\theta_L - 3\theta_L^2}{10\gamma},$$

which can be written as

$$-\frac{(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L]}{10\gamma} - \frac{\gamma(1 - 2a_H)(k_H + 3k_L) + \gamma(5k_H - k_L)(k_H - k_L) + \theta_H^2 - 2\theta_L^2}{5\gamma}.$$

From the condition in Proposition 4, $(3\theta_H - 7\theta_L)(\theta_H - \theta_L) - 2\gamma[(1 + 5a_L - 7a_H)k_H - (1 - 2a_L)k_L] > 0$. This condition also implies that $\theta_H^2 > 2\theta_L^2$. Therefore, for any $k_H \ge k_L$ and $a_H \le 1/2$,

$$\frac{\gamma(1 - 2a_H)(k_H + 3k_L) + \gamma(5k_H - k_L)(k_H - k_L) + \theta_H^2 - 2\theta_L^2}{5\gamma} > 0,$$

giving

$$\frac{2(-4 + 3a_L + 5a_H)\gamma k_L - 5\theta_H^2 + 10\theta_H\theta_L - 3\theta_L^2}{10\gamma} < 0.$$

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