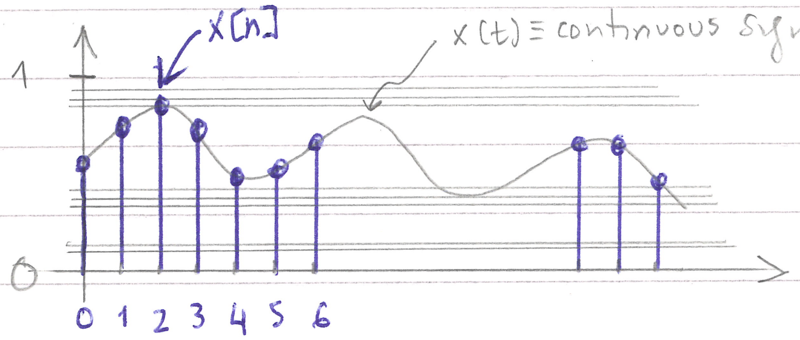
Biometric Recognition
SIGNALS IN 1D AND 2D

Lab exercise 1

* SIGNALS IN ONE DIMENSION (1D) : SAMPLING *

2) discrete amplitude



n (sampling)

T = Sampling period

1) discrete time
=
one sample each T seconds to create x[n]

$x(t) \xrightarrow{T} x[n] = x(nT)$

continuous signal

discrete signal ($n = 0, 1, 2 \dots$) integer

T = Sampling period

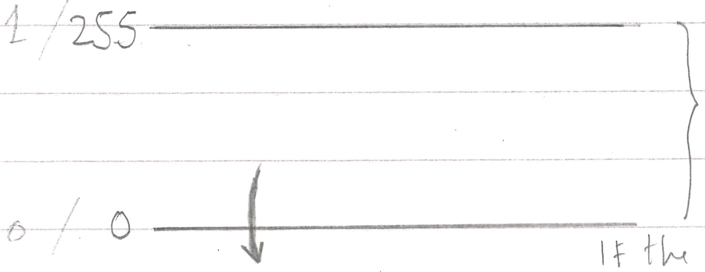
$F_S = \frac{1}{T}$ = sampling frequency

e.g. $F_S = 8000\,Hz = 8\,kHz$

$T = 1/8000 = 0.000125$ sec

2) discrete amplitude

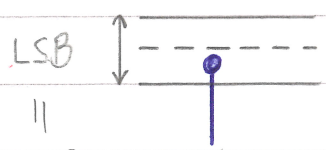
- only a finite number of values for x[n]
- Signal x(nT) at instant nT is discretized (i.e. rounded to the nearest possible value)



e.g resolution of 8 bits (binary signal)

$2^8 = 256$ possible values (0 to 255)

If the input signal is between 0 and 1, then:



- distance between consecutive values is $\boxed{\frac{1}{2^8 - 1} = LSB}$
- average error in the discretization is $\boxed{e_q = \pm \frac{1}{2} LSB}$

LSB = Least Significant Bit (minimum change in the input signal required to guarantee a change in the discretized signal)

→ Signals from the "real" world change with time in a continuous way

$x(t)$ $\begin{cases} \text{- have values all the time} \\ \text{- can take any value between two limits, e.g } [0,1] \end{cases}$

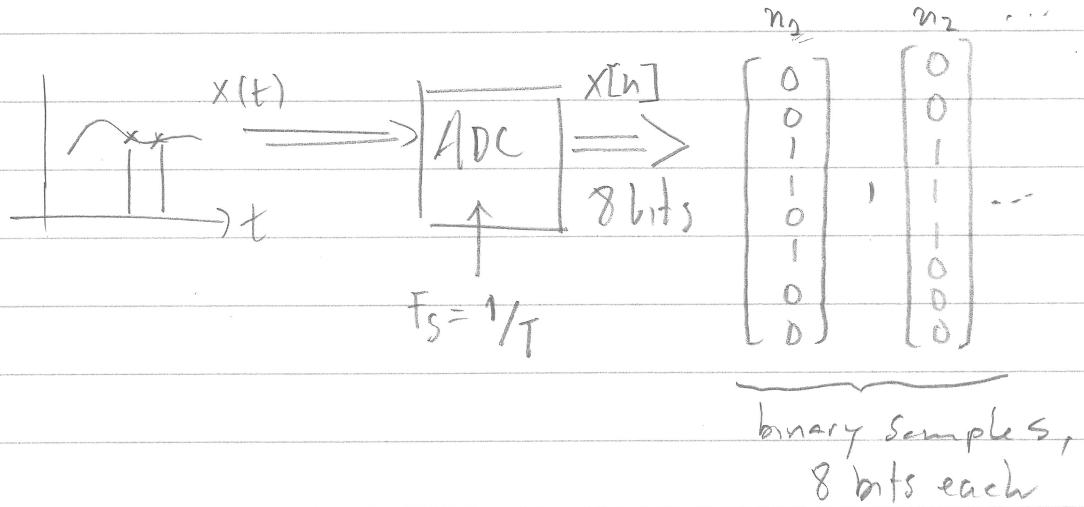
→ This cannot be represented with a computer! (would consume infinite memory)

e.g. Signals from $\begin{cases} \text{microphone (voice)} \\ \text{sensors (temperature, pressure...)} \\ \vdots \end{cases}$

→ Making a signal discrete both in amplitude and time ⟷ "sampling" (ADC)

↑ Analog to Digital Conversion

$x[n] = [\underset{x[0]}{x}, \underset{x[1]}{x}, \underset{x[2]}{x} \; .... \; x]$ ≡ vector (array of values)



* COSINE / SINE SIGNALS * (SINUSOIDS)

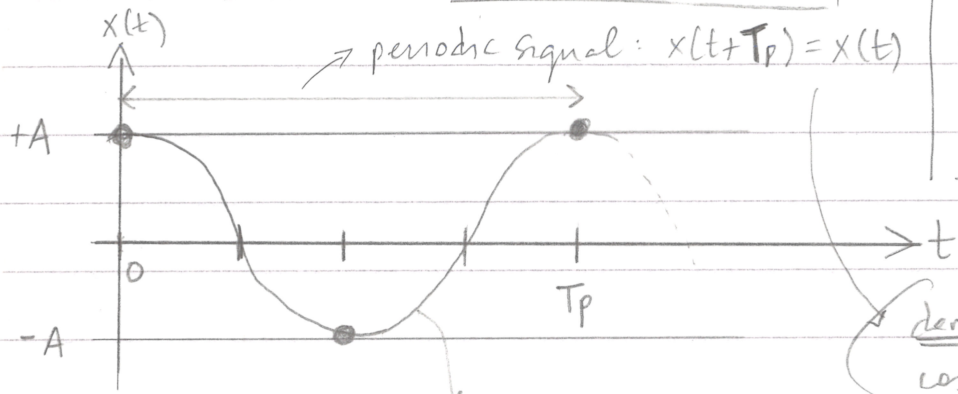
$$x(t) = A \cdot \cos(\Omega t + \theta)$$

- A → amplitude
- Ω → frequency (rad/s)
- θ → phase angle (rad/s)

$$\Omega = 2\pi \cdot F = \frac{2\pi}{T_p}$$

$$F = \frac{1}{T_p} \quad \text{(hertzs)}$$

periodic signal: $x(t+T_p) = x(t)$



$A \cdot \cos(\Omega t)$, with $\theta = 0$

$\Omega = \frac{2\pi}{T_p}$ (rad/s)

$\Omega = 2\pi F$

$F = 1/T_p$ (Hz) = frequency — number of oscillations per second

T_p (sec.) = period — time until the signal has the same value

dem ($\theta = 0$):

$\cos(\Omega(t+T_p)) = \cos\left(2\pi F\left(t + \frac{1}{F}\right)\right) =$

$\cos(2\pi F t + 2\pi) =$

$\cos(\Omega t)\underbrace{\cos(2\pi)}_{1} - \sin(\Omega t)\underbrace{\sin(2\pi)}_{0} =$

$= \cos(\Omega t)$

Discretization of cosine signals

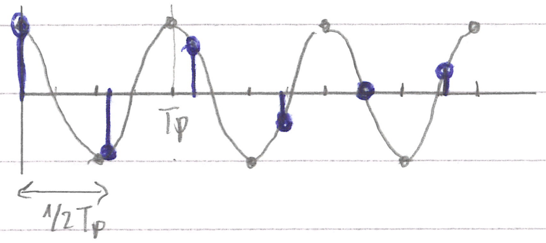
$$x(t) = A\cos(2\pi F \cdot t)$$

↓ take samples with frequency F_s (number of samples per second that I take), i.e. $\left\{ t = \frac{n}{F_s},\ t = n \cdot T_s \right\}$ or every $T_s = \frac{1}{F_s}$ seconds

$$x[n] = A\cos\left(2\pi \frac{F}{F_s} n\right) = A\cos(2\pi f n) \qquad \boxed{f = \frac{F}{F_s}}$$

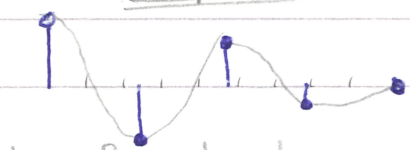
$f \equiv$ "frequency" of the discretized signal
- $F \equiv$ frequency of original (continuous) signal
- $F_s \equiv$ sampling frequency to discretize the signal

Is there a relation between F (frequency of the signal I want to discretize) and Fs (sampling frequency that I choose to discretize) ??

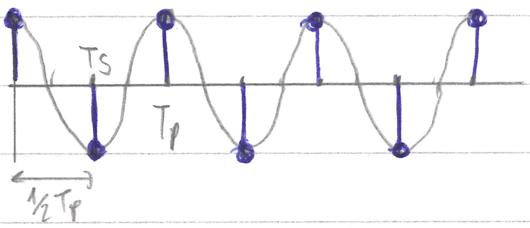


$$\frac{F_s < 2F}{T_s > \frac{1}{2} T_p}$$

bad representation!



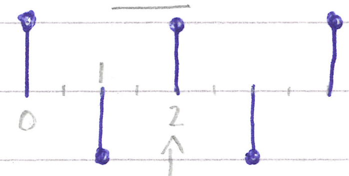
lose information!



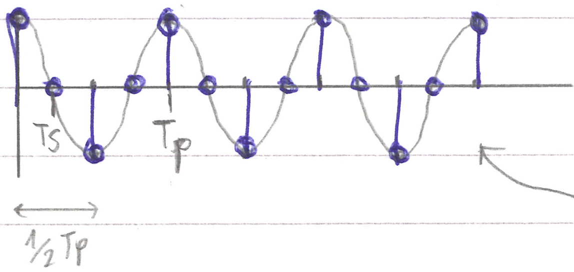
$$\frac{F_s = 2F}{T_s = \frac{1}{2} T_p}$$

$N = 2$

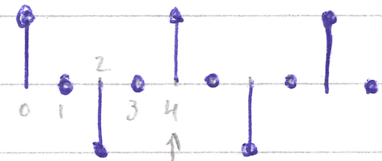
limit!



periodic each
N=2 samples



$$\frac{F_s > 2F}{T_s < \frac{1}{2} T_p}$$



e.g. $T_s = \frac{T_p}{4} \Rightarrow F_s = 4F \Rightarrow N = 4$

periodic each
N=4 samples

therefore...

Relation: $\boxed{F_s > 2F}$ in order to be able to "keep" the relevant information in the sampled sinusoid

F_s: Sampling frequency that I choose

F: Frequency of the signal that I want to discretize

And for this reason: $\boxed{f = \frac{F}{F_s} \in [0, \frac{1}{2}]}$

And the sampled signal is periodic with period $\boxed{N = \frac{1}{f} = \frac{F_s}{F}}$

Compromise with the sampling frequency for signals of the "real world"

- Voice: $F_{MAX} \cong 4\,kHz \Rightarrow$ sampling $F_s = 8\,kHz$ in telephony
- Hearing range of humans: $F_{MAX} \cong 20\,kHz \Rightarrow$ Sampling $F_s = 44{,}1\,kHz > 20\,kHz$ used in digital audio (CD) or $F_s = 48\,kHz$ in professional music

Example (lab exercise)

$f = \frac{F}{F_s} = \frac{200}{10000} = \frac{2}{100}$

```
>> n=0:199;
   F=200;     freq. of the sinusoid
   Fs=10000;  sampling rate
```

N=50 (see figure obtained in your exercise!)

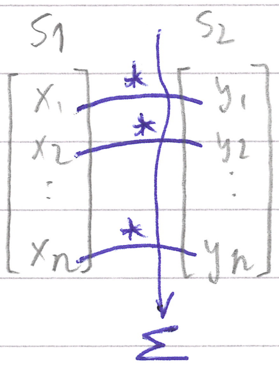
```
>> cos = 1 * cos(2*pi*(F/Fs)*(n+Ø))
   figure(21); subplot(311); stem(n, cos)
```

Show in the same figure

1) change in frequency of the sinusoid F=300; F=100
2) change in phase angle Ø
3) change in amplitude A=2, A=0.5

* SCALAR PRODUCT: COMPARISON OF TWO SIGNALS *

– Compare two signals with the normalized scalar product (must have the same length!)



1) element-wise multiplication (.* in Matlab)
2) sum of all multiplications

$\boxed{\langle s_1, s_2 \rangle = x_1 y_1 + x_2 y_2 + \dots + x_n \cdot y_n}$ SCALAR PRODUCT

the result is a number!

– To make the result independent of the "energy" of the signal, it is divided by the "length"/"norm" of s_1 and s_2

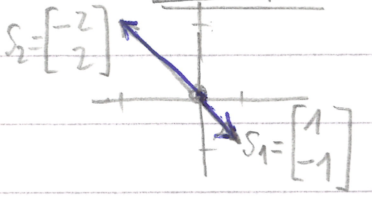
$\|s_1\| = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$

$\|s_2\| = \sqrt{y_1^2 + y_2^2 + \dots + y_n^2}$

$\Rightarrow \boxed{\frac{\langle s_1, s_2 \rangle}{\|s_1\| \, \|s_2\|}}$ NORMALIZED SCALAR PRODUCT

and due to this normalization, the result is always in [-1, 1]

Examples



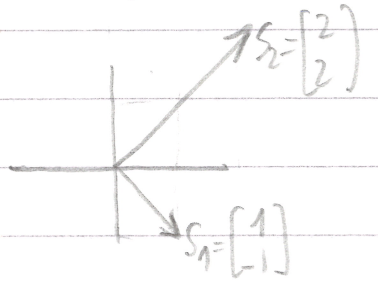
$$\boxed{\langle s_1, s_2 \rangle = -2 \cdot 1 + 2 \cdot (-1) = -2 - 2 = -4}$$

$$\|s_1\| = \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = \sqrt{8}$$

$$\|s_2\| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

$$\boxed{\frac{\langle s_1, s_2 \rangle}{\|s_1\| \, \|s_2\|} = \frac{-4}{\sqrt{8}\sqrt{2}} = \frac{-4}{\sqrt{16}} = \frac{-4}{4} = -1}$$

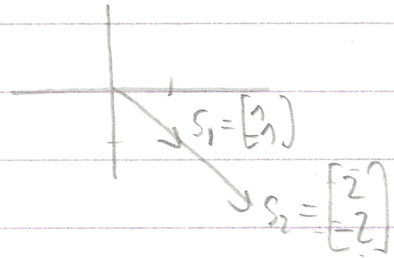
completely opposite vectors



$$\langle s_1, s_2 \rangle = 2 - 2 = 0$$

$$\boxed{\frac{\langle s_1, s_2 \rangle}{\|s_1\| \, \|s_2\|} = 0}$$

perpendicular



$$\langle s_1, s_2 \rangle = 2 + 2 = 4$$

$$\|s_1\| = \sqrt{1+1} = \sqrt{2}$$

$$\|s_2\| = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$\boxed{\frac{\langle s_1, s_2 \rangle}{\|s_1\| \, \|s_2\|} = \frac{4}{\sqrt{2}\sqrt{8}} = 1}$$

Same direction

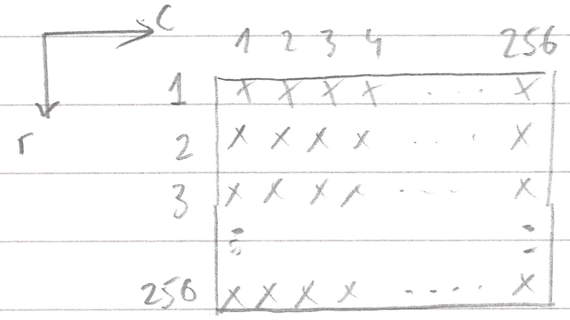
Another interpretation of $\frac{\langle s_1, s_2 \rangle}{\|s_1\| \, \|s_2\|}$ is as $\cos(\varphi)$

where φ is the angle between s_1 and s_2

* SIGNALS IN TWO DIMENSIONS (2D) : IMAGES *

image $\longleftrightarrow$ $\underbrace{\text{matrix}}_{\text{data structure}}$ (array in two dimensions)

Example 256x256 photo

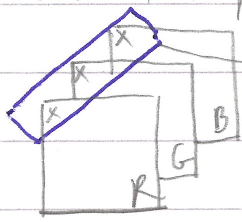


X = point of the image (pixel)

black white

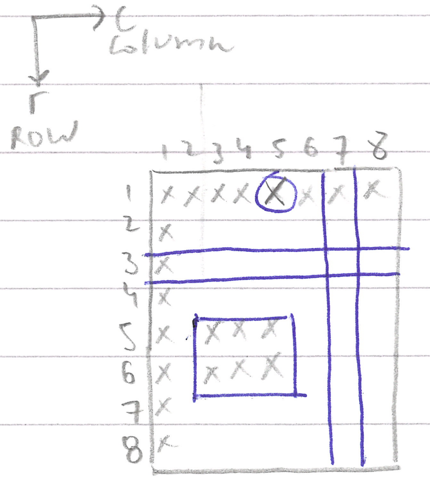
- Gray-scale images : $x \in [0, 255]$
- Color images (RGB) : three values per pixel

i.e. "three images"



$$\text{each pixel} = \begin{bmatrix} R \\ G \\ B \end{bmatrix} \text{ with } R, G, B \in [0, 255]$$

In Matlab : how to index (access to pictures)



I

8x8

One pixel : $I(r,c)$ e.g. $I(1,5)$

A row of pixels : $I(r,:)$ e.g. $I(3,:)$

A column : $I(:,c)$ e.g. $I(:,7)$

An area of pixels : $I(r_1:r_2, c_1:c_2)$ e.g. $I(5:6,3:5)$

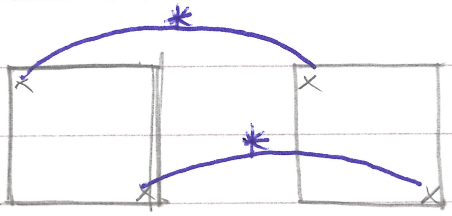
In Matlab: assign values to an image

I = ones(8,8) ⇒ image of 8x8 with all 1's

$\underbrace{I(5:6, 3:5)}_{\text{area}}$ = zeros(2,3) ⇒ modify an area and set it to all 0's

* COMPARISON OF IMAGES WITH SCALAR PRODUCT *

Same idea as before



B_1 $\qquad$ B_2

1) pixel-wise multiplication
2) sum of all products

$$\boxed{\frac{\langle B_1, B_2 \rangle}{\|B_1\| \cdot \|B_2\|}}$$

In Matlab

$\langle B_1, B_2 \rangle$ ⇒ sum(sum(B1.*B2))

$\|B_1\|$ ⇒ norm(B1)

Example

```
I = ones(8,8);
I1 = I
I1(5:6, 3:5) = zeros(2,3);
Skalprod = sum(sum(I.*I1));
NormSP = SkalProd/norm(I)/norm(I1);
```