(2009 年预赛第一(4)题) 设函数$y=y(x)$由方程$x\mathrm{e}^{f(y)}=\mathrm{e}^{y}\ln 29$确定，其中$f$具有二阶导数，且$f'\neq 1$，则$\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}=$______________.

解析：直接对等式两边关于变量x求导比较麻烦，更何况要求二阶导数！因为$\ln 29>0$，所以$x=\dfrac{\mathrm{e}^{y}\ln 29}{\mathrm{e}^{f(y)}}>0$. 等式两边取ln，得

$$\ln x+f(y)=y+\ln(\ln 29). \qquad (1)$$

等式(1)两边关于变量x求导，得$\dfrac{1}{x}+f'(y)\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{\mathrm{d}y}{\mathrm{d}x}$, (2)

解得$\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1}{x[1-f'(y)]}$.

等式(2)两边关于变量x求导，得

$$-\frac{1}{x^2}+f''(y)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+f'(y)\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}=\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} \qquad (3)$$

解得

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}=\frac{-\frac{1}{x^2}+f''(y)\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{1-f'(y)}=\frac{-\frac{1}{x^2}+\frac{f''(y)}{x^2[1-f'(y)]^2}}{1-f'(y)}=\frac{f''(y)-[1-f'(y)]^2}{x^2[1-f'(y)]^3}$$