1. $\lim\limits_{n\to\infty}\left(\dfrac{1^k+2^k+\cdots+n^k}{n^k}-\dfrac{n}{k+1}\right)$.

解析：

$$\lim_{n\to\infty}\left(\frac{1^k+2^k+\cdots+n^k}{n^k}-\frac{n}{k+1}\right)=\lim_{n\to\infty}n\left[\sum_{k=1}^{n}\frac{1}{n}\left(\frac{k}{n}\right)^k-\frac{1}{k+1}\right],$$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\left(\frac{k}{n}\right)^k=\int_0^1 x^k\mathrm{d}x=\frac{1}{k+1}.$$

由结论

$$\lim_{n\to\infty}n\left[\int_a^b f(x)dx-\frac{b-a}{n}\sum_{k=1}^{n}f\left(a+\frac{k}{n}(b-a)\right)\right]=\frac{(a-b)\left[f(b)-f(a)\right]}{2}$$

$$\lim_{n\to\infty}n\left[\sum_{k=1}^{n}\frac{1}{n}\left(\frac{k}{n}\right)^k-\frac{1}{k+1}\right]=\frac{(1-0)\left(1^k-0^k\right)}{2}=\frac{1}{2}.$$

2. $\lim\limits_{n\to\infty}\left(b^{\frac{1}{n}}-1\right)\sum\limits_{i=0}^{n-1}b^{\frac{i}{n}}\sin b^{\frac{2i+1}{n}}$，$(b>1)$.

解析：

$$\lim_{n\to\infty}\left(b^{\frac{1}{n}}-1\right)\sum_{i=0}^{n-1}b^{\frac{i}{n}}\sin b^{\frac{2i+1}{n}}$$

$$=\lim_{n\to\infty}\frac{b^{\frac{1}{n}}-1}{\frac{1}{n}}\sum_{i=0}^{n-1}\frac{1}{n}b^{\frac{i}{n}}\sin b^{\frac{2i+1}{n}}=\ln b\lim_{n\to\infty}b^{-\frac{1}{2n}}\sum_{i=0}^{n-1}\frac{1}{n}b^{\frac{i+\frac{1}{2}}{n}}\sin b^{2\left(\frac{i+\frac{1}{2}}{n}\right)}$$

$$=\ln b\lim_{n\to\infty}\sum_{i=0}^{n-1}\frac{1}{n}b^{\frac{i+\frac{1}{2}}{n}}\sin b^{2\left(\frac{i+\frac{1}{2}}{n}\right)}=\ln b\int_0^1 b^x\sin b^{2x}\mathrm{d}x$$

$$\int_0^1 b^x\sin b^{2x}\mathrm{d}x=\frac{1}{\ln b}\int_0^1\sin b^{2x}\mathrm{d}b^x=\frac{1}{\ln b}\int_1^b\sin u^2\mathrm{d}u$$

$$=\frac{1}{\ln b}\int_1^b\left(\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}u^{4n+2}\right)\mathrm{d}u=\frac{1}{\ln b}\sum_{n=0}^{\infty}\int_1^b\frac{(-1)^n}{(2n+1)!}u^{4n+2}\mathrm{d}u$$

$$=\frac{1}{\ln b}\sum_{n=0}^{\infty}\frac{(-1)^{n}(b^{4n+3}-1)}{(2n+1)!(4n+3)}$$

$$\lim_{n\to\infty}\left(b^{\frac{1}{n}}-1\right)\sum_{i=0}^{n-1}b^{\frac{i}{n}}\sin b^{\frac{2i+1}{n}}=\int_{1}^{b}\sin u^{2}\mathrm{d}u=\sum_{n=0}^{\infty}\frac{(-1)^{n}(b^{4n+3}-1)}{(2n+1)!(4n+3)}$$