(2017 年预赛第三题) 设曲线Γ为在$x^2+y^2+z^2=1$，$x+z=1$，$x\geqslant 0$，$y\geqslant 0$，$z\geqslant 0$，上从$A(1,0,0)$到$B(0,0,1)$的一段，求曲线积分$I=\int_\Gamma y\,\mathrm{d}x+z\,\mathrm{d}y+x\,\mathrm{d}z$.

解析：由$\begin{cases}x^2+y^2+z^2=1\\x+z=1\end{cases}$，得$x^2+y^2+(1-x)^2=1$，整理得$\dfrac{\left(x-\frac{1}{2}\right)^2}{\frac{1}{4}}+\dfrac{y^2}{\frac{1}{2}}=1$. 因此，得到曲线$\Gamma$的参数方程

$$\begin{cases}x=\dfrac{1}{2}+\dfrac{1}{2}\cos\theta\\ y=\dfrac{\sqrt{2}}{2}\sin\theta\\ z=\dfrac{1}{2}-\dfrac{1}{2}\cos\theta\end{cases}$$，其中θ：$0\to\pi$. 于是，

$$\begin{aligned}I&=\int_\Gamma y\,\mathrm{d}x+z\,\mathrm{d}y+x\,\mathrm{d}z\\&=\int_0^\pi\left[\frac{\sqrt{2}}{2}\sin\theta\left(-\frac{1}{2}\sin\theta\right)+\left(\frac{1}{2}-\frac{1}{2}\cos\theta\right)\frac{\sqrt{2}}{2}\cos\theta+\left(\frac{1}{2}+\frac{1}{2}\cos\theta\right)\frac{1}{2}\sin\theta\right]\mathrm{d}\theta\\&=\int_0^\pi\left(-\frac{\sqrt{2}}{4}+\frac{\sqrt{2}}{4}\cos\theta+\frac{1}{4}\sin\theta+\frac{1}{4}\sin\theta\cos\theta\right)\mathrm{d}\theta\\&=-\frac{\sqrt{2}}{4}\pi+\frac{1}{2}\end{aligned}$$