(2015 年预赛第一(5)题)设区间 $(0,+\infty)$ 上的函数u(x)定义为

$$u(x) = \int_0^{+\infty} \mathrm{e}^{-xt^2} \mathrm{d}t$$
,则 $u(x)$ 的初等函数表达式为_______.

解析: 已知概率积分
$$\int_0^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx = \frac{1}{2}$$
,所以 $\int_0^{+\infty} e^{-\frac{x^2}{2}} dx = \sqrt{\frac{\pi}{2}}$,

$$\sqrt{2}\!\int_0^{+\infty}\!\mathrm{e}^{-\left(\!rac{x}{\sqrt{2}}\!
ight)^{\!2}}d\!\left(\!rac{x}{\sqrt{2}}\!
ight)\!=\!\sqrt{rac{\pi}{2}}\;,\;\;\int_0^{+\infty}\!\mathrm{e}^{-u^2}\mathrm{d}u=rac{\sqrt{\pi}}{2}\,.$$

$$\begin{split} \sqrt{2} \int_0^{+\infty} \mathrm{e}^{-\left(\frac{x}{\sqrt{2}}\right)^2} d\left(\frac{x}{\sqrt{2}}\right) &= \sqrt{\frac{\pi}{2}} \;, \quad \int_0^{+\infty} \mathrm{e}^{-u^2} \mathrm{d}u = \frac{\sqrt{\pi}}{2} \,. \\ \mathbb{b} 此, \quad u(x) &= \int_0^{+\infty} \mathrm{e}^{-xt^2} \mathrm{d}t = \frac{1}{\sqrt{x}} \int_0^{+\infty} \mathrm{e}^{-(\sqrt{x}\,t)^2} \mathrm{d}\left(\sqrt{x}\,t\right) = \frac{\sqrt{\pi}}{2\sqrt{x}} \,. \end{split}$$

