$(2016 年预赛第一(1) 题) 若 f(x) 在点 x = a 可导,且 f(a) \neq 0,则$

$$\lim_{n o\infty}\left\lceilrac{f\!\left(a+rac{1}{n}
ight)}{f\!\left(a
ight)}
ight
ceil^n=$$
______.

解析: 幂指函数的典型做法,

$$\lim_{n o\infty}\!\left[rac{f\!\left(a+rac{1}{n}
ight)}{f\!\left(a
ight)}
ight]^n = \lim_{n o\infty}\!e^{n\ln\!\left[rac{f\!\left(a+rac{1}{n}
ight)}{f\!\left(a
ight)}
ight]} \!\!=\! e^{\lim_{n o\infty}n\ln\!\left[rac{f\!\left(a+rac{1}{n}
ight)}{f\!\left(a
ight)}
ight]},$$

$$\lim_{n o \infty} n \ln \left[rac{f\left(a + rac{1}{n}
ight)}{f(a)}
ight] = \lim_{n o \infty} rac{\ln \left| f\left(a + rac{1}{n}
ight)
ight| - \ln |f(a)|}{rac{1}{n}}$$
 $= \left(\ln |f(x)|\right)' \Big|_{x = a}$
 $= rac{1}{f(a)} \cdot f'(a)$
禁止, $\lim_{n o \infty} \left[rac{f\left(a + rac{1}{n}
ight)}{f(a)}
ight]^n = e^{rac{f'(a)}{f(a)}}.$

注: 拜公式 $(\ln|x|)' = \frac{1}{x}$ 所赐,我们不用讨论f(a)的正负了.

1908