



1. [10 points] Design a DFA for  $L = \{w \in \{0,1\}^* \mid w \text{ has exactly three 0s.}\}$

2. [10 points] Design an NFA for the language:

$$L = \{w \in \{a, b, c\}^* \mid w \text{ starts with } ac \text{ and ends with } cb.\}$$

ac

3. [10 points] Design regular expressions for languages over  $\Sigma = \{a, b\}$ .

(1) All strings that do not end with aba.

(2)  $L = \{w \mid w \text{ has no more than 5 } a\text{'s.}\}$

4. [10 points] Prove that the language  $L = \{w \in \{a, b\}^* \mid w = w^R\}$  is not regular with pumping lemma.

or

5. [10 points] Consider the following  $\varepsilon$ -NFA.

	$\varepsilon$	$a$	$b$	$c$
$\rightarrow p$	$\{q, r\}$	$\emptyset$	$\{q\}$	$\{r\}$
$q$	$\emptyset$	$\{p\}$	$\{r\}$	$\{p, q\}$
$*r$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

(1) Compute the  $\varepsilon$ -closure of each state.

(2) Give all the strings of length three or less accepted by the automaton.

(3) Convert the automaton to a DFA by subset construction. (diagram of transition function)

6. [10 points] Give a CFG for  $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j + k\}$ .

7. [10 points] Find a grammar equivalent to

$$S \rightarrow AB \mid CA$$

$$A \rightarrow a$$

$$B \rightarrow BC \mid AB$$

$$C \rightarrow aB \mid b$$

with no useless symbols.

8. [10 points] Design a PDA for  $L_{eq} = \{w \in \{0,1\}^* \mid w \text{ contains the same number of 0's and 1's}\}$ .

9. [10 points] Prove or disprove: if  $L_1$  is CFL and  $L_1 \cup L_2$  is also CFL, then  $L_2$  must be CFL.

10. [10 points] Design Turing machine for the language  $\{0^{2n}1^n \mid n \geq 0\}$ .

$$\Sigma = \{0, 1\}$$

$$L_1 = \{w \mid w \in \Sigma^* \text{ (0+1)}^*\}$$

not ww

$$L_2 = \Sigma$$

$$L_1 \cup L_2 = \Sigma^*$$

start  $\rightarrow 0$

OP

$$L_1 \text{ OP } L_2 = L_3$$

$$L_1 \cup L_2 = L_3$$

$$L_1 \cup L_2 = \Sigma^*$$

$$L_1 \cup L_2 = \Sigma^*$$

1. [10 points] Design a DFA for  $L = \{w \in \{0,1\}^* \mid w \text{ has exactly three 0s.}\}$

2. [10 points] Design an NFA for the language:

$$L = \{w \in \{a, b, c\}^* \mid w \text{ starts with } ac \text{ and ends with } cb.\}$$

3. [10 points] Design regular expressions for languages over  $\Sigma = \{a, b\}$ .

(1) All strings that do not end with  $aba$ .

(2)  $L = \{w \mid w \text{ has no more than 5 } a\text{'s.}\}$

4. [10 points] Prove that the language  $L = \{w \in \{a, b\}^* \mid w = w^R\}$  is not regular with pumping lemma.

5. [10 points] Consider the following  $\varepsilon$ -NFA.

	$\varepsilon$	$a$	$b$	$c$
$\rightarrow p$	$\{q, r\}$	$\emptyset$	$\{q\}$	$\{r\}$
$q$	$\emptyset$	$\{p\}$	$\{r\}$	$\{p, q\}$
$*r$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

(1) Compute the  $\varepsilon$ -closure of each state.

(2) Give all the strings of length three or less accepted by the automaton.

(3) Convert the automaton to a DFA by subset construction. (diagram of transition function)

6. [10 points] Give a CFG for  $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j + k\}$ .

7. [10 points] Find a grammar equivalent to

$$S \rightarrow AB \mid CA$$

$$A \rightarrow a$$

$$B \rightarrow BC \mid AB$$

$$C \rightarrow aB \mid b$$

with no useless symbols.

8. [10 points] Design a PDA for  $L_{eq} = \{w \in \{0,1\}^* \mid w \text{ contains the same number of 0's and 1's}\}$ .

9. [10 points] Prove or disprove: if  $L_1$  is CFL and  $L_1 \cup L_2$  is also CFL, then  $L_2$  must be CFL.

10. [10 points] Design Turing machine for the language  $\{0^{2n}1^n \mid n \geq 0\}$ .

1. [10 points] Design a DFA for  $L = \{w \in \{0,1\}^* \mid w \text{ has exactly three 0s.}\}$

2. [10 points] Design an NFA for the language:

$$L = \{w \in \{a, b, c\}^* \mid w \text{ starts with } ac \text{ and ends with } cb.\}$$

3. [10 points] Design regular expressions for languages over  $\Sigma = \{a, b\}$ .

(1) All strings that do not end with  $aba$ .

(2)  $L = \{w \mid w \text{ has no more than 5 } a\text{'s.}\}$

4. [10 points] Prove that the language  $L = \{w \in \{a, b\}^* \mid w = w^R\}$  is not regular with pumping lemma.

5. [10 points] Consider the following  $\varepsilon$ -NFA.

	$\varepsilon$	$a$	$b$	$c$
$\rightarrow p$	$\{q, r\}$	$\emptyset$	$\{q\}$	$\{r\}$
$q$	$\emptyset$	$\{p\}$	$\{r\}$	$\{p, q\}$
$*r$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

(1) Compute the  $\varepsilon$ -closure of each state.

(2) Give all the strings of length three or less accepted by the automaton.

(3) Convert the automaton to a DFA by subset construction. (diagram of transition function)

6. [10 points] Give a CFG for  $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j + k\}$ .

7. [10 points] Find a grammar equivalent to

$$S \rightarrow AB \mid CA$$

$$A \rightarrow a$$

$$B \rightarrow BC \mid AB$$

$$C \rightarrow aB \mid b$$

with no useless symbols.

8. [10 points] Design a PDA for  $L_{eq} = \{w \in \{0,1\}^* \mid w \text{ contains the same number of 0's and 1's}\}$ .

9. [10 points] Prove or disprove: if  $L_1$  is CFL and  $L_1 \cup L_2$  is also CFL, then  $L_2$  must be CFL.

10. [10 points] Design Turing machine for the language  $\{0^{2n}1^n \mid n \geq 0\}$ .



Design a DFA for the language  $L = \{w \in \{0,1\}^* \mid w \text{ contains both } 01 \text{ and } 10 \text{ as substrings}\}$ .

Design a NFA within four states for the language  $\{a\}^* \cup \{ab\}^*$ .

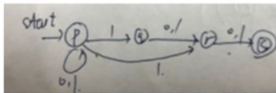
Design regular expressions for language over  $\Sigma = \{0,1\}$ .

(1). All strings contain the substring 001.

(2). All strings except the string 001.

Prove that  $L = \{0^m 1^n \mid m/n \text{ is an integer}\}$  is not regular with pumping lemma.

Convert the following NFA into DFA with subset construction.



Give a context-free grammar for  $L = \{a^i b^j c^{i+j} \mid i, j \geq 0\}$

Let  $L$  be the language generated by the grammar  $G$  below

$S \rightarrow AB|BBB$

$A \rightarrow Bb|\epsilon$

$B \rightarrow aB|A$

(1). 消除空产生式

(2). 消除单元产生式

(3). 转换到 CNF

Design a PDA for  $L = \{w \in \{a,b\}^* \mid w \text{ has more } a\text{'s than } b\text{'s}\}$

Prove : for every context free language  $L$ , the language  $L' = \{0^{|w|} \mid w \in L\}$  is also context free.

Design a Turing Machine that computes the following function  $f: 0^n \rightarrow \text{Binary}(n)$

Where integer  $n \geq 1$  and  $\text{binary}(n)$  is the binary representation of  $n$ .

For example:  $f(0^3) = 11$   $f(0^5) = 101$ .

Prove : for every context free language  $L$ , the language  $L' = \{0^{|w|} | w \in L\}$  is also context free.

构造映射  $h: \Sigma \rightarrow \Sigma^*$   $\Sigma = \{0, 1\}$

$$\text{令 } h(0) = 0 \quad h(1) = \emptyset$$

$$\underline{h(L) = \{0^{|w|} \mid w \in L\} = L'}$$

归纳前提  $|w| = 1$  时

$$h(0) = 0, \quad h(1) = \emptyset$$

$$\text{故 } |w| = 1 \text{ 时, } h(w) = \{0\}$$

假设  $|w| = k$  时, 命题成立,  $w = x$

$$w = xa \quad a = 0 \text{ 或 } 1$$

$$h(xa) = h(x)h(a)$$

$$= \{0^{|x|} \mid 0'\}$$

$$= \{0^{|x|+1}\} = \{0^{k+1}\}$$

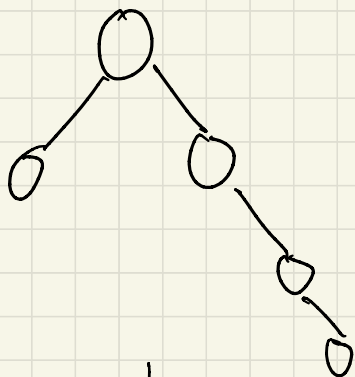
$|0|$

$$\therefore h(L) = L'$$

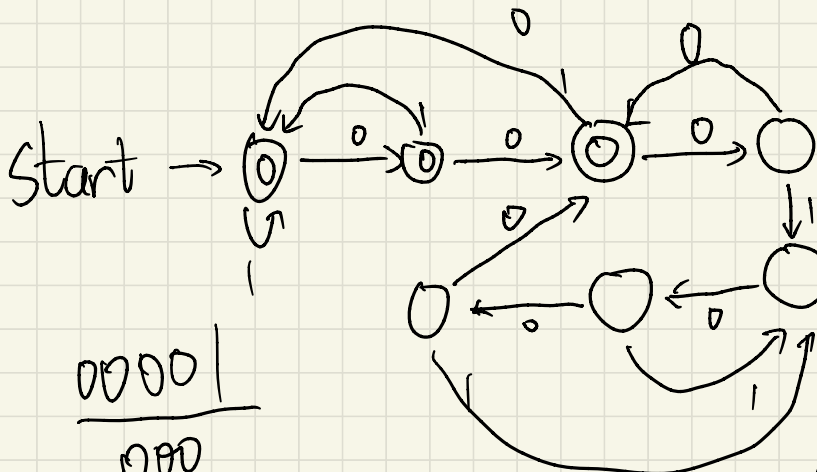
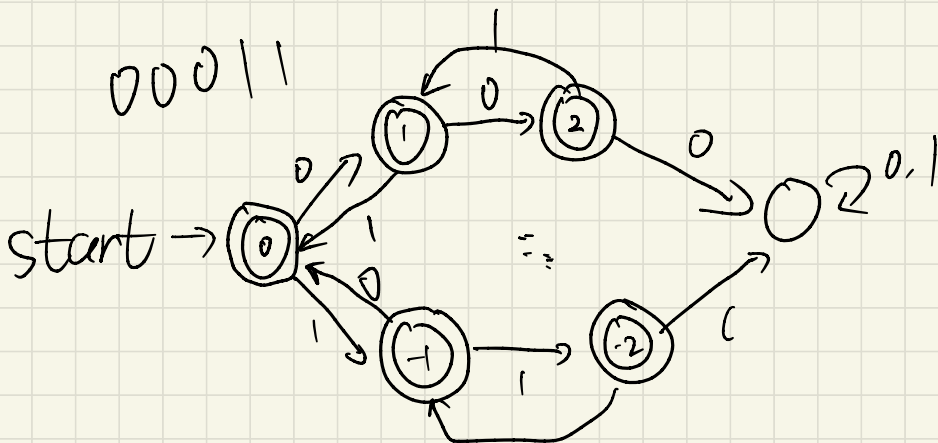
$$\underbrace{(0^1 + 1^0)^* (0^1 + 1^2)}$$

0111

01107



00011



11 000 790

$$\begin{array}{r} 0000 \\ \hline 000 \end{array}$$

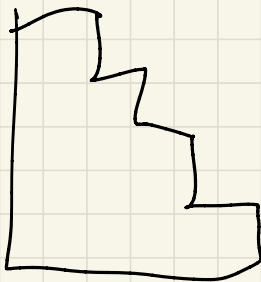
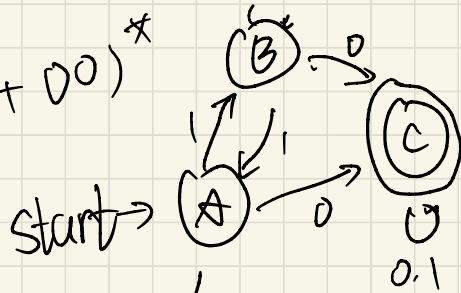
$\frac{000}{\text{---}}$   
 ~~$1000 + 0000 + 00010 + 00100 + 01000 + 10000$~~   
 $(0+10)^* (1+10)^*$



start  $\rightarrow$  @

1001101

ob + 11

$$(a+b+c)^x a (a+b+c)^x b (a+b+c)^x$$
$$f \sim b \sim a \sim (of)^8 / (of)^9$$
$$(10+0)^* (11+\epsilon) (01+0)^*$$
$$1^* (1^* 0 1^* 0 1^* 0 1^* 0 1^* 0 1^*)^* 1^*$$
$$(1 + 00)^*$$


B	<del>X</del>	<del>B</del>
C	X	X
	A	B

$$\checkmark A: \checkmark C \Rightarrow \checkmark D: \checkmark E$$

$$D: E \Rightarrow A: C$$