UNIT 3 : JOINT PROBABILITY DISTRIBUTION

* IA

1) The coin is tossed a thrice ($x = 0$ or 1 if tail or head occurs on the first tossed) respectively y = No. of tails determine.

a) Marginal distribution of x and y
b) $E(x)$, $E(y)$, $E(xy)$
c) variance of x and $\sigma^2(y)$ -- (variance: σ^2)
d) co-variance of xy
e) co-relation of xy

Note :-

$$S = \{ \overset{0}{HHH}, \overset{3}{TTT}, \overset{2}{THT}, \overset{2}{HTT}, \overset{2}{TTH}, \overset{1}{THH}, \overset{1}{HTH}, \overset{1}{HHT} \}$$

$x \rightarrow$ 1 0 0 1 0 0 1 1

x = if first is H then 1 & if not then 0.

$x = \{0, 1\}$

x	0	1
$p(x)$	4/8	4/8

— (Marginal distribution)

num of occurence of 0 " " of 1

both are joint

y = No. Tail

$y = \{0, 1, 2, 3\}$

y	0	1	2	3
$p(y)$	1/8	3/8	3/8	1/8

— (Marginal distribution)

b) joint probability distribution

x \ y	0	1	2	3
0	0/8	1/8	2/8	1/8
1	1/8	2/8	1/8	0/8

– combination of x and y
ex:- when x is 0 and y is occurence of this state

Sometimes we may get quation like by joint probability distribution find marginal distribution
that time if x represent column the add column of, if represents rows add rows
similary do for y

x \ y	0	1	2	3	
x	0/8	+ 2/8	+ 2/8	+ 1/8	→ add rows for x 4/8
y	1/8	+ 1/8	+ 1/8	+ 0/8	4/8
↓ add columns for y	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

x	0	1
$p(x)$	4/8	4/8

y	0	1	2	3
$p(y)$	1/8	3/8	3/8	1/8

c) Expectations

- When join only joint distribution is given first find marginal distribution.

- $E(x) = \Sigma x\, p(x)$ — mean

(shift + setup → ↓ 4 : STAT → Frequency ON → MODE → 3 : STAT → 1 : 1-VAR → Enter x and $p(x)$ [Freq] values in calcy) → AC → shift + 1 → 4 : VAR → Find $\bar{x}$, σx and σx^2)

$\bar{x} = 0.5$

$\sigma x = 0.5$

$\sigma x^2 = 0.25$ — $\Sigma x^2 (p(x)) - M^2$

- $E(y) = \Sigma y (p(y))$

$\bar{y} = 1.5$

$\sigma y = 0.8660$

$\sigma y^2 = 0.75$

- $E(xy) = \Sigma\, xy P_{ij}$ ← comb of xy from joint dis.

$(0)(0)(0/8) + (0)(1)(1/8) + (0)(2)(2/8) + (0)(3)(1/8) + (1)(0)(1/8) + (1)(1)(2/8) + (1)(2)(1/8) + (1)(3)(0/8)$

$\frac{2}{8} + \frac{2}{8} = \frac{4}{8} = \frac{1}{2}$

e) $cov(xy) = E xy - E x E y$ — covarience

$0.5 - (0.5)(1.5)$

$= -0.25$

$= -\frac{1}{4}$

correlation $(xy) = \int = \frac{cov(xy)}{\sigma x \sigma y}$

$= \frac{-0.25}{(0.5)(0.8660)} = -0.5773$

2. A joint PDF is given by

x \ y	-3	2	4
1	0.1	0.2	0.2
3	0.3	0.1	0.1

Find

a) $E(x)$, $E(y)$, $E(xy)$

b) varience of x, varience of y

c) coverience of xy

d) Correlation of xy

x	1	3
p(x)	0.5	0.5

y	-3	2	4
p(y)	0.4	0.3	0.3

a) $E(x) = \Sigma x\, p(x)$ $E(y) = \Sigma y\, p(y)$ $E(xy) = \Sigma xy P_{ij}$

$\bar{x} = 2$ $\bar{y} = 0.6$

$\sigma x = 1$ $\sigma y = 3.0397$

b) $\sigma x^2 = 1$ $\sigma y^2 = 9.24$

$\sum xy P_{ij} = Exy$

$(1)(-3)(0.1) + (1)(2)(0.2) + (1)(4)(0.2) + (3)(3)(0.3) +$
$(3)(2)(0.1) + (3)(4)(0.1)$

$= \underline{0}$

c) covarience of xy

$\text{cov}(xy) = Exy - Ex\,Ey$
$= 0 - 2(0.6)$
$= \underline{-1.2}$

d) correlation $(xy) = \dfrac{\text{cov}(xy)}{\sigma_x \sigma_y}$

$= \dfrac{-1.2}{(1)(3.0397)}$

$= \underline{-0.3947}$

3. The joint PDF of 2 random variable x and y is given by.

x \ y	2	3	4
1	0.06 P_{11}	0.15 P_{12}	0.09 P_{13}
2	0.14 P_{21}	0.35 P_{22}	0.21 P_{23}

a) ditermine the marginal distribution of x and y
b) verify whether x and y are independent

x	1	2
$p(x)$	0.3	0.7

y	2	3	4
$p(y)$	0.2	0.5	0.3

$P_{11} = p_1 q_1 = (0.3)(0.2) = 0.06$
$P_{12} = p_1 q_2 = (0.3)(0.5) = 0.15$
$P_{13} = p_1 q_3 = (0.3)(0.3) = 0.09$
$P_{21} = p_2 q_1 = (0.7)(0.2) = 0.14$
$P_{22} = p_2 q_2 = (0.7)(0.5) = 0.35$
$P_{23} = p_2 q_3 = (0.7)(0.3) = 0.21$

Here all are equals therefore these are independent

Note :-
- x and y are independent if co-varience of x and y is 0

4. The joint PDF is given by

x \ y	0	1
0	0.1 P_{11}	0.2 P_{12}
1	0.4 P_{21}	0.2 P_{22}
2	0.1 P_{31}	0 P_{32}

a) ditermine marginal distribution of x and y.
b) Are they independent?
c) ditermine $p(x + y > 1)$

a)

x	0	1	2
$p(x)$	0.3	0.6	0.7

y	0	1
$p(y)$	0.6	0.4

b)

$P_{11} = (0.3)(0.6) = 0.18$

it's not equals to $P_{11} \neq p_1 q_1 - (0.1)$

∴ these variables are not indipendent

c) $p(x+y \geq 1)$

$\{\underbrace{(1,1),(2,0),(2,1)}_{\text{combination of these pairs.}}\}$

$\overbrace{0.2+0.1+0}= \underline{0.3}$

- Conditional joint probability distribution

- Conditional joint probability distribution of y is given that

$x = x$ is

$h(y|x)$ — probability of y when x is given

Joint of x and y

$$h(y|x) = \frac{h(x,y)}{f(x)}$$

Marginal only of x

- Conditional probability distribution of x is given that

$y = y$

$$h(x/y) = \frac{h(x,y)}{f(y)}$$

1. Given

y \ x	1	2	3
1	0.05	0.05	0.1
2	0.05	0.1	0.35
3	0	0.2	0.1

$h(2,3)$ (x, y)

a) Find the marginal distribution of x and y

b) Find probability of $p(y=3 | x=2)$

a) marginal distribution

x	1	2	3
$p(x)$	0.1	0.35	0.55

$f(2)$

y	1	2	3
p(y)	0.2	0.5	0.3

b) $p(y=3 \mid x=2)$

$\frac{h(2,3)}{f(2)}$ ——— $\frac{h(x,y)}{f(x)}$

$= \frac{0.2}{0.35}$

$= 0.5714$

2. Given that

y \ x	0	1	2
0	0.1 p_{11}	0.4	0.1
1	0.2	0.2	0

a) Find the marginal distribution of x and y
b) Are they independent?
c) Find condition probability distribution $h(x \mid y=1)$

a) Marginal distribution

x	0	1	2
p(x)	0.3 p_1	0.6	0.1

y	0	1
p(y)	0.6 q_1	0.4

b) $p_{11} = p_1 q_1 = (0.3)(0.6) = 0.18$

$p_{11} = 0.1$

$\therefore p_{11} \neq p_1 q_1$

$\therefore$ they are not independent.

c) condition probability distribution

$h(x \mid y=1)$ ——— when x or y value is not given we have to do all combinations.

$\frac{h(x,y)}{f(y)} =$

1) $\frac{h(0,1)}{f(1)} = \frac{0.2}{0.4}$

2) $\frac{h(1,1)}{f(1)} = \frac{0.2}{0.4}$

3) $\frac{h(2,1)}{f(1)} = \frac{0}{0.4}$

- Stocastic process and marco chain

1. Probability vector :- It's a tuble col such that where sum of it's all element is 1.

2. Stocastic Matrix :- Matrix such that where sum of row elements is 1.

3. Reguler stocastic Matrix :- where all the row element are greater than 0.

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1/2 & 1/2 \\ 3/4 & 1/4 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0.4 & 0.5 & 0.1 \\ 0.25 & 0.25 & 0.5 \\ 0.3 & 0.6 & 0.1 \end{bmatrix}$$

stocastic Matrix   Regular stocastic Matrix

1) Find the unique fixed probability vector of regular stocastic matrix. ($VA = V$)

$$A = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$$

$VA = V$; $x + y = 1$ — $\boxed{\text{fixed equation}}$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

2 variable ~~because 2×2~~ Matrix

$$\begin{bmatrix} 3/4x + 1/2y & 1/4x + 1/2y \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

$3/4x + 1/2y = x$ — $(x = 1)$

$1 - \frac{3}{4}x - \frac{1}{2}y = 0$ — $\frac{4-3}{4} = \frac{1}{4}$

$\frac{1}{4}x - \frac{1}{2}y = 0$ — ①

$1/4x + 1/2y = y$

$1 - \frac{1}{4}x - \frac{1}{2}y = 0$ — $1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$

$-\frac{1}{4}x + \frac{1}{2}y = 0$ — ②

Calcy: Mode → equ → 1 → Enter fixed equation i.e $x + y = 1$ (1, 1, 1) → Enter any equation from eq ① or ② → Find x and y

$x = \frac{2}{3}$ $y = \frac{1}{3}$

H.W

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix}$$

$VA = V$, $x + y + z = 1$

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix}$$

$$\begin{bmatrix} 0x + 1/6y + 0z & 1x + 1/2y + 2/3z & 0x + 1/3y + 1/3z \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix}$$

$0x + 1/6y + 0z = x$

$1 - 0x - \frac{1}{6}y - 0z = 0$

$x - \frac{1}{6}y - 0z = 0$ — ①

$1x + 1/2y + 2/3z = y$

$1 - 1x + 1/2y + 2/3z = 0$ — $1 - \frac{1}{2} = \frac{2-1}{2} = \frac{1}{2}$

$1x - \frac{1}{2}y + \frac{2}{3}z = 0$ — ②

$0x + \frac{1}{3}y + \frac{1}{3}z = z$

$0x + \frac{1}{3}y + \frac{1}{3}z - 1 = 0$ — $\frac{1}{3} - 1 = \frac{1-3}{3} = \frac{-2}{3}$

$0x + \frac{1}{3}y - \frac{2}{3}z = 0$

Ans: $\frac{1}{10}$; $\frac{6}{10}$; $\frac{3}{10}$

1) A student study habits are as follows (if he studies one night he is sure not to study next night) and (if he does not study one night he is 60% sure not to study next night). 70% ...
In long run how often does the student study.

unique fixed probability vector

$$S = \begin{array}{c|cc} & S & NS \\ \hline S & 0.3 & 0.7 \\ NS & 0.4 & 0.6 \end{array}$$

In long run $vS = v$; $x + y = 1$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

$$\begin{bmatrix} 0.3x + 0.4y & 0.7x + 0.6y \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

$0.3x + 0.4y = x$

$1 - 0.3x \mp 0.4y = 0$

$0.7x - 0.4y = 0$ — ①

$0.7x + 0.6y = y$

$0.7x + 0.6y - 1 = 0$

$0.7x - 0.4y = 0$ — ②

$x = \frac{4}{11}$ $\quad y = \frac{7}{11}$

2. A sales man has 3 territories A, B and C (he never visit same cities on two consiqutive days) (if he is in city A he is sure to be in city B the next day) (if he is in city B or C he is twice as likely to visit city A as in other cities.
In long run what is probability of he visiting city A, B, C.

$$S = \begin{array}{c|ccc} & A & B & C \\ \hline A & 0 & 1 & 0 \\ B & \frac{2}{3} & 0 & \frac{1}{3} \\ C & \frac{2}{3} & \frac{1}{3} & 0 \end{array}$$

In long run $vA = v$, $x + y = 1$

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 2/3 & 0 & 1/3 \\ 2/3 & 1/3 & 0 \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix}$$

$$\begin{bmatrix} 0x + 2/3y + 2/3z & 1x + 0y + 1/3z & 0x + 1/3y + 0z \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix}$$

$0x + 2/3y + 2/3z = x$

$1 - 0x \mp \frac{2}{3}y - \frac{2}{3}z = 0$

$1x - \frac{2}{3}y - \frac{2}{3}z = 0$ — ①

$1x + 0y + \frac{1}{3}z = y$

$1x + 0y + \frac{1}{3}z - 1 = 0$

$1x - y + \frac{1}{3}z = 0$ — ②

$0x + \frac{1}{3}y + 0z = z$

$0x + \frac{1}{3}y + 0z - 1 = 0$

$0x + \frac{1}{3}y - z = 0$ —— ③

$x = \frac{2}{5}$ $\quad y = \frac{9}{20}$ $\quad z = \frac{3}{20}$

3. Habitual gambler visit clubs A and B he never misses visiting the club for playing cards.

- if he visit club A he is sure to visit club B on the next day
- If he visit club B he is as likely to visit club A as to B.

-

1. Find the transition matrix
2. Show that the transition matrix is regular stocastic matrix.
3. Find the unique fixed probability vector. (In long run how often does he visit club A or B)
4. If he visits club B on monday what is probability that he visit club A on thursday.

1. $$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ Q = \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix} \end{array}$$

2. Multiply the matrix to itself till you get all non-zero number to get regular stocastic matrix, in exam maximum p^6 time we have to do.

$$p^2 = \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 \\ 0.25 & 0.75 \end{bmatrix}$$

Since all the entries are positive (>0) $\therefore$ it's a regular stocastic matrix

3. In long run $vA = v$; $x + y = 1$

$$\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

$$\begin{bmatrix} 0x + 0.5y & 1x + 0.5y \end{bmatrix} = \begin{bmatrix} x & y \end{bmatrix}$$

$0x + 0.5y = x$

$1 - 0x + 0.5y = 0$

$1x - 0.5y = 0$ —— ①

$1x + 0.5y = y$

$1x + 0.5y - 1 = 0$

$1x - 0.5y = 0$ —— ②

$x = \frac{1}{3}$; $y = \frac{2}{3}$

4. Initial vector $- p^{(0)} = \begin{array}{cc} A & B \\ [\,0 & 1\,] \end{array}$ — He visits club B on monday

$p^{(0)}$ $p^{(1)}$ $p^{(2)}$ $p^{(3)}$

M Tu Wed Thus

$p^{(3)} = p^{(0)} p^{(3)}$ → put main matrix from start.

$$\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0.25 & 0.75 \\ 0.375 & 0.625 \end{bmatrix} = \begin{bmatrix} 0.375 & 0.625 \end{bmatrix}$$

1. 3 boys A, B and C are throwing balls to each other A always throws balls to B and B always throws balls to C. C is just as likely to throw the ball as to A if C was the 1st person to throw the ball find the probability that after 3 throws A as the ball B as the ball and C as the ball

$$P = \begin{array}{c|ccc} & A & B & C \\ A & 0 & 1 & 0 \\ B & 0 & 0 & 1 \\ C & 0.5 & 0.5 & 0 \end{array}$$

$$\begin{array}{cccc} & A & B & C \\ p^{(0)} = & [0 & 0 & 1] \end{array}$$

After the third throw

$$p^{(3)} = p^{(0)} p^{(3)}$$

$$p^{(3)} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

$$p^{(3)} = \begin{bmatrix} 0.25 & 0.25 & 0.5 \end{bmatrix}$$

2. Prove that the marco chain whose transition matrix is

$$P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

is irredusable

mode → MATRIX → MATA → 1 → AC → shift + 4 →

$p^{(2)}$

To make all element greater than 0 we have to keep on multipling the matrix till all elements become positive.

$$P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

$$P^{(2)} = \begin{bmatrix} 1/2 & 1/6 & 1/3 \\ 1/4 & 7/12 & 1/6 \\ 1/4 & 1/3 & 5/12 \end{bmatrix}$$

∴ Since all the entries are greater than 0 therefore this matrix is ~~irredusable~~ regular stocastic matrix.

3. Show that the matrix $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$ is regular stocastic matrix.

also find associative unique fix probability vector.

In exam solve max 6

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \qquad P^3 = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 \end{bmatrix} \qquad P^4 = \begin{bmatrix} 0 & 0.5 & 0.5 \\ 0.25 & 0.25 & 0.5 \\ 0.25 & 0.5 & 0.25 \end{bmatrix}$$

$$P^5 = \begin{bmatrix} 0.25 & 0.25 & 0.5 \\ 0.25 & 0.25 & 0.25 \\ 0.125 & 0.375 & 0.5 \end{bmatrix}$$

∴ Since all the entries are positive it's a regular stocastic matrix.

b) $vA = v$ $\qquad x + y + z = 1$

$$\begin{bmatrix} x & y & z \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix}$$

Note :- Make sure that unique fixed probability vector you have to take matrix from question $p^{(1)}$

$$\begin{bmatrix} 0x + 0y + 1/2 z & 1x + 0y + 1/2 z & 0x + 1y + 0z \end{bmatrix} = \begin{bmatrix} x & y & z \end{bmatrix}$$

$0x + 0y + 1/2 z = x$

$1 - 0x + 0y + 1/2 z = 0$

$1x - 0y - 1/2 z = 0$ ——— (1)

$1x + 0y + 1/2 z = y$

$1 - 1x - 0y - 1/2 z = 0$

$-1x + 1y - 1/2 z = 0$ ——— (2)

$0x + 1y + 0z = z$

$0x + 1y + 0z - 1 = 0$

$0x + 1y - 1z = 0$ ——— (3)

$x = \frac{1}{5} \qquad y = \frac{2}{5} \qquad z = \frac{2}{5}$

UNIT 4 : TESTING OF HYPOTHESIS

Definition :-

- Population :- An aggregate of objects under study is called population or units.

- Sample :- A finite subset of a universe or a small part of the universe

- Sample size :- The no. of individuals or members in the sample.

- Sampling :- The process of selecting the sample from the universe.

- Parameter :- The statistical constant of the population such as mean, variance

- Statistic :- The statistical concept from the sample from the members of the sample.

Note :- Population mean and variance are μ & σ^2
sample mean and variance are $\bar{x}$ and s^2

- Level of significance Table

	1% (0.01)	5% (0.05)
Two tailed test	$\lvert Z_\alpha \rvert = 2.58$	$\lvert Z_\alpha \rvert = 1.9666$
One tailed test Right: Left:	 $Z_\alpha = 2.33$ $Z_\alpha = -2.33$	 $Z_\alpha = 1.645$ $Z_\alpha = -1.645$

Note:- If the sample size is greater than 30 it is a large sample.

- Testing of significance for single propotion.

Formula :- $z = \dfrac{p - P}{\sqrt{\dfrac{P \cdot Q}{n}}}$ ← Propability of success

where $\boxed{p = \frac{x}{n}}$ is the propotion of success i.e x is number of successes in n independent trials with constant probability P of success of each trial and $\boxed{Q = 1 - P}$

~~Q.1~~ Testing of significance for single Propotion

Q.1. A coin was tossed 400 times and head turned up 216 times. Test the hypothesis that the coin is unbised. [z = 1.6]

H_0 : Coin is unbised $(P = 1/2)$
H_1 : Coin is bised $(P \neq 1/2)$ {two tailed test}

Given, $n = 400$ $x = 216$

$p = \frac{x}{n} = \frac{216}{400} = 0.54$

$P = 1/2$ $Q = 1 - \cancel{0.5}\,P = 1 - 0.5$
$Q = 0.5$

$$z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{400}}} = 1.6$$

$z_{cal} = 1.6 < 2.58$
$\therefore$ H_0 is accepted at 1% level of significanse.

$z_{cal} = 1.6 < 1.9666$
$\therefore$ H_0 is accepted at 5% level of significance.

Q.2. A certain cubical die was thrown 9000 times and 5 or 6 was obtained 3240 times. on the assumption of certain throwing do the data indicate an ~~realiased~~ unbised die? [rejected]

Ans.
H_0 : The die is unbised $(P = 1/6)$
H_1 : The die is bised $(P \neq 1/6)$ {two tailed test}

Given, $n = 9000$ $x = 3240$ — (for 5 or 6)

$P = \frac{1}{6} + \frac{1}{6}$ — because (5 and 6, 2 nums)

$= \frac{2}{6} = \frac{1}{3} = 0.33$

$p = \frac{x}{n} = \frac{3240}{9000} = 0.36$

$Q = 1 - 0.33$
$= 0.67$

$$z = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{0.36 - 0.33}{\sqrt{\frac{(0.33)(0.67)}{9000}}} = 6.0526$$

$z_{cal} = 6.0526 > 2.58$
$\therefore$ H_0 is rejected at 1% level of significance

$z_{cal} = 6.0526 > 1.9666$
$\therefore$ H_0 is rejected at 5% level of significance

Q.3. Twenty people were attacked by a disease and only 18 servived. will you reject for hypothesis that the survival rate, if attacked by this disease, is 85% in favore of the hypothesis that it is more. Test at 5% level (use large sample test) [z = 0.633]

H_0: The survival rate is 85%. $(P = 0.85)$
H_1: The survival rate is more than 85% $(P > 0.85)$

Given $n = 20$ $\quad x = 18$
$P = 0.85$ $\quad Q = 1 - 0.85 = 0.15$

$$p = \frac{x}{n} = \frac{18}{20} = 0.9$$

$$Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.9 - 0.85}{\sqrt{\frac{(0.85)(0.15)}{20}}} = 0.626$$

$Z_{cal} = 0.626 < 1.9666$

$\therefore$ H_0 is accepted at 5% level of significance.

Q.4. In a sample of 1000 people in Karnataka 540 are rice eaters and rest are wheat eaters. Can we assume that both rice and wheat eaters are equally popular in the state at 1% level of significance.

H_0: both rice and wheat eaters are equally popular $(P = 1/2)$
H_1: both rice and wheat eaters are not equally popular $(P \neq 1/2)$

Given $n = 1000$ $\quad x = 540$

$$p = \frac{x}{n} = \frac{540}{1000} = 0.54$$

$P = \frac{1}{2} = 0.5$ $\quad Q = 1 - P = 1 - \frac{1}{2} = 0.5$

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.54 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{1000}}} = 2.52$$

$Z_{cal} = 2.52 < 2.58$

$\therefore$ H_0 is accepted at 1% level of significance.

? do we have to write hypothesis

Q.5 A random sample of 500 apples was taken from a large consignment and 60 were found bad. obtain the 99% confidence limit for the percentage of bad apple in consignment? [8.61, 15.38]

confidence limit $= p \pm Z_\alpha \sqrt{\frac{PQ}{n}}$ ← ? is this small p or Capital P

Given, $n = 500$ $\quad x = 60$

$$p = \frac{60}{500} = 0.12$$

$q = 1 - p = 1 - 0.12 = 0.88$

Confidence limit $= p \pm Z_\alpha \sqrt{\frac{pq}{n}}$

because for 1%.

$$0.12 \pm Z_{0.01} \sqrt{\frac{(0.12)(0.88)}{500}}$$

$$0.12 \pm 2.58 \sqrt{\frac{0.12 \times 0.88}{500}}$$

for (+) $\quad$ for (−)
(0.1574, 0.0825)
(15.7%, 82%)

Till it's not mentioned always use two tailed test for Z_α.

- Test of difference between propotion.

let x_1 and x_2 be 2 sample with sizes n_1 and n_2 under the NULL hypothesis that there is no significance difference between propotion. The test statistics is given by

$$z = \frac{p_1 - p_2}{\sqrt{\hat{P}\hat{Q}\left\{\frac{1}{n_1} + \frac{1}{n_2}\right\}}}$$

where,

$$p_1 = \frac{x_1}{n_1} \quad ; \quad p_2 = \frac{x_2}{n_2}$$

and $\hat{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ and $\hat{Q} = 1 - \hat{P}$

Q.1 Before an increase in excise duty on tea, 800 people out of sample of 1000 persons were found to be tea drinkers, After an increase in the duty, 800 persons were ~~800 people~~ become to tea drinkers in a sample of 1200. Do you think that there has been a significant decrease in the consumption of tea drinkers after increase in the excise duty. [z = 6.84']

Ans.

H_0: There is no significance difference in propotion ($P_1 = P_2$)

H_1: There is decrease in propotion of teadrinkers ($P_1 > P_2$)

{Right tailed}

$n_1 = 1000$ $\qquad$ $n_2 = 1200$

$x_1 = 800$ $\qquad$ $x_2 = 800$

$$p_1 = \frac{x_1}{n_1} = \frac{800}{1000} = 0.8 \qquad p_2 = \frac{x_2}{n_2} = \frac{800}{1200} = 0.66$$

$$\hat{P} = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{(1000)(0.8) + (1200)(0.6)}{1000 + 1200} = 0.7236$$

$$\hat{Q} = 1 - \hat{P} = 1 - 0.7236 = 0.2764$$

$$z = \frac{P_1 - P_2}{\sqrt{\hat{P}\hat{Q}\left\{\frac{1}{n_1} + \frac{1}{n_2}\right\}}} = \frac{0.8 - 0.66}{\sqrt{(0.7236)(0.2764)\left\{\frac{1}{1000} + \frac{1}{1200}\right\}}} = 7.31$$

$Z_{cal} = 7.31 > 2.33$

$\therefore$ H_0 is rejected at 1% level of significance.

$Z_{cal} = 7.31 > 1.645$

$\therefore$ H_0 is rejected at 5% level of significance

Q.2 Random sample of 400 men and 600 women were asked whether they would like to have flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that propotion of men and women in favour of proposal are same against that they are not at 5% level of significance.

[z = -1.269]

H_0: There is no significance difference in propotions ($P_1 = P_2$)

H_0: There is significance difference in propotions ($P_1 \neq P_2$)

{two tailed test}

Given

$n_1 = 400$ $n_2 = 600$

$x_1 = 200$ $x_2 = 325$

$p_1 = \frac{x_1}{n_1} = \frac{200}{400} = 0.5$ $p_2 = \frac{x_2}{n_2} = \frac{325}{600} = 0.541$

$\hat{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{(400)(0.5) + (600)(0.54)}{400 + 600} = 0.524$

$\hat{Q} = 1 - \hat{P} = 1 - 0.524 = 0.476$

$Z = \frac{p_1 - p_2}{\sqrt{\hat{P}\hat{Q}\left\{\frac{1}{n_1} + \frac{1}{n_2}\right\}}} = \frac{0.5 - 0.54}{\sqrt{(0.524)(0.476)\left\{\frac{1}{400} + \frac{1}{600}\right\}}} = -1.240$

Zcal = $-1.240 < 2.58$

$\therefore$ H_0 is accepted at 1% level of significance.

Zcal = $-1.240 < 1.9666$

$\therefore$ H_0 is accepted at 5% level of significance

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Q.4 On basis of their total scores 200 candidates of a civil services examination are divided into 2 groups the upper 30% and remaining 70% consider the first operation of the examination. among the first group 40 had the correct answer whereas among the second group 80 had correct answers. On the basis of these realisation one conclude that the first question is good or dis ability of the type being examined here [z = 1.258]

H_0 : There is no significant difference in proportions ($P_1 = P_2$)
H_1 : There is significant difference in proportion ($P_1 \neq P_2$)
{two tailed test}

Given, 200 → candidates

2 groups 30 % 70 %

$n_1 = \frac{200}{100} \times 30 = 60$ $\quad$ $n_2 = \frac{200}{100} \times 70 = 140$

$x_1 = 40$ $\quad$ $x_2 = 80$

$p_1 = \frac{x_1}{n_1} = \frac{40}{60} = 0.66$ $\quad$ $p_2 = \frac{x_2}{n_2} = \frac{80}{140} = 0.57$

$$\hat{P} = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{(60)(0.66) + (140)(0.57)}{60 + 140} = 0.597$$

$$\hat{Q} = 1 - \hat{P} = 1 - 0.597 = 0.403$$

$$z = \frac{p_1 - p_2}{\sqrt{\hat{P}\hat{Q}\left\{\frac{1}{n_1} + \frac{1}{n_2}\right\}}} = \frac{0.66 - 0.57}{\sqrt{(0.597)(0.403)\left(\frac{1}{60} + \frac{1}{140}\right)}} = 1.189$$

$z_{cal} = 1.189 \ngtr 2.58$ ∴ H_0 is accepted at 1% level of significance
$z_{cal} = 1.189 < 1.9666$ ∴ H_0 is accepted at 5% level of significance

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- Test of significance of single mean

- If $\bar{x}$ is the sample **mean**
- μ is the population mean
- σ is the standard deviation of the ~~sample~~ population
- s is standard deviation of sample.

then under the Null hypothesis that there is no significant difference in the mean
The test statistics is

$$z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

Note:- If standard deviation of population (σ) is unknown then $z = \frac{\bar{x} - \mu}{\left(\frac{s}{\sqrt{n}}\right)}$ — only for large sample.

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Q.1. A normal population has a mean of 6.8 and standard deviation of 1.5. A sample of 400 members gave a mean of 6.75. Is the difference significant?

[~~z=0~~ $z = -0.67$]